LINCOS

DESIGN OF A LANGUAGE
FOR COSMIC INTERCOURSE

PART I

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INTRODUCTION

0 01. Scientists, artists and artisans tend to develop a terminology of their own. They use common language as a vernacular that will be enriched, impoverished, and modified in order to serve special purposes. The transformation may affect not only the vocabulary but also the syntax of the vernacular, though essential syntactical modifications are rather unusual.

It is certain, though far from generally admitted, that the meaning of a linguistic term should be determined by its contexts, it is a matter of fact that there cannot be any reasonably uniform opinion about the meaning of a word, if people cannot agree about the truth of the majority of contexts in which the word occurs. It is a common historical feature of many sciences, that their first representatives tried to create a terminology before the stock of known facts was large enough to provide a sufficiently large context for the elements of that terminology. Even now this is a serious drawback for philosophy and the arts. When people do not understand each other, this is usually said to be because 'they speak different languages'. This may be correct as long as "understanding" means a mere linguistic phenomenon. If understanding means intelligence, one may posit the inverse thesis with at least as much justification: People speak different languages because they do not understand each other. (Note that the word language has here a rather unusual meaning.)

As long as there is no reasonable agreement about facts and terminology among those who work in a given field, it seems wise to stick as closely as possible to the vernacular, and not to create any new term until it has become possible to define it in a satisfactory way by a sufficiently rich context. In any case, one should try to abstain from playing upon words as contexts.

In the following pages I shall use common English as a vernacular, and I shall endeavour not to use technical terms that are not generally agreed upon or else to provide for a sufficiently rich context or even something of a definition if I use technical terms of my own. So verbs like "to designate" and "to mean" will have the same meaning as they have in the vernacular and not that given them in recent publications which might be characterized by the term "formalist semantics". The use of a few words will be explained below.

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0 02. The meaning of the word "language" is fairly unambiguous as long as there is no reference to any special language. Generally people agree that somebody who uses English words when he explains some astronomical experiments, is speaking English, but sometimes, especially if one wishes to stress some divergences between the vernacular and the more technical idiom, one says that he uses a special astronomical language. Though borderlines may always be disputed (one may quarrel about the question whether as early as 1250 the monk Robert of Gloucester wrote English and whether nowadays "Beach-la-mar" is still English) "language" in the first sense is one of the best defined notions of contemporary English, whereas in the second sense it is extremely vague. So I will provisionally stick to the customary unsophisticated use of the word "language".

0 03. We shall have occasions to speak of natural languages. This is a term which may give rise to misunderstanding, as it seems to evoke the obsolete idea of one natural language in the dawn of mankind, as the germ of our present languages. When nature is opposed to society, then language is rather a social then a natural phenomenon. "Natural language", however, as meant by us, is opposed to "artificial language", and as the best-known example of such an "unnatural" language, we may mention Esperanto. Of course natural languages are not absolutely natural, any more than artificial languages are absolutely artificial. Sanskrit is a noteworthy transitional form, though it is perhaps more artificial than natural, but even in our present-day languages conscious influences have directed the spontaneous growth. Orthography in particular has often been the object of attack and the source of a conscious linguistic policy. On the other hand spontaneous growth has been observed in Esperanto, the only living artificial language.

0 04. When dealing with artificial languages we must decide whether codes are to be counted among languages. First of all we shall explain the sense in which we will use the word "code". In the European languages "code" may denote a collection of laws of jurisprudence, and in some of them "code" is also the system of rules of etiquette. This circumstance may give rise to a terminology in which codes are distinguished from languages by their highly conventional character. In this terminology every artificial language would be a code.

I shall not accept this terminology. I shall distinguish codes from languages not by a genetic, but rather by a formal criterion. If the meaning of a message is to be kept secret, sender and receiver may use a language that is unknown to other people. Inventing and using brand-new languages is difficult. So they will try to parasitize on a common language. By a well-

chosen system of superficial transformations they will change that common language into something like a language, that can only be understood by people who are acquainted with the system of transformation rules. This collection of rules is a code in the proper sense (as a juridical code is a collection of juridical rules). If a code X is employed to transform plain English into a less understandable text, we say that this text is written "in Code X", just as the original text was written "in English". So "Code X", which was the name of a system of "coding" rules, becomes the name of the linguistic matter which arises from transforming plain English according to the transformation rules of "Code X". I do not object to this ambiguous use of "Code X". The strong dependence of "code language" on ordinary language is stressed by the fact that the same word is used for both the system of "translation" rules and for the result of applying those rules. There is, however, still another word that means only the system of coding rules, viz. the word "key".

The relation between a natural language and a code is not the same as that between two languages. Translating from one language into another is quite different from coding and decoding. Coding and decoding can be done by formal substitution, whereas translating presupposes understanding. Of course this is again a gradual difference. The relation between spoken and written language is similar to that between a language and a code. Writing is like coding, and reading like decoding. Yet the rules that govern the relations between a spoken and a written language, are much more complicated than the rules of any cryptography. The rules of writing and reading are not purely formal such as those of coding and decoding. They can hardly be handled by people who do not understand the language in question, even in languages with a nearly phonetic spelling as Italian and Dutch. Coding and decoding machines can be much simpler than machines for writing down spoken language and for reading written language. Furthermore written language is much more independent of its spoken foster-mother than coded language is of its plain substratum. Spoken and written language may widely differ, and spoken language may undergo profound influences from written language. So it is not surprising that one speaks of spoken English and of written English, as though they were really two languages. The relation between plain English and coded English is quite different from this. Calling coded English a language would not match our customary use of the word "language". Yet I would agree to coded English being called a *coded* language.

0 05. A few other linguistic or quasi linguistic phenomena need to be considered. If a traffic sign shows the picture of two children going to school, or that of a fence, the car driver understands these pictures as if

they were linguistic announcements of a school and of a level-crossing gate. The skull on a bottle is to be read as "poison", an arrow or a fingerpost may mean "this way", a stroke of lightning is a warning against high tension wires. Pictures of children, fences, skulls, arrows, strokes of lightnings do not belong to customary English, but everybody knows how to translate them. The analogy with a code is striking. Of course there are differences: codes proper are to be secret, and they are fitted for more general purposes than the traffic-signs code. The common feature of these codes and the codes proper is the existence of a key for coding and decoding. Coding and decoding do not presuppose any understanding, but only the mechanical use of coding and decoding rules which can be collected in some list. An astrologer who casts someone's horoscope is considering the constellations as coded messages which can be transferred into plain language by adepts who possess the key. Telling the cards, interpreting dreams, looking for the "sigilla" of herbs as practised by herbalists of earlier times in order to recognize the medicinal value of plants, are methods of decoding something that is considered as a message.

0 06. We rejected the denotations "astronomical language", "medical language", and so on for the idiom used by astronomers, physicians, and so on. We now resume this discussion because there is at least one exceptional case: the case of mathematics. Is there something that, in the frame of a reasonable terminology, may be called mathematical language?

A strong argument for an affirmative answer may be found in what happens if a book be translated from one language into another, let us say from English into French. If this book is a mathematical textbook or treatise, there will be parts that need not be conveyed into, viz. the mathematical expressions and formulae. If we seek for other examples of texts that are excempt from translation, the result will be meagre. First of all: the occurrences of Arabic figures. Yet Arabic figures are, to say least of it, so near to mathematical expressions that they can hardly be considered as a new instance. The proper names Shakespeare, Oxford, and Portugal will reappear in the French translation in their original form, but "Pliny", "London" and "England" will be changed into "Pline", "Londres", and "Angleterre". Clearly it is not on principle that proper names are not translated. Or rather: the French translation of "Shakespeare" is again "Shakespeare", whereas the English and the French name of Pliny differ from each other. (Note that in a Russian translation all proper names would at least be transcribed, whereas mathematical expressions and formulae do not undergo any change.)

Another example: it is highly probable that a translator will leave quotations unchanged, especially if the language of such a quotation

differs from that of the surrounding text. The sentence "Before crossing the Rubicon Caesar said: Alea est iacta" will be translated "Avant de traverser le Rubicon, César dit: Alea est iacta". It is, however, not sure that a quotation of Hamlet's "to be or nor to be" would again read "to be or not to be" in a French translation. (Perhaps adherents of formalist semantics might say that, as compared with proper names, quotations are not a new instance of texts not to be translated, because a quotation is a proper name, namely of the thing quoted. Yet I do not feel the need to use the term "proper name" in a broader sense than usual.)

Even if mathematical expressions and formulae were proper names or quotations (I am sure that they are not), it would be a strange thing that they are never involved in the effective translation procedure. There are not many other examples of so strong an immunity. The best-known case is that of the formulae of chemistry. Physics is less striking. The resistant parts of a textbook on a subject of physics will be mathematical formulae, denotations of measures and units, and some reaction formulae like those of chemistry. The last instance of invariance with respect to translation I will mention, are staves.

I shall restrict myself to invariant parts that are neither proper names nor quotations, and I shall ask the question what is the reason for this invariance, e.g. why the mathematical formulae of an English textbook reappear unchanged in its French version. A priori two different answers are possible: The mathematical formulae are neither English nor French, or they are both. In the case of chemistry and music we may likewise choose between these two solutions.

We prefer the first answer: that the mathematical expressions and formulae belong to a language different from that of the surrounding context, because there are more reasons to adhere to this interpretation. The syntactical structure of "mathematical language" differs enormously from that of all natural languages. The main points of departure are "punctuation" and "treatment of variables".

0 07. One may fairly well maintain that punctuation is one of the numerous redundant elements in natural languages. There is hardly any interdependence between punctuation in spoken and in written language. Punctuation signs do not match the means of spoken punctuation, pauses and intonation. The question mark is the only punctuation sign that might reflect a kind of intonation, but its actual use is much more syntactic than phonetic. The punctuation signs which might mark pauses are placed according to conventional — in the main syntactic — rules which do not cover the use of pauses in spoken language. Attempts at phonetic punctuation may be found in the t'amim (accents) of the masoretic bible text,

and in the neumata added to the Greek gospel, but both were adjusted to
the special use of language in liturgy and they developed into rather
musical systems not related to spoken but to sung language.

   In speech punctuation still accomplishes syntactic tasks. Perhaps the
inability of written language to reproduce spoken punctuation has been
one of the reasons why nowadays spoken and written language differ.
Complicated syntactic patterns took the place of spoken punctuation.
Sumerian reflects a linguistic stage of mankind where connectives were
nearly unknown, and nevertheless syntactic patterns of subordination were
possible by chain constructions in which a whole sentence could be subject
or object of a superordinated sentence — this is still a characteristic of
infant language. Such patterns are unthinkable without a well-developed
punctuation system. Since then this system has been abandoned in favour
of a system of subordinating by means of connectives. The invention of
relative clauses has been the most important step in this direction, though
originally relative clauses are merely interpolations independent of the
sentence on which they seem to depend; indeed the relative pronoun is
etymologically interrogative or demonstrative. Relative clauses in the
proper sense are still unknown in Accadian and in Biblical Hebrew. The
words sha and asher that correspond to our relative pronouns cannot
literally be translated by relative pronouns. Sentences such as "a nation
whose language you do not understand", reads in a literal translation
"a nation *asher* you do not understand its language". So Accadian "sha"
and Hebrew "asher" are still rather punctuations than connectives. It is
true that contemporary English (and Scandinavian languages) also know
relative clauses without connectives, but this case is quite different from
that of the ancient Semitic languages. "The man I have seen . . ." would
read in an interlinear version of Biblical Hebrew "the man *asher* I have
seen *him* . . ."; in English the absence of the personal object "him" betrays
a dependence of the second clause; in Hebrew it is syntactically independent.

   The principle of syntactic structure by means of punctuation has been
abandoned in natural languages (at least in those of Western civilisation),
but it has been revived in the language of mathematical formulae. As
punctuation marks pairs of brackets are used. Pairs of brackets tell us
that in

$$\tfrac{1}{4}\,[(a + b)^2 - (a - b)^2]$$

we start by calculating the sum and difference of a and b, that these
numbers are then squared, that the squares are subtracted, and that the
difference is to be divided by 4. We should arrive at clumsy linguistic
structures, if we tried to translate such expressions into the vernacular.
Of course we can read the above expression as "one fourth of square
bracket open bracket open a plus b bracket closed square . . ." but this

would be to infringe lexicological and syntactic laws of the vernacular. It is noteworthy that in the vernacular the mathematical process of putting in brackets is described by a reference to the graphical means (the brackets) by which this process is represented in written formulae. Compare this situation with the one we would meet if the vernacular did not possess a word meaning "equal" and speaking about equality were only possible by using graphical descriptions of the mathematical equality sign such as "pair of horizontal lines".

Brackets are not the only means of punctuation used in mathematical expressions. By mere convention algebraic connectives are at the same time punctuation marks. The separative power of the different connectives is fixed by a few rules, some of which are explicitly stated in the textbooks, whereas others are tacitly assumed. The signs of equality and inequality, and of "smaller than" and "larger than" are the strongest separators; the separative power of a plus-sign is greater than that of a times-sign; algebraic sums (such as $a+b-c-d+e$) are to be read as if all open brackets stand at the utmost lefthand; and so on.

In the vernacular there are no rules about the use of connectives as punctuation signs. In the sentences "we visited Dutch towns and villages" and "we got lessons in Dutch history and mathematics" the apparent syntactic pattern of "Dutch towns and villages" is the same as that of "Dutch history and mathematics" though on semantic grounds we are to bracket "towns and villages" in the first case and "Dutch history" in the second. The sentence "He studied the history of medecine in Bologna" does not tell us whether he was in Bologna when studying history of medecine or whether he studied that part of the history of medecine that was played in Bologna. Again the decisive point is the place of the brackets.

Little attention is paid to the need of formal punctuation in the vernacular, whereas in the language of mathematical expressions it is the backbone of the syntactic structure.

0 08.   The other feature that distinguishes mathematical language from natural languages is to be found in the treatment of variables. Philologists usually ignore the notion of variable though they are reasonably acquainted with other notions discovered by logistical analysis of language such as constant, connective, relation, function and so on. Indeed the variable as a syntactic element is not easy to uncover in natural languages, precisely because in natural languages almost all syntactic units may be used as variables though mostly ranging over a very restricted domain; it depends on the context whether a unit is to be considered as a variable or not. Furthermore in almost all instances the variable or at least its binding is hidden and can be recognized only by logical analysis.

It is probable that generic names (appellatives) were originally (ambiguous) proper names, i.e. every appellative was really a proper name of every individual belonging to the genus designated by that name, e.g. ant is a proper name of every individual ant. This conception is reflected in the Bible when "Adam" or "Israel" or "Kanaan" is used not only for one single person, but also for a group of people and for every member of this group. Every man bears the name Adam = Human, or "Son of Adam" = "Son of Human" in virtue of his descent from the first person that bore the name Adam.

The semantic and syntactic change from proper name to generic name has been made in an early stage of linguistic development, as it is still made by children in their early youth (only a few vocatives such as "father", and "mother" will strongly resist the transformation from proper name into appellative).

The semantic transformation of proper names into ambiguous proper names and of ambiguous proper names into appellatives has not been accompanied by systematic syntactical transformations. "Man" will be an ambiguous proper name in the sentence "He killed a man" and a generic name in "man is mortal". Many languages have suffixes, prefixes or connectives that transform ambiguous proper names into generic names, but their use is neither obligatory nor always possible. I may say "man is mortal" though there is a "mankind", which could be used in this case, but I have to say "a mouse is a rodent", because there is no "mousekind". There are suffixes which in one case can transform an ambiguous name into a generic name and which in other cases will alter the sense. "Priesthood" may be the genus "priest", but "brotherhood" will never be the genus "brother".

Ambiguous proper names are an instance of what logical analysis calls variables. In the sentence "I bought a car" we recognize "a car" as a variable that has been bound by the fact that there is some car that I have bought, thus by existential binding. In the sentence "A car is a carriage" one can interpret "a car" as a generic name or as a variable that has been generalized over the genus "car". There are other kinds of binding, usually not considered in logical analysis, but as important as the preceding ones, e.g. the interrogative binding such as that met in "Which car did you buy?"

The linguistic equivalent of a variable may be still more complicated. In "I bought three apples" and in "three apples (of a certain kind) go to a pound" we may consider "three apples" as a variable ranging over all triples of apples; in the first case the binding is existential, in the second case it is generalizing. The use of "little money" in "I have little money" and "with little money you can succeed at Smith's" is quite analogous.

Pronouns are still other instances of variables. "I" is a variable bound

to the person who utters it by the fact that he does so. "You" is a variable bound to the person that the speaker is addressing. "Now" is a variable bound to the moment at which it is pronounced. "Here" is a variable bound to the place where it is pronounced. One might speak of demonstrative binding in these instances, because the demonstrative pronouns "this" and "that" are variables of this kind with an unusually large domain. Formally it is, however, possible to reduce demonstrative bindings to existential bindings.

These examples show that natural languages are equipped with a large stock of variables which all have different domains. Natural languages are unsystematic in dealing with variables; and they are far from a clear-cut solution of the problem of indicating the means by which a variable is bound. The makers of language acted instinctively, uninfluenced by rational analysis. The use of ambiguous proper names as generic names was a discovery. Rational analysis has turned it into a problem. The eleatics and sophists were troubled by the "one that can be many", but the first who consciously justified generic names as a new kind, was Socrates – in Aristotle's appraisal of Socrates' work stress is laid on this very fact. In Socrates' tracks Plato and Aristotle arrived at a frankly ontological solution of the problem. A generic name is said to be the proper name of a thing that is no less real than individuals are. These things are called ideas by Plato and genera by Aristotle. Historically this ontological interpretation seems to be provoked by the rather superficial and incidental methods which natural languages have applied in order to settle the relation between ambiguous proper names and generic names.

It would be an enormous task to analyse how the various natural languages operate variables. We have already noticed that every variable has its own domain, "car" cannot be used for other things than cars, "he" ranges over the set of males only, "yesterday" is a temporal variable, "there" varies over places and "thither" over directions. With the help of numerals one can form variables such as "five fingers" and "one hundred people". "One gallon" can hardly be interpreted as a variable, but "one gallon of milk" can. "One shilling" behaves differently in "this costs one shilling" and in "I gave him one shilling". In the first case it is not a variable, but something else, in the second case it will be a variable (at least if the sentence really means that he got a shilling, and not that the thing he got was worth one shilling). It is unnecessary to point once more to the indifference natural languages show to the character of binding, whether it be existential or generalizing.

We shall now compare this use of variables in natural languages with that in the language of mathematical expressions. It is a striking principle in this language that all variables have the same domain and are freely

interchangeable. Incidentally the domain may be restricted and the interchangeability may be partially blocked. This can be done in a formal way, e.g. if in the first chapter of a book on linear algebra the author announces that Latin letters will be used for vectors and Greek letters for scalars. The reader of a paper on elementary geometry is assumed to know that variables written as Latin capitals range over the set of points of three-space and variables written as Greek capitals range over the set of angles. The reader of a paper on complex functions may conclude from the context which letters are related to complex numbers and which to functions. If a theorem is pronounced, it is understood that all restrictions imposed on the variables belong to the premises and may easily be added, if they do not occur in the text as it stands. This does not affect the principle of uniform domain and free interchangeability.

It is true that this principle is not always strictly observed. The best-known infringement is related to a notation of function that in thirty years may be considered old-fashioned though it is now very common. If a function f is called $f(x)$, this may have been done in order to state a special relation of this function f with the particular variable x, so that $f(y)$ will be another function than $f(x)$ though y and x may range over the same domain. In the case of functions of more variables this notation will be a means of distinguishing between "the functions $f(x, y)$ and $f(y, x)$". It is well-known that this notation leads to serious inconsistencies, especially when partial derivatives occur. It is a remarkable feature in the development of mathematical language that mathematics are becoming more and more repugnant to such infringements.

By the device of free interchangeability of variables mathematical language avoids a difficulty that is vividly felt in natural languages. In natural languages there is not more than one variable that ranges over the set of stones (books, trees, numbers and so on) viz. the variable "stone" ("book", "tree", "number", and so on). So if I wish to speak about two or more stones (books, trees, numbers, and so on) in the same context, I am committed to clumsy circumlocutions such as "this stone, and that stone", "one stone and another", "the former and the latter", "the first, the second, the third" (and analogous constructions). Incidentally such variables may be distinguished by letters, "stone A, stone B, stone C", and this is just what mathematics has done systematically, because the simultaneous occurrence of many variables meaning variable points or numbers has been a not unusual phenomenon in mathematical texts from the oldest times. Finally the need for a large stock of variables has led to the principle of free interchangeability.

It will be clear from this exposition that natural languages and mathematical language differ widely as to the management of variables.

0 09.   In 7 and 8 we have seen that mathematical expressions have a syntax of their own which, owing to a rapid evolution, may successfully deny its descent from the syntaxes of natural languages. So I feel justified when I speak of a "mathematical language" as opposed to natural languages and to medical or juridical idioms which lack almost all characteristics of an independent language. Even in the case of chemical formulas one may wonder whether it would be reasonable to state that they constitute a more or less independent language. The syntax of these formulas, though very poor, is patterned after that of natural languages; for every chemical element there is only one variable that may vary through atoms of that kind (called H for the hydrogen atom), and the only respect in which the syntax of chemical formulas is superior to syntaxes of natural languages is a good system of punctuation.

It is true that mathematical language as written in textbooks and papers still parasitizes on natural languages. The text surrounding the mathematical formulas is usually written in an idiom that bears the characteristics of the vernacular, to which it belongs in the ordinary sense. This is why I spoke of a language of mathematical expressions. The mathematical expressions about which the author reasons are written in this language, whereas the reasonings are usually worded in the vernacular.

So far I have not yet mentioned the logistic language created by G. Peano and perfected by B. Russell and A. N. Whitehead. There have been earlier attempts, but Peano was the first to design a linguistic pattern more adequate to mathematical reasoning than common language. Logistic language used and developed consciously the peculiar features of the language of mathematical expressions. Of course words that mean logical connectives had to be introduced, but their number could be cut down, as syntactic structure is mainly got by means of punctuation: variables are treated as in the language of mathematical expressions, systematically and according to the principle of free interchangeability. Further, words are created to account for the kind of binding that has been applied to a variable, whether existential or generalizing.

Russell and Whitehead's purpose was different from Peano's. Peano's idea had been to create a workable language that should supersede the vernacular in mathematical texts. Yet in the work of Russell and White-head the stress shifted to the foundations of mathematics. Logistical language proved useful in foundational research, and improvements tended to increase that utility. Instead of the colloquial language of mathematicians logistics became the subject-matter of foundational exploration. Mathematicians not specialized in this domain did not mind whether logistic language could serve any other purpose than the very special ones of foundational research. Many mathematicians even distrusted

logistics, because practicians of this discipline used to link it to some
school of philosophy and because certain details created an impression of
hairsplitting and clumsiness. For years I too shared this false impression.
It is true that the actual systems are a little too rigid. Though logistics
has influenced the idiom of mathematical texts we still have little expe-
rience at putting mathematical expressions into a frankly logistical frame.
Mathematics, particularly in its youngest branches, has become an
intricate machinery. I think logistics may be a great help to better under-
standing.

   0 10.   As logistical language has not been much applied to mathematics,
it is not to be wondered at that no attempts have been made (at least as
far as I know) to apply logistics on a reasonable scale in some other domain,
let us say physics or daily life. Before logistics came into existence, people
were more optimistic. Leibniz fostered the idea of a "characteristica
universalis", and under his influence as well as independently of him,
17th and 18th-century scholars developed systems of this kind. At that
time the syntactic problems were not yet posed, still less solved. The
attempts did not lead to languages but rather to rational systems of
concepts, forerunners of the decimal vocabulary of our days.
   At present affairs would appear to be much more promising. Thanks
to logistics the syntactic problems are solved or, in any case, elucidated.
One can imagine numerous ways of tackling the subject. After several
unsuccessful attempts I finally became convinced that it is just the
difficulty of choice which causes the trouble, and that the only thing
which matters is to find a starting point. Seeking in history how analogous
situations were met, I came to the conclusion that one should start with a
concrete, sharply-defined and rather narrow problem.

   0 11.   In spite of Peano's original idea, logistical language has never
been used as a means of communication. If there was something to be
communicated, even logisticians kept to the vernacular. Logistical lan-
guage became a subject-matter. For the sake of a better understanding
of its intrinsic properties one tried to isolate it as strictly as possible. The
bonds with reality were cut. It was held that language should be treated
and handled as if its expressions were meaningless. Thanks to a reinter-
pretation, "meaning" became an intrinsic linguistic relation, not an
extrinsic one that could link language to reality.
   A language that is to operate without regard to the meaning of its
expressions, e.g. by people who do not understand it, must fulfil very
high requirements. It must be fully formalized. Formal rules of constitution
and transformation will determine the structure and handling of the
linguistic expressions.

Formalization of language is a highly important problem. Coding as treated in 0 04 is an example of formal translation. Indeed, in such cases it is imperative that a code works without any regard to the meaning of the text to be coded or decoded.

This is a rather unproblematic example, because inside the coded text there is no need for operations, any more than for rules of operation. The coded language has a static character. The best-known example of an "individual" really operating within a fully formualized language is the computing machine. Independently of any closer examination of the concept of meaning, we may assert that the machine does not understand the meaning of the linguistic expressions on which it is operating, in the sense in which we understand them. The machine works according to formal rules, whereas the human being who runs the machine knows the meaning of the input and output text. The subject matter on which the machine operates is language only from the point of view of the human being, just because it is he who attaches some meaning to it.

0 12. In view of the logistic tendency towards formalization it is not surprising that logisticians have paid less attention to language as a means of communication. In 0 10 I have explained why in the construction of a characteristica universalis we have to start with a concrete, sharply-defined, and rather narrow problem. After these explanations it will be clear why I think that this problem must be a problem of communication, and more precisely communication ab ovo. My purpose is to design a language that can be understood by a person not acquainted with any of our natural languages or even their syntactic structures. The messages communicated by means of this language will contain not only mathematics, but in principle the whole bulk of our knowledge. I shall assume that the receiver of these messages has understood their language if he is able to operate on it. He will be able to do so only if he has grasped the meaning of the expressions of this language, because it will be a moderately formalized language that cannot be handled on the ground of formal rules only.

The problem is not yet sufficiently outlined. Communicating with individuals who are not acquainted with our natural languages is a problem that is daily solved in the intercourse with babies and infants. We succeed in teaching them our language though we have started with a tabula rasa of lexicologic and syntactic knowledge. This linguistic education takes place in surroundings where things and situations are shown, and named at the same time, by people who are acquainted with the language. Showing will mostly be unintentional; the act of naming will mostly include that of showing, even if the thing or the situation is not named for the purpose of being shown.

In order to narrow the problem, I propose to exclude or at least to restrict excessively the opportunities of showing. I shall use showing as little as possible as a means of explaining the meaning of linguistic expressions.

On the other hand I shall suppose that the person who is to receive my messages is human or at least humanlike as to his mental state and experiences. I should not know how to communicate with an individual who does not fulfil these requirements. Yet I shall not suppose that the receivers of my messages must be humans or humanlike in the sense of anatomy and physiology.

0 13.   It was in this way that I arrived at the problem of designing a language for cosmic intercourse. Lincos, the name of this language, is an abbreviation of "lingua cosmica". As a linguistic vehicle I propose to use radio signals of various duration and wave length. These two dimensions will suffice. As compared with our acoustic vehicle, intensity as a third dimension is lacking. I rejected it because I think it less fit for radiographical use. But this is not a serious loss, because the wealth of information is much greater in the ether than in the acoustic medium, even if we dispose of the dimensions of duration and wave-length only.

I have not made a study of communication techniques, so I shall not dwell on technical details. When I designed Lincos, I had in mind using unmodulated waves. Perhaps modulated waves would be better. It would require only slight changes to adapt the program of messages to the use of modulated waves.

I have not examined which distances can actually be covered by radio messages. It is very probable that we can reach other planets of our solar system by means of radio signals, but I doubt whether with the powers now available we can communicate with inhabitants of other solar systems. Nevertheless I have taken into account the possibility that the receiver will belong to one of the neighbouring solar systems.

Of course I do not know whether there is any humanlike being on other celestial bodies, and even if there were millions of planets in the universe inhabited by humanlike beings, it is possible that our nearest neighbour lives at a distance of a million light-years and, as a consequence, beyond our reach.

On the other hand it is not unthinkable that inhabitants of other planets have anticipated this project. A language for cosmic intercourse might already exist. Messages in that language might unceasingly travel through the universe, maybe on wavelengths that are intercepted by the atmosphere of the earth and the ionosphere, but which could be received on a station outside. On such an outpost we could try to switch into the

cosmic conversation. Long ago I thought cosmic radiation to be a linguistic phenomenon, but at the present state of our knowledge this seems to be very improbable. It is not easy to state a priori how to distinguish messages from purely physical phenomena. But should the case really arise, we shall know how to answer the question: we should try to understand the message.

This, I suppose, intelligent beings in the universe will do if they receive our messages. They will try to decipher them and to translate them into their own language. This task might be easier than that of terrestrial decipherers who have to discover the key of a code. Indeed our objective is just the opposite of that of the sender of coded information. We want to communicate with everybody who might receive our messages, whereas the sender of a coded message wishes to keep secret the information contained in his message. But in spite of our efforts even intelligent receivers might interpret our messages as physical phenomena or as music of the spheres.

0 14. It has been one of our presumptions that we cannot start cosmic intercourse by showing concrete things or images of concrete things. One might observe that television images are sent by radio and that we could try to do likewise. Television is founded on the principle that an area is decomposed into a set of spots and that these spots are arranged in a simply ordered (temporal) sequence according to a fixed rule. As long as we cannot tell the receiver the principle and the code of decomposition and arrangements, we cannot expect that he will understand television messages and transform them into images. Synthesizing television messages into images may be taught by means of analytic geometry. In our Lincos program analytic geometry is a rather advanced stage. When this is reached, television, even three-dimensional, will be possible. But by then our linguistic proficiency will be great enough to dispense with television. I tend to believe that verbal description is superior to showing by television. I have not however paid sufficient attention to this problem as I am mainly interested in linguistic problems.

0 15. Throughout our exposition we will talk of Lincos as if it were speech (not writing). We shall say that Lincos phonemes (not letters) are radio-signals of varying wave-length and duration, and that Lincos words are constituted of phonemes (not of letters). If we aim at this constitution, we shall speak of phonetica (not of spelling).

This is no more than a convention, but it will prove very useful. We shall need to consider two versions of Lincos, Lincos proper, and a coded version. It will be convenient to use the term spoken Lincos for Lincos

proper, and the term written Lincos for coded Lincos. Moreover this choice of names is justified by the fact that the broadcast language will be "spoken Lincos" whereas in printed expositions such as the present book "written Lincos" will be used.

Written Lincos will not bear a one-to-one graphic-phonetic relation to spoken Lincos. On the contrary the relation will be global; every spoken Lincos word is reflected in written Lincos by a rather arbitrary graphical sign or group of graphical signs. This is a realistic procedure. A phonetic spelling of Lincos would be rather clumsy and illegible. Moreover for the time being there is no point in knowing how the different Lincos words will be pronounced (i.e. translated into groups of radio signals) when they are broadcast. The reader of the present book, who meets with words of written Lincos such as 10, $+$, $\wedge$, Num, PAN, $\female$, and so on will understand that these graphical complexes are codings of spoken Lincos words. It is quite unimportant how these spoken Lincos words are constituted, and I have never concerned myself with this question. In any case the spoken Lincos word corresponding to Num need not be composed of phonetic groups which correspond to N, u, and m. In this book we shall stick to the convention that e.g. 'Num' is not the proper Lincos word but the written (or printed) image of it. But we shall not always stress this feature, and sometimes, in order to avoid clumsy language we shall even behave as if the Lincos word were 'Num'.

0 16.   The notations in written Lincos are borrowed from mathematics (10, $+$), from logistics ($\wedge$), from other sciences ($\female$), and from Latin (e.g. Num is an abbreviation of Latin numerus). As a rule such abbreviations will be of three letters; we also use contractions of groups written of Lincos words, such as PAN (from Pau Ant Nnc). Finally there are Lincos words written in entirely arbitrary fashion, such as Ha, Hb, Hc,. . . which mean names of humans.

As pointed out before the spoken Lincos correlates of the written Lincos words will be rather arbitrary. A certain restriction, however, should be made. Lincos will be more easily understood if it has a systematic phonetics. It would be pleasant if the various syntactic and semantic categories could be distinguished by certain striking phonetic characteristics. A receiver who has to interpret the Lincos messages and who has made some progress in understanding them, will be grateful if he can see at first glance whether a word means a variable, a connective, a set, a human, a chemical element, and so on. Probably this will even help deciphering. Disconnected fragments of such a systematization exist in all natural languages. A noteworthy attempt to amalgamate these fragments has been made in Esperanto. There the syntactic and semantic categories emerging from Western

languages have been accepted without much criticism and have been characterized by the traditional means of stems and inflections.

Establishing the principles of an adequate Lincos phonetics would be premature. No more than a rough systematization would be available at the present time. A few syntactic and semantic categories can be clearly distinguished at the moment, but their number is rather small. We have still to wait for a more extensive vocabulary of Lincos. Then we can resume this question.

0 17. In a more essential way the choice of Lincos phonetic means is restricted on account of the use of ideophonetic words. We have agreed to abstain as much as possible from showing, but we cannot entirely abstain from it. Our first messages will show numerals, as an introduction to mathematics. Such an ostensive numeral meaning the natural number n consists of n peeps with regular intervals; from the context the receiver will conclude that it aims at showing just the natural number n. Ostensive numerals will soon be superseded by algorithmic ones. An ostensive numeral is ideophonetic (as a hieroglyph both showing and meaning an eye is ideogrammatic). Other ideophonetic words are the so called time-signals introduced in the second chapter. These words have no simple correlates in other languages. One might even doubt whether they are really words, or rather signs, that mean nothing but themselves. More things will be shown in the Lincos broadcasting program, but then the means of showing are still less linguistic. With a common name such occurrences might be called noises. Some of them mean nothing but a noise, others mean unintelligible speaking, and still others mean events which are usually accompanied by a noise, such as in a radio play the noise of smashing a door means smashing a door.

The phonetic structure of an ideophonetic word is strongly determined by its meaning. It cannot be arbitrarily changed. This is even true of the less linguistic pieces of our program. Of course a noise meaning a noise may be replaced by another noise, but in any case the substitute ought to be again a noise.

There is still another kind of truly linguistic phenomena which might be reckoned among ideophonetics: the means of punctuation. We have already had occasion to point out the importance of punctuation in mathematical and logistical language. Punctuation will be the principal means of showing Lincos syntactic structure. Lincos punctuation will be explained in 1 05 0. As punctuation marks we will use pauses, because pauses are self-explanatory to a high degree. It is a debatable point whether purposive pauses are to be considered as words. If we agree to this terminology, we may count Lincos punctuation marks among the

ideophonetic words. From a syntactic point of view they are a special
kind of connectives.

For punctuation in *written* Lincos we have adopted the Russell-White-
head system of square dots, though in a more economical version (see 1 05 0).

0 18.   The syntax and the lexicon of connectives of Lincos have been
mainly borrowed from logistical language, but at the same time we have
tried to mitigate the rigidity and clumsiness of logistical language. Looking
back at the result, I must confess that I am not satisfied, mainly because
I often lacked the courage to go far enough. (Formalists will surely claim
that I have gone too far.)

Lincos should be a means of communication. Lincos syntax is not an
aim in itself, but it should serve the general aim of Lincos. Lincos will be
far from fully formalized. It will be used by people who are supposed to
understand what they are saying, and to try to understand what they
perceive, not by machines which must operate according to formal rules.
So we can dispense with full formalization, even where this is possible.
To a high extent comprehension may be favoured by formalization, but
it may also be hampered by needless exaggerations.

The liberties we have taken are very usual in common language, the
common idiom of mathematics included. Words and clauses which can
readily be supplied are often omitted, especially when pieces from a
question would have to be repeated in the answer to this question. Some-
times we account for the omitted words or clauses by a word meaning
"and so on", but this is not compulsory. Variables bound by a definite
or indefinite article are repeated without the clause by means of which
they have been bound. In the common mathematical idiom an existentially
bound variable is allowed to reappear as an instance out of the set to
which it has been bound. We have adopted this habit, sometimes justifying
it by a more formal device (see 3 05 2), but sometimes disregarding even
this formality. Looking back I regret that I did not take more care for
uniformity in this matter. It would have been easy to introduce words
meaning "this", "that", "the afore mentioned", and so on, and it would
have been a real gain. On the other hand it is not so important a question
as to justify a total revision of the project.

Definite and indefinite articles, and other words which conceal exist-
entially bound variables, may only be used if it is clear where the binding
takes place. In many instances of our Lincos program this will be decided
not on syntactic but on semantic grounds.

Lincos speech has been assimilated to common language by introducing
words meaning "thus" (3 04 5), "suppose that" (3 05 2, 3 05 3), "because"
(3 12 0), and others, and by using them in a not too formal way. "And"

and "or" as used in "John and Mary have come" and in "John or Mary will come" are represented by Lincos words, of course different from the usual syntactic connectives meaning "and" and "or" (1 29 2). A rather verbal translation of "three people" such as occurring in "three people have come" is possible in Lincos (3 28 1). Modalities have been dealt with in a quite unformal way, disregarding the highly developed formal theory of modality (3 35 1–3 35 7). Nor have I respected the formalism of quotations and quasi-quotations; in Lincos one may speak about things and events in a recklessly autonymous fashion, i.e. by using copies of these things or events as names. I have decided on this conduct only after a thorough analysis which led to the result that the usual precautions are unsatisfactory and superfluous (2 04 0–2 06 0). The reader will have remarked already that we have not adopted the rules of formalist semantics in the use of quotation marks, or any other conventions of this kind.

I have lowered the degree of formalization of Lincos purposely and deliberately though without any intention of abusing this argument. It is not meant as a means of evading criticisms. In all cases where more formalization would have favoured comprehension and comprehensibility, I shall plead guilty. I feel convinced that this will happen rather often, and that even very serious mistakes may have occurred.

0 19. I have dwelt upon divergences from the syntax of logistical language. This might mislead the reader. In fact these divergences are far from essential, surely less essential than the concurrences and the additions which have proved indispensable.

One of this additions is concerned with the syntactic structure of oblique or reported speech. As far as I know logisticians have always treated oblique speech in another context, viz. that of the problem of intentionality. In my conviction this is a pseudo-problem of formalist semantics which is unable to deal with language as a means of communication adequately.

The syntactic problem of oblique speech arises if a certain person A says that a certain person B said that p, and if p contains variables which do not belong to B's but to A's discourse (e.g. if B said "I am ill" and A changes the variable "I" into "he"). Syntactical means have to be devised in order to ascertain whether such a variable is genuine or belongs to the discourse of another person. The devices used in common·languages are too unformal; they cannot serve in Lincos. Our solution is quite different. In Lincos hidden variables and words that look like variables will always be genuine, whereas overt variables belong to the discourse in which they are bound. We shall see that such variables cannot be avoided. The problem has been fully treated in 2 04 0–2 06 0, 3 01 5 and 3 07 0.

Hitherto in logistics little attention has been paid to sentence patterns other than the propositional one. The interrogative mode must however be accepted if one considers mathematics not as a stock of true propositions, but as an art of discovering. Moreover language as a means of communication cannot dispense with the interrogative mode. Interrogative sentences will be formed by a new kind of binding variables, the interrogative binding. We introduce a connective '?', which prefixed to a variable x in a context '? x . . .' produces a question that may be translated "wanted an x with the property . . ." (see 1 12 1 and 3 01 1, 3 02 1, 3 02 2 and so on).

Of course many more problems of syntax have had to be solved. In fact, whenever a new word is introduced, its syntactic status has to be settled. This requires a careful analysis in every single case. Fortunately the variety of syntactic status is rather small, thanks to the existence of a few syntactic paradigms, such as the words meaning "to happen", "to say", "to know", "second". For instance the unit of time is a paradigm for all units of measure; "second" is handled as if it were a (linear) function from the set of real numbers to the set of durations.

I have tried to outline the syntactic status of every word sharply, but I am not sure whether I have always strictly observed these syntactic rules. In any case it is still too early to write a syntax of Lincos.

0 20.   Lexicology is much more embarrassing than syntax. In a field which can be organized by simple rules, lexicology reduces as it were to syntax. A vocabulary of natural numbers can easily be designed on the basis of some positional system. Number vocabularies of soldiers are known in modern armies, and it would be easy to draw up similar vocabularies comprising all humans. Geographical coordinates can serve to set up a vocabulary of geographical positions; a pair of integer coordinates can be used to name chemical elements. But these are exceptional cases. We are still far from constructing the rational system of notions of which Plato and Leibniz might have dreamed, perhaps we are even farther than they were. We have not the faintest notion of the character of the enormous preparatory work that we ought to do if we want to realize those dreams. The system of stems and inflexion of our natural languages is more historical than rational, and even in languages with a relatively simple history such as German, this system is a logical chaos. In Esperanto inflexion has been intentionally organized, but not on logical grounds. (This is not at all a reproach. Nobody can tell us what conditions good means of communication are to fulfil. Under exceptional circumstances a rational structure may be advantageous. In daily life common languages have proved to be extremely useful in spite of their chaotic structure.)

As long as we have not the faintest idea of a rational system of notions,

lexicological engineering will be a mere succession of haphazard attempts. When introducing a few hundred Lincos words successively, I have not been guided by lexicological rules. I have made a rather sporadic use of inflexion as a means of formation of words. Two of the three affixes I have introduced are unproblematic; they transform a cardinal into a ordinal number and into a multiplicative number respectively. With the third I feel less satisfied. Its function is to transform a verb into a nomen actionis. If such a word really means the action and not also the result of the action as is often the case in common languages, it can be probably dispensed with.

0 21. Lincos has to be taught to the receiver. Therefore our program to be broadcast cannot have the character of a newsreel. On the contrary, in the beginning we shall communicate facts which may be supposed to be known to the receiver. In this way we can try to facilitate the work of deciphering.

As our means of showing are heavily restricted, we cannot start with a concrete subject matter. Mathematics is the most abstract subject we know and at the same time a subject that may be supposed to be universally known to humanlike intelligent beings. So we have decided to start our program with mathematics. Natural numbers are introduced by ostensive numerals in contexts of elementary arithmetics. The first texts of this kind will be true formulas of the type $a+b=c$, $a=b$, $a>b$, $a<b$, where a, b, c are to be replaced by natural numbers.

Of course we cannot communicate all formulas of this type because of their infinite number. We shall communicate a number of such formulas that is large enough to justify the supposition that an intelligent receiver can conclude from these texts what the words (complexes of signs) $+$, $=$, $>$, $<$ mean. By a finite number of instances the relation between natural numbers described by $a+b=c$, or $a=b$, or $a>b$, or $a<b$ is not uniquely fixed from a formalist point of view. A machine that does not know any more about the relation $a+b=c$, but its truth value for natural a, b, c with $1 \leq a$, b, $c \leq 1000$ cannot solve similar problems with numbers a, b or $c > 1000$. Yet the receiver will be supposed not to be a machine but humanlike. Children learn arithmetics by a finite number of instances, and a set of rather unformal rules. They succeed in generalizing the instances in a unique way. Arithmetics yields unique results though the definitions leave room for an infinite diversity. If I ask anybody to continue the sequence 1, 3, 5, 7, 9, 11, I may be sure that he will do it with 13, 15, 17 and so on. So I am sure that a humanlike being will conclude from a sufficiently large number of texts like $3>2$, $7>1$, $9>3$, that '$>$' has its usual meaning.

It is perhaps superfluous to insist on this point. A hundred years ago such assertions would have been self-evident. Since then we have got into the habit of giving formal general definitions and proofs even for arithmetical and algebraical motions and formulas which could be defined and proved in a less formal way. It is not long ago that in arithmetics and algebra a common method of defining and proving was the method I have called elsewhere the quasi-general definition and the quasi-general proof. Euclid proves the existence of infinitely many prime numbers by showing that three prime numbers being given, one can construct a fourth different from them. The proof is formulated in such a way that everybody understands immediately that it intends to show how n prime numbers being given, a $(n+1)$-th prime number can be constructed. Euclid could not sufficiently formalize such a proof or even such an assertion because he lacked a functional symbol which meant numbering a general finite set, such as our system of numbering by indices, which we owe to a gradual development that started after Vieta and lasted nearly three hundred years.

Euclid's proof for the assertion A_n is a formal proof only for $n = 3$. For A_n it is "quasi-general", i.e. every reader can be expected to be able to recognize the general pattern of the proof, and to generalize it for all natural numbers.

Up to the end of the 19th century it was commonly held that even geometrical proofs are quasi-general: a geometrical theorem is proved by means of one figure in such a way that the proof can be transferred to any other figure of the same kind.

Even now the quasi-general proof is still indispensable as a didactic tool, and the same is true of the quasi-general definition. In school mathematics, in sciences, and in daily life most definitions are quasi-general. One defines by means of instances, though in such a way that the other can generalize the definition as desired by the person who has given the definition. The method of quasi-general definitions (and even of quasi-general proofs) has been a fundamental principle in the construction and presentation of the Lincos vocabulary, in the chapter "Mathematics" as well as in the later chapters. The reader will meet a good many examples of this method. However, we may remark that our definitions will never have the logical form of a definition. Things will not be defined, but described. We shall not say "this is called a clock", but "this is a clock".

I will mention another didactic principle which has often been applied: If a set (a predicate, a relation) is to be introduced and named, we shall start not with a formal definition, but with a quasi-general definition, displaying a sufficiently large stock both of elements belonging to the set and of elements not belonging to the set (possessing the predicate, fulfilling the relation). So we shall not introduce the predicate "integer" until other

predicates can be named. We shall not introduce a word meaning action, until a large number of actions is known, and so on.

From the aforesaid it is clear that mathematical notions will not be introduced axiomatically. Of course when the Lincos vocabulary has developed far enough, one can add an axiomatic, and one can join this axiomatic to the preceding nonformalist mathematics by a semantic: We have renounced this, because it would not touch our main problem. In the chapter "Mathematics" we shall restrict ourselves to the fundamental notions. We shall even drop notions of calculus which are needed in other chapters, because at present it cannot be our aim to write a textbook of calculus in Lincos.

0 22. This is closely connected to another feature of our project.

The definite program to be broadcast will consist of a large number of pieces called program texts. The collection of texts to be found in the present book is to be considered as an abstract from the definite program. It would be no use publishing soemthing like the definite program in full. We have restricted ourselves to one example for every kind of program text which will occur. A kind such as $a+b=c$ (a, b, c, natural numbers) is represented by the single example $4+2=6$, though actually the program will contain twenty or more of this kind. If variables are introduced as a new kind of word, every text containing variables will be repeated with other variables, so that a sufficiently large stock of words meaning variables and interpreted as variables by the receiver becomes available. Program texts in which certain humans display a certain behaviour are to be followed by similar texts in which a similar behaviour is displayed by other humans, so that the receiver can understand that this is a general behaviour; and so on.

Written program texts are enclosed in pairs of signs # which do not belong to the program text itself.

0 23. For the convenience of the reader the program texts have been collected in paragraphs, and the paragraphs in chapters. The first chapters bear the titles

I. Mathematics, II. Time, III. Behaviour, IV. Space, Motion, Mass.

Further chapters entitled "Matter", "Earth", "Life", and a second chapter "Behaviour" have been planned for the second volume of this project. The reader should not attach too much importance to the division into chapters and to the titles of the chapters. Systematics is a passion not only of mathematicians. According to an ancient pattern, teaching a language is subdivided into chapters, running from the article and the

noun to conjunctions and interjections. This system is to be condemned not because it is illogical, but because it is a system. Systematics is an antididactic principle. To the same degree that it is a comfort to the teacher, it can be a discomfort to the pupil. There will be many people who are keen on systematics and who will judge the present work to be rather chaotic. My own feeling is rather that I have too often preferred systematics to didactics.

0 24.   A short abstract will give an idea of the subjects dealt with in the present volume, and of the method which has been followed. All details will be omitted.

Chapter I. Mathematics.

Ostensive numerals, quickly superseded by algorithmic ones, written in the dyadic positional system. '$+$', '$>$', '$=$', '$<$' introduced by numerical examples. Introduction of variables. Means of punctuation. The logical connectives '$\rightarrow$' '$\leftrightarrow$', '$\vee$', '$\wedge$' are introduced as connectives between arithmetical propositions. Problems are set, using the interrogation mark. Zero and negative integers, further arithmetical operations and relations. Dyadic fractions, and especially periodic dyadic fractions, which give the opportunity to introduce the word "Etc" meaning "and so on". The first predicates (sets) are 'Num' and 'Int' (natural numbers, and integers). The connectives '$\in$', '$\notin$'. The quantors '$\wedge$' and '$\vee$' arise as infinite conjunctors and disjunctors. More predicates and relations are defined, especially 'Pri' (prime numbers), 'Rat' (rationals), 'Rea' (reals), 'Com' (complex numbers), in order to prepare the more general notion 'Agg' (sets). Real numbers appear originally as infinite fractions; afterwards a more systematic theory is given. Set theory is dealt with in a rather unsophisticated way. Functions are introduced as we usually do in elementary courses; afterwards a formal definition is given. Algebraic notions such as group and field. Propositional calculus; the words 'Ver', 'Fal', '$\neg$' (true, false, non).

Chapter II. Time.

Duration measured by seconds is introduced by means of "time signals". A clock is installed. Mentioning events by means of time data.

Chapter III. Behaviour.

Actors appear. The fundamental action, 'Inq', is "saying". Actions are valuated by the judgements 'Ben' (good) and 'Mal' (bad). Conversations start with mathematics. One actor sets a problem, another solves it, and the first states whether the solution was good or bad. (If the problem is to find the x with $6\,x = 4$, the solution 4/6 will be bad.) Typical behaviour

words, such as 'Tan' and 'Sed', meaning "but". A great many interrogative pronouns and adverbs can be formed by means of the interrogation mark. Ordinals, multiplicative numerals, and words meaning "counting", "computing", "proving", "seaking", "finding", "analyzing", "responding", "changing", are introduced by showing the related actions. "Supposing that", "in the way of", "thus", "instead of", "omitting", "adding", "why", "because" appear. In common languages "why" and "because" are polysemantical. Provisionally we have adopted this use in Lincos. Later on it will be possible to distinguish the various meanings of "why" and "because" by means of indices.

Much care has to be bestowed on introducing words meaning "knowing", "perceiving", "understanding", "thinking".

An entirely new element appears with a word meaning "nearly". "Error", "much", "little", "too" (such as in "too large") are akin to it. "Now" is defined as "nearly the moment of pronouncing now". "Short time back", "long ago", and so on are akin to "now".

"Necessary" and "possible" are the first modalities that appear. Their meaning is borrowed from common language, not from modal logic.

The next topics are "age" (of humans), "existence" (actual, not logical), "birth", "death", "man", "animal", demographic statistics of actual mankind, average development of humans.

Modality is continued with words meaning "wishing", "allowing", "promising", "being obliged", "being allowed", "it is courteous", "it is convenient", "difficult".

Mathematical games are used as a means of showing human behaviour. Words meaning "giving", "penny", "fortune", law"", "game", "playing", "winning", "losing", "probability", "utility", "fair", "aiding", "ally" appear in this connection.

At the end the paradox of the liar is dealt with.

Chapter IV. Space, motion, mass.

Mechanics is not drawn up as an axiomatic system. The fundamental mechanical notions are introduced by acts of behaviour, approximatively in the way schoolboys usually learn them. Afterwards they are embedded into a mathematical system.

Difference of place is characterized by the delay of messages; distance is said to be proportional with the time of delay; the length unit is called centimeter, but its value is still unknown. Space is the union of all places. Special properties of Euclidian space are mentioned, and finally a description of Euclidian space as a three-dimensional vector space is given. Angle and volume are defined. Measures of humans and animals are mentioned. Movements are studied. Causing movements, carrying, throwing, mean

velocity, oscillation, vibration, communicating through undulating me-
diums, acoustical velocity, light velocity — this sequence of words leads
to the numerical definition of the length-unit which is sharpened by
communicating Rydberg's constant.

Mass is introduced as a property of things that determines the difficulty
of moving the things. More precisely mass ratio is defined by the law of
elastic collision. The mass unit remains unknown. Density, especially that
of human bodies. Things, humans, and animals have bodies. The existence
of the body of a human begins earlier and finishes later than the human
itself. Origin and place of a human body before the existence of the
human; notions "mother", "father", "female", "male".

Velocity, acceleration, free fall, Newton's attraction law; this fixes the
mass unit. Planetary motions, revolution, rotation. Masses, sizes, and some
movement elements of members of our solar system, and of a few stars in
our neighbourhood.

Solid bodies (their shape is difficult to change), liquid bodies (their
shape is easy to change, and their volume is difficult to change), gaseous
bodies (their volume is easy to change). Measuring: length by means of
measuring-staffs, time by means of a clock, angles by means of theodolites,
mass by means of balances — the instruments are described in a schematic
way. Measuring length by means of light signals leads to relativistic
mechanics. The fundamental relativistic notions are introduced. This has
been indispensable, because of the previous introduction of the velocity
of light as a universal constant.

0 25.   In the remainder of this introduction we shall continue the
analysis of linguistic and semantic notions interrupted by the exposition
of the main features of Lincos.

When designing Lincos the main problem I had to solve was how to
get rid of Russell's imposing theory of "description". This analysis is
particularly illustrative of Russell's work and, more generally, of formalist
semantics. Its background is not linguistic in the usual sense of the word.
No arguments are taken from language as a means of communication.
Description as it originates from this analysis, is not a means of describing,
but a mere technical device for reformulating and abbreviating certain
logistic expressions. Description in the usual sense has been eliminated by
Russell's analysis.

This is in accordance with the goal of formalist semantics. Its main
problem is to find formal rules by which the meaningfulness or even the
truth of linguistic expressions can be decided, independently of any
context. This is an important problem, but it is not ours. So an analysis
of sentences like "the present king of France is bald" cannot serve our

aims, firstly because they are too simple, but mainly because it is hard to find any context in which they could occur.

Even for Russell's standard example "the author of *Waverley*" I cannot imagine contexts in which it could be conceived as a description of Walter Scott. (Of course from Russell's standpoint this is not at all a disadvantage.) "The author of *Waverley*" is rather akin to "the Stagirite", "the laughing philosopher", "the king of beasts", "the Queen of the Adriatic", "the rock called the key of the Mediterranean", "the roof of the world", "the poet who is claimed by seven cities as their native", "the knight of the rueful countenance", "der Führer", "wo der Zimmermann das Loch gelassen hat", "l'Incorruptible", "l'Aiglon". These expressions might be called appositions with an omitted support. The name or appellative against which they are supposed to lean is omitted, because it is uniquely determined in a certain context and it is well-known. Appositions with an omitted support are used as a device of poetical or rhetorical expression. At the present stage we are allowed to disregard them. They can also be used as a means of imputation though expressions more akin to real descriptions may better serve this purpose.

Real descriptions can be found in the sentences

> The gentleman living over the way is ill.
> I have lost the pipe I bought at Smith's.
> The first book of the Bible is Genesis.
> I did not grasp what he said.

A variable (gentleman, pipe, book of the Bible) is bound by means of a restrictive clause (living over the way, I bought at Smith's) or an adjective (first). The uniqueness of the result is indicated by the definite article (the). Yet sometimes both the variable and the article can be hidden; in the last sentence the variable would be "the thing" and the restriction "that he said". Binding a variable with the definite article only can produce unambiguous descriptions of one thing. In a suitable context

> He opened the door

may mean "to open one sharply defined door". Here the definite article is not only the symptom but also the instrument of binding. In another context the same result might be got even by the indefinite article:

> I asked a policeman the way

does not point to the existence of policemen that I asked the way ("there is a policeman whom I asked the way"), but at an event of asking one well-defined policeman the way. I could also read the last sentence

> I asked the way of the policeman I happened to meet

but it would be as good an interpretation to assert that in this sentence

        a policeman

means

        the policeman whom I asked the way.

Then the sentence would run

        I asked the way of the policeman whom I asked the way

which is not at all a tautology. On the contrary I think that generally
in a sentence like

        The x with the property A has the property B

the property B may play an essential part in the description that restricts
the x; in other words that in many cases such a sentence should be read

        The x with the properties A and B has the property B.

This is not surprising, because in factual assertions the explicit description
will never be sufficient to bind the variable unambiguously. All the examples
we have given can only be understood in a broader context. Yet we cannot
admit the broader context as a part of the description, and at the same time
reject the nearer context, i.e. postulate the sentence in which the variable
occurs formally, shall not have any influence. So even the construction of
binding by means of the indefinite article can only be wholly understood
if due regard is paid to the context.

    As to the use of the article as a means or a symptom of binding (whether
the definite article or the indefinite article or no article shall be used)
natural languages disagree. There are even languages without articles.
People educated in a non-article language find it enormously difficult to
learn the use of articles in foreign languages, and users of an article language
are often embarassed by ambiguous readings caused by the lack of articles
in non-article language text, especially in short contexts.[1]) Non-article
languages prove that natural languages can do without articles, though
article substitutes may occur in non-article languages, such as the Greek
article $\tau\acute{o}$, sometimes used in medieval Latin (e.g. in $\tau\acute{o}$ esse, the being) or
Russian "to že x" which would probably be used to translate "the x" in

        The x with the property A has the property B.

    But such occurrences are rather unusual or artificial.

    In non-article languages binding of a variable to one value is formalized
by means of relative pronouns, adverbs, superlatives, ordinals, demon-
strative pronouns, adjectives, and so on. Since in Lincos these categories
are not available, we cannot dispense with a general device which formalizes
this kind of binding. We have called it article, though this word might

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    [1]) I once translated a Russian book-title "Istoriya zakona" by "History of Law"
instead of The history of a law (viz. the law of conservation of energy).

remind the reader too strongly of idiomatic peculiarities of some natural language. Perhaps neutral terms like description-connective and description-binding would have been better than article and article-binding. But this is relatively unimportant. My distinguishing definite and indefinite articles will perhaps evoke more serious criticism. I shall revert to this point.

According to Russell

> The x with the property A has the property B

is equivalent to

> There is one and only one x with the property A,
> and all x with the property A have the property B.

By this equivalence article binding is reduced to quantification ($\vee$- and $\wedge$-binding). Special devices are needed in order to ascertain how this reduction is to be carried out when "the property B" is a complex expression containing negations, quantifications and further article-bound variables. It may be doubtful where the existential quantifier that arises from the reduction of the article is to be put. So according to Russell

> The present king of France is bald

is false, though

> All present kings of France are bald

is true, and

> The present king of France is not bald

is false if it is parallelized to

> All present kings of France are not bald,

which is true,
and it is true if it is parallelized to

> Not all present kings of France are bald,

which is false.

Paradoxical results like this refute neither Russell nor English. Nevertheless Russell's procedure is misleading. As it has been often imitated, but never questioned, it is worthwhile to analyze it.

Russell's exposition is formulated as if it were an analysis of some phenomena of the English language. But in fact it is an explanation and justification of some habits adopted in *Principia Mathematica*. Here an extensive use is made of interlinear versions into English. Interlinear versions may be very useful when teaching foreign languages, but the part they play in Russell's exposition is not easy to understand. They seem to be used as arguments though it is not clear what they are supposed to prove, especially when they are accompanied by ontological arguments.

Nevertheless they are extremely instructive. Though most of his examples can hardly be put into a reasonable context, Russell does face the problem of lending them a definite meaning or a definite truth value. This is a consequence of formalization.

If problems of meaningfulness must be evaded as much as possible, the ideal solution will be to give some meaning to any expression whatso-ever. Formulas clearly borrowed from mathematics like "since so far the expression such and such is meaningless, we are free to endow it with the meaning so-and-so" are not unusual in formalist semantics, though Russell prefers more dogmatic statements. Without denying the impor-tance of eliminating problems of meaningfulness we may stress that our problem is just the inverse, i.e. not to invent some meaning for a given linguistic expression, but to invent linguistic means for sending certain messages.

Russell's analysis of description turns around the problem what

the x with the property A

means if there is no x or if there is more than one x with the property A. According to Russell, who gives ontological arguments, that expression is admissible, but meaningless, and a sentence in which it appears is (roughly speaking) false; according to Hilbert-Bernays both are forbidden; according to Quine it means the void class. All these solutions are possible, though Russell strongly advocates his.

I could not decide in favour of any of them. If a language is designed as a means of communication, it is better not to restrict its possibilities as long as there is no urgent need to do so. Furthermore it is not certain that void descriptions can be dispensed with in communication. Void descriptions are at least an important means of imputation, and since imputation is a not unusual form of social behaviour, we cannot but provide linguistic means to show it. In a certain context "the arrant rogue" may be an apposition with omitted support. In another context it might aim at a real description, though it is not one. In any case it is not clear how such expressions can be avoided, if human behaviour is to be shown. Of course one can imagine a language in which every question beginning with "whether" would have to be answered by "yes" or "no", and where it is not allowed to ask a husband whether he has stopped beating his wife, unless he has been convicted of a conduct like the imputed one in the past. Unfortunately such a language would not reflect human behaviour.

Yet this is not the main point. Closer inspection will show that we cannot apply any formalist theory of description. Such a theory would presuppose the possibility of formal descriptions, but at this stage of

Lincos formal descriptions are lacking. The best we can do is to try descriptions that can be understood by people who endeavour to understand what they are told. Formalist languages have always been dealt with by means of highly-developed non-formalist metalanguages. At a more advanced stage of Lincos we can try to do likewise, but at present it would be rather a waste of time.

The descriptions we shall give of concrete things will possibly be somewhat sharper than they are in natural languages, but they will remain extremely vague as compared with descriptions of mathematical entities. Every speaker will strongly rely on the context and on the good will of the listener.

Our rule for article-binding will not be purely syntactical. "The x", when honestly used, will mean one x; which one must appear from the formal description ("with the property A") together with the nearer and farther context. By article-binding the variable is intentionally, not formally bound to one instance, but the listener may always claim that the intentions of the speaker are clear enough. The result of article-binding must be as sharply defined as it has been in the examples at the beginning of this paragraph, put into a reasonable context. Of course article-binding interpreted in this way cannot be reduced to quantification, which is much more formal.

Mention was made above of a distinction between **definite** and **indefinite** article. I cannot deny that this distinction will be somewhat artificial. I shall prefer the indefinite to the definite article ("an x with the property A" to "the x with the property A") if the one and only one x that satisfies the context may be replaced by many formally equivalent other instances so that it does not matter which of them actually satisfies the context. In a suitable context this distinction between indefinite and definite article may be reflected by the pair of English sentences

> I opened a door,
> I opened the door of my bedroom,

or by the pair

> He wrote with a pencil,
> He wrote with the pencil I had lent him,

though indefinite articles like these will often be omitted in English

> I went by car,
> I went by the car of my friend.

It is a natural consequence of this policy that in the case of the definite article the description will be much more formal than in that of the indefinite article.

I may take the liberty of explaining how I arrived at this solution. In the first elaboration of this project I tended to avoid existential binding unless the existence had to be stressed.

I perceived a noise

was transformed into

There is a noise I perceived,

only if the existence of the noise was the main point. I made an extensive and sometimes reckless use of article-binding such as we do in the vernacular. Definite and indefinite article were not distinguished. Article binding is often a much less exact means of expression than quantification. When I looked the work over, I was assailed with doubts. For conscience sake, anxious about exactness of expression in Lincos, I undertook to eliminate the article in many occurrences, especially what could be called indefinite articles. Obviously I was still under the influence of Russell's theory of description. In any case I had not yet adequately clarified my ideas on this matter. Afterwards I happened to notice that a device for indefinite article-binding already existed in Lincos though I had failed to use it. Finally a number of texts have been transformed back by using this device. (Discoveries like this have been made more than once. Lincos proves as it were its independence. One suddenly sees that things not dreamt of beforehand can be expressed by devices created for other purposes. This is a strange sensation. Such a piece of good luck is not easily refused.)

Our rejection of Russell's theory of description is not absolute. Our dissenting from formalist semantics is only a consequence of our dealing with language as a means of communication. A language such as Russell's in which "Walter Scott" and "the author of *Waverley*" may be freely substituted for each other, cannot serve our purposes, because it does not allow to communicate the *fact* that Scott is the author of *Waverley* and to ask the *question* whether Scott is the author of *Waverley*. Nor can we do this with a language like Carnap's in which the incidental admissibility of this substitution is decided on the strength of formal criteria. The reader will have remarked a strong influence of intuitionism in our exposition. Against the formalists Brouwer was the first to stress the communicational character of language; but as mathematics can be formalized to a degree, examples drawn from it have failed to provide a convincing argument for who in daily life and in discussing mathematics use unformalized language as a means of communication though they wish to renounce this liberty, when formalization seems to be possible. I can only hope that extra mathematical examples for language as a means of communication will be more convincing.

0 26. The next notion we shall deal with is "convention". In the vernacular this has a fairly definite meaning. In some idioms, especially in that of analysis of science, it has undergone a development which is typical for many misused words. An initially aphoristic use has been interpreted as if it were seriously meant.

The original meaning of "convention" is "agreement" i.e. the act of agreeing as well as the result of agreeing, the thing agreed on. Convention may mean treaty (as in "Geneva convention") but mostly "convention" is less formal than "treaty". This informality is stressed, when "convention" is applied to forms of social behaviour which could have been the result of some explicit engagement (though actually they are the result of education). Fifty years ago it was a convention that women did not smoke in public. Convention now allows women to smoke in public. People more or less tacitly agreed to this change. Smoking or not smoking are not conventional, but smoking or not smoking under certain circumstances may be conventional. It is not conventional as long as it is not a form of *social* behaviour. There is no convention that people eat, because eating as such is biological, not social behaviour. But meal-times, manners of eating, dishes may be conventional. "Convention" seems always to refer to forms of *social* behaviour, but it is not true that all forms of social behaviour are conventional. That people do not kill each other is probably not a convention; rather is it true that many conventions have been adopted in order to ensure individual indemnity. Rousseau claimed that social organization was the result of a social contract. Of course this is not true, but it is even less true that the fact of social organization (though a symptom of social behaviour) is a convention. On the other hand many existing means of social organization may be highly conventional.

H. Poincaré in his *La Science et l'Hypothèse* (1st ed. 1902?) claimed that all general judgements of natural sciences were conventional. At least this proves to be the purport of his argumentation. He deals with different fields of physics, but constantly uses one pattern of argumentation which can meet many more general needs. So later conventionalists declared all general (not mathematical and not purely phenomenological) judgements to be conventional.

Conventionalists undoubtedly hold that their assertion "general judgements are conventional" is true, not merely conventional. Thus the first question we may raise is: "Why does the judgement 'general judgements are conventional' not apply to itself?" A provisional answer might be: This assertion is not conventional, because most people will reject it. This answer shows that the thesis of conventionalists should rather read: General (not mathematical and not purely phenomenological) judgements which are generally believed to be true, are merely conventional.

This gives rise to a second question: How about general judgements put forward by individuals and not generally approved? They are not conventional, but are they true? Was the Copernican world-system true as long as it was not generally admitted, did it become a convention afterwards, and did the Ptolemean system reach the status of truth as a consequence of Copernicus' triumph? We need not answer this question, but if we adopt the conventionalist view, we are obliged to say what kind of thing a convention, e.g. the theory of relativity, was before it was generally agreed on. Clearly conventionalists do not intend to hold that it was a truth. They have probably never considered this problem. Maybe they would answer that before being agreed on, it was a hypothesis (note that "hypothèse" occurs in the title of Poincaré's work, though it is not often used in the book itself). But this solution is impossible. Poincaré and other conventionalists continually speak about *linguistic* conventions; the counterpart of a linguistic convention would be a linguistic hypothesis, but it is evident that relativity might have been a hypothesis but not a linguistic hypothesis before it became a convention. Perhaps the predecessor of a convention could be called a proposal.

Poincaré and other conventionalists deal with "convention" and "linguistic convention" (convention de language) as if they were synonyms. This is unreasonable. Calling the metric unit of length "metre" was a linguistic convention. Adopting the metric system is not a linguistic, but a metrological convention. Adopting the name "metre" commits us to use the word "metre" under certain conditions, whereas adopting the metric system does not result primarily in certain linguistic behaviour, but in the behaviour of using metric measuring-staffs.

To elaborate: making the first metre-rod from platinum was a convention (a rheological convention); the choice of a solid as the material of the rod was not conventional, but implied by the aim, namely to get a measuring-staff. The actual depository of the standard metre in Sèvres is conventional; it is not a convention that the standard metre is held at a place where the temperature can be kept constant.

Poincaré's starting point was Euclidian geometry. He considered geometry as a physical science ("si donc il n'y avait pas de corps solides, il n'y aurait pas de géométrie"). Poincaré is far from Hilbert's axiomatic point of view. Interpreting Poincaré's work in the sense of axiomatics would be anachronistic. It appeared after Hilbert's *Grundlagen der Geometrie*, but it is a collection of essays published separately long before. Poincaré became acquainted with the idea of axiomatics only through Hilbert's *Grundlagen*. This is testified by his review of Hilbert's work where he was struck by Hilbert's questioning other axioms than that of parallels. He was not even familiar with the development of axiomatics

from Helmholtz to Hilbert. The sentence quoted above proves that he agreed with Helmholtz' erroneous criticism of Riemann's work; his ideas about geometry are mainly influenced by Helmholtz. In any case Poincaré's thesis that Euclidian geometry is conventional cannot be interpreted as an adhesion to axiomatics.

Poincaré argues that physical space could be described as well by non-Euclidian as by Euclidian geometry. This is correct as long as the curvature of geometric space is assumed equal to that of physical space (with due account to the precision of measurements). The argument still holds, if one agrees on notions of straight line, congruence and distance which differ from the usual notions. But then it ceases to be an argument. Any true theory will be replaced by another as soon as words used for one kind of things are used for a different kind. Poincaré's answer to this objection would run: In that case the physical notions of straight line, congruence and distance are conventional. To a certain extent this might be correct, but as a matter of fact the notions straight line, congruence and distance, and our physical methods of measuring distances and angles, are implied by our everyday experiences of space, in this sense that the physical notions are refinements of the everyday notions. In his discussion with Poincaré Russell pointed out that even if physical geometry is very loosely related to a very crude everyday geometry, it would not be possible to attach a curvature radius as small as 1 centimeter to the universe. Poincaré would answer once more that everyday geometry is conventional. Yet this is still more improbable. Everyday geometry is exhibited mainly by non-linguistic means, by bodily movements and by handicraft. The straight-line shape of arrows, of fold lines, of walking sticks, of paths and furrows, the circular shape of wheels and millstones, the rectangular shape of doors, the rotational symmetry of vases can hardly believed to be conventional. Of course the words denoting straight lines, circles and so on have been conventional, but as soon as a certain language is agreed upon, they lose this conventional character.

Other fields of physics have been analyzed by Poincaré with the same arguments as geometry. It is a common feature of all these analyses (but in the case of geometry it is less striking) that the rational character of physical theories is overstressed and their roots in everyday physics are rather neglected. He holds that for example the laws of mechanics are conventional definitions for the physical magnitudes related by means of these laws. Clearly such views are rooted in the same soil as definitions in French textbooks of rational mechanics like "mass is nothing but the ratio of force and acceleration". Of course "the number of pages of a sheet" may be said to be a definition of the number 16, but then the fact that such a clause defines a natural number, is still a law, and likewise

the constancy of the said ratio is a non-trivial law. Natural definitions of mechanical notions have to arise from notions of everyday mechanics by a process of refinement. For the mass of material points such a definition would run as follows:

1.   The mass of a material point is a positive number connected with a unit symbol. It does not depend on the place and velocity of that point.

2.   Two material points have equal masses if they are interchangeable in mechanical experiments (e.g. interchangeability in a symmetric arrangement).

3.   The mass of the union of two material points is the sum of the masses of the two points.

By these principles (supplemented with some principle of continuity) mass is fixed, logically and operatively. In mechanical problems these principles are continually used, but textbooks of mechanics do not deign to state such evident truths. The term "axiomatic system" (of mechanics or of electromagnetism) is often used for a collection of fundamental mechanical laws or for Maxwell's equations respectively. If by physical axioms we understand something like the above-mentioned principles for the notion of mass, we cannot but state the lack of an axiomatic system for any field of physics whatsoever. This lack may be made responsible for the usual misunderstandings as to the logical and epistemological status of mechanics and other fields of physics. As long as a few mechanical laws or Maxwell's equations can be believed to provide a sufficient basis for mechanics or electromagnetism, it is also possible to believe that physics is linked to experience by convention. In the chapter "Mechanics" a more careful analysis has been attempted, but the solution there suggested is a rather artificial one owing to the artificial character of our general problem of cosmic communication. A more natural approach is still required.

Physics isolated from experience has been responsible for Poincaré's conventionalism; if physical notions and laws are not viewed as having arisen from everyday physics, it is not astonishing that the bond between both is thought to be conventional. Yet Poincaré's difficulties are still greater. He is puzzled by the phenomenon that no isolated physical law can yield a satisfactory definition of any physical magnitude. So all notions and all laws are declared to be conventional. Poincaré disregards the possibility that nevertheless the total system could fix these notions, logically and operatively. If we remember that Poincaré wrote *La Science et l'Hypothèse* before Hilbert's *Grundlagen*, we can easily understand his perplexity. The idea of implicit definition by means of an axiomatic system

dominates Hilbert's work, but even after the publication of the *Grundlagen* it is still rejected by many mathematicians; even Frege believes it to be wrong. So we can understand why Poincaré makes no use of the concept of implicit definition by means of an axiomatic system.

How conventionalism arises from isolating the notions or the facts of a field of knowledge from each other, may be shown by many examples. When isolating one single phenomenon, one is right to hold that asserting the existence of microbes in a drop of water on microscopic evidence is a linguistic convention, and that without denying any fact we could assert the microscope to have produced the spots called microbes in the water. If, however, the action of looking through the microscope and interpreting the image seen, is taken in its usual context of biology and optics, and if that assertion as a linguistic phenomenon is to be taken as plain English, then this verdict is not allowed. Of course the conventionalist may declare biology, optics and English to be conventional and replaceable by some biology*, optics*, and English*, so that in the asterisk context it is true that it is the microscope that has produced the spots called microbes in the drop. Yet this is sooner said than done. As long as nobody has tried building up a biology* and an optics*, calling on them as arguments is premature. English* is an easier case. We should agree on saying "B produces A" instead of "A becomes perceivable by means of B". Then we are committed to introducing a new word "produce*" in order to translate the original "produce" in all its occurrences. English* would be merely coded English. Of course this is a poor result. By the same method one could prove "two and two make four" to be conventional, for it becomes false as soon as I interchange the English words three and four.

Let us imagine somebody who holds that it is a convention to consider all dogs as one species and all cats as another; one could as well agree on all male dogs and male cats as one species and on all female dogs and female cats as another. As long as no other notions and facts are affected, this may be correct. In the context of systematic zoology that assertion is false because it conflicts with the usual sense of "species". The conventionalist can evade this by declaring the usual notion of species to be conventional. He could insist on using "species" in such a way that in the case of dogs and cats it differs from the usual "species". Such a new notion of species would conflict with the bulk of our scientific habits, but for the conventionalist our scientific habits are as conventional as the others. If science were isolated from life, this might be true. But to the extent that science is one of our vital functions and means of subsistence, it is as little conventional as eating and sleeping are.

Isolating some parts of knowledge or language from their context is not bad in itself. It may even be very useful for analysis, just as considering

artificially-isolated physical systems may be useful in physics. But in physics one tries to account for the exterior disturbances when applying the results to a non-isolated real system. It is not only in conventionalist analysis that disregard of context is a weakness. It is a danger that threatens all attempts at analysis.

So far we have not given a proper answer to the first question of his paragraph: Why does the conventionalist not apply his thesis "all general assertions are conventional" to the sentence included within quotation marks? In this form the question asks for a motivation of conventionalist behaviour, which in fact is only a special case of a more general behaviour. People who analyze languages or idioms or theories is in order to discover features common to them, often state or postulate general rules which are belied by the language or the idiom they use or by the theory they develop when dealing with languages or idioms or theories (e.g. nominalists forbid general concepts while using a great many general concepts, especially the general concept "general concept"). More or less artificial idealizations, abstractions, dissections, isolations which are applied (perhaps because they cannot but be applied) to languages, idioms, theories as a subject matter, are lacking in the language, the idiom, the theory which is used as a working medium. We may disregard the communicating character of a subject language and the connection of a subject theory with reality, but in the meantime we still use the working language (metalanguage) as a means of communication, and we still believe that the working theory (metatheory) is related to some reality.

This explanation can probably not be applied to Poincaré's behaviour. In fact it has been Poincaré's merit to turn against two interpretations of geometry and mechanics, the aprioristic and the empiristic. He pointed out that geometry and mechanics were neither a priori nor empirical. Initially conventionalism was a merely aphoristic answer to the question what geometry and mechanics are, if they are neither apriori nor empirical. This aphorism has shared the fate of many aphorisms in being taken seriously and repeated more emphatically. While Poincaré discovered "conventions" and "linguistic conventions" everywhere, other conventionalists preferred the terms "arbitrary (linguistic) conventions" and "purely arbitrary (linguistic) conventions", but I doubt whether they sincerely meant that e.g. geometry is purely arbitrary; it seems more probable that they only wished to express their very strong conviction that geometry was conventional.

There is still another way of explaining conventionalist behaviour. In physics and other natural sciences it has often happened that in order to explain some observational facts investigators proposed two or more theories which seemed to fit equally well. Experiments were carried out

in order to decide for one of the competing theories. Gradually one of them became more and more probable, and finally the evidence for this one and against the other one became overwhelming. A certain theory which is now taken to be the true theory, might have been valued only as better or as not worse than other competing theories. So it would be wise to consider it not as a true theory, but as much the best theory now available. This remark is quite reasonable. It matches very well the actual behaviour of investigators who usually work with the best theory as if it were true. If you want to know the value of the elementary charge, a a physicist will answer you: the best value now available is such and such. (Usually the accuracy of a value will be measured by the observational error.) Such an answer does not mean: The value such and such is the present conventional value. It is possible that the only thing conventionalists wish to state is that theories are always questionable. This is correct, but in this respect observational facts are no better.

Valuations like "the best theory now available", though disregarded by analysts, are very important in science. Two correct mathematical theories in the same field need not be equally good; one of them may give more insight into the subject-matter. Among several definitions of the same mathematical object there may be one which is better than the others. Physicists might discuss the best method of arranging a certain experiment. People who work in analysis of scientific knowledge or of scientific language (idiom) are just trying out explanations which organize the subject matter somewhat better than other explanations do. Mostly there is no overwhelming evidence for one of them. So the explanation "a true theory is a theory which is much better than competing theories" is itself at most only much better than other explanations.

The reader might wonder why so much attention has been paid to the conventionalist thesis. If this thesis were correct, our Lincos project would be doomed to failure. If e.g. comparison of time intervals were purely conventional, we could not expect our messages containing a time signal together with the numerical value of its length to be interpreted as such. If Euclidean space as a good approximation of real space were conventional, we could not expect the receiver of our messages about space to interpret the intended object as space. In my opinion physical notions are not arbitrary. This belief is based on the history of science. In earlier times some discoveries might have been accidental. But theories which are current now have been the result of systematic attempts. Kepler and Planck tried a great many hypotheses before they succeeded with the one which has been accepted, not only by logical coercion, but also on the strength of overwhelming evidence. I suppose that these historical facts are little known.

The opinion that scientific theories are completely arbitrary is also held by irrationalists, as a consequence of their fundamental conviction that truth cannot be reached by rational methods, but only by intuition. I refrain from discussing this attitude.

0 27.   Finally we shall try to justify our use of the word "meaning" and of related words.

We start with a few artificial dialogues in which the verb "to mean" occurs.

1. A. I saw a robot at the exhibition.
   B. What is a robot?
   A. "Robot" means something like "golem".

2. A. I saw a robot at the exhibition.
   B. What is a robot?
   A. "Robot" means a machine which behaves like a human.

3. A. What does "Zoll" mean?
   B. Where did you read it?
   A. At Hamburg airport.
   B. There it means the Customs.

4. A. Miss Parker is ill.
   B. Who is Miss Parker?
   A. I mean our nurse.

5. A. She is a nice girl.
   B. Do you mean the blonde or the brunette?

6. A. She is a nice girl.
   B. Do you mean Miss Parker?

7. A. She is a nice girl.
   B. Do you mean this girl?

8. A. This is a robot.
   B. What do you mean by "robot"?

9. A. What did you pay for the car?
   B. It was very cheap.
   A. What do you mean by "cheap"?

I have avoided examples like "this cloud means rain", "I mean you no harm", "he meant what he said", in which we meet a semantically quite different verb "to mean".

There can be little doubt about the meaning of "to mean" in our examples. Words, sentences, and other linguistic phenomena may mean something, and what they mean can be shown, or said, by definition or

circumscription in more intelligible terms or by translation into a better-known language. Two slightly different syntactical forms are possible: "X means Y" and "by X somebody means Y". The first looks more objective, but in fact whenever the word "to mean" is used, it is bound to a special situation in a special way. In the 8th example "Do you mean . . ." refers in a more explicit way to the user of the word "robot" than "to mean" does in the 2nd or 3rd examples. If a more explicit reference to the actual situation is wanted in the explanation of the 2nd example, A could say: If "robot" is used in one context together with "exhibition", it means a machine which behaves like a human. Or: people speaking about an exhibition mean by "robot" a machine which behaves like a human. Or in the 3rd example: By "Zoll" people in Hamburg airport mean "customs".

(The double syntactic pattern of English "to mean" is neither reflected by French "signifier", nor by German "bedeuten". "Vouloir dire" and "bezeichnen" show the same feature as "to mean".)

So far "to mean" raises no special problems. Even the noun "meaning" would not cause difficulties if we used "the meaning of X is Y" as synonymous with "X means Y". Yet there is a strong tendency to assert that a thing which means something has a definite *meaning*, just as a thing which ends somewhere has a definite *ending*. "Meaning" (or "denotation" or "nominatum") is used as if it were a function from the set of linguistic entities to that of real entities — somehow this is the leading idea of any attempt of formalist semantics. I would not object to this use of "meaning" if it were suitably restricted and if it did not falsely suggest a much too simple description of the relation between language and reality.

I concede that linguistic expressions generally have some meaning, but I cannot imagine general rules of meaningfulness, and I do not know how to attach just one meaning to every meaningful linguistic expression. I think that in every not-too-poor language the meaning of expressions will strongly depend on the context in which they occur. The importance of the context is often disregarded as soon as language is not primarily considered as a means of communication.

The ancient belief that everything has a true name and that the system of the true names yields the true language, has given way to the insight that there is essentially a variety of languages, and that the meaning of a linguistic expression depends on the language in which it is met. Moreover nobody will seriously deny that in natural languages there are no general rules for stating whether some expression is meaningful and that the meaning of a meaningful expression may depend on its context. If someone asserts the contrary or if he acts as though the contrary were true, it is probable that he has in view not a natural language but some artificial

language and that he has tacitly asked permission to take his examples
from a natural language pending the existence of that artificial language.
If formalized mathematics is destined to play the part of that artificial
language, one may ask why the examples have not been taken from
mathematics. Formalized mathematics might be a suitable field to which
the formalist semantical notions could be applied, but I doubt whether
this is still true if a richer language with more essential relations to reality
is to be discussed. I think Carnap is right when he confines his analysis to
an artificial language given in advance and precisely described, though
I have the impression that his languages are just not rich enough to
disclose essential problems.

Formalist semantics is atomistic. It makes the assumption that some
elementary expressions have a definite meaning, and that the meaning
of composed expressions is derived by certain rules from that of the
elementary expressions. If this could be realized, the meaning of any
expression could be made independent of the context. In Lincos I have not
pursued this aim, mainly because I do not believe in it. Nevertheless
I admit that it might be important to fix one's attention on this aim as
long as possible.

Proper names seem to be a strong argument in favour of atomistic
semantics. But this argument cannot be maintained on closer inspection.
Proper names with a definite meaning are extremely rare (I disregard
mathematical notions). Socrates, Rembrandt, Walter Scott are good
examples, but what about John Smith and Mary Young? William
Shakespeare is never mentioned in this connection. In common English
it will depend on the context whether "William Shakespeare" means some
person known by certain portraits or a certain playwright or both. It is
an advantage that in common English one can raise questions like that
of the identity of Shakespeare even if they are whimsical. Formalist
semantics aim at a language in which this freedom is not allowed.

A moment ago we tackled a well-known problem. Usually it is connected
to the name of Walter Scott: Is the same meaning to be ascribed to
"Walter Scott" and "the author of *Waverley*"? This case looks somewhat
easier because Scott's authorship of Waverley is usually admitted to be
objective truth. The so-called problem of intension in a language with
formalist semantics is how to account for the fact that there might be
people who are not acquainted with that objective truth or who do not
believe it. In fact the case of Walter Scott is not essentially different
from that of Shakespeare. In some epoch the identity of the author of
Waverley was more obscure than that of the author of *Hamlet* (e.g. before
Scott published *Waverley*). At some moment Scott's authorship became
known. A language in which it will be possible to account for discoveries

of new facts cannot do with expressions which have a definite meaning. Formalist semantics will even not match mathematics, if mathematics is considered not as a stock of true propositions but as a field of invention.

The convention about the meaning of propositions is the strangest feature of formalist semantics. According to Frege a true proposition means truth, a false proposition means falsehood. As opposed to incomplete expressions such as subjects, predicates, and so on, propositions are syntactically independent. Therefore it is easier to put an isolated word "red" or "now" into a virtual context than to do the same with a proposition. The context of incomplete sentences like "it is raining" can also be guessed with relative ease. But what about propositions like "Socrates is a man" or "It rained at Utrecht on December 20th, 1957, at 15 o'clock"? The only context in which "Socrates is a man" can be imagined is the famous syllogism where it is the minor, and in this context the only thing that matters is just to know whether the minor is true. The second sentence might be met with in the report on a bet or something like this, and in this place its meaning might be truth or falsehood indeed. Such arguments may have been responsible for Frege's convention. Moreover if "meaning" is agreed to be a non-trivial function from the set of linguistic expressions which does not depend on the context, the truth function defined for propositions will be welcomed, because it seems to fit very well the requirements of the "meaning" function. In my opinion this does not justify infringing the terminology of the vernacular with respect to "meaning" as radically as Frege did, even if "meaning" is replaced by "denotation" or "nominatum". It is dangerous to create the impression that difficult semantic problems could be solved by a mere convention.

0 28.   Attempts to construct symbolic languages have often been linked to some philosophy, though the linking usually proved to be less close afterwards than the author believed. Dealing with language as a means of communication has been the only philosophy I have espoused in the present project. Though devised for a special aim Lincos must be a general-purpose language in which all sayable things can be said. By formal linguistic means we cannot forbid any religious, ethical, philosophical, political or social faith or even any nonsense. So it is better to abstain from such attempts. Our method has been determined by linguistic considerations only. Our pragmatism has been that of an engineer, not of a philosopher. The use of both predicates and sets from the beginning does not prove any partiality in favour of nominalism or realism. The use of the word behaviour does not involve support for behaviorism. Introducing human bodies much later than humans themselves is not to be interpreted as a spiritualist attitude. Defining life as the possibility of

perceiving is not the consequence of any form of sensualism. Lincos words for "good", "bad", "allowed", "forbidden" do not reflect any ethical theory.

Lincos is moderately formalized, but we do not object to fully-formalized language. Foolproof languages have an importance of their own. I hope Lincos will be wise-proof. Formalized languages appeal to the malevolent reader. The present book is addressed: lectori benevolenti.

Utrecht, December 23rd, 1957.

MATHEMATICS

1 00 0. Pairs of signs ⁜ will enclose the printed image of a program text.

A (metatextual) "and so on" after a text indicates that this text is an exemplary extract from the factual program. When carrying out the program we will replace this text by a large number of texts similar to the text we have printed. If the number of examples is large enough, we may expect that the receiver can generalize the quasi-general definition or proposition that is intended by the program text.

1 01 0. ⁜ $>$ I $<$ I $=$ I $+$ I $-$ I $\neq$ I $\leqq$ I $\geqq$ I

. I . . I . . . I I

1 I 10 I 11 I 111 I

a I b I c I

$\rightarrow$ I $\vee$ I $\wedge$ I ? I $\leftrightarrow$ ⁜ •

Loose Lincos words are presented, without any context, in order to stress their individuality. So it will be somewhat easier for the receiver to recognize them when they occur in a certain context.

The bold-faced strokes mean pauses.

1 01 1. ⁜ • • • • • $>$ • • • ⁜ and so on.

1 01 2. ⁜ • • • $<$ • • • • • ⁜ and so on.

1 01 3. ⁜ • • • • $=$ • • • • ⁜ and so on.

1 01 4. ⁜ • • • • $+$ • • $=$ • • • • • • ⁜ and so on.

1 01 5. ⁜ • • • • • • $-$ • • $=$ • • • • ⁜ and so on.

In these texts the Lincos phoneme that corresponds to the round dot is a short radio-signal (a peep). A Lincos word that consists of n successive phonemes of this kind, separated by short and equal intervals, is written as a group of n round dots. It both means and shows the *natural number n*. It is an *ideophonetic* word, which has the power of an image as well as that of a word. We also call it an *ostensive numeral*. The greater part of the Lincos vocabulary will be purely conventional; words may be permutated at pleasure.

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This is not true of ideophonetic words. Their essential features must not be changed.

The Lincos words written $>$, $<$, $=$, $+$, $-$, and so on designate connectives with the usual meaning. The receiver should guess their meaning from the context. Therefore each of the first program texts contains one unknown word only.

1 02 1. # $\cdot = 1$

$\cdot \cdot = 10$

$\cdot \cdot \cdot = 11$

$\cdot \cdot \cdot \cdot = 100$

$\cdot \cdot \cdot \cdot \cdot = 101$

$\cdot \cdot \cdot \cdot \cdot \cdot = 110$

$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot = 1101$ #

and so on.

Ostensive numerals are superseded by *algorithmic* ones, composed of *syllables* written 0 and 1 according to the rules of the *dyadic positional system*. For the convenience of the terrestrial reader we shall sometimes use the decimal code, but as a matter of fact such occurrences should be translated into the dyadic code.

1 02 2. Texts as those of **1** 01 1-**1** 01 5 will be repeated, using algorithmic numerals instead of ostensive ones.

1 03 1. # $111 = 110 + 1 = 101 + 10 = 100 + 11 = 11 + 100 =$
$10 + 101 = 1 + 110$ # and so on.

1 03 2. # $111 + 11 > 11 + 101 > 1 + 100 = 101$ # and so on.

In elementary arithmetics expressions such as "$4 + 3$" are often considered as problems, not as numbers; "$4 + 3$" is read as if it were "add four and three", "$4 + 3 = 7$" as if it were "if I add four and three, I get seven". Our program texts will prevent such interpretations. The receiver will learn the *relational symmetry* of the Lincos word written $=$, as well as the *"static"* character of expressions as '$111 + 11$'.

1 04 1. # $100 > 10$
$100 + 11 > 10 + 11$
$100 + 1101 > 10 + 1101$
$100 + 1 > 10 + 1$
$100 + 110 > 10 + 110$
$100 + 11111 > 10 + 11111$
$100 + a > 10 + a$ #

and so on.

By such texts we introduce Lincos words that mean *variables*. In Lincos speech words meaning variables will be distinguished from other words by certain phonetic characteristics.

1 04 2. $* 100 + 111 = 111 + 100$
$100 + 1 = 1 + 100$
$100 + 1101 = 1101 + 100$
$100 + 11 = 11 + 100$
$100 + a = a + 100 *$
and so on.

1 04 3. $* a + 1110 = 1110 + a$
$a + 11 = 11 + a$
$a + 11011 = 11011 + a$
$a + 1 = 1 + a$
$a + b = b + a *$
and so on.

1 04 4. Texts as those of 1 04 1-1 04 3 will be repeated using *other* words that mean variables. A sufficiently large stock of such words should be provided for.

1 05 0. So far no explicit attention has been paid to *punctuation*. In fact we can dispense with an exposition of the means and rules of punctuation. In speech, we mark punctuation mainly by *pauses*, and we will do so in Lincos speech as well. Pauses will be used as Lincos punctuations. It is understood that the longer pause will have the stronger power of separation, and the receiver will understand this tacitly imposed rule.

In the printed image spaces would be the equivalent of pauses, but such texts would annoy the reader. Russell-Whitehead square dots are much better as long as the number of dots in one punctuation mark does not exceed six. Unfortunately seven- and eight-point marks will not be unusual in our texts; we shall even meet with eleven- and twelve-points marks. So we have devised a variation on the Russell-Whitehead method:

A square dot will account for 1, 2, 4 *punctuation units* according as the level of its placing is the bottom, the middle, or the top of the letters b, d, h and so on. A short vertical bar will account for 8 units, a long vertical bar for 16 units, and a long horizontal bar for 32 units. Every group of such elements represents one punctuation mark. In each group the units are added, in order to get the *punctuation value* of the corresponding mark. The *separative power* of a mark will be a monotonously increasing function of its punctuation value.

We give a list of punctuations with their values:

$$. \; 1 \quad \cdot \; 2 \quad : \; 3 \quad \dot{} \; 4 \quad \vdots \; 5 \quad \vdots \; 6 \quad \vdots \; 7$$

$$\iota \; 8 \quad \iota. \; 9 \quad \iota\cdot 10 \quad \iota: 11 \quad \iota\dot{}\; 12 \quad \iota\vdots 13 \quad \iota\vdots 14 \quad \iota\vdots 15$$

$$| \; 16 \quad |. \; 17 \quad |\cdot 18 \quad |: 19 \quad |\dot{}\; 20 \quad |\colon 21 \quad |\vdots 22 \quad |\vdots 23$$

$$|\iota \; 24 \quad |\iota. \; 25 \quad |\iota\cdot 26 \quad |\iota: 27 \quad |\iota\dot{}\; 28 \quad |\iota\colon 29 \quad |\iota\vdots 30 \quad |\iota\vdots 31$$

$$\overline{\hspace{2cm}} \; 32$$

We shall not use all these marks. In *algebraic* formulas we shall often stick to the classical procedure (brackets with the usual conventions), but this is rather a concession to the terrestrial reader.

In our system there is a great variety of punctuation marks. So there is no need to economise punctuations, as there is in the Russell-Whitehead system. We can also avoid another drawback of that system and of other systems: the use of logical connectives as punctuation marks, and artificial conventions regarding their separative power.

Let us emphasize once more that these considerations do not affect Lincos speech, but only written Lincos.

1 05 1. $\# \; 101+11.+111\cdot=\cdot 101+.11+111=.101+11+111:$
     $101+11.+1\cdot=\cdot 101+.11+1\cdot=.101+11+1:$
     $101+11.+101\cdot=\cdot 101+.11+101\cdot=.101+11+101:$
     $101+11.+c\cdot=\cdot 101+.11+c\cdot=.101+11+c:$
     $101+1001.+c\cdot=\cdot 101+.1001+c\cdot=.101+1001+c:$
     $101+b.+c\cdot=\cdot 101+.b+c\cdot=.101+b+c:$
     $1101+b.+c\cdot=\cdot 1101+.b+c\cdot=.1101+b+c:$
     $a+b.+c\cdot=\cdot a+.b+c\cdot=.a+b+c \; \#$

1 05 2. $\# \; a+\cdot b+.c+d:=:a+\cdot b+c.+d:=:a+b.+.c+d:=$
     $:a+.b+c\cdot+d:=:a+b.+c\cdot+d:=.a+b+c+d \; \#$

1 06 1. $\# \; a>100.\rightarrow.a>10\cdot$
     $a>1101.\rightarrow.a>1\cdot$
     $a>11.\rightarrow.a>11 \; \#$
     and so on.

1 06 2. $\# \; a<10.\rightarrow.a<11101\cdot$
     $a<110.\rightarrow.a<110\cdot$
     $a<111.\rightarrow.a<1110 \; \#$
     and so on.

1 06 3. $\# \; 11<a.\rightarrow.1<a\cdot$
     $10<a.\rightarrow.10<a\cdot$
     $111<a.\rightarrow.a>10\cdot$
     $a>111.\rightarrow.10<a \; \#$
     and so on.

1 06 4. $* a > b . \rightarrow \cdot a + 1 . > b :$
$a > b . \rightarrow \cdot a + 10 . > b :$
$a > b . \rightarrow \cdot a + 110 . > b *$
and so on.

1 06 5. $* a < b . \rightarrow \cdot a < . b + 1 :$
$a < b . \rightarrow \cdot a < . b + 111 :$
$a < b . \rightarrow \cdot a < . b + 10 *$
and so on.

1 06 6. $* a = b . \rightarrow . b = a \cdot$
$a > b . \rightarrow . b < a \cdot$
$a < b . \rightarrow . b > a :$
$a > b . \rightarrow \cdot b < . a + 101 *$
and so on.

1 06 7. $* a + b . = c \cdot \rightarrow \cdot a + b + 1 . > c :$
$a + b . = c \cdot \rightarrow \cdot a + b + 11 . > c :$
$a + b . = c \cdot \rightarrow \cdot a + b . < . c + 1 :$
$a + b . = c \cdot \rightarrow \cdot a + b . < . c + 110 *$
and so on.

1 06 8. $* a + b . = c \cdot \rightarrow \cdot a = . c - b :$
$a + b . = c \cdot \rightarrow \cdot b = . c - a :$
$a = . c - b . \rightarrow \cdot a + b . = c *$
and so on.

At 1 06 1 the first connective with a logical meaning has occurred. In order to avoid misinterpretations we have inserted texts in which the '$\rightarrow$' could be replaced by '$\leftrightarrow$'. We have deliberately not admitted texts like

$10 + 1 . = 11 \cdot \rightarrow \cdot 11 + 1 . = 100 :$
$10 + 1 . = 101 \cdot \rightarrow \cdot 11 + 1 . = 100 :$
$10 + 1 . = 101 \cdot \rightarrow \cdot 11 + 10 . = 100 :$
$1 + 1 . = 10 \cdot \rightarrow \cdot a + b . = . b + a$

and so on. Such texts are meaningless for any receiver who is not acquainted with the formal use of implication.

1 06 9. $* a > 11 . \rightarrow : b > 10 . \rightarrow \cdot a + b . > 110 \cdot$
$a > c . \rightarrow : b > d . \rightarrow \cdot a + b . > . c + d \cdot$
$a > b . \rightarrow \cdot b > c . \rightarrow . a > c \cdot$
$a = c . \rightarrow : b = d . \rightarrow \cdot a + b . = . c + d \cdot$
$a = b . \rightarrow \cdot b = c . \rightarrow . a = c *$
and so on.

1 07 1.   # $a>100.\leftrightarrow.a>100$:
     $a>100.\leftrightarrow.100<a$:
     $a>b.\leftrightarrow.b<a$:
     $a>b.\leftrightarrow\cdot a+1.>.b+1$:
     $a>b.\leftrightarrow\cdot a+c.>.b+c$:
     $a+b.=c\cdot\leftrightarrow\cdot a=.c-b$ #

and so on.

1 07 2.   # $a=b.\rightarrow.c=d\cdot\rightarrow:c=d.\rightarrow.a=b\cdot\rightarrow\cdot a=b.\leftrightarrow.c=d$ #

The tautology $p\rightarrow q.\rightarrow\cdot q\rightarrow p.\rightarrow.p\leftrightarrow q$ has been stated in an exemplary way. As long as we have no words that designate propositions, this will be the way of formulating tautologies.

1 08 1.   # $1\neq10.1\neq11.1\neq100.1\neq110111.$
     $10\neq1.10\neq11.10\neq1010.$
     $11\neq1.11\neq10.11\neq111.$
     $1101\neq11011.110\neq111.111010\neq1100$ #

and so on.

1 08 2.   # $a>b.\rightarrow.a\neq b\cdot$
     $a<b.\rightarrow.a\neq b\cdot$
     $a\neq b.\rightarrow\cdot a+c.\neq.b+c$ #

and so on.

We try to avoid formulas that no longer hold true when the domain of the variables is extended.

1 09 1.   # $a>1.\vee.a<10\cdot$
     $a>10.\vee.a<11\cdot$
     $a>111.\vee.a<1000\cdot$
     $a>1.\vee.a<11\cdot$
     $a>10.\vee.a<100\cdot$
     $a>1110.\vee.a<110001$ #

and so on.

1 09 2.   # $a\neq101.\rightarrow\cdot a<101.\vee.a>101$:
     $a\neq b.\rightarrow\cdot a<b.\vee.a>b$:
     $a=b.\vee.a<b.\vee.a>b$:
     $a=10.\vee.a=1001.\vee.a=111\cdot\rightarrow.a<1011$:
     $a=10.\vee.a=1001.\vee.a=111\cdot\rightarrow.a>1$:
     $a=10.\rightarrow\cdot a=10.\vee.a=111$ #

and so on.

1 09 3. ${}^{\#}a=b.\to\cdot a=b.\lor.c=d:$
   $a=b.\lor.c=d\cdot\leftrightarrow\cdot c=d.\lor.a=b:$
   $a=b.\lor.c=d\cdot\leftrightarrow\cdot a\neq b.\to.c=d\,\dot{}$
   $a=b.\lor.c=d\cdot\lor.e=f:\leftrightarrow:a=b.\lor\cdot c=d.\lor.e=f\,\dot{}$
   $a=b.\lor.c=d\cdot\lor.e=f:\leftrightarrow\cdot a=b.\lor.c=d.\lor.e=f\,{}^{\#}$
   and so on.

   Exemplary tautologies. See 1 07 2.

1 10 1. ${}^{\#}1\leqq1.1\leqq10.1\leqq11.1\leqq10101.$
   $10\leqq10.10\leqq11.10\leqq111.$
   $11\leqq11.11\leqq11011.$
   $1\geqq1.10\geqq1.11\geqq1.1100\geqq1.$
   $10\geqq10.111\geqq10.$
   $1110\geqq101\ {}^{\#}$
   and so on.

1 10 2. ${}^{\#}a\leqq b.\leftrightarrow\cdot a=b.\lor.a<b:$
   $a\geqq b.\leftrightarrow\cdot a=b.\lor.a>b:$
   $a\leqq b.\lor.a\geqq b\ {}^{\#}$
   and so on.

1 11 1. ${}^{\#}a=111.\to.a\geqq10:$
   $a=111.\to.a\leqq11001:$
   $a=111.\to\cdot a\geqq10.\land.a\leqq11001:$
   $a\geqq10.\land.a\leqq111\cdot\leftrightarrow\cdot a=10.\lor.a=11.$
   $\lor.a=100.\lor.a=101.\lor.a=110.\lor.a=111\ {}^{\#}$
   and so on.

1 11 2. ${}^{\#}a=111.\leftrightarrow\cdot a\leqq111.\land.a\geqq111:$
   $a=b.\leftrightarrow\cdot a\leqq b.\land.b\leqq a\ {}^{\#}$
   and so on.

1 11 3. ${}^{\#}a\leqq111.\land.b\leqq10\cdot\to\cdot a+b.\leqq1001:$
   $a>110.\land.b\geqq10\cdot\to\cdot a+b.>1000:$
   $a\leqq b.\land.c\leqq d\cdot\to\cdot a+c.\leqq.b+d:$
   $a>b.\land.c\geqq d\cdot\to\cdot a+c.>.b+d\ {}^{\#}$

1 11 4. ${}^{\#}a=b.\land.c=d\cdot\to.a=b\,\dot{}$
   $a=b.\land.c=d\cdot\to\cdot c=d.\land.a=b\,\dot{}$
   $a=b.\land.c=d\cdot\land.e=f\cdot\leftrightarrow:a=b.\land\cdot c=d.\land e=f\,\dot{}$
   $a=b.\land.c=d\cdot\land.e=f:\leftrightarrow:a=b.\land.c=d.\land.e=f\,\dot{}$
   $a=b.\to.e=f\cdot\land\cdot c=d.\to.e=f:\to:a=b.\lor.c=d\cdot\to.e=f\,\dot{}$
   $a=b.\to.c=d\cdot\land\cdot c=d.\to.e=f:\to\cdot a=b.\to.e=f\,\dot{}$
   $a=b.\to\cdot c=d.\to.e=f:\leftrightarrow:a=b.\land.c=d\cdot\to.e=f\,{}^{\#}$
   and so on.

1 12 1. $*\,?\,x\,.\,x+10=111\,\cdot$
         $x+10=111\,.\to.\,x=101\,\cdot$
         $?\,x:a<b\,.\,\wedge\cdot x+a\,.=b\,\cdot$
         $a<b\,.\,\wedge\cdot x+a\,.=b:\to\cdot x=.\,b-a\;*$

and so on.

In symbolic logic little attention has been paid to sentential forms other than propositional ones. Problems and questions, though indispensable in mathematics and life, cannot be formalized by the usual symbolic methods. The philosophy that mathematics is rather a stock of true propositions than an art of discovering is responsible for this omission.

We have introduced a word written as an interrogation mark in order to formalize *questions*.

1 13 1. $*\,?\,x\cdot x+101\,.=11\;*$

and so on. No answer is given.

1 13 2. $*\;11-1\,.=10:$
         $11-10\,.=1:$
         $11-11\,.=0:$
         $11-100\,.=.-1:$
         $11-101\,.=.-10:$
         $11-110=.-11:$
         $11-11001\,.=.-10110:$
         $11101-1101101\,.=.-1010000\;*$

1 13 3. $*\,a-a\,.=0\,\cdot$
         $a<b\,.\to:a-b=\cdot-.\,b-a\;*$

and so on.

1 13 4. $*\,?\,x\cdot x+a\,.=b:$
         $x+a\,.=b\,\cdot\to\cdot x=.\,b-a\;*$

and so on.

1 13 5. $*\;10+11=101:$
         $1+11\,.=100:$
         $0+11\,.=11:$
         $-1\,.+11\cdot=10:$
         $-10\,.+11\cdot=1:$
         $-11\,.+11\cdot=0:$
         $-100\,.+11\cdot=-1\;*$

and so on.

* $-100.+11\cdot=.-1:$
    $-100.+10\cdot=.-10:$
    $-100.+1\cdot=.-11:$
    $-100.+0\cdot=.-100:$
    $-100.+.-1\cdot=.-101:$
    $-100.+.-10\cdot=.-110$ *

Of course different words must be used for the "minus" operating on one object (in $-a$) and the "minus" operating on a pair of objects (in $a-b$).

Addition and substraction laws for integers will be developed in detail by the method of quasi-induction. We omit these texts.

1 13 6. * $0\neq1:0\neq10:0\neq11:0\neq1101:$
    $-1.\neq0:-1.\neq1:-1.\neq10:-1.\neq11101:$
    $-10.\neq0:-10.\neq101:$
    $-100.\neq.-1010$ *

and so on.

1 13 7. * $-10001.<.-100.<.-11.<-10.<0<1<110$ *

1 13 8. * $a<b.\leftrightarrow\cdot-b.>.-a:$
    $a>0.\leftrightarrow\cdot-a.<0:$
    $a>b.\leftrightarrow\cdot a-b.>0:$
    $-.-a\cdot=a$ *

and so on.

1 13 9. Some earlier formulas will be repeated in order to suggest that variables now range through the system of all integers.

1 14 1. * $101\times1=101.$
    $101\times10=1010.$
    $101\times11=1111.$
    $101\times100=10100.$
    $101\times101=11001.$
    $101\times1010=110010$ *

and so on.

1 14 2. * $101\times0=0\cdot$
    $101\times.-1=.-101\cdot$
    $101\times.-10=.-1010\cdot$
    $101\times.-1010=.-110010$ *

and so on.

1 14 3.　$* \, a \times 0 = 0 :$
　　　　$a \times . \, b + 1 \cdot = \cdot a \times b . + a :$
　　　　$a \times . \, b - 1 \cdot = \cdot a \times b . - a \, *$
　　and so on.

1 14 4.　$* \, a \times b = b \times a :$
　　　　$a \times . \, b \times c \cdot = \cdot a \times b . \times c = . \, a \times b \times c :$
　　　　$a + b . \times c \cdot = \cdot a \times c . + . \, b \times c \, *$
　　and so on.

　　For the convenience of the reader of written Lincos we shall
now adopt the habit of *omitting the multiplication sign* and of ·
using brackets in algebraic contexts.

1 14 5.　$* \, ab = 0 . \leftrightarrow \cdot a = 0 . \lor . b = 0 \, \dot{}$
　　　　$ab \neq 0 . \leftrightarrow \cdot a \neq 0 . \land . b \neq 0 \, \dot{}$
　　　　$ab > 0 . \leftrightarrow : a > 0 . \land . b > 0 \cdot \lor \cdot a < 0 . \land . b < 0 \, \dot{}$
　　　　$aa \geqq 0 \, *$
　　and so on.

1 15 1.　$? \, x . \, 11 \, x = 110 \cdot$
　　　　$11 \, x = 110 . \rightarrow . \, x = 10 :$
　　　　$? \, x . \, 11 \, x = 1100 \cdot$
　　　　$11 \, x = 1100 . \rightarrow . \, x = 100 :$
　　　　$? \, x . \, 11 \, x = 100 \cdot$
　　　　$11 \, x = 100 . \rightarrow . \, x = 100/11 \, *$
　　and so on.

1 15 2.　$* \, 11 \, x = 100 . \leftrightarrow . \, 110 \, x = 1000 \cdot$
　　　　$100/11 = 1000/110 = 1100/1001 = 11000/10010$
　　　　　　$= (- 100) / (- 11) = (- 1000) / (- 110) \, *$
　　and so on.

1 15 3.　$* \, ? \, x \cdot a \neq 0 . \land . ax = b :$
　　　　$a \neq 0 . \land . ax = b \cdot \rightarrow . \, x = b/a \, *$

1 15 4.　$* \, ax = b . \rightarrow . \, cax = cb \cdot$
　　　　$c \neq 0 . \rightarrow . \, b/a = (cb)/(ca) \, *$

1 15 5.　$* \, a/1 = a \, *$

1 15 6.　$* \, 110/101 . > 0 \cdot$
　　　　$110/(- 101) . < 0 \cdot$
　　　　$ab > 0 . \leftrightarrow . \, a/b > 0 \cdot$
　　　　$ab < 0 . \leftrightarrow . \, a/b < 0 \, *$

1 15 7. In an analogous way arithmetics of fractions will be exposed in all necessary details.

1 16 1. 　＊ $1100/10 = 110$.

　　　　$110/10 = 11$.

　　　　$11/10 = 1,1$.

　　　　$1,1/10 = 0,11$ ＊

　　and so on.

1 16 2. 　＊ $1/10 = 0,1$.

　　　　$1/100 = 0,01$.

　　　　$1/100000 = 0,00001$ ＊

　　and so on.

1 16 3. 　＊ $1,101101 = 1 + 1/10 + 1/1000 + 1/10000 + 1/1000000$ ＊

　　and so on.

1 16 4. 　＊ $1/11 = 0,01010101$ Etc.

　　　　$1/101 = 0,0011001100110011$ Etc.

　　　　$1/110 = 0,001010101$ Etc.

　　　　$1/111 = 0,001001001001$ Etc.

　　　　$1/1001 = 0,000111000111000111$ Etc.

　　　　$10/11 = 0,1010101010$ Etc.

　　　　$1000/11 = 10,1010101010$ Etc:

　　　　$(1000/11) - (10/11) = \cdot 10,101010$ Etc. $-.0,101010$ Etc $\cdot =$

　　　　　　$10,0000$ Etc $= 10$:

　　　　$11/111 = 0,011011011011$ Etc.

　　　　$(11/111) - (1/1001) = 0,010100010100010100$ Etc.

　　　　$(11/111) + (1/1001) = 0,100010100010100010$ Etc.

　　　　$10 (11/111) = 0,110110110110$ Etc.

This is an important step. A Lincos word meaning "and so on" and written Etc is introduced. I doubt whether it could be done at an earlier stage, if we wish to avoid ambiguous interpretations. The child that computes its first periodic fraction faces *infinity*. By showing periodic fractions we get the most natural opportunity of introducing 'Etc'.

'Etc' is to correspond to the "..." of common mathematical language. Note that 'Etc' should not be confused with the meta-textual "and so on".

A few examples will illustrate the use of 'Etc'.

1 16 5. $\;\#\; 1+10=0,1\times 10\times 11.$

$1+10+11=0,1\times 11\times 100.$

$1+10+11+100=0,1\times 100\times 101.$

Etc.

$1+10+11+\text{Etc}+(a-1)+a=0,1\times a\,(a+1)\;\#\;.$

1 16 6. $\;\#\; a^0=1.$

$a^1=aa^0=a.$

$a^{10}=aa^1=aa.$

$a^{11}=aa^{10}=aaa.$

Etc.

$a^{n+1}=aa^n=aa\times\text{Etc}\times a:$

$a^{-n}=1/a^n\;\#$

1 16 7. $\;\#\; a\neq 1.\to.\,1+a+a^{10}+a^{11}+\text{Etc}+a^n=(1-a^{n+1})/(1-a)\cdot$

$-1<a<1.\to.\,1+a+a^{10}+a^{11}+\text{Etc}=1/(1-a)\;\#$

1 17 1. $\;\#\; 1\in\text{Num}.\,10\in\text{Num}.\,11\in\text{Num}.\,100\in\text{Num}.\,101\in\text{Num}.\,\text{Etc}\cdot$

$0\in\text{Int}.\,1\in\text{Int}.-1\in\text{Int}.\,10\in\text{Int}.-10\in\text{Int}.\,11\in\text{Int}.\,\text{Etc}\;\#$

The Lincos word written Num ($\int L$[1] numerus) means "natural numbers". The Lincos word written Int ($\int L$ integer) means "integers". Words designating sets have been introduced when at last two sets can be shown. As a rule we shall not name a particular kind of thing unless things not of this kind are known.

1 17 2. $\;\#\; a\in\text{Num}.\leftrightarrow\cdot a=1.\vee.a=10.\vee.a=11.\vee.a=100.$

$\vee.a=101.\vee\text{Etc}:$

$a\in\text{Int}.\leftrightarrow\cdot a=0.\vee.a=1.\vee.a=-1.\vee.a=10.$

$\vee.a=-10.\vee.a=11.\vee.\text{Etc}\;\#$

1 17 3. $\;\#\; a\in\text{Num}.\leftrightarrow\cdot a\in\text{Int}.\wedge.a>0\cdot$

$a\in\text{Num}.\to\cdot a+1.\in\text{Num}\cdot$

$a\in\text{Int}.\leftrightarrow:a+1.\in\text{Int}\cdot\wedge\cdot a-1.\in\text{Int}\cdot$

$a\in\text{Int}.\leftrightarrow\cdot-a.\in\text{Int}\cdot$

$a\in\text{Num}.\wedge.b\in\text{Num}\cdot\to\cdot a+b.\in\text{Num}\cdot$

$a\in\text{Num}.\wedge.b\in\text{Num}:\to:a-b.\in\text{Num}\cdot\leftrightarrow.a>b\;\#$

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[1] "$\int L$" means "from Latin". Most written Lincos words are abbreviations of Latin words or word-groups.

1 18 1. $*\, x/1 . \in \text{Int} \cdot \leftrightarrow \cdot x = 0 . \vee . x = 1 . \vee . x = -1 . \vee . \text{Etc} :$
$x/10 . \in \text{Int} \cdot \leftrightarrow \cdot x = 0 . \vee . x = 10 . \vee x = -10 . \vee . \text{Etc} :$
$x/11 . \in \text{Int} \cdot \leftrightarrow \cdot x = 0 . \vee . x = 11 . \vee . x = -11 . \vee . \text{Etc} :$
$\text{Etc} \cdot$
$x/1 . \in \text{Int} \cdot \wedge \cdot x/10 . \in \text{Int} \cdot \wedge \cdot x/11 . \in \text{Int} :$
$\quad \leftrightarrow \cdot x = 0 . \vee . x = 110 . \vee . x = -110 . \vee . \text{Etc} \cdot$
$x/1 . \in \text{Int} \cdot \wedge \cdot x/10 . \in \text{Int} \cdot \wedge \cdot x/11 . \in \text{Int} \cdot \wedge \cdot x/100 . \in \text{Int} :$
$\quad \leftrightarrow \cdot x = 0 . \vee . x = 1100 . \vee . x = -1100 . \vee . \text{Etc} \cdot$
$\text{Etc} \cdot$
$x/1 . \in \text{Int} \cdot \wedge \cdot x/10 . \in \text{Int} \cdot \wedge \cdot x/11 . \in \text{Int} \wedge \cdot x/100 . \in \text{Int} \cdot \wedge \cdot$
$\quad x/101 . \in \text{Int} \cdot \wedge \cdot \text{Etc} : \leftrightarrow . x = 0 \vdots$
$a = 1 . \rightarrow \cdot x/a . \in \text{Int} : \wedge : a = 10 . \rightarrow \cdot x/a . \in \text{Int} :$
$\quad \wedge : a = 11 . \rightarrow \cdot x/a . \in \text{Int} : \wedge : \text{Etc} \cdot$
$\quad \leftrightarrow : a \in \text{Num} . \rightarrow \cdot x/a . \in \text{Int} \vdots$
$x/1 . \in \text{Int} \cdot \wedge \cdot x/10 . \in \text{Int} \cdot \wedge \cdot x/11 . \in \text{Int} \cdot \wedge \cdot \text{Etc} :$
$\quad \leftrightarrow \cdot \wedge a : a \in \text{Num} . \rightarrow \cdot x/a . \in \text{Int} \vdots$
$\wedge a : a \in \text{Num} . \rightarrow \cdot x/a . \in \text{Int} \cdot \leftrightarrow . x = 0 \,*$

A connective written $\wedge$ and meaning "for every" has arisen from an infinite conjunction. This seems to be the most natural way of introduction. Some phonetic resemblance between '$\wedge$' and '$\wedge$' would be desirable.

Texts such as

$*\, a = 1 . \rightarrow . a^{10} > 0 \cdot \wedge \cdot a = 10 . \rightarrow . a^{10} > 10 \cdot \wedge \cdot a = 11 .$
$\quad \rightarrow . a^{10} > 0 \cdot \wedge \cdot \text{Etc} :$
$\wedge a \cdot a \in \text{Num} . \rightarrow . a^{10} > 0 \,*$

could hardly serve to introduce '$\wedge$'. Remember we have always been considering

$$a \in \text{Num} . \rightarrow . a^{10} > 0$$

as a (true) proposition. This means that we have tacitly agreed to *generalize over free variables*. (It would be difficult to avoid this convention.) If we should start with the last program text, the receiver would not understand why we at once add the word written '$\wedge$', and he would be unable to guess its meaning. We are obliged to introduce the word written $\wedge$ by means of a proposition, in which it is not syntactically superordinated to the rest of the proposition. Upon this point of view we choose our example. After this we are allowed to superordinate the connective written $\wedge$ to a whole sentence too.

It is advisable to give more examples:

1 18 2. $* \ 0 \notin \mathrm{Num} : - 1 . \notin \mathrm{Num} \cdot - 10 . \notin \mathrm{Num} \cdot - 11 . \notin \mathrm{Num} \cdot$
$- 100 . \notin \mathrm{Num} \cdot \mathrm{Etc} \cdot 11/111 . \notin \mathrm{Num} \cdot - 111/10 . \notin \mathrm{Num} \cdot \mathrm{Etc} \,\dot{}$
$a \notin \mathrm{Num} . \leftrightarrow \cdot a \neq 1 . \wedge . a \neq 10 . \wedge . a \neq 11 . \wedge . a \neq 100 . \wedge . \mathrm{Etc} \,\dot{}$
$a \notin \mathrm{Num} . \to : x = 1 . \to . a \neq x \cdot \wedge \cdot x = 10 . \to . a \neq x \cdot$
$\wedge \cdot x = 11 . \to . a \neq x \cdot \wedge \cdot x = 100 . \to . a \neq x \cdot \wedge \cdot \mathrm{Etc} \,\dot{}$
$a \notin \mathrm{Num} . \leftrightarrow : \wedge x \cdot x \in \mathrm{Num} . \to . a \neq x \ *$

1 18 3. $* \ \wedge x \cdot x \neq a . \vee . x \neq b : \to . a \neq b \ *$

1 18 4. $* \ \wedge x \cdot \mathrm{Etc} . x . \mathrm{Etc} :$
$\to : \mathrm{Etc} . 0 . \mathrm{Etc} \cdot \wedge \cdot \mathrm{Etc} . - 1 . \mathrm{Etc} \cdot \wedge \cdot \mathrm{Etc} . 100 . \mathrm{Etc} \cdot$
$\wedge \cdot \mathrm{Etc} . 1001/100 . \mathrm{Etc} \cdot \wedge \cdot \mathrm{Etc} \,\dot{}$
$\wedge x \cdot x \in \mathrm{Num} . \wedge . \mathrm{Etc} \, x \, \mathrm{Etc} :$
$\leftrightarrow \cdot \mathrm{Etc} \, 1 \, \mathrm{Etc} . \wedge . \mathrm{Etc} \, 10 \, \mathrm{Etc} . \wedge . \mathrm{Etc} \, 11 \, \mathrm{Etc} . \wedge . \mathrm{Etc} \,\dot{}$
$\wedge x \cdot x \in \mathrm{Int} . \wedge . \mathrm{Etc} \, x \, \mathrm{Etc} :$
$\leftrightarrow \cdot \mathrm{Etc} \, 0 \, \mathrm{Etc} . \wedge . \mathrm{Etc} \, 1 \, \mathrm{Etc} .$
$\wedge . \mathrm{Etc} - 1 \, \mathrm{Etc} . \wedge . \mathrm{Etc} \, 10 \, \mathrm{Etc} . \wedge . \mathrm{Etc} \,\dot{}$
$\wedge x \cdot \mathrm{Etc} . x . \mathrm{Etc} : \leftrightarrow : \wedge y \cdot \mathrm{Etc} . y . \mathrm{Etc} \ *$

1 18 5. $* \ a = 1 . \wedge . x = 10a \cdot \vee \cdot a = 10 . \wedge . x = 10a \cdot$
$\vee \cdot a = 11 . \wedge . x = 10a \cdot \vee \cdot a = 100 . \wedge . x = 10a \cdot \vee \cdot \mathrm{Etc} :$
$\leftrightarrow \cdot a \in \mathrm{Num} . \wedge . x = 10a \,\dot{}$
$x = 10 \times 1 . \vee . x = 10 \times 10 . \vee . x = 10 \times 11 . \vee . x = 10 \times 100 . \vee . \mathrm{Etc} \cdot$
$\leftrightarrow : \vee a \cdot a \in \mathrm{Num} . \wedge . x = 10a \ *$

A connective written $\vee$ and meaning "there is a" has arisen from an infinite disjunction. The case is entirely analogous to that of **1 18 1**.

1 18 6. $* \ y = 1 . \wedge . x = y^{10} \cdot \vee \cdot y = 10 . \wedge . x = y^{10} \cdot \vee \cdot y = 11 . \wedge . x = y^{10}$
$\cdot \vee \cdot y = 100 . \wedge . x = y^{10} \cdot \vee \cdot \mathrm{Etc} : \leftrightarrow \cdot y \in \mathrm{Num} . \wedge . x = y^{10} \,\dot{}$
$x = 1 . \vee . x = 100 . \vee . x = 1001 . \vee . x = 10000 . \vee . \mathrm{Etc} \cdot \leftrightarrow :$
$\vee y \cdot y \in \mathrm{Num} . \wedge . x = y^{10} \overset{.}{:}$
$y = 0 . \wedge . x = y^{10} \cdot \vee \cdot y = 1 . \wedge . x = y^{10} \cdot \vee \cdot y = - 1 . \wedge . x = y^{10} \cdot$
$\vee \cdot y = 10 . \wedge . x = y^{10} \cdot \vee \cdot y = - 10 . \wedge . x = y^{10} \cdot \vee \cdot \mathrm{Etc} :$
$\leftrightarrow \cdot y \in \mathrm{Int} . \wedge . x = y^{10} \,\dot{}$
$x = 0 . \vee . x = 100 . \vee . x = 1001 . \vee . x = 10000 . \vee . \mathrm{Etc} \cdot$
$\leftrightarrow : \vee y \cdot y \in \mathrm{Int} . \wedge . x = y^{10} \ *$

The last example is to prevent the misinterpretation that the '$\vee$' would meand "there is one".

1 18 7. $* \ \vee x \cdot x = a . \wedge . x = b : \to . a = b \ *$

1 18 8. $*\ \forall x\cdot\text{Etc}.x.\text{Etc}:\rightarrow:\text{Etc}.0.\text{Etc}\cdot\vee\cdot\text{Etc}.1.\text{Etc}\cdot\vee\cdot$
$\text{Etc}.-1.\text{Etc}\cdot\vee\cdot\text{Etc}.101.\text{Etc}\cdot$
$\vee\cdot\text{Etc}.10/111.\text{Etc}\cdot\vee\cdot\text{Etc}.-101/11.\text{Etc}\cdot\vee\cdot\text{Etc}^{\cdot}$
$\forall x\cdot x\in\text{Num}.\wedge.\text{Etc}\,x\,\text{Etc}:$
$\leftrightarrow\cdot\text{Etc}\,1\,\text{Etc}.\vee.\text{Etc}\,10\,\text{Etc}.\vee.\text{Etc}\,11\,\text{Etc}.\vee.\text{Etc}^{\cdot}$
$\forall x\cdot x\in\text{Int}.\wedge.\text{Etc}\,x\,\text{Etc}:$
$\leftrightarrow\cdot\text{Etc}\,0\,\text{Etc}.\vee.\text{Etc}\,1\,\text{Etc}.\vee.\text{Etc}-1\,\text{Etc}.\vee.\text{Etc}\,10\,\text{Etc}.\vee.\text{Etc}^{\cdot}$
$\forall x\cdot\text{Etc}.x.\text{Etc}:\leftrightarrow:\forall y\cdot\text{Etc}.y.\text{Etc}\ *$

1 19 1. $a\in\text{Int}.\wedge.b\in\text{Int}.\wedge:\wedge x\cdot x\in\text{Int}.\rightarrow.x^{10}+10\,a\,x+b>0^{\cdot}$
$\rightarrow:z=1.\rightarrow.z^{10}+10\,a\,z+b>0^{\cdot}$
$\wedge\cdot z=10.\rightarrow.z^{10}+10\,a\,z+b>0^{\cdot}$
$\wedge\cdot\text{Etc}\cdot\wedge\cdot z=-a.\rightarrow.z^{10}+10\,a\,z+b>0\colon$
$a\in\text{Int}.\wedge.b\in\text{Int}.\wedge:\wedge x\cdot x\in\text{Int}.\rightarrow.x^{10}+10\,a\,x+b>0^{\cdot}$
$\rightarrow.b>a^{10}\colon$
$a\in\text{Int}.\wedge.b\in\text{Int}^{\cdot}$
$\rightarrow\vdots\wedge x\cdot x\in\text{Int}.\rightarrow.x^{10}+10\,a\,x+b>0^{\cdot}\rightarrow.b>a^{10}\colon$
$a\in\text{Int}.\wedge.b\in\text{Int}.\wedge.b>a^{10}\cdot\rightarrow^{\cdot}\wedge x\colon x\in\text{Int}.$
$\rightarrow\cdot x^{10}+10\,a\,x+b>x^{10}+10\,a\,x+a^{10}=(x+a)^{10}\geqq a^{10}\geqq 0\colon$
$a\in\text{Int}.\wedge.b\in\text{Int}^{\cdot}$
$\rightarrow:b>a^{10}.\rightarrow^{\cdot}\wedge x\colon x\in\text{Int}.\rightarrow\cdot x^{10}+10\,a\,x+b>0\colon$
$a\in\text{Int}.\wedge.b\in\text{Int}^{\cdot}$
$\rightarrow\vdots b>a^{10}.\leftrightarrow^{\cdot}\wedge x\colon x\in\text{Int}.\rightarrow^{\cdot}x^{10}+10\,a\,x+b>0\ *$

1 19 2. $*\ a\in\text{Num}.\rightarrow:\forall x\cdot x\in\text{Num}.\wedge.a\leqq x^{10}\leqq 10\,a\ *$

1 20 0.   A few new words are introduced, in order to provide for a larger
stock of sets.

1 20 1. $*\ 10\,\text{Div}\,10.10\,\text{Div}\,100.10\,\text{Div}\,110.\text{Etc}^{\cdot}$
$11\,\text{Div}\,11.11\,\text{Div}\,110.11\,\text{Div}\,1001.\text{Etc}^{\cdot}$
$100\,\text{Div}\,100.100\,\text{Div}\,1000.\text{Etc}^{\cdot}$
$1011\,\text{Div}\,1001101.\text{Etc}^{\cdot}$
$10\,\text{Div}\,a.\leftrightarrow\cdot a/10.\in\text{Num}:$
$11\,\text{Div}\,a.\leftrightarrow\cdot a/11.\in\text{Num}:$
$\text{Etc}\colon$
$b\,\text{Div}\,a.\leftrightarrow:b\in\text{Num}.\wedge\cdot a/b.\in\text{Num}\colon$
$a\in\text{Num}.\rightarrow\cdot 1\,\text{Div}\,a.\wedge.a\,\text{Div}\,a\ *$

The word written Div (*ʃL* dividit) means "divides".

1 20 2.   ⚹ $10 \in \text{Pri} . 11 \in \text{Pri} . 101 \in \text{Pri} . 111 \in \text{Pri} . 1011 \in \text{Pri} . \text{Etc} \cdot$

$1 \notin \text{Pri} . 100 \notin \text{Pri} . 110 \notin \text{Pri} . 1000 \notin \text{Pri} . 1001 \notin \text{Pri} . \text{Etc} \cdot$

$a \in \text{Pri} . \leftrightarrow \cdot a = 10 . \lor . a = 11 . \lor . 101 . \lor . a = 111 . \lor . \text{Etc} \vcentcolon$

$10 \neq a . \land . 10 \operatorname{Div} a \cdot \rightarrow . a \notin \text{Pri} \vcentcolon$

$11 \neq a . \land . 11 \operatorname{Div} a \cdot \rightarrow . a \notin \text{Pri} \vcentcolon$

$100 \neq a . \land . 100 \operatorname{Div} a \cdot \rightarrow . a \notin \text{Pri} \vcentcolon$

Etc$\vcentcolon$

$b \neq a . \land . b \neq 1 . \land . b \operatorname{Div} a \cdot \rightarrow . a \notin \text{Pri} \vcentcolon$

$a \in \text{Pri} . \rightarrow \vcentcolon b \operatorname{Div} a . \rightarrow \cdot b = a . \lor . b = 1 \vcentcolon$

$a \in \text{Pri} . \rightarrow \cdot \land b \vcentcolon b \operatorname{Div} a . \rightarrow \cdot b = a . \lor . b = 1 \vcentcolon$

$a \in \text{Pri} . \leftrightarrow \vcentcolon a - 1 . \in \text{Num} \cdot \land \cdot \land b \vcentcolon b \operatorname{Div} a . \rightarrow \cdot b = a . \lor . b = 1$ ⚹

The word written Pri means "prime numbers".

1 21 1.   ⚹ $a \in \text{Num} . \rightarrow . a \in \text{Int} \cdot$

$\text{Num} \subset \text{Int} \vcentcolon$

$a \in \text{Pri} . \rightarrow . a \in \text{Num} \cdot$

$\text{Pri} \subset \text{Num} \vcentcolon$

$a \in \text{Pri} . \rightarrow . a \in \text{Int} \cdot$

$\text{Pri} \subset \text{Int} \vcentcolon$

$a \in \text{Num} . \rightarrow . a \in \text{Num} \cdot$

$\text{Num} \subset \text{Num} \vcentcolon$

$a \in \text{Pri} . \rightarrow . a \in \text{Pri} \cdot$

$\text{Pri} \subset \text{Pri} \vcentcolon$

$\text{Int} \subset \text{Int} \vcentcolon$

$\text{Num} \supset \text{Pri} .$

$\text{Int} \supset \text{Num} .$

$\text{Int} \supset \text{Pri} \cdot$

$\text{Pri} \subset \text{Num} \subset \text{Int} .$

$\text{Int} \supset \text{Num} \supset \text{Pri} \cdot$

$\text{Pri} = \text{Pri} . \text{Num} = \text{Num} . \text{Int} = \text{Int}$ ⚹

1 21 2.   ⚹ $\land a \vcentcolon a \in A . \leftrightarrow \cdot a = 11 . \lor . a = 111 . \lor . a = 10011 \cdot \rightarrow . A \subset \text{Pri} \vcentcolon$

$\land a \vcentcolon a \in B . \leftrightarrow \cdot a = 11 . \lor . a = 111 . \lor . a = 10011 \cdot \rightarrow . B \subset \text{Pri} \vcentcolon$

$\land a \vcentcolon a \in A . \leftrightarrow \cdot a = 11 . \lor . a = 111 . \lor . a = 10011 \cdot \rightarrow . A \subset \text{Num} \vcentcolon$

$\land a \vcentcolon a \in A . \leftrightarrow \cdot a = 11 . \lor . a = 111 . \lor . a = 10011 \cdot \rightarrow . A \subset \text{Int} \vcentcolon$

$\land a \vcentcolon a \in A . \leftrightarrow \cdot a = 100 . \lor . a = 101 . \lor . a = 10000 \cdot \rightarrow . A \subset \text{Num} \vcentcolon$

$\land a \vcentcolon a \in A . \leftrightarrow \cdot a = 100 . \lor . a = 101 . \lor . a = 10000 \cdot \rightarrow . A \subset \text{Int} \vcentcolon$

$\land a \vcentcolon a \in A . \leftrightarrow \cdot a = -11 . \lor . a = 101 . \lor . a = 0 \cdot \rightarrow . A \subset \text{Int}$ ⚹

and so on.

1 21 3.   ⚹ $\land a \cdot a \in A . \rightarrow . a \in \text{Pri} \vcentcolon \leftrightarrow . A \subset \text{Pri} \cdot$

$\land a \cdot a \in \text{Pri} . \rightarrow . a \in A \vcentcolon \leftrightarrow . \text{Pri} \subset A \cdot$

$\land a \cdot a \in \text{Pri} . \leftrightarrow . a \in A \vcentcolon \leftrightarrow . A = \text{Pri}$ ⚹

and so on.

1 21 4. $*\,b\in\mathrm{Num}.\to\dot{}\,\wedge a\cdot a\in A.\leftrightarrow.a\,\mathrm{Div}\,b\colon\to.A\subset\mathrm{Int}\colon$
$b\in\mathrm{Num}.\to\dot{}\,\wedge a\cdot a\in A.\leftrightarrow.b\,\mathrm{Div}\,a\colon\to.A\subset\mathrm{Int}\colon$
$\wedge a\colon a\in A.\leftrightarrow\dot{}\,a\in\mathrm{Int}.\wedge.a>111\dot{}\to.A\subset\mathrm{Num}\;*$
and so on.

1 21 5. $*\,\wedge a\colon a\in A.\leftrightarrow\dot{}\,a=-1.\vee.a=111.\vee.a=1000\dot{}\wedge\dot{}$
$\wedge a\colon a\in B.\leftrightarrow\dot{}\,a=-1.\vee.a=0.\vee.a=1.\vee.a=111.\vee.a=1000\colon$
$\to.A\subset B\colon$
$\wedge a\cdot a\in A.\leftrightarrow.a\,\mathrm{Div}\,c\colon\wedge\colon\wedge a\cdot a\in B.\leftrightarrow.a\,\mathrm{Div}\,d\colon\wedge.c\in\mathrm{Num}\dot{}$
$\to\dot{}\,A\subset B.\leftrightarrow.c\,\mathrm{Div}\,d\;*$
and so on.

1 21 6. $*\,A\subset\mathrm{Pri}.\to A\subset\mathrm{Num}\cdot\mathrm{Etc}\;*$

1 22 1. $*\,?\,x\cdot x^{10}=10.\wedge.x>0\colon$
$x^{10}=10.\wedge.x>0\cdot\to.x=\sqrt{10}\colon$
$\sqrt{10}=1,0110101000001\,\mathrm{Etc}\colon$
$?\,x.x^{10}=-1\colon$
$x^{10}=-1.\leftrightarrow\dot{}\,x=i.\vee.x=-i\;*$

1 22 2. $*\,\pi=100\times(1-1/11+1/101-1/111+1/1001-\mathrm{Etc}).$
$e=1+1/1+1/10+1/(10\times11)+1/(10\times11\times100)+\mathrm{Etc}.$
$e^{10}=1+10/1+100/10+1000/(10\times11)+10000/(10\times11\times100)+$
$\mathrm{Etc}\;*$

Of course i, e, π are constants with the usual meaning. In written Lincos we shall not try to distinguish such constants from variables by typographical characteristics. In Lincos speech they will be carefully distinguished by phonetic means.

1 22 3. $*\,0\in\mathrm{Rat}.1\in\mathrm{Rat}.-1\in\mathrm{Rat}.10\in\mathrm{Rat}.\mathrm{Etc}\cdot$
$1/10\in\mathrm{Rat}.-1/10\in\mathrm{Rat}.101/11\in\mathrm{Rat}.-1010/1011\in\mathrm{Rat}.\mathrm{Etc}\cdot$
$\sqrt{10}\notin\mathrm{Rat}.\pi\notin\mathrm{Rat}.e\notin\mathrm{Rat}.e^{10}\notin\mathrm{Rat}\cdot111\sqrt{10}-1.\notin\mathrm{Rat}\cdot$
$\pi+e.\notin\mathrm{Rat}\cdot111\,\pi-0,1\,e.\notin\mathrm{Rat}\cdot10\sqrt{10}-0,01\,\pi+e^{10}.\notin\mathrm{Rat}\cdot$
$i\notin\mathrm{Rat}.\pi\,i\notin\mathrm{Rat}\cdot11+10i.\notin\mathrm{Rat}\cdot\mathrm{Etc}\;*$

1 22 4. $*\,a\in\mathrm{Rat}.\leftrightarrow\dot{}\,\vee b\colon\vee c\cdot b\in\mathrm{Int}.\wedge.c\in\mathrm{Num}.\wedge.a=b/c\;*$

The word written Rat ($\int L$ numerus rationalis) means "rational numbers".

1 22 5. $*\,a\in\mathrm{Rat}.\wedge.b\in\mathrm{Rat}\cdot\to\colon a+b.\in\mathrm{Rat}\cdot\wedge\cdot a-b.\in\mathrm{Rat}\cdot\wedge\cdot a\,b.\in\mathrm{Rat}\dot{}$
$a\in\mathrm{Rat}.\wedge.b\in\mathrm{Rat}.\wedge.b\neq0\cdot\to\cdot a/b.\in\mathrm{Rat}\;*$

1 22 6. $*\,x\in\mathrm{Rat}.\wedge.x^{10}\in\mathrm{Int}\cdot\to.x\in\mathrm{Int}\colon$
$x\in\mathrm{Rat}.\wedge.x^{11}\in\mathrm{Int}\cdot\to.x\in\mathrm{Int}\;*$

1 22 7. $a \in \mathrm{Int} . \rightarrow . a \in \mathrm{Rat} \cdot$

$\mathrm{Int} \subset \mathrm{Rat} :$

$a \in \mathrm{Num} . \rightarrow a \in \mathrm{Rat} \cdot$

$\mathrm{Num} \subset \mathrm{Rat} :$

$\mathrm{Pri} \subset \mathrm{Rat} :$

$\mathrm{Rat} \supset \mathrm{Int} \supset \mathrm{Num} \supset \mathrm{Pri} :$

$A \subset \mathrm{Int} . \rightarrow . A \subset \mathrm{Rat} \colon$

$\wedge a \, ^\bullet a \in A . \leftrightarrow : \vee x \, ^\bullet x \in \mathrm{Num} . \wedge . a = x/11 \colon \rightarrow . A \subset \mathrm{Rat} \colon$

$\wedge a \, ^\bullet a \in A . \leftrightarrow : \vee x \, ^\bullet x \in \mathrm{Int} . \wedge . a = 1/x \colon \rightarrow . A \subset \mathrm{Rat} \ |$

$A \subset \mathrm{Rat} . \wedge \colon \wedge a \, ^\bullet a \in B . \leftrightarrow : \vee x \, ^\bullet x \in A . \wedge . x^{10} + x = a \colon \rightarrow . B \subset \mathrm{Rat} \ \#$

1 23 1. $\# \ 0 \in \mathrm{Rea} : 1 \in \mathrm{Rea} : -10 . \in \mathrm{Rea} : 111/11 . \in \mathrm{Rea} : \sqrt{10} \in \mathrm{Rea} :$

      $\pi \in \mathrm{Rea} : e \in \mathrm{Rea} : e^{10} \in \mathrm{Rea} : 11 \sqrt{10} - 101 + 0,1 \pi e . \in \mathrm{Rea} : \mathrm{Etc} :$

      $110,101001110110001 \ \mathrm{Etc} . \in \mathrm{Rea} : \mathrm{Etc} :$

      $i \notin \mathrm{Rea} : 111 - 11 \, i . \notin \mathrm{Rea} : \mathrm{Etc} :$

      $a \in \mathrm{Rea} . \wedge . b \in \mathrm{Rea} . \wedge . b \neq 0 \cdot \rightarrow . a + b \, i . \notin \mathrm{Rea} :$

      $a \in \mathrm{Rea} . \wedge . b \in \mathrm{Rea} \cdot \rightarrow \cdot a + b \, i . \in \mathrm{Com} \ \#$

The word written 'Rea' (*fL* numerus realis) means "real numbers". The word written 'Com' means "complex numbers". Though arithmetics of real and complex numbers have not yet been dealt with, we may venture to give a list of examples in which sums and products of real numbers occur. In similar light-hearted fashion we write down infinite sums, though limits have not yet been defined. Every one of us knew real numbers and limits long before he learned an exact definition of these notions.

1 23 2. $\# \ a \in \mathrm{Rat} . \rightarrow . a \in \mathrm{Rea} \cdot$

      $\mathrm{Rat} \subset \mathrm{Rea} :$

      $a \in \mathrm{Rea} . \rightarrow . a \in \mathrm{Com} \cdot$

      $\mathrm{Rea} \subset \mathrm{Com} :$

      $a \in \mathrm{Pos} . \leftrightarrow \cdot a \in \mathrm{Rea} . \wedge . a > 0 :$

      $\mathrm{Pos} \subset \mathrm{Rea} :$

      $\mathrm{Com} \supset \mathrm{Rea} \supset \mathrm{Rat} \supset \mathrm{Int} \supset \mathrm{Num} \supset \mathrm{Pri} :$

      $A \subset \mathrm{Rat} . \rightarrow . A \subset \mathrm{Rea} \colon$

      $\wedge a : a \in A . \leftrightarrow \cdot a \in \mathrm{Rea} . \wedge . a \notin \mathrm{Rat} \cdot \rightarrow . A \subset \mathrm{Rea} \ \#$

The word written Pos means "positive real numbers".

1 23 3. $\#$ $0,1101010001 \ \mathrm{Etc}$

      $+ \, 0,1011010010 \ \mathrm{Etc}$

      $= 1,1000100 \ \mathrm{Etc} :$

        $1,00100111101 \ \mathrm{Etc}$

      $+ \, 0,10011110010 \ \mathrm{Etc}$

      $= 1,110001 \ \mathrm{Etc} :$

$a = 0,101 \,.\, \wedge \,.\, b = 0,00010101 \text{ Etc} \,.\, \to \,.\, (a-b)+b =$
  $0,1000101 \text{ Etc}$
$+ 0,00010101 \text{ Etc}$
$= 0,10011111 \text{ Etc}$
$= 0,10100000 \text{ Etc}$
$= 0,101 \,\#$

and so on.

A typical difficulty met with in computations with infinite fractions has been shown. This was quite unavoidable. In intuitionistic mathematics it is not true that every real number can be developed in an infinite fraction.

1 23 4. $11,01010010 \leq 11,01010010 \text{ Etc} \leq 11,01010011 \,\vdots$
   $-11,01010011 \leq -11,01010010 \text{ Etc} \leq -11,01010010 \,\vdots$
   $0,100101111 \leq 0,100101111 \text{ Etc} \leq 0,10011 \,\vdots$
   $a \in \text{Rea} \,.\, \wedge \,.\, n \in \text{Num} \cdot \to \,\cdot\, \vee b : \vee c \cdot b \in \text{Rat} \,.\, \wedge \,.\, c \in \text{Rat} \,.$
     $\wedge \,.\, c - b < 10^{-n} \,.\, \wedge \,.\, b \leq a \leq c \,\vdots$
   $x \in \text{Rea} \,.\, \wedge \,.\, y \in \text{Rea} \,.\, \wedge \,.\, a \in \text{Rat} \,.\, \wedge \,.\, b \in \text{Rat} \,.\, \wedge \,.\, c \in \text{Rat} \,.\, \wedge \,.\, d \in \text{Rat} \,.$
     $\wedge \,.\, a \leq x \leq b \,.\, \wedge \,.\, c \leq y \leq d \cdot \to \,.\, a + c \leq x + y \leq b + d \,\vdots$
   $1,0110101000001 \leq \sqrt{10} \leq 1,011010100001 \,\vdots$
   $10,1101010000010 \leq \sqrt{10} + \sqrt{10} \leq 10,110101000010 \,\#$

1 23 5. $\# \; x \in \text{Rea} \,.\, \wedge \,.\, y \in \text{Rea} \,.\, \wedge \,.\, a \in \text{Rat} \,.\, \wedge \,.\, b \in \text{Rat} \,.\, \wedge \,.\, c \in \text{Rat} \,.\, \wedge \,.\, d \in \text{Rat} \,.$
     $\wedge \,.\, 0 \leq a \leq x \leq b \,.\, \wedge \,.\, 0 \leq c \leq y \leq d \cdot \to \,.\, a c \leq x y \leq b d \,:$
   $1,1111111111110101010000001 \leq \sqrt{10} \times \sqrt{10} \leq$
     $10,0000000000001000101000001 \,\#$

1 23 6. $\# \; x \in \text{Rea} \,.\, \wedge \,.\, y \in \text{Rea} \cdot \to \,\vdots\, x \leq y \,.$
     $\leftrightarrow \cdot \wedge a : a \in \text{Rat} \,.\, \wedge \,.\, a \leq x \cdot \to \,.\, a \leq y \,\vdots$
   $x \in \text{Rea} \,.\, \wedge \,.\, y \in \text{Rea} \cdot \to \,\cdot\, x = y \,.\, \leftrightarrow \,.\, x \leq y \leq x \,\#$

By 1 23 4–1 23 6 real numbers as well as their sums and products (for non-negative factors) are formally defined. The definition, which is essentially Dedekind's, fits into intuitionistic mathematics also. It does not rely on infinite fractions.

The remaining portion of real arithmetics should be dealt with in an analogous way.

1 23 7.   We do not dwell on complex numbers and their arithmetics as they do not cause any difficulty.

1 23 8. $* \; a \in \text{Rea}. \wedge . b \in \text{Rea} \cdot \to \dot{} \, a+b. \in \text{Rea} \cdot \wedge \cdot a-b. \in \text{Rea} \cdot \wedge \mathbf{\iota} \, ab. \in \text{Rea} \cdot$

   $\wedge : b \neq 0. \to \cdot a/b. \in \text{Rea} \ddot{.}$

   $a \in \text{Com}. \wedge . b \in \text{Com} \cdot$

   $\to \dot{} \, a+b. \in \text{Com} \cdot \wedge \cdot a-b. \in \text{Com} \cdot \wedge \cdot ab. \in \text{Com} \cdot$

   $\wedge : b \neq 0. \to \cdot a/b. \in \text{Com} \ddot{.}$

   $a \in \text{Com}. \wedge . b \in \text{Com}. \wedge . c \in \text{Com}. \wedge . a \neq 0 \cdot$

   $\to : \vee x \cdot x \in \text{Com}. \wedge . ax^{10}+bx+c = 0 \ddot{.}$

   $a \in \text{Rea}. \wedge . b \in \text{Rea}. \wedge . c \in \text{Rea}. \wedge . a > 0 \cdot$

   $\to \dot{} \, \vee x \cdot x \in \text{Rea}. \wedge . ax^{10}+bx+c = 0 : \leftrightarrow . b^{10}-100\,ac \geqq 0 \; *$

1 23 9. $* \; \wedge x \cdot x \in A . \leftrightarrow . a \leqq x \leqq b : \wedge : \wedge x \cdot x \in B . \leftrightarrow . c \leqq x \leqq d :$

   $\wedge : a \in \text{Rea}. \wedge . b \in \text{Rea}. \wedge . c \in \text{Rea}. \wedge . d \in \text{Rea}. \wedge . a \leqq b. \wedge . c \leqq d \dot{}$

   $\to \dot{} \, A \subset B . \leftrightarrow . c \leqq a. \wedge . b \leqq d \ddot{.}$

   $\wedge x : x \in A . \leftrightarrow . x \in \text{Com}. \wedge . x > 0 \dot{} \to . A \subset \text{Rea} \; *$

1 24 0.   In accordance with our general policy sets and relations between
sets (or classes) have preceded the appearance of the *word* "set".
Of course we cannot formally tell what sets are. Even in axiomatics
of set-theory we learn no more than how to expand a given stock
of sets.

   We shall teach the same thing, but we are bound to do it in a
less formal way. The working of axiomatic systems has always
been explained within highly developed meta-languages, whereas
for the present we do not know any Lincos word that we could use
if we were tempted to speak about the mathematics we are showing.

   Set theory as such is not very important in our program, but
as a tool it is indispensable. We have tackled this subject in
traditional fashion. One point that should be mentioned, is our
adoption of Quine's "$a \in a$" for one-element-sets a.

   In the Introduction we explained why the design of a language
like Lincos should be so liberal as not to impose opinions and
convictions by means of linguistic devices. Our attitude towards
the foundations of set theory and of mathematics has been largely
determined by this policy. No solution may seriously be excluded
by peculiarities of Lincos. As a consequence we cannot yet tackle
the set-theory paradoxes nor try to devise means of avoiding them.
At a more advanced stage of Lincos we can start dialogues between
representatives of different thoughts, in order to give an impartial
account of the various solutions. Nevertheless we cannot avoid
taking a few decisions even at this early stage. We are still free to
abrogate them as soon as it becomes desirable.

   We have adopted Quine's $a \in a$ for certain one-element-sets a,
but this is a mere convention without serious consequences. We

have adopted a name (Agg) for "sets" (classes), though only after much hesitation. A name is useless if it is not subjected to syntactical rules, which may anticipate certain solutions and exclude other possibilities. The question may be raised whether the adherence of Num to Agg is of the same kind as that of 3 to Num, or whether it is so different as to justify the use of a new copula instead of '∈'. We have decided to write Num ∈ Agg provisionally.

Introducing 'Agg' in this way commits us to indicating a few entities not belonging to Agg, if we are to stick to our general principles. We could state $A \notin$ Agg for some sets A which give rise to the well-known set-theory paradoxes (e.g. for Agg itself). It should be noted that there is not much sense in this kind of statement unless it serves to kill the paradoxes. As far as I can see adopting $A \notin$ Agg for such sets A is as dangerous as admitting $A \in$ Agg, if the paradoxes are not prevented by other means. If we wish to exclude $A \in$ Agg, we should do it by syntactical means, i.e. by asserting that it is void of sense. At present means of this kind are not yet available.

We therefore decided to indicate predicates (relations) and connectives like 'Div' and '∧' as things not belonging to Agg. I believe this is a honest solution because the average mathematician, when asked for instances of non-sets, is likely to answer in the same sense. I admit it is rather unsatisfactory because it implies that expressions like Div ∈ Agg are meaningful (though false). Of course no meaning may be attached to $x \in$ Div and so on; otherwise Div would be equal to the void set.

Some expressions void of sense will be mentioned in 1 36 3.

1 24 1. ＊ Num ∈ Agg . Int ∈ Agg . Pri ∈ Agg . Rat ∈ Agg . Com ∈ Agg :
$101 \in$ Agg ⋅ -10 . ∈ Agg ⋅ $11/111$ ⋅ ∈ Agg ⋅ $\sqrt{10} \in$ Agg ⋅ $i \in$ Agg ⋅ Etc :
Div $\notin$ Agg . $+ \notin$ Agg . $> \notin$ Agg . $\wedge \notin$ Agg . Etc $\notin$ Agg . Etc ＊

The word written Agg (∤L aggregatum) means "sets" ("classes").

1 24 2. ＊ $\wedge a \colon a \in A$. $\leftrightarrow$ ⋅ $a \in$ Com . ∧ : $\vee b$ ⋅ $b \in$ Num . ∧ . $a = 11 + bi$ ⋮
$\rightarrow$. $A \in$ Agg ❘
$\wedge x$ ⋅ $x \in B$. $\leftrightarrow :$ $x \in$ Rat . ∧ ⋅ $111 x^{10} - 10 x + 0,1$. ∈ Num ⋮
$\rightarrow$. $B \in$ Agg ❘
$\wedge a$ ⋅ $a \in C$. $\leftrightarrow :$ $a \in$ Pri . ∧ ⋅ $a + 10$. ∈ Pri ⋮ $\rightarrow$. $C \in$ Agg ❘
$\wedge n \colon n \in N$. $\leftrightarrow$ ⋮ $n \in$ Num . ∧ ⋮ $\vee x$ ⋅ $\vee y \colon \vee z$ ⋅
$x \in$ Num . ∧ . $y \in$ Num . ∧ . $z \in$ Num .
∧ . $x^{n+10} + y^{n+10} = z^{n+10}$ ❙ $\rightarrow$. $N \in$ Agg ＊

1 24 3. # $x \in 101 . \leftrightarrow . x = 101$:
$a \in \mathrm{Com} . \to \cdot x \in a . \leftrightarrow . x = a$:
$a \in A . \to . a \in \mathrm{Agg}$ #

1 24 4. # $\mathrm{Num} \notin \mathrm{Num} . \wedge . \mathrm{Num} = \mathrm{Num} \cdot \mathrm{Etc}$:
$\mathrm{Pri} \notin \mathrm{Num} . \mathrm{Com} \notin \mathrm{Rat} . \mathrm{Int} \notin \mathrm{Rea} . \mathrm{Etc}$:
$\wedge a : a \in A . \leftrightarrow \cdot a = 11 . \vee . a = 101 \cdot$
$\wedge \cdot \wedge a : a \in B . \leftrightarrow \cdot a = 11 . \vee . a = 101 . \vee . a = 0$:
$\to \cdot 11 \in B . \wedge . 101 \in B . \wedge . A \notin B . \wedge . B \notin B$ #

1 24 5. # $\wedge a : a \in A . \leftrightarrow \cdot a = 0 . \vee . a = 1 \cdot$
$\wedge \cdot \wedge a : a \in B . \leftrightarrow . a = 0 : \wedge \cdot \wedge x : x \in C . \leftrightarrow \cdot x = A . \vee . x = B$:
$\to \cdot A \neq B . \wedge . B \neq C . \wedge . A \neq C . \wedge . A \notin A .$
$\wedge . A \notin B . \wedge . A \in C . \wedge . B \in A . \wedge . B \in B . \wedge . B \in C .$
$\wedge . C \notin A . \wedge . C \notin B . \wedge . C \notin C$ #

1 24 6. # $\wedge x : x \in C . \leftrightarrow \cdot x = \mathrm{Pri} . \vee . x = \mathrm{Num} . \vee . x = \mathrm{Int} \cdot$
$\to \cdot \mathrm{Pri} \in C . \wedge . \mathrm{Num} \in C . \wedge . \mathrm{Int} \in C . \wedge . C \neq \mathrm{Pri} . \wedge . C \neq \mathrm{Num} .$
$\wedge . C \neq \mathrm{Int} . \wedge . 1010 \notin C$ #

1 24 7. # $\vee C \cdot C \in \mathrm{Agg} . \wedge . \mathrm{Pri} \in C$:
$\wedge x \cdot x \in C . \leftrightarrow . x = \mathrm{Pri} : \to . C \in \mathrm{Agg}$:
$\wedge x \cdot x \in C . \leftrightarrow . x = \mathrm{Pri} : \leftrightarrow . C = \ulcorner \mathrm{Pri} \urcorner$:
$A \in \mathrm{Agg} . \to \cdot \wedge x \cdot x \in \ulcorner A \urcorner . \leftrightarrow . x = A : \wedge \cdot \ulcorner A \urcorner \in \mathrm{Agg}$:
$x \in \ulcorner x \urcorner$:
$y \in \ulcorner x \urcorner . \leftrightarrow . x = y$ #

The connective $\ulcorner \; \urcorner$ transforms a set into a one-element-set containing just the given set as an element.

1 25 1. # $A \subset B . \leftrightarrow : \wedge x \cdot x \in A . \to . x \in B \cdot$
$A \subset B . \leftrightarrow . B \supset A \cdot$
$A \subset B . \wedge . B \subset C \cdot \to . A \subset C \cdot$
$A = B . \leftrightarrow : \wedge x \cdot x \in A . \leftrightarrow . x \in B \cdot$
$A = B . \leftrightarrow \cdot A \subset B . \wedge . B \subset A$ #

1 25 2. # $\wedge a : a \in A . \leftrightarrow \cdot a \in \mathrm{Pri} . \wedge . 10 \, \mathrm{Div} \, a \cdot \to . A = 10$:
$\wedge a : a \in A . \leftrightarrow \cdot a \in \mathrm{Rat} . \wedge . a < \sqrt{10} \cdot \wedge \cdot$
$\quad \wedge a : a \in B . \leftrightarrow \cdot a \in \mathrm{Rat} . \wedge . a \leq \sqrt{10} : \to . A = B$:
$\wedge a : a \in A . \leftrightarrow \cdot a \in \mathrm{Rea} . \wedge . -1 < a < 1 \cdot \wedge$:
$\quad \wedge a \cdot a \in B . \leftrightarrow : \vee x \cdot x \in A . \wedge . x^{11} = a : \to . A = B$ #

1 25 3. # $A \subset \mathrm{Num} . \wedge . 1 \in A \cdot \wedge \cdot \wedge x : x \in A . \to \cdot x + 1 . \in A : \to . A = \mathrm{Num}$:
$A \subset \mathrm{Int} . \wedge . 1 \in A \cdot \wedge : \wedge x \cdot x \in A . \to : x + 1 . \in A \cdot \wedge \cdot x - 1 . \in A$:
$\to . A = \mathrm{Int}$ #

1 25 4.　$\ast\ \ulcorner x\urcorner=\ulcorner y\urcorner.\leftrightarrow.x=y$:

$\ulcorner x\urcorner=x.\leftrightarrow:\wedge y\cdot y\in x.\leftrightarrow.y=x\ \ast$

1 26 1.　$\ast\ \wedge a:a\in A.\leftrightarrow\cdot a\in\mathrm{Rea}.\wedge.a\geqq 11\ \cdot$

$\wedge\cdot\wedge a:a\in B.\leftrightarrow a\in\mathrm{Rea}.\wedge.a<101$:

$\to:A\cup B.=\mathrm{Rea}\cdot\wedge\cdot\wedge a:a\in.A\cap B\cdot\leftrightarrow.11\leqq a<101$:

$\wedge\cdot\wedge a:a\in A\backslash B\cdot\leftrightarrow.a\geqq 101\ \ast$

1 26 2.　$\ast\ x\in.A\cap B\cdot\leftrightarrow\cdot x\in A.\wedge.x\in B\ \cdot$

$x\in.A\cup B\cdot\leftrightarrow\cdot x\in A.\vee.x\in B\ \cdot$

$x\in.A\backslash B\cdot\leftrightarrow\cdot x\in A.\wedge.x\notin B\ \cdot$

$A\in\mathrm{Agg}.\wedge.B\in\mathrm{Agg}\cdot\to:A\cap B.\in\mathrm{Agg}\ \cdot$

$\wedge\cdot A\cup B.\in\mathrm{Agg}\cdot\wedge\cdot A\backslash B.\in\mathrm{Agg}\ \ast$

1 26 3.　$\ast\ \wedge a:a\in A.\leftrightarrow\cdot a=10.\vee.a=101.\vee.a=-1\ \cdot$

$\leftrightarrow:A=\cdot 10\cup 101\cup.-1\ \cdot$

$A=.1\cup 10\cup 101\cdot\wedge\cdot B=.1\cup 0\cup 111$:

$\to:A\cup B.=.0\cup 1\cup 10\cup 101\cup 111\ \cdot$

$\wedge\cdot A\cap B.=1\ \cdot$

$\wedge\cdot A\backslash B.=.10\cup 101\ \ast$

1 26 4.　$\ast\ A\backslash A=\ulcorner\ \urcorner\ \ast$

The void set.

$\ast\ x\neq y.\leftrightarrow\cdot\ulcorner x\urcorner\cap\ulcorner y\urcorner.=\ulcorner\ \urcorner\ \ast$

1 27 1.　$\wedge x:x\in A.\leftrightarrow\cdot x=a.\vee.x=b.\vee.x=c\ \cdot$

$\wedge\cdot a\neq b.\wedge.a\neq c.\wedge.b\neq c$:

$\to\cdot\mathrm{Car}\,A.=11$:

$\wedge x:x\in A.\leftrightarrow.x=a\ \cdot\to\cdot\mathrm{Car}\,A.=1$:

$a\neq b.\to:\wedge x:x\in A.\leftrightarrow\cdot x=a.\vee.x=b\ \cdot\to\cdot\mathrm{Car}\,A.=10$:

$\mathrm{Car}\,\ulcorner x\urcorner.=1$:

$\mathrm{Car}\,\ulcorner\ \urcorner.=0\ \ast$

The word written Car means "cardinal number of . . .".

1 27 2.　$\ast\ \vee m\cdot m\in\mathrm{Int}.\wedge.m\geqq 0.\wedge.\mathrm{Car}\,A=m:\wedge.x\notin A\ \cdot$

$\to:\mathrm{Car}.A\cup\ulcorner x\urcorner\cdot=\cdot\mathrm{Car}\,A.+1$:

$\vee m\cdot m\in\mathrm{Int}.\wedge.m\geqq 0.\wedge.\mathrm{Car}\,A=m$:

$\wedge:\vee n\cdot n\in\mathrm{Int}.\wedge.n\geqq 0.\wedge.\mathrm{Car}\,B=n$:

$\wedge\cdot A\cap B.=\ulcorner\ \urcorner\ \cdot$

$\to:\mathrm{Car}.A\cup B\cdot=\cdot\mathrm{Car}\,A.+.\mathrm{Car}\,B$:

$\vee m\cdot m\in\mathrm{Int}.\wedge.m\geqq 0.\wedge.\mathrm{Car}\,A=m:\wedge.B\subset A\ \cdot$

$\to:\mathrm{Car}\,B\in\mathrm{Int}.\wedge\cdot 0\leqq.\mathrm{Car}\,B.\leqq.\mathrm{Car}\,A$:

$\vee m\cdot m\in\mathrm{Int}.\wedge.m\geqq 0.\wedge.\mathrm{Car}\,A=m$:

$\wedge:\vee n\cdot n\in\mathrm{Int}.\wedge.n\geqq 0.\wedge.\mathrm{Car}\,B=n\ \cdot$

$\to\cdot\mathrm{Car}.A\cup B\cdot+\cdot\mathrm{Car}.A\cap B:=\cdot\mathrm{Car}\,A.+.\mathrm{Car}\,B\ \ast$

1 27 3. $*\,n+1\,.\in\mathrm{Num}\cdot\wedge^{\cdot}\,\wedge x:x\in A\,.\leftrightarrow\cdot x\in\mathrm{Num}\,.\wedge.\,1\leqq x\leqq n\,\colon$
$\to.\,\mathrm{Car}\,A=n\,\colon$
$n+1\,.\in\mathrm{Num}\cdot\wedge^{\cdot}\,\wedge x:x\in A\,.\leftrightarrow\cdot x\in\mathrm{Num}\,.\wedge.\,111\leqq x\leqq n+110\,\colon$
$\to.\,\mathrm{Car}\,A=\boldsymbol{n}\,\colon$
$n+1\,.\in\mathrm{Num}\cdot$
$\wedge\,\colon\wedge x^{\cdot}\,x\in A\,.\leftrightarrow:\vee y\cdot y\in\mathrm{Num}\,.\wedge.\,1\leqq y\leqq n\,.\wedge.\,x=y^{10}\,\colon$
$\to.\,\mathrm{Car}\,A=n\,*$

1 27 4. $*\,\mathrm{Car}\,\mathrm{Pri}=\mathrm{Car}\,\mathrm{Num}=\mathrm{Car}\,\mathrm{Int}=\mathrm{Car}\,\mathrm{Rat}$
$=\aleph_0\neq\mathrm{Car}\,\mathrm{Rea}=\mathrm{Car}\,\mathrm{Com}\,*$

1 27 5. $*\,\mathrm{Car}\notin\mathrm{Agg}\,*$

1 28 1. $*\,\mathrm{Car}\,A=11\,.\wedge.\,\mathrm{Car}\,B=10\cdot\to.\,\mathrm{Car}\,\ulcorner A\,.\,B\urcorner=110\,*$

1 28 2. $*\,A=.\,\ulcorner a\urcorner\cup\ulcorner b\urcorner\cup\ulcorner c\urcorner\cdot\wedge.\,\mathrm{Car}\,A=11\,.$
$\wedge\cdot B=.\,\ulcorner u\urcorner\cup\ulcorner v\urcorner\cdot\wedge.\,\mathrm{Car}\,B=10\,\colon$
$\to\,\colon\ulcorner A\,.\,B\urcorner=.\,\ulcorner a\,.\,u\urcorner\cup\ulcorner a\,.\,v\urcorner\cup\ulcorner b\,.\,u\urcorner\cup\ulcorner b\,.\,v\urcorner\cup\ulcorner c\,.\,u\urcorner\cup\ulcorner c\,.\,v\urcorner\cdot$
$\wedge.\,\mathrm{Car}\,\ulcorner A\,.\,B\urcorner=11\times10=110\,*$

Pairs of sets, especially of one-element-sets.

$\ulcorner\,\urcorner$ and $\ulcorner\,\urcorner$ must not be confused.

1 28 3. $*\,\vee m:m+1\,.\in\mathrm{Num}\cdot\wedge.\,\mathrm{Car}\,A=m^{\cdot}$
$\wedge^{\cdot}\,\vee n:n+1\,.\in\mathrm{Num}\cdot\wedge.\,\mathrm{Car}\,B=n\,\colon$
$\to\cdot\mathrm{Car}\,\ulcorner A\,.\,B\urcorner=.\,\mathrm{Car}\,A\times\mathrm{Car}\,B\,*$

1 28 4. $*\,A\in\mathrm{Agg}\,.\wedge.\,B\in\mathrm{Agg}\cdot\to.\,\ulcorner A\,.\,B\urcorner\in\mathrm{Agg}\,\colon\wedge^{\cdot}\,\wedge x:x\in\ulcorner A\,.\,B\urcorner\,.$
$\leftrightarrow^{\cdot}\vee a\,\colon\vee b\cdot a\in A\,.\wedge.\,b\in B\,.\wedge.\,x=\ulcorner a\,.\,b\urcorner\,*$

1 28 5. $*\,a\in\mathrm{Agg}\,.\wedge.\,b\in\mathrm{Agg}\cdot\to.\,\ulcorner a\,.\,b\urcorner\in\mathrm{Agg}\,\colon$
$x\in\ulcorner a\,.\,b\urcorner\,.\leftrightarrow.\,x=\ulcorner a\,.\,b\urcorner\,\colon$
$\ulcorner a\,.\,b\urcorner=\ulcorner c\,.\,d\urcorner\,.\leftrightarrow\cdot a=b\,.\wedge.\,c=d\,*$

1 28 6. $*\,\ulcorner A\,.\,B\urcorner\subset\ulcorner C\,.\,D\urcorner\,.\leftrightarrow\cdot A\subset C\,.\wedge.\,B\subset D\,\colon$
$\ulcorner A\,.\,B\urcorner=\ulcorner C\,.\,D\urcorner\,.\leftrightarrow\cdot A=C\,.\wedge.\,B=D\,*$

1 28 7. $*\,a\in\mathrm{Agg}\,.\wedge.\,b\in\mathrm{Agg}\,.\wedge.\,c\in\mathrm{Agg}\cdot\to.\,\ulcorner a\,.\,b\,.\,c\urcorner\in\mathrm{Agg}\,|$
$x\in\ulcorner a\,.\,b\,.\,c\urcorner\,.\leftrightarrow.\,x=\ulcorner a\,.\,b\,.\,c\urcorner\,|$
$\ulcorner a\,.\,b\,.\,c\urcorner=\ulcorner u\,.\,v\,.\,w\urcorner\,.\leftrightarrow\cdot a=u\,.\wedge.\,b=v\,.\wedge.\,c=w\,|$
$A\in\mathrm{Agg}\,.\wedge.\,B\in\mathrm{Agg}\,.\wedge.\,C\in\mathrm{Agg}\cdot\to.\,\ulcorner A\,.\,B\,.\,C\urcorner\in\mathrm{Agg}\,\imath$
$\wedge\,\colon\wedge x\,\colon x\in\ulcorner A\,.\,B\,.\,C\urcorner\,.$
$\leftrightarrow\,\colon\vee a^{\cdot}\,\vee b\,\colon\vee c\cdot a\in A\,.\wedge.\,b\in B\,.\wedge.\,c\in C\,.\wedge.\,x=\ulcorner a\,.\,b\,.\,c\urcorner\,|$
$1+\mathrm{Car}\,A\,.\in\mathrm{Num}\cdot\wedge\cdot 1+\mathrm{Car}\,B\,.\in\mathrm{Num}\cdot\wedge\cdot 1+\mathrm{Car}\,C\,.\in\mathrm{Num}\,\colon$
$\to.\,\mathrm{Car}\,\ulcorner A\,.\,B\,.\,C\urcorner=\mathrm{Car}\,A\times\mathrm{Car}\,B\times\mathrm{Car}\,C\,*$

Triples of sets, especially of one-element-sets.

1 29 0. Notations may be simplified:

1 29 1. ✳ $x \in \text{Rat} . \leftrightarrow : \vee \ulcorner a . b \urcorner \cdot \ulcorner a . b \urcorner \in \ulcorner \text{Int} . \text{Int} \urcorner . \wedge . b \neq 0 . \wedge . x = a/b :$

$x \in \text{Rat} . \leftrightarrow : \vee y \cdot y \in \ulcorner \text{Int} . \text{Num} \urcorner . \wedge . y = \ulcorner a . b \urcorner . \wedge . x = a/b :$

$x \in \text{Com} . \leftrightarrow : \vee \ulcorner a . b \urcorner \cdot \ulcorner a . b \urcorner \in \ulcorner \text{Rea} . \text{Rea} \urcorner . \wedge . x = a + bi :$

$\text{Etc} : \vee a \cdot \vee b . \vee c \ \text{Etc}^{\cdot} \leftrightarrow . \text{Etc} . \vee \ulcorner a . b . c \urcorner \ \text{Etc} :$

$x \in \ulcorner A . B . C \urcorner . \leftrightarrow : \vee \ulcorner a . b . c \urcorner \cdot a \in A . \wedge . b \in B . \wedge . c \in C .$
$\quad \wedge . x = \ulcorner a . b . c \urcorner :$

$\ulcorner A \urcorner = A . \leftrightarrow : \text{Car} \ A = 1 . \wedge^{\cdot} \wedge \ulcorner x . y \urcorner \cdot \ulcorner x . y \urcorner \in \ulcorner A . A \urcorner . \rightarrow . x = y :$

$\text{Etc} : \wedge x \cdot \wedge y . \wedge z \ \text{Etc}^{\cdot} \leftrightarrow \cdot \text{Etc} . \wedge \ulcorner x . y . z \urcorner \ \text{Etc} \ ✳$

1 29 2. ✳ $x = a . \vee . x = b . \vee . x = c . \vee . x = d \cdot \leftrightarrow \cdot x = . a \smile b \smile c \smile d :$

$a \in X . \wedge . b \in X . \wedge . c \in X . \wedge . d \in X \cdot \leftrightarrow \cdot a \frown b \frown c \frown d . \in X :$

$a \in X . \vee . b \in X . \vee . c \in X . \vee . d \in X \cdot \leftrightarrow \cdot a \smile b \smile c \smile d . \in X \ ✳$

This is to legalize a familiar use of "and" and "or" in common mathematical and natural language.

1 29 3. ✳ $\wedge x \cdot x \in A . \leftrightarrow . a \leqq x \leqq b : \leftrightarrow : A = \cdot {}^{\uparrow}x . a \leqq x \leqq b \ ^{\vdots}$

$\wedge a \cdot a \in B . \leftrightarrow : a \in \text{Pri} . \wedge \cdot a + 10 . \in \text{Pri} \ ^{\vdots}$

$\quad \leftrightarrow : B = \cdot {}^{\uparrow}y : y \in \text{Pri} . \wedge . y + 10 . \in \text{Pri} \ ^{\vdots}$

$10 \cup 111 \cup 1000 . = \cdot {}^{\uparrow}a . a = 10 \smile 111 \smile 1000 :$

$A \cup B . = : {}^{\uparrow}x \cdot x \in A . \vee . x \in B \ ^{\cdot}$

$A \cap B . = : {}^{\uparrow}x \cdot x \in A . \wedge . x \in B \ ^{\cdot}$

$A \backslash B . = : {}^{\uparrow}x \cdot x \in A . \wedge . x \notin B \ ^{\cdot}$

$\ulcorner \ \urcorner = \cdot {}^{\uparrow}x . x \neq x \ ^{\vdots}$

$\ulcorner A . B \urcorner = \cdot {}^{\uparrow}x : \vee \ulcorner a . b \urcorner \cdot x = \ulcorner a . b \urcorner . \wedge . a \in A . \wedge . b \in B \ ^{\vdots}$

$\ulcorner A \urcorner = \cdot {}^{\uparrow}x . x = A :$

$A = \cdot {}^{\uparrow}x . x \in A \ ✳$

A set-forming connective, written $^{\uparrow}$ has been introduced. In an earlier version I put a circumflex (a small quantifier of generalization) over the letter representing the variable to be bound by set-forming. Typographical considerations led me to put the circumflex over a vertical stroke before the letter.

1 29 4. ✳ $a \in : {}^{\uparrow}x \cdot \text{Etc} . x . \text{Etc}^{\cdot} \leftrightarrow \cdot \text{Etc} . a . \text{Etc} \ ^{\vdots}$

$A \in \text{Agg} . \rightarrow : {}^{\uparrow}x : x \in A . \wedge \cdot \text{Etc} . x . \text{Etc}^{\cdot} \in \text{Agg} \ ✳$

1 29 5. ✳ $1 + n . \in \text{Num} \ ^{\vdots} \rightarrow : \text{Car} : {}^{\uparrow}x \cdot x \in \text{Num} . \wedge . 1 \leqq x \leqq n \ ^{\cdot} = n \ ^{\vdots}$

${}^{\uparrow}x \cdot x \in \text{Rat} . \wedge . x < \sqrt{10} . = {}^{\uparrow}x \cdot x \in \text{Rat} . \wedge . x \leqq \sqrt{10} \ ^{\vdots}$

${}^{\uparrow}x : x \in \text{Rea} . \wedge . x < 101 : \cap : {}^{\uparrow}x \cdot x \in \text{Rea} . \wedge . x > 11 \ ^{\cdot}$

$\quad = : {}^{\uparrow}x \cdot x \in \text{Rea} . \wedge . 11 < x < 101 \ ✳$

1 30 1. * $A =.1 \cup 10 \cup 11 . \wedge . B \subset A :\rightarrow: B = . \ulcorner \urcorner \smile 1 \smile 10 \smile 11 \smile .1 \cup 10 .$
$\smile .1 \cup 11 . \smile .10 \cup 11 . \smile A \vdots$
$\mathrm{Car}\, A = 11 . \rightarrow \cdot \mathrm{Car} \uparrow B . B \subset A := 1000 \vdots$
$A \in \mathrm{Agg} . \wedge \cdot 1 + \mathrm{Car}\, A . \in \mathrm{Num} :$
$\rightarrow \vdots \uparrow B . B \subset A . \in \mathrm{Agg} : \wedge \cdot \mathrm{Car} \cdot \uparrow B . B \subset A := . 10^{\mathrm{Car}\, A} :$
$\uparrow B . B \subset \ulcorner \urcorner \cdot \neq \ulcorner \urcorner \vdots$
$\uparrow B . B \subset \ulcorner \urcorner \cdot = \ulcorner \ulcorner \urcorner \urcorner \#$

1 30 2. * $\uparrow x . 10\, \mathrm{Div}\, x \cdot \cap \cdot \uparrow x . 11\, \mathrm{Div}\, x \cdot$
$\cap \cdot \uparrow x . 100\, \mathrm{Div}\, x \cdot \cap \cdot \uparrow x . 101\, \mathrm{Div}\, x \cdot \cap \cdot \mathrm{Etc} : = \ulcorner \urcorner \vdots$
$\uparrow x . x \in \mathrm{Rat} . \wedge . x \leqq 0,1 : \cap : \uparrow x \cdot x \in \mathrm{Rat} . \wedge . x \leqq 0,01 :$
$\cap : \uparrow x . x \in \mathrm{Rat} . \wedge . x \leqq 0,001 : \cap : \mathrm{Etc} \cdot = \cdot \uparrow x \cdot x \in \mathrm{Rat} . \wedge . x \leqq 0 :$
$\wedge X \cdot X \in M . \leftrightarrow . X = A \smile B \smile C \smile \mathrm{Etc} :$
$\rightarrow \cdot \bigcap M . = . A \cap B \cap C \cap \mathrm{Etc} \vdots$
$M \in \mathrm{Agg} . \rightarrow \cdot \bigcap M . \in \mathrm{Agg} :$
$\wedge \vdots \wedge x \cdot x \in . \bigcap M \cdot \leftrightarrow : \wedge A \cdot A \in M . \rightarrow . x \in A$ *

1 30 3. * $\wedge A \vdots A \in M . \leftrightarrow : \vee n \cdot n \in \mathrm{Num} . \wedge : A = \cdot \uparrow x . n\, \mathrm{Div}\, x \vdots$
$\rightarrow \vdots \bigcap M . = \cdot \uparrow x : \wedge n \cdot n \in \mathrm{Num} . \rightarrow . n\, \mathrm{Div}\, x \cdot = \ulcorner \urcorner \mid$
$\bigcap \vdots \uparrow y \vdots \vee n \cdot n \in \mathrm{Num} . \wedge . y = \cdot \uparrow x . n\, \mathrm{Div}\, x \cdot = \ulcorner \urcorner \mid$
$c \in \mathrm{Rea} . \rightarrow \vdots \bigcap \mathord{\mid} \uparrow y \vdots \vee a \vdots a \in \mathrm{Rea} . \wedge . a < c .$
$\wedge \vdots y \cdot = \cdot \uparrow x \cdot x \in \mathrm{Rea} . \wedge . x > a \mid . = \cdot \uparrow x \cdot x \in \mathrm{Rea} . \wedge . x \geqq c \mid$
$A \subset B . \rightarrow \cdot \bigcap \cdot \uparrow C . A \subset C \subset B := A$ *

1 30 4. $\uparrow x \cdot x \in \mathrm{Rea} . \wedge . x \leqq 1 : \cup : \uparrow x \cdot x \in \mathrm{Rea} . \wedge . x \leqq 10 :$
$\cup : \uparrow x \cdot x \in \mathrm{Rea} . \wedge . x \leqq 11 : \cup : \mathrm{Etc} \cdot = \mathrm{Rea} \vdots$
$\wedge X \cdot X \in M . \leftrightarrow . X = A \smile B \smile C \smile \mathrm{Etc} :$
$\rightarrow \cdot \bigcup M . = . A \cup B \cup C \cup \mathrm{Etc} \vdots$
$M \in \mathrm{Agg} . \rightarrow \cdot \bigcup M . \in \mathrm{Agg} :$
$\wedge \vdots \wedge x \cdot x \in . \bigcup M \cdot \leftrightarrow : \vee A \cdot A \in M . \wedge . x \in A$ *

1 30 5. * $\bigcup \mathord{\mid} . \uparrow y \mathord{\mid} \vee m \vdots m \in \mathrm{Num} .$
$\wedge \vdots y = \vdots \uparrow n \cdot \mathrm{Car} \cdot \uparrow x . x\, \mathrm{Div}\, n := m \mid \cdot = \mathrm{Num} \mid$
$\bigcup \cdot \uparrow B . B \subset A := A$ *

1 31 1. * $\curlyvee x . 11\, x = 10 \cdot = 10/11 \vdots$
$\curlyvee x \cdot x^{10} = 1001 . \wedge . x > 0 := 11 \vdots$
$\curlyvee x \cdot x \in \mathrm{Num} . \wedge . 0 \leqq \sqrt{10} - x < 1 := 1 \vdots$
$\curlyvee x \vdots x\, \mathrm{Div}\, 1100 . \wedge . x\, \mathrm{Div}\, 10010 .$
$\wedge \cdot \wedge y : y\, \mathrm{Div}\, 1100 . \wedge . y\, \mathrm{Div}\, 10010 \cdot \rightarrow . y\, \mathrm{Div}\, x \cdot = 110 :$
$\curlyvee x \cdot x^{10} = 1001 . \wedge . 0 \leqq x : \mathrm{Div}\, 1001 \vdots$
$a \in \mathrm{Num} . \wedge . a > 1 . \wedge . a \notin \mathrm{Pri} . \wedge \cdot A = \cdot \uparrow y \cdot y\, \mathrm{Div}\, a . \wedge . 1 < y \vdots$
$\rightarrow \vdots \curlyvee x \cdot x \in A . \wedge : \wedge y \cdot y \in A . \rightarrow . x \leqq y \vdots \leqq \sqrt{a} \vdots$

$a \in A . \wedge . \mathrm{Car}\, A = 1 \cdot \to : {}^{\curlyvee}x . x \in A \cdot = a \vdots$

${}^{\curlyvee}x . x \in A \cdot = a : \leftrightarrow . A = \ulcorner a \urcorner \vdots$

${}^{\curlyvee}x . \mathrm{Etc}\, x\, \mathrm{Etc} \cdot = a : \leftrightarrow \vdots \mathrm{Etc}\, a\, \mathrm{Etc} . \wedge \cdot \mathrm{Car} \cdot {}^{\uparrow}x . \mathrm{Etc}\, x\, \mathrm{Etc} : = 1 \vdots$

$\mathrm{Etc} \cdot {}^{\curlyvee}x . x \in A \cdot \mathrm{Etc} : \leftrightarrow \cdot \mathrm{Car}\, A = 1 . \wedge : \wedge a \cdot a \in A . \to . \mathrm{Etc}\, a\, \mathrm{Etc}$ #

The Lincos connective written as a "bird" on a vertical stroke, ${}^{\curlyvee}$, has been used as definite article. (Originally the bird was put over the letter representing the variable to be bound. See 1 29 3.)

These examples are insufficient. The factual use of the article can be learnt only from non-mathematical contexts. Our general ideas about description and the use of the article have been set forth in **0** 25. When regularly used, ${}^{\curlyvee}x . x \in A$ cannot occur unless Car $A = 1$. But this does not mean that ${}^{\curlyvee}x . x \in A$ occurs only if Car $A = 1$. Irregular between occurences of ${}^{\curlyvee}x . x \in A$ are needed if only as examples of irregular occurence.

An indefinite article will be introduced later.

As a consequence of our use of the definite article, we cannot decide in a mechanical way or on purely syntactic grounds whether certain expressions are meaningful or not. But this is no disadvantage. Lincos has been designed for the purpose of being used by people who know what they say, and who endeavour to utter meaningful speech.

1 32 1. # $\wedge x \cdot x \in \mathrm{Rea} . \to . f x = x^{10} + 11\, x + 1 :$

$\to : f\, 0 = 1 . \wedge . f\, 1 = 101 . \wedge \cdot f . - 1 = - 1 \cdot \wedge . f\, 0{,}1 = 10{,}11 .$

$\wedge \cdot f . - 0{,}1 = - 0{,}01 \cdot \wedge . f\, 0{,}01 = 1{,}1101 . \wedge \cdot f - 0{,}01 = 0{,}0101 .$

$\wedge . f \in \mathrm{Rea} \frown \mathrm{Rea} . \wedge : f = \cdot {}^{\curlyvee}x . x^{10} + 11\, x + 1 \vdots \mathrm{Rea}\,\iota$

$\wedge x \cdot x \in \mathrm{Rea} . \wedge . a \leq x \leq b \cdot$

$\to : \vee y \cdot c \leq y \leq d . \wedge . y = (101\, x - 1)/(x^{10} + 11\, x + 1) \vdots$

$\to \vdots {}^{\curlyvee}x . (101\, x - 1)/(x^{10} + 11\, x + 1) \vdots {}^{\uparrow}x . a \leq x \leq b \cdot$

$\in \vdots {}^{\uparrow}x . a \leq x \leq b \cdot \frown \mathrm{Rea} \cdot$

$\wedge \vdots \wedge z \cdot a \leq z \leq b . \wedge . z \in \mathrm{Rea} \cdot$

$\to : {}^{\curlyvee}x . (101\, x - 1)/(x^{10} + 11\, x + 1) \vdots {}^{\uparrow}x . a \leq x \leq b \cdot z$

$= (101\, z - 1)/(z^{10} + 11\, z + 1)\,\iota$

$\wedge x \vdots x \in \mathrm{Rea} . \wedge . a \leq x \leq b \cdot$

$\to \vdots \mathrm{Car} \vdots {}^{\uparrow}y \cdot c \leq y \leq d . \wedge . y^{10} = x^{10} + 11\, x + 1 \cdot = 1 \vdots$

$\to \vdots \vee f \vdots f \in \vdots {}^{\uparrow}x . a \leq x \leq b \cdot \frown \cdot {}^{\uparrow}y . c \leq x \leq d \cdot$

$\wedge \vdots \wedge x \cdot a \leq x \leq b .$

$\to . f x = \sqrt{x^{10} + 11\, x + 1} \smile - \sqrt{x^{10} + 11\, x + 1} :$

$\wedge \vdots f = \cdot {}^{\curlyvee}x \vdots {}^{\curlyvee}y \cdot c \leq y \leq d . \wedge . y^{10} = x^{10} + 11\, x + 1 \vdots {}^{\uparrow}x . a \leq x \leq b\,\iota$

$f \in A \frown B . \leftrightarrow \vdots f \subset \ulcorner A . B \urcorner .$

$\wedge \vdots \wedge x \vdots x \in A . \to \cdot \mathrm{Car} \cdot {}^{\uparrow}y . \ulcorner x . y \urcorner \in f \vdots = 1\,\iota$

$f \in A \frown B . \to \cdot x \in A . \to : f x = \cdot {}^{\curlyvee}y . \ulcorner x . y \urcorner \in f\,\iota$

$$\Lambda x \colon x \in A \ . \to \colon \mathrm{Car} \colon {}^{\curvearrowright}y \cdot y = \mathrm{Etc}\, x\, \mathrm{Etc} \ . \ \wedge \ . \ y \in B \ \dot{} = 1 \colon$$
$$\to \colon {}^{\vee}x \ . \ \mathrm{Etc}\, x\, \mathrm{Etc} \ | \ A \cdot \in \mathrm{A} \frown B \colon$$
$$\wedge \cdot {}^{\vee}x \ . \ \mathrm{Etc}\, x\, \mathrm{Etc} \ | \ A \cdot a \colon = \mathrm{Etc}\, a\, \mathrm{Etc} \ *$$

With the set-theory means now available we are allowed to con-sider *functions* as special sets, especially a function from A to B as a certain subset of $\ulcorner A \ . \ B \urcorner$ (a graphic). Yet this would not be the point to start from. When in an earlier period of our mathema-tical life we learned the notion of function and became familiar with it, a function was, as it were, a kinematical law, by which to every value of a variable, say x, corresponds one value of another variable, say y. Whenever we use functions, this idea predominates, even if we know some more formal definition. Psychologically and didactically the definition of a function as a special set would be inadequate. Nobody would grasp the sense and the importance of the concept of function. It is even unlikely that users of this concept will recognize any well-known idea in the formalized definition of function, if there is nobody to tell them what is really meant.

Perhaps it would have been better to delay the introduction of the concept of function, and to deal with it in the Chapter "Behavi-our" (say after the notion "law"); for our purpose didactic argu-ments should be more decisive than systematic ones. Nevertheless I have tried to introduce the notion of function at this point. The exact definition has been prepared by a number of examples that might be multiplied ad lib. The examples are rather clumsy, but this can hardly be avoided. For instance a text such as

$$\Lambda x \colon x \in \mathrm{Rea} \ . \to \colon \mathrm{Car} \cdot {}^{\curvearrowright}y \ . \ y = x^{10} + 11\,x + 1 \colon = 1 \ \dot{}$$
$$\wedge \ . \ fx = x^{10} + 11\,x + 1 \ | \to \ . \ f \in \mathrm{Rea} \frown \mathrm{Rea}$$

would be of no use. Indeed, it has been our principle to avoid implications with an always true antecedent.

The class of functions from A to B is written $A \frown B$ instead of B^A, in order to avoid punctuations in exponents, and ambiguities if A and B are numbers. The "cup" upon the stroke is essentially Church's "λ". (Its typographical history is analogous to those related in 1 29 3 and 1 31 1.) It transforms an expression into a function of the "cupped" variable. The ' $|$ ' is not a punctuation. It is a reminder of the vertical stroke as used in some versions of common mathematical language; after this sign the domain of the intended function is mentioned.

1 32 2. $\#\ f\in.A\frown\star\cdot\leftrightarrow:\vee B\cdot f\in.A\frown B\cdot$

$f\in.\star\frown B\cdot\leftrightarrow:\vee A\cdot f\in.A\frown B\cdot$

$f\in.\star\frown\star\cdot\leftrightarrow:\vee\ulcorner A.B\urcorner\cdot f\in.A\frown B\ \#$

1 32 3. $\#\ f\in.A\frown B\cdot\rightarrow.f^\wedge=A\vdots$

$f\in.A\frown B\cdot\rightarrow\vdots{}^\wedge f={}^{\cdot\wedge}y:\vee x\cdot x\in A\ .\wedge.\ulcorner x.y\urcorner\in f\ \#$

1 32 4. $\#\ ^\vee n\vdots{}^\vee y\cdot y=0\smile1\ .\wedge:\vee x\cdot x\in\mathrm{Int}\ .\wedge.\ n=2\,x+y\vdots\mathrm{Int}\cdot=f\vdots$

$\rightarrow{}^\cdot f\in.\mathrm{Int}\frown\mathrm{Int}\cdot\wedge:f\in{}^\cdot\mathrm{Int}\frown.0\cup1:\wedge{}^\cdot{}^\wedge f=.0\cup1\,\centerdot$

$^\vee x:{}^\vee y\cdot y\in\mathrm{Int}\ .\wedge.y\leq x<y+1\vdots\mathrm{Rea}\ {}^\cdot\in.\mathrm{Rea}\frown\mathrm{Int}\,\centerdot$

$^\vee\ulcorner x.y\urcorner.x+y\vdots\ulcorner\mathrm{Rea}.\mathrm{Rea}\urcorner\cdot=f:\rightarrow{}^\cdot f\in.\ulcorner\mathrm{Rea}.\mathrm{Rea}\urcorner\frown\mathrm{Rea}\cdot$

$\wedge:\wedge\ulcorner x.y\urcorner\cdot x\frown y\in\mathrm{Rea}\ .\rightarrow.f\ulcorner x.y\urcorner=x+y\,\centerdot$

$^\vee x.\ulcorner x^{10}.11\,x\urcorner\vdots\mathrm{Rea}\ {}^\cdot\in.\mathrm{Rea}\frown\ulcorner\mathrm{Rea}.\mathrm{Rea}\urcorner\ \#$

1 32 5. $\#\ \omega_1\ulcorner x.y\urcorner=x\cdot\omega_2\ulcorner x.y\urcorner=y\vdots$

$\omega_1\ulcorner x.y.z\urcorner=x\cdot\omega_2\ulcorner x.y.z\urcorner=y\cdot\omega_3\ulcorner x.y.z\urcorner=z\vdots$

Etc $\#$

For the convenience of the terrestrial reader we use decadic numbers as indices. The words written ω_1, ω_2, ω_3, and so on mean certain "universal" constants.

1 32 6. $\#\ f={}^\cdot{}^\vee x.x+1\vdots\mathrm{Rea}:\wedge:g={}^\cdot{}^\vee x.x^{10}\vdots\mathrm{Rea}\cdot$

$\rightarrow{}^\cdot fg={}^\cdot{}^\vee x.x^{10}+1\vdots\mathrm{Rea}:\wedge:gf={}^\cdot{}^\vee x.(x+1)^{10}\vdots\mathrm{Rea}\vdots$

$f\in.A\frown B\cdot\wedge\cdot g\in.B\frown C\vdots$

$\rightarrow\vdots gf\in.A\frown C\cdot\wedge\vdots\wedge x\cdot x\in A\ .\rightarrow\vdots gf.x\,{}^\cdot={}^\cdot g.fx\vdots$

$f={}^\cdot{}^\vee x.1/(x+1)\vdots\mathrm{Rea}\backslash\mathrm{Rat}:\wedge:g={}^\cdot{}^\vee x.(1/x)-1\vdots\mathrm{Rea}\backslash\mathrm{Rat}\cdot$

$\rightarrow{}^\cdot fg={}^\cdot{}^\vee x.x\vdots\mathrm{Rea}\backslash\mathrm{Rat}:\wedge\cdot gf={}^\vee x.x\vdots\mathrm{Rea}\backslash\mathrm{Rat}\ \#$

1 32 7. $\#\ n\in\mathrm{Num}\ .\rightarrow:\vee p\cdot p\in\mathrm{Pri}\ .\wedge.p\geqq n\,\centerdot$

$\wedge f\cdot f\in F\ .\leftrightarrow:f\in.\mathrm{Num}\frown\mathrm{Pri}\cdot\wedge.f\,1=10\,\centerdot$

$\wedge.f\,10=11\ .\wedge.f\,11=101\ .\wedge.f\,100=111\ .\wedge.f\,101=1011\ .\wedge.\mathrm{Etc}\vdots$

$\rightarrow\,\centerdot\mathrm{Car}\,F=1\ .\wedge\vdots\vee g\cdot g\in.\mathrm{Pri}\frown\mathrm{Num}\cdot\wedge:\wedge f\cdot f\in F\ .$

$\rightarrow:gf={}^\vee x.x\vdots\mathrm{Num}\cdot\wedge\cdot fg={}^\vee x.x\vdots\mathrm{Pri}\,\centerdot$

$\wedge f\vdots f\in F\ .\leftrightarrow\vdots f\in.\mathrm{Num}\frown\mathrm{Pri}\cdot\wedge.f\,1=10\,\wedge\vdots$

$\wedge\ulcorner n.x\urcorner\,{}^\cdot n\in\mathrm{Num}\ .\rightarrow:fn<f\,(n+1)\,\centerdot$

$\wedge\cdot fn<x<f\,(n+1)\ .\rightarrow.x\notin\mathrm{Pri}\,\centerdot$

$\rightarrow\vdots\mathrm{Car}\,F=1\ .\wedge\vdots\vee g\vdots g\in.\mathrm{Pri}\frown\mathrm{Num}\cdot$

$\wedge\cdot f\in F\ .\rightarrow:gf={}^\vee x.x\vdots\mathrm{Num}\cdot\wedge\cdot fg={}^\vee x.x\vdots\mathrm{Pri}\,\centerdot$

$\mathrm{Car}\,A=\mathrm{Car}\,B\ .\leftrightarrow\vdots\vee\ulcorner f.g\urcorner\,{}^\cdot f\in.A\frown B\cdot\wedge\cdot g\in.B\frown A\cdot$

$\wedge:gf={}^\cdot{}^\vee x.x\vdots A:\wedge:fg\,{}^\cdot={}^\vee x.x\vdots B\,\centerdot$

$\aleph_0=\mathrm{Car}\,\mathrm{Pri}=\mathrm{Car}\,\mathrm{Num}=\mathrm{Car}\,\mathrm{Int}=\mathrm{Car}\,\mathrm{Rat}$

$\neq\mathrm{Car}\,\mathrm{Rea}=\mathrm{Car}\,\mathrm{Com}\,\centerdot$

$A\subset\mathrm{Num}\ .\wedge\vdots\wedge n\cdot n\in\mathrm{Num}\ .\rightarrow:\vee a\cdot a\in A\ .\wedge.a\geqq n\vdots$

$\rightarrow.\mathrm{Car}\,A=\aleph_0\ \#$

1 32 8. $*\ 1 + \mathrm{Car}\, A . \in \mathrm{Num} \cdot \wedge \cdot 1 + \mathrm{Car}\, B . \in \mathrm{Num} :$
$\rightarrow : \mathrm{Car} . A \frown B \cdot = \mathrm{Car}\, B^{\mathrm{Car}\, A}\ *$

1 32 9. $*\ f = \cdot \,{}^{\vee}x . x^{10}\, |\, \mathrm{Com} . \wedge : g = \cdot \,{}^{\vee}x . x^{10}\, |\, \mathrm{Rea} \cdot \rightarrow . g = f\, |\, \mathrm{Rea} :$
$f \in . A \frown C \cdot \wedge . B \subset A :$
$\rightarrow : g = f\, |\, B . \leftrightarrow \cdot g \in . B \frown C \cdot \wedge : g = \cdot \,{}^{\vee}x . fx\, |\, B :$
$B \subset A . \rightarrow : {}^{\vee}x . \mathrm{Etc}\, x\, \mathrm{Etc}\, |\, A\, |\, B \cdot = \cdot \,{}^{\vee}x . \mathrm{Etc}\, x\, \mathrm{Etc}\, |\, B\ *$

1 33 1. $*\ A = \cdot \,{}^{\wedge}a . a > -11 : \rightarrow . \inf A = -11\, {}_{\text{I}}$
$A = \cdot \,{}^{\wedge}a . a \geqq -11 : \rightarrow . \inf A = -11\, {}_{\text{I}}$
$A = : {}^{\wedge}a \cdot a \in \mathrm{Rat} . \wedge . a^{10} < 10 \cdot \rightarrow . \inf A = -\sqrt{10}\, {}_{\text{I}}$
$A \subset \mathrm{Rea} . \wedge . B \subset \mathrm{Rea} . \wedge . A \neq \ulcorner\urcorner . \wedge . B \neq \ulcorner\urcorner \cdot$
$\rightarrow : A \leqq B . \leftrightarrow \cdot \wedge \ulcorner x . y \urcorner : x \in A . \wedge . y \in B \cdot \rightarrow . x \leqq y\, {}_{\text{I}}$
$A \subset \mathrm{Rea} . \wedge : {}^{\wedge}b . A \geqq b \cdot \neq \ulcorner\urcorner \cdot$
$\rightarrow \cdot \inf A \in \mathrm{Rea} . \wedge : A \geqq \inf A \geqq \cdot \,{}^{\wedge}b . A \geqq b\ *$

In an analogous way sup A will be defined.

1 34 1. $\ulcorner A . B \urcorner \in \mathrm{Agg}\tfrac{1}{2}\mathrm{Ord} . \leftrightarrow : A \in \mathrm{Agg} . \wedge . B \subset \ulcorner A . A \urcorner .$
$\wedge \ulcorner x . y . z \urcorner : \ulcorner x . y . z \urcorner \in \ulcorner A . A . A \urcorner . \rightarrow \cdot \ulcorner x . y \urcorner \in B . \rightarrow : \ulcorner y . x \urcorner \notin B \cdot$
$\wedge \cdot \ulcorner y . z \urcorner \in B . \rightarrow . \ulcorner x . z \urcorner \in B\, {}_{\text{I}}$
$\ulcorner A . B \urcorner \in \mathrm{Agg}\, \mathrm{Ord} . \leftrightarrow : \ulcorner A . B \urcorner \in \mathrm{Agg}\tfrac{1}{2}\mathrm{Ord} .$
$\wedge \cdot \wedge \ulcorner x . y \urcorner : \ulcorner x . y \urcorner \in \ulcorner A . A \urcorner .$
$\rightarrow \cdot x = y . \vee . \ulcorner x . y \urcorner \in B . \vee . \ulcorner y . x \urcorner \in B\, {}_{\text{I}}$
$B \cdot = : {}^{\wedge}z \cdot z = \ulcorner x . y \urcorner . \wedge \cdot x \frown y . \in \mathrm{Rea} \cdot \wedge . x < y :$
$\rightarrow \ulcorner \mathrm{Rea} . B \urcorner \in \mathrm{Agg}\, \mathrm{Ord}\ *$

The word written Agg Ord (fL aggregatum ordinatum) means "ordered sets". The word written Agg $\tfrac{1}{2}$ Ord means "partially ordered sets".

1 34 2. $*\ \ulcorner A . B \urcorner \in \mathrm{Agg}\, \mathrm{Ord}\, \mathrm{Ded} . \leftrightarrow {}_{\text{I}} \ulcorner A . B \urcorner \in \mathrm{Agg}\, \mathrm{Ord} .$
$\wedge : \wedge C : C \subset A . \wedge . C \neq A . \wedge . C \neq \ulcorner\urcorner .$
$\wedge \cdot \wedge \ulcorner x . y \urcorner : y \in C . \wedge . \ulcorner x . y \urcorner \in B \cdot \rightarrow . x \in C :$
$\rightarrow : \vee c \cdot C \backslash c . = {}^{\wedge}x . \ulcorner x . c \urcorner \in B\, {}_{\text{I}}$
$B = : {}^{\wedge}z \cdot z = \ulcorner x . y \urcorner . \wedge . x \frown y \in \mathrm{Rea} . \wedge . x < y \cdot$
$\rightarrow . \ulcorner \mathrm{Rea} . B \urcorner \in \mathrm{Agg}\, \mathrm{Ord}\, \mathrm{Ded}\ *$

The definition of Dedekind-ordered sets.

1 34 3. $*\ \ulcorner A . f \urcorner \in \mathrm{Gru} . \leftrightarrow {}_{\text{I}} \cdot A \in \mathrm{Agg} . \wedge \cdot f \in . \ulcorner A . A \urcorner \frown A \cdot$
$\wedge {}_{\text{I}} . \wedge \ulcorner x . y \urcorner \cdot \ulcorner x . y \urcorner \in \ulcorner A . A \urcorner . \rightarrow . x \oplus y = f \ulcorner x . y \urcorner :$
$\rightarrow {}_{\text{I}} \wedge \ulcorner x . y . z \urcorner : \ulcorner x . y . z \urcorner \in \ulcorner A . A . A \urcorner .$
$\rightarrow : (x \oplus y) \oplus z = x \oplus (y \oplus z) .$
$\wedge \cdot \mathrm{Car} \cdot {}^{\wedge}u . x \oplus u = y := 1 \cdot \wedge \cdot \mathrm{Car} \cdot {}^{\wedge}u . u \oplus x = y := 1\, {}_{\text{I}}$
$\wedge \ulcorner x . y \urcorner : \ulcorner x . y \urcorner \in \ulcorner \mathrm{Rea} . \mathrm{Rea} \urcorner . \rightarrow \cdot f \ulcorner x . y \urcorner . = x + y \cdot$
$\rightarrow . \ulcorner \mathrm{Rea} . f \urcorner \in \mathrm{Gru}\ *$

The word written Gru means "groups".

1 34 4. $*\ulcorner A.f\urcorner \in \text{Gru Abe} . \leftrightarrow \colon \ulcorner A.f\urcorner \in \text{Gru}. \wedge\dot{} \wedge \ulcorner x.y\urcorner \colon \ulcorner x.y\urcorner \in \ulcorner A.A\urcorner .$
$\rightarrow \cdot f\ulcorner x.y\urcorner .= .f\ulcorner y.x\urcorner *$

Abelian groups.

1 34 5. $*\ulcorner A.f.g\urcorner \in \text{Cam}. \leftrightarrow \iota \ulcorner A.f\urcorner \in \text{Gru Abe}.$
$\wedge \vdots \wedge z \colon z \in A . \wedge \cdot f\ulcorner z.z\urcorner .=z\cdot \rightarrow \colon \ulcorner A\backslash z.g\urcorner \in \text{Gru Abe}.$
$\wedge \dot{} \wedge x \colon x \in A . \rightarrow \cdot g\ulcorner x.z\urcorner .= .g\ulcorner z.x\urcorner =z\vdots$
$\wedge \dot{} \wedge \ulcorner x.y.z\urcorner \colon \ulcorner x.y.z\urcorner \in \ulcorner A.A.A\urcorner .$
$\rightarrow \cdot g\ulcorner f\ulcorner x.y\urcorner .z\urcorner .= .f\ulcorner g\ulcorner x.z\urcorner .g\ulcorner y.z\urcorner \urcorner \mid$
$\wedge \ulcorner x.y\urcorner \dot{} \ulcorner x.y\urcorner \in \ulcorner \text{Rea}.\text{Rea}\urcorner .$
$\rightarrow \colon f\ulcorner x.y\urcorner .= .x+y\cdot \wedge \cdot g\ulcorner x.y\urcorner .=xy\colon \rightarrow . \ulcorner \text{Rea}.f.g\urcorner \in \text{Cam}\mid$
$\wedge \ulcorner x.y\urcorner \dot{} \ulcorner x.y\urcorner \in \ulcorner \text{Com}.\text{Com}\urcorner .$
$\rightarrow \colon f\ulcorner x.y\urcorner .= .x+y\cdot \wedge \cdot g\ulcorner x.y\urcorner .=xy\colon$
$\rightarrow . \ulcorner \text{Com}.f.g\urcorner \in \text{Cam} *$

The word written Cam (fL "campus") means "fields".

1 34 6. $*\ulcorner A.B.f.g\urcorner \in \text{Cam Ord}.$
$\leftrightarrow \colon \ulcorner A.f.g\urcorner \in \text{Cam}. \wedge . \ulcorner A.B\urcorner \in \text{Agg Ord}.$
$\wedge \dot{} \wedge \colon \ulcorner x.y.z.u.a\urcorner \dot{} \ulcorner x.y\urcorner \in B . \wedge . \ulcorner z.u\urcorner \in \ulcorner A.A\urcorner . \wedge\cdot$
$f\ulcorner a.a\urcorner .=a\cdot \wedge . \ulcorner a.u\urcorner \in B\colon$
$\rightarrow \cdot \ulcorner f\ulcorner x.z\urcorner .f\ulcorner y.z\urcorner \urcorner \in B . \wedge . \ulcorner g\ulcorner x.u\urcorner .g\ulcorner y.u\urcorner \urcorner \in B\mid$
$\ulcorner A.B.f.g\urcorner \in \text{Cam Ord Ded}.$
$\leftrightarrow \cdot \ulcorner A.B.f.g\urcorner \in \text{Cam Ord}. \wedge . \ulcorner A.B\urcorner \in \text{Agg Ord Ded}\mid$
$B=\colon \hat{z}\cdot z= \ulcorner x.y\urcorner . \wedge .x \frown y \in \text{Rea}. \wedge .x<y\cdot$
$\wedge \colon \wedge \ulcorner x.y\urcorner \dot{} \ulcorner x.y\urcorner \in \ulcorner \text{Rea}.\text{Rea}\urcorner .$
$\rightarrow \colon f\ulcorner x.y\urcorner .=x+y\cdot \wedge \cdot g\ulcorner x.y\urcorner .=xy\colon$
$\rightarrow . \ulcorner \text{Rea}.B.f.g\urcorner \in \text{Cam Ord Ded} *$

1 35 1. $*\,\text{Car} \in \star \frown \star .$
$\sup \in \star \frown \star .$
$\inf \in \star \frown \star .$
$\omega_i \in \star \frown \star .$
Etc $*$

1 35 2. $*\, n \in \text{Num}. \wedge \dot{} A= \colon \hat{x}\cdot x \in \text{Num}. \wedge .x \leqq n\colon$
$\rightarrow \dot{} \Sigma .x \in A .x\cdot =n(n+1)/10$
$\colon \wedge \colon \Sigma .x \in A .x^{10}\cdot =n^{11}/11+n^{10}/10+n/110\colon$
$\text{Car } A = 11. \wedge .A= \ulcorner a\urcorner \cup \ulcorner b\urcorner \cup \ulcorner c\urcorner . \wedge \cdot f \in .A \frown \text{Com}\colon$
$\rightarrow \colon \Sigma .x \in A .fx\cdot =fa+fb+fc\colon$
$f \in .\star \frown \text{Com}\cdot \wedge .A= \ulcorner \urcorner \colon \rightarrow \colon \Sigma .x \in A .fx\cdot =0\colon$
$1+\text{Car } B. \in \text{Num}\cdot \wedge \cdot f \in .B \frown \text{Com}\colon$
$\rightarrow \dot{} \Sigma .x \in A .fx\cdot = \colon \Sigma \cdot x \in .A \cap B\cdot fx\colon$

$$1 + \operatorname{Car} C . \in \operatorname{Num} \cdot \wedge \cdot f \in . C \frown \operatorname{Com} \cdot$$
$$\wedge . A \subset B \subset C . \wedge \cdot \ulcorner a \urcorner = . B \backslash A :$$
$$\rightarrow : \varSigma . x \in B . fx \cdot = \cdot fa + \varSigma . x \in A . fx *$$

The use of the sum symbol (for finite sums only).

1 35 3. Finite and infinite sequences will be considered and written as functions. For the convenience of the reader we shall often use the customary index notation.

1 35 4. The definitive program will contain an exposition of Calculus. Though we shall use Calculus in the next chapters, we have omitted this exposition, because it cannot be our task to publish a text-book of Calculus in Lincos. But this argument cannot completely justify our silence in this matter. We should at least have dealt with notations. It is a fact that the usual system of notations of Calculus is inconsistent to a degree, especially in the field of functions of two and more variables. It is easy to remove these defects, but a notational system which preserves the obvious advantages of Leibniz's notation is still lacking. I have tried several methods but have been so far unable to decide for any one. I have therefore preferred to postpone the decision.

1 36 0. We now introduce a few words that will enable the user of Lincos to make statements in Lincos about some kinds of Lincos expressions. They are the words written

Ver (fL verum $=$ "true") meaning "true",

Fal (fL falsum $=$ "false") meaning "false",

Prp (fL propositio $=$ "proposition") meaning "propositions".

Qus (fL quaestio $=$ "question") meaning "questions".

Iud (fL iudicium $=$ "judgement") meaning "truth value".

We shall also meet with the Lincos connective of negation, written $\neg$.

1 36 1. $* \; 1 + 1 = 10 . \in \operatorname{Ver} :$

$1 + 1 = 11 . \in \operatorname{Fal} :$

$a + b = b + a . \in \operatorname{Ver} :$

$\wedge \ulcorner a . b \urcorner . a + b = b + a \cdot \in \operatorname{Ver} :$

$a + b \neq b + a . \in \operatorname{Fal} :$

$1 \in \operatorname{Int} . \in \operatorname{Ver} :$

$1 \notin \operatorname{Int} . \in \operatorname{Fal} :$

$\sqrt{10} \in \operatorname{Rea} . \in \operatorname{Ver} :$

$\sqrt{10} \in \operatorname{Rat} . \in \operatorname{Fal} :$

$\operatorname{Rat} \subset \operatorname{Rea} . \in \operatorname{Ver} :$

$\operatorname{Rea} \subset \operatorname{Rat} . \in \operatorname{Fal} :$

Etc $*$

1 36 2. $*$ $a \in A . \in \text{Ver} \cdot \leftrightarrow \cdot a \notin A . \in \text{Fal}$:
          $a \in A . \in \text{Fal} \cdot \leftrightarrow \cdot a \notin A . \in \text{Ver}$ ⵚ
          $\wedge n \cdot n \in \text{Num} . \rightarrow : \vee p \cdot p \in \text{Pri} . \wedge . p \geq n \vdots \in \text{Ver}$ ⵚ
          $\vee n \cdot n \in \text{Num} . \wedge : \wedge p \cdot p \in \text{Pri} . \rightarrow . p \geq n \vdots \in \text{Fal}$ ⵚ
          Etc $*$

Some people might feel shocked by the use of the '∈' in such
sentences, though I think it to be relatively harmless. Propositions
have not so far occurred as elements of sets. We are therefore
free to use the '∈' after a proposition as here. It would be more
risky to admit 'Ver' and 'Fal' as elements of sets (especially 'Ver
∈ Agg' and 'Fal ∈ Agg').

We need not be afraid of the paradox of the liar and of other
paradoxes. We have not yet dealt with human behaviour. We have
not spoken about acts, nor even about the acts of pronouncing,
proving and refuting propositions. In due course we shall learn
that by behaviour rules some kinds of human acts are forbidden.
One of these forbidden acts might be the famous act of the liar.

Note that our lexicological and syntactical apparatus does not
suffice for a treatment of Lincos syntaxis. If we wish to pronounce
both "x is an integer", and "x is a variable" in a common context,
we are obliged to invent some means of distinguishing either
between the two "x"-s or between the two copulas. (The first
way out has become classical.)

1 36 3. $*$ $p \in \text{Ver} . \rightarrow . p \in \text{Prp} \cdot$
          $p \in \text{Fal} . \rightarrow . p \in \text{Prp} \cdot$
          $10 \rightarrow 1001 . \notin \text{Prp} \cdot$
          $a \in \text{Div} . \notin \text{Prp} \cdot$
          $x \notin = . \notin \text{Prp} \cdot$
          $+ \wedge \in \rightarrow . \notin \text{Prp} \cdot$
          $\vee \text{Num} \, x \in 10 . \notin \text{Prp}$:
          $\text{Car} \, A \neq 1 . \rightarrow \cdot {}^{\vee}x . x \in A \cdot \in B : \notin \text{Prp}$.
          $\text{Car} \, A \neq 1 . \rightarrow \cdot {}^{\vee}x . x \in A \cdot \text{Div} \, b : \notin \text{Prp}$:
          $\text{Car} \, A \neq 1 . \rightarrow \cdot {}^{\vee}x . x \in A \cdot = 11 : \notin \text{Prp}$:
          $f \in A \frown B . \wedge . a \notin A \cdot \rightarrow : \text{Etc} . f a . \text{Etc} \cdot \notin \text{Prp} \cdot$
          Etc $\cdot$
          $\text{Ver} \in \text{Int} . \notin \text{Prp} \cdot$
          $\text{Ver} \notin \text{Fal} . \notin \text{Prp} \cdot$
          Etc $*$

1 36 4. $*$ $? \cdot x \in \text{Rea} . \wedge . x^{10} = 10 : \notin \text{Prp} \cdot$
          $? \cdot x \in \text{Rea} . \wedge . x^{10} = 10 : \in \text{Qus}$:
          $? \cdot x \in \text{Rea} . \wedge . x^{10} = 10 : \in \text{Qus} \cdot \in \text{Ver}$:
          Etc $*$

1 36 5. $^{\#}\, p \in \mathrm{Ver}\,.\,\leftrightarrow\,.\,\mathrm{Iud}\; p = \mathrm{Ver}\,\cdot$
$\quad\quad\quad\quad p \in \mathrm{Fal}\,.\,\leftrightarrow\,.\,\mathrm{Iud}\; p = \mathrm{Fal}\; ^{\#}$

1 36 6. $p \leftrightarrow\,.\,p \in \mathrm{Ver}\,\cdot$
$\quad\quad\quad p \leftrightarrow\,.\,p \notin \mathrm{Fal}\,\cdot$
$\quad\quad\quad \neg\, p\,.\,\leftrightarrow\,.\,p \in \mathrm{Fal}\,\cdot$
$\quad\quad\quad \neg\, p\,.\,\leftrightarrow\,.\,p \notin \mathrm{Ver}\,\cdot$
$\quad\quad\quad \neg\,.\,1 + 1 = 11\,\cdot$
$\quad\quad\quad \neg\,.\,\mathrm{Rea} \subset \mathrm{Rat}:$
$\quad\quad\quad a \in A\,.\,\leftrightarrow\,\cdot\,\neg\,.\,a \notin A:$
$\quad\quad\quad a \notin A\,.\,\leftrightarrow\,\cdot\,\neg\,.\,a \in A\; ^{\#}$

1 36 7. $^{\#}\, p \wedge q\,.\,\in \mathrm{Ver}\,\cdot\,\leftrightarrow\,\cdot\,p \in \mathrm{Ver}\,.\,\wedge\,.\,p \in \mathrm{Ver}:$
$\quad\quad\quad p \wedge q\,.\,\in \mathrm{Fal}\,\cdot\,\leftrightarrow\,\cdot\,p \in \mathrm{Fal}\,.\,\vee\,.\,q \in \mathrm{Fal}:$
$\quad\quad\quad p \vee q\,.\,\in \mathrm{Ver}\,\cdot\,\leftrightarrow\,\cdot\,p \in \mathrm{Ver}\,.\,\vee\,.\,q \in \mathrm{Ver}:$
$\quad\quad\quad p \vee q\,.\,\in \mathrm{Fal}\,\cdot\,\leftrightarrow\,\cdot\,p \in \mathrm{Fal}\,.\,\wedge\,.\,q \in \mathrm{Fal}:$
$\quad\quad\quad p \rightarrow q\,.\,\in \mathrm{Ver}\,\cdot\,\leftrightarrow\,\cdot\,p \in \mathrm{Fal}\,.\,\vee\,.\,q \in \mathrm{Ver}:$
$\quad\quad\quad p \rightarrow q\,.\,\in \mathrm{Fal}\,\cdot\,\leftrightarrow\,\cdot\,p \in \mathrm{Ver}\,.\,\wedge\,.\,q \in \mathrm{Fal}\; ^{\#}$

1 36 8. $^{\#}\, \neg\,.\,p \wedge q\,\cdot\,\leftrightarrow\,\cdot\,\neg\, p\,.\,\vee\,.\,\neg\, q:$
$\quad\quad\quad \neg\,.\,p \vee q\,\cdot\,\leftrightarrow\,\cdot\,\neg\, p\,.\,\wedge\,.\,\neg\, q:$
$\quad\quad\quad p \rightarrow q\,\cdot\,\leftrightarrow\,\cdot\,\neg\, p\,.\,\vee\, q:$
$\quad\quad\quad p \leftrightarrow\,.\,\neg\,\neg\, p\; ^{\#}$

1 36 9. $^{\#}\, \neg:\vee n\,\text{·}\, n \in \mathrm{Num}\,.\,\wedge:\wedge x\,\text{·}\, x \in \mathrm{Pri}\,.\,\rightarrow\,.\,x \leqq n:$
$\quad\quad\quad \wedge n\,\text{·}\, n \in \mathrm{Num}\,.\,\rightarrow\,\text{·}\,\neg:\wedge x\,\text{·}\, x \in \mathrm{Pri}\,.\,\rightarrow\,.\,x \leqq n\,\colon$
$\quad\quad\quad \neg\,\text{·}\,\wedge x\,.\,\mathrm{Etc}\; x\; \mathrm{Etc}:\leftrightarrow:\vee x\,\text{·}\,\neg\,.\,\mathrm{Etc}\; x\; \mathrm{Etc}\,\colon$
$\quad\quad\quad \neg\,\text{·}\,\vee x\,.\,\mathrm{Etc}\; x\; \mathrm{Etc}:\leftrightarrow:\wedge x\,\text{·}\,\neg\,.\,\mathrm{Etc}\; x\; \mathrm{Etc}\; ^{\#}$

TIME

2 00 0. Again we start with ostensive, ideophonetic signs, the so-called time-signals. They are even more ideophonetic than the peeps (written as dots) we used in Chapter I to introduce the natural numbers. While the peeps showed and meant arithmetical units, the new signs will not mean anything but themselves. So they can hardly be called words.

2 01 0. The new signs are radio-signals — time-signals — of various duration and wave-length. They are written as horizontal lines.

2 01 1. $*\,\mathrm{Dur}\;\overline{\quad\quad} = \mathrm{Sec}\,\mathfrak{a}\;*$

and so on.

The 'a' as it stands does not belong to our program text in the proper sense. It is a meta-text variable, used as a substitute for a Lincos constant. Generally *Gothic letters* will represent such substitutes.

Eventually this 'a' should be replaced by a Lincos word meaning a positive real number a such that the sentence

"The duration of the factual time-signal indicated by the horizontal line is of a seconds"

is true.

The Lincos word written Dur (fL duratio = duration) means "duration". Syntactically it is to be handled as a function to the set of durations. The domain of this function is not exactly the set of time-signals. It is much broader, but at this stage it would be unwise and even impossible to circumscribe it in a too definite way. This is symptomatic of many functions we shall deal with.

The Lincos word written Sec means the time unit second. Syntactically it behaves as a function from 'Pos' to the set of durations. So it is a paradigm of Lincos syntaxis for physical units. 'Cmt' and 'Gra' (centimeter and gramme) will occur as symbols for functions from 'Pos' to the set of lengths and the set of masses respectively.

2 01 2.   The program text of 2 01 1 will be repeated on different wave-
lengths and with different durations of their time-signals until
the receiver may be expected to remark that the numbers indicated
by a in the different texts are proportional to the durations he
observes, independently of their wave-length. So he will guess that
the numbers indicated by a are to mean the durations of those
time-signals, and that the Lincos word 'Dur' is to mean "dura-
tion". He will learn our time-unit as well.

Note that his knowledge of our time-unit may be corrupted by
a factor $1 - \frac{v}{c}$ (unrelativistically) as long as he does not know the
relative velocity v of sender and receiver.

If, e.g., he is moving away from us, the waves of the end of a
signal will travel a longer way to the sender than those of the
beginning, and so the signal will be stretched. If he is moving
towards us, it will be shortened (the "Olav Römer" effect). The
receiver may know v from direct observation of our planet or of
our solar system, but we cannot rely on this possibility. In due
course we shall learn how to free time-reckoning from that un-
known factor.

We could have used time-signals in order to introduce 'Rea'.
But then we would have deprived ourselves of a means of showing
duration.

It is important to send time-signals on different wave-lengths.
Otherwise the number indicated by a could be understood as
something relating to the number of waves or oscillations con-
tained in the time-signal.

2 01 3. $* \text{Nos} \overline{} = a *$

and so on.

Now the a is a substitute for the positive number that counts
the set of ether oscillations contained in the time-signal of the left-
hand member of the text-equation.

The Lincos word written 'Nos' means "number of oscillations".
It behaves as a function (from the set of time-signals or a larger
set) to 'Pos'.

As the number indicated by a is different for time-signals of
equal duration, but different wave-length, 'Nos' cannot be inter-
preted as "duration".

Intelligent beings who can receive electro-magnetic waves while
ignorant of any theory of electromagnetic vibrations (e.g. beings
who receive radio-signals by their senses as we do with light and

sound) will not immediately understand the program texts of
2 01 3. But if they are not colour-blind (i.e. if they can distinguish
wave-lenghts), they will perhaps learn from our information, and
they will build some theory of radio-signals.

Note that, independently of the state of motion the number of
oscillations in a signal given in our text will be equal to that
observed by the receiver himself. within the limits of observation
errors. All oscillations, from the beginning to the end of the
signal, must reach the receiver, however distorted the signal may
be.

2 01 4.   # Fre ‾‾‾ $= \mathrm{Sec}^{-1}\,a$ #

and so on.

The word written Fre (fL frequentia) means "frequency" (of
an oscillation). 'Fre' and 'Sec^{-1}' ("Hertz") are treated as 'Dur'
and 'Sec'. Again a is a substitute for some real number that gives
the frequency in Hertz of the time-signal in question.

2 01 5.   # $\mathrm{Dur}\,x = \mathrm{Sec}\,a\,.\to.\,a > 0\,\cdot$
$\mathrm{Fre}\,x = \mathrm{Sec}^{-1}\,b\,.\to.\,b > 0\,\cdot$
$\mathrm{Nos}\,x = c\,.\to.\,c > 0:$
$\mathrm{Dur}\,x = \mathrm{Sec}\,a\,.\wedge.\,\mathrm{Fre}\,x = \mathrm{Sec}^{-1}\,b\,.\wedge.\,\mathrm{Nos}\,x = c\,.\to.\,ab = c:$
$\mathrm{Sec}\,a \times \mathrm{Sec}^{-1}\,b = ab\,.$
$c/\mathrm{Sec}\,a = \mathrm{Sec}^{-1}\,c/a\,.$
$c/\mathrm{Sec}^{-1}\,b = \mathrm{Sec}\,c/b$ #

In a quite natural way we have switched over to variables as
arguments of the new functions.

An example of arithmetics of concrete numbers has been given.

2 01 6.   # Fin ____ $= \mathrm{Ini}$ ‾‾‾ #

and so on.

When the first time-signal finishes the second begins. The same
fact is stated by the text within the "picture". The words written
Fin (fL finis = end) and Ini (fL initium = beginning) mean "end"
and "beginning" respectively. The two-time signals are sent on
different wave-lengths. The receiver can therefore separate them
and observe that the end of the one coincides with the beginning
of the other. The text words are sent on still other wave-lengths.
So the word group '= Ini' and the first signal will not disturb
each other.

The difference of wave-length has been depicted by a difference
in height.

2 01 7. $*$ Ecc _____ ‾ **:**

$\qquad$ $\vee\ulcorner x\,.\,y\urcorner\cdot\operatorname{Fre}x=\mathfrak{a}\,.\,\wedge\,.\,\operatorname{Fre}y=\mathfrak{b}\,.\,\wedge\,.\operatorname{Ini}y=\operatorname{Fin}x\ *$

and so on.

$\qquad$ The audio-pictorial part of 2 01 5 has been repeated, but now together with a verbal description of the picture outside the picture. Of couse $\mathfrak{a}$ and $\mathfrak{b}$ are substitutes for such frequencies that the text becomes true when related to the picture.

$\qquad$ The word written Ecc (fL ecce = see) will always announce an audio-picture the legend of which is to be found outside the picture.

$\qquad$ Former texts may be repeated in order to explain this pattern:

2 01 8. $*$ Ecc ‾‾ ‾ **:**

$\qquad$ $\vee x\cdot\operatorname{Dur}x=\mathfrak{a}\,.\,\wedge\,.\,\operatorname{Nos}x=\mathfrak{b}\,.\,\wedge\,.\operatorname{Fre}x=\mathfrak{c}\ *$

and so on.

2 02 1. $*$ Ecc __

$\qquad$ $\vee\ulcorner x\,.\,y\urcorner\cdot\operatorname{Fre}x=\mathfrak{a}\,.\,\wedge\,.\,\operatorname{Fre}y=\mathfrak{b}\,.\,\wedge\,.x\operatorname{Ant}y\ *$

and so on.

$\qquad$ The word written Ant (fL ante = before) means "before". The time-signals are separated by a pause. In the text they are named by their frequencies.

2 02 2. $*\ x\operatorname{Ant}y\,.\leftrightarrow.\,y\operatorname{Pst}x\ *$

$\qquad$ The word written Pst (fL post = after) means "after".

2 02 3. $*\ x\operatorname{Pre}y\,.\leftrightarrow\colon\operatorname{Fre}x\,.=\mathfrak{w}\cdot\wedge\cdot\operatorname{Fre}y\,.=\mathfrak{w}\cdot\wedge\,.x\operatorname{Ant}y\,.$

$\qquad$ $\wedge\,.\rightharpoondown\colon\vee z\colon\operatorname{Fre}z\,.=\mathfrak{w}\cdot\wedge\,.x\operatorname{Ant}z\,.\wedge\,.z\operatorname{Ant}y\ *$

$\qquad$ The word written Pre (fL precedit = precedes) means the relation of immediate succession, restricted to the case of time-signals of frequency $\mathfrak{w}$, where $\mathfrak{w}$ is a meta-textual constant (the same in all Lincos texts).

$\qquad$ On this frequency a clock will be installed. It will tick, one peep a second, through all Lincos programs from now on. The receiver will be instructed on how to read the clock:

2 02 4. $*\ \operatorname{Ini}x\,.=\operatorname{Mom}n\colon\wedge\,.x\operatorname{Pre}y\colon\rightarrow\colon\operatorname{Ini}y\cdot=\operatorname{Mom}(n+1)\cdot$

$\qquad$ $\operatorname{Ini}^-=\operatorname{Mom}0\ *$

$\qquad$ The word written Mom means moment.

$\qquad$ The moment Mom 0 is shown.

$\qquad$ The rule will also be interpolated and extrapolated.

We may not use Sec n as a denotation of the moment Mom n, because Sec n means a duration.

Time references will be used in order to mention past (or even future) events.

The Lincos program clock will also possess hands for longer intervals (e.g. for 2^8 sec. and 2^{16} sec.).

By means of clock-reading, events like Ini x and Fin x may be considered as elements of the set of all moments Mom a (a real, even negative).

2 02 5.　✳ Ini $x \in$ Tem . Fin $x \in$ Tem ·

$a \in$ Rea . → . Mom $a \in$ Tem ·

$a \frown b \in$ Rea . → : Mom a . Ant . Mom b · ↔ . $a < b$ ·

Mom a . Ant . Mom b · ↔ . Mom $a <$ Mom b :

Mom $a =$ Mom b . ↔ . $a = b$ ·

Mom $a <$ Mom b . → : Mom $b -$ Mom a . $=$ · Sec . $b - a$ ·

$a \in$ Tem . ∧ . $b \in$ Tem . ∧ . $a < b$ · → · $b - a$. $\in$ ˆDur ·

Fin $x -$ Ini $x =$ Dur x :

Fin $x =$ Ini y . → · Fin $y -$ Ini x . $=$. Dur $x +$ Dur y :

Sec . $a + b =$ · Sec $a +$ Sec b :

Sec . $a - b =$ · Sec $a -$ Sec b :

$a < b$. → . Sec $a <$ Sec b :

$a = b$. → . Sec $a =$ Sec b ✳

The word written Tem (fL tempus $=$ time) means "time" (the time-axis). For elements of Tem a subtraction has been defined, but the differences are elements of ˆDur, not of Tem.

2 02 6.　✳ Sec $\in$ Pos $\frown$ ˆDur .

Dur $\in \star \frown$ ˆDur .

Nos $\in \star \frown$ Pos .

Fre $\in \star \frown$ ˆFre .

Sec$^{-1} \in$ Pos $\frown$ ˆFre .

Fin $\in \star \frown$ Tem .

Ini $\in \star \frown$ Tem .

Mom $\in \star \frown$ Tem ✳

2 03 1.　Beside the verbal text the program may contain events. In broad-casting, the weather-forecast, the newsreel, reports and lectures are verbal program texts. A musical performance is an event, a radio play is a sequence of events most of which (but not all) consist of pronouncing a verbal text.

We will be able to mention past (and even future) events. Such

a mention is a proposition (but mentioning is again an event). We now exercise mentioning past events.

$$\ast \, ^{t_1}\mathrm{p}\, ^{t_2} \, . \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, \mathrm{p} \, \ast$$

p is a metatextual substitute for an event, e.g. for a noise — i.e. a wild complex of radio signals that cannot be interpreted as a verbal text. The t_1 and t_2 are metatextual substitutes for Lincos words that designate some moments. The t_1 and t_2 above the line, surrounding the first p, do not relate to any part of the eventual program text, but to the activity of the clock. They should tell the terrestrial reader at which moments the event written p has begun and has ended.

The word written Usq ($/L$ usque ad = till) means "till". The word written Fit ($/L$ fit = happens) means "happens".[1]) After the 'Fit' a copy of the first p occurs.

The first part of the text is an event that happened from t_1 to t_2, namely the noise written p. The second part is a report that should be translated: From t_1 to t_2 p happened. We shall see that reports need not be literal.

2 03 2. $\quad \ast \, ^{t_1}\mathrm{p}\, ^{t_2} : \vee \, x \, . \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, x : {}^{\curlyvee}x \, . \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, x \, \cdot = \mathrm{p} \, \ast$

2 03 3. $\quad \ast \, ^{t_1}\mathrm{p}\, ^{t_2} \mathrm{q}\, ^{t_3} \cdot t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, \mathrm{p} \, . \, \wedge \, . \, t_2 \, \mathrm{Usq} \, t_3 \, \mathrm{Fit} \, \mathrm{q} \, .$
$\qquad \wedge : t_1 \, \mathrm{Usq} \, t_3 \, \mathrm{Fit} \cdot \mathrm{p} \, . \, \mathrm{q} \, \ast$

2 03 4. $\quad \ast \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, \mathrm{p} \, . \, \wedge \, . \, t_2 \, \mathrm{Usq} \, t_3 \, \mathrm{Fit} \, \mathrm{q} \, . \, \rightarrow : t_1 \, \mathrm{Usq} \, t_3 \, \mathrm{Fit} \cdot \mathrm{p} \, . \, \mathrm{q} \, \ast$

Note that the last t_1, t_2, t_3 are textual (written images of Lincos variables).

2 03 5. $\quad \ast \, \mathrm{Ecc}\, ^{t_1}\text{———}^{t_2} \cdot \mathrm{Dur} \cdot {}^{\curlyvee}x \, . \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, x : = t_2 - t_1 \, \cdot$
$\qquad \mathrm{Fre} \cdot {}^{\curlyvee}x \, . \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, x : = \mathfrak{a} \, \ast$

The horizontal line is again the picture of a time-signal. $\mathfrak{a}$ is a substitute for its frequency.

2 03 6. $\quad \ast \, ^{t_1}\mathrm{p}\, ^{t_2} : t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, \mathrm{p} \, . \in \mathrm{Prp} :$
$\qquad t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, \mathrm{p} \, . \in \mathrm{Ver} :$
$\qquad {}^{\curlyvee}x \, . \, t_1 \, \mathrm{Usq} \, t_2 \, \mathrm{Fit} \, x \, \cdot \notin \mathrm{Prp} \, \ast$

As a rule propositions about events will mention the date of the event. An event does not contain its date; the date of an event can be read on the clock. For the terrestrial reader it is mentioned above the line.

For the sake of brevity only we shall often omit the 'Usq' in the date of an event which is mentioned in a proposition about that event.

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[1]) Verbs are always considered to be used in the 3rd person singular. (Note that "verb" or "noun" are not Lincos syntactical or semantical categories. They refer to usual translations of Lincos words into English.)

2 03 7. The rule that propositions about events should specify their date was not observed when we introduced the functions Dur and so on, and even later on, when we used the words written 'Ant' and 'Post'. We now give an explicit definition of 'Ant' that conforms to that rule:

$$* \, x \, \text{Ant} \, y \, . \leftrightarrow : \vee \, \ulcorner t.t'.t''.t'''\urcorner \cdot t < t' < t'' < t''' \, .$$
$$\wedge . \, t \, t' \, \text{Fit} \, x \, . \wedge . \, t'' \, t''' \, \text{Fit} \, y \, *$$

2 03 8. $* \, x \, . \, h \, \text{Ant} \, . \, y \, \cdot \leftrightarrow : \vee \, \ulcorner t.t'.t''.t'''\urcorner \cdot t < t' < t'' < t''' \, .$
$$\wedge . \, t'' = t' + h \, . \wedge . \, t \, t' \, \text{Fit} \, x \, . \wedge . \, t'' \, t''' \, \text{Fit} \, y \, *$$

"x antedates y by h".

2 03 9. We shall also say

$$* \, x \, \text{Ant} \, y \, . \leftrightarrow \cdot \text{Ant} \, y \, . \, \text{Fit} \, x \, \cdot$$
$$x \, . \, h \, \text{Ant} \, . \, y \, \cdot \leftrightarrow : h \, \text{Ant} \, . \, y \, \cdot \text{Fit} \, x \, *$$

so using 'Ant y' and 'h Ant . y', as it were, as a date dating x.

$$* \, y \, \text{Pst} \, x \, . \leftrightarrow \cdot \text{Pst} \, x \, . \, \text{Fit} \, y \, \cdot$$
$$x \, . \, h \, \text{Ant} \, . \, y \leftrightarrow : y \, \cdot \, h \, \text{Pst} \, . \, x \, \cdot$$
$$y \, . \, h \, \text{Pst} \, . \, x \, \cdot \leftrightarrow : h \, . \, \text{Pst} \, x \, \cdot \text{Fit} \, y \, *$$

2 04 0. In order to speak about an event $^{t_1} \mathfrak{p} \, ^{t_2}$, we have used a copy of the event, whereas in formalist semantics it is agreed that things should not be used as words until they have been earmarked in a suitable way, e.g. by putting them between quotation marks. I concede our procedure is risky, but I do not believe that it is more so than any other method.

2 04 1. Let us first analyze what quotation marks can do.

In due course we shall meet with formulas such as

$$t_1 \, t_2 \, \text{Fit} \, \mathfrak{p} \, . \rightarrow \cdot \vee x . \, t_1 \, t_2 \, \text{Fit} \, x \, . \tag{$\star$}$$

Now, there may be events that look like x. Then it could happen that

$$^{t_1} x \, ^{t_2} \, ,$$

so that

$$t_1 \, t_2 \, \text{Fit} \, x$$

would be true. as well as

$$\vee x . \, t_1 \, t_2 \, \text{Fit} \, x \, ,$$

though we should wish that no more than one of these formulas were propositions.

This can be avoided by using an earmarked $\mathfrak{p}$ as a name of $\mathfrak{p}$.

Instead of quotation marks we may use a dot in the "lefthand ear" of $\mathfrak{p}$. When

$$^{t_1}\mathfrak{p}^{t_2},$$

we will speak about the $\mathfrak{p}$ while using $^\bullet\mathfrak{p}$. So

$$t_1\,t_2\,\text{Fit}\,^\bullet\mathfrak{p}.$$

The dot would be considered as belonging to the 'Fit' and it could even be incorporated into the 'Fit', if there were no formulas without a dot after the 'Fit'. Yet our formula $(\star)$ will now read

$$t_1\,t_2\,\text{Fit}\,^\bullet\mathfrak{p}.\leftrightarrow\cdot\lor x.t_1\,t_2\,\text{Fit}\,x.$$

As x is an (ambiguous) name for the event in question, we have observed the rule that demands the use of names after 'Fit'. If now by chance

$$^{t_1}x^{t_2},$$

we have the opportunity to state

$$t_1\,t_2\,\text{Fit}\,^\bullet x,$$

which is not inconsistent with

$$\lor x.t_1\,t_2\,\text{Fit}\,x.$$

It is a pity that in almost all cases the situation is much more complicated.

2 04 2. Let us suppose that there is a kind of events that may be characterized by some partial similarity, for instance events in which one can discover the algorithmic Lincos words for natural numbers as a part. These events may look like

$$^{t_1}\mathfrak{p}\,1010\,\mathfrak{q}^{t_2}$$

where $\mathfrak{p}$ and $\mathfrak{q}$ are substitutes for some constant parts, whereas instead of 1010 we can meet with 101 or 11011 or some other numeral. Such events should be named

$$^\bullet\mathfrak{p}\,1010\,\mathfrak{q},$$

but if we restricted ourselves to this kind of name, speech about those events would become unusually clumsy. For instance we should be confined to a purely arbitrary name for the class of all these events, instead of an algorithmic name which tells us immediately which are the members of that class.

It is evident what we are to do. We shall name events looking like

$$^{t_1}\mathfrak{p}\,1010\,\mathfrak{q}^{t_2}$$

by ambiguous names

$$^\bullet\mathfrak{p}\,x\,\mathfrak{q}$$

where x is a variable (not only: looks like a variable).

So, if

$$^{t_1}\mathfrak{p}\,1010\,\mathfrak{q}\,^{t_2},$$

we are entitled to state

$$\forall x \cdot x \in \mathrm{Num} \cdot \wedge \cdot t_1\, t_2\, \mathrm{Fit} \cdot \mathfrak{p}\, x\, \mathfrak{q}\,.$$

Unfortunately this is not completely correct.

$$\cdot\,\mathfrak{p}\,x\,\mathfrak{q}$$

is really the name of some

$$^{t_1}\mathfrak{p}\,x\,\mathfrak{q}\,^{t_2}$$

whereas we intended to name events such as

$$^{t_1}\mathfrak{p}\,1010\,\mathfrak{q}\,^{t_2}.$$

So it will be better not to use quotation, but quasi-quotation (as does Quine). The earmark • is to be changed into a quasi-earmark, let us say °. So

$$\forall x \cdot x \in \mathrm{Num} \cdot \wedge \cdot t_1\, t_2\, \mathrm{Fit} \,°\, \mathfrak{p}\, x\, \mathfrak{q}\,.$$

The quasi-earmark will tell us that some parts of the text covered by it mean variables (not only look like variables), but it will not tell us which parts are exempted from its working. It will be merely a warning signal, which is due at almost all appearances of 'Fit'. So it is superfluous and unsatisfactory.

2 04 3. The only thing that matters is how to know which parts of '. . .' in 'Fit . . .' are genuine, and which are substitutes. The latter should be anti-earmarked. As the substitutes both look like variables and are variables, one is tempted to use an extra-ordinary kind of variable in places like this. For instance such variables would be written as Gothic letters, whereas the genuine parts of the '. . .' in 'Fit . . .' should be in italic. But this is a cumbersome procedure. We cannot prevent Gothic written words from appearing in authentic events such as

$$t_1 \, \ldots \, t_2.$$

So we should be burdened with hypergothics and so on.

Moreover there is a much simpler device which works without earmarks and anti-earmarks.

2 04 4. Most of our program texts will be well poised speech. Parts of these texts may look like a hotchpotch if we try to read them as it were well poised speech. This is not important as long as we know that this hotchpotch is genuine. Fortunately we can easily recognize whether this is true or not. As soon as in a hotchpotch controlled by a 'Fit' substitutions have been made, so that

the text controlled by 'Fit' cannot be considered as the genuine
name of some event, the text controlled by the 'Fit' will no longer
look like a hotchpotch, at least not with respect to its relations
with the surrounding context. A single non-genuine piece will
mean a variable that does not truly belong to the text inside the
working sphere of the 'Fit', but is imposed from outside. So
quantification over such variables will take place outside. Con-
versely, if a variable suffers quantification outside the working
sphere of the 'Fit', it is certain that this variable is not genuine
inside. Moreover people from outside the 'Fit' who impose some
substitution inside, will be well advised not to choose a variable x,
if inside there is something that looks like x.

So, under our former assumptions

$$t_1 t_2 \text{ Fit } \mathfrak{p} \, x \, \mathfrak{q}$$

reports on an event

$$^{t_1} \mathfrak{p} \, x \, \mathfrak{q} \, ^{t_2}$$

because there is no quantification over x, and consequently x is
genuine.

$$t_1 t_2 \text{ Fit} \cdot \forall x . \mathfrak{p} \, x \, \mathfrak{q}$$

reports an event

$$^{t_1} \forall x . \mathfrak{p} \, x \, \mathfrak{q} \, ^{t_2}$$

because quantification took place inside the 'Fit' (it does not
matter whether such an event tells us something or is merely a
hotchpotch). In

$$\forall x \cdot x \in \text{Num} . \wedge . t_1 t_2 \text{ Fit } \mathfrak{p} \, x \, \mathfrak{q} \, ,$$

the x after the 'Fit' is not genuine because quantification has
taken place outside.

The part under the influence of 'Fit' may be arbitrary. For
instance

$$\forall x \cdot x \in \text{Num} . \wedge : t_1 t_2 \text{ Fit} \cdot \forall x . x$$

is a meaningful text, though the event mentioned in the text may
be merely a noise.

2 04 5.   I do not claim that this solves all problems that arise from men-
tioning an event, nor do I endeavour to solve them in future.
The only thing I have tried is to get a working pattern.

The exposition has been somewhat abstract. Perhaps the reader
will understand it better, if he re-reads it in connection with our
analysis of oblique speech. Oblique speech in natural languages
is a striking example of the occurrence of binding variables outside
the speech. (See 3 01 5.)

2 04 6. Nothing would need to be added to the preceding exposition, if
we had not developed the bad habit of dropping the '∧' quantifica-
tion word in a large numbers of its occurrences. Speech would
become more clumsy, if we were to abandon this habit, but
fortunately we need not. The mere appearance of the variable x
outside the working sphere of a 'Fit' betrays the non-genuine
character of every piece inside that looks like x. (To add a remark
that may seem unintelligible at the moment: that appearance
need not be explicit.)

2 05 0. Autonymity is the basis of our system of description. As a name
of an event þ we use a copy of þ. But the name þ will not unambigu-
ously name the event þ, unless þ has happened (and will happen)
only once. This ambiguity cannot always be removed by mention-
ing the event þ that has happened from t_1 to t_2 by means of

$$t_1\ t_2\ \text{Fit}\ \text{þ}$$

(for instance if the date of the event þ is to be concealed or asked
for). Nevertheless þ can be used as an unambiguous name of some
event þ as soon as it is clear from the context which event þ is
meant. This conforms to our general principles. In the sentence

$$\text{At ``off'' the racers start}$$

we meet with the name "off" of an event of pronouncing "off",
which in a certain context can be denoted unambiguously by this
name. Under analogous conditions we shall use þ as name of an
event þ in Lincos.

We can also mention an event through description, e.g. the
event þ that has happened from t_1 to t_2 by means of

$$\check{\ }x\ .\ t_1\ t_2\ \text{Fit}\ x.$$

From a formalist point of view such a description would be
ambiguous, or rather, we should not be allowed to use the article
because in every interval more than one event will happen. This
formalist objection would even hold true if we restricted the use
of 'Fit' to program events. For program events will be perceived,
and perceiving a program event can also be a program event.
This will yield at least two contemporary events. According to
what is said in 025 we can use the expression

$$\check{\ }x\ .\ t_1\ t_2\ \text{Fit}\ x$$

as an unambiguous description of that event, which is singled
out by the date $t_1\ t_2$ together with the nearer and farther context
of the above expression. As long as we do not pursue formalization
but communication, we can rest satisfied with this procedure.

BEHAVIOUR

3 00 1. For the time being it would be premature to try to describe
human behaviour by a system of general rules like the mathemat-
ical and chronometric rules of the preceding chapters and some
of the mechanical laws of the next chapter. Instead we shall *show*
behaviour by quasi-general examples, from which the receiver may
derive as many general behaviour rules as he pleases.

The things to be shown are events. The program of showing
events could be transformed into a program of reporting on events.

Of course the receiver will understand that the events we are
showing cannot have happened literally in the way we use to show
them. He will understand that normal terrestrial events are not
likely to be groups of radio-signals of such an intensity as to reach
far-remote worlds. Nevertheless we shall behave as though this
part of the program consists of events that are not only shown,
but also happen, and we expect the receiver to do likewise.

Such fictions are not unusual. Spectators are expected to take
it for granted that the events shown on the stage really happen.
For this purpose it is essential that the events shown are not
literally the same as those supposed to happen; for instance
whispering is played in such a way that the discourse is heard in
the house, though some people on the stage do not perceive it.

The aforesaid fiction is an essential tool in our program. The
things shown are events that really happen though they do not
happen literally in this way. This is not too unrealistic a view.
Even in daily life the literal happening of an event is a fiction.
All we know about an event is that we remember its happening,
and though we do not remember it twice in precisely the same
way, we state that it is the same event whenever we remember it.

3 00 2. As the program events are to display behaviour, it is necessary
for at least a part of them to be acts, i.e. caused by persons. Our
Lincos vocabulary is still far from sufficient for introducing the
bodies of the acting persons. So the only kind of act that can be

displayed immediately is the act of speaking. The Lincos word that designates this activity, is written Inq (*fL* inquit = says).

The terrestrial reader should guard against a too narrow interpretation of this 'Inq'. In the present chapter the physical background of the Inq-events, whether it be accoustical or optical or tactile or anything else, will remain undiscernible. 'Inq' will aim at communication acts in the broadest sense. After all we have said about literal happenings, it is clear that the same event can be played in different ways, but at the moment there is no sense in asking the question in which way it "really" happens.

3 00 3. The names of the dramatis personae will be written *Ha, Hb, Hc,* and so on. In due course we will state that these persons are members of the set called Hom (*fL* homo = man) in written Lincos.

3 00 4. Our theatre is still incomplete. Besides persons and acts a third thing is needed. We have been able to build a vocabulary of mathematics without valuating our propositions. We had only to confine ourselves to true propositions. The falsehood of $1 = 2$ could be formulated as $1 \neq 2$. Yet we cannot show behaviour by good actions only. We must stage bad ones too, if we wish to condemn them. We have to create a vocabulary that contains words meaning *"good"* and *"bad"* and intermediate valuations.

To begin with, we shall stick to two *values*. Of course they cannot be 'Ver' and 'Fal', which are values of propositions. What we wish to valuate are acts, not propositions. (We are not here considering value-judgements of esthetics.)

Our valuating words will be written Ben (*fL* bene = well) and Mal (*fL* male = badly). They mean "good" and "bad" respectively. These words can be introduced in a surprisingly simple manner. (This may be a striking refutation of a prejudice which is very common among representatives of *Geisteswissenschaften*: *Werturteile* other than "true" and "false" are said to be beyond the reach of mathematical methods.)

As we are neither inclined nor able to settle behaviour rules, we shall not pronounce valuating judgements otherwise than by the mouth of an acting person. 'Ben' and 'Mal' will be no more than expressions used by some people in order to approve or disapprove of some events. Occasionally pronouncing a true proposition may be judged to be a bad action, and pronouncing a false proposition a good one.

3 00 5.   The elementary events are written

$$Ha \operatorname{Inq} Hb \, \mathfrak{p} \, .$$

This reads "*Ha* tells (says) *Hb* the $\mathfrak{p}$". Here $\mathfrak{p}$ is again a metatextual substitute for something that can be said. This may also be a question, an incomplete answer, an order, a nonsensical group of words, a noise, and so on; it might even be an air sung by *Ha*.

3 00 6.   The last point to be discussed is to know which kind of saying is the best to start with. As far as I have tried it, mathematics seems to provide the most appropriate texts. So we shall construct dialogues about mathematical topics, as it were classes in mathematics.

3 00 7.   We have not endeavoured to give every acting person a constant character. The persons named *Ha* act independently in the different talks. This is a serious disadvantage. In a final redaction of our project the person named *Ha* (and likewise the others) should really be one and the same person during the whole program.

3 01 1.   * $Ha \notin \mathrm{Num} \, . \, Ha \notin \mathrm{Com} \, . \, Ha \in Ha \, .$
       $Hb \notin \mathrm{Num} \, . \, Hb \notin \mathrm{Com} \, . \, Hb \in Hb \, .$
       Etc *

       $\mathrm{Inq} \notin \mathrm{Agg}$ *

3 01 2.   * $Ha \operatorname{Inq} Hb \cdot ? \, x \, . \, 10 \, x = 101 :$
       $^{t_1} Hb \operatorname{Inq} Ha \, . \, 101/10^{t_2} :$
       $Ha \operatorname{Inq} Hb$ Ben *

and so on.

The incompleteness of answers is intentional. By means of this device the connexion with the preceding part of the text is stressed. The receiver will more easily understand that it is a dialogue. Yet one will also send:

* $^{t_1} Ha \operatorname{Inq} Hb \cdot ? \, x \, . \, 10 \, x = 101^{t_2} :$
       $Hb \operatorname{Inq} Ha \cdot 10 \, x = 101 \, . \rightarrow . \, x = 101/10^{t_3} :$
       $Ha \operatorname{Inq} Hb$ Ben^{t_4} *

* $Ha \operatorname{Inq} Hb \cdot ? \, x \, . \, 10 \, x = 101 :$
       $^{t_1} Hb \operatorname{Inq} Ha \, . \, x = 101/10^{t_2} \cdot$
       $Ha \operatorname{Inq} Hb : \, ^{\curlyvee} x \, . \, t_1 \, t_2 \operatorname{Fit} x \, . \in \operatorname{Ben}^{t_3}$ *

Ha's last answer seems to be still incomplete. When '$^{\curlyvee} x \, . \, t_1 \, t_2 \operatorname{Fit} x$' is replaced by '$Hb \operatorname{Inq} Ha \, . \, x = 101/10$' the answer will run '$Hb \operatorname{Inq} Ha \, . \, x = 101/10 \cdot \in \operatorname{Ben}$', and this is in fact incomplete, for it

mentions an event without the date that links the event to its context. This remark, however, is not correct. The aforesaid substitution is illegal. It is a fact that Ha said what he said, and not what we have got from it by some substitution. Quoting freely is permitted as long as it does not affect the meaning.

We have treated this question because it is the forerunner of more serious ones, not because the text could be a source of mis-interpretations. Nevertheless we may add some comment to the talks under this number in order to ascertain which kind of sub-stitutions are legal. We shall revert to this point in 3 01 8.

It is in our intention that the receiver should understand these sequences of questions and answers as dialogues. Yet we must also enable him to separate and to recognize the three components of 'Ha Inq Hb'. Therefore all talks will be repeated with other actors, so that 'Ha Inq Hb' cannot be read as a constant complex.

3 01 3.   ⚹ Ha Inq $Hb \cdot ? \, x . 100 \, x = 1010 :$
            Hb Inq $Ha . 1010/100 :$
            Ha Inq Hb Mal :
            Hb Inq $Ha . 1/10 :$
            Ha Inq Hb Mal :
            Hb Inq $Ha . 101/10 :$
            Ha Inq Hb Ben ⚹

        ⚹ Ha Inq $Hb \cdot ? \, x . x = 10 + 10 :$
            Hb Inq $Ha . 10 + 10 :$
            Ha Inq Hb Mal :
            Hb Inq $Ha \, 100 :$
            Ha Inq Hb Ben ⚹

        ⚹ Ha Inq $Hb \cdot ? \, x . x^{10} = 11001 \stackrel{.}{.}$
            Hb Inq $Ha . 101 \times 101 = 11001 \stackrel{.}{.}$
            Ha Inq Hb Mal $\stackrel{.}{.}$
            Hb Inq $Ha \cdot 101 \times 101 = 11001 . \in \text{Ver} \stackrel{.}{.}$
            Ha Inq $Hb \, \dot{.} \, \text{Ver Tan Mal} : \neg \cdot x^{10} = 11001 . \rightarrow . x = 101 \stackrel{.}{.}$
            Hb Inq $Ha . 101 \backsim - 101 \stackrel{.}{.}$
            Ha Inq Hb Ben ⚹

        ⚹ Ha Inq $Hb \cdot ? \, {}^{\curlyvee}x . x^{10} = 11001 :$
            Hb Inq $Ha \, 101 :$
            Ha Inq Hb Ben ⚹

        ⚹ Ha Inq $Hb \cdot ? \, {}^{\curlyvee}x . x^{10} = 11001 \, \dot{.}$
            Hb Inq $Ha : {}^{\curlyvee}x . x^{10} = 11001 \cdot \in \cdot {}^{\wedge}x . x^{10} = 11001 \, \dot{.}$
            Ha Inq Hb Mal ⚹

? $^\curvearrowright x$ asks for an x (not for all x) satisfying the context. This notation is rooted in an older version of Lincos (compare 025). Though it does not fit in our present notational system, we do not feel an overwhelming need to change it.

Note the appearance of a word written Tan (/L tamen = nevertheless) meaning "but". Logical analysis seems to prove it to be superfluous. In the talk, as it runs, it could be replaced by '∧'. Yet as an expression of behaviour it can hardly be dispensed with. What "but" means in behaviour might be illustrated by the two sentences

"He is a mathematician, but he is dull"

and

"He is a mathematician, but he is clever".

Both sentences contain a thought that is not explicitly stated, "mathematicians are usually clever" in the first case, "mathematicians are usually dull" in the second.

It will be advantageous to translate the adversative "but" in a somewhat more circumstantial way. We shall also say instead of

⚹ Ver Tan Mal ⚹

⚹ Qqm Ver Tan Mal ⚹ ,

with a redundant 'Qqm' (/L quamquam = though). Then the last sentence reads "Ver, it it true, but Mal", or though Ver still Mal. Compare German "zwar" (zwar Ver, aber Mal). In the last pattern we may interchange the two components:

⚹ Tan Mal Qqm Ver ⚹ ,

which reads "still Mal though Ver"; or more shortly

⚹ Mal Qqm Ver ⚹ .

All these patterns will be introduced in the same context, suitably repeated, as the original 'Tan'.

Here I must warn the reader against confusing 'Tan' with 'Sed', which will be introduced in 3 04 6. Compare the "but" of

He is not a clever man, but (Tan) he is very kind,

and

He is not clever, but (Sed) dull,

or of

He did not work hard, but (Tan) he passed the examination very well,

and

He did not work hard, but (Sed) wasted his time,

or of

He is 16 years old, but (Tan) he looks like 18,

and

He is not 16 years old, but (Sed) 18.

Though in many natural languages these two buts are usually identical, there is no reason whatsoever to stick to this idiomatic oddity in an artificial language. The *true adversative* 'Sed' and the *pseudo-adversative* 'Tan' are so well distinguished that one cannot expect these two notions to be identified in the thought of a being not influenced by our natural languages.

In German the difference between 'Tan' and 'Sed' is well-known. In many cases (not in all) it can be expressed by "aber (Tan)", and "sondern" (Sed). In written Dutch a similar distinction existed, but it fell into disuse.

3 01 4.　$* \, t_1 t_2 \, Ha \, \mathrm{Inq} \, Hb \, p \, . \leftrightarrow \cdot t_1 t_2 \, \mathrm{Fit} . \, Ha \, \mathrm{Inq} \, Hb \, p \, *$

An abbreviation has been introduced. Do not confuse

$^{t_1} Ha \, \mathrm{Inq} \, Hb \, \mathfrak{p} \, ^{t_2}$ with $t_1 t_2 \, Ha \, \mathrm{Inq} \, Hb \, \mathfrak{p}$.

In the first case an event has been sent, in the second a report on this event.

3 01 5.　$* \, Hc \, \mathrm{Inq} \, Hc \vdots \wedge \ulcorner t_1 . t_2 . t_3 \urcorner \vdots \vee t_4 \vdots t_1 t_2 \, Ha \, \mathrm{Inq} \, Hb \cdot ? \, x \, . \, 100 \, x = 1010 \vdots$
　　　　　$\rightarrow \cdot t_2 t_3 \, Hb \, \mathrm{Inq} \, Ha \, . \, 1010/100 \cdot \rightarrow . t_3 t_4 \, Ha \, \mathrm{Inq} \, Hb \, \mathrm{Mal} \vdots$
　　　　　$\wedge \vdots t_2 t_3 \, Hb \, \mathrm{Inq} \, Ha \, . \, 101/10 \cdot \rightarrow . t_3 t_4 \, Ha \, \mathrm{Inq} \, Hb \, \mathrm{Ben} \, *$

The person named Hc states a behaviour rule — perhaps he witnessed the dialogue of 3 01 3. It does not matter whether this rule is true or not. In any case Hc could be advised to suppose something about the lengths of the time-intervals within which the questions and answers should be comprised.

$* \, Hc \, \mathrm{Inq} \, Hc \vdots t_1 t_2 \, Ha \, \mathrm{Inq} \, Hb \cdot ? \, x \, . \, 100 \, x = 1010 \vdots$
　　　$\wedge . t_2 t_3 \, Hb \, \mathrm{Inq} \, Ha \, u/v \cdot \rightarrow \vdots \vee \ulcorner t_4 w \urcorner \cdot t_3 t_4 \, Ha \, \mathrm{Inq} \, Hb \, w \, . \wedge \vdots$
　　　$u = 1010 . \wedge . v = 100 \cdot \rightarrow . w = \mathrm{Mal} \vdots$
　　　$u = 101 . \wedge . v = 10 \cdot \rightarrow . w = \mathrm{Ben} \, *$

This is only a paraphrase of the preceding program text. Nevertheless it has an importance of its own, because it shows how to use variables in a context like this. We may not use one variable s instead of u and v, and we are not allowed to write

$t_2\,t_3\,Hb\,\mathrm{Inq}\,Ha\,s$ instead of $t_2\,t_3\,Hb\,\mathrm{Inq}\,Ha\,u/v$,

and $s = 1010/100$ instead of $u = 1010\,.\wedge.\,v = 100$,

and $s = 101/10$ instead of $u = 101\,.\wedge.\,v = 10$,

for e.g. $s = 101/10$ would be true if $s = 1010/100$,

and $t_2\,t_3\,Hb\,\mathrm{Inq}\,Ha\,101/10$ if $s = 1010/100$. In our actual formulation $t_2\,t_3\,Hb\,\mathrm{Inq}\,Ha\,101/10$ is only true if $u = 101$ and $v = 10$.

In this way more general behaviour rules may be stated:

$⧾\ Hc\,\mathrm{Inq}\,Hc \vdots t_1\,t_2\,Ha\,\mathrm{Inq}\,Hb \cdot \mathord{?}\,x\,.\,ax = b \vdots$

$\wedge \cdot a \cup b \cup c \cup d \cup e \cup f\,.\mathord{\subset}\,\mathrm{Num} \cdot \wedge.\,b/a = d/c\,.\wedge \vdots$

$\neg\cdot \vee p : p-1\,.\in\mathrm{Num} \cdot \wedge.\,p\,\mathrm{Div}\,c\,.\wedge.\,p\,\mathrm{Div}\,d \vdots \wedge \cdot t_2\,t_3\,Hb\,\mathrm{Inq}.\,f/e \vdots$

$\rightarrow \vdots \vee\ulcorner t_4\,.\,v\urcorner\cdot t_3\,t_4\,Ha\,\mathrm{Inq}\,Hb\,v\,.$

$\wedge \vdots f = d\,.\wedge.\,e = c \cdot \rightarrow.\,v = \mathrm{Ben} \vdots \wedge \vdots f \neq d\,.\vee.\,e \neq c \cdot \rightarrow.\,v = \mathrm{Mal}\ ⧾$

We face the problem of oblique speech here. Direct and oblique speech differ syntactically, not semantically, even in natural languages. In any case the semantic difference between

John said to me: "I am ill"

and

John told me that he was ill

is insignificant if compared with the syntactical transformation of "I" into "he" and "am" into "was". It is only exceptionally that a judge insists on a direct version of some witnessed statement talk, but even then he will not urge that the version be literal. It would be difficult to maintain that direct speech claims to be more literal than oblique speech.

In the oblique version of our example a variable (I) that belongs to John's discourse has been replaced by a variable (he) belonging to the discourse of the person who reports on John's speech.

In the examples

John said to me: "Mary is ill"

and

John told me that you were ill

(if said to Mary), it is a constant (Mary) that is replaced by a variable (you).

The "was" or "were" instead of "am" or "is" reflects the change from a (non-explicit) time variable "now" to another (non-explicit) time variable "recently".

It is an interlinguistic feature of oblique speech that inside the speech we find variables that are not genuine, but that belong to the discourse of the reporter. By synonymy this phenomenon may be obscured. Nevertheless it is clear that in

I bought a car, but John said that it was too expensive
the "it" belongs to my discourse and not to John's.

The pattern of oblique speech is a special case of that of mention-
ing events (in an oblique way), namely the case where the event
itself is speech. The last two program texts are good examples
for the working pattern I have proposed in 2 04 0–2 04 6. In the
last but one talk of Hc the variables u and v occurring in
'$t_2 t_3$ Hb Inq Ha u' and '$t_3 t_4$ Hb Inq Ha v' cannot be variables used
by Hb and Ha respectively, because they have been used by Hc
previously ("previously" is not to be taken in the sense of temporal
succession, but in that of logical structure; the '→' has coordinating
power). We could add $\wedge u$ to the discourse of Hc, but even if we
abstain from doing so, it is evident that binding u must take
place in Hc's discourse, at least if Hc is considered to be making
a meaningful statement.

In the last talk there are still more variables in the oblique
speeches of Ha and Hb that belong to the discourse of Hc, namely
a, b, e, f and v. The subject of Hc's talk is not one question
$? x . ax = b$ (for all numerals a and b) imputed to Ha, but any
question of the kind $? x . ax = b$ (where a and b are numerals),
e.g. $? x . 100 x = 1000$, $? x . 10 x = 11010$, and so on. The
$? x . ax = b$ is only a common name for all these questions, but
it is not the question that is aimed at by Hc. And it is the same
with Hb's answer. All these variables can be bound in Hc's dis-
course only.

3 01 6. There are more criteria that permit a distinction between 'Ben'
and 'Mal':

⚹ Ha Inq Hb · $? x . 100 x = 1010$:
 Hc Inq Ha . $101/10$:
 Ha Inq Hc Mal:
 Hb Inq Ha $101/10$:
 Ha Inq Hb Ben ⚹

Some comment of a fourth person on Ha's behaviour might
be added.

3 01 7. The talk of 3 01 6 is perhaps the first from which the receiver
may conclude that 'Ha Inq Hb ...' means "Ha says to Hb ..."
and not "Hb says to Ha ...". But discourses like

⚹ Ha Inq Hb . $t_1 t_2$ Fit $\mathfrak{p}$: Hb Inq Hc . $t_1 t_2$ Fit $\mathfrak{p}$: Hc Inq Hd . $t_1 t_2$ Fit $\mathfrak{p}$ ⚹

or

⚹ ^{t_1}Ha Inq Hb . $t_1\,t_2$ Fit $\mathfrak{p}^{t_1}$: Hb Inq Hc · $t_3\,t_4\,Ha$ Inq Hb . $t_1\,t_2$ Fit $\mathfrak{p}^{t_1}$:
　　Hc Inq Hd ˙ $t_4\,t_5\,Hb$ Inq Hc : $t_3\,t_4\,Ha$ Inq Hb . $t_1\,t_2$ Fit $\mathfrak{p}$ ⚹

are better suited for this purpose. We shall send a great many texts from which this will become still more evident.

3 01 8. Comments on some texts may be very useful. E.g. on the first talk of 3 01 2:

⚹ Hc Inq Hd : $t_1\,t_2\,Hb$ Inq Ha · $10\,x = 101$. → . $x = 101/10$ ˙
　　Hd Inq Hc Ben ⚹

A comment on the second talk of 3 01 2:

⚹ Hc Inq Hd ˙
　　$t_1\,t_2\,Ha$ Inq Hb · ? x . $10\,x = 101$: ∧ : $t_2\,t_3\,Hb$ Inq Ha . $101/10$:
　　Hd Inq Hc Ben ⚹

Another comment on the second talk of 3 01 2:

⚹ Hc Inq Hd ˙ $t_3\,t_4\,Ha$ Inq Hb : $^{\curlyvee}x$. $t_2\,t_3$ Fit x · ∈ Ben :
　　Hd Inq Hc Ben ⚹

A comment on the third talk of 3 01 2:

⚹ Hc Inq Hd ˙ $t_2\,t_3\,Ha$ Inq Hb : Hb Inq Ha $101/10$. ∈ Ben ,
　　Hd Inq Hc : Fal : $t_2\,t_3\,Ha$ Inq Hb : $^{\curlyvee}x$. $t_1\,t_2$ Fit x · ∈ Ben :
　　¬ : $t_2\,t_3\,Ha$ Inq Hb : Hb Inq Ha $101/10$. ∈ Ben ˙
　　↔ ˙ $t_2\,t_3\,Ha$ Inq Hb : $^{\curlyvee}x$. $t_1\,t_2$ Fit x · ∈ Ben ⚹

From these comments the receiver will learn what liberties a person may take when quoting other people. One could add a hypercomment put into the mouth of still other persons and containing behaviour rules on honest quoting.

In a former version of Lincos we distinguished between literal and free quotations by means of a special notation which was dropped later on. Literal quotation is a rather unimportant limit position. We shall develop a means of comparing the exactness of quotations ('Err', 3 09 1, 3 19 1). This will prove to be more useful. If needed, literal quotations may be characterized by 'Err . . . $= 0$'.

3 02 0. We shall here treat interrogative sentences:

3 02 1. Many interrogative pronouns and adverbs can be treated in the following manner:

⚹ ^{t_1}Ha Inq Hb · ? x . $100\,x = 1010^{t_1}$:
　　Hb Inq Hc ˙ ? y : $t_1\,t_2\,y$ Inq Hb · ? x . $100\,x = 1010$:
　　Hc Inq Hb Ha ⚹

$$* ^{t_1}Ha \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010^{t_2} \vdots$$
$$Hb \operatorname{Inq} Hc \cdot ? y : t_1 t_2 Ha \operatorname{Inq} y \cdot ? x . 100\,x = 1010 \vdots$$
$$Hc \operatorname{Inq} Hb \, Hb \; *$$

$$* ^{t_1}Ha \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010^{t_2} \vdots$$
$$Hb \operatorname{Inq} Hc : ? y \cdot t_1 t_2 Ha \operatorname{Inq} Hb\, y \vdots$$
$$Hc \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010 \; *$$

$$* ^{t_1}Ha \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010^{t_2} \vdots$$
$$Hb \operatorname{Inq} Hc \cdot ? y : y\, Ha \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010 \vdots$$
$$Hc \operatorname{Inq} Hb . t_1 t_2 \; *$$

These are the questions "who?", "to whom?", "what?", "when?". In the same way one can express the factual question:

$$* ^{t_1}Ha \operatorname{Inq} Hb . ? x . 100\,x = 1010^{t_2} \vdots$$
$$Hb \operatorname{Inq} Hc \vdots ? y \cdot y = \operatorname{Iud} : t_1 t_2 Ha \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010 \vdots$$
$$Hc \operatorname{Inq} Hb \operatorname{Ver} \; *$$

$$* ^{t_1}Ha \operatorname{Inq} Hb \cdot ? x . 100\,x = 1010^{t_2} \vdots$$
$$Hb \operatorname{Inq} Hc \vdots ? y \cdot y = \operatorname{Iud} : t_1 t_2 Ha \operatorname{Inq} Hb \cdot ? x . 10\,x = 101 \vdots$$
$$Hc \operatorname{Inq} Hb \operatorname{Fal} \; *$$

The question "whether?" has been formulated. For the sake of brevity we introduce a single word Utr (fL utrum $=$ whether) meaning "whether", by a behaviour rule:

$$* \, Hd \operatorname{Inq} Hd \vdots t t' \, Hb \operatorname{Inq} Hc : ? y \cdot y = . \operatorname{Iud} \operatorname{Etc} \cdot$$
$$\leftrightarrow \cdot t t' \, Hb \operatorname{Inq} Hc . \operatorname{Utr} \operatorname{Etc} \; *$$

Likewise we can introduce another abbreviation:

$$* \, ? = \operatorname{Etc} \; * \quad \text{instead of} \quad * \, ? y . y = \operatorname{Etc} \; *$$

and so on.

With the means now available we can stress once more that in "$Ha \operatorname{Inq} Hb \ldots$" Ha is the person who is speaking:

$$* \, Ha \operatorname{Inq} Hb \cdot ? x . t_1 t_2 \operatorname{Fit} x :$$
$$Hb \operatorname{Inq} Hc \cdot ? x . t_1 t_2 \operatorname{Fit} x :$$
$$Hc \operatorname{Inq} Hd \cdot ? x . t_1 t_2 \operatorname{Fit} x :$$
$$Hd \operatorname{Inq} Hc . t_1 t_2 \operatorname{Fit} \mathfrak{p} :$$
$$Hc \operatorname{Inq} Hb . t_1 t_2 \operatorname{Fit} \mathfrak{p} :$$
$$Hb \operatorname{Inq} Ha . t_1 t_2 \operatorname{Fit} \mathfrak{p} :$$
$$Ha \operatorname{Inq} Hb . \operatorname{Ben} \; *$$

$$* \, Ha \operatorname{Inq} Hb \cdot \operatorname{Utr} . 10001111 \in \operatorname{Pri} \cdot$$
$$Hb \operatorname{Inq} Hb : \lnot . 10 \operatorname{Div} \operatorname{Etc} \cdot \lnot 11 \operatorname{Div} \operatorname{Etc} \cdot \lnot . 101 \operatorname{Div} \operatorname{Etc} \cdot$$
$$\lnot 111 \operatorname{Div} \operatorname{Etc} \cdot 1011 \operatorname{Div} \operatorname{Etc} \cdot$$
$$Hb \operatorname{Inq} Ha . \notin \operatorname{Pri} \cdot$$
$$Ha \operatorname{Inq} Hb \operatorname{Ben} \; *$$

3 02 2. It is somewhat more difficult to treat the identity question, e.g. questions such as

"who is Caesar?", "what is an integer?".

It would be of no use replacing them by

"what does the name Caesar mean?"

"what does the word integer mean?"

At this stage we are not able to introduce a word that means "means", and moreover, it has been our policy not to define, but to describe.

Yet the difficulty is not as great as one might believe. A text such as

$\ast\ Ha \operatorname{Inq} Hb \cdot ?\, x \,.\, x = Hc :$
$\ ^{t_1}Hb \operatorname{Inq} Hc\, Hc\,^{t_1} :$
$\ Hb \operatorname{Inq} Ha \cdot\ ^{\curlyvee}x \,.\, t_1\, t_2\, Hb \operatorname{Inq} x\, Hc :$
$\ Ha \operatorname{Inq} Hb \operatorname{Ben} \ast$

is clear enough. Caesar (Hc) has been introduced to Ha in accordance with etiquette. If Caesar is absent, the dialogue runs as follows:

$\ast\ Ha \operatorname{Inq} Hb \cdot ?\, x \,.\, x = Hc \cdot$
$\ Hb \operatorname{Inq} Ha \cdot Hc = :\ ^{\curlyvee}x \cdot t_1\, t_2\, x \operatorname{Inq} x \,.\, \text{Alea est iacta} \cdot$
$\ Ha \operatorname{Inq} Hb \operatorname{Ben} \ast$

Here $t_1\, t_2$ is a substitute for some time-interval just before the crossing of the Rubicon. As we cannot name any other action but speaking, we have had to identify Caesar by one of his sayings — at the moment all his sayings are meaningless, so it does not matter which we choose.

The same pattern may be used for the question "what is an integer?"

$\ast\ Ha \operatorname{Inq} Hb \cdot ?\, x \,.\, x = \operatorname{Int} \cdot$
$\ Hb \operatorname{Inq} Ha \cdot \operatorname{Exg} \cdot 0 \in \operatorname{Int}. 1 \in \operatorname{Int}. 1011 \in \operatorname{Int}. -101 \in \operatorname{Int}. \text{Etc}.$
$\quad 11/101 \notin \operatorname{Int}. \sqrt{10} \notin \operatorname{Int}. \text{Etc}:$
$\quad x \in \operatorname{Int}. \leftrightarrow. x = 0 \smile 1 \smile -1 \smile 10 \smile -10 \smile \text{Etc} \cdot$
$\quad 0 \in A. \wedge \wedge x \cdot x \in A. \rightarrow : x+1. \cup. x-1 \cdot \mathsf{C} A \cdot \rightarrow. \operatorname{Int} \mathsf{C} A \cdot$
$\ Ha \operatorname{Inq} Hb \operatorname{Ben} \ast$

Exg (fL exempli gratia) means "for instance".

3 02 3. Our solution cannot be invalidated by the remark that the person named Hb could have answered 'Int = Int'. For then Ha should have replied 'Mal' as he would have done if ? $= 10+10$ had been answered $10+10 = 10+10$.

Another remark is more serious: How to formulate the identity question with respect to a thing Z for which the equation $x=Z$ is void of sense? We may not argue that such events are impossible. It is true that if Lincos is endowed with a rational phonetic system, there will be a clear distinction between words Z for which the equation $x=Z$ has some meaning, and words Z for which it is meaningless. So one should argue that the person named Ha would never ask e.g.

$$\text{'? } x . x = \in \text{'}, \text{ but rather '? } \ulcorner a . A \urcorner . a \in \text{A'}.$$

Yet this is not correct. We cannot forbid anyone asking meaningless questions. Moreover, the question '? $x . x = \in$' is not at all meaningless, though it contains a syntactically bad part. Questions like this can be the adequate expression of some behaviour, e.g. they can have some didactical importance. By means of a syntactically wrong question Ha will be able to examine whether Hb has understood Lincos syntaxis.

$*\ Ha\,\mathrm{Inq}\,Hb\,.\,?=\ \in\ \vdots$
   $Hb\,\mathrm{Inq}\,Ha\,\vdots\ \neg\,\vdots\ \vee x\,\cdot x=\in\,.\in\mathrm{Prp}\,\cdot\,\vee\ulcorner a.A\urcorner.a\in A\,\cdot$
     $\mathrm{Exg}\cdot 1\in\mathrm{Num}\,.\sqrt{10}\in\mathrm{Rea}\,.\mathrm{Dur}\in\star\frown\star\,.\mathrm{Etc}\vdots$
   $Ha\,\mathrm{Inq}\,Hb\,\mathrm{Ben}\,*$

If Hb had restricted himself to the negative part of his answer, Ha would have shown his dissatisfaction by saying 'Mal'. For clearly Ha had asked for definite information about the use of '$\in$', and such information is not given in the first part of the answer.

3 02 4. Other interrogative sentential forms are treated later on.

3 03 1. We send four separate time-signals of length $\mathfrak{h}$ on the frequency $\mathfrak{a}$ within the period from t_0 to t_1. The frequency $\mathfrak{a}$ is different from any frequency used for verbal texts (and from $\mathfrak{w}$). As usual the written image of a time-signal is a horizontal line.

$*\ {}^{t_0}\overline{Ha}\,\mathrm{Inq}\,Ha\,1^{\mathrm{um}}\,.\,\overline{Ha}\,\mathrm{Inq}\,Ha\,10^{\mathrm{um}}\,.\,\overline{Ha}\,\mathrm{Inq}\,Ha\,11^{\mathrm{um}}\,.$
   $\overline{Ha}\,\mathrm{Inq}\,Ha\,100^{\mathrm{um}\,t_1}\vdots$
   $Hb\,\mathrm{Inq}\,Ha\,\vdots\ ?=\,\cdot\,\mathrm{Car}\,{}^{\wedge}u\,:$
     $\mathrm{Fre}\,u=\mathfrak{a}\,.\,\wedge\,\cdot\,\mathrm{Ini}\,u\,.\,\mathrm{Pst}\,t_0\,\cdot\,\wedge\,\cdot\,\mathrm{Fin}\,u\,.\,\mathrm{Ant}\,t_1\vdots$
   $Ha\,\mathrm{Inq}\,Hb\,100\vdots$
   $Hb\,\mathrm{Inq}\,Hb\,\vdots\,t_0\,t_1\,Ha\,\mathrm{Enu}\,\vdots$
     $?=\,\cdot\,\mathrm{Car}\,{}^{\wedge}u\,:\mathrm{Fre}\,u=\mathfrak{a}\,.\,\wedge\,\cdot\,\mathrm{Ini}\,u\,.\,\mathrm{Pst}\,t_0\,\cdot\,\wedge\,\cdot\,\mathrm{Fin}\,u\,.\,\mathrm{Ant}\,t_1\vdots$
   $Hb\,\mathrm{Inq}\,Ha\,\mathrm{Ben}\,*$

The word written Enu (fL enumerat = counts) means "counts". We shall introduce many words that mean special kinds of speak-

ing. In some respects they will behave syntactically like 'Inq', though some of them will not have a personal object. The direct object of 'Enu' (the thing counted) will have to have the form of an interrogative sentence. Later on we shall adduce the reasons of this rule.

Ha uses ordinal numbers when counting. This seems to be the simplest way of introducing ordinal numbers. The termination of the ordinals is written ^{um}, in analogy with the Latin ordinals.

3 03 2. $*$ *Ha* Inq *Hb* ⫶? = $^\bullet$Car: $^\wedge x \cdot x \in$ Pri. $\wedge . x \leqq 100000$⫶

^{t_0}*Hb* Inq *Hb* $^\bullet$ 10 . 1^{um} : 11 . 10^{um} : 101 . 11^{um} : 111 . 100^{um} :

1011 . 101^{um} : 1101 . 110^{um} : 10001 . 111^{um} : 10011 . 1000^{um} :

10111 . 1001^{um} : 11101 . 1010^{um} : 11111 . 1011^{um}⫶^{t_1}

Hb Inq *Ha* 1011⫶

Ha Inq *Ha* ⫶t_0 t_1 *Ha* Enu⫶? = $^\bullet$Car: $^\wedge x \cdot x \in$ Pri. $\wedge . x \leqq 100000$⫶

Ha Inq *Hb* Ben $*$

3 03 3. $*$ *Ha* Inq *Hb* ⫶? = $^\bullet$Car: $^\wedge x \cdot x \in$ Pri. $\wedge . x \leqq 100000$⫶

^{t_1}*Hb* Enu . ? = Etc^{t_2}⫶

Hb Inq *Hb* 1011⫶

Ha Inq *Ha* $\cdot$ t_1 t_2 *Hb* Enu . ? = Etc⫶

Ha Inq *Hb* Ben $*$

The last program text is not simply a paraphrase of the last but one. It would be, if the whole program is considered as a report spoken by an anonymous person ("the sender"). If it is felt as a play (and the receiver should do this), the second text is rather a "parapraxis" of the first. In the first play the actor plays counting by pronouncing a sequence of numerals following the rules of counting. In the second play he shows another (not specified) behaviour that is characteristic of counting. For instance he whispers, he moves his fingers or a pencil, and so on. In the first paragraphs of the present chapter we announced that sometimes one and the same play will be performed in different versions.

The last text serves to free 't_1 t_2 *Hb* Enu . ? = ...' from the 't_1 t_2' that makes it a proposition. We now dispose of '*Hb* Enu . ? = ...' which is an event like '*Ha* Inq *Hb* ...'. This transformation can be stressed by saying

$*$ t_1 t_2 *Hb* Enu . ? = Etc . $\leftrightarrow$: t_1 t_2 Fit $\cdot$ *Hb* Enu . ? = Etc $*$

Numerous verbs will be treated in the same way.

By means of 'Etc' parts of the program text have been abbreviated. Gradually we shall form habits of omitting parts that are more or less evident:

$* \, Ha \, \mathrm{Inq} \, Hb \mathbin{:} ? = {}^\cdot \mathrm{Car} \mathbin{:} {}^\frown x \mathbin{:} x \in \mathrm{Pri} . \wedge . x \leqq 100000 \mathbf{:}$
$^{\mathrm{t_1}} Hb \, \mathrm{Enu^{t_2}} \mathbf{:}$
$Hb \, \mathrm{Inq} \, Ha \, 1011 \mathbf{:}$
$Ha \, \mathrm{Inq} \, Ha . \mathrm{t_1} \, \mathrm{t_2} \, Hb \, \mathrm{Enu} \mathbf{:}$
$Ha \, \mathrm{Inq} \, Hb \, \mathrm{Ben} \, *$

3 03 4. We send time-signals as in 3 03 1, but of different lengths, $\mathfrak{h}_1$, $\mathfrak{h}_2$, $\mathfrak{h}_3$, $\mathfrak{h}_4$.

$* \, \mathrm{t_0} \, \underline{\qquad\qquad} \, \mathrm{t_1} \mathbf{:}$
$\quad Ha \, \mathrm{Inq} \, Hb \mathbin{:} ? = {}^\cdot {}^\vee u . \mathrm{Dur} \, u \, \vdots$
$\qquad {}^\frown u \cdot \mathrm{Fre} \, u = \mathfrak{a} . \wedge . u \, \mathrm{Pst} \, \mathrm{t_0} . \wedge . u \, \mathrm{Ant} \, \mathrm{t_1} \mathbf{:}$
$\quad Hb \, \mathrm{Inq} \, Ha \mathbf{:} A = \mathbin{:} {}^\frown u \cdot \mathrm{Fre} \, u = \mathfrak{a} . \wedge . u \, \mathrm{Pst} \, \mathrm{t_0} . \wedge . u \, \mathrm{Ant} \, \mathrm{t_1} {}^\cdot$
$\qquad \rightarrow \mathbin{:} \mathrm{Dur} \cdot 1^{\mathrm{um}} {}^\vee u . u \in A \mathbin{:} = \mathfrak{h}_1 {}^\cdot \wedge {}^\cdot \mathrm{Dur} \cdot 10^{\mathrm{um}} {}^\vee u . u \in A \mathbin{:} = \mathfrak{h}_2 {}^\cdot$
$\qquad \wedge {}^\cdot \mathrm{Dur} \cdot 11^{\mathrm{um}} {}^\vee u . u \in A \mathbin{:} = \mathfrak{h}_3 {}^\cdot \wedge {}^\cdot \mathrm{Dur} \cdot \mathrm{Ult}^{\mathrm{um}} {}^\vee u . u \in A \mathbin{:} = \mathfrak{h}_4 \mathbf{:}$
$\quad Ha \, \mathrm{Inq} \, Hb . ? = i^{\mathrm{um}} \mathbf{:}$
$\quad Hb \, \mathrm{Inq} \, Ha \mathbf{:}$
$\qquad i^{\mathrm{um}} {}^\vee u . u \in A \cdot \mathrm{Ant} \cdot (i+1)^{\mathrm{um}} {}^\vee u . u \in A \mathbin{:} \wedge {}^\cdot i^{\mathrm{um}} {}^\vee u . u \in A \mathbf{:}$
$\qquad = {}^\cdot \mathrm{Ult}^{\mathrm{um}} {}^\vee u . u \in A \smile \mathbin{:} \mathrm{Ant} \cdot \mathrm{Ult}^{\mathrm{um}} {}^\vee u . u \in A \, *$

The syntaxis of 'n^{um}' has been stated in a more definite way[1]). $\mathrm{Ult}^{\mathrm{um}}$ (*fL* ultimum = last) meaning "last", is a fairly important word. Its importance does not yet appear from this text.

The text may be repeated in abbreviated form:

$* \, \mathrm{t_0} \, \underline{\qquad\quad} \, \mathrm{t_1} \mathbf{:}$
$\quad Ha \, \mathrm{Inq} \, Hb \mathbin{:} ? = {}^\vee u . \mathrm{Dur} \, u \, \vdots \, {}^\frown u . \mathrm{Fre} \, u = \mathfrak{a} . \wedge . u \, \mathrm{Pst} \, \mathrm{t_0} . \wedge . u \, \mathrm{Ant} \, \mathrm{t_1} \mathbf{:}$
$\quad Hb \, \mathrm{Inq} \, Ha \, {}^\cdot \mathrm{Dur} . 1^{\mathrm{um}} {}^\vee u \cdot = \mathfrak{h}_1 \mathbin{:} \wedge \mathbin{:} \mathrm{Dur} . 10^{\mathrm{um}} {}^\vee u \cdot = \mathfrak{h}_2 \mathbf{:}$
$\quad \wedge \mathbin{:} \mathrm{Dur} . 11^{\mathrm{um}} {}^\vee u \cdot = \mathfrak{h}_3 \mathbin{:} \wedge \mathbin{:} \mathrm{Dur} . 100^{\mathrm{um}} {}^\vee u \cdot = \mathfrak{h}_4 \, *$

and so on.

$* \, \underline{\qquad\quad} \, \mathrm{t_1} \quad \mathrm{t_2}$
$\quad Ha \, \mathrm{Inq} \, Hb \mathbin{:} ? \, v \, {}^\cdot v \, \mathrm{Fit} \mathbin{:} \mathrm{Ult}^{\mathrm{um}} {}^\vee u \cdot \mathrm{Fre} \, u = \mathfrak{a} . \wedge . u \, \mathrm{Ant} \, \mathrm{t_3} \, {}^{\mathrm{t_4}} \mathbf{:}$
$\quad Hb \, \mathrm{Inq} \, Ha \cdot \mathrm{t_1} \, \mathrm{t_2} \, \mathrm{Fit} . \mathrm{Ult}^{\mathrm{um}} {}^\vee u \, *$

3 04 1. $* \, Ha \, \mathrm{Inq} \, Hb . ? = 101101 + 11101 \mathbf{:}$
$\quad ^{\mathrm{t_1}} Hb \, \mathrm{Inq} \, Hb \cdot 1 + 1 = 10 . 1 + 0 + 0 = 1 . 1 + 1 = 10 .$
$\qquad 1 + 1 + 1 = 11 . 1 + 0 + 1 = 10 . 1 + 1 + 0 = 10^{\mathrm{t_2}} \mathbf{:}$
$\quad Hb \, \mathrm{Inq} \, Ha \, 1001010 \mathbf{:}$
$\quad Ha \, \mathrm{Inq} \, Ha \cdot \mathrm{t_1} \, \mathrm{t_2} \, Hb \, \mathrm{Cpu} . ? = 101101 + 11101 \mathbf{:}$
$\quad Ha \, \mathrm{Inq} \, Hb \, \mathrm{Ben} \, *$

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[1]) Later on we mostly write '$i^{\mathrm{um}} \, u$' instead of '$i^{\mathrm{um}} \, {}^\vee u$'. The second notation reflects an earlier version of Lincos. On a few places the notation has not been changed.

The word written Cpu (fL computat = computes) means "computes". Like 'Enu' it is constructed with an interrogative sub-sentence.

3 04 2. $^{\#} Ha \operatorname{Inq} Hb . ? = 11010 \times 1011 \cdot$
     $^{t_1} Hb \operatorname{Inq} Hb . 11010 + 110100 + 11010000 = 100011110 \cdot {}^{t_2}$
     $Hb \operatorname{Inq} Ha \, 100011110 \cdot$
     $Ha \operatorname{Inq} Ha . t_1 t_2 Hb \operatorname{Cpu} \cdot$
     $Ha \operatorname{Inq} Hb \operatorname{Ben} {}^{\#}$

3 04 3. $^{\#} Ha \operatorname{Inq} Hb \cdot ? x . x^{10} - 1011 \, x + 11110 = 0 :$
     $^{t_1} Hb \operatorname{Inq} Hb \cdot$
       $(x^{10} - 10 \times 101,1 \, x + 101,1^{10}) + (11110 - 11110,01) = 0 :$
       $\operatorname{Erg} . (x - 101,1)^{10} = 0,01 : \operatorname{Erg} \cdot x - 101,1 = . 0,1 \smallsmile - 0,1^{t_2} :$
     $Hb \operatorname{Inq} Ha . x = 101 \smallsmile 110 :$
     $Ha \operatorname{Inq} Ha . t_1 t_2 Hb \operatorname{Cpu} :$
     $Ha \operatorname{Inq} Hb \operatorname{Ben} {}^{\#}$

3 04 4. $^{\#} Ha \operatorname{Inq} Hb \cdot ? x . x^{10} - 1011 \, x + 11110 = 0 :$
     $^{t_1} Hb \operatorname{Cpu} \cdot ? x . x^{10} - 1011 \, x + 11110 = 0^{t_2} :$
     $Hb \operatorname{Inq} Ha . 101 \smallsmile 110 :$
     $Ha \operatorname{Inq} Ha . t_1 t_2 Hb \operatorname{Cpu} :$
     $Ha \operatorname{Inq} Hb \operatorname{Ben} {}^{\#}$

The '$t_1 t_2 Hb \operatorname{Cpu} \ldots$' has been freed from the '$t_1 t_2$'.

3 04 5. In 3 04 3 a word written Erg (fL ergo = thus) and meaning "thus" has been used. It is an important word that assimilates Lincos to common mathematical language. A person named Hc might give comment on the behaviour of Hb with respect to 'Erg'.

   $^{\#} Hc \operatorname{Inq} Hc \colon t_3 t_4 Hb \operatorname{Inq} u . \operatorname{Erg} q \cdot \rightarrow {}^{\cdot} \vee \ulcorner t_1 . t_2 . p \urcorner : t_2 < t_3 . \wedge . p \rightarrow q .$
     $\wedge \cdot t_1 t_2 Hb \operatorname{Inq} u . p {}^{\#}$

3 04 6. During, say, 8 seconds, from t_1 to t_2, four short time signals a second will be given on the frequency $\mathfrak{a}$; $t_2 = t_1 + \operatorname{Sec} 1000$.

   $^{\#} Ha \operatorname{Inq} Hb \colon ? = {}^{\cdot} \operatorname{Car} \colon {}^{\curvearrowright} u \cdot \operatorname{Fre} u = \mathfrak{a} . \wedge . t_1 \operatorname{Ant} u \operatorname{Ant} t_2 \colon$
   $\operatorname{Ecc} {}_{\mid}$
   $t_1 \ldots\ldots\ldots\ldots t_2$
   $Hb \operatorname{Inq} Hb \colon \operatorname{Car} \colon {}^{\curvearrowright} u \cdot \operatorname{Fre} u = \mathfrak{a} .$
     $\wedge . t_1 \leqq t \operatorname{Ant} u \operatorname{Ant} (t + \operatorname{Sec} 1) \leqq t_2 {}^{\cdot} = 100 \colon$
     $\wedge . t_2 - t_1 = \operatorname{Sec} 1000 . \wedge . 100 \times 1000 = 100000 \colon \operatorname{Erg} . 100000^{t_3} {}_{\mid}$
   $Hb \operatorname{Inq} Ha \, 100000 {}_{\mid}$
   $Ha \operatorname{Inq} Ha {}^{\cdot} \neg \cdot t_1 t_3 Hb \operatorname{Enu} . ? = . \operatorname{Car} \operatorname{Etc} :$
     $\operatorname{Sed} t_1 t_3 Hb \operatorname{Cpu} \cdot ? = . \operatorname{Car} \operatorname{Etc} {}_{\mid}$
   $Ha \operatorname{Inq} Hb \operatorname{Ben} {}^{\#}$

'Sed' (fL sed = but) means "but", but in another sense than 'Tan' (see 3 01 3).

'Sed' is used when one proposition is contrasted with another that is contradictory to the first one. In a more specific way it can serve to single out that part of the two propositions that embodies the opposition between them:

$$\# \: Ha \: \mathrm{Inq} \: Ha \cdot \neg . \: t_1 \: t_3 \: Hb \: \mathrm{Enu} \cdot \mathrm{Sed} . \: t_1 \: t_3 \: Hb \: \mathrm{Cpu} : ? = . \: \mathrm{Car} \: \mathrm{Etc} \: \#$$

or even

$$\# \: Ha \: \mathrm{Inq} \: Ha \cdot t_1 \: t_3 \: Hb \cdot \neg \: \mathrm{Enu} . \: \mathrm{Sed} \: \mathrm{Cpu} : ? = . \: \mathrm{Car} \: \mathrm{Etc} \: \#$$

The original sentence has been transformed as though we had applied a law of distributivity. Of course word groups such as $\neg$ Enu may only be used in contexts like the preceding program text. It makes no sense at all to interpret '$\neg$ Enu' as the negation of an *event*. Negation can only be applied to *propositions*.

The aforesaid transformations can be exhibited by repeated talks and confirmed by behaviour rules.

3 05 1.　$\# \: Ha \: \mathrm{Inq} \: Hb \: \vdots \: \mathrm{Utr} \: \wedge n \cdot n \in \mathrm{Num} . \to : \vee p \cdot p \in Pri . \wedge . p > n \:$ ı

$^{t_1}Hb \: \mathrm{Inq} \: Ha \: \vdots$

$\quad n \in \mathrm{Num} . \to : \vee a . \mathrm{Put} \: m . a = 10 \times 11 \times 10 \times 101 \times \mathrm{Etc} \times n + 1 \: \cdot$

$\quad \vee a \cdot \mathrm{Put} \: p \cdot a \in \mathrm{Pri} . \wedge . a \: \mathrm{Div} \: m \: \cdot$

$\quad \wedge x : x \leq n . \to \cdot \neg . x \: \mathrm{Div} \: m \: \cdot$

$\quad \mathrm{Sed} . p \: \mathrm{Div} \: m \: \cdot \mathrm{Erg} : \wedge x \cdot x \leq n . \to . p \neq x \: \cdot$

$\quad \mathrm{Erg} . p > n \: \cdot$

$\quad \mathrm{Erg} \cdot p \in \mathrm{Pri} . \wedge . p > n^{t_1} \:$ ı

$Ha \: \mathrm{Inq} \: Ha \: \vdots$

$\quad t_1 \: t_2 \: Hb \: \mathrm{Cpu} \cdot ? \: ^\curlyvee f : f \in . \mathrm{Num} \frown \mathrm{Pri} \cdot \wedge \cdot \wedge n . fn > n \: \vdots$

$\quad t_1 \: t_2 \: Hb \: \mathrm{Dem} \: \vdots \: \wedge n \cdot n \in \mathrm{Num} . \to : \vee p \cdot p \in \mathrm{Pri} . \wedge . p > n \: \#$

The word written Dem (fL demonstrat = proves) means "proves". It behaves like 'Inq', though the indirect object is lacking. The direct object is a proposition.

3 05 2.　The word written Put (fL puta = namely) means "namely". The use of 'Put' like that of 'Erg' tends to assimilate Lincos to colloquial mathematics. In mathematical texts it is even not unusual to designate the instance for an existential assertion with the same letter as that designating the existentially bound variable, without any comment. So a proof of the inverse of Cauchy's convergence criterion may run as follows:

"$\lim a_n = a$. Thus given an $\varepsilon > 0$, there is such an n_0 that

$$|a - a_n| < \tfrac{1}{2} \varepsilon \: \text{ for all } \: n > n_0.$$

Hence for all $m > n_0$, and $n > n_0$

$$|a_m - a_n| \leqq |a - a_m| + |a - a_n| < \tfrac{1}{2}\varepsilon + \tfrac{1}{2}\varepsilon = \varepsilon".$$

In the second sentence of this proof n_0 was a bound variable, in the third sentence it is something like a constant.

By means of 'Put' we try to legalize this habit.

The syntactic features of 'Put' will become clear, when new variables occur inside the working sphere of the '$\vee$' to which 'Put' belongs. In

$$\vee x . \operatorname{Put} a . \operatorname{Etc}$$

or

$$\vee x . \operatorname{Put} x . \operatorname{Etc}$$

the variable x is bound to a constant value. So it will not depend on not-explicitly-bound variables that might occur inside the working sphere of '$\vee x$'. This means that these variables must be supposed to be bound by generalizing inside, not outside. Of course x may depend on data which have occurred before.

3 05 3.   The same goal can be reached by the use of a word written 'Pon' ($\int L$ pone = put), meaning "put". One can introduce it by repeating the last program text with

$\ast \operatorname{Pon} m = \operatorname{Etc} \ast$ instead of $\ast \vee a . \operatorname{Put} m . a = \operatorname{Etc} \ast$

Later on 'Pon' will be used in a generalized sense.

3 06 1. $\ast Ha \operatorname{Inq} Hb \cdot ? x . t_1 t_2 x \operatorname{Inq} Ha \mathfrak{p} :$
      $Hb \operatorname{Inq} Ha \cdot \neg . t_1 t_2 Hb \operatorname{Inq} Ha \mathfrak{p} :$
      $^{t_s} Hb \operatorname{Inq} Hc \cdot \operatorname{Utr} . t_1 t_2 Hc \operatorname{Inq} Ha \mathfrak{p} :$
      $Hc \operatorname{Inq} Hb \operatorname{Fal} :$
      $Hb \operatorname{Inq} Hd \cdot \operatorname{Utr} . t_1 t_2 Hd \operatorname{Inq} Ha \mathfrak{p} :$
      $Hd \operatorname{Inq} Hb \operatorname{Fal} :$
      $Hb \operatorname{Inq} He \cdot \operatorname{Utr} . t_1 t_2 He \operatorname{Inq} Ha \mathfrak{p} :$
      $He \operatorname{Inq} Hb \operatorname{Fal} :$
      $Hb \operatorname{Inq} Hf \cdot \operatorname{Utr} . t_1 t_2 Hf \operatorname{Inq} Ha \mathfrak{p} :$
      $^{t_s} Hf \operatorname{Inq} Hb \operatorname{Ver}^{t_s} :$
      $Hb \operatorname{Inq} Ha . t_1 t_2 Hf \operatorname{Inq} Ha \mathfrak{p} :$
      $Ha \operatorname{Inq} Ha \cdot t_3 t_5 Hb \operatorname{Pet} \cdot ? x . t_1 t_2 x \operatorname{Inq} Ha \mathfrak{p} :$
         $\wedge : t_4 t_5 Hb \operatorname{Rep} \cdot ? x . t_1 t_2 x \operatorname{Inq} Ha \mathfrak{p} :$
      $Ha \operatorname{Inq} Hb \operatorname{Ben} \ast$

The words written Pet and Rep ($\int L$ petit = seeks and reperit = finds) mean "seeks" and "finds" respectively. Syntactically they behave as 'Enu' and 'Cpu'.

3 06 2. ∗ Ha Inq Hb : ? . 1^{um} $^{\curlyvee}x \cdot x \in$ Pri . $\wedge$. $x > 10^{1001}$ ˙

^{t_1}Hb Inq Hb ˙ 11 Div $(10^{1001} + 1)$. 10 Div $(10^{1001} + 10)$.

101 Div $(10^{1001} + 11)$. 10 Div $(10^{1001} + 100)$.

1011 Div $(10^{1001} + 101)$. 10 Div $(10^{1001} + 110)$.

11 Div $(10^{1001} + 111)$. 10 Div $(10^{1001} + 1000)^{t_2}$.

$(10^{1001} + 1001) \in$ Pri^{t_3} ˙

Hb Inq Ha . $1000001001 \in$ Pri ˙

Ha Inq Ha : $t_1 t_3$ Hb Pet $\frown$ $t_3 t_4$ Hb Rep ˙ ? . 1^{um} $^{\curlyvee}x$ Etc ˙

Ha Inq Hb Ben ∗

3 06 3. ∗ Ha Inq Hb : ? . 1^{um} $^{\curlyvee}x \cdot x \in$ Pri . $\wedge$. $x > 10^{1001}$ ˙

^{t_1}Hb Pet . ^{t_2} Hb Rep . ^{t_3}Hb Inq Ha $10^{1001} + 1001$ ˙

Ha Inq Ha ˙ $t_1 t_2$ Hb Pet . $\wedge$. $t_2 t_3$ Hb Rep ˙

Ha Inq Hb Ben ∗

This is now a well-known pattern of transformation (see 3 03 2–3 03 3, and 3 04 3–3 04 4).

Examples of 'Pet $\frown$. $\neg$ Rep' can easily be added.

3 07 1. $\mathfrak{p}$, $\mathfrak{q}$, $\mathfrak{r}$ will be metatextual substitutes for three different noises.

∗ $^{t_1}\mathfrak{p}$. $\mathfrak{q}$. $\mathfrak{r}$ ⁝ ^{t_2}

Ha Inq Hb ˙ ? u . $t_1 t_2$ Fit u ⁝

Hb Inq Ha ˙

$\mathfrak{p}$ Par u . $\wedge$. $\mathfrak{q}$ Par u . $\wedge$. $\mathfrak{r}$ Par u . $\wedge$: u Uni ˙ $\mathfrak{p}$. $\mathfrak{q}$. $\mathfrak{r}$ ⁝

Ha Inq Ha : $t_3 t_4$ Hb Anl ˙ $^{\curlyvee}u$. $t_1 t_2$ Fit u ⁝

Ha Inq Hb : ? $x \cdot x$. 1^{um} Par . u ⁝

^{t_1}Hb Inq Ha ⁝ $\mathfrak{p}$ Ant $\mathfrak{q}$ Ant $\mathfrak{r}$ ˙ Erg : $\mathfrak{p}$. 1^{um} Par . u ˙ $\wedge$ ˙ $\mathfrak{q}$. 10^{um} Par u ˙

$\wedge$ ˙ $\mathfrak{r}$. Ult^{um} Par . u^{t_2} ⁝

Ha Inq Ha : $t_3 t_4$ $\frown$ $t_5 t_6$. Hb Anl ˙ $^{\curlyvee}u$. $t_1 t_2$ Fit u ⁝

Ha Inq Hb Ben ∗

The word written Anl (fL analyzat) has a somewhat complex meaning. It is used for the act of structurally describing a structure of events (later on also with respect to spatial structures). Other translations might be "analyzes", 'describes", "sketches".

The word written Par (fL pars = part) means "part". It should not be confused with the set-theory "inclusion", '$\subset$'.

The word written Uni (fL unio = union) means "union". It should not be confused with the set-theory "union", '$\cup$'.

We use these words if structural parts and their structural union are considered.

More contexts are needed for a better understanding of these new words.

Later on we shall also use a slightly different syntax of Uni.

We shall say

$$u \, \text{Uni} \, A$$

instead of

$$u \, \text{Uni} \, z_1 \, z_2 \, \text{Etc} \, z_n ,$$

if

$$A = \ulcorner z_1 \urcorner \cup \ulcorner z_2 \urcorner \cup \text{Etc} \cup \ulcorner z_n \urcorner .$$

This can be stated by a behaviour rule.

3 07 2. $\#^{t_1} \mathfrak{p} . \mathfrak{p} . \mathfrak{p}^{t_2}$:

     $Ha \, \text{Inq} \, Hb \cdot ? \, u . t_1 \, t_2 \, \text{Fit} \, u$:

     $^{t_3} Hb \, \text{Inq} \, Ha \cdot t_1 \, t_2 \, 11^{es} \, \text{Fit} \, \mathfrak{p}^{t_4}$:

     $Ha \, \text{Inq} \, Ha \cdot t_3 \, t_4 \, Hb \, \text{Anl} \cdot ^{\curlyvee} u . t_1 \, t_2 \, \text{Fit} \, u$:

         $\mathfrak{p} . 11^{es} \, \text{Par} . u : \wedge \cdot u \, \text{Uni} . 11^{es} \, \mathfrak{p}$:

     $Ha \, \text{Inq} \, Hb \, \text{Ben} \, \#$

Multiplicative numerals ("n times") are formed with a termination ^{es} (from the Latin termination of almost all multiplicative numerals).

3 07 3. $\#^{t_1} \mathfrak{p} . \mathfrak{q} . \mathfrak{p} . \mathfrak{r} . \mathfrak{p} . \mathfrak{p} . \mathfrak{r} . \mathfrak{q} . \mathfrak{r}^{t_2}$:

     $Ha \, \text{Inq} \, Hb \cdot ? \, u . t_1 \, t_2 \, \text{Fit} \, u$:

     $^{t_3} Hb \, \text{Inq} \, Ha \cdot t_1 \, t_2 . 100^{es} \, \text{Fit} \, \mathfrak{p} \frown 10^{es} \, \text{Fit} \, \mathfrak{q} \frown 11^{es} \, \text{Fit} \, \mathfrak{r}^{t_4}$:

     $Ha \, \text{Inq} \, Ha : t_3 \, t_4 \, Hb \, \text{Anl} \cdot ^{\curlyvee} u . t_1 \, t_2 \, \text{Fit} \, u$:

     $Ha \, \text{Inq} \, Hb \cdot \text{Utr} : t_1 \, t_2 \, \text{Fit} \cdot \mathfrak{p} . \mathfrak{p} . \mathfrak{p} . \mathfrak{p} . \mathfrak{q} . \mathfrak{q} . \mathfrak{r} . \mathfrak{r} . \mathfrak{r}$:

     $Hb \, \text{Inq} \, Ha \cdot \text{Fal} : \text{Sed} \cdot \mathfrak{p} . \mathfrak{q} . \mathfrak{p} . \mathfrak{r} . \mathfrak{p} . \mathfrak{p} . \mathfrak{r} . \mathfrak{p} . \mathfrak{r}$:

     $Ha \, \text{Inq} \, Hb \, \text{Ben} \, \#$

3 07 4. $^{t_1} \overline{Ha} \, \text{Inq} \, Ha \, 1^{um} . \overline{Ha} \, \text{Inq} \, Ha \, 10^{um}$.

     $\overline{Ha} \, \text{Inq} \, Ha \, 11^{um} . \overline{Ha} \, \text{Inq} \, Ha \, 100^{um} : ^{t_2}$

     $Hb \, \text{Inq} \, Ha : ? \, x \cdot t_1 \, t_2 . x^{es} \, \text{Fit} \, \overline{}$:

     $Ha \, \text{Inq} \, Hb \, 100^{es}$:

     $Hc \, \text{Inq} \, Hb \cdot ? \, y . t_1 \, t_2 \, Ha \, \text{Inq} \, Ha \, y$:

     $^{t_3} Hb \, \text{Inq} \, Hc \cdot t_1 \, t_2 \, Ha \, \text{Enu} : ? \, x \cdot t_1 \, t_2 . x^{es} \, \text{Fit} \, \overline{} : ^{t_4}$

     $Hc \, \text{Inq} \, Hc : t_3 \, t_4 \, Hb \, \text{Anl} \cdot ^{\curlyvee} z . t_1 \, t_2 \, Ha \, \text{Inq} \, Ha \, z$:

     $Hc \, \text{Inq} \, Hb : ? \, ^{\curlyvee} u \cdot t_1 \, t_2 \, Ha \, \text{Enu} : ? \, x \cdot t_1 \, t_2 . x^{es} \, \text{Fit} \, \overline{} : \text{Mod} \, u$:

     $Hb \, \text{Inq} \, Hc : t_1 \, t_2 \, Ha \, \text{Enu} \cdot \text{Etc}$ ·

         $\text{Mod} : Ha \, \text{Inq} \, Ha \cdot 1^{um} . 10^{um} . 11^{um} . 100^{um}$:

     $Hc \, \text{Inq} \, Hb \, \text{Ben} \, \#$

According to this dialogue designating an act by 'Enu' may be a structural description of this act.

Furthermore we have learned the word written Mod (fL modo = in the manner). It is to conjoin a verb with a sentence that states in what manner the action described by the verb takes place (a

"modal subsentence"). We have also learned how to translate the modal interrogative adverb "how"? by '? u .Etc .Mod u'.

3 07 5.   \# $^{t_1}Ha\,\overline{\mathrm{Inq}}\,\bar{H}a\,100^{\mathrm{um}}$. $Ha\,\overline{\mathrm{Inq}}\,\bar{H}a\,1000^{\mathrm{um}}$.
$Ha\,\overline{\mathrm{Inq}}\,\bar{H}a\,1100^{\mathrm{um}\,t_1}$ ·
$Hb\,\mathrm{Inq}\,Ha$: ? x · $t_1\,t_2$. $x^{es}\,\mathrm{Fit}\,\overline{}$ ·
$Ha\,\mathrm{Inq}\,Hb\,1100^{es}$ ·
$^{t_1}Hb\,\mathrm{Inq}\,Hc$: $t_1\,t_2\,Ha\,\mathrm{Enu}$ · ? x . $t_1\,t_2\,x^{es}\,\mathrm{Fit}\,\overline{}{}^{t_1}$ ·
$Hc\,\mathrm{Inq}\,Hb$: ? $^{\curlyvee}u$ · $t_1\,t_2\,Ha\,\mathrm{Enu}$. Etc . Mod u ·
$^{t_1}Hb\,\mathrm{Inq}\,Hc$: Mod · Inq 100^{um} . 1000^{um} . $1100^{\mathrm{um}\,t_6}$ ·
$Hc\,\mathrm{Inq}\,Hc$: $t_3\,t_4\,Hb\,\mathrm{Anl}\,^{\curlyvee}v$. $t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,v$ ·
$\wedge$ · $t_5\,t_6\,Hb\,\mathrm{Anl}\,^{\curlyvee}v$. $t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,v$ \#

3 07 6.   \# $Ha\,\mathrm{Inq}\,Hb$. ? $= 11010 \times 1011$ ·
$^{t_1}Hb\,\mathrm{Inq}\,Ha$. $11010 + 110100 + 11010000 = 100011110^{t_2}$ ·
$Ha\,\mathrm{Inq}\,Ha$ · $t_1\,t_2\,Hb\,\mathrm{Cpu}$. ? $= 11010 \times 1011^{t_3}$ ·
$Hc\,\mathrm{Inq}\,Ha$: $t_2\,t_3\,Ha\,\mathrm{Anl}$ · $^{\curlyvee}u$. $t_1\,t_2\,Hb\,\mathrm{Inq}\,Ha\,u$ \#

3 07 7.   \# $Ha\,\mathrm{Inq}\,Hb$. ? $= 111111^{10}$ ·
$^{t_1}Hb\,\mathrm{Cpu}$. ? $= 111111^{10}$ · ^{t_2}
$Hb\,\mathrm{Inq}\,Ha\,111110000001$ ·
$Ha\,\mathrm{Inq}\,Hb$ · ? $^{\curlyvee}u$. $Hb\,\mathrm{Cpu}\,\mathrm{Mod}\,u$ ·
$^{t_1}Hb\,\mathrm{Inq}\,Ha$:
    Mod · $(x+y)^{10} = x^{10} + 10\,x\,y + y^{10}$. $\wedge$. $x = 10^{110}$. $\wedge$. $y = -1$ · ^{t_4}
$Hc\,\mathrm{Inq}\,Ha$: $t_2\,t_3\,Ha\,\mathrm{Anl}$ · $^{\curlyvee}u$. $t_1\,t_2\,Hb\,\mathrm{Inq}\,Ha\,u$ \#

3 07 8.   \# $Ha\,\mathrm{Inq}\,Hb$: Utr ‼ $\wedge n$ · $n \in \mathrm{Num}$. $\rightarrow$: $\vee p$ · $p \in \mathrm{Pri}$. $\wedge$. $p > n$ ‼
$^{t_1}Hb\,\mathrm{Dem}$ ‼ $\wedge n$ · $n \in \mathrm{Num}$. $\rightarrow$: $\vee p$ · $p \in \mathrm{Pri}$. $\wedge$. $p > n^{t_2}$ ‼
$Hc\,\mathrm{Inq}\,Ha$: ? $^{\curlyvee}u$ · $^{t_1 t_2}Hb\,\mathrm{Dem}$. Etc . Mod u ‼
$^{t_1}Ha\,\mathrm{Inq}\,Hc$ ‼
Mod · $Hb\,\mathrm{Pet}$: ? $^{\curlyvee}p$ · $p \in \mathrm{Pri}$. $\wedge$. $p\,\mathrm{Div}\,(10 \times 11 \times 100 \times \mathrm{Etc} \times n + 1)^{t_4}$ ‼
$Hc\,\mathrm{Inq}\,Hc$: $t_3\,t_4\,Ha\,\mathrm{Anl}$ · $^{\curlyvee}v$. $t_1\,t_2\,Hb\,\mathrm{Inq}\,Hb\,v$ \#

Likewise one should exhibit the use of 'Anl' with respect to 'Pet' and 'Rep'.

3 07 9.   \# $^{t_1}\mathfrak{p}$. $\mathfrak{q}$. $\mathfrak{r}^{t_4}$ ‼
$Ha\,\mathrm{Inq}\,Hb$ · ? u . $t_1\,t_2\,\mathrm{Fit}\,u$ ‼
$^{t_1}Hb\,\mathrm{Inq}\,Ha$ ·
$\mathfrak{p}\,\mathrm{Par}\,u$. $\wedge$. $\mathfrak{q}\,\mathrm{Par}\,u$. $\wedge$. $\mathfrak{r}\,\mathrm{Par}\,u$. $\wedge$: $u\,\mathrm{Uni}$ · $\mathfrak{p}$. $\mathfrak{q}$. $\mathfrak{r}$: ^{t_4}
$Ha\,\mathrm{Inq}\,Hc$: $t_3\,t_4\,Hb\,\mathrm{Anl}$ · $^{\curlyvee}u$. $t_1\,t_2\,\mathrm{Fit}\,u$ ‼
$Hc\,\mathrm{Inq}\,Ha$ · ? $^{\curlyvee}v$: $t_3\,t_4\,Hb\,\mathrm{Anl}$ · $^{\curlyvee}u$. $t_1\,t_2\,\mathrm{Fit}\,u$ · Mod v ‼
$^{t_1}Ha\,\mathrm{Inq}\,Hc$ · Mod $Hb\,\mathrm{Pet}$: ? $= $ · $^{\curlywedge}z$. $z\,\mathrm{Par}\,u$ ‼ ^{t_4}
$Hc\,\mathrm{Inq}\,Hc$: $t_5\,t_6\,Ha\,\mathrm{Anl}$ · $^{\curlyvee}w$. $t_3\,t_4\,Hb\,\mathrm{Inq}\,Ha\,w$ \#

3 08 1.   We now build nouns (nomina actionis) from verbal stems by means of a termination ^{io} (a Latin termination for building nomina actionis).

> Remark: No importance should be attached to the words verb and noun in this connection. They do not describe Lincos categories whether syntactical or semantical. They refer rather to conventional translations of Lincos texts into common English. It would be possible to translate Inq, Enu, Cpu, Dem, Pet, Rep, Anl not by verbal forms, but by nomina agentis, e.g. '$t_1 t_2 Ha$ Cpu x' by "Ha is the computer of x during $t_1 t_2$".

3 08 2. $\#\,^{t_1}\overline{Ha}\,\mathrm{Inq}\,Ha\,1^{\mathrm{um}}\,.\,\overline{Ha}\,\mathrm{Inq}\,Ha\,10^{\mathrm{um}}\,.\,\overline{Ha}\,\mathrm{Inq}\,Ha\,11^{\mathrm{um}}\,.$
     $\overline{Ha}\,\mathrm{Inq}\,Ha\,100^{\mathrm{um}}\,^{t_2}\,\vdots$
     $Hb\,\mathrm{Inq}\,Hc\,\vdots$
        $^\curlyvee y\,.\,t_1 t_2\,Ha\,\mathrm{Inq}\,Ha\,y\cdot\in\,{}^{\cdot}\,Ha\,\mathrm{Enu}^{io}:?\,z\cdot t_1 t_2\,.\,z^{es}\,\mathrm{Fit}\,\overline{}\,\vdots$
     $Hc\,\mathrm{Inq}\,Hc\,{}^{\cdot}\,tt'\,Ha\,\mathrm{Enu}\,?\,\mathrm{Etc}\,.$
        $\rightarrow\,:\,{}^\curlyvee u\,.\,tt'\,\mathrm{Fit}\,u\cdot\in\,{}^{\cdot}\,Ha\,\mathrm{Enu}^{io}\,.\,?\,\mathrm{Etc}\cdot\subset\cdot\mathrm{Enu}^{io}\,.\,?\,\mathrm{Etc}\,\#$

3 08 3. $\#\,Ha\,\mathrm{Inq}\,Hb\,.\,?=\mathrm{Cpu}^{io}\,\vdots$
     $Hb\,\mathrm{Inq}\,Ha\,\vdots\,\mathrm{Exg}\,{}^{\cdot}\,tt'\,Hc\,\mathrm{Inq}\,Hd\,.\,?\,\mathrm{Etc}\cdot\wedge\cdot t't''\,Hd\,\mathrm{Cpu}\,.\,?\,\mathrm{Etc}\,:$
        $\rightarrow\,:\,{}^\curlyvee u\,.\,t't''\,Hd\,\mathrm{Inq}\,Hd\,u\cdot\in\cdot\mathrm{Cpu}^{io}\,.\,?\,\mathrm{Etc}\,\#$

Likewise one should introduce Dem^{io}, Pet^{io}, Rep^{io}, Anl^{io}.

3 09 1. $\#\,^{t_1}\mathfrak{p}\,.\,\mathfrak{q}\,.\,\mathfrak{r}^{t_4}\,\vdots$
     $Ha\,\mathrm{Inq}\,Hb\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\vdots$
     $^{t_3}Hb\,\mathrm{Inq}\,Ha\,\vdots\,\mathrm{Car}\,{}^{\cdot}\,^\curlyvee y:y\cdot\mathrm{Par}\,.\,^\curlyvee x\,\mathrm{Etc}\,\vdots\,=11^{t_3}\,\vdots$
     $Ha\,\mathrm{Inq}\,Hb\,\vdots\,^\curlyvee z\,.\,t_3\,t_4\,Hb\,\mathrm{Inq}\,Ha\cdot\in\mathrm{Anl}^{io}\,.\,^\curlyvee x\,\mathrm{Etc}\,:$
        $\mathrm{Tan}\,\vdots\,?\,^\curlyvee u\,{}^{\cdot}\,u\in\mathrm{Anl}^{io}\,^\curlyvee x\,\mathrm{Etc}\,.\wedge:\mathrm{Err}\,u<\,{}^{\cdot}\,\mathrm{Err}\,.\,^\curlyvee z\,\mathrm{Etc}\,\vdots$
     $^{t_4}Hb\,\mathrm{Inq}\,Ha\,{}^{\cdot}\,\mathfrak{p}\,\mathrm{Par}\,^\curlyvee x\,.\,\wedge\,.\,\mathfrak{q}\,\mathrm{Par}\,^\curlyvee x\,.\,\wedge\,.\,\mathfrak{r}\,\mathrm{Par}\,^\curlyvee x\,.$
        $\wedge\,\vdots\,^\curlyvee x\,\mathrm{Etc}\,.\,\mathrm{Uni}\cdot\mathfrak{p}\,.\,\mathfrak{q}\,.\,\mathfrak{r}\,\vdots\,^{t_4}$
     $Ha\,\mathrm{Inq}\,Hb\,\vdots\,^\curlyvee u\,.\,t_5\,t_6\,Hb\,\mathrm{Inq}\,Ha\cdot\in\mathrm{Anl}^{io}\,^\curlyvee x\,\mathrm{Etc}\,:$
        $\wedge\,\vdots\,\mathrm{Err}\,.\,^\curlyvee u\,\mathrm{Etc}\cdot<\,{}^{\cdot}\,\mathrm{Err}\,.\,^\curlyvee z\,\mathrm{Etc}\,:$
        $\mathrm{Tan}\,\vdots\,?\,^\curlyvee v\,{}^{\cdot}\,v\in\cdot\mathrm{Anl}^{io}\,.\,^\curlyvee x\,\mathrm{Etc}\,\vdots\,\wedge:\mathrm{Err}\,v\,.<\,{}^{\cdot}\,\mathrm{Err}\,.\,^\curlyvee u\,\mathrm{Etc}\,\vdots$
     $Hb\,\mathrm{Inq}\,Ha\,{}^{\cdot}\,\mathfrak{p}\,\mathrm{Par}\,^\curlyvee x\,.\,\wedge\,.\,\mathfrak{q}\,\mathrm{Par}\,^\curlyvee x\,.\,\wedge\,.\,\mathfrak{r}\,\mathrm{Par}\,^\curlyvee x\,.$
        $\wedge\,\vdots\,^\curlyvee x\,\mathrm{Etc}\,.\,\mathrm{Uni}\cdot\mathfrak{p}\,.\,\mathfrak{q}\,.\,\mathfrak{r}:\wedge\,.\,\mathfrak{p}\,\mathrm{Ant}\,\mathfrak{q}\,\mathrm{Ant}\,\mathfrak{r}\,\vdots$
     $Ha\,\mathrm{Inq}\,Hb\,\mathrm{Ben}\,\#$

The word written Err ($\textit{fL}$ error = error) is used when the exactness of analyses (Anl^{io}) or other statements are to be compared. The three analyses to be found in the program text go in the

direction of increasing exactness and decreasing inexactness (Err).
Later on we shall meet with 'Err' in more numerical contexts.

Of course the use of Err does not at all imply the existence of
a simple ordering of statements with respect to exactness.

One should repeat former analyses in order to compare their
exactness by means of 'Err'.

3 10 1. $\ast\ ^{t_1}Ha\,\text{Inq}\,Hb\cdot\mathord{?}\,x\,.\,1010\,x = 101^{t_2}\,\vdots$
$Hb\,\text{Inq}\,Ha\,.\,1/10^{t_3}\,\vdots$
$Hc\,\text{Inq}\,Hd\,.\,t_2\,t_3\,Hb\,\text{Rsp}\,Ha\,\vdots$
$Hd\,\text{Inq}\,Hc\,.\,\mathord{?} = \text{Rsp}\,\vdots$
$Hc\,\text{Inq}\,Hd\,\vdots\,\text{Exg}\,\vdots\,tt'\,Ha\,\text{Inq}\,Hb\cdot\mathord{?}\,x\,.\,\text{Etc}\,x\,\text{Etc}\,\vdots$
$\wedge\cdot t't''\,Hb\,\text{Inq}\,Ha\,.\,\text{Etc}\,1/10\,\text{Etc}\,\cdot\to.\,t't''\,Hb\,\text{Rsp}\,Ha\,\ast$

The word written Rsp (fL respondit $=$ answers) means (the verb)
"answers".

3 10 2. $\ast\ ^{t_1}Ha\,\text{Inq}\,Hb\cdot\mathord{?}\,x\,.\,1010\,x = 101^{t_2}\,\colon$
$Hb\,\text{Inq}\,Ha\,101/1010^{t_3}\,\colon$
$Hc\,\text{Inq}\,Hd\,.\,t_2\,t_3\,Hb\,\text{Rsp}\,Ha\,\ast$

3 10 3. $\ast\ ^{t_1}Ha\,\text{Inq}\,Hb\cdot\mathord{?}\,x\,.\,1010\,x = 101^{t_2}\,\colon$
$Hb\,\text{Inq}\,Ha\,10^{t_3}\,\colon$
$Hc\,\text{Inq}\,Hd\cdot t_2\,t_3\,Hb\,\text{Rsp}\,Ha\,.\,\text{Tan Fal}\,\ast$

3 10 4. $\ast\ ^{t_1}Ha\,\text{Inq}\,Hb\,.\,1011111 \in \text{Pri}\,\cdot$
$^{t_2}Hb\,\text{Inq}\,Ha\,\text{Fal}\ ^{t_3}\cdot$
$Hc\,\text{Inq}\,Hd\,.\,t_2\,t_3\,Hb\,\text{Rsp}\,Ha\,\ast$

3 11 1. $\ast\ ^{t_1}\mathfrak{p}\,.\,\mathfrak{q}\,.\,\mathfrak{r}^{t_2}\,\vdots$
$Ha\,\text{Inq}\,Hb\cdot\mathord{?}\,x\,.\,t_1\,t_2\,\text{Fit}\,x\,\vdots$
$^{t_3}Hb\,\text{Inq}\,Ha\,\vdots\,t_1\,t_2\,\text{Fit}\cdot\mathfrak{p}'\,.\,\mathfrak{q}\,.\,\mathfrak{r}^{t_4}\,\vdots$
$Ha\,\text{Inq}\,Hb\,\cdot\,\text{Fal}\,\colon t_3\,t_4\,Hb\,\text{Mut}\cdot\lor x\,.\,t_1\,t_2\,\text{Fit}\,x\cdot\text{Mod}\,.\,\mathfrak{p}'\,\text{Ilo}\,\mathfrak{p}\,\ast$

The word written Mut (fL mutat $=$ changes) means "changes".
The word written Ilo (fL in loco $=$ instead of) means "instead of".

3 11 2. $\ast\ ^{t_1}\mathfrak{p}\,.\,\mathfrak{q}\,.\,\mathfrak{r}^{t_2}\,\vdots$
$Ha\,\text{Inq}\,Hb\cdot\mathord{?}\,x\,.\,t_1\,t_2\,\text{Fit}\,x\,\vdots$
$^{t_3}Hb\,\text{Inq}\,Ha\cdot\mathfrak{q}\,.\,\mathfrak{r}^{t_4}\,\vdots$
$Ha\,\text{Inq}\,Hb\,\cdot\,\text{Fal}\,\colon t_3\,t_4\,Hb\,\text{Mut}\cdot\lor x\,.\,t_1\,t_2\,\text{Fit}\,x\cdot\text{Mod}\,.\,\text{Sin}\,\mathfrak{p}\,\ast$

The word written Sin (fL sine $=$ without) means "omits".

3 11 3. $\ast\,{}^{t_1}\mathfrak{p}\,.\,\mathfrak{q}\,.\,\mathfrak{r}^{t_2}\,\colon$

$\quad$ Ha Inq $Hb\cdot?\,x\,.\,t_1\,t_2$ Fit $x\,\colon$

$\quad$ ${}^{t_1}Hb$ Inq $Ha\cdot\mathfrak{p}\,.\,\hat{s}\,.\,\mathfrak{q}\,.\,\mathfrak{r}^{t_4}$

$\quad$ Ha Inq $Hb\cdot$ Fal$\colon t_3\,t_4\,Hb$ Mut$\cdot{}^{\curlyvee}x\,.\,t_1\,t_2$ Fit $x\cdot$ Mod. Add $\hat{s}\,\ast$

$\quad$ The word written Add (fL addit $=$ adds) means "adds".

3 11 4. $\ast\,Ha$ Inq $Hb\,.\,? = 111111^{10}\,\colon$

$\quad$ Hb Cpu. $? = 111111^{10}\,\colon$

$\quad$ Hb Inq $Ha\,111110000001\,\colon$

$\quad$ Ha Inq $Hb\cdot?\,u\,.\,Hb$ Cpu Mod $u\,\colon$

$\quad$ Hb Inq $Ha\cdot$ Mod$\colon Ha$ Mut$\cdot(x+y)^{10} = x^{10} + 10\,x\,y + y^{10}\cdot$

$\quad\quad$ Mod. 1000000 Ilo $x\frown(-1)$ Ilo $y\,\colon$

$\quad$ Ha Inq Hb Ben $\ast$

3 12 0. After the interrogative "how?" we now treat "why?". Its Lincos equivalent will be written Cur (fL cur $=$ why). Answers to questions 'Cur Etc' may begin with a word written Qia (fL quia $=$ because). The latter word will also be used independently, as an inverse 'Erg' as it were. In this case it introduces an answer to a non-explicit question.

$\quad$ The polysignificance of "why" and its equivalents in other languages is a commonplace. One should at least distinguish "whys" asking for the reason, for the cause, and for the aim. If one tries to examine these notions more closely one runs the risk of getting entangled in a muddle of relations. From experiences of this kind one ultimately learns that in some cases it may be better to avoid such analyses or at least to delay them, if progress is to be made. An interlinguistic homonymy can have deep roots in human behaviour. Perhaps the questioner uses a polysemantic "why" because he often does not know which kind of "because" he is asking for. I think it is a useful rule to discriminate homonyms where this is easy, and to justify them when not. In the latter case one should invent syntactic rules that cover the polysemantic use of the word.

$\quad$ The question 'Cur' as we shall put it, will ask for a justification of the fact that some event or some series of events has happened within a larger series of events. These events may also be of a linguistic kind, i.e. occurrences of certain words or groups of words in a certain context. In any case the things to be justified will be mentioned by means of the word Cur.

3 12 1.　 ✳ Ha Inq Hb . $\sqrt{10} \notin$ Rat ⌐

　　　　Hb Inq Ha Cur ⌐

　　　　Ha Inq Hb ⌐ . Qia ⌐ Qqm . $\sqrt{10} \notin$ Rat · Pon . $\sqrt{10} \in$ Rat ⫶

　　　　　　Erg ⫶ ∨ ⌐$p . q$⌐ · Put ⌐$p . q$⌐ · $p \cup q$. ⊂ Num ·

　　　　　　∧ ⫶ ⌐ · 10 Div p . ∧ . 10 Div q ⫶ ∧ . $p/q = \sqrt{10}$ ⫶

　　　　　　Erg . $p^{10}/q^{10} = 10$ ⫶ Erg . $p^{10} = 10 \, q^{10}$ ⫶

　　　　　　Erg . 10 Div p ⫶ Erg . 100 Div p^{10} ⫶

　　　　　　Erg . 100 Div $(10 \, q^{10})$ ⫶ Erg . 10 Div q^{10} ⫶ Erg 10 Div q ⫶

　　　　　　Erg · 10 Div p . ∧ . 10 Div q . ∧ ⫶ ⌐ · 10 Div p . ∧ . 10 Div q ⫶

　　　　　　Erg ⫶ ∨s · $s \in$ Prp . ∧ . s ∧ . ⌐ s ⫶ s ∧ . ⌐ s · ∈ Fal ⫶

　　　　　　Erg · $\sqrt{10} \in$ Rat . ∈ Fal ⌐

　　　　Hb Inq Ha Ben ✳

The 'Cur' of this text asks for the reason.

The question, could also have been asked in the form 'Cur $\notin$' or 'Cur . $\notin$ Rat' or 'Cur . $\sqrt{10} \notin$ Rat' or 'Cur Ver', and so on. The talk should be repeated with these variations.

3 12 2.　 ✳ ^{t_0}Ha Inq Hb · $^{t_1} p \in$ Pri^{t_1} . ∧ · $a \cup b$. ⊂ Num · ∧ . p Div (ab) ⫶

　　　　　　→ · p Div a . ∨ . p Div b^{t_1} ⌐

　　　　Hb Inq Ha ⫶ Cur · $t_1 t_2$ Ha Inq Hb . $p \in$ Pri ⌐

　　　　Ha Inq Hb ⫶ Qia ·

　　　　　　110 Div (11×100) . ∧ ⫶ ⌐ · 110 Div 11 . ∨ . 110 Div 100 ⫶

　　　　　　tt' Ha Mut ⫶ ⌐x . $t_0 t_3$ Ha Inq Hb x ⫶ Mod · Sin . $p \in$ Pri ·

　　　　　　→ ⫶ ⌐y . tt' Ha Inq Hb y · ∈ Fal ⫶ Ncs · $t_1 t_2$ Ha Inq Hb . $p \in$ Pri ⌐

　　　　Hb Inq Ha ⌐ Cur ⫶ $t_1 t_2$ Ha Inq Hb . $p \in$ Pri ⫶

　　　　　　Sed ⫶ ⌐ · Etc · ∨ ⌐$q . n$⌐ · $q \in$ Pri . ∧ . $n \in$ Num . ∧ . $p = q^n$ ⌐

　　　　Ha Inq Hb ⫶ Qia ·

　　　　　　100 Div (10×110) . ∧ ⫶ ⌐ · 100 Div 10 . ∨ . 100 Div 110 ⫶

　　　　　　tt' Ha Mut ⫶ ⌐x . $t_0 t_3$ Ha Inq Hb x ⫶

　　　　　　Mod · ∨ ⌐$q . n$⌐ Etc . Ilo . $p \in$ Pri ·

　　　　　　→ ⫶ ⌐y . tt' Ha Inq Hb y · ∈ Fal ⫶ Ncs · $t_1 t_2$ Ha Inq Hb . $p \in$ Pri ⌐

　　　　Hb Inq Ha ⫶ Cur · ⌐ ⫶ $t_1 t_2$ Ha Inq Hb · $p \in$ Pri . ∨ . $p = 1$ ⌐

　　　　^{t_4}Ha Inq Hb ⫶ ⌐ ⫶ ∨s · $t_4 t_5$ Ha Inq Hb . Qia s ⫶

　　　　　　$^{t_5}tt'$ Ha Mut · ⌐x Etc · Mod ⫶ $p \in$ Pri . ∨ . $p = 1$ · Ilo . $p \in$ Pri ⫶

　　　　　　→ ⫶ ⌐y . tt' Ha Inq Hb y · ∈ Ver ⫶ ⌐ Ncs . Etc . $p \in$ Pri ✳

The 'Cur' of this talk asks what is aimed at by some act? *Ha* is giving a justification for having made some supposition.

We have incidentally learned the word written Ncs (*fL* necesse = necessary) that means "it is necessary that". The vocabulary of modality will be dealt with in detail later on.

3 12 3. $\# ^{t_1} \mathfrak{p}^{t_2}$:

  $Ha \operatorname{Inq} Hb \cdot ? \, x . t_1 t_2 \operatorname{Fit} x$:

  $^{t_1} Hb \operatorname{Inq} Ha \, \mathfrak{p}^{t_1}$:

  $Hc \operatorname{Inq} Hb \cdot \operatorname{Cur} . t_3 t_4 Hb \operatorname{Inq} Ha \, \mathfrak{p}$:

  $Hb \operatorname{Inq} Hc$: $\operatorname{Qia} \cdot t_2 t_3 Ha \operatorname{Inq} Hb \cdot ? \, x . t_1 t_2 \operatorname{Fit} x : \wedge . t_1 t_2 \operatorname{Fit} \mathfrak{p}$:

  $Hc \operatorname{Inq} Hb \operatorname{Ben} \#$

  This is a 'Cur' asking for something like the cause of an event.

3 13 0.   We now turn to the word written Sci (fL scit = knows). In many
languages the semantical domain of "to know" is claimed by two
words (or clans of words) though the division between them is
made rather on syntactical than on semantic grounds. In English
this distinction may be circumscribed as that between "to know
a fact" and "to know a thing", Latin "scire" and "noscere",
French "savoir" and "connaître", German "wissen" and "kennen".

  On the same grounds as given in the case of "why" we will stick
to one word 'Sci'. This use will be legalized by the rule: A sentence
beginning with 'Sci' is to be considered as an answer to a (not
necessarily explicit) question. Or more syntactically: the sentence
that depends on 'Sci' (the thing known) is to be interrogative
(perhaps elliptical).

  We now give a few examples of intermediate translations of
sentences to be translated into Lincos:

  "Ich weiss, wo mich der Schuh drückt" becomes "Ich weiss:
"wo drückt mich der Schuh?" "

  "Ich weiss nicht, was soll es bedeuten" becomes "Ich weiss
nicht: "was soll es bedeuten?" "

  "Kennst du das Land, wo die Zitronen blühn?" becomes
"Weisst du: "in welchem Lande blühen die Zitronen?" "

  "I know Caesar" becomes "there is an x with the properties
that x is Caesar and that I know: "is this x Caesar?" "

  "Eritis sicut Deus scientes bonum et malum" becomes "Like
God for all x you will know: "which is the value of x?" "

  "Scio nil scire" becomes "There is no p such that I know:
"is p true?" and I know: "is there any p such that I know:
"is p true?"?" "

  "Γνωτι σε αυτον" becomes "You shall know: "who are you?" "

  "I know a Roman citizen" may not be translated by "I know:
"who is a Roman citizen?" ", but "There is an x such that x is
a Roman citizen, and such that I know: "is x a Roman citizen?" ".

  The following is to justify the syntactical rule we have imposed
on 'Sci'.

Let us suppose we wish to say that Ha knows the solution x of $100\,x = 1010$. As '$Hb\cdot\text{Sci}\ 100\,x = 1010$' clearly does not fit, we could try $Hb\ \text{Sci}\cdot\ ^\curlyvee x\ .\ 100\,x = 1010$. But then it would not be clear what Hb is said to know. It may be the "bad" solution $1010/100$, i.e. the fact that $100\,x = 1010\ .\rightarrow.\ x = 1010/100$, whereas we wish to assert that he knows the good one, i.e. $100\,x = 1010\ .\rightarrow.\ x = 11/10$. Hb's ignorance may be still worse; perhaps he does not know any more than that $^\curlyvee x\ .\ 100\,x = 1010$ is a solution of $100\,x = 1010$, i.e. he can apply a syntactical pattern, but he has no notion of algebra.

Fortunately, we have created the means and established the habit of distinguishing between good and bad solutions. The approved method of asking for good solutions is to use the word written '?'. So '$Hb\ \text{Sci}\cdot\ ?\ x\ .\ 100\,x = 1010$' expresses fairly well what we wish to say. More fully 'Sci' then reads: "knows the good answer to ...".

As soon as a syntactical rule for 'Sci' is agreed on in one case, the best we can do is to stick to this rule generally. Thus we were obliged to transform "Ha knows that p" into the interrogative mode. The inverse way is not practicable. Indeed, we are not allowed to translate

$$Ha \text{ knows the solution of } ?x\ .\ 100\,x = 1010$$

by

$$Ha\,\text{Sci}\cdot 100\,x = 1010\ .\rightarrow.\ x = 101/10,$$

because this translation would convey not only the fact that Ha knows the solution, but also the solution itself. We must be able to say that Ha knows something without betraying the precise structure of the thing known.

It might have been useful to introduce a word that announces the x that is considered to be a good answer to a question. If this word is written as a note of exclamation, we should get, e.g.

$$!x\ .\ 100\,x = 1010 := 101/10.$$

Under this condition 'Sci' should be subjected to the syntactical rule that the thing known has to have the form of an answer to a (not necessarily explicit) question. We shall not treat the problems that would arise from this alternative convention, but we may mention that they are closely akin to some problems to be dealt with in 3 17 0.

As 'Sci' is always followed by an interrogative sentence one might enquire whether we cannot get rid of the question mark after 'Sci' by means of a fusion with the preceding 'Sci'. But this

is not possible, for we met with interrogative sentences without an introductory question mark, viz. those that start with 'Utr' or 'Cur'. Apart from this fortuitous circumstance, I would not favour dropping the question mark linking 'Sci' to the activity of solving problems.

As a final remark it is perhaps not superfluous to mention that our 'Sci' does not aim at a (more or less mysterious) mental state of the person knowing, but simply at a certain behaviour that will be exhibited by means of a number of examples.

As to the syntactic rules for 'Enu', 'Cpu' and others, the reader will understand why it has been useful to pattern them on that of 'Sci'.

It also goes without saying that 'Ha Sci p' is the written picture of an event, whereas '$t_1 t_2 Ha$ Sci p' is a report about an event. This report can also be written '$t_1 t_2$ Fit . Ha Sci p'.

3 13 1.   ⧉ ^{t_0}Ha Inq Hb · ? x . $100 x = 1010$:
            ^{t_1}Hb Inq Ha $101/10^{t_1}$:
            Ha Inq Hc : $t_1 t_2 Hb$ Sci · ? x . $100 x = 1010$:
            Hc Inq Ha . Cur Ver :
            Ha Inq Hc : Qia · $t_0 t_1 Ha$ Inq Hb · ? x . $100 x = 1010$:
                ∧ · $t_1 t_2 Hb$ Inq Ha . $101/10$:
            Hc Inq Ha Ben ⧉

        ⧉ ^{t_0}Ha Inq Hb · ? x . $100 x = 1010$:
            'Hb Inq Ha $1010/100$:^{t_1}
            Ha Inq Hc · ⌐ : $t_1 t_2 Hb$ Sci · ? x . $100 x = 1010$:
            Hc Inq Ha . Cur ⌐ :
            Ha Inq Hc · Qia : ⌐ . $t_1 t_2 Hb$ Inq Ha $101/10$ · Sed $1010/100$ ⧉

3 13 2.   From the last talks the receiver will not yet guess the true meaning of 'Sci'. 'Hb Sci p' could mean "Hb communicates the solution of p". Knowing has been recognized on the strength of symptoms that might appear to be too direct.

        ⧉ ^{t_0}Ha Inq Hb · ? x . $100 x = 1010$:
            ^{t_1}Hb Inq Ha $1010/100^{t_1}$:
            Ha Inq Hc : $t_1 t_2 Hb$ Sci · ? u . $t_0 t_1 Ha$ Inq Hb u :
            Hc Inq Ha Cur :
            ^{t_2}Ha Inq Hc : Qia · $t_0 t_1 Ha$ Inq Hb · ? x . $100 x = 1010$:
                ∧ : $^{\curlyvee}x$. $100 x = 1010$ · = · $^{\curlyvee}u$. $t_1 t_2 Hb$ Inq Ha u^{t_1} :
            Hd Inq He · ? v . $t_3 t_4 Ha$ Inq Hc v :
            He Inq Hd : $t_3 t_4 Ha$ Inq Hc : $t_1 t_2 Hb$ Rsp Ha ·
                Erg : $t_1 t_2 Hb$ Sci · ? u . $t_0 t_1 Ha$ Inq Hb u :

Ha Inq Hc : $t_1 t_2 Hb$ Sci $\cdot$? y : $t_0 t_1 y$ Inq $Hb \cdot$? x . $100 x = 1010$:

Hc Inq Ha Cur :

Ha Inq Hc : Qia . $t_1 t_2 Hb$ Rsp Ha :

   $t_1 t_2 Hb$ Sci $\cdot$? y : $t_0 t_1 Ha$ Inq y $\cdot$? Etc :

   $t_1 t_2 Hb$ Sci $\cdot$? z : $z Ha$ Inq $Hb \cdot$? Etc :

   $t_1 t_2 Hb$ Sci $\cdot$? w . $t_0 t_1$ Fit w #

3 13 3. # ^{t_0} p $^{t_1} \cdot$

  Ha Inq $Hb \cdot$? x . $t_0 t_1$ Fit $x \cdot$

  $^{t_1} Hb$ Inq Ha p$^{t_2} \cdot$

  Ha Inq Hc : $t_2 t_3 Hb$ Sci $\cdot$? y . y Fit p #

3 13 4. # $^{t_0} Ha$ Inq $Hb \cdot$? x . $100 x = 1010^{t_1}$ |

  Hc Inq $Hb \cdot$? u . $t_0 t_1$ Fit u^{t_2} |

  Hb Inq Ha $101/101$ |

  Hb Inq Hc : $\neg \cdot \vee u$. $t_0 t_1$ Fit u^{t_3} |

  Hc Inq Ha ι $t_2 t_3 Hb$ Sci :

   ? $y \cdot \vee \ulcorner t . t' \urcorner$: $tt' y$ Inq $Hb \cdot$? x . $100 x = 1010$:

   $\neg$: $t_2 t_3 Hb$ Sci $\cdot$? u . $t_0 t_1 Ha$ Inq $Hb u$:

   Qia $\cdot \neg$: $t_2 t_3 Hb$ Sci $\cdot$? z . $z Ha$ Inq Hb Etc :

   $\neg$: $t_2 t_3 Hb$ Sci $\cdot$? v : $t_1 t_2 v$ Inq $Hb \cdot$? u . $t_0 t_1$ Fit u :

   $t_2 t_3 Hb$ Sci : Utr $\cdot \vee \ulcorner w . t . t' \urcorner$: $tt' w$ Inq $Hb \cdot$? u . $t_0 t_1$ Fit u #

After these talks it is unlikely that the receiver could understand 'Sci' in the sense of "communicates the solution of . . .". Yet the evidence for 'Sci' may be still more indirect.

3 13 5. # ^{t_0} p . q . r$^{t_1} \cdot$

  Hb Inq Hb : p . q . r $\cdot$ p . q . r $\cdot$ p . q . r ^{t_2}

  Ha Inq Hc : $t_1 t_2 Hb$ Sci $\cdot$? x . $t_0 t_1$ Fit x #

$^{t_0} Ha$ Inq $Ha \cdot 1$. 100 . 1001 . 10000^{t_1} :

  Hb Inq Hb 11001^{t_2} :

  Ha Inq $Hc \cdot t_1 t_2 Hb$ Sci : ? $x \cdot \vee y$. $t_0 t_1 y$ Inq $y x$ #

$^{t_0} Ha$ Inq Hc . ? $= 10 \times 10 \cdot$

  Hc Inq Ha $101 \cdot$

  $^{t_1} Hb$ Inq Hc Fal$^{t_2} \cdot$

  Ha Inq Hc : $t_1 t_2 Hb$ Sci $\cdot$? x . $t_0 t_1$ Fit x #

$^{t_0} Ha$ Inq $Ha \cdot 10$. 11 . 101 . 111 . 1011^{t_1} :

  Hb Inq $Hb \cdot t_0 t_1 Ha$ Sci . ? $=$ Pri #

3 13 6. Knowledge may begin, last and end :

  $(t_1 < t_2 < t_3 < t_4 < t_5)$

  # Ha Inq Hb : ? 1^{um} $x \cdot x \in$ Pri . $\wedge$. $x > 10^{1001}$:

   $^{t_1} Hb$ Inq Ha 1000001001^{t_5} :

Ha Inq Ha . $t_4\,t_5\,Hb$ Sci Etc $\vdots$

Ha Inq Hb $\cdot$ Utr : $t_4\,t_5\,Hb$ Pet $\cdot$? $1^{um}\,x$. Etc $\vdots$

Hb Inq Ha Fal $\vdots$

Ha Inq $Hb\cdot$ Cur . $t_4\,t_5\,Hb$ Sci Etc $\vdots$

Hb Inq Ha $\cdot$ $t_1\,t_2\,Hb$ Pet . ? $1^{um}\,x$ Etc :

    $t_2\,t_3\,Hb$ Rep : Erg . $t_2\,t_3\,Hb$ Sci : Erg . $t_4\,t_5\,Hb$ Sci :

    $t < t' < t'' < t'''$. $\wedge$. $tt'\,Hb$ Sci Etc $\cdot \to$. $t''\,t'''\,Hb$ Sci Etc $\ast$

3 13 7. $(t_1 < t_2 < t_3 < t_4 < t_5 < t_6)$

    $\ast$ ^{t_1}Ha Inq $Hb\cdot$? x . $t_1\,t_2$ Fit x^{t_2} $\vdots$

    Hb Inq $Ha\cdot$ Cur . Ha Inq $\vdots$

    Ha Inq Hb $\vdots$ Qia $\cdot \neg$: $t_5\,t_6\,Ha$ Sci $\cdot$? x . $t_1\,t_2$ Fit x $\vdots$

    Hb Inq Ha $\cdot$ Tan $\cdot$ $t_3\,t_4\,Ha$ Inq Hb . $t_1\,t_2$ Fit a :

      Erg . $t_3\,t_4\,Ha$ Sci Etc $\vdots$

    Ha Inq Hb $\cdot$ Qqm . $t_3\,t_4\,Ha$ Sci : Tan $\cdot \neg$. $t_5\,t_6\,Ha$ Sci $\ast$

    $\ast$ Ha Inq $Hb\cdot$? x . $t_1\,t_2$ Fit x $\vdots$

    ^{t_1}Hb Inq Ha $\vdots$ Qqm $\cdot$ $t_3\,t_4\,Hb$ Sci : Utr $\cdot \vee p$. $t_1\,t_2\,Ha$ Inq $Hb\,p$ $\vdots$

    Tan : $\neg \cdot t_3\,t_4\,Hb$ Sci . ? $= p$ Etc^{t_2} $\vdots$

    Ha Inq Hb . $t_1\,t_2\,Ha$ Inq $Hb\,a$ $\vdots$

    ^{t_1}Hb Inq Ha $\cdot$ $\wedge\ulcorner t\,.\,t'\urcorner$: $t_5 < t < t'$. $\to \cdot tt'\,Hb$ Sci . ? x . $t_1\,t_2$ Fit x $\ast$

3 13 8. $\ast$ Ha Inq Hb : ? $1^{um}\,x\cdot x \in$ Pri . $\wedge$. $x > 10^{1001}$ $\vdots$

    ^{t_1}Hb Inq Ha $\cdot$ Qqm : $\neg$. $t_1\,t_2\,Hb$ Sci Etc $\cdot$ Tan . $t_3\,t_4\,Hb$ Sci^{t_2} $\vdots$

    Hb Pet : ? $1^{um}\,x\cdot x \in$ Pri . $\wedge$. $x > 10^{1001}$ ^{t_3} $\vdots$

    Hb Inq Ha 1000001001 : ^{t_4}

    Ha Inq Ha . $t_3\,t_4\,Hb$ Sci $\vdots$

    Ha Inq Hb . Cur Sci $\vdots$

    Hb Inq Ha : $t_2\,t_3\,Hb$ Pet $\frown$ Rep $\cdot$ Erg $t_3 < t' < t''$. $\to$. $t'\,t''\,Hb$ Sci $\ast$

3 13 9. $(t_1 < t_2 < t_3 < t_4 < t_5 < t_6)$

    $\ast$ ^{t_5}Ha Inq Hb $\cdot$ Utr : $t_5\,t_6\,Hb$ Sci $\cdot$? x . $t_1\,t_2$ Fit x^{t_6} $\vdots$

    Hb Inq Ha Ver $\vdots$

    Ha Inq Hb $\cdot$ Utr : $t_1\,t_2\,Hb$ Sci $\cdot$? x . $t_1\,t_2$ Fit x $\vdots$

    Hb Inq Ha $\vdots$ Fal $\cdot$ $t_3\,t_4\,Hc$ Inq Hb . $t_1\,t_2$ Fit a $\cdot$

    $t < t' < t'' < t''' < t^{(4)} < t^{(5)}$. $\wedge \cdot tt'$ Fit x . $\wedge \cdot t''\,t'''\,Hc$ Inq Hb . tt' Fit x :

      $\to$: $t^{(4)}\,t^{(5)}\,Hb$ Sci $\cdot$? y . tt' Fit y $\ast$

3 14 1. $\ast$ ^{t_3}Ha Inq Hb $\cdot$ Utr : $t_3\,t_4\,Hb$ Sci $\cdot$? x . $t_1\,t_2$ Fit x^{t_4} $\vdots$

    Hb Inq Ha Ver $\vdots$

    Ha Inq Hb : Utr $\cdot \vee \ulcorner t\,.\,t'\,.\,x\,.\,y\urcorner$: $t' < t_3$. $\wedge\,tt'\,y$ Inq Hb . $t_1\,t_2$ Fit x $\vdots$

    Hb Inq Ha Fal $\vdots$

    Ha Inq Hb $\cdot$ Utr : $t_1\,t_2\,Hb$ Sci $\cdot$? x . $t_1\,t_2$ Fit x $\vdots$

Hb Inq Ha Ver⁚

Ha Inq Hb Cur⁚

Hb Inq Ha꞉$t_1 t_2 Hb$ Ani $\cdot$? x . $t_1 t_2$ Fit x ＊

Hb knows that something has happened though nobody told him about it. He knew it as soon as it had happened. The reason he assigns is that he witnessed that event.

The word written Ani (fL animadvertit = perceives, is aware of) means "witnesses". Syntactically it behaves as 'Sci'. Of course 'Hb Ani Etc' is the written picture of an event that is reported on by '$t_1 t_2 Hb$ Ani Etc'.

$(t_1 < t_2 < t_3 < t_4 < t_5 < t_6)$

Ha Inq $Hb \cdot$? x . $t_1 t_2$ Fit x꞉

^{t_5}Hb Inq Ha . $t_1 t_2$ Fit $\mathfrak{a}^{t_6}$꞉

Ha Inq $Hb \cdot$ Utr . $t_1 t_2 Hb$ Ani Etc꞉

Hb Inq Ha Fal꞉

Ha Inq Hb꞉Utr $^{\cdot} \vee \ulcorner t . t' . y \urcorner$꞉$t_2 < t . \wedge \cdot tt' y$ Inq Hb . $t_1 t_2$ Fit $\mathfrak{a}$꞉

Hb Inq Ha Fal꞉

Ha Inq $Hb ^{\cdot}$ Cur꞉$t_5 t_6 Hb$ Sci $\cdot$? x . $t_1 t_2$ Fit x꞉

Hb Inq Ha꞉$t_3 t_4 Hc$ Inq Hd . $t_1 t_2$ Fit $\mathfrak{a} \cdot \wedge \cdot t_3 t_4 Hb$ Ani . Utr $t_3 t_4$ Etc ＊

3 14 2.　　$(t_1 < t_2 < t_3)$

^{t_1}Ha Inq $Hb ^{\cdot}$ Utr꞉$t_3 t_4 Hb$ Sci $\cdot$? x . $t_1 t_2$ Fit x^{t_4} |

^{t_5}Hb Inq Ha ꞏ . Fal ꞏ Qia꞉$\neg$꞉$t_1 t_2$ Ani $\cdot$? x . $t_1 t_2$ Fit $x^{\cdot}$

　　$\wedge^{\cdot} \neg$꞉$\vee \ulcorner t . t' . x . y \urcorner \cdot tt' y$ Inq Hb . $t_1 t_2$ Fit x꞉

　　　Erg $\cdot \neg$. $t_3 t_4 Hb$ Sci^{t_6} |

Ha Inq $Hb ^{\cdot} \neg \vee$ Etc . $\in$ Fal꞉$\vee x \cdot t_7 t_8 Ha$ Inq Hb . $t_1 t_2$ Fit x |

Hb Inq Ha ꞏ $t'' t''' Hb$ Mut꞉$^{\curlyvee}z$. $t_5 t_6 Hb$ Inq $Ha z$꞉

　　Mod $^{\cdot} \vee \ulcorner t . t' . x . y \urcorner \cdot t' < t_5 . \wedge$. Etc꞉Ilo . $\vee \ulcorner t . t' . x . y \urcorner$ Etc꞉

　　$\rightarrow$꞉$^{\curlyvee}z$. $t'' t''' Hb$ Inq $Ha z \cdot \in$ Ver |

^{t_9}Ha Inq Hb . $t_1 t_2$ Fit $\mathfrak{a}^{t_8}$ ＊

3 14 3.　　$(t_1 < t_2 < t_3 < t_4 < t_5)$

＊ ^{t_5}Ha Inq $Hb ^{\cdot}$ Utr꞉$t_5 t_6 Hb$ Sci $\cdot$? x . $t_1 t_2$ Fit x^{t_6}꞉

Hb Inq Ha Ver꞉

Ha Inq $Hb \cdot$ Utr . $t_1 t_2 Hb$ Ani Etc꞉

Hb Inq $Ha ^{\cdot}$ Fal꞉Sed $\cdot t_3 t_4 Hc$ Inq Ha . $t_1 t_2$ Fit $\mathfrak{a}$꞉

Ha Inq $Hb ^{\cdot}$ Cur꞉$t_3 t_4 Hc$ Sci $\cdot$? x . $t_1 t_2$ Fit x꞉

^{t_7}Hb Inq Ha꞉$\neg$. $t_7 t_8 Hb$ Sci Cur $\cdot$ Tan . $t_9 t_{10} Hb$ Sci Cur^{t_8}꞉

Hb Inq $Hc ^{\cdot}$ Utr꞉$t_1 t_2 Hc$ Ani $\cdot$? x . $t_1 t_2$ Fit x꞉

Hc Inq Hb Ver꞉

Hb Inq Ha . $t_1 t_2 Hc$ Ani Etc ＊

3 14 4. The talk 3 13 8 will be repeated. Then we continue.

   ✳ Ha Inq Hc : ? $1^{um} x \cdot x \in$ Pri $. \wedge . x > 10^{1001}$ |

    ^{t_1}Hc Inq Ha 1000001001^{t_2} |

    Ha Inq Hc . Cur Sci |

    Hc Inq Ha : $t_3 t_4 Hc$ Ani $\cdot$? y . $t_3 t_4 Hb$ Inq $Ha\, y$ |

    Ha Inq Hc ı : Mal ı . $t_5 t_6 Hb$ Sci $\cdot$? u . $t_3 t_4$ Fit u ı

      Tan ⫶ ⌐ ⫶ $t_5 t_6 Hb$ Sci ⫶ Utr ˙

      $\vee$ ⌜$t . t'$⌝ : $t < t' < t_3$. $\wedge \cdot tt' Ha$ Inq Hb . ? $1^{um} x$ Etc |

    Ha Inq Ha ˙ $\wedge$: $t_1 t_2 Hc$ Ani $\cdot$? u . $t_1 t_2$ Fit u |

    Ha Inq Hc ⫶ Mal ⫶ ⌐ ˙ $t_5 t_6 Hc$ Sci ⫶ Utr ⌃u . $t_3 t_4$ Fit u ˙ $\in$ Ben |

    Hc Inq Ha ˙ $\wedge$: $t_2 t_3 Hc$ Ani $\cdot$? u . $t_2 t_3$ Fit u : $\wedge$: ⌃u . $t_2 t_3$ Fit u ˙ $\in$ Ben |

    Ha Inq Hc Ben ✳

3 14 5.   ($t_1 < t_2 < t_3$)

   ✳ Ha Inq $Hb \cdot$? x . $t_1 t_2 x$ Inq Hb p |

    ^{t_1}Hb Inq Ha ı . ⌐ . $t_3 t_4 Hb$ Sci ı

      Qia ⫶ $t_1 t_2 Hb$ Ani : ? $y \cdot \vee x$. $t_1 t_2 x$ Inq $Hb\, y$ ˙

      Tan ⫶ ⌐ ˙ Ani $\cdot$? $x \cdot \vee y$. $t_1 t_2 x$ Inq $Hb\, y^{t_4}$ ✳

   ✳ Ha Inq $Hb \cdot$? x . $t_1 t_2$ Fit x |

    Hb Inq Ha ı ⌐ Sci ⫶ Qqm ˙ $t_1 t_2 Hb$ Ani : Utr $\cdot \vee x$. $t_1 t_2$ Fit x :

      Tan ˙ ⌐ : Ani $\cdot$? x . $t_1 t_2$ Fit x ✳

3 14 6. $^{t_1}\overline{Ha}$ Inq Ha 1^{um} . Ha Inq Ha 10^{um} .

    $\overline{Ha}$ Inq Ha 11^{um} . $\overline{Ha}$ Inq Ha $100^{um\ t_2}$ ⫶

    Hb Inq Hc ˙ $t_1 t_2 Ha$ Ani : ? $x \cdot$ Fre $x = a$. $\wedge$. t_1 Ant x Ant t_2 ⫶

    Hc Inq Hb . Cur Sci ⫶

    Hb Inq Hc : Qia $\cdot$ $t_1 t_2 Ha$ Enu . ? Etc ✳

3 14 7. ✳ ^{t_1}p . q . r^{t_2} ⫶

    Ha Inq Ha : p . q . r $\cdot$ p . q . r $\cdot$ p . q . r^{t_1} ⫶

    Hb Inq Hc : $t_1 t_2 Ha$ Ani $\cdot$? x . $t_1 t_2$ Fit x ⫶

    Hc Inq Hb Cur Sci ⫶

    Hb Inq Hc ⫶ Qia ⫶ $t_2 t_3 Ha$ Sci $\cdot$

      ? x . $t_1 t_2$ Fit x : $\wedge$ ˙ ⌐ : $\vee$ ⌜$t . t' . x . y$⌝ $\cdot tt' y$ Inq Ha . $t_1 t_2$ Fit x ✳

3 14 8. ✳ Ha Inq $Hb \cdot$? x . $t_1 t_2 Hc$ Inq $Hd\, x$ ⫶

    Hb Inq Ha . $t_1 t_2 Hc$ Inq Hd Fal ⫶

    Ha Inq Hb ˙ Utr : $t_1 t_2 Hb$ Ani $\cdot$ Utr . $t_1 t_2 Hc$ Inq Hd Fal ⫶

    Hb Inq $Ha \cdot$ ⌐ . $t_1 t_2 Hb$ Ani Etc ⫶

    ^{t_1}Ha Inq Hb ⫶ Utr ˙ $\vee$ ⌜$t . t' . x$⌝ : $t_2 < t < t' < t_3$.

      $\wedge \cdot tt' x$ Inq Hb . $t_1 t_2 Hc$ Inq Hd Fal ⫶

    Hb Inq $Ha \cdot$ ⌐ . $\vee$ ⌜$t . t' . x$⌝ Etc ⫶

    Ha Inq Hb . Cur Sci ⫶

Hb Inq Ha ˙ Qia · $t_0 t_1$ Hd Inq Hc . $10 \times 10 = 101$:
　　Erg . $t_1 t_2$ Hc Inq Hd Fal :
　　Erg . $t_1 t_2$ Hc Inq Hd Fal : tt' Hd Inq $Hc\,y$. ∧ . $y \in$ Fal ·
　　→ · ∨t'' . $t' t''$ Hc Inq Hd Fal ✳

Knowledge about events by inference.

3 14 9.　　($t_0 < t_1 < t_2 < t_3 < t_4$)

　　✳ Ha Inq Hb · ? x . $t_1 t_2$ Fit x ╷
　　Hb Inq Ha : ∨y · $t_1 t_2$ Hd Inq $Hc\,y$. ∧ . $y \in$ Fal ╷
　　Ha Inq Hb · Utr . $t_1 t_2$ Hb Ani Etc ╷
　　Hb Inq Ha Fal ╷
　　^{L}Ha Inq Hb : Utr · ∨ ⌜t' . t'' . x . u . v⌝ : $t_2 < t' < t'' < t_4$.
　　　　∧ ˙ $t' t''$ Hb Ani : Utr · u Inq v . $t_1 t_2$ Fit x ╷
　　Hb Inq Ha ┇ Fal ┇ Sed ˙ $t_2 t_3$ Hb Ani · Utr . $t_2 t_3$ Hc Inq Hd Fal :
　　　　∧ . $t_2 t_3$ Hc Inq Hd Fal ┇ Erg : ∨y · $t_1 t_2$ Hd Inq $Hc\,y$. ∧ . $y \in$ Fal ╷
　　Hc Inq Hb ┇ Fal ┇ Sed ┇ $t_0 t_1$ Hc Inq Hd : Utr · $10^{1001} + 1$. ∈ Pri ˙
　　　　∧ . $t_1 t_2$ Hd Inq Hc Fal ✳

3 15 1.　　We repeat 3 07 6, and then we continue:

　　✳ Ha Inq Hd . ? $= 11010 \times 1011$ ┇
　　Hd Inq Ha 100011110 ┇
　　Ha Inq Hd ˙ Utr : ∨ ⌜t . t'⌝ · tt' Hd Ani . ? $= 11010 \times 1011$ ┇
　　Hd Inq Ha ˙ Fal : tt' Hd Ani . ? $= 11010 \times 1011$ · ∉ Prp ┇
　　Ha Inq Hd ˙ Utr : ∨ ⌜t . t'⌝ · tt' Hd Cpu . ? $=$ Etc ┇
　　Hd Inq Ha Fal ┇
　　Ha Inq Hd . Cur Sci ┇
　　Hd Inq Ha ˙ $t_1 t_2$ Hd Ani :
　　　　Utr · Hb Inq Ha . $11010 \times 1011 = 10001110$ ✳

There is no perception of *mathematical* truth.

3 15 2.　　✳ Ha Inq Hb · ? x . $t_1 t_2$ Fit x ╷
　　Hb Inq Ha . $t_1 t_2$ Fit a ╷
　　Ha Inq Hb ┇ ? ⌜t . t'⌝ : tt' Hb Sci · ? x . $t_1 t_2$ Fit x :
　　　　∧ ˙ ¬ : ∨ ⌜u . u'⌝ · $u < u' < t$. ∧ . uu' Hb Sci Etc ╷
　　Hb Inq Ha : $t_3 t_4$ Hb Cog · ? x . $t_1 t_2$ Fit x ╷
　　Ha Inq Hb . ? $=$ Cog ╷
　　Hb Inq Ha ┇ Exg ˙ ¬ . tt' Hb Sci Etc
　　　· ∧ . $t' t''$ Hb Sci Etc : → . $t' t''$ Hb Cog Etc ┇
　　Ini . Hb Sci Etc · $=$ · Ini Hb Cog Etc ✳

The word written Cog (*fL* cognoscit = recognizes) means "comes
to know", "finds out". Syntactically it behaves like 'Sci'.

3 15 3.   ⚹ Ha Inq $Hb \cdot ? x . t_1 t_2$ Fit x ⫶

     Hb Inq Ha ɑ ⫶

     Ha Inq Hb . Cur Sci ⫶

     Hb Inq Ha . $t_3 t_4 Hb$ Cog Etc ⫶

     Ha Inq Hb ⸱ Utr : $t_3 t_4 Hb$ Cog ⸱ Etc ⸱ Mod . $t_3 t_4$ Ani ⫶

     Hb Inq Ha Fal ⫶

     Ha Inq Hb ⸱ Utr : Etc ⸱ Mod ⸱ $\lor x . t_3 t_4 x$ Inq Hb Etc ⫶

     Hb Inq Ha Fal ⫶

     Ha Inq Hb ⸱ ? u : $t_3 t_4 Hb$ Cog ⸱ Etc ⸱ Mod u ⫶

     Hb Inq Ha ⫶ Mod ⸱ Hb Ani : Utr ⸱ Hc Inq Hd . $t_1 t_2$ Fit ɑ ⚹

3 15 4.   We repeat 3 07 6, and then we continue:

     ⚹ Ha Inq Hd . ? $= 11010 \times 1011$ ⫶

     Hd Inq Ha 100011110 ⫶

     Ha Inq Hd . Cur Sci ⫶

     Hd Inq Ha . $t_1 t_2 Hd$ Cog Etc ⫶

     Ha Inq Hd Utr Mod Cpu ⫶

     Hd Inq Ha ⸱ Fal : Mod ⸱ Hb Cpu . ∧ . Hd Ani ⚹

3 15 5.   The greater part of 3 05 1 is repeated, till the moment t_2. Then one continues:

     ⚹ Ha Inq Hc ⫶ Utr ⫶ $\land n \cdot n \in$ Num . → ⫶ $\lor p \cdot p \in$ Pri . ∧ . $p > n$ ∣

     Hc Inq Ha Ver ∣

     Ha Inq Hc Cur Sci ∣

     Hc Inq Ha . $t_1 t_2 Hc$ Cog ∣

     Ha Inq Hc : ? $^\lor z \cdot t_1 t_2 Hc$ Cog . Etc . Mod z ∣

     Hc Inq Ha ⫶ $t_1 t_2 Hc$ Ani . Utr $t_1 t_2 Hb$ Dem ⸱ ∧ . $t_1 t_2 Hb$ Dem ⸱

       Erg . Ver ⸱ ∧ ⌐$t . t'$⌐ : $t_2 < t < t'$. → ⸱ tt' Hc Sci . Utr Ver ∣

     Ha Inq Hc ⫶ ? $=$ ⫶ Dem$^\mathrm{io}$ ⫶ $\land n \cdot n \in$ Num . → ⫶ $\lor p \cdot p \in$ Pri . ∧ . $p > n$ ∣

     $^{t_1} Hc$ Inq Ha ∣ ⌐ ⫶ $\lor v \cdot t_3 t_4$ Sci : Utr ⸱ $v \in$. Dem$^\mathrm{io}$ Etc ⫶

       Tan ⫶ $\lor v \cdot t_5 t_6$ Sci Etc ⫶

       Qia ⫶ $\lor$ ⌐$u . v$⌐ ⫶ $v \in$ Dem$^\mathrm{io}$ Etc ⸱ ∧ ⸱ $u \in$. Anl$^\mathrm{io}$ v ⸱ ∧

       ⸱ $t_3 t_4 Hc$ Sci : Utr ⸱ $u \in$. Anl$^\mathrm{io}$ v^{t_4} ⚹

3 15 6. Ha Inq $Hb \cdot ? x . x^{10} - 100 x - 111111001111 = 0$ ∣

     $^{t_1} Hb$ Inq Ha ∣ Qqm ⸱ ⌐ . $t_1 t_2 Hb$ Sci : Tan . $t_3 t_4 Hb$ Sci ⫶

       Qia ⫶ $t_3 t_4 Hb$ Sci ⫶ ? $^\lor u \cdot \lor v : v \in$ ⸱ Cpu$^\mathrm{io}$. ? x Etc . Mod u^{t_4} ∣

     Ha Inq Hb ∣ ⸱ Utr ∣ . ∧ ⌐$a . b . c$⌐ ∣ $a \frown b \frown c$. ∈ Com ⸱

       → ⫶ $t_1 t_2 Hb$ Sci ⫶ ? $^\lor u$ ⫶ $\lor v \cdot v \in$

       ⫶ Cpu$^\mathrm{io}$ ⸱ ? $x . ax^{10} + bx + c = 0$ ⸱ Mod u ∣

     Hb Inq Ha ∣ Ver ⫶ Eti ⫶ $t_1 t_2 Hb$ Sci ⫶ ? $=$ ⸱ $^\lor$ ⌐$a . b . c$⌐

       $^\frown x \cdot ax^{10} + bx + c = 0$. ∧ . $x \in$ Com ⫶ ⌐Com . Com . Com⌐ ∣

$Ha \operatorname{Inq} Hb : ? = \cdot^{\curlyvee} \ulcorner a . b . c \urcorner . \widehat{} x \operatorname{Etc} |$

$Hb \operatorname{Inq} Ha \cdot a \neq 0 . \rightarrow \cdot y^2 = b^{10} - 100 \, ac .$

$\quad \rightarrow . x = (y - b)/10 \, a \smallsmile (-y - b)/10 \, a :$

$\quad a = 0 . \wedge . b \neq 0 \cdot \rightarrow . x = - c/b :$

$\quad a = b = 0 . \wedge . c \neq 0 \cdot \rightarrow \cdot \neg . \vee x \operatorname{Etc} :$

$\quad a = b = c = 0 . \rightarrow \cdot \wedge x . \operatorname{Etc} \#$

Note that with respect to oblique speech 'Sci' (and also 'Ani' and so on) behaves as 'Inq'. (See 3 01 5, and 2 04 0–2 04 6.) Ha asked whether Hb can solve the quadratic equation $ax^2 + bx + c = 0$ for every value of a, b, c (quantification outside the working sphere of 'Sci'). Hb, however, stresses that he can solve even the general quadratic equation.

The word written Eti (*fL* etiam = also) means "even".

3 16 1. Since we interpret

$$Ha \operatorname{Sci} \cdot ? x . t_1 t_2 Hb \operatorname{Inq} Hc$$

as though Ha is answering the question ? x Etc, it cannot happen under this formulation that Ha knows what was said, while ignorant of who said it, or to whom it was said, or when it was said. If we wish to express such an occurrence, we can avail ourselves of the pattern of 3 14 4–3 14 5:

$\#\ Ha \operatorname{Inq} Hb \cdot ? x . t_1 t_2 Hb \operatorname{Inq} Hd x |$

$\quad ^{t_1}Hb \operatorname{Inq} Ha \shortmid t_3 t_4 Hb \operatorname{Sci} : ? x \cdot \vee y . t_1 t_2 y \operatorname{Inq} Hd x !$

$\quad \operatorname{Tan} : \neg \cdot \operatorname{Sci} : ? y \cdot \vee x . t_1 t_2 y \operatorname{Inq} Hd x^{t_4} |$

$Ha \operatorname{Inq} Ha \cdot t_3 t_4 Hb \operatorname{Sci} : ? x \cdot t_1 t_2 : ^{\curlyvee}y \cdot t_3 t_4 Ha \operatorname{Sci} . ? = y : \operatorname{Inq} Hc x \#$

At some stage of a case under trial the evidence may look like that reported by Ha in his last sentence or something even more intricate:

A murder has been committed, and the detective has traced a person, who knows the identity of the victim (though he does not know that this person has been murdered); the detective will also have found somebody who knows the author of the crime (though he does not know that a crime has been committed); he has discovered the possessor of the weapon (who is as little in the secret); and so on. The pattern of such evidence looks like:

$\#\ ^{\curlyvee}u \cdot t_1 t_2 Ha \operatorname{Sci} . ? = u : ^{\curlyvee}x \cdot t_1 t_2 Hb \operatorname{Sci} . ? = x :$

$\quad \operatorname{Inq} : ^{\curlyvee}y \cdot t_1 t_2 Hc \operatorname{Sci} . ? = y : ^{\curlyvee}p \cdot t_1 t_2 Hd \operatorname{Sci} . ? = p \#$

At a time u known to Ha a person x known to Hb said to a person y known to Hc a saying p known to Hd. The use of the definite article can be avoided by saying:

$*\ \vee\ \ulcorner u . x . y . p \urcorner : ux \operatorname{Inq} y p . \wedge \cdot t_1 t_2 Ha \operatorname{Sci} . ? = u \cdot$
$\wedge \cdot t_1 t_2 Hb \operatorname{Sci} . ? = x \cdot \wedge \cdot t_1 t_2 Hc \operatorname{Sci} . ? = y \cdot \wedge \cdot t_1 t_2 Hd \operatorname{Sci} . ? = p\ *$

3 17 0. In 3 01 5 we remarked that in the affirmative mode the direct and oblique speech are semantically equivalent. In the interrogative mode, however, the difference is enormous.

For instance, in French:

Il me disait: "où sont les neiges d'antan?"

Il me disait où sont les neiges d'antan.

In German:

Ich sagte ihm: "wer kommt?"

Ich sagte ihm, wer kommt.

In English this paradox is less striking on account of the use of the verb "to tell" in the second case:

I said to him: "who is coming?"

I told him who is coming.

It is clear that the second mode is not truly interrogative.

How shall we treat pseudo-interrogative oblique speech in Lincos? Of course we cannot translate

"*Ha* tells *Hb* what has happened from t_1 to t_2"

by

$Ha \operatorname{Inq} Hb \cdot ? x . t_1 t_2 \operatorname{Fit} x$.

because the latter sentence means

"*Ha* says to *Hb*: "what has happened from t_1 to t_2?""

The translation

$Ha \operatorname{Inq} Hb \cdot {}^{\curlyvee} x . t_1 t_2 \operatorname{Fit} x$

would be just as impossible (or at least unsatisfactory) because this means (or could mean)

"*Ha* says to *Hb*: ${}^{\curlyvee} x . t_1 t_2 \operatorname{Fit} x$".

If we were to translate not the event

"*Ha* tells *Hb* what has happened from t_1 to t_2",

but the report

"From t_3 to t_4 *Ha* told *Hb* what has happened from t_1 to t_2",

we could use the syntactical device of outside binding. Then we should translate:

$t_1 t_2 \operatorname{Fit} x . \rightarrow . t_3 t_4 Ha \operatorname{Inq} Hb x$.

But this way out is blocked in the present situation.

We need a new device for pseudo-interrogative oblique speech. This will be the exclamation mark, borrowed from the alternative method alluded to in 3 13 0.

"Ha tells Hb what has happened from t_1 to t_2"

could be translated by

$Ha \operatorname{Inq} Hb \cdot !\, ?\, x \cdot t_1\, t_2 \operatorname{Fit} x$.

But this is still unsatisfactory. The receiver might be tempted to believe that

$!\, ?\, x \cdot t_1\, t_2 \operatorname{Fit} x$

was something that could be said. Moreover, we could not prevent the actors from saying it.

I have therefore decided to write

$Ha \operatorname{Inq} ! \, Hb\, ?\, x \cdot t_1\, t_2 \operatorname{Fit} x$

with the exclamation mark after the 'Inq'.

3 17 1. $*\; Ha \operatorname{Inq} Hb \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x \vdots$
$Hb \operatorname{Inq} Ha \cdot {}^\vee x \cdot t_1\, t_2 \operatorname{Fit} x \vdots$
$Ha \operatorname{Inq} Hb \colon \operatorname{Qqm} \operatorname{Ver} . \operatorname{Tan} \operatorname{Mal} \cdot t_1\, t_2 \operatorname{Fit} \cdot {}^\vee x \cdot t_1\, t_2 \operatorname{Fit} x \colon \in \operatorname{Mal} \vdots$
$Hb \operatorname{Inq} Ha\, \mathfrak{a} \vdots$
$Ha \operatorname{Inq} Hb \operatorname{Ben} *$

3 17 2. $(t_1 < t_2 < t_3 .)$
$Ha \operatorname{Inq} Hb \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x \cdot$
${}^{t_3}Hb \operatorname{Inq} Ha\, \mathfrak{a}^{t_4} \cdot$
$Hc \operatorname{Inq} Hc \colon t_3\, t_4 Hb \operatorname{Inq} ! \, Ha \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x *$

3 17 3. $(t_1 < t_2 < t_3)$
$*\; Ha \operatorname{Inq} Hb \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x \;{\textbf{\i}}$
${}^{t_3}Hb \operatorname{Inq} Ha \cdot {}^\vee x \cdot t_1\, t_2 \operatorname{Fit} x^{t_4}\;{\textbf{\i}}$
$Ha \operatorname{Inq} Hb \operatorname{Mal} {\textbf{\i}}$
$Hc \operatorname{Inq} Hc \colon t_3\, t_4 Hb \operatorname{Inq} Ha \cdot {}^\vee x \cdot t_1\, t_2 \operatorname{Fit} x \vdots$
        $\operatorname{Sed} \cdot \neg \colon t_3\, t_4 Hb \operatorname{Inq} ! \, Ha \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x *$

3 17 4. $*\; {}^{t_1} \mathfrak{p} \; {}^{t_2} \cdot$
$Ha \operatorname{Inq} Hb . t_1\, t_2 \operatorname{Fit} \mathfrak{p}^{t_3} \cdot$
$Hc \operatorname{Inq} Hc \colon t_2\, t_3 Ha \operatorname{Inq} ! \, Hb \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x *$

3 17 5. $*\; {}^{t_1} \mathfrak{p} \; {}^{t_2} \vdots$
$Ha \operatorname{Inq} Hb\, \mathfrak{p} \vdots {}^{t_3}$
$Hc \operatorname{Inq} Hc \cdot \neg \colon t_2\, t_3 Ha \operatorname{Inq} ! \, Hb \cdot ?\, x \cdot t_1\, t_2 \operatorname{Fit} x *$

3 17 6. $\#^{t_1} \mathfrak{p}^{t_2}$:

     Ha Inq ! $Hb \cdot ? x . t_1 t_2$ Fit x^{t_3}:

     Hc Inq $Hd \cdot ? y . t_2 t_3 Ha$ Inq $Hb y$:

     Hd Inq $Hc \cdot t_2 t_3 Ha$ Inq $Hb . t_1 t_2$ Fit $\mathfrak{p}$ $\#$

3 18 1.   The (intentional) misprint in the following text is to reflect a slip of the speaker's tongue.

     $(t_1 < t_2 < t_3)$

     $\#^{t_3} Ha$ Inq $Hb \cdot ? x . t_1 t_2$ Ftt $x |^{t_4}$

     Hb Inq $Ha^{\cdot} \neg : t_4 t_5 Hb$ Itg $\cdot ? y . t_3 t_4 Ha$ Inq $Hb y |^{t_5}$

     Ha Inq $Hb : t_3 t_4 Ha$ Inq $Hb \cdot ? x . t_1 t_2$ Fit $x |$

     Hb Inq $Ha^{\cdot} t_1 t_2$ Fit $\mathfrak{a}$

         $: \neg^{\cdot} t_3 t_4 Ha$ Inq $Hb : ? x . t_1 t_2$ Fit . Sed Ftt . $x |$

     Hc Inq Ha:

         $\vee u^{\cdot} u \in$ Mal $. \wedge : t_3 t_4 Ha$ Inq $Hb \cdot ? x . t_1 t_2$ Fit $x \cdot$ Mod u:

         $\vee v^{\cdot} v \in$ Ben $. \wedge : t_3 t_4 Hb$ Ani $\cdot ? y . t_3 t_4$ Fit $y \cdot$ Mod v:

         $tt' Ha$ Inq $Hb p$. Mod Mal $\cdot$

         $\rightarrow^{\cdot} \neg : t' t'' Hb$ Itg $\cdot ? q . tt' Ha$ Inq $Hb q$ $\#$

     The word written Itg (fL intelligit = understands) means "understands". 'Inq Mod Mal (Mod Ben)' is an abbreviation of '$\vee \cdot u \ldots$ Inq Mod $u . \wedge . u \in$ Mal (Ben)'. This may be stated by a behaviour rule.

3 18 2. $\#^{t_3} Ha$ Inq $Hb \cdot ? x . t_1 t_2$ Fit x:^{t_4}

     Hb Inq $Ha . \neg$ Itg:

     Ha Inq $Hb : t_3 t_4 Ha$ Inq $Hb \cdot ? x . t_1 t_2$ Fit x:

     Hb Inq Ha: $\mathfrak{a}^{\cdot} t_3 t_4 Hb$ Ani $\cdot ? y . t_3 t_4$ Fit y:

         $\wedge : t_3 t_4 Ha$ Inq $Hb \cdot ? x . t_1 t_2$ Ftt:

     Hc Inq $Hb : t_3 t_4 Ha$ Inq Hb Etc . Mod Ben $\cdot$

     Tan $\cdot t_3 t_4 Hb$ Ani . ? Etc . Mod Mal $\#$

3 18 3. $\#^{t_1} Ha$ Inq $Hb \cdot$ Utr . $100001 \in$ Pri:^{t_2}

     Hb Inq Hc Fal:

     $^{t_3} Hc$ Inq $Hb^{\cdot} \neg : t_3 t_4 Hc$ Itg $\cdot ? x . t_2 t_3 Hb$ Inq $Hc x$:^{t_4}

     Hb Inq $Hc^{\cdot} \neg \cdot \vee x . t_2 t_3 Hb$ Inq $Hc x :$ Sed . $t_2 t_3 Hb$ Inq Ha Fal :

         Qia : $t_1 t_2 Ha$ Inq $Hb \cdot$ Utr . $100001 \in$ Pri:

     Ha Inq $Hb \cdot \neg . t_2 t_3 Hb$ Inq Ha Fal:

     Hd Inq $Ha \frown Hb \frown Hc^{\cdot} t_2 t_3 Hb$ Inq Hc Fal .

         $\wedge \cdot t_2 t_3 Hb$ Inq Ha Fal . Mod Mal $\cdot$

         $\wedge : t_2 t_3 Hb$ Inq Ha Fal $\cdot$ Mod . Hb Inq Hc Fal $\#$

3 18 4.   ⚹ ^{t_1}Ha Inq Hb · Utr $100001 \in$ Pri $:^{t_2}$

Hb Inq Ha Fal :

^{t_3}Hc Inq Hb ˙ ¬ : $t_3 t_4 Hc$ Itg · ? x . $t_2 t_3 Hb$ Inq $Hc\,x:^{t_4}$

Hb Inq Hc ˙ ¬ · $\vee x$. $t_2 t_3 Hb$ Inq $Hc\,x$: Sed . $t_2 t_3 Hb$ Inq Ha Fal :

Hd Inq Hb ⌢ Hc · $t_2 t_3 Hc$ Ani . Etc . Mod Mal ⚹

3 18 5. $(t_1 < t_2 < t_3)$

⚹ ^{t_2}Ha Inq Hb $\mathfrak{a}^{t_4}$:

Hb Inq Ha . ¬ Itg :

^{t_5}Ha Inq Hb · ˘x . $t_1 t_2$ Fit x^{t_6} :

Hb Inq Ha . ¬ Itg :

Ha Inq Hb · $t_3 t_4 Ha$ Inq ! Hb . ? x . $t_1 t_2$ Fit x :

Hb Inq Ha · Itg . $t_1 t_2$ Fit $\mathfrak{a}$:

Hc Inq Ha ⌢ Hb · $t_3 t_4$ ⌢ $t_5 t_6$. Ha Inq . Mod Mal ⚹

3 18 6. $(t_1 < t_2 < t_3)$

⚹ ^{t_3}Ha Inq Hb . $Hc:^{t_4}$

Hb Inq Ha . ¬ Itg :

Ha Inq Hb ˙ $t_3 t_4 Ha$ Inq ! Hb : ? x · $t_1 t_2 x$ Inq Ha . $10 \times 10 = 100$:

Hb Inq Ha : Itg · $t_1 t_2 Hc$ Inq Ha . $10 \times 10 = 100$ ⚹

3 18 7. $(t_1 < t_2 < t_3)$

⚹ ^{t_3}Ha Inq Hb ¬ ˙^{t_4}

Hb Inq Ha . ¬ Itg ˙

Ha Inq Hb : $t_3 t_4 Ha$ Inq ! Hb · Utr . $100001 \in$ Pri ˙

Hb Inq Ha : Itg · ¬ . $100001 \in$ Pri ⚹

3 18 8. $(t_1 < t_2 < t_3)$

⚹ ^{t_3}Ha Inq Hb · Qia . 11 Div 100001 ˙^{t_4}

Hb Inq Ha . ¬ Itg ˙

Ha Inq Hb : $t_3 t_4 Ha$ Inq ! Hb · Cur . $100001 \notin$ Pri ˙

Hb Inq Ha : 11 Div 100001 · Erg . $100001 \notin$ Pri ⚹

3 18 9.   ⚹ Ha Inq Hb · Utr . Hc Inq Hb Pri ˙

Hb Inq Ha ˙ ¬ Itg : Qia · Hc Inq Hb Pri · $\notin$ Prp ˙

Ha Inq Hb : Utr · $\vee$ ⌜tt'⌝ . tt' Hc Inq Hb Pri ˙

Hb Inq Ha · Itg . Tan . ¬ Sci ⚹

⚹ Ha Inq Hb . $t_1 t_2 Hc$ Inq Pri Hb ·

Hb Inq Ha . ¬ Itg ·

Ha Inq Hb . $t_1 t_2 Hc$ Inq Hb Pri ·

Hb Inq Ha . Itg ⚹

⚹ Ha Inq Hb : Utr · $\vee x$. $x \in$ Div ɪ

Hb Inq Ha ⫶ ¬ Itg ⫶ Qia ⫶ ¬ : $\vee x$ · $x \in$ Div . $\in$ Prp ˙

∧˙ $\vee x$. $x \in$ Div · $\notin$ Prp ˙ ∧ ˙ ¬ · $\vee x$. $x \in$ Div : $\notin$ Prp ⚹

These are a few examples for the use of '$\neg$ Itg' as a means of rejecting meaningless speech.

3 19 1. The time-signal in the next program texts will have a length of little more than 111,01 seconds and a frequency of about 10^{11111} Hertz.

$*$ Ha Inq Hb . Ecc^{t_1}———^{t_2} :

    Ha Inq Hb $\cdot$? $=$: Fre $\cdot$ $^\curlyvee x$. t_1 t_2 Fit x :

    Hb Inq Ha $\cdot$ $>$ $\cdot$ Sec^{-1} . 10^{10000} : $\frown$: $<$ $\cdot$ Sec^{-1} . 10^{100000} $\cdot$

        Fre $\cdot$ $^\curlyvee x$. t_1 t_2 Fit x : $=$ Cca $\cdot$ Sec^{-1} . 10^{11100} :

    Hc Inq Ha : Cca $\cdot$ Sec^{-1} . 10^{11111} :

    Ha Inq . Hb $\frown$ Hc $\cdot$ Err 10^{11111} . $<$. Err 10^{11100} :

    Ha Inq Hb $\cdot$? $=$: Dur $\cdot$ $^\curlyvee x$. t_1 t_2 Fit x :

    Hb Inq Ha . Sec 111 :

    Ha Inq Hb Mal :

    Hb Inq Ha $\cdot$ Cca . Sec 111 :

    Ha Inq Hb : ? $^\curlyvee a$ $\cdot$ Dur $^\curlyvee x$ Etc . $=$ Cca . a

      $\cdot$ $\wedge$: Err a . $<$ $\cdot$ Err . Sec 111 :

    Hb Inq Ha . Sec 111,01 :

    Ha Inq Hb $\cdot$ Ben : Dur $^\curlyvee x$ Etc . Pau $>$. Sec 111,01

      $\wedge$ $\cdot$ Dur $^\curlyvee x$ Etc . Mul $<$. Sec 111,10 :

    Err . Sec 111,01 $\cdot$ Mul $<$ $\cdot$ Err . Sec 111 $*$

The word written Err has been known from 3 09 1. 'Err' is a function from the set of propositions to the (partially ordered) set of errors. We have used and shall often use this abbreviated construction. The complete sentences would be

$*$ Err $\cdot$ Fre $\cdot$ $^\curlyvee x$. t_1 t_2 Fit x : $=$ Cca $\cdot$ Sec^{-1} . 10^{11111} :

    $<$ Err $\cdot$ Fre $\cdot$ $^\curlyvee x$. t_1 t_2 Fit x : $=$ Cca $\cdot$ Sec^{-1} . 10^{11100} $*$

and so on. These formulations should be linked together by a behaviour rule.

Cca (fL circa $=$ about, nearly) means "about, approximately". Later on it will also be used in less numerical contexts, in the sense of "nearly".

The words written Mul (fL multo $=$ much) and Pau (fL paulo $=$ little) mean "much" and "little" respectively. They behave like adverbs added to '$>$' and '$<$'.

3 19 2. $*$ Ha Inq Hb $\cdot$? $^\curlyvee x$: $x \in$ Rat . $\wedge \cdot x$. $=$ Cca . $\sqrt{10}$:

    Hb Inq Ha 1010/111 :

    Ha Inq Hb $\cdot$? $^\curlyvee x$: $x \in$ Rat . $\wedge \cdot x$. $=$ Cca . $\sqrt{10}$ $\cdot$

      $\wedge \cdot$ Err x . Mul $<$. Err 1010/11 :

    Hb Inq Ha 111/101 :

Ha Inq $Hb \cdot$ Err 111/101 . Pau $<$. Err 1010/11 $\vdots$
Hb Inq Ha 1100011/1000110 $\vdots$
Ha Inq Hb Ben #

3 19 3. About 250 time-signals of frequency $\mathfrak{a}$ and length $\mathfrak{h}$ are given in
a relatively short time, from t_1 to t_2.

Ha Inq Hb Ecc$^{t_1\cdots\cdots t_2}$ |
Ha Inq Hb � ? = $\vdots$ Car $\vdots\ ^\curlyvee t \cdot t_1 \leqq t \leqq t_2 - \mathfrak{h}$.
$\quad \wedge \vdots \vee x \cdot t\, t + \mathfrak{h}$ Fit x . $\wedge$. Fre $x = \mathfrak{a}$. $\wedge$. Dur $x = \mathfrak{h}$ |
Hb Inq $Ha \cdot$ Mul $>$. 1 |
Ha Inq Hb $\vdots$? $^\curlyvee y \cdot$ Err $y \cdot$ Mul $<$ $\vdots$ Err $\cdot$ Mul $>$. 1 |
Hb Inq $Ha \cdot$ Pau $<$. 100000000 #

3 19 4. $\mathfrak{p}$ will be the metatextual substitute for some noise, $\mathfrak{p}'$ that for
a bad copy of $\mathfrak{p}$, $\mathfrak{p}''$ that for a better copy.

Ha Inq $Hb \cdot$ Ecc . $^{t_1}\mathfrak{p}^{t_2}$ �
Ha Inq $Hb \cdot$? x . t_1 t_2 Fit x �
Hb Inq Ha $^{t_1}\mathfrak{p}'^{t_4}$ �
Ha Inq Hb $\vdots$ Mal $\vdots$? $^\curlyvee y \vdots y$. $=$ Cca $\cdot$ $^\curlyvee x$. t_1 t_2 Fit x $\cdot$
$\quad \wedge \cdot$ Err y . $<$ $\vdots$ Err $\cdot$ $^\curlyvee z$. $t_3 t_4$ Fit z �
Hb Inq Ha $\mathfrak{p}''$ ⠀
Ha Inq Hb Ben ⠀
Hc Inq Hd . $\mathfrak{p}$ Ant $\mathfrak{p}'$ ⠀
Ha Inq Hc $\vdots$? $x \cdot \mathfrak{p}$. x Ant . $\mathfrak{p}'$ ⠀
Hc Inq Ha Pau ⠀
Ha Inq Hc $\vdots$? $^\curlyvee y \vdots$ Err $\vdots \mathfrak{p} \cdot$ Cca y . Ant $\cdot$ $\mathfrak{p}'$ $\cdot$ $<$ $\cdot$ Err $\vdots \mathfrak{p}$. Pau Ant . $\mathfrak{p}'$ ⠀
Hc Inq $Hd \cdot \mathfrak{p}$. $(t_3 - t_2)$ Ant . $\mathfrak{p}'$ ⠀
Hd Inq Hc Ben #

3 19 5. # Ha Inq $Hb \cdot$? x . $x^{10} - 1110\,x - 111111001111 = 0$ $\vdots$
Hb Inq Ha . $x = 1000111 \backsim - 111001$ $\vdots$
Ha Inq Hb Cur $\vdots$
Hb Inq Ha . Qia Sci $\vdots$
Ha Inq $Hb \cdot$? u . Cog Mod u $\vdots$
Hb Inq $Ha \cdot$ Hb Cpu $\cdot$? x . $x^{10} - 1110\,x - 111111001111 = 0$ $\vdots$
$\quad$ Mul Ant . Ha Inq Hb Etc $\vdots$
Ha Inq Hb $\vdots$? $^\curlyvee t \cdot$ Err t . $<$. Err Mul $\vdots$
Hb Inq Ha . Sec 10^{11001} $\vdots$
Ha Inq Hb Ben #

3 20 1. The first time Ha pronounces the word written Nnc (fL nunc =
now) meaning "now", a time-signal of frequency $\mathfrak{a}$ is given.

$\#$ Ha Inq $Hb \cdot$ Ecc $: \overline{\text{Nnc}}$ Fit $\cdot {}^\curlyvee x$. Fre $x = \mathfrak{a}$ $\raisebox{-0.3ex}{:}$

Hb Inq Ha Ver $\raisebox{-0.3ex}{:}$

Ha Inq $Hb :$ Nnc Fit $\cdot {}^\curlyvee x$. Fre $x = \mathfrak{a}$ $\raisebox{-0.3ex}{:}$

Hb Inq Ha Fal $\raisebox{-0.3ex}{:}$

Hc Inq Hc $\raisebox{0.3ex}{:}$ tt' Ha Inq Hb . Nnc Fit $y \cdot$

$\qquad \to$ $\raisebox{0.3ex}{:}$ ${}^\curlyvee p$. tt' Ha Inq Hb $p \cdot \leftrightarrow \cdot$ Cca $\cdot {}^\curlyvee u$. u Ha Inq Hb Nnc $:$ Fit y $\#$

'Nnc Fit $\mathfrak{p}$' is a proposition the truth-value of which depends on time, whereas the value of '$t_1 t_2$ Fit $\mathfrak{p}$' does not depend on time. Extensionally they cannot be equivalent. In an intentional context they can.

Words such as "now", "here", "I", "this", "that" can be avoided. But for this economy we should have to pay clearly in the form of clumsy circumlocutions. We have not introduced a word meaning "I". Up to now, when speaking about himself, everybody used his proper name. Of course there may be situations where the speaker wants to conceal his name though speaking about himself. Then he can use bound variables, e.g.

$\#$ $t_1 t_2 :$ ${}^\curlyvee x \cdot \vee p$. Nnc x Inq Ha $p :$ Inq Ha q $\#$,

but if a word meaning "I" is available, he can simplify his speech considerably. As long as we are not interested in situations like this, we can dispense with "I". Likewise we have not introduced words meaning "this" and "that" though a more sophisticated analysis may reveal their occurrence in our texts: often the 'Etc' is used as if it were a demonstrative pronoun. It would be worthwhile considering whether this use should not be formalized by the introduction of formal demonstrative pronouns.

In any case it would not be wise to avoid words like "now" and "here". We will now deal with "now", and we shall see that its use is a real advantage.

"Now", like "I" and the other words mentioned, conceals a variable. So we must agree upon the place where this variable is considered to be bound, when Nnc occurs in a report such as

$\#$ Ha Inq $Hb :$ $t_1 t_2$ Hc Inq $Hb \cdot$ Nnc Hd Sci . ? Etc $\#$

In all occurrences of hidden variables we shall observe the rule of binding inside. The 'Nnc' of the example is (approximately) the moment of Hc's pronouncing Nnc (i.e. $t_1 t_2$) not that of Ha's reporting on Hc's pronouncing Nnc. Likewise a word meaning "I", if available, would mean the direct user, not the person reporting about its use. Thus syntactically oblique speech with respect to hidden variables, which is a well-known feature of common languages, is unknown in Lincos.

3 20 2. Parts of our former programm will be repeated, with 'Nnc' instead of an explicit date.

3 20 3. $(t_1 < t_2 < t_3)$

⚹ $t_5 Ha$ Inq Hb ˙ ⌐ : Nnc Ha Sci ‧ ? x . t_1 t_2 Fit x^{t_4} ⦂

Hb Inq Ha . t_1 t_2 Fit α ⦂

$t_6 Ha$ Inq Hb ⦂ ⌐ : Pau Ant . Nnc ‧ Ha Sci . ? Etc ˙

Tan ‧ Nnc Ha Sci . ? Etc. ⦂ Nnc Ha Cog ‧ ? x . t_1 t_2 Fit x ⦂

Hc Inq Hc ⦂ t_3 t_4 Ha Inq Hb : ⌐ ‧ t_3 t_4 Ha Sci . ? Etc ˙

t_5 t_6 Ha Inq Hb ⦂ ⌐ t_3 t_4 . Tan t_5 t_6 ‧ Ha Sci . ? Etc ⚹

Compare 2 03 9 on this use of 'h Ant . y' for dating.

3 20 4. $(t_1 < t_2 < t_3)$

⚹ $t_5 Ha$ Inq Hb ˙ ⌐ : t_3 t_4 Ha Sci ‧ ? x . t_1 t_2 Fit x^{t_4} |

Hb Inq Ha . t_1 t_2 Fit α |

$t_6 Ha$ Inq Hb ˙ ⌐ t_3 t_4 . Tan t_5 t_6 ‧ Ha Sci^{t_6} |

Hc Inq Hc ⦂ t_3 t_4 Hc Inq Hb . ⌐ Nnc Ha Sci ⦂

t_5 t_6 Ha Inq Hb ⦂ ⌐ ‧ Pau Ant . Nnc ⦂ Tan ⦂ Nnc ˙ Ha Sci |

Hd Inq Hd ╷ Pau Ant . Nnc ‧ Ha Inq Hb ⦂

⌐ ⦂ Pau Ant . Nnc ‧ Ha Sci ˙ Tan Nnc Sci ⦂

Pau Ant . Nnc ‧ Ha Cog ⚹

3 20 5.   ⚹ Ha Inq Hb ⦂ ? . 1^{um} x ‧ $x ∈$ Pri . ∧ . $x >$ 10^{1001} ⦂

Hb Inq Ha ⦂ ⌐ ‧ Nnc Hb Sci . ? Etc ˙

Tan ⦂ Pau Pst . Nnc ‧ Hb Sci . ? Etc ⦂

Ha Inq Ha ‧ Nnc Hb Pet . ? Etc ⦂

Hb Inq Ha . 10^{1001} + 1001 ⦂

Hb Inq Hb ⦂ Pau Ant . Nnc ‧ Hb Pet . ? Etc ⚹

3 20 6.   ⚹ Ha Inq Hb ⦂ ? . 1^{um} x ‧ $x ∈$ Pri . ∧ . $x >$ 10^{1001} ⦂

Hb Inq Ha . 10^{1001} + 1001 ⦂

Ha Inq Hb ‧ Cur . Nnc Hb Sci ⦂

Hb Inq Ha ⦂ Qia ⦂ Mul Ant . Nnc ‧ Hb Cpu ⦂

$h > 0$. → ˙ h Pst ‧ Mul Ant . Nnc ⦂ Hb Sci ⚹

Clearly this is a syntactically new appearance of 'Mul Ant . Nnc' and such expressions. In 'Mul Ant . Nnc ‧ Fit p' an existentially-bound variable is hidden in 'Mul Ant . Nnc'. In expressions like h Pst ‧ Mul Ant . Nnc ⦂ Fit q this existentially bound variable is used as an instance out of the set to which it has been bound. In other cases we have accounted for this use in common mathematical and everyday language by the word 'Put' (see 3 05 2). We could justify the new use of 'Mul Ant . Nnc' by a similar artifice, but we had perhaps better avoid subtleties of this kind. A behaviour rule stressing the new use could be added.

3 20 7. $*\, \mathfrak{p}\, \vdots$

     $Ha\, \mathrm{Inq}\, Hb \cdot ?\, x : \mathrm{Pau}\, \mathrm{Ant} . \mathrm{Nnc} \cdot \mathrm{Fit}\, x \vdots$

     $Hb\, \mathrm{Inq}\, Ha \cdot \mathrm{Pau}\, \mathrm{Ant} . Ha\, \mathrm{Inq}\, Hb\, \mathrm{Etc} \cdot \mathrm{Fit}\, \mathfrak{p} :$

       $\mathrm{Pau}\, \mathrm{Pst} . \mathfrak{p} . Ha\, \mathrm{Inq}\, Hb\, \mathrm{Etc}\, *$

3 21 1.   We introduce some abbreviations:

     $*\, \mathrm{Pau}\, \mathrm{Ant} . \mathrm{Nnc} \cdot \mathrm{Etc} : \leftrightarrow . \mathrm{PAN}\, \mathrm{Etc} \cdot$

     $\mathrm{Pau}\, \mathrm{Pst} . \mathrm{Nnc} \cdot \mathrm{Etc} : \leftrightarrow . \mathrm{PPN}\, \mathrm{Etc} \cdot$

     $\mathrm{Mul}\, \mathrm{Ant} . \mathrm{Nnc} \cdot \mathrm{Etc} : \leftrightarrow . \mathrm{MAN}\, \mathrm{Etc} \cdot$

     $\mathrm{Mul}\, \mathrm{Pst} . \mathrm{Nnc} \cdot \mathrm{Etc} : \leftrightarrow \mathrm{MPN}\, \mathrm{Etc}\, *$

3 21 2. $*\, \mathrm{Unq}\, \mathrm{Ant} . u \cdot \mathrm{Fit}\, p : \leftrightarrow . p\, \mathrm{Ant}\, u \cdot$

     $\mathrm{Unq}\, \mathrm{Pst} . u \cdot \mathrm{Fit}\, p : \leftrightarrow . p\, \mathrm{Pst}\, u \cdot$

     $\mathrm{Unq}\, \mathrm{Ant} . \mathrm{Nnc} \cdot \mathrm{Etc} : \leftrightarrow \mathrm{UAN}\, \mathrm{Etc} \cdot$

     $\mathrm{Unq}\, \mathrm{Pst} . \mathrm{Nnc} \cdot \mathrm{Etc} : \leftrightarrow \mathrm{UPN}\, \mathrm{Etc} \cdot$

     $\mathrm{Unq}\, \mathrm{Fit}\, p . \leftrightarrow \cdot \vee u . u\, \mathrm{Fit}\, p\, *$

The word written Unq (fL unquam = ever) means "at some time".

3 21 3. $*\, Ha\, \mathrm{Inq}\, Hb \cdot ?\, x . t_1\, t_2\, \mathrm{Fit}\, x \vdots$

     $^{t_b} Hb\, \mathrm{Inq}\, Ha \vdots t_1\, t_2\, Hb\, \mathrm{Ani} . ?\, \mathrm{Etc} \cdot \mathrm{Tan} : \neg \cdot t_3\, t_4\, Hb\, \mathrm{Sci} . ?\, \mathrm{Etc} \vdots^{t_b}$

     $Ha\, \mathrm{Inq}\, Ha \vdots \neg \cdot t_3\, t_4\, Hb\, \mathrm{Sci}?\, \mathrm{Etc} : \mathrm{Qqm} . \mathrm{Unq}\, \mathrm{Ant} \cdot t_3\, t_4 : Hb\, \mathrm{Sci}?\, \mathrm{Etc} \cdot$

     $\neg \cdot t_3\, t_4 . \mathrm{QUA} . Hb\, \mathrm{Sci} . ?\, \mathrm{Etc}\, *$

3 21 4. $*\, {}^{t_b} Ha\, \mathrm{Inq}\, Hb \cdot ?\, x . t_3\, t_4\, \mathrm{Fit}\, x^{t_1}{}_1$

     $Hb\, \mathrm{Inq}\, Ha \vdots \neg \cdot t_3\, t_4\, Hb\, \mathrm{Sci} . ?\, \mathrm{Etc} : \mathrm{Qia} \cdot t_0 < t_1 < t_2 < t_3 < t_4 \blacksquare$

     $\wedge : t_0\, t_1\, Ha\, \mathrm{Inq}\, Hb \cdot ?\, x . t_3\, t_4\, \mathrm{Fit}\, x \vdots$

     $\neg : t_1\, t_2\, Hb\, \mathrm{Sci} \cdot \mathrm{Tan} . t_3\, t_4\, Hb\, \mathrm{Sci} \cdot ?\, x . t_3\, t_4\, \mathrm{Fit}\, x \vdots$

     $\neg : t_1\, t_2 \cdot \mathrm{Qqm} . \mathrm{Unq}\, \mathrm{Pst} \cdot Hb\, \mathrm{Sci} . ?\, \mathrm{Etc} \vdots$

     $\neg \cdot t_1\, t_2 . \mathrm{QUP} . Hb\, \mathrm{Sci} . ?\, \mathrm{Etc}\, *$

In 3 21 3 Hb has forgotten, in 3 21 4 Hb does not yet know. So QUA means "longer" as used in "no longer"; QUP means "yet" as used in "not yet".

3 22 1. $\mathfrak{p}$ is a metatextual substitute for a noise that lasts longer than $t_2 - t_1$; $\mathfrak{q}$ is a metatextual substitute for a noise that lasts less than $t_2 - t_1$.

     $*\, Ha\, \mathrm{Inq}\, Hb . t_1\, t_2\, \mathrm{Fit}\, \mathfrak{p} \vdots$

     $Hb\, \mathrm{Inq}\, Ha\, \mathrm{Fal} \vdots$

     $Ha\, \mathrm{Inq}\, Hb \cdot ?\, x . t_1\, t_2\, \mathrm{Fit}\, x \vdots$

     $Hb\, \mathrm{Inq}\, Ha \vdots \neg : \mathrm{Nnc}\, Hb\, \mathrm{Sci} \cdot ?\, x . t_1\, t_2\, \mathrm{Fit}\, x \cdot \neg \cdot \mathrm{Pot} . t_1\, t_2\, \mathrm{Fit}\, \mathfrak{p} \vdots$

     $Ha\, \mathrm{Inq}\, Hb \cdot \mathrm{Cur} . \neg \mathrm{Pot} \vdots$

     $Hb\, \mathrm{Inq}\, Ha : \mathrm{Qia} \cdot \mathrm{Dur}\, \mathfrak{p} . > t_2 - t_1 \vdots$

     $Ha\, \mathrm{Inq}\, Hb . t_1\, t_2\, \mathrm{Fit}\, \mathfrak{q} \vdots$

     $Hb\, \mathrm{Inq}\, Ha : \mathrm{Dur}\, \mathfrak{q} . < t_2 - t_1 \cdot \mathrm{Erg} \cdot \mathrm{Pot} . t_1\, t_2\, \mathrm{Fit}\, \mathfrak{q} \cdot$

       $\mathrm{Tan} : \neg \cdot \mathrm{Ncs} . t_1\, t_2\, \mathrm{Fit}\, \mathfrak{q} \cdot \neg \cdot \mathrm{Nnc}\, Hb\, \mathrm{Sci} . \mathrm{Utr}\, \mathrm{Ver}\, *$

The word written Pot (*fL* potest = can) means "it is possible that". 'Ncs' is known from 3 12 2. Both 'Pot' and 'Ncs' are followed by propositions.

3 22 2. $(t_1 < t_2 < t_3)$

^{t_3}Hc Inq Hb . $t_1 t_2 Ha$ Inq Hc p^{t_4} :

Hd Inq Hb . $t_1 t_2 Ha$ Inq Hd q^{t_5} :

Hb Inq Hb : $^\curvearrowright x$. $t_3 t_4 Hc$ Inq $Hb x$: $\in$ Fal :

    v : $^\curvearrowright y$. $t_4 t_5 Hd$ Inq $Hb y$ · $\in$ Fal : Qia : $\neg$ ˙ Pot : $\vee$ $\ulcorner x . y . u . v . w \urcorner$ ·

  $u Ha$ Inq $v x$. $\wedge$. $u Ha$ Inq $w y$. $\wedge$. $x \neq y$ #

3 22 3. $(t_1 < t_2 < t_3 < t_4)$

  # Ha Inq Hb : $t_1 t_2 Ha$ Ani · ? x . $t_3 t_4$ Fit x ı

  Hb Inq Ha : Fal · Qia . $t_2 < t_3$:

    $\neg$: Pot ˙ $t_2 < t_3$. $\wedge$: $t_1 t_2$ Ani · ? x . $t_3 t_4$ Fit x :

    Pot ˙ $t_4 < t_1$. $\wedge$: $t_1 t_2 Ha$ Ani · ? x . $t_3 t_4$ Fit x :

    Tan : $\neg$ ˙ Ncs : $t_4 < t_1$. $\rightarrow$ · $t_1 t_2 Ha$ Ani . ? $t_3 t_4$ Fit x #

3 22 4. # Ha Inq Hb : Utr ˙ $t_3 t_4 Hc$ Sci · ? x . $t_1 t_2$ Fit x :

    $\rightarrow$: $t_1 t_2 Hc$ Ani · ? x . $t_1 t_2$ Fit x :

  Hb Inq Ha · Pot . Tan . $\neg$ Ncs :

  Ha Inq Hb ˙ Utr : $\vee$ $\ulcorner t . t' \urcorner$ · tt' Hc Ani . ? Etc :

  Hb Inq Ha : Pot . Tan . $\neg$ Ncs :

    Qia ˙ Pot : UAN Hd Inq ! Hc · ? x . $t_1 t_2$ Fit x #

3 22 5. # Ha Inq Hb : Utr · $10^{10000} + 1$. $\in$ Pri ı

  ^{t_1}Hb Inq Ha : $10^{10000} + 1$. $\in$ Pri^{t_2} ı

  Ha Inq Hb . Cur Ver ı

  Hb Inq Ha : $t_1 t_2 Hb$ Pet : ? x · x Div . $10^{10000} + 1$ ı

  Ha Inq Hb : Fal : Qia :

    Pot : tt' Hb Pet · ? x . Etc ˙ $\leftrightarrow$ · $t' - t$. Mul $>$. $t_2 - t_1$ ı

  Hb Inq Ha : $t_1 t_2 Hb$ Sci . Utr Etc · Qia · UAN Hb Pet . ? Etc ı

  Ha Inq Hb : Pot . Tan . $\neg$ Ncs :

    Pot : $\vee y$ · UAN y Inq ! Hb . Utr Etc :

    Eti : Pot : Qqm : $\neg$ · $t_1 t_2 Hb$ Sci . Utr Etc ˙

      Tan : $t_1 t_2 Hb$ Inq · $10^{10000} + 1$. $\in$ Pri #

3 22 6. # Ha Inq Hb ˙ PAN Ha Inq Hc · ? x . $t_1 t_2$ Fit x :

    Tan · $\neg$. PAN Hc Rsp Ha ı

  Hb Inq Ha : $\neg$ · Pot . PAN Hb Rsp Ha :

    Qia : $\neg$ ˙ PAN Hb Ani : Utr · $\vee$ $\ulcorner x . y \urcorner$. y Inq $Hc x$ ı

  Ha Inq Hc · ? x . $t_1 t_2$ Ftt x ı

  Hb Inq Ha ˙ $\neg$ · Pot . Nnc Hb Rsp Ha :

    Qia · PAN Ha Inq Hc . ? Etc . Mod Mal ı

Hc Inq Ha . $\neg$ Itg ۱
Hb Inq Ha ⁚ $\neg$. Pot . Nnc Hc Rsp Ha ⁚
    Qia · $\neg$: Nnc Ha Itg · ? y . PAN Ha Inq $Hc\,y$ ✳

3 22 7. ✳ Ha Inq Hb ⁚ Utr · $\wedge n$: $\vee p$ · $p \in$ Pri . $\wedge$. $p > n$ ۱
    Hb Inq Ha · $\neg$ · Pot . Nnc Hb Rsp Ha · Qia : $\neg$ · Nnc Sci . ? $=$ Pri ✳

3 22 8. ✳ Ha Inq Hb : ? $1^{\mathrm{um}} x$ · $x \in$ Pri . $\wedge$. $x > 10^{1001}$ ۱
    Hb Inq Ha ۱ $\neg$: Pot · Nnc Hb Inq ! Ha . ? Etc ⁚
    Ncs · Nnc Hb Pet . ? Etc ⁚ $\neg$ · Nnc . QUP . Hb Sci . ? Etc ⁚
    $\neg$ ⁚ Pot · Unq · Ant . ƅ Pst . Nnc : Hb Rsp Ha ✳

Compare the remark in 3 20 6.

3 23 1. ✳ ᵗHa Inq Hb · Utr : Nnc Hb Sci · ? x . $t_1 t_2$ Fit x ⁚
    Hb Inq Ha ⁚ $\neg$ · Pot : Nnc Hb Inq ! Ha · ? x . $t_1 t_2$ Fit x ⁚
    Qia ⁚ $\neg$: $t_1 t_2$ Hb Ani · ? x . $t_1 t_2$ Fit x ⁚
    Ha Inq Hc · Utr : Nnc Hc Sci · ? x . $t_1 t_2$ Fit x ⁚
    Hc Inq Ha ⁚ Pot : Nnc Hc Inq ! Ha · ? x . $t_1 t_2$ Fit x ·
    Qia : $t_1 t_2$ Hc Ani · ? x . $t_1 t_2$ Fit x ⁚
    Ha Inq Hc · Utr : MAN Hc Sci · ? x . $t_1 t_2$ Fit x ⁚
    Hc Inq Ha : Pot Etc . $\wedge$ · Pot : $\neg$ · MAN . QUA . Hc Sci Etc ✳

3 23 2. Ha Inq Hb ⁚ Utr ⁚ $\wedge n$ · $n \in$ Num . $\rightarrow$: $\vee p$ · $p \in$ Pri . $\wedge$. $p > n$ ۱
    Hb Inq Ha Ver ۱
    Ha Inq Hb : Utr · Pot . Nnc Hb Dem Etc ۱
    Hb Inq Ha ⁚ Qqm · MAN : Hd Dem Etc
      ⌢ Hb Ani · Utr . Nnc Hd Dem Etc ⁚
    Tan : $\neg$ · Pot . Nnc Hd Dem Etc ⁚
    Qia ⁚ $\neg$ · MAN Hb Itg : ? u · Nnc Hd Dem . Etc . Mod u ۱
    Ha Inq Hc : Utr · Pot . Nnc Hc Dem Etc ۱
    Hc Inq Ha ۱ Pot : $\neg$ Nnc . Qqm . PPN · Hc Dem Etc ⁚
    Qia ⁚ Nnc Hc Sci ⁚ ? ˘u · $\wedge$ ⌜t . t' . x⌝ : Pot · tt' x Dem . Etc . Mod u ✳

Reformulation of 3 15 5 with the new means of expression.

3 23 3. Ha Inq Hb · ? x . $x^{10} - 100\,x - 111111001111 = 0$ ۱
    Hb Inq Ha ⁚ Qqm · $\neg$. $t_1 t_2$: Tan . $t_3 t_4$ · Hb Sci ۱
    Ha Inq Hb Cur ۱
    Hb Inq Ha ⁚ Qia · $\wedge$ ⌜a . b . c⌝ : a ⌢ b ⌢ c . $\in$ Com ·
      $\rightarrow$ · Pot : Nnc Hb Cpu · ? x . $ax^{10} + bx + c = 0$ ✳

Reformulation of 3 15 6 with the new means of expression.

3 23 4. ✳ Ha Inq Ha · tt' x Sci . ? Etc · $\leftrightarrow$: Pot · tt' x Inq ! y . ? Etc ✳

3 24 1.　＊ *Ha* Inq *Hb* . Ecc |^{t_1}—　—　—　—^{t_2} |
　　　　Ha Inq *Hb* : ? *x* · PAN . *x^{es}* Fit — |
　　　　Hb Inq *Ha* 100 |
　　　　Ha Inq *Hb* . Cur Sci |
　　　　Hb Inq *Ha* · PAN *Hb* Ani · ? *y* . Nnc Fit *y* :
　　　　　　∧ · PAN *Hb* Enu . ? Etc |
　　　　Ha Inq *Hb* : $t_1 t_2$ *Ha* Ani : Utr · ∨*z* . $t_1 t_2$ *Hb* Inq *Hb z* ·
　　　　　　Tan : ⌐ · ∨*z* . $t_1 t_2$ *Hb* Inq *Hb z* |
　　　　Hb Inq *Ha* | $t_1 t_2$ *Hb* Enu ⦂ ? Etc ⦂
　　　　　　Mod ⦂ ∧ ⌈*t* . *t′* . *z*⌉ ⦂ *z* ≠ *Hb* . → · ⌐ Pot : *tt′ z* Ani ·
　　　　　　? *y* . $t_1 t_2$ *Hb* Inq *Hb y* ＊

　　Imperceptible speaking (thinking).

3 24 2.　＊ *Ha* Inq *Hb* · PAN *Ha* Inq *Hc* · ? *x* . $t_1 t_2$ Fit *x* :
　　　　　　Tan · ⌐ . PAN *Hc* Rsp *Ha* |
　　　　Hb Inq *Ha* | ⌐ · Pot . PAN *Hb* Rsp *Ha* ⦂
　　　　　　Qia ⦂ PAN *Ha* Inq *Hc* Etc . Mod ⦂ ⌐ · Pot :
　　　　　　Nnc *Hc* Ani · ? *y* . *Ha* Inq *Hc y* ＊

　　This may be varied to convey that *Hc* could not understand
whether *Ha* said something, or to whom *Ha* spoke.

3 24 3.　＊ *Ha* Inq *Hb* · ? *x* . $t_1 t_2$ Fit *x* ⦂
　　　　Hb Inq *Hc* · ? *x* . $t_1 t_2$ Fit *x* ⦂
　　　　Hc Inq *Hb* a ⦂
　　　　Hb Inq *Ha* a ⦂
　　　　Ha Inq *Hb* . Cur Sci ⦂
　　　　Hb Inq *Ha* · PAN *Hb* Inq *Hc* · ? *x* . $t_1 t_2$ Fit *x* :
　　　　　　∧ : PAN *Hc* Inq ! *Hb* · ? *x* . $t_1 t_2$ Fit *x* ⦂
　　　　Ha Inq *Hb* · Cur : Qqm *Hc* . Tan · ⌐ . *Hb* Sci ⦂
　　　　Hb Inq *Ha* ⦂ Qia ⦂ $t_1 t_2$ Fit a ·
　　　　　　Mod · Qqm · Pot . *Hc* Ani · Tan : ⌐ · Pot . *Hb* Ani ＊

3 24 4.　We repeat 3 22 1 with the following variation.
　　　　＊ *Hb* Inq *Ha* · ⌐ . Pot . $t_1 t_2$ Fit ℘ : Qia · Dur ℘ . > Nim |
　　　　Ha Inq *Hb* . ? = Nim |
　　　　Hb Inq *Ha* | Exg ⦂ PAN *Ha* Inq *Hb* . $t_1 t_2$ Fit ℘ ⦂
　　　　　　Sed ⦂ Dur ℘ > · sup : ˄ . Dur ↑*x* · Pot . $t_1 t_2$ Fit *x* ＊

　　The word written Nim (∫L nimis = too (as used in "too large"))
means "what is admissible". '> Nim' may be translated "too
large"; ' < Nim', "too small".

3 24 5.   Likewise we may vary 3 22 5.

$*$ Hb Inq Ha । $\neg$ $\cdot$ Pot $t_1 t_2$ Hb Pet $.$? Etc ⦂

     Qia ⦂ $t_2 - t_1 <$ ⦂ inf $\cdot$ $\wedge$ Dur ⁞ $^\wedge x : x =$ $\cdot$ Pet^{io} $.$? Etc ।

Ha Inq Hb : Erg $\cdot$ Qia $.$ $t_2 - t_1 <$ Nim $*$

3 24 6.   I is the written picture of a very large series of time-signals sent within a relatively short time.

$*$ Ha Inq Hb Ecc । ^{t_1}I^{t_2} ।

   Ha Inq Hb : ? $x \cdot t_1 t_2 . x^{es} .$ Fit $\overline{}$ ।

   Hb Inq Ha । $.$ $^\vee x \cdot t_1 t_2 . x^{es} .$ Fit x : $>$ ⦂ sup ⦂ $\wedge$ $.$ Car ⁞

     $^\wedge z$: Pot $\cdot$ Hb Enu $.$? $=$ Car z । $^\vee x . t_1 t_2 x^{es}$ Fit $x \cdot >$ Nim $*$

3 25 1. $*$ Ha Inq Ha ⦂ Pot Etc $. \leftrightarrow :$ $\neg$ $\cdot$ Ncs $.$ $\neg$ Etc $\cdot$

     $\neg$ $.$ Pot Etc $\cdot \leftrightarrow \cdot$ Ncs $.$ $\neg$ Etc $\cdot$

       $\neg$ $\cdot$ Pot $.$ $\neg$ Etc : $\leftrightarrow .$ Ncs Etc $\cdot$

     Pot $.$ $\neg$ Etc $\cdot \leftrightarrow \cdot$ $\neg$ $.$ Ncs Etc $*$

3 25 2.   We shall now adopt the habit of dropping punctuation between $\neg$ and words like $\wedge$, $\vee$, Ncs, Pot, and so on. This may be stated by some behaviour rule

$*$ Ha Inq Ha : $\neg$ $.$ Pot Etc $\cdot \leftrightarrow \cdot$ $\neg$ Pot $.$ Etc $*$

3 26 1. $*$ Ha Inq $Hb \cdot$? $u . u$ Fit $\mathfrak{p}$ ⦂

   Hb Inq Ha MAN ⦂

   Ha Inq Hb Cur ⦂

   Hb Inq $Ha \cdot$ Qia : $\mathfrak{p}$ Ant $\mathfrak{q}$ $. \wedge \cdot \mathfrak{q} .$ Mul Ant $.$ Nnc ⦂

   Ha Inq Hb $.$ Cur Mul ⦂

   Hb Inq Ha ⦂ $\neg$ UAN ⦂ $\mathfrak{q} \wedge \cdot$ Hb Ani $.$ Utr $\mathfrak{q}$ $\cdot$ Erg $.$ MAN ⦂

   Hc Inq Hd : Erg $\cdot$ Nnc Aet $Hb . < .$ Sec 10^{11101} ⦂

   Hd Inq Hc Cur ⦂

   Hc Inq $Hd \cdot$ Qia ⦂ $\neg$ UAN $\cdot$ Hb Ani $.$ Utr $\mathfrak{q}$:

     $\wedge \cdot$ Cca $.$ Sec $10^{11101} \cdot$ Ant Nnc : Fit $\mathfrak{q}$ ⦂

   Hd Inq Hc : Sec $10^{11101} .$ Ant Nnc $\cdot$ Hd Ani $.$ Utr $\mathfrak{q}$ ⦂

   Hc Inq Hd : Erg $\cdot$ Nnc Aet $Hd . > .$ Sec 10^{11101} ⦂

   Hd Inq Hc : $^\vee t . t$ Fit $\mathfrak{q}$ $\cdot$ Aet $Hd . =$ Cca $.$ Sec 10^{11110} ⦂

   Hc Inq Hd : Erg $\cdot$ Nnc Aet $Hd . =$ Cca $.$ Sec $(10^{11101} + 10^{11110})$ ⦂

   Hd Inq Hc : Nnc Aet $Hh .$ Mul $> .$

     Nnc Aet Ha $\frown$ Nnc Aet Hb $\frown$ Nnc Aet $Hc \cdot$

     Nnc Aet $Hh . =$ Cca $.$ Sec 10^{11111} $*$

In this program text $\mathfrak{p}$ and $\mathfrak{q}$ are metatextual substitutes for some words the meaning of which is still unknown; it might be "earthquake" and "total eclipse of the sun".

The word written Aet (*ʃL* aetas = age) means "age". *t* Aet *Ha* means "*Ha*'s age at the moment *t*". The beginning of individual human life has been identified with beginning of individual faculty of perception. Of course this is not correct, but it will be relatively easy to set things right when more refined considerations are needed.

3 26 2.   ⚹ *Ha* Inq *Ha* ₁ . ∨ ⌜*a . b*⌝ ₁*a* ⌢ *b* . ∈ Tem ⋅ ∧ ⦂ ∧ ⌜*A . B*⌝ ⦂

$$A = \cdot^{⌃}t . a \leqq t \leqq b \colon \wedge \colon B = \cdot^{⌃}t . 0 \leqq t \leqq b - a \cdot$$

$$\rightarrow \colon^{⌃}t . t \text{ Aet } x \cdot \in . A \frown B \colon \wedge \cdot$$

$$\wedge t \colon t \in A . \rightarrow \cdot t \text{ Aet } x . = . t - a \cdot \wedge \colon$$

$$\wedge t \cdot t \in A . \rightarrow . t x \text{ Ext} \colon \wedge \cdot$$

$$\wedge t \colon t < a . \vee . t > b \cdot \rightarrow \cdot \neg . t x \text{ Ext} \cdot$$

$$\wedge \cdot a = \cdot \text{ Ini} . x \text{ Ext} \colon \wedge \colon b = \cdot \text{ Fin} . x \text{ Ext } ⚹$$

The word written Ext (*ʃL* extat = exists) means "exists". It should not be confused with the word ∨. There is ('∨') a Roman general who crossed the Rubicon in 49 B.C., but he does not actually exist ('Ext').

It would be premature to analyze the difference between these two notions.

3 26 3.   ⚹ *Ha* Inq *Ha* ⦂

Pot ⦂ ∨*y* ⋅ *t*₁ *t*₂ *x* Ani . ⁇ *y* ⋅ ↔ ⦂ Ini . *x* Ext ⋅ < *t*₁ < *t*₂ < ⋅ Fin . *x* Ext ⦂

Pot ⦂ ∨*y* ⋅ *t*₁ *t*₂ *x* Inq *y* . Nnc ⌣ *y* ⌣ *Hc* ⌣ ¬ ⌣ Etc ⋅

    ↔ ⋅ Ini . *x* Ext ⋅ + ⋅ Cca . Sec 10^{11001} ⦂ < *t*₁ < *t*₂ < ⋅ Fin . *x* Ext ⦂

Pot ⦂ *t*₁ *t*₂ *x* Enu . ⁇ Etc . Mod 1 . 10 . 11 . 100 ⋅

    ↔ ⋅ Ini . *x* Ext ⋅ + ⋅ Cca . Sec 100 × 10^{11001} ⦂ < *t*₁

    < *t*₂ < ⋅ Fin *x* Ext ⦂

Pot ⋅ *t*₁ *t*₂ *x* Cpu . ⁇ = 101 × 101 ⋅

    ↔ ⋅ Ini . *x* Ext ⋅ + ⋅ Cca . Sec 111 × 10^{11001} ⦂ < *t*₁

    < *t*₂ < ⋅ Fin *x* Ext ⦂

Pot ⦂ *t*₁ *t*₂ *x* Cpu ⋅ ⁇ *x* . *x*^{10} + 111 *x* − 1000 ⋅

    ↔ ⋅ Ini . *x* Ext ⋅ + Cca . Sec 1110 × 10^{11001} ⦂ < *t*₁

    < *t*₂ < ⋅ Fin *x* Ext ⚹

Average development of humans has been sketched.

3 26 4.   𝔭 is again a metatextual substitute for some noise.

⚹ *Ha* Inq *Ha* ₁

Pot ⋅ ∨ ⌜*t*₁ . *t*₂ . *x*⌝ . *t*₁ *t*₂ *Ba* Inq *x* 𝔭 ⦂

¬ Pot ⦂ ∨ ⌜*t*₁ . *t*₂ . *x*⌝ ⋅ *t*₁ *t*₂ *Ba* Inq *x* .

    Nnc ⌣ *x* ⌣ *Hc* ⌣ ¬ ⌣ Etc ⦂

Pot ⦂ ∨ ⌜*t*₁ . *t*₂ . *y*⌝ ⋅ *t*₁ *t*₂ *Ba* Ani . ⁇ *y* ⦂

¬ Pot ⦂ ∨ ⌜*t*₁ . *t*₂ . *t*₃ . *t*₄ . *y*⌝ ⋅ *t*₁ *t*₂ *y* Inq *Ba* . 10 × 10 = 100 ⋅

    ∧ ⦂ *t*₃ *t*₄ *Ba* Itg ⋅ ⁇ *x* . *t*₁ *t*₂ *y* Inq *Ba x* ⚹

3 26 5.   ＊ $Ha \in \mathrm{Hom} . Hb \in \mathrm{Hom} . Hc \in \mathrm{Hom} . \mathrm{Etc} \cdot$

     $Ba \in \mathrm{Bes} . Bb \in \mathrm{Bes} . Bc \in \mathrm{Bes} . \mathrm{Etc} \cdot$

     $Ha \notin \mathrm{Bes} . \mathrm{Etc} . Ba \notin \mathrm{Hom} . \mathrm{Etc} \cdot$

     $\mathrm{Hom} \cap \mathrm{Bes} = \ulcorner \urcorner$ ＊

The words written Hom (fL homo = man) and Bes (fL bestia = animal) mean "human" and "animal" (not human) respectively. The words written Ba, Bb, Bc and so on are names of animals.

3 26 6.   ＊ $Ha \operatorname{Inq} Ha \cdot$

     $\neg \lor x : x \in \mathrm{Hom} . \land . \mathrm{Nnc}\, x\, \mathrm{Ext} . \land \cdot \mathrm{Nnc}\, \mathrm{Aet}\, x > . \mathrm{Sec}\, 10^{100000}$ ＊

3 26 7.   ＊ $Ha \operatorname{Inq} Ha :$

     $F = : \lor m : \mathrm{Car} \cdot {}^{\frown} x : x \in \mathrm{Hom} . \land . \mathrm{Nnc}\, x\, \mathrm{Ext} .$

       $\land \cdot \mathrm{Nnc}\, \mathrm{Aet}\, x . \leqq m : \land :$

     $a = \sup {}^{\frown} F . = \cdot F . \mathrm{Sec}\, 10^{100000} \mid \rightarrow : a = \mathrm{Cca} . 11 \times 10^{11110} \cdot$

     $\land \cdot F . \mathrm{Sec}\, 10^{11001} \cdot / a : = \mathrm{Cca} . 0,00001 \cdot$

     $\land \cdot F . \mathrm{Sec}\, 101 \times 10^{11011} \cdot / a : = \mathrm{Cca} . 0,1 \cdot$

     $\land \cdot F . \mathrm{Sec}\, 101 \times 10^{11100} \cdot / a : = \mathrm{Cca} . 0,11 \cdot$

     $\land \cdot F . \mathrm{Sec}\, 101 \times 10^{11101} \cdot / a : = \mathrm{Cca} . 1$ ＊

Short demographic statistic of actually existing mankind.

3 26 8.   ＊ $Ha \operatorname{Inq} Ha :$

     $\mathrm{Car} : {}^{\frown} x \cdot x \in \mathrm{Bes} . \land . \mathrm{Nnc}\, x\, \mathrm{Ext} \cdot \mathrm{Mul} > . 10^{100100}$ ＊

3 27 1.   ＊ $Ha \operatorname{Inq} Ha \mid$

     $\land n : Fn . \leftrightarrow \cdot \lor \ulcorner a . b . c \urcorner : a \cup b \cup c . \subset \mathrm{Num} \cdot \land . a^n + b^n = c^n :$

     $\land : \land n : n \in \mathrm{Pri} . \rightarrow : F' n . \leftrightarrow \cdot \lor \ulcorner a . b . c \urcorner : a \cup b \cup c . \subset \mathrm{Num} \cdot$

       $\land \cdot \neg . n\, \mathrm{Div}\, a \cdot \land \cdot \neg . n\, \mathrm{Div}\, b \cdot \land \cdot \neg . n\, \mathrm{Div}\, c \cdot \land . a^n + b^n = c^n :$

     $\land : G \leftrightarrow \cdot \neg \lor n : n - 10 . \in \mathrm{Num} \cdot \land . Fn \mid$

     $\rightarrow \mid : \neg \lor x : x \in \mathrm{Hom} . \land . \mathrm{Nnc}\, x\, \mathrm{Ext} . \land \cdot \mathrm{Nnc}\, x\, \mathrm{Sci} . \mathrm{Utr}\, G \cdot$

     $\mathrm{Tan} \mid . \lor x \mid x \in \mathrm{Hom} . \land : \mathrm{Unq} \cdot \mathrm{Sec}\, 101 \times 10^{11111} . \mathrm{Ant} \cdot \mathrm{Nnc} : x\, \mathrm{Ext}$

       $\frown \mathrm{Inq}\, x : \lor y : y \in . \mathrm{Dem}^{10} G \cdot$

       $\land \cdot \mathrm{Nnc}\, x\, \mathrm{Sci} : \mathrm{Utr} \cdot y \in . \mathrm{Dem}^{10} G \mid \cdot \land \mid \cdot$

     $\lor x : x \in \mathrm{Hom} . \land : \mathrm{Unq} \cdot \mathrm{Sec}\, 111 \times 10^{11101} . \mathrm{Ant} \cdot \mathrm{Nnc}\, x\, \mathrm{Ext} :$

       $\frown : x\, \mathrm{Dem} \cdot \land n : n - 10 \in \mathrm{Num} . \land . n < 1100100 \cdot \rightarrow . \neg Fn \mid \cdot \land \mid \cdot$

     $\lor x : x \in \mathrm{Hom} . \land . \mathrm{Nnc}\, x\, \mathrm{Ext} . \land : \mathrm{PAN}\, x\, \mathrm{Dem} \cdot$

       $\land n : n - 10 \in \mathrm{Num} . \land . n < 1001101100 \cdot \rightarrow . \neg Fn \mid \land \mid$

     $\lor x : x \in \mathrm{Hom} . \land . \mathrm{Nnc}\, x\, \mathrm{Ext} . \land : \mathrm{PAN}\, x\, \mathrm{Dem} \cdot$

       $\land n : n \in \mathrm{Pri} . \land . n \neq 10 . \land . n < 10^{11100} \cdot \rightarrow . \neg F' n$ ＊

Sketch of the history of Fermat's theorem.

3 28 1. $Ha \operatorname{Inq} Ha \mid$

     $t_1 t_2 Ha \operatorname{Inq} Hb \frown Hc \frown Hd \frown He \frown Hf . p \cdot$

       $\rightarrow \cdot t_1 t_2 Ha \operatorname{Inq} . 101 \,{}^{\mathbf{e}} \mathrm{Hom} . p :$

$$t_1 t_2 . Ha \frown Hb \frown Hc . Ani \, p \cdot \rightarrow \cdot t_1 t_2 . 11^\epsilon Hom . Ani \, p \vdots$$

$$t_1 t_2 \, Ha \, Inq . n^\epsilon Hom . p \cdot \leftrightarrow \vdots \vee A \vdots A \subset Hom . \wedge \cdot Car \, A .$$

$$= n \cdot \wedge \vdots \wedge x \cdot x \in A . \rightarrow . t_1 t_2 \, Ha \, Inq \, x \, p \vdots$$

$$t_1 t_2 . n^\epsilon Hom . Ani \, p \cdot \leftrightarrow \vdots \vee A \vdots A \subset Hom . \wedge \cdot Car \, A .$$

$$= n \cdot \wedge \vdots \wedge x \cdot x \in A . \rightarrow . t_1 t_2 \, x \, Ani \, p \vdots$$

$$t_1 t_2 \, Ha \, Inq \, Mul^\epsilon Hom \, p \cdot \leftrightarrow \cdot \vee n : n . Mul > . 1 \cdot \wedge \cdot$$

$$t_1 t_2 \, Ha \, Inq . n^\epsilon Hom . p \vdots$$

$$Etc . n^\epsilon Rea . Etc \cdot$$

$$\leftrightarrow \vdots \vee A \cdot A \subset Rea . \wedge \cdot Car \, A . = n \cdot \wedge \vdots \wedge x \cdot x \in A . \rightarrow . Etc \, x \, Etc \vdots$$

$$Etc \, ^\#$$

By the use of the pattern '$n^\epsilon A$', meaning "n things from A", Lincos speech becomes still more akin to common languages which admit e.g. "three people" as a subject or an object of a sentence.

Common languages do not distinguish between the "three apples" of

I bought three apples,

and of

Three apples of this kind go to a pound,

or between the "little money" of

I have little money,

and of

With little money you can succeed at Smith's,

though the variable that ranges over the triples of apples or over the small quantities of money has undergone essentially different binding.

We shall carefully use the pattern '$n^\epsilon A$', always remembering that it has been defined by existential binding.

1^ϵ can be used as *indefinite article*.

3 28 2. $^\# Ha \, Inq \, Hb \cdot ? x . x = 110/100 \vdots$

$Hb \, Inq \, Ha \, 11/10 \vdots$

$Hc \, Inq \, Hd \cdot ? y . PAN \, Fit \, y \vdots$

$Hd \, Inq \, Hc \cdot PAN : Ha \, Inq \, Hb \cdot ? x . Etc \frown Hb \, Rsp \, Ha \vdots$

$Hc \, Inq \, Hd . PAN \, Ha \, Hb \, Ise \, Inq \vdots$

$Hd \, Inq \, Hc \, Ben \, ^\#$

The word written Ise ($\int L$ inter se $=$ among themselves) means "to each other". '$Ha \, Hb \, Ise \, Inq$' is to be translated "Ha and Hb have a talk".

3 28 3. $*$ Ha Inq $Hb\,\mathfrak{p}$. Hb Inq $Hc\,\mathfrak{q}$. Hc Inq $Ha\,\mathfrak{r}$.
          Ha Inq $Hb\smile Hc\smile Hd\,\mathfrak{s}$. Hd Inq $Ha\,\mathfrak{t}$:
          He Inq $Hf\cdot\mathfrak{?}\,x$. PAN Fit x :
          Hf Inq He . PAN Ha Hb Hc Hd Ise Inq $*$

3 28 4. $*$ Ha Inq $Hb\,\mathfrak{p}$. Hb Inq $Hc\,\mathfrak{q}$. Hc Inq $Ha\,\mathfrak{r}$.
          Ha Inq $Hb\,\mathfrak{s}$. Hb Inq $Hc\,\mathfrak{t}$. Etc $\cdot$
          He Inq $Hf\cdot$ PAN Ha Hb Hc Alt Inq $*$

The word written Alt (fL alternando = alternating) means "alternating".

3 29 1. $*$ $^{\mathrm{h}}Ha$ Inq $Hb\cdot\mathfrak{?}\,x$. 101 $x=1111^{\mathrm{h}}$— — —:
          Ha Inq Hc : $\neg$ Nnc Hb Sci $\cdot\mathfrak{?}\,x$. 101 $x=1111$:
          Hc Inq Ha Cur :
          Ha Inq Hc . $\neg$ PAN Hb Rsp :
          Hc Inq $Ha\cdot$ MAN Usq Nnc . Hb Sci : Tan $\cdot$ $\neg$ Pot . Nnc Hb Rsp :
          Ha Inq Hc Cur :
          Hc Inq $Ha\cdot$ $\neg\,t_1\,t_2\,Hb$ Ani : $\mathfrak{?}\,u$. Nnc Ha Inq $Hb\,u$ $*$

Compare the remark in 3 20 6.

3 29 2.   A variation on 3 29 1 with $\neg$ Itg instead of $\neg$ Ani.

3 29 3. $*$ Ha Inq $Hb\cdot\mathfrak{?}\,x$. 101 $x=1111$ — — —
          Ha Inq $Hb\cdot$ Utr . Nnc Hb Sci Etc :
          Hb Inq Ha . Nnc Hb Sci Etc :
          Ha Inq Hb : Utr $\cdot$ $\neg$ Pot . Nnc Hb Rsp :
          Hb Inq $Ha\cdot$ Pot . Nnc Hb Rsp :
          Ha Inq Hb : Cur $\cdot\neg$. Nnc Hb Rsp :
          Hb Inq Ha : Qia $\cdot$ Nnc Hb Vul :
          $\neg$ Nnc Hb Inq ! $Ha\cdot\mathfrak{?}\,x$. 101 $x=1111$ $*$

The word written Vul (fL vult = wishes) means (the verb) "wishes". More texts are needed in order to grasp this meaning, but even now the receiver can conclude that somebody might refuse to answer a question though able to do so.

'Vul' obeys the following syntactical rules: As in the case of 'Pot' and 'Ncs' the object (the thing wanted) is to be a (possibly elliptical) proposition, not an event. Yet the use of 'Vul' differs from that of 'Pot' and 'Ncs' insofar as Ha Vul p is to be treated as an event. Of course '$t_1\,t_2\,Ha$ Vul p' is a report, hence a proposition.

By the use of 'Vul' neither a (more or less mysterious) state of mind of the person wishing nor an introspection of the person saying "I wish" is aimed at. 'Vul' describes the behaviour of

people who use this word explicitly in the first texts that follow. Later on it will appear that a person may show the behaviour of wishing though he does not explicitly state "I wish". In the case of 'Vul' we shall proceed in a way inverse to that we followed when treating 'Enu', 'Cpu', and so on. There we started with instances of the behaviour of counting and computing, and afterwards introduced words to describe these activities. Now we use 'Vul' from the beginning, whereas in a more advanced stage we shall learn to do without it.

Note that as a symptom the explicit use of 'Vul' is not sufficient to prove the fact of wishing. It may happen that someone claims to wish something though it is clear by circumstancial evidence that he does not wish at all or that he wishes something else. In this respect statements preceded by "I wish" do not differ from other statements about other people or about the speaker himself. Statements may always be false, intentionally or unintentionally.

$*$ *Ha* Inq *Hb* $\cdot$? *x* . $t_1 t_2$ Fit *x* ⫶
　Hb Inq *Ha* ⫶ ⫟ ⫶ ⋁*y* ⫶ $t_1 t_2$ *y* Ani $\cdot$? *x* . $t_1 t_2$ Fit *x* ⫶ ∧ . *y* ≠ *Hc* ⫶
　Ha Inq *Hc* $\cdot$? *x* . $t_1 t_2$ Fit *x* ⫶
　Hc Inq *Ha* ⫶ ⫟ $\cdot$ Nnc *Hc* Vul ⫶ Nnc *Ha* Inq ! *Hc* $\cdot$? *x* . $t_1 t_2$ Fit *x* ⫶
　Ha Inq *Hb* ⫶ ⫟ ⫶ Nnc *Hc* Vul $\cdot$ Nnc *Ha* Sci . ? Etc ⫶
　Hc Inq *Ha* ⫶ Eti $\cdot$ Nnc *Hc* Vul ⫶ ⫟ $\cdot$ Nnc *Ha* Sci . ? Etc ⫶
　Ha Inq *Hb* ⫶ Erg $\cdot$ ⫟ ⫶ Pot $\cdot$ Nnc *Ha* Sci . ? Etc $*$

3 29 4.　$*$ *Ha* Inq *Hc* $\cdot$ Nnc *Ha* Vul . PPN *Hc* Inq *Hd* 𝔭 ⎸
　Hc Inq *Hd* 𝔭 ⎸
　Ha Inq *Hc* Ben ⎸
　Hb Inq *Ha* $\cdot$ Nnc *Ha* Vul . PPN *Hc* Etc $\cdot$ ⟶ . PPN *Hc* Etc ⫶
　　Nnc *Ha* Vul *q* . ⟶ . PPN *Hc* Vul *q* ⎸
　Ha Inq *Hb* $\cdot$ Nnc *Ha* Vul . PPN *Hc* Inq *Hb* 𝔯 $\cdot$
　　Tan . ⫟ PPN *Hc* Inq *Hb* 𝔯 ⫶
　　Qia ⫶ ⫟ Nnc *Hb* Ani $\cdot$? *x* . Nnc *Ha* Inq *Hc* *x* ⎸
　Hb Inq *Ha* ⫶ Nnc *Ha* Vul *q* . ∧ ⫶ Nnc *Hb* Sci $\cdot$
　　Utr . Nnc *Ha* Vul *q* $\cdot$ ⟶ . PPN *Hc* Vul *q* ⎸
　Hb Inq *Hc* ⫶ Cur $\cdot$ *t* *Ha* Vul *q* . ⟶ ⫶ Pau Pst . *t* $\cdot$ *Hc* Vul *q* ⎸
　Hc Inq *Hb* ⫶ *t* *Ha* Vul . PPN Fit *s* $\cdot$ ⟶ $\cdot$ Pau Pst . *t* $\cdot$ Fit *s* ⫶ ⟶ . *s* ∈ Ben ⎸
　Ha Inq *Hc* ⫶ Nnc *Ha* Vul $\cdot$ PPN *Hc* Inq *Hb* . 10 × 10 = 101 ⎸
　Hc Inq *Hb* . 10 × 10 ≠ 101 ⎸
　Ha Inq *Hc* Mal ⎸
　Hc Inq *Ha* ⫶ ⫟ Pot $\cdot$ *t* *Ha* Vul ⫶ Pau Pst . *t* $\cdot$ *Hc* Inq *Hb* . 1$^{\text{ε}}$ Fal ⫶
　　Erg $\cdot$ ⫟ Nnc *Ha* Vul $\cdot$ PPN *Hc* Inq *Hb* . 10 × 10 = 101 ⫶

Sed Vul Etc $\neq$ ᛁ

Ha Inq *Hc* $\cdot$ Pot $. t$ *Ha* Vul ᛁ

Hc Inq *Hb* ˙ PAN *Ha* Vul $\cdot$ PPN *Hc* Inq *Hb* . $10 \times 10 = 101$:

    Erg $\cdot$ Nnc *Hc* Inq *Hb* . $10 \times 10 = 101$: Tan Fal ᛁ

Ha Inq *Hc* $\cdot$ Nnc *Ha* Vul . $10 \times 10 = 101$ ᛁ

Hc Inq *Ha* ⦂ $\neg$ Pot ⦂ Qqm $\cdot$ PAN *Ha* Inq *Hc* . Nnc *Ha* Vul :

    Tan . $\neg$ Nnc *Ha* Vul ⦂

    Qia ˙ $^\curlyvee w$ $\cdot$ PAN *Ha* Inq *Hc* . Nnc *Ha* Vul w ⦂ $\in$ Fal ᛁ

Ha Inq *Hc* Ben ᛁ

Hc Inq *Ha* ˙ Utr ⦂ Nnc *Ha* Vul $\cdot$ $\vee x$. Nnc *Hc* Inq *Hb* x ᛁ

Ha Inq *Hc* . $\neg$ Nnc *Ha* Vul ᛁ

Hc Inq *Ha* ˙ Utr ⦂ Nnc *Ha* Vul $\cdot$ $\neg$ $\vee x$. Nnc *Hc* Inq *Hb* x ᛁ

Ha Inq *Hc* . $\neg$ Nnc *Ha* Vul ᛁ

Hb Inq *Hb* $\cdot$ $\neg$ $\vee x$. Nnc *Ha* Vul x ᛁ

Ha Inq *Hb* . PPN *Hc* Inq *Hd* ꝑ ᛁ

Hb Inq *Ha* ˙ ? u ⦂ Nnc *Ha* Ani $\cdot$ Utr . PPN *Hc* Inq *Hd* ꝑ $\cdot$ Mod u ᛁ

Ha Inq *Hb* ⦂ $\neg$ $\vee u$ Etc Mod u ˙

    Qia ⦂ $\neg$ Pot $\cdot$ Nnc *Ha* Ani . Utr PPN Etc ᛁ

Hb Inq *Ha* ˙ ? u ⦂ Nnc *Ha* Sci $\cdot$ Utr . PPN *Hc* Inq *Hd* ꝑ $\cdot$ Mod u ᛁ

Ha Inq *Hb* ˙ Mod $\cdot$ Nnc *Ha* Vul . PPN *Hc* Inq *Hd* ꝑ :

    Qia $\cdot$ Nnc *Ha* Vul q . $\rightarrow$. PPN *Hc* Vul q ᛁ

Ha Inq *Hc* $\cdot$ Nnc *Ha* Vul . PPN *Hc* Inq *Hd* ꝑ ᛁ

Hc Inq *Hd* ꝑ ᛁ

He Inq *Hb* . PPN *Hc* Inq *Hd* ᚱ ᛁ

Ha Inq *Hb* . $\neg$ PPN *Hc* Inq *Hd* ᚱ ᛁ

He Inq *Hc* $\cdot$ Nnc *He* Vul . PPN *Hc* Inq *Hd* ᚱ ᛁ

Ha Inq *Hc* $\cdot$ Nnc *Ha* Vul . $\neg$ PPN *Hc* Inq *Hd* ᚱ ᛁ

Hc Inq *Hb* ˙ $^\curlyvee x$ $\cdot$ Nnc *He* Vul . PPN Fit x :

      $\neq$ ⦂ $^\curlyvee x$ $\cdot$ Nnc *Ha* Vul . PPN Fit x ˙ Erg $\cdot$ *He* Vul Etc . $\in$ Mal ᛁ

Hb Inq *Hc* $\cdot$ Nnc *Hb* Vul . PPN *Hc* Inq *Hd* ɑ ᛁ

Hc Inq *Ha* ⦂ Utr $\cdot$ Nnc *Ha* Vul . $\neg$ PPN *Hc* Inq *Hd* ɑ ᛁ

Ha Inq *Hc* . $\neg$ Nnc *Ha* Vul ᛁ

Hc Inq *Ha* ⦂ Utr $\cdot$ Nnc *Ha* Vul . PPN *Hc* Inq *Hd* ɑ ᛁ

Ha Inq *Hc* . $\neg$ Nnc *Ha* Vul ᛁ

Hb Inq *Hc* ˙ Nnc *Ha* Ced ⦂ Nnc *Hc* Vul $\cdot$ $^\curlyvee x$. Nnc *Hb* Vul x ᛁ

Hc Inq *Hb* ˙ $\wedge$ Ced ⦂ $\neg$ Nnc *Hc* Vul $\cdot$ $^\curlyvee x$. Nnc *Hb* Vul x ᛁ

Hc Inq *Hd* ɑ *

The word written Ced (*ʃL* concedit = allows) means "allows", "suffers". Syntactically it behaves as 'Vul'. The thing allowed is to be a proposition, though '*Ha* Ced Etc' is to be treated as an event. Again 't_1 t_2 *Ha* Ced Etc' is a proposition. The relation between 'Vul' and 'Ced' is akin to that between 'Nes' and 'Pot'.

3 29 5.   * Hb Inq Hb ˙

  $tt'\,Ha$ Ccd Etc . $\leftrightarrow$ ˙ $\neg\,tt'\,Ha$ Vul . $\neg$ Etc :

  $\neg\,tt'\,Ha$ Ccd Etc . $\leftrightarrow$ ˙ $tt'\,Ha$ Vul . $\neg$ Etc :

  $\neg\,tt'\,Ha$ Ccd . $\neg$ Etc ˙ $\leftrightarrow$. $tt'\,Ha$ Vul Etc :

  $tt'\,Ha$ Ccd . $\neg$ Etc ˙ $\leftrightarrow$ ˙ $\neg\,tt'\,Ha$ Vul Etc *

3 29 6. Ha Inq Hd ˙ Nnc Ha Vul . Nnc Hd Inq Hc ꝑ ⦂

  Hd Inq Ha . $\neg$ Nnc Hd Inq Hc ꝑ ⦂

  Ha Inq Hd Cur ⦂

  Hd Inq Ha . $\neg$ Ncs ⦂

  Ha Inq Hd ˙ Tan . Nnc Ha Vul ⦂

  Hd Inq Ha Cur ⦂

  Ha Inq Hd : Nnc Ha Vul ˙ Nnc Ha Sci . Utr ꝑ ⦂

    Qqm ˙ $\neg$ Pot : Nnc Hc Ani ˙ ? x . Nnc Ha Inq $Hc\,x$ ˙

    Tan ˙ Pot : Nnc Hc Ani ˙ ? x . Nnc Hd Inq $Hc\,x$ ⦂

  Hd Inq Hc ꝑ ⦂

  Hc Inq Hd : ? u ˙ Unq . Ant Nnc . Hd Cog . Utr ꝑ . Mod u ⦂

  Hd Inq Hc ˙ PAN Ha Vul . PPN Hd Inq Hc ꝑ ⦂

  Hc Inq Hd : Nnc Hc Vul ˙ PPN Hd Inq Ha . PAN Hc Cog Utr ꝑ ⦂

  Hd Inq Ha . Nnc Hc Sci Utr ꝑ *

3 29 7.   * Ha Inq He ˙ Nnc Ha Vul . Nnc He Inq Hc ꝑ ⦂

  He Inq Ha . $\neg$ Nnc He Vul ⦂

  Ha Inq He ⦂ $\neg$ Pot : Nnc Hc Ani ˙ ? x . Nnc Ha Inq $Hc\,x$ ˙

    Erg ˙ Nnc Ha Vul . Nnc He Inq Hc ꝑ ⦂

  He Inq Ha ˙ Tan : $\neg$ Nnc He Vul ˙ ⌐x . PPN Ha Vul x ⦂

  Ha Inq Hd ⦂ Utr ˙ Nnc Hd Vul : Nnc Hd Inq He ˙

    Nnc Hd Vul . Nnc He Inq Hc ꝑ ⦂

  Hd Inq Ha Ver ⦂

  Hd Inq He ˙ Utr Nnc He Vul . Nnc He Inq Hc ꝑ ⦂

  He Inq Hc ꝑ *

3 29 8.   * Ha Inq Hb . ? $=$ Itg ⦙

  Hb Inq Ha ⦂ Exg ⦂ Nnc Hc Inq $Hd\,p$.

    $\rightarrow$ ˙ Nnc Hd Itg ˙ ? x . Nnc Hc Inq $Hd\,x$:

      $\leftrightarrow$: Nnc Hd Sci ˙ ? x . Nnc Hc Inq $Hd\,x$ ⦙

  Ha Inq Hb ⦂ PAN Hc Inq Hd ˙ ? x . $t_1\,t_2$ Ftt x :

    $\wedge$: Pau Pst . PAN ˙ Hd Inq Hc . $t_1\,t_2$ Fit ɑ ˙

    Utr : PAN Hd Itg ˙ ? y . PAN Hc Inq $Hd\,y$ ⦙

  Hb Inq Ha ⦂ Ver : Nnc Hd Itg ˙ ? x . Nnc Hc Inq $Hd\,x$:

    $\leftrightarrow$ ˙ Nnc Hd Sci : ? x ˙ Nnc Hc Vul . Nnc Hc Inq $Hd\,x$ *

3 29 9.   Comments on 3 23 1–3 23 4 in order to state that somebody "can
    if he will". E.g. on 3 23 4:

$*$ *Ha* Inq *Ha* ⦂ *tt′* *x* Sci . ? Etc ·
    ↔ ˙ *tt′* *x* Vul · Nnc *x* Inq ! *y* . ? Etc : → · *tt′* *x* Inq ! *y* . ? Etc $*$

3 30 1. $*$ ᵗ°*Ha* Inq *Hb* ˙ Utr : Nnc *Hb* Sci · ? *x* . t₁ t₂ Fit *x* |
    Hb Inq *Ha* ı Ver ⦂ Tan ˸ Nnc *Hb* Vul ⦂
      ∧ ⌜*t* . *t′*⌝ ˙ ⌐ : *tt′* *Hc* Sci · ? *x* . t₁ t₂ Fit *x* |
    Ha Inq *Hb* : Nnc *Ha* Vul ˙ PPN *Hb* Inq ! *Ha* · ? *x* . t₃ t₂ Fit *x* ⦂
      → ˸ ∧ ⌜*t* . *t′*⌝ ˙ ⌐ : *tt′* *Ha* Inq ! *Hc* · ? *x* . t₁ t₂ Fit *x* |
    Hb Inq *Ha* ˸ ˇ*v* . Nnc *Hb* Vul *v* · Mul > · ˇ*u* . Nnc *Ha* Vul ⦂
      Ha Vul ˙ ⌐ : *tt′* *Ha* Inq ! *Hc* Etc ⦂
      Eti : *Hb* Vul · ⌐ ∨ ⌜*t* . *t′* . *y*⌝ . *Ha* Inq ! *y* Etc ⦂
      ⌐ Sat · ⌐ ∨ ⌜*t* . *t′*⌝ *Ha* Inq ! *Hc* Etc ⦂
      Qia ˙ Pot : ∨*x* · *x* ≠ *Ha* . ∧ . UPN *x* Inq ! *Hc* Etc |
    Ha Inq *Hb* ˙ Nnc *Ha* Pol *Hb* . ⌐ ∨ ⌜*t* . *t′* . *y*⌝ Etc |
    Hb Inq *Ha* . t₁ t₂ Fit α |
    Hd Inq *Ha* ˙ Utr : Nnc *Ha* Sci · ? *x* . t₁ t₂ Fit *x* |
    ᵗ°*Ha* Inq *Hd* ˸ Ver : Tan ˙ PAN *Ha* Pol *Hb* ⦂
      ⌐ UPN *Ha* Inq ! *Hd* · ? *x* . t₁ t₂ Fit *x* |
    Hd Inq *Ha* · Utr . Nnc *Ha* Inq ! *He* Etc |
    Ha Inq *Hd* : Eti : PAN *Ha* Pol *Hb* ·
      ⌐ ∨ ⌜*t* . *t′* . *y*⌝ . *tt′* *Ha* Inq ! *y* Etc ˙
      Erg : Nnc *Ha* Vul · ⌐ ∨ ⌜*t* . *t′* . *y*⌝ . *tt′* *Ha* Inq ! *y* Etc $*$

The word written Pol (*∫L* pollicetur = promises) means the verb "promises". Syntactically it behaves as 'Inq'.

The word written Sat (*∫L* satis = enough) means "enough".

3 30 2. $*$ *Ha* Inq *Ha* ⦂ *tt′* *Ha* Inq *Hb* ⦂
      Nnc *Ha* Vul ˙ Etc : *h* Pst . *t′* · *Ha* Inq Etc ⦂
    ↔ ⦂ *tt′* *Ha* Pol *Hb* ˙ Etc : *h* Pst . *t′* · *Ha* Inq Etc $*$

Explicit definition of 'Pol'.

3 30 3.   (t₁ < t₂ < t₃ < t₄)
    $*$ *Ha* Inq *Hb* : Utr Nnc *Hb* Sci · ? *y* . t₃ t₄ Fit *y* ⦂
    Hb Inq *Ha* Ver ⦂
    Ha Inq *Hb* ⦂ Nnc *Ha* Vul ˙
      Nnc *Hb* Inq ! *Ha* · ? *y* . t₃ t₄ Fit *y* ⦂
      → : PPN *Ha* Inq ! *Hb* · ? *x* . t₁ t₂ Fit *x* ⦂
    Hb Inq *Ha* : Nnc *Hb* Vul · ˇ*z* . PAN *Ha* Vul *z* ⦂
    Ha Inq *Hb* Ben ⦂
    Hb Inq *Ha* : t₃ t₄ Fit ƀ · ? *x* . t₁ t₂ Fit *x* ⦂
    Ha Inq *Hb* . t₁ t₂ Fit α ⦂
    Hc Inq *Ha* ˙ Cur : *Ha* Inq ! *Hb* · ? *x* . t₁ t₂ Fit *x* ⦂
    Ha Inq *Hc* : Qia · PAN *Ha* Pol *Hb* . Etc $*$

3 30 4. We repeat the greater part of 3 30 3, but now Ha refuses to answer.

Ha Inq $Hb \cdot \neg$ Nnc Ha Vul . Nnc Ha Inq ! Hb Etc $\vdots$
Hb Inq $Ha \cdot$ PAN Ha Pol Hb . PPN Ha Inq ! Hb Etc $\vdots$
Ha Inq $Hb \vdots$ Fal $\vdots$ PAN Ha Pol $\cdot$ Nnc Hb Inq ! $Ha \cdot ? y . t_3 t_4$ Fit y :
$\quad \rightarrow :$ PPN Ha Inq ! $Hb \cdot ? x . t_1 t_2$ Fit $x \vdots$
Hb Inq $Ha \cdot$ PAN Hb Inq $Ha . t_3 t_4$ Fit $b \vdots$
$^{t_5} Ha$ Inq $Hb \cdot \neg$ PAN Ha Ani $\cdot$ Utr . Hb Inq Ha Etc $:^{t_6}$
$\quad t_1 t_2$ Fit $a \vdots$
Hb Inq Ha Ben $\vdots$
Hc Inq $Hc \cdot t_5 t_6 Ha$ Inq $Hb u . \rightarrow . u \in$ Fal $*$

3 31 1. We introduce the abbreviations:

$* t . p \rightarrow q *$ instead of $* t$ Fit $p . \rightarrow . t$ Fit $q *$

$* t \cdot p . \rightarrow h$ Pst $. q *$ instead of $* t$ Fit $p . \rightarrow : h$ Pst $. t \cdot$ Fit $q *$

$* t \cdot p . \rightarrow h$ Ant $. q *$ instead of $* t$ Fit $p . \rightarrow : h$ Ant $. t \cdot$ Fit $q *$

This can be done by means of repetitions of former program texts and finally by means of a behaviour rule.

As a matter of course the arrow in '$t . p \rightarrow q$' does not state an implication from p to q, because p and q do not mean propositions, but events. If any doubt about the character of p and q is possible, it is better not to use this abbreviation. Likewise the "retarded and retroactive implications" '$\rightarrow h$ Pst' and '$\rightarrow h$ Ant' are not proper implications.

Note that

$$'t \cdot p . \rightarrow h \text{ Pst} . q'$$

cannot be expressed by

$$'q . h \text{ Pst} . p' \text{ or } h \text{ Pst} . p \cdot \text{Fit} q'$$

because in the latter sentences all variables are existentially bound, whereas the t in the first sentence is bound by generalizing.

3 32 1. A comment on 3 30 3 and 3 30 4:

$* Hb$ Inq $Ha \vdots tt' Ha$ Pol $Hb : h$ Pst $. t' \cdot Ha$ Inq Etc $\cdot$
$\quad \rightarrow \cdot$ Deb $: h$ Pst $. t' \cdot Ha$ Inq Etc $\vdots$
Ha Inq $Hb \vdots tt' Ha$ Pol $Hb \cdot$
$\quad t''$ Fit $p . \rightarrow : h$ Pst $. t'' \cdot Ha$ Inq Etc $\vdots$
$\quad \rightarrow \vdots$ Deb $\cdot t''$ Fit $p . \rightarrow : h$ Pst $. t'' \cdot Ha$ Inq Etc $*$

The word written Deb (*fL* debet = is obliged, ought) means "obliged". Syntactically it behaves as 'Ncs' and 'Pot'. In 'Deb Etc', 'Etc' is to mean a proposition. 'Deb Etc' itself means a proposition. Semantically 'Ncs' and 'Deb' differ. This difference will be analyzed later on.

An analogous comment may be added to 3 30 1.

3 32 2. The text 3 30 1 from t_3 onwards is changed into

$$* Ha \operatorname{Inq} Hd : \operatorname{Ver} : \operatorname{Tan} \cdot \neg \operatorname{Lic} : \operatorname{UPN} Ha \operatorname{Inq} ! Hd \cdot ? x . t_1 t_2 \operatorname{Fit} x :$$
$$Hd \operatorname{Inq} Ha \cdot \operatorname{Cur} . \neg \operatorname{Lic} :$$
$$Ha \operatorname{Inq} Hd : \operatorname{Qia} \cdot \operatorname{PAN} Ha \operatorname{Pol} Hb . \neg \operatorname{UPN} \operatorname{Etc} *$$

and so on.

The word written Lic (*fL* licet = it is allowed that) means "it is allowed (not forbidden) that". Syntactically it behaves like 'Ncs', 'Pot', 'Deb'. Semantically it differs from 'Pot'. 'Ccd' (allows) is both semantically and syntactically different from 'Lic' (it is allowed).

3 32 3. $* Ha \operatorname{Inq} Ha \cdot$
$$\operatorname{Lic} \operatorname{Etc} . \leftrightarrow \cdot \neg \operatorname{Deb} . \neg \operatorname{Etc} :$$
$$\neg \operatorname{Lic} \operatorname{Etc} . \leftrightarrow \cdot \operatorname{Deb} . \neg \operatorname{Etc} :$$
$$\neg \operatorname{Lic} . \neg \operatorname{Etc} \cdot \leftrightarrow . \operatorname{Deb} \operatorname{Etc} :$$
$$\operatorname{Lic} . \neg \operatorname{Etc} \cdot \leftrightarrow . \neg \operatorname{Deb} \operatorname{Etc} *$$

3 32 4. $* Ha \operatorname{Inq} Ha :$
$$\neg \operatorname{Lic} \operatorname{Etc} . \rightarrow \cdot x \operatorname{Vul} \operatorname{Etc} . \in \operatorname{Mal} *$$

3 32 5. The receiver will not grasp the meaning of 'Deb' as long as promises (Pol) seem to be the only sources of obligations (Deb). Other instances are indispensable.

The next talk is related to that of 3 29 4.

$$* {}^{t_2}Hb \operatorname{Inq} Ha : t : Ha \operatorname{Vul} q . \wedge : Hc \operatorname{Sci} \cdot \operatorname{Utr} . \operatorname{Nnc} Ha \operatorname{Vul} q \cdot$$
$$\rightarrow \cdot \operatorname{Pau} \operatorname{Pst} . Hc \operatorname{Vul} q \mid$$
$$Hc \operatorname{Inq} Ha \cdot ? x . t_1 t_2 \operatorname{Fit} x \mid$$
$$Ha \operatorname{Inq} Hc \iota t_1 t_2 \operatorname{Fit} \mathfrak{a} : \operatorname{Tan} : \operatorname{Nnc} Ha \operatorname{Vul} \cdot$$
$$\wedge \ulcorner t . t' . y \urcorner : tt' \cdot y \operatorname{Inq} Hc \cdot ? x . t_1 t_2 \operatorname{Fit} x :$$
$$\rightarrow \operatorname{Pau} \operatorname{Pst} . Hc \operatorname{Inq} y \mathfrak{b} \mid$$
$$Hc \operatorname{Inq} Ha : \neg \operatorname{Pot} \cdot {}^{\curlyvee} w . \operatorname{PAN} Ha \operatorname{Inq} Hc w \mid$$
$$Ha \operatorname{Inq} Hc \cdot \operatorname{Cur} . \neg \operatorname{Pot} \mid$$
$$Hc \operatorname{Inq} Ha : t_1 t_2 \operatorname{Fit} \mathfrak{a} . \wedge . \mathfrak{a} \neq \mathfrak{b} : \operatorname{Erg} \cdot t_1 t_2 \operatorname{Fit} \mathfrak{b} . \in \operatorname{Fal} \cdot$$
$$\operatorname{Erg} \cdot \neg \operatorname{Lic} : \vee \ulcorner t . t' . y \urcorner \cdot tt' Hc \operatorname{Inq} y . t_1 t_2 \operatorname{Fit} \mathfrak{b} :$$
$$\neg \operatorname{Lic} v . \rightarrow \cdot \neg \operatorname{Pot} . \operatorname{Unq} Ha \operatorname{Vul} v :$$

Erg $\vdots \neg$ Pot $\cdot$ PAN Ha Vul $: \vee \ulcorner t \cdot t' \cdot y \urcorner \cdot Hc$ Inq $y \cdot t_1 t_2$ Fit $\flat$ ❙

Ha Inq $Hc \vdots$ Pot $\cdot$ Qqm $. \neg$ Lic $\vdots$

  $\neg$ Lic $u . \rightarrow \dot{} \neg$ Lic $.$ Nnc Ha Vul $u \cdot$

  $\wedge : \neg$ Lic $\cdot$ Nnc Ha Inq $Hc .$ Nnc Ha Vul $u \vdots$

  Tan $:$ Lic $\cdot tt'$ Hc Inq $Hy \cdot t_1 t_2$ Fit $\flat$ ❙

Hc Inq $Ha \dot{} \neg$ Nnc Hc Itg $: t_1 t_2$ Fit $\flat . \in$ Fal $:$ Erg Etc $:$ Erg $. \neg$ Lic Etc ❙

Ha Inq $Hc \cdot t_1 t_2$ Fit $\flat . \in$ Ver ❙

Hc Inq $Ha :$ Qqm $\cdot$ PAN Ha Inq $Hc . t_1 t_2$ Fit α ❙

Ha Inq $Hc \dot{}$ Deb $\cdot$ PAN Ha Inq $Hc . t_1 t_2$ Fit $\flat :$

  PAN Ha Vul $.$ Nnc Ha Inq Hc $\flat :$

  Tan $.$ PAN Ha Inq Hc $\alpha :$

  PAN Ha Inq $Hc . \flat .$ Mod Mal ❙

Hd Inq $Hd \vdots$ Pot $\cdot$ PAN Ha Inq $Hc . \flat .$ Mod Mal $:$

  Tan $. \neg$ Ncs Etc $\dot{}$

  Pot $\cdot$ PAN Ha Vul $.$ Nnc Ha Inq Hc $\alpha :$ Qqm $. \neg$ Lic ✳

3 32 6. ✳ Ha Inq $Hb : ? 1^{um} x \cdot x \in$ Pri $. \wedge . x > 10^{10010} \vdots$

  Hb Inq $Ha \cdot$ Cur $. Ha$ Inq $Hb :$

  Ha Inq $Hb \dot{}$ Qia $: \neg$ Nnc Ha Sci $.$ Tan $\cdot$ Nnc Ha Vul $.$ Nnc Ha Sci $\vdots$

  Hb Inq $Hc : ? 1^{um} x \cdot x \in$ Pri $. \wedge . x > 10^{1001} \vdots$

  Hc Inq Hb 1000001001 $\vdots$

  Hb Inq Ha 1000001001 $\vdots$

  Hd Inq $Hd \cdot$ Lic $.$ PAN Hb Inq Hc Etc ✳

3 32 7. ✳ Ha Inq $Hb : ? 1^{um} x \cdot x \in$ Pri $. \wedge . x > 10^{1001} \vdots$

  Hb Inq $Hc : ? 1^{um} x \cdot x \in$ Pri $. \wedge . x > 10^{1001} \vdots$

  Ha Inq $Hc \dot{}$ Nnc Ha Vul $. \neg$ Nnc Hc Rsp $Hb \cdot$

    $\neg$ Lic $.$ PAN Hb Inq Hc Etc $\vdots$

  Hc Inq $Ha :$ Utr $\cdot$ PAN Ha Sci $. ? 1^{um}$ Etc $\vdots$

  Ha Inq Hc Ver $\vdots$

  Hc Inq $Ha \dot{}$ Cur $.$ PAN Ha Inq Hb Etc $\vdots$

  Ha Inq $Hc \vdots \neg$ PAN Ha Vul $\cdot$ Nnc Ha Sci $. ? 1^{um}$ Etc $\vdots$

    Sed $\dot{}$ PAN Ha Vul $:$ Nnc Ha Sci $\cdot$ Utr Nnc Hb Sci $. ? 1^{um}$ Etc $\vdots$

    Erg $\cdot \neg$ Lic $.$ PAN Hb Inq Hc Etc ✳

3 32 8. ✳ Ha Inq $Hb : ? 1^{um} x \cdot x \in$ Pri $. \wedge . x > 10^{1001} \vdots$

  Hc Inq $Hb \cdot$ 1000001001 $\cdot$ Mod $. \neg Ha$ Ani $\vdots$

  Hb Inq Ha 1000001001 $\vdots$

  Ha Inq Hb Ben $\vdots$

  Hd Inq $Hd \dot{}$ Nnc Hb Vul $:$ Nnc Ha Inq $Ha \cdot$ Nnc Hb Sci $. ? 1^{um}$ Etc $:$

    Qqm $\neg$ Sci $\vdots$

  Hd Inq $Hb :$ Mal $\dot{} \neg$ Lic $:$ Nnc Hb Vul $\cdot$ Nnc Ha Inq $Ha . 1^{e}$ Fal ✳

3 32 9. $(t_1 < t_2 < t_3 < t_4 < t_5)$

     # ^{t_5}Ha Inq $Hb \cdot ? x . t_3 t_4$ Fit x ⌇

        Hb Inq $Ha \cdot \neg$ Nnc Hb Vul : Nnc Hb Inq ! $Ha \cdot ? x . t_3 t_4$ Fit x ⌇

        Ha Inq Hb : Nnc Ha Vul . Nnc Ha Sci Etc :

           PAN Hb Inq ! $Ha \cdot ? y . t_1 t_2$ Fit y :

           Qqm · Pau Ant . PAN · Hb Pol Hc :

              $\neg$ UPN Hb Inq ! $Ha \cdot ? y . t_1 t_2$ Fit y :

              $\neg$ Nnc Hb Vul : Nnc Hb Inq ! $Ha \cdot ? x . t_3 t_4$ Fit x ·

                $\rightarrow$: PPN Ha Inq ! Hc · Utr : UAN Hb Inq ! $Ha \cdot ? y . t_1 t_2$ Fit y :

                  ∧ : Pau Pst . PPN · Hc Inq Hb Mal ⌇

        Hd Inq $Ha \cdot \neg$ Lic : Nnc Ha Inq $Hb \cdot {}^\curlyvee w$. Nnc Ha Inq Hb w #

Many examples like this may be constructed in order to explain the use of 'Deb' and 'Lic'.

3 33 1.   # ^{t_1}Ha Inq $Hb . ? = 10 \times 10^{t_2}$ ⌇

        Hb Inq Ha 100 ⌇

        Hc Inq $Hb \cdot ? x . t_1 t_2$ Fit x ⌇

        Hb Inq Hc : $t_1 t_2$ Ha Vul · Nnc Ha Sci . $? = 10 \times 10$ ⌇

        Hc Inq Hb : Fal . Qia · $t_1 t_2$ Ha Sci . $? = 10 \times 10$:

           $\neg$ $t_1 t_2$ Ha Vul . Nnc Ha Sci Etc ·

           Sed : $t_1 t_2$ Ha Vul · Nnc Ha Sci . Utr Nnc Hb Sci Etc ⌇

        Hb Inq Hc : Etc : Pot : $t_1 t_2$ Ha Vul Sci ·

           Utr : $t_1 t_2$ Hb Ani · $? y . t_1 t_2$ Ha Inq Hb y ⌇

        Hc Inq Hb Ben #

From this text and the next ones the receiver may learn how to prove 'Ha Vul Etc' by circumstantial evidence though Ha did not use the word Vul.

3 33 2. $(t_1 < t_2 < t_3)$

     # ^{t_5}Ha Inq $Hb \cdot ? x . t_1 t_2$ Fit x^{t_4} ⎮

        Hb Inq Ha α ⎮

        Hc Inq $Hb \cdot ? y . t_3 t_4$ Fit y ⎮

        Hb Inq Hc · $t_3 t_4$ Ha Vul : Nnc Ha Sci · $? x . t_1 t_2$ Fit x ⎮

        Hc Inq Hb : $? u$: PAN Hb Cog : Utr · $t_3 t_4$ Ha Vul :

           Nnc Ha Sci · $? x . t_1 t_2$ Fit x : Mod u ⎮

        Hb Inq Hc : $t_3 t_4$ Ha Inq $Hb \cdot ? x . t_1 t_2$ Fit x ⎮

        Hc Inq Hb ⌇ $\neg$ Ncs . $t_3 t_4$ Ha Vul Etc :

           Sed : Pot : $t_3 t_4$ Ha Sci Etc . ∧ · $t_3 t_4$ Ha Vul :

           Nnc Ha Sci · Utr . Nnc Hb Sci Etc #

3 33 3.   # ^{t_1}Ha Inq Hb Hb^{t_2} :

        Hb Inq Ha : Nnc Hb Ani · $? x . t_1 t_2$ Fit x :

Hc Inq $Hb \cdot \text{?} x . t_1 t_2$ Fit x :
Hb Inq Hc : $t_1 t_2$ Ha Vul : Nnc Ha Sci ·
　　Utr : Nnc Hb Ani · ? $x . t_1 t_2$ Fit x :
Hc Inq Hb · ? u : PAN Hb Cog · ? $p . t_1 t_2$ Ha Vul p : Mod u :
Hb Inq $Hc . t_1 t_2$ Ha Inq Hb Hb :
Hc Inq Hb Ben ⚹

3 33 4.　⚹ ^{t_1}Ha Inq Hb : Utr · Nnc Hb Vul . Nnc Hb Inq Hd ℈^{t_2} ǀ
Hb Inq Ha Ver ǀ
Hb Inq Hd ℈ ǀ
Hc Inq Hb · ? $x . t_1 t_2$ Fit x ǀ
Hb Inq Hc · $t_1 t_2$ Ha Vul . PPN Hb Inq Hd ℈ ǀ
Hc Inq Hb . Cur Sci ǀ
Hb Inq Hc : $t_1 t_2$ Ha Inq Hb : Utr · Nnc Hb Vul . Nnc Hb Inq Hd ℈
　　· Erg · $t_1 t_2$ Ha Vul . PPN Hb Inq Hd ℈ ǀ
Hc Inq Hb : Cur : ¬ $t_1 t_2$ Ha Inq Hb . Nnc Ha Vul Etc ·
　　Sed : $t_1 t_2$ Ha Inq Hb · Utr . Hb Vul Etc ǀ
Hc Inq Hc ι . Pot ι Pau Ant . t_1 · Ha Inq Ha :
　　Pot : t : Ha Inq Hb . Nnc Ha Vul r ·
　　　　→ Pau Pst · Hb Inq Ha . ¬ Nnc Ha Vul r ·
　　⌢ · Ha Inq Hb · Utr . Nnc Hb Vul r :
　　　　→ Pau Pst · Hb Inq Ha . Nnc Ha Vul r ⚹

3 34 1.　Continuation of the last talk:

　⚹ Hb Inq Hc ι . Qia : Qqm · Lic :
　　　　tt' Ha Inq Hb · Nnc Ha Vul . Nnc Hb Etc :
　　　Tan : Dec · tt' Ha Inq Hb : Utr · Hb Vul . Nnc Hb Etc ι
　　tt' Ha Vul . Nnc Hb Etc ·
　　　　→ : Dec · Ha Inq Hb : Utr · Hb Vul . Nnc Hb Etc ι
　　　¬ Deb . Sed Dec ǀ
　Hc Inq Hb ι Utr : Dec : Eti : Ha Inq Hb ·
　　Utr : Nnc Hb Ccd · Nnc Ha Vul . Nnc Hb Etc ǀ
　Hb Inq Hc : Sat : Ha Inq Hb Utr · Nnc Hb Vul . Nnc Hb Etc :
　　Tan : Plt : Ha Inq Hb ·
　　　Utr : Nnc Hb Ccd · Nnc Ha Vul . Nnc Hb Etc ⚹

The words written Dec (ʃL decet) and Plt (ʃL placet) mean modalities of decency. I have not succeeded in finding unambiguous translations of these words. Perhaps "it is courteous" (French "courtois", German "höflich") for 'Plt', and "it is convenient" (French "il convient", German "es schickt sich") for 'Dec' will do. The relationship between the two levels of decency 'Plt' and 'Dec' is akin to that between the two levels of duty, 'Deb', and 'Lic',

and to that between the two levels of necessity, 'Ncs' and 'Pot'.

The receiver will learn from the last talk that there may be some reason of uttering one's wishes in a more circumstantial way, not for instance saying "I wish that you . . ." but rather "do you wish to . . ." or even "do you allow me to ask you whether you . . .". As early as in 3 29 7 one of the partners has displayed this more polite behaviour. A comment analogous to that on 3 33 4 could be added to 3 29 7.

Though we have still little use for refined behaviour modalities like 'Plt' and 'Dec', a few more texts might be welcome.

3 34 2. $\#\,^{t_3}Ha\,\mathrm{Inq}\,Hb\,\dot{}\,\mathrm{Utr}:\mathrm{Nnc}\,Hb\,\mathrm{Sci}\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x^{t_4}\,|$
      $Hb\,\mathrm{Inq}\,Ha\,.\,t_1\,t_2\,\mathrm{Fit}\,\alpha^{t_5}\,|$
      $Hc\,\mathrm{Inq}\,Hb\,\dot{}\,\mathrm{Cur}:\,\neg\,t_4\,t_5\,Hb\,\mathrm{Inq}\,Ha\cdot\alpha\,.\,\mathrm{Sed}\,.\,t_1\,t_2\,\mathrm{Fit}\,\alpha\,|$
      $Hb\,\mathrm{Inq}\,Hc\,.\,\mathrm{Qia}\,\mathrm{Plt}\,|$
      $Hc\,\mathrm{Inq}\,Hb\,\vdots\,\mathrm{Cur}\,\colon\,\neg\,t_3\,t_4\,Ha\,\mathrm{Inq}\,Hb\,\vdots$
          $?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\dot{}\mathrm{Sed}\,\dot{}\,\mathrm{Utr}:\mathrm{Nnc}\,Hb\,\mathrm{Sci}\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,|$
      $Hb\,\mathrm{Inq}\,Hc\,\imath:\mathrm{Qia}\,\mathrm{Dec}\,\imath\cdot\mathrm{Eti}\,\imath\,.\,\mathrm{Plt}\,\imath$
        $t\vdots Ha\,\mathrm{Vul}:\mathrm{Nnc}\,Ha\,\mathrm{Sci}\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\dot{}$
        $\rightarrow\vdots Ha\,\mathrm{Inq}\,Hb\vdots\mathrm{Utr}\,\dot{}\,\mathrm{Nnc}\,Hb\,\mathrm{Ccd}:$
          $Ha\,\mathrm{Inq}\,Hb\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\#$

3 34 3.   Likewise one can treat talks such as:
      $\#\,Ha\,\mathrm{Inq}\,Hb\,\dot{}\,\mathrm{Utr}:\mathrm{Nnc}\,Hb\,\mathrm{Sci}\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\vdots$
      $Hb\,\mathrm{Inq}\,Ha\vdots\mathrm{Utr}:\mathrm{Nnc}\,Ha\,\mathrm{Ccd}\,\dot{}$
        $\mathrm{Nnc}\,Hb\,\mathrm{Inq}\,Ha:\,\neg\,\mathrm{Nnc}\,Hb\,\mathrm{Sci}\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\#$

      and

      $\#\,Ha\,\mathrm{Inq}\,Hb\,\dot{}\,\mathrm{Utr}:\mathrm{Nnc}\,Hb\,\mathrm{Sci}\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\vdots$
      $Hb\,\mathrm{Inq}\,Ha\vdots\mathrm{Ver}\,.\,\mathrm{Tan}\vdots\mathrm{Utr}\,\dot{}\,\mathrm{Nnc}\,Ha\,\mathrm{Ccd}:$
        $\neg\,\mathrm{Nnc}\,Hb\,\mathrm{Inq}\,!\,Ha\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\#$

3 34 4. $\#\,Ha\,\mathrm{Inq}\,Hb\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,\dot{}$
      $Hb\,\mathrm{Inq}\,Ha\,\alpha\,\dot{}$
      $Ha\,\mathrm{Inq}\,Hb\,x\,\dot{}$
      $Hc\,\mathrm{Inq}\,Hb:\neg\,\mathrm{Dec}\cdot Hb\,\mathrm{Inq}\,Ha\,\alpha\,.\,\mathrm{Ant}\,.\,Ha\,\mathrm{Inq}\,Hb\,x\,\#$

3 34 5. $\#\,Ha\,\mathrm{Inq}\,Hb\cdot ?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\vdots$
      $Hc\,\mathrm{Inq}\,Ha\,\alpha\vdots$
      $Hd\,\mathrm{Inq}\,Hc\vdots\mathrm{Dec}\,\dot{}\,t:Ha\,\mathrm{Inq}\,Hb\,.\,?\,\mathrm{Etc}\cdot$
        $\rightarrow\mathrm{Pau}\,\mathrm{Pst}\cdot Hc\,\mathrm{Ccd}\,.\,\mathrm{Nnc}\,Hb\,\mathrm{Rsp}\,Ha\,\#$

3 34 6. * Ha Inq Hb ꝑ ⅰ

    Hc Inq Hd ˙Utr : Nnc Hd Sci · ? x . PAN Ha Inq Hb x ⅰ

    Hd Inq Hc . ⌐ Sci ⅰ

    Hc Inq Hd · Utr . ⌐ PAN Hd Ani Etc ⅰ

    Hd Inq Hc Ver ⅰ

    Hc Inq Hd : Utr · ⌐ Pot . PAN Hd Inq Etc ⅰ

    Hd Inq Hc ꞉ Pot . Tan . ⌐ PAN Vul ꞉

        ⌐ PAN Hd Vul . Nnc Hd Ani Etc

        ꞉ Qia ꞉ ∨x ˙ PAN Ha Inq Hb : x : Mod · Ha Vul . ⌐ Nnc Hd Ani ꞉

        Erg ˙ Plt · PAN Hd Vul . ⌐ Nnc Hd Ani Etc :

            ∧ : Dec · ⌐ PAN Hd Vul . Nnc Hd Ani Etc *

3 34 7. * Ha Inq Hb ꝑ ⅼ

    Hc Inq Hd ˙ Utr : Nnc Hd Sci · ? x . PAN Ha Inq Hb x ⅼ

    Hd Inq Hc ⅰ Qqm · Nnc Hd Sci :

        Tan : Nnc Hd Vul · ⌐ . Unq Pst PAN . Hd Sci ꞉

        Qia ꞉ ∨x ˙ PAN Ha Inq Hb : x : Mod · Ha Vul . ⌐ Nnc Hd Ani ꞉

        ⌐ Dec : Nnc Hd Inq ! Hc · ? x . Ha Inq Hb x *

3 34 8. * Ha Inq Ha ˙

        Dec Etc . ↔ · ⌐ Plt . ⌐ Etc :

        ⌐ Dec Etc . ↔ · Plt . ⌐ Etc :

        ⌐ Dec . ⌐ Etc · ↔ . Plt Etc :

        Dec . ⌐ Etc · ↔ . ⌐ Plt Etc *

3 34 9. * Ha Inq Ha ·

        Ncs Etc . → . Deb Etc . → . Plt Etc . → .

        Dec Etc . → . Lic Etc . → . Pot Etc *

Perhaps we had better give incidental comments on former texts instead of this general behaviour rule which may become discredited when 'Etc' is a proposition such that e.g. Ncs Etc is true whereas Deb Etc is void of sense, and so on.

3 35 1. I have tried to design a vocabulary of modalities. It is still far from satisfactory. I found little help in the literature on this topic. I believe that modalities cannot be treated successfully, if not linked to human behaviour.

3 35 2. Modalities allude to some constraint. This constraint may be logical or physical (Ncs), legal or moral (Deb), it may be for decency's sake (Plt), and finally the constraint may be exercised by some person saying "I wish" (Vul). Even in the first three cases the author may be personified: Logic and nature (Ncs), Law (Deb), Decency (Plt).

3 35 3. In the simplest way such a modality is used when somebody justifies an action of his or other people by saying: I did it because I had to. As early as 3 12 2 *Ha* acted in this way. Compare also 3 29 4 and following texts, 3 34 1 and following texts. The motivation may be necessity, duty, decency or the will of another person.

Note that in our terminology Ncs need not be prohibitive.

Likewise somebody may justify the fact that he or other people abstain from some action by saying that there was a constraint to abstain. See 3 22 6, 3 22 7, 3 23 1, 3 23 3, 3 32 5, 3 32 7, 3 32 8, 3 29 4, 3 30 2 and so on.

Finally some action or the lack of some action may be justified by saying that there was no constraint to commit it or to abstain from it. See 3 22 3, 3 22 4, 3 22 9 and many others.

3 35 4. In a more complicated context modalities are used if somebody is accused of having acted against a constraint or of not having acted in spite of a constraint.

As this cannot be done with impunity, punishments should be shown. If in 3 12 2 *Ha* omits '$p \in \mathrm{Pri}$' the punishment will be: falsehood of his assertion. Physical constraints cannot be dealt with, because we do not know anything about the bodies of the acting persons; they are sheer spirits that cannot be punished by physical means.

A man who kills another man in self-defence, acts under the constraint of self-preservation. This constraint is not prohibitive. He could have done otherwise, but then he would have been punished with the loss of his own life. In our system there is no logical implication 'Ncs $p \,.\, \to p$'.

Punishments against moral transgressors will be treated in 3 36 1.

Justifications and accusations may be combined in the same text.

3 35 5. In all these cases the actual event p (or its non-actualization) has been the starting point. It has either been justified or condemned. Yet the inverse process is also possible. The event p (or its non-actualization) is proved or forecast by the argument that there is some constraint to provoke or to prevent it. See 3 22 1, 3 29 4 and many others.

3 35 6. 'Pot' is also used to designate some disposition. From potentiality as absence of certain constraints the stress is shifted to potentiality as the presence of certain faculties. If there are no constraints to prevent somebody from committing a certain action, nothing but

his own will is needed for this action to take place. His *own* will is counter-balanced by his *own* potentiality. As will is an activity, ability becomes an activity too.

In Lincos syntax we have not accounted for this semantic change from an impersonal "it is possible" to a personal "he can" by any corresponding syntactical device. See 3 23 1, 3, 3 29 8.

3 35 7. Naive physics uses human behaviour-patterns for the under-standing of nature. (The sun will not transgress its measures; otherwise the Erinys, Justice's bailiffs, will hunt it down — Herakleitos.) For centuries Aristotle was the philosopher of anthropomorphic physics. Potentiality, will, activity, and passivity are key notions in his natural philosophy. In spite of the law of action and reaction, Newtonian forces (when really applied) still have many features in common with constraints and manifesta-tions of will in human behaviour. Anthropomorphic physics as a first phase of men's intellectual development cannot be discarded. Even in the laboratory, and wherever human actions influence extrahuman nature or are influenced by it, we cannot dispense with an anthropomorphic terminology. When we say that the wind moves the branches of a tree, or can move them, we divide nature into a passive background of potentiality and a causa efficiens that actualizes some potentiality as individual will actualizes the potentialities of the individual. By overstressing this feature we can construct and understand machines. Here the points where the causae efficientes apply are sharply marked by taps, switches, handles and so on.

Physics has not yet been a subject of our texts, but when we turn to it, we shall apply human behaviour terminology in essen-tially the same way as we do in daily life.

3 36 1. * Ha Inq Hb . $t_1 t_2$ Fit a |
        Hc Inq Ha : Cur $\cdot$ PAN Ha Inq Hb . $t_1 t_2$ Fit a |
        Ha Inq Hc : Qia $\cdot$ Pau Ant . PAN $\cdot$ Ha Pol Hb :
            PPN Ha Inq ! Hb $\cdot$? x . $t_1 t_2$ Fit x : $t_1 t_2$ Fit a :
            Erg : Deb $\cdot$ PAN Ha Inq Hb . $t_1 t_2$ Fit a :
            Deb p . $\rightarrow$ $\cdot$ Ha Vul p . $\in$ Ben :
            Deb . $\neg p$ $\cdot$ $\rightarrow$ $\cdot$ Ha Vul p . $\in$ Mal |
        Hc Inq Ha ι ? q : t : Ha Vul : 1^ϵ $\cdot$ $\hat{}p$. Deb p $\cdot$ $\rightarrow$ Pau Pst . Fit q :
            ? r : t : Ha Vul : 1^ϵ $\cdot$ $\hat{}p$ $\cdot$ Deb $\neg p$ $\cdot$ $\rightarrow$ Pau Pst . Fit r |
        Ha Inq Hc : $q = $ $\cdot$ $\wedge$ $\ulcorner x . y \urcorner$: x $\frown$ y . $\in$ Hom $\cdot$ $\rightarrow$. xy Ise Inq Ben :
            $r = $ $\cdot$ $\wedge$ $\ulcorner x . y \urcorner$: x $\frown$ y . $\in$ Hom $\cdot$ $\rightarrow$. xy Ise Inq Mal :
            tt' Ha Vul $\cdot$ $\wedge p$: Nnc $\cdot$ Ha Vul p . $\rightarrow$ Pau Pst . xy Ise Inq Ben |

$Hc\,\mathrm{Inq}\,Ha$ । Utr ꞉ Etc ꞉ t ꞉ $Ha\,\mathrm{Vul}$ ꞉ 1^{ϵ} . $\mathrm{Deb}\,p$ ˙

    $\rightarrow \mathrm{Pau\,Pst}$ · $Hi\,Hj$. $\mathrm{Ise\,Inq\,Ben}$ ।

$Ha\,\mathrm{Inq}\,Hc$ । · Pot · $Hi\,Hj$. $\mathrm{Ise\,Inq\,Mal}$ । .

    $\forall x$ ꞉ $x \in \mathrm{Hom}$. $\wedge$ ˙ $\wedge$ ⌜$t.t'.p$⌝ ꞉ $tt'\,x\,\mathrm{Vul}\,p$ · $\rightarrow$ · Deb . $\neg\,p$ ꞉ $\wedge$ ꞉

    $\forall x$ ꞉ $x \in \mathrm{Hom}$. $\wedge$ ˙ $\vee$ ⌜$t.t'.p$⌝ ꞉ $tt'\,x\,\mathrm{Vul}\,p$. $\wedge$ · Deb . $\neg\,p$ ꞉ $\wedge$ ꞉

    $\forall x$ ꞉ $x \in \mathrm{Hom}$. $\wedge$ ˙ $\vee$ ⌜$t.t'.p$⌝ ꞉ $tt'\,x\,\mathrm{Vul}\,p$. $\wedge$ · $\mathrm{Deb}\,p$ ꞉ $\wedge$ ꞉

    $\forall x$ ꞉ $x \in \mathrm{Hom}$. $\wedge$ ˙ $\wedge$ ⌜$t.t'.p$⌝ ꞉ $tt'\,x\,\mathrm{Vul}\,p$. $\rightarrow$ · $\mathrm{Deb}\,p$ । .

    $A = $ ꞉ ↑x ꞉ $x \in \mathrm{Hom}$. $\wedge$ ˙ $\wedge$ ⌜$t.t'$. p⌝ ꞉ $tt'\,x\,\mathrm{Vul}\,p$. $\rightarrow$ · $\mathrm{Deb}\,p$ ꞉

     $\rightarrow$ ৷ t'' ꞉ $u\,\mathrm{Vul}$ ꞉ 1^{ϵ} . ↑q . $\mathrm{Deb}\,q$ ˙ $\rightarrow \mathrm{Pau\,Pst}$ · $\mathrm{Mul}\,{}^{\epsilon}A$. $\mathrm{Ise\,Inq\,Ben}$

     ꞉ $\wedge$ ꞉ t'' ꞉ $u\,\mathrm{Vul}$ ˙ 1^{ϵ} ꞉ ↑q · Deb . $\neg\,q$ ꞉ $\rightarrow \mathrm{Pau\,Pst}$ · $\mathrm{Mul}\,{}^{\epsilon}A$.

     $\mathrm{Ise\,Inq\,Mal}$ ✳

Clearly this is childish reasoning. But it is no more childish than any conversation with children about fundamentals of ethics or than the categorical imperative when translated into a language lacking the equivalents of "categoric", "imperative", "maxim", "principle". It is also no more childish than the introduction of natural numbers in the first chapter and of time in the second. We may subsequently reformulate the definition of natural number by means of Peano's axioms, though it is not yet certain that Peano's numbers are superior to simple counters. People have sound ideas about practical ethics because they once knew its principles in childish terms. On a higher linguistic level more sophisticated conversations about ethics will be possible, and if there be any rational system for ethics like Peano's axioms for arithmetics, we may try to translate it. At the present stage of our project this would be premature.

After all, the last talk is not as childish as it looks. *Ha* defines the set of good people. It is those who wish only good things to happen. Then *Ha* asserts that good people call good actions good and bad actions bad. If we apply to this their own actions, we may conclude that good peoples are committed to those actions they call good themselves. There is some sense in this assertion, though *Ha* may be wrong in saying that the set of people like this is not void. It is a criterion of goodness though it is not Kant's categorical imperative. It looks more like Shakespeare's

> This above all: to thine own self be true.
> And it must follow, as the night the day,
> Thou canst not then be false to any man,

but this is a mere accident. I have made no attempt to inquire into the fundamentals of ethics.

Of course the ethical principle stated by *Ha* is liable to many

variations, e.g. by using Lic instead of Deb or by inverting implication arrows. We do not enter into details.

A conversation about decency could be carried on in a similar way.

3 36 2. When reading the printer's proof, I remark that the analysis in 3 35 e has been too limited. The levels of modality cannot be localized by punishments only. We have had to use rewards too. 'Deb' and 'Plt' have been distinguished from 'Lic' and 'Dec' by means of their relation to the judgements "praiseworthy" (Ben) and "blameless" ($\neg$Mal).

3 37 1. $*\ Ha\,\mathrm{Inq}\,Hb\,.\,?=10\times10\,|$

    $Hb\,\mathrm{Inq}\,Ha\,100\,|$

    $Ha\,\mathrm{Inq}\,Hb\,.\,?=1010111\times100101\,|$

    $Hb\,\mathrm{Inq}\,Ha\,.\,\neg\,\mathrm{Sci}\,|$

    $Ha\,\mathrm{Inq}\,Hb\,\vdots\,\mathrm{Cur}\,{}^{\bullet}\,\mathrm{Nnc}\,Hb\,\mathrm{Sci}:$

        $\mathrm{Qqm}\,.\,?=10\times10\cdot\mathrm{Tan}\,\neg\,.\,?=1010111\times100101\,|$

    $Hb\,\mathrm{Inq}\,Ha\,\vdots\,\mathrm{Qia}\,{}^{\bullet}\,\mathrm{Dif}\cdot\mathrm{Nnc}\,Hb\,\mathrm{Pet}\,.\,?=1010111\times100101:$

      $>\,:\mathrm{Dif}\cdot\mathrm{Nnc}\,Hb\,\mathrm{Pet}\,.\,?=10\times10\,|$

    $Hc\,\mathrm{Inq}\,Ha\,\vdots\,\mathrm{Pot}\cdot\mathrm{Nnc}\,x\,\mathrm{Rep}\,.\,?=1010111\times100101:$

      $\mathrm{Qqm}\rightarrow.\,\mathrm{Tan}\,.\,\neg\,\leftrightarrow:\mathrm{Pot}\cdot\mathrm{Nnc}\,x\,\mathrm{Rep}\,.\,?=10\times10\,|$

    $Hd\,\mathrm{Inq}\,Ha\,.\,?=\mathrm{Dif}\,|$

    $Ha\,\mathrm{Inq}\,Hd\,\vdots\,\mathrm{Exg}\,{}^{\bullet}\,\mathrm{Dif}\cdot?\,x\,.\,x^{10}-101\,x+110\doteq0:$

      $>\,:\mathrm{Dif}\,.\,?\doteq1010111\times100101\,|$

    $Hc\,\mathrm{Inq}\,Hd\,\lvert\,\mathrm{Fal}\colon\mathrm{Nnc}\,y\,\mathrm{Sci}\,\colon?\,{}^{\smile}u\,{}^{\bullet}\,\vee v\colon v\in\cdot\mathrm{Cpu}^{10}\,.\,?\,x\,\mathrm{Etc}\,.\,\mathrm{Mod}\,u\,\vdots$

      $\rightarrow:\mathrm{Dif}\,.\,?\,x\,\mathrm{Etc}\,{}^{\bullet}<\,\cdot\,\mathrm{Dif}\,.\,?=1010111\times100101\,|$

    $Ha\,\mathrm{Inq}\,Hc\,{}^{\bullet}\,\mathrm{Tan}\colon\neg\,\mathrm{Nnc}\,y\,\mathrm{Sci}\,\mathrm{Etc}\,.\rightarrow\cdot\,\mathrm{Dif}\,\mathrm{Etc}\,.>\,.\,\mathrm{Dif}\,\mathrm{Etc}\,|$

    $Hc\,\mathrm{Inq}\,Ha\,\mathrm{Ver}\,*$

The word written Dif ($\!/\!L$ difficultas = difficulty) means "difficulty (of . . .)". It might be considered as a modality quite different from the modalities treated above. The syntax of 'Dif' is clear from its occurrence in Hb's third answer. Later on we use an abbreviated notation that can be explained by some behaviour rule.

3 37 2. $*\ Ha\,\mathrm{Inq}\,Hb\,\colon\mathrm{Utr}\colon\wedge n\,{}^{\bullet}\,n\in\mathrm{Num}\,.$

      $\rightarrow:\vee\ulcorner p\,.\,m\urcorner\cdot p\in\mathrm{Pri}\,.\,\wedge\,.\,m\in\mathrm{Num}\,.\,\wedge\,.\,p=100\,m-1\,.\,\wedge\,.\,p>n\,|$

    $Hb\,\mathrm{Inq}\,Ha\,\lvert:\mathrm{PPN}\,.\,\mathrm{Qqm}\,.\,\neg\,\mathrm{Nnc}\colon Hb\,\mathrm{Sci}\,.\,\mathrm{Utr}\,\mathrm{Etc}\,\lvert$

    $\mathrm{Qia}\colon\mathrm{Pot}\,\colon\mathrm{Nnc}\,Hb\,\mathrm{Dem}\colon$

      $\wedge n\,{}^{\bullet}\,n\in\mathrm{Num}\,.\rightarrow:\vee p\cdot p\in\mathrm{Pri}\,.\,\wedge\,.\,p>n\,\lvert\,.$

    $\mathrm{Tan}\,\lvert\,\mathrm{Dif}\colon\mathrm{Utr}\colon\wedge n\,{}^{\bullet}\,n\in\mathrm{Num}\,.\rightarrow:\vee\ulcorner p\,.\,m\urcorner\,\mathrm{Etc}\colon$

      $>\,\colon\mathrm{Dif}\colon\mathrm{Utr}\colon\wedge n\,{}^{\bullet}\,n\in\mathrm{Num}\,.\rightarrow:\vee p\,\mathrm{Etc}\,*$

More examples may be added, e.g. comparisions of the difficulty of

\# Utr $\vdots \wedge n\cdot n\in\mathrm{Num}\,.$

$\quad\to\colon\vee\ulcorner p\,.\,m\urcorner\cdot p\in\mathrm{Pri}\,.\wedge.\,m\in\mathrm{Num}\,.\wedge.\,p=100\,m+1\,.\wedge.\,p>n\ \#$

\# Utr $\vdots \wedge\ulcorner a\,.\,b\,.\,n\urcorner\vdots$

$\quad a\frown b\frown n\,.\in\mathrm{Num}\cdot\wedge\cdot\wedge q\colon q\,\mathrm{Div}\,a\,.\wedge.\,q\,\mathrm{Div}\,b\cdot\to.\,q=1\vdots$

$\quad\to\colon\vee\ulcorner p\,.\,m\urcorner\cdot p\in\mathrm{Pri}\,.\wedge.\,m\in\mathrm{Num}\,.\wedge.\,p=am+b\,.\wedge.\,p>n\ \#$

3 37 3. A comment on 3 27 1:

$\quad$ \# $\wedge\ulcorner tt'\,x\urcorner\colon\mathrm{Dif}\,.\,tt'\,x\,\mathrm{Pet}\,\mathrm{Utr}\,G\cdot>\mathrm{Vld}$ \#

The word written Vld (fL valde = very) is akin to 'Nim' (see 3 24 4). '$>$ Vld' may be translated "very large", '$<$ Vld' "very small".

$\quad$ \# $\mathrm{Dif}\cdot tt'\,x\,\mathrm{Pet}\,.\,?\,\mathrm{Etc}\colon>\mathrm{Vld}\cdot\leftrightarrow\cdot\mathrm{Cca}\colon\neg\,\mathrm{Pot}\cdot tt'\,x\,\mathrm{Rep}\,.\,?\,\mathrm{Etc}$ \#

3 37 4. A comment on 3 37 1:

$\quad$ \# $\wedge\ulcorner t\,.\,t'\,.\,x\urcorner\mathrm{Dif}\cdot tt'\,x\,\mathrm{Pet}\,.\,?=10\times10\colon<\mathrm{Vld}$ \#

$\quad$ \# $\mathrm{Dif}\cdot tt'\,x\,\mathrm{Pet}\,.\,?\,\mathrm{Etc}\colon<\mathrm{Vld}\cdot\leftrightarrow\cdot\mathrm{Cca}\colon\mathrm{Ncs}\cdot tt'\,x\,\mathrm{Rep}\,.\,?\,\mathrm{Etc}$ \#

3 37 5. There is a great variety of contexts in which 'Dif' can appear.

Let $\mathfrak{p}$, $\mathfrak{q}$, $\mathfrak{r}$ be sequences of 4, 32 and 256 short signals respectively within a relatively short time.

\# $t_1\,\mathfrak{p}\,t_2\,\mathfrak{q}\,t_3\,\mathfrak{r}\,t_4\vdots$

$\quad Ha\,\mathrm{Inq}\,Hb\vdots\mathrm{Dif}\cdot?\,x\,.\,t_1\,t_2\,x^{es}\,\mathrm{Fit}\bullet\colon<\mathrm{Vld}\vdots$

$\quad\quad\mathrm{Dif}\cdot?\,x\,.\,t_1\,t_2\,x^{es}\,\mathrm{Fit}\bullet\colon<\colon\mathrm{Dif}\cdot?\,y\,.\,t_2\,t_3\,y^{es}\,\mathrm{Fit}\bullet\colon$

$\quad\quad\quad<\colon\mathrm{Dif}\cdot?\,z\,.\,t_3\,t_4\,z^{es}\,\mathrm{Fit}\bullet\vdots$

$\quad\quad\mathrm{Dif}\cdot?\,z\,.\,t_3\,t_4\,z^{es}\,\mathrm{Fit}\bullet\colon>\mathrm{Vld}\cdot\mathrm{Qia}\colon{}^{\vee}z\,.\,t_3\,t_4\,z^{es}\,\mathrm{Fit}\bullet\cdot\mathrm{Mul}>.\,1$ \#

$\quad$ \# $Ha\,\mathrm{Inq}\,Hb\cdot\neg\,\mathrm{Nnc}\,Ha\,\mathrm{Sci}\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\colon$

$\quad\quad\wedge\colon\neg\,\mathrm{Nnc}\,Ha\,\mathrm{Sci}\cdot?\,y\,.\,t_3\,t_4\,\mathrm{Fit}\,y\,\shortmid$

$\quad Hb\,\mathrm{Inq}\,Ha\cdot\mathrm{Nnc}\,.\,\mathrm{Pau}^\epsilon\,\mathrm{Hom}\,.\,\mathrm{Sci}\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\colon$

$\quad\quad\mathrm{Tan}\colon\mathrm{Nnc}\,.\,\mathrm{Mul}^\epsilon\,\mathrm{Hom}\,.\,\mathrm{Sci}\cdot?\,y\,.\,t_1\,t_2\,\mathrm{Fit}\,y\,\shortmid$

$\quad Ha\,\mathrm{Inq}\,Hb\colon\mathrm{Erg}\colon\mathrm{Dif}\colon\mathrm{Nnc}\,Ha\,\mathrm{Pet}\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\mathrm{Etc}$

$\quad\quad\cdot>\cdot\mathrm{Dif}\colon\mathrm{Nnc}\,Ha\,\mathrm{Pet}\cdot?\,y\,.\,t_1\,t_2\,\mathrm{Fit}\,y\,\mathrm{Etc}$ \#

$\quad$ \# $Ha\,\mathrm{Inq}\,Hb\cdot?\,x\colon t_1\,t_2\,x\,\mathrm{Ani}\cdot?\,y\,.\,t_1\,t_2\,\mathrm{Fit}\,y\vdots$

$\quad Hb\,\mathrm{Inq}\,Ha\cdot\neg\,\vee x\,.\,t_1\,t_2\,x\,\mathrm{Ani}\,\mathrm{Etc}\vdots$

$\quad Ha\,\mathrm{Inq}\,Hb\colon\mathrm{Erg}\cdot\mathrm{Dif}\cdot?\,y\,.\,t_1\,t_2\,\mathrm{Fit}\,y\colon>\mathrm{Vld}\vdots$

$\quad Ha\,\mathrm{Inq}\,Hb\cdot?\,x\colon t_3\,t_4\,x\,\mathrm{Ani}\cdot?\,y\,.\,t_3\,t_4\,\mathrm{Fit}\,y\vdots$

$\quad Hb\,\mathrm{Inq}\,Ha\vdots t_3\,t_4\,x\,\mathrm{Ani}\cdot?\,y\,.\,t_3\,t_4\,\mathrm{Fit}\,y\colon\leftrightarrow.\,x=Hc\vdots$

$\quad\quad\mathrm{Tan}\vdots\wedge\ulcorner t\,.\,t'\,.\,z\urcorner\mathrm{Nnc}\,Hc\,\mathrm{Vul}\colon\neg\,tt'\,Hc\,\mathrm{Inq}\,!\,z\cdot?\,x\,.\,t_3\,t_4\,\mathrm{Fit}\,x\vdots$

$\quad Ha\,\mathrm{Inq}\,Hb\colon\mathrm{Dif}\colon\mathrm{Nnc}\,Ha\,\mathrm{Cog}\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\cdot$

$\quad\quad>\cdot\mathrm{Dif}\colon\mathrm{Nnc}\,Ha\,\mathrm{Cog}\cdot?\,x\,.\,t_3\,t_4\,\mathrm{Fit}\,x\vdots$

$\quad\quad\mathrm{Tan}\vdots\mathrm{Dif}\colon\mathrm{Nnc}\,Ha\,\mathrm{Cog}\cdot?\,x\,.\,t_3\,t_4\,\mathrm{Fit}\,x\cdot>\mathrm{Vld}$ \#

3 37 6. $*\, ^{t_1}Ha\,\mathrm{Inq}\,Hb\,\mathfrak{p}\,^{t_2}$ |

$Ha\,\mathrm{Ani}\cdot\mathrm{Utr}\,.\,\mathrm{Pot}\,Hc\,\mathrm{Ani}$ |

$Ha\,\mathrm{Inq}\,Hc\,\dot{\,}\,\mathrm{Utr}:\mathrm{PAN}\,Hc\,\mathrm{Ani}\cdot?\,x\,.\,\mathrm{Nnc}\,Ha\,\mathrm{Inq}\,Hb\,x$ |

$Hc\,\mathrm{Inq}\,Ha\,\mathrm{Ver}$ |

$Ha\,\mathrm{Inq}\,Hc\,\iota\,\mathrm{Utr}:\mathrm{Nnc}\,Hc\,\mathrm{Ccd}:\mathrm{Nnc}\,Ha\,\mathrm{Vul}:\mathrm{Nnc}\,Hc\,\mathrm{Vul}\,\dot{\,}$
  $\neg:\mathrm{Unq}\,\mathrm{Pst}\,\mathrm{PPN}\,.\,Hc\,\mathrm{Sci}\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x$ |

$Hc\,\mathrm{Inq}\,Ha\,\iota:\mathrm{Qqm}\,.\,\mathrm{Nnc}\,Hb\,\mathrm{Vul}:\mathrm{Tan}:\neg\,\mathrm{Nnc}\,Hb\,\mathrm{Sci}:$
  $\mathrm{Utr}:\mathrm{Pot}\,\dot{\,}\,\neg:\mathrm{Unq}\,\mathrm{Pst}\,\mathrm{PPN}\,.\,Hc\,\mathrm{Sci}\cdot?\,x\,.\,t_1\,t_2\,\mathrm{Fit}\,x\,\iota:$
  $\mathrm{Qia}\,\iota\,\dot{\,}\,\wedge y\,\iota:y\in\mathrm{Hom}\,.\to\iota\,\cdot$
    $\mathrm{Qqm}\,\iota\,\mathrm{Car}:\hat{\,}x:\vee t\,\dot{\,}\,t\,y\,\mathrm{Cog}\,x\,.\,\wedge:$
      $\neg\,\mathrm{Unq}\,.\,\mathrm{Pau}\,\mathrm{Pst}\,.\,t\cdot y\,\mathrm{Sci}\,x:>\mathrm{Vld}\,\iota\,.$
    $\mathrm{Tan}:\vee\ulcorner x\,.\,t\urcorner:t\,y\,\mathrm{Cog}\,x\,.\,\wedge:$
      $\wedge t'\,\dot{\,}\,t<t'<\cdot\,\mathrm{Fin}\,.\,y\,\mathrm{Ext}:t'\,y\,\mathrm{Sci}\,x\,\iota:$
  $\mathrm{Exg}:t\,Hb\,\mathrm{Cog}\,x\,.\,\wedge\,.\,t'\,Hb\,\mathrm{Cog}\,x'\cdot$
    $\wedge:v=\,\cdot\,\neg\,\mathrm{Unq}\,\mathrm{Pst}\,\mathrm{PPN}\,.\,Ha\,\mathrm{Sci}\,x:$
    $\wedge:v'=\,\cdot\,\neg\,\mathrm{Unq}\,\mathrm{Pst}\,\mathrm{PPN}\,.\,Ha\,\mathrm{Sci}\,x':$
    $\wedge:\mathrm{Pau}\,\mathrm{Pst}\,.\,t\cdot Hb\,\mathrm{Vul}\,v:$
    $\wedge:\neg\,\mathrm{Pau}\,\mathrm{Pst}\,.\,t'\cdot Hb\,\mathrm{Vul}\,v'\,\dot{\,}$
    $\to\cdot\mathrm{Dif}\,v\,.\,\mathrm{Mul}>\,.\,\mathrm{Dif}\,v'\,*$

3 38 0. Our treatment of the word written Vul is still unsatisfactory. The wishes uttered by the acting persons are rather whims, and even if a motivation is given, it is no less "whimsical". Punishments have been displayed, but I am not sure whether they are persuasive. So I do not believe that the receiver will really grasp the meaning of 'Vul'.

Stories usually start with a wish of one of the acting persons. As long as we do not know his past, we can hardly appreciate his wish. One could remove this difficulty by telling longer stories or rather by giving a text that is one connected story. However, in this tentative stage of Lincos construction I do not feel prepared for such a task. I have therefore renounced any attempt to show diversity of human individuals, though not for reasons of principle.

We can compensate this want of personality by a more solid general background of human wishes. This will be done in the remaining paragraphs of this chapter.

3 38 1. $Ha\,\mathrm{Inq}\,Hb:\mathrm{Nnc}\,Ha\,\mathrm{Vul}:\mathrm{PPN}\,Ha\,\mathrm{Inq}\,Hb\,.\,1^\varepsilon\,\mathrm{Qus}\cdot$
  $\wedge\,\dot{\,}\,\mathrm{Pau}\,\mathrm{Pst}\,.\,\mathrm{PPN}:Hb\,\mathrm{Rsp}\,\mathrm{Mod}\,\mathrm{Ben}\,.$
    $\to\mathrm{Pau}\,\mathrm{Pst}\cdot Ha\,\mathrm{Dat}\,Hb\,.\,\mathrm{Den}\,10000$
  $\dot{\,}\wedge\,\dot{\,}\,\mathrm{Pau}\,\mathrm{Pst}\,.\,\mathrm{PPN}:Hb\,\mathrm{Rsp}\,\mathrm{Mod}\,\mathrm{Mal}\,.$
    $\to\mathrm{Pau}\,\mathrm{Pst}\cdot Hb\,\mathrm{Dat}\,Ha\,.\,\mathrm{Den}\,10000$ |

Hb Inq $Ha \cdot$ Nnc Hb Vul $^\curlyvee w$. PAN Ha Vul w |

Ha Inq $Hb \cdot ? x . x^{11} - 11\,x - 1000{,}001 = 0$ |

Hb Inq Hb ı $\cdot$ Dif $>$ Vld ı . Qia ı $\neg$ Nnc Hb Sci ⦂

     $? ^\curlyvee y$ ⦂ $y \in$ ⦂ Anl$^{\mathrm{io}}$ ⦂ 1^ϵ Cpu$^{\mathrm{io}}$. ? Etc |

Hb Inq Hc ⦂ Nnc Hb Vul ⦂ PPN Hb Sci $\cdot$

     $? x . x^{11} - 11\,x - 1000{,}001 = 0 \dot{}$

    Nnc ⦂ Hc Inq ! Hb . ? Etc $\cdot \to$ Pau Pst $\cdot$ Hb Dat Hc . Den 1000 |

$Hc\,Hd\,He$ Ise Inq |

Hc Inq Hb ⦂ $\neg \lor y \cdot y \in$ Hom . $\land$. Nnc y Sci Etc |

Hb Inq Ha . $\neg$ Nnc Hb Sci |

Ha Inq Hb 10,1 |

Hb Inq Ha Mal |

Ha Inq Hb . $10{,}1^{11} - 11 \times 10{,}1 - 1000{,}001 = 0$ |

Hb Inq Ha ⦂ Ver . Tan . $\neg$ Sat $\dot{}$

    Deb ⦂ $\lor$ u $\cdot$ $u \in$. Cpu$^{\mathrm{io}}$ Etc . $\land$. Nnc Ha Inq Hb u |

Ha Inq Hb ⦂ Nnc Ha Sci ⦂ $? = u \cdot u \in$. Cpu$^{\mathrm{io}}$ Etc ⦂

    Tan $\dot{}$ $\neg$ Nnc Ha Vul ⦂ Nnc Ha Inq ! Hb $\cdot$ $? ^\curlyvee u . u \in$ Cpu$^{\mathrm{io}}$ Etc |

Hb Inq Ha ı $\neg$ Deb $\cdot$ Nnc Hb Dat Ha . Den 10000 ⦂

    t ⦂ Ha Inq ! Hb $\dot{}$ $? ^\curlyvee u$ ⦂ Pot $\cdot$ Nnc Hb Cpu . Etc . Mod u ⦂ $\leftrightarrow$ Pau Pst ⦂

    Hb Dat Ha . Den 10000 |

Ha Inq Hb Deb |

Hb Inq Ha ⦂ $\land$ $\ulcorner t . t' . p \urcorner \cdot tt'$ Hg Sci . Utr Deb p ⦂

    Erg ⦂ Nnc Hb Vul ⦂ Nnc $Ha \frown Hb$. Inq Hg $\dot{}$ Utr ⦂ Deb $\cdot$

     Nnc Hb Dat Ha . Den 10000 |

Ha Inq Hb Ben |

$Ha \frown Hb$. Inq ! $Hg \cdot ? w$. PAN $Ha\,Hb$ Ise Inq w |

Hg Inq . $Ha \frown Hb$ $\dot{}$ $\neg$ Deb $\cdot$ Nnc Hb Dat Ha . Den 10000 ⦂

    Deb $\cdot$ Nnc Ha Cpu . ? Etc |

Ha Inq Hb $\dot{}$ Pon . $x = u + v$ ⦂

    Erg . $u^{11} + v^{11} + (11\,uv - 11)(u + v) = 1000{,}001$ ⦂

    Pon $\cdot$ $uv = 1 \frown u^{11} + v^{11} = 1000{,}001 \frown$

$u^{11} = p$ ⦂ Erg . $p + 1/p = 1000{,}001$ |

Hb Inq Ha ⦂ Ben ⦂ $\land$ $\ulcorner a . b \urcorner$ $\dot{}$ $a \frown b$. $\in$ Rea $\cdot$

    $\to$ ⦂ Nnc Hb Sci $\cdot$ $? x . x^{11} + ax + b = 0$ ⦂

    Pot $\cdot$ Nnc Hb Cpu . ? Etc |

^{t_1} ⨍ ^{t_2} |

Hc Inq Hc $\cdot$ $t_1\,t_2$ Hb Dat Ha . Den 10000^{t_3} |

Hd Inq Hd ⦂ $t_2\,t_3$ Hc Anl $\cdot$ $^\curlyvee z$. $t_1\,t_2$ Fit z ⁜

The word written Dat (fL dat = gives) means "gives". The word
written Den (fL denarius = penny) means some unit of value.

    ⨍ is a metatextual substitute for a sequence of 16 striking sounds.

It means the payment of the 16 units. As this is the first action that cannot be derived from the fundamental action Inq, we did not stage it in the form '*Hb* Dat *Ha* . Den 10000', but rather by means of a sequence of characteristic sounds. One could repeat the text with '*Hb* Dat *Ha* . Den 10000' instead of 'ℓ'.

Of course it is not certain that the receiver will guess more details of the event of paying.

3 38 2. $*\ Ha\ \mathrm{Inq}\ Ha\ ^{\bullet}\ tt'\ x\ \mathrm{Dat}\ y\ .\ \mathrm{Den}\ z\ ^{\bullet}$

$\rightarrow .\ t'\ \mathrm{Pec}\ y - t\ \mathrm{Pec}\ y = t\ \mathrm{Pec}\ x - t'\ \mathrm{Pec}\ x = \mathrm{Den}\ z\ ^{\bullet}_{\bullet}$

$t \in \mathrm{Tem}\ .\ \wedge .\ x \in \mathrm{Hom}\ ^{\bullet} \rightarrow ^{\bullet}\ \vee u : u \in \mathrm{Rea}\ .\ \wedge \cdot t\ \mathrm{Pec}\ x\ . = .\ \mathrm{Den}\ u\ *$

The word written Pec (*ƒL* pecunia = fortune) means "fortune", '*t* Pec *x*' means the fortune of *x* at the moment *t*.

3 38 3.   The next talk continues those of 3 29 4 and 3 29 7.

$Hd\ \mathrm{Inq}\ He\ ^{\bullet}\ \mathrm{Cur} : t\ \cdot\ Ha\ \mathrm{Vul}\ q\ . \rightarrow \mathrm{Pau}\ \mathrm{Pst}\ .\ Hc\ \mathrm{Vul}\ q\ |$

$He\ \mathrm{Inq}\ Hd\ _{\shortmid} : \mathrm{Qia}\ _{\shortmid}.\ \mathrm{MAN}\ Ha\ \mathrm{Pol}\ Hc\ _{\shortmid}$

$\wedge t\ ^{\bullet}\ t\ \mathrm{Pst}\ \mathrm{Nnc}\ . \rightarrow : t\ \cdot\ Ha\ \mathrm{Vul}\ q\ . \rightarrow \mathrm{Pau}\ \mathrm{Pst}\ .\ Hc\ \mathrm{Vul}\ q\ ^{\bullet}_{\bullet}$

$\rightarrow ^{\bullet}_{\bullet}\ \wedge t\ ^{\bullet}\ \vee n\ ^{\bullet}\ n \in \mathrm{Num}\ .\ \wedge : t - \mathrm{Nnc}\ . = ^{\bullet}\ \mathrm{Sec}\ .\ n \times 10^{10100}\ ^{\bullet}_{\bullet}$

$\rightarrow \cdot t\ Ha\ \mathrm{Dat}\ Hc\ .\ \mathrm{Den}\ 10^{1100}\ |$

$Hd\ \mathrm{Inq}\ He\ ^{\bullet}_{\bullet}\ \mathrm{Cur}\ ^{\bullet}\ \neg\ \mathrm{PAN}\ ^{\bullet}_{\bullet}$

$Ha\ \mathrm{Inq}\ He\ \cdot\ \mathrm{Nnc}\ Ha\ \mathrm{Vul}\ q\ . \rightarrow \mathrm{Pau}\ \mathrm{Pst}\ .\ He\ \mathrm{Vul}\ q\ |$

$He\ \mathrm{Inq}\ Hd\ ^{\bullet}_{\bullet}\ \mathrm{Qia}\ ^{\bullet}_{\bullet}\ \vee x\ ^{\bullet}\ \mathrm{Deb}\ \cdot\ Ha\ \mathrm{Dat}\ He\ .\ \mathrm{Den}\ x\ ^{\bullet}_{\bullet}$

$\mathrm{Tan} : \neg\ \mathrm{Nnc}\ Ha\ \mathrm{Vul}\ \cdot\ \mathrm{Nnc}\ Ha\ \mathrm{Dat}\ He\ .\ \mathrm{Den}\ x\ |$

$Hd\ \mathrm{Inq}\ He\ .\ \mathrm{Cur}\ \mathrm{Deb}\ |$

$He\ \mathrm{Inq}\ Hd\ ^{\bullet}_{\bullet}\ \mathrm{Qia}\ ^{\bullet}\ \vee x\ ^{\bullet}_{\bullet}\ \mathrm{MAN}\ Ha\ \mathrm{Pol}\ He\ \cdot\ \mathrm{PPN}\ Ha\ \mathrm{Dat}\ He\ .\ \mathrm{Den}\ x\ ^{\bullet}_{\bullet}$

$\wedge\ ^{\bullet}\ \mathrm{MAN}\ Ha\ \mathrm{Vul} : \neg\ \mathrm{Unq}\ \mathrm{Pst}\ \mathrm{MAN}\ .\ Ha\ \mathrm{Dat}\ He\ .\ \mathrm{Den}\ x\ *$

3 39 1. $^{t_1} Ha\ Hb\ \mathrm{Alt}\ \mathrm{Inq}\ \cdot\ 1\ .\ 1\ .\ 11\ .\ 1001\ .\ 110\ .\ 100100\ .\ 10\ .\ 100\ .$

$110\ .\ 100100\ .\ 1101\ .\ 10101001\ ^{t_2}\ ^{\bullet}_{\bullet}$

$Hc\ \mathrm{Inq}\ Hd\ ^{\bullet}_{\bullet}\ t_1 \leqq t \leqq t_2\ . \rightarrow : t\ \cdot\ Ha\ \mathrm{Inq}\ Hb\ x\ .$

$\rightarrow \mathrm{Pau}\ \mathrm{Pst}\ .\ Hb\ \mathrm{Inq}\ Ha\ x^{10}\ ^{\bullet}_{\bullet}$

$t\ Ha\ \mathrm{Inq}\ Hb\ x\ . \rightarrow ^{\bullet}\ \mathrm{Deb} : \mathrm{Pau}\ \mathrm{Pst}\ .\ t\ \cdot\ Hb\ \mathrm{Inq}\ Ha\ x^{10}\ ^{\bullet}_{\bullet}$

$\mathrm{Lic}\ ^{\bullet}\ t\ Ha\ \mathrm{Inq}\ Hb\ ^{\bullet}_{\bullet}\ ^{\vee} y\ \cdot\ \mathrm{Nnc}\ Ha\ \mathrm{Vul}\ .\ \mathrm{Nnc}\ Ha\ \mathrm{Inq}\ Hb\ y\ ^{t_3}\ ^{\bullet}_{\bullet}$

$Hd\ \mathrm{Inq}\ Hc\ ^{\bullet}\ ^{\vee} v\ .\ t_2\ t_3\ Hc\ \mathrm{Inq}\ Hd\ v\ \cdot \in : \mathrm{Lex}\ \cdot\ ^{\vee} z\ .\ t_1\ t_2\ \mathrm{Fit}\ z\ *$

The word written Lex (*ƒL* lex = law) means "law". The law of a discourse has been stated.

3 39 2. $*\ ^{t_1} Ha\ Hb\ \mathrm{Alt}\ \mathrm{Inq}\ \cdot\ 1\ .\ 1\ .\ 11\ .\ 1001\ .\ 101\ .\ 11001\ .\ 111\ .$

$110001\ .\ 1001\ .\ 1010001\ ^{t_2}\ ^{\bullet}_{\bullet}$

$Hc\ \mathrm{Inq}\ Hd\ ^{\bullet}_{\bullet}\ t_1 \leqq t \leqq t_2\ . \rightarrow ^{\bullet}\ t\ \cdot\ Ha\ \mathrm{Inq}\ Hb\ .\ x\ . \rightarrow \mathrm{Pau}\ \mathrm{Pst}\ .\ Hb\ \mathrm{Inq}\ Ha\ x^{10}\ ^{\bullet}_{\bullet}$

$\wedge : t\ \cdot\ Hb\ \mathrm{Inq}\ Ha\ y\ . \rightarrow \mathrm{Pau}\ \mathrm{Pst}\ .\ Ha\ \mathrm{Inq}\ Hb\ .\ \sqrt{y} + 10\ ^{t_3}\ ^{\bullet}_{\bullet}$

$Hd\ \mathrm{Inq}\ Hc\ ^{\bullet}\ ^{\vee} v\ .\ t_2\ t_3\ Hc\ \mathrm{Inq}\ Hd\ v\ \cdot \in : \mathrm{Lex}\ \cdot\ ^{\vee} z\ .\ t_1\ t_2\ \mathrm{Fit}\ z\ *$

3 39 3. $*^{t_1}Ha\,Hb$ Alt Inq $\cdot\,1.1.10.11.101.1000.1101$ Etc^{t_2}⁞

Hc Inq Hd⁞ $x \cup y\,.=\,.Ha \cup Hb \cdot \wedge \cdot t_1 \leqq t \leqq t_2 : \to\,\dot{}$

$t : x$ Inq $y\,u\,.\,y$ Inq $x\,v \cdot \to$ Pau Pst $\cdot\,x$ Inq $y\,.\,u + v^{t_2}$⁞

Hd Inq $Hc \cdot\,{}^{\vee}v\,.\,t_2\,t_3\,Hc$ Inq $Hd\,v \cdot \in :$ Lex $\cdot\,{}^{\vee}z\,.\,t_1\,t_2$ Fit z $*$

3 40 1. $^{t_1}Ha\,Hb$ Alt Inq $\cdot\,110.1101.10010.10100.10111.$

     11000.100000^{t_2} |

Hb Dat $Ha\,.$ Den 1000^{t_3} |

Hc Inq $Hd\,.$ Cur Dat |

Hd Inq $Hc \cdot$ Qia . PAN Ha Vin |

Hc Inq $Hd\,.\,? =$ Vin |

Hd Inq Hc⁞ $t_1\,t_2\,Ha\,Hb$ Lud$_1$

    Deb $\cdot\,t_2\,t_3\,Hb$ Dat $Ha\,.$ Den $1000\,\dot{}$ Qia : Pau Ant $.\,t_3 \cdot Ha$ Vin $\dot{}$

    Pau Ant $.\,t_3 \cdot Ha$ Vin : Qia : Pau Ant $.\,t_3 \cdot Ha$ Inq $Hb\,100000$ |

Hc Inq Hd⁞ Cur $\dot{}\,x\,.\,1^{um}$ Par $\cdot\,{}^{\vee}v\,.\,t_1\,t_2\,Ha$ Inq $Hb\,v :$

    $\to\,.\,x = 110 :$ Sed $\cdot\,\neg\,.\,\to\,.\,x = 100000$⁞

    $\wedge$⁞ Cur $\dot{}\,x\,.\,1^{um}$ Par $\cdot\,{}^{\vee}v\,.\,t_1\,t_2\,Hb$ Inq $Ha\,v :$

    $\to\,.\,x = 1101 :$ Sed $\cdot\,\neg\,.\,\to\,.\,x = 100000 :$ Etc |

Hd Inq Hc ᵢ $: \neg$ Lic $: x\,.\,1^{um}$ Par $.\,{}^{\vee}v$ Etc $\cdot\,\to\,.\,x = 100000$ ᵢ $.$

    Sed ᵢ . Lic $\dot{}\,x\,.\,1^{um}$ Par $\cdot\,{}^{\vee}v\,.\,t_1\,t_2\,Ha$ Inq $Hb\,v :$

     $\to\,\cdot\,x \in$ Num $.\,\wedge\,.\,x \leqq 1000$ ᵢ

    $\wedge$ ᵢ Lic⁞ $y\,.\,1^{um}$ Par $\cdot\,{}^{\vee}w\,.\,t_1\,t_2\,Hb$ Inq $Ha\,w :$

     $\to\,\dot{}\,y \in$ Num $.\,\wedge$⁞ $y \leqq\,\dot{}\,{}^{\vee}x\cdot x\,.\,1^{um}$ Par $.\,{}^{\vee}v$ Etc $\cdot + 1000$ ᵢ

    $\wedge$ ᵢ Lic⁞ $z\,.\,10^{um}$ Par $\cdot\,{}^{\vee}v\,.\,t_1\,t_2\,Ha$ Inq $Hb\,v :$

     $\to\,\dot{}\,z \in$ Num $.\,\wedge\,.\,z \leqq\,\dot{}\,{}^{\vee}y\cdot y\,.\,1^{um}$ Par $.\,{}^{\vee}w$ Etc $\cdot + 1000$ ᵢ Etc ᵢ

    $\wedge$⁞ $\ulcorner x \urcorner \cup \ulcorner y \urcorner\,.\,= Ha \cup Hb \cdot \to$⁞ $x :$ Pau Pst $\cdot\,t_1 \cdot x$ Inq $y\,100000\,\dot{}$ Vin |

Hc Inq Hd⁞ Pot⁞ $tt'\,x$ Inq $y\,\dot{}\,1^{\epsilon}$⁞ $^{\wedge}r \cdot \neg$ Lic $.\,tt'\,x$ Inq $y\,r$ |

Hd Inq Hc⁞ Qqm Pot . Tan $\neg$ Lic⁞

    Qia⁞ $tt'\,x$ Inq $y\,\dot{}\,1^{\epsilon}$⁞ $^{\wedge}r \cdot \neg$ Lic $.\,tt'\,x$ Inq $y\,r\,\dot{}$

     $\to\,\cdot$ Pau Pst $t'\,.\,x$ Prd $\frown y$ Vin |

Hc Inq Hd⁞ Utr⁞ Lic⁞ $\neg \vee w\,\dot{}$

    Pau Pst $.\,1^{um}\,{}^{\vee}v\,.\,t_1\,t_2\,Ha$ Inq $Hb\,v :$ Hb Inq $Ha\,w$ |

Hd Inq Hc ᵢ $:$ Fal ᵢ $\cdot$ Qia⁞ Deb⁞ Pau Pst $.\,x$ Inq $y\,p \cdot y$ Inq $x\,\dot{}$

    1^{ϵ}⁞ $^{\wedge}q \cdot q \in$ Num $.\,\wedge\,.\,p < q \leqq p + 1000$ ᵢ $\cdot$

    ^{t_1}Cca $t\,.\,Ha\,Hb\,.$ Lud$_1 \cdot \to$ ᵢ . Deb ᵢ $\vee k$⁞ $\vee \ulcorner p_1\,.\,p_2\,.$ Etc $.\,p_k \urcorner$⁞

     $\wedge i : i \in$ Num $.\,\wedge\,.\,1 \leqq i \leqq k - 1 \cdot \to\,\cdot\,p_i < p_{i+1} \leqq p_i + 1000\,.\,\wedge\,.$

     $p_i \in$ Num⁞ $.\,\wedge\,.\,p_1 \leqq 1000\,.\,\wedge\,.\,p_k = 100000\,.$

     $\wedge :$ Cca $t\,.\,Ha \frown Hb$ Alt Inq $\cdot\,p_1\,.\,p_2\,.$ Etc $.\,p_k :$

     $\wedge$⁞ $\ulcorner x \urcorner \cup \ulcorner y \urcorner\,.\,=\,.\,Ha \cup Hb \cdot \to\,\dot{}$ Cca $t :\,{}^{\vee}x \cdot$ Cca $t\,.\,x$ Inq y

     $100000 :$ Vin^{t_2} |

Hc Inq Hd ᵢ $:\,{}^{\vee}r\,.\,t_4\,t_5\,Hd$ Inq $Hc\,r \cdot \in \cdot$ Lex $.\,Ha\,Hb$ Lud$_1$ ᵢ $\cdot$

    $w \in \cdot$ Cca $t_1\,.\,xy$ Lud^{io}⁞ $\to$ ᵢ . Deb ᵢ $\vee k$⁞ $\vee \ulcorner p_1\,.\,p_2\,.$ Etc $.\,p_k \urcorner$⁞

$p_1 \frown p_2 \frown \text{Etc} \frown p_k . \in \text{Num} \cdot$

$\wedge \cdot p_1 \leqq 1000 . \wedge . p_1 < p_2 \leqq p_1 + 1000 . \wedge . \text{Etc} . \wedge . p_k = 100000 \cdot$

$\wedge \cdot 1^{\text{um}} \text{Ict } w . = . x \text{ Inq } y \, p_1 \cdot$

$\wedge \cdot 10^{\text{um}} \text{Ict } w . = . y \text{ Inq } x \, p_2 \cdot$

$\wedge \cdot \text{Etc} \cdot$

$\wedge : \text{Ult}^{\text{um}} \text{Ict } w . = : x \text{ Inq } y \, p_k . x \text{ Vin} . y \text{ Prd} \cdot y \text{ Dat } x . \text{ Den } 1000 \cdot$

$\smile = : y \text{ Inq } x \, p_k . y \text{ Vin} . x \text{ Prd} \cdot x \text{ Dat } y . \text{ Den } 1000 \,|$

$Hd \text{ Inq } Hc . ? = \text{Ict} \,|$

$Hc \text{ Inq } Hd \cdot w = . xy \text{ Lud} : \rightarrow : z = . i^{\text{um}} \text{ Ict } w \cdot \leftrightarrow \cdot z . i^{\text{um}} \text{ Par} . w \; \#$

The word written Lud (fL ludit = plays) means "to play a game". We shall become acquainted with a variety of games, which will be distinguished by indices.

The words written Vin (fL vincit = wins) and Prd (fL perdit = loses) mean "wins" and "loses" respectively.

The word written Ict (fL ictus = blow) means (the substantive) "move".

3 40 2. $\#$ $Ha \text{ Inq } Hb \, \iota . \text{Nnc } Ha \text{ Vul } \iota \vee \ulcorner a . b \urcorner \mathbf{:} a \frown b \in \text{Pos} .$

$\wedge \mathbf{:} \text{PPN} \cdot \text{Fit } \mathfrak{p} . \rightarrow \mathbf{:} \text{Pau Pst} \cdot Ha \text{ Dat } Hb . \text{Den } a \mathbf{:}$

$\wedge \mathbf{:} \text{PPN} \cdot \text{Fit } \mathfrak{q} . \rightarrow \mathbf{:} \text{Pau Pst} \cdot Hb \text{ Dat } Ha . \text{Den } b \,|$

$Hb \text{ Inq } Ha \text{ Ben} \,|$

$Hc \text{ Inq } Hc \cdot \text{Nnc } Ha \frown Hb \text{ Vul} . \text{Nnc } Ha \, Hb \text{ Lud} :$

$\quad \text{PPN} \cdot \text{Fit } \mathfrak{q} . \rightarrow . Ha \text{ Vin} :$

$\quad \text{PPN} \cdot \text{Fit } \mathfrak{p} . \rightarrow . Hb \text{ Vin} :$

$\quad t \, Ha \text{ Vin} . \leftrightarrow . t \, Hb \text{ Prd} :$

$\quad t \, Hb \text{ Vin} . \leftrightarrow . t \, Ha \text{ Prd} \; \#$

3 40 3. A continuation of 3 40 1.

$Hc \text{ Inq } Hd \mathbf{:} \text{Nnc } Hc \text{ Vul} . \text{Nnc } Hc \, Hd \text{ Lud}_1 \cdot$

$\quad \text{Tan} \mathbf{:} \,^{\curlyvee} x . \text{PPN } x \text{ Prd} \cdot \text{Dat} \cdot \,^{\curlyvee} y . \text{PPN } y \text{ Vin} \cdot \text{Den } 10000 \mathbf{:}$

$Ha \text{ Inq } Hc \text{ Ben} \mathbf{:}$

$Hc \, Hd \text{ Alt Inq} \cdot 1000 . 1110 . 1111 . 10111 \mathbf{:}$

$Hc \text{ Inq } Hd . \text{Nnc } Hc \text{ Prd} \mathbf{:}$

$Hc \text{ Dat } Hd . \text{Den } 10000 \mathbf{:}$

$Hc \text{ Inq } Hd \mathbf{:} \text{Nnc } Hc \text{ Vul} . \text{Nnc } Hc \, Hd \text{ Lud}_1 \cdot$

$\quad \text{PPN} \mathbf{:} \,^{\curlyvee} x . x \text{ Prd} \cdot \text{Dat} \cdot \,^{\curlyvee} y . y \text{ Vin} \cdot \text{Den } 10000 \mathbf{:}$

$Hd \text{ Inq } Hc \text{ Ben} \mathbf{:}$

$Hc \, Hd \text{ Alt Inq} \cdot 101 . 1101 . 1110 . 1111 . 10111 \mathbf{:}$

$Hd \text{ Inq } Hc . \text{Nnc } Hd \text{ Prd} \mathbf{:}$

$Hd \text{ Dat } Hc . \text{Den } 10000 \mathbf{:}$

$Hd \text{ Inq } Hc \cdot \text{PAN } Hc \text{ Lud}_1 \text{ Mod Mal} . \text{Nnc } Hc \text{ Lud}_1 \text{ Mod Ben} \mathbf{:}$

$Hc \text{ Inq } Hd \mathbf{:} \text{Nnc } Hc \text{ Sci} \mathbf{:} ? \, p \cdot Hc \text{ Inq } Hd \, p . \in \text{Ben} \cdot$

Ncs : 1^{um} Ict . = 101 $\cdot \wedge \cdot$ 11^{um} Ict . = 1110 $\cdot \wedge \cdot$ 101^{um} Ict . = 10111 $\cdot$

Nnc Hc Vul . Nnc Hc Vin $\cdot \rightarrow$: Ncs $\cdot 1^{um}$ Ict . = 101 : Etc $\cdot$

Cca $t . xy$ Lud$_1 \cdot \wedge \cdot x$ Inq $y\,p. \in$ Ben : $\rightarrow \cdot$ Cca $t . x$ Vin $\cdot$

Erg : $\neg$ Pot $\cdot$ Cca $t .y$ Vin $\cdot$ Ncs $\cdot$ Cca $t .y$ Prd $\cdot$

$t : x$ Vul . Nnc x Vin $\cdot \rightarrow$ Pau Pst $\cdot x$ Vin $\cdot$

Lex . xy Lud$_1 \cdot \notin$ Ius $*$

The word written Ius (fL iustum = righteous) means "fair".

3 40 4.　$*$ Ha Inq Hb . 10 . Mod $\neg$ Pot Hb Ani |

Hb Inq Ha . 11 . Mod $\neg$ Pot Ha Ani |

Hc Inq $Ha \frown Hb \cdot$ PAN Ha Inq Hc 10 $\frown Hb$ Inq Hc 11 .

Erg Ha Vin |

Ha Inq Hc . 11 . Mod Etc |

Hb Inq Hc . 1 . Mod Etc |

Hc Inq $Ha \frown Hb$: 11 . 1 $\cdot$ Erg . Ha Vin |

Ha Inq Hc . 100 . Mod Etc |

Hb Inq Hc . 100 . Mod Etc |

Hc Inq $Ha \frown Hb$: 100 . 100 $\cdot$ Erg . Hb Vin |

Hd Inq He ı . PAN $Ha\ Hb$ Lud$_2$ ı

　　Cca $t . xy$ Lud$_2 \cdot \rightarrow$: Deb ⫶ $\vee \ulcorner z . p . q \urcorner$ ⫶

　　　Cca $t . x$ Inq $z . p$. Mod $\neg y$ Ani $\cdot \wedge \cdot$ Cca $t .y$ Inq $z . q$

　　　 . Mod $\neg x$ Ani $\cdot$

　　　$\wedge \cdot \ulcorner p \urcorner \cup \ulcorner q \urcorner . \mathsf{C} . 1 \cup 10 \cup 11 \cup 100$

　　　 . $\wedge \cdot p \neq q . \rightarrow$: Cca $t \cdot z$ Inq $x \frown y$. Nnc x Vin $\cdot$

　　　$\wedge \cdot p = q . \rightarrow$: Cca $t \cdot z$ Inq $x \frown y$. Nnc y Vin ı

　　$\ulcorner v$. PAN Hd Inq $He\ v \cdot \in$: Lex $\cdot xy$ Lud$_2$ |

Hc Inq $Hd \cdot$ Lex Etc . $\notin$ Ius |

Hd Inq Hc ı Lex Etc . $\in$ Ius ⫶

　　Qia ⫶ Deb ⫶ Nnc : x Vin . $\rightarrow$ Pau Pst $\cdot y$ Dat x . Den 1

　　　$\wedge \cdot$ Nnc : y Vin . $\rightarrow$ Pau Pst $\cdot x$ Dat y . Den 100 ⫶

　　Cca $t . xy$ Lud$_2 \cdot$

　　　$\rightarrow$ ⫶ Prb $\cdot$ Cca $t . x$ Vin ⫶ = 0,11 $\cdot \wedge \cdot$ Prb $\cdot$ Cca $t .y$ Vin ⫶ = 0,01 |

Hc Inq Hd ⫶ Cur $\cdot \neg$: Cca $t . Hb$ Inq $Hc \cdot \ulcorner u$. PAN Ha Inq $Hc\ u$ |

Hd Inq Hc ⫶ Cca $t . Hb$ Vin $\cdot$

　　　$\leftrightarrow$: Cca $t . Hb$ Inq ! $Hc \cdot ? u$. PAN Ha Inq $Hc\ u \cdot$

Tan . $\neg Hb$ Sci Etc . Qia $\neg$ Ani $\cdot \neg$ Lic . Hb Ani $\cdot$

$\neg$ Lic : Pau Ant . Hb Inq $Hc\ q \cdot Hc$ Inq ! $Hb \cdot ? p$. PAN Ha Inq $Hc\ p \cdot$

Pau Ant . Hb Inq $Hc\ q \cdot Hb$ Sci Etc :

　　　$\rightarrow$: Cca $t . Ha\ Hb$ Lud$_2 \cdot \notin$ Ius $\cdot$

Erg $\cdot$ Deb . $\neg$ Sci $*$

The word written Prb (fL probabilitas = probability) means "probability". Though mechanical stochastic devices are not

available, this simple game (akin to "matching pennies") gives us the opportunity to introduce numerical probability. Of course it should be repeated with other numerical constants.

From the next text the syntax of Prb will become clearer.

3 40 5.　　⁑ $Ha\,Hb$ Lud$_i$ ⏐
　　　　Hc Inq Hd · Nnc Hc Vul · Nnc $Hc\,Hd$ Lud$_i$ ⏐
　　　　Hd Inq Hc ⦂ ? = ˙ Prb ⦂ Fin . xy Lud$_i$ · x Vin ⏐
　　　　Hc Inq Hd ⏐ . Prb ⦂ Fin . xy Lud$_i$ · x Vin = 101/111 ⏐
　　　　　　Erg ˙ Prb ⦂ Fin . xy Lud$_i\,y$ · Vin = 10/111 ⦂
　　　　　　Qia ⦂ Prb ⦂ Fin . xy Lud · x Vin ˙ + ˙ Prb ⦂ Vin . xy Lud · y Vin ⦂ = 1 ⏐
　　　　Hd Inq Hc ⏐ ? ⌈⌐p . ⌐q⌉ ⦂ Deb ˙ t ⦂ x Vin . → ˙ y Dat x . Den p ˙
　　　　　　∧ ˙ t ⦂ y Vin . → ˙ x Dat y . Den q ⏐
　　　　Hc Inq Hd · p = 11 . ∧ . q = 100 ⏐
　　　　Hd Inq Hd ⦂ x Uti . xy Lud$_i$ · = ⦂ Den p × ˙ Prb ⦂ Fin . xy Lud$_i$ · x Vin ⦂
　　　　　　− ⦂ Den q × ˙ Prb ⦂ Fin . xy Lud$_i$ · y Vin ⦂ = Den 1 ⏐
　　　　Hd Inq Hc ⦂ Nnc Hd Vul ˙
　　　　　　t ⦂ $Hc\,Hd$ Lud$_i$. ↔ Pau Ant · Hc Dat Hd . Den 1 ⦂
　　　　　　Qia · Hc Uti . = . Den 1 · ∧ · Hd Uti . = . Den − 1 ⁑

The word written Uti (*fL* utilitas = utility) means "utility", "expectation".

3 40 6.　　⁑ Hd Inq Hd ⏐
　　　　Prb ⦂ Fin . xy Lud · x Vin ˙ = a ⦂
　　　　∧ ⦂ Prb ⦂ Fin . xy Lud · y Vin ˙ = b = 1 − a ⦂
　　　　∧ ⦂ w ∈ · Lex . xy Lud ⦂
　　　　∧ ˙ v = ⦂ t ⦂ x Vin . → ˙ y Dat x . Den p ˙
　　　　　　∧ ˙ t ⦂ y Vin . → ˙ x Dat y . Den q ⦂ ∧ . v Par w ⦂
　　　　→ ˙ x Uti . xy Lud · = · Den . $ap − bq$ ⦂
　　　　　　∧ ⦂ y Uti . xy Lud · = · Den . $bq − ap$ ⦂
　　　　　　∧ ⦂ w ∈ Ius . ↔ · x Uti . = . y Uti . = . Den 0 ⁑

3 40 7.　　⁑ Ha Inq Hc ⦂ 11 ⦂ Mod · −⏐ Pot . Hb Ani ⏐
　　　　Hb Inq Hc ⦂ 1 ⦂ Mod · ¬ Pot . Ha Ani ⏐
　　　　Hc Inq Ha ⌒ Hb ⦂ 11 . 1 · Erg . Ha Vin ⏐
　　　　Ha Inq Hc . 1 . Mod Etc ⏐
　　　　Hb Inq Hc . 10 . Mod Etc ⏐
　　　　Hc Inq Ha ⌒ Hb ⦂ 1 . 10 · Erg . Ha Vin ⏐
　　　　Ha Inq Hc . 10 . Mod Etc ⏐
　　　　Hc Inq Ha . 10 . Mod Etc ⏐
　　　　Hc Inq Ha ⌒ Hb ⦂ 10 . 10 · Erg . Hb Vin ⏐
　　　　Ha Dat Hb . Den 10 ⏐
　　　　Hd Inq He . PAN $Ha\,Hb$ Lud$_3$ ⏐

He Inq Hd ˙ ? $^\vee w : w \in$ ˙ Lex . xy Lud$_3$ |

Hd Inq He ፥ Deb ፥ ∨ ⌐$a.b.z.t$⌐ $a \frown b . \in . 1 \cup 10 \cup 11 \cup 100$ ˙ ∧

    ˙ Cca t . Ha Inq $z : a :$ Mod ˙ ¬ Pot . Hb Ani ˙ ∧

    ˙ Cca t . Hb Inq $z : b :$ Mod ˙ ¬ Pot . Ha Ani ˙ ∧

    ፥ Cca t . Hb Vin ˙ ↔ . $a = b :$ ∧

    ˙ Cca t : Hb Vin . → ˙ Ha Dat Hb . Den a |

He Inq Hd ˙ Nnc He Vul . Nnc Hd He Lud$_3$ |

Hd Inq He ፥ Nnc Hd Vul ˙ Nnc ፥ Hd He Lud$_3$.

    ↔ Pau Ant ˙ He Dat Hd . Den 0,101 |

He Inq Hd . Cur 0,101 |

Hd Inq He ፥ Qia ፥ Prb . Hd Inq Hc a ˙ = ˙ Prb . He Inq Hc b : = 0,01 ˙

    Erg ፥ Prb ˙ Hd Inq Hc i ⌢ He Inq Hc i = 0,0001 ˙

    Erg ˙ He Uti . = . Den 0,101 |

He Inq Hd ፥ Utr ˙ Deb : Prb . Hd Inq Hc i ˙ = 0,01 |

Hd Inq He ፥ ¬ Deb ፥ Qia ፥ ¬ ˙ Prb = 0,01 . Par : $^\vee w$ ˙

    $w \in$. Lex Lud$_3$ ፥ Tan : Prb = 0,01 . → ˙

    He Uti . = . Den 0,101 |

He Inq Hd : ? = ˙ Prb . Hd Inq He i |

Hd Inq He : ¬ Deb ˙ Nnc Hd Inq ! He . ? = Etc |

He Inq He ⌐ Pon ˙ Prb . Hd Inq Hc i ˙ = c_i :

      ∧ : Prb . He Inq Hc j ˙ = d_i ፥

    Erg ˙ He Uti . = . $c_1 d_1 + 10 \, c_2 d_2 + 11 \, c_3 d_3 + 100 \, c_4 d_4$ ፥

    Exg ፥ Nnc ˙ Hd Vul . 11 $c_3 \geqq 1 \, c_1 \frown \geqq 10 \, c_2 \frown \geqq 100 \, c_4$ ˙ → Pau Pst :

      He Vul . $d_3 = 1$ ˙ He Uti = Den 11 c_3 ፥ Etc ፥

    Erg ፥ Nnc Hd Vul . He Uti ˙ ≦ . Den e : →

      ፥ Ncs ፥ Nnc Hd Vul ˙ ∧ $i : i = 1 \smile 10 \smile 11 \smile 100$. → ˙ $i c_i \leqq e$.

      Erg . $c_i \leqq e / i$. Erg . $1 = c_1 + c_2 + c_3 + c_4 \leqq 11001 \, e / 1100$.

      Erg . $e \geqq 1100 / 11001$ ፥

    ¬ Pot : Nnc Hd Vul ˙ He Uti . < 1100/11001 ፥

    Ncs : Nnc Hd Ccd ˙ He Uti . ≧ 1100/11001 ፥

    Tan ˙ Pot : Nnc Hd Vul ˙ He Uti . = 1100/11001 |

He Inq Hd ˙ Nnc He Ccd :

    Ant . Hd He Lud$_3$ ˙ He Dat Hd . Den 1100/11001 |

Hd Inq He ˙ Ben . Qia Ius #

von Neumann's minimax principle.

3 40 8.   # Ha Hb Alt Inq Hc ፥ 1 . 10 . 1 . 1 . 10 . 10 . 10 ፥

      Mod ፥ ¬ Pot ˙ Hb Ani ˙ ? $^\vee u$. Ha Inq Hc u :

        ⌣ : Ha Ani ˙ ? $^\vee v$. Hb Inq Hc v |

     Hc Inq Ha ⌢ Hb ˙ $1 + 10 + 1 + 1 + 10 + 10 + 10 \geqq 1010$. Ha Vin |

     Hd Inq He . PAN Ha Hb Lud$_4$ |

     He Inq Hd ˙ ? $^\vee w : w \in$ ˙ Lex . Ha Hb Lud$_4$ |

Hd Inq He ⦂ Deb ⦂ $\vee k$ ⦂ $\vee \ulcorner p_1 . p_2 .$ Etc $. p_k \urcorner$ ⦂

  $p_1 \frown p_2 \frown$ Etc $\frown p_k . \in . 1 \cup 10 : \wedge : Ha$ Hb Alt Inq ·

  $p_1 . p_2 .$ Etc $. p_k : \wedge : k^{um}$ Ict $. = .$ Ult^{um} Ict $\cdot \leftrightarrow .$

  $p_1 + p_2 + $ Etc $+ p_k \geqq 1010 :$

  $\wedge \dot{}\ x \in . Ha \cup Hb \cdot \rightarrow :$ Ult^{um} Ict $. x$ Vin $\cdot \leftrightarrow \cdot$

    Ult^{um} Ict $. = . x$ Inq Hc $p_k \dot{}$

  $\wedge \dot{}\ t : Ha$ Vin $. \rightarrow$ Pau Pst $\cdot Hb$ Dat $Ha .$ Den 1 $\dot{}$

  $\wedge \dot{}\ t : Hb$ Vin $. \rightarrow$ Pau Pst $\cdot Ha$ Dat $Hb .$ Den 1 ⎮

He Inq $Hd \frown Hc \cdot$ Nnc He Vul $.$ Nnc Hd He Lud$_4$ ⎮

$Hd \frown Hc$ Inq He Ben ⎮

Hd $He .$ Alt Inq $Hc \cdot 1 . 10 . 1 . 10 . 10 . 1 . 10 \cdot$ Mod Etc ⎮

Hc Inq $Hd \frown He \cdot 1 + 10 + 1 + 10 + 10 + 1 + 10 \geqq 1010 . Hd$ Vin ⎮

He Inq $Hd \frown Hc \cdot$ Nnc He Vul $.$ Etc ⎮

$Hd \frown Hc$ Inq He Ben ⎮

Hd $He .$ Alt Inq $Hc \cdot 1 . 1 . 1 . 1 . 10 . 10 . 10 \cdot$ Mod Etc ⎮

Hc Inq $Hd \frown He \cdot 1 + 1 + 1 + 1 + 10 + 10 + 10 \geqq 1010 . Hd$ Vin ⎮

He Inq He ⦂ Utr $\dot{}\ \neg$ Pot $: \vee t \cdot$ Cca $t . He$ Vin ⎮

Hd He Lud$_4$ ⦂ Mod $: 1 . 1 . 1 . 1 . 10 . 1 . 10 . 1 \cdot He$ Vin $\dot{}$

  Mod $: 10 . 1 . 10 . 1 . 10 . 1 . 10 \cdot Hd$ Vin $\dot{}$

  Mod $: 10 . 1 . 10 . 1 . 10 . 10 \cdot He$ Vin $\dot{}$

  Mod $: 1 . 1 . 1 . 1 . 10 . 10 . 10 \cdot Hd$ Vin ⎮

He Inq He ⦂ PAN $He .$ Lud$_4$ $.$ Mod Mal $\cdot$

  $\wedge \cdot$ PPN $He .$ Lud$_4$ $.$ Mod Ben ⦂

  Nnc He Sci ⦂ ? $\check{}w \dot{}$ Cca $t : He$ Vul $.$ Nnc He Vin $\cdot$

    $\rightarrow \cdot$ Ncs $.$ Nnc He Inq $Hc \check{}w$ ⎮

Hd He Lud$_4$ ⦂ Mod $: 10 . 1 . 10 . 1 . 10 . 10 \cdot He$ Vin $\dot{}$

  Mod $: 10 . 10 . 10 . 10 . 10 \cdot Hd$ Vin $\dot{}$

  Mod $: 1 . 10 . 1 . 10 . 10 . 10 \cdot He$ Vin $\dot{}$

  Mod $: 10 . 1 . 10 . 1 . 10 . 10 \cdot He$ Vin $\dot{}$

  Mod $: 1 . 1 . 1 . 1 . 10 . 10 . 10 \cdot Hd$ Vin ⎮

Hf Inq Hf ⦂ Nnc Hd He Lud$_4$ ⦂ Mod ⦂

  Prb $: x$ Inq $Hc \cdot 1 . 1 . 10 . 10 . 10 .$ Etc $\dot{}$

  $= \dot{}$ Prb $: x$ Inq $Hc \cdot 10 . 10 . 10 .$ Etc ⦂

  x Lud$_4$ $.$ Mod $.$ Nnc Hd He Lud$_4$ $\cdot \rightarrow \cdot x$ Lud$_4$ $\in$ Ben ⦂

  Lex Lud$_4$ $. \in$ Ius ⎮

He Dat $Hd \cdot$ Den $110 -$ Den $101 . = .$ Den 1 $^{\#}$

3 40 9. $^{\#} Ha$ Hb $Hc .$ Alt Inq $\cdot 1 . 11 . 101 : Hc$ Vin $:$

  Ha Hb $Hc .$ Alt Inq $\cdot 1 . 10 . 11 . 101 : Ha$ Vin $:$

  Ha Hb $Hc .$ Alt Inq $\cdot 10 . 11 . 101 : Hc$ Vin $:$

  Ha Hb $Hc .$ Alt Inq $\cdot 1 . 10 . 11 . 101 : Ha$ Vin ⎮

  Hd Inq He ⎮ $.$ Nnc Ha Hb Hc Lud$_5$ ⎮

Deb $⦂ \vee \ulcorner t.n \urcorner ⦂ \vee \ulcorner p_1.p_2. \text{Etc} . p_n \urcorner ⦂$

   $p_1 \frown p_2 \frown \text{Etc} \frown p_n . \in . 1 \cup 10 \cup 11 \cup 100 \cup 101 \cdot$

   $\wedge . p_1 \leqq 10 . \wedge . p_2 - p_1 = 1 \vee 10 . \wedge . p_3 - p_2 = 1 \vee 10 . \wedge . \text{Etc} .$

   $\wedge ⦂ \text{Cca}\, t . Ha\, Hb\, Hc . \text{Alt Inq} \cdot p_1 . p_2 . \text{Etc} . p_n ⦂$

   $\wedge ⦂ n^{\text{um}} \text{Ict} . = . \text{Ult}^{\text{um}} \text{Ict} \cdot \leftrightarrow . p_n \geqq 101 ⦂$

   $\wedge \cdot \ulcorner x \urcorner \cup \ulcorner y \urcorner \cup \ulcorner z \urcorner . = . Ha \cup Hb \cup Hc \cdot \to ⦂$

     Pau Pst . Ult$^{\text{um}}$ Ict . x Vin $\cdot$

       $\leftrightarrow \cdot$ Ult$^{\text{um}}$ Ict . $= . x$ Inq $yz\,p_n \cdot \wedge \cdot$

   Cca $t . Ha$ Vin $\cdot \to$ Pau Pst $⦂ Hb$ Dat Ha . Den 1000 $\cdot$

     Hc Dat Ha . Den 1000 $\cdot \wedge \cdot$

   Cca $t . Hb$ Vin $\cdot \to$ Pau Pst . Etc $\cdot \wedge \cdot$

   Cca $t . Hc$ Vin $\cdot \to$ Pau Pst . Etc |

He Inq $Hd ⦂ \notin$ Ius $⦂$ Qia $⦂ \neg$ Pot $⦂ \vee t \cdot$ Cca $t . Hb$ Vin $\cdot$

   $t \cdot Ha$ Inq $Hb \frown Hc\, 10 . \to$ Pau Pst . Hc Vin $\cdot$

   $t \cdot Ha$ Inq $Hb \frown Hc\, 1 . \to$ Pau Pst . $Ha \vee Hc$ Vin |

Hd Inq $He ⦂$

   $t\, Ha$ Inq $1 . \wedge . z = Ha \vee Hc . \wedge ⦂$ Pau Pst . $t \cdot Hb$ Vul . Nnc z Vin $\cdot$

     $\to ⦂$ Pau Pst . $t \cdot z$ Vin $⦂$

   Cca $t . y$ Vul $⦂$ Pau Pst . $t \cdot z$ Vin $\cdot \leftrightarrow \cdot$ Cca $t . y$ Aux $z ⦂$

   Pot . Hb Aux $Ha ⦂$ Pot . Hb Aux Hc |

$Ha\, Hb\, Hc \cdot$ Vul $\cdot$ Nnc $Ha\, Hb\, Hc$. Lud$_5$ |

Ha Inq $Hb ⦂$ Nnc Ha Pol $Hb \cdot$

   Nnc Hb Vul . PPN Ha Vin $\cdot \to ⦂$ Nnc Ha Ccd $\cdot$

     PPN Hb Dat Ha . Den 0 |

Hc Inq Hb ‖ Nnc Hc Pol $Hb ⦂$

     Nnc Hb Aux $Hc . \to \cdot$ Nnc Hc Ccd $⦂$

     $\neg \cdot$ PPN Hb Dat Hc . Den 1000 $⦂$ Sed $\cdot Hc$ Dat Hb . Den 10 $⦂$

   Hc Uti Hb Aux $Hc =$ Den 1100 $⦂$

$\vee a ⦂$ Nnc Hb Aux $Hc . \to \cdot$ PPN Pec $Hc . = a \cdot$

     $- \vee b ⦂$ Nnc Hb Aux $Ha . \to \cdot$ PPN Pec $Hc . = b ⦂ =$ Den 1100 $⦂$

   Erg $⦂ Hc$ Pol $Hb . 0,1 \times$ Den 1100 $\cdot \in$ Ius |

Ha Inq $Hb ⦂$ Nnc Ha Pol $Hb ⦂$

   PPN Ha Dat $Hb \cdot$ Den $⦂ 10 + \vee x \cdot$ PAN Hc Pol Hb . Den x |

Hb Inq Ha Ben |

Ha Inq $Hb \frown Hc$ 1 |

Hb Inq $Hc \frown Ha$ 10 |

Hc Inq $Ha \frown Hb$. Nnc Hc Prd |

Hc Dat Ha . Den 1000 |

Ha Dat Hb . Den 100 |

Hd Inq $He \cdot$ PAN Hb Aux $Ha . Ha\, Hb$ Soc ✳

The word written Aux ($\it fL$ auxiliatur = aids) means (the verb) "aids". Soc ($\it fL$ socii = allies) means "allies".

3 41 1. $*\,^{t_1}Ha\,\mathrm{Inq}\,Ha\cdot\neg\vee x.t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,x^{t_2}$ ⫶

       $Hb\,\mathrm{Inq}\,Hc\cdot?\,p.t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,p$ ⫶

       $Hc\,\mathrm{Inq}\,Hb\cdot\neg\vee x.t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,x$ ⫶

       $Hb\,\mathrm{Inq}\,Hc\,\mathrm{Mal}$ ⫶

       $Hc\,\mathrm{Inq}\,Hb\cdot t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha:\neg\vee x.t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,x$ ⫶

       $Hb\,\mathrm{Inq}\,Hc\,\mathrm{Ben}$ ⫶

       $Hc\,\mathrm{Inq}\,Hb\cdot\mathrm{Tan}:{}^\curlyvee p.t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\cdot\in\mathrm{Fal}$ ⫶

       $Hb\,\mathrm{Inq}\,Hc\,\mathrm{Ben}\,*$

A text that prepares the way for the statement of the liar:

3 41 2. $*\,^{t_1}Ha\,\mathrm{Inq}\,Ha:{}^\curlyvee x.t_1\,t_2\,Ha\,\mathrm{Inq}\,Ha\,x\cdot\in\mathrm{Fal}^{t_2}$ |

       $Hb\,\mathrm{Inq}\,Ha:t_1\,t_2\,Ha\,\mathrm{Vul}:\mathrm{Nnc}\,Ha\,\mathrm{Inq}\,Ha\cdot1^\epsilon\mathrm{Ver}\,{}^\bullet$

           $\rightarrow{}^\bullet t_1\,t_2\,Ha\,\mathrm{Vul}:\mathrm{Nnc}\,Ha\,\mathrm{Inq}\,Ha\cdot1^\epsilon\mathrm{Fal}$ ⫶

         $\wedge:t_1\,t_2\,Ha\,\mathrm{Vul}:\mathrm{Nnc}\,Ha\,\mathrm{Inq}\,Ha\cdot1^\epsilon\mathrm{Fal}\,{}^\bullet$

           $\rightarrow{}^\bullet t_1\,t_2\,Ha\,\mathrm{Vul}:\mathrm{Nnc}\,Ha\,\mathrm{Inq}\,Ha\cdot1^\epsilon\mathrm{Ver}$ ⫶

       $\mathrm{Utr}\,{}^\bullet t_1\,t_2\,Ha\,\mathrm{Vul}:{}^\curlyvee p.\mathrm{Nnc}\,Ha\,\mathrm{Inq}\,Ha\cdot\in\mathrm{Ver}:\smile:\in\mathrm{Fal}$ |

       $Ha\,\mathrm{Inq}\,Hb:t_1\,t_2\,Ha\,\mathrm{Vul}:t_1\,t_2\,Ha\,\mathrm{Inq}\,Hb\,p\,.$

         $\rightarrow{}^\bullet p\in\mathrm{Ver}.\rightarrow.p\in\mathrm{Fal}\cdot\wedge\cdot p\in\mathrm{Fal}.\rightarrow.p\in\mathrm{Ver}$ |

       $Hb\,\mathrm{Inq}\,Ha\cdot t_1\,t_2\,\mathrm{Fit}\,q.\rightarrow.q\in\mathrm{Mal}\,*$

The talk of the liar.

SPACE, MOTION, MASS

4 00 0. So far the members of the class 'Hom' might be ghosts. The only extension we needed, was time. We shall now introduce space, motion, mass, and other notions of mechanics. We could do so by axioms, but such a procedure would be unsatisfactory. We prefer the behaviouristic approach. Afterwards the crude ideas we have acquired will be refined and settled with more precision by means of an axiomatic system.

4 01 1. $Ha\,\mathrm{Inq}\,Hb\,\vdots\,\vee h\,\dot{\cdot}\,h\in\mathrm{Pos}\,.$

$\quad\wedge\dot{:}\,\mathrm{Sec}\,h\,.\,\mathrm{Pst}\cdot Hc\,\mathrm{Inq}\,Hd\,\mathfrak{p}:Hd\,\mathrm{Ani}:\mathrm{Utr}\,.\,\mathrm{PAN}\,Hc\,\mathrm{Inq}\,Hd\,\mathfrak{p}\dot{}$

$\quad\wedge\dot{}\,\mathrm{Sec}\,h\,.\,\mathrm{Pst}\cdot Hd\,\mathrm{Inq}\,Hc\,\mathfrak{q}:Hc\,\mathrm{Ani}:\mathrm{Utr}\,.\,\mathrm{PAN}\,Hd\,\mathrm{Inq}\,Hc\,\mathfrak{q}\,\iota$

$Hb\,\mathrm{Inq}\,Ha\cdot\mathrm{Cur}\,.\,h>0\,\iota$

$Ha\,\mathrm{Inq}\,Hb\cdot\mathrm{Qia}\cdot\mathrm{Loc}\,Hc\,.\neq\,.\,\mathrm{Loc}\,Hd\,\dot{:}$

$\quad\mathrm{Loc}\,Hc\,.=\mathrm{Cca}\,.\,\mathrm{Loc}\,Hd\cdot\to\dot{}\mathrm{Pot}:t_1\,t_2\,Hc\,\mathrm{Ani}\cdot\tfrac{?}{}p\,.\,t_1\,t_2\,Hd\,\mathrm{Inq}\,Hc\,\dot{:}$

$\quad\mathrm{Loc}\,Hc\,.\neq\,.\,\mathrm{Loc}\,Hd\cdot\to\,.\,\neg\,\mathrm{Pot}\,\mathrm{Etc}\,\dot{:}$

$\quad\mathrm{Loc}\,Hc\subset\mathrm{Spa}\dot{:}\,\mathrm{Loc}\,Hd\subset\mathrm{Spa}\dot{:}$

$\quad a\in\mathrm{Spa}\,.\to\,.\,a\in a\dot{:}$

$\quad\wedge X:X\in\,.\,\mathrm{Hom}\cup\mathrm{Bes}\cdot\to\cdot\mathrm{Loc}\,X\,.\subset\mathrm{Spa}\dot{:}$

$\quad\vee Z\dot{}Z\subset\mathrm{Spa}\,.\,\wedge\dot{:}\,\neg\,\vee X\cdot Z=\,.\,\mathrm{Loc}\,X\dot{\vdots}$

$\quad\mathrm{Sec}\,h\,.\,\mathrm{Pst}\cdot X\,\mathrm{Inq}\,Y\,p:Y\,\mathrm{Ani}\,\mathrm{Etc}\dot{}\wedge$

$\quad\mathrm{Sec}\,h'\,.\,\mathrm{Pst}\cdot X'\,\mathrm{Inq}\,Y'\,p':Y'\,\mathrm{Ani}\,\mathrm{Etc}\dot{}\wedge\cdot\mathrm{Loc}\,X'\,.\neq\,.\,\mathrm{Loc}\,Y'\dot{:}$

$\quad\to:\mathrm{Dst}\,XY\,.\,/\,.\,\mathrm{Dst}\,X'\,Y'\cdot=\mathrm{Cca}\,.\,h/h'\dot{\vdots}$

$\quad\vee p:p\in\mathrm{Pos}\,.\,\wedge\cdot\mathrm{Dst}\,XY\,.=\,.\,\mathrm{Cmt}\,p\dot{:}$

$\quad\mathrm{Dst}\,XY\,.=\cdot\mathrm{Dst}\,.\,\mathrm{Loc}\,X\,.\,\mathrm{Loc}\,Y\dot{:}$

$\quad a\in\mathrm{Loc}\,X\,.\,\wedge\,.\,b\in\mathrm{Loc}\,Y\cdot\to\cdot\mathrm{Dst}\,XY\,.=\mathrm{Cca}\,.\,\mathrm{Dst}\,ab\,\ast$

The word written Loc (*ƒL* locus = place) means "place" (of). Delay of signals is symptomatic for difference of place.

The word written Spa (*ƒL* spatium = space) means "space" (or, more precisely: set of spatial points). All places are in space.

The word written Dst (*ƒL* distantia = distance) means "distance". 'Dst' is said to be proportional to the time of delay. Of course this definition is provisional. It can be applied as long as there is not more than one method of signalizing. In due course we shall learn that there are several kinds.

The word written 'Cmt' means "centimeter". The length unit is still unknown. 'Cmt' behaves syntactically like 'Sec'.

4 01 2. $*\,a \frown b \in \mathrm{Rea}\,.$

$\to \cdot \mathrm{Cmt}\,.\,a+b\cdot = \,.\,\mathrm{Cmt}\,a+\mathrm{Cmt}\,b:$

$\wedge:\mathrm{Cmt}\,.\,a-b\cdot = \,.\,\mathrm{Cmt}\,a-\mathrm{Cmt}\,b:$

$\wedge:\mathrm{Cmt}\,a\,.\leqq\,.\,\mathrm{Cmt}\,b\cdot\leftrightarrow.\,a\leqq b:$

$\wedge:\mathrm{Cmt}\,a\,.=\mathrm{Cca}\,.\,\mathrm{Cmt}\,b\cdot\leftrightarrow\cdot a\,.=\mathrm{Cca}\,.\,b:$

$\wedge:a\times.\,\mathrm{Cmt}\,b\cdot = \cdot\mathrm{Cmt}\,.\,a\times b:\wedge.$

Etc $*$

4 01 3. $*\,a \frown b \frown c\,.\in \mathrm{Spa}\,.$

$\to\,\vdots\,\vee p:p\in\mathrm{Rea}\,.\wedge.\,p\geqq 0\,.\wedge\cdot\mathrm{Dst}\,ab\,.=\,.\,\mathrm{Cmt}\,p\cdot$

$\wedge\cdot\mathrm{Dst}\,ab\,.=\,.\,\mathrm{Dst}\,ba\cdot$

$\wedge:\mathrm{Dst}\,ab\,.=\,.\,\mathrm{Cmt}\,0\cdot\leftrightarrow.\,a=b:$

$\wedge:\mathrm{Dst}\,ab\,.+\,.\,\mathrm{Dst}\,bc\cdot\geqq\,.\,\mathrm{Dst}\,ac$ $*$

Axioms of metric space.

4 01 4. $*\,a\frown b\in\mathrm{Spa}\,.\wedge.\,\alpha\frown\beta\in{}^\wedge\mathrm{Dst}\,.\wedge.\,\alpha\geqq\mathrm{Cmt}\,0\,.\wedge.\,\beta\geqq\mathrm{Cmt}\,0\,.\wedge$

$\quad\cdot\alpha+\beta = \,.\,\mathrm{Dst}\,ab\cdot$

$\wedge\gamma\frown\delta\in{}^\wedge\mathrm{Dst}\,.\wedge.\,\gamma\geqq\mathrm{Cmt}\,0\,.\wedge.\,\delta\geqq\mathrm{Cmt}\,0\,.\wedge\cdot\gamma-\delta=\,.\,\mathrm{Dst}\,ab:$

$\to\,\vdots\,\mathrm{Car}\cdot{}^\wedge c:c\in\mathrm{Spa}\,.\wedge\cdot\mathrm{Dst}\,ac\,.=\alpha\cdot\wedge\cdot\mathrm{Dst}\,bc\,.=\beta\,\vdots\,=1\,\vdots$

$\wedge\,\vdots\,\mathrm{Car}\cdot{}^\wedge c:c\in\mathrm{Spa}\cdot\wedge\cdot\mathrm{Dst}\,ac\,.=\gamma\cdot\wedge\cdot\mathrm{Dst}\,bc\,.=\delta\,\vdots\,=1$ $*$

Special properties of the metrics of Euclidian space.

4 01 5. $*\,a\frown b\in\mathrm{Spa}\,.\to\,\vdots$

$\mathrm{Cvx}\,.\,a\cup b\cdot = \cdot{}^\wedge c:\mathrm{Dst}\,ac\,.+\,.\,\mathrm{Dst}\,bc\cdot = \,.\,\mathrm{Dst}\,ab\,|$

$A\subset\mathrm{Spa}\,.\to\,\iota\,\mathrm{Cvx}\,A\,.$

$\quad = \,\vdots\,\bigcap{}^\wedge B:A\subset B\subset\mathrm{Spa}\,.\wedge:\wedge\ulcorner x.y\urcorner\cdot x\frown y\in B\,.$

$\quad\to:\mathrm{Cvx}\,.\,x\cup y\cdot\subset B$ $*$

The word written Cvx means "convex envelope".

4 01 6. $*\,r=\,.\,\mathrm{Rct}\,ab\cdot\leftrightarrow:a\frown b\in\mathrm{Spa}\,.\wedge.\,a\neq b\,.\wedge\,\vdots\,r=\,\vdots\,{}^\wedge c\cdot$

$\quad c\in\cdot\mathrm{Cvx}\,.\,a\cup b:\vee:a\in\cdot\mathrm{Cvx}\,.\,b\cup c:\vee:b\in\cdot\mathrm{Cvx}\,.\,a\cup c\,\iota$

$\quad r\in\mathrm{Rct}\cdot\leftrightarrow:\vee\ulcorner a.b\urcorner\cdot r=\,.\,\mathrm{Rct}\,ab$ $*$

The word written Rct (*fL* recta = straight line) means "straight lines". 'Rct *ab*' means the straight line through *a* and *b*.

4 01 7. $*\,p\in\mathrm{Pla}\,.\leftrightarrow\,\vdots\,p\subset\mathrm{Spa}\,.\wedge:\vee\ulcorner r.s\urcorner\cdot r\neq s\,.$

$\quad\wedge.\,r\frown s\in\mathrm{Rct}\,.\wedge\cdot r\cap s\,.\neq\ulcorner\,\urcorner\cdot\wedge:p=\,\cdot\mathrm{Cvx}\,.\,r\cup s$ $*$

The word written Pla (*fL* planum = plane) means "planes".

4·01 8. $*\,a \frown b \in \mathrm{Spa}\,.\,\wedge.\,a \neq b\,.\,\rightarrow\,\colon \mathrm{Car}\,\cdot\,{}^{\curlywedge}r : r \in \mathrm{Rct}\,.\,\wedge\,.\,a \cup b\,.\,\subset r\,\colon = 1\,\colon$

     $a \in \mathrm{Spa}\,.\,\wedge.\,r \in \mathrm{Rct}\,.\,\wedge.\,a \notin r\,.\,\rightarrow\,\colon \mathrm{Car}\,\cdot\,{}^{\curlywedge}p : p \in \mathrm{Pla}\,.\,\wedge\,.\,a \cup r\,.\,\subset p\,\colon = 1\,\colon$

     $r \frown s \in \mathrm{Rct}\,.\,\wedge\,\cdot\,r \cap s \neq \ulcorner\,\urcorner\,.\,\wedge.\,r \neq s\,\colon$

         $\rightarrow\,\colon \mathrm{Car}\,\cdot\,{}^{\curlywedge}p : p \in \mathrm{Pla}\,.\,\wedge\,.\,r \cup s\,.\,\subset p\,\colon = 1\,\colon$

     $r \in \mathrm{Rct}\,.\,\wedge\,.\,p \in \mathrm{Pla}\,.\,\wedge\,\cdot\,p \cap r\,.\neq r \frown\,\neq\,\ulcorner\,\urcorner\,\colon \rightarrow\,\colon \mathrm{Cvx}\,.\,r \cup p\,\cdot = \mathrm{Spa}\,\colon$

     $r \frown s \in \mathrm{Rct}\,.\,\wedge\,.\,r \neq s\,\cdot\,\rightarrow\,\colon \mathrm{Car}\,.\,r \cap s\,\cdot \leqq 1\,\colon$

     $p \frown q \in \mathrm{Pla}\,.\,\wedge\,.\,p \neq q\,\cdot\,\rightarrow\,\colon p \cap q\,.= \ulcorner\,\urcorner\,\cdot\,\vee\,\cdot\,p \cap q\,.\,\in \mathrm{Rct}\,\colon$

     $p \in \mathrm{Pla}\,.\,\wedge\,.\,r \in \mathrm{Rct}\,\cdot\,\rightarrow\,\cdot\,p \supset r\,.\,\vee\,\colon \mathrm{Car}\,.\,p \cap r\,\cdot \leqq 1\,\ast$

4·01 9. $r \frown s \in \mathrm{Rct}\,.\,\rightarrow\,\colon r\|s\,.$

     $\leftrightarrow\,\cdot\,\vee p\,\cdot\,p \in \mathrm{Pla}\,.\,\wedge\,.\,r \subset p\,.\,\wedge\,.\,s \subset p\,\colon \wedge : r = s\,.\,\vee\,\cdot\,r \cap s\,.= \ulcorner\,\urcorner\,\colon$

     $p \frown q \in \mathrm{Pla}\,.\,\rightarrow\,\cdot\,p\|q\,.\,\leftrightarrow\,\colon p = q\,.\,\vee\,\cdot\,p \cap q\,.= \ulcorner\,\urcorner\,\colon$

     $p \in \mathrm{Pla}\,.\,\wedge\,.\,r \in \mathrm{Rct}\,\cdot\,\rightarrow\,\cdot\,p\|r\,.\,\leftrightarrow\,\colon p \supset r\,.\,\vee\,\cdot\,p \cap r\,.= \ulcorner\,\urcorner\,\colon$

     $r \in \mathrm{Rct}\,.\,\wedge\,.\,a \in \mathrm{Spa}\,\cdot\,\rightarrow\,\colon \mathrm{Car}\,\colon {}^{\curlywedge}s\,\cdot\,s \in \mathrm{Rct}\,.\,\wedge\,.\,a \in s\,.\,\wedge\,.\,s\|r\,\cdot = 1\,\ast$

     '$\|$' means "parallel".

4·02 1. $*\,a \in \mathrm{Spa}\,.\,\wedge\,.\,\varrho \in \mathrm{Pos}\,\cdot\,\rightarrow\,\colon \mathrm{Sph}\,a\,\varrho\,.= {}^{\cdot\curlywedge}x : x \in \mathrm{Spa}\,.\,\wedge\,\cdot\,\mathrm{Dst}\,ax\,.= \varrho\,\colon$

     $\wedge\,\colon \mathrm{Bul}\,a\,\varrho\,.= {}^{\cdot\curlywedge}x : x \in \mathrm{Spa}\,.\,\wedge\,\cdot\,\mathrm{Dst}\,ax\,.\leqq \varrho\,\colon$

     $s \in \mathrm{Sph}\,.\,\leftrightarrow\,\colon \vee\,\ulcorner a\,.\,\varrho\,\urcorner\,\cdot\,s = .\,\mathrm{Sph}\,a\varrho\,\colon$

     $s \in \mathrm{Bul}\,.\,\leftrightarrow\,\colon \vee\,\ulcorner a\,.\,\varrho\,\urcorner\,\cdot\,s = .\,\mathrm{Bul}\,a\varrho\,\ast$

The words Sph ($\int L$ sphaera $=$ sphere) and Bul ($\int L$ bulla $=$ bubble) mean "spheres" and "solid spheres" respectively.

4·02 2. $*\,a \frown b \in \mathrm{Spa}\,.\,\wedge\,.\,a \neq b\,\cdot\,\rightarrow\,{}^{\cdot\curlywedge}c\,\cdot\,\mathrm{Dst}\,ac\,.= .\,\mathrm{Dst}\,bc\,\colon \in \mathrm{Pla}\,\colon$

     $a \frown b \in \mathrm{Spa}\,.\,\wedge\,.\,p \in \mathrm{Pla}\,\cdot\,\rightarrow\,{}^{\cdot}p \perp ab\,.\,\leftrightarrow\,\colon p = {}^{\curlywedge}c\,\cdot\,\mathrm{Dst}\,ac\,.= \mathrm{Dst}\,bc\,\colon$

     $p \frown q \in \mathrm{Pla}\,.\,\rightarrow\,\colon p \perp q\,.\,\rightarrow\,\cdot\,\vee\,\ulcorner a\,.\,b\,\urcorner : a \neq b\,.\,\wedge\,.\,a \frown b \in q\,.\,\wedge\,.\,p \perp ab\,\colon$

     $p \frown q \in \mathrm{Pla}\,.\,\rightarrow\,\cdot\,p \perp q\,.\,\rightarrow\,.\,q \perp p\,\colon$

     $p \in \mathrm{Pla}\,.\,\wedge\,.\,r \in \mathrm{Rct}\,\cdot\,\rightarrow\,\colon p \perp r\,.\,\leftrightarrow\,\cdot\,\wedge q : q \in \mathrm{Pla}\,.\,\wedge\,.\,q \supset r\,\cdot\,\rightarrow\,.\,p \perp q\,\colon$

     $r \frown s \in \mathrm{Rct}\,.\,\rightarrow\,{}^{\cdot}r \perp s\,.\,\leftrightarrow\,\colon \vee p\,\cdot\,p \in \mathrm{Pla}\,.\,\wedge\,.\,p \supset s\,.\,\wedge\,.\,p \perp r\,\colon$

     $r \frown s \in \mathrm{Rct}\,.\,\rightarrow\,{}^{\cdot}r \perp s\,.\,\leftrightarrow\,.\,s \perp r\,\ast$

     '$\perp$' means "orthogonal".

4·02 3. $*\,r \frown s \in \mathrm{Rct}\,.\,\wedge\,.\,a \frown b \frown c \in \mathrm{Spa}\,.\,\wedge\,\cdot\,c = .\,r \cap s\,\cdot$

     $\wedge\,.\,a \in r\,.\,\wedge\,.\,b \in s\,.\,\wedge\,.\,r \perp s\,.$

     $\wedge\,\cdot\,\mathrm{Dst}\,bc\,.= \mathrm{Cmt}\,\alpha\,\cdot\,\wedge\,\cdot\,\mathrm{Dst}\,ca\,.= \mathrm{Cmt}\,\beta\,\cdot\,\wedge\,\cdot\,\mathrm{Dst}\,ab\,.= \mathrm{Cmt}\,\gamma\,\colon$

     $\rightarrow\,.\,\alpha^{10} + \beta^{10} = \gamma^{10}\,\ast$

4·02 4. $*\,\varphi \in .\,\mathrm{Spa} \frown \mathrm{Spa}\,\cdot\,\rightarrow\,\colon \varphi \in \mathrm{Rfc}\,.$

     $\leftrightarrow\,\colon \wedge\,\ulcorner x\,.\,y\,\urcorner\,\cdot\,x \frown y \in \mathrm{Spa}\,.\,\rightarrow\,\colon \mathrm{Dst}\,xy\,.= \cdot\,\mathrm{Dst}\,.\,\varphi x\,.\,\varphi y\,\colon$

     $\wedge\,\colon {}^{\curlywedge}x\,\cdot\,x \in \mathrm{Spa}\,.\,\wedge\,.\,x = \varphi x\,\colon \in \mathrm{Pla}\,|$

     $\mathrm{Rfc} \neq \ulcorner\,\urcorner\,\ast$

The word written Rfc means "reflections". Translations and rotations could be defined as products of reflections.

4 03 1. $*\,a \frown b \in \mathrm{Spa}\,.\wedge.\alpha \frown \beta \in \mathrm{Rea}\,.\wedge.\alpha+\beta=1\,\cdot$
$\rightarrow\!\mathbf{\cdot}\,a=b\,.\rightarrow.\alpha a+\beta b=a\,\mathbf{\cdot}$
$\wedge\mathbf{\cdot}\,a\neq b\,.\rightarrow\mathbf{\cdot}\,\alpha a+\beta b\,.\in.\mathrm{Rct}\,ab\,\mathbf{\cdot}$
$\wedge\mathbf{\cdot}\,a\neq b\,.\wedge.\alpha\geqq 0\,.\wedge.\beta\geqq 0\,\cdot\rightarrow\mathbf{\cdot}\,\alpha a+\beta b\,.\in\mathrm{Cvx}\,a\cup b:$
$\wedge\mathbf{\cdot}\alpha\times \mathrm{Dst}\,.a\,.\alpha a+\beta b\,\mathbf{\cdot}=\,\cdot\,\beta\times \mathrm{Dst}\,.b\,.\alpha a+\beta b\,\mathbf{\cdot}$
$\wedge\mathbf{\cdot}\,a\neq b\,.\wedge.\alpha\times\beta<0\,\cdot\rightarrow\mathbf{\cdot}\,\alpha a+\beta b\,.\notin\mathrm{Cvx}\,a\cup b:$
$\wedge\mathbf{\cdot}\,|\,\alpha\,|\times \mathrm{Dst}\,.a\,.\alpha a+\beta b\,\mathbf{\cdot}=\,\cdot\,|\,\beta\,|\times \mathrm{Dst}\,.b\,.\alpha a+\beta b\,\mathbf{\cdot}$
$a\frown b \in \mathrm{Spa}\,.\wedge.\alpha\frown\beta\in \mathrm{Rea}\,.\wedge.\alpha+\beta=1\,\cdot\rightarrow.\alpha a+\beta b \in \mathrm{Spa}\ *\ \ ^{1})$

4 03 2. $*\,a\frown b\in\mathrm{Spa}\,.\rightarrow\!:b-a\,.\in\mathrm{Cmt}\,\mathrm{Vco}\,\cdot\wedge\cdot b-a\,.\notin\mathrm{Spa}\,\mathbf{:}$
$u\in\mathrm{Cmt}\,\mathrm{Vco}\,.\rightarrow.u\in u\ *$

'$u\in\mathrm{Vco}$' means u is a vector. 'Cmt Vco' is the set of $u\in\mathrm{Vco}$ labelled with the unit Cmt.

4 03 3. $*\,a\frown b\frown c\frown d\in\mathrm{Spa}\,.\rightarrow\cdot a-b=c-d\,.$
$\leftrightarrow.\,0{,}1\times a+0{,}1\times d=0{,}1\times b+0{,}1\times c\,\mathbf{:}$
$a\frown b\frown c\frown d\in\mathrm{Spa}\,.\rightarrow\!\mathbf{:}\,a-b=c-d\,.$
$\rightarrow\cdot\mathrm{Dst}\,ab\,.=\,.\,\mathrm{Dst}\,cd\,.\wedge\mathbf{:}\,a=b\,.\wedge.c=d\,\cdot\vee\cdot\mathrm{Rct}\,ab\,.\!/\!/\,.\,\mathrm{Rct}\,cd\ *$

4 03 4. $*\,u\in\mathrm{Cmt}\,\mathrm{Vco}\,.\wedge.a\in\mathrm{Spa}\,\cdot$
$\rightarrow\mathbf{:}\,\mathrm{Car}\mathbf{:}\,^{\curvearrowright}b\cdot b\in\mathrm{Spa}\,.\wedge.b-a=u\,\overset{\cdot}{}=1\ *$

4 03 5. $*\,a\frown b\in\mathrm{Spa}\,.\wedge.u=b-a\,\cdot\rightarrow.b=a+u=u+a\ *$

4 03 6. $*\,a\frown b\frown c\in\mathrm{Spa}\,.\wedge.u=b-a\,.\wedge.v=c-b\,\cdot$
$\rightarrow\cdot u+v=c-a\,.\wedge.-v=b-c\,.\wedge.u+(-v)=u-v\mathbf{:}$
$u\in\mathrm{Cmt}\,\mathrm{Vco}\,.\rightarrow.u-u=0_{\mathrm{Cmt}\,\mathrm{Vco}}=\mathrm{Cmt}\,0_{\mathrm{Vco}}\ *$

4 03 7. $*\,a\frown b\in\mathrm{Spa}\,.\wedge.\gamma\in\mathrm{Rea}\,.\wedge.u=b-a\,\cdot$
$\rightarrow.\gamma u=((1-\gamma)\,a+\gamma b)-a\ *$

4 04 1. $*\,\mathrm{Rea}^3=\ulcorner\mathrm{Rea}\,.\,\mathrm{Rea}\,.\,\mathrm{Rea}\urcorner\ *$

4 04 2. $*\,\gamma\in\mathrm{Rea}\,.\wedge.a\frown b\in\mathrm{Rea}^3\,.\wedge.a=\ulcorner\alpha_1.\alpha_2.\alpha_3\urcorner\,.\wedge.b=\ulcorner\beta_1.\beta_2.\beta_3\urcorner\,\cdot$
$\rightarrow\mathbf{:}\,\gamma a=\ulcorner\gamma\alpha_1.\gamma\alpha_2.\gamma\alpha_3\urcorner\,.\wedge.a+b=\ulcorner\alpha_1+\beta_1.\alpha_2+\beta_2.\alpha_3+\beta_3\urcorner\,.$
$\wedge.a-b=\ulcorner\alpha_1-\beta_1.\alpha_2-\beta_2.\alpha_3-\beta_3\urcorner\,.\wedge.\ulcorner 0\,.\,0\,.\,0\urcorner=0_3\,.$
$\wedge.\mathrm{aIb}=\alpha_1\beta_1+\alpha_2\beta_2+\alpha_3\beta_3\,.\wedge\cdot a\perp b\,.\leftrightarrow.\mathrm{aIb}=0\,\cdot$
$\wedge.\,|\,a\,|=\sqrt{a\,\mathrm{I}\,a}\,.\wedge.\mathrm{Dst}\,ab\,.=\,|\,a-b\,|\,.$
$\wedge\cdot\wedge\varphi\mathbf{:}\,\sphericalangle\,ab\,.=\varphi\,\cdot\leftrightarrow\cdot 0\leqq\varphi\leqq\pi\,.\wedge.\cos\varphi=\mathrm{aIb}\,/\,|\,a\,|\,|\,b\,|\ *$

'aIb' means the scalar product of the numerical vectors a and b; $\sphericalangle$ is the symbol of the angle-function.

$^{1})$ We have forgotten to define the absolute value of a number.

4 04 3. $^*\,\gamma \in \mathrm{Rea} . \wedge . \, a \in \mathrm{Rea}^3 . \wedge . \, |\,a\,| = 1\,\cdot$
$\to\,\colon \mathrm{Pla}_* a\gamma . = \,^\uparrow x \colon x \in \mathrm{Rea}^3 . \wedge . \, a\mathrm{I}x = \gamma \,\colon\!\vdots$
$a \frown b \in \mathrm{Rea}^3 . \wedge . \, |\,a\,| = 1\,\cdot$
$\to\,\colon \mathrm{Rct}_* ab = \,^\uparrow x\,^\cdot x \in \mathrm{Rea}^3 . \wedge \colon \vee \tau\,\cdot \tau \in \mathrm{Rea} . \wedge . \, x = \tau a + b\,\colon\!\vdots$
$p \in \mathrm{Pla}_* . \leftrightarrow \colon \vee \ulcorner a . \gamma \urcorner \cdot p = . \, \mathrm{Pla}_* a\gamma \,\colon\!\vdots$
$r \in \mathrm{Rct}_* . \leftrightarrow \colon \vee \ulcorner a . b \urcorner \cdot r = . \, \mathrm{Rct}_* ab \,\colon\!\vdots$
$\mathrm{Pla}_* a\gamma . \perp . \, \mathrm{Pla}_* a'\gamma' \cdot \leftrightarrow \cdot a\mathrm{I}a' . = 0 \,\colon\!\vdots$
$\mathrm{Rct}_* ab . \perp . \, \mathrm{Rct}_* a'b' \cdot \leftrightarrow \cdot a\mathrm{I}a' . = 0 \,\colon\!\vdots$
$\mathrm{Pla}_* a\gamma . \perp . \, \mathrm{Rct}_* a'b' \cdot \leftrightarrow \cdot a' = a . \vee a' = -a\ ^*$

4 04 4. $^*\, f \in \mathrm{Coo} . \leftrightarrow \colon f \in \star \frown \star . \wedge . \,^\wedge f\,\vdots \mathrm{Spa} = \mathrm{Rea}^3 .$
$\wedge . \wedge \ulcorner a . b \urcorner a \frown b \in \mathrm{Spa} . \to \,^\cdot \mathrm{Dst}\, ab = \colon \mathrm{Cmt} \cdot \mathrm{Dst} . \, fa . fb\,\vert$
$\vee f . f \in \mathrm{Coo}\ ^*$

The word written Coo means "coordinate system", or rather "coordinatization".

4 04 5. $^*\, f \in \mathrm{Coo} . \to \,\colon \wedge \ulcorner a . b \urcorner \colon a \frown b \in \mathrm{Spa} . \to \,^\cdot a = b . \leftrightarrow . \, fa = fb\,^\cdot$
$\wedge \colon \wedge p\,^\cdot p \in \mathrm{Pla} . \leftrightarrow \colon ^\wedge . f\,\vdots p \cdot \in \mathrm{Pla}_* \colon$
$\wedge \colon \wedge r\,^\cdot r \in \mathrm{Rct} . \leftrightarrow \colon ^\wedge . f\,\vdots r \cdot \in \mathrm{Rct}_* \colon$
$\wedge \colon \wedge \ulcorner r . s \urcorner\,^\cdot r \perp s . \leftrightarrow \colon ^\wedge . f\,\vdots r \cdot\,^\cdot{}^\wedge . f\,\vdots s\ ^*$

4 04 6. $^*\, f \in \mathrm{Coo} . \wedge . \, a \frown b \in \mathrm{Spa} . \wedge . \, \alpha \frown \beta \in \mathrm{Rea} . \wedge . \, \alpha + \beta = 1\,\cdot$
$\to\,^\cdot f . \alpha a + \beta b \cdot = . \, \alpha\, fa + \beta\, fb \colon \wedge \colon f . a - b \cdot = . \, fa - fb\ ^*$

4 04 7. $^*\, \mathrm{Cmt}\, \alpha \times \mathrm{Cmt}\, \beta = \mathrm{Cmt}^2 \alpha \times \beta\ ^*$

4 04 8. $^*\, u \frown v \in \mathrm{Cmt}\, \mathrm{Vco} . \wedge . \, \gamma \in \mathrm{Rea} . \wedge . \, f \in \mathrm{Coo} \colon$
$\to\,\colon f . u + v \cdot = . \, fu + fv \colon \wedge \colon f . \alpha u \cdot = . \, \alpha\, fu \colon$
$\wedge . \, |\,u\,| = \mathrm{Cmt}\, |\,fu\,| . \wedge \,^\cdot u\mathrm{I}v = \colon \mathrm{Cmt}^2 \cdot fu . \mathrm{I} . fv\,^\cdot$
$\wedge \colon \lessgtr uv . = \cdot \lessgtr . fu . fv\ ^*$

4 04 9. $^*\, \mathrm{Coo} = . \, \mathrm{Coo}^+ \cup \mathrm{Coo}^- \cdot \wedge \cdot \ulcorner \urcorner = . \, \mathrm{Coo}^+ \cap \mathrm{Coo}^- .$
$\wedge \,^\cdot \varphi \in \mathrm{Rfc} .$
$\to\,\colon f \in \mathrm{Coo}^+ . \to . \, f\varphi \in \mathrm{Coo}^- \cdot \wedge \cdot f \in \mathrm{Coo}^- . \to . \, f\varphi \in \mathrm{Coo}^+\,\vert$
$\mathrm{Coo}^+ \neq \ulcorner \urcorner . \wedge . \, \mathrm{Coo}^- \neq \ulcorner \urcorner\ ^*$

Coo$^+$ and Coo$^-$ mean "positive" and "negative" coordinate systems (lefthand and righthand). The distinction between Coo$^+$ and Coo$^-$ is purely verbal. At present we cannot do more than we have done, in spite of the efforts of a good many textbook writers. In **4 24 6** we shall fasten one element of Coo$^+$ to real space. From that moment onwards Coo$^+$ and Coo$^-$ will be materially distinguishable.

4 05 1.　$^\#\,u \in \mathrm{Vco} . \to . \mathrm{Cmt}\, u . \in . \mathrm{Cmt}\, \mathrm{Vco} :$

　　　$u \in \mathrm{Vco} . \to . u \in u :$

　　　$\mathrm{Cmt}\, u = \mathrm{Cmt}\, v . \to . u = v\ ^\#$

4 05 2.　$^\#\,u \frown v \in \mathrm{Vco} . \wedge . \gamma \in \mathrm{Rea} . \to . \mathrm{Cmt} . u + v . = . \mathrm{Cmt}\, u + \mathrm{Cmt}\, v :$

　　　$\wedge . \mathrm{Cmt}\, \gamma\, u = \gamma \, \mathrm{Cmt}\, u . \wedge . \mathrm{Cmt}\, u . \mathrm{I} . \mathrm{Cmt}\, v . = \mathrm{Cmt}^2 . u \mathrm{I} v :$

　　　$\wedge . \lessgtr uv . = . \lessgtr . \mathrm{Cmt}\, u . \mathrm{Cmt}\, v . \wedge . |\, \mathrm{Cmt}\, u\, | = \mathrm{Cmt}\, |\, u\, |\ ^\#$

4 05 3.　$^\#\,u \in \mathrm{Vco} . \to . u - u = 0_{\mathrm{Vco}}\ ^\#$

4 06 1.　$^\#\,A \in . \mathrm{Lap}\, a_1 a_2 a_3 .$

　　　$\leftrightarrow . A \subset \mathrm{Spa} . \wedge . a_1 \frown a_2 \frown a_3 \in \mathrm{Pos} .$

　　　$\wedge . \vee f . f \in \mathrm{Coo}^+ . \wedge . \wedge x . x \in A .$

　　　$\leftrightarrow . \wedge i . i \in 1 \cup 2 \cup 3 . \to . 0 \leqq \omega_i\, fx \leqq a_i \,|$

　　　$A \in \mathrm{Lap} . \leftrightarrow . \vee \ulcorner a_1 . a_2 . a_3 \urcorner . A \in \mathrm{Lap}\, a_1 a_2 a_3\ ^\#$

The word written Lap (fL lapis = stone) means "rectangular parallelepiped"; a_1, a_2, a_3 mean the lengths of its edges.

4 06 2.　$^\#\,\mathrm{Vol} . \mathrm{Lap}\, a_1 a_2 a_3 . = \mathrm{Cmt}^3 a_1 \times a_2 \times a_3\ ^\#$

4 06 3.　$^\#\,\mathrm{Cmt}^3 \gamma . + . \mathrm{Cmt}^3 \delta = . \mathrm{Cmt}^3 . \gamma + \delta : \mathrm{Etc}\ ^\#$

4 06 4.　$^\#\,\vee B . B \in \mathrm{Lap} . \wedge . A \subset B . \to . \alpha = \mathrm{Vol}\, A . \leftrightarrow . \wedge \ulcorner M . \varphi \urcorner :$

　　　$M = . {}^\wedge X . \mathrm{Car}\, X \in \mathrm{Num} . \wedge . X \subset \mathrm{Lap} . \wedge . A \subset \bigcup X . \wedge .$

　　　$\varphi = . {}^\vee X . \varSigma . x \in X . \mathrm{Vol}\, x \,\vdots\, M :$

　　　$\to . \alpha = . \inf\, {}^\wedge \varphi\ ^\#$

'Vol' means volume. We gave the definition of exterior content.

4 06 5.　$^\#\,\pi = 100 \times (1 - 1/11 + 1/101 - 1/111 + \mathrm{Etc})$

　　　$= 11,0010010000111111011010101000100011110011001\ \mathrm{Etc} :$

　　　$A \in . \mathrm{Bul}\, a\varrho . \to . \mathrm{Vol}\, A = 100/11 \times \pi \varrho^{11}\ ^\#$

4 07 1.　$^\#\,x \in . \mathrm{Hom} \cup \mathrm{Bes} . \to . \mathrm{Vol}\, x . = . \mathrm{Vol}\, \mathrm{Loc}\, x :$

　　　$x \in \mathrm{Hom} . \wedge . \mathrm{Aet}\, x . = . \mathrm{Sec}\, 10^{11001} :$

　　　$\to . \mathrm{Vol}\, x . = \mathrm{Cca} . \mathrm{Cmt}^3\, 10^{1101} :$

　　　$x \in \mathrm{Hom} . \wedge . \mathrm{Aet}\, x . \geqq . \mathrm{Sec}\, 101 \times 10^{11011} :$

　　　$\to . \mathrm{Vol}\, x . = \mathrm{Cca} . \mathrm{Cmt}^3\, 10^{10000} :$

　　　$\vee x : x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Vol}\, x . = \mathrm{Cca} . \mathrm{Cmt}^3\, 11 \times 10^{11001} :$

　　　$\neg \vee x : x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Vol}\, x . > \mathrm{Cmt}^3\, 11 \times 10^{11001} :$

　　　$\mathrm{Car} . {}^\wedge x : x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Vol}\, x . \geqq . \mathrm{Cmt}^3\, 10^{10000}$

　　　　$\vdots / . \mathrm{Car} . {}^\wedge x : x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Vol}\, x . < . \mathrm{Cmt}^3\, 10^{10000} :$

　　　　$\mathrm{Mul} < . 1\ ^\#$

Volume of humans and animals. — Perhaps the receiver will arrive at some crude estimation of our length unit on the strength of these data and the next.

4 07 2. $^\ast \vee B \cdot B \in \mathrm{Lap} . \wedge . A \subset B \colon$
$$\rightarrow \vdots \mathrm{Lon}\, A . = {}^{\mathsf{r}}\mathrm{sup} \colon {}^{\uparrow}x {}^{\boldsymbol{\cdot}} \vee \ulcorner a . b \urcorner \colon a \frown b \in A . \wedge . x = . \mathrm{Dst}\, ab \; {}^\ast$$

The word written Lon (fL longitudo $=$ length) means "length, diameter".

4 07 3. $x \in . \mathrm{Hom} \cup \mathrm{Bes} {}^{\boldsymbol{\cdot}} \rightarrow {}^{\boldsymbol{\cdot}} \mathrm{Lon}\, x . = . \mathrm{Lon}\, \mathrm{Loc}\, x \vdots$
$x \in \mathrm{Hom} . \wedge . \mathrm{Aet}\, x . = . \mathrm{Sec}\, 10^{11001} \colon \rightarrow {}^{\boldsymbol{\cdot}} \mathrm{Lon}\, x . = \mathrm{Cca} . \mathrm{Cmt}\, 10^{110} \vdots$
$x \in \mathrm{Hom} . \wedge . \mathrm{Aet}\, x . \geqq . \mathrm{Sec}\, 101 \times 10^{11011} \colon \rightarrow$
    ${}^{\boldsymbol{\cdot}} \mathrm{Lon}\, x . = \mathrm{Cca} . \mathrm{Cmt}\, 101 \times 10^{101} \vdots$
$\vee x \colon x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Lon}\, x . = \mathrm{Cca} . \mathrm{Cmt}\, 11 \times 10^{1010} \vdots$
$\neg \; \vee x \colon x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Lon}\, x . > . \mathrm{Cmt}\, 10^{1100} \vdots$
$\mathrm{Car} {}^{\boldsymbol{\cdot}} {}^{\uparrow}x \colon x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Lon}\, x . \geqq . \mathrm{Cmt}\, 101 \times 10^{101}$
   $\vdots / \colon \mathrm{Car} {}^{\boldsymbol{\cdot}} {}^{\uparrow}x \colon x \in \mathrm{Bes} . \wedge . \mathrm{Nnc}\, x\, \mathrm{Ext} . \wedge . \mathrm{Lon}\, x . < . \mathrm{Cmt}\, 101 \times 10^{101} \colon$
   $\mathrm{Mul} < . 1 \; {}^\ast$

4 08 1. $^\ast Ha\, \mathrm{Inq}\, Hb {}^{\boldsymbol{\cdot}}$
$t + h\, \mathrm{Usq}\, t' + h' . Hd\, \mathrm{Ani} \cdot \mathrm{Utr} . tt'\, Hc\, \mathrm{Inq}\, Hd\, p \colon \wedge . h \neq h' {}^{\boldsymbol{\cdot}}$
   $\rightarrow {}^{\boldsymbol{\cdot}} t\, \mathrm{Loc}\, Hd \neq t'\, \mathrm{Loc}\, Hd . \vee . t\, \mathrm{Loc}\, Hc \neq t'\, \mathrm{Loc}\, Hc \; {}^\ast$

4 08 2. $^\ast t\, \mathrm{Dst}\, XY . = \colon \mathrm{Dst} \cdot t . \mathrm{Loc}\, X \cdot t . \mathrm{Loc}\, Y \; {}^\ast$

4 08 3. $^\ast t\, X\, \mathrm{Mov} . \leftrightarrow \vdots \wedge \alpha \colon \alpha \in \mathrm{Pos} . \rightarrow {}^{\boldsymbol{\cdot}} \vee \ulcorner t' . t'' \urcorner \colon$
   $t - \mathrm{Sec}\, \alpha \leqq t' \leqq t \leqq t'' \leqq t + \mathrm{Sec}\, \alpha .$
    $\wedge . t\, \mathrm{Loc}\, X \neq t'\, \mathrm{Loc}\, X . \wedge . t\, \mathrm{Loc}\, X \neq t''\, \mathrm{Loc}\, X \; {}^\ast$

The word written Mov (fL movetur $=$ moves) means the neutre verb "moves".

4 08 4. $^\ast t_1\, t_2\, X\, \mathrm{Mov} . \leftrightarrow \colon \wedge t \cdot t_1 < t < t_2 . \rightarrow . tx\, \mathrm{Mov} {}^{\boldsymbol{\cdot}}$
$t\, \mathrm{Usq}\, t' . a\, \mathrm{Usd}\, a' . X\, \mathrm{Mov} {}^{\boldsymbol{\cdot}}$
   $\leftrightarrow {}^{\boldsymbol{\cdot}} tt'\, X\, \mathrm{Mov} . \wedge . t\, \mathrm{Loc}\, X = a . \wedge . t'\, \mathrm{Loc}\, X = a' \; {}^\ast$

The word written Usd (fL usque ad $=$ up to) is the spatial analogue of the temporal Usq.

4 08 5. $^\ast t\, \mathrm{Usq}\, t' . a\, \mathrm{Usd}\, a' . X\, \mathrm{Mov} \cdot \wedge . t'\, \mathrm{Usq}\, t'' . a'\, \mathrm{Usd}\, a'' . X\, \mathrm{Mov} \colon$
   $\rightarrow {}^{\boldsymbol{\cdot}} t\, \mathrm{Usq}\, t'' . a\, \mathrm{Usd}\, a'' . X\, \mathrm{Mov} \; {}^\ast$

4 09 1. $^\ast {}^{t_1} Ha\, \mathrm{Inq}\, Hb {}^{\boldsymbol{\cdot}} \mathrm{Nnc}\, Ha\, \mathrm{Vul} \colon \mathrm{Sec}\, 10 . \mathrm{Pst}\, \mathrm{Nnc} \cdot Hb\, \mathrm{Apu}\, Ha \vdots$
$Hb\, \mathrm{Inq}\, Ha . ? = \mathrm{Apu} \vdots$
$Ha\, \mathrm{Inq}\, Hb {}^{\boldsymbol{\cdot}} \mathrm{Nnc}\, Ha\, \mathrm{Vul} \colon \mathrm{Sec}\, 10 . \mathrm{Pst}\, \mathrm{Nnc} \cdot \mathrm{Loc}\, Hb . = \mathrm{Cca} . \mathrm{Loc}\, Ha \vdots$
$t_1\, \mathrm{Loc}\, Hb . \mathrm{Usd} . t_1\, \mathrm{Loc}\, Ha . Hb\, \mathrm{Mov} \vdots$
$Ha\, \mathrm{Inq}\, Ha \colon \mathrm{Nnc}\, \mathrm{Loc}\, Hb . = \mathrm{Cca} . \mathrm{Loc}\, Ha \cdot Hb\, \mathrm{Apu}\, Ha \cdot$
   $\mathrm{PAN} . Hb . \mathrm{Usd} . \mathrm{Apu}\, Ha . Hb\, \mathrm{Mov} \; {}^\ast$

The word written Apu (fL apud = at) means (the local) "at, near". There is a notable difference between the use of Apu in the first and in the last sentence. This transition is akin to that in the use of words like MAN (see 3 20 6).

4 09 2.   ✳ *Ha* Inq *Hb*˙Nnc *Ha* Vul : Sec 10 . Pst Nnc ˙ *Hb* Apu *Ha* ⦂
           Hb Inq *Ha* ⦂ ⌐ Pot Etc : Qia ˙ Nnc Dst *Ha Hb* . > Nim ˙
               Pot : Sec 1000 . Pst Nnc ˙ *Hb* Apu *Ha* ✳

4 09 3.   ✳ ^{t_1}*Ha* Inq *Hb*˙Nnc *Ha* Vul : Sec 10 . Pst Nnc ˙ *Hb* Apu *Ha* ⦂
           Hb Inq *Ha* ⦂ Nnc *Hb* Vul ˇp . PAN *Ha* Vul p ˙
               Tan ⦂ ⌐ Nnc *Hb* Vul . Nnc *Hb* Mov ˙
               Sed : Nnc *Hb* Vul ˙ t_1 Loc *Ha* . Usd . t_1 Loc *Hb* . *Ha* Mov ✳

4 09 4.   ✳ *Ha* Inq *Hb* : Nnc *Ha* Vul ⦂ Nnc ˙ Nnc Loc *Hb* . Usd . Apu *Hc* . *Hb* Mov :
           ∧ : PPN *Hb* Inq *Hc* ˙ Nnc *Ha* Vul :
               Nnc ˙ Nnc Apu *Hc* . Usd . Apu *Ha* . *Hb* ⌢ *Hc* Mov ❙
           Hd Inq *Hd* : PAN *Ha* Vul ˙ PPN . Apu *Hc* . *Hb* Inq *Hc* Etc ❙
           Usd . Apu *Hc* ˙ *Hb* Mov ❙
           Apu *Hc* . Usd . Apu *Ha* : ˇw . w Uni *Hb Hc* ˙ Mov ❙
           Ha Inq *Ha* : Nnc ˙ *Hb* Apu *Ha* . ∧ . *Hc* Apu *Ha* ✳

           'PPN . Apu *Hc* . *Hb* Inq *Hc* Etc' is a paradigm which shows how to say that a certain event happens at a certain place. The general pattern would be

$$t\ u\ \text{Fit}\ p,$$

where t is a temporal and u a spatial adjunct. More examples could be given.

4 10 1. p and q in the next program texts will represent definite noises.
           ✳ ^{t_0}*Ha* Inq *Ba* p ⦂
               ^{t_1}Usd . Apu *Ha* ˙ *Ba* Mov^{t_2} ⦂
               Hb Inq *Hc* ˙ ? x . t_1 t_2 Fit x ⦂
               Hc Inq *Hb* ⦂ t_0 t_1 *Ha* Vul . PPN *Ba* Apu *Ha* ˙
                   Pau Pst . *Ha* Vul Etc : *Ba* Ani ˙ Utr . PAN *Ha* Vul Etc ˙
                   t_1 Usq t_2 ˙ t_1 Loc *Ba* . Usd . t_1 Loc *Ha* ˙ *Ba* Mov ✳
           Whistling for one's dog.

4 10 2.   (t_1 < t_2 < t_3 < t_4)
           ✳ ^{t_1}*Ha* Inq *Ba* p^{t_2} ❙
               Ba Inq *Ha* q^{t_3} ❙
               Hb Inq *Hc* ˙ ? x . t_1 t_3 Fit x ❙
               Hc Inq *Hb* ⦂ t_1 t_2 *Ha* Vul ˙ t_2 . Usq ˙ Unq . Pau Pst . t_2 :
                   t_1 Loc *Ba* . Usd . t_1 Loc *Ha* ˙ *Ba* Mov ⦂
                   Sed ⦂ ⌐ t_1 t_3 *Ba* Vul ˙ ˇq . Nnc *Ha* Vul q ˙
                       Erg . Nnc *Ha* Mov ❙

Hb Inq Hc ˙ ? u : Nnc ˙ t_1 Loc Ha . Usd u ˙ Ha Mov |

Hc Inq Hb ˙ Nnc Ha Vul ˙ PPN . Ha Apu Ba :

   Erg ˙ Nnc . Adv Ba . Ha Mov |

Hb Inq Hc . ? = Adv |ᵗ⁴

Hc Inq Hb ı ˙ Nnc Ha Vul ˙ PPN Dst Ha Ba . < . Nnc Dst Ha Ba ı .

   Erg ı . Ncs ı ∧t ː t . = Cca . Nnc ˙ →ː ∨A :

   A ⊂ : Cvx ˙ t Loc Ha . ∪ . t Loc Ba ˙

   ∧ : Cca t ˙ t Loc Ha . Usd A ˙ Ha Mov |

Hb Inq Hc ˙ Utr . Nnc Ha Apu Ba |

Hc Inq Hb ı Fal ː Qia ː Cca t_4 ː Ba Ani : Utr ˙ Nnc . Adv Ba . Ha Mov ˙

   ⌢ ˙ Vul . ¬ PPN Ha Apu Ba ː

   Erg ˙ Pau Pst t_4 . Usq Nnc . Dev Ha . Ba Mov |

Hb Inq Hc . ? = Dev |

Hc Inq Hb ı . Nnc Ba Vul . ¬ UPN Ha Apu Ba : Eti ː

   Nnc Ba Vul ˙ PPN Dst Ha Ba . > . Nnc Dst Ha Ba ı

   Erg ı Ncs ː ∧t ː t . = Cca . Nnc ˙ →ː ∨ ⌈a . b . c⌉ ˙

     a ∈ Spa . ∧ . b ∈ Loc Ba . ∧ . c ∈ Loc Ha . ∧ : b ∈ ˙ Cvx . a ∪ c : ∧ :

     Cca t ˙ t Loc Ba . Usd a . Ha Mov ı

   Nnc . Dev Ha . Ba Mov ＊

The words written Adv (∫L adversus = towards) and Dev (∫L deversus = away) mean "towards" and "away" respectively.

4 10 3. ＊ Ha Inq Ha ː Car ː ↑x ˙ x ∈ Bes . ∧ . Nnc x Ext . ∧ :

   ∨p ˙ Pot . Nnc x Ani p :

   ∧ : ∨p ˙ Pot . Nnc x Vul p : ∧ : Pot . Nnc x Mov ː

   Mul > ˙ Car : ↑x ˙ x ∈ Hom . ∧ . Nnc x Ext ＊

4 10 4. ＊ ᵗ¹Ha Inq Hb . Ha ː

   Hb Inq Ha . Hbᵗ² ː

   Hc Inq Hd ˙ ? x . t_1 t_2 Fit x ː

   Hd Inq Hc ː Cca t_1 ˙ Ha Ani . ? = Loc Hb :

     ⌢ : Ha Vul ˙ Nnc Hb Ani . ? = Loc Ha ː

   Pau Pst . t_1 ˙ Obv Ha Hb Mov ː

   Hc Inq Hd . ? = Obv ː

   Hd Inq Hc ˙ Pau Pst t_1 : Loc Ha . Adv . Loc Hb ˙ Ha Mov :

     ⌢ : Loc Hb . Adv . Loc Ha ˙ Hb Mov ＊

The word written Obv (∫L obviam = going to meet) means "going to meet".

4 10 5. ＊ ᵗ¹t_1 Loc Ba . Usd . t_1 Loc Ha ˙ Ba Movᵗ² |

   Hb Inq Hc ˙ ? x . t_1 t_2 Fit x |

   Hc Inq Hb ˙ PAN Ba Vul . PPN Ba Apu Ha |

   Hb Inq Hc Cur |

Ha Inq Hb ⁚ Qia ⁚ PAN Ba Sci ⁚

⁇ y ˙ Nnc ⁚ Adv Ha . Ba Mov ˙ → Pau Pst . Ha Inq Hb y |

Hb Inq Hc . ⁇ y Etc |

Hc Inq Hb . Etc Ha Inq Hb Ben ⚹

4 11 1.	⚬ ^{t_1} ℘ ^{t_2} |

Hb Inq Hc ˙ ⁇ x . t_1 t_2 Fit x |

Hc Inq Hb ⁚ ∨y ˙ Put Ra ˙ PAN Ha Vul ⁚

Nnc Usq PPN · Nnc Loc y . Usd . Nnc Loc Ha · y Mov ⁚

Mod . Nnc Ha Inq y ℘ ⁚

Qqm ⁚ ⌐ Pot ⁚ ∨t · Cca t Ra Vul . Nnc Ra Mov ˙

Qia ⁚ Ra ∈ Res . Sed · Ra ∉ . Hom ∪ Bes ⁚

⌐ t_1 t_2 Ha Sci · Utr . Ra ∈ Res ⁚

Tan ⁚ Cca t_1 Ha Inq Ha . Ra ∈ Bes · Qqm Fal ⁚

Qqm · Loc Ra . ⊂ Spa ⁚ Tan · Ra ∉ . Hom ∪ Bes ⁚

⌐ Pot · ∨ ⌜u . p⌝ . u Ra Vul p ⁚

y ∈ Res . → ⁚ ⌐ Pot · y Inq . ∨ . y Sci . ∨ . y Ani . ∨ . y Vul |

^{t}Hb Inq Hc ⁚ Nnc · PAN Loc Ra . Usd . PAN Loc Ha · Ra Mov |

Hc Inq Hb ι ⌢ . Ha Mov ⁚ Cca t_1 . Hd Apu Ra ⁚

Pau Pst . t_2 · Hd Ani ˙ Utr ⁚ Nnc Ha Vul ·

Nnc . Usd . Apu Ha . Ra Mov ⁚

∨D ˙ D Uni Hd Ra . ∧ ⁚ t_3 Usq t_4 · t_3 Loc D . Usd . t_3 Loc Ha · D Mov ⁚

t_3 Usq t_4 · Loc D . = . Loc Hd . ∪ . Loc Ra ⁚

t_3 t_4 D Ext ⁚ Pau Pst . t_4 · t_4 Loc Hd . Usd . t_3 Loc Hd · Hd Mov ⁚

Tan . ⌐ Nnc Ra Mov ⁚ t_3 t_4 Hd Cau . Ra Mov ⁚

y ∈ Res . → ⁚ ⌐ Pot · y Vul . y Mov ⁚

Sed ⁚ Pot ˙ ∨x ⁚ x ∈ Hom . ∧ · x Vul . y Mov ⁚

x ∈ Hom . ∧ . y ∈ Res ·

→ ⁚ Pot ˙ t ⁚ x Vul . Nnc y Mov · → Pau Pst . y Mov ⚹

The word written Res (*fL* res = thing) means "concrete things". *Ra* is the name of a special thing. 'Cau' (*fL* causa = cause) means (the verb) "causes".

With respect to variables controlled by it Cau behaves as do words like Inq. (Oblique speech.)

A person can cause a thing to move.

4 11 2.	⚹ Ra ⌢ Rb ⌢ Rc ⌢ Rd ⌢ Etc ∈ Res ˙

x ∈ Res . → ⁚ Loc x . ⊂ Spa · ∧ · Vol x . = . Vol Loc x ⚹

4 11 3.	⚹ Ha Inq Hb ˙ Nnc Ha Vul ⁚ PPN · ˅x . Nnc x Apu Hb · Apu Ha ⁚

^{t}Cca . t_1 Loc Rb · Usd · Cca . t_1 Loc Ha · ˅y . y Uni Hb Rb · Mov^{t_2} ⁚

Hc Inq Hd · ⁇ z . t_1 t_2 Fit z ⁚

Hd Inq Hc ˙ t_1 t_2 Hb Cau · Rb Mov · Mod . Hb Fer Rb :

     t_1 t_2 · Cca . t_1 Loc Rb · Usd · Cca . t_1 Loc Ha · Hb Fer Rb ✳

The word written Fer (fL fert = carries) means (the verb) "carries".

4 11 4. Ha Inq Hb ˙ Nnc Ha Vul : PPN · ˇx . Nnc x Apu Hb · Apu Ha ⦂

     ^{t_1}Cca . t_1 Loc Rc · Usd · Cca t_1 . Loc Ha · Rc Mov^{t_2} ⦂

     Hc Inq Hd · ? z . t_1 t_2 Fit z ⦂

     Hd Inq Hc ˙ t_1 t_2 Hb Cau . Rc Mov · Tan : ¬ Mod . Hb Fer Rc ·

       Sed · Mod . Hb Iac Rc · Qia . ¬ t_1 t_2 Hb Mov ✳

The word written Iac (fL iactat = throws) means "throws, pushes".

4 11 5.   ✳ Ha Inq Hb ˙ Nnc Ha Vul : Nnc · Apu Hb . Usd . Apu Ha · Hb Fer Rd ⦂

     Hb Inq Ha ⦂ ¬ Pot ˙ Qia : Vol Rd . > Vld · Vol Rd . Mul > . Nim ˙

       Pot ⦂ ˇD · D Uni . 11^ε Hom · Fer Rd ✳

4 11 6.   ✳ ^{t_1}t_1 Loc Rf . Usd . t_1 Loc Ha · Rf Mov^{t_2} ⦂

     Hc Inq Hd · ? z . t_1 t_2 Fit z ⦂

     Hd Inq Hc ⦂ t_1 Hb Vul : Nnc · Apu Hb . Usd . Apu He · Rf Mov ·

       ∧ · t_1 . Adv He . Hb Iac Rf ⦂

     Tan ˙ t_2 Ha Cau : ¬ Unq PPN . Qqm Nnc · Rf Mov ✳

The Nnc after Cau is bound to approximately t_2, not to the moment when Hd is speaking. See the remark in **4 11 1**.

4 11 7.   ✳ ^{t_1}t_1 Loc Rf . Usd . t_1 Loc He · Rf Mov^{t_2} ⦂

     Hc Inq Hd · ? z . t_1 t_2 Fit z ⦂

     Hd Inq Hc ⦂ t_1 Hb Vul : Nnc · Apu Hb . Usd . Apu He · Rf Mov ·

       ∧ · t_1 . Adv He . Hb Iac Rf ⦂

     Hc Inq Hd ˙ Cur : ¬ PAN Ha Cau · ¬ Nnc Rf Mov ⦂

     Hd Inq Hc ⦂ Qqm · PAN Ha Vul . Nnc Ha Cau Etc ⦂

       Tan ˙ ∧t : t_1 ≤ t ≤ t_2 . → · t Dst Ha Rf . > Nim ✳

4 11 8.   ✳ ^{t_1}Ha Hb Lud$_6$^{t_4} ⫯

     Hc Inq Hd · ? x . t_1 t_4 Fit x ⫯

     Hd Inq Hc ⦂ t_1 t_4 Ha Hb Lud$_6$ ˙

       Deb ⦂ t_1 . Adv Hb . Ha Iac Rg ˙

         t_2 Hb Cau · ¬ Pau Pst t_2 . Rg Mov ˙

         t_3 Adv Ha . Hb Iac Rg ˙

         t_4 Hb Cau · ¬ Pau Pst t_4 . Rg Mov ˙

         t_2 : ¬ . Hb Cau Etc · → Pau Pst · Hb Dat Ha . Den 10 ˙

         t_4 : ¬ . Ha Cau Etc · → Pau Pst · Ha Dat Hb . Den 10 ✳

Playing ball.

4 12 1. $^\#$ $\mathrm{Dst}.t_1 \operatorname{Loc} x \cdot t_2 \operatorname{Loc} x \cdot = \mathrm{Cmt}\, \delta : \wedge . t_2 - t_1 = \operatorname{Sec} \tau . \wedge . \tau > 0 \cdot$
       $\rightarrow : t_1 \operatorname{Usq} t_2 \operatorname{Cel} \operatorname{Med} x \operatorname{Mov} . = \cdot \mathrm{Cmt}/\mathrm{Sec}.\, \delta/\tau :$
       $\delta \in \mathrm{Rea}. \wedge . \tau \in \mathrm{Pos} \cdot \rightarrow : \mathrm{Cmt}\, \delta/\mathrm{Sec}\, \tau . = \cdot \mathrm{Cmt}/\mathrm{Sec}.\, \delta/\tau :$
       $\alpha \frown \tau \in \mathrm{Rea} . \rightarrow \cdot \mathrm{Cmt}/\mathrm{Sec}\, \alpha \times \operatorname{Sec} \tau . = . \mathrm{Cmt}\, \alpha \tau :$
       $\alpha \frown \beta \in \mathrm{Rea} . \rightarrow \cdot \mathrm{Cmt}/\mathrm{Sec}\, \alpha . = . \mathrm{Cmt}/\mathrm{Sec}\, \beta : \leftrightarrow . \alpha = \beta :$
       Etc $^\#$

     The word written Cel Med (fL celeritas media $=$ mean velocity)
means "mean velocity".

4 12 2. $^\#$ $\tau \frown \nu \in \mathrm{Pos} . \rightarrow \cdot \operatorname{Sec}^{-1} \nu \times \operatorname{Sec} \tau . = \nu \tau :$
       $\operatorname{Sec}^{-1} \nu = \operatorname{Sec}^{-1} \nu_1 . \leftrightarrow . \nu = \nu_1 :$
       Etc $^\#$

4 13 1. $^\#$ $z \in \mathrm{Res} . \rightarrow {\scriptstyle\mathrm{I}} : t_1 \operatorname{Usq} t_2 . a . z \operatorname{Osc}_{\tau \alpha \nu} \cdot \leftrightarrow {\scriptstyle\mathrm{I}} \cdot$
       $a \in \mathrm{Spa} . \wedge . t_1 \frown t_2 \in \mathrm{Tem} . \wedge {\scriptstyle\mathrm{I}} . \vee \ulcorner \tau' . \alpha' . \nu' \urcorner {\scriptstyle\mathrm{I}} \tau' \frown \alpha' \frown \nu' \in \mathrm{Pos} .$
         $\wedge . \tau = \operatorname{Sec} \tau' . \wedge . \alpha = \mathrm{Cmt}\, \alpha' . \wedge . \nu = \operatorname{Sec}^{-1} \nu' . \wedge . 10\, \tau \nu < 1 . \wedge :$
         $\vee f : f \in \mathrm{Coo} . \wedge . f a = 0_3 . \wedge : \wedge t \cdot t_1 \leq t \leq t_2 .$
           $\rightarrow : {}^\wedge f : t \operatorname{Loc} z . = \operatorname{Cca} . \ulcorner \alpha \cos 10\, \pi \nu\, (t - \tau) . 0 . 0 \urcorner$ $^\#$

     The word written Osc (fL oscillat $=$ oscillates) means "oscillates".

4 13 2. $^\#$ $\mathrm{Aml} \cdot t_1 \operatorname{Usq} t_2 . a . z \operatorname{Osc}_{\tau \alpha \nu} := \alpha \cdot$
       $\mathrm{Fre} \cdot t_1 \operatorname{Usq} t_2 . a . z \operatorname{Osc}_{\tau \alpha \nu} := \nu \cdot$
       $\mathrm{Pha} \cdot t_1 \operatorname{Usq} t_2 . a . z \operatorname{Osc}_{\tau \alpha \nu} := \tau \cdot$
       $\mathrm{Eql} \cdot t_1 \operatorname{Usq} t_2 . a . z \operatorname{Osc}_{\tau \alpha \nu} := a$ $^\#$

     The words written Aml (fL amplitudo), Fre (fL frequentia),
Pha, Eql (fL aequilibrium) mean "amplitude", "frequency",
"phase", and "equilibrium" respectively.

4 13 3. $^\#$ $t . z \operatorname{Osc} \cdot \leftrightarrow \cdot \vee \ulcorner a . t_1 . t_2 . \tau . \alpha . \nu \urcorner : t_1 \leq t < t_2 . \wedge \cdot$
       $t_1 \operatorname{Usq} t_2 . a . z \operatorname{Osc}_{\tau \alpha \nu}$ $^\#$

     And, Fre, Pha, Eql are defined as in **4 13 2**.

4 13 4. $^\#$ $R \in \mathrm{Res} . \rightarrow {\scriptstyle\mathrm{I}} : t_1 \operatorname{Usq} t_2 R \operatorname{Vib}_\nu . \leftrightarrow {\scriptstyle\mathrm{I}} \cdot$
       $\vee A {\scriptstyle\mathrm{I}} : \operatorname{Car} A . \operatorname{Mul} > . 1 \cdot \wedge . R \operatorname{Uni} A . \wedge {\scriptstyle\mathrm{I}} \cdot$
         $\wedge z {\scriptstyle\mathrm{I}} . z \in A . \rightarrow {\scriptstyle\mathrm{I}} z \operatorname{Par} R \cdot \wedge \cdot \operatorname{Vol} z . \operatorname{Mul} < . \mathrm{Cmt}^3\, 1 \cdot$
           $\wedge : \wedge t \cdot t_1 \leq t \leq t_2 . \rightarrow : t . z \operatorname{Osc} \cdot \wedge \cdot \operatorname{Fre} \cdot t . z \operatorname{Osc} := \nu \cdot$
             $\wedge \cdot \operatorname{Aml} \cdot t . z \operatorname{Osc} : \operatorname{Mul} < . \operatorname{Lon} R \,|$
     $\operatorname{Exg} {\scriptstyle\mathrm{I}} : R a \in \mathrm{Res} . \wedge . \nu > \operatorname{Sec}^{-1} 0 . \wedge . t_0 \in \mathrm{Tem} .$
       $\wedge {\scriptstyle\mathrm{I}} . \vee f {\scriptstyle\mathrm{I}} f \in \mathrm{Coo} . \wedge : \wedge z : z \operatorname{Par} R a .$
         $\rightarrow : \vee \ulcorner x . \alpha \urcorner \cdot \alpha > 0 . \wedge . \wedge t : t R a \operatorname{Ext} . \rightarrow \cdot {}^\wedge f : t \operatorname{Loc} z . = \operatorname{Cca} .$
           $\ulcorner x + \alpha \cos 10\, \pi \nu\, (t - t_0) . 0 . 0 \urcorner {\scriptstyle\mathrm{I}} \cdot$
       $\rightarrow \cdot \wedge t : t R a \operatorname{Ext} . \rightarrow . t R a \operatorname{Vib}_\nu \,|$

Exg ι : $Ra \in \text{Res} . \wedge . l \frown \varepsilon \frown \gamma \frown \alpha \in \text{Pos} . \wedge . \varepsilon < \text{Vld} .$
$\wedge . t_0 \frown t_1 \in \text{Tem} . \wedge . 10 \pi\nu (t_1 - t_0) \in \text{Num} . \wedge \iota :$
$\vee f \iota \cdot f \in \text{Coo} . \wedge : {}^\wedge f \,\vdots\, t_0 \text{ Loc } Ra = : {}^\wedge x \cdot x = \ulcorner \xi_1 . \xi_2 . \xi_3 \urcorner . \wedge .$
$0 \leqq \xi_1 \leqq l . \wedge . |\xi_2| \leqq \varepsilon . \wedge . |\xi_3| \leqq \varepsilon^\cdot \wedge \iota .$
$\wedge z \iota z \text{ Par } Ra . \rightarrow \vdots \vee y \vdots \wedge t \vdots t \in \text{Tem} . \rightarrow^\cdot$
$t_0 \leqq t - \text{Sec } \gamma y \leqq t_1 . \rightarrow \cdot {}^\wedge f \,\vdots\, t \text{ Loc } z . = \text{Cca} .$
$\ulcorner y + \alpha \sin 10 \pi\nu (t - t_0 - \text{Sec } \gamma y) . 0 . 0 \urcorner :$
$\wedge : \neg . t_0 \leqq t - \text{Sec } \gamma y \leqq t_1 \cdot \rightarrow \cdot {}^\wedge f \,\vdots\, t \text{ Loc } z . = \text{Cca} . \ulcorner y . 0 . 0 \urcorner \iota^\cdot$
$\rightarrow \vdots \wedge t^\cdot t_0 \leqq t \leqq t_0 + \text{Sec } \gamma l . \rightarrow \vdots \vee w \cdot w \text{ Par } Ra . \wedge . \text{Cca } t w \text{ Vib}\nu$ #

The word written Vib (fL vibrat = vibrates) means "vibrates". Examples of stationary and progressive waves have been given.

4 14 1. # $Ha \text{ Inq } Ha^\cdot X \in \text{Hom} . \wedge . R \in \text{Res} . \wedge^\cdot$
$t X \text{ Cau} : t \text{ Usq } t' . a \text{ Usd } a' . R \text{ Mov} \cdot \text{Mod } X \text{ Iac} \vdots$
$\rightarrow \cdot \text{Cel Med } R \text{ Mov} . \text{Mul} < . \text{Cmt/Sec } 10^{1111}$ #

4 14 2. # $Ha \text{ Inq } Hb \vdots \text{Cur}^\cdot X \in . \text{Hom} \cup \text{Bes} \cdot \rightarrow : \text{Pot} \cdot t \text{ Loc } X . \neq . t' \text{ Loc } X \vdots$
$Hb \text{ Inq } Ha^\cdot \text{Qia} : \text{Pot} \cdot X \text{ Vul} . tt' X \text{ Mov} \vdots$
$Ha \text{ Inq } Hb \vdots \text{Cur}^\cdot R \in \text{Res} . \rightarrow : \text{Pot} \cdot t \text{ Loc } R . \neq t' \text{ Loc } R \vdots$
$Hb \text{ Inq } Ha : \text{Qia} \cdot \text{Pot} . tt' R \text{ Mov}$ #

The last text is to prepare the way for the next, enabling the receiver to understand the next 'Cur' in the sense of physically explaining causality.

4 14 3. # $Ha \text{ Inq } Hb \iota . \text{Pon } X \frown Y \in \text{Hom } \iota$
$\quad \text{Cur} : \text{Pot} \vdots t^\cdot X \text{ Inq } Y p . \rightarrow \text{Pau Pst} :$
$\quad\quad Y \text{ Ani} \cdot \text{Utr} . \text{PAN } X \text{ Inq } Y p$
$\quad \vdots \text{Qqm} \cdot t \text{ Dst } XY . \text{Mul} > . \text{Cmt } 0 \,|$
$\quad Hb \text{ Inq } Ha \iota \cdot \text{Qia } \iota . \vee w \iota \text{ Put } w \iota \wedge t \vdots t \in \text{Tem} .$
$\quad\quad \rightarrow \vdots \text{Pot} \vdots \text{Cca } t . X \text{ Cau}^\cdot \text{Nnc} . \text{Apu } X . \text{Usd} . \text{Apu } Y . w \text{ Mov} \cdot$
$\quad\quad\quad \frown : w \text{ Fer} \cdot {}^\curvearrowleft x . \text{Nnc } X \text{ Inq } Yx :$
$\quad\quad\quad \frown : Y \text{ Ani} \cdot \text{Utr} . \text{Nnc } w \text{ Apu } Y \,|$
$\quad Ha \text{ Inq } Hb . ? = w \,|$
$\quad Hb \text{ Inq } Ha \iota \cdot \text{Pon} : \text{Cel Med} . w \text{ Mov} \cdot \text{Mul} < . \text{Cmt/Sec } 10^{1111}{}^\cdot$
$\quad\quad \text{Erg}^\cdot w \in \text{Res} . \wedge : \text{Pot} \cdot \text{Apu } X . \text{Usd} . \text{Apu } Y . w \text{ Mov } \iota .$
$\quad\quad \text{Pon } \iota . \text{Cel Med} . w \text{ Mov} \cdot = \text{Cca} . \text{Cmt/Sec } 10^{1111} \iota$
$\quad\quad \neg \vee A \vdots A \in \text{Res} . \wedge^\cdot \text{Pot Apu } X . \text{Usd} . \text{Apu } Y . A \text{ Mov} \cdot$
$\quad\quad\quad \text{Mod} : \text{Cel Med} . A \text{ Mov} \cdot = \text{Cca} . \text{Cmt/Sec } 10^{1111} \iota$
$\quad\quad \text{Sed } \iota \vee R \vdots \text{Put Aer} \vdots R \in \text{Res} . \wedge^\cdot \text{Pot} \vdots t R \text{ Uul} . \wedge^\cdot t X \text{ Cau} :$
$\quad\quad\quad \text{Pau Pst} . t \cdot R \text{ Uul}^\cdot \wedge : \text{Pau Pst} . t \cdot Y \text{ Ani} \cdot \text{Utr} . \text{Nnc } R \text{ Uul } |$
$\quad Ha \text{ Inq } Hb : \text{Utr} \cdot w = \text{Aer} . \vee . w \text{ Par Aer } |$

Hb Inq Ha : Fal · Qia : ¬ Pot · v Par Aer . ∧ · tv Fer . ˅x Etc · Mod

: t Cel Med . v Mov = · Cel Med . w Mov |

Ha Inq Hb : Utr · t Loc w . ⊂ . t Loc Aer |

Hb Inq Ha ι · Ver ι . ˅ M ι Put M ι Car M > Vld . ∧ :

∧Z ∶ Z ∈ . Hom ∪ Bes · → : Car : ˄z · z ∈ M . ∧ . z Apu Z · > Vld ∶ ∧ ·

∧z : z ∈ M . → · z Par Aer . ∧ . Vol z < Vld · ∧ :

∧ ⌜z . z' . t⌝ · z ⌢ z' ∈ M . ∧ . z Apu z' . ∧ . t ∈ Tem ·

→ : t · z Mov . → Pau Pst . z' Mov ι :

Pon · z_1 ⌢ z_2 ⌢ Etc ⌢ z_N ∈ M . ∧ ·

z_1 Apu X . ∧ . z_2 Apu z_1 . ∧ . z_3 Apu z_2 . ∧ . Etc . ∧ . Y Apu z_N . ∧

· Loc z_1 . ∪ . Loc z_2 . ∪ . Etc . ∪ . Loc z_N · ⊂

: Cvx · Loc X . ∪ . Loc Y ι .

Erg ι . t · z_i Mov . → Pau Pst . z_{i+1} Mov : ∧ι

˅h ∶ ∧t ∶ t ∈ Tem . ∧ ∶ t · X Cau . $t z_1$ Mov · → h Pst :

z_N Mov . ⌢ · Y Ani . Utr Etc ι :

Erg ι : Pot : t Aer Uul . ∧ · t X Cau . Nnc Aer Uul · ∧ ι ·

˅α ι . ∧ ⌜X . Y . t⌝ ι X ⌢ Y ∈ Hom . ∧ . t ∈ Tem · → :

˅h ∶ h = α × t Dst XY . ∧ :

t X Cau . Nnc Aer Uul · → h Pst · Pot : Y Ani ·

Utr . Nnc Aer Uul |

Ha Inq Hb . ? = w |

ᵗ⁴Hb Inq Ha ι w = ∶ ˅t ∶ ˅v · v Uni : ˄z · z ∈ M . ∧ . tz Mov ∣ Tem ∶

t Loc w . = ∶ t Loc ∶ ˅v · v Uni : ˄z · z ∈ M . ∧ . tz Mov ∶ ᵗ⁴

w ∉ Res . Sed : w ∈ . Und Aer . ⊂ Und ∶

u ∈ Und Aer . → : Cel Med . u Mov · = Cca . Cmt/Sec 10^{1111} ∶

Eti ∶ ˅T ∶ T ∈ Res . ∧ · Pot . T Uul · ∧ ∶ ∧u · u ∈ Und T ·

→ : Cel Med . u Mov · = Cca . Cmt/Sec 10^{10011} |

Ha Inq Hb ∶ ? p · R ∈ Res . ∧ · Pot . R Uul · ∧ . z Par R :

→ : Pot · z Mov . Mod p |

Hb Inq Ha ∶ Exg · Ra ∈ Res . ∧ · Pot . Ra Uul · ∧ . z Par R :

→ : Pot · z Osc . Mod {last part of **4 13 4**} |

Ha Inq Hb ∶ ? u · Cca t_0 . Ra Uul . Mod {last part of **4 13 4**} ·

→ : u ∈ Und Ra . ∧ · Cca t_0 . u Mov |

Hb Inq Ha ∶ ∧t ∶ t_0 ≦ t ≦ t_0 + Sec γ l .

→ · ˄ ƒ ∣ t Loc u . = ∶ ˄x · x = ⌜ξ$_1$. ξ$_2$. ξ$_3$⌝ . ∧ . 0 ≦ Sec^{-1} (t − t_1)/γ

≦ ξ$_1$ ≦ Sec^{-1} (t − t_0)/γ ≦ l . ∧ . | ξ$_2$ | ≦ ε . ∧ . | ξ$_3$ | ≦ ε ∶

→ : u ∈ . Und Ra . ∧ · Cca t_0 . u Mov · ∧

· Cca t_0 . Cel Med · u Mov ∶ = . Cmt/Sec γ |

Ha Inq Hb ∶ Pon : Cel Med . w Mov ·

Mul > · Cmt/Sec 10^{10011} · Cur · Pot . w Mov |

Hb Inq Ha ⏌ ˙ Pon ⫶ Cel Med . w Mov ˙ $= c$. $=$ Cca .

  Cmt/Sec $11011111010111001111,1 \times 10^{1111}$ ⫶ ∧

  ˙ Err ˙ c . $=$ Cca . Etc ⫶ $=$ Cca ⫶ Cmt/Sec 10^{1111} ⏌ ⫶

  Pot ⫶ t ˙ w Mov . � ⫶ Fer ˙ ˇx . t X Inq Yx ⏌ ˙ Qia ⏌ ˙

  VS ⏌ . Put Ath ⏌ . $S \notin$. Res $\cup$ Hom $\cup$ Bes ˙

    ∧ ˙ Pot ⫶ Cca t . S Uul ˙ ∧ ˙ t X Cau . Nnc S Uul ˙

    ∧ ⏌ t X Cau . $t S$ Uul ˙ → ⫶

      ∨ ⌐w . t'⌐ ⫶ $w \in$. Und S ˙ ∧ ⫶ t Usq t' ˙ Apu X . Usd . Apu Y ˙

      w Mov ˙ Mod ⫶ Cel Med . w Mov ˙ $= c$ ⫶

      ∧ ˙ Pot ⫶ t' Y Ani ˙ Utr . Nnc S Uul ⏌ ⫶

  Loc Ath . $=$ Spa #

The word written Uul (fL undulat $=$ undulates) means "undulates" in the original sense such as Huygens took it (no underlying oscillation being supposed). The word written Und (fL unda $=$ wave) means "wave". (This notion cannot be expressed by Uul[io].) The words written Aer and Ath (fL aer $=$ air, and aether $=$ ether) mean "air" and "field" respectively. The precise meaning of these notions may remain unknown to the reveicer. The only thing that matters is the existence of some media that admit progressing waves with certain specified velocities.

'c' is a constant, the velocity of light in vacuo.

'{. . .}' stands for a quotation or repetition of the program text indicated between the braces.

In the next paragraph we shall account for the purpose of the last talk.

4 14 4. Introducing length is as difficult as introducing time was easy. In the beginning of the present chapter a word Cmt appeared. The receiver could guess that it means some length unit, but no indication was given of its value. It has been fixed by the text given in the last paragraph. This goal has been reached after long detours. Most of the developments of this chapter have merely been steps on this lengthy way.

Of course there are other possible contexts for presenting our length unit. Natural yardstick of astronomic size are the curvature radius of the universe, which is still extremely uncertain, and the parsec ($= 1296000/2 \pi$ times the distance between sun and earth), which is much better known, though not with great precision, but which can be significant only for a receiver inside our solar system. There are many microscopic yardsticks, but as long as we have not composed a vocabulary of chemistry or atomic physics, we cannot present them. As far as I can see, the only exception is the

phenomenological Rydberg constant R of hydrogen (or some other hydrogen-like element). It has the dimension of the inverse of a length, and it is one of the best-known physical constants (if not the best of all), with a precision of 10^{-7}. We could introduce it by its historical pre-Bohrian definition. We could say that there is a line spectrum with the reciprocal wave-lengths

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right); \; m < n, \text{ integers.}$$

But this cannot be done until something has been said about light. At this stage of the program the most striking feature of light, through which it could be characterized, is its tremendous velocity. On the other hand, as soon as we can say what light is, we can also present a reasonably satisfactory yardstick: the light-second, i.e. the distance light travels in the time-unit (known by mere presentation).

My original idea was to define velocity of light merely as the upper bound of the velocities of corpuscular inertial motions. Yet serious objections may be raised against this procedure. It would not be wise to start with relativistic mechanics. In the beginning we shall be committed to classical mechanics. But in classical inertial systems an upper bound for velocities would be an intolerable stumbling-block.

So I conceived the idea of introducing the velocity of light by a less physical and more behaviouristic procedure. Light waves, and more generally, electromagnetic waves are a means of communication. Fortunately communicating is a well-known phenomenon in our program. Start and finish are marked by the word Inq and Ani respectively. In the last paragraph Ha has asked the question what the intermediate link is. The answer is that there is something propagating from the speaker to the person who becomes aware of the spoken communication. Three cases have been dealt with by Hb, the first two cases as a background for the third, the only one that matters. The first case is that of mere convection; this is a rather slow method. Wavepropagation is much quicker. There is an undulating medium, characterized by sonic velocities. Finally there is wave propagation through an non-corporal medium ($\notin$ Res) which is characterized by a much higher velocity (that of light). If we should state that there is no quicker means of communication, we should give one more piece of information. This information would not be fundamental, and it would be harmless, because at this stage of our program mechanics is still in statu nascendi, and because that statement would not exclude any

quicker motion, as long as it cannot serve as a means of communication.[1])

We have quoted the numerical value of the velocity of light with the greatest precision now available.[2]) Of course it is not unlikely that the mere appearance of a number that is to mean the numerical value of some velocity and that bears a high degree of precision, would make the receiver assume that this number is to mean the numerical value of the velocity of light. Then our behaviouristic introduction would be redundant. But it is better not to rely on this possibility. Moreover it has been our policy to give multiple tautologous information.

As soon as the receiver has recognized that the numerical constant c means the velocity of light, he may use his knowledge of our time-unit in order to find out our length-unit. The precision of the result will mainly depend on five components: 1° and 2° the precision of time-measurements at the transmitter and the receiver, 3° and 4° the precision of measurements of the velocity of light at the transmitter and the receiver, 5° the precision of the receiver's information about the relative velocity of transmitter and receiver.

As to the last source of error we know that some knowledge about this relative velocity is needed for the elimination of the Ole Römer effect (see 2 01 2). If the program lasts long enough this elimination can be done on the strength of the knowledge of the *mean* relative velocity of transmitter and receiver. Inside our solar system this mean velocity is zero; if the receiver belongs to another solar system it is equal to the relative velocity of the two central bodies. A receiver who understands that the program is sent from our solar system can eliminate the unknown Ole Römer effect by means of an observed Doppler effect. The precision of this determination depends on that of wave-length measurements at the receiver's end.

The receiver may reduce the error from the first and second source by observing the program clock over a long period. In any case a lower bound for the possible relative error in the receiver's

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[1]) From our text the receiver may get the impression that human bodies can generate electromagnetic communication waves. This misunderstanding would be fairly harmless. Later on we can remove it.

[2]) This is a sore point. The most trustworthy values of fundamental physical constants quoted by the most trustworthy authors (from unquoted sources) at about the same time differ significantly from each other; even the values given in one and the same work at different places can differ.

knowledge about our length-unit will be given by the relative error of that value for the velocity of light we have inserted in our program. In fact the velocity of light seems to be known with a precision of three in one million, and it is with this precision that we can present our length-unit.

But afterwards this precision may be increased (and we could have done so even if it had been much worse) by presenting Rydberg's constant for hydrogen, which is known with a precision of one in ten millions. As we cannot yet convey in a more definite way what hydrogen is, it is possible that the receiver may interpret "hydrogen" in our context as "ionized helium", or as the name of some other hydrogen-like element. This can be prevented by a list of the Rydberg constants of all hydrogen-like elements or rather by a list of the Rydberg constants for the light elements and their limit for infinite nuclear mass. In these texts we have confined ourselves to hydrogen (see 4 14 6).

4 14 5. Continuation of **4 14 3**:

$*$ Ha Inq Hb $\cdot$ Pot : t Pst $\cdot$ Nnc Z Cpu . ? $=$ Cmt $*$

4 14 6. Further continuation of **4 14 3**:

$*$ Hb Inq Ha ι . $\lor T\iota$ Put Ato $H\iota T \subset$ Res . $\land \cdot$ Car T . $>$ Vld $\cdot$
$\qquad \land^{\cdot} \land x : x \in T$. $\to \cdot$ Vol x . Mul $<$. Cmt^3 1 $\cdot$ $\land$: $\lor R$: Put R :
$\qquad R$. $=$ Cca . Cmt^{-1} 11010110001101101,100100111
$\qquad \cdot \land$: Err $\cdot R$. $=$ Cca . Etc $\cdot$ $=$ Cca $\cdot$ Cmt^{-1} 10^{-110}
$\qquad$: $\land$: $\land$ $\ulcorner m . n . x . \nu . t \urcorner \cdot m \frown n \in$ Num . $\land . m < n$. $\land . x \in T$.
$\qquad \quad \land . \nu = cR \, (m^{-10} - n^{-10})$. $\land . t \in$ Tem $\cdot$
$\qquad \quad \to$: Pot $\cdot tx$ Cau . Nnc Ath Vib$_\nu$ |
$\quad$ Ha Inq Hb ι : Err ι . Cmt . $=$ Cca ι $^\lor x$: x . $=$ Cca .Cmt $\cdot \land$
$\qquad$: $t Z$ Sci $\cdot$ Utr . $R =$ Etc :
$\qquad \quad \to$: Pot t $\cdot Z$ Sci : Utr $\cdot$ Cmt . $=$ Cca . x ι $\cdot$ Mul $<$ ι $\cdot$
$\quad$ Err ι . Cmt . $=$ Cca ι $^\lor x$: x . $=$ Cca . Cmt $\cdot \land$: $t Z$ Sci $\cdot$ Utr $c =$ Etc :
$\qquad \quad \to$: t Pot $\cdot Z$ Sci : Utr $\cdot$ Cmt . $=$ Cca . x $*$

This is a description of the hydrogen spectrum. 'Ato H' means the genus "hydrogen atom".

4 18 1. The following text is a continuation of that of **4 11 5**.

$*$ Ha Inq Hb $\cdot$ Nnc Ha Vul :
$\quad$ Nnc $\cdot$ Apu Hb . Usd . Apu Ha $\cdot$ Hb Fer Rh |
$\quad$ Hb Inq Ha $\cdot$ $\lnot$ Pot |
$\quad$ Ha Inq Hb : Utr $\cdot$ Vol Rh . $>$ Nim |

Hb Inq Ha ⋅ Fal ┇ Vol Rh . $=$ Cca . Cmt³ 11×10^{1010} ⋅

   Tan ⋅ Mas Rh . $=$ Cca . Gra 11011×10^{1011} ┇

   Mas Rh . $>$ Nim ⋅

   Tan ⋅ $\wedge D : D$ Uni . 10^{ϵ} Hom ⋅ $\rightarrow$ ⋅ Pot . $t\,D$ Fer Rh ┇

   Dns Rh . $=$ Cca ⋅ Gra 11011×10^{1011} . $/$. Cmt³ 11×10^{1010} ⋅

   $=$ Cca . Gra/Cmt³ $10010,1011$ ∣

Ha Inq Hb ┇ Utr ┇ Dif ⋅ Nnc Hb Cau : Nnc ⋅ Apu Hb . Usd .

   Apu Ha ⋅ Ri Mov ┇ $=$ ┇ Dif ⋅ Nnc . Etc . Rj Mov ∣

Hb Inq Ha ┇ Qqm ⋅ Vol Ri . $=$. Vol Rj :

   Tan : Dif . Nnc Hb Fer Ri ⋅ $>$ ⋅ Dif . Nnc Hb Fer Rj :

   $\wedge$: Dif . Nnc Hb Iac Ri ⋅ $>$ ⋅ Dif . Nnc Hb Iac Rj ⋅

   Qia ⋅ Mas Ri . $>$. Mas Rj ⋅

   Dif ⋅ Nnc Hb . Fer $\smile$ Iac . Rh : $>$ Vld #

The words written Mas, Gra, Dns mean "mass", "gramme", "density" respectively. 'Mas' has been introduced by a behaviouristic definition. A crude estimation of the mass-unit would be possible on the strength of the data of the last talk.

'Gra' behaves syntactically like 'Sec', 'Cmt' and so on.

4 18 2. # $yc =$ Gra x . $\rightarrow$ ⋅ $x \in$ Rea . $\wedge$. $x \geqq 0$ ┇

   Gra $\in$ ⋅ $\hat{\ }x$ ⋅ $x \in$ Rea . $\wedge$. $x \geqq 0$: $\frown$. $\hat{\ }$ Mas ┇

   Gra . $a + b$ ⋅ $=$. Gra $a \dot{+}$ Gra b ┇

   Gra . $a - b$ ⋅ $=$. Gra $a \dot{-}$ Gra b ┇

   $\alpha \geqq 0$. $\rightarrow$. Gra $\alpha a = \alpha$ Gra a ┇

   Gra a . $\leqq$. Gra b ⋅ $\leftrightarrow$. $a \leqq b$ ┇

   Gra a . $=$. Gra b ⋅ $\leftrightarrow$. $a = b$ ┇

   Gra a . $=$ Cca . Gra b ⋅ $\leftrightarrow$ ⋅ a . $=$ Cca . b ┇

   Gra a/Cmt³ b . $=$ ⋅ Gra/Cmt³ . a/b ┇

   Gra $a \times$ Cmt/Sec . b ⋅ $=$ ⋅ Gra Cmt/Sec . $a \times b$ ┇

   Etc #

4 18 3. # Vol $x \neq$ Cmt³ 0 . $\rightarrow$ ⋅ Dns x . $=$. Mas x/Vol x :

   $x \in$. Hom $\cup$ Bes ⋅ $\rightarrow$ ⋅ Dns x . $=$ Cca . Gra/Cmt³ 1 #

4 18 4. # z Uni xy . $\wedge$ ⋅ Mas x . $=$. Gra α ⋅ $\wedge$ ⋅ Mas y . $=$. Gra β : $\rightarrow$:

   Mas $z = $ ⋅ Mas x . $\dot{+}$. Mas y ⋅ $=$ ⋅ Gra α . $\dot{+}$. Gra $\beta = $ ⋅ Gra $(\alpha + \beta)$ ⋅

   z Uni xy . $\wedge$ ⋅ Mas x . $=$. Gra α ⋅ $\wedge$ ⋅ Mas z . $=$. Gra γ : $\rightarrow$:

   Mas $y = $ ⋅ Mas z . $\dot{-}$. Mas x ⋅ $=$ ⋅ Gra γ . $\dot{-}$. Gra α ⋅ $=$. Gra $(\gamma - \alpha)$ #

4 18 5. # $x \in$. Res $\cup$ Hom $\cup$ Bes ⋅ $\rightarrow$ ┇ $\wedge t$ ┇ $t \in$ Tem . $\wedge$. tx Ext ⋅

   $\rightarrow$ ⋅ $\vee \alpha$: $\alpha > 0$. $\wedge$ ⋅ t Mas x . $=$. Gra α #

4 18 6. $^{\#} x \in$. Res $\cup$ Hom $\cup$ Bes $\cdot \rightarrow \dot{\cdot} \wedge t\dot{}\, t \in$ Tem $. \wedge . tx$ Ext $\cdot$

$\rightarrow \dot{\cdot} \vee y \cdot y = . t$ Cor $x \dot{:}$

t Cor $x . = . t$ Cor $y \cdot \leftrightarrow . x = y \dot{:}$

t Loc $x . = \cdot$ Loc $. t$ Cor $x \dot{:}$

t Vol $x . = \cdot$ Vol $. t$ Cor $x \dot{:}$

t Lon $x . = \cdot$ Lon $. t$ Cor $x \dot{:}$

t Mas $x . = \cdot$ Mas $. t$ Cor $x \dot{:}$

ι Dns $x . = \cdot$ Dns $. t$ Cor x $^{\#}$

The word written Cor ($\int L$ corpus = body) means "body". Though it has been introduced in a mere formal way, the receiver may guess that the bodies of men, animals and things may depend on time. In 4 07 1 the volume of humans as a function of time has been communicated. An air wave may be considered as a thing ($\in$ Res) with a varying body:

4 18 7. A comment on 4 14 3, especially on the part from t_3 to t_4:

$^{\#} Hc$ Inq Hd $\iota \cdot$ Qqm $\dot{:}$ PAN Hb Inq $Ha \cdot$ Cel Med $. w$ Mov $\cdot$

$= \cdot$ Cca $.$ Cmt/Sec $10^{1111} \dot{:} \rightarrow \cdot w \notin$ Res $. \wedge . w \neq$ Aer $. \wedge . \neg w$ Par Aer $\dot{:}$

Tan $\dot{:}$ Nnc Hc Inq $Hd \dot{:} w \in$ Res $.$

$\wedge \dot{} \wedge t : tw$ Ext $. \rightarrow \cdot t$ Cor $w .$ Par Aer $\iota .$

Qqm $\iota t_3 t_4 Hb$ Inq $Ha \cdot w = . \ulcorner t$ Etc ι

Tan ι Nnc Hc Inq $Hd \dot{:}$ Cor $w = \dot{:}$ Cor $\dot{:} \ulcorner v \cdot v$ Uni $\dot{:} \uparrow z \cdot z \in M .$

$\wedge . tz$ Mov $\iota .$

$\vee A \dot{:} A \in$ Res $. \wedge \dot{:}$ Pot $\dot{:} \vee \ulcorner t_1 . t_2 \urcorner \cdot$ Put $\ulcorner t_1 . t_2 \urcorner \cdot$

t_1 Usq $t_2 \cdot$ Apu $X .$ Usd $.$ Apu $Y \cdot A$ Mov $\dot{:}$

$\wedge \cdot$ Dst $XY/(t_2 - t_1) . = $ Cca $.$ Cmt/Sec $10^{1111} \cdot$

$\wedge \dot{} \neg \wedge \ulcorner t . t' \urcorner \dot{:} t_1 \leq t \leq t' \leq t_2 . \rightarrow \cdot t$ Cor $A . = . t'$ Cor $A \cdot$

Sed $\dot{} \wedge \ulcorner t . t' \urcorner \dot{:} t_1 \leq t < t' \leq t_1 . \rightarrow \cdot t$ Cor $A . \neq . t'$ Cor A $^{\#}$

4 18 8. *Ha* Inq *Ha* ι

$x \in$ Hom $. \rightarrow \dot{:}$ Ini $. x$ Ext $\cdot - \dot{:}$ Ini $\cdot$ Cor $x .$ Ext $\dot{} =$

Cca $.$ Sec $11 \times 10^{10111} \dot{:}$

$\vee x \dot{:} x \in$ Bes $. \wedge \dot{:}$ Ini $. x$ Ext $\cdot - \dot{:}$ Ini $\cdot$ Cor $x .$ Ext $\dot{} >$ Sec $0 \dot{:}$

$x \in$ Hom $. \rightarrow \dot{} \vee \ulcorner y . z \urcorner \dot{:} y \frown z \in$ Hom $. \wedge \cdot y = .$ Mat $x \cdot \wedge \cdot z = .$ Pat $x \dot{:}$

$\vee x \dot{:} x \in$ Bes $. \wedge \dot{} \vee \ulcorner y . z \urcorner \dot{:} y \frown z \in$ Bes $. \wedge \cdot y = .$ Mat $x \cdot \wedge \cdot z = .$ Pat $x \dot{:}$

$x \in$ Hom $. \rightarrow \dot{:} \wedge t \dot{:}$ Ini $\cdot$ Cor $x .$ Ext $:$ Ant $: t :$ Ant $:$ Ini $. x$ Ext $\cdot$

$\rightarrow \dot{:} t$ Cor $x .$ Par $\cdot t$ Cor $.$ Mat $x \dot{:}$

$\vee x : x \in$ Bes $. \wedge \cdot \wedge t .$ Etc $\dot{:}$

$x \in$ Hom $. \wedge \dot{:} s =$ Ini $\cdot$ Cor $x .$ Ext $\dot{}$

$\rightarrow \dot{:} \vee \ulcorner u . v \urcorner s$ Cor $u .$ Par $\cdot s$ Cor $.$ Mat $x :$

$\wedge \dot{}$ Pau Ant $. s \cdot$ Cor $v :$ Par $:$ Pau Ant $. s \cdot$ Cor $.$ Pat $x \cdot$

$\wedge \dot{:} s$ Cor $x .$ Uni $\cdot s$ Cor $u . s$ Cor $v \dot{:}$

$\vee x : x \in \text{Bes} . \wedge . \text{Etc}$ ⁞

$\text{Hom} = \text{Hom Fem} . \cup . \text{Hom Msc}$ ⁞

$\text{Hom Fem} \cap \text{Hom Msc} = \ulcorner \urcorner$ ⁞

$\text{Car} : \hat{}x \cdot \text{Nnc } x \text{ Ext} . \wedge . x \in \text{Hom Fem} \cdot$

$\quad \text{Pau} > \cdot \text{Car} : \hat{}x \cdot \text{Nnc } x \text{ Ext} . \wedge . x \in \text{Hom Msc}$ ⁞

$y = \text{Mat } x . \wedge . y \in \text{Hom} \cdot \rightarrow . y \in \text{Hom Fem} : \wedge :$

$y = \text{Pat } x . \wedge . y \in \text{Hom} \cdot \rightarrow . y \in \text{Hom Msc}$ ⁞

$x \in \text{Hom} \cup \text{Bes} : \rightarrow : \text{Fin} . \text{Cor } x \cdot \text{Pst} . \text{Fin } x$ ⁕

The words written Mat (*fL* mater = mother), Pat (*fL* pater = father), Fem (*fL* femininus = female), Msc (*fL* masculinus = male) mean "mother", "father", "female", "male" respectively. The last paragraph is somewhat premature. Its notions will not be used in this volume.

4 19 1.　⁕ $\vee a \cdot \wedge \ulcorner t . t' \urcorner : t_1 \leq t \leq t' \leq t_2 . \rightarrow \cdot t \text{ Usq } t' \text{ Cel Med } x . = a :$

$\quad \rightarrow : \wedge \ulcorner t . t' \urcorner \cdot t_1 < t \leq t' < t_2 . \rightarrow : \text{Cel } x . t \cdot = \cdot \text{Cel } x . t'$ ⁕

The word written Cel (*fL* celeritas = velocity) means "velocity". Here it has been introduced for uniform motions only.

4 19 2.　⁕ $\wedge \ulcorner t . t' \urcorner : t_1 < t \leq t' < t_2 .$

$\quad \rightarrow \cdot \text{Cel } x . t \cdot = \cdot \text{Cel } x . t' : \wedge : \text{Cel } y . t \cdot = \cdot \text{Cel } y . t'$ ⁞

$\quad \rightarrow : \wedge t : t_1 < t < t_2 . \rightarrow : \text{Cel } x . t \cdot = \cdot \text{Cel } y . t \cdot$

$\quad \leftrightarrow \cdot \wedge t : t_1 \leq t \leq t' \leq t_2 . \rightarrow \cdot t \text{ Dst } xy . = . t' \text{ Dst } xy$ ⁕

4 19 3.　⁕ $\tau \in \text{Rea} . \wedge . \tau \neq 0 . \wedge . a \in \text{Vco} \cdot$

$\quad \rightarrow : \text{Cmt}/\text{Sec} . a/\tau \cdot = \cdot \text{Cmt } a/\text{Sec } \tau . \in \cdot \text{Cmt}/\text{Sec} . \text{Vco}$ ⁞

$\tau \in \text{Rea} . \wedge . b \in \text{Vco} \cdot \rightarrow . \text{Sec } \tau \times \text{Cmt}/\text{Sec } b . = \text{Cmt } \tau b$ ⁞

$\mu \in \text{Pos} . \wedge . b \in \text{Vco} \cdot \rightarrow . \text{Gra } \mu \times \text{Cmt}/\text{Sec } b . = \text{Gra Cmt}/\text{Sec } \mu b$ ⁞

Etc ⁕

4 19 4.　⁕ $v \in \cdot \text{Cmt}/\text{Sec} . \text{Vco} : \leftrightarrow \cdot \vee w : w \in \text{Vco} . \wedge \cdot v = . \text{Cmt}/\text{Sec } w .$

$v \in \cdot \text{Gra Cmt}/\text{Sec} . \text{Vco} : \leftrightarrow \cdot \vee w : w \in \text{Vco} . \wedge \cdot v = . \text{Gra Cmt}/\text{Sec } w :$

Etc ⁕

4 19 5.　⁕ $a \frown b \in \text{Vco} . \wedge . \gamma \in \text{Rea} \cdot \rightarrow \cdot \text{Cmt}/\text{Sec } a . = . \text{Cmt}/\text{Sec } b \cdot \leftrightarrow$

$\quad . a = b : \wedge : \text{Cmt}/\text{Sec } a + \text{Cmt}/\text{Sec } b . = \cdot \text{Cmt}/\text{Sec} . a + b : \wedge$

$\quad \cdot \gamma \text{ Cmt}/\text{Sec } a . = . \text{Cmt}/\text{Sec } \gamma a \cdot \wedge$

$\quad : \text{Cmt}/\text{Sec } a . I . \text{Cmt}/\text{Sec } b \cdot = \text{Cmt}^2/\text{Sec}^2 . a I b : \wedge$

$\quad : \text{Cmt}/\text{Sec } a . = \text{Cca} . \text{Cmt}/\text{Sec } b \cdot \leftrightarrow \cdot a . = \text{Cca} . b : \wedge$

$\quad . \text{Etc}$ ⁕

4 19 6.　⁕ $f \in \text{Coo} . \rightarrow \cdot a \in \text{Vco} . \rightarrow : f . \text{Cmt}/\text{Sec } a \cdot = \cdot f . \text{Gra Cmt}/\text{Sec } a \cdot = f a :$

Etc ⁕

4 19 7. $* \wedge t^{\cdot} t_1 < t < t_2 . \rightarrow : \mathrm{Cel}\, x . t^{\cdot} = v \dot{:} \rightarrow \dot{:} v \in \cdot \mathrm{Cmt/Sec} . \mathrm{Vco} :$

$\wedge \dot{:} \wedge \ulcorner t . t'\urcorner \dot{:} t \frown t' \in \mathrm{Tem} . \wedge . t_1 \leqq t < t' \leqq t_2 :$

$\rightarrow {}^{\cdot} v^{\cdot} = . (t' \operatorname{Loc} x - t \operatorname{Loc} x)/(t - t')$

$\wedge \cdot t \operatorname{Loc} x . = . t_1 \operatorname{Loc} x + (t - t_1)\, v\ *$

4 19 8. $* \wedge \ulcorner tt'\urcorner {}^{\cdot} t_1 \leqq t < t' \leqq t_2 . \rightarrow : t \operatorname{Usq} t' . \operatorname{Cel} \operatorname{Med} x^{\cdot} = . \operatorname{Cmt} \gamma \dot{:}$

$\rightarrow {}^{\cdot} \wedge t : t_1 < t < t_2 . \rightarrow \cdot | \operatorname{Cel} x . t | = . \operatorname{Cmt} \gamma\ *$

4 20 1. In the following texts $\mathfrak{p}$ is a substitute for the noise of a collision.

$*^{t_1} \mathfrak{p}^{t_2} \dot{:}$

$\quad Ha \operatorname{Inq} Hb \cdot ? z . t_1 t_2 \operatorname{Fit} z \dot{:}$

$\quad Hb \operatorname{Inq} Ha \cdot \vee \ulcorner x . y \urcorner \cdot \operatorname{Put} \ulcorner x . y \urcorner \cdot x \in \operatorname{Res} . \wedge . y \in \operatorname{Res} . \wedge . t_1 xy \operatorname{Ccu} \dot{:}$

$\quad Ha \operatorname{Inq} Hb . ? = \operatorname{Ccu} \dot{:}$

$\quad Hb \operatorname{Inq} Ha \dot{:} \operatorname{Cca} t_1 : y \operatorname{Mov} . \frown \cdot \neg x \operatorname{Mov}^{\cdot} \wedge . t_1 y \operatorname{Apu} x \dot{:}$

$\quad\quad t < t_1 < t' . \rightarrow : \operatorname{Cel} y . t'^{\cdot} = \cdot - \operatorname{Cel} y . t^{\cdot} \neq \cdot \operatorname{Cmt/Sec} 0_{\mathrm{Vco}} \dot{:} \wedge$

$\quad\quad {}^{\cdot} t_1 x \operatorname{Cau} : \operatorname{Cel} y . t'^{\cdot} \neq \cdot \operatorname{Cel} y . t^{\cdot}$

$\quad\quad\quad \wedge {}^{\cdot} t_1 x \operatorname{Mut} : \operatorname{Cel} y . \operatorname{Cca} t_1 : \operatorname{Mod} . - a \operatorname{Ilo} a \dot{:}$

$\quad Ha \operatorname{Inq} Hb . \operatorname{Cur} \operatorname{Pot} \dot{:}$

$\quad Hb \operatorname{Inq} Ha : \operatorname{Qia} \cdot \operatorname{Mas} x . \operatorname{Mul} > . \operatorname{Mas} y\ *$

The word written Ccu (fL concutiunt = collide) means "collide".

4 20 2. $*^{t_1} \mathfrak{p}\,|$

$\quad Ha \operatorname{Inq} Hb^{\cdot} \operatorname{Utr} : \vee \ulcorner x . y \urcorner \cdot x \frown y \in \operatorname{Res} . \wedge . t_1 xy \operatorname{Ccu} |$

$\quad Hb \operatorname{Inq} Ha \cdot \operatorname{Ver} . t_1 x \operatorname{Apu} y\, |$

$\quad Ha \operatorname{Inq} Hb^{\cdot} \operatorname{Utr} \cdot \operatorname{Mas} x . \operatorname{Mul} > . \operatorname{Mas} y\, |$

$\quad Hb \operatorname{Inq} Ha \cdot \operatorname{Fal} \dot{:} \operatorname{Mas} x = \operatorname{Mas} y \dot{:} \operatorname{Erg} \dot{:} t < t_1 < t' . \rightarrow$

$\quad\quad : \operatorname{Cel} x . t'^{\cdot} = \cdot \operatorname{Cel} y . t : \wedge : \operatorname{Cel} y . t'^{\cdot} = \cdot \operatorname{Cel} x . t : \wedge$

$\quad\quad {}^{\cdot} t_1 x \operatorname{Cau} : \operatorname{Cel} y . t'^{\cdot} \neq \cdot \operatorname{Cel} y . t^{\cdot} \wedge$

$\quad\quad {}^{\cdot} t_1 y \operatorname{Cau} : \operatorname{Cel} x . t'^{\cdot} \neq \cdot \operatorname{Cel} x . t^{\cdot} \wedge : t_1 xy \operatorname{Ccu} . \operatorname{Cau} \cdot \operatorname{Etc}\ *$

4 20 3. $*^{t_1} \mathfrak{p}\,|$

$\quad Ha \operatorname{Inq} Hb^{\cdot} \operatorname{Utr} \vee \ulcorner x . y \urcorner \cdot \operatorname{Put} \ulcorner x . y \urcorner \cdot x \frown y \in \operatorname{Res} . \wedge . t_1 xy \operatorname{Ccu} |$

$\quad Hb \operatorname{Inq} Ha \operatorname{Ver} |$

$\quad Ha \operatorname{Inq} Hb : \operatorname{Utr} \cdot \operatorname{Mas} x . = . \operatorname{Mas} y\, |$

$\quad Hb \operatorname{Inq} Ha \cdot \neg \operatorname{Ncs} \dot{:} \operatorname{Sed}^{t_1} \dot{:} t < t_1 < t' . \rightarrow \dot{:}$

$\quad\quad \operatorname{Cel} x . t^{\cdot} = a : \wedge : \operatorname{Cel} x . t' = a' : \wedge : \operatorname{Cel} y . t = b : \wedge : \operatorname{Cel} y . t' = b' : \wedge .$

$\quad\quad \operatorname{Mas} x = \mu . \wedge . \operatorname{Mas} y = v^{\cdot} \rightarrow . \mu a + vb = \mu a' + vb' . \wedge .$

$\quad\quad\quad a' \neq a . \wedge . \mu a \operatorname{Ia} + vb \operatorname{Ib} = \mu a' \operatorname{Ia}' + vb' \operatorname{Ib}' \cdot$

$\quad\quad \operatorname{Erg} \cdot \mu a - \mu a' = vb' - vb . \wedge .$

$\quad\quad\quad \mu a \operatorname{Ia} - \mu a' \operatorname{Ia}' = vb' \operatorname{Ib}' - vb \operatorname{Ib} \dot{:}$

$\quad\quad {}^{t_1}\operatorname{Pon} : \vee \ulcorner \alpha . \beta . e \urcorner \cdot \operatorname{Put} e \cdot \alpha \frown \beta \in \operatorname{Rea} . \wedge .$

$\quad\quad\quad e \in \operatorname{Cmt/Sec} \operatorname{Vco} . \wedge . a = \alpha e . \wedge . b = \beta e^{\cdot}$

$\quad\quad \operatorname{Erg} : \vee \ulcorner \alpha' . \beta' \urcorner \cdot a' = \alpha' e . \wedge . b' = \beta' e^{\cdot}$

$\mathrm{Erg}^{t_1} \cdot a + a' = b + b' \cdot \wedge \cdot$

$\quad a' = [\mu a + \nu b + \nu(b-a)]/(\mu+\nu) \cdot \wedge \cdot$

$\quad b' = [\mu a + \nu b + \mu(a-b)]/(\mu+\nu)^{t_5} \vdots$

$\mathrm{Nnc}\, Hb\, \mathrm{Mut} \cdot {}^{\curlyvee} p \cdot t_2\, t_5\, Hb\, \mathrm{Inq}\, Ha\, p$

$\quad \cdot \mathrm{Mod} : \mathrm{Sin} \cdot {}^{\curlyvee} q \cdot t_3\, t_4\, Hb\, \mathrm{Inq}\, Ha\, q \vdots$

$\quad \rightarrow : {}^{\curlyvee} r \cdot \mathrm{Nnc}\, Hb\, \mathrm{Inq}\, Ha\, r \cdot \in \mathrm{Ver} \frown \in \mathrm{Lex}\, \mathrm{Nat} \vdots$

$t''\, Hb\, \mathrm{Sci} : \mathord{?} = \mathrm{Cel}\, x \cdot t \cdot \frown \cdot \mathrm{Cel}\, y \cdot t \cdot$

$\quad \rightarrow : \mathrm{Pot} \cdot t''\, Hb\, \mathrm{Cpu} : \mathord{?} = \mathrm{Cel}\, x \cdot t' \cdot \frown \cdot \mathrm{Cel}\, y \cdot t' \mid$

$Ha\, \mathrm{Inq}\, Hb \cdot x \frown y \in \mathrm{Res} \cdot \rightarrow : \mathrm{Nnc}\, Ha\, \mathrm{Sci} \cdot \mathord{?} = {}^{\curlyvee} \varrho \cdot$

$\quad \mathrm{Mas}\, y = \varrho\, \mathrm{Mas}\, x \; {}^{+}$

Hb has formulated the law of elastic collision. *Ha* states that this law implies an unambiguous definition of mass ratio. The receiver can conclude that 'Mas' means something that is proportional with mass. The mass unit, however, remains unknown.

Lex Nat (*fL* lex naturalis = law of nature) means "law of nature".

It might be remarked that an ideal elastic collision cannot produce any noise. This is true. Nevertheless I think if there were some danger in showing the noise of collision, it is more than counterbalanced by the obvious gain of demonstrativeness.

4 20 4. ${}^{*} Ha\, \mathrm{Inq}\, Ha\, \mathfrak{\iota}$

$\quad \mathrm{Ctr}\, x \cdot \in \cdot \mathrm{Cvx} \cdot \mathrm{Loc}\, x \vdots$

$\quad \mathrm{Mas}\, x = \mu \cdot \wedge \cdot \mathrm{Mas}\, y = \nu \cdot \wedge \cdot z\, \mathrm{Uni}\, xy \cdot$

$\quad \rightarrow \cdot \mathrm{Mas}\, z = \mu + \nu \cdot \wedge \cdot (\mu+\nu)\, \mathrm{Ctr}\, z = \mu\, \mathrm{Ctr}\, x + \nu\, \mathrm{Ctr}\, y \cdot$

$\quad\quad \wedge \cdot (\mu + \nu)\, \mathrm{Cel}\, \mathrm{Ctr}\, z = \mu\, \mathrm{Cel}\, \mathrm{Ctr}\, x + \nu\, \mathrm{Cel}\, \mathrm{Ctr}\, y \vdots$

$\quad \mathrm{Ipu}\, x = \mathrm{Mas}\, x \times \mathrm{Cel}\, \mathrm{Ctr}\, x \vdots$

$\quad \mathrm{Ipu}\, x \in \cdot \mathrm{Gra}\, \mathrm{Cmt}/\mathrm{Sec} \cdot \mathrm{Vco} \vdots$

$\quad t < t_1 < t' \cdot \wedge \cdot t_1\, xy\, \mathrm{Ccu} \cdot \rightarrow \cdot {}^{!} t_1\, x\, \mathrm{Apu}\, y \cdot$

$\quad \wedge \colon \mathrm{Cel} : \mathrm{Ctr} \cdot {}^{\curlyvee} z \cdot z\, \mathrm{Uni}\, xy : t \cdot = \cdot \mathrm{Cel} : \mathrm{Ctr} \cdot {}^{\curlyvee} z \cdot z\, \mathrm{Uni}\, xy : t' \vdots$

$\quad\quad \wedge \cdot \mathrm{Ipu}\, x \cdot t \cdot + \cdot \mathrm{Ipu}\, y \cdot t \colon = \colon \mathrm{Ipu}\, x \cdot t' \cdot + \cdot \mathrm{Ipu}\, y \cdot t' \vdots$

$\quad \mathrm{Kin}\, x = 0,1 \times \mathrm{Mas}\, x \,|\, \mathrm{Cel}\, \mathrm{Ctr}\, x \,|^{10} \vdots$

$\quad z\, \mathrm{Uni}\, xy \cdot \rightarrow \cdot \mathrm{Kin}\, z = \mathrm{Kin}\, x + \mathrm{Kin}\, y \vdots$

$\quad t < t_1 < t' \cdot \wedge \cdot t_1\, xy\, \mathrm{Ccu} \cdot \rightarrow \cdot {}^{!} z\, \mathrm{Uni}\, xy \cdot$

$\quad \rightarrow : \mathrm{Kin}\, z \cdot t \cdot = \cdot \mathrm{Kin}\, z \cdot t' \; {}^{+}$

The word written Ctr (*fL* centrum = centre) means "mass centre". The words written Ipu and Kin mean impulse and kinetic energy respectively.

It is understood that in 4 20 1–4 20 3 we dealt with mass points, whereas in 4 20 4 the objects may be bodies with positive extensions. This could be stressed by means of a comment.

4 21 1. $*$ Via $x\,.\,t\,\boldsymbol{\cdot}=\,.\,t$ Loc Ctr x $*$

The word written Via (fL via $=$ path) means the path function of some body.

4 21 2. $*$ Cel $x\,.\,t\,\boldsymbol{\cdot}=\,:\dfrac{d}{dt}\,.$ Via $x\boldsymbol{\cdot}t:$

$\quad=\,\vdots\,\lim\,\boldsymbol{\cdot}h\to 0\,\boldsymbol{\cdot}$ Via $x\,.\,t+h\,\boldsymbol{\cdot}-$ Via $x\,.\,t:/h\,\vdots$

$\quad$ Cel $x\,.\,t\,\boldsymbol{\cdot}\in\,\boldsymbol{\cdot}$ Cmt/Sec $.$ Vco $*$

4 21 3. $*$ Acc $x\,.\,t\,\boldsymbol{\cdot}=\,:\dfrac{d}{dt}\,.$ Cel $x\boldsymbol{\cdot}t:$

$\quad=\,\vdots\,\lim\,\boldsymbol{\cdot}h\to 0\,\boldsymbol{\cdot}$ Cel $x\,.\,t+h\,\boldsymbol{\cdot}-$ Cel $x\,.\,t:/h\,\vdots$

$\quad$ Acc $x\,.\,t\,\boldsymbol{\cdot}\in\,\boldsymbol{\cdot}$ Cmt/Sec$^2\,.$ Vco $*$

The word written Acc (fL acceleratio $=$ acceleration) means "acceleration".

The use of the traditional notation of calculus is rather provisional (see **1 35 4**).

4 22 1. $*\,\oplus\in$ Res $:$

Loc $\oplus\,.\in$ Cca $.$ Bul $:$

Loc $\oplus\,.=$ Cca $.$ Bul $a\,\varrho\,\boldsymbol{\cdot}\to a\,.=$ Cca $.$ Ctr $\oplus\,\boldsymbol{\cdot}\wedge\,.\,\varrho=$ Rad $\oplus\,.$

$=$ Cca $.$ Cmt $101\times 10^{11011}:$

Dns $\oplus\,.=$ Cca $.$ Gra/Cmt^3 $101,1$ $*$

The word written $\oplus$ (the astronomical symbol for "earth") means "earth".

4 22 2. $*$ Loc $\oplus\,.=$ Cca $.$ Bul $a\varrho\,\boldsymbol{\cdot}\to\,\boldsymbol{\cdot}$ Loc Sfi $\oplus\,.=$ Cca $.$ Sph $a\varrho$ $*$

The word written Sfi (fL superficies $=$ surface) means "surface".

4 22 3. $*\,x\in\,.$ Hom $\cup$ Bes $\boldsymbol{\cdot}\to:$ Dst $.\,x\,.$ Sfi $\oplus\,\boldsymbol{\cdot}<$ Vld $*$

4 22 4. $\mathfrak{p}$ is again a substitute for a noise of some collision.

$*\,{}^{t_1}\mathfrak{p}\,|$

Ha Inq $Hb:?\,x\,\boldsymbol{\cdot}$ Cca $t_1\,.$ Fit $x\,|$

Hb Inq $Ha\,\boldsymbol{\cdot}\,\vee y\,\boldsymbol{\cdot}$ Put $y\,\boldsymbol{\cdot}y\in$ Res $.\,\wedge\,.\,t_1\,y\,\oplus$ Ccu $:$

$\quad t\,.$ Pau Ant $.\,t_1\,\boldsymbol{\cdot}\to\,\boldsymbol{\cdot}ty$ Mov $.\,\wedge\,.\,ty$ Cad $|$

Ha Inq $Hb\,.\,?=$ Cad $|$

Hb Inq $Ha:ty$ Cad $.\leftrightarrow\,\boldsymbol{\cdot}t\,.$ Adv Sfi $\oplus\,.\,y$ Mov $|$

Ha Inq $Hb:?\,z\,\boldsymbol{\cdot}$ Cca $t_1\,.\,z$ Cau $.$ Nnc y Cad $|$

Hb Inq $Ha\,.\,\oplus$ Cau $|$

Ha Inq Hb Cur $|$

Hb Inq $Ha\,\boldsymbol{\cdot}\,\neg\,\boldsymbol{\cdot}$ Dns $y\,.<$ Vld $:\wedge\,\boldsymbol{\cdot}t\,\boldsymbol{\cdot}$ Dst $.\,y\,.$ Sfi $\oplus:$

$\quad$ Mul $<\,.$ Rad $\oplus:\to\,\boldsymbol{\cdot}$ Ncs $.\,ty$ Cad $|$

Ha Inq Hb Cur $|$

Hb Inq $Ha:\,{}^{\vee}\mathfrak{p}\,.$ PAN Ha Inq Hb $\mathfrak{p}\,\boldsymbol{\cdot}\in$ Lex Nat $|$

Ha Inq $Hb\,.\,?=$ Cel $y\,|$

Hb Inq Ha ⏵ Pon ⦂ Dst . y . Sfi $\dot{\circ}$ ⋅ Mul $<$. Rad $\dot{\circ}$:

$\qquad \wedge \cdot t_0 \leqq t \leqq t_1$. $\wedge$. ty Cad ⋅

$\qquad \wedge \vcentcolon t_0$ Loc Ctr $\dot{\circ}$. $-$. t_0 Loc y ⋅ $=$ ⋅ t_0 ⋅ Dst . Ctr $\dot{\circ}$. y ⦂ $\times$ e ⦂

$\qquad$ Erg ⦂ Cel y . t ⋅ $-$ ⋅ Cel y . $t_0 =$. g $(t - t_0)$ e ⦂ $\wedge$ ⦂

$\qquad$ Via y . t ⋅ $=$ ⋅ Via y . t_0 ⋅ $+$ ⦂ $(t - t_0)$ ⋅ Cel y . t_0 ⦂ $+ 0{,}1 \times g$ $(t - t_0)^{10}$ e ⦂

$\qquad \wedge \cdot t$ Acc y . $= ge$ ⦂

$\qquad \curlyvee p$. PAN Hb Inq Ha ⋅ $\in$ Lex Nat ⏐

Ha Inq Hb . ? $= g$ ⏐

Hb Inq Ha ⋅ g . $=$ Cca . Cmt/Sec^2 1111010101 ⏐

Ha Inq Hb ⦂ Cur ⋅ PAN Hb Inq Ha ⦂ Pon ⋅ Etc . ty Cad . Etc ⏐

Hb Inq Ha ⏵ $\neg$ Nes ⦂ Erg ⦂ Nnc Hb Mut ⋅ $\curlyvee p$. PAN Hb Inq Ha p ⋅

$\qquad$ Mod ⦂ Pot . ty Cad ⋅ Ilo . ty Cad ⏐

Ha Inq Hb ⋅ Utr ⦂ $t \dot{\circ}$ Cau ⋅ t Acc y . $= ge$ ⏐

Hb Inq Ha Ver ⏐

Ha Inq Hb ⦂ Utr ⋅ Mod . Fer $\smile$ Iac ⏐

Hb Inq Ha ⋅ $\neg$ Mod Fer . $\wedge$. $\neg$ Mod Iac ⦂

$\qquad$ Pon ⦂ $t z$ Ext . $\wedge$. $z \in$ Res ⋅ $\leftrightarrow$. $z = x \smile y$ ⦂ $\wedge$. $x \neq y$.

$\qquad \wedge \cdot$ Via x . $t -$ Via y . t ⋅ $= \cdot t$ Dst $xy \times e$ ⦂

$\qquad$ Erg ⦂ tx Cau ⦂ Acc y . t ⋅ $= \cdot x$ Acc y . t ⋅ $\wedge$ ⋅

$\qquad ty$ Cau ⦂ Acc x . t ⋅ $= \cdot y$ Acc x . t ⦂

$\qquad\qquad \wedge \vcentcolon \vee \gamma \cdot$ Put γ ⋅

$\qquad\qquad x$ Acc y . t ⋅ $= \gamma$ Mas $x/($Dst $xy)^{10}$. $\times e$ ⦂ $\wedge$ ⦂

$\qquad\qquad y$ Acc x . t ⋅ $= -\gamma$ Mas $y/($Dst $xy)^{10}$. $\times e$ ⦂ $\wedge$ ⦂

$\qquad\qquad \gamma = \cdot$ Cmt^3/Sec^2 Gra . 100011110,01 $\times 10^{-100000}$ ⦂ $\wedge$

$\qquad\qquad$ ⦂ Err . $\gamma =$ Etc ⋅ $=$ Cca . Cmt^3/Sec^2 Gra $10^{-100010}$ ⏐

Ha Inq Hb Cur ⏐

Hb Inq Ha ⦂ $\curlyvee p$. PAN Hb Inq Ha ⋅ $\in$ Lex Nat ⏐

Ha Inq Hb ⋅ Nnc Ha Sci . ? $=$ Gra ✳

The law of free fall and the universal attraction law. Cad (*fL* cadit $=$ falls) means "falls". As stated by Ha, the receiver can now express our mass unit by his own unit, if he compares our value of the universal gravitation constant γ with his. Unfortunately the precision of the value of γ we can communicate is rather poor. His knowledge about our mass-unit will be as poor, but we can increase the precision by indicating the mass of the hydrogen atom. This is still far from satisfactory. For the present we must content ourselves with this result. If we want to give the receiver the opportunity to acquire more precise knowledge about our mass-unit, we ought to introduce chemical substances and a few physical notions such as pressure and temperature.

4 22 5. ✳ $x \in$ Ato H . $\rightarrow$ ⋅ Mas x . $=$ Cca . Gra 1,000000111010001 $\times 10^{-1001111}$

$\qquad$ ⋅ $\wedge$ ⦂ Err . Mas $x =$ Etc ⋅ $=$ Cca . Gra $10^{-1011110}$ ✳

4 22 6. Continuation of the text of 4 22 4.

$*\,Ha\,\mathrm{Inq}\,Hb\stackrel{.}{.}\mathrm{Pot}\stackrel{.}{.}\mathrm{Car}\stackrel{.}{.}\widehat{\ }x\cdot tx\,\mathrm{Ext}\,.\wedge.x\in\mathrm{Res}\stackrel{.}{\ }>10\,|$

$Hb\,\mathrm{Inq}\,Ha\,\imath\,\mathrm{Acc}\,x\,.\,t\stackrel{.}{\ }=\,\stackrel{.}{.}\Sigma\cdot z\in\mathrm{Res}\,.z\neq x\,.\wedge.\,tz\,\mathrm{Ext}\cdot z\,\mathrm{Acc}\,x\,.\,t\stackrel{.}{.}$

$\wedge z:z\neq x\,.\rightarrow\cdot\mathrm{Dst}\,xz\,.<\mathrm{Vld}\stackrel{.}{\ }\rightarrow\stackrel{.}{.}\mathrm{Acc}\,x\,.\,t\stackrel{.}{\ }=\mathrm{Cca}\,.\mathrm{Cmt/Sec^2}\,0_{\mathrm{Vco}}\stackrel{.}{.}\wedge\stackrel{.}{.}$

$\wedge z:z\neq x\,.\wedge.z\neq y\stackrel{.}{\ }\rightarrow\cdot\mathrm{Dst}\,xz\,.<\mathrm{Vld}\stackrel{.}{\ }\rightarrow\stackrel{.}{.}$

$\mathrm{Acc}\,x\,.\,t\stackrel{.}{\ }=\mathrm{Cca}\cdot y\,\mathrm{Acc}\,x\,.\,t\,*$

4 23 1. $*\,p>0\,.\wedge.e\geqq0\,.\wedge.p\,\widehat{\frown}\,e\in\mathrm{Rea}\cdot\rightarrow\stackrel{.}{.}$

$\mathrm{Ssc}_*\,pe\,.=\,\stackrel{.}{.}\widehat{\ }x\stackrel{.}{.}x\in\mathrm{Rea}^3\,.\wedge\stackrel{.}{\ }\wedge\ulcorner\xi_1\,.\,\xi_2\,.\,\xi_3\urcorner:x=\ulcorner\xi_1\,.\,\xi_2\,.\,\xi_3\urcorner\,.$

$\rightarrow\cdot\xi_3=0\,.\wedge.\sqrt{\xi_1{}^{10}+\xi_2{}^{10}}=p+e\,\xi_1\,\imath$

$p>0\,.\wedge.e\geqq0\rightarrow\stackrel{.}{.}z\in.\mathrm{Ssc}\,pe\cdot\wedge\cdot y\in.\mathrm{Foc}\,z\stackrel{.}{.}$

$\leftrightarrow\stackrel{.}{.}z\subset\mathrm{Spa}\,.\wedge\stackrel{.}{\ }\vee f\stackrel{.}{.}f\in\mathrm{Coo}\,.\wedge\cdot\widehat{\ }f\stackrel{.}{.}z\,.=\,.\mathrm{Ssc}_*\,pe\cdot\wedge\cdot fy\,.=\ulcorner0\,.0\,.0\urcorner\stackrel{.}{.}$

$z\in\mathrm{Ssc}\,.\leftrightarrow\stackrel{.}{.}\vee\ulcorner p\,.\,e\urcorner\cdot z\in.\mathrm{Ssc}\,pe\,*$

The word written Ssc $p\,e$ (fL sectio semiconi) means "ellipses or parabolae or semi-hyperbolae with parameter p and eccentricity e". (The eccentricity is 1 in the case of the parabola; if Ssc is an ellipse with the axes a, b, and $a>b$, then $e=\sqrt{a^2-b^2}/a<1$, and $p=b^2/a$; in the case of the hyperbola with "real" axis a and "imaginary" axis b, we have $e=\sqrt{a^2+b^2}/a>1$, and $p=b^2/a$.)

The word written Foc means "focus".

4 23 2. $*\,Ha\,\mathrm{Inq}\,Hb\,|$

$\mathrm{Pon}\stackrel{.}{.}A=\stackrel{.}{.}\mathrm{Mas}\,x\,.\mathrm{Mul}>\,.\mathrm{Mas}\,y\cdot\wedge\cdot\mathrm{Cel}\,x=\mathrm{Cmt/Sec}\,0_{\mathrm{Vco}}\,.$

$\wedge\stackrel{.}{.}\rightharpoondown\vee t\,.\,t\,\mathrm{Dst}\,xy=\mathrm{Cmt}\,0\stackrel{.}{.}\wedge\cdot\mathrm{Acc}\,y\,.=\,.x\,\mathrm{Acc}\,y\cdot$

$\wedge\stackrel{.}{\ }\rightharpoondown\vee u\stackrel{.}{.}u\in\mathrm{Rct}\,.\wedge\cdot\widehat{\ }\mathrm{Via}\,y\,.\,\mathcal{C}\,u\stackrel{.}{\ }\stackrel{.}{\ }$

$\mathrm{Erg}\,\imath\stackrel{.}{.}A\rightarrow\stackrel{.}{.}\widehat{\ }\mathrm{Via}\,y\,.\in\mathrm{Ssc}\cdot\wedge\cdot\mathrm{Ctr}\,x\,.\in\,.\mathrm{Foc}\,\widehat{\ }\mathrm{Via}\,y\,\imath\cdot$

$A\wedge\stackrel{.}{\ }\vee\ulcorner p\,.\,e\urcorner:e<1\,.\wedge\cdot\widehat{\ }\mathrm{Via}\,y\,.\in.\mathrm{Ssc}\,pe\stackrel{.}{.}$

$\rightarrow\imath.\,\vee T\,\imath\,\mathrm{Put}\,\mathrm{Rev}\,y\,\imath\,T=\stackrel{.}{.}\inf\stackrel{.}{.}\widehat{\ }\tau\stackrel{.}{.}\tau>\mathrm{Sec}\,0\,.$

$\wedge\stackrel{.}{\ }\wedge t\stackrel{.}{.}\mathrm{Via}\,y\,.\,t+\tau\stackrel{.}{\ }=\,\cdot\mathrm{Via}\,y\,.\,t\,*$

The word written Rev (fL revolutio) means "period of revolution".

4 23 3. $*\,tx\,\mathrm{Rot}\,a\,d\,.\leftrightarrow\imath\,a\,.=\mathrm{Cca}\,.\mathrm{Ctr}\,x\,.\wedge.d\in\mathrm{Sec^{-1}}\,\mathrm{Vco}\,.$

$\wedge\stackrel{.}{.}\vee f\stackrel{.}{.}f\in\mathrm{Coo^+}\,.\wedge.fa=0_3\,.\wedge.fd=\ulcorner0\,.0\,.\mathrm{Sec}\,|\,d\,|\urcorner\,.$

$\wedge\stackrel{.}{.}\wedge\ulcorner y\,.\,\zeta_1\,.\,\zeta_2\,.\,\zeta_3\urcorner\stackrel{.}{\ }y\,\mathrm{Par}\,x\,.\wedge\cdot f\,.\,t\,\mathrm{Loc}\,\mathrm{Ctr}\,y=\ulcorner\zeta_1\,.\,\zeta_2\,.\,\zeta_3\urcorner\stackrel{.}{.}$

$\rightarrow\cdot f\cdot\mathrm{Cel}\,y\,.\,t\stackrel{.}{.}=\ulcorner-\mathrm{Sec}\,|\,d\,|\,\zeta_2\,.\mathrm{Sec}\,|\,d\,|\,\zeta_1\,.\,0\urcorner\,*$

The word written Rot (fL rotat = rotates) means "rotates". d is the vector of angular velocity.

4 23 4. $*\,t_1\,\mathrm{Usq}\,t_2\,x\,\mathrm{Rot}\,a\,d\,.\leftrightarrow\stackrel{.}{.}\wedge t\stackrel{.}{.}t_1\leqq t\leqq t_2\,.\rightarrow.\,tx\,\mathrm{Rot}\,a\,d\,*$

4 23 5. $*\,t_1\,\mathrm{Usq}\,t_2\,x\,\mathrm{Rot}\,a\,d\,.\wedge.f\in\mathrm{Coo^+}\,.\wedge.fa=0_3\,.$

$\wedge.fd=\ulcorner0\,.0\,.\mathrm{Sec}\,|\,d\,|\urcorner\,.\wedge.y\,\mathrm{Par}\,x\,.\wedge\stackrel{.}{\ }f\cdot\mathrm{Via}\,y\,.\,t_1:=\ulcorner\zeta_1\,.\,\zeta_2\,.\,\zeta_3\urcorner\stackrel{.}{.}$

$\rightarrow\cdot f\cdot\mathrm{Via}\,y\,.\,t_2:=\ulcorner\zeta_1\cos\,|\,d\,|\,(t_2-t_1)-\zeta_2\sin\,|\,d\,|\,(t_2-t_1)\,.$

$\zeta_1\sin\,|\,d\,|\,(t_2-t_1)+\zeta_2\cos\,|\,d\,|\,(t_2-t_1)\,.\,\zeta_3\urcorner\,*$

4 23 6. $^{\#} t_1 \operatorname{Usq} t_2 . x \operatorname{Rot} a\, d . \wedge . t_2 - t_1 = 10\,\pi/|d| \cdot \wedge . y \operatorname{Par} x :$
$\qquad \to {}^{\cdot} \operatorname{Via} y . t_2 {}^{\cdot} = {}^{\cdot} \operatorname{Via} y . t_1 : \wedge . \operatorname{Rtt} x = t_2 - t_1 {}^{\#}$

The word written Rtt means "period of (uniform) rotation".

4 23 7. $^{\#} d/|d| . = {}^{\cdot} \operatorname{Axs} . t\, x \operatorname{Rot} a\, d : \wedge : a = {}^{\cdot} \operatorname{Ctr} . t\, x \operatorname{Rot} a\, d {}^{\#}$

'Axs' and 'Ctr' mean axis and centre of the rotation. [1]

4 24 1. $^{\#} \odot \in \operatorname{Ste} . \ \text{☿} \in \operatorname{Ste} . \ \text{♀} \in \operatorname{Ste} . \ \text{♁} \in \operatorname{Ste} . \ \text{♂} \in \operatorname{Ste} . \operatorname{Aoi} \subset \operatorname{Ste} . \ \text{♃} \in \operatorname{Ste} .$
$\qquad \text{♄} \in \operatorname{Ste} . \ \text{⛢} \in \operatorname{Ste} . \ \text{♆} \in \operatorname{Ste} . \ \text{♇} \in \operatorname{Ste} : \text{☾} \in \operatorname{Ste} :$
$\qquad \alpha\, \text{Centauri} \in \operatorname{Ste} . \operatorname{Munich} 15040 \in \operatorname{Ste} . \operatorname{Wolf} 359 \in \operatorname{Ste} .$
$\qquad \operatorname{Luyten} 726\text{--}8 \in \operatorname{Ste} . \operatorname{Lalande} 21185 \in \operatorname{Ste} . \operatorname{Sirius} \in \operatorname{Ste} .$
$\qquad \operatorname{Procyon} \in \operatorname{Ste} . \operatorname{Altair} \in \operatorname{Ste} :$
$\qquad \alpha\, \text{Centauri} \cdot \operatorname{Uni} \cdot \alpha\, \text{Centauri}\, A . \alpha\, \text{Centauri}\, B . \alpha\, \text{Centauri}\, C :$
$\qquad \operatorname{Munich} 15040 \cdot \operatorname{Uni} \cdot \operatorname{Munich} 15040\, A . \operatorname{Munich} 15040\, B :$
$\qquad \operatorname{Luyten} 726\text{--}8 \cdot \operatorname{Uni} \cdot \operatorname{Luyten} 726\text{--}8\, A . \operatorname{Luyten} 726\text{--}8\, B :$
$\qquad \operatorname{Lalande} 21185 \cdot \operatorname{Uni} \cdot \operatorname{Lalande} 21185\, A . \operatorname{Lalande} 21185\, B :$
$\qquad \operatorname{Sirius} \cdot \operatorname{Uni} \cdot \operatorname{Sirius}\, A . \operatorname{Sirius}\, P :$
$\qquad \operatorname{Procyon} \cdot \operatorname{Uni} \cdot \operatorname{Procyon}\, A . \operatorname{Procyon}\, B :$
$\qquad \operatorname{Car} \operatorname{Aoi} > 10^{1010} {}^{\#}$

The word written Ste (fL stella = star) means "celestial body". A few near celestial bodies have been mentioned. The usual astronomical names have been used. Aoi means the asteroids. Components of double and triple star systems are distinguished by the letters A, B, C.

4 24 2. $^{\#} x \in \operatorname{Ste} . \to : \operatorname{Loc} x \subset \operatorname{Spa} . \wedge \cdot \operatorname{Loc} x . \operatorname{Cca} \in . \operatorname{Bul} \cdot$
$\qquad \wedge : \operatorname{Loc} x . = \operatorname{Cca} . \operatorname{Bul} a \varrho \cdot \to {}^{\cdot} a . = \operatorname{Cca} . \operatorname{Ctr} x \cdot$
$\qquad \wedge \cdot \varrho . = \operatorname{Cca} . \operatorname{Rad} x \cdot \wedge \cdot \operatorname{Sfi} x . = \operatorname{Cca} . \operatorname{Sph} a\, \varrho {}^{\#}$

4 24 3. $^{\#} \operatorname{Mas} \odot . = \operatorname{Cca} . \operatorname{Gra} 11 \times 10^{1101101} :$
$\qquad \operatorname{Mas} \text{♁} . = \operatorname{Cca} . \operatorname{Gra} 101 \times 10^{1011010} :$
$\qquad \operatorname{Mas} \text{☾} . = \operatorname{Cca} . \operatorname{Gra} 10^{1010110} :$
$\qquad x \in \operatorname{Aoi} . \to \cdot \operatorname{Mas} x . < . \operatorname{Gra} 10^{1010000} :$
$\qquad \operatorname{Mas} \operatorname{Sirius} A . = \operatorname{Cca} . 10,1 \times \operatorname{Mas} \odot :$
$\qquad \operatorname{Mas} \operatorname{Procyon} A . = \operatorname{Cca} . 1,1 \times \operatorname{Mas} \odot :$
$\qquad \operatorname{Mas} \alpha\, \operatorname{Centauri} A . = \operatorname{Cca} . \operatorname{Mas} \odot :$
$\qquad \operatorname{Mas} \alpha\, \operatorname{Centauri} B . = \operatorname{Cca} . 0,11 \times \operatorname{Mas} \odot :$
$\qquad \operatorname{Mas} \alpha\, \operatorname{Centauri} C . \operatorname{Mul} < . \operatorname{Mas} \odot :$
$\qquad \operatorname{Rad} \odot . = \operatorname{Cca} . \operatorname{Cmt} 10^{100100} :$
$\qquad \operatorname{Rad} \text{♁} . = \operatorname{Cca} . \operatorname{Cmt} 10011 \times 10^{11001} :$
$\qquad \operatorname{Rad} \text{☾} . = \operatorname{Cca} . \operatorname{Cmt} 101,0011 \times 10^{11001} :$
$\qquad \operatorname{Rtt} \odot . = \operatorname{Cca} . \operatorname{Sec} 10^{10101} :$
$\qquad \operatorname{Rtt} \text{♁} . = \operatorname{Cca} . \operatorname{Sec} 1010100001001100,00011 :$

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[1] The use of the same word Ctr as in **4 20 4** is not correct.

Dst ⊙♂.=Cca.Cmt 1101,10011 × 10^{101000} :

Dst ♂☾.=Cca.Cmt 1000,11110011 × 10^{100000} :

Dst ⊙x.=Cca.Cmt 10^{111000} × a·→:

  x = α Centauri.→.a = 111010·∧·

  x = Munich 15040.→.a = 1001110·∧·

  x = Wolf 359.→.a = 1100100·∧·

  x = Luyten 726–8.→.a = 1100111·∧·

  x = Lalande 21185.→.a = 1101011·∧·

  x = Sirius.→.a = 1110001·∧·

  x = Procyon.→.a = 10010011·∧·

  x = Altair.→.a = 11010111 #

Numerical data on the celestial bodies mentioned. The order of magnitude of masses, radii, and distances will be an indication for any receiver that 'Ste' means "celestial body".

We have restricted ourselves to a few instances. More numerical data of this kind should be given, especially on the planets. A receiver within our solar system might then possibly identify some bodies. A receiver outside our solar system will guess that the smaller bodies are members of our solar system.

4 24 4. # Pon:Cel ⊙.t=Cmt/Sec 0$_{vco}$ ı·

  Erg ı·x=☿ ⌣ ♀ ⌣ Etc ⌣ ♆ ⌣ ♇.→·

    ^Via x.Cca∈.Ssc·∧:1^{ε}Foc·^Via x.=Cca.Ctr ⊙∶∧ı.

    A = ∶^ˇx∶^ˇa·∨⌐p.e⌐∶^Via x.Cca∈.Ssc pe·∧.p=a(1−e^2)⦙

    ☿∪♀∪Etc∪♆∪♇∶∧∶

    E = ∶^ˇx·^ˇe∶∨p·^Via x.Cca∈.Ssc pe⦙Etc∶∧∶

    ∧x·x Etc.→∶$A'x$=.Ax/A♂·∧·$R'x$=.Rev x/Rev ♂ı→∶

    A♂.=1101,10011 × 10^{101000}·∧·

    Rev♂.=Sec 111100001101100010001 0101,10000001 Etc·∧·

    A'☿.=0,0110001100011 Etc·∧·

    E☿.=0,0011011100110 Etc·∧·

    R'☿.=0,00111101100001 Etc·∧·

    A'♀.=0,1011100100101 Etc·∧·

    E♀.=0,0000000111000 Etc·∧·

    R'♀.=0,10011101011111 Etc·∧·

    E♂.=0,00000100010001 Etc #

and so on.

The Kepler motion of planets, the astronomical unit (A ♂), the sideral year (Rev ♂), mean distances from sun ($A'x$), mean periods ($R'x$), and eccentricities (Ex). Other elements should be added, yet the data of 4 24 3–4 24 4 will suffice to identify the various bodies within our solar system.

4 24 5.   ⚬ Pon : Cel ♂ . t = Cmt/Sec 0$_\text{Vco}$ ·

     Erg · ˄ Via ☾ . Cca ∈ Ssc · ˄ : 1ᶜ Foc · ˄ Via ☾ . = Cca . Ctr ♂ :

       ˄ · Rev ☾ . = Sec 100100,0000010100001111011011001 × 10^{10000} ⚬

The motion of the moon. In the same way satellites of other planets could be dealt with.

4 24 6.   ⚬ f ∈ t$_0$ Coo Aqo . ↔ ι

     f ∈ Coo⁺ . ˄ · f · Via ♀ . t$_0$: = ⌈0 . 0 . 0⌉ · ˄ ⁚

     ˄x ⁚ x = α Centauri ⌣ Munnich 15040 ⌣ Etc ⌣ Altair ⌣ Etc .

       → ⁚ | f · Cel x . t$_0$ | ⁚ / ⁚ | f · Via x . t$_0$ | · < Vld ⁚ ˄ ⁚

       ∨ ⌈α . β . γ . α′⌉ ⁚ α ⌢ β ⌢ γ ∈ Pos . ˄ ·

         f · Via ☉ . t$_0$: = ⌈α . 0 . 0⌉ · ˄ ·

         f · Cel ☉ . t$_0$: = ⌈α′ . β . γ⌉ · ˄ ·

         f · Axs . t$_0$ ♀ Rot Etc : = ⌈0 . 0 . 1⌉ ⚬

't$_0$ Coo Aqo' means the equatorial coordinate system at (the moment) t$_0$. Here, and in 4 24 7–4 24 9 t$_0$ is a substitute for something that means the moment of the vernal equinox of the year 1950.

The equatorial coordinate system has been defined by the following properties:

     The fixed stars are approximately at rest in it.

     The centre of the Earth is the origin.

     The first axis is directed towards the first point of Aries.[1])

     At t$_0$ the Sun moves in the direction of increasing second and third coordinates.

     The third axis is the directed rotation axis of the Earth.

The conditions are redundant. They yield the supplementary information that the annual (apparent) revolution of the Sun and the daily revolution of the Earth take place in the same sense.

By the condition f ∈ Coo⁺, which seems to be redundant, we have fixed the meaning of Coo⁺[2]).

Of course it does not matter which coordinate system we use in our communications.

     ⚬ Car . t$_0$ Coo Aqo · = 1 ⚬

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[1]) The true equinox of March 21, which is slightly different from the mean equinox of January 1, as used in catalogues.

[2]) Coo⁺ cannot be defined by means of the right hand rule of electromagnetics without referring to geographical data (the terrestrial north pole). In our program text we offered cosmographical data. Any oriented pair of stars with their velocity vectors are sufficient to settle space orientation. Nowadays thanks to the non-invariance of parity we could define Coo⁺ without any reference to geographical or cosmographical data.

4 24 7.　$^*f \in t_0 \operatorname{Coo} \operatorname{Aqo} . \wedge . fx = \ulcorner \xi_1 . \xi_2 . \xi_3 \urcorner . \wedge . \xi_1{}^2 + \xi_2{}^2 = \varrho^2 . \wedge . \varrho > 0 \cdot \rightarrow \cdot$
　　　　$\xi_1 = \varrho \cos \operatorname{Ras} x . \wedge . \xi_2 = \varrho \sin \operatorname{Ras} x . \wedge . \xi_3 = \varrho \operatorname{tg} \operatorname{Del} x . \wedge .$
　　　　$0 \leqq \operatorname{Ras} x < 2\pi . \wedge . -\tfrac{1}{2}\pi < \operatorname{Del} x < \tfrac{1}{2}\pi$　*

The words written Ras (*fL* rectascensio) and Del (*fL* declinatio)
mean "right ascension" and "declination" respectively.

4 24 8.　$^* x \in \operatorname{Spa} . \wedge . f \in t_0 \operatorname{Coo} \operatorname{Aqo} \cdot \wedge . (\omega_1 fx)^2 + (\omega_2 fx)^2 \neq 0 : \rightarrow :$
　　　　$x_{dcl} = \ulcorner -\sin \operatorname{Ras} x . \cos \operatorname{Ras} x . 0 \urcorner . \wedge .$
　　　　$x_{ras} = \ulcorner -\cos \operatorname{Ras} x \sin \operatorname{Del} x . - \sin \operatorname{Ras} x \sin \operatorname{Del} x . \cos \operatorname{Del} x \urcorner . \wedge .$
　　　　$x_{rad} = (fx) / |fx|$　*

x_{dcl} and x_{ras} mean the tangential directions at x to the declina-
tion parallel and the hour circle respectively, taken in the sense
of increasing right ascension and declination respectively (the
directions called eastwards and northwards in the jargon of the
astronomer who tacitly assumes himself to observe the southern
sky).
The meaning of x_{rad} is evident.

* $x_{dcl} \perp x_{ras} . \wedge . x_{dcl} \perp x_{rad} . \wedge . x_{ras} \perp x_{ras} .$
　$|x_{dcl}| = |x_{ras}| = |x_{rad}| = 1$　*

4 24 9.　$^* \operatorname{Pon} \vdots \operatorname{Cel} \tfrac{+}{\circ} . t_0 \cdot = . \operatorname{Cmt/Sec} 0_{\mathrm{Vco}} : \wedge . s \in \operatorname{Ste} . \wedge :$
　　　　$\operatorname{Via} s . t_0 \cdot = x : \wedge . \operatorname{Del} x = \tfrac{1}{2}\pi \delta . \wedge . \operatorname{Ras} x = 2\pi \lambda . \wedge \cdot \alpha > 0 \cdot \wedge \cdot$
　　　　$f \cdot \operatorname{Cel} s . t_0 := 10^{10000} \times [\alpha (\cos 2\pi \mu\, x_{ras} + \sin 2\pi \mu\, x_{dcl}) + \beta\, x_{rad}] \vdots$
　　　　$\operatorname{Erg} \cdot$

$s = \alpha \operatorname{Centauri} . \rightarrow \cdot \lambda = 0,1001101111000 . \wedge .$
　　　　　　　　　　$\delta = -0,1010110001101 . \wedge .$
　　　　　　　　　　$\alpha = 100011 . \wedge .$
　　　　　　　　　　$\mu = 0,110010000 . \wedge .$
　　　　　　　　　　$\beta = -100110 : \wedge :$

$s = \operatorname{Munich} 15040 . \rightarrow \cdot \lambda = 0,1011111100011 . \wedge .$
　　　　　　　　　　$\delta = 0,0000110011110 . \wedge .$
　　　　　　　　　　$\alpha = 10001001 . \wedge .$
　　　　　　　　　　$\mu = 0,111111010 . \wedge .$
　　　　　　　　　　$\beta = -10100101 : \wedge :$

$s = \operatorname{Wolf} 359 . \rightarrow \cdot \lambda = 0,0111010001010 . \wedge .$
　　　　　　　　　　$\delta = 0,0001010011111 . \wedge .$
　　　　　　　　　　$\alpha = 1010010 . \wedge .$
　　　　　　　　　　$\mu = 0,101001110 . \wedge .$
　　　　　　　　　　$\beta = 10100 : \wedge :$

$s = \text{Luyten } 726\text{–}8 . \to \cdot \lambda = 0,0001000100101 . \wedge .$
$\qquad \delta = -0,0011001111010 . \wedge .$
$\qquad \alpha = 101010 . \wedge .$
$\qquad \mu = 0,001000010 . \wedge .$
$\qquad \beta = 100100 : \wedge :$

$s = \text{Lalande } 21185 . \to \cdot \lambda = 0,0111010101110 . \wedge .$
$\qquad \delta = 0,0110011100000 . \wedge .$
$\qquad \alpha = 1011001 . \wedge .$
$\qquad \mu = 0,100001010 . \wedge .$
$\qquad \beta = 10000011 . \wedge :$

$s = \text{Sirius} . \to \cdot \lambda = 0,0100011110100 . \wedge .$
$\qquad \delta = -0,0010111111011 . \wedge .$
$\qquad \alpha = 11000 . \wedge .$
$\qquad \mu = 0,100100010 . \wedge .$
$\qquad \beta = -1100 . \wedge :$

$s = \text{Procyon} . \to \cdot \lambda = 0,0101000100110 . \wedge .$
$\qquad \delta = 0.0001001000010 . \wedge .$
$\qquad \alpha = 11111 . \wedge .$
$\qquad \mu = 0,100110000 . \wedge .$
$\qquad \beta = -101 . \wedge :$

$s = \text{Altair} . \to \cdot \lambda = 0,1101001101000 . \wedge .$
$\qquad \delta = 0,0001100001000 . \wedge .$
$\qquad \alpha = 11000 . \wedge .$
$\qquad \mu = 0,001001101 . \wedge .$
$\qquad \beta = -101000 \; {}^{*}$

$2 \pi \lambda$ and $\tfrac{1}{2} \pi \delta$ are the right ascension and the declination of the star, α and β are the numerical values (in 2^{16} Cmt/Sec) of its cross and radial velocities, μ is the so-called position angle of its proper motion. λ and δ together with the distances (in 4 24 3) determine the position, and the last three data determine the velocity in space. On the strength of this information the receiver will be able to identify the objects mentioned and the position of the sender. The information is highly redundant.

4 25 1. * *Ha* Inq *Hb* ⋅ Utr ⋅ Cel $\odot$. $=$. Cmt/Sec 0_{Vco} :
$\qquad$ Utr Cel $\underset{\circ}{\top}$. $=$. Cmt/Sec 0_{Vco} ı
$\qquad$ *Hb* Inq *Ha* ⋮ Acc x . $=$ Cca . Cmt/Sec^2 0_{Vco} ⋅ $\wedge$. $y \in$ Hom . $\wedge$. $t \in$ Tem :
$\qquad \to$ ⋮ Nnc *Hb* Ccd ⋅ Cca t . y Inq *Hb* : Pon ⋅ Cel x . $=$. Cmt/Sec 0_{Vco} ı
$\qquad$ *Ha* Inq *Hb* ⋮ Nnc *Hc* Inq *Hb* ⋅ Cel $x = a$. $\wedge$. Cel $y = b$:
$\qquad \wedge$ ⋮ Nnc *Hd* Inq *Hb* ⋅ Cel x . $=$. Cmt/Sec 0_{Vco} ⋅
$\qquad \to$ ⋮ Ncs ⋅ Nnc *Hd* Inq *Hb* . Cel $y = b - a$ ı

Hb Inq $Ha \cdot |a| \frown |l| \frown |b-a|$. Mul $< . c \cdot$
→ : $^\vee p$. PAN Ha Inq Hb $p \cdot$ Cca ∈ . Ver ⌐
Ha Inq $Hb : x \in$ Res . ∧ . $y \in$ Und Ath . ∧ : Nnc Hd Vul
$\cdot$ Nnc Hc Inq Hb : ∨ ⌐$e . \alpha$⌐ $\cdot e \in$ Vco . ∧ . Cel $x = \alpha e$. ∧ . Cel $y = c e \cdot$
∧ : Nnc Hd Inq $Hb \cdot$ Cel $x . = .$ Cmt/Sec 0_{vco} :
→ : Ncs $\cdot$ Nnc Hd Inq Hb . Cel Med $y = c - \alpha$ ⌐
Hb Inq Ha : Fal : ¬ ∨ ⌐$t . x$⌐ : t Hb Ccd $\cdot$ Nnc x Inq Hb :
Pon $\cdot y \in$ Und Ath . ∧ . Cel Med $y \neq c$ ⌐
Ha Inq Hb Cur .
Hb Inq $Ha \cdot$ Nnc Hb Vul : † Pst . Nnc $\cdot Ha$ Inq ! Hb Cur ✳

Hb will answer this question in the talks 4 28 2–4 28 6.

4 26 1. ✳ For $x =$ For $y . \leftrightarrow \cdot$ ∨ ⌐$f . g$⌐ : $f \frown g \in$ Coo⁺ .
∧ $\cdot$ ^f ⁞ Loc $x . = .$ ^g ⁞ Loc y :
Loc $x . \in .$ Lap $a b c \cdot$ ∧ $\cdot$ Loc $x' . \in .$ Lap $a' b' c' . ∧ . a \leq b \leq c . ∧ . a' \leq b' \leq c'$:
→ : For $x =$ For $y . \leftrightarrow \cdot a = a' . ∧ . b = b' . ∧ . c = c'$ ✳

The word written For (fL forma) means "shape".

4 26 2. ✳ t_1 Usq t_2 X Mut For $x . \leftrightarrow \cdot t_1$ Usq t_2 X Cau . t_1 For $x \neq t_2$ For x ✳

4 26 3. $x \in$ Rig . $\leftrightarrow$: ∧ ⌐$t . X$⌐ : $X \in$ Hom . ∧ . $t x$ Ext $\cdot$
→ : ∨$y \cdot y =$ Cor x : ∧ $\cdot$ Dif $\cdot$ Cca t . X Mut For $x :>$ Vld ⌐
$x \in$ Lqd . $\leftrightarrow$ ⌐ ∧ ⌐$t . X$⌐ : $X \in$ Hom . ∧ . $t x$ Ext $\cdot$
→ : ∨$y \cdot y =$ Cor x : ∧ : Dif $\cdot$ Cca t . X Mut For $x :<$ Vld $\cdot$
∧ $\cdot$ Dif $\cdot$ Cca t . X Mut Vol $x :>$ Vld ⌐
$x \in$ Gas . $\leftrightarrow$ ⌐ ∧ ⌐$t . X$⌐ : $X \in$ Hom . ∧ . $t x$ Ext $\cdot$
→ : ∨$y \cdot y =$ Cor x : ∧ $\cdot$ Dif $\cdot$ Cca t . X Mut Vol $x :<$ Vld ⌐
Exg . Aer $\subset$ Gas ✳

The words written Rig (fL rigidus = rigid), Lqd (fL liquidus = liquid), and Gas mean "rigid", "liquid" and "gaseous" respectively.

4 26 4. ✳ $x \in$. Rig ∪ Lqd $\cdot$ → $\cdot$ Dns x . Mul $> .$ Gra/Cmt³ 10^{-1000} :
$x \in$ Gas . → $\cdot$ Dns x . Mul $< .$ Gra/Cmt³ 10^{-1000} ✳

4 27 1. t_1———t_2
Ha Inq $Hb \cdot ? = :$ Dur $\cdot ^\vee x$. PAN Fit x ⌐
Hb Inq Ha . Sec 100 ⌐
Ha Inq Hb : ? $u :$ PAN Hb Cog $\cdot ? = :$ Dur $\cdot ^\vee x$. Nnc Fit $x \cdot$ Mod u ⌐
Hb Inq Ha : $t_1 t_2$ Hb Met $\cdot ? = :$ Dur $\cdot ^\vee x$. Nnc Fit x ⌐
Ha Inq Hb . ? = Met ⌐
Hb Inq Ha ⌐ : PAN Hb Ani : ? ⌐$z_1 z_2$⌐ $\cdot$ Fre $z_1 = \mathfrak{w}$. ∧ . Fre $z_2 = \mathfrak{w}$.
∧ : Ini $z_1 . = \cdot$ Ini $. ^\vee x$ Etc . ∧ : Ini $z_2 . = \cdot$ Fin $. ^\vee x$ Etc :
∧ : PAN Hb Cpu $\cdot ? = .$ Ini $z_2 -$ Ini z_1 ⌐ .

w Uni $\cdot$ $^{\uparrow}z$. Fre $z = \omega : \to \cdot w \in$. Sca Sec 1 ｜.

Ecc $\cdots\cdots\cdots$ ｜. $^{\vee}y$. PAN Fit $y \cdot \in$. Sca Sec 10^{-100} ｜.

$\vee A : w$ Uni A . $\wedge \cdot$ Car A . Mul $> . 1 \cdot$

$\wedge^{\centerdot} \wedge i : i \in$ Num . $\wedge . i <$ Car $A \cdot \wedge \cdot x \, (i-1)^{\text{um}}$ Par w . $\wedge$. $y \, i^{\text{um}}$ Par：

$\to :$ Ini $x \cdot$ Sec 10^{-100} . Ant $\cdot$ Ini y ｜

$\to . w \in$ Sca Sec 10^{-100} ｜.

$K \in$ Hor . $\wedge . t \, K$ Ext $\cdot$

$\leftrightarrow^{\centerdot} \vee \ulcorner w . \sigma \urcorner : w \in$. Sca Sec $\sigma \cdot \wedge :$ Pot $\cdot$ Ccа t . K Cau w ｜.

PAN Hb Met $^{\centerdot}? = :$ Dur $\cdot ^{\vee}x$. Nnc Fit $x \cdot$ Per . 1^{ϵ} Hor $^{\#}$

The word written Met (fL metitur = measures) means the verb "measures". The word written Sca (fL scala = scale) means "scales"; after 'Sca' the unit of the scales has been mentioned. The word written Hor (fL horologium = clock) means instruments that can produce time-scales. 'Met . . . Per. 1^{ϵ} Hor' means "measuring by means of a clock".

The time-signal in the beginning lasts 4 seconds. The sequence of time signals after 'Ecc' consists of peeps with intervals of a sixteenth of a second.

The ｍ refers to the program clock (see 2 02 4).

4 27 2.　$^{\#}Ha$ Inq $Hb : ? = :$ Dst $^{\centerdot\,\vee}a ^{\vee}b : a \frown b \in Ra$. $\wedge$. Dst $ab =$ Lon Ra ｜

Hb Inq $Ha :$ Dst $\cdot ^{\vee}a ^{\vee}b$ Etc $\cdot =$ Cmt 11001001 ｜

Ha Inq $Hb : ? u \cdot$ PAN Hb Cog . $? =$ Dst Etc . Mod u ｜

Hb Inq $Ha \cdot$ PAN Hb Met $: ? =$ Dst Etc : Per . 1^{ϵ} Rgu ｜

Ha Inq $Hb . ? =$ Rgu ｜

Hb Inq Ha ｜$: w \in$ Rig . $\wedge : \vee y \cdot$ Put $y \cdot y$ Par w . $\wedge$.

$y \in$ Sca Cmt 1 $^{\centerdot} \to . w \in$ Rgu ｜$\cdot$

Exg ｜. Pon $^{\centerdot}A = :^{\uparrow}x \cdot x \in$ Rea^3 .

$\wedge . 0 \leq \omega_1 x \leq 10^{1001}$. $\wedge . 0 \leq \omega_2 x \leq 10$. $\wedge . 0 \leq \omega_3 x \leq 10 :$

Pon $^{\centerdot} \wedge n : n \in$ Num . $\wedge . 0 \leq n \leq 10^{1001}$.

$\to^{\centerdot} B_n = :^{\uparrow}x \cdot x \in$ Rea^3 . $\wedge . n - 10^{-100} \leq \omega_1 x \leq n + 10^{-100}$.

$\wedge . 0 \leq \omega_1 x \leq 10^{1001}$. $\wedge . 0 \leq \omega_2 x \leq 10$. $\wedge . 10 \leq \omega_3 x \leq 10 + 10^{-100} :$

Pon $^{\centerdot}C = :^{\uparrow}x \cdot \vee n . x \in B_n$ ｜

Erg $: \vee f : f \in$ Coo . $\wedge \cdot^{\uparrow}f \vdots$ Loc w . $= . A \cup C \cdot \wedge \cdot^{\uparrow}f \vdots$ Loc y . $= C \cdot$

$\wedge \cdot^{\vee}v \cdot^{\uparrow}f \vdots$ Loc v . $= B_n : n^{\text{um}}$ Par y ｜

$y \in \cdot$ Sca Cmt 1 . w ｜

Ha Inq $Hb : ? u \cdot$ Pot : Nnc Hb Met $\cdot$ Per . 1^{ϵ} Rgu $\cdot$ Mod u ｜

Hb Inq Ha ｜$\cdot t$ ｜$\cdot X$ Ani ｜Utr : Nnc a Apu $^{\vee}v : v . n^{\text{um}}$ Par $^{\centerdot}$

$^{\vee}y : y \in \cdot$ Sca . Cmt 1 . $w :$

$\wedge :$ Nnc b Apu $^{\vee}v : v . n^{\text{um}}$ Par $^{\vee}y$ Etc ｜.

$\to^{\centerdot} X$ Sci : Utr $\cdot$ Dst $a b$. $=$ Cca . Cmt $| m - n |$ ｜：

Nnc X Vul $\cdot$ Nnc X Sci . ? $=$ Dst $a\,b$:

    $\rightarrow$: Nnc X Cau : Nnc : $^\curlyvee v \cdot v$. 0^{um} Par . y Etc : Apu $a\,^\cdot$

     $\wedge$: $\vee m\,^\cdot$ Nnc : $^\curlyvee v \cdot v$. m^{um} Par . y : Apu b |

Ha Inq Hb : Pon : Dst $a\,b\,^\cdot$ Qqm : $>$: Tan $\cdot \neg$. Mul $>\,^\cdot$ Lon w |

Hb Inq Ha $\bullet$ $\vee k$ $\bullet$. $k \in$ Num . $\wedge$ $\bullet$ $\vee$ $^\ulcorner z_1$ Etc $z_k^\urcorner$:

    $z_1 = a$. $\wedge$. $z_k = b$. $\wedge$: $\wedge i$: $i \in$ Num . $\wedge$. $10 \leq i \leq k-1\,^\cdot$

    $\rightarrow$ $^\cdot z \in$ $\cdot$ Cvx . $z_{i-1} \cup z_{i+1}$: $\wedge$: Cca t . X Met $\cdot$? $=$. Dst $z_i\,z_{i+1}$ |

Ha Inq Hb : Pon $\cdot$ Dst $a\,b$. Mul $>$. Lon w |

Hb Inq Ha $\bullet$ $\vee d$: $d \in$ Spa . $\wedge$: Cca t X Met :

    ? $=$ Dst $a\,d$. $\frown$ $^\cdot$? $=$: $\not\lessgtr$ $\cdot b - a$. $d - a\,^\cdot$ $\frown$ $^\cdot$? $=$: $\not\lessgtr$ $\cdot b - d$. $a - d$:

    $\wedge$: Cca t . X Cpu $\cdot$? $=$. Dst $a\,b$ |

Ha Inq Hb : ? $u\,^\cdot$ Pot : t X Met $\cdot$? $=$. $\not\lessgtr$ Etc $\cdot$ Mod u |

Hb Inq Ha $\cdot$ Per . 1$^\epsilon$ Tdo #

The word written Rgu (*fL* regula = ruler), means "ruler, measuring-staff". 'Sca' now appears in connection with a length-unit.

The word written Tdo means "theodolite". We have dropped its description which would be similar to that of the ruler. (Roughly speaking it is a thin rigid cylinder with a hole around its axis and a ring of teeth on its convex surface serving as an angular scale.)

4 27 3.   # Dst . x . Sfi $\updownarrow$ $\cdot$ Mul $<$. Rad $\updownarrow$: $\wedge$: Dst . y . Sfi $\updownarrow$ $\cdot$ Mul $<$. Rad $\updownarrow$ $^\cdot$

     $\rightarrow$: x Ifa y . $\leftrightarrow$ $\cdot$ Loc $x \subset$ Cvx . Loc y . Loc Ctr $\updownarrow$ $\cdot$ $\leftrightarrow$. y Sua x #

The words written Ifa (*fL* infra = below) and Sua (*fL* supra = above) mean "below" and "above" respectively.

4 27 4.   # w Apu . Sfi $\updownarrow$ $\cdot$ $\wedge$. $w \in$ Rig . $\wedge$. z Par w . $\wedge$. Vol $z <$ Vld .

     $\wedge$ $^\cdot$ $\wedge t$: $t\,w$ Ext . $\rightarrow$ $\cdot \neg$ Pot . $t\,z$ Mov $^\cdot$

     $\wedge$ $^\cdot$ $\wedge$ $^\ulcorner t$. $v^\urcorner$: v Par w . $\wedge$. $v \neq z\,^\cdot$ $\rightarrow$ $^\cdot$ Pot . $t\,v$ Mov :

     $\rightarrow$: $\neg$ $\vee v$ $\cdot$ v Par w . $\wedge$. $t\,v$ Mov :

     $\leftrightarrow$ $^\cdot$ t $\cdot$ Ctr w . Ifa z : $\vee$: t $\cdot$ Ctr w . Sua z #

4 27 5.   # Ha Inq Hb $\cdot$? $=$. Mas Ra |

     Hb Inq Ha . Gra 11×10^{111} |

     Ha Inq Hb $^\cdot$? u : PAN Hb Cog $\cdot$? $=$. Mas Ra $\cdot$ Mod u |

     Hb Inq Ha : PAN Hb Met $\cdot$? $=$. Mas Ra $\cdot$ Per . 1$^\epsilon$ Lib |

     Ha Inq Hb . ? $=$ Lib |

     Hb Inq Ha $\bullet$ Pon : $w \in$ Rig . $\wedge$: w $\cdot$ Apu $\cdot$ Sfi $\updownarrow$:

       $\wedge$. z_0 $\frown$ z_1 $\frown$ z_2 $\frown$ z_3 Par w . $\wedge$. Vol z_0 $\frown$ z_1 $\frown$ z_2 $\frown$ $z_3 <$ Vld .

       $\wedge$: t Loc z_0 . $\subset$ $\cdot$ Cvx . t Loc z_1 . t Loc z_2 : $\wedge$:

        t Loc z_0 . $\subset$ $\cdot$ Cvx . t Loc z_3 . t Loc Ctr w :

$$\wedge \vdots \wedge \ulcorner r.s \urcorner \ulcorner r \frown s \in \mathrm{Rct} . \wedge \cdot t \operatorname{Loc} z_0 \frown t \operatorname{Loc} z_1 \frown t \operatorname{Loc} z_2 . \mathrm{Cca} \subset . r \cdot$$

$$\wedge \cdot t \operatorname{Loc} z_0 \frown t \operatorname{Loc} z_3 . \mathrm{Cca} \subset . s \vdots$$

$$\rightarrow . r \operatorname{Cca} \perp s \vdots$$

$$\wedge \vdots \vee p \vdots p \in \mathrm{Pla} . \wedge \cdot \mathrm{Ctr} \dotplus . \in p \cdot \wedge \cdot \wedge t \colon t \, w \, \mathrm{Ext} .$$

$$\rightarrow \cdot \operatorname{Loc} z_0 \frown z_1 \frown z_2 \frown z_3 . \mathrm{Cca} \subset . p \vdots$$

$$\wedge \cdot \neg \operatorname{Pot} \colon \vee t \cdot t \, w \, \mathrm{Ext} . \wedge . t \, z_0 \, \mathrm{Mov} \cdot$$

$$\wedge \cdot \wedge t \colon t \, w \, \mathrm{Ext} . \rightarrow \cdot \operatorname{Pot} . t \, z_1 \frown z_2 \frown z_3 \, \mathrm{Mov} \, \shortmid$$

$\mathrm{Erg} . w \in \mathrm{Lib} \, \shortmid$

$\mathrm{PAN} \, Hb \, \mathrm{Met} . ? = \mathrm{Mas} \, Ra . \operatorname{Per} w \cdot$

$$\leftrightarrow \shortmid \vee \ulcorner u . z^1{}_1 . z^1{}_2 \urcorner \vdots u \operatorname{Uni} w \, Ra \, Rb . \wedge . Ra \operatorname{Apu} z^1{}_1 . \wedge . Rb \operatorname{Apu} z^1{}_2 . \wedge .$$

$$z^1{}_1 \operatorname{Par} w . \wedge . z^1{}_2 \operatorname{Par} w .$$

$$\wedge \vdots \wedge t \colon t \, u \, \mathrm{Ext} . \rightarrow \cdot \operatorname{Loc} z^1{}_1 . \subset \cdot \mathrm{Cvx} . t \operatorname{Loc} z_1 . t \operatorname{Loc} z_0 \vdots$$

$$\wedge \colon t \operatorname{Loc} z^1{}_2 . \subset \cdot \mathrm{Cvx} . t \operatorname{Loc} z_2 . t \operatorname{Loc} z_0 \vdots \wedge \colon$$

$\mathrm{PAN} \, Hb \, \mathrm{Cau} \cdot \mathrm{Nnc} . u \, \mathrm{Ext} \frown z_3 \operatorname{Sua} z_0 \vdots \wedge$

$\cdot \mathrm{PAN} \, Hb \, \mathrm{Cog} \colon \mathrm{Utr} \cdot \mathrm{Mas} \, Ra \times \mathrm{Dst} \, z_0 \, z^1{}_1 .$

$$= . \mathrm{Mas} \, Rb \times \mathrm{Dst} \, z_0 \, z^1{}_2 \cdot \wedge \colon$$

$\mathrm{PAN} \, Hb \, \mathrm{Met} . ? = \ulcorner \mathrm{Dst} \, z_0 \, z^1{}_1 . \mathrm{Dst} \, z_0 \, z^1{}_2 \urcorner \vdots \wedge \colon$

$\mathrm{MAN} \, Hb \, \mathrm{Cog} . ? = \mathrm{Mas} \, Rb \vdots \wedge \colon$

$\mathrm{PAN} \, Hb \, \mathrm{Cpu} . ? = \mathrm{Mas} \, Ra \, ^\#$

The word written Lib (*∫L* libra = balance) means "balance". 'Met Per . 1$^\epsilon$ Lib' means "weighs".

4 28 1. $^\#$ $Ha \operatorname{Inq} Hb \cdot ? = . \mathrm{Dst} \, Hb \, Ra \, \shortmid$

$^{t_1} Hb \operatorname{Inq} Ha . \mathrm{Cmt} \, 1101111 \times 10^{10010} \shortmid$

$Ha \operatorname{Inq} Hb \cdot ? \, u . \mathrm{PAN} \, Hb \, \mathrm{Cog} \, \mathrm{Etc} \, \mathrm{Mod} \, u \, \shortmid$

$Hb \operatorname{Inq} Ha \vdots t_1 \, Hb \, \mathrm{Met} \colon ? = \mathrm{Dst} \, Hb \, Ra \colon \operatorname{Per} . 1^\epsilon \, \mathrm{Und} \, \mathrm{Ath} \vdots$

$$\vee \ulcorner w . w' . t . t' \urcorner \cdot w \frown w' \in \mathrm{Und} \, \mathrm{Ath} .$$

$$\wedge \colon t \, Hb \, \mathrm{Cau} \cdot \mathrm{Cca} \, t . \mathrm{Adv} \, Ra . w \, \mathrm{Mov} \colon \wedge . t' \, w \operatorname{Apu} Ra .$$

$$\wedge \colon t' \, Ra \, \mathrm{Cau} \cdot \mathrm{Cca} \, t' . \mathrm{Adv} \, Hb . w' \, \mathrm{Mov} \colon$$

$\mathrm{Sec} \, 10^{-1001} \, \mathrm{Pst} . Hb \, \mathrm{Cau} \, \mathrm{Etc} \cdot {}^\curlyvee w' \, \mathrm{Etc} . \operatorname{Apu} Hb \vdots$

$\mathrm{Erg} \cdot t_1 \, \mathrm{Dst} \, Hb \, Ra . = \mathrm{Cca} . \mathrm{Sec} \, 0,1 \times 10^{-1001} \times c$

$$= \mathrm{Cmt} \, 1101111 \times 10^{10010} \shortmid$$

$Ha \operatorname{Inq} Hb \cdot \mathrm{Utr} . \mathrm{Nnc} \, Ra \, \mathrm{Mov} \, \shortmid$

$Hb \operatorname{Inq} Ha \cdot \mathrm{Nnc} . \mathrm{Dev} \, Hb . Ra \, \mathrm{Mov} \cdot$

$\mathrm{Qia} \cdot \mathrm{PAN} \, \mathrm{Loc} \, Ra . \subset \colon \mathrm{Cvx} \cdot \mathrm{Loc} \, Hb . \mathrm{Nnc} \, \mathrm{Loc} \, Ra \cdot \wedge \colon$

$\mathrm{Sec} \, 10^{110} . \mathrm{Ant} \, \mathrm{Nnc} \cdot \mathrm{Dst} \, Hb \, Ra = . \mathrm{Cmt} \, 1101111 \times 10^{10010} \colon$

$\wedge \cdot \mathrm{Nnc} \, \mathrm{Dst} \, Hb \, Ra . = . \mathrm{Cmt} \, 1101111 \times 10^{10011} \vdots$

$\mathrm{Erg} \cdot \mathrm{Nnc} \, \mathrm{Cel} \, \mathrm{Med} \, Ra . = \mathrm{Cca} . \mathrm{Cmt} / \mathrm{Sec} \, 1101111 \times 10^{1100} \, ^\#$

'Met . . . Per .1$^\epsilon$ Und Ath' means "measuring by means of radar".

4 28 2.　⚹ Hb Inq Ha ।

Pon । $X \frown Y \in$ Hom . ∧ ! ∨ ⌜$a . b$⌝ ! ∧t!$t . =$ Cca . t_1 ·

→ · Cel $X . t$ · $= a$: ∧ : Cel $Y . t$ · $= b$ ।·

Erg । : Cca t_1 . X Inq X ! $t =$ Nnc .

→ · Cel $X . t$ · $= .$ Cmt/Sec 0_{Vco} : ∧ : Cel $Y . t$ · $= y$! → ।·

∨x ।. $|x| = |y|$.

∧ । Ncs ! Cca t_1 . X Ccd ! Cca t . Y Inq Y ! $t' =$ Nnc .

→ · Cel $Y . t'$ · $= .$ Cmt/Sec 0_{Vco} : ∧ : Cel $X . t'$ · $= x$ ⚹

As we wish to initiate the receiver into relativity, we are committed to a careful treatment of the time variables. In 'Cca t_1 . X Inq X ! $t =$ Nnc Etc' the variable t_1 relates to Hb's clock, because Hb is the direct user of t_1. The variable t, however, though belonging to Hb's discourse and bound through quantification over by Hb, is to be considered as being used by X and relating to X's clock. Finally 'Nnc' belongs to X in any respect. In 'Cca t_1 . X Ccd ! Cca t . Y Inq Y ! $t' =$ Nnc Etc' new time variables 't'' and 'Nnc' occur; they relate to Y's clock. Many such examples can be found in the following paragraphs.

4 28 3.　⚹ Hb Inq Ha ।

Pon । $X \frown Y \in$ Hom . ∧ ! ∨ ⌜$a . b$⌝ ! ∧t!$t . =$ Cca . t_1 ·

→ · Cel $X . t$ · $= a$: ∧ : Cel $Y . t$ · $= b$ ।·

Erg । Cca t_1 . X Inq X ! $t =$ Nnc .

→ · Cel $X . t$ · $= .$ Cmt/Sec 0_{Vco} : ∧ : Cel $Y . t$ · $= y$:

∧ · Nnc . Dev $X . Y$ Mov ! → !

Ncs ! Cca t_1 . X Ccd · Nnc Hb Inq Ha : Cel Rel $X Y . t_1$ · $= |y|$ ⚹

The word written 'Cel Rel' means relative velocity. If X and Y approach, it will be considered to be negative. In order to say this, we should repeat the last program text with 'Adv' instead of 'Dev' and $- |y|$ instead of $|y|$.

4 28 4.　⚹ Hb Inq Ha ।

Pon । $X \frown Y \in$ Hom . ∧ δ . Mul $>$. Sec 1 ·

∧ ! ∨ ⌜$a . b$⌝ ! ∧t!$t_1 - \delta \leq t \leq t_1 + \delta$.

→ · Cel $X . t$ · $= a$: ∧ : Cel $Y . t$ · $= b$: ∧ : Cel Rel $X Y . t$ · $= v$! ∧ .

$\alpha \in$ Pos . ∧ · $W \cup W'$. ⊂ Und Ath · ∧ . $U \in$ Sca Sec α .

∧ ! ∧ ⌜$u . u' . u''$⌝ ! u Par U . ∧ . u' Par U' . ∧ . u'' Par U'' ·

↔ ! ∨ ⌜$w . w' . t . t' . t''$⌝ · $w \in W$. ∧ . $w' \in W'$.

∧ : t Usq t' · Apu X . Usd . Apu Y · w Mov :

∧ : t' Usq t'' · Apu Y . Usd . Apu X · w' Mov :

∧ · $u = . w$ Apu X · ∧ · $u' = . w$ Apu Y · ∧ · $u'' = . w'$ Apu X ।:

Erg ।· ∨ ⌜$\alpha' . \alpha''$⌝ · $\alpha' \frown \alpha'' \in$ Pos . ∧ . $U' \in$ Sca Sec α' . ∧ . $U'' \in$

Sca Sec α'' . ∧ ।.

$\vee \varrho$ ɪ Put ϱ ɪ $\wedge \ulcorner \beta . \gamma \urcorner \vdots \beta \frown \gamma \in$ Pos.

$\rightarrow \vdots$ Cca t_1 . X Inq X . $U \in$ Sca Sec β ·

$\rightarrow$ ˙Ncs : Cca t_1 . X Ccd · Nnc Y Inq Y . $U' \in$ Sca Sec $\beta \varrho$:

$\wedge \vdots$ Cca t_1 . Y Inq Y . $U' \in$ Sca Sec γ ·

$\rightarrow$ ˙Ncs : Cca t_1 . Y Ccd · Nnc X Inq X . $U'' \in$ Sca Sec $\gamma \varrho$:

Eti ˙Cca t_1 . X Inq X . $U \in$ Sca Sec β ·

$\rightarrow$ ˙Ncs : Cca t_1 . X Ccd · Nnc X Inq X . $U'' \in$ Sca Sec $\beta \varrho^{10}$ ɪ . $\wedge \vdots$

Cca t_1 . X Inq X ˙$U \in$ Sca Sec 1 . $\wedge . w \frown w_1 \in W$. $\wedge . w' \frown w_1' \in W'$. $\wedge :$

w_1 Apu X · Sec 1 . Pst · w Apu $X : \wedge :$

w_1 Apu Y · t Pst · w Apu $Y : \wedge :$

w_1' Apu Y · t Pst · w' Apu $Y : \wedge :$

w_1' Apu X · t' Pst · w' Apu $X :$

$\rightarrow \vdots$ Ncs ˙ Cca t_1 . X Ccd : t' − Sec 1 = 10 $(t$ − Sec 1$)$.

$\wedge . c$ $(t$ − Sec 1$) = vt$. $\wedge . t =$ Sec $c/(c-v)$. $\wedge . t' =$ Sec $(c+v)/c-v)$

· Erg $U' \in$ Sca Sec $c/(c-v)$. $\wedge . U'' \in$ Sca Sec $(c+v)/(c-v)$.

$\wedge . \varrho = \sqrt{(c+v)/(c-v)}$. $\wedge$

· Nnc Y Inq Y . $U' \in$ Sca Sec $\sqrt{(c+v)/(c-v)}$ ɪ :

Erg : Cca t_1 . X Inq X . $U' \in$ Sca Sec 1 ·

$\rightarrow$ ˙Ncs : Cca t_1 . X Ccd · Nnc Y Inq Y .

$U' \in$ Sca Sec $\sqrt{(c+v)/(c-v)}/(c/(c-v)) =$ Sca Sec $\sqrt{1 - v^{10}/c^{10}}$ #

Relativistic time dilatation has been proved in the most elementary and direct way.

Hb Inq Ha ɪ

Pon ɪ $X \frown Y \in$ Hom . $\wedge$ · δ . Mul $>$. Sec 1 · $\wedge \vdots$

$\vee \ulcorner a . b \urcorner \vdots \wedge t \vdots t_1 - \delta \leqq t \leqq t_1 + \delta$. $\rightarrow$

˙Cel X . t ˙ = $a : \wedge :$ Cel Y . t ˙ = $b : \wedge :$ Cel Rel X Y . t ˙ = $v \vdots \wedge$.

$\alpha \in$ Pos . $\wedge$. $V \in$ Sca Sec α . $\wedge : \wedge t \vdots t_1 - \delta \leqq t \leqq t + \delta$. $\rightarrow$.

t V Apu Y ɪ .

Erg ˙Cca t_1 . X Vul . $V \in$ Sca Sec 1 .

$\rightarrow$ ˙Ncs : Cca t_1 . X Ccd . Nnc Y Vul . $V \in$ Sca Sec $\sqrt{1 - v^{10}/c^{10}}$:

$\wedge \vdots$ Cca t_1 . Y Vul . $V \in$ Sca Sec 1 .

$\rightarrow$ ˙Ncs : Cca t_1 . Y Ccd · Nnc X Vul . $V \in$ Sca Sec $1/\sqrt{1 - v^{10}/c^{10}}$ #

This is only a paraphrase of the result of the preceding dialogue.

4 28 5. # Hb Inq Ha ɪ

Pon ɪ $X \frown Y \in$ Hom . $\wedge . \alpha \in$ Pos . $\wedge . A \in$ Sca Cmt α .

$\wedge \vdots \wedge n$ ˙ $1 + n$. $\in$ Num · $\wedge . n \leqq N : \rightarrow$ · z_n . n^{um} Par $A :$

$\wedge$ · z_N . Ult^{um} Par A · $\wedge$ · δ . Mul $>$. Sec 1 ·

$\wedge \vdots \vee \ulcorner a . b \urcorner \vdots \wedge t \vdots t_1 - \delta \leqq t \leqq t_1 + \delta$.

$\rightarrow$ ˙Cel X . t ˙ = $a : \wedge :$ Cel Y . t ˙ = $b : \wedge :$ Cel Rel X Y . t ˙ = $v :$

$\wedge . t$ z_0 Apu X . $\wedge : t$ Loc X . $\subset :$ Cvx · t Loc A . $\cup$. t Loc Y ɪ .

Erg $:$ Cca t_1 . X Vul $\cdot$ $A \in$ Sca Cmt 1 .

$\quad \rightarrow \cdot$ Ncs $:$ Cca t_1 . X Ccd $\cdot$

$\quad\quad$ Nnc Y Vul . $A \in$ Sca Cmt $\sqrt{1 - \overline{v^{10}/c^{10}}}$ $\cdot$:

Qia $\iota \cdot$ Pon ι $w \in$ Und Ath .

$\quad \wedge : \vee \ulcorner t . t'\urcorner \cdot t \frown t' . = $ Cca $. t_1 \cdot \wedge : t$ Usq $t' \cdot$

$\quad\quad$ Apu Y . Usd . Apu $z_N \cdot w$ Mov $:$

$\quad \wedge : \wedge n : n + 1 . \in$ Num $\cdot \wedge . n \leqq N . \rightarrow : w_n \in$ Und Ath .

$\quad\quad \wedge : \vee \ulcorner t . t'\urcorner \cdot t \frown t' = $ Cca $t_1 \cdot \wedge : t$ Usq $t' \cdot$

$\quad\quad\quad$ Apu z_n . Usd . Apu $Y \cdot w_n$ Mov $:$

$\quad \wedge : \wedge t \cdot t w$ Apu $z_n . \leftrightarrow . t w_n$ Apu z_n :

$\quad \wedge : \wedge n \cdot z_n$ Par $A . \rightarrow : u_n = . w_n$ Apu $z_0 \cdot \wedge \cdot u'_n = . w_n$ Apu $Y :$

$\quad \wedge : U$ Uni $. u_1 . u_2$. Etc $: \wedge : U'$ Uni $\cdot u_1' . u_2'$. Etc ι .

Erg $:$ Ncs $\cdot$ Cca t_1 . X Ccd . $U \in$ Sca Cmt $10 \, c^{-1}$:

$\quad$ Erg $:$ Ncs $\cdot$ Cca t_1 . Y Ccd . $U' \in$ Sca Cmt $10 \, c^{-1} \sqrt{(c+v)/(c-v)}$ $\cdot$

$\quad\quad \wedge :$ Cca t_1 . Y Inq $Y : \beta \in$ Pos . $\wedge . A \in$ Sca Cmt β .

$\quad\quad\quad \wedge \cdot w$ Apu $z_n . t_n$ Pst . w Apu $Y \cdot \wedge \cdot$ $\quad\quad$ [1]

$\quad\quad\quad w_n$ Apu $Y . t'_n$ Pst . w Apu $Y \cdot$

$\quad\quad \rightarrow :$ Ncs $\cdot$ Cca t_1 . Y Ccd $: t'_n = 10 \, t_n$.

$\quad\quad \wedge .$ Cmt $\beta = (t_{n+1} - t_n)(c - v) . \wedge .$ Cmt $10 \beta = (t'_{n+1} - t'_n)(c-v) \cdot$

$\quad\quad$ Erg . Cmt $10 \beta = 10 \, c^{-1} \sqrt{(c+v)/(c-v)} (c-v) \cdot$ Erg . β

$\quad\quad\quad = \sqrt{1 - v^{10}/c^{10}}$ $\#$

A short proof of Lorentz contraction.

4 28 6. $\#$ Hb Inq Ha $\mathbf{I}$

$\quad$ Pon $\iota . X \frown Y \frown Z \in$ Hom $. \wedge \cdot \delta .$ Mul $> .$ Sec 1 $\cdot$

$\quad\quad \wedge \iota \vee \ulcorner a . b . c \urcorner : \wedge t : t_1 - \delta \leqq t \leqq t_1 + \delta . \rightarrow :$ Cel $X . t \cdot = a :$

$\quad\quad \wedge :$ Cel $Y . t \cdot = b : \wedge :$ Cel $Z . t \cdot = c : \wedge :$ Cel Rel $X Y . t \cdot = x :$

$\quad\quad \wedge :$ Cel Rel $Y Z . t \cdot = y : \wedge :$ Cel $X Z . t \cdot = z .$

$\quad\quad \wedge \cdot t$ Loc $Y . \subset :$ Cvx $\cdot t$ Loc $X . \cup . t$ Loc $Z \iota \cdot$

$\quad$ Erg . $z = (x + y)/(1 + xy/c^{10})$ ι :

Qia $\iota \cdot$ Pon ι $W \subset$ Und Ath $. \wedge . \alpha \frown \beta \frown \gamma \in$ Pos .

$\quad \wedge . U \in$ Sca Sec $\alpha . \wedge . U' \in$ Sca Sec $\beta . \wedge . U'' \in$ Sca Sec γ .

$\quad \wedge : \wedge \ulcorner u . u' . u'' \urcorner : u$ Par $U . \wedge . u'$ Par $U' . \wedge . u''$ Par $U'' \cdot$

$\quad\quad \leftrightarrow : \vee \ulcorner w . t . t'' \urcorner \cdot w \in W . \wedge : t$ Usq $t'' \cdot$ Apu X . Usd .

$\quad\quad\quad$ Apu $Z \cdot w$ Mov $:$

$\quad\quad \wedge \cdot u = . w$ Apu $X \cdot \wedge \cdot u' = . w$ Apu $Y \cdot \wedge \cdot u'' = w$ Apu $Z \iota$.

Erg $:$ Cca t_1 . X Vul $\cdot$ $U \in$ Sca Sec 1 :

$\quad \rightarrow :$ Ncs $\cdot$ Cca t_1 . X Ccd $:$ Nnc Y Vul . $U' \in$ Sca Sec

$$\sqrt{(c+x)/(c-x)} \cdot$$

$\quad\quad \wedge \cdot$ Nnc Z Vul . $U'' \in$ Sca Sec $\sqrt{(c+z)/(c-z)}$:

$\quad \wedge :$ Cca t_1 . Y Vul $\cdot$ $U' \in$ Sca Sec 1 :

---

[1]) Inadvertently I introduced a t_n, which for $n = 1$ does not mean the explicit t_1.

$\rightarrow$ ˙ Ncs : Cca t_1 . Y Ccd $\cdot$ Nnc Z Vul . $U''\in$ Sca Sec

$\sqrt{(c+y)/(c-y)}$:

$\wedge$: Cca t_1 . Y Vul $\cdot$ $U'\in$ Sca Sec $\sqrt{(c+x)/(c-x)}$:

$\rightarrow$ ˙ Ncs : Cca t_1 Y Ccd $\cdot$ Nnc Z Vul . $U''\in$ Sca Sec

$\sqrt{(c+x)/(c-x)} \times \sqrt{(c+y)/(c-y)}$:

Erg : Cca t_1 . X Vul $\cdot$ $U\in$ Sca Sec 1 :

$\rightarrow$ ˙ Ncs : Cca t_1 . X Ccd $\cdot$ Nnc Z Vul . $U''\in$ Sca Sec

$\sqrt{(c+x)/(c-x)} \times \sqrt{(c+y)/(c-y)}$:

Erg . $\sqrt{(c+z)/(c-z)} = \sqrt{(c+x)/(c-x)} \sqrt{(c+y)/(c-y)}$:

Erg . $z = (x+y)/(1+xy/c^{10})$ *

An easy proof of the relativistic addition-theorem of velocities.

4 28 7.   After this behaviouristic introduction into relativity, Lorentz transformations should be derived. We omit those purely mathematical developments.

4 28 8.   * Hb Inq Ha ▮ ˙

$\bigvee\sigma$ ▮ . Put σ ▮ . $\sigma\in$: $^\frown v$. $-c<v<c$ $\cdot$ $\frown$ Pos ˙ $\wedge$ ▮

$\wedge$ ⌜ X . Y . z . t . a . v . μ ⌝ ː X $\frown$ $Y\in$ Hom . $\wedge$. $z\in$ Res . $\wedge$. z Apu X .

$\wedge$: t X Vul ˙

Cel z . Nnc ˙ $=$ Cmt/Sec 0_{Vco} : $\wedge$:

Cel Y . Nnc ˙ $=a$: $\wedge$:

Cel Rel X Y . Nnc ˙ $=v$: $\wedge$.

Mas $z=\mu$ ˙

$\dashrightarrow$ ː Ncs ː t X Ccd ˙ Nnc Y Vul :

Cel Y . Nnc ˙ $=$ Cmt/Sec 0_{Vco} . $\wedge$. Mas $z=\mu$ σ (v) ▮

Ha Inq Hb . ? $=\sigma$ ▮

Hb Inq Ha . σ $(v) = 1/\sqrt{1-v^{10}/c^{10}}$ ▮

Ha Inq Hb Cur ▮

Hb Inq Ha ▮ :

Pon ː x $\frown$ $z\in$ Res . $\wedge$ ː Nnc Hb Vul ˙

Cel Hb . Nnc ˙ $=$ Cmt/Sec 0_{Vco} : $\wedge$:

Cel x . Nnc ˙ $= -$ Cel z . Nnc $= -a$: $\wedge$:

Cel Rel Hb x . Nnc ˙ $=$ Cel Rel Hb z . Nnc ˙ $=v$: $\wedge$ ˙

Mas $x=$ Mas $z=\mu$ σ (v) . $\wedge$.

y Uni x z ː

Erg ▮ . Ncs ▮ Nnc Hb Ccd ː

Cel y . Nnc $=$ Cmt/Sec 0_{Vco} ˙ $\wedge$.

Mas $y = 10$ μ σ (v) . $\wedge$ ː

Nnc Ha Vul ː

Cel x . Nnc ˙ $=$ Cmt/Sec 0_{Vco} : $\wedge$:

Cel z . Nnc ˙ $= 10$ $a/(1+v^{10}/c^{10})$: $\wedge$:

Cel y . Nnc ˙ $=a$: $\wedge$.

$$\text{Mas}\, x = \mu \cdot \wedge .$$
$$\text{Mas}\, z = \mu\, \sigma\, (10\, v/(1 + v^{10}/c^{10})) \cdot \wedge .$$
$$\text{Mas}\, y = 10\, \mu\, \sigma\, (v)^{10} \cdot \wedge .$$
$$\text{Mas}\, y = \text{Mas}\, x + \text{Mas}\, z \cdot \wedge .$$
$$\text{Mas}\, y \times \text{Cel}\, y = \text{Mas}\, x \times \text{Cel}\, x + \text{Mas}\, z \times \text{Cel}\, z \; \dot{}$$
$$\text{Erg} \; \dot{}\; 10\, \sigma\, (v)^{10} = 1 + \sigma\, (10\, v/(1 + v^{10}/c^{10})) \cdot \wedge .$$
$$10\, \sigma\, (v)^{10}\, a = \sigma\, (10\, v/(1 + v^{10}/c^{10})) \times 10\, a/(1 + v^{10}/c^{10}) :$$
$$\text{Erg} : \sigma\, (v)^{10}\, (1 + v^{10}/c^{10}) = 10\, \sigma\, (v)^{10} - 1 .$$
$$\text{Erg} \cdot \sigma\, (v) = 1/\sqrt{1 - v^{10}/c^{10}} \; \divideontimes$$

Proofs given in textbooks of relativistic mechanics for the mass increase formula appeared on closer inspection to be sham proofs. This led me to believe that a mechanical proof (not using electromagnetics or quantum theory) was impossible. In rational relativistics the movement after an elastic collision is indeterminate [1] — I thought this was the reason why the mass increase formula could not be proved. This is not correct because the scalar features of a collision are determinate, and mass is a scalar. A proof was found, and finally it was even possible to eliminate collision. The proof is extremely simple now. The only essential assumption we have made is that the classical procedure of calculating the mass and the velocity of the union of two material points holds true in every inertial system, in other words that the four-impulses are additive (we have in practicue sed it only in a very special case.)

We have omitted the proof that the function σ defined by $\sigma\,(v) = 1/\sqrt{1 - v^{10}/c^{10}}$ satisfies the requirement that the relational system

$$\sum_{1}^{3} \mu_i\, \sigma\, (x_i) = 0, \quad \sum_{1}^{3} \mu_i\, \sigma\, (x_i)\, x_i = 0$$

remains valid if the velocities x_i are simultaneously replaced by $(x_i + v)/(1 + x_i\, v/c^{10})$. This can easily be proved by using the transformation law of the expression $(c + x)/(c - x)$ which gets the factor $(c + v)/(c - v)$. From the given relations we can derive

$$\sum_{1}^{3} \mu_i\, \sqrt{c^{10} - x_i^{10}}^{\,-1}\, (c \pm x_i) = 0$$

which appears to multiply with $\sqrt{(c \pm v)/(c \mp v)}$.

---

[1]   In reality classical and relativistic mechanics do not diverge from each other in this respect. In classical mechanics the result of an elastic collision cannot be predicted from laws of conversation. But it can be as soon as we add the postulates: 1° the solution is unique, 2° it is Galileo-invariant, 3° by the collision the velocities are changed (in order to exclude the "trivial" solution). The case of relativity is not different. Galileo-invariance is superseded by Lorentz-invariance.

3 37 3 – 3 37 4   > Vld, < Vld, very large, very small.

3 37 5   It is very difficult to know something about an event, if
                 the only witness refuses to tell anything about it. It is still
                 more difficult if nobody witnessed it.

3 37 6   It is very difficult to forget something especially if somebody
                 asks you expressly to forget it.

3 38 0   A more solid general background of human wishes is needed.

3 38 1   A bet on solving a cubic equation. The winner refuses to
                 tell the general method of solution. The loser refuses to pay
                 up. An arbiter is called in. The winner tells his method, the
                 loser pays. Dat, giving. Den, penny.

3 38 2   Pec, fortune. How one's fortune changes under the influence
                 of payments.

3 38 3   This talk is related to 3 32 5 and 3 29 4. It appears that
                 the obedient servant is regularly paid by his master.
                 Another person is disobedient, because the master did not
                 pay him his due.

3 39 1 – 3 39 3 Regular dialogues, e.g. one actor pronounces a number,
                 and the other pronounces its square. A third person states
                 the law, Lex, of the dialogue.

3 40 1   Two persons play a simple arithmetical game (adding a
                 number ≤ 8; he who reaches 32, wins).
                 Lud, playing, Vin, winning. Prd, losing. Ict, move (of a
                 game).

3 40 2   General definition of game.

3 40 3   The same game played by other players. Good and bad
                 strategies. The game is unfair, because the beginner can
                 always win. Ius, fair.

3 40 4   Another game. One player chooses one of the numbers 1
                 2, 3, 4, and the other tries to guess it. His chance of winning
                 is only 1/4, but the stake of the first player is 3 against 1 of
                 the second. So the game is fair. Prb, probability.

3 40 5 – 3 40 6 General notions about probability, expectation (Uti).

3 40 7   One player chooses one of the numbers 1, 2, 3, 4, and the
                 other tries to guess it. If he hits the chosen number, the
                 first player pays as many pennies as the chosen number
                 has units. If he fails, nothing is paid. The players discuss
                 the stake. Minimax principle.

3 40 8   A still more complicated two-person game.

3 40 9   A three-person game. Aux, aiding. Soc, allies.

3 41 1   A paradox: I do not say anything.

3 41 2   The paradox of the liar.

4 09 1	One actor calls another. He moves. Apu, at, near.
4 09 2	In too short a time a person cannot go from one place to another.
4 09 3	*Hb* refuses to walk to *Ha*. *Hb* wants *Ha* to walk to him.
4 09 4	*Ha* wants *Hb* to fetch *Hc* and to walk with him to *Ha*. They obey.
4 10 1	Whistling for one's dog.
4 10 2	The dog refuses. He persues the fleeing dog. Adv, towards. Dev, away.
4 10 3	Number of animals which can perceive, wish and move; much larger than that of humans.
4 10 4	One actor perceives another, he calls him, they go to meet.
4 10 5	A dog joins his master in order to please him.
4 11 1	*Ha* was mistaken, he called to a thing, thinking that it was a human or an animal. Things cannot move themselves. Now this thing is moved by *Hd* towards *Ha*. Humans can cause things to move. Res, things. Cau, causing.
4 11 2	Things are in space.
4 11 3	Fer, carrying.
4 11 4	Iac, throwing.
4 11 5	If an object is too large it cannot be carried by a single person, though perhaps by more persons together.
4 11 6	An object thrown and intercepted.
4 11 7	An actor tries to intercept a thrown object, but does not succeed because the distance is too large.
4 11 8	Playing ball.
4 12 1	Mean velocity.
4 12 2	Sec^{-1}.
4 13 1	Harmonic oscillation of a small particle. Osc, oscillating.
4 13 2 – 4 13 3	Aml, Fre, Pha, Eql, Amplitude, frequency, phase, point of equilibrium of an oscillation.
4 13 4	Vib, an object vibrates, if its parts oscillate. Examples for stationary and progressive waves.
4 14 1	Movements of objects caused by humans are rather slow.
4 14 2	Why can a human or an animal change its place? Because he want to move. Why can a thing change its place? Because it moves.
4 14 3	Why can a person receive a message sent from another place? There are three ways. The first, through mere convection of the message, is rather slow. Wave propagation is much quicker. There is an undulating medium, Aer, characterized by sonic velocities. Finally there is wave propagation through

4 26 4   Density of solids, liquids, gases.

4 27 1   One of the actors tells how time is measured. Met, measuring. Sca, scale. Hor, clock. Met . . . Per . . . Hor, measuring by means of a clock.

4 27 2   One of the actors describes a measuring staff and measuring distances by means of a measuring staff, and by trigonometrical means. Rgu, measuring staff. Tdo, theodolite. Met ... Per ... Rgu, measuring by means of a measuring staff.

4 27 3   Ifa, below. Sua, above.

4 27 4   Equilibrium of solid bodies.

4 27 5   One of the actors describes a balance and weighing.

4 28 1   One of the actors describes measuring distances and velocities by means of radar. Met . . . Per . . . Und Ath, measuring by means of radar.

4 28 2   *Hb* says that if X and Y move uniformly and if X asserts that he is at rest and that Y has the velocity y, X is comitted to allowing that Y can assert that he is at rest and that X has a velocity x which is absolutely equal to y.

4 28 3   In this way the relative velocity, Cel Rel XY, of X and Y is defined; if X and Y approach it will be considered to be negative.

4 28 4   Observation of moved clocks leads to a proof of relativistic time dilatation.

4 28 5   Observation of measuring staffs by means of radar leads to a proof of Lorentz contraction.

4 28 6   The relativistic addition theorem of velocities.

4 28 7   Lorentz transformations.

4 28 8   Relativistic mass increase formula.

REGISTER

$\neq$	**1 00 0**	$\notin$	**1 18 2**
$>$	**1 01 1**	$\lor$	**1 18 5**
$<$	**1 01 2**	n^ϵ	**3 28 1**
$=$	**1 01 3**	$\subset$	**1 21 1**
$+$	**1 01 4**	$\supset$	**1 21 1**
$-$	**1 01 5**	$\not{V}$	**1 22 1**
1	**1 02 1**	i	**1 22 1**
10	**1 02 1**	π	**1 22 2, 4 06 5**
11	**1 02 1**	e	**1 22 2**
$a, b, c, \ldots$		$\ulcorner \urcorner$	**1 24 7, 1 26 4**
$x, y, z, \ldots$		$\cup$	**1 26 1**
(variables)	**1 04 1 – 1 04 4**	$\cap$	**1 26 1**
$\left. \begin{array}{c} \cdot, : \\ {}_\shortmid, \shortmid \end{array} \right\}$	**1 05 0**	$\setminus$	**1 26 1**
		$\aleph$	**1 27 4**
$\rightarrow$	**1 06 1, 3 31 1**	$\lceil . \rceil$	**1 28 2**
$\leftrightarrow$	**1 07 1**	$\smile$	**1 29 2**
$\neq$	**1 08 1**	$\frown$	**1 29 2**
$\lor$	**1 09 1**	$\hat{} x$	**1 29 3**
$\leqq$	**1 10 1**	$\bigcap$	**1 30 2**
$\geqq$	**1 10 1**	$\bigcup$	**1 30 4**
$\land$	**1 11 1**	$\breve{} x$	**1 31 1, 3 01 3**
?	**1 12 1, 3 02 1, 3 01 2,**	$\breve{} x$	**1 32 1**
	3 01 3	$\vdots$	**1 32 1, 1 32 9**
0	**1 13 2**	$\frown$	**1 32 1, 1 32 2**
-1	**1 13 2**	$\star$	**1 32 2**
-10	**1 13 2**	$f^{\hat{}}$	**1 32 3**
$\times$	**1 14 1**	$\hat{} f$	**1 32 3**
a/b	**1 15 2**	$\omega_1, \omega_2, \omega_3$	**1 32 5**
1,1	**1 16 1**	Σ	**1 35 2**
0,1	**1 16 2**	$\neg$	**1 36 0, 1 36 6**
1,101 101	**1 16 3**	———	**2 01 1**
a^n	**1 16 6**	$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \ldots$	**2 01 1**
$\in$	**1 17 1**	!	**3 17 0**
$\land$	**1 18 1**	$/\!/$	**4 01 9**