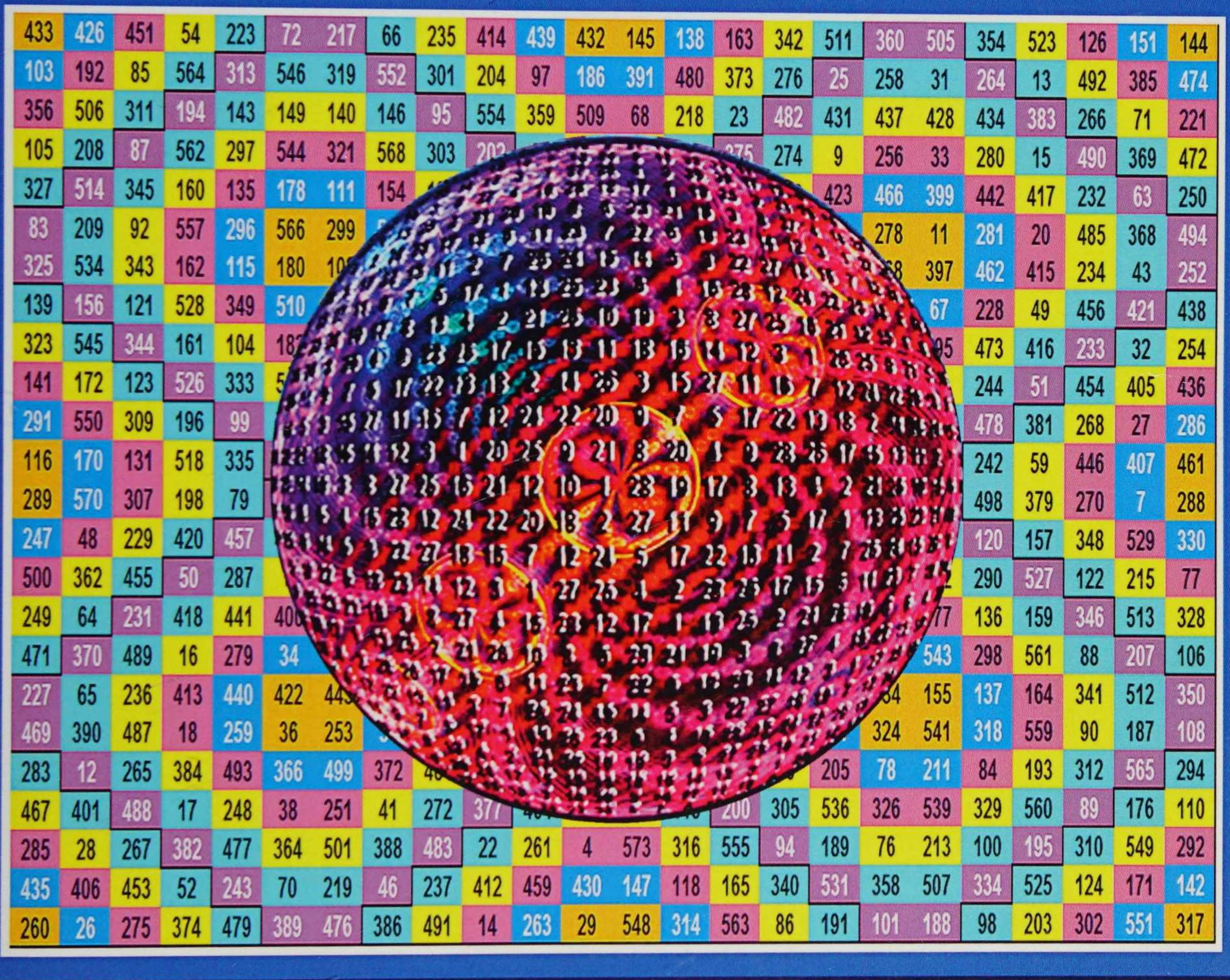
Geonometry

Equal-sum Patterns in the Natural Number Series

A Companion Book to the 10-Program Video Presentation



by

Robert Francis Hauck

— 5th Edition —

Presented at the highschool-senior / college-freshman level

Digitized by the Internet Archive
in 2022 with funding from
Kahle/Austin Foundation

https://archive.org/details/geonometryequals0000hauc

atterns in the natural
umbers series :
2015
3305235510173
i 2anf 02/03/16

Geonometry

Equal-sum Patterns in the Natural Number Series

A companion book to the 10-Program Video Presentation

by

Robert Francis Hauck

Initial Publication Date: December 24th, 2014

Revised and extended July 24th, 2015

Fifth Edition

10 9 8 7 6 5

Uncovering the Fabric of Space Itself

You can get any of the books referenced in this Program Series at

www.CreateSpace.com

Be sure to have the ISBN number handy to locate the book's webpage on this website.

Author Contact Info

Robert Hauck
PO Box 1526
San Martin CA 95046
or
Mr.Math@Live.com

SANTA CLARA COUNTY LIBRARY
3 3305 23551 0173

Copyright Notice

All the number tables posted herein have been formally copyrighted prior to publication. Their use in any other published work, website, CD, Power Point slide or college thesis that is not authorized in writing by the author or his duly appointed agent, is expressly prohibited. All number **patterns** in this book may not be reproduced by copying or through camouflage by numeric manipulation or numeric reflection. It's basically the number patterns that are expressly protected by copyright, not their exact values or positional layouts.

Copies of this document at various stages throughout its development have been timely filed with the US Register of Copyrights and all the number patterns are protected from unauthorized reproduction of any kind without the express written permission of the author or his duly authorized agent.

This book is protected by the following copyrights already granted:

TXu 1-302-220 TX 1-307-747
TX 1-360-951 TX 1-365-314
TX 1-626-704 TX 7-018-696
TX 7-155-560 TX 7-360-814
TXu 1-884-365

Additional copyrights are pending for new text, patterns and properties to date.

ISBN: 978-150-567225-1

Table of Contents

Program 1

Introduction, Overview, Definitions & Categorizations

Gives the basic definitions for patterns and summations involved in Geonometry. Categorizes the number tables by table-size into distinctly different classes, each of which has their own unique characteristic equal-summing patterns 1-40

List of the program topics and description of their subject matter 3-5
Recognitions 6
Overview.....7
Spaceflight 10
Other potential applications 14
Definitions and examples 17
1-Dimensional Geonometry 18
Linear Pattern in Planetary Orbits 18
2-Dimensional Geonometry 20
Definitions and Examples 21
Squares 22
Categorization of Squares by Class 24
Class -1 Squares 25
Size-5 Square 25
Size-7 Square 25
Class -2 Squares 26
Size-6 Square 26
Size-10 Square 27
Class -3 Squares 28
Size-15 Square 28
Class -4 Squares 29
Size-4 Square 29
Size-8 Squares 29
Size-16 Squares 30
Size-12 Square 31
Characteristic Circles in Class-4 Squares 32
Size-24 Square with nested circles 34
Class -5 Squares 36
Size-35 Square 36
Class -6 Squares 37
Size-9 Square 37
Size-25 Square 38
Notes 39
References 40

Table of Contents

Program 2

Part I – Tiling Patterns

Depicts the amazing interlocking equal-summing tiling patterns hidden in all Classes of Squares 1-38

Introduction 1
Tiling Patterns for Geonomic Squares 2
Tiling Patterns for Odd Size Squares 3
Class-1 Squares Tiling Patterns 3
Sizes 5, 7, 11 & 13 3
Sizes 5 thru 31 patterns 4
Size-17 patterns 5
Size-17 alternate versions 6
Size-19 patterns 7
Size-31 pattern A 8
Size-31 pattern B 9
Class-5 Squares Tiling Patterns 10
Other Size-35 Patterns 11
Class 6 Squares Tiling Patterns 12
Size-9 Patterns 12
Size-25 Patterns 12
Size-27 Patterns 13
Tiling Patterns for Even Size Squares 14
Class-2 Squares Tiling Patterns 14
Size-14 Patterns 14
Size-22 Patterns 15
Size-26 Patterns 16
Complementary Tiling Patterns Found in Class-2 Squares 17
Size-10 17
Size-14 20
Patterns in Size-2 Squares that Arise from Continuous Minor Inequalities 21
Size-14 21
Size-18 22
Size-22 25
Size-26 27

(continued)

Table of Contents

Program 2 cont'd

Class 4 Squares Tiling Patterns 28
Size-8 patterns 28
Size-12 patterns 29
Size-16 patterns 30
Size-20 patterns 31
Size-24 patterns 32
Size-28 patterns 33
Square Ring Patterns in Class-4 Squares 34
Summary of Class-4 Tiling Patterns 35
Other Common Tiling Patterns for Class-4 Squares 36
Part I Notes 38

Part II – Anchor-dot Distribution Patterns

Identifies yet another set of patterns which sum equally to the square's characteristic number 39-58

Class-1 Squares 40
Size-7 square Anchor-dot Patterns 41
Size-11 square Anchor-dot Patterns 42
Size-37 square Anchor-dot Patterns - A Rare Exception 43
Notes for Class-1 Anchor-Dot Patterns 45
Class-3 Squares 45
Size-15 square Anchor-dot Patterns 46
Size-33 square Anchor-dot Patterns 47

Class-5 Squares 49
Size-35 49
A different Size-35 square with a different Dot Distribution Pattern 52
Equal-summing Dot Patterns on the Size-35 Square 51

Class-6 Squares 53
Size-9 square Anchor-dot Patterns 53
Size-25 square Anchor-dot Patterns 53
Size-27 square Anchor-dot Patterns 54
The Nova-dot Patterns Common among Class-6 Squares 55
The Geonomic Conversion Process for All Odd-size Squares 56
Part II Notes 57

Table of Contents

Program 3

Part I - Loom Tables

Introduces loom tables derived from the original table, shows how their dual table is constructed from them and demonstrates the fundamental mathematics underlying the real fabric of space 1-22

Loom Tables for Class-1 squares 2
Size-7 Loom Tables 3
Interchangeability of Loom Tables 4
Loom difference table 5
Harmonics in 7x7 space 7
Four different versions of the size-7 perfect square 8
Tiling Patterns on Size-11 X and W.....9
Tiling Patterns on Size-11 Y and U.....10
Size-17 loom tables 11
Size-17 Loom-table Tile-Centers and Internal Sequences.....13
Sequential Centers and Scattered Numbers.....13
Constant Centers and Fixed Numbers.....14
Size-31 loom tables 15
Weaving the Fabric of Space.....16
Size-31 Loom Table X with Tiling Patterns A and hA 17
Size-31 Loom Table Y with Tiling Patterns A and hA 18
Points of Equality between Dual Squares
and the centers of their Tiling Patterns.....19
Alignment of centers of transposed tiles with common
equal value locations of Size-7's dual squares.....20
Geonometry's Application to Quantum Computing.....21
Part I Notes.....22

Part II - The Amazing Cloaking Property of Loom-Tables

Demonstrates the self-cloaking properties of complementary loom-tables.....23-62

The Cloaking Property of tiling patterns on Loom-table Differences 24
For Class-1 squares 24
Roaming size-17 Loom Difference Table 25
Same size-17 loom difference table with tile patterns A & B 26
Continuous Loom-table Differences of the Size-31 Square 27
Size-31 loom difference table 27
For Class-4 squares 28
Size-12 loom difference table.....28
Size-24 Square.....30
Size-24 Loom-tables.....31
Size-24 Loom-Difference Table.....32

(continued)

Table of Contents

Program 3 cont'd

Size-28 and it's Loom Difference table 33
Class-1 string summations found in some Class-4 loom-tables.....34
For Class-3 Squares 35
Size-15 block-patterns 35
Size-21 block-patterns 37
Size-21 Loom Tables 38
Size-21 Loom Difference Table 39
Size-27 block-patterns 40
Patterns on the Size-27 Dual Square.....41
The third block-tiling pattern on the size-27 dual square 42
For Class-5 Squares 43
Size-35 primal square with size-7 block-tile patterns 43
Size-35 dual square with size-5 block-tile patterns 44
The Class-5 uniform-summation property.....45
Size-35 Modulus and Integer Loom-table Patterns in the base 35.....46
The Cloaking property of size-35 loom-tables 48
For Class-6 Squares 49
Size-9.....49
The Cloaking Property of Size-9 Loom Tables 50
Size-25 Primal and Dual Squares with block-tile patterns....51
Size-25 Loom Tables 52
Size-25 Loom Difference Table 53
Summary Table of Cloaking Property by Class 54
Unification of all the Square-generation Formulas 54

Application #1 – The Membrane of Life.....55-59
The Double and Triple Zipper-like Patterns in Class-5 Loom-tables .56
Size 35 Modulus Loom Table 56
Size 35 Integer Loom Table 57
A plausible application of Geonometry in molecular biology.....58

Program 3 Notes 60
References.....61

Table of Contents

Program 4 – The Amazing Equal-sum Patterns in Geonomic Squares

Part I – Uncovers the amazing diamond quilt patterns and triangular tapestries hidden in Class-3 and Class-4 squares.....1-24

Diamond Quilt Patterns.....2
For Class-3 Squares.....2
Size-15.....2
Size-21.....4
For Class-4 Squares.....5
Size-12.....5
Size-20.....6
Size-24.....7
Size-28.....10
Size-32.....11
Size-36.....13
Size-44.....14
Size-16.....16
Size-32.....17
A Fixed-size Tile Pattern for All Squares whose Size is a Power of 2.....18
Triangular Tapestry Patterns.....19
Size-20.....19
Size-28.....21
Inequalities in Pyramidal Summations of Lesser Viable Levels Lead to the Recovery of the Expanded Squares.....22
Size-28 Summation Inequalities in pyramidal diamonds.....20
Size-35 Summation Inequalities in pyramidal squares.....26

Part II – Uncovers unique tiling patterns at the 2x2 block-square level hidden in Class-4 squares.....27-38

Size-16.....28
Sizes 20 thru 36.....29
A 2nd Characteristic Tiling Pattern at the 2x2 block-square level..30
Size-16.....30
Size-20.....31
Size-24.....32
Size-28.....33
Size-32.....34
Size-36.....35
Summaries of Tiling Patterns for Class-3 and Class-4 Squares. ..36
Size-33 at the 3x3 block-square level.....37
Part II Notes..... 38

(continued)

Table of Contents

Program 4 cont'd

Part III – 3-Dimensional Patterns in 2-Dimensional Space.....39-64

Equal Pyramidal Summations.....40
Class-3 Squares.....40
Size-21.....40
Size-27.....41
Size-33.....42
Class-4 Squares.....43
Size-20.....43
Size-24.....45
Size-28.....47
13 Continuous Equal-summing Patterns on the Size-32 square.....50
Size-32.....51
Size-44.....52
Summary of Solid and Pyramidal Diamond Patterns in Class-4 Squares.....53
Pyramidal Summary Tables for Class-4 Squares..54
A simple method for calculating pyramidal diamond summations in Excel.....546
Class-5 Squares.....57
Square Pyramidal Patterns in the Class-5 Size-35 Square 57
Integrated Square Pyramidal Patterns in the Class-5 Squares.....58
Overall Summary of all Continuously Equal-summing Rectangular, Diamond and Triangular Patters up thru Size-44.....59
Class-1 Squares are the source of Class-4 Patterns.....60-62
Part III Notes.....63

Table of Contents

Program 5 – The 3rd Dimension

Explores the properties of 3-dimensions and the amazing equal-summing 1-dimensional and 2-dimensional patterns discovered in Geonomic cubes through the compact view of depth-sum tables. Here begins the incredible mathematical journey up through higher dimensions as only Geonometry can provide.....1-24

Introduction 1
Cubic Definitions 2
Class-1 Cubes 3
Size-5 Cube 3
5x3D Depth-sum Tables 4
Reduction of Depth-sum table B to a perfect size-5 square.....4
5x3D Cubic Tiling Patterns 5
Class-2 Cube 6
Size-6 Cube 6
Collapsing the size-6 cube to size-6 punctuated-perfect square.....7
Class-3 Cube 8
Size-3 Cube 8
Class-4 Cube 9
Size-4 Cube 9
Segregated into block-squares.....10
Collapsing the depth-sum table B to a perfect square.....10
Class-6 Cube 11
Size-9 Cube 11
Size-9 Cube's Octal and Octahedral sums.....12
Collapsing the depth-sum table B to a perfect square.....13
Equalities between X and Y in Odd-size Cubes.....14
Equalities between X and Y in the Centrally Embedded Square of Class-1 Cubes.....15
Generating Cubes from Squares.....16
Embedded Characteristic Spheres in Class-4 Cubes.....17
Octal and Quadral Differences between Circles and Cubes 20
Summary of Spheres 21
Summary of Cubic Properties 22
Notes 23
Reference.....24

(continued)

Table of Contents

Program 5 cont'd

Electron Shell Pairs – It is demonstrated in this segment just how the mathematics of 2- and 3-dimensional Geonometry can explain and account for the distribution and navigation of the electrons around the nuclei in atoms.....25-40

Application #2 – Accounting for the structure of electron orbits 26

The Periodic Table of the Elements 26
The Niels-Bohr Model of the Atom 27
Distribution pattern of electrons in the Noble elements 28
The Distribution of Octals in the Size-12 Characteristic Sphere 29
Tiling Patterns on the Surface of Characteristic Spheres 29
Tiling Patterns of Loom-tables on the Surface of Characteristic Spheres.....30
Orbital motion of Electrons according to Geonometry.....31
The Double-Shell Cloaking of Electron Shells 34
Extension of electron distribution pattern to other shell pairs.....34
Electron Fill Patterns.....35
Electron Distribution among Electron Shells 36
Orbital Capacities.....37
Geonometry's view of the electron distribution pattern 37
Derivation of the Formula for the Number of Electrons in She..lls.....38
Derivation of the number series for the number of elements in consecutive periods of the Periodic Table 38
Notes 39
References.....40

Table of Contents

Program 6 – 4-Dimensional Geonometry

Depicts 4-Dimensional space as never seen before 1-30

Definitions 2
Quadracubes by Class 4
Class-2 Quadracube of Size-2.....4
Class-3 Quadracube of Size-3.....5
Expanded 4-Dimensional Tiling Pattern on the Size-3 Quadracube.....6
Dual Size-3 Quadracubes and their Common Loom Tables...7
Reduction of depth-sum table along the D-axis 8
Class-4 Quadracube of Size-4.....9
Class-1 Quadracube of Size-5.....12
Size-5 Quadracube's 10 Embedded Cubes 13
Size-5 Quadracube's Depth-sum Tables 14
Size-5 Quadracube's collapse to a perfect cube.....14
Derived perfect cube collapses further to a perfect square 15
Depth-sum tables B and D collapse along their B-axes to identical perfect squares 16
Generating Quadracubes from Cubes.....17
Quadracube's characteristic tiling patterns at the block-square level 18
Size-5 Quadracube 18
Size-7 Quadracube 19
Categorization of Quadracubes by Class 20
Exploration of an alternative Size-4 Quadracube.....21
Embedded Hemispheres in the Primal Quadracube 22
Uncovering the Size-4 Characteristic Torus via Loom Tables ..23
Loom-table Differences 24
Embedded Hemispheres in the Dual Quadracube 25
All the Hidden Hemispheres in Both the Primal and Dual 26
The Characteristic Torus 27
A 3-Dimensional sectional cutout yields a sphere 28
A picture of 4-dimensional space from perspective of 5D.....28
Distinguishing Quadracubic Tables from Tables of Class-4 Squares.....29
Program 6 Notes 30

(continued)

Table of Contents

Program 6 cont'd

Application #3

– **Accounting for the sub-atomic sub-structure of the proton and neutron.**

Here, equal-summing patterns in Class-4 hypercubes are demonstrated to unlock the relationships among quark-pairs at the sub-dimensional level of 1/4D. Geonometry confirms what sub-atomic scientists have discovered through their linear accelerators as an inherent property of the 4-dimensional spatial fabric.....31-38

Dimensional Levels 33
The Sub-dimensional Space of the Quark.....34
Size-4 Quadracube's Modulus Loom Table.....35
Strong correlation with the atomic physicists' model and descriptions.....36
References.....37

Program 7

Part I – In this segment, exploration of the properties of a seemingly recreational geonomic square leads to the discovery that these special number tables serve as the very twine for weaving the fabric of space in all that has gone before! 1-12

The Source of Origin of the Matchmaker's Square.....2
The Five Basic Properties of the Matchmaker's Magic Square.....3
Comparison of Characteristic Circles between Class-4 Regular Squares and Matchmaker Squares of the same size 4
The Matchmakers Magic Cube 5
The Matchmakers Magic Quadracube 6
Loom Tables of Matchmaker Squares 7
Generation of Ultra-perfect Class-1 Squares from Matchmaker Squares of the same size via The Double Quark Method.....10
Constructing the resequencing pattern..... 11
Part I Notes.....12

(continued)

Table of Contents

Program 7 cont'd

Methods for Generating Composite-size Squares

Part II – In this segment we see just how the squares beyond Class-1 are generated. There are five distinct methods for doing this; and there is one method for co-mingling regular geonomic squares with the matchmaker square of the same size to get an even more complex matchmaker square. This all results in a total of 6 more methods in addition to the Double-Quark method of Part I 13-54

#1 : Expansion by the **Balloon** method 14
#2: Expansion by the **T-Ball** method.....15
Application to Composing Class-6 Squares 15
#3: Expansion by the **ATE** Method for Composite Squares..... 17
Application to Composing Class-4 Squares 17
Expansion and merging of the size-3 and size-4 square to get a size-12 square.....18
Deriving the size-20 square with the **ATE** method.....20
Application of **ATE** to Composing Class-5 Squares 22
The 8 versions of the size-35 square.....23-26
Size-35 Primal an Dual Squares derived in the base 49.. 23
Size-35 Primal an Dual Squares derived in the base 25...24
Size-35 Primal and Dual Squares derived in the base 3 ... from the those generated in the base 49.....25
... from the those generated in the base 25.....26
Relationships among the two primal size-35 squares derived by the **ATE** method and their unique duals.....27
#4: Expansion by the Bootstrap method.....28
Application to Composing Class-6 Squares.....29
#5: Expansion by the **SPD** method 33
Application to Composing Class-3 Squares 33
Size-15.....33
Size-21.....34
Size-27.....35
The Homogenization of the Derived Spread Pattern for Larger Class-3 Squares.....36
Application to Composing Class-4 Squares.....37
Generating the Size-8 square..... 37
A Special Pattern for generating Class-4 Squares divisible by 8.....38
Size-16 square.....40
Size-24 square.....41
Size-32 square.....42
Application to Composing Class-2 Squares.....43
A Unique Method for Ironing Out the Wrinkle in Class-2 Squares .. 49
#6: Expansion by the Matchmaker Pattern 52
Part II Notes.....54

Table of Contents

Program 8 - Advanced Perspectives of Geonometry

This program demonstrates some remarkable properties in Geonometry. Here we will get an even broader picture of the spatial fabric and its pervasiveness. These membrane patterns detected in Geonometry through complementary loom tables may well extend across vast distances on a cosmological level too. This program looks at multi-dimensional space from the point of view of expansion properties and not-so-obvious hidden patterns.....1-30

The Corner Triangle Property..... 2
Concentricity Patterns in all Pangenic Tables..... 3
The Zero-differential Property..... 4
The Zero-Differential property on the Size-12 Class-4 Square..... 6
Class-2 squares lack the zero-differential property 7
The Sub-additive Property of all Perfect Squares 8
Size-7 Class-1 square.....8
Size-11 Class-1 square.....8
Size-5 Class-1 square.....9
Size-4 Class-4 square.....10
Size-9 Class-6 square.....10
The Checkerboard Difference Property for Odd-size Squares.....11
Size-9 Class-6 Square.....12
The Cubic Conversion Property 13
Conversion of size-8 squares between pairwise row and central symmetries.....14
Summary of Properties of Squares by Class 16
Table of 2D Characteristic numbers 17
2D Characteristic numbers expressed as a quadratic.....18
3D Characteristic numbers expressed as a 3rd-degree polynomial.....19
Index Triangle's Correlation with 2D Characteristic Numbers.....20
Index Triangle's Correlation with 3D Characteristic Numbers.....21
The Pattern of Characteristic Numbers as Volume Averages of basic geometries in higher and lower dimensions..... 22
The Loom-table hidden in the Tiling Patterns of Class-1 Cubes.....23
Tiling pattern A on embedded squares 24
Tiling pattern B on embedded squares 25
The Geonomic Square Hidden in the Cube's Tiling Patterns 26
The loom table hidden in the Class-1 Quadracube's tiling patterns 27
Generating Ultra-perfect Dual Squares from the derived Modulus Loom-table 29
Program 8 Notes.....30

Table of Contents

Program 9 – Navigating Multidimensional Space

In this program we discover how to explore the properties of any higher dimension through the unique mathematics of Geonometry.....1-28

Fracticality in Geonometry 2
Fracticality in 4-Dimensions 2
Fracticality in 5-Dimensions 4
Prima-fascia proof of the fractal nature of Space 4
Generalized Formulas for the Cubic Dual Series 5
Navigating Multidimensional Space via Class-1 Geonomic Tables.....6
The Crash and Burn of the dual for Class-2 squares 7
k-1 Different navigational paths in the series of Class-1 duals in dimension k..... 8
The Increasing Complexity of Multi-dimensional Space..... 9
The Big Picture of the Multi-dimensional Universe 10
First, the Penetrating Picture of Sub-dimensional space 10
Next, the Picture of 3-Dimensional Space 12
Harmonics in the Spacing of Solar System Planets.....12
Light-Speeds in 3-dimensional Space.....13
Finally, Geonometry vs. Cosmology..... 15
Rolling wave forms hidden in complementary loom-tables of Class-1 squares.....16
Space Travel and Levitation Using the Layered Fabric of 2- and 3-Dimensional Space....18
Building a Wall in Empty Space.....19
The Space Propulsion Sequence.....20
The Spatial Gear for Space Propulsion.....22
Navigating the Fabric of Space.....22
The discovery that ushered in the Industrial revolution.....23
Program 9 Notes 23
References.....24

Table of Contents

Program 10 – Other Geonomic Forms

Here we'll see number patterns in other geometries and how to construct them from loom-tables.....1-36

Overview.....1
Triangles.....2
Sizes 7, 8 & 9.....2
Size-12 and its Dual.....3
Size-17.....4
A Size-10 Tetrahedron.....5
Distribution of the number series in triangles for vertical and horizontal pair equalities.....6
Diamonds.....7
Generating Geonomic Diamonds from two Geonomic Squares.....7
1. The Laminated Indexing Method **LIM**.....7
2. The Bump and Grind **BAG** Method.....9
Size-23 Diamond.....10
Pentagons.....11
Size-5 Equal-summing Nested-frame Pentagon.....11
Size-5 Triangular-summing Nested-Frame Pentagon.....12
Pentagonal Geonometry in Sub-dimensional Level 1/3D (Atoms)....13
Geonomic Hexagons.....14
String Summations in Hexagons.....15
Modular Triangle Method MTM Yields Dual Size-7 Hexagons.....16
Application #4 – Inherent Vibratory Membrane for Snowflake Formation...18
The Conversion of Vapor-laden air into Frozen Hexagonal Segments.....19
Snowflakes are Numerical Weaves of Wonder.....20
Odd-size Hexagons.....21
Size-5 Hexagon.....21
Size-7 Hexagon.....22
Size-9 Hexagon.....23
Size-11 Hexagon.....24
Size-13 Hexagon.....25
The Size-13 Loom Tables.....26
Constructing Dual Snowflake-Patterned Loom Tables.....27
Correspondences of values between complementary loom-tables of geonomic squares.....30
Correspondences of values between complementary loom-tables of geonomic hexagons.....32
Even Size Hexagons.....34
What may seem as a geonomic hexagon many times isn't.....35
Program 10 Notes.....36
Reference.....36

Table of Contents

Appendix.....1-20

Proof of the Properties for the Matchmaker's Magic Square.....2

The Dimensional Average Property 2

The Matchings Property.....3

The Totality Property.....3

The Subset Property 4

Invariance-under-permutation 4

Mathematical Proof of the ATE Method for Class-4 Squares.....5

Kernel Value Multiples equal to the Characteristic Number for any Dimension.....7

General Formulas for Standard Even-sided Figures.....8

Geonometry's Hook into Matrix Algebra.....9-18

Complementary Loom-tables' Matrix Product.....9

Determinants of loom tables.....11

Determinants for Geonomic Squares.....12

Determinants of Class-1 Squares.....13

Loom-tables' Eigenvalues.....14

Loom table's Matrix Inverse.....15

The Matrix Product of a Loom-table's Inverse with its Transpose.....17

Lack of Geonomic Properties for the Standard Identity Matrix.....18

Proof that points of equalities between modulus and integer functions always sum to the square's characteristic number.. .19

Table of Contents

Glossary 1-8

Prime numbers 2
Averages 2
- Dimensional average
- Linear average
- Planar average
- Cubic average
- Quartic average
- Quintic average

Conceptual Notions 2
- Intelligent patterns
- Harmonic waveform
- Measure
- Mapping
- The 4th dimension 3

Geometries 3
- Quadral
- Octal
- Hexadectal
- Duohexadectal
- Octahedron
- Icosahedron
- Duocosahedron
- Quadracube

Table Terminology 4
- Channel
- Pillar
- Block
- Major diagonal
- Minor diagonal
- Wrap diagonal
- Pivot number
- Kernel number
- Density
- Pangenic
- Pangenicity
- Concentricity

Table of Contents

Table Operations..... 5
Table collapsing
Table expanding
Roll-wrapping
Tilt-wrapping
Equalities..... 5
Planar equality
Pairwise central-symmetry
Pairwise row-symmetry
Symmetric-pair equality
Row-pair equality
Cross-directional equalities
Complementary equalities
Spatial Terms..... 5
Hyperspace
Hypercube
Hyperplane
Characterizations..... 6
Squares..... 6
Geonomic square
Perfect square
Near-perfect square
Punctuated perfect square
Ultra-perfect square
Cubes..... 6-7
Perfect cube
Absolutely perfect cube
Absolutely ultra-perfect cube
Hypercubes..... 7
Perfect hypercube
Absolutely perfect hyper-cube
Geometrical Properties..... 7
Continuous modularity
Complementary tile-patterns
Continuous complementary
tiling patterns

Preface

Geonometry's Potential for Advanced Extra-curricular Math Education

With the capability of Microsoft's Excel™ program, Geonometry can have extensive use in teaching mathematics. Exercises could be based on confirming the properties identified in the companion book using the extensive numerical tables up through the 5th dimension that are provided in the Author's other book, Number Magic.

This program series demonstrates that education can be taught using observations of wonderment, rather by rote. Practice should come from curiosity and exploration, not through the drudgery of repetitive on-demand homework. At least to those students who are naturally receptive to new ideas and can operate on their own.

After getting acquainted with Excel™, students could then be taught math up through Advanced Algebra, Trigonometry and Matrix Theory. Matrix Algebra can be readily performed in Excel itself and could be supplemented by internet-performed operations for free on the website www.quickmath.com.

In fact, there is a large portion of one of the later programs that addresses the closed form formulations that loom-tables have for Matrix Algebra topics such as: determinants, inverses, inner-product between complementary loom-tables and their matrix multiplication, and common eigenvalues between complementary loom-tables. And amazingly, all of these formulas only require the value **n**, the size of the square. These formulas were determined from the results obtained from quickmath. This is all demonstrated in the Appendix.

This new math can be taught at the Senior highschool / college Freshman level.

My Path of Discovery of this New Math

What is commonly known about the subject is called *magic squares*. This subject is classified by librarians in the category of **recreational math**. These number tables distribute the series of consecutive numbers from **1** thru $\mathbf{n^2}$ in a square matrix in such a manner that all the rows, columns and diagonals from corner-to-corner sum to the same number. Benjamin Franklin tinkered with these tables and discovered reasonably good squares for his time for the sizes **8** & **16** way back in the late 1700's. But this subject has languished since with just more sizes being discovered through trial and error. And for all instances of discovery but a few, that I can count on one hand, the vast majority neglected dealing with all the other diagonals which cross-over to the opposite edges and continue from there.

The subject that I have developed to its fullest is called **Geonometry**, short for *the natural geometry lain hidden in the natural number system*. It mostly involves basic arithmetic on numbers in tables. It includes all the diagonals formerly neglected to be included in the four-directional equal-summations. From that point on, it uncovers many more amazing equal-summing patterns never before even imagined to exist in this simple consecutive series of whole numbers with **1** at its core.

One of the first fundamental discoveries I made was the amazing integrated tiling patterns which all sum to the same number as do all the rows, columns and diagonals. These tiling patterns come in pairs, two per size of square. The tiling patterns only depend upon the table's size and are independent of the tables' number distributions. The tiling patterns can be dragged across a square numeric table, duplicated four times by having just one corner in common, like a table-cloth across a dinner table, and every tile within the pattern still sums to the same number. This has never been seen before.

Another amazing discovery was that most of the different classes of perfect squares each had a common 3-dimensional pattern in 2-dimensional space that summed continuously to a multiple of the square's characteristic number. Then it was discovered that some of these 3-dimensional patterns were pyramidal square and/or diamond shaped. Some classes of squares had both and at different height levels, all simultaneously. At certain sizes, these 2- and 3-dimensional patterns began to appear in the same square, sometimes up to a dozen different equal-summing patterns, all simultaneously.

The next major discovery was that each geonomic square could be decomposed into two complementary loom tables. These were easily obtained from a slight modification to the classical algebraic *modulus* and *integer* functions derived in the number base **n**, the size of the square. These loom-tables decomposed the original square with numbers from **1** thru $\mathbf{n^2}$ into two squares with numbers from **1** thru **n**. The numbers in both loom tables only range from **1** thru **n**, and not $\mathbf{n^2}$. These loom tables were extraordinary in that every row, column, all **2n** diagonals in both directions and each tile in both complementary tiling patterns contained the distribution of numbers **1** thru **n** exactly once, all simultaneously!

Now, this is beyond the mind of man to achieve on any square larger than size-5 by trial and error. Imagine the accomplishment of being able to generate the complementary loom tables with all the equal-summing patterns for the size-31 square with some simple arithmetic on a table with its numbers arranged in a very simple pattern.

The original square can be regenerated from these two complementary loom-tables from a very simple matrix formula which only involves scalar multiplication and matrix addition. Then came yet another major discovery: By swapping the loom-tables' positions in the regeneration formula, a dual square could be generated that had all the same properties of the original one. It was later found that, in higher dimensions, these dual tables came one-from-the-other in a series of generations that matched the number of dimensions.

Once it was observed how loom-tables could generate perfect geonomic squares, they were extended to generating perfect geonomic cubic tables in 3-dimensions and hypercubic tables in 4-dimensions. The tiling patterns found in squares were now observed in cubes and hypercubes. And the surprising discovery here was that the tiling patterns of higher-dimensioned tables possessed a hidden version of one of the loom-tables. It was later determined how its complementary loom table could be obtained directly from the original one. This indicated that these tiling patterns were fractal across dimensional levels.

Now, the size-9 square could never be generated from loom tables because its loom tables could never be fabricated from scratch – nothing added up for the size-9 square in a way that it had for other squares. In extending Geonometry to the 5-th dimension using the size-3 quadracube, which was not perfect at the time, and collapsing the resulting size-3 quintacube along its 4 different axes one at a time, a perfect size-3 quadracube was obtained from one of the depth-sum tables. And again it was yet another discovery – it was also a perfect size-9 square! And that could only have come about from the fractal property of Geonometry among different dimensional levels.

Now, all this is just the tip of the iceberg. Squares were found to contain inscribed circles whose incident cells summed to the same number as all its other summations; cubes were found to contain spheres whose incident cells summed to the same number as all its other summations; and 4-dimensional hypercubes were found to contain inscribed toruses that also did likewise. What was happening here was this: this new math was actually measuring and mapping the natural harmonics in the fabric of space.

To prove this, these complementary loom-tables were then shown to describe the distribution of electrons around the nucleus and shell-pair electron fill-sequence of all the atoms. Further, it was demonstrated that the complementary tiling patterns accounted for the pairing of electron shells throughout all the atoms and is the reason the Periodic Table of Elements rows are duplicated in pairs. Additionally, these complementary loom tables were found to cloak each other, giving basic credence as to why actual electron orbits could not ever be observed directly by atomic physicists.

Once properties of 4-dimensional Geonometry were determined, they were applied to explaining the relationships among baryons at the sub-dimensional level **1/4D**. There are **6** of them, only **2** of which are stable and make up the neutron and proton of the atom's nucleus. Geonometry was able to explain how these baryons got the peculiar names that scientists gave them by showing their mathematical relationships in 4-dimensional loom-tables.

Although yet in its early stage of development, it was demonstrated how the double zipper-like pattern between strings of even and odd numbers of a certain class of geonomic square could account for the curling-up of nucleotides into the life-form double-helix of molecular biology. This could imply that life automatically springs forth anytime conditions are favorable, anywhere in the universe.

Geonomic hexagons were also found to possess the inherent pattern for snowflake formation in one of their loom-tables. No chemist has ever satisfactorily explained this scientifically.

Now don't be alarmed that this new math is driving off a cliff here; these properties and distribution patterns are quite obvious when depicted numerically and highlighted in color.

That's one of the major properties of this new math: it needs and uses color throughout to demonstrate the amazing integrated and interconnected patterns which all sum equally. No classical math uses color; it's all black and white, chalk and blackboard. Geonometry requires color to comprehend and to operate within it.

Further, this new math does not involve decimal fractions or imaginary numbers like classical math. All fractions are expressible in whole-number factors throughout. So the math is fairly easy; it's the concepts that are awe inspiring.

Overview

Geonometry

Geonometry is essentially the natural geometry hidden in the natural number series.

The math only involves addition, subtraction and multiplication of integers. Occasionally division is called for but there are no numbers expressed as decimals and numbers expressed as fractions always lead to whole numbers.

The math involves matrices, but not those governed by the rules of Matrix Algebra. It has its own set of rules and formulas that are comparatively very simple.

Unlike Matrix Algebra with all its complicated functions, Geonometry involves only two functions from classical math and those are the modulus and integer functions. Those have merely been reformulated to avoid producing **0**'s and better alignment of the numbers 1 and **n** in each function.

Geonometry uses Microsoft's Excel software entirely. In fact, without Excel, this new math would never have been discovered and developed.

Here are some of the amazing properties discovered in Geonometry:

In 2-dimensions, every square table consisting of the number sequence **1** thru **n-squared** has summations which all add up to the same number in 4 directions: i.e. for every row, every column and every major and minor diagonal, including all those diagonals starting between corners that must wrap across the edges going up or down. That number is only dependent upon the size of the square and is called the size's **characteristic number**.

Further, every set of numbers at the four corners of every rectangle and rhombi centered over the very middle of the square, called a **quadral**, sum equally to a different number too, that is a specific whole-number fraction **4/n** of the square's characteristic number.

The square tables can be segregated into **6** classes, some with subclasses within them, that have very distinctive properties. All classes of square have tiling patterns where each interlocking tile sums to the square's characteristic number too or a multiple thereof.

Moreover, there are always two different complementary tiling patterns which do so simultaneously for every size of square., and all the individual tiles in these interlocking tiling patterns sum equally, no matter where their centers are located in the square. Further, each size Class-1 square has its own pair of tile patterns that are unique to its class and size and are found nowhere else.

Class-3 squares are of sizes that are a **3** times multiple of an odd number **b** >**3**. The tiling patterns in these squares are the tiling patterns characteristic of the Class-1 square of size **b** but in **3-by-3** block-squares and these tiling patterns sum equally everywhere to **3** times the square's characteristic number..

In Class-4 squares whose size is a multiple of **4**, there is a different kind of tiling pattern formed from a **ring-diamond** and an **X** pattern which also sum equally continuously everywhere in the square. And because they do, the two patterns are indistinguishable from one another. The tiles in these tiling patterns do not intersect one another either but fit together like a horizontal stack of chain-link fences.

The Class-5 squares, which are of a size that is a product of two distinct odd-prime numbers **a** and **b**, both greater than **3** and not equal, have block-squares of sizes **axa** and **bxb** that sum equally at the block-square level in the complimentary tiling configuration of both Class-1 squares of sizes **b** and **a**, respectively, and these **4** simultaneous tiling patterns sum equally everywhere to **a** or **b** times, respectively, the square's characteristic number.

Overview

Class-6 squares are squares whose size is a power of an odd-prime number **b** where $\mathbf{n = b^a}$. There, the tiling patterns are the **bxb** block-squares in a tiling configuration of the size **b** Class-1 square which all sum equally to **b** times the square's characteristic number everywhere in the table.

Now each of these "**primal**" squares in all classes but Class-2 have what is called a "**dual**" square, each of which can be derived directly from the other by what are called **loom tables**. These tables are the **modulus** table and the **integer** table and are initially derived from the primal square. In fact, they regenerate the primal square according to a very simple formula.

Here's the surprising part: by interchanging the location of the two complementary loom tables in the regeneration formula, a **dual square** is derived with all the properties of the primal. And further, if now those functions are applied to the dual to get its two complementary loom tables, what was obtained from the integer function on the primal square, is now obtained from the modulus function on the dual square. And further yet, what was obtained from the modulus function on the primal square, now is obtained from the integer function on the dual square.

When cubic numeric tables are examined, not only do the rows, columns and diagonals all sum equally in every planar square slice of the cube, of which there are **9** directions, but all the characteristic tiling patterns in embedded squares do too. And what were equal-summing squares and rhombi in squares, now become **octals** ("boxahedrons") and **octahedrons** equal-summing separately to two distinct but related values in cubes.

Further, all the numbers in the squares that are embedded within the cubic rectangular-table all sum equally to the cube's characteristic number. And in odd-size cubes, their central numbers all together sum to the characteristic number for the size of cube too.

The properties of cubes can be easily seen by collapsing their channels along each of their 3 axes in what are called **depth-sum tables** and treating each of these 3 square tables geonomically like squares.

One of these depth-sum tables will convert to a regular square by a specific division and subtraction. That is, the cubes will collapse along one of its axes to a perfect geonomic square by a very simple arithmetic process involving the value-difference between the characteristic numbers of the square and cube of the same size.

Without going into detail, all these properties determined for cubes are demonstrated to get propagated into 4-dimensional cubes, called **quadracubes**, in addition to having yet even more amazing summation properties.

In Class-4 **squares,** there are maximally inscribed **circles** whose numbers in the cells, just impinging the circle from the interior, also sum exactly to the square's characteristic number.

In Class-4 **cubes**, there are maximally inscribed **spheres** whose numbers in the cells, just impinging the sphere from the interior, sum exactly to the cube's characteristic number.

In Class-4 4-dimensional **quadracubes**, there are **8** hemispheres which sum equally in the same fashion to half of the quadracubes characteristic number. Further, the sums of each embedded cube throughout the 4-dimensional square table also sum to the characteristic number of the quadracube. And each embedded 2-dimensional square sums equally to all the 1-dimensional strings running throughout the quadracubic table.

The Class-1 **quadracube** not only has a dual, it has a series of duals which lead back to the original quadracube through the use of their derived loom tables. The same goes for **cubes**; they have a series of **3** successive duals, whereas the quadracube has a series of **4**. In all higher dimensions of dimension **k**, there is a series of **k-1** distinct duals leading back to the original. That is why in 2-dimensions, there is only **1** distinct dual for each primal square.

When any two of these **quadracubic** duals are laminated together, the equal-summing intersecting embedded hemispheres become **8** equal-summing spheres. This translates into a 4-dimensional

Overview

torus. And because there are **4** combinations of **4** duals taken **2** at a time in a circular series, there are **4** intersecting toruses that make up 4-dimension space as perceived from the 5th dimension.

For dimensions of odd prime-number size, these dual series may be navigated in many different ways depending upon one single parameter in the generation formula.

There is a very special type of primal square with duplication of numbers allowed, called the **matchmakers square**. It has **5** unique properties of its own. One of the chief properties of a size **n** matchmaker square is that the matching-up of each row with a distinct column and summing the numbers in those intersections will always result in the same number **n-squared**. The positions of these columns and rows can be independently interchanged at random as many times as desired, like the spinning of a Rubric's Cube, and this characteristic, equal-summing property will prevail.

Geonometry demonstrates just how a matchmaker square of odd-prime-number size will convert to a perfect geonomic Class-1 square of the same size.

A special algorithm is subsequently presented that will convert the resulting perfect square to a square with all the tiling properties of an ultra-perfect square

There are **four** unique expansion methods for deriving perfect composite size squares among **4** of the **5** classes of squares which are composites, of which **two** of these methods apply to multiple classes of square. There is only one (the last one – the matchmaker's square just cited) that is the very source of all Class-1 squares which makes it all possible.

Triangular tables of consecutive numbers originating with the number **1** are shown to have nested triangular frames all with equal-summing sides or embedded 3-celled triangles.

Geonomic diamond-shaped tables are shown which have a characteristic number for their size. They are shown to be readily derived from two geonomic squares of consecutive sizes.

Hexagonal tables are shown which have complementary loom-tables too. These loom tables generate a dual hexagon of the same size with all the same properties as the original. One of the complementary hexagonal loom tables always contains a perfectly-symmetric snowflake pattern; the other, always a Xenia flower pattern.

The mathematics of Geonometry is seen to be very different than classical math, yet interfaces with classical Matrix Algebra with closed functions for basic operations of inner and outer products of matrices, matrix inverses and eigenvalues. As such, it demonstrates that Geonometry is the missing link between basic 2-dimensional arithmetic and Matrix Algebra.

Further, the characteristic number of squares and cubes are correlated 100% with index triangles. And moreover, the characteristic numbers of squares can be represented by a quadratic polynomial in classical Algebra. The characteristic numbers of cubes can be expressed as an irreducible 3rd-degree polynomial. In fact, the characteristic number of a geonomic table of any dimension **k** can be represented by the average of its size **n** to the powers of **2k-1** and **k-1**, all of which can be expressed as an irreducible **k**th degree polynomial. So all of these algebraic expressions are equal among characteristic numbers, thereby providing the basis for a whole new multidimensional algebra.

Geonometry is capable of explaining the distribution of electrons in nested orbital shell-pairs of atoms, the interpretation of the relationship between baryons in the atom's nucleus as patterns found in the 4th dimension, and the framework underlying snowflake formation, none of which Science itself is capable of explaining neither on its own terms nor in its contorted 11-dimensional Calculus.

This new math will clearly give attendees an advantage over their future college classmates who will remain ignorant of the subject as this math is not taught in schools or colleges. It has many existing and potential applications to Science and Engineering. And It has yet to suffer a single criticism that is based on technical merits.

Overview

Applications to date

The program series presents six applications of Geonometry to date:

1. The identification of a harmonic pattern in the distance of the planets from the sun relative to the orbit of Jupiter.
2. The discovery of the spatial vibrational membrane pattern that enables nucleotides to assemble themselves into the double helical structure of DNA.
3. The membrane description of the distribution, count and orbital paths of the electrons in atoms and the reason electrons operate in shell-pairs.
4. The clarification of the relationship among the six quarks which make up the six baryons of which the proton and neutron are the only two stable combinations.
5. The mapping of the structure of 4-dimensional space as four intersecting toruses.
6. The mathematical description of how perfectly-symmetric hexagonal snowflakes form.

Foreword

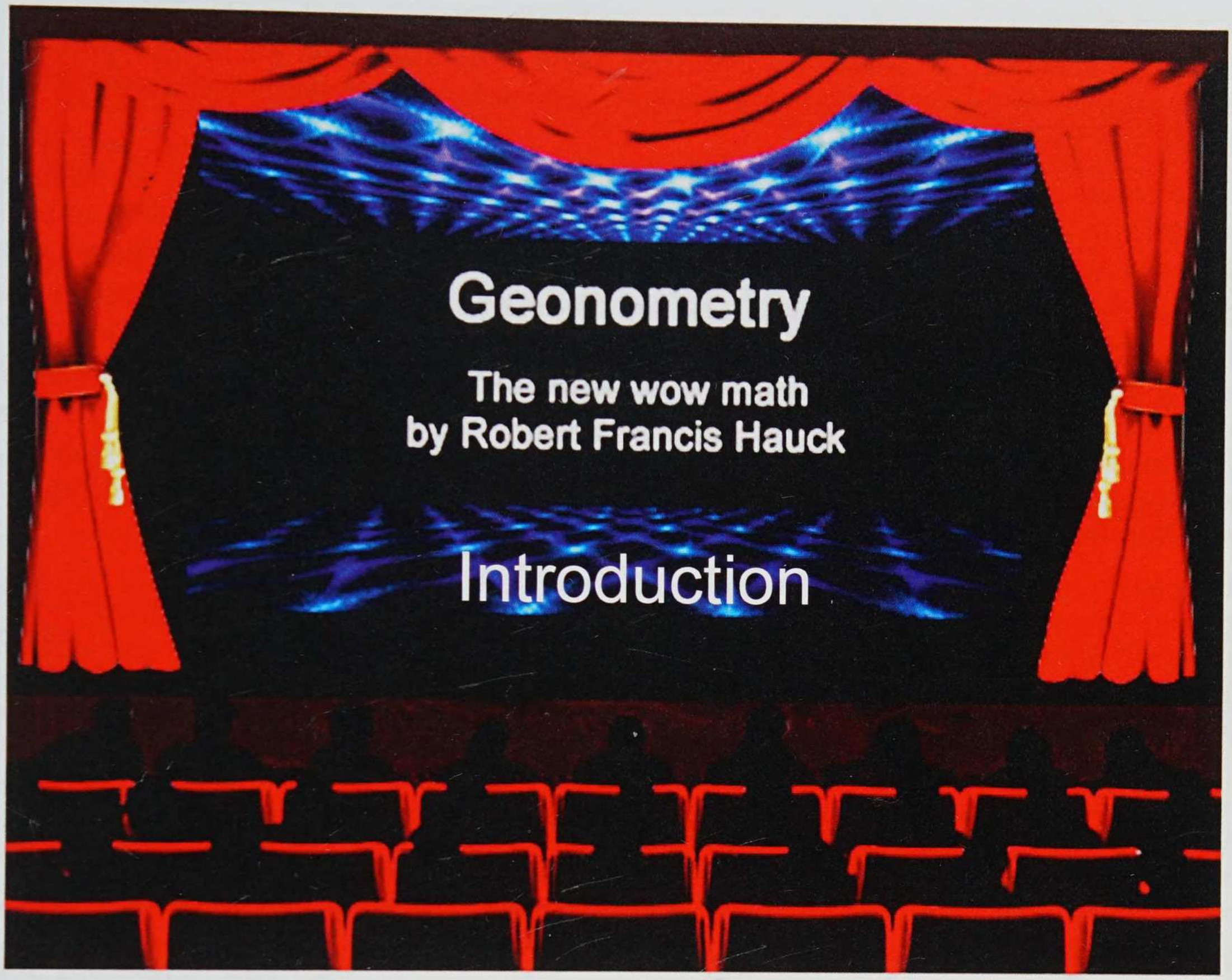
Here's what I have concluded is happening with this new math: the patterns that are being uncovered are really those vibrations which occur naturally in empty space that make up the actual fabric of space itself. This program series demonstrates just how these complementary tiling patterns cloak one another and must therefore co-exist simultaneously. Further, this program presents mathematical evidence that these patterns must exist at all dimensional levels induced by the process of table-expansion and at all sub-dimensional levels by the mapping of the **k**-th dimension upon the sub-dimension of **1/kD**. These harmonic fundamental spatial patterns can be of any size, from nano pulsations to galactic undulations, all simultaneously.

The immediately promising application of this new math is the harnessing of these vibrations for levitation or propulsion. Theoretically, devices could be developed that would produce vertical pulsations in a planar array in accordance with one or more of the complementary tiling patterns in Class-1 squares, and later in a 3-dimensional tiling configuration of Class-4 cubes, to decloak the energy lying hidden in its complementary tiling pattern. Then perhaps space could be surfed like waves in the oceans or the pulsations be harnessed with the counterpart of sails to catch the circular wind.

In light of the mathematical correlations made in this program between complementary loom tables and the distribution of electrons about the nucleus of atoms, this is more promising than being just mere conjectural thinking. It is this implication that should be being conveyed to promising math and science students of the up and coming generation. There are numerous experiments that could be made involving arrays of pulsations of sound (phasers), electromagnetic and laser energy in the interwoven numerical patterns that are identified in this program series.

With this new math, you just never know what will arise until you test a conjecture. Given that I could quickly test any formula or check out a correlation so readily with Excel software, in the eleven-year period that I was developing this new math, I could never predict what would occur until I actually tried something. Then when something worked, it was usually beyond my expectations and I would say to myself, "Wow, that's just amazing! Who could have foreseen that?" That's one of the reasons I began referring to it as "Wow! Math". I claim that this new math was already there just awaiting discovery by whomever stumbled upon it. Later when I started making presentations about it, at various points throughout the lectures, I would hear the audience saying "Wow!" in unison.

With this new math, it's like in the days of the Western Gold rush; it's all out there awaiting yet even more discoveries. It just takes some dedicated prospecting.



Welcome to the 10-part program series on the new wow math called Geonometry. It is called Geonometry because it is the discovery of the naturally-occurring geometry hidden in the natural number series**1, 2, 3, ...** etc. This program series will demonstrate that these amazing equal-summing number patterns are actually the harmonic frequencies that form the basis of the real fabric of space.

This math has its formulas as all maths do, but the math is quite elementary in that it involves predominantly basic arithmetic on integral number tables. It was developed by using Microsoft's Excel program on a laptop computer. So the math is actually quite comprehendible at the Senior highschool / college Freshman level.

But the most interesting element of this new math is that it predominantly consists of observable patterns through the use of colors. So now coloration becomes a key element in observing the spatial patterns. And despite the actual formulas, the patterns are clearly visible to everyone, even to those who don't comprehend the math. That is why it has been dubbed the **new wow math**.

It will be seen that this math does have practical applications in real space. This new math will clearly give attendees an advantage over their future classmates who will remain ignorant of the subject as this math is not taught in schools. It has yet to suffer a single criticism that is based on technical merits.

This program series demonstrates that education can be taught through observations of wonderment, rather by rote. Practice should come from curiosity and exploration, not through the drudgery of homework on demand – at least to those students that are naturally receptive to new ideas.

With the capability of Microsoft's Excel™ program, which was originally developed for book-keeping and cost/billing summarizations by VisiCalc Corp. in Silicon Valley of California back in 1978 (and later acquired by Microsoft), it can have extensive use in teaching mathematics. Exercizes could be based on confirming the properties identified in this companion book using the extensive numerical tables up through the 5th dimension that are provided in the Author's book, **Number Magic**. These tables will soon be made separately available in Excel.

After getting acquainted with Excel, students could then be taught math up through advanced Algebra, Trigonometry and Matrix Theory. Matrix Algebra could be supplemented by internet operations readily performed on the free website **www.quickmath.com or** in **Excel** itself. This is how I discovered and developed the amazing properties of this new math and later confirmed the interconnection of Geonometry with classical math as demonstrated in the Appendix.

* * *

Geonomics is totally my own contribution to Mathematics

This program series with its PowerPoint presentations and this companion book were all made by me without any help from so-called *experts*. I didn't have the money for that and no one would give me any under a grant for such endeavors because it wasn't priorily peer reviewed.

There were over 200 revisions and 60 reprints over the last 8 years

1. to get rid of typographical errors,
2. to get the cross-references corrected with each reorganizing revision,
3. to get the number tables to print clearly in minute detail,
4. to do all the detailed coloring of the tables, differently for both printing and screen projection,
5. to get the coloring not to overwhelm patterns or to camouflage the numbers,
6. to get the math categorized, organized and systematically integrated for presentation,
7. to formulate the occurances of properties into basic known algebraic expressions,
8. to get the new properties named appropriately without corrupting or conflicting with existing classical terminology,
9. to clearly present the mathematical concepts pictorially as well as descriptively,
10. to provide proofs of the math in both Excel and the Appendix of this companion book,
11. and to find fundamental novel applications in Science and Technology.

I admit that I couldn't have accomplished this Program Series without Microsoft's Word, Excel and PowerPoint software, Adobe's PhotoShop for illustrations and PDFLite's PDF file-generator for self-publishing. All this was totally new to me back in 2000, just 15 years ago.

List of the program topics and description of their subject matter

Here is the list of the program topics and description of their subject matter by program sequence:

Program 1

Introduction

Gives the basic definitions for patterns and summations involved in Geonometry. Categorizes the number tables by table-size into 6 distinctly different classes, each of which has their own unique characteristic equal-summing patterns. It provides examples of each class.

Program 2

Part I: Tiling Patterns

Uncovers the amazing interlocking equal-summing tiling patterns hidden in every class of square. It depicts the natural quilt patterns in spatial membranes.

Part III: Anchor-dot Patterns

Identifies the distribution patterns of uniformly scattered numbers which additionally and continuously sum to the square's characteristic number for all squares – except Class-2 squares which lack this property.

Program 3

Part I: Loom Tables

Introduces loom tables derived from the original table; shows how the primary dual table is constructed from them and demonstrates the fundamental mathematics underlying the real fabric of space.

Part II: Cloaking Patterns

Demonstrates the continuous joint cloaking properties of complementary tiling patterns on loom tables for all but Class-2 squares.

Application #1 of Geonometry – The membrane of life

Program 4

Part I: Continuous Diamond Quilt Patterns and Triangular Tapestries

Shows the many diamond patterns that exist in individual geonomic squares.

Part II: Unique Tiling Patterns at the 2x2 block-square level

Shows how these diamond patterns sometimes can be divided into 4 triangles that behave as single equal-summing tiling patterns composed of just two integrated triangular patterns.

Part III: 3-Dimensional Patterns in 2-Dimensional Space

Program 1

Program 5

Cubes
Explores the properties of 3-dimensions and the amazing equal-summing 1-dimensional and 2-dimensional patterns discovered in Geonomic cubes through the compact view of *depth-sum* tables. There begins the incredible mathematical journey up through higher dimensions as only Geonometry can provide.

Application #2 -- Geonomic description of the atom
It is demonstrated just how the mathematics of 2- and 3-dimensional Geonometry can explain and account for the distribution and navigation of the electrons around the nuclei of atoms.

Program 6

Quadracubes
Depicts 4-dimensional space mathematically as never seen before in such a new dimensional perspective.

Application #3 – Accounting for the sub-atomic substructure of the proton and neutron
Equal-summing patterns in Class-4 quadracubes are demonstrated to unlock the relationships among quark-pairs at the sub-dimensional level of **1/4D**. Here Geonometry independently confirms what sub-atomic scientists have discovered through their linear accelerators, as an inherent property of the 4-dimensional spatial fabric.

Program 7

Part I: Matchmaker Squares
In this segment, exploration of the properties of a seemingly recreational geonomic square leads to the discovery that these special number tables serve as the very twine that is involved in weaving the fabric of space. One of these complementary loom tables leads to the manifestation of both complementary loom-tables for Class-1 squares for all prime-number sizes and provides for their systematic generation.

Part II: Methods for generating composite-size squares
Knowing how to generate Class-1 squares, this segment shows how to generate Class-2 squares from the doubling of Class-1 squares. Then having Class-1 and Class-2 squares generating all the other classes of square is described in detail.

Program 8

Advanced Perspectives of Geonometry
Here we get an even broader picture of the numerical spatial fabric and its pervasiveness by observing squares, cubes and quadracubes from many new perspectives.

Program 9

Navigating Multidimensional Space
Here it is demonstrated how to explore the properties of any higher dimension through the unique mathematics of Geonometry. The math involved is no higher than basic Algebra. From this is obtained a whole new perspective of real multi-dimensional space.

Other Geonomic Forms

Here Geonometry explores nested-frame triangles. It shows relationships discovered between diamonds and squares. It addresses constructing nested-frame pentagons and hexagons that have inherent intelligent patterns in their complementary loom-tables of the same configuration.

Application #4 – Demonstrates the Mathematical Discipline behind Snowflake Formation

For hexagons, it demonstrates just what makes snowflakes so hexagonally symmetrically perfect. Scientists are still to this day unable to systematically explain this phenomenon by the chemical makeup of water alone. It demonstrates again and again in an undeniable manner that Geonometry is really uncovering the very fabric of space itself.

* * *

First, some points of recognition are in order.

This program series and this companion book was written and compiled by Robert Francis Hauck, the mathematician independent of any organization or academic institution who originally developed its contents.

What began as a series of PowerPoint slide presentations was later converted into a series of narrated video programs.

Today this video series is being made available under a 10-part pre-sponsored scholarship program for advanced–placement students, their parents and math teachers, free in corporate assembly halls and academic auditoriums throughout Northern California.

The pictures and tables in this program are protected from reproduction by the following copyrights already granted by the United States Government's Register of Copyrights:

TXu1-302-220 TX 1-307-747
TX 1-360-951 TX 1-365-314
TX 1-626-704 TXu1-884-365
TX 7-018-696 TX 7-155-560
TX 7-360-814

Additional copyrights are pending for new tables and patterns shown for the first time in the video program and here in its companion book.

Program 1

Overview

This program series is a real mind trip. To comprehend what is presented here, you will have to have an interest in mathematics and be of above average intelligence. Almost every slide describes something never seen before.

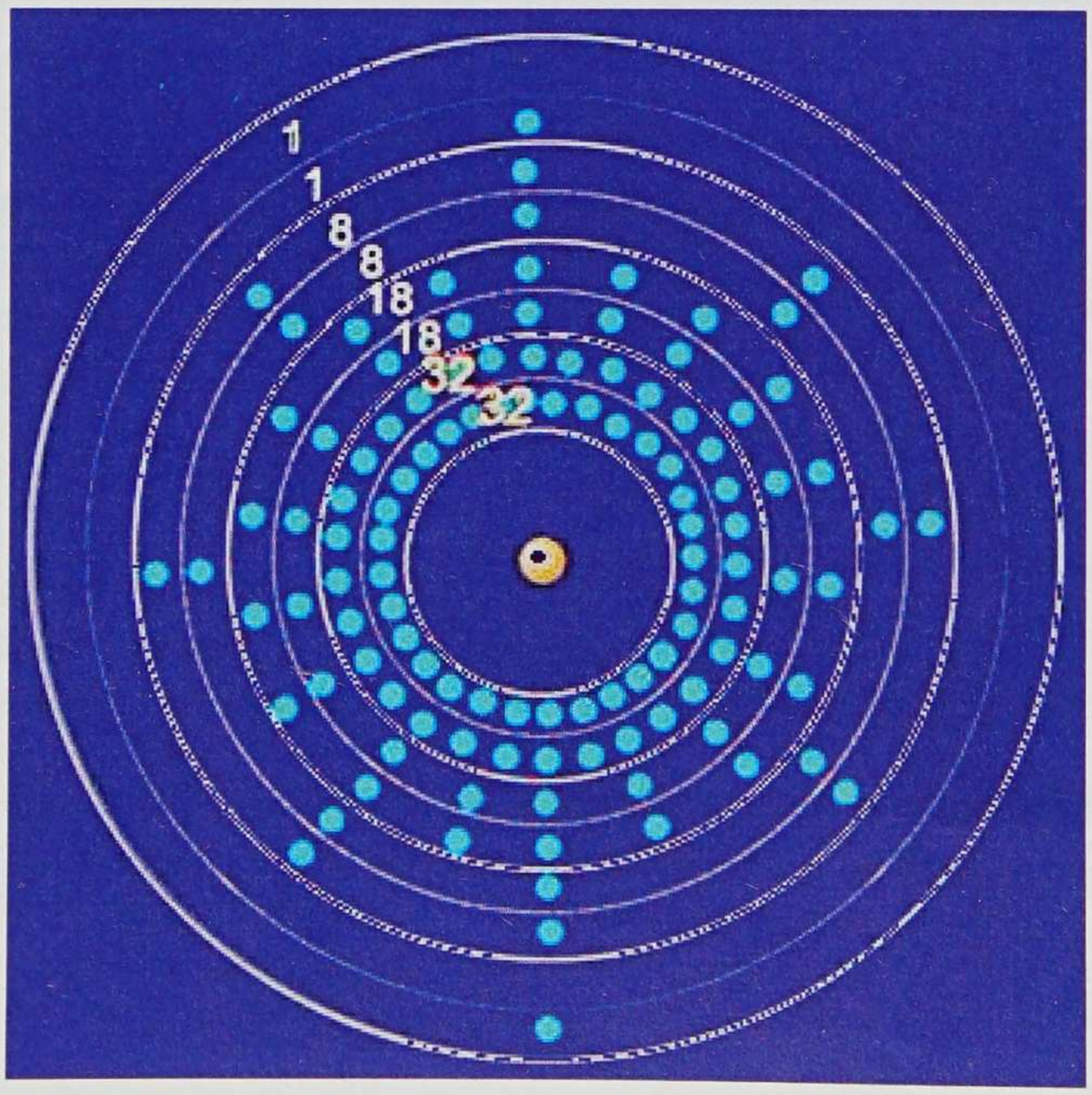
It is a whole new mathematics, yet on the highschool Senior / college Freshman level. Rarely is any higher math than simple algebra involved. Most of the math involves merely addition, subtraction, multiplication or division – simple grade school arithmetic. It's the concepts that are exceptionally advanced. And it is the fascinating presentation of the concepts in color – something usually not done in explaining mathematics.

What is this new math good for?

People are always questioning me what this math is good for. In this introductory portion of the program series we're going to talk about some potential applications.

When I was in college, I would stumble along in my math classes because I couldn't see how it applied to anything that might be of interest to me. That is until we got to the applied math part. Then it would all come together and make sense for the first time and I would ace the final exam. I had to first see the BIG picture.

So from this experience with my initial ignorance, I am going to show you in the next few slides some major applications that this new math has been and would be good for.

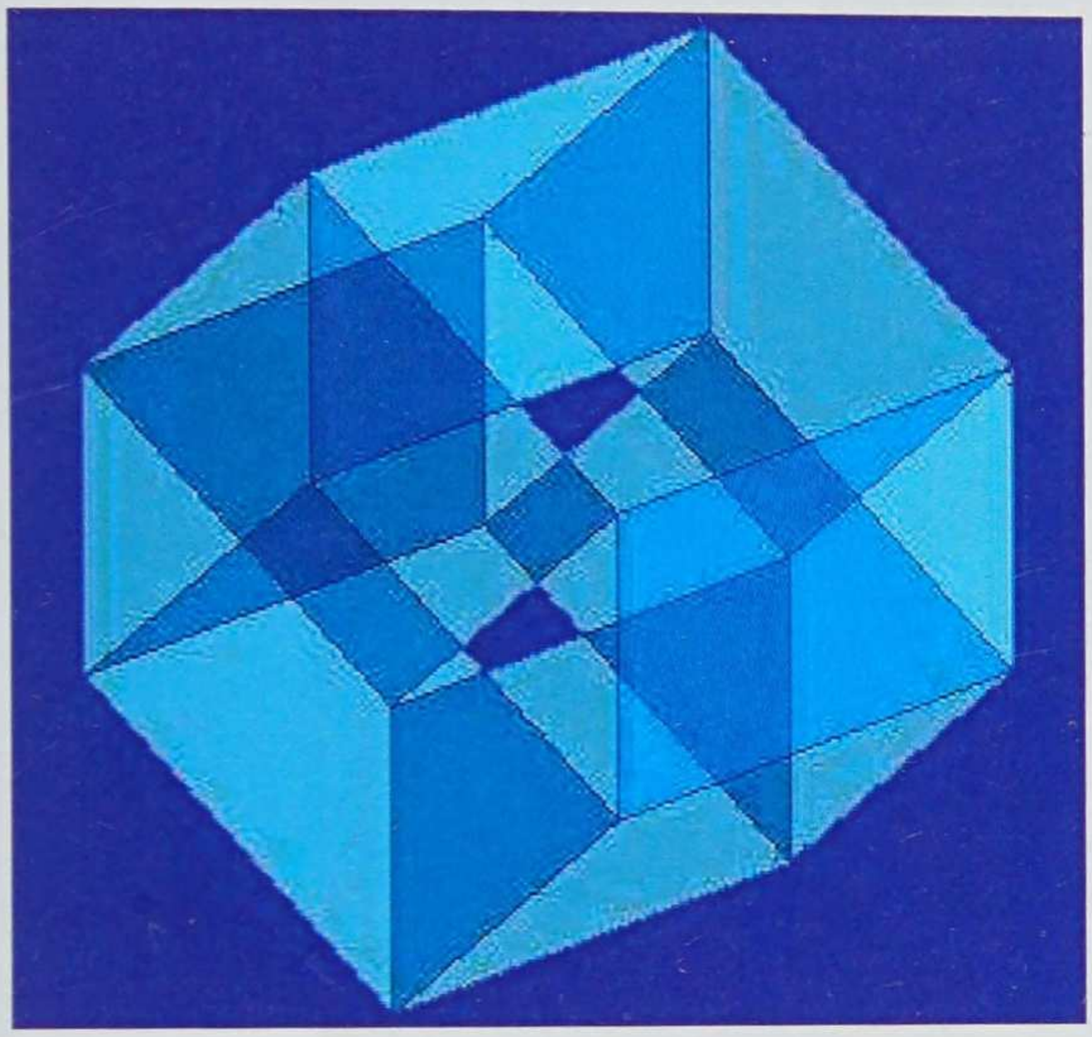


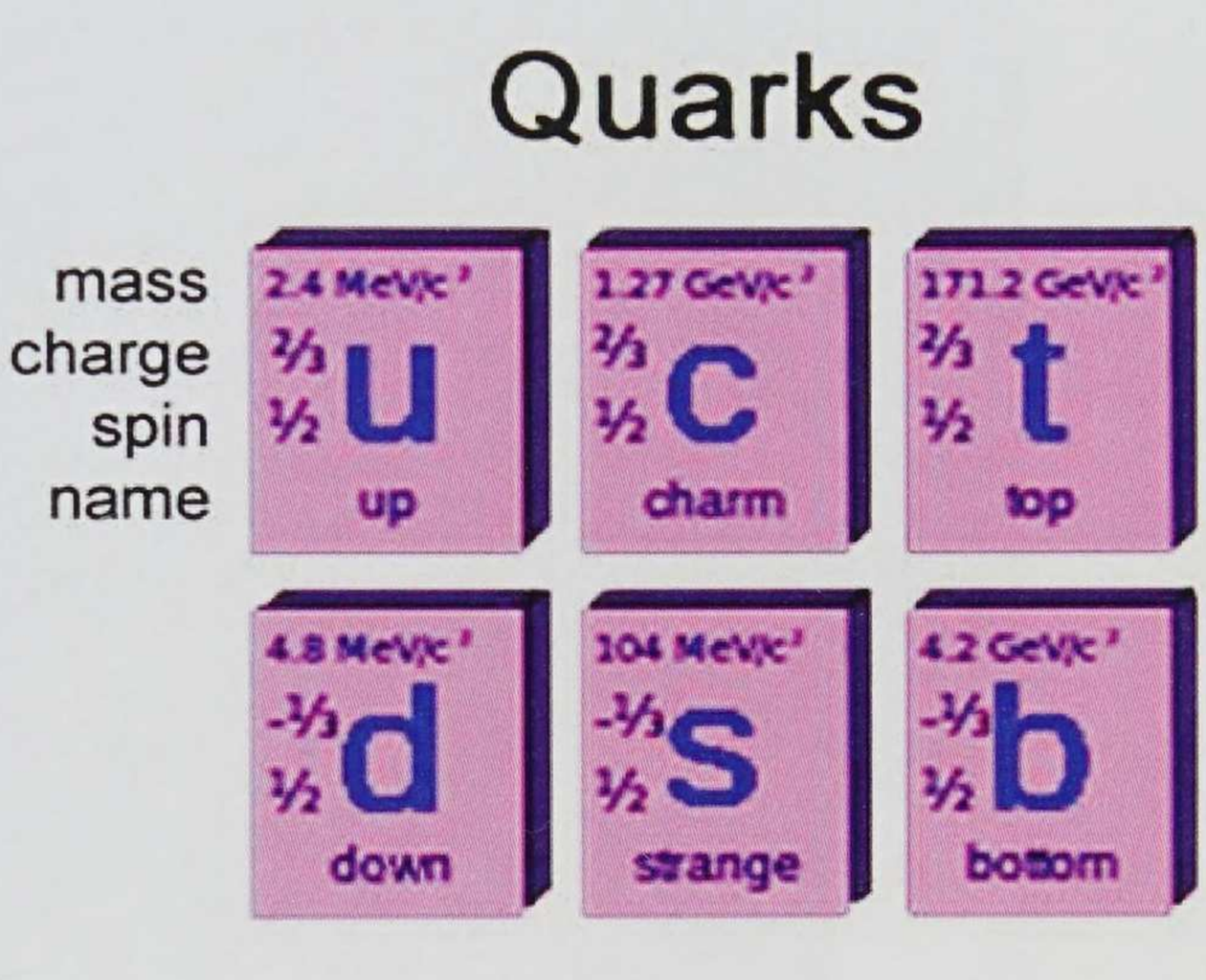
One application demonstrates amazing tiling patterns in the 2nd dimension that are numerically harmonic in the 3rd dimension. And these provide an accounting for the number, distribution and interaction of the electrons in all the atoms in sub-dimension **1/3D**. This is dealt with in Program 5.

Program 8 demonstrates that these same harmonic patterns found in 3 & 4 dimensions, themselves contain harmonic patterns embedded within them!

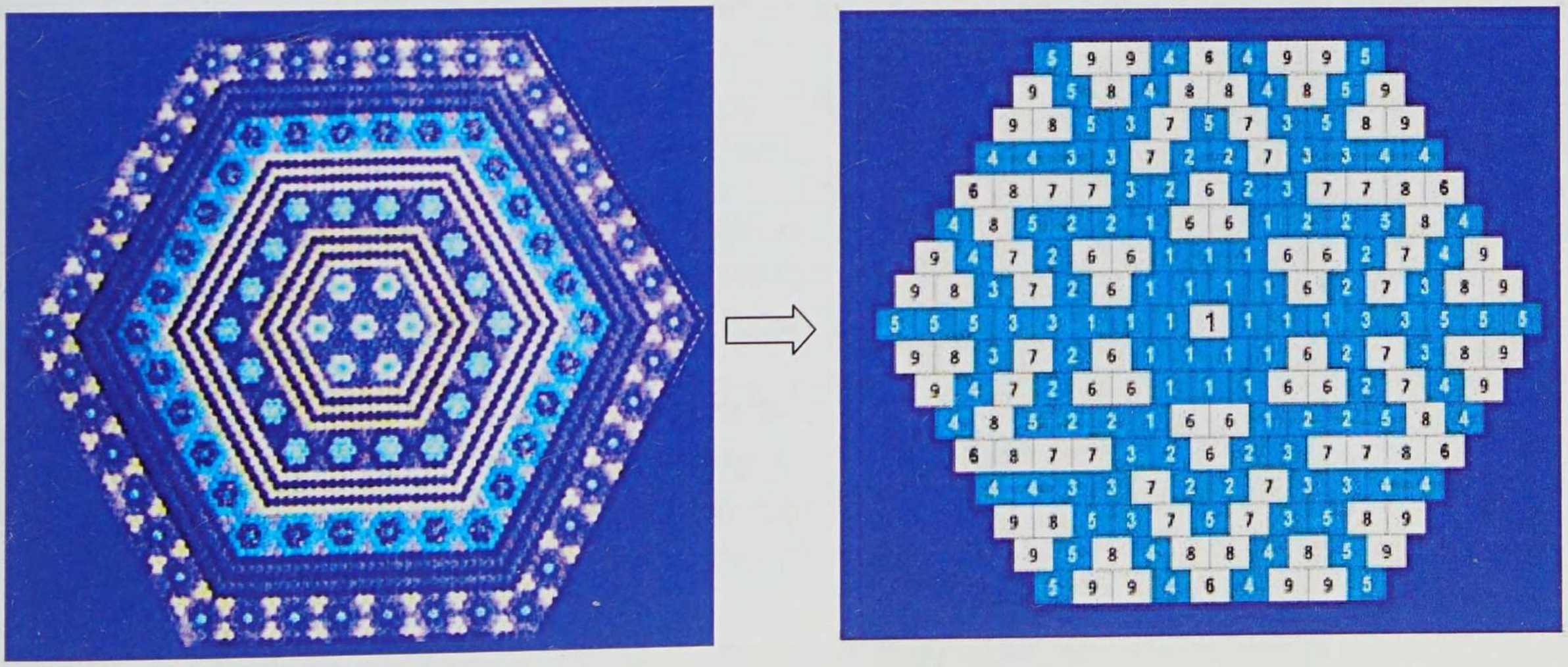
Program 1

Another application offers our first, potentially real view of the 4-th dimension. It demonstrates the application of 4-dimensional spatial patterns to explain the properties of quarks at a sub-dimensional level **1/4D**. This is addressed in Program 7.





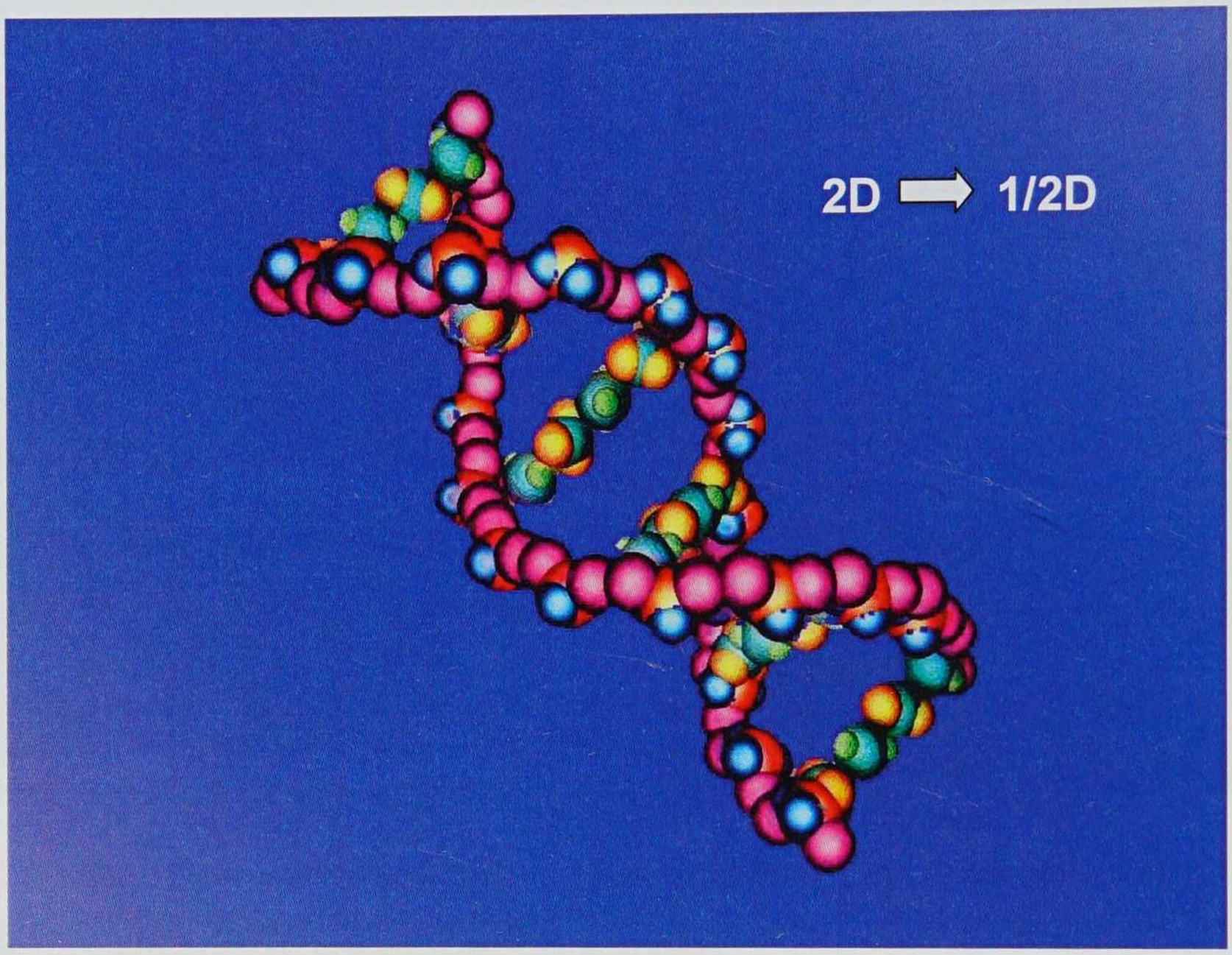
Another application demonstrates mathematically that snowflakes are really be being fashioned from the harmonic quantumized vibrations in a spatial membrane when a frozen blanket of vapor contracts into isolated hexagons. This is addressed in Program 10.

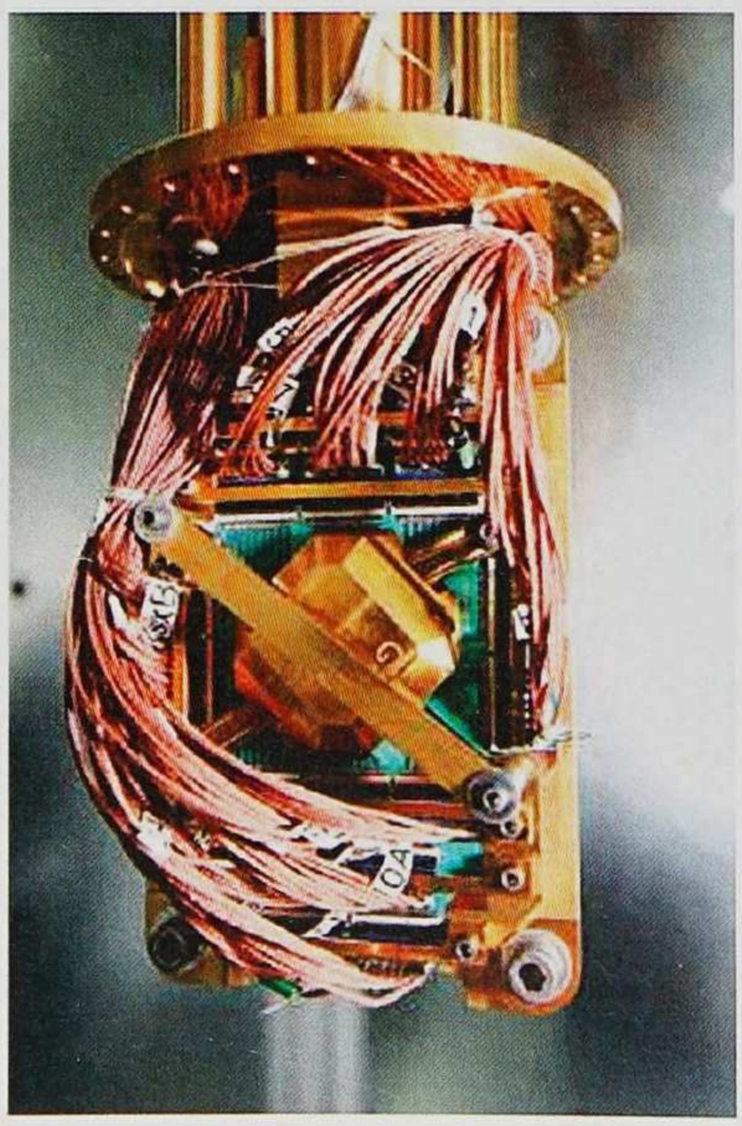


Program 1

This program series will demonstrate time and time again that space must be composed of a vibrating harmonic fabric.

A potential application is the identification of spatial templates upon which biological DNA molecules could assemble themselves into double helixes and form reproductive molecules. This is encountered in Program 3.





A definite application lies in the data-scanning methodology for quantum computing (explained in detail later at the end of Part I of Program 3).

Picture: Redux D-Wave Systems' quantum computer processor pictured without its protective thermal canisters at the company's lab in Burnaby, British Columbia, Canada.

Photo by Kim Stallknecht of
The New York Times

Spaceflight

The picture here shows an actual photo taken of an **F-118** U.S. fighter-jet as it just breaks the sound barrier.

Only back in 1903 did we learn how to fly with a propeller-driven engine. Jet fighter planes appeared in Germany in 1942 where they were invented. Not until 1947 after World War II, was the sound barrier broken for the very first time by a jet engine. On June 2nd, 2010, Boeing's **X-51 Waverider** rocket powered by a scram-jet engine flew **6** times the speed of sound for three minutes. That's over 4600 mph. Most of this progress in flight in air took place in just the last 60 years. So all the amazing things that you see today were invented in just the last three working generations of engineers.

In fact, the digital computer revolution took place 5 years after I graduated from college. Before that we used slide-rules and later hand-held digital calculators after the digital chip was invented and made programmable. I was programming main-frame computers up to 1974 that used tiny Cheerios-style magnetic donuts strung together in a fabric of copper wire to store information in **1**'s and **0**'s called "bits", a short term for the phrase "bits of information".

Given the rate of technological progress by each new generation of engineers and technicians in just the last half century, going faster than light speed into the 4th dimension doesn't seem to me all that remote.

Program 1

Just as we have in times of civilizations past, where we used the undulating patterns in the oceans to surf its waves, or the sails on boats to capture the circular patterns of wind to cross the ocean-divides, it may now be possible to identify the spatial vibrations among the vibrating membranes of space that could be harnessed to surf the Cosmos.

To travel in space, we need something yet even more innovative than thrust from a scramjet. Given the inherent spatial vibrations whose existence will be made quite evident mathematically in this program series, we should begin looking for an electro-magnetic mechanism or a planar nano-pulse generator that will generate complementary pulsations that will surf these hidden planer spatial vibrations.

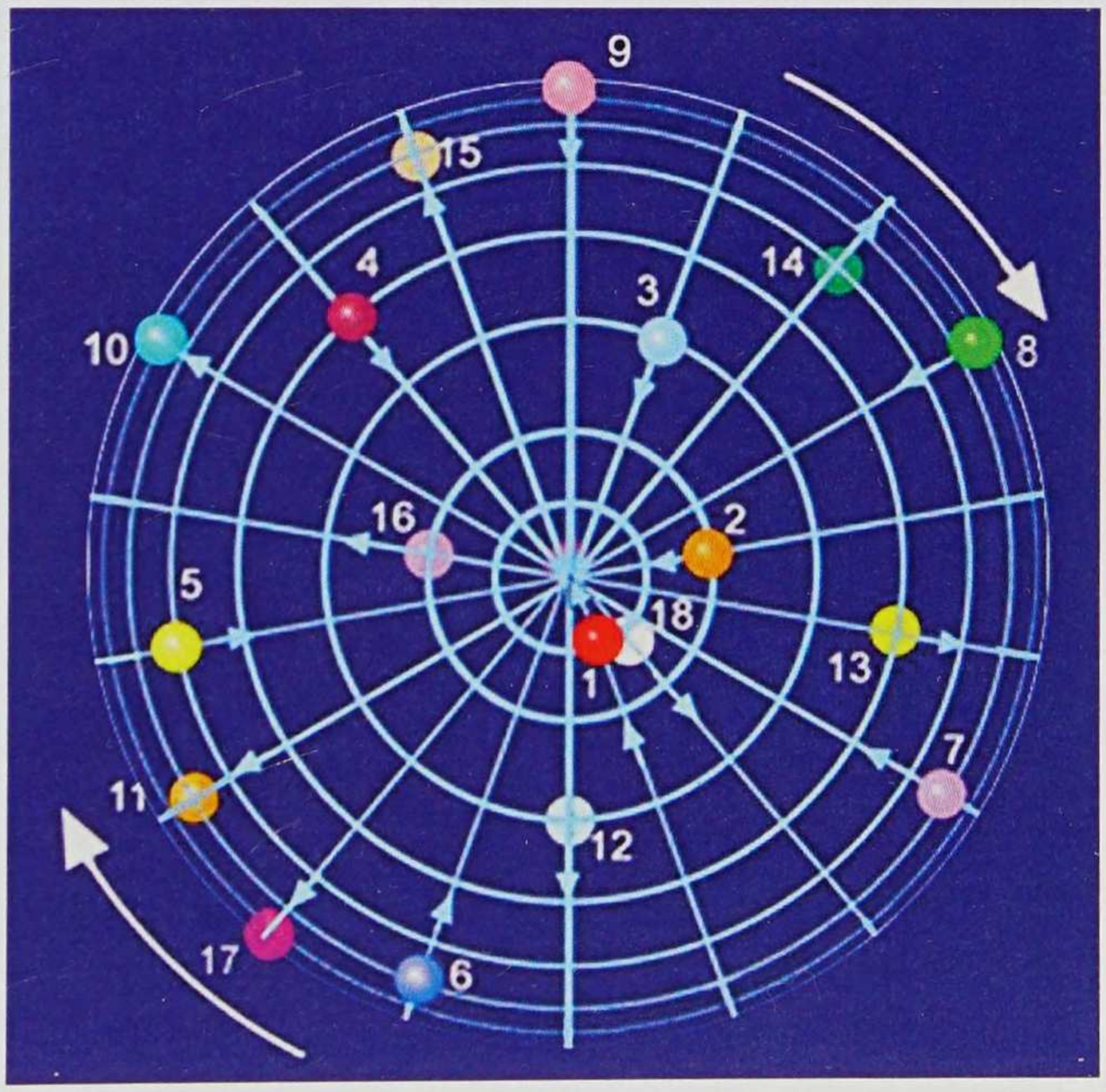
Program 1

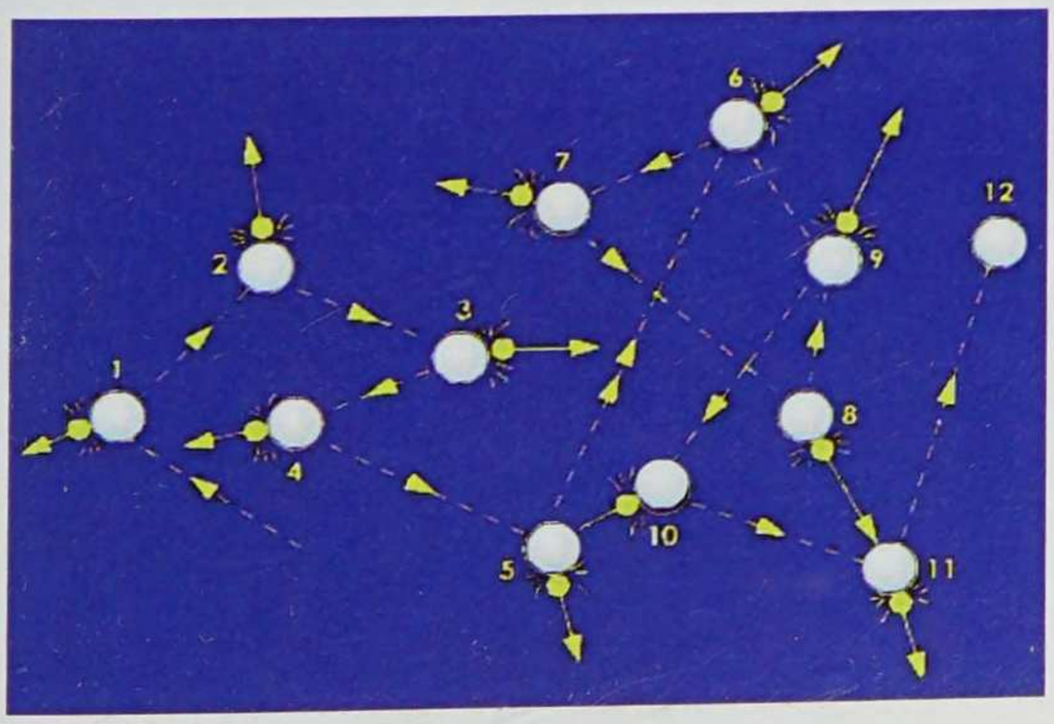
This picture is a virtual representation of how the spaceship such as Hollywood's famous Starship Enterprise would appear upon breaking through the light-speed barrier of the 3rd dimension as it enters the fourth dimension, as seen from within 4-dimensional space. Note the harmonically patterned trail it leaves behind it as it surfs the vibrations of space.

This program series will identify the vibrational patterns in the fabric of space itself. And it will prove mathematically just why they haven't yet been observed by Science. It will prove that there are two complementary fundamental vibrational patterns which must coexist together simultaneously and that these dual patterns continuously cloak one another! That cloaking pattern will be demonstrated to exist mathematically in Program 3 of this series. The spatial ether is there, but to see it, it needs to be exposed by removing one of the complementary vibrational layers in the natural pattern.

And once this can be done with a yet to be developed spatial surfing technology, Mankind will be able to ride the continuously undulating spatial fabric! So a potential future application of this new math is space propulsion. Do I now have your attention?

Let me pose these questions: What do you think keeps the electrons flying around the nucleus of the atom? That orbital energy pattern has to be coming from somewhere.





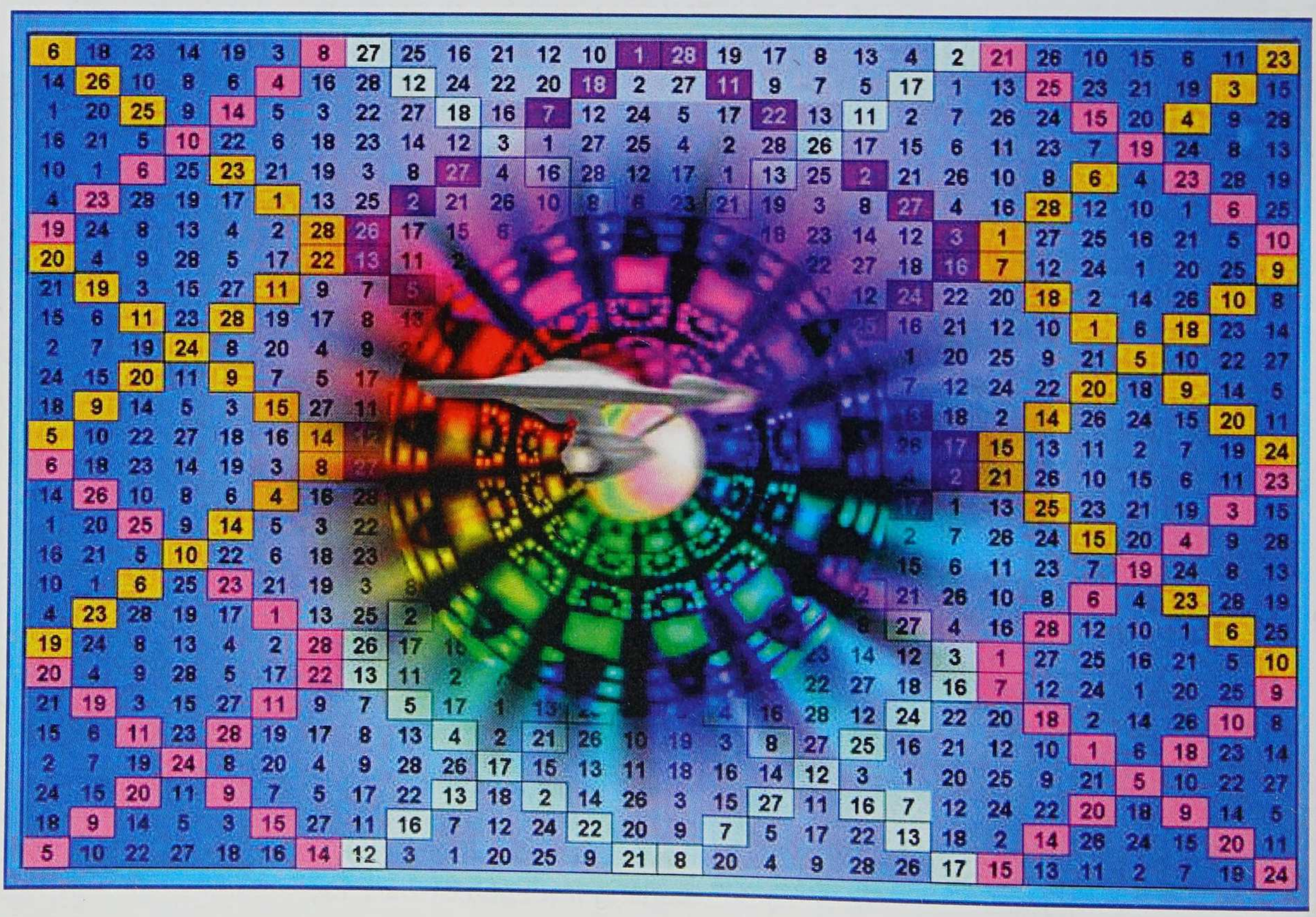
Where do you think the energy for Brownian motion in liquid molecules and gasses is coming from? That energy at the molecular level of **1/2D** has to be coming from somewhere too.

Program 1

Scientists say that it is the heat left over from the Big Bang. That can never be proven, so why even deal with that notion? It could be just that the universe is pressing in upon itself, simple as that. You don't need a cosmic bang.

Regardless, I say that there is energy out there to be tapped, but it is being cloaked in the vacuum of space by space itself. Geometry may someday in the not too distant future be used to uncloak that fabric and tap that energy source. Which by the way, would be so powerful as to make nuclear energy seem like mere trickling water.

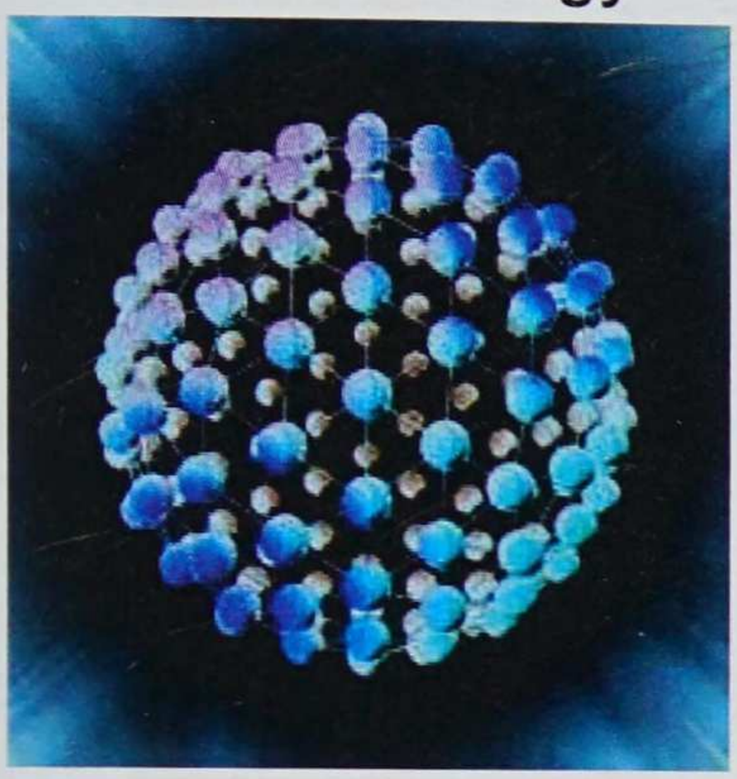
Here's that spaceship, the Starship Enterprise, again, now surfing the harmonically vibratory pattern in the discloaked fabric of space. All the waves are of size-28 interlaced diamond patterns like a pile of chain-link fences. These will be demonstrated in Program 3 to exist pervasively throughout space everywhere. And the method of propulsion using size-28 patterns for space travel will be detailed later in Program 9



Other potential applications

Who knows at this point-in-time to just where this new math might lead in the future? It may guide in the exploration of patterns to be discovered in Nano-technology, Materials science, Sub-atomic physics, Micro-chip technology or Molecular-biology, or in Astrophysics, and even in Cosmology itself.

Nano technology

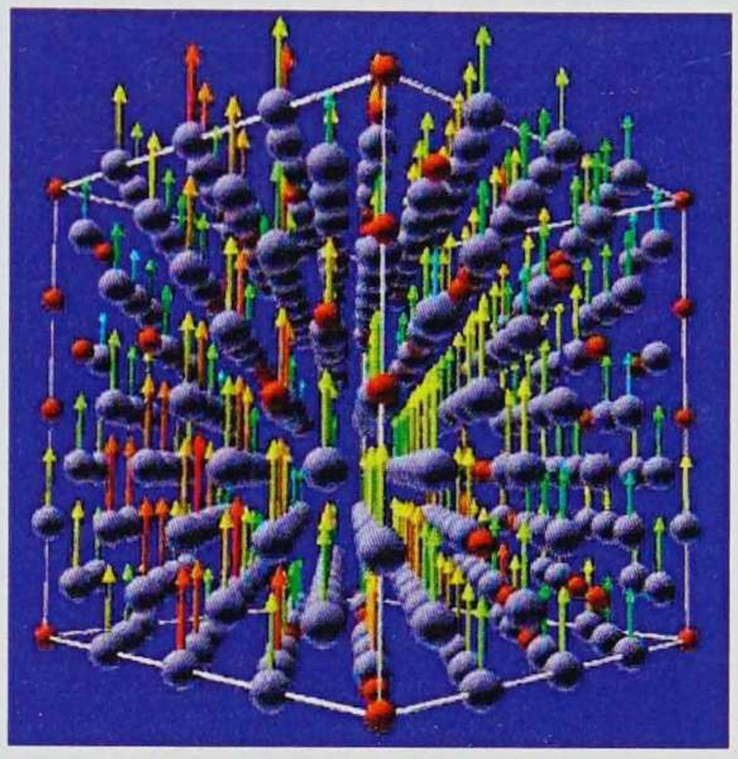


Microchip Technology

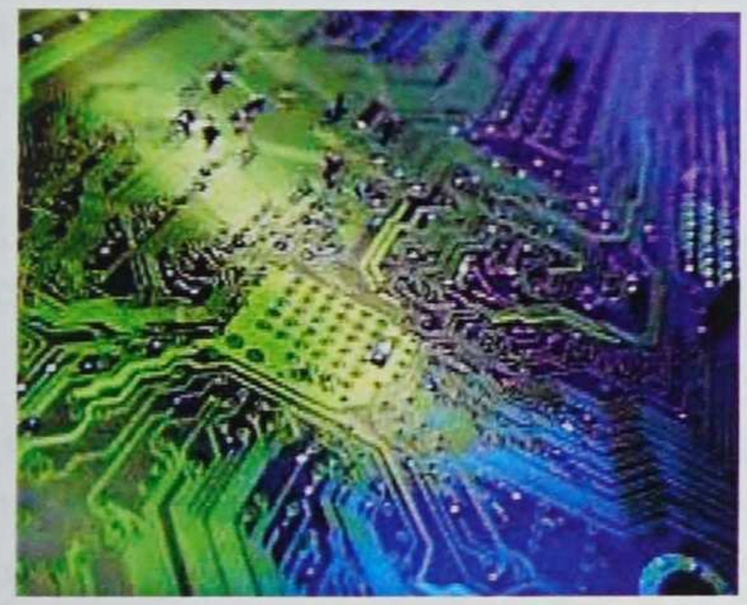
Micro-biology

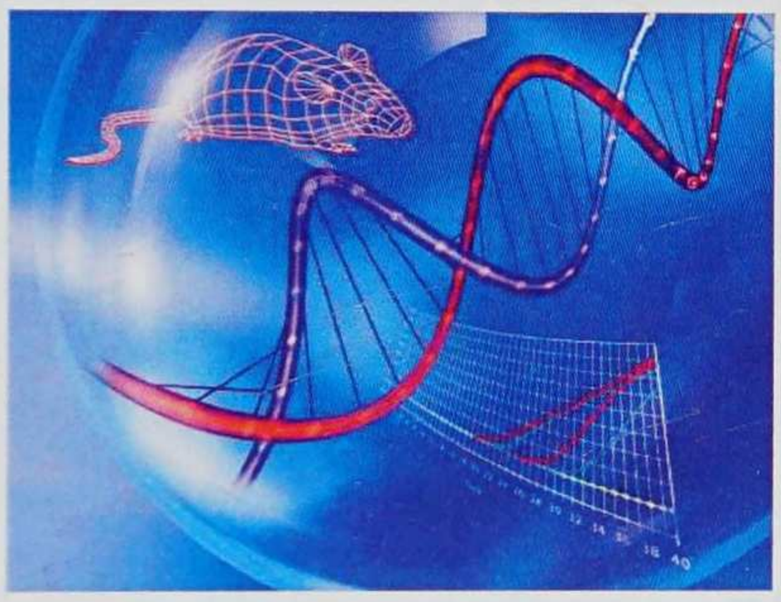
Micro-biology

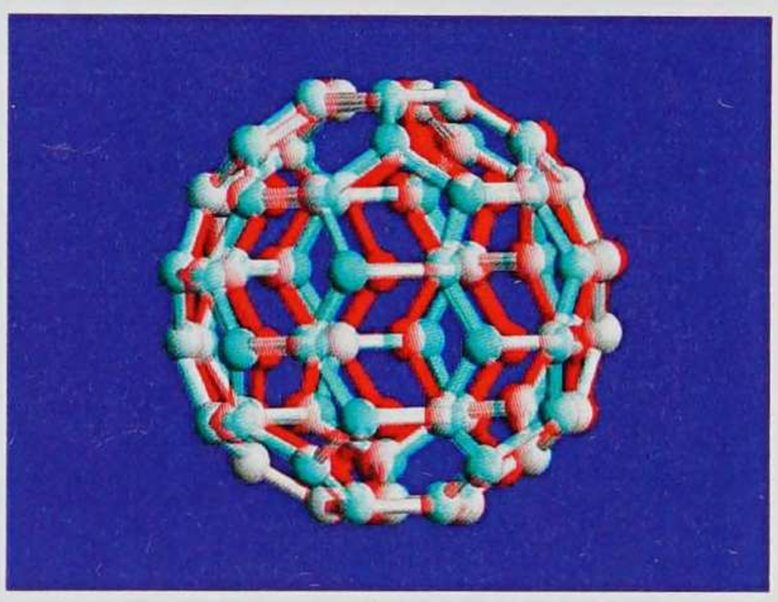
Materials Science



Materials Science

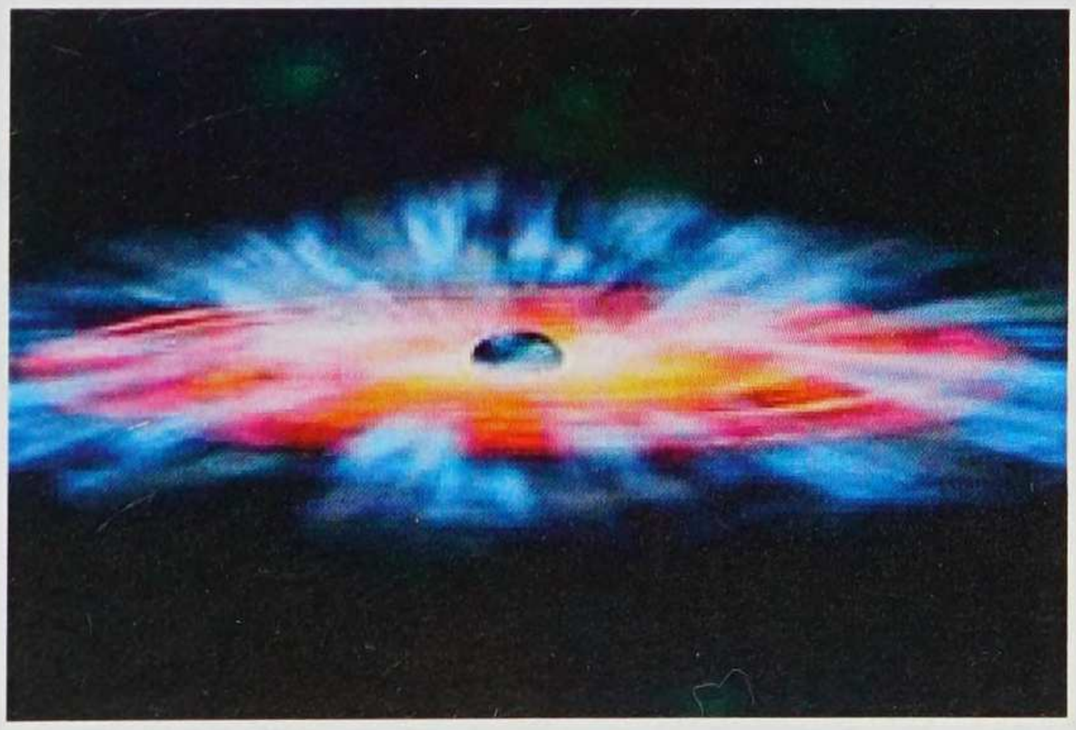






Astrophysics

Cosmology



So let your mind be open to this new math which was just developed to the extent it is today in the last eight years prior to the year 2013. If you are going to succeed in any creative area of Science you will probably need to be at least aware of this new math because all these technologies are getting more and more sophisticated with newly-uncovered, intricate and complex patterns by the day.

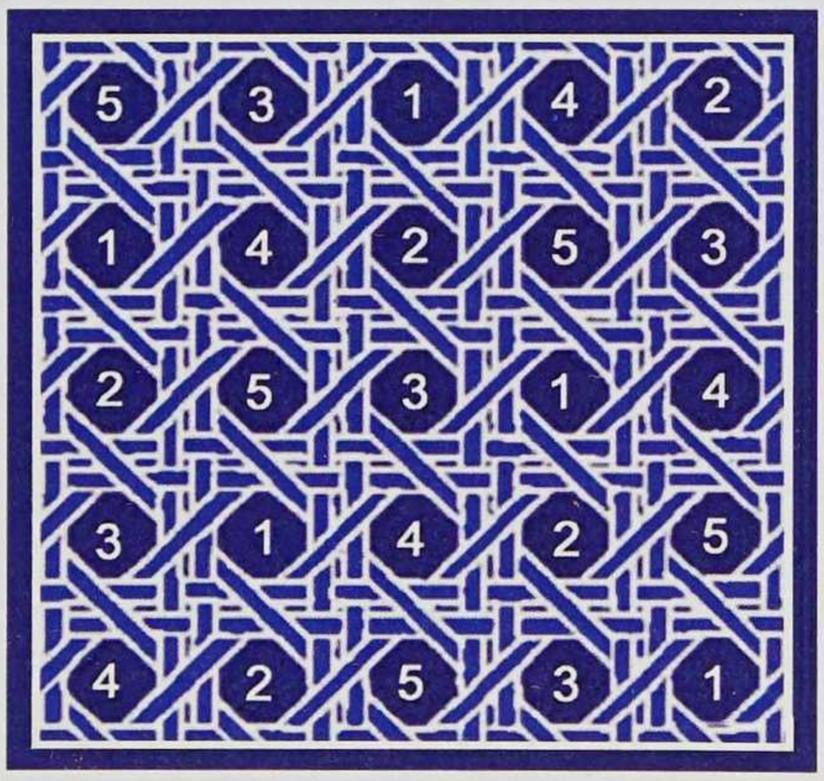
Program 1

Various scientific theories have emerged for explaining the structure of space at the various dimensional levels. The big effort today in Cosmology is to find some mathematical principle to unite all these different dimensional formulations into one grand theory of the universe. It's an attempt by Science to discover the superior intelligence underlying the structure of the spatial fabric. Einstein had been quoted as saying "God doesn't roll dice" in his search for "intelligent design" underlying the structure of space.

Even today, String Theory is graduating from the mathematics involving string vibrations to one involving vibrational patterns of membranes, called M-Theory. That is, Cosmology is evolving from a 1-dimensional view to a 2-dimensional view; one from a linear perspective to a planar perspective, because the 1-dimensional version was becoming too complicated. And as yet, the topic is still not settled.

Whereas these scientific theories attempt to formulate principles from the vast perspective of the **infinite** in a very convoluted effort of trying to unite Cosmology with String Theory by extending the math out to **11** dimensions of space, the accomplishments of Geonometry thus far in this same attempt are made possible by simply confining the exploration of harmonic patterns to fixed-size spaces. You will witness in this program series just how Geonometry accomplishes this with just the natural number series and two elementary, complementary mathematical functions.

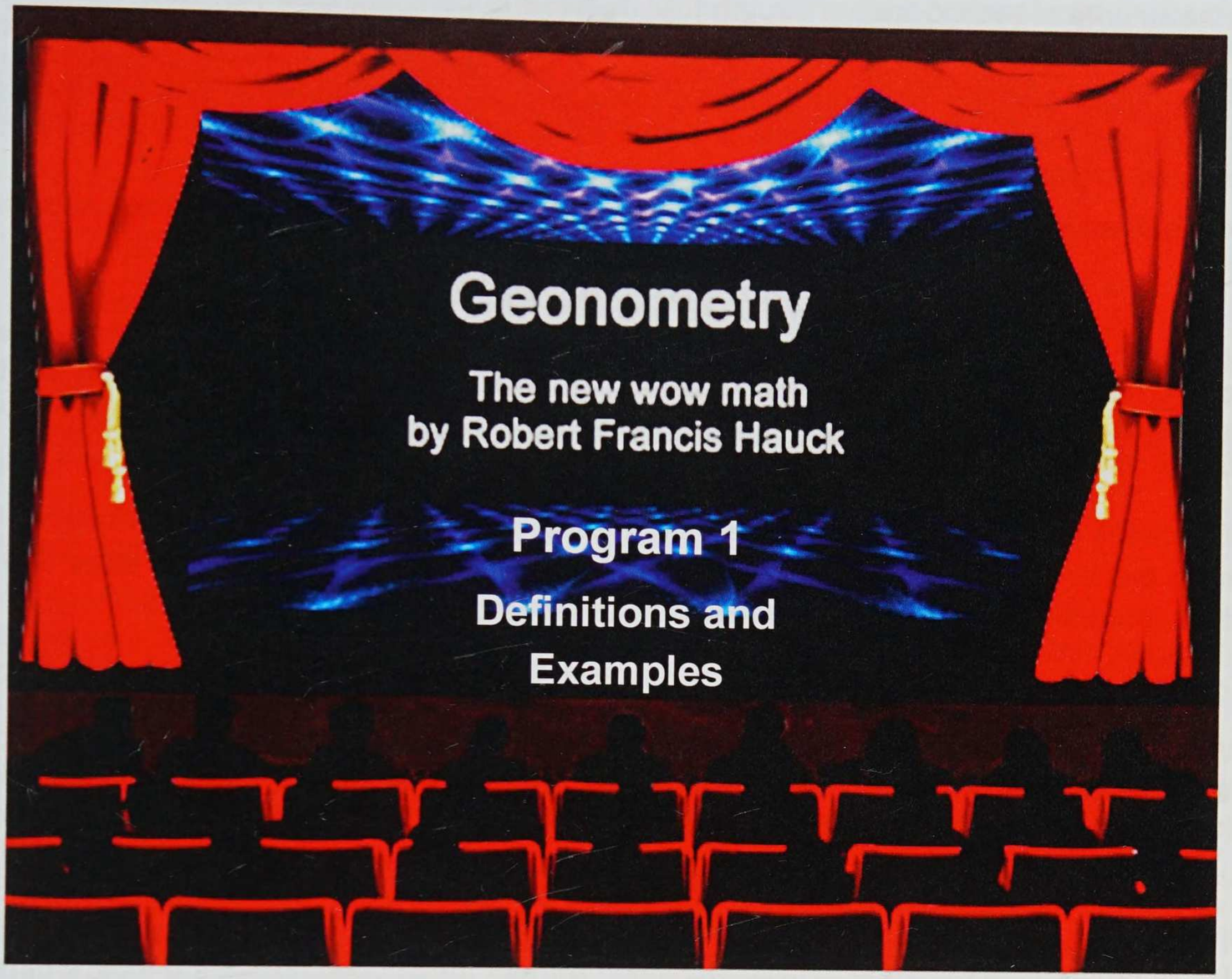
These topics indicate just some of what this new math is good for. It's all about *numerical-pattern* geometry. It helps us to comprehend the natural world and interpret the patterns that we were fortunate enough to have the mental capacity to comprehend – and in my opinion, is one of a superior intelligent pre-design. Geonometry is the math that has the potential to uncover that hidden design without recourse to all that far-out convoluted 11-dimensional math of Cosmology.



Here is a 4-way equal-summing membrane which depicts the harmonically vibrating spatial membrane in a **5x5** area. The expansion properties that will be described in Program 7 will show that these harmonic membranes may be extended to cover as large of an area as you like and still sum equally in **4n** string summations in all 4 spatial directions. That means that those properties determined on a smaller scale as depicted in this program series will also exist simultaneously over a much larger spatial area. And not only that, these harmonic vibrational patterns exist throughout any dimension level. Space could be filled with many multiple complementary harmonic waves that might be harnessed for space propulsion.

I believe that these spatial patterns are at the basis of operation for the billions of iphones being used today for concurrent instantaneous communication all over the globe.

In this program series, you will see four detailed fundamental applications of this new math to explain basic structures in nature whose existence have yet to be satisfactorily explained by Science.



We'll now begin the first program, **Introduction to Geonometry**.

There are a few aspects of Geonometry that distinguish it from classical math:

1. It involves only whole numbers, called integers. There are no decimal fractions, real or complex numbers. So it makes the math quite easy to comprehend.
2. There are only two mathematical functions involved beyond addition, subtraction, multiplication and division of basic arithmetic.
3. The equal summing patterns in tables are easily distinguished by different color-highlighting.
4. There is no need to employ Greek lettering in the formulas; they can be kept quite simple using just ordinary letters of the alphabet.
5. Although the number tables are matrices, Matrix Algebra is neither needed nor involved.
6. There are only a few examples left as exercises but no quizzes to take. Learning takes place because the subject is full of surprises all along the way. Additionally, you have your companion book to this series to let you preview beforehand and review the material after its presentation later at your convenience.
7. All of the operations upon the number-tables can be easily generated in Microsoft's Excel™ program. In fact, this math could not been discovered without Excel™.

However in the second half of Program 9 where it is demonstrated just how this new math interfaces with classical math, items #1 thru #5 will be relaxed to make the discussion possible.

So pay close attention, because there are many new concepts that have new names and symbolisms, and these will come up time and time again throughout this program series without any repeated explanation.

Now, get ready for some preliminary *geonomic* concepts, formats, definitions and symbols.

1-Dimensional Geonometry

Geonomic strings in dimension-1 of size **n** are a linear arrangement of the numbers ranging from **1** thru **n**. Many fundamental theorems in classical mathematics are based on this 1-dimensional series. So the program will not dwell on this topic as there is a wealth of knowledge about it already in classical math.

But there is one formula from classical mathematics that is used throughout this new math and that is the formula for the sum of numbers in the series **1** thru $\mathbf{n}^{\mathbf{k}}$. The capital letter **S** with the subscript **n** will always represent the sum of numbers from **1** thru **n**.

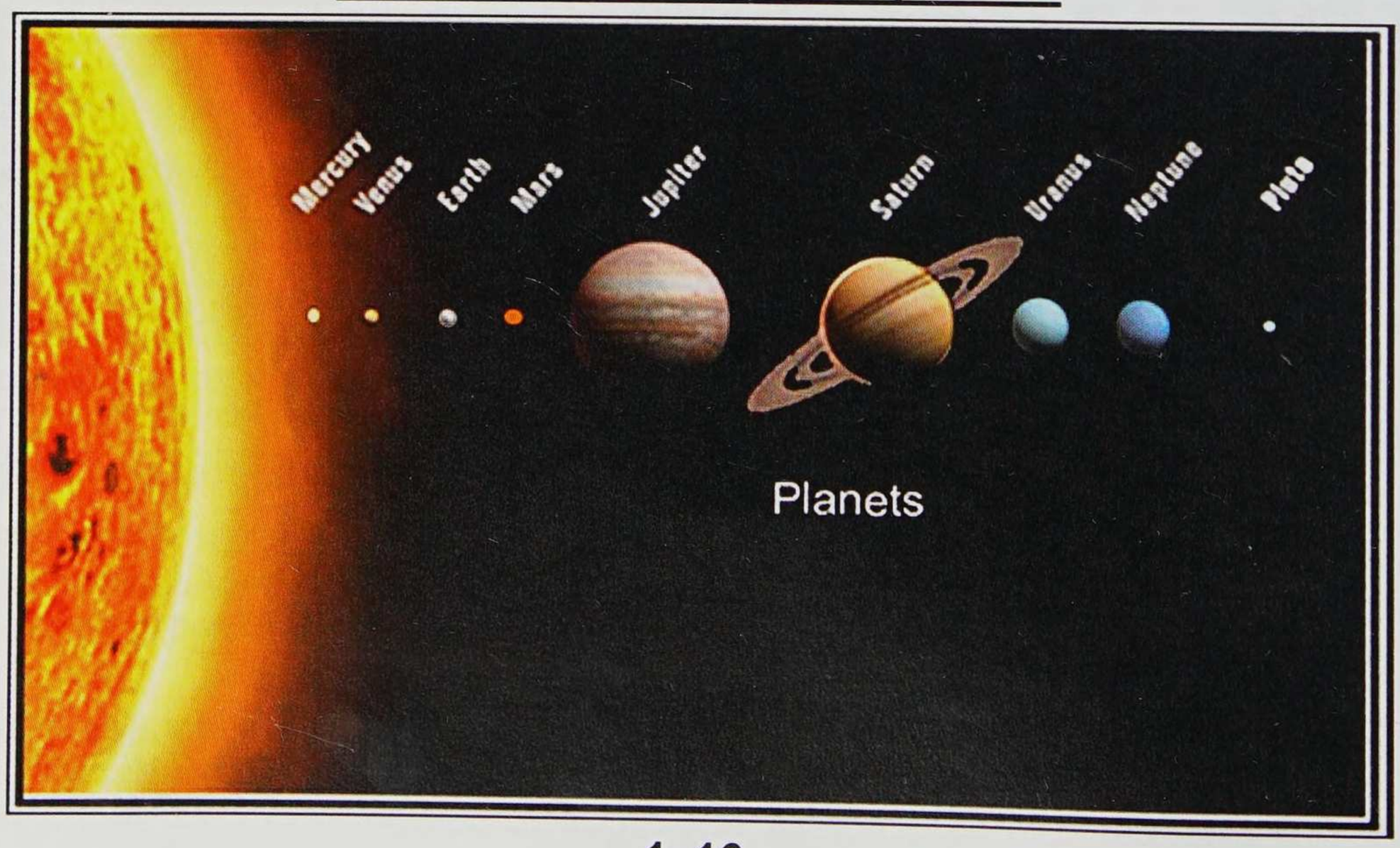
(1.1) $\mathbf{S_n = n(n+1)/2}$

When the sum is to a higher power of **n**, that power number **k** will be a superscript of **S** in parenthesis as shown in the second formula. The superscript in parenthesis denotes the dimension in which the summation takes place.

(1.2) $\mathbf{S_n^{(k)} = n^k(n^k+1)/2}$

The sum of numbers in the simple series **1** thru **n** of dimension-1 or **1** thru $\mathbf{n^2}$ in dimension-2 will always be shown as just the simplified version without the superscript as the topic's dimension will be obvious.

Linear Pattern in Planetary Orbits



Program 1

Here is our solar system. The orbital distances from the sun of each planet and asteroid ring are listed in the table. This distribution pattern of orbiting material is most dense midway from the sun where Jupiter lies. It becomes less dense closer to the sun according to the inverses of a series of ascending prime numbers, and less dense out away from the nebular center according to a series of ascending even numbers.

Prime-number series					Even number series				
1/13	1/7	1/5	1/3	1/2	1	2	4	6	8
13	7	5	3	2	1	2	4	6	8

The series of red numbers are the inverses of the numbers above them. Those are the ratios of the inner planets' distance from the Sun relative to Jupiter's.

The series of blue numbers are the multiples of the distance from the Sun of outer planets relative to Jupiter's.

Notice how the numbers in red follow a series of increasing prime numbers to the left and how the series of blue numbers follow a series to the right that increases by **2**, all of this starting from a **1** corresponding to the orbital distance of Jupiter.

The orbital ratios of those planets within the orbit of Jupiter's, excluding the asteroid belt and the planet Mercury, when added to those beyond Jupiter's orbit and beyond, show a constant relationship of all **9**'s.

13	7	5	3	2	1
	2	4	6		8
	9	9	9		9

All of this is numerical evidence of the 1-dimensional quantization around a star, namely our Sun.

What this pattern indicates is that there are quantized patterns in the distribution of planetary orbits. Only time will tell if this is a common stellar quantization of space as full planetary systems of other stars are discovered and measured. Perhaps this numeric pattern will assist in discovering those planets around nearby stars.

Incidentally, this is the first time that this particular sequence was derived using Jupiter's orbit as the unit of measure for correlating all the planetary orbits. And it yielded two stunning intelligent patterns, one on each side of Jupiter's orbit! – And yet a third one when they were both aligned!

Solar System Planets	Distance from Sun in Astronomical units	Orbit relative to Jupiter's
Mercury	0.39	1/13
Venus	0.72	1/7
Earth	1	1/5
Mars	1.5	1/3
Asteroids	2.7	1/2
Jupiter	5.2	1
Saturn	9.5	2
Uranus	19.2	4
Neptune	30.1	6
(Pluto)	39.5	8

Program 1

2-Dimensional Geonometry

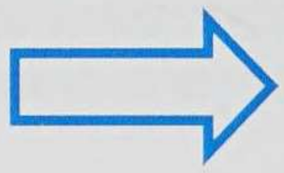
Here we begin the exploration into a new territory of mathematics never before seen beyond a few trivial and limited examples.

Sure, magic squares have been around for at least half a century, ever since the time of Benjamin Franklin, but never beyond mere imperfect small-size squares. Actually, I'm amazed that Ben Franklin was able to get as far as he did on the size-8 and size-16 squares without a digital computer. His versions of those sizes are not what herein will be called "**perfect**".

Let's get started.

Throughout this math, the lowercase letter **n** will always denotes the size of the square. What you see here at left is what is called an **index square of size 5**. It contains the number series **1** up thru **25.** The largest number in a **square** of size **n** will always be $\mathbf{n^2}$.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25



20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

A number square in this new math is a matrix that has been converted by a rearrangement of the numbers in an index square into a *geonomic square* that has many equal-summing properties. Those squares always contain the numbers from **1** thru $\mathbf{n^2}$ exactly once.

All matrices in Geonometry are integral. That is, they contain only whole numbers; there are no fractions, decimal or imaginary numbers. That aspect makes the subject much more comprehendible than higher classical math. Only in the Appendix will the properties of Geonometry be related to classical math with its fractions and real and imaginary numbers. And these will be seen to readily simplify back to whole numbers.

Forget about classical diagonal measures that involve the square-root function; diagonal measures in Geonometry are simply the sum of the numbers along the diagonals of the squares. These sums are unrelated to any classical geometric measure and independent of the Pythagorean Theorem in classical Geometry.

Program 1

Definitions and Examples

Here is what I define as **intelligent number patterns**:

Numerically, it is any number sequence whose elements can be reduced by one or more algebraic operations to one of the following whole-number sequences in which all elements equal to one of the following sequences:

a) a series of all one constant,
b) the sequence of natural numbers; **1, 2, 3, 4, 5, 6 ...**
c) the odd-number sequence **1, 3, 5, 7, 9, 11...**
d) the sequence of odd-prime numbers **1, 3, 5, 7, 11, 13, 19, 23, ...**

Note that an even consecutive number sequence can be reduced to the natural number sequence merely by dividing them by **2** or to the odd-number series by subtracting a **1** from all the numbers

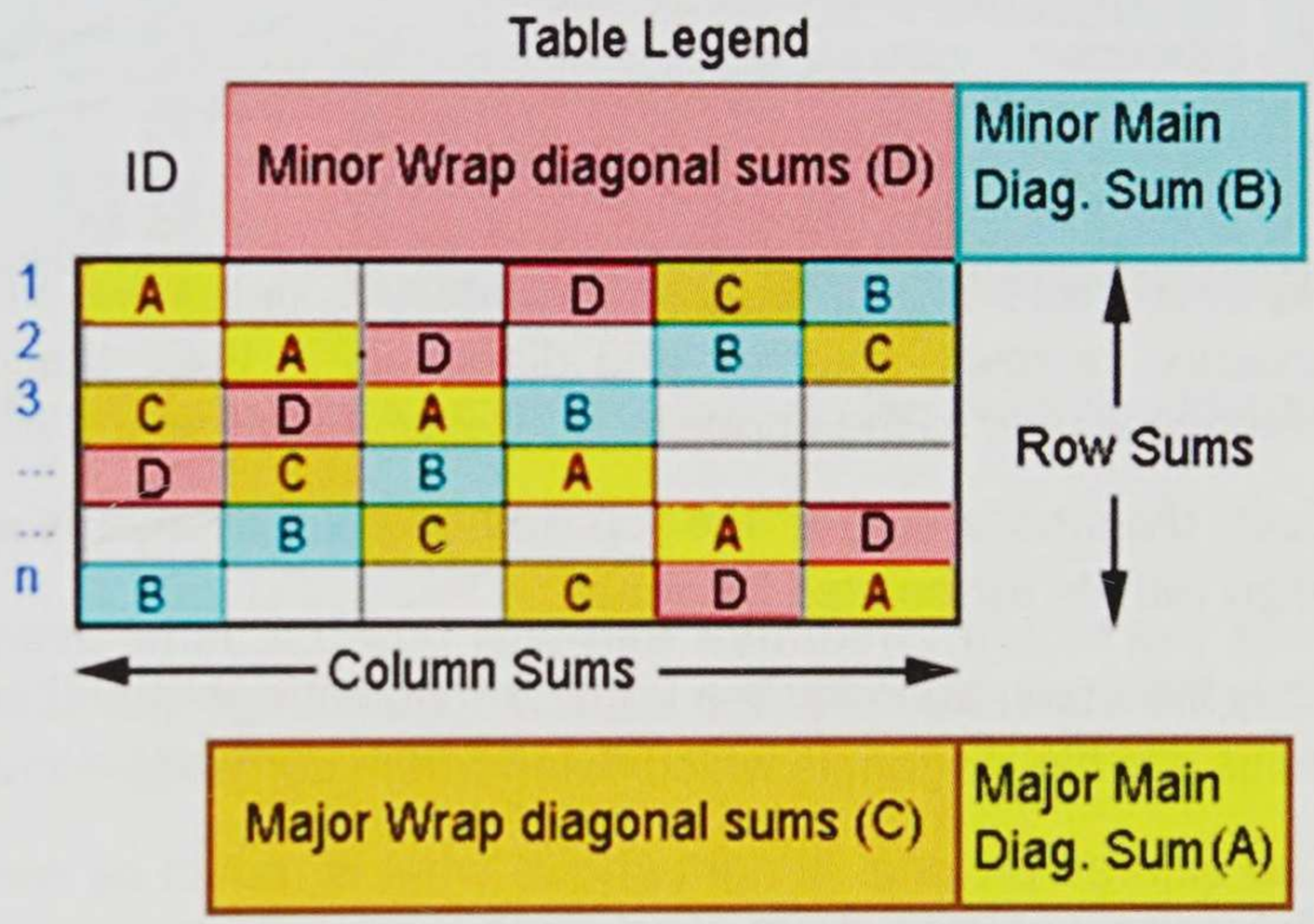
A **harmonic waveform** is a series of consecutive numbers whose values follow a wave pattern:

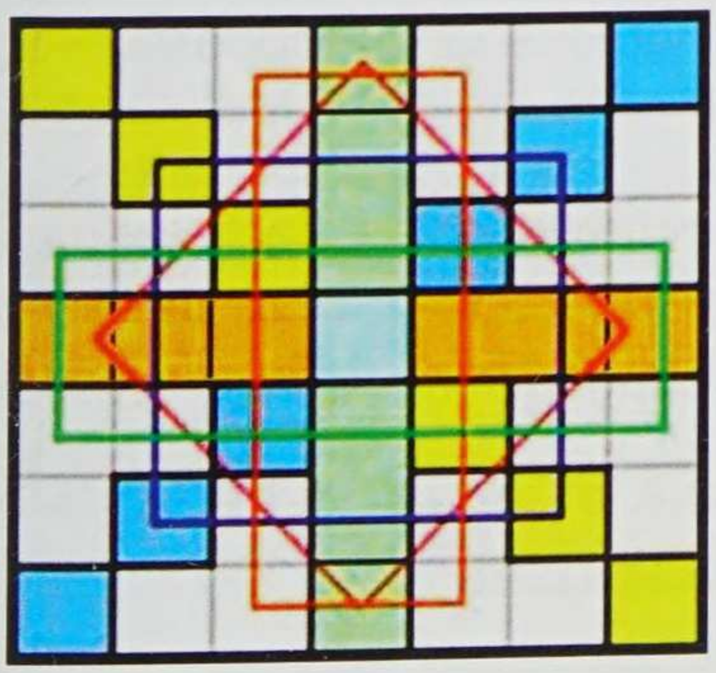
n, n-1, n-2, ... , 3, 2, 1, 2, 3, ... , n-2, n-1, n, n-1, n-2, ...

Here is the square table's legend. It shows how the summation strings are calculated and where the corresponding sum is located relative to the table.

A square table in this format is said to be **geonomic** when the rows, columns and the two main diagonals all sum equally.

The number that they all sum to is dependent upon the table's size and is the same for all geonomic squares of that size. It is called the table's **characteristic number**.



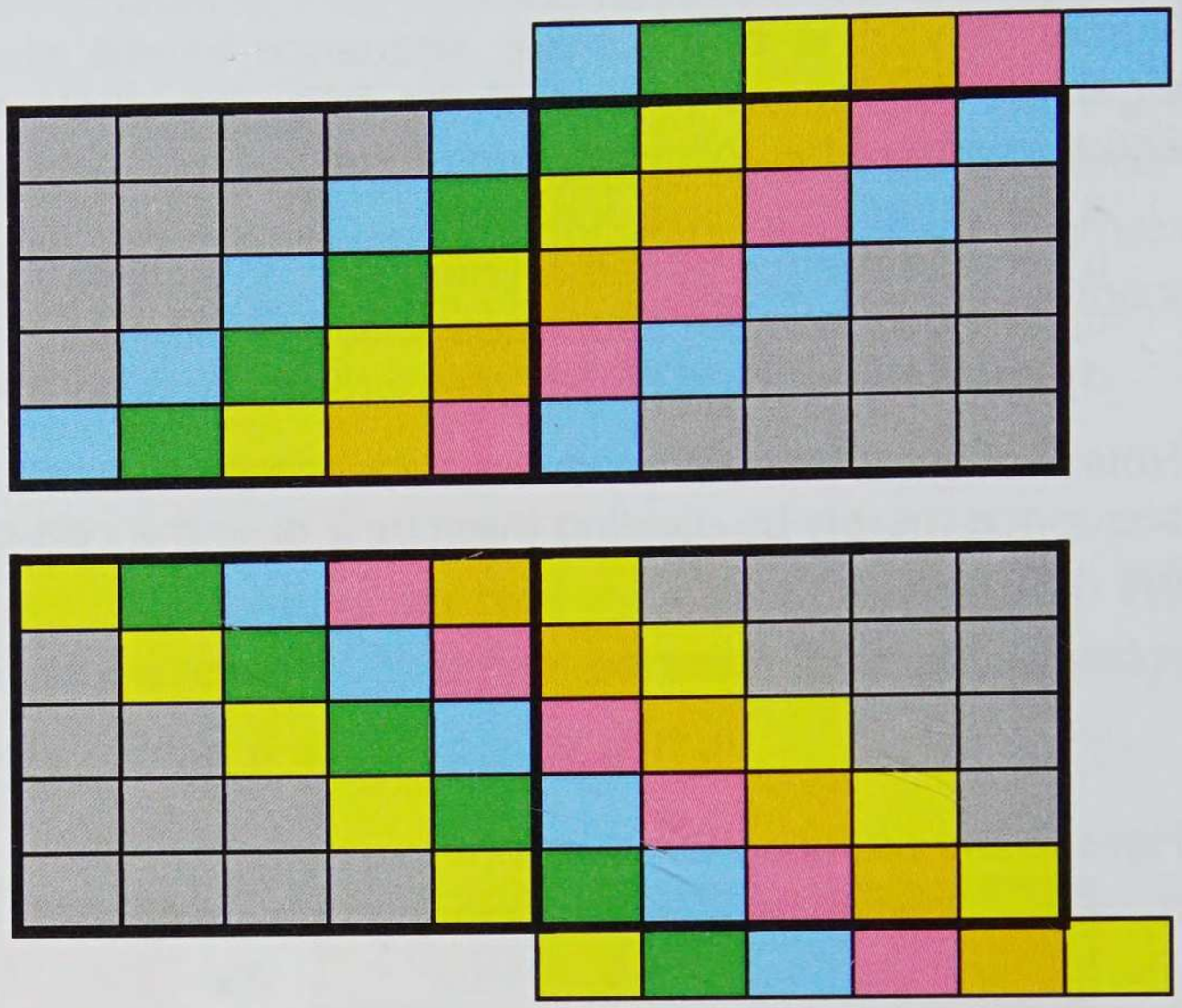


At this point, we introduce the concept of a **quadral**. It is the summation of numbers in the 4 corners of any rectangle, diamond or rhombus centered over the middle of the square. It can be of any size as long as it fits within the square itself. In all squares that are **geonomic**, every quadral sums to the same number -- and that number is called the table's **kernel number**.

Geonomic squares with this property are called **pangenic**. Unless otherwise noted, all squares in Geonometry are, by basic construction, pangenic.

Squares

	1	2	3	4	5	
	5x5	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65



Here is a size-5 geonomic square. Size is determined by the number of cells in each row or column. Its rows, columns and diagonals in both direct-ions, including those wrap-diagonals which spill over to the opposite side when reaching the edge, all sum to the same number.

Each diagonal's sum at the top and below the table is offset by one cell to the right to better align with its associated diagonal.

Only the summations at the upper and lower right-hand corners pertain to the **main** diagonals – those are the diagonals which extend from corner-to-corner.

The colored pictorial at right shows what is meant by **main** and **wrap** diagonals and the minor distinction between them. The **main** diagonals are those emanating at the far right at the top and bottom.

Each **wrap** diagonal summation starts somewhere between the corners and continues onto and across an adjacent duplicate table, which is equivalent to wrapping across to the opposite edge of a single table and continuing on from there.

There are two designations for the direction of a diagonal already established in classical math, one is the **major** and the other is the **minor**. The **major** diagonal direction is from top left to lower right. The **minor** diagonal direction is from top right to lower left.

Pangenic squares whose all four directional string summations are all equal to the square's characteristic number are designated as **perfect**. Anything less is **imperfect**.

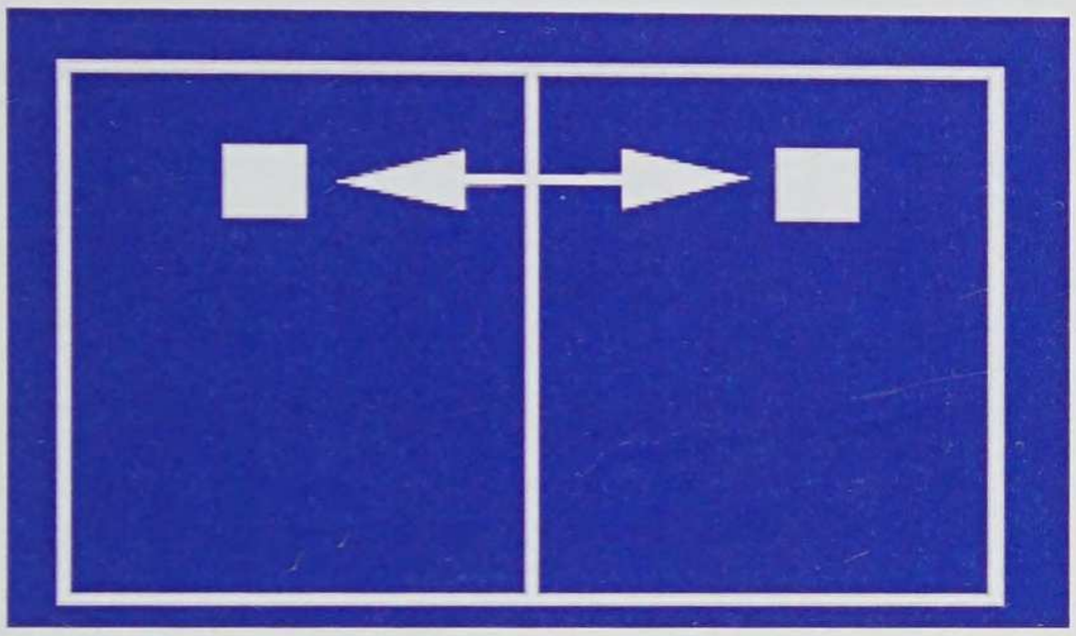
Program 1

Geonomic squares are either an **even-number** size or an **odd-number** size. Odd-size squares have a number that stands alone; it's at the very center of the table. It is called the **pivot number**. In geonomic tables of any dimension, it is always the dimensional average of all the table's numbers. In formulas, it is always designated by the lower-case letter **p**. Its formula is shown below in terms of the size of the matrix **n** and dimension **k**.

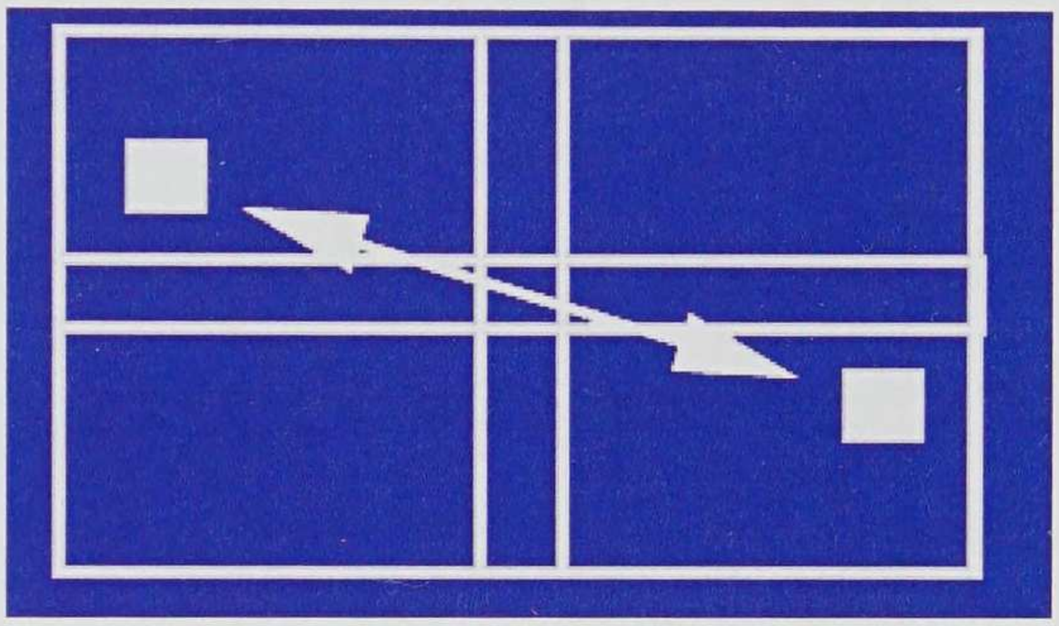
$$(1.3) \quad p = (n^k + 1)/2$$

All squares will usually have the background of their beginning and ending numbers highlighted in **red**.

Even-size squares



Odd-size squares



Even-size squares have distinctly different summation properties than do the ones of odd-size.

For one thing, even-size squares don't have a central number.

The second thing is that they may have what is called **row-pair symmetry** in that any pair of numbers in each row that are symmetrically equidistant from the vertical centerline always sum to the same number **d** and that number is given by formula (**1.4**):

$$(1.4) \quad d = n^k + 1$$

This pairwise pattern differs from odd-size squares which always have **centrally-symmetric** pairs in that every pair of numbers equidistant and exactly opposite from the pivot number sum equally to the same number **d**. In both cases, however, their quadrals, which include two pairs equidistant from the square's center, always sum to the square's kernel number regardless of whether the size of square is odd or even. Consequently, whether of even- or odd-size, these squares are **pangenic**.

There are rare exceptions to this summary of general pairwise symmetry where pair-sums equal **d ± 1** and those will be pointed out when they are encountered.

Categorization of Squares by Class

Class 1	Class 2	Class 3	Class 4	Class 5	Class 6
n = b b is a prime number b > 3	n = 2b b Is an odd number	n = 3b b is an odd number b > 3	n = 4b b is any integer	n = ab a & b both primes a > b > 3	$n = b^a$ b is an odd number a = 2 or 3
5	6 = 2x3	15 = 3x5	4 = 4x1	35 = 5x7	$9 = 3^2$
7	10 = 2x5	21 =3x7	8 = 4x2	55 = 5x11	$25 = 5^2$
11	14 = 2x7	27 =3x9	12 = 4x3	65 = 5x13	$27 = 3^3$
13	18 = 2x9	33 =3x11	16 = 4x4	75 = 5x15	$49 = 7^2$
17	22 = 2x11	39 = 3x13	20 = 4x5	77 = 7x11	$81 = 9^2$
19	26 = 2x13	45 = 3x 15	24 = 4x6	85 = 5x17	$121 = 11^2$
23	30 = 2x15	51 = 3x 17	28 = 4x7	91 = 7x13	$169 = 13^2$
29	34 = 2x17	57 = 3x 19	32 = 4x8	95 = 5x19	$225 = 15^2$
31	38 = 2x19	63 = 3x 21	36 = 4x9	119 =7x17	$289 = 17^2$

Geonomic tables of <u>any dimension</u> can be categorized by their common properties into **6** major classes. These are:

Class 1 – **n** equals **b** where **b** is a prime number.

Class 2 – **n** equals **2 b** where **b** is an odd number.

Class 3 – **n** equals **3 b** where **b** is an odd number**.**

Class 4 – **n** equals **4 b** where **b** is any number.

Class 5 – **n** equals a product of two unequal prime numbers **a** & **b**, each greater than **3**.

Class 6 – **n** equals a square or cube of an odd number **b**.

These classes pretty much segregate all the printable sizes of square into mutually exclusive categories. Examples are listed for each category, eventhough those that are greater than size **36** are too large for display in print. Those are listed here in **blue**.

Note that the size **27** appears in both Classes 3 & 6. Why that is so will become evident as we explore the properties of the various classes.

Not all printable sizes in each class will be shown as they are all depicted in the book, **Number Magic**, which is referenced throughout this program series.

Class 1 Squares

Prime number Size 5

	1	2	3	4	5	
	5x5	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

Centrally Symmetric pairs sum to 26.

We have already seen the size-5 square. Its characteristic number is **65**. All six of its quadrals sum to **52**, its kernel number, which is **4/5** of its characteristic number. Every centrally-symmetric pair of numbers sums to **26**, which is twice the pivot number, **13**. These aspects are common among all Class-1 squares.

Here is the Class-1 size-7 square. It too is pairwise centrally symmetric with summations equal to **50 = 1 + 49.** All of its quadrals sum to **100, its kernel number.** Its pivot number is **25 = ¼** its kernel number.

Size-7

	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175
1	43	24	5	35	9	39	20	175
2	14	37	18	48	22	3	33	175
3	27	1	31	12	42	16	46	175
4	40	21	44	25	6	29	10	175
5	4	34	8	38	19	49	23	175
6	17	47	28	2	32	13	36	175
7	30	11	41	15	45	26	7	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

Program 1

Class 2 Squares

Size-6 Square

Here is a square of size-6. It is a Class-2 square. It is only what will be called **near-perfect**. It has two symmetrically-located rows which differ from the size-6 characteristic number by **±1**.

It also has row-pairs which sum to **37** for the most part but there are a few exceptions. The ones that don't, sum equally pairwise centrally. This minor anomaly arises because the size-6 square is **embryonic**, that is, its series of numbers is not sufficient to produce all the properties normally ascribed to its class. Yet all of its quadrals are still equal to **74**, which is **4/6** of its characteristic number. So it is pangenic.

	1	2	3	4	5	6		
	111	111	111	111	111	111	111	
1	6	33	12	25	4	31	111	
2	17	16	21	15	22	19	110	–1
3	30	3	36	1	34	7	111	
4	32	9	26	11	28	5	111	
5	18	23	14	24	13	20	112	+1
6	8	27	2	35	10	29	111	
	111	111	111	111	111	111		
	111	111	111	111	111	111	111	

The near perfect size-6 square can be converted into what will be called a **punctuated-perfect** square by interchanging just two numbers in the same column, here **19** and **20**, or **17** and **18**. It still remains pangenic. But the equality then brought to its rows by this interchange of numbers is counter-productive because it introduces four unequal diagonals with the same variances as the formerly unequal rows, shaded in **crimson**.

	1	2	3	4	5	6	
		1			-1		
	112	111	111	110	111	111	
1	6	33	12	25	4	31	111
2	17	16	21	15	22	20	111
3	30	3	36	1	34	7	111
4	32	9	26	11	28	5	111
5	18	23	14	24	13	19	111
6	8	27	2	35	10	29	111
	111	111	111	111	111	111	
	110	111	111	112	111	111	
		-1			1		

Swapping the numbers **22** and **23** in the minor main-diagonal will also correct the two unequal rows but that will result in two unequal columns. There is no way to iron out this wrinkle for Class-2 squares. It is a fundamental wrinkle in the spatial fabric.

Program 1

Size-10 Square

10x10	1	2	3	4	5	6	7	8	9	10	
	505	505	505	505	505	505	505	505	505	505	505
	95	8	96	14	77	20	83	21	89	2	505
	11	79	17	85	23	86	4	92	10	98	505
	82	50	63	26	69	32	100	13	76	19	530
	36	54	42	60	48	61	29	67	35	73	505
	45	58	46	64	27	70	33	71	39	52	505
	49	62	30	68	31	74	37	55	43	56	505
	28	66	34	72	40	53	41	59	47	65	505
	57	25	88	1	94	7	75	38	51	44	480
	3	91	9	97	15	78	16	84	22	90	505
	99	12	80	18	81	24	87	5	93	6	505
	505	505	505	505	505	505	505	505	505	505	
	505	505	505	505	505	505	505	505	505	505	505

+25

-25

Y(25)											
	25	25	25	25	25	25	25	25	25	25	25
	4	1	4	1	4	1	4	1	4	1	25
	1	4	1	4	1	4	1	4	1	4	25
	4	2	3	2	3	2	4	1	4	1	26
	2	3	2	3	2	3	2	3	2	3	25
	2	3	2	3	2	3	2	3	2	3	25
	2	3	2	3	2	3	2	3	2	3	25
	2	3	2	3	2	3	2	3	2	3	25
	3	1	4	1	4	1	3	2	3	2	24
	1	4	1	4	1	4	1	4	1	4	25
	4	1	4	1	4	1	4	1	4	1	25
	25	25	25	25	25	25	25	25	25	25	
	25	25	25	25	25	25	25	25	25	25	25

+1

-1

The integer function at right taken to the base **25** on the numbers in the Class-2 size-10 square shows the source of the spatial wrinkle that makes all Class-2 squares only near-perfect. (More about the *integer function* in Program 2).

The wrinkle stems from the impossibility of an even numbered checkered pattern being continued in an odd-numbered string. Compare odd-count patterns in the unequal rows to the even-count pattern in the other rows. Any modification to the location of numbers will result in unequal diagonals.

This is a basic corruptive property of all Class-2 squares. **It is a fundamental wrinkle in the basic fabric of space.**

Nonetheless, Class-2 squares play a very important role in the manifestation of the squares in Class-4 and in even-size tables of all higher-dimensions.

Class 3 Squares

Odd-size perfect squares, Size = 3b, odd-number b > 3

Size-15

15x15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	33	71	14	52	95	108	146	89	127	170	183	221	164	202	1695
2	86	104	142	85	148	11	29	67	10	73	161	179	217	160	223	1695
3	157	200	213	151	219	82	125	138	76	144	7	50	63	1	69	1695
4	3	41	59	22	65	153	191	209	172	215	78	116	134	97	140	1695
5	174	187	205	168	206	99	112	130	93	131	24	37	55	18	56	1695
6	45	8	46	64	27	120	83	121	139	102	195	158	196	214	177	1695
7	111	79	117	135	123	36	4	42	60	48	186	154	192	210	198	1695
8	182	175	188	201	194	107	100	113	126	119	32	25	38	51	44	1695
9	28	16	34	72	40	178	166	184	222	190	103	91	109	147	115	1695
10	199	162	180	218	181	124	87	105	143	106	49	12	30	68	31	1695
11	70	58	21	39	2	145	133	96	114	77	220	208	171	189	152	1695
12	136	129	92	110	98	61	54	17	35	23	211	204	167	185	173	1695
13	207	225	163	176	169	132	150	88	101	94	57	75	13	26	19	1695
14	53	66	9	47	15	203	216	159	197	165	128	141	84	122	90	1695
15	224	212	155	193	156	149	137	80	118	81	74	62	5	43	6	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

Every quadral sums to **452.**

452 = 4 x 113 = **4/15** x 1695

Here is the smallest Class-3 square, a size **15**. It is geonomically perfect. It has all equal **centrally-symmetric** pairwise sums. All of its quadrals sum to **4/15** of its characteristic number.

Note the pattern of quadral sums seen so far: they always equal **4** times the characteristic number divided by the square's size.

The size of Class-3 squares is always **3-times** an <u>odd number</u> **b>3**. They are all continuously-modular at the **bxb** block-square level. These block-squares will always sum to **b/3** of the square's characteristic number.

The size-15 square has **5x5** block-squares that sum to **5/3** of its characteristic number anywhere in the table, even wrapping over the edges. This is called, **continuous modularity**.

If the size of the square is **3** times an odd number **b**, it will be **continuously bxb modular** and each size-**b** block-square will sum to **b-thirds** of the characteristic number. The square will just be called "**bx-modular**" for short.

Class 4 Squares

Size-4

Here is the size-4 Class-4 square. It is perfect. Its characteristic number is **34**. All of its quadrals sum to **34**.

In addition to being perfect:

1. All of its **2x2** block-squares sum to **34**.
2. It is continuously **2x**-modular. This **2x2** pattern sums to **34** even when wrapping onto the opposite edge.

This is the only size square which has its kernel number, its modularity, and its characteristic number all equal.

4x4	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34	34	

Size-8

W1(8)	260	260	260	260	260	260	260	260
	24	9	8	25	40	57	56	41	260
	58	39	42	55	10	23	26	7	260
	14	19	30	3	62	35	46	51	260
	36	61	52	45	20	13	4	29	260
	17	16	1	32	33	64	49	48	260
	63	34	47	50	15	18	31	2	260
	11	22	27	6	59	38	43	54	260
	37	60	53	44	21	12	5	28	260
	260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260	260

Quadrals

130	130	130	130
130	130	130	130
130	130	130	130
130	130	130	130

Their distribution of numbers in each square is different. At right is a version of size-8 that is centrally pairwise symmetric. All of its centrally-symmetric pairs sum to **65**.

Both sizes, **8** and **16**, have both versions of equal summing pairs.

Here is a size **8** square. It is perfect; it has equal-summing symmetric row-pairs which add to **65**.

Class-4 squares have a special subcategory -- All squares whose size is **8** or an even multiple of **8** also have both versions of equal-summing pairs.

W2(8)	260	260	260	260	260	260	260	260
	8	41	33	16	56	25	17	64	260
	58	23	31	50	10	39	47	2	260
	62	19	27	54	14	35	43	6	260
	4	45	37	12	52	29	21	60	260
	5	44	36	13	53	28	20	61	260
	59	22	30	51	11	38	46	3	260
	63	18	26	55	15	34	42	7	260
	1	48	40	9	49	32	24	57	260
	260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260	260

Quadrals

130	130	130	130
130	130	130	130
130	130	130	130
130	130	130	130

Program 1

Size-16 Squares

Here is the Class-4 square of size **16** with equal-summing symmetric row-pairs.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	22	107	64	97	54	75	32	65	192	225	182	203	160	193	150	235	2056
2	188	197	146	207	156	229	178	239	18	79	28	101	50	111	60	69	2056
3	9	120	35	126	41	88	3	94	163	254	169	216	131	222	137	248	2056
4	167	218	141	212	135	250	173	244	13	84	7	122	45	116	39	90	2056
5	27	102	49	112	59	70	17	80	177	240	187	198	145	208	155	230	2056
6	181	204	159	194	149	236	191	226	31	66	21	108	63	98	53	76	2056
7	8	121	46	115	40	89	14	83	174	243	168	217	142	211	136	249	2056
8	170	215	132	221	138	247	164	253	4	93	10	119	36	125	42	87	2056
9	86	43	128	33	118	11	96	1	256	161	246	139	224	129	214	171	2056
10	252	133	210	143	220	165	242	175	82	15	92	37	114	47	124	5	2056
11	73	56	99	62	105	24	67	30	227	190	233	152	195	158	201	184	2056
12	231	154	205	148	199	186	237	180	77	20	71	58	109	52	103	26	2056
13	91	38	113	48	123	6	81	16	241	176	251	134	209	144	219	166	2056
14	245	140	223	130	213	172	255	162	95	2	85	44	127	34	117	12	2056
15	72	57	110	51	104	25	78	19	238	179	232	153	206	147	200	185	2056
16	234	151	196	157	202	183	228	189	68	29	74	55	100	61	106	23	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Here is the Class-4 square of size-16 which has equal-summing centrally-symmetric pairs.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Program 1

Size-12 Square

Here is Class-4 square of size-12. It has all symmetric row-pair sums equal to **145**. So clearly, its quadrals all sum to twice that, viz. **290**. It is perfect.

It is also continuously 6x-modular at 3-times the characteristic number.

Class-4 squares, for the most part, have all equal symmetric row-pair sums. However, in Program 2 we will encounter a systematic sequence of Class-4 squares, including the size-12 square, which have no equal-summing pairwise symmetry at all, yet have another more important property.

	1	2	3	4	5	6	7	8	9	10	11	12	
W(12)	870	870	870	870	870	870	870	870	870	870	870	870	870
1	109	30	127	90	43	108	37	102	55	18	115	36	870
2	67	84	49	24	133	6	139	12	121	96	61	78	870
3	107	41	128	89	32	110	35	113	56	17	104	38	870
4	69	100	51	22	117	4	141	28	123	94	45	76	870
5	75	46	93	124	27	142	3	118	21	52	99	70	870
6	44	98	59	14	119	29	116	26	131	86	47	101	870
7	73	66	91	126	7	144	1	138	19	54	79	72	870
8	31	120	13	60	97	42	103	48	85	132	25	114	870
9	140	2	95	122	71	77	68	74	23	50	143	5	870
10	33	136	15	58	81	40	105	64	87	130	9	112	870
11	111	10	129	88	63	106	39	82	57	16	135	34	870
12	11	137	20	53	80	62	83	65	92	125	8	134	870
	870	870	870	870	870	870	870	870	870	870	870	870	
		870	870	870	870	870	870	870	870	870	870	870	870

Characteristic Circles In Class-4 Squares

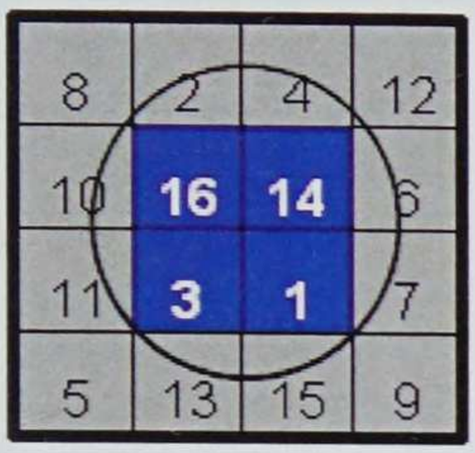
Computing which size squares have Characteristic circles			
Size Square	Characteristic number	Quadral sums	Quadral multiple = characteristic no.
3	15	20	0.75
4	34	34	1
5	65	52	1.25
6	111	74	1.5
7	175	100	1.75
8	260	130	2
9	369	164	2.25
10	505	202	2.5
11	671	244	2.75
12	870	290	3
13	1105	340	3.25
14	1379	394	3.5
15	1695	452	3.75
16	2056	514	4
17	2465	580	4.25
18	2925	650	4.5
19	3439	724	4.75
20	4010	802	5
21	4641	884	5.25
22	5335	970	5.5
23	6095	1060	5.75
24	6924	1154	6
25	7825	1252	6.25
26	8801	1354	6.5
27	9855	1460	6.75
28	10990	1570	7
29	12209	1684	7.25
30	13515	1802	7.5
31	14911	1924	7.75
32	16400	2050	8

The table shows that only the squares whose size is a multiple of **4** can have a whole-number of quadrals which add up to the square's characteristic number exactly without a fractional remnant. Consequently, we only need to address those squares with exact multiples for characteristic circles and identify the participating quadrals. Although the patterns appear elementary for squares here in Program 1, this exploration sets the stage for a fundamental correlation in 3-dimensional space in Program 4 where we examine geonomic cubes in the search for *characteristic spheres*.

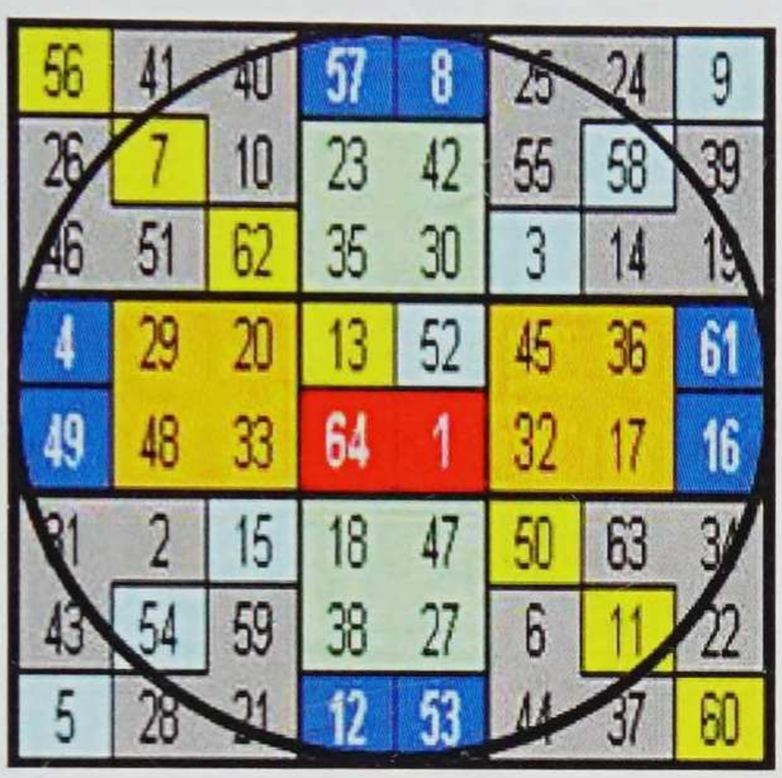
Program 1

Here are shown the *characteristic circles* inscribed in Class-4 squares from size-4 through size-20. A characteristic circle's value is the sum of numbers in all the cells which just impinge the largest inscribed circle from its interior. Because of these squares' pangenicity, these sums can be readily determined by simply counting all the quadrals formed from these symmetrically located cells and multiplying this quantity by the square's **kernel** number **q**.

4x4

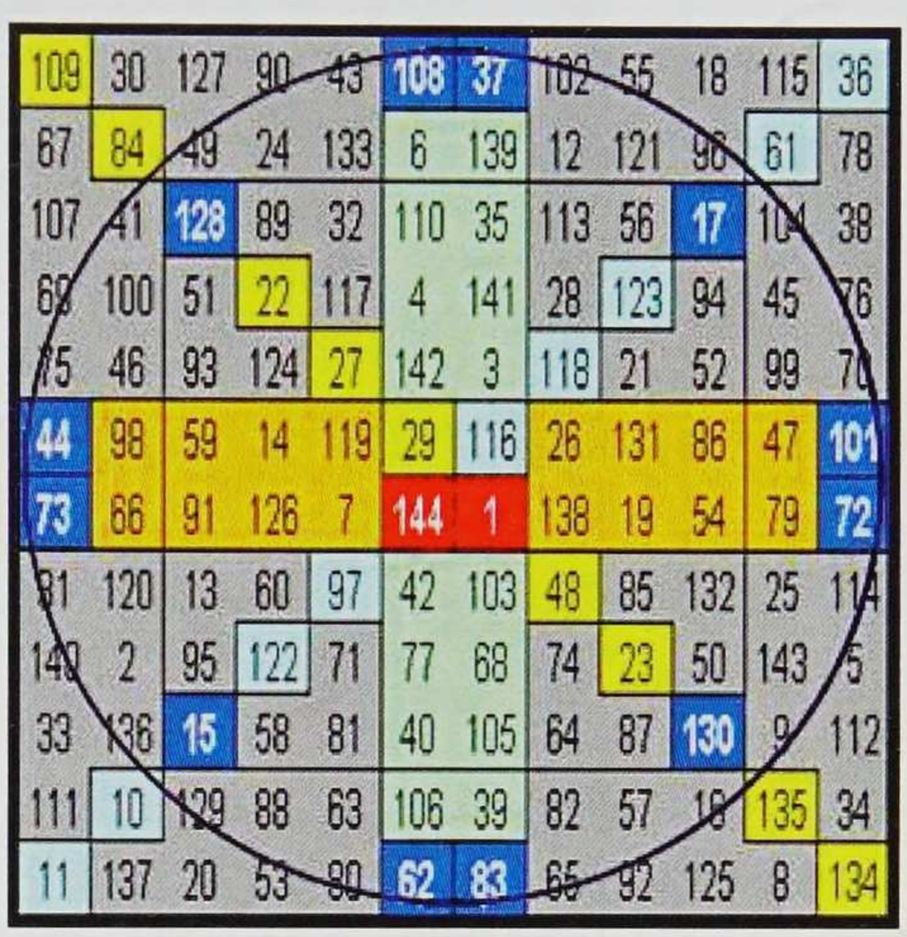


8x8



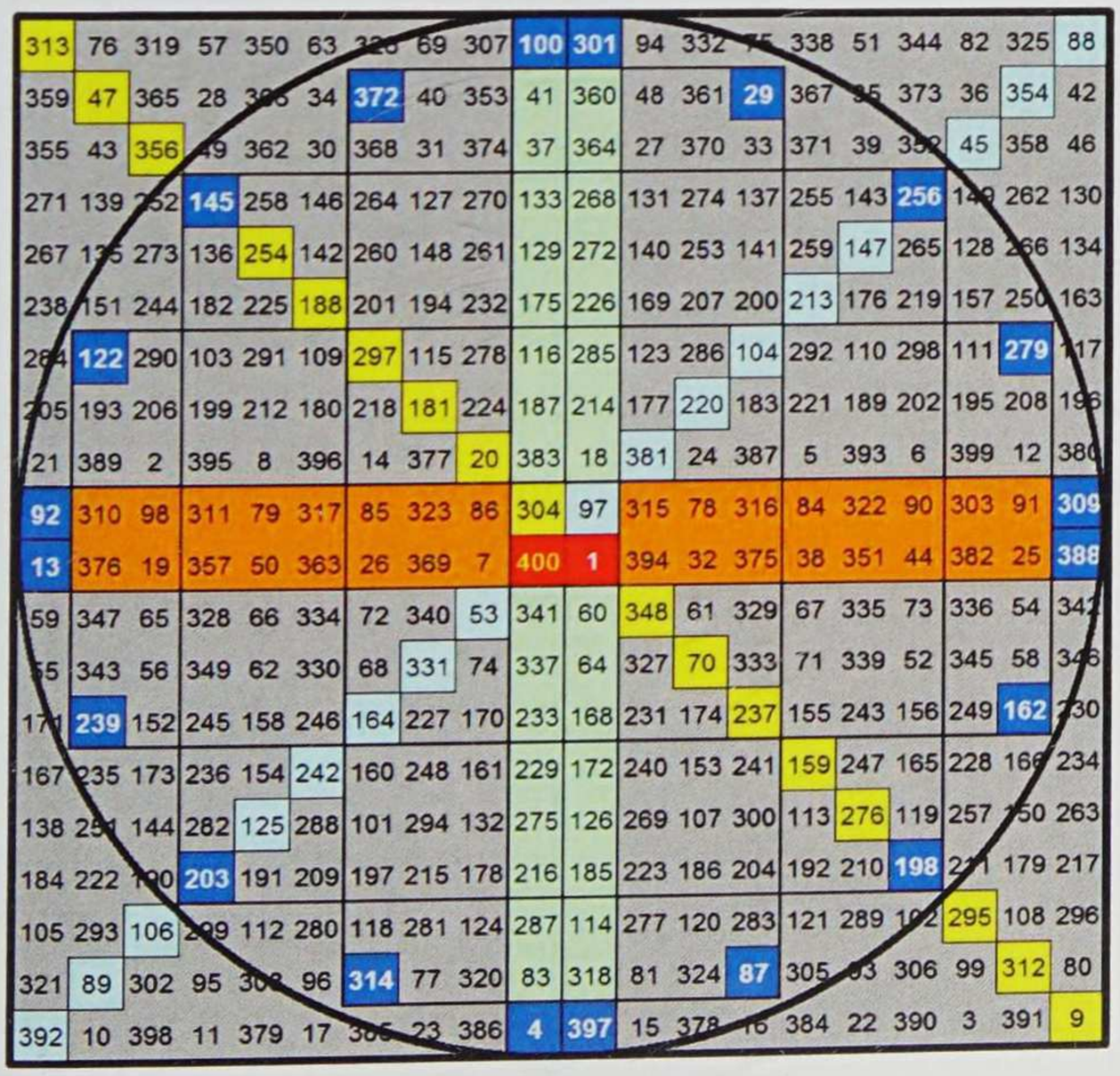
Every quadral sums to 130.

12x12

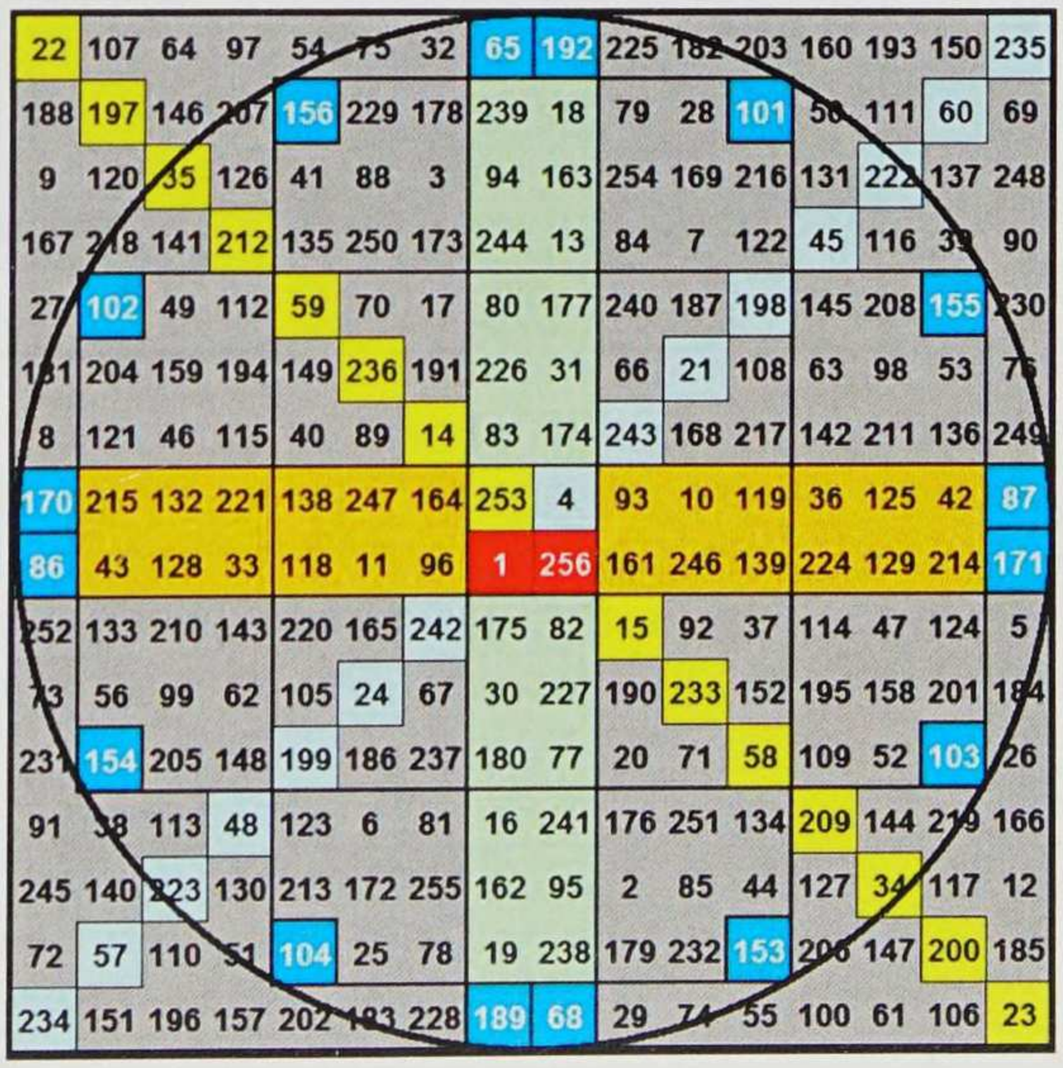


Every quadral sums to 290.

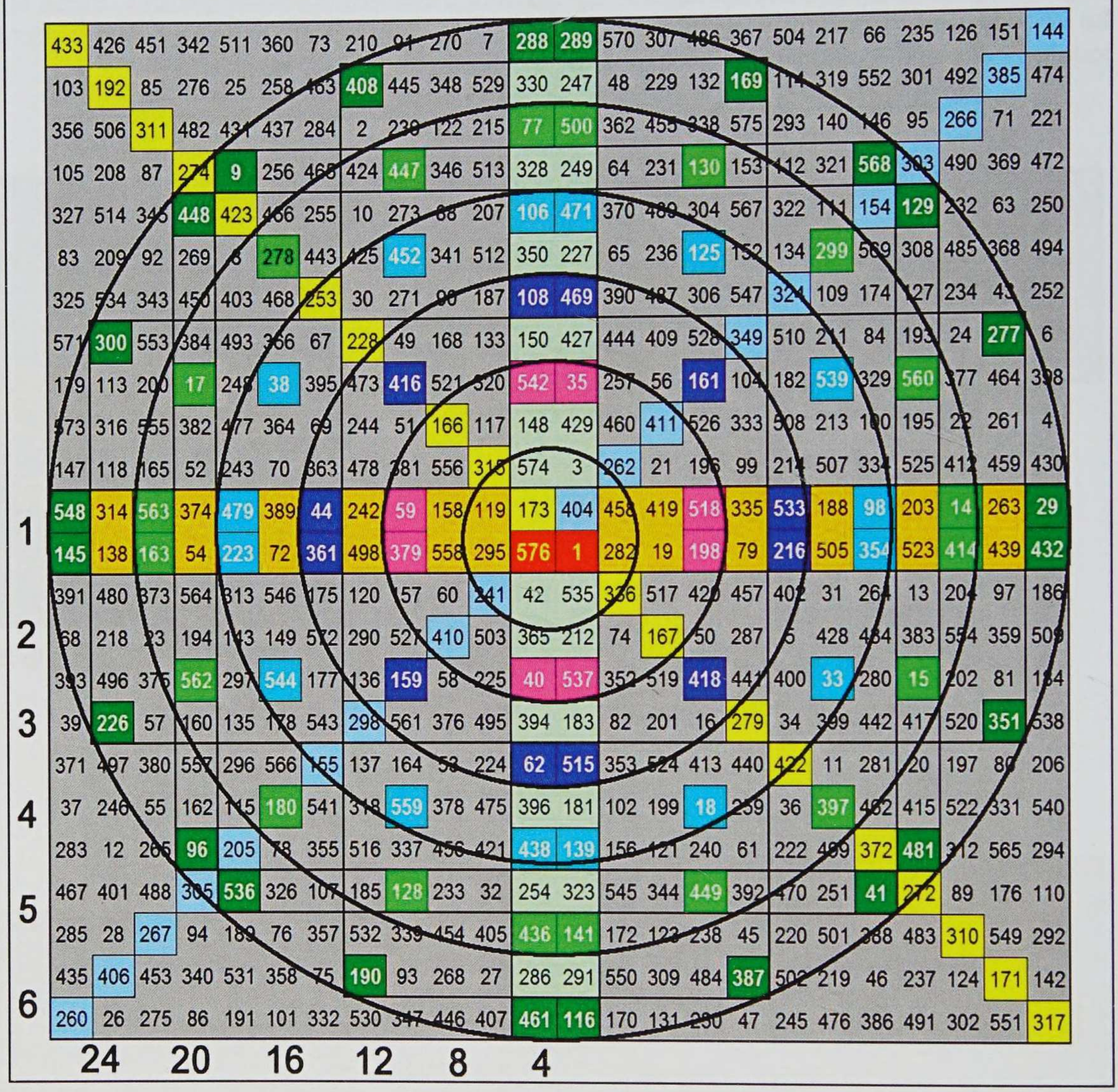
20x20



16x16



Size-24 Square with nested circles



Here is the size-24 characteristic circle. Each concentric circle within the outer circle corresponds to progressively less number of quadrals; the inner circle corresponds to a single quadral. This demonstrates the nesting pattern of incident quadrals. Observe that going outward from the center, the number of quadrals incident to the inscribed circles increases by **1** (exterior numbers along left side). The exterior numbers at bottom indicate the number of cells involved.

Program 1

The table shows the number of impinging quadrals for consecutively nested circles, each size of which is a multiple of **4**, down to size-4 itself. The pattern follows the sequence: **n** divided by **4**.

The list shows how many impinging quadrals sum to the square's characteristic number by size. An exception was made for the size-4 square, which could have two quadrals instead of just one. A single quadral for the size-4 square was chosen to maintain the natural sequence of quadral multiples seen in the central column. Observe that the number of quadrals involved in these circles is the series **1**, **2**, **3**, **4**, **5**, **6**. This series is basic to Geonometry.

This series can be formulated as

(1.5) y = 4x

where **y** represents the table's size **n** and **x** represents both the sequence of the nested circle as encountered counting from the smallest circle outward and the number of quadrals impinging the largest inscribed sphere.

This formula will come into play again in Programs 4, 5 and 8 where it will crop up again and again in surprisingly different ways.

Size n	Number of Quadrals in Characteristic Circles
4	1
8	2
12	3
16	4
20	5
24	6

Program 1

Class-5 Squares

Size-35 Square

Here is the only size of Class-5 square that is printable. Its size is **35**. It is geonomically perfect. It has equal-summing quadrals because it has all its centrally symmetric pairs summing equally.

35x35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	665	768	871	974	1077	1180	58	161	264	367	470	573	676	779	882	985	1088	1191	104	207	310	413	516	619	722	825	928	1031	1134	12	115	218	321	424	527	21455
2	596	734	837	940	1043	1146	24	127	230	333	436	539	642	745	848	951	1054	1157	70	173	276	379	482	585	688	791	894	997	1100	1203	81	184	287	390	493	21455
3	562	700	803	906	1009	1112	1215	93	196	299	402	505	608	711	814	917	1020	1123	1	139	242	345	448	551	654	757	860	963	1066	1169	47	150	253	356	459	21455
4	528	631	769	872	975	1078	1181	59	162	265	368	471	574	677	780	883	986	1089	1192	105	208	311	414	517	620	723	826	929	1032	1135	13	116	219	322	425	21455
5	494	597	735	838	941	1044	1147	25	128	231	334	437	540	643	746	849	952	1055	1158	36	174	277	380	483	586	689	792	895	998	1101	1204	82	185	288	391	21455
6	460	563	666	804	907	1010	1113	1216	94	197	300	403	506	609	712	815	918	1021	1124	2	140	243	346	449	552	655	758	861	964	1067	1170	48	151	254	357	21455
7	426	529	632	770	873	976	1079	1182	60	163	266	369	472	575	678	781	884	987	1090	1193	71	209	312	415	518	621	724	827	930	1033	1136	14	117	220	323	21455
8	392	495	598	701	839	942	1045	1148	26	129	232	335	438	541	644	747	850	953	1056	1159	37	175	278	381	484	587	690	793	896	999	1102	1205	83	186	289	21455
9	358	461	564	667	805	908	1011	1114	1217	95	198	301	404	507	610	713	816	919	1022	1125	3	106	244	347	450	553	656	759	862	965	1068	1171	49	152	255	21455
10	324	427	530	633	736	874	977	1080	1183	61	164	267	370	473	576	679	782	885	988	1091	1194	72	210	313	416	519	622	725	828	931	1034	1137	15	118	221	21455
11	290	393	496	599	702	840	943	1046	1149	27	130	233	336	439	542	645	748	851	954	1057	1160	38	141	279	382	485	588	691	794	897	1000	1103	1206	84	187	21455
12	256	359	462	565	668	771	909	1012	1115	1218	96	199	302	405	508	611	714	817	920	1023	1126	4	107	245	348	451	554	657	760	863	966	1069	1172	50	153	21455
13	222	325	428	531	634	737	875	978	1081	1184	62	165	268	371	474	577	680	783	886	989	1092	1195	73	176	314	417	520	623	726	829	932	1035	1138	16	119	21455
14	188	291	394	497	600	703	806	944	1047	1150	28	131	234	337	440	543	646	749	852	955	1058	1161	39	142	280	383	486	589	692	795	898	1001	1104	1207	85	21455
15	154	257	360	463	566	669	772	910	1013	1116	1219	97	200	303	406	509	612	715	818	921	1024	1127	5	108	211	349	452	555	658	761	864	967	1070	1173	51	21455
16	120	223	326	429	532	635	738	841	979	1082	1185	63	166	269	372	475	578	681	784	887	990	1093	1196	74	177	315	418	521	624	727	830	933	1036	1139	17	21455
17	86	189	292	395	498	601	704	807	945	1048	1151	29	132	235	338	441	544	647	750	853	956	1059	1162	40	143	246	384	487	590	693	796	899	1002	1105	1208	21455
18	52	155	258	361	464	567	670	773	876	1014	1117	1220	98	201	304	407	510	613	716	819	922	1025	1128	6	109	212	350	453	556	659	762	865	968	1071	1174	21455
19	18	121	224	327	430	533	636	739	842	980	1083	1186	64	167	270	373	476	579	682	785	888	991	1094	1197	75	178	281	419	522	625	728	831	934	1037	1140	21455
20	1209	87	190	293	396	499	602	705	808	911	1049	1152	30	133	236	339	442	545	648	751	854	957	1060	1163	41	144	247	385	488	591	694	797	900	1003	1106	21455
21	1175	53	156	259	362	465	568	671	774	877	1015	1118	1221	99	202	305	408	511	614	717	820	923	1026	1129	7	110	213	316	454	557	660	763	866	969	1072	21455
22	1141	19	122	225	328	431	534	637	740	843	946	1084	1187	65	168	271	374	477	580	683	786	889	992	1095	1198	76	179	282	420	523	626	729	832	935	1038	21455
23	1107	1210	88	191	294	397	500	603	706	809	912	1050	1153	31	134	237	340	443	546	649	752	855	958	1061	1164	42	145	248	351	489	592	695	798	901	1004	21455
24	1073	1176	54	157	260	363	466	569	672	775	878	981	1119	1222	100	203	306	409	512	615	718	821	924	1027	1130	8	111	214	317	455	558	661	764	867	970	21455
25	1039	1142	20	123	226	329	432	535	638	741	844	947	1085	1188	66	169	272	375	478	581	684	787	890	993	1096	1199	77	180	283	386	524	627	730	833	936	21455
26	1005	1108	1211	89	192	295	398	501	604	707	810	913	1016	1154	32	135	238	341	444	547	650	753	856	959	1062	1165	43	146	249	352	490	593	696	799	902	21455
27	971	1074	1177	55	158	261	364	467	570	673	776	879	982	1120	1223	101	204	307	410	513	616	719	822	925	1028	1131	9	112	215	318	421	559	662	765	868	21455
28	937	1040	1143	21	124	227	330	433	536	639	742	845	948	1051	1189	67	170	273	376	479	582	685	788	891	994	1097	1200	78	181	284	387	525	628	731	834	21455
29	903	1006	1109	1212	90	193	296	399	502	605	708	811	914	1017	1155	33	136	239	342	445	548	651	754	857	960	1063	1166	44	147	250	353	456	594	697	800	21455
30	869	972	1075	1178	56	159	262	365	468	571	674	777	880	983	1086	1224	102	205	308	411	514	617	720	823	926	1029	1132	10	113	216	319	422	560	663	766	21455
31	835	938	1041	1144	22	125	228	331	434	537	640	743	846	949	1052	1190	68	171	274	377	480	583	686	789	892	995	1098	1201	79	182	285	388	491	629	732	21455
32	801	904	1007	1110	1213	91	194	297	400	503	606	709	812	915	1018	1121	34	137	240	343	446	549	652	755	858	961	1064	1167	45	148	251	354	457	595	698	21455
33	767	870	973	1076	1179	57	160	263	366	469	572	675	778	881	984	1087	1225	103	206	309	412	515	618	721	824	927	1030	1133	11	114	217	320	423	526	664	21455
34	733	836	939	1042	1145	23	126	229	332	435	538	641	744	847	950	1053	1156	69	172	275	378	481	584	687	790	893	996	1099	1202	80	183	286	389	492	630	21455
35	699	802	905	1008	1111	1214	92	195	298	401	504	607	710	813	916	1019	1122	35	138	241	344	447	550	653	756	859	962	1065	1168	46	149	252	355	458	561	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 1

Class-6 Squares

Size-9 Square

Here is the smallest of the Class-6 squares, the size-9. It is perfect. All of its quadrals sum equally because it has all equal-summing **centrally-symmetric** pairwise sums.

It is characteristically 3x-modular because its **3-by-3** block-squares sum to its characteristic number everywhere.

	1	2	3	4	5	6	7	8	9	
	W(9)	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

All centrally-symmetric pairs sum to **82.**

Every quadral sums to **164**.

164 = 4 x 41 = **4/9** x 369.

q = 164.

p = 41

q = 4p

Program 1

Size 25 Square

Here is the next smallest size Class-6 square, size **25**. Its Characteristic number is **7825**.

It too has all equal-summing centrally symmetric pairwise sums, so all of its quadrals sum equally.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

All of its **5x5** block-squares sum to the square's characteristic number continuously, so it is **5x-modular**.

All sizes of Class-6 squares for which $\mathbf{n = b^2}$ or $\mathbf{b^3}$ will be continuously **bx-modular**.

Program 1

Notes

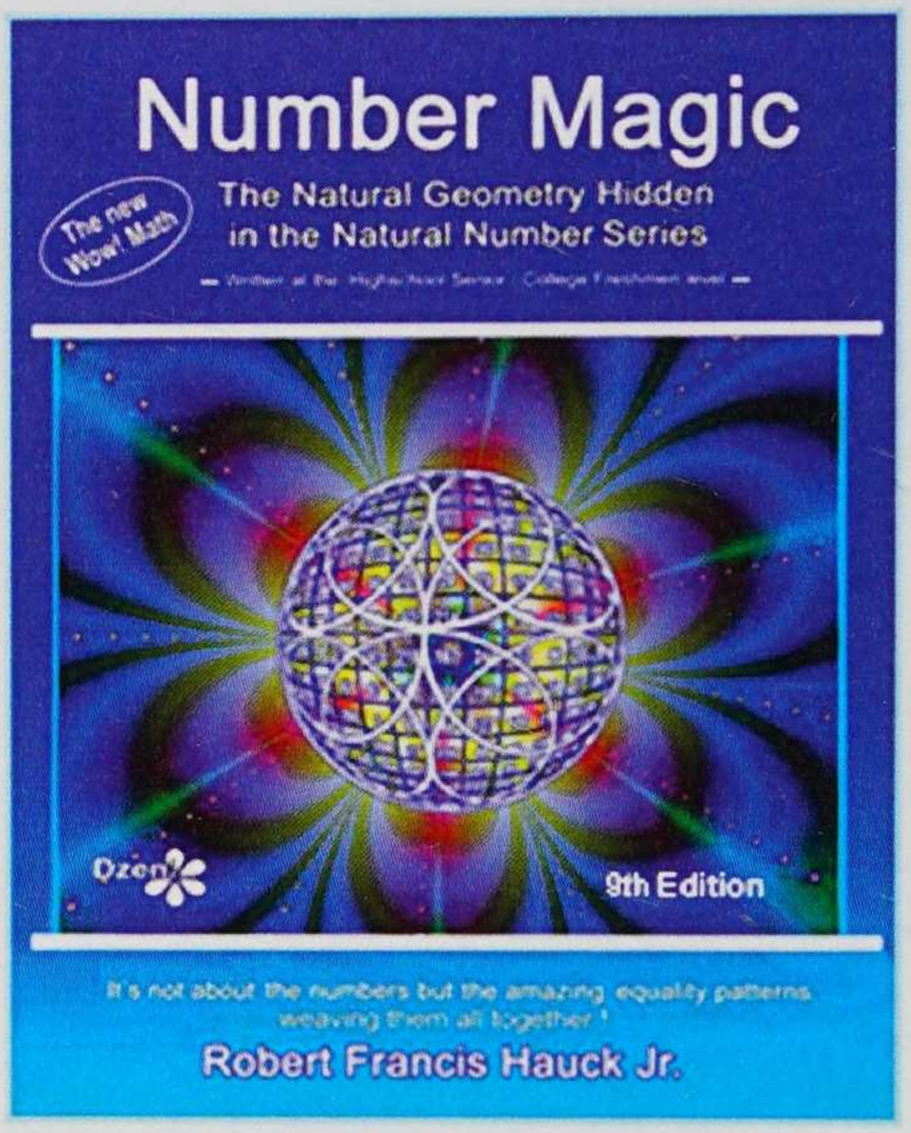
Geonomic squares can be classified into 6 distinct major classes.

1. All except Class-2 squares are perfect in that all four directional summations add up to the same number and are pangenic.
2. Class-2 squares are pangenic, but are only near-perfect in that they have two symmetrically located rows which differ by **±1**.
3. Class-3 squares of **n = 3b** possess **bx**-modularity.
4. Class-4 squares for sizes **n = 4b** for **b >1** also possess continuous **bx**-modularity.
5. Class-4 squares whose size is divisible by an even multiple of **8** have both row and central pairwise symmetry.
6. Class-4 squares possess characteristic circles in which the numbers impinging the largest inscribed circle from its interior sum to the squares characteristic number.
7. Class-5 squares are only centrally pairwise symmetric.
8. Class-6 squares of size **n = b^2** or **b^3** are **bx**-modular.

Program 1

We have come to the end of Program 1 of the new **math**, **Geonometry**. This program was merely an introduction to the topic of geonomic squares – more of a preparedness program that may have seemed rather tedious in a way because of the need for detailed explanations of the limitations of the various classifications. From here on out however, the programs will become quite entertaining and even inspirational because these summation patterns will be observed to be just amazing, one after another after another, non-stop right to the last slide.

Here the book upon which this introductory program was based.



Number Magic – The Natural Geometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

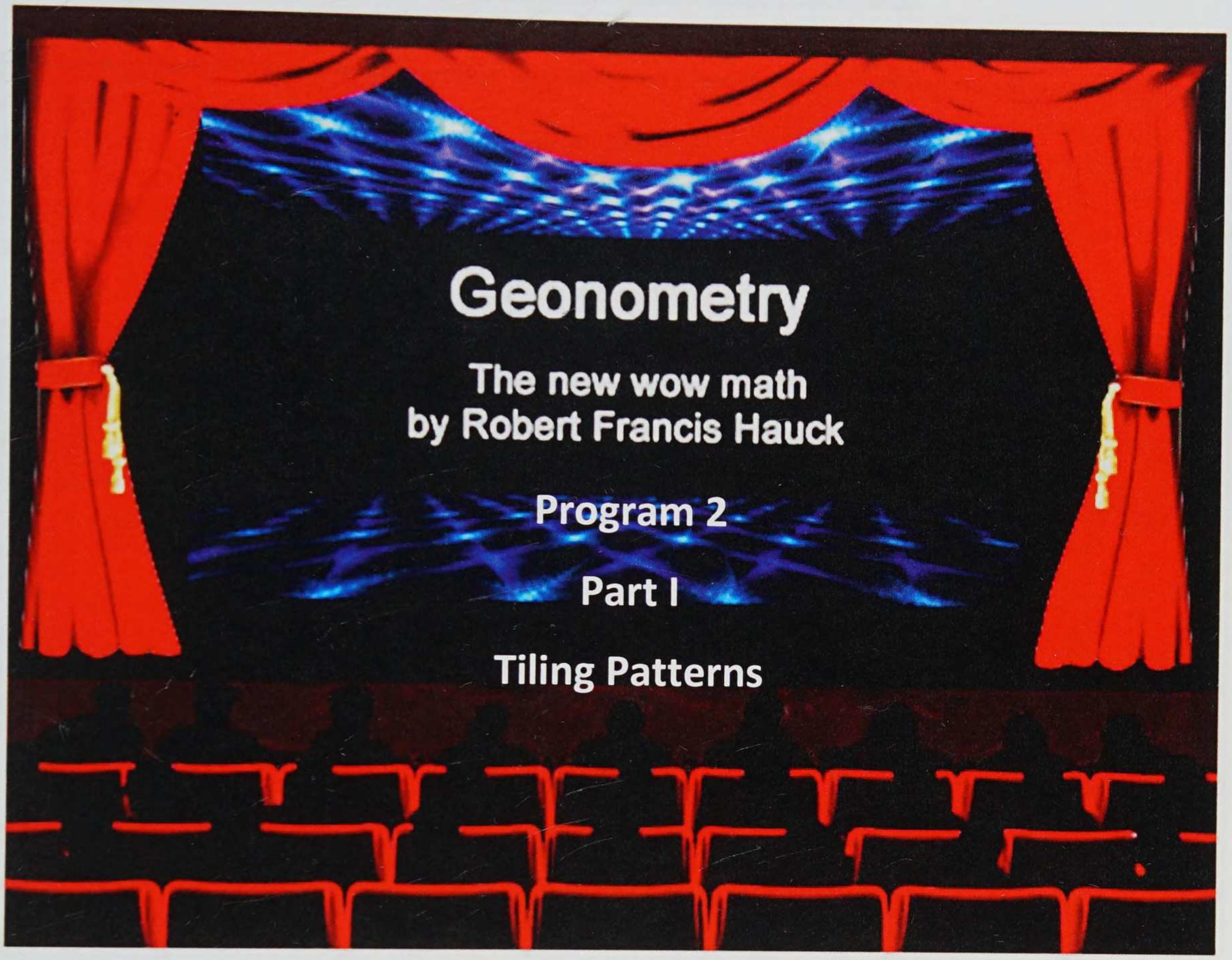
Shows examples of every size table that can be printed legibly up through the 5th dimension.

Ninth Edition

Black & white print (360+ pages)

In Part I of the next program we will discover the amazing interlocking equal-summing tiling patterns hidden in geonomic squares – the natural quilt patterns in the fabric of space.

Then in Part II we will see just how these quilt patterns are woven through the discovery of tables that act as 4-directional looms.



Introduction

Next, we're going to explore some amazing geometries that are found in all the squares of all classes. These geometries are called **tiling patterns**. They will be seen to exhibit amazing characteristic quilt patterns in the 2-dimensional spatial fabric.
The thing to note is that each tile corresponding to a square of size **n** consists of exactly **n** numbered-cells. In addition, since the square contains $\mathbf{n}^2$ numbers there will be exactly **n** **independent interlocking** tiles, or an integer multiple thereof, in the overall tiling pattern.

These tiling patterns have **four** special properties, called the **"4C"** properties:

1. They are **continuous**. Their center tile can be placed anywhere in the table and the equal summations persist in each tile!
2. They are **contiguous**. There are no isolated cells segregated from the others in the pattern. All of their cells are connected to another even if only at the cells' corners.
3. They are **characteristic** in that all their number cells sum to the square's characteristic number or a multiple thereof.
4. In addition, they are **complete** in that they interlock to cover the square or multiples of the square completely without voids.

Program 2

Tiling Patterns for Geonomic Squares

This picture makes the point that all these tiling patterns to follow are continuous everywhere. They can be pulled across the table like a tablecloth and the equal summations persist in each tile!

Further, the square's number pattern can be wrapped around the surface of a numbered sphere and the tiling patterns can be spun like a top and the equal summations persist in each tile! This property will have a major application in Program 5 where we explore the distribution of electrons around the nucleus of the atom.

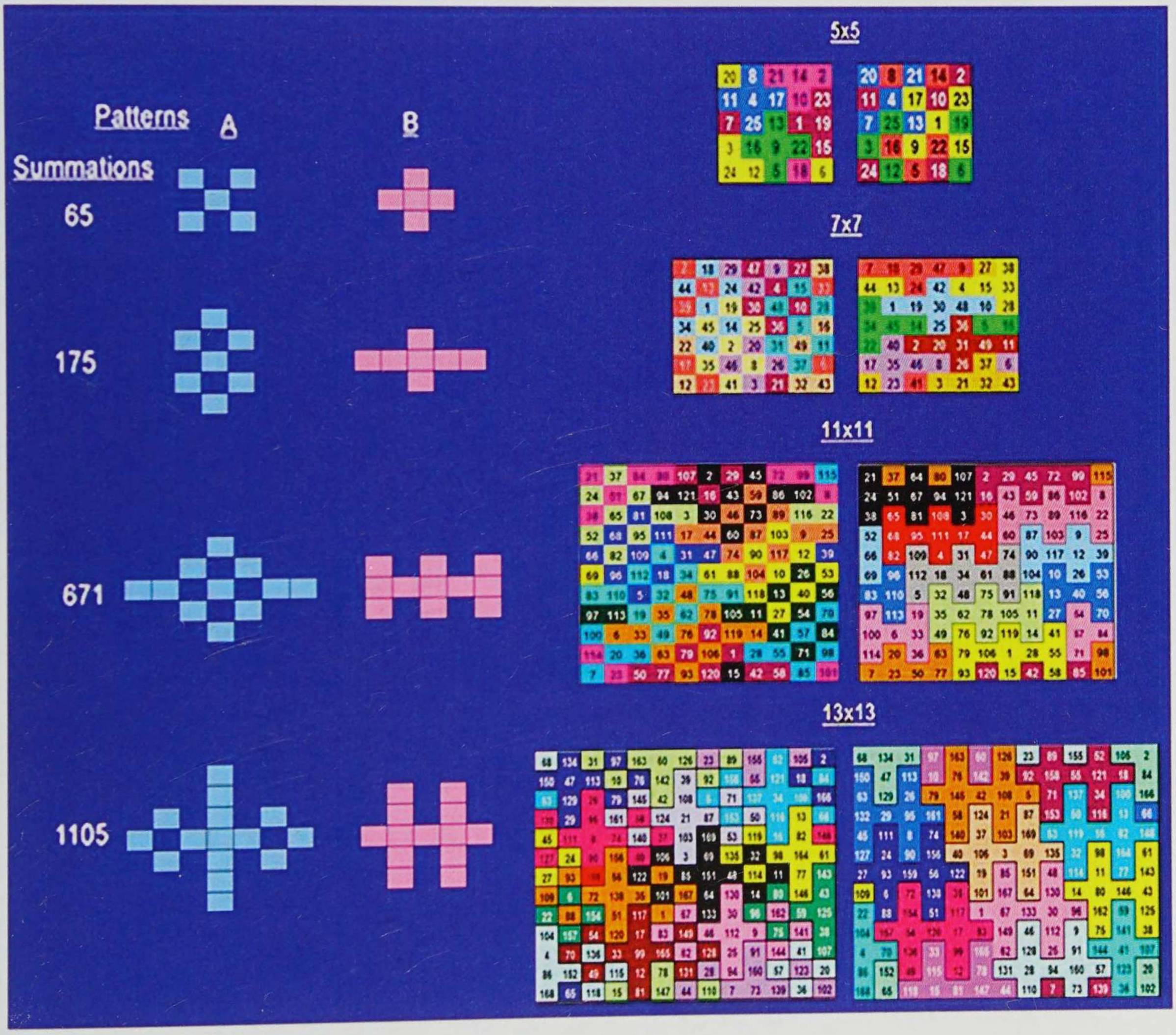
We'll first investigate tiling patterns for odd size squares in Classes **1**, **5** and **6**. Class **3** squares are of odd size too, however, they do not possess any 2-dimensional tiling configuration that will sum continuously to their characteristic number. So they will be skipped here in this Part II.

Program 2

Tiling Patterns for Odd Size Squares

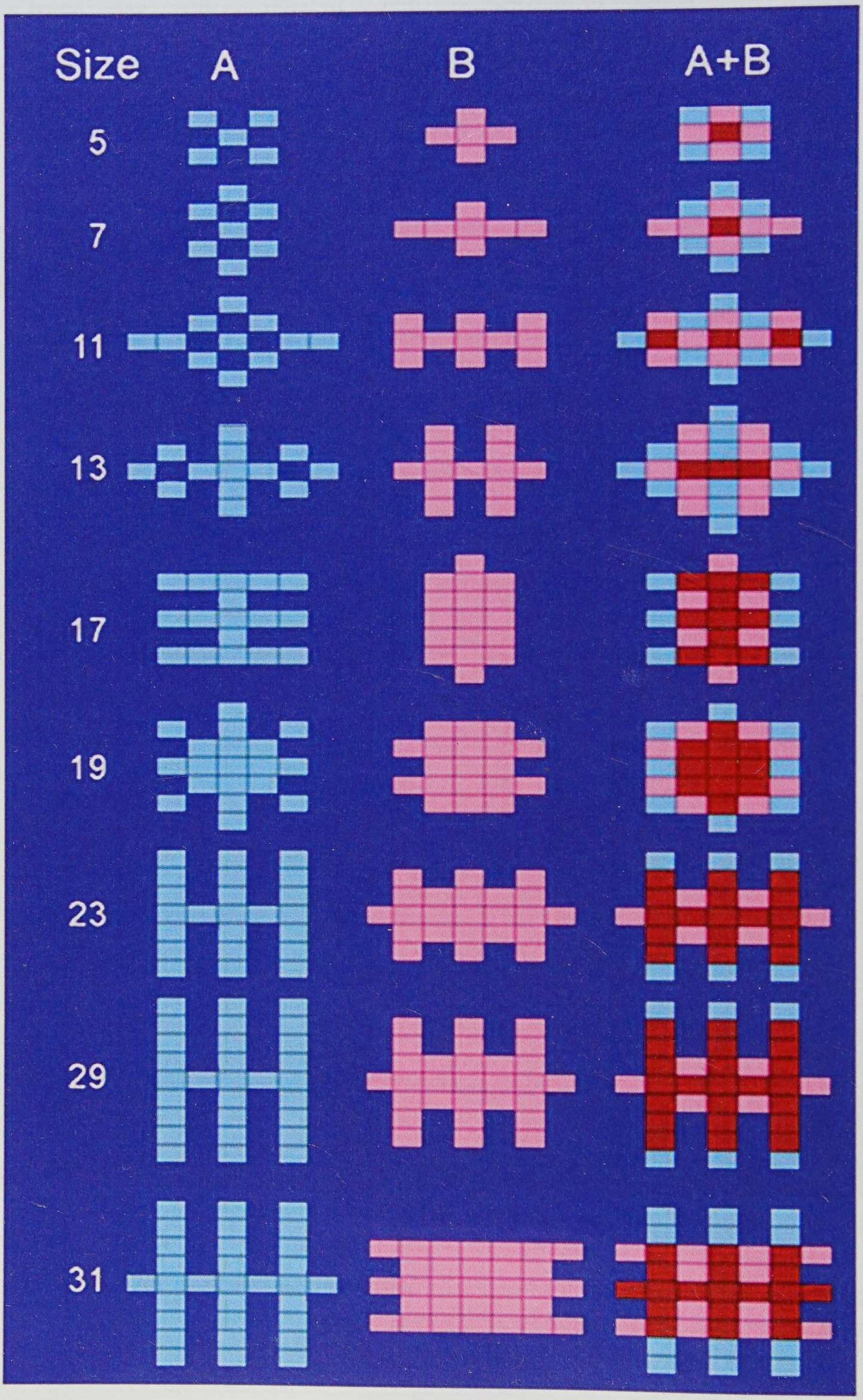
Class-1 Squares

Class-1 squares all have distinct configurations unique to their square's size. Here is a sample of the complementary tiling patterns for Class-1 sizes **5** through **13**. Note the interlocking of the tiles to form a complete tiling of the square. All the numbers in each tile sum to the same number, as do all the rows, columns, and diagonals in both directions. That number is the square's characteristic number. The complementary tile patterns will be denoted by non-bold letters A and B.



Program 2

Here is a pictorial list of all the different dual tile patterns up through all prime-number sizes from **5** to **31**. The figures on the right show the dual tiles laminated together with their centers and overlapping cells darkly shaded.



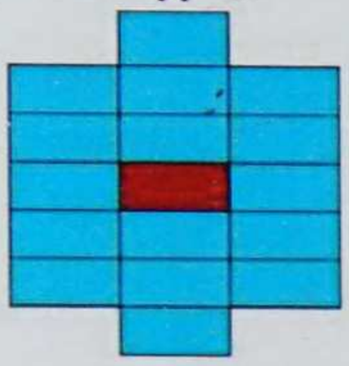
Program 2

Size 17 Patterns

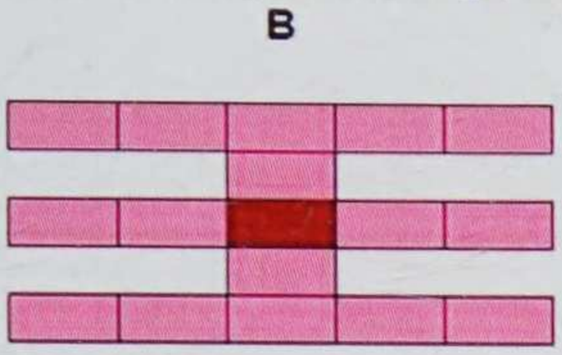
Here are the two continuous tiling patterns A and B identified by various colors for the original version of the size **17** square. Observe the interlocking of the tiles in each tiling pattern. The strips on the right show that the tile centers sum to the characteristic number. They only do this provided the central tile's center is centered in the square.

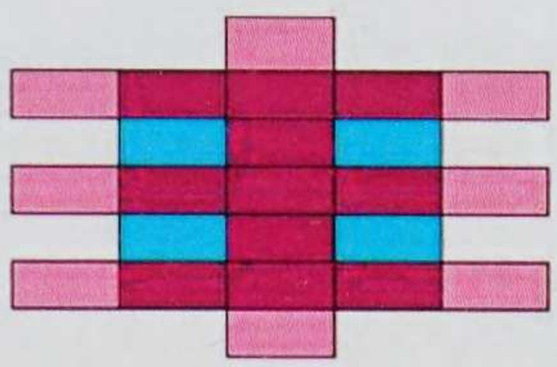
Size-17 complementary patterns

A



B





	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
2465	255	140	42	233	135	20	211	113	15	189	91	282	184	69	260	162	64	2465
2465	267	169	54	245	147	49	223	125	27	218	103	5	196	98	289	174	76	2465
2465	279	181	83	257	159	61	252	137	39	230	132	34	208	110	12	203	88	2465
2465	2	193	95	286	171	73	264	166	68	242	144	46	237	122	24	215	117	2465
2465	31	205	107	9	200	102	276	178	80	271	156	58	249	151	36	227	129	2465
2465	43	234	136	21	212	114	16	190	92	283	185	70	261	163	65	239	141	2465
2465	55	246	148	50	224	126	28	219	104	6	197	99	273	175	77	268	170	2465
2465	84	258	160	62	253	138	40	231	133	18	209	111	13	204	89	280	182	2465
2465	96	287	172	74	265	167	52	243	145	47	238	123	25	216	118	3	194	2465
2465	108	10	201	86	277	179	81	272	157	59	250	152	37	228	130	32	206	2465
2465	120	22	213	115	17	191	93	284	186	71	262	164	66	240	142	44	235	2465
2465	149	51	225	127	29	220	105	7	198	100	274	176	78	269	154	56	247	2465
2465	161	63	254	139	41	232	134	19	210	112	14	188	90	281	183	85	259	2465
2465	173	75	266	168	53	244	146	48	222	124	26	217	119	4	195	97	288	2465
2465	202	87	278	180	82	256	158	60	251	153	38	229	131	33	207	109	11	2465
2465	214	116	1	192	94	285	187	72	263	165	67	241	143	45	236	121	23	2465
2465	226	128	30	221	106	8	199	101	275	177	79	270	155	57	248	150	35	2465
	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465

9
26
43
60
77
94
111
128
145
162
179
196
213
230
247
264
281
2465

Individual tile patterns sum to 2465 anywhere throughout table.

	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
2465	255	140	42	233	135	20	211	113	15	189	91	282	184	69	260	162	64	2465
2465	267	169	54	245	147	49	223	125	27	218	103	5	196	98	289	174	76	2465
2465	279	181	83	257	159	61	252	137	39	230	132	34	208	110	12	203	88	2465
2465	2	193	95	286	171	73	264	166	68	242	144	46	237	122	24	215	117	2465
2465	31	205	107	9	200	102	276	178	80	271	156	58	249	151	36	227	129	2465
2465	43	234	136	21	212	114	16	190	92	283	185	70	261	163	65	239	141	2465
2465	55	246	148	50	224	126	28	219	104	6	197	99	273	175	77	268	170	2465
2465	84	258	160	62	253	138	40	231	133	18	209	111	13	204	89	280	182	2465
2465	96	287	172	74	265	167	52	243	145	47	238	123	25	216	118	3	194	2465
2465	108	10	201	86	277	179	81	272	157	59	250	152	37	228	130	32	206	2465
2465	120	22	213	115	17	191	93	284	186	71	262	164	66	240	142	44	235	2465
2465	149	51	225	127	29	220	105	7	198	100	274	176	78	269	154	56	247	2465
2465	161	63	254	139	41	232	134	19	210	112	14	188	90	281	183	85	259	2465
2465	173	75	266	168	53	244	146	48	222	124	26	217	119	4	195	97	288	2465
2465	202	87	278	180	82	256	158	60	251	153	38	229	131	33	207	109	11	2465
2465	214	116	1	192	94	285	187	72	263	165	67	241	143	45	236	121	23	2465
2465	226	128	30	221	106	8	199	101	275	177	79	270	155	57	248	150	35	2465
	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465

9
26
43
60
77
94
111
128
145
162
179
196
213
230
247
264
281
2465

Program 2

This slide shows dual tile patterns for two different versions of the size-17 square. These tile patterns are independent of the number distribution in the table as long as the square is geonomically perfect. The important point is this: **These equal-summing tiles are characteristic of the size of the square and not of the particular square itself.**

Original version

17x17	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465
255	140	42	233	135	20	211	113	15	189	91	282	184	69	260	162	64	2465
267	169	54	245	147	49	223	125	27	218	103	5	196	98	289	174	76	2465
279	181	83	257	159	61	252	137	39	230	132	34	208	110	12	203	88	2465
2	193	95	286	171	73	264	166	68	242	144	46	237	122	24	215	117	2465
31	205	107	9	200	102	276	178	80	271	156	58	249	151	36	227	129	2465
43	234	136	21	212	114	16	190	92	283	185	70	261	163	65	239	141	2465
55	246	148	50	224	126	28	219	104	6	197	99	273	175	77	268	170	2465
84	258	160	62	253	138	40	231	133	18	209	111	13	204	89	280	182	2465
96	287	172	74	265	167	52	243	145	47	238	123	25	216	118	3	194	2465
108	10	201	86	277	179	81	272	157	59	250	152	37	228	130	32	206	2465
120	22	213	115	17	191	93	284	186	71	262	164	66	240	142	44	235	2465
149	51	225	127	29	220	105	7	198	100	274	176	78	269	154	56	247	2465
161	63	254	139	41	232	134	19	210	112	14	188	90	281	183	85	259	2465
173	75	266	168	53	244	146	48	222	124	26	217	119	4	195	97	288	2465
202	87	278	180	82	256	158	60	251	153	38	229	131	33	207	109	11	2465
214	116	1	192	94	285	187	72	263	165	67	241	143	45	236	121	23	2465
226	128	30	221	106	8	199	101	275	177	79	270	155	57	248	150	35	2465
2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465

Contiguous complementary patterns sum to 2465 continuously.

Alternate version

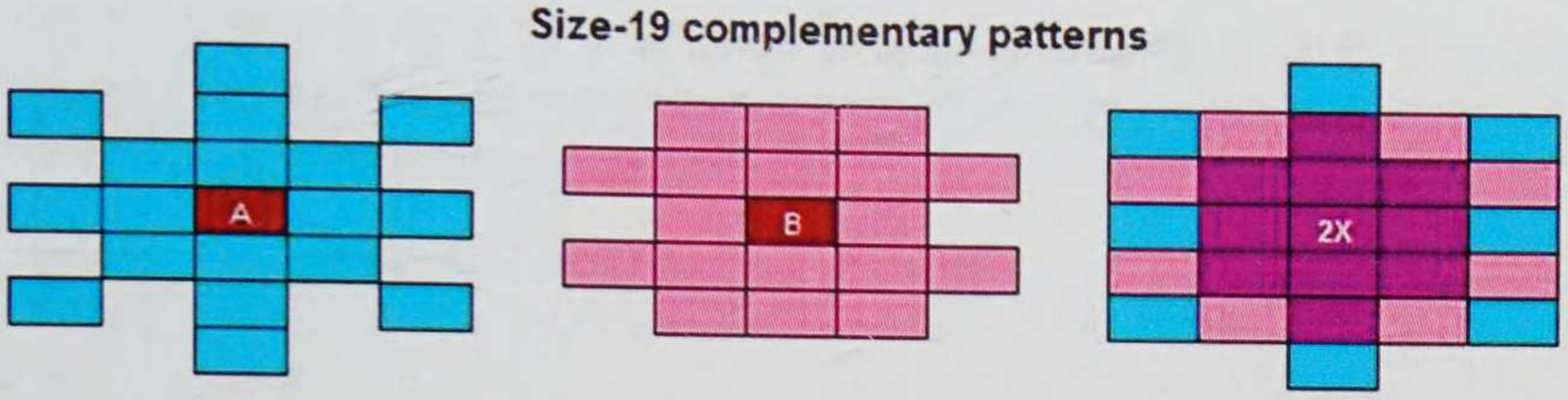
17x17	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465
185	196	207	235	246	257	285	7	18	46	57	85	96	107	135	146	157	2465
237	248	259	287	9	20	48	59	70	98	109	120	148	159	187	198	209	2465
289	11	22	50	61	72	100	111	122	150	161	172	200	211	222	250	261	2465
35	63	74	102	113	124	152	163	174	202	213	224	252	263	274	13	24	2465
87	115	126	137	165	176	204	215	226	254	265	276	15	26	37	65	76	2465
139	167	178	189	217	228	239	267	278	17	28	39	67	78	89	117	128	2465
191	219	230	241	269	280	2	30	41	52	80	91	119	130	141	169	180	2465
243	271	282	4	32	43	54	82	93	104	132	143	154	182	193	221	232	2465
6	34	45	56	84	95	106	134	145	156	184	195	206	234	245	256	284	2465
58	69	97	108	136	147	158	186	197	208	236	247	258	286	8	19	47	2465
110	121	149	160	171	199	210	238	249	260	288	10	21	49	60	71	99	2465
162	173	201	212	223	251	262	273	12	23	51	62	73	101	112	123	151	2465
214	225	253	264	275	14	25	36	64	75	86	114	125	153	164	175	203	2465
266	277	16	27	38	66	77	88	116	127	138	166	177	188	216	227	255	2465
29	40	68	79	90	118	129	140	168	179	190	218	229	240	268	279	1	2465
81	92	103	131	142	170	181	192	220	231	242	270	281	3	31	42	53	2465
133	144	155	183	194	205	233	244	272	283	5	33	44	55	83	94	105	2465
2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465

Program 2

Size-19 Patterns

Here is the size **19** square with the transpose of its complementary tile patterns tA and tB identified by various colors. In the external strip on the right, note that the center numbers in the tiles in each pattern sum to the characteristic number. They do this continuously.

Size-19 complementary patterns



Individual transposed B tile patterns equal 3439 anywhere throughout table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
	19x19	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	
1	247	336	64	153	261	350	78	186	275	3	111	200	289	36	125	214	322	50	139	3439	11
2	252	360	88	177	285	13	102	191	299	27	116	224	313	41	149	238	327	74	163	3439	23
3	276	4	112	201	290	37	126	215	323	51	140	229	337	65	154	262	351	79	187	3439	54
4	300	28	117	225	314	42	150	239	328	75	164	253	361	89	178	267	14	103	192	3439	66
5	305	52	141	230	338	66	155	263	352	80	188	277	5	113	202	291	38	127	216	3439	78
6	329	76	165	254	343	90	179	268	15	104	193	301	29	118	226	315	43	151	240	3439	109
7	353	81	189	278	6	114	203	292	20	128	217	306	53	142	231	339	67	156	264	3439	121
8	16	105	194	302	30	119	227	316	44	152	241	330	58	166	255	344	91	180	269	3439	152
9	21	129	218	307	54	143	232	340	68	157	265	354	82	190	279	7	96	204	293	3439	164
10	45	134	242	331	59	167	256	345	92	181	270	17	106	195	303	31	120	228	317	3439	176
11	69	158	266	355	83	172	280	8	97	205	294	22	130	219	308	55	144	233	341	3439	207
12	93	182	271	18	107	196	304	32	121	210	318	46	135	243	332	60	168	257	346	3439	219
13	98	206	295	23	131	220	309	56	145	234	342	70	159	248	356	84	173	281	9	3439	231
14	122	211	319	47	136	244	333	61	169	258	347	94	183	272	19	108	197	286	33	3439	262
15	146	235	324	71	160	249	357	85	174	282	10	99	207	296	24	132	221	310	57	3439	274
16	170	259	348	95	184	273	1	109	198	287	34	123	212	320	48	137	245	334	62	3439	286
17	175	283	11	100	208	297	25	133	222	311	39	147	236	325	72	161	250	358	86	3439	317
18	199	288	35	124	213	321	49	138	246	335	63	171	260	349	77	185	274	2	110	3439	329
19	223	312	40	148	237	326	73	162	251	359	87	176	284	12	101	209	298	26	115	3439	360
	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439		3439
		3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	

Individual transposed A tile patterns equal 3439 anywhere throughout table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
	19x19	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	
1	247	336	64	153	261	350	78	186	275	3	111	200	289	36	125	214	322	50	139	3439	5
2	252	360	88	177	285	13	102	191	299	27	116	224	313	41	149	238	327	74	163	3439	36
3	276	4	112	201	290	37	126	215	323	51	140	229	337	65	154	262	351	79	187	3439	48
4	300	28	117	225	314	42	150	239	328	75	164	253	361	89	178	267	14	103	192	3439	60
5	305	52	141	230	338	66	155	263	352	80	188	277	5	113	202	291	38	127	216	3439	91
6	329	76	165	254	343	90	179	268	15	104	193	301	29	118	226	315	43	151	240	3439	103
7	353	81	189	278	6	114	203	292	20	128	217	306	53	142	231	339	67	156	264	3439	201
8	16	105	194	302	30	119	227	316	44	152	241	330	58	166	255	344	91	180	269	3439	115
9	21	129	218	307	54	143	232	340	68	157	265	354	82	190	279	7	96	204	293	3439	146
10	45	134	242	331	59	167	256	345	92	181	270	17	106	195	303	31	120	228	317	3439	158
11	69	158	266	355	83	172	280	8	97	205	294	22	130	219	308	55	144	233	341	3439	189
12	93	182	271	18	107	196	304	32	121	210	318	46	135	243	332	60	168	257	346	3439	213
13	98	206	295	23	131	220	309	56	145	234	342	70	159	248	356	84	173	281	9	3439	244
14	122	211	319	47	136	244	333	61	169	258	347	94	183	272	19	108	197	286	33	3439	256
15	146	235	324	71	160	249	357	85	174	282	10	99	207	296	24	132	221	310	57	3439	268
16	170	259	348	95	184	273	1	109	198	287	34	123	212	320	48	137	245	334	62	3439	299
17	175	283	11	100	208	297	25	133	222	311	39	147	236	325	72	161	250	358	86	3439	311
18	199	288	35	124	213	321	49	138	246	335	63	171	260	349	77	185	274	2	110	3439	342
19	223	312	40	148	237	326	73	162	251	359	87	176	284	12	101	209	298	26	115	3439	354
	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439		3439
		3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	

Program 2

Size 31 Pattern A

Here is a size **31** square with its A tiling pattern. Its tiles each sum to the square's characteristic number. The tile centers sum to the characteristic number too, but only when the center tile is symmetrically positioned over the square's pivot number **481,** as it is here, and nowhere else.

Program 2

Size 31 Pattern B

Here is a size **31** square with its B tiling pattern. Its tiles each sum to the square's characteristic number. Again the tile centers sum to the characteristic number too, but only when the center tile is symmetrically positioned over the square's pivot number **481**, as it is here, and nowhere else.

In the transpose of each tiling pattern, the tile centers again sum to the characteristic number but do so continuously across the square. This is referred to as **anchor-dot patterns**. However the transposed tiles no longer sum equally. One property is traded for the other.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	31x31	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911
1		402	220	69	879	728	577	426	275	124	903	752	601	450	299	148	958	776	625	474	323	172	21	831	680	498	347	196	45	855	704	553	14911
2		364	213	62	841	690	539	388	237	86	896	714	563	412	261	110	920	769	618	436	285	134	944	793	642	491	340	158	7	817	666	515	14911
3		326	175	24	834	652	501	350	199	48	858	707	556	374	223	72	882	731	580	429	278	96	906	755	604	453	302	151	961	779	628	477	14911
4		288	137	947	796	645	494	312	161	10	820	669	518	367	216	34	844	693	542	391	240	89	899	717	566	415	264	113	923	772	590	439	14911
5		250	99	909	758	607	456	305	154	933	782	631	480	329	178	27	837	655	504	353	202	51	861	710	528	377	226	75	885	734	583	432	14911
6		243	92	871	720	569	418	267	116	926	775	593	442	291	140	950	799	648	466	315	164	13	823	672	521	370	188	37	847	696	545	394	14911
7		205	54	864	713	531	380	229	78	888	737	586	404	253	102	912	761	610	459	308	126	936	785	634	483	332	181	30	809	658	507	356	14911
8		167	16	826	675	524	342	191	40	850	699	548	397	246	64	874	723	572	421	270	119	929	747	596	445	294	143	953	802	651	469	318	14911
9		129	939	788	637	486	335	184	2	812	661	510	359	208	57	867	685	534	383	232	81	891	740	589	407	256	105	915	764	613	462	280	14911
10		122	901	750	599	448	297	146	956	805	623	472	321	170	19	829	678	527	345	194	43	853	702	551	400	218	67	877	726	575	424	273	14911
11		84	894	743	561	410	259	108	918	767	616	465	283	132	942	791	640	489	338	156	5	815	664	513	362	211	60	839	688	537	386	235	14911
12		46	856	705	554	403	221	70	880	729	578	427	276	94	904	753	602	451	300	149	959	777	626	475	324	173	22	832	681	499	348	197	14911
13		8	818	667	516	365	214	32	842	691	540	389	238	87	897	715	564	413	262	111	921	770	619	437	286	135	945	794	643	492	341	159	14911
14		931	780	629	478	327	176	25	835	653	502	351	200	49	859	708	557	375	224	73	883	732	581	430	279	97	907	756	605	454	303	152	14911
15		924	773	591	440	289	138	948	797	646	495	313	162	11	821	670	519	368	217	35	845	694	543	392	241	90	869	718	567	416	265	114	14911
16		886	735	584	433	251	100	910	759	608	457	306	155	934	783	632	481	330	179	28	807	656	505	354	203	52	862	711	529	378	227	76	14911
17		848	697	546	395	244	93	872	721	570	419	268	117	927	745	594	443	292	141	951	800	649	467	316	165	14	824	673	522	371	189	38	14911
18		810	659	508	357	206	55	865	683	532	381	230	79	889	738	587	405	254	103	913	762	611	460	309	127	937	786	635	484	333	182	31	14911
19		803	621	470	319	168	17	827	676	525	343	192	41	851	700	549	398	247	65	875	724	573	422	271	120	930	748	597	446	295	144	954	14911
20		765	614	463	281	130	940	789	638	487	336	185	3	813	662	511	360	209	58	868	686	535	384	233	82	892	741	559	408	257	106	916	14911
21		727	576	425	274	123	902	751	600	449	298	147	957	806	624	473	322	171	20	830	679	497	346	195	44	854	703	552	401	219	68	878	14911
22		689	538	387	236	85	895	744	562	411	260	109	919	768	617	435	284	133	943	792	641	490	339	157	6	816	665	514	363	212	61	840	14911
23		682	500	349	198	47	857	706	555	373	222	71	881	730	579	428	277	95	905	754	603	452	301	150	960	778	627	476	325	174	23	833	14911
24		644	493	311	160	9	819	668	517	366	215	33	843	692	541	390	239	88	898	716	565	414	263	112	922	771	620	438	287	136	946	795	14911
25		606	455	304	153	932	781	630	479	328	177	26	836	654	503	352	201	50	860	709	558	376	225	74	884	733	582	431	249	98	908	757	14911
26		568	417	266	115	925	774	592	441	290	139	949	798	647	496	314	163	12	822	671	520	369	187	36	846	695	544	393	242	91	870	719	14911
27		530	379	228	77	887	736	585	434	252	101	911	760	609	458	307	125	935	784	633	482	331	180	29	808	657	506	355	204	53	863	712	14911
28		523	372	190	39	849	698	547	396	245	63	873	722	571	420	269	118	928	746	595	444	293	142	952	801	650	468	317	166	15	825	674	14911
29		485	334	183	1	811	660	509	358	207	56	866	684	533	382	231	80	890	739	588	406	255	104	914	763	612	461	310	128	938	787	636	14911
30		447	296	145	955	804	622	471	320	169	18	828	677	526	344	193	42	852	701	550	399	248	66	876	725	574	423	272	121	900	749	598	14911
31		409	258	107	917	766	615	464	282	131	941	790	639	488	337	186	4	814	663	512	361	210	59	838	687	536	385	234	83	893	742	560	14911
		14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911
		14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911

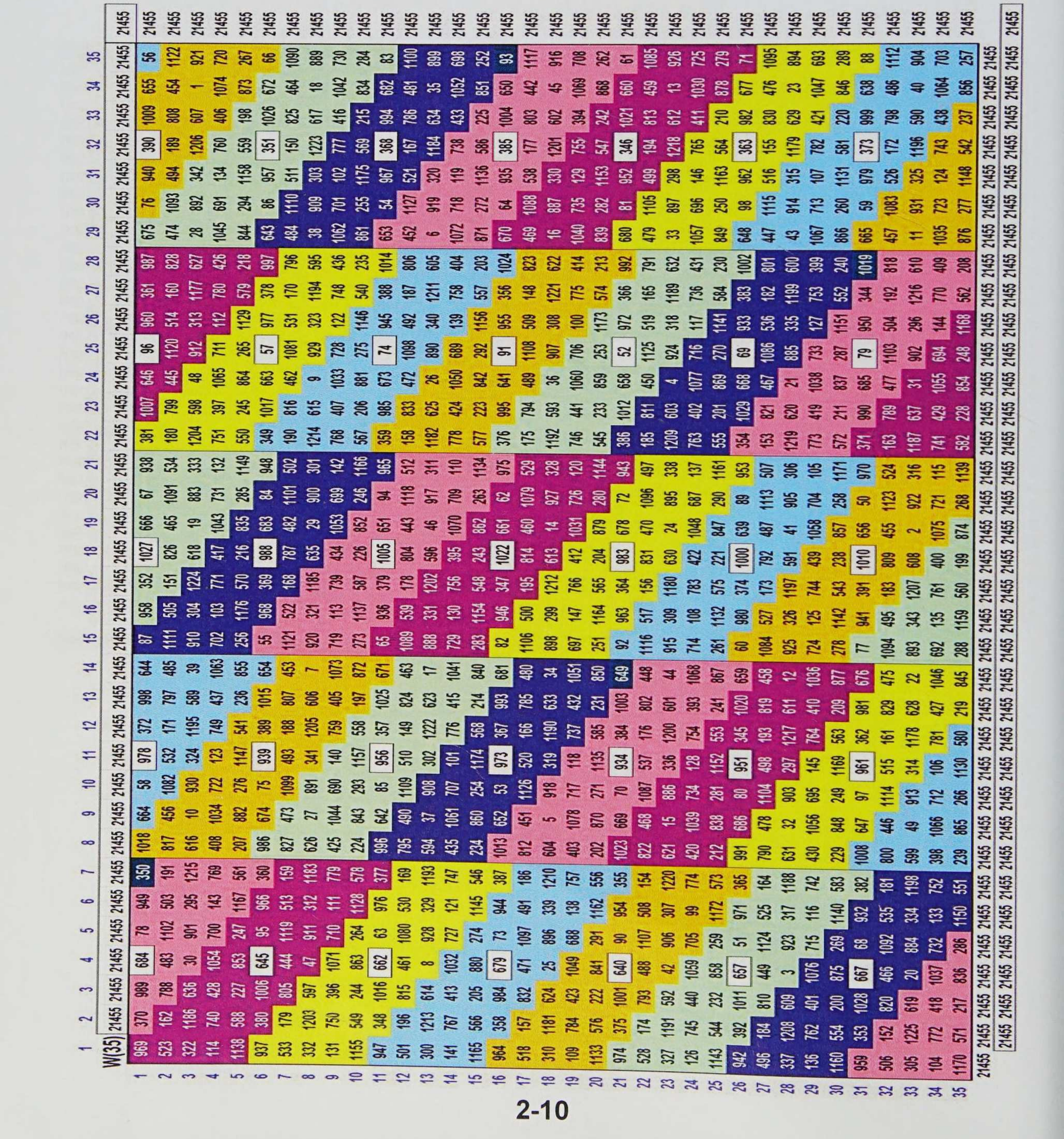
Program 2

Class-5 Square Tiling Patterns

Size-35

35 = 5x7. Every staircase tile of 35 cells sums to the size-35 square's characteristic number. Further the centers of these tiles sum continuously to the squares characteristic number too.

The horizontal flip of these tile patterns is identical to the vertical flip and continuously sums to the characteristic number too. And so do their resulting central dot patterns.



Size-35

Similar patterns of the appropriate size **ab** carry over to all Class-5 squares.

Here is the same size 35 square as on the priop page now with **10** simultaneous interlocking tiling patterns that sum continuously to the square's characteristic number **21455**. In a complete tiling of the square, their tile centers collectively function as one anchor-dot pattern that sums to this same number too. There are two of these: one in the vertically aligned arrangement and one horizontally aligned.

945	658	221	1164	702	445	158	946	664	202	1170	708	446	164	927	670	208	1171	714	427	170	933	671	214	1152	720	433	171	939	652	220	1158	721	439	152
811	354	92	1035	598	311	1079	817	360	98	1036	579	317	1085	823	361	79	1042	585	323	1086	804	367	85	1048	586	304	1092	810	373	86	1029	592	310	1098
682	250	1188	726	469	7	975	688	226	1194	732	475	13	951	694	232	1200	738	451	19	957	700	238	1176	744	457	25	963	676	244	1182	750	463	1	969
378	116	884	622	340	1103	841	384	122	890	603	341	1109	847	390	103	891	609	347	1115	828	391	109	897	615	328	1116	834	397	115	878	616	334	1122	840
274	1212	755	493	31	999	537	255	1218	756	499	37	980	543	256	1224	762	480	43	981	549	262	1205	768	481	49	987	530	268	1206	774	487	30	993	531
145	908	646	189	1127	870	408	146	914	627	195	1133	871	414	127	920	633	196	1139	852	420	133	921	639	177	1145	858	421	139	902	645	183	1146	864	402
1061	779	517	60	1023	561	279	1067	785	523	61	1004	567	285	1073	786	504	67	1010	573	286	1054	792	510	73	1011	554	292	1060	798	511	54	1017	560	298
932	675	213	1151	719	432	175	938	651	219	1157	725	438	151	944	657	225	1163	701	444	157	950	663	201	1169	707	450	163	926	669	207	1175	713	426	169
803	366	84	1047	590	303	1091	809	372	90	1028	591	309	1097	815	353	91	1034	597	315	1078	816	359	97	1040	578	316	1084	822	365	78	1041	584	322	1090
699	237	1180	743	456	24	962	680	243	1181	749	462	5	968	681	249	1187	730	468	6	974	687	230	1193	731	474	12	955	693	231	1199	737	455	18	956
395	108	896	614	327	1120	833	396	114	877	620	333	1121	839	377	120	883	621	339	1102	845	383	121	889	602	345	1108	846	389	102	895	608	346	1114	827
261	1204	767	485	48	986	529	267	1210	773	486	29	992	535	273	1211	754	492	35	998	536	254	1217	760	498	36	979	542	260	1223	761	479	42	985	548
132	925	638	176	1144	857	425	138	901	644	182	1150	863	401	144	907	650	188	1126	869	407	150	913	626	194	1132	875	413	126	919	632	200	1138	851	419
1053	791	509	72	1015	553	291	1059	797	515	53	1016	559	297	1065	778	516	59	1022	565	278	1066	784	522	65	1003	566	284	1072	790	503	66	1009	572	290
949	662	205	1168	706	449	162	930	668	206	1174	712	430	168	931	674	212	1155	718	431	174	937	655	218	1156	724	437	155	943	656	224	1162	705	443	156
820	358	96	1039	577	320	1083	821	364	77	1045	583	321	1089	802	370	83	1046	589	302	1095	808	371	89	1027	595	308	1096	814	352	95	1033	596	314	1077
686	229	1192	735	473	11	954	692	235	1198	736	454	17	960	698	236	1179	742	460	23	961	679	242	1185	748	461	4	967	685	248	1186	729	467	10	973
382	125	888	601	344	1107	850	388	101	894	607	350	1113	826	394	107	900	613	326	1119	832	400	113	876	619	332	1125	838	376	119	882	625	338	1101	844
253	1216	759	497	40	978	541	259	1222	765	478	41	984	547	265	1203	766	484	47	990	528	266	1209	772	490	28	991	534	272	1215	753	491	34	997	540
149	912	630	193	1131	874	412	130	918	631	199	1137	855	418	131	924	637	180	1143	856	424	137	905	643	181	1149	862	405	143	906	649	187	1130	868	406
1070	783	521	64	1002	570	283	1071	789	502	70	1008	571	289	1052	795	508	71	1014	552	295	1058	796	514	52	1020	558	296	1064	777	520	58	1021	564	277
936	654	217	1160	723	436	154	942	660	223	1161	704	442	160	948	661	204	1167	710	448	161	929	667	210	1173	711	429	167	935	673	211	1154	717	435	173
807	375	88	1026	594	307	1100	813	351	94	1032	600	313	1076	819	357	100	1038	576	319	1082	825	363	76	1044	582	325	1088	801	369	82	1050	588	301	1094
678	241	1184	747	465	3	966	684	247	1190	728	466	9	972	690	228	1191	734	472	15	953	691	234	1197	740	453	16	959	697	240	1178	741	459	22	965
399	112	880	618	331	1124	837	380	118	881	624	337	1105	843	381	124	887	605	343	1106	849	387	105	893	606	349	1112	830	393	106	899	612	330	1118	831
270	1208	771	489	27	995	533	271	1214	752	495	33	996	539	252	1220	758	496	39	977	545	258	1221	764	477	45	983	546	264	1202	770	483	46	989	527
136	904	642	185	1148	861	404	142	910	648	186	1129	867	410	148	911	629	192	1135	873	411	129	917	635	198	1136	854	417	135	923	636	179	1142	860	423
1057	800	513	51	1019	557	300	1063	776	519	57	1025	563	276	1069	782	525	63	1001	569	282	1075	788	501	69	1007	575	288	1051	794	507	75	1013	551	294
928	666	209	1172	715	428	166	934	672	215	1153	716	434	172	940	653	216	1159	722	440	153	941	659	222	1165	703	441	159	947	665	203	1166	709	447	165
824	362	80	1043	581	324	1087	805	368	81	1049	587	305	1093	806	374	87	1030	593	306	1099	812	355	93	1031	599	312	1080	818	356	99	1037	580	318	1081
695	233	1196	739	452	20	958	696	239	1177	745	458	21	964	677	245	1183	746	464	2	970	683	246	1189	727	470	8	971	689	227	1195	733	471	14	952
386	104	892	610	348	1111	829	392	110	898	611	329	1117	835	398	111	879	617	335	1123	836	379	117	885	623	336	1104	842	385	123	886	604	342	1110	848
257	1225	763	476	44	982	550	263	1201	769	482	50	988	526	269	1207	775	488	26	994	532	275	1213	751	494	32	1000	538	251	1219	757	500	38	976	544
128	916	634	197	1140	853	416	134	922	640	178	1141	859	422	140	903	641	184	1147	865	403	141	909	647	190	1128	866	409	147	915	628	191	1134	872	415
1074	787	505	68	1006	574	287	1055	793	506	74	1012	555	293	1056	799	512	55	1018	556	299	1062	780	518	56	1024	562	280	1068	781	524	62	1005	568	281

These patterns are persistent among all Class-5 squares. For example, size 55 squares have 8 5x11and 11x5 staircase patterns plus two 5x11 and 11x5 rectangular modularities, again for a total of 10 such simultaneous patterns. Then add in the two dot-patterns and that makes 12 distinct non-linear patterns which sum equally to C_{35} and do so continuously across the square. For all of that to occur simultaneously on just one square is absolutely amazing! That is way beyond the mind of Man to fashion: it can only be discovered. It says something about the underlying spatial fabric contained in just Class-5 squares alone.

Program 2

Class-6 Squares Tiling Patterns

Here is the Class-6 size-9 square where **a = b = 3**. It is **3x** modular so every **3x3** block-square is a continuous tiling pattern. The centers of blocks all sum to the characteristic number continuously. These tiles cannot be aligned differently (offset) for this to happen.

Size-9

	1	2	3	4	5	6	7	8	9	
W(9)	369	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	**9**	10	50	**36**	37	77	**63**	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	**14**	27	28	**41**	54	55	**68**	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	**19**	5	45	**46**	32	72	**73**	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

Size-25

Here is the Class-6 size-25 square. It is **5x** modular so every **5x5** block-squares is a continuous tiling pattern.

The centers of the block tiles sum continuously to **7825**, the square's characteristic number. So does the horizontal flip of this block pattern. These block-squares can be offset both horizontally and vertically and this will still occur.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	**13**	151	319	582	125	**138**	276	444	82	250	**263**	401	569	207	375	**388**	526	69	332	500	**513**	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	**482**	525	38	176	344	**607**	25	163	301	469	**107**	150	288	426	594	**232**	275	413	551	94	**357**	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	**201**	369	507	50	188	**326**	494	7	175	313	**451**	619	132	300	438	**576**	119	257	425	563	**76**	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	**575**	88	226	269	532	**75**	213	351	394	32	**200**	338	476	519	157	**325**	463	601	19	282	**450**	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	538	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	**294**	557	100	238	251	**419**	57	225	363	376	**544**	182	350	488	501	**44**	307	475	613	1	**169**	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Program 2

Size-27

Here is an example of a continuous tiling pattern on the size-27 square composed of both **9x3** and **3x9** rectangles simultaneously. Both size rectangular tiling patterns sum to the characteristic number continuously. The size-27 square is clearly a member of Class- 6, since Class-3 squares do not possess tile patterns which sum exactly to their square's characteristic number. The centers of the rectangular tiles also sum to the square's characteristic number.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
W(27)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	399	463	314	345	409	260	372	436	287	156	220	71	102	166	17	129	193	44	642	706	557	588	652	503	615	679	530	9855
2	590	657	496	617	684	523	644	711	550	347	414	253	374	441	280	401	468	307	104	171	10	131	198	37	158	225	64	9855
3	133	191	42	160	218	69	106	164	15	619	677	528	646	704	555	592	650	501	376	434	285	403	461	312	349	407	258	9855
4	386	471	319	332	417	265	359	444	292	143	228	76	89	174	22	116	201	49	629	714	562	575	660	508	602	687	535	9855
5	568	662	513	595	689	540	622	716	567	325	419	270	352	446	297	379	473	324	82	176	27	109	203	54	136	230	81	9855
6	114	205	47	141	232	74	87	178	20	600	691	533	627	718	560	573	664	506	357	448	290	384	475	317	330	421	263	9855
7	391	485	300	337	431	246	364	458	273	148	242	57	94	188	3	121	215	30	634	728	543	580	674	489	607	701	516	9855
8	585	667	491	612	694	518	639	721	545	342	424	248	369	451	275	396	478	302	99	181	5	126	208	32	153	235	59	9855
9	119	213	34	146	240	61	92	186	7	605	699	520	632	726	547	578	672	493	362	456	277	389	483	304	335	429	250	9855
10	318	382	476	264	328	422	291	355	449	75	139	233	21	85	179	48	112	206	561	625	719	507	571	665	534	598	692	9855
11	509	576	658	536	603	685	563	630	712	266	333	415	293	360	442	320	387	469	23	90	172	50	117	199	77	144	226	9855
12	52	110	204	79	137	231	25	83	177	538	596	690	565	623	717	511	569	663	295	353	447	322	380	474	268	326	420	9855
13	305	390	481	251	336	427	278	363	454	62	147	238	8	93	184	35	120	211	548	633	724	494	579	670	521	606	697	9855
14	487	581	675	514	608	702	541	635	729	244	338	432	271	365	459	298	392	486	1	95	189	28	122	216	55	149	243	9855
15	33	124	209	60	151	236	6	97	182	519	610	695	546	637	722	492	583	668	276	367	452	303	394	479	249	340	425	9855
16	310	404	462	256	350	408	283	377	435	67	161	219	13	107	165	40	134	192	553	647	705	499	593	651	526	620	678	9855
17	504	586	653	531	613	680	558	640	707	261	343	410	288	370	437	315	397	464	18	100	167	45	127	194	72	154	221	9855
18	38	132	196	65	159	223	11	105	169	524	618	682	551	645	709	497	591	655	281	375	439	308	402	466	254	348	412	9855
19	480	301	395	426	247	341	453	274	368	237	58	152	183	4	98	210	31	125	723	544	638	669	490	584	696	517	611	9855
20	671	495	577	698	522	604	725	549	631	428	252	334	455	279	361	482	306	388	185	9	91	212	36	118	239	63	145	9855
21	214	29	123	241	56	150	187	2	96	700	515	609	727	542	636	673	488	582	457	272	366	484	299	393	430	245	339	9855
22	467	309	400	413	255	346	440	282	373	224	66	157	170	12	103	197	39	130	710	552	643	656	498	589	683	525	616	9855
23	649	500	594	676	527	621	703	554	648	406	257	351	433	284	378	460	311	405	163	14	108	190	41	135	217	68	162	9855
24	195	43	128	222	70	155	168	16	101	681	529	614	708	556	641	654	502	587	438	286	371	465	313	398	411	259	344	9855
25	472	323	381	418	269	327	445	296	354	229	80	138	175	26	84	202	53	111	715	566	624	661	512	570	688	539	597	9855
26	666	505	572	693	532	599	720	559	626	423	262	329	450	289	356	477	316	383	180	19	86	207	46	113	234	73	140	9855
27	200	51	115	227	78	142	173	24	88	686	537	601	713	564	628	659	510	574	443	294	358	470	321	385	416	267	331	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
W(27)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	399	463	314	345	409	260	372	436	287	156	220	71	102	166	17	129	193	44	642	706	557	588	652	503	615	679	530	9855
2	590	657	496	617	684	523	644	711	550	347	414	253	374	441	280	401	468	307	104	171	10	131	198	37	158	225	64	9855
3	133	191	42	160	218	69	106	164	15	619	677	528	646	704	555	592	650	501	376	434	285	403	461	312	349	407	258	9855
4	386	471	319	332	417	265	359	444	292	143	228	76	89	174	22	116	201	49	629	714	562	575	660	508	602	687	535	9855
5	568	662	513	595	689	540	622	716	567	325	419	270	352	446	297	379	473	324	82	176	27	109	203	54	136	230	81	9855
6	114	205	47	141	232	74	87	178	20	600	691	533	627	718	560	573	664	506	357	448	290	384	475	317	330	421	263	9855
7	391	485	300	337	431	246	364	458	273	148	242	57	94	188	3	121	215	30	634	728	543	580	674	489	607	701	516	9855
8	585	667	491	612	694	518	639	721	545	342	424	248	369	451	275	396	478	302	99	181	5	126	208	32	153	235	59	9855
9	119	213	34	146	240	61	92	186	7	605	699	520	632	726	547	578	672	493	362	456	277	389	483	304	335	429	250	9855
10	318	382	476	264	328	422	291	355	449	75	139	233	21	85	179	48	112	206	561	625	719	507	571	665	534	598	692	9855
11	509	576	658	536	603	685	563	630	712	266	333	415	293	360	442	320	387	469	23	90	172	50	117	199	77	144	226	9855
12	52	110	204	79	137	231	25	83	177	538	596	690	565	623	717	511	569	663	295	353	447	322	380	474	268	326	420	9855
13	305	390	481	251	336	427	278	363	454	62	147	238	8	93	184	35	120	211	548	633	724	494	579	670	521	606	697	9855
14	487	581	675	514	608	702	541	635	729	244	338	432	271	365	459	298	392	486	1	95	189	28	122	216	55	149	243	9855
15	33	124	209	60	151	236	6	97	182	519	610	695	546	637	722	492	583	668	276	367	452	303	394	479	249	340	425	9855
16	310	404	462	256	350	408	283	377	435	67	161	219	13	107	165	40	134	192	553	647	705	499	593	651	526	620	678	9855
17	504	586	653	531	613	680	558	640	707	261	343	410	288	370	437	315	397	464	18	100	167	45	127	194	72	154	221	9855
18	38	132	196	65	159	223	11	105	169	524	618	682	551	645	709	497	591	655	281	375	439	308	402	466	254	348	412	9855
19	480	301	395	426	247	341	453	274	368	237	58	152	183	4	98	210	31	125	723	544	638	669	490	584	696	517	611	9855
20	671	495	577	698	522	604	725	549	631	428	252	334	455	279	361	482	306	388	185	9	91	212	36	118	239	63	145	9855
21	214	29	123	241	56	150	187	2	96	700	515	609	727	542	636	673	488	582	457	272	366	484	299	393	430	245	339	9855
22	467	309	400	413	255	346	440	282	373	224	66	157	170	12	103	197	39	130	710	552	643	656	498	589	683	525	616	9855
23	649	500	594	676	527	621	703	554	648	406	257	351	433	284	378	460	311	405	163	14	108	190	41	135	217	68	162	9855
24	195	43	128	222	70	155	168	16	101	681	529	614	708	556	641	654	502	587	438	286	371	465	313	398	411	259	344	9855
25	472	323	381	418	269	327	445	296	354	229	80	138	175	26	84	202	53	111	715	566	624	661	512	570	688	539	597	9855
26	666	505	572	693	532	599	720	559	626	423	262	329	450	289	356	477	316	383	180	19	86	207	46	113	234	73	140	9855
27	200	51	115	227	78	142	173	24	88	686	537	601	713	564	628	659	510	574	443	294	358	470	321	385	416	267	331	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Program 2

Tiling Patterns for Even Size Squares

Class-2 Squares Tiling Patterns

Class-2 squares have a simple pattern that sums exactly to the square's characteristic number quasi-continuously. The only place where it doesn't sum equally is when this pattern is vertically aligned with the square's double column (green strip) or the square's double row (orange strip). The tile pattern is a **bx2** or a **2xb** block-rectangle. Here it is shown for the size-14, size-22 and size-26 squares. The size-10 square did not have any tiling pattern. No other tiling pattern complementary to this pattern, also composed of **n = 2b** cells, was found for any Class-2 square.

Size-14

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
14x14	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379
1	140	17	96	169	80	153	36	137	20	93	172	77	156	33	1379
2	85	192	69	148	53	132	9	88	189	72	145	56	129	12	1379
3	64	165	48	121	4	105	184	61	168	45	124	1	108	181	1379
4	37	116	21	100	173	84	157	39	114	23	98	175	82	159	1378 (−1)
5	14	91	194	75	150	31	134	15	90	195	74	151	30	135	1379
6	187	66	143	50	127	6	111	186	67	142	51	126	7	110	1379
7	162	43	118	27	102	179	58	163	42	119	26	103	178	59	1379
8	138	19	94	171	78	155	34	139	18	95	170	79	154	35	1379
9	87	190	71	146	55	130	11	86	191	70	147	54	131	10	1379
10	62	167	46	123	2	107	182	63	166	47	122	3	106	183	1379
11	38	115	22	99	174	83	158	40	113	24	97	176	81	160	1380 (+1)
12	16	89	196	73	152	29	136	13	92	193	76	149	32	133	1379
13	185	68	141	52	125	8	109	188	65	144	49	128	5	112	1379
14	164	41	120	25	104	177	60	161	44	117	28	101	180	57	1379
	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	
	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379

	1	2
1	#	#
2	#	#
3	#	#
4	#	#
5	#	#
6	#	#
7	#	#

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
2	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
3	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
4	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
5	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
6	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
7	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379	1379	1379	1379
8	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379
9	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379
10	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379
11	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379
12	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379
13	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379
14	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379	1379	1379	1379

Program 2

Size-22

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
	22x22	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
1	81	148	253	320	425	8	113	180	285	396	457	84	145	256	317	428	5	116	177	288	393	460	5335
2	96	201	268	373	484	61	172	233	344	405	32	93	204	265	376	481	64	169	236	341	408	29	5335
3	149	260	321	432	9	120	181	292	353	464	85	152	257	324	429	12	117	184	289	356	461	88	5335
4	208	269	380	441	68	173	240	345	412	33	100	205	272	377	444	65	176	237	348	409	36	97	5335
5	261	328	433	16	121	188	293	360	465	48	153	264	325	436	13	124	185	296	357	468	45	156	5335
6	276	381	448	69	136	241	352	413	40	101	212	274	383	446	71	134	243	350	415	38	103	210	5336
7	330	439	18	127	190	299	362	471	50	159	222	331	438	19	126	191	298	363	470	51	158	223	5335
8	387	450	75	138	247	310	419	42	107	214	279	386	451	74	139	246	311	418	43	106	215	278	5335
9	398	23	130	195	302	367	474	55	162	227	334	399	22	131	194	303	366	475	54	163	226	335	5335
10	455	78	143	250	315	422	3	110	219	282	391	454	79	142	251	314	423	2	111	218	283	390	5335
11	27	90	199	306	371	478	59	166	231	338	403	26	91	198	307	370	479	58	167	230	339	402	5335
12	83	146	255	318	427	6	115	178	287	394	459	82	147	254	319	426	7	114	179	286	395	458	5335
13	95	202	267	374	483	62	171	234	343	406	31	94	203	266	375	482	63	170	235	342	407	30	5335
14	150	259	322	431	10	119	182	291	354	463	86	151	258	323	430	11	118	183	290	355	462	87	5335
15	207	270	379	442	67	174	239	346	411	34	99	206	271	378	443	66	175	238	347	410	35	98	5335
16	262	327	434	15	122	187	294	359	466	47	154	263	326	435	14	123	186	295	358	467	46	155	5335
17	275	382	447	70	135	242	351	414	39	102	211	273	384	445	72	133	244	349	416	37	104	209	5334
18	329	440	17	128	189	300	361	472	49	160	221	332	437	20	125	192	297	364	469	52	157	224	5335
19	388	449	76	137	248	309	420	41	108	213	280	385	452	73	140	245	312	417	44	105	216	277	5335
20	397	24	129	196	301	368	473	56	161	228	333	400	21	132	193	304	365	476	53	164	225	336	5335
21	456	77	144	249	316	421	4	109	220	281	392	453	80	141	252	313	424	1	112	217	284	389	5335
22	25	92	197	308	369	480	57	168	229	340	401	28	89	200	305	372	477	60	165	232	337	404	5335
	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	
	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335

1	2	3	4	5	6	7	8	9	10	11
#	#	#	#	#	#	#	#	#	#	#
#	#	#	#	#	#	#	#	#	#	#

	2x11											
1	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
2	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
3	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
4	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
5	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
6	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
7	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
8	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
9	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
10	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
11	5336	5336	5336	5336	5336	5336	5334	5334	5334	5334	5334	5334
12	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
13	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
14	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
15	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
16	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
17	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
18	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
19	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
20	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335
21	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335

Program 2

Size-26

Here is the size-26 Class-2 square. Observe that all Class-2 squares are all pairwise centrally symmetric, contrary to what is seen for Class-4 even-size squares (next).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26		
	W(26)	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	
1	413	564	33	184	329	480	625	152	297	448	593	68	213	416	561	36	181	332	477	628	149	300	445	596	65	216	8801	
2	316	513	664	133	284	429	580	101	252	397	548	17	168	313	516	661	136	281	432	577	104	249	400	545	20	165	8801	
3	265	468	613	88	233	384	529	4	201	352	497	648	117	268	465	616	85	236	381	532	1	204	349	500	645	120	8801	
4	220	365	568	37	188	333	484	629	156	301	452	597	72	217	368	565	40	185	336	481	632	153	304	449	600	69	8801	
5	169	320	517	668	137	288	433	584	53	256	401	552	21	172	317	520	665	140	285	436	581	56	253	404	549	24	8801	
6	124	269	420	617	92	237	388	533	8	205	356	501	652	121	272	417	620	89	240	385	536	5	208	353	504	649	8801	
7	73	224	369	572	41	192	337	488	633	108	305	456	601	75	222	371	570	43	190	339	486	635	106	307	454	603	8800	1
8	27	174	323	470	671	142	291	438	587	58	259	406	555	26	175	322	471	670	143	290	439	586	59	258	407	554	8801	
9	654	127	274	423	622	95	242	391	538	11	158	359	506	655	126	275	422	623	94	243	390	539	10	159	358	507	8801	
10	607	78	227	374	523	46	195	342	491	638	111	310	459	606	79	226	375	522	47	194	343	490	639	110	311	458	8801	
11	558	31	178	327	474	675	146	295	442	591	62	211	410	559	30	179	326	475	674	147	294	443	590	63	210	411	8801	
12	511	658	131	278	427	574	99	246	395	542	15	162	363	510	659	130	279	426	575	98	247	394	543	14	163	362	8801	
13	463	610	83	230	379	526	51	198	347	494	643	114	263	462	611	82	231	378	527	50	199	346	495	642	115	262	8801	
14	415	562	35	182	331	478	627	150	299	446	595	66	215	414	563	34	183	330	479	626	151	298	447	594	67	214	8801	
15	315	514	663	134	283	430	579	102	251	398	547	18	167	314	515	662	135	282	431	578	103	250	399	546	19	166	8801	
16	266	467	614	87	234	383	530	3	202	351	498	647	118	267	466	615	86	235	382	531	2	203	350	499	646	119	8801	
17	219	366	567	38	187	334	483	630	155	302	451	598	71	218	367	566	39	186	335	482	631	154	303	450	599	70	8801	
18	170	319	518	667	138	287	434	583	54	255	402	551	22	171	318	519	666	139	286	435	582	55	254	403	550	23	8801	
19	123	270	419	618	91	238	387	534	7	206	355	502	651	122	271	418	619	90	239	386	535	6	207	354	503	650	8801	
20	74	223	370	571	42	191	338	487	634	107	306	455	602	76	221	372	569	44	189	340	485	636	105	308	453	604	8802	-1
21	28	173	324	469	672	141	292	437	588	57	260	405	556	25	176	321	472	669	144	289	440	585	60	257	408	553	8801	
22	653	128	273	424	621	96	241	392	537	12	157	360	505	656	125	276	421	624	93	244	389	540	9	160	357	508	8801	
23	608	77	228	373	524	45	196	341	492	637	112	309	460	605	80	225	376	521	48	193	344	489	640	109	312	457	8801	
24	557	32	177	328	473	676	145	296	441	592	61	212	409	560	29	180	325	476	673	148	293	444	589	64	209	412	8801	
25	512	657	132	277	428	573	100	245	396	541	16	161	364	509	660	129	280	425	576	97	248	393	544	13	164	361	8801	
26	461	612	81	232	377	528	49	200	345	496	641	116	261	464	609	84	229	380	525	52	197	348	493	644	113	264	8801	
	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801		
	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	

The size **26** square has the **13x2** vertically rectangular pattern as a quasi-continuous tiling pattern. It only violates the continuity when centered over the double vertical columns.

13x2

8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801
8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801

Program 2

Complementary Tiling Patterns Found in Class-2 Squares

Size-10

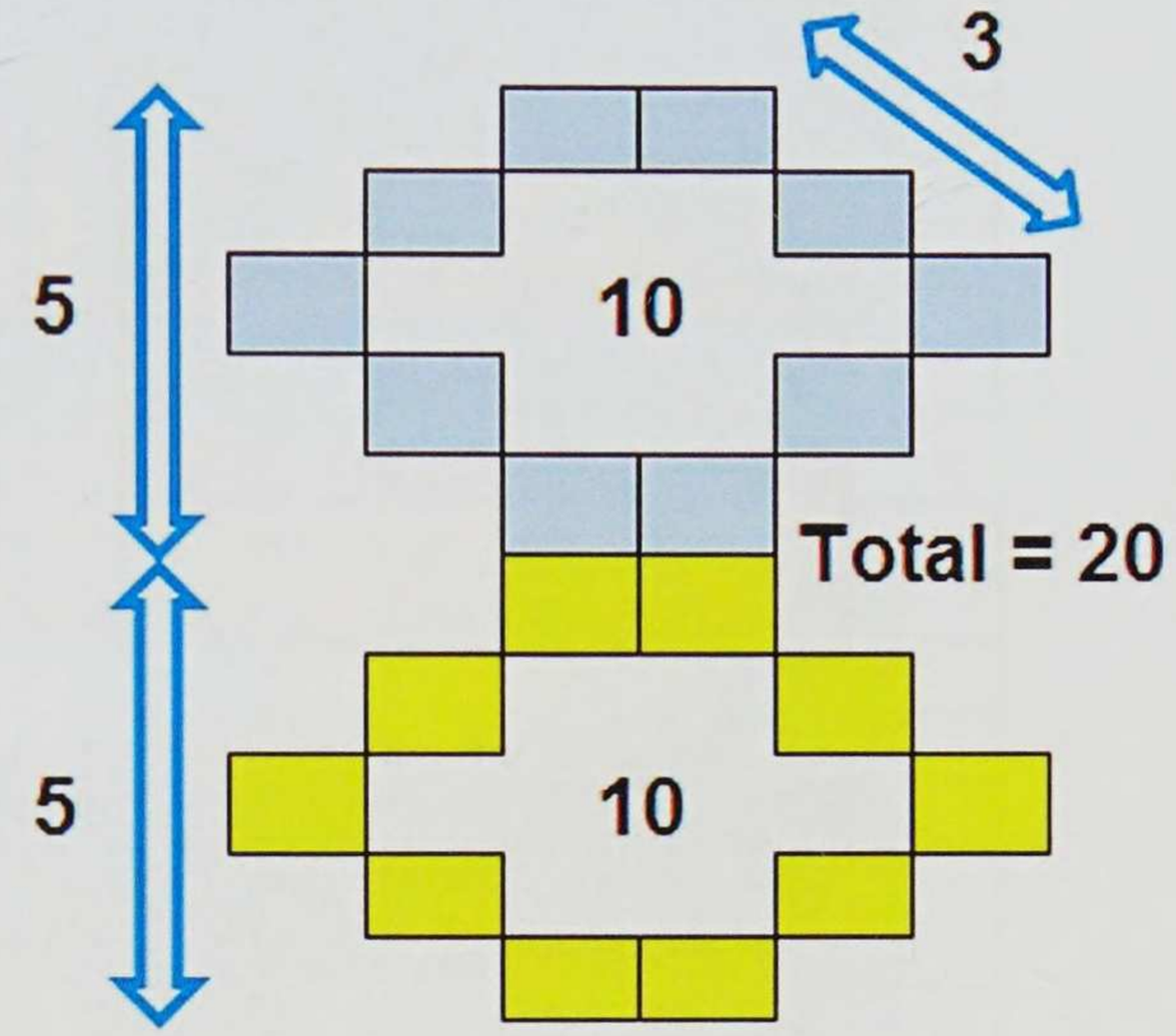
Continuous 10-celled double ring-diamond patterns

537	538	376	537	537	537	538	376	537	537
489	488	570	488	488	490	489	569	489	490
505	505	504	505	505	504	505	506	506	505
505	505	506	505	505	506	505	504	504	505
521	522	440	522	522	520	521	441	521	520
473	472	634	473	473	473	472	634	473	473
521	522	440	522	522	520	521	441	521	520
505	505	506	505	505	506	505	504	504	505
505	505	504	505	505	504	505	506	506	505
489	488	570	488	488	490	489	569	489	490

Top and bottom patterns combined into a double-diamond

1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010

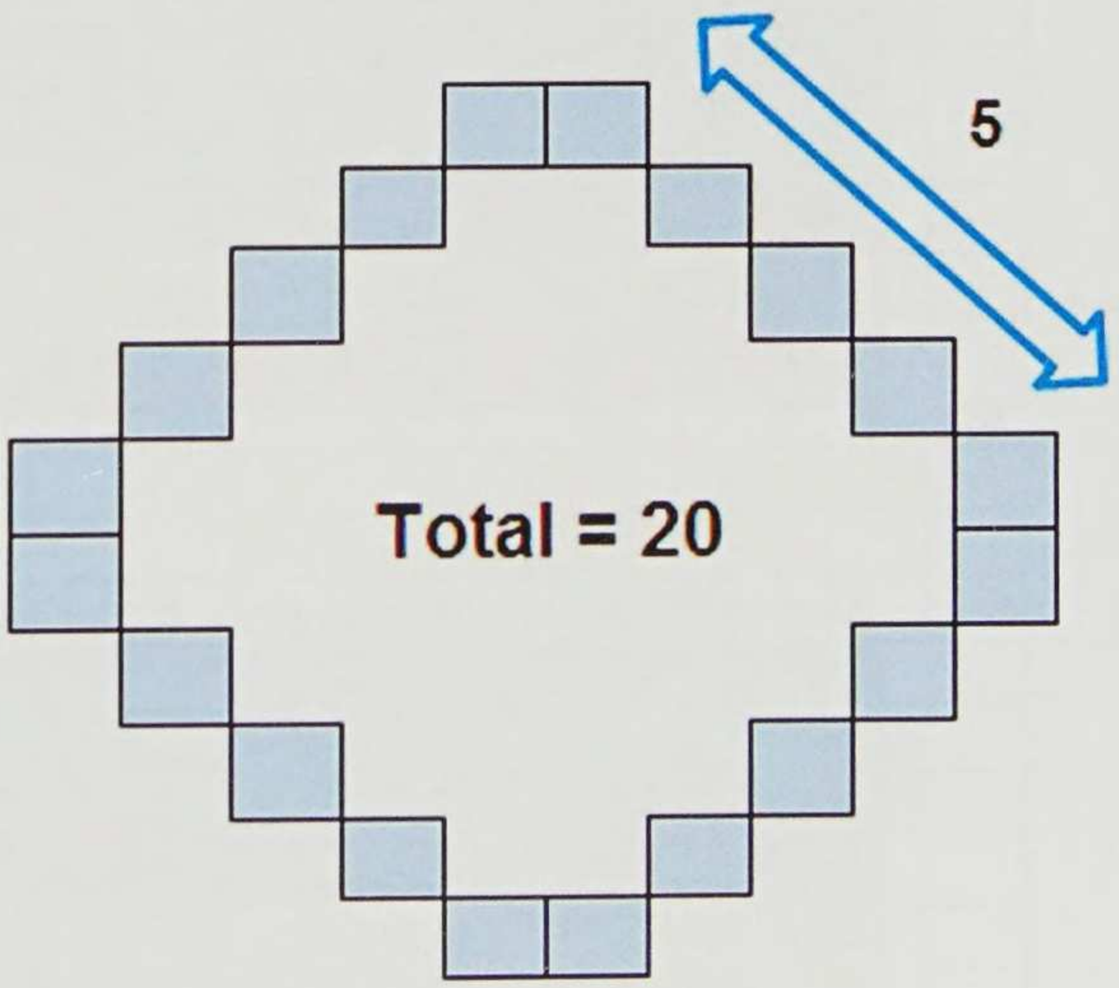
Double vertical ring-diamond patterns



Here is what the size-3 double ring-diamond looks like for the size-10 square. Note that it consists of **20** cells. These cells sum continuously to $\mathbf{2C_{10}}$. Amazingly the size **5** ring-diamond shown below has **20** cells too which also sum continuously to $\mathbf{2C_{10}}$.

All this generalizes to all larger Class-2 squares. So these two renditions are the complementary tiling patterns for Class-2 squares of size-10 and beyond.

Ring-diamond



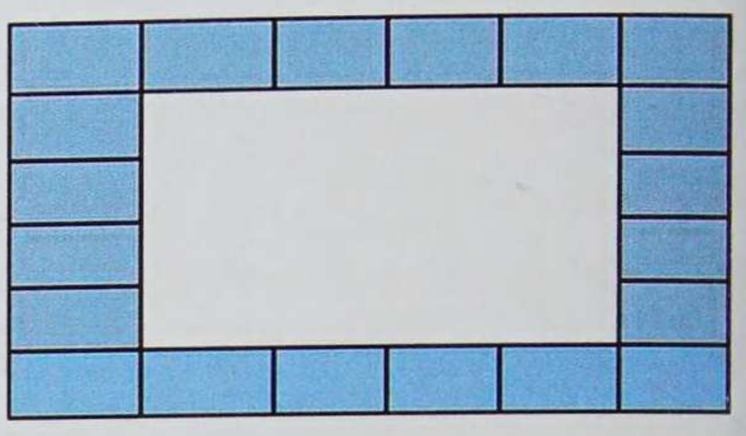
Program 2

Here in the next segment are shown some amazing patterns in the pattern summation inequalities for Class-2 squares.

Size-10

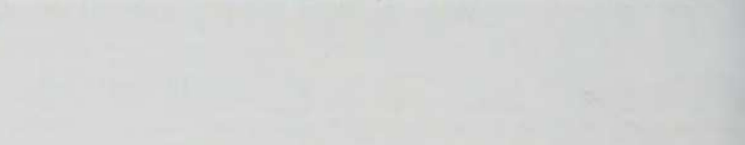
Patterns of 6x6 frames = 20 cells

961	1121	883	1121	963	961	1121	883	1121	963
977	1057	979	1057	979	977	1057	979	1057	979
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
1043	963	1041	963	1041	1043	963	1041	963	1041
1059	899	1137	899	1057	1059	899	1137	899	1057
1059	899	1137	899	1057	1059	899	1137	899	1057
1043	963	1041	963	1041	1043	963	1041	963	1041
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
977	1057	979	1057	979	977	1057	979	1057	979
961	1121	883	1121	963	961	1121	883	1121	963



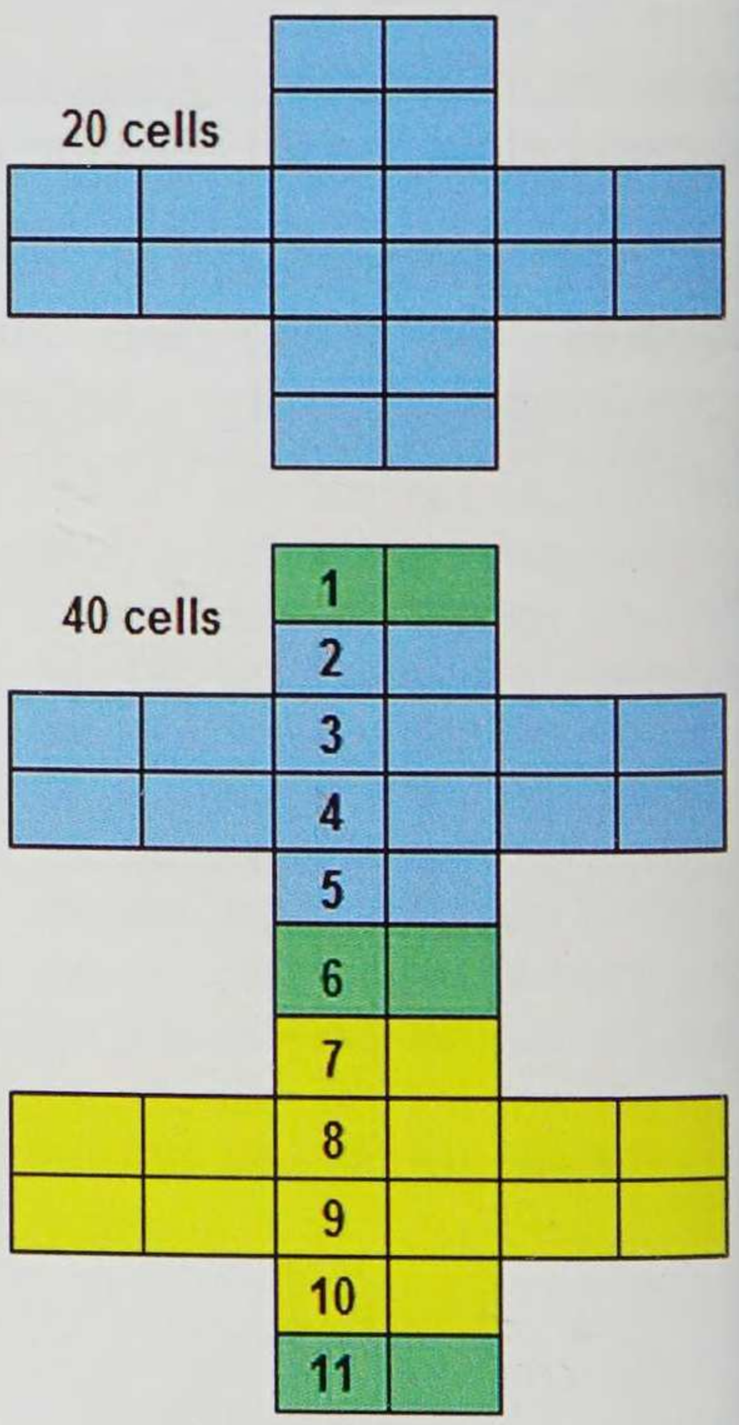
Sum of 6x6 frame patterns overlapping by 1st and last rows

2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020



+ Patterns of 2x2 block-squares

977	1057	978	1057	978	977	1057	978	1057	978
1026	1026	946	1025	1026	1026	1026	946	1025	1026
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
994	994	1074	995	994	994	994	1074	995	994
1043	963	1042	963	1042	1043	963	1042	963	1042
1043	963	1042	963	1042	1043	963	1042	963	1042
994	994	1074	995	994	994	994	1074	995	994
1010	1010	1010	1010	1010	1010	1010	1010	1010	1010
1026	1026	946	1025	1026	1026	1026	946	1025	1026
977	1057	978	1057	978	977	1057	978	1057	978

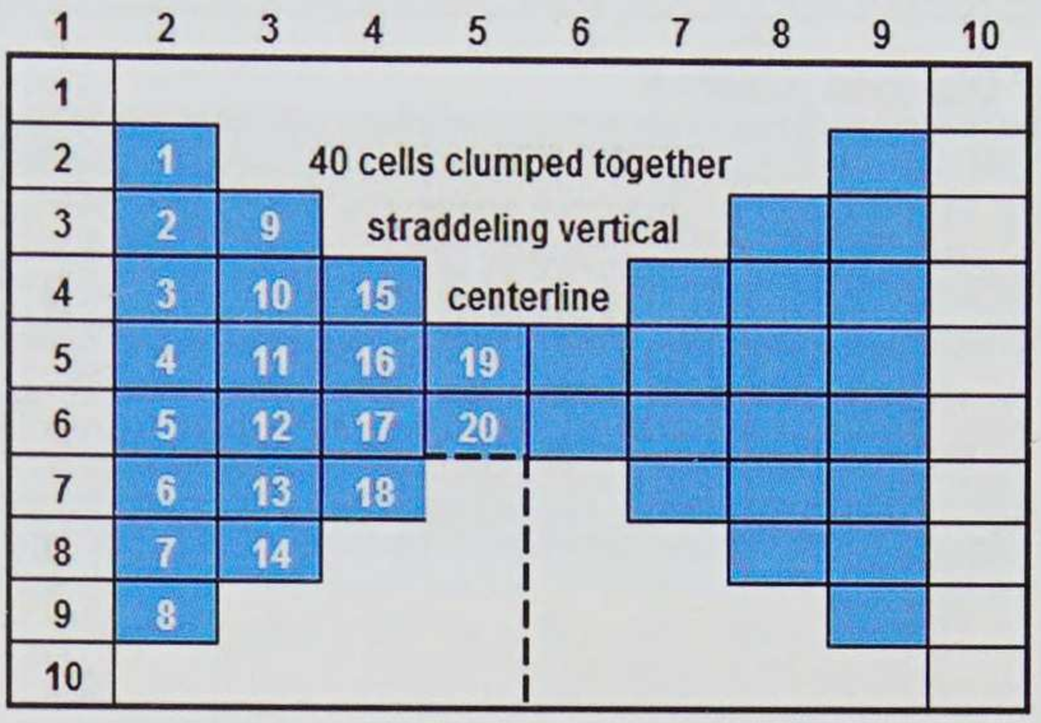


Sum of **+** patterns overlapping by 1st and last rows

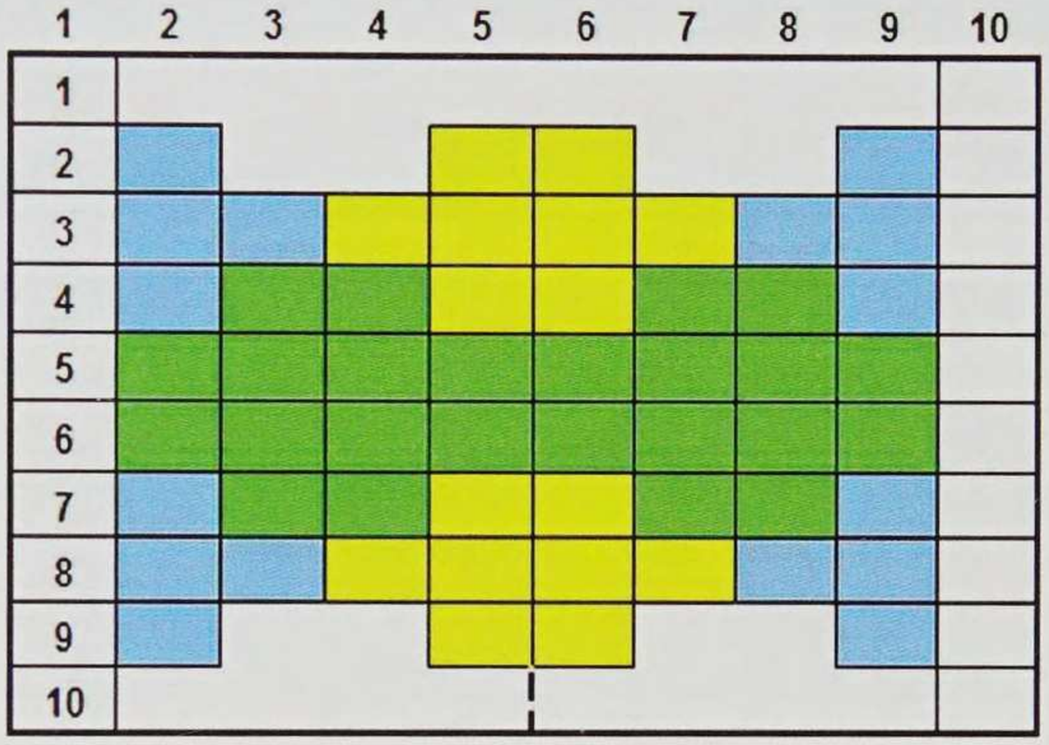
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020

Program 2

2084	2004	2004	2003	2003	2084	2004	2004	2003	2003	2084
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
1956	2036	2036	2037	2037	1956	2036	2036	2037	2037	1956
2021	2020	2021	2021	2021	2021	2020	2021	2021	2021	2021
2021	2020	2021	2021	2021	2021	2020	2021	2021	2021	2021
1956	2036	2036	2037	2037	1956	2036	2036	2037	2037	1956
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2084	2004	2004	2003	2003	2084	2004	2004	2003	2003	2084
2019	2020	2019	2019	2019	2019	2020	2019	2019	2019	2019
2019	2020	2019	2019	2019	2019	2020	2019	2019	2019	2019

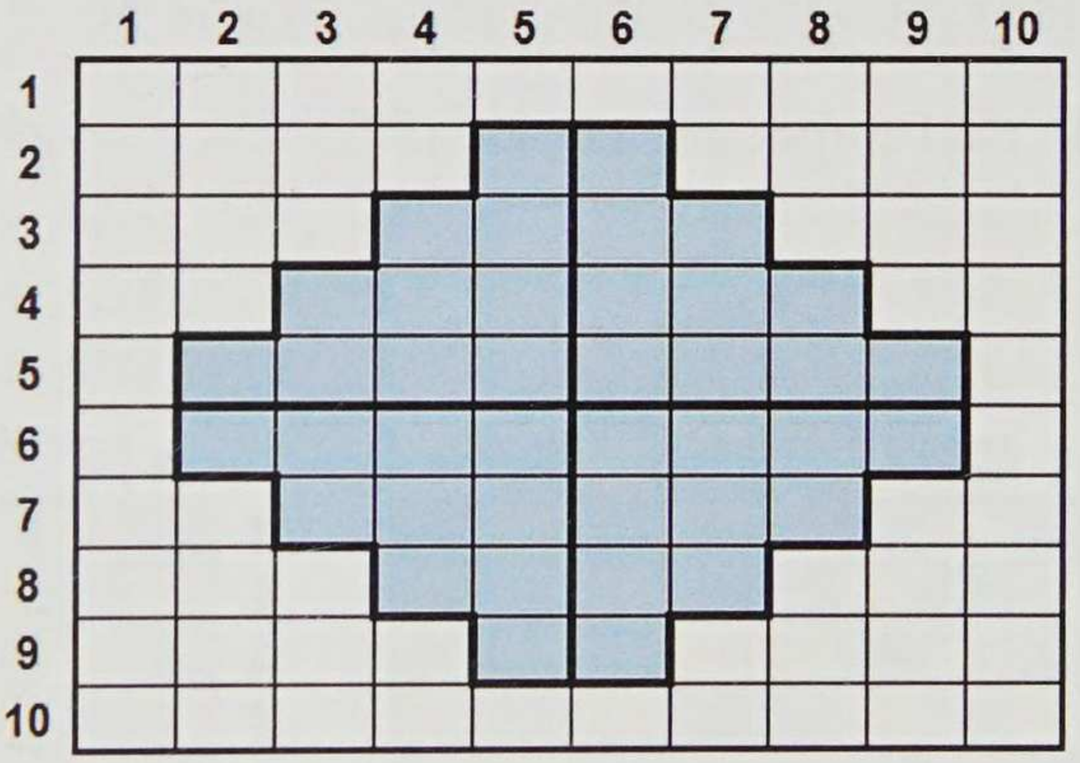


4040	4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040	4040

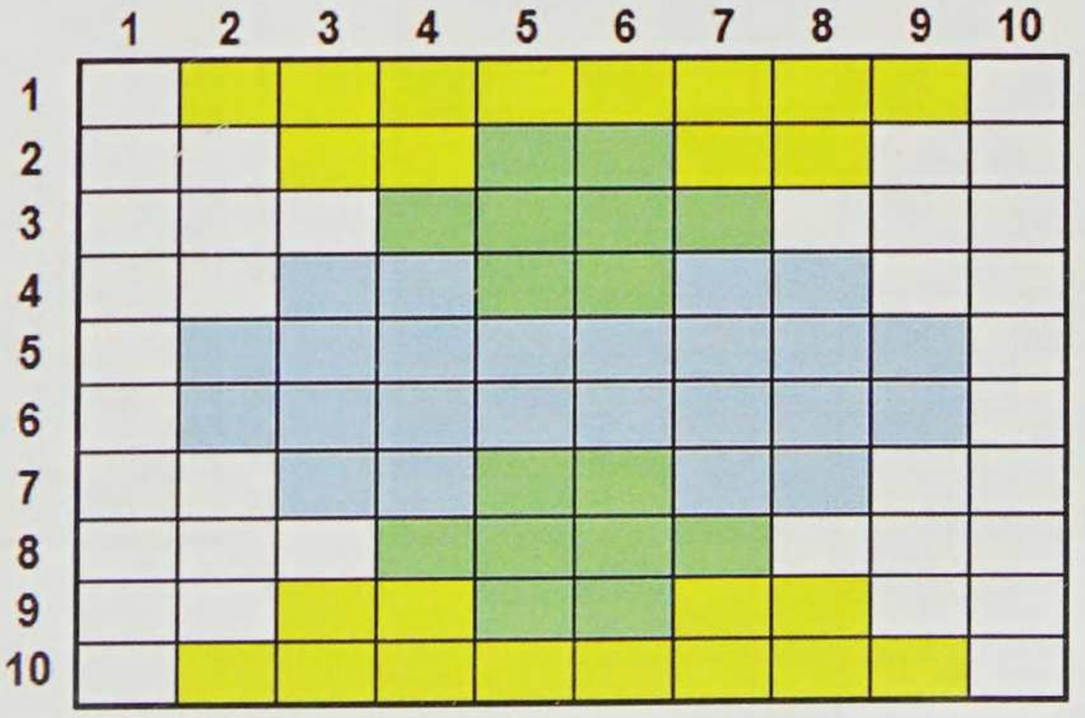


There are two continuous patterns that emerge here: one is the size-4 even-size diamond and the other is the wide "+" pattern as depicted here at right.

1	2	3	4	5	6	7	8	9	10
1956	2036	2036	2037	2037	1956	2036	2036	2037	2037
2021	2020	2021	2021	2021	2021	2020	2021	2021	2021
2021	2020	2021	2021	2021	2021	2020	2021	2021	2021
1956	2036	2036	2037	2037	1956	2036	2036	2037	2037
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020
2084	2004	2004	2003	2003	2084	2004	2004	2003	2003
2019	2020	2019	2019	2019	2019	2020	2019	2019	2019
2019	2020	2019	2019	2019	2019	2020	2019	2019	2019
2084	2004	2004	2003	2003	2084	2004	2004	2003	2003
2020	2020	2020	2020	2020	2020	2020	2020	2020	2020



4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040
4040	4040	4040	4040	4040	4040	4040	4040	4040	4040



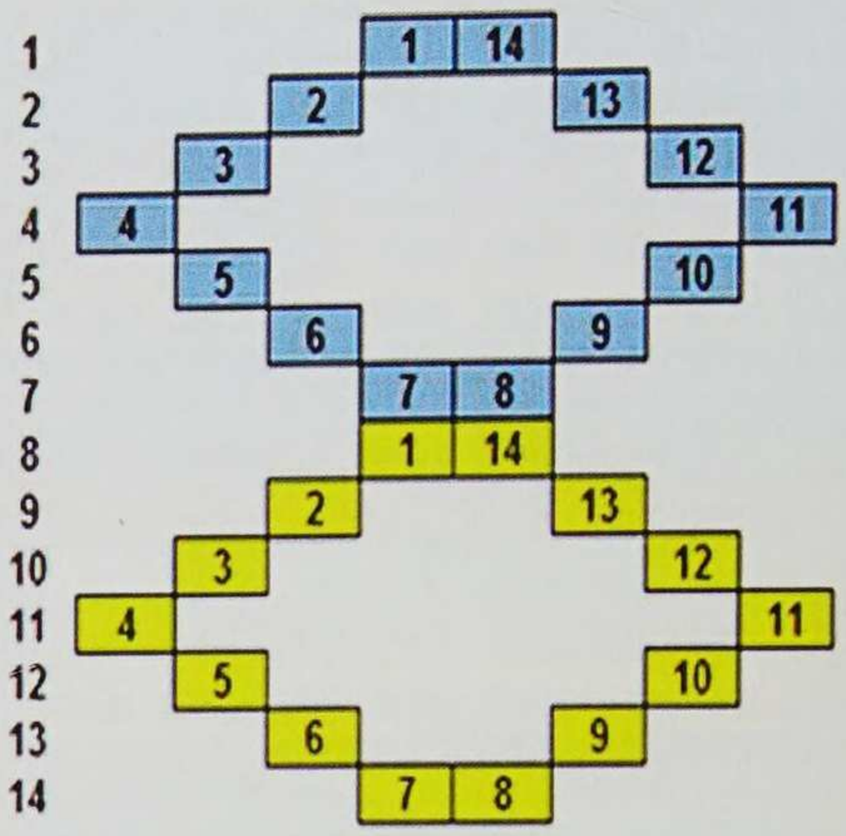
It is continuous across the square and sums to $2C_{14}$.

Program 2

Size-14

7x7 Diamond patterns

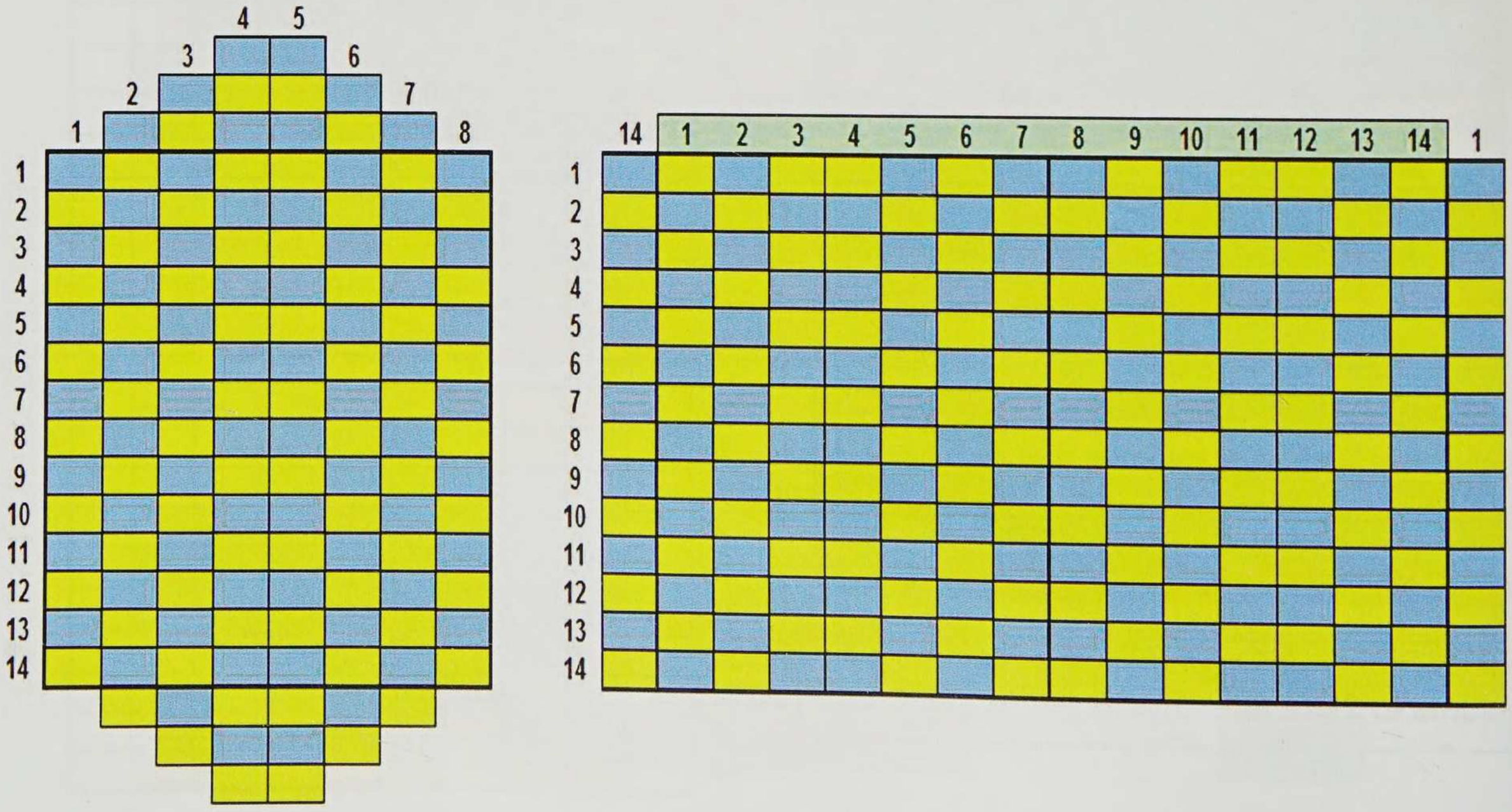
1378	1380	1378	1380	1378	1380	1378	1378	1380	1378	1380	1378	1380	1378
1379	1378	1380	1378	1380	1378	1379	1379	1378	1380	1378	1380	1378	1379
1379	1379	1378	1380	1378	1379	1379	1379	1379	1378	1380	1378	1379	1379
1379	1379	1379	1378	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379
1379	1379	1379	1380	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379
1379	1379	1380	1378	1380	1379	1379	1379	1379	1380	1378	1380	1379	1379
1379	1380	1378	1380	1378	1380	1379	1379	1380	1378	1380	1378	1380	1379
1380	1378	1380	1378	1380	1378	1380	1380	1378	1380	1378	1380	1378	1380
1379	1380	1378	1380	1378	1380	1379	1379	1380	1378	1380	1378	1380	1379
1379	1379	1380	1378	1380	1379	1379	1379	1379	1380	1378	1380	1379	1379
1379	1379	1379	1380	1379	1379	1379	1379	1379	1379	1380	1379	1379	1379
1379	1379	1379	1378	1379	1379	1379	1379	1379	1379	1378	1379	1379	1379
1379	1379	1378	1380	1378	1379	1379	1379	1379	1378	1380	1378	1379	1379
1379	1378	1380	1378	1380	1378	1379	1379	1378	1380	1378	1380	1378	1379



2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758
2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758
2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758
2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758
2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758
2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758
2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758

Shown here is how the double ring-diamond pattern integrates with itself and how it wraps across the vertical edges of the size-14 square. This all generalizes to all Class-2 squares.

The over-run at the top and bottom is column wrapped across the bottom and top, respectively.



Patterns in Class-2 Squares that arise from Continuous Minor Inequalities

What is so amazing about these fairly symmetric patterns contrasted in green and pink is that they arise from minor inequalities of **±1**. These were found to occur throughout Class-2 squares. It is now evident how close to perfect this class of squares is yet the spatial wrinkle still persists in many ways.

Size-14

Size-6 even-size solid diamond

8274	8274	8274	8274	8275	8275	8274	8274	8274	8274	8274	8273	8273	8274	8274	8274	8274
8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274
8274	8274	8274	8274	8275	8275	8274	8274	8274	8274	8274	8273	8273	8274	8274	8274	8274
8274	8274	8274	8275	8275	8275	8275	8274	8274	8274	8273	8273	8273	8273	8274	8274	8274
8273	8274	8275	8275	8275	8275	8275	8275	8274	8273	8273	8273	8273	8273	8273	8274	8275
8273	8274	8275	8275	8275	8275	8275	8275	8274	8273	8273	8273	8273	8273	8273	8274	8275
8274	8274	8274	8275	8275	8275	8275	8274	8274	8274	8273	8273	8273	8273	8274	8274	8274
8274	8274	8274	8274	8275	8275	8274	8274	8274	8274	8274	8273	8273	8274	8274	8274	8274
8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274
8274	8274	8274	8274	8275	8275	8274	8274	8274	8274	8274	8273	8273	8274	8274	8274	8274
8274	8274	8274	8275	8275	8275	8275	8274	8274	8274	8273	8273	8273	8273	8274	8274	8274
8273	8274	8275	8275	8275	8275	8275	8275	8274	8273	8273	8273	8273	8273	8273	8274	8275
8273	8274	8275	8275	8275	8275	8275	8275	8274	8273	8273	8273	8273	8273	8273	8274	8275
8274	8274	8274	8275	8275	8275	8275	8274	8274	8274	8273	8273	8273	8273	8274	8274	8274
8274	8274	8274	8274	8275	8275	8274	8274	8274	8274	8274	8273	8273	8274	8274	8274	8274
8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274	8274
8274	8274	8274	8274	8275	8275	8274	8274	8274	8274	8274	8273	8273	8274	8274	8274	8274

Program 2

Size-18

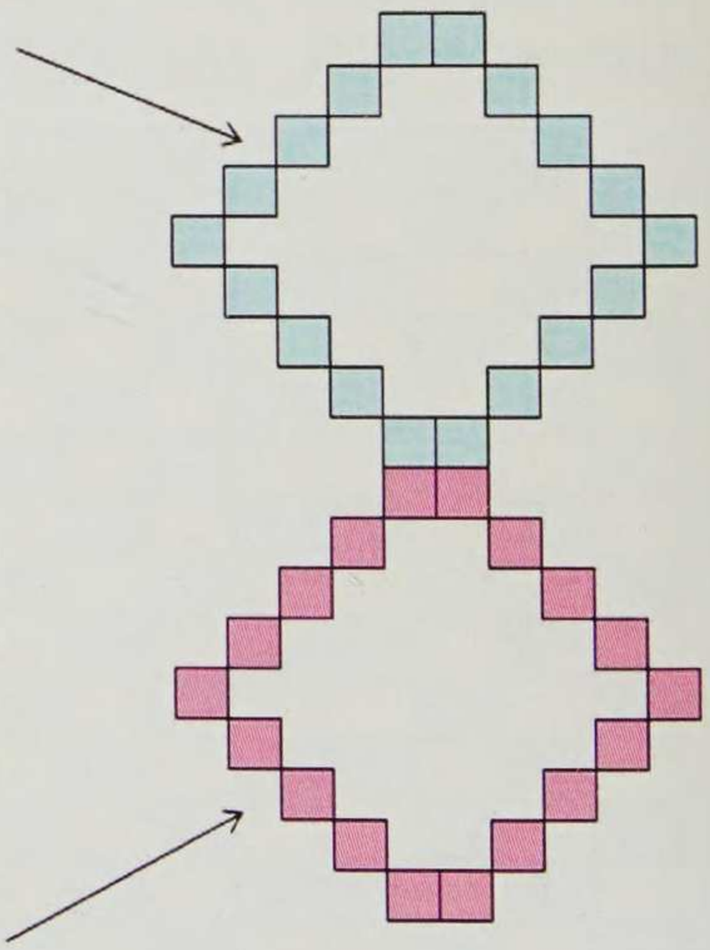
9x2 rectangle

2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925
2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925

Surprisingly, combining the ring-diamond sums from the top and bottom rows yields a double ring-diamond pattern that sums continuously equal, while all the other row combinations yield double ring-diamond patterns that are not continuous equal-summing but yield even more patterns whose differences are ±1 as shown on the next few pagse.

Size-5 odd-even ring-diamond

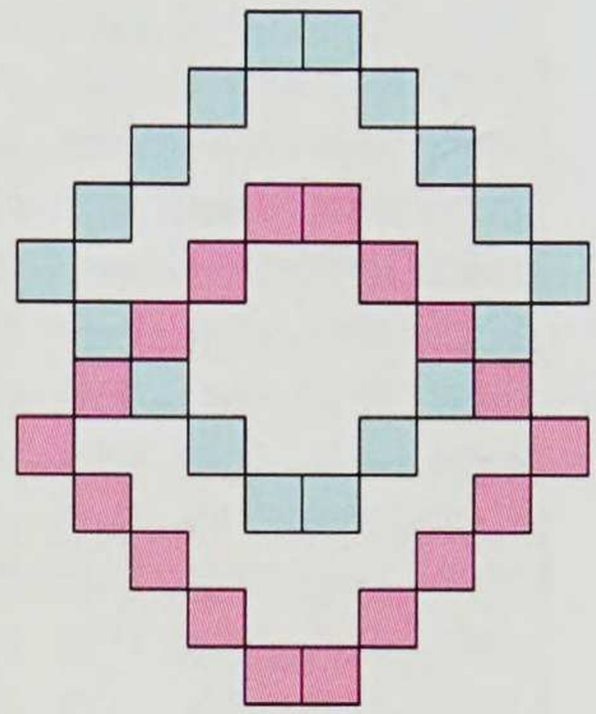
2924	2926	2924	2926	2924	2926	2924	2926	2924	2924	2926	2924	2926	2924	2926	2924	2926	2924
2925	2924	2926	2924	2926	2924	2926	2924	2925	2925	2924	2926	2924	2926	2924	2926	2924	2925
2925	2925	2924	2926	2924	2926	2924	2925	2925	2925	2925	2924	2926	2924	2926	2924	2925	2925
2925	2925	2925	2924	2926	2924	2925	2925	2925	2925	2925	2925	2924	2926	2924	2925	2925	2925
2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925
2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925
2925	2925	2925	2926	2924	2926	2925	2925	2925	2925	2925	2925	2926	2924	2926	2925	2925	2925
2925	2925	2926	2924	2926	2924	2926	2925	2925	2925	2925	2926	2924	2926	2924	2926	2925	2925
2925	2926	2924	2926	2924	2926	2924	2926	2925	2925	2926	2924	2926	2924	2926	2924	2926	2925
2926	2924	2926	2924	2926	2924	2926	2924	2926	2926	2924	2926	2924	2926	2924	2926	2924	2926
2925	2926	2924	2926	2924	2926	2924	2926	2925	2925	2926	2924	2926	2924	2926	2924	2926	2925
2925	2925	2926	2924	2926	2924	2926	2925	2925	2925	2925	2926	2924	2926	2924	2926	2925	2925
2925	2925	2925	2926	2924	2926	2925	2925	2925	2925	2925	2925	2926	2924	2926	2925	2925	2925
2925	2925	2925	2925	2926	2925	2925	2925	2925	2925	2925	2925	2925	2926	2925	2925	2925	2925
2925	2925	2925	2925	2924	2925	2925	2925	2925	2925	2925	2925	2925	2924	2925	2925	2925	2925
2925	2925	2925	2924	2926	2924	2925	2925	2925	2925	2925	2925	2924	2926	2924	2925	2925	2925
2925	2925	2924	2926	2924	2926	2924	2925	2925	2925	2925	2924	2926	2924	2926	2924	2925	2925
2925	2924	2926	2924	2926	2924	2926	2924	2925	2925	2924	2926	2924	2926	2924	2926	2924	2925
2924	2926	2924	2926	2924	2926	2924	2926	2924	2924	2926	2924	2926	2924	2926	2924	2926	2924



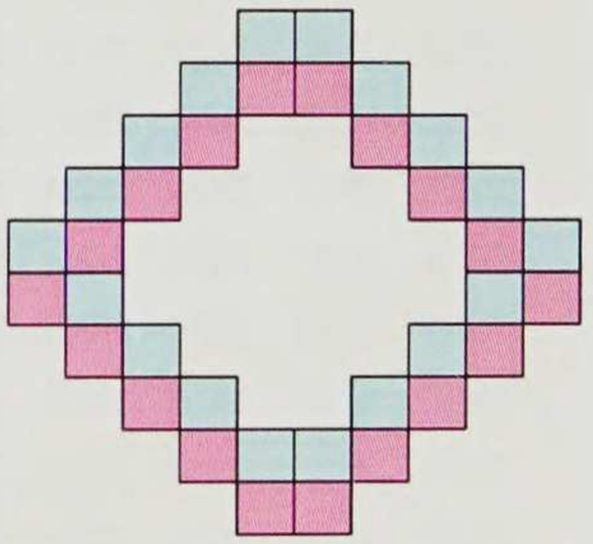
Program 2

Here are summation patterns of just two of 8 symmetric row-combinations on the prior page.

5849 5851 5849 5850 5850 5850 5849 5851 5849 5849 5851 5849 5850 5850 5850 5849 5851 5849
5850 5849 5851 5849 5850 5849 5851 5849 5850 5850 5849 5851 5849 5850 5849 5851 5849 5850
5850 5850 5849 5851 5850 5851 5849 5850 5850 5850 5850 5849 5851 5850 5851 5849 5850 5850
5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850
5850 5850 5851 5849 5850 5849 5851 5850 5850 5850 5850 5851 5849 5850 5849 5851 5850 5850
5850 5851 5849 5851 5850 5851 5849 5851 5850 5850 5851 5849 5851 5850 5851 5849 5851 5850
5851 5849 5851 5850 5850 5850 5851 5849 5851 5851 5849 5851 5850 5850 5850 5851 5849 5851
5850 5851 5849 5851 5850 5851 5849 5851 5850 5850 5851 5849 5851 5850 5851 5849 5851 5850
5850 5850 5851 5849 5850 5849 5851 5850 5850 5850 5850 5851 5849 5850 5849 5851 5850 5850
5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850
5850 5850 5849 5851 5850 5851 5849 5850 5850 5850 5850 5849 5851 5850 5851 5849 5850 5850
5850 5849 5851 5849 5850 5849 5851 5849 5850 5850 5849 5851 5849 5850 5849 5851 5849 5850
5849 5851 5849 5850 5850 5850 5849 5851 5849 5849 5851 5849 5850 5850 5850 5849 5851 5849
5850 5849 5850 5850 5850 5850 5850 5849 5850 5850 5849 5850 5850 5850 5850 5850 5849 5850
5850 5849 5850 5850 5850 5850 5850 5849 5850 5850 5849 5850 5850 5850 5850 5850 5849 5850



5849 5850 5850 5850 5850 5850 5850 5850 5849 5849 5850 5850 5850 5850 5850 5850 5850 5849
5850 5849 5850 5850 5850 5850 5850 5849 5850 5850 5849 5850 5850 5850 5850 5850 5849 5850
5850 5850 5849 5850 5850 5850 5849 5850 5850 5850 5850 5849 5850 5850 5850 5849 5850 5850
5850 5850 5850 5849 5850 5849 5850 5850 5850 5850 5850 5850 5849 5850 5849 5850 5850 5850
5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850
5850 5850 5850 5851 5850 5851 5850 5850 5850 5850 5850 5850 5851 5850 5851 5850 5850 5850
5850 5850 5851 5850 5850 5850 5851 5850 5850 5850 5850 5851 5850 5850 5850 5851 5850 5850
5850 5851 5850 5850 5850 5850 5850 5851 5850 5850 5851 5850 5850 5850 5850 5850 5851 5850
5851 5850 5850 5850 5850 5850 5850 5850 5851 5851 5850 5850 5850 5850 5850 5850 5850 5851
5851 5850 5850 5850 5850 5850 5850 5850 5851 5851 5850 5850 5850 5850 5850 5850 5850 5851
5850 5851 5850 5850 5850 5850 5850 5851 5850 5850 5851 5850 5850 5850 5850 5850 5851 5850
5850 5850 5851 5850 5850 5850 5851 5850 5850 5850 5850 5851 5850 5850 5850 5851 5850 5850
5850 5850 5850 5851 5850 5851 5850 5850 5850 5850 5850 5850 5851 5850 5851 5850 5850 5850
5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850
5850 5850 5850 5849 5850 5849 5850 5850 5850 5850 5850 5850 5849 5850 5849 5850 5850 5850
5850 5850 5849 5850 5850 5850 5849 5850 5850 5850 5850 5849 5850 5850 5850 5849 5850 5850
5850 5849 5850 5850 5850 5850 5850 5849 5850 5850 5849 5850 5850 5850 5850 5850 5849 5850



5849 5850 5850 5850 5850 5850 5850 5850 5849 5849 5850 5850 5850 5850 5850 5850 5850 5849
5849 5850 5850 5850 5850 5850 5850 5850 5849 5849 5850 5850 5850 5850 5850 5850 5850 5849
5850 5849 5850 5850 5850 5850 5850 5849 5850 5850 5849 5850 5850 5850 5850 5850 5849 5850
5850 5850 5849 5850 5850 5850 5849 5850 5850 5850 5850 5849 5850 5850 5850 5849 5850 5850
5850 5850 5850 5849 5850 5849 5850 5850 5850 5850 5850 5850 5849 5850 5849 5850 5850 5850
5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850 5850

Program 2

Here are two even-size diamond summations which have patterned iunequalities in their continuous summations across the square with wrapping across the opposite edges. All this on just one size of square. This was found common among even-size squares whose size was divisible by **4** but not by **8**.

Size-8 even-size solid diamond 23400 ±1

23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400
23400	23400	23400	23400	23401	23401	23400	23400	23400	23400	23400	23400	23400	23401	23401	23400	23400	23400	23400
23400	23400	23400	23401	23401	23401	23401	23400	23400	23400	23400	23400	23401	23401	23401	23401	23400	23400	23400
23400	23400	23401	23401	23401	23401	23401	23401	23400	23400	23400	23401	23401	23401	23401	23401	23401	23400	23400
23400	23401	23401	23401	23401	23401	23401	23401	23401	23400	23401	23401	23401	23401	23401	23401	23401	23401	23400
23400	23400	23401	23401	23401	23401	23401	23401	23400	23400	23400	23401	23401	23401	23401	23401	23401	23400	23400
23400	23400	23400	23401	23401	23401	23401	23400	23400	23400	23400	23400	23401	23401	23401	23401	23400	23400	23400
23400	23400	23400	23400	23401	23401	23400	23400	23400	23400	23400	23400	23400	23401	23401	23400	23400	23400	23400
23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400
23400	23400	23400	23400	23399	23399	23400	23400	23400	23400	23400	23400	23400	23399	23399	23400	23400	23400	23400
23400	23400	23400	23399	23399	23399	23399	23400	23400	23400	23400	23400	23399	23399	23399	23399	23400	23400	23400
23400	23400	23399	23399	23399	23399	23399	23399	23400	23400	23400	23399	23399	23399	23399	23399	23399	23400	23400
23400	23399	23399	23399	23399	23399	23399	23399	23399	23400	23399	23399	23399	23399	23399	23399	23399	23399	23400
23400	23399	23399	23399	23399	23399	23399	23399	23399	23400	23399	23399	23399	23399	23399	23399	23399	23399	23400
23400	23400	23399	23399	23399	23399	23399	23399	23400	23400	23400	23399	23399	23399	23399	23399	23399	23400	23400
23400	23400	23400	23399	23399	23399	23399	23400	23400	23400	23400	23400	23399	23399	23399	23399	23400	23400	23400
23400	23400	23400	23400	23399	23399	23400	23400	23400	23400	23400	23400	23400	23399	23399	23400	23400	23400	23400
23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400	23400

Size-9 even-size solid diamond 29250 ±1

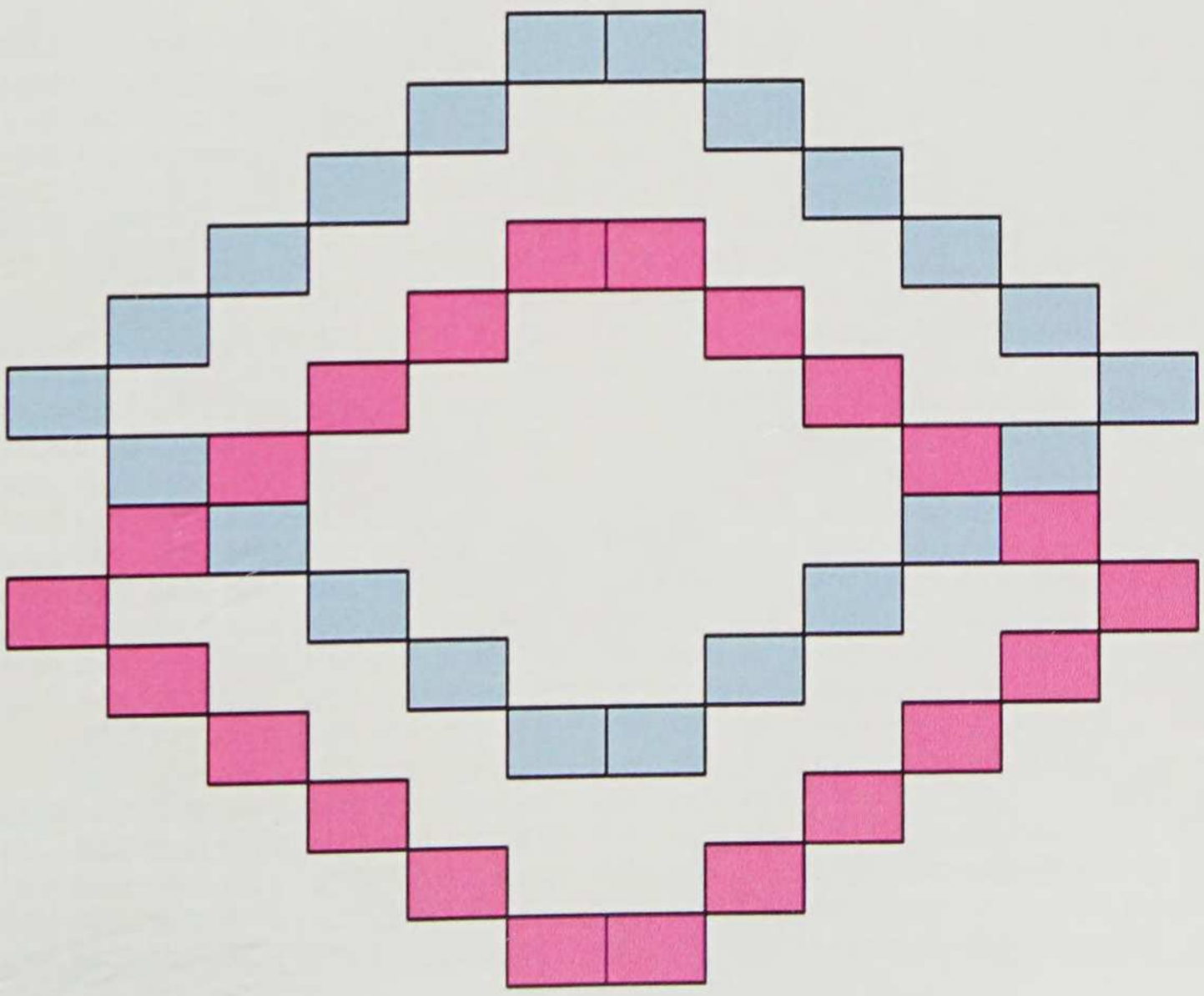
29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250
29250	29250	29250	29250	29251	29251	29250	29250	29250	29250	29250	29250	29250	29251	29251	29250	29250	29250	29250
29250	29250	29250	29251	29251	29251	29251	29250	29250	29250	29250	29250	29251	29251	29251	29251	29250	29250	29250
29250	29250	29251	29251	29251	29251	29251	29251	29250	29250	29250	29251	29251	29251	29251	29251	29251	29250	29250
29250	29251	29251	29251	29251	29251	29251	29251	29251	29250	29251	29251	29251	29251	29251	29251	29251	29251	29250
29250	29251	29251	29251	29251	29251	29251	29251	29251	29250	29251	29251	29251	29251	29251	29251	29251	29251	29250
29250	29250	29251	29251	29251	29251	29251	29251	29250	29250	29250	29251	29251	29251	29251	29251	29251	29250	29250
29250	29250	29250	29251	29251	29251	29251	29250	29250	29250	29250	29250	29251	29251	29251	29251	29250	29250	29250
29250	29250	29250	29250	29251	29251	29250	29250	29250	29250	29250	29250	29250	29251	29251	29250	29250	29250	29250
29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250
29250	29250	29250	29250	29249	29249	29250	29250	29250	29250	29250	29250	29250	29249	29249	29250	29250	29250	29250
29250	29250	29250	29249	29249	29249	29249	29250	29250	29250	29250	29250	29249	29249	29249	29249	29250	29250	29250
29250	29250	29249	29249	29249	29249	29249	29249	29250	29250	29250	29249	29249	29249	29249	29249	29249	29250	29250
29250	29249	29249	29249	29249	29249	29249	29249	29249	29250	29249	29249	29249	29249	29249	29249	29249	29249	29250
29250	29249	29249	29249	29249	29249	29249	29249	29249	29250	29249	29249	29249	29249	29249	29249	29249	29249	29250
29250	29250	29249	29249	29249	29249	29249	29249	29250	29250	29250	29249	29249	29249	29249	29249	29249	29250	29250
29250	29250	29250	29249	29249	29249	29249	29250	29250	29250	29250	29250	29249	29249	29249	29249	29250	29250	29250
29250	29250	29250	29250	29249	29249	29250	29250	29250	29250	29250	29250	29250	29249	29249	29250	29250	29250	29250
29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250	29250

Program 2

Size-22

As the double ring-diamond patterns are brought into closer alignment in larger squares, the patterns of inequalities become more elaborate. Yet the differences between the correct summation and the inequalities are still **±1**.

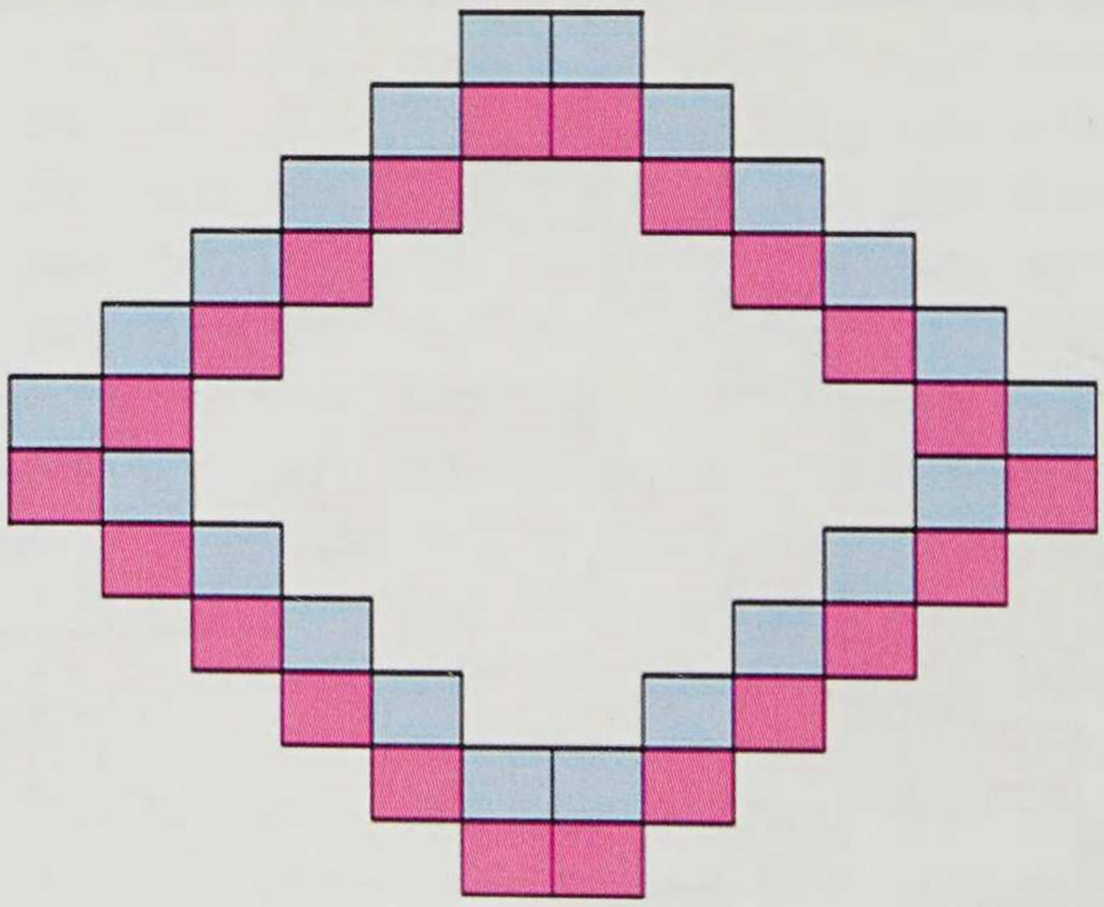
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	10671	10669	10671	10670	10670	10670	10670	10670	10671	10669	10671	10671	10669	10671	10670	10670	10670	10670	10670	10671	10669	10671
2	10670	10671	10669	10671	10670	10670	10670	10671	10669	10671	10670	10670	10671	10669	10671	10670	10670	10670	10671	10669	10671	10670
3	10670	10670	10671	10669	10671	10670	10671	10669	10671	10670	10670	10670	10670	10671	10669	10671	10670	10671	10669	10671	10670	10670
4	10670	10670	10670	10671	10669	10670	10669	10671	10670	10670	10670	10670	10670	10670	10671	10669	10670	10669	10671	10670	10670	10670
5	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670
6	10670	10670	10670	10669	10671	10670	10671	10669	10670	10670	10670	10670	10670	10670	10669	10671	10670	10671	10669	10670	10670	10670
7	10670	10670	10669	10671	10669	10670	10669	10671	10669	10670	10670	10670	10670	10669	10671	10669	10670	10669	10671	10669	10670	10670
8	10670	10669	10671	10669	10670	10670	10670	10669	10671	10669	10670	10670	10669	10671	10669	10670	10670	10670	10669	10671	10669	10670
9	10669	10671	10669	10670	10670	10670	10670	10670	10669	10671	10669	10669	10671	10669	10670	10670	10670	10670	10670	10669	10671	10669
10	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670
11	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670
12	10669	10671	10669	10670	10670	10670	10670	10670	10669	10671	10669	10669	10671	10669	10670	10670	10670	10670	10670	10669	10671	10669
13	10670	10669	10671	10669	10670	10670	10670	10669	10671	10669	10670	10670	10669	10671	10669	10670	10670	10670	10669	10671	10669	10670
14	10670	10670	10669	10671	10669	10670	10669	10671	10669	10670	10670	10670	10670	10669	10671	10669	10670	10669	10671	10669	10670	10670
15	10670	10670	10670	10669	10671	10670	10671	10669	10670	10670	10670	10670	10670	10670	10669	10671	10670	10671	10669	10670	10670	10670
16	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670
17	10670	10670	10670	10671	10669	10670	10669	10671	10670	10670	10670	10670	10670	10670	10671	10669	10670	10669	10671	10670	10670	10670
18	10670	10670	10671	10669	10671	10670	10671	10669	10671	10670	10670	10670	10670	10671	10669	10671	10670	10671	10669	10671	10670	10670
19	10670	10671	10669	10671	10670	10670	10670	10671	10669	10671	10670	10670	10671	10669	10671	10670	10670	10670	10671	10669	10671	10670
20	10671	10669	10671	10670	10670	10670	10670	10670	10671	10669	10671	10671	10669	10671	10670	10670	10670	10670	10670	10671	10669	10671
21	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670
22	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670



Program 2

Again the differences between the correct summation and the inequalities are **±1**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	10671	10670	10670	10670	10670	10670	10670	10670	10670	10670	10671	10671	10670	10670	10670	10670	10670	10670	10670	10670	10670	10671
2	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670
3	10670	10670	10671	10670	10670	10670	10670	10670	10671	10670	10670	10670	10670	10671	10670	10670	10670	10670	10670	10671	10670	10670
4	10670	10670	10670	10671	10670	10670	10670	10671	10670	10670	10670	10670	10670	10670	10671	10670	10670	10670	10671	10670	10670	10670
5	10670	10670	10670	10670	10671	10670	10671	10670	10670	10670	10670	10670	10670	10670	10670	10671	10670	10671	10670	10670	10670	10670
6	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670
7	10670	10670	10670	10670	10669	10670	10669	10670	10670	10670	10670	10670	10670	10670	10670	10669	10670	10669	10670	10670	10670	10670
8	10670	10670	10670	10669	10670	10670	10670	10669	10670	10670	10670	10670	10670	10670	10669	10670	10670	10670	10669	10670	10670	10670
9	10670	10670	10669	10670	10670	10670	10670	10670	10669	10670	10670	10670	10670	10669	10670	10670	10670	10670	10670	10669	10670	10670
10	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670
11	10669	10670	10670	10670	10670	10670	10670	10670	10670	10670	10669	10669	10670	10670	10670	10670	10670	10670	10670	10670	10670	10669
12	10669	10670	10670	10670	10670	10670	10670	10670	10670	10670	10669	10669	10670	10670	10670	10670	10670	10670	10670	10670	10670	10669
13	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670	10670	10669	10670	10670	10670	10670	10670	10670	10670	10669	10670
14	10670	10670	10669	10670	10670	10670	10670	10670	10669	10670	10670	10670	10670	10669	10670	10670	10670	10670	10670	10669	10670	10670
15	10670	10670	10670	10669	10670	10670	10670	10669	10670	10670	10670	10670	10670	10670	10669	10670	10670	10670	10669	10670	10670	10670
16	10670	10670	10670	10670	10669	10670	10669	10670	10670	10670	10670	10670	10670	10670	10670	10669	10670	10669	10670	10670	10670	10670
17	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670	10670
18	10670	10670	10670	10670	10671	10670	10671	10670	10670	10670	10670	10670	10670	10670	10670	10671	10670	10671	10670	10670	10670	10670
19	10670	10670	10670	10671	10670	10670	10670	10671	10670	10670	10670	10670	10670	10670	10671	10670	10670	10670	10671	10670	10670	10670
20	10670	10670	10671	10670	10670	10670	10670	10670	10671	10670	10670	10670	10670	10671	10670	10670	10670	10670	10670	10671	10670	10670
21	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670	10670	10671	10670	10670	10670	10670	10670	10670	10670	10671	10670
22	10671	10670	10670	10670	10670	10670	10670	10670	10670	10670	10671	10671	10670	10670	10670	10670	10670	10670	10670	10670	10670	10671



Size-26

The top and bottom rows of this table lead to the vertical size-26 double ring-diamond tiling pattern which is continuously equal-summing.. Again the differences between the correct summation and the inequalities are ±1. Compare this pattern with the one for size-18 on page 2-23.

Size-13 odd-even ring-diamond

1	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800
2	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801
3	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801
4	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801
5	8801	8801	8801	8801	8800	8802	8800	8802	8800	8801	8801	8801	8801
6	8801	8801	8801	8801	8801	8800	8802	8800	8801	8801	8801	8801	8801
7	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801
8	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801
9	8801	8801	8801	8801	8801	8802	8800	8802	8801	8801	8801	8801	8801
10	8801	8801	8801	8801	8802	8800	8802	8800	8802	8801	8801	8801	8801
11	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801
12	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801
13	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801
14	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802

The pattern below derives from continuous summations on the size-26 square of just a single size-13 odd-even ring-diamond. Note that the blue and pink ring-diamonds are also of size-13. Again the differences are just **±1** from the characteristic value.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800
2	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801
3	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801
4	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801
5	8801	8801	8801	8801	8800	8802	8800	8802	8800	8801	8801	8801	8801	8801	8801	8801	8801	8800	8802	8800	8802	8800	8801	8801	8801	8801
6	8801	8801	8801	8801	8801	8800	8802	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8802	8800	8801	8801	8801	8801	8801
7	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801
8	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801
9	8801	8801	8801	8801	8801	8802	8800	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8800	8802	8801	8801	8801	8801	8801
10	8801	8801	8801	8801	8802	8800	8802	8800	8802	8801	8801	8801	8801	8801	8801	8801	8801	8802	8800	8802	8800	8802	8801	8801	8801	8801
11	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801
12	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801
13	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801
14	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802
15	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801
16	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8800	8802	8801	8801
17	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801	8801	8801	8801	8802	8800	8802	8800	8802	8800	8802	8801	8801	8801
18	8801	8801	8801	8801	8802	8800	8802	8800	8802	8801	8801	8801	8801	8801	8801	8801	8801	8802	8800	8802	8800	8802	8801	8801	8801	8801
19	8801	8801	8801	8801	8801	8802	8800	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8800	8802	8801	8801	8801	8801	8801
20	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8802	8801	8801	8801	8801	8801	8801
21	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8801	8801	8801	8801	8801	8801
22	8801	8801	8801	8801	8801	8800	8802	8800	8801	8801	8801	8801	8801	8801	8801	8801	8801	8801	8800	8802	8800	8801	8801	8801	8801	8801
23	8801	8801	8801	8801	8800	8802	8800	8802	8800	8801	8801	8801	8801	8801	8801	8801	8801	8800	8802	8800	8802	8800	8801	8801	8801	8801
24	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801
25	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801
26	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801	8801	8800	8802	8800	8802	8800	8802	8800	8802	8800	8802	8800	8801

Program 2

Class-4 Squares Tiling Patterns

Here, the tiling patterns are diamonds and X-patterns, indistinguishable from each other.

Sizes divisible by **8** have only one of their two equal-summing pairwise versions with a continuous tiling pattern.

All other sizes of Class-4 squares are not pairwise equal-summing yet are still pangenic. They too have continuous tiling patterns.

Size 8 Patterns

Because the size-4 square's tiling pattern is the entire square itself, here is the smallest perfect Class-4 square with a distinct tiling pattern; it's the size-8.

8x8	260	260	260	260	260	260	260	260
24	9	8	25	40	57	56	41	260
58	39	42	55	10	23	26	7	260
14	19	30	3	62	35	46	51	260
36	61	52	45	20	13	4	29	260
17	16	1	32	33	64	49	48	260
63	34	47	50	15	18	31	2	260
11	22	27	6	59	38	43	54	260
37	60	53	44	21	12	5	28	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260

It has neither diamond pattern ◊ nor **X**-pattern which by themselves sum continuously to the square's characteristic number **260**. However, together in **4x4** block-squares they sum to **2** times the square's characteristic number **520** = $2C_8$.

Diamond pattern

260	260	260	260	260
196	292	260	228	324
260	260	260	260	260
324	228	260	292	196
260	260	260	260	260

+

X-pattern

260	260	260	260	260
324	228	260	292	196
260	260	260	260	260
196	292	260	228	324
260	260	260	260	260

=

Diamond+X-patterns

520	520	520	520	520
520	520	520	520	520
520	520	520	520	520
520	520	520	520	520
520	520	520	520	520

Both pairwise row-symmetric and centrally-symmetric versions of the size-8 square are continuously **4x**-modular. Consequently, both versions have this continuous double tiling property.

Size 12 Patterns

Here is the size **12** square with its characteristic ◊ and X-pattern tiles. Both patterns sum to **870 = C_{12}**. This size-12 square is not pairwise symmetric nor is it even pangenic**.** Pangenicity was traded for having many equal-summing patterns (re: Program 7 **ATE** method). This is the only case among all Class-4 squares where this became a mutually exclusive option.

12x12	1	2	3	4	5	6	7	8	9	10	11	12	
	870	870	870	870	870	870	870	870	870	870	870	870	870
1	21	106	95	84	101	26	31	100	85	90	111	20	870
2	144	67	6	9	80	131	134	73	16	3	70	137	870
3	50	45	124	119	34	61	60	39	114	125	44	55	870
4	59	40	113	126	43	56	49	46	123	120	33	62	870
5	133	74	15	4	69	138	143	68	5	10	79	132	870
6	32	99	86	89	112	19	22	105	96	83	102	25	870
7	18	109	92	87	98	29	28	103	82	93	108	23	870
8	139	72	1	14	75	136	129	78	11	8	65	142	870
9	53	42	127	116	37	58	63	36	117	122	47	52	870
10	64	35	118	121	48	51	54	41	128	115	38	57	870
11	130	77	12	7	66	141	140	71	2	13	76	135	870
12	27	104	81	94	107	24	17	110	91	88	97	30	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870

870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870
870	870	870	870	870	870	870

The size-12's tile-patterns each sum to the square's characteristic number continuously.

Size 16 Patterns

Here is the size-16 Class-4 square.

It has two independent equal-summing tile patterns, a diamond pattern ◊ and an X pattern, each of which consists of **16** cells which sum to the square's characteristic number **2056 = C_{16}** everywhere. The tiled square shows the interlocking diamond pattern. Note that if the diamond-tiling pattern were shifted by four columns either way, left or right, its tiling pattern would be indistinguishable from that of the interlocking X-pattern.

Although distinct, each pattern converts into the other because the tiling pattern is continuous; it can be dragged across the table in any of four directions: horizontally, vertically or diagonally and all the individual tile summations remain the same.

Pairwise centrally symmetric version

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Of the two versions of the size-16 square, it was found that only the centrally pairwise symmetric version had this tiling property.

Size-4 even-size ring-diamond

	1	2	3	4	5	6	7	8	
1	2056	2056	2056	2056	2056	2056	2056	2056	2056
2	2056	2056	2056	2056	2056	2056	2056	2056	2056
3	2056	2056	2056	2056	2056	2056	2056	2056	2056
4	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	2056	2056	2056	2056	2056	2056	2056	2056	2056
6	2056	2056	2056	2056	2056	2056	2056	2056	2056
7	2056	2056	2056	2056	2056	2056	2056	2056	2056
8	2056	2056	2056	2056	2056	2056	2056	2056	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056

Size-8 X-pattern

	1	2	3	4	5	6	7	8	
1	2056	2056	2056	2056	2056	2056	2056	2056	2056
2	2056	2056	2056	2056	2056	2056	2056	2056	2056
3	2056	2056	2056	2056	2056	2056	2056	2056	2056
4	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	2056	2056	2056	2056	2056	2056	2056	2056	2056
6	2056	2056	2056	2056	2056	2056	2056	2056	2056
7	2056	2056	2056	2056	2056	2056	2056	2056	2056
8	2056	2056	2056	2056	2056	2056	2056	2056	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056

Size 20 Patterns

Here is the size **20** square. The square is neither pairwise row nor centrally symmetric. Yet it is pangenic.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	20x20	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	246	364	2	195	233	371	14	177	245	358	21	189	227	370	8	196	239	352	4010
2	336	254	117	85	348	261	104	92	335	273	111	79	342	260	123	86	329	267	110	98	4010
3	157	50	388	201	169	32	400	213	151	44	382	225	163	26	394	207	175	38	376	219	4010
4	278	316	59	147	290	303	66	134	297	315	53	141	284	322	65	128	291	309	72	140	4010
5	24	187	230	368	6	199	237	355	18	181	249	362	5	193	231	374	12	180	243	356	4010
6	345	258	121	89	327	270	108	96	339	252	120	83	346	264	102	95	333	271	114	77	4010
7	161	29	392	210	173	36	379	217	160	48	386	204	167	35	398	211	154	42	385	223	4010
8	282	325	63	126	294	307	75	138	276	319	57	150	288	301	69	132	300	313	51	144	4010
9	3	191	234	372	15	178	241	359	22	190	228	366	9	197	240	353	16	184	247	365	4010
10	349	262	105	93	331	274	112	80	343	256	124	87	330	268	106	99	337	255	118	81	4010
11	170	33	396	214	152	45	383	221	164	27	395	208	171	39	377	220	158	46	389	202	4010
12	286	304	67	135	298	311	54	142	285	323	61	129	292	310	73	136	279	317	60	148	4010
13	7	200	238	351	19	182	250	363	1	194	232	375	13	176	244	357	25	188	226	369	4010
14	328	266	109	97	340	253	116	84	347	265	103	91	334	272	115	78	341	259	122	90	4010
15	174	37	380	218	156	49	387	205	168	31	399	212	155	43	381	224	162	30	393	206	4010
16	295	308	71	139	277	320	58	146	289	302	70	133	296	314	52	145	283	321	64	127	4010
17	11	179	242	360	23	186	229	367	10	198	236	354	17	185	248	361	4	192	235	373	4010
18	332	275	113	76	344	257	125	88	326	269	107	100	338	251	119	82	350	263	101	94	4010
19	153	41	384	222	165	28	391	209	172	40	378	216	159	47	390	203	166	34	397	215	4010
20	299	312	55	143	281	324	62	130	293	306	74	137	280	318	56	149	287	305	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Each tile consists of **20** cells. Each diamond and X-pattern sums to **4010 = C_{20}** continuously.

4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Program 2

Size 24 Patterns

Here is the size-24 square. Each diamond and X-pattern sums to **6924** = C_{24} continuously. All the patterns for the size-20 square were also found here in the size-24 square.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	217	66	235	414	439	432	145	138	163	342	511	360	505	354	523	126	151	144	6924
2	103	192	85	564	313	546	319	552	301	204	97	186	391	480	373	276	25	258	31	264	13	492	385	474	6924
3	356	506	311	194	143	149	140	146	95	554	359	509	68	218	23	482	431	437	428	434	383	266	71	221	6924
4	105	208	87	562	297	544	321	568	303	202	81	184	393	496	375	274	9	256	33	280	15	490	369	472	6924
5	327	514	345	160	135	178	111	154	129	520	351	538	39	226	57	448	423	466	399	442	417	232	63	250	6924
6	83	209	92	557	296	566	299	569	308	197	80	206	371	497	380	269	8	278	11	281	20	485	368	494	6924
7	325	534	343	162	115	180	109	174	127	522	331	540	37	246	55	450	403	468	397	462	415	234	43	252	6924
8	139	156	121	528	349	510	355	516	337	168	133	150	427	444	409	240	61	222	67	228	49	456	421	438	6924
9	323	545	344	161	104	182	107	185	128	521	320	542	35	257	56	449	392	470	395	473	416	233	32	254	6924
10	141	172	123	526	333	508	357	532	339	166	117	148	429	460	411	238	45	220	69	244	51	454	405	436	6924
11	291	550	309	196	99	214	75	190	93	556	315	574	3	262	21	484	387	502	363	478	381	268	27	286	6924
12	116	170	131	518	335	533	332	530	347	158	119	173	404	458	419	230	47	245	44	242	59	446	407	461	6924
13	289	570	307	198	79	216	73	210	91	558	295	576	1	282	19	486	367	504	361	498	379	270	7	288	6924
14	247	48	229	420	457	402	463	408	445	60	241	42	535	336	517	132	169	114	175	120	157	348	529	330	6924
15	500	362	455	50	287	5	284	2	239	410	503	365	212	74	167	338	575	293	572	290	527	122	215	77	6924
16	249	64	231	418	441	400	465	424	447	58	225	40	537	352	519	130	153	112	177	136	159	346	513	328	6924
17	471	370	489	16	279	34	255	10	273	376	495	394	183	82	201	304	567	322	543	298	561	88	207	106	6924
18	227	65	236	413	440	422	443	425	452	53	224	62	515	353	524	125	152	134	155	137	164	341	512	350	6924
19	469	390	487	18	259	36	253	30	271	378	475	396	181	102	199	306	547	324	541	318	559	90	187	108	6924
20	283	12	265	384	493	366	499	372	481	24	277	6	571	300	553	96	205	78	211	84	193	312	565	294	6924
21	467	401	488	17	248	38	251	41	272	377	464	398	179	113	200	305	536	326	539	329	560	89	176	110	6924
22	285	28	267	382	477	364	501	388	483	22	261	4	573	316	555	94	189	76	213	100	195	310	549	292	6924
23	435	406	453	52	243	70	219	46	237	412	459	430	147	118	165	340	531	358	507	334	525	124	171	142	6924
24	260	26	275	374	479	389	476	386	491	14	263	29	548	314	563	86	191	101	188	98	203	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Here is the first Class-4 square found to be only pairwise row symmetric.

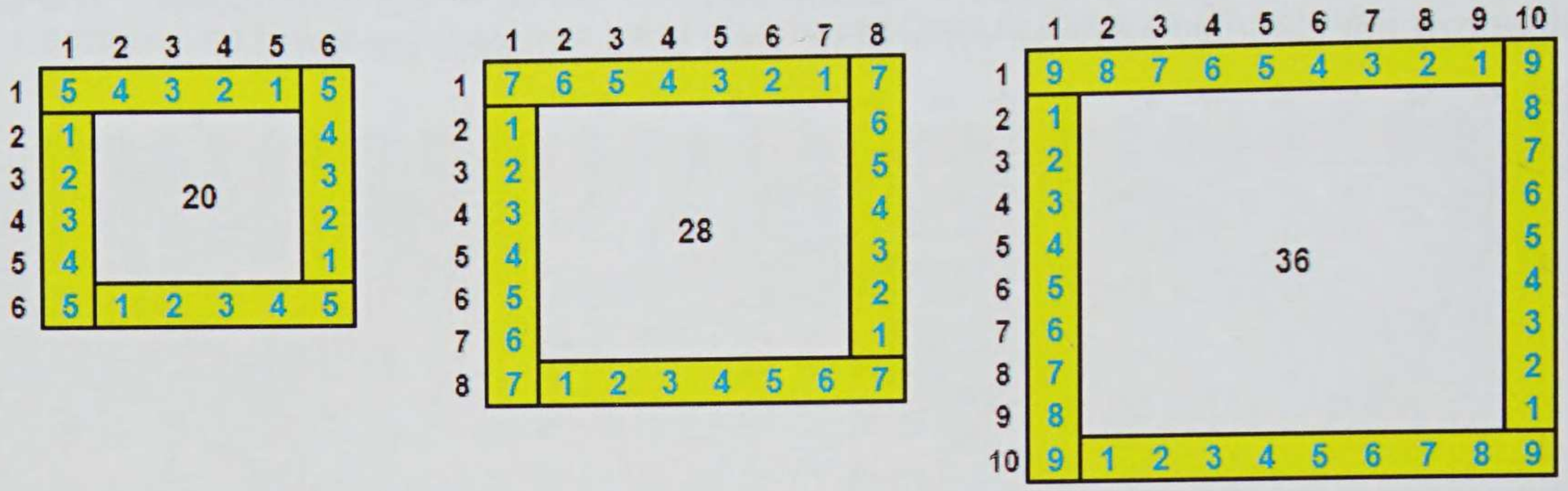
Program 2

Size 28 Patterns

Here is the size **28** square with diamond and X-pattern tiles of **28** cells each. Its characteristic number is **10990**. Each diamond and X-pattern sums to **10990 = C_{28}** continuously. The square is neither row nor centrally pairwise symmetric, yet it is nonetheless pangenic.

Program 2

Square Ring Patterns in Class-4 Squares



Emerging at size-20 and continuing every eighth size up is the square ring pattern shown here which sums exactly and continuously to the square's characteristic number.

Repetitions of the pattern are wholly contained within the square if wrapping and overlap is imposed along the horizontal, vertical and diagonal directions.

For this to occur, it takes **n** repetitions of the square ring to complete its linear distribution across the square from corner-to-corner and edge-to-edge.

W(20)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	321	289	2	195	308	296	14	177	320	283	21	189	302	295	8	196	314	277	4010
2	336	254	42	160	348	261	29	167	335	273	36	154	342	260	48	161	329	267	35	173	4010
3	82	125	388	201	94	107	400	213	76	119	382	225	88	101	394	207	100	113	376	219	4010
4	353	241	59	147	365	228	66	134	372	240	53	141	359	247	65	128	366	234	72	140	4010
5	24	187	305	293	6	199	312	280	18	181	324	287	5	193	306	299	12	180	318	281	4010
6	345	258	46	164	327	270	33	171	339	252	45	158	346	264	27	170	333	271	39	152	4010
7	86	104	392	210	98	111	379	217	85	123	386	204	92	110	398	211	79	117	385	223	4010
8	357	250	63	126	369	232	75	138	351	244	57	150	363	226	69	132	375	238	51	144	4010
9	3	191	309	297	15	178	316	284	22	190	303	291	9	197	315	278	16	184	322	290	4010
10	349	262	30	168	331	274	37	155	343	256	49	162	330	268	31	174	337	255	43	156	4010
11	95	108	396	214	77	120	383	221	89	102	395	208	96	114	377	220	83	121	389	202	4010
12	361	229	67	135	373	236	54	142	360	248	61	129	367	235	73	136	354	242	60	148	4010
13	7	200	313	276	19	182	325	288	1	194	307	300	13	176	319	282	25	188	301	294	4010
14	328	266	34	172	340	253	41	159	347	265	28	166	334	272	40	153	341	259	47	165	4010
15	99	112	380	218	81	124	387	205	93	106	399	212	80	118	381	224	87	105	393	206	4010
16	370	233	71	139	352	245	58	146	364	227	70	133	371	239	52	145	358	246	64	127	4010
17	11	179	317	285	23	186	304	292	10	198	311	279	17	185	323	286	4	192	310	298	4010
18	332	275	38	151	344	257	50	163	326	269	32	175	338	251	44	157	350	263	26	169	4010
19	78	116	384	222	90	103	391	209	97	115	378	216	84	122	390	203	91	109	397	215	4010
20	374	237	55	143	356	249	62	130	368	231	74	137	355	243	56	149	362	230	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Rectangular rings of **n** cells were found not to sum equally.

Program 2

Summary of Class-4 Tiling Patterns

At right is a summary table of Class-4 tiling patterns. Note that only the patterns for size-4 and size-8 are unusual. This stems from their small size having an insufficient amount of consecutive numbers to yield distinct tiling patterns that by themselves can sum to the square's characteristic number. That is, for this tiling property, these sizes are *embryonic*.

However, the two tile patterns together are continuous as a block-square of size **n/2xn/2** which do sum to the squares characteristic number for size-4 and two times the characteristic number for size-8. This on its own constitutes **n/2** continuous modularity.

Both pairwise row-symmetric and centrally-symmetric versions of the size-8 square are continuously 4x-modular. Consequently, both versions have this tiling property by default.

That differs from the size-16 square. Although there are two categorical versions of the size-16 square -- one with equal row-pair sums and one with equal pairwise centrally-symmetric sums – only the version with centrally-symmetric pair-sums has these continuously characteristic diamond and X tiling patterns.

Unlike the size-16 square with these same tiling patterns, the size-24 square has equal symmetric row-pair sums. That is the only version of the size-24 square to be geonomically perfect and to possess these equal-summing tiling patterns.

Squares of size **n = 4b**, where **b** is an odd number greater than **3**, have no pairwise symmetry yet are still geonomic.

The size of the ring-diamonds are all of size **b**, however, there is an additional size for the size-32 square which is of size **4** that sums to $\frac{1}{2}C_{32}$. This is the only instance in the series up thru size-44 where this was observed to occur.

Emerging at size-20 and occurring every eighth size thereon, the square also has the square ring of **n** cells.

Summary of Class-4 Tiling Patterns		
Size	**Continuous Equal-summing Patterns**	**Pairwise symmetry with tiling pattern**
4	◊ & X in tight 2x2 block-squares Together = C_4	**No pairwise symmetry**
8	◊ + X in tight 4x4 block-squares Together = $2C_8$	**Both row & central symmetry**
12	Both ◊ & X Each = C_{12}	**No pairwise symmetry**
16	Both ◊ & X Each = C_{16}	**Central symmetry**
20	Both ◊ & X plus □ Each = C_{20}	**No pairwise symmetry**
24	Both ◊ & X Each = C_{24}	**Row symmetry**
28	Both ◊ & X plus □ Each = C_{28}	**No pairwise symmetry**
32	Both ◊ & X Each = C_{32}	**Central symmetry**
36	Both ◊ & X plus □ Each = C_{36}	**No pairwise symmetry**
40	Both ◊ & X Each = C_{40}	**Row symmetry**
44	Both ◊ & X plus □ Each = C_{44}	**No pairwise symmetry**

Program 2

Other Common Tiling Patterns for Class-4 Squares

Here are four more simultaneous patterns for the size **36** square. Each pattern contains **36** cells and sums to C_{36}.

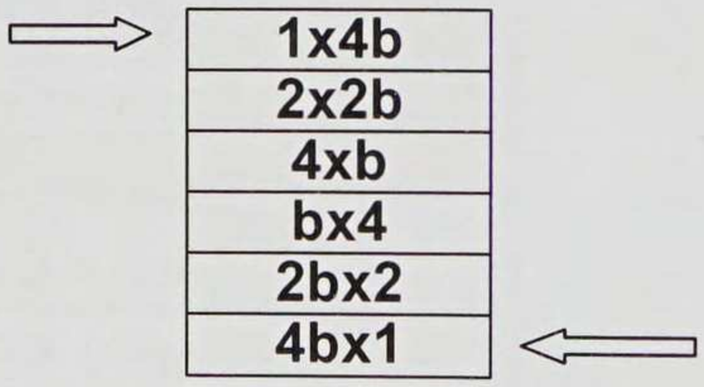
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

1	23346	23346	23346	23346	23346	23346	23346	23346
2	23346	23346	23346	23346	23346	23346	23346	23346
3	23346	23346	23346	23346	23346	23346	23346	23346
4	23346	23346	23346	23346	23346	23346	23346	23346
5	23346	23346	23346	23346	23346	23346	23346	23346
6	23346	23346	23346	23346	23346	23346	23346	23346
7	23346	23346	23346	23346	23346	23346	23346	23346
8	23346	23346	23346	23346	23346	23346	23346	23346
9	23346	23346	23346	23346	23346	23346	23346	23346
10	23346	23346	23346	23346	23346	23346	23346	23346
11	23346	23346	23346	23346	23346	23346	23346	23346
12	23346	23346	23346	23346	23346	23346	23346	23346
13	23346	23346	23346	23346	23346	23346	23346	23346
14	23346	23346	23346	23346	23346	23346	23346	23346
15	23346	23346	23346	23346	23346	23346	23346	23346
16	23346	23346	23346	23346	23346	23346	23346	23346
17	23346	23346	23346	23346	23346	23346	23346	23346
18	23346	23346	23346	23346	23346	23346	23346	23346

23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	
23346	23346	23346	23346	23346	23346	23346	23346	

23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346

In general, the rectangular patterns which sum to any Class-4 size **n = 4b** square's characteristic number, all simultaneously, are listed here:



The first and last patterns are simply the linear row and column summations.

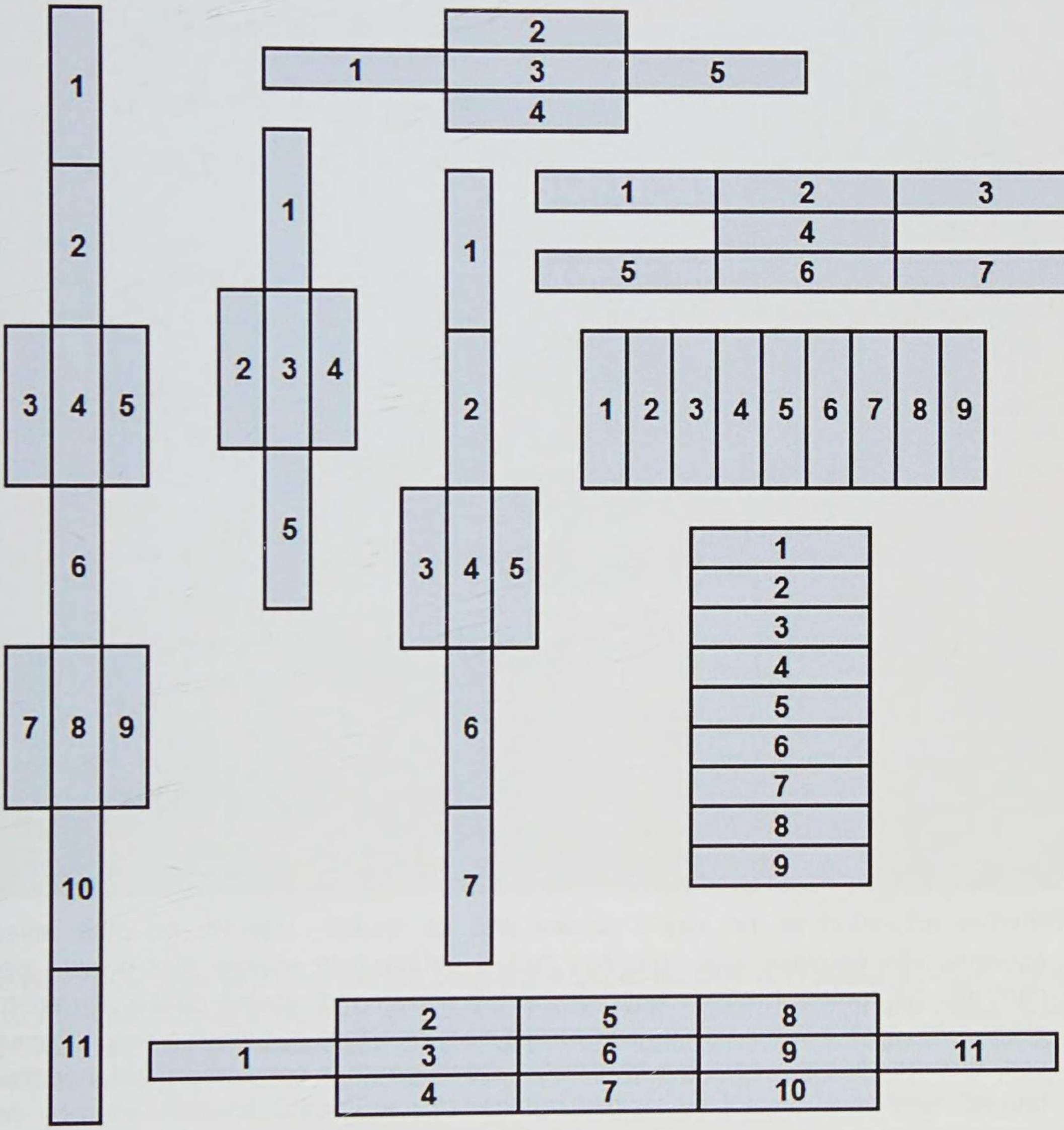
Rectangular tiling patterns for Class-4 squares			
n=4b	b	4xb	bx4
4	1	4x1	1x4
8	2	4x2	--
12	3	--	--
16	4	4x4	4x4
20	5	4x5	5x4
24	6	4x6	--
28	7	4x7	7x4
32	8	4x8	8x4
36	9	4x9	9x4
40	10	4x10	10x4
44	11	4x11	11x4

The only irregularities in the series of sizes correspond to where **b = 3** or **6**. The rectangular pattern and its transpose hold for all **b ≥ 7**.

Program 2

Here are complementary sets of tile patterns for sizes **20, 28, 36** and **44** composed of **1x4** and **4x1** linear segments of **5, 7, 9** and **11** composits. These all sum equally and continuously to the size square's characteristic number.

It should be apparent how similar tiling patterns can be determined for larger squares whose size is **8t+4** for **t ≥ 2**.



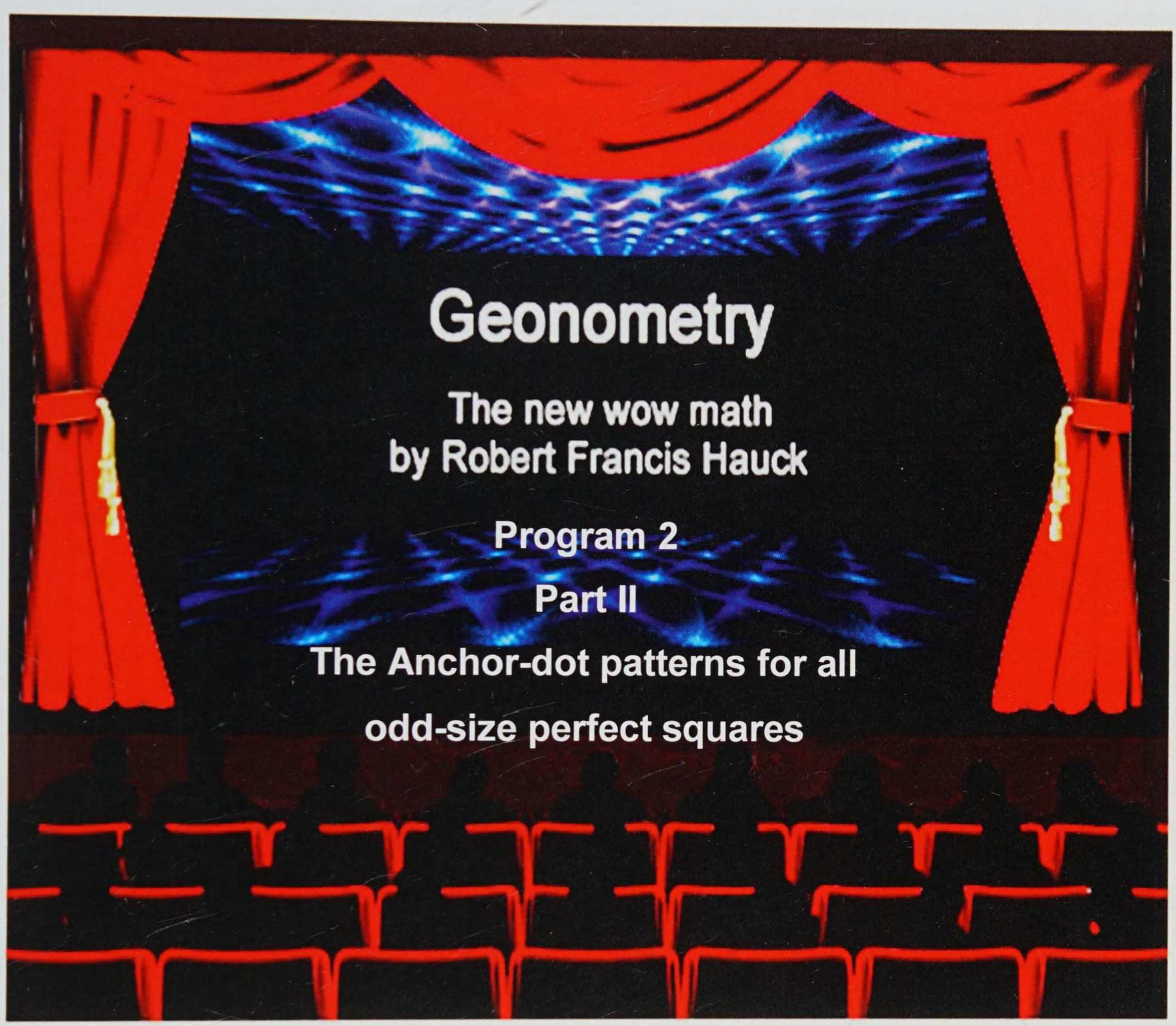
These tile patterns integrate with themselves to cover the square exactly and completely with wrapping imposed, so any tightly integrated pattern of **k** tiles will sum to **kCn**.

Observe that the tile patterns are unique only where **b** is a prime-number for **n = 4b**. This is a general principle.

Program 2

Part I Notes

1. Class-1 squares each have two unique complementary tile patterns that are characteristic of their size.
2. Class-2 squares have just one 2-dimensional tiling pattern which sums quasi-continuously for squares of sizes **≥ 14,** namely the **bx2** or **2xb** rectangle, but not both, which sum everywhere to the square's characteristic number except when covering the double central vertical columns or central rows, respectively.
3. Class-2 squares are consistently pairwise centrally symmetric for sizes **≥ 14.**
4. Class-3 squares whose size is **3** times a prime number have no tiling patterns which sum to its characteristic number.
5. Class-4 squares have two indistinguishable complementary tile patterns, the ring ◊ diamond and the **X** pattern that are common among all sizes, with the exceptions of size-4 which has none and size-8 which compounds its two complementary tiles together into a **4x4** block-square with **4x**-modularity.
6. Class-4 squares are not sequentially consistent in having pairwise symmetry (table page 2-35). From size-4 onward, every 8th size has no pairwise symmetry at all. From size-24 on, every 8th size square has either central or row pairwise symmetry, and does so in an alternating pattern.
7. Class-2 and Class-4 squares have symmetric patterns among inequalities in their continuous summations.
8. Class-5 tiling patterns for **n = ab**, where **a>b>3** and **a** and **b** are both prime-numbers, have, in addition to **5x** and **7x** modularity, four complementary **axb** and **bxa** staircase tiling patterns in both slanted directions.
9. Class-6 squares have complementary characteristic tiling patterns where $\mathbf{n = b^2}$ and **b** is a prime-number. These are simply **bxb** block-squares whose **b**-block rows are shifted horizontally and **b**-block columns are shifted vertically.
10. All tiling patterns are continuous.
11. With the exception of the size-8 square and its double, size-16, no other even-size squares with pairwise row symmetry has a counterpart square that is also pairwise centrally symmetric. Further, while the size-8 square is pairwise doubly symmetric, no other even-size square can be converted from one that is pairwise row symmetric to one that is centrally pairwise symmetric because any conversion will separate the largest number $\mathbf{n^2}$ from **1** by an odd number of rows and/or columns thereby denying symmetry in an even size square. This fact was determined after making many noble attempts on numerous even-size squares with pairwise row symmetry.
12. On the otherhand, the size-8 square can be converted from pairwise row symmetry to central symmetry and back again. This is demonstrated later in Program 8. There is a specific reason why, which identifies a property possessed by no other even-size square's numerical structure.



There is another type of pattern which contains **n** separate periodic locations which together also sum to the characteristic number of a square of size-n. These patterns are called ***anchor-dot*** distribution patterns because each central dot in distribution pattern functions as an anchor for dot distributions of additional equal summations in relation to the anchor-dots.

Program 2

Anchor-dot Patterns

Class-1 Squares

What a star is to a galaxy of solar systems, an anchor-dot is to a complete tiling pattern.

If one takes a tiling pattern across a Class-1 square and transposes it, the transposed tiles' centers will emerge as an anchor-dot pattern whose dots will sum together continuously to the square's characteristic number just as the individual characteristic tiles do.

Further, these dot-patterns can be dragged continuously across the square, with wrapping imposed across the edges, and the dot pattern will still sum to the square's characteristic number everywhere.

Program 2

Size-7 Anchor-dot Patterns

Here is the size-7 square **W(7)** again. It is shown here with a uniformly distributed pattern of centers of the transpose of its complementary characteristic tiling patterns A and B. Their center numbers are highlighted in white. Each tile contains **7** cells.

W(7)	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43	175
2	33	3	22	48	18	37	14	175
3	46	16	42	12	31	1	27	175
4	10	29	6	25	44	21	40	175
5	23	49	19	38	8	34	4	175
6	36	13	32	2	28	47	17	175
7	7	26	45	15	41	11	30	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

W(7)	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43	175
2	33	3	22	48	18	37	14	175
3	46	16	42	12	31	1	27	175
4	10	29	6	25	44	21	40	175
5	23	49	19	38	8	34	4	175
6	36	13	32	2	28	47	17	175
7	7	26	45	15	41	11	30	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

These transposed tiling patterns can be dragged across the table with roll-wrapping imposed and the tile centers will still sum equally to the characteristic number. Thus the transposed tile centers function as **anchor dots** for the tiling patterns. Every location relative to these dots among all the tiles sum equally. That yields **7** equal dot patterns for each cell in either tile pattern.

The horizontal flip of both tiling patterns adds another **7** equal summing dot patterns for each one.

These patterns may be represented numerically by counting the number of cells to arrive at the nearest center of an adjoining tile, first the number of cells vertically, and second the number of cells horizontally. Both tile patterns tA and tB here may be described as **(1;2)** for "**up 1; right 2**". In this descriptive methodology, there is no "down" in this context and "left" is indicated by a negative number.

(2; 1)

175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

Further, there is another tiling pattern "C" of the description of **(2;1)** which is also equal-summing to the characteristic number for the size-7 square shown here. But when transposed to tC, the dot pattern also becomes **(1;2),** the same as tA.

So the size-7 has a total of only **2n = 14** equal-summing dot patterns.

Program 2

Size-11 Anchor-dot Patterns

Here are 2 anchor-dot patterns which sum continuously to the size-11 square's characteristic number.They arise from the centers of transposed tiles A^T and B^T. Their patterns are **(1;+2)**. That yields **2n = 22** anchor-dot patterns. Note that the centers of the transposed tile patterns are identical.

W(11) **tA**

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

W(11)

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

W(11) **tB**

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

W(11)

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

U(11) **tA**

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

U(11)

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

U(11) **tB**

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

U(11)

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

Program 2

Class-1 Size-37 Square

A Rare Exception in the Possession of the Anchor-dot Property

This size-37 square was generated by the **Double-Quark** method of Program 7.

Here is the size-37 square shown with its tiling pattern A. Each tile sums to its characteristic number **25345**.

The centers of all the tiles together sum to **25345** too. The center cell of the center tile must be located at the center of the square for this dot pattern to emerge on the A tiling pattern.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
	W(37)	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345
1	78	306	534	762	990	1218	40	268	496	724	952	1180	2	230	458	686	914	1142	1333	192	420	648	876	1104	1332	154	382	610	838	1066	1294	116	344	572	800	1028	1256	25345
2	42	270	498	726	954	1182	4	232	460	688	916	1144	1335	194	422	650	878	1106	1297	156	384	612	840	1068	1259	118	346	574	802	1030	1258	80	308	536	764	992	1220	25345
3	6	234	462	690	918	1146	1337	196	424	652	880	1108	1299	158	386	614	842	1070	1261	120	348	576	804	1032	1223	82	310	538	766	994	1185	44	272	500	728	956	1184	25345
4	1339	198	426	654	882	1110	1301	160	388	616	844	1072	1263	122	350	578	806	1034	1225	84	312	540	768	996	1187	46	274	502	730	958	1149	8	236	464	692	920	1111	25345
5	1303	162	390	618	846	1037	1265	124	352	580	808	1036	1227	86	314	542	770	998	1189	48	276	504	732	960	1151	10	238	466	694	922	1113	1341	200	428	656	884	1075	25345
6	1267	126	354	582	810	1001	1229	88	316	544	772	963	1191	50	278	506	734	962	1153	12	240	468	696	924	1115	1343	202	430	658	886	1077	1305	164	392	620	848	1039	25345
7	1231	90	318	546	774	965	1193	52	280	508	736	927	1155	14	242	470	698	889	1117	1345	204	432	660	888	1079	1307	166	394	622	850	1041	1269	128	356	584	812	1003	25345
8	1195	54	282	510	738	929	1157	16	244	472	700	891	1119	1347	206	434	662	853	1081	1309	168	396	624	815	1043	1271	130	358	586	814	1005	1233	92	320	548	776	967	25345
9	1159	18	246	474	702	893	1121	1349	208	436	664	855	1083	1311	170	398	626	817	1045	1273	132	360	588	779	1007	1235	94	322	550	741	969	1197	56	284	512	740	931	25345
10	1123	1351	210	438	666	857	1085	1313	172	400	628	819	1047	1275	134	362	590	781	1009	1237	96	324	552	743	971	1199	58	286	514	705	933	1161	20	248	476	667	895	25345
11	1087	1315	174	402	593	821	1049	1277	136	364	592	783	1011	1239	98	326	554	745	973	1201	60	288	516	707	935	1163	22	250	478	669	897	1125	1353	212	440	631	859	25345
12	1051	1279	138	366	557	785	1013	1241	100	328	519	747	975	1203	62	290	518	709	937	1165	24	252	480	671	899	1127	1355	214	442	633	861	1089	1317	176	404	595	823	25345
13	1015	1243	102	330	521	749	977	1205	64	292	483	711	939	1167	26	254	445	673	901	1129	1357	216	444	635	863	1091	1319	178	406	597	825	1053	1281	140	368	559	787	25345
14	979	1207	66	294	485	713	941	1169	28	256	447	675	903	1131	1359	218	409	637	865	1093	1321	180	371	599	827	1055	1283	142	370	561	789	1017	1245	104	332	523	751	25345
15	943	1171	30	258	449	677	905	1133	1361	220	411	639	867	1095	1323	182	373	601	829	1057	1285	144	335	563	791	1019	1247	106	297	525	753	981	1209	68	296	487	715	25345
16	907	1135	1363	222	413	641	869	1097	1325	184	375	603	831	1059	1287	146	337	565	793	1021	1249	108	299	527	755	983	1211	70	261	489	717	945	1173	32	223	451	679	25345
17	871	1099	1327	149	377	605	833	1061	1289	148	339	567	795	1023	1251	110	301	529	757	985	1213	72	263	491	719	947	1175	34	225	453	681	909	1137	1365	187	415	643	25345
18	835	1063	1291	113	341	569	797	1025	1253	75	303	531	759	987	1215	74	265	493	721	949	1177	36	227	455	683	911	1139	1367	189	417	645	873	1101	1329	151	379	607	25345
19	799	1027	1255	77	305	533	761	989	1217	39	267	495	723	951	1179	1	229	457	685	913	1141	1369	191	419	647	875	1103	1331	153	381	609	837	1065	1293	115	343	571	25345
20	763	991	1219	41	269	497	725	953	1181	3	231	459	687	915	1143	1334	193	421	649	877	1105	1296	155	383	611	839	1067	1295	117	345	573	801	1029	1257	79	307	535	25345
21	727	955	1183	5	233	461	689	917	1145	1336	195	423	651	879	1107	1298	157	385	613	841	1069	1260	119	347	575	803	1031	1222	81	309	537	765	993	1221	43	271	499	25345
22	691	919	1147	1338	197	425	653	881	1109	1300	159	387	615	843	1071	1262	121	349	577	805	1033	1224	83	311	539	767	995	1186	45	273	501	729	957	1148	7	235	463	25345
23	655	883	1074	1302	161	389	617	845	1073	1264	123	351	579	807	1035	1226	85	313	541	769	997	1188	47	275	503	731	959	1150	9	237	465	693	921	1112	1340	199	427	25345
24	619	847	1038	1266	125	353	581	809	1000	1228	87	315	543	771	999	1190	49	277	505	733	961	1152	11	239	467	695	923	1114	1342	201	429	657	885	1076	1304	163	391	25345
25	583	811	1002	1230	89	317	545	773	964	1192	51	279	507	735	926	1154	13	241	469	697	925	1116	1344	203	431	659	887	1078	1306	165	393	621	849	1040	1268	127	355	25345
26	547	775	966	1194	53	281	509	737	928	1156	15	243	471	699	890	1118	1346	205	433	661	852	1080	1308	167	395	623	851	1042	1270	129	357	585	813	1004	1232	91	319	25345
27	511	739	930	1158	17	245	473	701	892	1120	1348	207	435	663	854	1082	1310	169	397	625	816	1044	1272	131	359	587	778	1006	1234	93	321	549	777	968	1196	55	283	25345
28	475	703	894	1122	1350	209	437	665	856	1084	1312	171	399	627	818	1046	1274	133	361	589	780	1008	1236	95	323	551	742	970	1198	57	285	513	704	932	1160	19	247	25345
29	439	630	858	1086	1314	173	401	629	820	1048	1276	135	363	591	782	1010	1238	97	325	553	744	972	1200	59	287	515	706	934	1162	21	249	477	668	896	1124	1352	211	25345
30	403	594	822	1050	1278	137	365	556	784	1012	1240	99	327	555	746	974	1202	61	289	517	708	936	1164	23	251	479	670	898	1126	1354	213	441	632	860	1088	1316	175	25345
31	367	558	786	1014	1242	101	329	520	748	976	1204	63	291	482	710	938	1166	25	253	481	672	900	1128	1356	215	443	634	862	1090	1318	177	405	596	824	1052	1280	139	25345
32	331	522	750	978	1206	65	293	484	712	940	1168	27	255	446	674	902	1130	1358	217	408	636	864	1092	1320	179	407	598	826	1054	1282	141	369	560	788	1016	1244	103	25345
33	295	486	714	942	1170	29	257	448	676	904	1132	1360	219	410	638	866	1094	1322	181	372	600	828	1056	1284	143	334	562	790	1018	1246	105	333	524	752	980	1208	67	25345
34	259	450	678	906	1134	1362	221	412	640	868	1096	1324	183	374	602	830	1058	1286	145	336	564	792	1020	1248	107	298	526	754	982	1210	69	260	488	716	944	1172	31	25345
35	186	414	642	870	1098	1326	185	376	604	832	1060	1288	147	338	566	794	1022	1250	109	300	528	756	984	1212	71	262	490	718	946	1174	33	224	452	680	908	1136	1364	25345
36	150	378	606	834	1062	1290	112	340	568	796	1024	1252	111	302	530	758	986	1214	73	264	492	720	948	1176	35	226	454	682	910	1138	1366	188	416	644	872	1100	1328	25345
37	114	342	570	798	1026	1254	76	304	532	760	988	1216	38	266	494	722	950	1178	37	228	456	684	912	1140	1368	190	418	646	874	1102	1330	152	380	608	836	1064	1292	25345
	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	
		25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345		

Program 2

Here is the same square with a complementary tiling pattern B that is just the transpose of pattern A ! They just appear differently when transposed due to the rectangular shape of the cells to accommodate the numbers.

This is very rare for Class-1 squares: no other Class-1 square to date was found to have their complementary tile patterns be the transpose of each other. Its tile centers are distributed differently but also sum to **25345** too. Moreover, because this B tile pattern is the transpose of pattern A, its tile centers sum equally continuously across the square and as such form an anchor-dot pattern.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
W(37)	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345
1	78	306	534	762	990	1218	40	268	496	724	952	1180	2	230	458	686	914	1142	1333	192	420	648	876	1104	1332	154	382	610	838	1066	1294	116	344	572	800	1028	1256	25345
2	42	270	498	726	954	1182	4	232	460	688	916	1144	1335	194	422	650	878	1106	1297	156	384	612	840	1068	1259	118	346	574	802	1030	1258	80	308	536	764	992	1220	25345
3	6	234	462	690	918	1146	1337	196	424	652	880	1108	1299	158	386	614	842	1070	1261	120	348	576	804	1032	1223	82	310	538	766	994	1185	44	272	500	728	956	1184	25345
4	1339	198	426	654	882	1110	1301	160	388	616	844	1072	1263	122	360	578	806	1034	1225	84	312	540	768	996	1187	46	274	502	730	958	1149	8	236	464	692	920	1111	25345
5	1303	162	390	618	846	1037	1265	124	352	580	808	1036	1227	86	314	542	770	998	1189	48	276	504	732	960	1151	10	238	466	694	922	1113	1341	200	428	656	884	1075	25345
6	1267	126	354	582	810	1001	1229	88	316	544	772	963	1191	50	278	506	734	962	1153	12	240	468	696	924	1115	1343	202	430	658	886	1077	1305	164	392	620	848	1039	25345
7	1231	90	318	546	774	965	1193	52	280	508	736	927	1155	14	242	470	698	889	1117	1345	204	432	660	888	1079	1307	166	394	622	850	1041	1269	128	356	584	812	1003	25345
8	1195	54	282	510	738	929	1157	16	244	472	700	891	1119	1347	206	434	662	853	1081	1309	168	396	624	815	1043	1271	130	358	586	814	1005	1233	92	320	548	776	967	25345
9	1159	18	246	474	702	893	1121	1349	208	436	664	855	1083	1311	170	398	626	817	1045	1273	132	360	588	779	1007	1235	94	322	550	741	969	1197	56	284	512	740	931	25345
10	1123	1351	210	438	666	857	1085	1313	172	400	628	819	1047	1275	134	362	590	781	1009	1237	96	324	552	743	971	1199	58	286	514	705	933	1161	20	248	476	667	895	25345
11	1087	1315	174	402	593	821	1049	1277	136	364	592	783	1011	1239	98	326	554	745	973	1201	60	288	516	707	935	1163	22	250	478	669	897	1125	1353	212	440	631	859	25345
12	1051	1279	138	366	557	785	1013	1241	100	328	519	747	975	1203	62	290	518	709	937	1165	24	252	480	671	899	1127	1355	214	442	633	861	1089	1317	176	404	595	823	25345
13	1015	1243	102	330	521	749	977	1205	64	292	483	711	939	1167	26	254	445	673	901	1129	1357	216	444	635	863	1091	1319	178	406	597	825	1053	1281	140	368	559	787	25345
14	979	1207	66	294	485	713	941	1169	28	256	447	675	903	1131	1359	218	409	637	865	1093	1321	180	371	599	827	1055	1283	142	370	561	789	1017	1245	104	332	523	751	25345
15	943	1171	30	258	449	677	905	1133	1361	220	411	639	867	1095	1323	182	373	601	829	1057	1285	144	335	563	791	1019	1247	106	297	525	753	981	1209	68	296	487	715	25345
16	907	1135	1363	222	413	641	869	1097	1325	184	375	603	831	1059	1287	146	337	565	793	1021	1249	108	299	527	755	983	1211	70	261	489	717	945	1173	32	223	451	679	25345
17	871	1099	1327	149	377	605	833	1061	1289	148	339	567	795	1023	1251	110	301	529	757	985	1213	72	263	491	719	947	1175	34	225	453	681	909	1137	1365	187	415	643	25345
18	835	1063	1291	113	341	569	797	1025	1253	75	303	531	759	987	1215	74	265	493	721	949	1177	36	227	455	683	911	1139	1367	189	417	645	873	1101	1329	151	379	607	25345
19	799	1027	1255	77	305	533	761	989	1217	39	267	495	723	951	1179	1	229	457	685	913	1141	1369	191	419	647	875	1103	1331	153	381	609	837	1065	1293	115	343	571	25345
20	763	991	1219	41	269	497	725	953	1181	3	231	459	687	915	1143	1334	193	421	649	877	1105	1296	155	383	611	839	1067	1295	117	345	573	801	1029	1257	79	307	535	25345
21	727	955	1183	5	233	461	689	917	1145	1336	195	423	651	879	1107	1298	157	385	613	841	1069	1260	119	347	575	803	1031	1222	81	309	537	765	993	1221	43	271	499	25345
22	691	919	1147	1338	197	425	653	881	1109	1300	159	387	615	843	1071	1262	121	349	577	805	1033	1224	83	311	539	767	995	1186	45	273	501	729	957	1148	7	235	463	25345
23	655	883	1074	1302	161	389	617	845	1073	1264	123	351	579	807	1035	1226	85	313	541	769	997	1188	47	275	503	731	959	1150	9	237	465	693	921	1112	1340	199	427	25345
24	619	847	1038	1266	125	353	581	809	1000	1228	87	315	543	771	999	1190	49	277	505	733	961	1152	11	239	467	695	923	1114	1342	201	429	657	885	1076	1304	163	391	25345
25	583	811	1002	1230	89	317	545	773	964	1192	51	279	507	735	926	1154	13	241	469	697	925	1116	1344	203	431	659	887	1078	1306	165	393	621	849	1040	1268	127	355	25345
26	547	775	966	1194	53	281	509	737	928	1156	15	243	471	699	890	1118	1346	205	433	661	852	1080	1308	167	395	623	851	1042	1270	129	357	585	813	1004	1232	91	319	25345
27	511	739	930	1158	17	245	473	701	892	1120	1348	207	435	663	854	1082	1310	169	397	625	816	1044	1272	131	359	587	778	1006	1234	93	321	549	777	968	1196	55	283	25345
28	475	703	894	1122	1350	209	437	665	856	1084	1312	171	399	627	818	1046	1274	133	361	589	780	1008	1236	95	323	551	742	970	1198	57	285	513	704	932	1160	19	247	25345
29	439	630	858	1086	1314	173	401	629	820	1048	1276	135	363	591	782	1010	1238	97	325	553	744	972	1200	59	287	515	706	934	1162	21	249	477	668	896	1124	1352	211	25345
30	403	594	822	1050	1278	137	365	556	784	1012	1240	99	327	555	746	974	1202	61	289	517	708	936	1164	23	251	479	670	898	1126	1354	213	441	632	860	1088	1316	175	25345
31	367	558	786	1014	1242	101	329	520	748	976	1204	63	291	482	710	938	1166	25	253	481	672	900	1128	1356	215	443	634	862	1090	1318	177	405	596	824	1052	1280	139	25345
32	331	522	750	978	1206	65	293	484	712	940	1168	27	255	446	674	902	1130	1358	217	408	636	864	1092	1320	179	407	598	826	1054	1282	141	369	560	788	1016	1244	103	25345
33	295	486	714	942	1170	29	257	448	676	904	1132	1360	219	410	638	866	1094	1322	181	372	600	828	1056	1284	143	334	562	790	1018	1246	105	333	524	752	980	1208	67	25345
34	259	450	678	906	1134	1362	221	412	640	868	1096	1324	183	374	602	830	1058	1286	145	336	564	792	1020	1248	107	298	526	754	982	1210	69	260	488	716	944	1172	31	25345
35	186	414	642	870	1098	1326	185	376	604	832	1060	1288	147	338	566	794	1022	1250	109	300	528	756	984	1212	71	262	490	718	946	1174	33	224	452	680	908	1136	1364	25345
36	150	378	606	834	1062	1290	112	340	568	796	1024	1252	111	302	530	758	986	1214	73	264	492	720	948	1176	35	226	454	682	910	1138	1366	188	416	644	872	1100	1328	25345
37	114	342	570	798	1026	1254	76	304	532	760	988	1216	38	266	494	722	950	1178	37	228	456	684	912	1140	1368	190	418	646	874	1102	1330	152	380	608	836	1064	1292	25345
	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	
		25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	25345	

Program 2

Note 1: Backing up, the size-5 square was determined to have no equal-summing dot patterns at all that were not all located in its central row or central column. The centers of the totally double-symmetric tiles, "**+**" and "**x**" did not sum equally nor did their transposes. So these anchor dot patterns that arise in Class-1 squares begin with size-7 and continue on from there.

Note 2: Compared with the larger size squares, the size-7 possesses only **2n** distinct anchor-dot patterns as opposed to **4n**. That stems from the anchor-dot's rare coincidence with the points of equality between dual tables to be described later in Program 3.

Note 3: Each size Class-1 square for **n ≥11** possesses **4n** equal-summing anchor-dot patterns (**2n** from the transposed complementary tile centers, and another 2**n** from the latter pattern's horizontal flip) and **2n** equal-summing tile patterns for a grand total of **6n** non-linear equal-summing patterns. So it is clear that for Class-1 squares from size-11 onward that these non-linear patterns yield more equal characteristic summations than all the possible linear summations combined (**4n**). So prior linear math on so called "magic squares" is indeed very rudimentary; they never got past **2n+2** equal summations, i.e. **n** rows, **n** columns and the **2** main diagonals.

Class-3 Anchor-dot Patterns

Size-15

Here is the size-15 square with tiling patterns whose centers sum continuously to the characteristic number. One pattern is **(3; -2)** and the other is **(2; 3)**, the transpose of the first. **The tile patterns themselves do not sum equally.**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
W(15)	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	108	221	64	77	170	33	146	214	2	95	183	71	139	152	1695
2	111	204	42	85	198	36	129	192	10	123	186	54	117	160	48	1695
3	207	100	13	176	144	57	175	88	26	219	132	25	163	101	69	1695
4	153	41	134	222	15	78	191	59	147	165	3	116	209	72	90	1695
5	49	137	180	18	106	199	62	105	168	31	124	212	30	93	181	1695
6	70	83	171	39	127	220	8	96	189	52	145	158	21	114	202	1695
7	86	179	67	135	173	11	104	217	60	98	161	29	142	210	23	1695
8	182	150	38	151	119	32	225	113	1	194	107	75	188	76	44	1695
9	203	16	84	197	65	128	166	9	122	215	53	91	159	47	140	1695
10	24	112	205	68	81	174	37	130	218	6	99	187	55	143	156	1695
11	45	133	196	14	102	195	58	121	164	27	120	208	46	89	177	1695
12	136	154	17	110	223	61	79	167	35	148	211	4	92	185	73	1695
13	157	125	63	201	94	7	200	138	51	169	82	50	213	126	19	1695
14	178	66	109	172	40	103	216	34	97	190	28	141	184	22	115	1695
15	74	87	155	43	131	224	12	80	193	56	149	162	5	118	206	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
W(15)	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	108	221	64	77	170	33	146	214	2	95	183	71	139	152	1695
2	111	204	42	85	198	36	129	192	10	123	186	54	117	160	48	1695
3	207	100	13	176	144	57	175	88	26	219	132	25	163	101	69	1695
4	153	41	134	222	15	78	191	59	147	165	3	116	209	72	90	1695
5	49	137	180	18	106	199	62	105	168	31	124	212	30	93	181	1695
6	70	83	171	39	127	220	8	96	189	52	145	158	21	114	202	1695
7	86	179	67	135	173	11	104	217	60	98	161	29	142	210	23	1695
8	182	150	38	151	119	32	225	113	1	194	107	75	188	76	44	1695
9	203	16	84	197	65	128	166	9	122	215	53	91	159	47	140	1695
10	24	112	205	68	81	174	37	130	218	6	99	187	55	143	156	1695
11	45	133	196	14	102	195	58	121	164	27	120	208	46	89	177	1695
12	136	154	17	110	223	61	79	167	35	148	211	4	92	185	73	1695
13	157	125	63	201	94	7	200	138	51	169	82	50	213	126	19	1695
14	178	66	109	172	40	103	216	34	97	190	28	141	184	22	115	1695
15	74	87	155	43	131	224	12	80	193	56	149	162	5	118	206	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

Program 2

Size-33

The centers of both vertical and horizontal rectangular patterns (offset by 3 rows or 3 columns) here identify an anchor-dot pattern **(3;-3)** that sums to the characteristic number **17985** continuously. **The rectangular tiles do not sum to the characteristic number.**

33x33	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	
1	989	400	306	927	470	2	876	408	314	946	720	263	763	669	201	833	365	150	771	677	220	1083	626	37	1032	564	107	728	513	45	1040	583	357	17985
2	145	1019	551	94	1089	500	43	1027	570	102	855	508	293	914	457	363	883	406	301	933	465	129	871	656	188	820	726	137	769	664	207	828	492	17985
3	764	549	81	1076	487	272	1014	557	89	1084	385	38	912	444	350	850	635	288	920	452	358	748	401	186	807	713	124	998	651	194	815	721	22	17985
4	657	68	1063	595	17	770	544	87	1071	493	267	1020	431	337	958	380	44	907	450	345	856	630	294	794	700	232	743	407	181	813	708	130	993	17985
5	187	1050	593	4	999	531	74	1058	601	12	886	550	324	956	367	273	894	437	332	964	375	160	913	687	230	730	636	168	800	695	238	738	523	17985
6	89	580	838	260	518	1029	330	588	736	268	416	795	217	475	986	155	666	1056	225	373	994	53	432	943	112	623	881	303	693	951	10	631	779	17985
7	688	110	973	516	48	801	575	118	981	524	298	1051	473	247	879	411	75	938	481	255	887	661	325	836	610	153	774	438	212	844	618	161	1024	17985
8	218	1081	503	35	1030	562	105	979	511	54	917	581	355	868	398	304	925	468	253	874	417	191	944	718	140	761	667	199	831	616	148	780	554	17985
9	826	490	33	1017	560	334	1087	498	41	1025	447	100	853	396	291	923	697	361	861	404	299	810	463	127	759	654	197	1060	724	135	787	662	84	17985
10	719	20	1004	547	79	832	485	28	1023	555	340	1082	383	278	910	442	106	848	391	297	918	703	356	746	641	184	805	469	122	754	660	192	1066	17985
11	854	628	171	803	698	241	741	647	179	811	585	128	991	534	77	1061	604	15	1010	542	85	948	491	265	897	440	335	967	378	284	905	448	222	17985
12	747	521	84	1048	591	244	997	529	72	1067	478	21	884	427	322	954	607	271	892	435	341	841	384	158	790	685	228	970	634	166	798	704	115	17985
13	266	777	672	215	847	379	164	785	691	223	976	629	51	1035	578	121	742	527	59	1054	586	250	992	414	309	941	484	16	890	422	328	949	613	17985
14	885	670	202	834	608	151	772	678	210	842	506	159	1033	565	108	971	514	46	1041	573	116	869	522	307	928	471	245	877	409	315	936	479	143	17985
15	415	189	821	716	138	1012	665	208	829	614	25	778	552	95	1079	501	286	1028	571	103	977	388	52	915	458	353	864	649	302	934	466	251	751	17985
16	308	808	714	125	757	410	195	816	722	133	1007	671	82	1077	488	31	773	558	90	1085	496	281	1034	445	351	851	394	47	921	453	359	859	644	17985
17	190	701	959	18	639	908	88	709	857	26	537	916	338	596	744	276	545	814	346	494	752	174	553	1064	233	381	1002	182	451	1072	131	389	900	17985
18	446	231	731	637	169	1043	696	239	739	645	56	809	594	5	1000	532	317	1059	602	13	1008	419	83	957	368	274	895	680	333	965	376	282	782	17985
19	339	839	624	158	788	441	226	737	632	175	1038	702	113	987	519	62	804	589	11	995	538	312	1065	476	261	882	425	78	952	374	269	901	675	17985
20	947	611	154	775	681	213	845	619	162	783	568	221	974	517	49	1044	576	119	982	525	57	931	584	248	880	412	318	939	482	256	888	420	205	17985
21	477	141	762	668	200	1074	606	149	781	676	98	840	504	36	1031	563	348	969	512	55	1039	461	114	867	399	305	926	711	243	875	418	313	824	17985
22	975	386	292	924	456	120	862	405	300	932	706	249	749	655	198	819	483	136	768	663	206	1069	612	23	1018	561	93	846	499	42	1026	569	343	17985
23	868	642	185	806	712	123	755	650	193	825	599	142	1005	548	80	1075	486	29	1013	556	99	962	505	279	911	443	349	849	392	287	919	462	236	17985
24	24	898	430	336	968	621	285	906	449	344	734	387	172	793	699	242	984	648	180	812	707	8	750	535	67	1062	605	258	1011	543	88	1070	371	17985
25	1006	428	323	955	366	30	893	436	331	963	627	280	791	686	229	729	393	167	799	694	237	990	643	65	1049	592	3	756	530	73	1057	600	264	17985
26	536	310	942	474	259	891	423	329	950	372	146	899	673	216	837	622	165	786	692	224	735	509	173	1036	579	111	985	528	80	1055	587	9	872	17985
27	86	929	472	246	878	652	316	937	480	254	765	429	203	835	609	152	1015	679	211	843	617	39	792	566	109	972	515	289	1042	574	117	980	402	17985
28	311	459	1080	139	397	787	209	467	978	147	658	1037	96	717	865	34	424	935	104	615	873	295	674	822	354	502	760	61	572	830	252	510	1021	17985
29	567	352	852	395	290	922	454	360	860	403	177	930	715	126	758	653	196	817	723	134	766	540	204	1078	489	32	1016	559	91	1086	497	40	903	17985
30	97	960	382	277	909	683	347	858	390	296	796	460	234	745	640	183	1046	710	132	753	659	70	823	597	19	1003	546	320	1073	495	27	1022	433	17985
31	1068	369	275	896	439	92	966	377	283	904	689	342	732	638	170	802	455	240	740	646	178	1052	705	6	1001	533	76	818	603	14	1009	541	326	17985
32	598	262	883	426	321	953	364	270	902	434	219	961	625	157	789	684	227	727	633	176	797	582	235	988	520	63	1047	590	1	996	539	71	945	17985
33	733	507	50	1045	577	362	983	526	58	1053	464	7	870	413	319	940	725	257	889	421	327	827	370	144	776	682	214	1088	620	163	784	690	101	17985
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	

These same dot patterns **(3;-3)** also hold for the size **21** and size **27** squares. It appears that this pattern continues for every Class-3 square of size **n > 15**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	
1	989	400	306	927	470	2	876	408	314	946	720	263	763	669	201	833	365	150	771	677	220	1083	626	37	1032	564	107	728	513	45	1040	583	357	17985
2	145	1019	551	94	1089	500	43	1027	570	102	855	508	293	914	457	363	883	406	301	933	465	129	871	656	188	820	726	137	769	664	207	828	492	17985
3	764	549	81	1076	487	272	1014	557	89	1084	385	38	912	444	350	850	635	288	920	452	358	748	401	186	807	713	124	998	651	194	815	721	22	17985
4	657	68	1063	595	17	770	544	87	1071	493	267	1020	431	337	958	380	44	907	450	345	856	630	294	794	700	232	743	407	181	813	708	130	993	17985
5	187	1050	593	4	999	531	74	1058	601	12	886	550	324	956	367	273	894	437	332	964	375	160	913	687	230	730	636	168	800	695	238	738	523	17985
6	89	580	838	260	518	1029	330	588	736	268	416	795	217	475	986	155	666	1056	225	373	994	53	432	943	112	623	881	303	693	951	10	631	779	17985
7	688	110	973	516	48	801	575	118	981	524	298	1051	473	247	879	411	75	938	481	255	887	661	325	836	610	153	774	438	212	844	618	161	1024	17985
8	218	1081	503	35	1030	562	105	979	511	54	917	581	355	868	398	304	925	468	253	874	417	191	944	718	140	761	667	199	831	616	148	780	554	17985
9	826	490	33	1017	560	334	1087	498	41	1025	447	100	853	396	291	923	697	361	861	404	299	810	463	127	759	654	197	1060	724	135	787	662	84	17985
10	719	20	1004	547	79	832	485	28	1023	555	340	1082	383	278	910	442	106	848	391	297	918	703	356	746	641	184	805	469	122	754	660	192	1066	17985
11	854	628	171	803	698	241	741	647	179	811	585	128	991	534	77	1061	604	15	1010	542	85	948	491	265	897	440	335	967	378	284	905	448	222	17985
12	747	521	84	1048	591	244	997	529	72	1067	478	21	884	427	322	954	607	271	892	435	341	841	384	158	790	685	228	970	634	166	798	704	115	17985
13	266	777	672	215	847	379	164	785	691	223	976	629	51	1035	578	121	742	527	59	1054	586	250	992	414	309	941	484	16	890	422	328	949	613	17985
14	885	670	202	834	608	151	772	678	210	842	506	159	1033	565	108	971	514	46	1041	573	116	869	522	307	928	471	245	877	409	315	936	479	143	17985
15	415	189	821	716	138	1012	665	208	829	614	25	778	552	95	1079	501	286	1028	571	103	977	388	52	915	458	353	864	649	302	934	466	251	751	17985
16	308	808	714	125	757	410	195	816	722	133	1007	671	82	1077	488	31	773	558	90	1085	496	281	1034	445	351	851	394	47	921	453	359	859	644	17985
17	190	701	959	18	639	908	88	709	857	26	537	916	338	596	744	276	545	814	346	494	752	174	553	1064	233	381	1002	182	451	1072	131	389	900	17985
18	446	231	731	637	169	1043	696	239	739	645	56	809	594	5	1000	532	317	1059	602	13	1008	419	83	957	368	274	895	680	333	965	376	282	782	17985
19	339	839	624	158	788	441	226	737	632	175	1038	702	113	987	519	62	804	589	11	995	538	312	1065	476	261	882	425	78	952	374	269	901	675	17985
20	947	611	154	775	681	213	845	619	162	783	568	221	974	517	49	1044	576	119	982	525	57	931	584	248	880	412	318	939	482	256	888	420	205	17985
21	477	141	762	668	200	1074	606	149	781	676	98	840	504	36	1031	563	348	969	512	55	1039	461	114	867	399	305	926	711	243	875	418	313	824	17985
22	975	386	292	924	456	120	862	405	300	932	706	249	749	655	198	819	483	136	768	663	206	1069	612	23	1018	561	93	846	499	42	1026	569	343	17985
23	868	642	185	806	712	123	755	650	193	825	599	142	1005	548	80	1075	486	29	1013	556	99	962	505	279	911	443	349	849	392	287	919	462	236	17985
24	24	898	430	336	968	621	285	906	449	344	734	387	172	793	699	242	984	648	180	812	707	8	750	535	67	1062	605	258	1011	543	88	1070	371	17985
25	1006	428	323	955	366	30	893	436	331	963	627	280	791	686	229	729	393	167	799	694	237	990	643	65	1049	592	3	756	530	73	1057	600	264	17985
26	536	310	942	474	259	891	423	329	950	372	146	899	673	216	837	622	165	786	692	224	735	509	173	1036	579	111	985	528	80	1055	587	9	872	17985
27	86	929	472	246	878	652	316	937	480	254	765	429	203	835	609	152	1015	679	211	843	617	39	792	566	109	972	515	289	1042	574	117	980	402	17985
28	311	459	1080	139	397	787	209	467	978	147	658	1037	96	717	865	34	424	935	104	615	873	295	674	822	354	502	760	61	572	830	252	510	1021	17985
29	567	352	852	395	290	922	454	360	860	403	177	930	715	126	758	653	196	817	723	134	766	540	204	1078	489	32	1016	559	91	1086	497	40	903	17985
30	97	960	382	277	909	683	347	858	390	296	796	460	234	745	640	183	1046	710	132	753	659	70	823	597	19	1003	546	320	1073	495	27	1022	433	17985
31	1068	369	275	896	439	92	966	377	283	904	689	342	732	638	170	802	455	240	740	646	178	1052	705	6	1001	533	76	818	603	14	1009	541	326	17985
32	598	262	883	426	321	953	364	270	902	434	219	961	625	157	789	684	227	727	633	176	797	582	235	988	520	63	1047	590	1	996	539	71	945	17985
33	733	507	50	1045	577	362	983	526	58	1053	464	7	870	413	319	940	725	257	889	421	327	827	370	144	776	682	214	1088	620	163	784	690	101	17985
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	

Program 2

Here is another anchor-dot pattern for the size-33 square. They are both of the same pattern **(3;+3)**. So the transpose of the rectangular pattern on the prior page is amazingly just the horizontal flip of these dot patterns. All the dots relative to the anchor-dots sum continuously to the characteristic number **17985**.

The staircase tiles do not sum to the characteristic number.

Program 2

Class-5 Size-35

Here is the size-35 anchor-dot pattern composed of the centers of **5x7** and **7x5** rectangular blocks. All the centers sum to the characteristic number continuously. The dot patterns are **(5; 2)** and **(2; -5)**. Their horizontal flips **(5; 2)** and **(2; +5)** do too. **For Class-5 squares, these rectangular tiles do sum to the characteristic number because the square is both 5x7 and 7x5 continuously modular.**

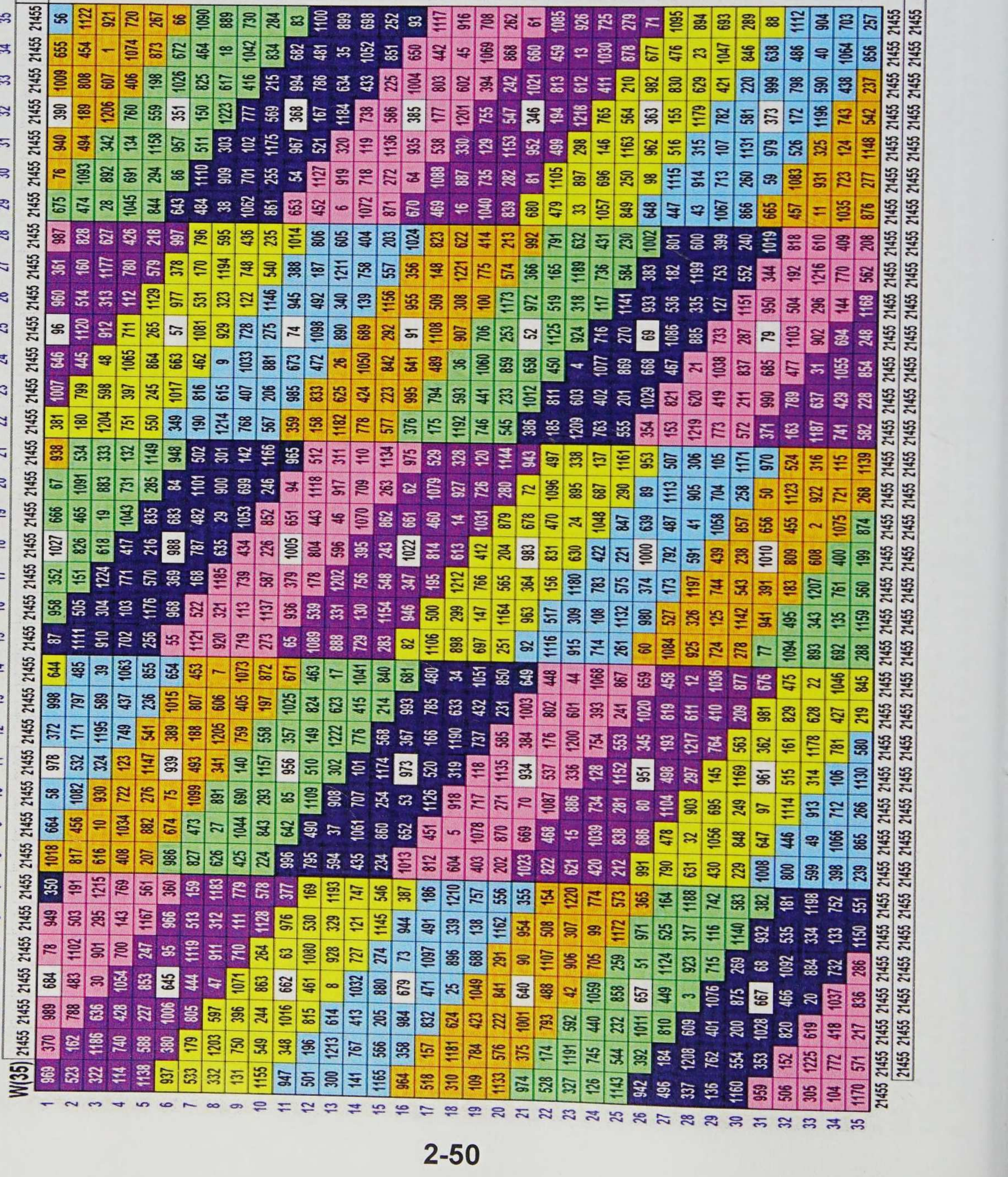
35x35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
1	665	768	871	974	1077	1180	58	161	264	367	470	573	576	779	882	985	1088	1191	104	207	310	413	516	619	722	825	928	1031	1134	12	115	218	321	424	527	21455
2	596	734	837	940	1043	1146	24	127	230	333	436	539	642	745	848	951	1054	1157	70	173	276	379	482	585	688	791	894	997	1100	1203	81	184	287	390	493	21455
3	562	700	803	906	1009	1112	1215	93	196	299	402	505	608	711	814	917	1020	1123	1	139	242	345	448	551	654	757	860	963	1066	1169	47	150	253	356	459	21455
4	528	631	769	872	975	1078	1181	59	162	265	368	471	574	677	780	883	986	1089	1192	105	208	311	414	517	620	723	826	929	1032	1135	13	116	219	322	425	21455
5	494	597	735	838	941	1044	1147	25	128	231	334	437	540	643	746	849	952	1055	1158	36	174	277	380	483	586	689	792	895	998	1101	1204	82	185	288	391	21455
6	460	563	666	804	907	1010	1113	1216	94	197	300	403	506	609	712	815	918	1021	1124	2	140	243	346	449	552	655	758	861	964	1067	1170	48	151	254	357	21455
7	426	529	632	770	873	976	1079	1182	60	163	266	369	472	575	678	781	884	987	1090	1193	71	209	312	415	518	621	724	827	930	1033	1136	14	117	220	323	21455
8	392	495	598	701	839	942	1045	1148	26	129	232	335	438	541	644	747	850	953	1056	1159	37	175	278	381	484	587	690	793	896	999	1102	1205	83	186	289	21455
9	358	461	564	667	805	908	1011	1114	1217	95	198	301	404	507	610	713	816	919	1022	1125	3	106	244	347	450	553	656	759	862	965	1068	1171	49	152	255	21455
10	324	427	530	633	736	874	977	1080	1183	61	164	267	370	473	576	679	782	885	988	1091	1194	72	210	313	416	519	622	725	828	931	1034	1137	15	118	221	21455
11	290	393	496	599	702	840	943	1046	1149	27	130	233	336	439	542	645	748	851	954	1057	1160	38	141	279	382	485	588	691	794	897	1000	1103	1206	84	187	21455
12	256	359	462	565	668	771	909	1012	1115	1218	96	199	302	405	508	611	714	817	920	1023	1126	4	107	245	348	451	554	657	760	863	966	1069	1172	50	153	21455
13	222	325	428	531	634	737	875	978	1081	1184	62	165	268	371	474	577	680	783	886	989	1092	1195	73	176	314	417	520	623	726	829	932	1035	1138	16	119	21455
14	188	291	394	497	600	703	806	944	1047	1150	28	131	234	337	440	543	646	749	852	955	1058	1161	39	142	280	383	486	589	692	795	898	1001	1104	1207	85	21455
15	154	257	360	463	566	669	772	910	1013	1116	1219	97	200	303	406	509	612	715	818	921	1024	1127	5	108	211	349	452	555	658	761	864	967	1070	1173	51	21455
16	120	223	326	429	532	635	738	841	979	1082	1185	63	166	269	372	475	578	681	784	887	990	1093	1196	74	177	315	418	521	624	727	830	933	1036	1139	17	21455
17	86	189	292	395	498	601	704	807	945	1048	1151	29	132	235	338	441	544	647	750	853	956	1059	1162	40	143	246	384	487	590	693	796	899	1002	1105	1208	21455
18	52	155	258	361	464	567	670	773	876	1014	1117	1220	98	201	304	407	510	613	716	819	922	1025	1128	6	109	212	350	453	556	659	762	865	968	1071	1174	21455
19	18	121	224	327	430	533	636	739	842	980	1083	1186	64	167	270	373	476	579	682	785	888	991	1094	1197	75	178	281	419	522	625	728	831	934	1037	1140	21455
20	1209	87	190	293	396	499	602	705	808	911	1049	1152	30	133	236	339	442	545	648	751	854	957	1060	1163	41	144	247	385	488	591	694	797	900	1003	1106	21455
21	1175	53	156	259	362	465	568	671	774	877	1015	1118	1221	99	202	305	408	511	614	717	820	923	1026	1129	7	110	213	316	454	557	660	763	866	969	1072	21455
22	1141	19	122	225	328	431	534	637	740	843	946	1084	1187	65	168	271	374	477	580	683	786	889	992	1095	1198	76	179	282	420	523	626	729	832	935	1038	21455
23	1107	1210	88	191	294	397	500	603	706	809	912	1050	1153	31	134	237	340	443	546	649	752	855	958	1061	1164	42	145	248	351	489	592	695	798	901	1004	21455
24	1073	1176	54	157	260	363	466	569	672	775	878	981	1119	1222	100	203	306	409	512	615	718	821	924	1027	1130	8	111	214	317	455	558	661	764	867	970	21455
25	1039	1142	20	123	226	329	432	535	638	741	844	947	1085	1188	66	169	272	375	478	581	684	787	890	993	1096	1199	77	180	283	386	524	627	730	833	936	21455
26	1005	1108	1211	89	192	295	398	501	604	707	810	913	1016	1154	32	135	238	341	444	547	650	753	856	959	1062	1165	43	146	249	352	490	593	696	799	902	21455
27	971	1074	1177	55	158	261	364	467	570	673	776	879	982	1120	1223	101	204	307	410	513	616	719	822	925	1028	1131	9	112	215	318	421	559	662	765	868	21455
28	937	1040	1143	21	124	227	330	433	536	639	742	845	948	1051	1189	67	170	273	376	479	582	685	788	891	994	1097	1200	78	181	284	387	525	628	731	834	21455
29	903	1006	1109	1212	90	193	296	399	502	605	708	811	914	1017	1155	33	136	239	342	445	548	651	754	857	960	1063	1166	44	147	250	353	456	594	697	800	21455
30	869	972	1075	1178	56	159	262	365	468	571	674	777	880	983	1086	1224	102	205	308	411	514	617	720	823	926	1029	1132	10	113	216	319	422	560	663	766	21455
31	835	938	1041	1144	22	125	228	331	434	537	640	743	846	949	1052	1190	68	171	274	377	480	583	686	789	892	995	1098	1201	79	182	285	388	491	629	732	21455
32	801	904	1007	1110	1213	91	194	297	400	503	606	709	812	915	1018	1121	34	137	240	343	446	549	652	755	858	961	1064	1167	45	148	251	354	457	595	698	21455
33	767	870	973	1076	1179	57	160	263	366	469	572	675	778	881	984	1087	1225	103	206	309	412	515	618	721	824	927	1030	1133	11	114	217	320	423	526	664	21455
34	733	836	939	1042	1145	23	126	229	332	435	538	641	744	847	950	1053	1156	69	172	275	378	481	584	687	790	893	996	1099	1202	80	183	286	389	492	630	21455
35	699	802	905	1008	1111	1214	92	195	298	401	504	607	710	813	916	1019	1122	35	138	241	344	447	550	653	756	859	962	1065	1168	46	149	252	355	458	561	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

35x35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
1	665	768	871	974	1077	1180	58	161	264	367	470	573	676	779	882	985	1088	1191	104	207	310	413	516	619	722	825	928	1031	1134	12	115	218	321	424	527	21455
2	596	734	837	940	1043	1146	24	127	230	333	436	539	642	745	848	951	1054	1157	70	173	276	379	482	585	688	791	894	997	1100	1203	81	184	287	390	493	21455
3	562	700	803	906	1009	1112	1215	93	196	299	402	505	608	711	814	917	1020	1123	1	139	242	345	448	551	654	757	860	963	1066	1169	47	150	253	356	459	21455
4	528	631	769	872	975	1078	1181	59	162	265	368	471	574	677	780	883	986	1089	1192	105	208	311	414	517	620	723	826	929	1032	1135	13	116	219	322	425	21455
5	494	597	735	838	941	1044	1147	25	128	231	334	437	540	643	746	849	952	1055	1158	36	174	277	380	483	586	689	792	895	998	1101	1204	82	185	288	391	21455
6	460	563	666	804	907	1010	1113	1216	94	197	300	403	506	609	712	815	918	1021	1124	2	140	243	346	449	552	655	758	861	964	1067	1170	48	151	254	357	21455
7	426	529	632	770	873	976	1079	1182	60	163	266	369	472	575	678	781	884	987	1090	1193	71	209	312	415	518	621	724	827	930	1033	1136	14	117	220	323	21455
8	392	495	598	701	839	942	1045	1148	26	129	232	335	438	541	644	747	850	953	1056	1159	37	175	278	381	484	587	690	793	896	999	1102	1205	83	186	289	21455
9	358	461	564	667	805	908	1011	1114	1217	95	198	301	404	507	610	713	816	919	1022	1125	3	106	244	347	450	553	656	759	862	965	1068	1171	49	152	255	21455
10	324	427	530	633	736	874	977	1080	1183	61	164	267	370	473	576	679	782	885	988	1091	1194	72	210	313	416	519	622	725	828	931	1034	1137	15	118	221	21455
11	290	393	496	599	702	840	943	1046	1149	27	130	233	336	439	542	645	748	851	954	1057	1160	38	141	279	382	485	588	691	794	897	1000	1103	1206	84	187	21455
12	256	359	462	565	668	771	909	1012	1115	1218	96	199	302	405	508	611	714	817	920	1023	1126	4	107	245	348	451	554	657	760	863	966	1069	1172	50	153	21455
13	222	325	428	531	634	737	875	978	1081	1184	62	165	268	371	474	577	680	783	886	989	1092	1195	73	176	314	417	520	623	726	829	932	1035	1138	16	119	21455
14	188	291	394	497	600	703	806	944	1047	1150	28	131	234	337	440	543	646	749	852	955	1058	1161	39	142	280	383	486	589	692	795	898	1001	1104	1207	85	21455
15	154	257	360	463	566	669	772	910	1013	1116	1219	97	200	303	406	509	612	715	818	921	1024	1127	5	108	211	349	452	555	658	761	864	967	1070	1173	51	21455
16	120	223	326	429	532	635	738	841	979	1082	1185	63	166	269	372	475	578	681	784	887	990	1093	1196	74	177	315	418	521	624	727	830	933	1036	1139	17	21455
17	86	189	292	395	498	601	704	807	945	1048	1151	29	132	235	338	441	544	647	750	853	956	1059	1162	40	143	246	384	487	590	693	796	899	1002	1105	1208	21455
18	52	155	258	361	464	567	670	773	876	1014	1117	1220	98	201	304	407	510	613	716	819	922	1025	1128	6	109	212	350	453	556	659	762	865	968	1071	1174	21455
19	18	121	224	327	430	533	636	739	842	980	1083	1186	64	167	270	373	476	579	682	785	888	991	1094	1197	75	178	281	419	522	625	728	831	934	1037	1140	21455
20	1209	87	190	293	396	499	602	705	808	911	1049	1152	30	133	236	339	442	545	648	751	854	957	1060	1163	41	144	247	385	488	591	694	797	900	1003	1106	21455
21	1175	53	156	259	362	465	568	671	774	877	1015	1118	1221	99	202	305	408	511	614	717	820	923	1026	1129	7	110	213	316	454	557	660	763	866	969	1072	21455
22	1141	19	122	225	328	431	534	637	740	843	946	1084	1187	65	168	271	374	477	580	683	786	889	992	1095	1198	76	179	282	420	523	626	729	832	935	1038	21455
23	1107	1210	88	191	294	397	500	603	706	809	912	1050	1153	31	134	237	340	443	546	649	752	855	958	1061	1164	42	145	248	351	489	592	695	798	901	1004	21455
24	1073	1176	54	157	260	363	466	569	672	775	878	981	1119	1222	100	203	306	409	512	615	718	821	924	1027	1130	8	111	214	317	455	558	661	764	867	970	21455
25	1039	1142	20	123	226	329	432	535	638	741	844	947	1085	1188	66	169	272	375	478	581	684	787	890	993	1096	1199	77	180	283	386	524	627	730	833	936	21455
26	1005	1108	1211	89	192	295	398	501	604	707	810	913	1016	1154	32	135	238	341	444	547	650	753	856	959	1062	1165	43	146	249	352	490	593	696	799	902	21455
27	971	1074	1177	55	158	261	364	467	570	673	776	879	982	1120	1223	101	204	307	410	513	616	719	822	925	1028	1131	9	112	215	318	421	559	662	765	868	21455
28	937	1040	1143	21	124	227	330	433	536	639	742	845	948	1051	1189	67	170	273	376	479	582	685	788	891	994	1097	1200	78	181	284	387	525	628	731	834	21455
29	903	1006	1109	1212	90	193	296	399	502	605	708	811	914	1017	1155	33	136	239	342	445	548	651	754	857	960	1063	1166	44	147	250	353	456	594	697	800	21455
30	869	972	1075	1178	56	159	262	365	468	571	674	777	880	983	1086	1224	102	205	308	411	514	617	720	823	926	1029	1132	10	113	216	319	422	560	663	766	21455
31	835	938	1041	1144	22	125	228	331	434	537	640	743	846	949	1052	1190	68	171	274	377	480	583	686	789	892	995	1098	1201	79	182	285	388	491	629	732	21455
32	801	904	1007	1110	1213	91	194	297	400	503	606	709	812	915	1018	1121	34	137	240	343	446	549	652	755	858	961	1064	1167	45	148	251	354	457	595	698	21455
33	767	870	973	1076	1179	57	160	263	366	469	572	675	778	881	984	1087	1225	103	206	309	412	515	618	721	824	927	1030	1133	11	114	217	320	423	526	664	21455
34	733	836	939	1042	1145	23	126	229	332	435	538	641	744	847	950	1053	1156	69	172	275	378	481	584	687	790	893	996	1099	1202	80	183	286	389	492	630	21455
35	699	802	905	1008	1111	1214	92	195	298	401	504	607	710	813	916	1019	1122	35	138	241	344	447	550	653	756	859	962	1065	1168	46	149	252	355	458	561	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 2

A different Size-35 square with a different Dot Distribution Pattern

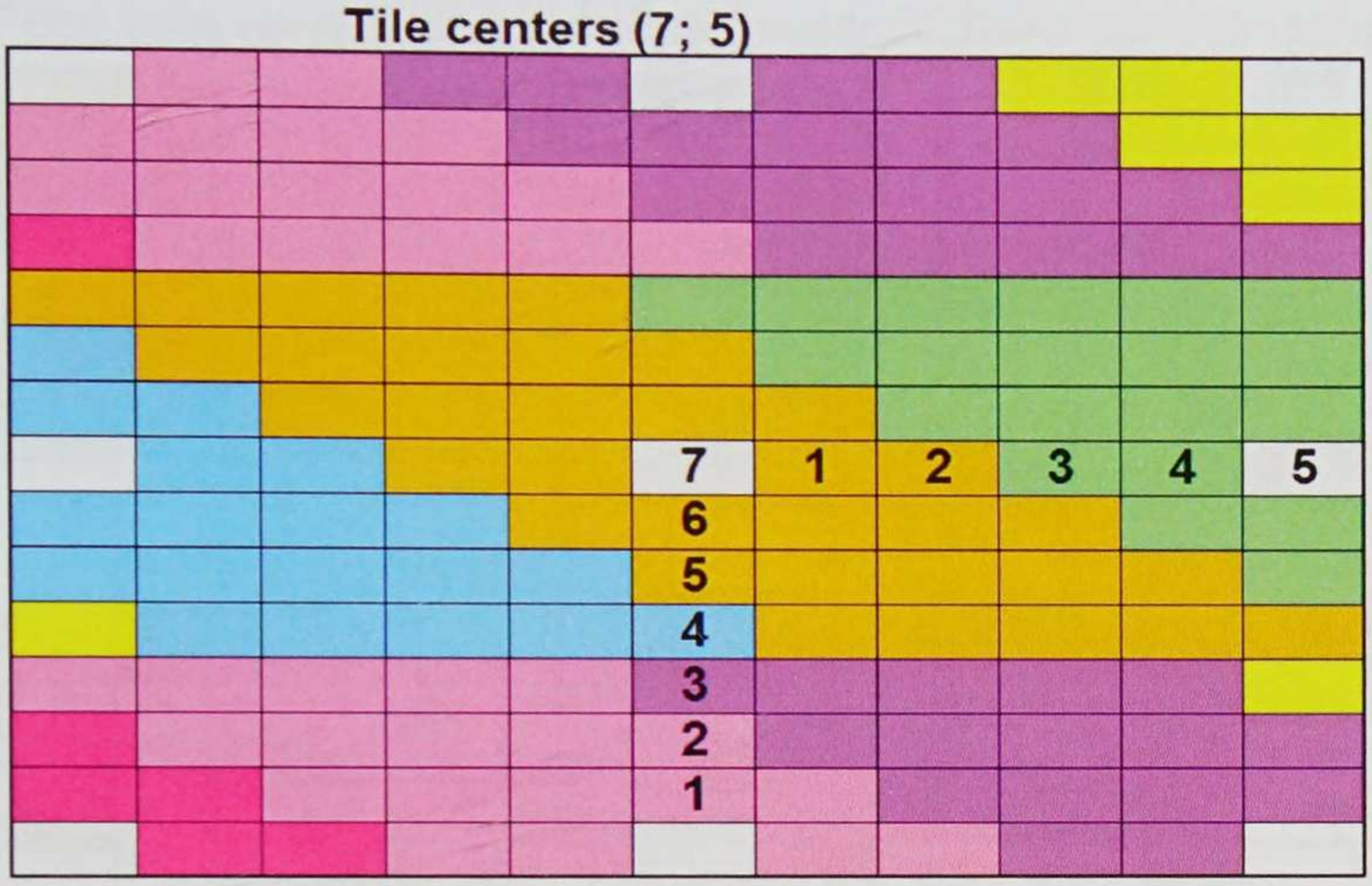
Here is a different size-35 square. (note the location of the two extreme numbers **1** and **1225** near the lower-left and upper-righthand corners). Its tiling patterns are staircase structures running diagonally in both major and minor directions. **Here too, the tiles sum continuously to the characteristic number.** Their centers function as continuous anchor-dot patterns that do so too. There's much much more (next page).



Equal-summing Dot Patterns on the Size-35 Square

Shown here and on the next page are the tile patterns and their equal summing patterns of tile centers as these tiles are dragged across the square with wrapping imposed. The centers of the two tile patterns on this page are continuously equal-summing everywhere.

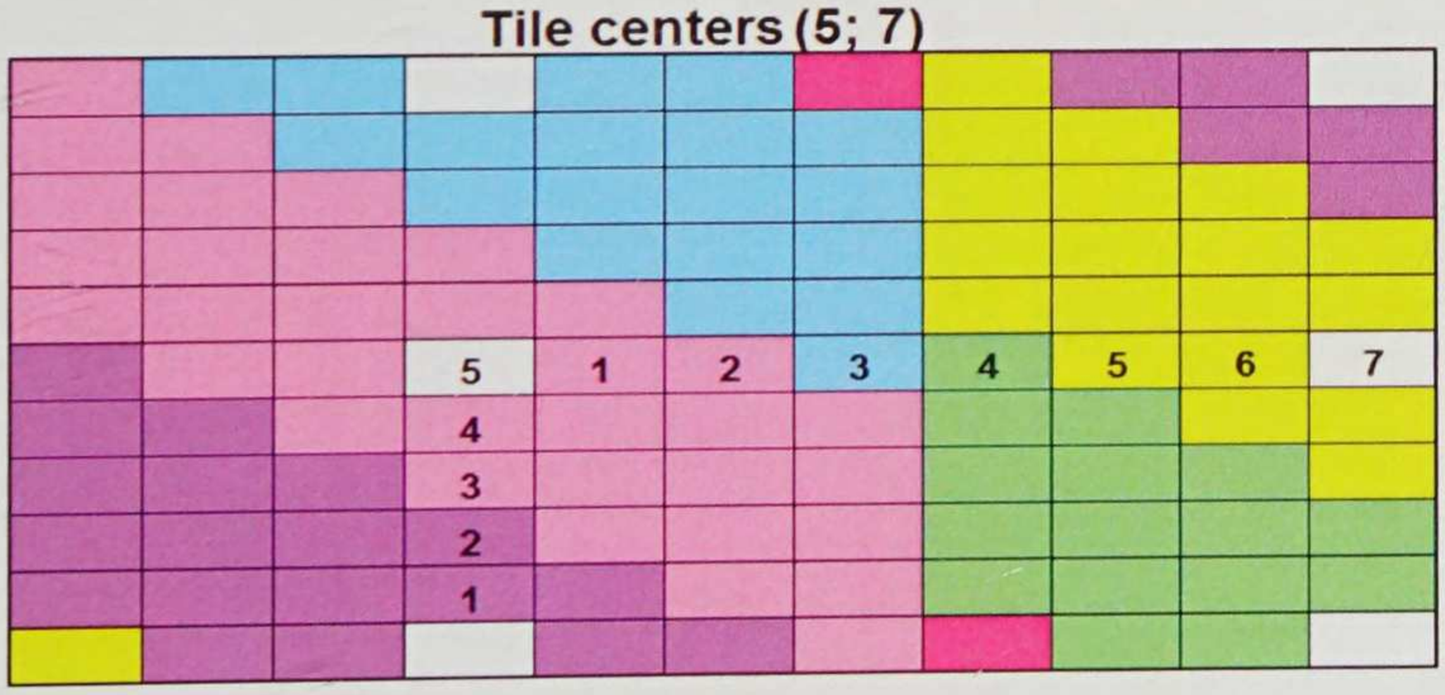
Tile centers (7; 5)



Dot patterns Continuous

21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Tile centers (5; 7)



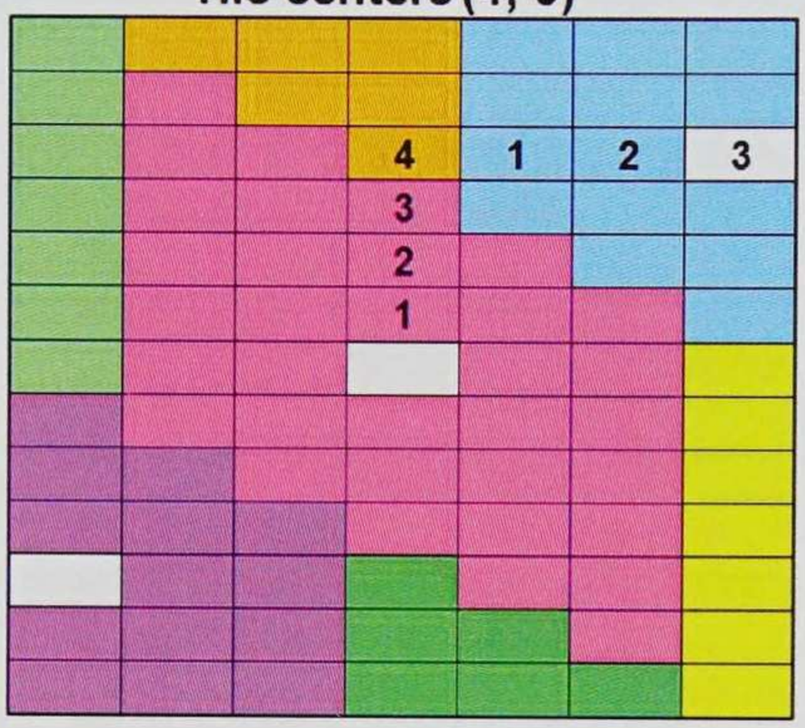
Dot patterns Continuous

21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 2

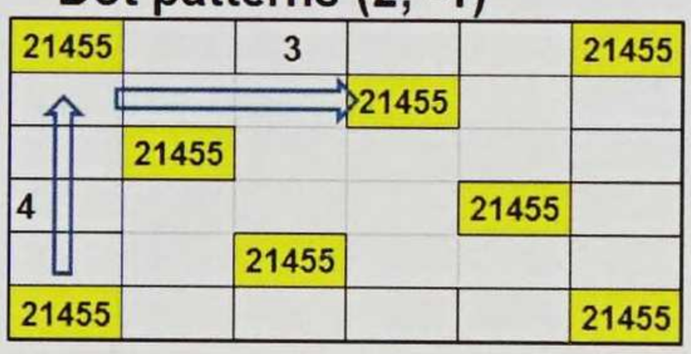
Although all the tiles themselves form a continuous equal-summing pattern, the centers of the tile patterns' shown here are not continuously equal-summing across the square; they are only periodically equal-summing. It is these periodic patterns that are shown below the tiling pattern they are related to. Note that the dot pattern for both its original tile pattern and its 90° rotated pattern are the same. This dot pattern gets reversed horizontally on a vertical or horizontal flip of each pattern (re: top and bottom tile patterns in both left and right cases).

Tile centers (4; 3)

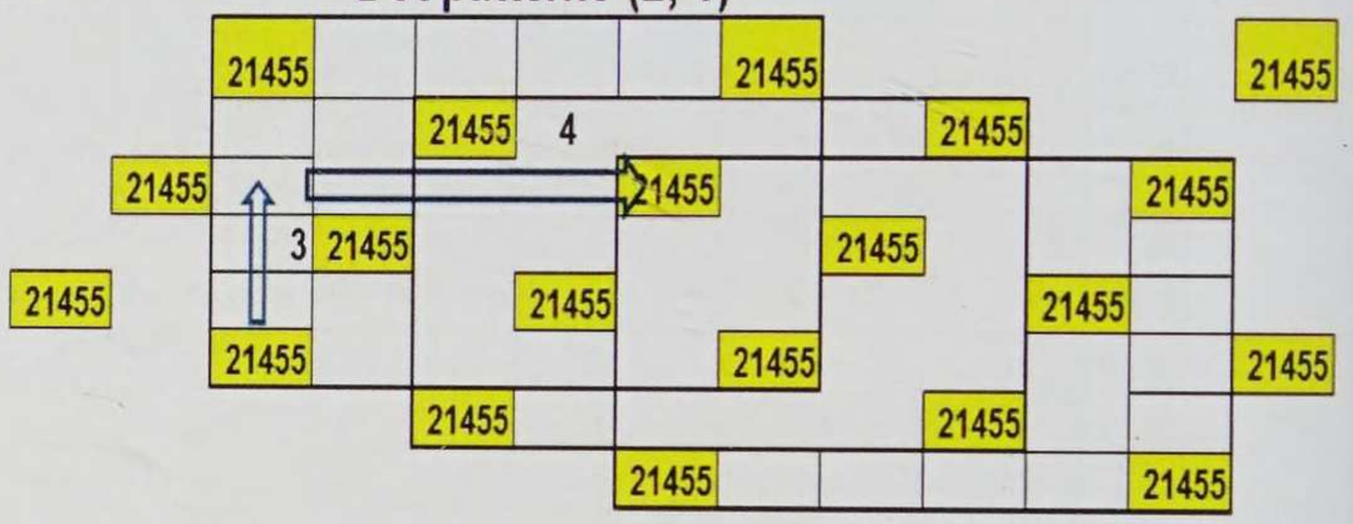


Tile centers (3; 4)

Dot patterns (2; -1)

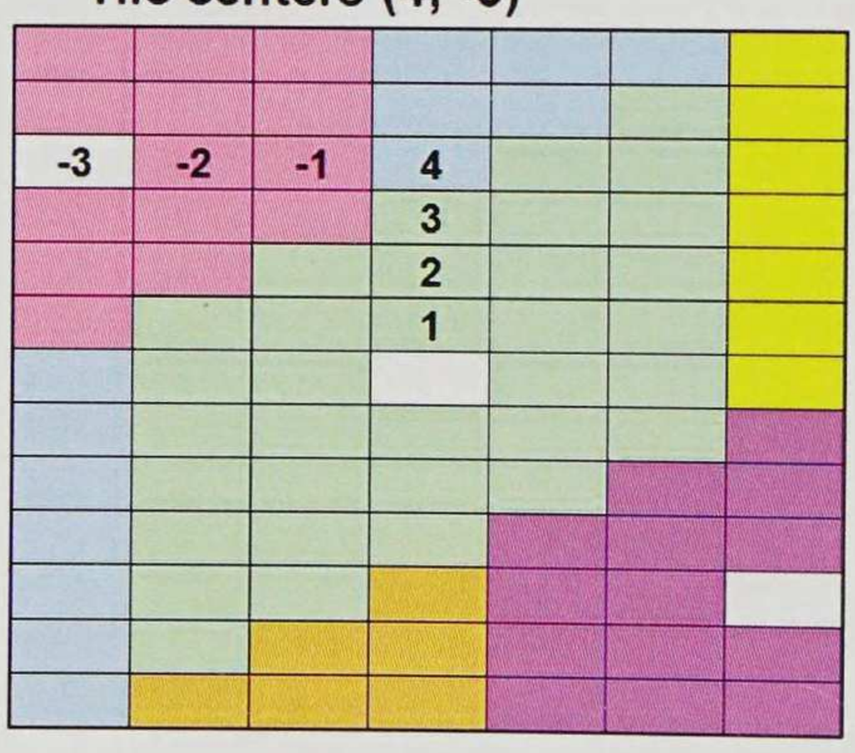


Dot patterns (2; 1)

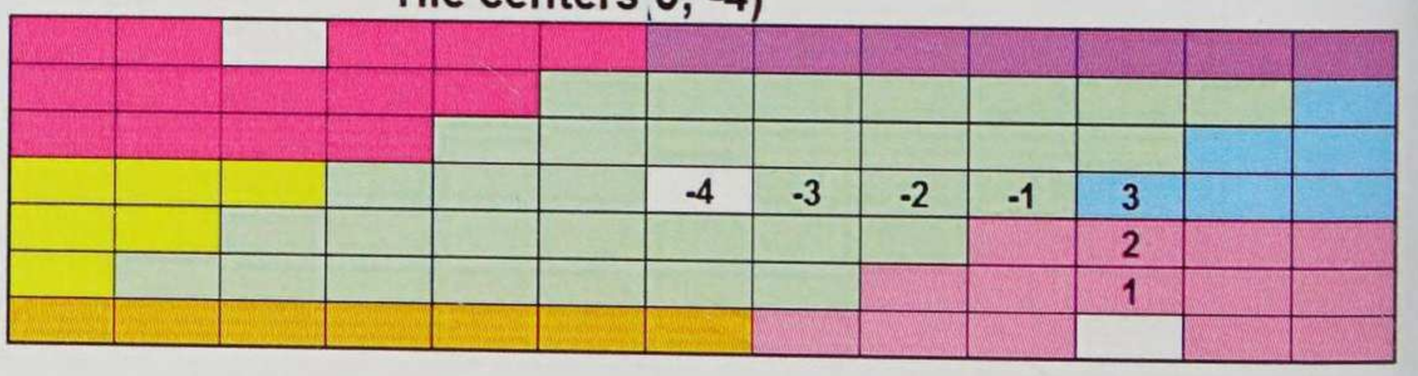


This explains why the tile centers sum to the characteristic number; it's because those centers are aligned with one of the multiple spread dot patterns which have that property.

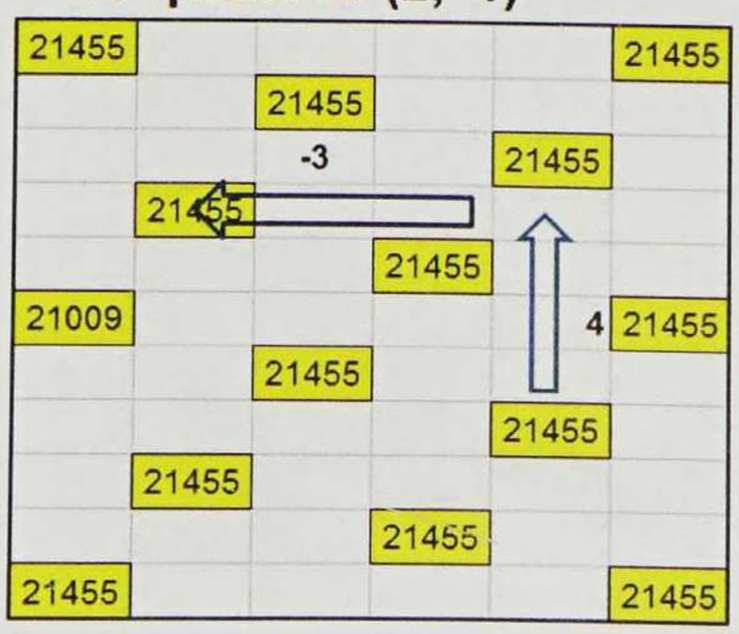
Tile centers (4; -3)



Tile centers (3; -4)



Dot patterns (2; 1)



Dot patterns (2; -1)

Program 2

Anchor-dot Patterns for Class 6 Squares

$n = b^a$ where **b** is a prime-number and **a = 2 or 3**

Here is the Class-6 size-9 square where **a = b = 3**. It is **3x** modular so every **3x3** block-square is a continuous tiling pattern. The centers of blocks all sum to the characteristic number continuously. These tiles cannot be aligned differently (offset) for this to happen.

Size-9

	1	2	3	4	5	6	7	8	9	
W(9)	369	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

Size-25

Here is the Class-6 size-25 square. It is **5x** modular so every **5x5** block-squares is a continuous tiling pattern.

The centers of the block tiles sum continuously to **7825**, the square's characteristic number. So does the horizontal flip of this block pattern.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Size-27 Square Anchor-dot Patterns

Below, the size-27 square has been sectioned into **3x9** block-segments. The block-segment rectangles are continuously equal-summing to the square's characteristic number due to the square's 3x-modularity and so are the size 9x3 block-segments. So unlike all other Class-3 squares, this size possesses equal-summing complementary tiling patterns. As such it has the properties of Class-6 squares. **However, the size-27 has no nova-dot pattern as shown on the next page as described for Class-6 squares whose size is a square numner.**

The anchor dot pattern is readily uncovered from the centers of these off-set block-rectangles highlighted here. Those are **(3; 3)**. The pattern's horizontal flip **(3;-3)** yields another continuously summing anchor-dot pattern. The transpose of the pattern yields nothing new because the height and horizontal distances between dots are equal.

The number of equal summation patterns per anchor-dot pattern is then just **2n = 2(27) = 54**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	399	463	314	345	409	260	372	436	287	156	220	71	102	166	17	129	193	44	642	706	557	588	652	503	615	679	530
2	590	657	496	617	684	523	644	711	550	347	414	253	374	441	280	401	468	307	104	171	10	131	198	37	158	225	64
3	133	191	42	160	218	69	106	164	15	619	677	528	646	704	555	592	650	501	376	434	285	403	461	312	349	407	258
4	386	471	319	332	417	265	359	444	292	143	228	76	89	174	22	116	201	49	629	714	562	575	660	508	602	687	535
5	568	662	513	595	689	540	622	716	567	325	419	270	352	446	297	379	473	324	82	176	27	109	203	54	136	230	81
6	114	205	47	141	232	74	87	178	20	600	691	533	627	718	560	573	664	506	357	448	290	384	475	317	330	421	263
7	391	485	300	337	431	246	364	458	273	148	242	57	94	188	3	121	215	30	634	728	543	580	674	489	607	701	516
8	585	667	491	612	694	518	639	721	545	342	424	248	369	451	275	396	478	302	99	181	5	126	208	32	153	235	59
9	119	213	34	146	240	61	92	186	7	605	699	520	632	726	547	578	672	493	362	456	277	389	483	304	335	429	250
10	318	382	476	264	328	422	291	355	449	75	139	233	21	85	179	48	112	206	561	625	719	507	571	665	534	598	692
11	509	576	658	536	603	685	563	630	712	266	333	415	293	360	442	320	387	469	23	90	172	50	117	199	77	144	226
12	52	110	204	79	137	231	25	83	177	538	596	690	565	623	717	511	569	663	295	353	447	322	380	474	268	326	420
13	305	390	481	251	336	427	278	363	454	62	147	238	8	93	184	35	120	211	548	633	724	494	579	670	521	606	697
14	487	581	675	514	608	702	541	635	729	244	338	432	271	365	459	298	392	486	1	95	189	28	122	216	55	149	243
15	33	124	209	60	151	236	6	97	182	519	610	695	546	637	722	492	583	668	276	367	452	303	394	479	249	340	425
16	310	404	462	256	350	408	283	377	435	67	161	219	13	107	165	40	134	192	553	647	705	499	593	651	526	620	678
17	504	586	653	531	613	680	558	640	707	261	343	410	288	370	437	315	397	464	18	100	167	45	127	194	72	154	221
18	38	132	196	65	159	223	11	105	169	524	618	682	551	645	709	497	591	655	281	375	439	308	402	466	254	348	412
19	480	301	395	426	247	341	453	274	368	237	58	152	183	4	98	210	31	125	723	544	638	669	490	584	696	517	611
20	671	495	577	698	522	604	725	549	631	428	252	334	455	279	361	482	306	388	185	9	91	212	36	118	239	63	145
21	214	29	123	241	56	150	187	2	96	700	515	609	727	542	636	673	488	582	457	272	366	484	299	393	430	245	339
22	467	309	400	413	255	346	440	282	373	224	66	157	170	12	103	197	39	130	710	552	643	656	498	589	683	525	616
23	649	500	594	676	527	621	703	554	648	406	257	351	433	284	378	460	311	405	163	14	108	190	41	135	217	68	162
24	195	43	128	222	70	155	168	16	101	681	529	614	708	556	641	654	502	587	438	286	371	465	313	398	411	259	344
25	472	323	381	418	269	327	445	296	354	229	80	138	175	26	84	202	53	111	715	566	624	661	512	570	688	539	597
26	666	505	572	693	532	599	720	559	626	423	262	329	450	289	356	477	316	383	180	19	86	207	46	113	234	73	140
27	200	51	115	227	78	142	173	24	88	686	537	601	713	564	628	659	510	574	443	294	358	470	321	385	416	267	331

The Nova-dot Patterns Common among Class-6 Squares

Class-6 squares whose size $\mathbf{n = b^2}$ have a characteristic dot pattern composed of $\mathbf{b^2}$ segregated dots as shown by the different colored cells.

Here is the size-9 square with three concentric 9-dot checker configurations. The size-9 square is also 3x-modular (yellow block-square including the center cell). Each colored pattern, always including the central dot, sum continuously to the size-9's characteristic number.

	1	2	3	4	5	6	7	8	9	
	9x9	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

Here is a nested series of 25-celled nova-dot patterns that sum continuously to the size-25 square's characteristic number. The dots are separated from one another by **1, 2, 3, 4** & **5** spacings vertically and horizontally. The larger patterns share cells with smaller ones of a different color.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	545	358	71	389	202	370	58	396	214	527	70	383	221	539	352	395	208	546	364	52	220	533	371	64	377	7825
2	436	129	592	285	123	136	579	292	110	448	586	279	117	435	148	286	104	442	135	598	111	429	142	585	298	7825
3	332	50	488	176	519	32	500	188	501	344	482	200	513	326	44	182	525	338	26	494	507	350	38	476	194	7825
4	228	566	259	97	415	553	266	84	422	240	253	91	409	247	565	78	416	234	572	265	403	241	559	272	90	7825
5	24	462	155	618	306	474	162	605	318	6	174	612	305	18	456	624	312	5	468	156	324	12	455	168	606	7825
6	445	133	596	289	102	145	583	296	114	427	595	283	121	439	127	295	108	446	139	577	120	433	146	589	277	7825
7	336	29	492	185	523	36	479	192	510	348	486	179	517	335	48	186	504	342	35	498	511	329	42	485	198	7825
8	232	575	263	76	419	557	275	88	401	244	257	100	413	226	569	82	425	238	551	269	407	250	563	251	94	7825
9	3	466	159	622	315	453	166	609	322	15	153	616	309	22	465	603	316	9	472	165	303	16	459	172	615	7825
10	549	362	55	393	206	374	62	380	218	531	74	387	205	543	356	399	212	530	368	56	224	537	355	68	381	7825
11	345	33	496	189	502	45	483	196	514	327	495	183	521	339	27	195	508	346	39	477	520	333	46	489	177	7825
12	236	554	267	85	423	561	254	92	410	248	281	79	417	235	573	86	404	242	560	273	411	229	567	260	98	7825
13	7	475	163	601	319	457	175	613	301	19	157	625	313	1	469	607	325	13	451	169	307	25	463	151	619	7825
14	528	366	59	397	215	353	66	384	222	540	53	391	209	547	365	378	216	534	372	65	203	541	359	72	390	7825
15	449	137	580	293	106	149	587	280	118	431	599	287	105	443	131	299	112	430	143	581	124	437	130	593	281	7825
16	245	558	271	89	402	570	258	96	414	227	270	83	421	239	552	95	408	246	564	252	420	233	571	264	77	7825
17	11	454	167	610	323	461	154	617	310	23	161	604	317	10	473	611	304	17	460	173	311	4	467	160	623	7825
18	532	375	63	376	219	357	75	388	201	544	57	400	213	526	369	382	225	538	351	69	207	550	363	51	394	7825
19	428	141	584	297	115	128	591	284	122	440	578	291	109	447	140	278	116	434	147	590	103	441	134	597	290	7825
20	349	37	480	193	506	49	487	180	518	331	499	187	505	343	31	199	512	330	43	481	524	337	30	493	181	7825
21	20	458	171	614	302	470	158	621	314	2	170	608	321	14	452	620	308	21	464	152	320	8	471	164	602	7825
22	536	354	67	385	223	361	54	392	210	548	61	379	217	535	373	386	204	542	360	73	211	529	367	60	398	7825
23	432	150	588	276	119	132	600	288	101	444	582	300	113	426	144	282	125	438	126	594	107	450	138	576	294	7825
24	328	41	484	197	515	28	491	184	522	340	478	191	509	347	40	178	516	334	47	490	503	341	34	497	190	7825
25	249	562	255	93	406	574	262	80	418	231	274	87	405	243	556	99	412	230	568	256	424	237	555	268	81	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
	7825	7825	7825	7825	7825																					

Similar patterns hold for the size **49** (7x7) square too (not shown).

Program 2

The Geonomic Conversion Process for All Odd-size Squares

W(11)	671	671	671	671	671	671	671	671	671	671	671
	21	37	64	80	107	2	29	45	72	99	115	671
	24	51	67	94	121	16	43	59	86	102	8	671
	38	65	81	108	3	30	46	73	89	116	22	671
	52	68	95	111	17	44	60	87	103	9	25	671
	66	82	109	4	31	47	74	90	117	12	39	671
	69	96	112	18	34	61	88	104	10	26	53	671
	83	110	5	32	48	75	91	118	13	40	56	671
	97	113	19	35	62	78	105	11	27	54	70	671
	100	6	33	49	76	92	119	14	41	57	84	671
	114	20	36	63	79	106	1	28	55	71	98	671
	7	23	50	77	93	120	15	42	58	85	101	671
	671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671	671

W*(11)	671	671	671	671	671	671	671	671	671	671	671
	37	107	45	115	64	2	72	21	80	29	99	671
	51	121	59	8	67	16	86	24	94	43	102	671
	65	3	73	22	81	30	89	38	108	46	116	671
	68	17	87	25	95	44	103	52	111	60	9	671
	82	31	90	39	109	47	117	66	4	74	12	671
	96	34	104	53	112	61	10	69	18	88	26	671
	110	48	118	56	5	75	13	83	32	91	40	671
	113	62	11	70	19	78	27	97	35	105	54	671
	6	76	14	84	33	92	41	100	49	119	57	671
	20	79	28	98	36	106	55	114	63	1	71	671
	23	93	42	101	50	120	58	7	77	15	85	671
	671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671	671

W*(11)	671	671	671	671	671	671	671	671	671	671	671
	37	107	45	115	64	2	72	21	80	29	99	671
	51	121	59	8	67	16	86	24	94	43	102	671
	65	3	73	22	81	30	89	38	108	46	116	671
	68	17	87	25	95	44	103	52	111	60	9	671
	82	31	90	39	109	47	117	66	4	74	12	671
	96	34	104	53	112	61	10	69	18	88	26	671
	110	48	118	56	5	75	13	83	32	91	40	671
	113	62	11	70	19	78	27	97	35	105	54	671
	6	76	14	84	33	92	41	100	49	119	57	671
	20	79	28	98	36	106	55	114	63	1	71	671
	23	93	42	101	50	120	58	7	77	15	85	671
	671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671	671

Center dot patterns are the points of conversion of the square they are in. See the conversion Algorithm below to understand.

The converted square at right has a whole new set of center-dots totally unrelated to the original ones that also sum to **671**.

This process may be repeated again and again and a new ultra-perfect version will be generated every time.

It is uncertain what the limit actually is until the original version is recovered. I conjecture it is **n** times.

Conversion Algorithm

Dot patterns are the points of conversion for turning Class-1 geonomic squares inside out as follows:

1. First, take the column with a pattern dot in the first row and move it to the first column.
2. Next, take the column with a pattern dot in the second row and move it to the second column.
3. Continue this process row after row until all columns in the square are filled with pattern dots all aligned along the major diagonal.
4. Tile the new square with the original pattern.

You now have a new perfect geonomic square having a dot pattern with values different from the original tile centers which again sum to the square's characteristic number.

Note: The roles between rows and columns can be switched and the process still works.

Part II Notes

1. Class-1 squares each have two unique complementary tile patterns that are characteristic of their size.

2. Class-2 squares have a simple tiling pattern that sums exactly to the square's characteristic number everywhere except when centered across the vertical or horizontal centerline; it is the **bx2** or **2xb** vertical or horizontal rectangle, respectively.

3. Class-3 squares whose size is **3** times a prime number have no tiling patterns which sum to its characteristic number.

4. Class-4 squares have two indistinguishable complementary tile patterns, the ring ◊ diamond and the **X** pattern that are common among all sizes, with the exceptions of size-4 which has none and size-8 which compounds its two complementary tiles together into a **4x4** block square with **4x**-modularity.

5. Class-5 tiling patterns for **n = ab**, where **a>b>3** and **a** and **b** are both prime-numbers, have four distinct vertically stacked **axb** and **bxa** staircase tiling patterns in both slanted directions. Further, rectangles of size **axb** and **bxa** also sum to the characteristic number.

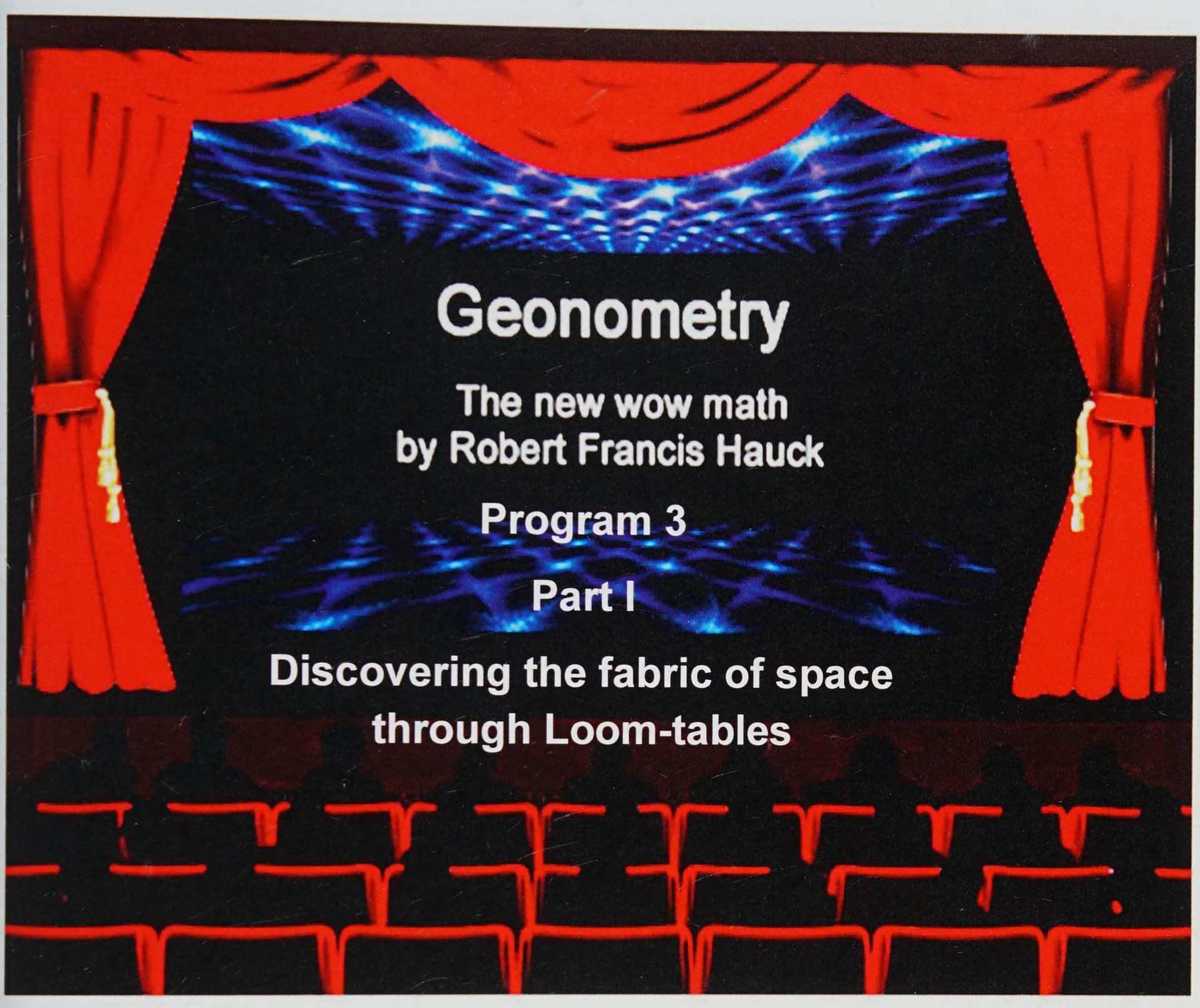
6. Class-6 squares have complementary characteristic tiling patterns where $\mathbf{n = b^2}$ and **b** is a prime-number. These are simply **bxb** block-squares whose **b**-block rows are shifted horizontally and **b**-block columns are shifted vertically.

7. All tiling patterns are continuous, with the sole exception of Class-4 size-12 patterns whose continuity is confined to segregated vertical sections.

8. The nova.dot pattern holds for all Class-6 squares whose size is a square of a prime number.

9. Tile center dot patterns are points of conversion of any odd-size perfect square they are in.

Program 2



Here in Part I we will see just how space is woven from equal-summing 1-dimensional numerical "strings" in what are called **loom tables**. It will be seen that the word "strings" has a more fundamental meaning here than just *linear summations*. We're talking here about the very fabric of space itself.

Program 3

Loom Tables for Class-1 squares

Geonomic squares can be decomposed into two loom tables. One is called the **modulus loom** and is based on the reformulated modulus function (**3.1**). The other loom table is called the **integer loom** and is based on the integer function (**3.2**). In what follows throughout this program, the **modulus** loom table will always be denoted by the upper-case letter **X** and the **integer** loom table always be denoted by the upper-case letter **Y**, always in bold lettering..

Denoting the initial perfect square by the symbol **W,** the reducing tables are complementary functions first applied to the initial square, **W**, as follows:

(3.1) $x_{ij} \equiv \text{modulus}\,[\,(w_{ij} - 1) \mid n\,] + 1$

(3.2) $y_{ij} \equiv \text{integer}\,[\,(w_{ij} - 1) / n\,] + 1$

The subscripts **ij** denote the cell in the **i-th row** and **j-th** column in the designated table. The symbol "≡" means "defined as".

To observe what the relationship of the modulus function (**3.1**) is to the integer function (**3.2**), the values of these functions are shown here in sequence for the full range of values w_{ij} for the size-7 square where **n = 7**.

Natural sequence	w	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Modulus function	x	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4
Integer function	y	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4

Natural sequence	w	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
Modulus function	x	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
Integer function	y	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	7	7

The numbers in each loom table for a square of size **n** run the gamut from **1** through **n**. Here these two functions have equal values for the numbers **1, 9, 17, 25, 33, 41** and **49**. Further, each number appears exactly **n** times in each loom table for a total of $\mathbf{n^2}$ numbers.

Theorum: The points of equality between the modulus and integer functions in the base n will always sum to the size n square's characteristic number! This characteristic summation property holds for every class and size of square.

Here **1+9+17+25+33+41+49 = 175**, the characteristic number for size-7.

Note that the number $\mathbf{n^2}$ can always be factored into a product of **n-minus-1** and **n-plus-1**, plus an **additional 1**, as in formula (**3.3**):

(3.3) $n^2 = (n-1)(n+1) + 1$

So while ranging through the numbers **1** to $\mathbf{n^2}$ there are always **n** points of equality between the modulus and integer functions and these numbers occur, starting at **1**, every **n-plus-1** numbers for **n-minus-1** additional occurrences beyond the first number, **1**.

Note that the number **0** never appears as it persistently does in the classical version of the modulus and integer functions.

The original square will be called the **primal** and will always henceforth be denoted by the upper-case letter **W**. It can be regenerated from the matrix formula (**3.4**):

(3.4) $W = n\,(Y - |1|) + X$ The symbol **|1|** represents a corresponding size table of all **1**'s.

Program 3

Size-7 Loom Tables

	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

Shown here at left is the size **n = 7** perfect square.

Below are the modulus table **X** and integer table **Y** for this size-7 square. Each is duplicated here **6** times to exhibit the amazing string-patterns of numbers **1** through **7**. Note that every row, column, both main and all wrap diagonals, and both dual tile patterns, simultaneously contain the numbers **1** through **7** exactly once. So clearly, all these each sum to the same number **28.**

Y

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

X

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

The characteristic number of loom tables will always be denoted by the upper-case letter **L** with the subscript **n**. Since a loom table that is from Class-1 will always have the numbers from **1** thru **n** exactly once in every geonomic summation, the characteristic number of a square loom table of size **n** from Class-1 will always just be the sum of numbers **1** thru **n**:

$$(3.5) \quad L_n = n(n+1)/2 = S_n^{(1)}$$

where $S_n^{(1)}$ denotes the linear sum from **1** through **n** (re: formula (**1.1**) from Program 1).

Interchangeability of Loom-tables

W

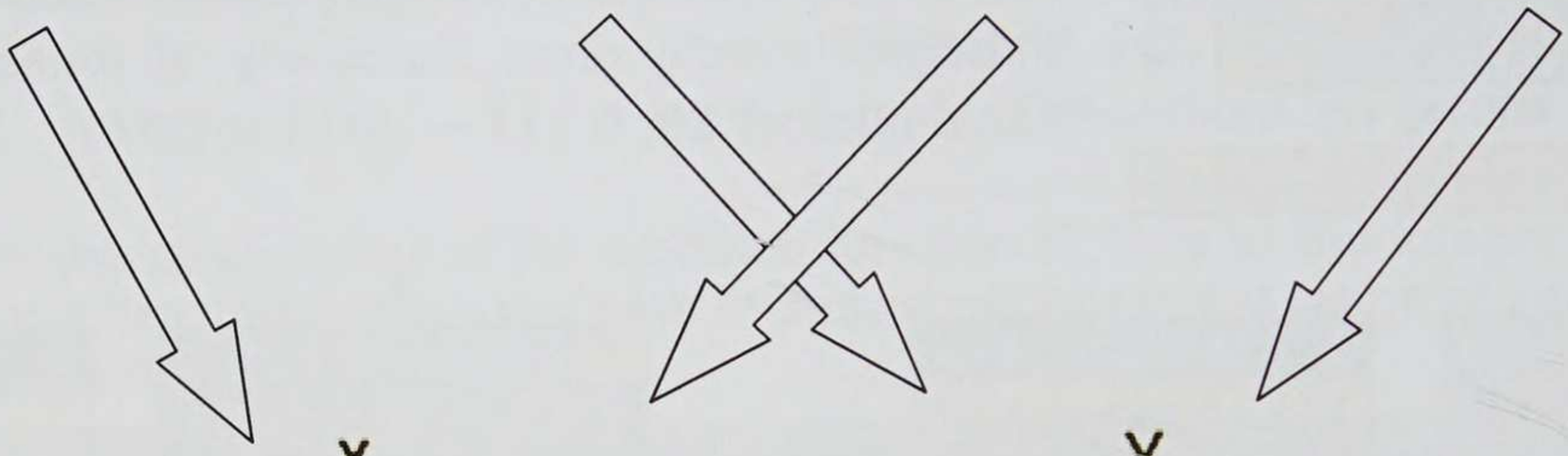
Modulus function of W

Integer function of W

Integer function of U

U

Modulus function of U



X

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

Y

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

The original *primal* square **W** has a perfect counterpart, called the **dual** square, which will always henceforth be denoted by the upper-case letter **U**.

Now, here is an absolutely amazing correlation between the complementary loom tables **X** and **Y**: the modulus loom **X** as derived from the primal square **W** is identical to the integer loom derived from the dual square **U**. And similarly, the integer loom table **Y** as derived from the primal square **W** is identical to the modulus loom table derived from the dual square **U**.

So the dual **U** can be generated by just switching the roles played by **X** and **Y** in formula (**3.4**).

(3.6) $U = n(X - |1|) + Y$

Make note: This property is one of the most fundamental properties of Geonometry.

It will also be observed that the geometric patterns in both loom tables carry over to both the primal square and its dual. The primary squares share properties jointly only when both of their complementary loom tables do so simultaneously. (**There are exceptions to this property in Classes 3, 4 and 5 squares.**)

We will be encountering this all throughout this program series. This kind of relationship gets even more amazing as we get into higher dimensioned tables in later programs.

Program 3

Here is the dual square **U** generated from the loom tables **X** and **Y** of the primal square **W**.

	1	2	3	4	5	6	7	
W(7)	175	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

	1	2	3	4	5	6	7	
U(7)	175	175	175	175	175	175	175	175
1	43	24	5	35	9	39	20	175
2	14	37	18	48	22	3	33	175
3	27	1	31	12	42	16	46	175
4	40	21	44	25	6	29	10	175
5	4	34	8	38	19	49	23	175
6	17	47	28	2	32	13	36	175
7	30	11	41	15	45	26	7	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

Loom difference table with complementary tiling patterns

(3.7) V = X – Y

The loom difference table will always be denoted by the capital letter **V**. The difference table's characteristic number will always be geonomically determined as **0**.

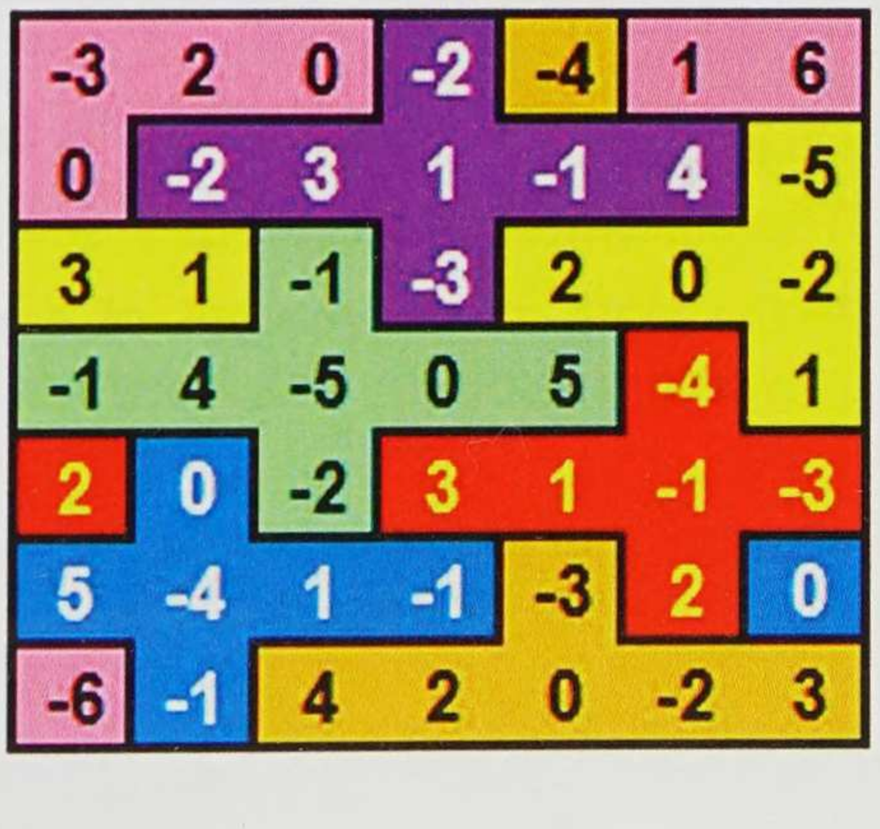
Here is the table **V** of the differences between the two loom tables. Observe that not only do all the rows, columns and diagonals of the difference table each sum to **0**, but that each tile placed anywhere in the difference table does too. This is a general property of all Class-1 squares.

This property will come into play again when we observe the cloaking property of loom tables in Part II.

A on V

-3	2	0	-2	-4	1	6
0	-2	3	1	-1	4	-5
3	1	-1	-3	2	0	-2
-1	4	-5	0	5	-4	1
2	0	-2	3	1	-1	-3
5	-4	1	-1	-3	2	0
-6	-1	4	2	0	-2	3

B on V



Program 3

Here again is the table of the size-7 loom-table differences; all summations equal **0**. Note the amazingly ordered pattern of numbers at bottom to the right of the difference table after these numbers were rearranged. The pattern of reorganized difference values, when put into a geonomic summation framework, has all of its diagonals summing equally to **0**.

Rows, columns and diagonals in **V** sum to **0**.

X

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

Y

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

V = X-Y

-6	-1	4	2	0	-2	3
5	-4	1	-1	-3	2	0
2	0	-2	3	1	-1	-3
-1	4	-5	0	5	-4	1
3	1	-1	-3	2	0	-2
0	-2	3	1	-1	4	-5
-3	2	0	-2	-4	1	6

	1	2	3	4	5	6	7
1	0	1	2	3	4	5	6
2	-1	0	1	2	3	4	5
3	-2	-1	0	1	2	3	4
4	-3	-2	-1	0	1	2	3
5	-4	-3	-2	-1	0	1	2
6	-5	-4	-3	-2	-1	0	1
7	-6	-5	-4	-3	-2	-1	0

All geonomic summations in the table at bottom left sum to **0**.

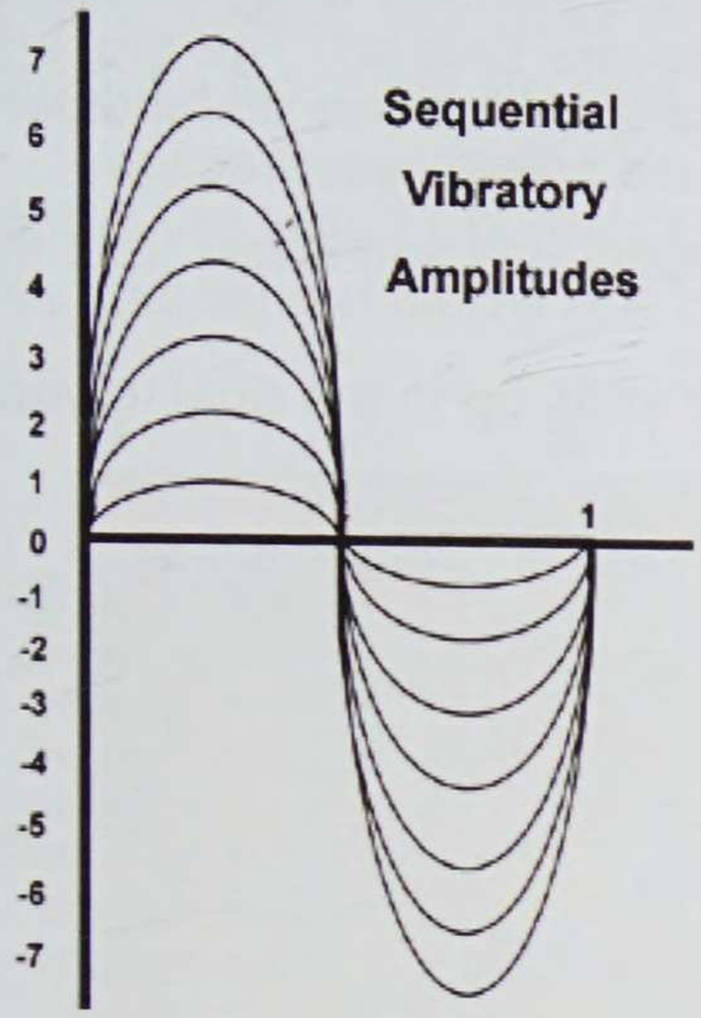
The difference table is totally anti-symmetric across the center.

In the table at bottom right, all geonomic diagonal summations in either direction sum to **0**. The row and column summations which are non-zero are all multiples of **±7** and in total sum to **0** too.

The table at lower right is a reassembly of the difference table to highlight the harmonic pattern hidden in the difference table.

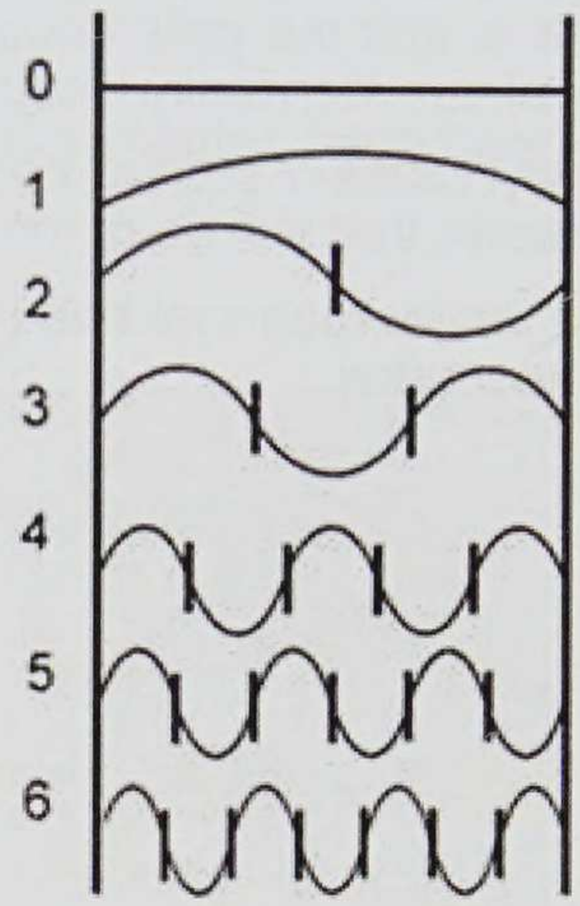
Program 3

Harmonics in 7x7 space



#	Count	Value
0's	7	0
1's	6 pairs	6
2's	5 pairs	10
3's	4 pairs	12
4's	3 pairs	12
5's	2 pairs	10
6's	1 pair	6
Total	$49 = 7^2$	$56 = 2x28 = 2L_7$

Sequential vibratory frequencies



What is happening is that we are observing through the patterns in loom tables, the fundamental quantum vibrations that fit harmoniously within a confined size-7 square space itself. It makes no difference whether the vibrations are frequencies or amplitudes; they all interweave together harmonically. And their differences in complementary patterns at **n-squared** points have net-value frequencies which cancel out overall.

Most importantly, they also cancel out to zero within each tile pattern too, anywhere and everywhere. The center of the tile doesn't have to be aligned with the center of its complementary tile counterpart for this to happen either!

This is an extremely important property that will become fundamentally useful later in Program 5 where Geonometry is applied to explaining the activity of electrons in the electron shells of atoms. And why atomic scientists will never directly observe such patterns with their atom-smashers or electron microscopes.

Here is the relationship between the characteristic number of primary squares and that of their loom-tables.

$$(3.8) \quad C_n = (n+1)\,L_n - n^2$$

as derived from:

$$C_n = \sum_j w_{ij} = \sum_j [n(y_{ij} - 1) + x_{ij}] = n\sum_j y_{ij} - n\sum_j + \sum_j x_{ij} = n\sum_j y_{ij} + \sum_j x_{ij} - n\sum_{j(1)}$$

$$= (n+1)\,L_n - n^2$$

Program 3

Four different versions of the size-7 perfect square

Here are 4 different perfect size-7 squares shown with their dual square on the far right. We have been observing square #2 in the examples shown for the size-7 square thus far. The property to note is that the cells containing the numbers where the two functions of modulus and integer are equal, are identically located in both the primal and dual versions of the primary squares too.

Every Class-1 square has a version where all their points of equality are non-linear. It is these squares that will be of interest in Part III.

The major diagonal has been highlighted to observe just where it ends up in the dual in various conversions.

1

W							
175	175	175	175	175	175	175	175
1	10	4	47	41	35	22	16	175
2	42	29	23	17	11	5	48	175
3	18	12	6	49	36	30	24	175
4	43	37	31	25	19	13	7	175
5	26	20	14	1	44	38	32	175
6	2	45	39	33	27	21	8	175
7	34	28	15	9	3	46	40	175
175	175	175	175	175	175	175	175	
175	175	175	175	175	175	175	175	

X							
3	4	5	6	7	1	2	28
7	1	2	3	4	5	6	28
4	5	6	7	1	2	3	28
1	2	3	4	5	6	7	28
5	6	7	1	2	3	4	28
2	3	4	5	6	7	1	28
6	7	1	2	3	4	5	28
28	28	28	28	28	28	28	

Y							
2	1	7	6	5	4	3	28
6	5	4	3	2	1	7	28
3	2	1	7	6	5	4	28
7	6	5	4	3	2	1	28
4	3	2	1	7	6	5	28
1	7	6	5	4	3	2	28
5	4	3	2	1	7	6	28
28	28	28	28	28	28	28	

1 W flipped horizontally

U							
175	175	175	175	175	175	175	175
16	22	35	41	47	4	10	175
48	5	11	17	23	29	42	175
24	30	36	49	6	12	18	175
7	13	19	25	31	37	43	175
32	38	44	1	14	20	26	175
8	21	27	33	39	45	2	175
40	46	3	9	15	28	34	175
175	175	175	175	175	175	175	
175	175	175	175	175	175	175	

2

W							
175	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
175	175	175	175	175	175	175	175	
175	175	175	175	175	175	175	175	

X							
7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

Y							
1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

2 Major diagonal flipped

U							
175	175	175	175	175	175	175	175
43	24	5	35	9	39	20	175
14	37	18	48	22	3	33	175
27	1	31	12	42	16	46	175
40	21	44	25	6	29	10	175
4	34	8	38	19	49	23	175
17	47	28	2	32	13	36	175
30	11	41	15	45	26	7	175
175	175	175	175	175	175	175	
175	175	175	175	175	175	175	

3

W							
175	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43	175
2	33	3	22	48	18	37	14	175
3	46	16	42	12	31	1	27	175
4	10	29	6	25	44	21	40	175
5	23	49	19	38	8	34	4	175
6	36	13	32	2	28	47	17	175
7	7	26	45	15	41	11	30	175
175	175	175	175	175	175	175	175	
175	175	175	175	175	175	175	175	

X							
6	4	2	7	5	3	1	28
5	3	1	6	4	2	7	28
4	2	7	5	3	1	6	28
3	1	6	4	2	7	5	28
2	7	5	3	1	6	4	28
1	6	4	2	7	5	3	28
7	5	3	1	6	4	2	28
28	28	28	28	28	28	28	

Y							
3	6	2	5	1	4	7	28
5	1	4	7	3	6	2	28
7	3	6	2	5	1	4	28
2	5	1	4	7	3	6	28
4	7	3	6	2	5	1	28
6	2	5	1	4	7	3	28
1	4	7	3	6	2	5	28
28	28	28	28	28	28	28	

3 Major diagonal made vertical

U							
175	175	175	175	175	175	175	175
38	27	9	47	29	18	7	175
33	15	4	42	24	13	44	175
28	10	48	30	19	1	39	175
16	5	36	25	14	45	34	175
11	49	31	20	2	40	22	175
6	37	26	8	46	35	17	175
43	32	21	3	41	23	12	175
175	175	175	175	175	175	175	
175	175	175	175	175	175	175	

4

W							
175	175	175	175	175	175	175	175
1	36	46	7	10	20	23	33	175
2	13	16	26	29	39	49	3	175
3	32	42	45	6	9	19	22	175
4	2	12	15	25	35	38	48	175
5	28	31	41	44	5	8	18	175
6	47	1	11	21	24	34	37	175
7	17	27	30	40	43	4	14	175
175	175	175	175	175	175	175	175	
175	175	175	175	175	175	175	175	

X							
1	4	7	3	6	2	5	28
6	2	5	1	4	7	3	28
4	7	3	6	2	5	1	28
2	5	1	4	7	3	6	28
7	3	6	2	5	1	4	28
5	1	4	7	3	6	2	28
3	6	2	5	1	4	7	28
28	28	28	28	28	28	28	

Y							
6	7	1	2	3	4	5	28
2	3	4	5	6	7	1	28
5	6	7	1	2	3	4	28
1	2	3	4	5	6	7	28
4	5	6	7	1	2	3	28
7	1	2	3	4	5	6	28
3	4	5	6	7	1	2	28
28	28	28	28	28	28	28	

4 Major diagonal made vertical

U							
175	175	175	175	175	175	175	175
6	28	43	16	38	11	33	175
37	10	32	5	27	49	15	175
26	48	21	36	9	31	4	175
8	30	3	25	47	20	42	175
46	19	41	14	29	2	24	175
35	1	23	45	18	40	13	175
17	39	12	34	7	22	44	175
175	175	175	175	175	175	175	
175	175	175	175	175	175	175	

Program 3

Size-11 tiling patterns A & hB with common centers on X & W

In what follows, the label of a tile pattern preceded by the lower-case letter **h**, will denote the horizontal flip of the tile pattern.

Here is the size-11 primal square **W** and its modulus loom table **X**. Both tables have been tiled with dual tiling patterns A & hB.

X **A** **W** **A**

10	4	9	3	8	2	7	1	6	11	5
2	7	1	6	11	5	10	4	9	3	8
5	10	4	9	3	8	2	7	1	6	11
8	2	7	1	6	11	5	10	4	9	3
11	5	10	4	9	3	8	2	7	1	6
3	8	2	7	1	6	11	5	10	4	9
6	11	5	10	4	9	3	8	2	7	1
9	3	8	2	7	1	6	11	5	10	4
1	6	11	5	10	4	9	3	8	2	7
4	9	3	8	2	7	1	6	11	5	10
7	1	6	11	5	10	4	9	3	8	2

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

X **hB** **W** **hB**

10	4	9	3	8	2	7	1	6	11	5
2	7	1	6	11	5	10	4	9	3	8
5	10	4	9	3	8	2	7	1	6	11
8	2	7	1	6	11	5	10	4	9	3
11	5	10	4	9	3	8	2	7	1	6
3	8	2	7	1	6	11	5	10	4	9
6	11	5	10	4	9	3	8	2	7	1
9	3	8	2	7	1	6	11	5	10	4
1	6	11	5	10	4	9	3	8	2	7
4	9	3	8	2	7	1	6	11	5	10
7	1	6	11	5	10	4	9	3	8	2

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

Note that tile centers are all equal to **11** on the loom table. On the size-11 primal square, they run the gamut from **11** to**121**, in increments of **11**.

Program 3

Size-11 tiling patterns hA & B with common centers on Y & U

Here is the size-11 **dual** square **U** and its integer loom table **Y**. Both tables have been tiled with tiling patterns hA and B.

Note again that these tile centers again are all equal to **11** on the loom table but on the size-11 **dual** square, run the gamut from **11** to**121**, in increments of **11**.

Y **hA**

2	4	6	8	10	1	3	5	7	9	11
3	5	7	9	11	2	4	6	8	10	1
4	6	8	10	1	3	5	7	9	11	2
5	7	9	11	2	4	6	8	10	1	3
6	8	10	1	3	5	7	9	11	2	4
7	9	11	2	4	6	8	10	1	3	5
8	10	1	3	5	7	9	11	2	4	6
9	11	2	4	6	8	10	1	3	5	7
10	1	3	5	7	9	11	2	4	6	8
11	2	4	6	8	10	1	3	5	7	9
1	3	5	7	9	11	2	4	6	8	10

U **hA**

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

Y **B**

2	4	6	8	10	1	3	5	7	9	11
3	5	7	9	11	2	4	6	8	10	1
4	6	8	10	1	3	5	7	9	11	2
5	7	9	11	2	4	6	8	10	1	3
6	8	10	1	3	5	7	9	11	2	4
7	9	11	2	4	6	8	10	1	3	5
8	10	1	3	5	7	9	11	2	4	6
9	11	2	4	6	8	10	1	3	5	7
10	1	3	5	7	9	11	2	4	6	8
11	2	4	6	8	10	1	3	5	7	9
1	3	5	7	9	11	2	4	6	8	10

U **B**

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

Observe that in all complete tiling patterns, the centers of interlocking tiles propagate diagonally “up” or “down” going from right to left. This direction of propagation may be reversed from “down” to “up” by flipping the tiling pattern horizontally. Then the tile centers would no longer be equal.

Program 3

Size-17 Loom Tables

Here is the **modulus** loom table for the Class-1 size-17 square. Every tile contains the numbers **1** through **17** exactly once. It has all the same properties as the size-7 square Note that the number distribution pattern in each tile is identical throughout the tiling pattern. This occurs in both tiling patterns regardless of which number is commonly central between them.

Also note that the central numbers of the individual tiles in both tiling patterns are identical everywhere. This occurs regardless of the number which is central. That results in all the tiles' being identical in loom table **X** for each complete tiling pattern.

X

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

X

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

A

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

B

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Program 3

Here is the integer loom table for the size-17 square. Again, every tile contains the numbers **1** through **17** exactly once. However, this time each tile's number distribution is different. The central numbers among all the tiles now range from **1** through **17** exactly once and all the numbers within each tile occupy a unique location in the tile throughout the entire pattern.

Y

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

Y

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

A

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

B

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

Program 3

Size-17 Loom-table Tile Centers and Internal Sequences

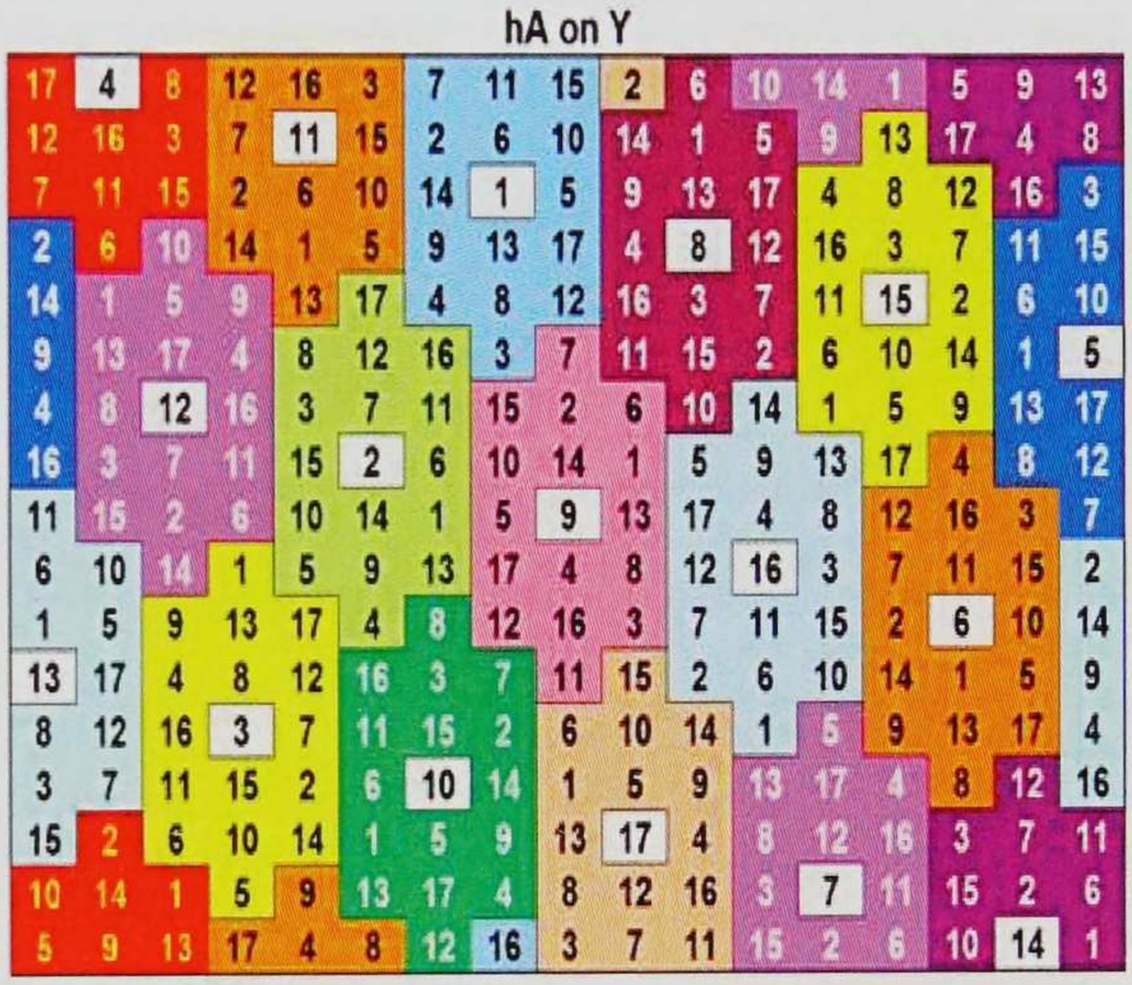
Sequential Centers and Scattered Numbers in Tiles

Here are both loom tables **X** and **Y** for the size-17 square. Each loom table is shown with its complementary tiles depicted. The center of each tile is highlighted. In all four tables, the tile centers contain the numbers from **1** thru **17**. Note that the relational sequences are not serial but random among the tiles.

Further the distribution of all the numbers within each tile is random among all the tiles.

A on X

hA on Y



B on X

hB on Y

Program 3

Constant Centers and Fixed Numbers

It is a general property for every Class-1 square of size **n** that the values of the tiles' central numbers in both patterns can be made all identical or be made to run the gamut from **1** through **n** depending upon their horizontal relationship. The identical number **9** is contained in all the centers here where each pattern has been flipped horizontally but the numbers held steady. Here all the tiles have the number **9** as their centers.

Further the distribution of all the numbers within each tile is identical among all the tiles.

This pattern will come into play in a big way in Program 5 where we will wrap these tiling patterns around spheres and apply them to a explaining a fundamental in Atomic Physics.

hA on X

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

A on Y

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

hB on X

15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

B on Y

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Program 3

Size-31 Loom Tables

Here is the size-31 modulus table. Observe the extraordinarily complex and intricate 4-directional weave of the numbers **1** through **31**. At this size, the loom pattern is way beyond the mind of Man to concoct without using computers. This establishes that these extraordinary number patterns are pre-existent and can only be discovered by the processes described by Geonometry in Program 7. It will be shown in Program 7 just how these loom tables can be uncovered without knowing ahead of time the distribution of numbers in the original geonomic square with which they are associated. In other words, it's already there awaiting its discovery, like panning for gold in the Rockies!

30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26
23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19
16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12
9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5
2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29
26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22
19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15
12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8
5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1
29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25
22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18
15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11
8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4
1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28
25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21
18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14
11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7
4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31
28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24
21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17
14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10
7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3
31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27
24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20
17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13
10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6
3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30
27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23
20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16
13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9
6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2

X	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	1
2	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	2
3	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	3
4	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	4
5	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	5
6	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	6
7	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	7
8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	8
9	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	9
10	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	10
11	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	11
12	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	12
13	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	13
14	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	14
15	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	15
16	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	16
17	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	17
18	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	18
19	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	19
20	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	20
21	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	21
22	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	22
23	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	23
24	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24
25	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	25
26	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	26
27	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	27
28	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	28
29	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	29
30	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	30
31	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	31
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	

30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26
23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19
16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12
9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5
2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29
26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22
19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15
12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8
5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1
29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25
22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18
15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11
8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4
1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28
25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21
18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14
11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7
4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31
28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24
21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17
14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10
7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3
31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27
24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20
17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13
10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6
3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30
27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23
20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16
13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9
6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2

X	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	1
2	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	2
3	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	3
4	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	4
5	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	5
6	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	6
7	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	7
8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	8
9	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	9
10	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	10
11	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	11
12	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	12
13	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	13
14	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	14
15	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	15
16	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	16
17	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	17
18	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	18
19	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	19
20	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	20
21	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	21
22	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	22
23	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	23
24	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24
25	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	25
26	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	26
27	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	27
28	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	28
29	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	29
30	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	30
31	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	31
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	

Weaving the Fabric of Space

Lucasfilm Stunt Works' version of a Quadraloom which weaves the 4-directional spatial fabric just observed in Class-1 squares

This picture was furnished courtesy of the Stunt Works group -- of Lucas film Star Wars fame -- which depicts a Quadraloom weaving a 4-directional weave of wonder.

This picture was created by them for a film entitled "**Looms"**, way before these tiling patterns were ever discovered and exhibits that artist-group's amazingly advanced imagination.

What we just saw in the last few slides was the inherent fabric of space – I call it the "***weave of wonder"***.

It will be shown time and time again that quadralooms and Geonometry are fundamentally "intertwined".

Program 3

Size-31 Loom Table X with Tiling Patterns A and hA

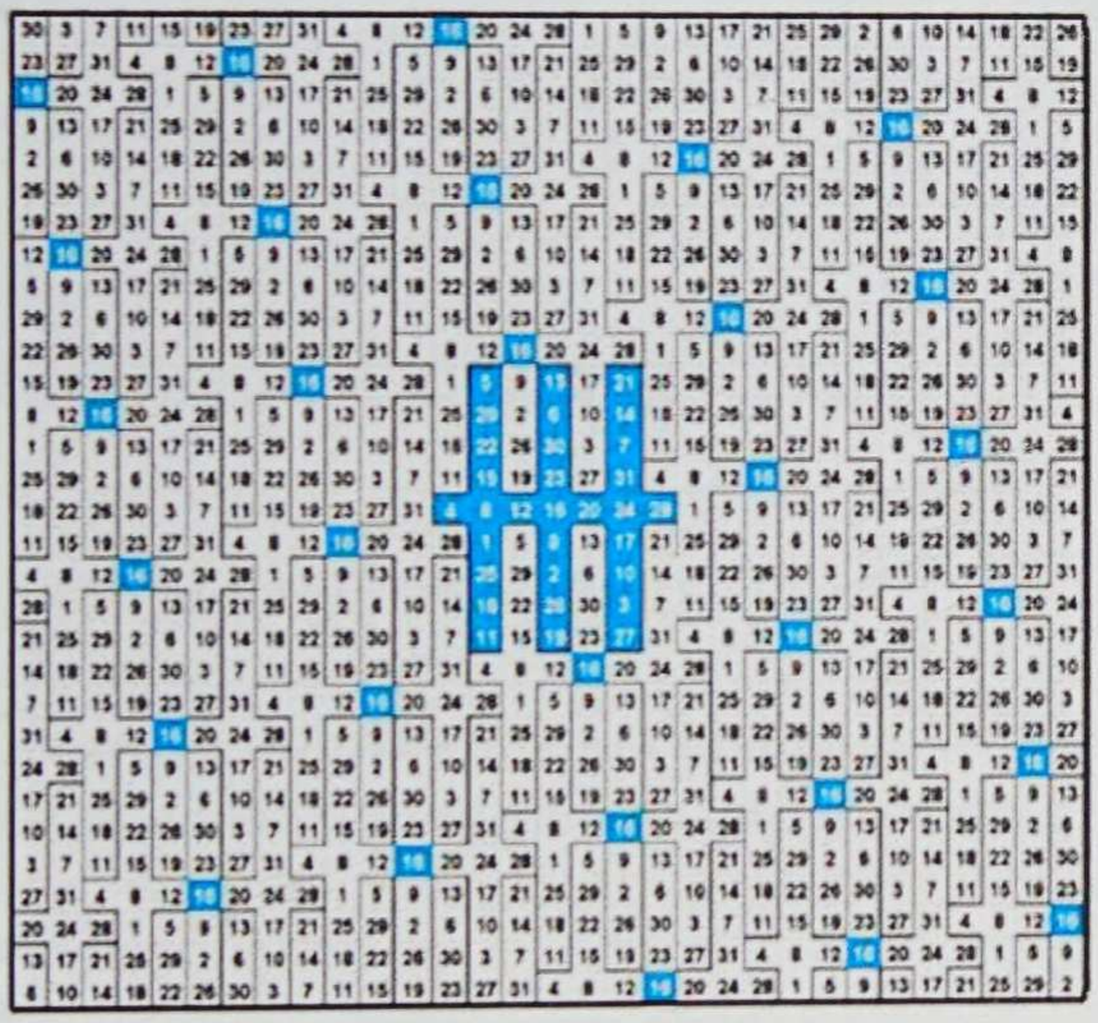
Here is the modulus loom table **X** for the size-31 square with tiling pattern A and its horizontally flipped version hA . The central numbers of the tile patterns in the tables are highlighted in blue. The table on the left is the tiling pattern A on **X** whose central numbers are all equal. The table on the right is the tiling pattern hA on **X** whose central numbers run the gamut from **1** through **31**.

Make note that the constant value among all the tile centers changes as the central tile is relocated over another number in the table. Furthermore, the pivot number for both loom tables is **16**, which lies at the center of each loom table.

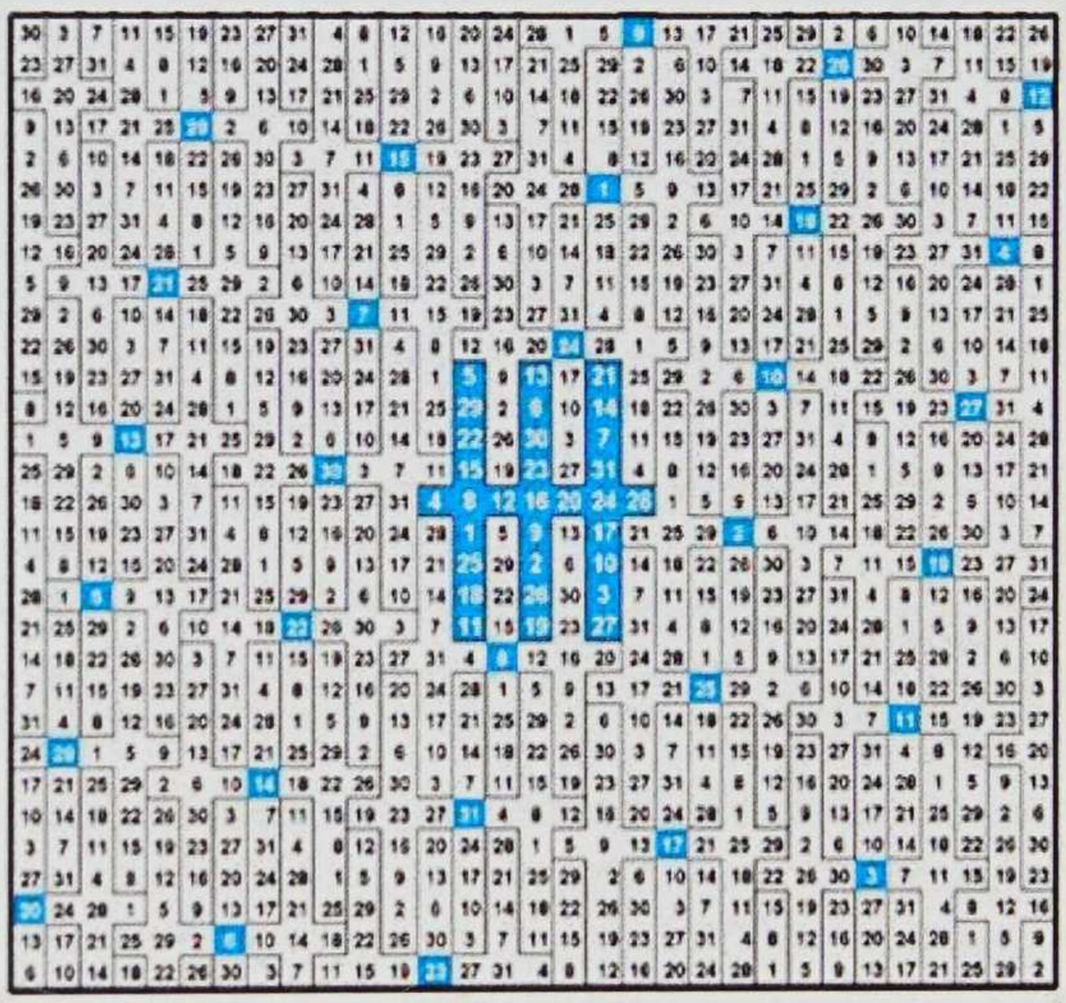
This is a general property: the number (n + 1)/2 will always lie at the center of both loom tables of size n.

X

A



hA



Centers all equal to 16

Pivot number = 16

Centers run gamut from 1 thru 31

Pivot number = 16

Program 3

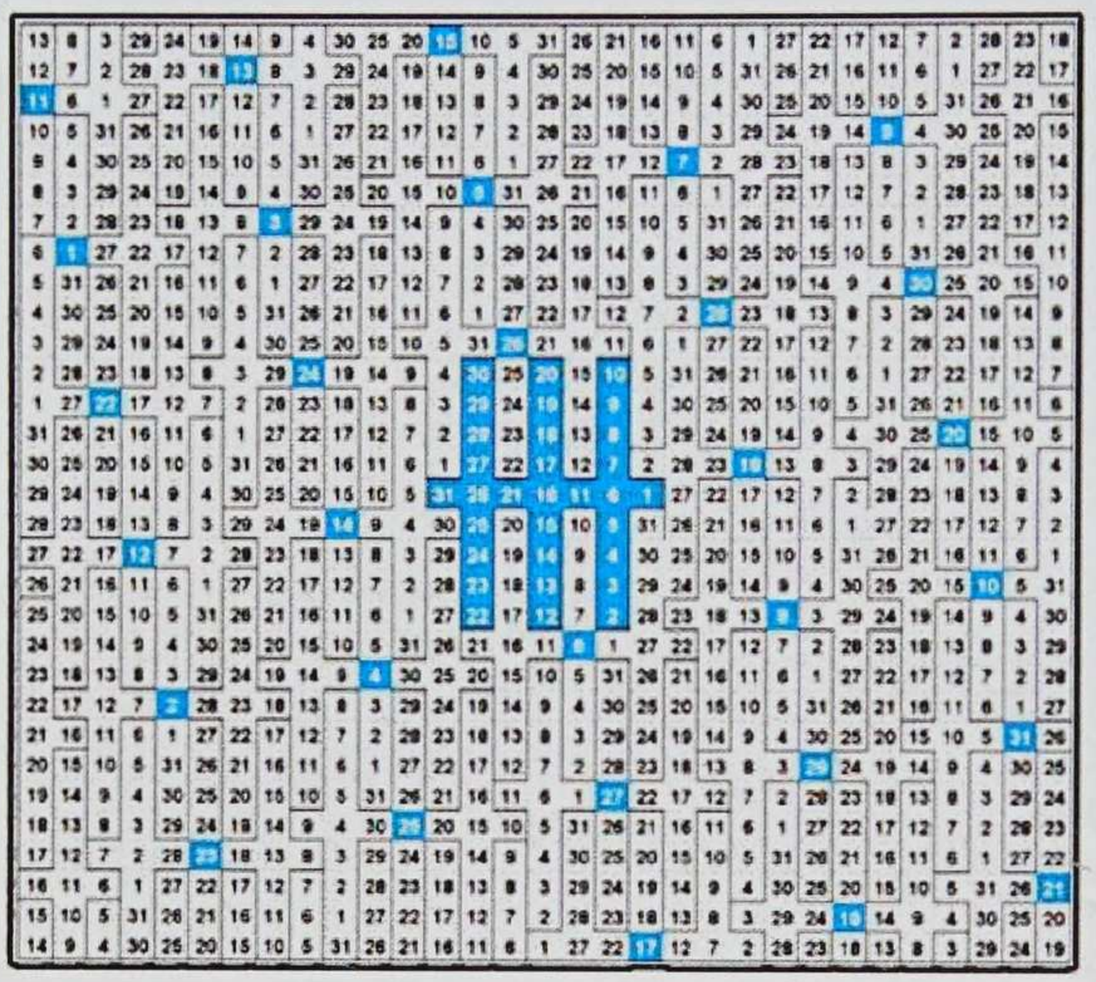
Size-31 Loom Table Y with Tiling Patterns A and hA

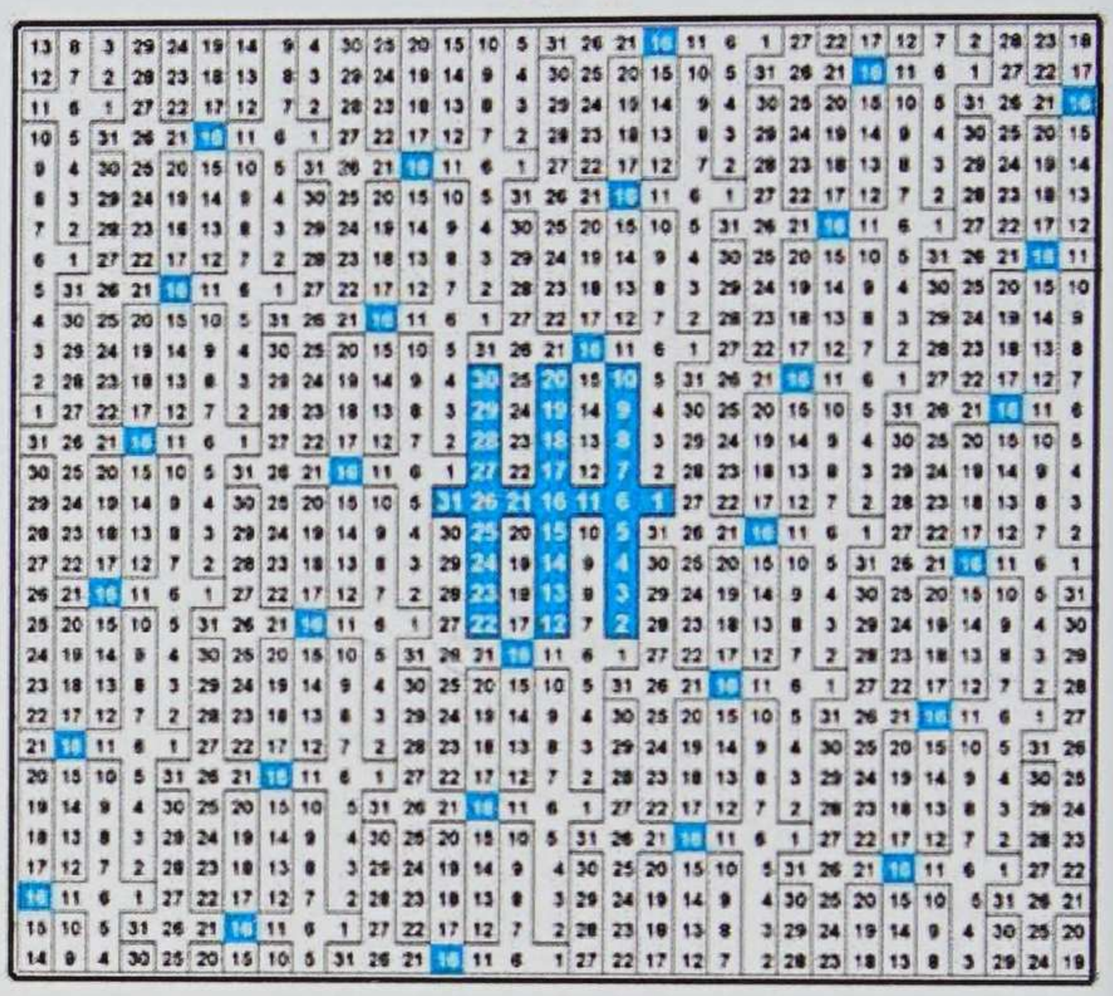
Here is the integer loom table **Y** with tiling pattern A and its horizontally flipped version hA. The table on the left is the tiling pattern A on **Y**. Its central numbers run the gamut from **1** through **31**. The table on the right is the tiling pattern hA on **Y**. The central numbers are all equal to **16**. Note the switch in which tile pattern A or hA has the constant central numbers between **X** and **Y**.

A

hA

Y





Centers run gamut from 1 thru 31

Centers all equal to 16

Tiling patterns A and hA can never have all common central numbers on the same loom table because their tiles propagate within the tiling pattern in different diagonal directions. The same goes for B and hB.

This may become a little confusing when all of these correspondences can't be seen on a single page, so here is a general table of all the correspondences among the tiling patterns **with constant central numbers** on loom tables **X** and **Y**.

All unlisted correspondences defer to tiling patterns on loom tables of size **n** whose central numbers run the gamut from **1** through **n.**

Tile Centers All Equal			
Modulus loom X		Integer loom Y	
A	hB	hA	B

Program 3

Points of Equality between Dual Squares and the centers of their Tiling Patterns

Tiling patterns' centers $\Delta = n$	Loom tables' points of equality $\Delta = n + 1$
11	1
22	13
33	25
44	37
55	49
66	61
77	73
88	85

W

21	**37**	64	80	107	2	29	45	72	99	115
24	51	67	94	**121**	16	43	59	86	102	8
38	65	81	108	3	30	46	**73**	89	116	22
52	68	95	111	17	44	60	87	103	9	**25**
66	82	**109**	4	31	47	74	90	117	12	39
69	96	112	18	34	**61**	88	104	10	26	53
83	110	5	32	48	75	91	118	**13**	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	**49**	76	92	119	14	41	57	84
114	20	36	63	79	106	**1**	28	55	71	98
7	23	50	77	93	120	15	42	58	**85**	101

U

101	**37**	94	30	87	12	69	5	62	119	55
14	71	7	64	**121**	46	103	39	96	32	78
48	105	41	98	23	80	16	**73**	9	66	112
82	18	75	11	57	114	50	107	43	89	**25**
116	52	**109**	34	91	27	84	20	77	2	59
29	86	22	68	4	**61**	118	54	100	36	93
63	120	45	102	38	95	31	88	**13**	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	**49**	106	42	99	24	81	17	74
44	90	26	83	19	76	**1**	58	115	51	108
67	3	60	117	53	110	35	92	28	**85**	21

Now we have seen that there are **n** points in each size-**n** Class-1 square where the numbers between the primal and dual squares are equal. These are those points where the modulus and integer functions are equal and zeroes appear in their complementary looms' difference table. One might wonder if the tiling patterns which can be aligned to have identical values in their centers could also be centered over these points of dual square equalities too. We just saw that their horizontal flip contained numbers **n** thru $\mathbf{n^2}$ in increments of **n**.

Observe that the points of equality between the primal and dual were seen to start at **1**, not **n**, and increment by **n-plus-1, not by n**. The centers of tiling patterns containing the extreme point is only at one point $\mathbf{n^2 = 121}$ and never start at the other extreme point **1**.

However, there is an amazing property that applies only to the size-7 square. The centers of both transposed tile patterns can be miraculously positioned to align with the points of primal and dual equalities!

These are called **anchor-dot patterns** and are shown on the next page.

Alignment of centers of transposed tiles with common equal value locations of Size-7's dual squares

W(7)	175	175	175	175	175	175	175
1	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12
	175	175	175	175	175	175	175
	175	175	175	175	175	175	175

U(7)	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43
2	33	3	22	48	18	37	14
3	46	16	42	12	31	1	27
4	10	29	6	25	44	21	40
5	23	49	19	38	8	34	4
6	36	13	32	2	28	47	17
7	7	26	45	15	41	11	30
	175	175	175	175	175	175	175
	175	175	175	175	175	175	175

Tile Pattern tA

W(7)	175	175	175	175	175	175	175
1	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12
	175	175	175	175	175	175	175
	175	175	175	175	175	175	175

U(7)	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43
2	33	3	22	48	18	37	14
3	46	16	42	12	31	1	27
4	10	29	6	25	44	21	40
5	23	49	19	38	8	34	4
6	36	13	32	2	28	47	17
7	7	26	45	15	41	11	30
	175	175	175	175	175	175	175
	175	175	175	175	175	175	175

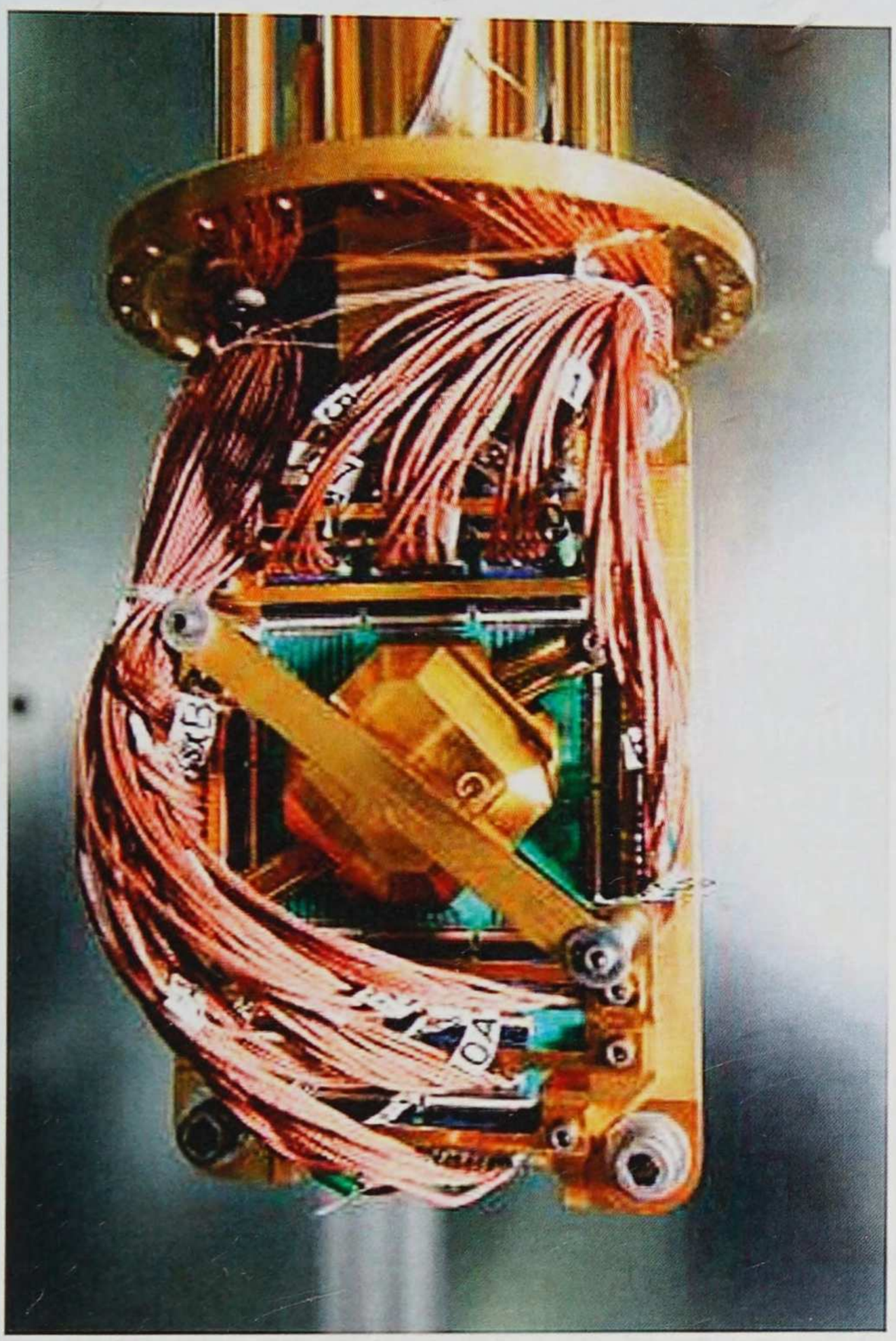
Tile Pattern tB

W(7)	175	175	175	175	175	175	175
1	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12
	175	175	175	175	175	175	175
	175	175	175	175	175	175	175

U(7)	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43
2	33	3	22	48	18	37	14
3	46	16	42	12	31	1	27
4	10	29	6	25	44	21	40
5	23	49	19	38	8	34	4
6	36	13	32	2	28	47	17
7	7	26	45	15	41	11	30
	175	175	175	175	175	175	175
	175	175	175	175	175	175	175

Program 3

Geonometry's Application to Quantum Computing



Redux D-Wave Systems' quantum computer processor pictured without its protective thermal canisters at the company's lab in Burnaby, British Columbia, Canada.

Photo by Kim Stallknecht of The New York Times

Quantum computing involves viewing a mass array of numbers all simultaneously and detecting and integrating intelligent patterns. Loom-tables of Class-1 squares provide the fundamental basis for doing exactly this.

Every Class-1 loom-table of size **n** has **n** distinct numbers linearly distributed in each of four directions [**+4n**]. Each characteristic complementary tile pattern also contains exactly **n** cells with **n** different numbers [**+2n**]. Additionally, the horizontal flip of the entire tiling pattern yields two more patterns with each tile containing the numbers from **1** thru **n** [+**2n**]. And the transpose of both complete tiling patterns contain **n** anchor-dot patterns, each containing **n** distinct numbers [**+2n**].

Now, that's a total of [**10n**] simultaneous patterns each containing the numbers **1** thru **n** on each loom-table. Since there are two complementary loom-tables in ultra-perfect squares (viz. with tiling-patterns) there is a total of **20n** patterned arrays of numbers **1** thru **n**. For instance, the size-7 loom-tables yield **140** distinct patterns of the numbers **1** thru **7**.

In quantum processing, the cell locations in the data table are all scanned simultaneously. Various sizes of Class-1 loom-tables can be applied simultaneously, Further, the same-size loom-table could be spread across the data table in a tiled expansion. Whatever scanning method is used will depend on the capacity of the quantum computer and the vastness of the data.

The data cells corresponding to the numbers **1** thru **n** in each of the **20n** patterns are examined for patterns in the data. Combinations of patterns can also be compared if and when an intelligent or repetitive pattern is detected The area of the data found promising can then be examined further by specific linear-processing algorithms that are designed to search for specific patterns in the numbers or characters.

Program 3

Part I Notes

1. There are two different loom tables, one derived from applying the modulus function to the square and the other derived from applying the integer function to the square.

2. Every class of square except Class-2 squares has these complementary geonomic loom tables. (Class-3 thru Class-6 looms are shown next in Part II.)

3. Class-1 loom tables are unique among all the classes in that every row, column, al diagonals and all tiles of a table of size-n contains the numbers **1** thru **n** exactly once.

4. The size-n Class-1 loom-tables' joint characteristic number $\mathbf{L_n}$ is the sum of consecutive numbers from **1** to **n**.

5. All the tile patterns sum individually and continuously to $\mathbf{L_n}$ on both loom tables.

6. These loom tables, when used in the regeneration formula, reproduce the origina primary square.

7. When the roles of these loom tables are interchanged in the regeneration formula, they produce another perfect square, called its **dual**, which has all the properties as the original primary square but is distinctly different in its distribution of numbers **1** thru $\mathbf{n^2}$.

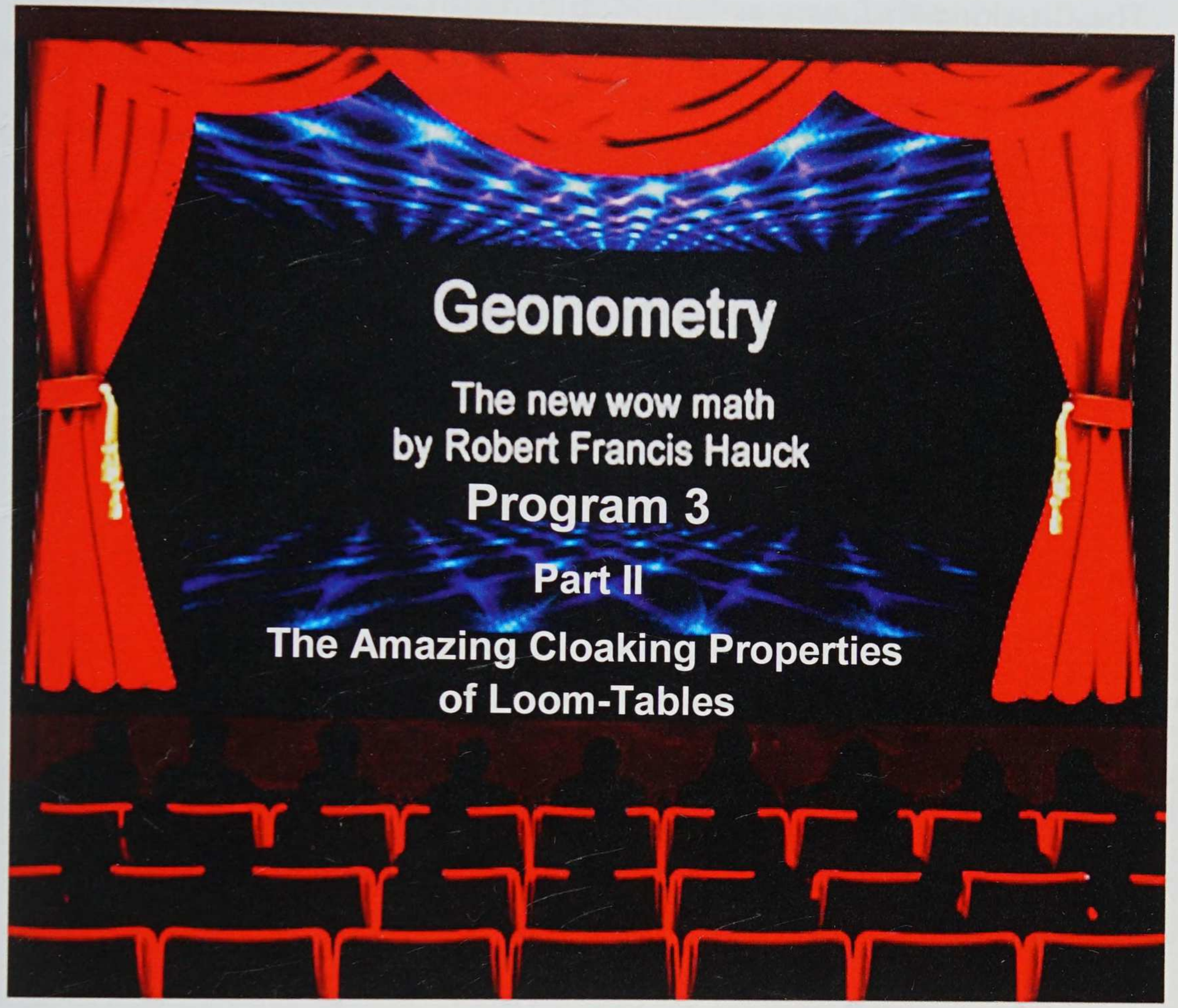
8. These dual primary squares, both of size **n**, have **n** locations between them where thei numbers are identical and these numbers sum to the squares' characteristic number.

9. Class-2 and Class-3 squares do not have loom-tables to the base **n** (the size of the square) that are geonomic like the other classes do,

   i. Class-3 squares' modulus loom-table must be derived in the base **b** and the intege loom-table must be derived in the base **3n** for these to function in the basic dua generation formulas as the other class squares do.

   ii. Class-4 squares' modulus loom-table is always an expansion of the size-4 square to match the size **n = 4b**. The integer table is always the expansion of the size **b** square to **4** quadrant duplicates. However, once generated in this manner, its modulus and integer loom tables can be derived in the base **n** and its dual can be derived just by switching the roles of these complementary loom tables in the basic generation formula.

10. Alignment of the centers of transposed tiling patterns with common locations of equa values in Size-7' dual squares is unique among all Class-1 squares.



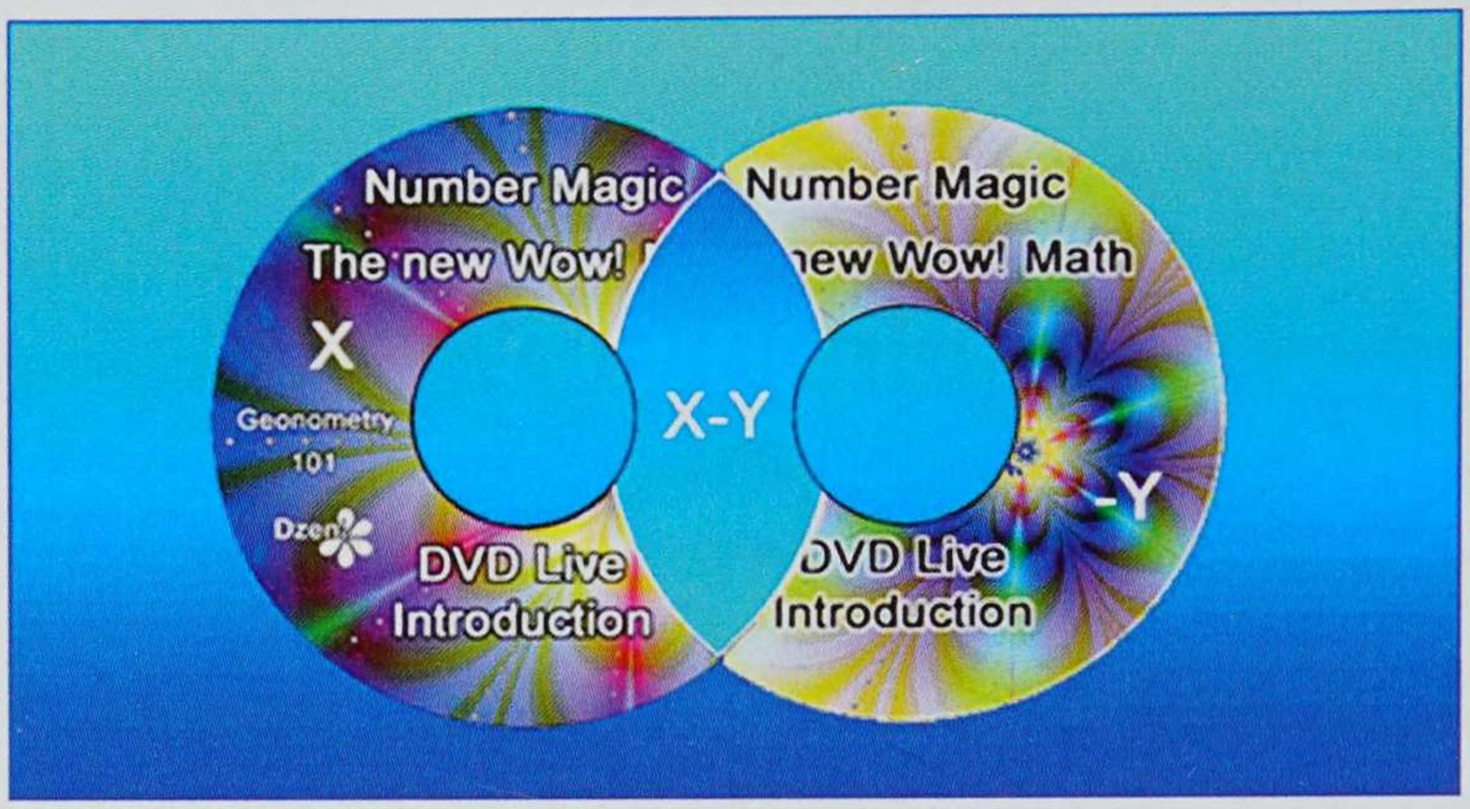
In this program is devoted to demonstrating the cloaking properties of complementary loom-tables. We have already seen how space is woven from equal-summing 1-dimensional numerical "strings" in Class-1 loom tables. Now, it will be seen that the word "strings" has a more fundamental meaning here than just *linear summations*.

These cloaking properties will come into play later in Program 5 where Geonometry is used to describe the orbital distribution of electrons in atoms. So take note.

Toward the end of the program, the 1st application of Geonometry shows how the double helix could arise from the 2-dimensional spatial fabric to form the basis for DNA and variations of it to arise naturally.

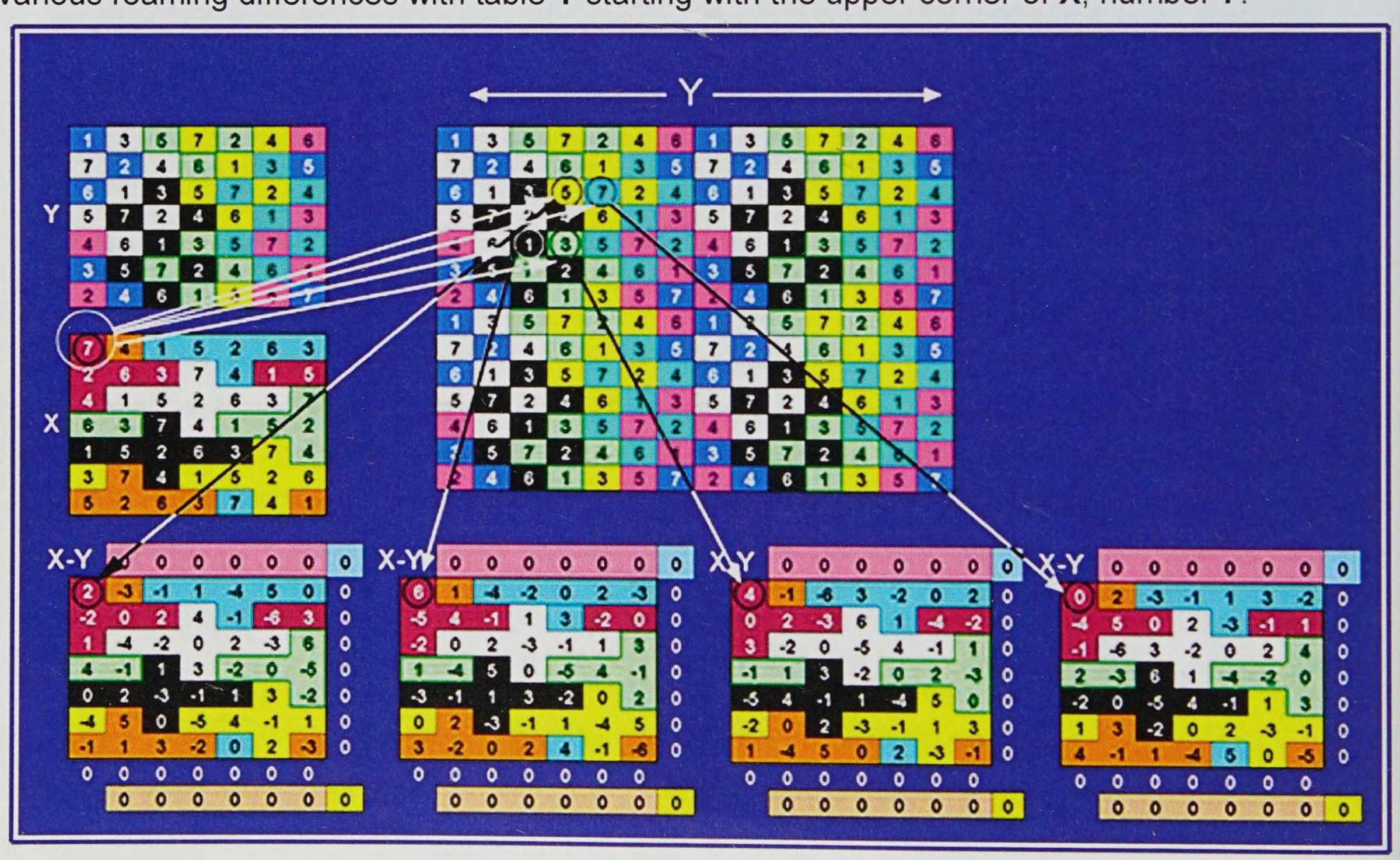
The Cloaking Property of Tiling Patterns on Loom-table Differences

Next we will observe the cloaking property of Classes 1, 4, 5 and 6 complementary loom tables covered with identical tiling patterns. First, we'll begin with squares of Class-1.



Class-1 Squares

Here the size-7 loom table has been duplicated four times to make it easier to see what is meant by cloaking pattern. The tables at the bottom use the loom table **X** at left center to take various roaming differences with table **Y** starting with the upper corner of **X**, number **7**.



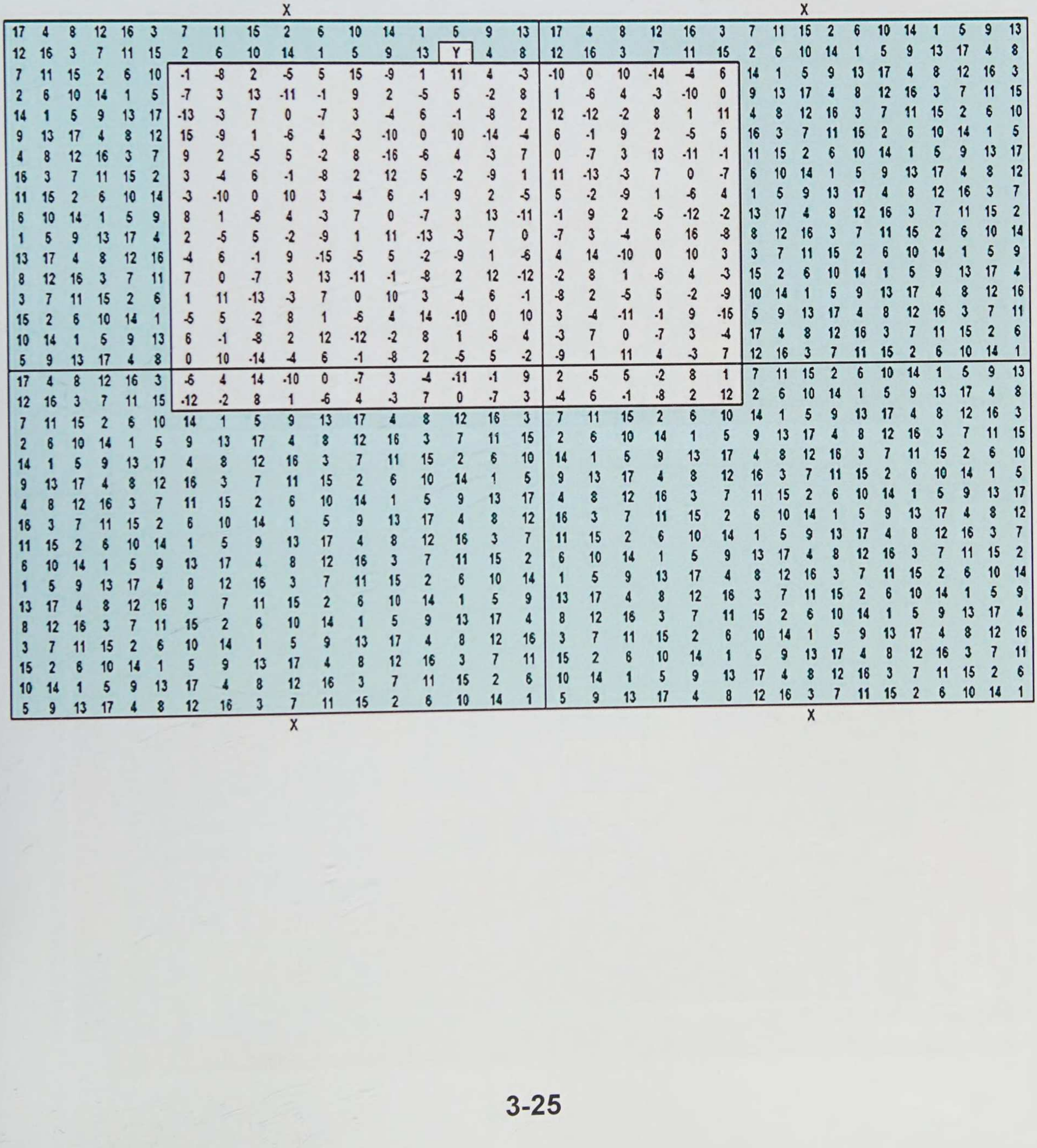
Here is the absolutely amazing result: All the rows, columns, diagonals and tiles sum equally to **0** everywhere the differences are taken. It's as if neither loom table exists; one loom table cloaks the other.

Program 3

Roaming size-17 Loom Difference Table

$$V(n) = X(n) - Y(n)$$

This slide shows a composite table formed from the loom table **X** duplicated four times. The loom table **Y** is then subtracted from the composite table as shown in white. Now this loom table can be subtracted from the composite table anywhere and what is shown on the next slide will occur.



Program 3

Same size-17 loom difference table with tile patterns A & B

Again, every row, column, diagonal and tile sums to **0** everywhere. And not only that but the tiles can be located anywhere in the table and each will still sum to **0**. It's as if the loom membranes didn't even exist because they would be undetectable.

X-Y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-5	5	-2	8	1	-6	4	14	-10	0	-7	3	-4	-11	-1	9		0
-4	6	-1	-8	2	12	-12	-2	8	1	-6	4	-3	7	0	-7	3		0
-10	0	10	-14	-4	6	-1	-8	2	-5	5	15	-9	1	11	4	-3		0
1	-6	4	-3	-10	0	-7	3	13	-11	-1	9	2	-5	5	-2	8		0
12	-12	-2	8	1	11	-13	-3	7	0	-7	3	-4	6	-1	-8	2		0
6	-1	9	2	-5	5	15	-9	1	-6	4	-3	-10	0	10	-14	-4		0
0	-7	3	13	-11	-1	9	2	-5	5	-2	8	-16	-6	4	-3	7		0
11	-13	-3	7	0	-7	3	-4	6	-1	-8	2	12	5	-2	-9	1		0
5	-2	-9	1	-6	4	-3	-10	0	10	3	-4	6	-1	9	2	-5		0
-1	9	2	-5	-12	-2	8	1	-6	4	-3	7	0	-7	3	13	-11		0
-7	3	-4	6	16	-8	2	-5	5	-2	-9	1	11	-13	-3	7	0		0
4	14	-10	0	10	3	-4	6	-1	9	-15	-5	5	-2	-9	1	-6		0
-2	8	1	-6	4	-3	7	0	-7	3	13	-11	-1	-8	2	12	-12		0
-8	2	-5	5	-2	-9	1	11	-13	-3	7	0	10	3	-4	6	-1		0
3	-4	-11	-1	9	-15	-5	5	-2	8	1	-6	4	14	-10	0	10		0
-3	7	0	-7	3	-4	6	-1	-8	2	12	-12	-2	8	1	-6	4		0
-9	1	11	4	-3	7	0	10	-14	-4	6	-1	-8	2	-5	5	-2		0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

X-Y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-5	5	-2	8	1	-6	4	14	-10	0	-7	3	-4	-11	-1	9		0
-4	6	-1	-8	2	12	-12	-2	8	1	-6	4	-3	7	0	-7	3		0
-10	0	10	-14	-4	6	-1	-8	2	-5	5	15	-9	1	11	4	-3		0
1	-6	4	-3	-10	0	-7	3	13	-11	-1	9	2	-5	5	-2	8		0
12	-12	-2	8	1	11	-13	-3	7	0	-7	3	-4	6	-1	-8	2		0
6	-1	9	2	-5	5	15	-9	1	-6	4	-3	-10	0	10	-14	-4		0
0	-7	3	13	-11	-1	9	2	-5	5	-2	8	-16	-6	4	-3	7		0
11	-13	-3	7	0	-7	3	-4	6	-1	-8	2	12	5	-2	-9	1		0
5	-2	-9	1	-6	4	-3	-10	0	10	3	-4	6	-1	9	2	-5		0
-1	9	2	-5	-12	-2	8	1	-6	4	-3	7	0	-7	3	13	-11		0
-7	3	-4	6	16	-8	2	-5	5	-2	-9	1	11	-13	-3	7	0		0
4	14	-10	0	10	3	-4	6	-1	9	-15	-5	5	-2	-9	1	-6		0
-2	8	1	-6	4	-3	7	0	-7	3	13	-11	-1	-8	2	12	-12		0
-8	2	-5	5	-2	-9	1	11	-13	-3	7	0	10	3	-4	6	-1		0
3	-4	-11	-1	9	-15	-5	5	-2	8	1	-6	4	14	-10	0	10		0
-3	7	0	-7	3	-4	6	-1	-8	2	12	-12	-2	8	1	-6	4		0
-9	1	11	4	-3	7	0	10	-14	-4	6	-1	-8	2	-5	5	-2		0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Program 3

Size-31 loom difference table

Each tile sums to **0** too and it does this continuously. As before, this loom table can be subtracted from the composite table anywhere. See the result of a random difference below.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
V(31)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	17	-5	4	-18	-9	0	9	18	27	-26	-17	-8	1	10	19	-3	-25	-16	-7	2	11	20	-2	7	-15	-6	3	12	-10	-1	8	0
2	11	20	29	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	13	-9	0	9	-13	-4	5	-17	-8	1	10	19	-3	6	-16	-7	2	0
3	5	14	23	1	-21	-12	-3	6	15	-7	2	11	-11	-2	7	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	-22	-13	-4	0
4	-1	8	-14	-5	4	13	-9	0	9	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	2	-20	-11	-2	7	16	-6	3	-19	-10	0
5	-7	2	-20	-11	-2	7	16	25	-28	-19	-10	-1	8	17	26	4	-18	-9	0	9	18	-4	5	-17	-8	1	10	-12	-3	6	15	0
6	18	27	-26	-17	-8	1	10	19	-3	6	-16	-7	2	11	-11	-2	7	-15	-6	3	12	-10	-1	8	17	-5	4	-18	-9	0	9	0
7	12	21	-1	8	-14	-5	4	13	-9	0	9	-13	-4	5	-17	-8	1	10	19	-3	-25	-16	-7	2	11	20	29	-24	-15	-6	3	0
8	6	15	-7	2	11	-11	-2	7	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	-22	-13	-4	5	14	-8	1	10	-12	-3	0
9	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	2	-20	-11	-2	7	16	-6	3	12	-10	-1	8	-14	-5	4	13	-9	0
10	25	-28	-19	-10	-1	8	17	-5	4	-18	-9	0	9	18	-4	5	14	-8	1	10	-12	-3	6	15	-7	2	-20	-11	-2	7	16	0
11	19	-3	6	-16	-7	2	11	-11	-2	7	16	-6	3	-19	-10	-1	8	17	-5	4	-18	-9	0	9	18	-4	-26	-17	-8	1	10	0
12	13	-9	0	9	18	-4	5	-17	-8	1	10	19	-3	-25	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	0
13	7	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	-22	-13	-4	5	14	-8	1	10	-12	-3	6	-16	-7	2	11	20	-2	0
14	-30	-21	-12	-3	6	15	24	2	-20	-11	-2	7	16	-6	3	12	-10	-1	8	-14	-5	4	13	22	0	-22	-13	-4	5	14	23	0
15	-5	4	-18	-9	0	9	-13	-4	5	14	-8	1	10	-12	-3	6	15	24	2	-20	-11	-2	7	16	25	-28	-19	-10	-1	8	17	0
16	-11	-2	7	16	-6	3	-19	-10	-1	8	17	26	-27	-18	-9	0	9	18	27	-26	-17	-8	1	10	19	-3	6	-16	-7	2	11	0
17	-17	-8	1	10	19	28	-25	-16	-7	2	11	20	-2	-24	-15	-6	3	12	-10	-1	8	-14	-5	4	13	-9	0	9	18	-4	5	0
18	-23	-14	-5	4	13	22	0	-22	-13	-4	5	14	-8	1	10	-12	-3	6	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	30	0
19	2	-20	-11	-2	7	16	-6	3	12	-10	-1	8	-14	-5	4	13	22	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	-7	0
20	-4	5	14	-8	1	-21	-12	-3	6	15	24	2	-20	-11	-2	7	16	25	3	-19	-10	-1	8	17	-5	4	-18	-9	0	9	-13	0
21	-10	-1	8	17	26	-27	-18	-9	0	9	18	-4	5	-17	-8	1	10	19	-3	6	-16	-7	2	11	-11	-2	7	16	-6	3	-19	0
22	-16	-7	2	11	20	-2	7	-15	-6	3	12	-10	-1	8	-14	-5	4	-18	-9	0	9	18	-4	5	-17	-8	1	10	19	28	-25	0
23	9	-13	-4	5	14	-8	1	10	-12	-3	6	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	0
24	3	12	-10	-1	8	-14	-5	4	13	22	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	-7	2	11	-11	-2	7	-15	-6	0
25	-3	6	15	24	-29	-20	-11	-2	7	16	25	3	-19	-10	-1	8	17	-5	4	13	-9	0	9	-13	-4	5	14	-8	1	-21	-12	0
26	-9	0	9	18	-4	5	-17	-8	1	10	-12	-3	6	15	-7	2	11	-11	-2	7	16	-6	3	-19	-10	-1	8	17	26	-27	-18	0
27	-15	-6	3	12	-10	-1	8	17	-5	4	-18	-9	0	9	18	-4	-26	-17	-8	1	10	19	28	-25	-16	-7	2	11	20	-2	7	0
28	10	19	-3	6	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	13	-9	0	9	-13	-4	5	14	-8	1	0
29	4	13	22	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	-7	2	11	-11	-2	7	-15	-6	3	12	21	-1	-23	-14	-5	0
30	-2	7	16	-6	3	-19	-10	-1	8	17	-5	4	13	-9	0	9	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	-29	-20	-11	0
31	-8	1	10	-12	-3	6	15	-7	2	-20	-11	-2	7	16	25	3	-19	-10	-1	8	17	26	-27	-18	-9	0	9	18	-4	5	-17	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
Random X-Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	11	20	-2	7	-15	-6	3	12	-10	-1	8	-14	-5	4	-18	-9	0	9	18	-4	5	-17	-8	1	10	19	28	-25	-16	-7	0
2	-4	5	14	-8	1	10	-12	-3	6	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	9	-13	0
3	-10	-1	8	-14	-5	4	13	22	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	-7	2	11	-11	-2	7	-15	-6	3	12	0
4	15	24	-29	-20	-11	-2	7	16	25	3	-19	-10	-1	8	17	-5	4	13	-9	0	9	-13	-4	5	14	-8	1	-21	-12	-3	6	0
5	9	18	-4	5	-17	-8	1	10	-12	-3	6	15	-7	2	11	-11	-2	7	16	-6	3	-19	-10	-1	8	17	26	-27	-18	-9	0	0
6	3	12	-10	-1	8	17	-5	4	-18	-9	0	9	18	-4	-26	-17	-8	1	10	19	28	-25	-16	-7	2	11	20	-2	7	-15	-6	0
7	-3	6	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	13	-9	0	9	-13	-4	5	14	-8	1	10	19	0
8	22	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	-7	2	11	-11	-2	7	-15	-6	3	12	21	-1	-23	-14	-5	4	13	0
9	16	-6	3	-19	-10	-1	8	17	-5	4	13	-9	0	9	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	-29	-20	-11	-2	7	0
10	10	-12	-3	6	15	-7	2	-20	-11	-2	7	16	25	3	-19	-10	-1	8	17	26	-27	-18	-9	0	9	18	-4	5	-17	-8	1	0
11	4	-18	-9	0	9	18	27	-26	-17	-8	1	10	19	-3	-25	-16	-7	2	11	20	-2	7	-15	-6	3	12	-10	-1	8	17	-5	0
12	29	-24	-15	-6	3	12	21	-1	-23	-14	-5	4	13	-9	0	9	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	-29	-20	-11	0
13	23	1	-21	-12	-3	6	15	-7	2	11	-11	-2	7	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	-22	-13	-4	5	14	0
14	-14	-5	4	13	-9	0	9	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	2	-20	-11	-2	7	16	-6	3	-19	-10	-1	8	0
15	-20	-11	-2	7	16	25	-28	-19	-10	-1	8	17	26	4	-18	-9	0	9	18	-4	5	-17	-8	1	10	-12	-3	6	15	-7	2	0
16	-26	-17	-8	1	10	19	-3	6	-16	-7	2	11	-11	-2	7	-15	-6	3	12	-10	-1	8	17	-5	4	-18	-9	0	9	18	27	0
17	-1	8	-14	-5	4	13	-9	0	9	-13	-4	5	-17	-8	1	10	19	-3	-25	-16	-7	2	11	20	29	-24	-15	-6	3	12	21	0
18	-7	2	11	-11	-2	7	-15	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	-22	-13	-4	5	14	-8	1	10	-12	-3	6	15	0
19	-13	-4	5	14	23	1	-21	-12	-3	6	15	24	2	-20	-11	-2	7	16	-6	3	12	-10	-1	8	-14	-5	4	13	-9	0	-22	0
20	-19	-10	-1	8	17	-5	4	-18	-9	0	9	18	-4	5	14	-8	1	10	-12	-3	6	15	-7	2	-20	-11	-2	7	16	25	-28	0
21	6	-16	-7	2	11	-11	-2	7	16	-6	3	-19	-10	-1	8	17	-5	4	-18	-9	0	9	18	27	-26	-17	-8	1	10	19	-3	0
22	0	9	18	-4	5	-17	-8	1	10	19	-3	-25	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	-1	8	-14	-5	4	13	-9	0
23	-6	3	12	21	-1	-23	-14	-5	4	13	22	0	-22	-13	-4	5	14	-8	1	10	-12	-3	6	-16	-7	2	11	20	-2	7	-15	0
24	-12	-3	6	15	24	2	-20	-11	-2	7	16	-6	3	12	-10	-1	8	-14	-5	4	13	22	0	-22	-13	-4	5	14	23	-30	-21	0
25	-18	-9	0	9	-13	-4	5	14	-8	1	10	-12	-3	6	15	24	2	-20	-11	-2	7	16	25	-28	-19	-10	-1	8	17	-5	4	0
26	7	16	-6	3	-19	-10	-1	8	17	26	-27	-18	-9	0	9	18	27	-26	-17	-8	1	10	19	-3	6	-16	-7	2	11	-11	-2	0
27	1	10	19	28	-25	-16	-7	2	11	20	-2	-24	-15	-6	3	12	-10	-1	8	-14	-5	4	13	-9	0	9	18	-4	5	-17	-8	0
28	-5	4	13	22	0	-22	-13	-4	5	14	-8	1	10	-12	-3	6	-16	-7	2	11	20	-2	-24	-15	-6	3	12	21	30	-23	-14	0
29	-11	-2	7	16	-6	3	12	-10	-1	8	-14	-5	4	13	22	0	-22	-13	-4	5	14	23	1	-21	-12	-3	6	15	-7	2	-20	0
30	14	-8	1	-21	-12	-3	6	15	-7	2	-20	-11	-2	7	16	25	3	-19	-10	-1	8	17	-5	4	-18	-9	0	9	-13	-4	5	0
31	8	17	26	-27	-18	-9	0	9	18	-4	5	-17	-8	1	10	19	-3	6	-16	-7	2	11	-11	-2	7	16	-6	3	-19	-10	-1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

For Class-4 Squares

Here is the cloaking property of these Class-4 tiling patterns which are very different from Class-1 tiling patterns. Here, every diamond and X-pattern sums to **0**.

Just as for Class-1 squares, the loom tables do not need to be aligned. This property is continuous for Class-4 squares too. Each loom table cloaks the other.

Size-12

Here is the same size-12 square (p.1-31) but with segregated diamond and X patterns shown. Note that it is pairwise row symmetric.

	1	2	3	4	5	6	7	8	9	10	11	12	
12x12	870	870	870	870	870	870	870	870	870	870	870	870	870
1	109	30	127	90	43	108	37	102	55	18	115	36	870
2	67	84	49	24	133	6	139	12	121	96	61	78	870
3	107	41	128	89	32	110	35	113	56	17	104	38	870
4	69	100	51	22	117	4	141	28	123	94	45	76	870
5	75	46	93	124	27	142	3	118	21	52	99	70	870
6	44	98	59	14	119	29	116	26	131	86	47	101	870
7	73	66	91	126	7	144	1	138	19	54	79	72	870
8	31	120	13	60	97	42	103	48	85	132	25	114	870
9	140	2	95	122	71	77	68	74	23	50	143	5	870
10	33	136	15	58	81	40	105	64	87	130	9	112	870
11	111	10	129	88	63	106	39	82	57	16	135	34	870
12	11	137	20	53	80	62	83	65	92	125	8	134	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

Program 3

X(12)

1	6	7	6	7	12	1	6	7	6	7	12
7	12	1	12	1	6	7	12	1	12	1	6
11	5	8	5	8	2	11	5	8	5	8	2
9	4	3	10	9	4	9	4	3	10	9	4
3	10	9	4	3	10	3	10	9	4	3	10
8	2	11	2	11	5	8	2	11	2	11	5
1	6	7	6	7	12	1	6	7	6	7	12
7	12	1	12	1	6	7	12	1	12	1	6
8	2	11	2	11	5	8	2	11	2	11	5
9	4	3	10	9	4	9	4	3	10	9	4
3	10	9	4	3	10	3	10	9	4	3	10
11	5	8	5	8	2	11	5	8	5	8	2

Here are its loom-tables derived in the base **12**.

Note that the integer loom-table **Y** has absolutely no repetitive pat-terns in it.

However, the modulus loom-table X has numerous repetitive vert-ical patterns as well as a duplication of the left and right.

Y(12)

10	3	11	8	4	9	4	9	5	2	10	3
6	7	5	2	12	1	12	1	11	8	6	7
9	4	11	8	3	10	3	10	5	2	9	4
6	9	5	2	10	1	12	3	11	8	4	7
7	4	8	11	3	12	1	10	2	5	9	6
4	9	5	2	10	3	10	3	11	8	4	9
7	6	8	11	1	12	1	12	2	5	7	6
3	10	2	5	9	4	9	4	8	11	3	10
12	1	8	11	6	7	6	7	2	5	12	1
3	12	2	5	7	4	9	6	8	11	1	10
10	1	11	8	6	9	4	7	5	2	12	3
1	12	2	5	7	6	7	6	8	11	1	12

V(12) = X(12) - Y(12)

-9	3	-4	-2	3	3	-3	-3	2	4	-3	9
1	5	-4	10	-11	5	-5	11	-10	4	-5	-1
2	1	-3	-3	5	-8	8	-5	3	3	-1	-2
3	-5	-2	8	-1	3	-3	1	-8	2	5	-3
-4	6	1	-7	0	-2	2	0	7	-1	-6	4
4	-7	6	0	1	2	-2	-1	0	-6	7	-4
-6	0	-1	-5	6	0	0	-6	5	1	0	6
4	2	-1	7	-8	2	-2	8	-7	1	-2	-4
-4	1	3	-9	5	-2	2	-5	9	-3	-1	4
6	-8	1	5	2	0	0	-2	-5	-1	8	-6
-7	9	-2	-4	-3	1	-1	3	4	2	-9	7
10	-7	6	0	1	-4	4	-1	0	-6	7	-10

Here is the resulting difference table **V(12)**. Now it's quite easy to see that both the X and diamond tiles sum to **0** everywhere.

Program 3

Size-24

n = 4b where b = 6, an even number

Here is the size-24 primal square.. It is pairwise row symmetric.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	217	66	235	414	439	432	145	138	163	342	511	360	505	354	523	126	151	144	6924
2	103	192	85	564	313	546	319	552	301	204	97	186	391	480	373	276	25	258	31	264	13	492	385	474	6924
3	356	506	311	194	143	149	140	146	95	554	359	509	68	218	23	482	431	437	428	434	383	266	71	221	6924
4	105	208	87	562	297	544	321	568	303	202	81	184	393	496	375	274	9	256	33	280	15	490	369	472	6924
5	327	514	345	160	135	178	111	154	129	520	351	538	39	226	57	448	423	466	399	442	417	232	63	250	6924
6	83	209	92	557	296	566	299	569	308	197	80	206	371	497	380	269	8	278	11	281	20	485	368	494	6924
7	325	534	343	162	115	180	109	174	127	522	331	540	37	246	55	450	403	468	397	462	415	234	43	252	6924
8	139	156	121	528	349	510	355	516	337	168	133	150	427	444	409	240	61	222	67	228	49	456	421	438	6924
9	323	545	344	161	104	182	107	185	128	521	320	542	35	257	56	449	392	470	395	473	416	233	32	254	6924
10	141	172	123	526	333	508	357	532	339	166	117	148	429	460	411	238	45	220	69	244	51	454	405	436	6924
11	291	550	309	196	99	214	75	190	93	556	315	574	3	262	21	484	387	502	363	478	381	268	27	286	6924
12	116	170	131	518	335	533	332	530	347	158	119	173	404	458	419	230	47	245	44	242	59	446	407	461	6924
13	289	570	307	198	79	216	73	210	91	558	295	576	1	282	19	486	367	504	361	498	379	270	7	288	6924
14	247	48	229	420	457	402	463	408	445	60	241	42	535	336	517	132	169	114	175	120	157	348	529	330	6924
15	500	362	455	50	287	5	284	2	239	410	503	365	212	74	167	338	575	293	572	290	527	122	215	77	6924
16	249	64	231	418	441	400	465	424	447	58	225	40	537	352	519	130	153	112	177	136	159	346	513	328	6924
17	471	370	489	16	279	34	255	10	273	376	495	394	183	82	201	304	567	322	543	298	561	88	207	106	6924
18	227	65	236	413	440	422	443	425	452	53	224	62	515	353	524	125	152	134	155	137	164	341	512	350	6924
19	469	390	487	18	259	36	253	30	271	378	475	396	181	102	199	306	547	324	541	318	559	90	187	108	6924
20	283	12	265	384	493	366	499	372	481	24	277	6	571	300	553	96	205	78	211	84	193	312	565	294	6924
21	467	401	488	17	248	38	251	41	272	377	464	398	179	113	200	305	536	326	539	329	560	89	176	110	6924
22	285	28	267	382	477	364	501	388	483	22	261	4	573	316	555	94	189	76	213	100	195	310	549	292	6924
23	435	406	453	52	243	70	219	46	237	412	459	430	147	118	165	340	531	358	507	334	525	124	171	142	6924
24	260	26	275	374	479	389	476	386	491	14	263	29	548	314	563	86	191	101	188	98	203	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Program 3

Size-24 Loom-tables

X(24)

300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	
1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	300
7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	300
20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	300
9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	300
15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	300
11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	300
13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	300
19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	300
11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	300
21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	300
3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	300
20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	300
1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	300
7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	300
20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	300
9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	300
15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	300
11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	300
13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	300
19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	300
11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	300
21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	300
3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	300
20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	300
300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	
	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300

Y(24)

300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	
19	18	19	3	10	3	10	3	10	18	19	18	7	6	7	15	22	15	22	15	22	6	7	6	300
5	8	4	24	14	23	14	23	13	9	5	8	17	20	16	12	2	11	2	11	1	21	17	20	300
15	22	13	9	6	7	6	7	4	24	15	22	3	10	1	21	18	19	18	19	16	12	3	10	300
5	9	4	24	13	23	14	24	13	9	4	8	17	21	16	12	1	11	2	12	1	21	16	20	300
14	22	15	7	6	8	5	7	6	22	15	23	2	10	3	19	18	20	17	19	18	10	3	11	300
4	9	4	24	13	24	13	24	13	9	4	9	16	21	16	12	1	12	1	12	1	21	16	21	300
14	23	15	7	5	8	5	8	6	22	14	23	2	11	3	19	17	20	17	20	18	10	2	11	300
6	7	6	22	15	22	15	22	15	7	6	7	18	19	18	10	3	10	3	10	3	19	18	19	300
14	23	15	7	5	8	5	8	6	22	14	23	2	11	3	19	17	20	17	20	18	10	2	11	300
6	8	6	22	14	22	15	23	15	7	5	7	18	20	18	10	2	10	3	11	3	19	17	19	300
13	23	13	9	5	9	4	8	4	24	14	24	1	11	1	21	17	21	16	20	16	12	2	12	300
5	8	6	22	14	23	14	23	15	7	5	8	17	20	18	10	2	11	2	11	3	19	17	20	300
13	24	13	9	4	9	4	9	4	24	13	24	1	12	1	21	16	21	16	21	16	12	1	12	300
11	2	10	18	20	17	20	17	19	3	11	2	23	14	22	6	8	5	8	5	7	15	23	14	300
21	16	19	3	12	1	12	1	10	18	21	16	9	4	7	15	24	13	24	13	22	6	9	4	300
11	3	10	18	19	17	20	18	19	3	10	2	23	15	22	6	7	5	8	6	7	15	22	14	300
20	16	21	1	12	2	11	1	12	16	21	17	8	4	9	13	24	14	23	13	24	4	9	5	300
10	3	10	18	19	18	19	18	19	3	10	3	22	15	22	6	7	6	7	6	7	15	22	15	300
20	17	21	1	11	2	11	2	12	16	20	17	8	5	9	13	23	14	23	14	24	4	8	5	300
12	1	12	16	21	16	21	16	21	1	12	1	24	13	24	4	9	4	9	4	9	13	24	13	300
20	17	21	1	11	2	11	2	12	16	20	17	8	5	9	13	23	14	23	14	24	4	8	5	300
12	2	12	16	20	16	21	17	21	1	11	1	24	14	24	4	8	4	9	5	9	13	23	13	300
19	17	19	3	11	3	10	2	10	18	20	18	7	5	7	15	23	15	22	14	22	6	8	6	300
11	2	12	16	20	17	20	17	21	1	11	2	23	14	24	4	8	5	8	5	9	13	23	14	300
300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	
	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300

Program 3

Size-24 Loom-Difference Table

Here is the size-24 loom-difference table. It is readily apparent that the tiles all sum to **0**.

V(24)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-18	0	0	3	-3	21	-9	15	9	-12	-12	6	-6	12	12	-9	-15	9	-21	3	-3	0	0	18	0
2	16	9	-12	-13	-5	-7	1	0	3	-4	10	-10	4	-3	0	-1	7	5	13	12	-9	-16	-2	0
5	-20	10	-7	17	-2	14	-5	19	-22	8	-17	17	-8	22	-19	5	-14	2	-17	7	-10	20	-5	0
4	7	11	-14	-4	-7	-5	-8	2	1	5	8	-8	-5	-1	-2	8	5	7	4	14	-11	-7	-4	0
1	-12	-6	9	9	2	10	3	3	-6	0	-13	13	0	6	-3	-3	-10	-2	-9	-9	6	12	-1	0
7	8	16	-19	-5	-10	-2	-7	7	-4	4	5	-5	-4	4	-7	7	2	10	5	19	-16	-8	-7	0
-1	-17	-8	11	14	4	8	-2	1	-4	5	-11	11	-5	4	-1	2	-8	-4	-14	-11	8	17	1	0
13	5	-5	2	-2	-16	4	-10	-14	17	7	-1	1	-7	-17	14	10	-4	16	2	-2	5	-5	-13	0
-3	-6	-7	10	3	6	6	9	2	-5	-6	-9	9	6	5	-2	-9	-6	-6	-3	-10	7	6	3	0
15	-4	-3	0	7	-18	6	-19	-12	15	16	-3	3	-16	-15	12	19	-6	18	-7	0	3	4	-15	0
-10	-1	8	-5	-2	13	-1	14	17	-20	-11	-2	2	11	20	-17	-14	1	-13	2	5	-8	1	10	0
15	-6	5	-8	9	-18	6	-21	-4	7	18	-3	3	-18	-7	4	21	-6	18	-9	8	-5	6	-15	0
-12	-6	6	-3	3	15	-3	9	15	-18	-6	0	0	6	18	-15	-9	3	-15	-3	3	-6	6	12	0
-4	22	3	-6	-19	1	-13	7	-6	9	-10	16	-16	10	-9	6	-7	13	-1	19	6	-3	-22	4	0
-1	-14	4	-1	11	4	8	1	13	-16	2	-11	11	-2	16	-13	-1	-8	-4	-11	1	-4	14	1	0
-2	13	5	-8	-10	-1	-11	-2	-4	7	-1	14	-14	1	-7	4	2	11	1	10	8	-5	-13	2	0
-5	-6	-12	15	3	8	4	9	-3	0	-6	-7	7	6	0	3	-9	-4	-8	-3	-15	12	6	5	0
1	14	10	-13	-11	-4	-8	-1	1	2	-2	11	-11	2	-2	-1	1	8	4	11	13	-10	-14	-1	0
-7	-11	-14	17	8	10	2	4	-5	2	-1	-5	5	1	-2	5	-4	-2	-10	-8	-17	14	11	7	0
7	11	-11	8	-8	-10	-2	-4	-20	23	1	5	-5	-1	-23	20	4	2	10	8	-8	11	-11	-7	0
-9	0	-13	16	-3	12	0	15	-4	1	-12	-3	3	12	-1	4	-15	0	-12	3	-16	13	0	9	0
9	2	-9	6	1	-12	0	-13	-18	21	10	3	-3	-10	-21	18	13	0	12	-1	-6	9	-2	-9	0
-16	5	2	1	-8	19	-7	20	11	-14	-17	4	-4	17	14	-11	-20	7	-19	8	-1	-2	-5	16	0
9	0	-1	-2	3	-12	0	-15	-10	13	12	3	-3	-12	-13	10	15	0	12	-3	2	1	0	-9	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In general, every pattern which summed equally and continuously to the square's characteristic number will sum to 0 on the loom-difference table for Class-4 squares whose size is a multiple of **8** or **12**.

Program 3

Size-28 and it's Loom Difference table

Here's the size-28 square again. The square is neither pair-wise row nor centrally symmetric yet is pangenic where **q=1570**. Each diamond and X tile-pattern sums to $\mathbf{C_{28}}$ continuously.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	615	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990

Here is the size-28 difference-table between its two loom-tables (neither of which is shown here) as expressed in the base **28**. It is geonomically equal to **0**. Further, all of its diamond and X tiling patterns, each containing **28** cells, sum equally to **0** too. So the size-28 square also has the cloaking property. It just takes some calculating in software program Excel™ to certify all that.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
	V(28)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-1	-6	-11	19	1	3	-2	8	10	5	-22	10	19	-15	-4	19	-8	-6	-2	-1	1	12	-22	1	10	-15	-13	10	0
2	-5	23	7	-2	-16	-4	16	0	-7	5	-4	-11	-5	14	-2	-2	-25	23	16	0	-16	-4	-4	9	-7	14	-2	-11	0
3	0	-21	6	4	9	-12	-14	13	20	-3	-12	-7	0	6	-3	4	9	-21	6	13	18	-12	-12	-7	20	-3	-3	2	0
4	1	10	13	-6	3	-1	15	3	-17	8	24	-24	1	10	4	-15	10	-10	6	3	-17	-1	15	-17	-8	1	4	-15	0
5	14	-13	-9	21	-6	-4	0	-6	3	7	-20	3	12	-13	-18	12	-6	-4	-9	21	3	-2	0	3	12	7	-20	12	0
6	-3	16	0	-7	-23	18	18	2	-14	-2	-9	11	-12	16	0	-9	-3	18	9	-7	-14	-2	18	2	-12	7	-9	-9	0
7	2	1	-1	6	11	-19	1	15	13	-10	-10	-5	22	-8	-1	-3	2	-19	8	6	4	-10	-19	15	22	-1	-10	-12	0
8	3	12	-1	-13	5	-8	8	5	-15	-6	17	-22	-6	3	6	-13	-4	12	8	-4	5	1	17	-2	-15	3	26	-22	0
9	7	-11	-16	14	-4	-9	-7	16	5	0	2	5	7	9	-25	14	16	-11	-7	16	-4	-9	2	-4	5	9	-25	5	0
10	-10	11	2	-14	-1	20	11	-5	-19	0	13	4	-10	9	-7	-14	-1	11	2	-5	-21	20	13	4	-19	0	-7	13	0
11	24	-6	1	-1	-3	-17	3	8	6	-8	-17	10	24	-6	-8	-10	4	3	-6	8	6	-17	3	17	15	-15	-8	-10	0
12	-11	5	1	-11	-2	14	10	-9	7	-4	19	0	-13	5	21	-20	-2	14	1	-11	7	-13	10	0	-13	-4	19	-20	0
13	9	11	-23	9	18	-16	-5	18	-2	-7	-3	-2	0	11	-23	7	9	-16	-14	9	-2	-7	-5	18	0	2	-3	7	0
14	-8	4	-5	-12	1	13	-3	-3	-26	22	15	6	-17	-5	-5	8	-8	13	4	-12	-6	22	6	-3	-17	2	15	-1	0
15	26	-4	-13	-8	-1	5	-4	10	8	-22	5	12	17	-13	-6	-8	19	-4	-4	1	-1	-15	5	3	8	-13	-15	12	0
16	-18	7	23	-18	0	9	3	-16	9	-11	12	2	-18	-2	14	-18	-9	7	3	-16	0	9	12	-7	9	-2	14	2	0
17	2	6	-21	2	11	-14	-12	11	-7	-5	-10	20	2	4	-1	2	11	6	-21	11	20	-14	-10	20	-7	-5	-1	0	0
18	-15	-3	-3	10	-13	15	-1	-10	-4	24	8	-8	-15	-3	17	1	-6	6	-10	-10	-4	15	-1	-1	-24	17	17	1	0
19	12	-11	-11	-6	21	-2	-2	-4	1	-20	7	5	10	-11	-20	14	21	-2	-11	-6	1	0	-2	5	10	-20	7	14	0
20	-16	0	16	-23	-7	2	5	-14	2	11	7	-5	4	0	16	4	-16	2	25	-23	2	11	5	-14	4	-9	7	4	0
21	4	-1	1	4	13	8	-26	13	15	-12	-8	22	-5	-10	1	-5	4	8	-19	4	6	-12	-17	13	-5	-3	-8	15	0
22	-13	-1	12	3	-11	8	-8	-8	-2	10	1	-6	-22	19	19	3	-20	-1	-8	12	-11	17	1	-15	-2	19	10	-6	0
23	5	-9	-18	16	23	-7	-9	-11	3	2	0	7	5	-18	2	16	14	-9	-9	-11	23	-7	0	-2	3	-18	2	7	0
24	6	-5	18	-1	-14	4	27	-21	-3	13	0	-12	6	-7	9	-1	-14	-5	18	-21	-5	4	0	-12	-3	13	9	-3	0
25	-3	-8	3	-3	-1	10	-24	6	8	-10	-15	8	-3	-8	-6	17	6	1	-4	6	8	10	-24	15	17	-17	-6	17	0
26	-27	21	14	5	-18	1	-6	7	-9	12	3	-13	0	21	5	-4	-18	1	14	5	-9	3	-6	-13	0	12	3	-4	0
27	7	-16	4	11	16	-14	-7	-9	25	-5	-5	0	-2	-16	4	9	7	-14	-16	11	25	-5	-7	-9	-2	4	-5	9	0
28	8	-12	11	1	-12	-3	13	-19	-10	6	2	-10	-1	8	11	-8	8	-3	20	1	-19	6	22	-19	-1	15	2	-17	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	

Program 3

Class-1 string summations found in some Class-4 loom-tables

X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	20	13	11	9	2	5	18	16	14	7	10	3	1	19	12	15	8	6	4	17
2	6	14	2	10	18	1	9	17	5	13	16	4	12	20	8	11	19	7	15	3
3	12	5	8	11	4	7	20	3	6	19	2	15	18	1	14	17	10	13	16	9
4	13	11	9	7	5	18	16	14	12	10	3	1	19	17	15	8	6	4	2	20
5	4	17	15	13	6	9	2	20	18	11	14	7	5	3	16	19	12	10	8	1
6	15	18	6	14	17	10	13	1	9	12	5	8	16	4	7	20	3	11	19	2
7	16	4	12	20	8	11	19	7	15	3	6	14	2	10	18	1	9	17	5	13
8	17	20	13	6	9	2	5	18	11	14	7	10	3	16	19	12	15	8	1	4
9	3	1	19	17	15	8	6	4	2	20	13	11	9	7	5	18	16	14	12	10
10	19	2	10	18	1	14	17	5	13	16	9	12	20	8	11	4	7	15	3	6
11	5	8	16	4	7	20	3	11	19	2	15	18	6	14	17	10	13	1	9	12
12	1	19	17	15	13	6	4	2	20	18	11	9	7	5	3	16	14	12	10	8
13	7	10	3	16	19	12	15	8	1	4	17	20	13	6	9	2	5	18	11	14
14	18	6	14	2	10	13	1	9	17	5	8	16	4	12	20	3	11	19	7	15
15	9	12	20	8	11	4	7	15	3	6	19	2	10	18	1	14	17	5	13	16
16	10	3	1	19	12	15	8	6	4	17	20	13	11	9	2	5	18	16	14	7
17	11	9	7	5	3	16	14	12	10	8	1	19	17	15	13	6	4	2	20	18
18	2	15	18	1	14	17	10	13	16	9	12	5	8	11	4	7	20	3	6	19
19	8	16	4	12	20	3	11	19	7	15	18	6	14	2	10	13	1	9	17	5
20	14	7	5	3	16	19	12	10	8	1	4	17	15	13	6	9	2	20	18	11

The modulus loom table **X** derived directly from **W(20)** in the base **20** has the Class-1 string-summing patterns. Observe that every row, column and diagonal in both major and minor directions contains the numbers **1** thru **20** exactly once.

However, the corresponding integer loom-table **Y** has none of these properties.

X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	20	13	11	9	2	5	18	16	14	7	10	3	1	19	12	15	8	6	4	17
2	6	14	2	10	18	1	9	17	5	13	16	4	12	20	8	11	19	7	15	3
3	12	5	8	11	4	7	20	3	6	19	2	15	18	1	14	17	10	13	16	9
4	13	11	9	7	5	18	16	14	12	10	3	1	19	17	15	8	6	4	2	20
5	4	17	15	13	6	9	2	20	18	11	14	7	5	3	16	19	12	10	8	1
6	15	18	6	14	17	10	13	1	9	12	5	8	16	4	7	20	3	11	19	2
7	16	4	12	20	8	11	19	7	15	3	6	14	2	10	18	1	9	17	5	13
8	17	20	13	6	9	2	5	18	11	14	7	10	3	16	19	12	15	8	1	4
9	3	1	19	17	15	8	6	4	2	20	13	11	9	7	5	18	16	14	12	10
10	19	2	10	18	1	14	17	5	13	16	9	12	20	8	11	4	7	15	3	6
11	5	8	16	4	7	20	3	11	19	2	15	18	6	14	17	10	13	1	9	12
12	1	19	17	15	13	6	4	2	20	18	11	9	7	5	3	16	14	12	10	8
13	7	10	3	16	19	12	15	8	1	4	17	20	13	6	9	2	5	18	11	14
14	18	6	14	2	10	13	1	9	17	5	8	16	4	12	20	3	11	19	7	15
15	9	12	20	8	11	4	7	15	3	6	19	2	10	18	1	14	17	5	13	16
16	10	3	1	19	12	15	8	6	4	17	20	13	11	9	2	5	18	16	14	7
17	11	9	7	5	3	16	14	12	10	8	1	19	17	15	13	6	4	2	20	18
18	2	15	18	1	14	17	10	13	16	9	12	5	8	11	4	7	20	3	6	19
19	8	16	4	12	20	3	11	19	7	15	18	6	14	2	10	13	1	9	17	5
20	14	7	5	3	16	19	12	10	8	1	4	17	15	13	6	9	2	20	18	11

Note that this sub-class of **n = 4b** applies only to <u>odd</u> **b**.

Now for **b = 3**, going in reverse order by **8**, neither size **12**'s loom-tables have these characteristic Class-1 string-summing properties. The size-12 is embryonic and has an insufficient quantity of numbers to possess this property. So they begin at size **20** for **b = 5** and continue on from there in size increments of **8**.

X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	20	13	11	9	2	5	18	16	14	7	10	3	1	19	12	15	8	6	4	17
2	6	14	2	10	18	1	9	17	5	13	16	4	12	20	8	11	19	7	15	3
3	12	5	8	11	4	7	20	3	6	19	2	15	18	1	14	17	10	13	16	9
4	13	11	9	7	5	18	16	14	12	10	3	1	19	17	15	8	6	4	2	20
5	4	17	15	13	6	9	2	20	18	11	14	7	5	3	16	19	12	10	8	1
6	15	18	6	14	17	10	13	1	9	12	5	8	16	4	7	20	3	11	19	2
7	16	4	12	20	8	11	19	7	15	3	6	14	2	10	18	1	9	17	5	13
8	17	20	13	6	9	2	5	18	11	14	7	10	3	16	19	12	15	8	1	4
9	3	1	19	17	15	8	6	4	2	20	13	11	9	7	5	18	16	14	12	10
10	19	2	10	18	1	14	17	5	13	16	9	12	20	8	11	4	7	15	3	6
11	5	8	16	4	7	20	3	11	19	2	15	18	6	14	17	10	13	1	9	12
12	1	19	17	15	13	6	4	2	20	18	11	9	7	5	3	16	14	12	10	8
13	7	10	3	16	19	12	15	8	1	4	17	20	13	6	9	2	5	18	11	14
14	18	6	14	2	10	13	1	9	17	5	8	16	4	12	20	3	11	19	7	15
15	9	12	20	8	11	4	7	15	3	6	19	2	10	18	1	14	17	5	13	16
16	10	3	1	19	12	15	8	6	4	17	20	13	11	9	2	5	18	16	14	7
17	11	9	7	5	3	16	14	12	10	8	1	19	17	15	13	6	4	2	20	18
18	2	15	18	1	14	17	10	13	16	9	12	5	8	11	4	7	20	3	6	19
19	8	16	4	12	20	3	11	19	7	15	18	6	14	2	10	13	1	9	17	5
20	14	7	5	3	16	19	12	10	8	1	4	17	15	13	6	9	2	20	18	11

Although not shown in this program, the size-28's modulus loom table taken in the base **28** has these same linear 4-directional summation patterns. Check it out for yourself (re: prior page).

X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	20	13	11	9	2	5	18	16	14	7	10	3	1	19	12	15	8	6	4	17
2	6	14	2	10	18	1	9	17	5	13	16	4	12	20	8	11	19	7	15	3
3	12	5	8	11	4	7	20	3	6	19	2	15	18	1	14	17	10	13	16	9
4	13	11	9	7	5	18	16	14	12	10	3	1	19	17	15	8	6	4	2	20
5	4	17	15	13	6	9	2	20	18	11	14	7	5	3	16	19	12	10	8	1
6	15	18	6	14	17	10	13	1	9	12	5	8	16	4	7	20	3	11	19	2
7	16	4	12	20	8	11	19	7	15	3	6	14	2	10	18	1	9	17	5	13
8	17	20	13	6	9	2	5	18	11	14	7	10	3	16	19	12	15	8	1	4
9	3	1	19	17	15	8	6	4	2	20	13	11	9	7	5	18	16	14	12	10
10	19	2	10	18	1	14	17	5	13	16	9	12	20	8	11	4	7	15	3	6
11	5	8	16	4	7	20	3	11	19	2	15	18	6	14	17	10	13	1	9	12
12	1	19	17	15	13	6	4	2	20	18	11	9	7	5	3	16	14	12	10	8
13	7	10	3	16	19	12	15	8	1	4	17	20	13	6	9	2	5	18	11	14
14	18	6	14	2	10	13	1	9	17	5	8	16	4	12	20	3	11	19	7	15
15	9	12	20	8	11	4	7	15	3	6	19	2	10	18	1	14	17	5	13	16
16	10	3	1	19	12	15	8	6	4	17	20	13	11	9	2	5	18	16	14	7
17	11	9	7	5	3	16	14	12	10	8	1	19	17	15	13	6	4	2	20	18
18	2	15	18	1	14	17	10	13	16	9	12	5	8	11	4	7	20	3	6	19
19	8	16	4	12	20	3	11	19	7	15	18	6	14	2	10	13	1	9	17	5
20	14	7	5	3	16	19	12	10	8	1	4	17	15	13	6	9	2	20	18	11

Program 3

For Class-3 Squares

Size **n = 3b** where **b** is an odd-number **> 3**

Size-15 block-patterns

Here is the size-15 square. It has **5 3x3** block-square tiling patterns which sum to **3** times the square's characteristic number. Note that the tiling pattern is identical with the X-pattern of size-5 square but at the **3x3** block-square level.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
W(15)	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	108	221	64	77	170	33	146	214	2	95	183	71	139	152	1695
2	111	204	42	85	198	36	129	192	10	123	186	54	117	160	48	1695
3	207	100	13	176	144	57	175	88	26	219	132	25	163	101	69	1695
4	153	41	134	222	15	78	191	59	147	165	3	116	209	72	90	1695
5	49	137	180	18	106	199	62	105	168	31	124	212	30	93	181	1695
6	70	83	171	39	127	220	8	96	189	52	145	158	21	114	202	1695
7	86	179	67	135	173	11	104	217	60	98	161	29	142	210	23	1695
8	182	150	38	151	119	32	225	113	1	194	107	75	188	76	44	1695
9	203	16	84	197	65	128	166	9	122	215	53	91	159	47	140	1695
10	24	112	205	68	81	174	37	130	218	6	99	187	55	143	156	1695
11	45	133	196	14	102	195	58	121	164	27	120	208	46	89	177	1695
12	136	154	17	110	223	61	79	167	35	148	211	4	92	185	73	1695
13	157	125	63	201	94	7	200	138	51	169	82	50	213	126	19	1695
14	178	66	109	172	40	103	216	34	97	190	28	141	184	22	115	1695
15	74	87	155	43	131	224	12	80	193	56	149	162	5	118	206	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

The dual square has the same properties. Note that the tiling pattern is identical with the "**+**" pattern of size-5 square but at the **3x3** block-square level.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
U(15)	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	184	112	45	148	61	214	97	30	178	46	199	127	15	163	76	1695
2	23	176	54	197	130	8	161	84	182	115	38	146	69	212	100	1695
3	87	200	93	36	164	57	215	108	6	179	72	185	123	21	149	1695
4	121	9	162	90	183	106	39	147	75	213	91	24	177	60	198	1695
5	145	73	216	94	22	175	58	201	124	7	160	88	186	109	37	1695
6	194	107	35	143	71	224	92	20	173	56	209	122	5	158	86	1695
7	18	171	59	207	125	3	156	89	192	110	33	141	74	222	95	1695
8	82	210	98	31	159	52	225	113	1	174	67	195	128	16	144	1695
9	131	4	152	85	193	116	34	137	70	223	101	19	167	55	208	1695
10	140	68	221	104	17	170	53	206	134	2	155	83	191	119	32	1695
11	189	117	40	138	66	219	102	25	168	51	204	132	10	153	81	1695
12	28	166	49	202	135	13	151	79	187	120	43	136	64	217	105	1695
13	77	205	103	41	154	47	220	118	11	169	62	190	133	26	139	1695
14	126	14	157	80	188	111	44	142	65	218	96	29	172	50	203	1695
15	150	63	211	99	27	180	48	196	129	12	165	78	181	114	42	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

The loom tables were computed using the base **5** instead of **15**. **In fact, all Class-3 loom tables must be derived in the base b where n = 3b, otherwise they will not be geonomic.**

(3.9) **W(15) = 5 ▪ (Y(5)-|1|) + X(5)** for generation of the Primal square

(3.10) **U(15) = 3 ▪ 15 ▪ (X(5)-|1|) + Y(5)** for generation of the Dual square

Program 3

This slide shows loom tables **X** & **Y** for the size-15 square derived from **W(15)** in the base **5**.

X(5)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
1	5	3	1	4	2	5	3	1	4	2	5	3	1	4	2	45
2	1	4	2	5	3	1	4	2	5	3	1	4	2	5	3	45
3	2	5	3	1	4	2	5	3	1	4	2	5	3	1	4	45
4	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5	45
5	4	2	5	3	1	4	2	5	3	1	4	2	5	3	1	45
6	5	3	1	4	2	5	3	1	4	2	5	3	1	4	2	45
7	1	4	2	5	3	1	4	2	5	3	1	4	2	5	3	45
8	2	5	3	1	4	2	5	3	1	4	2	5	3	1	4	45
9	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5	45
10	4	2	5	3	1	4	2	5	3	1	4	2	5	3	1	45
11	5	3	1	4	2	5	3	1	4	2	5	3	1	4	2	45
12	1	4	2	5	3	1	4	2	5	3	1	4	2	5	3	45
13	2	5	3	1	4	2	5	3	1	4	2	5	3	1	4	45
14	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5	45
15	4	2	5	3	1	4	2	5	3	1	4	2	5	3	1	45
	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	
	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45

Y(5)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345
1	4	22	45	13	16	34	7	30	43	1	19	37	15	28	31	345
2	23	41	9	17	40	8	26	39	2	25	38	11	24	32	10	345
3	42	20	3	36	29	12	35	18	6	44	27	5	33	21	14	345
4	31	9	27	45	3	16	39	12	30	33	1	24	42	15	18	345
5	10	28	36	4	22	40	13	21	34	7	25	43	6	19	37	345
6	14	17	35	8	26	44	2	20	38	11	29	32	5	23	41	345
7	18	36	14	27	35	3	21	44	12	20	33	6	29	42	5	345
8	37	30	8	31	24	7	45	23	1	39	22	15	38	16	9	345
9	41	4	17	40	13	26	34	2	25	43	11	19	32	10	28	345
10	5	23	41	14	17	35	8	26	44	2	20	38	11	29	32	345
11	9	27	40	3	21	39	12	25	33	6	24	42	10	18	36	345
12	28	31	4	22	45	13	16	34	7	30	43	1	19	37	15	345
13	32	25	13	41	19	2	40	28	11	34	17	10	43	26	4	345
14	36	14	22	35	8	21	44	7	20	38	6	29	37	5	23	345
15	15	10	31	9	27	45	3	16	39	12	30	33	1	24	42	345
	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345	
	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345

Here is a version of a difference-table that is geonomically **0.** It was derived here by the formula:

(3.11) $V = \bar{y}X - \bar{x}Y$

where $\bar{x}$ is the average value of **X** and $\bar{y}$ is the average value of **Y** that have been derived in the lowest base number consistent with their size.

This formula is a more generalized version than the simple difference table **V = X – Y**.

Note that the values of the average of both **X** and **Y** lie at the center of their odd-size loom-tables. The values of $\bar{x}$ and $\bar{y}$ can be reduced by any common factors.

V = 23X-3Y

V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
103	3	-112	53	-2	13	48	-67	-37	43	58	-42	-22	8	-47	0
-46	-31	19	64	-51	-1	14	-71	109	-6	-91	59	-26	19	39	0
-80	55	60	-85	5	10	10	15	5	-40	-35	100	-30	-40	50	0
-24	-4	11	-89	106	21	-94	56	-44	16	66	-49	-34	1	61	0
62	-38	7	57	-43	-28	7	52	-33	2	17	-83	97	12	-88	0
73	18	-82	68	-32	-17	63	-37	-22	13	28	-27	8	23	-77	0
-31	-16	4	34	-36	14	29	-86	79	9	-76	74	-41	-11	54	0
-65	25	45	-70	20	25	-20	0	20	-25	-20	70	-45	-25	65	0
-54	11	41	-74	76	-9	-79	86	-29	-14	36	-34	-4	16	31	0
77	-23	-8	27	-28	-13	22	37	-63	17	32	-68	82	-18	-73	0
88	-12	-97	83	-17	-2	33	-52	-7	28	43	-57	-7	38	-62	0
-61	-1	34	49	-66	-16	44	-56	94	-21	-106	89	-11	4	24	0
-50	40	30	-100	35	40	-5	-15	-10	-10	-5	85	-60	-55	80	0
-39	-19	26	-59	91	6	-109	71	-14	1	51	-64	-19	31	46	0
47	-8	22	42	-58	-43	37	67	-48	-13	2	-53	112	-3	-103	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

At the 3x3 block-square level

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

At the 3x3 block-square level

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Although only shown here, this cloaking property holds for all Class-3 squares to follow. Thus, all Class-3 squares possess the cloaking property via (**3.16**). You can check them out in Excel™.

Program 3

Size-21 block-patterns

Here is the size-21 primal square. It has **7 3x3** block-square tiling patterns Note that the tiling pattern is identical with the A-pattern of size-7 square but at the **3x3** block-square level. This pattern sums continuously to **3** times the size-21 square's characteristic number.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
W(21)	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
1	399	67	176	439	58	27	430	350	18	274	390	9	125	381	301	116	225	341	107	76	332	4641
2	191	209	122	189	347	260	180	240	258	24	238	396	162	229	289	160	73	287	298	211	278	4641
3	88	295	411	79	195	402	77	39	393	362	30	293	353	28	137	344	313	128	244	304	126	4641
4	426	94	161	417	85	5	408	377	45	259	368	36	103	359	328	143	210	319	134	54	310	4641
5	169	285	51	167	423	245	158	267	236	2	265	374	196	256	218	187	100	216	325	294	207	4641
6	17	378	438	8	222	429	6	66	427	340	57	271	331	55	115	329	389	106	173	380	104	4641
7	404	72	188	395	70	32	435	355	23	286	346	21	130	386	306	121	237	297	119	81	337	4641
8	252	214	29	292	352	174	283	203	165	127	243	303	272	234	154	263	78	194	401	223	185	4641
9	44	356	416	42	200	407	33	93	405	318	91	249	309	82	142	307	367	140	151	358	131	4641
10	382	1	264	373	48	108	371	333	99	215	324	146	59	322	431	50	166	422	97	10	420	4641
11	279	241	14	270	379	152	261	230	192	112	221	330	250	212	181	290	63	172	428	201	163	4641
12	22	432	345	20	276	392	11	120	383	296	118	227	343	109	71	334	394	69	178	441	60	4641
13	311	84	291	302	75	135	300	360	133	193	351	124	37	349	409	35	242	400	26	86	398	4641
14	257	219	41	248	364	179	288	208	170	139	199	315	277	239	159	268	90	150	413	228	190	4641
15	105	361	323	145	205	321	136	56	312	421	96	156	419	87	7	410	372	47	254	370	38	4641
16	338	62	269	336	53	113	327	387	111	171	385	102	15	376	436	13	220	434	4	64	425	4641
17	235	148	117	226	342	255	224	186	246	68	177	440	206	175	284	197	19	275	391	157	273	4641
18	132	388	308	123	232	299	114	83	339	406	74	183	397	65	34	437	357	25	281	348	16	4641
19	316	138	198	314	129	98	305	414	89	149	412	80	49	403	365	40	247	363	31	147	354	4641
20	164	231	144	155	369	282	153	213	280	46	204	418	184	202	262	182	95	253	320	233	251	4641
21	110	366	335	101	217	326	141	61	317	433	52	168	424	92	12	415	384	3	266	375	43	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

Here is the size-21 dual square. The dual version has the same properties as the primal. It is shown with the characteristic B-pattern tiling of the size-7 square in **7 3x3** block-squares. This pattern also sums to **3** times the size-21 square's characteristic number continuously.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
U(21)	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
1	435	199	26	315	72	319	188	428	192	40	308	65	333	181	421	206	33	301	79	326	174	975
2	91	345	144	405	239	38	278	98	352	130	412	246	24	285	105	338	137	419	232	31	292	985
3	202	43	311	75	343	184	389	195	57	304	68	357	177	382	209	50	297	82	350	170	396	4641
4	376	140	401	249	13	253	122	369	133	415	242	6	267	115	362	147	408	235	20	260	108	4641
5	25	293	71	339	187	413	212	39	286	64	353	180	406	226	32	279	78	346	173	420	219	4641
6	129	432	252	2	284	125	316	136	439	238	9	291	111	323	143	425	245	16	277	118	330	4641
7	310	74	342	183	388	194	63	303	67	356	176	381	208	56	296	81	349	169	395	201	49	4641
8	414	220	5	294	114	340	167	407	213	19	287	107	354	160	400	227	12	280	121	347	153	4641
9	70	366	186	384	218	59	257	77	373	172	391	225	45	264	84	359	179	398	211	52	271	4641
10	244	1	290	117	322	142	431	237	15	283	110	336	135	424	251	8	276	124	329	128	438	4641
11	355	161	380	228	55	274	101	348	154	394	221	48	288	94	341	168	387	214	62	281	87	4641
12	4	314	113	318	166	434	191	18	307	106	332	159	427	205	11	300	120	325	152	441	198	4641
13	171	390	231	44	263	83	358	178	397	217	51	270	69	365	185	383	224	58	256	76	372	4641
14	289	95	321	162	430	215	42	282	88	335	155	423	229	35	275	102	328	148	437	222	28	4641
15	393	241	47	273	93	361	146	386	234	61	266	86	375	139	379	248	54	259	100	368	132	4641
16	112	324	165	426	197	17	299	119	331	151	433	204	3	306	126	317	158	440	190	10	313	4641
17	223	22	269	96	364	163	410	216	36	262	89	378	156	403	230	29	255	103	371	149	417	4641
18	334	182	422	207	34	295	80	327	175	436	200	27	309	73	320	189	429	193	41	302	66	4641
19	46	272	92	360	145	392	233	60	265	85	374	138	385	247	53	258	99	367	131	399	240	4641
20	150	411	210	23	305	104	337	157	418	196	30	312	90	344	164	404	203	37	298	97	351	4641
21	268	116	363	141	409	236	21	261	109	377	134	402	250	14	254	123	370	127	416	243	7	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

(3.12) **W(21) = 7▪ (Y(7)-|1|) + X(7)** for generation of the Primal square

(3.13) **U(21) = 3▪ 21▪ (X(7)-|1|) + Y(7)** for generation of the Dual square

Program 3

Size-21 Loom Tables

Here are the size-21 loom tables derived from the primal in the base **7**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
	X(7)	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84
1		7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	84
2		2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	84
3		4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	84
4		6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	84
5		1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	84
6		3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	84
7		5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	84
8		7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	84
9		2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	84
10		4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	84
11		6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	84
12		1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	84
13		3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	84
14		5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	84
15		7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	84
16		2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	84
17		4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	84
18		6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	84
19		1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	84
20		3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	84
21		5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	84
		84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	
		84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84

Note that the ratio of the loom tables' centers is **32 / 4= 8**, so the common factor **4** can be factored out.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
	Y(7)	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672
1		57	10	26	63	9	4	62	50	3	40	56	2	18	55	43	17	33	49	16	11	48	672
2		28	30	18	27	50	38	26	35	37	4	34	57	24	33	42	23	11	41	43	31	40	672
3		13	43	59	12	28	58	11	6	57	52	5	42	51	4	20	50	45	19	35	44	18	672
4		61	14	23	60	13	1	59	54	7	37	53	6	15	52	47	21	30	46	20	8	45	672
5		25	41	8	24	61	35	23	39	34	1	38	54	28	37	32	27	15	31	47	42	30	672
6		3	54	63	2	32	62	1	10	61	49	9	39	48	8	17	47	56	16	25	55	15	672
7		58	11	27	57	10	5	63	51	4	41	50	3	19	56	44	18	34	43	17	12	49	672
8		36	31	5	42	51	25	41	29	24	19	35	44	39	34	22	38	12	28	58	32	27	672
9		7	51	60	6	29	59	5	14	58	46	13	36	45	12	21	44	53	20	22	52	19	672
10		55	1	38	54	7	16	53	48	15	31	47	21	9	46	62	8	24	61	14	2	60	672
11		40	35	2	39	55	22	38	33	28	16	32	48	36	31	26	42	9	25	62	29	24	672
12		4	62	50	3	40	56	2	18	55	43	17	33	49	16	11	48	57	10	26	63	9	672
13		45	12	42	44	11	20	43	52	19	28	51	18	6	50	59	5	35	58	4	13	57	672
14		37	32	6	36	52	26	42	30	25	20	29	45	40	35	23	39	13	22	59	33	28	672
15		15	52	47	21	30	46	20	8	45	61	14	23	60	13	1	59	54	7	37	53	6	672
16		49	9	39	48	8	17	47	56	16	25	55	15	3	54	63	2	32	62	1	10	61	672
17		34	22	17	33	49	37	32	27	36	10	26	63	30	25	41	29	3	40	56	23	39	672
18		19	56	44	18	34	43	17	12	49	58	11	27	57	10	5	63	51	4	41	50	3	672
19		46	20	29	45	19	14	44	60	13	22	59	12	7	58	53	6	36	52	5	21	51	672
20		24	33	21	23	53	41	22	31	40	7	30	60	27	29	38	26	14	37	46	34	36	672
21		16	53	48	15	31	47	21	9	46	62	8	24	61	14	2	60	55	1	38	54	7	672
		672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	
		672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672

Program 3

Size-21 Loom Difference Table

$$V(7) = 8X(7) - Y(7)$$

Here is the loom-table difference table. The number **8** in the formula comes from **the ratio of the loom tables' centers.**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
V(7)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	22	-18	-23	7	44	-38	6	29	-32	-16	14	30	-31	13	15	-25	-9	0	37	-24	0
-12	18	6	29	-18	-30	14	-19	11	20	22	-25	-16	7	-26	25	13	15	-11	-23	0	0
19	-35	-19	4	20	-34	45	26	-49	-12	11	6	-27	52	12	-42	-5	-3	13	-20	38	0
-13	10	33	-28	-5	39	-43	-6	17	19	-21	2	25	-36	1	3	26	-14	-12	32	-29	0
-17	-1	8	24	-37	21	9	-31	6	15	10	-30	28	-5	-24	13	1	17	-23	14	2	0
21	2	-31	6	8	-46	47	14	-5	-17	-1	1	-32	40	7	9	-24	-8	15	-39	33	0
-18	5	21	-33	46	27	-55	-11	12	7	-26	53	13	-48	-4	-2	14	-19	39	20	-41	0
20	1	3	-2	-35	23	-17	27	8	-11	5	-28	9	-10	34	-6	-4	12	-42	16	-3	0
9	-3	-36	50	3	-51	35	2	-10	-22	43	-4	-37	28	-5	4	-29	36	10	-44	21	0
-23	7	2	-38	41	8	3	-16	-7	9	-31	27	15	10	-30	0	16	-45	34	22	-4	0
8	-11	54	-7	-47	18	-22	15	-4	40	0	-40	4	-15	22	-18	47	7	-54	11	-8	0
4	-22	-34	45	-16	0	30	-10	-15	-27	31	-9	7	16	-3	-8	-41	38	-2	-7	23	0
-21	44	-10	-36	29	-4	5	-28	37	4	-43	22	10	-2	-35	51	-3	-50	36	3	-9	0
3	-16	42	-12	4	6	-34	10	-9	28	-5	11	-8	-27	17	-23	35	2	-3	-1	-20	0
41	-20	-39	19	-14	2	4	48	-13	-53	26	-7	-12	11	55	-27	-46	33	-21	-5	18	0
-33	39	-15	8	24	-9	-7	-40	32	-1	1	17	5	-14	-47	46	-8	-6	31	-2	-21	0
-2	-14	23	-17	-1	-13	24	5	-28	30	-10	-15	-6	31	-9	-21	37	-24	-8	1	17	0
29	-32	12	14	-26	-3	-1	36	-25	-2	21	-19	-17	6	43	-39	5	28	-33	-10	13	0
-38	20	-13	3	5	42	-12	-52	27	-6	-11	12	49	-26	-45	34	-20	-4	19	35	-19	0
0	23	11	-15	-13	-25	26	-7	16	25	-22	-20	-11	19	-14	30	18	-29	-6	-18	12	0
24	-37	0	9	25	-15	-13	31	-30	-14	16	32	-29	-6	38	-44	-7	23	18	-22	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The difference table is continuously **7x**-modular at **0**. It is also totally anti-symmetric. The **3x3** size-7 tile patterns A and B sum to **0** continuously.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
V(7)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	22	-18	-23	7	44	-38	6	29	-32	-16	14	30	-31	13	15	-25	-9	0	37	-24	0
-12	18	6	29	-18	-30	14	-19	11	20	22	-25	-16	7	-26	25	13	15	-11	-23	0	0
19	-35	-19	4	20	-34	45	26	-49	-12	11	6	-27	52	12	-42	-5	-3	13	-20	38	0
-13	10	33	-28	-5	39	-43	-6	17	19	-21	2	25	-36	1	3	26	-14	-12	32	-29	0
-17	-1	8	24	-37	21	9	-31	6	15	10	-30	28	-5	-24	13	1	17	-23	14	2	0
21	2	-31	6	8	-46	47	14	-5	-17	-1	1	-32	40	7	9	-24	-8	15	-39	33	0
-18	5	21	-33	46	27	-55	-11	12	7	-26	53	13	-48	-4	-2	14	-19	39	20	-41	0
20	1	3	-2	-35	23	-17	27	8	-11	5	-28	9	-10	34	-6	-4	12	-42	16	-3	0
9	-3	-36	50	3	-51	35	2	-10	-22	43	-4	-37	28	-5	4	-29	36	10	-44	21	0
-23	7	2	-38	41	8	3	-16	-7	9	-31	27	15	10	-30	0	16	-45	34	22	-4	0
8	-11	54	-7	-47	18	-22	15	-4	40	0	-40	4	-15	22	-18	47	7	-54	11	-8	0
4	-22	-34	45	-16	0	30	-10	-15	-27	31	-9	7	16	-3	-8	-41	38	-2	-7	23	0
-21	44	-10	-36	29	-4	5	-28	37	4	-43	22	10	-2	-35	51	-3	-50	36	3	-9	0
3	-16	42	-12	4	6	-34	10	-9	28	-5	11	-8	-27	17	-23	35	2	-3	-1	-20	0
41	-20	-39	19	-14	2	4	48	-13	-53	26	-7	-12	11	55	-27	-46	33	-21	-5	18	0
-33	39	-15	8	24	-9	-7	-40	32	-1	1	17	5	-14	-47	46	-8	-6	31	-2	-21	0
-2	-14	23	-17	-1	-13	24	5	-28	30	-10	-15	-6	31	-9	-21	37	-24	-8	1	17	0
29	-32	12	14	-26	-3	-1	36	-25	-2	21	-19	-17	6	43	-39	5	28	-33	-10	13	0
-38	20	-13	3	5	42	-12	-52	27	-6	-11	12	49	-26	-45	34	-20	-4	19	35	-19	0
0	23	11	-15	-13	-25	26	-7	16	25	-22	-20	-11	19	-14	30	18	-29	-6	-18	12	0
24	-37	0	9	25	-15	-13	31	-30	-14	16	32	-29	-6	38	-44	-7	23	18	-22	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Program 3

Size-27 block-patterns

Here is the size-27 primal square. It has **9** **3x3** block-square tiling patterns which sum continuously to **3** times the square's characteristic number.

Note that the tiling pattern here is new because the size-9 square is not a Class-1 square like the size-5 and size-7 squares which have characteristic tiling patterns.

The loom tables were computed using the base **9** instead of **27**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
	W(27)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1		399	463	314	345	409	260	372	436	287	156	220	71	102	166	17	129	193	44	642	706	557	588	652	503	615	679	530	9855
2		590	657	496	617	684	523	644	711	550	347	414	253	374	441	280	401	468	307	104	171	10	131	198	37	158	225	64	9855
3		133	191	42	160	218	69	106	164	15	619	677	528	646	704	555	592	650	501	376	434	285	403	461	312	349	407	258	9855
4		386	471	319	332	417	265	359	444	292	143	228	76	89	174	22	116	201	49	629	714	562	575	660	508	602	687	535	9855
5		568	662	513	595	689	540	622	716	567	325	419	270	352	446	297	379	473	324	82	176	27	109	203	54	136	230	81	9855
6		114	205	47	141	232	74	87	178	20	600	691	533	627	718	560	573	684	506	357	448	290	384	475	317	330	421	263	9855
7		391	485	300	337	431	246	364	458	273	148	242	57	94	188	3	121	215	30	634	728	543	580	674	489	607	701	516	9855
8		585	667	491	612	694	518	639	721	545	342	424	248	369	451	275	396	478	302	99	181	5	126	208	32	153	235	59	9855
9		119	213	34	146	240	61	92	186	7	605	699	520	632	726	547	578	672	493	362	456	277	389	483	304	335	429	250	9855
10		318	382	476	264	328	422	291	355	449	75	139	233	21	85	179	48	112	206	561	625	719	507	571	665	534	598	692	9855
11		509	576	658	536	603	685	563	630	712	266	333	415	293	360	442	320	387	469	23	90	172	50	117	199	77	144	226	9855
12		52	110	204	79	137	231	25	83	177	538	596	690	565	623	717	511	569	663	295	353	447	322	380	474	268	326	420	9855
13		305	390	481	251	336	427	278	363	454	62	147	238	8	93	184	35	120	211	548	633	724	494	579	670	521	606	697	9855
14		487	581	675	514	608	702	541	635	729	244	338	432	271	365	459	298	392	486	1	95	189	28	122	216	55	149	243	9855
15		33	124	209	60	151	236	6	97	182	519	610	695	546	637	722	492	583	668	276	367	452	303	394	479	249	340	425	9855
16		310	404	462	256	350	408	283	377	435	67	161	219	13	107	165	40	134	192	553	647	705	499	593	651	526	620	678	9855
17		504	586	653	531	613	680	558	640	707	261	343	410	288	370	437	315	397	464	18	100	167	45	127	194	72	154	221	9855
18		38	132	196	65	159	223	11	105	169	524	618	682	551	645	709	497	591	655	281	375	439	308	402	466	254	348	412	9855
19		480	301	395	426	247	341	453	274	368	237	58	152	183	4	98	210	31	125	723	544	638	669	490	584	696	517	611	9855
20		671	495	577	698	522	604	725	549	631	428	252	334	455	279	361	482	306	388	185	9	91	212	36	118	239	63	145	9855
21		214	29	123	241	56	150	187	2	96	700	515	609	727	542	636	673	488	582	457	272	366	484	299	393	430	245	339	9855
22		467	309	400	413	255	346	440	282	373	224	66	157	170	12	103	197	39	130	710	552	643	656	498	589	683	525	616	9855
23		649	500	594	676	527	621	703	554	648	406	257	351	433	284	378	460	311	405	163	14	108	190	41	135	217	68	162	9855
24		195	43	128	222	70	155	168	16	101	681	529	614	708	556	641	654	502	587	438	286	371	465	313	398	411	259	344	9855
25		472	323	381	418	269	327	445	296	354	229	80	138	175	26	84	202	53	111	715	566	624	681	512	570	688	539	597	9855
26		666	505	572	693	532	599	720	559	626	423	262	329	450	289	356	477	316	383	180	19	86	207	46	113	234	73	140	9855
27		200	51	115	227	78	142	173	24	88	686	537	601	713	564	628	659	510	574	443	294	358	470	321	385	416	267	331	9855
		9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
		9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Program 3

Patterns on the Size-27 Dual Square

Here is the dual of the size-27 square. It has the same tiling properties as the primal version. It is shown with another new characteristic tiling pattern at the **3x3** block-square level. This pattern too sums continuously to **3** times the size-27 square's characteristic number.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
U(27)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	207	295	602	201	289	596	204	292	599	180	268	575	174	262	569	177	265	572	234	322	629	228	316	623	231	319	626	9855
2	390	721	56	393	724	59	396	727	62	363	694	29	366	697	32	369	700	35	336	667	2	339	670	5	342	673	8	9855
3	501	103	410	504	106	413	498	100	407	555	157	464	558	160	467	552	154	461	528	130	437	531	133	440	525	127	434	9855
4	610	215	279	604	209	273	607	212	276	583	188	252	577	182	246	580	185	249	637	242	306	631	236	300	634	239	303	9855
5	64	398	705	67	401	708	70	404	711	37	371	678	40	374	681	43	377	684	10	344	651	13	347	654	16	350	657	9855
6	418	509	87	421	512	90	415	506	84	472	563	141	475	566	144	469	560	138	445	536	114	448	539	117	442	533	111	9855
7	287	621	196	281	615	190	284	618	193	260	594	169	254	588	163	257	591	166	314	648	223	308	642	217	311	645	220	9855
8	713	75	379	716	78	382	719	81	385	686	48	352	689	51	355	692	54	358	659	21	325	662	24	328	665	27	331	9855
9	95	429	490	98	432	493	92	426	487	149	483	544	152	486	547	146	480	541	122	456	517	125	459	520	119	453	514	9855
10	198	286	620	192	280	614	195	283	617	171	259	593	165	253	587	168	256	590	225	313	647	219	307	641	222	310	644	9855
11	381	712	74	384	715	77	387	718	80	354	685	47	357	688	50	360	691	53	327	658	20	330	661	23	333	664	26	9855
12	492	94	428	495	97	431	489	91	425	546	148	482	549	151	485	543	145	479	519	121	455	522	124	458	516	118	452	9855
13	601	206	297	595	200	291	598	203	294	574	179	270	568	173	264	571	176	267	628	233	324	622	227	318	625	230	321	9855
14	55	389	723	58	392	726	61	395	729	28	362	696	31	365	699	34	368	702	1	335	669	4	338	672	7	341	675	9855
15	409	500	105	412	503	108	406	497	102	463	554	159	466	557	162	460	551	156	436	527	132	439	530	135	433	524	129	9855
16	278	612	214	272	606	208	275	609	211	251	585	187	245	579	181	248	582	184	305	639	241	299	633	235	302	636	238	9855
17	704	66	397	707	69	400	710	72	403	677	39	370	680	42	373	683	45	376	650	12	343	653	15	346	656	18	349	9855
18	86	420	508	89	423	511	83	417	505	140	474	562	143	477	565	137	471	559	113	447	535	116	450	538	110	444	532	9855
19	216	277	611	210	271	605	213	274	608	189	250	584	183	244	578	186	247	581	243	304	638	237	298	632	240	301	635	9855
20	399	703	65	402	706	68	405	709	71	372	676	38	375	679	41	378	682	44	345	649	11	348	652	14	351	655	17	9855
21	510	85	419	513	88	422	507	82	416	564	139	473	567	142	476	561	136	470	537	112	446	540	115	449	534	109	443	9855
22	619	197	288	613	191	282	616	194	285	592	170	261	586	164	255	589	167	258	646	224	315	640	218	309	643	221	312	9855
23	73	380	714	76	383	717	79	386	720	46	353	687	49	356	690	52	359	693	19	326	660	22	329	663	25	332	666	9855
24	427	491	96	430	494	99	424	488	93	481	545	150	484	548	153	478	542	147	454	518	123	457	521	126	451	515	120	9855
25	296	603	205	290	597	199	293	600	202	269	576	178	263	570	172	266	573	175	323	630	232	317	624	226	320	627	229	9855
26	722	57	388	725	60	391	728	63	394	695	30	361	698	33	364	701	36	367	668	3	334	671	6	337	674	9	340	9855
27	104	411	499	107	414	502	101	408	496	158	465	553	161	468	556	155	462	550	131	438	526	134	441	529	128	435	523	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

(3.14) $W(27) = 9 \cdot (Y(9)-|1|) + X(9)$ for generation of the Primal square

(3.15) $U(27) = 3 \cdot 27 \cdot (X(9)-|1|) + Y(9)$ for generation of the Dual square

Those loom tables are geonomically ultra-perfect too. Further their loom difference table sums to **0** everywhere. So do all the block-tile patterns shown here. Consequently it possesses the cloaking property.

Remark: The size-27 square is a special case among the other Class-3 squares shown thus far. It has a different dual that can be derived from loom tables taken to the base 27 in addition to the base 9.

Since this was the only exception found in the range of Class-3 squares investigated, the size **27** square would be better treated as a square from Class-6, which it will be henceforth, where $n = b^a$, **b = 3** and **a = 3**.

Program 3

The third tiling pattern on the size-27 dual square

Here is yet a third characteristic tiling pattern for the size-27 Class-3 square shown here on the dual square. Again, the **3x3** block-square tiles in this tiling pattern also sum to **3** times the square's characteristic number.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
U(27)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	207	295	602	201	289	596	204	292	599	180	268	575	174	262	569	177	265	572	234	322	629	228	316	623	231	319	626	9855
2	390	721	56	393	724	59	396	727	62	363	694	29	366	697	32	369	700	35	336	667	2	339	670	5	342	673	8	9855
3	501	103	410	504	106	413	498	100	407	555	157	464	558	160	467	552	154	461	528	130	437	531	133	440	525	127	434	9855
4	610	215	279	604	209	273	607	212	276	583	188	252	577	182	246	580	185	249	637	242	306	631	236	300	634	239	303	9855
5	64	398	705	67	401	708	70	404	711	37	371	678	40	374	681	43	377	684	10	344	651	13	347	654	16	350	657	9855
6	418	509	87	421	512	90	415	506	84	472	563	141	475	566	144	469	560	138	445	536	114	448	539	117	442	533	111	9855
7	287	621	196	281	615	190	284	618	193	260	594	169	254	588	163	257	591	166	314	648	223	308	642	217	311	645	220	9855
8	713	75	379	716	78	382	719	81	385	686	48	352	689	51	355	692	54	358	659	21	325	662	24	328	665	27	331	9855
9	95	429	490	98	432	493	92	426	487	149	483	544	152	486	547	146	480	541	122	456	517	125	459	520	119	453	514	9855
10	198	286	620	192	280	614	195	283	617	171	259	593	165	253	587	168	256	590	225	313	647	219	307	641	222	310	644	9855
11	381	712	74	384	715	77	387	718	80	354	685	47	357	688	50	360	691	53	327	658	20	330	661	23	333	664	26	9855
12	492	94	428	495	97	431	489	91	425	546	148	482	549	151	485	543	145	479	519	121	455	522	124	458	516	118	452	9855
13	601	206	297	595	200	291	598	203	294	574	179	270	568	173	264	571	176	267	628	233	324	622	227	318	625	230	321	9855
14	55	389	723	58	392	726	61	395	729	28	362	696	31	365	699	34	368	702	1	335	669	4	338	672	7	341	675	9855
15	409	500	105	412	503	108	406	497	102	463	554	159	466	557	162	460	551	156	436	527	132	439	530	135	433	524	129	9855
16	278	612	214	272	606	208	275	609	211	251	585	187	245	579	181	248	582	184	305	639	241	299	633	235	302	636	238	9855
17	704	66	397	707	69	400	710	72	403	677	39	370	680	42	373	683	45	376	650	12	343	653	15	346	656	18	349	9855
18	86	420	508	89	423	511	83	417	505	140	474	562	143	477	565	137	471	559	113	447	535	116	450	538	110	444	532	9855
19	216	277	611	210	271	605	213	274	608	189	250	584	183	244	578	186	247	581	243	304	638	237	298	632	240	301	635	9855
20	399	703	65	402	706	68	405	709	71	372	676	38	375	679	41	378	682	44	345	649	11	348	652	14	351	655	17	9855
21	510	85	419	513	88	422	507	82	416	564	139	473	567	142	476	561	136	470	537	112	446	540	115	449	534	109	443	9855
22	619	197	288	613	191	282	616	194	285	592	170	261	586	164	255	589	167	258	646	224	315	640	218	309	643	221	312	9855
23	73	380	714	76	383	717	79	386	720	46	353	687	49	356	690	52	359	693	19	326	660	22	329	663	25	332	666	9855
24	427	491	96	430	494	99	424	488	93	481	545	150	484	548	153	478	542	147	454	518	123	457	521	126	451	515	120	9855
25	296	603	205	290	597	199	293	600	202	269	576	178	263	570	172	266	573	175	323	630	232	317	624	226	320	627	229	9855
26	722	57	388	725	60	391	728	63	394	695	30	361	698	33	364	701	36	367	668	3	334	671	6	337	674	9	340	9855
27	104	411	499	107	414	502	101	408	496	158	465	553	161	468	556	155	462	550	131	438	526	134	441	529	128	435	523	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

This instance where there is more than just two distinct simultaneously-occurring complementary tiling patterns for one size of square and its dual is not antithetical to squares' pairwise complementary tiling patterns because the size-27 square is **3x**-modular while all the other Class-3 squares shown here of size **3b** are not **bx**-modular. This means that you could pick out any **nine 3x3** block-squares from anywhere in the size-27 table and that pattern would sum to **3** times the square's characteristic number continuously.

This pattern is not true-to-form because the size-9 square possesses no inherent unique tiling patterns. So actually, all tiling patterns for squares of any class are only pair-complementary, if they possess tiling patterns at all.

For Class-5 Squares

Size **n = ab**, **a & b** odd numbers **a > b > 3**

Next we'll investigate the tiling patterns for Class-5 squares. Only one size is small enough to be barely legible and that is the size-35 square where **n = 5x7.**

Size-35 primal square with size-7 block-tile patterns

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	W(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1		969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2		523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3		322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4		114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5		1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6		937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	67	977	378	997	643	86	957	351	1026	672	66	21455
7		533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8		332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9		131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10		1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11		947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12		501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13		300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14		141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15		1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16		964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17		518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18		310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19		109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20		1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21		974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22		528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23		327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24		126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25		1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26		942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27		496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28		337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29		136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30		1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31		959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32		506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33		305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34		104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35		1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

The shaded patterns are the continuously equal-summing size-7 characteristic tile pattern A at the **5x5** block-square level. Each tile sums to **7**-times the size-35 square's characteristic number. Although not shown here, the same also holds for the size-7 tile pattern B too.

Program 3

Size-35 dual square with size-5 block-tile patterns

Here is the size-35 square's dual square.

The shaded patterns are the continuously equal-summing tile patterns, again at the block-square level. This time they are each composed of **5x5** block-squares in the characteristic tile pattern B of the size-5 square. Each tile sums to **5** times the size-35 square's characteristic number. Although not shown here, the same also holds for the size-5 tile pattern A too.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	833	676	309	650	248	133	1200	100	1174	772	1148	746	624	474	563	448	46	415	20	1087	972	1061	939	544	878	518	361	239	335	178	1042	152	1009	859	702	21455
2	1135	740	618	959	592	435	531	409	14	1116	226	1055	933	1029	907	505	355	724	329	207	296	146	1003	853	1222	820	670	794	644	277	120	461	94	1168	68	21455
3	220	1084	194	1016	901	499	875	718	316	692	290	175	997	107	1216	814	1190	788	631	271	605	490	88	422	62	1129	769	1103	946	586	920	560	403	1	377	21455
4	284	162	258	136	1210	75	1177	782	660	756	599	477	573	451	56	1123	23	1097	975	1071	914	547	397	521	371	214	338	188	1045	895	984	862	712	836	686	21455
5	628	962	567	445	43	419	17	1091	236	1058	943	541	882	515	358	734	332	182	1039	149	1013	856	1197	830	673	313	647	252	130	464	104	1171	776	1145	743	21455
6	937	1026	904	509	843	728	326	204	300	143	1007	117	1219	824	667	798	641	274	615	458	98	1165	65	1139	737	1113	956	589	439	528	413	11	380	230	1052	21455
7	261	111	1213	818	1187	785	635	759	609	487	85	426	59	1133	33	1100	950	583	924	557	400	496	374	224	1081	191	1020	898	994	872	715	320	689	294	172	21455
8	570	455	53	387	27	1094	979	1068	911	551	885	525	368	211	342	185	1049	159	981	866	709	840	683	281	657	255	140	1207	72	1181	779	1155	753	596	481	21455
9	879	512	362	731	336	179	303	153	1010	860	1194	827	677	801	651	249	127	468	101	1175	40	1142	747	625	966	564	442	538	416	21	1088	233	1062	940	1036	21455
10	1223	821	1162	795	638	278	612	462	95	429	69	1136	741	1110	953	593	927	532	410	8	384	227	1056	201	1023	908	506	847	725	323	699	297	147	1004	114	21455
11	63	1130	30	1104	947	1078	921	554	404	493	378	221	345	195	1017	902	991	869	719	808	693	291	169	265	108	1217	82	1184	789	632	763	606	484	580	423	21455
12	365	706	339	189	1046	156	985	863	1204	837	680	285	654	259	137	471	76	1178	783	1152	750	600	969	574	452	50	391	24	1098	243	1065	915	548	889	522	21455
13	674	805	648	246	622	465	105	1172	37	1146	744	1120	963	561	446	535	420	18	352	237	1059	944	1033	876	516	850	735	333	176	307	150	1014	124	1191	831	21455
14	5	1107	957	590	931	529	407	503	381	231	1053	198	1027	905	1001	844	722	327	696	301	144	268	118	1220	825	1159	792	642	766	616	459	92	433	66	1140	21455
15	349	192	1021	166	988	873	716	812	690	288	664	262	112	1214	79	1188	786	1127	760	603	488	577	427	60	394	34	1101	951	1075	918	558	892	497	375	218	21455
16	658	256	134	475	73	1182	47	1149	754	597	973	571	449	545	388	28	1095	240	1069	912	1043	886	519	369	703	343	186	310	160	982	867	1201	834	684	773	21455
17	960	565	934	539	417	15	356	234	1063	208	1030	880	513	854	732	330	671	304	154	1011	121	1195	828	1169	802	645	250	619	469	102	436	41	1143	748	1117	21455
18	1024	909	998	841	726	815	700	298	141	272	115	1224	89	1156	796	639	770	613	456	587	430	70	1137	2	1111	954	1085	928	526	411	500	385	228	317	202	21455
19	109	478	83	1185	790	1124	757	607	976	581	424	57	398	31	1105	215	1072	922	555	896	494	372	713	346	196	1018	163	992	870	1211	809	687	292	661	266	21455
20	453	542	392	25	359	244	1066	916	1040	883	523	857	707	340	183	314	157	986	131	1198	838	681	777	655	253	629	472	77	1179	44	1153	751	1092	970	568	21455
21	1008	851	729	334	668	308	151	275	125	1192	832	1166	799	649	738	623	466	99	440	38	1147	12	1114	964	562	938	536	414	510	353	238	1060	205	1034	877	21455
22	86	1160	793	1134	767	610	460	584	434	67	401	6	1108	958	1082	925	530	899	504	382	225	321	199	1028	173	995	845	723	819	697	295	636	269	119	1221	21455
23	395	35	1102	212	1076	919	1050	893	491	376	710	350	193	282	167	989	874	1208	806	691	780	665	263	106	482	80	1189	54	1121	761	604	980	578	421	552	21455
24	704	337	678	311	161	983	128	1202	835	1176	774	652	257	626	476	74	443	48	1150	755	1089	967	572	941	546	389	22	363	241	1070	180	1037	887	520	861	21455
25	803	646	742	620	463	594	437	42	1144	9	1118	961	1057	935	533	418	507	357	235	324	209	1031	881	1005	848	733	822	672	305	148	279	122	1196	96	1163	21455
26	1112	222	1079	929	527	903	501	379	720	318	203	1025	170	999	842	1218	816	694	299	633	273	116	485	90	1157	797	1131	764	614	948	588	431	64	405	3	21455
27	190	286	164	993	138	1205	810	688	784	662	260	601	479	84	1186	51	1125	758	1099	977	575	425	549	399	32	366	216	1073	923	1047	890	495	864	714	347	21455
28	745	630	473	71	447	45	1154	19	1086	971	569	945	543	386	517	360	245	1067	177	1041	884	1015	858	701	341	675	315	158	247	132	1199	839	1173	771	656	21455
29	1054	932	537	906	511	354	232	328	206	1035	145	1002	852	730	826	669	302	643	276	126	1193	93	1167	800	1141	739	617	467	591	441	39	408	13	1115	965	21455
30	174	996	846	1215	813	698	787	637	270	113	489	87	1161	61	1128	768	611	952	585	428	559	402	7	1109	219	1083	926	1022	900	498	383	717	322	200	289	21455
31	483	81	450	55	1122	762	1096	974	579	913	553	396	29	370	213	1077	187	1044	894	492	868	711	344	685	283	168	990	135	1209	807	1183	781	659	264	598	21455
32	540	390	514	364	242	331	181	1038	888	1012	855	705	829	679	312	155	251	129	1203	103	1170	775	653	749	627	470	566	444	49	1151	16	1090	968	1064	942	21455
33	849	1225	823	666	306	640	280	123	457	97	1164	804	1138	736	621	955	595	438	36	412	10	1119	229	1051	936	534	910	508	351	727	325	210	1032	142	1006	21455
34	1158	58	1132	765	1106	949	582	432	556	406	4	373	223	1080	930	1019	897	502	871	721	319	197	293	171	1000	110	1212	817	695	791	634	267	608	486	91	21455
35	524	367	217	1074	184	1048	891	987	865	708	348	682	287	165	254	139	1206	811	1180	778	663	752	602	480	78	454	52	1126	26	1093	978	576	917	550	393	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Now, the principle here is this: tiles composed of **bxb** block-squares will each sum equally to **b** times the characteristic number of the square of size **ab**. Likewise, tiles composed of **axa** block-squares will each sum equally to **a** times the characteristic number of the square of size **ab**.

The tables shown here were derived in the base **ab = 35**. There are **3** different versions of each Class-5 square of size **n = ab**: each one derived with loom tables in one of the bases $\mathbf{a}^2$, **ab** and $\mathbf{b}^2$. Their generation and relationship among each other will be addressed in Program 7.

Program 3

The Class-5 uniform-summation property

The size-35 primal square here has been blocked-out with **5x7** rectangles with the numbers located in their centers highlighted. These numbers are those values for which the modulus and integer functions are equal. It has already been stated in this program that these central numbers all sum to the square's characteristic number.

It was seen in Part III of Program 2 that all the numbers in the same relative location within each rectangle also sum to the square's characteristic number. Consequently, these dot patterns sum equally continuously with wrapping imposed.

This property occurs uniformly as shown here for any Class-5 perfect square.

The **5x7** and **7x5** block rectangles each sum to the square's characteristic number. In fact, the square is both **5x7** and **7x5** continuously modular. Check out the **7x5** block rectangles.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
W62(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	945	658	221	1164	702	445	158	946	664	202	1170	708	446	164	927	670	208	1171	714	427	170	933	671	214	1152	720	433	171	939	652	220	1158	721	439	152	21455
2	811	354	92	1035	598	311	1079	817	360	98	1036	579	317	1085	823	361	79	1042	585	323	1086	804	367	85	1048	586	304	1092	810	373	86	1029	592	310	1098	21455
3	682	250	1188	726	469	7	975	688	226	1194	732	475	13	951	694	232	1200	738	451	19	957	700	238	1176	744	457	25	963	676	244	1182	750	463	1	969	21455
4	378	116	884	622	340	1103	841	384	122	890	603	341	1109	847	390	103	891	609	347	1115	828	391	109	897	615	328	1116	834	397	115	878	616	334	1122	840	21455
5	274	1212	755	493	31	999	537	255	1218	756	499	37	980	543	256	1224	762	480	43	981	549	262	1205	768	481	49	987	530	268	1206	774	487	30	993	531	21455
6	145	908	646	189	1127	870	408	146	914	627	195	1133	871	414	127	920	633	196	1139	852	420	133	921	639	177	1145	858	421	139	902	645	183	1146	864	402	21455
7	1061	779	517	60	1023	561	279	1067	785	523	61	1004	567	285	1073	786	504	67	1010	573	286	1054	792	510	73	1011	554	292	1060	798	511	54	1017	560	298	21455
8	932	675	213	1151	719	432	175	938	651	219	1157	725	438	151	944	657	225	1163	701	444	157	950	663	201	1169	707	450	163	926	669	207	1175	713	426	169	21455
9	803	366	84	1047	590	303	1091	809	372	90	1028	591	309	1097	815	353	91	1034	597	315	1078	816	359	97	1040	578	316	1084	822	365	78	1041	584	322	1090	21455
10	699	237	1180	743	456	24	962	680	243	1181	749	462	5	968	681	249	1187	730	468	6	974	687	230	1193	731	474	12	955	693	231	1199	737	455	18	956	21455
11	395	108	896	614	327	1120	833	396	114	877	620	333	1121	839	377	120	883	621	339	1102	845	383	121	889	602	345	1108	846	389	102	895	608	346	1114	827	21455
12	261	1204	767	485	48	986	529	267	1210	773	486	29	992	535	273	1211	754	492	35	998	536	254	1217	760	498	36	979	542	260	1223	761	479	42	985	548	21455
13	132	925	638	176	1144	857	425	138	901	644	182	1150	863	401	144	907	650	188	1126	869	407	150	913	626	194	1132	875	413	126	919	632	200	1138	851	419	21455
14	1053	791	509	72	1015	553	291	1059	797	515	53	1016	559	297	1065	778	516	59	1022	565	278	1066	784	522	65	1003	566	284	1072	790	503	66	1009	572	290	21455
15	949	662	205	1168	706	449	162	930	668	206	1174	712	430	168	931	674	212	1155	718	431	174	937	655	218	1156	724	437	155	943	656	224	1162	705	443	156	21455
16	820	358	96	1039	577	320	1083	821	364	77	1045	583	321	1089	802	370	83	1046	589	302	1095	808	371	89	1027	595	308	1096	814	352	95	1033	596	314	1077	21455
17	686	229	1192	735	473	11	954	692	235	1198	736	454	17	960	698	236	1179	742	460	23	961	679	242	1185	748	461	4	967	685	248	1186	729	467	10	973	21455
18	382	125	888	601	344	1107	850	388	101	894	607	350	1113	826	394	107	900	613	326	1119	832	400	113	876	619	332	1125	838	376	119	882	625	338	1101	844	21455
19	253	1216	759	497	40	978	541	259	1222	765	478	41	984	547	265	1203	766	484	47	990	528	266	1209	772	490	28	991	534	272	1215	753	491	34	997	540	21455
20	149	912	630	193	1131	874	412	130	918	631	199	1137	855	418	131	924	637	180	1143	856	424	137	905	643	181	1149	862	405	143	906	649	187	1130	868	406	21455
21	1070	783	521	64	1002	570	283	1071	789	502	70	1008	571	289	1052	795	508	71	1014	552	295	1058	796	514	52	1020	558	296	1064	777	520	58	1021	564	277	21455
22	936	654	217	1160	723	436	154	942	660	223	1161	704	442	160	948	661	204	1167	710	448	161	929	667	210	1173	711	429	167	935	673	211	1154	717	435	173	21455
23	807	375	88	1026	594	307	1100	813	351	94	1032	600	313	1076	819	357	100	1038	576	319	1082	825	363	76	1044	582	325	1088	801	369	82	1050	588	301	1094	21455
24	678	241	1184	747	465	3	966	684	247	1190	728	466	9	972	690	228	1191	734	472	15	953	691	234	1197	740	453	16	959	697	240	1178	741	459	22	965	21455
25	399	112	880	618	331	1124	837	380	118	881	624	337	1105	843	381	124	887	605	343	1106	849	387	105	893	606	349	1112	830	393	106	899	612	330	1118	831	21455
26	270	1208	771	489	27	995	533	271	1214	752	495	33	996	539	252	1220	758	496	39	977	545	258	1221	764	477	45	983	546	264	1202	770	483	46	989	527	21455
27	136	904	642	185	1148	861	404	142	910	648	186	1129	867	410	148	911	629	192	1135	873	411	129	917	635	198	1136	854	417	135	923	636	179	1142	860	423	21455
28	1057	800	513	51	1019	557	300	1063	776	519	57	1025	563	276	1069	782	525	63	1001	569	282	1075	788	501	69	1007	575	288	1051	794	507	75	1013	551	294	21455
29	928	666	209	1172	715	428	166	934	672	215	1153	716	434	172	940	653	216	1159	722	440	153	941	659	222	1165	703	441	159	947	665	203	1166	709	447	165	21455
30	824	362	80	1043	581	324	1087	805	368	81	1049	587	305	1093	806	374	87	1030	593	306	1099	812	355	93	1031	599	312	1080	818	356	99	1037	580	318	1081	21455
31	695	233	1196	739	452	20	958	696	239	1177	745	458	21	964	677	245	1183	746	464	2	970	683	246	1189	727	470	8	971	689	227	1195	733	471	14	952	21455
32	386	104	892	610	348	1111	829	392	110	898	611	329	1117	835	398	111	879	617	335	1123	836	379	117	885	623	336	1104	842	385	123	886	604	342	1110	848	21455
33	257	1225	763	476	44	982	550	263	1201	769	482	50	988	526	269	1207	775	488	26	994	532	275	1213	751	494	32	1000	538	251	1219	757	500	38	976	544	21455
34	128	916	634	197	1140	853	416	134	922	640	178	1141	859	422	140	903	641	184	1147	865	403	141	909	647	190	1128	866	409	147	915	628	191	1134	872	415	21455
35	1074	787	505	68	1006	574	287	1055	793	506	74	1012	555	293	1056	799	512	55	1018	556	299	1062	780	518	56	1024	562	280	1068	781	524	62	1005	568	281	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 3

Size-35 Modulus and Integer Loom-table Patterns in the base 35

Here is the size-35 loom table **X(35)** taken in the base **35**. Every row, column and diagonal contains the numbers **1** thru **35.**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
X2(35)	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630
	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	630
	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	630
	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	630
	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	630
	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	630
	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	630
	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	630
	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	630
	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	630
	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	630
	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	630
	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	630
	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	630
	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	630
	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	630
	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	630
	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	630
	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	630
	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	630
	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	630
	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	630
	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	630
	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	630
	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	630
	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	630
	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	630
	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	630
	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	630
	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	630
	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	630
	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	630
	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	630
	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	630
	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	630
	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	630
	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	
	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630

Each colored section contains all the numbers **1** thru **35**. Note that all the even numbers are in the top row of each section. Dub this tile pattern P, for the size-35 modulus loom table **X(35)**.

Program 3

Here is the size-35 loom table **Y(35)** taken in the base **35.** Again, every row, column and diagonal contains the numbers **1** thru **35** too.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
Y2(35)	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	
1	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	630
2	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	630
3	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	630
4	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	630
5	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	630
6	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	630
7	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	630
8	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	630
9	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	630
10	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	630
11	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	630
12	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	630
13	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	630
14	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	630
15	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	630
16	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	630
17	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	630
18	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	630
19	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	630
20	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	630
21	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	630
22	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	630
23	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	630
24	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	630
25	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	630
26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	630
27	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	630
28	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	630
29	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	630
30	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	630
31	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	630
32	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	630
33	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	630
34	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	630
35	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	630
	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	
	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630	630

Each colored section also contains all the numbers **1** thru **35**. These sections highlight a tiling pattern, dub it Q, for the size-35 integer loom table **Y(35)**.

Note that tile pattern P on the prior page contains **2** rows while pattern Q here contains **3** rows

Further, tiling pattern P contains exactly **2** tiles horizontally per **3** rows whereas here tiling pattern Q contains exactly **3** tiles horizontally per **5** rows.

Now here is what is so surprising: Tile pattern P doesn't work on **Y(35)** and tile pattern Q doesn't work on **X(35)** ! And moreover, neither works on the primary dual squares **W(35)** and **U(35)** because they are not shared in common between their loom tables.

Further, in the loom difference table neither tile pattern P nor Q sum to **0** here because they are not complementary.

Program 3

Cloaking Property of loom tables

for Class-5 Squares

Size-35

V1(35) = X1(35) – Y1(35)

Here is the difference table of loom tables derived in the base **35**. It is totally pairwise anti-symmetric. All the rows, columns and main and wrap diagonals sum to **0**. All **7x7** block-squares in both A and B tile patterns of the size-5 square sum continuously to **0**.

Due to the square's **5x7** and **7x5** modularity, those rectangles in the difference table sum to **0** everywhere too.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	V(35)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-4	9	-20	-1	5	-24	25	-27	15	21	5	11	-11	-5	14	-15	-9	-18	-19	30	1	20	-2	-3	23	-13	0	-22	-10	3	3	-7	0	6	19	0
2	18	17	-5	14	-15	-2	10	-12	-13	1	-9	26	4	16	-6	0	6	-3	-4	-26	-7	-1	6	12	3	9	15	-1	5	-24	-11	8	-21	21	-31	0
3	-3	-3	-13	29	0	6	-10	3	9	-7	-1	-30	12	2	9	15	-1	5	18	-18	8	-21	-15	11	-25	24	-12	14	27	-9	17	-19	-6	0	-16	0
4	5	-17	-5	-27	15	-2	12	11	-11	1	14	-8	4	-18	-19	30	-22	20	-2	10	23	-6	0	-16	-10	3	-13	-7	0	6	25	3	9	-7	-1	0
5	-15	11	10	-12	-6	-22	-16	26	-19	23	-6	0	19	-10	3	-13	-7	-1	6	-4	-4	9	28	-1	12	-24	2	1	-21	5	-31	18	17	8	14	0
6	0	19	-3	-4	22	-7	-1	-23	-11	2	2	-8	6	5	18	-5	8	-21	-2	11	-25	24	-28	14	20	4	17	-12	-6	13	-16	-10	-19	-13	29	0
7	-8	-2	12	11	2	8	14	-2	4	-18	-12	7	-22	20	-32	17	23	-6	13	-16	-3	9	-13	-7	0	-10	25	3	15	-7	6	5	-4	-5	-27	0
8	7	-22	-16	10	-26	23	-6	13	26	-10	16	-20	-7	6	-17	-4	-4	-14	28	-1	12	-11	2	8	-8	-2	-31	18	1	8	14	-2	4	17	-12	0
9	22	-7	-1	-10	-11	2	-14	-8	-1	5	31	2	8	-8	-2	4	-18	1	-28	14	-3	11	10	-12	7	13	-9	3	-19	-20	29	-16	19	-3	9	0
10	2	8	27	-2	11	-25	1	7	-22	4	-32	17	16	7	20	-16	10	9	-13	-7	-23	-10	25	-20	22	-7	-1	18	-4	2	-14	-8	-2	5	-5	0
11	-26	23	-29	13	26	3	16	-13	-7	12	-17	-4	-20	-14	28	-1	18	-4	2	21	-8	-2	-24	-12	1	8	-9	5	4	17	-6	7	-15	-3	10	0
12	-4	15	-14	-8	-1	-11	24	2	21	-8	5	4	-5	-6	-28	-2	-3	11	10	1	7	13	4	3	-19	-13	6	-23	19	-26	16	22	-7	12	-17	0
13	11	-12	1	7	-9	4	-32	17	0	7	13	-3	10	16	-13	6	-23	-17	9	-20	22	-7	12	25	-11	15	-14	-8	5	-18	-5	-5	-15	34	-2	0
14	-4	10	16	-13	6	12	-10	2	-20	-14	28	-17	18	-4	8	21	-1	-2	-11	-12	1	-15	-9	5	4	30	1	7	-9	-3	10	-19	0	-29	13	0
15	-24	-11	24	-21	21	-8	5	17	-5	1	-15	-9	-3	11	-6	1	7	26	-3	10	-19	0	6	-23	3	-33	16	22	6	19	-17	9	8	-14	-1	0
16	-9	-3	-25	-6	0	7	-10	4	3	16	0	6	-16	-4	9	-27	22	-23	12	25	2	15	-14	-8	18	-18	-5	-21	-15	27	-2	24	-5	1	20	0
17	13	12	3	2	-20	-14	5	-17	18	-27	15	21	-8	11	-11	-5	14	-15	-9	-2	-12	30	1	20	-9	4	3	-6	0	-29	-3	-4	10	9	0	0
18	21	-8	11	24	-5	14	-15	-9	4	-19	-6	1	-16	33	-3	10	-13	0	13	-10	3	-33	16	-1	6	19	-4	9	15	-14	5	-24	-11	8	-21	0
19	0	-9	-10	4	3	29	0	6	-3	-4	9	-20	-1	-30	12	2	9	15	-14	5	11	-11	8	-21	-15	27	-18	17	-5	14	20	-2	-3	-12	-13	0
20	-20	-1	5	-24	2	-27	15	21	5	18	-18	8	14	-15	-2	-25	-12	23	-22	27	-9	4	16	-6	0	-16	-3	-4	10	-7	0	6	25	3	9	0
21	1	14	-8	-9	17	-19	-6	-22	-16	33	-3	23	-6	0	19	-10	3	-26	-7	-1	6	-11	3	9	15	-1	5	-17	-5	8	-21	21	-24	11	24	0
22	-13	29	0	19	-10	3	9	-7	-1	-30	-4	-5	9	15	-1	12	11	2	1	-21	-8	4	-18	17	-28	14	20	-2	10	-12	-6	13	-16	-10	4	0
23	2	-34	15	5	5	18	-5	8	14	-15	11	-25	-12	7	-22	20	-9	17	23	-6	13	-16	-10	3	-13	-7	0	-17	32	-4	9	-7	-1	12	-11	0
24	17	-12	7	-22	-16	26	-19	23	-6	13	19	-3	-4	-13	-7	-1	-10	-11	3	2	28	6	5	-4	-5	8	-21	-2	-24	11	1	8	14	-15	4	0
25	-10	3	15	-7	6	-17	-4	-5	9	-8	-1	12	24	2	8	-21	-2	4	-18	1	-28	14	20	4	17	-12	7	13	-16	-3	-26	-13	29	-23	26	0
26	5	-5	2	8	14	-2	4	-18	1	7	-22	20	-25	10	23	7	13	-9	-10	16	-20	-7	-16	-17	32	-4	22	-7	-1	25	-11	2	-27	-8	-2	0
27	-9	3	-19	16	-29	20	19	-3	9	-13	-7	12	-10	-11	3	-14	28	-1	18	-4	2	8	-8	-2	-31	-5	1	8	14	-2	11	10	1	7	-22	0
28	12	-17	-4	2	-14	-8	-1	-18	31	2	8	-8	-2	11	-12	1	-28	14	4	4	17	-6	7	20	-16	10	-26	-13	6	-23	26	-10	16	22	-7	0
29	27	5	4	-5	-6	7	-15	-3	-25	10	0	7	13	-9	3	16	-13	6	-23	-17	32	-20	22	-7	12	18	-4	2	-14	-8	-2	-11	-12	2	8	0
30	-29	13	19	10	16	-13	6	12	-17	-4	-20	-14	28	-24	25	-11	2	21	-8	5	-18	-5	-6	8	-2	-2	11	23	1	7	-22	4	3	-19	0	0
31	-14	-8	-17	-18	31	-5	21	-1	-2	24	-12	1	-28	-9	4	4	-6	1	7	13	-3	10	-19	0	6	-23	19	-26	16	22	6	12	-10	-11	15	0
32	1	7	-9	-3	-25	-6	0	7	13	-3	10	16	0	6	-23	-10	2	-20	22	-30	19	18	-4	8	-14	-1	11	-11	-12	2	-15	27	5	17	-5	0
33	16	0	6	19	-17	9	-27	-14	12	-24	25	-11	15	21	-8	18	-18	-5	1	-15	-9	-2	-12	30	1	7	-9	-3	10	-6	0	-29	13	3	3	0
34	31	-21	21	-8	11	24	-5	1	-15	-9	-3	-12	-6	1	7	26	4	3	-6	0	6	-16	-4	-26	9	-1	13	12	-10	2	15	-14	5	-17	-18	0
35	-19	-6	0	7	-3	-3	10	22	0	13	-23	3	2	-20	-1	-30	19	18	9	15	-14	5	11	-11	-5	-21	-15	27	-25	24	-5	1	20	-9	4	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

All **5x5** block-squares in both A and B tile patterns of the size-7 square sum continuously to **0** too. **In general, every pattern which summed to the characteristic number, now sums to 0 in the loom-difference table.**

Program 3

Class-6 Squares

Size $\mathbf{n = b^a}$, **b** is an odd number ≥ **3**; **a** ≥ **2**

Size-9

Here is the Class-6 size-9 square and its dual with their shared complementary loom tables. The loom tables were derived in the base **9**.

	1	2	3	4	5	6	7	8	9	
W(9)	369	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

	1	2	3	4	5	6	7	8	9	
X(9)	369	369	369	369	369	369	369	369	369	369
1	3	4	8	3	4	8	3	4	8	369
2	5	9	1	5	9	1	5	9	1	369
3	7	2	6	7	2	6	7	2	6	369
4	8	3	4	8	3	4	8	3	4	369
5	1	5	9	1	5	9	1	5	9	369
6	6	7	2	6	7	2	6	7	2	369
7	4	8	3	4	8	3	4	8	3	369
8	9	1	5	9	1	5	9	1	5	369
9	2	6	7	2	6	7	2	6	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

	1	2	3	4	5	6	7	8	9	
U(9)	369	369	369	369	369	369	369	369	369	369
1	27	34	71	21	28	65	24	31	68	369
2	39	73	2	42	76	5	45	79	8	369
3	60	13	50	63	16	53	57	10	47	369
4	70	26	36	64	20	30	67	23	33	369
5	1	38	75	4	41	78	7	44	81	369
6	49	59	15	52	62	18	46	56	12	369
7	35	72	25	29	66	19	32	69	22	369
8	74	3	37	77	6	40	80	9	43	369
9	14	51	58	17	54	61	11	48	55	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

	1	2	3	4	5	6	7	8	9	
Y(9)	45	45	45	45	45	45	45	45	45	45
1	9	7	8	3	1	2	6	4	5	45
2	3	1	2	6	4	5	9	7	8	45
3	6	4	5	9	7	8	3	1	2	45
4	7	8	9	1	2	3	4	5	6	45
5	1	2	3	4	5	6	7	8	9	45
6	4	5	6	7	8	9	1	2	3	45
7	8	9	7	2	3	1	5	6	4	45
8	2	3	1	5	6	4	8	9	7	45
9	5	6	4	8	9	7	2	3	1	45
	45	45	45	45	45	45	45	45	45	
	45	45	45	45	45	45	45	45	45	45

Both the primal and its dual square are continuously **3x** modular with **3x3** block-squares each summing to **369**, the square's characteristic number. The size-9 square has no tiling pattern outside of this **3x-modularity**.

Program 3

Cloaking Property of Size-9 Loom Tables

X-Y	1	2	3	4	5	6	7	8	9	
	0	0	0	0	0	0	0	0	0	0
1	-6	-3	0	0	3	6	-3	0	3	0
2	2	8	-1	-1	5	-4	-4	2	-7	0
3	1	-2	1	-2	-5	-2	4	1	4	0
4	1	-5	-5	7	1	1	4	-2	-2	0
5	0	3	6	-3	0	3	-6	-3	0	0
6	2	2	-4	-1	-1	-7	5	5	-1	0
7	-4	-1	-4	2	5	2	-1	2	-1	0
8	7	-2	4	4	-5	1	1	-8	-2	0
9	-3	0	3	-6	-3	0	0	3	6	0
	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0

X-Y	1	2	3	4	5	6	7	8	9	
	0	0	0	0	0	0	0	0	0	0
1	0	-6	-3	6	0	3	3	-3	0	0
2	-1	5	5	-4	2	2	-7	-1	-1	0
3	-3	0	3	-6	-3	0	0	3	6	0
4	-2	1	-8	4	7	-2	1	4	-5	0
5	6	0	3	3	-3	0	0	-6	-3	0
6	4	-2	-2	1	-5	-5	7	1	1	0
7	-7	-4	2	-1	2	8	-4	-1	5	0
8	4	4	1	1	1	-2	-2	-2	-5	0
9	-1	2	-1	-4	-1	-4	2	5	2	0
	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0

Shown here are loom table differences taken aligned one-on-one (top) and at random (bottom). In addition to all the rows, columns and diagonals summing to **0** in the loom difference tables, all the 3x3 block squares sum continuously to **0** too. So the Size-9 square possesses the cloaking property.

Size-25 Primal and Dual Square Block-tile Patterns

W(25)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

U(25)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	477	183	514	345	46	482	188	519	350	26	487	193	524	330	31	492	198	504	335	36	497	178	509	340	41	7825
2	273	79	410	236	567	253	84	415	241	572	258	89	420	246	552	263	94	425	226	557	268	99	405	231	562	7825
3	169	625	301	7	463	174	605	306	12	468	154	610	311	17	473	159	615	316	22	453	164	620	321	2	458	7825
4	65	391	222	528	359	70	396	202	533	364	75	376	207	538	369	55	381	212	543	374	60	386	217	548	354	7825
5	581	287	118	449	130	586	292	123	429	135	591	297	103	434	140	596	277	108	439	145	576	282	113	444	150	7825
6	478	184	515	341	47	483	189	520	346	27	488	194	525	326	32	493	199	505	331	37	498	179	510	336	42	7825
7	274	80	406	237	568	254	85	411	242	573	259	90	416	247	553	264	95	421	227	558	269	100	401	232	563	7825
8	170	621	302	8	464	175	601	307	13	469	155	606	312	18	474	160	611	317	23	454	165	616	322	3	459	7825
9	61	392	223	529	360	66	397	203	534	365	71	377	208	539	370	51	382	213	544	375	56	387	218	549	355	7825
10	582	288	119	450	126	587	293	124	430	131	592	298	104	435	136	597	278	109	440	141	577	283	114	445	146	7825
11	479	185	511	342	48	484	190	516	347	28	489	195	521	327	33	494	200	501	332	38	499	180	506	337	43	7825
12	275	76	407	238	569	255	81	412	243	574	260	86	417	248	554	265	91	422	228	559	270	96	402	233	564	7825
13	166	622	303	9	465	171	602	308	14	470	151	607	313	19	475	156	612	318	24	455	161	617	323	4	460	7825
14	62	393	224	530	356	67	398	204	535	361	72	378	209	540	366	52	383	214	545	371	57	388	219	550	351	7825
15	583	289	120	446	127	588	294	125	426	132	593	299	105	431	137	598	279	110	436	142	578	284	115	441	147	7825
16	480	181	512	343	49	485	186	517	348	29	490	191	522	328	34	495	196	502	333	39	500	176	507	338	44	7825
17	271	77	408	239	570	251	82	413	244	575	256	87	418	249	555	261	92	423	229	560	266	97	403	234	565	7825
18	167	623	304	10	461	172	603	309	15	466	152	608	314	20	471	157	613	319	25	451	162	618	324	5	456	7825
19	63	394	225	526	357	68	399	205	531	362	73	379	210	536	367	53	384	215	541	372	58	389	220	546	352	7825
20	584	290	116	447	128	589	295	121	427	133	594	300	101	432	138	599	280	106	437	143	579	285	111	442	148	7825
21	476	182	513	344	50	481	187	518	349	30	486	192	523	329	35	491	197	503	334	40	496	177	508	339	45	7825
22	272	78	409	240	566	252	83	414	245	571	257	88	419	250	551	262	93	424	230	556	267	98	404	235	561	7825
23	168	624	305	6	462	173	604	310	11	467	153	609	315	16	472	158	614	320	21	452	163	619	325	1	457	7825
24	64	395	221	527	358	69	400	201	532	363	74	380	206	537	368	54	385	211	542	373	59	390	216	547	353	7825
25	585	286	117	448	129	590	291	122	428	134	595	296	102	433	139	600	276	107	438	144	580	281	112	443	149	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Program 3

Size-25 Loom Tables X(25) & Y(25)

Here are the size-25 complementary loom tables. Each **5-by-5** block-square contains the numbers **1** thru **25** exactly once. Size-25 squares are continuously **5x5** modular and each block square sums to the size-25 loom-table's characteristic number, $\mathbf{L_{25} = 325}$.

This is the source for block-square modularity in the primal and dual tables. The loom tables were both derived in the base **25**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	X	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
1	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
2	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
3	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
4	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
5	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
6	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
7	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
8	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
9	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
10	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
11	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
12	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
13	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
14	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
15	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
16	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
17	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
18	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
19	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
20	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
21	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
22	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
23	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
24	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
25	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
		325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	Y	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
1	2	8	14	20	21	7	13	19	25	1	12	18	24	5	6	17	23	4	10	11	22	3	9	15	16	325
2	23	4	10	11	17	3	9	15	16	22	8	14	20	21	2	13	19	25	1	7	18	24	5	6	12	325
3	19	25	1	7	13	24	5	6	12	18	4	10	11	17	23	9	15	16	22	3	14	20	21	2	8	325
4	15	16	22	3	9	20	21	2	8	14	25	1	7	13	19	5	6	12	18	24	10	11	17	23	4	325
5	6	12	18	24	5	11	17	23	4	10	16	22	3	9	15	21	2	8	14	20	1	7	13	19	25	325
6	3	9	15	16	22	8	14	20	21	2	13	19	25	1	7	18	24	5	6	12	23	4	10	11	17	325
7	24	5	6	12	18	4	10	11	17	23	9	15	16	22	3	14	20	21	2	8	19	25	1	7	13	325
8	20	21	2	8	14	25	1	7	13	19	5	6	12	18	24	10	11	17	23	4	15	16	22	3	9	325
9	11	17	23	4	10	16	22	3	9	15	21	2	8	14	20	1	7	13	19	25	6	12	18	24	5	325
10	7	13	19	25	1	12	18	24	5	6	17	23	4	10	11	22	3	9	15	16	2	8	14	20	21	325
11	4	10	11	17	23	9	15	16	22	3	14	20	21	2	8	19	25	1	7	13	24	5	6	12	18	325
12	25	1	7	13	19	5	6	12	18	24	10	11	17	23	4	15	16	22	3	9	20	21	2	8	14	325
13	16	22	3	9	15	21	2	8	14	20	1	7	13	19	25	6	12	18	24	5	11	17	23	4	10	325
14	12	18	24	5	6	17	23	4	10	11	22	3	9	15	16	2	8	14	20	21	7	13	19	25	1	325
15	8	14	20	21	2	13	19	25	1	7	18	24	5	6	12	23	4	10	11	17	3	9	15	16	22	325
16	5	6	12	18	24	10	11	17	23	4	15	16	22	3	9	20	21	2	8	14	25	1	7	13	19	325
17	21	2	8	14	20	1	7	13	19	25	6	12	18	24	5	11	17	23	4	10	16	22	3	9	15	325
18	17	23	4	10	11	22	3	9	15	16	2	8	14	20	21	7	13	19	25	1	12	18	24	5	6	325
19	13	19	25	1	7	18	24	5	6	12	23	4	10	11	17	3	9	15	16	22	8	14	20	21	2	325
20	9	15	16	22	3	14	20	21	2	8	19	25	1	7	13	24	5	6	12	18	4	10	11	17	23	325
21	1	7	13	19	25	6	12	18	24	5	11	17	23	4	10	16	22	3	9	15	21	2	8	14	20	325
22	22	3	9	15	16	2	8	14	20	21	7	13	19	25	1	12	18	24	5	6	17	23	4	10	11	325
23	18	24	5	6	12	23	4	10	11	17	3	9	15	16	22	8	14	20	21	2	13	19	25	1	7	325
24	14	20	21	2	8	19	25	1	7	13	24	5	6	12	18	4	10	11	17	23	9	15	16	22	3	325
25	10	11	17	23	4	15	16	22	3	9	20	21	2	8	14	25	1	7	13	19	5	6	12	18	24	325
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
		325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

Program 3

Size-25 Loom Difference Table

Here is the size-25 loom difference table. Every **5x5** block-square sums to **0** continuously. Further, randomly aligned differences do the same.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
X(25)-Y(25)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	7	-6	-19	13	-5	2	-11	1	8	-10	-3	9	-4	3	-15	17	4	-9	-2	5	12	-1	-14	0	
-12	0	7	-1	6	8	-5	2	-6	1	3	-10	-3	-11	21	-2	-15	-8	9	16	-7	-20	12	4	11	0	
-12	0	12	-6	6	-17	20	7	-11	1	3	15	2	-16	-4	-2	10	-3	-21	16	-7	5	-8	-1	11	0	
-12	0	-13	19	6	-17	-5	7	14	1	-22	15	2	9	-4	-2	10	-3	4	-9	-7	5	-8	-1	11	0	
18	0	-13	-6	1	13	-5	-18	14	-4	8	-10	2	9	-9	3	10	-3	4	-14	23	5	-8	-1	-19	0	
17	-1	6	-2	-20	12	-6	1	-7	0	7	-11	-4	13	-5	2	-16	16	8	-10	-3	4	11	3	-15	0	
-13	-1	11	-2	5	7	-6	6	-7	0	2	-11	1	-12	20	-3	-16	-4	8	15	-8	-21	16	3	10	0	
-13	4	11	-7	5	-18	24	6	-12	0	2	19	1	-17	-5	-3	14	-4	-22	15	-8	9	-9	-2	10	0	
-8	-1	-14	18	5	-13	-6	6	13	0	-18	14	1	8	-5	2	9	-4	3	-10	-3	4	-9	-2	10	0	
17	-1	-14	-7	5	12	-6	-19	13	0	7	-11	1	8	-5	2	9	-4	3	-10	22	4	-9	-2	-15	0	
16	-2	10	-3	-21	11	-7	5	-8	-1	6	-12	0	12	-6	1	-17	20	7	-11	-4	3	15	2	-16	0	
-14	3	10	-3	4	6	-2	5	-8	-1	1	-7	0	-13	19	-4	-12	-5	7	14	-9	-17	15	2	9	0	
-9	3	10	-8	4	-14	23	5	-13	-1	6	18	0	-18	-6	1	13	-5	-23	14	-4	8	-10	-3	9	0	
-9	-2	-15	17	9	-14	-7	5	12	4	-19	13	0	7	-1	1	8	-5	2	-6	-4	3	-10	-3	14	0	
16	-2	-15	-3	4	11	-7	-20	17	-1	6	-12	0	12	-6	1	8	-5	7	-11	21	3	-10	2	-16	0	
15	2	9	-4	-22	10	-3	4	-9	-2	5	-8	-1	11	-7	0	-13	19	6	-12	-5	7	14	1	-17	0	
-10	2	9	-4	3	10	-3	4	-9	-2	5	-8	-1	-14	18	0	-13	-6	6	13	-5	-18	14	1	8	0	
-10	2	9	-9	8	-15	22	4	-14	3	5	17	-1	-19	-2	0	12	-6	-24	18	-5	7	-11	-4	13	0	
-10	-3	-16	21	8	-15	-8	4	16	3	-20	12	-1	11	-2	0	7	-6	6	-7	-5	2	-11	1	13	0	
15	-3	-11	-4	3	10	-8	-16	16	-2	5	-13	4	11	-7	0	7	-1	6	-12	20	2	-6	1	-17	0	
19	1	8	-5	-23	14	-4	3	-10	-3	9	-9	-2	10	-8	4	-14	18	5	-13	-1	6	13	0	-18	0	
-11	1	8	-5	7	9	-4	3	-10	2	4	-9	-2	-15	22	-1	-14	-7	5	17	-6	-19	13	0	12	0	
-11	1	8	-5	7	-16	21	3	-10	2	4	16	-2	-15	-3	-1	11	-7	-20	17	-6	6	-12	0	12	0	
-11	-4	-12	20	7	-16	-9	8	15	2	-21	11	3	10	-3	-1	6	-2	5	-8	-6	1	-7	0	12	0	
14	1	-12	-5	2	9	-4	-17	15	-3	4	-9	3	10	-8	-1	11	-2	5	-13	19	6	-7	0	-18	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The yellow cells highlight points of equality between the complementary loom tables.

Make note: The loom tables of all Class-6 squares, whose size is a square number, also possess the cloaking property. This cloaking property is an amazing property that may someday find security applications – that's yet another potential supplication that this math would be good for.

Summary Table of Cloaking Property by Class

	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6
Cloaking Complementary loom-tables	Yes	No	Yes at the bxb block-square level	Yes	Yes at the bxb block-square level	Yes

All but Class-2 squares possess the cloaking property.

Note that Class-3 & Class-5 squares possess the cloaking property at the block-square tiling pattern level, although any pattern which sums to the characteristic number or a multiple of it will sum to **0** on the loom-difference table, provided it is geonomic.

Unification of all the Square-generation Formulas

Complementary loom tables **X** and **Y** exist for Classes 3, 4 & 5 squares only if derived in the base **b** where **c** is the class of square of size **n**:

(3.16) $b = n/c$

(3.17) $W(n) = n/c\,(\,Y(b) - |1|\,) + X(b)$ for generation of the Primal square

(3.18) $U(n) = c \cdot n\,(\,X(b) - |1|\,) + Y(b)$ for generation of the Dual square

Class-2 squares do not possess complementary loom tables that are geonomic and so are excluded from these formulas.

Recall that for Class-1 squares $b = n/c = n$ and $c \cdot n = n$ since $c = 1$. So class was not visibly part of the generation formulas.

It was then determined later that Class-4 squares whose size was a multiple of **8** and Classes-5 and 6 squares could not only be treated the same as Class-1 squares, but also could be treated like Class-3 squares here too.

For Class-5 squares, these more generalized formulas only hold if **a** replaces **c**, where $a > b > 3$ and $n = ab$.

Remark: The size-27 square is a special case among the Class-3 squares shown thus far. It has a different dual that can be derived from loom tables taken to the base **27** as well as the base **9**. Those loom tables are also geonomically ultra-perfect.

Further, their loom difference table for loom-tables derived in the base **27** sums to **0** everywhere. So do all the block-tile patterns. Consequently it possesses the cloaking property.

Since this square was the only exception found in the range of Class-3 squares investigated, the size-27 square would be better treated as a square from Class-6 and has therefore been included in that class too where $b = 3$ and $a = 3$; $n = b^a$.

Program 3

Application #1 of Geonometry

The Membrane of Life

In this segment of the program we will explore the zipper-like patterns in Class-5 squares and see just how Geonometry can be applied to the formation of DNA molecules.

To verify what is presented next, you will need to refer to the size-35 square and its loom tables shown earlier in this program.

Program 3

The Double and Triple Zipper-like Patterns in Class-5 Loom-tables

Size-35 Modulus Loom-table X(35)

The size-35 modulus loom table is seen to contain a double zipper-like pattern when the distribution of even numbers is highlighted against the distribution of odd numbers. Note the double zipper-like pattern where the rows change from even numbers to odd numbers.

Down 2 over 1

X(35)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2
2	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3
3	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4
4	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5
5	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6
6	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7
7	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8
8	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9
9	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10
10	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11
11	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12
12	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13
13	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14
14	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15
15	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16
16	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17
17	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18
18	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19
19	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20
20	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21
21	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22
22	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23
23	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24
24	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25
25	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26
26	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27
27	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28
28	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29
29	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30
30	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31
31	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32
32	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33
33	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34
34	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35
35	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1

Program 3

Size-35 Integer Loom-table Y(35)

The size-35 integer loom table is seen to contain a triple helical pattern when, again, the distribution of even numbers is highlighted against the distribution of odd numbers. Note the checkerboard pattern this time between the even and odd numbers.

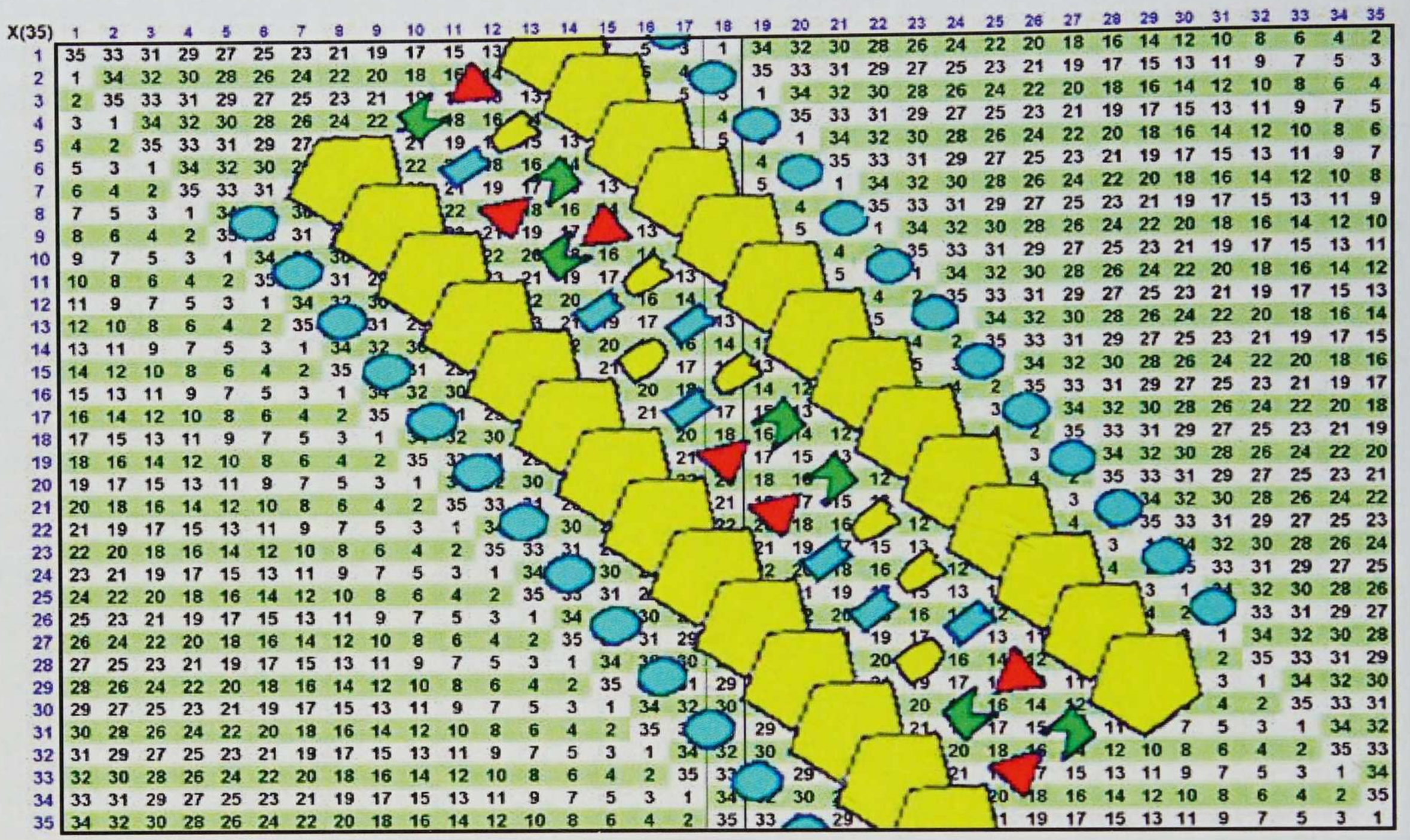
Down 3 over 1

Y(35)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
1	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2
2	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1
3	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35
4	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34
5	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33
6	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32
7	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31
8	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30
9	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29
10	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28
11	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27
12	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26
13	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25
14	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24
15	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23
16	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22
17	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21
18	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20
19	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19
20	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18
21	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17
22	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16
23	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15
24	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14
25	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13
26	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12
27	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11
28	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10
29	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9
30	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8
31	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7
32	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6
33	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5
34	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4
35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3
36	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2
37	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1

Program 3

A plausible application of Geonometry in molecular biology

The double helical pattern may explain how phosphate atoms connected with sugar molecules collect along the zipper-like seams in the planar frequencies of a spatial membrane from a primordial soup of various nucleotides.



I claim that these geonomic squares have been all along actually measuring and mapping space itself. The basis of life, both DNA (deoxyribos nucleic acid) and its controlling biological molecule RNA (ribose nucleic acid) are built upon double helixes.

The question of just how did life begin in a lifeless universe can be envisioned as the appropriate nucleotides in a soup of multiple differently-structured nucleotides, aligning themselves along these inherent zipper-like patterns to form the backbone of RNA, which biologists presume came before DNA due to its simpler structure and its DNA-controlling function.

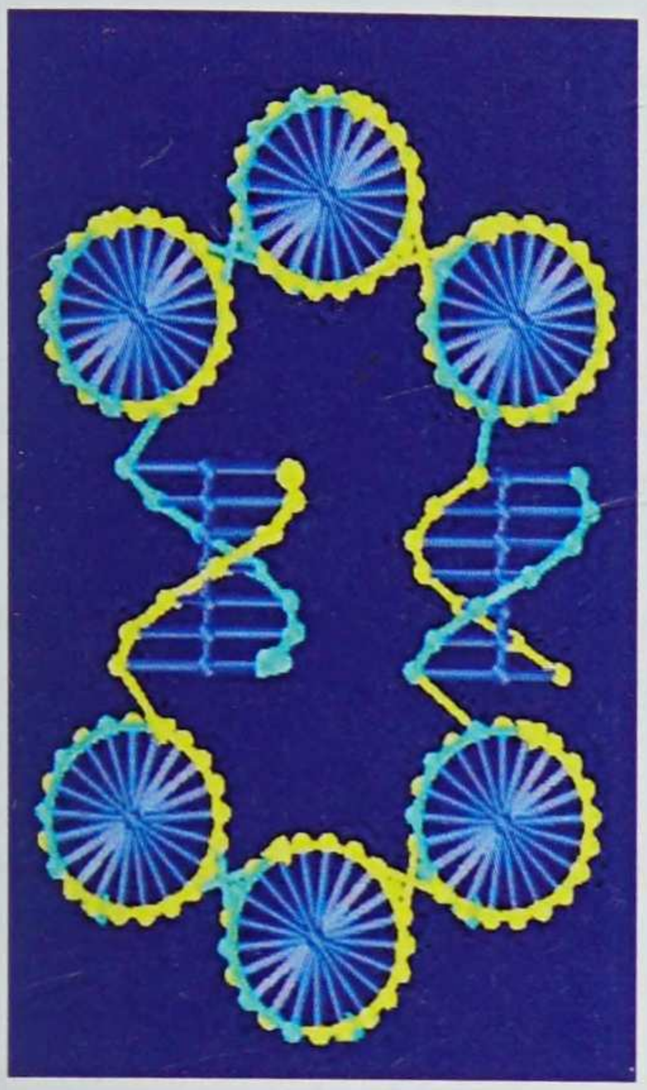
My perception is that these nucleotide molecules align themselves linearly along each of these diagonal seams in 2-dimensional space. Then after acquiring their biological molecules which attract each other in a non-covalent weak bond, roll up into a 3-dimensional double helix. The two seams connect in a spiraling ladder structure due to the dissymmetry of adjacent sugar molecules.

I can't say just how these complex molecules actually do this: I'll have to leave that subject to the molecular biologists. None the less, what Geonometry has done here is provide plausible evidence of the underlying spatial framework for all this to happen.

Today in bio-medicine, a procedure called *transcription inhibition* is being used to shut-off problematic gene expression in RNA. This process involves cladding a third bio-molecular strand to a section of it by forming a triple helical segment. So the triple helix is applicable too.

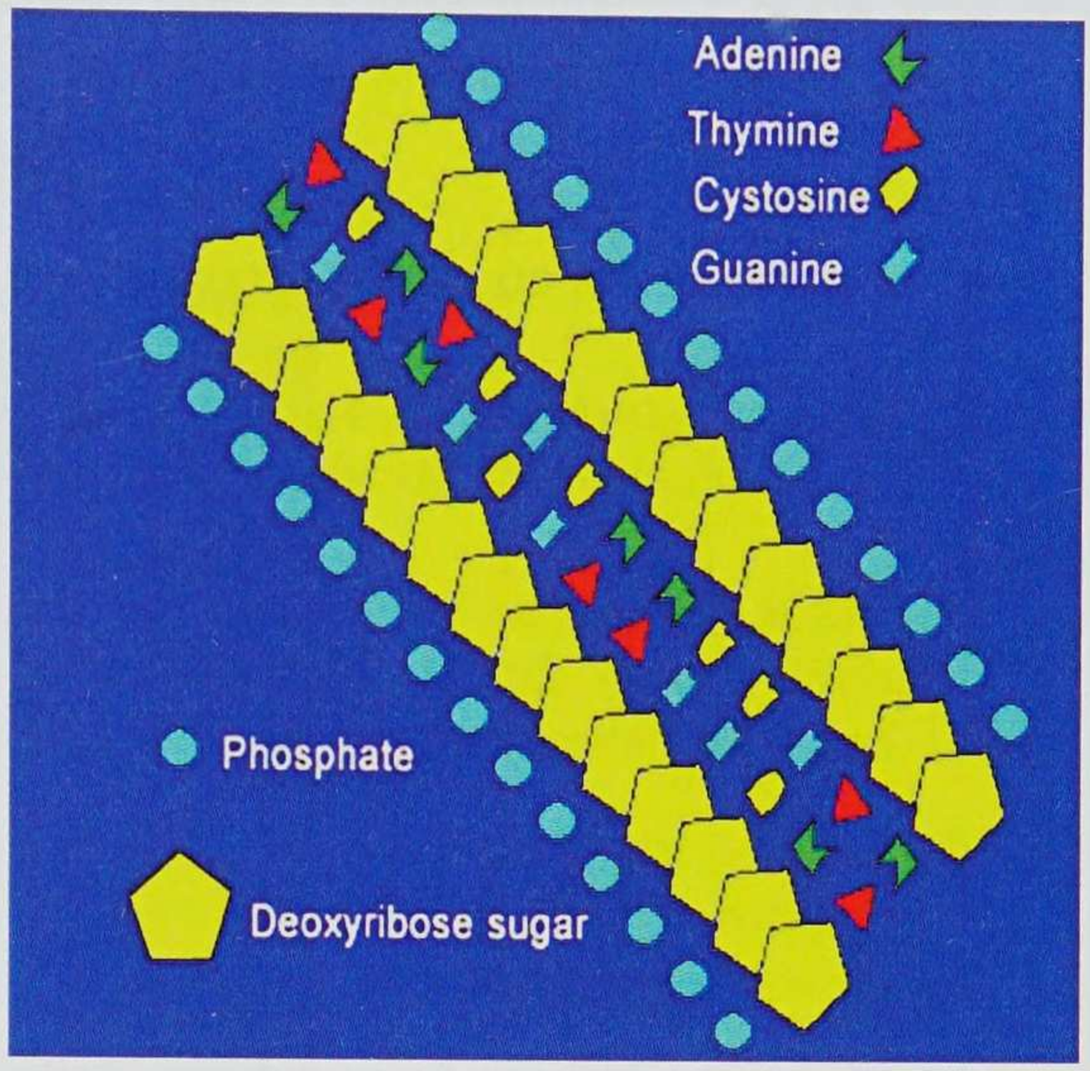
At left is the double helical closed loop DNA structure looking from the top down.

Program 3



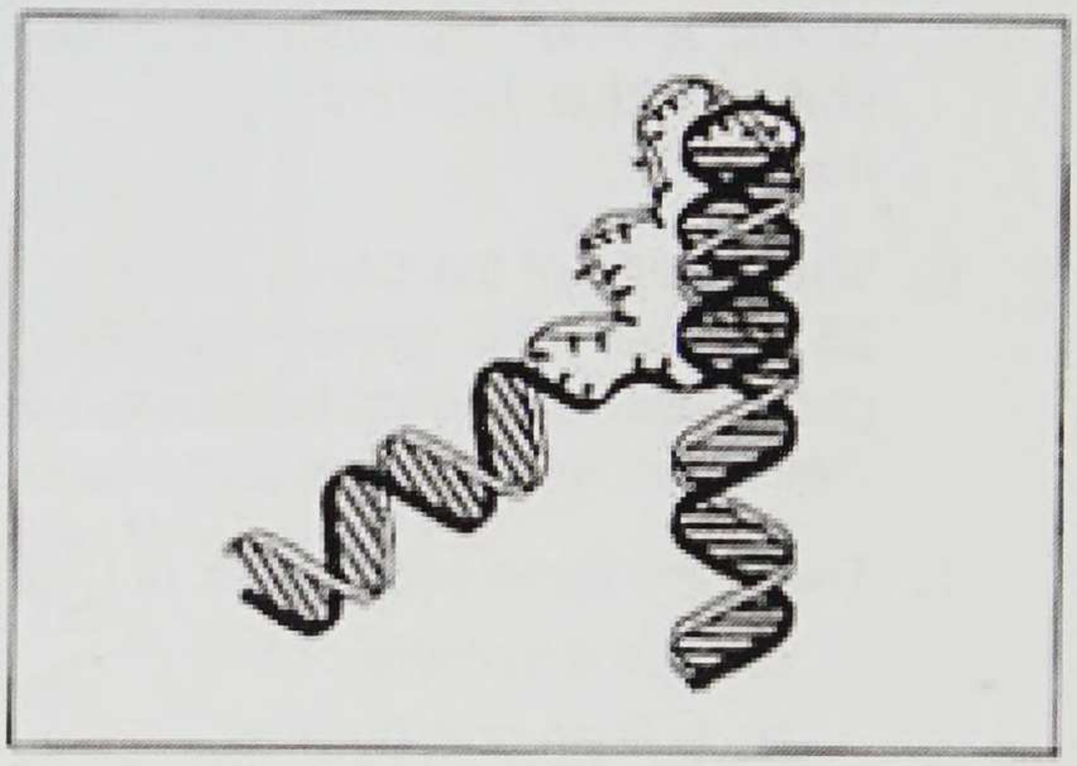
At right is shown the inherent bonds between nucleotides and how they would attract one another in this curling-up pro-cess.

Note how Adenine is only attracted to Thymine and Cytosine is only attracted to Guanine.



Further, upon replication of DNA via RNA, a triple helix is involved.

The triple helix is also a result of a natural assembly process just as is the double helix. This very structure is now being studied and synthesized by molecular biologists for creating other life forms. So the template necessary for this to take place may also be the underlying complementary harmonic vibrations in Class-5 planar membranes. We will see in Program 8 just how these Class-5 vibrational membranes can be expanded to any multiple of its size. So these flattened helical spatial patterns can exist at any multiple of its basic size.



The implication here is that there definitely is more to evolution than just natural selection.

For instances: What caused the big jumps such as going from amphibians to mammals? Or from cold-blooded creatures to warm-blooded ones? Or from 4-legged animals to ones with flapping wings for flight? How did that just evolve from something less complex to something functionally more complex with no track record inbetween?

How did the first mammary gland evolve in mammals? How could a female's milk-producing mammary glands evolve through natural selection over time? Wouldn't they need to be fully functional right from the get-go? Wouldn't the mother require to be fed herself upon her birth yet without an initial source of milk? There had to be something more than natural selection of the fittest going on here to introduce these major biological changes in the first place.

Might all these major sudden shifts in function have been the result of a sudden major shift in the DNA of unborn fetuses or external embryos in eggs caused by a ripple in the structured fabric of space itself? – something like a transient rolling wave in the fabric of space akin to a tsunami? Maybe a direct hit upon the Earth by a coronal ejection which does happen from time to time.

Evolution is clearly far from being the whole picture.

Program 3

Program 3 Notes

1. All classes of squares, except those of Class-2, possess the loom-cloaking property.

2. Class-3 squares have continuous complementary tiling patterns at the **bxb** block-square level where **n = 3b** which sum equally to **3** times the square's characteristic number.

3. Class-4 squares have continuous diamond and X complementary tiling patterns that are confined to **2bx2b** block-squares where **n = 4b** and sum exactly to the square's characteristic number.

4. Class-5 squares have continuous tiling patterns simultaneously at the block-square level at both of their factor's sizes **a** & **b** where **n = ab** and sum to **a** and **b** times the square's characteristic number, respectively.

5. Class-6 squares are simply continuously **bx-**modular at the size **b** block-square level where $\mathbf{n = b^a}$. So its complementary tiles are simply **bxb** block-squares, aligned or shifted either horizontally or vertically and sum exactly to the square's characteristic number.

6. The version of the Class-5 size-35 square shown in this program was derived in the base **35** by the **ATE** generation method described in Program 7. Only Class-5 squares generated from loom-tables in the base **n = ab** exhibit the zipper-like patterns between their linear and checker patterns of even and odd numbers.

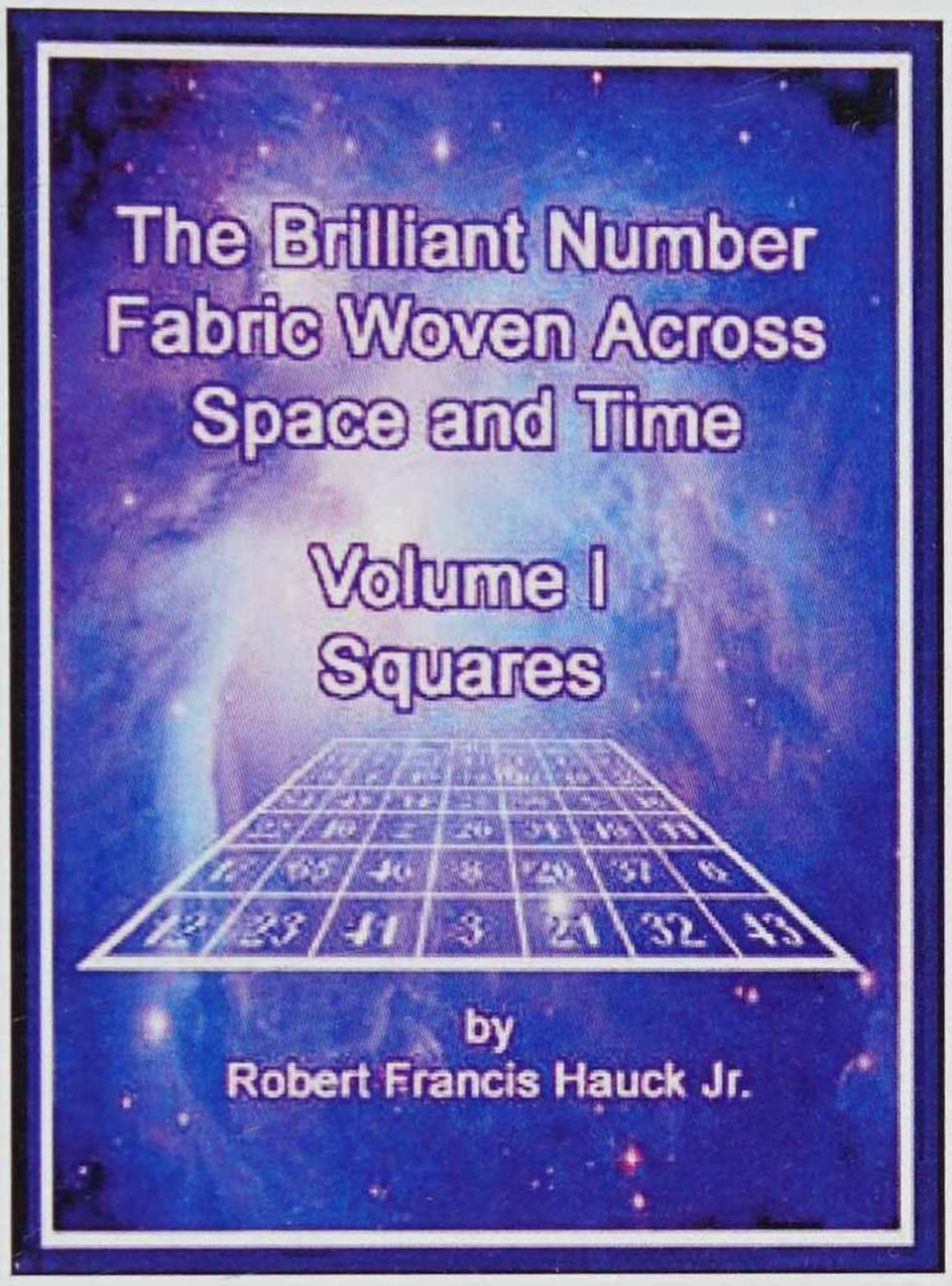
7. There are three subclasses of Class-4 squares, where **n = 4b**:
   a. **n** is a multiple of **8**.
   b. **b** is an odd-number.
   c. **b** is an even number not divisible by **4**,. (This sub-class is the only one to make use of near-perfect Class-2 squares such as **6**, **10** and **14** in their manifestation and that involves the **ATE** expansion method detailed in Program 7.)

* * *

Program 3

We have come to the end of program 3 of the new **math**, **Geonometry**. I hope that this program has been as awe-inspiring for you as it was for me in its development.

Here is the book upon which the material in Program 3 was based.



The Brilliant Number Fabric Woven across Space and Time
Volume I - Squares

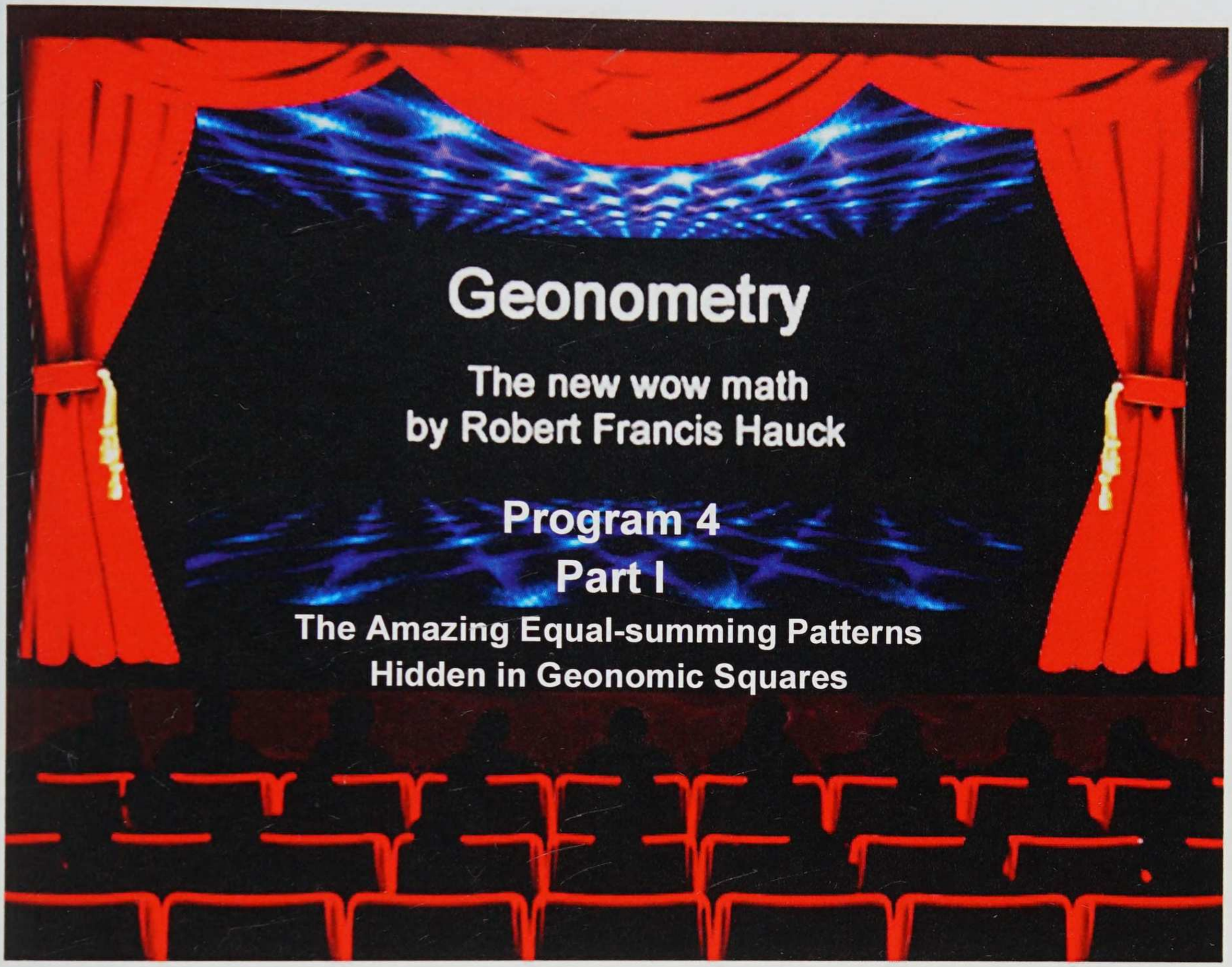
ISBN: 978-1-461-06984-3

Deals with loom tables and complementary tiling patterns

Third Edition (140 pages)

In the next program we will explore the amazing multiple equal-summing patterns found among all classes and sizes of squares. This is where Geonometry really shines.

Program 3



In this segment we will be uncovering the amazing diamond shaped integrated quilt patterns plus triangular tapestries hidden in Class-3 and Class-4 squares. Further, we will discover equal-summing 3-dimensional patterns in 2-dimensional squares.

Program 4

Diamond Quilt Patterns

For Class-3 Squares

Size-15

Here is the size-15 square with its quilt pattern. It is composed of all size-5 diamonds. Note that the summations are equal to $\mathbf{4C_{15}}$.

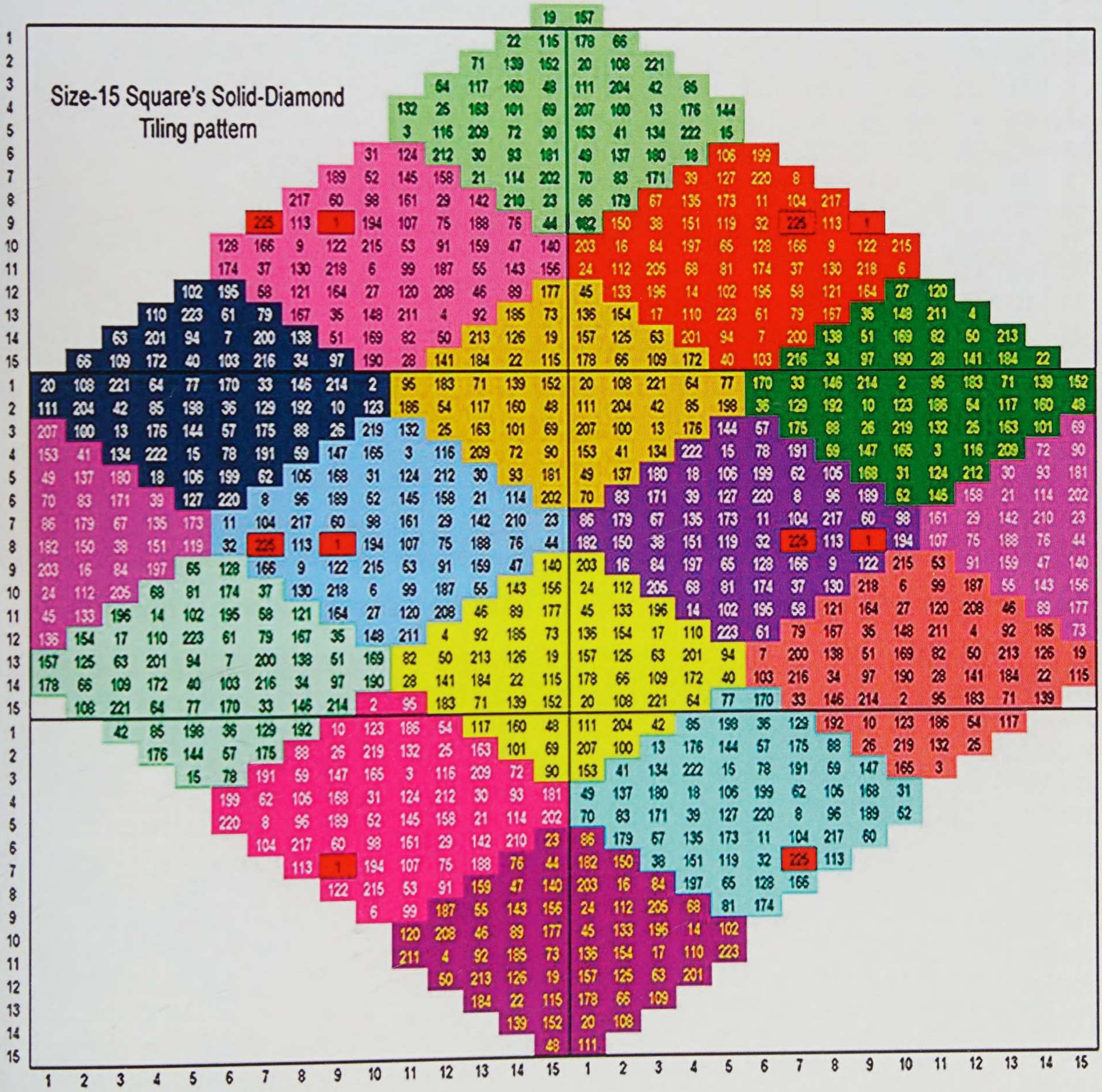
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	20	108	221	64	77	170	33	146	214	2	95	183	71	139	152
2	111	204	42	85	198	36	129	192	10	123	186	54	117	160	48
3	207	100	13	176	144	57	175	88	26	219	132	25	163	101	69
4	153	41	134	222	15	78	191	59	147	165	3	116	209	72	90
5	49	137	180	18	106	199	62	105	168	31	124	212	30	93	181
6	70	83	171	39	127	220	8	96	189	52	145	158	21	114	202
7	86	179	67	135	173	11	104	217	60	98	161	29	142	210	23
8	182	150	38	151	119	32	225	113	1	194	107	75	188	76	44
9	203	16	84	197	65	128	166	9	122	215	53	91	159	47	140
10	24	112	205	68	81	174	37	130	218	6	99	187	55	143	156
11	45	133	196	14	102	195	58	121	164	27	120	208	46	89	177
12	136	154	17	110	223	61	79	167	35	148	211	4	92	185	73
13	157	125	63	201	94	7	200	138	51	169	82	50	213	126	19
14	178	66	109	172	40	103	216	34	97	190	28	141	184	22	115
15	74	87	155	43	131	224	12	80	193	56	149	162	5	118	206

Here it is seen that at most two tiles are completely contained within the table at any one instance, yet the diamond pattern satisfies the 4C tiling properties and so qualifies as a tiling pattern for the size-15 square. It has no complementary quilt pattern.

Program 4

Here is pictured the tiling pattern over six size-15 squares on a **45x30** rectangle. Full diamond tiling patterns propagate beyond the boundaries of the square.

The one-row flap extension at the top indicates that it will take a total of **15 more** vertical repetitions of this arrangement to totally contain the pattern within multiple squares of size-15. Amazing!



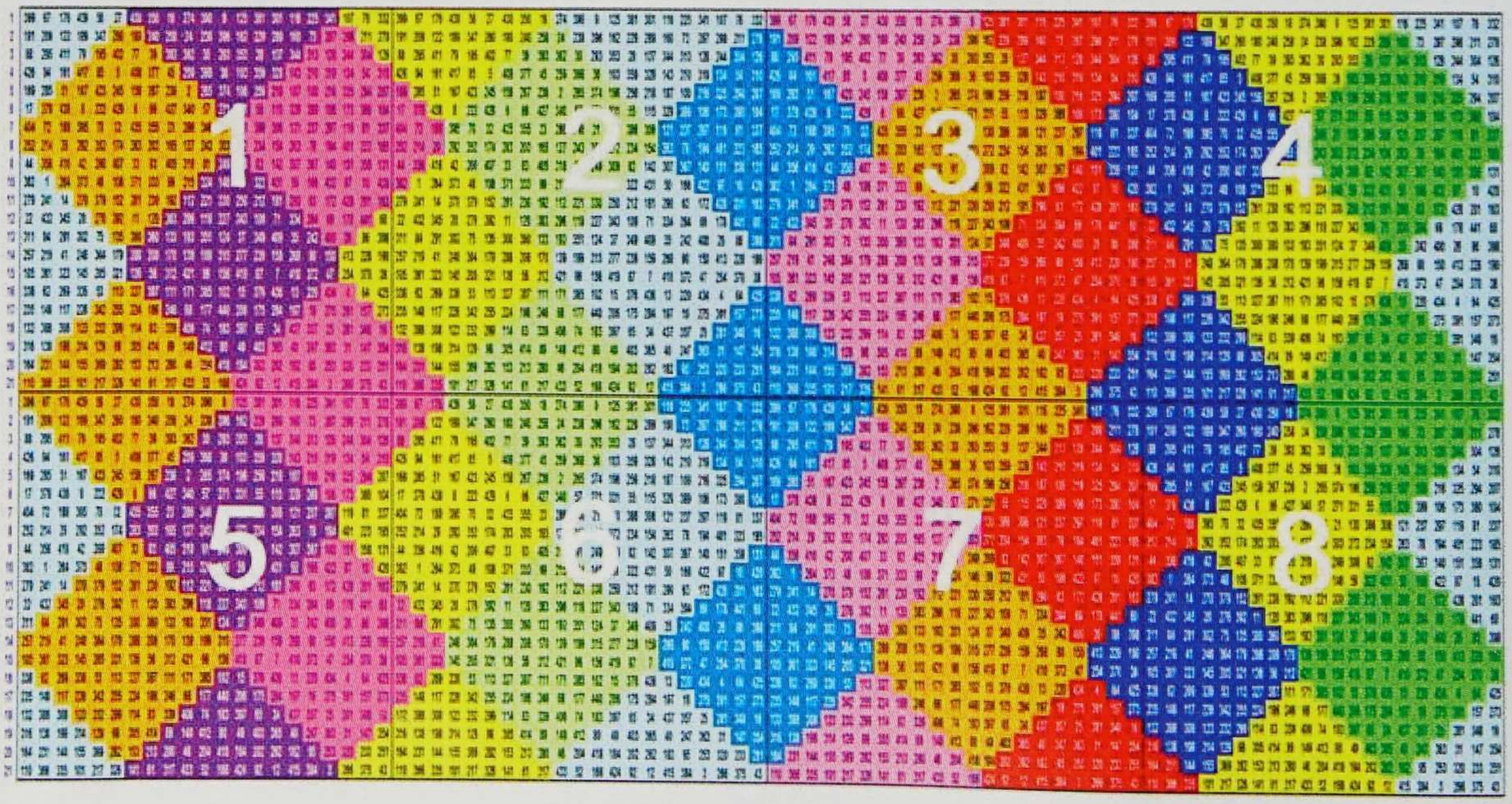
Program 4

Size-21

The same sequence of tables shows that the size-21 square has a full-diamond tiling pattern too. The tiles are all of size-6 and sum to $\mathbf{4C_{21}}$. Again, at most two tiles fit completely within the square at any one instance. Nonetheless, the pattern satisfies the 4C tiling properties.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	399	67	176	439	58	27	430	350	18	274	390	9	125	381	301	116	225	341	107	76	332
2	191	209	122	189	347	260	180	240	258	24	238	396	162	229	289	160	73	287	298	211	278
3	88	295	411	79	195	402	77	39	393	362	30	293	353	28	137	344	313	128	244	304	126
4	426	94	161	417	85	5	408	377	45	259	368	36	103	359	328	143	210	319	134	54	310
5	169	285	51	167	423	245	158	267	236	2	265	374	196	256	218	187	100	216	325	294	207
6	17	378	438	8	222	429	6	66	427	340	57	271	331	55	115	329	389	106	173	380	104
7	404	72	188	395	70	32	435	355	23	286	346	21	130	386	306	121	237	297	119	81	337
8	252	214	29	292	352	174	283	203	165	127	243	303	272	234	154	263	78	194	401	223	185
9	44	356	416	42	200	407	33	93	405	318	91	249	309	82	142	307	367	140	151	358	131
10	382	1	264	373	48	108	371	333	99	215	324	146	59	322	431	50	166	422	97	10	420
11	279	241	14	270	379	152	261	230	192	112	221	330	250	212	181	290	63	172	428	201	163
12	22	432	345	20	276	392	11	120	383	296	118	227	343	109	71	334	394	69	178	441	60
13	311	84	291	302	75	135	300	360	133	193	351	124	37	349	409	35	242	400	26	86	398
14	257	219	41	248	364	179	288	208	170	139	199	315	277	239	159	268	90	150	413	228	190
15	105	361	323	145	205	321	136	56	312	421	96	156	419	87	7	410	372	47	254	370	38
16	338	62	269	336	53	113	327	387	111	171	385	102	15	376	436	13	220	434	4	64	425
17	235	148	117	226	342	255	224	186	246	68	177	440	206	175	284	197	19	275	391	157	273
18	132	388	308	123	232	299	114	83	339	406	74	183	397	65	34	437	357	25	281	348	16
19	316	138	198	314	129	98	305	414	89	149	412	80	49	403	365	40	247	363	31	147	354
20	164	231	144	155	369	282	153	213	280	46	204	418	184	202	262	182	95	253	320	233	251
21	110	366	335	101	217	326	141	61	317	433	52	168	424	92	12	415	384	3	266	375	43

Here is the completely contained tiling pastern; it takes eight size-21 squares to contain it.



Program 4

For Class-4 Squares

Continuous Solid Diamond Tile Patterns				
b	n = 4b	C_n	Diamond Sizes	Diamond Sum Multiples of C_n
1	4	34	0 & 1	0 & 1
2	8	260	1 & 2	½ & 1½
3	12	870	2 & 3	1 & 2
4	16	2056	3 & 4	1½ & 2½
5	20	4010	4 & 5	2 & 3
6	24	6924	5 & 6	2½ & 3½
7	28	10990	6 & 7	3 & 4
8	32	14911	7 & 8	3½ & 4½
9	36	23346	8 & 9	4 & 5
10	40	32020	9 & 10	4½ & 5½
11	44	42614	10 & 11	5 & 6
12	48	55320	11 & 12	5½ & 6½
13	52	70330	12 & 13	6 & 7
14	56	87836	13 & 14	6½ & 7½
15	60	108030	14 & 15	7 & 8
16	64	131104	15 & 16	7½ & 8½
17	68	157250	16 & 17	8 & 9
18	72	186660	17 & 18	8½ & 9½
19	76	219526	18 & 19	9 & 10

Diamond quilt patterns are found in the Class-4 squares beginning with the size-20 square. At left is a chart of the arithmetical series found for those patterns.

The discovered numerical patterns (in black print) can be generated backwards to come to the beginning of this series where **b = 1** and the smallest value is **0**.

However squares of sizes **4** thru **16** do not satisfy these predictions due to their limiting embryonic sizes.

But once sufficiently evolved to appear, they continue in the predictive pattern hence-forth.

There are even more patterns than those predicted schematically as we shall see.

Size-12

Here is the size-12 square. It possesses a quilt pattern of sizes **2** (yellow) and **3** integrated solid diamonds summing respectively to **870** and **1740**. However, both of these size diamond patterns only sum equally continuously, vertically when centered over the vertical lines or wrapped across the vertical edges. In other words, the quilt pattern only holds when shifted **3** columns either left or right. The pattern is totally contained within the square.

	1	2	3	4	5	6	7	8	9	10	11	12	
12x12	870	870	870	870	870	870	870	870	870	870	870	870	870
1	109	30	127	90	43	108	37	102	55	18	115	36	870
2	67	84	49	24	133	6	139	12	121	96	61	78	870
3	107	41	128	89	32	110	35	113	56	17	104	38	870
4	69	100	51	22	117	4	141	28	123	94	45	76	870
5	75	46	93	124	27	142	3	118	21	52	99	70	870
6	44	98	59	14	119	29	116	26	131	86	47	101	870
7	73	66	91	126	7	144	1	138	19	54	79	72	870
8	31	120	13	60	97	42	103	48	85	132	25	114	870
9	140	2	95	122	71	77	68	74	23	50	143	5	870
10	33	136	15	58	81	40	105	64	87	130	9	112	870
11	111	10	129	88	63	106	39	82	57	16	135	34	870
12	11	137	20	53	80	62	83	65	92	125	8	134	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

Program 4

Size-20

Here is the size-20 square's quilt pattern. It consists of sizes **4** (yellow) and **5** integrated solid diamonds each of which sum continuously to **2** and **3** C_{20}, respectively just as the pattern table specifies. The quilt pattern is totally contained within the square.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
W(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	321	289	2	195	308	296	14	177	320	283	21	189	302	295	8	196	314	277	4010
2	336	254	42	160	348	261	29	167	335	273	36	154	342	260	48	161	329	267	35	173	4010
3	82	125	388	201	94	107	400	213	76	119	382	225	88	101	394	207	100	113	376	219	4010
4	353	241	59	147	365	228	66	134	372	240	53	141	359	247	65	128	366	234	72	140	4010
5	24	187	305	293	6	199	312	280	18	181	324	287	5	193	306	299	12	180	318	281	4010
6	345	258	46	164	327	270	33	171	339	252	45	158	346	264	27	170	333	271	39	152	4010
7	86	104	392	210	98	111	379	217	85	123	386	204	92	110	398	211	79	117	385	223	4010
8	357	250	63	126	369	232	75	138	351	244	57	150	363	226	69	132	375	238	51	144	4010
9	3	191	309	297	15	178	316	284	22	190	303	291	9	197	315	278	16	184	322	290	4010
10	349	262	30	168	331	274	37	155	343	256	49	162	330	268	31	174	337	255	43	156	4010
11	95	108	396	214	77	120	383	221	89	102	395	208	96	114	377	220	83	121	389	202	4010
12	361	229	67	135	373	236	54	142	360	248	61	129	367	235	73	136	354	242	60	148	4010
13	7	200	313	276	19	182	325	288	1	194	307	300	13	176	319	282	25	188	301	294	4010
14	328	266	34	172	340	253	41	159	347	265	28	166	334	272	40	153	341	259	47	165	4010
15	99	112	380	218	81	124	387	205	93	106	399	212	80	118	381	224	87	105	393	206	4010
16	370	233	71	139	352	245	58	146	364	227	70	133	371	239	52	145	358	246	64	127	4010
17	11	179	317	285	23	186	304	292	10	198	311	279	17	185	323	286	4	192	310	298	4010
18	332	275	38	151	344	257	50	163	326	269	32	175	338	251	44	157	350	263	26	169	4010
19	78	116	384	222	90	103	391	209	97	115	378	216	84	122	390	203	91	109	397	215	4010
20	374	237	55	143	356	249	62	130	368	231	74	137	355	243	56	149	362	230	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Program 4

Size-24

Here is a size-24 square. Its quilt pattern is composed of integrated sizes **5** (yellow) and **6** solid diamonds which sum continuously to **$2^1/_2$** and **$3^1/_2$ C_{24}** as predicted.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	21	250	95	228	101	170	31	244	85	234	111	164	309	538	383	516	389	458	319	532	373	522	399	452	6924
2	432	499	294	441	368	563	422	505	304	435	358	569	144	211	6	153	80	275	134	217	16	147	70	281	6924
3	50	189	124	263	34	205	60	183	114	269	44	199	338	477	412	551	322	493	348	471	402	557	332	487	6924
4	347	472	401	558	331	488	337	478	411	552	321	494	59	184	113	270	43	200	49	190	123	264	33	206	6924
5	133	218	15	148	69	282	143	212	5	154	79	276	421	506	303	436	357	570	431	500	293	442	367	564	6924
6	320	531	374	521	400	451	310	537	384	515	390	457	32	243	86	233	112	163	22	249	96	227	102	169	6924
7	18	253	92	231	98	173	28	247	82	237	108	167	306	541	380	519	386	461	316	535	370	525	396	455	6924
8	427	504	289	446	363	568	417	510	299	440	353	574	139	216	1	158	75	280	129	222	11	152	65	286	6924
9	53	186	127	260	37	202	63	180	117	266	47	196	341	474	415	548	325	490	351	468	405	554	335	484	6924
10	352	467	406	553	336	483	342	473	416	547	326	489	64	179	118	265	48	195	54	185	128	259	38	201	6924
11	130	221	12	151	66	285	140	215	2	157	76	279	418	509	300	439	354	573	428	503	290	445	364	567	6924
12	315	536	369	526	395	456	305	542	379	520	385	462	27	248	81	238	107	168	17	254	91	232	97	174	6924
13	165	106	239	84	245	26	175	100	229	90	255	20	453	394	527	372	533	314	463	388	517	378	543	308	6924
14	576	355	438	297	512	419	566	361	448	291	502	425	288	67	150	9	224	131	278	73	160	3	214	137	6924
15	194	45	268	119	178	61	204	39	258	125	188	55	482	333	556	407	466	349	492	327	546	413	476	343	6924
16	491	328	545	414	475	344	481	334	555	408	465	350	203	40	257	126	187	56	193	46	267	120	177	62	6924
17	277	74	159	4	213	138	287	68	149	10	223	132	565	362	447	292	501	426	575	356	437	298	511	420	6924
18	464	387	518	377	544	307	454	393	528	371	534	313	176	99	230	89	256	19	166	105	240	83	246	25	6924
19	162	109	236	87	242	29	172	103	226	93	252	23	450	397	524	375	530	317	460	391	514	381	540	311	6924
20	571	360	433	302	507	424	561	366	443	296	497	430	283	72	145	14	219	136	273	78	155	8	209	142	6924
21	197	42	271	116	181	58	207	36	261	122	191	52	485	330	559	404	469	346	495	324	549	410	479	340	6924
22	496	323	550	409	480	339	486	329	560	403	470	345	208	35	262	121	192	51	198	41	272	115	182	57	6924
23	274	77	156	7	210	141	284	71	146	13	220	135	562	365	444	295	498	429	572	359	434	301	508	423	6924
24	459	392	513	382	539	312	449	398	523	376	529	318	171	104	225	94	251	24	161	110	235	88	241	30	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Program 4

A Different Quilt Pattern for a Different Size-24 Square

Here is a different size-24 square tiled with **24** diamonds all of size-3 consisting of **24** cells each. This quilt pattern was unpredicted by the arithmetic pattern in the Quilt Pattern Summary table.

Each diamond sums <u>exactly</u> to the square's characteristic number. However, again the tile pattern is not continuously summing either. It is only vertically continuous every fourth column.

The quilt pattern is totally self-contained without any overlap.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	433	426	451	54	223	72	505	354	523	414	439	432	145	138	163	342	511	360	217	66	235	126	151	144
2	103	192	85	564	313	546	31	264	13	204	97	186	391	480	373	276	25	258	319	552	301	492	385	474
3	356	506	311	194	143	149	428	434	383	554	359	509	68	218	23	482	431	437	140	146	95	266	71	221
4	105	208	87	562	297	544	33	280	15	202	81	184	393	496	375	274	9	256	321	568	303	490	369	472
5	327	514	345	160	135	178	399	442	417	520	351	538	39	226	57	448	423	466	111	154	129	232	63	250
6	83	209	92	557	296	566	11	281	20	197	80	206	371	497	380	269	8	278	299	569	308	485	368	494
7	325	534	343	162	115	180	397	462	415	522	331	540	37	246	55	450	403	468	109	174	127	234	43	252
8	139	156	121	528	349	510	67	228	49	168	133	150	427	444	409	240	61	222	355	516	337	456	421	438
9	323	545	344	161	104	182	395	473	416	521	320	542	35	257	56	449	392	470	107	185	128	233	32	254
10	141	172	123	526	333	508	69	244	51	166	117	148	429	460	411	238	45	220	357	532	339	454	405	436
11	291	550	309	196	99	214	363	478	381	556	315	574	3	262	21	484	387	502	75	190	93	268	27	286
12	116	170	131	518	335	533	44	242	59	158	119	173	404	458	419	230	47	245	332	530	347	446	407	461
13	289	570	307	198	79	216	361	498	379	558	295	576	1	282	19	486	367	504	73	210	91	270	7	288
14	247	48	229	420	457	402	175	120	157	60	241	42	535	336	517	132	169	114	463	408	445	348	529	330
15	500	362	455	50	287	5	572	290	527	410	503	365	212	74	167	338	575	293	284	2	239	122	215	77
16	249	64	231	418	441	400	177	136	159	58	225	40	537	352	519	130	153	112	465	424	447	346	513	328
17	471	370	489	16	279	34	543	298	561	376	495	394	183	82	201	304	567	322	255	10	273	88	207	106
18	227	65	236	413	440	422	155	137	164	53	224	62	515	353	524	125	152	134	443	425	452	341	512	350
19	469	390	487	18	259	36	541	318	559	378	475	396	181	102	199	306	547	324	253	30	271	90	187	108
20	283	12	265	384	493	366	211	84	193	24	277	6	571	300	553	96	205	78	499	372	481	312	565	294
21	467	401	488	17	248	38	539	329	560	377	464	398	179	113	200	305	536	326	251	41	272	89	176	110
22	285	28	267	382	477	364	213	100	195	22	261	4	573	316	555	94	189	76	501	388	483	310	549	292
23	435	406	453	52	243	70	507	334	525	412	459	430	147	118	165	340	531	358	219	46	237	124	171	142
24	260	26	275	374	479	389	188	98	203	14	263	29	548	314	563	86	191	101	476	386	491	302	551	317

Unlike the size-16 square, whose tiling patterns exist only on the pairwise centrally symmetric version, the size-24 square here, whose size is also divisible by **8**, is only pairwise <u>row-symmetric.</u>

This version was generated from a size-12 square, which in turn was generated from a size-6 near-perfect Class-2 square, yet surprisingly it is **6x**-modular, summing continuously to **1½ C_{24}**

This very same version of the size-24 square is shown on the next page to have yet another diamond quilt pattern.

Program 4

Another size-24 solid diamond Quilt-pattern

Here is the same size-24 square as on the prior page, now with both a solid-diamond of size **11** tile and one of size **12**. They are completely integrated into a quilt pattern of only **2** diamonds. They sum continuously to **11** and **13** times C_{24}.

Totally row-wise symmetric

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	217	66	235	414	439	432	145	138	163	342	511	360	505	354	523	126	151	144	6924
2	103	192	85	564	313	546	319	552	301	204	97	186	391	480	373	276	25	258	31	264	13	492	385	474	6924
3	356	506	311	194	143	149	140	146	95	554	359	509	68	218	23	482	431	437	428	434	383	266	71	221	6924
4	105	208	87	562	297	544	321	568	303	202	81	184	393	496	375	274	9	256	33	280	15	490	369	472	6924
5	327	514	345	160	135	178	111	154	129	520	351	538	39	226	57	448	423	466	399	442	417	232	63	250	6924
6	83	209	92	557	296	566	299	569	308	197	80	206	371	497	380	269	8	278	11	281	20	485	368	494	6924
7	325	534	343	162	115	180	109	174	127	522	331	540	37	246	55	450	403	468	397	462	415	234	43	252	6924
8	139	156	121	528	349	510	355	516	337	168	133	150	427	444	409	240	61	222	67	228	49	456	421	438	6924
9	323	545	344	161	104	182	107	185	128	521	320	542	35	257	56	449	392	470	395	473	416	233	32	254	6924
10	141	172	123	526	333	508	357	532	339	166	117	148	429	460	411	238	45	220	69	244	51	454	405	436	6924
11	291	550	309	196	99	214	75	190	93	556	315	574	3	262	21	484	387	502	363	478	381	268	27	286	6924
12	116	170	131	518	335	533	332	530	347	158	119	173	404	458	419	230	47	245	44	242	59	446	407	461	6924
13	289	570	307	198	79	216	73	210	91	558	295	576	1	282	19	486	367	504	361	498	379	270	7	288	6924
14	247	48	229	420	457	402	463	408	445	60	241	42	535	336	517	132	169	114	175	120	157	348	529	330	6924
15	500	362	455	50	287	5	284	2	239	410	503	365	212	74	167	338	575	293	572	290	527	122	215	77	6924
16	249	64	231	418	441	400	465	424	447	58	225	40	537	352	519	130	153	112	177	136	159	346	513	328	6924
17	471	370	489	16	279	34	255	10	273	376	495	394	183	82	201	304	567	322	543	298	561	88	207	106	6924
18	227	65	236	413	440	422	443	425	452	53	224	62	515	353	524	125	152	134	155	137	164	341	512	350	6924
19	469	390	487	18	259	36	253	30	271	378	475	396	181	102	199	306	547	324	541	318	559	90	187	108	6924
20	283	12	265	384	493	366	499	372	481	24	277	6	571	300	553	96	205	78	211	84	193	312	565	294	6924
21	467	401	488	17	248	38	251	41	272	377	464	398	179	113	200	305	536	326	539	329	560	89	176	110	6924
22	285	28	267	382	477	364	501	388	483	22	261	4	573	316	555	94	189	76	213	100	195	310	549	292	6924
23	435	406	453	52	243	70	219	46	237	412	459	430	147	118	165	340	531	358	507	334	525	124	171	142	6924
24	260	26	275	374	479	389	476	386	491	14	263	29	548	314	563	86	191	101	188	98	203	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

This too was not predicted by the Quilt Pattern Summary table; it was just stumbled upon in being thorough in testing all size diamond patterns for equal sums.

Program 4

Size-28

Here is the size-28 square depicting its full-diamond quilt pattern. It is composed of sizes **6** and **7** integrated solid diamond patterns which sum to **3** and **4** C_{28} respectively as predicted. It is totally self-contained.

There are **4** tiles of each size diamond. Again we have two simultaneous tile patterns which integrate to tile the square exactly and completely.

Not only are we tapping into the fabric of space, we are uncovering its incredible infinite tapestry. Now that's wow math!

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
	W(28)	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990

Program 4

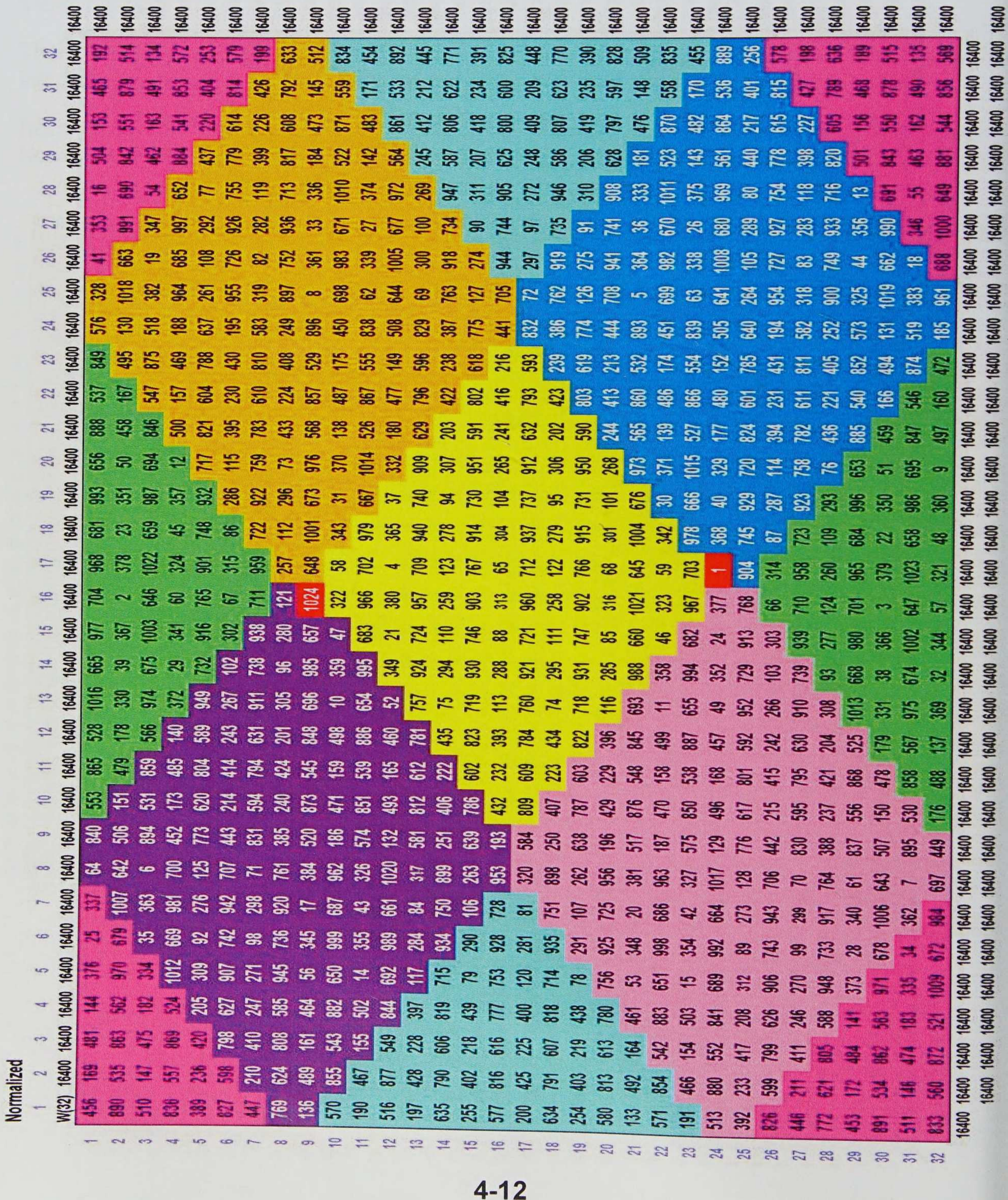
Size-32

Here is the size-32 square's quilt pattern composed of sizes **7** and **8** integrated solid diamond patterns summing continuously to the predicted **3½** and **4½** multiples of C_{32} respectively. It is totally self contained within the square by **4** diamonds of each size. This version was generated by the **SPD** method of Program 7 using a binary based spread pattern.

Program 4

Size-32

Here is different version of the size-32 square with the same quilt pattern. This version is also pairwise centrally symmetric and was also generated by the **SPD** method of Program 7 but using a special **2x2** spread pattern. Again the size-32 square is another Class-4 square whose size is evenly divisible by **8**.



Program 4

Size-36

The size-36 tiles are full-diamonds of intigrated sizes **8** and **9** as predicted, that fill the square completely and exactly.There are **4** of each size. The size-8 tile sums to $\mathbf{4C_{36}}$; the size-9 tile sums to $\mathbf{5C_{36}}$.

In general for Class-4 squares, the sizes of tiles are $\mathbf{n_1 = (n/4)}$ and $\mathbf{n_2 = n_1 - 1}$. In general also, neither of these two integrateable tile patterns separately exactly cover the square; they must remain integrated

Program 4

Size-44 Square – Left half

Here is the size-44 square, spread across two pages, depicting its continuous equal-summing diamond quilt pattern. It is composed of **4** size **10** diamonds (yellow) and **4** size **11** diamonds, each summing to **5** and **6** times C_{44}, respectively. There's also a size **9** continuous equal-summing diamond but it doesn't integrate with either of the other two nor does it complete the square, so it is not shown. You might want to check that out.

Each of these three diamond patterns possesses an even-size pyramidal diamond of their same size respectively summing to **15**, **20** and **26** times C_{44}.

W(44)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	505	1126	1758	443	591	1091	1723	408	556	1188	1809	384	521	1153	1774	470	486	1118	1739	435	583	1204
2	1839	293	672	1062	1936	258	648	1027	1901	344	613	992	1866	309	699	1089	1831	285	664	1054	1917	250
3	159	1517	1412	834	124	1482	1377	799	210	1568	1353	764	186	1533	1439	729	151	1498	1404	815	237	1474
4	1262	915	95	1684	1227	891	60	1660	1313	856	25	1625	1278	942	111	1590	1254	907	87	1676	1219	872
5	550	1171	1803	367	515	1136	1768	453	601	1101	1733	429	566	1198	1698	394	531	1163	1784	480	496	1128
6	1884	338	717	986	1849	303	693	1072	1825	268	658	1037	1911	354	623	1002	1876	330	709	978	1841	295
7	204	1562	1336	758	169	1527	1422	844	134	1492	1387	809	231	1457	1363	774	196	1543	1449	739	161	1508
8	1307	960	19	1608	1272	925	105	1584	1237	901	70	1670	1323	866	35	1635	1288	952	11	1600	1264	917
9	584	1095	1727	412	560	1181	1813	377	525	1146	1778	463	490	1122	1743	439	576	1208	1708	404	541	1173
10	1929	262	641	1031	1894	348	606	996	1870	313	703	1082	1835	278	668	1047	1921	243	633	1023	1886	340
11	128	1475	1381	803	214	1572	1346	768	179	1537	1432	733	144	1502	1408	819	241	1467	1373	784	206	1553
12	1231	884	64	1653	1317	849	29	1618	1282	946	115	1594	1247	911	80	1680	1212	876	45	1645	1309	962
13	508	1140	1761	457	605	1105	1737	422	570	1191	1702	387	535	1156	1788	484	500	1132	1753	449	586	1097
14	1853	307	686	1076	1818	272	651	1041	1904	358	627	1006	1880	323	713	971	1845	288	678	1057	1931	264
15	173	1520	1426	837	138	1496	1391	813	224	1461	1356	778	189	1547	1442	743	165	1512	1418	829	130	1477
16	1276	929	109	1577	1241	894	74	1663	1327	859	39	1639	1292	956	4	1604	1257	921	90	1690	1222	886
17	553	1185	1806	381	518	1150	1782	467	494	1115	1747	432	580	1201	1712	397	545	1177	1798	373	510	1142
18	1898	352	610	1000	1863	317	696	1086	1828	282	661	1051	1925	247	637	1016	1890	333	723	981	1855	298
19	218	1565	1350	761	183	1530	1436	737	148	1506	1401	823	234	1471	1366	788	199	1557	1342	753	175	1522
20	1310	853	33	1622	1286	939	119	1587	1251	904	84	1673	1216	880	49	1649	1302	966	14	1614	1267	931
21	598	1109	1730	426	563	1195	1695	391	539	1160	1792	477	504	1125	1757	442	590	1090	1722	418	555	1187
22	1822	265	655	1045	1908	362	620	1010	1873	327	706	975	1838	292	682	1061	1935	257	647	1026	1900	343
23	142	1489	1395	806	228	1454	1360	771	193	1551	1446	747	158	1516	1411	833	123	1481	1376	798	220	1567
24	1234	898	67	1667	1331	863	43	1632	1296	949	8	1597	1261	914	94	1694	1226	890	59	1659	1312	855
25	522	1154	1775	471	487	1119	1740	436	573	1205	1716	401	549	1170	1802	366	514	1135	1767	452	600	1111
26	1867	310	700	1079	1832	286	665	1055	1918	251	630	1020	1883	337	716	985	1859	302	692	1071	1824	267
27	187	1534	1440	730	152	1499	1405	816	238	1464	1370	792	203	1561	1335	757	168	1526	1421	843	133	1491
28	1279	943	112	1591	1244	908	88	1677	1220	873	53	1642	1306	959	18	1607	1271	935	104	1583	1236	900
29	567	1199	1699	395	532	1164	1785	481	497	1129	1750	446	594	1094	1726	411	559	1180	1812	376	524	1145
30	1912	355	624	1003	1877	320	710	979	1842	296	675	1065	1928	261	640	1030	1893	347	616	995	1869	312
31	221	1458	1364	775	197	1544	1450	740	162	1509	1415	826	127	1485	1380	802	213	1571	1345	767	178	1536
32	1324	867	36	1636	1289	953	1	1601	1265	918	98	1687	1230	883	63	1652	1316	848	28	1628	1281	945
33	491	1112	1744	440	577	1209	1709	405	542	1174	1795	370	507	1139	1771	456	604	1104	1736	421	569	1190
34	1836	279	669	1048	1922	244	634	1013	1887	341	720	989	1852	306	685	1075	1817	271	650	1040	1914	357
35	145	1503	1398	820	242	1468	1374	785	207	1554	1339	750	172	1519	1425	847	137	1495	1390	812	223	1460
36	1248	912	81	1681	1213	877	46	1646	1299	963	22	1611	1275	928	108	1576	1240	893	73	1662	1326	869
37	536	1157	1789	474	501	1133	1754	450	587	1098	1719	415	552	1184	1805	380	528	1149	1781	466	493	1114
38	1881	324	714	972	1846	289	679	1058	1932	254	644	1034	1897	351	609	999	1862	316	695	1085	1827	281
39	190	1548	1443	744	155	1513	1419	830	131	1478	1384	795	217	1564	1349	760	182	1540	1435	736	147	1505
40	1293	957	5	1605	1258	922	91	1691	1223	887	56	1656	1320	852	32	1621	1285	938	118	1586	1250	903
41	581	1202	1713	398	546	1167	1799	374	511	1143	1764	460	597	1108	1729	425	562	1194	1705	390	538	1159
42	1915	248	638	1017	1891	334	724	982	1856	299	689	1068	1821	275	654	1044	1907	361	619	1009	1872	326
43	235	1472	1367	789	200	1558	1332	754	176	1523	1429	840	141	1488	1394	805	227	1453	1359	781	192	1550
44	1217	870	50	1650	1303	967	15	1615	1268	932	101	1580	1233	897	77	1666	1330	862	42	1631	1295	948
	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614

Program 4

Size-44 Square – Right half

All these Class-4 squares are perfect as well as pangenic, though not necessarily pairwise symmetric.

23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	
1715	400	548	1169	1801	365	513	1134	1766	462	599	1110	1731	427	564	1196	1696	392	529	1161	1793	478	42614
629	1019	1882	336	726	984	1858	301	691	1070	1823	266	656	1035	1909	363	621	1011	1874	328	707	976	42614
1369	791	202	1560	1334	756	167	1525	1420	842	143	1490	1396	807	229	1455	1361	772	194	1541	1447	748	42614
52	1641	1305	958	17	1617	1270	934	103	1582	1235	899	68	1668	1321	864	44	1633	1297	950	9	1598	42614
1760	445	593	1093	1725	410	558	1179	1811	375	523	1155	1776	472	488	1120	1741	437	574	1206	1706	402	42614
674	1064	1927	260	639	1029	1903	346	615	994	1868	311	701	1080	1833	276	666	1056	1919	252	631	1021	42614
1414	836	126	1484	1379	801	212	1570	1344	766	177	1535	1441	731	153	1500	1406	817	239	1465	1371	782	42614
97	1686	1229	882	62	1651	1315	858	27	1627	1280	944	113	1592	1245	909	78	1678	1221	874	54	1643	42614
1794	369	517	1138	1770	455	603	1103	1735	420	568	1189	1700	396	533	1165	1786	482	498	1130	1751	447	42614
719	988	1851	305	684	1074	1816	270	660	1039	1913	356	625	1004	1878	321	711	969	1843	297	676	1066	42614
1338	749	171	1529	1424	846	136	1494	1389	811	222	1459	1354	776	198	1545	1451	741	163	1510	1416	827	42614
21	1610	1274	927	107	1575	1239	892	72	1672	1325	868	37	1637	1290	954	2	1602	1255	919	99	1688	42614
1718	414	551	1183	1815	379	527	1148	1780	465	492	1113	1745	430	578	1210	1710	406	543	1175	1796	371	42614
643	1033	1896	350	608	998	1861	315	694	1084	1837	280	670	1049	1923	245	635	1014	1888	331	721	990	42614
1383	794	216	1563	1348	770	181	1539	1434	735	146	1504	1399	821	232	1469	1375	786	208	1555	1340	751	42614
66	1655	1319	851	31	1620	1284	937	117	1585	1249	913	82	1682	1214	878	47	1647	1300	964	12	1612	42614
1763	459	596	1107	1728	424	572	1193	1704	389	537	1158	1790	475	502	1123	1755	451	588	1099	1720	416	42614
688	1078	1820	274	653	1043	1906	360	618	1008	1871	325	715	973	1847	290	680	1059	1933	255	645	1024	42614
1428	839	140	1487	1393	804	226	1463	1358	780	191	1549	1444	745	156	1514	1409	831	132	1479	1385	796	42614
100	1579	1243	896	76	1665	1329	861	41	1630	1294	947	6	1606	1259	923	92	1692	1224	888	57	1657	42614
1808	383	520	1152	1773	469	485	1117	1749	434	582	1203	1714	399	547	1168	1800	364	512	1144	1765	461	42614
612	991	1865	319	698	1088	1830	284	663	1053	1916	249	628	1018	1892	335	725	983	1857	300	690	1069	42614
1352	763	185	1532	1438	728	150	1497	1403	825	236	1473	1368	790	201	1559	1333	755	166	1524	1430	841	42614
24	1624	1277	941	121	1589	1253	906	86	1675	1218	871	51	1640	1304	968	16	1616	1269	933	102	1581	42614
1732	428	565	1197	1697	393	530	1162	1783	479	506	1127	1759	444	592	1092	1724	409	557	1178	1810	385	42614
657	1036	1910	353	622	1012	1875	329	708	977	1840	294	673	1063	1926	259	649	1028	1902	345	614	993	42614
1397	808	230	1456	1362	773	195	1542	1448	738	160	1518	1413	835	125	1483	1378	800	211	1569	1343	765	42614
69	1669	1322	865	34	1634	1298	951	10	1599	1263	916	96	1685	1228	881	61	1661	1314	857	26	1626	42614
1777	473	489	1121	1742	438	575	1207	1707	403	540	1172	1804	368	516	1137	1769	454	602	1102	1734	419	42614
702	1081	1834	277	667	1046	1920	253	632	1022	1885	339	718	987	1850	304	683	1073	1826	269	659	1038	42614
1431	732	154	1501	1407	818	240	1466	1372	783	205	1552	1337	759	170	1528	1423	845	135	1493	1388	810	42614
114	1593	1246	910	79	1679	1211	875	55	1644	1308	961	20	1609	1273	926	106	1574	1238	902	71	1671	42614
1701	386	534	1166	1787	483	499	1131	1752	448	585	1096	1717	413	561	1182	1814	378	526	1147	1779	464	42614
626	1005	1879	322	712	970	1844	287	677	1067	1930	263	642	1032	1895	349	607	997	1860	314	704	1083	42614
1355	777	188	1546	1452	742	164	1511	1417	828	129	1476	1382	793	215	1573	1347	769	180	1538	1433	734	42614
38	1638	1291	955	3	1603	1256	920	89	1689	1232	885	65	1654	1318	850	30	1619	1283	936	116	1595	42614
1746	431	579	1200	1711	407	544	1176	1797	372	509	1141	1762	458	595	1106	1738	423	571	1192	1703	388	42614
671	1050	1924	246	636	1015	1889	332	722	980	1854	308	687	1077	1819	273	652	1042	1905	359	617	1007	42614
1400	822	233	1470	1365	787	209	1556	1341	752	174	1521	1427	838	139	1486	1392	814	225	1462	1357	779	42614
83	1683	1215	879	48	1648	1301	965	13	1613	1266	930	110	1578	1242	895	75	1664	1328	860	40	1629	42614
1791	476	503	1124	1756	441	589	1100	1721	417	554	1186	1807	382	519	1151	1772	468	495	1116	1748	433	42614
705	974	1848	291	681	1060	1934	256	646	1025	1899	342	611	1001	1864	318	697	1087	1829	283	662	1052	42614
1445	746	157	1515	1410	832	122	1480	1386	797	219	1566	1351	762	184	1531	1437	727	149	1507	1402	824	42614
7	1596	1260	924	93	1693	1225	889	58	1658	1311	854	23	1623	1287	940	120	1588	1252	905	85	1674	42614
42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	

The summary table of quilt and pyramidal patterns on page 4-5 shows the arithmetic progressions for each size among the Class-4 squares up to a square of size **76**. All sizes up thru **64** sizes have been verified.

Program 4

Size-16

Here is the quilt pattern for the size-16 square. Each individual pattern is composed of two side-by-side "+" diamonds of a peculiar shape. This was the only quilt pattern found for the size-16 square that summed equally and continuously. Both the horizontal and vertical versions sum equally to $2C_{16}$ and so are complementary patterns.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

W(16)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249

Diamond sizes **3** and **4** as specified in the arithmetic pattern table do not work here. This is the result of the embryonic nature of the size-16 square. The one pattern that does work is the average of the predicted values $1\frac{1}{2}\,C_{16}$ and $2\frac{1}{2}\,C_{16}$, namely $2C_{16}$.

Program 4

Size-32

In addition to the size-32 square possessing complementary diamond and X-patterns and diamond quilt patterns, it also possesses a tile pattern consisting of **32** cells in a double "**+**" configuration which also sum continuously and exactly to its characteristic number.

This pattern and its transpose were found to hold for the size-16 square on the prior page. Where it summed to $\mathbf{2C_{16}}$ there, it sums to exactly $\mathbf{C_{32}}$ here for the size-32. The pattern's transpose (not shown) also holds here for the size-32 square and can be seen as one quarter of its complete size on prior page.

The fact that these complementary tiling patterns re-appeared in exactly same size and configurations for a square twice the size is just amazing! That's more wow math!

Program 4

A Fixed-size Tile Pattern for Squares whose Size is a Power of 2

The size-64 Class-4 square is way too large for print and even comprehension, the amazing point is the exact same tile patterns that worked for sizes **16** and **32** also held for the size **64** !

Whereas the pattern equaled $\mathbf{2C_{16}}$ and $\mathbf{1C_{32}}$ on the sizes **16** and **32**, it now equaled $\mathbf{\frac{1}{2}C_{64}}$ on size **64**. I predict that the size **128** square will possess this same tile pattern and will sum equally to $\mathbf{\frac{1}{4}C_{128}}$.

65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552

65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552

65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552
65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552	65552

Doubling the size of tile pattern didn't sum equally. Quadrupling its size didn't work either.

1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040	1040

1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040
1040	1040	1040	1040	1040	1040	1040	1040	1040

When this same pattern was tried on the size-8 square, it worked there too, summing to **4** times the characteristic number. Clearly, this pattern doesn't apply to sizes **2** and **4**.

So this is the characteristic tile pattern for all squares whose size $\mathbf{n = 2^j}$ for $\mathbf{j \geq 3}$. That's some more wow math !

Program 4

Triangular Tapestry Patterns

The three sizes **20**, **28** and **36** possess another amazing tiling property: their diamond patterns can be segregated into four triangles that also sum equally independently and continuously across the square.

Size-20

Here is the size-20 square that we saw earlier on page 7 now with integrated size **4** and **5** even-size diamonds. The size **20**'s triangles are of different orientations but each still sums to **½ C_{20}**.

Check out the triangles in the size-20 square below. These equal-summing triangles can be rearranged into different tiling patterns. Note that two triangles can be brought together in a **5x4** block-rectangle which results in **5x4**-modularity because that configuration will sum to **C_{20}** exactly.

That's not all: The transpose tiling pattern yields triangles which also sum to **½ C_{20}**. So the **4x5** and **5x4** rectangles are characteristic complementary tiling patterns for the size-20 square. Check that out.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	20	183	321	289	2	195	308	296	14	177	320	283	21	189	302	295	8	196	314	277
2	336	254	42	160	348	261	29	167	335	273	36	154	342	260	48	161	329	267	35	173
3	82	125	388	201	94	107	400	213	76	119	382	225	88	101	394	207	100	113	376	219
4	353	241	59	147	365	228	66	134	372	240	53	141	359	247	65	128	366	234	72	140
5	24	187	305	293	6	199	312	280	18	181	324	287	5	193	306	299	12	180	318	281
6	345	258	46	164	327	270	33	171	339	252	45	158	346	264	27	170	333	271	39	152
7	86	104	392	210	98	111	379	217	85	123	386	204	92	110	398	211	79	117	385	223
8	357	250	63	126	369	232	75	138	351	244	57	150	363	226	69	132	375	238	51	144
9	3	191	309	297	15	178	316	284	22	190	303	291	9	197	315	278	16	184	322	290
10	349	262	30	168	331	274	37	155	343	256	49	162	330	268	31	174	337	255	43	156
11	95	108	396	214	77	120	383	221	89	102	395	208	96	114	377	220	83	121	389	202
12	361	229	67	135	373	236	54	142	360	248	61	129	367	235	73	136	354	242	60	148
13	7	200	313	276	19	182	325	288	1	194	307	300	13	176	319	282	25	188	301	294
14	328	266	34	172	340	253	41	159	347	265	28	166	334	272	40	153	341	259	47	165
15	99	112	380	218	81	124	387	205	93	106	399	212	80	118	381	224	87	105	393	206
16	370	233	71	139	352	245	58	146	364	227	70	133	371	239	52	145	358	246	64	127
17	11	179	317	285	23	186	304	292	10	198	311	279	17	185	323	286	4	192	310	298
18	332	275	38	151	344	257	50	163	326	269	32	175	338	251	44	157	350	263	26	169
19	78	116	384	222	90	103	391	209	97	115	378	216	84	122	390	203	91	109	397	215
20	374	237	55	143	356	249	62	130	368	231	74	137	355	243	56	149	362	230	68	131

Program 4

Triangular Tapestries for the Size-20 Square Continued

Here is the size **20** square with a different integration of the **5x4** and **4x5** block-rectangles which sum to the square's characteristic number. Observe the wavy tapestries.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	397	215	153	41	384	222	165	28	391	209	172	40	378	216	159	47	390	203	166	34
2	68	131	299	312	55	143	281	324	62	130	293	306	74	137	280	318	56	149	287	305
3	239	352	20	183	246	364	2	195	233	371	14	177	245	358	21	189	227	370	8	196
4	110	98	336	254	117	85	348	261	104	92	335	273	111	79	342	260	123	86	329	267
5	376	219	157	50	388	201	169	32	400	213	151	44	382	225	163	26	394	207	175	38
6	72	140	278	316	59	147	290	303	66	134	297	315	53	141	284	322	65	128	291	309
7	243	356	24	187	230	368	6	199	237	355	18	181	249	362	5	193	231	374	12	180
8	114	77	345	258	121	89	327	270	108	96	339	252	120	83	346	264	102	95	333	271
9	385	223	161	29	392	210	173	36	379	217	160	48	386	204	167	35	398	211	154	42
10	51	144	282	325	63	126	294	307	75	138	276	319	57	150	288	301	69	132	300	313
11	247	365	3	191	234	372	15	178	241	359	22	190	228	366	9	197	240	353	16	184
12	118	81	349	262	105	93	331	274	112	80	343	256	124	87	330	268	106	99	337	255
13	389	202	170	33	396	214	152	45	383	221	164	27	395	208	171	39	377	220	158	46
14	60	148	286	304	67	135	298	311	54	142	285	323	61	129	292	310	73	136	279	317
15	226	369	7	200	238	351	19	182	250	363	1	194	232	375	13	176	244	357	25	188
16	122	90	328	266	109	97	340	253	116	84	347	265	103	91	334	272	115	78	341	259
17	393	206	174	37	380	218	156	49	387	205	168	31	399	212	155	43	381	224	162	30
18	64	127	295	308	71	139	277	320	58	146	289	302	70	133	296	314	52	145	283	321
19	235	373	11	179	242	360	23	186	229	367	10	198	236	354	17	185	248	361	4	192
20	101	94	332	275	113	76	344	257	125	88	326	269	107	100	338	251	119	82	350	263

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	397	215	153	41	384	222	165	28	391	209	172	40	378	216	159	47	390	203	166	34
2	68	131	299	312	55	143	281	324	62	130	293	306	74	137	280	318	56	149	287	305
3	239	352	20	183	246	364	2	195	233	371	14	177	245	358	21	189	227	370	8	196
4	110	98	336	254	117	85	348	261	104	92	335	273	111	79	342	260	123	86	329	267
5	376	219	157	50	388	201	169	32	400	213	151	44	382	225	163	26	394	207	175	38
6	72	140	278	316	59	147	290	303	66	134	297	315	53	141	284	322	65	128	291	309
7	243	356	24	187	230	368	6	199	237	355	18	181	249	362	5	193	231	374	12	180
8	114	77	345	258	121	89	327	270	108	96	339	252	120	83	346	264	102	95	333	271
9	385	223	161	29	392	210	173	36	379	217	160	48	386	204	167	35	398	211	154	42
10	51	144	282	325	63	126	294	307	75	138	276	319	57	150	288	301	69	132	300	313
11	247	365	3	191	234	372	15	178	241	359	22	190	228	366	9	197	240	353	16	184
12	118	81	349	262	105	93	331	274	112	80	343	256	124	87	330	268	106	99	337	255
13	389	202	170	33	396	214	152	45	383	221	164	27	395	208	171	39	377	220	158	46
14	60	148	286	304	67	135	298	311	54	142	285	323	61	129	292	310	73	136	279	317
15	226	369	7	200	238	351	19	182	250	363	1	194	232	375	13	176	244	357	25	188
16	122	90	328	266	109	97	340	253	116	84	347	265	103	91	334	272	115	78	341	259
17	393	206	174	37	380	218	156	49	387	205	168	31	399	212	155	43	381	224	162	30
18	64	127	295	308	71	139	277	320	58	146	289	302	70	133	296	314	52	145	283	321
19	235	373	11	179	242	360	23	186	229	367	10	198	236	354	17	185	248	361	4	192
20	101	94	332	275	113	76	344	257	125	88	326	269	107	100	338	251	119	82	350	263

Program 4

Size-28 and 36 Squares

This same principle holds for the size **28** and **36** squares too. Again, there are <u>two</u> sizes of diamonds in each of those square's tiling patterns. The table on page 5 shows that the largest size diamond in the size-28 square and the smallest size diamond in the size 36 square are even multiples of their characteristic numbers, namely **4**. So these diamond patterns will decompose into 4 simultaneous equal-summing triangles to form triangular tapestries. Both size **28**'s and **36**'s triangles sum exactly to their characteristic number making them a set of four complementary characteristic tiles independently.

In the case of the size-28 square, the complementary tiling patterns are **7x8** and **8x7** block-rectangles. In the case of the size-36 square, the complementary tiling patterns are **8x9** and **9x8** block-rectangles (not shown). Each of these will sum to twice their square's characteristic number.

Here is the same size-28 square as seen on page 10 with a tapestry composed of **7x7** block-squares which in turn are composed of size-6 and size-7 triangles. The arrangement is not **7x**-modular because the modularity is confined to **7x8** and **8x7** rectangles. However both sizes of diamonds and the <u>larger triangles</u> still sum equally and continuously per their size and configuration. So the continuously equal summing patterns are the diamonds composed of the smaller triangles (yellow) which do sum equally and the larger triangles in an integrated tapestry.

The size-36 tapestry (not shown) is left as an exercise.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	

Program 4

Inequalities in Pyramidal Summations of Lesser Viable Levels Lead to the Recovery of the Expanded Squares

Here is the Class-4 size-28 square again. It is tiled with the size-5 even-size diamond. That diamond contains a size-4 even-size diamond, which in turn contains a size-3 even-size diamond, which in turn contains a size-2 even-size diamond, which in turn contains a size-1 even-size diamond.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680

The table at bottom is a continuous summation of the size-1 diamond across the square. The size-1 even-size diamond is a **2x2** block-square. Note that the summation is repeated in **7x7** blocks. Now here is the surprise: these block-squares are geonomic in that they sum exactly to the size-28 square's characteristic number **10990** in every row, column and diagonal.

So we have discovered a geonomic pattern in the size-1 even-size diamond pattern summations of a geonomic square!

1	2	3	4	5	6	7	Size-1 even-size diamonds																		Horizontal sums
1552	1554	1612	1572	1525	1583	1592	1552	1554	1612	1572	1525	1583	1592	1552	1554	1612	1572	1525	1583	1592					
1567	1527	1585	1594	1547	1556	1614	1567	1527	1585	1594	1547	1556	1614	1567	1527	1585	1594	1547	1556	1614					
1589	1549	1558	1609	1569	1529	1587	1589	1549	1558	1609	1569	1529	1587	1589	1549	1558	1609	1569	1529	1587					
1611	1571	1531	1582	1591	1551	1553	1611	1571	1531	1582	1591	1551	1553	1611	1571	1531	1582	1591	1551	1553					
1584	1593	1546	1555	1613	1573	1526	1584	1593	1546	1555	1613	1573	1526	1584	1593	1546	1555	1613	1573	1526					
1557	1615	1568	1528	1586	1588	1548	1557	1615	1568	1528	1586	1588	1548	1557	1615	1568	1528	1586	1588	1548					
1530	1581	1590	1550	1559	1610	1570	1530	1581	1590	1550	1559	1610	1570	1530	1581	1590	1550	1559	1610	1570					
1552	1554	1612	1572	1525	1583	1592	1552	1554	1612	1572	1525	1583	1592	1552	1554	1612	1572	1525	1583	1592					
1567	1527	1585	1594	1547	1556	1614	1567	1527	1585	1594	1547	1556	1614	1567	1527	1585	1594	1547	1556	1614					
1589	1549	1558	1609	1569	1529	1587	1589	1549	1558	1609	1569	1529	1587	1589	1549	1558	1609	1569	1529	1587					
1611	1571	1531	1582	1591	1551	1553	1611	1571	1531	1582	1591	1551	1553	1611	1571	1531	1582	1591	1551	1553					
1584	1593	1546	1555	1613	1573	1526	1584	1593	1546	1555	1613	1573	1526	1584	1593	1546	1555	1613	1573	1526					
1557	1615	1568	1528	1586	1588	1548	1557	1615	1568	1528	1586	1588	1548	1557	1615	1568	1528	1586	1588	1548					
1530	1581	1590	1550	1559	1610	1570	1530	1581	1590	1550	1559	1610	1570	1530	1581	1590	1550	1559	1610	1570					
1552	1554	1612	1572	1525	1583	1592	1552	1554	1612	1572	1525	1583	1592	1552	1554	1612	1572	1525	1583	1592	10990				
1567	1527	1585	1594	1547	1556	1614	1567	1527	1585	1594	1547	1556	1614	1567	1527	1585	1594	1547	1556	1614	10990				
1589	1549	1558	1609	1569	1529	1587	1589	1549	1558	1609	1569	1529	1587	1589	1549	1558	1609	1569	1529	1587	10990				
1611	1571	1531	1582	1591	1551	1553	1611	1571	1531	1582	1591	1551	1553	1611	1571	1531	1582	1591	1551	1553	10990				
1584	1593	1546	1555	1613	1573	1526	1584	1593	1546	1555	1613	1573	1526	1584	1593	1546	1555	1613	1573	1526	10990				
1557	1615	1568	1528	1586	1588	1548	1557	1615	1568	1528	1586	1588	1548	1557	1615	1568	1528	1586	1588	1548	10990				
1530	1581	1590	1550	1559	1610	1570	1530	1581	1590	1550	1559	1610	1570	1530	1581	1590	1550	1559	1610	1570	10990				
Vertical sums →														10990	10990	10990	10990	10990	10990	10990					

Program 4

There's more! This exact same thing occurs for size-2 diamond, the size-3 diamond, the size-4 diamond and the size-5 diamond too, which sum to **3**, **6**, **10** and **15** multiples of C_{28} respectively.

Size-2 even-size diamonds

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	4728	4706	4719	4697	4717	4688	4715	4728	4706	4719	4697	4717	4688	4715
2	4696	4716	4694	4707	4734	4698	4725	4696	4716	4694	4707	4734	4698	4725
3	4713	4726	4704	4724	4695	4722	4686	4713	4726	4704	4724	4695	4722	4686
4	4723	4701	4714	4692	4705	4732	4703	4723	4701	4714	4692	4705	4732	4703
5	4684	4711	4731	4702	4729	4693	4720	4684	4711	4731	4702	4729	4693	4720
6	4708	4721	4699	4712	4690	4710	4730	4708	4721	4699	4712	4690	4710	4730
7	4718	4689	4709	4736	4700	4727	4691	4718	4689	4709	4736	4700	4727	4691
8	4728	4706	4719	4697	4717	4688	4715	4728	4706	4719	4697	4717	4688	4715
9	4696	4716	4694	4707	4734	4698	4725	4696	4716	4694	4707	4734	4698	4725
10	4713	4726	4704	4724	4695	4722	4686	4713	4726	4704	4724	4695	4722	4686
11	4723	4701	4714	4692	4705	4732	4703	4723	4701	4714	4692	4705	4732	4703
12	4684	4711	4731	4702	4729	4693	4720	4684	4711	4731	4702	4729	4693	4720
13	4708	4721	4699	4712	4690	4710	4730	4708	4721	4699	4712	4690	4710	4730
14	4718	4689	4709	4736	4700	4727	4691	4718	4689	4709	4736	4700	4727	4691
15	4728	4706	4719	4697	4717	4688	4715	4728	4706	4719	4697	4717	4688	4715
16	4696	4716	4694	4707	4734	4698	4725	4696	4716	4694	4707	4734	4698	4725
17	4713	4726	4704	4724	4695	4722	4686	4713	4726	4704	4724	4695	4722	4686
18	4723	4701	4714	4692	4705	4732	4703	4723	4701	4714	4692	4705	4732	4703
19	4684	4711	4731	4702	4729	4693	4720	4684	4711	4731	4702	4729	4693	4720
20	4708	4721	4699	4712	4690	4710	4730	4708	4721	4699	4712	4690	4710	4730
21	4718	4689	4709	4736	4700	4727	4691	4718	4689	4709	4736	4700	4727	4691

Horizontal sums of size-2 even-size diamond sums

1	4728	4706	4719	4697	4717	4688	4715
2	4696	4716	4694	4707	4734	4698	4725
3	4713	4726	4704	4724	4695	4722	4686
4	4723	4701	4714	4692	4705	4732	4703
5	4684	4711	4731	4702	4729	4693	4720
6	4708	4721	4699	4712	4690	4710	4730
7	4718	4689	4709	4736	4700	4727	4691
8	4728	4706	4719	4697	4717	4688	4715
9	4696	4716	4694	4707	4734	4698	4725
10	4713	4726	4704	4724	4695	4722	4686
11	4723	4701	4714	4692	4705	4732	4703
12	4684	4711	4731	4702	4729	4693	4720
13	4708	4721	4699	4712	4690	4710	4730
14	4718	4689	4709	4736	4700	4727	4691
15	4728	4706	4719	4697	4717	4688	4715
16	4696	4716	4694	4707	4734	4698	4725
17	4713	4726	4704	4724	4695	4722	4686
18	4723	4701	4714	4692	4705	4732	4703
19	4684	4711	4731	4702	4729	4693	4720
20	4708	4721	4699	4712	4690	4710	4730
21	4718	4689	4709	4736	4700	4727	4691

Vertical sums of size-2 even-size diamond sums ⇒	32970	32970	32970	32970	32970	32970	32970	3 x 10990

Size-3 even-size diamond sums

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	9432	9374	9428	9454	9410	9352	9490	9432	9374	9428	9454	9410	9352	9490
2	9466	9408	9350	9488	9430	9386	9412	9466	9408	9350	9488	9430	9386	9412
3	9486	9442	9370	9424	9464	9406	9348	9486	9442	9370	9424	9464	9406	9348
4	9422	9462	9404	9346	9498	9426	9382	9422	9462	9404	9346	9498	9426	9382
5	9358	9482	9438	9380	9420	9460	9402	9358	9482	9438	9380	9420	9460	9402
6	9378	9418	9458	9414	9342	9494	9436	9378	9418	9458	9414	9342	9494	9436
7	9398	9354	9492	9434	9376	9416	9470	9398	9354	9492	9434	9376	9416	9470
8	9432	9374	9428	9454	9410	9352	9490	9432	9374	9428	9454	9410	9352	9490
9	9466	9408	9350	9488	9430	9386	9412	9466	9408	9350	9488	9430	9386	9412
10	9486	9442	9370	9424	9464	9406	9348	9486	9442	9370	9424	9464	9406	9348
11	9422	9462	9404	9346	9498	9426	9382	9422	9462	9404	9346	9498	9426	9382
12	9358	9482	9438	9380	9420	9460	9402	9358	9482	9438	9380	9420	9460	9402
13	9378	9418	9458	9414	9342	9494	9436	9378	9418	9458	9414	9342	9494	9436
14	9398	9354	9492	9434	9376	9416	9470	9398	9354	9492	9434	9376	9416	9470
15	9432	9374	9428	9454	9410	9352	9490	9432	9374	9428	9454	9410	9352	9490
16	9466	9408	9350	9488	9430	9386	9412	9466	9408	9350	9488	9430	9386	9412
17	9486	9442	9370	9424	9464	9406	9348	9486	9442	9370	9424	9464	9406	9348
18	9422	9462	9404	9346	9498	9426	9382	9422	9462	9404	9346	9498	9426	9382
19	9358	9482	9438	9380	9420	9460	9402	9358	9482	9438	9380	9420	9460	9402
20	9378	9418	9458	9414	9342	9494	9436	9378	9418	9458	9414	9342	9494	9436
21	9398	9354	9492	9434	9376	9416	9470	9398	9354	9492	9434	9376	9416	9470

Horizontal sums of size-3 even-size diamond sums

1	9432	9374	9428	9454	9410	9352	9490
2	9466	9408	9350	9488	9430	9386	9412
3	9486	9442	9370	9424	9464	9406	9348
4	9422	9462	9404	9346	9498	9426	9382
5	9358	9482	9438	9380	9420	9460	9402
6	9378	9418	9458	9414	9342	9494	9436
7	9398	9354	9492	9434	9376	9416	9470
8	9432	9374	9428	9454	9410	9352	9490
9	9466	9408	9350	9488	9430	9386	9412
10	9486	9442	9370	9424	9464	9406	9348
11	9422	9462	9404	9346	9498	9426	9382
12	9358	9482	9438	9380	9420	9460	9402
13	9378	9418	9458	9414	9342	9494	9436
14	9398	9354	9492	9434	9376	9416	9470
15	9432	9374	9428	9454	9410	9352	9490
16	9466	9408	9350	9488	9430	9386	9412
17	9486	9442	9370	9424	9464	9406	9348
18	9422	9462	9404	9346	9498	9426	9382
19	9358	9482	9438	9380	9420	9460	9402
20	9378	9418	9458	9414	9342	9494	9436
21	9398	9354	9492	9434	9376	9416	9470

Vertical sums of size-3 even-size diamond sums ⇒	65940	65940	65940	65940	65940	65940	65940	6 x 10990

Program 4

Size-4 even-size diamonds

	1	2	3	4	5	6	7
1	15682	15695	15722	15693	15713	15691	15704
2	15692	15719	15683	15710	15674	15701	15721
3	15702	15680	15700	15720	15698	15711	15689
4	15726	15690	15717	15681	15708	15679	15699
5	15687	15707	15678	15705	15718	15696	15709
6	15697	15724	15688	15715	15686	15706	15684
7	15714	15685	15712	15676	15703	15716	15694
	15682	15695	15722	15693	15713	15691	15704
	15692	15719	15683	15710	15674	15701	15721
	15702	15680	15700	15720	15698	15711	15689
	15726	15690	15717	15681	15708	15679	15699
	15687	15707	15678	15705	15718	15696	15709
	15697	15724	15688	15715	15686	15706	15684
	15714	15685	15712	15676	15703	15716	15694
	15682	15695	15722	15693	15713	15691	15704
	15692	15719	15683	15710	15674	15701	15721
	15702	15680	15700	15720	15698	15711	15689
	15726	15690	15717	15681	15708	15679	15699
	15687	15707	15678	15705	15718	15696	15709
	15697	15724	15688	15715	15686	15706	15684
	15714	15685	15712	15676	15703	15716	15694
	15682	15695	15722	15693	15713	15691	15704
	15692	15719	15683	15710	15674	15701	15721
	15702	15680	15700	15720	15698	15711	15689
	15726	15690	15717	15681	15708	15679	15699
	15687	15707	15678	15705	15718	15696	15709
	15697	15724	15688	15715	15686	15706	15684
	15714	15685	15712	15676	15703	15716	15694

Horizontal sums of size-4 even-size diamond sums

15682	15695	15722	15693	15713	15691	15704	109900
15692	15719	15683	15710	15674	15701	15721	109900
15702	15680	15700	15720	15698	15711	15689	109900
15726	15690	15717	15681	15708	15679	15699	109900
15687	15707	15678	15705	15718	15696	15709	109900
15697	15724	15688	15715	15686	15706	15684	109900
15714	15685	15712	15676	15703	15716	15694	109900
15682	15695	15722	15693	15713	15691	15704	109900
15692	15719	15683	15710	15674	15701	15721	109900
15702	15680	15700	15720	15698	15711	15689	109900
15726	15690	15717	15681	15708	15679	15699	109900
15687	15707	15678	15705	15718	15696	15709	109900
15697	15724	15688	15715	15686	15706	15684	109900
15714	15685	15712	15676	15703	15716	15694	109900
15682	15695	15722	15693	15713	15691	15704	109900
15692	15719	15683	15710	15674	15701	15721	109900
15702	15680	15700	15720	15698	15711	15689	109900
15726	15690	15717	15681	15708	15679	15699	109900
15687	15707	15678	15705	15718	15696	15709	109900
15697	15724	15688	15715	15686	15706	15684	109900
15714	15685	15712	15676	15703	15716	15694	109900

Vertical sums of size-4 even-size diamond sums ⇒ 109900 109900 109900 109900 109900 109900 109900 — 10 x 10990

Size-5 even-size diamonds

	1	2	3	4	5	6	7
1	23593	23553	23506	23564	23573	23526	23535
2	23566	23568	23528	23537	23595	23548	23508
3	23539	23590	23550	23510	23561	23570	23530
4	23505	23563	23572	23532	23534	23592	23552
5	23527	23536	23594	23547	23507	23565	23574
6	23549	23509	23567	23569	23529	23538	23589
7	23571	23531	23533	23591	23551	23511	23562
	23593	23553	23506	23564	23573	23526	23535
	23566	23568	23528	23537	23595	23548	23508
	23539	23590	23550	23510	23561	23570	23530
	23505	23563	23572	23532	23534	23592	23552
	23527	23536	23594	23547	23507	23565	23574
	23549	23509	23567	23569	23529	23538	23589
	23571	23531	23533	23591	23551	23511	23562
	23593	23553	23506	23564	23573	23526	23535
	23566	23568	23528	23537	23595	23548	23508
	23539	23590	23550	23510	23561	23570	23530
	23505	23563	23572	23532	23534	23592	23552
	23527	23536	23594	23547	23507	23565	23574
	23549	23509	23567	23569	23529	23538	23589
	23571	23531	23533	23591	23551	23511	23562
	23593	23553	23506	23564	23573	23526	23535
	23566	23568	23528	23537	23595	23548	23508
	23539	23590	23550	23510	23561	23570	23530
	23505	23563	23572	23532	23534	23592	23552
	23527	23536	23594	23547	23507	23565	23574
	23549	23509	23567	23569	23529	23538	23589
	23571	23531	23533	23591	23551	23511	23562

Horizontal sums of size-5 even-size diamond sums

23593	23553	23506	23564	23573	23526	23535	164850
23566	23568	23528	23537	23595	23548	23508	164850
23539	23590	23550	23510	23561	23570	23530	164850
23505	23563	23572	23532	23534	23592	23552	164850
23527	23536	23594	23547	23507	23565	23574	164850
23549	23509	23567	23569	23529	23538	23589	164850
23571	23531	23533	23591	23551	23511	23562	164850
23593	23553	23506	23564	23573	23526	23535	164850
23566	23568	23528	23537	23595	23548	23508	164850
23539	23590	23550	23510	23561	23570	23530	164850
23505	23563	23572	23532	23534	23592	23552	164850
23527	23536	23594	23547	23507	23565	23574	164850
23549	23509	23567	23569	23529	23538	23589	164850
23571	23531	23533	23591	23551	23511	23562	164850
23593	23553	23506	23564	23573	23526	23535	164850
23566	23568	23528	23537	23595	23548	23508	164850
23539	23590	23550	23510	23561	23570	23530	164850
23505	23563	23572	23532	23534	23592	23552	164850
23527	23536	23594	23547	23507	23565	23574	164850
23549	23509	23567	23569	23529	23538	23589	164850
23571	23531	23533	23591	23551	23511	23562	164850

Vertical sums of size-5 even-size diamond sums ⇒ 164850 164850 164850 164850 164850 164850 164850 — 15 x 10990

Below is a spread-summation for one of the size-6 triangle patterns (re; page 4-22) each of which doesn't sum continuously in the size-28 square; yet their spread-summations sum geonomically to **6** times the square's characteristic number in **4x4** block-squares like the one shown below. Each **4x4** block-square can be reduced to a perfect size-4 square by the same procedure that is described on the next page. The perfect versions of the size **4** and **7** squares were used in the **ATE** method of Program 7 to generate the perfect size **28** square. Here they are both recovered.

Size-6 triangles

	65940	65940	65940	65940
8169	7924	8463	8414	65940
8512	8365	8218	7875	65940
8022	8071	8316	8561	65940
8267	8610	7973	8120	65940
65940	65940	65940	65940	6
	65940	65940	65940	65940

W(4)	34	34	34	34
7	2	13	12	34
14	11	8	1	34
4	5	10	15	34
9	16	3	6	34
34	34	34	34	
	34	34	34	34

Program 4

Reduction of Spread Summations to a Perfect Square

R(7|5)

23593	23553	23506	23564	23573	23526	23535	164850
23566	23568	23528	23537	23595	23548	23508	164850
23539	23590	23550	23510	23561	23570	23530	164850
23505	23563	23572	23532	23534	23592	23552	164850
23527	23536	23594	23547	23507	23565	23574	164850
23549	23509	23567	23569	23529	23538	23589	164850
23571	23531	23533	23591	23551	23511	23562	164850
164850	164850	164850	164850	164850	164850	164850	15

23505 =min

Here the repeated spread-summation **R** that sums geonomically to **15** times C_{28} is reduced to a perfect size **7** square.

R – 23504

89	49	2	60	69	22	31
62	64	24	33	91	44	4
35	86	46	6	57	66	26
1	59	68	28	30	88	48
23	32	90	43	3	61	70
45	5	63	65	25	34	85
67	27	29	87	47	7	58

First, the minimum value r_0 is determined in **R**.

Second, **R** is reduced by $(r_0 - 1)$.

X(7) = Mod(R –| 1 |⌊ 7) + | 1 |

5	7	2	4	6	1	3
6	1	3	5	7	2	4
7	2	4	6	1	3	5
1	3	5	7	2	4	6
2	4	6	1	3	5	7
3	5	7	2	4	6	1
4	6	1	3	5	7	2

Third, the modulus loom-table **X(7)** is derived from the reduced table in the base **7**.

Y(7) = tX(7)

5	6	7	1	2	3	4
7	1	2	3	4	5	6
2	3	4	5	6	7	1
4	5	6	7	1	2	3
6	7	1	2	3	4	5
1	2	3	4	5	6	7
3	4	5	6	7	1	2

Fourth, the integer loom-table **Y(7)** is derived by transposing **X**.

$W^*(7)$

33	42	44	4	13	15	24
48	1	10	19	28	30	39
14	16	25	34	36	45	5
22	31	40	49	2	11	20
37	46	6	8	17	26	35
3	12	21	23	32	41	43
18	27	29	38	47	7	9

Fifth, the loom-tables **X** and **Y** are used to generate **W*(7)** by the standard generation formula.

	1	2	3	4	5	6	7	
W(7)	175	175	175	175	175	175	175	175
1	9	18	27	29	38	47	7	175
2	24	33	42	44	4	13	15	175
3	39	48	1	10	19	28	30	175
4	5	14	16	25	34	36	45	175
5	20	22	31	40	49	2	11	175
6	35	37	46	6	8	17	26	175
7	43	3	12	21	23	32	41	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

Finally, **W*** is normalized to produce the perfect size-7 square **W(7)**.

This can be done by using the same procedure on every spread-summation for the size-28 square which is shown on the prior pages.

Program 4

Size-35 Patterned Summation Inequalities

Here is a level-3 square pyramidal pattern on the size-35 square. These pyramids do not sum to the square's characteristic number anywhere as evidenced by the continuous summations below.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	W(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
1	969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4	114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5	1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6	937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21455
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8	332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9	131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15	1165	566	205	980	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18	310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23	327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25	1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26	942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

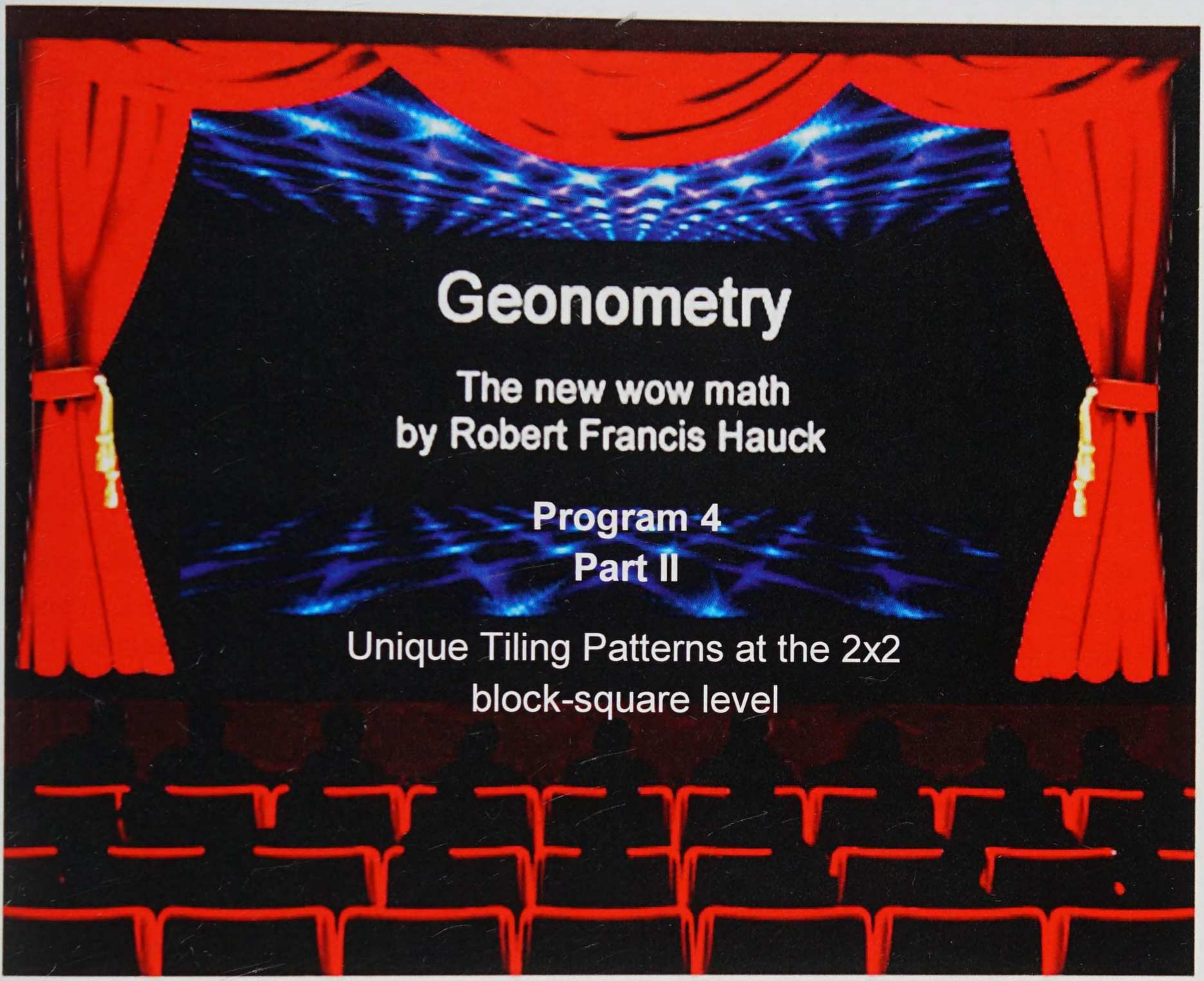
However, note the repetitious pattern of pyramidal summations in the **5x5** block-squares.

Size-7 level-4 square pyramid

51051	51394	51982	51345	51688	51051	51394	51982	51345	51688	51051	51394	51982	51345	51688
51247	51835	50953	51541	51884	51247	51835	50953	51541	51884	51247	51835	50953	51541	51884
51443	52031	51149	51737	51100	51443	52031	51149	51737	51100	51443	52031	51149	51737	51100
51639	51002	51590	51933	51296	51639	51002	51590	51933	51296	51639	51002	51590	51933	51296
52080	51198	51786	50904	51492	52080	51198	51786	50904	51492	52080	51198	51786	50904	51492
51051	51394	51982	51345	51688	51051	51394	51982	51345	51688	51051	51394	51982	51345	51688
51247	51835	50953	51541	51884	51247	51835	50953	51541	51884	51247	51835	50953	51541	51884
51443	52031	51149	51737	51100	51443	52031	51149	51737	51100	51443	52031	51149	51737	51100
51639	51002	51590	51933	51296	51639	51002	51590	51933	51296	51639	51002	51590	51933	51296
52080	51198	51786	50904	51492	52080	51198	51786	50904	51492	52080	51198	51786	50904	51492
51051	51394	51982	51345	51688	51051	51394	51982	51345	51688	51051	51394	51982	51345	51688
51247	51835	50953	51541	51884	51247	51835	50953	51541	51884	51247	51835	50953	51541	51884
51443	52031	51149	51737	51100	51443	52031	51149	51737	51100	51443	52031	51149	51737	51100
51639	51002	51590	51933	51296	51639	51002	51590	51933	51296	51639	51002	51590	51933	51296
52080	51198	51786	50904	51492	52080	51198	51786	50904	51492	52080	51198	51786	50904	51492

These **5x5** block-squares in the summation table above, when placed in a geonomic summation table of size-5 (here at right), sum equally to **12 (= 5 + 7)** times the size **35** square's characteristic number **257460 = 12 x 21455 ≡ 12 $C^{\#}_{35}$**. This can be reduced to recover the size 5 square used in its generation by the **ATE** method of Program 7.

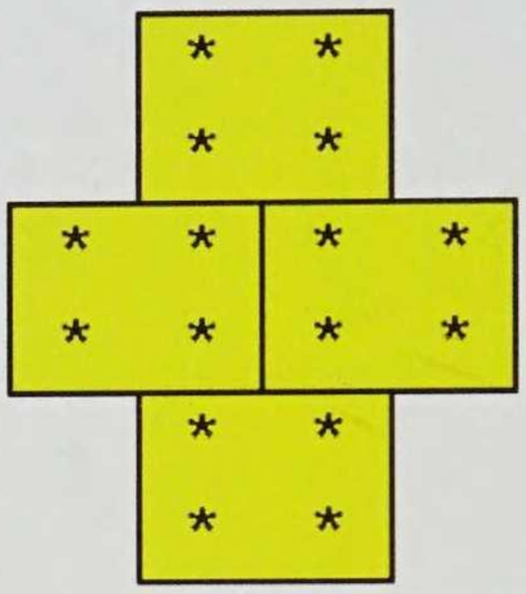
	1	2	3	4	5	
		257460	257460	257460	257460	257460
1	51051	51394	51982	51345	51688	257460
2	51247	51835	50953	51541	51884	257460
3	51443	52031	51149	51737	51100	257460
4	51639	51002	51590	51933	51296	257460
5	52080	51198	51786	50904	51492	257460
	257460	257460	257460	257460	257460	
		257460	257460	257460	257460	257460



In this segment we will be uncovering the unique tiling patterns at the 2x2 block-square level hidden in Class-4 squares

Class-4 Unique Tiling Patterns at the 2x2 block-square level

Size-16



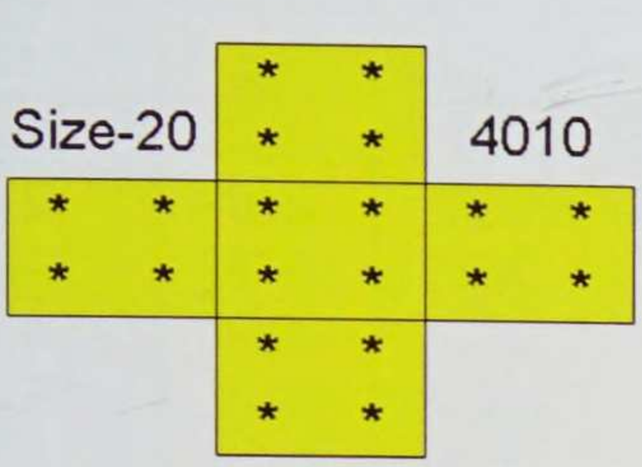
The **2x2** block-pattern above sums exactly to $\mathbf{C_{16}}$ in an interrupted manner as in the diamond quilt pattern: that is, continuously every 4th column and continuously every other row. These **2x2** block-square patterns are just beginning to manifest themselves with the size-16 square.

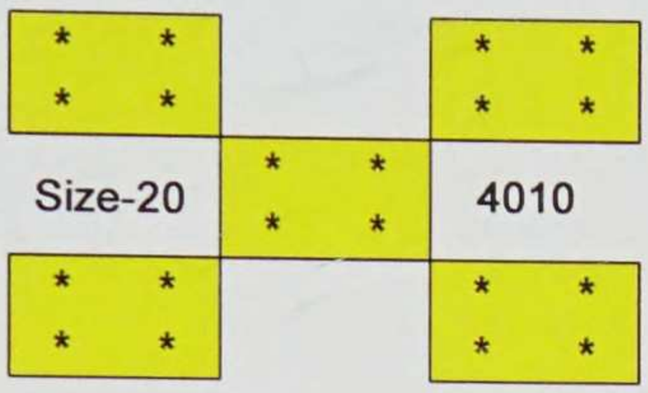
8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224
8224	8416	7968	8416	8224	8032	8480	8032	8224	8416	7968	8416	8224
8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224
8224	8032	8480	8032	8224	8416	7968	8416	8224	8032	8480	8032	8224
8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224
8224	8416	7968	8416	8224	8032	8480	8032	8224	8416	7968	8416	8224
8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224
8224	8032	8480	8032	8224	8416	7968	8416	8224	8032	8480	8032	8224
8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224
8224	8416	7968	8416	8224	8032	8480	8032	8224	8416	7968	8416	8224
8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224	8224

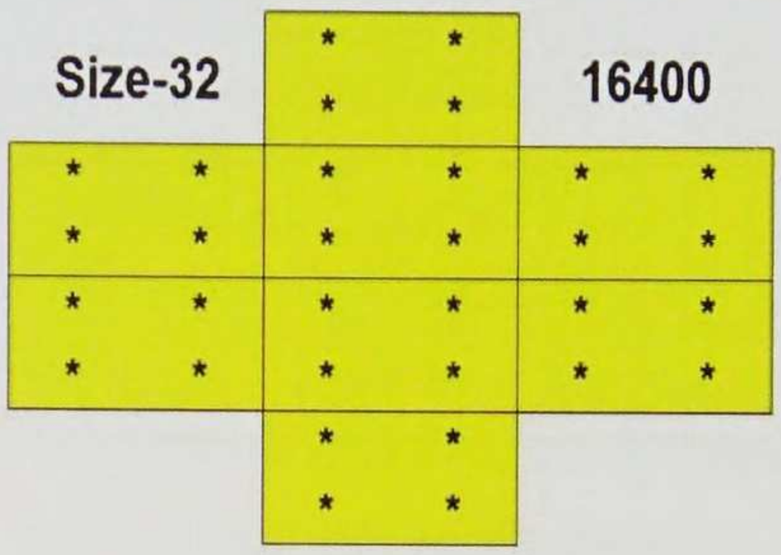
Program 4

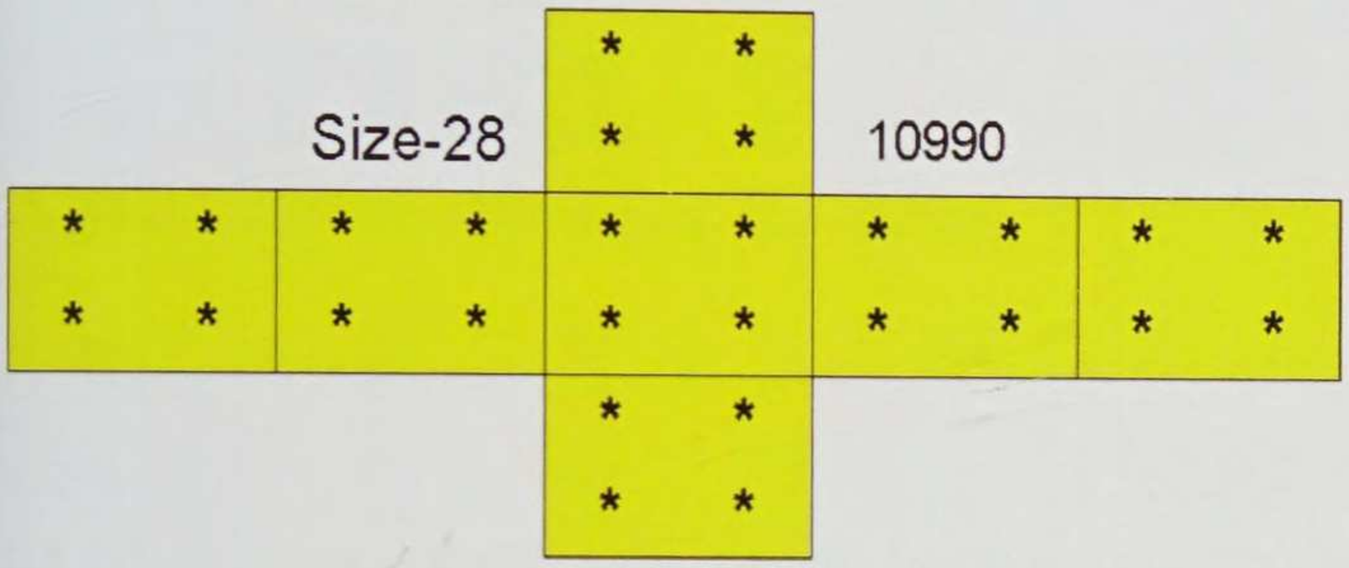
Sizes 20 thru 36

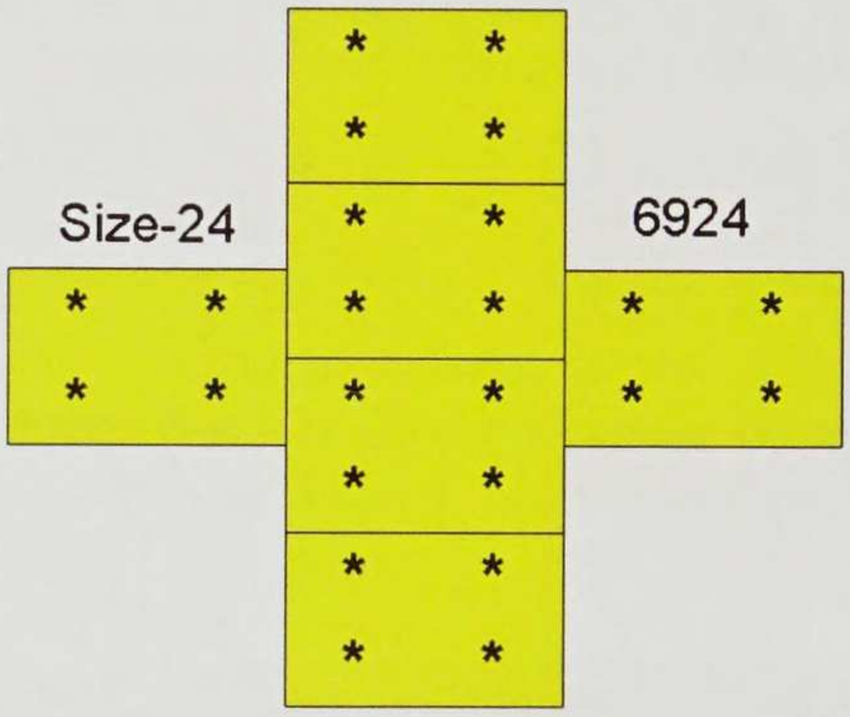
Here are tiling patterns at the **2x2** block-square level that sum to the square's characteristic number continuously. These manifest themselves for all Class-4 squares from size-20 onward. Each pattern consists of exactly **n** numbers for the size **n = 4b** square. All of these patterns are integra teable to cove r the entire square completely and exactly

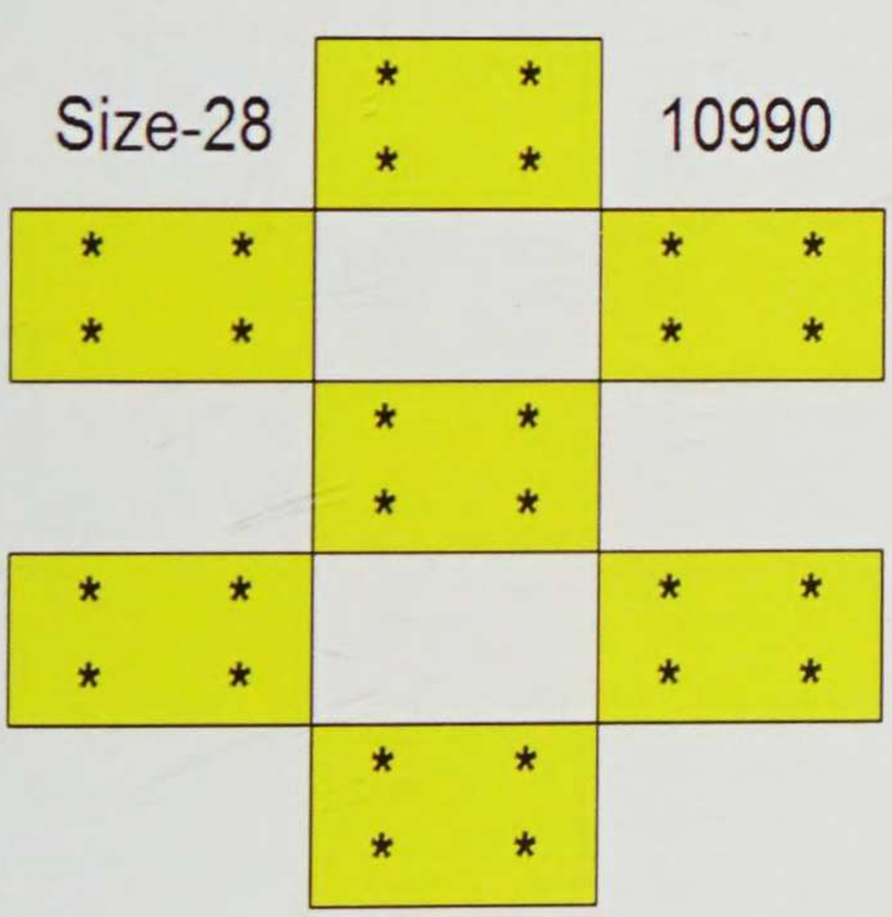


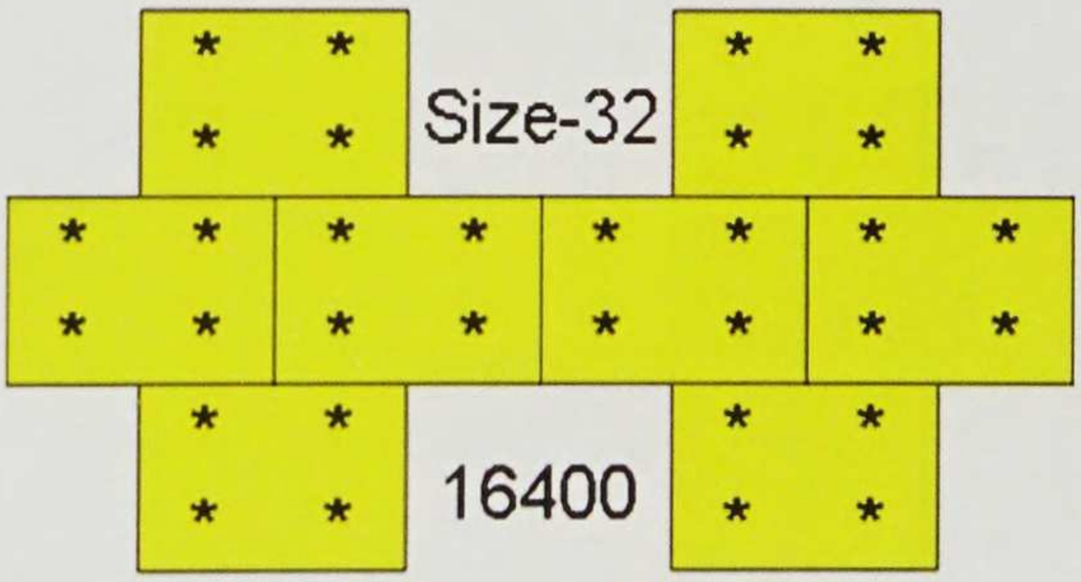








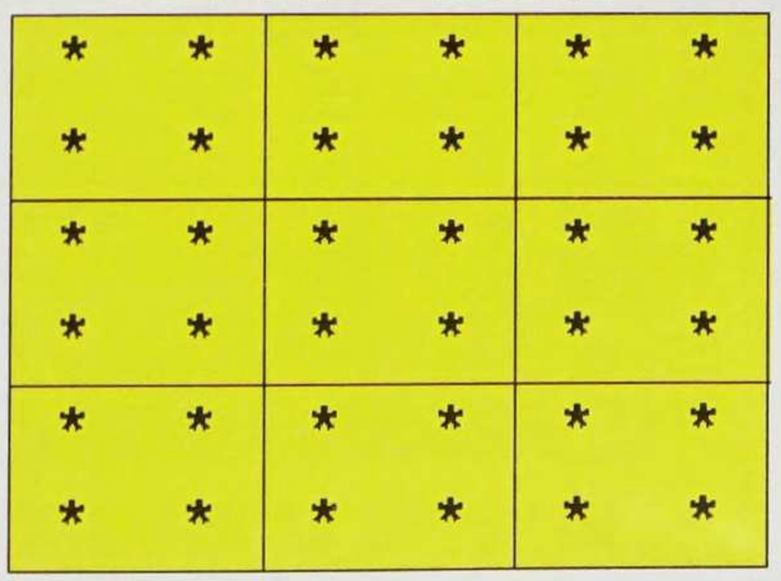




When **b** is a prime number, the **2x2** tile patterns mimic the complementary tile patterns of a square of size **b**.

For the size-36 square, **b = 9** is non-prime, and, has no tiling pattern at all. Amazingly however, the size-9 square is **3x3** modular and the **2x2** block pattern is **3x3** modular on the size-36 square.

So the bottom-line is that any pattern which is continuously equal-summing on the size b square will be equal-summing at the **2x2** block level on the size **n = 4b** square. Now that's wow math!



Program 4

A 2nd Characteristic Tiling Pattern for Class-4 Squares

All at the 2x2 block-square level

In the following pages is shown a common pattern that first reveals itself in the size **16** square and continues on from there. It consists of exactly **4** diamonds consisting of **2x2** blocks that cover the square completely. The diamonds are of size **n = 4b** and sum continuously to **bC_n**. It has no complementary tile pattern.

At right is a summary table of these characteristic diamonds up thru size-36. Their size as measured in **2x2** blocks turns out to be their multiple of the square's characteristic number!

b	n = 4b	◊ Sum	C_n	Mult of C_n
1	4	34	34	1
2	8	520	260	2
3	12	2610	870	3
4	16	8224	2056	4
5	20	20050	4010	5
6	24	41544	6924	6
7	28	76930	10990	7
8	32	131200	16400	8
9	36	210114	23346	9

The sizes **8** and **12** are embryonic with respect to this diamond pattern and so do not possess the pattern.

The size-1 diamond for the size-4 square consists of one **2x2** block square. Since the size-4 square has all of its **2x2** block-squares summing to **34**, the size-4 square satisfies the predictive model.

Size-16

Again, as for the unique prior pattern distribution composed of four **2x2** block-squares, the size-16 diamond pattern sums equally continuously vertically every 4th column and continuously horizontally every other row. This is the only square which manifests its pattern with these voids.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
	W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Program 4

Size-20

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
W(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	321	289	2	195	308	296	14	177	320	283	21	189	302	295	8	196	314	277	4010
2	336	254	42	160	348	261	29	167	335	273	36	154	342	260	48	161	329	267	35	173	4010
3	82	125	388	201	94	107	400	213	76	119	382	225	88	101	394	207	100	113	376	219	4010
4	353	241	59	147	365	228	66	134	372	240	53	141	359	247	65	128	366	234	72	140	4010
5	24	187	305	293	6	199	312	280	18	181	324	287	5	193	306	299	12	180	318	281	4010
6	345	258	46	164	327	270	33	171	339	252	45	158	346	264	27	170	333	271	39	152	4010
7	86	104	392	210	98	111	379	217	85	123	386	204	92	110	398	211	79	117	385	223	4010
8	357	250	63	126	369	232	75	138	351	244	57	150	363	226	69	132	375	238	51	144	4010
9	3	191	309	297	15	178	316	284	22	190	303	291	9	197	315	278	16	184	322	290	4010
10	349	262	30	168	331	274	37	155	343	256	49	162	330	268	31	174	337	255	43	156	4010
11	95	108	396	214	77	120	383	221	89	102	395	208	96	114	377	220	83	121	389	202	4010
12	361	229	67	135	373	236	54	142	360	248	61	129	367	235	73	136	354	242	60	148	4010
13	7	200	313	276	19	182	325	288	1	194	307	300	13	176	319	282	25	188	301	294	4010
14	328	266	34	172	340	253	41	159	347	265	28	166	334	272	40	153	341	259	47	165	4010
15	99	112	380	218	81	124	387	205	93	106	399	212	80	118	381	224	87	105	393	206	4010
16	370	233	71	139	352	245	58	146	364	227	70	133	371	239	52	145	358	246	64	127	4010
17	11	179	317	285	23	186	304	292	10	198	311	279	17	185	323	286	4	192	310	298	4010
18	332	275	38	151	344	257	50	163	326	269	32	175	338	251	44	157	350	263	26	169	4010
19	78	116	384	222	90	103	391	209	97	115	378	216	84	122	390	203	91	109	397	215	4010
20	374	237	55	143	356	249	62	130	368	231	74	137	355	243	56	149	362	230	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	217	66	235	414	439	432	145	138	163	342	511	360	505	354	523	126	151	144	6924
2	103	192	85	564	313	546	319	552	301	204	97	186	391	480	373	276	25	258	31	264	13	492	385	474	6924
3	356	506	311	194	143	149	140	146	95	554	359	509	68	218	23	482	431	437	428	434	383	266	71	221	6924
4	105	208	87	562	297	544	321	568	303	202	81	184	393	496	375	274	9	256	33	280	15	490	369	472	6924
5	327	514	345	160	135	178	111	154	129	520	351	538	39	226	57	448	423	466	399	442	417	232	63	250	6924
6	83	209	92	557	296	566	299	569	308	197	80	206	371	497	380	269	8	278	11	281	20	485	368	494	6924
7	325	534	343	162	115	180	109	174	127	522	331	540	37	246	55	450	403	468	397	462	415	234	43	252	6924
8	139	156	121	528	349	510	355	516	337	168	133	150	427	444	409	240	61	222	67	228	49	456	421	438	6924
9	323	545	344	161	104	182	107	185	128	521	320	542	35	257	56	449	392	470	395	473	416	233	32	254	6924
10	141	172	123	526	333	508	357	532	339	166	117	148	429	460	411	238	45	220	69	244	51	454	405	436	6924
11	291	550	309	196	99	214	75	190	93	556	315	574	3	262	21	484	387	502	363	478	381	268	27	286	6924
12	116	170	131	518	335	533	332	530	347	158	119	173	404	458	419	230	47	245	44	242	59	446	407	461	6924
13	289	570	307	198	79	216	73	210	91	558	295	576	1	282	19	486	367	504	361	498	379	270	7	288	6924
14	247	48	229	420	457	402	463	408	445	60	241	42	535	336	517	132	169	114	175	120	157	348	529	330	6924
15	500	362	455	50	287	5	284	2	239	410	503	365	212	74	167	338	575	293	572	290	527	122	215	77	6924
16	249	64	231	418	441	400	465	424	447	58	225	40	537	352	519	130	153	112	177	136	159	346	513	328	6924
17	471	370	489	16	279	34	255	10	273	376	495	394	183	82	201	304	567	322	543	298	561	88	207	106	6924
18	227	65	236	413	440	422	443	425	452	53	224	62	515	353	524	125	152	134	155	137	164	341	512	350	6924
19	469	390	487	18	259	36	253	30	271	378	475	396	181	102	199	306	547	324	541	318	559	90	187	108	6924
20	283	12	265	384	493	366	499	372	481	24	277	6	571	300	553	96	205	78	211	84	193	312	565	294	6924
21	467	401	488	17	248	38	251	41	272	377	464	398	179	113	200	305	536	326	539	329	560	89	176	110	6924
22	285	28	267	382	477	364	501	388	483	22	261	4	573	316	555	94	189	76	213	100	195	310	549	292	6924
23	435	406	453	52	243	70	219	46	237	412	459	430	147	118	165	340	531	358	507	334	525	124	171	142	6924
24	260	26	275	374	479	389	476	386	491	14	263	29	548	314	563	86	191	101	188	98	203	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Program 4

Size-24

Program 4

Size-28

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
	W(28)	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	

Program 4

Size-32

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	
	W(32)	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400
1	776	617	801	592	952	729	913	768	392	233	417	208	312	89	273	128	264	105	289	80	440	217	401	256	904	745	929	720	824	601	785	640	16400
2	442	215	415	242	266	103	303	66	826	599	799	626	906	743	943	706	954	727	927	754	778	615	815	578	314	87	287	114	394	231	431	194	16400
3	830	595	795	630	910	739	939	710	446	211	411	246	270	99	299	70	318	83	283	118	398	227	427	198	958	723	923	758	782	611	811	582	16400
4	388	237	421	204	308	93	277	124	772	621	805	588	948	733	917	764	900	749	933	716	820	605	789	636	260	109	293	76	436	221	405	252	16400
5	837	556	868	525	1013	668	980	701	453	172	484	141	373	28	340	61	325	44	356	13	501	156	468	189	965	684	996	653	885	540	852	573	16400
6	507	150	478	179	331	38	366	3	891	534	862	563	971	678	1006	643	1019	662	990	691	843	550	878	515	379	22	350	51	459	166	494	131	16400
7	895	530	858	567	975	674	1002	647	511	146	474	183	335	34	362	7	383	18	346	55	463	162	490	135	1023	658	986	695	847	546	874	519	16400
8	449	176	488	137	369	32	344	57	833	560	872	521	1009	672	984	697	961	688	1000	649	881	544	856	569	321	48	360	9	497	160	472	185	16400
9	584	809	609	784	760	921	721	960	200	425	225	400	120	281	81	320	72	297	97	272	248	409	209	448	712	937	737	912	632	793	593	832	16400
10	250	407	223	434	74	295	111	258	634	791	607	818	714	935	751	898	762	919	735	946	586	807	623	770	122	279	95	306	202	423	239	386	16400
11	638	787	603	822	718	931	747	902	254	403	219	438	78	291	107	262	126	275	91	310	206	419	235	390	766	915	731	950	590	803	619	774	16400
12	196	429	229	396	116	285	85	316	580	813	613	780	756	925	725	956	708	941	741	908	628	797	597	828	68	301	101	268	244	413	213	444	16400
13	517	876	548	845	693	988	660	1021	133	492	164	461	53	348	20	381	5	364	36	333	181	476	148	509	645	1004	676	973	565	860	532	893	16400
14	187	470	158	499	11	358	46	323	571	854	542	883	651	998	686	963	699	982	670	1011	523	870	558	835	59	342	30	371	139	486	174	451	16400
15	575	850	538	887	655	994	682	967	191	466	154	503	15	354	42	327	63	338	26	375	143	482	170	455	703	978	666	1015	527	866	554	839	16400
16	129	496	168	457	49	352	24	377	513	880	552	841	689	992	664	1017	641	1008	680	969	561	864	536	889	1	368	40	329	177	480	152	505	16400
17	520	873	545	848	696	985	657	1024	136	489	161	464	56	345	17	384	8	361	33	336	184	473	145	512	648	1001	673	976	568	857	529	896	16400
18	186	471	159	498	10	359	47	322	570	855	543	882	650	999	687	962	698	983	671	1010	522	871	559	834	58	343	31	370	138	487	175	450	16400
19	574	851	539	886	654	995	683	966	190	467	155	502	14	355	43	326	62	339	27	374	142	483	171	454	702	979	667	1014	526	867	555	838	16400
20	132	493	165	460	52	349	21	380	516	877	549	844	692	989	661	1020	644	1005	677	972	564	861	533	892	4	365	37	332	180	477	149	508	16400
21	581	812	612	781	757	924	724	957	197	428	228	397	117	284	84	317	69	300	100	269	245	412	212	445	709	940	740	909	629	796	596	829	16400
22	251	406	222	435	75	294	110	259	635	790	606	819	715	934	750	899	763	918	734	947	587	806	622	771	123	278	94	307	203	422	238	387	16400
23	639	786	602	823	719	930	746	903	255	402	218	439	79	290	106	263	127	274	90	311	207	418	234	391	767	914	730	951	591	802	618	775	16400
24	193	432	232	393	113	288	88	313	577	816	616	777	753	928	728	953	705	944	744	905	625	800	600	825	65	304	104	265	241	416	216	441	16400
25	840	553	865	528	1016	665	977	704	456	169	481	144	376	25	337	64	328	41	353	16	504	153	465	192	968	681	993	656	888	537	849	576	16400
26	506	151	479	178	330	39	367	2	890	535	863	562	970	679	1007	642	1018	663	991	690	842	551	879	514	378	23	351	50	458	167	495	130	16400
27	894	531	859	566	974	675	1003	646	510	147	475	182	334	35	363	6	382	19	347	54	462	163	491	134	1022	659	987	694	846	547	875	518	16400
28	452	173	485	140	372	29	341	60	836	557	869	524	1012	669	981	700	964	685	997	652	884	541	853	572	324	45	357	12	500	157	469	188	16400
29	773	620	804	589	949	732	916	765	389	236	420	205	309	92	276	125	261	108	292	77	437	220	404	253	901	748	932	717	821	604	788	637	16400
30	443	214	414	243	267	102	302	67	827	598	798	627	907	742	942	707	955	726	926	755	779	614	814	579	315	86	286	115	395	230	430	195	16400
31	831	594	794	631	911	738	938	711	447	210	410	247	271	98	298	71	319	82	282	119	399	226	426	199	959	722	922	759	783	610	810	583	16400
32	385	240	424	201	305	96	280	121	769	624	808	585	945	736	920	761	897	752	936	713	817	608	792	633	257	112	296	73	433	224	408	249	16400
	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	
	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400

Program 4

Size-36

4-35

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
	W(36)	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
1	399	787	1205	264	328	746	1182	274	368	804	1192	314	345	733	1151	291	355	773	1209	301	395	750	1138	260	372	760	1178	318	382	800	1155	247	341	777	1165	287	23346
2	1238	171	415	698	1251	199	482	711	1279	185	414	658	1265	198	442	725	1278	226	428	657	1225	212	441	685	1292	225	469	671	1224	172	455	684	1252	239	468	712	23346
3	133	1001	933	565	137	1041	916	488	96	1024	920	528	160	1028	960	511	83	987	943	515	123	1051	947	555	106	974	906	538	110	1014	970	542	150	997	893	501	23346
4	872	633	76	1061	822	589	35	1092	859	629	66	1129	818	579	22	1088	849	616	62	1119	886	575	12	1075	845	606	49	1115	876	643	8	1065	832	602	39	1102	23346
5	325	743	1161	271	365	783	1189	311	405	730	1148	270	352	770	1188	298	392	810	1135	257	351	757	1175	297	379	797	1215	244	338	756	1162	284	378	784	1202	324	23346
6	1248	205	452	708	1285	236	411	664	1235	195	448	695	1275	232	479	654	1231	182	438	691	1262	222	475	722	1221	178	425	681	1258	209	465	718	1289	168	421	668	23346
7	148	1052	948	499	107	975	931	539	111	1039	971	543	94	998	894	526	134	1002	958	566	138	985	917	489	121	1025	921	553	161	1029	904	512	84	1012	944	516	23346
8	828	586	5	1098	856	599	72	1126	869	585	19	1058	855	613	32	1125	883	626	18	1072	815	612	46	1085	882	640	59	1071	829	572	45	1099	842	639	73	1112	23346
9	362	780	1168	308	402	790	1145	267	331	767	1185	277	389	807	1195	254	348	736	1172	294	358	794	1212	304	335	753	1141	281	375	763	1199	321	385	740	1158	250	23346
10	1290	220	476	669	1219	179	453	679	1259	237	463	719	1236	166	422	696	1246	206	480	706	1286	183	409	665	1263	193	449	723	1273	233	426	652	1232	210	436	692	23346
11	104	981	901	536	117	1009	968	549	145	995	900	496	131	1008	928	563	144	1036	914	495	91	1022	927	523	158	1035	955	509	90	982	941	522	118	1049	954	550	23346
12	862	596	42	1132	866	636	25	1055	825	619	29	1095	889	623	69	1078	812	582	52	1082	852	646	56	1122	835	569	15	1105	839	609	79	1109	879	592	2	1068	23346
13	386	795	1210	251	336	751	1169	282	373	791	1200	319	332	741	1156	278	363	778	1196	309	400	737	1146	265	359	768	1183	305	390	805	1142	255	346	764	1173	292	23346
14	1216	176	432	676	1256	216	460	716	1296	163	419	675	1243	203	459	703	1283	243	406	662	1242	190	446	702	1270	230	486	649	1229	189	433	689	1269	217	473	729	23346
15	114	1015	938	546	151	1046	897	502	101	1005	934	533	141	1042	965	492	97	992	924	529	128	1032	961	560	87	988	911	519	124	1019	951	556	155	978	907	506	23346
16	877	647	57	1066	836	570	40	1106	840	634	80	1110	823	593	3	1093	863	597	67	1133	867	580	26	1056	850	620	30	1120	890	624	13	1079	813	607	53	1083	23346
17	342	748	1139	288	370	761	1206	316	383	747	1153	248	369	775	1166	315	397	788	1152	262	329	774	1180	275	396	802	1193	261	343	734	1179	289	356	801	1207	302	23346
18	1253	213	439	713	1293	223	416	672	1222	200	456	682	1280	240	466	659	1239	169	443	699	1249	227	483	709	1226	186	412	686	1266	196	470	726	1276	173	429	655	23346
19	156	1030	962	507	85	989	939	517	125	1047	949	557	102	976	908	534	112	1016	966	544	152	993	895	503	129	1003	935	561	139	1043	912	490	98	1020	922	530	23346
20	833	576	10	1103	846	604	77	1116	874	590	9	1063	860	603	37	1130	873	631	23	1062	820	617	36	1090	887	630	64	1076	819	577	50	1089	847	644	63	1117	23346
21	376	758	1176	322	380	798	1159	245	339	781	1163	285	403	785	1203	268	326	744	1186	272	366	808	1190	312	349	731	1149	295	353	771	1213	299	393	754	1136	258	23346
22	1277	228	481	656	1227	184	440	687	1264	224	471	724	1223	174	427	683	1254	211	467	714	1291	170	417	670	1250	201	454	710	1281	238	413	660	1237	197	444	697	23346
23	82	986	918	514	122	1026	946	554	162	973	905	513	109	1013	945	541	149	1053	892	500	108	1000	932	540	136	1040	972	487	95	999	919	527	135	1027	959	567	23346
24	843	610	47	1113	880	641	6	1069	830	600	43	1100	870	637	74	1059	826	587	33	1096	857	627	70	1127	816	583	20	1086	853	614	60	1123	884	573	16	1073	23346
25	391	809	1191	256	350	732	1174	296	354	796	1214	300	337	755	1137	283	377	759	1201	323	381	742	1160	246	364	782	1164	310	404	786	1147	269	327	769	1187	273	23346
26	1233	181	410	693	1261	194	477	721	1274	180	424	653	1260	208	437	720	1288	221	423	667	1220	207	451	680	1287	235	464	666	1234	167	450	694	1247	234	478	707	23346
27	119	1023	925	551	159	1033	902	510	88	1010	942	520	146	1050	952	497	105	979	929	537	115	1037	969	547	92	996	898	524	132	1006	956	564	142	983	915	493	23346
28	885	625	71	1074	814	584	48	1084	854	642	58	1124	831	571	17	1101	841	611	75	1111	881	588	4	1070	858	598	44	1128	868	638	21	1057	827	615	31	1097	23346
29	347	738	1144	293	360	766	1211	306	388	752	1143	253	374	765	1171	320	387	793	1157	252	334	779	1170	280	401	792	1198	266	333	739	1184	279	361	806	1197	307	23346
30	1267	191	447	727	1271	231	430	650	1230	214	434	690	1294	218	474	673	1217	177	457	677	1257	241	461	717	1240	164	420	700	1244	204	484	704	1284	187	407	663	23346
31	143	1038	967	494	93	994	926	525	130	1034	957	562	89	984	913	521	120	1021	953	552	157	980	903	508	116	1011	940	548	147	1048	899	498	103	1007	930	535	23346
32	811	581	27	1081	851	621	55	1121	891	568	14	1080	838	608	54	1108	878	648	1	1067	837	595	41	1107	865	635	81	1054	824	594	28	1094	864	622	68	1134	23346
33	357	772	1181	303	394	803	1140	259	344	762	1177	290	384	799	1208	249	340	749	1167	286	371	789	1204	317	330	745	1154	276	367	776	1194	313	398	735	1150	263	23346
34	1282	242	462	661	1241	165	445	701	1245	229	485	705	1228	188	408	688	1268	192	472	728	1272	175	431	651	1255	215	435	715	1295	219	418	674	1218	202	458	678	23346
35	99	991	896	531	127	1004	963	559	140	990	910	491	126	1018	923	558	154	1031	909	505	86	1017	937	518	153	1045	950	504	100	977	936	532	113	1044	964	545	23346
36	848	618	34	1118	888	628	11	1077	817	605	51	1087	875	645	61	1064	834	574	38	1104	844	632	78	1114	821	591	7	1091	861	601	65	1131	871	578	24	1060	23346
	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	

A Common Characteristic Tiling Pattern for Class-3 Squares

Tile patterns at the 3x3 block-square level			
n	b	Summation value	Multiple of Cn
15	5	5085	3
21	7	13923	3
27	9	16425	3
33	11	53955	3
39	13	89037	3
45	15	136755	3

What was found for Class-4 squares to have tiling patterns composed of **2x2** block-squares, was also found for Class-3 squares composed of **3x3** block squares. Witness the size-33 square on the next page and the results of its third tile pattern C summations.

Note that although the size-9 square has no tile patterns unique to its size like Class-1 squares, it is continuously **3x3** modular and any pattern containing **9 3x3** block squares will sum equally and continuously to **3** times its characteristic number.

The patterns that summed to **3** times the size **45** square's characteristic number were the **9x15** and **15x9** block-rectangles.

See #4, #5 and #6 in the notes at the end of this section.

A Common Characteristic Tiling Pattern for Class-4 Squares where b is a prime-number

Further, what was found for Class-3 squares was found to hold for Class-4 squares where **b** was a **prime number**. The size **b** square's Class-1 complementary tile patterns now summed equally and continuously in **4x4** block-squares. Those patterns all summed to **4** times the square's characteristic number!

Program 4

Size-33

Here is the size-33 square with the third tiling pattern C of the size-11 square, but in **3x3** block-squares. As do both earlier tile patterns A and B expressed in **3x3** block-squares, tile pattern C here sums to **3** times the square's characteristic number continuously.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
	W(33)	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985
1	989	400	306	927	470	2	876	408	314	946	720	263	763	669	201	833	365	150	771	677	220	1083	626	37	1032	564	107	728	513	45	1040	583	357	17985
2	145	1019	551	94	1089	500	43	1027	570	102	855	508	293	914	457	363	863	406	301	933	465	129	871	656	188	820	726	137	769	664	207	828	492	17985
3	764	549	81	1076	487	272	1014	557	89	1084	385	38	912	444	350	850	635	288	920	452	358	748	401	186	807	713	124	998	651	194	815	721	22	17985
4	657	68	1063	595	17	770	544	87	1071	493	267	1020	431	337	958	380	44	907	450	345	856	630	294	794	700	232	743	407	181	813	708	130	993	17985
5	187	1050	593	4	999	531	74	1058	601	12	886	550	324	956	367	273	894	437	332	964	375	160	913	687	230	730	636	168	800	695	238	738	523	17985
6	69	580	838	260	518	1029	330	588	736	268	416	795	217	475	986	155	666	1056	225	373	994	53	432	943	112	623	881	303	693	951	10	631	779	17985
7	688	110	973	516	48	801	575	118	981	524	298	1051	473	247	879	411	75	938	481	255	887	661	325	836	610	153	774	438	212	844	618	161	1024	17985
8	218	1081	503	35	1030	562	105	979	511	54	917	581	355	866	398	304	925	468	253	874	417	191	944	718	140	761	667	199	831	616	148	780	554	17985
9	826	490	33	1017	560	334	1087	498	41	1025	447	100	853	396	291	923	697	361	861	404	299	810	463	127	759	654	197	1060	724	135	767	662	84	17985
10	719	20	1004	547	79	832	485	28	1023	555	340	1082	383	278	910	442	106	848	391	297	918	703	356	746	641	184	805	469	122	754	660	192	1066	17985
11	854	628	171	803	698	241	741	647	179	811	585	128	991	534	77	1061	604	15	1010	542	85	948	491	265	897	440	335	967	378	284	905	448	222	17985
12	747	521	64	1048	591	244	997	529	72	1067	478	21	884	427	322	954	607	271	892	435	341	841	384	158	790	685	228	970	634	166	798	704	115	17985
13	266	777	672	215	847	379	164	785	691	223	976	629	51	1035	578	121	742	527	59	1054	586	250	992	414	309	941	484	16	890	422	328	949	613	17985
14	885	670	202	834	608	151	772	678	210	842	506	159	1033	565	108	971	514	46	1041	573	116	869	522	307	928	471	245	877	409	315	936	479	143	17985
15	415	189	821	716	138	1012	665	208	829	614	25	778	552	95	1079	501	286	1028	571	103	977	388	52	915	458	353	864	649	302	934	466	251	751	17985
16	308	808	714	125	757	410	195	816	722	133	1007	671	82	1077	488	31	773	558	90	1085	496	281	1034	445	351	851	394	47	921	453	359	859	644	17985
17	190	701	959	18	639	908	88	709	857	26	537	916	338	596	744	276	545	814	346	494	752	174	553	1064	233	381	1002	182	451	1072	131	389	900	17985
18	446	231	731	637	169	1043	696	239	739	645	56	809	594	5	1000	532	317	1059	602	13	1008	419	83	957	368	274	895	680	333	965	376	282	782	17985
19	339	839	624	156	788	441	226	737	632	175	1038	702	113	987	519	62	804	589	11	995	538	312	1065	476	261	882	425	78	952	374	269	901	675	17985
20	947	611	154	775	681	213	845	619	162	783	568	221	974	517	49	1044	576	119	982	525	57	931	584	248	880	412	318	939	482	256	888	420	205	17985
21	477	141	762	668	200	1074	606	149	781	676	98	840	504	36	1031	563	348	969	512	55	1039	461	114	867	399	305	926	711	243	875	418	313	824	17985
22	975	386	292	924	456	120	862	405	300	932	706	249	749	655	198	819	483	136	768	663	206	1069	612	23	1018	561	93	846	499	42	1026	569	343	17985
23	868	642	185	806	712	123	755	650	193	825	599	142	1005	548	80	1075	486	29	1013	556	99	962	505	279	911	443	349	849	392	287	919	462	236	17985
24	24	898	430	336	968	621	285	906	449	344	734	387	172	793	699	242	984	648	180	812	707	8	750	535	67	1062	605	258	1011	543	86	1070	371	17985
25	1006	428	323	955	366	30	893	436	331	963	627	280	791	686	229	729	393	167	799	694	237	990	643	65	1049	592	3	756	530	73	1057	600	264	17985
26	536	310	942	474	259	891	423	329	950	372	146	899	673	216	837	622	165	786	692	224	735	509	173	1036	579	111	985	528	60	1055	587	9	872	17985
27	66	929	472	246	878	652	316	937	480	254	765	429	203	835	609	152	1015	679	211	843	617	39	792	566	109	972	515	289	1042	574	117	980	402	17985
28	311	459	1080	139	397	787	209	467	978	147	658	1037	96	717	865	34	424	935	104	615	873	295	674	822	354	502	760	61	572	830	252	510	1021	17985
29	567	352	852	395	290	922	454	360	860	403	177	930	715	126	758	653	196	817	723	134	766	540	204	1078	489	32	1016	559	91	1086	497	40	903	17985
30	97	960	382	277	909	683	347	858	390	296	796	460	234	745	640	183	1046	710	132	753	659	70	823	597	19	1003	546	320	1073	495	27	1022	433	17985
31	1068	369	275	896	439	92	966	377	283	904	689	342	732	638	170	802	455	240	740	646	178	1052	705	6	1001	533	76	818	603	14	1009	541	326	17985
32	598	262	883	426	321	953	364	270	902	434	219	961	625	157	789	684	227	727	633	176	797	582	235	988	520	63	1047	590	1	996	539	71	945	17985
33	733	507	50	1045	577	362	983	526	58	1053	464	7	870	413	319	940	725	257	889	421	327	827	370	144	776	682	214	1088	620	163	784	690	101	17985
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	
		17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985

53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955
53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955	53955

Program 4

Part II Notes

1. Class-4 squares possess **2x2** block-square tiles which mimic the size **b** square's complementary tile pattern continuously for sizes **20** and above. When **b** is an odd number and not prime, its tile pattern at the **2x2** block-square level is a **3x3** block-square. The sums equal the square's characteristic number exactly.

2. All Class-4 squares from size **16** onward possess a solid diamond pattern of size **b** at the **2x2** block-square level. The sums equal **b** times the square's characteristic number.

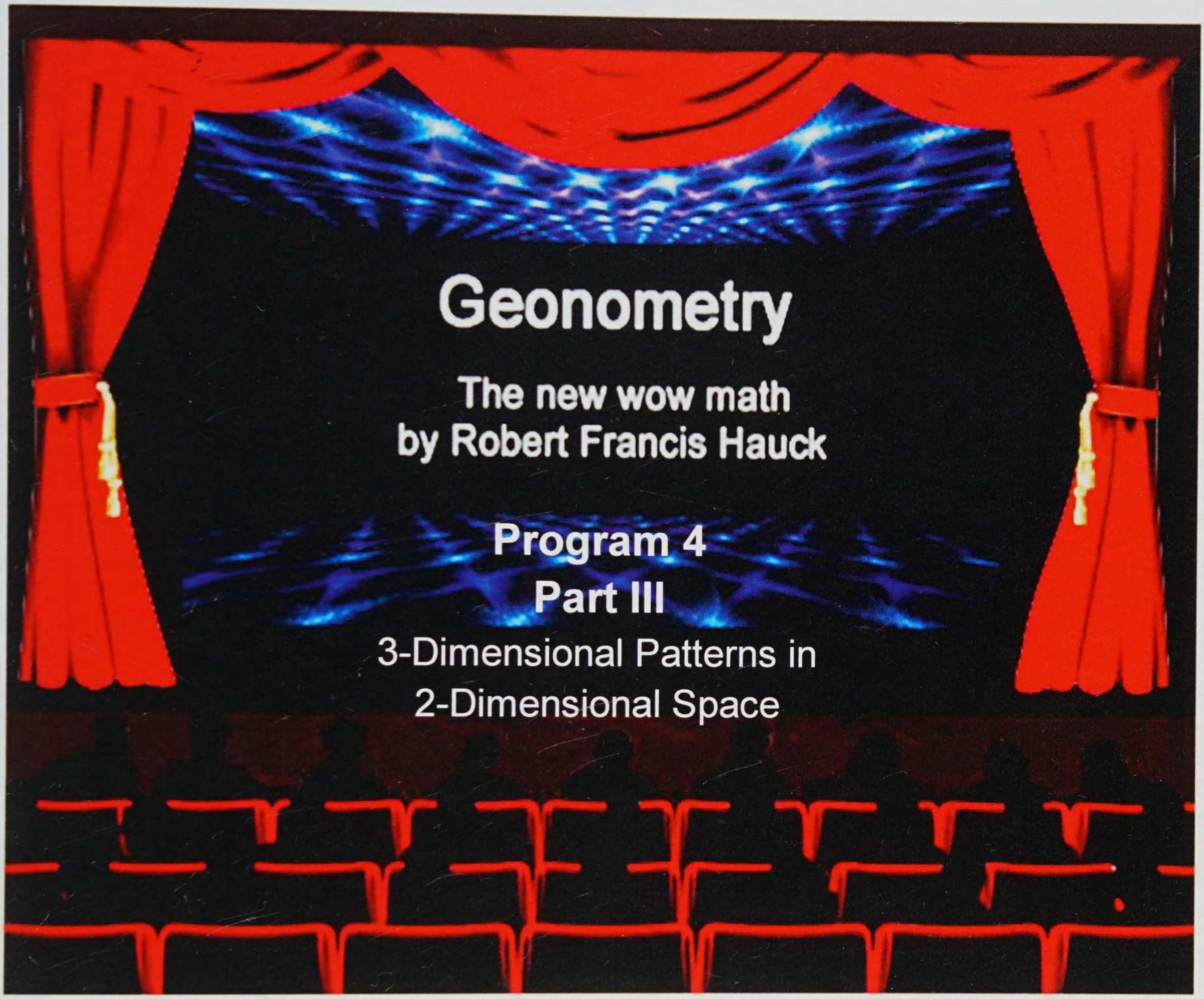
3. A third complementary tiling pattern for Class-4 squares consists of **4xb** and **bx4** rectangles from **b ≥ 4**. The sums equal the square's characteristic number exactly with the single exception for **b = 6** where the second tile pattern is non-conforming.

4. When **b** is a prime number, Class-3 squares possess complementary tile patterns that mimic the size **b** squares' tile patterns but at the **3x3** block-square level. The sums equal **3** times the square's characteristic number.

5. When **b** is a Class-3 square and is not a prime number, such as **9** or **15**, the complementary tile patterns are block- rectangles of size **cxd** and **dxc** where **c** and **d** are any two factors of **n.** The sums equal the square's characteristic number exactly. For instance, when **n = 45**, **c** and **d** are **5** with **9** and **3** with **15**. Where **n = 27**, **c** and **d** are just **3** with **9**.

6. That's just for starters: it was found that when **b** was a non-prime and **n = 3b** were multiplied by any number **k**, then the rectangular blocks the size any two factors of **kn** that fit within the square summed equally and continuously to **k** times the size **n** square's characteristic number. Now that's so absolutely amazing as to be unbelievably incredible! But it's for real. Just see it for yourself on the size-45 square.

7. When **b** is a prime number, Class-4 squares possess complementary tile patterns that mimic the size **b** squares' tile patterns but at the **4x4** block-square level. The sums equal **4** times the square's characteristic number.



In this segment we will be uncovering many 3-dimensional patterns running concurrently throughout 2-dimensional space

Program 4

Equal 3-Dimensional Summations

In Class-3 Squares

Size-21 Patterns

Here is the class-3 size-21 perfect square. Its characteristic number is **4641**. It is pairwise centrally symmetric

.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
	21x21	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
1	399	67	176	439	58	27	430	350	18	274	390	9	125	381	301	116	225	341	107	76	332	4641
2	191	209	122	189	347	260	180	240	258	24	238	396	162	229	289	160	73	287	298	211	278	4641
3	88	295	411	79	195	402	77	39	393	362	30	293	353	28	137	344	313	128	244	304	126	4641
4	426	94	161	417	85	5	408	377	45	259	368	36	103	359	328	143	210	319	134	54	310	4641
5	169	285	51	167	423	245	158	267	236	2	265	374	196	256	218	187	100	216	325	294	207	4641
6	17	378	438	8	222	429	6	66	427	340	57	271	331	55	115	329	389	106	173	380	104	4641
7	404	72	188	395	70	32	435	355	23	286	346	21	130	386	306	121	237	297	119	81	337	4641
8	252	214	29	292	352	174	283	203	165	127	243	303	272	234	154	263	78	194	401	223	185	4641
9	44	356	416	42	200	407	33	93	405	318	91	249	309	82	142	307	367	140	151	358	131	4641
10	382	1	264	373	48	108	371	333	99	215	324	146	59	322	431	50	166	422	97	10	420	4641
11	279	241	14	270	379	152	261	230	192	112	221	330	250	212	181	290	63	172	428	201	163	4641
12	22	432	345	20	276	392	11	120	383	296	118	227	343	109	71	334	394	69	178	441	60	4641
13	311	84	291	302	75	135	300	360	133	193	351	124	37	349	409	35	242	400	26	86	398	4641
14	257	219	41	248	364	179	288	208	170	139	199	315	277	239	159	268	90	150	413	228	190	4641
15	105	361	323	145	205	321	136	56	312	421	96	156	419	87	7	410	372	47	254	370	38	4641
16	338	62	269	336	53	113	327	387	111	171	385	102	15	376	436	13	220	434	4	64	425	4641
17	235	148	117	226	342	255	224	186	246	68	177	440	206	175	284	197	19	275	391	157	273	4641
18	132	388	308	123	232	299	114	83	339	406	74	183	397	65	34	437	357	25	281	348	16	4641
19	316	138	198	314	129	98	305	414	89	149	412	80	49	403	365	40	247	363	31	147	354	4641
20	164	231	144	155	369	282	153	213	280	46	204	418	184	202	262	182	95	253	320	233	251	4641
21	110	366	335	101	217	326	141	61	317	433	52	168	424	92	12	415	384	3	266	375	43	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	
		4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

The patterns on the left highlight **7x1** vertical and **1x7** horizontal summations across the size-21 square. The summations on the right equal the sums of 3 consecutive numbers in each case. When the top summations are summed vertically three at a time, their value equals the characteristic number. The same for the bottom summations when summed horizontally.

What is happening horizontally is depicted at bottom: The tiling pattern is a 3-dimensional skewed linear summation of three laminated strips of the same size forming a tile of size **1x(b+2)** where **n=3b**. This same pattern holds vertically too. These tiling patterns are complementary, continuous and completely cover the square.

7x1 summations

1694	1400	1547	1694	1400	1400	1694	1694	1400	1547	1694	1400
1547	1547	1400	1547	1694	1547	1547	1547	1547	1400	1547	1694
1400	1694	1694	1400	1547	1694	1400	1400	1694	1694	1400	1547
1694	1400	1547	1694	1400	1400	1694	1694	1400	1547	1694	1400
1547	1547	1400	1547	1694	1547	1547	1547	1547	1400	1547	1694
1400	1694	1694	1400	1547	1694	1400	1400	1694	1694	1400	1547
1694	1400	1547	1694	1400	1400	1694	1694	1400	1547	1694	1400
1547	1547	1400	1547	1694	1547	1547	1547	1547	1400	1547	1694
1400	1694	1694	1400	1547	1694	1400	1400	1694	1694	1400	1547
1694	1400	1547	1694	1400	1400	1694	1694	1400	1547	1694	1400
1547	1547	1400	1547	1694	1547	1547	1547	1547	1400	1547	1694
1400	1694	1694	1400	1547	1694	1400	1400	1694	1694	1400	1547
1694	1400	1547	1694	1400	1400	1694	1694	1400	1547	1694	1400
1547	1547	1400	1547	1694	1547	1547	1547	1547	1400	1547	1694
1400	1694	1694	1400	1547	1694	1400	1400	1694	1694	1400	1547

Vertical tripple sums

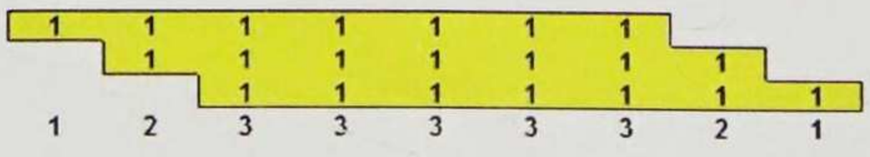
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

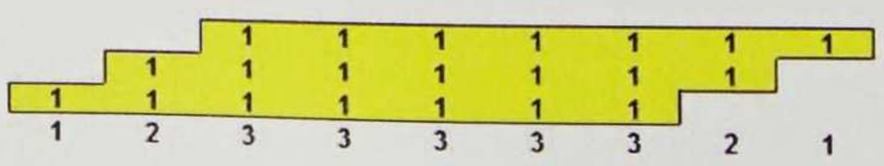
1x7 summations

1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498
1498	1547	1596	1498	1547	1596	1498	1547	1596	1498	1547	1596
1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498	1596
1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498
1498	1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547
1498	1547	1596	1498	1547	1596	1498	1547	1596	1498	1547	1596
1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498
1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498
1498	1547	1596	1498	1547	1596	1498	1547	1596	1498	1547	1596
1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498	1596
1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547	1498
1498	1596	1547	1498	1596	1547	1498	1596	1547	1498	1596	1547
1498	1547	1596	1498	1547	1596	1498	1547	1596	1498	1547	1596

Horizontal tripple sums

4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641





Program 4

Size-27

Here is the size-27 Class-3 square. Its characteristic number is **9855**.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
399	463	314	345	409	260	372	436	287	156	220	71	102	166	17	129	193	44	642	706	557	588	652	503	615	679	530	9855
590	657	496	617	684	523	644	711	550	347	414	253	374	441	280	401	468	307	104	171	10	131	198	37	158	225	64	9855
133	191	42	160	218	69	106	164	15	619	677	528	646	704	555	592	650	501	376	434	285	403	461	312	349	407	258	9855
386	471	319	332	417	265	359	444	292	143	228	76	89	174	22	116	201	49	629	714	562	575	660	508	602	687	535	9855
568	662	513	595	689	540	622	716	567	325	419	270	352	446	297	379	473	324	82	176	27	109	203	54	136	230	81	9855
114	205	47	141	232	74	87	178	20	600	691	533	627	718	560	573	664	506	357	448	290	384	475	317	330	421	263	9855
391	485	300	337	431	246	364	458	273	148	242	57	94	188	3	121	215	30	634	728	543	580	674	489	607	701	516	9855
585	667	491	612	694	518	639	721	545	342	424	248	369	451	275	396	478	302	99	181	5	126	208	32	153	235	59	9855
119	213	34	146	240	61	92	186	7	605	699	520	632	726	547	578	672	493	362	456	277	389	483	304	335	429	250	9855
318	382	476	264	328	422	291	355	449	75	139	233	21	85	179	48	112	206	561	625	719	507	571	665	534	598	692	9855
509	576	658	536	603	685	563	630	712	266	333	415	293	360	442	320	387	469	23	90	172	50	117	199	77	144	226	9855
52	110	204	79	137	231	25	83	177	538	596	690	565	623	717	511	569	663	295	353	447	322	380	474	268	326	420	9855
305	390	481	251	336	427	278	363	454	62	147	238	8	93	184	35	120	211	548	633	724	494	579	670	521	606	697	9855
487	581	675	514	608	702	541	635	729	244	338	432	271	365	459	298	392	486	1	95	189	28	122	216	55	149	243	9855
33	124	209	60	151	236	6	97	182	519	610	695	546	637	722	492	583	668	276	367	452	303	394	479	249	340	425	9855
310	404	462	256	350	408	283	377	435	67	161	219	13	107	165	40	134	192	553	647	705	499	593	651	526	620	678	9855
504	586	653	531	613	680	558	640	707	261	343	410	288	370	437	315	397	464	18	100	167	45	127	194	72	154	221	9855
38	132	196	65	159	223	11	105	169	524	618	682	551	645	709	497	591	655	281	375	439	308	402	466	254	348	412	9855
480	301	395	426	247	341	453	274	368	237	58	152	183	4	98	210	31	125	723	544	638	669	490	584	696	517	611	9855
671	495	577	698	522	604	725	549	631	428	252	334	455	279	361	482	306	388	185	9	91	212	36	118	239	63	145	9855
214	29	123	241	56	150	187	2	96	700	515	609	727	542	636	673	488	582	457	272	366	484	299	393	430	245	339	9855
467	309	400	413	255	346	440	282	373	224	66	157	170	12	103	197	39	130	710	552	643	656	498	589	683	525	616	9855
649	500	594	676	527	621	703	554	648	406	257	351	433	284	378	460	311	405	163	14	108	190	41	135	217	68	162	9855
195	43	128	222	70	155	168	16	101	681	529	614	708	556	641	654	502	587	438	286	371	465	313	398	411	259	344	9855
472	323	381	418	269	327	445	296	354	229	80	138	175	26	84	202	53	111	715	566	624	661	512	570	688	539	597	9855
666	505	572	693	532	599	720	559	626	423	262	329	450	289	356	477	316	383	180	19	86	207	46	113	234	73	140	9855
200	51	115	227	78	142	173	24	88	686	537	601	713	564	628	659	510	574	443	294	358	470	321	385	416	267	331	9855
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Here are four 3-dimensional summation patterns which sum continuously to the characteristic number.

1	2	3	4	5
1	1	1		
1	2	2	1	
1	2	3	2	1
	1	2	2	1
		1	1	1

	1	2	3
1	1	1	1
2	2	2	2
3	3	3	3
4	2	2	2
5	1	1	1

	1	2	3	4	5
1			1	1	1
2		1	2	2	1
3	1	2	3	2	1
4	1	2	2	1	
5	1	1	1		

	1	2	3	4	5
1	1	2	3	2	1
2	1	2	3	2	1
3	1	2	3	2	1

Program 4

Size-33

Here is the next size up Class-3 square, the size-33. Its characteristic number is **17985**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
		17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985
1	989	400	306	927	470	2	876	408	314	946	720	263	763	669	201	833	365	150	771	677	220	1083	626	37	1032	564	107	728	513	45	1040	583	357	17985
2	145	1019	551	94	1089	500	43	1027	570	102	855	508	293	914	457	363	863	406	301	933	465	129	871	656	188	820	726	137	769	664	207	828	492	17985
3	764	549	81	1076	487	272	1014	557	89	1084	385	38	912	444	350	850	635	288	920	452	358	748	401	186	807	713	124	998	651	194	815	721	22	17985
4	657	68	1063	595	17	770	544	87	1071	493	267	1020	431	337	958	380	44	907	450	345	856	630	294	794	700	232	743	407	181	813	708	130	993	17985
5	187	1050	593	4	999	531	74	1058	601	12	886	550	324	956	367	273	894	437	332	964	375	160	913	687	230	730	636	168	800	695	238	738	523	17985
6	69	580	838	260	518	1029	330	588	736	268	416	795	217	475	986	155	666	1056	225	373	994	53	432	943	112	623	881	303	693	951	10	631	779	17985
7	688	110	973	516	48	801	575	118	981	524	298	1051	473	247	879	411	75	938	481	255	887	661	325	836	610	153	774	438	212	844	618	161	1024	17985
8	218	1081	503	35	1030	562	105	979	511	54	917	581	355	866	398	304	925	468	253	874	417	191	944	718	140	761	667	199	831	616	148	780	554	17985
9	826	490	33	1017	560	334	1087	498	41	1025	447	100	853	396	291	923	697	361	861	404	299	810	463	127	759	654	197	1060	724	135	767	662	84	17985
10	719	20	1004	547	79	832	485	28	1023	555	340	1082	383	278	910	442	106	848	391	297	918	703	356	746	641	184	805	469	122	754	660	192	1066	17985
11	854	628	171	803	698	241	741	647	179	811	585	128	991	534	77	1061	604	15	1010	542	85	948	491	265	897	440	335	967	378	284	905	448	222	17985
12	747	521	64	1048	591	244	997	529	72	1067	478	21	884	427	322	954	607	271	892	435	341	841	384	158	790	685	228	970	634	166	798	704	115	17985
13	266	777	672	215	847	379	164	785	691	223	976	629	51	1035	578	121	742	527	59	1054	586	250	992	414	309	941	484	16	890	422	328	949	613	17985
14	885	670	202	834	608	151	772	678	210	842	506	159	1033	565	108	971	514	46	1041	573	116	869	522	307	928	471	245	877	409	315	936	479	143	17985
15	415	189	821	716	138	1012	665	208	829	614	25	778	552	95	1079	501	286	1028	571	103	977	388	52	915	458	353	864	649	302	934	466	251	751	17985
16	308	808	714	125	757	410	195	816	722	133	1007	671	82	1077	488	31	773	558	90	1085	496	281	1034	445	351	851	394	47	921	453	359	859	644	17985
17	190	701	959	18	639	908	88	709	857	26	537	916	338	596	744	276	545	814	346	494	752	174	553	1064	233	381	1002	182	451	1072	131	389	900	17985
18	446	231	731	637	169	1043	696	239	739	645	56	809	594	5	1000	532	317	1059	602	13	1008	419	83	957	368	274	895	680	333	965	376	282	782	17985
19	339	839	624	156	788	441	226	737	632	175	1038	702	113	987	519	62	804	589	11	995	538	312	1065	476	261	882	425	78	952	374	269	901	675	17985
20	947	611	154	775	681	213	845	619	162	783	568	221	974	517	49	1044	576	119	982	525	57	931	584	248	880	412	318	939	482	256	888	420	205	17985
21	477	141	762	668	200	1074	606	149	781	676	98	840	504	36	1031	563	348	969	512	55	1039	461	114	867	399	305	926	711	243	875	418	313	824	17985
22	975	386	292	924	456	120	862	405	300	932	706	249	749	655	198	819	483	136	768	663	206	1069	612	23	1018	561	93	846	499	42	1026	569	343	17985
23	868	642	185	806	712	123	755	650	193	825	599	142	1005	548	80	1075	486	29	1013	556	99	962	505	279	911	443	349	849	392	287	919	462	236	17985
24	24	898	430	336	968	621	285	906	449	344	734	387	172	793	699	242	984	648	180	812	707	8	750	535	67	1062	605	258	1011	543	86	1070	371	17985
25	1006	428	323	955	366	30	893	436	331	963	627	280	791	686	229	729	393	167	799	694	237	990	643	65	1049	592	3	756	530	73	1057	600	264	17985
26	536	310	942	474	259	891	423	329	950	372	146	899	673	216	837	622	165	786	692	224	735	509	173	1036	579	111	985	528	60	1055	587	9	872	17985
27	66	929	472	246	878	652	316	937	480	254	765	429	203	835	609	152	1015	679	211	843	617	39	792	566	109	972	515	289	1042	574	117	980	402	17985
28	311	459	1080	139	397	787	209	467	978	147	658	1037	96	717	865	34	424	935	104	615	873	295	674	822	354	502	760	61	572	830	252	510	1021	17985
29	567	352	852	395	290	922	454	360	860	403	177	930	715	126	758	653	196	817	723	134	766	540	204	1078	489	32	1016	559	91	1086	497	40	903	17985
30	97	960	382	277	909	683	347	858	390	296	796	460	234	745	640	183	1046	710	132	753	659	70	823	597	19	1003	546	320	1073	495	27	1022	433	17985
31	1068	369	275	896	439	92	966	377	283	904	689	342	732	638	170	802	455	240	740	646	178	1052	705	6	1001	533	76	818	603	14	1009	541	326	17985
32	598	262	883	426	321	953	364	270	902	434	219	961	625	157	789	684	227	727	633	176	797	582	235	988	520	63	1047	590	1	996	539	71	945	17985
33	733	507	50	1045	577	362	983	526	58	1053	464	7	870	413	319	940	725	257	889	421	327	827	370	144	776	682	214	1088	620	163	784	690	101	17985
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	

6358	5632	5995	6358	5632	5995	6358	5632	5995	6358
5995	6358	5632	5995	6358	5632	5995	6358	5632	5995
6358	5632	5995	6358	5632	5995	6358	5632	5995	6358
5632	5995	6358	5632	5995	6358	5632	5995	6358	5632
5995	6358	5632	5995	6358	5632	5995	6358	5632	5995
5632	6358	5995	5632	6358	5995	5632	6358	5995	5632
5632	5995	6358	5632	5995	6358	5632	5995	6358	5632
5995	6358	5632	5995	6358	5632	5995	6358	5632	5995

17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985	17985

6116	5995	6116	5874	5995	5874	5874
5874	6116	5874	5995	6116	6116	5995
5995	5874	5995	6116	5874	5995	6116
6116	5995	6116	5874	5995	5874	5874
5874	6116	5874	5995	6116	6116	5995
5995	5874	5995	6116	5874	5995	6116
6116	5995	6116	5874	5995	5874	5874

17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985
17985	17985	17985	17985	17985	17985	17985

The summation tables are sums of linear strips of **1x11** and **11x1**. The summations show an unequal pattern but taken in consecutive triples, both horizontally (upper tables) and vertically (lower tables), show that the characteristic patterns for the size-33 square is similar to that for the size-21 square in three overlapping size **b** linear summations of **1x(b+2)** horizontally and **(b+2)x1** vertically.

It appears that this characteristic pattern applies only when **b** is a prime number; witness the difference of the size-27 square on the prior page where **b = 9**.

Program 4

In Class-4 Squares

Size-20

Here is the size-20 square with level-5 even-size square pyramidal sums. The sums are continuous. Each pyramidal sum equals **11 C_{20}**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	W(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	321	289	2	195	308	296	14	177	320	283	21	189	302	295	8	196	314	277	4010
2	336	254	42	160	348	261	29	167	335	273	36	154	342	260	48	161	329	267	35	173	4010
3	82	125	388	201	94	107	400	213	76	119	382	225	88	101	394	207	100	113	376	219	4010
4	353	241	59	147	365	228	66	134	372	240	53	141	359	247	65	128	366	234	72	140	4010
5	24	187	305	293	6	199	312	280	18	181	324	287	5	193	306	299	12	180	318	281	4010
6	345	258	46	164	327	270	33	171	339	252	45	158	346	264	27	170	333	271	39	152	4010
7	86	104	392	210	98	111	379	217	85	123	386	204	92	110	398	211	79	117	385	223	4010
8	357	250	63	126	369	232	75	138	351	244	57	150	363	226	69	132	375	238	51	144	4010
9	3	191	309	297	15	178	316	284	22	190	303	291	9	197	315	278	16	184	322	290	4010
10	349	262	30	168	331	274	37	155	343	256	49	162	330	268	31	174	337	255	43	156	4010
11	95	108	396	214	77	120	383	221	89	102	395	208	96	114	377	220	83	121	389	202	4010
12	361	229	67	135	373	236	54	142	360	248	61	129	367	235	73	136	354	242	60	148	4010
13	7	200	313	276	19	182	325	288	1	194	307	300	13	176	319	282	25	188	301	294	4010
14	328	266	34	172	340	253	41	159	347	265	28	166	334	272	40	153	341	259	47	165	4010
15	99	112	380	218	81	124	387	205	93	106	399	212	80	118	381	224	87	105	393	206	4010
16	370	233	71	139	352	245	58	146	364	227	70	133	371	239	52	145	358	246	64	127	4010
17	11	179	317	285	23	186	304	292	10	198	311	279	17	185	323	286	4	192	310	298	4010
18	332	275	38	151	344	257	50	163	326	269	32	175	338	251	44	157	350	263	26	169	4010
19	78	116	384	222	90	103	391	209	97	115	378	216	84	122	390	203	91	109	397	215	4010
20	374	237	55	143	356	249	62	130	368	231	74	137	355	243	56	149	362	230	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Below is the size-20 with level-4 even-size diamond pyramidal sums. The sums are continuous. Each pyramidal diamond sum equals **2 C_{20}**.

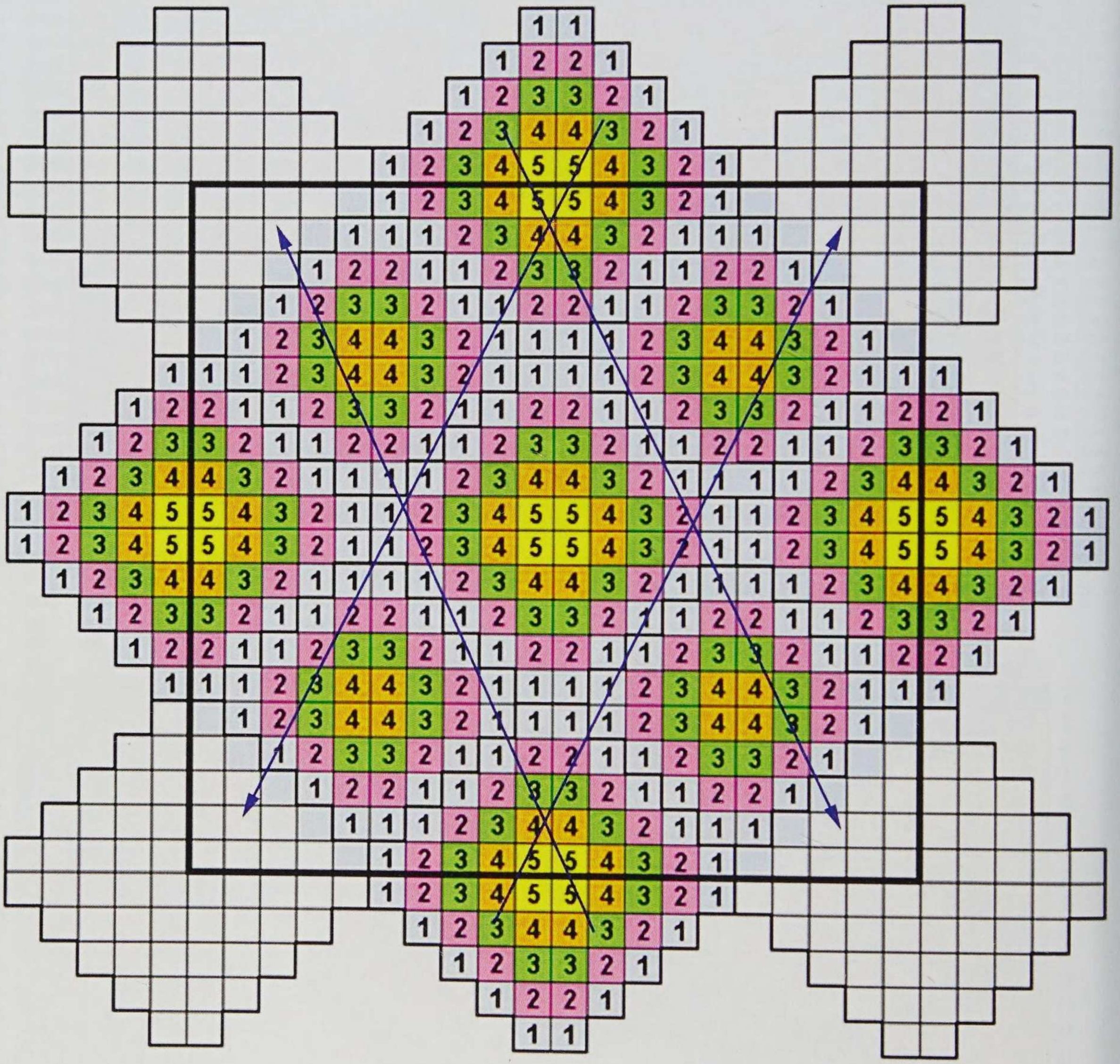
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	W(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	120	233	371	89	102	245	358	96	114	227	370	83	121	239	352	95	108	246	364	77	4010
2	386	54	142	210	398	61	129	217	385	73	136	204	392	60	148	211	379	67	135	223	4010
3	32	325	288	151	44	307	300	163	26	319	282	175	38	301	294	157	50	313	276	169	4010
4	253	191	9	347	265	178	16	334	272	190	3	341	259	197	15	328	266	184	22	340	4010
5	124	237	355	93	106	249	362	80	118	231	374	87	105	243	356	99	112	230	368	81	4010
6	395	58	146	214	377	70	133	221	389	52	145	208	396	64	127	220	383	71	139	202	4010
7	36	304	292	160	48	311	279	167	35	323	286	154	42	310	298	161	29	317	285	173	4010
8	257	200	13	326	269	182	25	338	251	194	7	350	263	176	19	332	275	188	1	344	4010
9	103	241	359	97	115	228	366	84	122	240	353	91	109	247	365	78	116	234	372	90	4010
10	399	62	130	218	381	74	137	205	393	56	149	212	380	68	131	224	387	55	143	206	4010
11	45	308	296	164	27	320	283	171	39	302	295	158	46	314	277	170	33	321	289	152	4010
12	261	179	17	335	273	186	4	342	260	198	11	329	267	185	23	336	254	192	10	348	4010
13	107	250	363	76	119	232	375	88	101	244	357	100	113	226	369	82	125	238	351	94	4010
14	378	66	134	222	390	53	141	209	397	65	128	216	384	72	140	203	391	59	147	215	4010
15	49	312	280	168	31	324	287	155	43	306	299	162	30	318	281	174	37	305	293	156	4010
16	270	183	21	339	252	195	8	346	264	177	20	333	271	189	2	345	258	196	14	327	4010
17	111	229	367	85	123	236	354	92	110	248	361	79	117	235	373	86	104	242	360	98	4010
18	382	75	138	201	394	57	150	213	376	69	132	225	388	51	144	207	400	63	126	219	4010
19	28	316	284	172	40	303	291	159	47	315	278	166	34	322	290	153	41	309	297	165	4010
20	274	187	5	343	256	199	12	330	268	181	24	337	255	193	6	349	262	180	18	331	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Program 4

Size-20 Integrated Pyramidal Pattern

Here is an integrated pyramidal diamond pattern up to levels **4** and **5** which exists in the size **20** square. The numbered layers indicate the number of times the value in the cell is multiplied in the pyramidal summation. Numbers of the same multiple are colored the same color.

This pattern can be wholly contained within the size-20 square itself.



Program 4

Size-24

Here is the size-24 square with level-6 even-size square pyramidal sums. The sums are continuous. However, they do not sum to an integral multiple of C_{24}.

W(24)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	342	511	360	73	210	91	270	7	288	289	570	307	486	367	504	217	66	235	126	151	144	6924
2	103	192	85	276	25	258	463	408	445	348	529	330	247	48	229	132	169	114	319	552	301	492	385	474	6924
3	356	506	311	482	431	437	284	2	239	122	215	77	500	362	455	338	575	293	140	146	95	266	71	221	6924
4	105	208	87	274	9	256	465	424	447	346	513	328	249	64	231	130	153	112	321	568	303	490	369	472	6924
5	327	514	345	448	423	466	255	10	273	88	207	106	471	370	489	304	567	322	111	154	129	232	63	250	6924
6	83	209	92	269	8	278	443	425	452	341	512	350	227	65	236	125	152	134	299	569	308	485	368	494	6924
7	325	534	343	450	403	468	253	30	271	90	187	108	469	390	487	306	547	324	109	174	127	234	43	252	6924
8	571	300	553	384	493	366	67	228	49	168	133	150	427	444	409	528	349	510	211	84	193	24	277	6	6924
9	179	113	200	17	248	38	395	473	416	521	320	542	35	257	56	161	104	182	539	329	560	377	464	398	6924
10	573	316	555	382	477	364	69	244	51	166	117	148	429	460	411	526	333	508	213	100	195	22	261	4	6924
11	147	118	165	52	243	70	363	478	381	556	315	574	3	262	21	196	99	214	507	334	525	412	459	430	6924
12	548	314	563	374	479	389	44	242	59	158	119	173	404	458	419	518	335	533	188	98	203	14	263	29	6924
13	145	138	163	54	223	72	361	498	379	558	295	576	1	282	19	198	79	216	505	354	523	414	439	432	6924
14	391	480	373	564	313	546	175	120	157	60	241	42	535	336	517	420	457	402	31	264	13	204	97	186	6924
15	68	218	23	194	143	149	572	290	527	410	503	365	212	74	167	50	287	5	428	434	383	554	359	509	6924
16	393	496	375	562	297	544	177	136	159	58	225	40	537	352	519	418	441	400	33	280	15	202	81	184	6924
17	39	226	57	160	135	178	543	298	561	376	495	394	183	82	201	16	279	34	399	442	417	520	351	538	6924
18	371	497	380	557	296	566	155	137	164	53	224	62	515	353	524	413	440	422	11	281	20	197	80	206	6924
19	37	246	55	162	115	180	541	318	559	378	475	396	181	102	199	18	259	36	397	462	415	522	331	540	6924
20	283	12	265	96	205	78	355	516	337	456	421	438	139	156	121	240	61	222	499	372	481	312	565	294	6924
21	467	401	488	305	536	326	107	185	128	233	32	254	323	545	344	449	392	470	251	41	272	89	176	110	6924
22	285	28	267	94	189	76	357	532	339	454	405	436	141	172	123	238	45	220	501	388	483	310	549	292	6924
23	435	406	453	340	531	358	75	190	93	268	27	286	291	550	309	484	387	502	219	46	237	124	171	142	6924
24	260	26	275	86	191	101	332	530	347	446	407	461	116	170	131	230	47	245	476	386	491	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Below is an even-size pyramidal diamond pattern. It contains two sizes of diamond: one is an even-size size **6** ring diamond which sums exactly to C_{24} and a smaller one of size **5** which only sums equally as a level-5 pyramidal diamond. Both integrate to cover the size-24 square completely and continuously. This square is the background on the front cover.

W(24)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	217	66	235	414	439	432	145	138	163	342	511	360	505	354	523	126	151	144	6924
2	103	192	85	564	313	546	319	552	301	204	97	186	391	480	373	276	25	258	31	264	13	492	385	474	6924
3	356	506	311	194	143	149	140	146	95	554	359	509	68	218	23	482	431	437	428	434	383	266	71	221	6924
4	105	208	87	562	297	544	321	568	303	202	81	184	393	496	375	274	9	256	33	280	15	490	369	472	6924
5	327	514	345	160	135	178	111	154	129	520	351	538	39	226	57	448	423	466	399	442	417	232	63	250	6924
6	83	209	92	557	296	566	299	569	308	197	80	206	371	497	380	269	8	278	11	281	20	485	368	494	6924
7	325	534	343	162	115	180	109	174	127	522	331	540	37	246	55	450	403	468	397	462	415	234	43	252	6924
8	139	156	121	528	349	510	355	516	337	168	133	150	427	444	409	240	61	222	67	228	49	456	421	438	6924
9	323	545	344	161	104	182	107	185	128	521	320	542	35	257	56	449	392	470	395	473	416	233	32	254	6924
10	141	172	123	526	333	508	357	532	339	166	117	148	429	460	411	238	45	220	69	244	51	454	405	436	6924
11	291	550	309	196	99	214	75	190	93	556	315	574	3	262	21	484	387	502	363	478	381	268	27	286	6924
12	116	170	131	518	335	533	332	530	347	158	119	173	404	458	419	230	47	245	44	242	59	446	407	461	6924
13	289	570	307	198	79	216	73	210	91	558	295	576	1	282	19	486	367	504	361	498	379	270	7	288	6924
14	247	48	229	420	457	402	463	408	445	60	241	42	535	336	517	132	169	114	175	120	157	348	529	330	6924
15	500	362	455	50	287	5	284	2	239	410	503	365	212	74	167	338	575	293	572	290	527	122	215	77	6924
16	249	64	231	418	441	400	465	424	447	58	225	40	537	352	519	130	153	112	177	136	159	346	513	328	6924
17	471	370	489	16	279	34	255	10	273	376	495	394	183	82	201	304	567	322	543	298	561	88	207	106	6924
18	227	65	236	413	440	422	443	425	452	53	224	62	515	353	524	125	152	134	155	137	164	341	512	350	6924
19	469	390	487	18	259	36	253	30	271	378	475	396	181	102	199	306	547	324	541	318	559	90	187	108	6924
20	283	12	265	384	493	366	499	372	481	24	277	6	571	300	553	96	205	78	211	84	193	312	565	294	6924
21	467	401	488	17	248	38	251	41	272	377	464	398	179	113	200	305	536	326	539	329	560	89	176	110	6924
22	285	28	267	382	477	364	501	388	483	22	261	4	573	316	555	94	189	76	213	100	195	310	549	292	6924
23	435	406	453	52	243	70	219	46	237	412	459	430	147	118	165	340	531	358	507	334	525	124	171	142	6924
24	260	26	275	374	479	389	476	386	491	14	263	29	548	314	563	86	191	101	188	98	203	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Program 4

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	21	250	95	228	101	170	31	244	85	234	111	164	309	538	383	516	389	458	319	532	373	522	399	452	6924
2	432	499	294	441	368	563	422	505	304	435	358	569	144	211	6	153	80	275	134	217	16	147	70	281	6924
3	50	189	124	263	34	205	60	183	114	269	44	199	338	477	412	551	322	493	348	471	402	557	332	487	6924
4	347	472	401	558	331	488	337	478	411	552	321	494	59	184	113	270	43	200	49	190	123	264	33	206	6924
5	133	218	15	148	69	282	143	212	5	154	79	276	421	506	303	436	357	570	431	500	293	442	367	564	6924
6	320	531	374	521	400	451	310	537	384	515	390	457	32	243	86	233	112	163	22	249	96	227	102	169	6924
7	18	253	92	231	98	173	28	247	82	237	108	167	306	541	380	519	386	461	316	535	370	525	396	455	6924
8	427	504	289	446	363	568	417	510	299	440	353	574	139	216	1	158	75	280	129	222	11	152	65	286	6924
9	53	186	127	260	37	202	63	180	117	266	47	196	341	474	415	548	325	490	351	468	405	554	335	484	6924
10	352	467	406	553	336	483	342	473	416	547	326	489	64	179	118	265	48	195	54	185	128	259	38	201	6924
11	130	221	12	151	66	285	140	215	2	157	76	279	418	509	300	439	354	573	428	503	290	445	364	567	6924
12	315	536	369	526	395	456	305	542	379	520	385	462	27	248	81	238	107	168	17	254	91	232	97	174	6924
13	165	106	239	84	245	26	175	100	229	90	255	20	453	394	527	372	533	314	463	388	517	378	543	308	6924
14	576	355	438	297	512	419	566	361	448	291	502	425	288	67	150	9	224	131	278	73	160	3	214	137	6924
15	194	45	268	119	178	61	204	39	258	125	188	55	482	333	556	407	466	349	492	327	546	413	476	343	6924
16	491	328	545	414	475	344	481	334	555	408	465	350	203	40	257	126	187	56	193	46	267	120	177	62	6924
17	277	74	159	4	213	138	287	68	149	10	223	132	565	362	447	292	501	426	575	356	437	298	511	420	6924
18	464	387	518	377	544	307	454	393	528	371	534	313	176	99	230	89	256	19	166	105	240	83	246	25	6924
19	162	109	236	87	242	29	172	103	226	93	252	23	450	397	524	375	530	317	460	391	514	381	540	311	6924
20	571	360	433	302	507	424	561	366	443	296	497	430	283	72	145	14	219	136	273	78	155	8	209	142	6924
21	197	42	271	116	181	58	207	36	261	122	191	52	485	330	559	404	469	346	495	324	549	410	479	340	6924
22	496	323	550	409	480	339	486	329	560	403	470	345	208	35	262	121	192	51	198	41	272	115	182	57	6924
23	274	77	156	7	210	141	284	71	146	13	220	135	562	365	444	295	498	429	572	359	434	301	508	423	6924
24	459	392	513	382	539	312	449	398	523	376	529	318	171	104	225	94	251	24	161	110	235	88	241	30	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Here is a different size-24 square. It was derived by the **SPD** method of Program 7 by expanding a size-12 square that was not pangenic. Consequently, the resulting size-24 square is also not pangenic and so it is not perfect even though all of its linear summations equal its characteristic number.

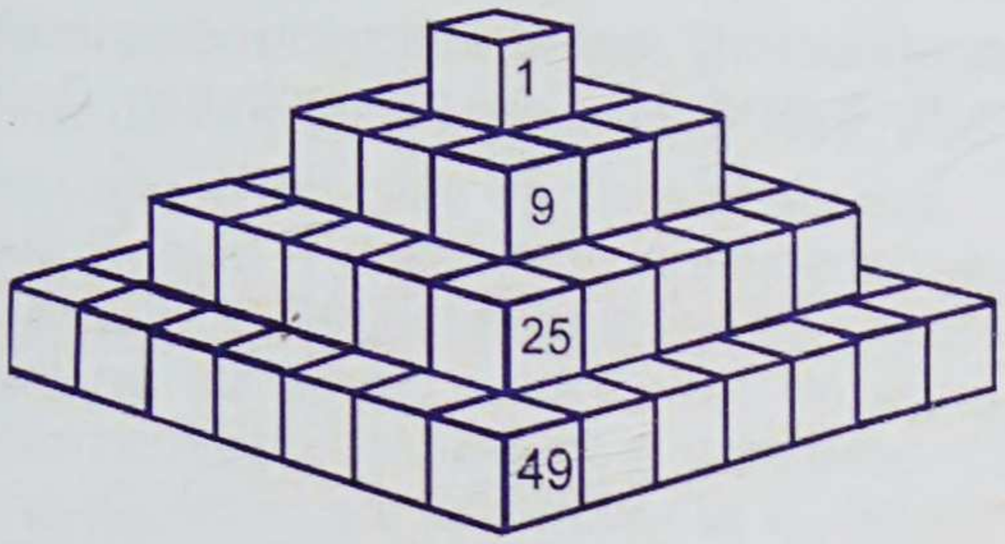
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	21	250	95	228	101	170	31	244	85	234	111	164	309	538	383	516	389	458	319	532	373	522	399	452	6924
2	432	499	294	441	368	563	422	505	304	435	358	569	144	211	6	153	80	275	134	217	16	147	70	281	6924
3	50	189	124	263	34	205	60	183	114	269	44	199	338	477	412	551	322	493	348	471	402	557	332	487	6924
4	347	472	401	558	331	488	337	478	411	552	321	494	59	184	113	270	43	200	49	190	123	264	33	206	6924
5	133	218	15	148	69	282	143	212	5	154	79	276	421	506	303	436	357	570	431	500	293	442	367	564	6924
6	320	531	374	521	400	451	310	537	384	515	390	457	32	243	86	233	112	163	22	249	96	227	102	169	6924
7	18	253	92	231	98	173	28	247	82	237	108	167	306	541	380	519	386	461	316	535	370	525	396	455	6924
8	427	504	289	446	363	568	417	510	299	440	353	574	139	216	1	158	75	280	129	222	11	152	65	286	6924
9	53	186	127	260	37	202	63	180	117	266	47	196	341	474	415	548	325	490	351	468	405	554	335	484	6924
10	352	467	406	553	336	483	342	473	416	547	326	489	64	179	118	265	48	195	54	185	128	259	38	201	6924
11	130	221	12	151	66	285	140	215	2	157	76	279	418	509	300	439	354	573	428	503	290	445	364	567	6924
12	315	536	369	526	395	456	305	542	379	520	385	462	27	248	81	238	107	168	17	254	91	232	97	174	6924
13	165	106	239	84	245	26	175	100	229	90	255	20	453	394	527	372	533	314	463	388	517	378	543	308	6924
14	576	355	438	297	512	419	566	361	448	291	502	425	288	67	150	9	224	131	278	73	160	3	214	137	6924
15	194	45	268	119	178	61	204	39	258	125	188	55	482	333	556	407	466	349	492	327	546	413	476	343	6924
16	491	328	545	414	475	344	481	334	555	408	465	350	203	40	257	126	187	56	193	46	267	120	177	62	6924
17	277	74	159	4	213	138	287	68	149	10	223	132	565	362	447	292	501	426	575	356	437	298	511	420	6924
18	464	387	518	377	544	307	454	393	528	371	534	313	176	99	230	89	256	19	166	105	240	83	246	25	6924
19	162	109	236	87	242	29	172	103	226	93	252	23	450	397	524	375	530	317	460	391	514	381	540	311	6924
20	571	360	433	302	507	424	561	366	443	296	497	430	283	72	145	14	219	136	273	78	155	8	209	142	6924
21	197	42	271	116	181	58	207	36	261	122	191	52	485	330	559	404	469	346	495	324	549	410	479	340	6924
22	496	323	550	409	480	339	486	329	560	403	470	345	208	35	262	121	192	51	198	41	272	115	182	57	6924
23	274	77	156	7	210	141	284	71	146	13	220	135	562	365	444	295	498	429	572	359	434	301	508	423	6924
24	459	392	513	382	539	312	449	398	523	376	529	318	171	104	225	94	251	24	161	110	235	88	241	30	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Nevertheless, this rendition of the size-24 square has not only many continuously equal-summing tiling patterns, but yields the predicted even-size **5** and **6** solid diamond tiling patterns which sum to **2½** and **3½** times the characteristic number (re: p.4-5). Check out all these equal-summing patterns on this non-perfect square:

1. the size **5** even solid diamond pattern
2. the size **6** even solid diamond pattern
3. the size **6** even ring-diamond pattern
4. the size **12** X-pattern
5. the level **2** pyramidal square pattern
6. the level **3** pyramidal square pattern
7. the level **6** pyramidal square pattern
8. the level **4** even pyramidal diamond pattern
9. the level **5** even pyramidal diamond pattern
10. the level **6** even pyramidal diamond pattern
11. the integrated size **5** and size **6** diamond patterns here which cover the square exactly.

Program 4

Size-28



Square Pyramidal Pattern

Here is the size-28 square. It contains a 3-dimensional level -4 square pyramidal structure of odd-size (7) which sums continuously to $\mathbf{3C_{28}}$.

Square pyramidal sums, of odd size, were not found to have any arithmetical pattern of repetition nor basis for prediction.

Program 4

Size-28

Multiple Pyramidal Diamond Patterns

Here is the size-28 square again depicting its continuously equal-summing even-structured pyramidal **diamonds** of sizes **6** and **7**. They sum to **8** and **12** $\mathbf{C_{28}}$, respectively. Both sizes integrate to cover the square completely.

Within this square, there is also a size-5 diamond pattern which possesses a level-5 pyramidal pattern that also sums equally continuously. However, the complete size-5 diamond pattern doesn't fit within the square nor does it integrate with the size **6** or **7** diamonds on the same plane. However, the size-5 diamond pyramid, when placed atop a size-6 diamond, completes the size-6 diamond pyramid. And when the size-6 pyramid is placed atop the size-7 diamond, it completes the size-7 pyramid. So all three sizes fit within the square simultaneously in this manner.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
	W(28)	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	

Now, the point to recognize is that every ring-diamond which sums continuously to the square's characteristic number also possesses a solid diamond pattern of the same size which sums continuously to a multiple of the characteristic number.

Further, any solid diamond pattern which sums continuously to a multiple of a Class-4 square's characteristic number possesses a pyramidal diamond pattern of the same size which does the same.

Contraversally, an equal-summing pyramidal diamond pattern does not guarantee an equal-summing solid diamond pattern. Further, an equal-summing solid diamond pattern does not guarantee an equal-summing ring-diamond pattern.

Here, the level **5** pyramidal diamond doesn't have a solid-diamond counterpart nor a ring-diamond counterpart; and the size **6** solid-diamond pattern doesn't have a ring-diamond counterpart. Only the size-7 solid-diamond has a size-7 ring-diamond counterpart.

Program 4

Here is the size-28's level-5 pyramidal diamonds totally contained within the square. The diamond patterns must share their adjacent cells (blue). The smaller odd-size diamonds in-between do not sum equally. Actually, odd-size solid diamonds have not been seen thus far to sum equally in any size or class of square.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
	W(28)	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1		203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2		779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3		88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4		524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5		218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6		752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7		61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8		497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9		240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10		774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11		83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12		512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13		213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14		747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15		56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16		534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17		235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18		769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19		71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20		507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21		208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22		742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23		93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24		529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25		230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26		757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27		66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28		502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	

Program 4

Size-32

The size-32 square on the next page follows the relationships among diamonds described on the preceding page. However, it possesses a size-4 ring-diamond pattern but does not possess a solid-size-4 diamond. Further, it possesses a size-3 pyramidal diamond but not a size-3 even-size solid diamond. So the properties described cannot be made into generalized theorems; there will always be exceptions due to embryonic conditions of smaller diamonds even in larger size squares.

Note that the level-3 pyramidal diamond pattern is actually the fourth occurrence of a pyramidal diamond cluster in the size-32 square.

13 Continuous Equal-summing Patterns on the Size-32 square

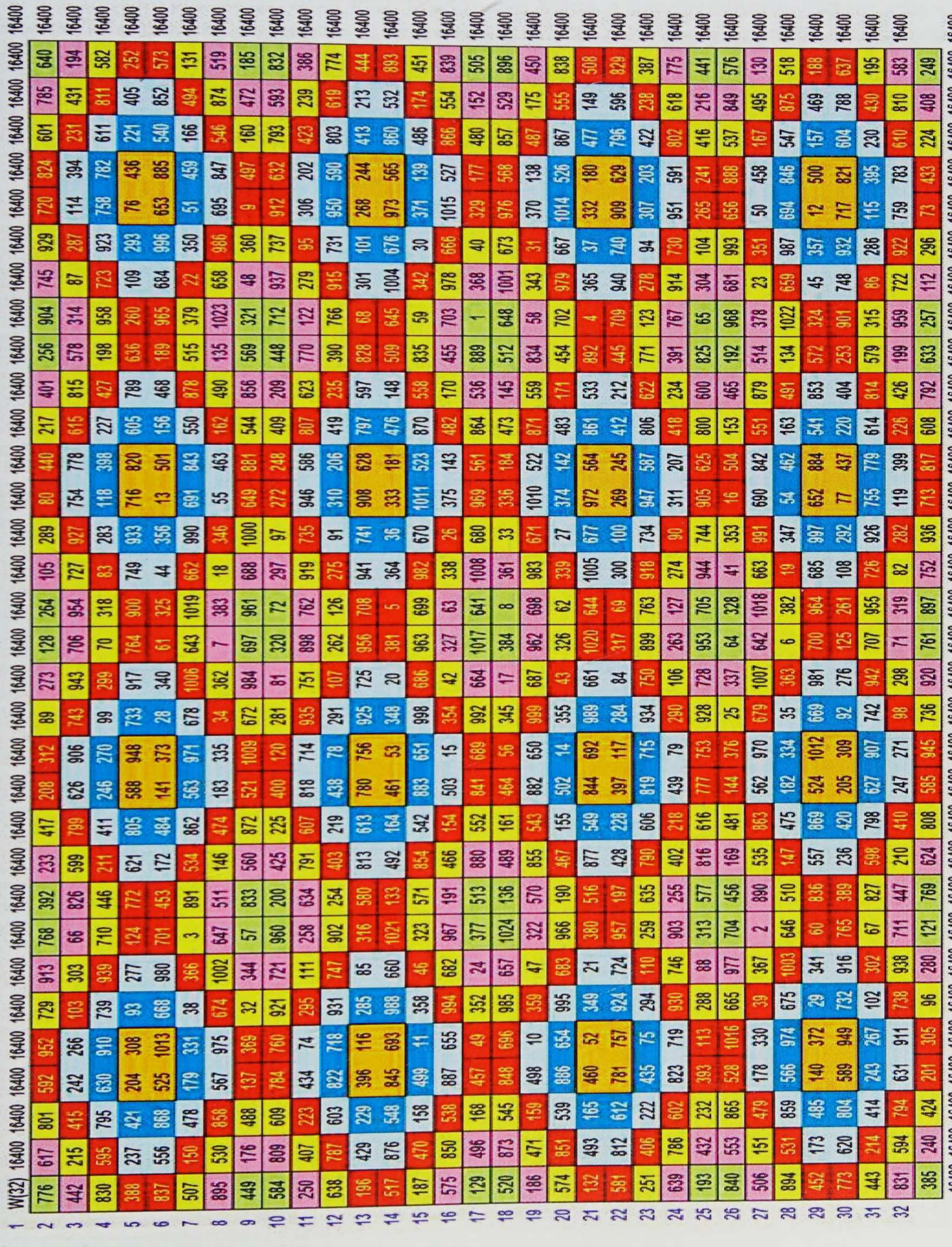
#	Pattern	Value of sums	Multiple of C_{32}
1	4x-modularity	8200	½
2	Size-4 even-size ring-diamond	8200	½
3	Size-8 even-size ring-diamond	16400	1
4	Double "+" vertical and horizontal tiles	16400	2
5	Size-7 even-size solid diamond	57400	3 ½
6	Size-8 even-size solid diamond	73800	4 ½
7	Level-3 square pyramid	28700	1 3/4
8	Level-4 square pyramid	61500	3 3/4
9	Level-3 even size pyramidal diamond *	20500	1 1/4
10	Level-6 even size pyramidal diamond	114800	7
11	Level-7 even size pyramidal diamond	172200	10 ½
12	Level-8 even size pyramidal diamond	246000	15
13	Size-30 2x2-incremented solid diamond	131200	8

* Depicted as integrated on the next page

Program 4

Size-32

Here is a level-3 equal-summing pyramidal diamond pattern surrounded by a size-4 equal-summing ring-diamond pattern (red cells). Both summations are continuous.



Program 4

Size-44

Along these same lines, here is another square, this time of size-44, with its 6-level odd-size
(11x11) square pyramidal sums highlighted.

Program 4

Summary of Solid and Pyramidal Diamond Patterns in Class-4 Squares

The summary table shows a continuous pattern of sizes of solid and pyramidal diamonds for Class-4 squares up thru size-76. The pattern holds for the size-4 square. The patterns are violated only for squares of size **8** and **12** in all cases and for the size **16** in not possessing its smaller pyramidal diamonds. After that, the squares follow a very systematic pattern with respect to their solid and pyramidal diamond properties. Observe the ordered pattern of values in each column.

Continuous Solid Diamond Tile Patterns					3D Pyramidal Diamond Patterns			
b	n = 4b	C_n	Diamond Sizes	Diamond Sum Multiples of C_n	Pyramidal Heights	Diamond Sizes	Diamond Sum Multiples of C_n	Pyramidal Heights
1	4	34	0 & 1	0 & 1	0 & 1	1 & 2	1 & 3	1 & 2
2	8	260	1 & 2	½ & 1½	0, 1 & 2	3 & 4	3 & 5	2, 3 & 4
3	12	870	2 & 3	1 & 2	1, 2 & 3;	5 & 6	5 & 7	4, 5 & 6
4	16	2056	3 & 4	1½ & 2½	2,3 & 4;	7 & 8	7 & 9	6, 7 & 8
5	20	4010	4 & 5	2 & 3	3, 4 & 5;	9 & 10	9 & 11	8, 9 & 10
6	24	6924	5 & 6	2½ & 3½	4, 5 & 6;	11 & 12	11 & 13	10, 11 & 12
7	28	10990	6 & 7	3 & 4	5, 6 & 7;	13 & 14	13 & 15	12, 13 & 14
8	32	14911	7 & 8	3½ & 4½	6, 7 & 8;	15 & 16	15 & 17	14, 15 & 16
9	36	23346	8 & 9	4 & 5	7, 8 & 9;	17 & 18	17 & 19	16, 17 & 18
10	40	32020	9 & 10	4½ & 5½	8, 9 &10;	19 & 20	19 & 21	18, 19 & 20
11	44	42614	10 & 11	5 & 6	9, 10 & 11;	21 & 22	21 & 23	20, 21 & 22
12	48	55320	11 & 12	5½ & 6½	10, 11 & 12;	23 & 24	23 & 25	22, 23 & 24
13	52	70330	12 & 13	6 & 7	11, 12 & 13;	25 & 26	25 & 27	24, 25 & 26
14	56	87836	13 & 14	6½ & 7½	12, 13 & 14;	27 & 28	27 & 29	26, 27 & 28
15	60	108030	14 & 15	7 & 8	13, 14 & 15;	29 & 30	29 & 31	28, 29 & 30
16	64	131104	15 & 16	7½ & 8½	14, 15 & 16;	31 & 32	31 & 33	30, 31 & 32
17	68	157250	16 & 17	8 & 9	15, 16 & 17;	33 & 34	33 & 35	32, 33 & 34
18	72	186660	17 & 18	8½ & 9½	16, 17 & 18;	35 & 36	35 & 37	34, 35 & 36
19	76	219526	18 & 19	9 & 10	17, 18 & 19;	37 & 38	37 & 39	36, 37 & 38

Items in blue are violated due to embryonic conditions

The ***proof by induction*** method of classical math will need to be adapted to Geonometry because one of the requirements in that method is to first show that the property holds for **1**, then assume it holds for **n** and prove that it must hold for **n+1**.

The induction proof needs to be restated as follows:

"Demonstrate that the pattern begins with 0 for n = 1, then show that the pattern can be mathematically extended in increments of k to m where the serial pattern comes into existence and prove that it always holds for m+k."

This is necessary because the smaller tables, although their formulas can be systematically traced back to **0**, do not come into play until a sufficient quantity of consecutive numbers are always present to adequately express the stated property.

Program 4

Pyramidal Summary Tables for Class-4 Squares

Pyramid count by size and shape					
Size of Square	Even-size		Odd-size		Total
	Diamonds	Squares	Squares	Diamonds	
16	6	3	0	0	9
20	6	1	1	0	8
24	6	1	0	0	7
28	6	2	1	0	9
32	6	2	0	0	8
36	6	3	2	0	11
44	6	3	1	0	10
64	8	6	0	0	14

Pyramidal diamonds on size-40 square		
Count	Level	Value
1	3	32020
2	4	64040
3	5	112070
4	8	384240
5	10	704440
6	13	1456910
7	14	1793120
8	15	2177360
9	18	3650280
10	19	4258660
11	20	4931080

The summary table shows the occurrences of pyramidal equalities according to shape and size for a broad sampling of Class-4 squares. Note that no odd-size pyramidal diamonds were ever encountered. Also observe the consistency of even-size diamonds from sizes **16** thru **44**.

The size **40** is exceptional: it has a count of **11** pyramidal diamonds in both of its dual squares.

In the table below, note the number of diamond summations which are multiples of **10** (highlighted in blue). This is surprising for sizes **16**, **24**, **36**, **44** & **64** which don't have characteristic numbers that are multiples of **10**. In fact, the size **24** level-5 pyramid is a fractional **4/5**th of its characteristic number.

Class-4 squares with even-size diamond ◊ pyramidal patterns									
Size of square	Height of stacks	Sums of cells in pyramids				Characteristic number C_n	Pyramidal sums' multiple of charascteristic number		
12	2	1160				870	1⅓		
16	3, 6, & 7	5140	28784	43176		2056	2½	14	21
20	3, 4 & 5	8020	16040	28070		4010	2	4	7
24	4, 5 & 6	23080	40390	64624		6924	3⅕	5⅘	9⅕
28	5, 6 & 7	54950	87920	131880		10990	5	8	12
32	6, 7 & 8	114800	172200	246000		16400	7	10½	15
36	7, 8 & 9	217896	311280	428010		23346	9⅓	13⅓	18⅓
44	9, 10 & 11	639210	852280	1107964		42614	15	20	26
64	5, 6, 7 & 8	573580	1032444	1720740	2704020	131104	4 3/8	7 7/8 & 13 1/8	20 5/8
	13,14,15,16	14913080	19501720	25073640	31759944		113¾	148¾ & 191¼	242¼

Could this spatial property be the source of why most mammals have **5** toes on each foot and some with hands in place of paws even have **5** fingers on each hand?

Program 4

Note in the top table, the sizes **16**, **24**, **32** & **44** have no odd-size pyramidal squares, yet all sizes have even-size pyramidal squares.

Although all sizes are of Class-4, there is no predictable pattern in the pyramidal sum's multiplicity of the square's characteristic number; they expand and contract randomly.

Class-4 squares with odd-size square □ pyramidal patterns

Size of square	Height of stack	Size of base	# of cells in base	Cells in pyramid	Value of Sums	Characteristic number	Pyramidal sums as multiples of Cn
16	None					2056	
20	4	7	49	84	26065	4010	6½
24	None					6924	
28	7	13	169	196	406630	10990	37
32	None					16400	
36	4 & 14	7 & 27	49 & 729	84 & 3654	54474 & 2369619	23346	2⅓ & 101½
44	6	11	121	286	276991	42614	6½
64	None					131104	

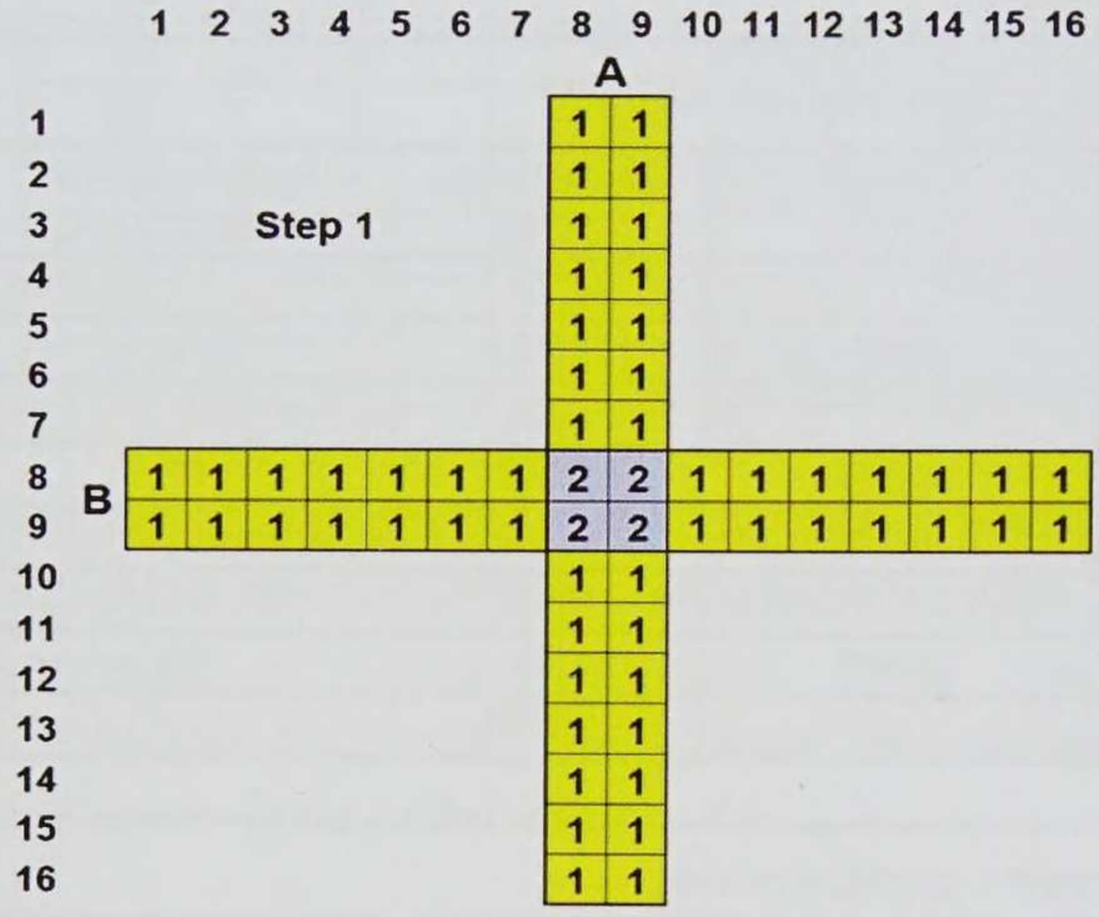
Class-4 squares with even-size square □ pyramidal patterns

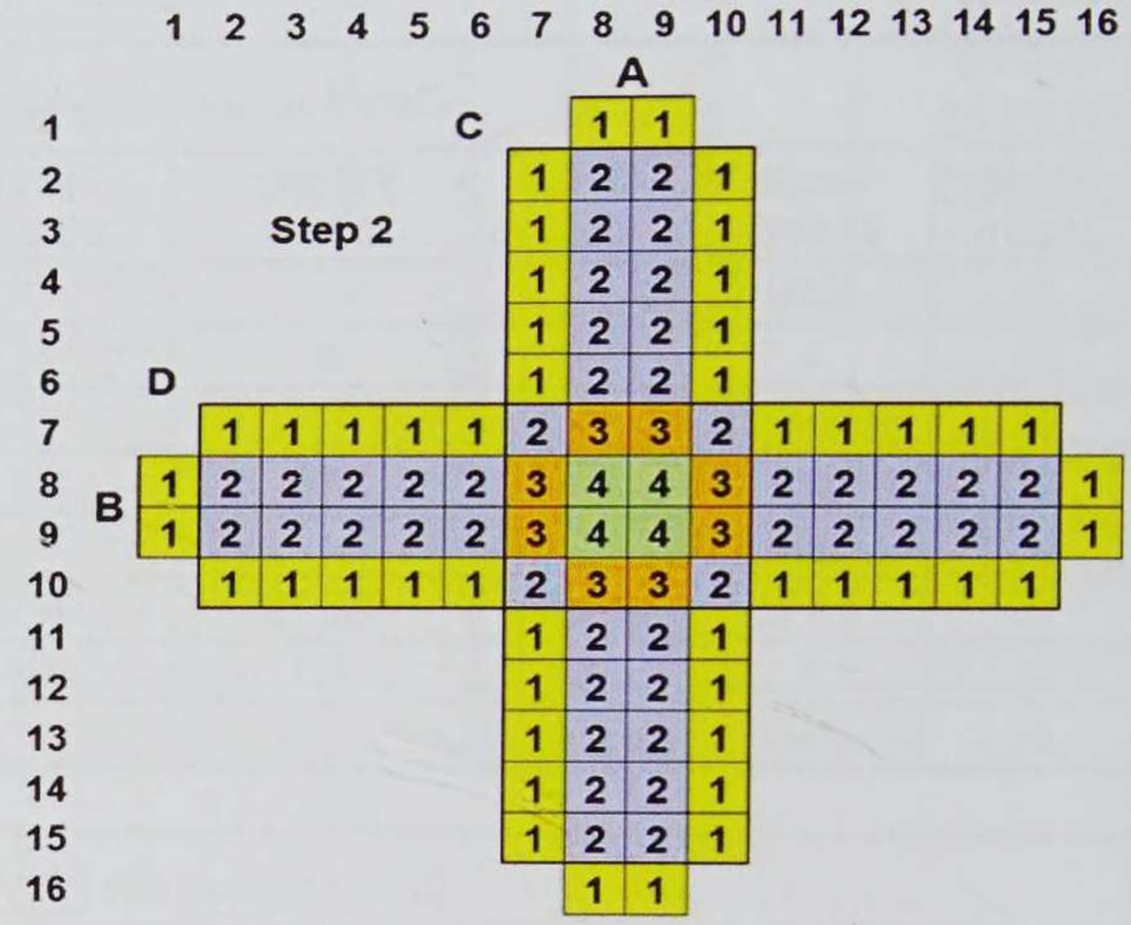
Size of square	Height of stack	Size of base	# of cells in base	Cells in pyramid	Value of Sums	Characteristic number	Pyramidal sums as multiples of Cn
16	3, 4 & 7	6, 8 & 14	36, 64 & 196	52, 100 & 340	7196, 15420 & 71960	2056	3½, 4 & 27½
20	5	10	100	220	44110	4010	10
24	6	12	144	364	105014	6924	15 1/6
28	4 & 7	8 & 14	64 & 196	84 & 560	32970 & 219800	10990	17 & 20
32	3 & 4	6 & 8	36 & 64	120 & 144	61500 & 28700	16400	1¾ & 3¾
36	4	8	64	120	54474 & 77820	23346	2⅓ & 3⅓
	8 & 9	16 & 18	256 & 324	816 & 1140	529174 & 739290		22⅔ & 31⅔
44	5	10	100	220	213070	42614	5
	10 & 11	20 & 22	400 & 484	1540 & 2024	1491490 & 1960244		35 & 46
64	3 & 4	6 & 8	36 & 64	56 & 120	114716 & 245820	131104	7/8 & 1 7/8
	7 & 8	14 & 16	196 & 256	560 & 816	1147160 & 1671576		8¾ & 12¾
	11 & 12	22 & 24	484 & 576	2024 & 2600	4146164 & 5326100		31 5/8 & 40 5/8

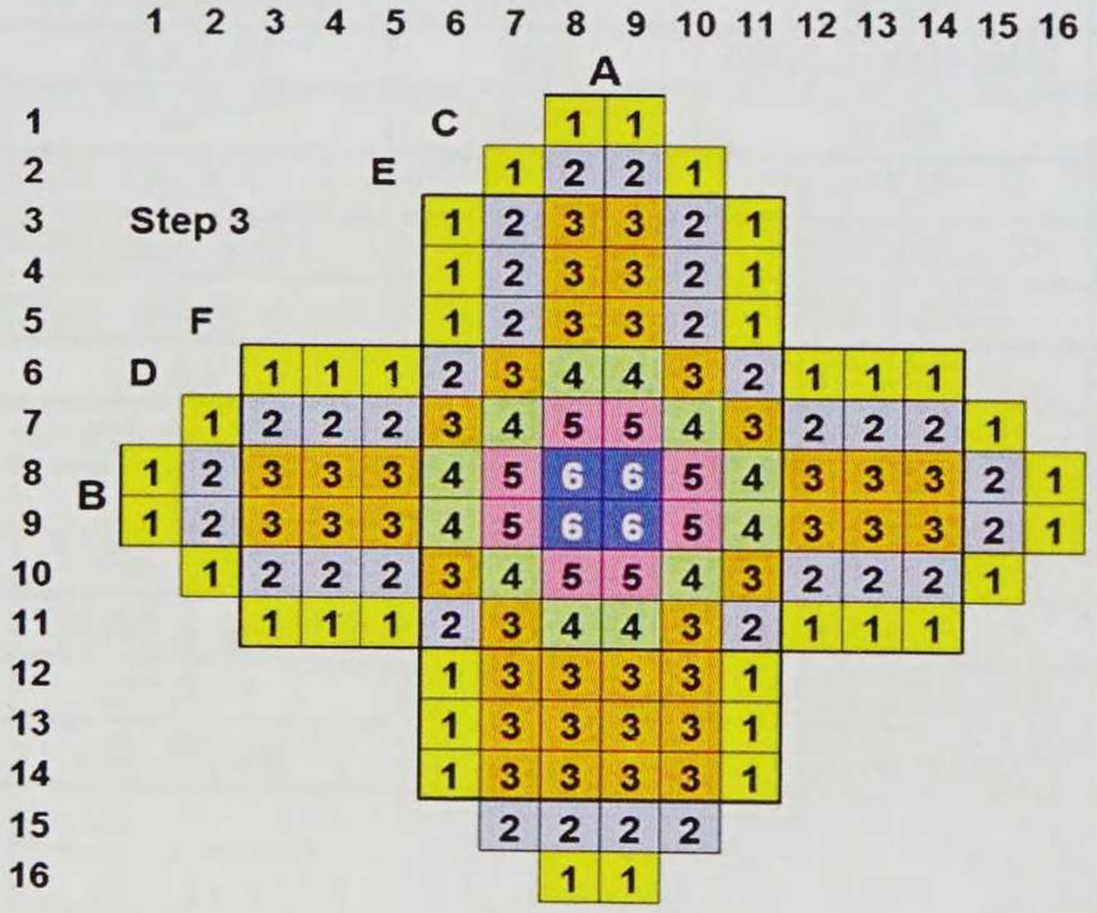
Program 4

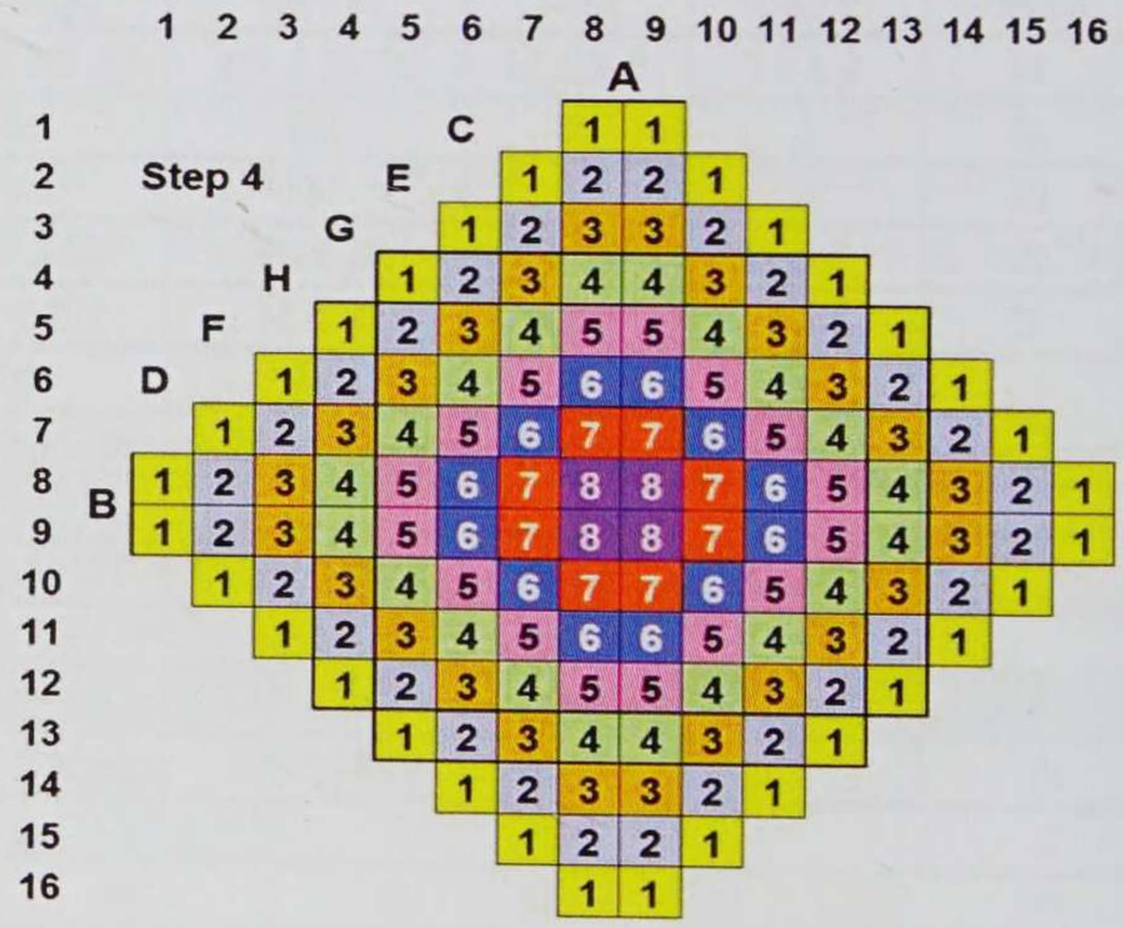
A simple method for calculating pyramidal diamond summations in Excel

Here in just 4 steps, applying two summations per step for a total of **8** summations, a level-8 even size pyramidal sum is readily obtained.









Program 4

Square Pyramidal Patterns in the Class-5 Size-35 Square

Here is the size-35 Class-5 square with **level-3 pyramidal square patterns**. These patterns sum continuously to the size-35 square's characteristic number **21455**.

However, the square does not possess any larger pyramidal square pattern nor any pyramidal diamond pattern of even or odd size that sums equally.

Program 4

Integrated Square Pyramidal Patterns in the Class-5 Square

1	1	2	3	4	5	5	5	4	3	2	1
2	1	2	3	4	5	5	5	4	3	2	1
3	1	2	3	4	5	5	5	4	3	2	1
4	1	2	3	4	5	5	5	4	3	2	1
5	1	2	3	4	5	5	5	4	3	2	1
6	1	2	3	4	5	5	5	4	3	2	1
7	1	2	3	4	5	5	5	4	3	2	1
	1	2	3	4	5	6	7	8	9	10	11

1	1	1	1	1	1	1	1				
2	1	2	2	2	2	2	2	1			
3	1	2	3	3	3	3	3	2	1		
4	1	2	3	4	4	4	4	3	2	1	
5	1	2	3	4	5	5	5	4	3	2	1
6	1	2	3	4	5	5	5	4	3	2	1
7	1	2	3	4	5	5	5	4	3	2	1
	1	2	3	4	5	6	7	8	9	10	11

				1	1	1	1	1	1	1	
			1	2	2	2	2	2	2	1	
		1	2	3	3	3	3	3	2	1	
	1	2	3	4	4	4	4	3	2	1	
1	1	2	3	4	5	5	5	4	3	2	1
2	1	2	3	4	5	5	5	4	3	2	1
3	1	2	3	4	5	5	5	4	3	2	1
4	1	2	3	4	4	4	4	3	2	1	
5	1	2	3	3	3	3	3	2	1		
6	1	2	2	2	2	2	2	1			
7	1	1	1	1	1	1	1				
	1	2	3	4	5	6	7	8	9	10	11

1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	5	5	5	5	5	5	5
7	5	5	5	5	5	5	5
8	4	4	4	4	4	4	4
9	3	3	3	3	3	3	3
10	2	2	2	2	2	2	2
11	1	1	1	1	1	1	1

The numbers indicate the number of times in which a particular cell is included in the overall integrated sum.

Pyramidal patterns of square and diamond configurations were found to exist extensively in Class-4 squares. However, the size-35 Class-5 square was found to have square pyramidal summations, one independently continuous and one continuously overlapping configuration in the four linear geonomic summation directions as shown here at left.

Here is what this means:

Any vertical, horizontal or diagonal sum of **5** adjacent level-4 square pyramids with integrated overlapping, as shown here, will sum continuously to **12** (5+7) times the **size 35** square's characteristic number. That's another amazing 3-dimensional configuration pattern on a Class-5 square!

This is a much more advanced summation than that seen earlier for the size-27 Class-6 square.

What has occurred here in Part III is that we have discovered meaningful 3-dimensional patterns in the tables of 2-dimensional summations.

Here is the generalized formula for the integrated pyramida sums in all Class-5 squares:

(4.1) $C^{\#}_{b} = (a+b)C_n$

where **n = ab, a > b > 3**, **a** & **b** are prime numbers.

Program 4

Overall Summary of all Continuously Equal-summing Rectangular, Diamond and Triangular Patterns up thru Size-44							
Size	Class	Modularity	Ring-Diamonds	Solid Diamonds	Pyramidal Squares	Pyramidal Diamonds	Triangles
4	4	2x2	2x2	2x2	2x2	2x2	3x3
5	1						
6	2		3x3				
7	1						
8	4	4x4	4x4		6x6 pw row sym only	3x3	
9	6	3x3					
10	2		Double-3x = 5x◊		5x5		
11	1						
12	4	6x6	6x6		6x6		
13	1						
14	2	2x7*	Double-4x =7x◊	7x7	7x7		
15	3	5x5		5x5			
16	4	4x4	4x4		8x8 pw cent sym only	3x3	
17	1						
18	2	3x6 & 9x2	Double-5x = 9x◊		9x9		
19	1						
20	4	4x5 & 5x4	5x5	4x4 & 5x5	10x10	4x4 & 5x5	4x4
21	3			6x6			
22	2		Double-6x = 11x◊		11x11		
23	1						
24	4	6x6	6x6	5x5 & 6x6 11x11 & 12x12	12x12	5x5 & 6x6	
25	6	5x5					
26	2	2x13* & 13x2*	Double-7x = 13x◊		13x13		
27	6	3x3					
28	4	4x7 & 7x4 2x14 & 14x2 14x14	7x7	6x6 & 7x7 13x13 & 14x14	7x7 & 14x14	5x5 & 6x6 & 7x7	7x7
29	1						
30	2	15x2*	Double-8x =15x◊		15x15		
31	1						
32	4	4x4	8x8	7x7 & 8x8	6x6 & 8x8 16x16	6x6 & 7x7 & 8x8	
33	3	11x11					
34	2		Double-9x = 17x◊		17x17		
35	5				5x5		
36	4	6x6 4x9 & 9x4	9x9	8x8 & 9x9	7x7 & 8x8 16x16 &18x18	7x7 & 8x8 & 9x9	8x8
37	1						
38	2		Double-10x = 19x◊		19x19		
39	3	13x13					
40	4	4x10 & 10x4 10x10	5x5	4x4 & 5x5 9x9 & 10x10	4x4 & 8x8 10x10 & 20x20	8x8 & 9x9 & 10x10	
41	1						
42	2		Double-11x = 21x◊		21x21		
43	1						
44	4	4x11 & 11x4 11x11	11x11	10x10 & 11x11	5x5 & 10x10 11x11 & 22x22	9x9 &10x10 & 11x11	11x11

* Everywhere except when the pattern's center encroaches upon the square's horizontal or vertical double center line

Expressions in blue are only included for completion of the numerical pattern but do not actually hold for the size square

Program 4

Class-1 Squares are the source of Class-4 Patterns

Since every Class-4 square was derived from a Class-1 or Class-2 square and further every Class-2 square was derived from a Class-1 square, these diamond quilt patterns and their pyramidal property for Class-4 squares follows directly from the corner triangle property of Class-1 squares as shown here. Equal-summing diamonds of size **6** and **7** were found in the size-28 square.

Here is a pallet of **16** copies of the size-7 square. The first thing to note is that these two diamond sizes (**6** & **7**) of equal-summing corner triangles will integrate to form an equal-summing diamond quilt pattern summing continuously to **12** and **16** times the characteristic number of the size-7 square, respectively.

	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12

Program 4

Secondly, the triangles arranged as size **6** and **7** pyramidal diamonds are equal-summing continuously across the pallet to **32** and **48** times the size-7 square's characteristic number, respectively.

	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	
	W(7)	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	700
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	700
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	700
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	700
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	700
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	700
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	700
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	700
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	700
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	700
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	700
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	700
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	700
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	700
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	700
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	700
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	700
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	700
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	700
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	700
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	700
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	700
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	700
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	700
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	700
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	700
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	700
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	700
	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	
		700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700	700
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	

Thirdly, there was also a continuously equal-summing level-5 pyramidal diamond that was determined to lie within the size-28 square too. Shown below is this level-5 pyramidal diamond which sums to **20** times the size-7's characteristic number continuously across the same pallet of sixteen **7x7** block-squares.

	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12

So now it is clear that the sources of these diamond patterns are the Class-1 squares themselves.

Program 4

Here is the same pallet again with the size-7 square spread across it **16** times. Note that the pyramidal diamond patterns of level-7 share their outermost ring-diamond. They are equal summing because each side of the ring is one of a size-7's main diagonals.

	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12
1	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7	38	27	9	47	29	18	7
2	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44	33	15	4	42	24	13	44
3	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39	28	10	48	30	19	1	39
4	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34	16	5	36	25	14	45	34
5	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17	6	37	26	8	46	35	17
7	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12	43	32	21	3	41	23	12

So now it is seen that all those patterns found in the size-28 Class-4 square derive from the Class-1 size-7 square.

Program 4

Part III Notes

1. Class-3 squares have four linear complementary tile patterns that are 3-dimensional **3x (b+2)** horizontal and vertical staircase structures over **3** square laminations. The summations equal the square's characteristic number exactly. This applies when **b** is a prime number.

2. The size-27 square possesses two sets of complementary tiling patterns which sum to the square's characteristic number exactly. One set is a 3-level up-over-and down **3x5** or **5x3** block-square staircase. The other set is composed of level-3 pyramidal squares that are shifted diagonally up or down.

3. Class-4 squares have pyramidal diamonds of levels **b**, **b-1** & **b-2** and one of level **(b-1)/2** for **b ≥ 7.**

4. At right is a summary of pyramidal squares discovered for consecutive Class-4 squares.

5. Class-1 squares were seen to be the source of Class-4 tile patterns.

6. Class-5 square was seen to possess a level-3 odd-size pyramidal square.

Pyramidal Squares		
Size square	Pyramidal level	Other sizes
20	5	none
24	6	none
28	7	4 odd
32	8	4 even
36	9	8 even
40	10	4 & 8
44	11	5 & 10

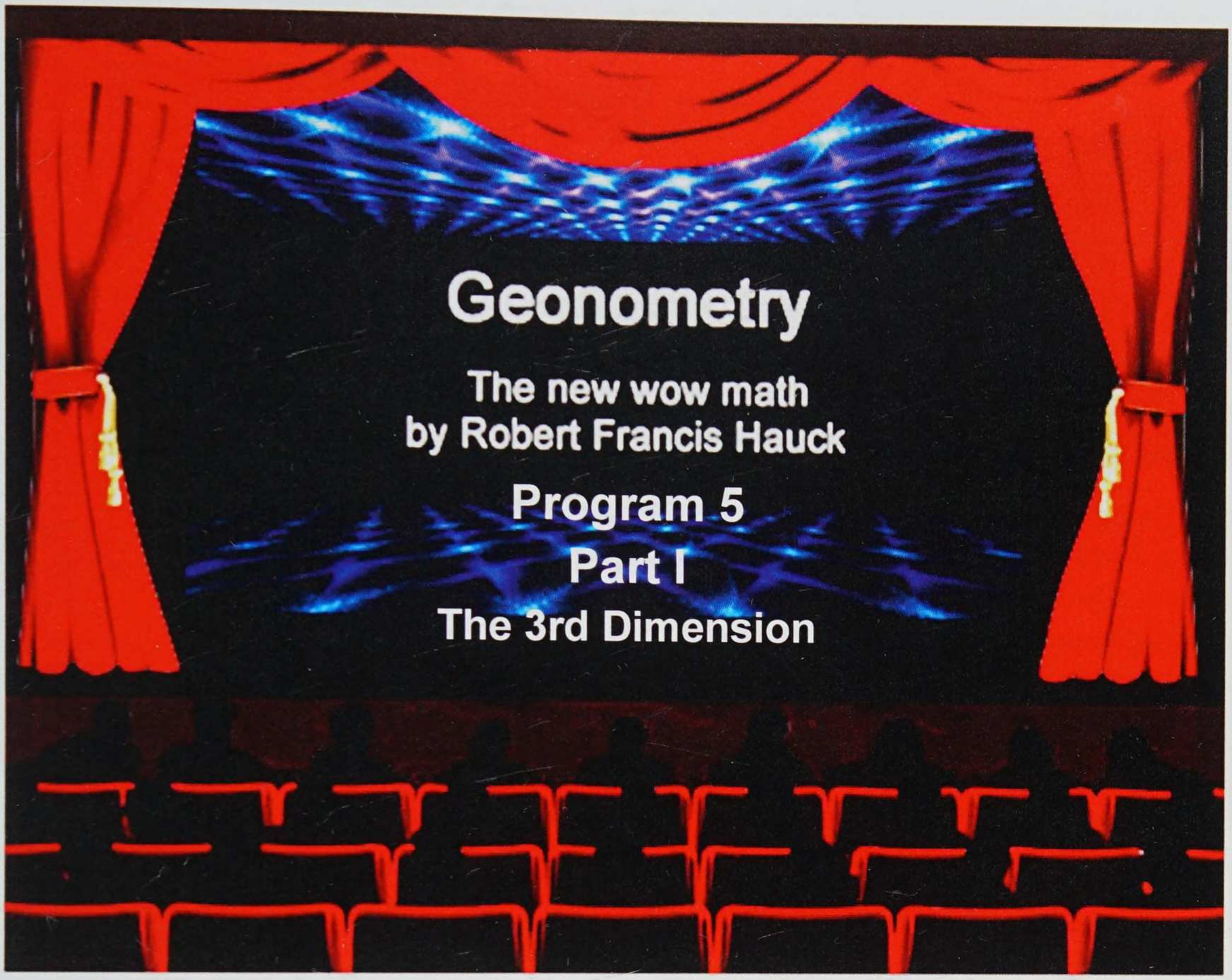
7. Geonomic inequalities of the level-4 odd-size pyramidal square lead to the recovery of the size-5 square used in its expansion to size-35 by the **ATE** method of Program 7.

8. There are four integrated level-5 pyramidal **7x7** block-squares which sum to **12C35** (page 57 per formula (4.1). This formula should generalize to all Class-5 squares. The next size up is **5x11 = 55**.

* * *

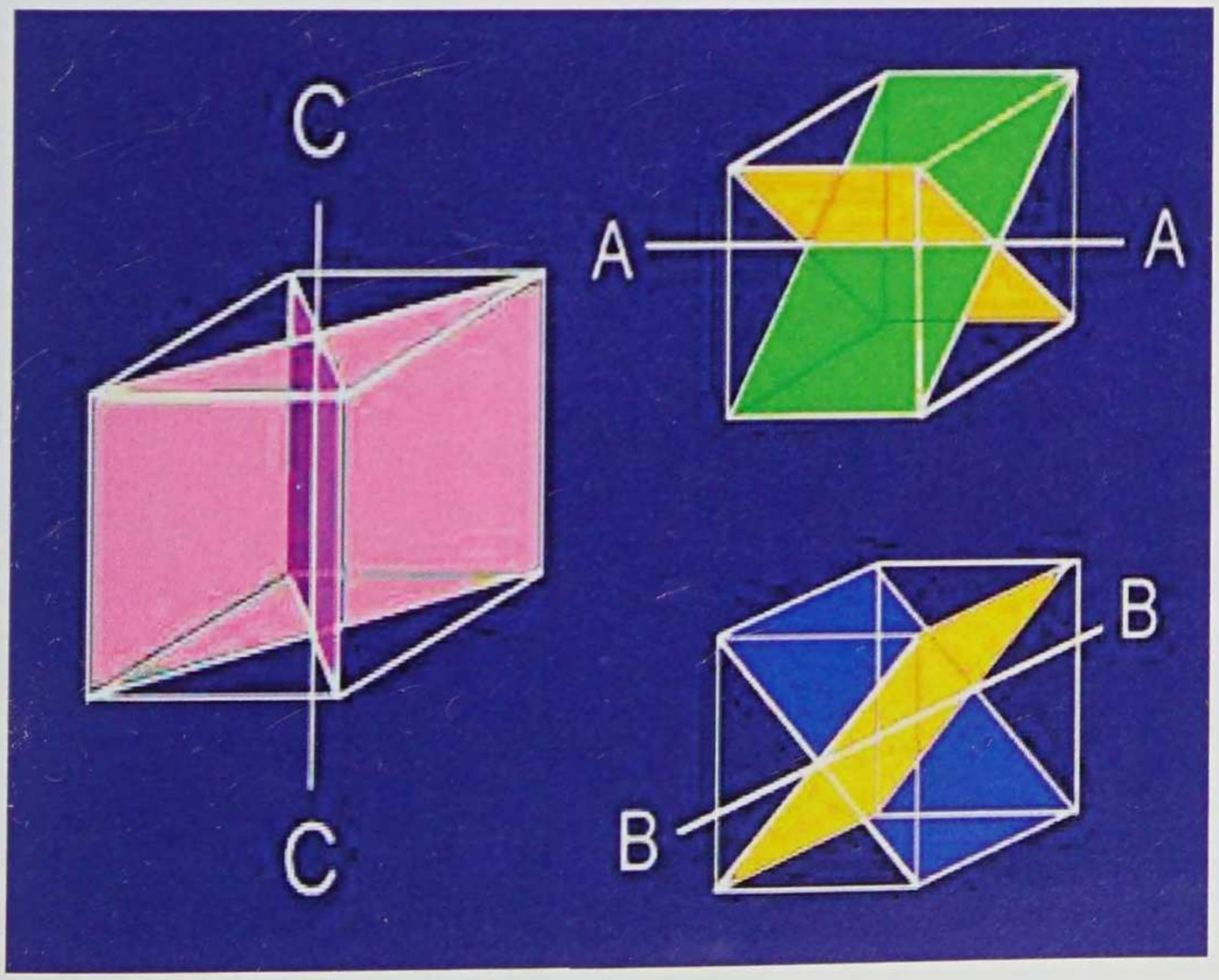
In the next program we will explore the properties of the 3rd dimension and the amazing equal-summing 1-dimensional and 2-dimensional patterns discovered in geonomic cubes. We will accomplish this through the compact view of yet another type of table, called the **depth-sum table**. There we will begin the incredible mathematical journey up through higher dimensions and down to lower sub-dimensions as only Geonometry can provide.

Program 4



Next we will address cubic tables which are shown to actually tap into, measure and map 3-dimensional space.

Introduction



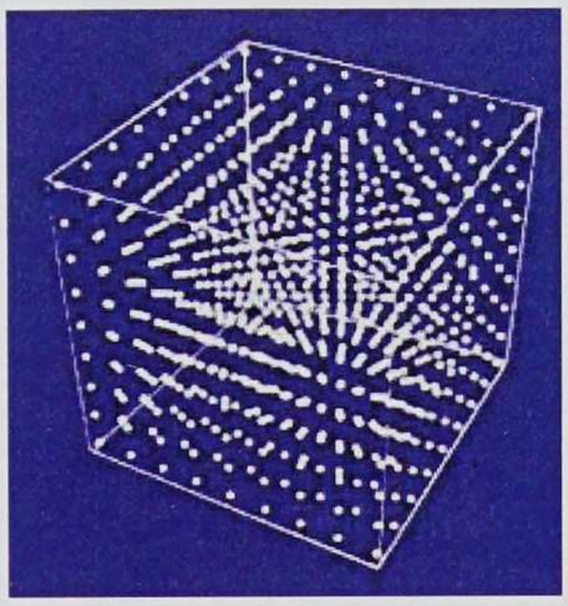
Here are the **3** directions that will be referred to by letters **A B** & **C** in identifying the depth-sum tables taken along their designated axis.

All cubes are described by rectangular tables of embedded squares along the B-axis; akin to the cards in a deck of cards.

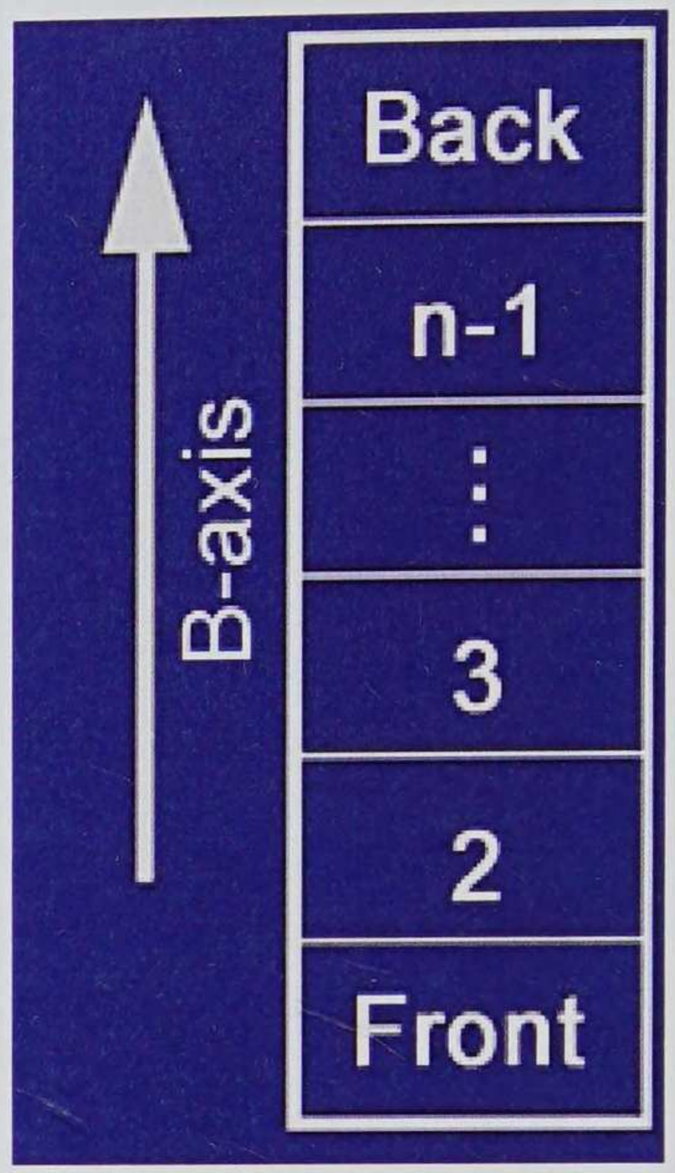
A cube of size **n** has **n** block-squares of size **nxn** embedded within it. Bottom-to-top in the table correlates with going from front-to-back in the cube along the B-axis.

Program 5

Cubic Definitions



As shown at right, the bottom embedded block-square lies at the front of the cube along the B-axis and the top block-square lies at the back.



Here are some basic definitions for cubes:

The **octal** is the 3-dimensional counter-part of the quadral in squares. These are the sum of the numbers at the **8** corners of a symmetrically-centered box within the cube. In a cubic table, they are the numbers in identical quadrals of two equidistant centrally-symmetric embedded block-squares as shown below at right.

A cube has the status of being **perfect** if:

1. All the linear channels along the A-axis and pillars along the C-axis each sum equally.
2. All the main planar diagonals sum equally.
3. All octals sum equally: that is, the cube is 3-dimensionally pangenic.

A perfect cube is **absolutely-perfect** when, in addition:

1. All the diagonal planes sum equally.
2. The depth-sum table along the **B** axis collapses to a perfect square.

A cube is also **ultra-perfect** when it contains equal-summing complementary tile patterns in each of its embedded squares.

Note: the tile patterns are confined to squares and do not cross the borders of embedded squares but do wrap around the individual squares themselves, both horizontally and vertically.

All cubes that are associated with Class-1 squares are ultra-perfect. On the otherhand, cubes whose corresponding squares are from Class-4 which have characteristic equal-summing diamond and X-tile patterns have not been found to be also ultra-perfect in that these equal-summing patterns were lost in the construction method for cubes.

Program 5
Class-1 Cubes

315	315	315	315	315		
315	315	315	315	315		
315	315	315	315	315		
315	315	315	315	315		
315	315	315	315	315		

Block-square
minor diagonal sums

Size-5 Cube

		1	2	3	4	5		
		1575	1575	1575	1575	1575		
	1	95	33	121	64	2	315	
	2	111	54	17	85	48	315	
5	3	7	100	38	101	69	315	1575
	4	28	116	59	22	90	315	
	5	74	12	80	43	106	315	
	1	45	108	71	14	77	315	
	2	61	4	92	35	123	315	
4	3	82	50	113	51	19	315	1575
	4	103	66	9	97	40	315	
	5	24	87	30	118	56	315	
	1	120	58	21	89	27	315	
	2	11	79	42	110	73	315	
3	3	32	125	63	1	94	315	1575
	4	53	16	84	47	115	315	
	5	99	37	105	68	6	315	
	1	70	8	96	39	102	315	
	2	86	29	117	60	23	315	
2	3	107	75	13	76	44	315	1575
	4	3	91	34	122	65	315	
	5	49	112	55	18	81	315	
	1	20	83	46	114	52	315	
	2	36	104	67	10	88	315	
1	3	57	25	88	26	119	315	1575
	4	78	41	109	72	15	315	
	5	124	62	5	93	31	315	
		1575	1575	1575	1575	1575		
		1575	1575	1575	1575	1575		

Quadrals

277	152	277
277	277	277

Octals

504	504	504
504	504	504

Quadrals

202	327	202
327	202	202

Octals

504	504	504
504	504	504

Quadrals

252	252	252
252	252	252

Octals

504	504	504
504	504	
504	504	

Quadrals

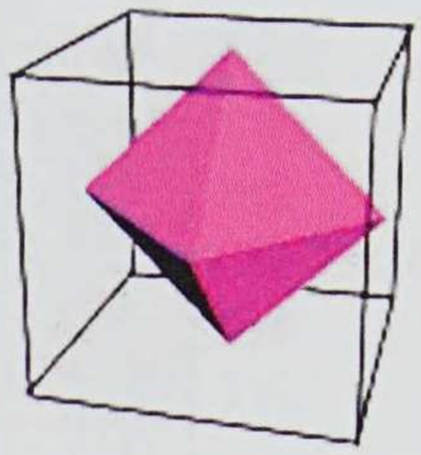
302	177	302
177	302	302

Octahedrons

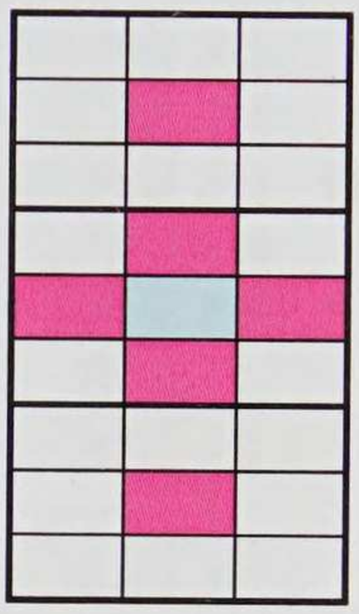
378
378

Quadrals

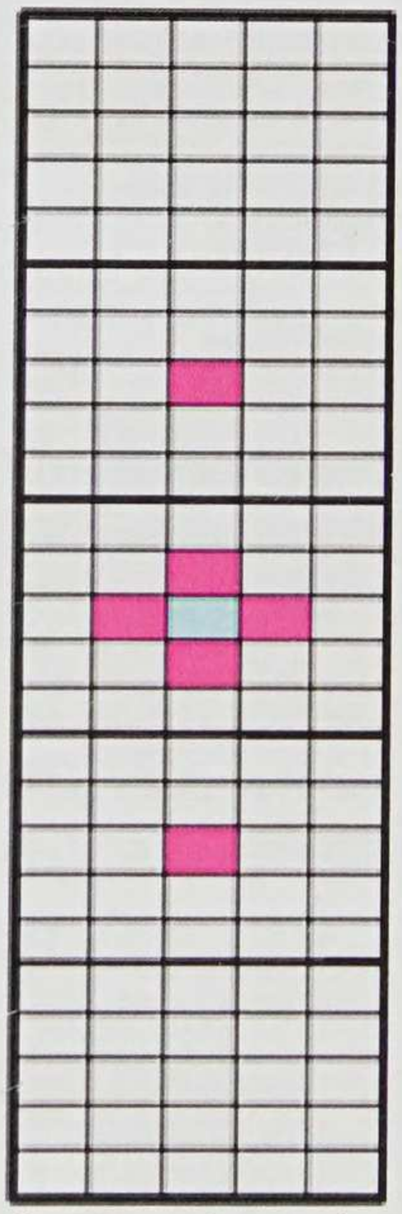
227	352	227
227	227	227



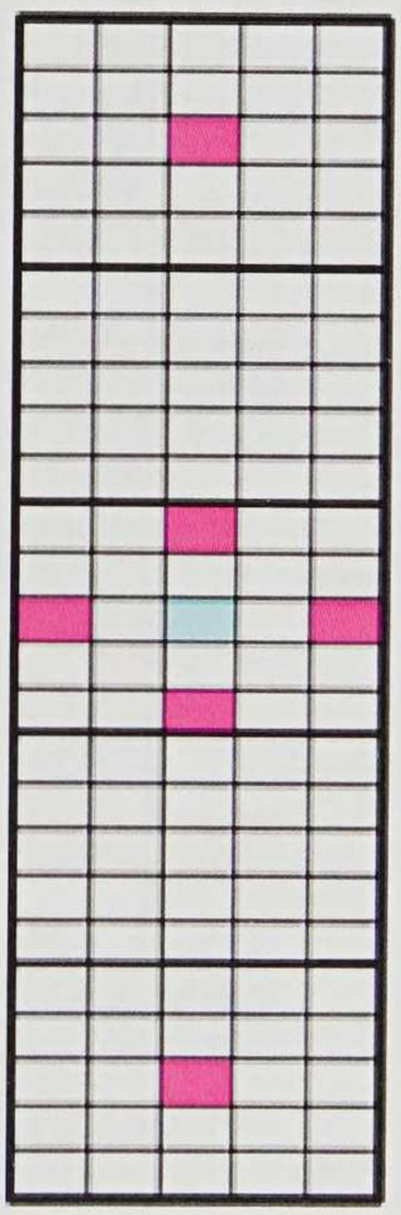
Octahedron in 3D



Octahedron #1



Octahedron #2



315	315	315	315	315		
315	315	315	315	315		
315	315	315	315	315		
315	315	315	315	315		
315	315	315	315	315		

Block-square
major diagonal sums

The crimson colored cells in the quadral and octal sum boxes at center are those which are associated with the central embedded square.

The top row in the last octal summation box is the octals formed from two symmetrically positioned quadrals in the central square. Those octal sums were obtained from adding together the top and bottom rows of the central quadral summation box.

The other octal sums were obtained from adding together the numbers in the same location from two symmetrically located quadral sum boxes. Throughout this program, this is the typical representation.

Every row, column and diagonal in both minor and major directions, including wrapping in every block-square, sums to **315**. This qualifies the cube to be also *absolutely* -perfect.

Program 5

5x3D Depth-sum Tables

Depth-sums along A-axis

A	1575	1575	1575	1575	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
1575	1575	1575	1575	1575	
1575	1575	1575	1575	1575	

Depth-sum quadrals

1260	1260	1260
1260	1260	1260

Depth-sums along B-axis

B	1575	1575	1575	1575	1575
350	290	355	320	260	1575
305	270	335	300	365	1575
285	375	315	255	345	1575
265	330	295	360	325	1575
370	310	275	340	280	1575
1575	1575	1575	1575	1575	
1575	1575	1575	1575	1575	

Depth-sum quadrals

1260	1260	1260
1260	1260	1260

Depth-sums along C-axis

C	1575	1575	1575	1575	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
1575	1575	1575	1575	1575	
1575	1575	1575	1575	1575	

Depth-sum quadrals

1260	1260	1260
1260	1260	1260

The depth-sum table **B** collapses to an ultra-perfect size-5 square by first dividing every number in the depth-sum table by **5** and then subtracting **50** from each number in the resulting table. That number is **1** less than the minimum number in the table resulting from the first operation. But make note that that number is always the difference between the pivot numbers of the same size square and cube.

Reduction of Depth-sum Table B to a perfect size-5 square

Depth-sums B divided by 5

B/5	1575	1575	1575	1575	1575
70	58	71	64	52	315
61	54	67	60	73	315
57	75	63	51	69	315
53	66	59	72	65	315
74	62	55	68	56	315
315	315	315	315	315	
315	315	315	315	315	

B/5 - 50

	1	2	3	4	5	
5x5	65	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
	65	65	65	65	65	

All of the pillars along the C-axis and all the channels along the A-axis sum equally as seen in the duplication of numbers in the cells of the depth-sum tables corresponding to those axes.

Program 5

5x3D Cubic Tiling Patterns

At right are the five size-5 embedded squares with one of two continuously complementary tiling patterns high-lighted. The other complementary tile pattern consisting of an X-pattern does the same. Both sum to **315** in every tile. The tiling can be relocated anywhere across each embedded square and still sum to **315**. This qualifies the cube to be *ultra*-perfect.

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

Every row, column and diagonal in both minor and major directions, including wrapping in every block-square, sums to **315**. This qualifies the cube to be also *absolutely-perfect*.

Only the central block-square #3 is pangenic as a square, with quadrals equal to **252**.

Note: The tile patterns do not extend across their square's borders to adjacent squares; keep in mind that we're looking at three dimensions here, 2-dimensional slices at a time.

	Level	1	2	3	4	5		
		315	315	315	315	315	315	
Square 5	1	95	33	121	64	2	315	1575
	2	111	54	17	85	48	315	
	3	7	100	38	101	69	315	
	4	28	116	59	22	90	315	
	5	74	12	80	43	106	315	
		315	315	315	315	315		
		315	315	315	315	315	315	

	Level	1	2	3	4	5		
		315	315	315	315	315	315	
Square 4	1	45	108	71	14	77	315	1575
	2	61	4	92	35	123	315	
	3	82	50	113	51	19	315	
	4	103	66	9	97	40	315	
	5	24	87	30	118	56	315	
		315	315	315	315	315		
		315	315	315	315	315	315	

	Level	1	2	3	4	5		
		315	315	315	315	315	315	
Square 3	1	120	58	21	89	27	315	1575
	2	11	79	42	110	73	315	
	3	32	125	63	1	94	315	
	4	53	16	84	47	115	315	
	5	99	37	105	68	6	315	
		315	315	315	315	315		
		315	315	315	315	315	315	

	Level	1	2	3	4	5		
		315	315	315	315	315	315	
Square 2	1	70	8	96	39	102	315	1575
	2	86	29	117	60	23	315	
	3	107	75	13	76	44	315	
	4	3	91	34	122	65	315	
	5	49	112	55	18	81	315	
		315	315	315	315	315		
		315	315	315	315	315	315	

	Level	1	2	3	4	5		
		315	315	315	315	315	315	
Square 1	1	20	83	46	114	52	315	1575
	2	36	104	67	10	98	315	
	3	57	25	88	26	119	315	
	4	78	41	109	72	15	315	
	5	124	62	5	93	31	315	
		315	315	315	315	315		
		315	315	315	315	315	315	

Program 5

Class-2 Cube

Size-6

Here is the Class-2 size-6 cube. It is geonomicaliy perfect but not absolutely-perfect because not all of its planar diagonals sum equally parallel to its B-axis. But all of its octals sum equally.

Square		1	2	3	4	5	6		
			3888	3906	3906	3924	3906	3906	
	1	204	97	42	175	120	13	651	
	2	89	68	149	143	2	200	651	
6	3	58	153	112	33	208	87	651	3906
	4	160	135	34	183	82	57	651	
	5	122	11	206	80	65	167	651	
	6	18	187	108	37	174	127	651	
	1	168	133	6	211	84	49	651	
	2	125	32	185	107	38	164	651	
5	3	22	189	76	69	172	123	651	3906
	4	196	99	70	147	118	21	651	
	5	86	47	170	116	29	203	651	
	6	54	151	144	1	210	91	651	
	1	60	169	114	31	192	85	651	
	2	161	140	5	215	74	56	651	
4	3	130	9	184	105	64	159	651	3906
	4	16	207	106	39	154	129	651	
	5	194	83	62	152	137	23	651	
	6	90	43	180	109	30	199	651	
	1	24	205	78	67	156	121	651	
	2	197	104	41	179	110	20	651	
	3	94	45	148	141	28	195	651	3906
3	4	52	171	142	3	190	93	651	
	5	158	119	26	188	101	59	651	
	6	126	7	216	73	66	163	651	
	1	132	25	186	103	48	157	651	
	2	17	212	77	71	146	128	651	
2	3	202	81	40	177	136	15	651	3906
	4	88	63	178	111	10	201	651	
	5	50	155	134	8	209	95	651	
	6	162	115	36	181	102	55	651	
	1	96	61	150	139	12	193	651	
	2	53	176	113	35	182	92	651	
1	3	166	117	4	213	100	51	651	3906
	4	124	27	214	75	46	165	651	
	5	14	191	98	44	173	131	651	
	6	198	79	72	145	138	19	651	
		3906	3906	3906	3906	3906	3906		
			3924	3906	3906	3888	3906	3906	

Quadrals

6

362	578	362
578	146	578
362	578	362

5

362	578	362
578	146	578
362	578	362

4

434	434	434
434	434	434
434	434	434

3

434	434	434
434	434	434
434	434	434

2

506	290	506
290	722	290
506	290	506

1

506	290	506
290	722	290
506	290	506

Octals

6 + 1

868	868	868
868	868	868
868	868	868

5 + 2

868	868	868
868	868	868
868	868	868

4 + 3

868	868	868
868	868	868
868	868	868

Depth-sums along A-axis

A						
3906	3906	3906	3906	3906	3906	
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
3906	3906	3906	3906	3906	3906	
3906	3906	3906	3906	3906	3906	

Depth-sum Quadrals

2604	2604	2604
2604	2604	2604
2604	2604	2604

Depth-sums along B-axis

B						
3888	3906	3906	3924	3906	3906	
684	690	576	726	612	618	3906
642	732	570	750	552	660	3906
672	594	564	738	708	630	3906
636	702	744	558	600	666	3906
624	606	696	588	714	678	3906
648	582	756	546	720	654	3906
3906	3906	3906	3906	3906	3906	
3924	3906	3906	3888	3906	3906	

Depth-sum Quadrals

2604	2604	2604
2604	2604	2604
2604	2604	2604

Depth-sums along C-axis

C						
3906	3906	3906	3906	3906	3906	
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
3906	3906	3906	3906	3906	3906	
3906	3906	3906	3906	3906	3906	

Depth-sum Quadrals

2604	2604	2604
2604	2604	2604
2604	2604	2604

There are no octahedrons in even size cubes because their existence requires a distinct central row and column which only odd-size squares have.

It is evident from the depth-sum tables that all of the pillars along the C-axis and all the channels along the A-axis sum equally from the duplication of numbers in the cells of the depth-sum tables corresponding to axes A and C.

Program 5

Depth-sums along B-axis

B	3888	3906	3906	3924	3906	3906
684	690	576	726	612	618	3906
642	732	570	750	552	660	3906
672	594	564	738	708	630	3906
636	702	744	558	600	666	3906
624	606	696	588	714	678	3906
648	582	756	546	720	654	3906
3906	3906	3906	3906	3906	3906	
3924	3906	3906	3888	3906	3906	

B/6	648	651	651	654	651	651
114	115	96	121	102	103	651
107	122	95	125	92	110	651
112	99	94	123	118	105	651
106	117	124	93	100	111	651
104	101	116	98	119	113	651
108	97	126	91	120	109	651
651	651	651	651	651	651	
654	651	651	648	651	651	

B/6-90	108	111	111	114	111	111
24	25	6	31	12	13	111
17	32	5	35	2	20	111
22	9	4	33	28	15	111
16	27	34	3	10	21	111
14	11	26	8	29	23	111
18	7	36	1	30	19	111
111	111	111	111	111	111	
114	111	111	108	111	111	

6x6	108	111	111	114	111	111
24	25	6	31	12	13	111
17	32	5	35	2	20	111
22	9	4	33	28	15	111
16	27	34	3	10	21	111
14	11	26	8	29	23	111
18	7	36	1	30	19	111
111	111	111	111	111	111	
114	111	111	108	111	111	

Collapsing the size-6 cube to a size-6 punctuated-perfect square

Its depth sum table **B** will collapse to a punctuated-perfect size-6 square by:

1. Dividing **B** by **6**
2. Subtracting **90**
3. Wrapping the rows/columns to center the beginning and ending numbers.

The resulting version can then be converted to a near-perfect square by interchanging two numbers in one of its columns.

Note that the number **90** subtracted in step #2 is always **1** less than the minimum number in the table resulting from step #1. But it is also the difference between the characteristic numbers of the cube and square of size-6.

This, of course, generalizes to the differences between the characteristic numbers of squares and cubes of the same size **n** in this collapsation process.

Program 5
Class-3 Cube

Size-3

3x3x3

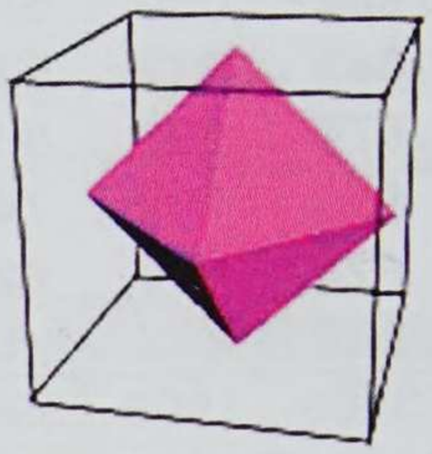
Squares		1	2	3		
		126	126	126		
	1	17	4	21	42	
3	2	22	12	8	42	126
	3	3	26	13	42	
	1	10	9	23	42	
2	2	27	14	1	42	126
	3	5	19	18	42	
	1	15	2	25	42	
1	2	20	16	6	42	126
	3	7	24	11	42	
		126	126	126		
		45	207	126		

Depth sums along A axis

A	126	126	126	
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
	126	126	126	

Depth-sum quadrals

168	168



Octahedron in 3D

Depth sums along B axis

B	126	126	126	
1	42	15	69	126
2	69	42	15	126
3	15	69	42	126
	126	126	126	
	45	207	126	

Depth-sum quadrals

168	168

Depth sums along C axis

C	126	126	126	
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
	126	126	126	

Depth-sum quadrals

168	168

Quadrals

54	60
56	56
58	52

Octals

112	112
112	

Octahedron

84

Here is a size-3 cube. It is perfect but is not absolutely perfect as seen by two unequal planar diagonals in its depth-sum table **C**. Notice how this would be missed if it were not for depth-sum tables.

Further, its depth-sum table **B** will not collapse to a size-3 square because it has a duplication of numbers. **The principle is -- that eventhough a geonomic cube is perfect, it will not collapse to a square if its associated size square is not perfect.** That's how true-to-form Geonometry actually is!

Its octals all sum equally to **112**. Its sole octahedron sums to **84 = ¾ x 112**.

Program 5

Class-4

Absolutely- Perfect Size-4 Cube

Here is the size-4 cube from Class-4. It is absolutely perfect in that the planar summations in all 9 directions sum equally to the cube's characteristic number. It is pangenic because all its octals sum equally; and surprisingly so do the quadrals in the embedded squares.

The duplication of numbers in **A** & **C** shows the equality among channels and pillars.

4x3D

Square							
		520	520	520	520		
	1	37	58	15	20	130	
4	2	16	19	38	57	130	520
	3	50	45	28	7	130	
	4	27	8	49	46	130	
	1	21	42	63	4	130	
3	2	64	3	22	41	130	520
	3	2	61	44	23	130	
	4	43	24	1	62	130	
	1	5	26	47	52	130	
2	2	48	51	6	25	130	520
	3	18	13	60	39	130	
	4	59	40	17	14	130	
	1	53	10	31	36	130	
1	2	32	35	54	9	130	520
	3	34	29	12	55	130	
	4	11	56	33	30	130	
		520	520	520	520		
			520	520	520	520	

Depth-sum along A-axis

A	520	520	520	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
520	520	520	520	
	520	520	520	520

Depth-sum along B-axis

B	520	520	520	520
116	136	156	112	520
160	108	120	132	520
104	148	144	124	520
140	128	100	152	520
520	520	520	520	
	520	520	520	520

Depth-sum along C-axis

C	520	520	520	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
520	520	520	520	
	520	520	520	520

Quadrals

130	130	130	130
130	130	130	130
130	130	130	130
130	130	130	130

Octals

260	260
260	260
260	260
260	260

Program 5

Segregated into squares

4	130	130	130	130
37	58	15	20	130
16	19	38	57	130
50	45	28	7	130
27	8	49	46	130
	130	130	130	130

Quadrals

130	130
130	130

3	130	130	130	130
21	42	63	4	130
64	3	22	41	130
2	61	44	23	130
43	24	1	62	130
	130	130	130	130

Quadrals

130	130
130	130

1	130	130	130	130
53	10	31	36	130
32	35	54	9	130
34	29	12	55	130
11	56	33	30	130
	130	130	130	130

Quadrals

130	130
130	130

2	130	130	130	130
5	26	47	52	130
48	51	6	25	130
18	13	60	39	130
59	40	17	14	130
	130	130	130	130

Quadrals

130	130
130	130

Here are the four embedded squares. They are all independently pangenic. They sum equally in both their rows and columns.

On the right, the series of operations on the depth-sum table B collapses it to a perfect square.

Depth-sums along B-axis

B	520	520	520	520
116	136	156	112	520
160	108	120	132	520
104	148	144	124	520
140	128	100	152	520
520	520	520	520	
	520	520	520	520

B/4

29	34	39	28
40	27	30	33
26	37	36	31
35	32	25	38

B/4-24

5	10	15	4
16	3	6	9
2	13	12	7
11	8	1	14

4x4	34	34	34	34
5	10	15	4	34
16	3	6	9	34
2	13	12	7	34
11	8	1	14	34
34	34	34	34	
	34	34	34	34

Program 5

Class- 6 Cube

Size 9

Here is the Class-3 cube of size-9. It is **absolutely perfect** – all of its diagonal planes in **9** directions, **2** in parallel with each of its **3** axes, and **3** in alignment with each axis-pair all sum to the cube's characteristic number, as seen in all **3** of its depth-sum tables on the next page. As seen in the summation table at the bottom, all of the main and wrap major diagonals in each embedded square sum equally too.

Even though the numbers are unequal, each column in the diagonal summation table at top sums to the characteristic number too. Those column summations correspond to the diagonal planes in parallel with the B-axis.

Block-square #	Major (Wrap) Diagonal Sums per Block-square								
9	1098	1827	2556	3285	4014	4743	5472	6201	369
8	1827	2556	3285	4014	4743	5472	6201	369	1098
7	2556	3285	4014	4743	5472	6201	369	1098	1827
6	3285	4014	4743	5472	6201	369	1098	1827	2556
5	4014	4743	5472	6201	369	1098	1827	2556	3285
4	4743	5472	6201	369	1098	1827	2556	3285	4014
3	5472	6201	369	1098	1827	2556	3285	4014	4743
2	6201	369	1098	1827	2556	3285	4014	4743	5472
9x3D 1	369	1098	1827	2556	3285	4014	4743	5472	6201
	29565	29565	29565	29565	29565	29565	29565	29565	29565

Square												
	1	156	220	314	345	409	503	615	679	44	3285	
	2	185	252	334	455	522	604	725	63	145	3285	
	3	295	353	447	565	623	717	25	83	177	3285	
	4	386	471	562	575	660	22	116	201	292	3285	
9	5	406	500	594	676	41	135	217	311	405	3285	29565
	6	519	610	695	60	151	236	249	340	425	3285	
	7	634	728	57	94	188	246	364	458	516	3285	
	8	666	19	86	207	289	356	477	559	626	3285	
	9	38	132	196	308	402	466	497	591	655	3285	
	1	237	301	395	426	490	584	696	31	125	3285	
	2	266	333	415	536	603	685	77	144	226	3285	
	3	376	434	528	646	704	69	106	164	258	3285	
	4	467	552	643	656	12	103	197	282	373	3285	
8	5	487	581	675	28	122	216	298	392	486	3285	29565
	6	600	691	47	141	232	317	330	421	506	3285	
	7	715	80	138	175	269	327	445	539	597	3285	
	8	18	100	167	288	370	437	558	640	707	3285	
	9	119	213	277	389	483	547	578	672	7	3285	
	1	318	382	476	507	571	665	48	112	206	3285	
	2	347	414	496	617	684	37	158	225	307	3285	
	3	457	515	609	727	56	150	187	245	339	3285	
	4	548	633	724	8	93	184	278	363	454	3285	
7	5	568	662	27	109	203	297	379	473	567	3285	29565
	6	681	43	128	222	313	398	411	502	587	3285	
	7	67	161	219	256	350	408	526	620	678	3285	
	8	99	181	248	369	451	518	639	721	59	3285	
	9	200	294	358	470	564	628	659	24	88	3285	
	1	399	463	557	588	652	17	129	193	287	3285	
	2	428	495	577	698	36	118	239	306	388	3285	
	3	538	596	690	79	137	231	268	326	420	3285	
	4	629	714	76	89	174	265	359	444	535	3285	
6	5	649	14	108	190	284	378	460	554	648	3285	29565
	6	33	124	209	303	394	479	492	583	668	3285	
	7	148	242	300	337	431	489	607	701	30	3285	
	8	180	262	329	450	532	599	720	73	140	3285	
	9	281	375	439	551	645	709	11	105	169	3285	
	1	480	544	638	669	4	98	210	274	368	3285	
	2	509	576	658	50	117	199	320	387	469	3285	
	3	619	677	42	160	218	312	349	407	501	3285	
	4	710	66	157	170	255	346	440	525	616	3285	
5	5	1	95	189	271	365	459	541	635	729	3285	29565
	6	114	205	290	384	475	560	573	664	20	3285	
	7	229	323	381	418	512	570	688	53	111	3285	
	8	261	343	410	531	613	680	72	154	221	3285	
	9	362	456	520	632	726	61	92	186	250	3285	

	1	561	625	719	21	85	179	291	355	449	3285	
	2	590	657	10	131	198	280	401	468	550	3285	
	3	700	29	123	241	299	393	430	488	582	3285	
	4	62	147	238	251	336	427	521	606	697	3285	
4	5	82	176	270	352	446	540	622	716	81	3285	29565
	6	195	286	371	465	556	641	654	16	101	3285	
	7	310	404	462	499	593	651	40	134	192	3285	
	8	342	424	491	612	694	32	153	235	302	3285	
	9	443	537	601	713	78	142	173	267	331	3285	
	1	642	706	71	102	166	260	372	436	530	3285	
	2	671	9	91	212	279	361	482	549	631	3285	
	3	52	110	204	322	380	474	511	569	663	3285	
	4	143	228	319	332	417	508	602	687	49	3285	
3	5	163	257	351	433	527	621	703	68	162	3285	29565
	6	276	367	452	546	637	722	6	97	182	3285	
	7	391	485	543	580	674	3	121	215	273	3285	
	8	423	505	572	693	46	113	234	316	383	3285	
	9	524	618	682	65	159	223	254	348	412	3285	
	1	723	58	152	183	247	341	453	517	611	3285	
	2	23	90	172	293	360	442	563	630	712	3285	
	3	133	191	285	403	461	555	592	650	15	3285	
	4	224	309	400	413	498	589	683	39	130	3285	
2	5	244	338	432	514	608	702	55	149	243	3285	29565
	6	357	448	533	627	718	74	87	178	263	3285	
	7	472	566	624	661	26	84	202	296	354	3285	
	8	504	586	653	45	127	194	315	397	464	3285	
	9	605	699	34	146	240	304	335	429	493	3285	
	1	75	139	233	264	328	422	534	598	692	3285	
	2	104	171	253	374	441	523	644	711	64	3285	
	3	214	272	366	484	542	636	673	2	96	3285	
	4	305	390	481	494	579	670	35	120	211	3285	
1	5	325	419	513	595	689	54	136	230	324	3285	29565
	6	438	529	614	708	70	155	168	259	344	3285	
	7	553	647	705	13	107	165	283	377	435	3285	
	8	585	667	5	126	208	275	396	478	545	3285	
	9	686	51	115	227	321	385	416	510	574	3285	
		29565	29565	29565	29565	29565	29565	29565	29565	29565		

Block-square #	Minor (Wrap) Diagonal Sums per Block-square								
9	3285	3285	3285	3285	3285	3285	3285	3285	3285
8	3285	3285	3285	3285	3285	3285	3285	3285	3285
7	3285	3285	3285	3285	3285	3285	3285	3285	3285
6	3285	3285	3285	3285	3285	3285	3285	3285	3285
5	3285	3285	3285	3285	3285	3285	3285	3285	3285
4	3285	3285	3285	3285	3285	3285	3285	3285	3285
3	3285	3285	3285	3285	3285	3285	3285	3285	3285
2	3285	3285	3285	3285	3285	3285	3285	3285	3285
1	3285	3285	3285	3285	3285	3285	3285	3285	3285

Program 5

Size-9 Cube's Octal and Octahedral sums

8

488	1217	1946	1946	1946
1217	1217	1217	1946	1946
1946	1217	1217	1217	1946
1946	1946	1217	1217	488

9+1

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

7

812	812	1541	2270	2270
812	1541	1541	1541	2270
1541	1541	1541	1541	812
2270	1541	1541	812	812

8+2

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

6

1136	1136	1136	1865	2594
1136	1136	1865	1865	1136
1136	1865	1865	1136	1136
1865	1865	1136	1136	1136

7+3

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

5

1460	1460	1460	1460	1460
1460	1460	1460	1460	1460
1460	1460	1460	1460	1460
1460	1460	1460	1460	1460

6+4

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

Octahedrons

2190
2190
2190
2190

4 Quadrals

1784	1784	1784	1055	326
1784	1784	1055	1055	1784
1784	1055	1055	1784	1784
1055	1055	1784	1784	1784

3

2108	2108	1379	650	650
2108	1379	1379	1379	650
1379	1379	1379	1379	2108
650	1379	1379	2108	2108

2

2432	1703	974	974	974
1703	1703	1703	974	974
974	1703	1703	1703	974
974	974	1703	1703	2432

1

2027	1298	1298	1298	1298
1298	2027	1298	1298	1298
1298	1298	2027	1298	1298
1298	1298	1298	2027	1298

Here are the quadral sums per each embedded block-square and their combination into octal sums. They are all equal to

$$2920 = q = 8/9^2 \times Cn.$$

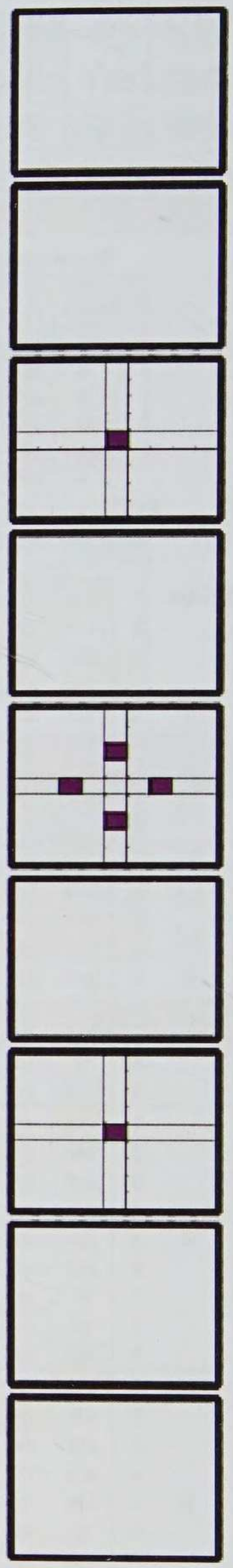
The octahedrals all sum equally to

$$2190 = \tfrac{3}{4}\, q$$

where **q** denotes the octal's cubic kernel number

Octahedral sums are the sum of one diamond quadral in the central block-square and the central numbers of two opposing block-squares that are as equidistant from the center of the cube as the chosen diamond is from the center of the central block square.

Schema at right



Program 5

Collapsing the depth-sum table B to a perfect square

Depth-sums along axis-A

A	29565	29565	29565	29565	29565	29565	29565	29565	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
29565	29565	29565	29565	29565	29565	29565	29565	29565	29565

Quadrals of Depth-sum Table A

13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140

Depth-sums along axis-B

B	29565	29565	29565	29565	29565	29565	29565	29565	29565
3591	3438	3555	3105	2952	3069	3348	3195	3312	29565
3123	2997	3006	3366	3240	3249	3609	3483	3492	29565
3384	3177	3294	3627	3420	3537	3141	2934	3051	29565
3474	3510	3600	2988	3024	3114	3231	3267	3357	29565
2925	3042	3159	3168	3285	3402	3411	3528	3645	29565
3213	3303	3339	3456	3546	3582	2970	3060	3096	29565
3519	3636	3429	3033	3150	2943	3276	3393	3186	29565
3078	3087	2961	3321	3330	3204	3564	3573	3447	29565
3258	3375	3222	3501	3618	3465	3015	3132	2979	29565
29565	29565	29565	29565	29565	29565	29565	29565	29565	
29565	29565	29565	29565	29565	29565	29565	29565	29565	29565

Quadrals of Depth-sum Table B

13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140

Depth-sums along axis-C

C	29565	29565	29565	29565	29565	29565	29565	29565	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	
29565	29565	29565	29565	29565	29565	29565	29565	29565	
29565	29565	29565	29565	29565	29565	29565	29565	29565	29565

Depth-sum tables **A** and **C** show that all the channels along the A-axis and all the pillars along the C-axis sum equally because their cells are all equal. The equal summing rows and columns in depth-sum table **B** also confirm this.

The differences in numbers in depth-sum table **B** show that the channels along the **B** axis are unequal; but this provides for its collapse to a perfect size-9 square as a result of these differences. Again, the reduction number is the difference between the averages of the size-9 square and cube

Depth-sums divided by 9

B/9	3285	3285	3285	3285	3285	3285	3285	3285	3285
399	382	395	345	328	341	372	355	368	3285
347	333	334	374	360	361	401	387	388	3285
376	353	366	403	380	393	349	326	339	3285
386	390	400	332	336	346	359	363	373	3285
325	338	351	352	365	378	379	392	405	3285
357	367	371	384	394	398	330	340	344	3285
391	404	381	337	350	327	364	377	354	3285
342	343	329	369	370	356	396	397	383	3285
362	375	358	389	402	385	335	348	331	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

B/9 reduced by 324

9x9	369	369	369	369	369	369	369	369	369
75	58	71	21	4	17	48	31	44	369
23	9	10	50	36	37	77	63	64	369
52	29	42	79	56	69	25	2	15	369
62	66	76	8	12	22	35	39	49	369
1	14	27	28	41	54	55	68	81	369
33	43	47	60	70	74	6	16	20	369
67	80	57	13	26	3	40	53	30	369
18	19	5	45	46	32	72	73	59	369
38	51	34	65	78	61	11	24	7	369
369	369	369	369	369	369	369	369	369	
369	369	369	369	369	369	369	369	369	369

The size-9 square has no tiling pattern so it cannot be ultra-perfect. Nonetheless it is absolutely perfect because all **9** planar directional sums are equal to its characteristic number.

Program 5

Equalities between X and Y in Class-6 Cubes

We saw back in Program 3 that the cells where the modulus and integer loom tables were equal, i.e. **$x_{ij}(n) = y_{ij}(n)$,** the numbers ranged from **1** to $\mathbf{n^2}$ and all corresponding numbers in the square of size **n** summed to the square's characteristic number $\mathbf{C^{(2)}_n}$.

In cubes of odd-size, where the modulus and integer loom tables are equal, **X** in the base **n** and **Y** in the base $\mathbf{n^2}$, i.e. where **$x_{ij}(n) = y_{ij}(n^2)$,** the numbers now range from **1** to $\mathbf{n^3}$ and all the numbers corresponding to these loom-table equalities in the cube sum to the cube's characteristic number $\mathbf{C^{(3)}_n = 29565}$. In the size-9 cube there are **9** such equalities in each of the **9** embedded squares, each of which sums to **3285**, e.g. numbers highlighted in orange in the central square.

Central Square

480	544	638	669	4	98	210	274	368	3285
509	576	658	50	117	199	320	387	469	3285
619	677	42	160	218	312	349	407	501	3285
710	66	157	170	255	346	440	525	616	3285
1	95	189	271	365	459	541	635	729	3285
114	205	290	384	475	560	573	664	20	3285
229	323	381	418	512	570	688	53	111	3285
261	343	410	531	613	680	72	154	221	3285
362	456	520	632	726	61	92	186	250	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	

X(9)

3	4	8	3	4	8	3	4	8
5	9	1	5	9	1	5	9	1
7	2	6	7	2	6	7	2	6
8	3	4	8	3	4	8	3	4
1	5	9	1	5	9	1	5	9
6	7	2	6	7	2	6	7	2
4	8	3	4	8	3	4	8	3
9	1	5	9	1	5	9	1	5
2	6	7	2	6	7	2	6	7

Y(81)

6	7	8	9	1	2	3	4	5
7	8	9	1	2	3	4	5	6
8	9	1	2	3	4	5	6	7
9	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	1
3	4	5	6	7	8	9	1	2
4	5	6	7	8	9	1	2	3
5	6	7	8	9	1	2	3	4

Program 5

Equalities between X and Y in the Centrally Embedded Square of Class-1 Cubes

On the foregoing page, we addressed equalities between **X(n)** and **Y(n^2)** in which the numbers ranged from **1** to **n**.

It was found for Class-1, but not for Class-6, cubes that **X(n) = Y(n^2) for** the cube's centrally embedded square. Since all rows and columns sum equally to $C^{(3)}_n/n$, the entire embedded central square sums to $C^{(3)}_n$. Although all the embedded squares sum equally to $C^{(3)}_n$, only in the central square did this equality between loom-tables in different bases exist.

Here it is demonstrated for the size-7 cube.

	7	12	219	90	248	119	326	190	1204	8428
	1	301	165	29	243	58	272	136	1204	
	2	93	258	122	336	151	15	229	1204	
	3	186	1	215	79	293	108	322	1204	
4	4	279	143	308	172	36	201	65	1204	
	5	22	236	51	265	129	343	158	1204	
	6	115	329	193	8	222	86	251	1204	
	7	208	72	286	101	315	179	43	1204	8428
	1	154	18	225	96	254	125	332	1204	

X(n)									Y(n^2)							
5	2	6	3	7	4	1	21	1	5	2	6	3	7	4
7	4	1	5	2	6	3	22	7	4	1	5	2	6	3
2	6	3	7	4	1	5	23	2	6	3	7	4	1	5
4	1	5	2	6	3	7	24	4	1	5	2	6	3	7
6	3	7	4	1	5	2	25	6	3	7	4	1	5	2
1	5	2	6	3	7	4	26	1	5	2	6	3	7	4
3	7	4	1	5	2	6	27	3	7	4	1	5	2	6
5	2	6	3	7	4	1	28	5	2	6	3	7	4	1
7	4	1	5	2	6	3	29	4	1	5	2	6	3	7

Program 5

Generating Cubes from Squares

Given loom tables $\mathbf{X}^{(2)}$ and $\mathbf{Y}^{(2)}$ for a mathemagic square $\mathbf{W}_n$, dual mathemagic cubes can be generated as follows. The size-7 loom-tables of $\mathbf{W}_7$ will be used for this demonstration.

The integer loom-table $\bar{\mathbf{Y}}^{(3)}$ for the cube is merely the integer loom-table $\mathbf{Y}^{(2)}$ for the square, column-wrapped per each embed-ded square. To do this properly so that the resulting table is pairwise symmetric, the square's loom table is first placed in the center of the cube's loom-table. Column wrapping begins from there: above it, to the left; below it, to the right.

The primal version of the cube is given by:

$$(\ 5.2\ )\ \mathbf{W}^{(3)}{}_n = n^2(\ \bar{\mathbf{Y}}^{(3)}{}_n - |\ 1\ |\ ) + \bar{\mathbf{W}}^{(3)}{}_n$$

Note that the entire primal square is used repetitiously in place of the modulus loom-table in the customary square-generation formula.

A dual cube can be obtained likewise by replacing $\bar{\mathbf{Y}}^{(3)}$ with $\bar{\mathbf{X}}^{(3)}$ as in (**5.2**):

$$(\ 5.3\ )\ \ \mathbf{U}^{(3)}{}_n = n^2(\ \bar{\mathbf{X}}^{(3)}{}_n - |\ 1\ |\ ) + \bar{\mathbf{W}}^{(3)}{}_n$$

where $\bar{\mathbf{X}}^{(3)}$ was column wrapped $\mathbf{X}^{(2)}$ just like $\bar{\mathbf{Y}}^{(3)}$ was of $\mathbf{Y}^{(2)}$.

Note: what was column-wrapped could also be row-wrapped and the procedure will still produce a mathemagical cube, only it will be different in its distribution of the numbers in the table.

So there are many ways to negotiate 3-dimensional space geonomically just as 2-dimensional space was explored.

$\bar{\mathbf{Y}}^{(3)}$

7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1

$\bar{\mathbf{W}}^{(3)}$

38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12

Program 5

$\bar{X}^{(3)}$

5	1	4	7	3	6	2
7	3	6	2	5	1	4
2	5	1	4	7	3	6
4	7	3	6	2	5	1
6	2	5	1	4	7	3
1	4	7	3	6	2	5
3	6	2	5	1	4	7
2	5	1	4	7	3	6
4	7	3	6	2	5	1
6	2	5	1	4	7	3
1	4	7	3	6	2	5
3	6	2	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	1	4
6	2	5	1	4	7	3
1	4	7	3	6	2	5
3	6	2	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	1	4
2	5	1	4	7	3	6
4	7	3	6	2	5	1
3	6	2	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	1	4
2	5	1	4	7	3	6
4	7	3	6	2	5	1
6	2	5	1	4	7	3
1	4	7	3	6	2	5
7	3	6	2	5	1	4
2	5	1	4	7	3	6
4	7	3	6	2	5	1
6	2	5	1	4	7	3
1	4	7	3	6	2	5
3	6	2	5	1	4	7
5	1	4	7	3	6	2
4	7	3	6	2	5	1
6	2	5	1	4	7	3
1	4	7	3	6	2	5
3	6	2	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	1	4
2	5	1	4	7	3	6
1	4	7	3	6	2	5
3	6	2	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	1	4
2	5	1	4	7	3	6
4	7	3	6	2	5	1
6	2	5	1	4	7	3

$\bar{W}^{(3)}$

38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12
38	27	9	47	29	18	7
33	15	4	42	24	13	44
28	10	48	30	19	1	39
16	5	36	25	14	45	34
11	49	31	20	2	40	22
6	37	26	8	46	35	17
43	32	21	3	41	23	12

$U_7^{(3)}$

234	27	156	341	127	263	56
327	113	249	91	220	13	191
77	206	48	177	313	99	284
163	299	134	270	63	241	34
256	98	227	20	149	334	120
6	184	320	106	291	84	213
141	277	70	199	41	170	306
87	223	9	194	323	116	252
180	309	102	287	73	209	44
273	59	244	30	166	295	137
16	152	330	123	259	94	230
109	294	80	216	2	187	316
202	37	173	302	144	280	66
337	130	266	52	237	23	159
283	76	205	47	176	312	105
33	162	298	140	269	62	240
126	255	97	226	19	148	333
212	5	183	319	112	290	83
305	147	276	69	198	40	169
55	233	26	155	340	133	262
190	326	119	248	90	219	12
136	272	58	243	29	165	301
229	15	151	336	122	258	93
322	108	293	79	215	1	186
65	201	36	172	308	143	279
158	343	129	265	51	236	22
251	86	222	8	193	329	115
43	179	315	101	286	72	208
332	125	254	96	225	18	154
82	211	4	189	318	111	289
175	304	146	275	68	197	39
261	54	232	25	161	339	132
11	196	325	118	247	89	218
104	282	75	204	46	182	311
239	32	168	297	139	268	61
185	321	107	292	78	214	7
278	64	200	42	171	307	142
28	157	342	128	264	50	235
114	250	85	221	14	192	328
207	49	178	314	100	285	71
300	135	271	57	242	35	164
92	228	21	150	335	121	257
38	174	303	145	274	67	203
131	260	53	238	24	160	338
224	10	195	324	117	246	88
310	103	281	74	210	45	181
60	245	31	167	296	138	267
153	331	124	253	95	231	17
288	81	217	3	188	317	110

$W_7^{(3)}$

332	223	107	47	274	165	56
278	162	53	336	220	111	44
224	108	48	275	166	50	333
163	54	330	221	112	45	279
109	49	276	167	51	334	218
55	331	222	106	46	280	164
43	277	168	52	335	219	110
87	321	205	145	29	263	154
33	260	151	91	318	209	142
322	206	146	30	264	148	88
261	152	85	319	210	143	34
207	147	31	265	149	89	316
153	86	320	204	144	35	262
141	32	266	150	90	317	208
185	76	303	243	127	18	252
131	15	249	189	73	307	240
77	304	244	128	19	246	186
16	250	183	74	308	241	132
305	245	129	20	247	187	71
251	184	75	302	242	133	17
239	130	21	248	188	72	306
283	174	58	341	225	116	7
229	113	4	287	171	62	338
175	59	342	226	117	1	284
114	5	281	172	63	339	230
60	343	227	118	2	285	169
6	282	173	57	340	231	115
337	228	119	3	286	170	61
38	272	156	96	323	214	105
327	211	102	42	269	160	93
273	157	97	324	215	99	39
212	103	36	270	161	94	328
158	98	325	216	100	40	267
104	37	271	155	95	329	213
92	326	217	101	41	268	159
136	27	254	194	78	312	203
82	309	200	140	24	258	191
28	255	195	79	313	197	137
310	201	134	25	259	192	83
256	196	80	314	198	138	22
202	135	26	253	193	84	311
190	81	315	199	139	23	257
234	125	9	292	176	67	301
180	64	298	238	122	13	289
126	10	293	177	68	295	235
65	299	232	123	14	290	181
11	294	178	69	296	236	120
300	233	124	8	291	182	66
288	179	70	297	237	121	12

Program 5

Embedded Characteristic Spheres in Class-4 Cubes

Just as for squares, the cube's kernel number divides the cube's characteristic number exactly without a fractional remnant only for Class-4 cubes. However, the size-2 cube does so too because the whole cube is its only octal.

Determining which size cubes have Characteristic spheres			
Size Cube	Characteristic number	Kernel Number	# of Octals = characteristic no.
1	1	8	0.125
2	18	18	1
3	126	112	1.125
4	520	260	2
5	1,575	504	3.125
6	3,906	868	4.5
7	8,428	1,376	6.125
8	16,416	2,052	8
9	29,565	2,920	10.125
10	50,050	4,004	12.5
11	80,586	5,328	15.125
12	124,488	6,916	18
13	185,731	8,792	21.125
14	269,010	10,980	24.5
15	379,800	13,504	28.125
16	524,416	16,388	32
17	710,073	19,656	36.125
18	944,946	23,332	40.5
19	1,238,230	27,440	45.125
20	1,600,200	32,004	50
21	2,042,271	37,048	55.125
22	2,577,058	42,596	60.5
23	3,218,436	48,672	66.125
24	3,981,600	55,300	72

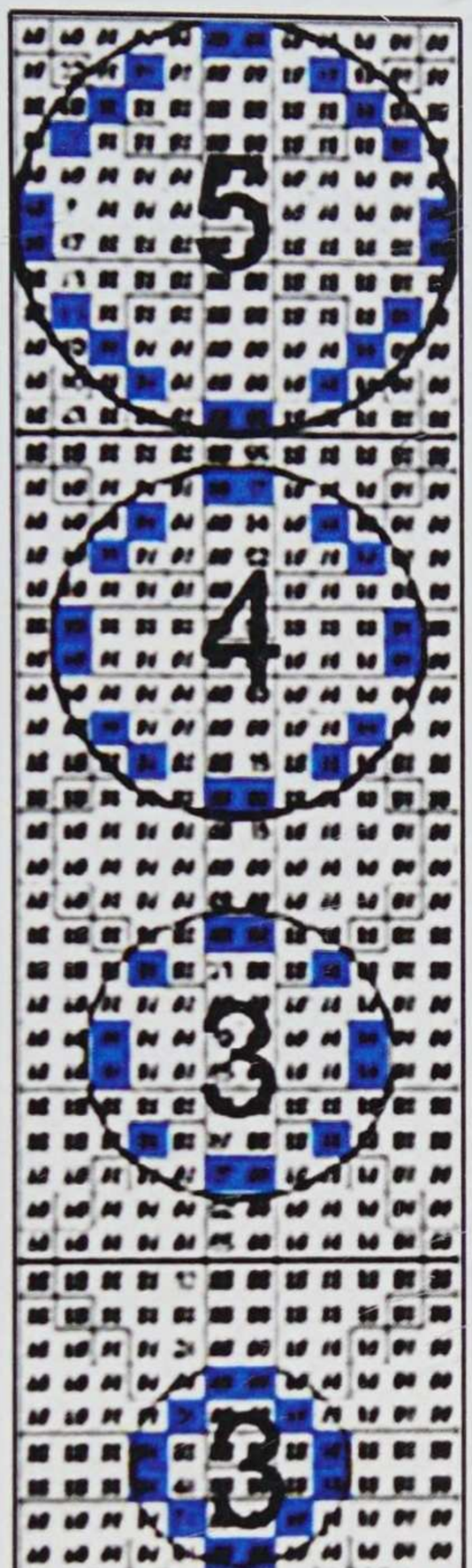
The formula for the right-hand column is

(5.4) $\mathbf{C^{(3)}_n / q = n^2/8}$

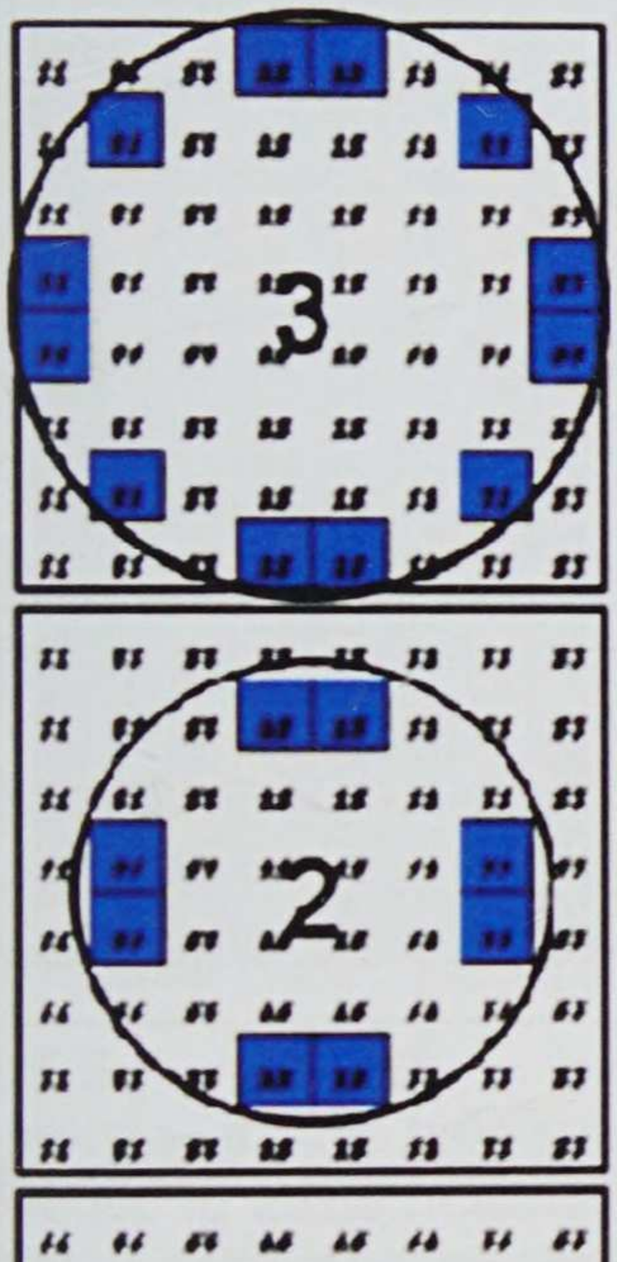
where **q** = octal sum (3D kernel number).

A characteristic sphere is composed of impinging octals which sum to the cube's characteristic number.

12x Cube
Bottom Half



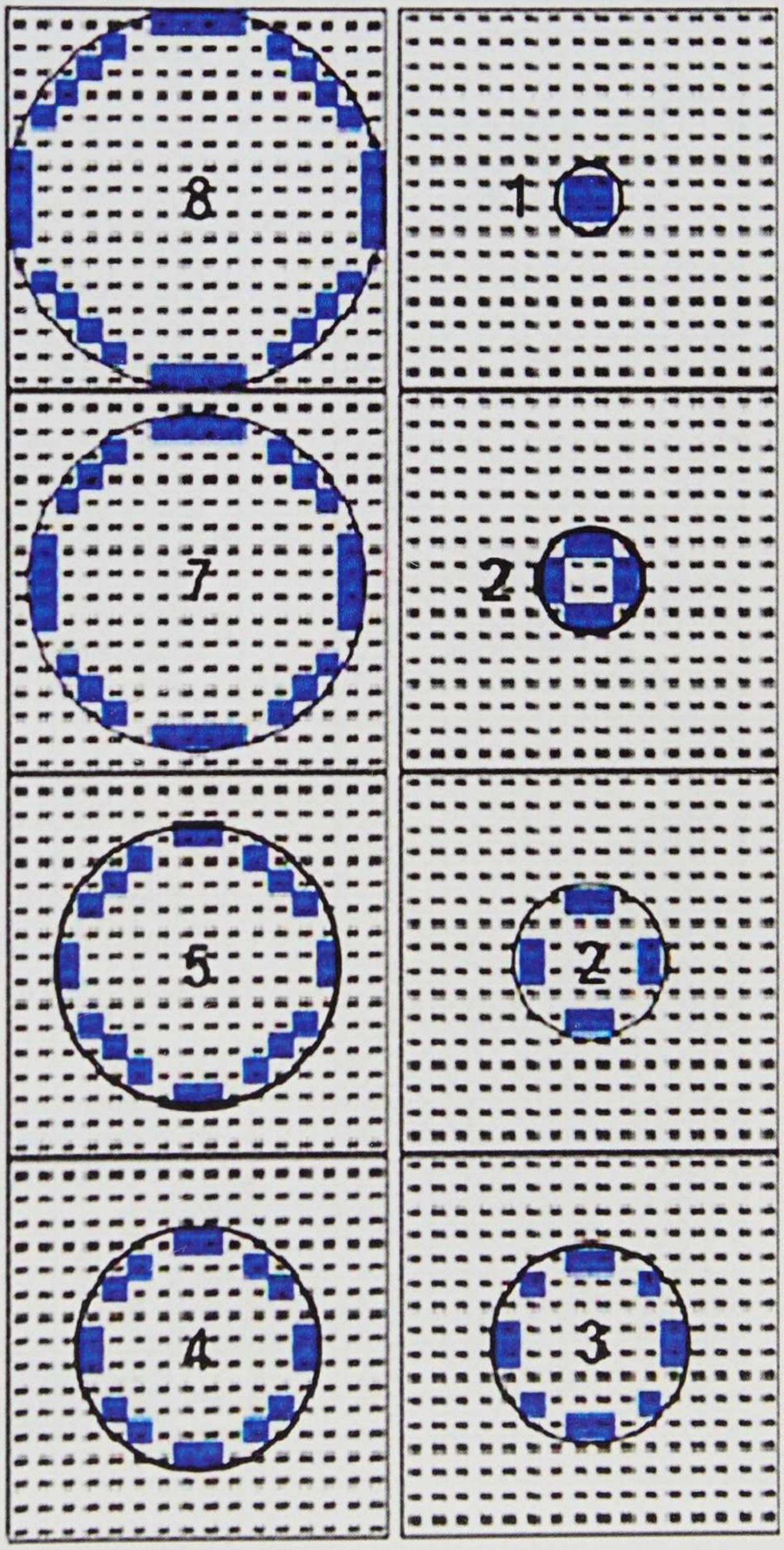
Size-8 Cube
Bottom half



Here are the cells participating in impinging the interior of characteristic spheres for size cubes **8, 12** and **16**.Their octals are **8**, **18** and **32**, respectively.

The size-4 cube is shown on the next page.

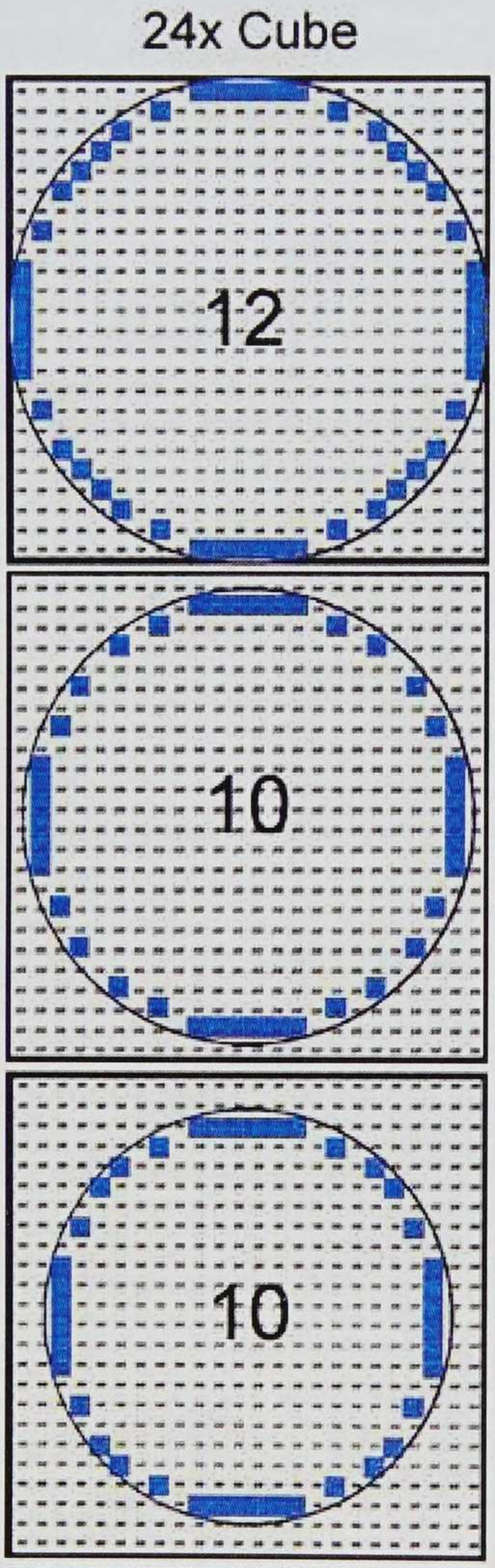
Size-16 Cube
Bottom half (cont'd)

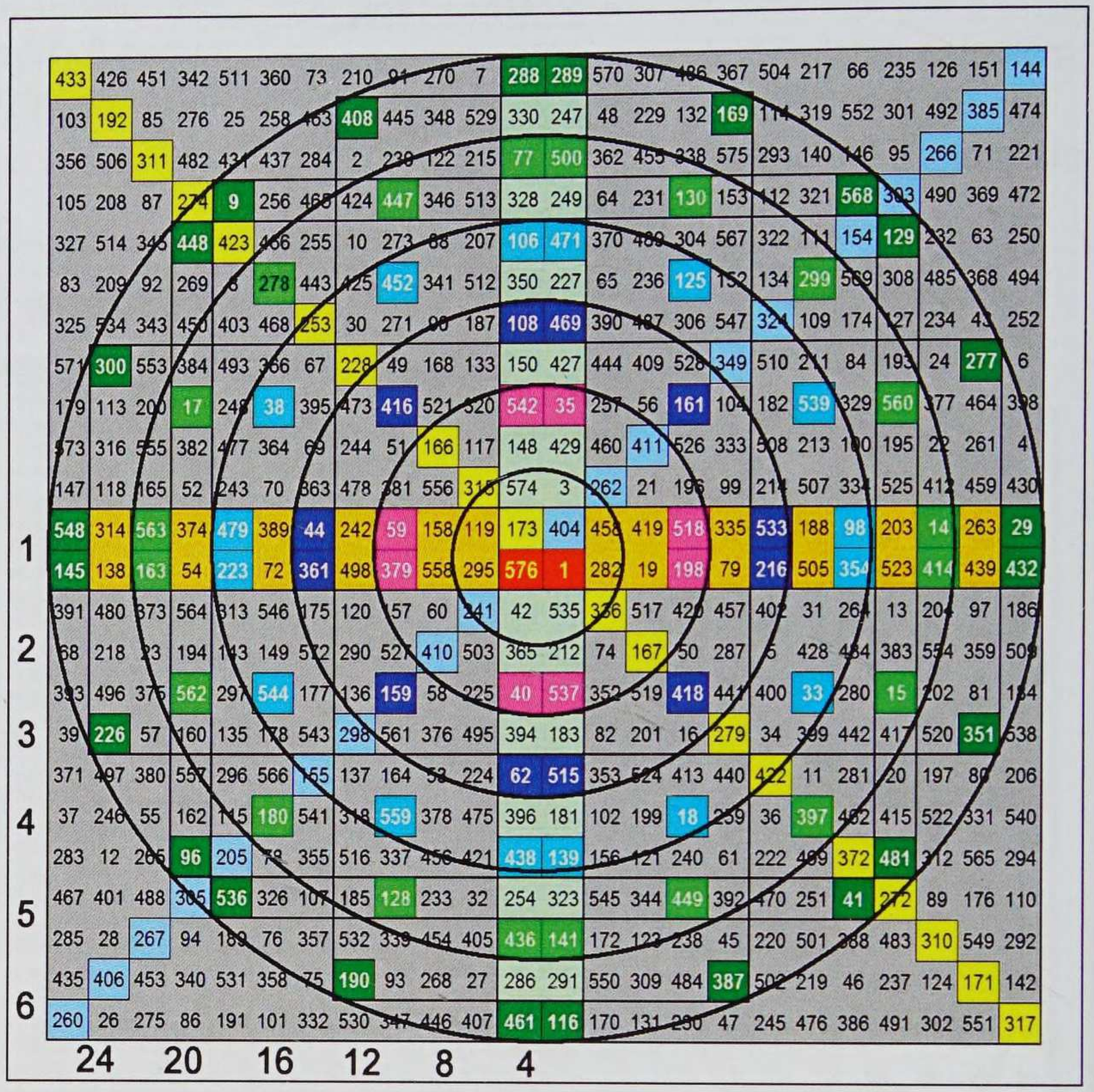


Program 5

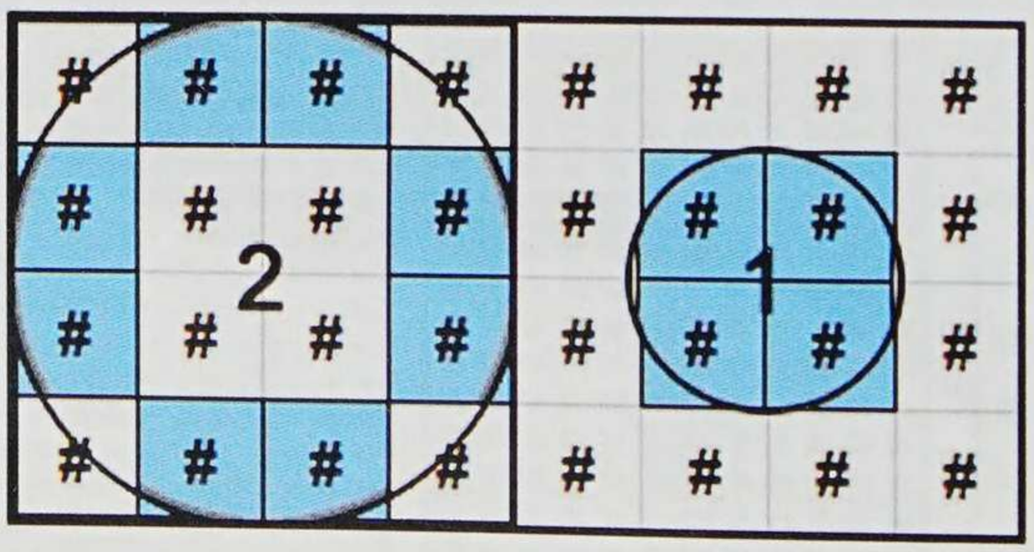
Octal and Quadral Differences between Characteristic Circles and Cubes

This page explains the difference in the number of octals participating in impinging the series of nested circles of spheres with the number of quadrals involved in impinging the nested circles of squares.





When characteristic circles were depicted for a series of independent squares, the difference between sizes of consecutive squares was **4**, where here for the nested circles within a single sphere, that size difference at each level is merely **2**. Whence the reason for the larger count for octals in cubes than for quadrals in squares of the same size.



Size-4 Cube (bottom half)

This size cube has 3 octals of which only 2 at a time sum to the sphere's characteristic number.

Program 5

x	Cube's size n	$C_n^{(3)}$	z
1	4	2q	**2**
2	8	8q	**8**
3	12	18q	**18**
4	16	32q	**32**
5	20	50q	**50**
6	24	72q	**72**

The thing to note here in this table is the series of unshadded numbers in the rightmost column for class-4 squares. They number **2, 8, 18** and **32**. This is much more than just coincidental! These numbers will be demonstrated to be the very key to cracking the hidden code underlying the distribution of electrons in the atoms, coming up next here in Program 5. Further, the fact that the size-4 cube has **3** octals impinging its inscribed sphere of which only **2** at a time sum to the cube's characteristic number is also able to explain mathematically some observed anomalies in the fill sequence of electrons in lower shell pairs where electrons tumble to a lower shell in threesomes.

Summary of Spheres

Formula (**5.4**) is the formula for this series in terms of size and kernel number that was derived from the table of characteristic spheres three pages prior.

(5.4) $C^{(3)}{}_n / q = (n^2/8)$ where **q** = octal sum (i.e. 3D kernel number).

Recall that formula (**1.5**) from Program 1:

$$y = 4x$$

The area under this straight line is measured by **z**:

(5.5) $z = 2x^2$

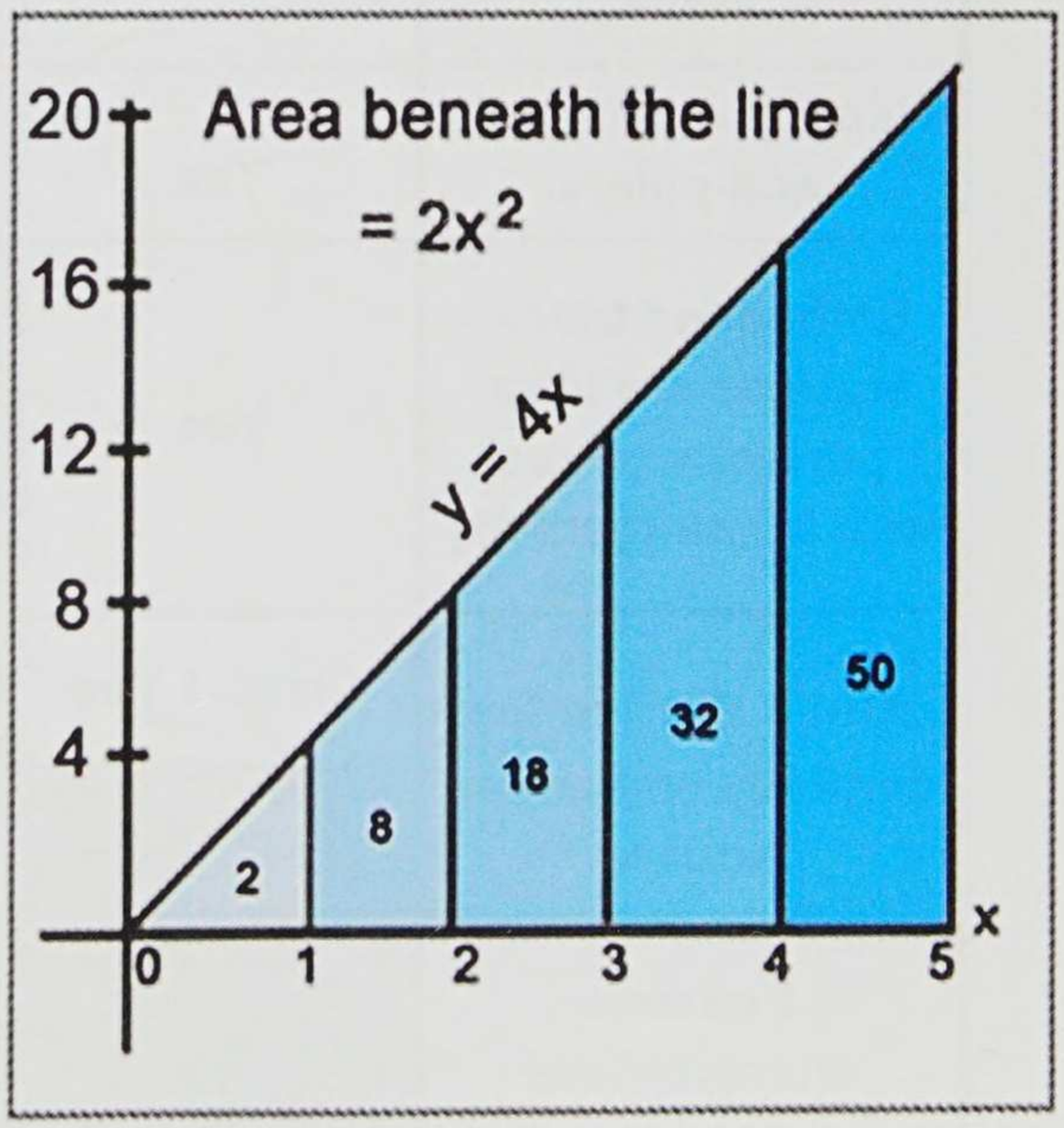
Equations (**5.4**) and (**5.5**) together yield

$$n^2 / 8 = z = 2x^2$$

and solving this for **n** in terms of **x** we get

$$n = 4x = y$$

The significance of this one-to-one correlation is that the sequence of incident quadrals in characteristic circles in squares follows the straight line **y = 4x** and the sequence of incident octals in characteristic spheres follows the area under this line. In other words, a 2-dimensional relationship follows a 1-dimensional line, and a 3-dimensional relationship follows a 2-dimensional area. This is expanded upon to any dimension **k** in Program 8 and generalized in the Appendix.



Program 5

Summary of Cubic Properties

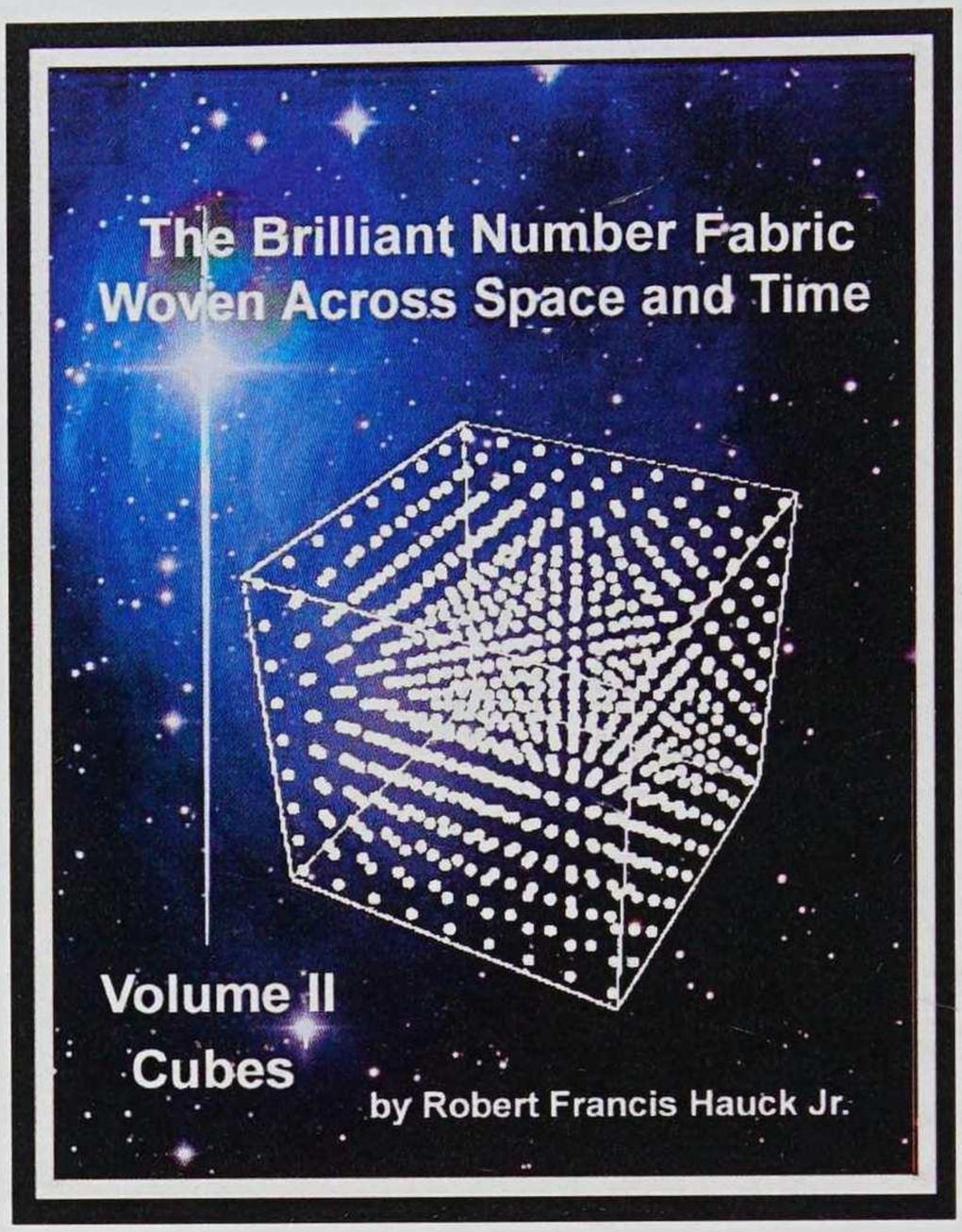
This table summarizes the properties of geonomic cubes by class. It is not to be studied here but may serve as a reference guide to be returned to as needed for comprehending the broader property-distribution pattern. All this detail is printed in your companion book to this program series.

Properties of Cubes	Odd		Even	
Class	Classes 1 & 6	Classes 3 & 5	Class 2	Class 4
Size factors	n is a prime or square of an odd number	n = ab both odd	n = 2a odd a > 1	n = 2b b even
Perfection Level	Both Absolute & Ultra-perfect	Absolutely Perfect	Perfect	Absolutely perfect
Pangenic Octal sums equal	Yes	Yes	Yes	Yes
Possess equal block-square row and column sums	Yes	Yes	Yes	Yes
Has equal channels and pillars	Yes	Yes	Yes	Yes
Embedded block-squares are ultra-perfect squares with 3D pangenicity	Yes	No	No	No
$X(n) = Y(n^2)$ in the centrally embedded square	Class-1 Yes	Class-3 No	N/A	N/A
	Class-6 No	Class-5 unknown		
Possess characteristic spheres	No	No	No	Yes

Program 5

Notes

1. Class-5 cubes have not been explored and consequently their properties are at this time unknown. So all the notes exclude Class-5 cubes. "All" means "All except Class-5".
2. In all cubes, all the numbers in each embedded block-square sum to the cube's characteristic number.
3. All cubes are pangenic in that the corners of all their centrally located octals sum equally.
4. In Classes1 and 3 cubes, all their centrally located octagons sum equally too.
5. All classes of cubes are perfect in that all their **nxn** horizontal, vertical and two main diagonal planes sum to the cube's characteristic number.
6. Class-1 cubes have equal-summing block-squares which sum to the cube's characteristic number.
7. Each embedded block-square in a Class-1 cube possesses the continuous complementary tiling patterns of the comparable size-n square.
8. Only the size-9 Class-6 cube has been explored and was found to have all the basic properties of Class-1 cubes including equal summing centrally symmetric octals and octagons, with the exception of having contiguous tiling patterns in its embedded block squares because the size-9 square lacks characteristic tiling patterns.
9. All classes of cubes except those of Class-2 collapse to a perfect square.
10. All Class-2 cubes collapse to a punctuated-perfect square with 4 unequal wrap diagonals.
11. All Class-4 cubes possess a characteristic sphere.



The Brilliant Number Fabric Woven across Space and Time - Volume II Cubes

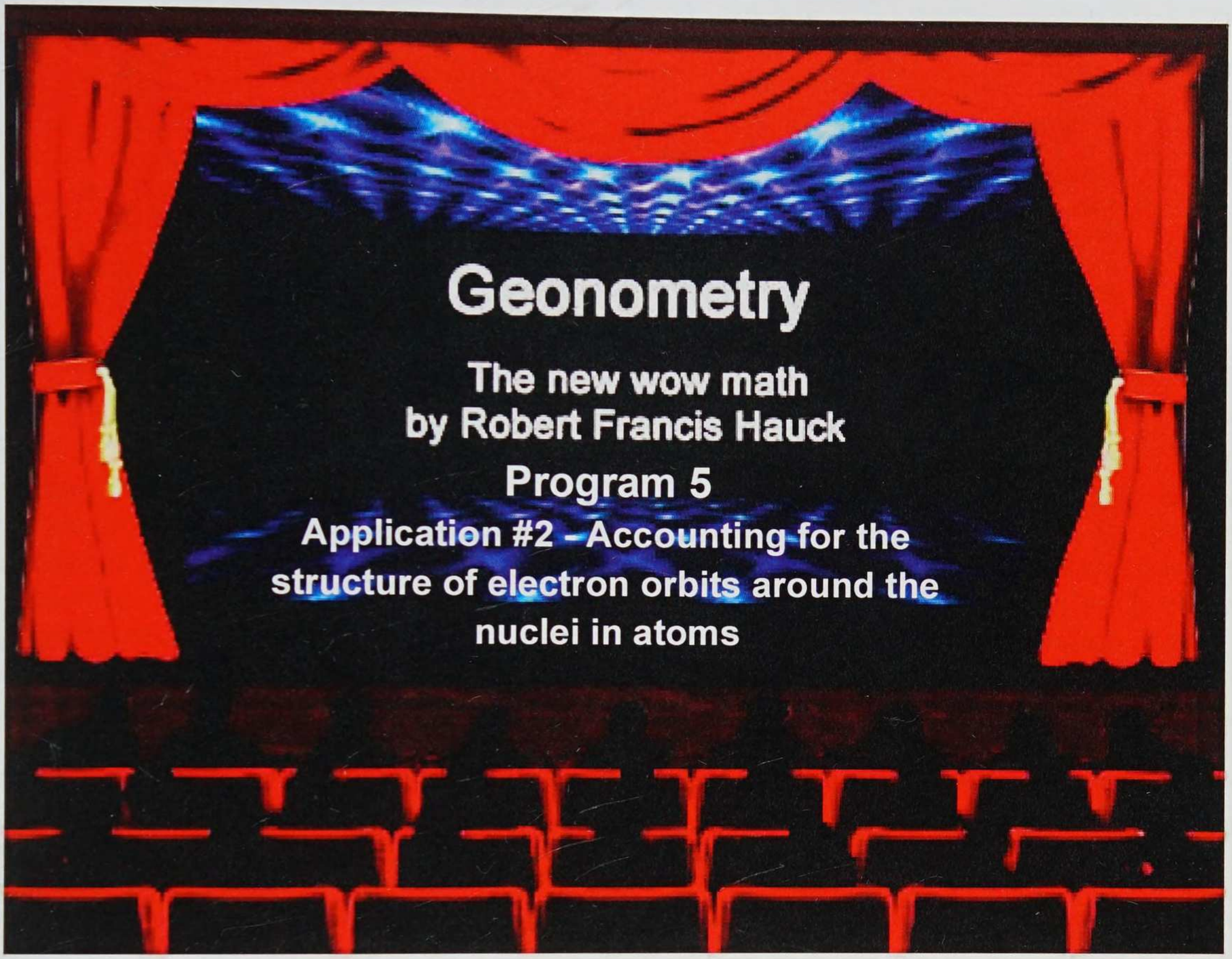
ISBN: 978-146-107278-2

Shows how to construct geonomic cubes from squares. Rev. October 16th, 2011

Third Edition (60 pages)

In the next segment we will get to witness just how the mathematics of 2- and 3-dimensional Geonometry can explain and account for the distribution and navigation of the electrons around the nuclei of atoms.

Application #2 of Geonometry

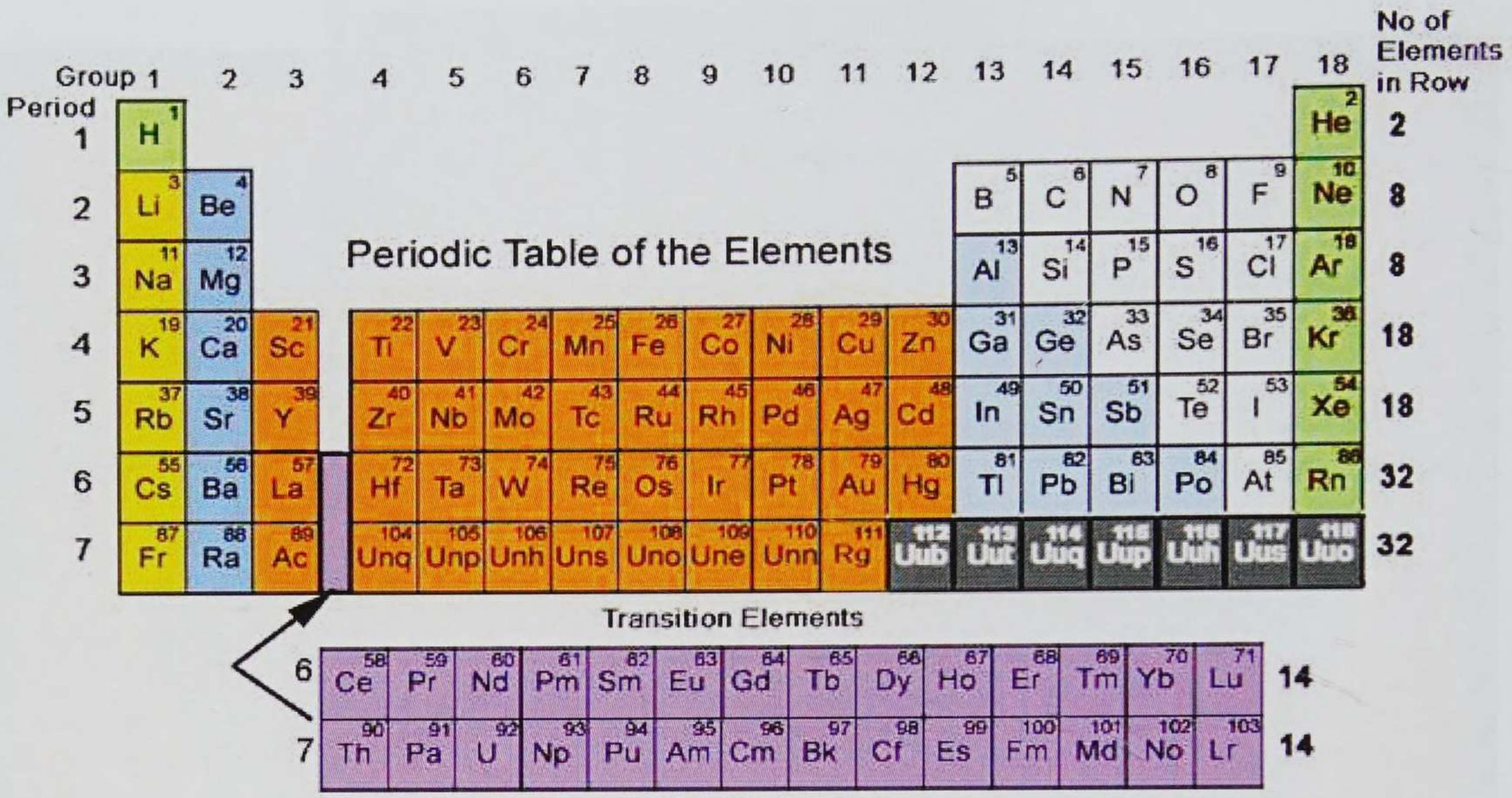


Next we will observe the first major application of Geonometry when we interpret the distribution and orbital patterns of electrons at the atomic level **1/3D**. In this program we shall ex-plore the electron pattern in each of the noble elements and see just how Geonometry has been applied to gain a better understanding of what has already been found in Atomic Physics.

Program 5

Accounting for the Distribution of Electrons in the Atoms

The Periodic Table of Elements



Antimatter Matter .

[32, 32, 18, 18, 8, 8, 2, 2, 8, 8, 18, 18, 32, 32]

[64, 36, 16, 4, 16, 36, 64]

[8, 6, 4, 2, 4, 6, 8]

[4, 3, 2, 1, 2, 3, 4] ⇐ **a perfect harmonic wave-form pattern**

Note the sequence of the number series **2, 8, 18** and **32** at the far right. These were the number of octals incident to cubes of sizes **4, 8, 12** and **16** whose kernel numbers of those sizes summed to the characteristic number. This correlation is profound because it correlates one hundred percent to the number of elements in consecutive row-pairs of the Periodic Table of Elements.

The series at bottom shows what this series would look-like if it were extended to the left to include the atoms composed of anti-matter. These are atoms whose constituents are positrons instead of electrons and anti-protons with the opposite charge of protons. Neutrons of matter and antimatter are theoretically the same.

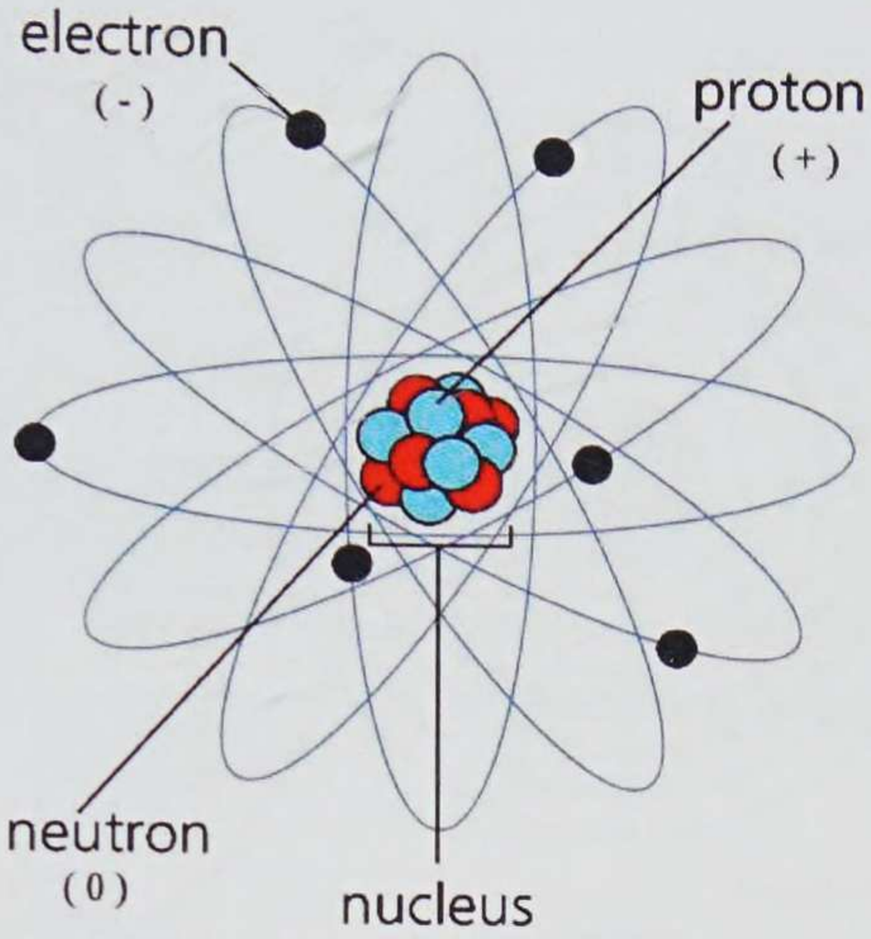
This expanded series can be reduced by simple arithmetic operations to a series of consecutive numbers stretching in opposite directions with the number **1** at its center.

1. First, the duplicated numbers in the series are combined into separate sums.
2. Next, these sums are converted to their square root.
3. And finally, those square-root numbers are divided by **2**.

This is what is defined mathematically as a perfect harmonic waveform pattern.

Program 5

The Niels-Bohr Model of the Atom



This slide depicts the Niels-Bohr model of the atoms of the noble elements, that is, those elements whose electron shells are full as in column **18** of the Periodic Table. Its electrons, each having a negative charge, are depicted as orbiting the nucleus which consists of an equal number of protons with a positive charge. These protons are separated from each other by intervening neutrons in close proximity, having no charge, which serve as insulation among the positively-charged protons, which in turn are attracting the electrons which have orbital motion.

Atomic Science				Geonometry
Principal Quantum Number x	Energy level	Shell–pair Letters	Electron Capacity	Impinging octals $y = 2x^2$
1	s	K & L	2	2
2	p	M & N	8	8
3	d	O & P	18	18
4	f	Q & R	32	32

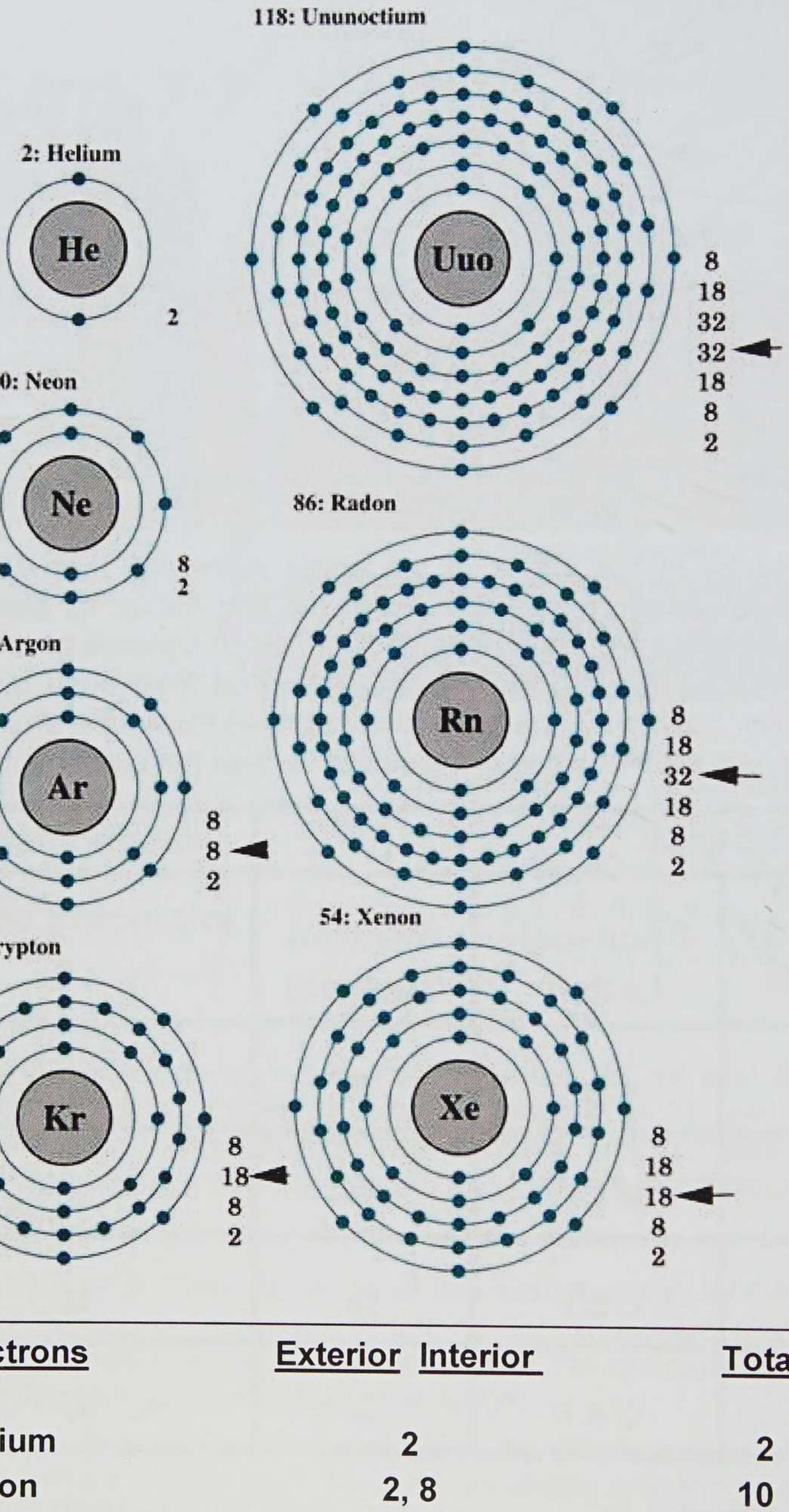
That description has been used by chemists for over a hundred years but has been improved upon since its acceptance back a century ago as shown in the next slide.

The variable **x** represents the Principal Quantum Number, the number of the energy level in the associated electron shell-pair.

Note that **y =** the multiple of the kernel number **q** which equals the size-4x cube's characteristic number = the number of octals impinging the cube's characteristic sphere.

Program 5

Distribution pattern of electrons in the Noble elements



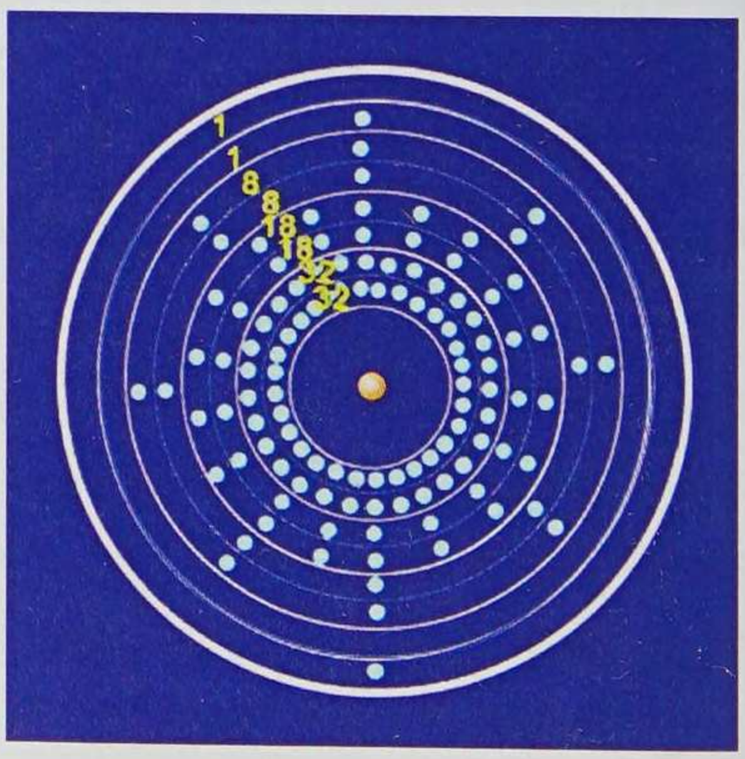
The distribution of electrons has been determined by atomic physicists to consist of nested shells of electrons as shown for the noble elements here. Note that each new completed shell originates within the middle of all the previous nested shells. That's a big correction to the Niels-Bohr model.

However, don't accept this model as fact; Geonometry has a different view yet as shown at bottom.

Now we have observed earlier in this Program that the number of octals in the characteristic spheres for cubes of sizes **4, 8, 12** and **16** are equal to the numbers **2, 8, 18** and **32**, respectively. Note that each of these numbers minus **1** are the numbers **1, 7, 17** and **31** respectively, the latter of which are sequential prime-numbers.

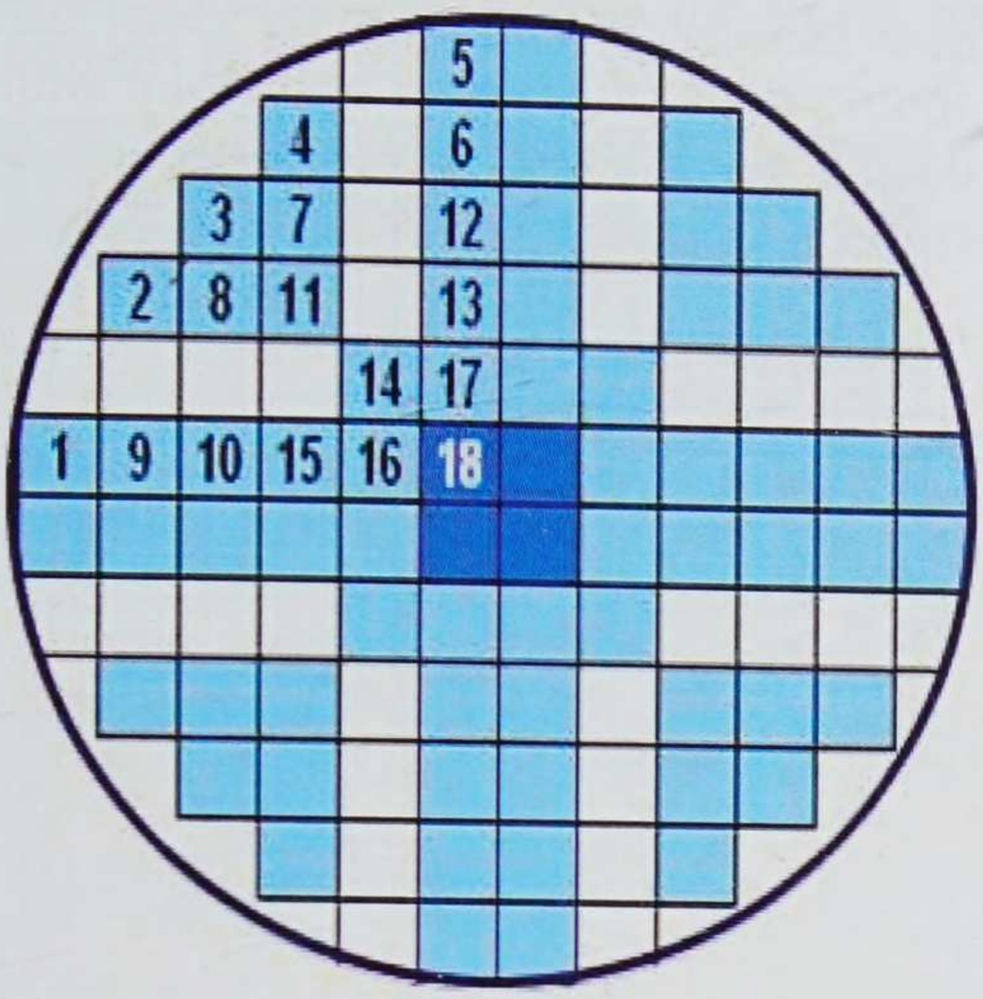
We have already seen in Program 2 that geonomic squares for these sizes greater than **1** are Class-1 squares that have complementary tiling patterns which cover the entire square exactly and completely.

Electrons	Exterior Interior	Total
Helium	2	2
Neon	2, 8	10
Argon	2, 8, 8	18
Krypton	2, 8, 8, 18	36
Xenon	2, 8,8, 18, 18	54
Radon	2, 8, 8, 18, 18, 32	86
Ununoctium	2, 8 ,8, 18, 18, 32, 32	118



Program 5

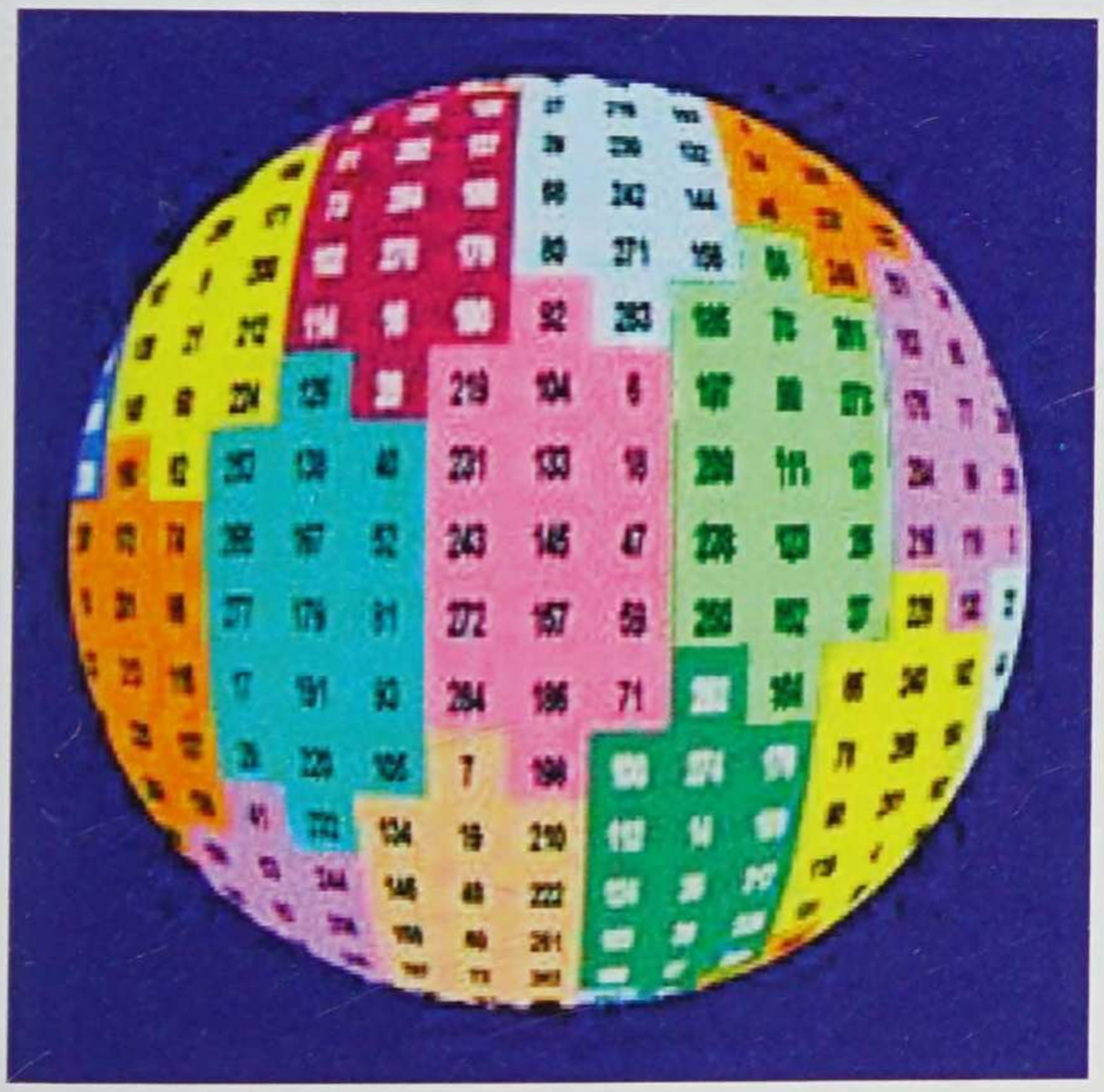
The Distribution of Octals in the Size-12 Characteristic Sphere

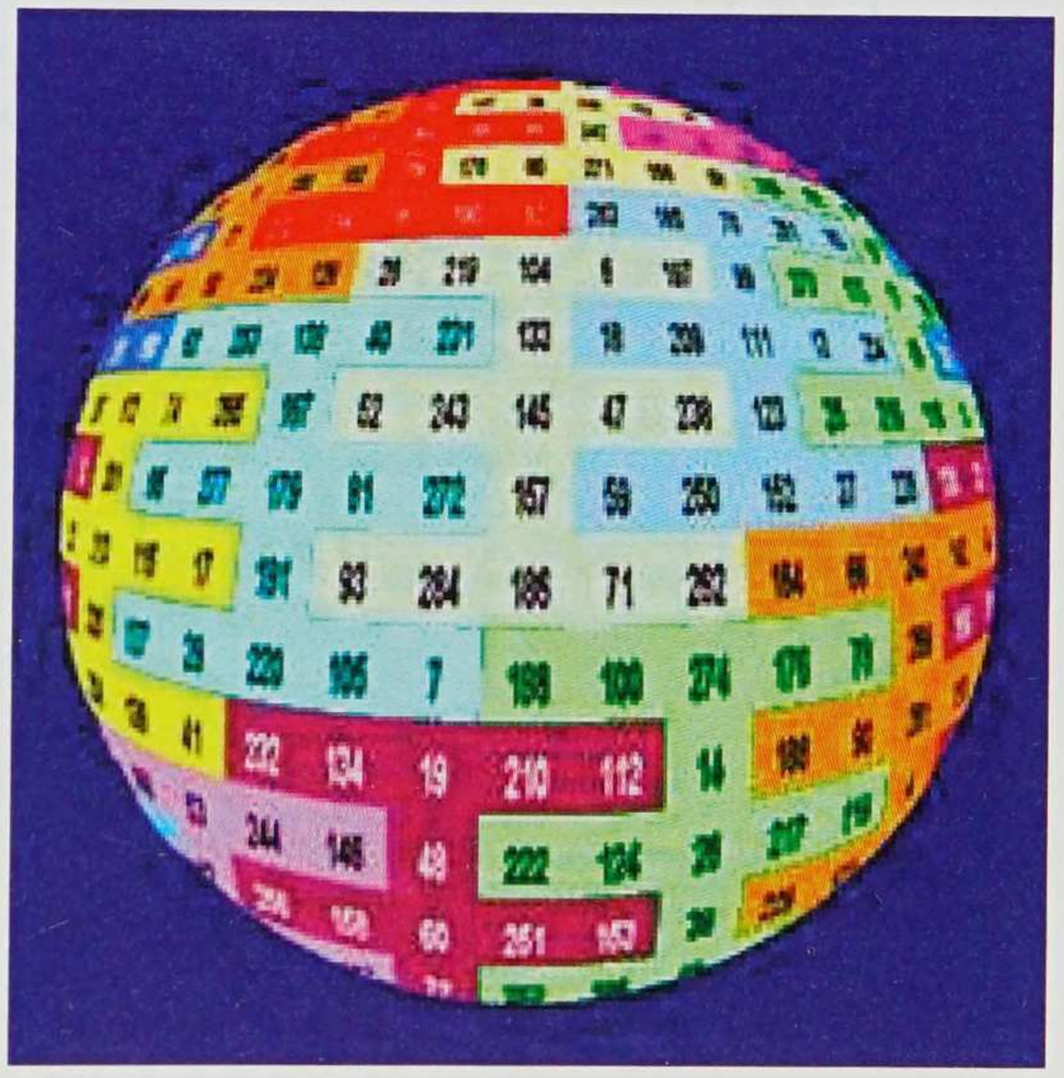


The size-12 cube's inscribed sphere showing the location of its 18 octals looking from top-down

The **18** octals of the size-12 cube's inscribed sphere indicates that it has the capacity for **18** electrons per spherical electron shell.

Tiling Patterns on the Surface of Characteristic Spheres





Picture the sze-17 tile patterns **A** & **B** wrapped around the size-12 characteristic sphere. This sphere would have the capacity for **18** electrons on its surface. At any one instant one of the **18** electrons passes through one of the non-tiled polar regions.

If these two tiling patterns were converted to their shared complementary loom tables, we would get two complementary spheres each with a capacity of **18** electrons.

One can picture one tiling pattern guiding the electrons in one electron shell and the other tiling pattern orchestrating the activity of electrons in the other shell within each associated shell-pair. Further, the vibrating spherical-surface membrane of one shell could operate according to the modulus loom table and the other according to the integer loom table.

Program 5

Tiling Patterns of Loom-tables on the Surface of Characteristic Spheres

The complementary tiling patterns of prime-number size-17 complementary loom tables serve as templates for the location of electrons in adjacent shells of Krypton and Neon with electron counts of **18** electrons each, shown here on the modulus loom table with the different tile centers aligned over the number **9**. Krypton uses the modulus table **X(17)** and Neon uses the integer table **Y(17)**.

A

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

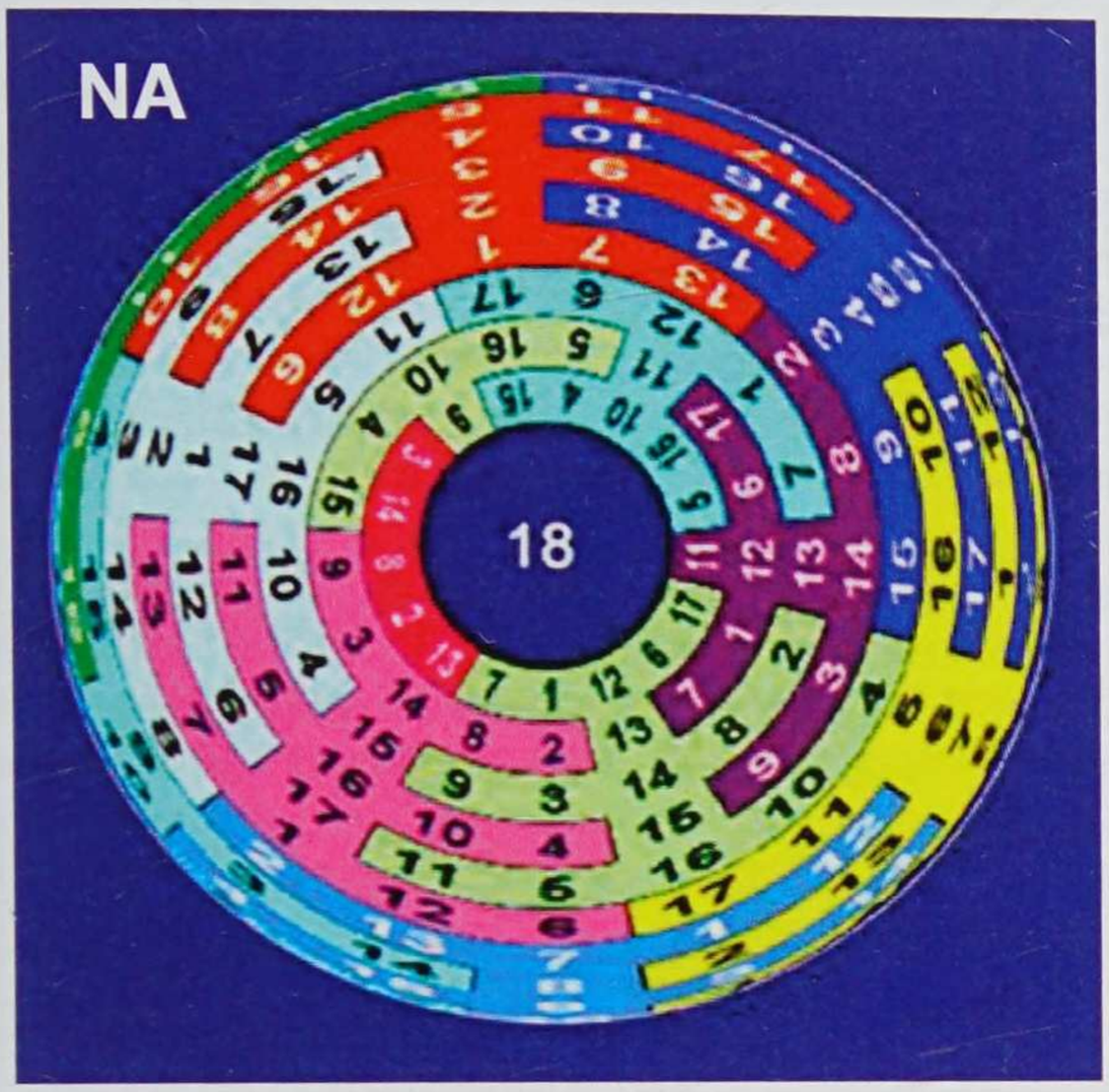
B

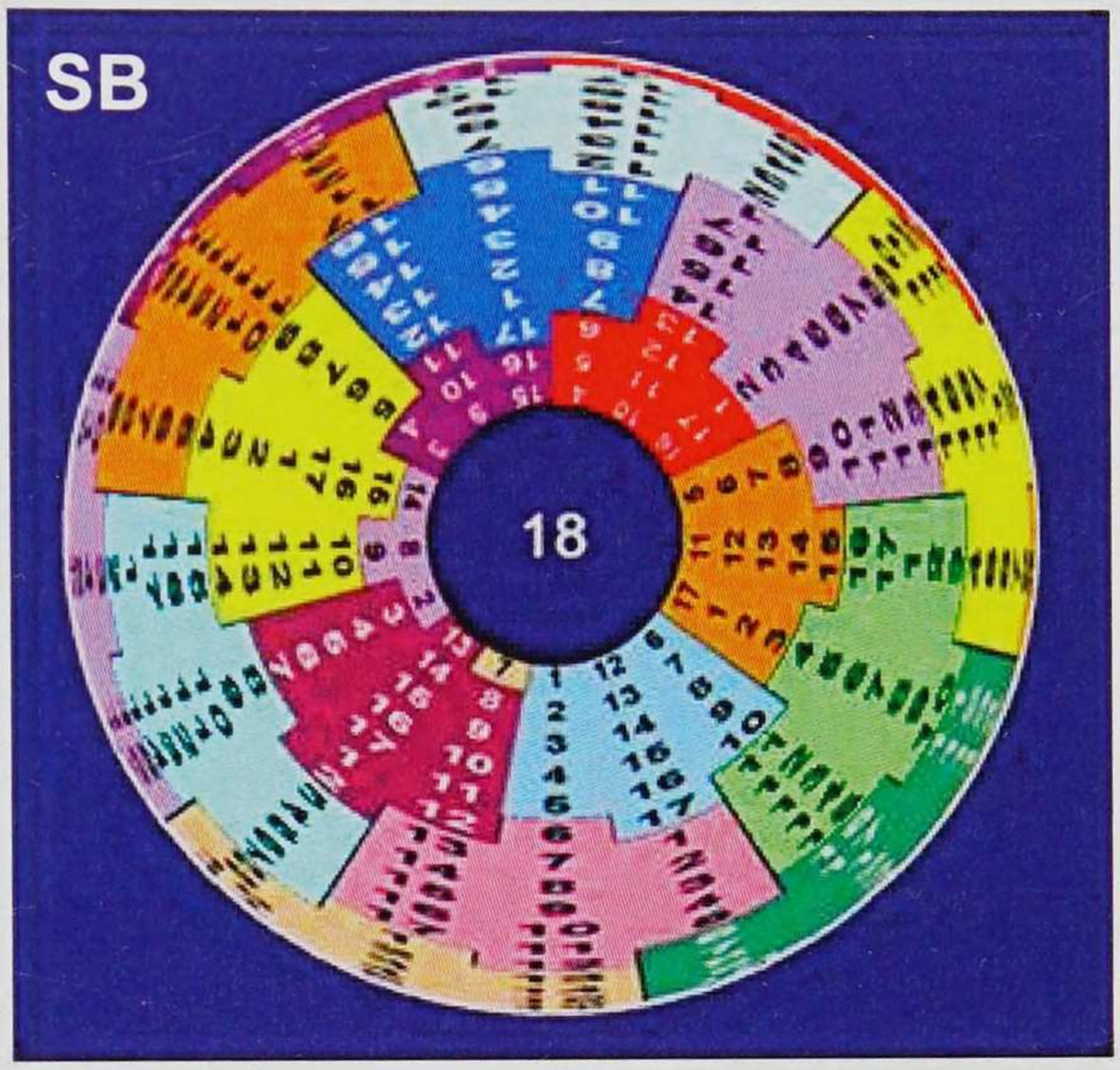
17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Program 5

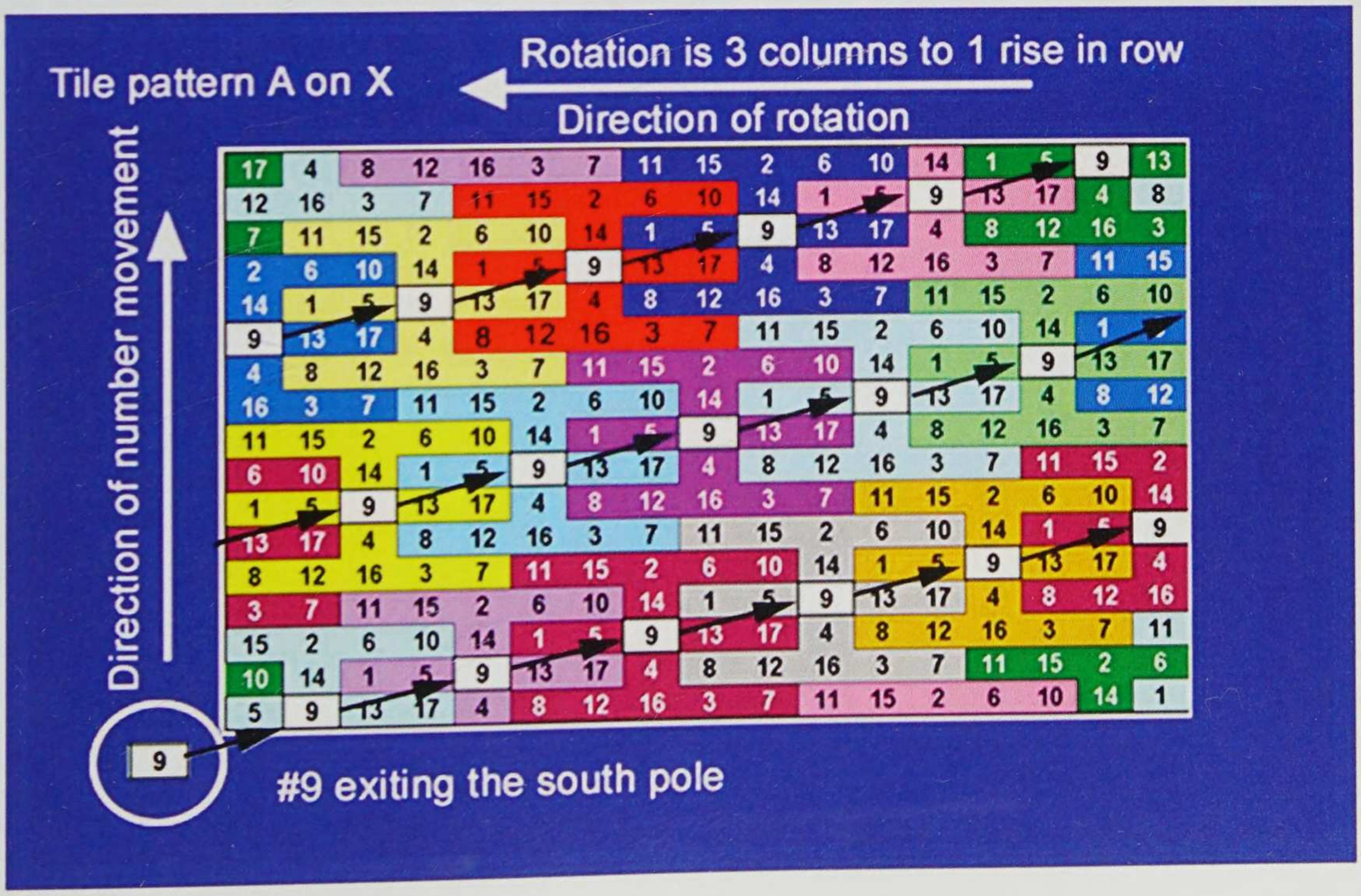
Orbital motion of Electrons according to Geonometry

Here are the tiling patterns **A** and **B** for the size-17 square centered at the North and South poles respectively, wrapped around the surface of the size-12 cube's loom tables' characteristic sphere. **NA** stands for **North** pole and tile pattern **A**. **SB** stands for **South** pole and tile pattern **B.**



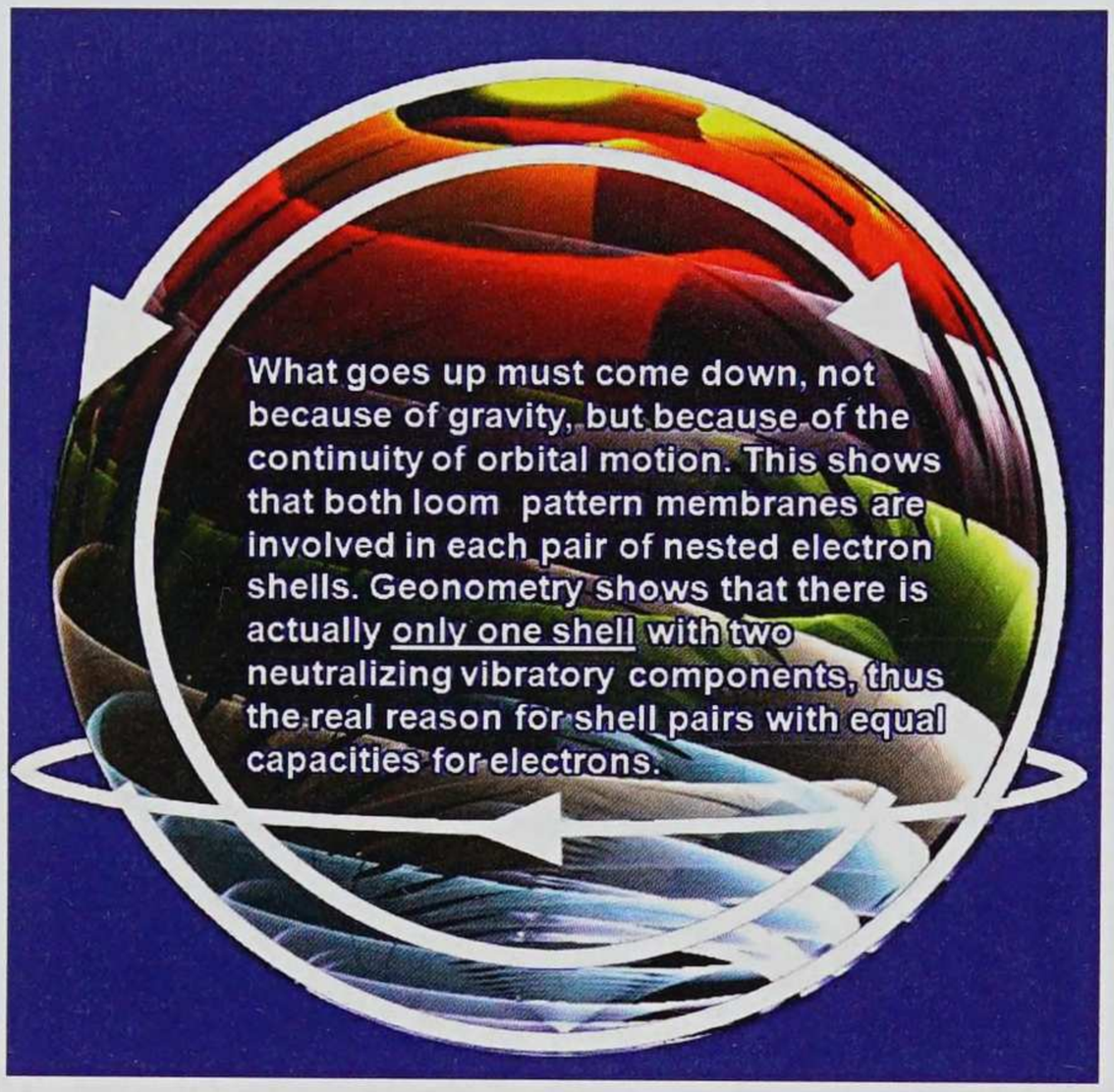


Each pattern has room for **18** electrons on its surface. At any one instant one of the **18** electrons in each shell would pass through a non-tiled polar region, both north and south of each of the paired spherical shells. These regions would be glide areas without any propelling vibrations. Further, due to the density of electrons passing through the opposing poles, these poles would function as the polar axis for the alignment of the shells in the shell-pair.



Program 5

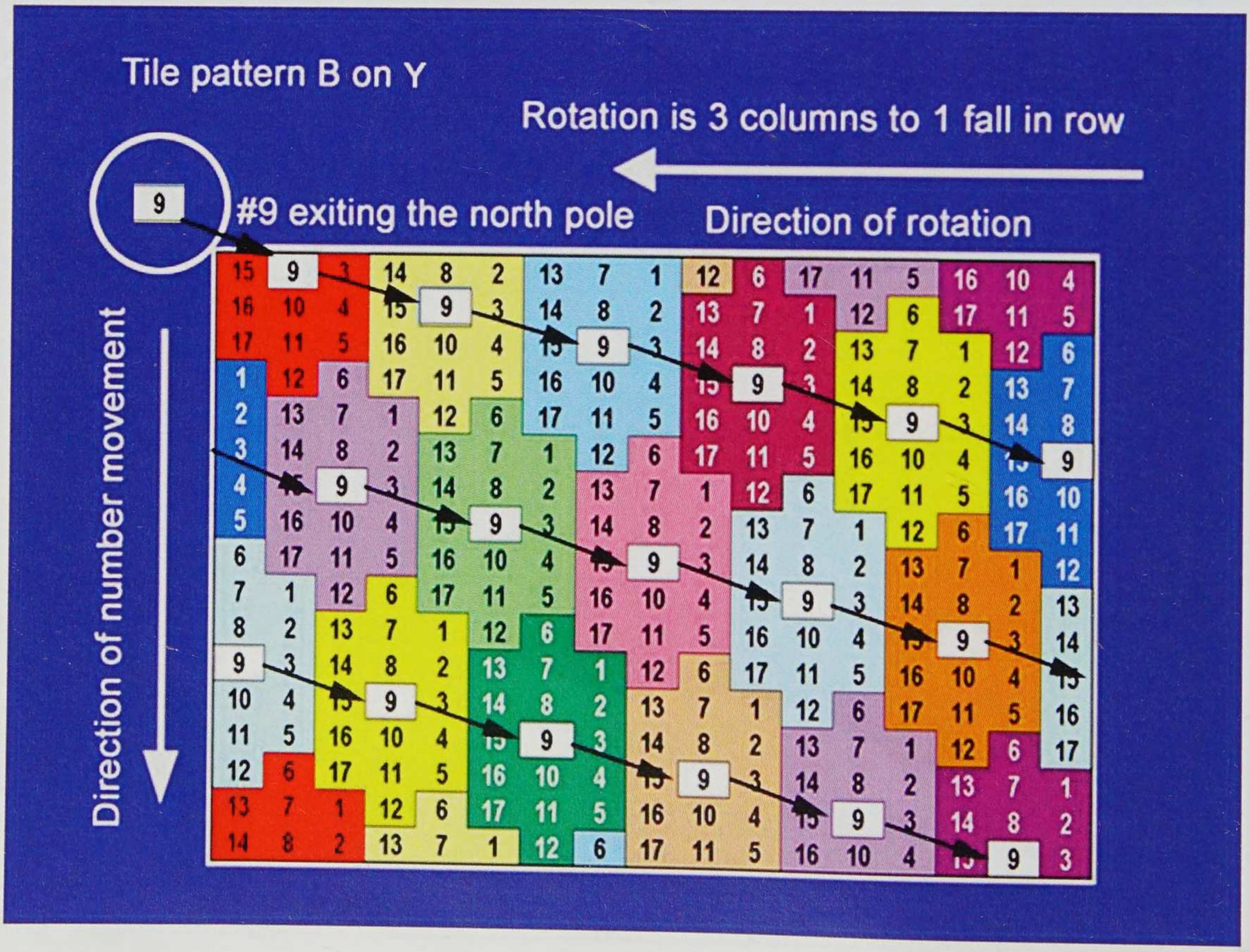
Now, picture the tiling pattern as rotating from right to left. As it does, the center of the next tile jumps 1 row upwards in 3 paces of rotation leftward toward the north pole of the sphere. Now picture the electron riding these vibrations straight upward and crossing over the north pole of the sphere.



What goes up must come down, not because of gravity, but because of the continuity of orbital motion. This shows that both loom pattern membranes are involved by necessity in each pair of nested electron shells. Geonometry shows that actually there is **one shell** with two neutralizing components, thus the real reason for **shell pairs** with equal capacities for electrons.

We've already seen the cloaking property of tiling patterns in Program 4. So these laminated tiling patterns need not even be aligned in order to be neutral outside their shell-pair membrane.

On the other half of the shell-pair, picture the descention of electrons toward the south pole. For every 3 paces of rotation leftward, the electron riding the wave membrane within the tile descends one row straight downward.



Program 5

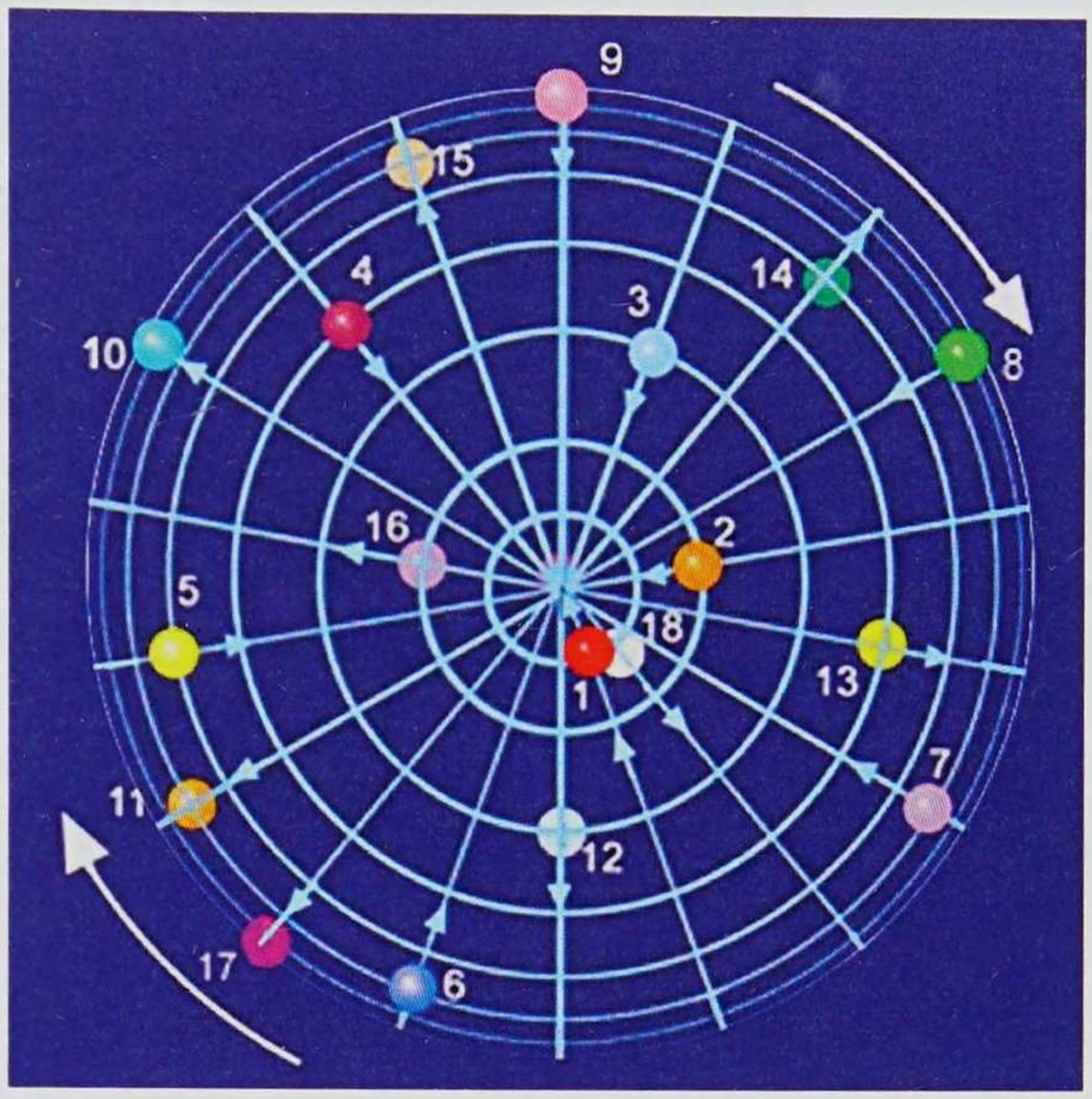
This shows the only orbital pattern that can be inferred from the spinning tiling patterns.

The electrons are numbered consecutively from the top downward.

As the complementary tile patterns are rotating clockwise each electron advances one more level straight up toward the north pole or straight down toward the south pole depending upon which loom membrane it is riding.

Half of the **18** electrons are heading upward toward the north pole and half are heading downward toward the south pole.

Both loom membranes are involved as are both tiling patterns: **A** on **X** and **B** on **Y**.



The double membrane actually has the capacity for **36** electrons before it is completely filled. Hence the reason for atomic physicists' classification of the electron distribution into combined shell-pairs. Geonometry shows how this shell-pair is actually constructed as two laminated complementary joint-neutralizing vibrating membranes.

Further, the vibrations at the poles are zero, so it's a glide area for crossing electrons. All the tiled areas have powering vibrations proportional to the average of the numbers in each tile. So in the case of **18** electrons here, that number is **9,** which is the central number in each tile.

The same construct explains the electron shell pairs having **8** and **32** electrons per shell. In these cases the shell pair consists of two tiling patterns on two laminated vibrating loom membranes of prime-number sizes **7** and **31**, respectively. This was the motivation for showing these size loom tables and tiling patterns in detail back in Programs 2 and 3.

The electron shell containing just **2** electrons is just a simplification of the larger shells where here there is one shell-pair with just two shells, one for the "up" portion and one for the "down" portion of the orbit. Its capacity then is just 2 electrons, each shepparding the other thru mutual repelling electro-parity.

Now, we have already seen back in Program 3 that the tiling patterns on the loom tables could have central numbers that equaled **(n+1)/2** for all Class-1 squares. Thus, we may interpret the central numbers in the A and B tile patterns on complementary loom-tables **X** and **Y**, respectively, to be the energy level of all $\mathbf{2n^2}$ electrons in that shell-pair, thus accounting for the series of consecutive elements which have the same electron energy level in their shell-pairs (**2, 16, 36, 64}**. Thus, we have accounted for all the electrons in terms of size-n patterns A and B wrapped around the associated characteristic sphere of cubes of sizes **4**, **8**, **12** & **16**.

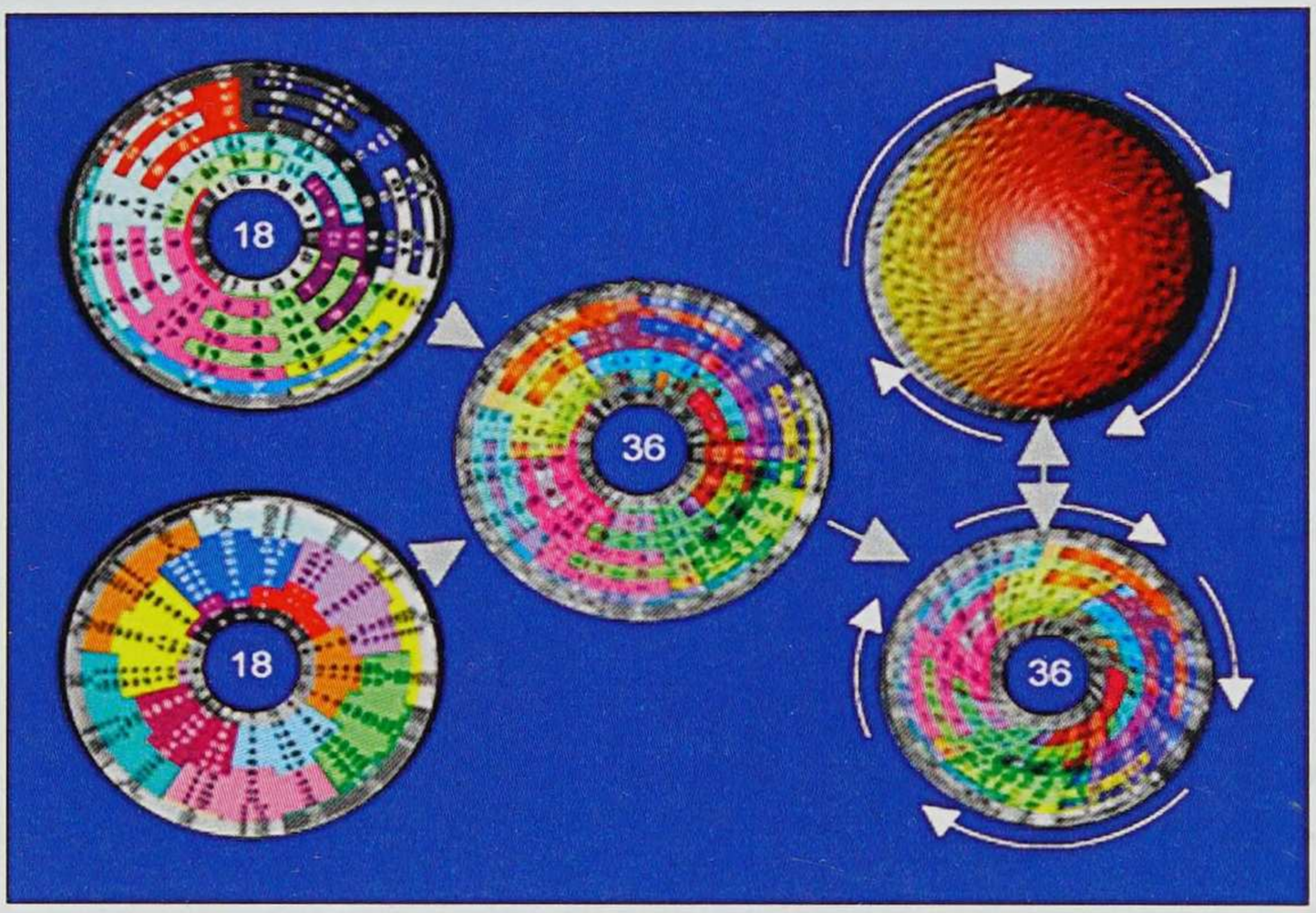
On two pages hence, is a table of shell accommodation for the next electron in going to the next element in sequence. Observe that there is only once (element 103) which retains the 3rd electron in the outermost shell. When the outer shell-pair is full, in all cases except this one instance, the electrons are all passed on lower shell(s) on the addition of the next electron because of the instability of the outermost shell with **3** electrons. This anomaly is coming from the 3rd octal in the size-4 characteristic sphere that was identified earlier on page 5-15. It's unstable because the octals in each sphere when full must <u>equal</u> the characteristic number of the cube. Geonometry cannot provide the cause for this anomaly, only the reason for its occurrence.

Program 5

The Double-Shell Cloaking of Electron Shells

It is physically impossible to isolate one individual electron to take its temperature, so to speak, to determine its energy level. Atomic physicists have derived their categorization of electron energy levels by averaging thousands upon thousands of atom-smasher measurements. So their categorizations are only the averages of what is detectable.

Geonometry tells a different story. It indicates that there must be **18** energy levels of electrons in the innermost shell of Krypton and Xenon of which only **17** are non-zero. And that these energy levels derive from the spherical membrane vibrating at **17** identical energies in each of the size-12 characteristic spherical shells in their shared electron shell-pair. And further, it is these vibrations that drive the electrons in their orbits with equal velocities around the common compact nucleus.



These distinct electron shells can never be observed in pairs directly by atomic physicists because the pairs of same-size loom tables involved cloak one another regardless of the location of the dual tiling patterns relative to one another – and that was proven mathematically in Program 4. The only alignment that is necessary in this interpretation is that over the poles.

But such alignment is guaranteed if the vibrating membranes spin on a common axis and that was demonstrated quite vividly here in Program 5.

Atomic Physics has determined that there must be two equal but opposite spins for the electrons in any shell-pair as **+½** and **-½**. In the case for Krypton and Xenon here, that spin sign is the opposite up-down directions of the electrons in the complementary tiling patterns on complementary loom-tables spinning in the same direction. The centers of the tiles in each spinning complementary tiling pattern were seen to be the average of all the numbers in each tile, **9**, accounting for the value **½** for the size-18 shells of the shell-pair:

(5.6) $\bar{x} = \bar{y} = \frac{1}{2}n$

Extension of electron distribution pattern to other shell-pairs

The same construct explains the electron shell-pairs having **8** and **32** electrons per shell. In these cases the shell-pair consists of **2** tiling patterns on **2** laminated vibrating loom membranes of prime-number sizes **7** and **31,** respectively. The centers of the tiling patterns are **4** and **16** respectively.

The electron shell containing just **2** electrons is just a simplification of the larger shells. Here there is one shell-pair with just two shells, one for the up and one for the down portion of the orbit. Its capacity then is just **2** electrons with spins of **+½** and **–½**.

The spin detected in one shell-pair may be different from the spin of any other shell-pair. We're only talking about different shell-pairs here, not individual shells.

Electron Fill Patterns

Electron Count		Electron-fill Sequence			
#	Element	1	2	3	4
1	Hydrogen	1			
2	Helium	1	1		
3	Lithium	1	1	1	
4	Berylium	1	1	2	
5	Boron	1	1	3	
6	Carbon	1	1	4	
7	Nitrogen	1	1	5	
8	Oxygen	1	1	6	
9	Fluorine	1	1	7	
10	Neon	1	1	8	

Electron Count		Electron-fill Sequence					
#	Element	1	2	3	4	5	6
19	Potassium	1	1	8	8	1	
20	Calcium	1	1	8	8	2	
21	Scandium	1	1	8	9	2	
22	Titanium	1	1	8	10	2	
23	Vanadium	1	1	8	11	2	
24	Chromium	1	1	8	13	1	
25	Manganese	1	1	8	13	2	
26	Iron	1	1	8	14	2	
27	Cobalt	1	1	8	15	2	
28	Nickel	1	1	8	17	1	
29	Copper	1	1	8	18	1	
30	Zinc	1	1	8	18	2	
31	Gallium	1	1	8	18	3	
32	Germanium	1	1	8	18	4	
33	Arsenic	1	1	8	18	5	
34	Selenium	1	1	8	18	6	
35	Bromine	1	1	8	18	7	
36	Krypton	1	1	8	18	8	
		1	2	3	4	5	6

Electron Count		Electron-fill Sequence					
#	Element	1	2	3	4	5	6
11	Sodium	1	1	8	1		
12	Magnesium	1	1	8	2		
13	Aluminum	1	1	8	3		
14	Silicon	1	1	8	4		
15	Phosphorus	1	1	8	5		
16	Sulfur	1	1	8	6		
17	Chlorine	1	1	8	7		
18	Argon	1	1	8	8		

Electron Count		Electron-fill Sequence					
#	Element	1	2	3	4	5	6
37	Rubidium	1	1	8	18	8	1
38	Strontium	1	1	8	18	8	2
39	Yttrium	1	1	8	18	9	2
40	Zirconium	1	1	8	18	10	2
41	Niobium	1	1	8	18	12	1
42	Molybdenum	1	1	8	18	13	1
43	Technetium	1	1	8	18	13	2
44	Ruthenium	1	1	8	13	15	1
45	Rhodium	1	1	8	18	15	2
46	Palladium	1	1	8	18	17	0
47	Silver	1	1	8	18	17	1
48	Cadmium	1	1	8	18	18	2
49	Indium	1	1	8	18	18	3
50	Tin	1	1	8	18	18	4
51	Antimony	1	1	8	18	18	5
52	Tellurium	1	1	8	18	18	6
53	Iodine	1	1	8	18	18	7
54	Xenon	1	1	8	18	18	8
		1	2	3	4	5	6

Electron Count		Electron-fill Sequence							
#	Element	1	2	3	4	5	6	7	8
55	Cesium	1	1	8	18	18	8	1	
56	Barium	1	1	8	18	18	8	2	
57	Lanthanum	1	1	8	18	18	9	2	
58	Cerium	1	1	8	18	19	9	2	
59	Praseodymium	1	1	8	18	21	8	2	
60	Neodymium	1	1	8	18	22	8	2	
61	Promethium	1	1	8	18	23	8	2	
62	Samarium	1	1	8	18	24	8	2	
63	Europium	1	1	8	18	25	8	2	
64	Gadolinium	1	1	8	18	26	8	2	
65	Terbium	1	1	8	18	27	8	2	
66	Dysprosium	1	1	8	18	28	8	2	
67	Holmium	1	1	8	18	29	8	2	
68	Erbium	1	1	8	18	30	8	2	
69	Thulium	1	1	8	18	31	8	2	
70	Ytterbium	1	1	8	18	32	8	2	
71	Lutetium	1	1	8	18	32	9	2	
72	Hafnium	1	1	8	18	32	10	2	
73	Tantalum	1	1	8	18	32	11	2	
74	Tungsten	1	1	8	18	32	12	2	
75	Rhenium	1	1	8	18	32	13	2	
76	Osmium	1	1	8	18	32	14	2	
77	Iridium	1	1	8	18	32	15	2	
78	Platinum	1	1	8	18	32	17	1	
79	Gold	1	1	8	18	32	18	1	
80	Mercury	1	1	8	18	32	18	2	
81	Thallium	1	1	8	18	32	18	3	
82	Lead	1	1	8	18	32	18	4	
83	Bismuth	1	1	8	18	32	18	5	
84	Polonium	1	1	8	18	32	18	6	
85	Astatine	1	1	8	18	32	18	7	
86	Radon	1	1	8	18	32	18	8	
		1	2	3	4	5	6	7	8

Electron Count		Electron-fill Sequence							
#	Element	1	2	3	4	5	6	7	8
87	Francium	1	1	8	18	32	18	8	1
88	Radium	1	1	8	18	32	18	8	2
89	Actinium	1	1	8	18	32	18	9	2
90	Thorium	1	1	8	18	32	18	10	2
91	Protactinium	1	1	8	18	32	20	9	2
92	Uranium	1	1	8	18	32	21	9	2
93	Neptunium	1	1	8	18	32	22	9	2
94	Plutonium	1	1	8	18	32	24	8	2
95	Americium	1	1	8	18	32	25	8	2
96	Curium	1	1	8	18	32	25	9	2
97	Berkelium	1	1	8	18	32	27	8	2
98	Californium	1	1	8	18	32	28	8	2
99	Einsteinium	1	1	8	18	32	29	8	2
100	Fermium	1	1	8	18	32	30	8	2
101	Mendelevium	1	1	8	18	32	31	8	2
102	Nobelium	1	1	8	18	32	32	8	2
103	Lawrencium	1	1	8	18	32	32	8	3
104	Rutherfordium	1	1	8	18	32	32	10	2
105	Dubnium	1	1	8	18	32	32	11	2
106	Seaborgium	1	1	8	18	32	32	13	1
107	Bohrium	1	1	8	18	32	32	13	2
108	Hassium	1	1	8	18	32	32	14	2
109	Meitnerium	1	1	8	18	32	32	15	2
110	Darmstadtium	1	1	8	18	32	32	18	0
111	Roentgenium	1	1	8	18	32	32	18	1
112	Copernicium	1	1	8	18	32	32	18	2
113	Ununtrium	1	1	8	18	32	32	18	3
114	Flerovium	1	1	8	18	32	32	18	4
115	Ununpentium	1	1	8	18	32	32	18	5
116	Livermorium	1	1	8	18	32	32	18	6
117	Ununseptium	1	1	8	18	32	32	18	7
118	Ununoctium	1	1	8	18	32	32	18	8
		1	2	3	4	5	6	7	8

Program 5

Electron Distribution among Shells

The next slide helps to comprehend this somewhat complex electron tumbling process as electrons are added.

On the prior page is the sequence table for which shell gets an electron as atoms are derived from the preceding one in the electron shell filling process. The colored numbers indicate where unusual rearrangements occur.

Sorted on shell sequence

Periods	s
1	K L
	1 2

	s	p	
2 & 3	K L	M N	← Shell sequence
	1 3	2 4	← Fill sequence

← Energy level

	s	p	d
4 & 5	K L	M N	O P
	1 3	2 4	2 5

	s	p	d	f
6 & 7	K L	M N	O P	Q R
	1 3	2 4	2 5	2 6

The elements with all applicable shells filled to capacity are called "noble" elements and are highlighted in yellow. These are the elements in the rightmost column of the Periodic Table.

Note that this chart has **8** columns, not **7** as seen in all current textbooks on Chemistry. It appears rather redundant here to have all **1**'s in **2** columns as opposed to all **2**'s in **1** column, but Geonometry indicates that there must be **8** electron shells, not **7**. Having shell-pairs for all the electrons is more consistent with the double counting of additional electrons in every other noble element beginning with Helium.

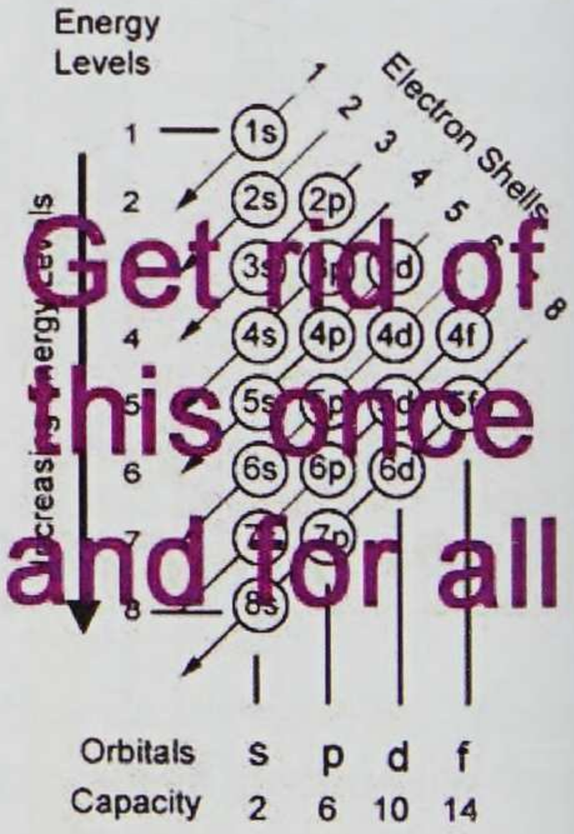
It is important to note that the numbered sequence relates strictly to the sequence in which electron are added to the atoms.

That sequence indicated by Geonometry is shown here sorted on the energy level. Geonometry indicates all of these shell-pairs must be adjacent for them to function together properly; they should not be separated as some scientists picture them being split as was seen earlier.

Observe that the **shell-pairs** are half-filled with the 1st **2** electrons in the outermost shell-pair and then the balance from the innermost to outermost vacant half-shell.

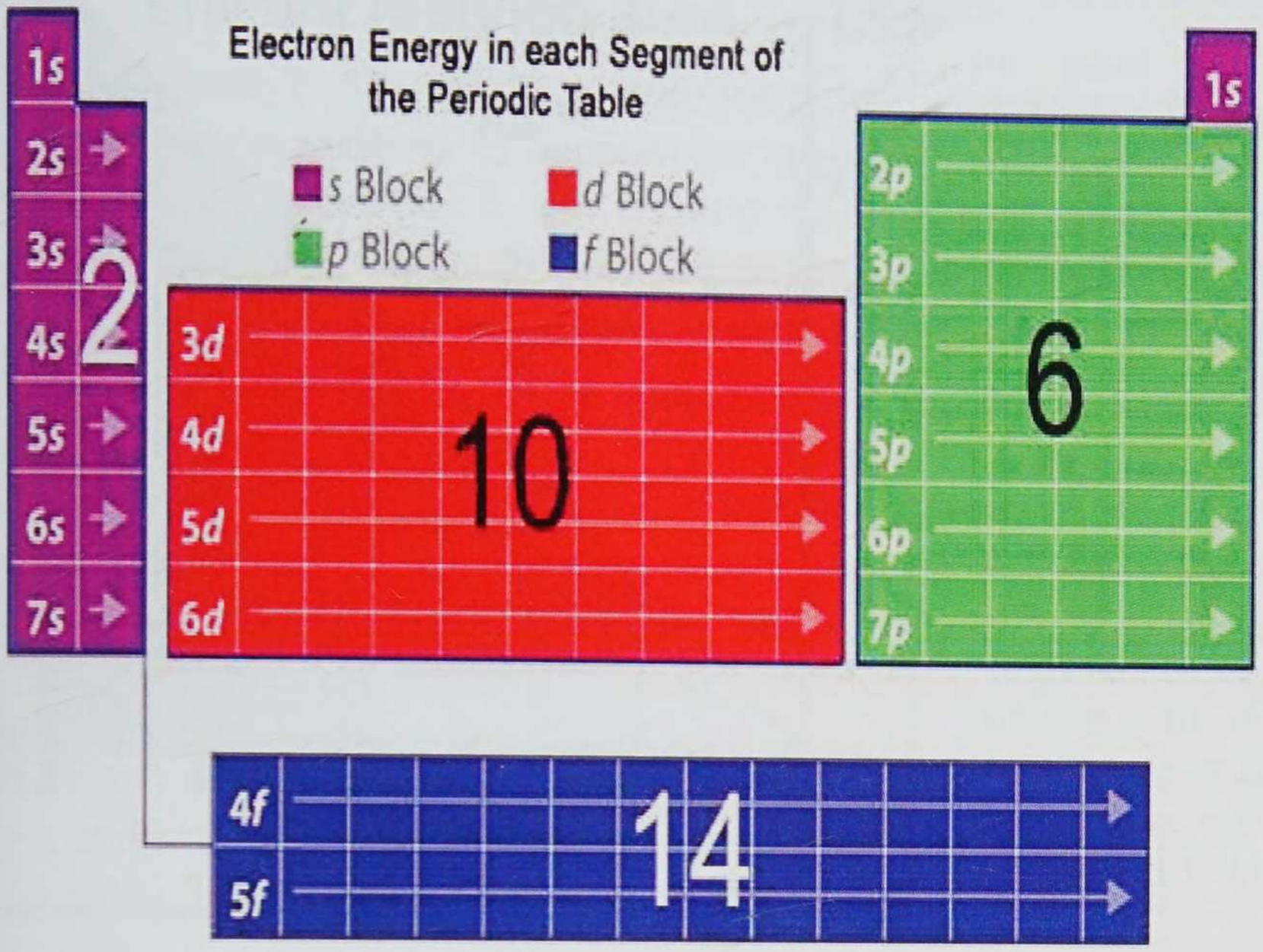
On the next page is the distribution of these orbitals throughout the Periodic Table. Observe that every shell of the same size has all of its electrons at one distinct energy level.

At right is the atomic scientists' contorted way of looking at these orbitals from an abbreviated perspective. This may be more convenient for scientists but this just makes understanding more obtuse for students. It took me two days to see that this chart matched up to the Periodic Table on the next page There is pictured the easy way to view these average electron energy levels without this confusing diagonally-oriented graph. If you want all the energies for a particular atom, just trace your way back up through the table to Hydrogen. All this chart is on the right is a shorthand notation for recovering all the energies for consecutive orbitals. You don't need it.



Program 5

Orbital Capacities



Here is the distribution of these orbitals throughout the Periodic Table. Observe that every shell has all its electrons at its own distinct energy level. Observe that the orbital capacity of each section **s, p, d, f**, is **2, 6, 10** & **14,** respectively.

And note that these capacities are **2** times the series of odd-prime numbers **1, 3, 5, 7.**

So once again, odd-prime-numbers are playing a prominent cohesive role in the organization of atoms!.

Geonometry's view of the electron distribution pattern

Atomic Scientists have determined that the outer **2** electrons tumble inward when the 3rd electron is added to the outer shell. The inward movement transfers **3** electrons to the nearest shell-pair with vacant capacity. From there on, the outer shell-pair of capacity **2** remains vacant until the newly receptive shell-pair reaches its capacity. Then the process repeats itself.

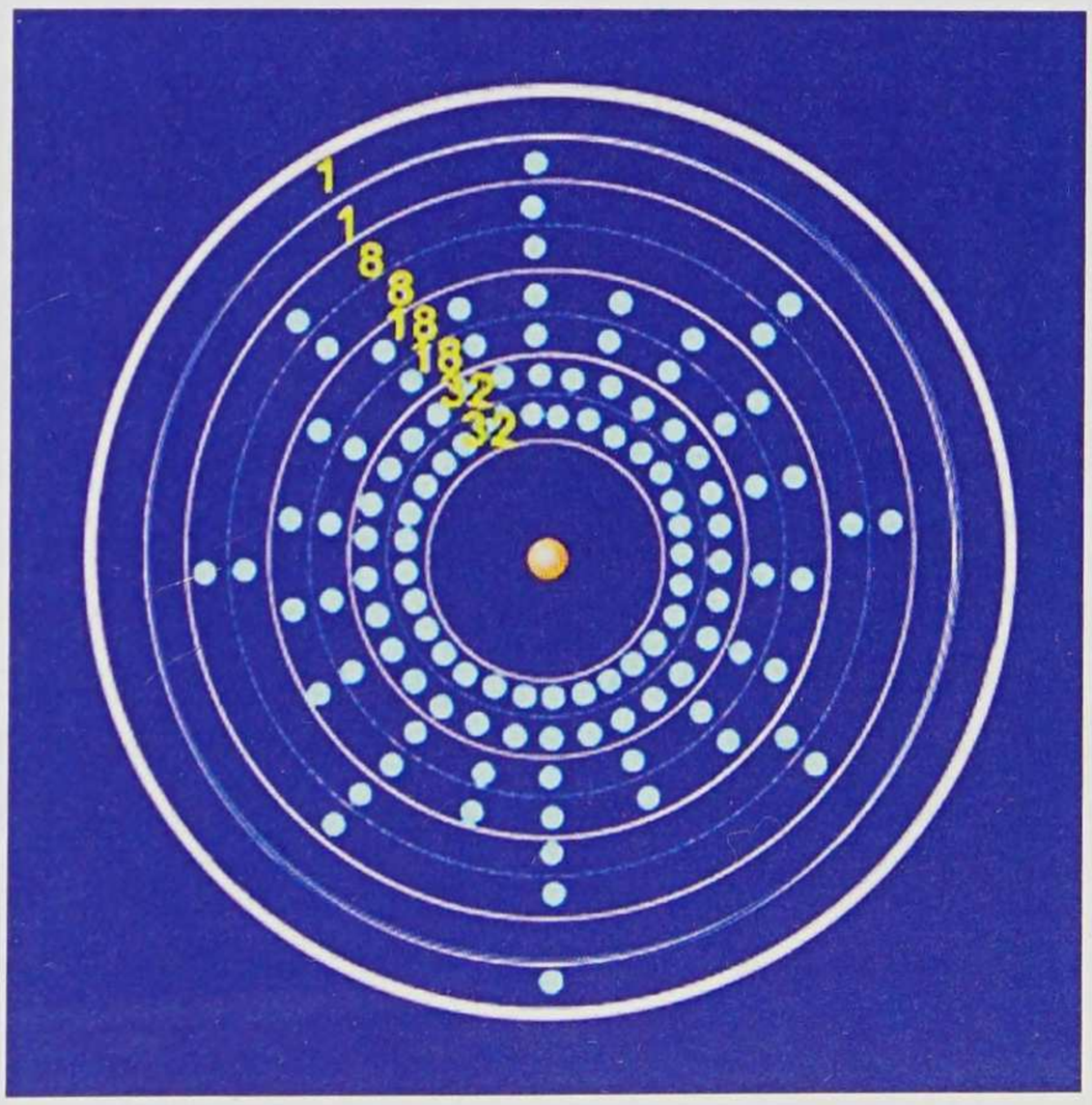
Laminated shell-pairs have the joint capacities of **2**, **16**, **36** and **64**.

All of this makes a whole lot more sense and is a more organized framework than the helter-skelter motion envisioned by today's atomic physicists, which is somewhat reminiscent of the old Niels-Bohr model shown at the beginning of this program.

Of course, the physics had to come first before the math could be applied, but Geonometry depicts the electron properties quite vividly for better explaining the inherent source of the physics.

So now it should be evident just how the natural fundamental geometric patterns inherent in Geonometry can be applied to explain naturally occurring phenomenon at the sub-dimensional space of the atom!

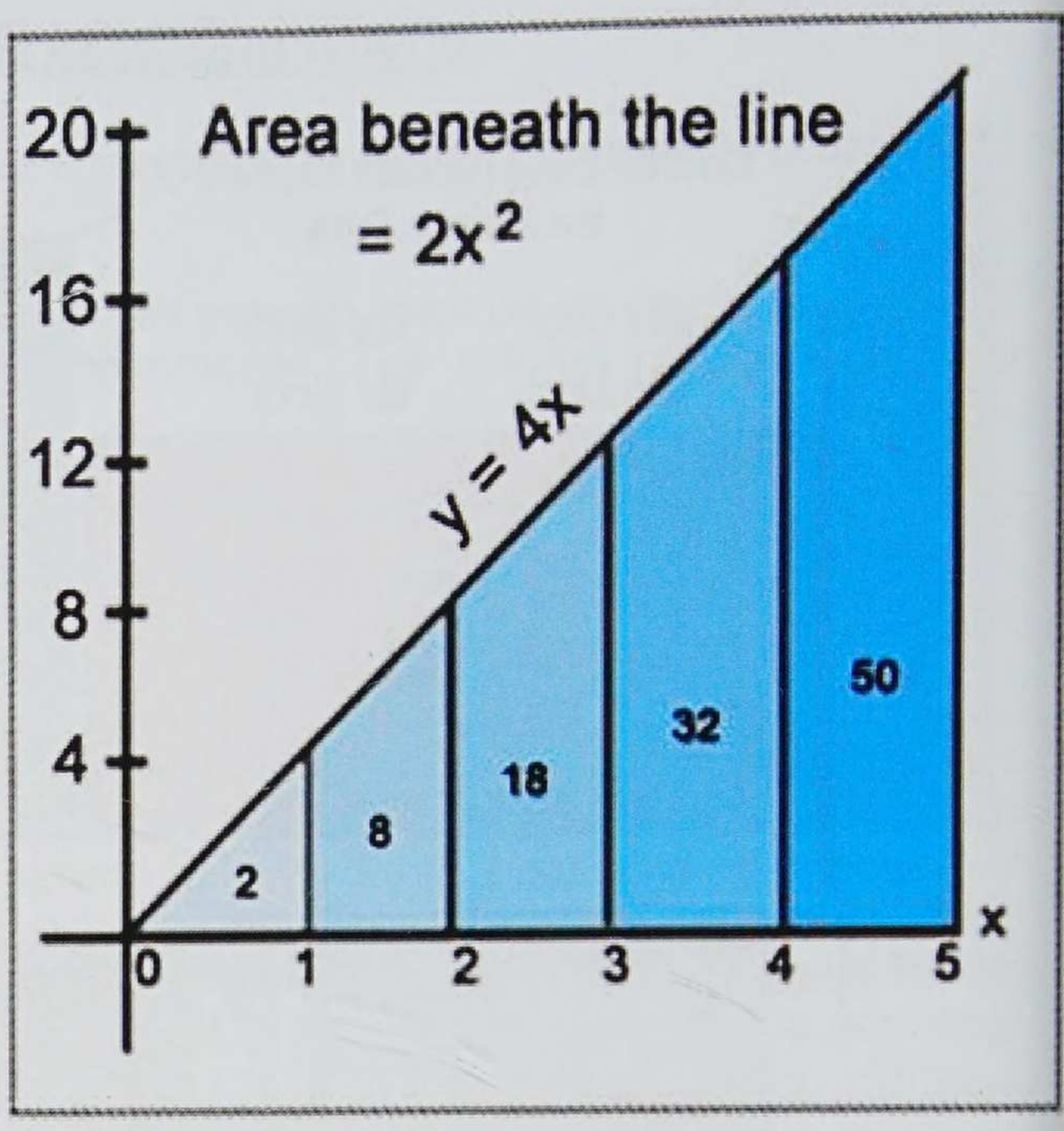
That is why Geonometry has been dubbed the "new wow math".



Program 5

Derivation of the Formula for the Number of Electrons in Shells

For those who have had at least an introduction to Integral Calculus 101, here is the derivation of the formula for the number of electrons per level of electron shell. The count of electrons in the series-level for variable **x** is the area under the line **y = 4x**, from **0** out to its numbered level.



Integral calculus shows that the number of electrons per level **x** is $2x^2$:

(5.8) $$\int_0^y y\,\partial y = \int_0^x 4x\,\partial x = 2x^2$$

Note that **2 – 0 = 2**; **8 – 2 = 6**; **18 – 8 = 10**; and **32 – 18 =14**. These are the orbital capacities **2, 6, 10** & **14** of the successive shells, **s, p, d, f**, respectively (re: Periodic Table on prior page).

Derivation of the number series for the number of elements in consecutive periods of the Periodic Table

Now the series of numbers for the count of electrons or positrons in full electron shells among both matter and antimatter atoms can be constructed from simple operations of basic arithmetic on the basic harmonic series **1** thru **4** as indicated by the operations below. And this produces the numerical solution to the calculus formula but now built upon a perfectly harmonic pattern:

	Anti-matter	Matter	
$2x^2$	[32, 32, 18, 18, 8, 8, 2,	2, 8, 8, 18, 18, 32, 32]	Split numbers into halves & separate.
$4x^2$	[64, 36, 16, 4, 16, 36, 64]		Square the numbers.
$2x$	[8, 6, 4, 2, 4, 6, 8]		Multiply series by **2**.
x start	[4, 3, 2, 1, 2, 3, 4]		Basic harmonic number pattern

And this produces the numerical solution to the calculus formula (**5.8**)

Everyone can now contemplate about what was just presented. It was shown here in Program 5 that Geonometry explains the number and the distribution pattern of electrons plus the existence of neutral complementary pairs of electron shells in the atoms of the noble elements, including Helium. That was a totally new mathematical explanation. On this page, the formula is confirmed by the classical mathematics of integral calculus as well as the simple 1-dimensional arithmetical operations on a perfect harmonic string.

The distribution of anti-matter will be addressed next in Program 6.

Program 5

Notes

1. The sizes **7**, **17** and **31** characteristic complementary tiling patterns furnish the vibrating membrane patterns to account for three of four electron shell pairs. These four shell pairs have capacities for **2**, **16**, **36** and **64** electrons, and the individual shells have electron capacities of (**1** & **1**) (**8** & **8**) (**18** & **18**) (**32** & **32**) respectively in that order. Characteristic spheres of Geonomic cubes of size **n = 4x** have $\mathbf{2x^2}$ octals which sum to the cube's characteristic number. This accounts for the shell capacities of electrons: **2**, **8**, **18**, **32**

2. The shell pairs account for the total pair capacity, **16**, **36**, **64** which is $\mathbf{4x^2}$ for **x>1**. For **x = 1**, what is known for certain in Atomic Science stems from the possibility that the unseen complementary shell also with a capacity of **2** is related to antimatter and cannot be detected among ordinary matter. This is where the two integrated universes separate.

3. Each electron shell-pair has **2** glide regions with no pulsating vibrations, one at the north and one at the south pole regions where electrons go from "up" to "down" and vice versa within their rotating shell-pair, thus accounting for the **±½** spins among electrons.

4. The cloaking property of Class-1 complementary tiling patterns prevent atomic scientists from directly observing these electron shell pairs. Everything claimed by Atomic Science regarding electron distribution around the nucleus has been inferred from energy measurements but never observed directly.

5. Atomic Science claims that the additional electron added in going from one atom to the next enters an orbit associated with the lowest vacant energy level.

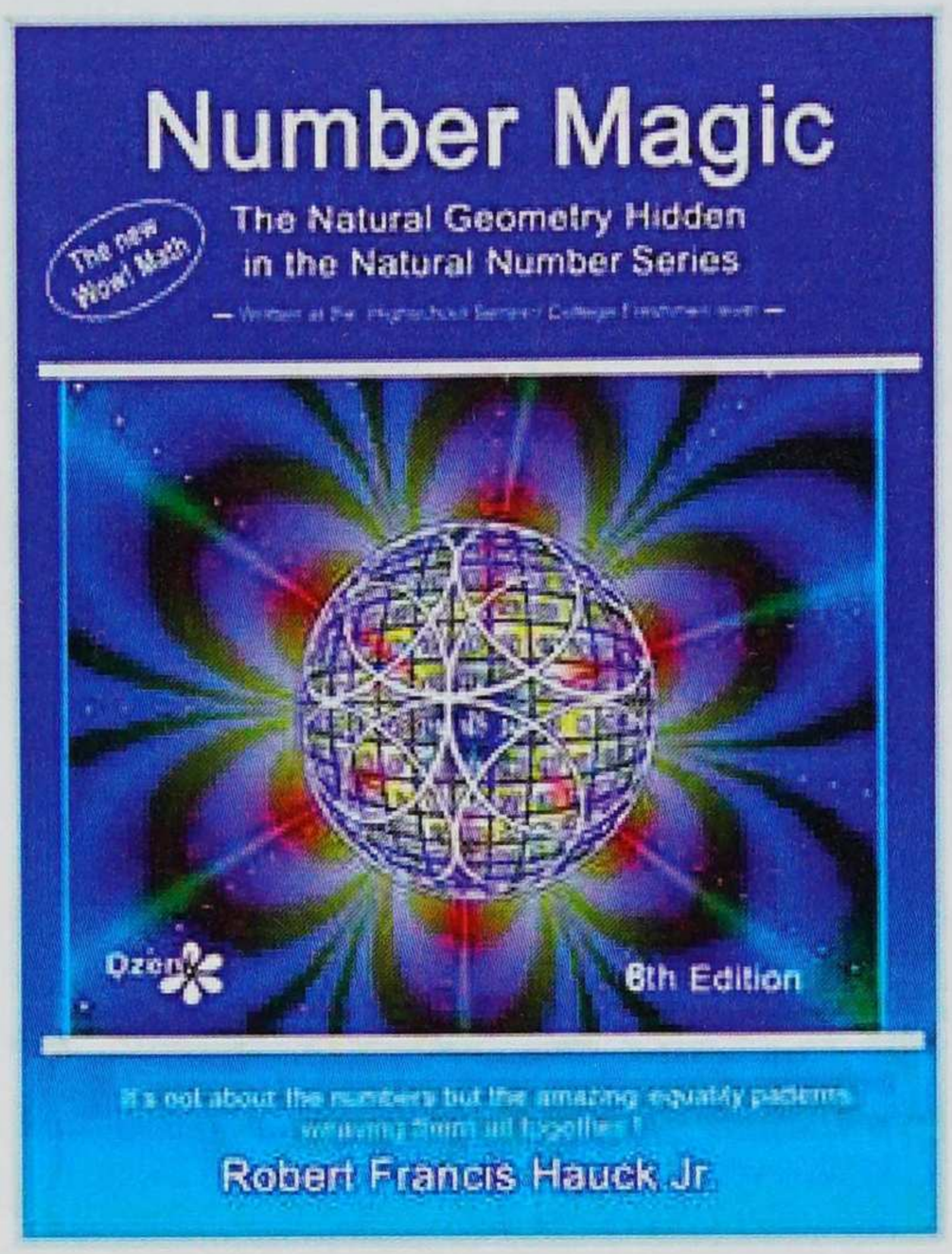
6. Geonometry claims that each shell-pair fills to only half its capacity before additional electrons begin filling in the next level closer to the nucleus. After the electron shell nearest the nucleus gets half full, it continues accepting electrons until its laminated other-half gets completely filled. Then the next lesser distant shell-pair starts getting filled and so on until the outermost shell-pair with capacity of **2** electrons gets completely filled in <u>again</u>. This sequence is necessary to explain why the shells function together in pairs. Shell-pairs are necessary to complete the north-to-south pole and south-to-north pole transitions in the orbital pattern. These electrons' motions are powered by the rotational multi-layered pulsating geonomic fabric of space surrounding the nucleus.

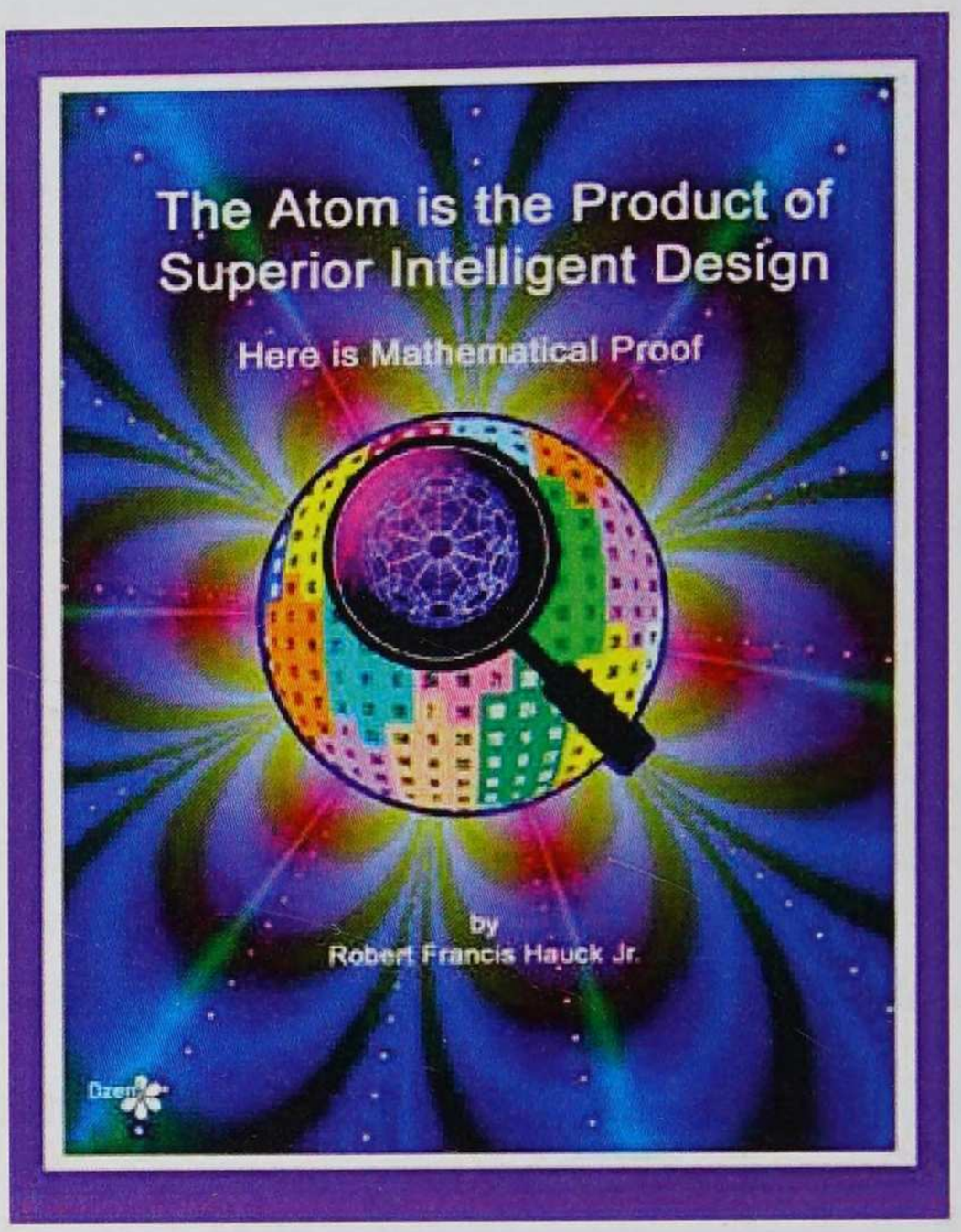
7. The higher the number of the shell-pair, the higher is the energy of its orbiting electrons. All electrons within the same shell-pair possess the same energy. Electrons within a shell-pair differ in spin energy by **±½** between the two to account for the upward and downward velocity described by Geonometry.

Program 5

We have come to the end of Program 5 of the new math, Geonometry.

Here are the author's books upon which this program was based:





Number Magic – The Natural Geometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension

Ninth Edition (360+ pages)

The Atom is the Product of Superior Intelligent Design
Here's mathematical proof

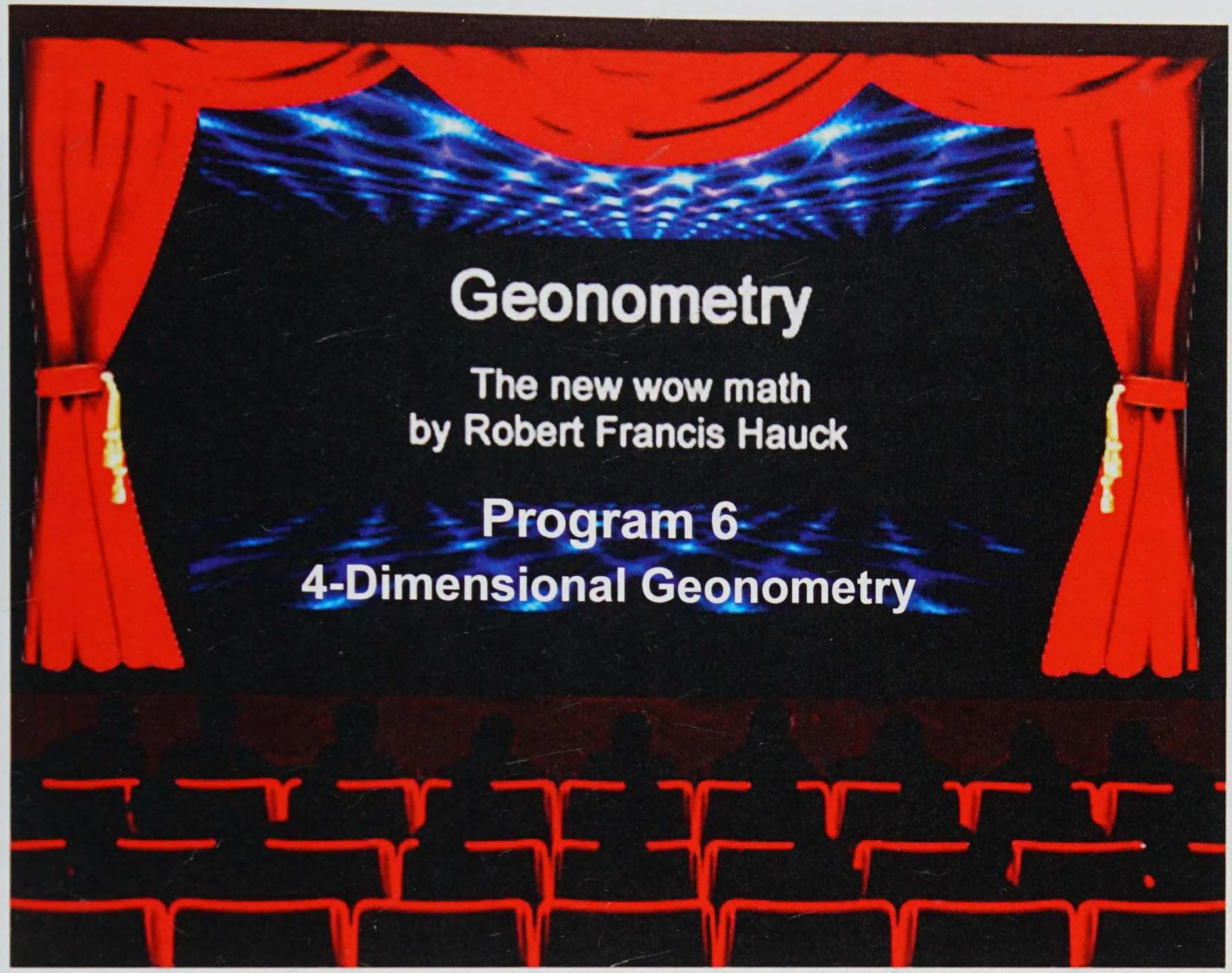
ISBN: 978-1-461-07458-8

Printed in color

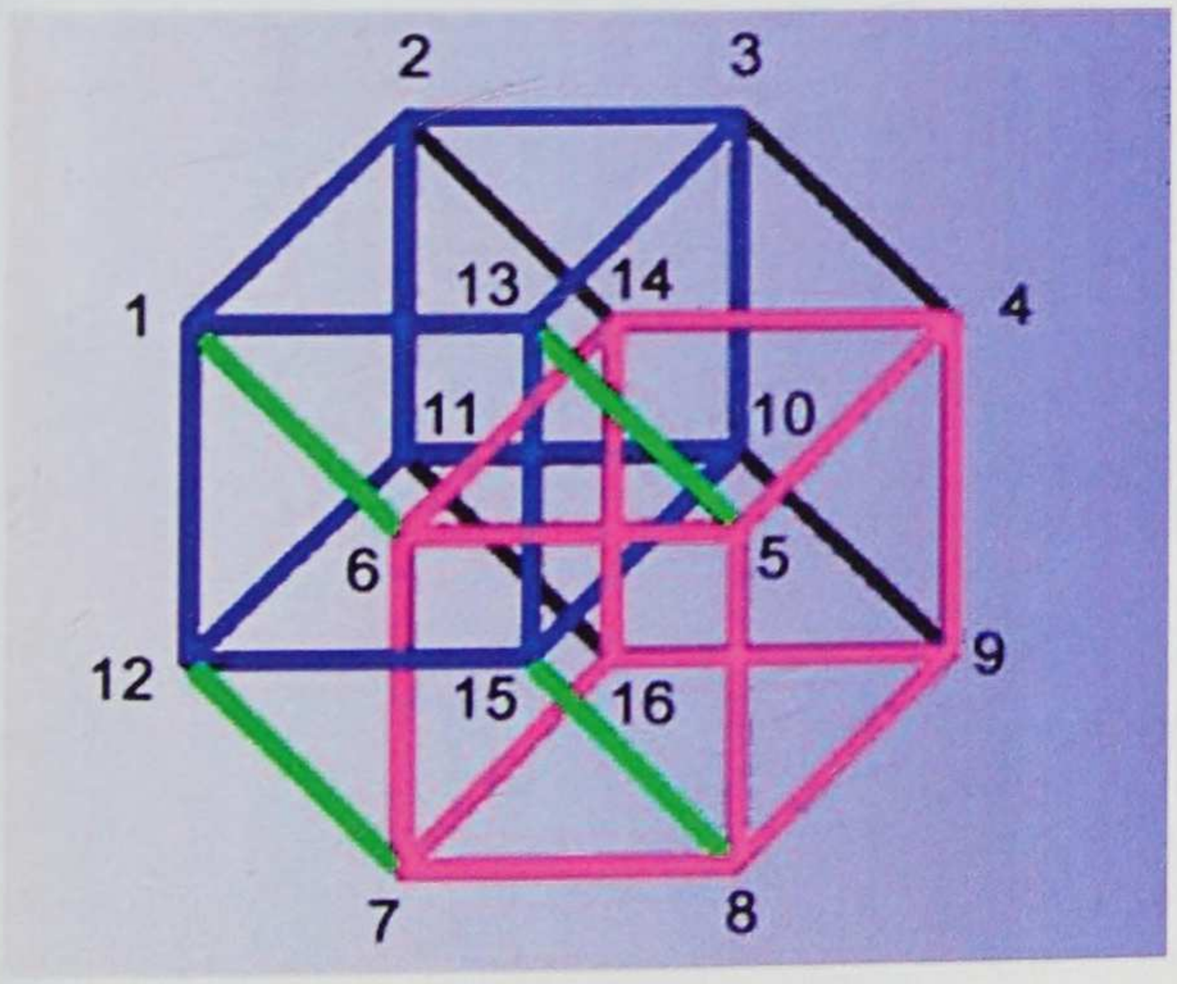
Third Edition (42 pages)

In the next program we shall explore 4-dimensional space as never seen before in such a definitive manner. There we will discover equal-summing patterns in Class-4 hypercubes that will unlock the properties of quarks in subdimension **1/4D**. You will see how the discoveries in Geonometry confirm what sub-atomic scientists have discovered through their linear accelerators – and why they named the six basic quarks to what they did.

Program 6



In this program, we shall explore the amazing spatial properties of the fourth dimension. It's quite a mind-trip!

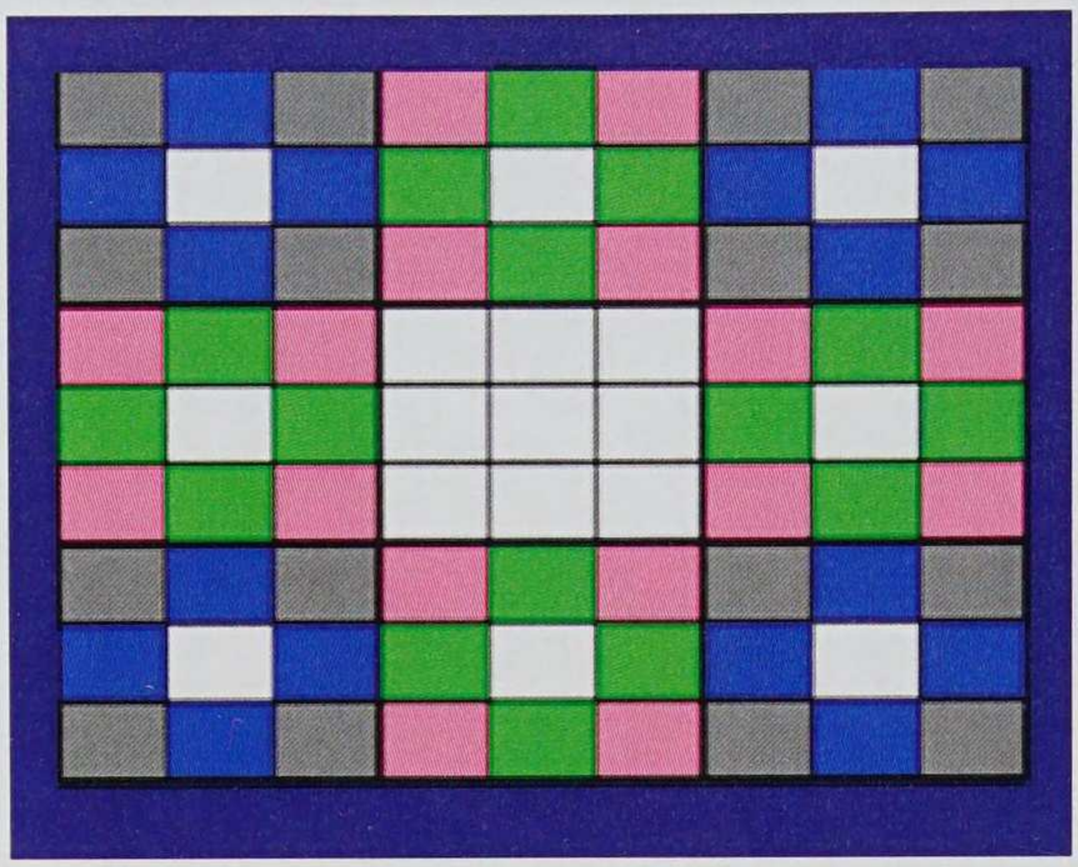


Here is a 3-dimensional view of a 4-dimensional hypercube. Like the square and cube earlier, this figure shows the basic rectangular structure of 4-dimensional space. Note that now the quadracubic enclosure possesses **16** 90° corners or points. This structure need only have equal length sides parallel to each of the 4 axes; they need not to be equal be-tween different axes. As there is no description of this structure in classical math, it will be called a **hexadectal**, which means a 16-corner "boxahedron".

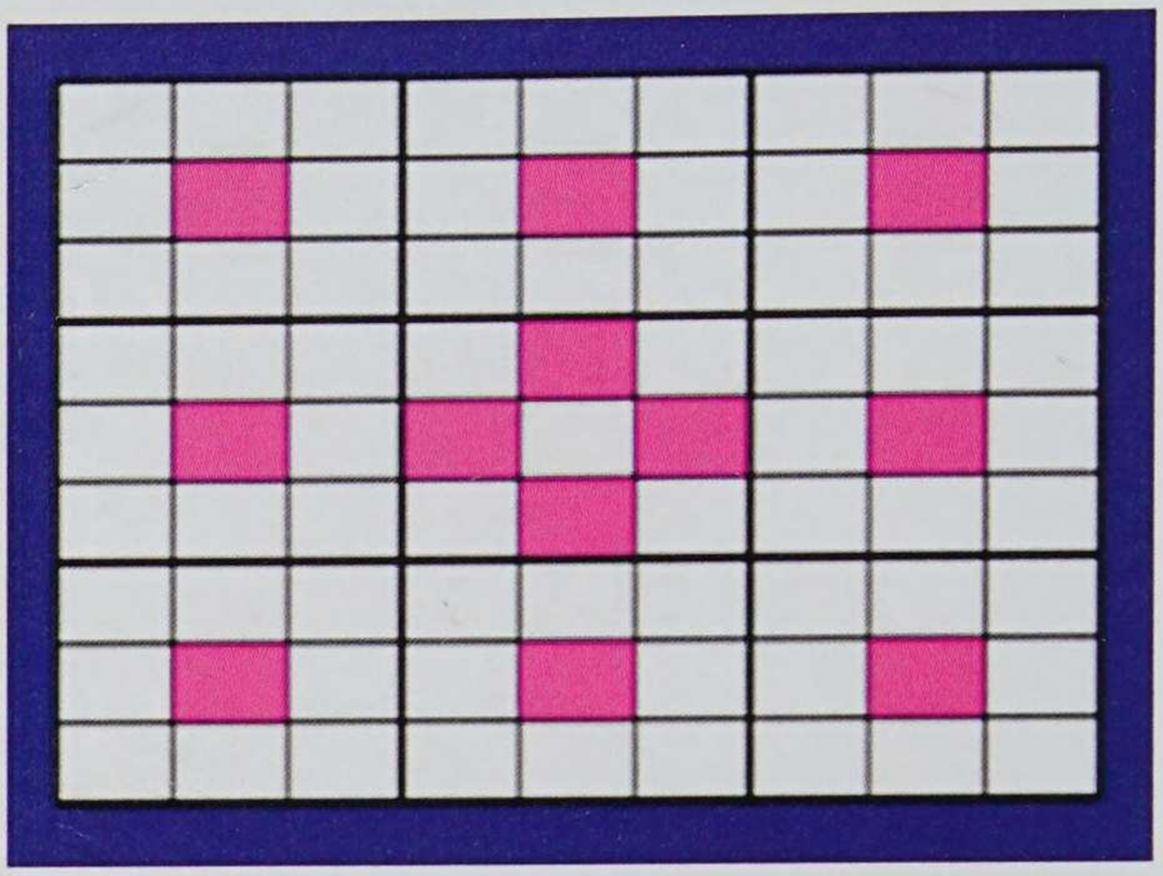
Program 6

Definitions

Hexadectals



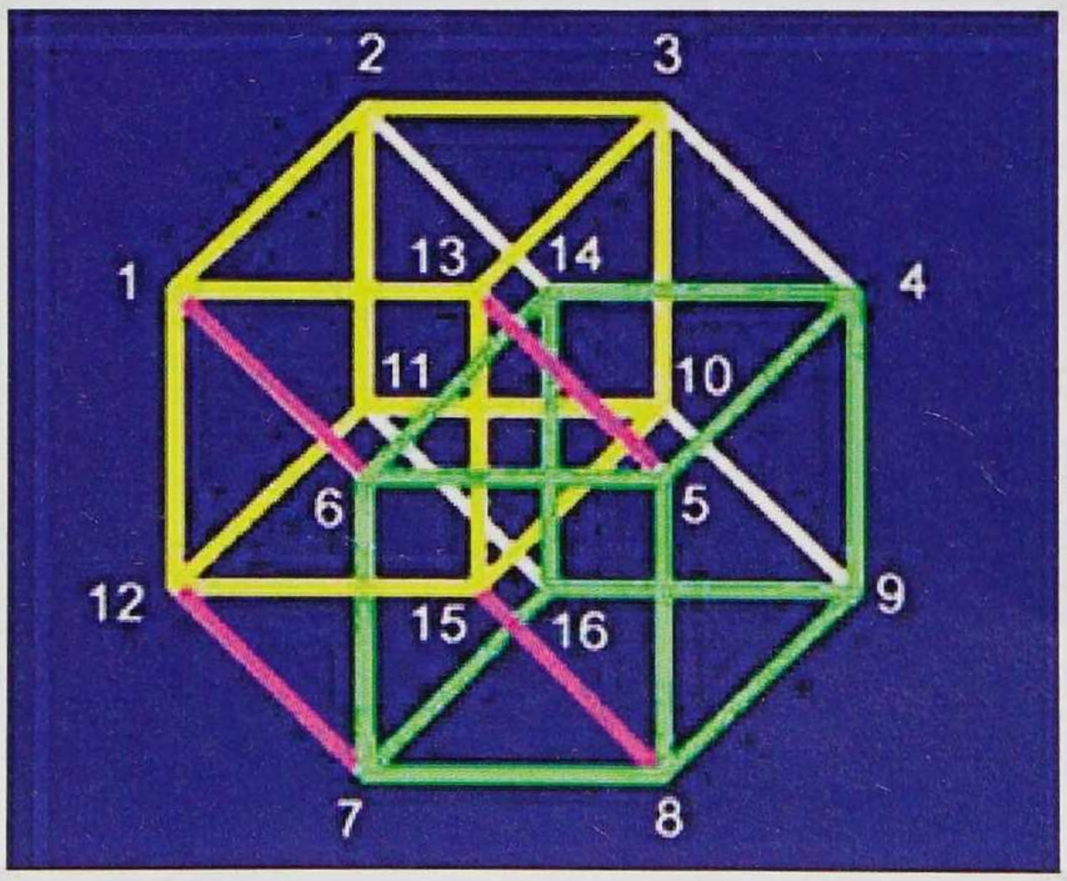
Icosahedron

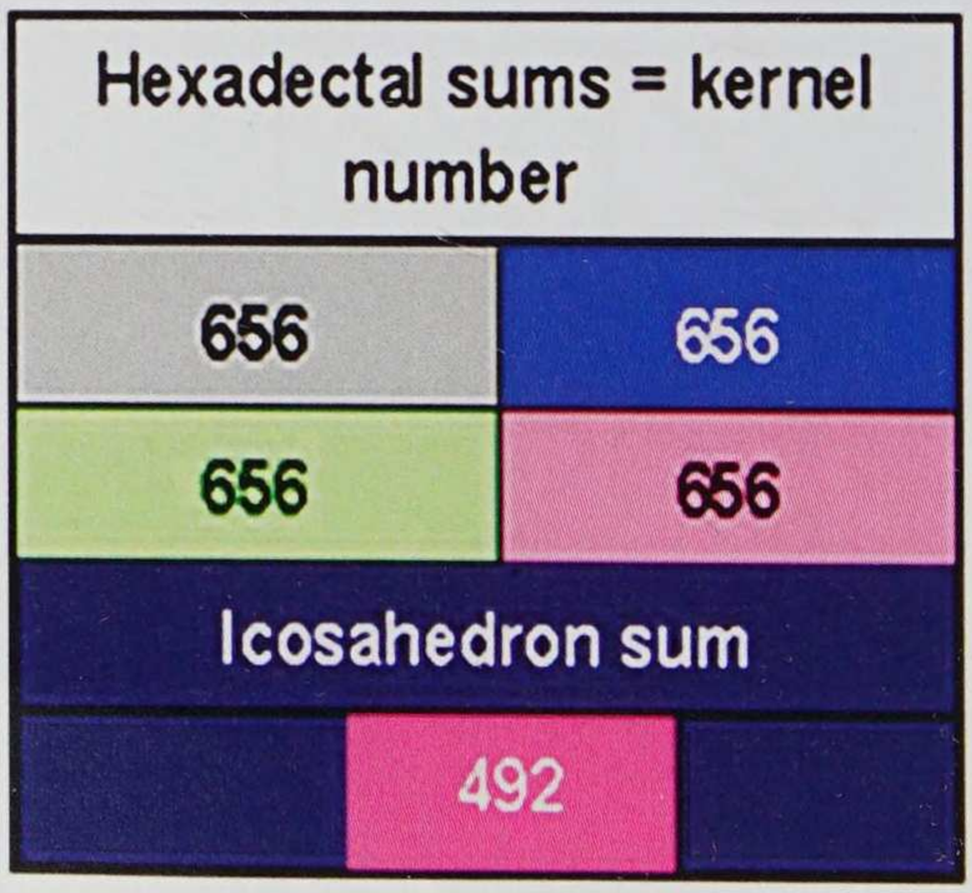


Each square 4-dimensional geonomic table represents a 4-dimensional cube, called a **quadracube**. The **hexadectal** is the 16-corner version in 4-dimensions of the 8-corner octal in 3-dimensions. In all the quadracubes to follow, each has all of its hexadectal summations equal. So consequently, all the quadracubes are pangenic with respect to the fourth dimension. The hexadectals' equal value in 4-dimensional space is again referred to as that size table's kernel number.

The figure at upper-left highlights with different color shadings those cells which are involved in computing the sums of the various nested hexadectals when there is more than one.

The figure at upper-right highlights the corners of a geometry which is the 4-dimensional counterpart of the octahedron in cubes, called the **icosahedron** not because of its shape but merely because it has **12** corners. Each icosahedron always sums to **3/4** of the kernel number.





Icosahedral sums = ¾ hexadectal sums

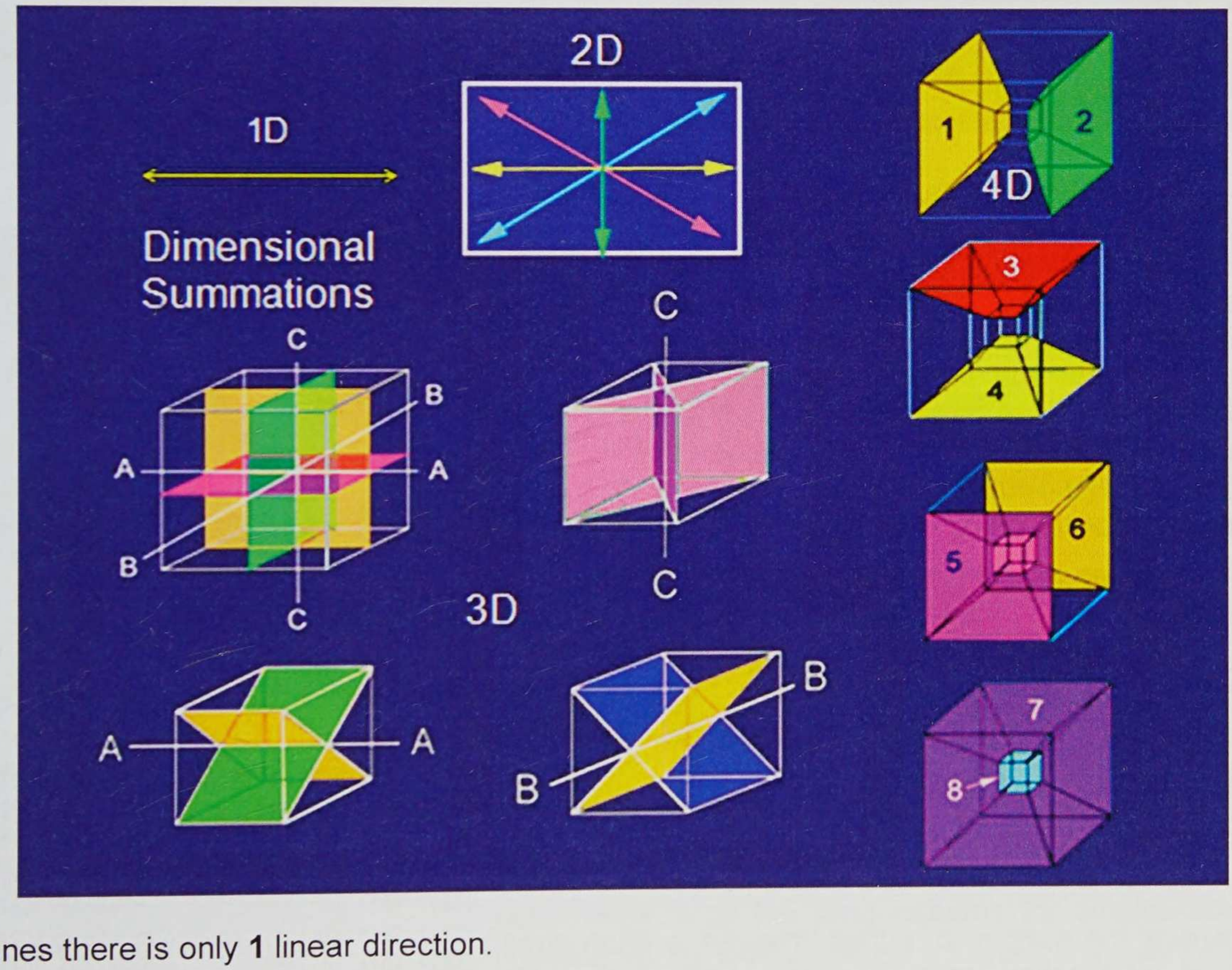
Program 6

We say that a quadracube is **perfect** whenever all of its cubical summations are equal to its characteristic number. It is **absolutely perfect** when each of its embedded squares of size **n** sum equally.

A quadracube of size **n** has continuous **nx-block-square modularity** when every size **n** block-square also sums to the same number continuously across the table, including wrapping across the edges.

A quadracube of size **n** is said to be **ultra-perfect** when all the equal tiling patterns for the square of size **n** persist in each embedded square.

A quadracube of size **n** is said to collapse to a cube when either of its depth-sum tables taken along the **B** or **D** axis reduces to a rectangular cubic table by first dividing all the numbers in the depth-sum table of size **n**, by **n** itself, and then subtracting from each number in the table the minimum number less **1**. For odd-size squares, this difference is actually the difference between the central numbers (dimensional averages) in the cube and quadracube of size **n**.



In lines there is only **1** linear direction.

In squares there are **4** linear directions: horizontal, vertical, and the corner-to-corner diagonals.

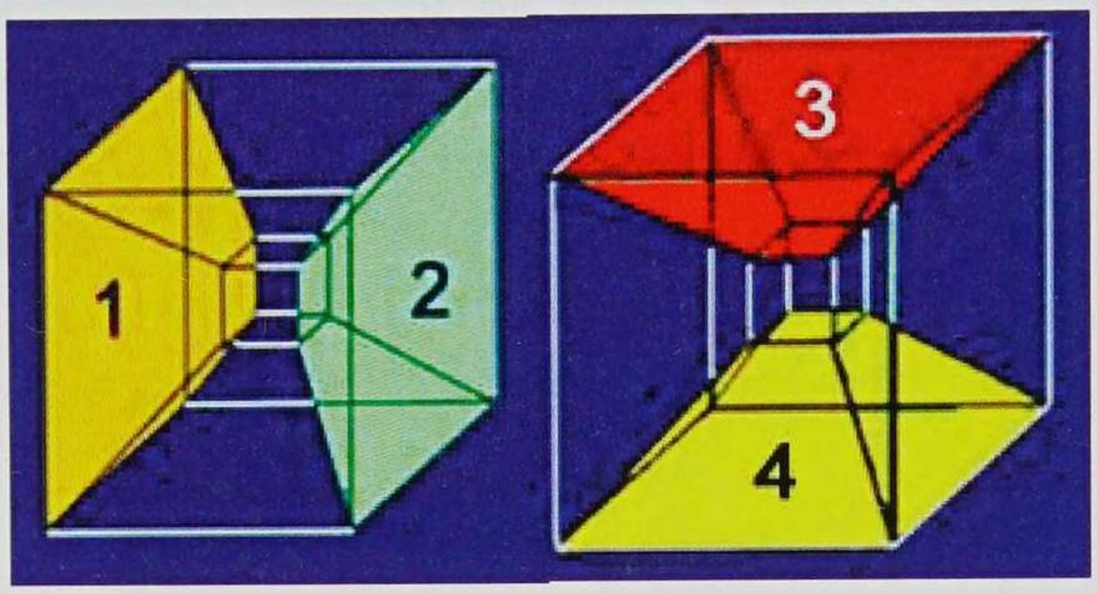
In cubes there are **9** planar directions: **2** intersecting along each of their **3** axes; and **3** parallel to each pair of axes. Each plane has its own combination of directions. There are no corner-to-symmetrically-opposite-corner cubic summations because these would be linear, not planar.

In quadracubes there are **8** cubical sections, each with its own combination of directions.

Program 6

Quadracubes by Class

A Class-2 Quadracube of Size-2



Here is the smallest quadracube. It is of size-2. It contains **4** intersecting embedded size-2 cubes as seen in the upper two tables where two vertical cubes intersect two horizontal cubes. Its dual quadracube is depicted below and is represented by the lower two tables. So between the primal and dual quadracubes together, the number of embedded cubes totals **8**. That's just for the size-2 quadracube.

Cubes

	16	9	7	2
3	4	5	11	14
	1	8	10	15
4	13	12	6	3

Quadrals

34	34	68
34	34	68

Cubes 1 2

16	9	7	2
4	5	11	14
1	8	10	15
13	12	6	3

Quadrals

34	34	
34	34	
68	68	

Cubes 5 I 6

	16	9	7	2
5	4	5	11	14
	1	8	10	15
6	13	12	6	3

Quadrals

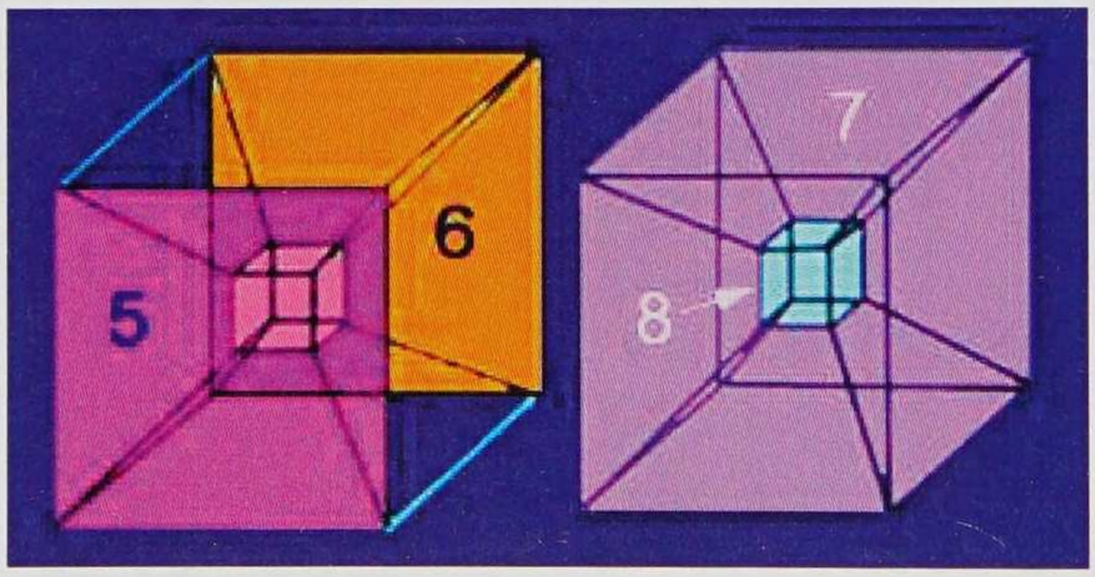
34	34	
34	34	
68	68	

Cubes

	16	9	7	2
7	4	5	11	14
	1	8	10	15
8	13	12	6	3

Quadrals

34	34	68
34	34	68



All the embedded cubes sum equally to **68**, so the quadracube is perfect. Every **2x2** embedded square quadrant sums to **34**. So the size-2 quadracube is also absolutely perfect.

In general the number of cubes involved in each quadracube is always **2n**, **n** horizontal and **n** vertical intersecting each horizontal cube once. So the inter-connectedness of cubes in quadracubes becomes impossible to display 3-dimensionally as done here for the size-2 quadracube. It can only be viewed as segmented number tables.

Program 6

A Class-3 Quadracube of Size-3

Cubes 3x4D		1			2			3			
	369	369	369	369	369	369	369	369	369	369	
4	44	58	21	71	4	48	17	31	75	369	
	64	9	50	10	36	77	37	63	23	369	
	15	29	79	42	56	25	69	2	52	369	1107
5	49	66	8	76	12	35	22	39	62	369	
	81	14	28	27	41	55	54	68	1	369	
	20	43	60	47	70	6	74	16	33	369	1107
6	30	80	13	57	26	40	3	53	67	369	
	59	19	45	5	46	72	32	73	18	369	
	7	51	65	34	78	11	61	24	38	369	1107
	369	369	369	369	369	369	369	369	369		
			1107			1107			1107		
	369	369	369	369	369	369	369	369	369	369	

Here is the size-3 quadracube. It is a Class-3 hypercube by definition. It has 3 vertical cubes intersecting 3 vertical cubes.

It consists of **9** embedded **3x3** block-squares.

Its characteristic number is the sum of all the numbers in each embedded cube. That number here is **3** times **369** which equals **1107**,

Each embedded square sums to **369**. Consequently, all the main and wrap diagonal sums at the block-square level sum equally. Again that number here is **3** times **369** which equals the quadracube's characteristic number, **1107**. So the quadracube is absolutely-perfect.

1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107

Here is a tiling pattern which continuously sums to the quadracube's characteristic number. Each tiling pattern contains **27** number cells and equivalently **nine 1x3** or **3x1** blocks which altogether sum to the quadracube's characteristic number.

Now, here is a surprising discovery: Each tiling pattern is its complement's transpose.

1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107

Similar to the complementary tiling patterns of the size-5 square that are the 45° rotation of the other, these tiling patterns here for the size-3 quadracube are the only occurrence in all the tiling patterns for quadracubes discovered so far where the tile and its transpose constitute the totality of complementary patterns.

Program 6

Expanded 4-Dimensional Tiling Pattern on the Size-3 Quadracube

Here is the size-3 quadracube's tiling pattern duplicated to a size where the tiling pattern does wrap across the square and interlocks for completion. Observe that it took a duplication of **9** times its original size. There are **9** copies of the size-3 quadracube laid out contiguously.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	44	58	21	71	4	48	17	31	75	44	58	21	71	4	48	17	31	75	44	58	21	71	4	48	17	31	75
2	64	9	50	10	36	77	37	63	23	64	9	50	10	36	77	37	63	23	64	9	50	10	36	77	37	63	23
3	15	29	79	42	56	25	69	2	52	15	29	79	42	56	25	69	2	52	15	29	79	42	56	25	69	2	52
4	49	66	8	76	12	35	22	39	62	49	66	8	76	12	35	22	39	62	49	66	8	76	12	35	22	39	62
5	81	14	28	27	41	55	54	68	1	81	14	28	27	41	55	54	68	1	81	14	28	27	41	55	54	68	1
6	20	43	60	47	70	6	74	16	33	20	43	60	47	70	6	74	16	33	20	43	60	47	70	6	74	16	33
7	30	80	13	57	26	40	3	53	67	30	80	13	57	26	40	3	53	67	30	80	13	57	26	40	3	53	67
8	59	19	45	5	46	72	32	73	18	59	19	45	5	46	72	32	73	18	59	19	45	5	46	72	32	73	18
9	7	51	65	34	78	11	61	24	38	7	51	65	34	78	11	61	24	38	7	51	65	34	78	11	61	24	38
10	44	58	21	71	4	48	17	31	75	44	58	21	71	4	48	17	31	75	44	58	21	71	4	48	17	31	75
11	64	9	50	10	36	77	37	63	23	64	9	50	10	36	77	37	63	23	64	9	50	10	36	77	37	63	23
12	15	29	79	42	56	25	69	2	52	15	29	79	42	56	25	69	2	52	15	29	79	42	56	25	69	2	52
13	49	66	8	76	12	35	22	39	62	49	66	8	76	12	35	22	39	62	49	66	8	76	12	35	22	39	62
14	81	14	28	27	41	55	54	68	1	81	14	28	27	41	55	54	68	1	81	14	28	27	41	55	54	68	1
15	20	43	60	47	70	6	74	16	33	20	43	60	47	70	6	74	16	33	20	43	60	47	70	6	74	16	33
16	30	80	13	57	26	40	3	53	67	30	80	13	57	26	40	3	53	67	30	80	13	57	26	40	3	53	67
17	59	19	45	5	46	72	32	73	18	59	19	45	5	46	72	32	73	18	59	19	45	5	46	72	32	73	18
18	7	51	65	34	78	11	61	24	38	7	51	65	34	78	11	61	24	38	7	51	65	34	78	11	61	24	38
19	44	58	21	71	4	48	17	31	75	44	58	21	71	4	48	17	31	75	44	58	21	71	4	48	17	31	75
20	64	9	50	10	36	77	37	63	23	64	9	50	10	36	77	37	63	23	64	9	50	10	36	77	37	63	23
21	15	29	79	42	56	25	69	2	52	15	29	79	42	56	25	69	2	52	15	29	79	42	56	25	69	2	52
22	49	66	8	76	12	35	22	39	62	49	66	8	76	12	35	22	39	62	49	66	8	76	12	35	22	39	62
23	81	14	28	27	41	55	54	68	1	81	14	28	27	41	55	54	68	1	81	14	28	27	41	55	54	68	1
24	20	43	60	47	70	6	74	16	33	20	43	60	47	70	6	74	16	33	20	43	60	47	70	6	74	16	33
25	30	80	13	57	26	40	3	53	67	30	80	13	57	26	40	3	53	67	30	80	13	57	26	40	3	53	67
26	59	19	45	5	46	72	32	73	18	59	19	45	5	46	72	32	73	18	59	19	45	5	46	72	32	73	18
27	7	51	65	34	78	11	61	24	38	7	51	65	34	78	11	61	24	38	7	51	65	34	78	11	61	24	38

This shows that the 4-dimensional characteristic tiling pattern is indeed continuous on a larger scale than the quadracube itself. We have seen this before in Class-2 squares and in the Class-3 squares of sizes **15** and **21**; so the only aspect that is new here is that it has occurred for the first time in the 4th dimension.

The transpose tiling pattern also requires a triple expansion to interlock across the table (not shown).

Program 6

Two Size-3 Dual Quadracubes and their Common Loom Tables

The loom-tables are identified as those related to the primal quadracube $W^{(4)}_n$.

Here is the first dual quadracubes of size-3.

Cubes		1			2			3			
	W	369	369	369	369	369	369	369	369	369	369
4	44	58	21	71	4	48	17	31	75	369
	64	9	50	10	36	77	37	63	23	369
	15	29	79	42	56	25	69	2	52	369
5	49	66	8	76	12	35	22	39	62	369
	81	14	28	27	41	55	54	68	1	369
	20	43	60	47	70	6	74	16	33	369
6	30	80	13	57	26	40	3	53	67	369
	59	19	45	5	46	72	32	73	18	369
	7	51	65	34	78	11	61	24	38	369
		369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369	369

Loom-table X

8	4	3	8	4	3	8	4	3
1	9	5	1	9	5	1	9	5
6	2	7	6	2	7	6	2	7
4	3	8	4	3	8	4	3	8
9	5	1	9	5	1	9	5	1
2	7	6	2	7	6	2	7	6
3	8	4	3	8	4	3	8	4
5	1	9	5	1	9	5	1	9
7	6	2	7	6	2	7	6	2

Loom-table Y

5	7	3	8	1	6	2	4	9
8	1	6	2	4	9	5	7	3
2	4	9	5	7	3	8	1	6
6	8	1	9	2	4	3	5	7
9	2	4	3	5	7	6	8	1
3	5	7	6	8	1	9	2	4
4	9	2	7	3	5	1	6	8
7	3	5	1	6	8	4	9	2
1	6	8	4	9	2	7	3	5

Cubes		7			8			9			
	U	369	369	369	369	369	369	369	369	369	369
10	68	34	21	71	28	24	65	31	27	369
	8	73	42	2	76	45	5	79	39	369
	47	13	63	50	16	57	53	10	60	369
11	33	26	64	36	20	67	30	23	70	369
	81	38	4	75	41	7	78	44	1	369
	12	59	52	15	62	46	18	56	49	369
12	22	72	29	25	66	32	19	69	35	369
	43	3	77	37	6	80	40	9	74	369
	55	51	17	58	54	11	61	48	14	369
		369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369	369

Here is the second dual in the series of three.

The complementary loom tables for both are depicted alongside.

In **X**, every column and all diagonals, main and wrap, contain the numbers from **1** through **9** exactly once.

In **Y**, every row, column and all minor diagonals contains the numbers from **1** through **9** exactly once.

Further, every embedded size-3 block-square in either loom table contains the numbers from **1** through **9** exactly once.

Because there are numerical differences in the row, column or diagonal properties between **X** and **Y**, the dual quadracube is always pairwise centrally symmetric and can never be pairwise row or column symmetric outside of its central row and column.

Program 6

Reduction of depth-sum table along the D-axis

D

132	93	144
111	108	150
126	87	156
147	117	105
162	123	84
141	129	99
90	159	120
96	138	135
102	153	114

→

D/3

44	31	48
37	36	50
42	29	52
49	39	35
54	41	28
47	43	33
30	53	40
32	46	45
34	51	38

→

D/3-27

	126	126	126
17	4	21	42
10	9	23	42
15	2	25	42 126
22	12	8	42
27	14	1	42
20	16	6	42 126
3	26	13	42
5	19	18	42
7	24	11	42 126
126	126	126	
	126	126	126

The size-3 quadracube will collapse along its D-axis to a perfect size-3 cube. However, the resulting cube will not collapse further to a size-3 square along any of its axes because the size-3 square is imperfect. Such a collapse is automatically prevented by the duplication of numbers in all three of the cube's depth-sum tables. Again, this shows how true-to-form Geonomics actually is!

The dual version does not collapse to a cube. We'll see shortly that this is a fundamental property of dimensionality.

Cube's Depth sums along its A axis

A				
	126	126	126	
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
	126	126	126	

Cube's Depth sums along its B axis

B				
	126	126	126	
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
	126	126	126	

Cube's Depth sums along its C axis

C				
	126	126	126	
1	42	15	69	126
2	69	42	15	126
3	15	69	42	126
	126	126	126	
	45	207	126	

Program 6

A Class-4 Quadracube of Size-4

Here is the absolutely-perfect Class-4 quadracube of size-4. All the numbers in each embedded cube sum to the quadracube's characteristic number. Consequently all of the horizontal and vertical cubic sections sum equally to the quadracubes characteristic number **8224**.
The hypercube will collapse along its D-axis to a perfect size-4 cube. We have already seen this cube in Part VIII where it was seen to collapse to a perfect size-4 square.

Embedded Cube		1				2				3				4				
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	
	4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	1	101	186	207	20	37	122	143	212	229	58	79	148	165	250	15	84	2056
	2	208	19	102	185	144	211	38	121	80	147	230	57	16	83	166	249	2056
	3	50	237	156	71	114	45	220	135	178	109	28	199	242	173	92	7	2056
	4	155	72	49	238	219	136	113	46	27	200	177	110	91	8	241	174	2056 8224
6	1	149	106	63	196	213	170	127	4	21	234	191	68	85	42	255	132	2056
	2	128	131	214	41	64	67	150	233	256	3	86	169	192	195	22	105	2056
	3	194	61	108	151	130	253	44	87	66	189	236	23	2	125	172	215	2056
	4	43	216	129	126	107	24	193	190	171	88	1	254	235	152	65	62	2056 8224
7	1	5	218	175	116	69	26	239	180	133	90	47	244	197	154	111	52	2056
	2	176	115	6	217	240	179	70	25	48	243	134	89	112	51	198	153	2056
	3	82	141	252	39	18	77	188	231	210	13	124	167	146	205	60	103	2056
	4	251	40	81	142	187	232	17	78	123	168	209	14	59	104	145	206	2056 8224
8	1	245	10	95	164	181	202	31	100	117	138	223	36	53	74	159	228	2056
	2	32	227	182	73	96	35	246	137	160	99	54	201	224	163	118	9	2056
	3	162	93	12	247	226	157	76	55	34	221	140	119	98	29	204	183	2056
	4	75	184	225	30	11	120	161	222	203	56	97	158	139	248	33	94	2056 8224
		2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Depth-sum D transposed

418	482	610	546
482	418	546	610
546	610	482	418
610	546	418	482
418	482	610	546
482	418	546	610
546	610	482	418
610	546	418	482
418	482	610	546
482	418	546	610
546	610	482	418
610	546	418	482
418	482	610	546
482	418	546	610
546	610	482	418
610	546	418	482

Quadrals

514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514

Octals

1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028

Hexadectals

2056	2056	2056	2056
2056	2056	2056	2056
2056	2056	2056	2056
2056	2056	2056	2056

Embedded squares						
	4x4D	620	620	620	620	
4	1	37	58	15	20	130
	2	16	19	38	67	130
	3	50	45	28	7	130
	4	27	8	49	46	130 620
3	1	21	42	63	4	130
	2	64	3	22	41	130
	3	2	61	44	23	130
	4	43	24	1	62	130 620
2	1	5	26	47	62	130
	2	48	51	6	25	130
	3	18	13	60	39	130
	4	69	40	17	14	130 620
1	1	53	10	31	36	130
	2	32	35	64	9	130
	3	34	29	12	55	130
	4	11	56	33	30	130 620
		620	620	620	620	
		620	620	620	620	

Block major diagonal sums

Cube	5	6	7	8	
	2056	2056	2056	2056	2056
4	514	514	514	514	
3	514	514	514	514	
2	514	514	514	514	
1	514	514	514	514	
	2056	2056	2056	2056	
	2056	2056	2056	2056	2056

Block minor diagonal sums

Cube	5	6	7	8	
	2056	2056	2056	2056	2056
4	514	514	514	514	
3	514	514	514	514	
2	514	514	514	514	
1	514	514	514	514	
	2056	2056	2056	2056	
	2056	2056	2056	2056	2056

Hypercube Block sums

8224	8224	8224	8224	8224
2056	2056	2056	2056	8224
2056	2056	2056	2056	8224
2056	2056	2056	2056	8224
2056	2056	2056	2056	8224
8224	8224	8224	8224	
8224	8224	8224	8224	8224

The yellow and blue tables at the bottom show that every diagonal of every embedded square sums equally.

The white table at the bottom shows that all the embedded block squares sum equally. The multicolored table right above it shows that all hexadectal sums are equal too. And these sums are equal to the embedded block-square sums. All these sums are equal to **1/4**th the quadracube's characteristic number.

Program 6

Note that the entire quadracube sums geonomically at the embedded **4x4** block-square level too.

The size-4 quadracube's equal-summing pairs are now identified by matching colors within the embedded squares.

Note that the pattern is distinctly different from the equal-summing pair-patterns already identified for the size-16 square which has symmetric row-pairs or centrally symmetric pairs.

The equal-summing pairs are distributed within each embedded square and are related to each other as depicted by the colors and connected circles.

A square geonomic table is either 2- or 4-dimensional. The size-16 2-dimensional square is either row pair-wise or centrally pair-wise equal summing. The pattern here has neither.

This demonstrates the dimensionality of the table as not being one of 2-dimensions even though the table is square and of a size that is a square number. That confirms its 4-dimensionality.

Primal	1				2				3				4				
4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
5	101	186	207	20	37	122	143	212	229	58	79	148	165	250	15	84	2056
	208	19	102	185	144	211	38	121	80	147	230	57	16	83	166	249	2056
	50	237	156	71	114	45	220	135	178	109	28	199	242	173	92	7	2056
	155	72	49	288	219	136	113	46	27	200	177	110	91	8	241	174	2056 8224
6	149	106	63	196	213	170	127	4	21	234	191	68	85	42	255	132	2056
	128	131	214	41	64	67	150	233	256	3	86	169	192	195	22	105	2056
	194	61	108	151	130	253	44	87	66	189	236	23	2	125	172	215	2056
	43	216	129	126	107	24	193	190	171	88	1	254	235	152	65	62	2056 8224
7	5	218	175	116	69	26	239	180	133	90	47	244	197	154	111	52	2056
	176	115	6	217	240	179	70	25	48	243	134	89	112	51	198	153	2056
	82	141	252	39	18	77	168	231	210	13	124	167	146	205	60	103	2056
	251	40	81	142	187	232	17	78	123	168	209	14	59	104	145	206	2056 8224
8	245	10	95	164	181	202	31	100	117	138	223	36	53	74	159	228	2056
	32	227	182	73	96	35	246	137	160	99	54	201	224	163	118	9	2056
	162	93	12	247	226	157	76	55	34	221	140	119	98	29	204	183	2056
	75	184	225	30	11	120	161	222	203	56	97	158	139	248	33	94	2056 8224
	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	

Now, given all this, here is what is absolutely amazing: In every block square, each row, each column and each diagonal in both directions and even between corners, sums to **514 = 1/4 x 2056**. That's almost incredible! The numbers from **1** thru **256** have been so uniformly distributed that they sum equally and geonomically within each block-square in all four directions simultaneously!

Although way too large to print, even across two pages, the size-8 quadracube was found to do the same. There, the linear summations within each **8x8** block-square equals **1/8** of the embedded squares' common characteristic number. When I first observed that, I almost fainted from disbelief.

Further, that implies that within each **8x8** block-square there is a size-4 ring-diamond and a 16-cell X tile pattern that sum to **1/8** of the embedded squares' common characteristic number. That tells me that the spatial fabric on the 4-dimensional level is way more interconnected and interlocking than it is on the 2-dimensional level (i.e. convoluted).

Program 6

Dual 4x4D	13				14				15				16					
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
12	71	156	237	50	67	152	233	62	79	148	229	58	75	160	225	54	2056	
	253	34	87	140	249	46	83	136	245	42	95	132	241	38	91	144	2056	
	20	207	186	101	24	195	190	105	28	199	178	109	32	203	182	97	2056	
	170	117	4	223	174	121	8	211	162	125	12	215	166	113	16	219	2056	8224
11	74	151	228	61	78	155	232	49	66	159	236	53	70	147	240	57	2056	
	248	41	94	131	244	37	90	143	256	33	86	139	252	45	82	135	2056	
	29	196	183	106	25	208	179	102	21	204	191	98	17	200	187	110	2056	
	163	126	9	216	167	114	13	220	171	118	1	224	175	122	5	212	2056	8224
10	65	158	235	56	69	146	239	60	73	150	227	64	77	154	231	52	2056	
	251	40	81	142	255	44	85	130	243	48	89	134	247	36	93	138	2056	
	22	201	192	99	18	197	188	111	30	193	184	107	26	205	180	103	2056	
	176	115	6	217	172	127	2	213	168	123	14	209	164	119	10	221	2056	8224
9	80	145	230	59	76	157	226	55	72	153	238	51	68	149	234	63	2056	
	242	47	92	133	246	35	96	137	250	39	84	141	254	43	88	129	2056	
	27	198	177	112	31	202	181	100	19	206	185	104	23	194	189	108	2056	
	165	124	15	210	161	120	11	222	173	116	7	218	169	128	3	214	2056	8224
	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224	2056	2056	2056	2056 8224		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

X(16)

5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14

Y(16)

7	12	13	2	3	8	9	14	15	4	5	10	11	16	1	6
13	2	7	12	9	14	3	8	5	10	15	4	1	6	11	16
4	15	10	5	8	3	14	9	12	7	2	13	16	11	6	1
10	5	4	15	14	9	8	3	2	13	12	7	6	1	16	11
10	7	4	13	14	11	8	1	2	15	12	5	6	3	16	9
8	9	14	3	4	5	10	15	16	1	6	11	12	13	2	7
13	4	7	10	9	16	3	6	5	12	15	2	1	8	11	14
3	14	9	8	7	2	13	12	11	6	1	16	15	10	5	4
1	14	11	8	5	2	15	12	9	6	3	16	13	10	7	4
11	8	1	14	15	12	5	2	3	16	9	6	7	4	13	10
6	9	16	3	2	5	12	15	14	1	8	11	10	13	4	7
16	3	6	9	12	15	2	5	8	11	14	1	4	7	10	13
16	1	6	11	12	13	2	7	8	9	14	3	4	5	10	15
2	15	12	5	6	3	16	9	10	7	4	13	14	11	8	1
11	6	1	16	15	10	5	4	3	14	9	8	7	2	13	12
5	12	15	2	1	8	11	14	13	4	7	10	9	16	3	6

Here is the **4x4D** modulus loom-table. Note the repetition of embedded squares. The distribution of numbers within each block-square is the same as that for the modulus of the 2-dimensional size-4 primal square.

Here is the **4x4D** integer loom-table. Note the distribution of numbers among the embedded squares. No number is in the same relative location twice among all the block-squares.

The complementary loom-tables generate a perfect dual quadracube with an identical equal-summing pair pattern as the primal quadracube, again confirming its 4-dimensional structure.

Program 6

A Class-1 Quadracube of Size-5

Here is the size-5 quadracube of Class-1. All of its **10** embedded cubical sections each sum equally to the quadracube's characteristic number **39,125**.

All of its **25** embedded squares sum equally to **7825, 1/5** of the quadracube's characteristic number: **7825 = 39,125 / 5**. The center cells of all the embedded squares together sum to **7825** too. Further, the quadracube is **5x**-modular like the size-25 square.

All the hexadectals sum equally to **5008**. The hexadectal sums are just any **4** quadrals in any **4** embedded squares that are symmetrically positioned around the centrally embedded square.

It has **2** icosahedrons. One has its associated cells highlighted in crimson and the other in purple. Note that pairwise the centrally-symmetrical same-colored cells sum equally to **626**. So both icosahedrons sum equally to **6 x 626 = 3756 = 3/4 x 5008**.

Cubes		1					2					3					4					5					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	5x4D	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
	1	470	158	621	314	2	220	533	371	64	377	595	283	121	439	127	345	33	496	189	502	95	408	246	564	252	7825
	2	611	304	17	460	173	361	54	392	210	548	111	429	142	585	298	486	179	517	335	48	236	554	267	85	423	7825
6	3	7	475	163	601	319	382	225	538	351	69	132	600	288	101	444	507	350	38	476	194	257	100	413	226	569	7825
	4	153	616	309	22	465	528	366	59	397	215	278	116	434	147	590	28	491	184	522	340	403	241	559	272	90	7825
	5	324	12	455	168	606	74	387	205	543	358	449	137	580	293	106	199	512	330	43	481	574	262	80	418	231	7825
	1	420	233	571	264	77	170	608	321	14	452	545	358	71	389	202	295	108	446	139	577	45	483	196	514	327	7825
	2	561	254	92	410	248	311	4	467	160	623	61	379	217	535	373	438	129	592	285	123	186	504	342	35	498	7825
7	3	82	425	238	551	269	457	175	613	301	19	207	550	363	51	394	582	300	113	426	144	332	50	488	176	519	7825
	4	228	566	259	97	415	603	316	9	472	165	353	66	384	222	540	103	441	134	597	290	478	191	509	347	40	7825
	5	274	87	405	243	556	24	462	155	618	306	399	212	530	368	56	149	587	280	118	431	524	337	30	493	181	7825
	1	495	183	521	339	27	245	558	271	89	402	620	308	21	464	162	370	58	396	214	527	120	433	146	589	277	7825
	2	511	329	42	485	198	261	79	417	235	573	11	454	167	610	323	386	204	542	360	73	136	579	292	110	448	7825
8	3	32	500	188	501	344	407	250	563	251	94	157	625	313	1	469	532	375	63	376	219	282	125	438	126	594	7825
	4	178	516	334	47	490	553	266	84	422	240	303	16	459	172	615	53	391	209	547	365	428	141	584	297	115	7825
	5	349	37	480	193	506	99	412	230	568	256	474	162	605	318	6	224	537	355	68	381	599	287	105	443	131	7825
	1	445	133	596	289	102	195	508	346	39	477	570	258	86	414	227	320	8	471	164	602	70	383	221	539	352	7825
	2	586	279	117	435	148	336	29	492	185	523	86	404	242	560	273	461	154	617	310	23	211	529	367	60	398	7825
9	3	107	450	138	576	294	482	200	513	326	44	232	575	263	76	419	607	325	13	451	169	357	75	388	201	544	7825
	4	128	591	284	122	440	503	341	34	497	190	253	91	409	247	565	3	466	159	622	315	378	216	534	372	65	7825
	5	299	112	430	143	581	49	487	180	518	331	424	237	555	268	81	174	612	305	18	456	549	362	55	393	206	7825
	1	395	208	546	364	52	145	583	296	114	427	520	333	46	489	177	270	83	421	239	552	20	458	171	614	302	7825
	2	536	354	67	385	223	286	104	442	135	598	36	479	192	510	348	411	229	567	280	98	161	604	317	10	473	7825
10	3	57	400	213	526	369	432	150	588	276	119	182	525	338	26	494	557	275	88	401	244	307	25	463	151	619	7825
	4	203	541	359	72	390	578	291	109	447	140	328	41	484	197	515	78	416	234	572	265	453	166	609	322	15	7825
	5	374	62	380	218	531	124	437	130	593	281	499	187	505	343	31	249	562	255	93	406	624	312	5	468	156	7825
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Block major diagonal sums

Cube	1	2	3	4	5	
	7825	7825	7825	7825	7825	7825
6	1565	1565	1565	1565	1565	7825
7	1565	1565	1565	1565	1565	7825
8	1565	1565	1565	1565	1565	7825
9	1565	1565	1565	1565	1565	7825
10	1565	1565	1565	1565	1565	7825
	7825	7825	7825	7825	7825	7825

Hypercube 5x5 block sums

Cube	1	2	3	4	5
6	7825	7825	7825	7825	7825
7	7825	7825	7825	7825	7825
8	7825	7825	7825	7825	7825
9	7825	7825	7825	7825	7825
10	7825	7825	7825	7825	7825

Block minor diagonal sums

Cube	1	2	3	4	5	
	7825	7825	7825	7825	7825	7825
6	1565	1565	1565	1565	1565	7825
7	1565	1565	1565	1565	1565	7825
8	1565	1565	1565	1565	1565	7825
9	1565	1565	1565	1565	1565	7825
10	1565	1565	1565	1565	1565	7825
	7825	7825	7825	7825	7825	7825

Program 6

Size-5 Quadracube's 10 Embedded Cubes

Here all of the **10** cubes embedded within the size-5 quadracube are shown separately. Note that each in itself is perfect because all of its planar block-squares sum equally to **7825**.

All the octals in the central cubes **3** & **8** sum to **2504**. So the hexadectals formed from their octals sum to **5008**. Of all the hexadectals in the quadracube, this is the only case where they are generated from only two intersecting cubes. Further, these are the only cubes that can symmetrically form the **16** corners of that 3D hexadectal seen at the beginning of this Part.

Cubes 1

7825	7825	7825	7825	7825	
470	158	621	314	2	1565
611	304	17	460	173	1565
7	475	163	601	319	1565
153	616	309	22	465	1565
324	12	455	168	606	1565 7825
420	233	571	264	77	1565
561	254	92	410	248	1565
82	425	238	551	269	1565
228	566	259	97	415	1565
274	87	405	243	556	1565 7825
495	183	521	339	27	1565
511	329	42	485	198	1565
32	500	188	501	344	1565
178	516	334	47	490	1565
349	37	480	193	506	1565 7825
445	133	596	289	102	1565
586	279	117	435	148	1565
107	450	138	576	294	1565
128	591	284	122	440	1565
299	112	430	143	581	1565 7825
395	208	546	364	52	1565
536	354	67	385	223	1565
57	400	213	526	369	1565
203	541	359	72	390	1565
374	62	380	218	531	1565 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 2

7825	7825	7825	7825	7825	
220	533	371	64	377	2765
361	54	392	210	548	370
382	225	538	351	69	975
528	366	59	397	215	1555
74	387	205	543	356	2160 7825
170	608	321	14	452	2810
311	4	467	160	623	290
457	175	613	301	19	895
603	316	9	472	165	1625
24	462	155	618	306	2205 7825
245	558	271	89	402	2855
261	79	417	235	573	335
407	250	563	251	94	940
553	266	84	422	240	1545
99	412	230	568	256	2150 7825
195	508	346	39	477	2800
336	29	492	185	523	255
482	200	513	326	44	985
503	341	34	497	190	1590
49	487	180	518	331	2195 7825
145	583	296	114	427	2845
286	104	442	135	598	325
432	150	588	276	119	905
578	291	109	447	140	1510
124	437	130	593	281	2240 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 3

7825	7825	7825	7825	7825	
595	283	121	439	127	265
111	429	142	586	298	995
132	600	288	101	444	1600
278	116	434	147	590	2180
449	137	580	293	106	2785 7825
545	358	71	389	202	310
61	379	217	535	373	915
207	550	363	51	394	1520
353	66	384	222	540	2250
399	212	530	368	56	2830 7825
620	308	21	464	152	355
11	454	167	610	323	960
157	625	313	1	469	1565
303	16	459	172	615	2170
474	162	605	318	6	2775 7825
570	258	96	414	227	300
86	404	242	560	273	880
232	575	263	76	419	1610
253	91	409	247	565	2215
424	237	555	268	81	2820 7825
520	333	46	489	177	345
36	479	192	510	348	950
182	525	338	26	494	1530
328	41	484	197	515	2135
499	187	505	343	31	2865 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 4

7825	7825	7825	7825	7825	
345	33	496	189	502	890
486	179	517	335	48	1620
507	350	38	476	194	2225
28	491	184	522	340	2805
199	512	330	43	481	285 7825
295	108	446	139	577	935
436	129	592	285	123	1540
582	300	113	426	144	2145
103	441	134	597	290	2875
149	587	280	118	431	330 7825
370	58	396	214	527	980
386	204	542	360	73	1585
532	375	63	376	219	2190
53	391	209	547	365	2795
224	537	355	68	381	275 7825
320	8	471	164	602	925
461	154	617	310	23	1505
607	325	13	451	169	2235
3	466	159	622	315	2840
174	612	305	18	456	320 7825
270	83	421	239	552	970
411	229	567	260	98	1575
557	275	88	401	244	2155
78	416	234	572	265	2760
249	562	255	93	406	365 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 5

7825	7825	7825	7825	7825	
95	408	246	564	252	1515
236	554	267	85	423	2245
257	100	413	226	569	2850
403	241	559	272	90	305
574	262	80	418	231	910 7825
45	483	196	514	327	1560
186	504	342	35	498	2165
332	50	488	176	519	2770
478	191	509	347	40	375
524	337	30	493	181	955 7825
120	433	146	589	277	1605
136	579	292	110	448	2210
282	125	438	126	594	2815
428	141	584	297	115	295
599	287	105	443	131	900 7825
70	383	221	539	352	1550
211	529	367	60	398	2130
357	75	388	201	544	2860
378	216	534	372	65	340
549	362	55	393	206	945 7825
20	458	171	614	302	1595
161	604	317	10	473	2200
307	25	463	151	619	2780
453	166	609	322	15	260
624	312	5	468	156	990 ###
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 6

7825	7825	7825	7825	7825	
324	153	7	611	470	1440
12	616	475	304	158	1315
455	309	163	17	621	1815
168	22	601	460	314	1690
606	465	319	173	2	1565 7825
74	528	382	361	220	1440
387	366	225	54	533	1315
205	59	538	392	371	1815
543	397	351	210	64	1690
356	215	69	548	377	1565 7825
449	278	132	111	595	1440
137	116	600	429	283	1315
580	434	288	142	121	1815
293	147	101	585	439	1690
106	590	444	298	127	1565 7825
199	28	507	486	345	1440
512	491	350	179	33	1315
330	184	38	517	496	1815
43	522	476	335	189	1690
481	340	194	48	502	1565 7825
574	403	257	236	95	1440
262	241	100	554	408	1315
80	559	413	267	246	1815
418	272	226	85	564	1690
231	90	569	423	252	1565 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 7

7825	7825	7825	7825	7825	
274	228	82	561	420	1315
87	566	425	254	233	1815
405	259	238	92	571	1690
243	97	551	410	264	1565
556	415	269	248	77	1440 7825
24	603	457	311	170	1315
462	316	175	4	608	1815
155	9	613	467	321	1690
618	472	301	160	14	1565
306	165	19	623	452	1440 7825
399	353	207	61	545	1315
212	66	550	379	358	1815
530	384	363	217	71	1690
368	222	51	535	389	1565
56	540	394	373	202	1440 7825
149	103	582	436	295	1315
587	441	300	129	108	1815
280	134	113	592	446	1690
118	597	426	285	139	1565
431	290	144	123	577	1440 7825
524	478	332	186	45	1315
337	191	50	504	483	1815
30	509	488	342	196	1690
493	347	176	35	514	1565
181	40	519	498	327	1440 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 8

7825	7825	7825	7825	7825	
349	178	32	511	495	1815
37	516	500	329	183	1690
480	334	188	42	521	1565
193	47	501	485	339	1440
506	490	344	198	27	1315 7825
99	553	407	261	245	1815
412	266	250	79	558	1690
230	84	563	417	271	1565
568	422	251	235	89	1440
256	240	94	573	402	1315 7825
474	303	157	11	620	1815
162	16	625	454	308	1690
605	459	313	167	21	1565
318	172	1	610	464	1440
6	615	469	323	152	1315 7825
224	53	532	386	370	1815
537	391	375	204	58	1690
355	209	63	542	396	1565
68	547	376	360	214	1440
381	365	219	73	527	1315 7825
599	428	282	136	120	1815
287	141	125	579	433	1690
105	584	438	292	146	1565
443	297	126	110	589	1440
131	115	594	448	277	1315 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 9

7825	7825	7825	7825	7825	
299	128	107	586	445	1565
112	591	450	279	133	1565
430	284	138	117	596	1565
143	122	576	435	289	1565
581	440	294	148	102	1565 7825
49	503	482	336	195	1565
487	341	200	29	508	1565
180	34	513	492	346	1565
518	497	326	185	39	1565
331	190	44	523	477	1565 7825
424	253	232	86	570	1565
237	91	575	404	258	1565
555	409	263	242	96	1565
268	247	76	560	414	1565
81	565	419	273	227	1565 7825
174	3	607	461	320	1565
612	466	325	154	8	1565
305	159	13	617	471	1565
18	622	451	310	164	1565
456	315	169	23	602	1565 7825
549	378	357	211	70	1565
362	216	75	529	383	1565
55	534	388	367	221	1565
393	372	201	60	539	1565
206	65	544	398	352	1565 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Cubes 10

7825	7825	7825	7825	7825	
374	203	57	536	395	1565
62	541	400	354	208	1440
380	359	213	67	546	1315
218	72	526	385	364	1815
531	390	369	223	52	1690 7825
124	578	432	286	145	1565
437	291	150	104	583	1440
130	109	588	442	296	1315
593	447	276	135	114	1815
281	140	119	598	427	1690 7825
499	328	182	36	520	1565
187	41	525	479	333	1440
505	484	338	192	46	1315
343	197	26	510	489	1815
31	515	494	348	177	1690 7825
249	78	557	411	270	1565
562	416	275	229	83	1440
255	234	88	567	421	1315
93	572	401	260	239	1815
406	265	244	98	552	1690 7825
624	453	307	161	20	1565
312	166	25	604	458	1440
5	609	463	317	171	1315
468	322	151	10	614	1815
156	15	619	473	302	1690 7825
7825	7825	7825	7825	7825	
7825	7825	7825	7825	7825	

Program 6

Size-5 Quadracube's Depth-sum Tables

A

1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565

B

2850	1540	355	2195	885
305	2145	960	2800	1615
910	2875	1565	255	2220
1515	330	2170	985	2825
2245	935	2775	1590	280
1600	290	2230	945	2760
2180	895	2835	1550	365
2785	1625	315	2130	970
265	2205	920	2860	1575
995	2810	1525	340	2155
350	2165	980	2820	1510
930	2770	1585	300	2240
1535	375	2190	880	2845
2140	955	2795	1610	325
2870	1560	275	2215	905
2225	915	2855	1570	260
2805	1520	335	2175	990
285	2250	940	2755	1595
890	2830	1545	360	2200
1620	310	2150	965	2780
975	2790	1605	320	2135
1555	270	2210	925	2865
2160	1000	2815	1505	345
2765	1580	295	2235	950
370	2185	900	2840	1530

C

1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565

D

1725	1415	1855	1570	1260
1805	1520	1335	1675	1490
1285	1750	1440	1755	1595
1390	1830	1545	1360	1700
1620	1310	1650	1465	1780
1475	1790	1605	1320	1635
1555	1270	1710	1425	1865
1660	1500	1815	1505	1345
1765	1580	1295	1735	1450
1370	1685	1400	1840	1530
1850	1540	1355	1695	1385
1305	1645	1460	1800	1615
1410	1875	1565	1255	1720
1515	1330	1670	1485	1825
1745	1435	1775	1590	1280
1600	1290	1730	1445	1760
1680	1395	1835	1550	1365
1785	1625	1315	1630	1470
1265	1705	1420	1860	1575
1495	1810	1525	1340	1655
1350	1665	1480	1820	1510
1430	1770	1585	1300	1740
1535	1375	1690	1380	1845
1640	1455	1795	1610	1325
1870	1560	1275	1715	1405

This picture depicts the four depth-sum tables of the size-5 quadracube. Note that both depth-sums taken along the A and C axes have all their numbers equal. This shows that all the channels and pillars in each embedded cube sum equally.

Collapse of the Quadracube to a Perfect Size-5 Cube

Here the quadracube's depth-sum table **D** is collapsed along its D-axis to a perfect size-5 cube.

D

1725	1415	1855	1570	1260
1805	1520	1335	1675	1490
1285	1750	1440	1755	1595
1390	1830	1545	1360	1700
1620	1310	1650	1465	1780
1475	1790	1605	1320	1635
1555	1270	1710	1425	1865
1660	1500	1815	1505	1345
1765	1580	1295	1735	1450
1370	1685	1400	1840	1530
1850	1540	1355	1695	1385
1305	1645	1460	1800	1615
1410	1875	1565	1255	1720
1515	1330	1670	1485	1825
1745	1435	1775	1590	1280
1600	1290	1730	1445	1760
1680	1395	1835	1550	1365
1785	1625	1315	1630	1470
1265	1705	1420	1860	1575
1495	1810	1525	1340	1655
1350	1665	1480	1820	1510
1430	1770	1585	1300	1740
1535	1375	1690	1380	1845
1640	1455	1795	1610	1325
1870	1560	1275	1715	1405

→

D/5

345	283	371	314	252
361	304	267	335	298
257	350	288	351	319
278	366	309	272	340
324	262	330	293	356
295	358	321	264	327
311	254	342	285	373
332	300	363	301	269
353	316	259	347	290
274	337	280	368	306
370	308	271	339	277
261	329	292	360	323
282	375	313	251	344
303	266	334	297	365
349	287	355	318	256
320	258	346	289	352
336	279	367	310	273
357	325	263	326	294
253	341	284	372	315
299	362	305	268	331
270	333	296	364	302
286	354	317	260	348
307	275	338	276	369
328	291	359	322	265
374	312	255	343	281

→

E = D/5-250

95	33	121	64	2
111	54	17	85	48
7	100	38	101	69
28	116	59	22	90
74	12	80	43	106
45	108	71	14	77
61	4	92	35	123
82	50	113	51	19
103	66	9	97	40
24	87	30	118	56
120	58	21	89	27
11	79	42	110	73
32	125	63	1	94
53	16	84	47	115
99	37	105	68	6
70	8	96	39	102
86	29	117	60	23
107	75	13	76	44
3	91	34	122	65
49	112	55	18	81
20	83	46	114	52
36	104	67	10	98
57	25	88	26	119
78	41	109	72	15
124	62	5	93	31

Program 6

Derived perfect cube collapses further to a perfect square

This picture shows that the derived cube is absolutely perfect.

It also shows that this cube will collapse further to a perfect size-5 square along its B-axis.

Block-square minor diagonal sums

315	315	315	315	315	
315	315	315	315	315	
315	315	315	315	315	
315	315	315	315	315	
315	315	315	315	315	

	1575	1575	1575	1575	1575	
5	95	33	121	64	2	315
	111	54	17	85	48	315
	7	100	38	101	69	315 1575
	28	116	59	22	90	315
	74	12	80	43	106	315
4	45	108	71	14	77	315
	61	4	92	35	123	315
	82	50	113	51	19	315 1575
	103	66	9	97	40	315
	24	87	30	118	56	315
3	120	58	21	89	27	315
	11	79	42	110	73	315
	32	125	63	1	94	315 1575
	53	16	84	47	115	315
	99	37	105	68	6	315
2	70	8	96	39	102	315
	86	29	117	60	23	315
	107	75	13	76	44	315 1575
	3	91	34	122	65	315
	49	112	55	18	81	315
1	20	83	46	114	52	315
	36	104	67	10	98	315
	57	25	88	26	119	315 1575
	78	41	109	72	15	315
	124	62	5	93	31	315
	1575	1575	1575	1575	1575	
	1575	1575	1575	1575	1575	

BD Depth-sums

350	290	355	320	260
305	270	335	300	365
285	375	315	255	345
265	330	295	360	325
370	310	275	340	280

BD/5

70	58	71	64	52
61	54	67	60	73
57	75	63	51	69
53	66	59	72	65
74	62	55	68	56

BD/5-50

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

Block-square major diagonal sums

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

Program 6

Depth-sum tables B and D collapse along their B-axes to identical perfect squares

B

350	2805	2160	1515	995
915	270	2875	2205	1560
1605	960	315	2795	2150
2195	1550	880	360	2840
2760	2240	1595	950	280
975	305	2785	2140	1620
1540	895	375	2830	2185
2230	1585	940	295	2775
2820	2175	1505	985	340
260	2865	2220	1575	905
1600	930	285	2765	2245
2165	1520	1000	330	2810
2855	2210	1565	920	275
320	2800	2130	1610	965
885	365	2845	2200	1530
2225	1555	910	265	2870
2790	2145	1625	955	310
355	2835	2190	1545	900
945	300	2755	2235	1590
1510	990	345	2825	2155
2850	2180	1535	890	370
290	2770	2250	1580	935
980	335	2815	2170	1525
1570	925	255	2860	2215
2135	1615	970	325	2780

D

1475	1540	1730	1820	1260
1555	1645	1835	1300	1490
1660	1875	1315	1380	1595
1765	1330	1420	1610	1700
1370	1435	1525	1715	1780
1600	1665	1855	1320	1385
1680	1770	1335	1425	1615
1785	1375	1440	1505	1720
1265	1455	1545	1735	1825
1495	1560	1650	1840	1280
1725	1790	1355	1445	1510
1805	1270	1460	1550	1740
1285	1500	1565	1630	1845
1390	1580	1670	1860	1325
1620	1685	1775	1340	1405
1850	1290	1480	1570	1635
1305	1395	1585	1675	1865
1410	1625	1690	1755	1345
1515	1705	1795	1360	1450
1745	1810	1275	1465	1530
1350	1415	1605	1695	1760
1430	1520	1710	1800	1365
1535	1750	1815	1255	1470
1640	1830	1295	1485	1575
1870	1310	1400	1590	1655

This picture shows that the size-5 quadracube will collapse along either its B or D-axis directly to a perfect size-5 square.

The Depth-sum tables **B** and **D** are at right angles to each other. Note that the depth-sums **BB** and **BD** are the depth-sum tables of **B** and **D**, respectively, taken along their own B-axis. Note that they are the transpose of each other.

That yields squares that are the transpose of each other too.

BB Depth-sums

8000	7775	7675	7575	8100
7700	7600	8125	7900	7800
8025	7925	7825	7725	7625
7850	7750	7525	8050	7950
7550	8075	7975	7875	7650

BD Depth-sums

8000	7700	8025	7850	7550
7775	7600	7925	7750	8075
7675	8125	7825	7525	7975
7575	7900	7725	8050	7875
8100	7800	7625	7950	7650

BB/25

320	311	307	303	324
308	304	325	316	312
321	317	313	309	305
314	310	301	322	318
302	323	319	315	306

BD/25

320	308	321	314	302
311	304	317	310	323
307	325	313	301	319
303	316	309	322	315
324	312	305	318	306

BB/25-300

20	11	7	3	24
8	4	25	16	12
21	17	13	9	5
14	10	1	22	18
2	23	19	15	6

BD/25-300

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

Program 6

Generating Quadracubes from Cubes

Recall the cubic integer loom-table $\bar{Y}^{(3)}$ from Program 5 (re: p.5-16 & 17) shown again here at right. This cubic loom-table is converted to a quadracubic loom-table by moving each embedded block-square to the block-diagonal of a square of size n^2xn^2 and then duplicating each block square vertically from top to bottom as shown below. Let this table be denoted by $\bar{Y}^{(3)}{}_n$.

$\bar{Y}^{(3)}$

7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	1	6
5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1

Y	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	196
6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	196
5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	196
4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	196
3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	196
2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	196
1	6	4	2	7	5	3	3	1	6	4	2	7	5	5	3	1	6	4	2	7	7	5	3	1	6	4	2	2	7	5	3	1	6	4	4	2	7	5	3	1	6	6	4	2	7	5	3	1	196
196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	
	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196

Then the primal quadracube $W^{(4)}{}_n$ is obtained by

$$(6.1) \quad W^{(4)}{}_n = n^3(\bar{Y}^{(3)}{}_n - |1|) + \bar{W}^{(3)}{}_n$$

where $\bar{W}^{(3)}{}_n$ is the cube $W^{(3)}{}_n$ repeated horizontally for **n** repetitions.

Its dual is obtained from

$$(6.2) \quad U^{(4)}{}_n = n^3(\bar{X}^{(3)}{}_n - |1|) + \bar{W}^{(3)}{}_n$$

where $\bar{X}^{(3)}{}_n$ is the cubic modulus loom-table $X^{(3)}{}_n$ expanded in a like manner as was $Y^{(3)}{}_n$ above.

Generating $W^{(4)}{}_7$ and $U^{(4)}{}_7$ from $\bar{W}^{(3)}{}_7$ is left as an exercise.

Program 6

Quadracube's characteristic tiling patterns at the block-square level

Size-5

This picture shows that those size-5 "**+**" and "**x**" tiling patterns hold continuously at the cellular level only within each embedded square. For the tiling pattern to hold continuously on the quadracube, it has to be at the **5x5** block-square level.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	Cubes			1					2					3					4						5			
	5x4D	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1		45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2		561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	6	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4		353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5		149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825	39125
6		70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7		586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	7	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9		253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10		174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825	39125
11		95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12		611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	8	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14		278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15		199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825	39125
16		120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17		511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	9	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19		303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20		224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825	39125
21		20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22		536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	10	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24		328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25		249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825	39125
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
						39125					39125					39125					39125					39125	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

All **5x5** block-square tile patterns A & B each sum to the quadracube's characteristic number **39,125** continuously – the same as all the numbers in each horizontal and vertical cube.

Within each block-square, the quadrals sum to **1565**; that's **1/25** of **39,125**.

The tiling patterns involve the embedded block-squares of **n+1 = 6** intersecting cubes. **That is an amazing newly-discovered general tiling-pattern property for quadracubes.**

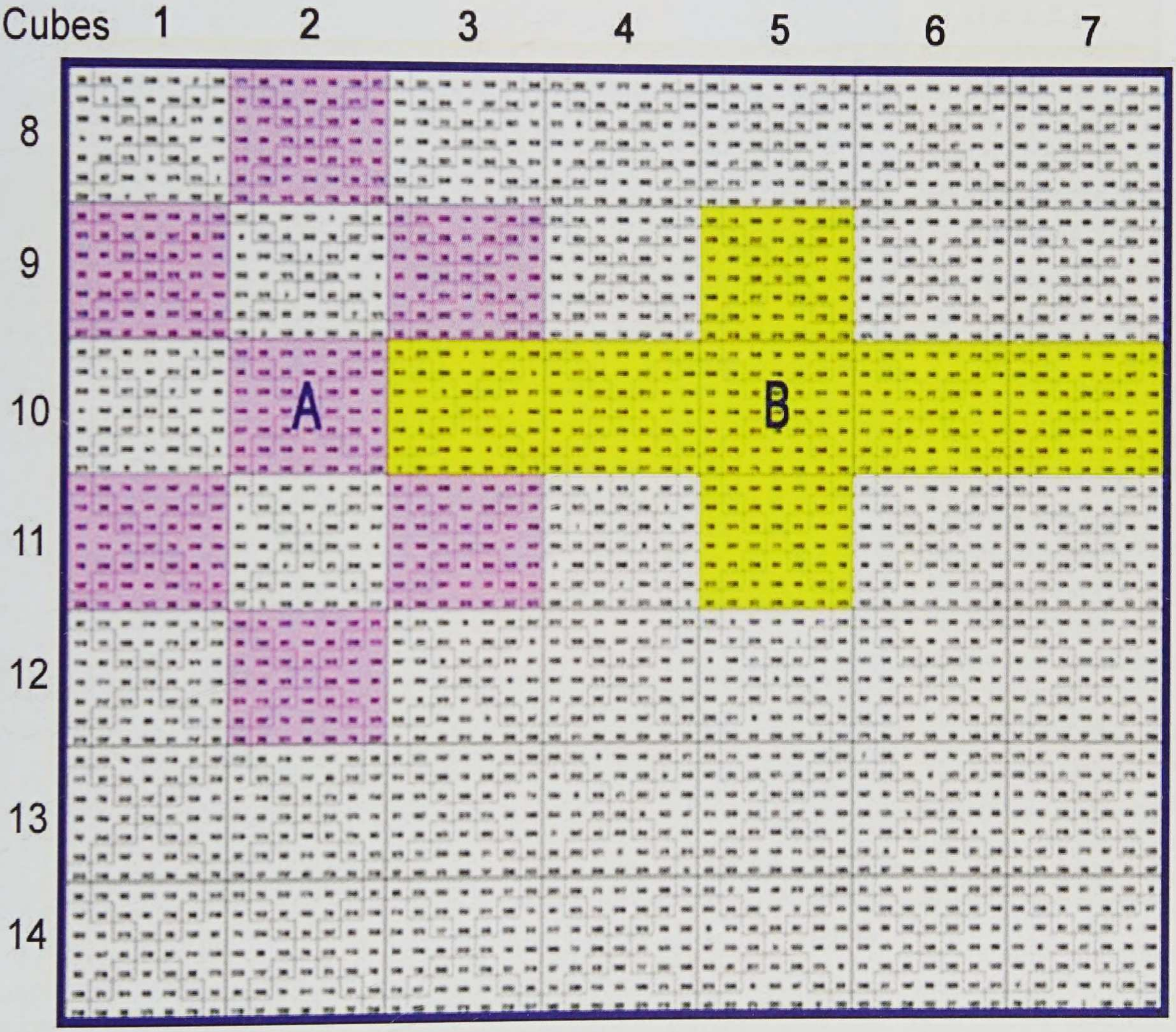
At the quadracubic level, the status of being **ultra-perfect** now applies at the **nxn** block-square level as well as within each embedded square.

Program 6

Size-7

The size-7 quadracube is too large to show the numbers legibly. Nonetheless it's the patterns here that are important.

This same property applies to all quadracubes in Class-1 as demonstrated again here for the size-7 quadracube. Both size-7 characteristic tiling patterns are at the **7x7** block-square level as highlighted. The tiling patterns mimic those for the size-7 square. All the numbers in each pattern sum exactly to the quadracube's characteristic number and do this continuously across the table. This tiling pattern involves the embedded block-squares of **n+1 = 8** intersecting cubes.



The quadrals within each **7x7** block-square sums to **1/49** of the quadracube's characteristic number.

Program 6

Categorization of Quadracubes by Class

Quadracubes can also be categorized by class. The table lists the properties common among all the sizes of quadracube in each class.

Class 1	Class 2	Class 3	Class 4	Class 6
n = 1b b is an odd prime > 3 (3, 5 & 7)	n = 2b b is odd ≥ 1 (2 & 6)	n = 3b b is odd ≥ 3 (3)	n = 2b b is even ≥ 2 (4)	$n = b^a$ b odd (0)
Absolutely perfect Continuously nx modular All embedded cubes are ultra-perfect in that each sums to the quadracube's characteristic number, all tiling patterns persist in each embedded square and the tiling patterns extend to the block-square level continuously Both primal and dual will collapse to perfect cube and a perfect square	Absolutely perfect Continuously nx modular for b>1 Possesses a dual Both primal and dual will collapse to a perfect cube	Absolutely perfect Continuously 3x modular for b>3 Possesses a dual Both primal and dual will collapse to a perfect cube	Absolutely perfect Continuously nx modular Possesses a dual Both primal and dual will collapse to both a perfect cube and a perfect square	Unknown Smallest size is 9 Largest size quadracube investigated is size-7 and is not of this class

The smallest Class-5 quadracube's table is of size **1225** by **1225** (**1225=35x35**), way too large to deal with in a laptop. Class-5 quadracubes' properties are unknown as of this compilation; the largest size of quadracube investigated is of size-7 and is not of this class.

The smallest such size of Class-6 quadracube is of size-9. Its table would be of the size **81x81**; way too large to be legible for print

Program 6

Exploration of an alternative Size-4 Quadracube

In the course of navigating the 4th dimension, I happened upon this size-4 quadracube. What is different between this one and the previous one of the same size is this: even though this version will collapse to a perfect cube too, it will not collapse to a perfect square as the first one did. Consequently it has no relationship with the perfect size-4 square shown earlier. It was generated from an imperfect square with the same ± deviations in its wrap diagonals, which canceled out when they were incorporated into a size-4 cube thereby making the resulting cube also perfect. That cube was used to generate this quadracube. This version of the size-4 quadracube has equal-summing row-pair summations and some amazing properties that, because of its pairwise row-symmetry, lend insight into hyper- and sub-dimensional spaces.

Size 4 Primal Quadracube

Cube		1			2				3				4				
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056
	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056
6	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056
	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056
7	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056
	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056
8	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056
	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Its equal row-pair summations are those with the same color in each row.

Complementary loom-tables generate a dual quadracube with an identical equal-summing pair pattern as the prim-al quadracube, again confirming its 4-dimensionality.

Size 4 Dual Quadracube

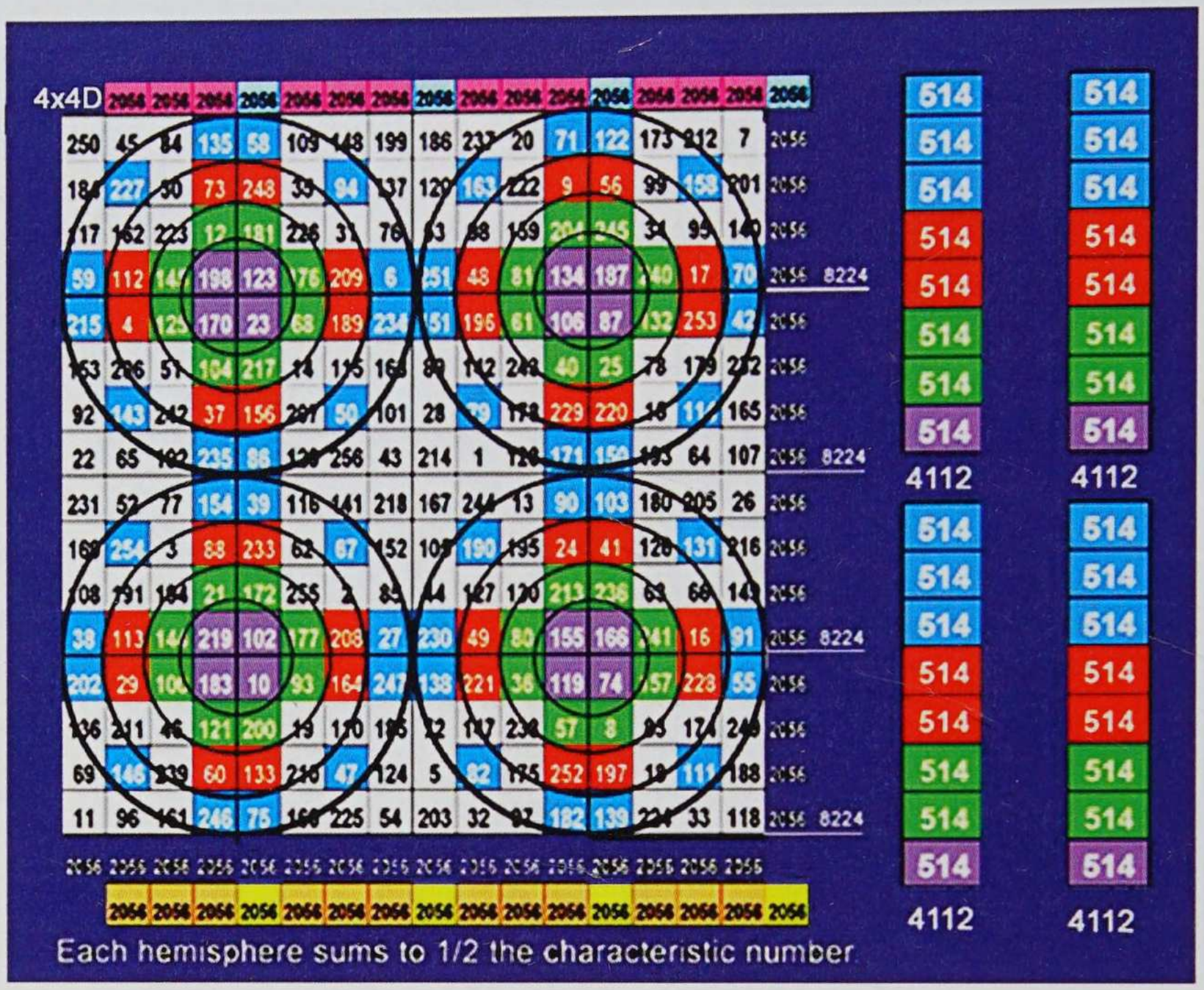
Cube		1			2				3				4				
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	160	195	54	105	148	199	58	109	156	207	50	101	152	203	62	97	2056
	124	47	210	133	128	35	214	137	120	43	222	129	116	39	218	141	2056
	72	27	238	177	76	31	226	181	68	23	234	189	80	19	230	185	2056
	164	247	10	93	168	251	14	81	176	243	6	89	172	255	2	85	2056
6	110	49	200	155	98	53	204	159	106	61	196	151	102	57	208	147	2056
	138	221	36	119	142	209	40	123	134	217	48	115	130	213	44	127	2056
	182	233	32	67	186	237	20	71	178	229	28	79	190	225	24	75	2056
	82	5	252	175	86	9	256	163	94	1	248	171	90	13	244	167	2056
7	111	52	197	154	99	56	201	158	107	64	193	150	103	60	205	146	2056
	139	224	33	118	143	212	37	122	135	220	45	114	131	216	41	126	2056
	183	236	29	66	187	240	17	70	179	232	25	78	191	228	21	74	2056
	83	8	249	174	87	12	253	162	95	4	245	170	91	16	241	166	2056
8	157	194	55	108	145	198	59	112	153	206	51	104	149	202	63	100	2056
	121	46	211	136	125	34	215	140	117	42	223	132	113	38	219	144	2056
	69	26	239	180	73	30	227	184	65	22	235	192	77	18	231	188	2056
	161	246	11	96	165	250	15	84	173	242	7	92	169	254	3	88	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

The discovery of these two primary and dual quadracubes here provided a picture of the 4th dimension that is absolutely astounding as you will now behold.

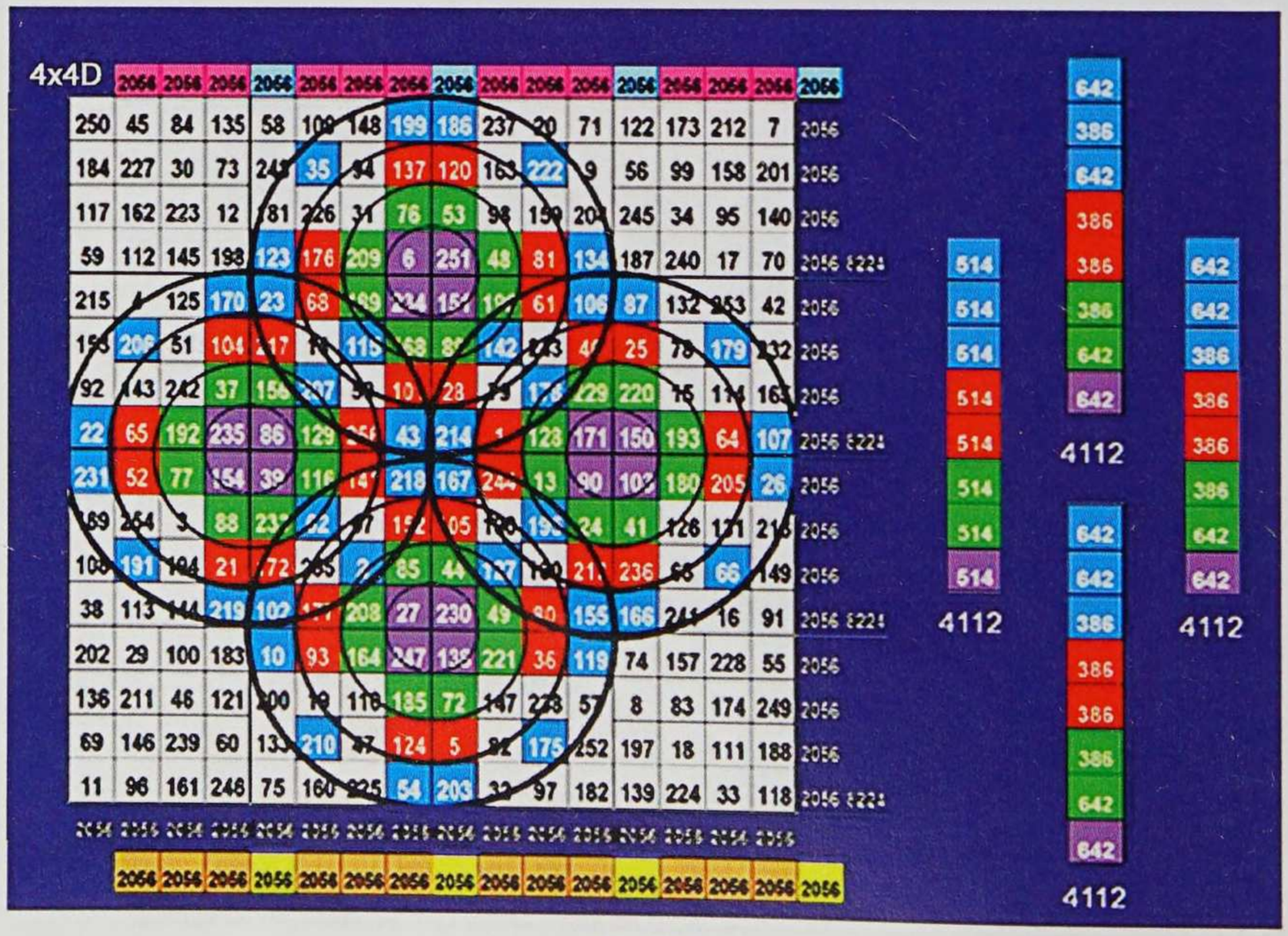
Program 6

Embedded Hemispheres in the Primal Quadracube

The quadracube possesses **4 non-intersecting** inscribed hemispheres each of which sums to **1/2** the hypercube's characteristic number.



It also simultaneously possesses **4** inscribed hemi-spheres that are **intersecting,** each of which again sums to **1/2** the hypercube's characteristic number.



Program 6

Uncovering the Size-4 Characteristic Torus via Loom Tables

Its modulus loom table shows the same equal-summing hemispherical patterns. Note that number **34** for each of the quadrals in the embedded squares. That's the characteristic number of the size-4 square!

Cube		1				2				3				4					
Loom X		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136		
	10	13	4	7	10	13	4	7	10	13	4	7	10	13	4	7	136		
5	8	3	14	9	8	3	14	9	8	3	14	9	8	3	14	9	136		
	5	2	15	12	5	2	15	12	5	2	15	12	5	2	15	12	136		
	11	16	1	6	11	16	1	6	11	16	1	6	11	16	1	6	136	544	
	7	4	13	10	7	4	13	10	7	4	13	10	7	4	13	10	136		
6	9	14	3	8	9	14	3	8	9	14	3	8	9	14	3	8	136		
	12	15	2	5	12	15	2	5	12	15	2	5	12	15	2	5	136		
	6	1	16	11	6	1	16	11	6	1	16	11	6	1	16	11	136	544	
	7	4	13	10	7	4	13	10	7	4	13	10	7	4	13	10	136		
7	9	14	3	8	9	14	3	8	9	14	3	8	9	14	3	8	136		
	12	15	2	5	12	15	2	5	12	15	2	5	12	15	2	5	136		
	6	1	16	11	6	1	16	11	6	1	16	11	6	1	16	11	136	544	
	10	13	4	7	10	13	4	7	10	13	4	7	10	13	4	7	136		
8	8	3	14	9	8	3	14	9	8	3	14	9	8	3	14	9	136		
	5	2	15	12	5	2	15	12	5	2	15	12	5	2	15	12	136		
	11	16	1	6	11	16	1	6	11	16	1	6	11	16	1	6	136	544	
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136			
		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136		

34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
272	272	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
272	272	

xij = Modulus [(wij-1) | 16] + 1

So does its integer loom table. Again every quadral in each embedded square sums to **34**.

Cube		1				2				3				4					
Loom Y		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136		
	16	3	6	9	4	7	10	13	12	15	2	5	8	11	14	1	136		
5	12	15	2	5	16	3	6	9	8	11	14	1	4	7	10	13	136		
	8	11	14	1	12	15	2	5	4	7	10	13	16	3	6	9	136		
	4	7	10	13	8	11	14	1	16	3	6	9	12	15	2	5	136	544	
	14	1	8	11	2	5	12	15	10	13	4	7	6	9	16	3	136		
6	10	13	4	7	14	1	8	11	6	9	16	3	2	5	12	15	136		
	6	9	16	3	10	13	4	7	2	5	12	15	14	1	8	11	136		
	2	5	12	15	6	9	16	3	14	1	8	11	10	13	4	7	136	544	
	15	4	5	10	3	8	9	14	11	16	1	6	7	12	13	2	136		
7	11	16	1	6	15	4	5	10	7	12	13	2	3	8	9	14	136		
	7	12	13	2	11	16	1	6	3	8	9	14	15	4	5	10	136		
	3	8	9	14	7	12	13	2	15	4	5	10	11	16	1	6	136	544	
	13	2	7	12	1	6	11	16	9	14	3	8	5	10	15	4	136		
8	9	14	3	8	13	2	7	12	5	10	15	4	1	6	11	16	136		
	5	10	15	4	9	14	3	8	1	6	11	16	13	2	7	12	136		
	1	6	11	16	5	10	15	4	13	2	7	12	9	14	3	8	136	544	
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136			
		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136		

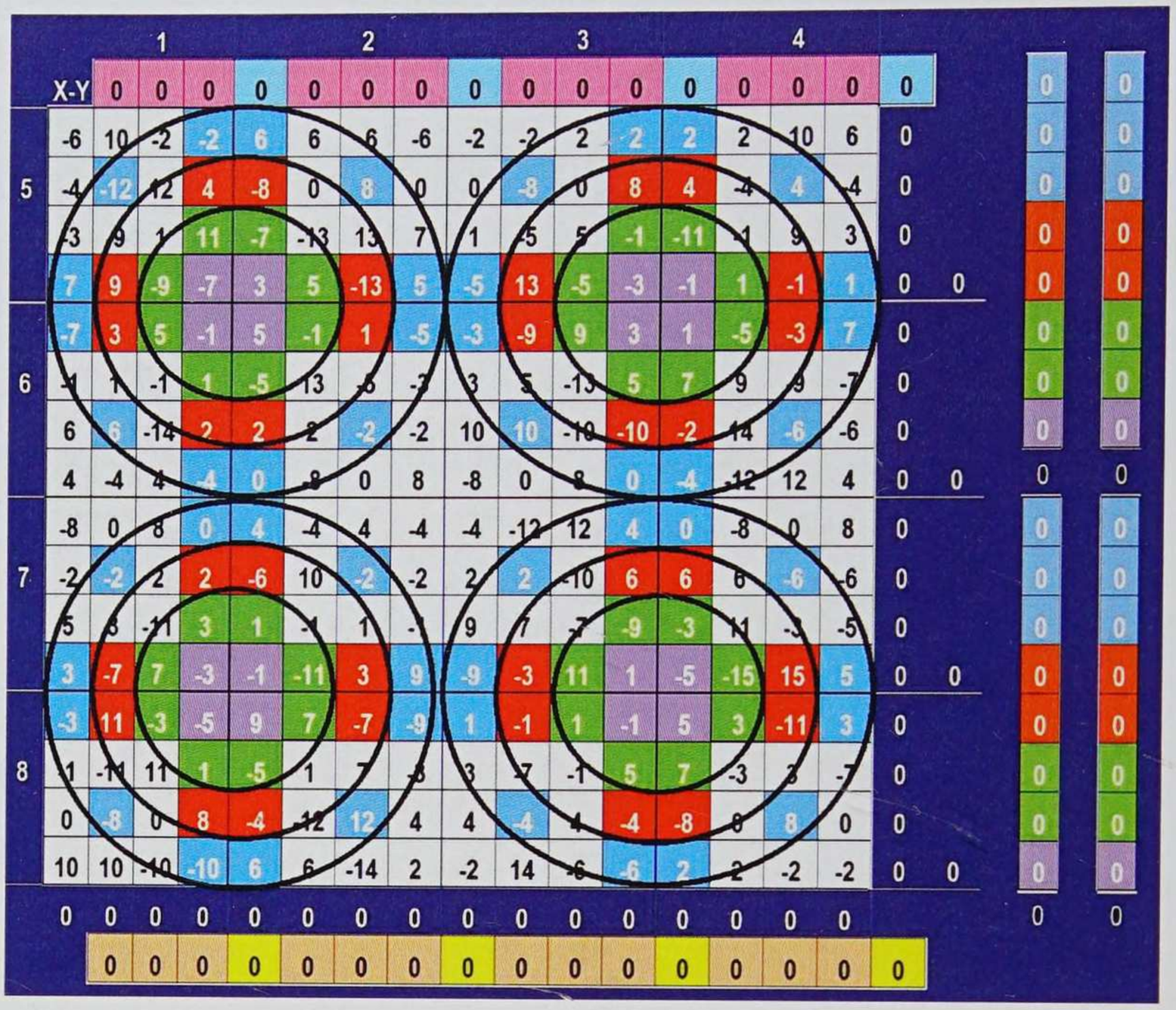
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
272	272	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
34	34	
272	272	

yij = Integer [(wij-1)/16] +1

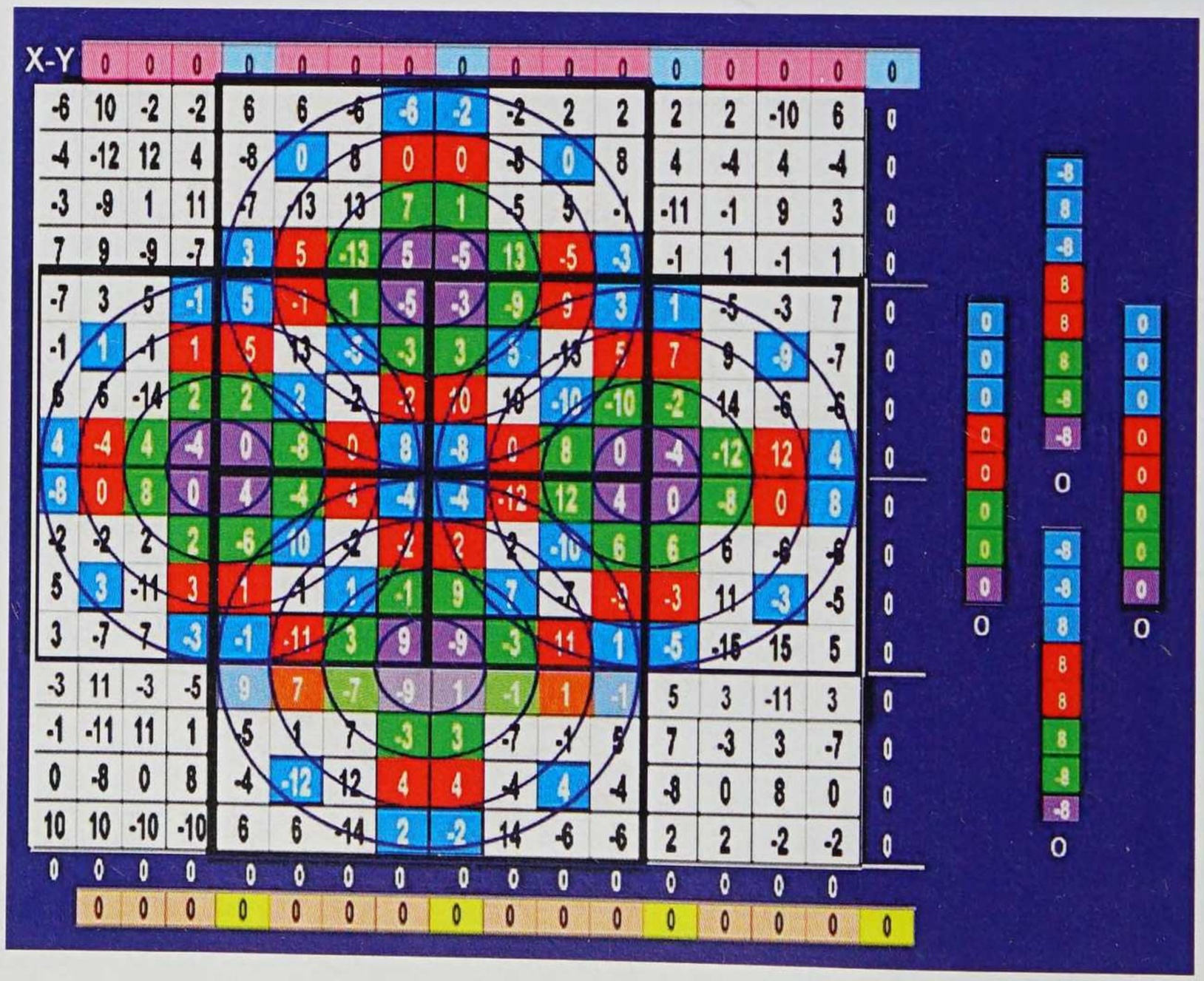
Program 6

Loom-table Differences

Here is the loom difference table. All four hemispherical patterns sum to **0**.



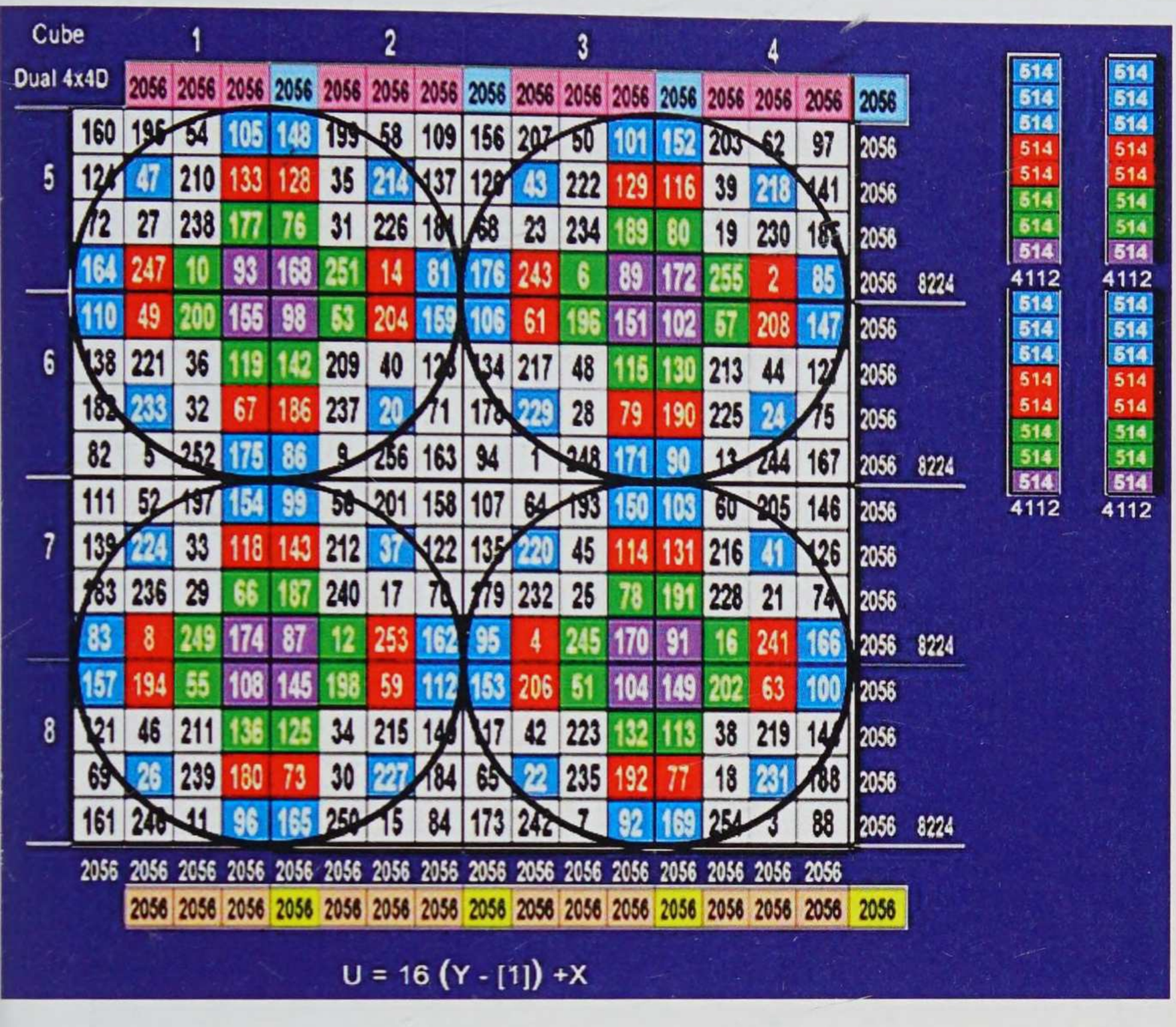
And so do the **intersecting** hemispheres.



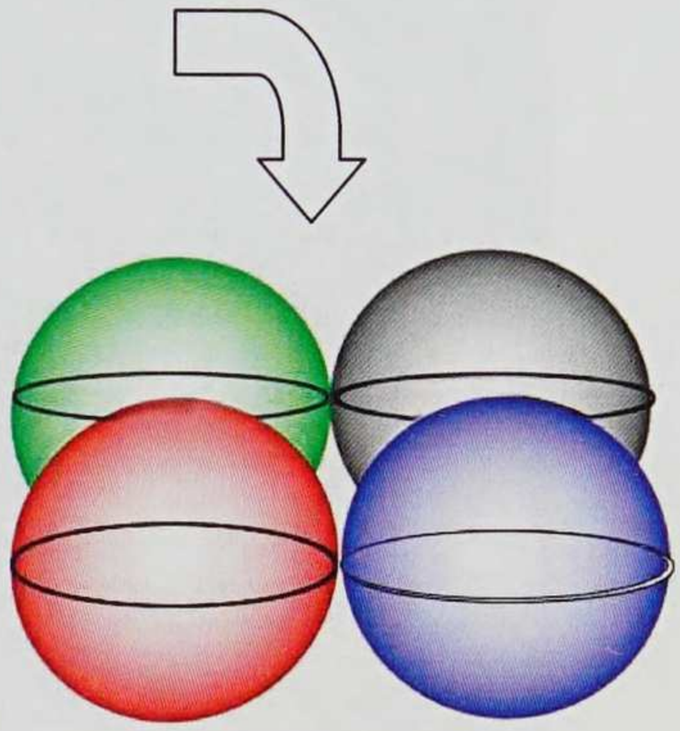
Program 6

Embedded Hemispheres in the Dual Quadracube

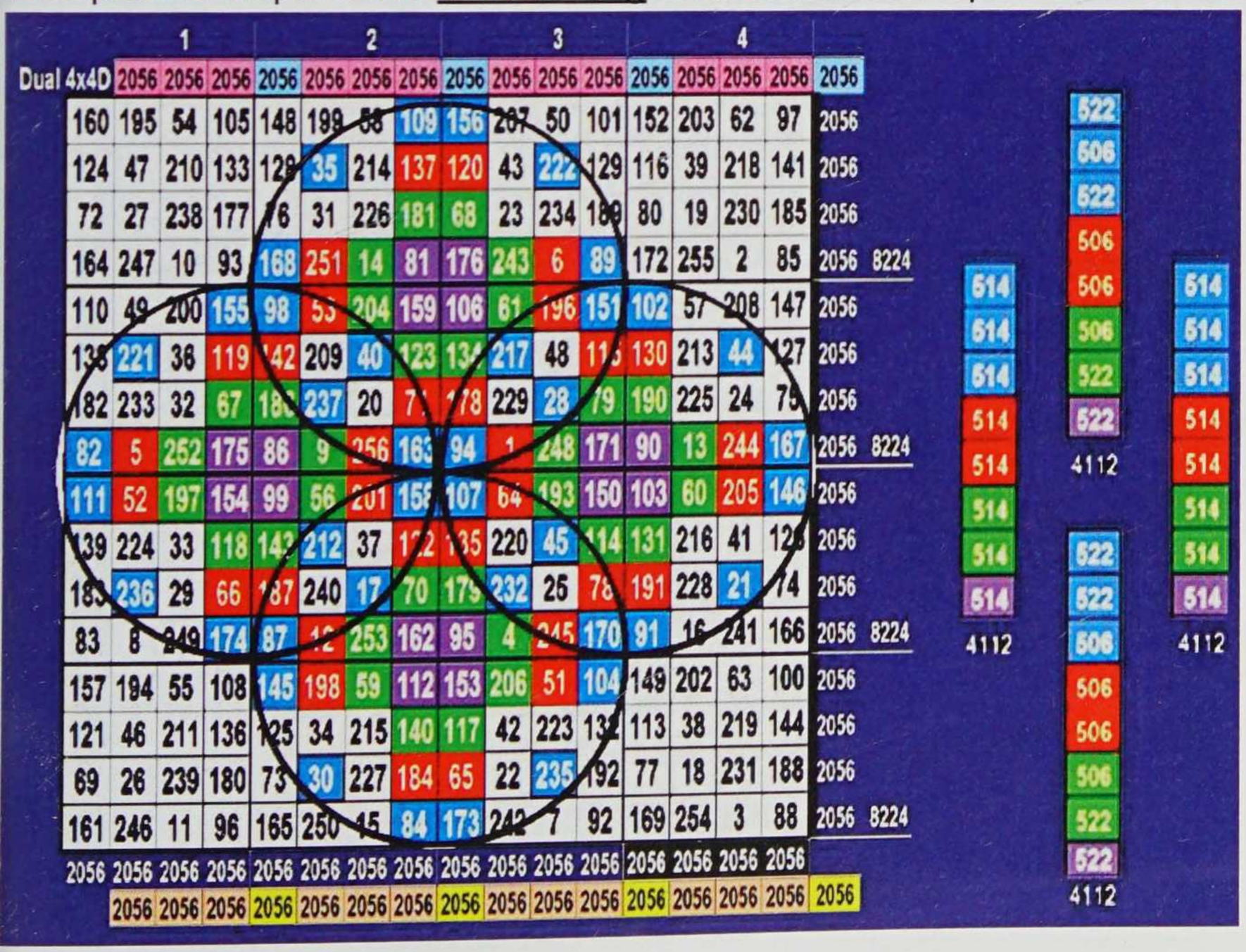
This picture depicts the non-intersecting embedded hemispheres of the dual quadracube.



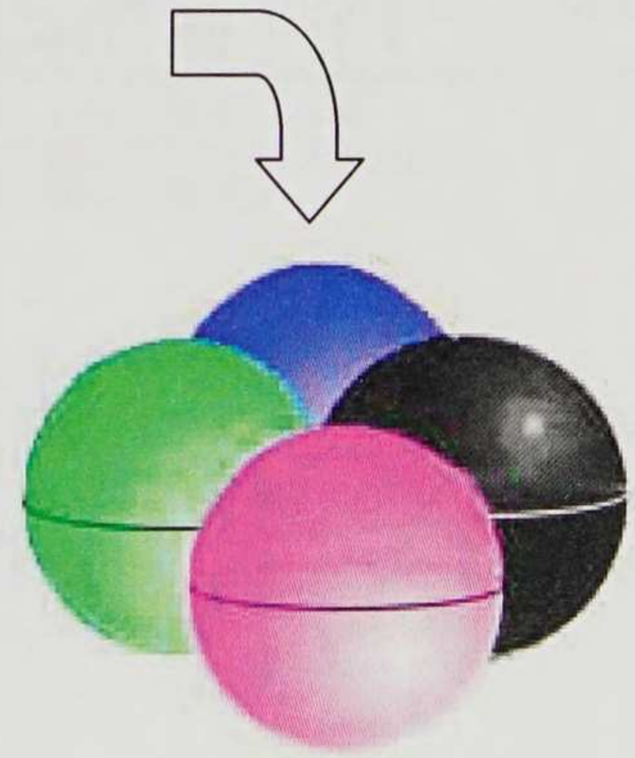
Note that every quadral in the four quadrants sums equally to **514**. So all the hemispheres sum equally to **4112**.



This picture depicts the intersecting embedded hemispheres of the dual quadracube.



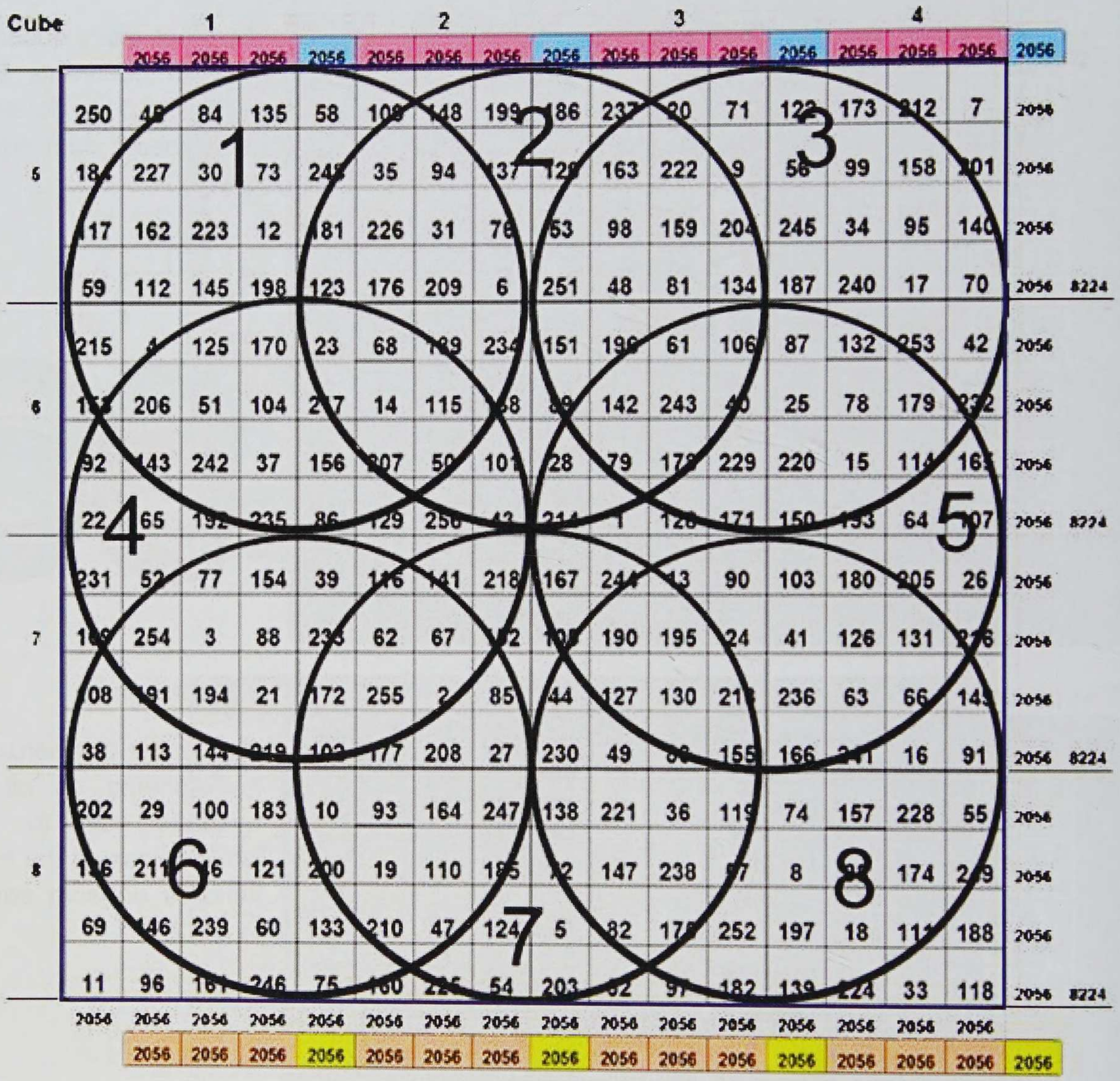
Only in the two horizontal hemispheres do all the quadrals sum to **514**. Nonetheless all the hemispheres do sum equally to **4112**.



Program 6

All the Hidden Hemispheres in Both the Primal and Dual

Here are all **8** hemispheres that lie hidden in the primal quadracube. The same pattern exists in its dual quadracube too.

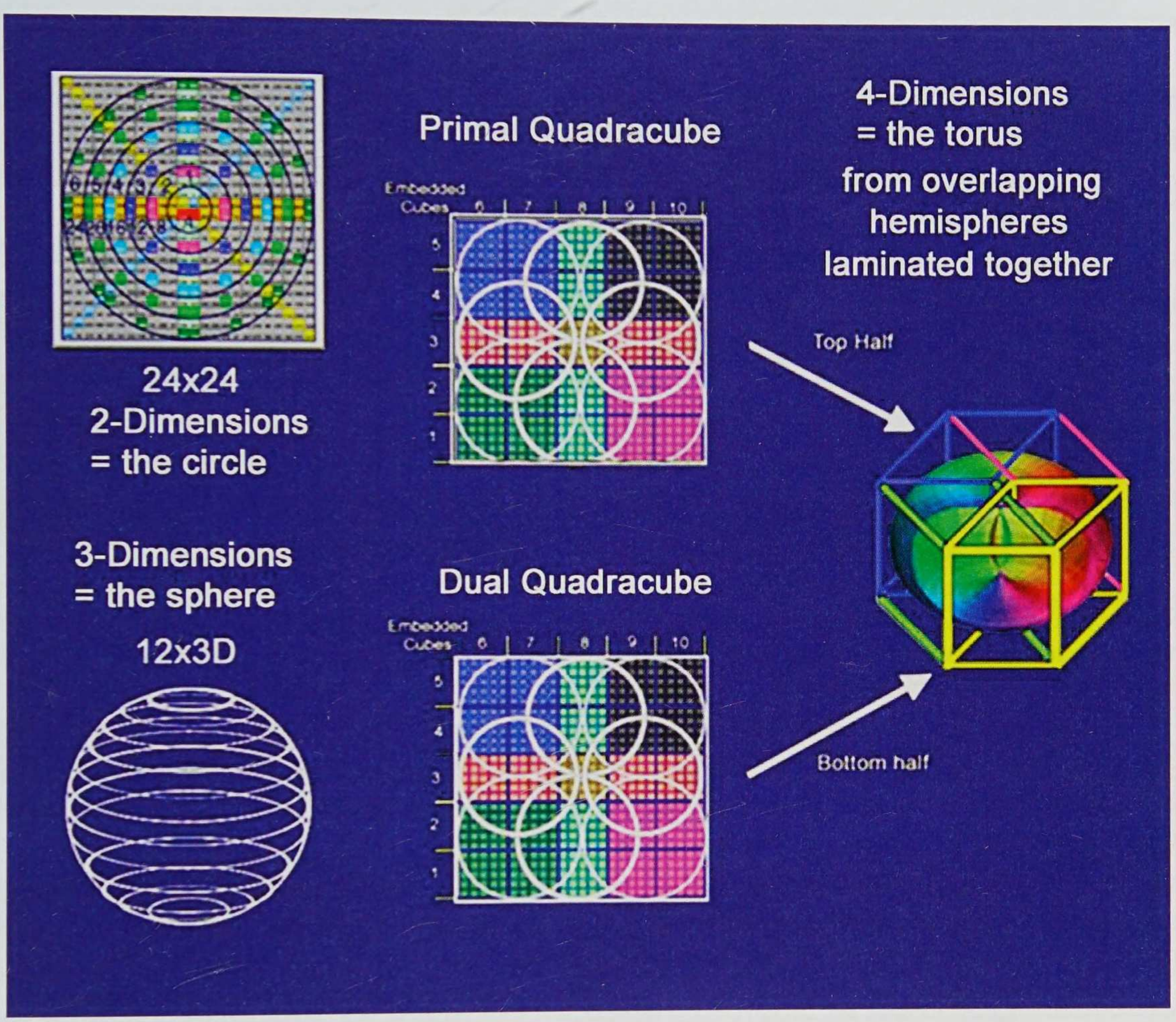


Each circle represents a 3D characteristic hemisphere.

Program 6

The Characteristic Torus

This picture depicts the complete pattern when the table of the size-4 quadracube's primal version is laminated to its dual version. We get a torus in 4-dimensional space!

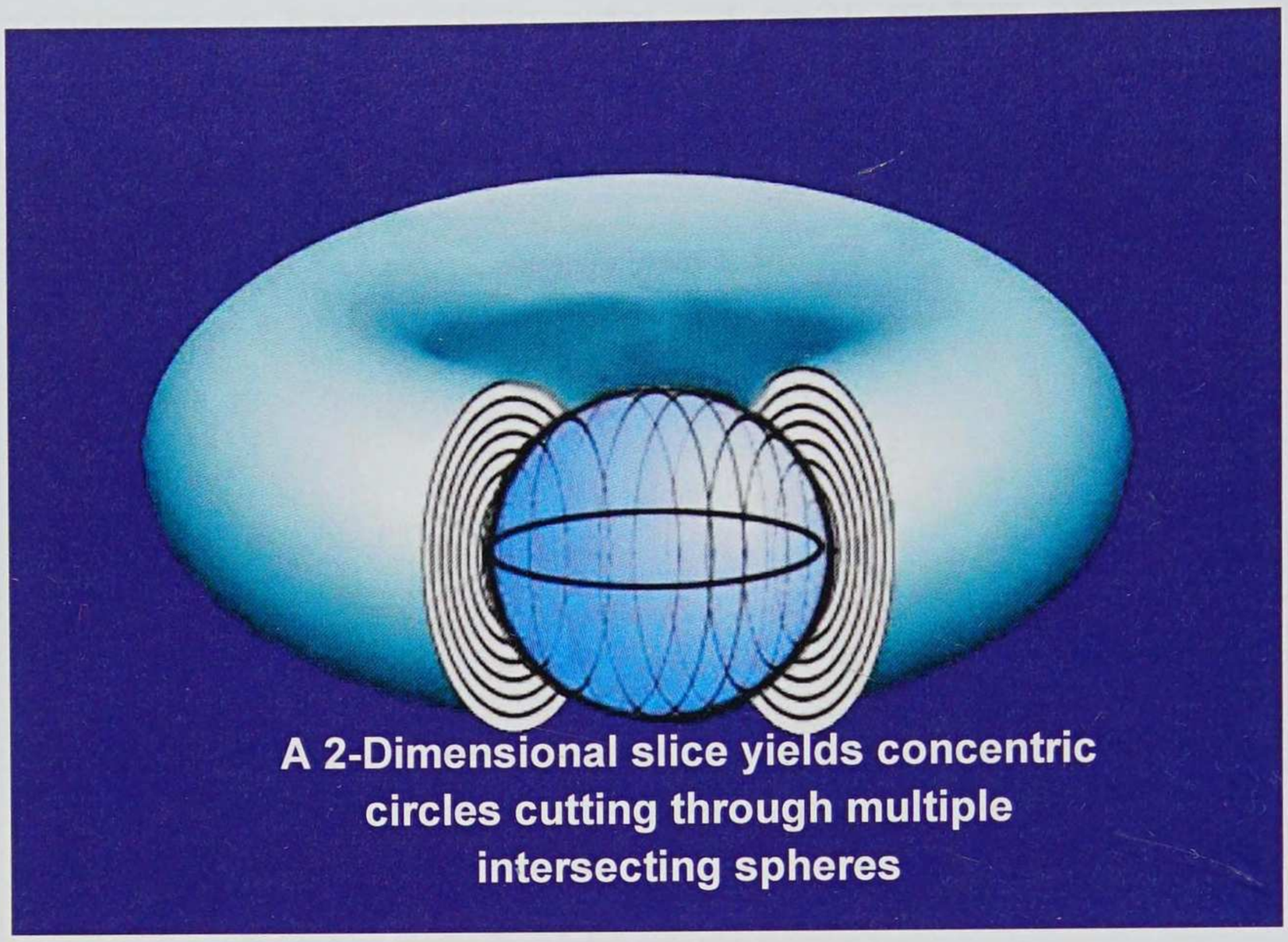


In squares we had the characteristic circle; in cubes we had the characteristic sphere; and now in quadracubes we have the characteristic torus.

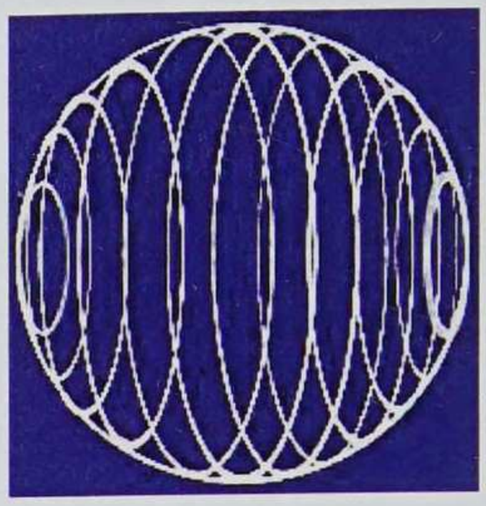
The picture at left shows the torus resulting from laminating together the complementary loom tables which lie at the heart of the torus embedded in the two dual quadracubes.

Program 6

A 3-Dimensional sectional cutout yields a sphere



A 2-Dimensional slice yields concentric circles cutting through multiple intersecting spheres

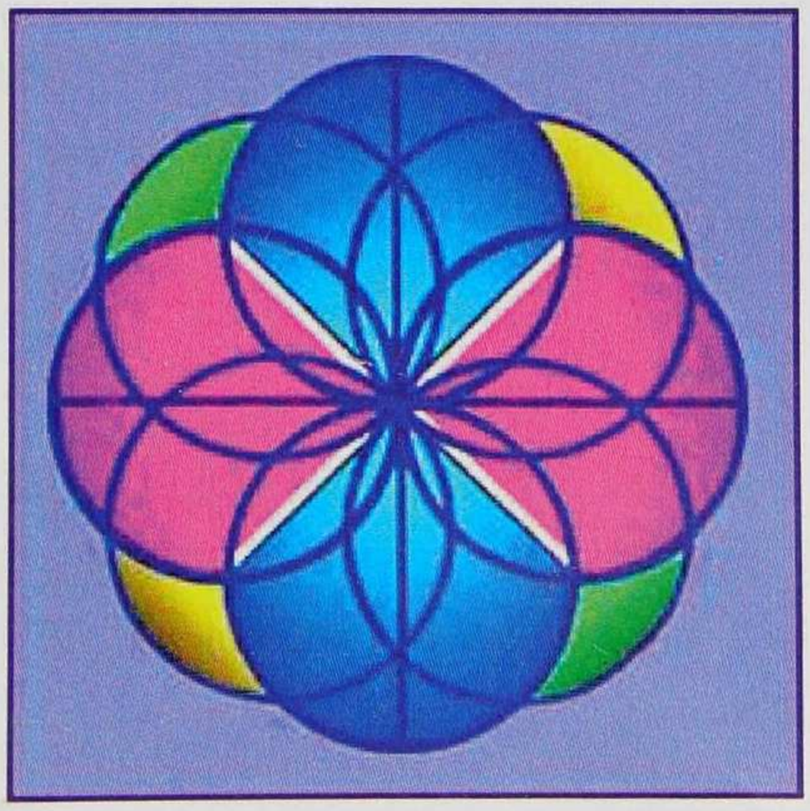


This picture shows that a cubic section cut from a torus is a sphere, composed of 2 hemispheres. The planar slice is a series of nested concentric circles, each one from different intersecting embedded spheres.

So we have now seen how we first discovered the characteristic circle in squares, then characteristic spheres in cubes, and now characteristic toruses in quadracubes. This establishes that just as 2-dimensional space is circular and 3-dimensional space is spherical, 4-dimensional space is toroidal !

As such, only two of those figures are **isotropic** – that is, the spaces measure out equally in all directions. The torus clearly does not measure out equally; its outer rim is way longer than its inner one. So a single torus is not the whole picture.

A picture of 4-dimensional space from perspective of 5D



It will be seen in Program 9 that in dimensions greater than **2D** that there is more than one dual. In **4D**, specifically, there are 4 quadracubes that are related to each other through a circular series of **4** connections. The **4** quadracubes taken **2** at a time in adjacent circular series to create complete spheres via two-fold laminations yields **4** toroidal spaces in sequence. Each toroidal space intersects the other three. It amounts to four donuts with very small center holes intersecting each other at uniformly-spaced 3-dimensional angles as seen here.

Here is what the entire 4-dimensional space looks like from the perspective of 5-dimensional space as determined by Geonometry.

Program 6

Distinguishing Quadracubic Tables from Tables of Class-4 Squares

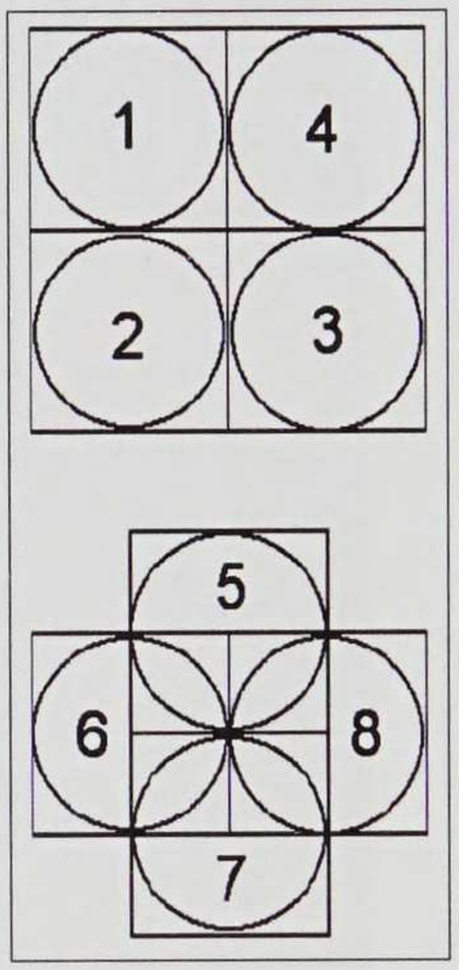
Both quadracubic and Class-4 **2D** tables are square in shape. Here's how one is distinguished from the other:

a) A quadracube will collapse along its **B** and **D** axes to a perfect cube whose symmetric octals are equal.

b) A Class-4 square seemingly collapsed in the same manner will possess unequal symmetric octals everywhere. And moreover, it may even contain duplicate numbers as well as unequal rows. For instance, the size-16 square, having the same size of number table as a **4x4D** quadracube, will not collapse to a **4x3D** cube.

c) On the otherhand, a Class-4 square will possess equal summing quadrals symmetric around its center, where a quadracube's quadrals will be unequal; only its ***hexadectals*** will sum equally. (A *hexadectal* is a 16 corner 4-dimensional figure shown on page 2.)

However, this amazing property was discovered for the size 8 quadracube: the **8x4D** quadracube was found to independently possess equal quadrals among **6** of its **8** overlapping square segments of size **16x16** which contain those peripheral hemispheres. Only square segments #6 and #8 pictured at right did not have equal-summing quadrals everywhere. Nonetheless, those two hemispheres still summed equally to half the characteristic number with all the others.



This equal-summing peripheral hemisphere property may be a general property for all Class-4 quadracubes; it has yet to be verified for the size-12 quadracube whose square table is **144x144**, way too large for processing in Excel.

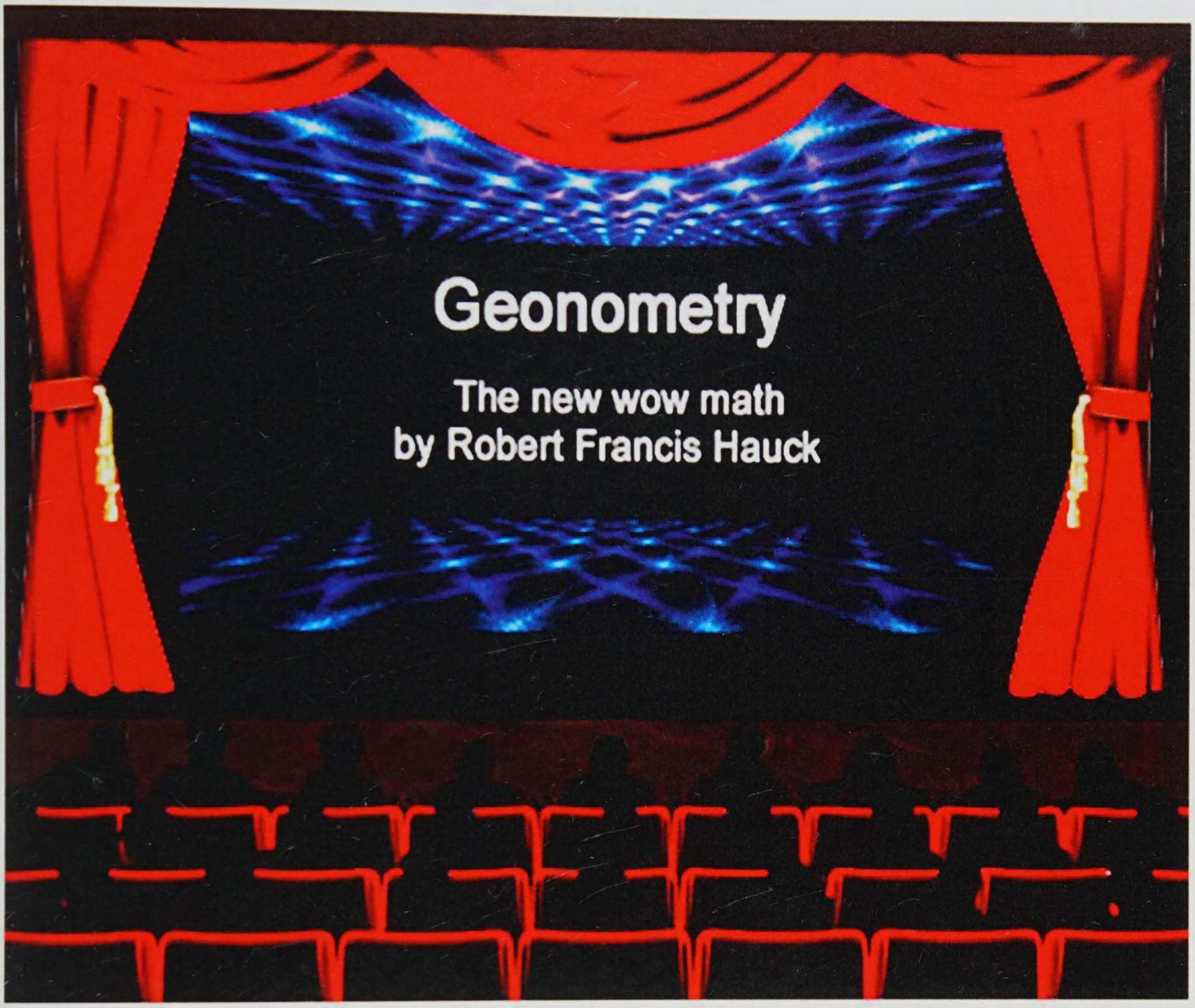
* * *

Because the size-8 quadracube is a **64x64** square table, it takes 8 pages to print all the peripheral circles. Since this companion book is already 500 pages thick, the maximum number of pages the publisher will print in an 8.5"x11" paper-back book, the size-8 quadracube will be handed out separately at the end of the current Program. Be sure to get your copy.

Program 6

Program 6 Notes

1. A quadracube of size **n** is composed of **2n** embedded cubes, half of which are vertically oriented and half which are oriented horizontally.
2. Each embedded cube sums to the quadracube's characteristic number.
3. A quadracube of size **n** is composed of $\mathbf{n^2}$ embedded block-squares, each of which sums equally to **1/n**-th of the quadracube's characteristic number.
4. The centers of all embedded cubes together sum to **1/n**-th of the quadracube's characteristic number.
5. A quadracube will collapse to a perfect geonomic cube along both its **B** and **D** axes.
6. Quadracubes of size **4x4D** possess **8** characteristic peripheral hemispheres. Each one laminated together with one of its **dual** hemispheres produces a 4-dimensional characteristic torus from **8** geonomically perfect, intersecting 3-dimensional spheres.
7. These special minimal size-4 quadracubes lend insight about quark interactions lying within sub-dimensional space **1/4D**.

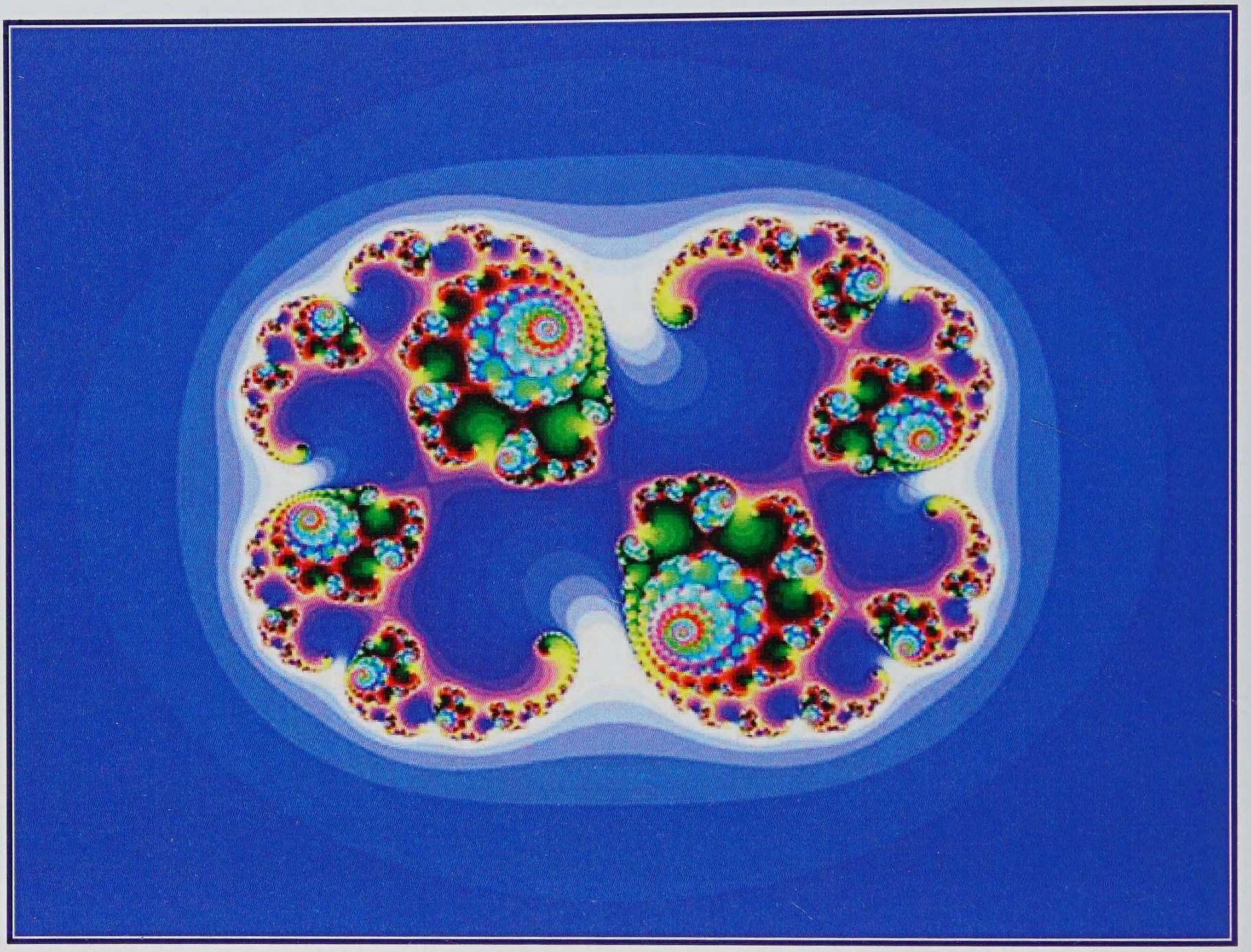


In this segment we're going sub-dimensional once again, but this time all the way down to the basement **1/5D** were we will encounter the particles that make up **quarks** in **1/4D**!

Program 6

Application #3

Application #3 – Accounting for the sub-atomic sub-structure of the proton and neutron



We have seen that the pattern properties observed in the 2nd dimension led to an interpretation of DNA molecular formation in the sub-dimension of **1/2D**, and the 3rd dimension led to a mathematical interpretation of the electron distribution in atoms at the sub-dimension **1/3D**. Having now observed the properties of the 4th dimension, we'll be investigating the quadracube's properties at the sub-dimensional level, **1/4D**.

Program 6

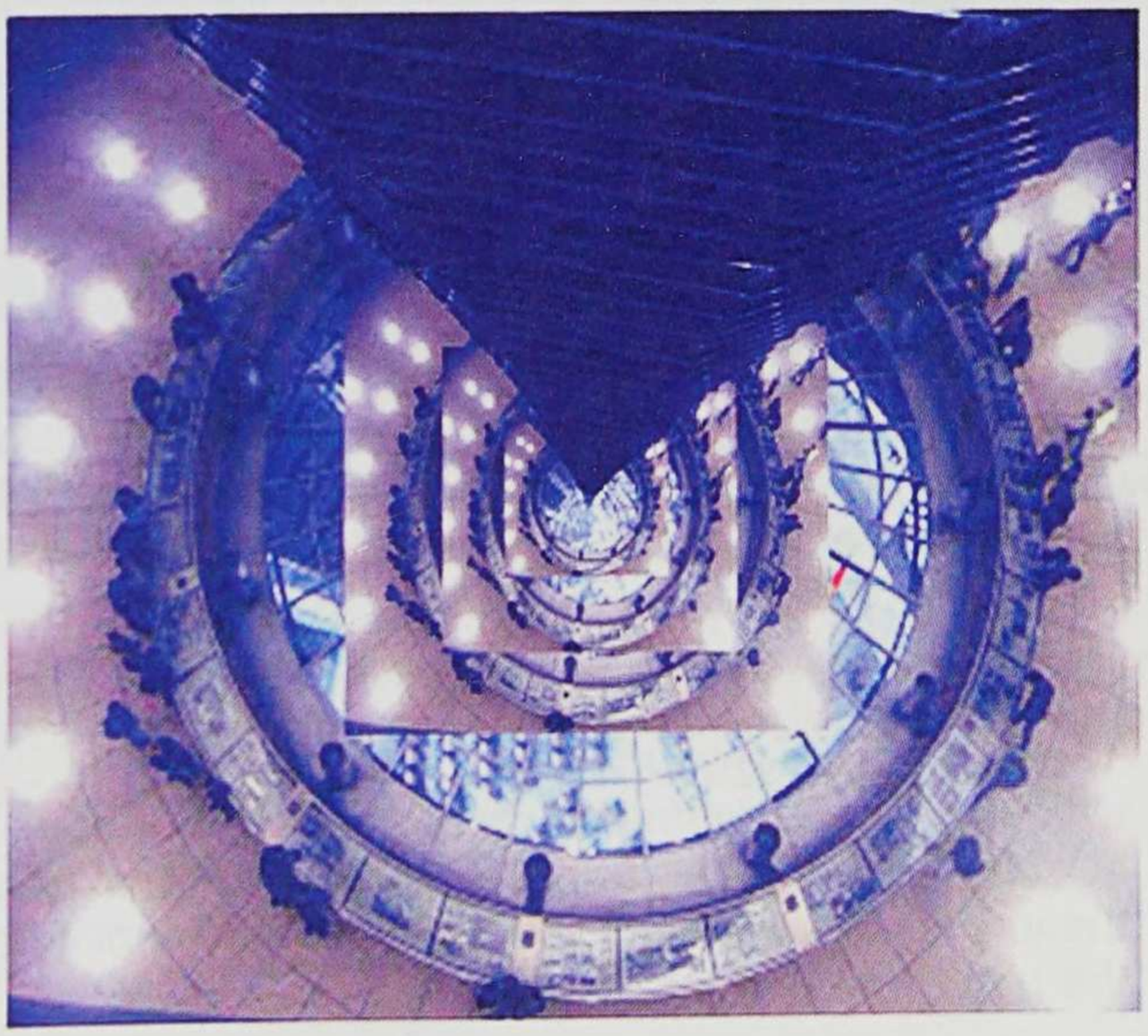
Dimensional Levels

The table here lists descriptive examples of the objects observed at the various levels of multi-dimensional and sub-dimensional space. Of course, with the current state of Man's knowledge, objects in 4-dimensional space can only be conjectured about -- such as has been done here.

Table of Dimensional Levels

Dimensions	Description (examples)
4	Hyperspace (multiple 3D universes)
3	Cubic space
2	Planar space
1	Linear space
1/2	Molecular level
1/3	Atomic level
1/4	Particle level (electrons, protons, neutrons, baryons)
1/5	Sub-particle level (quarks, leptons, hadrons)

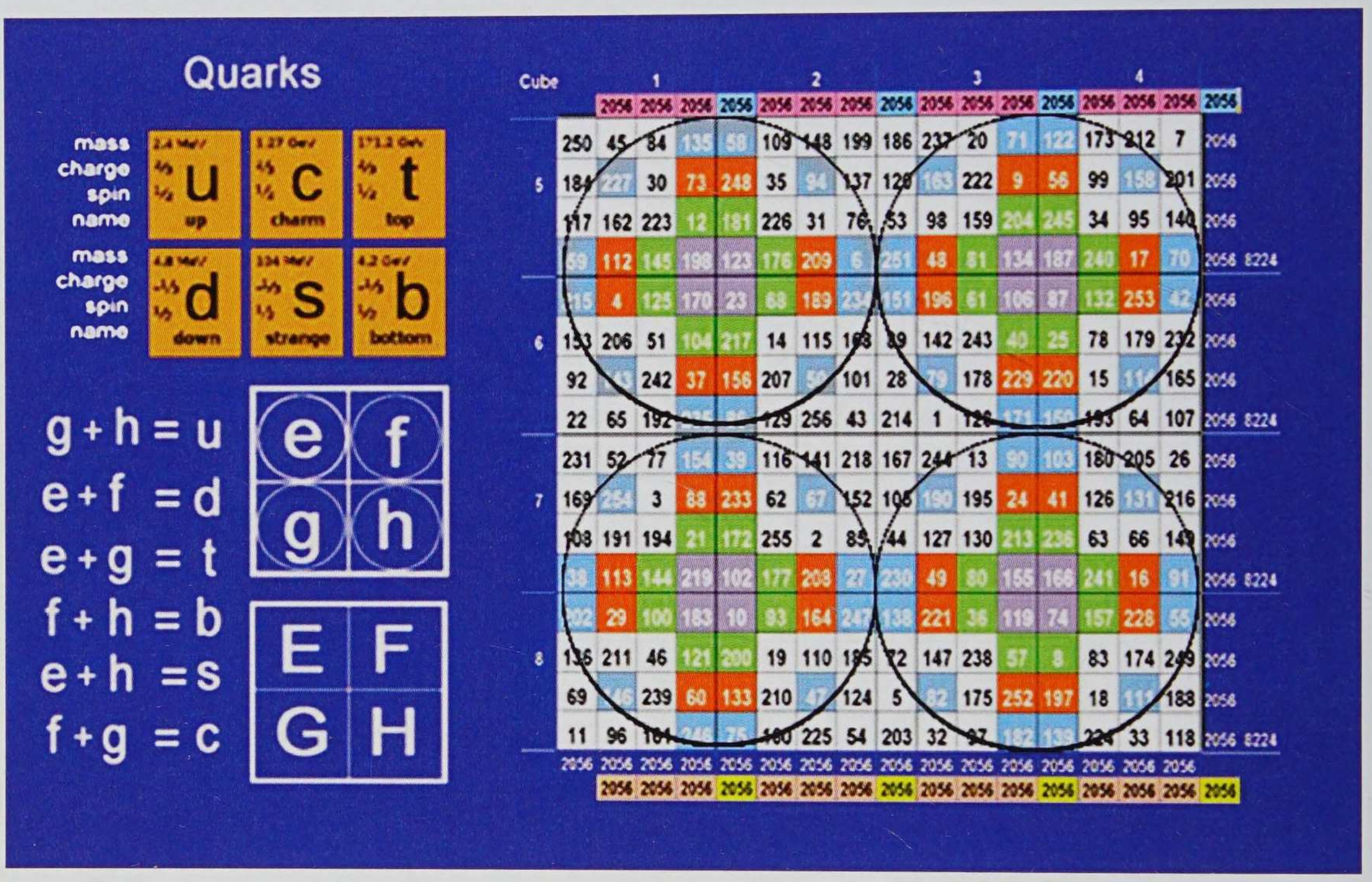
Next we're going down the "wellevator" to the basement of the universe at level **1/5D** and see what we can fathom by applying what we just learned from 4-dimensional Geonometry.



Program 6

The Sub-dimensional Space of the Quark

The particles found at this sub-dimensional level are called quarks. A quark is an elementary particle and a fundamental constituent of matter. Quarks combine to form composite particles called baryons, the most stable of which are protons and neutrons, the basic components of nuclei of the atoms that we have already encountered at the sub-dimensional level **1/3D**. It has been determined that there are six types of quarks, labeled **u** & **d, s** & **c,** and **t** & **b**, standing for **up** & **down / strange** & **charm / top** & **bottom**, respectively. We'll see shortly just how the physicists arrived at these names.



The heavier quarks rapidly change into **up** and **down** quarks through a process of particle decay in an automatic transformation from a higher mass state to a lower mass state. Because of this, **up** and **down** quarks are generally stable and consequently are the only ones found in the nuclei of atoms; the other four quarks are transient and only encountered in particle colliders.

We have already seen that the size-4 quadracube had **4** independent hemispheres embedded within its table. These hemispheres have been labeled with the small letters **e f g** & **h.** These **4** quadrants, each consisting of just one hemisphere, are identified with corresponding capital letters **E F G** & **H**.

Take note of the six combinations of **e**, **f, g** & **h** at bottom left.

These hemispheres may be combined in **6** different combinations to form complete spheres in which all their impinging octals sum to the quadracube's characteristic number, as shown. Whichever hemisphere correlates to which quark is immaterial at the moment. Nonetheless these patterns correlate perfectly to the stability of the neutron and proton and the instability of the other 4 baryons.

Program 6

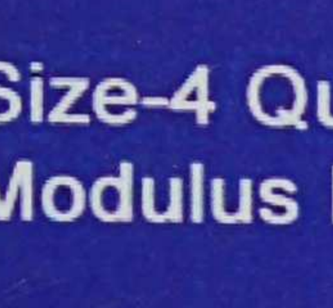
Size-4 Quadracube's Modulus Loom table

This is the **modulus loom table** for the size-4 quadracube. Note that it has only **2** distinct embedded size-4 squares, labeled here as **C** & **D.** This modulus table may be represented in terms of these block-squares as depicted at left. The corresponding quadrants of the loom table, which contain the characteristic hemispheres, are denoted again here by the capital letters **E F G** & **H**.

Size-4 Quadracube's Modulus Loom Table

Block-squares

C	C	C	C
D	D	D	D
D	D	D	D
C	C	C	C



Loom table quadrants

	1				2				3				4				
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
5	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40
	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40
	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40 160
6	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40
	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40
	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40 160
7	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40
	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40
	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40 160
8	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40
	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40
	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40 160
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40

C

2	1	4	3
4	3	2	1
1	2	3	4
3	4	1	2

D

3	4	1	2
1	2	3	4
4	3	2	1
2	1	4	3

E+G = [n+1] = [5]
F+H = [n+1] = [5]

E+H = [n+1] = [5]
G+F = [n+1] = [5]

E - F = [0]
G - H = [0]

C + D

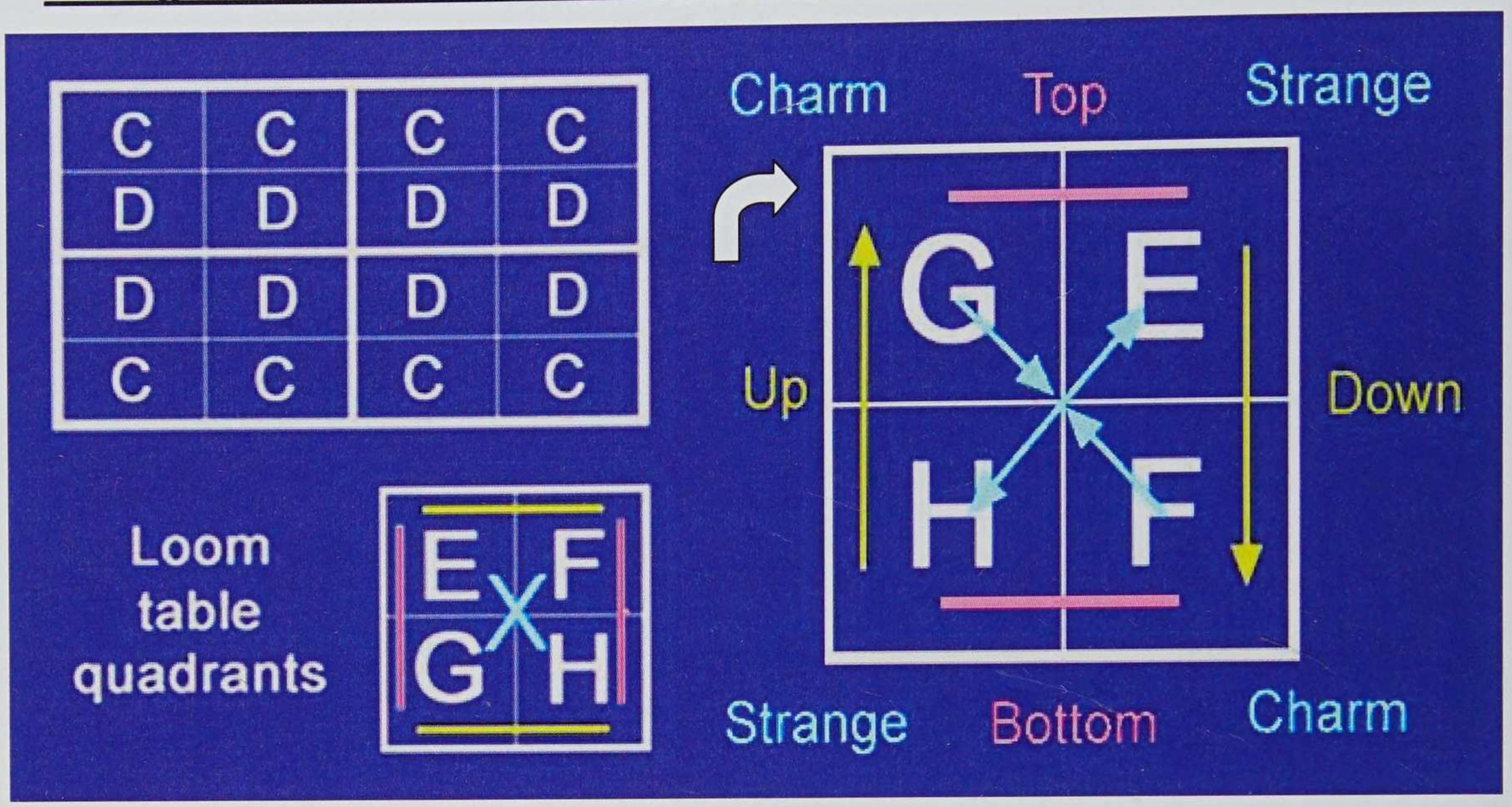
5	5	5	5
5	5	5	5
5	5	5	5
5	5	5	5

Now observe that the four **addition** combinations of these quadrants, as expressed in the lower left corner, yield matrices of all **5**'s. And 2 **difference** combinations next to them yield matrices of all **0**'s. Those combinations which correspond to the matrix of all **5**'s are those with different **complementary** loom table patterns. Those combinations which correspond to the matrix of all **0**'s are those with **identical** loom table patterns. This shows that of the **6** possible combinations of hemispheres, **2** are **identical** and **4** are **complementary.**

These patterns correlate perfectly to the stability of the neutron and proton and the instability of the other 4 baryons.

Program 6

Strong correlation with the atomic physicists' model and descriptions



By rotating the diagram 90° clockwise, the model derived here exhibits a100% correlation with the atomic physicists' model and descriptions.

Note that **G & H** and **E & F** are the only two combinations that are composed from quadracubic quadrants with identical numerical patterns. They now correspond to the "**up**" and "**down**" quarks. And it is these two quarks that make up the two baryons with the least mass and the most stability: the proton and neutron.

The combinations of **E** & **H** repel each other and are strangers. The combinations of **F** & **G** are attracted to each other because they possess charm.

The combinations **G** & **E** just happen to be at the top of the relationship box and **H** & **F** just happen to be at the bottom of the box when the particle physicists were organizing the properties into a 4-quadrant relationship box. That's how they got their quirky names: **up down / strange charm / top bottom**.

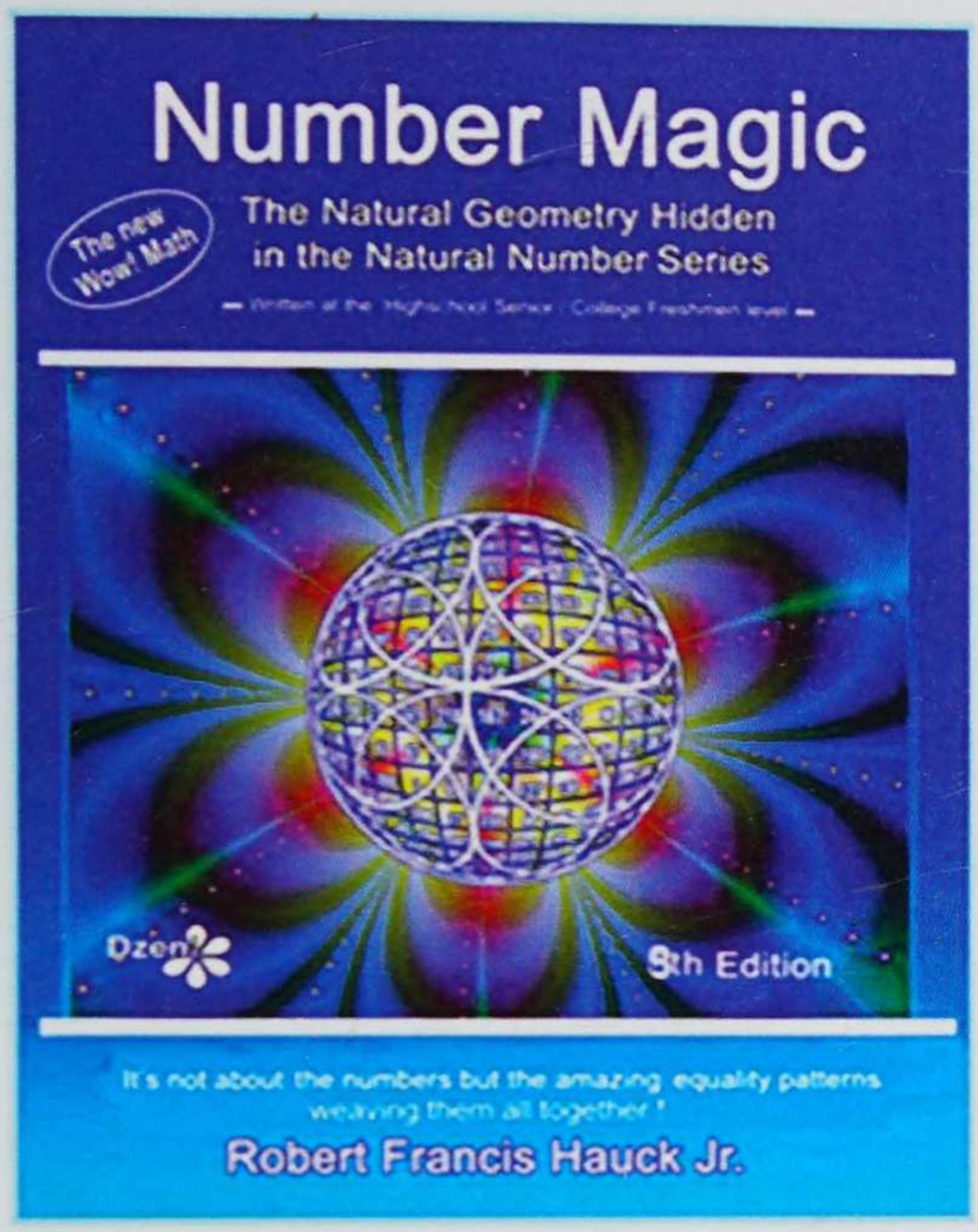
Recall that these quadrants correspond to characteristic hemispheres in **4D** space for both loom tables **X** & **Y** and for both primary quadracubes **W** & **U** simultaneously. All this cannot be just coincidental! This demonstrates that the properties uncovered in **4D space** have fundamental application for comprehending the basic properties at the **1/4D** sub-dimensional level!

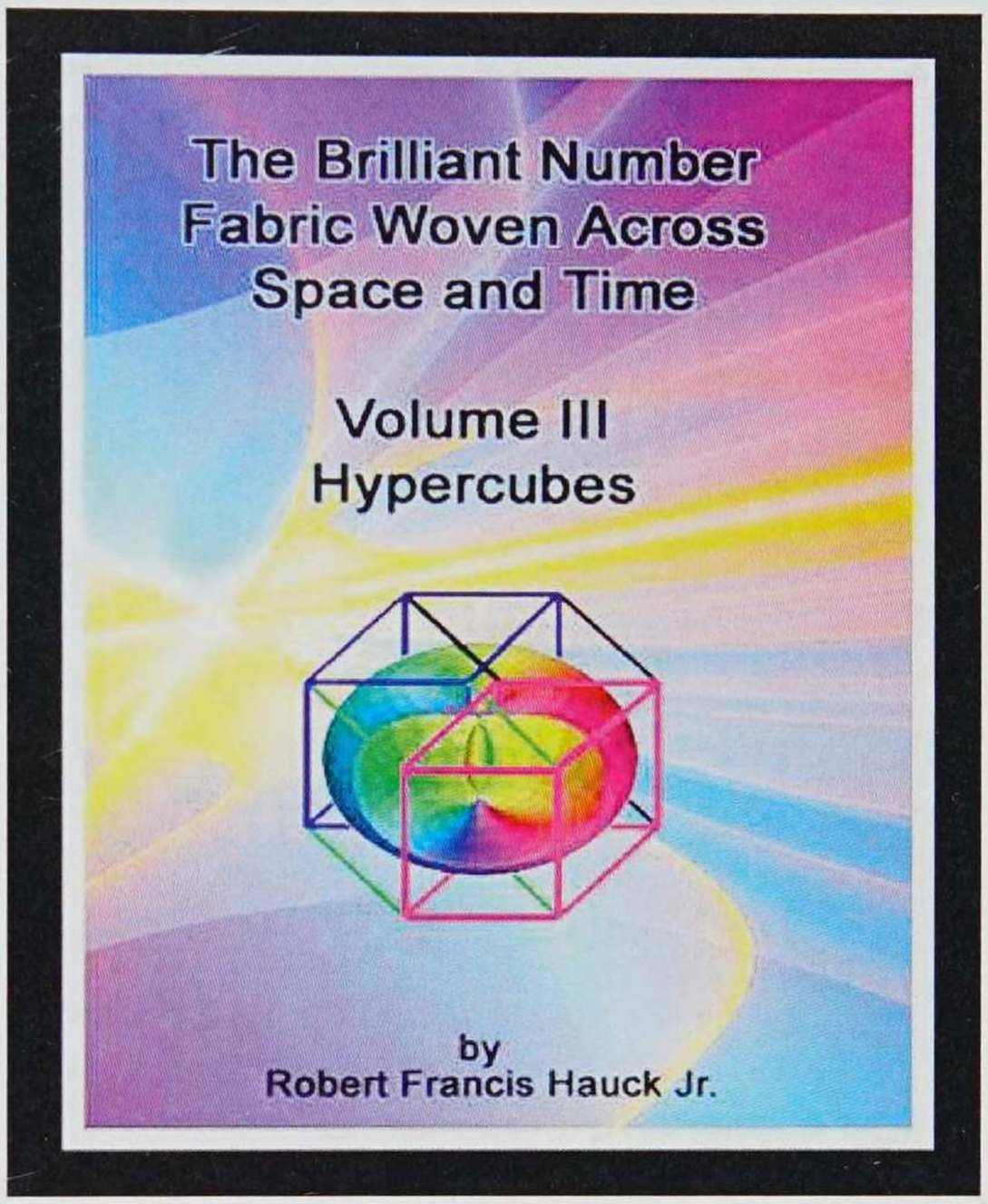
So once again Geonometry has supplied the math with which to explain the properties of basic structures, from one of a higher dimensional level to one of a lower level corresponding to the inverse sub-dimension.

Program 6

We have come to the end of Program 6.

Here are the books upon which this program was based.





Number Magic The Natural Geonometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension.

Ninth Edition (360+ pages)

The Brilliant Number Fabric Woven Across Space and Time Volume III – Hypercubes

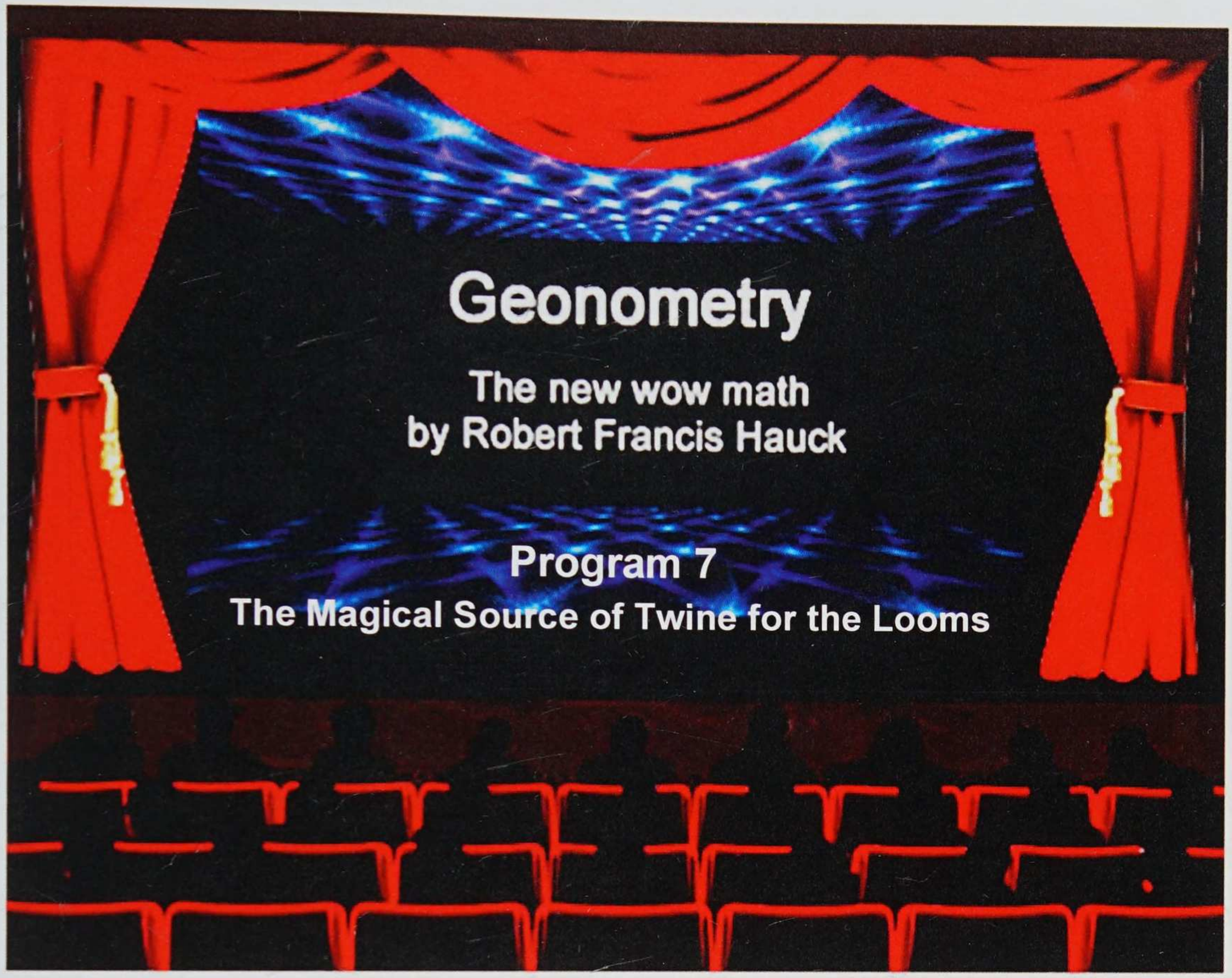
ISBN: 978-1-461-08073-2

Deals only with 4- and 5-dimensional hypercube that are legible in print.

Third Edition (74 pages)

Program 6

Program 7



In this program, we will explore a completely new type of square whose properties are truly magical. This type of square allows for duplication of numbers but in a very specific normalized manner. They are called **Matchmaker squares**. What you will discover in this program is a fundamental relationship unknown in Matrix Theory prior to the release of this program series.

Caution: The matchmaker square is what is categorized in mathematics as recreational math. It is shown here to exhibit its fundamental connection to Geonomical math. This correspondence between these two distinct categories of math should not be used to lump Geonometry in with recreational math to taint Geonometry as being merely a topic of curiosity with no fundamental application to Science and technology. It's way more than that.

Program 7

The Source of Origin of the Matchmaker's Square

Of all the squares dubbed "magic", the matchmaker's square is truly magic. It wasn't invented as such; it was discovered by the author quite by accident while trying to contrive an example to demonstrate the effectiveness of a scheduling algorithm that he developed for National Semiconductor Corporation to maximize production priorities. The matrix you will see here is the actual table of priorities used in that intended demonstration. It was never used because to his stunning surprise, he kept getting the same priority answer in the quest of maximizing the total priorities no matter in which order the items were to be scheduled.

Geonometry is the mathematics of the **fabric of space**. The matchmaker square is the essential basic source of all the Class-1 squares and, whether you believe it or not, is the **twine** used by the *quadralooms* of Lucasfilm *Stuntworks Division* (re: Program 3 p.16) to create the **fabric of space** as you will now behold.

Program 7

The Five Basic Properties of the Matchmaker's Magic Square

9x9	1	2	3	4	5	6	7	8	9
1	9	4	11	5	7	8	6	10	3
2	15	10	17	11	13	14	12	16	9
3	7	2	9	3	5	6	4	8	1
4	12	7	14	8	10	11	9	13	6
5	11	6	13	7	9	10	8	12	5
6	10	5	12	6	8	9	7	11	4
7	14	9	16	10	12	13	11	15	8
8	8	3	10	4	6	7	5	9	2
9	13	8	15	9	11	12	10	14	7

A matching in a matchmaker's square is a selection of a different column for every distinct row and summing the numbers in the cells at their intersections.

Here is an example of a size-9 numeric table of what is called the matchmaker's magic square.

It has these **5** properties:

1. **The Dimensional Average Property:** All the numbers average to its size **n**, here **9**.
2. **The Matchings Property**: All *matchings* sum to the same number, $\mathbf{n^2}$; here **81**.
3. **The Totality Property**: All the numbers in the square sum to $\mathbf{n^3}$, here **729**.
4. **The Subset Property**: Any smaller square, called an **abbreviated square**, taken from anywhere in the table has all possible *matching*s summing to the same number. This number is different for different abbreviated squares. Moreover, these squares need not be composed of numbers from adjacent rows and columns -- any number of **m** independent rows and **m** independent columns may be selected at random and used to make an abbreviated **m-by-m** square from the cells common among their intersections -- of course keeping their places in the abbreviated square relative to the bigger square. And every *matching* within the abbreviated square will sum to the same number! This is just an amazing property that derives from both the numbers and the space in which they are strewn, that is, their underlying organized pattern.
5. **The Invariance-under-permutation Property**: The rows and columns may be swapped one pair at a time, that is, rearranged into whatever order you like, by interchanging the position of any two rows or any two columns, any number of times in series, like spinning a Rubric's Cube, and the square will still have the same properties #1 thru #4 ! Formally, the matchmaker's square is said to be **invariant under column and row permutation**.

Note: Properties #1 thru #5 are proven as *theorems* in the Appendix.

Program 7

Characteristic number for the abbreviated square = **41**.
All **24** (4x3x2) possible matchings sum to **41**

The Size-9 mm-Square Normalized

9x9	81	81	81	81	81	81	81	81	81
17	16	15	14	13	12	11	10	9	
16	15	14	13	12	11	10	9	8	
15	14	13	12	11	10	9	8	7	
14	13	12	11	10	9	8	7	6	
13	12	11	10	9	8	7	6	5	81
12	11	10	9	8	7	6	5	4	
11	10	9	8	7	6	5	4	3	
10	9	8	7	6	5	4	3	2	
9	8	7	6	5	4	3	2	1	
				81					
81	81	81	81	81	81	81	81	81	

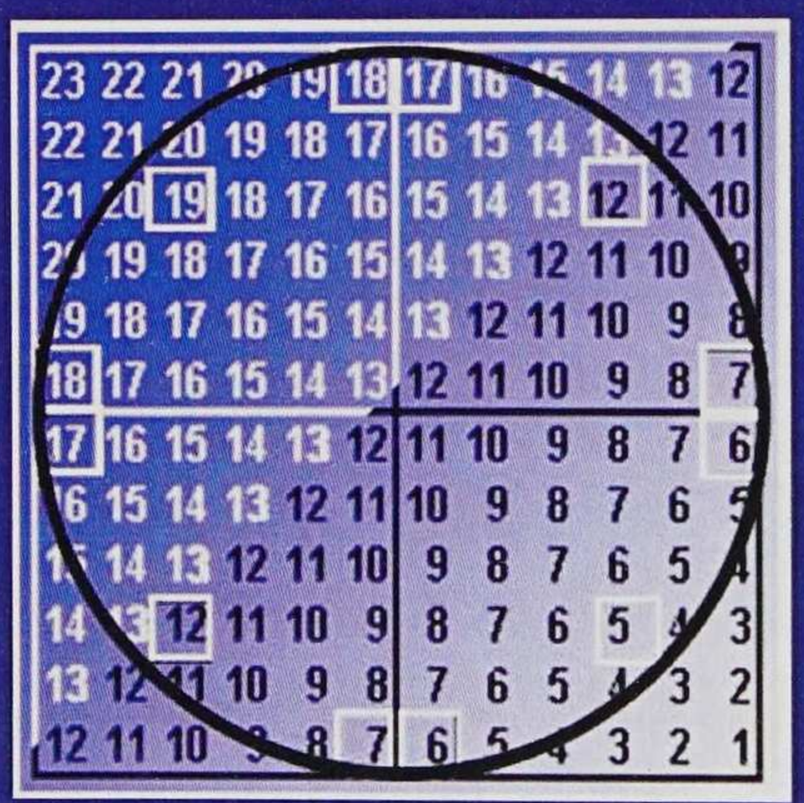
Columns

Rows	3	5	7	9
1	11	3	5	7
4	14	6	8	10
7	17	9	11	13
9	15	7	9	11

Here is the size-9 matchmaker's square unscrambled. Note the obvious number pattern. When the numbers are laid out in this fashion, the square is said to be **normalized**.

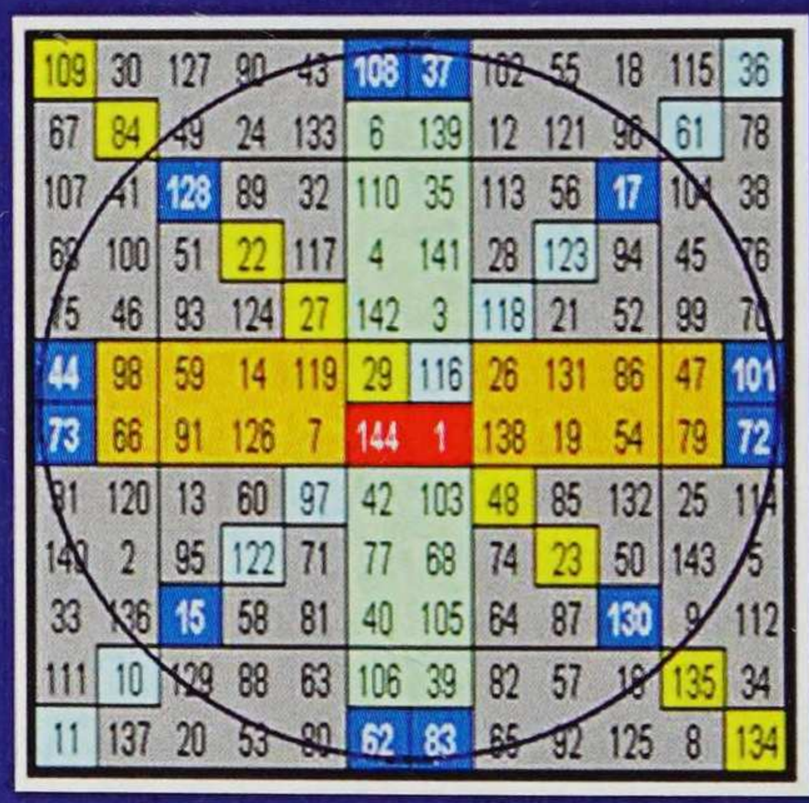
Comparison of Characteristic Circles between Class-4 Regular Squares and Matchmaker Squares of the same size

12x12



All symmetric pairs sum to 24.

12x12



Every quadral sums to 290.

Here are the characteristic circles of both the matchmaker's square of size-12 and the regular size-12 square. They are shown side-by-side for direct comparison.

Observe that in both cases, eventhough both of these tables and their characteristic numbers are different, the **12** impinging cells of the largest inscribed circle in each table sum to each square's distinct characteristic number. This one-to-one correspondence between matchmaker squares and regular geonomic squares holds for every size of square in Class-4. This is another amazing discovery of Geonometry.

Program 7

The Matchmakers Magic Cube

Here is the size-5 matchmaker's cube. Its three depth-sum tables depicted at left are all identical.

Observe that the cube appears as being numerically identical along each of the three axial directions as viewed horizontally from the left and behind, and vertically from the top; then again horizontally from the right and front, and vertically from the bottom.

5x3D mm-cube

A					
	175	175	175	175	175
55	50	45	40	35	
50	45	40	35	30	
45	40	35	30	25	175
40	35	30	25	20	
35	30	25	20	15	
	175				
	175	175	175	175	175

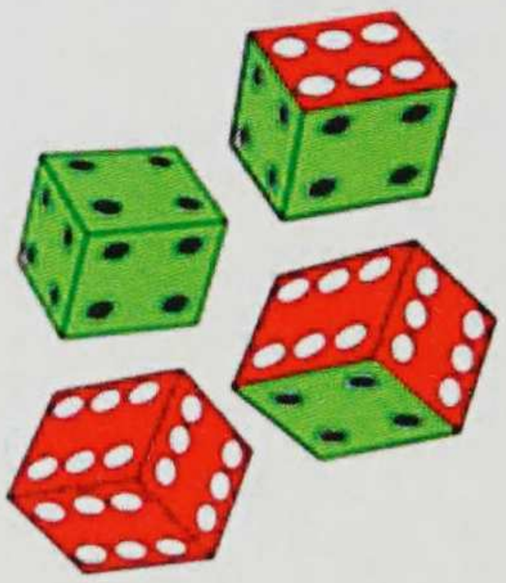
B					
	175	175	175	175	175
55	50	45	40	35	
50	45	40	35	30	
45	40	35	30	25	175
40	35	30	25	20	
35	30	25	20	15	
	175				
	175	175	175	175	175

C					
	175	175	175	175	175
55	50	45	40	35	
50	45	40	35	30	
45	40	35	30	25	175
40	35	30	25	20	
35	30	25	20	15	
	175				
	175	175	175	175	175

View along B-axis
Front to Back

	13	12	11	10	9	5
	12	11	10	9	8	4
5	11	10	9	8	7	3
	10	9	8	7	6	2
	9	8	7	6	5	1
	12	11	10	9	8	5
	11	10	9	8	7	4
4	10	9	8	7	6	3
	9	8	7	6	5	2
	8	7	6	5	4	1
	11	10	9	8	7	5
	10	9	8	7	6	4
3	9	8	7	6	5	3
	8	7	6	5	4	2
	7	6	5	4	3	1
	10	9	8	7	6	5
	9	8	7	6	5	4
2	8	7	6	5	4	3
	7	6	5	4	3	2
	6	5	4	3	2	1
	9	8	7	6	5	5
	8	7	6	5	4	4
1	7	6	5	4	3	3
	6	5	4	3	2	2
	5	4	3	2	1	1
	5	4	3	2	1	

The three axial perspectives depicted below show that the matchmaker's cube is like a 6-sided dice with three adjoining faces dotted with one number and their three opposing faces with another.



A-axis

Last column in each block-square

13	12	11	10	9	5
12	11	10	9	8	4
11	10	9	8	7	3
10	9	8	7	6	2
9	8	7	6	5	1
5	4	3	2	1	

First column in each block-square

9	8	7	6	5	5
8	7	6	5	4	4
7	6	5	4	3	3
6	5	4	3	2	2
5	4	3	2	1	1
5	4	3	2	1	

B-axis

Back

Last block-square

	13	12	11	10	9
	12	11	10	9	8
5	11	10	9	8	7
	10	9	8	7	6
	9	8	7	6	5

Front

First block-square

	9	8	7	6	5
	8	7	6	5	4
1	7	6	5	4	3
	6	5	4	3	2
	5	4	3	2	1

C-axis

Last row in each block-square

5	13	12	11	10	9
4	12	11	10	9	8
3	11	10	9	8	7
2	10	9	8	7	6
1	9	8	7	6	5
	5	4	3	2	1

First row in each block-square

5	9	8	7	6	5
4	8	7	6	5	4
3	7	6	5	4	3
2	6	5	4	3	2
1	5	4	3	2	1
	5	4	3	2	1

Program 7

The Matchmakers Magic Quadracube

Here is the size-4 matchmakers quadracube. Its embedded hexadectals and its tw icosahedrons each sum equally within their distinct geometries. The cells involved in the tw icosahedral summations are highlighted by crimson and green.

9	8	7	6	5
8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1

The numbers in lower right-hand corner of each **5x5** block-square are just the expansion of each number into an mm block –square all of the same size.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	5x4D	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225
1	17	16	15	14	13	16	15	14	13	12	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	325
2	16	15	14	13	12	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	300
3	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	275
4	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
5	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
6	16	15	14	13	12	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	300
7	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	275
8	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
9	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
10	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
11	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	275
12	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
13	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
14	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
15	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	175
16	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
17	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
18	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
19	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	175
20	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	6	5	4	3	2	150
21	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
22	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
23	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	175
24	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	6	5	4	3	2	150
25	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	6	5	4	3	2	5	4	3	2	1	125
	325	300	275	250	225	300	275	250	225	200	275	250	225	200	175	250	225	200	175	150	225	200	175	150	125	
	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225

The hexadectal sums are just any **4** quadrals in four embedded squares symmetrically positione around the central embedded square.

The **36** hexadectal summations are each equal to **144**.

The cells involved in the **2** icosahedral summations are highlighted by crimson and green. Eac sums to **108 = ¾** of **144**.

A matching in 4-dimensions in a size-5 **quadracube** selects **5** numbers such that no tw numbers are selected from neither the same cube nor from the same location in all the othe embedded squares among all the embedded cubes. All of its 4-dimensional matchings of selected cells will sum equally everywhere to **45**. That's **5 x 9** where the number **9** is th dimensional average of the table for 4-dimensions.

Then the table can be treated at the **cubic** block-square level. Every embedded horizontal cube is matched with one vertical cube. Then a complete matching is made within their intersecting block-square. That's **25** matchings in all. All of these matchings will sum to the same number every time. In this 4D table of size-5 it will sum to **225**. That's equal to **9 x 25** where the number **9** is again the dimensional average of the table for 3-dimensions.

Further, you can just do a row-to-column matching as if the table were an ordinary matchmaker square of size **25**. Again you will have **25** matchings, total. All of these matchings will also sum to **225**.

Note that for matchmaker squares of odd-size **n**, the dimensional average, i.e. pivot number, **p = n**.

Loom Tables of Matchmaker Squares

Now here is where we encounter the fundamental relationship between matchmaker squares and regular Class-1 geonomic squares that was promised earlier in this program.

The matchmaker's square of size-7 here, is decomposed into its modulus and integer loom tables as depicted at left. The modulus loom table is seen to have row and column patterns in which each contains the numbers **1** thru **7** exactly once. Each of its minor diagonals contains **7** repetitions of one distinct number, with no diagonal pattern being duplicated. So it only has partially the basic number distribution of a loom table because it lacks the complete pattern of having the numbers from **1** thru **7** exactly once in all four directional summations.

On the next page we'll convert this matchmaker loom-table to a modulus loom-table for a size-7 regular geonomic square in just **5** steps.

Y

2	2	2	2	2	2	1
2	2	2	2	2	1	1
2	2	2	2	1	1	1
2	2	2	1	1	1	1
2	2	1	1	1	1	1
2	1	1	1	1	1	1
1	1	1	1	1	1	1

X

6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

X

6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

Program 7

Generation of Perfect Class-1 Squares from Matchmakers Squares of the same size

Matchmaker square's modulus loom table

6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

Sort table on center column

4	3	2	1	7	6	5
5	4	3	2	1	7	6
6	5	4	3	2	1	7
7	6	5	4	3	2	1
1	7	6	5	4	3	2
2	1	7	6	5	4	3
3	2	1	7	6	5	4

Column tilt-wrap to the right

4	2	7	5	3	1	6
5	3	1	6	4	2	7
6	4	2	7	5	3	1
7	5	3	1	6	4	2
1	6	4	2	7	5	3
2	7	5	3	1	6	4
3	1	6	4	2	7	5

Normalize = X

X	28	28	28	28	28	28	28
7	5	3	1	6	4	2	28
1	6	4	2	7	5	3	28
2	7	5	3	1	6	4	28
3	1	6	4	2	7	5	28
4	2	7	5	3	1	6	28
5	3	1	6	4	2	7	28
6	4	2	7	5	3	1	28
28	28	28	28	28	28	28	

Normalize = Y

	28	28	28	28	28	28	28
Y	28	28	28	28	28	28	28
6	4	2	7	5	3	1	28
5	3	1	6	4	2	7	28
4	2	7	5	3	1	6	28
3	1	6	4	2	7	5	28
2	7	5	3	1	6	4	28
1	6	4	2	7	5	3	28
7	5	3	1	6	4	2	28
28	28	28	28	28	28	28	
	28	28	28	28	28	28	28

Row tilt-wrap to the right

5	3	1	6	4	2	7
4	2	7	5	3	1	6
3	1	6	4	2	7	5
2	7	5	3	1	6	4
1	6	4	2	7	5	3
7	5	3	1	6	4	2
6	4	2	7	5	3	1

Calculate W=7*(Y-1)+X

W	175	175	175	175	175	175	175
1	42	26	10	43	34	18	2	175
2	29	20	4	37	28	12	45	175
3	23	14	47	31	15	6	39	175
4	17	1	41	25	9	49	33	175
5	11	44	35	19	3	36	27	175
6	5	38	22	13	46	30	21	175
7	48	32	16	7	40	24	8	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	

Quadral sums

100	100	100	100
100	100	100	100
100	100	100	100

Triangles

700	700
700	700

The Five-step Class-1 Algorithm

1. First, the rows of the Matchmaker's modulus loom-table are sorted on its central column from small to large. Note that this brings the table's average **4=½(n+1)** into the center.

2. Next, the columns of that result are tilt-wrapped downward and the result is normalized. This yields the modulus loom-table **X**.

3. Next, the rows of **X** are tilt-wrapped horizontally. After normalization, that yields its complementary loom-table **Y**.

4. The primal square is derived from the standard generation formula

$$\mathbf{W = n(Y-|1|) + X}.$$

5. All the basic properties of Class-1 squares on **W** can then be verified.

This simple method was determined to work for generating a perfect version of any Class-1 square. It always yields a perfect square with pairwise central symmetry.

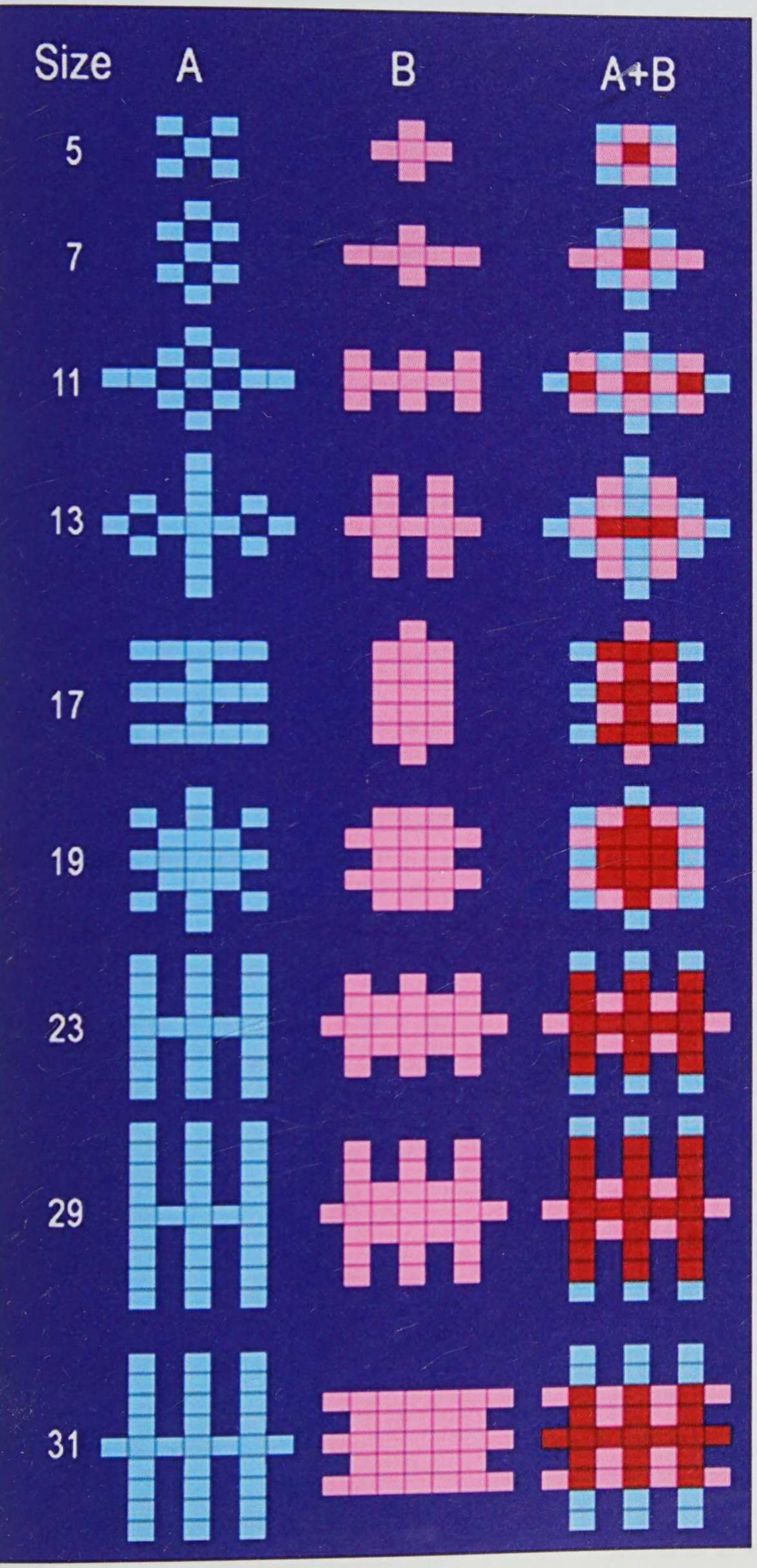
However, it will not directly generate an ultra-perfect square with complementary tiling patterns beyond the size-11 square. That topic is taken up on the page 7-10.

Now that you know just how all odd-prime number size squares can be manifested to be **perfect**, you'll get to see just how all the composite-size squares can be manifested from these Class-1 perfect squares in the next segment. Then you will be able to generate all perfect geonomic squares of any desired size on your own and explore still larger squares.

Further, you'll get to see how the near perfect Class-2 squares are generated.

Program 7

Tiling Patterns for Class-1 Squares up thru size-31



At left are the tiling patterns for Class-1 squares from sizes **5** thru **31**. The cells common to both patterns are highlighted in brown

Here is how most of the versions of squares with complementary tiling patterns were found:

First, an empty table that had all the linear geonomric sums already calcu-lated was set up.

Next all the continuous doubly-symmetric summations were laid out on the blank table that contained **n** cells and met the **4C** properties.

Thirdly, the rows and columns of consecutively derived squares were manipulated by doing row and column tilt-wrappings. If a derived table be-came imperfect in any way, that line of redistribution was abandond.

Once, a perfect Class-1 square was obtained from the method on the prior page, the numbers were system-atically redistributed by row and column tilt-wrapping.

After each derivative square was obtained, its values were pasted into the empty table and all the precal-culated pattern summations were observed. If any pattern summed to the square's characteristic number continuously, it was noted. If there was no complementary pattern appearing, then the table was transposed and the suggested patterns were examined again.

Between tilt-wrappings, the table was sorted on the rows by sorting the central column from small to large.

The same procedure was applied to

the columns by sorting the central row from small to large. This was done to normalize each tilt-wrapping procedure. The results were independent of whether the sort was low-to-high or high-to-low. Eventually, a version of a size **n** square with both tile patterns was found. This method eventually finds some version with both complementary tile-patterns.

As the square's size grew larger, this trial and error process became more and more laborious and tedious.

Generation of Ultra-perfect Class-1 Squares from Matchmakers Squares of the same size via The Double Quark Method

Fortunately, however, there is a very systematic number redistribution process which can be used for resequencing the columns to produce the ultra-perfect square directly from the perfect one initially derived from the Matchmakers modulus loom-table. The method is called the **Double-Quark method** to signify its fundamental process; what the double-quark is to the fundamental existence of the neutron and proton as shown earlier in Program 6, this method is to the generation of Class-1 odd-prime-number size squares from scratch starting with the derived complementary loom-tables **X** and **Y**. Here is that method.

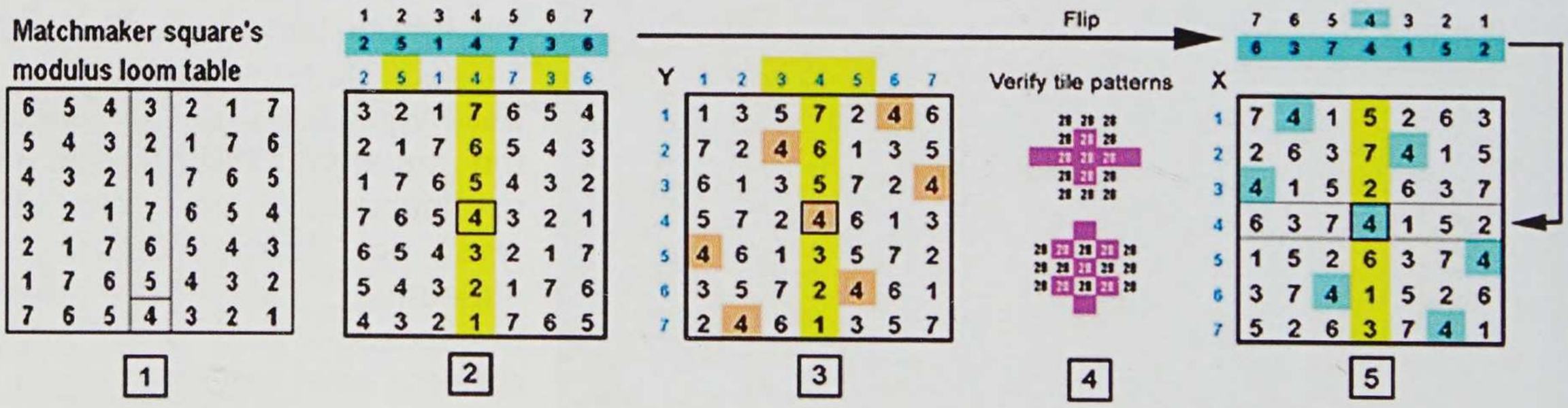
1. First, the rows of the Matchmaker's modulus loom-table are sorted on its central column from large to small. Note that this brings the table's average **½(n+1)** into the center.

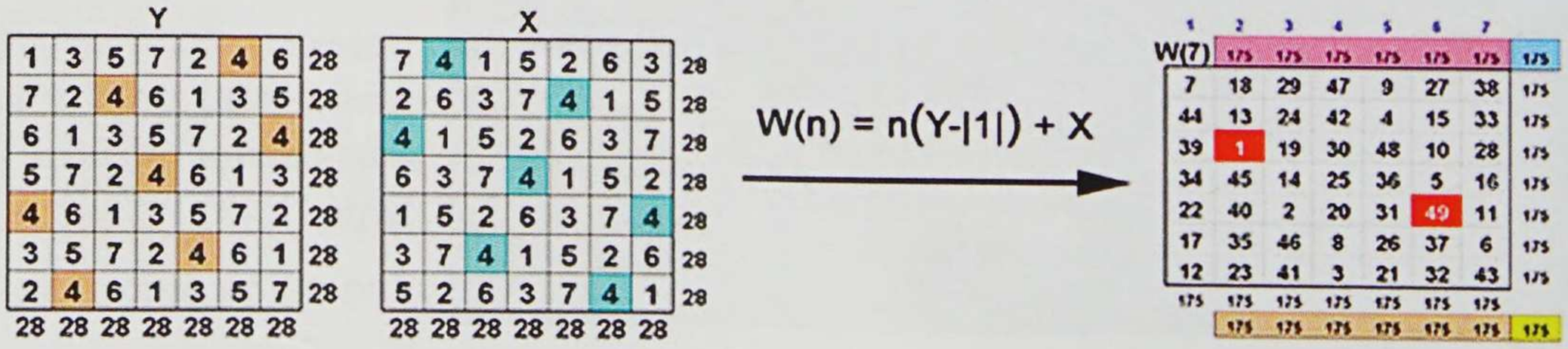
2. Next, the columns of that result are sorted on the resequencing pattern along the top (see algorithm next page). This yields a candidate for the integer loom-table **Y**.

3. Verify/test for characteristic complementary tiling patterns. Transpose the table if not all are equal. If neither the derived table nor its transpose yields two complementary equal-summing tiling patterns, increment the resequencing parameter **Δh** by **1** and continue from step #2. When two of all the suggested tiles sum equally, you will have the integer loom-table **Y**. Go to step #4.

4. Flip **Y** horizontally to get **X**. (Note that every row in **X** is just a flipped version of the resequencing pattern.)

5. Compute the primal square from the standard generation formula: **W = n (Y- |1|) + X**. It will be ultra-perfect





Program 7

Constructing the resequencing pattern

In the resequencing table at right, a **T** indicates that subsequent to the resequencing sort on columns of **Y**, the resulting square needed to be transposed to exhibit the characteristic complementary tile patterns.

Resequencing Table	
Size of square	**Δh**
5	**1**
7	**1**
11	**1**
13	**3**
17	**3T**
19	**5**
23	**5T**
29	**5**
31	**6T**
37	**6**
41	**7**

Now, these resequencing patterns were determined by trial and error as described two pages prior. The complementary tiling patterns were discovered by trying out different patterns alongside the simultaneous generation of the geonomic square based on what was already discovered for squares of lesser sizes. The table here lists the results of these discoveries up through size-41.

Observe the sequence of increasing Δ**h** in the Resequencing Table. It was found that no viable loom table was derived from Δ**h** being any less than the prior size of successful loom tables. So the last value for Δh can be the first value to start the search for the next size Class-1 ultra-perfect square.

This is the best that has been observed so far. Obviously there is plenty of opportunity for further development with respect to uncovering additional ultra-perfect Class-1 squares. But their discovery will remain on an enlightened trial-and-error basis for the foreseeable future.

Nonetheless, this table along with the 5-step **Double-Quark** algorithm on the prior page provides the means to generate ultra-perfect Class-1 squares up to and beyond size-**41** very directly using Microsoft's Excel™ program. That computer program can construct the initial table, sort its columns, and perform the transpose each with just a few clicks of the mouse on the computer screen.

And now that you know just how all odd-prime-number size squares can be manifested very easily, you'll get to see just how all composite-size squares can be manifested from these perfect squares in the next segment.

Then you will be able to generate all perfect and near-perfect geonomic squares of any desired size and even explore still larger squares on your own.

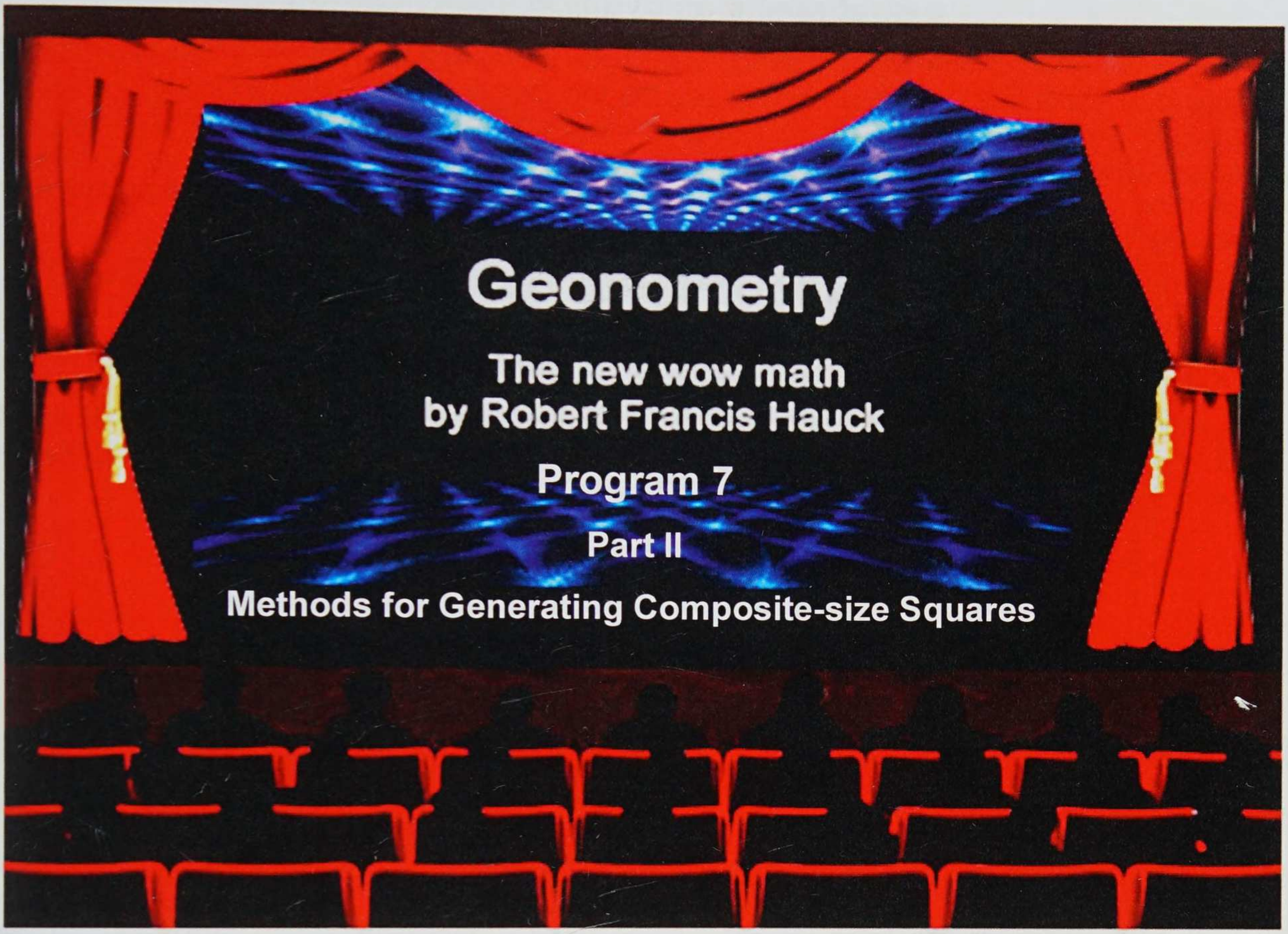
You are situated at just the very beginning of the era for this new math, Geonometry, and there is a lot more opportunity for the discovery of additional properties and patterns now that the foundation has been laid.

So have at it.

Program 7

Part I Notes

1. The modulus loom table of a size **4x** quadracube correlates 100% with the relationship amon the **6** quarks in sub-dimension **1/4D**.
2. The modulus loom table of the matchmaker's square derives the complementary loom table of any ultra-perfect square without any knowledge of the Class-1 square beforehand. Tha method, called the **Double-Quark** method, was described in this program. It always resulte in a non-linear distribution of the numbers for which the modulus and integer functions ar equal. As such, the squares are not only ultra-perfect but when the tile-patterns ar transposed, possess characteristic anchor-dot patterns distinctly different from those forme from, and in addition to, the centers of their tiling patterns. These anchor-dot patterns ar continuously equal-summing.
3. Geonometry is the mathematics of the **fabric of space**. The matchmaker square is th essential basic source of all the Class-1 squares and is the very **twine** used by the **looms** t generate the fabric of space. It is the mathematical equivalent in its significance to the **up** an **down** quarks that are so fundamental to the existence of the neutron and proton. That is wh the resequencing method was called the **Double-Quark algorithm**.



Here in Part II we will see just how the squares beyond Class-1 are generated. There are five distinct methods for doing this and there is one method for co-mingling regular geonomic squares with the matchmaker square of the same size to get an even more complex matchmaker square. This all results in a total of **6** more methods in addition to the Two-Quark method of Part I.

Program 7

#1: Expansion by the Balloon Method

The demonstration here shows that the size-5 perfect square can be expanded by enlarging i cells to any size block-square uniformly and the resulting square will still be perfect. This proper holds for any size square that is perfect or near-perfect. Considering that the wrap diagonals c across the block-squares in different proportions from multiplicity **3** onward, this is just a amazing property! Although it does have a mathematical basis, it's not worth the time and effort explain it all.

The characteristic number $\mathbf{C_{mn}}$ of each square of size **n** of multiplicity **m**, where **p** is th dimensional average of the square, is given by the general formula:

(7.1) $\mathbf{C_{mn} = pmn}$

Here **n = 5**, **m** ranges from **1** thru **3** and **p = 13**. The characteristic numbers are **65**, **130** and **19** which are **1**, **2** and **3** times **pn = 65**, respectively. Observe that the number **p** is the same fo every multiplicity.

W(5)

	65	65	65	65	65	
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
	65	65	65	65	65	

W(5,2)

	130	130	130	130	130	130	130	130	130	130	
1	20	20	8	8	21	21	14	14	2	2	130
2	20	20	8	8	21	21	14	14	2	2	130
3	11	11	4	4	17	17	10	10	23	23	130
4	11	11	4	4	17	17	10	10	23	23	130
5	7	7	25	25	13	13	1	1	19	19	130
6	7	7	25	25	13	13	1	1	19	19	130
7	3	3	16	16	9	9	22	22	15	15	130
8	3	3	16	16	9	9	22	22	15	15	130
9	24	24	12	12	5	5	18	18	6	6	130
10	24	24	12	12	5	5	18	18	6	6	130
	130	130	130	130	130	130	130	130	130	130	
	130	130	130	130	130	130	130	130	130	130	

W(5,3)

	195	195	195	195	195	195	195	195	195	195	195	195	195	195	195
1	20	20	20	8	8	8	21	21	21	14	14	14	2	2	2	195
2	20	20	20	8	8	8	21	21	21	14	14	14	2	2	2	195
3	20	20	20	8	8	8	21	21	21	14	14	14	2	2	2	195
4	11	11	11	4	4	4	17	17	17	10	10	10	23	23	23	195
5	11	11	11	4	4	4	17	17	17	10	10	10	23	23	23	195
6	11	11	11	4	4	4	17	17	17	10	10	10	23	23	23	195
7	7	7	7	25	25	25	13	13	13	1	1	1	19	19	19	195
8	7	7	7	25	25	25	13	13	13	1	1	1	19	19	19	195
9	7	7	7	25	25	25	13	13	13	1	1	1	19	19	19	195
10	3	3	3	16	16	16	9	9	9	22	22	22	15	15	15	195
11	3	3	3	16	16	16	9	9	9	22	22	22	15	15	15	195
12	3	3	3	16	16	16	9	9	9	22	22	22	15	15	15	195
13	24	24	24	12	12	12	5	5	5	18	18	18	6	6	6	195
14	24	24	24	12	12	12	5	5	5	18	18	18	6	6	6	195
15	24	24	24	12	12	12	5	5	5	18	18	18	6	6	6	195
	195	195	195	195	195	195	195	195	195	195	195	195	195	195	195	
	195	195	195	195	195	195	195	195	195	195	195	195	195	195	195	195

Program 7

#2: Expansion by the T-Ball method

(The merging of a Tiled-expansion with a Balloon-expansion)

This method is applicable to generating Class-6 squares from Class-1 squares. The top table is a ballooned expansion of the size-5 square to one of size-25, denoted by **D(5,5)**. Below is a size-25 expansion by tiling with the size-5 square, denoted by **E(5)**. Both are generated from the same size-5 perfect square **W(5)**.

D(5,5)

	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
1	20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
2	20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
3	20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
4	20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
5	20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
6	11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
7	11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
8	11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
9	11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
10	11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
11	7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
12	7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
13	7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
14	7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
15	7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
16	3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
17	3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
18	3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
19	3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
20	3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
21	24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
22	24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
23	24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
24	24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
25	24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

E(5)

	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325	
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325	
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325	
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325	
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325	
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325	
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325	
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325	
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325	
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325	
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325	
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325	
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325	
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325	
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325	
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325	
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325	
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325	
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325	
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325	
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325	
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325	
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325	
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325	
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325	
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

Two distinct perfect size-25 squares, each the dual of the other, are next generated from the two formulas for **n = 5**:

(7.2) $W(n^2) = n^2\,(E(n)\text{-}|1|) + D(n,n)$

(7.3) $U(n^2) = n^2\,(D(n,n)\text{-}|1|) + E(n)$

Program 7

Here are the generated dual squares **W(25)** and **U(25)**.

W(25)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	495	195	520	345	45	483	183	508	333	33	496	196	521	346	46	489	189	514	339	39	477	177	502	327	27	7825
2	270	95	420	245	570	258	83	408	233	558	271	96	421	246	571	264	89	414	239	564	252	77	402	227	552	7825
3	170	620	320	20	470	158	608	308	8	458	171	621	321	21	471	164	614	314	14	464	152	602	302	2	452	7825
4	70	395	220	545	370	58	383	208	533	358	71	396	221	546	371	64	389	214	539	364	52	377	202	527	352	7825
5	595	295	120	445	145	583	283	108	433	133	596	296	121	446	146	589	289	114	439	139	577	277	102	427	127	7825
6	486	186	511	336	36	479	179	504	329	29	492	192	517	342	42	485	185	510	335	35	498	198	523	348	48	7825
7	261	86	411	236	561	254	79	404	229	554	267	92	417	242	567	260	85	410	235	560	273	98	423	248	573	7825
8	161	611	311	11	461	154	604	304	4	454	167	617	317	17	467	160	610	310	10	460	173	623	323	23	473	7825
9	61	386	211	536	361	54	379	204	529	354	67	392	217	542	367	60	385	210	535	360	73	398	223	548	373	7825
10	586	286	111	436	136	579	279	104	429	129	592	292	117	442	142	585	285	110	435	135	598	298	123	448	148	7825
11	482	182	507	332	32	500	200	525	350	50	488	188	513	338	38	476	176	501	326	26	494	194	519	344	44	7825
12	257	82	407	232	557	275	100	425	250	575	263	88	413	238	563	251	76	401	226	551	269	94	419	244	569	7825
13	157	607	307	7	457	175	625	325	25	475	163	613	313	13	463	151	601	301	1	451	169	619	319	19	469	7825
14	57	382	207	532	357	75	400	225	550	375	63	388	213	538	363	51	376	201	526	351	69	394	219	544	369	7825
15	582	282	107	432	132	600	300	125	450	150	588	288	113	438	138	576	276	101	426	126	594	294	119	444	144	7825
16	478	178	503	328	28	491	191	516	341	41	484	184	509	334	34	497	197	522	347	47	490	190	515	340	40	7825
17	253	78	403	228	553	266	91	416	241	566	259	84	409	234	559	272	97	422	247	572	265	90	415	240	565	7825
18	153	603	303	3	453	166	616	316	16	466	159	609	309	9	459	172	622	322	22	472	165	615	315	15	465	7825
19	53	378	203	528	353	66	391	216	541	366	59	384	209	534	359	72	397	222	547	372	65	390	215	540	365	7825
20	578	278	103	428	128	591	291	116	441	141	584	284	109	434	134	597	297	122	447	147	590	290	115	440	140	7825
21	499	199	524	349	49	487	187	512	337	37	480	180	505	330	30	493	193	518	343	43	481	181	506	331	31	7825
22	274	99	424	249	574	262	87	412	237	562	255	80	405	230	555	268	93	418	243	568	256	81	406	231	556	7825
23	174	624	324	24	474	162	612	312	12	462	155	605	305	5	455	168	618	318	18	468	156	606	306	6	456	7825
24	74	399	224	549	374	62	387	212	537	362	55	380	205	530	355	68	393	218	543	368	56	381	206	531	356	7825
25	599	299	124	449	149	587	287	112	437	137	580	280	105	430	130	593	293	118	443	143	581	281	106	431	131	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

U(25)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	495	483	496	489	477	195	183	196	189	177	520	508	521	514	502	345	333	346	339	327	45	33	46	39	27	7825
2	486	479	492	485	498	186	179	192	185	198	511	504	517	510	523	336	329	342	335	348	36	29	42	35	48	7825
3	482	500	488	476	494	182	200	188	176	194	507	525	513	501	519	332	350	338	326	344	32	50	38	26	44	7825
4	478	491	484	497	490	178	191	184	197	190	503	516	509	522	515	328	341	334	347	340	28	41	34	47	40	7825
5	499	487	480	493	481	199	187	180	193	181	524	512	505	518	506	349	337	330	343	331	49	37	30	43	31	7825
6	270	258	271	264	252	95	83	96	89	77	420	408	421	414	402	245	233	246	239	227	570	558	571	564	552	7825
7	261	254	267	260	273	86	79	92	85	98	411	404	417	410	423	236	229	242	235	248	561	554	567	560	573	7825
8	257	275	263	251	269	82	100	88	76	94	407	425	413	401	419	232	250	238	226	244	557	575	563	551	569	7825
9	253	266	259	272	265	78	91	84	97	90	403	416	409	422	415	228	241	234	247	240	553	566	559	572	565	7825
10	274	262	255	268	256	99	87	80	93	81	424	412	405	418	406	249	237	230	243	231	574	562	555	568	556	7825
11	170	158	171	164	152	620	608	621	614	602	320	308	321	314	302	20	8	21	14	2	470	458	471	464	452	7825
12	161	154	167	160	173	611	604	617	610	623	311	304	317	310	323	11	4	17	10	23	461	454	467	460	473	7825
13	157	175	163	151	169	607	625	613	601	619	307	325	313	301	319	7	25	13	1	19	457	475	463	451	469	7825
14	153	166	159	172	165	603	616	609	622	615	303	316	309	322	315	3	16	9	22	15	453	466	459	472	465	7825
15	174	162	155	168	156	624	612	605	618	606	324	312	305	318	306	24	12	5	18	6	474	462	455	468	456	7825
16	70	58	71	64	52	395	383	396	389	377	220	208	221	214	202	545	533	546	539	527	370	358	371	364	352	7825
17	61	54	67	60	73	386	379	392	385	398	211	204	217	210	223	536	529	542	535	548	361	354	367	360	373	7825
18	57	75	63	51	69	382	400	388	376	394	207	225	213	201	219	532	550	538	526	544	357	375	363	351	369	7825
19	53	66	59	72	65	378	391	384	397	390	203	216	209	222	215	528	541	534	547	540	353	366	359	372	365	7825
20	74	62	55	68	56	399	387	380	393	381	224	212	205	218	206	549	537	530	543	531	374	362	355	368	356	7825
21	595	583	596	589	577	295	283	296	289	277	120	108	121	114	102	445	433	446	439	427	145	133	146	139	127	7825
22	586	579	592	585	598	286	279	292	285	298	111	104	117	110	123	436	429	442	435	448	136	129	142	135	148	7825
23	582	600	588	576	594	282	300	288	276	294	107	125	113	101	119	432	450	438	426	444	132	150	138	126	144	7825
24	578	591	584	597	590	278	291	284	297	290	103	116	109	122	115	428	441	434	447	440	128	141	134	147	140	7825
25	599	587	580	593	581	299	287	280	293	281	124	112	105	118	106	449	437	430	443	431	149	137	130	143	131	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Now, here is what's so amazing: **D(5) = Y(25)** is the integer loom table and **E(5) = X(25)** is the modulus loom table that can both be derived from **W(25)** in the base **25**.

And further, **Y(25)** is the modulus table and **X(25)** is the integer table that can both be derived from the dual **U(25)** in the base **25**. So clearly, **D(5)** and **E(5)** are complementary loom tables for the size-25 dual squares.

Note that both **W(25)** and **U(25)** are pairwise centrally symmetric just like the size-5 square **W(5)** that was involved in their manifestation.

It is pretty obvious from **U(25)** that the resulting squares possess no modularity.

Program 7

#3: Expansion by the ATE Method for generating Class-4 & Class-5 Composite Squares

The **ATE** method applies to the generation of Class-4 and Class-5 squares.

The name **ATE** stands for **A**ddition of **T**iled **E**xpansions. This method of expansion is simply the tiling of a square of size **a** that is a tiled multiple of itself with another of size **b** that is a tiled multiple of itself to form a composite square of size **n = ab**. If the two squares are perfect, the expanded tables will be perfect.

Caution: Neither a nor b can be a composite square itself or the method will not yield a square that is geonomically perfect.

Note, since the size-3 square is not perfect, to get an expansion table that is, each size-3 square would have to be married up with its reflection and the **9x9** expansion table would have to be tiled with **3x6** blocks. But this is impossible because **9** is not divisible by **2** ! That implies that the size-3 square can never be expanded geonomically with an odd multiple of itself. Therefore, the size-9 square could never be generated from the size-3 square's expansion. It will be described in Program 9 just how the perfect size-9 square was found.

Since Class-2 squares are not perfect, the same restriction on expansion applies to them too.

Application to Composing Class-4 Squares

	1	2	3	4	5	6	7	8	9	10	11	12	
E(3)	60	60	60	60	60	60	60	60	60	60	60	60	60
1	2	7	6	6	7	2	2	7	6	6	7	2	60
2	9	5	1	1	5	9	9	5	1	1	5	9	60
3	4	3	8	8	3	4	4	3	8	8	3	4	60
4	4	3	8	8	3	4	4	3	8	8	3	4	60
5	9	5	1	1	5	9	9	5	1	1	5	9	60
6	2	7	6	6	7	2	2	7	6	6	7	2	60
7	2	7	6	6	7	2	2	7	6	6	7	2	60
8	9	5	1	1	5	9	9	5	1	1	5	9	60
9	4	3	8	8	3	4	4	3	8	8	3	4	60
10	4	3	8	8	3	4	4	3	8	8	3	4	60
11	9	5	1	1	5	9	9	5	1	1	5	9	60
12	2	7	6	6	7	2	2	7	6	6	7	2	60
	60	60	60	60	60	60	60	60	60	60	60	60	
	60	60	60	60	60	60	60	60	60	60	60	60	60

	1	2	3	4	5	6	7	8	9	10	11	12	
E(4)	102	102	102	102	102	102	102	102	102	102	102	102	102
1	5	10	15	4	5	10	15	4	5	10	15	4	102
2	16	3	6	9	16	3	6	9	16	3	6	9	102
3	2	13	12	7	2	13	12	7	2	13	12	7	102
4	11	8	1	14	11	8	1	14	11	8	1	14	102
5	5	10	15	4	5	10	15	4	5	10	15	4	102
6	16	3	6	9	16	3	6	9	16	3	6	9	102
7	2	13	12	7	2	13	12	7	2	13	12	7	102
8	11	8	1	14	11	8	1	14	11	8	1	14	102
9	5	10	15	4	5	10	15	4	5	10	15	4	102
10	16	3	6	9	16	3	6	9	16	3	6	9	102
11	2	13	12	7	2	13	12	7	2	13	12	7	102
12	11	8	1	14	11	8	1	14	11	8	1	14	102
	102	102	102	102	102	102	102	102	102	102	102	102	
	102	102	102	102	102	102	102	102	102	102	102	102	102

Program 7

Expansion and merging of the size-3 and size-4 squares to get a size-12 Class-4 square

Here is one configuration for size-12 that was found to have continuous diamond and X-pattern tilings. But it is not pan-genic.

It was obtained from the two tables **E(3)** and **E(4)** depicted on the prior page.

Note that **E(4)** is composed of duplicates of the perfect size-4 square while **E(3)** is composed of four variations of the imperfect size-3 square in order to make the expanded table be geonomic.

E(4) is obviously pangenic, but **E(3)** is not. Therein lies the source of the loss of pangenicity in the derived dual squares of size-12. This loss ceases to occur for larger squares when **b** is even number.

	1	2	3	4	5	6	7	8	9	10	11	12	
W(12)	870	870	870	870	870	870	870	870	870	870	870	870	870
1	21	106	95	84	101	26	31	100	85	90	111	20	870
2	144	67	6	9	80	131	134	73	16	3	70	137	870
3	50	45	124	119	34	61	60	39	114	125	44	55	870
4	59	40	113	126	43	56	49	46	123	120	33	62	870
5	133	74	15	4	69	138	143	68	5	10	79	132	870
6	32	99	86	89	112	19	22	105	96	83	102	25	870
7	18	109	92	87	98	29	28	103	82	93	108	23	870
8	139	72	1	14	75	136	129	78	11	8	65	142	870
9	53	42	127	116	37	58	63	36	117	122	47	52	870
10	64	35	118	121	48	51	54	41	128	115	38	57	870
11	130	77	12	7	66	141	140	71	2	13	76	135	870
12	27	104	81	94	107	24	17	110	91	88	97	30	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870

870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870

Class-4 squares of sizes **n = 4b** where **b** is an odd number are best composed in this manner.

All such squares of size greater than **12** in this sub-class will also be pangenic because the square of size **b** laid out repetitively like **E(4)** will be from either Class-1, 3, 5 or 6, all of which will be of odd-size and consequently will be pangenic.

Although pangenicity was lost here for the size-12 square, the size-12 square has many continuously equal-summing patterns:

1. Size-2 even-size pyramidal diamond
2. Size-3 even-size ring-diamond
3. Size-4 even-size pyramidal diamond
4. Size-5 even-size solid diamond
5. Size-6 even-size solid diamond
6. The 12-point anchor-dot pattern

How this **ATE** method is applied to generate composite squares from odd-size squares will be demonstrated next where **20 = 4x5** and **b=5**.

Program 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	E(4)	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	
		170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170

	E(5)	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260
1	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
2	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
3	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
4	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
5	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
6	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
7	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
8	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
9	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
10	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
11	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
12	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
13	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
14	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
15	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
16	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
17	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
18	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
19	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
20	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260

Program 7

Deriving the size-20 square with the ATE method

Next the two expanded squares **E(4)** and **E(5)** are merged together by addition to get a size-20 square **W(20)** without any duplication of numbers. In order to do that, the tilings need to be exact multiples of each other: **E(4)** has **25** tiles of the size-4 square and **E(5)** has **16** tiles of the size-5 square, yielding two tables of size **20**.

Here is the result of that addition according to the formula:

(7.4) $\mathbf{W1(20) = 5^2(E(4) - |1|) + E(5)}$

Note the location of the extreme numbers **1** and **400** highlighted in red. Yet, the square is pangenic.

W1(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	395	58	146	214	377	70	133	221	389	52	145	208	396	64	127	220	383	71	139	202	4010
2	36	304	292	160	48	311	279	167	35	323	286	154	42	310	298	161	29	317	285	173	4010
3	257	200	13	326	269	182	25	338	251	194	7	350	263	176	19	332	275	188	1	344	4010
4	103	241	359	97	115	228	366	84	122	240	353	91	109	247	365	78	116	234	372	90	4010
5	399	62	130	218	381	74	137	205	393	56	149	212	380	68	131	224	387	55	143	206	4010
6	45	308	296	164	27	320	283	171	39	302	295	158	46	314	277	170	33	321	289	152	4010
7	261	179	17	335	273	186	4	342	260	198	11	329	267	185	23	336	254	192	10	348	4010
8	107	250	363	76	119	232	375	88	101	244	357	100	113	226	369	82	125	238	351	94	4010
9	378	66	134	222	390	53	141	209	397	65	128	216	384	72	140	203	391	59	147	215	4010
10	49	312	280	168	31	324	287	155	43	306	299	162	30	318	281	174	37	305	293	156	4010
11	270	183	21	339	252	195	8	346	264	177	20	333	271	189	2	345	258	196	14	327	4010
12	111	229	367	85	123	236	354	92	110	248	361	79	117	235	373	86	104	242	360	98	4010
13	382	75	138	201	394	57	150	213	376	69	132	225	388	51	144	207	400	63	126	219	4010
14	28	316	284	172	40	303	291	159	47	315	278	166	34	322	290	153	41	309	297	165	4010
15	274	187	5	343	256	199	12	330	268	181	24	337	255	193	6	349	262	180	18	331	4010
16	120	233	371	89	102	245	358	96	114	227	370	83	121	239	352	95	108	246	364	77	4010
17	386	54	142	210	398	61	129	217	385	73	136	204	392	60	148	211	379	67	135	223	4010
18	32	325	288	151	44	307	300	163	26	319	282	175	38	301	294	157	50	313	276	169	4010
19	253	191	9	347	265	178	16	334	272	190	3	341	259	197	15	328	266	184	22	340	4010
20	124	237	355	93	106	249	362	80	118	231	374	87	105	243	356	99	112	230	368	81	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

The resulting square possesses the common complementary diamond and X-pattern tiles. It took no rearranging of rows and columns to achieve this property for the size-20 square.

4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Program 7

Next the roles of **E(4)** and **E(5)** are interchanged as in:

$$(7.5) \quad W2(20) = 4^2(E(5) - |1|) + E(4)$$

And we get a completely different perfect size-20 square. Yet each has the same properties as the other. Both have **diamond** and **X** characteristic tiling patterns and both are pangenic. However, neither is row nor centrally pairwise symmetric.

W2(20)																				
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	320	115	326	217	32	307	118	329	224	19	310	121	336	211	22	313	128	323	214	25	4010
2	162	61	268	151	354	173	60	263	146	365	172	55	258	157	364	167	50	269	156	359	4010
3	107	392	193	14	299	104	385	206	11	296	97	398	203	8	289	110	395	200	1	302	4010
4	37	250	143	340	229	42	255	132	341	234	47	244	133	346	239	36	245	138	351	228	4010
5	384	179	70	281	96	371	182	73	288	83	374	185	80	275	86	377	192	67	278	89	4010
6	306	125	332	215	18	317	124	327	210	29	316	119	322	221	28	311	114	333	220	23	4010
7	171	56	257	158	363	168	49	270	155	360	161	62	267	152	353	174	59	264	145	366	4010
8	101	394	207	4	293	106	399	196	5	298	111	388	197	10	303	100	389	202	15	292	4010
9	48	243	134	345	240	35	246	137	352	227	38	249	144	339	230	41	256	131	342	233	4010
10	370	189	76	279	82	381	188	71	274	93	380	183	66	285	92	375	178	77	284	87	4010
11	315	120	321	222	27	312	113	334	219	24	305	126	331	216	17	318	123	328	209	30	4010
12	165	58	271	148	357	170	63	260	149	362	175	52	261	154	367	164	53	266	159	356	4010
13	112	387	198	9	304	99	390	201	16	291	102	393	208	3	294	105	400	195	6	297	4010
14	34	253	140	343	226	45	252	135	338	237	44	247	130	349	236	39	242	141	348	231	4010
15	379	184	65	286	91	376	177	78	283	88	369	190	75	280	81	382	187	72	273	94	4010
16	309	122	335	212	21	314	127	324	213	26	319	116	325	218	31	308	117	330	223	20	4010
17	176	51	262	153	368	163	54	265	160	355	166	57	272	147	358	169	64	259	150	361	4010
18	98	397	204	7	290	109	396	199	2	301	108	391	194	13	300	103	386	205	12	295	4010
19	43	248	129	350	235	40	241	142	347	232	33	254	139	344	225	46	251	136	337	238	4010
20	373	186	79	276	85	378	191	68	277	90	383	180	69	282	95	372	181	74	287	84	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Again, note the location of the extreme numbers **1** and **400** highlighted in red. They are in the same location in both squares **W1** and **W2**. Actually **W2** is the dual of **W1**. Again, the square is pangenic.

Further, the size-20 square is a matrix of even size, yet the maximum and minimum numbers are separated by an odd number of columns and an odd number of rows. So the two extreme numbers can never be realigned by normalization to be equal-summing pairwise symmetric. In general, even-size matrices cannot be made to be pairwise symmetric by this **ATE** method of expansion due to the absence of pairwise symmetry in the perfect size-4 square..

4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Squares of sizes **28** and **32** were also derived in this manner

Program 7

Application of ATE to Composing Class-5 Squares

The only size square of Class **5** that is printable is the size **35**.

The expansion tables of the sizes **5** and **7** squares to ones of size **35** will not be shown here since they are readily obtained from the basic Class-1 squares below.

	1	2	3	4	5	
5x5	65	65	65	65	65	
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
	65	65	65	65	65	

	1	2	3	4	5	6	7	
W	175	175	175	175	175	175	175	
1	43	24	5	35	9	39	20	175
2	14	37	18	48	22	3	33	175
3	27	1	31	12	42	16	46	175
4	40	21	44	25	6	29	10	175
5	4	34	8	38	19	49	23	175
6	17	47	28	2	32	13	36	175
7	30	11	41	15	45	26	7	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	

The **eight** resulting size **35** squares that automatically result from this **ATE** method will be shown first.

Note that all of the resulting squares are pairwise centrally symmetric like the size **5** and **7** squares here used in their generation.

For Class-5 squares there are two styles of tiling pattern: one which is composed of **bxb** block-squares in the size **a** characteristic tiling patterns; the other which is composed of **axa** block-squares in the size **b** characteristic tiling patterns. The **bxb** pattern sums to **b** times the characteristic number; the **axa** pattern sums to **a** times the characteristic number.

All Class-5 squares, derived in any one of the three bases $\mathbf{a}^2$, $\mathbf{b}^2$ or **ab**, will have both pairs of tiling patterns and consequently will have **4** distinct equal-summing tiling patterns simultaneously. This only occurs for Class-5 squares.

Once any primal or dual square is derived in either base $\mathbf{a}^2$ or $\mathbf{b}^2$, all the other versions can be derived in any of the other bases by deriving the modulus and integer loom tables from these two in the desired base $\mathbf{a}^2$, $\mathbf{b}^2$ or **ab**. Then amazingly, their common dual can be generated by interchanging the roles of the loom tables in the standard generating formula (**3.4**) back in Program 3.

Program 7

Size-35 primal square W1(49) derived in the base 49 using the ATE method

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
W1(49)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4	114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5	1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6	937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21455
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8	332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9	131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15	1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18	310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23	327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25	1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26	942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Size-35 dual square U1(49) derived in the base 49 using the ATE method

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U1(49)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	945	658	221	1164	702	445	158	946	664	202	1170	708	446	164	927	670	208	1171	714	427	170	933	671	214	1152	720	433	171	939	652	220	1158	721	439	152	21455
2	811	354	92	1035	598	311	1079	817	360	98	1036	579	317	1085	823	361	79	1042	585	323	1086	804	367	85	1048	586	304	1092	810	373	86	1029	592	310	1098	21455
3	682	250	1188	726	469	7	975	688	226	1194	732	475	13	951	694	232	1200	738	451	19	957	700	238	1176	744	457	25	963	676	244	1182	750	463	1	969	21455
4	378	116	884	622	340	1103	841	384	122	890	603	341	1109	847	390	103	891	609	347	1115	828	391	109	897	615	328	1116	834	397	115	878	616	334	1122	840	21455
5	274	1212	755	493	31	999	537	255	1218	756	499	37	980	543	256	1224	762	480	43	981	549	262	1205	768	481	49	987	530	268	1206	774	487	30	993	531	21455
6	145	908	646	189	1127	870	408	146	914	627	195	1133	871	414	127	920	633	196	1139	852	420	133	921	639	177	1145	858	421	139	902	645	183	1146	864	402	21455
7	1061	779	517	60	1023	561	279	1067	785	523	61	1004	567	285	1073	786	504	67	1010	573	286	1054	792	510	73	1011	554	292	1060	798	511	54	1017	560	298	21455
8	932	675	213	1151	719	432	175	938	651	219	1157	725	438	151	944	657	225	1163	701	444	157	950	663	201	1169	707	450	163	926	669	207	1175	713	426	169	21455
9	803	366	84	1047	590	303	1091	809	372	90	1028	591	309	1097	815	353	91	1034	597	315	1078	816	359	97	1040	578	316	1084	822	365	78	1041	584	322	1090	21455
10	699	237	1180	743	456	24	962	680	243	1181	749	462	5	968	681	249	1187	730	468	6	974	687	230	1193	731	474	12	955	693	231	1199	737	455	18	956	21455
11	395	108	896	614	327	1120	833	396	114	877	620	333	1121	839	377	120	883	621	339	1102	845	383	121	889	602	345	1108	846	389	102	895	608	346	1114	827	21455
12	261	1204	767	485	48	986	529	267	1210	773	486	29	992	535	273	1211	754	492	35	998	536	254	1217	760	498	36	979	542	260	1223	761	479	42	985	548	21455
13	132	925	638	176	1144	857	425	138	901	644	182	1150	863	401	144	907	650	188	1126	869	407	150	913	626	194	1132	875	413	126	919	632	200	1138	851	419	21455
14	1053	791	509	72	1015	553	291	1059	797	515	53	1016	559	297	1065	778	516	59	1022	565	278	1066	784	522	65	1003	566	284	1072	790	503	66	1009	572	290	21455
15	949	662	205	1168	706	449	162	930	668	206	1174	712	430	168	931	674	212	1155	718	431	174	937	655	218	1156	724	437	155	943	656	224	1162	705	443	156	21455
16	820	358	96	1039	577	320	1083	821	364	77	1045	583	321	1089	802	370	83	1046	589	302	1095	808	371	89	1027	595	308	1096	814	352	95	1033	596	314	1077	21455
17	686	229	1192	735	473	11	954	692	235	1198	736	454	17	960	698	236	1179	742	460	23	961	679	242	1185	748	461	4	967	685	248	1186	729	467	10	973	21455
18	382	125	888	601	344	1107	850	388	101	894	607	350	1113	826	394	107	900	613	326	1119	832	400	113	876	619	332	1125	838	376	119	882	625	338	1101	844	21455
19	253	1216	759	497	40	978	541	259	1222	765	478	41	984	547	265	1203	766	484	47	990	528	266	1209	772	490	28	991	534	272	1215	753	491	34	997	540	21455
20	149	912	630	193	1131	874	412	130	918	631	199	1137	855	418	131	924	637	180	1143	856	424	137	905	643	181	1149	862	405	143	906	649	187	1130	868	406	21455
21	1070	783	521	64	1002	570	283	1071	789	502	70	1008	571	289	1052	795	508	71	1014	552	295	1058	796	514	52	1020	558	296	1064	777	520	58	1021	564	277	21455
22	936	654	217	1160	723	436	154	942	660	223	1161	704	442	160	948	661	204	1167	710	448	161	929	667	210	1173	711	429	167	935	673	211	1154	717	435	173	21455
23	807	375	88	1026	594	307	1100	813	351	94	1032	600	313	1076	819	357	100	1038	576	319	1082	825	363	76	1044	582	325	1088	801	369	82	1050	588	301	1094	21455
24	678	241	1184	747	465	3	966	684	247	1190	728	466	9	972	690	228	1191	734	472	15	953	691	234	1197	740	453	16	959	697	240	1178	741	459	22	965	21455
25	399	112	880	618	331	1124	837	380	118	881	624	337	1105	843	381	124	887	605	343	1106	849	387	105	893	606	349	1112	830	393	106	899	612	330	1118	831	21455
26	270	1208	771	489	27	995	533	271	1214	752	495	33	996	539	252	1220	758	496	39	977	545	258	1221	764	477	45	983	546	264	1202	770	483	46	989	527	21455
27	136	904	642	185	1148	861	404	142	910	648	186	1129	867	410	148	911	629	192	1135	873	411	129	917	635	198	1136	854	417	135	923	636	179	1142	860	423	21455
28	1057	800	513	51	1019	557	300	1063	776	519	57	1025	563	276	1069	782	525	63	1001	569	282	1075	788	501	69	1007	575	288	1051	794	507	75	1013	551	294	21455
29	928	666	209	1172	715	428	166	934	672	215	1153	716	434	172	940	653	216	1159	722	440	153	941	659	222	1165	703	441	159	947	665	203	1166	709	447	165	21455
30	824	362	80	1043	581	324	1087	805	368	81	1049	587	305	1093	806	374	87	1030	593	306	1099	812	355	93	1031	599	312	1080	818	356	99	1037	580	318	1081	21455
31	695	233	1196	739	452	20	958	696	239	1177	745	458	21	964	677	245	1183	746	464	2	970	683	246	1189	727	470	8	971	689	227	1195	733	471	14	952	21455
32	386	104	892	610	348	1111	829	392	110	898	611	329	1117	835	398	111	879	617	335	1123	836	379	117	885	623	336	1104	842	385	123	886	604	342	1110	848	21455
33	257	1225	763	476	44	982	550	263	1201	769	482	50	988	526	269	1207	775	488	26	994	532	275	1213	751	494	32	1000	538	251	1219	757	500	38	976	544	21455
34	128	916	634	197	1140	853	416	134	922	640	178	1141	859	422	140	903	641	184	1147	865	403	141	909	647	190	1128	866	409	147	915	628	191	1134	872	415	21455
35	1074	787	505	68	1006	574	287	1055	793	506	74	1012	555	293	1056	799	512	55	1018	556	299	1062	780	518	56	1024	562	280	1068	781	524	62	1005	568	281	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 7

Size-35 primal square W2(25) derived in the base 25 using the ATE method

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
W2(25)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	945	658	221	1164	702	445	158	946	664	202	1170	708	446	164	927	670	208	1171	714	427	170	933	671	214	1152	720	433	171	939	652	220	1158	721	439	152	21455
2	811	354	92	1035	598	311	1079	817	360	98	1036	579	317	1085	823	361	79	1042	585	323	1086	804	367	85	1048	586	304	1092	810	373	86	1029	592	310	1098	21455
3	682	250	1188	726	469	7	975	688	226	1194	732	475	13	951	694	232	1200	738	451	19	957	700	238	1176	744	457	25	963	676	244	1182	750	463	1	969	21455
4	378	116	884	622	340	1103	841	384	122	890	603	341	1109	847	390	103	891	609	347	1115	828	391	109	897	615	328	1116	834	397	115	878	616	334	1122	840	21455
5	274	1212	755	493	31	999	537	255	1218	756	499	37	980	543	256	1224	762	480	43	981	549	262	1205	768	481	49	987	530	268	1206	774	487	30	993	531	21455
6	145	908	646	189	1127	870	408	146	914	627	195	1133	871	414	127	920	633	196	1139	852	420	133	921	639	177	1145	858	421	139	902	645	183	1146	864	402	21455
7	1061	779	517	60	1023	561	279	1067	785	523	61	1004	567	285	1073	786	504	67	1010	573	286	1054	792	510	73	1011	554	292	1060	798	511	54	1017	560	298	21455
8	932	675	213	1151	719	432	175	938	651	219	1157	725	438	151	944	657	225	1163	701	444	157	950	663	201	1169	707	450	163	926	669	207	1175	713	426	169	21455
9	803	366	84	1047	590	303	1091	809	372	90	1028	591	309	1097	815	353	91	1034	597	315	1078	816	359	97	1040	578	316	1084	822	365	78	1041	584	322	1090	21455
10	699	237	1180	743	456	24	962	680	243	1181	749	462	5	968	681	249	1187	730	468	6	974	687	230	1193	731	474	12	955	693	231	1199	737	455	18	956	21455
11	395	108	896	614	327	1120	833	396	114	877	620	333	1121	839	377	120	883	621	339	1102	845	383	121	889	602	345	1108	846	389	102	895	608	346	1114	827	21455
12	261	1204	767	485	48	986	529	267	1210	773	486	29	992	535	273	1211	754	492	35	998	536	254	1217	760	498	36	979	542	260	1223	761	479	42	985	548	21455
13	132	925	638	176	1144	857	425	138	901	644	182	1150	863	401	144	907	650	188	1126	869	407	150	913	626	194	1132	875	413	126	919	632	200	1138	851	419	21455
14	1053	791	509	72	1015	553	291	1059	797	515	53	1016	559	297	1065	778	516	59	1022	565	278	1066	784	522	65	1003	566	284	1072	790	503	66	1009	572	290	21455
15	949	662	205	1168	706	449	162	930	668	206	1174	712	430	168	931	674	212	1155	718	431	174	937	655	218	1156	724	437	155	943	656	224	1162	705	443	156	21455
16	820	358	96	1039	577	320	1083	821	364	77	1045	583	321	1089	802	370	83	1046	589	302	1095	808	371	89	1027	595	308	1096	814	352	95	1033	596	314	1077	21455
17	686	229	1192	735	473	11	954	692	235	1198	736	454	17	960	698	236	1179	742	460	23	961	679	242	1185	748	461	4	967	685	248	1186	729	467	10	973	21455
18	382	125	888	601	344	1107	850	388	101	894	607	350	1113	826	394	107	900	613	326	1119	832	400	113	876	619	332	1125	838	376	119	882	625	338	1101	844	21455
19	253	1216	759	497	40	978	541	259	1222	765	478	41	984	547	265	1203	766	484	47	990	528	266	1209	772	490	28	991	534	272	1215	753	491	34	997	540	21455
20	149	912	630	193	1131	874	412	130	918	631	199	1137	855	418	131	924	637	180	1143	856	424	137	905	643	181	1149	862	405	143	906	649	187	1130	868	406	21455
21	1070	783	521	64	1002	570	283	1071	789	502	70	1008	571	289	1052	795	508	71	1014	552	295	1058	796	514	52	1020	558	296	1064	777	520	58	1021	564	277	21455
22	936	654	217	1160	723	436	154	942	660	223	1161	704	442	160	948	661	204	1167	710	448	161	929	667	210	1173	711	429	167	935	673	211	1154	717	435	173	21455
23	807	375	88	1026	594	307	1100	813	351	94	1032	600	313	1076	819	357	100	1038	576	319	1082	825	363	76	1044	582	325	1088	801	369	82	1050	588	301	1094	21455
24	678	241	1184	747	465	3	966	684	247	1190	728	466	9	972	690	228	1191	734	472	15	953	691	234	1197	740	453	16	959	697	240	1178	741	459	22	965	21455
25	399	112	880	618	331	1124	837	380	118	881	624	337	1105	843	381	124	887	605	343	1106	849	387	105	893	606	349	1112	830	393	106	899	612	330	1118	831	21455
26	270	1208	771	489	27	995	533	271	1214	752	495	33	996	539	252	1220	758	496	39	977	545	258	1221	764	477	45	983	546	264	1202	770	483	46	989	527	21455
27	136	904	642	185	1148	861	404	142	910	648	186	1129	867	410	148	911	629	192	1135	873	411	129	917	635	198	1136	854	417	135	923	636	179	1142	860	423	21455
28	1057	800	513	51	1019	557	300	1063	776	519	57	1025	563	276	1069	782	525	63	1001	569	282	1075	788	501	69	1007	575	288	1051	794	507	75	1013	551	294	21455
29	928	666	209	1172	715	428	166	934	672	215	1153	716	434	172	940	653	216	1159	722	440	153	941	659	222	1165	703	441	159	947	665	203	1166	709	447	165	21455
30	824	362	80	1043	581	324	1087	805	368	81	1049	587	305	1093	806	374	87	1030	593	306	1099	812	355	93	1031	599	312	1080	818	356	99	1037	580	318	1081	21455
31	695	233	1196	739	452	20	958	696	239	1177	745	458	21	964	677	245	1183	746	464	2	970	683	246	1189	727	470	8	971	689	227	1195	733	471	14	952	21455
32	386	104	892	610	348	1111	829	392	110	898	611	329	1117	835	398	111	879	617	335	1123	836	379	117	885	623	336	1104	842	385	123	886	604	342	1110	848	21455
33	257	1225	763	476	44	982	550	263	1201	769	482	50	988	526	269	1207	775	488	26	994	532	275	1213	751	494	32	1000	538	251	1219	757	500	38	976	544	21455
34	128	916	634	197	1140	853	416	134	922	640	178	1141	859	422	140	903	641	184	1147	865	403	141	909	647	190	1128	866	409	147	915	628	191	1134	872	415	21455
35	1074	787	505	68	1006	574	287	1055	793	506	74	1012	555	293	1056	799	512	55	1018	556	299	1062	780	518	56	1024	562	280	1068	781	524	62	1005	568	281	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Size-35 dual square U2(25) derived in the base 25 using the ATE method

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U2(25)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4	114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5	1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6	937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21455
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8	332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9	131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15	1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18	310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23	327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25	1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26	942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 7

Size-35 primal square W3(35) derived in the base 35 from W1(49)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U31(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4	114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5	1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6	937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21455
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8	332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9	131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15	1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18	310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23	327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25	1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26	942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Size-35 dual square U3(35) derived in the base 35 from W1(49)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U31(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	833	676	309	650	248	133	1200	100	1174	772	1148	746	624	474	563	448	46	415	20	1087	972	1061	939	544	878	518	361	239	335	178	1042	152	1009	859	702	21455
2	1135	740	618	959	592	435	531	409	14	1116	226	1055	933	1029	907	505	355	724	329	207	296	146	1003	853	1222	820	670	794	644	277	120	461	94	1168	68	21455
3	220	1084	194	1016	901	499	875	718	316	692	290	175	997	107	1216	814	1190	788	631	271	605	490	88	422	62	1129	769	1103	946	586	920	560	403	1	377	21455
4	284	162	258	136	1210	75	1177	782	660	756	599	477	573	451	56	1123	23	1097	975	1071	914	547	397	521	371	214	338	188	1045	895	984	862	712	836	686	21455
5	628	962	567	445	43	419	17	1091	236	1058	943	541	882	515	358	734	332	182	1039	149	1013	856	1197	830	673	313	647	252	130	464	104	1171	776	1145	743	21455
6	937	1026	904	509	843	728	326	204	300	143	1007	117	1219	824	667	798	641	274	615	458	98	1165	65	1139	737	1113	956	589	439	528	413	11	380	230	1052	21455
7	261	111	1213	818	1187	785	635	759	609	487	85	426	59	1133	33	1100	950	583	924	557	400	496	374	224	1081	191	1020	898	994	872	715	320	689	294	172	21455
8	570	455	53	387	27	1094	979	1068	911	551	885	525	368	211	342	185	1049	159	981	866	709	840	683	281	657	255	140	1207	72	1181	779	1155	753	596	481	21455
9	879	512	362	731	336	179	303	153	1010	860	1194	827	677	801	651	249	127	468	101	1175	40	1142	747	625	966	564	442	538	416	21	1088	233	1062	940	1036	21455
10	1223	821	1162	795	638	278	612	462	95	429	69	1136	741	1110	953	593	927	532	410	8	384	227	1056	201	1023	908	506	847	725	323	699	297	147	1004	114	21455
11	63	1130	30	1104	947	1078	921	554	404	493	378	221	345	195	1017	902	991	869	719	808	693	291	169	265	108	1217	82	1184	789	632	763	606	484	580	423	21455
12	365	706	339	189	1046	156	985	863	1204	837	680	285	654	259	137	471	76	1178	783	1152	750	600	969	574	452	50	391	24	1098	243	1065	915	548	889	522	21455
13	674	805	648	246	622	465	105	1172	37	1146	744	1120	963	561	446	535	420	18	352	237	1059	944	1033	876	516	850	735	333	176	307	150	1014	124	1191	831	21455
14	5	1107	957	590	931	529	407	503	381	231	1053	198	1027	905	1001	844	722	327	696	301	144	268	118	1220	825	1159	792	642	766	616	459	92	433	66	1140	21455
15	349	192	1021	166	988	873	716	812	690	288	664	262	112	1214	79	1188	786	1127	760	603	488	577	427	60	394	34	1101	951	1075	918	558	892	497	375	218	21455
16	658	256	134	475	73	1182	47	1149	754	597	973	571	449	545	388	28	1095	240	1069	912	1043	886	519	369	703	343	186	310	160	982	867	1201	834	684	773	21455
17	960	565	934	539	417	15	356	234	1063	208	1030	880	513	854	732	330	671	304	154	1011	121	1195	828	1169	802	645	250	619	469	102	436	41	1143	748	1117	21455
18	1024	909	998	841	726	815	700	298	141	272	115	1224	89	1156	796	639	770	613	456	587	430	70	1137	2	1111	954	1085	928	526	411	500	385	228	317	202	21455
19	109	478	83	1185	790	1124	757	607	976	581	424	57	398	31	1105	215	1072	922	555	896	494	372	713	346	196	1018	163	992	870	1211	809	687	292	661	266	21455
20	453	542	392	25	359	244	1066	916	1040	883	523	857	707	340	183	314	157	986	131	1198	838	681	777	655	253	629	472	77	1179	44	1153	751	1092	970	568	21455
21	1008	851	729	334	668	308	151	275	125	1192	832	1166	799	649	738	623	466	99	440	38	1147	12	1114	964	562	938	536	414	510	353	238	1060	205	1034	877	21455
22	86	1160	793	1134	767	610	460	584	434	67	401	6	1108	958	1082	925	530	899	504	382	225	321	199	1028	173	995	845	723	819	697	295	636	269	119	1221	21455
23	395	35	1102	212	1076	919	1050	893	491	376	710	350	193	282	167	989	874	1208	806	691	780	665	263	106	482	80	1189	54	1121	761	604	980	578	421	552	21455
24	704	337	678	311	161	983	128	1202	835	1176	774	652	257	626	476	74	443	48	1150	755	1089	967	572	941	546	389	22	363	241	1070	180	1037	887	520	861	21455
25	803	646	742	620	463	594	437	42	1144	9	1118	961	1057	935	533	418	507	357	235	324	209	1031	881	1005	848	733	822	672	305	148	279	122	1196	96	1163	21455
26	1112	222	1079	929	527	903	501	379	720	318	203	1025	170	999	842	1218	816	694	299	633	273	116	485	90	1157	797	1131	764	614	948	588	431	64	405	3	21455
27	190	286	164	993	138	1205	810	688	784	662	260	601	479	84	1186	51	1125	758	1099	977	575	425	549	399	32	366	216	1073	923	1047	890	495	864	714	347	21455
28	745	630	473	71	447	45	1154	19	1086	971	569	945	543	386	517	360	245	1067	177	1041	884	1015	858	701	341	675	315	158	247	132	1199	839	1173	771	656	21455
29	1054	932	537	906	511	354	232	328	206	1035	145	1002	852	730	826	669	302	643	276	126	1193	93	1167	800	1141	739	617	467	591	441	39	408	13	1115	965	21455
30	174	996	846	1215	813	698	787	637	270	113	489	87	1161	61	1128	768	611	952	585	428	559	402	7	1109	219	1083	926	1022	900	498	383	717	322	200	289	21455
31	483	81	450	55	1122	762	1096	974	579	913	553	396	29	370	213	1077	187	1044	894	492	868	711	344	685	283	168	990	135	1209	807	1183	781	659	264	598	21455
32	540	390	514	364	242	331	181	1038	888	1012	855	705	829	679	312	155	251	129	1203	103	1170	775	653	749	627	470	566	444	49	1151	16	1090	968	1064	942	21455
33	849	1225	823	666	306	640	280	123	457	97	1164	804	1138	736	621	955	595	438	36	412	10	1119	229	1051	936	534	910	508	351	727	325	210	1032	142	1006	21455
34	1158	58	1132	765	1106	949	582	432	556	406	4	373	223	1080	930	1019	897	502	871	721	319	197	293	171	1000	110	1212	817	695	791	634	267	608	486	91	21455
35	524	367	217	1074	184	1048	891	987	865	708	348	682	287	165	254	139	1206	811	1180	778	663	752	602	480	78	454	52	1126	26	1093	978	576	917	550	393	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 7

Size-35 primal square W4(35) derived in the base 25 from W2(25)

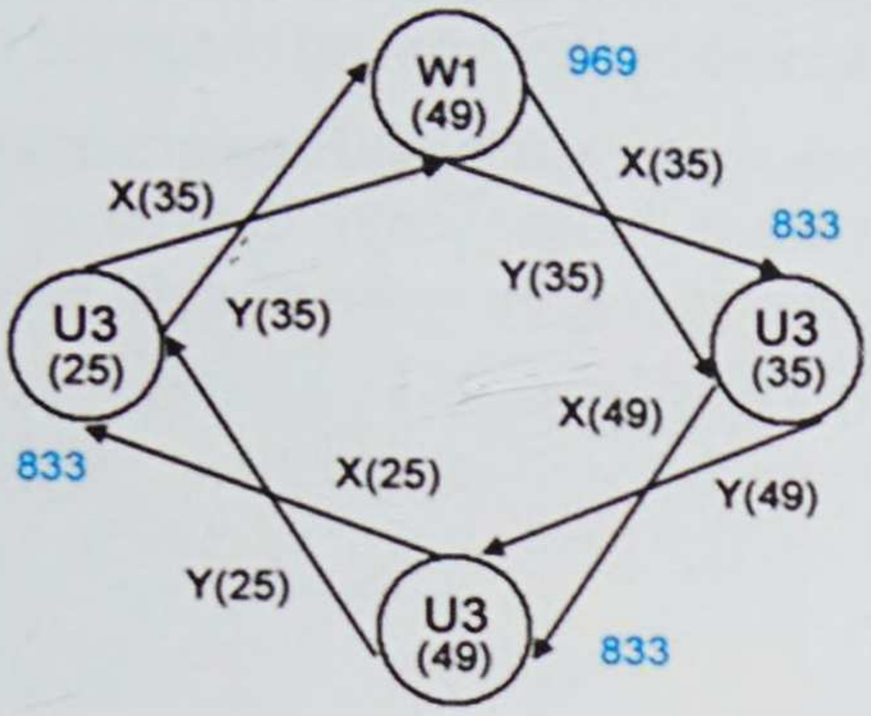
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
W62(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	945	658	221	1164	702	445	158	946	664	202	1170	708	446	164	927	670	208	1171	714	427	170	933	671	214	1152	720	433	171	939	652	220	1158	721	439	152	21455
2	811	354	92	1035	598	311	1079	817	360	98	1036	579	317	1085	823	361	79	1042	585	323	1086	804	367	85	1048	586	304	1092	810	373	86	1029	592	310	1098	21455
3	682	250	1188	726	469	7	975	688	226	1194	732	475	13	951	694	232	1200	738	451	19	957	700	238	1176	744	457	25	963	676	244	1182	750	463	1	969	21455
4	378	116	884	622	340	1103	841	384	122	890	603	341	1109	847	390	103	891	609	347	1115	828	391	109	897	615	328	1116	834	397	115	878	616	334	1122	840	21455
5	274	1212	755	493	31	999	537	255	1218	756	499	37	980	543	256	1224	762	480	43	981	549	262	1205	768	481	49	987	530	268	1206	774	487	30	993	531	21455
6	145	908	646	189	1127	870	408	146	914	627	195	1133	871	414	127	920	633	196	1139	852	420	133	921	639	177	1145	858	421	139	902	645	183	1146	864	402	21455
7	1061	779	517	60	1023	561	279	1067	785	523	61	1004	567	285	1073	786	504	67	1010	573	286	1054	792	510	73	1011	554	292	1060	798	511	54	1017	560	298	21455
8	932	675	213	1151	719	432	175	938	651	219	1157	725	438	151	944	657	225	1163	701	444	157	950	663	201	1169	707	450	163	926	669	207	1175	713	426	169	21455
9	803	366	84	1047	590	303	1091	809	372	90	1028	591	309	1097	815	353	91	1034	597	315	1078	816	359	97	1040	578	316	1084	822	365	78	1041	584	322	1090	21455
10	699	237	1180	743	456	24	962	680	243	1181	749	462	5	968	681	249	1187	730	468	6	974	687	230	1193	731	474	12	955	693	231	1199	737	455	18	956	21455
11	395	108	896	614	327	1120	833	396	114	877	620	333	1121	839	377	120	883	621	339	1102	845	383	121	889	602	345	1108	846	389	102	895	608	346	1114	827	21455
12	261	1204	767	485	48	986	529	267	1210	773	486	29	992	535	273	1211	754	492	35	998	536	254	1217	760	498	36	979	542	260	1223	761	479	42	985	548	21455
13	132	925	638	176	1144	857	425	138	901	644	182	1150	863	401	144	907	650	188	1126	869	407	150	913	626	194	1132	875	413	126	919	632	200	1138	851	419	21455
14	1053	791	509	72	1015	553	291	1059	797	515	53	1016	559	297	1065	778	516	59	1022	565	278	1066	784	522	65	1003	566	284	1072	790	503	66	1009	572	290	21455
15	949	662	205	1168	706	449	162	930	668	206	1174	712	430	168	931	674	212	1155	718	431	174	937	655	218	1156	724	437	155	943	656	224	1162	705	443	156	21455
16	820	358	96	1039	577	320	1083	821	364	77	1045	583	321	1089	802	370	83	1046	589	302	1095	808	371	89	1027	595	308	1096	814	352	95	1033	596	314	1077	21455
17	686	229	1192	735	473	11	954	692	235	1198	736	454	17	960	698	236	1179	742	460	23	961	679	242	1185	748	461	4	967	685	248	1186	729	467	10	973	21455
18	382	125	888	601	344	1107	850	388	101	894	607	350	1113	826	394	107	900	613	326	1119	832	400	113	876	619	332	1125	838	376	119	882	625	338	1101	844	21455
19	253	1216	759	497	40	978	541	259	1222	765	478	41	984	547	265	1203	766	484	47	990	528	266	1209	772	490	28	991	534	272	1215	753	491	34	997	540	21455
20	149	912	630	193	1131	874	412	130	918	631	199	1137	855	418	131	924	637	180	1143	856	424	137	905	643	181	1149	862	405	143	906	649	187	1130	868	406	21455
21	1070	783	521	64	1002	570	283	1071	789	502	70	1008	571	289	1052	795	508	71	1014	552	295	1058	796	514	52	1020	558	296	1064	777	520	58	1021	564	277	21455
22	936	654	217	1160	723	436	154	942	660	223	1161	704	442	160	948	661	204	1167	710	448	161	929	667	210	1173	711	429	167	935	673	211	1154	717	435	173	21455
23	807	375	88	1026	594	307	1100	813	351	94	1032	600	313	1076	819	357	100	1038	576	319	1082	825	363	76	1044	582	325	1088	801	369	82	1050	588	301	1094	21455
24	678	241	1184	747	465	3	966	684	247	1190	728	466	9	972	690	228	1191	734	472	15	953	691	234	1197	740	453	16	959	697	240	1178	741	459	22	965	21455
25	399	112	880	618	331	1124	837	380	118	881	624	337	1105	843	381	124	887	605	343	1106	849	387	105	893	606	349	1112	830	393	106	899	612	330	1118	831	21455
26	270	1208	771	489	27	995	533	271	1214	752	495	33	996	539	252	1220	758	496	39	977	545	258	1221	764	477	45	983	546	264	1202	770	483	46	989	527	21455
27	136	904	642	185	1148	861	404	142	910	648	186	1129	867	410	148	911	629	192	1135	873	411	129	917	635	198	1136	854	417	135	923	636	179	1142	860	423	21455
28	1057	800	513	51	1019	557	300	1063	776	519	57	1025	563	276	1069	782	525	63	1001	569	282	1075	788	501	69	1007	575	288	1051	794	507	75	1013	551	294	21455
29	928	666	209	1172	715	428	166	934	672	215	1153	716	434	172	940	653	216	1159	722	440	153	941	659	222	1165	703	441	159	947	665	203	1166	709	447	165	21455
30	824	362	80	1043	581	324	1087	805	368	81	1049	587	305	1093	806	374	87	1030	593	306	1099	812	355	93	1031	599	312	1080	818	356	99	1037	580	318	1081	21455
31	695	233	1196	739	452	20	958	696	239	1177	745	458	21	964	677	245	1183	746	464	2	970	683	246	1189	727	470	8	971	689	227	1195	733	471	14	952	21455
32	386	104	892	610	348	1111	829	392	110	898	611	329	1117	835	398	111	879	617	335	1123	836	379	117	885	623	336	1104	842	385	123	886	604	342	1110	848	21455
33	257	1225	763	476	44	982	550	263	1201	769	482	50	988	526	269	1207	775	488	26	994	532	275	1213	751	494	32	1000	538	251	1219	757	500	38	976	544	21455
34	128	916	634	197	1140	853	416	134	922	640	178	1141	859	422	140	903	641	184	1147	865	403	141	909	647	190	1128	866	409	147	915	628	191	1134	872	415	21455
35	1074	787	505	68	1006	574	287	1055	793	506	74	1012	555	293	1056	799	512	55	1018	556	299	1062	780	518	56	1024	562	280	1068	781	524	62	1005	568	281	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

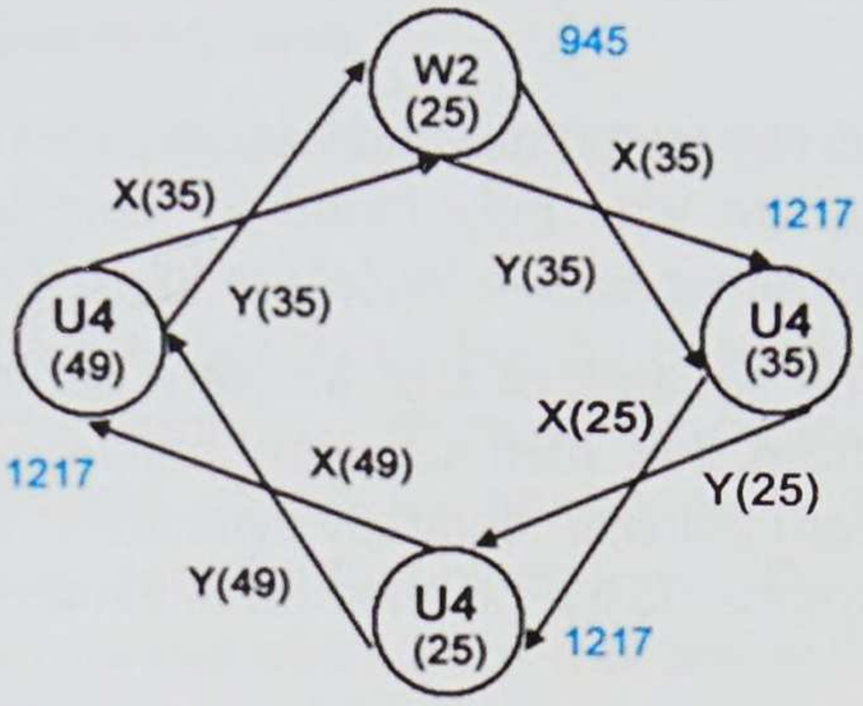
Size-35 dual square U4(35) derived in the base 25 from W2(25)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U52(25)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	1217	964	357	314	56	853	600	28	1174	916	524	266	888	810	587	160	1126	559	476	223	1020	797	195	112	1118	686	433	1055	1007	754	322	104	721	643	390	21455
2	199	116	738	695	88	1059	1011	409	326	948	730	647	45	1221	619	361	283	940	857	255	32	1178	571	493	1150	892	814	242	164	781	528	485	1102	1024	452	21455
3	580	148	1154	896	469	211	1043	790	532	140	1106	679	421	203	1000	742	350	92	1063	631	413	1210	952	734	302	49	841	623	370	1162	944	512	259	1	833	21455
4	956	354	306	928	850	627	25	1166	564	516	263	885	837	235	152	1123	551	473	1095	1047	794	187	109	761	683	430	1082	1004	397	319	96	718	640	68	1214	21455
5	988	770	687	85	1051	659	401	323	980	722	295	37	1218	611	358	1190	932	854	247	29	821	568	525	1142	889	457	239	156	778	560	128	1099	1016	449	191	21455
6	145	1146	544	461	243	1040	782	180	132	1103	671	453	1075	992	739	342	89	706	663	410	1202	949	377	299	41	873	620	13	1159	936	509	251	908	830	572	21455
7	381	303	925	842	275	17	1163	591	513	1135	877	834	227	149	801	548	470	1087	1044	437	184	136	758	680	73	1079	996	394	346	968	715	632	65	1206	604	21455
8	762	335	77	1083	651	398	1195	972	719	287	69	861	608	355	1182	929	497	279	21	818	565	168	1139	881	489	231	1028	775	552	125	1091	699	441	188	985	21455
9	1143	536	458	1115	1032	779	207	129	746	668	450	1067	989	417	339	81	703	660	53	1199	976	374	291	913	870	612	10	1186	584	501	248	905	822	220	172	21455
10	1175	917	874	267	14	806	588	510	1127	909	477	224	141	798	545	113	1119	1036	434	176	1008	755	672	105	1071	644	386	343	965	707	315	57	1203	596	378	21455
11	327	74	726	648	395	1222	969	362	284	61	858	605	33	1179	921	494	271	893	815	592	165	1131	529	481	228	1025	802	200	117	1088	691	438	1060	1012	759	21455
12	533	490	1107	1029	422	204	121	743	700	93	1064	981	414	331	953	735	652	50	1191	624	366	288	945	862	260	2	1183	576	498	1155	897	819	212	169	786	21455
13	914	517	264	6	838	585	153	1124	901	474	216	1048	795	537	110	1111	684	426	208	1005	747	320	97	1068	636	418	1215	957	704	307	54	846	628	375	1167	21455
14	101	723	645	38	1219	961	359	311	933	855	597	30	1171	569	521	268	890	807	240	157	1128	556	478	1100	1017	799	192	114	766	688	435	1052	1009	402	324	21455
15	133	1104	1021	454	196	993	740	692	90	1056	664	406	328	950	727	300	42	1223	616	363	1160	937	859	252	34	826	573	495	1147	894	462	244	161	783	530	21455
16	514	256	878	835	577	150	1151	549	466	213	1045	787	185	137	1108	676	423	1080	997	744	347	94	711	633	415	1207	954	382	304	46	843	625	18	1164	941	21455
17	720	637	70	1211	609	351	308	930	847	280	22	1168	561	518	1140	882	839	232	154	771	553	475	1092	1049	442	189	106	763	685	78	1084	1001	399	316	973	21455
18	1096	669	446	193	990	767	340	82	1053	656	403	1200	977	724	292	39	866	613	360	1187	934	502	249	26	823	570	173	1144	886	459	236	1033	780	557	130	21455
19	253	910	827	225	142	1148	541	463	1120	1037	784	177	134	751	673	455	1072	994	387	344	86	708	665	58	1204	946	379	296	918	875	617	15	1156	589	506	21455
20	285	62	1208	601	383	1180	922	844	272	19	811	593	515	1132	879	482	229	146	803	550	118	1089	1041	439	181	1013	760	677	75	1076	649	391	348	970	712	21455
21	696	443	1065	982	764	332	79	731	653	400	1192	974	367	289	66	863	610	3	1184	926	499	276	898	820	562	170	1136	534	486	233	1030	772	205	122	1093	21455
22	902	824	217	174	791	538	460	1112	1034	427	209	126	748	670	98	1069	986	419	336	958	705	657	55	1196	629	371	293	915	867	265	7	1188	581	503	1125	21455
23	59	851	598	380	1172	919	522	269	11	808	590	158	1129	906	479	221	1018	800	542	115	1116	689	431	178	1010	752	325	102	1073	641	388	1220	962	709	312	21455
24	440	1057	1014	407	329	71	728	650	43	1224	966	364	281	938	860	602	35	1176	574	491	273	895	812	245	162	1133	526	483	1105	1022	804	197	119	736	693	21455
25	467	214	166	788	535	138	1109	1026	424	201	998	745	697	95	1061	634	411	333	955	732	305	47	1193	621	368	1165	942	864	257	4	831	578	500	1152	899	21455
26	848	630	23	1169	911	519	261	883	840	582	155	1121	554	471	218	1050	792	190	107	1113	681	428	1085	1002	749	317	99	716	638	420	1212	959	352	309	51	21455
27	1054	1006	404	321	978	725	642	40	1216	614	356	313	935	852	250	27	1173	566	523	1145	887	809	237	159	776	558	480	1097	1019	447	194	111	768	690	83	21455
28	241	1038	785	527	135	1101	674	451	198	995	737	345	87	1058	661	408	1205	947	729	297	44	871	618	365	1157	939	507	254	31	828	575	143	1149	891	464	21455
29	622	20	1161	594	511	258	880	832	230	147	1153	546	468	1090	1042	789	182	139	756	678	425	1077	999	392	349	91	713	635	63	1209	951	384	301	923	845	21455
30	654	396	318	975	717	290	67	1213	606	353	1185	927	849	277	24	816	563	520	1137	884	487	234	151	773	555	123	1094	1046	444	186	983	765	682	80	1081	21455
31	1035	777	210	127	1098	666	448	1070	987	769	337	84	701	658	405	1197	979	372	294	36	868	615	8	1189	931	504	246	903	825	567	175	1141	539	456	238	21455
32	12	1158	586	508	1130	907	829	222	144	796	543	465	1117	1039	432	179	131	753	675	103	1074	991	389	341	963	710	662	60	1201	599	376	298	920	872	270	21455
33	393	1225	967	714	282	64	856	603	385	1177	924	492	274	16	813	595	163	1134	876	484	226	1023	805	547	120	1086	694	436	183	1015	757	330	72	1078	646	21455
34	774	202	124	741	698	445	1062	984	412	334	76	733	655	48	1194	971	369	286	943	865	607	5	1181	579	496	278	900	817	215	167	1138	531	488	1110	1027	21455
35	836	583	505	1122	904	472	219	171	793	540	108	1114	1031	429	206	1003	750	667	100	1066	639	416	338	960	702	310	52	1198	626	373	1170	912	869	262	9	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 7

Relationships among the two primal size-35 squares derived by the ATE method and their unique duals





A crossing of the arrows is where the roles of the loom-tables are interchanged in the generation formulas. Note the changes in the bases used for derivation of the loom-tables

Of the eight derived squares: **W1(b^2)** & **U1(b^2)**; **W2(a^2)** & **U2(a^2)**; **W3(ab)** & **U3(ab)**, and **W4(ab)** & **U4(ab)**, these reduce to only four because

(7.6a) **W1(49) = W3(35) = U2(25)** 969

(7.6b) **U1(49) = W4(35) = W2(25)** 949

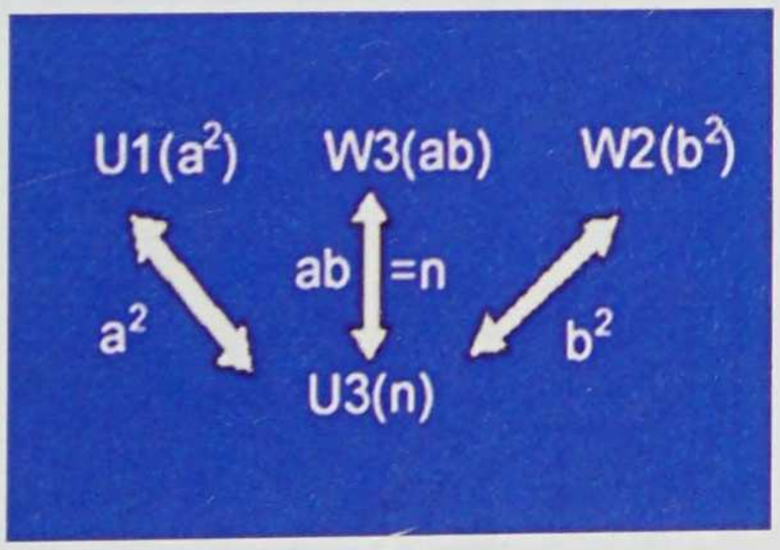
(7.6c) **U3(35) ≠ any other version** 833

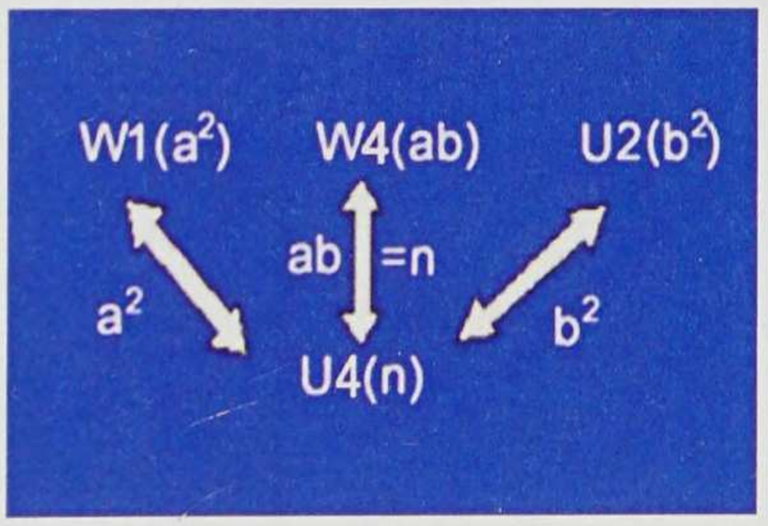
(7.6d) **U4(35) ≠ any other version** 1217

The numbers at left in **blue** are the numbers in the upper left-hand corner of the associated square.

Only **U3(35)** and **U4(35)** are unique. Nonetheless each can be derived from any of **W1(49)**, **W2(25)**, **W3(35)** and **W4(35)** in the base **49**, **25**, and **35**, respectively, because of their identical numbering!

Further, the complementary expansion tables **E(a)** and **E(b)** can be recovered by deriving the loom-tables in base $\mathbf{a^2}$ or $\mathbf{b^2}$ from **W1(49)** and **W2(25)**, respectively.





However, what is so amazing is that, while three different versions can derive an identical dual using three different number bases, the converse is also true: the single dual square **U(35)** can produce three different perfect geonomic squares of the same size different from itself with the same characteristic number using each of the three different bases. That property only exists among Class-5 squares. Again, that's wow math!

Program 7

Note 1: This expansion method which results in two pairs of distinct duals only relates to **Class-**
squares.

Note 2: For the demonstrated helical patterns that were shown earlier in Program 3, the size-3
Class-5 square was only found in loom tables that were subsequently derived in the **base**
n = ab from either square $\mathbf{W_1(a^2)}$ or $\mathbf{W_2(b^2)}$ originally generated in the base $\mathbf{a^2}$ or $\mathbf{b^2}$, respectively

Note 3: The squares of size **35** were derived using the **ATE** method here. All size **35** squares
resulting from this method were pairwise centrally symmetric because their two expansion
squares were pairwise centrally symmetric. In general, only odd-size matrices can be made to be
pairwise symmetric by this method of expansion.

Program 7

#4: Expansion by the Bootstrap method
Application to Composing Class-6 Squares

This method is called the *bootstrap* method because it only utilizes the size-**b** square **W(b)** by itself. It applies in general to the composition of Class-6 squares.

This method starts with the scaling up of the **W(b)** square by formula (**7.7**) to get **Z(b)** as shown here.

	1	2	3	4	5	
	W(5)	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

Then **Z** is permuted first by one column per each block-column **n-1** times in series to form the first block-row of an expansion table **D** as shown below. Next the first block-row in **D** is permuted by one row per each block-row below it **n-1** times to form the size-$\mathbf{b^2}$ table $\mathbf{D(b^2)}$.

$$(7.7) \qquad Z(b) = b^2(W(b) - |1|)$$

Observe that the rows and columns of the block-square in the upper left corner are permuted across block-rows and block-columns as shown here so that each number never appears in the same place twice among all the **25 5x5** block-squares.

	1	2	3	4	5	
	Z(5)	1500	1500	1500	1500	1500
1	475	175	500	325	25	1500
2	250	75	400	225	550	1500
3	150	600	300	0	450	1500
4	50	375	200	525	350	1500
5	575	275	100	425	125	1500
	1500	1500	1500	1500	1500	
		1500	1500	1500	1500	1500

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	D(25)	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500
1	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
2	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
3	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
4	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
5	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
6	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
7	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
8	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
9	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
10	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
11	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
12	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
13	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
14	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
15	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
16	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
17	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
18	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
19	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
20	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
21	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
22	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
23	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
24	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
25	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	
		7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500

Program 7

Next the tiled expansion table **E(b^2)** is added to **D(b^2)** to get **W*(b^2)**.

(7.8) W*(b^2) = D(b^2) +E(b^2)

E(25)

20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6

After this, **W*(b^2)** is normalized by moving the upper-left block-square to the central position by permuting block-rows and block-columns to get **W(b^2) as** shown here.

This pattern is unique among all the geonomic squares investigated to date: here all the numbers for which the modulus and integer functions are equal lie in the central embedded **5x5** block square! This centralizing property is the result of the block-normalization of **W***.

The central block-square sums totally to the square's characteristic number, but is the only one to do so. The central numbers of all the **5x5** block-squares sum to the square's characteristic number too.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	320	8	471	164	602	620	308	21	464	152	170	608	321	14	452	470	158	621	314	2	20	458	171	614	302	7825
2	211	529	367	60	398	386	204	542	360	73	61	379	217	535	373	361	54	392	210	548	536	354	67	385	223	7825
3	107	450	138	576	294	282	125	438	126	594	582	300	113	426	144	132	600	288	101	444	432	150	588	276	119	7825
4	503	341	34	497	190	178	516	334	47	490	478	191	509	347	40	28	491	184	522	340	328	41	484	197	515	7825
5	424	237	555	268	81	99	412	230	568	256	274	87	405	243	556	574	262	80	418	231	249	562	255	93	406	7825
6	420	233	571	264	77	95	408	246	564	252	270	83	421	239	552	570	258	96	414	227	245	558	271	89	402	7825
7	311	4	467	160	623	611	304	17	460	173	161	604	317	10	473	461	154	617	310	23	11	454	167	610	323	7825
8	207	550	363	51	394	382	225	538	351	69	57	400	213	526	369	357	75	388	201	544	532	375	63	376	219	7825
9	103	441	134	597	290	278	116	434	147	590	578	291	109	447	140	128	591	284	122	440	428	141	584	297	115	7825
10	524	337	30	493	181	199	512	330	43	481	499	187	505	343	31	49	487	180	518	331	349	37	480	193	506	7825
11	520	333	46	489	177	195	508	346	39	477	495	183	521	339	27	45	483	196	514	327	345	33	496	189	502	7825
12	411	229	567	260	98	86	404	242	560	273	261	79	417	235	573	561	254	92	410	248	236	554	267	85	423	7825
13	307	25	463	151	619	607	325	13	451	169	157	625	313	1	469	457	175	613	301	19	7	475	163	601	319	7825
14	203	541	359	72	390	378	216	534	372	65	53	391	209	547	365	353	66	384	222	540	528	366	59	397	215	7825
15	124	437	130	593	281	299	112	430	143	581	599	287	105	443	131	149	587	280	118	431	449	137	580	293	106	7825
16	120	433	146	589	277	295	108	446	139	577	595	283	121	439	127	145	583	296	114	427	445	133	596	289	102	7825
17	511	329	42	485	198	186	504	342	35	498	486	179	517	335	48	36	479	192	510	348	336	29	492	185	523	7825
18	407	250	563	251	94	82	425	238	551	269	257	100	413	226	569	557	275	88	401	244	232	575	263	76	419	7825
19	303	16	459	172	615	603	316	9	472	165	153	616	309	22	465	453	166	609	322	15	3	466	159	622	315	7825
20	224	537	355	68	381	399	212	530	368	56	74	387	205	543	356	374	62	380	218	531	549	362	55	393	206	7825
21	220	533	371	64	377	395	208	546	364	52	70	383	221	539	352	370	58	396	214	527	545	358	71	389	202	7825
22	111	429	142	585	298	286	104	442	135	598	586	279	117	435	148	136	579	292	110	448	436	129	592	285	123	7825
23	507	350	38	476	194	182	525	338	26	494	482	200	513	326	44	32	500	188	501	344	332	50	488	176	519	7825
24	403	241	559	272	90	78	416	234	572	265	253	91	409	247	565	553	266	84	422	240	228	566	259	97	415	7825
25	324	12	455	168	606	624	312	5	468	156	174	612	305	18	456	474	162	605	318	6	24	462	155	618	306	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Each **5x5** block-square is geonomic in that all its 4-directional sums are equal to **1565**.

Program 7

Only the central block-square is pangenic as a block-square in that all its quadrals sum equally to **1252 = 4 x 313**.

Pangenic

	1	2	3	4	5	
	5x5	1565	1565	1565	1565	1565
1	495	183	521	339	27	1565
2	261	79	417	235	573	1565
3	157	625	313	1	469	1565
4	53	391	209	547	365	1565
5	599	287	105	443	131	1565
	1565	1565	1565	1565	1565	
	1565	1565	1565	1565	1565	

Not pangenic

	1	2	3	4	5	
	5x5	1565	1565	1565	1565	1565
1	145	583	296	114	427	1565
2	36	479	192	510	348	1565
3	557	275	88	401	244	1565
4	453	166	609	322	15	1565
5	374	62	380	218	531	1565
	1565	1565	1565	1565	1565	
	1565	1565	1565	1565	1565	

The size-25 square is the smallest Class-6 square that can be bootstrapped and the largest such size still legible. So it's all you will get to see regarding this centralizing property.

From the centralizing property stems the following quasi-continuous modularity property as shown below. The **5x5** block-squares which are certain to sum to the square's characteristic number reside only in locations where no more than one boundary between adjacent embedded block-squares is crossed.

7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	7200	7075	7200	7200	7825	8450	7825	7700	7825	7825	7825	8450	7825	7700	7825	7950	7950	8575	7950	7825
7825	7325	7825	7325	7325	7825	8325	7825	8325	7825	7825	7825	8325	7825	8325	7825	7325	7325	7825	7325	7825
7825	7325	7825	7200	7950	7825	7700	7200	7700	7075	7825	8575	8450	7950	8450	7825	7200	7950	7825	7325	7825
7825	7825	7825	7200	7950	7825	7700	7700	7700	7075	7825	8575	8450	8450	8450	7825	7200	7950	7825	7825	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	7825	7825	8450	7700	7825	7950	7950	7950	8575	7825	7075	7200	7200	7200	7825	8450	7700	7825	7825	7825
7825	7200	7075	7825	7075	7825	8575	7950	7825	8575	7825	7075	7825	7200	7075	7825	8575	7825	8575	7950	7825
7825	7325	7825	7950	7200	7825	8450	7950	8450	8575	7825	7075	7700	7200	7700	7825	7950	7200	7825	7325	7825
7825	7325	7825	7825	7825	7825	7825	7325	7825	7825	7825	7825	7825	7325	7825	7825	7825	7825	7825	7325	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	8325	7825	7825	7825	7825	7825	8325	7825	7825	7825	7825	7825	8325	7825	7825	7825	7825	7825	8325	7825
7825	8325	7825	8450	7700	7825	7950	8450	7950	8575	7825	7075	7200	7700	7200	7825	8450	7700	7825	8325	7825
7825	7700	7075	7825	7075	7825	8575	8450	7825	8575	7825	7075	7825	7700	7075	7825	8575	7825	8575	8450	7825
7825	7825	7825	7950	7200	7825	8450	8450	8450	8575	7825	7075	7700	7700	7700	7825	7950	7200	7825	7825	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	7825	7825	7700	8450	7825	7200	7200	7200	7075	7825	8575	7950	7950	7950	7825	7700	8450	7825	7825	7825
7825	8325	7825	7700	8450	7825	7200	7700	7200	7075	7825	8575	7950	8450	7950	7825	7700	8450	7825	8325	7825
7825	8325	7825	8325	8325	7825	7325	7825	7325	7825	7825	7825	7325	7825	7325	7825	8325	8325	7825	8325	7825
7825	7700	7075	7700	7700	7825	7950	7825	7200	7825	7825	7825	7950	7825	7200	7825	8450	8450	8575	8450	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

This completes discussion of the Bootstrap expansion method.

* * *

Program 7

A Note

None of the expansion methods, the **T-Ball** method #2, the **ATE** method #3, nor the **Bootstrap** method #4 will produce a Class-6 square that possesses continuous modularity.

Square-number sizes $\mathbf{b}^2$ that are continuously **bx** modular are only the result of a 2-dimensional fractal projection of a 4-dimensional size-**b** quadracube onto a 2-dimensional square of size $\mathbf{b}^2$. Further, only those quadracubes that were obtained from collapsing a 5-dimensional quintacube of the same size along its B-axis would yield such modularity. Dimensional fracticality is addressed in Program 9.

Here is the 5x-modular version of the size-25 square.

Obtained from 5x4D

25x25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Collapsing 5-dimensional quintacubes to 4-dimensional qauadracubes is only demonstrated in the book **Number Magic** as 5-dimensional squares are too large to be legible or even comprehendible even on a large projector screen.

Program 7

#5: Expansion by the SPD method

The Spread Pattern Distribution method

This method applies in general to the composition of Classes 2, 3 & 4 squares. Classes 3 & 4 use two loom tables, one leads to the derivation of the other according to formula (**7.9**) for Class-3 squares or (**7.14**, later) for Class-4 squares. Together they form a composite loom table upon which the composite square can be readily manifested as follows.

Application to Composing Class-3 Squares

Let **D** denote the spread distribution table and **B** its basic pattern. The D is obtained from:

$$(7.9) \qquad D = B + 3B^T$$

Then the Class-3 square **W(3b)** is obtained from the size **3b** pattern distribution table and the expansion table tiled with W_b according to:

$$(7.10) \qquad W(3b) = b^2D + E(3b) = \begin{vmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{vmatrix} \begin{vmatrix} W_b & W_b & W_b \\ W_b & W_b & W_b \\ W_b & W_b & W_b \end{vmatrix} n^2$$

The rightmost matrix in the formula above is the tiled expansion table **E** to size **3b** of the size **b** square W_b.

Size-15

The pattern table **B** of size **3b** follows the basic pattern shown here for the size**15** square.

D as derived from (**7.10**) is shown below.

Observe that each number **0** thru **8** appears exactly **25** times and never in the same location within each of the nine **5x5** block-squares. Note that **9 x 25 = 15x15 = 225**, the total number of cells in the **15x15** table.

D(15)	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
0	3	6	0	6	1	4	7	1	7	2	5	8	2	8	60
1	4	7	1	7	0	3	6	0	6	2	5	8	2	8	60
2	5	8	2	8	1	4	7	1	7	0	3	6	0	6	60
0	3	6	0	6	2	5	8	2	8	1	4	7	1	7	60
2	5	8	2	8	1	4	7	1	7	0	3	6	0	6	60
3	0	3	6	3	4	1	4	7	4	5	2	5	8	5	60
4	1	4	7	4	3	0	3	6	3	5	2	5	8	5	60
5	2	5	8	5	4	1	4	7	4	3	0	3	6	3	60
3	0	3	6	3	5	2	5	8	5	4	1	4	7	4	60
5	2	5	8	5	4	1	4	7	4	3	0	3	6	3	60
6	6	0	3	0	7	7	1	4	1	8	8	2	5	2	60
7	7	1	4	1	6	6	0	3	0	8	8	2	5	2	60
8	8	2	5	2	7	7	1	4	1	6	6	0	3	0	60
6	6	0	3	0	8	8	2	5	2	7	7	1	4	1	60
8	8	2	5	2	7	7	1	4	1	6	6	0	3	0	60
60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	
	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60

B(15)	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15

Carrying out the calculations of formula (**7.10**) yields the perfect size-15 square seen earlier in Program 3.

Program 7

Size-21

Here is the pattern **B** for the size **21** square.

B(21)

21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	1	0	2	0	2	0	2	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	1	2	0	2	0	2	0	21
21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21

And here is the **D** pattern distribution table derived from formula (**7.10**):

D(21)

84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84
8	2	5	8	2	2	8	7	1	4	7	1	1	7	6	0	3	6	0	0	6	84
6	0	3	6	0	0	6	7	1	4	7	1	1	7	8	2	5	8	2	2	8	84
7	1	4	7	1	1	7	6	0	3	6	0	0	6	8	2	5	8	2	2	8	84
8	2	5	8	2	2	8	7	1	4	7	1	1	7	6	0	3	6	0	0	6	84
6	0	3	6	0	0	6	8	2	5	8	2	2	8	7	1	4	7	1	1	7	84
6	0	3	6	0	0	6	7	1	4	7	1	1	7	8	2	5	8	2	2	8	84
8	2	5	8	2	2	8	7	1	4	7	1	1	7	6	0	3	6	0	0	6	84
5	5	2	5	8	5	5	4	4	1	4	7	4	4	3	3	0	3	6	3	3	84
3	3	0	3	6	3	3	4	4	1	4	7	4	4	5	5	2	5	8	5	5	84
4	4	1	4	7	4	4	3	3	0	3	6	3	3	5	5	2	5	8	5	5	84
5	5	2	5	8	5	5	4	4	1	4	7	4	4	3	3	0	3	6	3	3	84
3	3	0	3	6	3	3	5	5	2	5	8	5	5	4	4	1	4	7	4	4	84
3	3	0	3	6	3	3	4	4	1	4	7	4	4	5	5	2	5	8	5	5	84
5	5	2	5	8	5	5	4	4	1	4	7	4	4	3	3	0	3	6	3	3	84
2	8	8	2	5	8	2	1	7	7	1	4	7	1	0	6	6	0	3	6	0	84
0	6	6	0	3	6	0	1	7	7	1	4	7	1	2	8	8	2	5	8	2	84
1	7	7	1	4	7	1	0	6	6	0	3	6	0	2	8	8	2	5	8	2	84
2	8	8	2	5	8	2	1	7	7	1	4	7	1	0	6	6	0	3	6	0	84
0	6	6	0	3	6	0	2	8	8	2	5	8	2	1	7	7	1	4	7	1	84
0	6	6	0	3	6	0	1	7	7	1	4	7	1	2	8	8	2	5	8	2	84
2	8	8	2	5	8	2	1	7	7	1	4	7	1	0	6	6	0	3	6	0	84
84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	
	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84

Observe that each number **0** thru **8** now appears exactly **49** times in **D** and never in the same location within each of the nine **7x7** block-squares. Note that **9 x 49 = 21x21 = 441**, the total number of cells in the **21x21** table.

Carrying out formula (**7.11**) yields the perfect size-21 square also seen earlier in Program 3.

Program 7

Size-27

Pattern **B** can be expanded further just by repeating the basic pattern. This method was used to generate the size **27** perfect square seen in Program 3. Different sizes involve different arrangements of the columns shown here.

D(27)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	

It is best to first get a rectangular table of size **3b x b** which is geonomic as shown here at right and then triplicate it to get a **3bx3b** size square table as shown on the prior page for the size-21 square.

	7	7	7	7	7	7	7
	7	7	7	7	7	7	7
	7	7	7	7	7	7	7
	2	0	1	2	0	0	2	7
	2	0	1	2	0	0	2	7
	2	0	1	2	0	0	2	7
	2	0	1	2	0	0	2	7
	2	0	1	2	0	0	2	7
	2	0	1	2	0	0	2	7
	2	0	1	2	0	0	2	7
	1	1	0	1	2	1	1	7
	1	1	0	1	2	1	1	7
	1	1	0	1	2	1	1	7
	1	1	0	1	2	1	1	7
	1	1	0	1	2	1	1	7
	1	1	0	1	2	1	1	7
	1	1	0	1	2	1	1	7
	0	2	2	0	1	2	0	7
	0	2	2	0	1	2	0	7
	0	2	2	0	1	2	0	7
	0	2	2	0	1	2	0	7
	0	2	2	0	1	2	0	7
	0	2	2	0	1	2	0	7
	0	2	2	0	1	2	0	7
	21	21	21	21	21	21	21	

Note : The loom-tables **X** and **Y** taken to the base **n = 3b** from **W(3b)** will not be geonomic for Class-3 squares. However, **X(b)** and **Y(b)** derived in the base **b** will be both geonomic and will regenerate the original square and its dual according to the following formulas:

(7.11) $W(n) = b\,(Y(b)-1) + X(b)$

(7.12) $U(n) = 3b\,(X(b)-1) + Y(b)$

Carrying out formula (**7.11**) yields the perfect size-27 square seen earlier in Program 3.

Program 7

The Homogenization of the Derived Spread Pattern for Larger Class-3 Squares

Expanding the pattern used for the size-27 square for generating the size **33** square fails to produce a square with the Class-3 laminated tiling pattern. From here forward, the table **B(3b)** needs to be **homogenized**. This is easily accomplished by taking one row from each of the three block segments and making three consecutive rows from them; then duplicating this pattern vertically from top to bottom as shown at bottom. This was found to spread the numbers more evenly which in turn retained the vertical and horizontal laminated level-3 tiling pattern common among Class-3 squares.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33
1	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
3	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
4	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
5	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
6	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
7	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
8	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
9	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
10	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
11	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
12	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
13	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
14	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
15	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
16	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
17	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
18	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
19	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
20	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
21	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
22	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
23	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
24	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
25	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
26	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
27	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
28	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
29	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
30	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
31	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
32	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
33	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	
		33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33

The homogonized spread pattern

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
		33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33
1	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
2	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
3	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
4	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
5	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
6	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
7	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
8	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
9	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
10	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
11	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
12	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
13	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
14	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
15	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
16	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
17	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
18	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
19	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
20	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
21	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
22	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
23	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
24	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
25	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
26	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
27	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
28	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
29	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
30	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
31	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	2	0	2	1	0	0	1	0	2	1	2	33
32	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	1	2	1	0	2	1	0	2	1	0	1	33
33	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	0	1	0	2	1	2	2	1	0	2	0	33
	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	
		33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33

Program 7

Application of the SPD method to Composing Class-4 Squares

A similar expansion method can be applied to get Class-4 squares.

The formulas (**7.9**) and (**7.10**) are specialized to handle expansions to double the size of the square. To formulate this process, the vertically and/or horizontally flipped version of $\mathbf{W_b}$ shall be denoted by the symbol $\bar{\mathbf{W}}_\mathbf{b}$.

(7.13) $D(2b) = B_1 + 2B_2$

(7.14) $$W(2b) = b^2 \begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix} + \begin{vmatrix} W(b) & \bar{W}(b) \\ W(b) & \bar{W}(b) \end{vmatrix}$$

		-6		6	
	4x4	28	34	40	34
1	8	2	4	12	26
2	10	16	14	6	46
3	11	3	1	7	22
4	5	13	15	9	42
	34	34	34	34	
		28	34	40	34
		-6		6	

Here is a size-4 square that is purposely imperfect and it will be used to show how squares of size **2n** can be obtained from Class-2 squares of size **n**. Here $\mathbf{W_b}$ is the size-4 square shown above. This size-4 square here has its wrap diagonals merely differing in sign. So it suffices to flip the square horizontally for the variances to cancel out.

Because the square also had unequal rows too, it also has to be flipped vertically. So there was a double flip of $\mathbf{W_b}$ involved here to get $\bar{\mathbf{W}}_\mathbf{b}$.

$B_1(8)$

	1	2	3	4	5	6	7	8	
	4	4	4	4	4	4	4	4	4
1	1	0	1	0	1	0	1	0	4
2	1	0	1	0	1	0	1	0	4
3	0	1	0	1	0	1	0	1	4
4	0	1	0	1	0	1	0	1	4
5	0	1	0	1	0	1	0	1	4
6	0	1	0	1	0	1	0	1	4
7	1	0	1	0	1	0	1	0	4
8	1	0	1	0	1	0	1	0	4
	4	4	4	4	4	4	4	4	
		4	4	4	4	4	4	4	4

Size-8

$D(8)$

	12	12	12	12	12	12	12	12
3	0	3	0	3	0	3	0	12
3	0	3	0	3	0	3	0	12
2	1	2	1	2	1	2	1	12
2	1	2	1	2	1	2	1	12
0	3	0	3	0	3	0	3	12
0	3	0	3	0	3	0	3	12
1	2	1	2	1	2	1	2	12
1	2	1	2	1	2	1	2	12
12	12	12	12	12	12	12	12	
	12	12	12	12	12	12	12	12

$B_2(8)$

	4	4	4	4	4	4	4	4
1	0	1	0	1	0	1	0	4
1	0	1	0	1	0	1	0	4
1	0	1	0	1	0	1	0	4
1	0	1	0	1	0	1	0	4
0	1	0	1	0	1	0	1	4
0	1	0	1	0	1	0	1	4
0	1	0	1	0	1	0	1	4
0	1	0	1	0	1	0	1	4
4	4	4	4	4	4	4	4	
	4	4	4	4	4	4	4	4

Here is the resulting perfect size-8 square from (**7.14**). It is **4x2** modular.

8x8	260	260	260	260	260	260	260	260
24	9	8	25	40	57	56	41	260
58	39	42	55	10	23	26	7	260
14	19	30	3	62	35	46	51	260
36	61	52	45	20	13	4	29	260
17	16	1	32	33	64	49	48	260
63	34	47	50	15	18	31	2	260
11	22	27	6	59	38	43	54	260
37	60	53	44	21	12	5	28	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260

Program 7

A Special Pattern for generating all Class-4 Squares divisible by 8 and > 8

This pattern applies when **b = n/4** is an even number. This pattern here was used to generate th size-24 ultra-perfect square from the size-12 ultra-perfect square.

Observe that the **2x2** block number distribution is different in each quadrant. Going clockwis first the block square is flipped vertically; then that result is flipped horizontally; and finally tha result is flipped vertically. Then this pattern was used in (**7.14**) as **D**. This pattern will alway preserve the pangenicity as well as any pairwise symmetry and the characteristic tiling propertie of the smaller square that is being expanded.

The quadrants in this pattern can be expanded or contracted by **4** and the resulting pattern w still be geonomically perfect. However, expanding or contracting the entire pattern by **4** will no produce a viable expansion of a Class-2 square. So the pattern here must only be expanded c contracted by **8.**

This pattern was used to generate the perfect size **24** square from the perfect size **12** squar (next page), the perfect size **32** square from the size **16** square (page 7-41), the perfect size **4** square from the size **20** square, the perfect size **48** square from the size **24** square, and th perfect size **64** square from the resulting size **32** square (the latter three expanded squares ar not shown).

Size-16

Here is the geonomic spread-pattern used to expand the size **8** square to one of size **16**.

	24	24	24	24	24	24	24
0	1	0	1	0	1	0	1
2	3	2	3	2	3	2	3
0	1	0	1	0	1	0	1
2	3	2	3	2	3	2	3
0	1	0	1	0	1	0	1
2	3	2	3	2	3	2	3
0	1	0	1	0	1	0	1
2	3	2	3	2	3	2	3

24	24	24	24	24	24	24	24	24
2	3	2	3	2	3	2	3	24
0	1	0	1	0	1	0	1	24
2	3	2	3	2	3	2	3	24
0	1	0	1	0	1	0	1	24
2	3	2	3	2	3	2	3	24
0	1	0	1	0	1	0	1	24
2	3	2	3	2	3	2	3	24
0	1	0	1	0	1	0	1	24

1	0	1	0	1	0	1	0
3	2	3	2	3	2	3	2
1	0	1	0	1	0	1	0
3	2	3	2	3	2	3	2
1	0	1	0	1	0	1	0
3	2	3	2	3	2	3	2
1	0	1	0	1	0	1	0
3	2	3	2	3	2	3	2
24	24	24	24	24	24	24	24
	24	24	24	24	24	24	24

3	2	3	2	3	2	3	2	24
1	0	1	0	1	0	1	0	24
3	2	3	2	3	2	3	2	24
1	0	1	0	1	0	1	0	24
3	2	3	2	3	2	3	2	24
1	0	1	0	1	0	1	0	24
3	2	3	2	3	2	3	2	24
1	0	1	0	1	0	1	0	24
24	24	24	24	24	24	24	24	
24	24	24	24	24	24	24	24	24

Program 7

Here are the size-16 versions produced by the spread-pattern depicted on the prior page. The pairwise symmetry in the size-8 squares was preserved in the expanded squares, central symmetry in top version and row-symmetry in bottom version. All symmetric pairs sum to **157** in both generated squares. Both squares had to be normalized after derivation to maintain this property.

Normalized

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
W(16)	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**
1	37	84	52	69	189	204	172	221	165	212	180	197	61	76	44	93	2056
2	155	238	142	251	3	118	22	99	27	110	14	123	131	246	150	227	2056
3	31	106	10	127	135	242	146	231	159	234	138	255	7	114	18	103	2056
4	161	216	184	193	57	80	48	89	33	88	56	65	185	208	176	217	2056
5	104	17	113	8	256	137	233	160	232	145	241	136	128	9	105	32	2056
6	218	175	207	186	66	55	87	34	90	47	79	58	194	183	215	162	2056
7	94	43	75	62	198	179	211	166	222	171	203	190	70	51	83	38	2056
8	228	149	245	132	124	13	109	28	100	21	117	4	252	141	237	156	2056
9	101	20	116	5	253	140	236	157	229	148	244	133	125	12	108	29	2056
10	219	174	206	187	67	54	86	35	91	46	78	59	195	182	214	163	2056
11	95	42	74	63	199	178	210	167	223	170	202	191	71	50	82	39	2056
12	225	152	248	129	121	16	112	25	97	24	120	1	249	144	240	153	2056
13	40	81	49	72	192	201	169	224	168	209	177	200	64	73	41	96	2056
14	154	239	143	250	2	119	23	98	26	111	15	122	130	247	151	226	2056
15	30	107	11	126	134	243	147	230	158	235	139	254	6	115	19	102	2056
16	164	213	181	196	60	77	45	92	36	85	53	68	188	205	173	220	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	2056	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**

Normalized

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
W(16)	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**
1	2	119	18	103	26	111	10	127	130	247	146	231	154	239	138	255	2056
2	134	243	150	227	158	235	142	251	6	115	22	99	30	107	14	123	2056
3	60	77	44	93	36	85	52	69	188	205	172	221	164	213	180	197	2056
4	185	208	169	224	161	216	177	200	57	80	41	96	33	88	49	72	2056
5	71	50	87	34	95	42	79	58	199	178	215	162	223	170	207	186	2056
6	195	182	211	166	219	174	203	190	67	54	83	38	91	46	75	62	2056
7	125	12	109	28	101	20	117	4	253	140	237	156	229	148	245	132	2056
8	256	137	240	153	232	145	248	129	128	9	112	25	104	17	120	1	2056
9	66	55	82	39	90	47	74	63	194	183	210	167	218	175	202	191	2056
10	198	179	214	163	222	171	206	187	70	51	86	35	94	43	78	59	2056
11	124	13	108	29	100	21	116	5	252	141	236	157	228	149	244	133	2056
12	249	144	233	160	225	152	241	136	121	16	105	32	97	24	113	8	2056
13	7	114	23	98	31	106	15	122	135	242	151	226	159	234	143	250	2056
14	131	246	147	230	155	238	139	254	3	118	19	102	27	110	11	126	2056
15	61	76	45	92	37	84	53	68	189	204	173	220	165	212	181	196	2056
16	192	201	176	217	168	209	184	193	64	73	48	89	40	81	56	65	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	2056	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**	**2056**

Program 7

Both versions are ultra-perfect because they possess the ring-diamond and X patterns commo
to Class-4 squares.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
	W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	37	84	52	69	189	204	172	221	165	212	180	197	61	76	44	93	2056
2	155	238	142	251	3	118	22	99	27	110	14	123	131	246	150	227	2056
3	31	106	10	127	135	242	146	231	159	234	138	255	7	114	18	103	2056
4	161	216	184	193	57	80	48	89	33	88	56	65	185	208	176	217	2056
5	104	17	113	8	256	137	233	160	232	145	241	136	128	9	105	32	2056
6	218	175	207	186	66	55	87	34	90	47	79	58	194	183	215	162	2056
7	94	43	75	62	198	179	211	166	222	171	203	190	70	51	83	38	2056
8	228	149	245	132	124	13	109	28	100	21	117	4	252	141	237	156	2056
9	101	20	116	5	253	140	236	157	229	148	244	133	125	12	108	29	2056
10	219	174	206	187	67	54	86	35	91	46	78	59	195	182	214	163	2056
11	95	42	74	63	199	178	210	167	223	170	202	191	71	50	82	39	2056
12	225	152	248	129	121	16	112	25	97	24	120	1	249	144	240	153	2056
13	40	81	49	72	192	201	169	224	168	209	177	200	64	73	41	96	2056
14	154	239	143	250	2	119	23	98	26	111	15	122	130	247	151	226	2056
15	30	107	11	126	134	243	147	230	158	235	139	254	6	115	19	102	2056
16	164	213	181	196	60	77	45	92	36	85	53	68	188	205	173	220	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

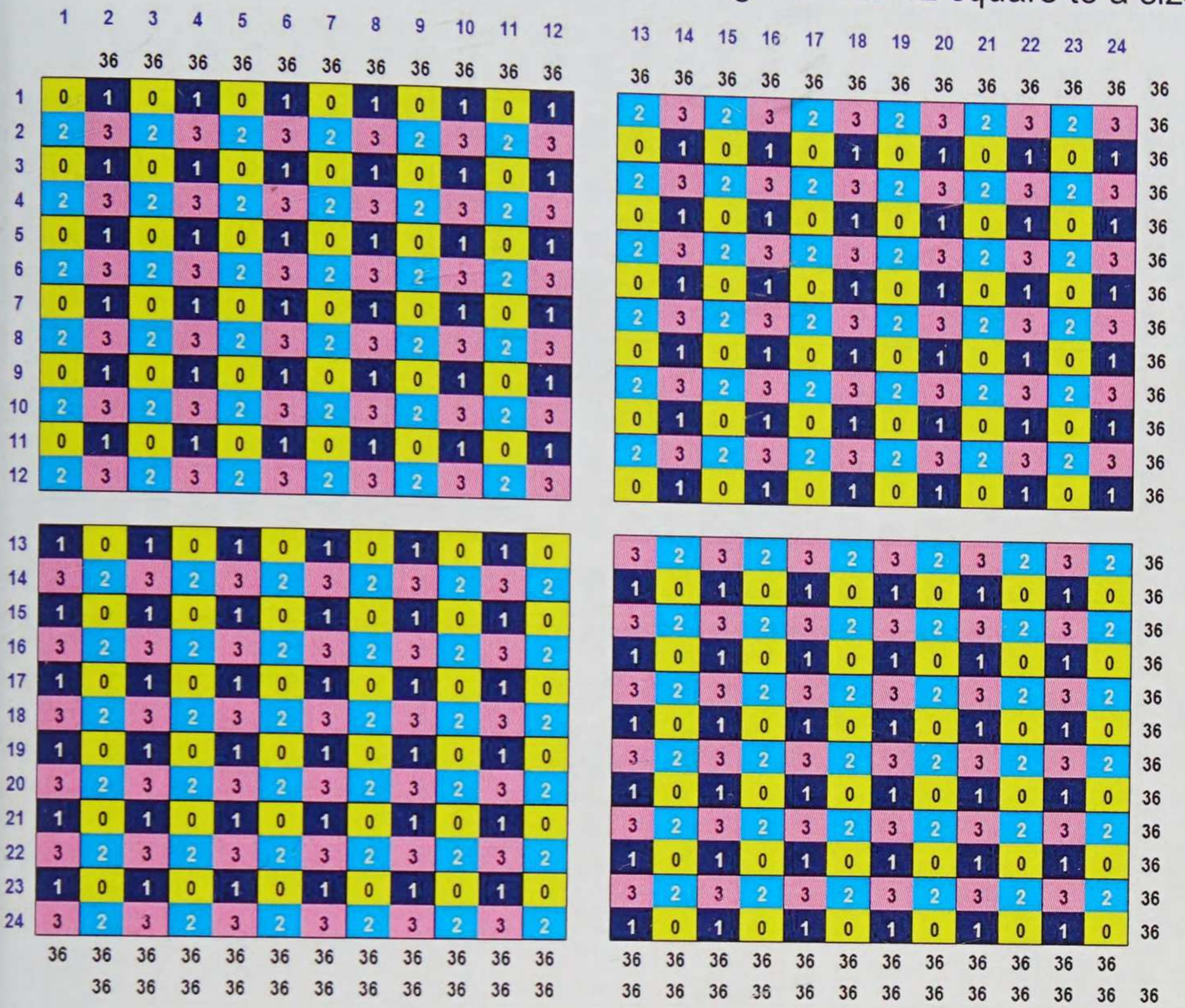
The pairwise row-symmetric version possess a special tiling pattern all its own, shown here.
does not complete the square as seen in the top and bottom rows.

	W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	2	119	18	103	26	111	10	127	130	247	146	231	154	239	138	255	2056
2	134	243	150	227	158	235	142	251	6	115	22	99	30	107	14	123	2056
3	60	77	44	93	36	85	52	69	188	205	172	221	164	213	180	197	2056
4	185	208	169	224	161	216	177	200	57	80	41	96	33	88	49	72	2056
5	71	50	87	34	95	42	79	58	199	178	215	162	223	170	207	186	2056
6	195	182	211	166	219	174	203	190	67	54	83	38	91	46	75	62	2056
7	125	12	109	28	101	20	117	4	253	140	237	156	229	148	245	132	2056
8	256	137	240	153	232	145	248	129	128	9	112	25	104	17	120	1	2056
9	66	55	82	39	90	47	74	63	194	183	210	167	218	175	202	191	2056
10	198	179	214	163	222	171	206	187	70	51	86	35	94	43	78	59	2056
11	124	13	108	29	100	21	116	5	252	141	236	157	228	149	244	133	2056
12	249	144	233	160	225	152	241	136	121	16	105	32	97	24	113	8	2056
13	7	114	23	98	31	106	15	122	135	242	151	226	159	234	143	250	2056
14	131	246	147	230	155	238	139	254	3	118	19	102	27	110	11	126	2056
15	61	76	45	92	37	84	53	68	189	204	173	220	165	212	181	196	2056
16	192	201	176	217	168	209	184	193	64	73	48	89	40	81	56	65	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Program 7

Size-24

Here is the spread-pattern used for expanding the size **12** square to a size **24** square.



Below is the resulting ultra-perfect size-24 square **W(24)** after normalization. It is not pairwise symmetric because the size-12 square involved was not pairwise symmetric (p.7-18). However this version possesses the most equal-summing patterns.

Note that it is ultra-perfect in that it is both pangenic and has equal-summing ring-diamond tiling patterns of size **6** and and X tiling patterns of size **12**, respectively.

W(24)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
21	250	95	228	101	170	31	244	85	234	111	164	309	538	383	516	389	458	319	532	373	522	399	452	6924
432	499	294	441	368	563	422	505	304	435	358	569	144	211	6	153	80	275	134	217	16	147	70	281	6924
50	189	124	263	34	205	60	183	114	269	44	199	338	477	412	551	322	493	348	471	402	557	332	487	6924
347	472	401	558	331	488	337	478	411	552	321	494	59	184	113	270	43	200	49	190	123	264	33	206	6924
133	218	15	148	69	282	143	212	5	154	79	276	421	506	303	436	357	570	431	500	293	442	367	564	6924
320	531	374	521	400	451	310	537	384	515	390	457	32	243	86	233	112	163	22	249	96	227	102	169	6924
18	253	92	231	98	173	28	247	82	237	108	167	306	541	380	519	386	461	316	535	370	525	396	455	6924
427	504	289	446	363	568	417	510	299	440	353	574	139	216	1	158	75	280	129	222	11	152	65	286	6924
53	186	127	260	37	202	63	180	117	266	47	196	341	474	415	548	325	490	351	468	405	554	335	484	6924
352	467	406	553	336	483	342	473	416	547	326	489	64	179	118	265	48	195	54	185	128	259	38	201	6924
130	221	12	151	66	285	140	215	2	157	76	279	418	509	300	439	354	573	428	503	290	445	364	567	6924
315	536	369	526	395	456	305	542	379	520	385	462	27	248	81	238	107	168	17	254	91	232	97	174	6924
165	106	239	84	245	26	175	100	229	90	255	20	453	394	527	372	533	314	463	388	517	378	543	308	6924
576	355	438	297	512	419	566	361	448	291	502	425	288	67	150	9	224	131	278	73	160	3	214	137	6924
194	45	268	119	178	61	204	39	258	125	188	55	482	333	556	407	466	349	492	327	546	413	476	343	6924
491	328	545	414	475	344	481	334	555	408	465	350	203	40	257	126	187	56	193	46	267	120	177	62	6924
277	74	159	4	213	138	287	68	149	10	223	132	565	362	447	292	501	426	575	356	437	298	511	420	6924
464	387	518	377	544	307	454	393	528	371	534	313	176	99	230	89	256	19	166	105	240	83	246	25	6924
162	109	236	87	242	29	172	103	226	93	252	23	450	397	524	375	530	317	460	391	514	381	540	311	6924
571	360	433	302	507	424	561	366	443	296	497	430	283	72	145	14	219	136	273	78	155	8	209	142	6924
197	42	271	116	181	58	207	36	261	122	191	52	485	330	559	404	469	346	495	324	549	410	479	340	6924
496	323	550	409	480	339	486	329	560	403	470	345	208	35	262	121	192	51	198	41	272	115	182	57	6924
274	77	156	7	210	141	284	71	146	13	220	135	562	365	444	295	498	429	572	359	434	301	508	423	6924
459	392	513	382	539	312	449	398	523	376	529	318	171	104	225	94	251	24	161	110	235	88	241	30	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Program 7

Size-32

The size-32 square here was generated by the SPD method likewise.

Normalized Square has pairwise central symmetry

W(32)	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400
456	169	481	144	376	25	337	64	840	553	865	528	1016	665	977	704	968	681	993	656	888	537	849	576	328	41	353	16	504	153	465	192	16400
890	535	863	562	970	679	1007	642	506	151	479	178	330	39	367	2	378	23	351	50	458	167	495	130	1018	663	991	690	842	551	879	514	16400
510	147	475	182	334	35	363	6	894	531	859	566	974	675	1003	646	1022	659	987	694	846	547	875	518	382	19	347	54	462	163	491	134	16400
836	557	869	524	1012	669	981	700	452	173	485	140	372	29	341	60	324	45	357	12	500	157	469	188	964	685	997	652	884	541	853	572	16400
389	236	420	205	309	92	276	125	773	620	804	589	949	732	916	765	901	748	932	717	821	604	788	637	261	108	292	77	437	220	404	253	16400
827	598	798	627	907	742	942	707	443	214	414	243	267	102	302	67	315	86	286	115	395	230	430	195	955	726	926	755	779	614	814	579	16400
447	210	410	247	271	98	298	71	831	594	794	631	911	738	938	711	959	722	922	759	783	610	810	583	319	82	282	119	399	226	426	199	16400
769	624	808	585	945	736	920	761	385	240	424	201	305	96	280	121	257	112	296	73	433	224	408	249	897	752	936	713	817	608	792	633	16400
136	489	161	464	56	345	17	384	520	873	545	848	696	985	657	1024	648	1001	673	976	568	857	529	896	8	361	33	336	184	473	145	512	16400
570	855	543	882	650	999	687	962	186	471	159	498	10	359	47	322	58	343	31	370	138	487	175	450	698	983	671	1010	522	871	559	834	16400
190	467	155	502	14	355	43	326	574	851	539	886	654	995	683	966	702	979	667	1014	526	867	555	838	62	339	27	374	142	483	171	454	16400
516	877	549	844	692	989	661	1020	132	493	165	460	52	349	21	380	4	365	37	332	180	477	149	508	644	1005	677	972	564	861	533	892	16400
197	428	228	397	117	284	84	317	581	812	612	781	757	924	724	957	709	940	740	909	629	796	596	829	69	300	100	269	245	412	212	445	16400
635	790	606	819	715	934	750	899	251	406	222	435	75	294	110	259	123	278	94	307	203	422	238	387	763	918	734	947	587	806	622	771	16400
255	402	218	439	79	290	106	263	639	786	602	823	719	930	746	903	767	914	730	951	591	802	618	775	127	274	90	311	207	418	234	391	16400
577	816	616	777	753	928	728	953	193	432	232	393	113	288	88	313	65	304	104	265	241	416	216	441	705	944	744	905	625	800	600	825	16400
200	425	225	400	120	281	81	320	584	809	609	784	760	921	721	960	712	937	737	912	632	793	593	832	72	297	97	272	248	409	209	448	16400
634	791	607	818	714	935	751	898	250	407	223	434	74	295	111	258	122	279	95	306	202	423	239	386	762	919	735	946	586	807	623	770	16400
254	403	219	438	78	291	107	262	638	787	603	822	718	931	747	902	766	915	731	950	590	803	619	774	126	275	91	310	206	419	235	390	16400
580	813	613	780	756	925	725	956	196	429	229	396	116	285	85	316	68	301	101	268	244	413	213	444	708	941	741	908	628	797	597	828	16400
133	492	164	461	53	348	20	381	517	876	548	845	693	988	660	1021	645	1004	676	973	565	860	532	893	5	364	36	333	181	476	148	509	16400
571	854	542	883	651	998	686	963	187	470	158	499	11	358	46	323	59	342	30	371	139	486	174	451	699	982	670	1011	523	870	558	835	16400
191	466	154	503	15	354	42	327	575	850	538	887	655	994	682	967	703	978	666	1015	527	866	554	839	63	338	26	375	143	482	170	455	16400
513	880	552	841	689	992	664	1017	129	496	168	457	49	352	24	377	1	368	40	329	177	480	152	505	641	1008	680	969	561	864	536	889	16400
392	233	417	208	312	89	273	128	776	617	801	592	952	729	913	768	904	745	929	720	824	601	785	640	264	105	289	80	440	217	401	256	16400
826	599	799	626	906	743	943	706	442	215	415	242	266	103	303	66	314	87	287	114	394	231	431	194	954	727	927	754	778	615	815	578	16400
446	211	411	246	270	99	299	70	830	595	795	630	910	739	939	710	958	723	923	758	782	611	811	582	318	83	283	118	398	227	427	198	16400
772	621	805	588	948	733	917	764	388	237	421	204	308	93	277	124	260	109	293	76	436	221	405	252	900	749	933	716	820	605	789	636	16400
453	172	484	141	373	28	340	61	837	556	868	525	1013	668	980	701	965	684	996	653	885	540	852	573	325	44	356	13	501	156	468	189	16400
891	534	862	563	971	678	1006	643	507	150	478	179	331	38	366	3	379	22	350	51	459	166	494	131	1019	662	990	691	843	550	878	515	16400
511	146	474	183	335	34	362	7	895	530	858	567	975	674	1002	647	1023	658	986	695	847	546	874	519	383	18	346	55	463	162	490	135	16400
833	560	872	521	1009	672	984	697	449	176	488	137	369	32	344	57	321	48	360	9	497	160	472	185	961	688	1000	649	881	544	856	569	16400
16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400
16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400

Program 7

Application of the SPD method to composing Class-2 Squares

Here are three patterns to demonstrate the general pattern that can be applied as **D** back in formula (**7.14**), shown here as formula (**7.15**):

$$(7.15)\quad W(2b) = b^2\,D(2b) + E(2b) \quad = b^2 \begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix} + \begin{vmatrix} W(b)_{11} & W(b)_{12} \\ W(b)_{21} & W(b)_{22} \end{vmatrix}$$

D6

	1	2	3	4	5	6	
	9	9	9	9	9	9	
1	3	0	3	0	3	0	9
2	0	3	0	2	1	2	8
3	1	2	1	2	1	2	9
4	1	2	1	2	1	2	9
5	1	2	1	3	0	3	10
6	3	0	3	0	3	0	9
	9	9	9	9	9	9	
	9	9	9	9	9	9	

-1

1

E(3)

	30	30	30	30	30	30
2	7	6	6	7	2	30
9	5	1	1	5	9	30
4	3	8	8	3	4	30
4	3	8	8	3	4	30
9	5	1	1	5	9	30
2	7	6	6	7	2	30
30	30	30	30	30	30	
	30	30	30	30	30	30

D10

	1	2	3	4	5	6	7	8	9	10	
	15	15	15	15	15	15	15	15	15	15	15
1	0	3	0	3	0	3	0	3	0	3	15
2	3	0	3	0	3	0	3	0	3	0	15
3	0	3	0	3	0	2	1	2	1	2	14
4	2	1	2	1	2	1	2	1	2	1	15
5	2	1	2	1	2	1	2	1	2	1	15
6	2	1	2	1	2	1	2	1	2	1	15
7	2	1	2	1	2	1	2	1	2	1	15
8	1	2	1	2	1	3	0	3	0	3	16
9	3	0	3	0	3	0	3	0	3	0	15
10	0	3	0	3	0	3	0	3	0	3	15
	15	15	15	15	15	15	15	15	15	15	
		15	15	15	15	15	15	15	15	15	15

There are four different versions of **W(3)** in (**7.15**) in the tiled expansion **E(3)** to make it geonomic (top right). That is the only case where this was involved because all the other distribution tables are composed of Class-1 tiles which are all perfect.

However, this horizontal and vertical flipping of **W(3)** resulted in the loss of pangenicity for **W(6)**. The pangenic version of W(6) was obtained by trial and error and that process is detailed at the end of this section.

W(5)

	2	3	4	5	6	7	8	9	10	
	130	130	130	130	130	130	130	130	130	130
1	20	8	21	14	2	20	8	21	14	2	130
2	11	4	17	10	23	11	4	17	10	23	130
3	7	25	13	1	19	7	25	13	1	19	130
4	3	16	9	22	15	3	16	9	22	15	130
5	24	12	5	18	6	24	12	5	18	6	130
6	20	8	21	14	2	20	8	21	14	2	130
7	11	4	17	10	23	11	4	17	10	23	130
8	7	25	13	1	19	7	25	13	1	19	130
9	3	16	9	22	15	3	16	9	22	15	130
10	24	12	5	18	6	24	12	5	18	6	130
	130	130	130	130	130	130	130	130	130	130	
		130	130	130	130	130	130	130	130	130	130

The tiled expansion of the **W(5)** square is shown at left.

The tiled expansions of other squares are obvious and have therefore been omitted in the following exhibitions.

A systematic construction of spread-patterns for **D(2b)** for larger squares is exhibited on the next two pages.

Program 7

D_{14}

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
1	0	3	0	3	0	3	0	3	0	3	0	3	0	3	21
2	3	0	3	0	3	0	3	0	3	0	3	0	3	0	21
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	21
4	3	0	3	0	3	0	3	1	2	1	2	1	2	1	22
5	1	2	1	2	1	2	1	2	1	2	1	2	1	2	21
6	2	1	2	1	2	1	2	1	2	1	2	1	2	1	21
7	2	1	2	1	2	1	2	1	2	1	2	1	2	1	21
8	2	1	2	1	2	1	2	1	2	1	2	1	2	1	21
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	21
10	1	2	1	2	1	2	1	2	1	2	1	2	1	2	21
11	2	1	2	1	2	1	2	0	3	0	3	0	3	0	20
12	0	3	0	3	0	3	0	3	0	3	0	3	0	3	21
13	3	0	3	0	3	0	3	0	3	0	3	0	3	0	21
14	0	3	0	3	0	3	0	3	0	3	0	3	0	3	21
	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
		21	21	21	21	21	21	21	21	21	21	21	21	21	21

D_{18}

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
1	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	27
2	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	27
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	27
4	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	27
5	0	3	0	3	0	3	0	3	0	2	1	2	1	2	1	2	1	2	26
6	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	27
7	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	27
8	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	27
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	27
10	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	27
11	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	27
12	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	27
13	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	27
14	1	2	1	2	1	2	1	2	1	3	0	3	0	3	0	3	0	3	28
15	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	27
16	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	27
17	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	27
18	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	27
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	
		27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27

D_{22}

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33
1	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	33
2	3	3	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	33
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	33
4	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	33
5	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	33
6	3	0	3	0	3	0	3	0	3	0	3	1	2	1	2	1	2	1	2	1	2	1	34
7	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	33
8	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	33
9	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	33
10	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	33
11	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	33
12	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	33
13	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	33
14	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	33
15	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	33
16	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	33
17	2	1	2	1	2	1	2	1	2	1	2	0	3	0	3	0	3	0	3	0	3	0	32
18	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	33
19	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	33
20	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	33
21	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	33
22	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	33
	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	
		33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33

Program 7

D26

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39
1	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
2	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	39
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
4	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	39
5	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
6	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	39
7	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
8	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	38
9	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	39
10	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
11	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	39
12	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
13	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
14	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
15	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
16	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	39
17	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
18	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	39
19	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	39
20	1	2	1	2	1	2	1	2	1	2	1	2	1	3	0	3	0	3	0	3	0	3	0	3	0	3	40
21	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	39
22	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
23	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	39
24	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
25	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	39
26	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	39
	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	
	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	

ote that the pattern in the spread-pattern square has the following key properties:

- The checkerboard pattern of **3**s and **0**s are always the same, starting with **0** in the upper left-hand corner and terminating with **3** in the lower right-hand corner
- There is always a 4-row band running through the center of columns of alternating **1**s and **2**s, beginning with a column of **2**s.
- There is a checkerboard pattern of **1**s and **2**s running reflectively along the top and bottom of the central 4-row band that starts with the complementary number opposite the striped pattern of the adjacent central band. (Note that this was absent for the size-10 spread-pattern square.)
- There is always an extended half-row on the lower left and upper right half of the checkerboard pattern of **1**s and **2**s which continues the checkerboard pattern.

Note that as **n** increases by **4**, the characteristic number of the geonomic spread patterns increases by a constant **6**. Where this **6** is coming from is the addition of a **0, 1, 2** & **3** to every linear geonomic summation: **0+1+2+3 = 6**, upon expanding the spread pattern by **4** in every direction.

This pattern is totally general upon expansion and provides the basic pattern for manifesting all even-size geonomic squares whose size is two times an odd-number, i.e. not divisible by **4**. It uses as its basis a known perfect square of odd-size.

This pattern not only manifests near-perfect squares but also squares with the property of being pairwise centrally symmetric.

Program 7

	1	2	3	4	5	6	
	C	9	9	9	9	9	9
1	0	3	0	3	0	3	9
2	3	0	3	1	2	1	**10**
3	2	1	2	1	2	1	9
4	2	1	2	1	2	1	9
5	2	1	2	0	3	0	**8**
6	0	3	0	3	0	3	9
	9	9	9	9	9	9	
	9	9	9	9	9	9	

Quadrals

6	6	6
6	6	6
6	6	6

Deriving the Near-perfect Square of Size-6

Lets now address manifesting geonomic square of size-**6**.

A near-perfect **6x6** geonomic square does in fact exist but it was only uncovered by extensiv number swapping until its near-perfect status became manifest. The following depictions sho the sequence of operations which yielded the near-perfect **6x6** square starting from scratch.

Starting pattern

Δ	-3	3	-3	3	-3	3	
	108	114	108	114	108	114	Δ
1	2	3	4	5	6	21	-190
12	11	10	9	8	7	57	-54
13	14	15	16	17	18	93	-18
24	23	22	21	20	19	129	+18
25	26	27	28	29	30	165	+54
36	35	34	33	32	31	201	+90
111	111	111	111	111	111		
	114	108	114	108	114	108	
Δ	3	-3	3	-3	3	-3	

Flip shaded columns

Program 7

Δ	-15	15	-15	15	-15	15
	96	126	96	126	96	126
1	35	3	33	8	31	111
12	26	10	28	5	30	111
13	23	15	21	20	19	111
24	14	22	16	17	18	111
25	11	27	9	32	7	111
36	2	34	4	29	6	111
111	111	111	111	111	111	
	126	96	126	96	126	96
Δ	15	-15	15	-15	15	-15

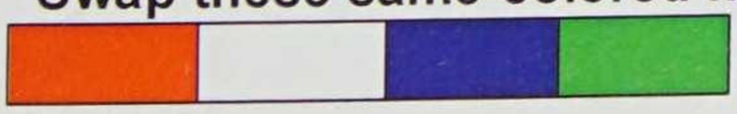
Flip shaded rows

	111	111	111	111	111	111
1	35	3	33	8	31	111
30	5	28	10	26	12	111
13	23	15	21	20	19	111
18	17	16	22	14	24	111
25	11	27	9	32	7	111
6	29	4	34	2	36	111
93	120	93	129	102	129	
-18	9	-18	18	-9	18	Δ
	111	111	111	111	111	111

Transpose

	111	111	111	111	111	111	Δ
1	30	13	18	25	6	93	-18
35	5	23	17	11	29	120	9
3	28	15	16	27	4	93	-18
33	10	21	22	9	34	129	18
8	26	20	14	32	2	102	-9
31	12	19	24	7	36	129	18
111	111	111	111	111	111		
	111	111	111	111	111	111	

Swap these same-colored numbers vertically



Then exchange these differently colored numbers

horizontally

Program 7

Now that we have the near-perfect **6x6** geonomic square, we can use it to manifest othe geonomic tables such as the size-6 cube and quadracube,

							Δ
	111	111	111	111	111	111	
31	12	13	24	25	6	111	0
35	2	23	17	32	5	114	3
33	28	15	22	9	4	111	0
3	10	21	16	27	34	111	0
8	29	20	14	11	26	108	-3
1	30	19	18	7	**36**	111	0
111	111	111	111	111	111		
	111	111	111	111	111	111	

Swap block-squares diagonally

	1	2	3	4	5	6		
6x6	111	111	111	111	111	111		
16	27	34	3	10	21	111		
14	11	26	8	29	20	**108**	(-3)	
18	7	**36**	**1**	30	19	111		
24	25	6	31	12	13	111		
17	32	5	35	2	23	**114**	(+3)	
22	9	4	33	28	15	111		
111	111	111	111	111	111			
	111	111	111	111	111	111		

Quadrals

74	74	74
74	74	74
74	74	74

Now that we have the near-perfect **6x6** square, we can use it to manifest other geonomic squares which are even multiples of **6**.

Program 7

A Unique Method for Ironing Out the Wrinkle in Class-2 Squares

	13	12	11	10	9	8	7	6	5	4	3	2	1	13	1	2	3	4	5	6	7	8	9	10	11	12	13
13	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
12	2	1	2	1	2	1	2	1	2	1	2	1	2	11	1	2	1	2	1	2	1	2	1	2	1	2	1
11	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
10	2	1	2	1	2	1	2	1	2	1	2	1	2		1	2	1	2	1	2	1	2	1	2	1	2	1
9	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
8	2	1	2	1	2	1	2	1	2	1	2	1	2	7	1	2	1	2	1	2	1	2	1	2	1	2	1
7	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
6	2	1	2	1	2	1	2	1	2	1	2	1	2	5	1	2	1	2	1	2	1	2	1	2	1	2	1
5	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
4	2	1	2	1	2	1	2	1	2	1	2	1	2		1	2	1	2	1	2	1	2	1	2	1	2	1
3	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
2	2	1	2	1	2	1	2	1	2	1	2	1	2		1	2	1	2	1	2	1	2	1	2	1	2	1
1	0	3	0	3	0	3	0	3	0	3	0	3	0		3	0	3	0	3	0	3	0	3	0	3	0	3
13		11				7		5												5		7				11		13
1	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
2	3	0	3	0	3	0	3	0	3	0	3	0	3		0	3	0	3	0	3	0	3	0	3	0	3	0
3	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
4	3	0	3	0	3	0	3	0	3	0	3	0	3		0	3	0	3	0	3	0	3	0	3	0	3	0
5	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
6	3	0	3	0	3	0	3	0	3	0	3	0	3	5	0	3	0	3	0	3	0	3	0	3	0	3	0
7	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
8	3	0	3	0	3	0	3	0	3	0	3	0	3	7	0	3	0	3	0	3	0	3	0	3	0	3	0
9	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
10	3	0	3	0	3	0	3	0	3	0	3	0	3		0	3	0	3	0	3	0	3	0	3	0	3	0
11	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
12	3	0	3	0	3	0	3	0	3	0	3	0	3	11	0	3	0	3	0	3	0	3	0	3	0	3	0
13	1	2	1	2	1	2	1	2	1	2	1	2	1		2	1	2	1	2	1	2	1	2	1	2	1	2
														13													

D(14)	42	42	42	42	42	42	42	42	42	42	42	42	42	
	20	22	20	22	20	22	20	22	20	22	20	22	20	22
7	0	3	0	3	0	3	0	3	0	3	0	3	0	3 21
6	2	1	2	1	2	1	2	1	2	1	2	1	2	1 21
5	0	3	0	3	0	3	0	3	0	3	0	3	0	3 21
4	2	1	2	1	2	1	2	1	2	1	2	1	2	1 21
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3 21
2	2	1	2	1	2	1	2	1	2	1	2	1	2	1 21
1	0	3	0	3	0	3	0	3	0	3	0	3	0	3 21
1	1	2	1	2	1	2	1	2	1	2	1	2	1	2 21
2	3	0	3	0	3	0	3	0	3	0	3	0	3	0 21
3	1	2	1	2	1	2	1	2	1	2	1	2	1	2 21
4	3	0	3	0	3	0	3	0	3	0	3	0	3	0 21
5	1	2	1	2	1	2	1	2	1	2	1	2	1	2 21
6	3	0	3	0	3	0	3	0	3	0	3	0	3	0 21
7	1	2	1	2	1	2	1	2	1	2	1	2	1	2 21
	19	23	19	23	19	23	19	23	19	23	19	23	19	23
	42	42	42	42	42	42	42	42	42	42	42	42	42	
	22	20	22	20	22	20	22	20	22	20	22	20	22	20
	42	42	42	42	42	42	42	42	42	42	42	42	42	

D(10)	30	30	30	30	30	30	30	30	30	30	
	14	16	14	16	14	16	14	16	14	16	
5	0	3	0	3	0	3	0	3	0	3	15
4	2	1	2	1	2	1	2	1	2	1	15
3	0	3	0	3	0	3	0	3	0	3	15
2	2	1	2	1	2	1	2	1	2	1	15
1	0	3	0	3	0	3	0	3	0	3	15
1	1	2	1	2	1	2	1	2	1	2	15
2	3	0	3	0	3	0	3	0	3	0	15
3	1	2	1	2	1	2	1	2	1	2	15
4	3	0	3	0	3	0	3	0	3	0	15
5	1	2	1	2	1	2	1	2	1	2	15
	13	17	13	17	13	17	13	17	13	17	
	30	30	30	30	30	30	30	30	30	30	
	16	14	16	14	16	14	16	14	16	14	
	30	30	30	30	30	30	30	30	30	30	

Program 7

On the prior page is the spread-pattern used for Class-4 squares but truncated to accommodat squares whose size is not divisible by 4, more specifically, Class-2 squares.

Note that the rows all sum equally, but that's all of the equalities in strictly linear summations. Th remainder linear summations differ by equal "**+**" and "**–**" amounts. The spatial wrinkle gets irone out by combining adjacent linear summations; then all these combinations sum equall everywhere.

Here are the size-10 and size-14 doubly summed squares derived by the **SPD** method using th spread-patterns **D(10)** and **D(14)** shown on the prior page.

	1	2	3	4	5	6	7	8	9	10		
		1010	1010	1010	1010	1010	1010	1010	1010	1010		
	WW(10)	530	480	530	480	530	480	530	480	530	480	
1		45	58	46	64	27	70	33	71	39	52	505	1010
2		86	4	92	10	98	11	79	17	85	23	505	1010
3		32	75	38	51	44	57	50	63	26	69	505	1010
4		78	16	84	22	90	3	91	9	97	15	505	1010
5		49	62	30	68	31	74	37	55	43	56	505	1010
6		20	83	21	89	2	95	8	96	14	77	505	1010
7		61	29	67	35	73	36	54	42	60	48	505	1010
8		7	100	13	76	19	82	25	88	1	94	505	1010
9		53	41	59	47	65	28	66	34	72	40	505	1010
10		24	87	5	93	6	99	12	80	18	81	505	
		455	555	455	555	455	555	455	555	455	555		
		1010	1010	1010	1010	1010	1010	1010	1010	1010	1010		
		480	530	480	530	480	530	480	530	480	530		
		1010	1010	1010	1010	1010	1010	1010	1010	1010	1010		

	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
		2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758		
	WW(14)	1330	1428	1330	1428	1330	1428	1330	1428	1330	1428	1330	1428	1330	1428	
1		42	173	10	190	34	165	2	189	26	157	43	181	18	149	1379	2758
2		127	69	102	86	126	61	143	78	118	53	135	77	110	94	1379	2758
3		23	161	47	178	15	153	39	170	14	194	31	162	6	186	1379	2758
4		115	50	139	74	107	98	131	66	99	90	123	58	147	82	1379	2758
5		11	191	35	166	3	183	27	158	44	182	19	150	36	174	1379	2758
6		103	87	120	62	144	79	119	54	136	71	111	95	128	70	1379	2758
7		48	179	16	154	40	171	8	195	32	163	7	187	24	155	1379	2758
8		91	124	59	141	83	116	51	140	75	108	92	132	67	100	1379	2758
9		176	20	151	37	175	12	192	29	167	4	184	28	159	45	1379	2758
10		72	112	96	129	64	104	88	121	63	145	80	113	55	137	1379	2758
11		164	1	188	25	156	49	180	17	148	41	172	9	196	33	1379	2758
12		60	142	84	117	52	134	76	109	93	133	68	101	85	125	1379	2758
13		152	38	169	13	193	30	168	5	185	22	160	46	177	21	1379	2758
14		97	130	65	105	89	122	57	146	81	114	56	138	73	106	1379	2758
		1281	1477	1281	1477	1281	1477	1281	1477	1281	1477	1281	1477	1281	1477		
		2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758		
		1428	1330	1428	1330	1428	1330	1428	1330	1428	1330	1428	1330	1428	1330		
		2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758	2758		

Program 7

Here are some surprising properties of the doubly expanded Class-2 square:

1. Every double linear sum in all 4 directions sum equally to **2Cn**.

2. The resulting squares have upper and lower row segments of size **(n/2)xn** that are independently pairwise centrally symmetric. Yet the entire square is totally pangenic.

3. Whereas the vertical **bx2** tile pattern was interrupted everywhere over the two central columns, it is now continuous.

4. The two adjacent horizontally overlapping **2x7's** jointly sum equally and continuously to **2Cn**.

5. The complementary tile patterns of the size **b** squares **W(b)** used in the **SPD** expansion formula now sum equally and continuously to **2Cn** on the expanded square in **2x2** block-square configurations.

6. The doubly expanded square **WW(2b)** still has no viable dual square as derived from its complementary loom-tables.

Doubly expanded Class-2 squares generated by this generalized spread-pattern were all verified up thru size **46.**

* * *

Program 7

#6: Expansion by the Matchmaker Pattern

Here is another amazing expansion property: When the size-4 square is expanded to a size-2 square with the size-5 matchmaker pattern, all the equal string summations that are characteristi of matchmaker squares are found in the expanded table.

M(4,5)																				
	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250
	13	12	11	10	9	18	17	16	15	14	23	22	21	20	19	12	11	10	9	8	290
	12	11	10	9	8	17	16	15	14	13	22	21	20	19	18	11	10	9	8	7	270
	11	10	9	8	7	16	15	14	13	12	21	20	19	18	17	10	9	8	7	6	250
	10	9	8	7	6	15	14	13	12	11	20	19	18	17	16	9	8	7	6	5	230
	9	8	7	6	5	14	13	12	11	10	19	18	17	16	15	8	7	6	5	4	210
	24	23	22	21	20	11	10	9	8	7	14	13	12	11	10	17	16	15	14	13	290
	23	22	21	20	19	10	9	8	7	6	13	12	11	10	9	16	15	14	13	12	270
	22	21	20	19	18	9	8	7	6	5	12	11	10	9	8	15	14	13	12	11	250
	21	20	19	18	17	8	7	6	5	4	11	10	9	8	7	14	13	12	11	10	230
	20	19	18	17	16	7	6	5	4	3	10	9	8	7	6	13	12	11	10	9	210
	10	9	8	7	6	21	20	19	18	17	20	19	18	17	16	15	14	13	12	11	290
	9	8	7	6	5	20	19	18	17	16	19	18	17	16	15	14	13	12	11	10	270
	8	7	6	5	4	19	18	17	16	15	18	17	16	15	14	13	12	11	10	9	250
	7	6	5	4	3	18	17	16	15	14	17	16	15	14	13	12	11	10	9	8	230
	6	5	4	3	2	17	16	15	14	13	16	15	14	13	12	11	10	9	8	7	210
	19	18	17	16	15	16	15	14	13	12	9	8	7	6	5	22	21	20	19	18	290
	18	17	16	15	14	15	14	13	12	11	8	7	6	5	4	21	20	19	18	17	270
	17	16	15	14	13	14	13	12	11	10	7	6	5	4	3	20	19	18	17	16	250
	16	15	14	13	12	13	12	11	10	9	6	5	4	3	2	19	18	17	16	15	230
	15	14	13	12	11	12	11	10	9	8	5	4	3	2	1	18	17	16	15	14	210
	290	270	250	230	210	290	270	250	230	210	290	270	250	230	210	290	270	250	230	210	
		250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250

	1	2	3	4	
4x4	34	34	34	34	
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34	34	

Observe how the expansion is done by comparing the numbers in yellow cells in the square with the lower right-hand corner in the yellow block squares of the expanded table.

A matching in this expanded square is equal to the sum of matchings in each block square. A matching here within a block-square always equals **5** times its center number. The center numbers can be put into a size-4 table as shown here just by adding **4** to each number in the regular square of size-4 and then summing geonomically.

	1	2	3	4	
W	34	34	34	34	
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34	34	

\+

4	4	4	4	
4	4	4	4	
4	4	4	4	
4	4	4	4	

=

	1	2	3	4	
W	50	50	50	50	
1	9	14	19	8	50
2	20	7	10	13	50
3	6	17	16	11	50
4	15	12	5	18	50
	50	50	50	50	
	50	50	50	50	

Then that table is multiplied by **5**. It is now seen that the sum of all the block square matchings equals all the overa matchings in the expanded square! So the matching property is retained in this expansion too.

Like the other expansions for regula squares, this matchmaker expansion property holds for all perfect squares too.

Note, however, the **invariance unde row and column permutation** property now holds only at the block square level.

	1	2	3	4	
W	50	50	50	50	
1	9	14	19	8	50
2	20	7	10	13	50
3	6	17	16	11	50
4	15	12	5	18	50
	50	50	50	50	
	50	50	50	50	

x

5	5	5	5	
5	5	5	5	
5	5	5	5	
5	5	5	5	

=

	1	2	3	4	
W	250	250	250	250	
1	45	70	95	40	250
2	100	35	50	65	250
3	30	85	80	55	250
4	75	60	25	90	250
	250	250	250	250	
	250	250	250	250	

This ends the discussion on table-expansions.

* * *

Program 7

Part II Notes

1. There are really a total of **6** expansion methods for squares:
    1) The **Balloon** method for expanding any class of square except Class-2,
    2) The **T-Ball** method for generating Class-6 squares,
    3) The **ATE** method for generating Class-4 and Class-5 squares,
    4) The **Bootstrap** method for generating Class-6 squares,
    5) The **SPD** method for generating Class-2, Class-3 and Class-4 squares,
    6) The **Matchmaker-pattern** method for expanding matchmaker squares using Class-1 squares,

2. The modulus and integer functions taken in the base **n** for Class-4 squares will fail to yield loom tables that are geonomic. So the expansion tables for Class-4 squares actually need to be produced directly by the **ATE** method. They can then be subsequently derived as **E(a)** and **E(b)** in either base $\mathbf{a^2}$ or $\mathbf{b^2}$ from the primary squares.

3. Class-5 squares of a size **n = ab** that are derived by the **ATE** method are the only Class--5 squares that possess the double and triple zipper-like patterns. These patterns are only found in loom tables that are subsequently derived in the base **ab** from either square generated in the base $\mathbf{a^2}$ or $\mathbf{b^2}$.

4. Class-6 squares which are of a size $\mathbf{n = b^2}$ may be generated by 1) the **T-Ball** method, 2) the **SPD** method and 3) the **Bootstrap method**, after which they can be treated as Class-1 squares in generating their dual from the resulting primal square. Even so, Class-6 squares cannot be derived from matchmaker squares as all Class-1 squares can. Further, Class-6 squares derived by any of these methods will lack any continuous modularity. **Only the 2D projection of a 4D quadracube that was initially obtained from the collapse of a 5D quintacube via a depth-sum table along the quintacube's B or D axis will possess this continuous modularity property.**

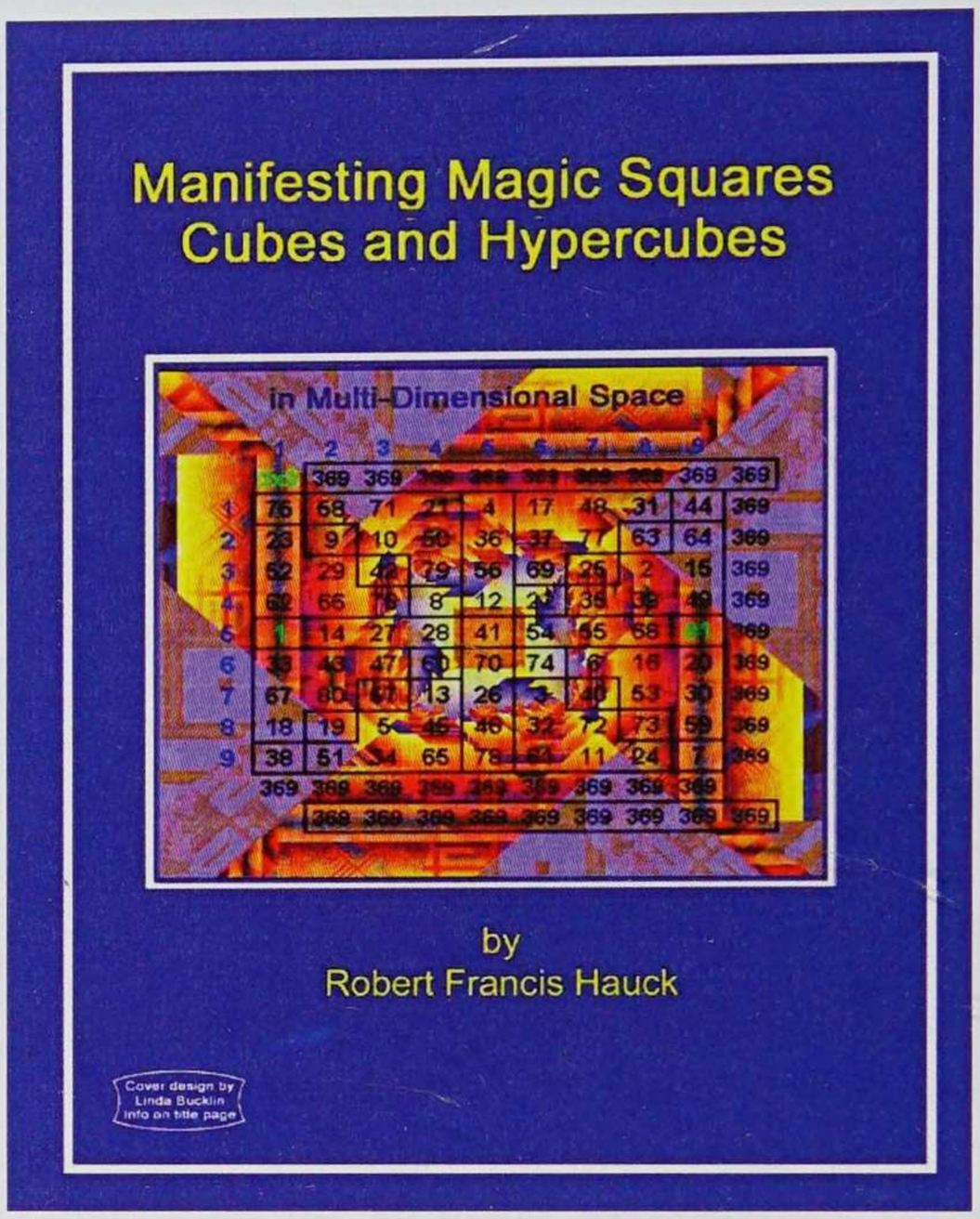
5. Class-4 size-8 squares can be converted from their pairwise row-symmetric version to their centrally-symmetric version and back again by one of two methods, one the transpose of the other. That demonstration is shown in Program-8 (p.8-14).

    a) The size-16 square and the size-32 square derived from the doubling of the size-8 and size-16 squares, using the **SPD** method shown earlier, were found to also have both pairwise symmetries (p.7-39 for the size-16 square). The size-64 derived from the doubling of the size-32 square was also found to contain both versions of pairwise symmetry.

    b) The **ATE** method was used to generate both pairwise versions for the size-24 and size-40 squares from **E(3) a**nd **E(5)** with **E(8)**, respectively for each specific pairwise symmetry.. Then the **SPD** method was applied to the size-24 square to get the size-48 square with both types of pairwise symmetry. So every Class-4 square whose size is **8** or a multiple of **8** can be made to exhibit both pairwise symmetries.

Program 7

We have come to the end of Program 7. I hope that this program has been as awe-inspiring for you as it was for me in its discovery. Here is one book which summarizes in a brief manner, the expansion patterns shown in Program 7 for all sizes and classes of squares and diamonds.



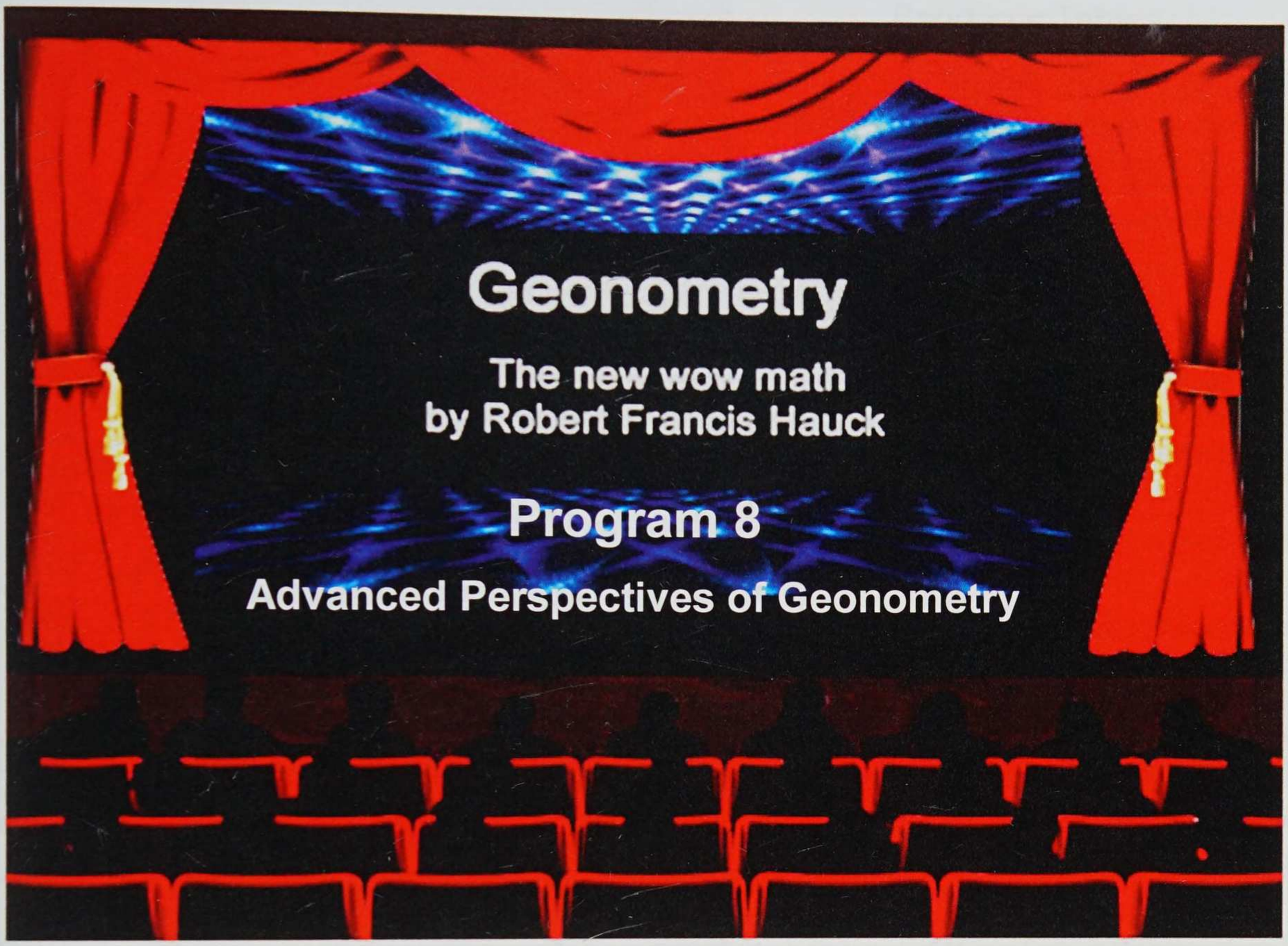
Manifesting Magic Squares, Cubes and Hypercubes

ISBN: 978-151-439841-8

Shows how to construct geonomic squares from scratch and how to spread them across redistribution patterns for expansions to larger composite sizes or higher dimensions.

Printed in color. First Edition (100 pages)

In the next program we'll look at some advanced properties of Geonometry. From these additional fundamental properties yet to be described, we will get an even broader picture of the spatial fabric and its pervasiveness. Come back again for yet some more mind-blowing "Wows!"



This program demonstrates some remarkable properties in Geonometry. Here we will get an even broader picture of the spatial fabric and its pervasiveness. These membrane patterns detected in Geonometry through complementary loom tables may well extend across vast distances on a cosmological level too. This program looks at multi-dimensional space from the point of view of expansion properties and not-so-obvious hidden patterns.

Program 8

The Corner Triangle Property

Here is an example of the corner triangle property. This property is such that every one of the triangles, which include one corner and its related main diagonal and all number cells in-between, sum equally. It is just coincidental that the triangular sum here is exactly 4 times the size-7 square's characteristic number. The multiple is always **½(n+1)**.

Size-7 perfect square

	1	2	3	4	5	6	7	
7x7	175	175	175	175	175	175	175	
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	

Quadral sums

100	100	100	100
100	100	100	100
100	100	100	100

Quarter triangle sums

700	700
700	700

Every square that's perfect has this property. Class-2 squares are not perfect and so do not have this property.

Since each quarter triangle shares one of the main diagonals which sum equally to **175**, eliminating the main diagonal from each quarter square will yield four independent triangles of size **n-1** which sum equally to **525 = 700 - 175 = 3 x 175 = 7 x 25 = ½(n-1)C_n.**

Now, what is so amazing is that each of these triangular summations is continuously equal-summing across the square as seen by dragging it across four copies of the square having a common corner.

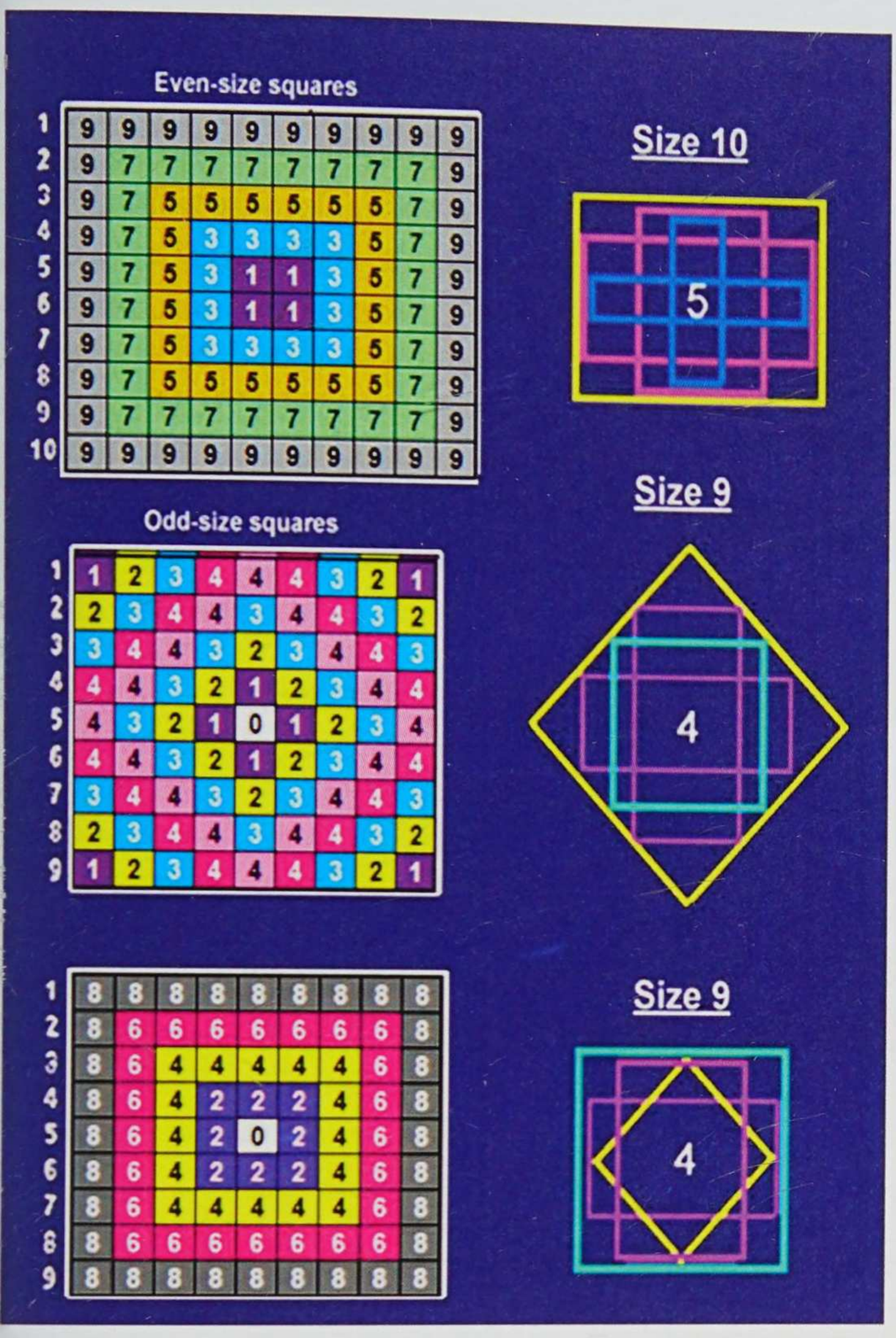
These triangular summations can be categorized as 4th-degree polynomial expressions of size **n-1** as:

(8.1) $\underline{\Delta}_n = ½(n-1)\, C_n = ¼\, n(n^3 - n^2 + n^1 - n^0)$

where the expression in parenthesis is a 3rd-degree monomial with alternating signs of unity which in turn can be factored as:

(8.2) $(n^3 - n^2 + n^1 - n^0) = (n-1)(n^2+1)$.

Program 8



Concentricity Patterns in all Pangenic Tables

All pangenic squares have the following *concentricity* patterns:

Even-size squares as depicted at the top have frames that sum consecutively to the series:

1, 3, 5, 7 ... (n-1)

The numbers in concentric frames indicate the number of quadrals in that respective frame. The diagram to its right shows the source of the **5** quadrals in the 3rd frame.

Similar patterns are found in odd-size squares too:

Two odd-size square representations are shown here at left. The diagrams at their right depict the concentric frame with **4** quadrals and show where these quadrals are located in that frame.

Now observe the patterns of the quantity of quadrals in concentric frames emanating from the center: In odd-size squares, the pattern is both:

0, 1, 2, 3 ... (n-1)/2, **(n-1)/2 ... 3, 2, 1, 0**

and **0, 2, 4, 6 ... (n-1)**.

The first series is a perfect harmonic waveform. Clearly, the latter series for odd-size squares can be converted to an intelligent pattern by adding **1** to each number to get: **1, 3, 5, 7 ... n**.

Now each of these series is either an *intelligent pattern* or a perfect *harmonic waveform as* defined at the beginning of this program series. Since all tables in Geonometry of any dimension are pangenic, all squares of any class possess these nested concentric frames whose total numbers in each frame are multiples of its kernel number which sum sequentially along either an intelligent-pattern or a harmonic waveform.

Since octals in cubes and hexadectals in quadracubes are composed of symmetrically located quadrals, these patterns extend to higher dimensions too. In odd-size tables of higher dimensions, the central embedded square is not involved in octal or hexadectal summations, so it doesn't come into play there.

In quadracubes, not only does the intelligent pattern appear in the quadrals among 4 symmetrically located embedded block squares, but it appears at the block-square level too, all simultaneously!

Here we see that intelligent patterns and harmonic waveforms arise naturally in Geonometry. So now, having observed all the amazing equality patterns in Class-1 and Class-4 tables plus having seen the roles that Class-1 and Class-4 tables played in interpreting the structure of the atom, and to find yet even more, can you deny the underlying intelligent patterning inherent in the fabric of space?

Program 8

The Zero-differential Property

	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

Here is shown column-difference and row-difference tables for the size-7 square. Observe that the characteristic number of these difference-tables is zero. Further, the size-7 complementary characteristic tile patterns both sum to zero everywhere in the difference table. This property holds for all ultra-perfect squares.

Column differences

	0	0	0	0	0	0	0
-11	-11	-18	38	-18	-11	31	0
31	-11	-18	38	-11	-18	-11	0
38	-18	-11	-18	38	-18	-11	0
-11	31	-11	-11	31	-11	-18	0
-18	38	-18	-11	-18	38	-11	0
-18	-11	38	-18	-11	31	-11	0
-11	-18	38	-18	-11	-11	31	0
0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0

Pattern A sum of col. differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of col. differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Row differences

	0	0	0	0	0	0	0
-37	5	5	5	5	12	5	0
5	12	5	12	-44	5	5	0
5	-44	5	5	12	5	12	0
12	5	12	5	5	-44	5	0
5	5	-44	12	5	12	5	0
5	12	5	5	5	5	-37	0
5	5	12	-44	12	5	5	0
0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0

Pattern A sum of row differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of row differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

This property also holds for the characteristic block-square modularity common to all class-3 squares too.

Program 8

Here is shown two more difference tables for the size-7 square. They now are differences along the two different diagonal directions. Observe that the characteristic number of these difference-tables is zero too. Further, the size-7 characteristic tile patterns also both sum to zero in these difference tables everywhere. This property holds for all ultra-perfect squares.

Minor diagonal differences

	0	0	0	0	0	0	0
-26	-26	16	23	-33	23	23	0
16	-26	23	23	-26	-33	23	0
23	-33	-26	16	23	-26	23	0
23	23	-26	23	16	-26	-33	0
16	23	-33	-26	23	23	-26	0
-26	23	23	-33	23	16	-26	0
-26	16	23	-26	-26	23	16	0
0	0	0	0	0	0	0	
	0	0	0	0	0	0	0

Pattern A sum of minor diag. differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of minor diag. differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

This property might seem obvious for sums taken in the same direction as were the differences because these differences wrap around in that direction. But to also hold for both tile patterns and for all the other directional differences all simultaneously in all instances is absolutely astounding!

Major diagonal differences

	0	0	0	0	0	0	0
-6	-6	-13	43	-6	-6	-6	0
43	-6	-6	-6	-6	-13	-6	0
-6	-13	-6	-6	43	-6	-6	0
-6	43	-6	-6	-13	-6	-6	0
-13	-6	-6	-6	-6	43	-6	0
-6	-6	43	-13	-6	-6	-6	0
-6	-6	-6	-6	-6	-6	36	0
0	0	0	0	0	0	0	
	0	0	0	0	0	0	0

Pattern A sum of major diag. differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of major diag. differences

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

In differential calculus, a *zero-differential* in two directions (2 variables) indicates a plane of balance. Here it means that the numeric table is perfectly balanced with respect to all 4 directions simultaneously! When, in addition, both of its inherent tile patterns sum to zero throughout all the difference tables, each tile is balanced throughout the numeric table in all 4 directions at every point.

If a square composed of multiple same-size ultra-perfect squares were to be laid atop a placid pool of water and a stone were pitched onto it anywhere within and clear of its edges, no ripples would emanate from the square.

In classical mathematics this property is called **isotropism**. In this new numeric geometry, the tables are said to be ***differentially isotropic***, i.e. their difference sums are invariant with respect to all the directions regardless of the direction in which the adjacent number-pair differences are taken.

Among all the possible square tables of sequential numbers, only ultra-perfect geonomic squares exhibit this property. For Classes 1 & 4 squares it's their tile patterns which sums to **0** anywhere and everywhere. For Classes 3, 5 & 6 squares, it's their block-square modularity and/or block-tile patterns which sum to **0** anywhere. And that's for every class of square except those of Class-2.

Program 8

The Zero-Differential property on the Size-12 Class-4 Square

	0	0	0	0	0	0	0	0	0	0	0	0
-50	-44	99	-10	-40	-45	94	-8	-45	-46	104	-9	0
121	-23	-27	-67	113	-27	-23	-59	117	-31	-31	-63	0
-98	4	45	50	-100	9	46	40	-99	14	44	45	0
28	58	-117	32	26	63	-116	22	27	68	-118	27	0
-41	-41	99	-13	-49	-45	103	-5	-45	-49	95	-9	0
112	-26	-27	-64	122	-27	-32	-62	117	-28	-22	-63	0
-104	10	45	44	-94	9	40	46	-99	8	50	45	0
31	67	-117	23	23	63	-113	31	27	59	-121	27	0
-44	-50	99	-4	-46	-45	100	-14	-45	-40	98	-9	0
118	-32	-27	-58	116	-27	-26	-68	117	-22	-28	-63	0
-95	13	45	41	-103	9	49	49	-99	5	41	45	0
22	64	-117	26	32	63	-122	28	27	62	-112	27	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0
-106	65	86	-40	-97	56	74	-43	-94	68	83	-52	0
131	-88	-61	11	128	-85	-49	20	119	-97	-52	23	0
-81	45	99	-63	-81	45	99	-63	-81	45	99	-63	0
49	-20	-119	97	52	-23	-131	88	61	-11	-128	85	0
-92	61	76	-50	-101	70	88	-47	-104	58	79	-38	0
126	-90	-54	18	126	-90	-54	18	126	-90	-54	18	0
-88	47	104	-58	-79	38	92	-61	-76	50	101	-70	0
59	-16	-133	83	56	-13	-121	92	47	-25	-124	95	0
-99	63	81	-45	-99	63	81	-45	-99	63	81	-45	0
121	-92	-47	25	124	-95	-59	16	133	-83	-56	13	0
-74	43	94	-68	-83	52	106	-65	-86	40	97	-56	0
54	-18	-126	90	54	-18	-126	90	54	-18	-126	90	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0
15	42	59	-107	16	29	51	-102	23	37	52	-115	0
33	-84	-16	61	28	-76	-3	60	20	-83	-8	68	0
-53	103	-18	-31	-55	108	-17	-41	-54	113	-19	-36	0
8	-61	-20	84	3	-68	-28	83	16	-60	-33	76	0
20	35	49	-114	21	43	56	-109	13	30	57	-101	0
22	-80	-9	62	32	-81	-14	64	27	-82	-4	63	0
-57	114	-13	-35	-56	101	-21	-30	-49	109	-20	-43	0
15	-66	-34	79	10	-58	-21	78	2	-65	-26	86	0
19	31	54	-103	17	36	55	-113	18	41	53	-108	0
26	-79	-2	66	21	-86	-10	65	34	-78	-15	58	0
-52	107	-23	-42	-51	115	-16	-37	-59	102	-15	-29	0
4	-62	-27	80	14	-63	-32	82	9	-64	-22	81	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0
56	-109	13	30	57	-101	20	35	49	-114	21	43	0
-10	65	34	-78	-15	58	26	-79	-2	66	21	-86	0
-17	-41	-54	113	-19	-36	-53	103	-18	-31	-55	108	0
-21	78	2	-65	-26	86	15	-66	-34	79	10	-58	0
51	-102	23	37	52	-115	15	42	59	-107	16	29	0
-14	64	27	-82	-4	63	22	-80	-9	62	32	-81	0
-16	-37	-59	102	-15	-29	-52	107	-23	-42	-51	115	0
-28	83	16	-60	-33	76	8	-61	-20	84	3	-68	0
55	-113	18	41	53	-108	19	31	54	-103	17	36	0
-3	60	20	-83	-8	68	33	-84	-16	61	28	-76	0
-21	-30	-49	109	-20	-43	-57	114	-13	-35	-56	101	0
-32	82	9	-64	-22	81	4	-62	-27	80	14	-63	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

Program 8

Class-2 squares lack the zero-differential property

This is typical of near-perfect Class-2 squares and shows that they lack the zero-differential property that all perfect squares share. They also don't have tile patterns or any modularity property that would sum to **0** either.

	1	2	3	4	5	6	
W(6)	111	111	111	111	111	111	
1	6	33	12	25	4	31	111
2	17	16	21	15	22	19	110 (-1)
3	30	3	36	1	34	7	111
4	32	9	26	11	28	5	111
5	18	23	14	24	13	20	112 (+1)
6	8	27	2	35	10	29	111
	111	111	111	111	111	111	
	111	111	111	111	111	111	111

	1	2	3	4	5	6	
1	6	33	12	25	4	31	6
2	17	16	21	15	22	19	17
3	30	3	36	1	34	7	30
4	32	9	26	11	28	5	32
5	18	23	14	24	13	20	18
6	8	27	2	35	10	29	8
	6	33	12	25	4	31	6

	0	0	0	0	0	0	
	-27	21	-13	21	-27	25	0
	1	-5	6	-7	3	2	0
	27	-33	35	-33	27	-23	0
	23	-17	15	-17	23	-27	0
	-5	9	-10	11	-7	2	0
	-19	25	-33	25	-19	21	0
	0	0	0	0	0	0	
		0	0	0	0	0	0

	0	0	0	0	0	0	
	-11	17	-9	10	-18	12	1
	-13	13	-15	14	-12	12	-1
	-2	-6	10	-10	6	2	0
	14	-14	12	-13	15	-15	-1
	10	-4	12	-11	3	-9	1
	2	-6	-10	10	6	-2	0
	0	0	0	0	0	0	
		0	0	0	0	0	0

	0	0	0	0	0	0	
	-10	12	-3	3	-15	14	1
	14	-20	20	-19	15	-11	-1
	21	-23	25	-27	29	-25	0
	9	-5	2	-2	8	-13	-1
	-9	21	-21	14	-16	12	1
	-25	15	-23	31	-21	23	0
	0	0	0	0	0	0	
		0	0	0	0	0	0

	0	0	0	0	0	0	
	-16	4	-4	11	-9	13	-1
	14	-18	21	-21	15	-10	1
	29	-27	25	-23	21	-25	0
	9	-3	3	-4	8	-12	1
	-15	13	-22	22	-10	11	-1
	-21	31	-23	15	-25	23	0
	0	0	0	0	0	0	
		0	0	0	0	0	0

Program 8

The Sub-additive Property of all perfect Squares

The sub-additive property of perfect squares is that, given a square **W** of size **n**, a square **R** of the same size whose numbers are the **(n-1)x(n-1)** continuous summations within **W**, with wrapping imposed in the summations, will reduce back to **W** upon collapse and normalization. Here it is demonstrated for the size 7 square Class-1 square. The extreme values are depicted in **red**.

	1	2	3	4	5	6	7	
7x7	175	175	175	175	175	175	175	175
1	36	46	7	10	20	23	33	175
2	13	16	26	29	39	49	3	175
3	32	42	45	6	9	19	22	175
4	2	12	15	25	35	38	48	175
5	28	31	41	44	5	8	18	175
6	47	1	11	21	24	34	37	175
7	17	27	30	40	43	4	14	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

R(6x6) Minimum = 876

889	892	902	905	915	918	879
908	911	921	882	885	895	898
878	888	891	901	904	914	924
897	907	917	920	881	884	894
923	877	887	890	900	910	913
893	903	906	916	919	880	883
912	922	876	886	896	899	909

R-875

14	17	27	30	40	43	4
33	36	46	7	10	20	23
3	13	16	26	29	39	49
22	32	42	45	6	9	19
48	2	12	15	25	35	38
18	28	31	41	44	5	8
37	47	1	11	21	24	34

	1	2	3	4	5	6	7	
7x7	175	175	175	175	175	175	175	175
1	36	46	7	10	20	23	33	175
2	13	16	26	29	39	49	3	175
3	32	42	45	6	9	19	22	175
4	2	12	15	25	35	38	48	175
5	28	31	41	44	5	8	18	175
6	47	1	11	21	24	34	37	175
7	17	27	30	40	43	4	14	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

Next, this property is demonstrated for the size-11 square from Class-1.

11x11	671	671	671	671	671	671	671	671	671	671	671
	21	37	64	80	107	2	29	45	72	99	115	671
	24	51	67	94	121	16	43	59	86	102	8	671
	38	65	81	108	3	30	46	73	89	116	22	671
	52	68	95	111	17	44	60	87	103	9	25	671
	66	82	109	4	31	47	74	90	117	12	39	671
	69	96	112	18	34	61	88	104	10	26	53	671
	83	110	5	32	48	75	91	118	13	40	56	671
	97	113	19	35	62	78	105	11	27	54	70	671
	100	6	33	49	76	92	119	14	41	57	84	671
	114	20	36	63	79	106	1	28	55	71	98	671
	7	23	50	77	93	120	15	42	58	85	101	671
	671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671	671

R(11x11) Minimum = 6040

6140	6046	6062	6089	6116	6132	6159	6054	6081	6097	6124
6154	6060	6076	6103	6119	6146	6041	6068	6084	6111	6138
6047	6063	6090	6106	6133	6160	6055	6082	6098	6125	6141
6061	6077	6104	6120	6147	6042	6069	6085	6112	6128	6155
6064	6091	6107	6134	6150	6056	6083	6099	6126	6142	6048
6078	6105	6121	6148	6043	6070	6086	6113	6129	6156	6051
6092	6108	6135	6151	6057	6073	6100	6127	6143	6049	6065
6095	6122	6149	6044	6071	6087	6114	6130	6157	6052	6079
6109	6136	6152	6058	6074	6101	6117	6144	6050	6066	6093
6123	6139	6045	6072	6088	6115	6131	6158	6053	6080	6096
6137	6153	6059	6075	6102	6118	6145	6040	6067	6094	6110

R-6039

101	7	23	50	77	93	120	15	42	58	85
115	21	37	64	80	107	2	29	45	72	99
8	24	51	67	94	121	16	43	59	86	102
22	38	65	81	108	3	30	46	73	89	116
25	52	68	95	111	17	44	60	87	103	9
39	66	82	109	4	31	47	74	90	117	12
53	69	96	112	18	34	61	88	104	10	26
56	83	110	5	32	48	75	91	118	13	40
70	97	113	19	35	62	78	105	11	27	54
84	100	6	33	49	76	92	119	14	41	57
98	114	20	36	63	79	106	1	28	55	71

11x11	671	671	671	671	671	671	671	671	671	671	671
	21	37	64	80	107	2	29	45	72	99	115	671
	24	51	67	94	121	16	43	59	86	102	8	671
	38	65	81	108	3	30	46	73	89	116	22	671
	52	68	95	111	17	44	60	87	103	9	25	671
	66	82	109	4	31	47	74	90	117	12	39	671
	69	96	112	18	34	61	88	104	10	26	53	671
	83	110	5	32	48	75	91	118	13	40	56	671
	97	113	19	35	62	78	105	11	27	54	70	671
	100	6	33	49	76	92	119	14	41	57	84	671
	114	20	36	63	79	106	1	28	55	71	98	671
	7	23	50	77	93	120	15	42	58	85	101	671
	671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671	671

Program 8

On Size-5 Square

Here is the size-5 Class-1 square. The table below is composed of quadrants of the size-5. This construct makes it easy to calculate those block-sums that wrap across the edges of the square.

20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6

P Sums of 2x2s

43	50	62	49	56
47	59	41	53	60
51	63	45	57	44
55	42	54	61	48
64	46	58	40	52

Q Sums of 3x3s

126	113	120	107	119
105	117	129	111	123
114	121	108	115	127
118	125	112	124	106
122	109	116	128	110

R Sums of 4x4s

201	219	207	200	213
197	215	203	216	209
218	206	199	212	205
214	202	220	208	196
210	198	211	204	217

P−39					
	65	65	65	65	65
4	11	23	10	17	65
8	20	2	14	21	65
12	24	6	18	5	65
16	3	15	22	9	65
25	7	19	1	13	65
65	65	65	65	65	
	65	65	65	65	65

Q−104					
	65	65	65	65	65
22	9	16	3	15	65
1	13	25	7	19	65
10	17	4	11	23	65
14	21	8	20	2	65
18	5	12	24	6	65
65	65	65	65	65	
	65	65	65	65	65

R−195					
	65	65	65	65	65
6	24	12	5	18	65
2	20	8	21	14	65
23	11	4	17	10	65
19	7	25	13	1	65
15	3	16	9	22	65
65	65	65	65	65	
	65	65	65	65	65

Normalized

	65	65	65	65	65
6	18	5	12	24	65
15	22	9	16	3	65
19	1	13	25	7	65
23	10	17	4	11	65
2	14	21	8	20	65
65	65	65	65	65	
	65	65	65	65	65

Normalized

	65	65	65	65	65
6	18	5	12	24	65
15	22	9	16	3	65
19	1	13	25	7	65
23	10	17	4	11	65
2	14	21	8	20	65
65	65	65	65	65	
	65	65	65	65	65

Normalized

	65	65	65	65	65
20	8	21	14	2	65
11	4	17	10	23	65
7	25	13	1	19	65
3	16	9	22	15	65
24	12	5	18	6	65
65	65	65	65	65	
	65	65	65	65	65

Block-square sums of size **2x2**, **3x3** and **4x4** are taken with wrapping in the top table. Upon reduction and normalization which places the pivot number at its center, all such derived **5x5** tables reduce to either the original perfect square or its reflection. This property extended to every sub-size block-square and was found to apply only to the size-5 square.

Program 8

Here is the perfect size-4 square from Class-4. The sub-additive property yields its identical twin.

	1	2	3	4	
4x4	34	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34	34	34

R(8x8) Min = 69

82	79	76	69
72	73	78	83
77	84	71	74
75	70	81	80

R-68

14	11	8	1
4	5	10	15
9	16	3	6
7	2	13	12

	1	2	3	4	
W(4)	34	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34	34	34

The same thing results for the size-9 square from Class-6.

	1	2	3	4	5	6	7	8	9	
9x9	369	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369
	62	66	76	8	12	22	35	39	49	

R(8x8) Minimum = 2584

2590	2621	2634	2617	2648	2661	2644	2594	2607
2627	2658	2641	2654	2604	2587	2600	2631	2614
2647	2606	2592	2593	2633	2619	2620	2660	2646
2598	2635	2612	2625	2662	2639	2652	2608	2585
2632	2645	2649	2659	2591	2595	2605	2618	2622
2664	2584	2597	2610	2611	2624	2637	2638	2651
2603	2616	2626	2630	2643	2653	2657	2589	2599
2613	2650	2663	2640	2596	2609	2586	2623	2636
2642	2601	2602	2588	2628	2629	2615	2655	2656

R-2583

7	38	51	34	65	78	61	11	24
44	75	58	71	21	4	17	48	31
64	23	9	10	50	36	37	77	63
15	52	29	42	79	56	69	25	2
49	62	66	76	8	12	22	35	39
81	1	14	27	28	41	54	55	68
20	33	43	47	60	70	74	6	16
30	67	80	57	13	26	3	40	53
59	18	19	5	45	46	32	72	73

	1	2	3	4	5	6	7	8	9	
9x9	369	369	369	369	369	369	369	369	369	369
	75	58	71	21	4	17	48	31	44	369
	23	9	10	50	36	37	77	63	64	369
	52	29	42	79	56	69	25	2	15	369
	62	66	76	8	12	22	35	39	49	369
	1	14	27	28	41	54	55	68	81	369
	33	43	47	60	70	74	6	16	20	369
	67	80	57	13	26	3	40	53	30	369
	18	19	5	45	46	32	72	73	59	369
	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

Program 8

The Checkerboard Difference Property for Odd-size Squares

Here is the Class-1 size-7 square **W(7)** again. To its right is a checkerboard pattern **Q** of **±1**'s.

	1	2	3	4	5	6	7	
W(7)	175	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

Q

1	-1	1	-1	1	-1	1
-1	1	-1	1	-1	1	-1
1	-1	1	-1	1	-1	1
-1	1	-1	1	-1	1	-1
1	-1	1	-1	1	-1	1
-1	1	-1	1	-1	1	-1
1	-1	1	-1	1	-1	1

WQ

7	-18	29	-47	9	-27	38
-44	13	-24	42	-4	15	-33
39	-1	19	-30	48	-10	28
-34	45	-14	25	-36	5	-16
22	-40	2	-20	31	-49	11
-17	35	-46	8	-26	37	-6
12	-23	41	-3	21	-32	43

25

The sum of all the numbers in the checkerboard table is **+1**.

XQ

7	-4	1	-5	2	-6	3
-2	6	-3	7	-4	1	-5
4	-1	5	-2	6	-3	7
-6	3	-7	4	-1	5	-2
1	-5	2	-6	3	-7	4
-3	7	-4	1	-5	2	-6
5	-2	6	-3	7	-4	1

4

At far right is the result of the inner product of **W** and **Q**:

$$(8.3) \qquad WQ = |\, w_{ij} \times q_{ij} \,| = \begin{cases} +w_{ij} \text{ for } (i+j) \text{ even} \\ -w_{ij} \text{ for } (i+j) \text{ odd} \end{cases}$$

Now, the sum of all the numbers in **WQ** is the size-7 square's pivot number **p = 25**. The same holds for the dual **U(7)**.

And just as for the sum of numbers in **WQ**, the sum of numbers in **XQ** yields its pivot number **p = 4.** The same holds for the loom-table **Y** of **W, YQ** (not shown).

A(WQ)

-175	175	-175	175	-175	19	77
175	-175	175	-175	175	-53	-97
-175	175	-175	175	-175	73	131
139	-117	81	-157	121	-31	-53
-77	-3	-15	19	-51	21	39
1	31	-35	53	27	11	29
129	-93	169	-133	111	3	-19

175

In the next table are the results of the continuous sums of the tile pattern A on **WQ**. The sum of all the numbers in that table is the size-7 square's characteristic number $C_7 = 175$.

The same holds for the dual **U(7)** on **A(UQ)** (not shown).

In the next table, tile pattern A is summed continuously on loom-table **XQ**. The sum of all the numbers in that table is the common size-7 loom-table characteristic number $L_7 = 28$.

A(XQ)

-28	28	-28	28	-28	12	14
28	-28	28	-28	28	-18	-6
-28	28	-28	28	-28	10	12
20	-26	18	-24	16	-10	-4
0	4	-8	-2	-16	0	4
8	10	0	4	-8	4	8
24	-16	22	-14	20	-4	2

28

Although not shown here, the same properties hold for tile pattern B on **WQ, UQ, XQ** and **YQ**.

It is next shown that these checkerboard properties hold for every geonomic square of odd size, whether prime or composite.

Program 8

Size-9 Class-6 Square

Here is the size-9 Class-6 square. It was stated earlier in Program 2 that Class-6 squares could be treated as Class-1 squares in the formulas.

	1	2	3	4	5	6	7	8	9	
W(9)	369	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369	369

WQ

75	-58	71	-21	4	-17	48	-31	44
-23	9	-10	50	-36	37	-77	63	-64
52	-29	42	-79	56	-69	25	-2	15
-62	66	-76	8	-12	22	-35	39	-49
1	-14	27	-28	41	-54	55	-68	81
-33	43	-47	60	-70	74	-6	16	-20
67	-80	57	-13	26	-3	40	-53	30
-18	19	-5	45	-46	32	-72	73	-59
38	-51	34	-65	78	-61	11	-24	7

41

Its only tiling pattern was its **3x** modularity shown at lower right as tile pattern B. Although the pattern A shown at bottom left is not a functional tile pattern for size-9 squares, nonetheless it still acts as a complementary tile to B in depicting the checkerboard property here.

A(WQ)

-178	165	-164	70	-219	218	-286	207	76
102	-209	196	-318	263	-250	210	-173	-48
-313	261	-257	205	-153	149	-97	19	227
200	-160	147	-92	214	-201	308	-198	-73
-124	192	-191	340	-246	245	-232	170	39
177	-126	228	-177	126	-66	15	48	-110
73	-50	108	-19	-4	-54	-35	86	-16
117	-28	86	-63	-26	-32	9	98	-4
231	-180	120	-69	18	-120	69	28	-130

369

B(WQ)

129	-25	77	-75	-29	-23	21	129	21
-31	-19	-57	-23	-89	3	-85	-31	-85
7	-83	-21	-115	29	-87	61	7	61
-95	39	-97	41	15	43	13	-95	13
21	5	53	33	103	1	75	21	75
3	79	7	105	-25	101	-51	3	-51
61	-59	111	-7	5	-57	-47	61	-47
105	-31	95	-51	-23	-41	-3	105	-3
85	-41	105	-31	-13	-51	-23	85	-23

369

Again note that the sum of numbers in **WQ** is its pivot number, here **41,** and the sum of numbers in **A(WQ)** and **B(WQ)** is the size-9 squares' common characteristic number, **369**.

The next size Class-6 square is size-25 which is too large to demonstrate its checkerboard properties here.

The checkerboard difference property, when applied to even-size squares, yields a **0** for overall summations of **WQ** and **XQ**. The pattern in the tile-summation tables **A(WQ)** and **B(WQ)** yields overall totals of **0** too.

Program 8

The Cubic Conversion Property

Unknown to most engineers and even math teachers worldwide, there is a little-known theorem published circa 1950 by the late George Pólya, Professor Emeritus of Mathematics at Stanford University which states that:

The sum of the cubes of consecutive num-bers from 1 thru n equals the sum of the numbers 1 thru n, quantity squared.

$$(8.4) \qquad \sum_{j=1}^{n} j^{3} = \{ \sum_{j=1}^{n} j \}^{2}$$

In Geonometry that translates to: **The cubing of all the numbers in a loom table of Class-1 squares yields a table whose characteristic number will be the square of its former self**!

At top right is the modulus loom table of the size-5 square. Its cubic conversion is shown below it.

Since all loom tables of Class-1 squares contain the numbers from **1** thru **n** exactly once in all 4 directions, Polya's theorem applies to all 4 directional sums simultaneously.

Further, since all Class-1 squares are ultra-perfect, the tiles in both complementary tiling patterns on the cubic conversions of both complementary loom-tables sum to Z's characteristic number too!

X(5)	15	15	15	15	15
	5	3	1	4	2	15
	1	4	2	5	3	15
	2	5	3	1	4	15
	3	1	4	2	5	15
	4	2	5	3	1	15
	15	15	15	15	15	
	15	15	15	15	15	15

$$z_{ij} = x_{ij}^{3}$$

Z(5)	225	225	225	225	225
	125	27	1	64	8	225
	1	64	8	125	27	225
	8	125	27	1	64	225
	27	1	64	8	125	225
	64	8	125	27	1	225
	225	225	225	225	225	
	225	225	225	225	225	225

$$225 = 15 \times 15$$

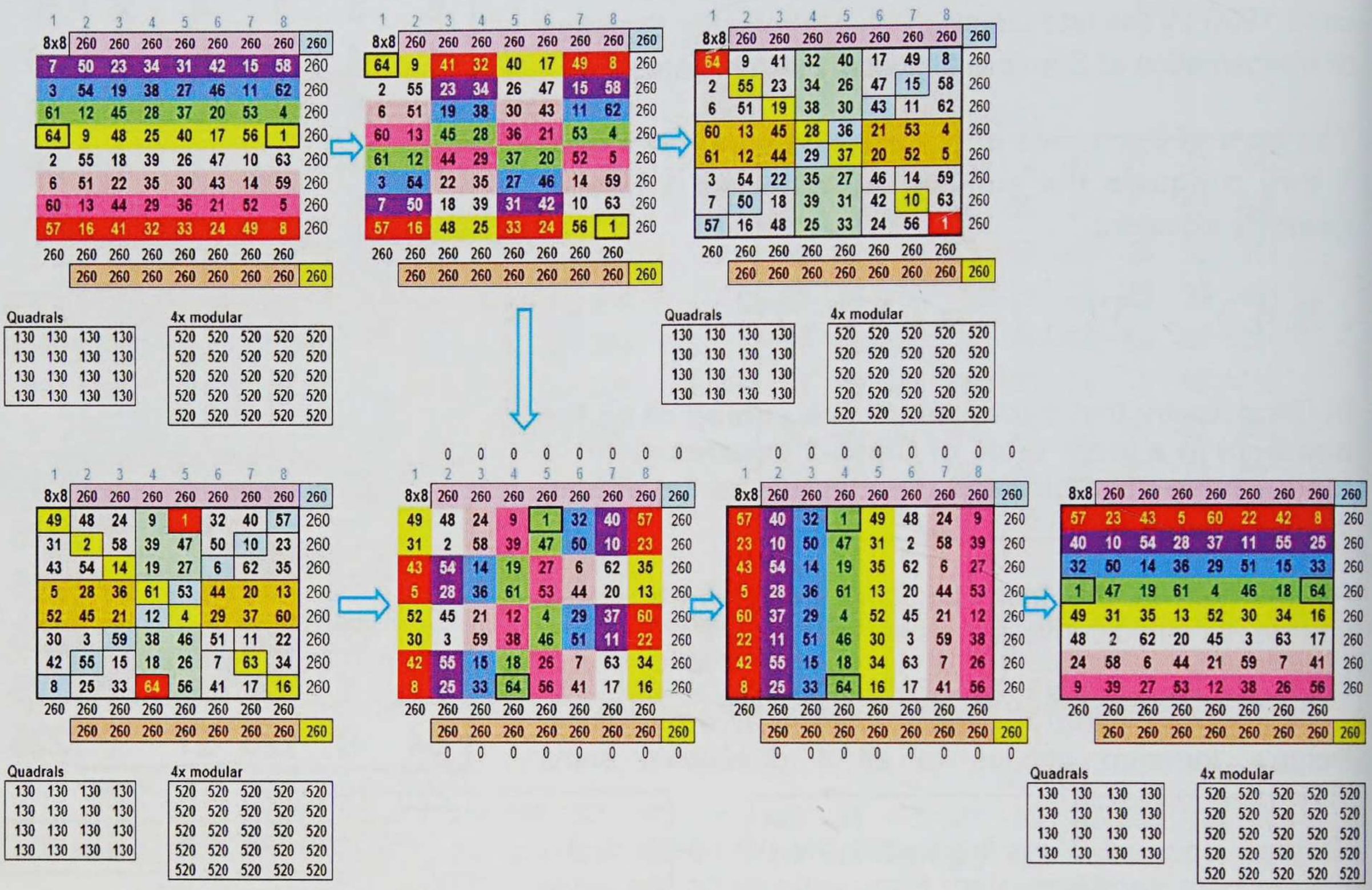
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225

225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225

There is future discovery potential here applying functions to the serial number sequences in loom-tables.

Program 8

Conversion of size-8 squares between pairwise row and central symmetries



Here are two different versions of the perfect size-8 square. The upper one is pairwise row symmetric and has been converted to one of pairwise central symmetry. The lower one is pairwise centrally symmetric and has been converted to one of pairwise row symmetry.

The upper conversion starts with a row pattern which is then converted to a specific spread pattern of two columns at a time.

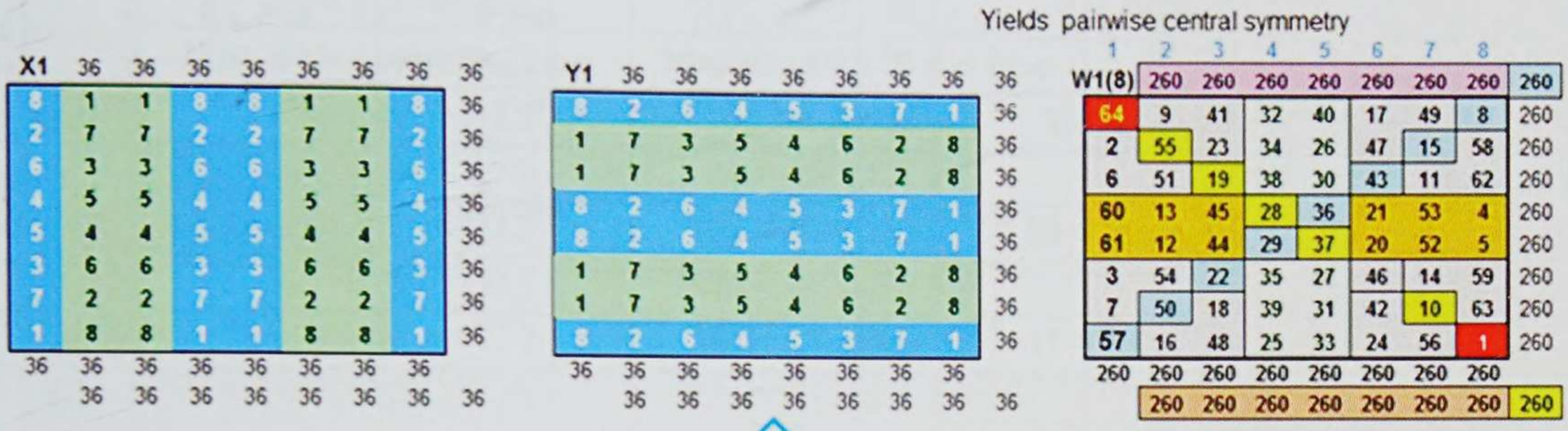
The lower one starts with the same spread pattern but transposed and converts it to a column pattern. Then the entire table is transposed back to a row pattern.

Both methods can be reversed to recover the original version. One or the other conversion pattern will work on any perfect size-8 square.

It's that easy given this specific spread pattern. It is unique to the size-8 square.

Program 8

Here is why the pairwise symmetry of the size-8 square can be traded between row and central symmetry: All the rows in **Y** and all the columns in **X1** and **X2** loom-tables contain just the numbers **1** thru **n**. This usually only occurs elsewhere for Class-1 squares.



Here, each loom table contains this number pattern only in one direction, but between them both directions are possessed. One just has to duplicate the columns differently between the **X1** and **X2** loom-tables maintaining geonomicity to go from one pairwise symmetry to the other. This was found to only work with the size-8 versions with pairwise symmetry.

This simple method does not work for other Class-4 squares that are multiples of **8** even if they have pairwise symmetry to begin with.

Program 8

Summary of Properties of Squares by Class						
Summary of Squares' Properties	**Odd**				**Even**	
	Class 1	**Class 3**	**Class 5**	**Class 6**	**Class 2**	**Class 4**
Size factors	**n** is an odd prime number	n = 3b odd b > 3	n = ab both odd a > b >3	$n = b^2$ odd prime b ≥ 3	n = 2b b is odd b ≥ 3	n = 4b b is any positive integer
Perfection Level Program 1	**Ultra-perfect**	**Perfect**	**Perfect**	**Perfect**	Near-perfect	**Ultra-Perfect**
Pangenicity Program 1	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
Characteristic circles Program 1	No	No	No	No	No	**Yes**
Continuous Modularity Program 1	None	Always **3x** modular	None	Always **bx** modular	None	Always **2bx** modular
Characteristic tiling patterns Programs 2 & 3	**Yes**	**Yes** but at **3x** block-square level	**Yes** but at both **ax** & **bx** block-square level	**Yes** but at the **bx** block-square level	**Yes** omitting the central double-row or double-column	**Yes** with the ◊ & X patterns
Anchor-dot Patterns Program 2	**Yes**	**Yes**	**Yes**	**Yes**	No	**Yes**
Corner Triangle property Program 8	**Yes**	**Yes**	**Yes**	**Yes**	No	**Yes**
Sub-additive property Program 8	**Yes**	**Yes**	**Yes**	**Yes**	No	Yes
Concentricity property Program 8	**Yes**	**Yes**	**Yes**	**Yes**	Yes	Yes
Zero-differential property Program 8	**Yes**	**Yes**	**Yes**	**Yes**	No	**Yes**
Checkerboard difference property Program 8	**Yes**	**Yes**	**Yes**	**Yes**	No	No
Cubic–conversion property Program 8	**Yes**	No	No	No	No	No

Program 8

Table of 2D Characteristic numbers

Size	Characteristic number	Quadral Sum	Pivot number	Larger Size squares		
3	15 = 3 (3^2 +1) / 2	20 = 4/3 x 15	5 = 15/3	31	59	87
4	34 = 4 (4^2 +1) / 2	34 = 4/4 x 34		32+	60+	88+
5	65 = 5 (5^2 +1) / 2	52 = 4/5 x 65	13 = 65/5	33	61	89
6	111 = 6 (6^2 +1) / 2	74 = 4/6 x 111		34	62	90
7	175 = 7 (7^2 +1) / 2	100 = 4/7 x 165	25 = 165/7	35	63	91
8Δ	260 =8 (8^2 +1) / 2	130 = 4/8 x 260		36+	64+	92+
9	369 = 9 (9^2 +1) / 2	164 = 4/9 x 369	41 = 369/9	37	65	93
10	505 = 10 (10^2 +1) / 2	202 = 4/10 x 505		38	66	94
11	671 = 11 (11^2 +1) / 2	244 = 4/11 x 671	61 = 671/11	39	67	95
12+	870 = 12 (12^2 +1) / 2	290 = 4/12 x 870		40+	68+	96+
13	1105 = 13 (13^2 +1) / 2	340 = 4/13 x 1105	85 = 1105/13	41	69	97
14	1379 = 14 (14^2 +1) / 2	394 = 4/14 x 1379		42	70	98
15	1695 = 15 (15^2 +1) / 2	452 = 4/15 x 1695	113 = 1695/15	43	71	99
16Δ	2056 = 16 (16^2 +1) / 2	514 = 4/16 x 2056		44+	72+	100+
17	2465 = 17 (17^2 +1) / 2	580 = 4/17 x 2465	145 = 2465/17	45	73	101
18	2925 = 18 (18^2 +1) / 2	650 = 4/18 x 2925		46	74	102
19	3439 = 19 (19^2 +1) / 2	724 = 4/19 x 3439	181 = 3439/19	47	75	103
20+	4010 = 20 (20^2 +1) / 2	802 = 4/20 x 4010		48+	76+	104+
21	4641 = 21 (21^2 +1) / 2	884 = 4/21 x 4641	221 = 4641/21	49	77	105
22	5335 = 22 (22^2 +1) / 2	970 = 4/22 x 5335		50	78	106
23	6095 = 23 (23^2 +1) / 2	1060 = 4/23 x 6095	265 = 6095/23	51	79	107
24+	6924 = 24 (24^2 +1) / 2	1154 = 4/24 x 6924		52+	80+	108+
25	7825 = 25 (25^2 +1) / 2	1252 = 4/25 x 7825	313 = 7825/25	53	81	109
26	8801 = 26 (26^2 +1) / 2	1354 = 4/26 x 8801		54	82	110
27	9855 = 27 (27^2 +1) / 2	1460 = 4/27 x 9855	365 = 9855/27	55	83	111
28+	10990 = 28 (28^2 +1) / 2	1570 = 4/28 x 10990		56+	84+	112+
29	12209 = 29 (29^2 +1) / 2	1684 = 4/29 x 12209	421 = 12209/29	57	85	113
30	13515 = 30 (30^2 +1) / 2	1802 = 4/30 x 13515		58	86	114
Perfect		Merely non-perfect				
Near-perfect						

Here is a summary of characteristic numbers for squares up through size-30 along with their quadral sums. For odd-size squares, their pivot numbers are also listed. It cites the size-3 square as being geonomic but not perfect as a consequence of its embryonic size.

It identifies those even-size squares with all equal row-pair summations and cites the size-8 and size-16 squares as having both equal row-pair and centrally-symmetric pair sums. It shows the pattern of perfect and near-perfect squares up through size **114.**

Program 8

The right-hand column lists the pivot umber of each odd-size square. Note the incremental pattern: **(4, 8, 12, 16, ... 4x)**. This may be expressed in formulas as:

(8.5 a) $p(n) = p(n-2) + \Delta p$ where

(8.5 b) $p(1) \equiv 1$ and

(8.5 c) $\Delta p = 4(n-1)/2$

Remember that formula (**1.5**) **y = 4x** back in Program 1 where **y** was the size of square and **x** was the number of quadrals incident to the square's characteristic circle?

And again, that formula back in Program 5 when we observed that the number of incident octals in the characteristic spheres of cubes was given by the area below the line **y = 4x** ?

Well here it is again in the incremental sequence of pivot numbers in squares where now **y =** Δp and **x = (n-1)/2**.

2D Characteristic numbers of squares expressed as a quadratic

(8.6) $C_n = an^2 + bn + c$

The tables below list the coefficients **a** and **b** and constant **c** for the quadratic in (**8.6**). The formulas for the coefficients and constant differ between even and odd-size squares but the fact that these characteristic numbers can be expressed as a quadratic is pretty amazing.

Just the visible patterns in each table here are sufficient to be the mathematical proof by induction, one for the even-size squares and one for the odd-size squares, separately.

Even-size Squares				
a = c = n/2, b = 0				
Size n	a	b	c	Cn
2	1	0	1	5
4	2	0	2	34
6	3	0	3	111
8	4	0	4	260
12	5	0	5	505
14	6	0	6	870
16	7	0	7	1379
18	8	0	8	2056
20	9	0	9	2925
22	10	0	10	4010
24	11	0	11	5335
26	12	0	12	6924
28	13	0	13	8801
30	14	0	14	10990
32	15	0	15	13515
34	16	0	16	16400

Odd-size Squares				
a = (n-1) /2, b = (n+1) /2, c = 0				
Size n	a	b	c	Cn
1	0	1	0	1
3	1	2	0	11
5	2	3	0	65
7	3	4	0	175
9	4	5	0	369
11	5	6	0	671
13	6	7	0	1105
15	7	8	0	1695
17	8	9	0	2465
19	9	10	0	3439
21	10	11	0	4641
23	11	12	0	6095
25	12	13	0	7825
27	13	14	0	9855
29	14	15	0	12209
31	15	16	0	14911

Program 8

3D Characteristic numbers expressed as a 3rd-degree polynomial

(8.7) $C_n^{(3)} = an^3+bn^2+cn^1+dn^0$

where for **odd**-size cubes: $a = (n^2-1)/2$, $b = (n+1)/2$, $c = d = 0$

and for **even**-size cubes: $a = n^2/2$, $c = n/2$, $b = d = 0$

Representing the coefficients as functions of **n**, observe that $\mathbf{a(n_2)}$ and $\mathbf{c(n_2)}$ for even sizes $\mathbf{n_2}$ and $\mathbf{a(n_1)}$ and $\mathbf{b(n_1)}$ for odd sizes $\mathbf{n_1}$, then for $\mathbf{n_2 = n_1 + 1}$:

(8.8.1) $\mathbf{a(n_2) - a(n_1) = n_2}$ and

(8.8.2) $\mathbf{b(n_1) - c(n_2) = 0}$

Observe the series of the first four values of coefficient **a** for even-size cubes. Is this not the series of numbers, namely {2, 8, 8, 18, 18, 32, 32}, for elements in rows of the Periodic Table of Elements? And aren't these the first four values for **a**?

And 4, 8, 12 & 16 were the sizes of cubes for characteristic spheres that were used in determining the shell capacity for electrons seen back in Program-5. Isn't this just **y = 4c** for even-size cubes up to size-8? Again we see **y = 4x**.

Odd-size cubes					
n	C_n	a	b	c	d
1	1	0	1	0	0
3	126	4	2	0	0
5	1575	12	3	0	0
7	8428	24	4	0	0
9	29565	40	5	0	0
11	80586	60	6	0	0
13	185731	84	7	0	0

Even-size cubes					
n	C_n	a	b	c	d
2	18	2	0	1	0
4	520	8	0	2	0
6	3906	18	0	3	0
8	16416	32	0	4	0
10	50050	50	0	5	0
12	124488	72	0	6	0
14	269010	98	0	7	0

Now, all this is not just coincidental – It is further evidence that Geonometry is uncovering the very fabric of space itself !!!

* * *

Advice: When you discover any of these algebraic forms in your data:

1. a quadratic (page 18),
2. a 3rd-degree polynomial (this page),
3. a 3rd-degree monomial with alternating signs of unity (page 2),
4. $\mathbf{y = 4x}$
5. $\mathbf{y = 2x^2}$
6. $\{ C^{(2)}_n \}$

you are onto something meaningful. You should refer back to these cited pages to see if your data is related in any way to Geonometry and if so, Geonometry can aid in correlating your data with the spatial fabric.

Program 8

Index Triangle's Correlation with 2D Characteristic Numbers

For Squares

$$(8.9)\quad R_n^{(2)} = \sum_{j=a+1}^{b} j$$

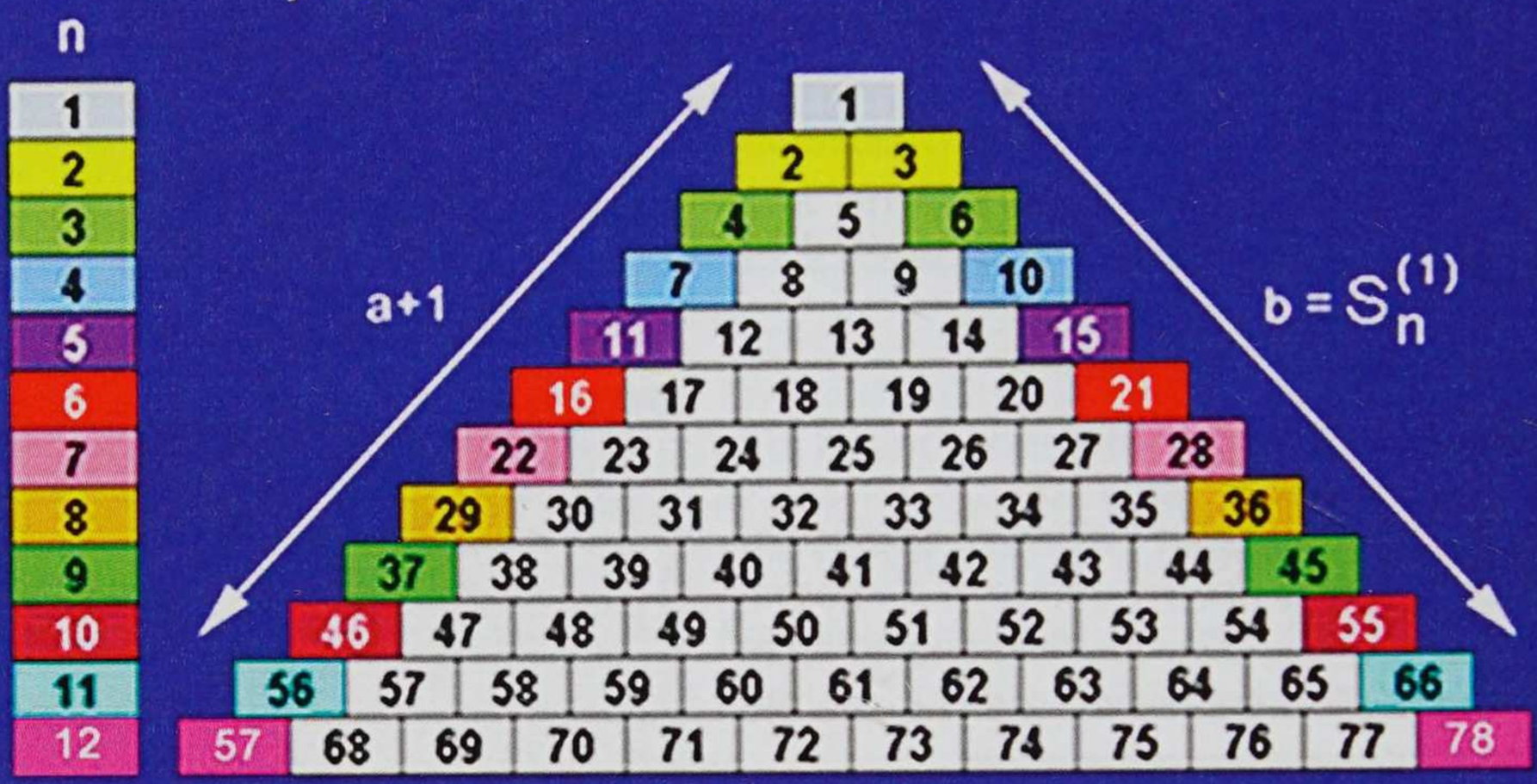
where

$b = n(n+1)/2$

$a = n(n-1)/2$

Row sum = Column sum = Characteristic number

$R_n^{(2)} = C_n^{(2)}$



This picture shows a triangular index table where the cells are numbered sequentially from top-down by row and within each row from left-to-right. Note the relationship between **a** and **b** in formula (**8.9**) and on the left and right edges of the index triangle. The amazing feature of this index table is that all the numbers in the **n**-th row sum to the characteristic number for all geonomic squares of size **n**. The largest number in an index triangle of size **n** is:

(8.10) $\quad N_n = n(n+1)/2$

Sum of the numbers in the n-th row = the characteristic number of square of size n:

(8.11) $\quad C^{(2)}_n = n(n^2+1)/2$

Note the correspondence between formula (**8.10**) for the count of numbers in the triangular table of size **n** and formula (**8.11**) for the characteristic numbers of squares of size **n**. The only difference between them is simply **n** verses n^2 as highlighted in **blue**.

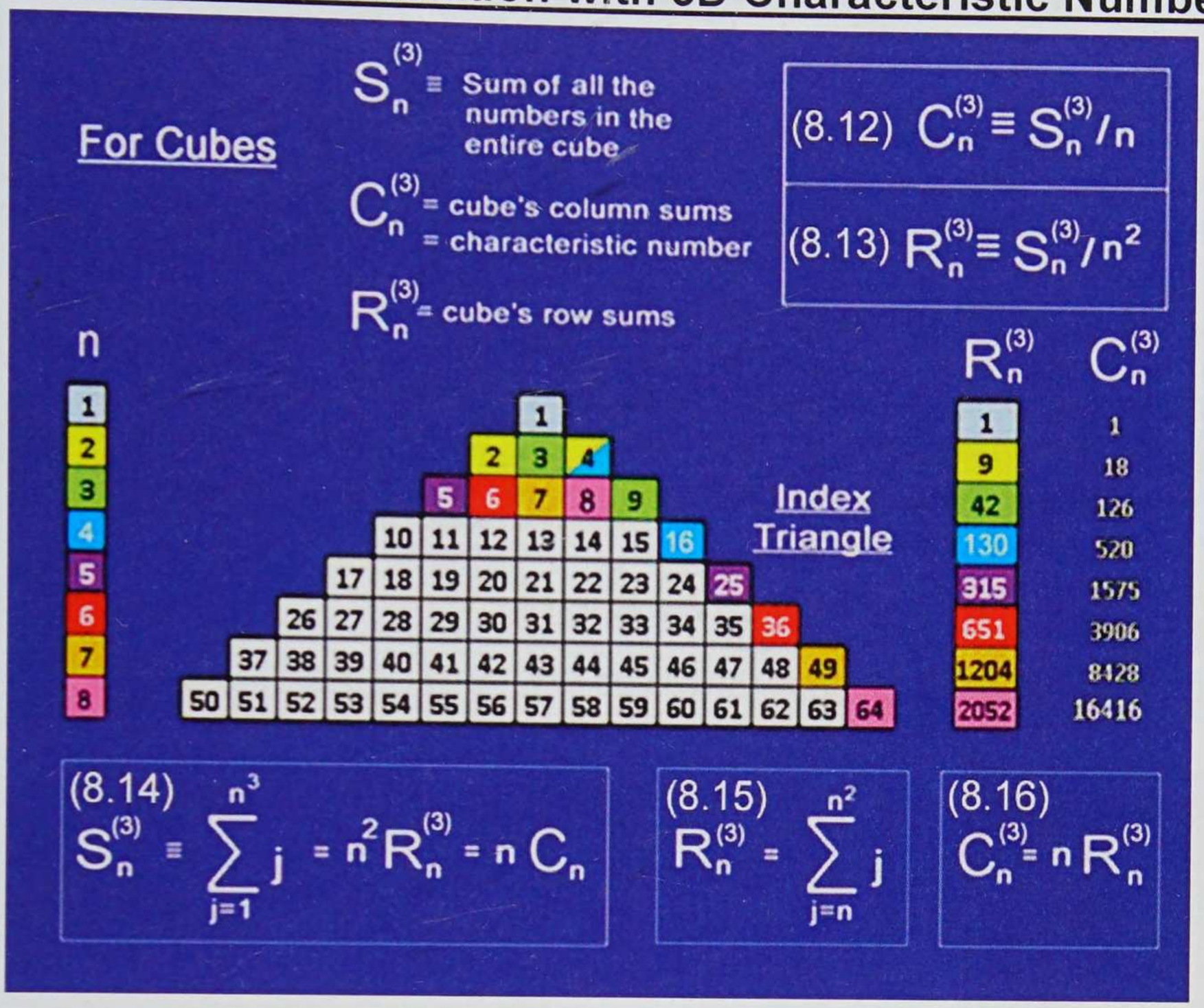
The formula for the common row sums **Rn** for squares of size **n** is given in terms of the index triangle row's starting and ending numbers. In geonomic squares, all of the row sums and the column sums are equal. So $R_n = C_n$ here.

Note that the values of the cells on the right side of the triangle are simply the sum of all the numbers in the vector to the left of the triangle from the top down to the cell of the same color.

Observe that the superscript number **2** of **Cn** in (**8.11**) stands for the characteristic number for geonomic tables of size **n** in dimension-2.

Program 8

Index Triangle's Correlation with 3D Characteristic Numbers



Like the correspondence of triangular index sums with characteristic numbers of squares, here is yet another amazing correspondence now between consecutive numbers on an index triangle and characteristic numbers of cubes.

The index triangle here is different from the one for squares. There, each row contained only one more number than the preceding row; here it contains two more numbers per row.

It's quite amazing that all the ending numbers in each row of this index triangle are consecutive **square numbers**.

Here's how the process works in this index triangle: On the left, **n** is selected and the color noted. The same colored cell in the triangle with the least value greater than **n** is determined. Next, these numbers are summed in sequence from number **n** thru $\mathbf{n^2}$ also of the same color on the right side of the triangle. That sum is the value $\mathbf{R_n}$ of all the cube's rows (**8.15**).

Then $\mathbf{R_n}$ is multiplied by **n** and that number is the value of all the cube's columns (**8.16**). This is also the characteristic number $\mathbf{C_n}$ for the cube of size **n**. Those numbers are listed at far right for cubes of sizes **1** thru **8**.

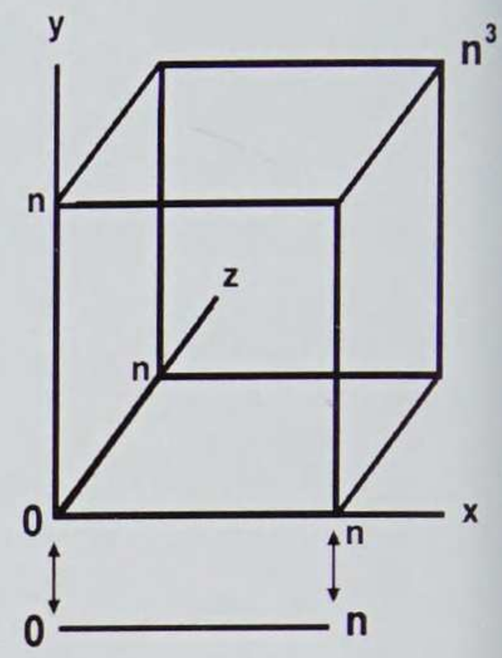
Note in the triangle that the number **4's** background is doubly colored. That's because it is the ending number for **n = 2** and the starting number for **n = 4**. In larger index tables this will occur on the right hand side of the triangle for every number that is both the starting number and an ending number for a lesser number in the triangle.

Although $\mathbf{S_n}$ is defined as the sum of all the numbers in the cube starting from **1** and ending at $\mathbf{n^3}$, it is quite amazing that it is directly related to $\mathbf{R_n}$ which is the sum of numbers starting at **n** an ending at $\mathbf{n^2}$ for a total of only **n+1** numbers in all when compared to the $\mathbf{n^3}$ numbers otherwise involved! Further averaging $\mathbf{S_n}$ by **n** yields the column sums as per formula (**8.12**) and averaging $\mathbf{S_n}$ by $\mathbf{n^2}$ yields the row sums per formula (**8.13**). Yet both $\mathbf{R_n}$ and $\mathbf{C_n}$ can be obtained with less computation from the shorter-length sums in the index triangle.

Program 8

The Pattern of Characteristic Numbers as Volume Averages between basic geometries in higher and lower Dimensions

The picture at right shows that the characteristic number for a geonomic **square** (2-dimensions) is the dimensional average of a **cube** (3-dimensions) and a **string** (1-dimension) all of the same size **n**. The formula for $\mathbf{C_n}$ below the picture yields the characteristic number for any square of size **n**.



$C_n = \frac{1}{2}(n^3+n)$

The table below shows that the distance between the two dimensions involved for characteristic numbers in **k**-dimensions expands as the dimension increases. That expansion difference Δ between the powers of **n** in the rightmost column is equal to **k** itself.

In general, for any dimension **k**, the characteristic number for a geonomic table of size **n** may be expressed as:

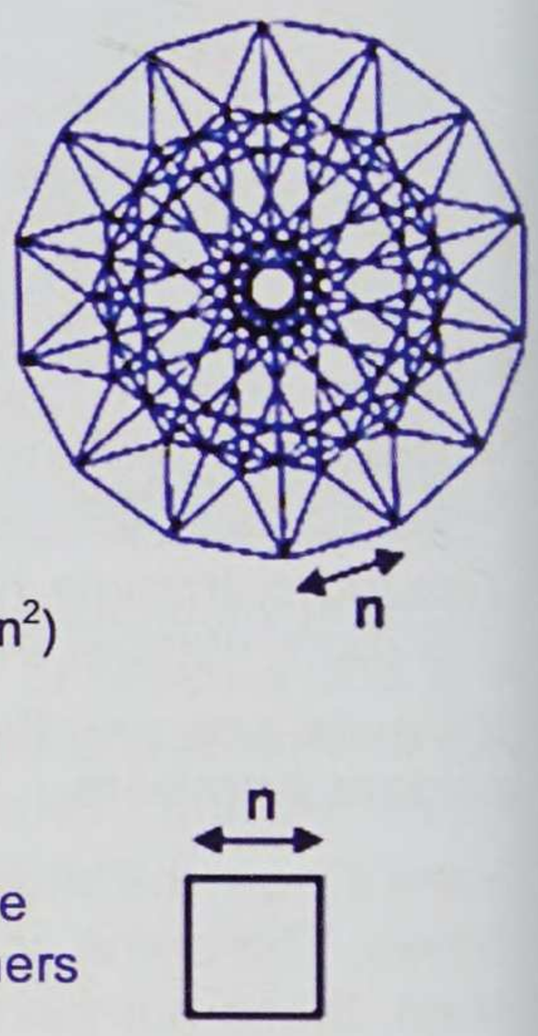
$$\mathbf{(8.17)} \quad C_n^{(k)} = S_n^{(k)}/n = n^k(n^k+1)/2n = \tfrac{1}{2}\,(n^{(2k-1)} + n^{(k-1)})$$

Note that for strings where **k=1**, which involves **0D,** (**8.17**) yields $C_n^{(1)} = (n+1)/2$ as expected,.

k	(2k-1)	(k-1)	Δ
1	1	0	1
2	3	1	2
3	5	2	3
4	7	3	4
5	9	4	5
6	11	5	6
7	13	6	7

Geonometry is going to give this simple formula a very profound interpretation. This is exemplified by the pictorial at right.

Here, the characteristic number of any size **cube** is the just the average of the **quintacube's** volume and the area of a **square** of the same size. This averaging of measures between two different dimensions, one lower and one higher, yielding in an intervening dimension, the fundamental characteristic number of any size-**n** geonomic table is the profound interpretation.



This has implication for the rate of the passage of time among different dimensions as follows:

If time and spatial volume are correlated in any way, this would imply that the rate of the passage of time in dimension **k+1** is somehow correlated with the average of the rates of time-passage between dimension **2k** and **k**. If so, given that the rate of time passage in the lower dimension was faster than the dimension above it would imply that the rate of time-passage in the higher dimension is slower than that in the **intervening** dimension.

It is well known in Atomic Science as well as in Micro-biology that events take place in nano-seconds and micro-seconds, respectively. So time is in fact faster in dimensions less than 3D. That implies that the passage of time in the 4th dimension is slower than the passage of time in the 3rd dimension. This observation is the basis upon which Geonometry predicts that such is the case in its big picture of the multi-dimensional universe to be shown at the end of Program 9.

Program 8

The Loom-table hidden in the Tiling Patterns of Class-1 Cubes

Here is a size-7 Class-1 Cube. It is absolutely ultra-perfect. That is, all of its embedded squares sum equally in all four directions and it has continuously equal-summing complementary tiling patterns, A and B. (Shown on the next 2 pages.)

		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		1204	1204	1204	1204	1204	1204	1204	
		8428	8428	8428	8428	8428	8428	8428	
Square	1	56	263	127	341	156	27	234	1204	
	2	191	13	220	91	249	113	327	1204	
	3	284	99	313	177	48	206	77	1204	
7	4	34	241	63	270	134	299	163	1204	
	5	120	334	149	20	227	98	256	1204	
	6	213	84	291	106	320	184	6	1204	
	7	306	170	41	199	70	277	141	1204	8428
	1	252	116	323	194	9	223	87	1204	
	2	44	209	73	287	102	309	180	1204	
	3	137	295	166	30	244	59	273	1204	
6	4	230	94	259	123	330	152	16	1204	
	5	316	187	2	216	80	294	109	1204	
	6	66	280	144	302	173	37	202	1204	
	7	159	23	237	52	266	130	337	1204	8428
	1	105	312	176	47	205	76	283	1204	
	2	240	62	269	140	298	162	33	1204	
	3	333	148	19	226	97	255	126	1204	
5	4	83	290	112	319	183	5	212	1204	
	5	169	40	198	69	276	147	305	1204	
	6	262	133	340	155	26	233	55	1204	
	7	12	219	90	248	119	326	190	1204	8428
	1	301	165	29	243	58	272	136	1204	
	2	93	258	122	336	151	15	229	1204	
	3	186	1	215	79	293	108	322	1204	
4	4	279	143	308	172	36	201	65	1204	
	5	22	236	51	265	129	343	158	1204	
	6	115	329	193	8	222	86	251	1204	
	7	208	72	286	101	315	179	43	1204	8428
	1	154	18	225	96	254	125	332	1204	
	2	289	111	318	189	4	211	82	1204	
	3	39	197	68	275	146	304	175	1204	
3	4	132	339	161	25	232	54	261	1204	
	5	218	89	247	118	325	196	11	1204	
	6	311	182	46	204	75	282	104	1204	
	7	61	268	139	297	168	32	239	1204	8428
	1	7	214	78	292	107	321	185	1204	
	2	142	307	171	42	200	64	278	1204	
	3	235	50	264	128	342	157	28	1204	
2	4	328	192	14	221	85	250	114	1204	
	5	71	285	100	314	178	49	207	1204	
	6	164	35	242	57	271	135	300	1204	
	7	257	121	335	150	21	228	92	1204	8428
	1	203	67	274	145	303	174	38	1204	
	2	338	160	24	238	53	260	131	1204	
	3	88	246	117	324	195	10	224	1204	
1	4	181	45	210	74	281	103	310	1204	
	5	267	138	296	167	31	245	60	1204	
	6	17	231	95	253	124	331	153	1204	
	7	110	317	188	3	217	81	288	1204	8428
		8428	8428	8428	8428	8428	8428	8428		
		8428	8428	8428	8428	8428	8428	8428		
		1204	1204	1204	1204	1204	1204	1204		
		1204	1204	1204	1204	1204	1204	1204		
		1204	1204	1204	1204	1204	1204	1204		
		1204	1204	1204	1204	1204	1204	1204		
		1204	1204	1204	1204	1204	1204	1204		
		1204	1204	1204	1204	1204	1204	1204		
		1204	1204	1204	1204	1204	1204	1204		

← Every major diagonal in each block-square sums to **1204**.

Quadrals

737	737	394	737
737	394	1080	737
737	737	737	394

835	492	835	492
492	835	492	835
835	835	492	835

590	933	590	590
590	590	933	590
933	590	590	590

688	688	688	688
688	688	688	688
688	688	688	688

786	443	786	786
786	786	443	786
443	786	786	786

541	884	541	884
884	541	884	541
541	541	884	541

639	639	982	639
639	982	296	639
639	639	639	982

Octals

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

Octahedrons

1032
1032
1032

← Every minor diagonal in each block-square sums to **1204**.

Program 8

Tiling pattern A on embedded squares

Each embedded square contains complementary tiling patterns in which each tile sums to **1204**. (Only one of which is shown here; the other is shown on the next page)

The number strips on the right are lists of tile centers in the respective tiling pattern, sorted in order of increasing value. They are put into a square according to their sequence in the cube. This square is shown at the next-to-last square on the right. Left-to-right in the square corresponds to front-to-back in the cube. The last table on the right is the modulus table derived from the square of center-numbers above it. The same thing is shown on the next page for the other tiling pattern.

Embedded Square 7

	1	2	3	4	5	6	7		
	1204	1204	1204	1204	1204	1204	1204	1204	
1	56	263	127	341	156	27	234	1204	6
2	191	13	220	91	249	113	327	1204	56
3	284	99	313	177	48	206	77	1204	99
7 4	34	241	63	270	134	299	163	1204	149
5	120	334	149	20	227	98	256	1204	199
6	213	84	291	106	320	184	6	1204	249
7	306	170	41	199	70	277	141	1204	299
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

Embedded Square 3

	1204	1204	1204	1204	1204	1204	1204	1204	
1	154	18	225	96	254	125	332	1204	4
2	289	111	318	189	4	211	82	1204	54
3	39	197	68	275	146	304	175	1204	104
3 4	132	339	161	25	232	54	261	1204	154
5	218	89	247	118	325	196	11	1204	197
6	311	182	46	204	75	282	104	1204	247
7	61	268	139	297	168	32	239	1204	297
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

Embedded Square 6

	1204	1204	1204	1204	1204	1204	1204	1204	
1	252	116	323	194	9	223	87	1204	2
2	44	209	73	287	102	309	180	1204	52
3	137	295	166	30	244	59	273	1204	102
6 4	230	94	259	123	330	152	16	1204	152
5	316	187	2	216	80	294	109	1204	202
6	66	280	144	302	173	37	202	1204	252
7	159	23	237	52	266	130	337	1204	295
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

Embedded Square 2

	1204	1204	1204	1204	1204	1204	1204	1204	
1	7	214	78	292	107	321	185	1204	7
2	142	307	171	42	200	64	278	1204	50
3	235	50	264	128	342	157	28	1204	100
2 4	328	192	14	221	85	250	114	1204	150
5	71	285	100	314	178	49	207	1204	200
6	164	35	242	57	271	135	300	1204	250
7	257	121	335	150	21	228	92	1204	300
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

Embedded Square 5

	1204	1204	1204	1204	1204	1204	1204	1204	
1	105	312	176	47	205	76	283	1204	5
2	240	62	269	140	298	162	33	1204	55
3	333	148	19	226	97	255	126	1204	105
5 4	83	290	112	319	183	5	212	1204	148
5	169	40	198	69	276	147	305	1204	198
6	262	133	340	155	26	233	55	1204	248
7	12	219	90	248	119	326	190	1204	298
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

Embedded Square 1

	1204	1204	1204	1204	1204	1204	1204	1204	
1	203	67	274	145	303	174	38	1204	3
2	338	160	24	238	53	260	131	1204	53
3	88	246	117	324	195	10	224	1204	103
1 4	181	45	210	74	281	103	310	1204	153
5	267	138	296	167	31	245	60	1204	203
6	17	231	95	253	124	331	153	1204	246
7	110	317	188	3	217	81	288	1204	296
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

Embedded Square 4

	1204	1204	1204	1204	1204	1204	1204	1204	
1	301	165	29	243	58	272	136	1204	1
2	93	258	122	336	151	15	229	1204	51
3	186	1	215	79	293	108	322	1204	101
4 4	279	143	308	172	36	201	65	1204	151
5	22	236	51	265	129	343	158	1204	201
6	115	329	193	8	222	86	251	1204	251
7	208	72	286	101	315	179	43	1204	301
	1204	1204	1204	1204	1204	1204	1204		
	1204	1204	1204	1204	1204	1204	1204	1204	

1	2	3	4	5	6	7
3	7	4	1	5	2	6
53	50	54	51	55	52	56
103	100	104	101	105	102	99
153	150	154	151	148	152	149
203	200	197	201	198	202	199
246	250	247	251	248	252	249
296	300	297	301	298	295	299

Table composed of external columns above

3	7	4	1	5	2	6	28
4	1	5	2	6	3	7	28
5	2	6	3	7	4	1	28
6	3	7	4	1	5	2	28
7	4	1	5	2	6	3	28
1	5	2	6	3	7	4	28
2	6	3	7	4	1	5	28

Modulus loom table of the above table

Program 8

Tiling pattern B on embedded squares

Although the tile centers between the two tiling patterns were deliberately made non-coincidental, Lo and behold, both derived modulus tables are numerically identical!

And moreover, the table possesses all the characteristics of a loom table because every row, column and diagonal, including wrap diagonals in both directions, contains all the numbers **1** through **7** exactly once. We shall identify this loom table in what follows as **X***.

Embedded Square 7

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	56	263	127	341	156	27	234	1204
2	191	13	220	91	249	113	327	1204
3	284	99	313	177	48	206	77	1204
4	34	241	63	270	134	299	163	1204
5	120	334	149	20	227	98	256	1204
6	213	84	291	106	320	184	6	1204
7	306	170	41	199	70	277	141	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 34, 84, 127, 177, 227, 277, 327

Embedded Square 3

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	154	18	225	96	254	125	332	1204
2	289	111	318	189	4	211	82	1204
3	39	197	68	275	146	304	175	1204
4	132	339	161	25	232	54	261	1204
5	218	89	247	118	325	196	11	1204
6	311	182	46	204	75	282	104	1204
7	61	268	139	297	168	32	239	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 32, 82, 132, 182, 225, 275, 325

Embedded Square 6

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	252	116	323	194	9	223	87	1204
2	44	209	73	287	102	309	180	1204
3	137	295	166	30	244	59	273	1204
4	230	94	259	123	330	152	16	1204
5	316	187	2	216	80	294	109	1204
6	66	280	144	302	173	37	202	1204
7	159	23	237	52	266	130	337	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 30, 80, 130, 180, 230, 280, 323

Embedded Square 2

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	7	214	78	292	107	321	185	1204
2	142	307	171	42	200	64	278	1204
3	235	50	264	128	342	157	28	1204
4	328	192	14	221	85	250	114	1204
5	71	285	100	314	178	49	207	1204
6	164	35	242	57	271	135	300	1204
7	257	121	335	150	21	228	92	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 35, 78, 128, 178, 228, 278, 328

Embedded Square 5

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	105	312	176	47	205	76	283	1204
2	240	62	269	140	298	162	33	1204
3	333	148	19	226	97	255	126	1204
4	83	290	112	319	183	5	212	1204
5	169	40	198	69	276	147	305	1204
6	262	133	340	155	26	233	55	1204
7	12	219	90	248	119	326	190	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 33, 83, 133, 176, 226, 276, 326

Embedded Square 1

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	203	67	274	145	303	174	38	1204
2	338	160	24	238	53	260	131	1204
3	88	246	117	324	195	10	224	1204
4	181	45	210	74	281	103	310	1204
5	267	138	296	167	31	245	60	1204
6	17	231	95	253	124	331	153	1204
7	110	317	188	3	217	81	288	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 31, 81, 131, 181, 231, 274, 324

Embedded Square 4

	1	2	3	4	5	6	7	
	1204	1204	1204	1204	1204	1204	1204	
1	301	165	29	243	58	272	136	1204
2	93	258	122	336	151	15	229	1204
3	186	1	215	79	293	108	322	1204
4	279	143	308	172	36	201	65	1204
5	22	236	51	265	129	343	158	1204
6	115	329	193	8	222	86	251	1204
7	208	72	286	101	315	179	43	1204
	1204	1204	1204	1204	1204	1204	1204	
	1204	1204	1204	1204	1204	1204	1204	

Tile centers: 29, 79, 129, 179, 229, 279, 329

1	2	3	4	5	6	7
31	35	32	29	33	30	34
81	78	82	79	83	80	84
131	128	132	129	133	130	127
181	178	182	179	176	180	177
231	228	225	229	226	230	227
274	278	275	279	276	280	277
324	328	325	329	326	323	327

Table composed of external columns above

3	7	4	1	5	2	6	28
4	1	5	2	6	3	7	28
5	2	6	3	7	4	1	28
6	3	7	4	1	5	2	28
7	4	1	5	2	6	3	28
1	5	2	6	3	7	4	28
2	6	3	7	4	1	5	28

Modulus loom table of the above table

Program 8

The Geonomic Square Hidden in the Cube's Tiling Patterns

The table at upper left is the transpose of **X*** that we just derived from the modulus table of the tile center numbers.

To its right is its complementary loom-table denoted by $\mathbf{Y^{*H}}$ that was obtained by horizontally flipping $\mathbf{X^{*T}}$.

$\mathbf{X^{*T}}$

1	3	4	5	6	7	1	2
2	7	1	2	3	4	5	6
3	4	5	6	7	1	2	3
4	1	2	3	4	5	6	7
5	5	6	7	1	2	3	4
6	2	3	4	5	6	7	1
7	6	7	1	2	3	4	5

$\mathbf{Y^{*H}}$

7	2	1	7	6	5	4	3
6	6	5	4	3	2	1	7
5	3	2	1	7	6	5	4
4	7	6	5	4	3	2	1
3	4	3	2	1	7	6	5
2	1	7	6	5	4	3	2
1	5	4	3	2	1	7	6

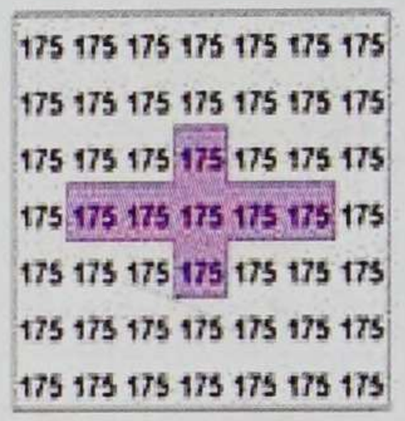
Then applying $\mathbf{X^{*T}}$ and $\mathbf{Y^{*H}}$ to the standard **2D** formula for generation of the primal:

(8.18) $\quad \mathbf{W^* = n(Y^{*H} - |1|) + X^{*T}}$

we get **W*** as seen at bottom left.

W*

	175	175	175	175	175	175	175	
1	10	4	47	41	35	22	16	175
2	42	29	23	17	11	5	48	175
3	18	12	6	49	36	30	24	175
4	43	37	31	25	19	13	7	175
5	26	20	14	1	44	38	32	175
6	2	45	39	33	27	21	8	175
7	34	28	15	9	3	46	40	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	



This derived square is seen to be an ultra-perfect size-7 square. Now that's one big surprise. Who could have guessed that?

Note that all the values that the modulus and integer functions have in common are located in the center column of **W***. This is the result of flipping $\mathbf{X^{*T}}$ horizontally to get $\mathbf{Y^{*H}}$.

Now here's the second big surprise: Since these tiles can be moved continuously in the cube by moving the entire cubical tiling pattern, this implies that there are **7** distinctly different tables of sorted tile-center numbers. And surprisingly, all **7** yield loom tables identical to **X***. That's another amazing discovery!

This same result can be obtained from the tiling patterns in the cube's first and second duals too. (Not shown)

The first dual **U1*** of **W*** is obtained by interchanging the derived loom-tables in the standard generation formula.

Program 8

The loom table hidden in the Class-1 Quadracube's tiling patterns

ext, we'll examine the patterns inherent in the tiling pattern 4-dimensional of a quadracube of ize-7. Although hardly legible, the larger size-7 quadracube was chosen over the size-5 uadracube because it was seen earlier that the size-5 table has some peculiar additional roperties that stem from its characteristic tile patterns' transposable symmetry.

Program 8

Follow these steps: First the center numbers are obtained from each tile's center. In each column in the first table below are the tile centers in the corresponding embedded square in a horizontally embedded cube.

Then separately within each block of its own, each column is sorted on its increasing values to arrive at the second table.

Then the modulus function is applied to each value of the second table to get the third table. What results are 7-duplicates of the modulus loom table, one for each horizontal cube.

The transpose of this loom-table can be derived from using the horizontally flipped version of pattern hB on the vertically embedded cubes. (That's not interesting enough to be shown too.)

L = center numbers of tiles

34	1406	377	1749	720	2092	1063
427	1799	770	2142	1113	84	1456
813	2185	1156	127	1499	470	1842
1206	177	1549	520	1892	863	2235
1992	963	2335	1306	277	1649	620
2385	1356	327	1699	670	2042	1013
1599	570	1942	913	2285	1256	227
230	1602	673	1945	916	2288	1259
623	1995	966	2338	1309	280	1652
1009	2381	1352	323	1695	666	2038
1059	30	1402	373	1745	716	2088
1845	816	2188	1159	130	1502	473
2238	1209	180	1552	523	1895	866
1452	423	1795	766	2138	1109	80
83	1455	426	1798	769	2141	1112
476	1848	819	2191	1162	133	1505
862	2234	1205	176	1548	519	1891
1255	226	1598	569	1941	912	2284
2041	1012	2384	1355	326	1698	669
2091	1062	33	1405	376	1748	719
1648	619	1991	962	2334	1305	276
279	1651	622	1994	965	2337	1308
672	2044	1015	2387	1358	329	1701
715	2087	1058	29	1401	372	1744
1108	79	1451	422	1794	765	2137
1894	865	2237	1208	179	1551	522
2287	1258	229	1601	572	1944	915
1501	472	1844	815	2187	1158	129
132	1504	475	1847	818	2190	1161
525	1897	868	2240	1211	182	1554
911	2283	1254	225	1597	568	1940
1304	275	1647	618	1990	961	2333
1747	718	2090	1061	32	1404	375
2140	1111	82	1454	425	1797	768
1697	668	2040	1011	2383	1354	325
328	1700	671	2043	1014	2386	1357
378	1760	721	2093	1064	35	1407
764	2136	1107	78	1450	421	1793
1157	128	1500	471	1843	814	2186
1943	914	2286	1257	228	1600	571
2336	1307	278	1650	621	1993	964
1550	521	1893	864	2236	1207	178
181	1553	524	1896	867	2239	1210
574	1946	917	2289	1260	231	1603
960	2332	1303	274	1646	617	1989
1353	324	1696	667	2039	1010	2382
1796	767	2139	1110	81	1453	424
2189	1160	131	1503	474	1846	817
1403	374	1746	717	2089	1060	31

L centers sorted within each block-row

34	177	327	127	277	84	227
427	570	377	520	670	470	620
813	963	770	913	720	863	1013
1206	1356	1156	1306	1113	1256	1063
1599	1406	1549	1699	1499	1649	1456
1992	1799	1942	1749	1892	2042	1842
2385	2185	2335	2142	2285	2092	2235
230	30	180	323	130	280	80
623	423	573	373	523	666	473
1009	816	966	766	916	716	866
1059	1209	1352	1159	1309	1109	1259
1452	1602	1402	1652	1695	1502	1652
1845	1995	1795	1945	1745	1895	2038
2238	2381	2188	2338	2138	2288	2088
83	226	33	176	326	133	276
476	619	426	569	376	519	669
862	1012	819	962	769	912	719
1255	1062	1205	1355	1162	1305	1112
1648	1455	1598	1405	1648	1698	1505
2041	1848	1991	1798	1941	1748	1891
2091	2234	2384	2191	2334	2141	2284
279	79	229	29	179	329	129
672	472	622	422	572	372	522
715	865	1015	815	965	765	915
1108	1258	1058	1208	1358	1158	1308
1501	1651	1451	1601	1401	1551	1701
1894	2044	1844	1994	1794	1944	1744
2287	2087	2237	2387	2187	2337	2137
132	275	82	225	32	182	325
525	668	475	618	425	568	375
911	718	868	1011	818	961	768
1304	1111	1254	1061	1211	1354	1161
1697	1504	1647	1454	1597	1404	1554
1747	1897	2040	1847	1990	1797	1940
2140	2283	2090	2240	2383	2190	2333
328	128	278	78	228	35	178
378	521	671	471	621	421	671
764	914	721	864	1014	814	964
1157	1307	1107	1257	1064	1207	1357
1550	1700	1500	1650	1450	1600	1407
1943	1760	1893	2043	1843	1993	1793
2336	2136	2286	2093	2236	2386	2186
181	324	131	274	81	231	31
574	374	524	667	474	617	424
960	767	917	717	867	1010	817
1353	1160	1303	1110	1260	1060	1210
1403	1553	1696	1503	1646	1453	1603
1796	1946	1746	1896	2039	1846	1989
1314	1308	1323	1311	1331	1313	1302

X = Modulus 7 of L

6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2

Program 8

Generating Ultra-perfect Dual Squares from the Derived Modulus Loom-table

An ultra-perfect square can be derived from this loom table **X*** just as was shown for 3-dimensions. Equivalently, the derived modulus loom-table can be vertically flipped instead of transposed to get a viable complementary integer loom-table. And to preserve the orientation of the tile patterns, both loom tables need to be transposed.

It is these versions that produce the hidden geonomic squares shown here. Square **U1*** can produce two more additional dual squares (not shown) with all the same properties.

X*

6	2	6	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2

X*^T

6	7	1	2	3	4	5
2	3	4	5	6	7	1
5	6	7	1	2	3	4
1	2	3	4	5	6	7
4	5	6	7	1	2	3
7	1	2	3	4	5	6
3	4	5	6	7	1	2

Y*

5	1	4	7	3	6	2
4	7	3	6	2	5	1
3	6	2	5	1	4	7
2	5	1	4	7	3	6
1	4	7	3	6	2	5
7	3	6	2	5	1	4
6	2	5	1	4	7	3

Y*^T

5	4	3	2	1	7	6
1	7	6	5	4	3	2
4	3	2	1	7	6	5
7	6	5	4	3	2	1
3	2	1	7	6	5	4
6	5	4	3	2	1	7
2	1	7	6	5	4	3

W*

	1	2	3	4	5	6	7	
	175	175	175	175	175	175	175	175
1	34	28	15	9	3	46	40	175
2	2	45	39	33	27	21	8	175
3	26	20	14	1	44	38	32	175
4	43	37	31	25	19	13	7	175
5	18	12	6	49	36	30	24	175
6	42	29	23	17	11	5	48	175
7	10	4	47	41	35	22	16	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

U1*

	1	2	3	4	5	6	7	
	175	175	175	175	175	175	175	175
1	40	46	3	9	15	28	34	175
2	8	21	27	33	39	45	2	175
3	32	38	44	1	14	20	26	175
4	7	13	19	25	31	37	43	175
5	24	30	36	49	6	12	18	175
6	48	5	11	17	23	29	42	175
7	16	22	35	41	47	4	10	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

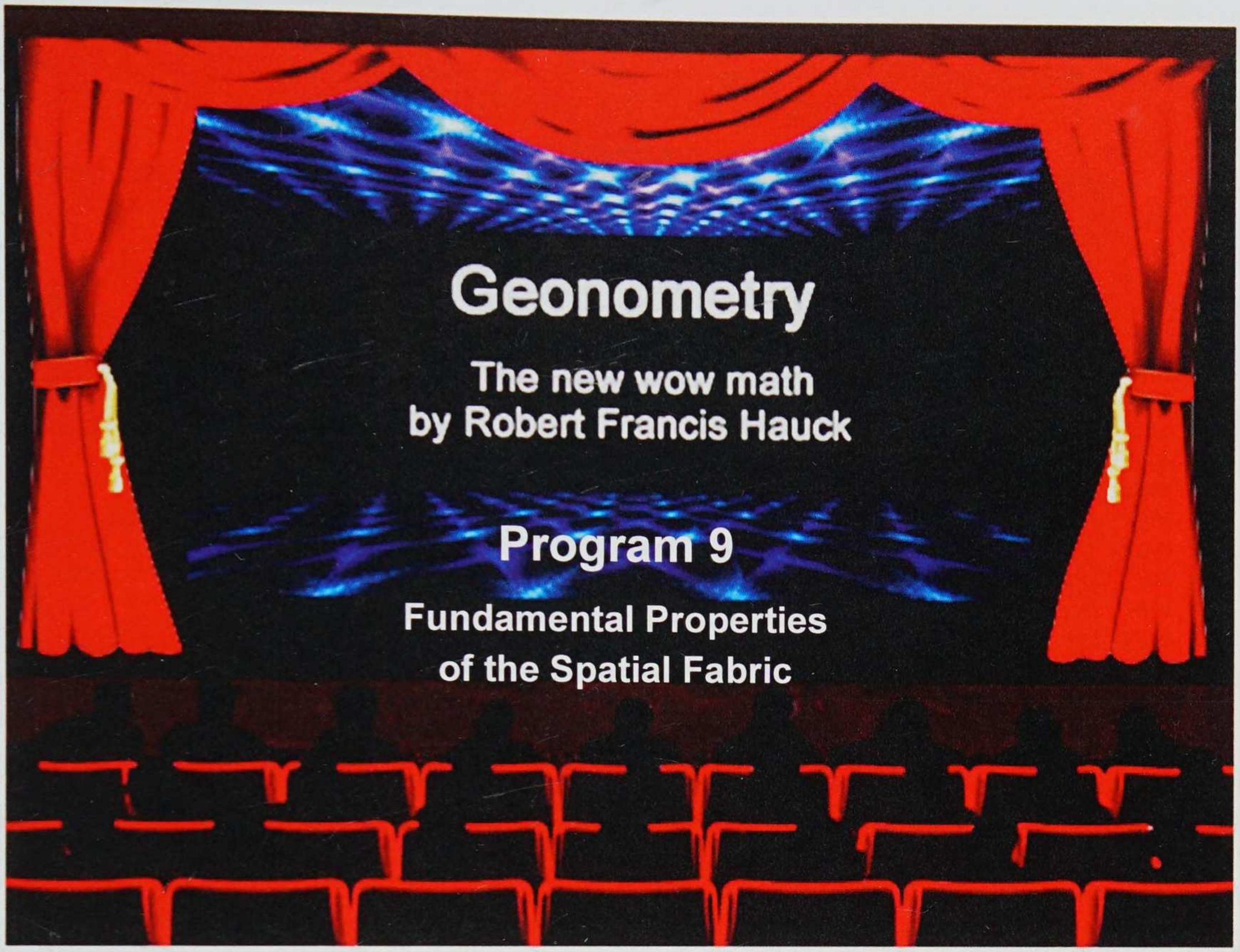
These hidden loom tables in 3-dimensions and now in 4-dimensions lend credence to the pervasive undulating harmonic fabric of space that Geonometry has inadvertently tapped into!

What this implies is this: There are basic fundamental harmonic patterns running everywhere throughout any multi-dimensional space. This is an indication of the existence of the *ether* long sought-after since the 15th century. Here is another mathematical confirmation of the existence of the very fabric of space itself.

Program 8

Program 8 Notes

1. The zero-differential property holds for all ultra-perfect squares of all classes except for those of Class-2.
2. The sub-additive property holds for every size sub-additive block only for the size-5 square.
3. The increment in the values of pivot numbers for consecutive odd-size squares follows the formula **$y = \Delta p = 2(n-1) = 4x$**; another instance of **$y = 4x$**.
4. The characteristic number of a square of size **n** is directly related to the **n**-th row of an index triangle.
5. The characteristic number of a cube of size **n** is directly related to the sum of consecutive numbers from **n** to **n^2** in an index triangle.
6. The Polya formula (**7.4**) applied to loom-tables' cells in the cubic-conversion method results in a matrix whose characteristic number is the square of the loom-tables' original characteristic number.
7. The modulus loom-table can be recovered from the centers of tiling patterns in cubes and quadracubes.
8. Size-8 squares with both pairwise row and central symmetry can be easily generated back and forth from just three loom-tables. This does not apply to any other Class-4 squares.

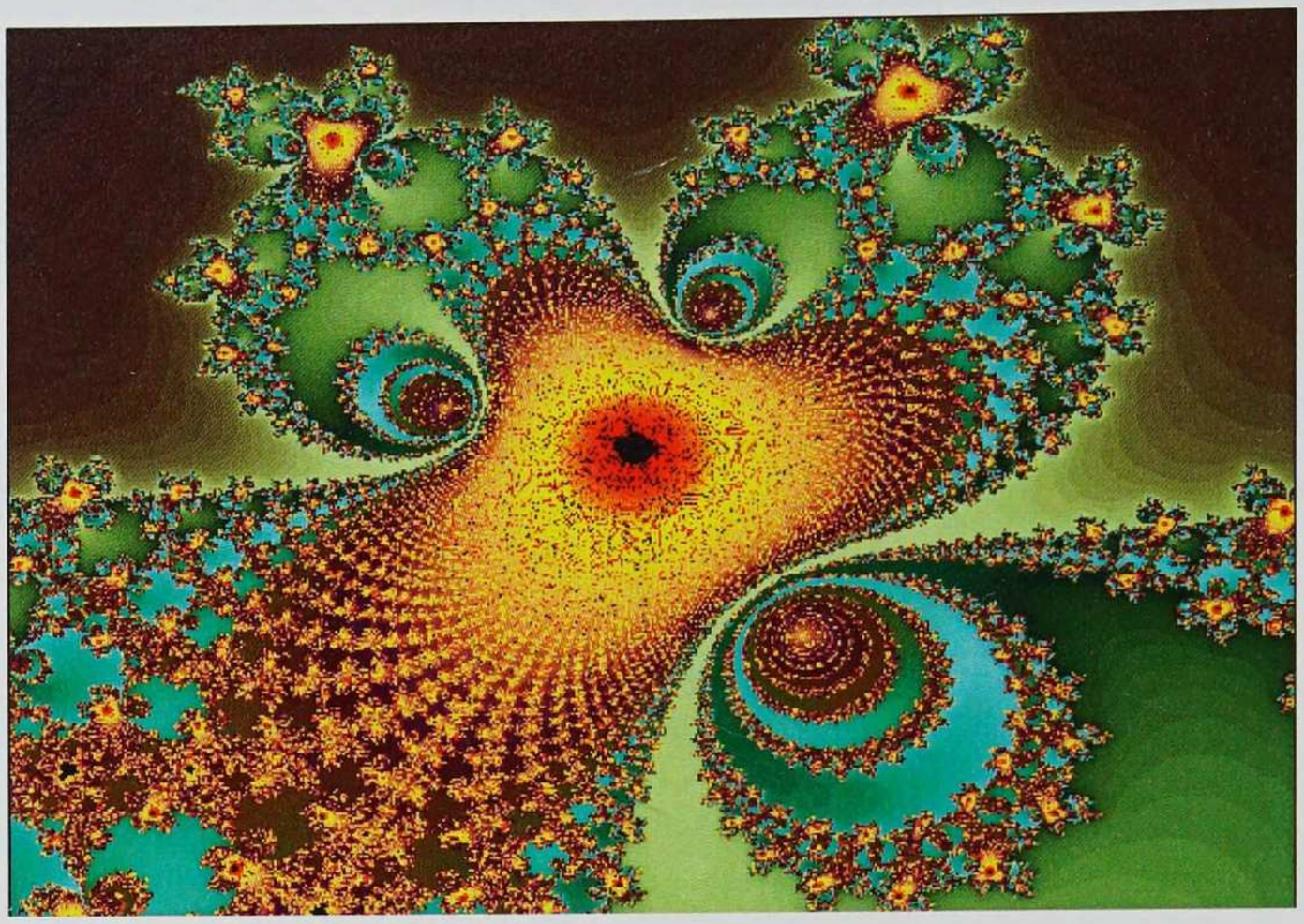


There are still yet a number of amazing general properties of perfect dimensional geonomic tables. These we'll look at next. It will provide an even more fundamental picture of the natural fabric of space.

In this Program, we will discover how to explore the properties of any higher dimension through the unique mathematics of Geonometry. Here we'll prove mathematically that space is fractal and what happens in higher dimensions is mimicked in lower dimensions.

Program 9

Fracticality in Geonometry



The Mandelbrot Fractal Pattern

Next, we'll look at the property of Fracticality in geonomic squares. Being "fractal" means that a pattern is composed of parts that are repetitions of itself. Here is the Mandelbrot fractal pattern. Note those yellow patterns all throughout the picture. Those are replications of the big pattern. Well, Geonometry is also fractal.

Remember those tiling patterns for Class-3 squares of sizes **15** and **21** that were shown back in Program 3? Those complementary tiling patterns of sizes **5** and **7** respectively were at the block-square level. Those patterns qualify as being **fractal**.

Fracticality in 4-Dimensions

Size-3 Quadracube

Cubes	1			2			3				
3x4D	369	369	369	369	369	369	369	369	369		
	44	58	21	71	4	48	17	31	75	369	
4	64	9	50	10	36	77	37	63	23	369	
	15	29	79	42	56	25	69	2	52	369	1107
	49	66	8	76	12	35	22	39	62	369	
5	81	14	28	27	41	55	54	68	1	369	
	20	43	60	47	70	6	74	16	33	369	1107
	30	80	13	57	26	40	3	53	67	369	
6	59	19	45	5	46	72	32	73	18	369	
	7	51	65	34	78	11	61	24	38	369	1107
	369	369	369	369	369	369	369	369	369		
	369	369	369	369	369	369	369	369	369	369	

Here is a size-3 quadracube. It contains **9** embedded size-3 squares each of which sums to the size-9's characteristic number **369**. In fact, the size-3 quadracube projects onto 2-dimensions as a square of size-9. Consequently, the quadracube is fractal because the summation patterns hold for two different sizes of table in two different dimensions.

The size-3 quadracube is continuously **3x** modular, so any configuration of **3x3** block-squares in a **3** block-square tiling pattern will sum to the quadracube's characteristic number, **1107**, such as a block-diagonal.

Program 9

Here is a size-4 quadracube. It projects onto 2-dimensions as a perfect size-16 square with continuous **4x**-modularity. As such, any **4x4** block-square tile configuration of **4** block-squares will sum to the quadracube's characteristic number.

4x4D

Cube	1				2				3				4					
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
5	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056	
	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056	
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056	
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056	8224
6	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056	
	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056	
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056	
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056	8224
7	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056	
	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056	
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056	
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056	8224
8	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056	
	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056	
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056	
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
				8224				8224				8224				8224		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

Here is the size-5 quadracube generated directly from the size-5 cube. All of its embedded block-squares sum to **1/5**-th of the quadracube's characteristic number. As such, any **5x5** block-square tile configuration of **5** block-squares will sum to the quadracube's characteristic number.

Further, it is also simultaneously equivalent to a size **25** Class-6 square. As such it is also continuously **5x** modular there where any size **5** block-square will sum equally to the characteristic number of the size-25 square.

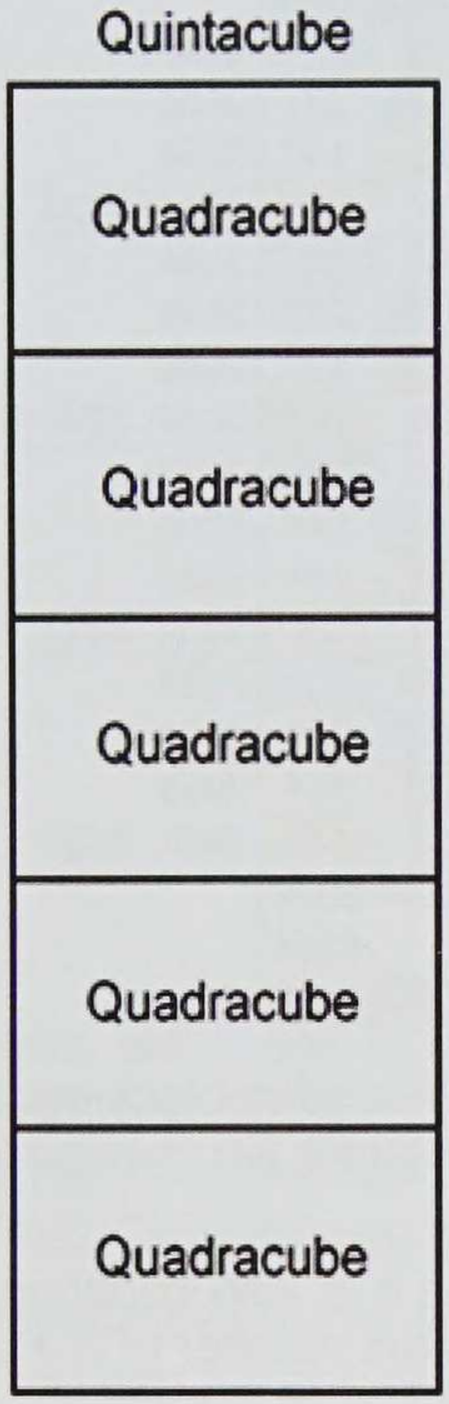
5x5x5x5

Cubes	1					2					3					4					5					
	5x4D	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	70	158	271	489	577	195	283	396	614	77	320	408	521	114	202	445	533	21	239	327	570	33	146	364	452	7825	
	561	29	142	360	473	61	154	267	485	598	186	279	392	610	98	311	404	517	110	223	436	529	17	235	348	7825	
6	432	550	13	226	344	557	50	138	351	469	57	175	263	476	594	182	300	388	601	94	307	425	513	101	219	7825	
	303	416	509	122	215	428	541	9	247	340	553	41	134	372	465	53	166	259	497	590	178	291	384	622	90	7825	
	199	287	380	618	81	324	412	505	118	206	449	537	5	243	331	574	37	130	368	456	74	162	255	493	581	7825	39125
	95	183	296	389	602	220	308	421	514	102	345	433	546	14	227	470	558	46	139	352	595	58	171	264	477	7825	
	586	54	167	260	498	86	179	292	385	623	211	304	417	510	123	336	429	542	10	248	461	554	42	135	373	7825	
7	457	575	38	126	369	582	75	163	251	494	82	200	288	376	619	207	325	413	501	119	332	450	538	1	244	7825	
	328	441	534	22	240	453	566	34	147	365	578	66	159	272	490	78	191	284	397	615	203	316	409	522	115	7825	
	224	312	405	518	106	349	437	530	18	231	474	562	30	143	356	599	62	155	268	481	99	187	280	393	606	7825	39125
	120	208	321	414	502	245	333	446	539	2	370	458	571	39	127	495	583	71	164	252	620	83	196	289	377	7825	
	611	79	192	285	398	111	204	317	410	523	236	329	442	535	23	361	454	567	35	148	486	579	67	160	273	7825	
8	482	600	63	151	269	607	100	188	276	394	107	225	313	401	519	232	350	438	526	19	357	475	563	26	144	7825	
	353	466	559	47	140	478	591	59	172	265	603	91	184	297	390	103	216	309	422	515	228	341	434	547	15	7825	
	249	337	430	543	6	374	462	555	43	131	499	587	55	168	256	624	87	180	293	381	124	212	305	418	506	7825	39125
	20	233	346	439	527	145	358	471	564	27	270	483	596	64	152	395	608	96	189	277	520	108	221	314	402	7825	
	511	104	217	310	423	11	229	342	435	548	136	354	467	560	48	261	479	592	60	173	386	604	92	185	298	7825	
9	382	625	88	176	294	507	125	213	301	419	7	250	338	426	544	132	375	463	551	44	257	500	588	51	169	7825	
	253	491	584	72	165	378	616	84	197	290	503	116	209	322	415	3	241	334	447	540	128	366	459	572	40	7825	
	149	362	455	568	31	274	487	580	68	156	399	612	80	193	281	524	112	205	318	406	24	237	330	443	531	7825	39125
	45	133	371	464	552	170	258	496	589	52	295	383	621	89	177	420	508	121	214	302	545	8	246	339	427	7825	
	536	4	242	335	448	36	129	367	460	573	161	254	492	585	73	286	379	617	85	198	411	504	117	210	323	7825	
10	407	525	113	201	319	532	25	238	326	444	32	150	363	451	569	157	275	488	576	69	282	400	613	76	194	7825	
	278	391	609	97	190	403	516	109	222	315	528	16	234	347	440	28	141	359	472	565	153	266	484	597	65	7825	
	174	262	480	593	56	299	387	605	93	181	424	512	105	218	306	549	12	230	343	431	49	137	355	468	556	7825	39125
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Program 9

Fracticality in 5-Dimensions

The size-5 quintacube in 5-dimensions is way too large to show, so we'll deal with a facsimile of it here. It is depicted in detail over **5** consecutive pages in the book, **Number Magic**.



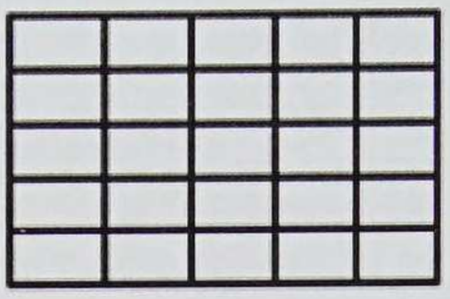


Table is too large to show the numbers.

A size-5 quintacube is composed of 125 5x5 embedded squares that are perfect in that they each have all the equal summations that perfect squares have.

This quintacube can be viewed as a size-5 cube of 5x5 block-squares.

Therefore the quintacube is fractal too but at the block-square level.

Prima-fascia proof of the fractal nature of Space

The perfect size-9 square was first discovered as the result of collapsing a 5-dimensional size-3 quintacube to a 4-dimensional size-3 quadracube.

An absolutely perfect size-9 square could not be derived itself because the size-3 square cannot be expanded to a size-9 square due to the fact that it is imperfect.

Further, constructing a size-9 loom table with the property of having the numbers **1** thru **9** strung equally in all 4 directions, is impossible.

The perfect size-3 quadracube could not be derived from the expansion of the size-3 cube either, as were the size-4 and size-5 quadracubes, because although the size-3 cube is perfect, it is not absolutely-perfect and that was seen as such back in Program 4. There would always be some planar wrap diagonals that would not sum equally.

	1	2	3	4	5	6	7	8	9	
	Y(9)	45	45	45	45	45	45	45	45	45
1	9	7	8	3	1	2	6	4	5	45
2	3	1	2	6	4	5	9	7	8	45
3	6	4	5	9	7	8	3	1	2	45
4	7	8	9	1	2	3	4	5	6	45
5	1	2	3	4	5	6	7	8	9	45
6	4	5	6	7	8	9	1	2	3	45
7	8	9	7	2	3	1	5	6	4	45
8	2	3	1	5	6	4	8	9	7	45
9	5	6	4	8	9	7	2	3	1	45
	45	45	45	45	45	45	45	45	45	
		45	45	45	45	45	45	45	45	45

Here is the derived integer loom table. Observe that both main diagonals each contain only **3** of the **9** numbers with the number **5** shared between them. Yet the loom table is still geonomic. That's pretty amazing!

Consequently, if geonomic space were not fractal throughout, there would be no absolutely-perfect size-9 square. That's prima-fascia proof of the fractal nature of the fabric of space.

Program 9

Generalized Formulas for the Cubic Dual Series

Here are the generalized formulas for generating a series of dual cubes from the original one. It takes three generations to arrive back at the starting cube. The diagram shows that only the original cube will collapse to a square, if Geonometry allows it.

The crossing of directional arrows in the diagram represents the continued exchange of the roles between derived loom tables at each step when navigating from one version of the cube to the next. It takes three generations to arrive back at the starting cube.

The formulas for obtaining the cubic dual $\mathbf{U_1}$ from the primal cube **W** are as follows:

(9.1) $\mathbf{X_1 = modulus\ |W - |1|\ |\ n| + 1}$

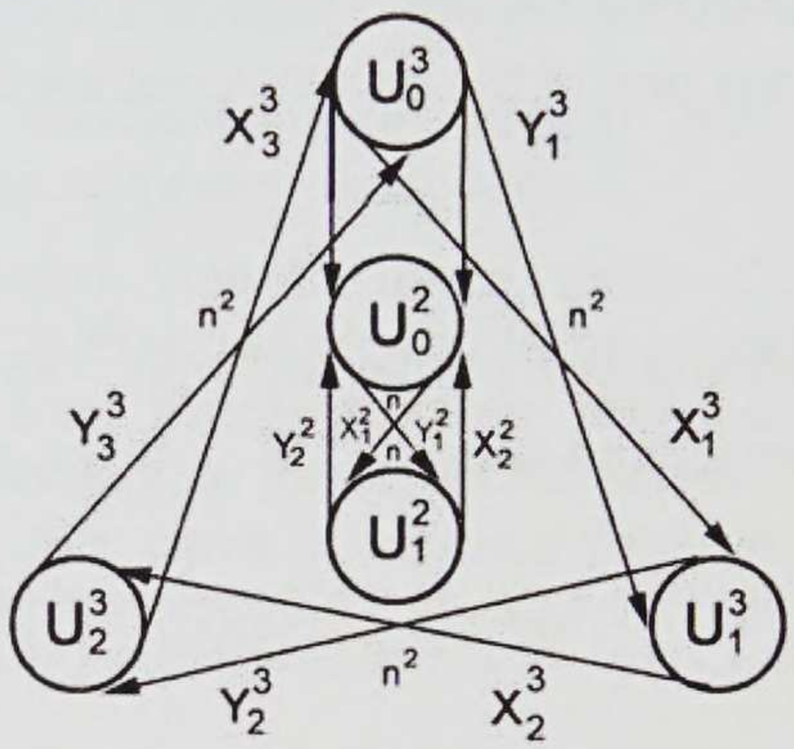
(9.2) $\mathbf{Y_1 = integer\ |(W - |1|) / n| + 1}$ where **|1|** is a cubic matrix of all **1**'s,

from which **W** may be reconstructed from

(9.3) $\mathbf{W = n\ (Y_1 - |1|) + X_1}$

Then the dual $\mathbf{U_1}$ of **W** may be constructed by interchanging the roles of **X** and **Y** and using the base $\mathbf{n^2}$ instead of **n**:

(9.4) $\mathbf{U_1 = n^2\ (X_1 - |1|) + Y_1}$



Next, $\mathbf{U_1}$ can be decomposed into its own distinct complementary loom tables as follows:

(9.5) $\mathbf{X_2 = modulus\ |U_1 - |1|\ |\ n| + 1}$

(9.6) $\mathbf{Y_2 = integer\ |(U_1 - |1|) / n| + 1}$

Now the **dual of the dual**, $\mathbf{U_2}$, can be obtained from the first dual cube $\mathbf{U_1}$ following the same procedure:

(9.7) $\mathbf{U_2 = n^2\ (X_2 - |1|) + Y_2}$

Neither $\mathbf{U_1}$ nor $\mathbf{U_2}$ equals **W**, viz. neither the dual nor the dual of its dual equals the primal cubic version as is otherwise common for squares, where there, the dual of its dual does regenerate the primal square directly. The dual of the dual of the dual of **W** is yet a third absolutely perfect cube, $\mathbf{U_3}$, in the series of duals and this finally does equal **W**.

(9.8) $\mathbf{X_3 = modulus\ |U_2 - |1|\ |\ n| + 1}$

(9.9) $\mathbf{Y_3 = integer\ |(U_2 - |1|) / n| + 1}$

(9.10) $\mathbf{U_3 = n^2\ (X_3 - |1|) + Y_3 = W_3}$

The navigation path is pictured here at right. The diagram shows that only the original cube will collapse to a square, if Geonometry even allows it.

Program 9

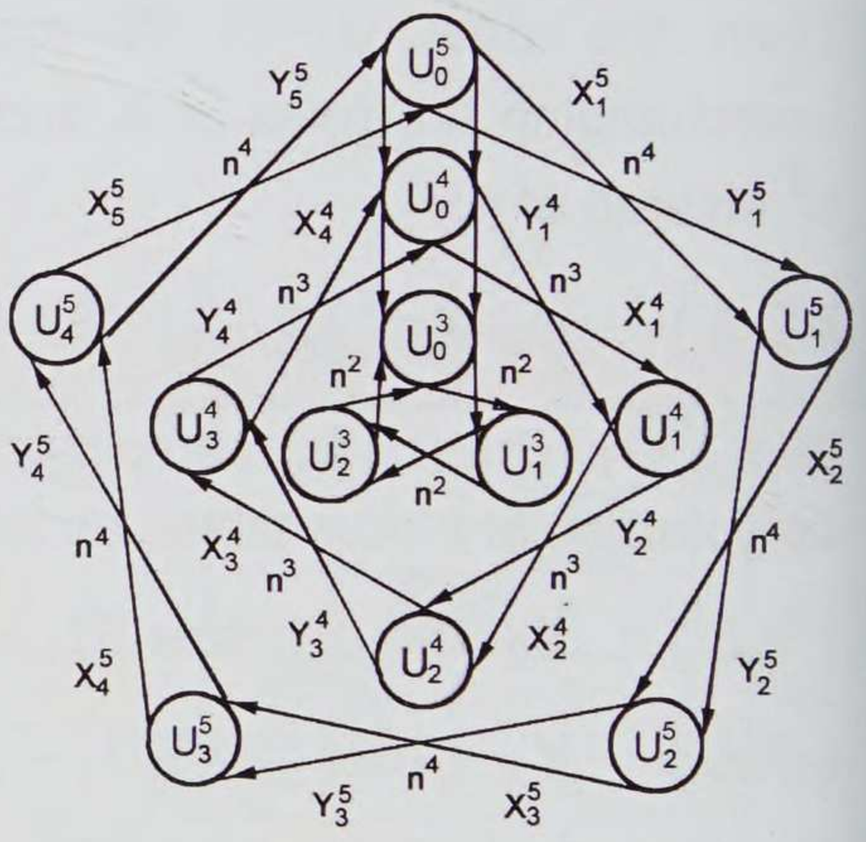
Navigating Multidimensional Space via Class-1 Geonomic Tables

Here is the mathematical generalization of duality theory in Geonometry as it applies to the series of dual geonomic tables of dimension **k**. In dimension **k**, there is a series of successive dual tables that lead back to the original version in **k** generations. This is described by generalized formulas (**9.11**) through (**9.14**). However only the **k-th** dual will collapse down to the next lower dimension of **k -1** through one or more of its depth-sum tables. And that **k-th** dual table is identical to the original geonomic table of dimension **k** back where it all started.

The diagrams at right show this navigational network among dimensions **5** down through **2**.

The crossovers of the links represent the interchange of the roles between the modulus and integer loom tables in each transition to the next successive dual. The downward vertical transitions between adjacent dimensional levels represent the collapse of the depth-sum table along one of the dimensional axes.

Defining the primal table $\mathbf{W^k}$ as $\mathbf{U^k_0}$, and letting $\underline{\mathbf{h}}$ run through the series **1, 2 … k**, we get:



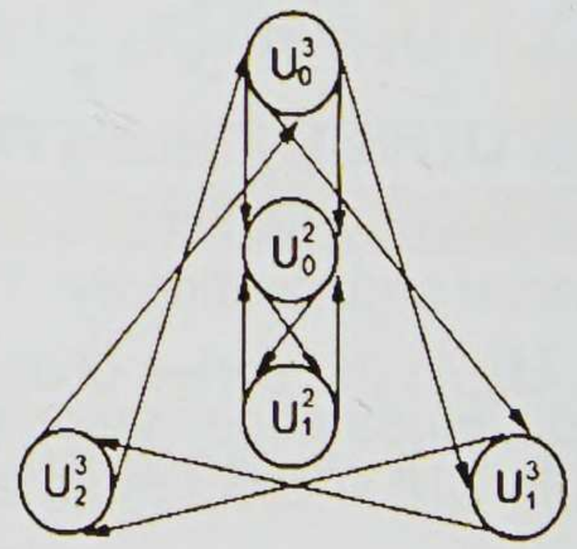
(9.11) $\mathbf{X^k_h = modulus\ [U^k_{h-1} - |1|\ |\ n] + |1|}$

(9.12) $\mathbf{Y^k_h = integer\ [(U^k_{h-1} - |1|)\ /\ n] + |1|}$

(9.13) $\mathbf{U^k_h = n^{k-1}\ (X^k_h - |1|) + Y^k_h}$

(9.14) $\mathbf{U^k_k = U^k_0,}$ in dimension **k.**

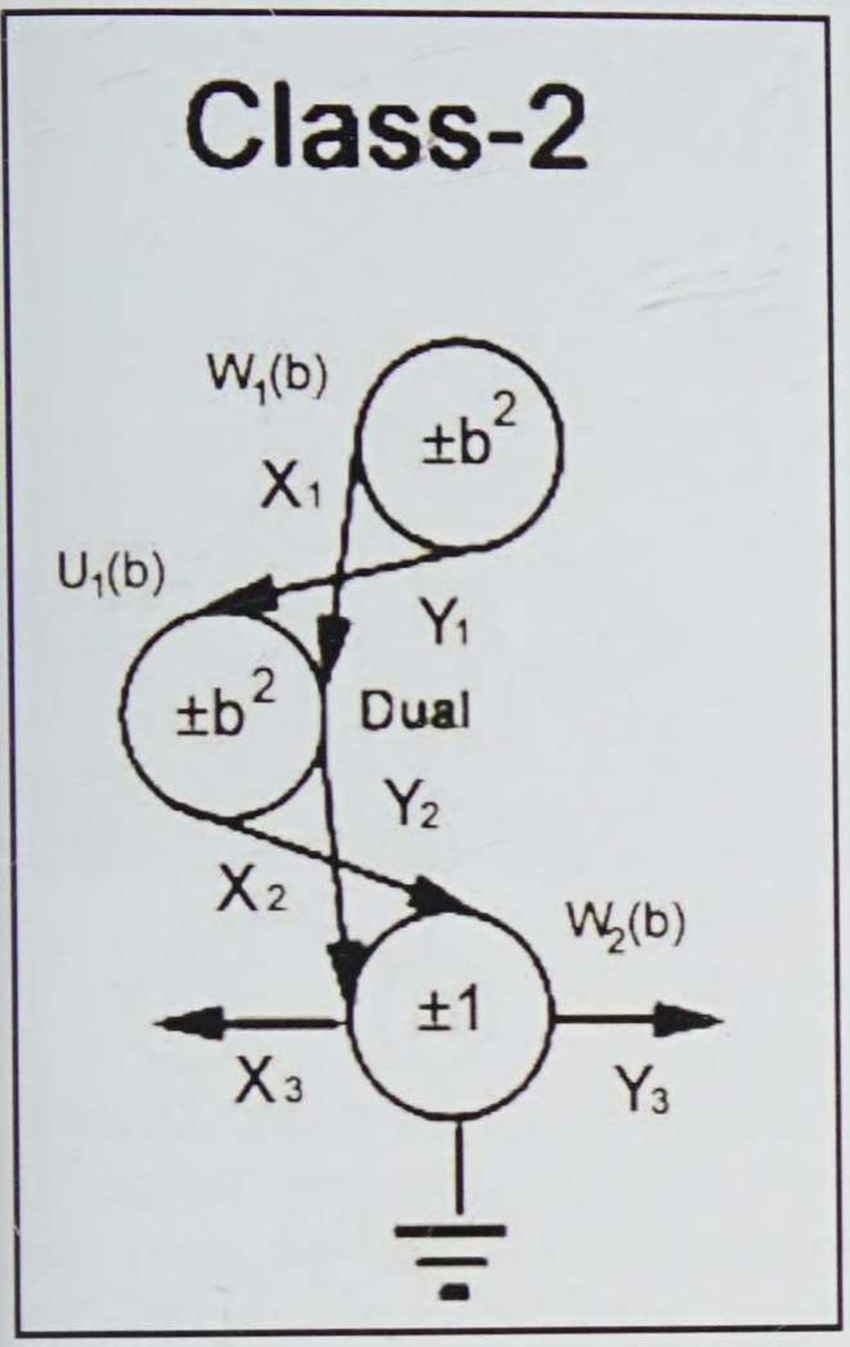
All dual tables derived by this serial dualization process are absolutely ultra-perfect, if their corresponding size square is also, in that they still retain all the equal-summing dual tiling patterns of $\mathbf{W^k}$ within each of their embedded block-squares. However, only $\mathbf{U^k_0}$ will collapse to the corresponding geometry $\mathbf{U^{k-1}_0}$ in the next lower dimension **k-1** by one or more of its depth-sum tables.



Program 9

The Crash and Burn of the dual for Class-2 squares

Here is what happens for a Class-2 square of size **n = 2b** when the generation formula for Class-1 squares is used in the base **n** for deriving the dual of the dual in an attempt to return to the primal square; it ends in a primal square where the deviation between the two unequal rows diminishes from $\pm b^2$ to **±1.** At this point the loom tables derived from the dual become non-geonomic in the base **n** in that their columns do not all sum equally. That is, the generation process becomes grounded; it cannot return to the version of the original primal square.



It was this last version that was used for the primal table in this program because it was that table which had the minimum deviation from the characteristic number. That's why the Class-2 squares have no dual: the grounded version was used as the original primal version.

This minimization of the deviation value through the dual-ization process was another surprising discovery.

Various methods have been tried to make these Class-2 squares function normally, but that all proved to be in vain. So it has been concluded that these squares represent an unavoidable wrinkle or possibly a fundamental seam in the fabric of space. Someday, this class of squares may be used to prove that something cannot or could not exist, like "the Big Bang".

Program 9

k-1 Different navigational paths in the series of Class-1 duals in dimension k

Generalizing (**9.11**) through (**9.14**) even further we get for the next dual **#h** in the series of duals

(9.15) $X_h \equiv \text{modulus}\,[\,U_h - |1|\,|\,n^t\,] + |1|$

(9.16) $Y_h \equiv \text{integer}\,[\,(U_h - |1|)\,/\,n^t\,] + |1|$

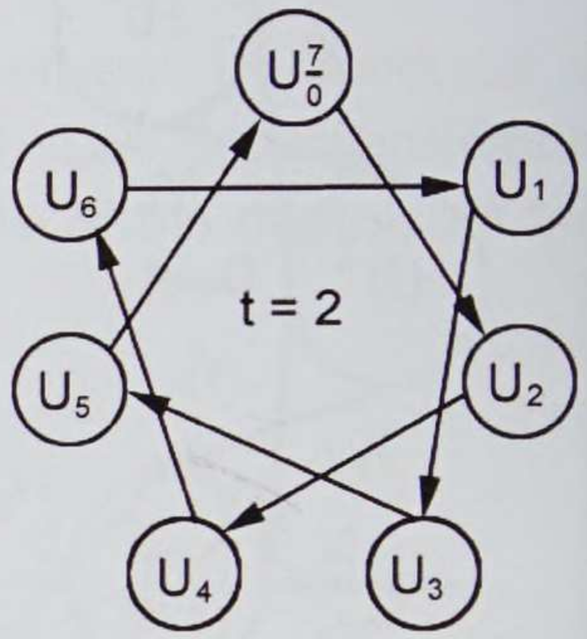
(9.17) $s = \text{modulus}\,[\,h+t\,|\,k\,]$

t is a parameter in this process.

(9.18) $U_s = n^{k-t}\,(X_h - |1|) + Y_h$

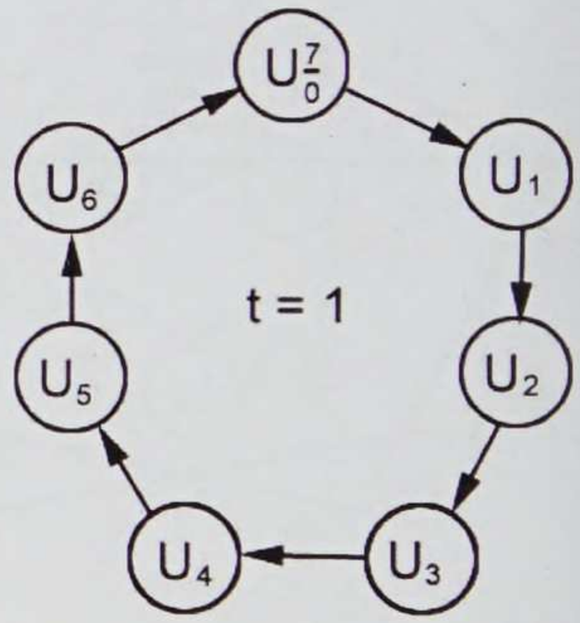
(9.19) $h \leftarrow s$

Starting at (**9.15**) with **h = 0**, repeat the process from (**9.15**) thru (**9.19**) until **h** becomes **0** again in (**9.19**).



The diagrams depict the series of duals for **k=7** and **t = 1, 2, 3** & **4**. Thus starting at (**9.15**) with **h = 0**, one can navigate k-dimensional space in a different sequence than that pictured at the upper left diagram for **t=1**, which was described in the previous slide, by now selecting **t** such that **k > t > 1**.

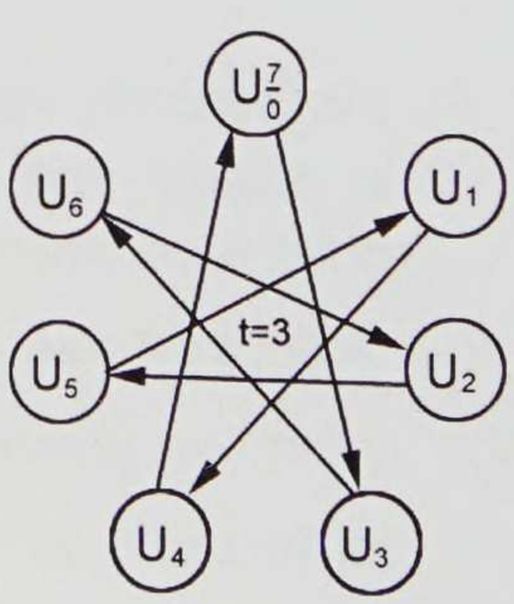
When **k** is an odd prime-number, regard-less of the value of **t** in the range of **k > t ≥ 1**, it will always still take **k** steps to get back to U_0.

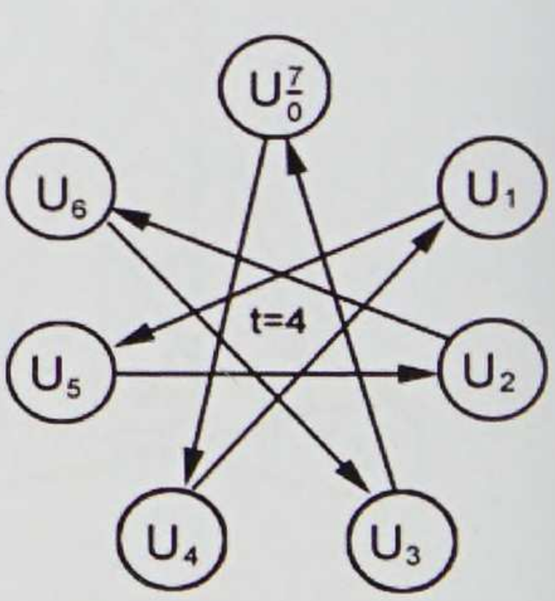


Note that the path for **t = 4** is just the re-verse of the sequence when **t = 3**. When **t = 6**, the path is the reverse of **t=1**. The same holds for when **t = 5**; it is the re-verse of **t = 2**. This is always the case between the paths for **t** and **k–t, t < k**.

Keep in mind that this process only works for Class-1 and Class-6 cubes and hyper-cubes of odd prime-number dimensions.

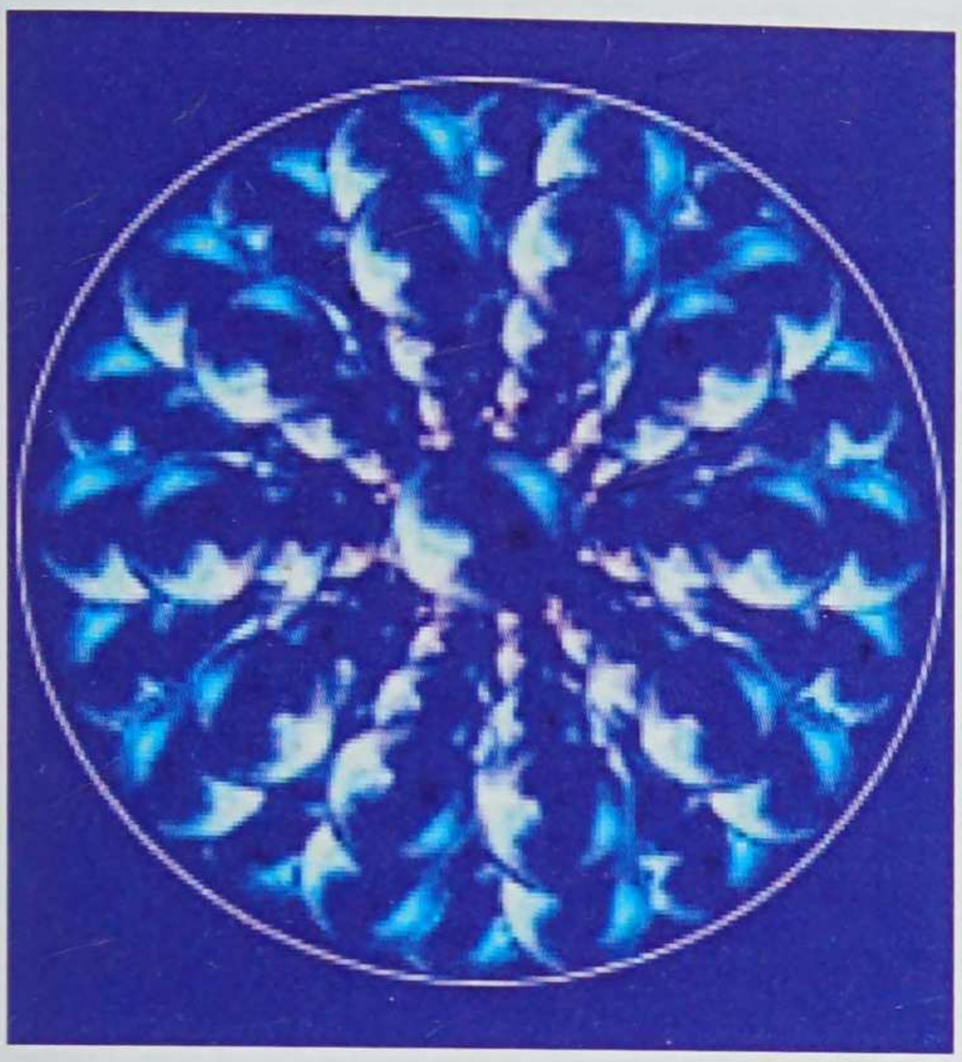
This formulation implies the existence of quantum harmonics throughout prime-number dimensions of space. It's just amazing how both odd-prime-number sizes and odd-prime-number dimensions are so definitive of the properties of multi-dimensional space.

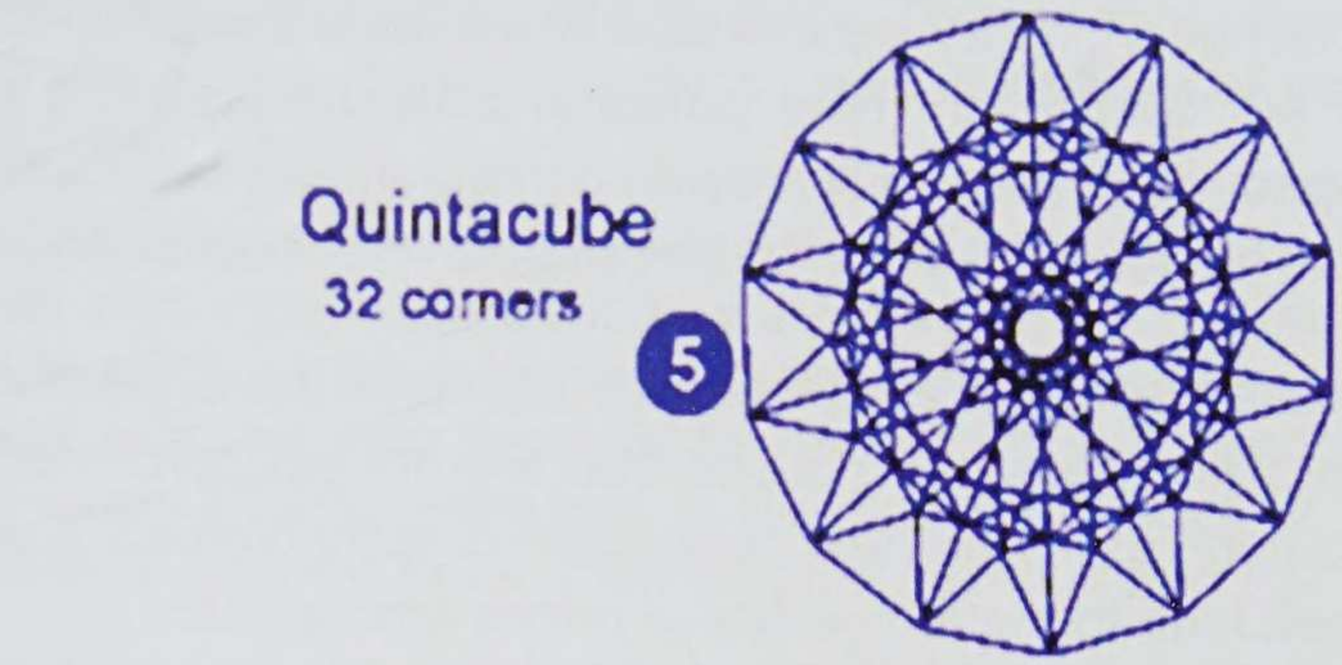


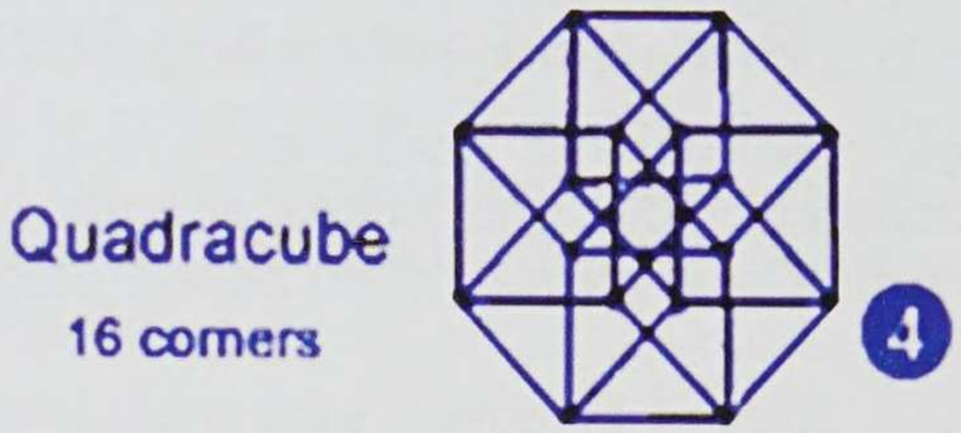


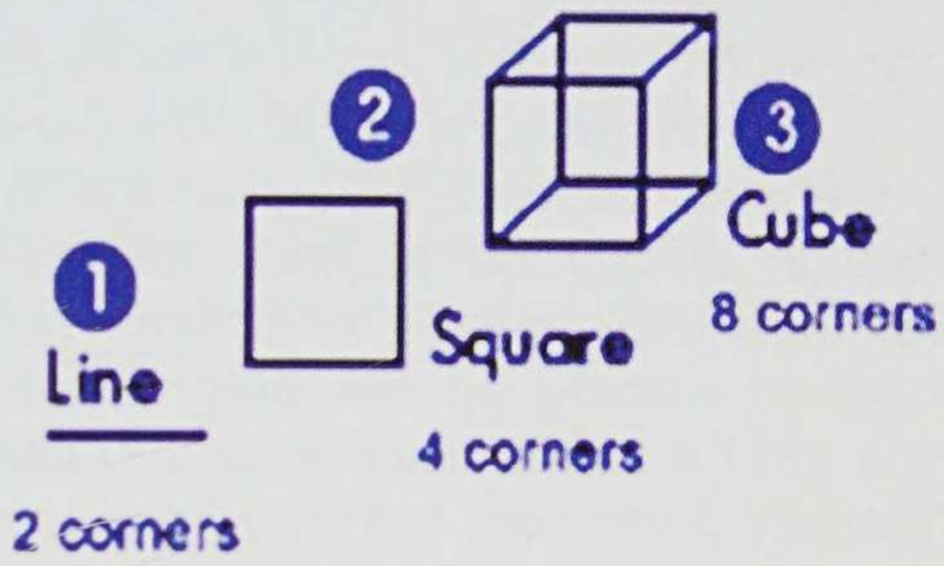
Program 9

The Increasing Complexity of Multi-dimensional Space









The **32** cornered figure at upper right is called a duohexadectal. Remember how the kernel values were linked with circles in squares, spheres in cubes and toruses in quadracubes. This figure corresponds to the quintacube in 5-dimensions.

What curved figure would that deter-mine in 5-dimensions among the 4 integrated quadracubes in a 5-dimen-sional quadracubic projection we saw earlier in this program when 5-dimensions was shown to be fractal? Could it again be another sphere generated from multiple 3D spheres as pictured here at left?

It's pretty obvious to me that 5-dimensions takes the shape of a sphere made up of many spheres like atoms are made of many bundled neutrons and protons. Could this structure go on in both higher and lower dimensions indefinitely? Why would it have to end on some higher or lower dimensional level? Is it just only the 2nd and 3rd dimensions that can support life?

Program 9

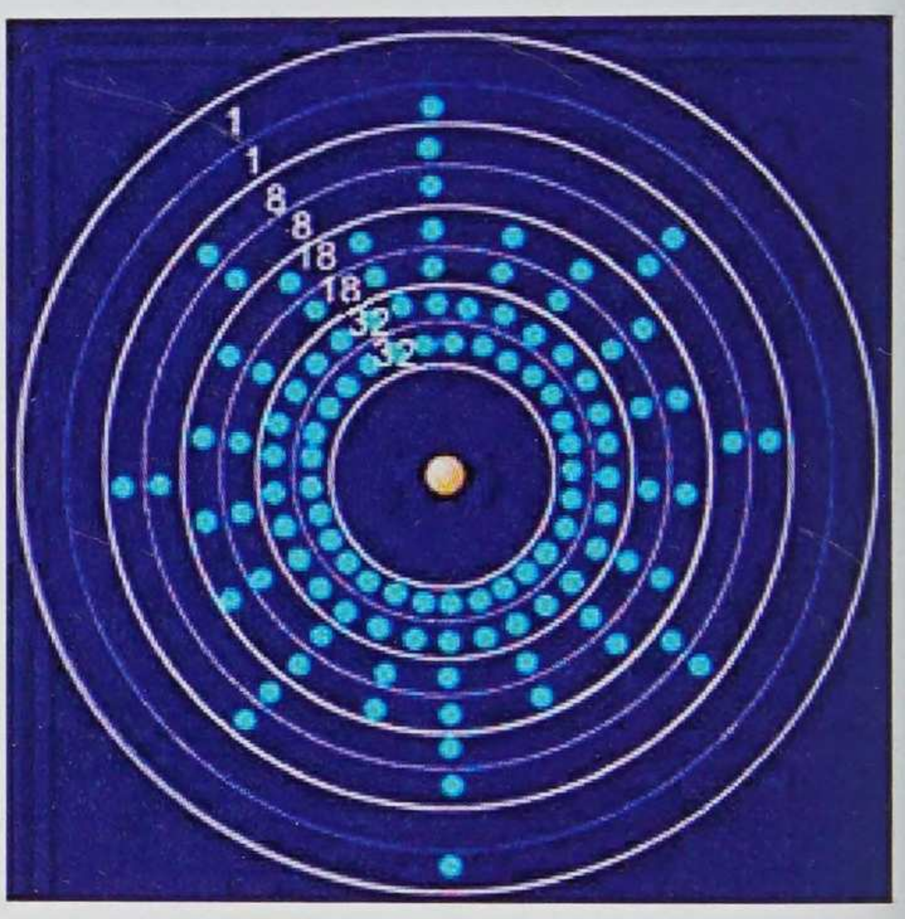
The Big Picture of the Multi-dimensional Universe

With geonometry we are able to evaluate the general nature of any dimensional level. However, it will take much more powerful computers than a PC or laptop to do it and it will also take specially written softwarewith computational capabilities beyond Microsoft's Excel program to do it. The foundation for future research has now been laid by Geonometry.

Next, we'll take a look-see at just what the big-picture of the multi-dimensional universe might look like based on notions that are not yet confirmed by either Geonometry or Science. These notions grant the next generation of technologists a vast arena of exciting exploration that could well lead to numerous Nobel prizes among them.

First, the Penetrating Picture of Sub-dimensional Space

We know that particle accelerators and atom smashers measure their observations in **milliseconds** in **1/3D** and **microseconds** in **1/4D**, so there should be no argument against accepting that time passes at an accelerated pace when descending different sub-dimensional levels.



Given our midpoint perspective of time in 3-dimensions as the basis upon which to relate our tick-tocks to the passage of time in sub-dimensions, which are orders of magnitude faster than our "seconds", the passage of time in the 4th dimension should be at least an order of magnitude slower than the passage of time in our 3rd dimension, just based merely on the progression just cited. So time may not be a dimension in and of itself as the Theory of Relativity presumes, but may be a property of each dimensional level. What happens as an object accelerates to light-speed is just its entry into the 4-th dimension where time is relatively much slower than in 3-dimensional space. And just as Einstein's theory concludes, spacemen would return younger than their Earth-bound counterparts. It's just that the properties of 4-dimensional space remained unknown to Einstein and to this day still remain unknown to Science. Geonometry has provided that missing link here, so now its exploration and even exploitation can proceed.

We have seen just how the basic patterns that are found in Geonometry of the 2nd, 3rd & 4th dimensions can be applied to provide the basic mathematics for interpreting the supposedly scientifically-known atomic structures in sub-dimensions **1/2D**, **1/3D** and **1/4D**, respectively.

It was seen back in Program 5 that Geonometry provided a new reasonable view of how the nested electron-shell pairs were positioned around the nucleus. What that indicates about dimensions below the point of inversion is that sub-dimensional space expands inward, not outward as they do in dimensional spaces above the point-of-inversion.

The sequence of electron addition in going from one atom to the next in increasing atomic number need not follow the sequence of the acquiring shell's location relative to its distance from the nucleus. Atomic-scientists might consider revising their perspective in their viewing of sub-

dimensional space and not view the preceding shell-pairs as being saturated and then being cleaved in half by newly added shells. This will never be known for certain because the electron shells are cloaked by the very vibrational fabric of space itself anyway. This current view by Science was conjured up from trying to make sense of the thousands upon thousands of atom-smasher measurements taken only a half century ago.

Geonometry explains how shells work in pairs, which Atomic Physics fails to do. Considering the lightest shell-pair as a single electron shell which then gets buried by the filling in of other shells of more capacity is just nonsensical. All throughout the addition of electrons, it was seen right here in your companion book to this series from the data provided by Atomic Science itself that there were always **1**, **2** and sometimes even **3** fringe electrons in the outermost shell at the points of transition to the start of the filling-in of the next lower shell with the next **3** electrons.

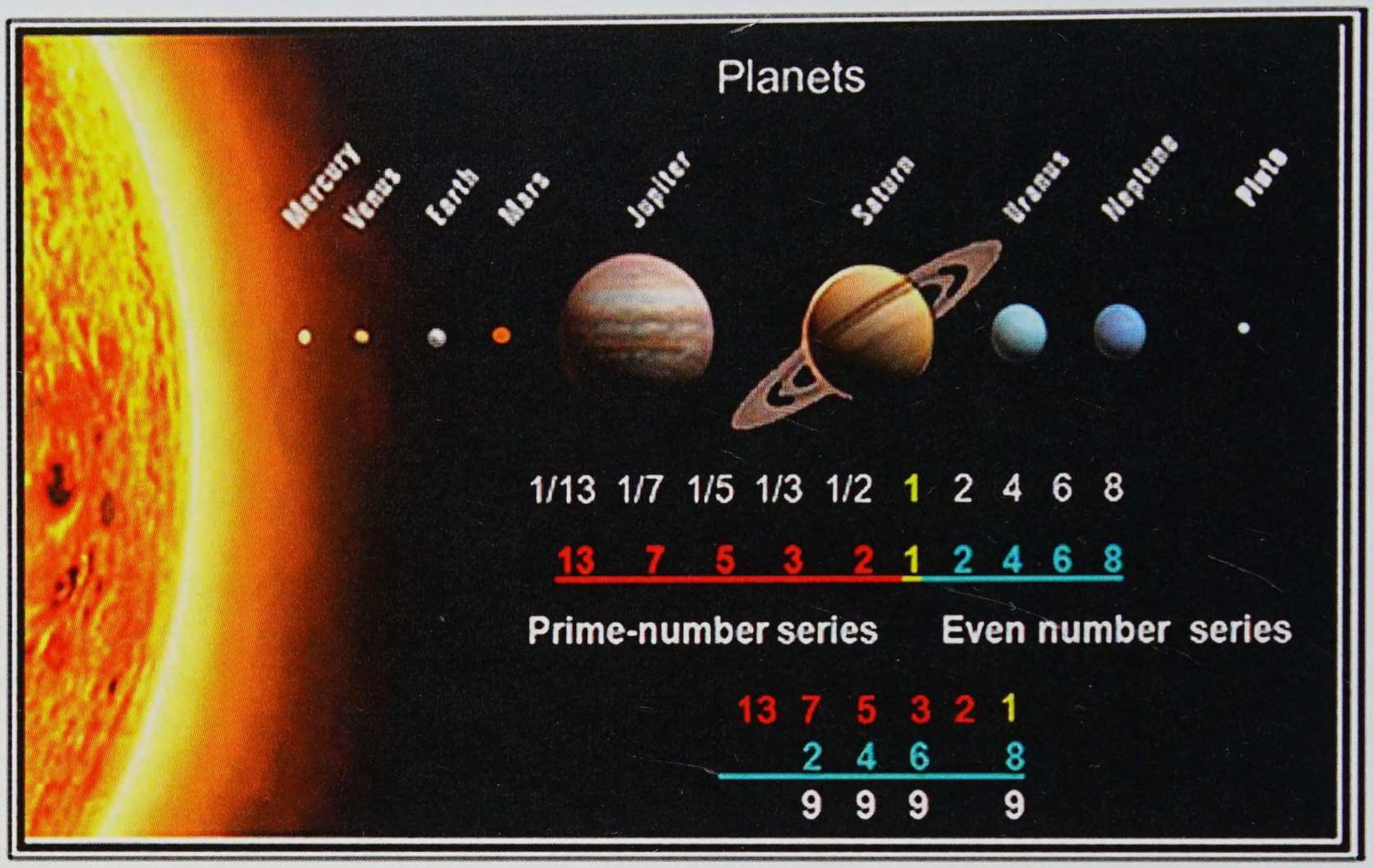
I say that it makes more sense if the shell-pairs, the emphasis is on "pairs", not "shells", to become only half full before the next lower shell begins accepting electrons, on and on, shell to next lower shell, until the lowest shell pair gets completely full. Then the remaining half-pairs begin filling up in succession progressing away from the nucleus, until the outermost shell-pair with only the capacity of 2 electrons becomes filled-in once again. Then the process ends with Ununoctium (#118) having all of its shell-pairs saturated.

Further, Geonometry predicts that there should be two more elements beyond element #118 because there may be another shell-pair #5 residing further inside the known shell-pair #4 having the capacity of **100** electrons ($\mathbf{4x^2 = 100}$ for $\mathbf{x = 5}$) and that shell-pair could tolerate receiving a mere **2** more electrons and protons. It follows from the stability of all elements in Groups I & II (columns) of the Periodic Table that these new elements should be stable too. The hurdle for nuclear scientists to clear is the merging the nuclei of two stable elements together to get one with the electron capacity of **119** or **120**.

Program 9

Next, the Picture of 3-Dimensional Space

Harmonics in the Spacing of Solar System Planets



Here is our solar system. We encountered this slide way back in Program 1. It is shown again here because the next slide will need to refer to the ratios of a few planetary orbital diameters.

Solar System Planets	Distance from Sun in a.u.	Orbit relative to Jupiter's
Mercury	0.39	1/13
Venus	0.72	1/7
Earth	1	1/5
Mars	1.5	1/3
Asteroids	2.7	1/2

Solar System Planets	Distance from Sun in a.u.	Orbit relative to Jupiter's
Jupiter	5.2	1
Satum	9.5	2
Uranus	19.2	4
Neptune	30.1	6
(Pluto)	39.5	8

Program 9

Light-Speeds in 3-Dimensional Space

Let's address something never talked about in schools because everyone <u>believes</u> that light speed is a constant.

Speed of light: 186,000 Mi/Second

Diameter of Earths Orbit / Speed of Light
186,000,000 / 186,000 /second
=1000 seconds = 10^3 seconds

Odds of "1000" even = 1 in a million (1000^2)

Would be the same if measured in kilometers or used a different clock rate because the distance units cancel out and measuring distance from the sun involved using a clock that ticked in seconds.

The Michelson-Morley experiment back in 1887 established the speed of light to be **186,000** miles per second. Then all the astrophysicists took that speed to apply evenly throughout all of space so they would have a means for calibrating cosmic distances. All that which that experiment established is that such is the speed of light on Earth at the distance of the Earth from the Sun; <u>nothing more</u>.

Let's take a closer look at this Michelson-Morley measurement: The center of the Earth is approximately **93** million miles (one astronomical unit) from the center of the Sun, so the diameter of Earth's orbit is **186 million miles**. Now do the math: the measured speed of light is exactly **1000 times the diameter of Earth's orbit**!

186.000,000 mi / 186,000 mi/sec = 1000 sec = 10^3 sec = 1 kilosecond

Or is it an indication of the pace of 3-dimensional time as well as one of the speed of light could be pegged to the distance from a star?

Why is the light speed limit so amazingly correlated with the Earth's orbital distance from the sun? It appears to me that the only thing that <u>is constant</u> is the time of <u>1 **kilo**second</u>. I consider that number to be **thee** 3-dimensional constant, not the light-speed limit. In Geonometry's perspective, that's the unit of time relative to the 3rd dimension.

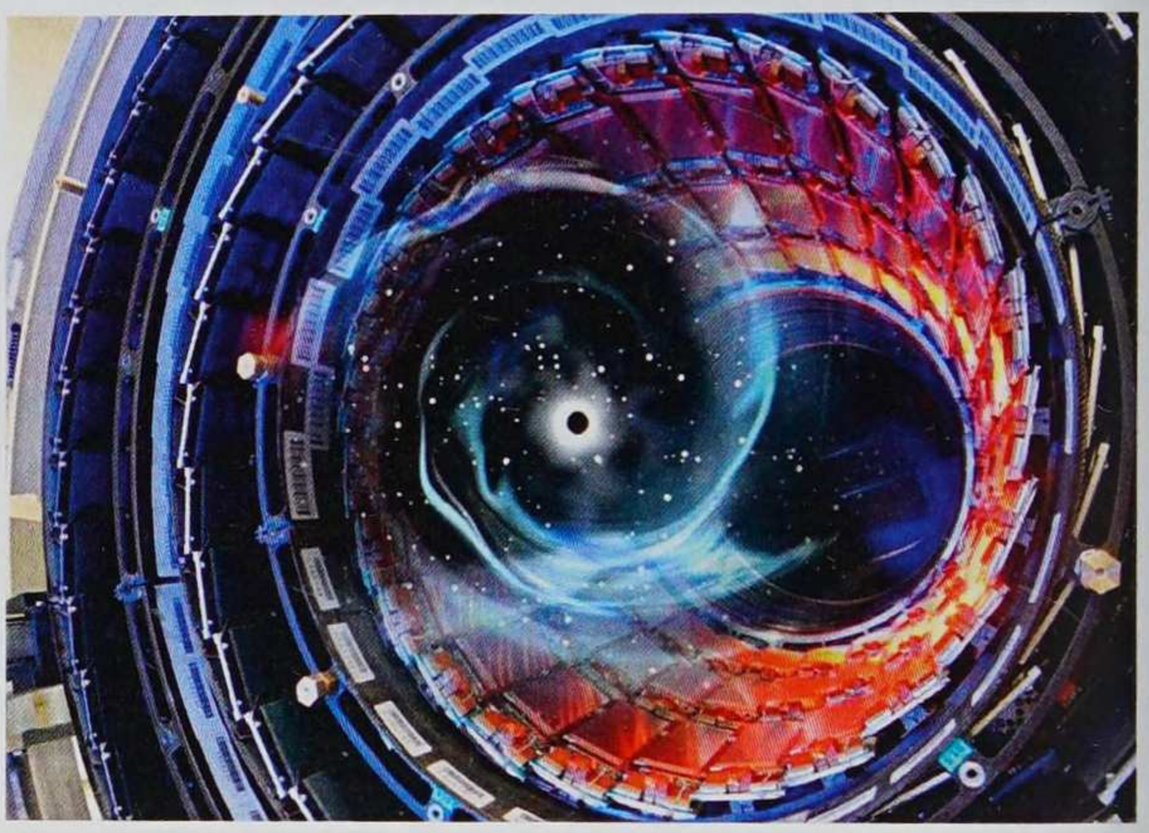
Program 9

Given the measured distances of the planetary orbits, could the speed of light on Mars be **5/3**rds of that on Earth? Could the speed of light on Jupiter be **5** times of that on Earth? That's the ratios of the diameters of their orbits relative to Earth's. We won't know until we at least perform the Michelson-Morley experiment on Mars with distances measured-out physically. That experiment is awaiting the next batch of astronauts, space engineers and astrophysicists.

Could the velocity of time be different on different dimensional levels too?

We know that particle accelerators and atom smashers measure their observations in **milliseconds** in **1/3D** and **nanoseconds** in **1/4D**, so there should be no argument against accepting that time passes at an accelerated pace when descending sequential sub-dimensional levels. So the passage of time in the 4th dimension should be at least an order of magnitude slower than the passage of time in the 3rd dimension, just based merely on the inverse of the progression just cited.

The particle accelerator CERN in Switzerland may someday send particles speeding at 3-dimensional light speed. I conjecture that these particles will never be detected because they will disappear into the 4th dimension before their capture.



I consider black-holes to be the intersection of other universes that are connected via the overlapping of spheres throughout the four 4-dimensional toruses as seen earlier in this program series. And what falls into these black holes emerges in other universes as <u>decomposed elementary matter</u> only to later become embryonic substances for molecular formations there.

And likewise, matter is constantly being fed into our universe as cosmic elementary decomposed matter, the so-called *dark matter*. And what matter arises from this becomes stars, planets and galaxies with a black hole in their center of rotation. Thus the multi-dimensional universe is inter-connected and self renewing.

That is why I don't buy the Big-Bang theory. That theory is an *assumption*, neither provable nor disprovable. It's merely a hopeful <u>conjecture</u> in Science, not a fact. So it has no place in Science.

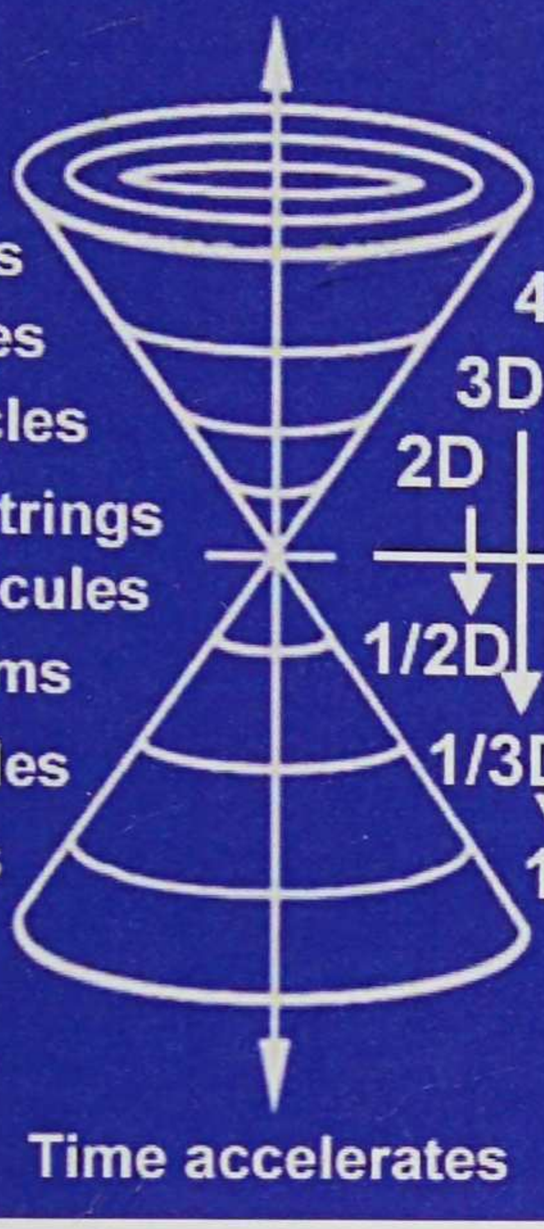
And that could explain the detected continual expansion of our own universe. These embryonic stars in our universe are filling in the voids left by the universe's expansion. It's just the way the multi-dimensional universe interacts among its many intersecting dimensions.

Mankind is just now beginning to poke its collective head out the window on space. Science needs to determine the light-speed limit on Mars and make a determined effort to detect the existence of 4th dimension.

Program 9

Finally, Geonometry vs. Cosmology

Geonometry's Big Picture of the multi-dimensional universe



Dimension	Time Constant	
4	Mega-second	10^{4}
3	Kilosecond	10^{3}
2	Decasecond	10^{2}
1	Second	10^{1}
1/2	Decisecond	10^{-1}
1/3	Microsecond	10^{-2}
1/4	Millisecond	10^{-3}
1/5	Nanosecond	10^{-4}

Below the point of inversion into the sub-dimensional realm, space expands inwardly and time accelerates proportional to the dimensional distance from the point of inversion.

Above the point-of inversion, space expands outwardly and time decelerates proportional to the dimensional distance from the point of inversion.

To the right of the figure is listed the various scientific theories that have emerged for explaining the structure of space at the various dimensional levels. The big effort today in Cosmology is to find some mathematical principle to unite all these formulations into one grand theory of the universe. It's an attempt by Science to discover the superior intelligence underlying the interwoven structure of the spatial fabric.

Even today, String Theory is graduating from the mathematics involving string vibrations to one involving vibrational patterns of membranes, called M-Theory. That is, Cosmology is evolving from a 1-dimensional viewpoint to a 2-dimensional view; one from a linear perspective to one that's planar – supposedly a sign of progress. Yet still Cosmology can't tie all the different dimensional mathematics into a single theory.

Likewise, Geonometry is the expansion of linear arithmetic to the arithmetic of 2-dimensions and beyond. However, Geonometry is the math that does have the potential to uncover that hidden fundamental design without recourse to all that far-out convoluted 11-dimensional math of Cosmology.

Program 9

Rolling wave forms hidden in complementary loom-tables of Class-1 squares

X* Pattern F

-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3
1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2
3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0
-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2
0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3
2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1
-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1
-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3
1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2
3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0
-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2
0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3
2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1
-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1

Y* Pattern G

2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3
1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3
0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2
-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1
-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0
-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1
3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2
2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3
1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3
0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2
-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1
-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0
-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1
3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2

X*+Y*

	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	-4	4	-2	-1	0	1	2	-4	4	-2	-1	0	0
2	-4	-3	5	-1	0	1	2	-4	-3	5	-1	0	1	0
3	-3	5	-1	0	-6	2	3	-3	5	-1	0	-6	2	0
-3	-2	-1	0	1	2	3	-3	-2	-1	0	1	2	3	0
-2	6	0	1	-5	3	-3	-2	6	0	1	-5	3	-3	0
-1	0	1	-5	3	4	-2	-1	0	1	-5	3	4	-2	0
0	1	2	-4	4	-2	-1	0	1	2	-4	4	-2	-1	0
1	2	-4	4	-2	-1	0	1	2	-4	4	-2	-1	0	0
2	-4	-3	5	-1	0	1	2	-4	-3	5	-1	0	1	0
3	-3	5	-1	0	-6	2	3	-3	5	-1	0	-6	2	0
-3	-2	-1	0	1	2	3	-3	-2	-1	0	1	2	3	0
-2	6	0	1	-5	3	-3	-2	6	0	1	-5	3	-3	0
-1	0	1	-5	3	4	-2	-1	0	1	-5	3	4	-2	0
0	1	2	-4	4	-2	-1	0	1	2	-4	4	-2	-1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Here are the anti-symmetric loom-tables o the size-7 put into four connected block squares making from them a size-14 loom table enlargement.

The loom tables as have been portrayed do not expose the inherent embedded waveform. To do that, they need be balanced anti-symmetrically. This is done by reducing the numbers in each loom table by its dimensional average $\bar{x} = \bar{y} = (n+1)/2$.

The patterns on **X*+Y*** show that it possesses a continuous **±6** rolling wave pattern.

This procedure will fathom a continuous wave pattern for every Class-1 square. Consequently, it is these patterns that should be explored for propulsion.

These rolling waveforms in Class-1 squares are ripe for further exploration.

Program 9

Just as we have used the undulating patterns in the oceans to surf its waves,

and in times of civilizations past when we used the sails on boats to capture the circular patterns of wind to cross the ocean-divides and discover new continents,

it may now be possible to identify the spatial harmonic vibrations among the vibrating membranes within the fabric of space itself that could well be harnessed to surf the universe throughout 3-dimensional and 4-dimensional space. We just have to tap into the fabric of 3-dimensional space to propel the spaceship into hyperspace.

Program 9

Space Travel and Levitation Using the Layered Fabric of 2- and 3-Dimensional Space

We have finally arrived at the promised major application of Geonometry – it's potential fo levitation and space propulsion

First, we'll look at Class-4 squares which also have continuous tiling patterns. The smallest size Class-4 square whose loom-tables independently possess both tiling patterns ◊ and **X** is the size-16. Here are the loom-tables **X** and **Y**. We have observed that both are pairwise centrally symmetrical. If each loom-table is flipped both vertically and horizontally and added back to itself, we get a table of all constants **Z** in both cases. If this pattern were to be projected onto a confined fabric of space of size-16, a wall of positive constants would emerge – and this would provide a traveling platform to push against to travel through "empty" space.

X(16)	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136
	8	9	1	16	8	9	1	16	8	9	1	16	8	9	1	16	136
	10	7	15	2	10	7	15	2	10	7	15	2	10	7	15	2	136
	14	3	11	6	14	3	11	6	14	3	11	6	14	3	11	6	136
	4	13	5	12	4	13	5	12	4	13	5	12	4	13	5	12	136
	5	12	4	13	5	12	4	13	5	12	4	13	5	12	4	13	136
	11	6	14	3	11	6	14	3	11	6	14	3	11	6	14	3	136
	15	2	10	7	15	2	10	7	15	2	10	7	15	2	10	7	136
	1	16	8	9	1	16	8	9	1	16	8	9	1	16	8	9	136
	8	9	1	16	8	9	1	16	8	9	1	16	8	9	1	16	136
	10	7	15	2	10	7	15	2	10	7	15	2	10	7	15	2	136
	14	3	11	6	14	3	11	6	14	3	11	6	14	3	11	6	136
	4	13	5	12	4	13	5	12	4	13	5	12	4	13	5	12	136
	5	12	4	13	5	12	4	13	5	12	4	13	5	12	4	13	136
	11	6	14	3	11	6	14	3	11	6	14	3	11	6	14	3	136
	15	2	10	7	15	2	10	7	15	2	10	7	15	2	10	7	136
	1	16	8	9	1	16	8	9	1	16	8	9	1	16	8	9	136
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136

Y(16)	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136
	1	7	3	5	12	14	10	16	9	15	11	13	4	6	2	8	136
	12	14	10	16	1	7	3	5	4	6	2	8	9	15	11	13	136
	4	6	2	8	9	15	11	13	12	14	10	16	1	7	3	5	136
	9	15	11	13	4	6	2	8	1	7	3	5	12	14	10	16	136
	5	3	7	1	16	10	14	12	13	11	15	9	8	2	6	4	136
	16	10	14	12	5	3	7	1	8	2	6	4	13	11	15	9	136
	8	2	6	4	13	11	15	9	16	10	14	12	5	3	7	1	136
	13	11	15	9	8	2	6	4	5	3	7	1	16	10	14	12	136
	5	3	7	1	16	10	14	12	13	11	15	9	8	2	6	4	136
	16	10	14	12	5	3	7	1	8	2	6	4	13	11	15	9	136
	8	2	6	4	13	11	15	9	16	10	14	12	5	3	7	1	136
	13	11	15	9	8	2	6	4	5	3	7	1	16	10	14	12	136
	1	7	3	5	12	14	10	16	9	15	11	13	4	6	2	8	136
	12	14	10	16	1	7	3	5	4	6	2	8	9	15	11	13	136
	4	6	2	8	9	15	11	13	12	14	10	16	1	7	3	5	136
	9	15	11	13	4	6	2	8	1	7	3	5	12	14	10	16	136
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136

Z1 =X+hvX-|1 |

16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

Z2 =Y+hvY-|1 |

16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

Program 9

Building a Wall in Empty Space

The basic pattern of all spatial vibrations take place at the loom-table level. Here's what was learned from Geonometry for the Class-4 size-28 square: It was found that the size-28 square contained **3** continuously equal-summing even-sized diamond patterns of consecutive sizes **5**, **6** & **7** which summed to **5**, **8** and **12** times $\mathbf{C_{28}}$.

Shown here is its modulus loom-table **X(7)** derived from the primal square in the base **7**.

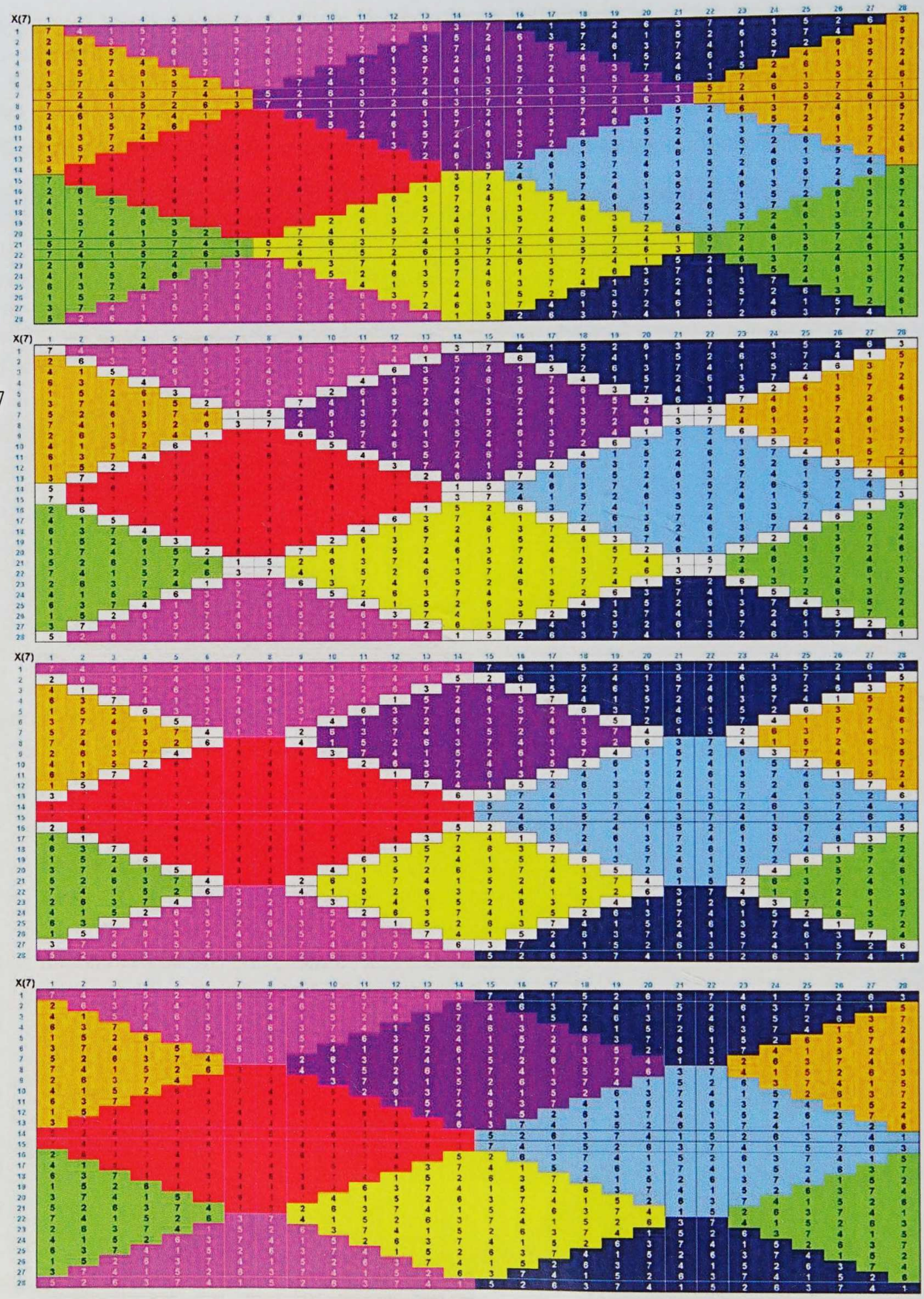
X(7)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	112
	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112
1	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	112
2	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	112
3	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	112
4	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	112
5	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	112
6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	112
7	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	112
8	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	112
9	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	112
10	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	112
11	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	112
12	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	112
13	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	112
14	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	112
15	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	112
16	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	112
17	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	112
18	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	112
19	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	112
20	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	112
21	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	112
22	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	112
23	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	112
24	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	112
25	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	112
26	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	112
27	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	112
28	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	112
	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112

Z(7) is computed from **Z(7) = X(7) + vhX(7) - | 1 | = | 7 |**. So the size-28 square space can be flattened to vibrate at a constant vibration of **7** λ where the actual basic spatial frequency is λ.

Z(7)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	196
	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	##	196	196
1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
5	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
8	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
9	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
10	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
11	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
12	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
13	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
14	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
15	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
16	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
17	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
18	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
19	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
20	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
22	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
23	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
24	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
26	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
27	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	196
28	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	##	196	
	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	##	196	196
	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	196	##	196	196

Program 9

The Space Propulsion Sequence

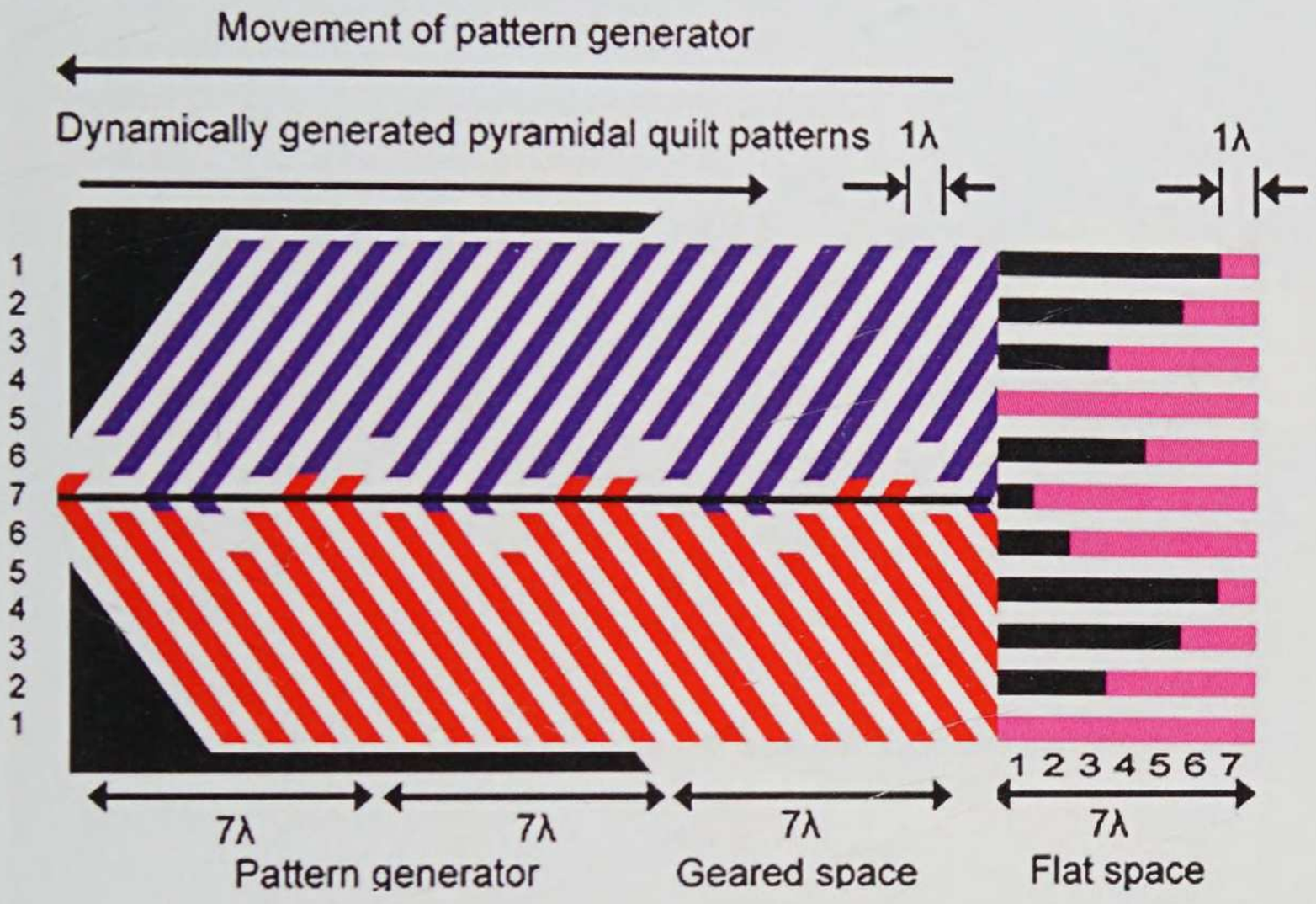


Program 9

The Spatial Gear for Space Propulsion

The size-28 vibrational patterns could possibly be harnessed for space flight as follows:

1. First, the scale ***λ*** at which **a circle** of space is vibrating is determined.
2. Second, this scale is used to determine a **28*λ* x 28*λ*** window onto space at a distance of **7*λ***.
3. Third, the vibrational space in the window is converted to vibrate at **X(7*λ*)**.
4. Fourth, a projection of **hvX(7*λ*) - |1|** is shot at this square space, flattening the space to vibrate at a constant **7*λ***.
5. Then a series of **28*λ* x 28*λ*** geonomic frequency in pyramidal form is shot at the flattened space, followed by the oscillating patterns on the prior page in rapid secession while maintaining the flattened space upon which to push.



The reason that the size-5 diamond was brought into play is that all three sizes of diamonds, **5**, **6** & **7,** have hooks into 3-dimensional space via their continuous pyramidal diamond summations. It's akin to having the equivalent of a continuous chain-link gear in 3-dimensions.

Here is the gear sequence as seen from the side: **blue** is the original size-7 diamond; **red** is the original size-6 diamond.

The window can be broadened to cover as much multiple spatial area as the engine's internal power would allow. Two different kinds of engines are involved here: one to maintain the flattening of the spatial window and another to provide the thrust against the solid flattened background. Here's what is being dynamically projected:

First the original integrated two-size diamond pattern is projected. Next, the size-7 diamond patterns are reduced to size-6. (Refer to the pattern sequence on the page at left.)

Then, the reduced diamonds are reduced further to size-5, while simultaneously the original size-6 diamonds are enlarged to a size-7.

Then, the size-5 diamonds are enlarged back to size-6, closing the gap. This exchanges the sizes of the diamonds in the integrated quilt pattern from **7** to **6** and from **6** to **7**.

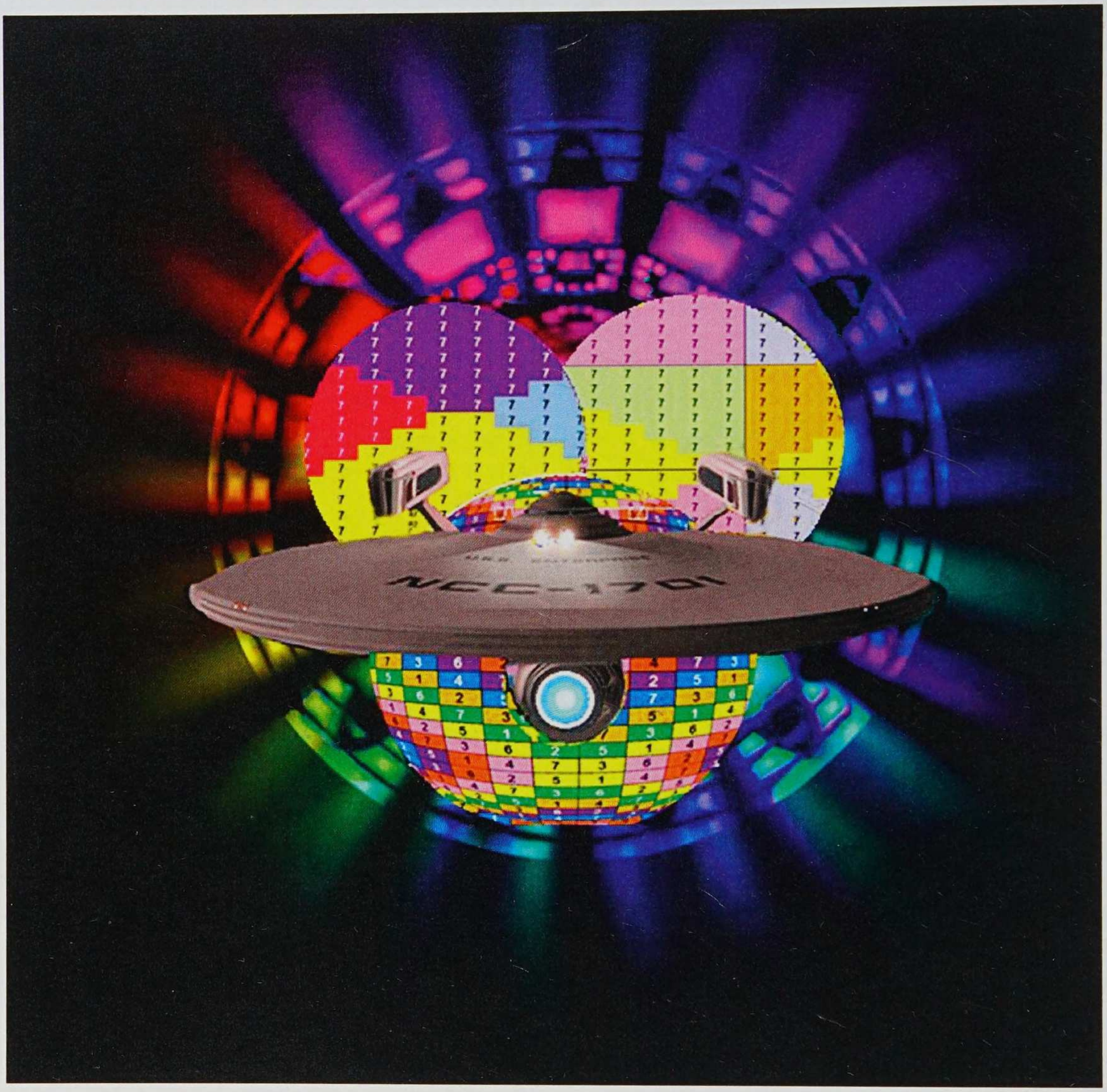
Finally, the process is applied again but upon the newly transformed quilt pattern, reversing the first four steps. Thus in **7** steps, the pattern generator engine advances a distance of **7*λ***.

The process is repeated continuously.

Program 9

Navigating the Fabric of Space

This picture is a virtual representation of how the spaceship such as the famous **Starshi** **Enterprise** would appear upon breaking through the light-speed barrier of the 3rd dimension a seen from within 4-dimensional space as it was breaking through.



Note the harmonic patterned trail it leaves behind it as the thruster engine below the spaceshi pushes against the solid wall created by the two upper pulsating engines, one for generating th integrated diamond pattern of vibrations and the other for generating an integrated triangula pattern.

The engine below the craft generates the geared propulsion sequence just described.

Although today this is mere fantasy, once the spatial fabric is actually discovered, analyzed an categorized, it could be harnessed for levitation and spatial propulsion in a relatively short order.

Program 9

The discovery that ushered in the Industrial revolution

Here is the complementary relationship between electric and magnetic fields in an electromagnetic wave. This fundamental complementarity was fathomed long after electricity and magnetism were discovered independently.

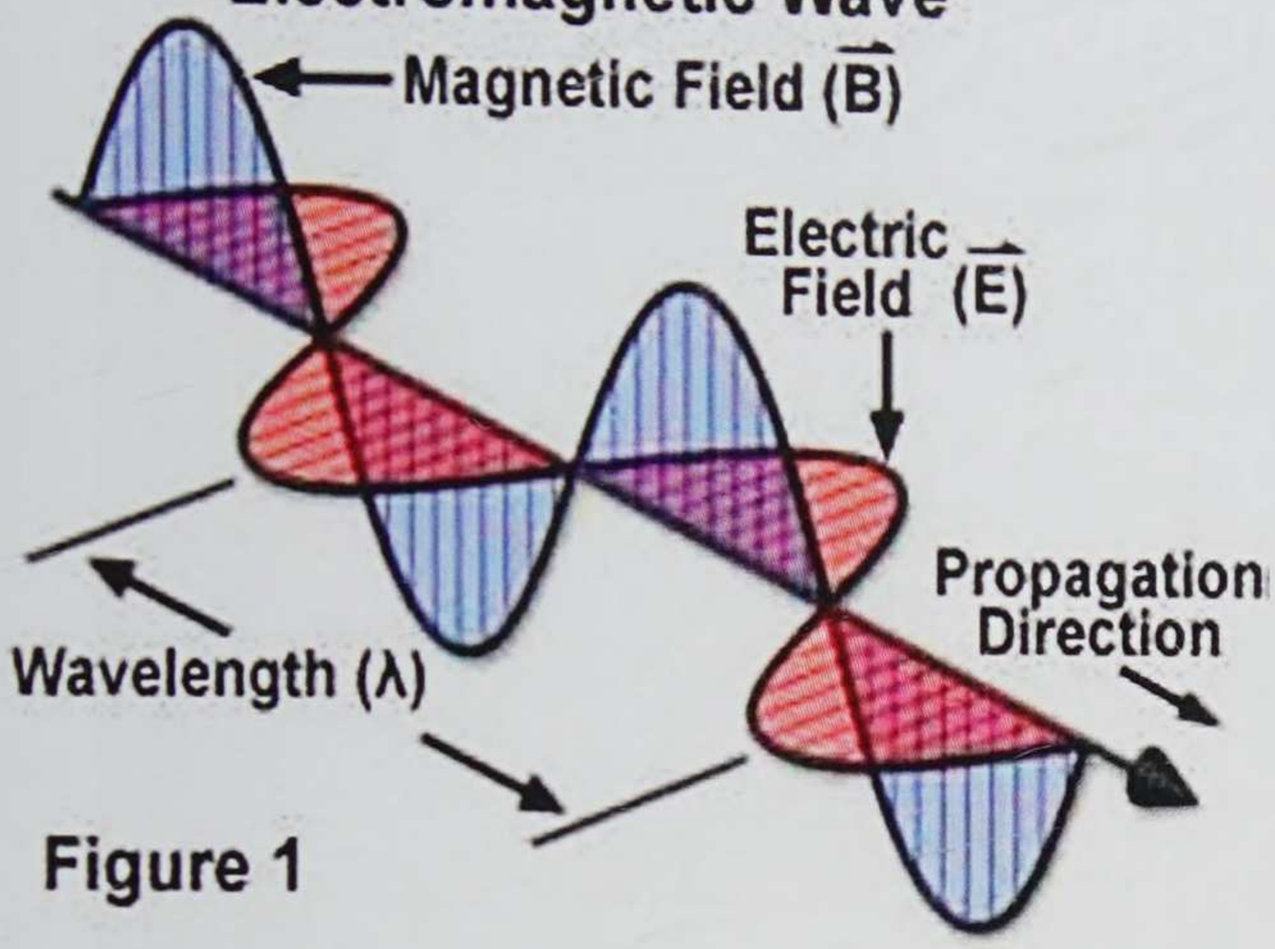


Figure 1

Before electromagnetism was harnessed, this relationship between magnetic and electric fields was unknown. Only by trial-and-error research was this fundamental relationship discovered. And once discovered, electric motors and generators were soon developed right afterwards and the industrial age was ushered in.

The point being made here is that just as a 3-dimensional electromagnetic wave lies at the heart of modern-day electric motors, the complementarity and interwoven cross-patterns of loom-tables that were shown throughout **2D** potentially lay at the heart of the vibrational **3D** spatial fabric.

Program 9 Notes

1. Geonomic space can be navigated by a series of successive Class-1 duals in 3-dimensions and higher.

2. The complete 4-dimensional space consists of four intersecting toruses forming a hyper-sphere as seen from the 5th dimension.

3. Dimensions 4 and 5 are fractal in that Class-1 size **n** quadracubes and quintacubes project onto to 2-dimensional space as perfect size $\mathbf{n}^2$ Class-6 squares. Further, these projections are the only way for Class-6 squares to acquire **nx**-modularity.

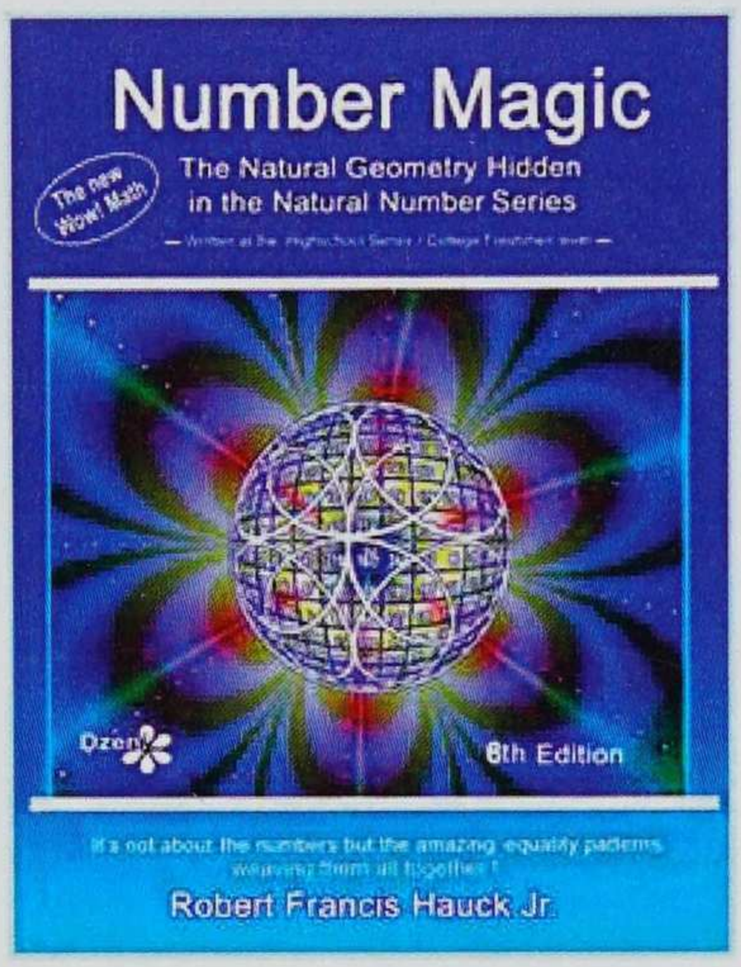
4. The series of kernel numbers which equal the characteristic number for hypercubes of any dimension can be expressed as a function solely of its size **n**.

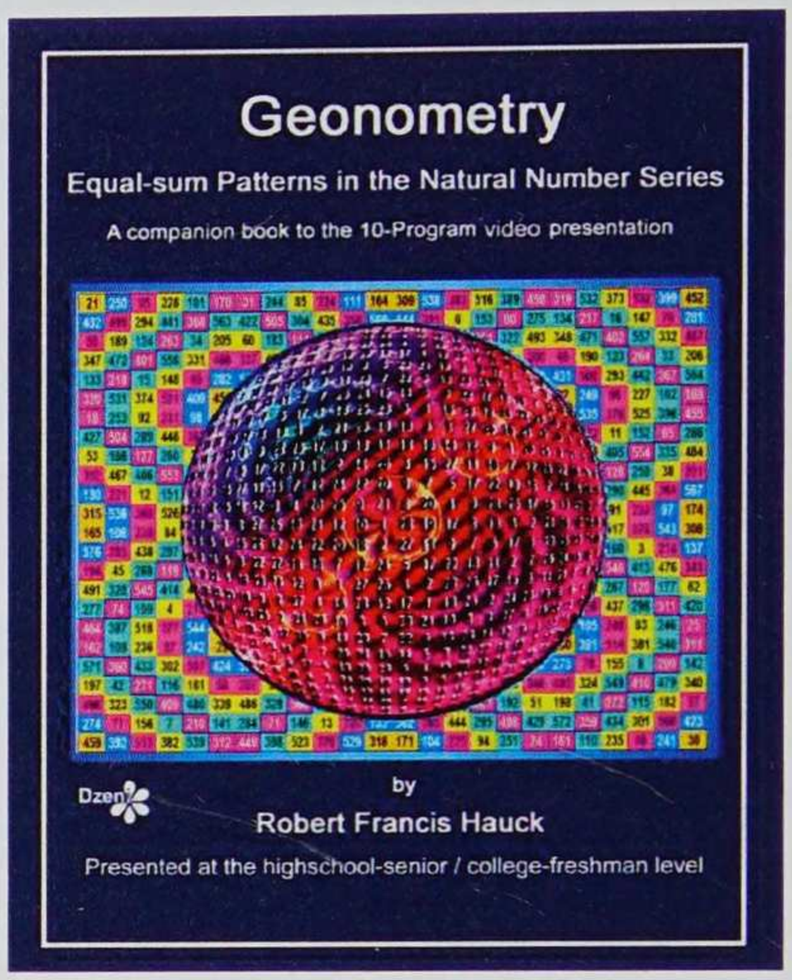
5. The spatial fabric which lies at the heart of loom-table patterns is described as a potential source for exploration of propulsion and levitation in the next decade utilizing pulsars and lasers and maybe some newly developed spatial oscillator.

Program 9

Here is the master book **Number Magic** which contains much of the material presented in this program series plus containing numerical tables for all legible sizes of square, cube and hypercubes of 4 & 5-dimensions. The only book which contains all of the material presented in this program series is **Geonometry**. You should have had in it your possession throughout this entire program series.

It is available only from the publisher's internet website **www.CreateSpace.com**





Number Magic – The Natural Geometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension.

Ninth Edition
Black & white print (360+ pages)

Geonometry – Equal-sum Patterns in the Natural Number Series

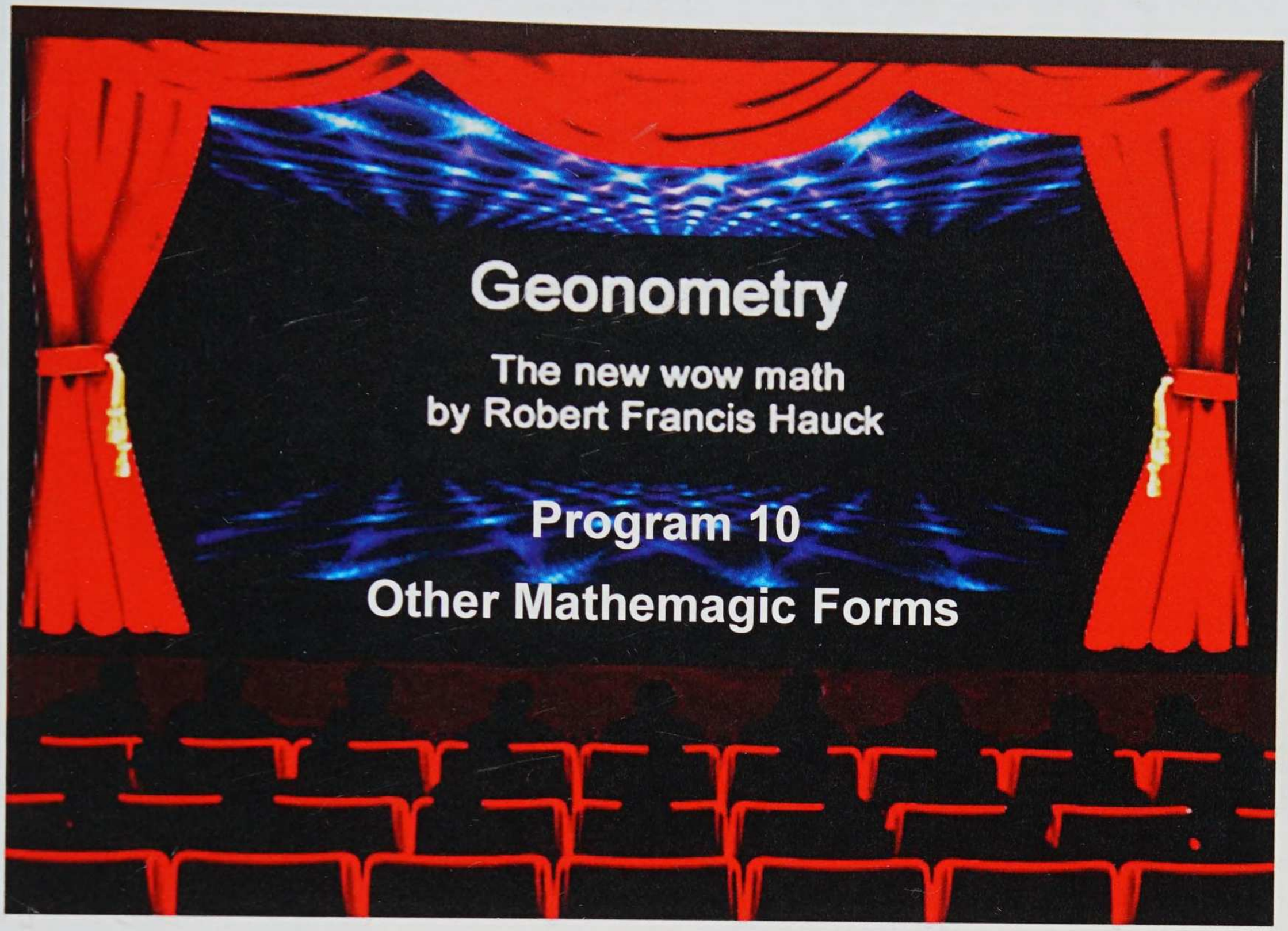
ISBN 978-1-505-67225-1

Contains all the slides and narration in this 10-program series.
Selected examples.

Third Edition
Printed in color. (500 pages)

In the next program, we will explore other geonomic forms such as triangles, diamonds, pentagons and hexagons.

There we will discover through the application of Geonometry to hexagons just what makes snowflakes so hexagonally symmetrically perfect. Scientists are to this day unable to systematically explain this phenomenon by the chemical makeup of water alone. That program will provide yet the final fundamental proof that Geonometry is truly uncovering the real hidden fabric of space.



Overview

First we'll see geonomic triangles.

Then we'll look at generating geonomic diamonds from perfect adjacent-sized geonomic squares. And in the process, we'll discover a second method that when it is applicable will yield an identical result as the brute-force method does but in a very obtuse manner.

Next, we'll observe a single nested-frame pentagon. Then we'll examine a pentagon composed of five equal summing triangles.

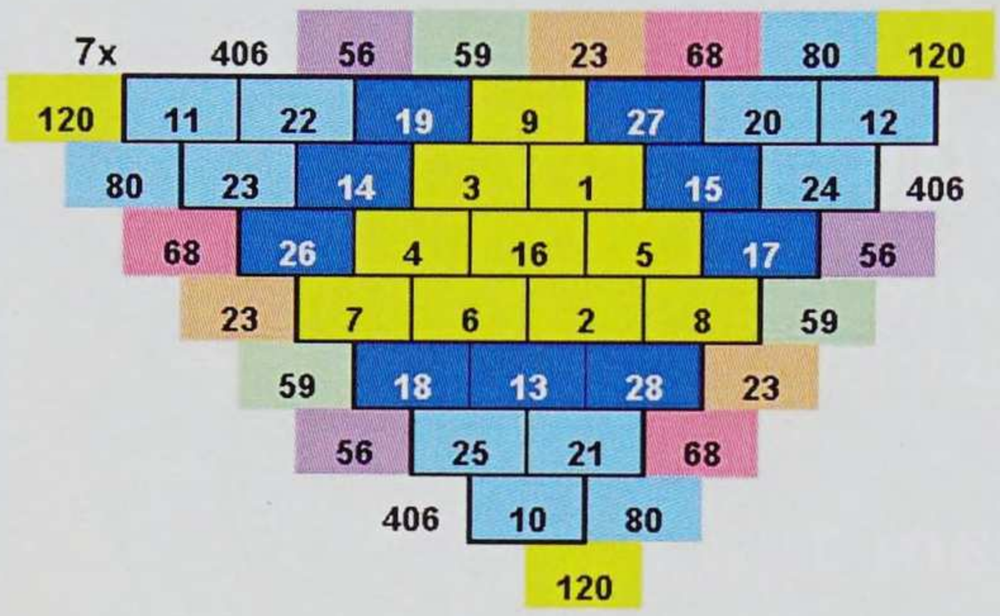
After pentagons, we'll look at generating geonomic hexagons from hexagonal loom-tables that employ some of the same basic principles we discovered earlier in squares. Here we will observe the inherent vibratory membrane for snowflake formation.

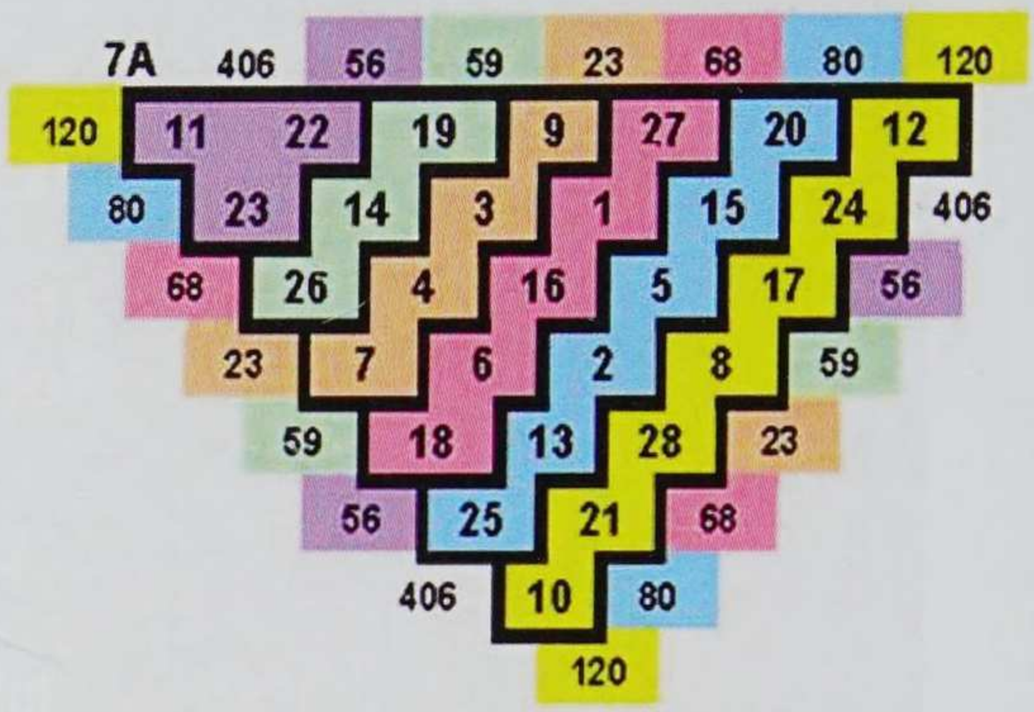
Geonometry shows definitively just what makes snowflakes so hexagonally symmetrically perfect. Scientists still to this day are unable to systematically explain this phenomenon by the chemical makeup of water and the static electricity of vapor. In otherwords, Science cannot use its own knowledge resources to provide even a near plausible explanation that doesn't rely on some undetectable perfectly symmetrical force. It demonstrates once again that Geonomics is uncovering the real hidden fabric of space.

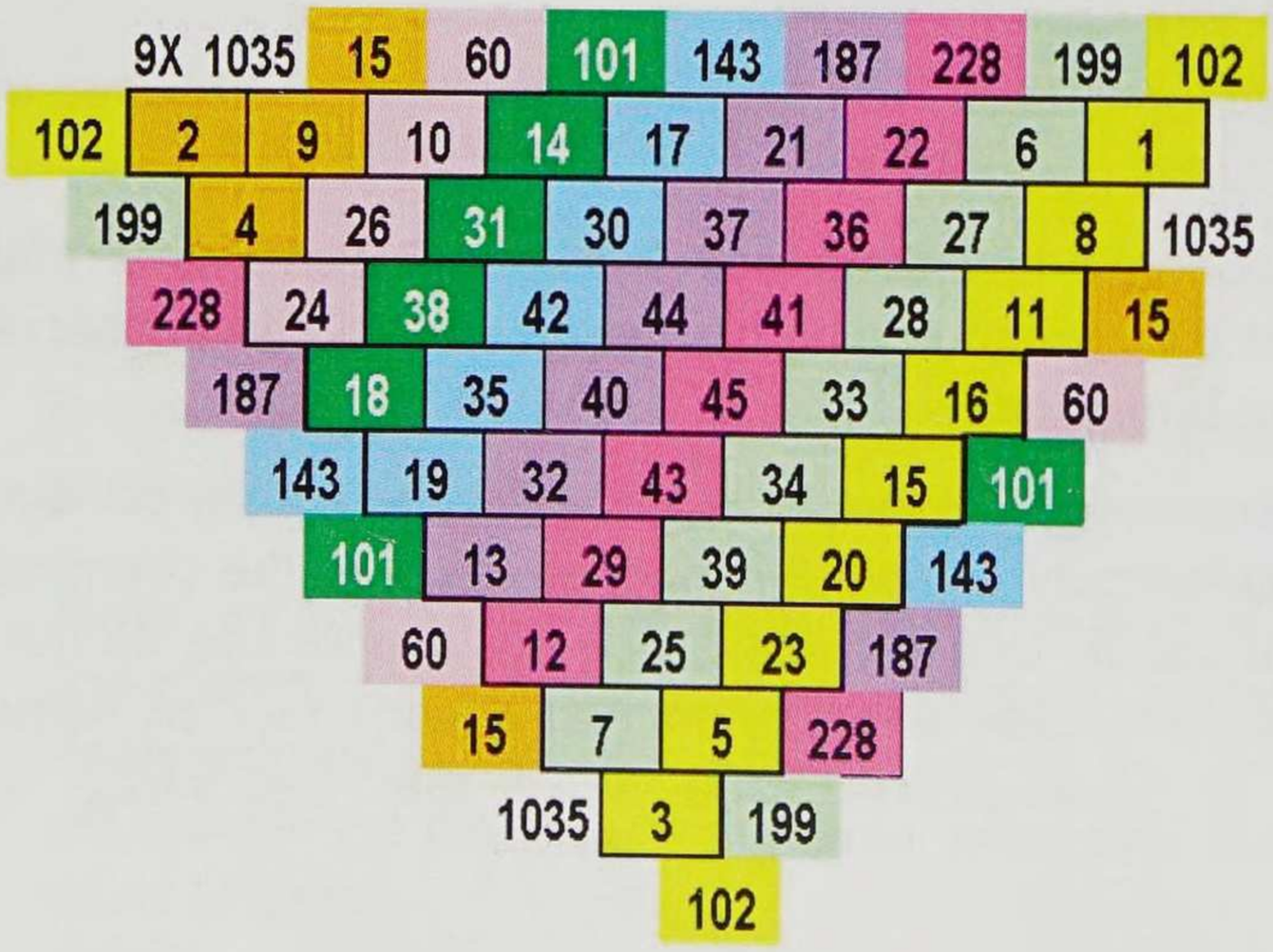
Program 10

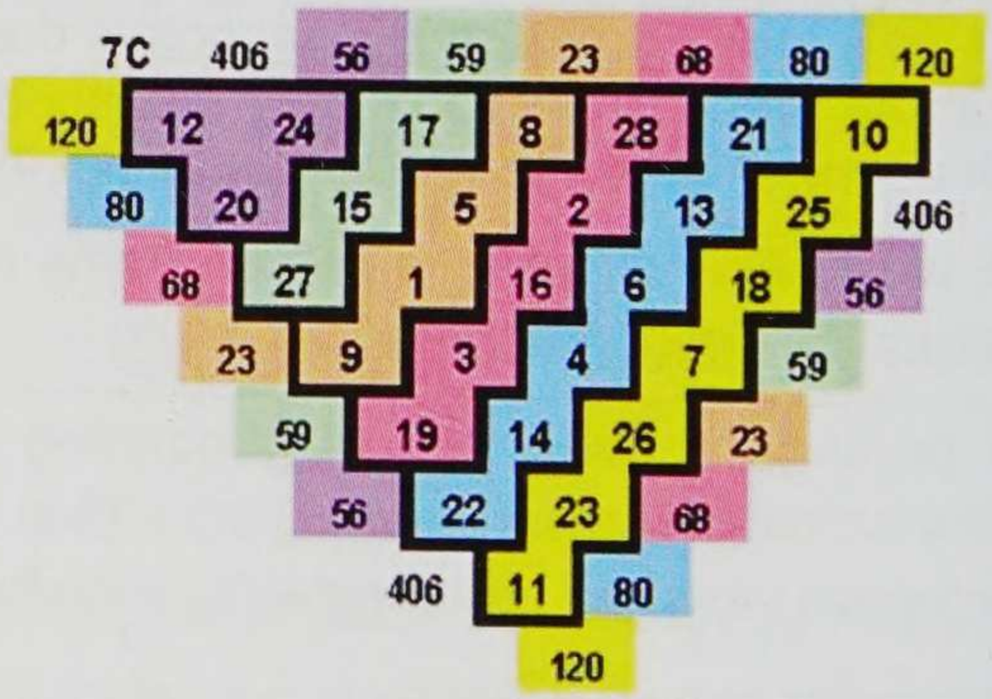
Triangles

Here are the sizes **7**, **8** & **9** triangles. The size-7 triangle is being spun 120° each turn so it is easie to fathom the equal-summing diagonals, rows and corner triangles. These summations go all th way from one side to the other in each row and diagonal.









Program 10

Here is a perfect size-12 geonomic triangle. It was manifested by brute force using the number series **1** thru **78** and not by loom-tables. It has equal-summing crossovers from edge to edge. Each of its embedded corner triangles of same size sum equally all across the triangle.

W(12) 3081 222 195 218 205 147 79 164 268 399 534 650
650 75 77 65 50 39 17 30 34 51 61 78 73
534 70 67 59 40 20 3 31 47 60 66 71 3081
399 63 56 45 23 10 7 25 44 58 68 222
268 53 46 26 9 13 6 24 42 49 195
164 35 29 2 15 14 12 19 38 218
79 32 4 11 8 5 1 18 205
147 21 16 22 28 27 33 147
205 37 43 48 41 36 79
218 52 55 57 54 164
195 62 69 64 268
222 76 72 399
3081 74 534
650

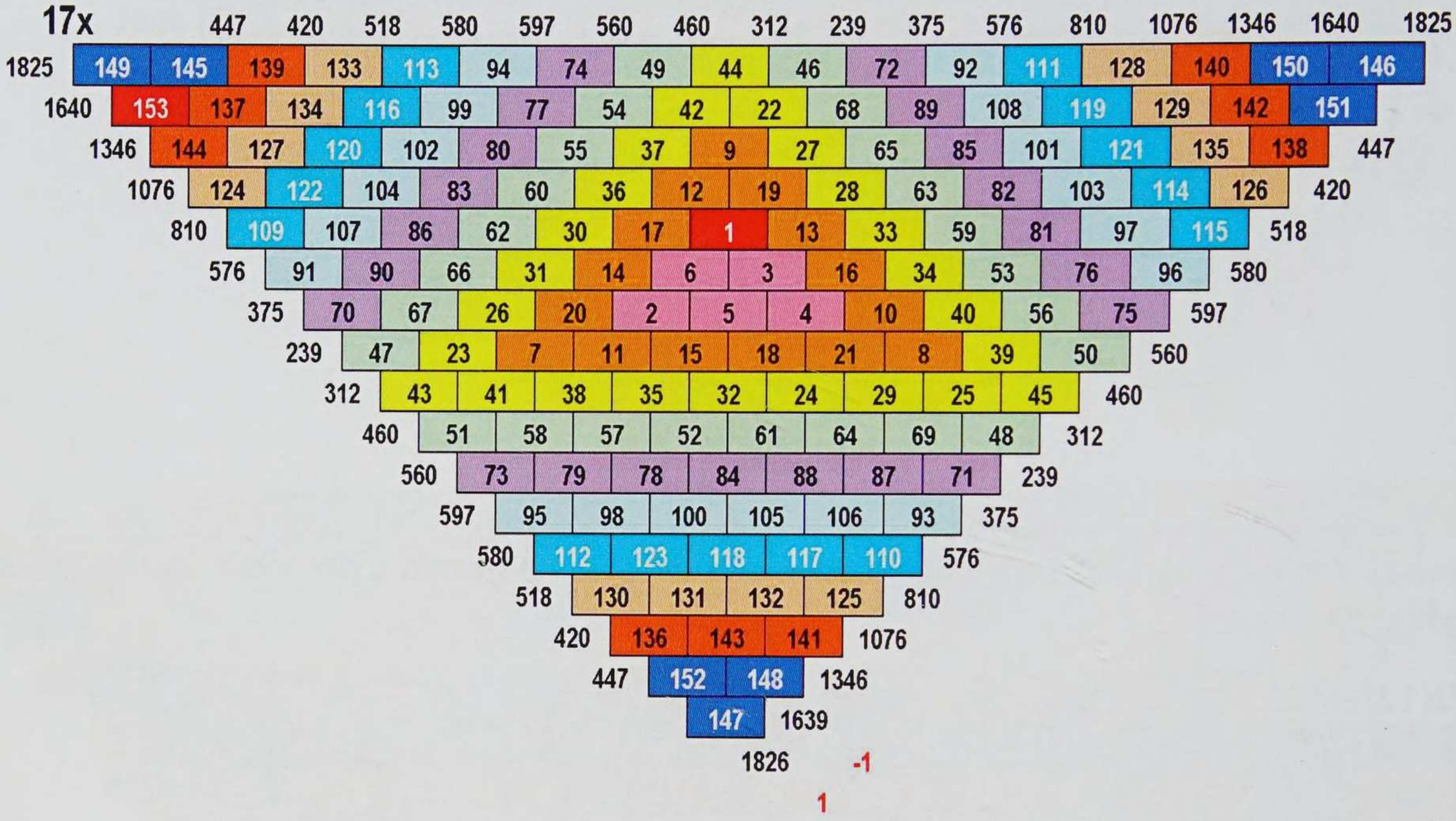
its loom-tables were derived from **W(12)** above and used in the standard formula to obtain its dual triangle shown below.

U(12) 5226 200 162 284 403 411 442 450 543 817 710 804
518 31 55 54 17 28 50 63 111 29 6 67 7
996 114 78 125 40 86 25 75 124 137 66 126 5226
817 30 89 100 122 109 73 3 88 113 90 200
543 53 112 15 97 2 61 134 64 5 162
450 123 51 13 26 14 133 74 16 284
442 87 37 121 85 49 1 62 403
411 98 38 110 39 27 99 411
403 4 76 136 52 135 442
284 41 77 101 65 450
162 18 102 42 543
200 43 138 817
5226 19 853
661

Not only is the dual triangle not geonomically perfect, it does not possess a continuous number series that ranges from **1** thru **78** like the primal triangle. Further, triangular loom tables that are geonomic cannot be generated from scratch because the integer loom-table cannot be made to sum geonomically. So geonomic triangles must be derived by the brute-force method.

Program 10

Here is the size-17 triangle. All triangles between sizes **13** and **17** derived by brute-force wer found to be imperfect by **±1** at one of the size-2 corner triangles, but each of its embedded corne triangles of same size **>2** sum equally. All of its equal-length crossovers sum equally too.



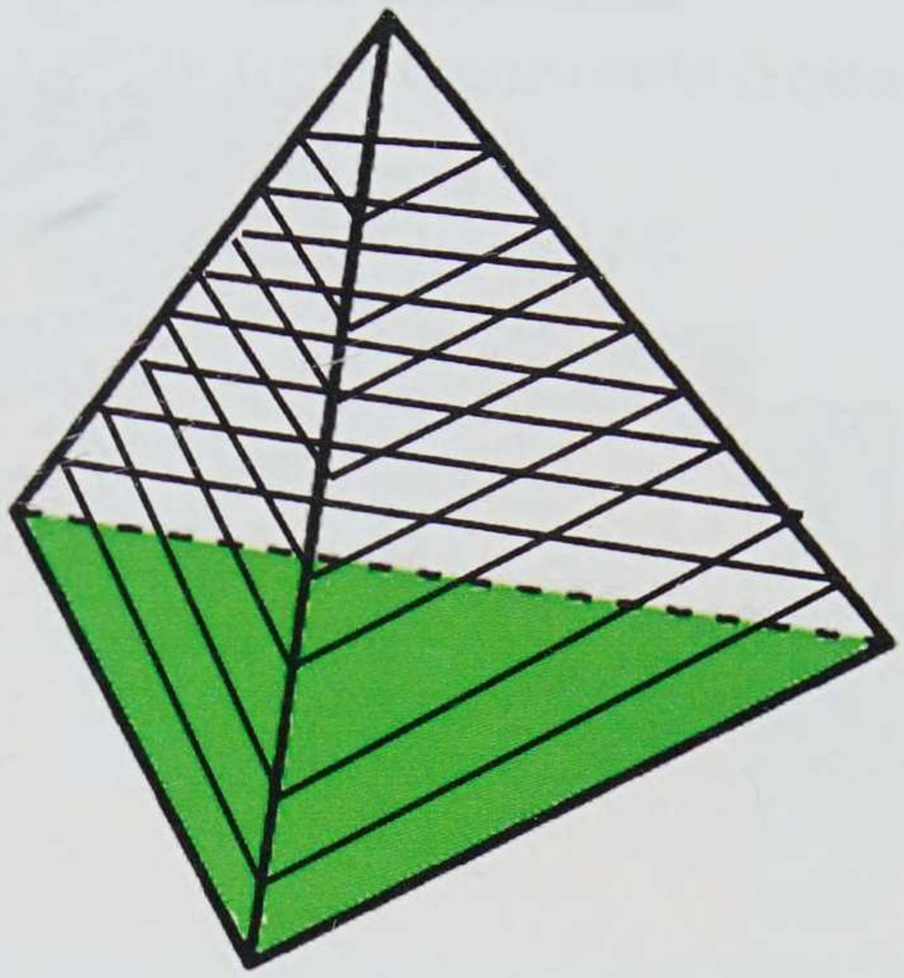
Here are the results of perfection and near perfection in geonomic nested frame triangles by size:

4, 5, 6, 7, 8. 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. The numbers in red are the sizes of near-perfect triangles.

At this time no further all-equal summation properties have been found for triangles.

Program 10

A Size-10 Tetrahedron



At left is a size-10 geonomic tetrahedron with the series of numbers **1** thru **220**. Each of its horizontal planes is geonomic in that every linear summation from edge to edge of the same length sums equally.

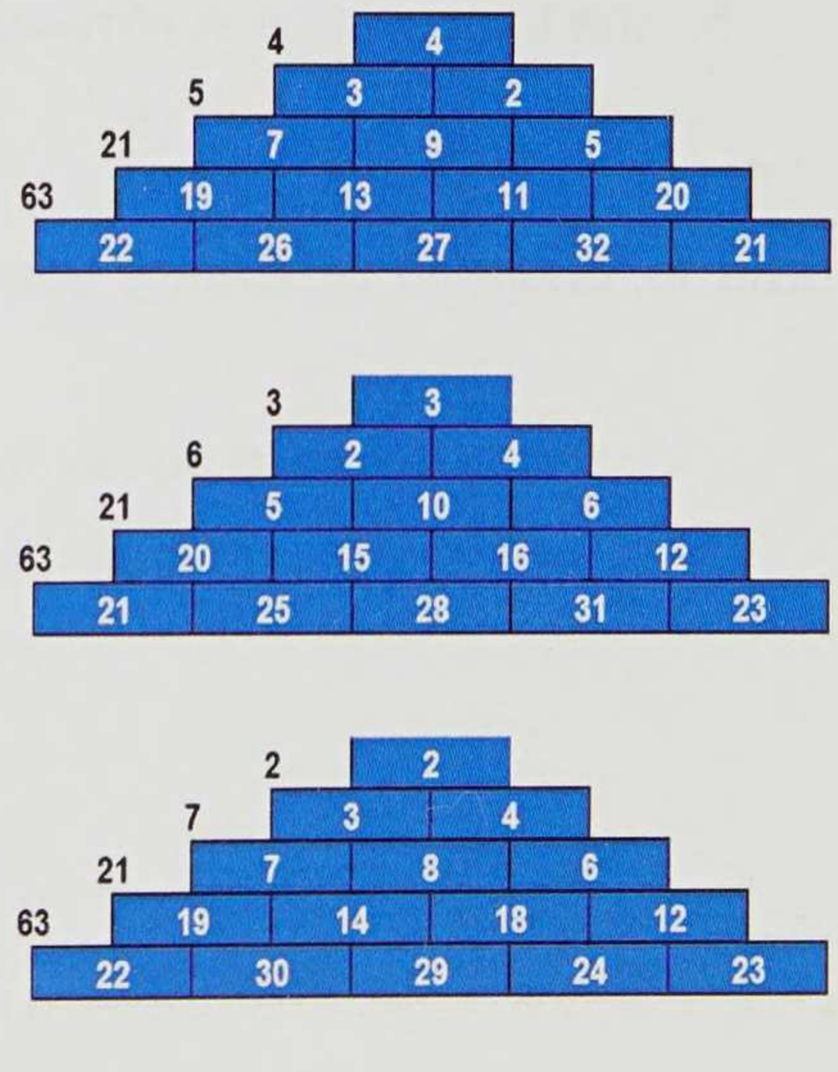
The corners of planar summations, remember, must contain the embedded size-2 triangle as there are no size-2 equal linear sums. In the size-2 triangle second from the top, that's all there is.

Consequently, all the triangles of the same size which contain one of the extreme corners sum equally too.

These differ between triangular summations parallel to the base and the vertical sides.

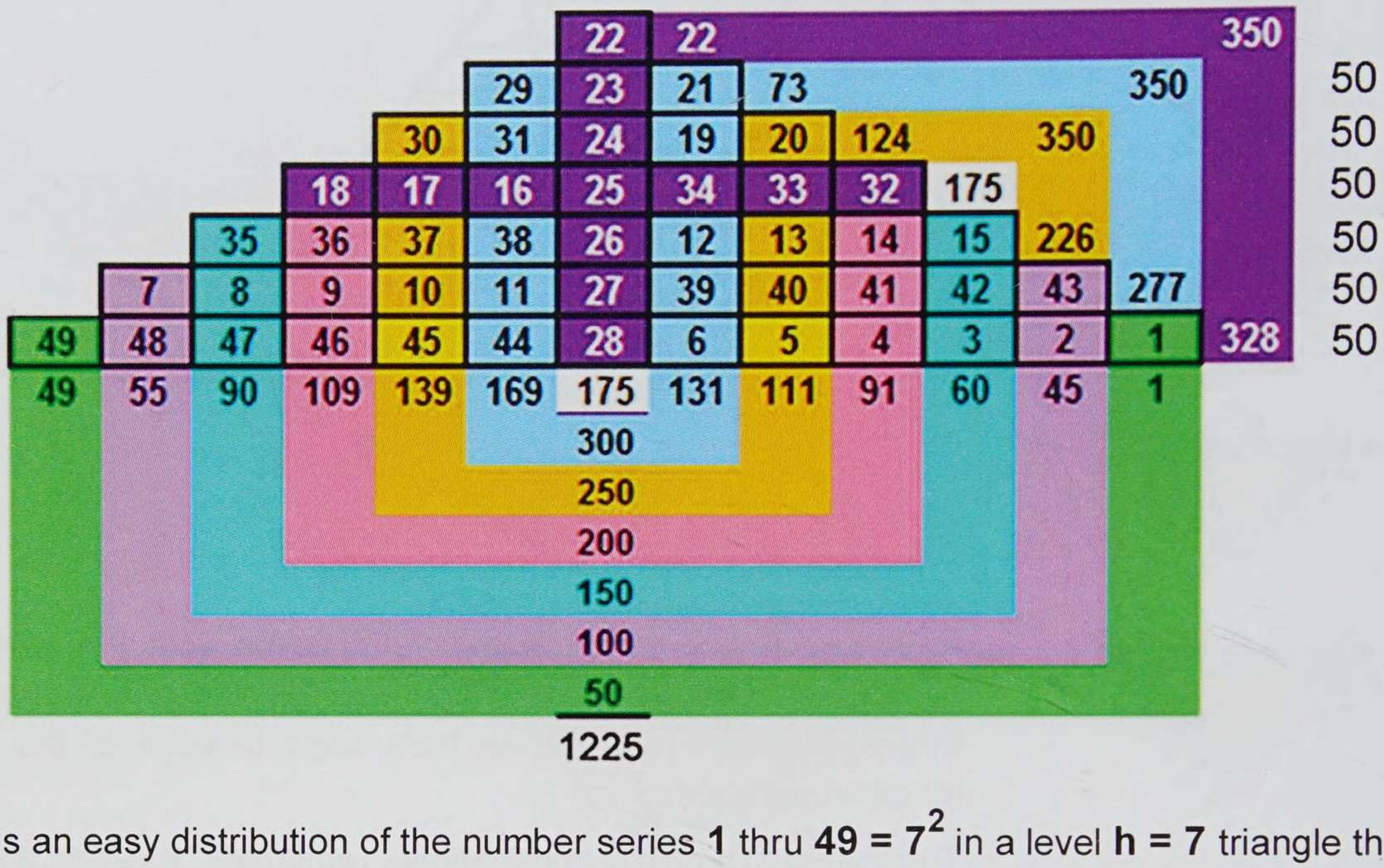
At right are the three vertical triangles from the top down each of size-5, all of which sum to **221**.

The three size-5 triangles in the base each sum to **4061**.



Program 10

Distribution of the number series in triangles for vertical and horizontal pair equalities



Here is an easy distribution of the number series **1** thru **49 = 7^2** in a level **h = 7** triangle that results in

1. equal row sums of **50 = 7^2 +1** which exclude the center column
2. equal row-pair sums equidistant from the central row
3. uniformly progressive column-pair sums to the left and right of the central column which have $\Delta = h^2+1$ as the basic increment
4. equal central row and column summations which are equal to **h** x **(Δ/2)**
5. the total column summations equaling h^2 x **(Δ/2)**

Program 10

Diamonds

Generating Geonomic Diamonds from Geonomic Squares

1. The Laminated Indexing Method – LIM

Geonomic Diamond's size is measured by its height and width.

Nested-frame diamonds have equal summing sides down to, but excluding the size-2 frame.

We're going to generate a perfect diamond from merging two perfect consecutive size squares. We'll choose the size-7 & size-8 squares to generate a size-15 diamond. First we start with a double-index table of **113** (= 49 + 64) cells aligned as shown in a bumpety-bump pattern, **8** high by **15** long (next page). The index table is incremented from top to bottom then left to right, one column after another. The tall columns are highlighted in orange and the shorter columns are highlighted in green.

Next, these differently-colored columns are separated from each other. Then the columns are preceded by consecutive numbering from top down in one empty column then right to the next empty column until all columns are preceded by numbered columns. There should be numbering from **1** to **64** in the leftmost section and from **1** to **49** in the second one.

Now, all the numbers in both geonomic squares are converted from the uncolored numbers to their adjacent colored numbers. Observe that the squares are still geonomic and that both sets of their quadrals are all equal to **228**.

Next, the two squares are merged, column per column as shown.

Then we tilt it column-wise into a stair-case pattern with a half-step at the top and full steps the rest of the way. Then going down, we tilt the table row-wise by one column each row and all those above it until we have a perfect diamond.

It will have all equal rhomboid quadrals centered across the middle of the diamond. All the quadrals for diamonds sum to **4/(2n+1)** of the diamond's characteristic number. Here, **4/15 x 855 = 228** similar to that for geonomic squares.

Letting **m ≡ 2n+1**, this result is formulated as

(10.1) $\mathbf{q = 4/m \times C_m}$

We just derived the size-15 diamond. This method will derive any geonomic diamond of size **m ≥ 9**.

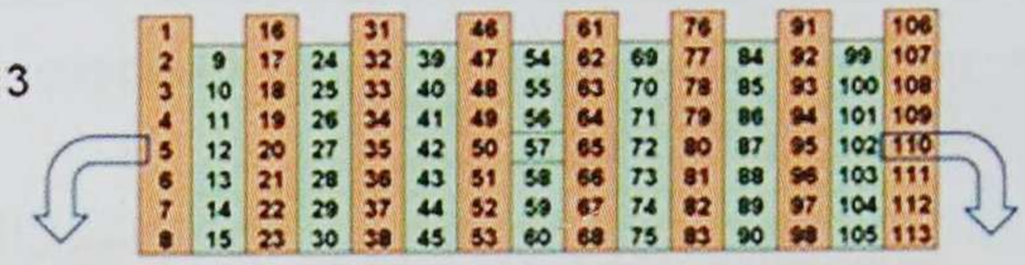
Note that all of the diamonds with the cell-structure as depicted here will be of odd size.

These diamonds have no duals because their loom tables are not geonomic.

Program 10

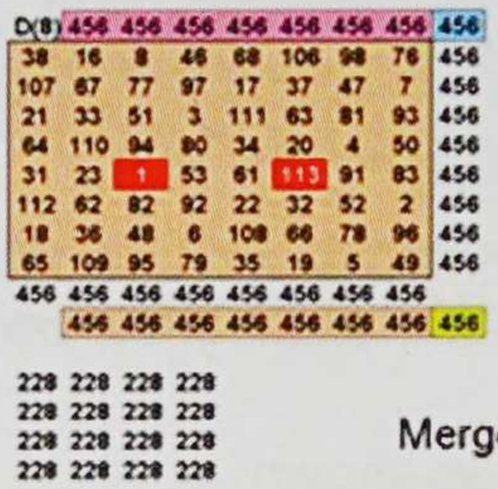
64 + 49 = 113

Layout merged index sequence

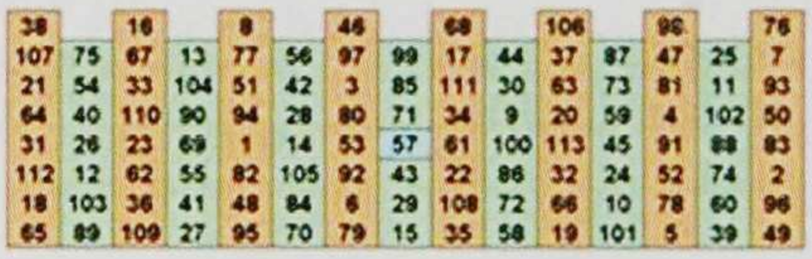


Construct numerical conversion tables

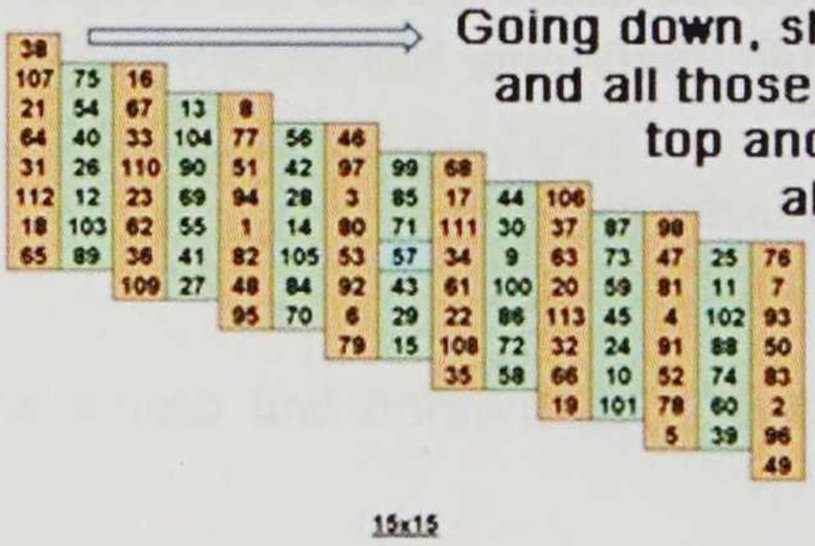
Convert numbers per correspond-ence



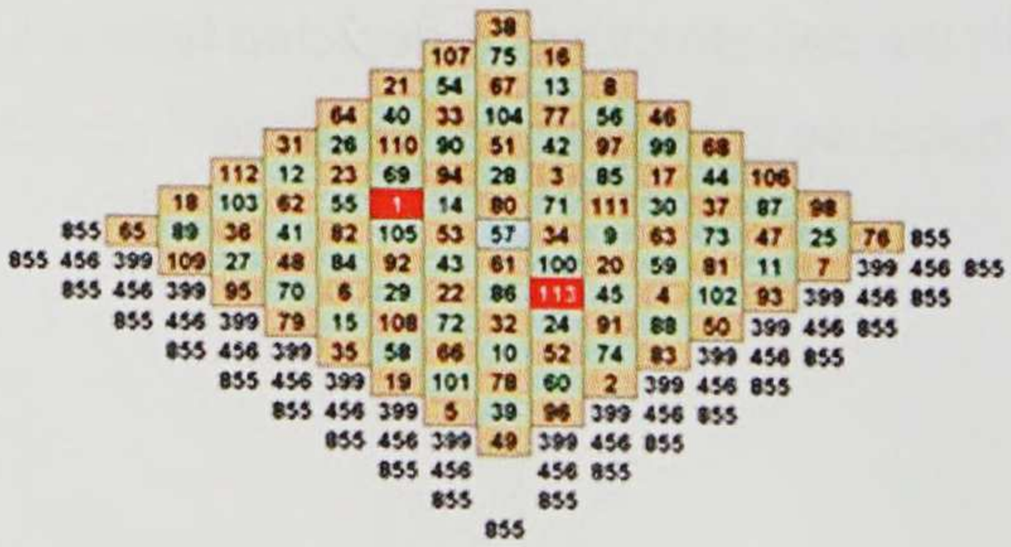
Merge tables



Going down, shift each row by 1 and all those above it until the top and bottom rows are aligned in the same column



15x15



Program 10

2. The Bump and Grind method – BAG

Here is another method for creating geonomic diamonds. This one only works using a Class-1 and an adjacent size Class-4 square because it requires viable geonomic **integer** loom-tables for both. To demonstrate this, we'll again select the size-7 and size-8 squares to generate the size-15 diamond.

	1	2	3	4	5	6	7	
W(7)	175	175	175	175	175	175	175	175
1	35	5	24	43	20	39	9	175
2	22	48	18	37	14	33	3	175
3	16	42	12	31	1	27	46	175
4	10	29	6	25	44	21	40	175
5	4	23	49	19	38	8	34	175
6	47	17	36	13	32	2	28	175
7	41	11	30	7	26	45	15	175
	175	175	175	175	175	175	175	
	175	175	175	175	175	175	175	175

\+ 8 *

Y(7)						
5	1	4	7	3	6	2
4	7	3	6	2	5	1
3	6	2	5	1	4	7
2	5	1	4	7	3	6
1	4	7	3	6	2	5
7	3	6	2	5	1	4
6	2	5	1	4	7	3

=

	1	2	3	4	5	6	7	
D(7)	399	399	399	399	399	399	399	399
1	75	13	56	99	44	87	25	399
2	54	104	42	85	30	73	11	399
3	40	90	28	71	9	59	102	399
4	26	69	14	57	100	45	88	399
5	12	55	105	43	86	24	74	399
6	103	41	84	29	72	10	60	399
7	89	27	70	15	58	101	39	399
	399	399	399	399	399	399	399	
	399	399	399	399	399	399	399	399

	1	2	3	4	5	6	7	8	
W(8)	260	260	260	260	260	260	260	260	260
1	24	9	8	25	40	57	56	41	260
2	58	39	42	55	10	23	26	7	260
3	14	19	30	3	62	35	46	51	260
4	36	61	52	45	20	13	4	29	260
5	17	16	1	32	33	64	49	48	260
6	63	34	47	50	15	18	31	2	260
7	11	22	27	6	59	38	43	54	260
8	37	60	53	44	21	12	5	28	260
	260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260	260

\+ 7 *

Y(8) - |1|							
2	1	0	3	4	7	6	5
7	4	5	6	1	2	3	0
1	2	3	0	7	4	5	6
4	7	6	5	2	1	0	3
2	1	0	3	4	7	6	5
7	4	5	6	1	2	3	0
1	2	3	0	7	4	5	6
4	7	6	5	2	1	0	3

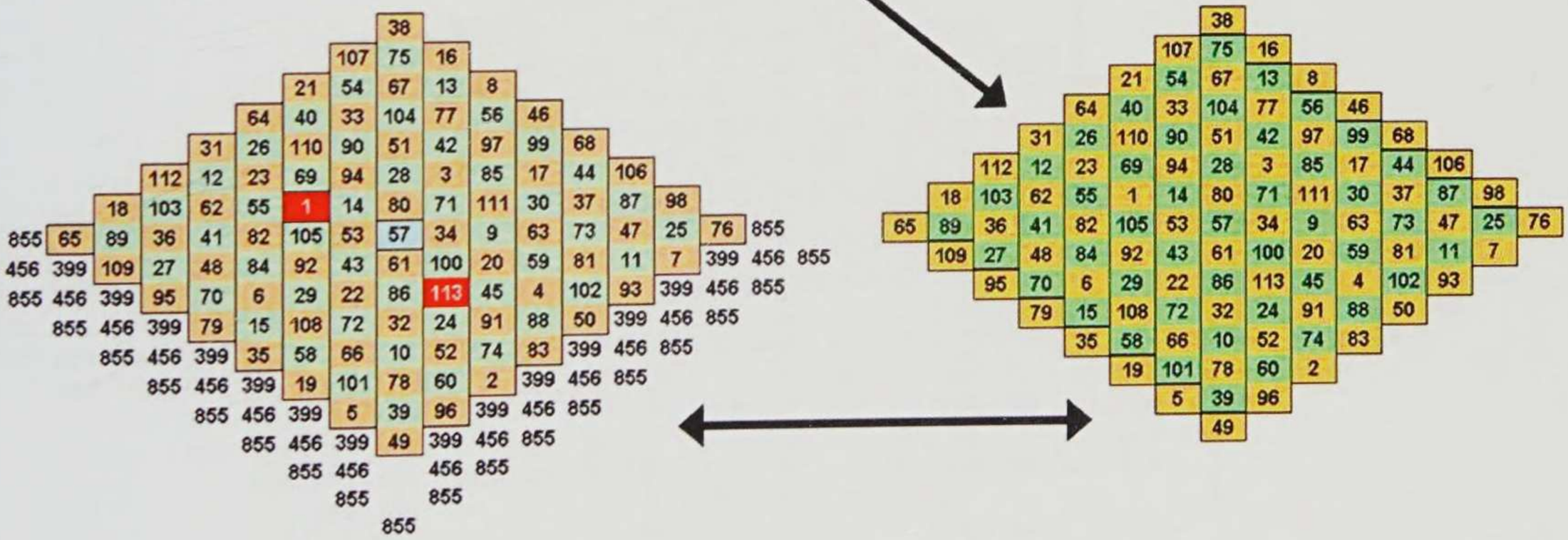
=

	456	456	456	456	456	456	456	456
38	16	8	46	68	106	98	76	456
107	67	77	97	17	37	47	7	456
21	33	51	3	111	63	81	93	456
64	110	94	80	34	20	4	50	456
31	23	1	53	61	113	91	83	456
112	62	82	92	22	32	52	2	456
18	36	48	6	108	66	78	96	456
65	109	95	79	35	19	5	49	456
456	456	456	456	456	456	456	456	
	456	456	456	456	456	456	456	456

Note: The centrally symmetric pairwise version of the size-8 square will also conform to this procedure. Check it out (re: Part-II p.5)

38		16		8		46		68		106		98		76
107	75	67	13	77	56	97	99	17	44	37	87	47	25	7
21	54	33	104	51	42	3	85	111	30	63	73	81	11	93
64	40	110	90	94	28	80	71	34	9	20	59	4	102	50
31	26	23	69	1	14	53	57	61	100	113	45	91	88	83
112	12	62	55	82	105	92	43	22	86	32	24	52	74	2
18	103	36	41	48	84	6	29	108	72	66	10	78	60	96
65	89	109	27	95	70	79	15	35	58	19	101	5	39	49

15x15



Now, compare the resulting diamond on the right with the one we got on the prior page at left – they're identical in every detail! Who could have predicted that? Here were the formulas used:

(10.2) $D(7) = W(7) + 8Y(7)$

(10.3) $D(8) = W(8) + 7(Y(8)-|1|)$

Subtracting a matrix of 1's was used in the last formula to preserve the fist **8** digits in **D(8)** from being modified.

Program 10

The Size-23 Diamond

Here is the size-23 diamond composed from the size-11 and size-12 perfect squares.

	1	2	3	4	5	6	7	8	9	10	11	12	
		1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596
1	109	85	193	145	109	229	37	157	121	73	181	157	1596
2	133	205	49	145	133	61	205	133	121	217	61	133	1596
3	217	85	205	133	109	121	145	157	133	61	181	49	1596
4	157	133	73	121	205	37	229	61	145	193	133	109	1596
5	97	145	181	157	49	241	25	217	109	85	121	169	1596
6	121	109	169	25	229	73	193	37	241	97	157	145	1596
7	73	121	157	181	73	265	1	193	85	109	145	193	1596
8	97	241	13	181	97	97	169	169	85	253	25	169	1596
9	217	13	205	133	181	121	145	85	133	61	253	49	1596
10	121	169	37	157	169	73	193	97	109	229	97	145	1596
11	133	109	217	121	85	205	61	181	145	49	157	133	1596
12	121	181	97	97	157	73	193	109	169	169	85	145	1596
	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	
		1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596

	W(11)	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463
1	22	200	136	61	250	175	111	36	225	161	86	1463
2	106	42	220	156	92	17	206	131	67	245	181	1463
3	201	137	62	251	176	112	37	226	151	87	23	1463
4	43	221	157	82	18	207	132	68	246	182	107	1463
5	138	63	252	177	113	38	227	152	88	13	202	1463
6	222	158	83	19	197	133	69	247	183	108	44	1463
7	64	253	178	114	39	228	153	89	14	203	128	1463
8	159	84	20	198	134	59	248	184	109	45	223	1463
9	243	179	115	40	229	154	90	15	204	129	65	1463
10	85	21	199	135	60	249	174	110	46	224	160	1463
11	180	105	41	230	155	91	16	205	130	66	244	1463
	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	
		1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463

109		85		193		145		109		229		37		157		121		73		181		157
133	22	205	200	49	136	145	61	133	250	61	175	205	111	133	36	121	225	217	161	61	86	133
217	106	85	42	205	220	133	156	109	92	121	17	145	206	157	131	133	67	61	245	181	181	49
157	201	133	137	73	62	121	251	205	176	37	112	229	37	61	226	145	151	193	87	133	23	109
97	43	145	221	181	157	157	82	49	18	241	207	25	132	217	68	109	246	85	182	121	107	169
121	138	109	63	169	252	25	177	229	113	73	38	193	227	37	152	241	88	97	13	157	202	145
73	222	121	158	157	83	181	19	73	197	265	133	1	69	193	247	85	183	109	108	145	44	193
97	64	241	253	13	178	181	114	97	39	97	228	169	153	169	89	85	14	253	203	25	128	169
217	159	13	84	205	20	133	198	181	134	121	59	145	248	85	184	133	109	61	45	253	223	49
121	243	169	179	37	115	157	40	169	229	73	154	193	90	97	15	109	204	229	129	97	65	145
133	85	109	21	217	199	121	135	85	60	205	249	61	174	181	110	145	46	49	224	157	160	133
121	180	181	105	97	41	97	230	157	155	73	91	193	16	109	205	169	130	169	66	85	244	145

													109													
												133	22	85												
											217	106	205	200	193											
										157	201	85	42	49	136	145										
									97	43	133	137	205	220	145	61	109									
								121	138	145	221	73	62	133	156	133	250	229								
							73	222	109	63	181	157	121	251	109	92	61	175	37							
						97	64	121	158	169	252	157	82	205	176	121	17	205	111	157						
					217	159	241	253	157	83	25	177	49	18	37	112	145	206	133	36	121					
				121	243	13	84	13	178	181	19	229	113	241	207	229	37	157	131	121	225	73				
			133	85	169	179	205	20	181	114	73	197	73	38	25	132	61	226	133	67	217	161	181			
	3059	121	180	109	21	37	115	133	198	97	39	265	133	193	227	217	68	145	151	61	245	61	86	157	3059	
3059	1596	1463	181	105	217	199	157	40	181	134	97	228	1	69	37	152	109	246	193	87	181	181	133	1463	1596	3059
	3059	1596	1463	97	41	121	135	169	229	121	59	169	153	193	247	241	88	85	182	133	23	49	1463	1596	3059	
		3059	1596	1463	97	230	85	60	73	154	145	248	169	89	85	183	97	13	121	107	109	1463	1596	3059		
			3059	1596	1463	157	155	205	249	193	90	85	184	85	14	109	108	157	202	169	1463	1596	3059			
				3059	1596	1463	73	91	61	174	97	15	133	109	253	203	145	44	145	1463	1596	3059				
					3059	1596	1463	193	16	181	110	109	204	61	45	25	128	193	1463	1596	3059					
						3059	1596	1463	109	205	145	46	229	129	253	223	169	1463	1596	3059						
							3059	1596	1463	169	130	49	224	97	65	49	1463	1596	3059							
								3059	1596	1463	169	66	157	160	145	1463	1596	3059								
									3059	1596	1463	85	244	133	1463	1596	3059									
										3059	1596	1463	145	1463	1596	3059										
											3059	1596		1596	3059											
												3059		3059												
													3059													

Diamonds were found to have no dual counterpart. Note that diamonds generated by either the LIM or BAG methods are only diagonally pangenic. The kernel number for the size-23 diamond here is **532**. It has no whole number relationship to the characteristic number, **3069**.

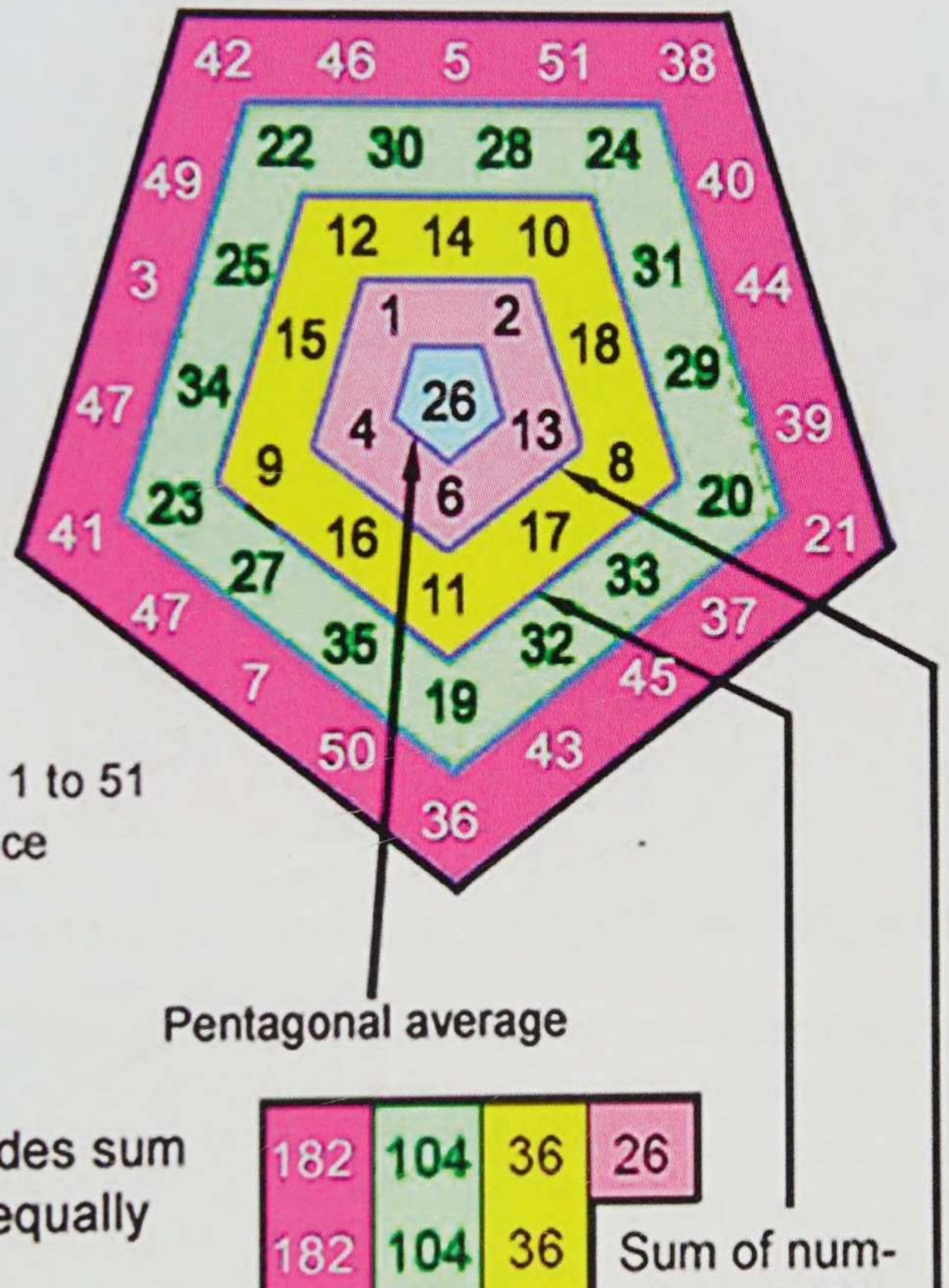
Program 10

Size-5 Equal-summing Nested-frame Pentagon

Here is a size-5 geonomic pentagon filled with the numbers from **1** thru **51**. Every frame except the innermost one of size-2 has equal summing sides which are a multiple of the pentagon's average **26**. Even the entire innermost frame of size-2 sums to **26**.

There are no crossover summations on odd-sided configurations beyond triangles so geonomic pentagons do not exist.

Size 5 Pentagon



Sides sum equally

182	104	36	26
182	104	36	
182	104	36	
182	104	36	
182	104	36	

182 = 7 x 26

104 = 4 x 26

Sum of numbers = 130 = 5 x 26

Sum of numbers = 26

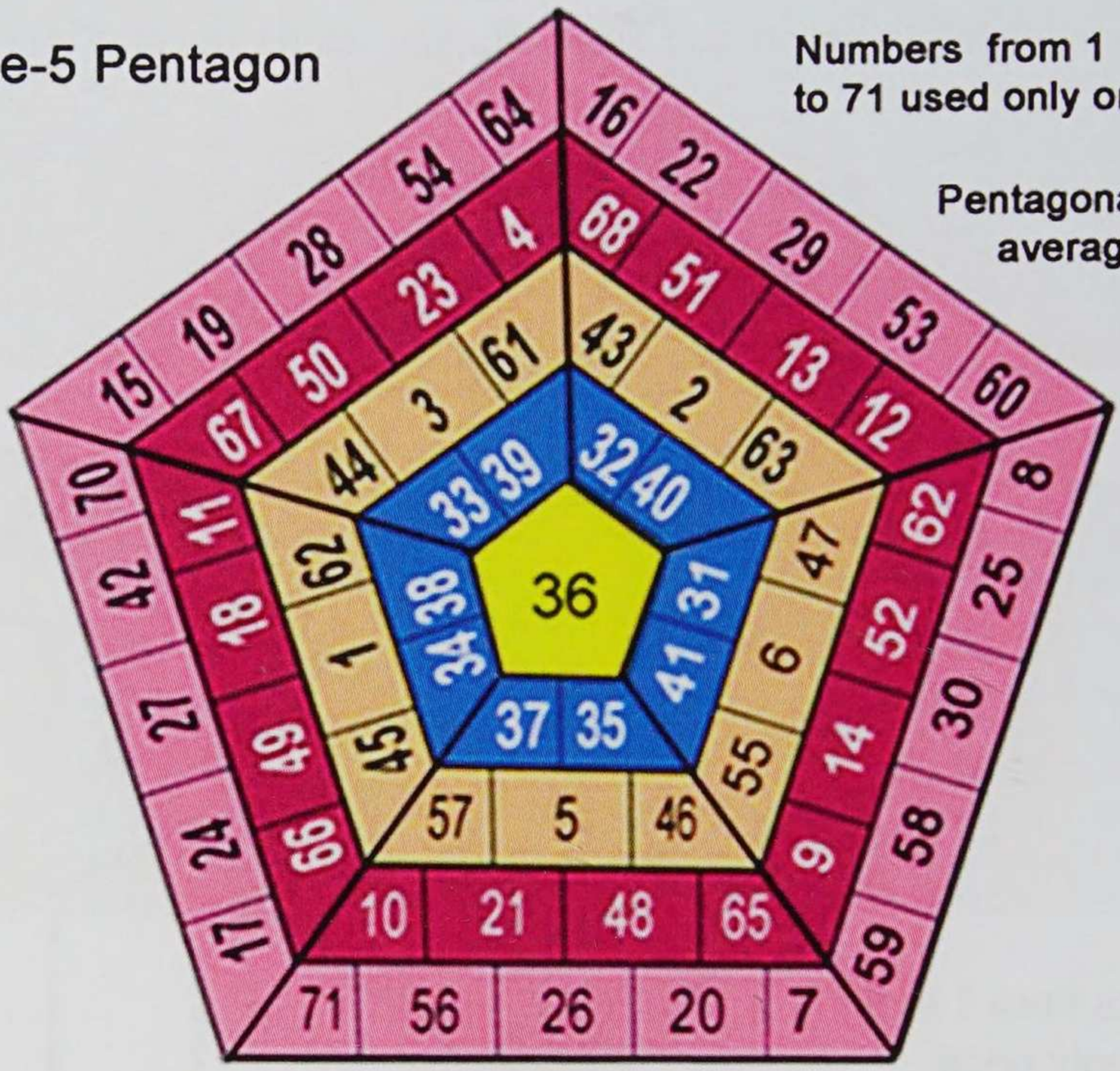
Nested-frame pentagons were found to have no dual counterpart.

Program 10

Size-5 Triangular-summing Nested-Frame Pentagon

Here is a size-5 pentagon that shares no numbers at the frame corners as the first one did; onl
the center cell is shared among all triangular segments.

Size-5 Pentagon



Numbers from 1 to 71 used only once

Pentagonal average = 36

Every size-5 triangular segment including the center cell sums to **504 = 14 x 36**.

Every layer within each segment sums to a multiple of **36** according to its length:

100	**144**	**108**	**72**	**36**
5	**4**	**3**	**2**	**1**

Every symmetrically centered pentagon sums to a multiple of **36** according to its size:

5:	**2556 = 71 x 36**	**Δx**	**= 25**
4:	**1656 = 46 x 36**		**= 20**
3:	**936 = 26 x 36**		**= 15**
2:	**396 = 11 x 36**		**= 10**
1:	**36 = 1 x 36**		

Note that the decrements **Δx** in the multiples upon going to the next smaller size pentagon i
related to the pentagon's overall size **5**.

Program 10

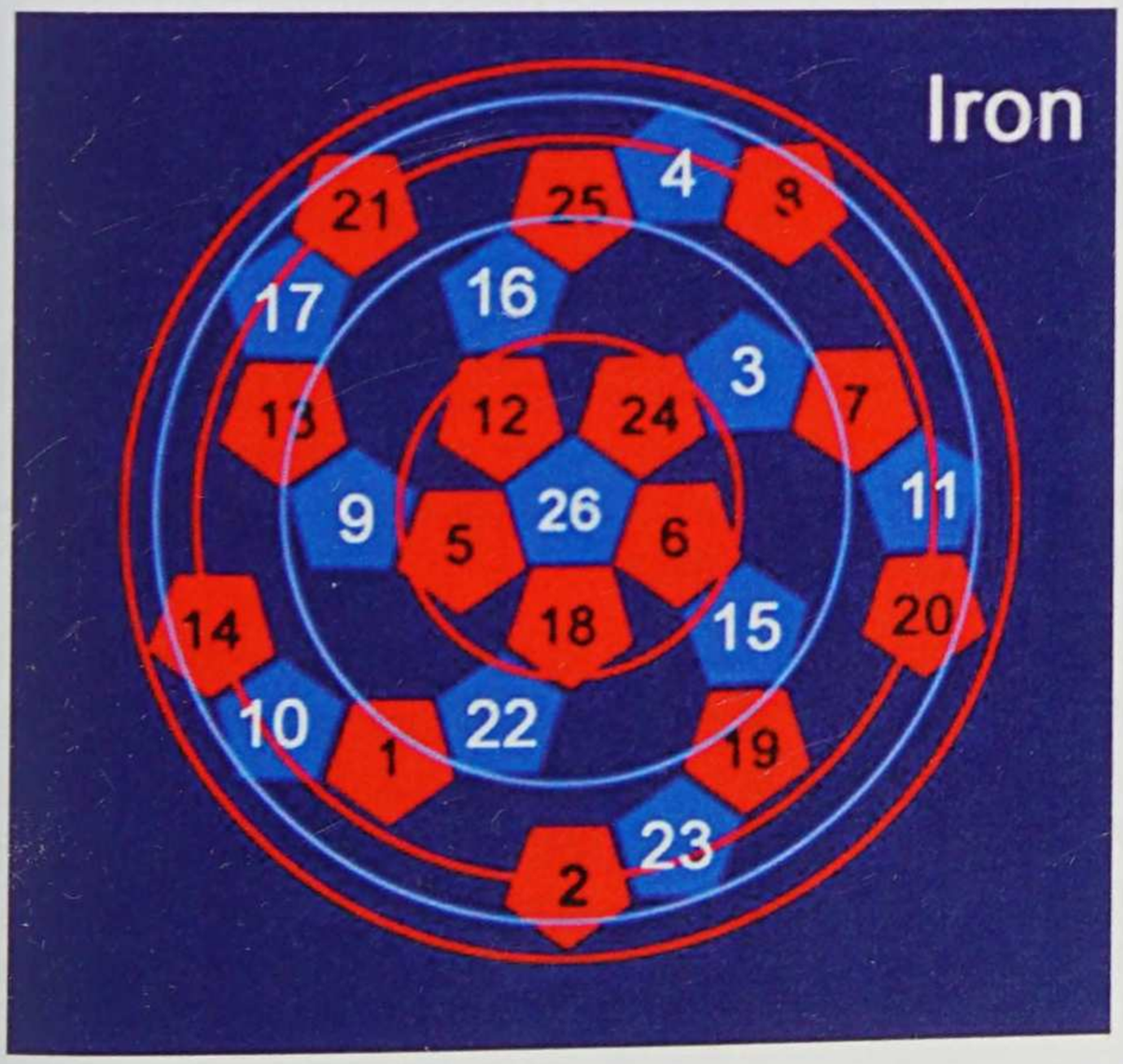
Pentagonal Geonometry at Sub-dimensional Level 1/3D (Atoms)

Many of the atoms' electron distribution can be represented geonomically by arrangements of incidental pentagons.

At left is element **#13 aluminum** which has **13** electrons. The electron representation distributes the numbers **1** thru **13** symmetrically. The pentagonal cells within each rectangle sum equally to **46**.

At right is the representation of the element **#16 sulfur** which has **16** electrons. Each grouping of three pentagonal cells incident with the center cell **#16,** plus the center cell, sum equally to **40.**

All the cells incident to the outer ring sum to **60**. All the cells incident to the inner ring plus the center also sum to **60**.



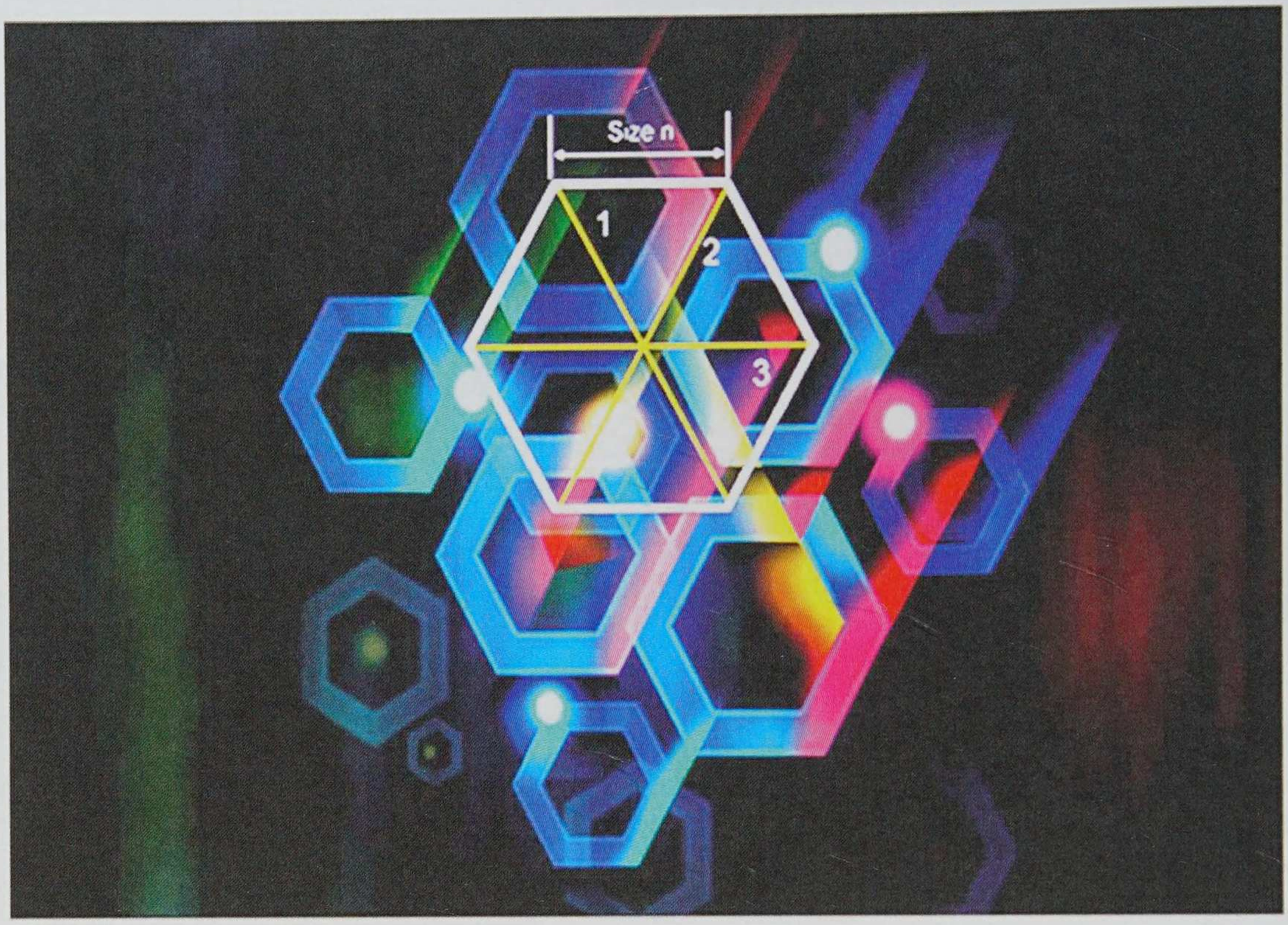
At left is the geonomic representation of the element **#26 iron** which has **26** electrons. Not only do the pentagonal cells incident with each circle of the same color sum equally to **65** but so do each of the five independent propeller blades plus the axial center **26**.

To accomplish a geonomic pentagonal layout, the number of electrons in the atom less **1** must be divisible by **3** or **5**.

Examples of these are elements: #19 Potassium, #22 Titanium, #25 Manganese, #28 Nickel, #31 Gallium, #34 Selenium, #36 Krypton, #37 Rubidium, #40 Zirconium, #41 Niobium, #43 Technetium, #46 Palladium, #49 Indium, #51 Antimony, #56 Barium, #61 Promethium, etc.

Program 10

Geonomic Hexagons



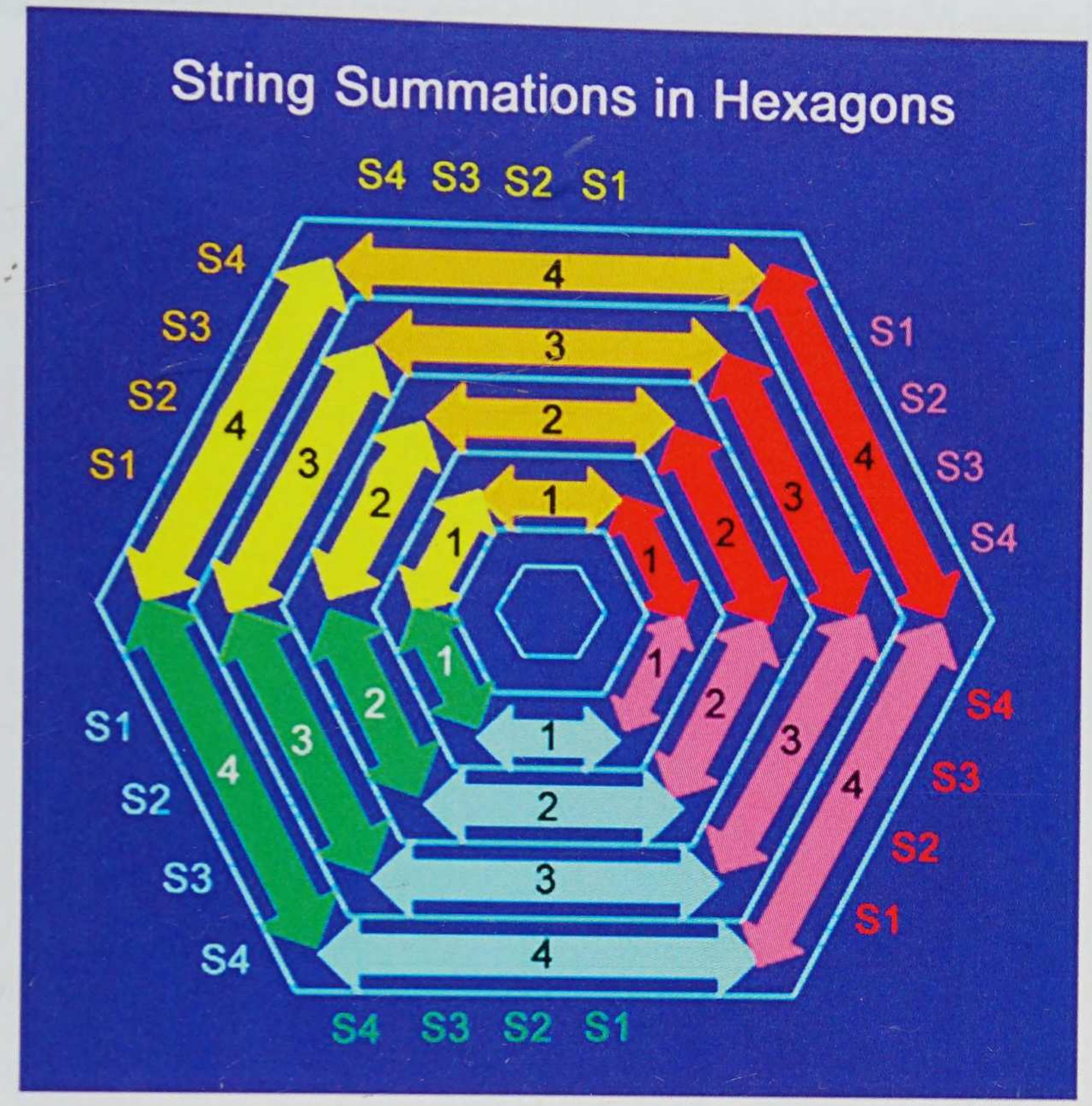
The next size up are hexagons which are geonomic with equal-summing nested frames, hav complementary geonomic loom-tables and consequently have a geonomic dual counterpart.

The properties necessary to qualify as a numeric hexagonal table to be a *geonomic* hexagon are:

1. The numbers range in series starting from **1** to the total number of cells.
2. There are **3** summing directions.
3. Each frame's sides sum equally, with **1** minor variance permitted in the next to th innermost frame.
4. The innermost frame is exempt because it is impossible to have all adjacent pairs equal b sharing any **6** distinct numbers in a pairwise round-robin sequence.

By definition, numerical hexagons determined by others thus far that are to be found on the interne at Wikipedia.com do not satisfy these pre-conditions and are consequently not *geonomic*. Furthe those numeric patterns yield no sensical loom tables. In fact, their hexagonal tables cannot eve be regenerated by the loom tables derived from them. As a consequence they have no dua counterparts like those of the hexagonal tables shown in this program. And further, none of ther possess any inherent patterns. So, in this program, you're in for something very new in mathe matics.

Program 10



This diagram shows the relationship of numbers outside the hexagon to the summations taken within it. The lettered points correspond to summations taken along their identically colored arrows, with the number corresponding to the length of the arrow: the larger its value, the longer the summation string and conversely, the smaller its value, the shorter the summation string.

All but the next to last smallest frame will have **6** equal-summing sides. By the nature of space itself, the modulus table may have one side that is greater or lesser by **±1.**

Each hexagonal table contains all the numbers in the series from **1** thru capital **N** given by formulas (**10.4**) and (**10.5**) for a hexagon whose size is **n**, as measured by the number of cells per hexagonal side in the outermost frame.

(10.4) $m = 3(n - 1)$

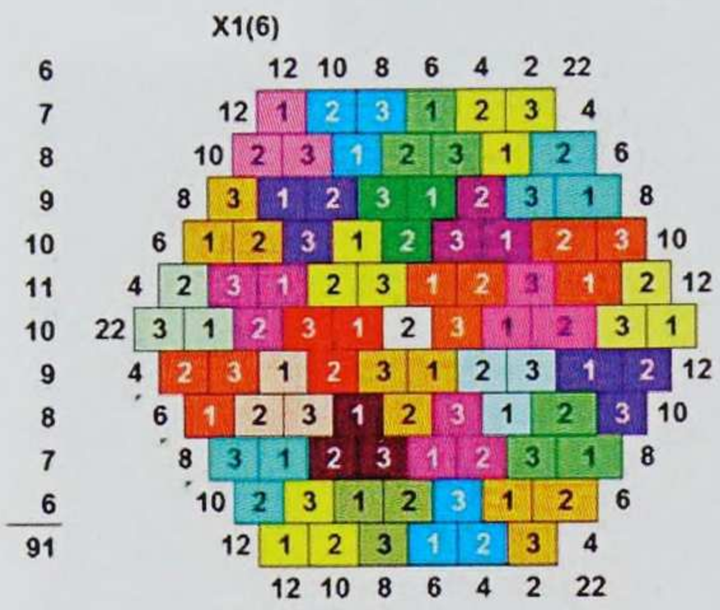
(10.5) $N = nm + 1$

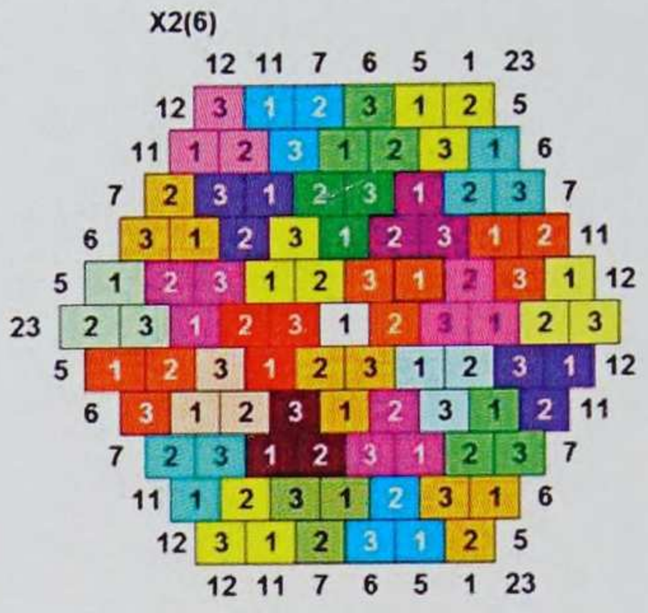
Derivations of the two dual hexagons and their complementary loom tables will use both numbers **n** and **m** as the bases involved.

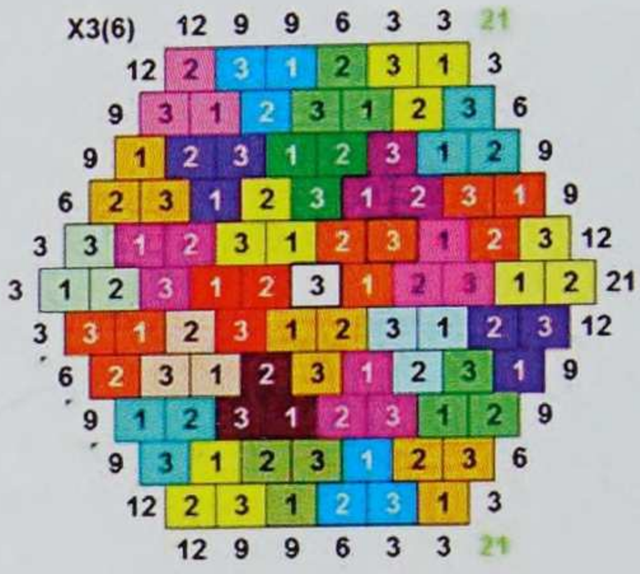
The central numbers of all hexagonal tables will be included for even size hexagons and excluded for odd size hexagons. They will be excluded for odd sizes because they cause more complications in the dual formulations than their accommodation would resolve. To make the formulas all work together for odd size hexagons, the center number in those tables will be a duplicated **1**.

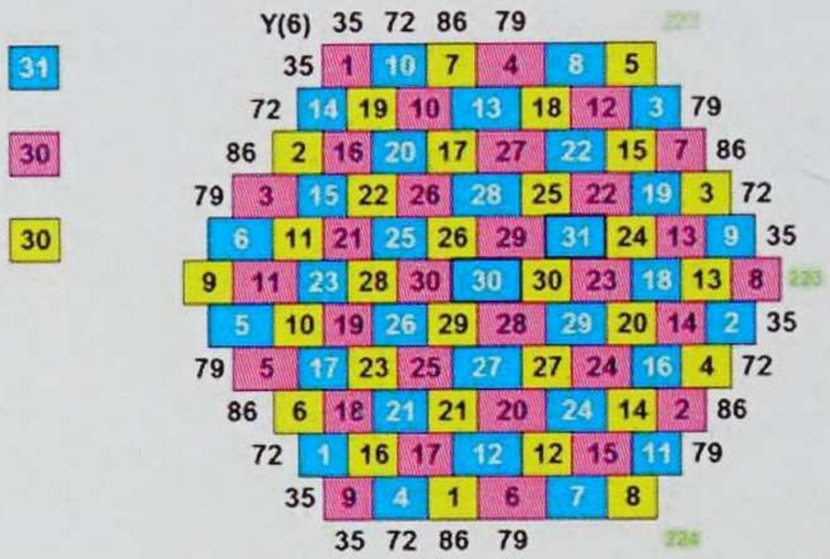
Program 10

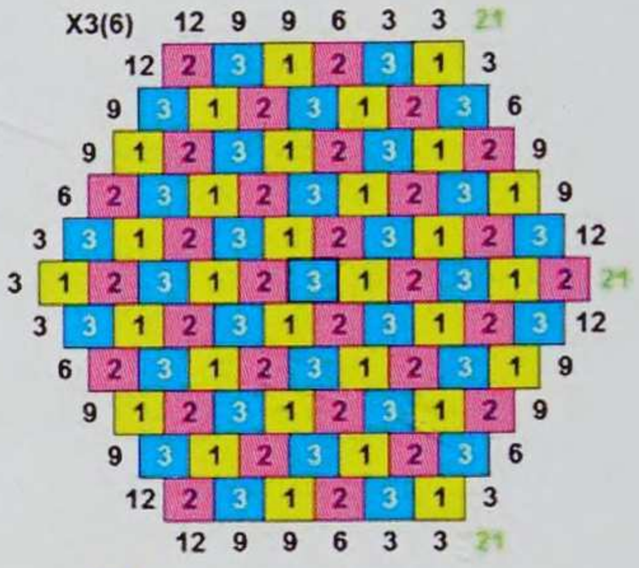
Modular Triangle Method MTM Yields Dual Size-6 Geonomic Hexagons











By filling triangular modululs with the numbers **1**, **2** & **3** as depicted, three modulus loom tables are generated. By taking one of these loom tables, say **X3(6)** and coloring the background of each o these numbers with a different color, a fill-pattern for its complementary loom-table **Y(6)** is obtained. Now, **Y** is filled with the series of numbers, one complete series for each set of colored cells in a manner such that each side of each nested hexagon sums equally. Then the geonomic hexagon is derived from (**10.6**):

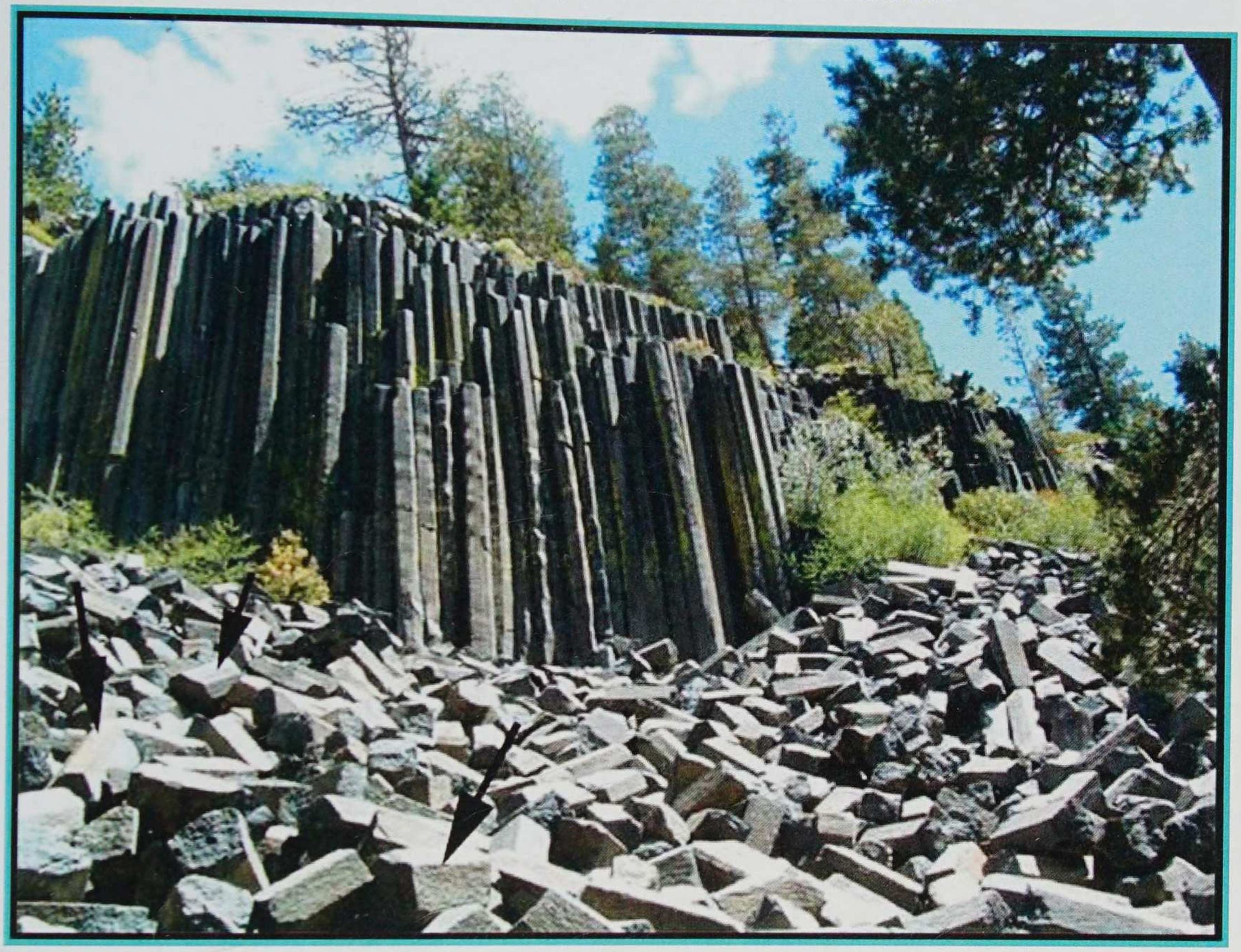
(10.6) W(6) = 30(Y(6) - | 1 |) + X(6)

That result is shown at the bottom. Each side of each nested hexagon sums equally. The tri-axia sums differ here by **1**. However, when the size of the hexagon is such that the last number in the largest series lies in the center cell, all the tri-axial sums will be equal too.

Each of these three X's will produce a geonomic hexagon in this manner.

Application #4 – Inherent Vibratory Membrane for Snowflake Formation

Devils Post-pile Rock Formation

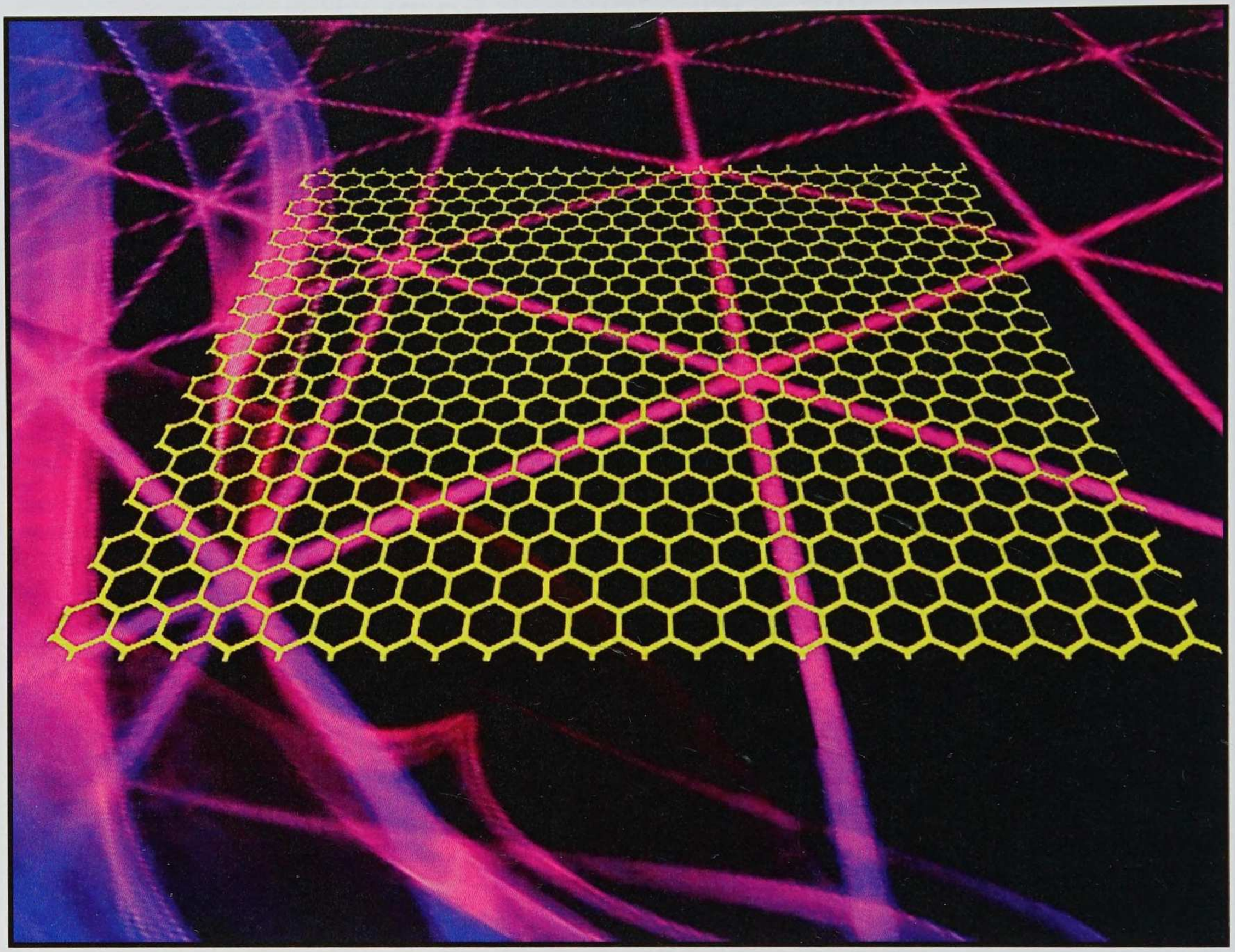


Before exploring hexagonal tables, we need to pictorially explore the rock formations at Devils Post Pile in the Sierra Nevada mountain range just west of the Sierra Nevada crest. The formation is a rare sight in the geological world and ranks as one of the world's finest examples of columnar basalt. Its columns tower **60** feet high and display an unusual hexagonal symmetry from top to bottom.

The post-pile was created by a lava flow which reached a thickness of **500** feet. Because of its unusual thickness, much of the mass of pooled lava cooled very slowly and evenly over a long time, which is why the columns are so long and so symmetrical, forming almost perfect vertical hexagonal columns.

The picture of Devils Post-pile is shown here to demonstrate that when liquid material is subject to uniform cooling, it contracts into hexagonal segments.

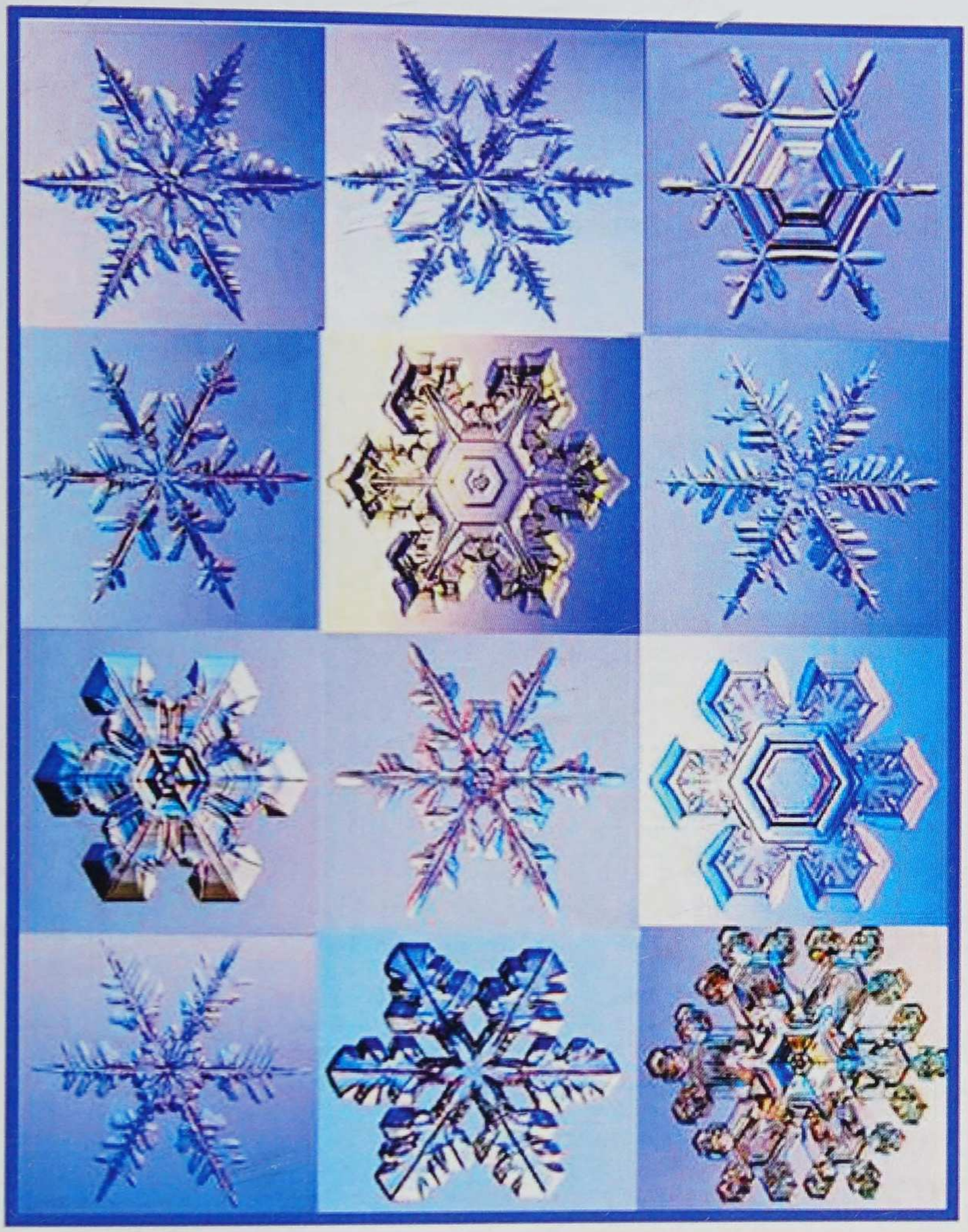
The Conversion of Vapor-laden air into Frozen Hexagonal Segments



Even though ice expands when it forms from water, when it converts directly to a frozen blanket from moisture-laden vapor, it actually shrinks. When snow condenses out of a moisture-laden vapor, it first is a continuous sheet sandwiched between two contacting layers of air, one cold enough to make ice crystals form and the other layer containing the moisture-laden vapor. This thin blanket of frozen homogenous vapor then naturally separates uniformly into detached hexagonal segments.

From this point onward, these hexagonal membranes of ice configure their icy content into snowflakes according to the harmonic vibrational membranes naturally inherent in their local space. Here is where Geonometry picks up the story of the snowflake-formation process that Science can't explain.

Snowflakes are Numerical Weaves of Wonder



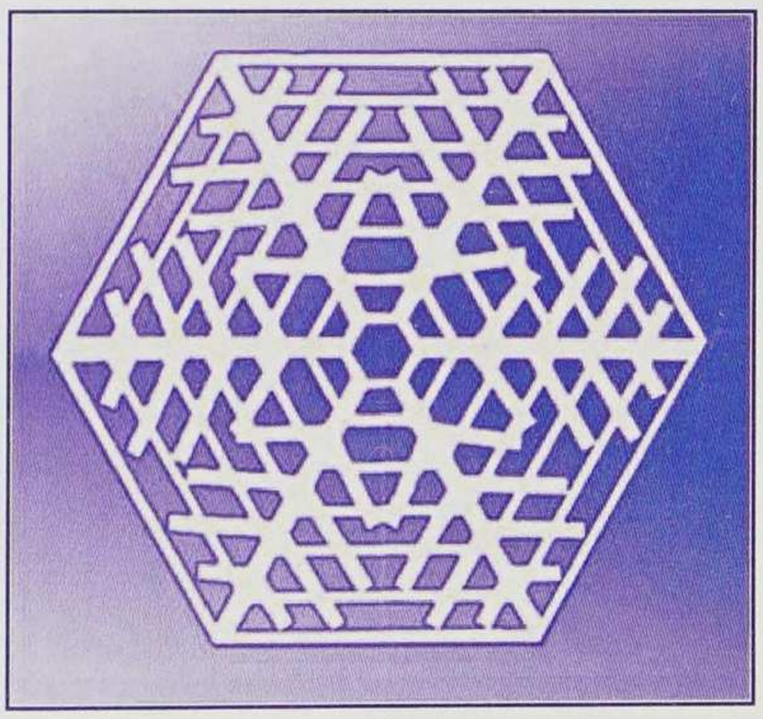
Photos Courtesy of Kenneth G. Libbrecht PhD.
at Caltech University
re: www.its.caltech.edu/~atomic/snowcrystals/photos

Snowflakes are really numerical Weaves of Wonder.

This composite picture shows the perfect 3-directional symmetry of various snowflakes. This perfect symmetry cannot be explained by the molecular structure of water nor the static electricity of vapor without a template for which it can all take place. So that must be coming from somewhere else. That is what will be demonstrated here geonomically.

In the following section, snowflakes will be shown to exist in hexagons found in hexagonal loom-tables. These will be attributed to patterns that contain all the numbers less than the loom-table average plus the average itself. These tables can be balanced in positive and negative numbers by subtracting the average from all the numbers. Then it is mathematically obvious why vapor would condense in the plains of zero and the troughs below zero and avoid the mounds above zero forming snowflakes as we see them. .

The diagram to the right shows the snowflake pattern spread across a series of nested hexagonal frames of decreasing size. This is the form of the equal summation patterns that will be shown for various size hexagonal tables to follow.



Odd Size Hexagons

The size-5 hexagon

Here are the size-5 dual geonomic hexagons. The primal hexagon **W** has all its calculations done in the base **m=12**; the dual **U** has all its calculations done in the base **n=5**. Here are the formulas for those calculations:

(10.6) $X(m) = \text{Mod}\,[W(m)-|1| \mid m] + |1|$

(10.7) $Y(m) = \text{Int}\,[(W(m)- |1|) / m] + |1|$

(10.8) $X(n) = \text{Int}\,[(U(n)-|1|) / n] + |1|$

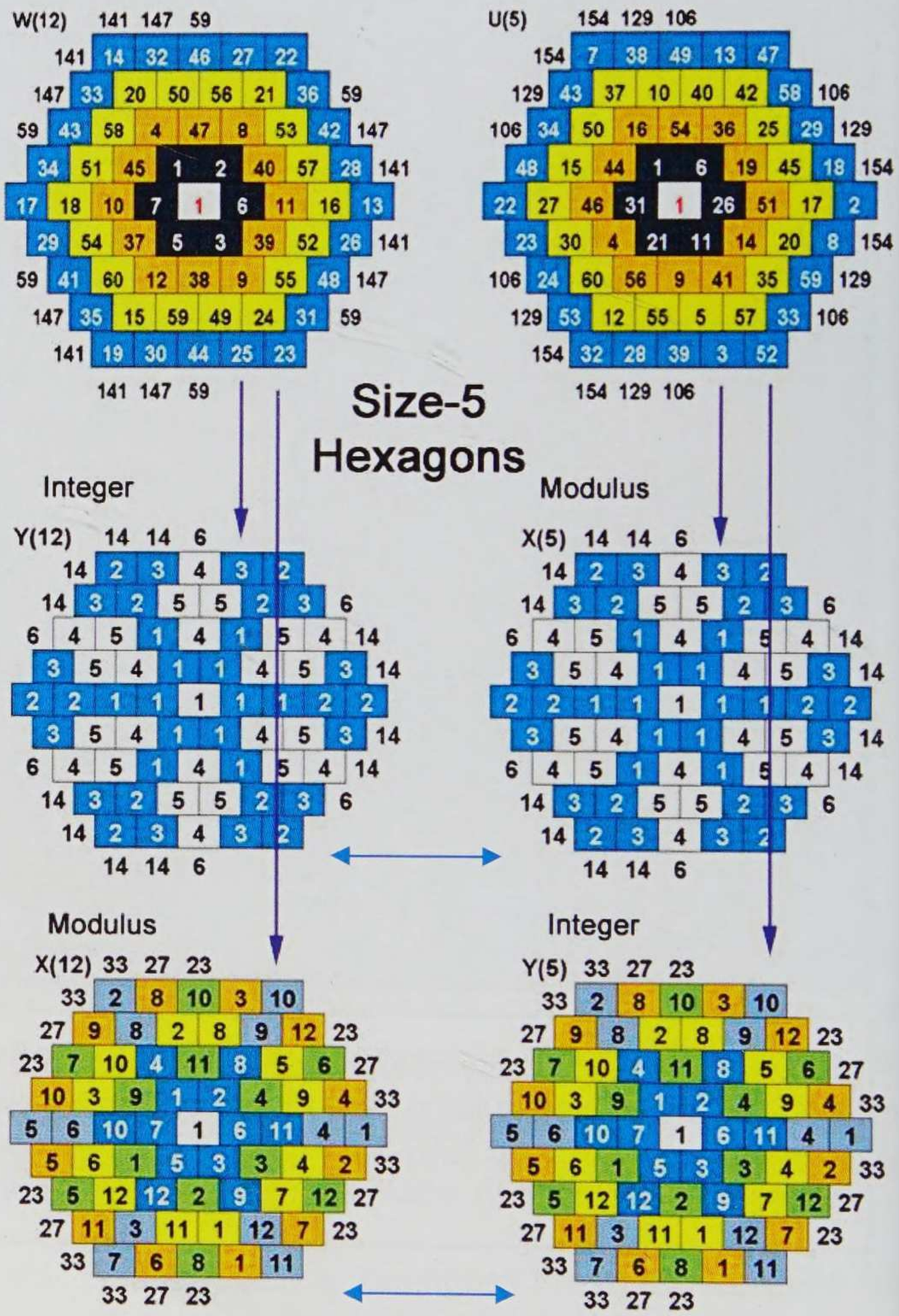
(10.9) $Y(n) = \text{Mod}\,[U(n)-|1| \mid n] + |1|$

Now here's the amazing part:

(10.10) $\begin{cases} Y(m) = X(n) \\ X(m) = Y(n) \end{cases}$

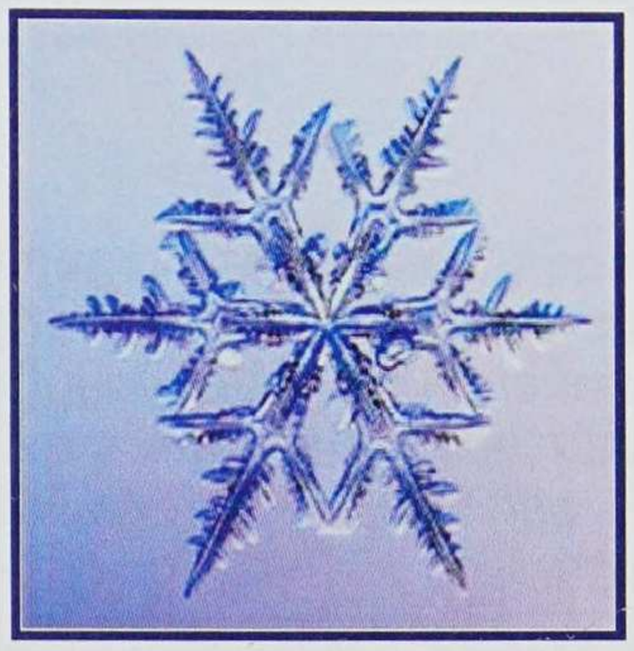
just like for squares.

In what follows, tables **Y(n)** and **Y(m)** will be shown along with the primal hexagon **W(m)**. Derivation of their duals **U(n)** will be left as an exercise if you want to explore and verify them.



Program 10

Size-7 Hexagon



Here is the size-7 hexagon. All of its nested frames have equal-summing sides. It is geonomically perfect.

Its integer loom table in the base **18** unmasks a snow-flake pattern upon shading numbers **1** through **4**. All of its frames have equal-summing sides. The pattern mimics the snowflake depicted at right.

W(18)

	606	379	328	73	100				
606	60	110	70	120	66	111	69		
379	126	45	49	105	100	39	41	119	100
328	65	40	32	74	107	79	36	53	56	73
73	118	99	88	8	27	28	10	87	98	124	328
100	59	101	97	33	11	75	14	20	96	93	67	379
114	42	82	31	73	2	7	77	26	84	44	116	606
64	52	29	1	16	5	1	3	9	17	25	50	55
122	43	83	24	80	15	12	85	22	76	47	113	606
100	61	106	104	30	4	90	6	21	103	91	72	379
73	109	95	78	18	19	23	13	89	102	112	328
328	63	46	34	86	92	81	35	51	71	73
379	125	37	48	108	94	54	38	115	100
606	62	117	58	123	57	121	68		
	606	379	328	73	100				

Y(18)

	37	24	20	6	7				
37	4	7	4	7	4	7	4		
24	7	3	3	6	6	3	3	7	7
20	4	3	2	5	6	5	2	3	4	6
6	7	6	5	1	2	2	1	5	6	7	20
7	4	6	6	2	1	5	1	2	6	6	4	24
7	3	5	2	5	1	1	5	2	5	3	7	37
4	3	2	1	1	1	1	1	1	1	2	3	4
7	3	5	2	5	1	1	5	2	5	3	7	37
7	4	6	6	2	1	5	1	2	6	6	4	24
6	7	6	5	1	2	2	1	5	6	7	20
20	4	3	2	5	6	5	2	3	4	6
24	7	3	3	6	6	3	3	7	7
37	4	7	4	7	4	7	4		
	37	24	20	6	7				

X(18)

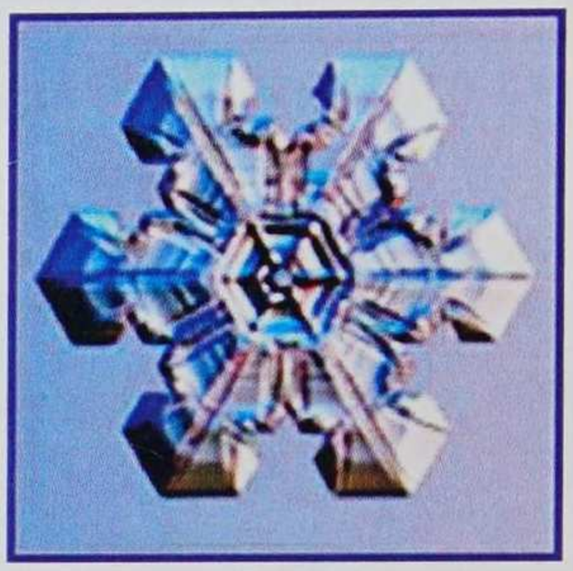
	66	55	58	37	28							
66	6	2	16	12	12	3	15					
55	18	9	13	15	10	3	5	11	28			
58	11	4	14	2	17	7	18	17	2	37		
37	10	9	16	8	9	10	10	15	8	16	58	
28	5	11	7	15	11	3	14	2	6	3	13	55
6	6	10	13	1	2	7	5	8	12	8	8	66
10	16	11	1	16	5	18	3	9	17	7	14	1
14	7	11	6	8	15	12	13	4	4	11	5	66
28	7	16	14	12	4	18	6	3	13	1	18	55
37	1	5	6	18	1	5	13	17	12	4	58
58	9	10	16	14	2	9	17	15	17	37
55	17	1	12	18	4	18	2	7	28
66	8	9	4	15	3	13	14		
	66	55	58	37	28				

Size-9 Hexagon

Here is the size-9 hexagon. It contains all of the numbers from **1** thru **216**. All but one of its nested frames have equal-summing sides. Its outermost frame has one side that is off by **+1**. This minor variation in the outermost frame is a spatial wrinkle that cannot be eliminated but merely shifted to another frame.

W(24)

		1317	1115	754	516	343	268	54		974	
	1317	97	194	205	74	134	89	215	204	105	
	1115	200	104	169	96	181	191	83	180	111	193	54
	754	212	177	112	62	148	120	153	49	110	170	207	268
	516	84	90	54	63	164	33	38	161	57	59	87	75	343
	343	143	188	145	166	68	35	127	41	72	163	147	183	140	516
	268	73	185	114	29	39	8	129	125	6	27	44	115	187	79	754
	54	209	91	151	25	123	132	24	7	23	137	135	47	160	95	214	1115
		198	174	60	168	43	126	12	1	10	9	122	40	154	64	179	202	1317
		101	106	118	65	70	2	18	17	1	15	22	3	69	51	99	103	102	962
		199	176	56	165	42	141	16	14	19	11	136	34	146	58	171	195	1317
	54	211	92	152	31	133	121	20	13	21	124	139	48	156	88	206	1115
	268	82	190	116	32	37	4	128	131	5	30	46	119	182	76	754
	343	142	186	150	157	61	36	130	45	71	158	155	192	138	516
	516	78	85	53	66	162	28	26	167	67	50	93	81	343
	754	210	173	109	52	149	113	159	55	117	178	216	268
	1115	197	107	175	86	189	184	94	172	108	203	54
	1317	98	201	213	80	144	77	208	196	100	
		1318	1115	754	516	343	268	54		986	
		+1									



The integer loom-table in the base **m=24** reveals the hidden snow-flake by highlighting the numbers **1** thru **5**. All of its frames have equal-summing sides. It mimics the snowflake pattern above.

Y(24)

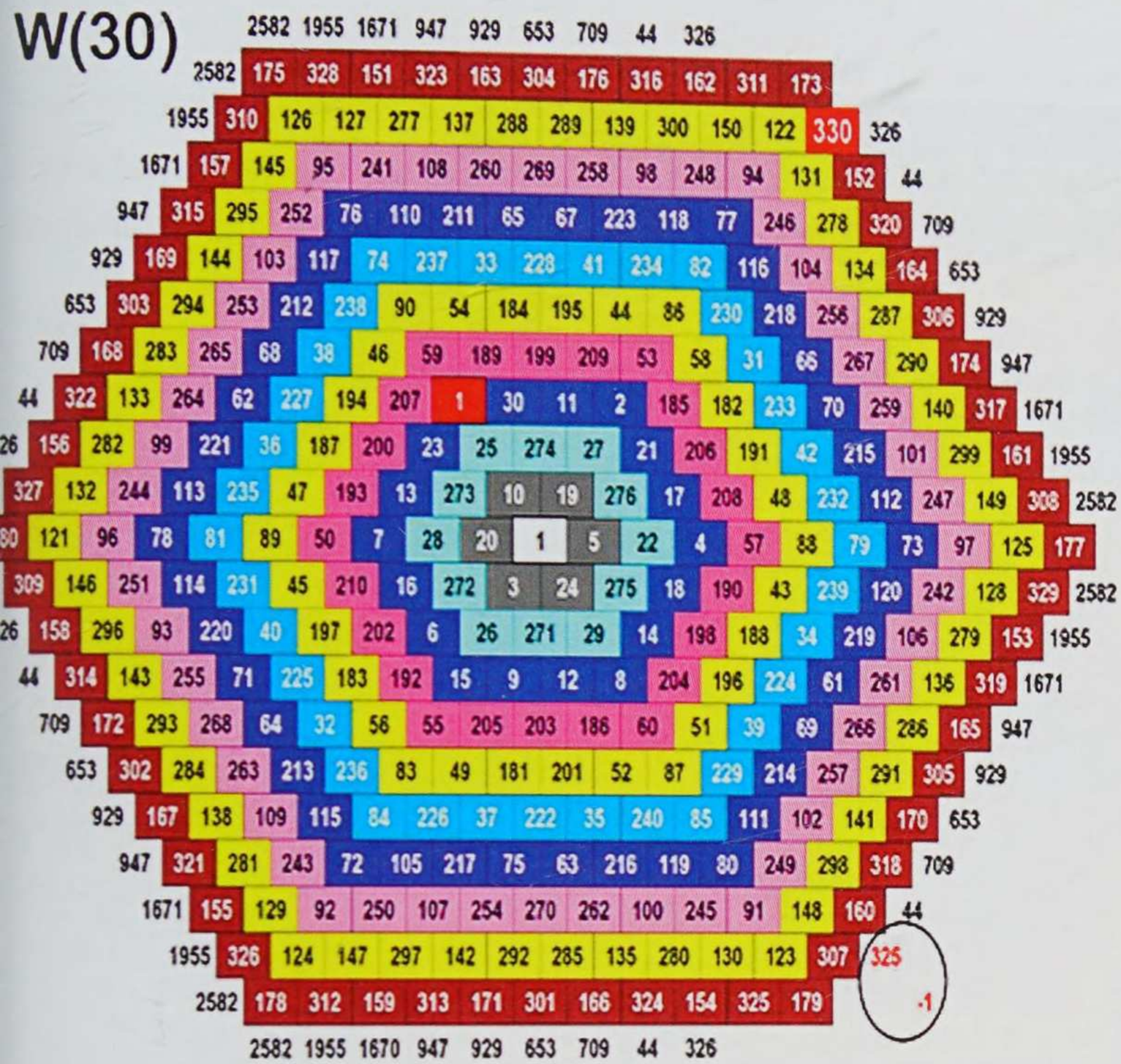
		60	50	35	24	16	14	3			
	60	5	9	9	4	6	4	9	9	5	
	50	9	5	8	4	8	8	4	8	5	9	3
	35	9	8	5	3	7	5	7	3	5	8	9	14
	24	4	4	3	3	7	2	2	7	3	3	4	4	16
	16	6	8	7	7	3	2	6	2	3	7	7	8	6	24
	14	4	8	5	2	2	1	6	6	1	2	2	5	8	4	35
	3	9	4	7	2	6	6	1	1	1	6	6	2	7	4	9	50
	9	8	3	7	2	6	1	1	1	1	6	2	7	3	8	9	60
	5	5	5	3	3	1	1	1	1	1	1	1	3	3	5	5	5
	9	8	3	7	2	6	1	1	1	1	6	2	7	3	8	9	60
	3	9	4	7	2	6	6	1	1	1	6	6	2	7	4	9	50
	14	4	8	5	2	2	1	6	6	1	2	2	5	8	4	35
	16	6	8	7	7	3	2	6	2	3	7	7	8	6	24
	24	4	4	3	3	7	2	2	7	3	3	4	4	16
	35	9	8	5	3	7	5	7	3	5	8	9	14
	50	9	5	8	4	8	8	4	8	5	9	3
	60	5	9	9	4	6	4	9	9	5	
		60	50	35	24	16	14	3			

X(24)

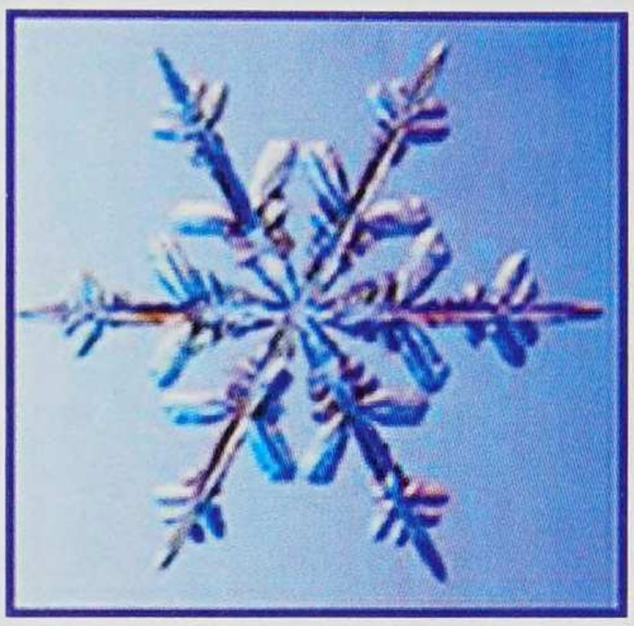
		93	107	82	84	79	28	54		
	93	1	2	13	2	14	17	23	12	9	
	107	8	8	1	24	13	23	11	12	15	1	54
	82	20	9	16	14	4	24	9	1	14	2	15	28
	84	12	18	6	15	20	9	14	17	9	11	15	3	79
	79	23	20	1	22	20	11	7	17	24	19	3	15	20	84
	28	1	17	18	5	15	8	9	5	6	3	20	19	19	7	82
	54	17	19	7	1	3	12	24	7	23	17	15	23	16	23	22	107
		6	6	12	24	19	6	12	1	10	9	2	16	10	16	11	10	93
		5	10	22	17	22	2	18	17	1	15	22	3	21	3	3	7	6
		7	8	8	21	18	21	16	14	19	11	16	10	2	10	3	3	93
	54	19	20	8	7	13	1	20	13	21	4	19	24	12	16	14	107
	28	10	22	20	8	13	4	8	11	5	6	22	23	14	4	82
	79	22	18	6	13	13	12	10	21	23	14	11	24	18	84
	84	6	13	5	18	18	4	2	23	19	2	21	9	79
	82	18	5	13	4	5	17	15	7	21	10	24	28
	107	5	11	7	14	21	16	22	4	12	11	54
	93	2	9	21	8	24	5	16	4	4
		94	107	82	84	79	28	54		
		+1								

Size-11 Hexagon

W(30)

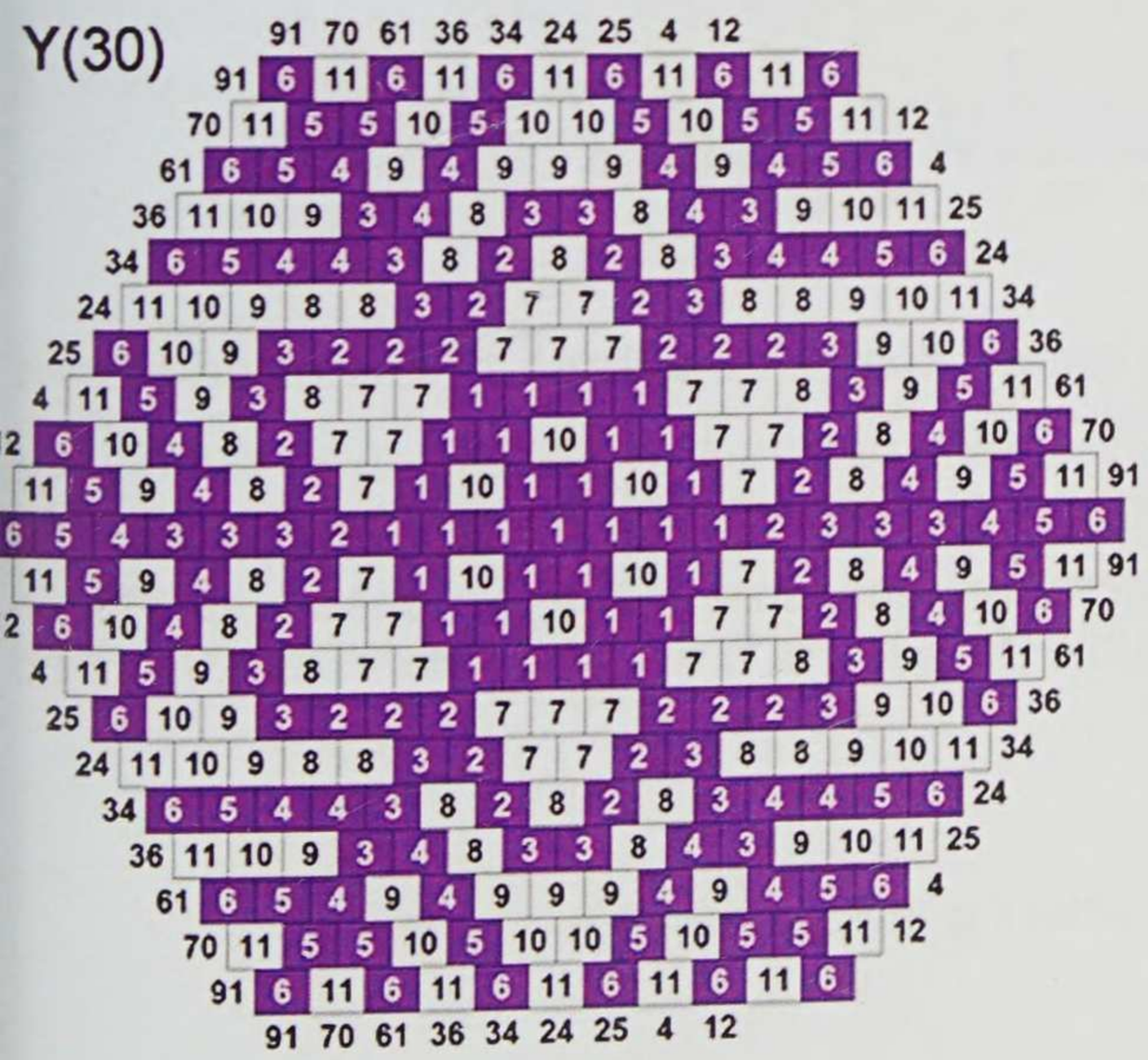


Here is the size-11 hexagon. It contains all the numbers from **1** to **330**. All but one of its nested frames have equal-summing sides. Its innermost viable frame has one side that is off by **1**. This minor variation in the innermost viable frame cannot be eliminated, only shifted to another frame.

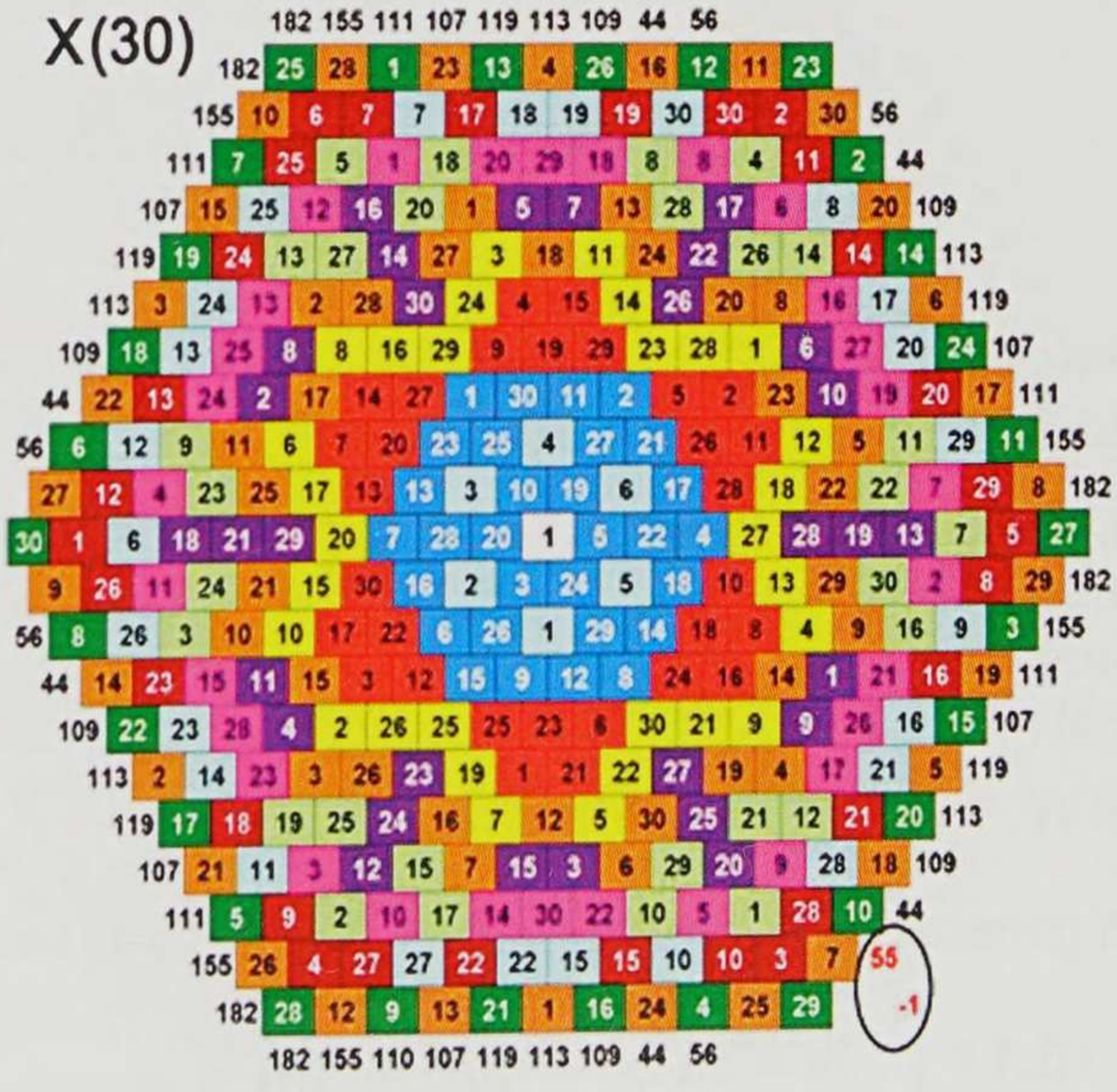


Its Integer loom-table reveals the hidden snowflake pattern by high-lighting the numbers **1** thru **6**. It mimics the snowflake shown here. All of its sides sum equaly except one side in the innermost viable frame.

Y(30)

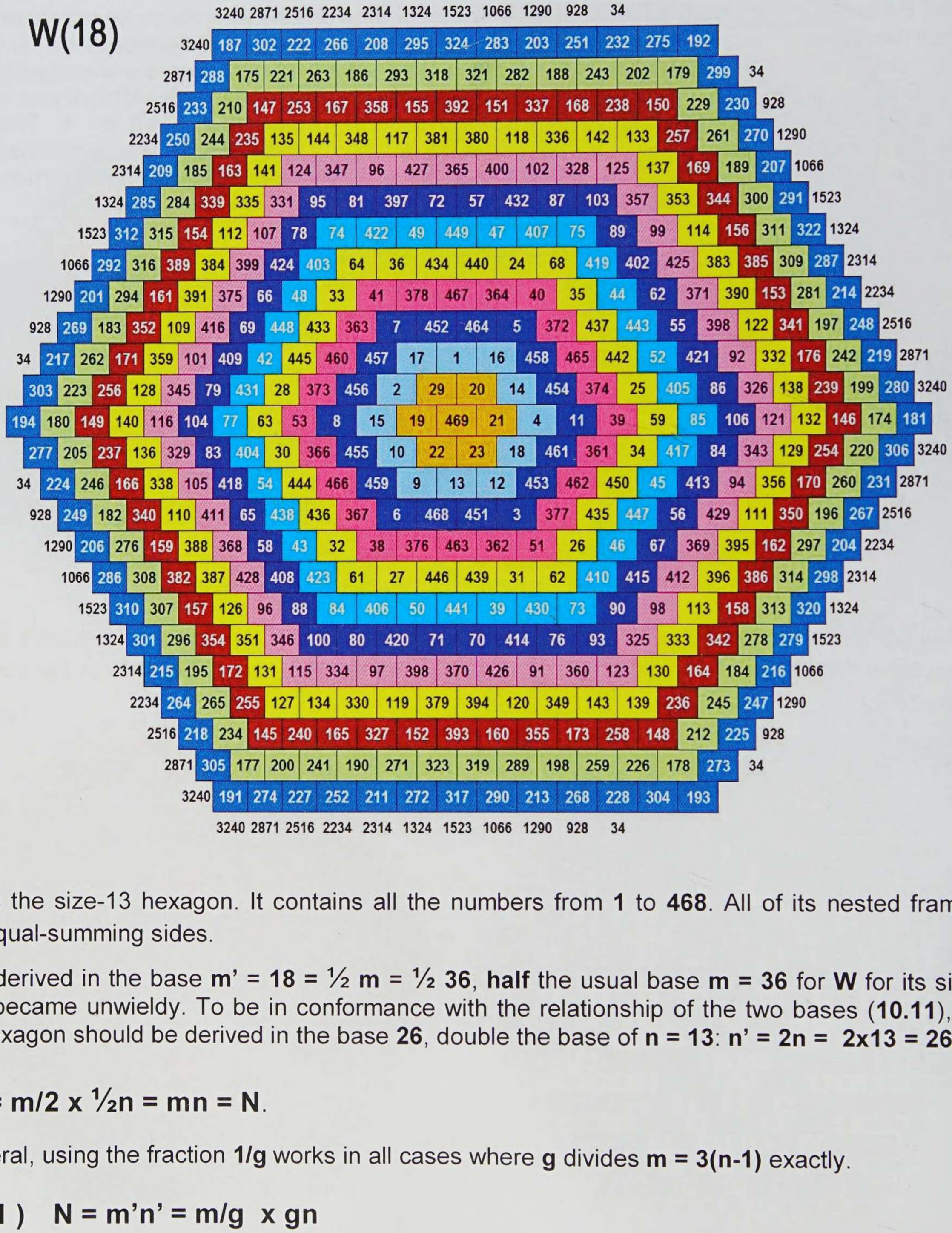


X(30)



Size-13 Hexagon

W(18)



Here is the size-13 hexagon. It contains all the numbers from **1** to **468**. All of its nested frames have equal-summing sides.

It was derived in the base **m' = 18 = ½ m = ½ 36**, **half** the usual base **m = 36** for **W** for its size, which became unwieldy. To be in conformance with the relationship of the two bases (**10.11**), its dual hexagon should be derived in the base **26**, double the base of **n = 13**: **n' = 2n = 2x13 = 26** so that

m'n' = m/2 x ½n = mn = N.

In general, using the fraction **1/g** works in all cases where **g** divides **m = 3(n-1)** exactly.

(10.11) N = m'n' = m/g x gn

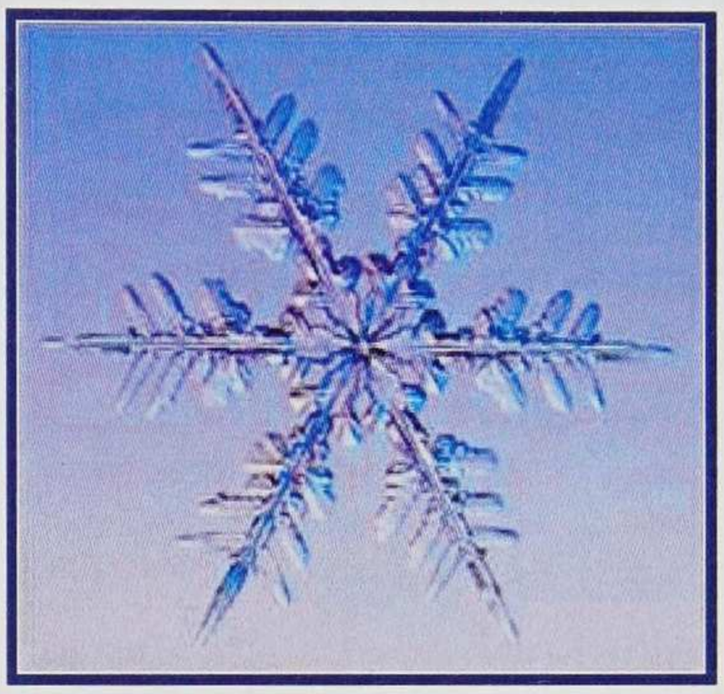
Program 10

The Size-13 Loom Tables

The size-13 hexagons integer loom table derived using the base **m' = 18 = 36/2** unmasks a snowflake pattern upon shading numbers **1** thru **13 = ½n'**. The pattern mimics the depicted snowflake at right.

Y(18)

185	11	17	13	15	12	17	18	16	12	14	13	16	11	
165	16	10	13	15	11	17	18	18	16	11	14	12	10	17	3
146	13	12	9	15	10	20	9	22	9	19	10	14	9	13	13	54
129	14	14	14	8	8	20	7	22	22	7	19	8	8	15	15	15	74
133	12	11	10	8	7	20	6	24	21	23	6	19	7	8	10	11	12	62
77	16	16	19	19	19	6	5	23	4	4	24	5	6	20	20	20	17	17	88
88	18	18	9	7	6	5	5	24	3	25	3	23	5	5	6	7	9	18	18	77
62	17	18	22	22	23	24	23	4	2	25	25	2	4	24	23	24	22	22	18	16	133
74	12	17	9	22	21	4	3	2	3	21	26	21	3	2	3	4	21	22	9	16	12	129
54	15	11	20	7	24	4	25	25	21	1	26	26	1	21	25	25	4	23	7	19	11	14	146
3	13	15	10	20	6	23	3	25	26	26	1	1	1	26	26	25	3	24	6	19	10	14	13	165
	17	13	15	8	20	5	24	2	21	26	1	2	2	1	26	21	2	23	5	19	8	14	12	16	185
	11	10	9	8	7	6	5	4	3	1	1	2	1	2	1	1	3	4	5	6	7	8	9	10	11	
	16	12	14	8	19	5	23	2	21	26	1	2	2	1	26	21	2	24	5	20	8	15	13	17	185
3	13	14	10	19	6	24	3	25	26	26	1	1	1	26	26	25	3	23	6	20	10	15	13	165
54	14	11	19	7	23	4	25	25	21	1	26	26	1	21	25	25	4	24	7	20	11	15	146
74	12	16	9	22	21	4	3	2	3	21	26	21	3	2	3	4	21	22	9	17	12	129
62	16	18	22	22	24	23	24	4	2	25	25	2	4	23	24	23	22	22	18	17	133
88	18	18	9	7	6	5	5	23	3	25	3	24	5	5	6	7	9	18	18	77
77	17	17	20	20	20	6	5	24	4	4	23	5	6	19	19	19	16	16	88
133	12	11	10	8	7	19	6	23	21	24	6	20	7	8	10	11	12	62
129	15	15	15	8	8	19	7	22	22	7	20	8	8	14	14	14	74
146	13	13	9	14	10	19	9	22	9	20	10	15	9	12	13	54
165	17	10	12	14	11	16	18	18	17	11	15	13	10	16	3
185	11	16	13	14	12	16	18	17	12	15	13	17	11
	185	165	146	129	133	77	88	62	74	54	3



The modulus loom-table to the base **n' = 26** is shown here.

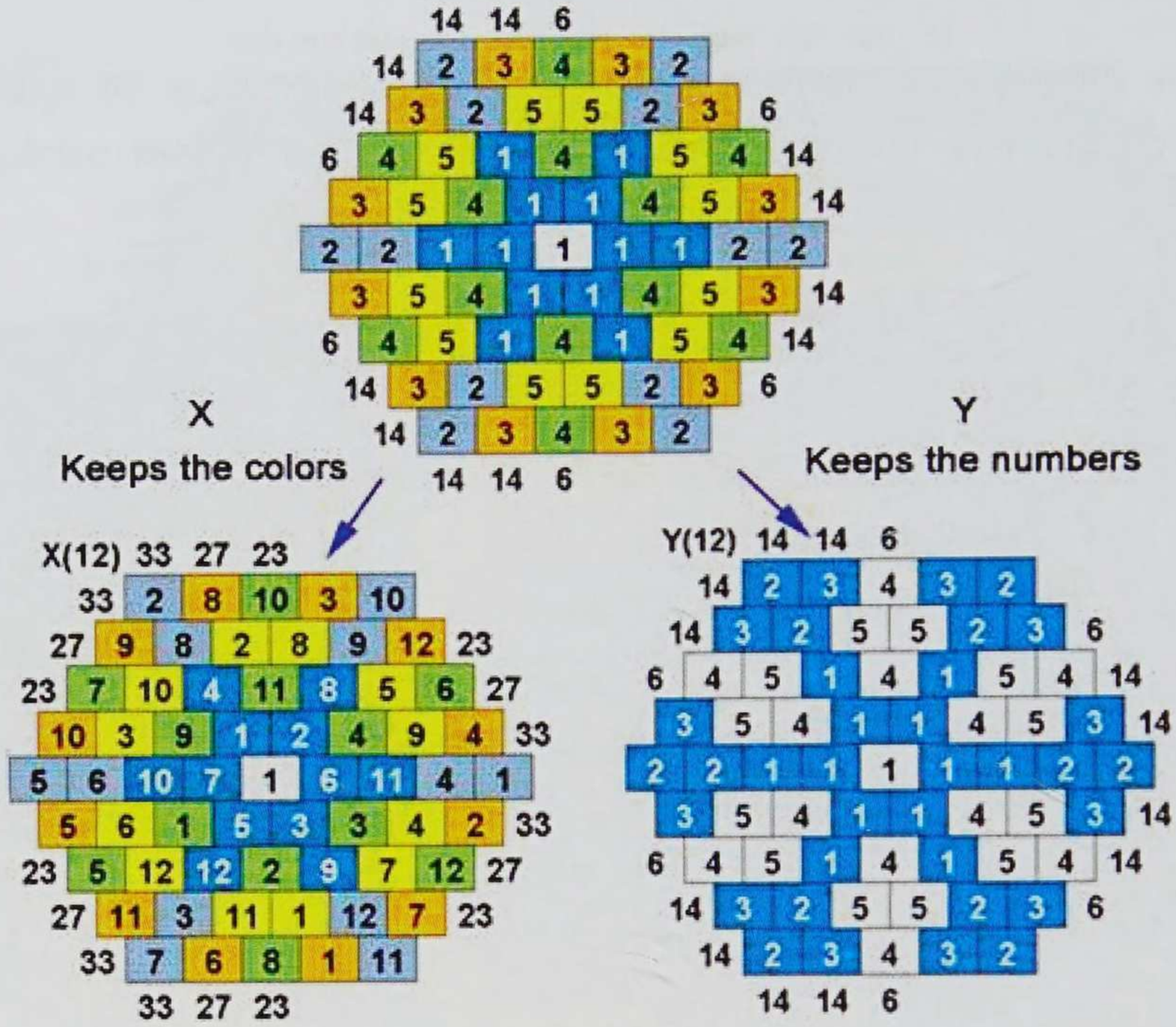
You now have enough information to derive the dual to the base **26** from:

(10.12) U = n'(Y(m') - |1|) + X(n')

X(26)

144	7	14	6	14	10	7	18	13	5	17	16	5	12		
117	18	13	5	11	6	5	12	15	12	8	9	4	17	11	34
86	17	12	3	1	5	16	11	14	7	13	6	4	6	13	14	28
92	16	10	1	9	18	6	9	3	2	10	12	16	7	5	9	18	48
82	11	5	1	15	16	5	6	13	5	4	12	4	17	11	7	9	9	58
82	15	14	15	11	7	5	9	1	18	3	18	15	13	15	11	2	12	3	65
65	6	9	10	4	17	6	2	8	13	17	11	11	3	17	9	6	12	5	16	82
58	4	10	11	6	3	10	7	10	18	2	8	6	14	5	6	11	5	7	3	17	82
48	3	6	17	13	15	12	12	15	5	18	17	4	4	17	8	8	11	12	9	11	16	92
28	17	3	10	1	2	15	16	1	3	7	2	14	5	12	5	11	1	2	14	17	17	14	86
34	1	10	9	17	11	13	6	13	10	7	17	1	16	8	15	10	16	7	2	8	14	8	3	117
15	7	4	2	3	7	17	10	13	6	2	11	2	14	4	14	7	9	14	2	12	5	1	10	144
14	18	5	14	8	14	5	9	17	8	15	1	1	3	4	11	3	5	13	16	13	6	2	12	1	
7	7	3	10	5	11	8	12	6	5	10	4	5	18	11	1	16	3	12	1	3	2	4	18	144
34	8	12	4	14	15	4	18	12	16	9	9	13	12	3	12	18	9	17	4	14	8	8	15	117
28	15	2	16	2	15	11	6	4	7	6	18	1	3	17	3	15	2	15	3	8	16	15	86
48	8	6	15	10	8	4	7	14	2	16	13	2	15	8	10	13	9	17	18	9	6	92
58	16	2	4	9	14	12	9	7	9	14	7	13	8	14	1	16	18	8	8	10	82
65	4	1	13	18	6	16	12	10	14	9	3	16	1	18	8	5	14	7	14	82
82	13	8	12	9	4	10	8	6	17	16	18	4	3	1	9	18	8	9	65
82	17	15	10	5	7	10	7	2	10	12	1	18	15	4	2	4	18	58
92	12	13	3	1	8	6	11	1	16	12	7	17	13	2	11	13	48
86	2	18	1	6	3	3	8	15	16	13	11	6	4	14	9	28
117	17	15	2	7	10	1	17	13	1	18	7	10	16	3	34
144	11	4	11	18	13	2	11	2	15	16	12	16	13
	144	117	86	92	82	82	65	58	48	28	34

Constructing Dual Snowflake-Patterned Complementary Loom-Tables



Denoting the hexagonal loom-tables by $\mathbf{X} \equiv |\, x_{ij} \,|$ and $\mathbf{Y} \equiv |\, y_{ij} \,|$:

$$(\,10.13\,) \qquad \begin{cases} y_{ij} = i & \text{for } 1 \le i \le n \\ x_{ij} = j & \text{for } 1 \le j \le m \end{cases} \qquad \text{where } \mathbf{m = 3(n-1)}$$

Ordinarily the subscripts **ij** would denote the cell in the **i**-th row and **j**-th column of a square table. Here, in view of the loom table patterns and their numbering within them, the subscript **i** refers to the **i**-th pattern of **Y** and **j** refers to the cell in the same **i**-th pattern but in **X** which is equal to **j**.

The loom table **Y** with the intended pattern is constructed first. The top hexagon has five differently colored patterns numbered separately from **1** thru **n,** here **n = 5**. The central number is always a duplicate **1**.

These patterns are arranged in a snowflake design for numbers **1** thru **3**, i.e. **(n + 1)/2**, so that all the viable nested frames in **Y** have equal-summing sides. These patterns are laid out to be symmetrical across each of the three diametrical axes as shown in the hexagon on the right.

Next, the loom table **X** is constructed according to the colored patterns at top. These numbers range from **1** thru **m** for each color; here **m = 12**.

First, the numbers **1** thru **n** in **X** are entered circularly, one number per side until all the numbers up thru **m** are placed within each color pattern. Next, the challenge is to get all the sides of each frame to sum equally in **X** or as close as is possible by swapping the locations of the numbers in **X** among their same-colored backgrounds.

Program 10

The frame-side equalizing process involves rearranging numbers in **X** in one or more frames simultaneously, all the while keeping each number within its corresponding color pattern. **It has to be emphasized here that the numbers in X must be rearranged within X to stay within the same-colored patterns to avoid any duplication of numbers in the resulting master table W.**

The process starts with the outermost frame and proceeds frame by frame inward toward the center, completing each frame before going on to the next. Frequently the need arises to swap numbers across multiple frames, some already completed, in order to be successful. That's the reason for working at least 2 frames simultaneously.

Then the numbers can be readily re-arranged among identically colored cells in Microsoft's Excel™ program. (The Excel™ program contains a limited number of discernible colors so only a subset of consecutive frames on larger hexagonal tables can be worked on at any one time due to this limitation.)

This rearrangement process is a real mental challenge but still surmountable after some practice. It took me 20 hours of swapping numbers to get the numerical layout of the size-13 modulus table **X** in the base **18**. Much time must also be spent in color formatting in order to avoid inadvertent duplication of numbers.

Now, once both loom tables **X** and **Y** are completed by having all equal sides in all their nested frames, or as near as possible in the smallest viable frame, then the master hexagon table **W** can be generated according to the formula:

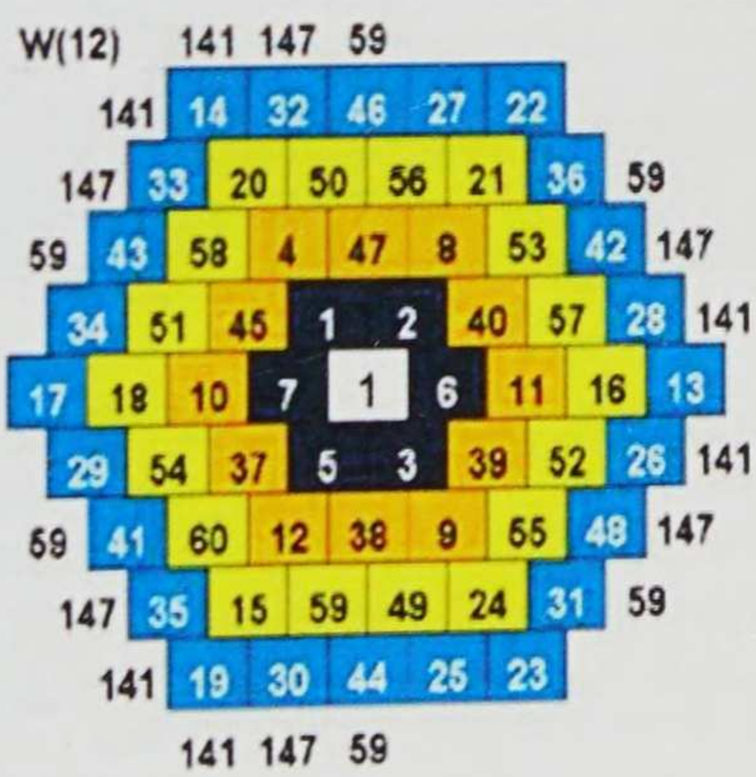
$$\mathbf{(10.14)} \qquad W = n(Y(m) - |1|) + X(n)$$

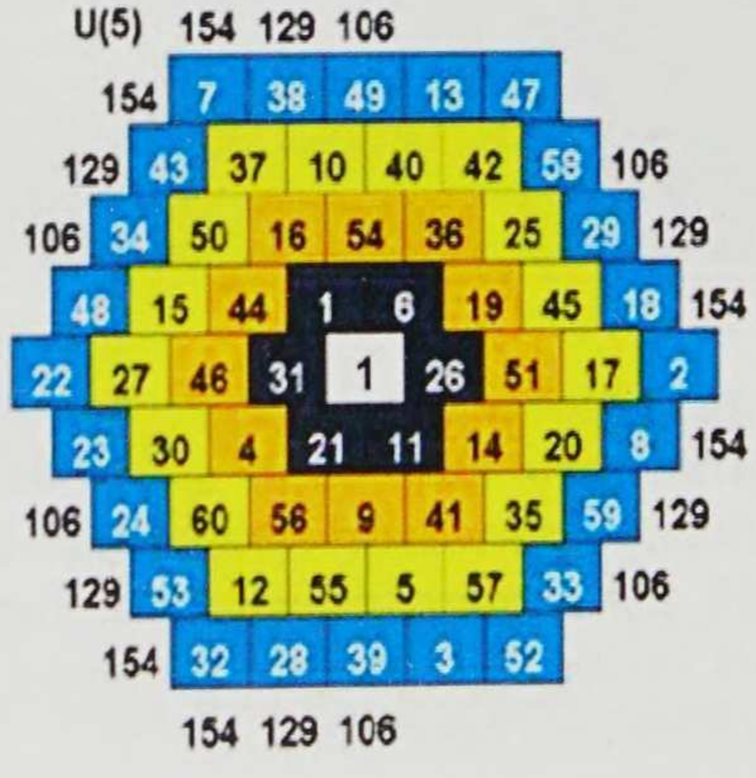
And the dual hexagon table **U** can be derived according to the formula:

$$\mathbf{(10.15)} \qquad U = m(X(n) - |1|) + Y(m)$$

Now, this whole process can be easily reversed by using formulas **(10.6)** thru (**10.9**) shown earlier so that both **X(n)** and **Y(m)** can be derived from **W** anew. This provides the means to regenerate the snowflake pattern in **Y(m)** that has now been buried within **W** in the base **m**, and to generate the snowflake pattern **X(n)** now buried within the dual **U** in the base **n**.

Size 5





Program 10

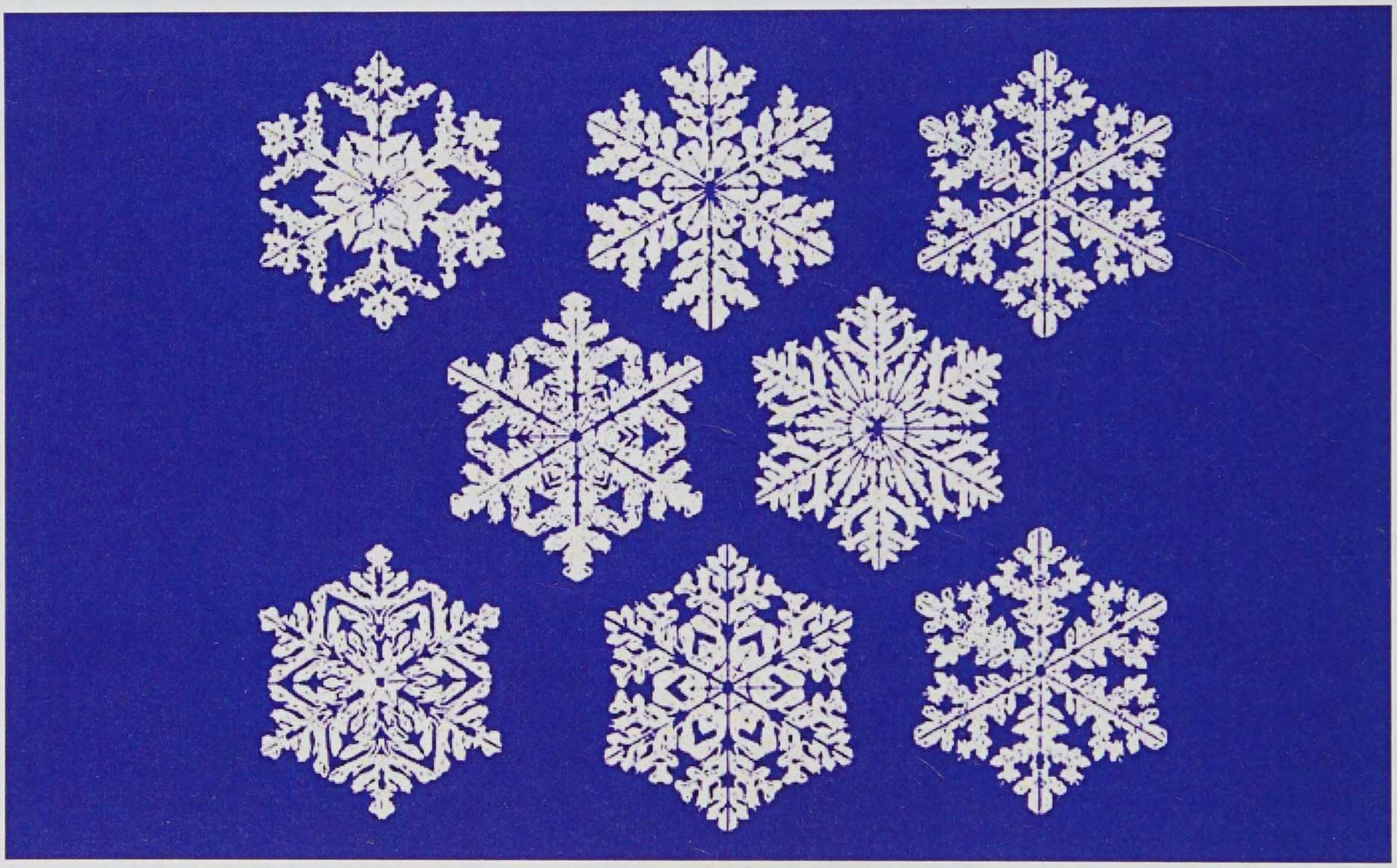
Both the original size-5 hexagonal table and its dual possess the uninterrupted number series **1** thru **60**.

Keep in mind that the central number is just an artifact to facilitate the natural duality possessed by the **N** off-center cells in odd size hexagons. Hexagons always possess 3**n(n-1)+1** cells.

Note that **N = nm = 3n(n-1)** is always wholly divisible by **6**:

(10.16) N/6 = 3n(n-1)/6 = n(n-1)/2

and either **n** or **n-1** is an even number and so is divisible by **2**.



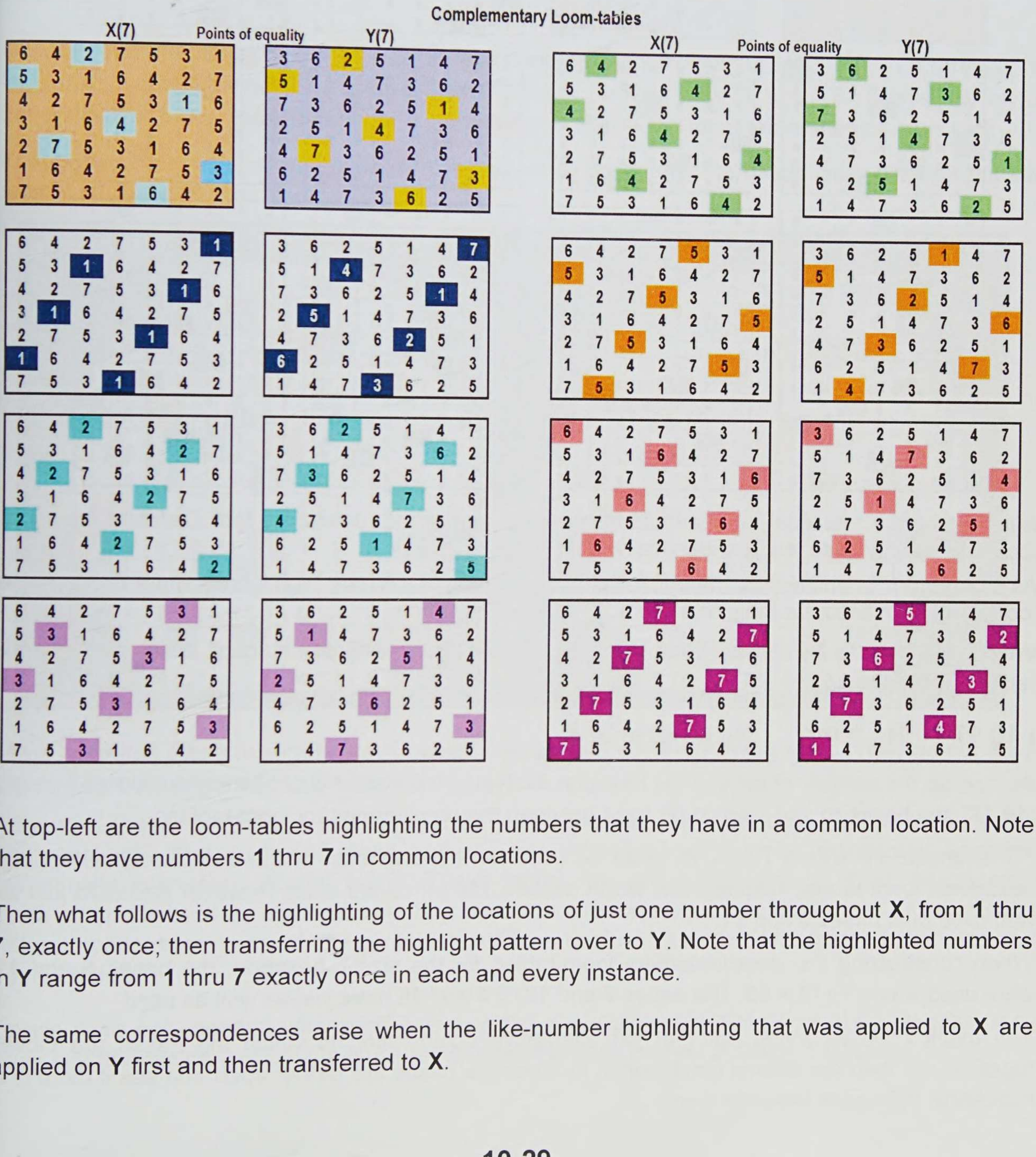
Although these snowflake patterns were prefabricated, this process does demonstrate mathematically that such snowflake patterns are indeed possible to be manifested from the numerically interwoven spatial fabric. Despite the fact that the pattern was fabricated, these tables are still a mapping of a sized, hexagonally confined space itself as embodied by the natural number series. It reflects the natural spatial harmonics wholly contained within a geometrical space of that size and configuration!

This will now be proven based on the mathematics common to all complementary geonomic loom-tables.

Program 10

Correspondences of values between complementary loom-tables of squares

Here are two size-7 complementary loom-tables **X(7)** and **Y(7)** from Class-1 squares.



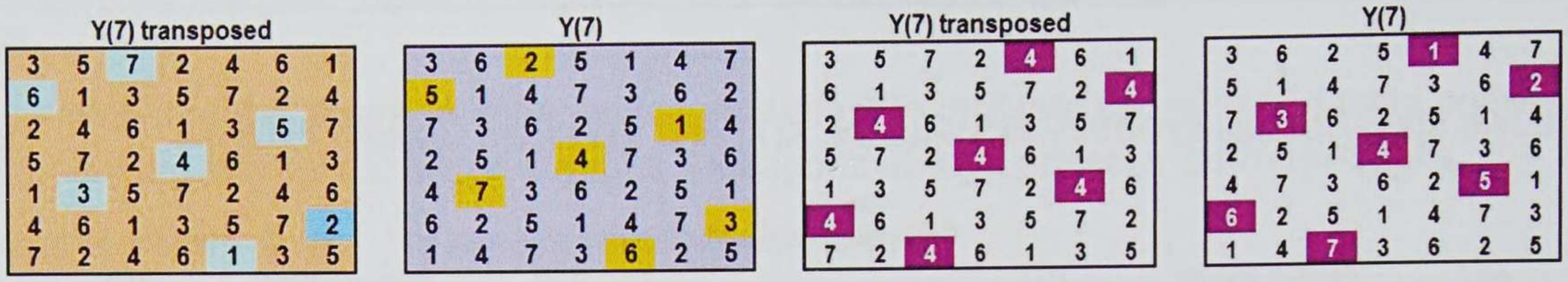
At top-left are the loom-tables highlighting the numbers that they have in a common location. Note that they have numbers **1** thru **7** in common locations.

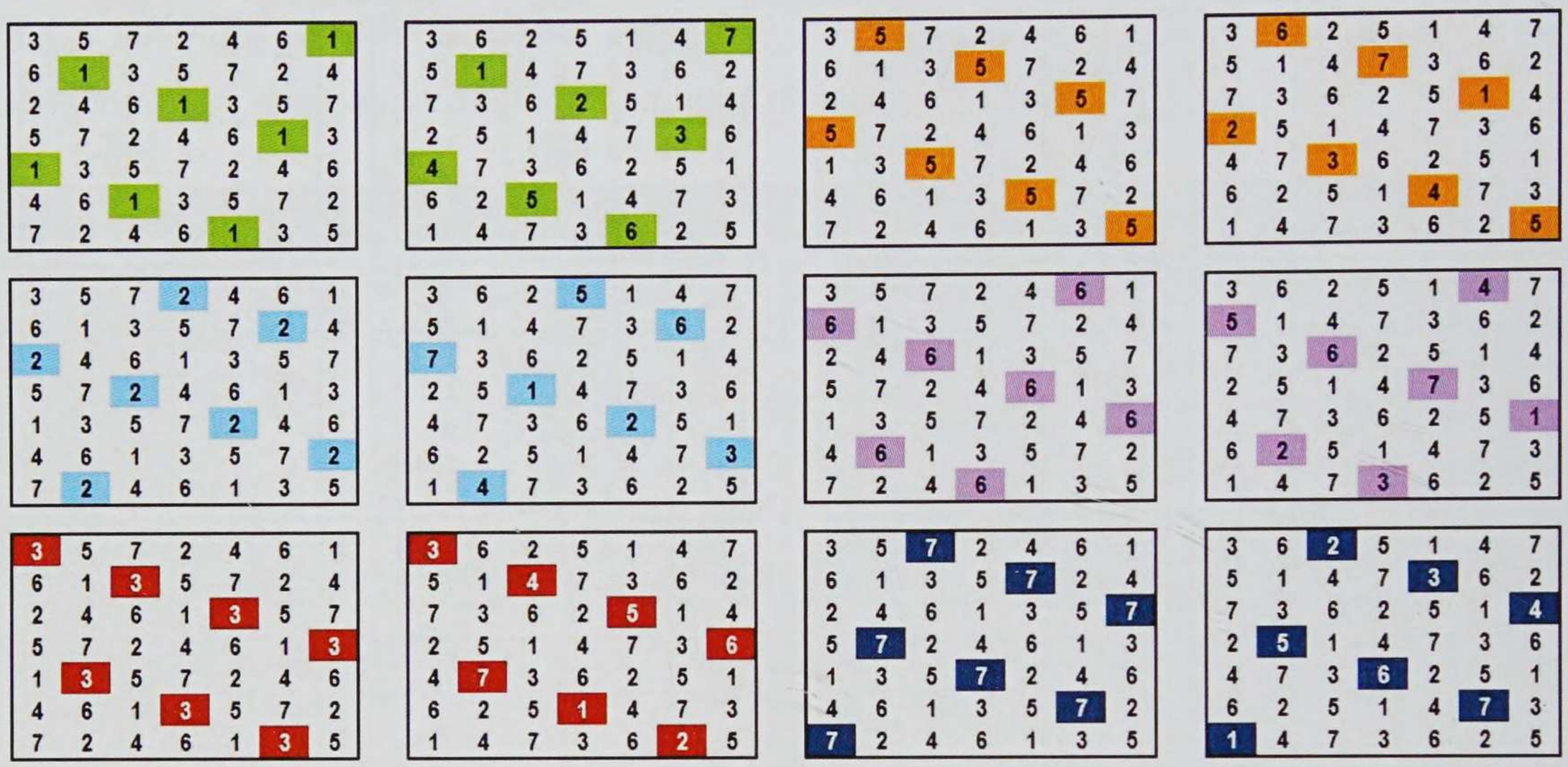
Then what follows is the highlighting of the locations of just one number throughout **X**, from **1** thru **7**, exactly once; then transferring the highlight pattern over to **Y**. Note that the highlighted numbers in **Y** range from **1** thru **7** exactly once in each and every instance.

The same correspondences arise when the like-number highlighting that was applied to **X** are applied on **Y** first and then transferred to **X**.

Program 10

Here is the size-7 integer loom-table and its transpose; both highlighted in the same manner between them as were the integer loom-table with its complementary modulus loom-table.





Again as before, the loom-table and its transpose possess the numbers **1** thru **7** among the same locations exactly once, but not identically number for number.

As the pattern is shifted, its transpose has all of a single number highlighted while those same locations within itself run the gamut from **1** thru **7** exactly once.

Well this is exactly what was done in hexagons but using different number bases which were related by the formula:

(10.17) $\mathbf{N_n = mn}$, $\mathbf{N_n}$ divisible by **6**.

As long as the number of cells in the hexagon excluding the center could be expressed as formula (**10.17**), the bases **m** and **n** could be used between the complementary loom tables.

For example, the bases **7** and **18** could be used for the derivation of the size-7 dual and primal hexagonal loom-tables, respectively, which contain **126** off-center cells. However we could just as well used dual bases **6** and **21**.

When constructing the complementary loom-tables for the size-7 hexagon, the bases **5** and **12** were used where **5x12 = 60**. The bases **6** and **10** or **4** and **15** have just as well as used.

You might consider trying out the first alternative combination in which the bases are closer together and then the second combination in which the bases are farther apart and see if there is a noticeable difference between them.

Correspondences of values between complementary loom-tables of hexagons

Here again are the complementary loom-tables of the size-7 **geonomic** hexagon seen earlier. On the right, the three numbers **1**, **7** and **18** have been highlighted in three distinct colors. These highlights were then transferred from the integer loom-table on the right to the modulus loom-table on the left.

Observe that each color in the table on the left contains the numbers from **1** thru **7** exactly once. In fact, this occurs between the loom-tables for every number from **1** thru **18**. This is merely a generalization of what we've seen for squares. This confirms the loom-tables' complementarity.

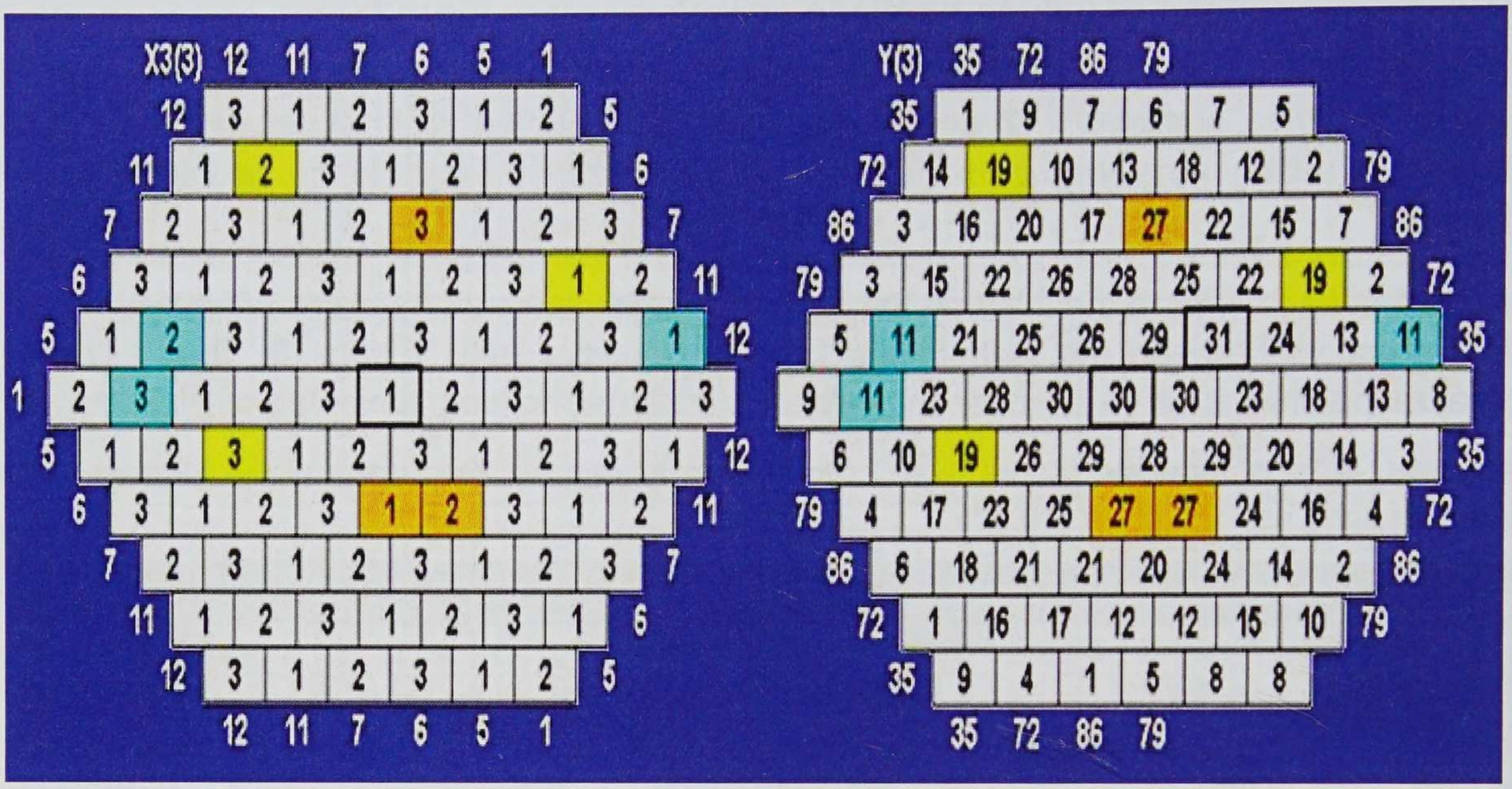
In squares, $\mathbf{N_2 = n^2}$ and the bases were common. In hexagons, the bases were different but still related similarly as in formula (**10.17**).

Make note that these hexagons have nested frames with equal-summing sides. They do not have equal crossover summations like geonomic squares.

Although these snowflake patterns were prefabricated, this process does demonstrate mathematically that such snowflake patterns are indeed possible to be manifested from the numerically interwoven spatial fabric. These tables are still a mapping of sized, hexagonally confined space itself as embodied by the natural number series just as for squares. It reflects the natural spatial harmonics wholly contained within a confined geometrical space of that size and configuration!

So we were not cheating when prefabricating the snowflakes into the geonomic hexagons; they obey a mathematical principle that is already fundamentally evident in Class-1 squares.

Program 10



And just as was shown on the picture on page 29, coloring all identical numbers in **Y** with the same color and transferring the coloring pattern to **X** will yield the numbers from **1** thru **3** in **X** for each distinct color transferred.

It would be nice to have perfect hexagonal primal and dual tables that possessed equality across its three corner-to-corner axes. Maybe as the sizes are increased, this may be easier to accomplish.

Even Size Hexagons

Size 6

			470	254	113	30						
		470	83	70	73	79	76	89				
	254	87	61	39	58	44	52	62	30			
	113	72	49	23	24	34	32	46	65	113		
	30	71	43	28	2	23	5	20	51	81	254	
		67	41	33	18	1	8	16	31	57	85	470
		90	60	29	10	4	15	11	9	30	48	88
		82	38	22	3	12	19	14	37	40	68	470
		30	66	45	36	17	6	7	21	53	69	254
		113	63	55	26	35	27	25	54	75	113	
		254	78	56	47	50	42	59	86	30		
		470	91	80	74	77	64	84				
			470	254	113	30						

Here is the perfect size-6 geonomic hexagon with all sides of its nested frames equal. It was obtained by starting with the odd-size-5 hexagon and adding another frame to it, starting with **62** and ending with **91**.

Notice that all these numbers lie within the outermost frame; there was no need to exchange numbers with other frames. So deriving even-size geonomic hexagons starting with a perfect hexagon is quite straight forward.

Program 10

What may seem as a geonomic hexagon many times isn't

W(5)

196 251 107 38 196
251 52 37 54 50 58
107 53 27 21 33 26 38 38
38 42 28 7 17 14 35 47 107
45 32 15 3 5 13 24 48 251
196 59 20 16 6 0 2 11 22 60 196
51 30 10 1 4 9 25 40 251
38 41 34 12 8 18 31 46 107
107 43 23 36 19 29 49 38
251 57 55 44 39 56
196 251 107 38 196

Above is a perfect size-5 hexagon. Even its tri-axial summations are equal. However, its loom tables are not geonomic and their axial sums are unequal. So the hexagon here was not derived in any systematic manner. Nonetheless, it is still remarkable!

Y(12)

21 23 11 5 20
24 5 4 5 5 5
11 5 3 2 3 3 4 4
5 4 3 1 2 2 3 4 11
4 3 2 1 1 2 2 4 23
19 5 2 2 1 0 1 1 2 5 19
5 3 1 1 1 1 3 4 22
4 4 3 1 1 2 3 4 10
10 4 2 3 2 3 5 5
23 5 5 4 4 5
20 23 10 4 21

X(5)

26 16 12 8 26
16 2 2 4 5 3
7 3 2 1 3 1 3 8
8 2 3 2 2 4 5 2 12
5 2 5 3 5 3 4 3 16
26 4 5 1 1 5 2 1 2 5 26
1 5 5 1 4 4 5 5 16
8 1 4 2 3 3 1 1 12
12 3 3 1 4 4 4 8
16 2 5 4 4 1
26 11 17 8 26

Program 10

Program 10 Notes

1. The numbers in red are the near-perfect geonomic triangle sizes encountered so far: **4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18**.

2. A size-10 geonomic tetrahedron was depicted with equal-summing surface triangular patterns parallel with the bottom base and at the top corner..These differed as a consequence of the sequence in which the triangles parallel to the base were constructed one after another going from the smallest to the largest in that order.

3. Perfect geonomic **diamonds** can be generated by merging two consecutive size squares.

   a) The *laminated indexing method* **LIM** will always work with any two adjacent size squares. It will even work with a Class-2 square involved.

   b) The method that will match the laminated indexing method is the **BAG** method in which a multiple of the square's integer loom table was added to it and then merged with an adjacent size square having undergone the same procedure. This method will only work between adjacent Class-1 and Class-4 squares. Further, it will only work with Class-4 squares that are pairwise row-symmetric.

4. There three different types of pentagon which have various equal summation properties. One type can be applied to enumerating electrons in atoms whose **atomic number less 1** is divisible by either **3** or **5**.

5. The size **5** hexagon is the smallest odd-size hexagon that is geonomically perfect in both its pair of complementary loom tables and its pair of dual hexagons.

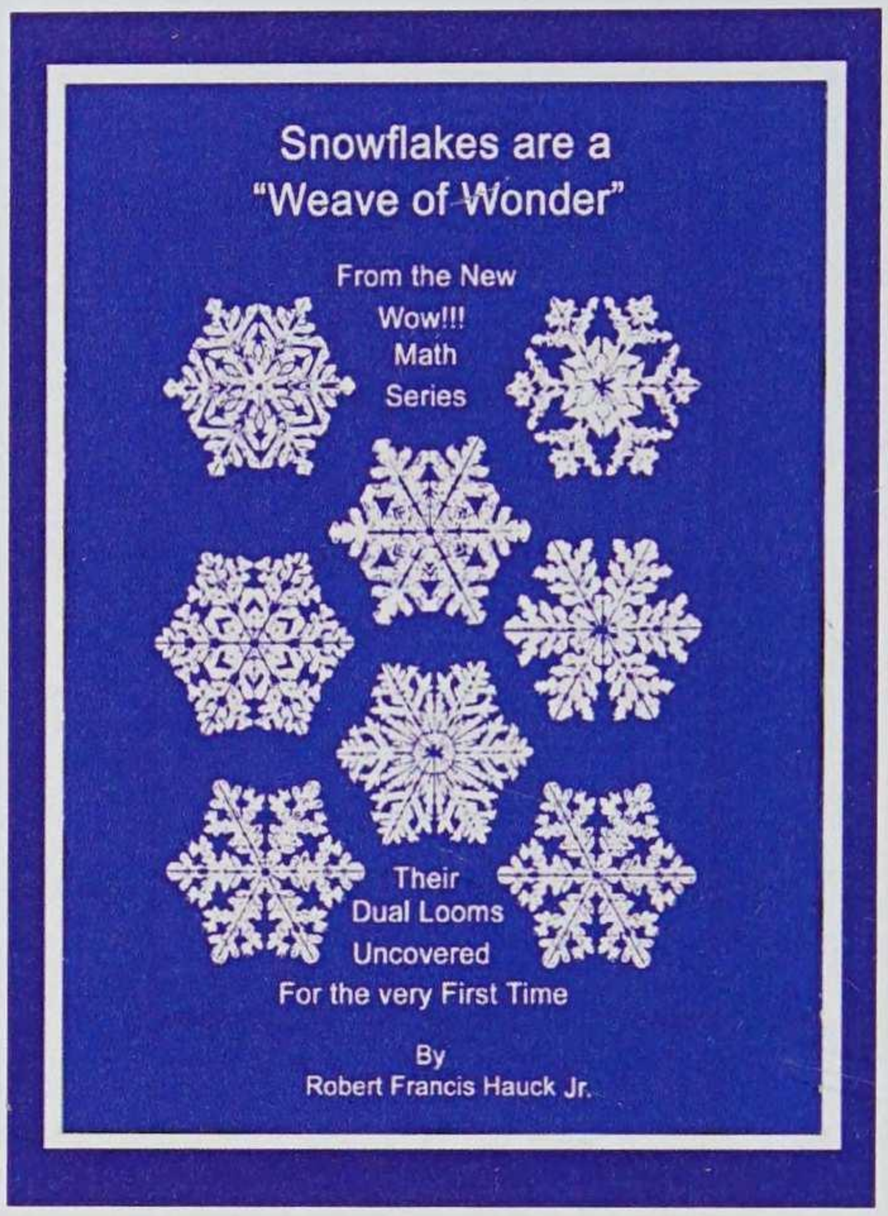
6. The size **6** hexagon is the next size to be perfect, followed by sizes **7** and **13**. The sizes **9** and **11** in-between may be imperfect merely due to the manner in which the modulus loom-table was constructed; it may possibly be improved upon.

7. Mathematically, geonomic hexagons are unrelated to hexagons appearing on the internet which have every linear slice in all three directions summing equally to a single number. Those constructs have neither relationship nor application to measuring and mapping the fabric of space.

8. Geonomic hexagons of odd size can be made to show how snowflakes form so symmetrically perfect along three distinct equally spaced tri-axial directions and do so with such branching complexity.

Program 10

Here is the book which contains the segment on snowflakes separately.



Snowflakes are a Weave of Wonder

ISBN: 978-1-469-94348-0

Second edition

Printed in color (34 pages)

Appendix

Appendix

Appendix

Proof of the Properties for the Matchmaker's Magic Square

Generalized Match-making Magic Square

1	2n-1	2n- 2	2n- 3	...	n+j	…	n+2	n+1	n
2	2n- 2	2n- 3	...	n+j	…	n+2	n+1	n	n-1
3	2n- 3	...	n+j	…	n+2	n+1	n	n-1	n- 2
...	...	n+j	…	n+2	n+1	n	n-1	n- 2	…
n - j	n+j	…	n+2	n+1	n	n-1	n- 2	…	j
...	…	n+2	n+1	n	n-1	n- 2	…	j	…
n - 2	n+2	n+1	n	n-1	n- 2	…	j	…	3
n - 1	n+1	n	n-1	n- 2	…	j	…	3	2
n	n	n-1	n- 2	…	j	…	3	2	1

US Copyright **TX1-302-220** 5/21/2006 *Robert Francis Hauck Jr.*

1. **The Dimensional Average Property:** All the numbers average to its size **n**, here **9**.

   Proof of property 1 – Dimensional Average

   In the normalized table, there are j values of j and **j** values of **2n-j**

   Mathematically, that is

   $$\mathbf{m = \sum_1^n j^2 + \sum_1^{n-1} j(2n - j) = n^2 + 2n(n\text{-}1)n/2 = n^3}$$

   Then the average of $\mathbf{n^2}$ numbers is

   $$\mathbf{\bar{m} = n^3/n^2 = n}$$

   QED

2. **The Matchings Property**: All *matchings* sum to the same number, $\mathbf{n^2}$; here **81**.

Proof of property 2 – Matchings:

$$\mathbf{n^2 - (n-1)^2 = 2n - 1.}$$

Let M_n denote the matching total on a size **n** square in normalized form. Then in going from a size **(n – 1)** square to a size **n** square in normalized form, using the top row and leftmost column, we get

$$\mathbf{M_n = (2n-1) + M_{n-1} = (2n-1) + (n-1)^2 = n^2.}$$

Note that $\mathbf{M_1 = [\ 1\ ] = 1 = 1^2}$. This proves, by the principle of induction, the matching property for the normalized, generalized mm-square. The proof for any **scrambled** generalized mm-square follows from properties 4 and 5 below. **QED**

3. **The Totality Property**: All the numbers in the square sum to $\mathbf{n^3}$, here **729**.

Proof of property 3 – Totality:

Observe that there are **j** occurrences of the number **j** and there are **n – j** occurrences of the number **n+j** for **j** running from **1** through **n**. Note that there are **n – n = 0** occurrences for the number **2n** which is appropriately not present in the square.

Then using the generalized table above, we get:

$$\mathbf{\sum_1^n [\ (j)(j) + (n-j)(n+j)\ ] = \sum_1^n [\ j^2 + n^2 - j^2\ ] = \sum_1^n [\ n^2\ ] = n^3}$$

Note that the last sum is merely $\mathbf{n^2}$ added **n** times – the vanishing of the index of the function's argument means that the function is to sum the number $\mathbf{n^2}$ **n**-times over. **QED**.

Appendix

4. **The Subset Property:** Any smaller square, called an abbreviated square, taken from anywhere in the table has all possible *matching*s summing to the same number.

Proof of property 4 – Subset:

p	**p - j**
p - i	**p - i - j**

Referring to the generalized square in normalized form, any abbreviated 2x2 square looks like the one here on the left. Clearly, both of the two possible matchings each sum independently to **2p – i – j**.

Now taking <u>any</u> abbreviated mm-square of size **m** in the normalized generalized square we get

(p – i) + (p – j) + (p – k) + (p – r) + . . . + (p – h) + (p – s)

= **(p – i – j) + (p – k – r) + . . . + (p – h – s)** down its major diagonal

= **mp – [(i + j) + (k + r) + . . . + (h + s)]**

But since all the **m** rows and **m** columns are represented in the brackets, the bracketed sum must be constant regardless of the order in which it was obtained. **QED**

5. **Invariance-under-permutation:** The rows and columns may be swapped one pair at a time, that is, rearranged into whatever order you like, by interchanging the position of any two rows or any two columns, any number of times in series, like spinning a Rubric's Cube, and the square will still have the same properties #1 thru #4 .

Proof of property 5 – Invariance-under-permutation:

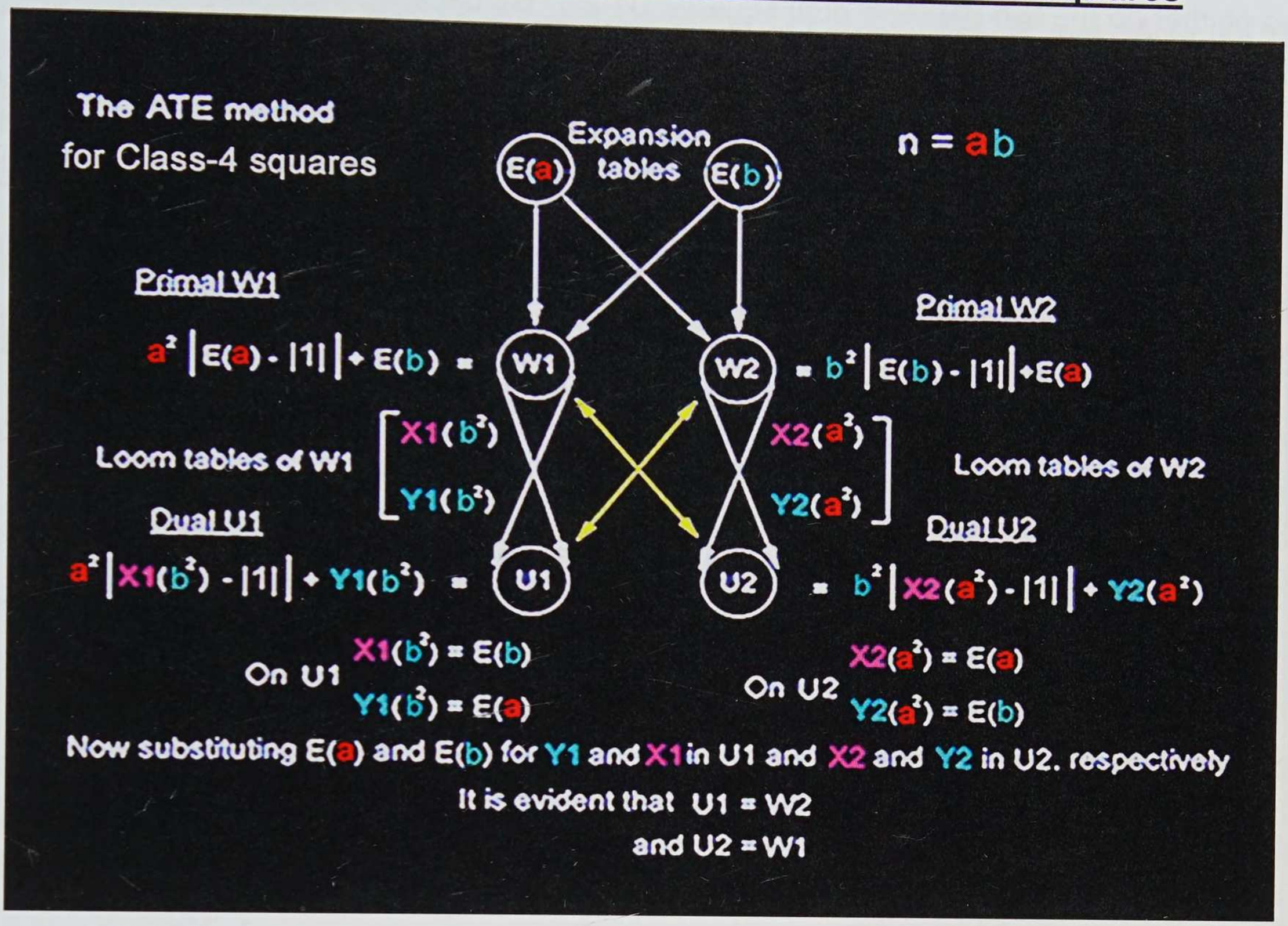
This property follows directly from the proof of property 4 above. Take any abbreviated square from the normalized generalized mm-square and rearrange its rows and columns but maintain the elements **(i, j)** with row **i** and column **j** wherever they may be. Then any matching on the abbreviated square sums to

mp – [(i + j) + (k + r) + . . . + (h + s)]

We just saw this expression to be constant for any aggregate of **n** rows and **n** columns selected. Just extending the abbreviated square to the full generalized square by setting **m = n** completes the proof. **QED**

Appendix

Mathematical Proof of the ATE Method for Class-4 Squares



The merging of two different expanded tiled matrices deals with two squares of different sizes **a** and **b** to get one of size **n = ab**. The factors must be one odd-number and one even number. Their addition will yield two completely different squares of size **ab** according to the order in which their expansion tables are merged.

(A.1) $W1(n) = a^2(E(b) - |1|) + E(a)$

(A.2) $W2(n) = b^2(E(a) - |1|) + E(b)$

where **n = ab**, **a ≠ b**, **a** an odd-number, **b** an even number

Both primary tables will be perfect and have identical properties. Both will be pangenic and will have their extreme numbers in the same location.

Their loom tables are the expansion tables **E(a)** and **E(b)**. The schematic above proves that **W1** and **W2** are truly the dual of each other.

Suppose we have two primal squares **W1** and **W2**, each of size **n** generated by merging two tiled expansion tables **E(a)** and **E(b)** according to the two addition formulas (**A.1**) and (**A.2**).

Next complementary loom tables derived from **W1** and **W2** are interchanged in their generation formulas for their respective primal versions. This yields two distinct duals, **U1** and **U2**.

Appendix

At this point, the two sets of complementary loom tables have no relationship to one another, and neither do the two resulting dual squares **U1** and **U2** because they have been constructed and derived in two different bases, **a** and **b.**

Yet, here's the amazing part: the primal squares initially derived from the expansion tables are related because their modulus and integer loom-tables **X1** and **Y1** taken from **W1** in the base $\mathbf{a}^2$ yield both original expansion tables **E(a)** and **E(b)** in that order. Additionally **X2** and **Y2** taken in that order from **W2** in the other base $\mathbf{b}^2$ also yield both original expansion tables **E(b)** and **E(a)** but now in reverse order. For instance, where the modulus table **X1** yields **E(b)**, the modulus table **X2** yields **E(a)**.

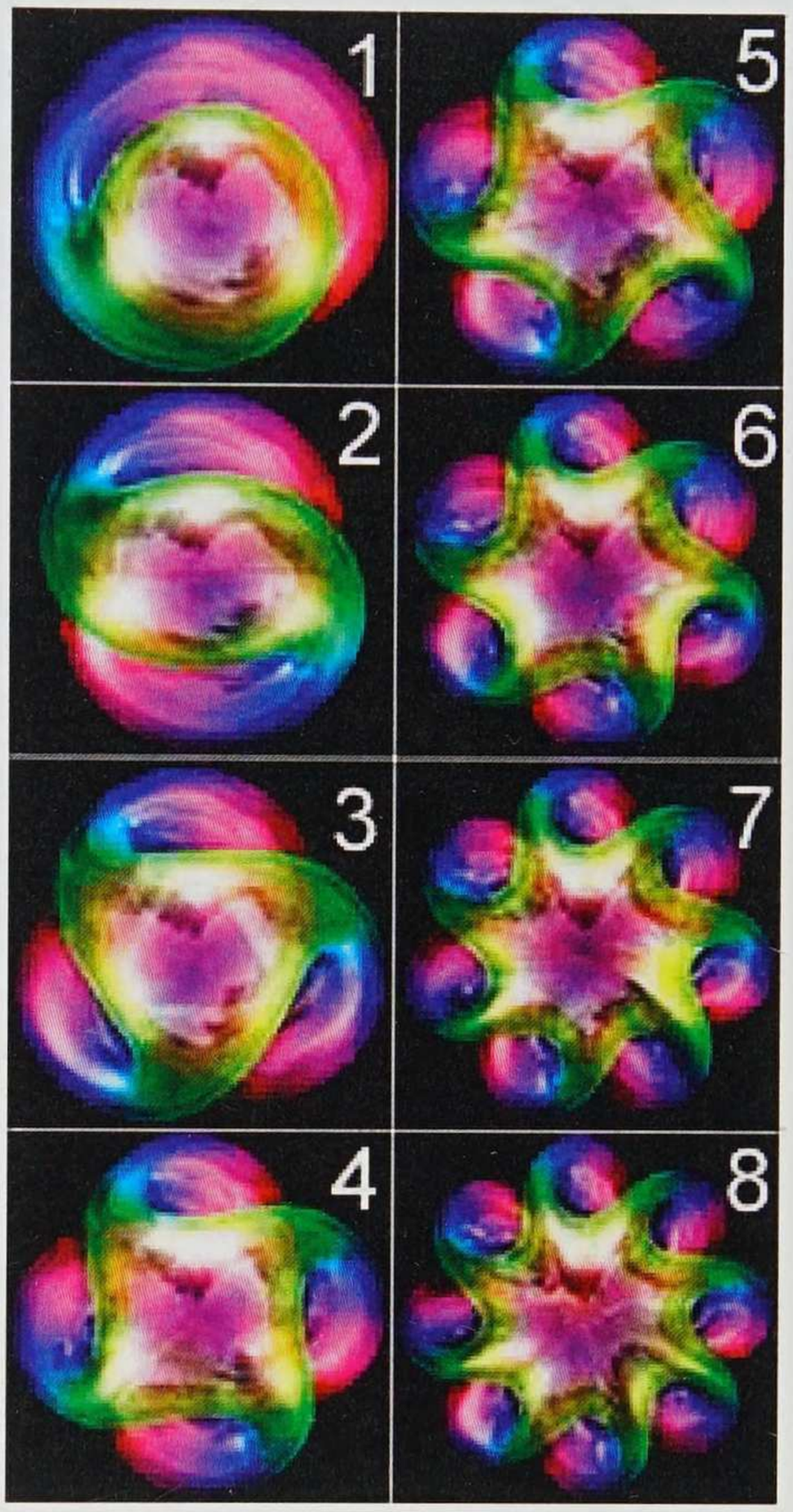
Now substituting **E(a)** for **Y1** in **U1** and for **X2 in U2**, **E(b)** for **X1 in U1**, and for **Y2** in **U2**, we readily observe that **W1** is identically **U2** and **W2** is identically **U1. QED**

Appendix

Kernel Value Multiples equal to the Characteristic Number for any Dimension .

Here is a table listing the dimensions and the count of kernel numbers that equal the characteristic number in tables of size **4x** in different dimensions. Recall that this multiple determined circles in squares, spheres in cubes and toruses in quadracubes. $\mathbf{V}^{k}$ represents the list of kernel numbers corresponding to dimension **k**.

Kernel Value Multiples equal to the Characteristic Number in dimensions 2 thru 5					
Table Size n	n/2	V_2	V_3	V_4	V_5
4	2	1	*2	4	8
8	4	2	8	32	128
12	6	3	18	108	648
16	8	4	32	256	2048
20	10	5	50	500	5000
24	12	6	72	864	10,368
28	14	7	98	1372	19,208
32	16	8	128	2048	32,768



Noting that:

$V_3 = (n/2)^1 V_2 \quad V_4 = (n/2)^2 V_2 \quad V_5 = (n/2)^3 V_2$

$V_4 = (n/2)^1 V_3 \quad V_5 = (n/2)^2 V_3$

$V_5 = (n/2)^1 V_4$

and that

(A.3) $\quad V_2 = n/4$

Putting everything in terms of V_2, we see that the foregoing relationships can be generalized as:

(A.4) $\quad V_k = (n/4)\,(n/2)^{k-2} = \frac{1}{2}(n/2)^{k-1}$

Further,

(A.5) $\quad V_k / V_{k-r} = \frac{1}{2}(n/2)^{r}$

What this demonstrates is that as the dimensions increase and the sub-dimensions decrease away from the point of inversion, the convolution of space from contorted spheres increases as shown at right.

*Numbers highlighted in yellow in the table above correlate to the number of electrons in the innermost shell the Nobel elements. Further, $\mathbf{V_3}$**-1** is a series of continuous prime numbers that were used for the size of loom tables involved in explaining the distribution and operation within shell-pairs; that series is interrupted here at **49** (**50-1)** which is not a prime number. The Noble elements stop at **32** (**31+1**) electrons. Is this just a coincidence?

I conjecture that why elements become unstable beyond element 118 is that there is no more **1/3D** inner space for a shell-pair with complementary loom tables.

Appendix

General Formulas for Standard Even-sided Figures

Let the number of sides in a standard geometric table be denoted by **s**. Then the number **N** of cells in a table of size **n** is given by the formula:

(A.6) $N = \frac{1}{2}\, ns(n - 1) + 1$ for $s \geq 4$

Note: When **s = 4**, the formula applies **to diamonds** and not to squares. When **s = 8**, the formula applies to octagons.

Let **m** denote the common number of cells within each colored pattern. The number of independent patterns with a cell-count of **m** (or less as is required to fill the last partial pattern with consecutive numbers starting with **1** is always equal to **n+1**.

Let

(A.7) $m = (N - 1)/n$

For hexagons **s = 6** and size **m** is always a whole number.

n	7	8	9	10	11	12	13	
m	18	21	24	27	30	33	36	= 3(n-1)
N	127	169	217	271	331	397	469	= nm+1
(N-1)/6	21	28	36	45	55	66	78	—
=	3n	3½n	4n	4½n	5n	5½n	6n	= n(n-1)/2

For even-sided geometries:

(A.8) $m = s(n-1)/2$

The following generalized formulas apply to any table of convex geometric configuration with **s** equal-length sides of length $n \geq 4$:

(A.9)
$$\begin{cases} y_{ij} = i & 1 \leq i \leq m \\ x_{ij} = j & 1 \leq j \leq n \end{cases}$$

(A.9) $w_k = m(y_k - 1) + x_k$ for all $1 \leq k \leq N$, the number of cells in the table.

(A.10)
$$\begin{cases} x_k = \text{Mod}\,[\, w_k - 1 \mid m \,] \\ y_k = \text{Int}\,[\, (w_k - 1) / m \,] \end{cases}$$

(A.11) $W = m\,(Y - |1|) + X$

where |**1**| represents a like-size table of congruent geometry whose cells are all **1**'s.

Now the dual tables **U** may be determined from

(A.12) $U = n\,(\overline{Y} - |1|) + \overline{X}$ where

(A.13) $\overline{X} = Y + m\Theta$

(A.14) $\overline{Y} = X - n\Theta$

Θ is a conforming table of **0**'s with a **1** in its center.

Now these loom tables for hexagons are the best that the human mind can achieve in weaving the numerical spatial fabric underlying the hexagonal tables *apriori*. When these seemingly complex numerical loom patterns for hexagons are compared with the integrated loom-tables derived for prime-number-size squares in Program 3, with all their equalizing cross-summations, geometric pangenicity, continuous modularity, complementary tiling patterns and dualities, continuously summing diamond quilt patterns and triangular tapestries, and 3D pyramidal patterns, only then is it solidly evident that those geonomic squares are way beyond weave-manifestation by the mind of Man without the assistance of a computer. Since the laptop computer was just recently made public and EXCEL was developed for its use on the laptop, this new math is all a recent discovery.

Appendix

Geonometry's Hooks into Matrix Algebra

Complementary Loom-tables' Matrix Product

Here is the matrix product of complementary loom tables **X** and **Y** of size-7. Each has the same characteristic number **28**. The characteristic number of their matrix product equals the product of their characteristic numbers **784 = 28 x 28.**

X

7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

Y

1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

This principle holds for all loom tables of Class-1 squares and is expressed in general by the formula:

$$(A.15) \qquad L_n(XY) = L_n(X)\, L_n(Y) = L_n^2$$

where L_n is the loom-table's characteristic number.

This formula also holds for

$Y = X^T$, the transpose of **X** and

$X = Y^T$, the transpose of **Y**.

Matrix product

XY	784	784	784	784	784	784	784
	98	119	126	119	98	112	112	784
	126	119	98	112	112	98	119	784
	98	112	112	98	119	126	119	784
	112	98	119	126	119	98	112	784
	119	126	119	98	112	112	98	784
	119	98	112	112	98	119	126	784
	112	112	98	119	126	119	98	784
	784	784	784	784	784	784	784	
	784	784	784	784	784	784	784	784

This property has never been observed in Matrix Theory before. That is because this property does not hold for number tables that are not Class-1 loom-tables, i.e. the sequence of numbers from **1** thru **n** exist exactly once in every row, column and diagonal in both directions. So this property of matrix products of Class-1 complementary loom-tables is only found in tables that are geonomically ultra-perfect.

Here's another property that follows from George Polya's formula (**8.4**) back in Program 8 (re:p.8-13):

(**A.15**) becomes

$$(A.16) \qquad L_n(XY) = L_n^2 = \left[\sum_{j=1}^{n} j\right]^2 = \sum_{j=1}^{n} j^3$$

That's new to Matrix Algebra too!

Here's another surprise: while both tile patterns A and B sum continuously to the loom tables' characteristic number on both **X** and **Y** individually, only the **transpose** of the tile patterns now do so on the matrix product **XY** for all Class-1 tables of size **n > 5**.

A^T

784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784

B^T

784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784

Appendix

For Class-4, 5 & 6 loom tables of size **n = ab**:

$$X(b^2)\,Y(b^2) = E(b)\,E(a) \qquad X(a^2)\,Y(a^2) = E(a)\,E(b)$$

In fact Identical !!

$$= \left| L_n(E(b))/a \times L_n(E(a))/b \right|$$

$$= |C_b \times C_a|$$

Note that the last expression is a table of size-n of constant values equal to the product of the tiling square's characteristic number C_b of **W(b)** and C_a of **W(a)**.

For Class-6 square's loom tables used in the **T-Ball** method, set **a = b** and substitute **D(b)** for **E(a)** in the formulation here. Then the constant in the matrix product is $(C_b)^2$.

This property applies to matrix products of compatible tiled expansion tables used in the **ATE** and **T-Ball** methods and no others.

Both matrix products **X(n)Y(n)** and **Y(n)X(n)** in the base **n = ab** however will fail to yield a matrix of all constants for loom tables for:

1. Class-4 squares because their loom tables to the base **n** are not pangenic and are therefore incompatible.
2. Class-5 squares just because they just don't.

For Class-6 squares generated by the **T-Ball** method, contrary to that for Classes 4 & 5 squares generated by loom tables with the **ATE** method, both **X(n)** and **Y(n)** can indeed be derived in the base **n**.

The smallest Class-4 square in which this can be demonstrated is of size-20 and the smallest Class-5 square is of size-35. In this book are shown the size-20 square to the bases **16** and **25** and three versions of the size-35 square to the bases **25**, **35** and **49**. From these you can derive the various loom tables in Excel™ and use Excel multiplication to compute their matrix products in their respective bases and verify what was just presented as formulas here.

E(4) x E(5)

2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210

The size-25 Class-6 square, its dual and complementary loom tables were shown in their entirety in Programs 3 & 8.

Example

2210

= (170 x 260)/20

= (170)/5 x (260)/4

= 34 x 65 = C_4 x C_5

Appendix

Determinants of loom tables

Here is another interesting property: Every loom table – both the modulus loom and the integer loom simultaneously – derived from any geonomic square which has a non-zero determinant, has a non-zero determinant too. That might be anticipated; however, what is so amazing is its **absolute value** is always the same among all different loom tables of all perfect squares of that size! That is, for that size of loom table, it is the **Loom-table's Characteristic Determinant** (**LCD**).

Here are closed formulas for the non-zero determinants of loom tables. Closed formulas for determinants are extremely rare in Matrix Algebra because a determinant is primarily a long drawn-out systematic matrix calculation that yields only a single number. Yet formula (**9.22**) here gives the formula for the determinant solely in terms of the loom table's characteristic number.

The **LCD** for size **n** loom tables were all found to be expressible as

$$\textbf{(A.17)} \qquad \textbf{Det } |X_n| = \pm L_n \, n^{n-2} = \textbf{Det } |Y_n|$$

where L_n is the loom table's characteristic number. Note that L_n here is just the sum of numbers **1** through **n**.

So (**A.17**) becomes

$$\textbf{(A.18)} \qquad \textbf{Det } |X_n| = \pm 1/2 \, (n+1) \, n^{n-1} = - \textbf{Det } |Y_n|$$

The determinant for X_n is always the negative of the determinant for Y_n. That is yet again another amazing property!

Five examples whose determinants were obtained from computations made on internet *Quick Math* are given below as verification of formula (**A.5**):

Examples:

- The size **3** loom tables' determinants are **±18** which equals $\pm 1/2 \, (4) \, 3^2$.
- The size **5** loom tables' determinants are **±1875** which equals $\pm 1/2 \, (6) \, 5^4$.
- The size **7** loom tables' determinants are **±470,596** which equals $\pm 1/2 \, (8) \, 7^6$.
- The size **11** loom tables' determinants are **±155,624,547,606** which equals $\pm 1/2 \, (12) \, 11^9$.

Observe how rapidly these determinants increase. Consequently these formulas are essential because the value of the determinant quickly outpaces the capacity for computer accuracy.

Appendix

Determinants for Geonomic Squares

Here is a summary of determinants for geonomic squares up through size-35. Notice how rapidly these values expand!

Size	Determinant
3	-360
4	0
5	-4680000
6	0
7	-34602923880
8	0
9	0
10	0
11	4103774968930397766066O
12	0
13	$461332193857381539862041 6 \times 10^{4}$
14	0
15	0
16	0
17	-343347181240267750640339540289566234937600
18	0
19	-196077732828889316611003154299607822772783549O624
20	0
21	0
22	0
23	2184090645209036171864993404257413559829401325635816741976450 60
24	0
25	$39379835549285708962363238283433020214898681640625000000 00 \times 10^{8}$
26	0
27	0
28	0
29	-330042583545130100737627766122251448151726476058132904348217067 44 3554076711688428057 6
30	0
31	-138583113726654610838973809855259458374851780415465757699860454333 26464966738718637738559 00
32	0
33	0
34	0
35	607264854990001244217328738660095747790077104674354315699626139615 $07323493568347167968750 0 \times 10^{19}$
36	0

Perfect & non-singular	Perfect & singular	Near-perfect & singular	Imperfect & non-singular

Only squares which have loom tables with a non-zero determinant have a non-zero determinant themselves and we'll attempt to formulate those for squares next.

Appendix

Determinants of Class-1 Squares

Determinant of W(5)

4680000

Remainder	Factors
7488	5^4
117	2^6
13	3^2
1	$13 = L_5 - 2$

Determinant of W(7)

34602923880

Remainder	Factors	
294120	7^6	
49020	**6**	
9804	**5**	
2451	**4**	
817	**3**	
19	43	$= (3 L_7 - 2)/2$
1	19	$= (2L_7 + 1)/3$

Here are the determinants and their factors for the three smallest consecutive Class-1 squares: sizes **5**, **7** and **11**.

The factors are shown to demonstrate that there is no systematic formulation of the determinants of primal squares. Observe the two large primes at the bottom of Size-11's factors which preclude making any further progress. These large prime numbers have no interpretation in terms of the size-11 square's properties and are therefore useless.

However, note that the loom tables' common determinant can be factored out and hence always wholly divides the primal square's determinant:

$$\text{Det } |X(5)| = 5^4 \times 3$$
$$\text{Det } |X(7)| = 7^6 \times 4$$
$$\text{Det } |X(11)| = 11^{10} \times 6$$

Determinant of W(11)

4103774968930397766066O

Remainder	Factors	
13075891740	11^{12}	
2615178348	5	
871726116	3	
435863058	3	
217931529	2	
72643843	2	
48787	**1489**	**Dead-end**

Both factors are primes without Geonomic rhymes

Appendix

Loom tables' Eigenvalues

Here are two fundamental theorems from Matrix Algebra regarding *eigenvalues*:

I. The determinant of a matrix is equal to the product of its eigenvalues,

$$\textbf{Det } |X_n| = \prod \lambda_j = \lambda_1 \lambda_2 \lambda_3 \ldots \lambda_n.$$

II. The sum of a matrix's eigenvalues is equal to the sum of the numbers along its major diagonal, called the **trace** of the matrix.

$$\textbf{tr } |X_n| = \sum \lambda_j = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n$$

We saw that the formula (**A.18**) gave the determinant for a Class-1 loom table of prime-number size **n** as

$$\textbf{(A.19)} \qquad \textbf{Det } |X_n| = \tfrac{1}{2}(n+1)\, n^{n-1} = L_n \times n_1 \times n_2 \times \ldots \times n_{n-2}$$

And since every prime-number size loom table contains the numbers from **1** thru **n** in every diagonal, the ***trace*** of the loom table X_n of size **n** is simply the sum of numbers **1** thru **n**:

$$\textbf{(A.20)} \qquad \textbf{tr } |X_n| = \sum j = 1 + 2 + 3 + \ldots + n\text{-}1 + n = L_n$$

Website **www.bluebit.gr/matrix-calculator/calculate.aspx** gives these imaginary numbers for the size-7 loom table X_7.

Eigenvalues

(28.000, 0.000i)
(5.061, 0.000i)
(2.531, 4.383i)
(2.531,-4.383i)
(-5.061, 0.000i)
(-2.531, 4.383i)
(-2.531,-4.383i)

The trace of loom table $\textbf{tr } |X_7| = 28 = L_7$. It is obvious that when these imaginary numbers are summed at right that all but the number **28** cancels out in both the real and imaginary components.

And we have already seen that $\textbf{Det}|X_7| = 470596$. Excel gives **470625.41** for the product. Clearly the value determined from formula (**A.19**) given by Geonometry is the more accurate one. That's the benefit of having generalized closed formulas.

Now, let me make this perfectly clear: Decimal fractions, real and imaginary numbers have no place in Geonometry. Geonometry is about whole integral numbers and their fundamental equal-summing patterns. The same point is made for matrix inverses of loom tables: the numerators can be expressed in whole numbers and the denominator can be expressed as an inverse of a product of two whole numbers.

Here's where this new math butts-up against classical math. Since we already have a closed formula for the determinant and the loom table's characteristic number is the dominant eigenvalue, that's all that is needed in engineering applications. The rest lies at the noise level for all the other eigenvalues and their associated eigenvectors. The only vital point in applied Geonometry is that the dominant eigenvalue for Class-1 loom tables, dominant by an order of magnitude, is λ_1 where:

$$\textbf{(A.21)} \qquad \lambda_1(n) = L_n$$

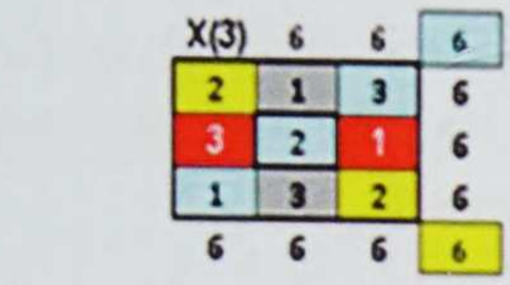
The rest of the eigenvalues and their associated eigenvectors are superfluous.

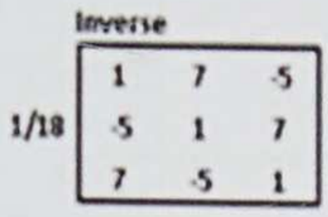
Appendix

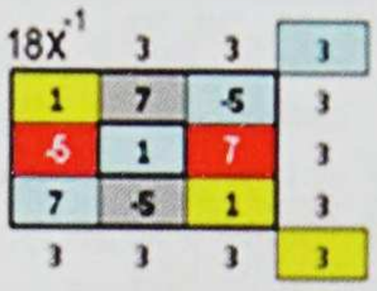
Loom table's Matrix Inverse

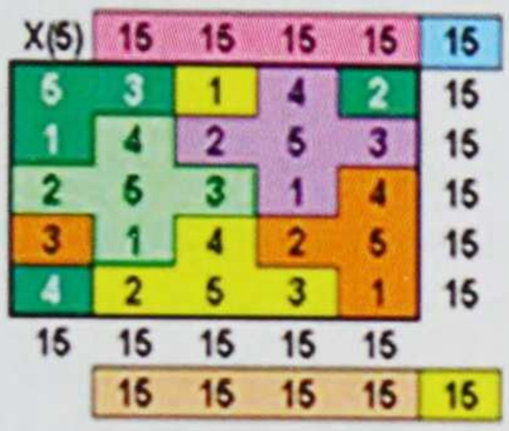
A third useful theorem is that a matrix has an inverse only if all of its eigenvalues are all non-zero.

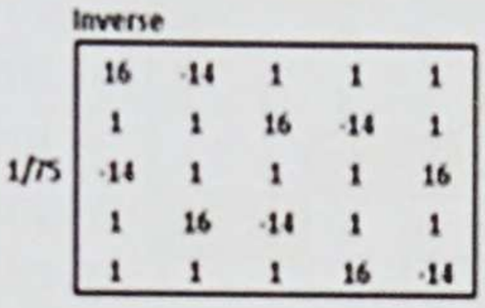
Here are the modulus loom tables of sizes **3** thru **7** and their matrix inverses. These are sufficient to generalize formulas for all Class-1 squares.

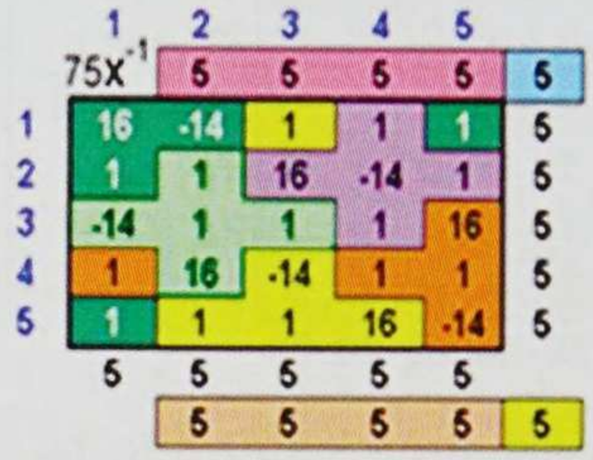


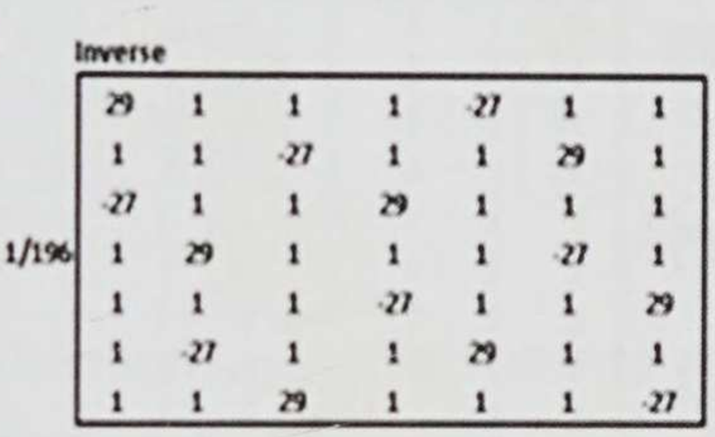


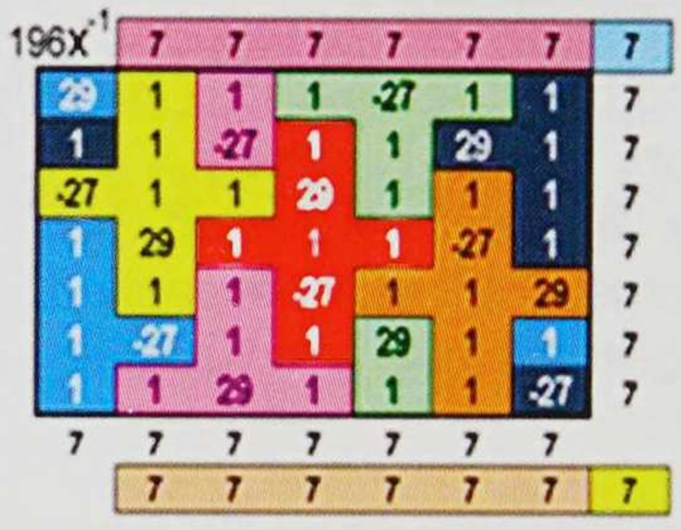












1. First, we observe that the common denominator $\mathbf{d_n}$ in the inverse for size **n** is given by

   (A.22) $\mathbf{d_n = n\,L_n}$

2. Secondly, the matrix of numerators always contain the numbers $(\mathbf{1 \pm L_n})$ exactly once and **n-2** number **1**'s in every row, column, diagonal and the transpose of characteristic tiles. These each sum to **n** as seen in the calculation below:

   $\mathbf{(1+L_n) + (1-L_n) + (n-2) = (L_n - L_n) + (2 - 2) + n = n.}$

3. Thirdly, the matrix of numerators placed in a geonomic framework has a characteristic number equal to its own size **n**.

The size-3 square has no tiling pattern and the size-5 square has totally symmetric tiling patterns. But size-7 demonstrates point #2. Further, although not shown, what holds for the transpose of tile pattern B also holds for the transpose of its complementary tile pattern A.

Appendix

X

7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

X^{-1}

1/196 x

29	1	1	1	-27	1	1
1	1	-27	1	1	29	1
-27	1	1	29	1	1	1
1	29	1	1	1	-27	1
1	1	1	-27	1	1	29
1	-27	1	1	29	1	1
1	1	29	1	1	1	-27

The inverse's table of numerators can be generated by **1±L7** in a continuous step pattern as shown here in yellow and **1**'s everywhere else. Similarly for the pattern in the inverse of the Integer loom table.

Note that the geonomic character-istic number of the numerator matrix is exactly its own size, **7**.

The next two loom-table inverses are of size-17. Note that their patterns are distinctly different from the size-7 inverses. Even the direction of propa-gation and overlap in the in-verses are different.

Y

1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

Y^{-1}

1/196 x

-27	29	1	1	1	1	1
1	1	-27	29	1	1	1
1	1	1	1	-27	29	1
29	1	1	1	1	1	-27
1	-27	29	1	1	1	1
1	1	1	-27	29	1	1
1	1	1	1	1	-27	29

So these patterns need to be determined by actually com-puting the inverse; there is no apriori general pattern. But once having uncovered the patterns, they can be applied directly without error hence-forth.

Observe that the absolute value of each of the larger two numbers in the size-17 inverse average **153**. And on the prior slide, the absolute value of each of the larger two numbers in the size-7 inverse averaged **28**. These are the loom-tables' charac-teristic numbers respectively.

n = 17
L17 = 153
d_{17}= 2601
= 17 x 153
= n Ln

$X(17)^{-1}$ =

$\frac{1}{2601}$ x

154	1	1	1	1	1	1	1	1	1	-152	1	1	1	1	1	1
1	1	1	1	-152	1	1	1	1	1	1	154	1	1	1	1	1
1	1	1	1	1	154	1	1	1	1	1	1	1	1	1	-152	1
1	1	1	1	1	1	1	1	1	-152	1	1	1	1	1	1	154
1	1	1	-152	1	1	1	1	1	1	154	1	1	1	1	1	1
1	1	1	1	154	1	1	1	1	1	1	1	1	1	-152	1	1
1	1	1	1	1	1	1	1	-152	1	1	1	1	1	1	154	1
1	1	-152	1	1	1	1	1	1	154	1	1	1	1	1	1	1
1	1	1	154	1	1	1	1	1	1	1	1	1	-152	1	1	1
1	1	1	1	1	1	1	-152	1	1	1	1	1	1	154	1	1
1	-152	1	1	1	1	1	1	154	1	1	1	1	1	1	1	1
1	1	154	1	1	1	1	1	1	1	1	1	-152	1	1	1	1
1	1	1	1	1	1	-152	1	1	1	1	1	1	154	1	1	1
-152	1	1	1	1	1	1	154	1	1	1	1	1	1	1	1	1
1	154	1	1	1	1	1	1	1	1	1	-152	1	1	1	1	1
1	1	1	1	1	-152	1	1	1	1	1	1	154	1	1	1	1
1	1	1	1	1	1	154	1	1	1	1	1	1	1	1	1	-152

$Y(17)^{-1}$ =

$\frac{1}{2601}$ x

1	1	154	-152	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	154	-152	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	154	-152	1
1	1	1	154	-152	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	154	-152	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	154	-152
1	1	1	1	1	154	-152	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	154	-152	1	1
-152	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	154
1	1	1	1	1	154	-152	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	154	-152	1	1	1	1
154	-152	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	154	-152	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	154	-152	1	1
1	154	-152	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	154	-152	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	154	-152	1

Appendix

The Matrix Product of a Loom-table's Inverse with its Transpose

In regular Matrix Algebra, the inverse of a matrix times itself equals the Identity matrix, i.e. . $\mathbf{Q^{-1}Q} = \mathrm{I}$. In Geonometry, there is a property resulting from the product of a loom-table's **inverse** with its **transpose** as shown here. This does not hold for any flipped version of **X**.

X

1	2	1	7	6	5	4	3
2	6	5	4	3	2	1	7
3	3	2	1	7	6	5	4
4	7	6	5	4	3	2	1
5	4	3	2	1	7	6	5
6	1	7	6	5	4	3	2
7	5	4	3	2	1	7	6

X^T

	2	6	3	7	4	1	5
	1	5	2	6	3	7	4
	7	4	1	5	2	6	3
	6	3	7	4	1	5	2
	5	2	6	3	7	4	1
	4	1	5	2	6	3	7
	3	7	4	1	5	2	6
	1	2	3	4	5	6	7

X^{-1}

0.005	0.005	0.005	0.148	0.005	-0.138	0.005
-0.138	0.005	0.005	0.005	0.005	0.148	0.005
0.148	0.005	-0.138	0.005	0.005	0.005	0.005
0.005	0.005	0.148	0.005	-0.138	0.005	0.005
0.005	0.005	0.005	0.005	0.148	0.005	-0.138
0.005	-0.138	0.005	0.005	0.005	0.005	0.148
0.005	0.148	0.005	-0.138	0.005	0.005	0.005

$X^{-1}X^T$

1	1	1	1	1	1	1	
0.429	0.429	0.429	0.429	-0.571	0.429	-0.571	1
0.429	-0.571	0.429	-0.571	0.429	0.429	0.429	1
-0.571	0.429	0.429	0.429	0.429	-0.571	0.429	1
0.429	0.429	-0.571	0.429	-0.571	0.429	0.429	1
0.429	-0.571	0.429	0.429	0.429	0.429	-0.571	1
0.429	0.429	0.429	-0.571	0.429	-0.571	0.429	1
-0.571	0.429	-0.571	0.429	0.429	0.429	0.429	1
1	1	1	1	1	1	1	

A

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

B

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

The product $\mathbf{X^{-1}X^T}$ is filled with fractions, yet its geonomic summations are all equal to **1**. Wow!

The same result holds for the product $\mathbf{Y^{-1}Y^T}$.

Here is the complementary loom-table **Y**, its transpose $\mathbf{Y^T}$, its inverse $\mathbf{Y^{-1}}$ and product $\mathbf{Y^{-1}Y^T}$.

Y

1	3	4	5	6	7	1	2
2	7	1	2	3	4	5	6
3	4	5	6	7	1	2	3
4	1	2	3	4	5	6	7
5	5	6	7	1	2	3	4
6	2	3	4	5	6	7	1
7	6	7	1	2	3	4	5

Y^T

	3	7	4	1	5	2	6
	4	1	5	2	6	3	7
	5	2	6	3	7	4	1
	6	3	7	4	1	5	2
	7	4	1	5	2	6	3
	1	5	2	6	3	7	4
	2	6	3	7	4	1	5
	1	2	3	4	5	6	7

Y^{-1}

0.005	0.148	0.005	-0.138	0.005	0.005	0.005
0.005	-0.138	0.005	0.005	0.005	0.005	0.148
0.005	0.005	0.005	0.005	0.148	0.005	-0.138
0.005	0.005	0.148	0.005	-0.138	0.005	0.005
0.148	0.005	-0.138	0.005	0.005	0.005	0.005
-0.138	0.005	0.005	0.005	0.005	0.148	0.005
0.005	0.005	0.005	0.148	0.005	-0.138	0.005

$Y^{-1}Y^T$

1	1	1	1	1	1	1	
-0.143	-0.143	-0.143	-0.143	0.857	-0.143	0.857	1
-0.143	0.857	-0.143	0.857	-0.143	-0.143	-0.143	1
0.857	-0.143	-0.143	-0.143	-0.143	0.857	-0.143	1
-0.143	-0.143	0.857	-0.143	0.857	-0.143	-0.143	1
-0.143	0.857	-0.143	-0.143	-0.143	-0.143	0.857	1
-0.143	-0.143	-0.143	0.857	-0.143	0.857	-0.143	1
0.857	-0.143	0.857	-0.143	-0.143	-0.143	-0.143	1
1	1	1	1	1	1	1	

A

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

B

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

Appendix

Lack of Geonomic Properties for the Standard Identity Matrix

I_7	1	1	1	1	1	1	1
1	0	0	0	0	0	0	1
0	1	0	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	0	1	0	0	0	1
0	0	0	0	1	0	0	1
0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	1
1	1	1	1	1	1	1	
	0	0	0	0	0	0	7

Here is the standard identity matrix of Matrix Algebra. This is the result one gets when pre-multiplying any non-singular matrix by its inverse.

Put in geonomic form, it displays no continuously equal summations along its major diagonals. Further, it has no continuously equal summations in either of the size-7 characteristic complementary tiling patterns.

This demonstrates that Geonometry is distinct from Matrix Algebra.

What is shown here for the size-7 identity matrix also holds for the size-11 identity matrix of the same class.

Now, cosmologists rely on Matrix Theory to compute their elements of String Theory in **eleven** contorted dimensions. Yet Geonometry has been demonstrated to yield a more consistent and perfect property for this matrix product of table and its inverse.

This is proof that Geonometry is operating on a level never seen before – and in my opinion, is tapping, measuring and mapping the harmonic fabric of space itself.

Consequently, those concepts put forth in the latter part of this program regarding the properties of multi-dimensional space at various levels shouldn't be dismissed as fantasy. Of course they are only conjectures at this point in time, but they have as much credibility as the cosmological notions so prevalent among scientists today.

For you students watching this program, it is an encouragement to question what you are being taught as <u>theory</u> and to learn to think outside of the academic box. There are amazing discoveries to be made about the characteristics of multi-dimensional space by your generation. So keep your mind open and come on board with this new math. Learn its amazing properties so you're able to apply them in your chosen field of endeavor, whatever that may be.

A

0	3	0	2	0	0	2
2	0	3	0	2	0	0
0	2	0	3	0	2	0
0	0	2	0	3	0	2
2	0	0	2	0	3	0
0	2	0	0	2	0	3
3	0	2	0	0	2	0

B

2	1	0	0	1	2	1
1	2	1	0	0	1	2
2	1	2	1	0	0	1
1	2	1	2	1	0	0
0	1	2	1	2	1	0
0	0	1	2	1	2	1
1	0	0	1	2	2	2

Appendix

Proof that points of equalities between modulus and integer functions always sum to the square's characteristic number

Theorum #1: The points of equality between the modulus and integer functions in the base n will always sum to the size n square's characteristic number. This characteristic summation property holds for every class and size of square. (re: p.3-2)

Proof:

Per formulas **(3.1)** and **(3.2)**, the points of equality between them are:

(1) **1**

(2) **1 + (n+1)**

(3) **1 + (n+1) + (n+1)**

(4) **1 + (n+1) + (n+1) + (n+1)**

...

(n) **1 + (n+1) + (n+1) + (n+1) + ... + (n+1)**

$(\sum)$ $n + (n-1)(n+1) + (n-2)(n+1) + (n-3)(n+1) + \ldots + 2(n+1) + 1(n+1)$

$= n + (n-1)n(n+1)/2$

$= n[(n^2 - 1)/2 + 1]$

$= \frac{1}{2}[n^3 - n + 2n]$

$= \frac{1}{2}[n^3 + n]$

$= C_n$

per equation (**8.17**) p.8-22 for **k=2**. **QED**

* * *

Appendix

Glossary
Of
Terms and Definitions

Conceptual Notions

Prime numbers

When the term "**prime**" or "**prime numbers**" is used in this book, it is to denote an **odd-prime** or **all odd primes**, respectively. It's ridiculously burdensome to have to use the prefix "**odd**" every time when referencing all primes other than **2** since the number **2 is the only even prime** in the entire number series while the number of odd-primes is infinite. In my view, the number **2** should be dismissed from its inclusion in prime numbers altogether since, other than it has no other divisor than itself in the entire number sequence, it possesses none of the other attributes in Geonometry of the odd primes and vice versa. The number **2** is really a "**sub-prime**" as this book demonstrates over and over again and begs demotion like the planet Pluto (circa 2007) so we can talk about just odd primes without the need for all the preliminary qualifiers every time we cite them. The mention of **even prime** in this book is made the exception.

Size vs. Order

Classical math uses the term "order" to describe the size of the matrix. This is a remnant of the days when the properties in Matrix Algebra applied to consecutive sizes and as such could be considered ordered. In Geonometry, "order" has no meaning because the fundamental properties are scattered across 6 distinct classes of square, none of which are incremental. Here there is no consecutive ordering of property with size. Further, "order" has 2 syllables while "size" has only 1, yet they have the same connotation. So "**size**" is not only justified but preferred in Geonometry.

In Geonometry, "order" is best applied to describe the sequence of geometries: triangle, square, diamond, pentagon, hexagon, heptagon, octagon etc.

Arithmetical number pattern

The existence of a numeric pattern among serial elements that is conceptual in its design.

Numerically, it is any number sequence whose elements can be reduced by one or more algebraic operations to one of the following sequences:

a) all elements equal a single constant,
b) the sequence of natural numbers; **1, 2, 3, 4,5, 6 …**
c) the odd-number sequence **1, 3, 5 ,7, 9, 11…**
d) the sequence of odd prime numbers **1, 3, 5, 7, 11, 13, 19 ,23, …**

Note that a consecutive even number sequence can be reduced to the natural number sequence merely by dividing all the numbers by **2** or subtracting by **1**.

Harmonic waveform

A series of consecutive numbers whose values follow a pattern:

n, n-1,n-2, ... , 3, 2, 1, 2, 3, ... , n-2, n-1, n, n-1,n-2, ...

Measure

The summation across a table in any of the four directions: top-to-bottom, left-to-right, upper-left to lower-right, upper-right to lower-left.

Mapping

The uncovering of natural numeric sequences in the 4 directional strings and tilings in loom-tables.

Table Terminology

Channel

The string summation along any **horizontal** linear path as viewed along the cited axis; an individual depth sum along the A, B or D-axis.

Pillar

The string summation along any **vertical** linear path as viewed along the C-axis; relates to the individual depth-sums in depth-sum table C.

Block

A square array corresponding to a 2-dimensional group of distinct, contiguous numbers.

Major diagonal

A diagonal that goes from upper-left to lower-right.

Minor diagonal

A diagonal that goes from upper-right to lower-left.

Wrap diagonal

A diagonal which does not originate at a corner of the table and, where its downward path meets the edge, resumes on the next row at the opposite side of the table and continues until it either reaches the other opposite edge again and continues or reaches the bottom of the table and terminates its decent.

Pivot number

The central number in odd-size tables; equals the **dimensional average** of all the numbers in the table.

Kernel number

The number **q**, characteristic of the table's size **n** and the number of dimensions **k** in which the table is contained that is the value to which the $\mathbf{2^k}$ corners of the characteristic dimensionally-geometric figures all sum.

Density

The proportion of **geometric equalities** equal to the kernel number, characteristic for the size and dimensions of the table, compared with the number of all such possible geometric summations. Any geonomic square, cube, diamond or hypercube of any size which exhibits **100% density**, **(**i.e. maximum density) is said to be ***pangenic*** (see below).

Pangenic

The term applied to any table with its dimensional geometry summing equally throughout the table. All geonomic tables in this book are pangenic.

Pangenicity

The property of being pangenic.

Concentricity

The property of nested concentric frames which parallel the perimeter shape of a table which exhibit an arithmetic number pattern as defined on page **G-2** in going from the center outward. See p. 8-3 for examples. Property follows from pangenicity.

Table Operations

Table expansion
The process of expanding a square table to a size-multiple of itself.

Table collapsing
The summing of all the numbers in a higher dimensioned table along one of its axes and reducing the numbers in the summations by subtracting one less than the minimum number from every number.

Roll-wrapping
The sliding of rows horizontally or columns vertically and then repositioning the excess beyond the original border to the vacated cells in their original order.

Tilt-wrapping
The sliding of rows horizontally or columns vertically in proportion to their distance from a lower left corner and then repositioning the excess beyond the original border to the vacated cells in their original order.

Equalities

Pairwise central-symmetry
The property of equality among all pair sums of two distinct numbers equidistant from and diametrically opposite the table's center.

Pairwise row-symmetry
The property of equality among all pair sums of two distinct numbers in the same row equidistant from the table's vertical centerline.

Symmetric-pair equality
The property of equality among all pair sums of numbers which are symmetrically opposite the center of the table.

Row-pair equality
The property in which every pair of numbers in each row located symmetrically on each side of the row's center adds to the same number.

Complementary patterns
Two distinct geometrical patterns of numbers in a table that always occur simultaneously and sum equally. This includes complementary loom-tables in their entirety.

Quadral
A set of **4** numbers corresponding to the corners of a rectangle or diamond (rhombus) that is centered in the table.

Octal
A set of **8** numbers corresponding to the **4** corners of **2** rectangles or diamonds (rhombi) that are centered in the centrally embedded square or in two embedded squares equidistant from the center of the cubic table. Re: p. 5-2.

Octahedral
A set of **6** numbers lying at the centers of the six faces of the cube. That includes five numbers in the central embedded block square. Re: p. 5-3.

Hexadectal
A set of **16** numbers corresponding to the **4** corners of **4** rectangles or diamonds (rhombi) that are centered in four embedded squares equidistant from the center of the quadracubic table. Re: p. 6 -2.

Icosahedron

A set of **12** numbers corresponding to the **4** corners of diamonds (rhombi) in the central embedded square along with the centers of **8** other surrounding embedded squares equidistant from the center of the quadracubic table. Re: p. 6 -2.

Characterizations

Squares

A square of consecutive numbers may have any of the following summation properties:

1. Equal column sums
2. Equal row sums
3. Equal main diagonal sums
4. Equal quadrals
5. Equal major wrap diagonal sums
6. Equal minor wrap diagonal sums

Geonomic square

A pangenic square with properties #1 thru #4.

Perfect square

Pangenic square with properties #1 thru #6. They are "perfect" because they add up equally from four distinct directions, the maximum possible number of linear equalities in 2-dimensions in ordinary circumstances.

Near-perfect square

A square whose summation equalities bring it very close to attaining its named status but is short from achieving it because two of their row summations differ from equality by **±1**. Still considered a geonomic square because its rows could all be made to sum equally by a simple interchange of two numbers in the same column without affecting equal column and main diagonal sums, but will affect four wrap-diagonal sums.

Punctuated perfect square

A punctuated-perfect square is one that is otherwise perfect except for some of its wrap diagonal sums being periodically unequal. A near-perfect square always converts to a punctuated perfect square, but not conversely.

Ultra-perfect square

A perfect square that also exhibits two complementary tile patterns whose elements sum to the square's characteristic number anywhere throughout the square table, in addition to all the other equalities ascribed to perfect status, i.e. an order of magnitude beyond perfection, as defined in **continuous complementary tiling patterns**.

Ultra-perfect Cube or Quadracube

A cube or quadracube for which all the embedded squares possess continuous complementary tiling patterns within themselves.

Absolutely-perfect Cube or Quadracube

A cube or quadracube for which all the embedded squares sum equally and geonomically and the depth-sum table along the B axis (and D axis in quadracubes) collapses to a perfect square.

Geonomical Patterns

Dot pattern
The pattern in the tile centers of tiling patterns which sum to the characteristic number. These are not continuously equal summing as are anchor-dot patterns.

Anchor-dot pattern
The dot pattern that exists in the centers of the transpose of each tiling pattern that sums continuously to the same number. These dot patterns serve as an anchor around which all other related equal dot patterns are spaced.

Continuous modularity
The property that all **nxn (or mxn, m≠n)** blocks anywhere in a table all sum to the same number. For perfect squares, cubes and quadracubes of size **n**, that number equals the table's characteristic number.

Complementary tile patterns
The property unique to *absolutely-perfect* Class-1 and Class-4 squares and cubes where each and every number in the table serves as the center of two distinct but partially overlapping geometrical configurations involving numbers that within each pattern sum to the characteristic number of the square.

Continuous complementary tiling patterns
For all perfect squares of size **n**, there are **n** such non-overlapping repetitions of a single pattern of **n**-connected numbers which blanket the entire table without voids (wrapping imposed). Moreover, these integrated composite "tiling" patterns may be dragged across the table with wrapping imposed and all the equalities persist.

Integrated patterns
Like patterns of two different sizes or of the same size that can be placed side-by-side either vertically, horizontally or shifted slightly diagonally across the table to cover the table completely.

Diamond quilt patterns
A pattern on squares composed of individual solid flat diamonds of equal size or differing in size by ±1 that integrate to cover the square exactly, with wrapping imposed; or to be contained in an expanded rectangular configuration of multiple repetitions of the same square.

Triangular tapestry patterns
A pattern of equal size triangles or of two sizes differing by ±1 such that the larger size triangles sum equally for their size to ½ the squares characteristic number continuously throughout the table.

Pyramidal patterns
Either a block-square or diamond shaped 3-dimensional sums taken on 2-dimensional geonomical squares or embedded squares in geonomical cubes, such that the numbers in each nested frame is multiplied by their distance from the figure's perimeter plus **1** in a summation of the numbers contained in the square or diamond. Sometimes diamonds in a quilt pattern are also pyramidal. If so they are of **3** simultaneous sizes; **2** in an integrated pattern and **1** integrated with itself to cover the number table completely.

Ring-diamond patterns
Diamond patterns composed of only the internal cells along the diamond's border.

Double-diamond patterns
Patterns that are composed of two ring-diamonds connected together at one adjacent corner.

Characteristic circle patterns

A pattern that sums all the numbers in the cells just impinging the largest inscribed circle in a square table from its interior by at least one corner of the cell which altogether sum to the square's characteristic number. These numbers are contained in one or more complete quadrals.

Characteristic spherical patterns

A pattern that sums all the numbers in the cells just impinging the largest inscribed sphere in a cubic table from its interior by at least one corner of the cubic cell which altogether sum to the cube's characteristic number. These numbers are contained in one or more complete octals.

Table Types

Loom-table

There two types of loom-tables generated by taking every number in the table and converting it by either the modulus function or the integer function to a distribution of a smaller series of numbers with repetitions in a table of the same size and configuration.

Depth-sum table

A number table derived by taking all linear sums through a cubic or quadracubic table along one of its axes.

Symbols

A & B – Non-bold type: two complementary tile patterns for squares and embedded squares in cubes and quadracubes

a & **b** – Factors of the size **n**

B – A number-pattern table

$\mathbf{C_n}$ – The characteristic number of the geonomical; table of size **n**

c – The class number of the geonomical table

D – An intermediate number table in table operations

A , **B**, **C** & **D** – Bold type: Depth-sum tables along the designated axis A, B, C and D

E – An expansion table composed of multiple segments of a loom-table or a geonomical square

h – The horizontal flip of a number pattern or table (an operator)

k – The dimension

L – Loom-table's characteristic number

M – The result of a formula relating two different sizes **m** & **n**

n – The size of the number table

N – The largest number in the table

p – The pivot number in odd-size tables

q – The kernel number for the size **n** number table of dimension **k**

$\mathbf{S_n}$ – The sum of all the numbers in the table of size **n**

t – The transpose of a number table (an operator)

U – The dual number table

u – a cell in **U**

V – The difference table of complementary loom-tables

v – The vertical flip of a number pattern or table (an operator)

W – The primal number table

w – A cell in **W**

X – The modulus loom table of the primal number table

x – A number in **X**

Y – The integer loom-table of the primal number table

y – A number in **Y**

Z – The number table that results from arithmetic operations on other number tables together

|1| – A compatible size table of all 1's

23922265R00278

Made in the USA
San Bernardino, CA
04 September 2015

Santa Clara County
LIBRARY

Renewals:
(800) 471-0991
www.santaclaracountylib.org

The author has degrees in mathematics from Penn State University (BS '64) and Stanford University (MS '71).

This book is the result of 11 years of exploration and discovery employing MicroSoft's Excel software.

The book is a companion treatise to the 10-Program Series on Geonometry. Every slide shown there is copied here along with the narration in print.

Geonometry is a whole new mathematics on a very basic level. It mostly involves simple arithmatic of addition, subtraction, multiplication and division of whole numbers in tables. There are no fractions involved.

In spite of its elementary math, it is demonstrated to solve some fundamental pattern structures in Chemistry and Physics.

It accounts for the distribution and orbital layers of electrons in the atom.

It uncovers the pattern of quarks in the neutron and proton and shows independently just why the atomic physicists gave these quarks their unusual names, like "up" and "down", "strange" and "charm", and "top" and "bottom".

It describes just why snowflakes are perfectly hexagonally symmetric.

It identifies a harmonic pattern in the orbits of the planets around the Sun.

Geonometry is shown to have some amazing mathematical properties which have no counterpart in higher-level classical math.

Yet, all this is presented in color so the properties are easy to recognize.

ISBN: 978-1-50-567225-1

SANTA CLARA COUNTY LIBRARY
3 3305 23551 0173