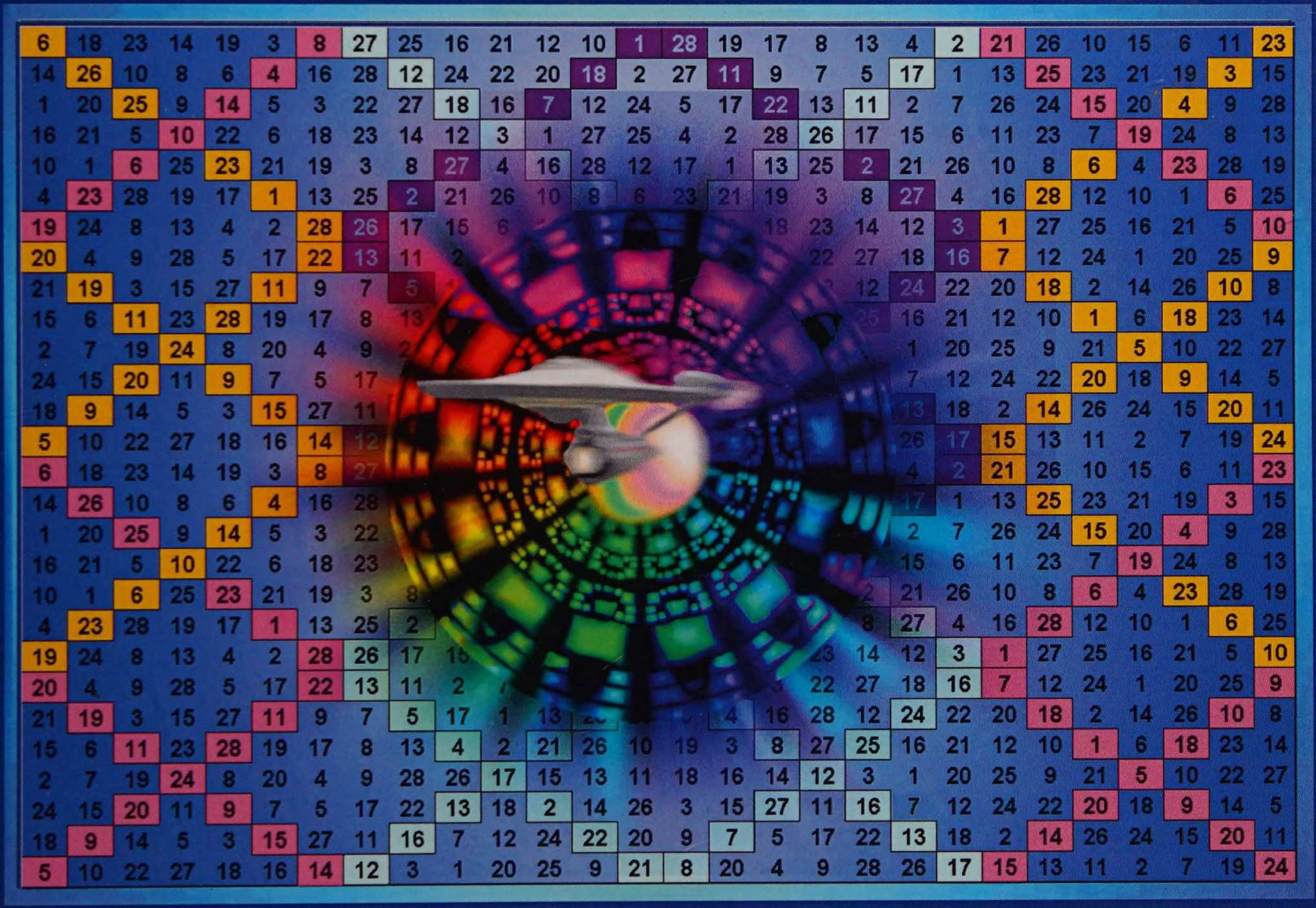
An Introduction to the new wow math

Geonometry

A companion book to the 10-Program video presentation



Uncovering the fabric of space itself

by

Robert Francis Hauck

An Introduction to the New Wow Math

Geonometry

A companion book to the 10-Program slide presentation

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Uncovering the Fabric of Space Itself

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Overview

Geonometry

Geonometry is essentially the natural geometry hidden in the natural number series.

The math only involves addition, subtraction and multiplication of integers. Occasionally division is called for but there are no numbers expressed as decimals and numbers expressed as fractions always lead to whole numbers.

The math involves matrices, but not those governed by the rules of Matrix Algebra. It has its own set of rules and formulas that are comparatively very simple.

Unlike Matrix Algebra with all its complicated functions, Geonometry involves only two functions from classical math and those are the modulus and integer functions. Those have merely been reformulated to avoid producing **0**'s.

Geonometry uses Microsoft's Excel software entirely. In fact, without Excel, this new math would never have been discovered and developed.

Here are some of the amazing properties discovered in Geonometry:

In 2-dimensions, every square table consisting of the number sequence **1** thru **n-squared** has summations which all add up to the same number in 4 directions: i.e. for every row, every column and every major and minor diagonal, including all those diagonals starting between corners that must wrap across the edges going up or down. That number is only dependent upon the size of the square and is called the size's **characteristic number**.

Further, every set of numbers at the four corners of every rectangle and rhombi centered over the very middle of the square, called a **quadral**, sum equally to a different number too, that is a specific whole-number fraction **4/n** of the square's characteristic number.

The square tables can be segregated into **6** classes, some with subclasses within them, which have very distinctive properties. All but Class-2 squares have tiling patterns where each interlocking tile sums to the square's characteristic number too or a multiple thereof.

Moreover, there are always two different complementary tiling patterns which do so simultaneously for every size of square except those in Class-2. And all the individual tiles in these interlocking tiling patterns sum equally, no matter where their centers are located in the square. Further, each size Class-1 square has its own pair of tile patterns that are unique to its class and size and are found nowhere else. And these carry over to other classes of square whose sizes are multiples of Class-1 squares and do so at the block level.

For example, Class-3 squares are of sizes that are a **3** times multiple of an odd-prime number **b** >**3**. The tiling patterns in these squares are the tiling patterns characteristic of the Class-1 square of size **b** but in **3-by-3** block-squares and these tiling patterns sum equally everywhere to **3** times the square's characteristic number..

In Class-4 squares whose size is a multiple of **4**, there is a different kind of tiling pattern formed from a **diamond** and an **X** pattern which also sum equally continuously everywhere in the square. And because they do, the complete tiling patterns are indistinguishable from one another. The tiles in these tiling patterns do not intersect one another either but fit together like a horizontal stack of chain-link fences.

The Class-5 squares, which are of a size that is a product of two distinct odd-prime numbers **a** and **b**, both greater than **3** and **a≠b**, have block-squares of sizes **a** and **b** that sum equally at the block-square level in the complimentary tiling configuration of both Class-1 squares of sizes **b** and **a**, respectively, and these **4** simultaneous tiling patterns sum equally everywhere to **b** or **a** times, respectively, the square's characteristic number.

Overview

Class-6 squares are squares whose size is a power of an odd-prime number **b** where **$n = b^a$**. There, the tiling patterns are the **bxb** block-squares in a tiling configuration of the size **b** Class-1 square which all sum equally to **b** times the square's characteristic number everywhere in the table.

Now each of these "**primal**" squares in all classes but Class-2 have what is called a "**dual**" square, each of which can be derived directly from the other by what are called **loom tables**. These tables are the **modulus** table and the **integer** table and are initially derived from the primal square. In fact, they regenerate the primal square according to a very simple formula.

Here's the surprising part: by interchanging the location of the two complementary loom tables in the regeneration formula, a **dual square** is derived with all the properties of the primal. And further, if now those functions are applied to the dual to get its two complementary loom tables, what was obtained from the integer function on the primal square, is now obtained from the modulus function on the dual square. And further yet, what was obtained from the modulus function on the primal square, now is obtained from the integer function on the dual square.

When cubic numeric tables are examined, not only do the rows, columns and diagonals all sum equally in every planar square slice of the cube, of which there are **9** directions, but all the characteristic tiling patterns in embedded squares do too. And what were equal-summing squares and rhombi in squares, now become "**boxahedrons**" and **octahedrons** equal-summing separately to two distinct but related values in cubes.

Further, all the numbers in the squares that are embedded within the cubic rectangular-table all sum equally to the cube's characteristic number. And their central numbers all together sum to the characteristic number for the size of cube too.

The properties of cubes can be easily seen by collapsing their channels along each of their 3 axes in what are called **depth-sum tables** and treating each of these 3 square tables geonomically like squares.

One of these depth-sum tables will convert to a regular square by a specific division and subtraction. That is, the cubes will collapse along one of its axes to a perfect geonomic square by a very simple arithmetic process involving the difference between the characteristic numbers of the square and cube of the same size.

Without going into detail, all these properties determined for cubes are demonstrated to get propagated into 4-dimensional cubes, called **quadracubes**, in addition to having yet even more amazing summation properties.

In Class-4 **squares,** there are maximally inscribed **circles** whose numbers in the cells, just impinging the circle from the interior, also sum exactly to the square's characteristic number.

In Class-4 **cubes**, there are maximally inscribed **spheres** whose numbers in the cells, just impinging the sphere from the interior, sum exactly to the cube's characteristic number.

In Class-4 4-dimensional **quadracubes**, there are **8** hemispheres which sum equally in the same fashion to half of the quadracubes characteristic number. Further, the sums of each embedded cube throughout the 4-dimensional square table also sum to the characteristic number of the quadracube. And each embedded 2-dimensional square sums equally to all the 1-dimensional strings running throughout the quadracubic table.

The Class-1 **quadracube** not only has a dual, it has a series of duals which lead back to the original quadracube through the use of their derived loom tables. The same goes for **cubes**; they have a series of **3** successive duals, whereas the quadracube has a series of **4**. In all higher dimensions of dimension **k**, there is a series of **k-1** distinct duals leading back to the original. That is why in 2-dimensions, there is only **1** distinct dual for each primal square.

Overview

When any two of these **quadracubic** duals are laminated together, the equal-summing intersecting embedded hemispheres become **8** equal-summing spheres. This translates into a 4-dimensional **torus**. And because there are **4** combinations of **4** duals taken **2** at a time in a circular series, there are **4** intersecting toruses that make up 4-dimension space as perceived from the 5th dimension.

For dimensions of odd prime-number size, these dual series may be navigated in many different ways depending upon one single parameter in the generation formula.

There is a very special type of primal square with duplication of numbers allowed, called the **matchmakers square**. It has 4 unique properties of its own. One of the chief properties of a size-n matchmaker square is that the matching-up of each row with a distinct column and summing the numbers in those intersections will always result in the same number: **n-squared**. The positions of these columns and rows can be independently interchanged at random as many times as desired, like the spinning of a Rubric's Cube, and this characteristic, equal-summing property will prevail.

There are **seven** unique expansion methods for deriving perfect composite size squares. Among **4** of the **5** classes of squares which are composites or multiples of Class-1 squares, **four** of these methods apply to multiple classes of square.

There is only one (the **eighth** one) that is the very source of all Class-1 squares which makes it all possible. Geonometry demonstrates just how a matchmaker square of odd-prime-number size will convert to a geonomic Class-1 square with all the tiling properties of an ultra-perfect square of the same size.

Triangular tables of consecutive numbers originating with the number **1** are shown to have nested triangular frames all with equal-summing sides or embedded 3-celled triangles.

Geonomic diamond-shaped tables are shown which have a characteristic number for their size. They are shown to be readily derived from two geonomic squares of consecutive size.

Just as for squares, hexagonal tables are shown which have complementary loom-tables too. And just as for squares, these loom tables generate a dual hexagon of the same size with all the same properties as the original. One of the complementary hexagonal loom tables always contains a perfectly-symmetric triple-axial snowflake pattern; the other, always a Xenia flower pattern.

The mathematics of Geonometry is seen to be very different than classical math, yet interfaces with classical Matrix Algebra with closed functions for basic operations of inner and outer products of matrices, matrix inverses and eigenvalues. As such, it demonstrates that Geonometry is the missing link between basic 2-dimensional arithmetic and Matrix Algebra.

Further, the characteristic number of squares and cubes are correlated 100% with index triangles.

And moreover, the characteristic numbers of squares can be represented by a quadratic polynomial in classical Algebra. The characteristic numbers of cubes can be expressed as an irreducible 3rd-degree polynomial. In fact, the characteristic number of a geonomic table of any dimension **k** can be represented by the average of its size **n** to the powers of **2k-1** and **k-1**, all of which can be expressed as an irreducible **k**-th degree polynomial. So all of these algebraic expressions are equal among characteristic numbers, thereby providing the basis for a whole new multidimensional algebra.

Geonometry is capable of explaining the distribution of electrons in nested orbital shell-pairs of atoms, the interpretation of the relationship between baryons in the atom's nucleus as patterns found in the 4th dimension, and the framework underlying snowflake formation, none of which Science itself is capable of explaining neither on its own terms nor in its contorted 11-dimensional Calculus, called String Thoery.

Overview

Applications to date

The program series presents six applications of Geonometry to date:

1. The identification of a harmonic pattern in the distance of the planets from the sun relative to the orbit of Jupiter.
2. The discovery of the spatial vibrational membrane pattern that enables nucleotides to assemble themselves into the double helical structure of DNA.
3. The membrane description of the distribution, count and orbital paths of the electrons in atoms.
4. The clarification of the relationship among the six quarks which make up the six baryons of which the proton and neutron are the only two stable combinations.
5. The mapping of the structure of 4-dimensional space as four intersecting toruses.
6. The mathematical description of how perfectly-symmetric hexagonal snowflakes form.

Foreword

Here's what I have concluded is happening with this new math: the patterns that are being uncovered are really those vibrations which occur naturally in empty space that make up the actual fabric of space itself. This program series demonstrates just how these complementary tiling patterns cloak one another and must therefore co-exist simultaneously. Further, this program presents mathematical evidence that these patterns must exist at all dimensional levels induced by the process of table-expansion and at all sub-dimensional levels by the mapping of the **k**-th dimension upon the sub-dimension of **1/kD**. These harmonic fundamental spatial patterns can be of any size, from nano pulsations to galactic undulations, all simultaneously.

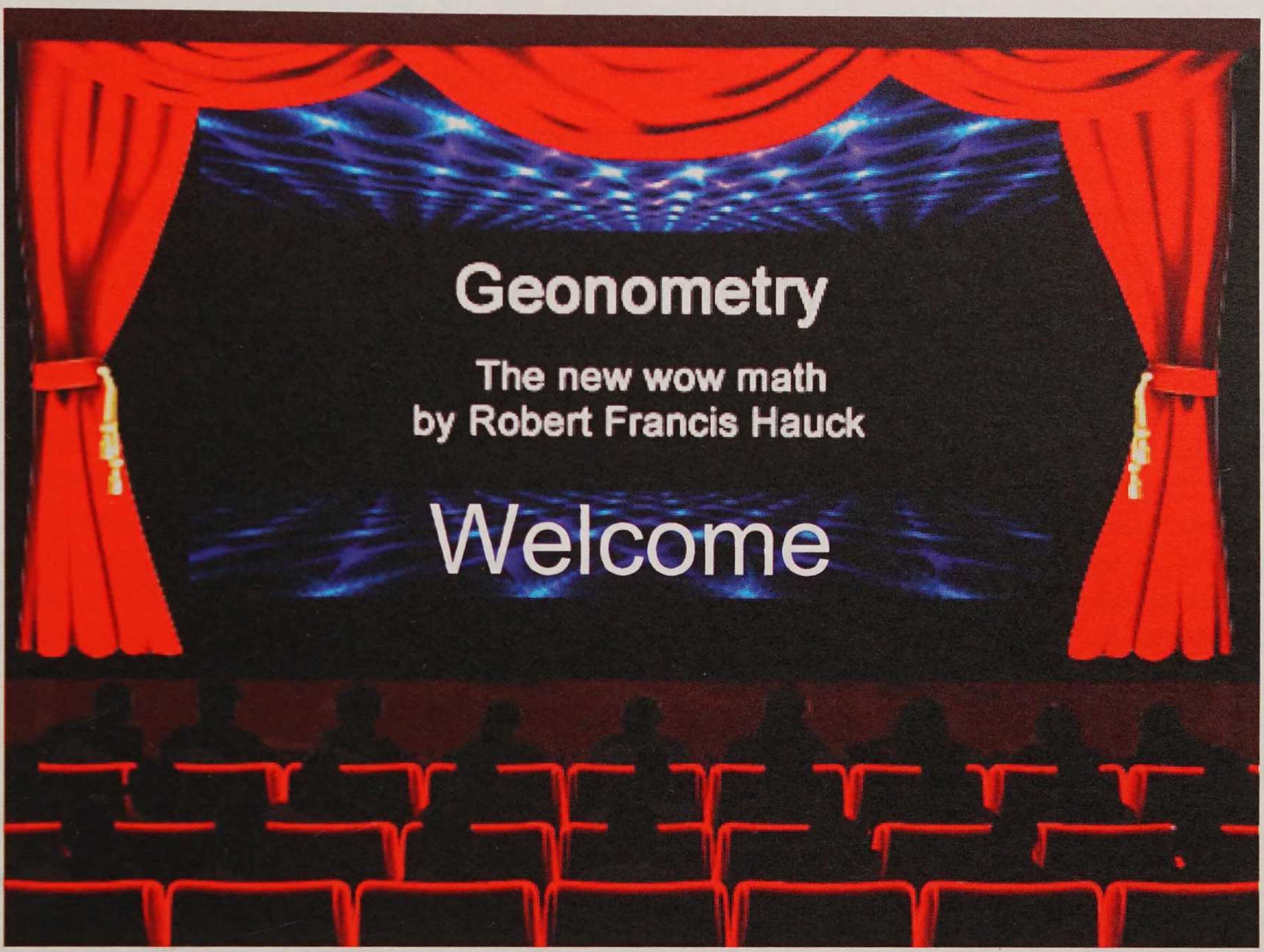
The immediately promising application of this new math is the harnessing of these vibrations for levitation or propulsion. Theoretically, devices could be developed that would produce vertical pulsations in a planar array in accordance with one or more of the complementary tiling patterns in Class-1 squares, and later in a 3-dimensional tiling configuration of Class-4 cubes, to decloak the energy lying hidden in its complementary tiling pattern. Then perhaps space could be surfed like waves in the oceans or the pulsations be harnessed with the counterpart of sails to catch the vibrational wind.

In light of the mathematical correlations made in this program between complementary loom tables and the distribution of electrons about the nucleus of atoms, this is more promising than being just mere conjectural thinking. It is this implication that should be being conveyed to promising math and science students of the up and coming generation. There are numerous experiments that could be made involving arrays of pulsations of sound (phasers), electromagnetic and laser energy in the interwoven numerical patterns that are identified in this program series.

With this new math, you just never know what will arise until you test a conjecture. Given that I could quickly test any formula or check out a correlation so readily with Excel software, in the eight-year period that I was developing this new math, I could never predict what would occur until I actually tried something. Then when something worked, it was usually beyond my expectations and I would say to myself, "Wow, that's just amazing! Who could have foreseen that?" That's one of the reasons I began referring to it as "Wow! Math". I claim that this new math was already there just awaiting discovery by whomever stumbled upon it. Later when I started making presentations about it, at various points throughout the lectures, I would hear the audience saying "Wow!" in unison.

With this new math, it's like in the days of the Western Gold rush; it's all out there awaiting yet even more discoveries. It just takes some dedicated prospecting.

Program 1



Geonometry

The new wow math
by Robert Francis Hauck

Welcome

Welcome to the 10-part program series on the new wow math called Geonometry. It is called Geonometry because it is the discovery of the naturally-occurring geometry hidden in the natural number series**1, 2, 3, ...** etc. This program series will demonstrate that these amazing equal-summing number patterns are actually the harmonic frequencies that form the basis of the real fabric of space.

This math has its formulas as all maths do, but the math is quite elementary in that it involves predominantly basic arithmetic on integral number tables. It was developed by using Microsoft's Excel program on a laptop computer. So the math is actually quite comprehendible at the Senior highschool / college Freshman level.

But the most interesting element of this new math is that it predominantly consists of observable patterns through the use of colors. So now coloration becomes a key element in observing the spatial patterns. And despite the actual formulas, the patterns are clearly visible to everyone, even to those who don't comprehend the math. That is why it has been dubbed the **new wow math**.

It will be seen that this math does have practical applications in real space. This new math will clearly give attendees an advantage over their future classmates who will remain ignorant of the subject as this math is not taught in schools. It has yet to suffer a single chriticism that is based on technical merits.

Program 1

This program series demonstrates that education can be taught through observations of wonderment, rather by rote. Practice should come from curiosity and exploration, not through the drudgery of homework – at least to those students that are naturally receptive to new ideas.

With the capability of Microsoft's Excel™ program, which was originally developed for book-keeping and cost/billing summerizations by VisiCalc Corp. in Silicon Valley of California back in 1978 (and later purchased by Microsoft), it can have extensive use in teaching mathematics. Exercizes could be based on confirming the properties identified in this companion book using the extensive numerical tables up through the 5th dimension that are provided in the Author's book, **Number Magic**.

After getting acquainted with Excel, students could then be taught math up through advanced Algebra, Trigonometry and Matrix Theory. Matrix Algebra could be supplimented by internet operations readily performed on the free website **www.quickmath.com** or in **Excel** itself. This is how I discovered and developed the amazing properties of this new math and later confifmed the interconnection of Geonometry with classical math as demonstrated in the latter half of Program 9. I admit that I couldn't have accomplished this Program Series without Microsoft's Word, Excel and PowerPoint software, Adobe's PhotoShop for illustrations and PDFLite's PDF file-generator for self-publishing. All this was totally new to me back in 1998, just 15 years ago.

List of the program topics and description of their subject matter

Here is the list of the program topics and description of their subject matter by program sequence:

Program 1

Introduction

Gives the basic definitions for patterns and summations involved in Geonometry. Categorizes the number tables by table-size into 6 distinctly different classes, each of which has their own unique characteristic equal-summing patterns. It provides examples of each class.

Program 2

Part I: Tiling Patterns

Uncovers the amazing interlocking equal-summing tiling patterns hidden in Class-1 and Class-4 Squares. It depicts the natural quilt patterns in spatial membranes.

Part II: Loom Tables

Introduces loom tables derived from the original table; shows how the primary dual table is constructed from them and demonstrates the fundamental mathematics underlying the real fabric of space.

Part III: Anchor-dot Patterns

Identifies the distribution patterns of uniformly scattered numbers which additionally and continuously sum to the square's characteristic number for all squares – except Class-2 squares which lack this property.

Program 3

Cloaking Patterns

Demonstrates the continuous joint cloaking properties of complementary tiling patterns on loom tables for all but Class-2 squares.

Program 4

Cubes

Explores the properties of 3-dimensions and the amazing equal-summing 1-dimensional and 2-dimensional patterns discovered in Geonomic cubes through the compact view of *depth-sum* tables. There begins the incredible mathematical journey up through higher dimensions as only Geonometry can provide.

Program 5

Geonomic description of the atom

It is demonstrated just how the mathematics of 2- and 3-dimensional Geonometry can explain and account for the distribution and navigation of the electrons around the nuclei of atoms.

Program 6

Quadracubes

4-Dimensional space is shown as never seen before in such a definitive manner.

Program 7

Part I: Quarks

Equal-summing patterns in Class-4 quadracubes are demonstrated to unlock the relationships among quark-pairs at the sub-dimensional level of **1/4D**. Here Geonometry independently confirms what sub-atomic scientists have discovered through their linear accelerators, as an inherent property of the 4-dimensional spatial fabric.

Program 1

Part II: Matchmaker Squares

In this segment, exploration of the properties of a seemingly recreational geonomic square leads to the discovery that these special number tables serve as the very twine that is involved in weaving the fabric of space. One of these complementary loom tables leads to the manifestation of both complementary loom-tables for Class-1 squares for all prime-number sizes and provides for their systematic generation.

Program 8

Additional Geonomic Properties

In this program, yet even more mathematical properties of Geonometry are revealed. Here it is detailed how all the squares of composite size can be generated from ones of prime-number size. Here we will get an even broader picture of the numerical spatial fabric and its pervasiveness by discovering loom-tables hidden in the tiling patterns of cubes and hypercubes.

Program 9

Part I: Navigating Multidimensional Space

Here it is demonstrated how to explore the properties of any higher dimension through the unique mathematics of Geonometry. Math higher than basic Algebra is not necessary for this segment.

Part II: Connections with Higher Classical Math

Later in this program some amazing mathematics from classical math are shown to be already embodied in Geonometry. For this portion it would be helpful if the viewer had an introductory course in Matrix Algebra.

Program10

Other Geonomic Forms

Here Geonometry explores nested-frame triangles. It shows relationships discovered between diamonds and squares. For hexagons, it demonstrates just what makes snowflakes so hexagonally symmetrically perfect. Scientists are still to this day unable to systematically explain this phenomenon by the chemical makeup of water alone. It demonstrates once again in an undeniable manner that Geonometry is really uncovering the very fabric of space itself.

Program 1

First, some points of recognition are in order.

This program series was written and compiled by Robert Francis Hauck, the mathematician who originally developed its contents.

What began as a series of PowerPoint slide presentations was later converted into a series of video programs by Larry Talbot of television station MHAT TV intended for initial broadcast over local Channel 19 in Morgan Hill, California.

Today this video series is being made available under pre-sponsored scholarship programs for advanced–placement to students, their parents and math teachers free in corporate assembly halls and academic auditoriums throughout California and across the nation.

Program 1

Overview

This program series is a real mind trip. To comprehend what is presented here, you will have to have an interest in mathematics and be of above average intelligence. Almost every slide describes something never seen before.

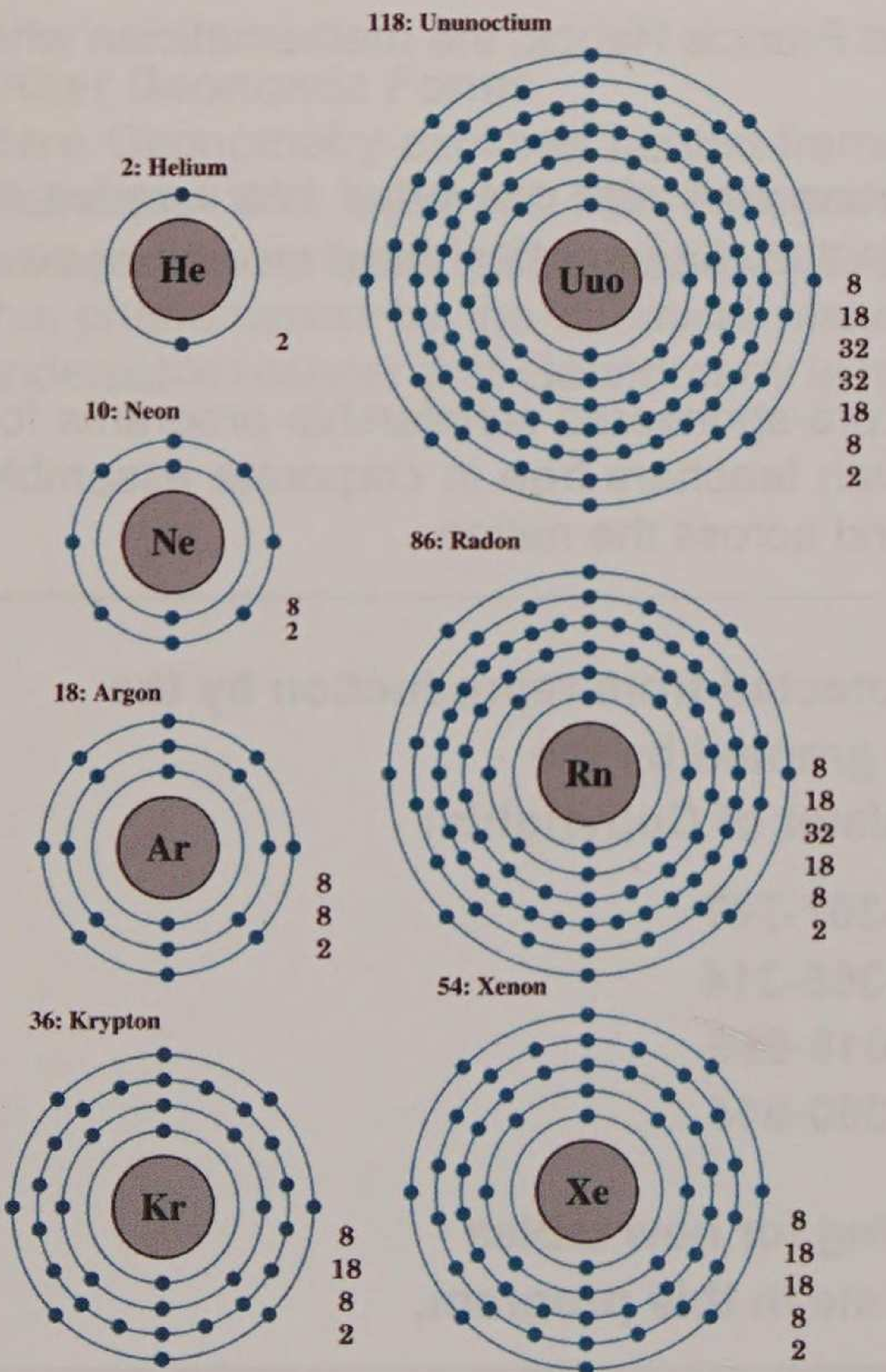
It is a whole new mathematics, yet on the highschool Senior / college Freshman level. Rarely is any higher math than simple algebra involved. Most of the math involves merely addition, subtraction, multiplication or division – simple grade school arithmetic. It's the concepts that are exceptionally advanced. And it is the fascinating presentation of the concepts in color – something usually not done in explaining mathematics.

What is this new math good for?

People are always questioning me what this math is good for. In this introductory portion of the program series we're going to talk about some potential applications.

When I was in college, I would stumble along in my math classes because I couldn't see how it applied to anything that might be of interest to me. That is until we got to the applied math part. Then it would all come together and make sense for the first time and I would ace the final exam. I had to first see the BIG picture.

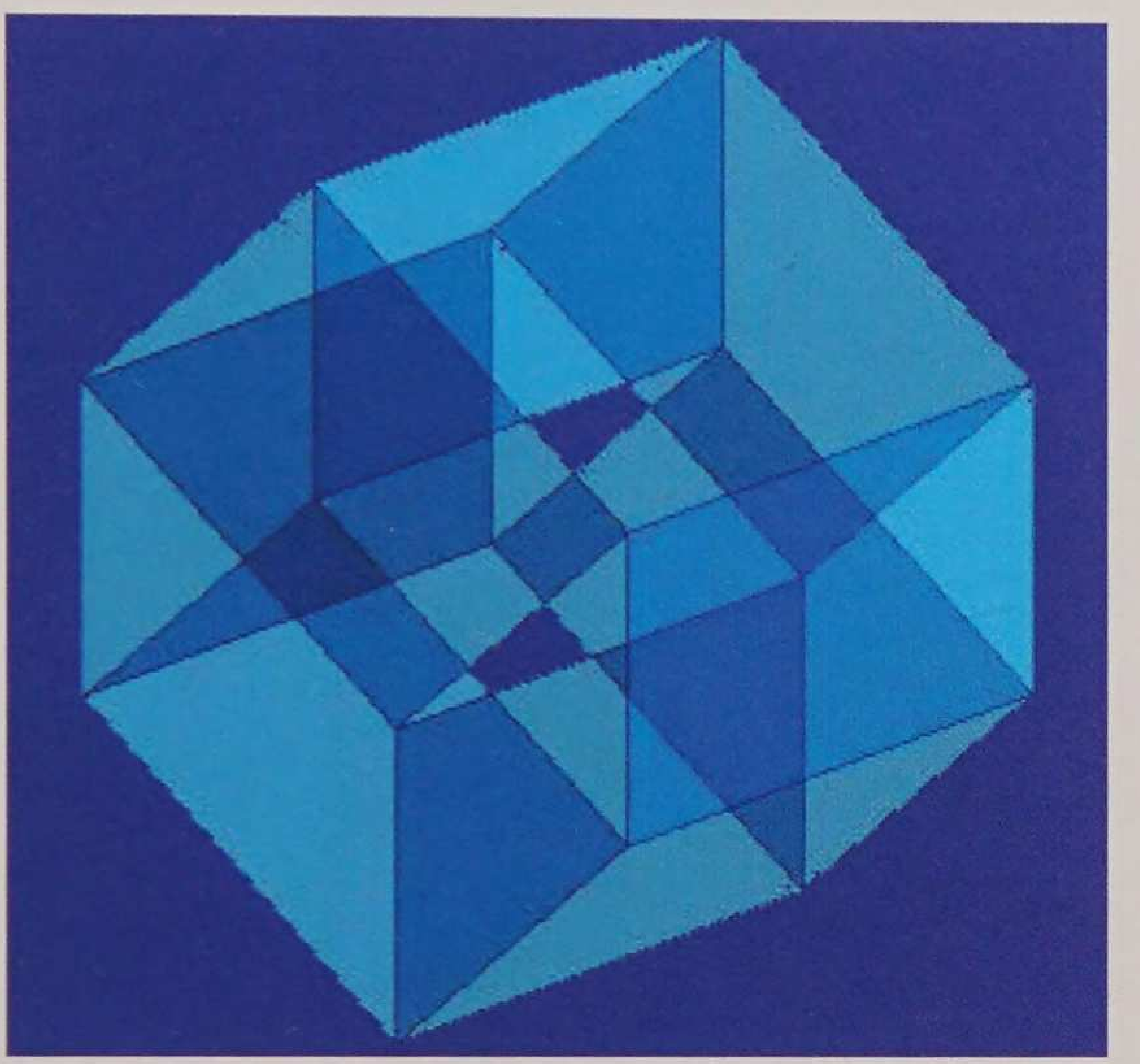
So from this experience with my initial ignorance, I am going to show you in the next few slides some major applications that this new math has been and would be good for.

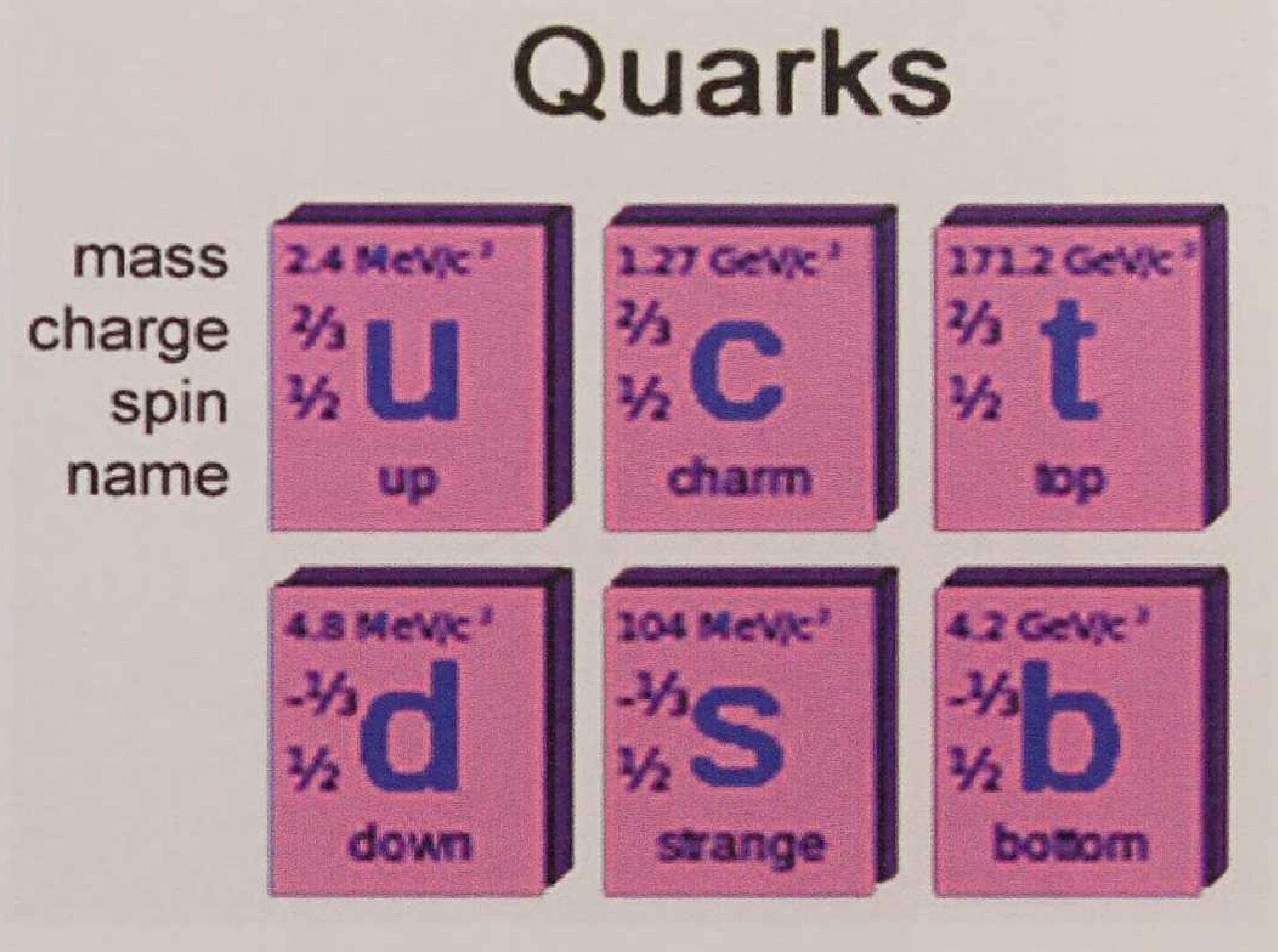


One application demonstrates amazing tiling patterns in the 2nd dimension that are numerically harmonic in the 3rd dimension. And these provide an accounting for the number, distribution and interaction of the electrons in all the atoms in sub-dimension **1/3D**. This is dealt with in Program 5.

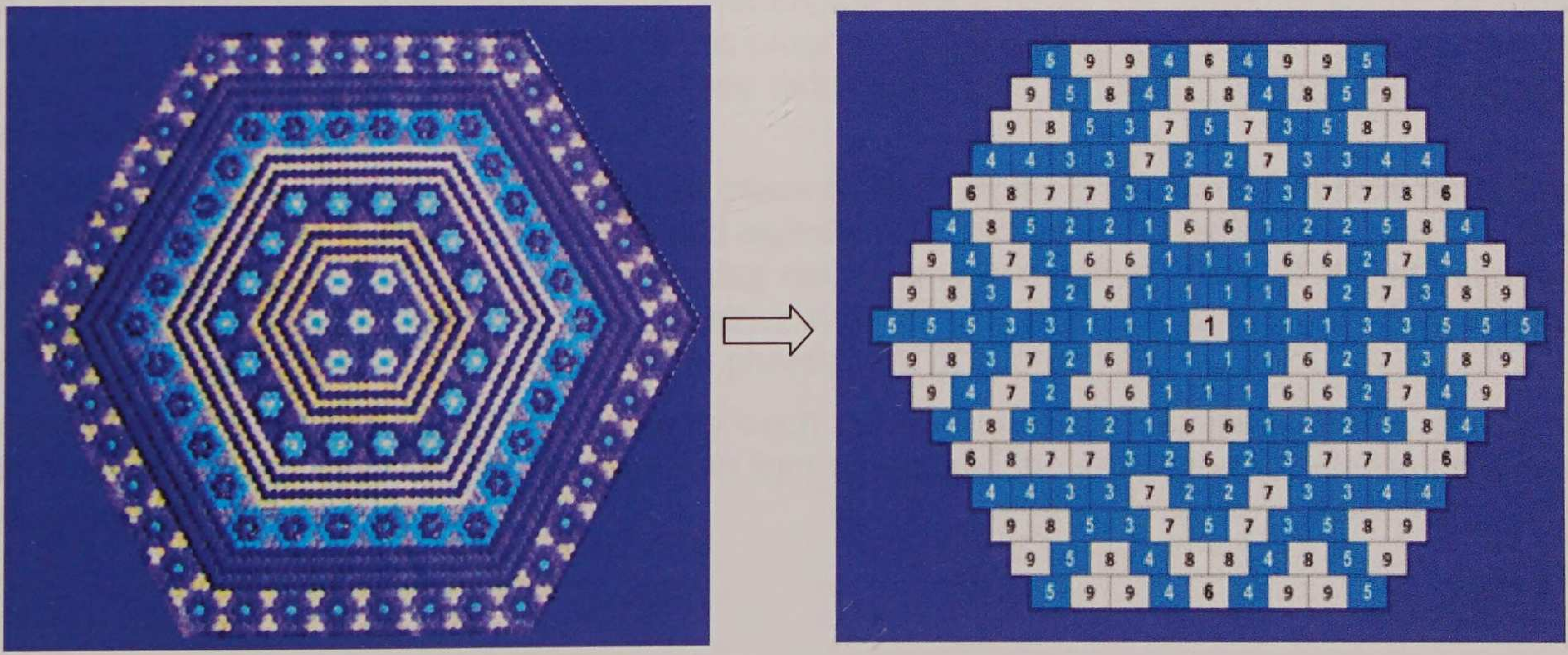
Program 8 demonstrates that these same harmonic patterns found in 3 & 4 dimensions, themselves contain harmonic patterns embedded within them!

Another application offers our first, potentially real view of the 4-th dimension. It demonstrates the application of 4-dimensional spatial patterns to explain the properties of quarks at a sub-dimensional level **1/4D**. This is addressed in Program 7.





Another application demonstrates mathematically that snowflakes are really be being fashioned from the harmonic quantumized vibrations in a spatial membrane when a frozen blanket of vapor contracts into isolated hexagons. This is addressed in Program 10.



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9 4 7 2 6 6 1 1 1 6 6 2 7 4 9
9 8 3 7 2 6 1 1 1 1 6 2 7 3 8 9
5 5 5 3 3 1 1 1 1 1 1 1 3 3 5 5 5
9 8 3 7 2 6 1 1 1 1 6 2 7 3 8 9
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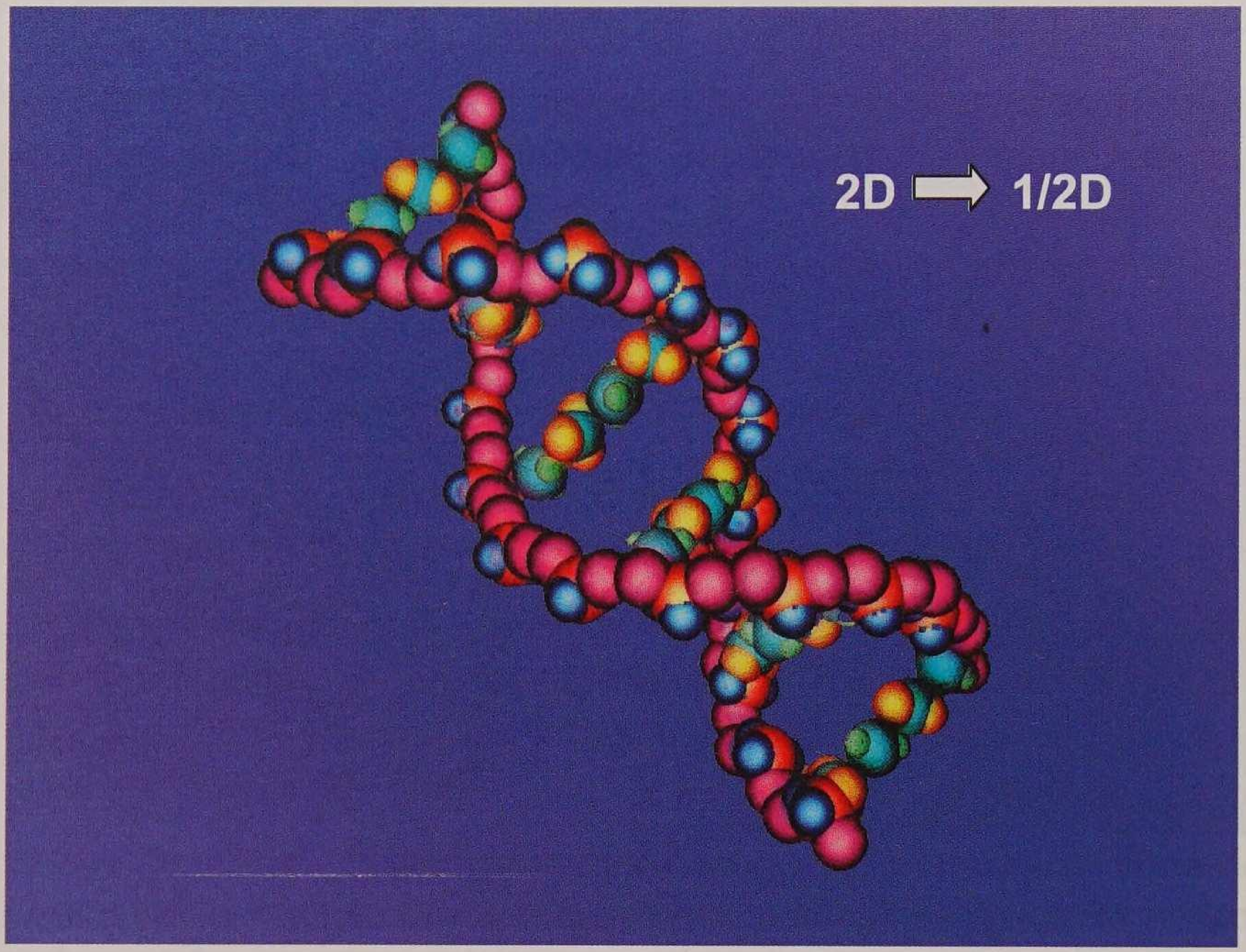
4 4 3 3 7 2 2 7 3 3 4 4
9 8 5 3 7 5 7 3 5 8 9
9 5 8 4 8 8 4 8 5 9
5 9 9 4 6 4 9 9 5

This program series will demonstrate time and time again that space must be composed of a vibrating harmonic fabric.

Program 1

A potential application is the identification of spatial templates upon which biological DNA molecules could assemble themselves into double helixes and form reproductive molecules. This is encountered in Program 3.



Spaceflight

The picture here shows an actual photo taken of an **F-118** U.S. fighter-jet as it just breaks the sound barrier.

Only back in 1903 did we learn how to fly with a propeller-driven engine. Jet fighter planes appeared in Germany in 1942 where they were invented. Not until 1947 after World War II, was the sound barrier broken for the very first time by a jet engine. On June 2nd, 2010, Boeing's **X-51 Waverider** rocket powered by a scram-jet engine flew **6** times the speed of sound for three minutes. That's over 4600 mph. Most of this progress in flight in air took place in just the last 60 years. So all the amazing things that you see today were invented in just the last three working generations of engineers.

In fact, the digital computer revolution took place 5 years after I graduated from college. Before that we used slide-rules and later hand-held digital calculators after the digital chip was invented and made programmable. I was programming main-frame computers up to 1974 that used tiny Cheerios-style magnetic donuts strung together in a fabric of copper wire to store information in **1**'s and **0**'s called "bits", a short term for the phrase "bits of information".

Given the rate of technological progress by each new generation of engineers and technicians in just the last half century, going faster than light speed into the 4th dimension doesn't seem to me all that remote.

Program 1

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	9x9	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23								64	369
3	52								15	369
4	62								49	369
5	1								81	369
6	33								20	369
7	67								30	369
8	18								59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

Just as we have in times of civilizations past, where we used the undulating patterns in the oceans to surf its waves, or the sails on boats to capture the circular patterns of wind to cross the ocean-divides, it may now be possible to identify the spatial vibrations among the vibrating membranes of space that could be harnessed to surf the Cosmos.

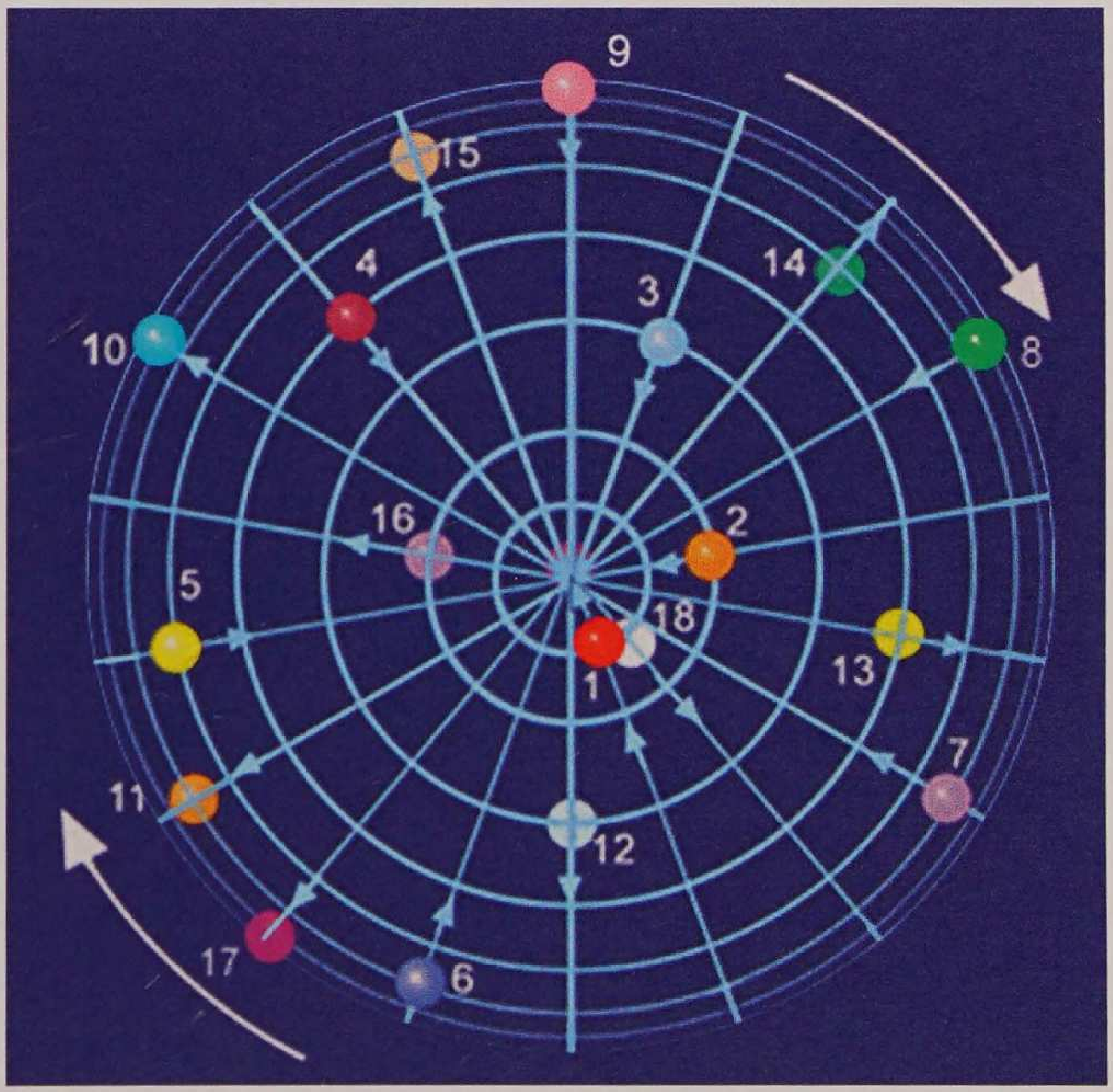
To travel in space, we need something yet even more innovative than thrust from a scramjet. Given the inherent spatial vibrations whose existence will be made quite evident mathematically in this program series, we should begin looking for an electro-magnetic mechanism or a planar nano-pulse generator that will generate complementary pulsations that will surf these hidden planer spatial vibrations.

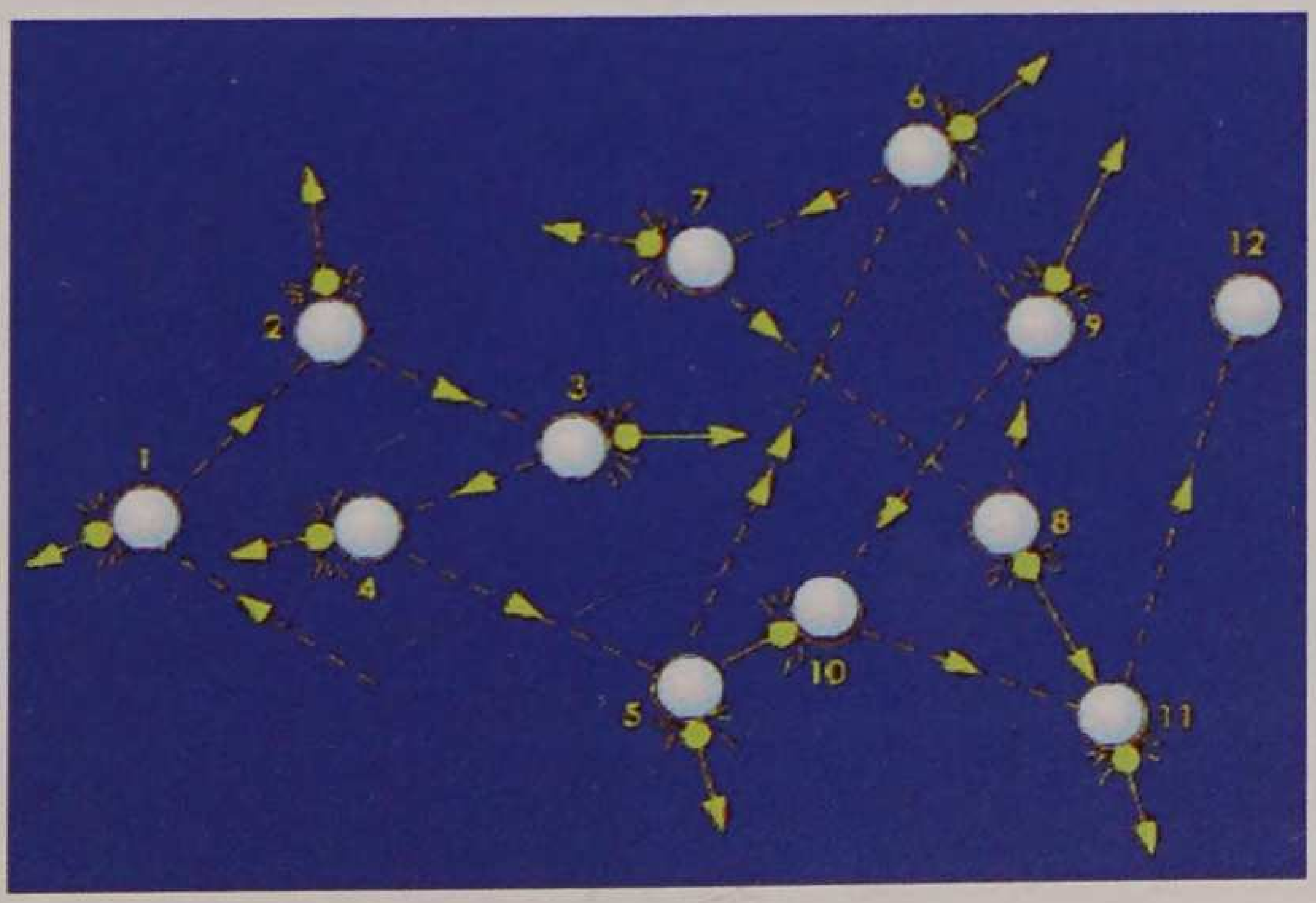
This picture is a virtual representation of how the spaceship such as Hollywood's famous Starship Enterprise would appear upon breaking through the light-speed barrier of the 3rd dimension as it enters the fourth dimension, as seen from within 4-dimensional space. Note the harmonically patterned trail it leaves behind it as it surfs the vibrations of space.

This program series will identify the vibrational patterns in the fabric of space itself. And it will prove mathematically just why they haven't yet been observed by Science. It will prove that there are two complementary fundamental vibrational patterns which must coexist together simultaneously and that these dual patterns continuously cloak one another! That cloaking pattern will be demonstrated to exist mathematically in Program 3 of this series. The spatial ether is there, but to see it, it needs to be exposed by removing one of the complementary vibrational layers in the natural pattern.

And once this can be done with a yet to be developed spatial surfing technology, Mankind will be able to ride the continuously undulating spatial fabric! So a potential future application of this new math is space propulsion. Do I now have your attention?

Let me pose these questions: What do you think keeps the electrons flying around the nucleus of the atom? That orbital energy pattern has to be coming from somewhere.



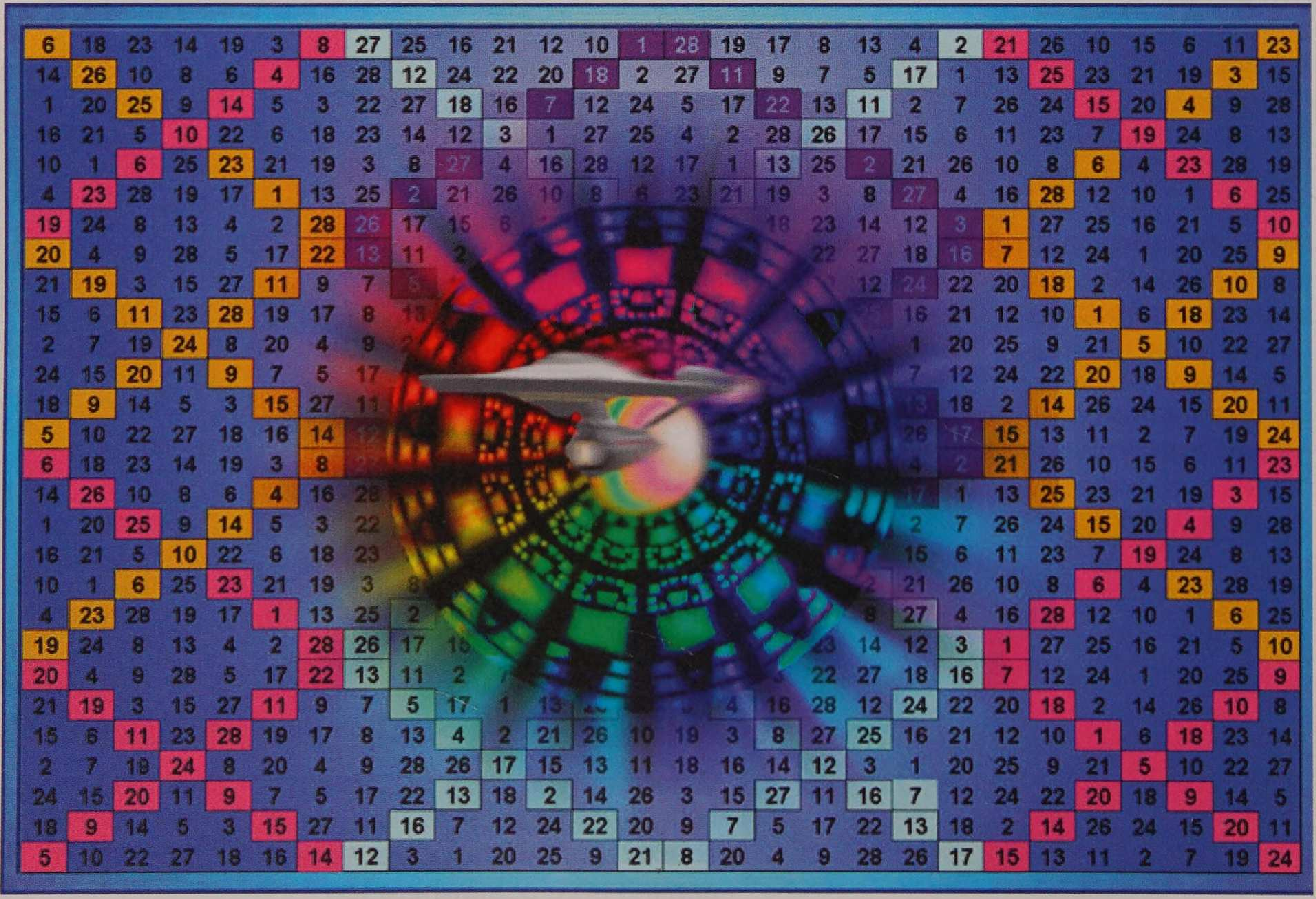


Where do you think the energy for Brownian motion in liquid molecules and gasses is coming from? That energy at the molecular level of **1/2D** has to be coming from somewhere too.

Scientists say that it is the heat left over from the Big Bang. That can never be proven, so why even deal with that notion? It could be the universe pressing in upon itself, simple as that. You don't need a bang.

Regardless, I say that there is energy out there to be tapped, but it is being cloaked in the vacuum of space by space itself. Geonometry may someday in the not too distant future be used to uncloak that fabric and tap that energy source. Which by the way, would be so powerful as to make nuclear energy seem like mere trickling water.

Here's that spaceship, the Starship Enterprise, again, now surfing the harmonically vibratory pattern in the discloaked fabric of space. All the waves are of size-28 interlaced diamond patterns like a pile of chain-link fences. These will be demonstrated in Program 3 to exist pervasively throughout space everywhere.



6	18	23	14	19	3	8	27	25	16	21	12	10	1	28	19	17	8	13	4	2	21	26	10	15	6	11	23
14	26	10	8	6	4	16	28	12	24	22	20	18	2	27	11	9	7	5	17	1	13	25	23	21	19	3	15
1	20	25	9	14	5	3	22	27	18	16	7	12	24	5	17	22	13	11	2	7	26	24	15	20	4	9	28
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4	23	28	19	17	1	13	25	2	21	26	10	8	6	23	21	19	3	8	27	4	16	28	12	10	1	6	25
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24	15	20	11	9	7	5	17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7	12	24	22	20	18	9	14	5
18	9	14	5	3	15	27	11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13	18	2	14	26	24	15	20	11
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6	18	23	14	19	3	8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	2	21	26	10	15	6	11	23
14	26	10	8	6	4	16	28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17	1	13	25	23	21	19	3	15
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10	1	6	25	23	21	19	3	8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	21	26	10	8	6	4	23	28	19
4	23	28	19	17	1	13	25	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8	27	4	16	28	12	10	1	6	25
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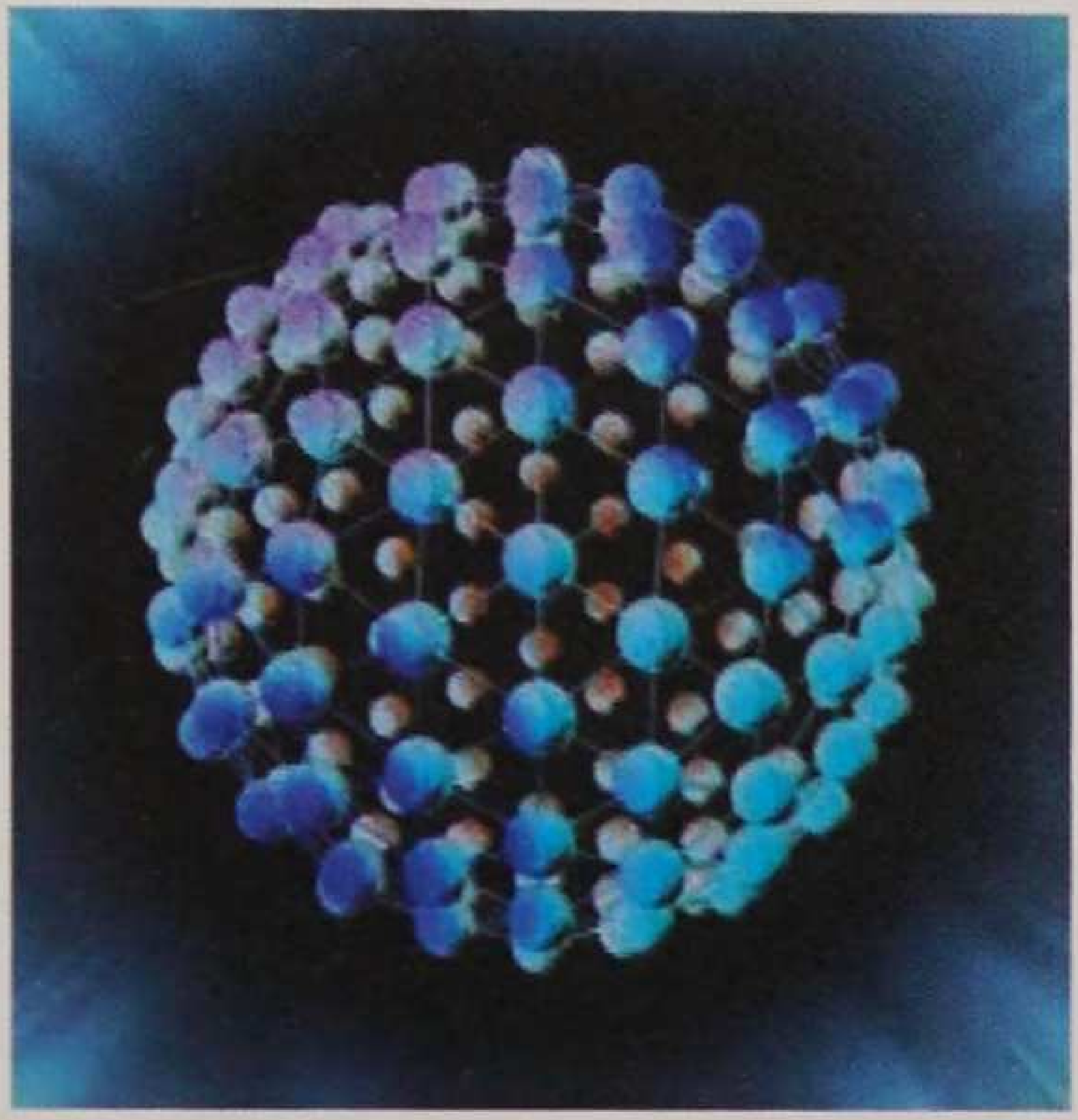
5	10	22	27	18	16	14	12	3	1	20	25	9	21	8	20	4	9	28	26	17	15	13	11	2	7	19	24

Other potential applications

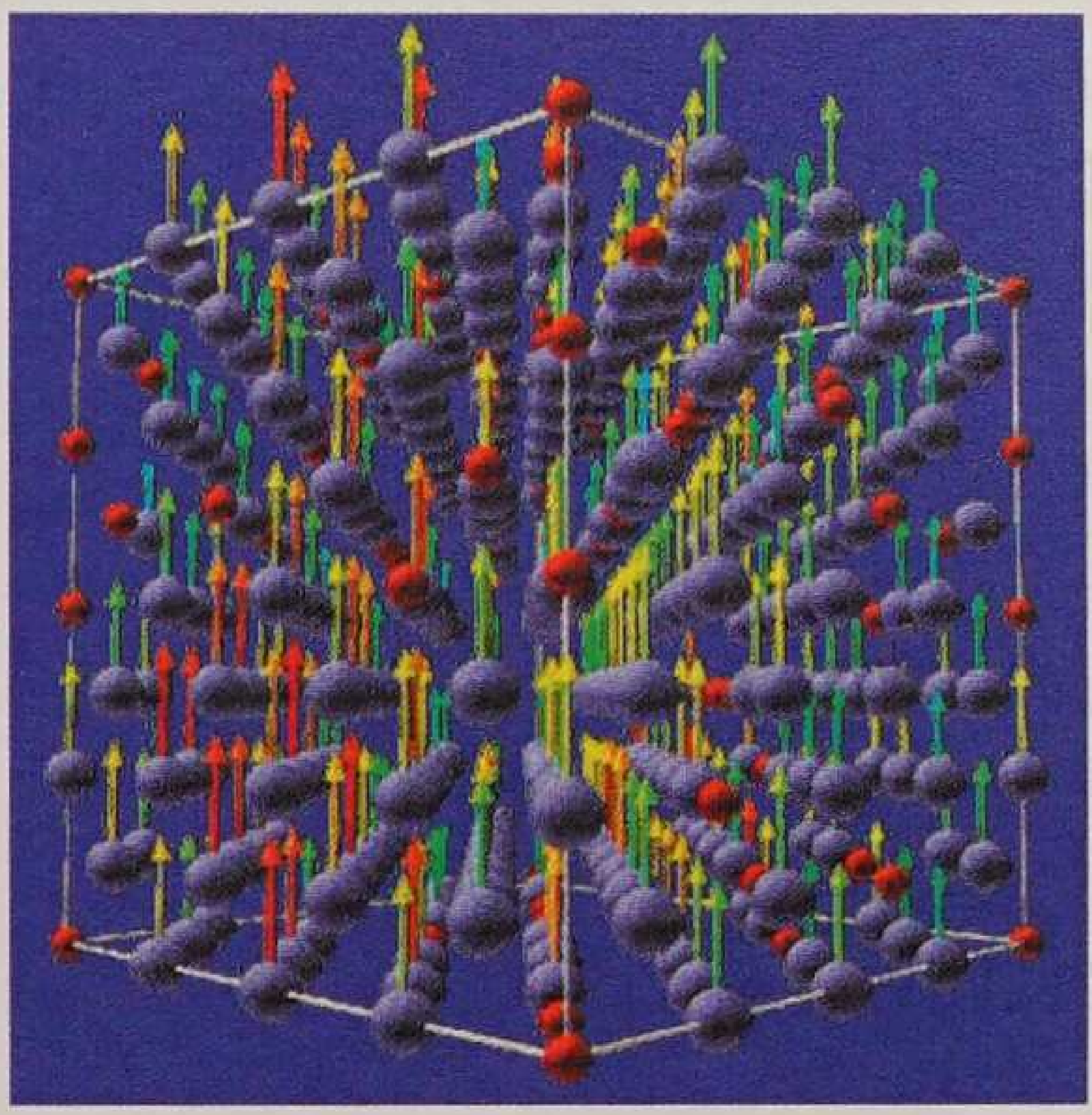
Who knows at this point-in-time to just where this new math might lead in the future? It may guide in the exploration of patterns to be discovered in Nano-technology, Materials science, Sub-atomic physics, Micro-chip technology or Molecular-biology, or in Astrophysics, and even in Cosmology itself.

Nano technology

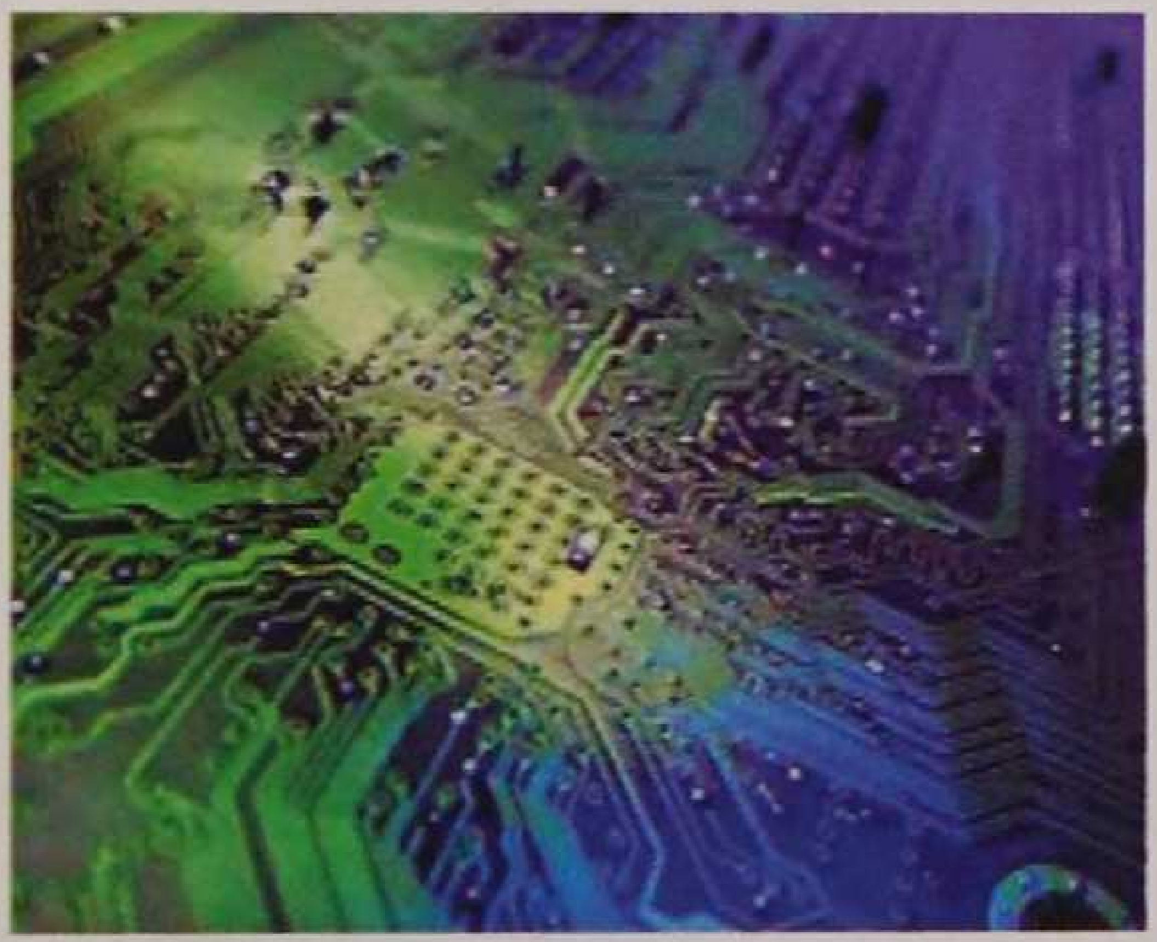


Micro-biology

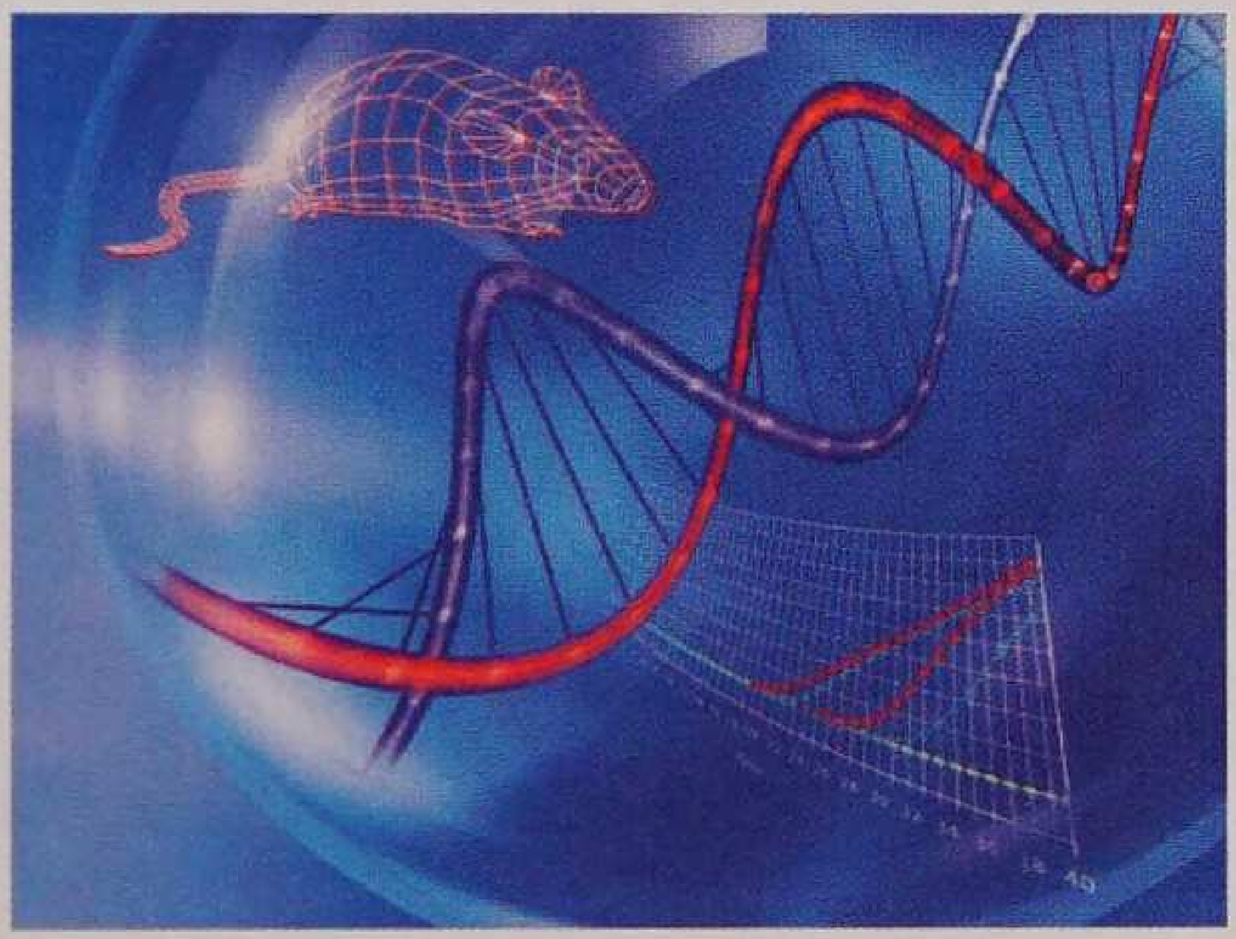
Materials Science



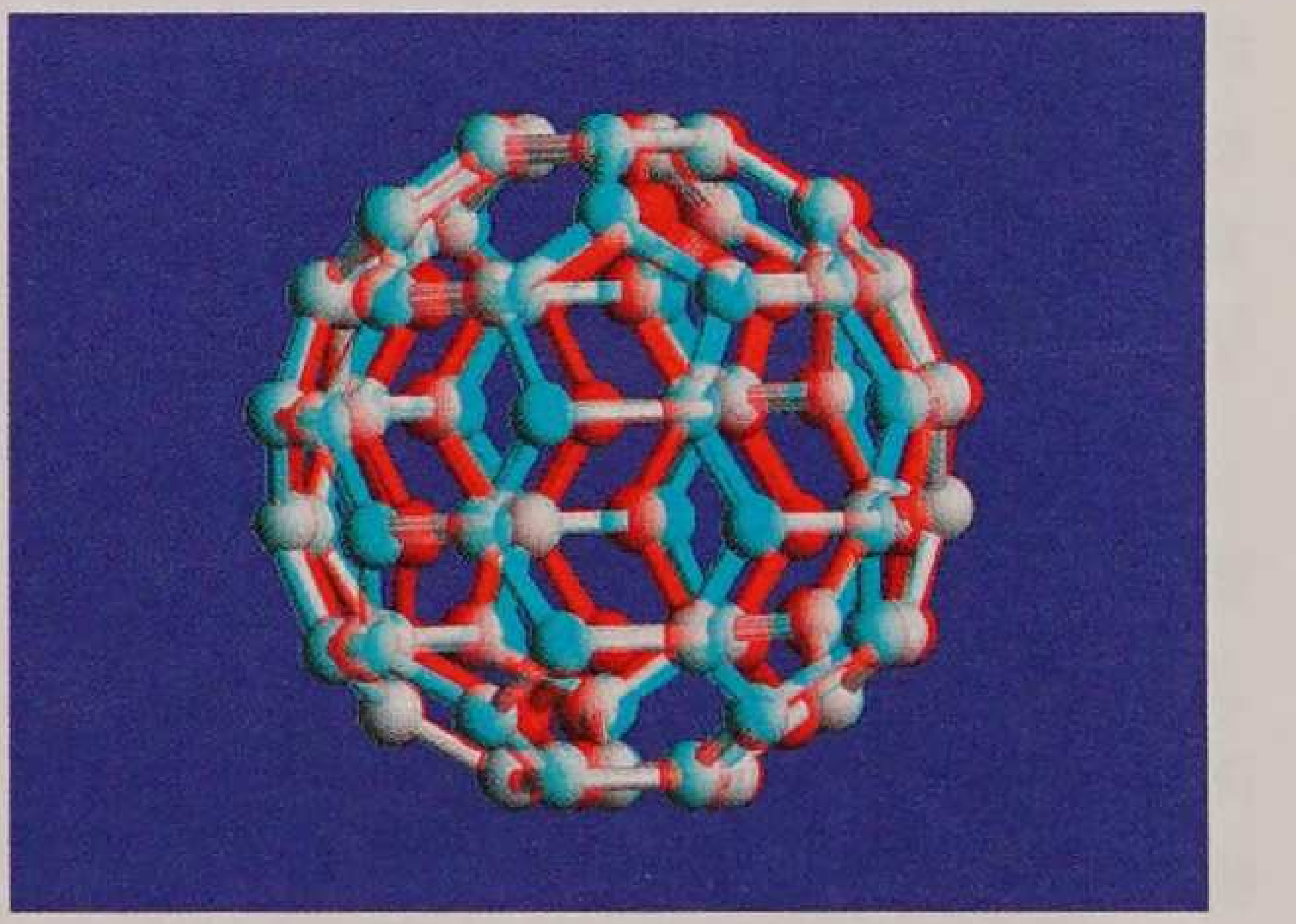
Microchip Technology



Micro-biology

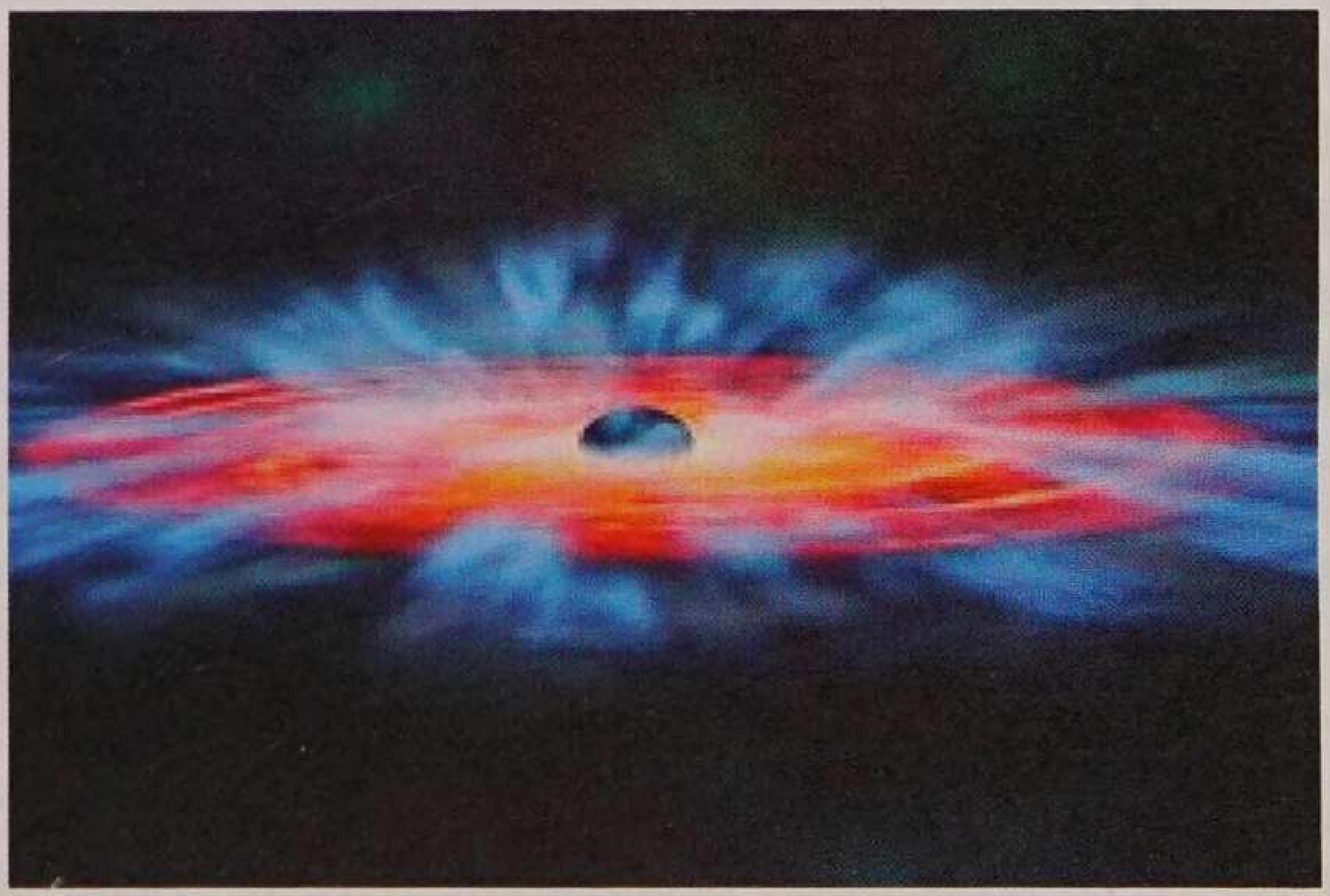


Materials Science



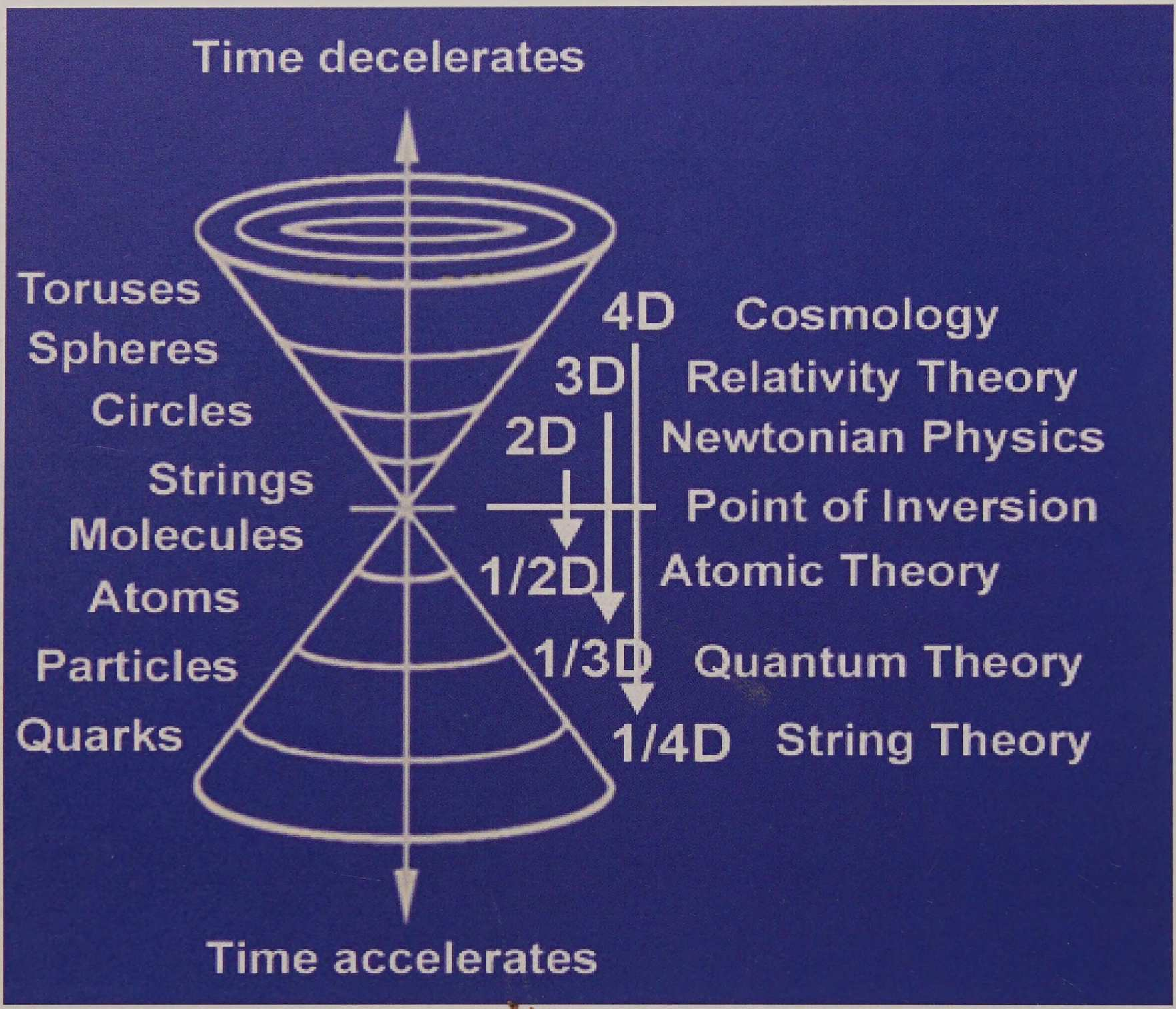
Astrophysics

Cosmology



So let your mind be open to this new math which was just developed to the extent it is today in the last eight years prior to the year 2013. If you are going to succeed in any creative area of Science you will probably need to be at least aware of this new math because all these technologies are getting more and more sophisticated with newly-uncovered, intricate and complex patterns by the day.

Here is Geonometry's Big Picture of the multi-dimensional universe



We know that particle accelerators and atom smashers measure their observations in **milliseconds** in **1/3D** and **nanoseconds** in **1/4D**, so there should be no argument against accepting that time passes at an accelerated pace when descending sequential sub-dimensional levels.

Given our midpoint perspective of time in 3-dimensions as the basis upon which to relate our seconds to the passage of time in sub-dimensions, which are orders of magnitude faster than our "seconds", the passage of time in the 4th dimension should be at least an order of magnitude slower than the passage of time in the 3rd dimension, just based merely on the inverse of the progression just cited. So time may not be a dimension in and of itself as the Theory of Relativity presumes, but is most likely a property of each dimensional level.

We will see just how the basic patterns that are found in Geonometry of the **2nd, 3rd** & **4th** dimensions can be applied to provide the basic mathematics for interpreting the already scientifically-known molecular and atomic structures in sub-dimensions **1/2**, **1/3** and **1/4D**, respectively.

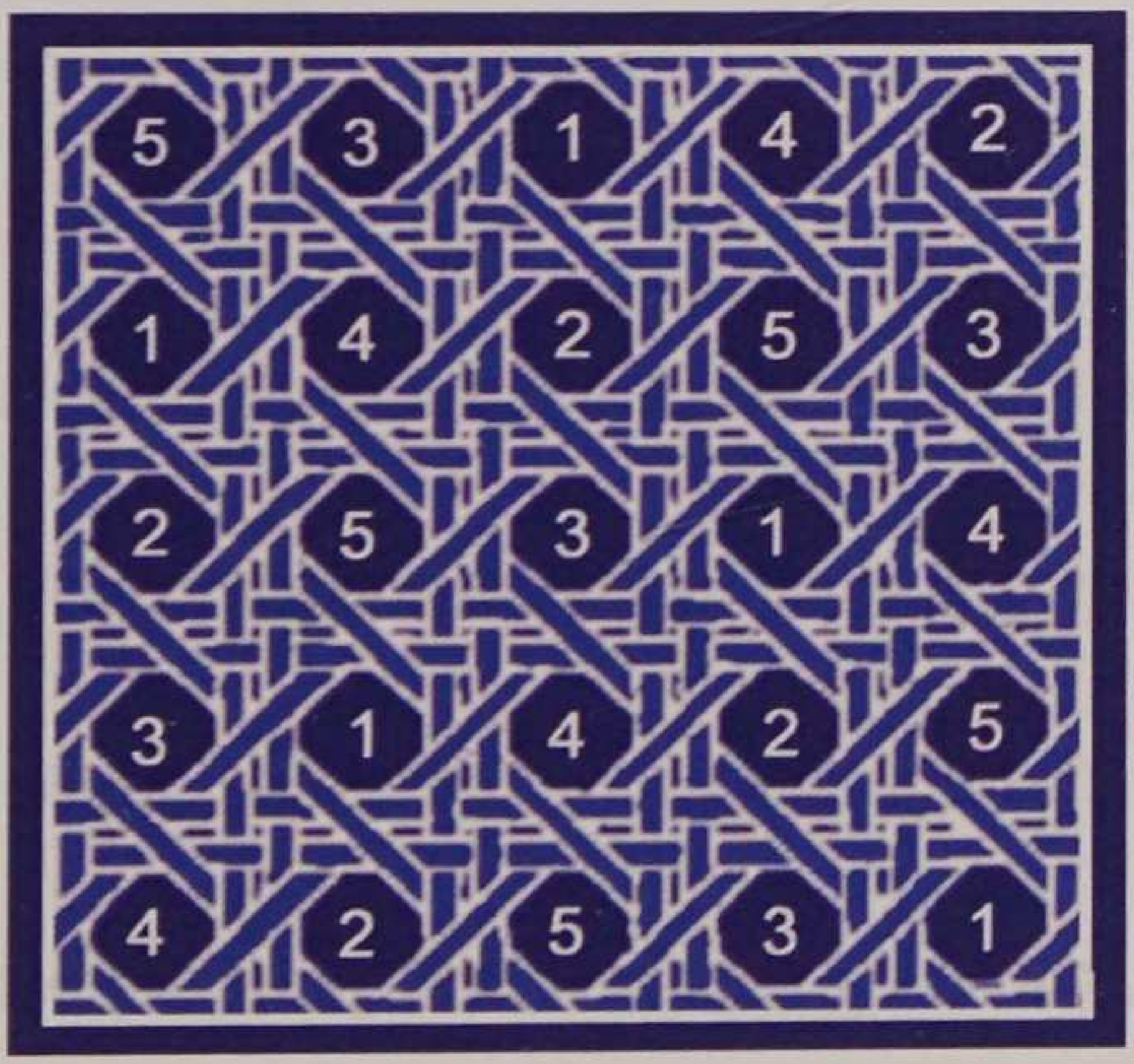
Program 1

The pictorial lists on the right the various scientific theories that have emerged for explaining the structure of space at the various dimensional levels. The big effort today in Cosmology is to find some mathematical principle to unite all these formulations into one grand theory of the universe. It's an attempt by Science to discover the superior intelligence underlying the structure of the spatial fabric. Einstein had been quoted as saying "God doesn't roll dice" in his search for the intelligent design behind the structure of space.

Even today, String Theory is graduating from the mathematics involving string vibrations to one involving vibrational patterns of membranes, called M-Theory. That is, Cosmology is evolving from a 1-dimensional view to a 2-dimensional view; one from a linear perspective to a planar one. As yet, the topic is still not settled.

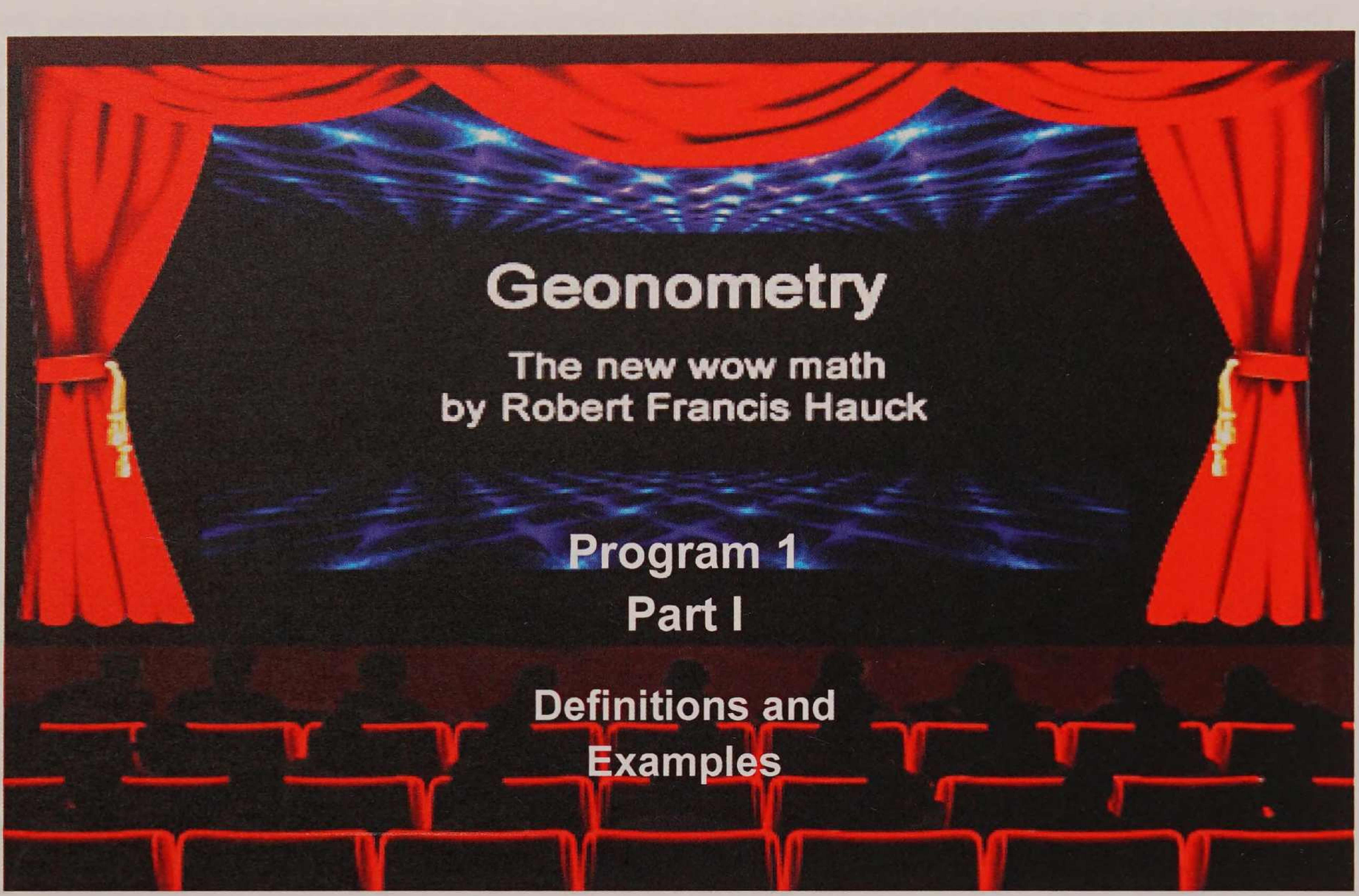
Whereas these scientific theories attempt to formulate principles from the vast perspective of the **infinite** in a very convoluted effort of trying to unite Cosmology with String Theory by extending the math out to **11** dimensions of space, the accomplishments of Geonometry thus far in this same attempt are made possible by simply confining the exploration of harmonic patterns to fixed-size spaces. You will witness in this program series just how Geonometry accomplishes this with just the natural number series and two elementary, complementary mathematical functions.

These topics indicate just some of what this new math is good for. It's all about *numerical-pattern* geometry. It helps us to comprehend the natural world and interpret the patterns that we were fortunate enough to have the mental capacity to comprehend – and in my opinion, is one of a superior intelligent pre-design. Geonometry is the math that has the potential to uncover that hidden design without recourse to all that far-out convoluted 11-dimensional math of Cosmology.



Here is a 4-way equal-summing membrane which depicts the harmonically vibrating spatial membrane in a **5x5** area. The expansion properties that will be described in Program 8 will show that these harmonic membranes may be extended to cover as large of an area as you like and still sum equally in **4n** string summations in all 4 spatial directions. That means that those properties determined on a smaller scale as depicted in this program series will also exist simultaneously over a much larger spatial area. And not only that: these harmonic vibrational patterns exist throughout any dimension level. Space could be filled with many multiple complementary harmonic waves that might be harnessed for space propulsion.

In this program series, you will see four detailed fundamental applications of this new math to explain basic structures in nature whose existence have yet to be satisfactorily explained by Science.



Geonometry

The new wow math
by Robert Francis Hauck

Program 1
Part I

Definitions and Examples

We'll now begin the first program, **Introduction to Geonometry**.

There are a few aspects of Geonometry that distinguish it from classical math:

1. It involves only whole numbers, called integers. There are no decimal fractions, real or complex numbers. So it makes the math quite easy to comprehend.
2. There are only two mathematical functions involved beyond addition, subtraction, multiplication and division of basic arithmetic.
3. The equal summing patterns in tables are easily distinguished by different color-highlighting.
4. There is no need to employ Greek lettering in the formulas; they can be kept quite simple using just ordinary letters of the alphabet.
5. Although the number tables are matrices, Matrix Algebra is neither needed nor involved.
6. There are only a few examples left as exercises but no quizzes to take. Learning takes place because the subject is full of surprises all along the way. Additionally, you have your companion book to this series to let you preview beforehand and review the material after its presentation later at your convenience.
7. All of the operations upon the number-tables can be easily generated in Microsoft's Excel™ program. In fact, this math could not been discovered without Excel™.

However in the second half of Program 9 where it is demonstrated just how this new math interfaces with classical math, items #1 thru #5 will be relaxed to make the discussion possible.

So pay close attention, because there are many new concepts that have new names and symbolisms, and these will come up time and time again throughout this program series without any repeated explanation.

Now, get ready for some preliminary *geonomic* concepts, formats, definitions and symbols.

1-Dimensional Geonometry

Geonomic strings in dimension-1 of size **n** are a linear arrangement of the numbers ranging from **1** thru **n**. Many fundamental theorems in classical mathematics are based on this 1-dimensional series. So the program will not dwell on this topic as there is a wealth of knowledge about it already in classical math.

But there is one formula from classical mathematics that is used throughout this new math and that is the formula for the sum of numbers in the series **1** thru $\mathbf{n}^{\mathbf{k}}$. The capital letter **S** with the subscript **n** will always represent the sum of numbers from **1** thru **n**.

(1.1) $\mathbf{S_n = n(n+1)/2}$

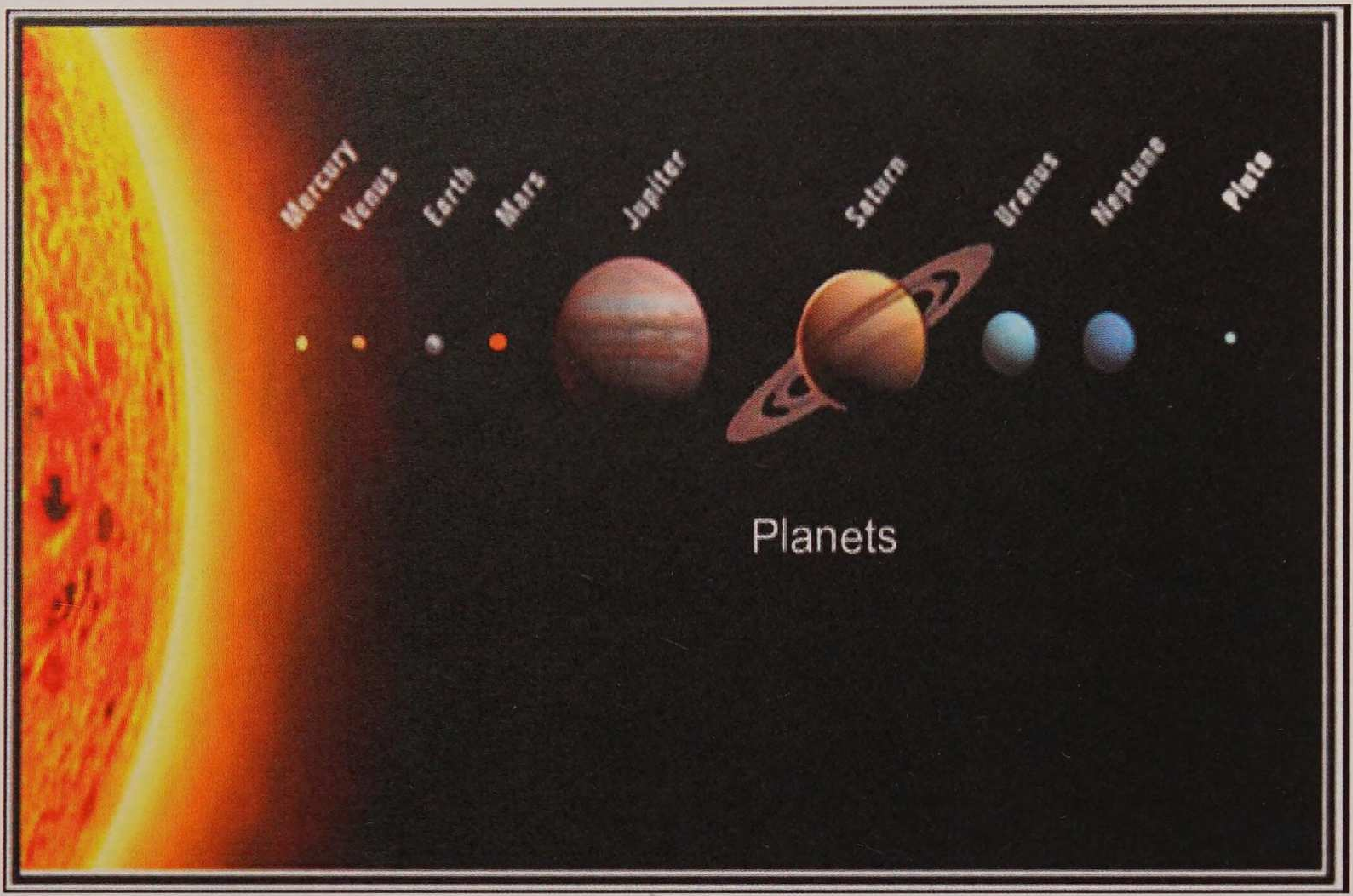
When the sum is to a higher power of **n**, that power number **k** will be a superscript of **S** in parenthesis as shown in the second formula. The superscript in parenthesis denotes the dimension in which the summation takes place.

(1.2) $\mathbf{S_n^{(k)} = n^k(n^k+1)/2}$

The sum of numbers in the simple series **1** thru **n** of dimension-1 or **1** thru $\mathbf{n}^{\mathbf{2}}$ in dimension-2 will always be shown as just the simplified version without the superscript as the topic's dimension will be obvious.

Program 1

Linear Pattern in Planetary Orbits



Here is our solar system. The orbital distances from the sun of each planet and asteroid ring are listed in the table. This distribution pattern of orbiting material is most dense midway from the sun where Jupiter lies. It becomes less dense closer to the sun according to the inverses of a series of ascending prime numbers, and less dense out away from the nebular center according to a series of ascending even numbers.

Prime-number series **Even number series**

1/13 1/7 1/5 1/3 1/2 1 2 4 6 8
13 7 5 3 2 1 2 4 6 8

The series of red numbers are the inverses of the numbers above them. Those are the ratios of the inner planets' distance from the Sun relative to Jupiter's.

The series of blue numbers are the multiples of the distance from the Sun of outer planets relative to Jupiter's.

Notice how the numbers in red follow a series of increasing prime numbers to the left and how the series of blue numbers follow a series to the right that increases by **2**, all of this starting from a **1** corresponding to the orbital distance of Jupiter.

Solar System Planets	Distance from Sun in Astronomical units	Orbit relative to Jupiter's
Mercury	0.39	1/13
Venus	0.72	1/7
Earth	1	1/5
Mars	1.5	1/3
Asteroids	2.7	1/2
Jupiter	5.2	1
Saturn	9.5	2
Uranus	19.2	4
Neptune	30.1	6
(Pluto)	39.5	8

Program 1

The orbital ratios of those planets within the orbit of Jupiter's, excluding the asteroid belt and the planet Mercury, when added to those of Jupiter's orbit and beyond, show a constant relationship of all **9**'s.

$$\begin{array}{cccccc} 13 & 7 & 5 & 3 & 2 & 1 \\ & 2 & 4 & 6 & & 8 \\ \hline & 9 & 9 & 9 & & 9 \end{array}$$

All of this is numerical evidence of the 1-dimensional quantization around a star, namely our Sun.

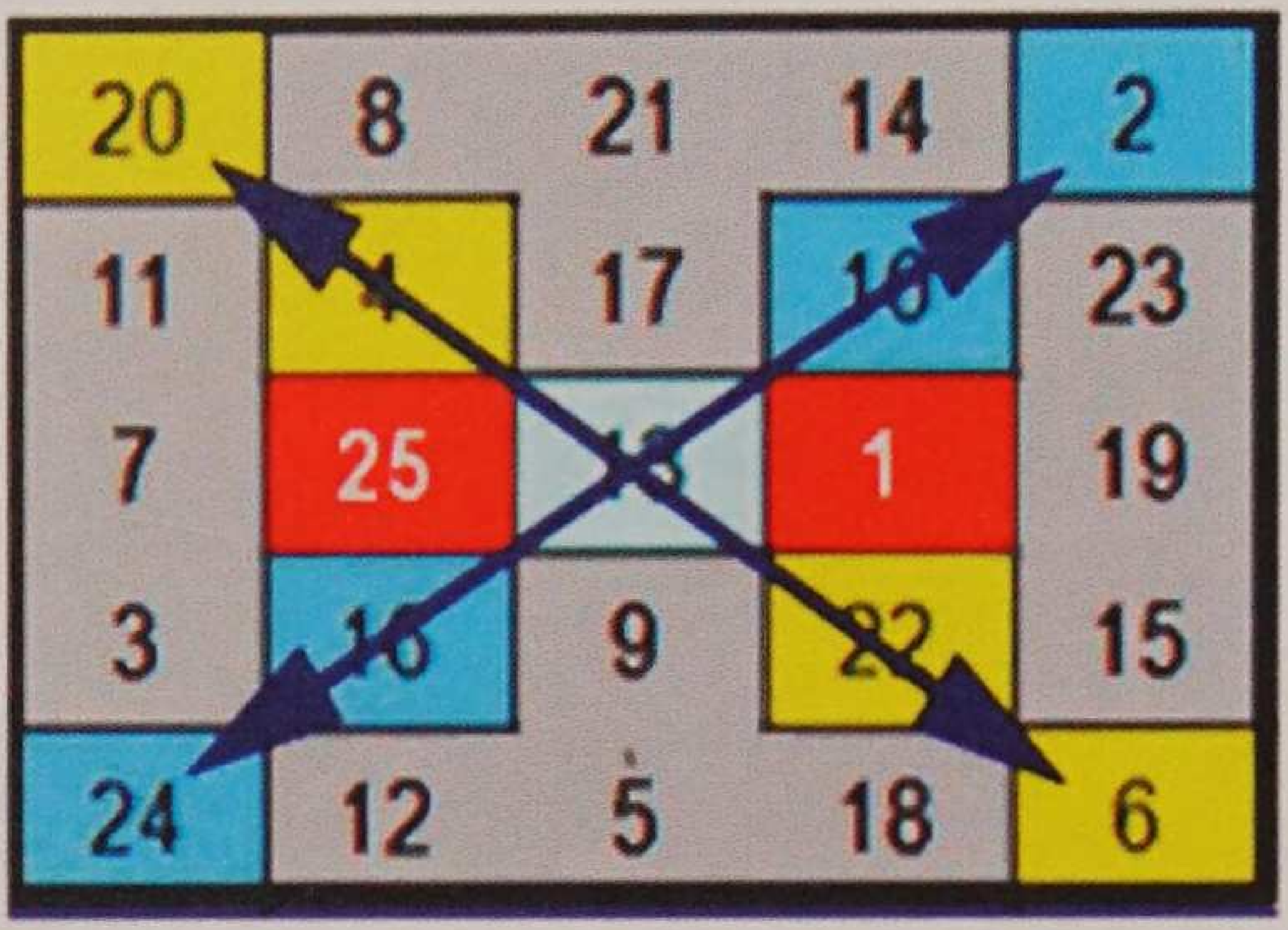
What this pattern indicates is that there are quantized patterns in the distribution of planetary orbits. Only time will tell if this is a common stellar quantization of space as full planetary systems of other stars are discovered and measured. Perhaps this numeric pattern will assist in discovering those planets around nearby stars.

Incidentally, this is the first time that this particular sequence was derived using Jupiter's orbit as the unit of measure for correlating all the planetary orbits. And it yielded two stunning intelligent patterns, one on each side of Jupiter's orbit! – And yet a third one when they were both aligned!

Program 1

2-Dimensional Geonometry

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25



20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

Here we begin the exploration into a new territory of mathematics never before seen beyond a few trivial and limited examples.

Sure, magic squares have been around for at least half a century, ever since the time of Benjamin Franklin, but never beyond mere imperfect small-size squares. Actually, I’m amazed that Ben Franklin was able to get as far as he did on the size-8 and size-16 squares without a digital computer. His versions of those sizes are not what herein will be called “**perfect**”.

Let’s get started.

Throughout this math, the lowercase letter **n** will always denotes the size of the square. What you see here at left is what is called an **index square of size 5**. It contains the number series **1** up thru **25.** The largest number in a **square** of size **n** will always be $\mathbf{n}^2$.

A number square in this new math is a matrix that has been converted by a rearrangement of the numbers in an index square into a *geonomic square* that has many equal-summing properties. Those squares always contain the numbers from **1** thru $\mathbf{n}^2$ exactly once.

All matrices in Geonometry are integral. That is, they contain only whole numbers; there are no fractions, decimal or imaginary numbers. That aspect makes the subject much more comprehendible than higher classical math. Only in Program 9 will the properties of Geonometry be related to classical math with its fractions and real and imaginary numbers. And these will be seen to readily simplify back to whole numbers.

Forget about classical diagonal measures that involve the square-root function; diagonal measures in Geonometry are simply the sum of the numbers along the diagonals of the squares. These sums are unrelated to any classical geometric measure and independent of the Pythagorean Theorem in classical Geometry.

Program 1

Definitions and Examples

Here is what I define as **intelligent number patterns**:

Numerically, it is any number sequence whose elements can be reduced by one or more algebraic operations to one of the following whole-number sequences in which all elements are equal to one of the following sequences:

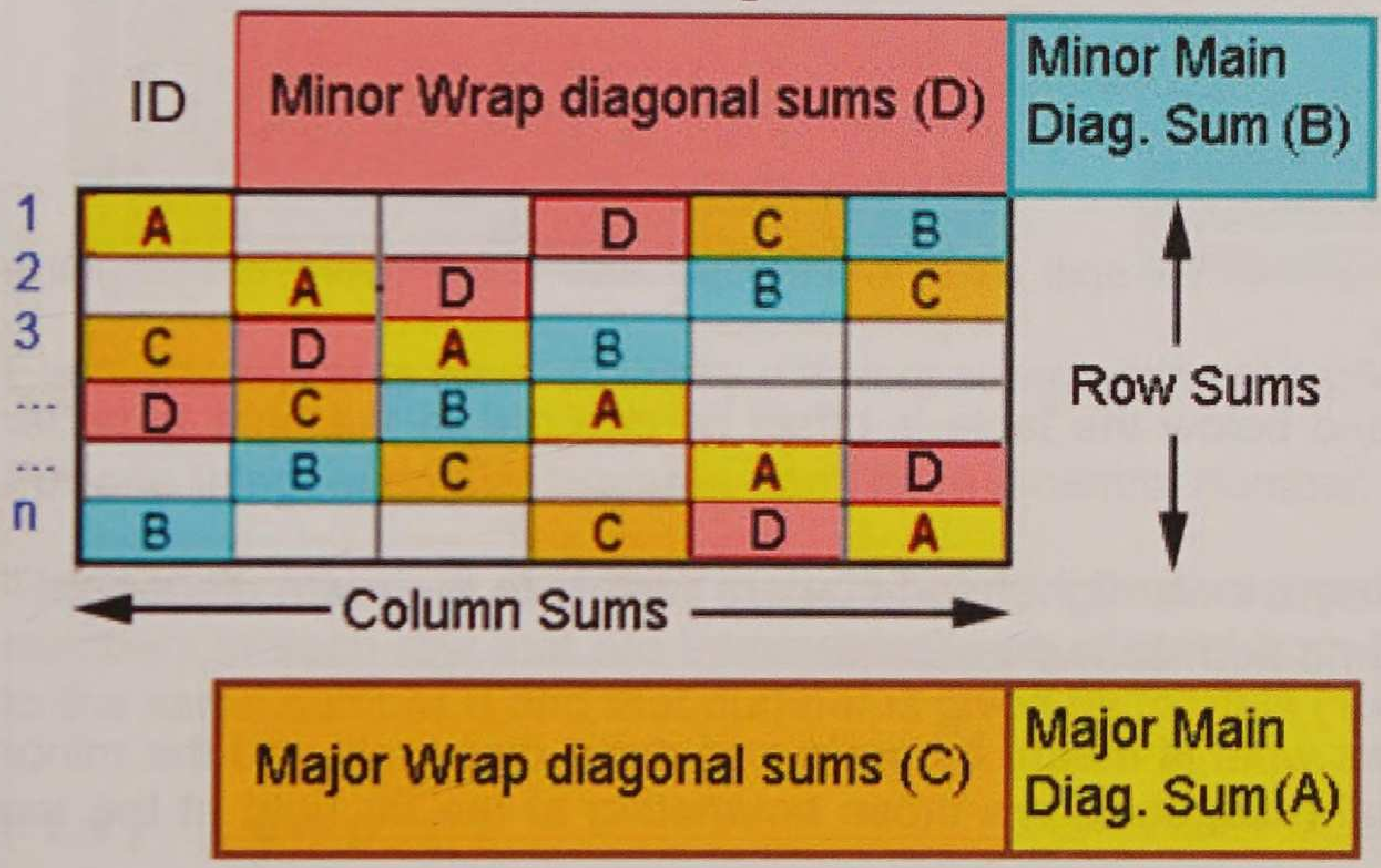
a) a series of all one constant,
b) the sequence of natural numbers; **1, 2, 3, 4, 5, 6 …**
c) the odd-number sequence **1, 3, 5, 7, 9, 11…**
d) the sequence of odd-prime numbers **1, 3, 5, 7, 11, 13, 19, 23, …**

Note that an even consecutive number sequence can be reduced to the natural number sequence merely by dividing all the numbers by **2** or by just subtracting **1**from each number.

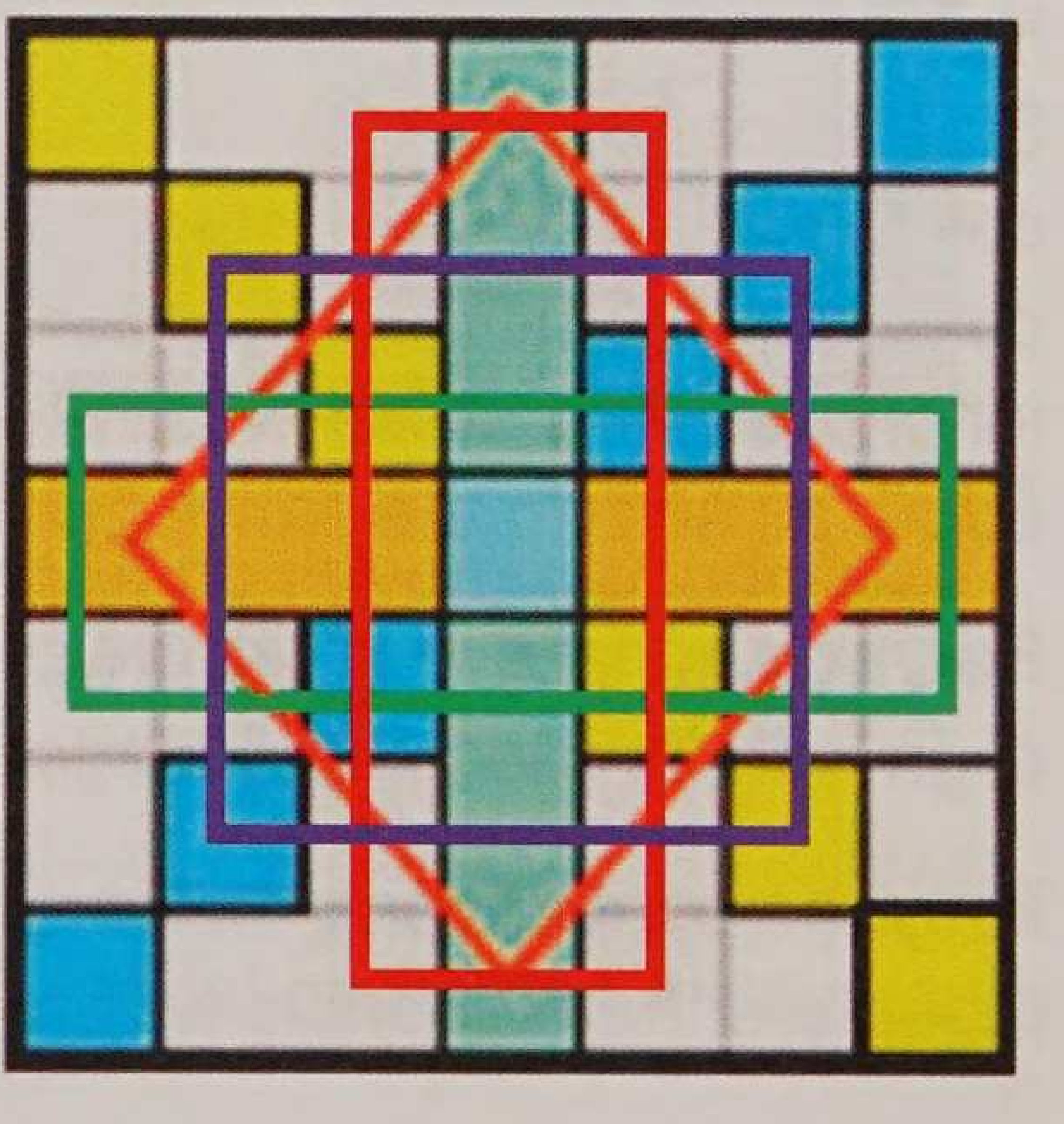
A **harmonic waveform** is a series of consecutive numbers whose values follow a wave pattern:

n, n-1, n-2, ... , 3, 2, 1, 2, 3, ... , n-2, n-1, n, n-1, n-2, ...

Table Legend



ID						
1	A			D	C	B
2		A	D		B	C
3	C	D	A	B		
...	D	C	B	A		
...		B	C		A	D
n	B			C	D	A



Here is the square table's legend. It shows how the summation strings are calculated and where the corresponding sum is located relative to the table.

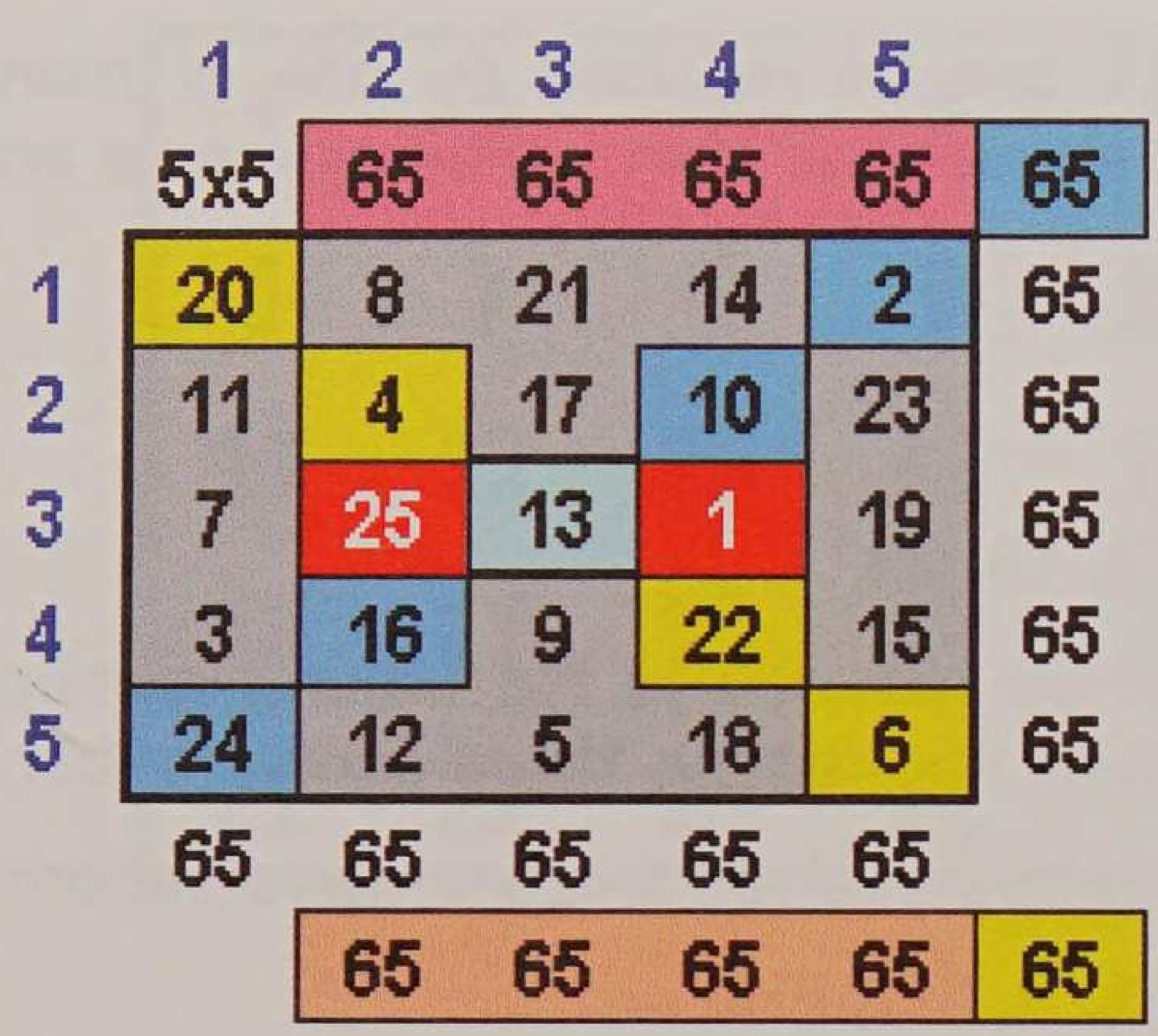
A square table <u>in this format</u> is said to be <u>**geonomic**</u> when the rows, columns and the two main diagonals all sum equally.

The number that they all sum to is dependent upon the table's size and is the same for all geonomic squares of that size. It is called the table's **characteristic number**.

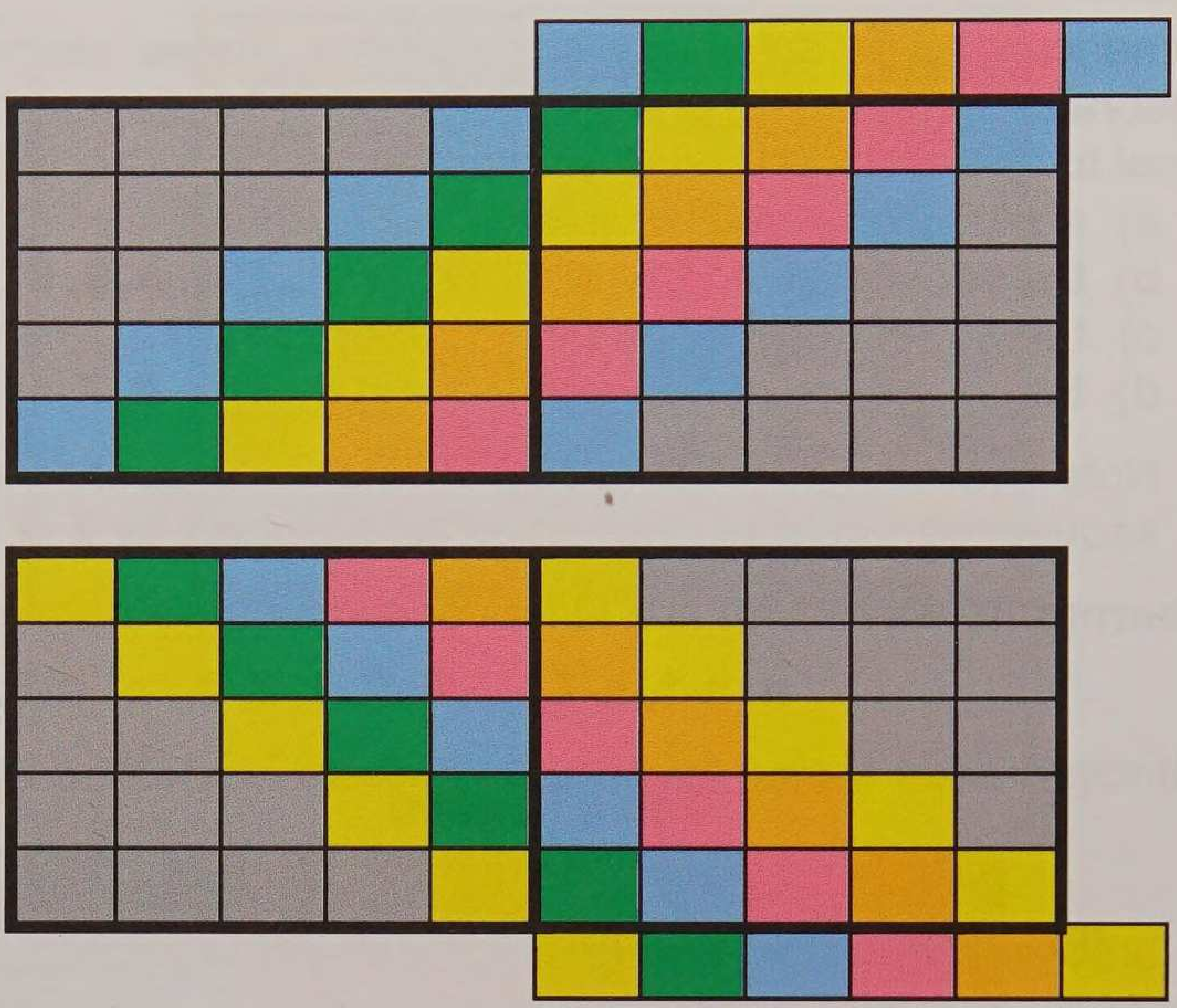
At this point, we introduce the concept of a <u>**quadral**</u>. It is the summation of numbers in the 4 corners of any rectangle, diamond or rhombus centered over the middle of the square. It can be of any size as long as it fits within the square itself. In all squares that are **geonomic**, every quadral sums to the same number -- and that number is called the table's <u>**kernel number**</u>.

Geonomic squares with this property are called <u>**pangenic**</u>. All squares in Geonometry are, by basic construction, pangenic, unless otherwise noted.

Squares



	1	2	3	4	5	
	5x5	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65



Here is a size-5 geonomic square. Size is determined by the number of cells in each row or column. Its rows, columns and diagonals in both directions, including those wrap-diagonals which spill over to the opposite side when reaching the edge, all sum to the same number.

Each diagonal's sum at the top and below the table is offset by one cell to the right to better align with its associated diagonal.

Only the summations at the upper and lower right-hand corners pertain to the **main** diagonals – those are the diagonals which extend from corner-to-corner.

The colored pictorial at right shows what is meant by **main** and **wrap** diagonals and the minor distinction between them. The **main** diagonals are those emanating at the far right at the top and bottom.

Each **wrap** diagonal summation starts somewhere between the corners and continues onto and across an adjacent duplicate table, which is equivalent to wrapping across to the opposite edge of a single table and continuing on from there.

There are two designations for the direction of a diagonal already established in classical math, one is the **major** and the other is the **minor**. The **major** diagonal direction is from top left to lower right. The **minor** diagonal direction is from top right to lower left.

Pangenic squares whose all four directional string summations are all equal to the square's characteristic number are designated as **perfect**. Anything less is **imperfect**.

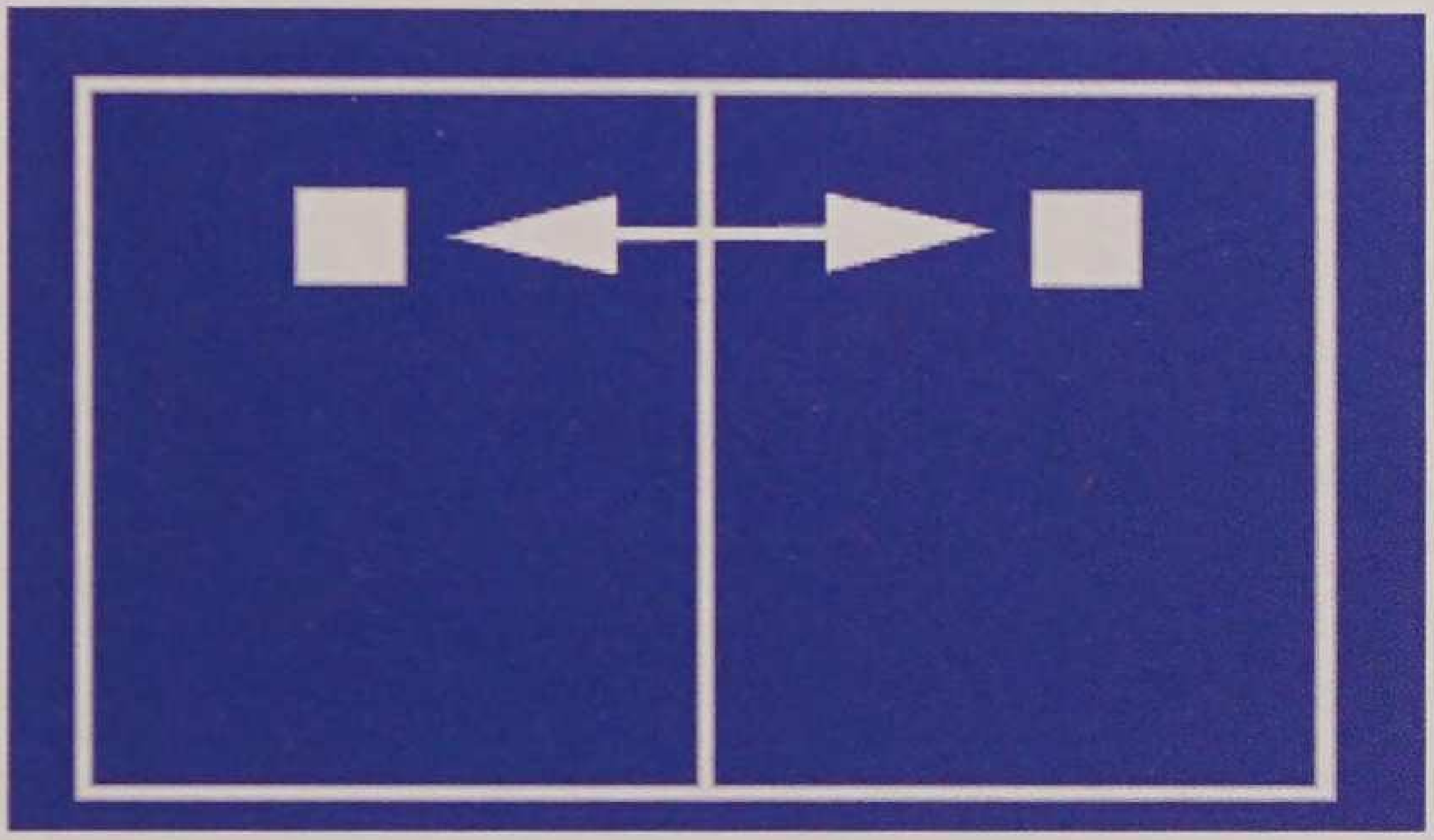
Program 1

Geonomic squares are either an **even-number** size or an **odd-number** size. Odd-size squares have a number that stands alone; it's at the very center of the table. It is called the **pivot number**. In geonomic tables of any dimension, it is always the dimensional average of all the table's numbers. In formulas, it is always designated by the lower-case letter **p**. Its formula is shown below in terms of the size of the matrix **n** and dimension **k**.

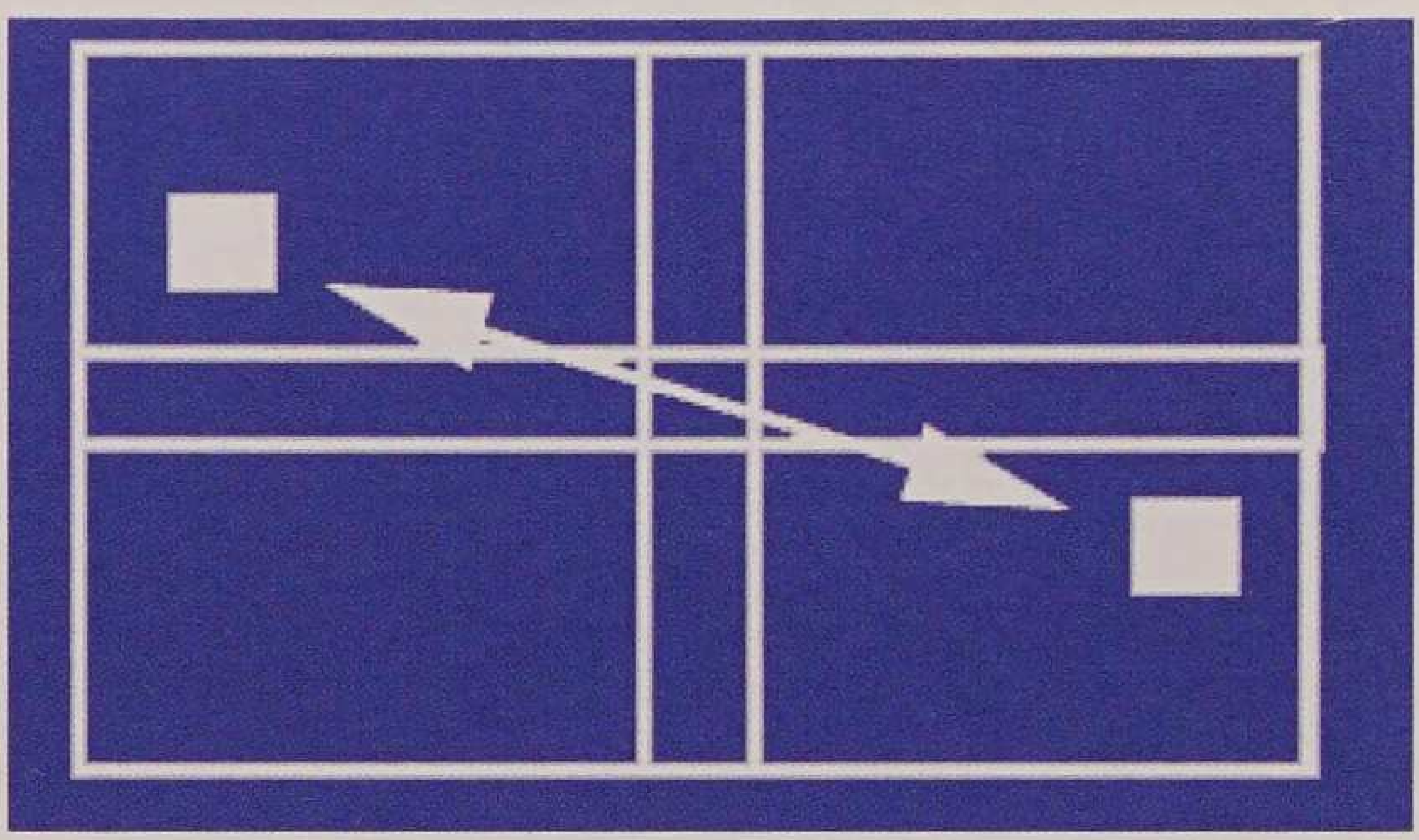
(1.3) $\mathbf{p = (n^k + 1)/2}$

All squares will usually have the background of their beginning and ending numbers highlighted in red.

Even-size squares



Odd-size squares



Even-size squares have distinctly different summation properties than do the ones of odd-size.

For one thing, even-size squares don't have a central number.

The second thing is that they have what is called **row-pair symmetry** in that any pair of numbers in each row that are symmetrically equidistant from the vertical centerline always sum to the same number **d** and that number is given by formula (**1.4**):

(1.4) $\mathbf{d = n^k + 1}$

This pairwise pattern differs from odd-size squares which have **centrally-symmetric** pairs in that every pair of numbers equidistant and exactly opposite from the pivot number sum equally to the same number **d**. In both cases, however, their quadrals, which include two pairs equidistant from the square's center, always sum to the square's <u>kernel number</u> regardless of whether the size of square is odd or even. Consequently, whether of even- or odd-size, these squares are **pangenic**.

There are some minor and rare exceptions to this general pairwise symmetry where pair-sums equal **d ± 1** and those will be pointed out when they are encountered.

Program 1

Categorization of Squares by Class

Class 1	Class 2	Class 3	Class 4	Class 5	Class 6
n = b b is a prime number	n = 2b b is an odd number	n = 3b b is an odd number	n = 4b b is any integer	n = ab a & b both odd a > b > 3	$n = b^2$ b is an odd number
5	6 = 2x3	15 = 3x5	4 = 4x1	35 = 5x7	$9 = 3^2$
7	10 = 2x5	21 = 3x7	8 = 4x2	45 = 5x9	$25 = 5^2$
11	14 = 2x7	27 = 3x9	12 = 4x3	55 = 5x11	$27 = 3^3$
13	18 = 2x9	33 = 3x11	16 = 4x4	65 = 5x13	$49 = 7^2$
17	22 = 2x11	39 = 3x13	20 = 4x5	75 = 5x15	$81 = 9^2$
19	26 = 2x13	45 = 3x 15	24 = 4x6	77 = 7x11	$121 = 11^2$
23	30 = 2x15	51 = 3x 17	28 = 4x7	85 = 5x17	$169 = 13^2$
29	34 = 2x17	57 = 3x 19	32 = 4x8	91 = 7x13	$225 = 15^2$
31	38 = 2x19	63 = 3x 21	36 = 4x9	95 = 5x19	$289 = 17^2$

Geonomic tables of <u>any dimension</u> can be categorized by their common properties into **6** major classes. These are:

Class 1 – **n** equals **b** where **b** is a prime number.

Class 2 – **n** equals **2 b** where **b** is an odd number.

Class 3 – **n** equals **3 b** where **b** is an odd number**.**

Class 4 – **n** equals **4 b** where **b** is any number.

Class 5 – **n** equals a product of two unequal odd numbers **a** & **b**, each greater than **3**.

Class 6 – **n** equals a square or cube of an odd number **b**.

These classes pretty much segregate all the printable sizes of square into mutually exclusive categories. Examples are listed for each category, eventhough those that are greater than size **35** are too large for display in print. Those are listed here in blue.

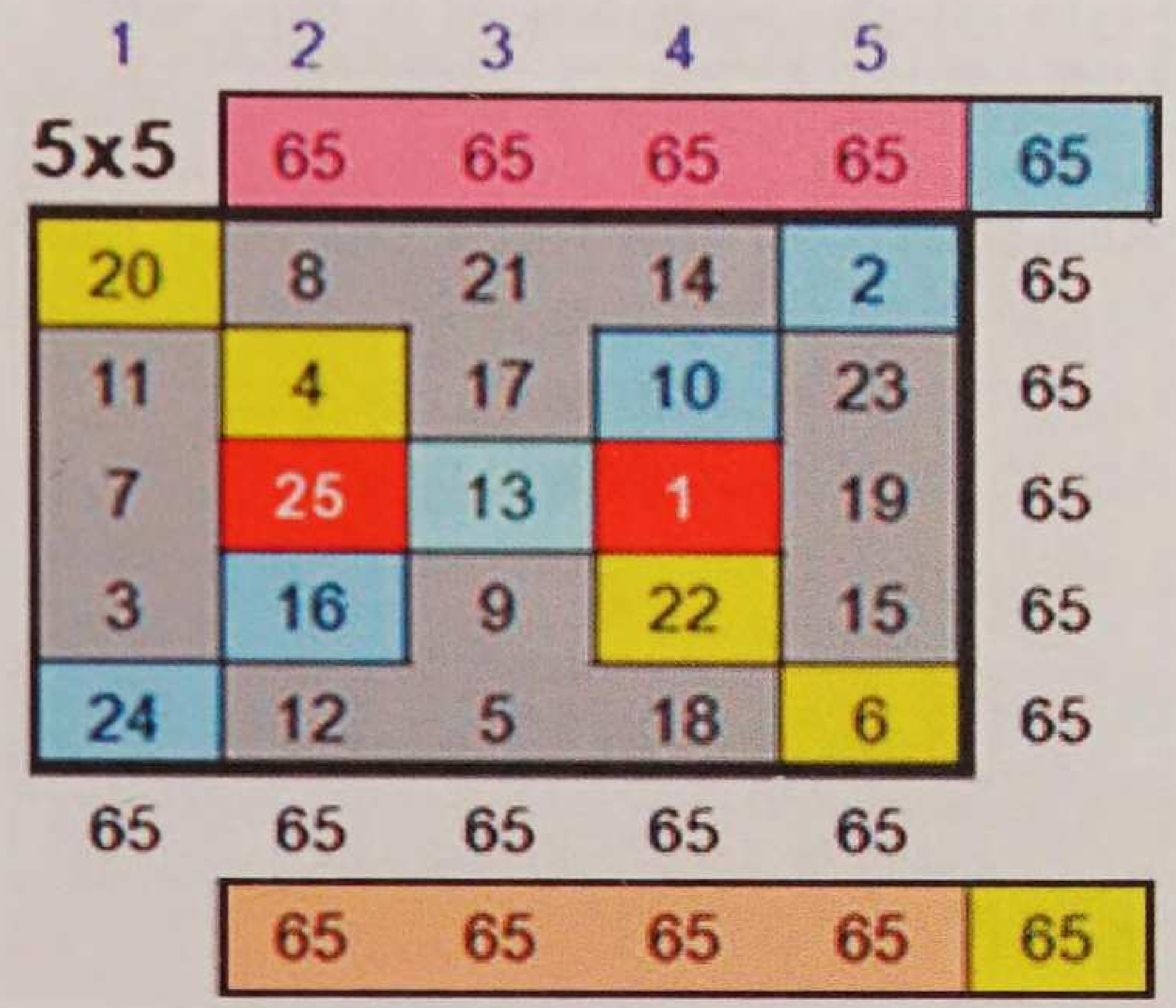
Note that the size **27** appears in both Classes 3 & 6. Why that is so will become evident as we explore the properties of the various classes.

Not all printable sizes in each class will be shown as they are all depicted in the book, **Number Magic**, which is referenced throughout this program series.

Program 1

Class 1 Squares

Prime number Size 5



	1	2	3	4	5	
5x5		65	65	65	65	65
	20	8	21	14	2	65
	11	4	17	10	23	65
	7	25	13	1	19	65
	3	16	9	22	15	65
	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

Centrally Symmetric pairs sum to 26.

We have already seen the size-5 square. Its characteristic number is **65**. All six of its quadrals sum to **52**, its kernel number, which is **4/5** of its characteristic number. Every centrally-symmetric pair of numbers sums to **26**, which is twice the pivot number, **13**. These aspects are common among all Class-1 squares.

Here is the Class-1 size-7 square. It too is pairwise centrally symmetric with summations equal to **50 = 1 + 49.** All of its quadrals sum to **100, its kernel number.** Its pivot number is **25 =** ¼ its kernel number.

Size-7

		1	2	3	4	5	6	7	
	7x7		175	175	175	175	175	175	175
1		7	18	29	47	9	27	38	175
2		44	13	24	42	4	15	33	175
3		39	1	19	30	48	10	28	175
4		34	45	14	25	36	5	16	175
5		22	40	2	20	31	49	11	175
6		17	35	46	8	26	37	6	175
7		12	23	41	3	21	32	43	175
		175	175	175	175	175	175	175	
			175	175	175	175	175	175	175

Class 2 Squares

Size-6 Square

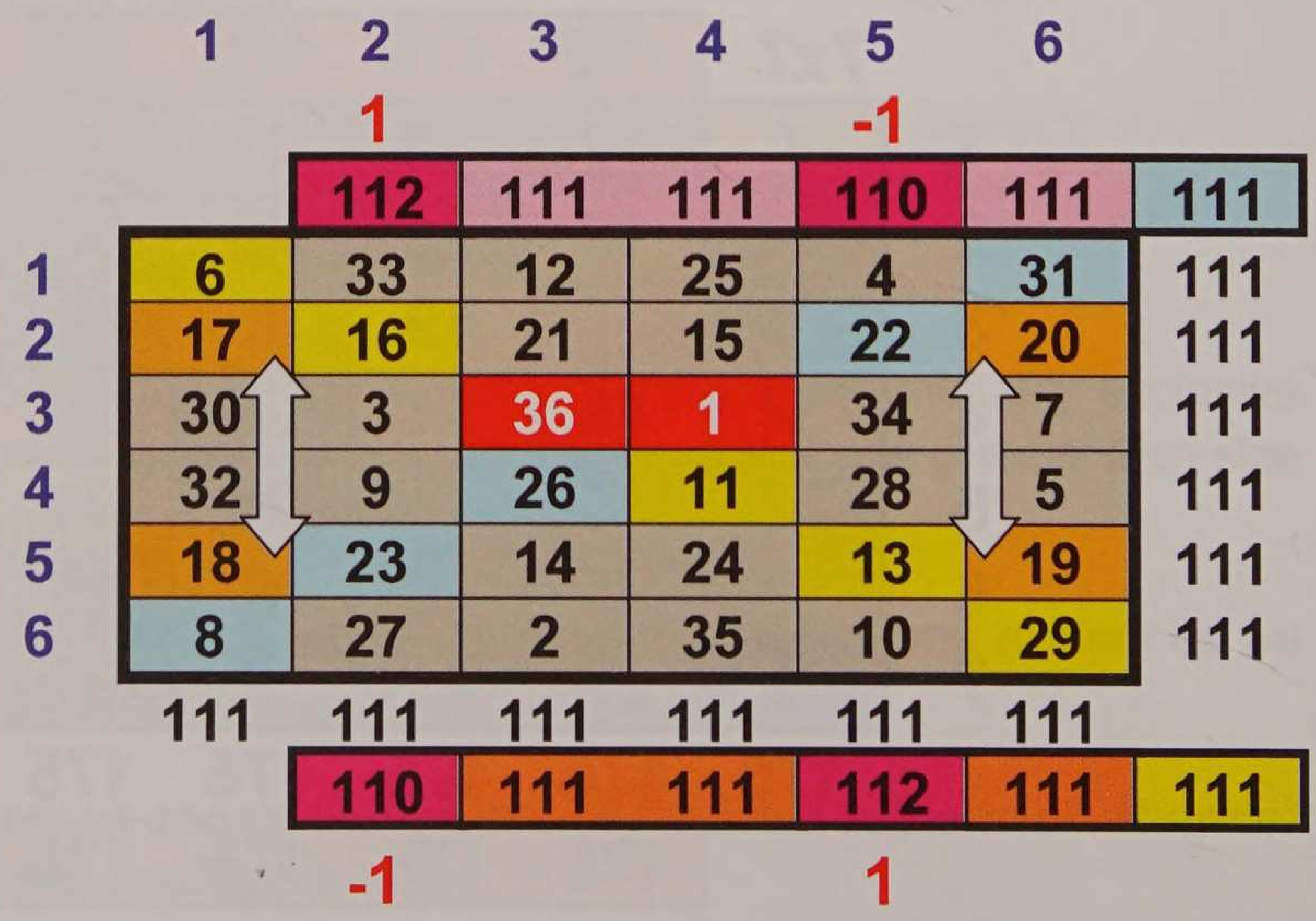
Here is a square of size-6. It is a Class-2 square. It is only what will be called **near-perfect**. It has two symmetrically-located rows which differ from the size-6 characteristic number by **±1**.

It also has row-pairs which sum to **37** for the most part but there are a few exceptions. This minor anomaly arises because the size-6 square is **embryonic**, that is, its series of numbers is not sufficient to produce all the properties normally ascribed to its class. Yet all of its quadrals are still equal to **74**, which is **4/6** of its characteristic number. So it is pangenic.

	1	2	3	4	5	6		
		111	111	111	111	111	111	
1	6	33	12	25	4	31	111	
2	17	16	21	15	22	19	110	–1
3	30	3	36	1	34	7	111	
4	32	9	26	11	28	5	111	
5	18	23	14	24	13	20	112	+1
6	8	27	2	35	10	29	111	
	111	111	111	111	111	111		
		111	111	111	111	111	111	

The near perfect size-6 square can be converted into what will be called a **punctuated-perfect** square by interchanging just two numbers in the same column, here **19** and **20**, or **17** and **18**. It still remains pangenic. But the equality then brought to its rows by this interchange of numbers is counter-productive because it introduces four unequal diagonals with the same variances as the formerly unequal rows, shaded in crimson.

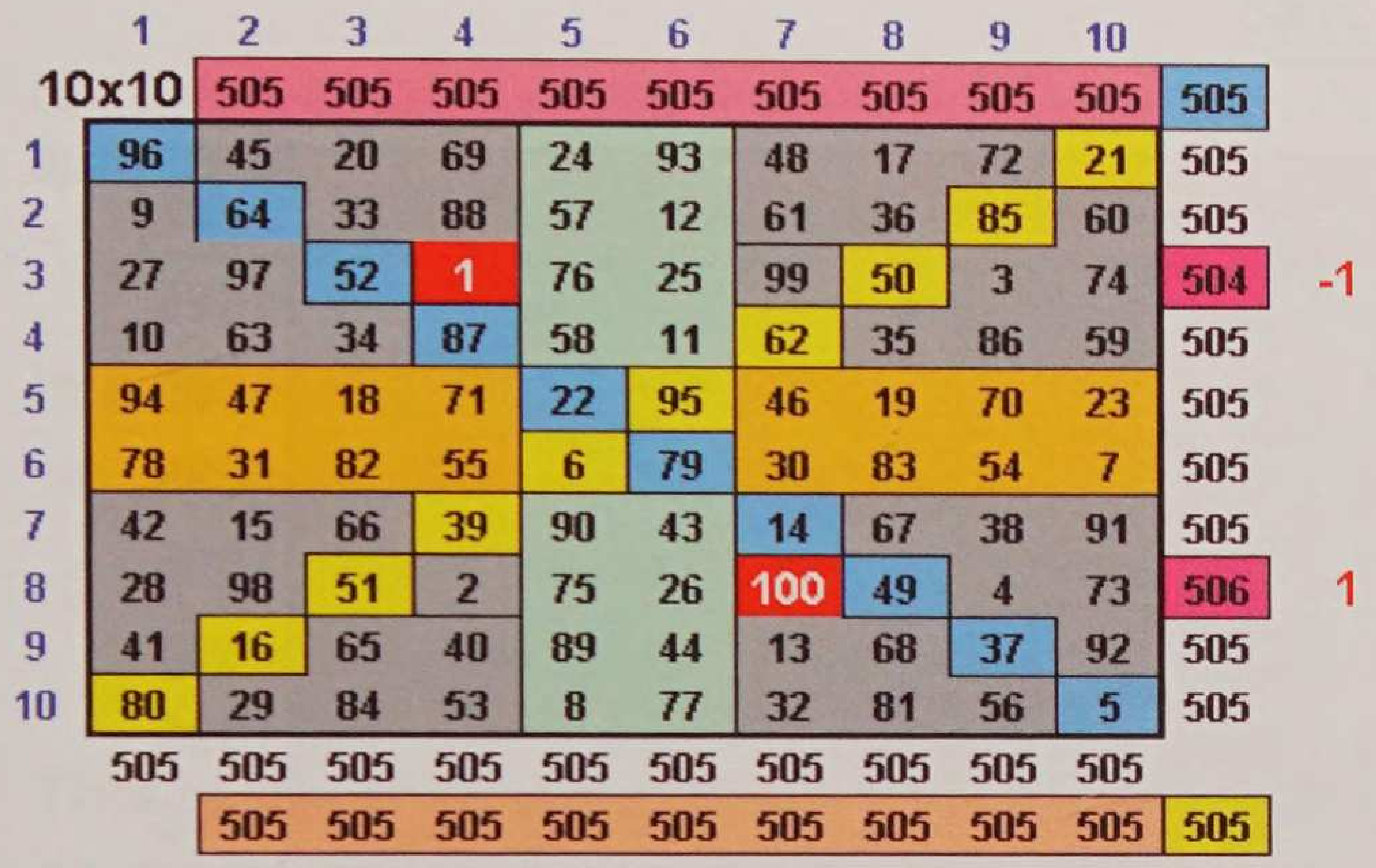
Swapping the numbers **22** and **23** in the minor main-diagonal will also correct the two unequal rows but that will result in two unequal columns. There is no way to iron out this wrinkle for Class-2 squares. It is a fundamental wrinkle in the spatial fabric.



	1	2	3	4	5	6	
		1			-1		
		112	111	111	110	111	111
1	6	33	12	25	4	31	111
2	17	16	21	15	22	20	111
3	30	3	36	1	34	7	111
4	32	9	26	11	28	5	111
5	18	23	14	24	13	19	111
6	8	27	2	35	10	29	111
	111	111	111	111	111	111	
		110	111	111	112	111	111
		-1			1		

Program 1

Size-10 Square



	1	2	3	4	5	6	7	8	9	10		
10x10		505	505	505	505	505	505	505	505	505	505	
1	96	45	20	69	24	93	48	17	72	21	505	
2	9	64	33	88	57	12	61	36	85	60	505	
3	27	97	52	1	76	25	99	50	3	74	504	-1
4	10	63	34	87	58	11	62	35	86	59	505	
5	94	47	18	71	22	95	46	19	70	23	505	
6	78	31	82	55	6	79	30	83	54	7	505	
7	42	15	66	39	90	43	14	67	38	91	505	
8	28	98	51	2	75	26	100	49	4	73	506	1
9	41	16	65	40	89	44	13	68	37	92	505	
10	80	29	84	53	8	77	32	81	56	5	505	
	505	505	505	505	505	505	505	505	505	505		
		505	505	505	505	505	505	505	505	505	505	

Y(25)	1	2	3	4	5	6	7	8	9	10		
		25	25	25	25	25	25	25	25	25	25	
1	4	1	4	1	4	1	4	1	4	1	25	
2	1	4	1	4	1	4	1	4	1	4	25	
3	3	2	3	2	3	1	4	1	4	1	24	-1
4	2	3	2	3	2	3	2	3	2	3	25	
5	2	3	2	3	2	3	2	3	2	3	25	
6	2	3	2	3	2	3	2	3	2	3	25	
7	2	3	2	3	2	3	2	3	2	3	25	
8	4	1	4	1	4	2	3	2	3	2	26	+1
9	1	4	1	4	1	4	1	4	1	4	25	
10	4	1	4	1	4	1	4	1	4	1	25	
	25	25	25	25	25	25	25	25	25	25		
		25	25	25	25	25	25	25	25	25	25	

The integer function at right taken to the base **25** on the numbers in the Class-2 size-10 square shows the source of the spatial wrinkle that makes all Class-2 squares only near-perfect. (More about the *integer function* in Program 2).

The wrinkle stems from the impossibility of an even numbered checkered pattern being continued in an odd-numbered string. Compare odd-count patterns in the unequal rows to the even-count pattern in the other rows. Any modification to the location of numbers will result in unequal diagonals.

This is a basic corruptive property of all Class-2 squares. It is a fundamental wrinkle in the basic fabric of space.

Nonetheless, Class-2 squares play a very important role in the manifestation of the squares in Class-4 and in even-size tables of all higher-dimensions.

Class 3 Squares

Odd-size perfect squares, Size = 3b, odd-number b > 3

Size-15

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	33	71	14	52	95	108	146	89	127	170	183	221	164	202	1695
2	86	104	142	85	148	11	29	67	10	73	161	179	217	160	223	1695
3	157	200	213	151	219	82	125	138	76	144	7	50	63	1	69	1695
4	3	41	59	22	65	153	191	209	172	215	78	116	134	97	140	1695
5	174	187	205	168	206	99	112	130	93	131	24	37	55	18	56	1695
6	45	8	46	64	27	120	83	121	139	102	195	158	196	214	177	1695
7	111	79	117	135	123	36	4	42	60	48	186	154	192	210	198	1695
8	182	175	188	201	194	107	100	113	126	119	32	25	38	51	44	1695
9	28	16	34	72	40	178	166	184	222	190	103	91	109	147	115	1695
10	199	162	180	218	181	124	87	105	143	106	49	12	30	68	31	1695
11	70	58	21	39	2	145	133	96	114	77	220	208	171	189	152	1695
12	136	129	92	110	98	61	54	17	35	23	211	204	167	185	173	1695
13	207	225	163	176	169	132	150	88	101	94	57	75	13	26	19	1695
14	53	66	9	47	15	203	216	159	197	165	128	141	84	122	90	1695
15	224	212	155	193	156	149	137	80	118	81	74	62	5	43	6	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
		1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

Every quadral sums to **452.**
452 = 4 x 113 = **4/15** x 1695

Here is the smallest Class-3 square, a size **15**. It is geonomically perfect. It has all equal **centrally-symmetric** pairwise sums. All of its quadrals sum to **4/15** of its characteristic number.

Note the pattern of quadral sums seen so far: they always equal **4** times the characteristic number divided by the square's size.

The size of Class-3 squares is always **3-times** an <u>odd number</u> **b>3**. They are all continuously-modular at the **bxb** block-square level. These block-squares will always sum to **b/3** of the square's characteristic number.

The size-15 square has **5x5** block-squares that sum to **5/3** of its characteristic number anywhere in the table, even wrapping over the edges. This is called, **continuous modularity**.

If the size of the square is **3** times an odd number **b**, it will be **continuously bxb modular** and each size-**b** block-square will sum to **b-thirds** of the characteristic number. The square will just be called "**bx-modular**" for short.

Program 1

Class 4 Squares

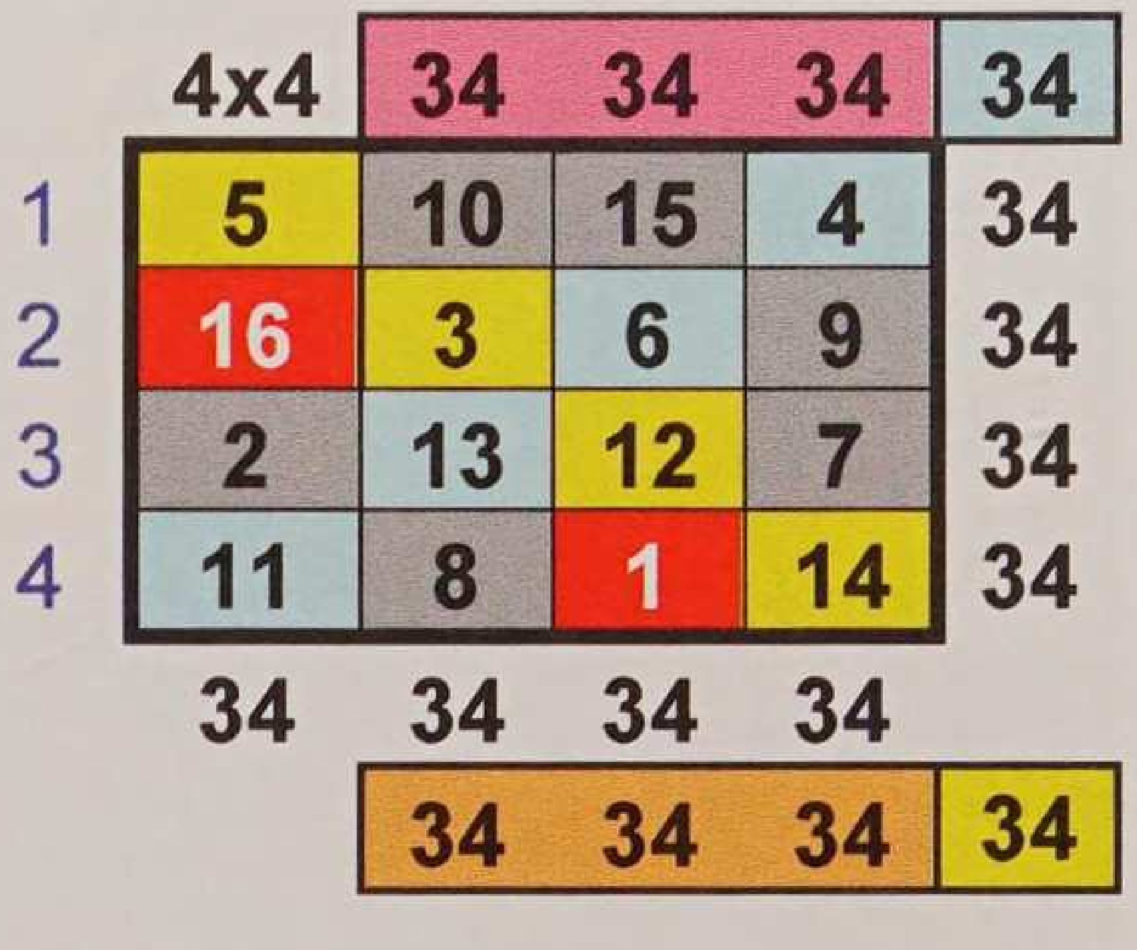
Size-4

Here is the size-4 Class-4 square. It is perfect. Its characteristic number is **34**. All of its quadrals sum to **34**.

In addition to being perfect:

1. All of its **2x2** block-squares sum to **34**.
2. It is continuously **2x**-modular. This **2x2** pattern sums to **34** even when wrapping onto the opposite edge.

This is the only size square which has its kernel number, its modularity, and its characteristic number all equal.



	4x4	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
		34	34	34	34

Size-8

	1	2	3	4	5	6	7	8	
		260	260	260	260	260	260	260	260
1	24	9	8	25	40	57	56	41	260
2	58	39	42	55	10	23	26	7	260
3	14	19	30	3	62	35	46	51	260
4	36	61	52	45	20	13	4	29	260
5	17	16	1	32	33	64	49	48	260
6	63	34	47	50	15	18	31	2	260
7	11	22	27	6	59	38	43	54	260
8	37	60	53	44	21	12	5	28	260
	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260

Quadrals

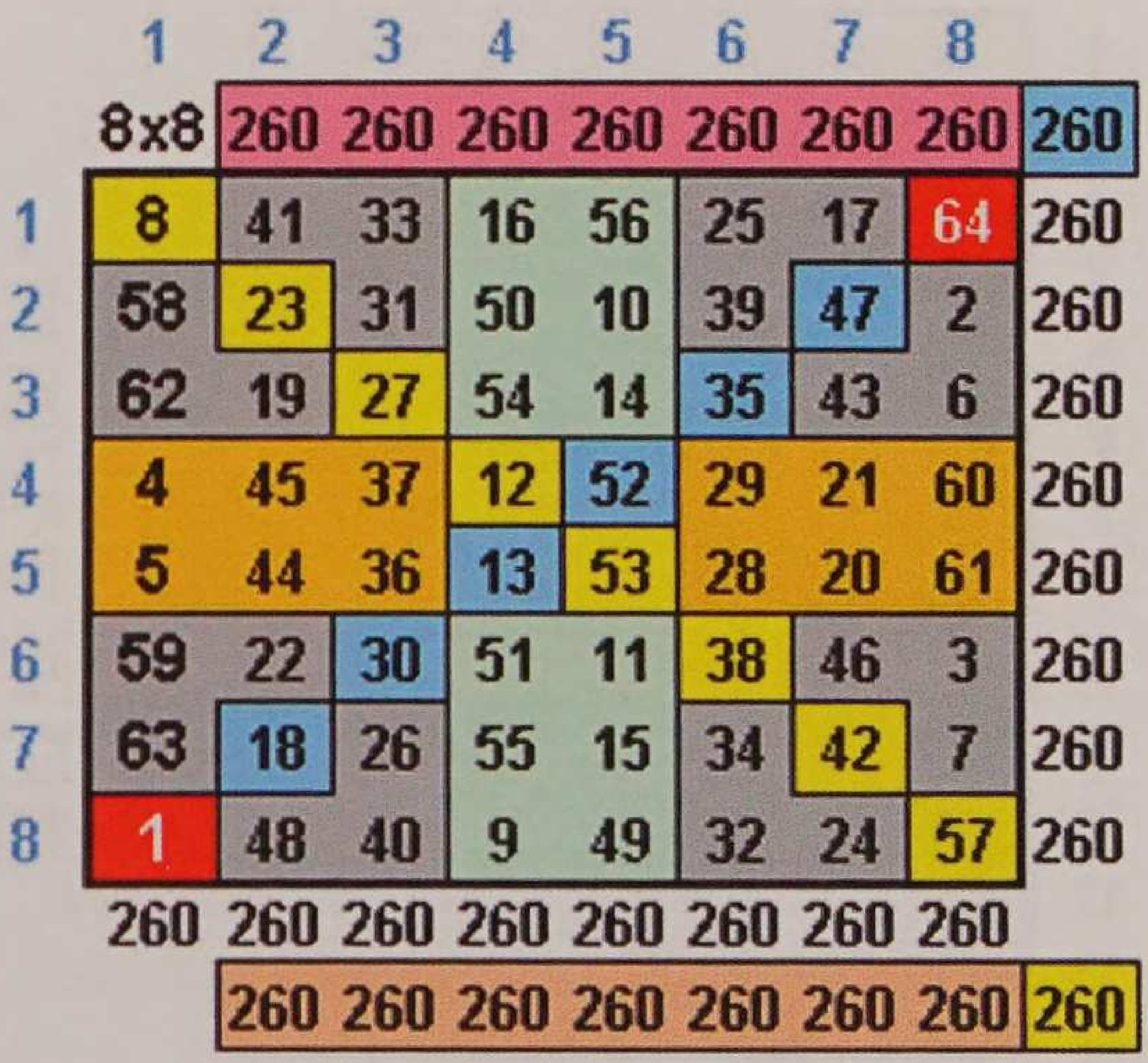
130	130	130	130
130	130	130	130
130	130	130	130
130	130	130	130

Here is a size **8** square. It is perfect; it has equal-summing symmetric row-pairs which add to **65**.

Class-4 squares have a special subcategory -- All squares whose size is **8** or an even multiple of **8** also have both versions of equal-summing pairs.

Their distribution of numbers in each square is different between each of these two versions. Both sizes, **8** and **16**, have both versions of equal summing pairs.

At right is a version of size-8 that is centrally pairwise symmetric. All centrally-symmetric pairs sum to **65**.



	1	2	3	4	5	6	7	8	
	8x8	260	260	260	260	260	260	260	260
1	8	41	33	16	56	25	17	64	260
2	58	23	31	50	10	39	47	2	260
3	62	19	27	54	14	35	43	6	260
4	4	45	37	12	52	29	21	60	260
5	5	44	36	13	53	28	20	61	260
6	59	22	30	51	11	38	46	3	260
7	63	18	26	55	15	34	42	7	260
8	1	48	40	9	49	32	24	57	260
	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260

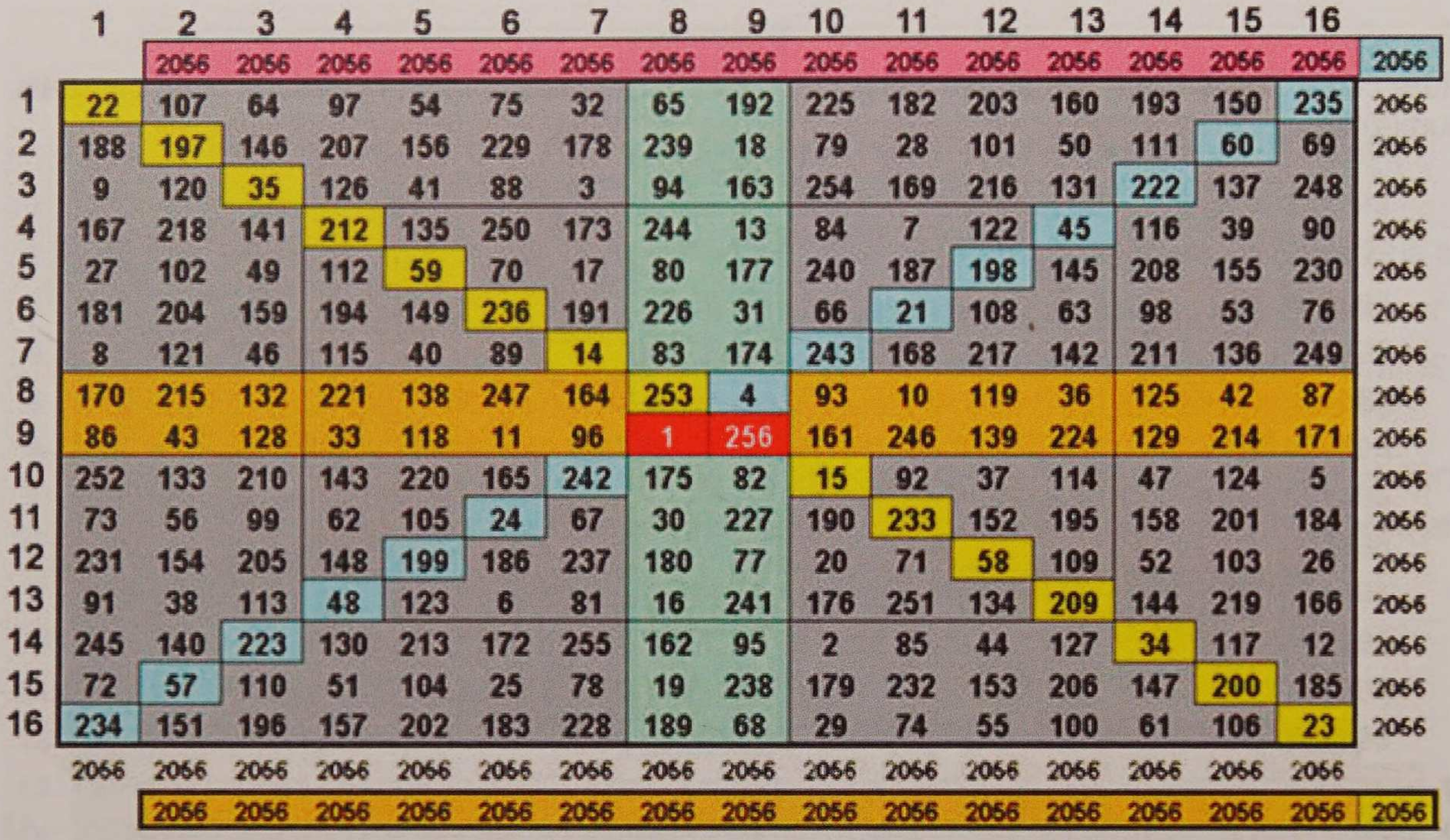
Quadrals

130	130	130	130
130	130	130	130
130	130	130	130
130	130	130	130

Program 1

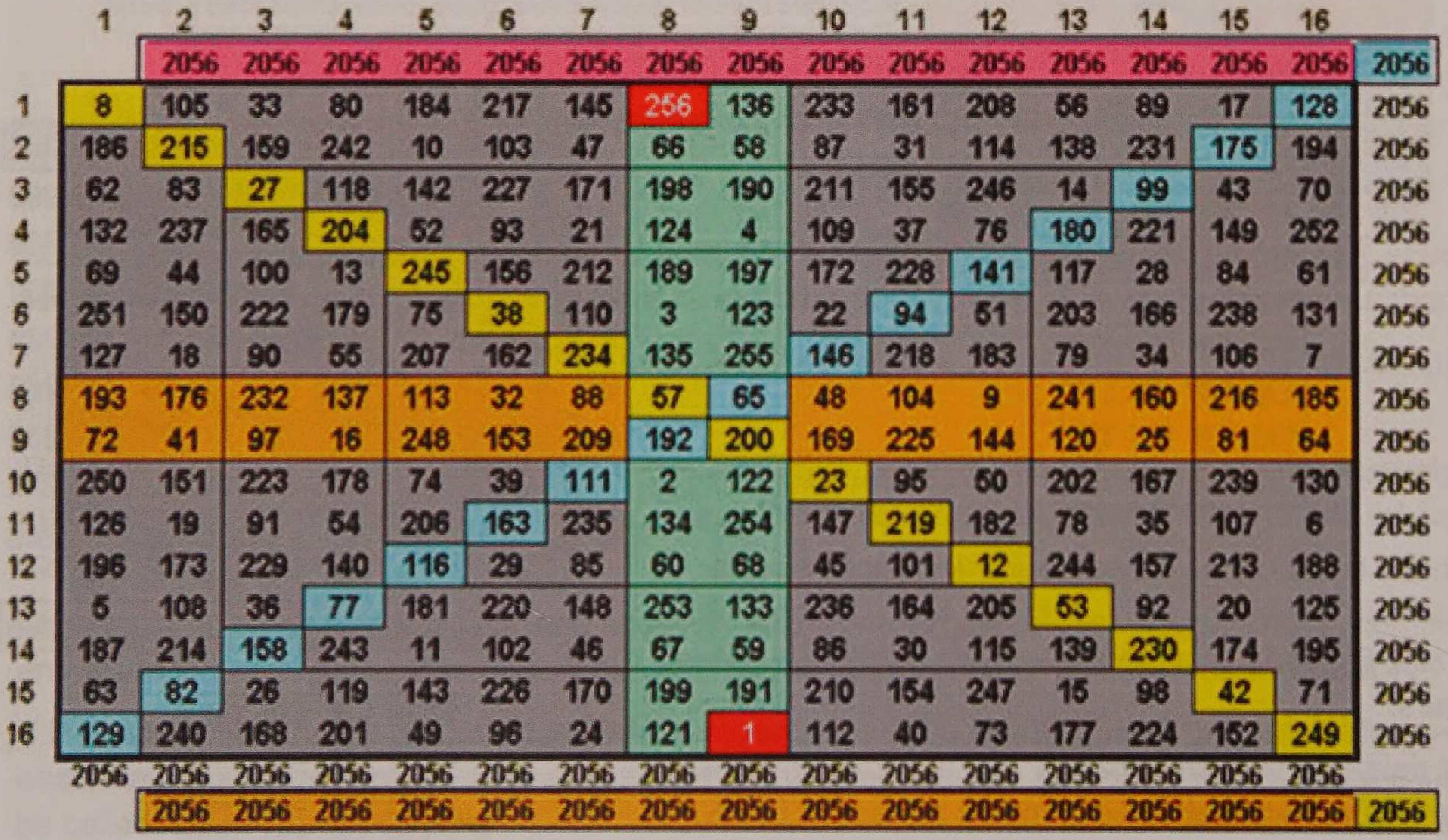
Size-16 Squares

Here is the Class-4 square of size **16** with equal-summing symmetric row-pairs.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	22	107	64	97	54	75	32	65	192	225	182	203	160	193	150	235	2056
2	188	197	146	207	156	229	178	239	18	79	28	101	50	111	60	69	2056
3	9	120	35	126	41	88	3	94	163	254	169	216	131	222	137	248	2056
4	167	218	141	212	135	250	173	244	13	84	7	122	45	116	39	90	2056
5	27	102	49	112	59	70	17	80	177	240	187	198	145	208	155	230	2056
6	181	204	159	194	149	236	191	226	31	66	21	108	63	98	53	76	2056
7	8	121	46	115	40	89	14	83	174	243	168	217	142	211	136	249	2056
8	170	215	132	221	138	247	164	253	4	93	10	119	36	125	42	87	2056
9	86	43	128	33	118	11	96	1	256	161	246	139	224	129	214	171	2056
10	252	133	210	143	220	165	242	175	82	15	92	37	114	47	124	5	2056
11	73	56	99	62	105	24	67	30	227	190	233	152	195	158	201	184	2056
12	231	154	205	148	199	186	237	180	77	20	71	58	109	52	103	26	2056
13	91	38	113	48	123	6	81	16	241	176	251	134	209	144	219	166	2056
14	245	140	223	130	213	172	255	162	95	2	85	44	127	34	117	12	2056
15	72	57	110	51	104	25	78	19	238	179	232	153	206	147	200	185	2056
16	234	151	196	157	202	183	228	189	68	29	74	55	100	61	106	23	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Here is the Class-4 square of size-16 which has equal-summing centrally-symmetric pairs.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

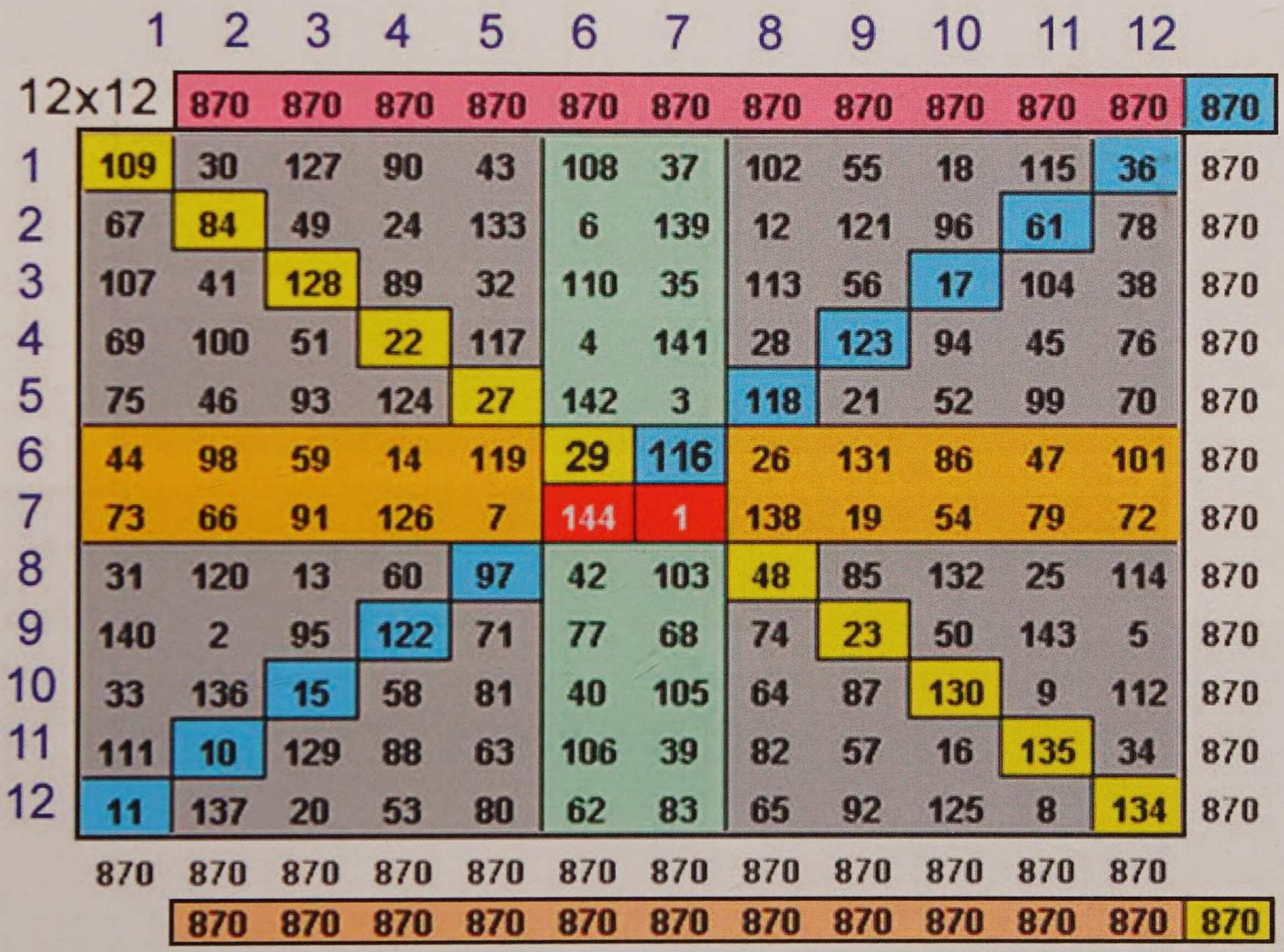
Program 1

Size-12 Square

Here is Class-4 square of size-12. It has all symmetric row-pair sums equal to **145**. So clearly, its quadrals all sum to twice that, viz. **290**. It is perfect.

It is also continuously 6x-modular at 3-times the characteristic number.

Class-4 squares, for the most part, have all equal symmetric row-pair sums. However, in Program 8 we will encounter some versions of Class-4 squares, including the size-12 square, which have no equal-summing pair-wise symmetry at all, yet has another more important property.



	1	2	3	4	5	6	7	8	9	10	11	12	
12x12	870	870	870	870	870	870	870	870	870	870	870	870	870
1	109	30	127	90	43	108	37	102	55	18	115	36	870
2	67	84	49	24	133	6	139	12	121	96	61	78	870
3	107	41	128	89	32	110	35	113	56	17	104	38	870
4	69	100	51	22	117	4	141	28	123	94	45	76	870
5	75	46	93	124	27	142	3	118	21	52	99	70	870
6	44	98	59	14	119	29	116	26	131	86	47	101	870
7	73	66	91	126	7	144	1	138	19	54	79	72	870
8	31	120	13	60	97	42	103	48	85	132	25	114	870
9	140	2	95	122	71	77	68	74	23	50	143	5	870
10	33	136	15	58	81	40	105	64	87	130	9	112	870
11	111	10	129	88	63	106	39	82	57	16	135	34	870
12	11	137	20	53	80	62	83	65	92	125	8	134	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

Characteristic Circles In Class-4 Squares

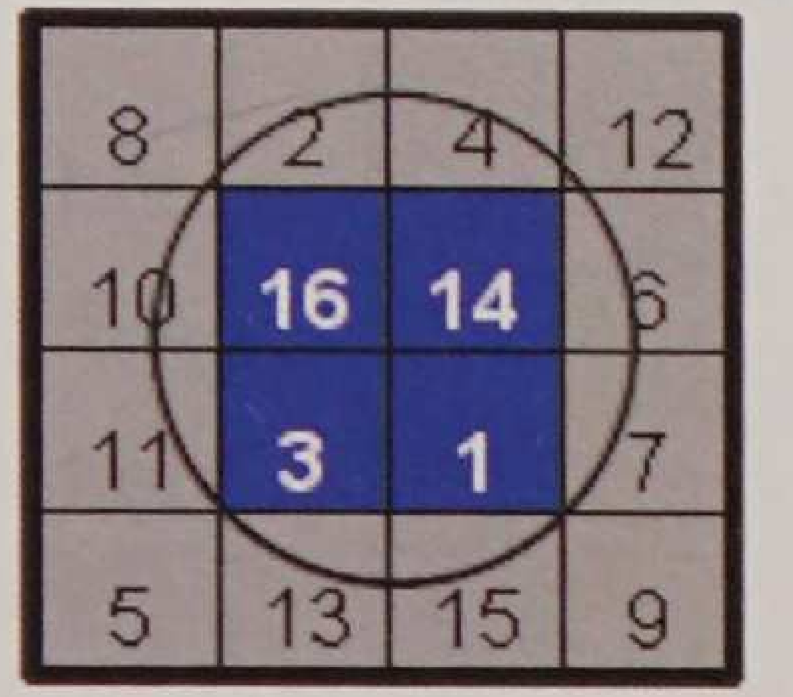
Computing which size squares have Characteristic circles			
Size	Characteristic	Quadral	Quadral multiple =
Square	number	sums	characteristic no.
3	15	20	0.75
4	34	34	1
5	65	52	1.25
6	111	74	1.5
7	175	100	1.75
8	260	130	2
9	369	164	2.25
10	505	202	2.5
11	671	244	2.75
12	870	290	3
13	1105	340	3.25
14	1379	394	3.5
15	1695	452	3.75
16	2056	514	4
17	2465	580	4.25
18	2925	650	4.5
19	3439	724	4.75
20	4010	802	5
21	4641	884	5.25
22	5335	970	5.5
23	6095	1060	5.75
24	6924	1154	6
25	7825	1252	6.25
26	8801	1354	6.5
27	9855	1460	6.75
28	10990	1570	7
29	12209	1684	7.25
30	13515	1802	7.5
31	14911	1924	7.75
32	16400	2050	8

The table shows that only the squares whose size is a multiple of **4** can have a whole-number of quadrals which add up to the square's characteristic number exactly without a fractional remnant. Consequently, we only need to address those squares with exact multiples for characteristic circles and identify the participating quadrals. Although the patterns appear elementary for squares here in Program 1, this exploration sets the stage for a fundamental correlation in 3-dimensional space in Program 4 where we examine geonomic cubes in the search for *characteristic spheres*.

Program 1

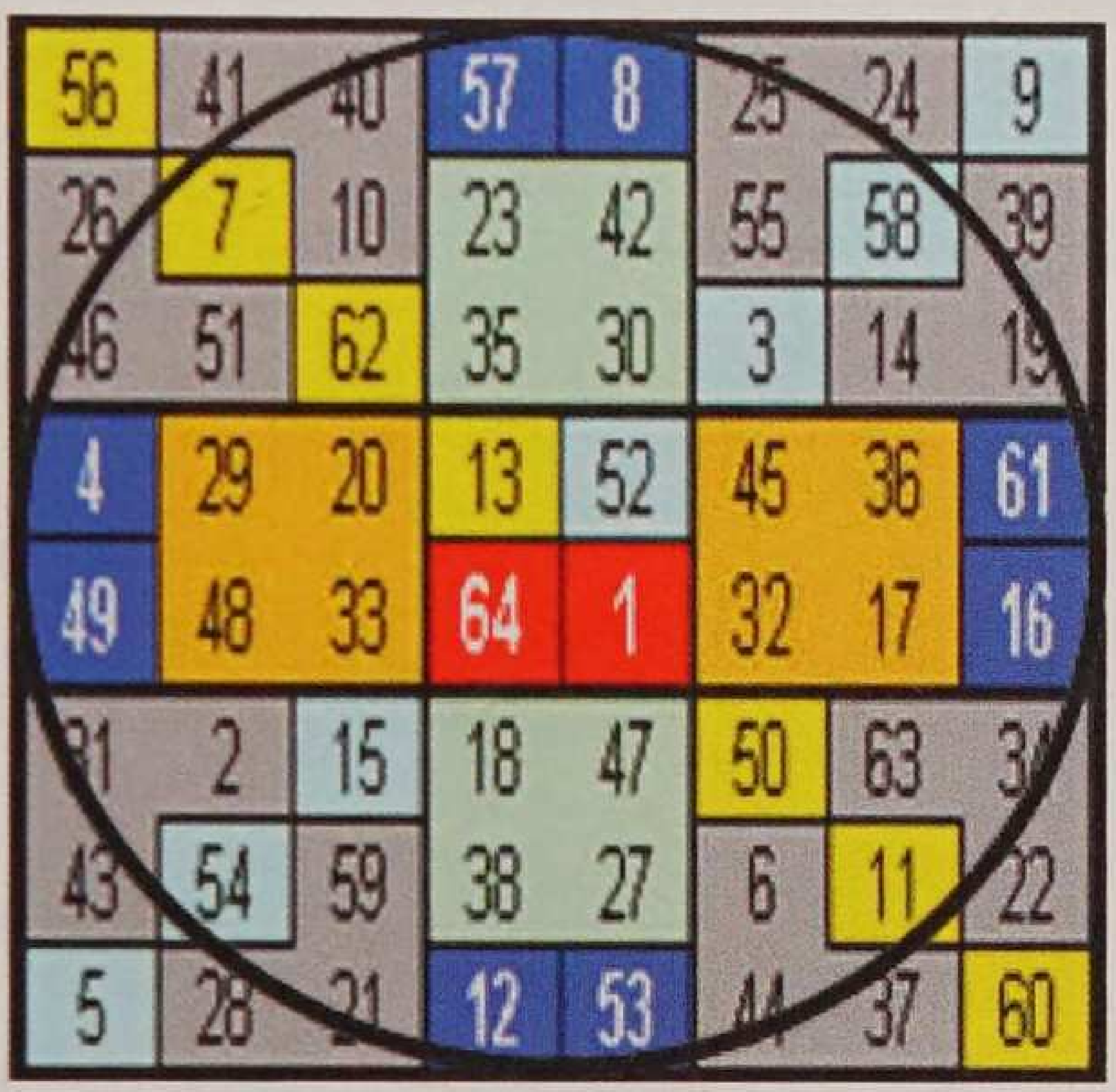
Here are shown the *characteristic circles* inscribed in Class-4 squares from size-4 through size-20. A characteristic circle's value is the sum of numbers in all the cells which just impinge the largest inscribed circle from its interior. Because of these squares' pangenicity, these sums can be readily determined by simply counting all the quadrals formed from these symmetrically located cells and multiplying this quantity by the square's **kernel** number **q**.

4x4



8	2	4	12
10	16	14	6
11	3	1	7
5	13	15	9

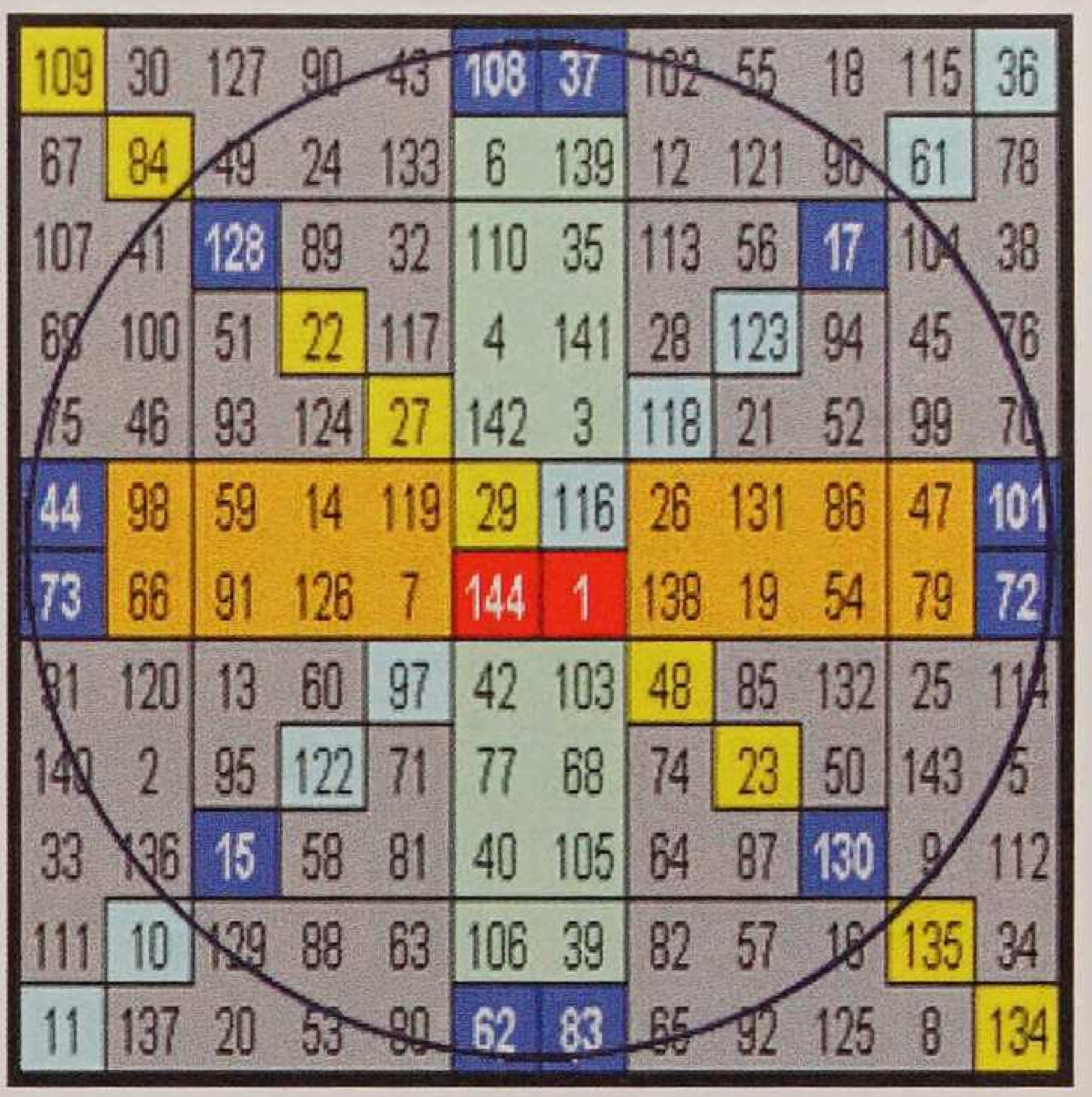
8x8



56	41	40	57	8	25	24	9
26	7	10	23	42	55	58	39
46	51	62	35	30	3	14	19
4	29	20	13	52	45	36	61
49	48	33	64	1	32	17	16
31	2	15	18	47	50	63	34
43	54	59	38	27	6	11	22
5	28	21	12	53	44	37	60

Every quadral sums to 130.

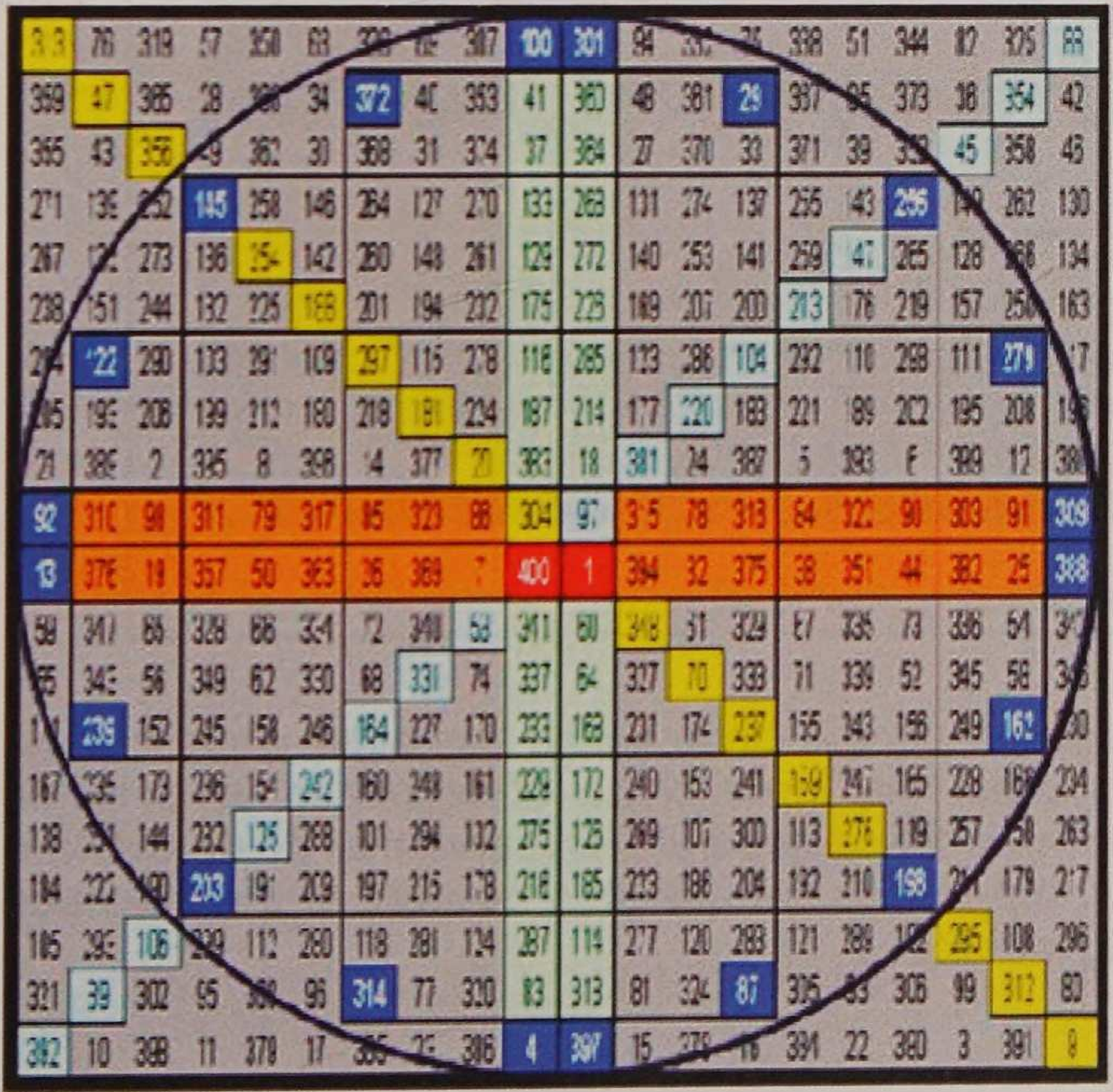
12x12



109	30	127	90	43	108	37	102	55	18	115	36
67	84	49	24	133	6	139	12	121	96	61	78
107	41	128	89	32	110	35	113	56	17	104	38
69	100	51	22	117	4	141	28	123	94	45	76
75	46	93	124	27	142	3	118	21	52	99	70
44	98	59	14	119	29	116	26	131	86	47	101
73	66	91	126	7	144	1	138	19	54	79	72
31	120	13	60	97	42	103	48	85	132	25	114
140	2	95	122	71	77	68	74	23	50	143	5
33	136	15	58	81	40	105	64	87	130	9	112
111	10	129	88	63	106	39	82	57	16	135	34
11	137	20	53	80	62	83	65	92	125	8	134

Every quadral sums to 290.

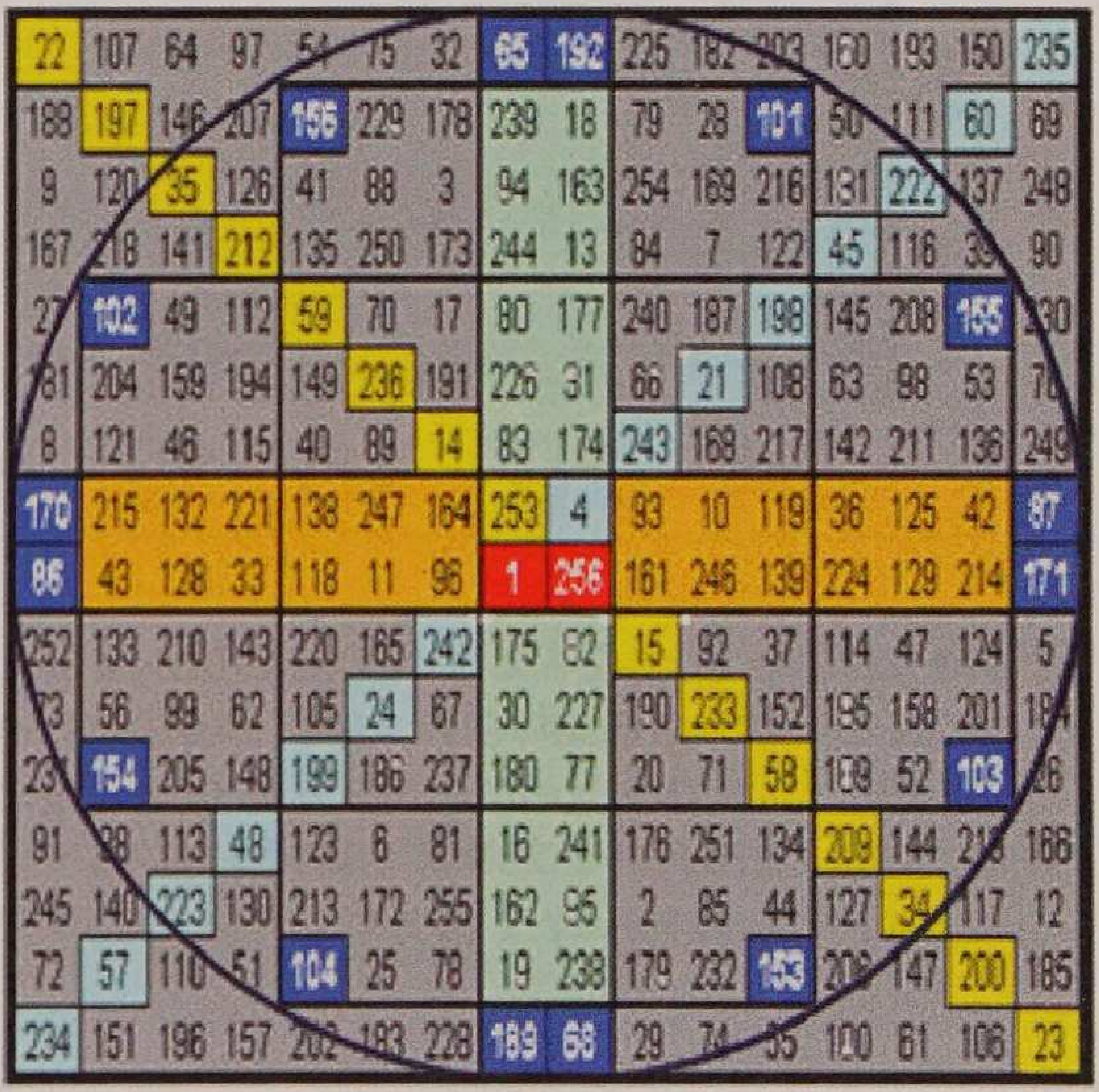
20x20



[illegible]	76	319	57	350	63	[illegible]	[illegible]	307	100	301	94	[illegible]	[illegible]	398	51	344	17	325	88
369	47	385	28	[illegible]	34	372	40	353	41	360	48	381	29	337	[illegible]	373	18	354	42
365	43	356	49	362	30	368	31	374	37	364	27	370	33	371	39	[illegible]	45	358	46
271	135	252	145	258	146	284	127	270	133	268	131	274	137	265	143	286	[illegible]	262	130
267	[illegible]	273	136	154	142	280	148	261	129	272	140	253	141	259	47	265	128	[illegible]	134
238	151	244	132	225	168	201	194	232	175	229	169	207	200	213	176	219	157	[illegible]	163
[illegible]	122	290	133	291	109	297	115	278	118	285	123	286	104	292	110	298	111	279	[illegible]
[illegible]	193	208	139	212	160	218	181	224	187	214	177	220	183	221	189	202	195	208	[illegible]
21	385	2	325	8	398	14	377	20	383	18	381	24	387	5	393	[illegible]	399	12	[illegible]
92	310	98	311	79	317	85	323	88	304	97	315	78	313	84	322	90	303	91	309
13	376	19	357	50	363	36	369	7	400	1	394	32	375	38	351	44	382	25	388
58	341	65	328	66	334	72	340	53	341	60	348	31	329	67	335	73	336	54	[illegible]
[illegible]	343	56	349	62	330	68	331	74	337	64	327	70	333	71	339	52	345	58	[illegible]
[illegible]	239	152	245	158	246	164	227	170	233	169	231	174	237	155	343	156	249	162	[illegible]
167	235	173	236	154	242	160	248	161	228	172	240	153	241	159	247	165	228	[illegible]	234
138	[illegible]	144	232	125	288	101	294	132	275	125	269	107	300	113	276	119	257	[illegible]	263
184	222	[illegible]	203	191	209	197	215	178	216	185	223	186	204	192	210	198	[illegible]	179	217
185	295	106	[illegible]	112	280	118	281	124	287	114	277	120	283	121	286	[illegible]	295	108	296
321	39	302	95	[illegible]	96	314	77	320	83	313	81	324	87	305	[illegible]	306	99	312	80
382	10	388	11	378	17	[illegible]	[illegible]	386	4	397	15	[illegible]	[illegible]	394	22	380	3	391	8

Every quadral sums to 802.

16x16

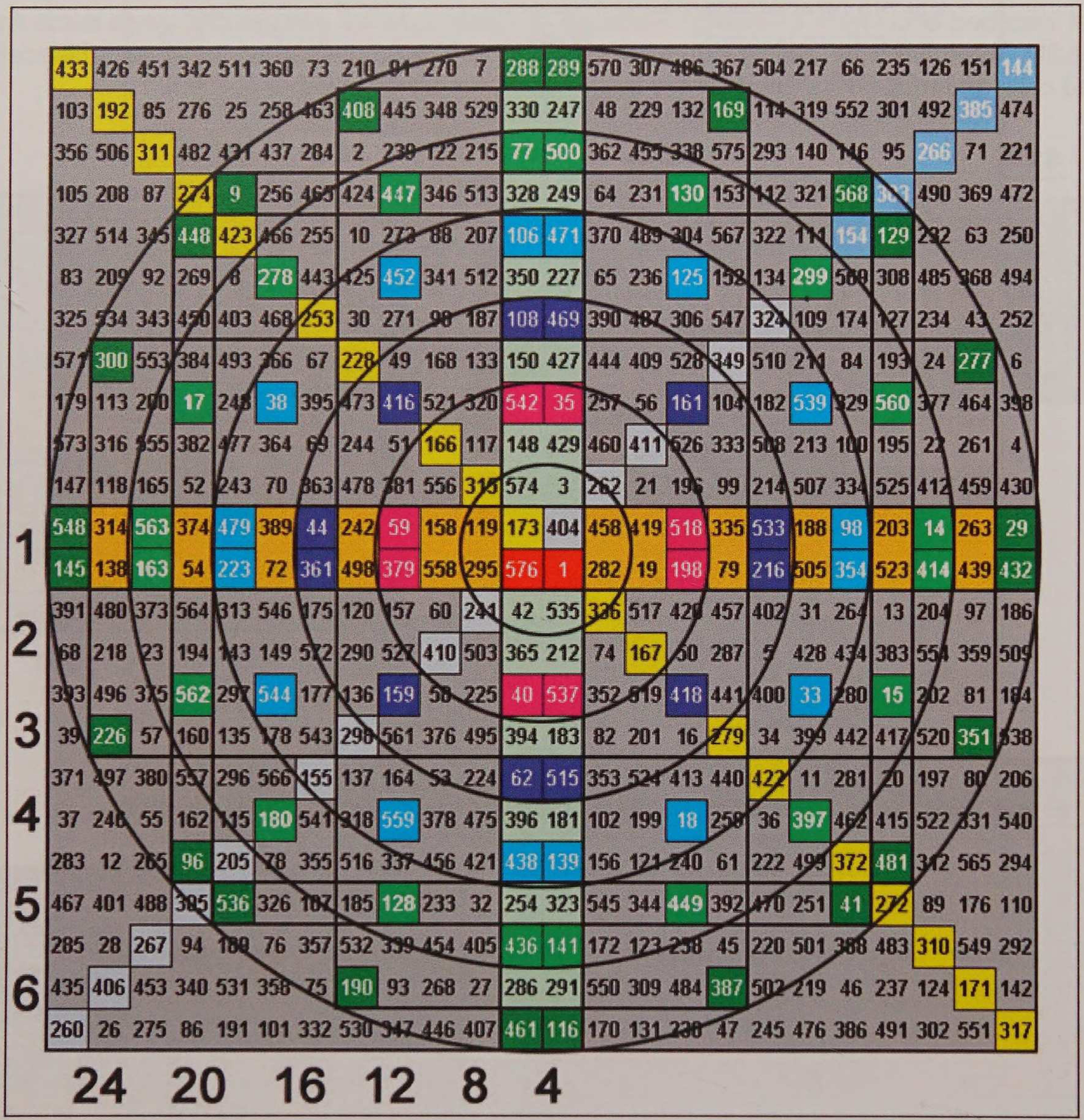


22	107	64	97	54	75	32	65	192	225	182	203	160	193	150	235
188	197	146	207	156	229	178	239	18	79	28	101	50	111	60	69
9	120	35	126	41	88	3	94	163	254	169	216	131	222	137	248
167	218	141	212	135	250	173	244	13	84	7	122	45	116	39	90
27	102	49	112	59	70	17	80	177	240	187	198	145	208	155	230
181	204	159	194	149	236	191	226	31	66	21	108	63	98	53	76
8	121	46	115	40	89	14	83	174	243	168	217	142	211	136	249
170	215	132	221	138	247	164	253	4	93	10	119	36	125	42	87
86	43	128	33	118	11	96	1	256	161	246	139	224	129	214	171
252	133	210	143	220	165	242	175	82	15	92	37	114	47	124	5
73	56	99	62	105	24	67	30	227	190	233	152	195	158	201	184
231	154	205	148	199	186	237	180	77	20	71	58	109	52	103	26
91	38	113	48	123	6	81	16	241	176	251	134	209	144	219	166
245	140	223	130	213	172	255	162	95	2	85	44	127	34	117	12
72	57	110	51	104	25	78	19	238	179	232	153	206	147	200	185
234	151	196	157	202	183	228	189	68	29	74	35	100	61	106	23

Every quadral sums to 514

Program 1

Size-24 Square with nested circles



433	426	451	342	511	360	73	210	91	270	7	288	289	570	307	486	367	504	217	66	235	126	151	144
103	192	85	276	25	258	463	408	445	348	529	330	247	48	229	132	169	114	319	552	301	492	385	474
356	506	311	482	431	437	284	2	239	122	215	77	500	362	455	338	575	293	140	146	95	266	71	221
105	208	87	274	9	256	465	424	447	346	513	328	249	64	231	130	153	112	321	568	303	490	369	472
327	514	345	448	423	466	255	10	273	88	207	106	471	370	489	304	567	322	111	154	129	232	63	250
83	209	92	269	8	278	443	425	452	341	512	350	227	65	236	125	152	134	299	569	308	485	368	494
325	534	343	450	403	468	253	30	271	90	187	108	469	390	487	306	547	324	109	174	127	234	43	252
571	300	553	384	493	366	67	228	49	168	133	150	427	444	409	528	349	510	211	84	193	24	277	6
179	113	200	17	248	38	395	473	416	521	320	542	35	257	56	161	104	182	539	329	560	377	464	398
573	316	555	382	477	364	69	244	51	166	117	148	429	460	411	526	333	508	213	100	195	22	261	4
147	118	165	52	243	70	363	478	381	556	315	574	3	262	21	196	99	214	507	334	525	412	459	430
548	314	563	374	479	389	44	242	59	158	119	173	404	458	419	518	335	533	188	98	203	14	263	29
145	138	163	54	223	72	361	498	379	558	295	576	1	282	19	198	79	216	505	354	523	414	439	432
391	480	373	564	313	546	175	120	157	60	241	42	535	336	517	420	457	402	31	264	13	204	97	186
68	218	23	194	143	149	572	290	527	410	503	365	212	74	167	50	287	5	428	434	383	554	359	509
393	496	375	562	297	544	177	136	159	58	225	40	537	352	519	418	441	400	33	280	15	202	81	184
39	226	57	160	135	178	543	298	561	376	495	394	183	82	201	16	279	34	399	442	417	520	351	538
371	497	380	557	296	566	155	137	164	53	224	62	515	353	524	413	440	422	11	281	20	197	80	206
37	246	55	162	115	180	541	318	559	378	475	396	181	102	199	18	259	36	397	462	415	522	331	540
283	12	265	96	205	78	355	516	337	456	421	438	139	156	121	240	61	222	499	372	481	312	565	294
467	401	488	305	536	326	107	185	128	233	32	254	323	545	344	449	392	470	251	41	272	89	176	110
285	28	267	94	189	76	357	532	339	454	405	436	141	172	123	238	45	220	501	388	483	310	549	292
435	406	453	340	531	358	75	190	93	268	27	286	291	550	309	484	387	502	219	46	237	124	171	142
260	26	275	86	191	101	332	530	347	446	407	461	116	170	131	230	47	245	476	386	491	302	551	317

Here is the size-24 characteristic circle. Each concentric circle within the outer circle corresponds to progressively less number of quadrals; the inner circle corresponds to a single quadral. This demonstrates the nesting pattern of incident quadrals. Observe that going outward from the center (exterior numbers at bottom), the number of quadrals incident to the inscribed circles increases by **1** (exterior numbers along left side).

Program 1

The table shows the number of impinging quadrals for consecutively nested circles, each size of which is a multiple of **4**, down to size-4 itself. The pattern follows the sequence: **n** divided by **4**.

The list shows how many impinging quadrals sum to the square's characteristic number by size. An exception was made for the size-4 square, which could have two quadrals instead of just one. A single quadral for the size-4 square was chosen to maintain the natural sequence of quadral multiples seen in the central column. Observe that the number of quadrals involved in these circles is the series **1**, **2**, **3**, **4**, **5**, **6**. This series is basic to Geonometry.

This series can be formulated as

(1.4) $$y = 4x$$

where **y** represents the table's size **n** and **x** represents both the sequence of the nested circle as encountered counting from the smallest circle outward and the number of quadrals impinging the largest inscribed sphere.

This formula will come into play again in Programs 4, 5 and 8 where it will crop up again and again in surprisingly different ways.

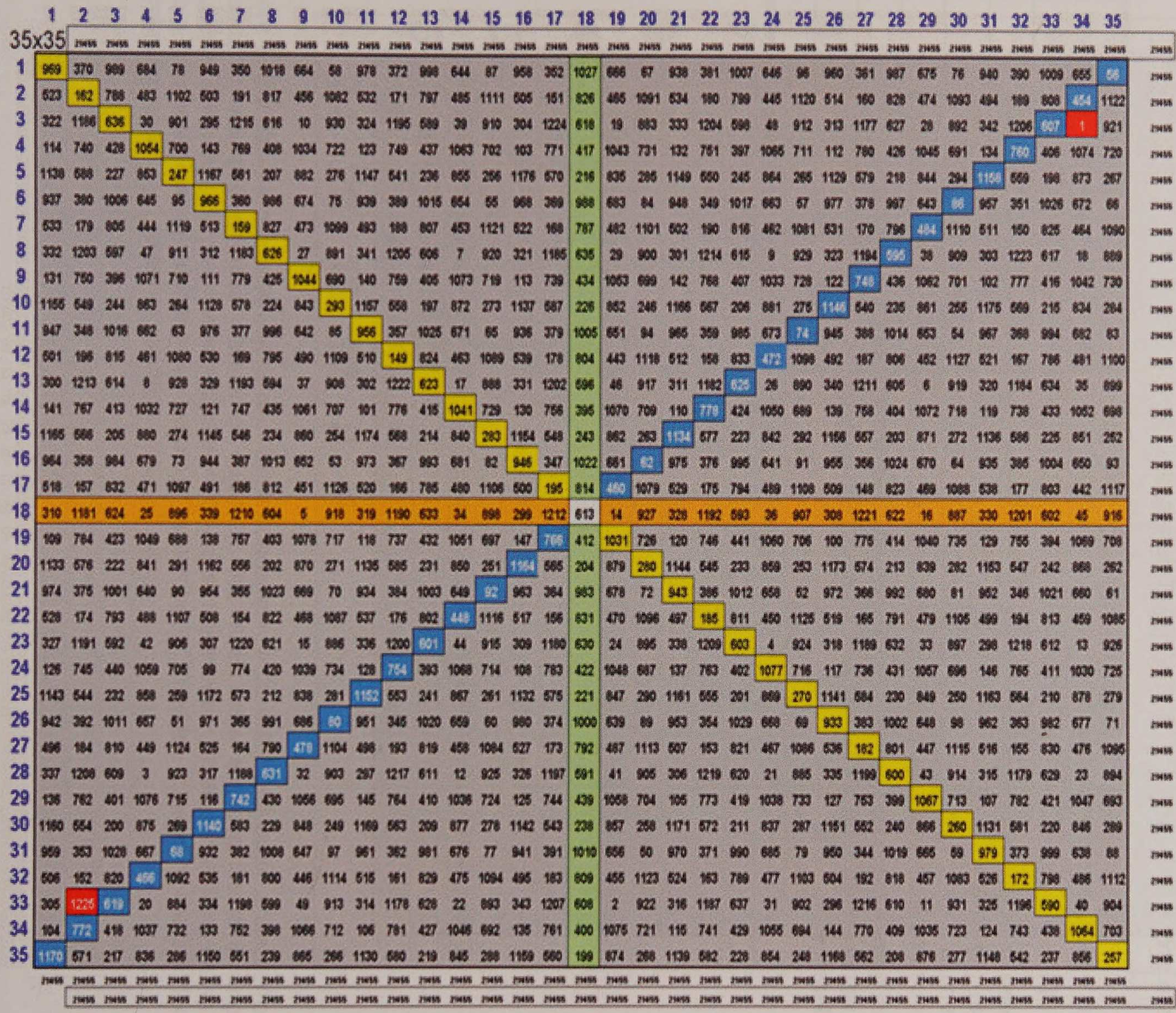
Size n	Number of Quadrals in Characteristic Circles
4	1
8	2
12	3
16	4
20	5
24	6

Program 1

Class-5 Squares

Size-35 Square

Here is the only size of Class-5 square that is printable. Its size is **35**. It is geonomically perfect. It has equal-summing quadrals because it has all its centrally symmetric pairs summing equally.



35x35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4	114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5	1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6	937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21455
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8	332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9	131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15	1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18	310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23	327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25	1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26	942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

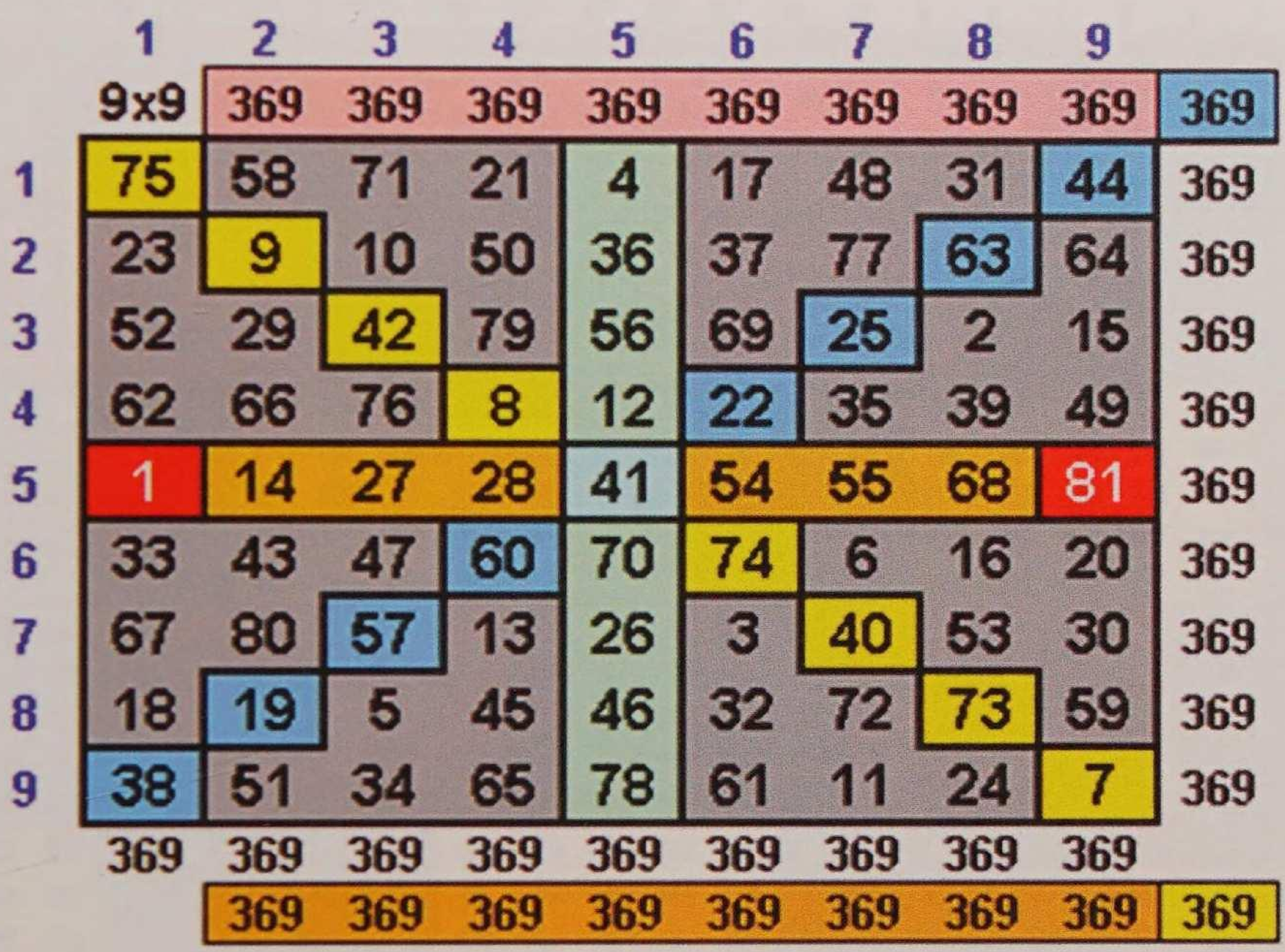
Program 1

Class-6 Squares

Size-9 Square

Here is the smallest of the Class-6 squares, the size-9. It is perfect. All of its quadrals sum equally because it has all equal-summing **centrally-symmetric** pairwise sums.

It is characteristically 3x-modular because its **3-by-3** block-squares sum to its characteristic number everywhere.



	1	2	3	4	5	6	7	8	9	
	9x9	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

All centrally-symmetric pairs sum to **82.**

Every quadral sums to **164**.

164 = 4 x 41 = **4/9** x 369.

q = 164.

p = 41

q = 4p

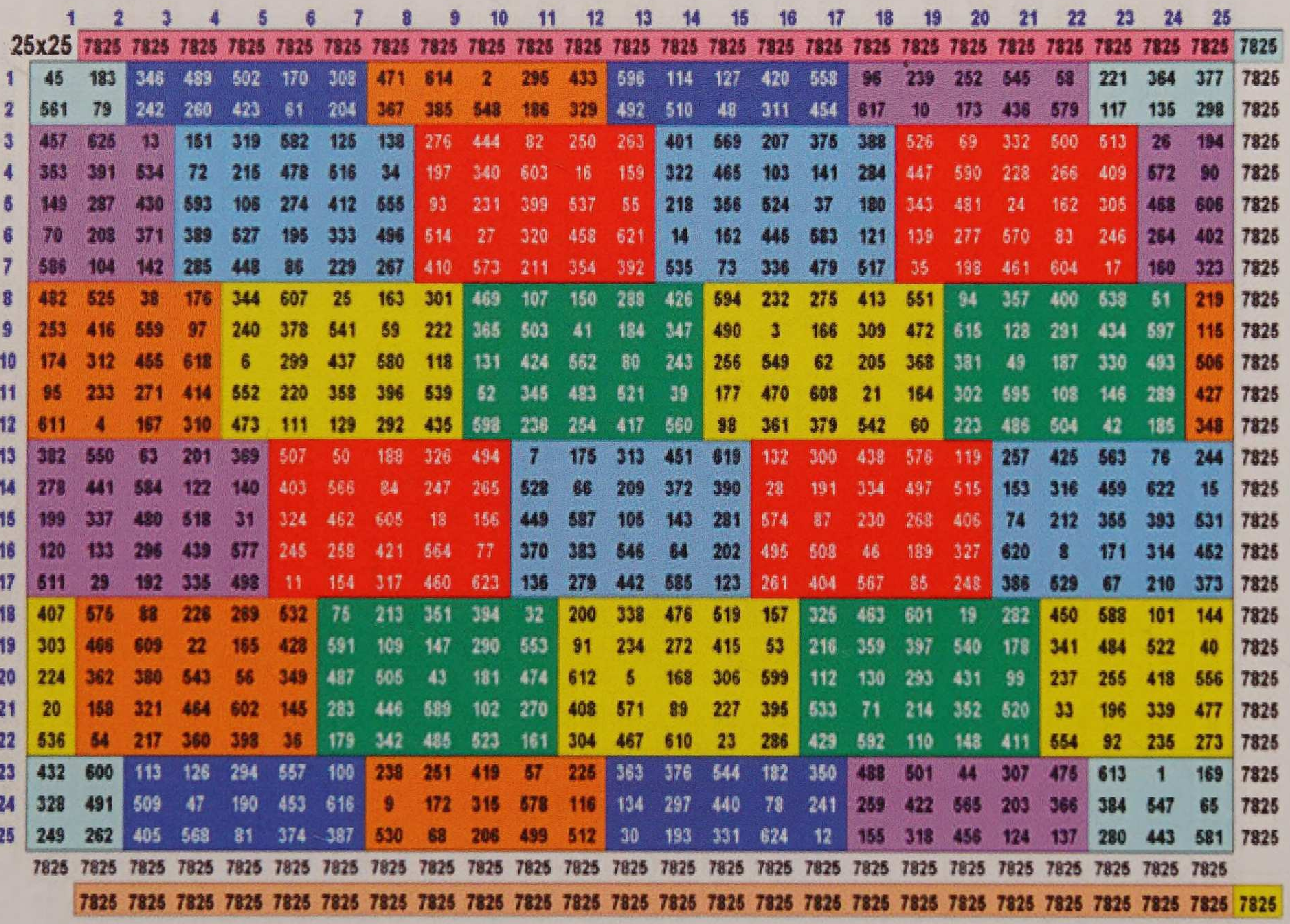
Program 1

Size 25 Square

Here is the next smallest size Class-6 square, size **25**. Its Characteristic number is **7825**.

It too has all equal-summing centrally symmetric pairwise sums, so all of its quadrals sum equally.

All of its **5x5** block-squares sum to the square's characteristic number continuously, so it is **5x-modular**.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
25x25		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

All sizes of Class-6 squares for which **n = b**^{2} or **b**^{3} will be continuously **bx-modular**.

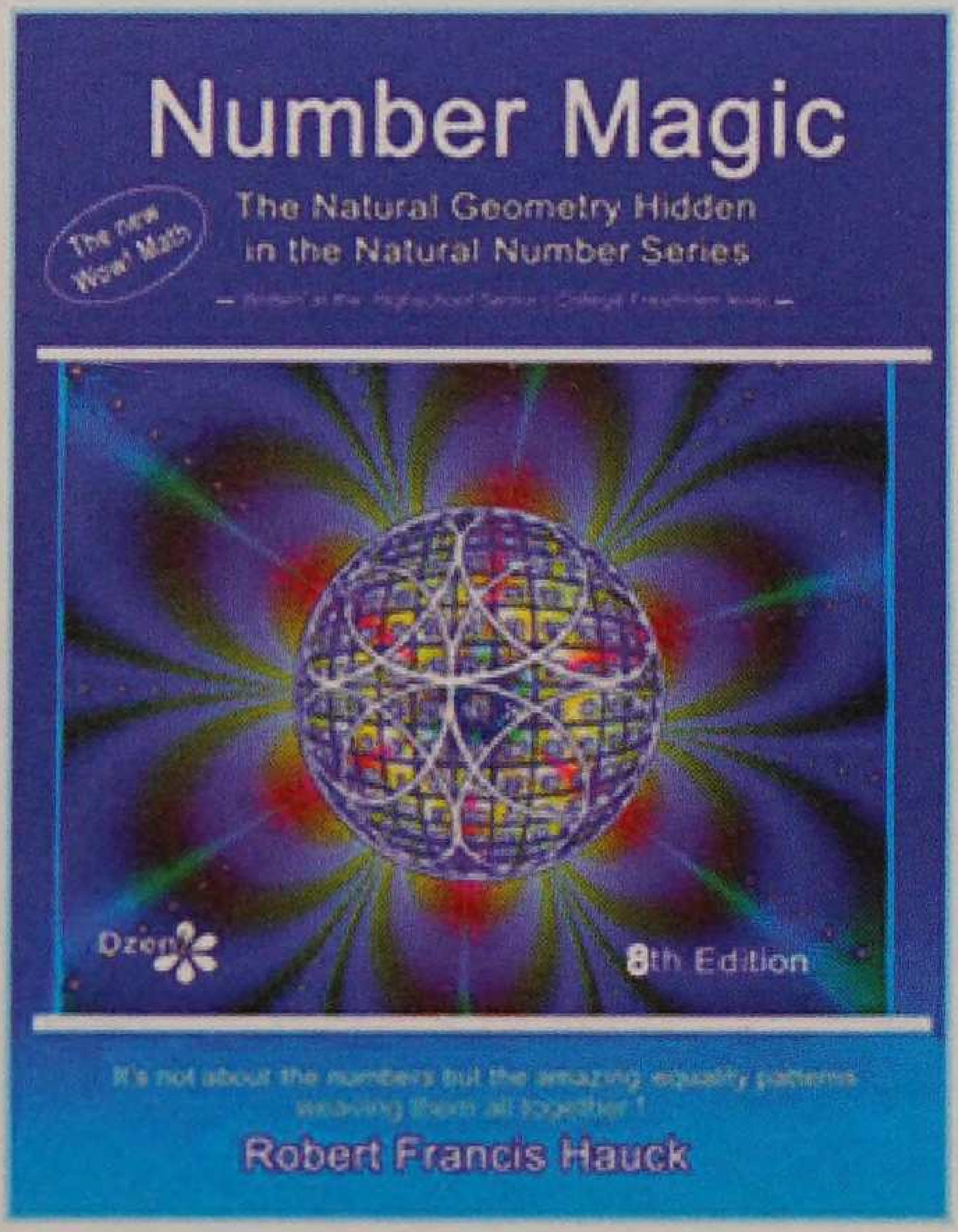
Notes

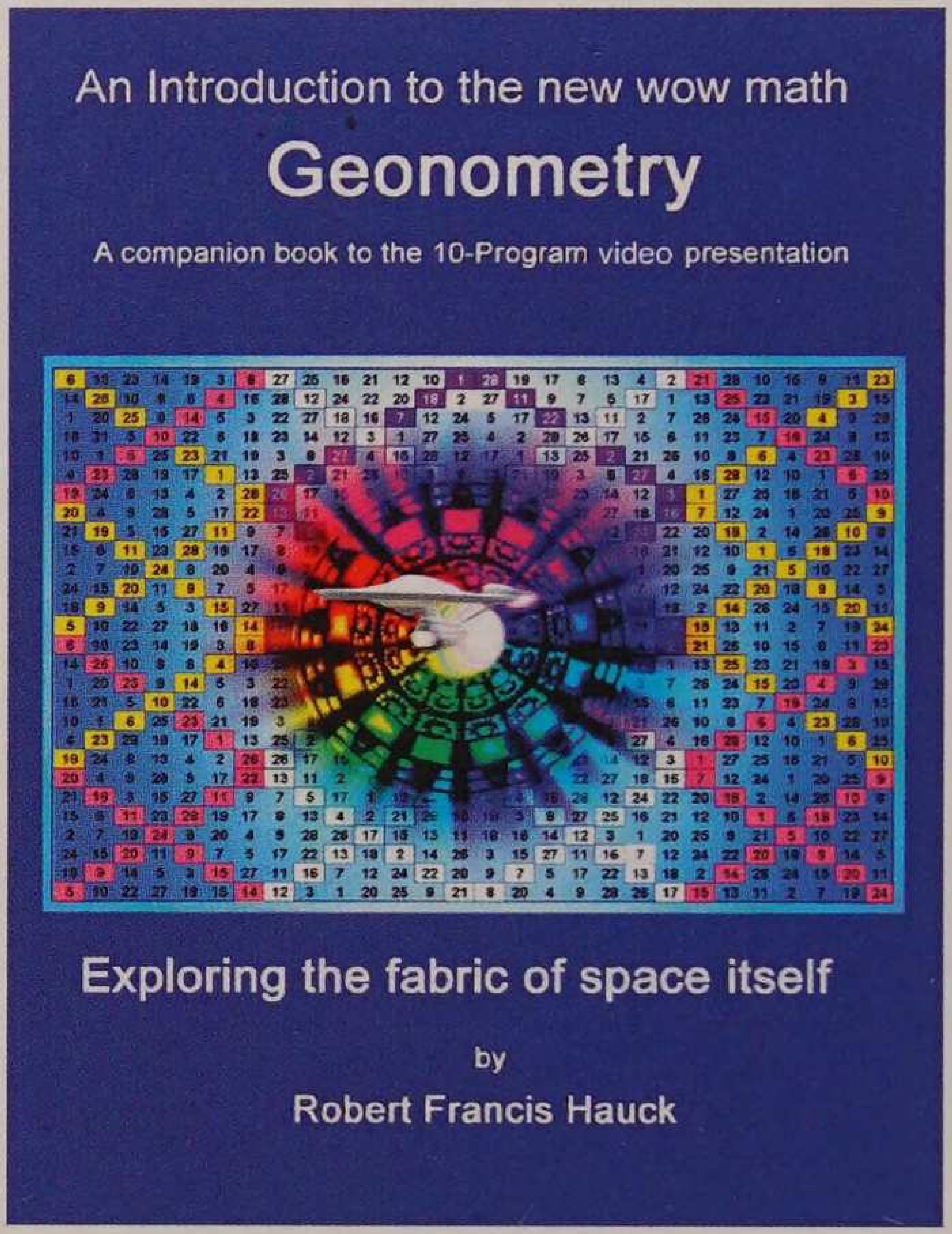
Geonomic squares can be classified into 6 distinct major classes.

1. All except Class-2 squares are perfect in that all four directional summations add up to the same number and are pangenic.
2. Class-2 squares are pangenic, but are only near-perfect in that they have two symmetrically located rows which differ by **±1**.
3. Class-3 squares of **n = 3b** possess **bx**-modularity.
4. Class-4 squares for sizes **n = 4b** for **b >1** also possess continuous **bx**-modularity.
5. Class-4 squares whose size is divisible by **8** have both row and central pairwise symmetry.
6. Class-4 squares possess characteristic circles in which the numbers impinging the largest inscribed circle from its interior sum to the squares characteristic number.
7. Class-5 squares are only centrally pairwise symmetric.
8. Class-6 squares of size **n = b^2** or **b^3** are **bx**-modular.

We have come to the end of Program 1 of the new **math**, **Geonometry**. This program was merely an introduction to the topic of geonomic squares – more of a preparedness program that may have seemed rather tedious in a way because of the need for detailed explanations of the limitations of the various classifications. From here on out however, the programs will become quite entertaining and even inspirational because these summation patterns will be observed to be just amazing, one after another after another, non-stop right to the last slide.

Here are the two books upon which this introductory program was based.





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ISBN 978-1-479-23823-1

Contains all the slides and narration in this 10-program video series.
Selected examples.

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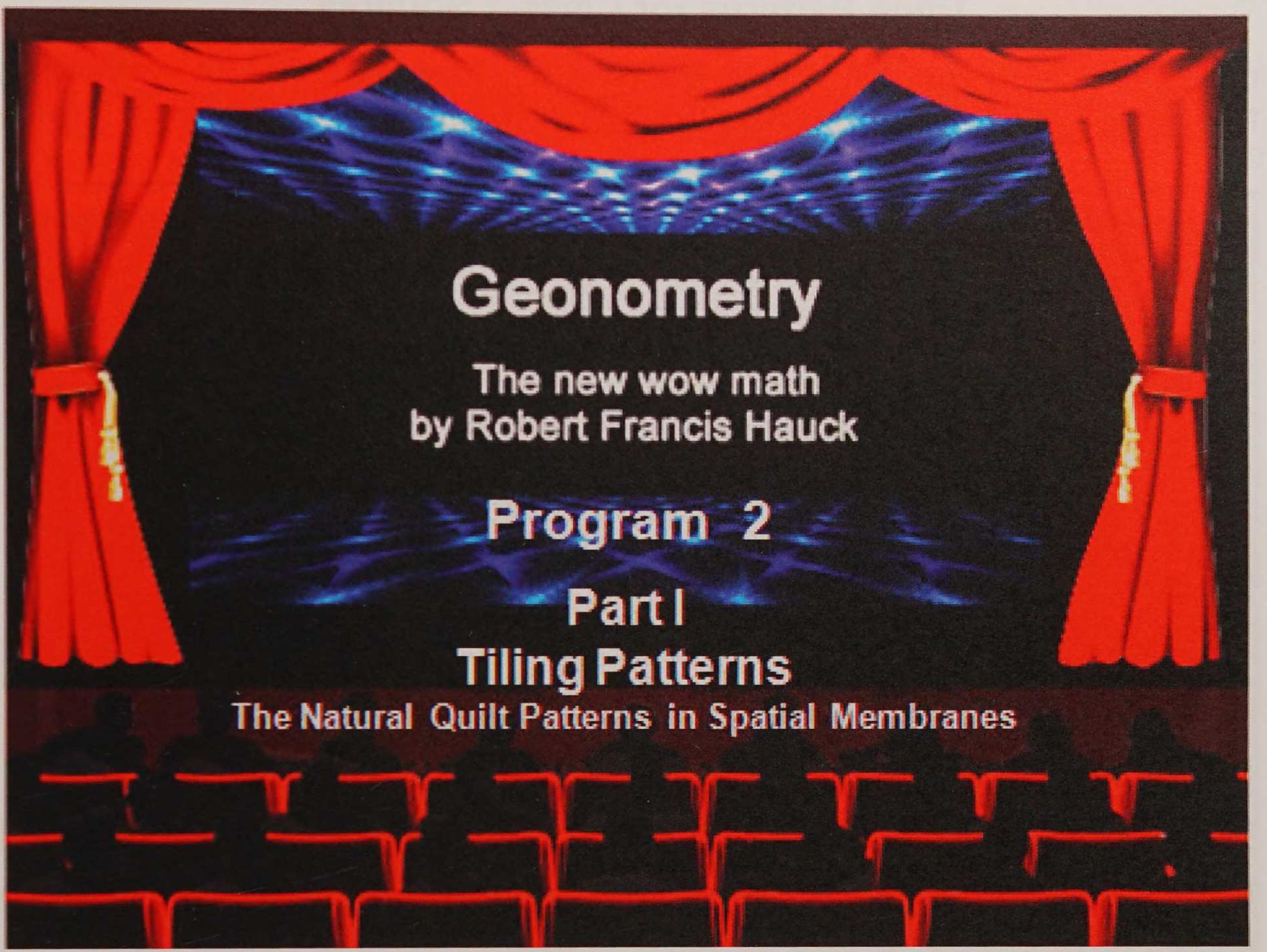
In Part I of the next program we will discover the amazing interlocking equal-summing tiling patterns hidden in geonomic squares – the natural quilt patterns in the fabric of space.

Then in Part II we will see just how these quilt patterns are woven through the discovery of tables that act as 4-directional looms.



Geonometry

The new wow math
by Robert Francis Hauck

Program 2

Part I
Tiling Patterns
The Natural Quilt Patterns in Spatial Membranes

Next, we're going to explore some amazing geometries that are found in all the squares of all classes except Class-2. These geometries are called **tiling patterns**. They will be seen to exhibit amazing characteristic quilt patterns in 2-dimensional spatial fabric.

The thing to note is that each tile corresponding to a square of size **n** consists of exactly **n** numbered-cells. In addition, since the square contains $\mathbf{n^2}$ numbers there will be exactly **n independent interlocking** tiles in the overall tiling pattern.

These tiling patterns have **five** special properties, called the **"5C"** properties:

1. They are **continuous**. Their center tile can be placed anywhere in the table and the equal summations persist in each tile!
2. They are **contiguous**. There are no isolated cells segregated from the others in the pattern. All of their cells are connected to another even if only at the cells' corners.
3. They are **characteristic** in that all their number cells sum to the square's characteristic number.
4. They are **complementary** in that there are **two** distinct tile patterns for each size of square occurring simultaneously.
5. In addition, they are **complete** in that they interlock to cover the square completely without voids or overlaps.

Program 2

Tiling Patterns for Class-1 Squares

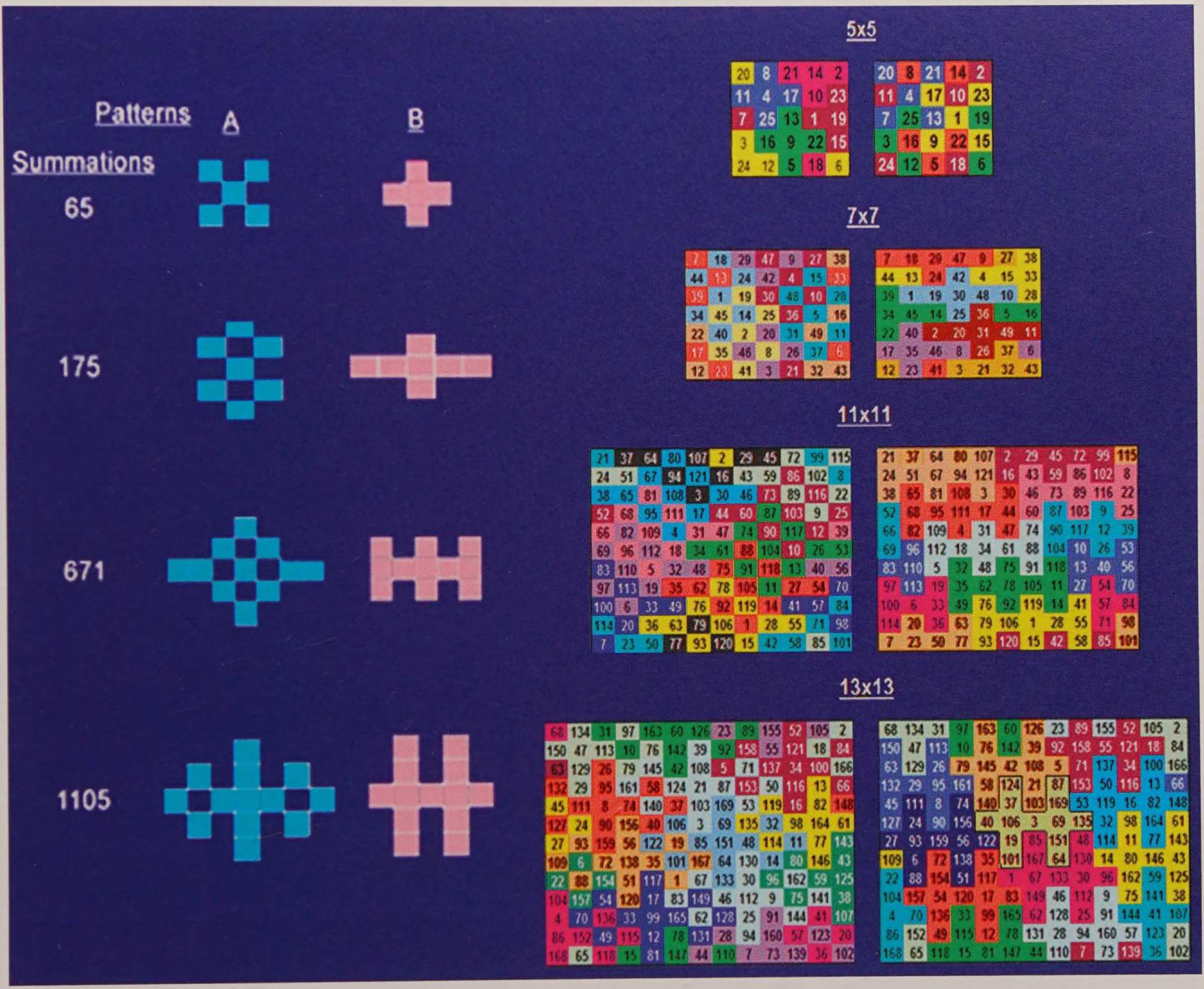
This slide makes the point that all these quilt patterns to follow are continuous everywhere. They can be pulled across the table like a tablecloth and the equal summations persist in each tile!

Further, the square's number pattern can be wrapped around the surface of a numbered sphere and the tiling patterns can be spun like a top and the equal summations persist in each tile! This property will have a major application in Program 5 where we explore the distribution of electrons around the nucleus of the atom.

We'll first investigate tiling patterns in Class-1 squares which have configurations unique to their square's size.

Program 2

Here is a sample of the complementary tiling patterns for class-1 sizes **5** through **13**. Note the interlocking of the tiles to form a complete tiling of the square. All the numbers in each tile sum to the same number, as do all the rows, columns, and diagonals in both directions. That number in is the square's characteristic number. The complementary tile patterns will be denoted by non-bold letters A and B.



5x5

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

7x7

7	18	29	47	9	27	38
44	13	24	42	4	15	33
39	1	19	30	48	10	28
34	45	14	25	36	5	16
22	40	2	20	31	49	11
17	35	46	8	26	37	6
12	23	41	3	21	32	43

7	18	29	47	9	27	38
44	13	24	42	4	15	33
39	1	19	30	48	10	28
34	45	14	25	36	5	16
22	40	2	20	31	49	11
17	35	46	8	26	37	6
12	23	41	3	21	32	43

11x11

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

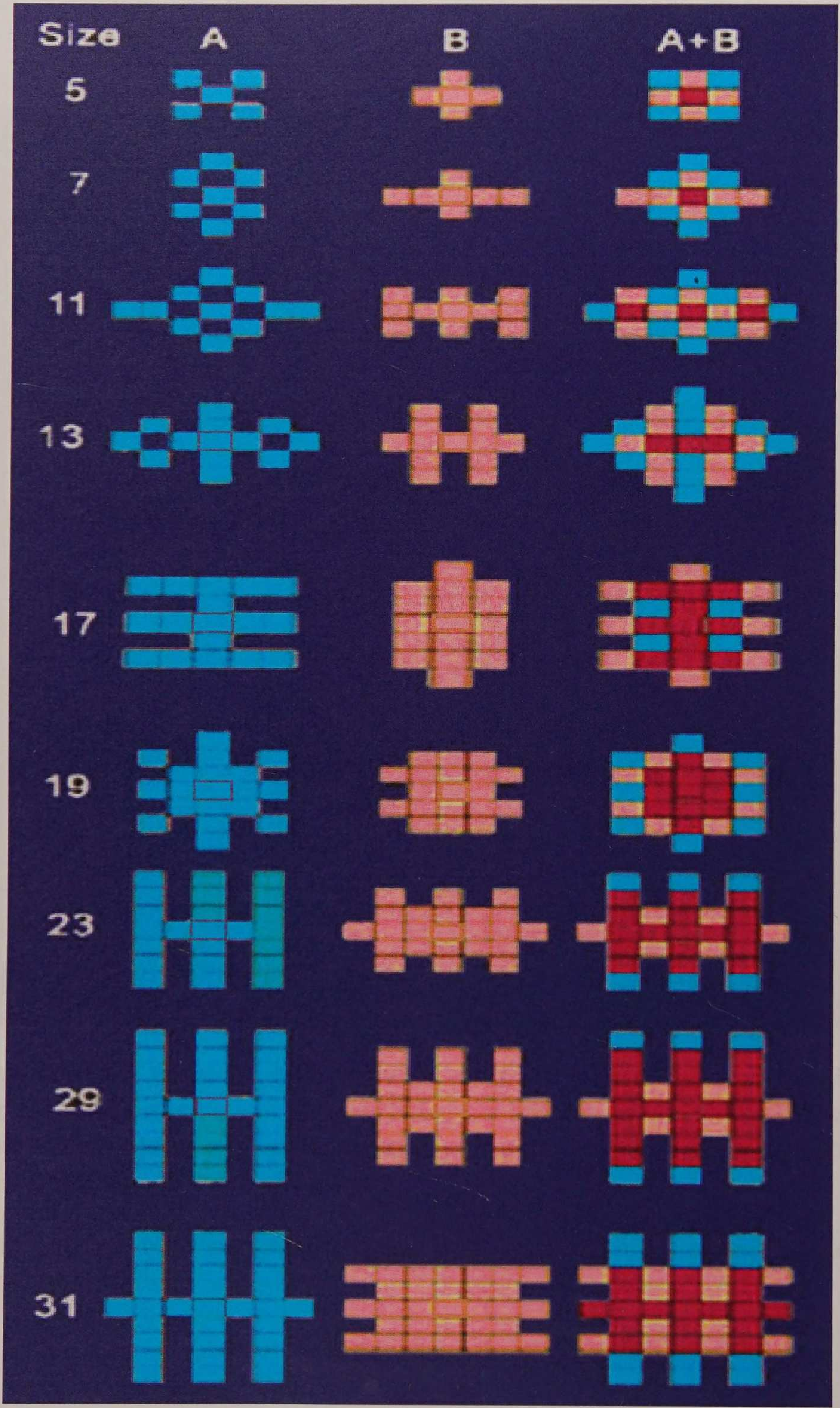
13x13

68	134	31	97	163	60	126	23	89	155	52	105	2
150	47	113	10	76	142	39	92	158	55	121	18	84
63	129	26	79	145	42	108	5	71	137	34	100	166
132	29	95	161	58	124	21	87	153	50	116	13	66
45	111	8	74	140	37	103	169	53	119	16	82	148
127	24	90	156	40	106	3	69	135	32	98	164	61
27	93	159	56	122	19	85	151	48	114	11	77	143
109	6	72	138	35	101	167	64	130	14	80	146	43
22	88	154	51	117	1	67	133	30	96	162	59	125
104	157	54	120	17	83	149	46	112	9	75	141	38
4	70	136	33	99	165	62	128	25	91	144	41	107
86	152	49	115	12	78	131	28	94	160	57	123	20
168	65	118	15	81	147	44	110	7	73	139	36	102

68	134	31	97	163	60	126	23	89	155	52	105	2
150	47	113	10	76	142	39	92	158	55	121	18	84
63	129	26	79	145	42	108	5	71	137	34	100	166
132	29	95	161	58	124	21	87	153	50	116	13	66
45	111	8	74	140	37	103	169	53	119	16	82	148
127	24	90	156	40	106	3	69	135	32	98	164	61
27	93	159	56	122	19	85	151	48	114	11	77	143
109	6	72	138	35	101	167	64	130	14	80	146	43
22	88	154	51	117	1	67	133	30	96	162	59	125
104	157	54	120	17	83	149	46	112	9	75	141	38
4	70	136	33	99	165	62	128	25	91	144	41	107
86	152	49	115	12	78	131	28	94	160	57	123	20
168	65	118	15	81	147	44	110	7	73	139	36	102

Program 2

Here is a pictorial list of all the different dual tile patterns up through all prime-number sizes from **5** to **31**. The figures on the right show the dual tiles laminated together with their centers and overlapping cells darkly shaded.

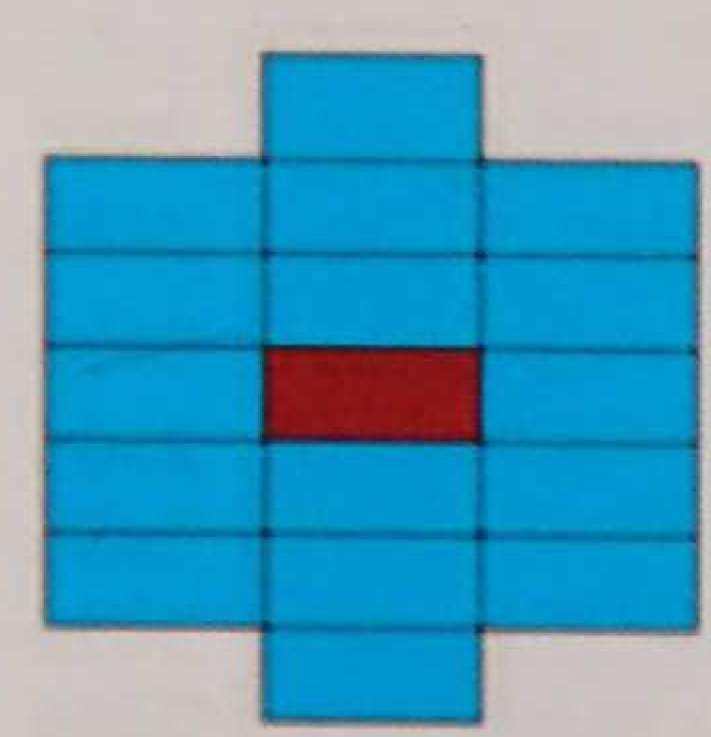


Program 2

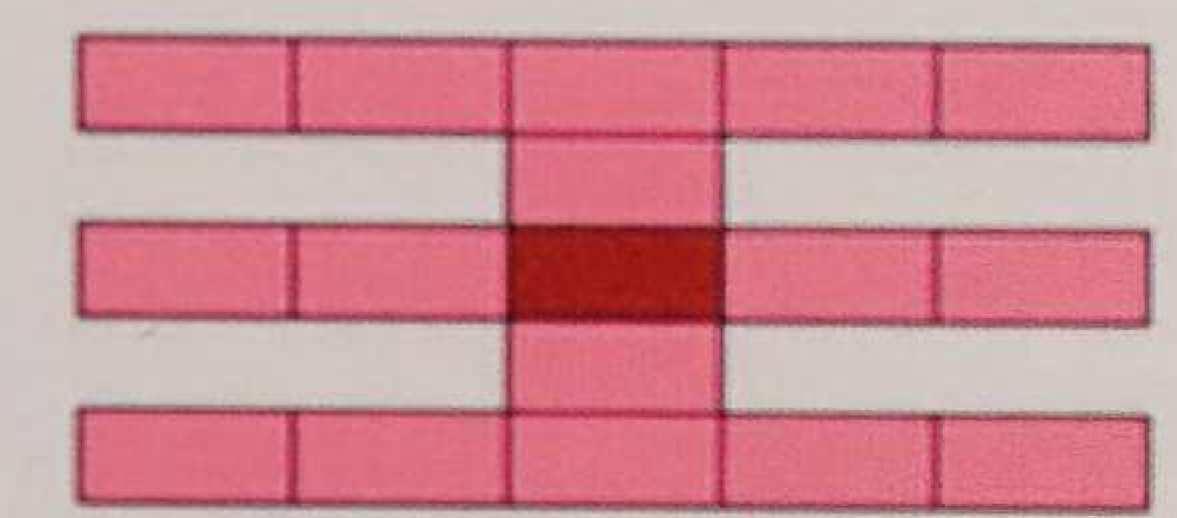
Size 17 Patterns

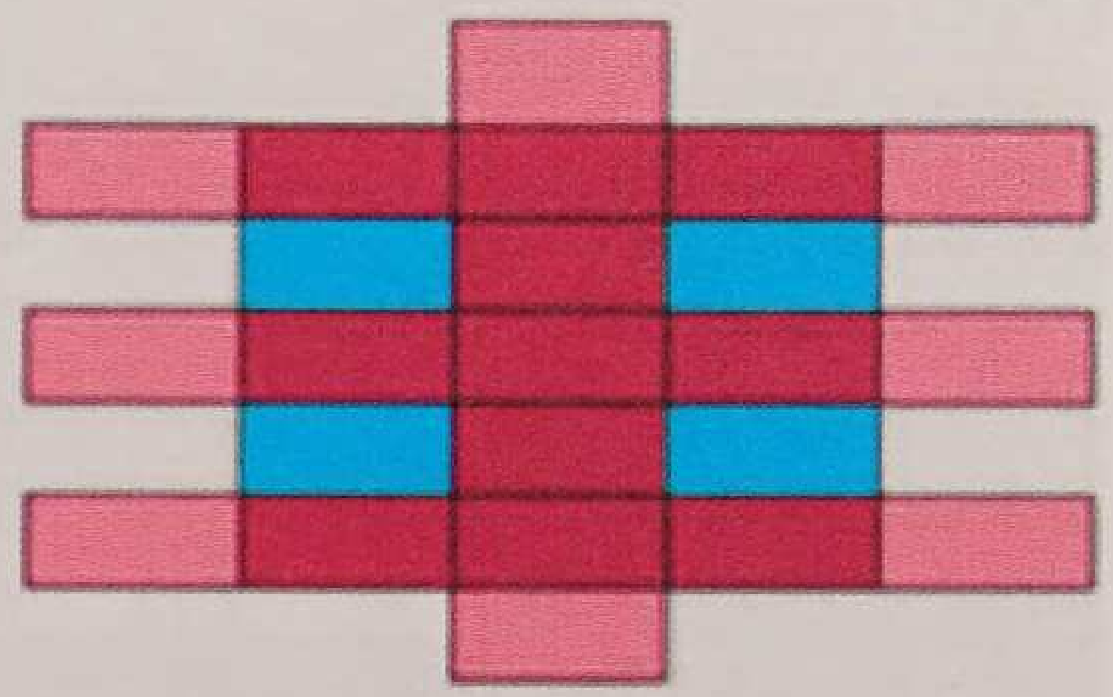
Here are the two continuous tiling patterns A and B identified by various colors for the original version of the size **17** square. Observe the interlocking of the tiles in each tiling pattern.

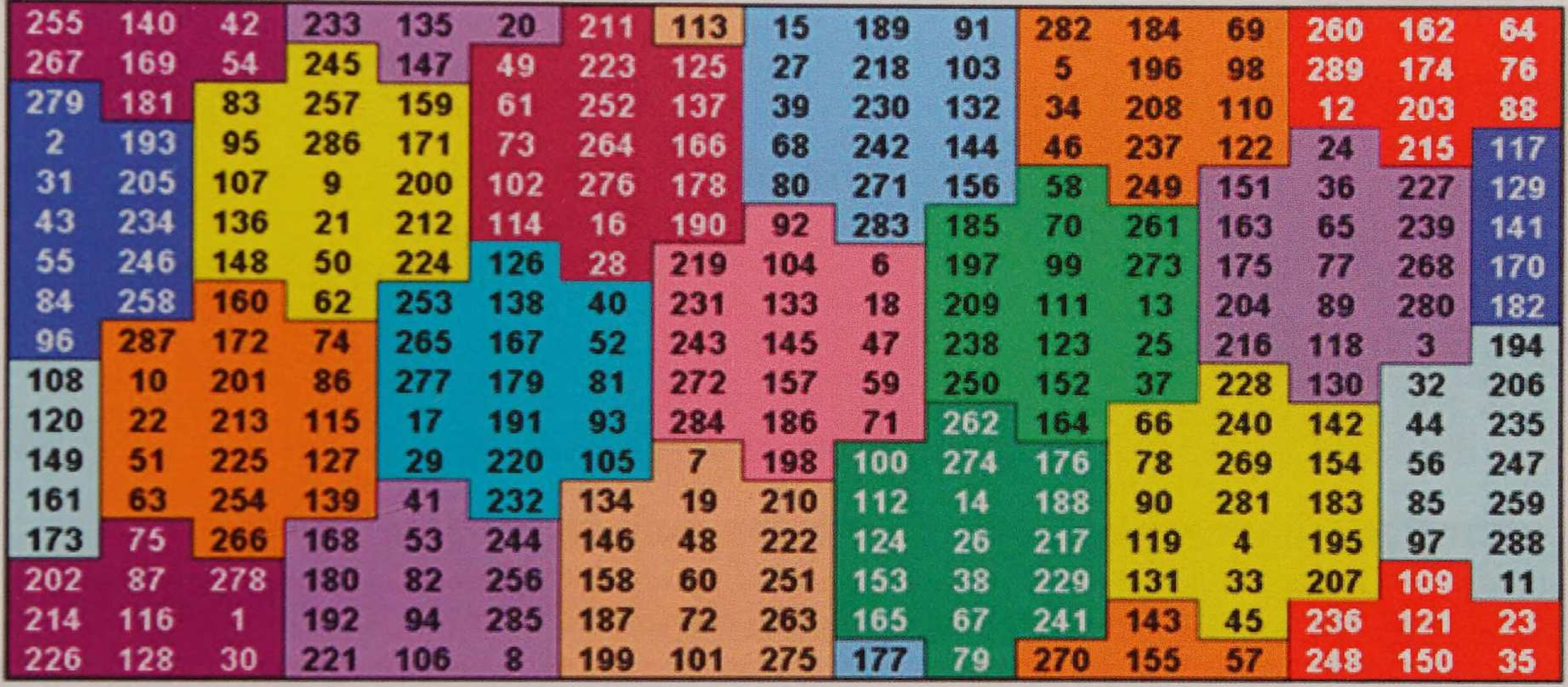
A



B







255	140	42	233	135	20	211	113	15	189	91	282	184	69	260	162	64
267	169	54	245	147	49	223	125	27	218	103	5	196	98	289	174	76
279	181	83	257	159	61	252	137	39	230	132	34	208	110	12	203	88
2	193	95	286	171	73	264	166	68	242	144	46	237	122	24	215	117
31	205	107	9	200	102	276	178	80	271	156	58	249	151	36	227	129
43	234	136	21	212	114	16	190	92	283	185	70	261	163	65	239	141
55	246	148	50	224	126	28	219	104	6	197	99	273	175	77	268	170
84	258	160	62	253	138	40	231	133	18	209	111	13	204	89	280	182
96	287	172	74	265	167	52	243	145	47	238	123	25	216	118	3	194
108	10	201	86	277	179	81	272	157	59	250	152	37	228	130	32	206
120	22	213	115	17	191	93	284	186	71	262	164	66	240	142	44	235
149	51	225	127	29	220	105	7	198	100	274	176	78	269	154	56	247
161	63	254	139	41	232	134	19	210	112	14	188	90	281	183	85	259
173	75	266	168	53	244	146	48	222	124	26	217	119	4	195	97	288
202	87	278	180	82	256	158	60	251	153	38	229	131	33	207	109	11
214	116	1	192	94	285	187	72	263	165	67	241	143	45	236	121	23
226	128	30	221	106	8	199	101	275	177	79	270	155	57	248	150	35

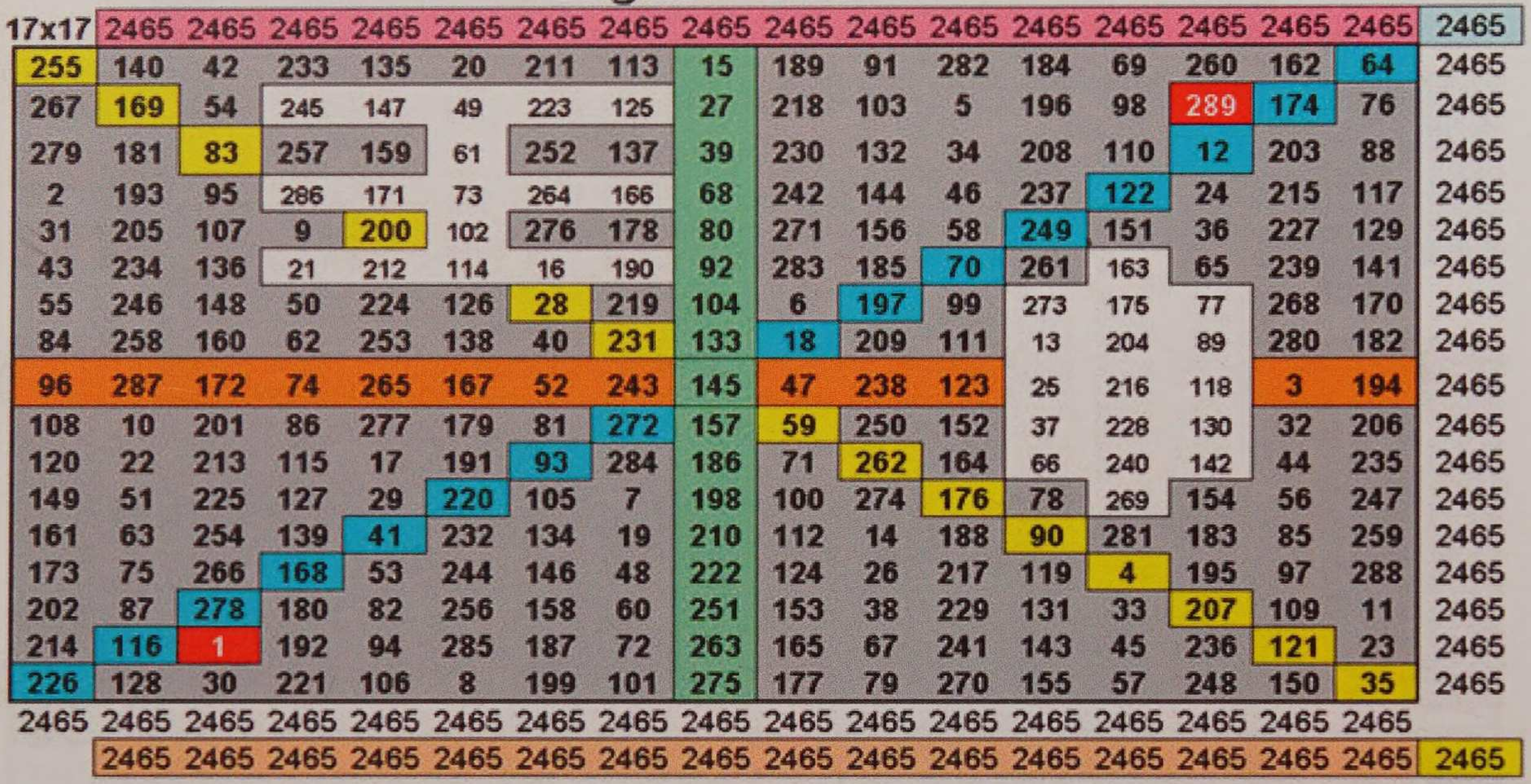
Individual patterns equal 2465 anywhere throughout table.

255	140	42	233	135	20	211	113	15	189	91	282	184	69	260	162	64
267	169	54	245	147	49	223	125	27	218	103	5	196	98	289	174	76
279	181	83	257	159	61	252	137	39	230	132	34	208	110	12	203	88
2	193	95	286	171	73	264	166	68	242	144	46	237	122	24	215	117
31	205	107	9	200	102	276	178	80	271	156	58	249	151	36	227	129
43	234	136	21	212	114	16	190	92	283	185	70	261	163	65	239	141
55	246	148	50	224	126	28	219	104	6	197	99	273	175	77	268	170
84	258	160	62	253	138	40	231	133	18	209	111	13	204	89	280	182
96	287	172	74	265	167	52	243	145	47	238	123	25	216	118	3	194
108	10	201	86	277	179	81	272	157	59	250	152	37	228	130	32	206
120	22	213	115	17	191	93	284	186	71	262	164	66	240	142	44	235
149	51	225	127	29	220	105	7	198	100	274	176	78	269	154	56	247
161	63	254	139	41	232	134	19	210	112	14	188	90	281	183	85	259
173	75	266	168	53	244	146	48	222	124	26	217	119	4	195	97	288
202	87	278	180	82	256	158	60	251	153	38	229	131	33	207	109	11
214	116	1	192	94	285	187	72	263	165	67	241	143	45	236	121	23
226	128	30	221	106	8	199	101	275	177	79	270	155	57	248	150	35

Program 2

This slide shows dual tile patterns for two different versions of the size-17 square. These tile patterns are independent of the number distribution in the table as long as the square is geonomically perfect. The important point is this: These equal-summing tiles are characteristic of the size of the square and not of the particular square itself.

Original version



17x17	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465
255	140	42	233	135	20	211	113	15	189	91	282	184	69	260	162	64	2465
267	169	54	245	147	49	223	125	27	218	103	5	196	98	289	174	76	2465
279	181	83	257	159	61	252	137	39	230	132	34	208	110	12	203	88	2465
2	193	95	286	171	73	264	166	68	242	144	46	237	122	24	215	117	2465
31	205	107	9	200	102	276	178	80	271	156	58	249	151	36	227	129	2465
43	234	136	21	212	114	16	190	92	283	185	70	261	163	65	239	141	2465
55	246	148	50	224	126	28	219	104	6	197	99	273	175	77	268	170	2465
84	258	160	62	253	138	40	231	133	18	209	111	13	204	89	280	182	2465
96	287	172	74	265	167	52	243	145	47	238	123	25	216	118	3	194	2465
108	10	201	86	277	179	81	272	157	59	250	152	37	228	130	32	206	2465
120	22	213	115	17	191	93	284	186	71	262	164	66	240	142	44	235	2465
149	51	225	127	29	220	105	7	198	100	274	176	78	269	154	56	247	2465
161	63	254	139	41	232	134	19	210	112	14	188	90	281	183	85	259	2465
173	75	266	168	53	244	146	48	222	124	26	217	119	4	195	97	288	2465
202	87	278	180	82	256	158	60	251	153	38	229	131	33	207	109	11	2465
214	116	1	192	94	285	187	72	263	165	67	241	143	45	236	121	23	2465
226	128	30	221	106	8	199	101	275	177	79	270	155	57	248	150	35	2465
2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465

Contiguous complementary patterns sum to 2465 continuously.

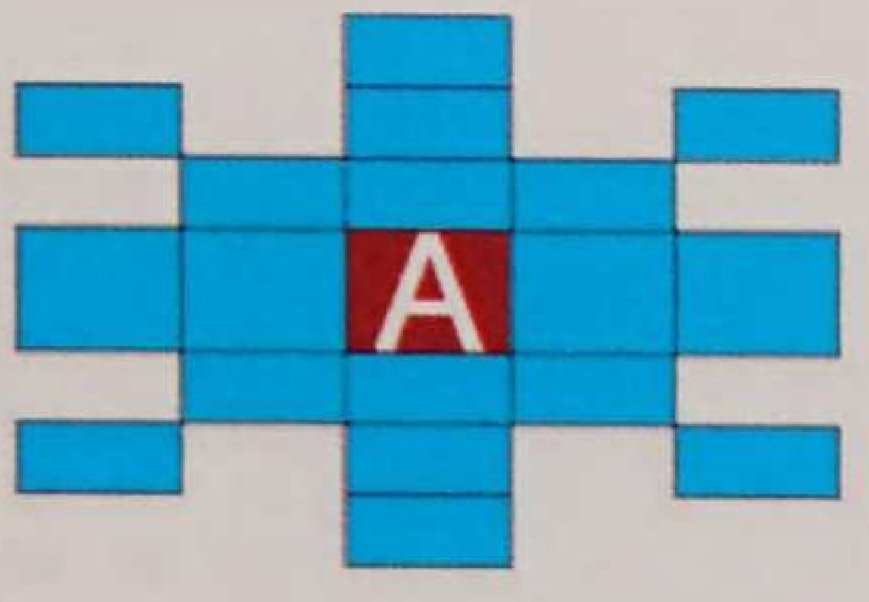
Alternate version

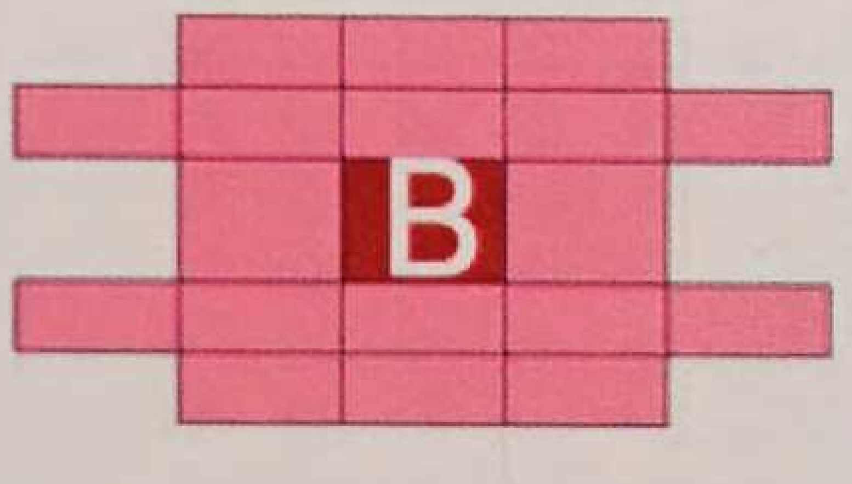
17x17	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465
185	196	207	235	246	257	285	7	18	46	57	85	96	107	135	146	157	2465
237	248	259	287	9	20	48	59	70	98	109	120	148	159	187	198	209	2465
289	11	22	50	61	72	100	111	122	150	161	172	200	211	222	250	261	2465
35	63	74	102	113	124	152	163	174	202	213	224	252	263	274	13	24	2465
87	115	126	137	165	176	204	215	226	254	265	276	15	26	37	65	76	2465
139	167	178	189	217	228	239	267	278	17	28	39	67	78	89	117	128	2465
191	219	230	241	269	280	2	30	41	52	80	91	119	130	141	169	180	2465
243	271	282	4	32	43	54	82	93	104	132	143	154	182	193	221	232	2465
6	34	45	56	84	95	106	134	145	156	184	195	206	234	245	256	284	2465
58	69	97	108	136	147	158	186	197	208	236	247	258	286	8	19	47	2465
110	121	149	160	171	199	210	238	249	260	288	10	21	49	60	71	99	2465
162	173	201	212	223	251	262	273	12	23	51	62	73	101	112	123	151	2465
214	225	253	264	275	14	25	36	64	75	86	114	125	153	164	175	203	2465
266	277	16	27	38	66	77	88	116	127	138	166	177	188	216	227	255	2465
29	40	68	79	90	118	129	140	168	179	190	218	229	240	268	279	1	2465
81	92	103	131	142	170	181	192	220	231	242	270	281	3	31	42	53	2465
133	144	155	183	194	205	233	244	272	283	5	33	44	55	83	94	105	2465
2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	
	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465	2465

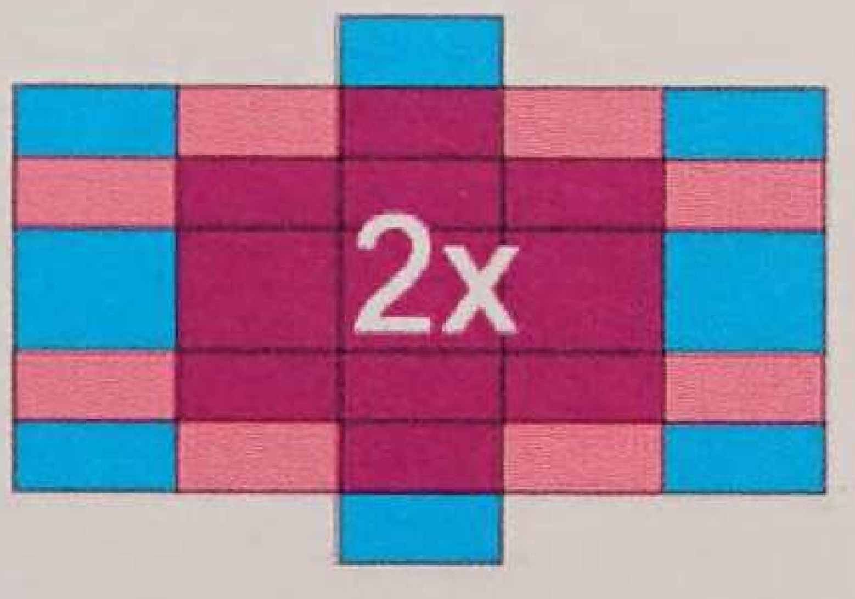
Program 2

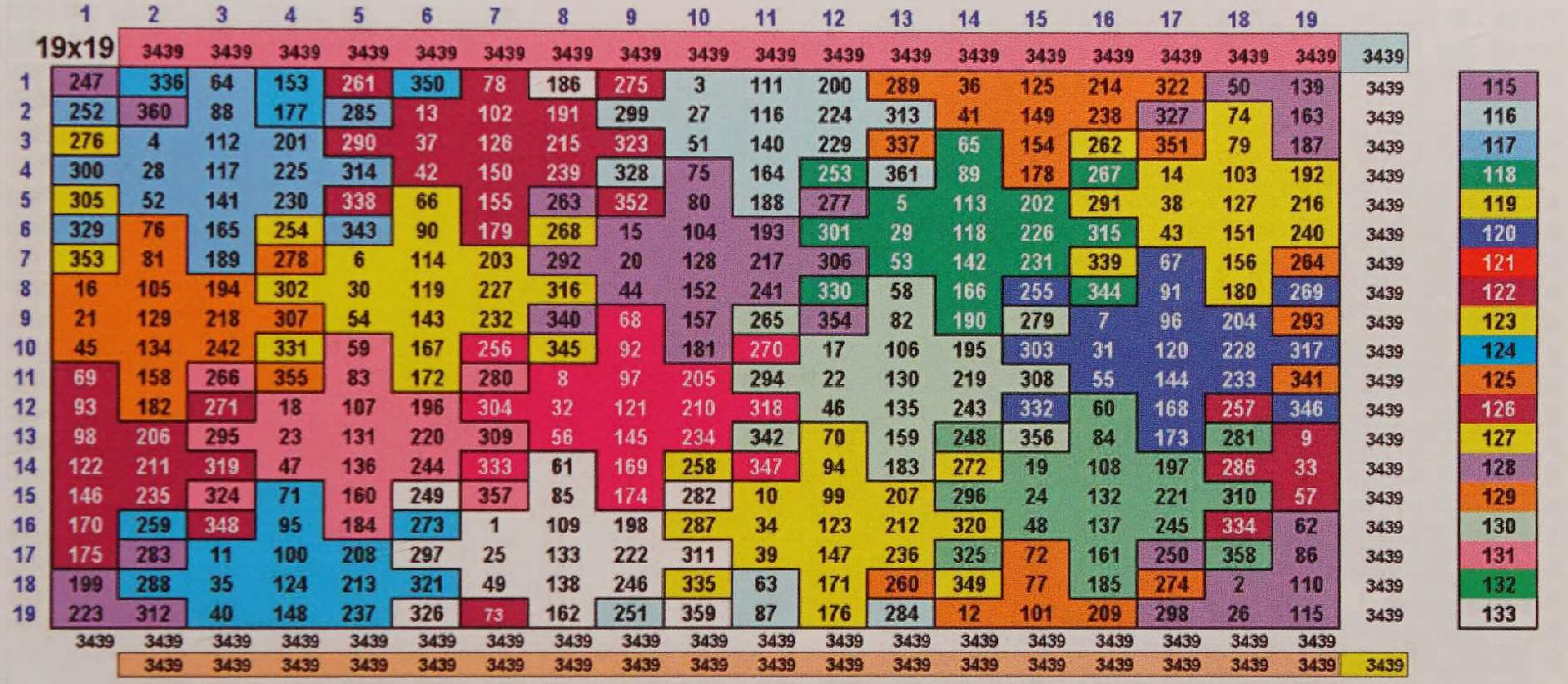
Size 19 Patterns

Here is the size **19** square with its dual tile patterns A and B identified by various colors. In the external strip on the right note that the center numbers in the tiles in each pattern run consecutively.



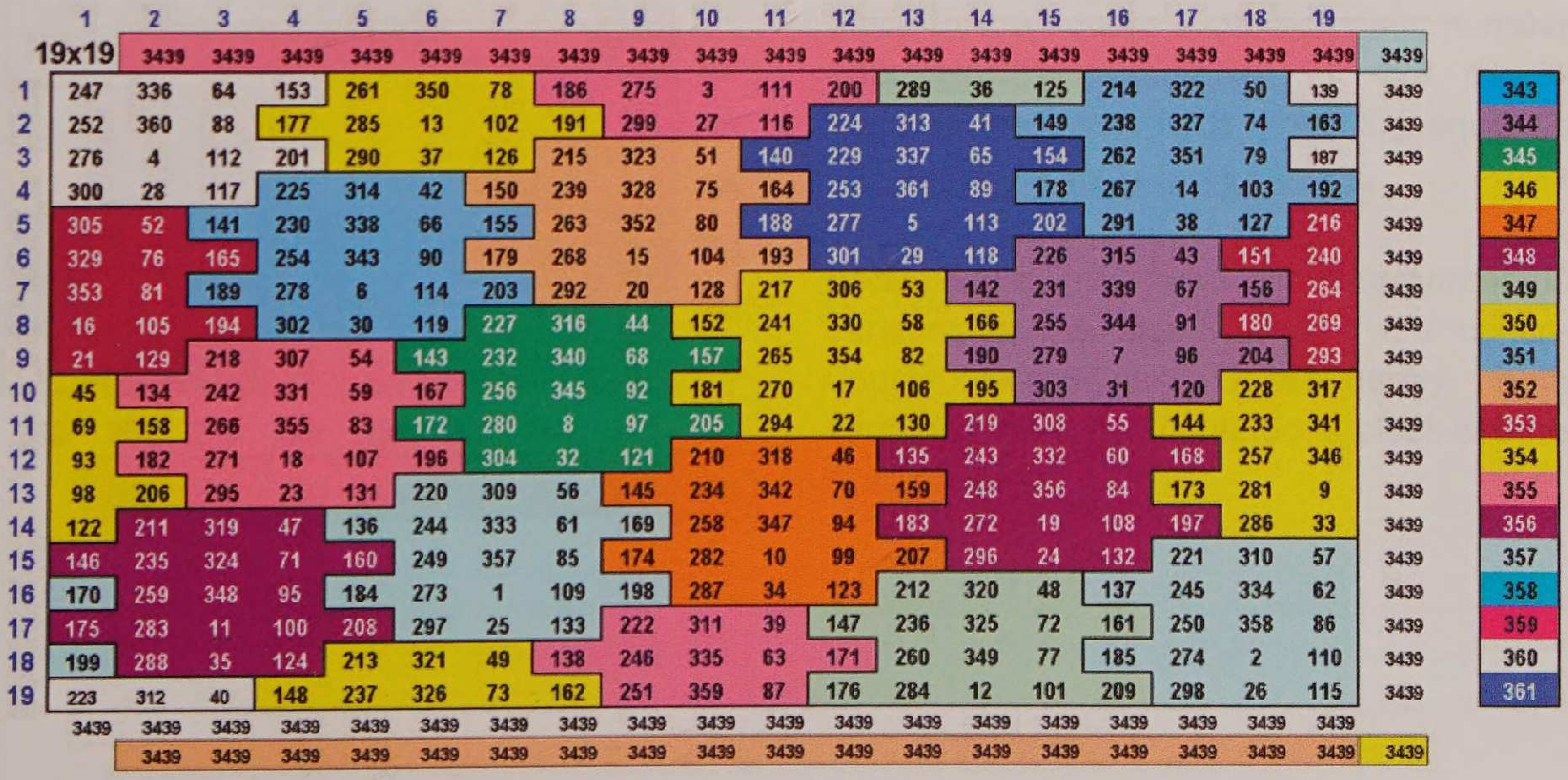






	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
19x19		3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	
1	247	336	64	153	261	350	78	186	275	3	111	200	289	36	125	214	322	50	139	3439	115
2	252	360	88	177	285	13	102	191	299	27	116	224	313	41	149	238	327	74	163	3439	116
3	276	4	112	201	290	37	126	215	323	51	140	229	337	65	154	262	351	79	187	3439	117
4	300	28	117	225	314	42	150	239	328	75	164	253	361	89	178	267	14	103	192	3439	118
5	305	52	141	230	338	66	155	263	352	80	188	277	5	113	202	291	38	127	216	3439	119
6	329	76	165	254	343	90	179	268	15	104	193	301	29	118	226	315	43	151	240	3439	120
7	353	81	189	278	6	114	203	292	20	128	217	306	53	142	231	339	67	156	264	3439	121
8	16	105	194	302	30	119	227	316	44	152	241	330	58	166	255	344	91	180	269	3439	122
9	21	129	218	307	54	143	232	340	68	157	265	354	82	190	279	7	96	204	293	3439	123
10	45	134	242	331	59	167	256	345	92	181	270	17	106	195	303	31	120	228	317	3439	124
11	69	158	266	355	83	172	280	8	97	205	294	22	130	219	308	55	144	233	341	3439	125
12	93	182	271	18	107	196	304	32	121	210	318	46	135	243	332	60	168	257	346	3439	126
13	98	206	295	23	131	220	309	56	145	234	342	70	159	248	356	84	173	281	9	3439	127
14	122	211	319	47	136	244	333	61	169	258	347	94	183	272	19	108	197	286	33	3439	128
15	146	235	324	71	160	249	357	85	174	282	10	99	207	296	24	132	221	310	57	3439	129
16	170	259	348	95	184	273	1	109	198	287	34	123	212	320	48	137	245	334	62	3439	130
17	175	283	11	100	208	297	25	133	222	311	39	147	236	325	72	161	250	358	86	3439	131
18	199	288	35	124	213	321	49	138	246	335	63	171	260	349	77	185	274	2	110	3439	132
19	223	312	40	148	237	326	73	162	251	359	87	176	284	12	101	209	298	26	115	3439	133
	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439		
		3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	

Individual patterns equal 3439 anywhere throughout table.

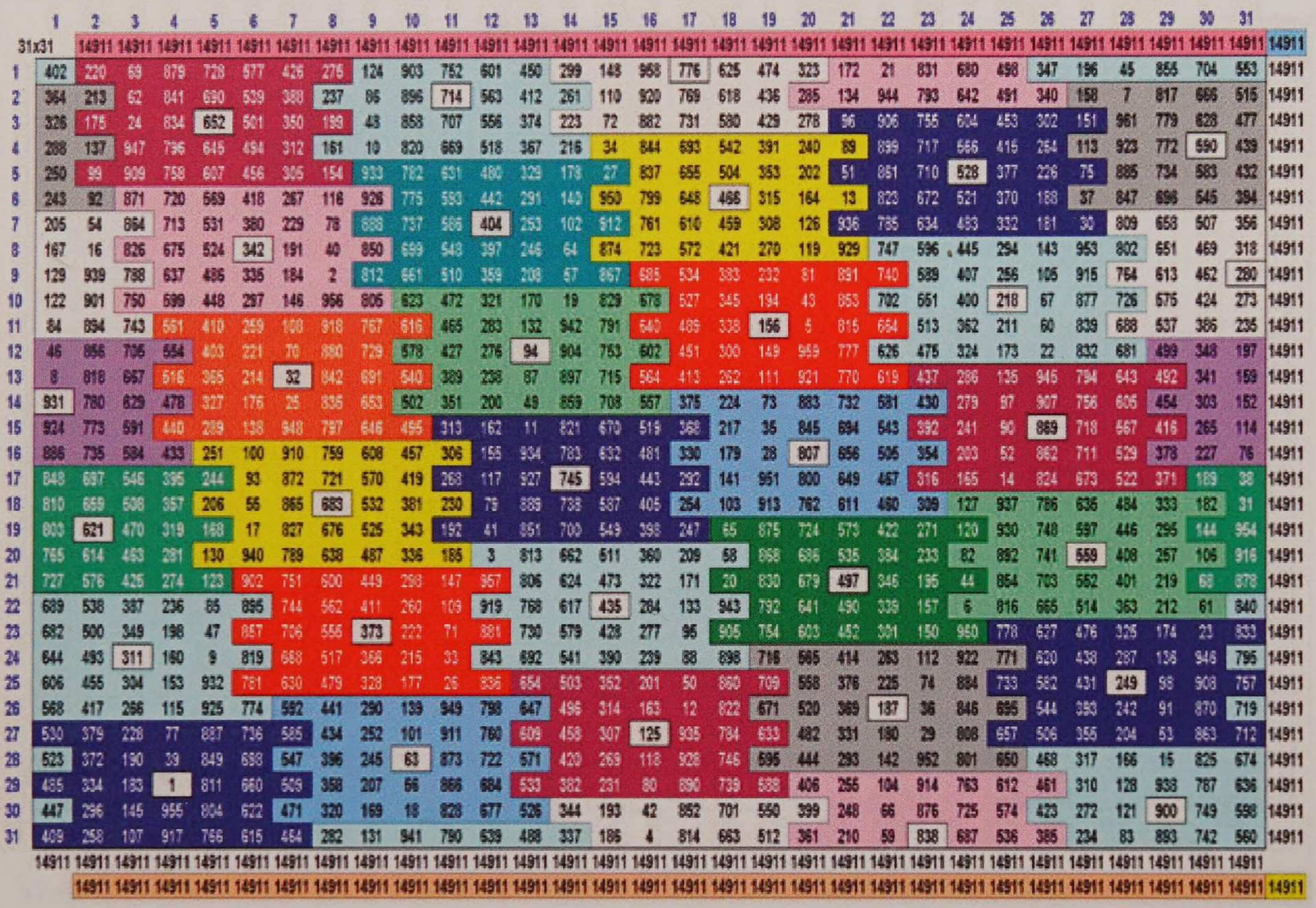


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		
19x19		3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	
1	247	336	64	153	261	350	78	186	275	3	111	200	289	36	125	214	322	50	139	3439	343
2	252	360	88	177	285	13	102	191	299	27	116	224	313	41	149	238	327	74	163	3439	344
3	276	4	112	201	290	37	126	215	323	51	140	229	337	65	154	262	351	79	187	3439	345
4	300	28	117	225	314	42	150	239	328	75	164	253	361	89	178	267	14	103	192	3439	346
5	305	52	141	230	338	66	155	263	352	80	188	277	5	113	202	291	38	127	216	3439	347
6	329	76	165	254	343	90	179	268	15	104	193	301	29	118	226	315	43	151	240	3439	348
7	353	81	189	278	6	114	203	292	20	128	217	306	53	142	231	339	67	156	264	3439	349
8	16	105	194	302	30	119	227	316	44	152	241	330	58	166	255	344	91	180	269	3439	350
9	21	129	218	307	54	143	232	340	68	157	265	354	82	190	279	7	96	204	293	3439	351
10	45	134	242	331	59	167	256	345	92	181	270	17	106	195	303	31	120	228	317	3439	352
11	69	158	266	355	83	172	280	8	97	205	294	22	130	219	308	55	144	233	341	3439	353
12	93	182	271	18	107	196	304	32	121	210	318	46	135	243	332	60	168	257	346	3439	354
13	98	206	295	23	131	220	309	56	145	234	342	70	159	248	356	84	173	281	9	3439	355
14	122	211	319	47	136	244	333	61	169	258	347	94	183	272	19	108	197	286	33	3439	356
15	146	235	324	71	160	249	357	85	174	282	10	99	207	296	24	132	221	310	57	3439	357
16	170	259	348	95	184	273	1	109	198	287	34	123	212	320	48	137	245	334	62	3439	358
17	175	283	11	100	208	297	25	133	222	311	39	147	236	325	72	161	250	358	86	3439	359
18	199	288	35	124	213	321	49	138	246	335	63	171	260	349	77	185	274	2	110	3439	360
19	223	312	40	148	237	326	73	162	251	359	87	176	284	12	101	209	298	26	115	3439	361
	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439		
		3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	3439	

Program 2

Size 31 Pattern B

Here is the size **31** square with its B tiling pattern. Pattern A will be shown in the Part II of this program.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
31x31		14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911
1	402	220	69	879	728	577	426	275	124	903	752	601	450	299	148	958	776	625	474	323	172	21	831	680	498	347	196	45	855	704	553	14911
2	364	213	62	841	690	539	388	237	86	896	714	563	412	261	110	920	769	618	436	285	134	944	793	642	491	340	158	7	817	666	515	14911
3	326	175	24	834	652	501	350	199	48	858	707	556	374	223	72	882	731	580	429	278	96	906	755	604	453	302	151	961	779	628	477	14911
4	288	137	947	796	645	494	312	161	10	820	669	518	367	216	34	844	693	542	391	240	89	899	717	566	415	264	113	923	772	590	439	14911
5	250	99	909	758	607	456	305	154	933	782	631	480	329	178	27	837	655	504	353	202	51	861	710	528	377	226	75	885	734	583	432	14911
6	243	92	871	720	569	418	267	116	926	775	593	442	291	140	950	799	648	466	315	164	13	823	672	521	370	188	37	847	696	545	394	14911
7	205	54	864	713	531	380	229	78	888	737	586	404	253	102	912	761	610	459	308	126	936	785	634	483	332	181	30	809	658	507	356	14911
8	167	16	826	675	524	342	191	40	850	699	548	397	246	64	874	723	572	421	270	119	929	747	596	445	294	143	953	802	651	469	318	14911
9	129	939	788	637	486	335	184	2	812	661	510	359	208	57	867	685	534	383	232	81	891	740	589	407	256	105	915	764	613	462	280	14911
10	122	901	750	599	448	297	146	956	805	623	472	321	170	19	829	678	527	345	194	43	853	702	551	400	218	67	877	726	575	424	273	14911
11	84	894	743	561	410	259	108	918	767	616	465	283	132	942	791	640	489	338	156	5	815	664	513	362	211	60	839	688	537	386	235	14911
12	46	856	705	554	403	221	70	880	729	578	427	276	94	904	753	602	451	300	149	959	777	626	475	324	173	22	832	681	499	348	197	14911
13	8	818	667	516	365	214	32	842	691	540	389	238	87	897	715	564	413	262	111	921	770	619	437	286	135	945	794	643	492	341	159	14911
14	931	780	629	478	327	176	25	835	653	502	351	200	49	859	708	557	375	224	73	883	732	581	430	279	97	907	756	605	454	303	152	14911
15	924	773	591	440	289	138	948	797	646	495	313	162	11	821	670	519	368	217	35	845	694	543	392	241	90	869	718	567	416	265	114	14911
16	886	735	584	433	251	100	910	759	608	457	306	155	934	783	632	481	330	179	28	807	656	505	354	203	52	862	711	529	378	227	76	14911
17	848	697	546	395	244	93	872	721	570	419	268	117	927	745	594	443	292	141	951	800	649	467	316	165	14	824	673	522	371	189	38	14911
18	810	659	508	357	206	55	865	683	532	381	230	79	889	738	587	405	254	103	913	762	611	460	309	127	937	786	635	484	333	182	31	14911
19	803	621	470	319	168	17	827	676	525	343	192	41	851	700	549	398	247	65	875	724	573	422	271	120	930	748	597	446	295	144	954	14911
20	765	614	463	281	130	940	789	638	487	336	185	3	813	662	511	360	209	58	868	686	535	384	233	82	892	741	559	408	257	106	916	14911
21	727	576	425	274	123	902	751	600	449	298	147	957	806	624	473	322	171	20	830	679	497	346	195	44	854	703	552	401	219	68	878	14911
22	689	538	387	236	85	895	744	562	411	260	109	919	768	617	435	284	133	943	792	641	490	339	157	6	816	665	514	363	212	61	840	14911
23	682	500	349	198	47	857	706	555	373	222	71	881	730	579	428	277	95	905	754	603	452	301	150	960	778	627	476	325	174	23	833	14911
24	644	493	311	160	9	819	668	517	366	215	33	843	692	541	390	239	88	898	716	565	414	263	112	922	771	620	438	287	136	946	795	14911
25	606	455	304	153	932	781	630	479	328	177	26	836	654	503	352	201	50	860	709	558	376	225	74	884	733	582	431	249	98	908	757	14911
26	568	417	266	115	925	774	592	441	290	139	949	798	647	496	314	163	12	822	671	520	369	187	36	846	695	544	393	242	91	870	719	14911
27	530	379	228	77	887	736	585	434	252	101	911	760	609	458	307	125	935	784	633	482	331	180	29	808	657	506	355	204	53	863	712	14911
28	523	372	190	39	849	698	547	396	245	63	873	722	571	420	269	118	928	746	595	444	293	142	952	801	650	468	317	166	15	825	674	14911
29	485	334	183	1	811	660	509	358	207	56	866	684	533	382	231	80	890	739	588	406	255	104	914	763	612	461	310	128	938	787	636	14911
30	447	296	145	955	804	622	471	320	169	18	828	677	526	344	193	42	852	701	550	399	248	66	876	725	574	423	272	121	900	749	598	14911
31	409	258	107	917	766	615	464	282	131	941	790	639	488	337	186	4	814	663	512	361	210	59	838	687	536	385	234	83	893	742	560	14911
	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	
		14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911

Program 2

Tiling Patterns for Class 4 Squares

These patterns are radically different from those for Class-1 squares.

Here, the tiling patterns are **diamonds** and **X-patterns**, indistinguishable from each other.

Sizes divisible by **8** have only one of their two equal-summing pairwise versions with a continuous tiling pattern.

All other sizes of Class-4 squares are not pairwise equal-summing yet are still pangenic. They too have continuous tiling patterns.

Size 8 Patterns

Because the size-4 square's tiling pattern is the entire square itself, here is the smallest perfect Class-4 square with a distinct tiling pattern; it's the size-8.

Diamond pattern

260	260	260	260	260
196	292	260	228	324
260	260	260	260	260
324	228	260	292	196
260	260	260	260	260

+

X-pattern

260	260	260	260	260
324	228	260	292	196
260	260	260	260	260
196	292	260	228	324
260	260	260	260	260

=

Diamond+X-patterns

520	520	520	520	520
520	520	520	520	520
520	520	520	520	520
520	520	520	520	520
520	520	520	520	520

Row-pair symmetric version

8x8	260	260	260	260	260	260	260	260
24	9	8	25	40	57	56	41	260
58	39	42	55	10	23	26	7	260
14	19	30	3	62	35	46	51	260
36	61	52	45	20	13	4	29	260
17	16	1	32	33	64	49	48	260
63	34	47	50	15	18	31	2	260
11	22	27	6	59	38	43	54	260
37	60	53	44	21	12	5	28	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260

It has neither diamond pattern ◊ nor X-pattern which by themselves sum continuously to the square's characteristic number **260**. However, together in **4x4** block-squares they sum to **2** times the square's characteristic number **520** = **$2C_8$**

Both pairwise row- and centrally-symmetric versions of the size-8 square are continuously 4x-modular. Consequently, both vers-ions have this continuous block-square tiling property.

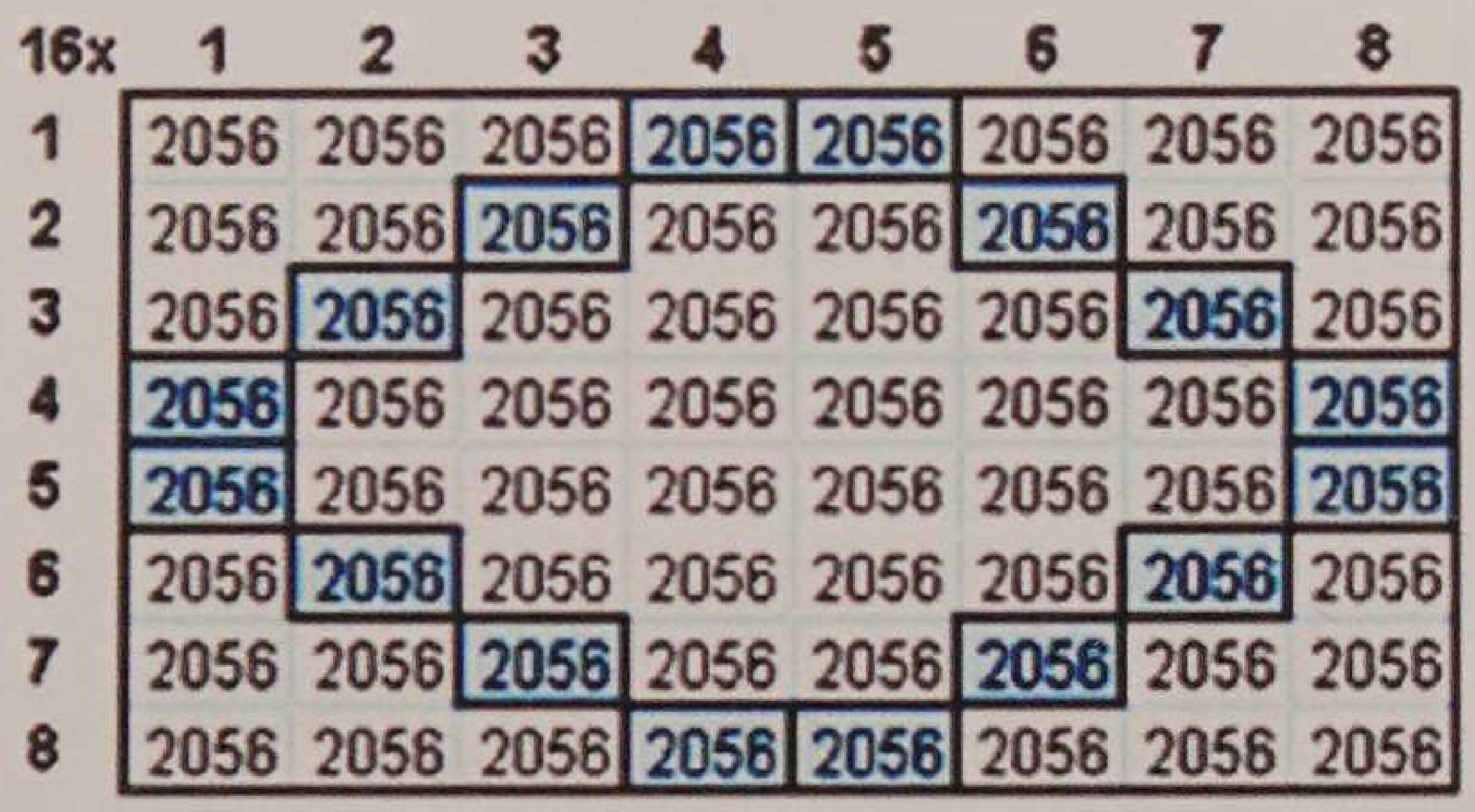
Program 2

Size 16 Patterns

Here is the size-16 square of Class-4.

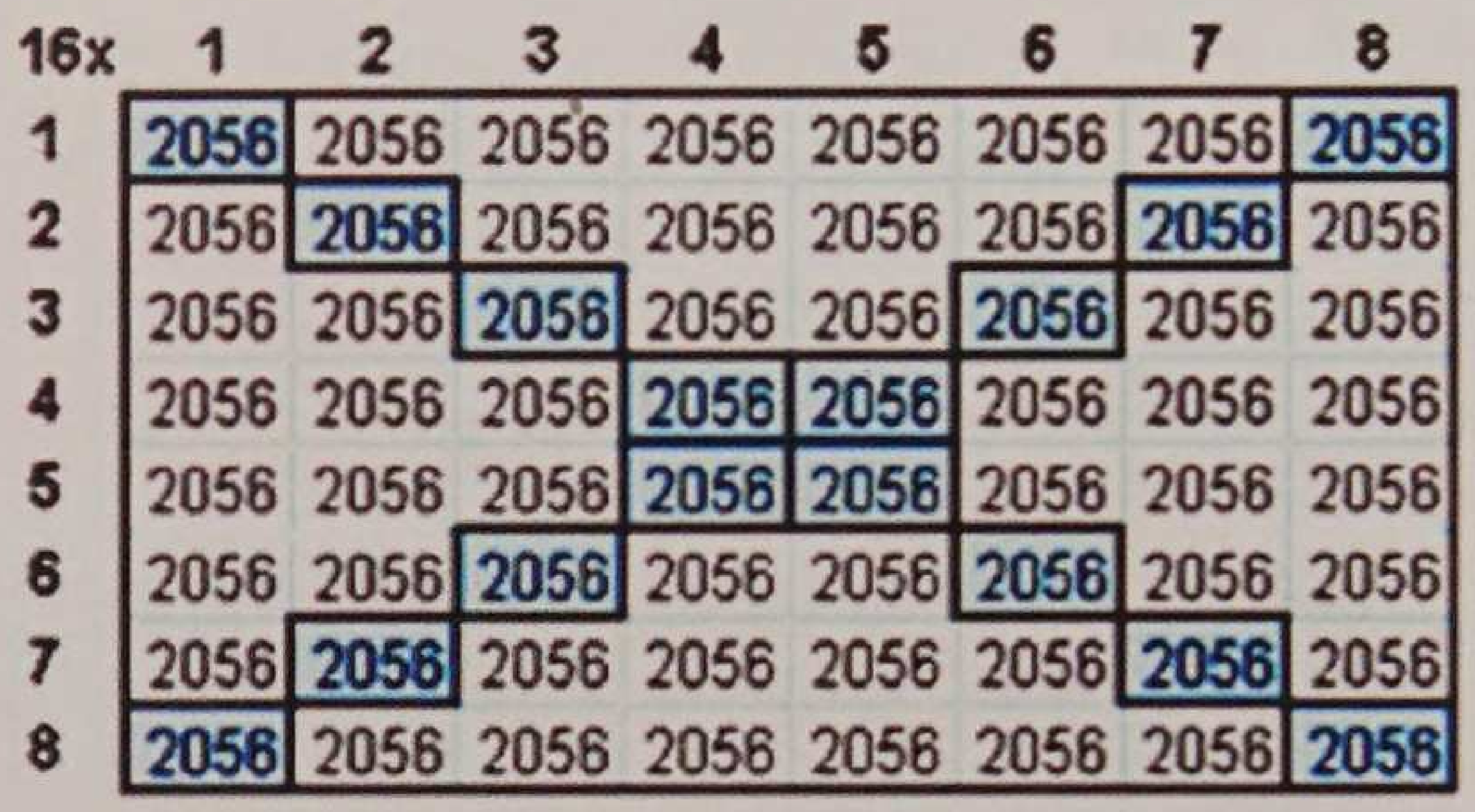
It has two independent equal-summing tile patterns, a diamond pattern ◊ and an X pattern, each of which consists of **16** cells which sum to the squares characteristic number, **2056 = C16**, everywhere. The tiled square shows the interlocking diamond pattern. Note that if the diamond-tiling pattern were shifted by four columns either way, left or right, its tiling pattern would be indistinguishable from that of the interlocking X-pattern.

◇ Pattern



16x	1	2	3	4	5	6	7	8
1	2056	2056	2056	2056	2056	2056	2056	2056
2	2056	2056	2056	2056	2056	2056	2056	2056
3	2056	2056	2056	2056	2056	2056	2056	2056
4	2056	2056	2056	2056	2056	2056	2056	2056
5	2056	2056	2056	2056	2056	2056	2056	2056
6	2056	2056	2056	2056	2056	2056	2056	2056
7	2056	2056	2056	2056	2056	2056	2056	2056
8	2056	2056	2056	2056	2056	2056	2056	2056

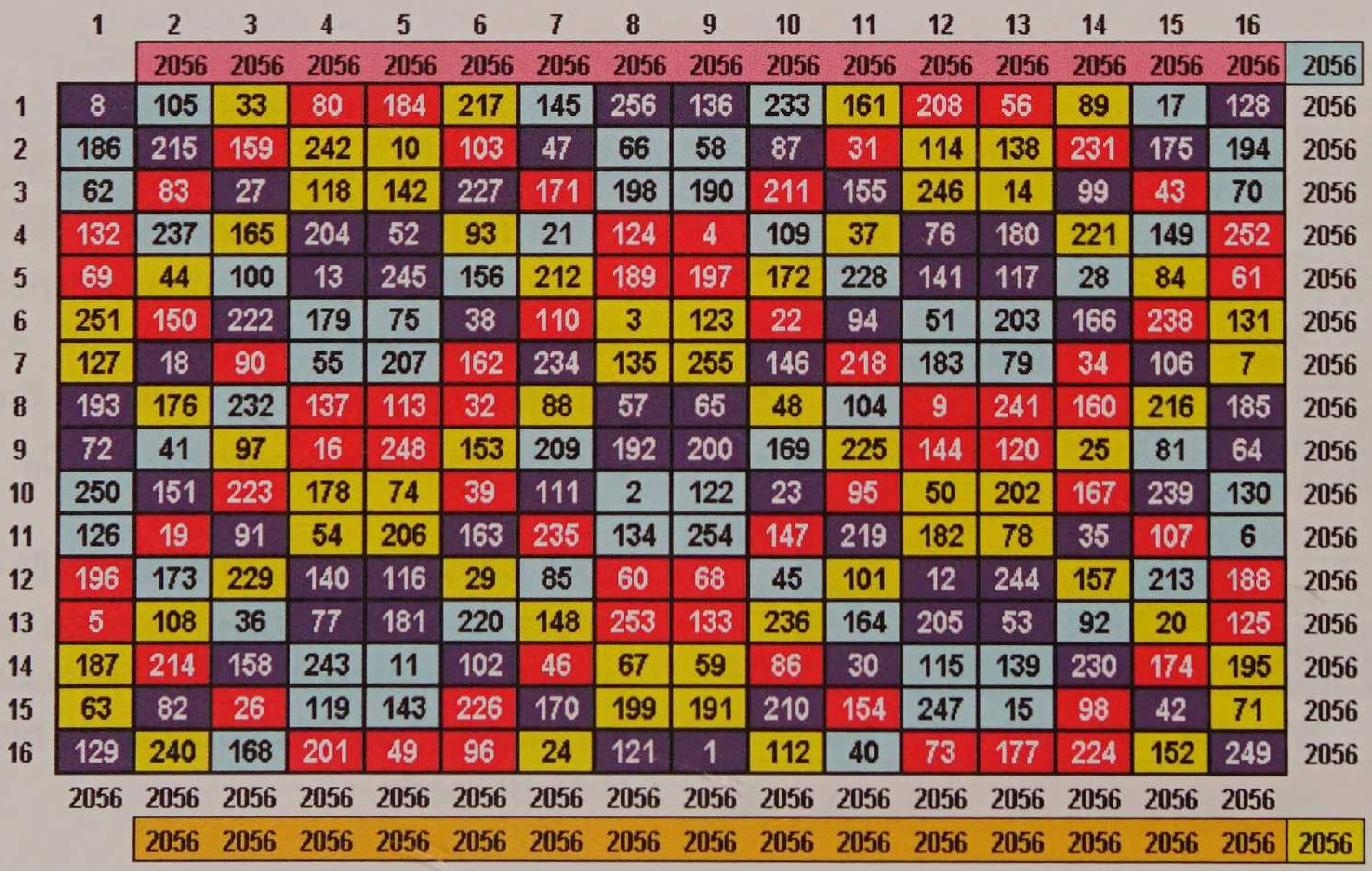
X Pattern



16x	1	2	3	4	5	6	7	8
1	2056	2056	2056	2056	2056	2056	2056	2056
2	2056	2056	2056	2056	2056	2056	2056	2056
3	2056	2056	2056	2056	2056	2056	2056	2056
4	2056	2056	2056	2056	2056	2056	2056	2056
5	2056	2056	2056	2056	2056	2056	2056	2056
6	2056	2056	2056	2056	2056	2056	2056	2056
7	2056	2056	2056	2056	2056	2056	2056	2056
8	2056	2056	2056	2056	2056	2056	2056	2056

Although distinct, each pattern converts into the other because the tiling pattern is continuous; it can be dragged across the table in any of four directions: horizontally, vertically or diagonally and all the individual tile summations remain the same.

Pairwise centrally symmetric version



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Of the two versions of the size-16 square, it was found that only the centrally pairwise symmetric version had this tiling property.

Program 2

Size 12 Patterns

Here is the size **12** square with its characteristic diamond and X-pattern tiles. Both patterns sum to **870 = C$_{12}$**. The size-12's tile-pattern continuity is confined to just four vertical sections, one straddling the middle vertical centerline, two abutting each edge and one straddling the left and right edges. This is the only Class-4 square with any segmented tiling patterns.

This size-12 square is pairwise row-symmetric and consequently pangenic.

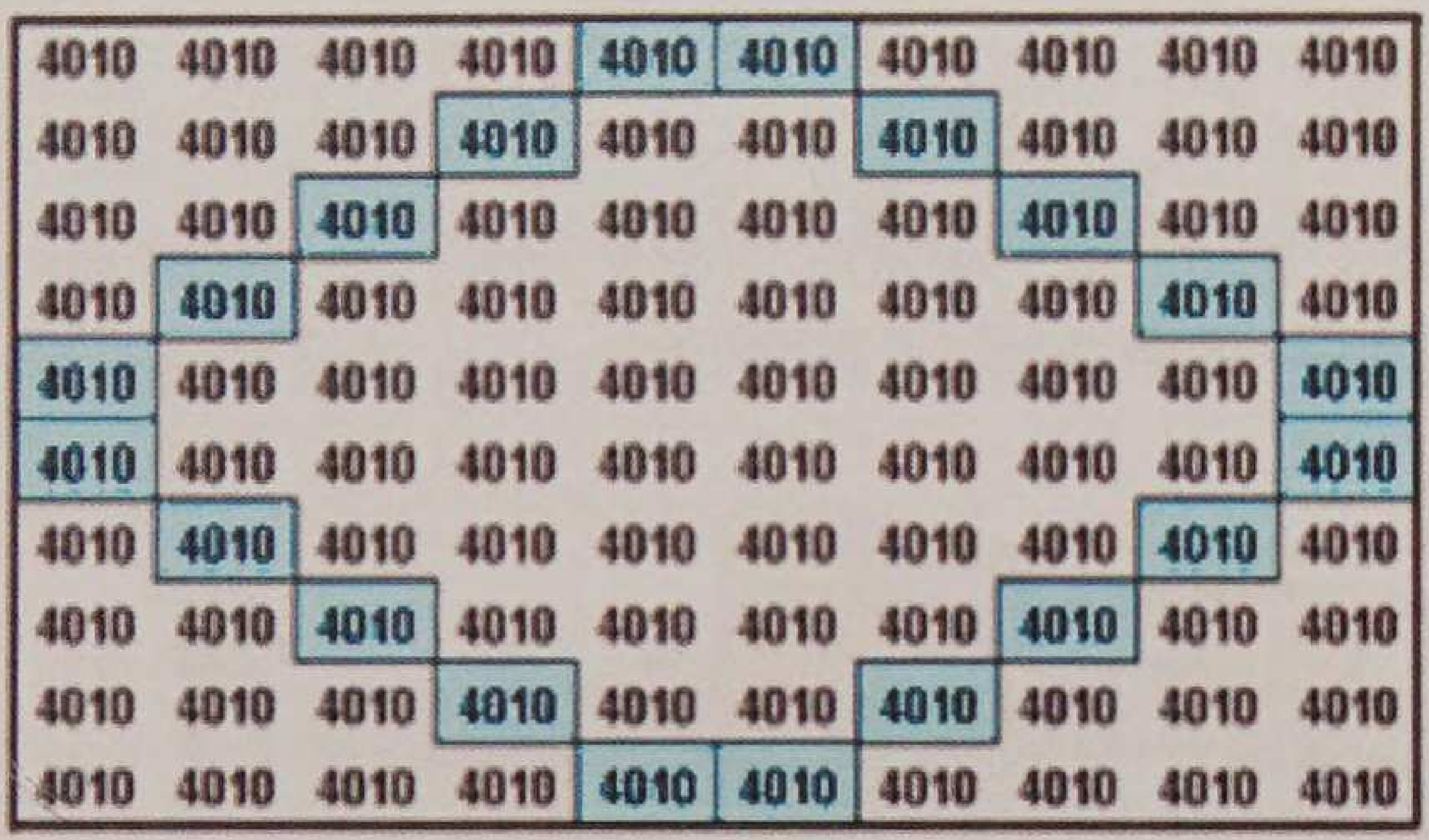
870	906	834	870	906	834	870	906	834	870
870	831	867	870	873	909	870	831	867	870
870	903	873	870	867	837	870	903	873	870
870	840	900	870	840	900	870	840	900	870
870	867	837	870	903	873	870	867	837	870
870	867	831	870	909	873	870	867	831	870
870	834	906	870	834	906	870	834	906	870

	1	2	3	4	5	6	7	8	9	10	11	12	
		870	870	870	870	870	870	870	870	870	870	870	870
1	67	84	49	24	133	6	139	12	121	96	61	78	870
2	107	41	128	89	32	110	35	113	56	17	104	38	870
3	69	100	51	22	117	4	141	28	123	94	45	76	870
4	75	46	93	124	27	142	3	118	21	52	99	70	870
5	44	98	59	14	119	29	116	26	131	86	47	101	870
6	73	66	91	126	7	144	1	138	19	54	79	72	870
7	31	120	13	60	97	42	103	48	85	132	25	114	870
8	140	2	95	122	71	77	68	74	23	50	143	5	870
9	33	136	15	58	81	40	105	64	87	130	9	112	870
10	111	10	129	88	63	106	39	82	57	16	135	34	870
11	11	137	20	53	80	62	83	65	92	125	8	134	870
12	109	30	127	90	43	108	37	102	55	18	115	36	870
	870	870	870	870	870	870	870	870	870	870	870	870	
		870	870	870	870	870	870	870	870	870	870	870	870

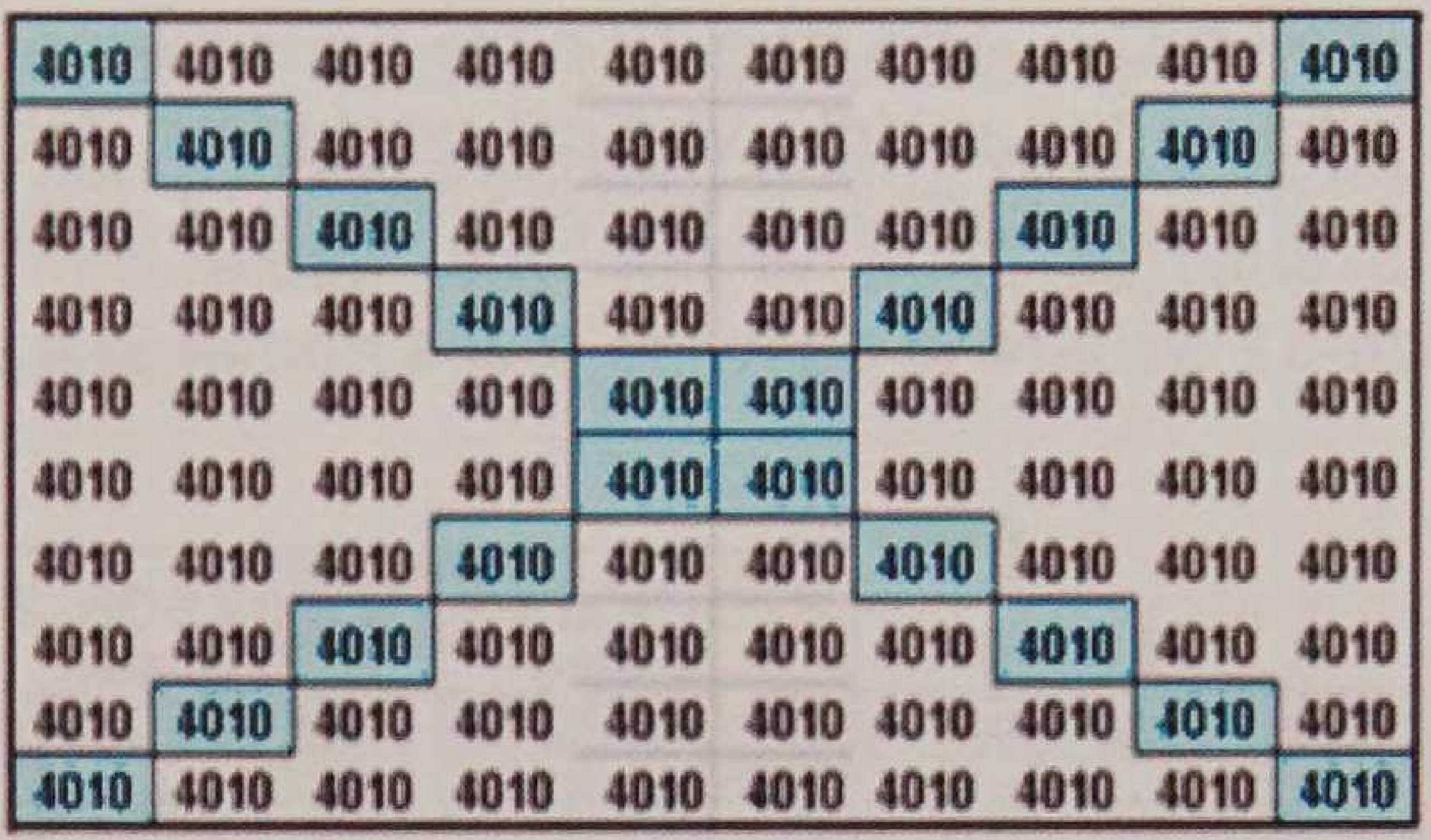
Program 2

Size 20 Patterns

Here is the size **20** square with diamond and X-pattern tiles. Each tile consists of **20** cells.

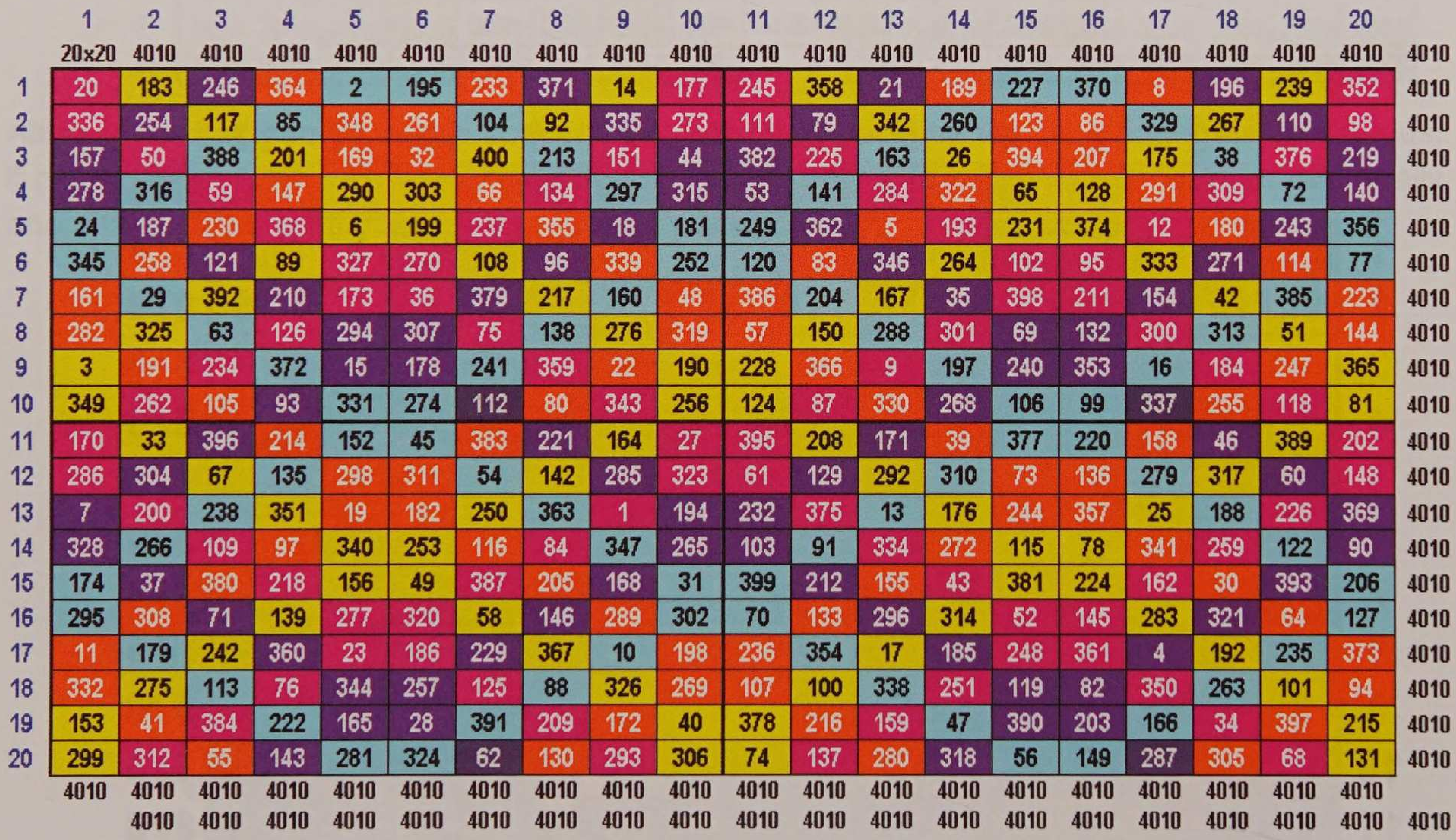


4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010



4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

The square is neither totally pairwise row nor centrally symmetric. Yet it is pangenic. Each diamond and X-pattern sums to **4010 = C_{20}** continuously.

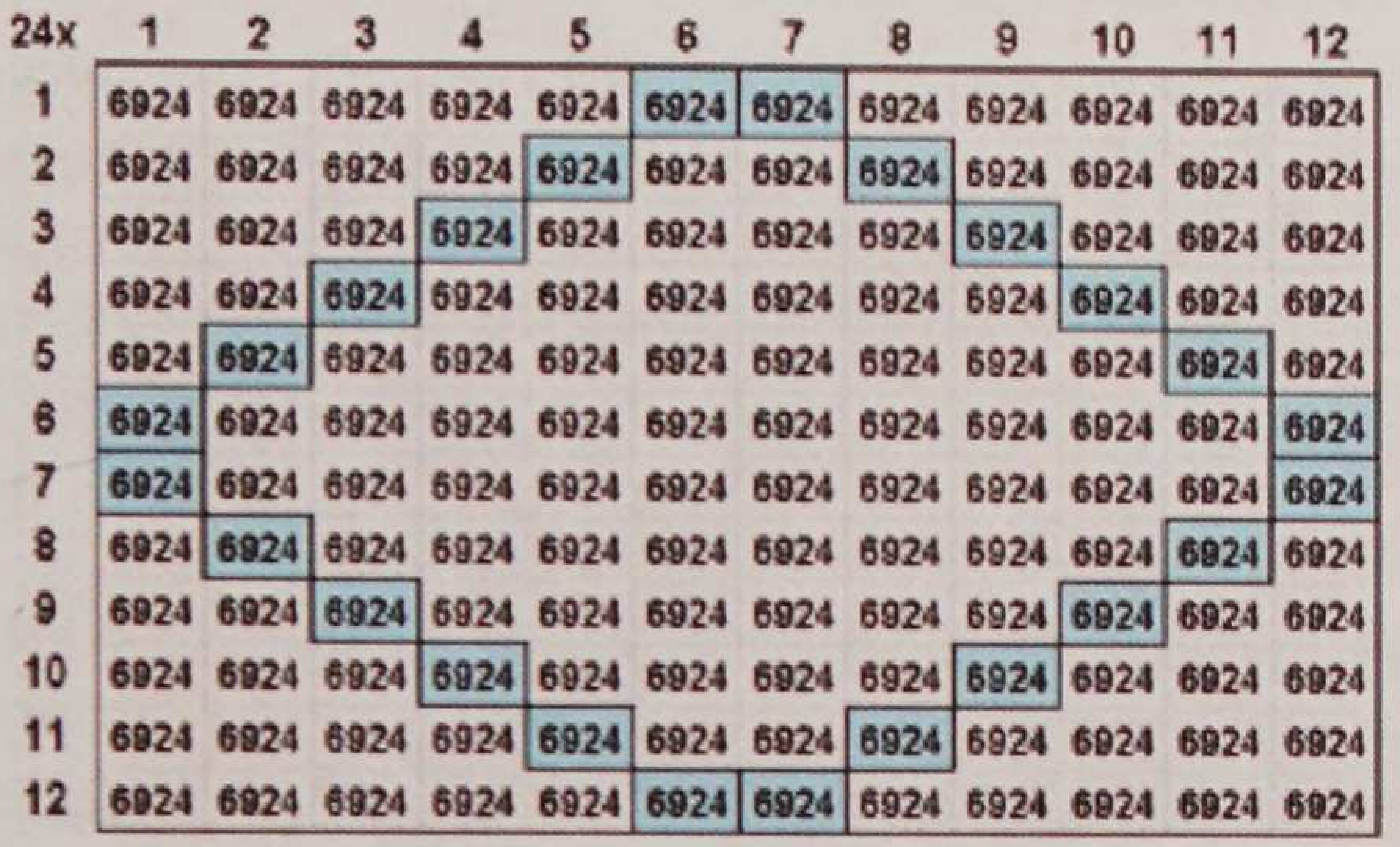


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	20x20	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	246	364	2	195	233	371	14	177	245	358	21	189	227	370	8	196	239	352	4010
2	336	254	117	85	348	261	104	92	335	273	111	79	342	260	123	86	329	267	110	98	4010
3	157	50	388	201	169	32	400	213	151	44	382	225	163	26	394	207	175	38	376	219	4010
4	278	316	59	147	290	303	66	134	297	315	53	141	284	322	65	128	291	309	72	140	4010
5	24	187	230	368	6	199	237	355	18	181	249	362	5	193	231	374	12	180	243	356	4010
6	345	258	121	89	327	270	108	96	339	252	120	83	346	264	102	95	333	271	114	77	4010
7	161	29	392	210	173	36	379	217	160	48	386	204	167	35	398	211	154	42	385	223	4010
8	282	325	63	126	294	307	75	138	276	319	57	150	288	301	69	132	300	313	51	144	4010
9	3	191	234	372	15	178	241	359	22	190	228	366	9	197	240	353	16	184	247	365	4010
10	349	262	105	93	331	274	112	80	343	256	124	87	330	268	106	99	337	255	118	81	4010
11	170	33	396	214	152	45	383	221	164	27	395	208	171	39	377	220	158	46	389	202	4010
12	286	304	67	135	298	311	54	142	285	323	61	129	292	310	73	136	279	317	60	148	4010
13	7	200	238	351	19	182	250	363	1	194	232	375	13	176	244	357	25	188	226	369	4010
14	328	266	109	97	340	253	116	84	347	265	103	91	334	272	115	78	341	259	122	90	4010
15	174	37	380	218	156	49	387	205	168	31	399	212	155	43	381	224	162	30	393	206	4010
16	295	308	71	139	277	320	58	146	289	302	70	133	296	314	52	145	283	321	64	127	4010
17	11	179	242	360	23	186	229	367	10	198	236	354	17	185	248	361	4	192	235	373	4010
18	332	275	113	76	344	257	125	88	326	269	107	100	338	251	119	82	350	263	101	94	4010
19	153	41	384	222	165	28	391	209	172	40	378	216	159	47	390	203	166	34	397	215	4010
20	299	312	55	143	281	324	62	130	293	306	74	137	280	318	56	149	287	305	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

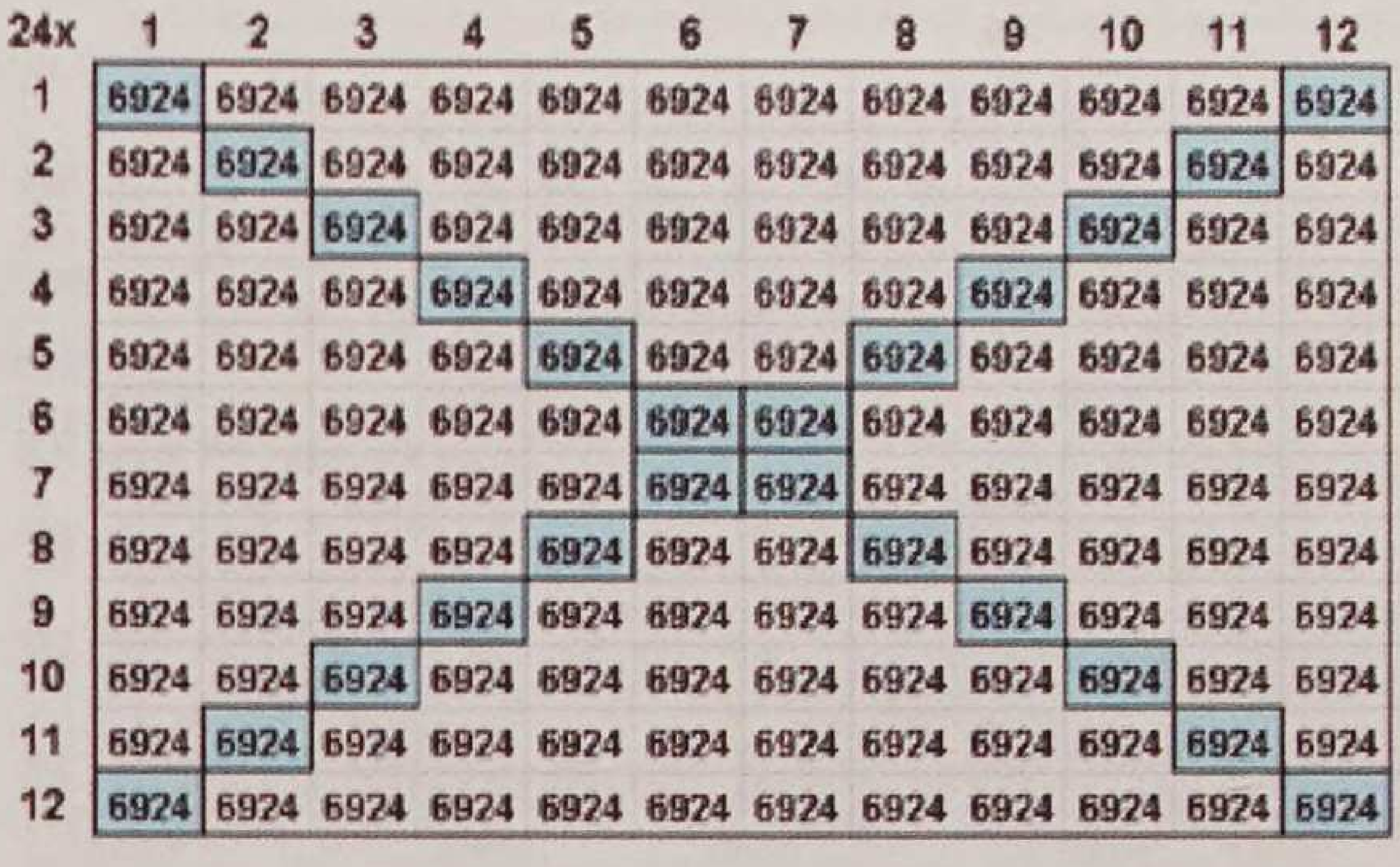
Program 2

Size 24 Patterns

These same **diamond** and **X** tile-patterns are found in the size-24 square too. Here, each tile consists of **24** cells.



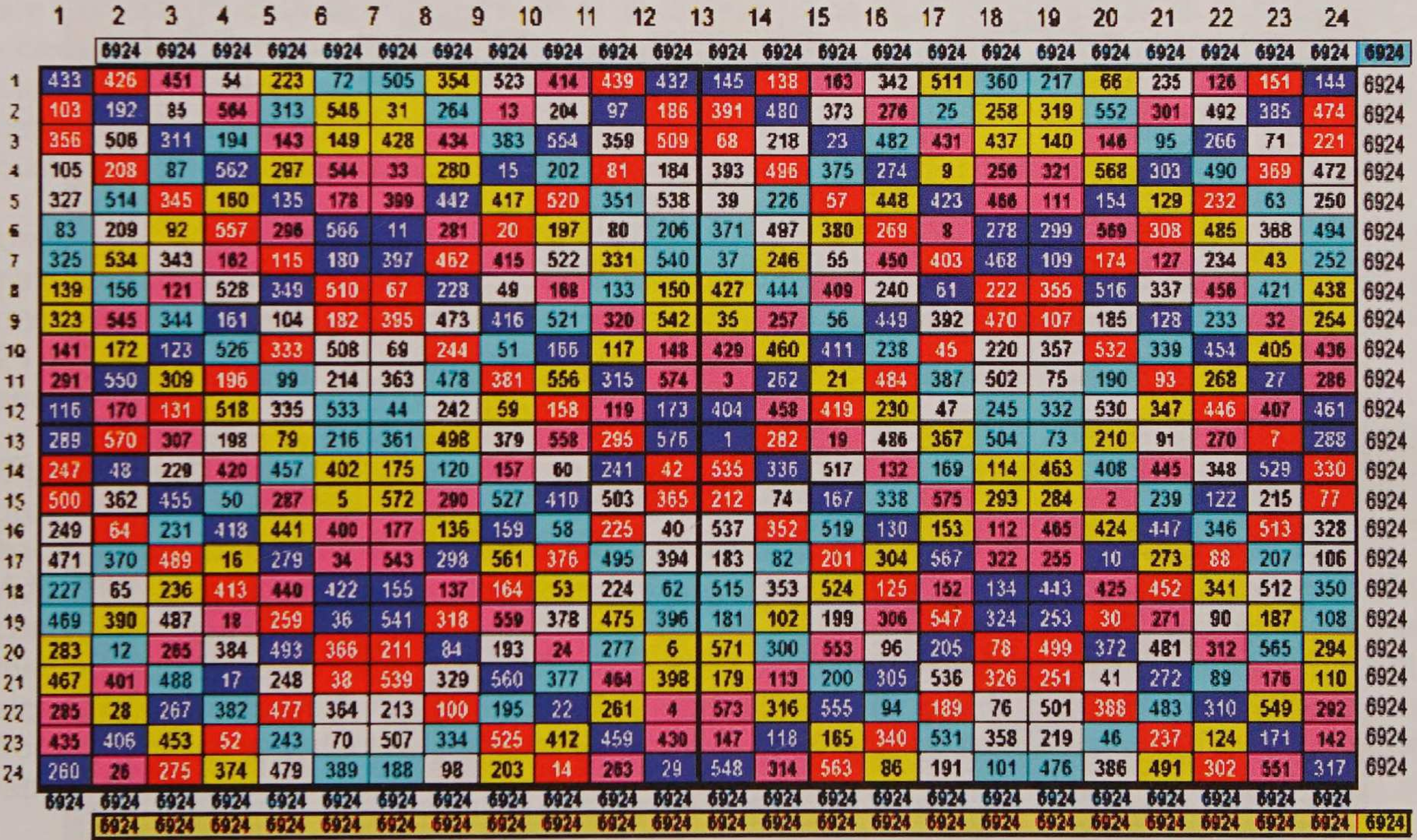
24x	1	2	3	4	5	6	7	8	9	10	11	12
1	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
2	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
3	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
4	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
5	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
6	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
7	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
8	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
9	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
10	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
11	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
12	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924



24x	1	2	3	4	5	6	7	8	9	10	11	12
1	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
2	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
3	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
4	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
5	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
6	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
7	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
8	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
9	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
10	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
11	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
12	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

The square is pairwise row-symmetric.

Each diamond and X-pattern sums to **6924** = C_{24} continuously.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	505	354	523	414	439	432	145	138	163	342	511	360	217	66	235	126	151	144	6924
2	103	192	85	564	313	548	31	264	13	204	97	186	391	480	373	276	25	258	319	552	301	492	385	474	6924
3	356	506	311	194	143	149	428	434	383	554	359	509	68	218	23	482	431	437	140	146	95	266	71	221	6924
4	105	208	87	562	297	544	33	280	15	202	81	184	393	496	375	274	9	256	321	568	303	490	369	472	6924
5	327	514	345	160	135	178	399	442	417	520	351	538	39	226	57	448	423	466	111	154	129	232	63	250	6924
6	83	209	92	557	296	566	11	281	20	197	80	206	371	487	380	269	8	278	299	569	308	485	368	494	6924
7	325	534	343	162	115	180	397	462	415	522	331	540	37	246	55	450	403	468	109	174	127	234	43	252	6924
8	139	156	121	528	349	510	67	228	48	168	133	150	427	444	409	240	61	222	355	516	337	456	421	438	6924
9	323	545	344	161	104	182	395	473	416	521	320	542	35	257	56	449	392	470	107	185	128	233	32	254	6924
10	141	172	123	526	333	508	69	244	51	166	117	148	429	460	411	238	45	220	357	532	339	454	405	436	6924
11	291	550	309	196	99	214	363	478	381	556	315	574	3	262	21	484	387	502	75	190	93	268	27	286	6924
12	116	170	131	518	335	533	44	242	59	158	119	173	404	458	419	230	47	245	332	530	347	446	407	461	6924
13	289	570	307	198	79	216	361	498	379	558	295	576	1	282	19	486	367	504	73	210	91	270	7	288	6924
14	247	48	229	420	457	402	175	120	157	60	241	42	535	336	517	132	169	114	463	408	445	348	529	330	6924
15	500	362	455	50	287	5	572	290	527	410	503	365	212	74	167	338	575	293	284	2	239	122	215	77	6924
16	249	64	231	418	441	400	177	136	159	58	225	40	537	352	519	130	153	112	465	424	447	346	513	328	6924
17	471	370	489	16	279	34	543	298	561	376	495	394	183	82	201	304	567	322	255	10	273	88	207	106	6924
18	227	65	236	413	440	422	155	137	164	53	224	62	515	353	524	125	152	134	443	425	452	341	512	350	6924
19	469	390	487	18	259	36	541	318	559	378	475	396	181	102	199	306	547	324	253	30	271	90	187	108	6924
20	283	12	265	384	493	366	211	84	193	24	277	6	571	300	553	96	205	78	499	372	481	312	565	294	6924
21	467	401	488	17	248	38	539	329	560	377	464	398	179	113	200	305	536	326	251	41	272	89	176	110	6924
22	285	28	267	382	477	364	213	100	195	22	261	4	573	316	555	94	189	76	501	388	483	310	549	292	6924
23	435	406	453	52	243	70	507	334	525	412	459	430	147	118	165	340	531	358	219	46	237	124	171	142	6924
24	260	26	275	374	479	389	188	98	203	14	263	29	548	314	563	86	191	101	476	386	491	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Unlike the size-16 square with these same tiling patterns that is only pairwise centrally-symmetric, the size-24 square here, whose size is also divisible by **8**, has equal pairwise <u>row-symmetric</u> sums. That is the only version of the size-24 square to be geonomically perfect and to possess these continuously equal-summing tiling patterns. There is a size-24 version with equal pairwise centrally-symmetric sums, but it is shy of being perfect because some of its wrap diagonals are unequal and consequently it lacks having either of these tile patterns summing equally continuously throughout the square.

Program 2

Size 28 Patterns

Here is the size **28** square with **diamond** and **X**-pattern tiles of **28** cells each. Its characteristic number is **10990**. Each diamond and X-pattern sums to **10990** continuously. The square is neither row nor centrally pairwise symmetric, yet it is nonetheless pangenic.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990

At right is a summary table of Class-4 tiling patterns. Note that only the patterns for size-4 and size-8 are unusual. This stems from its small size having an insufficient amount of consecutive numbers to yield distinct tiling patterns that by themselves can sum to the square's characteristic number. That is, for this tiling property, these sizes are *embryonic*.

Summary of Class-4 Tiling Patterns		
Size	**◊ & X Pattern**	**Pairwise symmetry with tiling pattern**
4	◊ & X in tight 2x2 block-squares	**No pair symmetry**
8	◊ + X in tight 4x4 block-squares equal 2x char. no.	**Both row & centrally symmetric**
12	**Both ◊ & X**	**Row symmetric**
16	**Both ◊ & X**	**Centrally symmetric**
20	**Both ◊ & X**	**No pair symmetry**
24	**Both ◊ & X**	**Row symmetric**
28	**Both ◊ & X**	**No pair symmetry**

However, the two tile patterns together are continuous as a block-square of size **n/2xn/2** which do sum to the squares characteristic number for size-4 and 2 times the characteristic number for size-8. This on its own constitutes **n/2**continuous modularity.

Both pairwise row-symmetric and centrally-symmetric versions of the size-8 square are continuously 4x-modular. Consequently, both versions have this tiling property by default.

That differs from the size-16 square. Although there are two categorical versions of the size-16 square -- one with equal row-pair sums and one with equal centrally-symmetric sums – only the version with centrally-symmetric pair-sums has these continuously characteristic diamond and X tiling patterns.

Unlike the size-16 square with these same tiling patterns, the size-24 square has equal symmetric row-pair sums. That is the only version of the size-24 square to be geonomically perfect and to possess these equal-summing tiling patterns.

Squares of size **n = 4b**, where **b** is an odd number greater than **3**, have no pairwise symmetry. That is because only those size squares obtained by the *Add Tile Expansions* (ATE) method, described in Program 3 and again in Program 8, have the continuous diamond tiling pattern. In addition, for all such squares greater than size-12, that method eliminates such pairwise symmetry while retaining the pangenicity of the two different-size squares involved in its composition. Those sizes have no pair-wise symmetry. This stems from the use of imperfect size squares such as size **10** and **14** as tiles in one of the expansion tables.

The size-12 square was derived from a different method, called the **TAP** method for *Tile And Pattern,* and consequently didn't suffer the loss of pairwise row-symmetry because the imperfect size-6 square was not involved. It is described in Program 8.

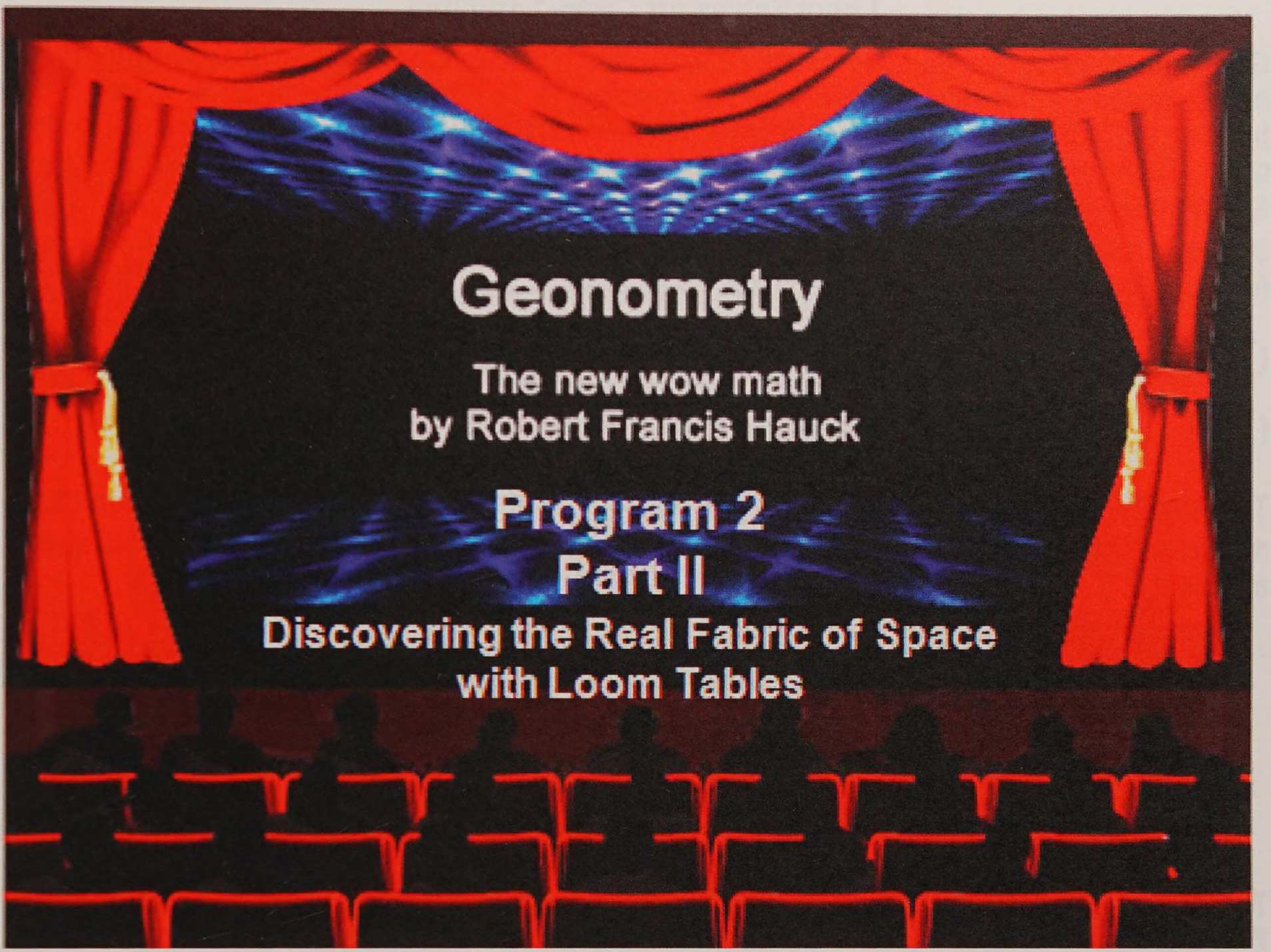
***

Next, we will be seeing how just space is woven from equal-summing 1-dimensional numerical "strings" in what are called **loom tables**. It will be seen that the word "*strings*" has a more fundamental meaning here than just linear summations. We're talking here about the very fabric of space itself.

Program 2

Part I Notes

1. Squares can be segregated into **6** mutually exclusive classes.
2. Odd-size squares have centrally symmetric equal-summing pairs.
3. Even-size squares have row-symmetric equal-summing pairs.
4. Class-3, Class-4 and Class-6 squares are continuously **bx**-modular.
5. Class-1 and Class-2 squares have no modularity at all.
6. Class-4 squares contain characteristic circles.
7. Class-1 squares have two unique complementary tile patterns that are characteristic of its size.
8. Class-4 squares have two indistinguishable complementary tile patterns that are common among all sizes, with the exceptions of size-4 which has none and size-8 which compounds its two complementary tiles together into **4x**-modularity.
9. Both complete tiling patterns for both Classes 1 & 4 are continuous, with the sole exception of size-12 whose continuity is confined to **4** vertical sections, **3** located centrally and **1** horizontal-wrap segment.



Geonometry

The new wow math
by Robert Francis Hauck

Program 2
Part II
Discovering the Real Fabric of Space
with Loom Tables

In Part II, we will see just how space is woven from equal-summing 1-dimensional numerical “strings” in what are called **loom tables**. It will be seen that the word “strings” has a more fundamental meaning here than just *linear summations*. We’re talking here about the very fabric of space itself.

Program 2

Loom Tables for Class-1 squares

Geonomic squares can be decomposed into two loom tables. One is called the **modulus loom** and is based on the reformulated modulus function (**2.1**). The other loom table is called the **integer loom** and is based on the integer function (**2.2**). In what follows throughout this program, the **modulus** loom table will always be denoted by the upper-case letter **X** and the **integer** loom table always be denoted by the upper-case letter **Y**.

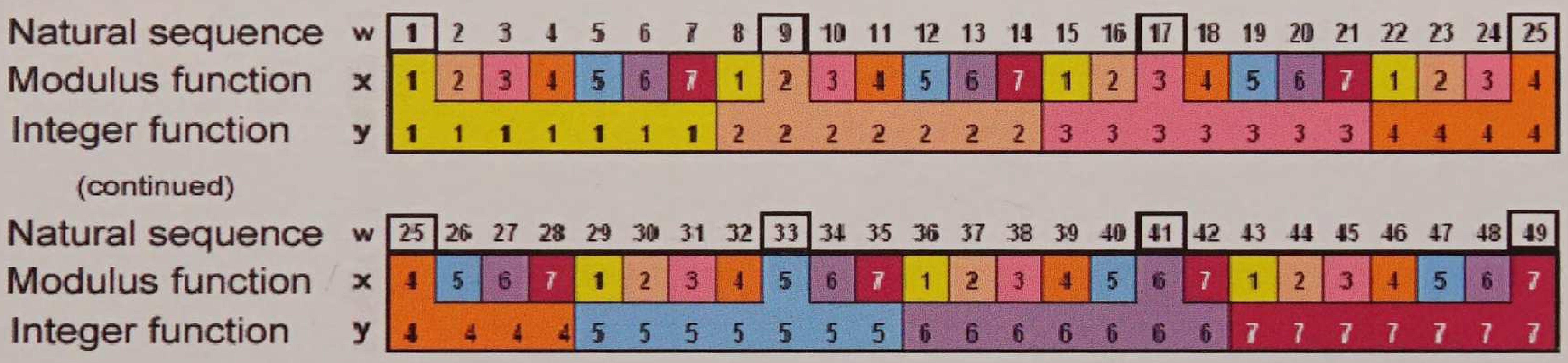
Denoting the initial perfect square by the symbol **W,** the reducing tables are complementary functions first applied to the initial square, **W**, as follows:

(2.1) $x_{ij} \equiv \text{modulus}\left[(w_{ij} - 1) \mid n \right] + 1$

(2.2) $y_{ij} \equiv \text{integer}\left[(w_{ij} - 1) / n \right] + 1$

The subscripts **ij** denote the cell in the **i-th row** and **j-th** column in the designated table. The symbol "≡" means "defined as".

To observe what the relationship of the modulus function (**2.1**) is to the integer function (**2.2**), the values of these functions are shown below in sequence for the values w_{ij} for the size-7 square:



Natural sequence	w	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Modulus function	x	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4
Integer function	y	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4

(continued)

Natural sequence	w	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
Modulus function	x	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
Integer function	y	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	7	7

The picture of values for x_{ij} and y_{ij} is shown above for the full range of numbers for the size 7square. The numbers in each loom table for a square of size **n** run the gamut from **1** through **n**. Here these two functions have equal values for the numbers **1, 9, 17, 25, 33, 41** and **49**. Further, each number appears exactly **n** times in each loom table for a total of n^2 numbers.

The points of equality between the functions always sum to the square's characteristic number! Here **1+9+17+25+33+41+49 = 175**, the characteristic number for the size-7 square.

Note that the number n^2 can always be factored into a product of **n-minus-1** and **n- plus-1**, plus an **additional 1**, as in formula (**2.3**):

(2.3) $n^2 = (n-1)\,(n+1) + 1$

So while ranging through the numbers **1** to n^2 there are always **n** points of equality between the modulus and integer functions and these numbers occur, starting at **1**, every **n-plus-1** numbers for **n-minus-1** additional occurrences beyond the first number, **1**. Note that the number **0** never appears as it persistently does in the classical version of the modulus and integer functions.

The original square will be called the **primal** and will always henceforth be denoted by the upper-case letter **W**. It can be regenerated from the matrix formula (**2.4**):

(2.4) $W = n\,(Y - |1|) + X$ The symbol **|1|** represents a corresponding size table of all **1**'s.

Program 2

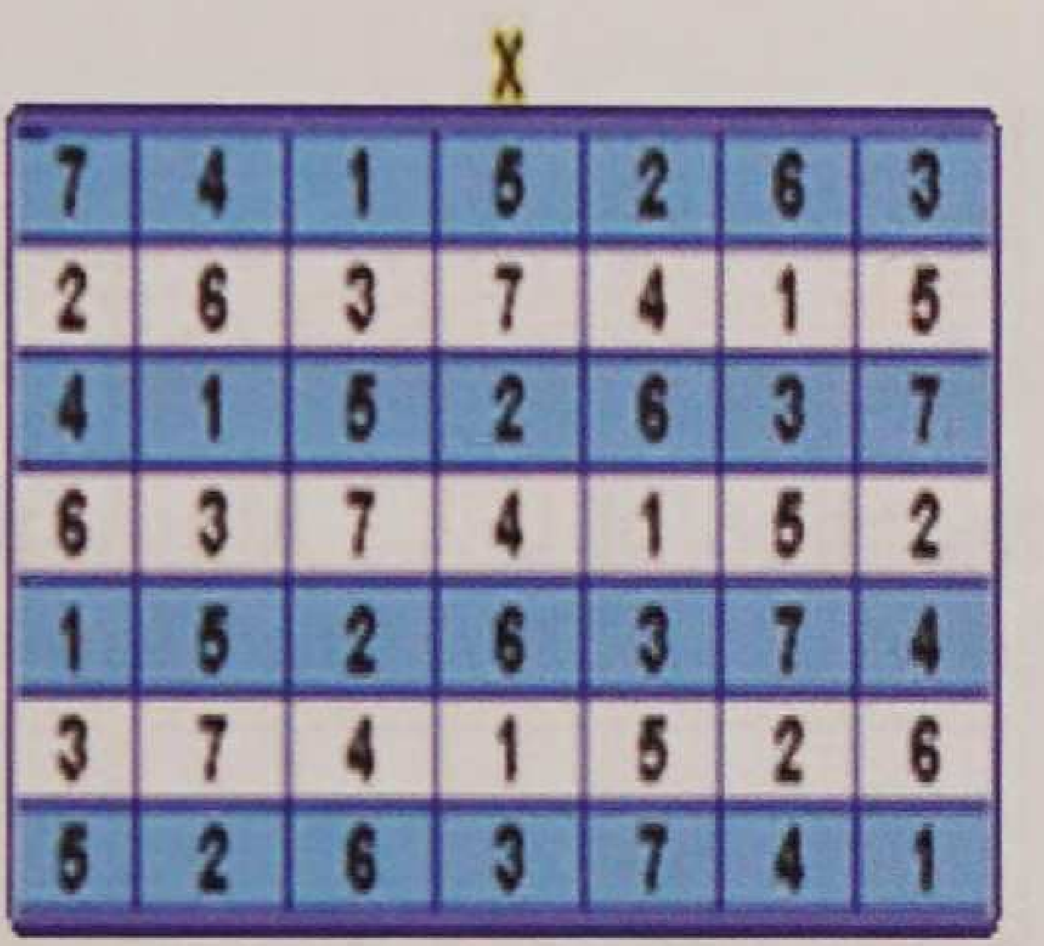
Size-7 Loom Tables

	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

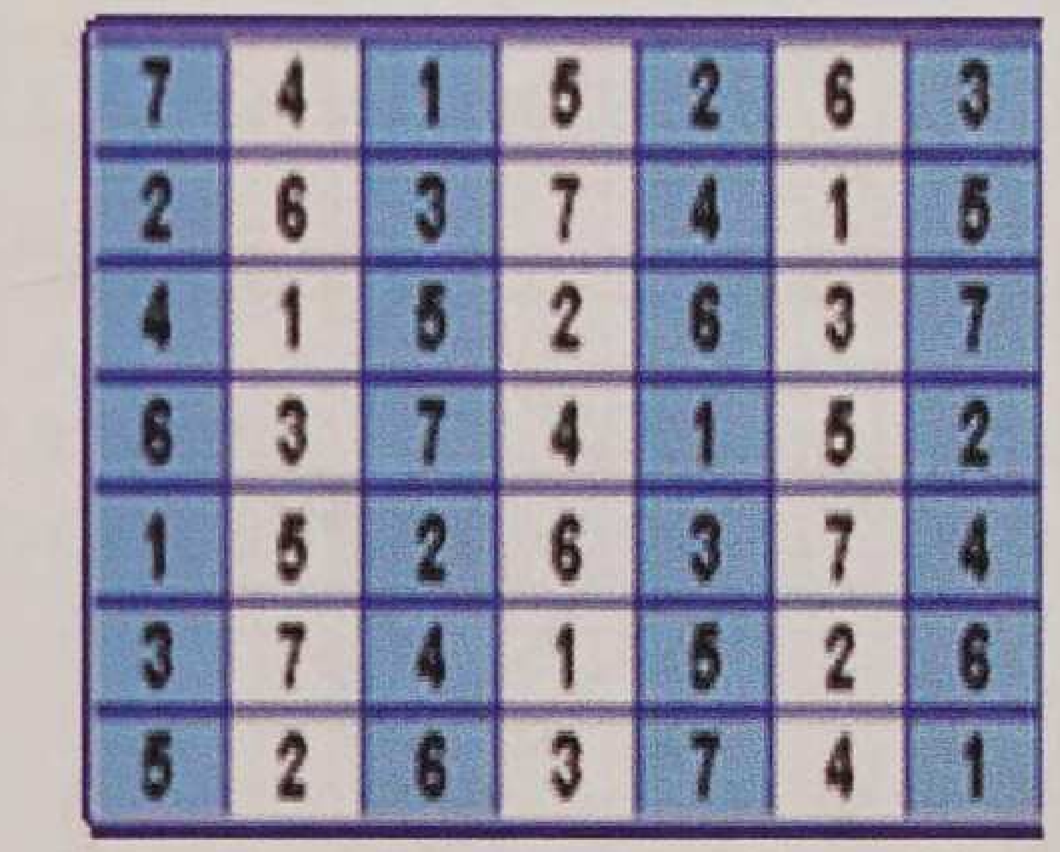
Shown here at left is the size **n = 7** perfect square.

Below are the modulus table **X** and integer table **Y** for this size-7 square. Each is duplicated here **6** times to exhibit the amazing string-patterns of numbers **1** through **7**. Note that every row, column, both main and all wrap diagonals, and both dual tile patterns, simultaneously contain the numbers **1** through **7** exactly once. So clearly, all these each sum to the same number **28.**

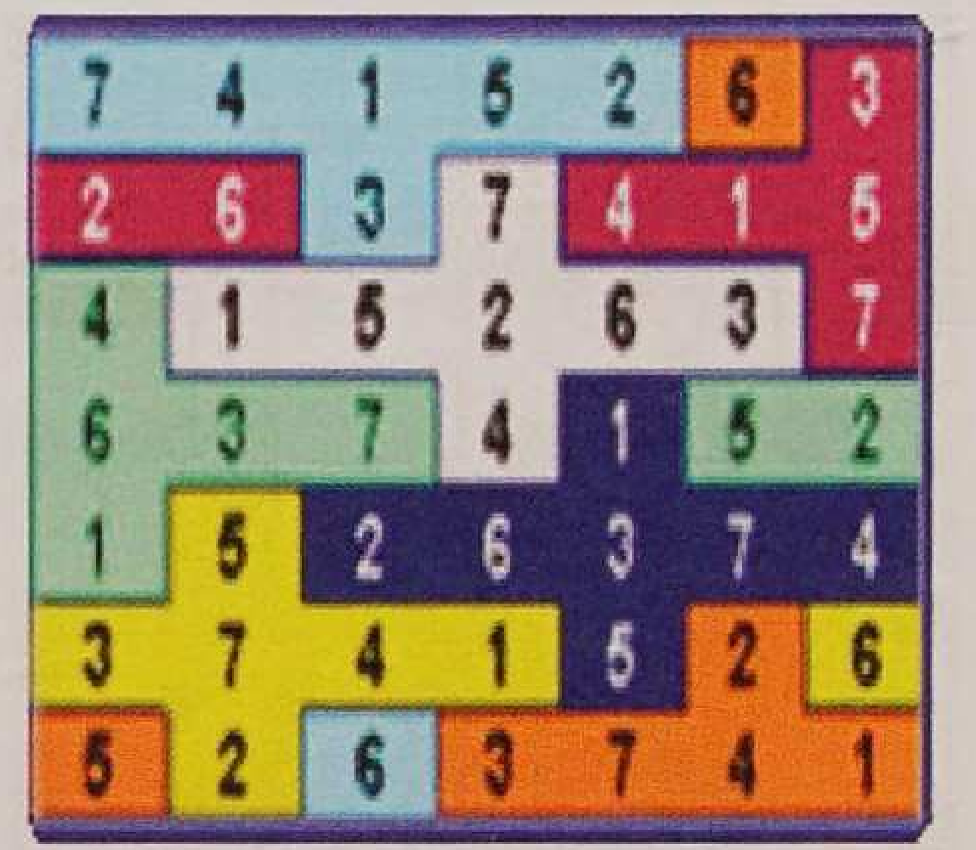
X



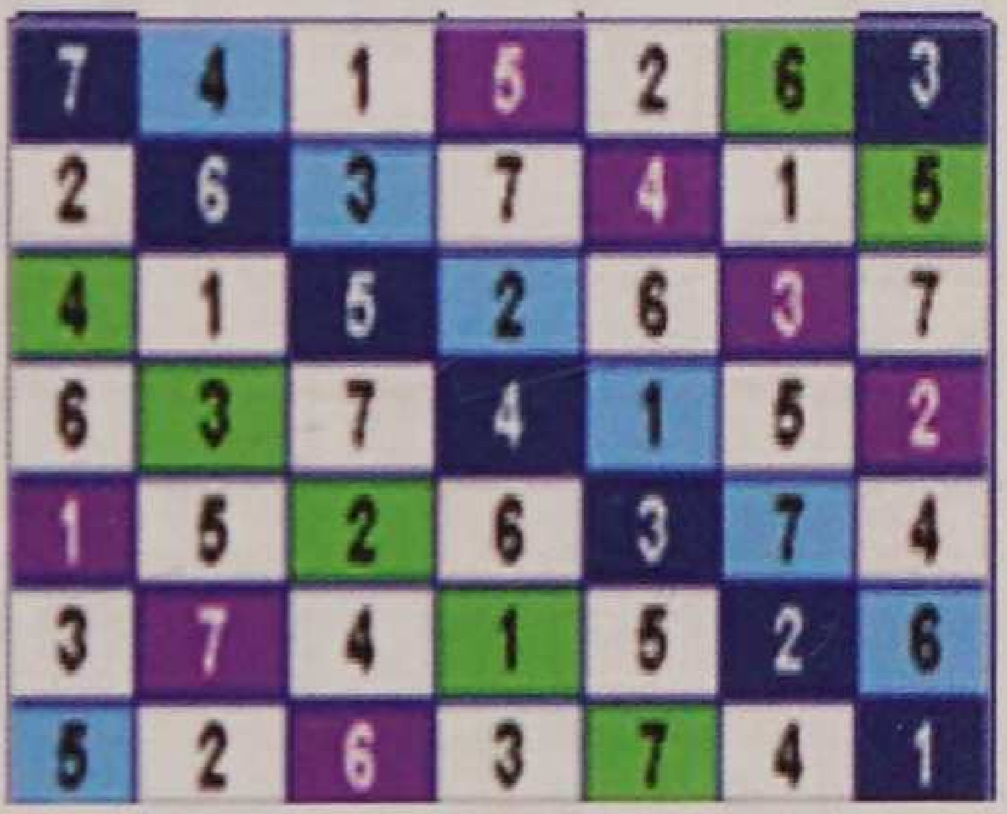
7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1



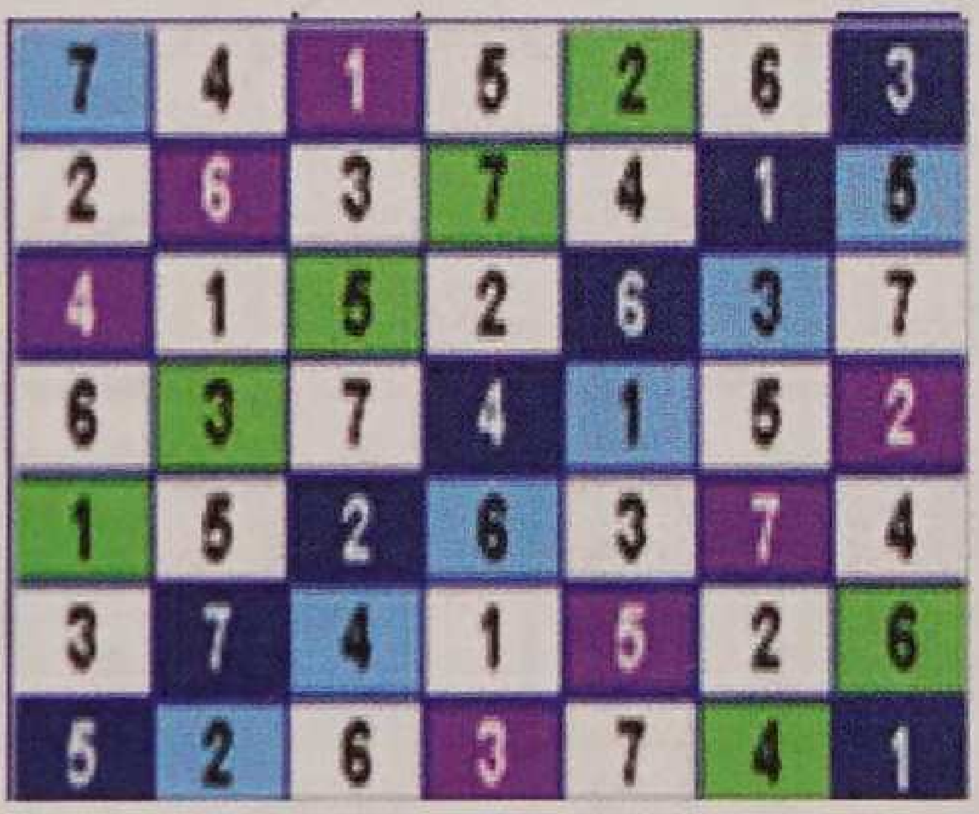
7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1



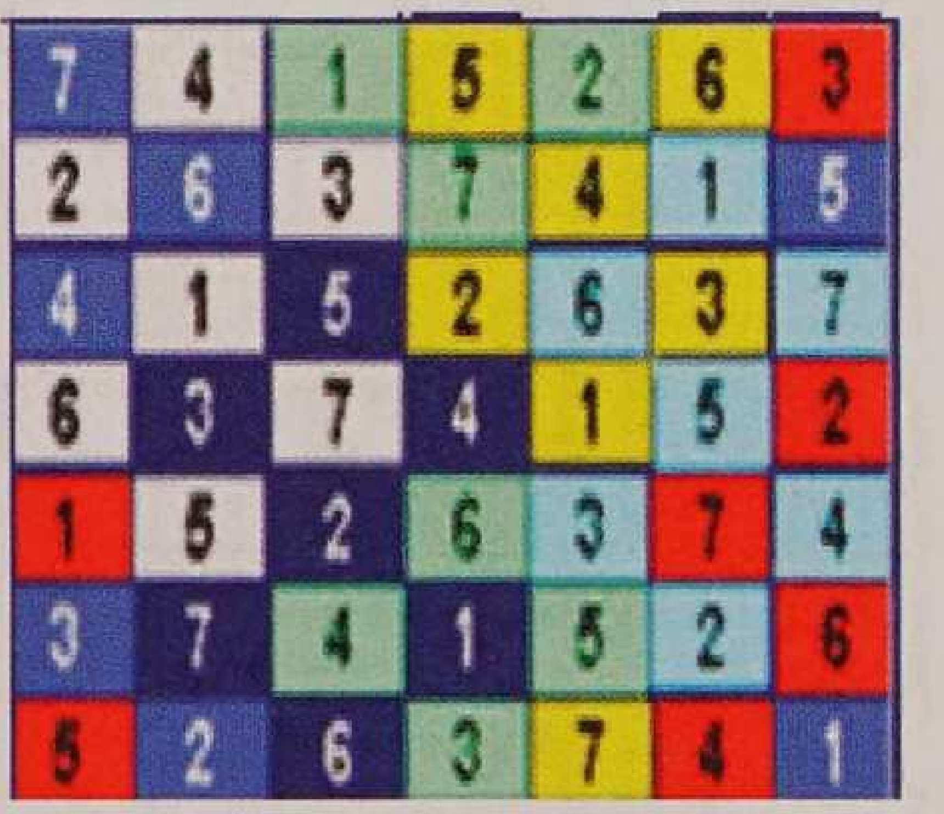
7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1



7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

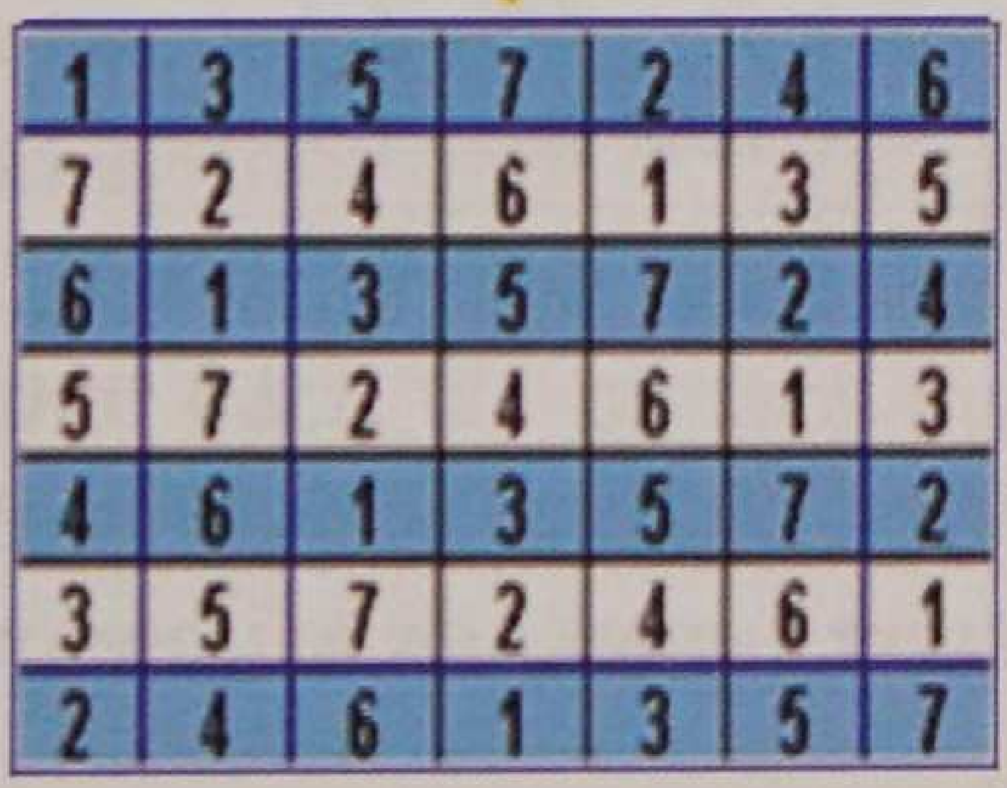


7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

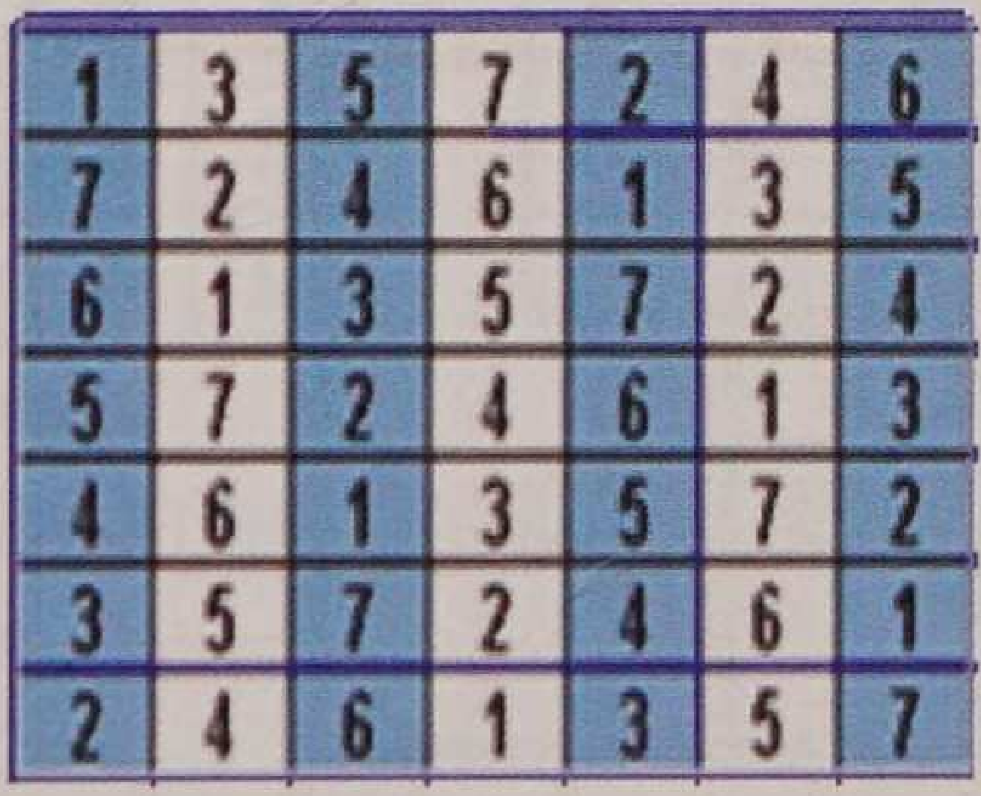


7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

Y



1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7



1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
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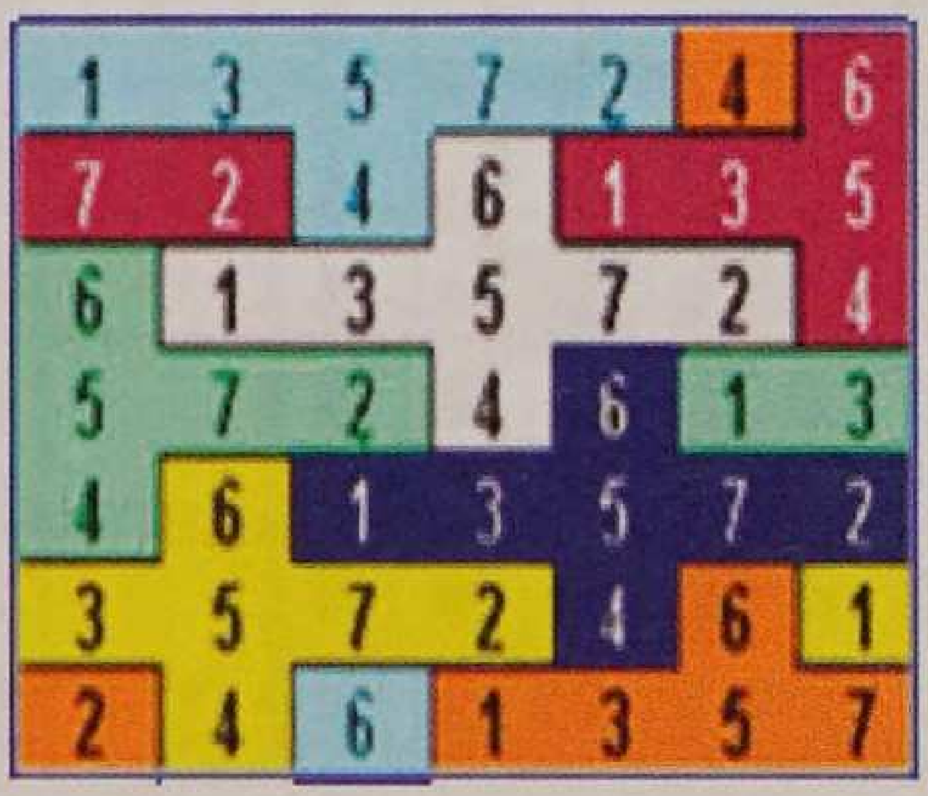
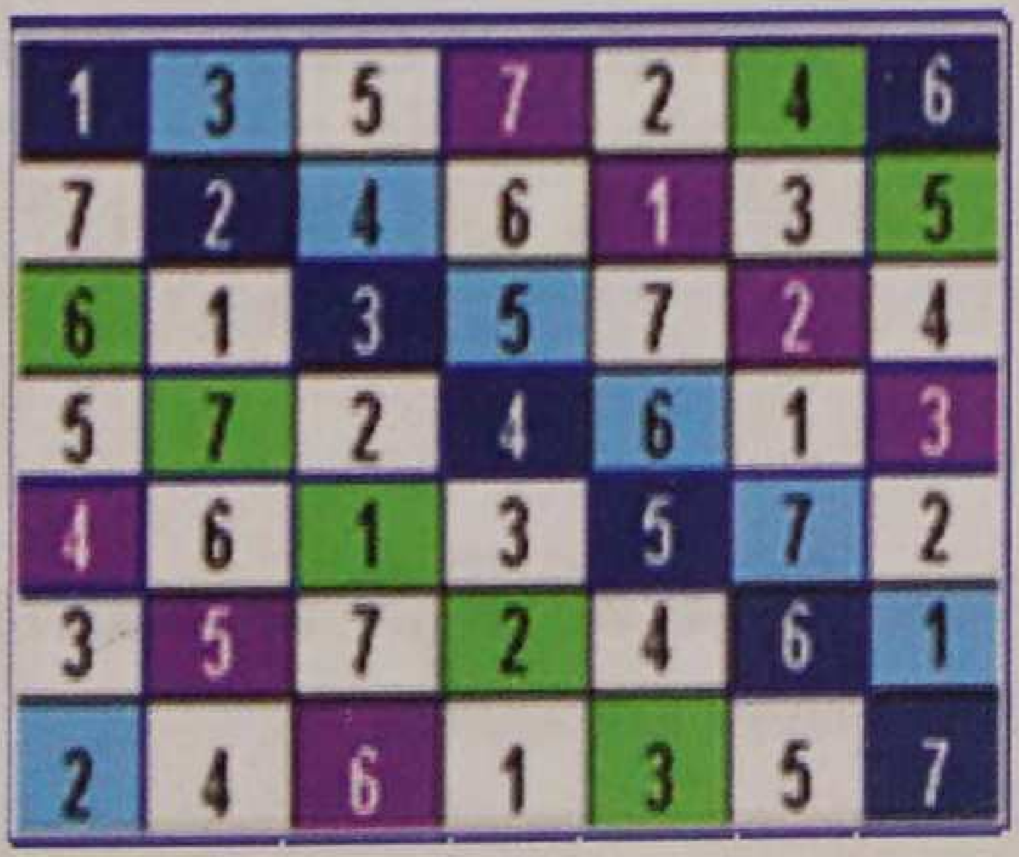
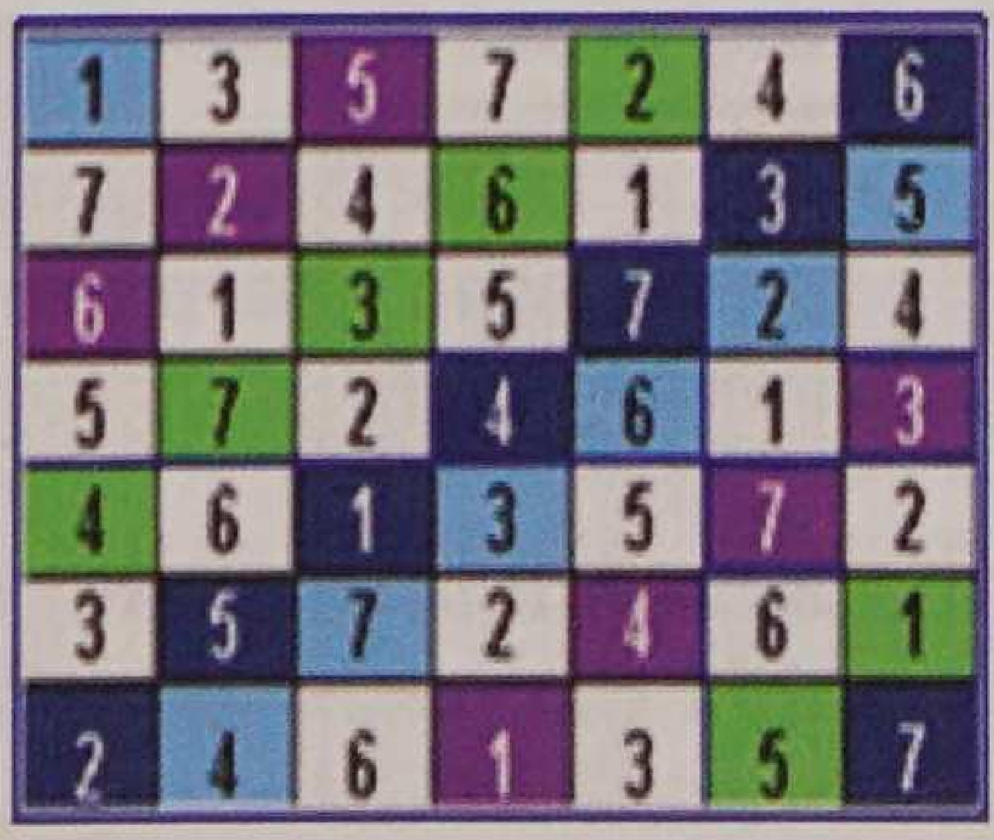
2	4	6	1	3	5	7



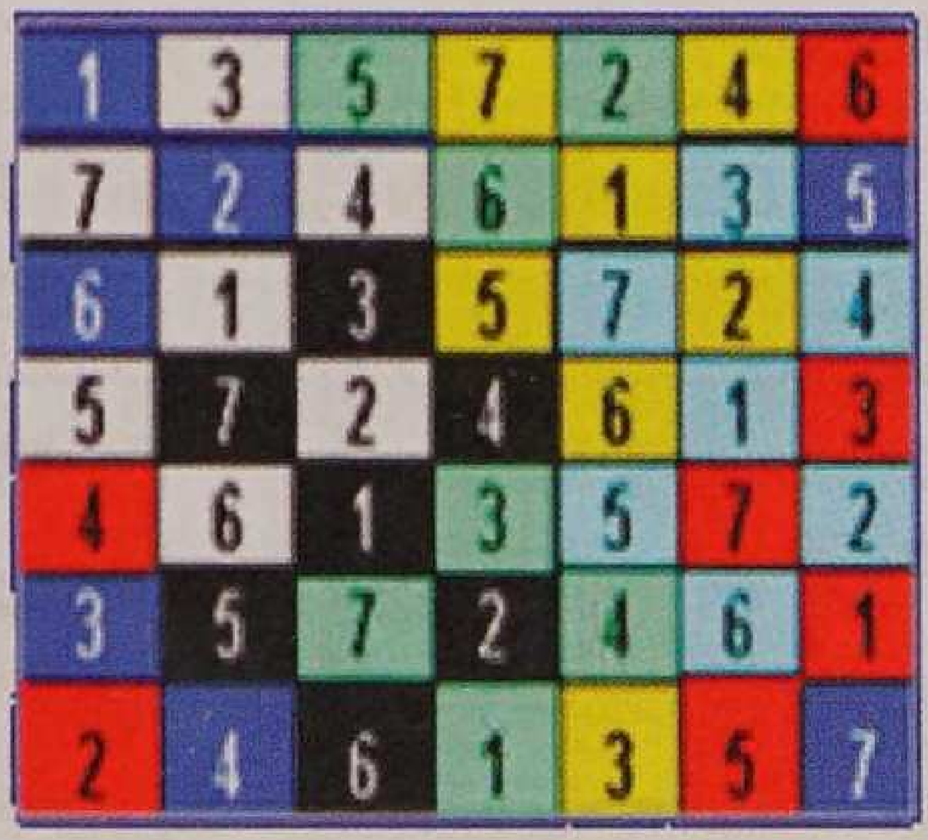
1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7



1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7



1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7



1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

The characteristic number of loom tables will always be denoted by the upper-case letter **L** with the subscript **n**. Since a loom table that is from Class-1 will always have the numbers from **1** thru **n** exactly once in every geonomic summation, the characteristic number of a square loom table of size **n** from Class-1 will always just be the sum of numbers **1** thru **n**:

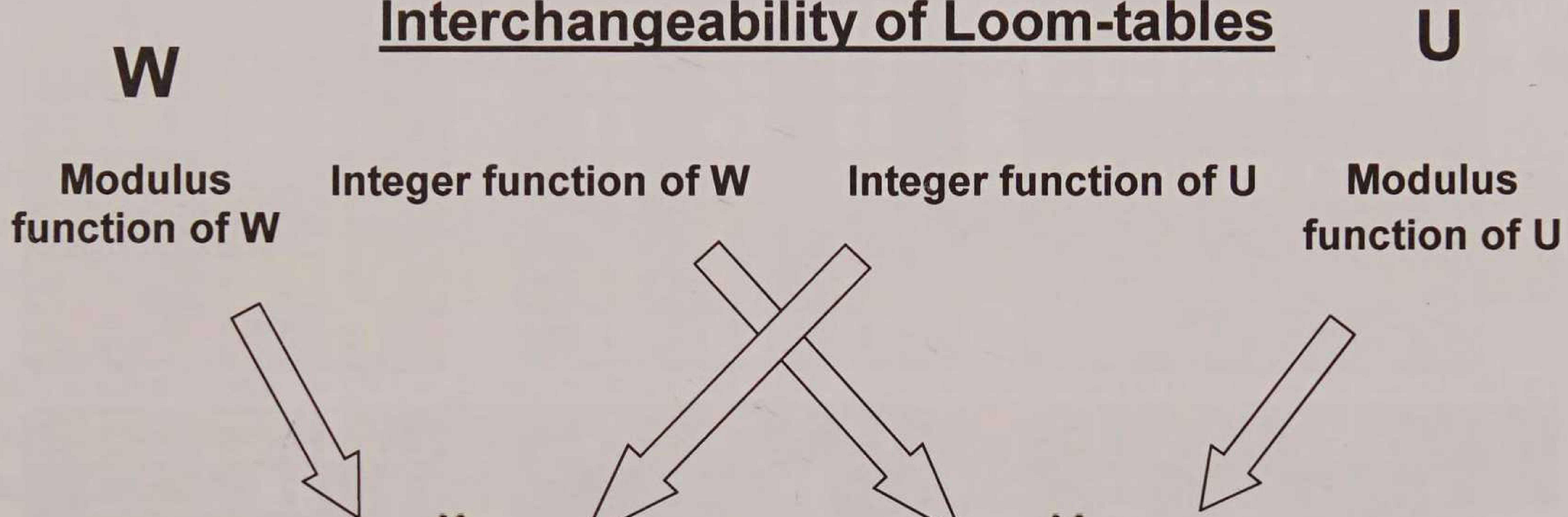
(2.5) $L_n = n(n+1)/2 = S_n$

where S_n denotes the linear sum from **1** through **n** (re: formula (**1.1**) from Program 1).

The original *primal* square **W** has a perfect counterpart, called the **dual** square, which will always henceforth be denoted by the upper-case letter **U**.

Now, here is an absolutely amazing correlation between the complementary loom tables **X** and **Y**: the modulus loom **X** as derived from the primal square **W** is identical to the integer loom derived from the dual square **U**. And similarly, the integer loom table **Y** as derived from the primal square **W** is identical to the modulus loom table derived from the dual square **U**.

Interchangeability of Loom-tables



X

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

Y

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

So the dual **U** can be generated by just switching the roles played by **X** and **Y** in formula (**2.4**).

(2.6) $U = n\left(X - |1|\right) + Y$

Make note: This property is one of the most fundamental properties of Geonometry.

It will also be observed that the geometric patterns in both loom tables carry over to both the primal square and its dual. The primary squares share properties jointly only when both of their complementary loom tables do so simultaneously

We will be encountering this all throughout this program series. This kind of relationship gets even more amazing as we get into higher dimensioned tables in later programs.

Program 2

Here is the dual square **U** generated from the loom tables **X** and **Y** of the primal square **W**.

	1	2	3	4	5	6	7	
W(7)	175	175	175	175	175	175		175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

	1	2	3	4	5	6	7	
U(7)	175	175	175	175	175	175		175
1	43	24	5	35	9	39	20	175
2	14	37	18	48	22	3	33	175
3	27	1	31	12	42	16	46	175
4	40	21	44	25	6	29	10	175
5	4	34	8	38	19	49	23	175
6	17	47	28	2	32	13	36	175
7	30	11	41	15	45	26	7	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

Loom difference table

(2.7) V = X - Y

	1	2	3	4	5	6	7	
V	0	0	0	0	0	0		0
1	6	1	-4	-2	0	2	-3	0
2	-5	4	-1	1	3	-2	0	0
3	-2	0	2	-3	-1	1	3	0
4	1	-4	5	0	-5	4	-1	0
5	-3	-1	1	3	-2	0	2	0
6	0	2	-3	-1	1	-4	5	0
7	3	-2	0	2	4	-1	-6	0
	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0

The loom difference table will always be denoted by the capital letter **V**. The difference table's characteristic number will always be geonomically determined as **0**.

Here is the table **V** of the differences between the two loom tables. Observe that not only do all the rows, columns and diagonals of the difference table each sum to **0**, but that each tile placed anywhere in the difference table does too. This a general property of all Class-1 squares.

This property will come into play again when we observe the cloaking property of loom tables in Program 3.

6	1	-4	-2	0	2	-3
-5	4	-1	1	3	-2	0
-2	0	2	-3	-1	1	3
1	-4	5	0	-5	4	-1
-3	-1	1	3	-2	0	2
0	2	-3	-1	1	-4	5
3	-2	0	2	4	-1	-6

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

6	1	-4	-2	0	2	-3
-5	4	-1	1	3	-2	0
-2	0	2	-3	-1	1	3
1	-4	5	0	-5	4	-1
-3	-1	1	3	-2	0	2
0	2	-3	-1	1	-4	5
3	-2	0	2	4	-1	-6

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Program 2

Here again is the table of the size-7 loom-table differences; all summations equal **0**. Note the amazingly ordered pattern of numbers at bottom to the right of the difference table after these numbers were rearranged. The pattern of reorganized difference values, when put into a geonomic summation framework, has all of its diagonals summing equally to **0**.

X

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

Y

1	3	5	7	2	4	6
7	2	4	6	1	3	5
6	1	3	5	7	2	4
5	7	2	4	6	1	3
4	6	1	3	5	7	2
3	5	7	2	4	6	1
2	4	6	1	3	5	7

	1	2	3	4	5	6	7	
	X-Y	0	0	0	0	0	0	0
1	6	1	-4	-2	0	2	-3	0
2	-5	4	-1	1	3	-2	0	0
3	-2	0	2	-3	-1	1	3	0
4	1	-4	5	0	-5	4	-1	0
5	-3	-1	1	3	-2	0	2	0
6	0	2	-3	-1	1	-4	5	0
7	3	-2	0	2	4	-1	-6	0
	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0

	1	2	3	4	5	6	7	
		0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	21
2	-1	0	1	2	3	4	5	14
3	-2	-1	0	1	2	3	4	7
4	-3	-2	-1	0	1	2	3	0
5	-4	-3	-2	-1	0	1	2	-7
6	-5	-4	-3	-2	-1	0	1	-14
7	-6	-5	-4	-3	-2	-1	0	-21
	-21	-14	-7	0	7	14	21	0
		0	0	0	0	0	0	0

Rows and columns that are anti-symmetric sum to **0** too. The row and column summations which are non-zero are all multiples of ±**7** and in toto sum to **0** too.

Program 2

Weaving the Fabric of Space

Lucasfilm Stunt Works' version of a Quadraloom which weaves the 4-directional spatial fabric just observed in Class-1 squares

This picture was furnished courtesy of the Stunt Works group -- of Lucas film Star Wars fame -- which depicts a Quadraloom weaving a 4-directional weave of wonder.

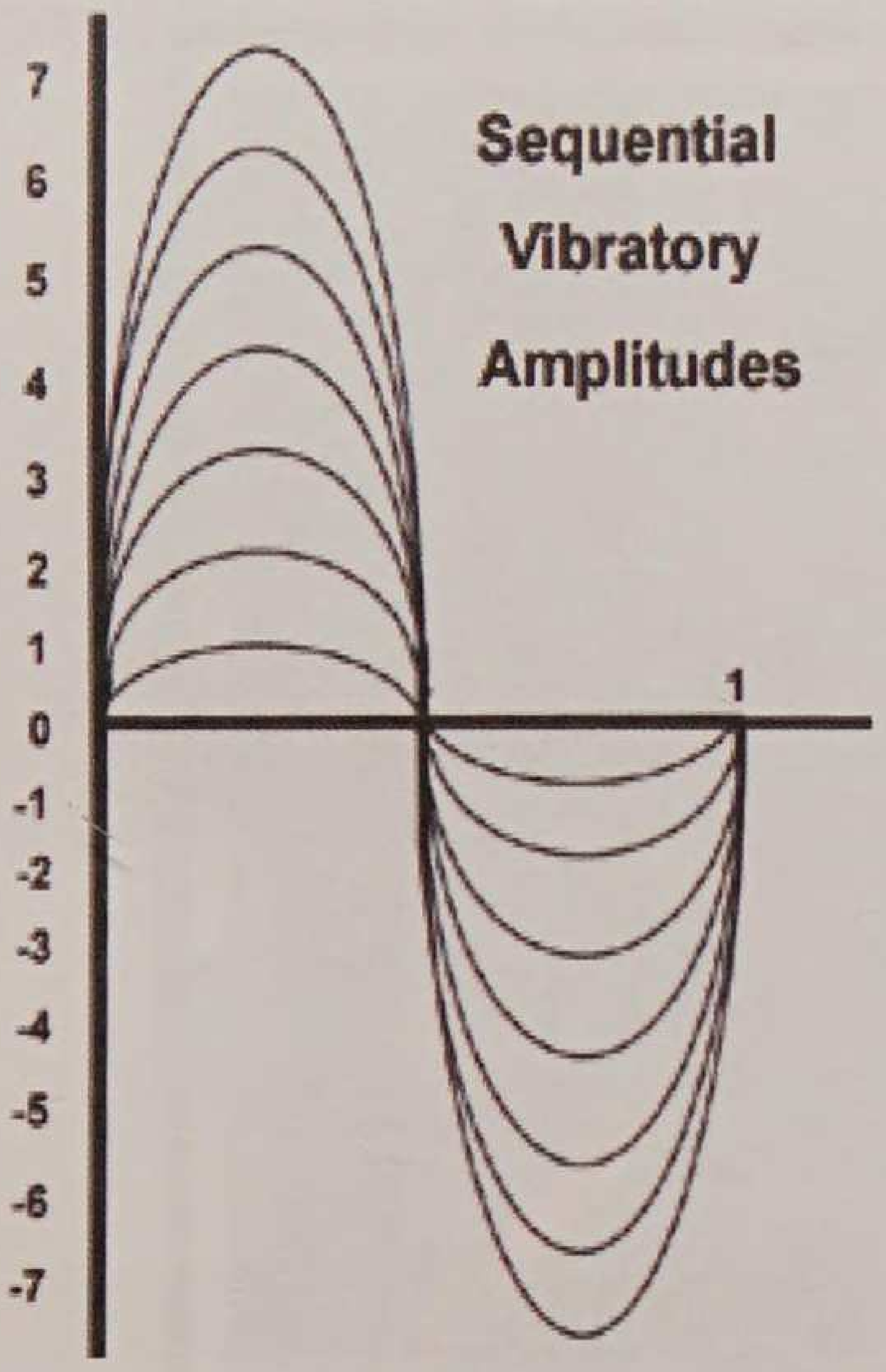
This picture was created by them for a film entitled "**Looms"**, way before these tiling patterns were ever discovered and exhibits that artist-group's amazingly advanced imagination.

What we just saw in the last few slides was the inherent fabric of space – I call it the "***weave of wonder"***.

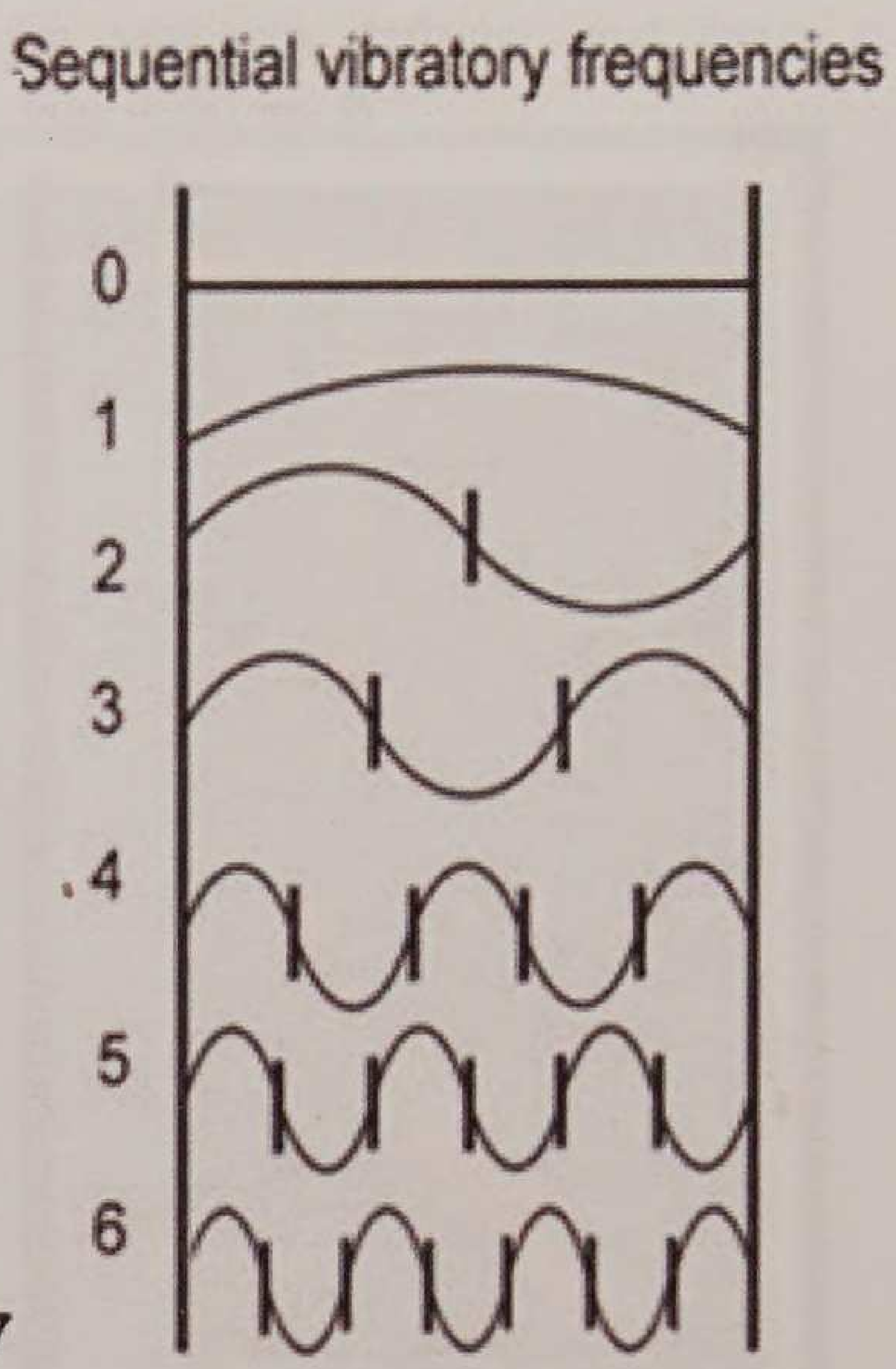
It will be shown time and time again that quadralooms and Geonometry are fundamentally "intertwined".

Program 2

Harmonics in 7x7 space



#	Count	Value
0's	7	0
1's	6 pairs	6
2's	5 pairs	10
3's	4 pairs	12
4's	3 pairs	12
5's	2 pairs	10
6's	1 pair	6
Total	$49 = 7^2$	56 = 2x28 =2L$_7$



What is happening is that we are observing through the patterns in loom tables, the fundamental quantum vibrations that fit harmoniously within a confined size-7 square space itself. It makes no difference whether the vibrations are frequencies or amplitudes; they all interweave together harmonically. And their differences in complementary patterns at n-squared points have net-value frequencies which cancel out overall.

Most importantly, they also cancel out to zero within each tile pattern too, anywhere and everywhere. The center of the tile doesn't have to be aligned with the center of its complementary tile counterpart for this to happen either!

This is an extremely important property that will become fundamentally useful later in Program 5 where Geonometry is applied to explaining the activity of electrons in the electron shells of atoms. And why atomic scientists will never directly observe such patterns with their atom-smashers or electron microscopes.

Note that the absolute values of the **49** numbers in the loom difference table sum to **56** which is **2** times the size-7 loom-tables' characteristic number. That's not a surprise: when we subtract two loom-tables and then sum their absolute values, it's as if we merely added the loom-tables together.

This is a general property of all Class-1 squares: The total of absolute-values of the numbers in their loom difference table **V(n)** always equals **2L$_n$**.

Here is the relationship between the characteristic number of primary squares and that of their loom-tables.

(2.8) $C_n = (n+1)\,L_n - n^2$

as derived from:

$$C_n = \sum_j w_{ij} = \sum_j \left[n(y_{ij}-1) + x_{ij}\right] = n\sum_j yij - n\sum_j + \sum_j x_{ij} = n\sum_j y_{ij} + \sum_j x_{ij} - n\sum_{j(1)}$$

$$= (n+1)\,L_n \; - \; n^2$$

Program 2

Four different versions of the size-7 perfect square

Here are 4 different perfect size-7 squares shown with their dual square on the far right. We have been observing square #2 in the examples shown for the size-7 square thus far. The property to note is that the cells containing the numbers where the two functions of modulus and integer are equal, are identically located in both the primal and dual versions of the primary squares too.

Every Class-1 square has a version where all their points of equality are non-linear. It is these squares that will be of interest in Part III.

1

	W	175	175	175	175	175	175	175
1	10	4	47	41	35	22	16	175
2	42	29	23	17	11	5	48	175
3	18	12	6	49	36	30	24	175
4	43	37	31	25	19	13	7	175
5	26	20	14	1	44	38	32	175
6	2	45	39	33	27	21	8	175
7	34	28	15	9	3	46	40	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

			X				
3	4	5	6	7	1	2	28
7	1	2	3	4	5	6	28
4	5	6	7	1	2	3	28
1	2	3	4	5	6	7	28
5	6	7	1	2	3	4	28
2	3	4	5	6	7	1	28
6	7	1	2	3	4	5	28
28	28	28	28	28	28	28	

			Y				
2	1	7	6	5	4	3	28
6	5	4	3	2	1	7	28
3	2	1	7	6	5	4	28
7	6	5	4	3	2	1	28
4	3	2	1	7	6	5	28
1	7	6	5	4	3	2	28
5	4	3	2	1	7	6	28
28	28	28	28	28	28	28	

1 **W flipped horizontally**

U	175	175	175	175	175	175	175
16	22	35	41	47	4	10	175
48	5	11	17	23	29	42	175
24	30	36	49	6	12	18	175
7	13	19	25	31	37	43	175
32	38	44	1	14	20	26	175
8	21	27	33	39	45	2	175
40	46	3	9	15	28	34	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

2

	W	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

			X				
7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

			Y				
1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

2 **Major diagonal flipped**

U	175	175	175	175	175	175	175
43	24	5	35	9	39	20	175
14	37	18	48	22	3	33	175
27	1	31	12	42	16	46	175
40	21	44	25	6	29	10	175
4	34	8	38	19	49	23	175
17	47	28	2	32	13	36	175
30	11	41	15	45	26	7	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

3

	W	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43	175
2	33	3	22	48	18	37	14	175
3	46	16	42	12	31	1	27	175
4	10	29	6	25	44	21	40	175
5	23	49	19	38	8	34	4	175
6	36	13	32	2	28	47	17	175
7	7	26	45	15	41	11	30	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

			X				
6	4	2	7	5	3	1	28
5	3	1	6	4	2	7	28
4	2	7	5	3	1	6	28
3	1	6	4	2	7	5	28
2	7	5	3	1	6	4	28
1	6	4	2	7	5	3	28
7	5	3	1	6	4	2	28
28	28	28	28	28	28	28	

			Y				
3	6	2	5	1	4	7	28
5	1	4	7	3	6	2	28
7	3	6	2	5	1	4	28
2	5	1	4	7	3	6	28
4	7	3	6	2	5	1	28
6	2	5	1	4	7	3	28
1	4	7	3	6	2	5	28
28	28	28	28	28	28	28	

3 **Major diagonal made vertical**

U	175	175	175	175	175	175	175
38	27	9	47	29	18	7	175
33	15	4	42	24	13	44	175
28	10	48	30	19	1	39	175
16	5	36	25	14	45	34	175
11	49	31	20	2	40	22	175
6	37	26	8	46	35	17	175
43	32	21	3	41	23	12	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

4

	W	175	175	175	175	175	175	175
1	36	46	7	10	20	23	33	175
2	13	16	26	29	39	49	3	175
3	32	42	45	6	9	19	22	175
4	2	12	15	25	35	38	48	175
5	28	31	41	44	5	8	18	175
6	47	1	11	21	24	34	37	175
7	17	27	30	40	43	4	14	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

			X				
1	4	7	3	6	2	5	28
6	2	5	1	4	7	3	28
4	7	3	6	2	5	1	28
2	5	1	4	7	3	6	28
7	3	6	2	5	1	4	28
5	1	4	7	3	6	2	28
3	6	2	5	1	4	7	28
28	28	28	28	28	28	28	

			Y				
6	7	1	2	3	4	5	28
2	3	4	5	6	7	1	28
5	6	7	1	2	3	4	28
1	2	3	4	5	6	7	28
4	5	6	7	1	2	3	28
7	1	2	3	4	5	6	28
3	4	5	6	7	1	2	28
28	28	28	28	28	28	28	

4 **Major diagonal made vertical**

U	175	175	175	175	175	175	175
6	28	43	16	38	11	33	175
37	10	32	5	27	49	15	175
26	48	21	36	9	31	4	175
8	30	3	25	47	20	42	175
46	19	41	14	29	2	24	175
35	1	23	45	18	40	13	175
17	39	12	34	7	22	44	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

The major diagonal has been highlighted to observe just where it ends up in the dual in various conversions.

Program 2

Size-11 tiling patterns A & hB with common centers on X & W

In what follows, the label of a tile pattern preceded by the lower-case letter **h**, will denote the horizontal flip of the tile pattern.

Here is the size-11 primal square **W** and its modulus loom table **X**. Both tables have been tiled with dual tiling patterns A & hB.

Note that tile centers are all equal to **11** on the loom table. On the size-11 primal square, they run the gamut from **11** to**121**, in increments of **11**.

X A

10	4	9	3	8	2	7	1	6	11	5
2	7	1	6	11	5	10	4	9	3	8
5	10	4	9	3	8	2	7	1	6	11
8	2	7	1	6	11	5	10	4	9	3
11	5	10	4	9	3	8	2	7	1	6
3	8	2	7	1	6	11	5	10	4	9
6	11	5	10	4	9	3	8	2	7	1
9	3	8	2	7	1	6	11	5	10	4
1	6	11	5	10	4	9	3	8	2	7
4	9	3	8	2	7	1	6	11	5	10
7	1	6	11	5	10	4	9	3	8	2

W A

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

X hB

10	4	9	3	8	2	7	1	6	11	5
2	7	1	6	11	5	10	4	9	3	8
5	10	4	9	3	8	2	7	1	6	11
8	2	7	1	6	11	5	10	4	9	3
11	5	10	4	9	3	8	2	7	1	6
3	8	2	7	1	6	11	5	10	4	9
6	11	5	10	4	9	3	8	2	7	1
9	3	8	2	7	1	6	11	5	10	4
1	6	11	5	10	4	9	3	8	2	7
4	9	3	8	2	7	1	6	11	5	10
7	1	6	11	5	10	4	9	3	8	2

W hB

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

Observe that in all complete tiling patterns, the centers of interlocking tiles propagate diagonally up or down going from right to left. This direction of propagation may be reversed from “down” to “up” and vise versa by flipping the tiling pattern horizontally.

Size-11 tiling patterns hA & B with common centers on Y & U

Here is the size-11 **dual** square **U** and its integer loom table **Y**. Both tables have been tiled with tiling patterns hA and B.

Y hA

2	4	6	8	10	1	3	5	7	9	11
3	5	7	9	11	2	4	6	8	10	1
4	6	8	10	1	3	5	7	9	11	2
5	7	9	11	2	4	6	8	10	1	3
6	8	10	1	3	5	7	9	11	2	4
7	9	11	2	4	6	8	10	1	3	5
8	10	1	3	5	7	9	11	2	4	6
9	11	2	4	6	8	10	1	3	5	7
10	1	3	5	7	9	11	2	4	6	8
11	2	4	6	8	10	1	3	5	7	9
1	3	5	7	9	11	2	4	6	8	10

U hA

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

Y B

2	4	6	8	10	1	3	5	7	9	11
3	5	7	9	11	2	4	6	8	10	1
4	6	8	10	1	3	5	7	9	11	2
5	7	9	11	2	4	6	8	10	1	3
6	8	10	1	3	5	7	9	11	2	4
7	9	11	2	4	6	8	10	1	3	5
8	10	1	3	5	7	9	11	2	4	6
9	11	2	4	6	8	10	1	3	5	7
10	1	3	5	7	9	11	2	4	6	8
11	2	4	6	8	10	1	3	5	7	9
1	3	5	7	9	11	2	4	6	8	10

U B

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

Note that these tile centers again are all equal to **11** on the loom table and on the size-11 **dual** square, run the gamut from **11** to**121**, in increments of **11**.

Program 2

Points of Equality between Dual Squares and the centers of their Tiling Patterns

Tiling patterns' centers	Loom tables' points of equality
Δ = n	Δ = n +1
11	1
22	13
33	25
44	37
55	49
66	61
77	73
88	85
99	97
110	109
121	121

W

21	37	64	80	107	2	29	45	72	99	115
24	51	67	94	121	16	43	59	86	102	8
38	65	81	108	3	30	46	73	89	116	22
52	68	95	111	17	44	60	87	103	9	25
66	82	109	4	31	47	74	90	117	12	39
69	96	112	18	34	61	88	104	10	26	53
83	110	5	32	48	75	91	118	13	40	56
97	113	19	35	62	78	105	11	27	54	70
100	6	33	49	76	92	119	14	41	57	84
114	20	36	63	79	106	1	28	55	71	98
7	23	50	77	93	120	15	42	58	85	101

U

101	37	94	30	87	12	69	5	62	119	55
14	71	7	64	121	46	103	39	96	32	78
48	105	41	98	23	80	16	73	9	66	112
82	18	75	11	57	114	50	107	43	89	25
116	52	109	34	91	27	84	20	77	2	59
29	86	22	68	4	61	118	54	100	36	93
63	120	45	102	38	95	31	88	13	70	6
97	33	79	15	72	8	65	111	47	104	40
10	56	113	49	106	42	99	24	81	17	74
44	90	26	83	19	76	1	58	115	51	108
67	3	60	117	53	110	35	92	28	85	21

Now we have seen that there are **n** points in each size-**n** Class-1 square where the numbers between the primal and dual squares are equal. These are those points where the modulus and integer functions are equal and zeroes appear in their complementary looms' difference table. One might wonder if the tiling patterns which can be aligned to have identical values in their centers could also be centered over these points of dual square equalities too. We just saw that their horizontal flip contained numbers **n** thru $\mathbf{n}^2$ in increments of **n**.

Observe that the points of equality between the primal and dual were seen to start at **1**, not **n**, and increment by **n-plus-1, not by n**. The centers of tiling patterns containing the extreme point is only at one point $\mathbf{n}^2$ **= 121** and never start at the other extreme point **1**.

However, there is one exception and that applies only to the Class-1 square of size-7. It will be seen shortly in Part III that the **7** centers of both transposed tile patterns miraculously do align with the 7 points of primal and dual equalities.

From the size-11 square here on out however, these points of equality were found to never again align with the tile centers of the transpose of either of their two characteristic tiling-patterns.

Program 2

Size-17 Loom Tables

Here is the **modulus** loom table for the Class-1 size-17 square. Every tile contains the numbers **1** through **17** exactly once. It has all the same properties as the size-7 square Note that the number distribution pattern in each tile is identical throughout the tiling pattern. This occurs in both tiling patterns regardless of which number is commonly central between them.

Also note that the central numbers of the individual tiles in both tiling patterns are identical everywhere. This occurs regardless of the number which is central. That results in all the tiles' being identical in loom table **X** for each complete tiling pattern.

X

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

X

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
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16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
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17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
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13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
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17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
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7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
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3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
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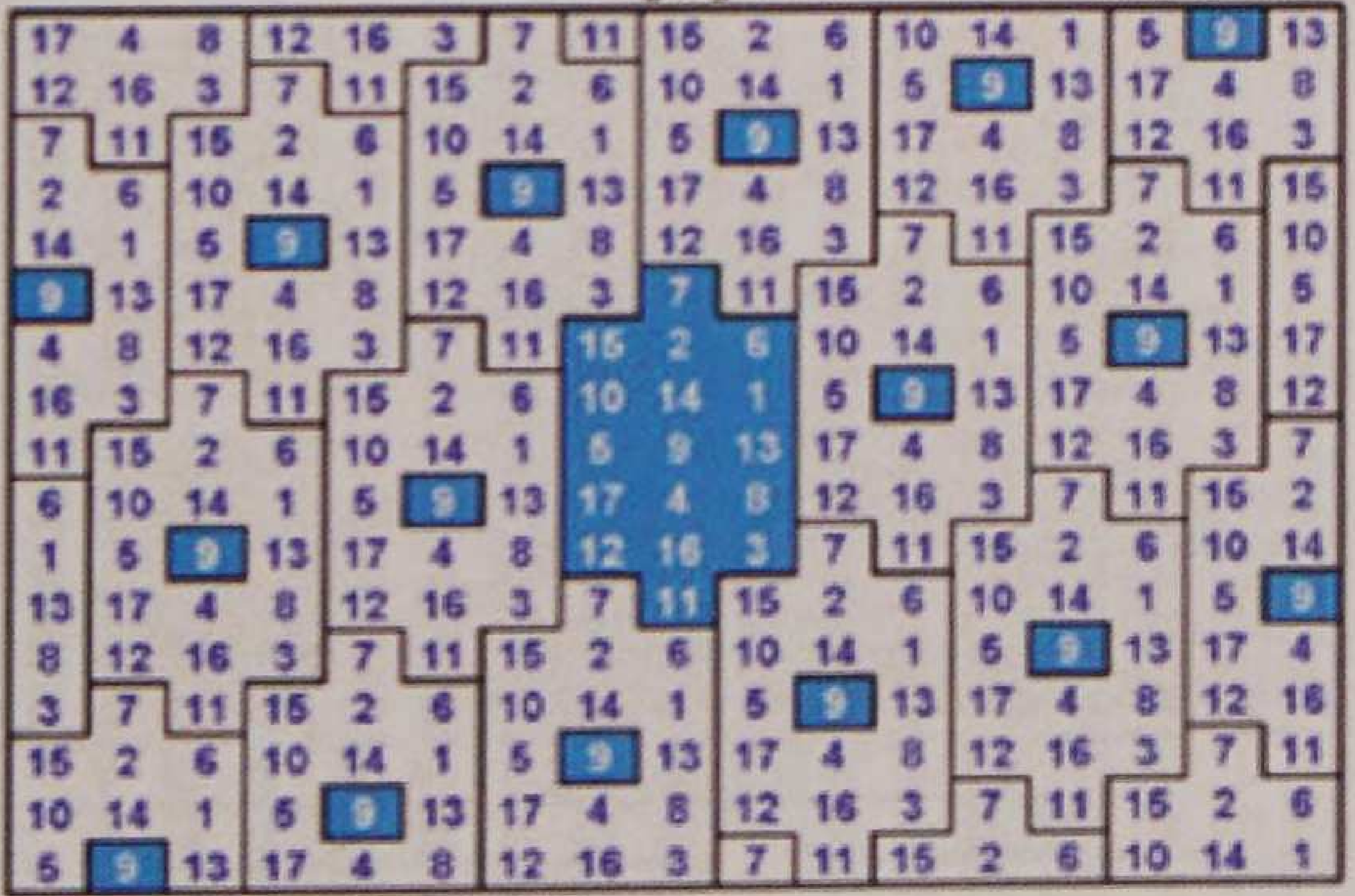
A



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7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
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3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
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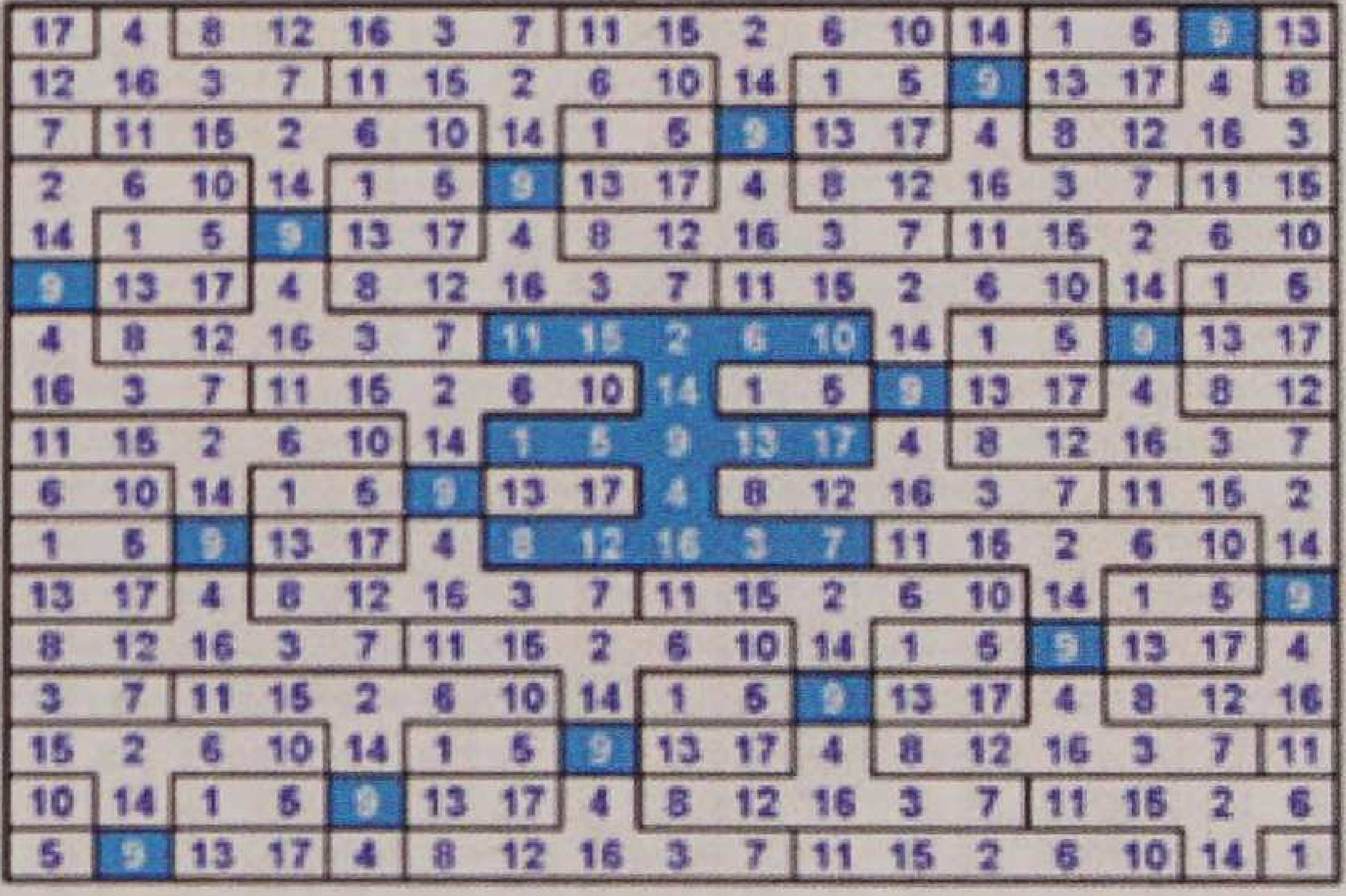
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

B



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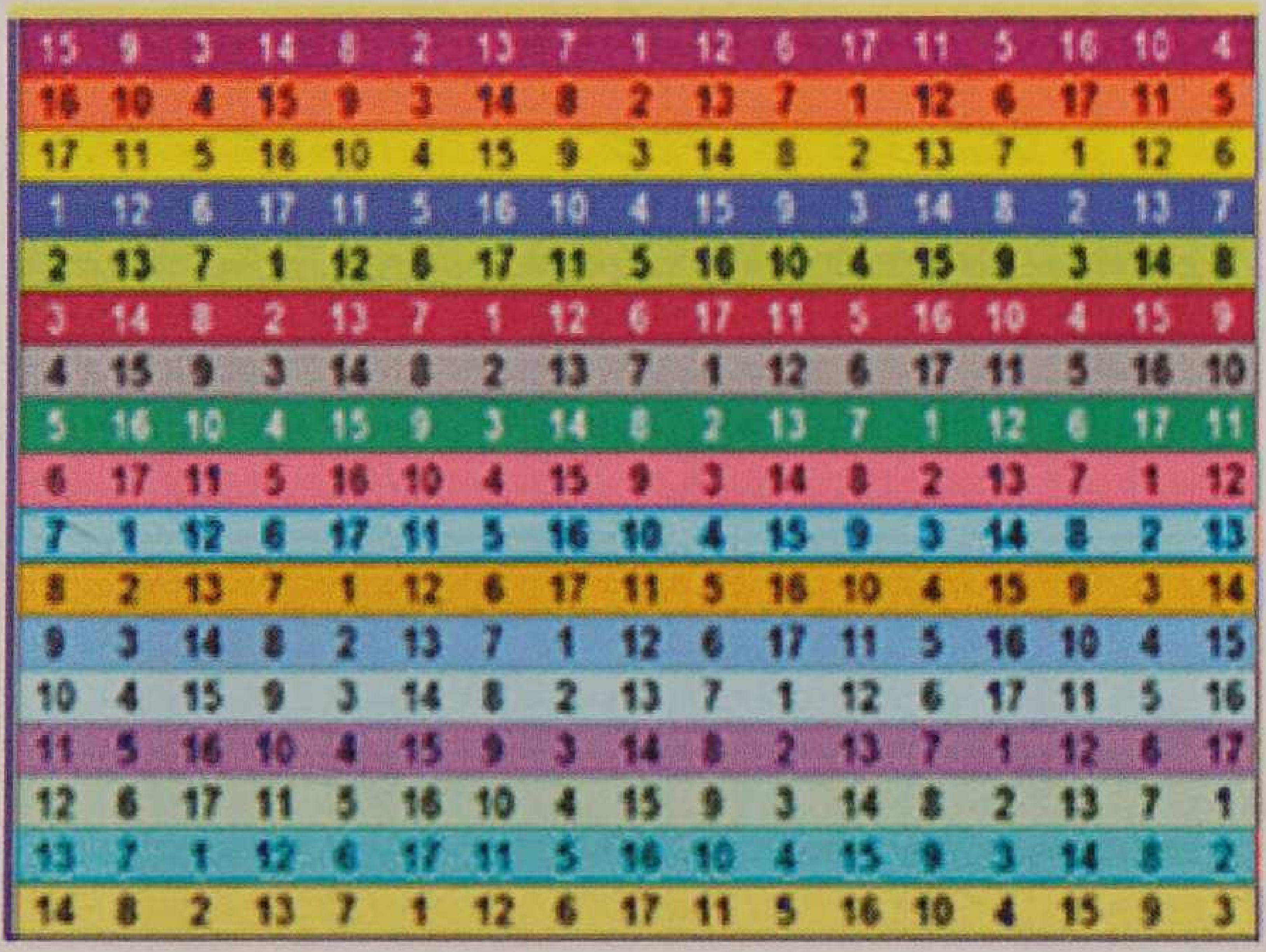
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Program 2

Here is the integer loom table for the size-17 square. Again, every tile contains the numbers **1** through **17** exactly once. However, this time each tile's number distribution is different. The central numbers among all the tiles now range from **1** through **17** exactly once and all the numbers within each tile occupy a unique location in the tile throughout the entire pattern.

Y



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13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
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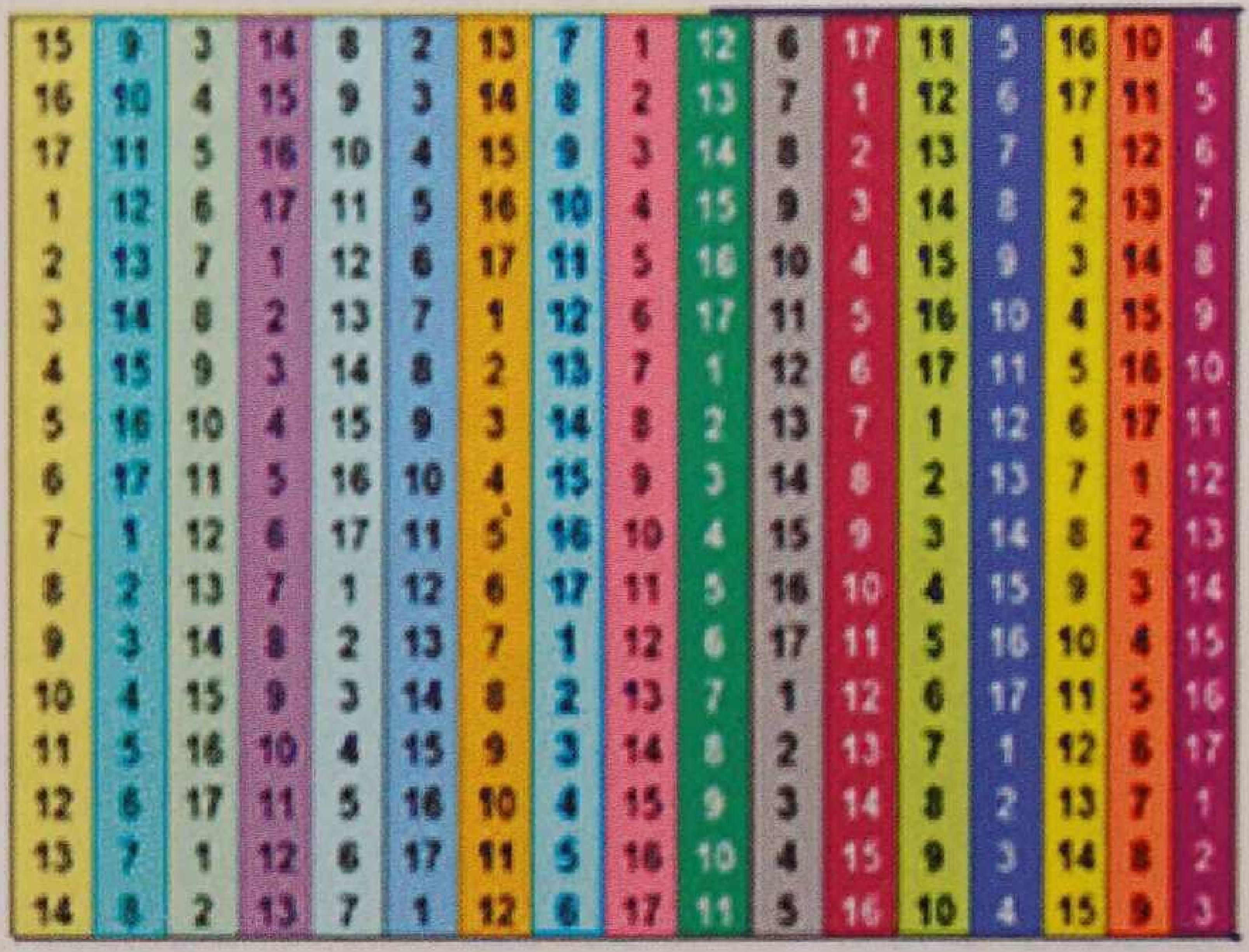
Y



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13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

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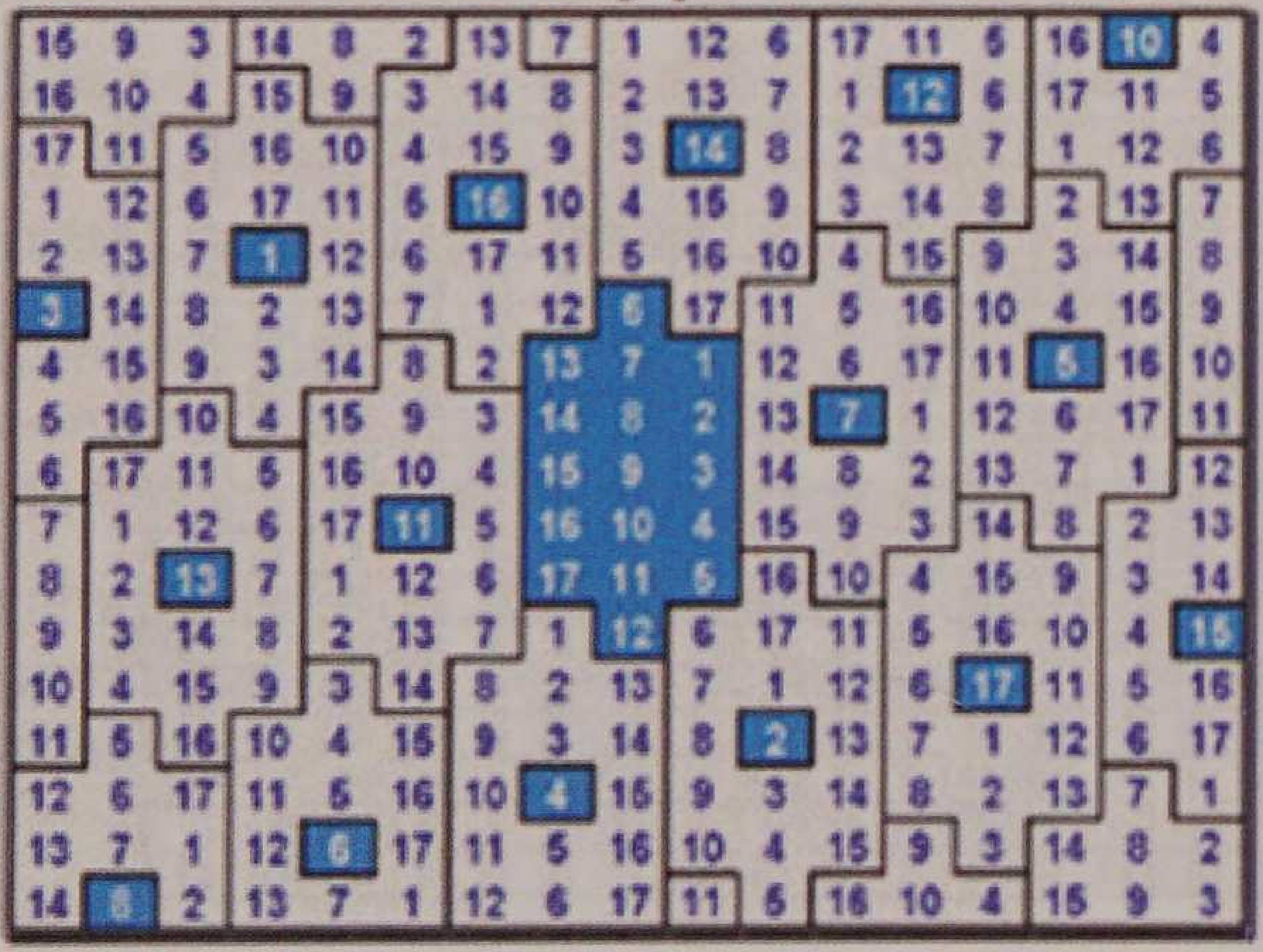
A



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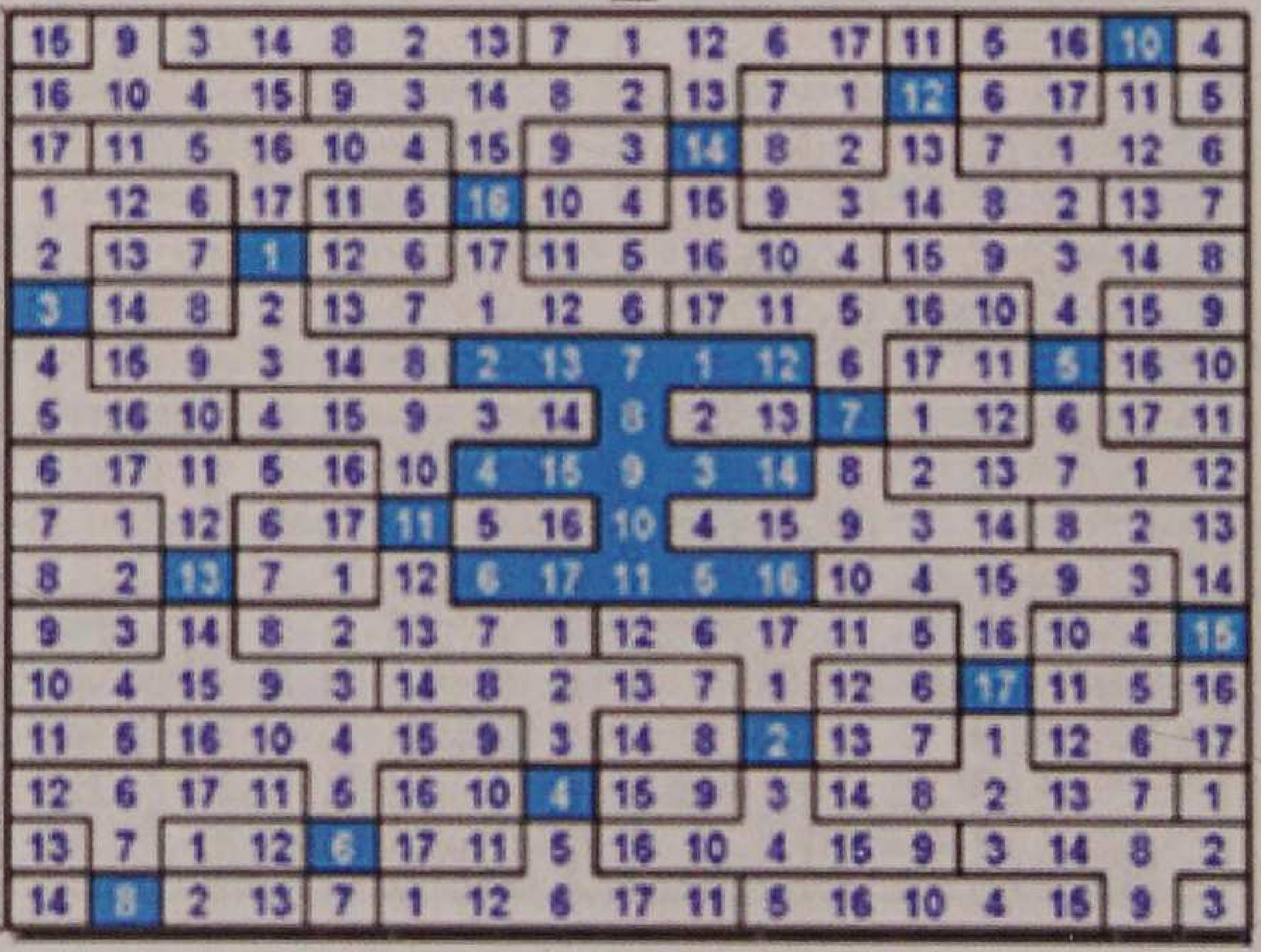
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

B



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17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
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2	13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8
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6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
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9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
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11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

Program 2

Size-17 Loom Tile-Centers

Here are both loom tables **X** and **Y** for the size-17 square. Each loom table is shown with its complementary tiles depicted. The center of each tile is highlighted. In all four tables, the tiles are identically centered with the center of their central tile located in the exact middle of each loom table.

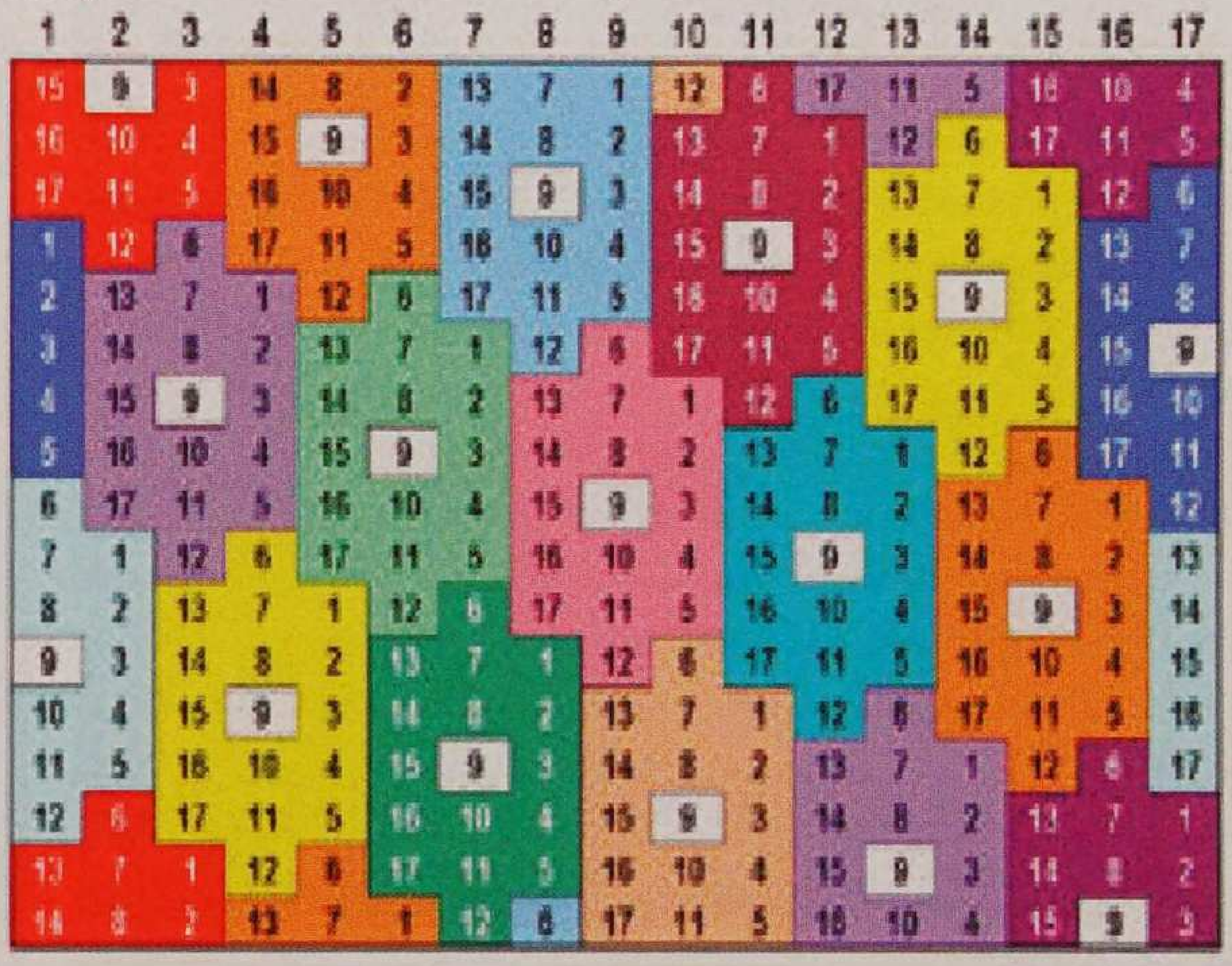
Note that all of the tiles on both **X** and **Y** at the top have a **9** for their central number. The central numbers of the tile patterns at the bottom run the gamut from **1** through **17** on both **X** and **Y**.

It is a general property for every Class-1 square of size n that the values of all the tiles' central numbers in both patterns can be made identical or made to run the gamut from 1 through n depending upon their horizontal relationship to each other.

X(17) A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Y(17) B



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16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
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3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
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9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
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11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

X(17) hA

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
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9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
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13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
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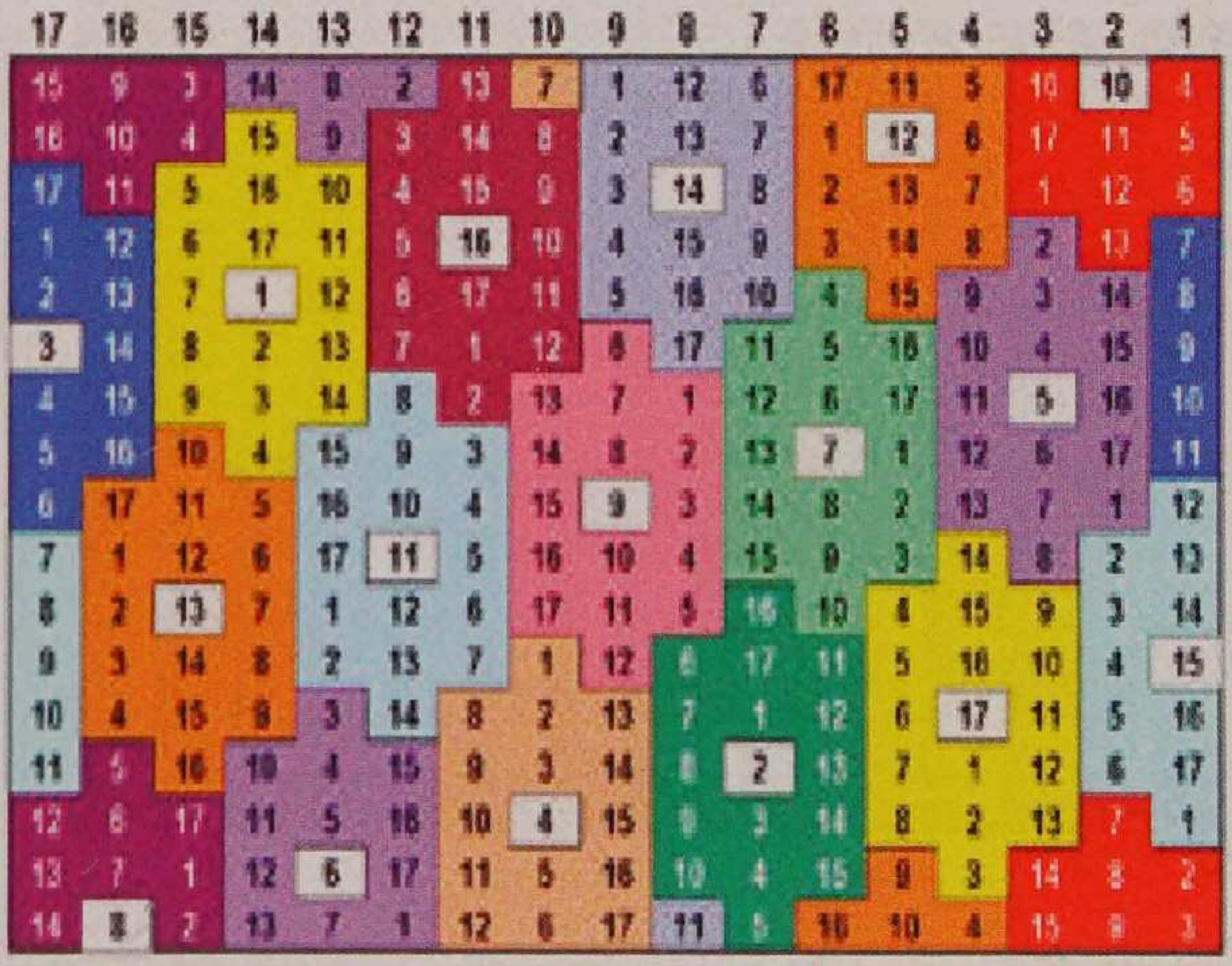
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Y(17) hB



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16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
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3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
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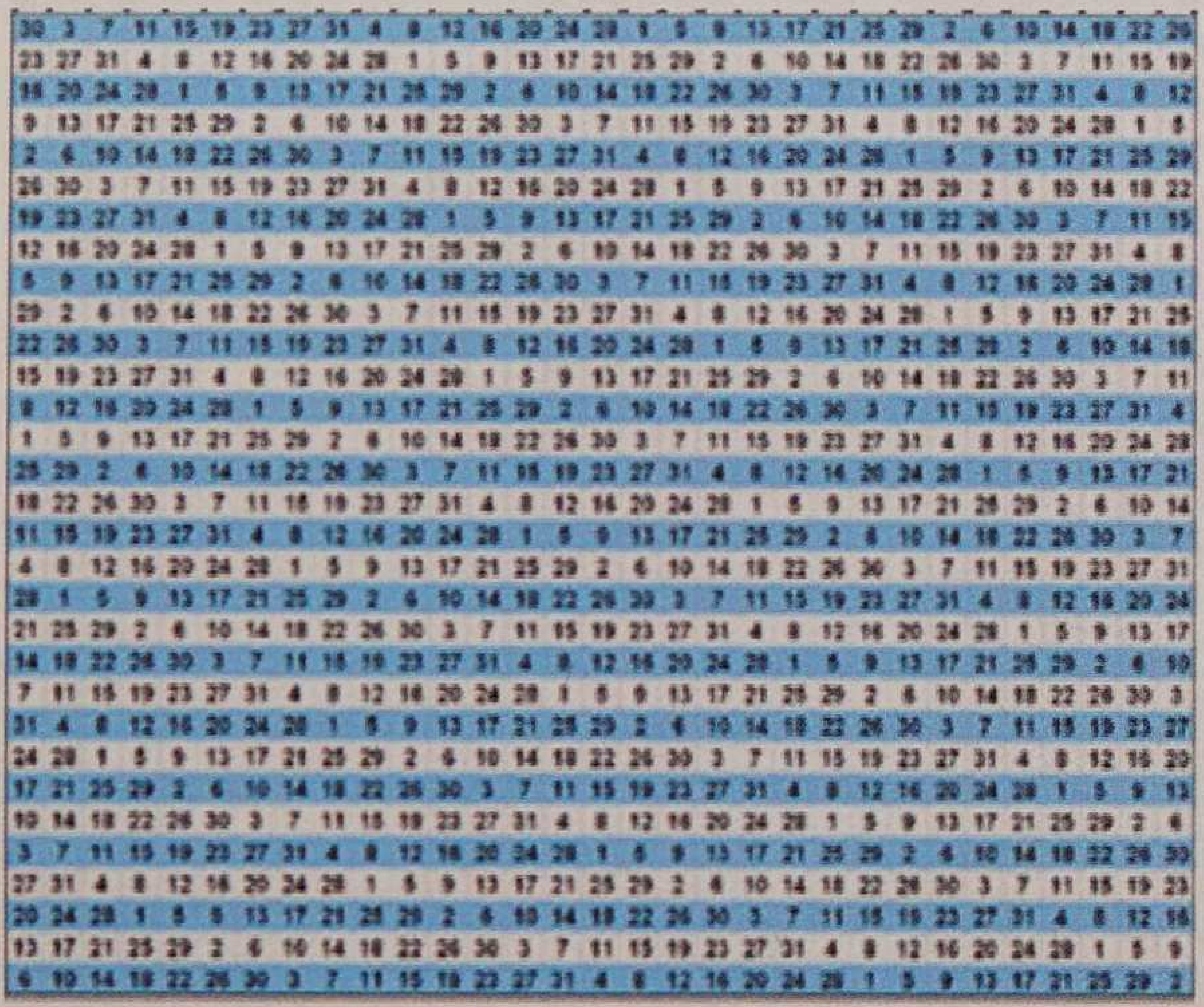
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9	3

Program 2

Size-31 Loom Tables

Here is the size-31 modulus table. Observe the extraordinarily complex and intricate 4-directional weave of the numbers **1** through **31**. At this size, the loom pattern is way beyond the mind of Man to concoct without using computers. This establishes that these extraordinary number patterns are pre-existent and can only be discovered by the processes described by Geonometry in Program-7. It will be shown in Program 7 just how these loom tables can be uncovered without knowing ahead of time the distribution of numbers in the original geonomic square with which they are associated. In other words, it's already there awaiting its discovery, like panning for gold in the Rockies!



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23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19
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9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5
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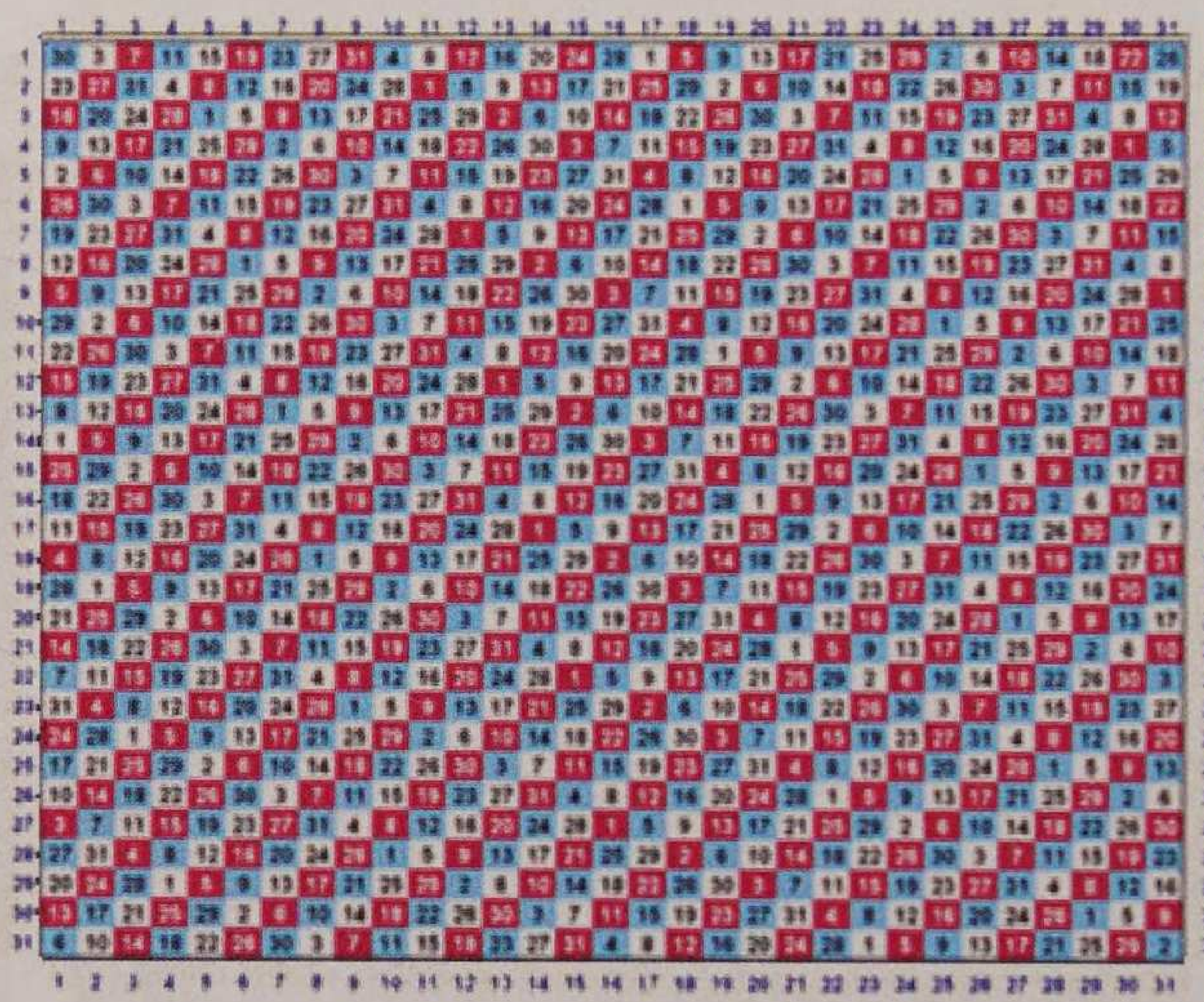

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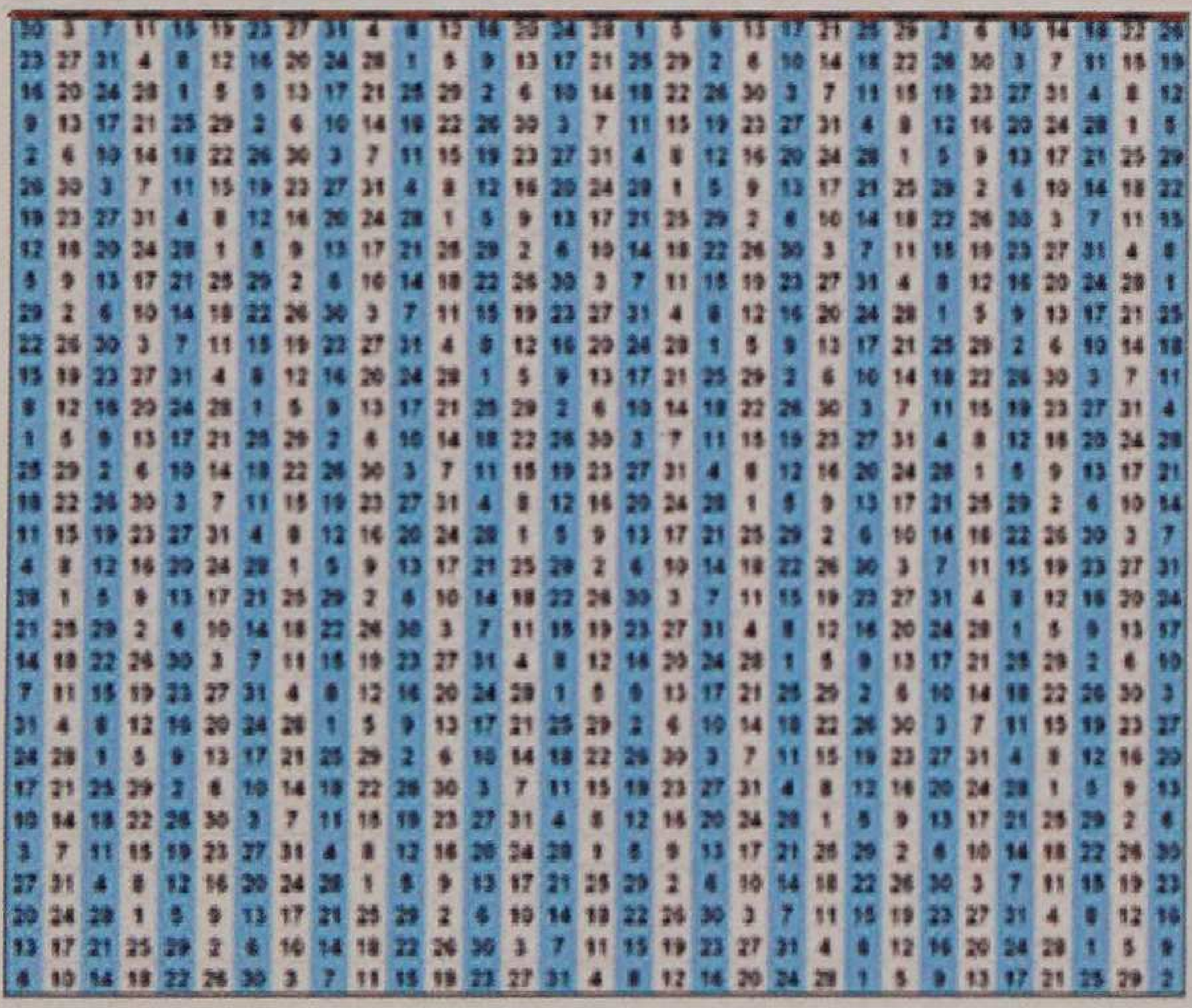

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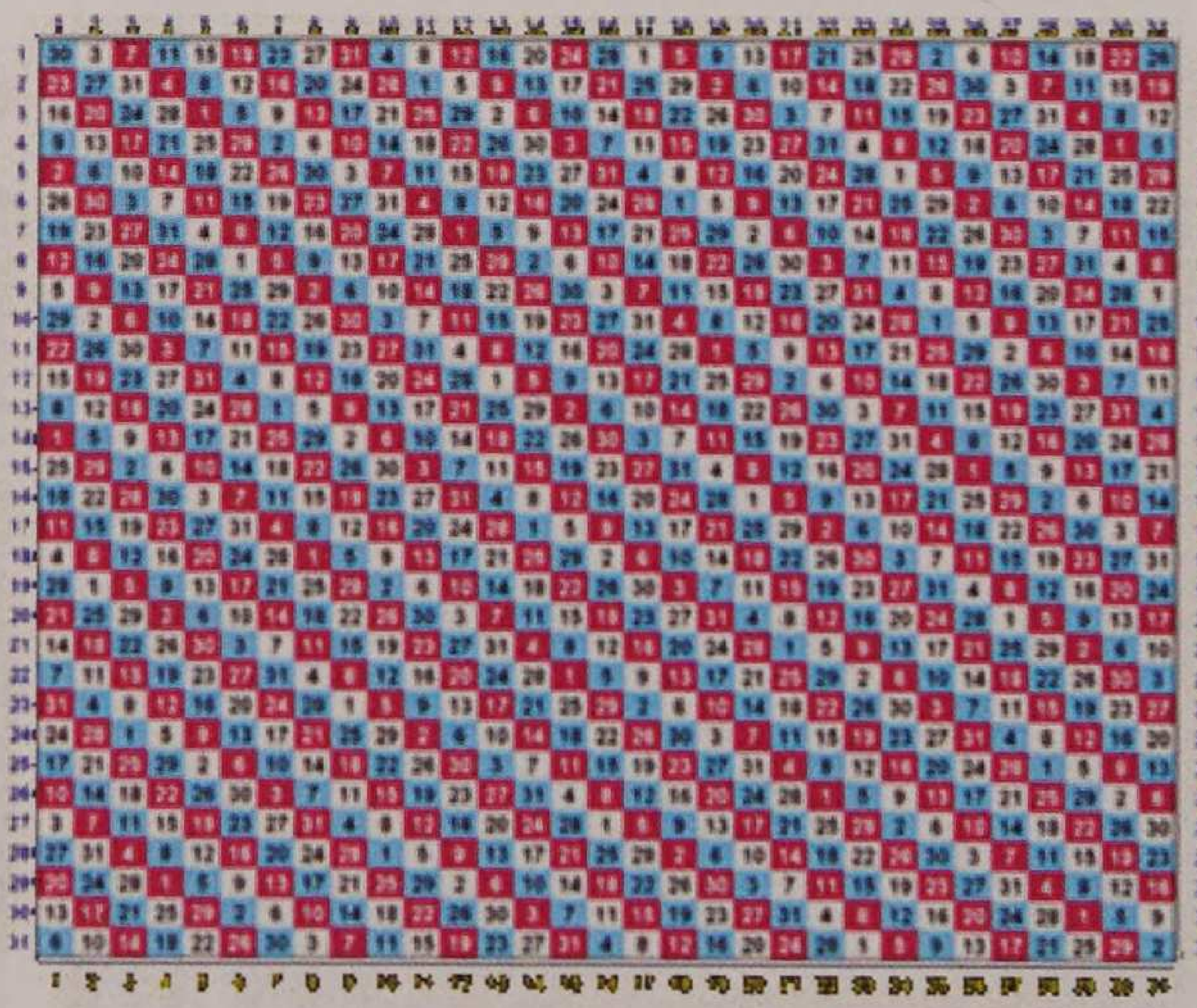

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Size-31 Loom Table X with Tiling Patterns A and hA

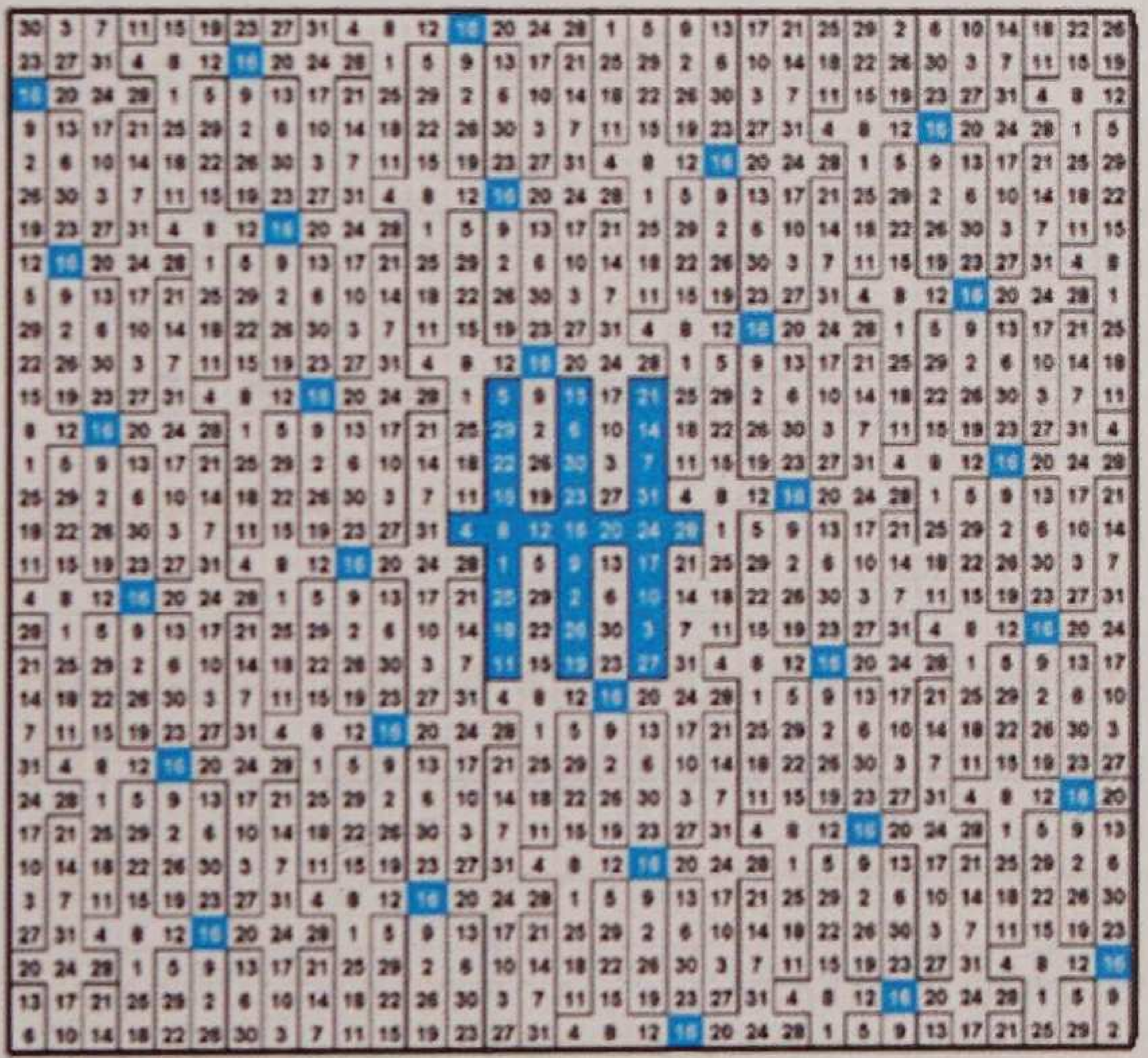
Here is the modulus loom table **X** for the size-31 square with tiling pattern A and its horizontally flipped version hA . The central numbers of the tile patterns in the tables are highlighted in blue. The table on the left is the tiling pattern A on **X** whose central numbers are all equal. The table on the right is the tiling pattern hA on **X** whose central numbers run the gamut from **1** through **31**.

Make note that the constant value among all the tile centers changes as the central tile is relocated over another number in the table. Furthermore, the pivot number for both loom tables is **16**, which lies at the center of each loom table.

This is a general property: the pivot number p = $(n^2 + 1)/2$ will always lie at the center of both loom tables of size n.

X

A



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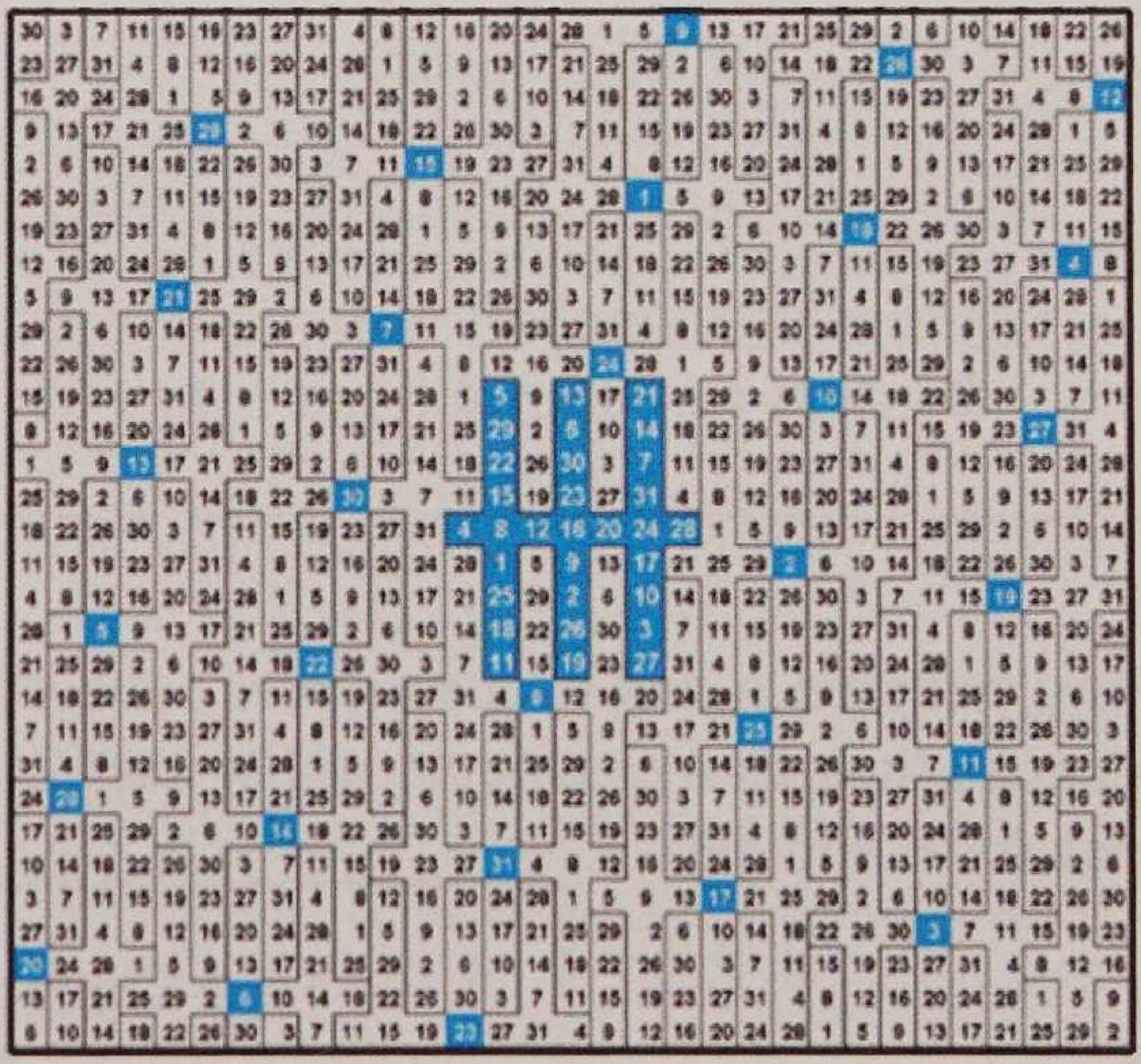
Centers all equal to 16
Pivot number = 16

hA



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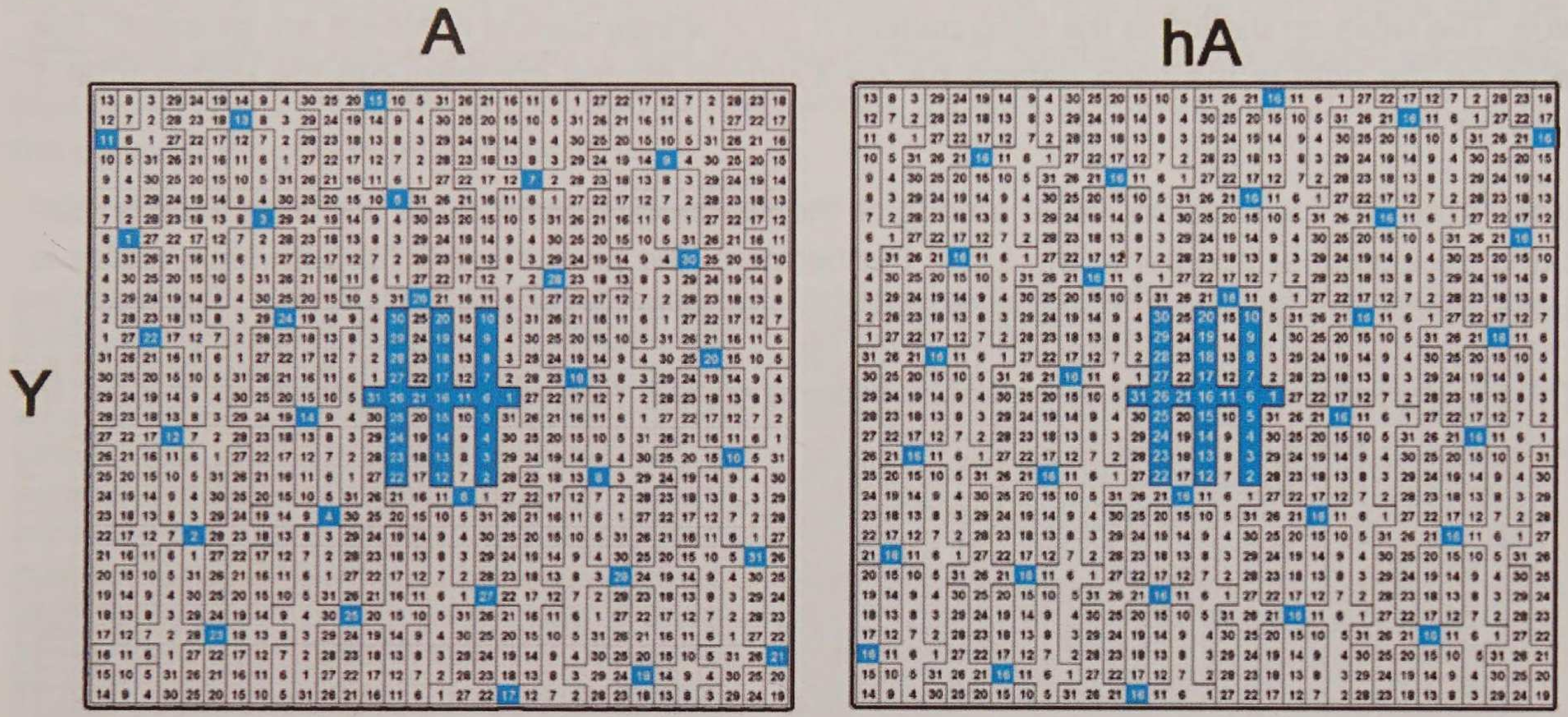
25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21
18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14
11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7
4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31
28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24
21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17
14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10
7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3
31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27
24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20
17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13
10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6
3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30
27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23
20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16
13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9
6	10	14	18	22	26	30	3	7	11	15	19	23	27	31	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2

Centers run gamut from 1 thru 31
Pivot number = 16

Program 2

Size-31 Loom Table Y with Tiling Patterns A and hA

Here is the integer loom table **Y** with tiling pattern A and its horizontally flipped version hA**.** The table on the left is the tiling pattern A on **Y**. Its central numbers run the gamut from **1** through **31**. The table on the right is the tiling pattern hA on **Y**. The central numbers are all equal to **16**. Note the switch in which tile pattern A or hA has the constant central numbers between **X** and **Y**.



Centers run gamut from 1 thru 31

Centers all equal to 16

Tiling patterns A and hA can never have all common central numbers on the same loom table because their tiles propagate within the tiling pattern in different diagonal directions. The same goes for B and hB.

Tile Centers All Equal			
Modulus loom X		Integer loom Y	
A	hB	hA	B

This may become a little confusing when all of these correspondences can't be seen on a single slide, so here is a general table of all the correspondences among the tiling patterns **with constant central numbers** on loom tables **X** and **Y**.

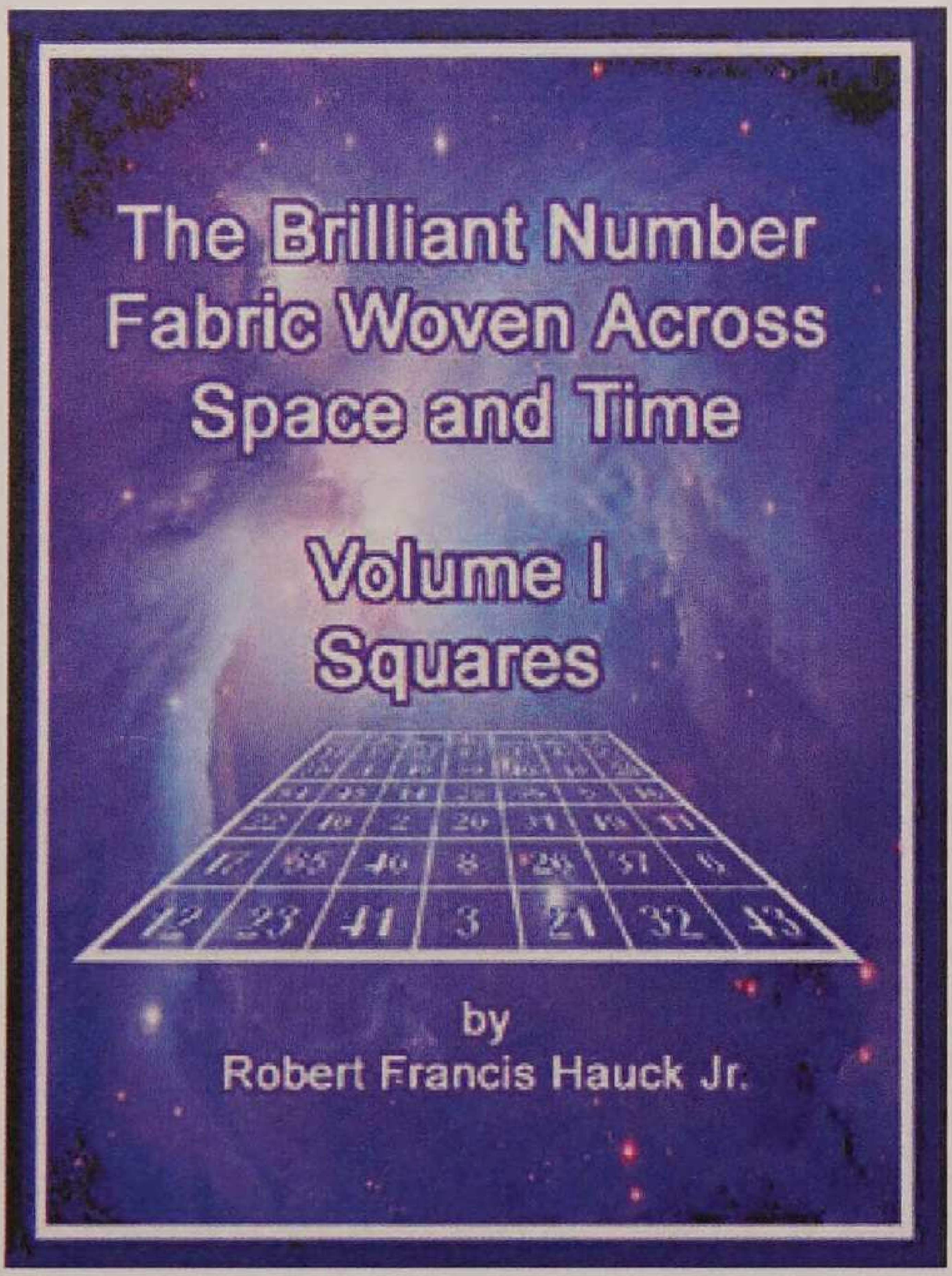
All unlisted correspondences defer to tiling patterns on loom tables of size **n** whose central numbers run the gamut from **1** through **n.**

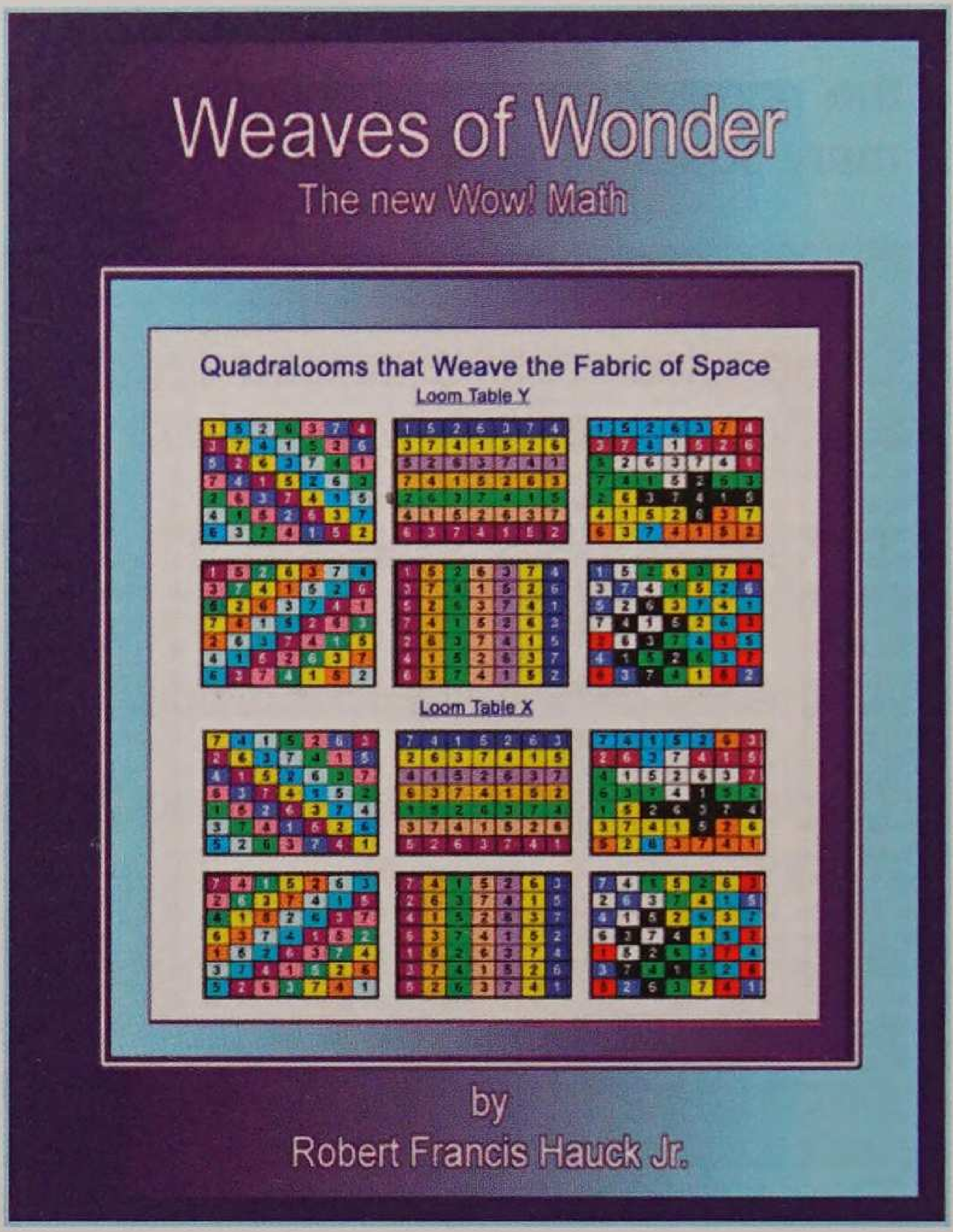
Part II Notes

1. There are two different loom tables, one derived from applying the modulus function to the primary square and the other derived from applying the integer function to the primary square.
2. Every Class-1 and Class-4 square has these complementary geonomic loom tables.
3. Class-1 loom tables are unique among all the classes in that every row, column, all diagonals and all tiles of a table of size-n contains the numbers **1** thru **n** exactly once.
4. The size-n Class-1 loom-tables' joint characteristic number $\mathbf{L_n}$ is the sum of consecutive numbers from **1** to **n**.
5. All the tile patterns sum individually and continuously to $\mathbf{L_n}$ on both loom tables.
6. These loom tables, when used in the regeneration formula, reproduce the original primary square.
7. When the roles of these loom tables are interchanged in the regeneration formula, they produce another perfect square, called its **dual**, which has all the properties as the original primary square but is distinctly different in its distribution of numbers **1** thru $\mathbf{n^2}$.
8. These dual primary squares, both of size **n**, have **n** locations between them where their numbers are identical and these numbers sum to the squares' characteristic number.

We have come to the end of Part II of Program 2 of the new **math**, **Geonometry**.

Here are the two books upon related to this program.





The Brilliant Number Fabric Woven across Space and Time Volume I - Squares

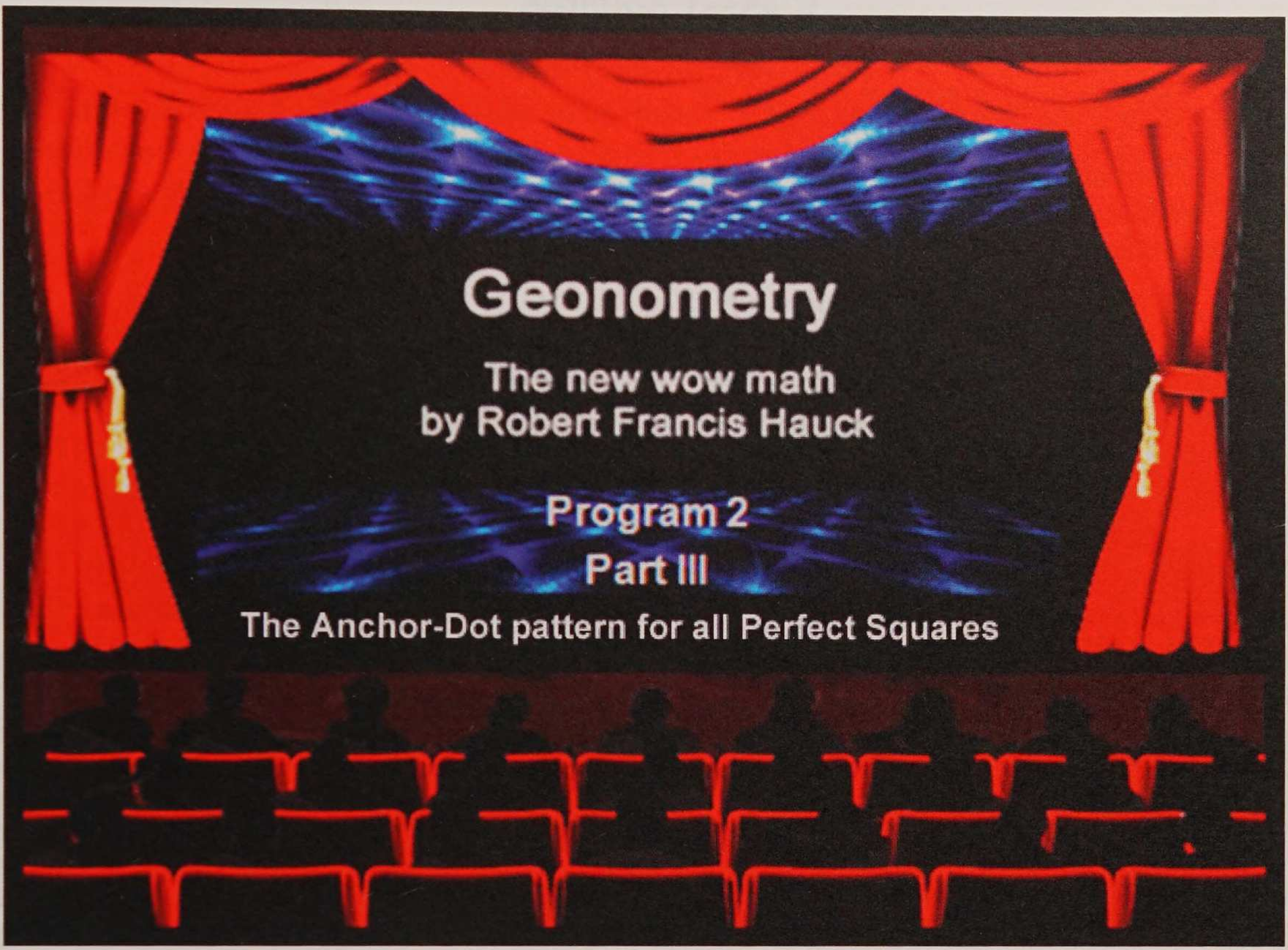
(Rev. October 17th, 2011, Third Edition

ISBN: 978-1-461-06984-3

Weaves of Wonder –The New Wow Math

Shows how to construct geonomic squares from loom tables; January 24th, 2012 First Edition (128 pages)

ISBN: 978-1-469-93296-5

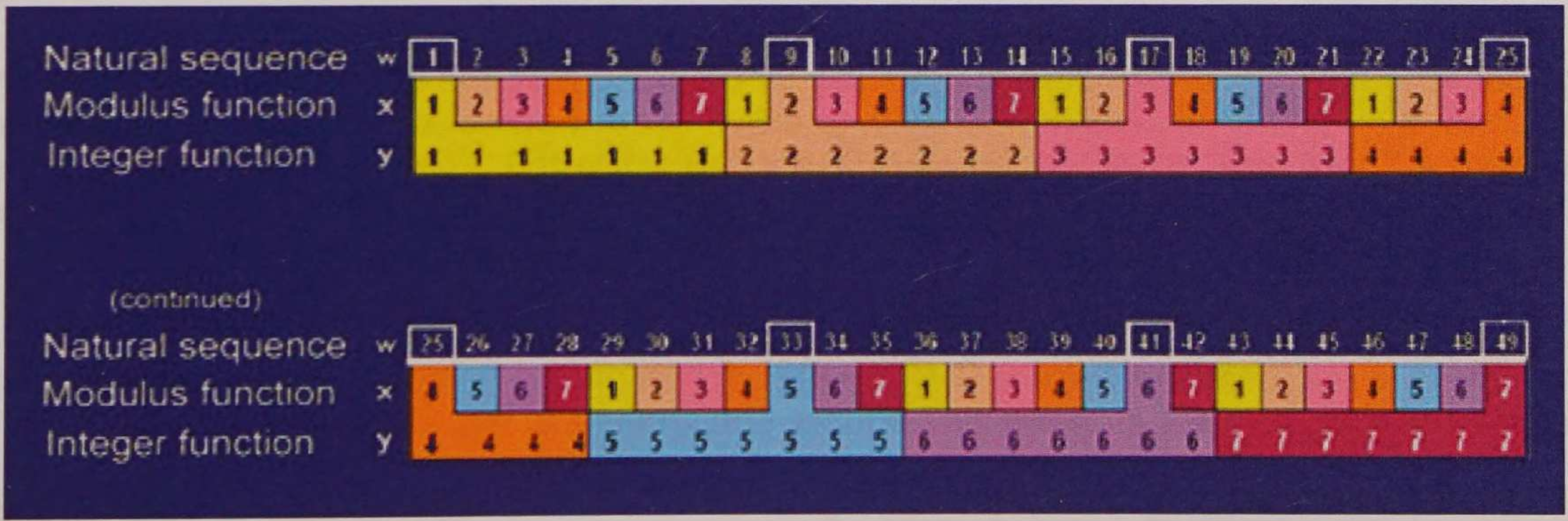


Geonometry

The new wow math
by Robert Francis Hauck

Program 2
Part III
The Anchor-Dot pattern for all Perfect Squares

Next we'll uncover yet another basic pattern that sums continuously to the square's characteristic number. We have seen in Part II that the locations of the values in the primary square where the modulus and integer functions are equal sum to the square's characteristic number. And we also saw that these points of modulus-integer equalities could never be aligned with the centers of characteristic tiling patterns.



Natural sequence	w	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Modulus function	x	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4
Integer function	y	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4

(continued)

Natural sequence	w	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
Modulus function	x	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
Integer function	y	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	7	7

Whereas these points of modulus-integer equalities form a consistently linked dot pattern, there is yet another large set of 4**n** dotted summations which also sum to the square's characteristic number. These will be demonstrated here in Part III.

These patterns are called *anchor-dot* distribution patterns because each central dot in the tiling pattern functions as an anchor for the distribution of additional equal summations in relation to it among all the tiles in the tiling pattern.

Class-1 Squares

Size-7 Anchor-dot Patterns

Here is the size-7 square **W(7)** again. It is shown here with a uniformly distributed pattern of centers of the transpose of its complementary characteristic tiling patterns. Each tile contains **7** cells. Their centers are printed in white. Observe that the centers of both patterns are the only cells which the two tiles have in common.

	1	2	3	4	5	6	7	
	W	175	175	175	175	175	175	175
1	38	27	9	47	29	18	7	175
2	33	15	4	42	24	13	44	175
3	28	10	48	30	19	1	39	175
4	16	5	36	25	14	45	34	175
5	11	49	31	20	2	40	22	175
6	6	37	26	8	46	35	17	175
7	43	32	21	3	41	23	12	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

	1	2	3	4	5	6	7	
	U	175	175	175	175	175	175	175
1	20	39	9	35	5	24	43	175
2	33	3	22	48	18	37	14	175
3	46	16	42	12	31	1	27	175
4	10	29	6	25	44	21	40	175
5	23	49	19	38	8	34	4	175
6	36	13	32	2	28	47	17	175
7	7	26	45	15	41	11	30	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

Anchor-dot pattern = original points of loom table equalities.

175 175 175 175 175 175 175

175 175 175 175 175 175 175

Further, their center numbers are those for which the two squares share a common location and they are the locations where the complementary loom-tables are equal. The same holds for all the transposed tile patterns on the dual square **U(7)** too.

It was already shown that the numbers in the primary and dual squares where the loom-tables are equal sum to the squares' characteristic number.

Here's what is surprising: these transposed tiling patterns can be dragged across the table with roll-wrapping imposed and the tile centers will still sum equally to the characteristic number. Thus the transposed tile centers can function as **anchor dots** for the tiling patterns. Every location relative to these dots among all the tiles sum equally. That yields **7** equal dot patterns for the size-7 squares for one transposed tile pattern and 6 more from the other, for 13.

These patterns may be represented numerically by counting first, the number of cells vertically, and second, the number of cells horizontally, to arrive at the nearest center of an adjoining tile. Tile pattern $\mathbf{A}^T$ here may be described as **(1;2)** for "**up 1; right 2**". Tile pattern $\mathbf{B}^T$ is described as **(1;2)** also. In this methodology, there is no "down" and "left" is indicated by a negative number.

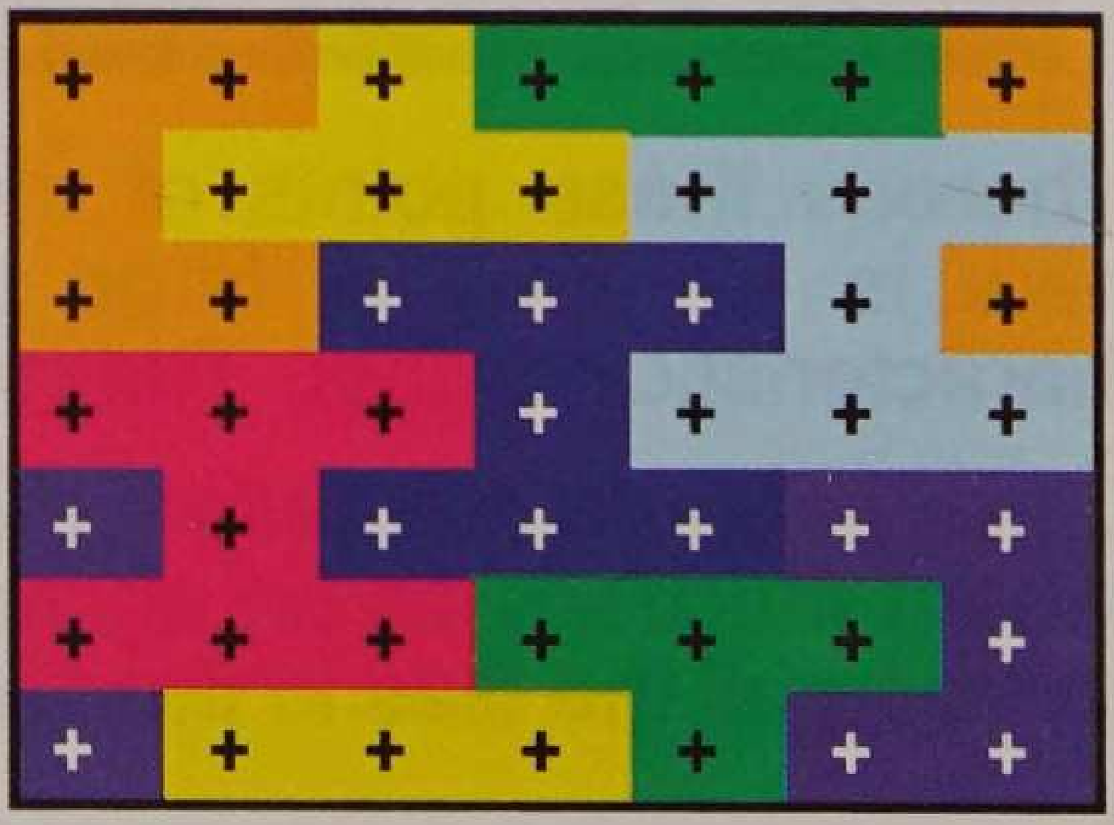
Here is another surprise: The horizontal flip of both patterns adds another **13** equal sums.

Further, there is another tiling pattern **C** of the description of **(2;1)** which is also equal-summing to the characteristic number for the size-7 square shown here at left. Check it out.

(2;1)

175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

However, the transpose of this tiling pattern $\mathbf{C}^T$ is also described numerically by **(1;2)** and yields nothing new that isn't already accounted for jointly by $\mathbf{A}^T$ and $\mathbf{B}^T$. So the size-7 has only **26** equal-summing dot patterns.



+	+	+	+	+	+	+
+	+	+	+	+	+	+
+	+	+	+	+	+	+
+	+	+	+	+	+	+
+	+	+	+	+	+	+
+	+	+	+	+	+	+
+	+	+	+	+	+	+

Program 2

Transforming the anchor-dot pattern into an alternative ultra-perfect square

			X							Y			
3	6	**2**	5	1	4	7	6	4	**2**	7	5	3	1
5	1	4	7	3	6	2	**5**	3	1	6	4	2	7
7	3	6	2	5	**1**	4	4	2	7	5	3	**1**	6
2	5	1	**4**	7	3	6	3	1	6	**4**	2	7	5
4	**7**	3	6	2	5	1	2	**7**	5	3	1	6	4
6	2	5	1	4	7	**3**	1	6	4	2	7	5	**3**
1	4	7	3	**6**	2	5	7	5	3	1	**6**	4	2
3	6	**2**	5	1	4	7	6	4	**2**	7	5	3	1
5	1	4	7	3	6	2	**5**	3	1	6	4	2	7
7	3	6	2	5	**1**	4	4	2	7	5	3	**1**	6
2	5	1	**4**	7	3	6	3	1	6	**4**	2	7	5
4	**7**	3	6	2	5	1	2	**7**	5	3	1	6	4
6	2	5	1	4	7	**3**	1	6	4	2	7	5	**3**
1	4	7	3	**6**	2	5	7	5	3	1	**6**	4	2

At left are the loom tables **X** and **Y** derived from the primal size-7 square **W** on the prior page. Each has been duplicated once below them to facilitate the derivation of loom tables from the anchor-dot pattern.

The highlighted numbers are the points of equality between them. Below that are the loom tables **X*** and **Y*** derived by merely taking the numbers highlighted in each column in the first block and making a row from their values in each table.

Then this pattern is dragged down vertically recording the values as subsequent rows in **X*** and **Y*** until the last row in each has been completed.

X*	28	28	28	28	28	28	28
2	5	1	4	7	3	6	28
4	7	3	6	2	5	1	28
6	2	5	1	4	7	3	28
1	4	7	3	6	2	5	28
3	6	2	5	1	4	7	28
5	1	4	7	3	6	2	28
7	3	6	2	5	1	4	28
28	28	28	28	28	28	28	
	28	28	28	28	28	28	28

Y*						
2	5	1	4	7	3	6
1	4	7	3	6	2	5
7	3	6	2	5	1	4
6	2	5	1	4	7	3
5	1	4	7	3	6	2
4	7	3	6	2	5	1
3	6	2	5	1	4	7

Y^{*T}	28	28	28	28	28	28	28
2	1	7	6	5	4	3	28
5	4	3	2	1	7	6	28
1	7	6	5	4	3	2	28
4	3	2	1	7	6	5	28
7	6	5	4	3	2	1	28
3	2	1	7	6	5	4	28
6	5	4	3	2	1	7	28
28	28	28	28	28	28	28	
	28	28	28	28	28	28	28

Next **Y*** is transposed to get **Y^{*T}**. The new square is derived from the standard generation formula using the reconstructed modulus loom-table and the transpose of the reconstructed integer loom-table to get **W***. Then **W*** is normalized and summed geonomically to produce another ultra-perfect size-7 square **W(7)** as shown here. Check it out.

$$W^* = 7*(Y^* - |\,1\,|) + X^*$$

9	5	43	39	35	24	20
32	28	17	13	2	47	36
6	44	40	29	25	21	10
22	18	14	3	48	37	33
45	41	30	26	15	11	7
19	8	4	**49**	38	34	23
42	31	27	16	12	**1**	46

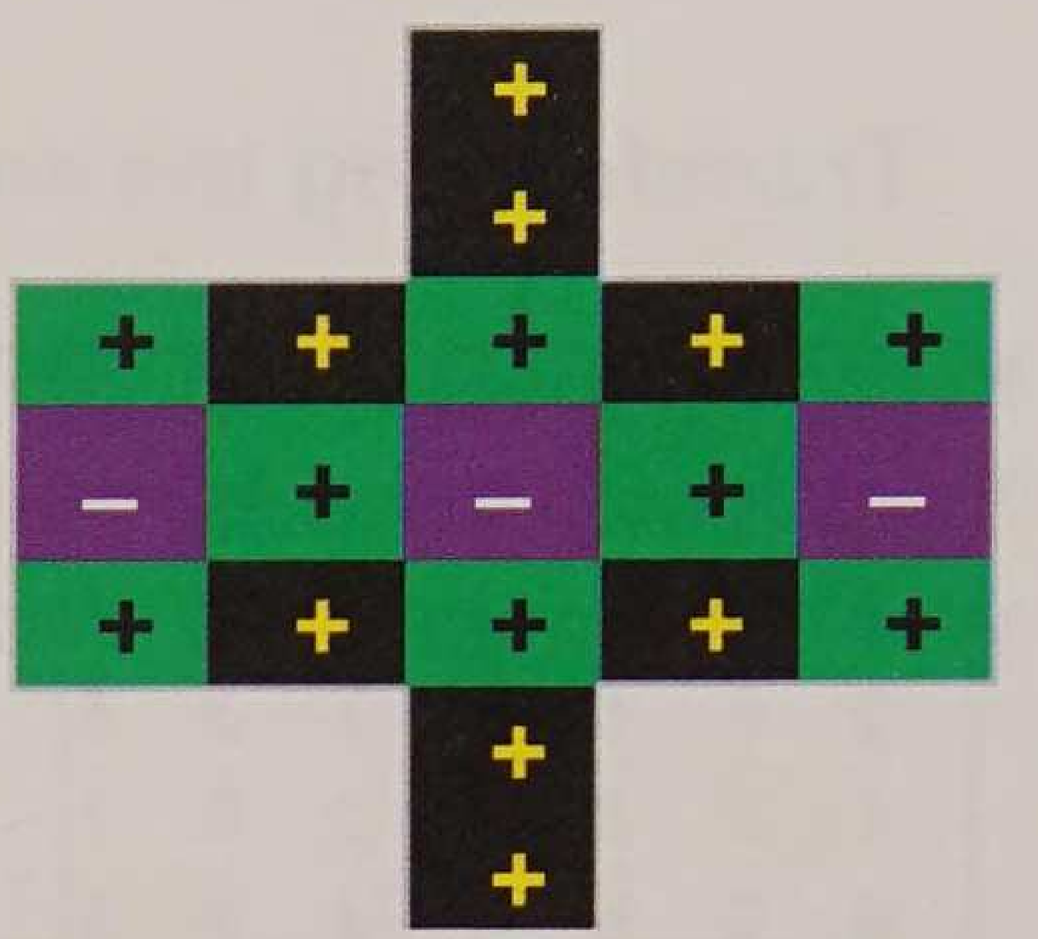
W*(7) After normalization

	175	175	175	175	175	175	175
31	27	16	12	**1**	46	42	175
5	43	39	35	24	20	9	175
28	17	13	2	47	36	32	175
44	40	29	25	21	10	6	175
18	14	3	48	37	33	22	175
41	30	26	15	11	7	45	175
8	4	**49**	38	34	23	19	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

This method is a general method that can be applied to any size Class-1 square whose points of equality between its own loom-tables are not linearly distributed such as along a main diagonal or across a row or column.

Program 2

Size-11 Anchor-dot Patterns

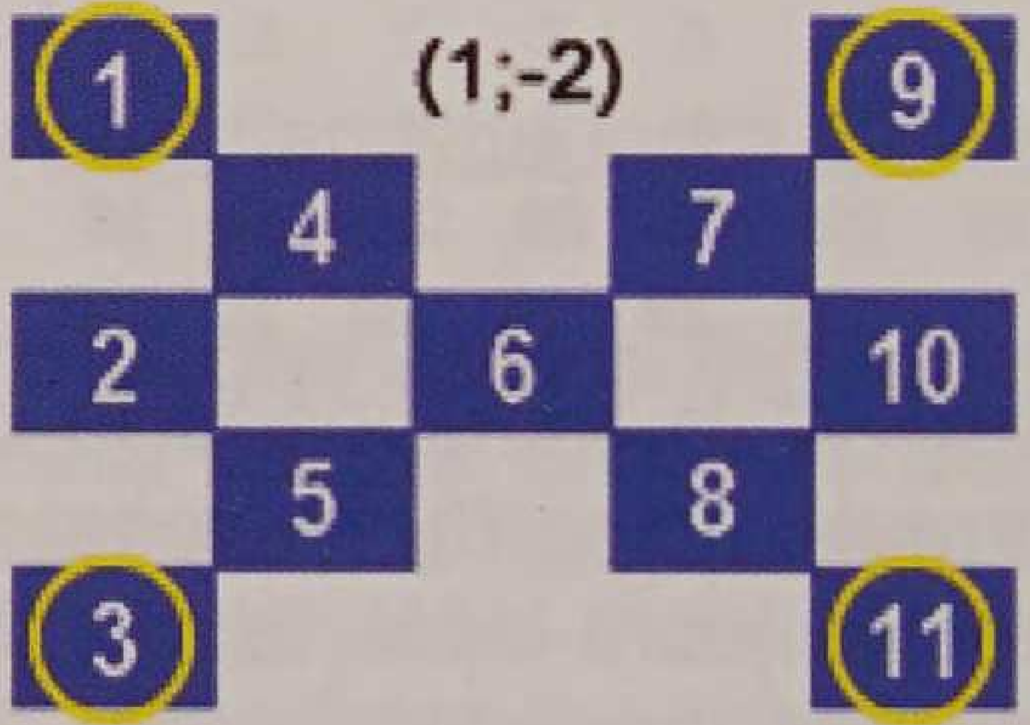


Here are the size-11 primary squares with their common characteristic tiling patterns transposed. Again, the centers are aligned between the tiles and are shown with a distributed anchor-dot pattern. On the left is pattern **A^T @ (1;-2)** and on the right is pattern **B^T @ (1;-2)**. And again, the sum of numbers in the same location among the **11** tiling patterns sum continuously to **671**, the size-11's characteristic number.

W(11)	671	671	671	671	671	671	671	671	671	671	671
21	37	64	80	107	2	29	45	72	99	115	671
24	51	67	94	121	16	43	59	86	102	8	671
38	65	81	108	3	30	46	73	89	116	22	671
52	68	95	111	17	44	60	87	103	9	25	671
66	82	109	4	31	47	74	90	117	12	39	671
69	96	112	18	34	61	88	104	10	26	53	671
83	110	5	32	48	75	91	118	13	40	56	671
97	113	19	35	62	78	105	11	27	54	70	671
100	6	33	49	76	92	119	14	41	57	84	671
114	20	36	63	79	106	1	28	55	71	98	671
7	23	50	77	93	120	15	42	58	85	101	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671

U(11)	671	671	671	671	671	671	671	671	671	671	671
101	37	94	30	87	12	69	5	62	119	55	671
14	71	7	64	121	46	103	39	96	32	78	671
48	105	41	98	23	80	16	73	9	66	112	671
82	18	75	11	57	114	50	107	43	89	25	671
116	52	109	34	91	27	84	20	77	2	59	671
29	86	22	68	4	61	118	54	100	36	93	671
63	120	45	102	38	95	31	88	13	70	6	671
97	33	79	15	72	8	65	111	47	104	40	671
10	56	113	49	106	42	99	24	81	17	74	671
44	90	26	83	19	76	1	58	115	51	108	671
67	3	60	117	53	110	35	92	28	85	21	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671

But there's more! The distribution patterns of **hA^T @ (1;2)** and **hB^T @ (1;2)** do too. Together each of these compound patterns adds **2x11–3 = 19 additional** equal summations for a total so far of **2x19=38**.



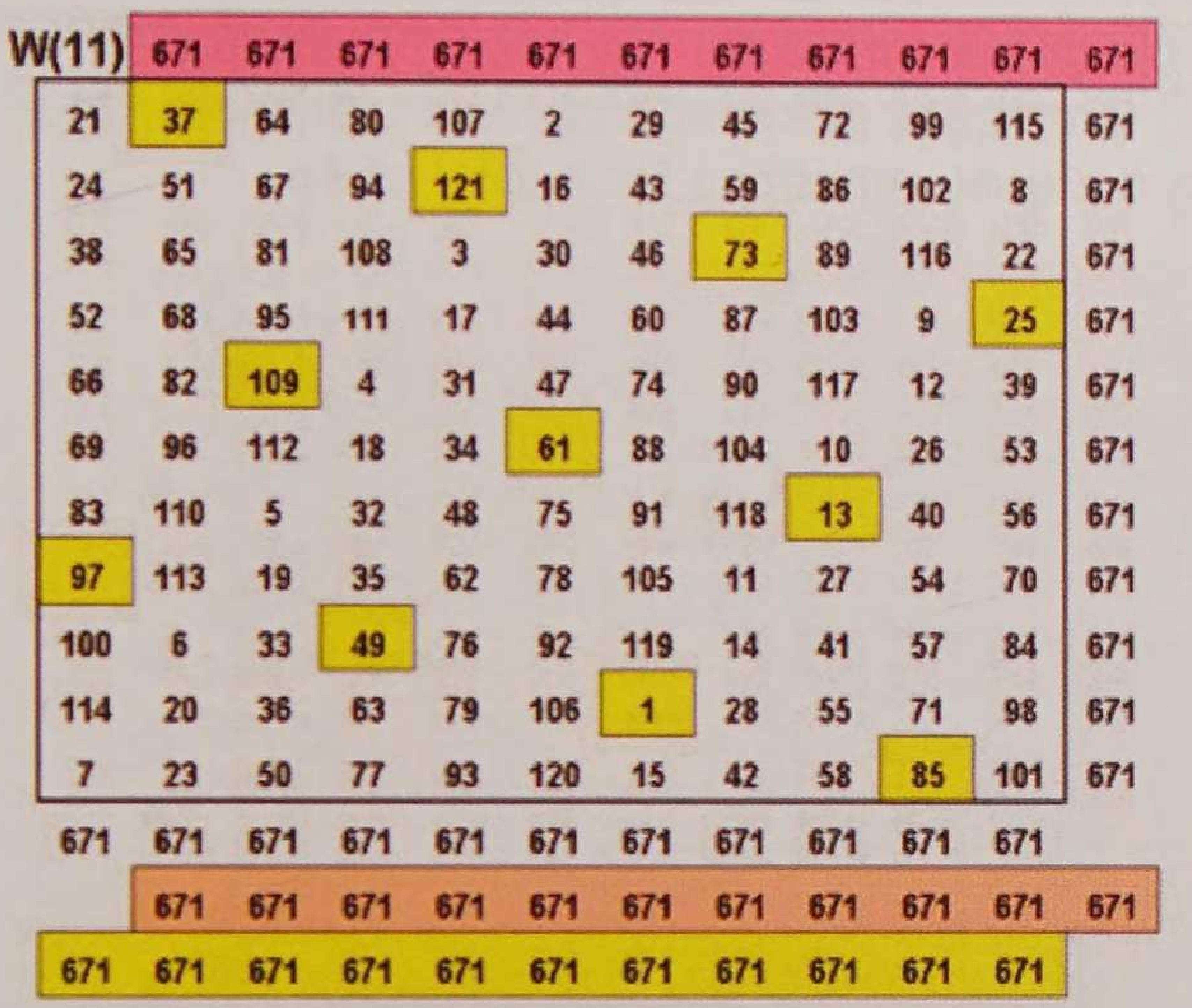
There is a **3rd** continuously summing contiguous tiling pattern **C @ (1;-2)** at right for which its transpose **C^T** yields anchor-dots **@ (1;2)** in their centers and possesses the continuously equal summing dot pattern as shown here on the primal and dual squares below. Because the horizontal flip **hC^T @ (1;-2)** does the same, that pattern adds another **2x4 = 8** equal summing patterns to the mix for a total of **46**.

W(11)	671	671	671	671	671	671	671	671	671	671	671
21	37	64	80	107	2	29	45	72	99	115	671
24	51	67	94	121	16	43	59	86	102	8	671
38	65	81	108	3	30	46	73	89	116	22	671
52	68	95	111	17	44	60	87	103	9	25	671
66	82	109	4	31	47	74	90	117	12	39	671
69	96	112	18	34	61	88	104	10	26	53	671
83	110	5	32	48	75	91	118	13	40	56	671
97	113	19	35	62	78	105	11	27	54	70	671
100	6	33	49	76	92	119	14	41	57	84	671
114	20	36	63	79	106	1	28	55	71	98	671
7	23	50	77	93	120	15	42	58	85	101	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671

U(11)	671	671	671	671	671	671	671	671	671	671	671
101	37	94	30	87	12	69	5	62	119	55	671
14	71	7	64	121	46	103	39	96	32	78	671
48	105	41	98	23	80	16	73	9	66	112	671
82	18	75	11	57	114	50	107	43	89	25	671
116	52	109	34	91	27	84	20	77	2	59	671
29	86	22	68	4	61	118	54	100	36	93	671
63	120	45	102	38	95	31	88	13	70	6	671
97	33	79	15	72	8	65	111	47	104	40	671
10	56	113	49	106	42	99	24	81	17	74	671
44	90	26	83	19	76	1	58	115	51	108	671
67	3	60	117	53	110	35	92	28	85	21	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671

Program 2

We saw for the size-7 squares that one set of anchor-dots are the very locations where their primary squares and their common loom tables are equal. Those points are depicted here for the size-11 square. However, none of the transposed tile centers align with these points @ **(1;-3)** nor the horizontally flipped version @ **(1;3)**. So these dot-patterns do not conflict with any tiling patterns.



W(11)	671	671	671	671	671	671	671	671	671	671	671
21	37	64	80	107	2	29	45	72	99	115	671
24	51	67	94	121	16	43	59	86	102	8	671
38	65	81	108	3	30	46	73	89	116	22	671
52	68	95	111	17	44	60	87	103	9	25	671
66	82	109	4	31	47	74	90	117	12	39	671
69	96	112	18	34	61	88	104	10	26	53	671
83	110	5	32	48	75	91	118	13	40	56	671
97	113	19	35	62	78	105	11	27	54	70	671
100	6	33	49	76	92	119	14	41	57	84	671
114	20	36	63	79	106	1	28	55	71	98	671
7	23	50	77	93	120	15	42	58	85	101	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671
671	671	671	671	671	671	671	671	671	671	671	

U(11)	671	671	671	671	671	671	671	671	671	671	671
101	37	94	30	87	12	69	5	62	119	55	671
14	71	7	64	121	46	103	39	96	32	78	671
48	105	41	98	23	80	16	73	9	66	112	671
82	18	75	11	57	114	50	107	43	89	25	671
116	52	109	34	91	27	84	20	77	2	59	671
29	86	22	68	4	61	118	54	100	36	93	671
63	120	45	102	38	95	31	88	13	70	6	671
97	33	79	15	72	8	65	111	47	104	40	671
10	56	113	49	106	42	99	24	81	17	74	671
44	90	26	83	19	76	1	58	115	51	108	671
67	3	60	117	53	110	35	92	28	85	21	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671
671	671	671	671	671	671	671	671	671	671	671	

So these equal summing dot patterns contribute **11** equal summations. The horizontal flip of this pattern does too. That brings the total count to **68= 46+2x11** for just the dot patterns alone.

Now there are **3** distinct contiguous tiling patterns here, each of which have **11** tiles summing equally to the characteristic number. Since they do so continuously, they add **$3n^2$=363** equal summations to the mix. That brings the total equal tiling-pattern summations to **68+363=431**. That's nearly **10** times all the linear summations combined, **4n = 44** !

So what started out as a seemingly improbable number of linear equalities for perfect squares was merely the tip of the icegerg, **9/10**ths of which lies below the water's surface. In otherwords, the **5C** tiling patterns alone yield characteristic summations an order of magnitude greater than do all the other characteristic summations combined.

Note 1: Of all the Class-1 squares for sizes **7** thru **31**, the size-7 square was the only one whose points of dual and loom equalities could be aligned with the centers of its transposed tiling patterns.

Note 2: Backing up, the size-5 square was determined to have no equal-summing dot patterns at all that were not all located in its central row or central column. The centers of the totally double-symmetric tiles, "**+**" & "**x**" did not sum equally. So these anchor dot patterns arise in Class-1 squares beginning with size-7 and continue on from there.

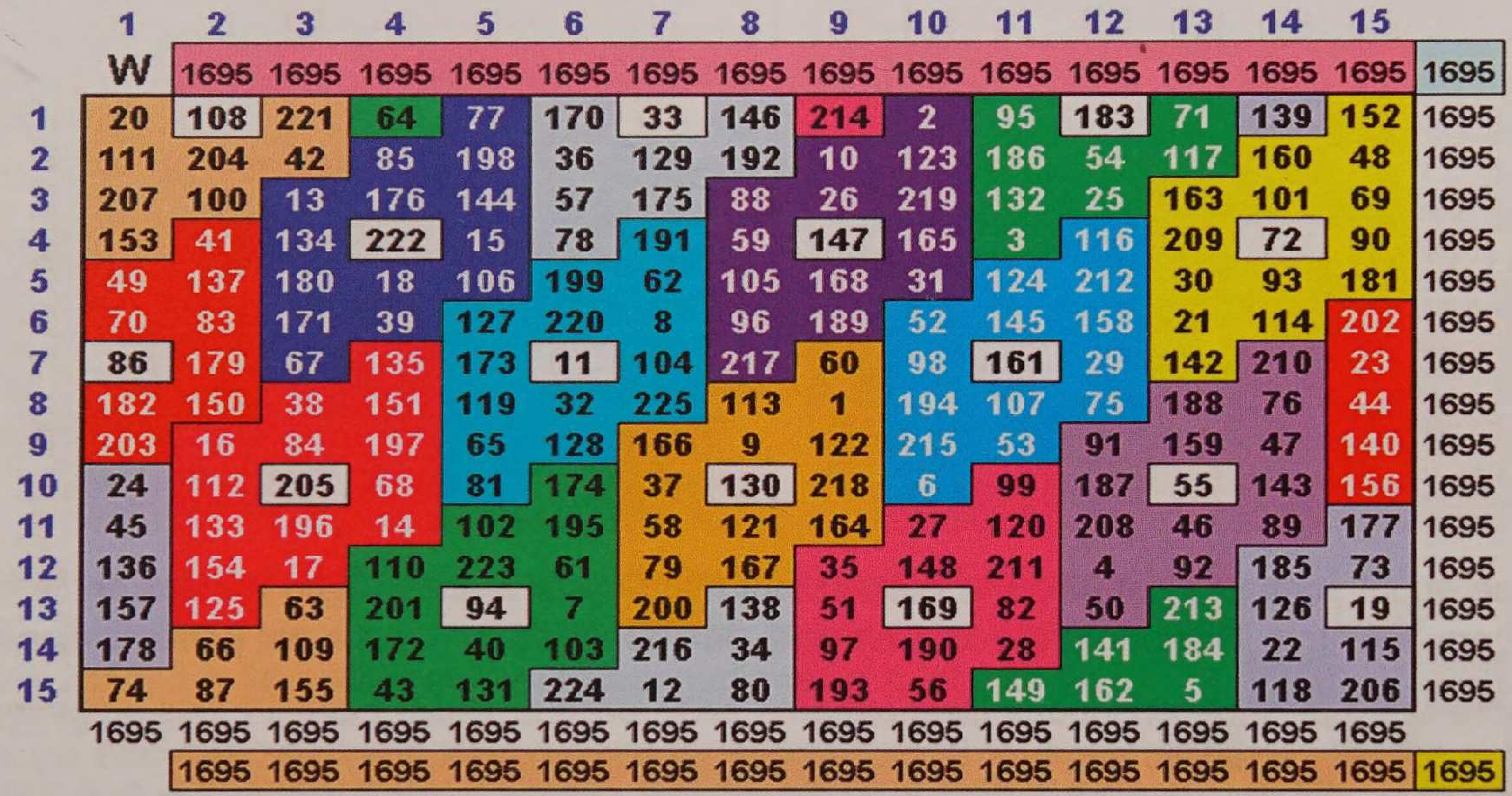
Note 3: Each size Class-1 square beyond size-7 possesses **5n** equal-summing dot patterns (**4n** from the tiles themselves and **1n** from the points of equality between the loom-tables) and **$2n^2$** equal-summing tile patterns. So it is clear that tile patterns yield more equal characteristic summations than all the linear summations combined from size-7 squares on.

Program 2

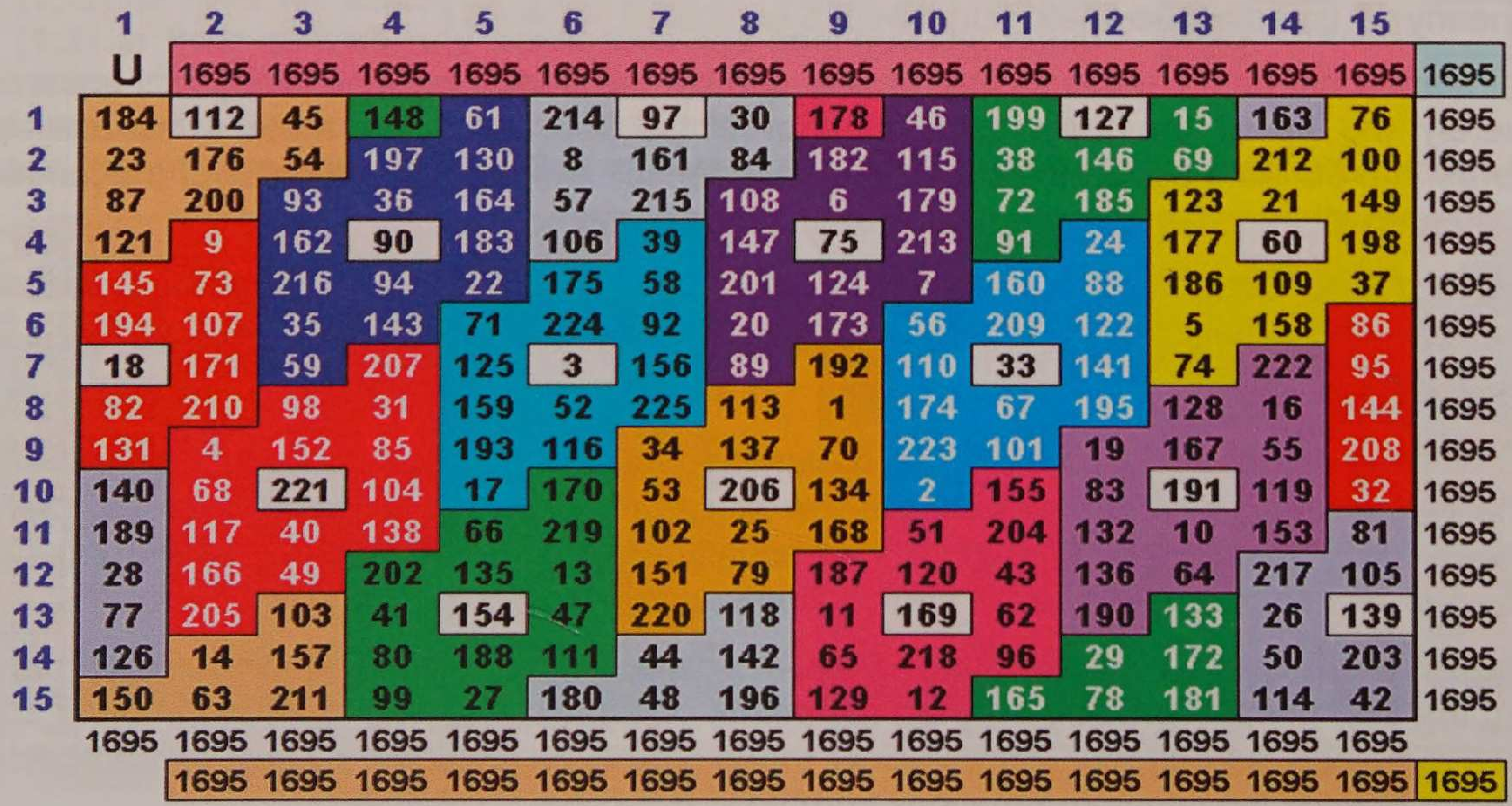
Class-3 Squares

Size-15 Anchor-dot Patterns

Here are the size-15 primal and dual squares. Each has been segregated into tiles. These tiles do not have the numbers for which the modulus and integer functions are equal at their centers. Nonetheless, the same anchor-dot property holds: all the numbers in the same location among the tiles sum to the characteristic number **1695** for the size-15 square. The transpose of this pattern does the same, yielding **2n** equal summations. And still more, the horizontal flip of these patterns add yet another **2n** equal summations for a total of **4n**.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	W	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	108	221	64	77	170	33	146	214	2	95	183	71	139	152	1695
2	111	204	42	85	198	36	129	192	10	123	186	54	117	160	48	1695
3	207	100	13	176	144	57	175	88	26	219	132	25	163	101	69	1695
4	153	41	134	222	15	78	191	59	147	165	3	116	209	72	90	1695
5	49	137	180	18	106	199	62	105	168	31	124	212	30	93	181	1695
6	70	83	171	39	127	220	8	96	189	52	145	158	21	114	202	1695
7	86	179	67	135	173	11	104	217	60	98	161	29	142	210	23	1695
8	182	150	38	151	119	32	225	113	1	194	107	75	188	76	44	1695
9	203	16	84	197	65	128	166	9	122	215	53	91	159	47	140	1695
10	24	112	205	68	81	174	37	130	218	6	99	187	55	143	156	1695
11	45	133	196	14	102	195	58	121	164	27	120	208	46	89	177	1695
12	136	154	17	110	223	61	79	167	35	148	211	4	92	185	73	1695
13	157	125	63	201	94	7	200	138	51	169	82	50	213	126	19	1695
14	178	66	109	172	40	103	216	34	97	190	28	141	184	22	115	1695
15	74	87	155	43	131	224	12	80	193	56	149	162	5	118	206	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
		1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	U	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	184	112	45	148	61	214	97	30	178	46	199	127	15	163	76	1695
2	23	176	54	197	130	8	161	84	182	115	38	146	69	212	100	1695
3	87	200	93	36	164	57	215	108	6	179	72	185	123	21	149	1695
4	121	9	162	90	183	106	39	147	75	213	91	24	177	60	198	1695
5	145	73	216	94	22	175	58	201	124	7	160	88	186	109	37	1695
6	194	107	35	143	71	224	92	20	173	56	209	122	5	158	86	1695
7	18	171	59	207	125	3	156	89	192	110	33	141	74	222	95	1695
8	82	210	98	31	159	52	225	113	1	174	67	195	128	16	144	1695
9	131	4	152	85	193	116	34	137	70	223	101	19	167	55	208	1695
10	140	68	221	104	17	170	53	206	134	2	155	83	191	119	32	1695
11	189	117	40	138	66	219	102	25	168	51	204	132	10	153	81	1695
12	28	166	49	202	135	13	151	79	187	120	43	136	64	217	105	1695
13	77	205	103	41	154	47	220	118	11	169	62	190	133	26	139	1695
14	126	14	157	80	188	111	44	142	65	218	96	29	172	50	203	1695
15	150	63	211	99	27	180	48	196	129	12	165	78	181	114	42	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
		1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

The same anchor-dot property holds for the size **21** and **27** squares segregated into **21** and **27** cell offset tiles, respectively (not shown).

Program 2

Class-4 Squares

Size-12 square Anchor-dot Patterns

Here are the size-12 primal and dual squares. Each has been segregated into **3x4** off-set blocks; one with the blocks transposed. The same anchor-dot property holds: all the numbers in the same location among all the blocks sum to **870**, the characteristic number for the size-12 square. Both initial and transposed block patterns work on either squares. The horizontal flip of these patterns add another **2n** equal-summations for a total of **4n**.

	1	2	3	4	5	6	7	8	9	10	11	12	
W(12)	870	870	870	870	870	870	870	870	870	870	870	870	870
1	21	106	95	84	101	26	31	100	85	90	111	20	870
2	144	67	6	9	80	131	134	73	16	3	70	137	870
3	50	45	124	119	34	61	60	39	114	125	44	55	870
4	59	40	113	126	43	56	49	46	123	120	33	62	870
5	133	74	15	4	69	138	143	68	5	10	79	132	870
6	32	99	86	89	112	19	22	105	96	83	102	25	870
7	18	109	92	87	98	29	28	103	82	93	108	23	870
8	139	72	1	14	75	136	129	78	11	8	65	142	870
9	53	42	127	116	37	58	63	36	117	122	47	52	870
10	64	35	118	121	48	51	54	41	128	115	38	57	870
11	130	77	12	7	66	141	140	71	2	13	76	135	870
12	27	104	81	94	107	24	17	110	91	88	97	30	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

	1	2	3	4	5	6	7	8	9	10	11	12	
U(12)	870	870	870	870	870	870	870	870	870	870	870	870	870
1	38	88	132	33	43	83	128	34	42	87	133	29	870
2	144	23	46	73	140	27	54	77	136	19	50	81	870
3	13	111	107	62	12	112	103	57	17	116	102	58	870
4	94	66	8	125	93	67	4	120	98	71	3	121	870
5	45	86	127	28	41	90	135	32	37	82	131	36	870
6	137	25	51	78	142	20	47	79	141	24	52	74	870
7	11	115	105	60	16	110	101	61	15	114	106	56	870
8	99	68	1	118	95	72	9	122	91	64	5	126	870
9	40	84	134	35	39	85	130	30	44	89	129	31	870
10	139	21	53	80	138	22	49	75	143	26	48	76	870
11	18	113	100	55	14	117	108	59	10	109	104	63	870
12	92	70	6	123	97	65	2	124	96	69	7	119	870
	870	870	870	870	870	870	870	870	870	870	870	870	
	870	870	870	870	870	870	870	870	870	870	870	870	870

Program 2

Size-16 square Anchor-dot Patterns

Here are the size-16 primal and dual squares. Each has been segregated into **4x4** off-set block-squares. All the numbers in the same location among the blocks sum to **2056**, the characteristic number for the size-16 square. This same property also holds for the transpose of these blocks too (shown on the dual) for a total of **32 = 2n** additional equal summations to the characteristic number per square. The horizontal flip of these patterns add another **2n**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
	W(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	8	105	33	80	184	217	145	256	136	233	161	208	56	89	17	128	2056
2	186	215	159	242	10	103	47	66	58	87	31	114	138	231	175	194	2056
3	62	83	27	118	142	227	171	198	190	211	155	246	14	99	43	70	2056
4	132	237	165	204	52	93	21	124	4	109	37	76	180	221	149	252	2056
5	69	44	100	13	245	156	212	189	197	172	228	141	117	28	84	61	2056
6	251	150	222	179	75	38	110	3	123	22	94	51	203	166	238	131	2056
7	127	18	90	55	207	162	234	135	255	146	218	183	79	34	106	7	2056
8	193	176	232	137	113	32	88	57	65	48	104	9	241	160	216	185	2056
9	72	41	97	16	248	153	209	192	200	169	225	144	120	25	81	64	2056
10	250	151	223	178	74	39	111	2	122	23	95	50	202	167	239	130	2056
11	126	19	91	54	206	163	235	134	254	147	219	182	78	35	107	6	2056
12	196	173	229	140	116	29	85	60	68	45	101	12	244	157	213	188	2056
13	5	108	36	77	181	220	148	253	133	236	164	205	53	92	20	125	2056
14	187	214	158	243	11	102	46	67	59	86	30	115	139	230	174	195	2056
15	63	82	26	119	143	226	170	199	191	210	154	247	15	98	42	71	2056
16	129	240	168	201	49	96	24	121	1	112	40	73	177	224	152	249	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

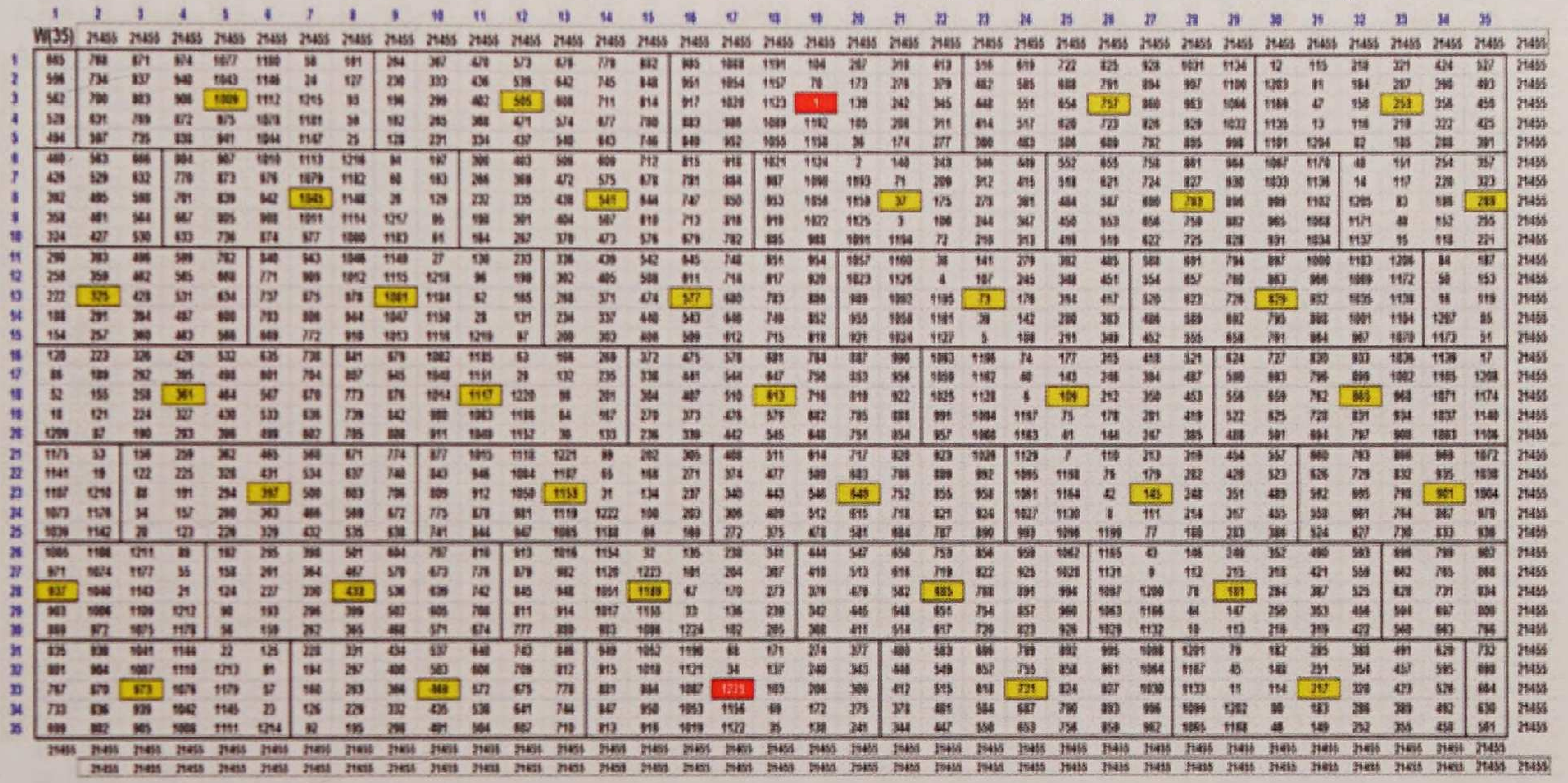
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
	U(16)	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
1	113	135	3	245	124	142	10	256	121	143	11	253	116	134	2	248	2056
2	156	110	234	32	145	103	227	21	148	102	226	24	153	111	235	29	2056
3	212	38	162	88	217	47	171	93	220	46	170	96	209	39	163	85	2056
4	57	207	75	189	52	198	66	184	49	199	67	181	60	206	74	192	2056
5	69	179	55	193	80	186	62	204	77	187	63	201	72	178	54	196	2056
6	176	90	222	44	165	83	215	33	168	82	214	36	173	91	223	41	2056
7	232	18	150	100	237	27	159	105	240	26	158	108	229	19	151	97	2056
8	13	251	127	137	8	242	118	132	5	243	119	129	16	250	126	140	2056
9	117	131	7	241	128	138	14	252	125	139	15	249	120	130	6	244	2056
10	160	106	238	28	149	99	231	17	152	98	230	20	157	107	239	25	2056
11	216	34	166	84	221	43	175	89	224	42	174	92	213	35	167	81	2056
12	61	203	79	185	56	194	70	180	53	195	71	177	64	202	78	188	2056
13	65	183	51	197	76	190	58	208	73	191	59	205	68	182	50	200	2056
14	172	94	218	48	161	87	211	37	164	86	210	40	169	95	219	45	2056
15	228	22	146	104	233	31	155	109	236	30	154	112	225	23	147	101	2056
16	9	255	123	141	4	246	114	136	1	247	115	133	12	254	122	144	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Program 2

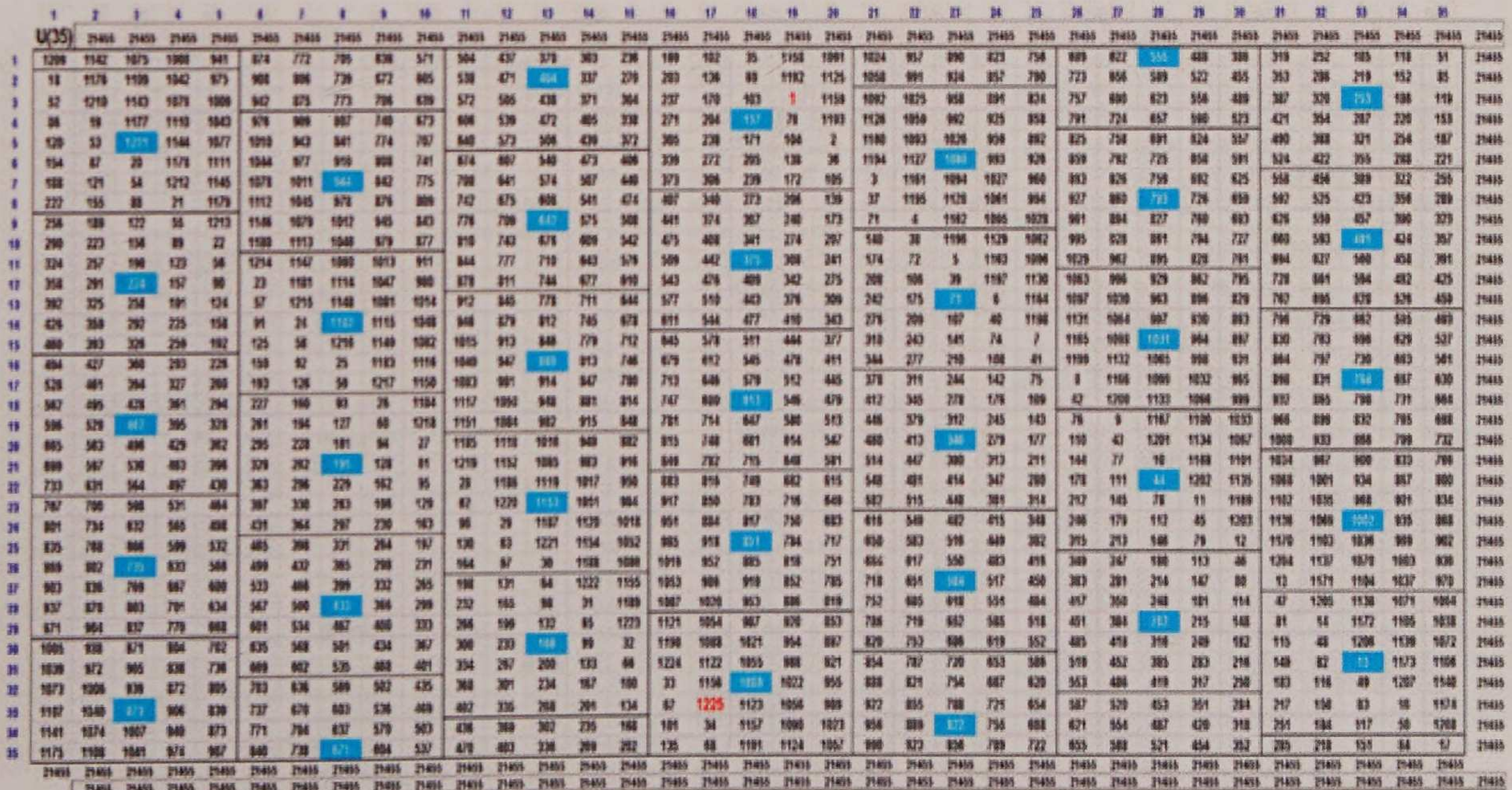
Class-5 Squares

Size-35 square Anchor-dot Patterns

Here is the only legible Class-5 square: size-35. It is segregated into **5x7** blocks centered around the locations where the values of its primal and dual squares and their modulus and integer functions are equal. Just as for Class-1 squares, every anchor-dot pattern relative to one cell in the same location @ **(5;-2)** in each **5x7** block all sum to the square's characteristic number of **21455**. All patterns which hold for the primal square apply to the dual square too.



The bottom pattern is the **7x5** block transpose of the top pattern @ **(2;-5)**. Again there are **n=35** equal sums. Now, the same also holds when the top and bottom patterns are flipped horizontally. That's now **4n=140**. In all we have **(5;-2), (2;-5), (5;2) & (2;5)** dot distributions.



Class-6 Squares

Size-9 square Anchor-dot Pattern

Here are the size-9 primal and dual squares. They are patterned with size-3 tilt-tiles. Just as for all the prior anchor-dot patterns, the summmation of the numbers in the same location among all the tiles sum to **369**, the size-9's characteristic number. The transposed tiles yield nothing new because their tile centers coincide with the initial ones. So the tiles here only total **9=n** additional equal anchor-dot summations.

	1	2	3	4	5	6	7	8	9	
	W(9)	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

	1	2	3	4	5	6	7	8	9	
	U(9)	369	369	369	369	369	369	369	369	369
1	27	34	71	21	28	65	24	31	68	369
2	39	73	2	42	76	5	45	79	8	369
3	60	13	50	63	16	53	57	10	47	369
4	70	26	36	64	20	30	67	23	33	369
5	1	38	75	4	41	78	7	44	81	369
6	49	59	15	52	62	18	46	56	12	369
7	35	72	25	29	66	19	32	69	22	369
8	74	3	37	77	6	40	80	9	43	369
9	14	51	58	17	54	61	11	48	55	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

Program 2

Size-25 square Anchor-dot Patterns

Here is the size-25 primal square. Unlike Class-5 squares, these anchor points are not the locations where the primal and dual and their loom-tables are equal. They were determined by trial and error. The sum of the numbers in the same location (top) among all the square **5x5** tiles sum to **7825**, the size-25's characteristic number. Similarly, the transpose of these patterns (bottom) do the same: **(5;2)** vs. **(2;5)**. Continuously they each add **25=n** equal sums for **2n = 50**.

Further, the horizontal flip of each of these dot-patterns also sum equally. That's another **50** for a total of **100=4n** equal summations. The same holds for the size-25 dual (not shown here).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	**13**	151	319	582	125	**138**	276	444	82	250	**263**	401	569	207	375	**388**	526	69	332	500	**513**	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	**482**	525	38	176	344	**607**	25	163	301	469	**107**	150	288	426	594	**232**	275	413	551	94	**357**	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	**201**	369	507	50	188	**326**	494	7	175	313	**451**	619	132	300	438	**576**	119	257	425	563	**76**	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	**575**	88	226	269	532	**75**	213	351	394	32	**200**	338	476	519	157	**325**	463	601	19	282	**450**	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	**294**	557	100	238	251	**419**	57	225	363	376	**544**	182	350	488	501	**44**	307	475	613	1	**169**	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	**471**	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	**617**	10	173	436	579	117	135	298	7825
3	457	625	**13**	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	**159**	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	**305**	468	606	7825
6	70	208	371	389	527	195	333	**496**	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	**517**	35	198	461	604	17	160	323	7825
8	482	525	**38**	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	**184**	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	**330**	493	506	7825
11	95	233	271	414	552	220	358	**396**	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	**542**	60	223	486	504	42	185	348	7825
13	382	550	**63**	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	**209**	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	**355**	393	531	7825
16	120	133	296	439	577	245	258	**421**	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	**567**	85	248	386	529	67	210	373	7825
18	407	575	**88**	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	**234**	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	**255**	418	556	7825
21	20	158	321	464	602	145	283	**446**	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	**592**	110	148	411	554	92	235	273	7825
23	432	600	**113**	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	**134**	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	**280**	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Program 2

Part III Notes

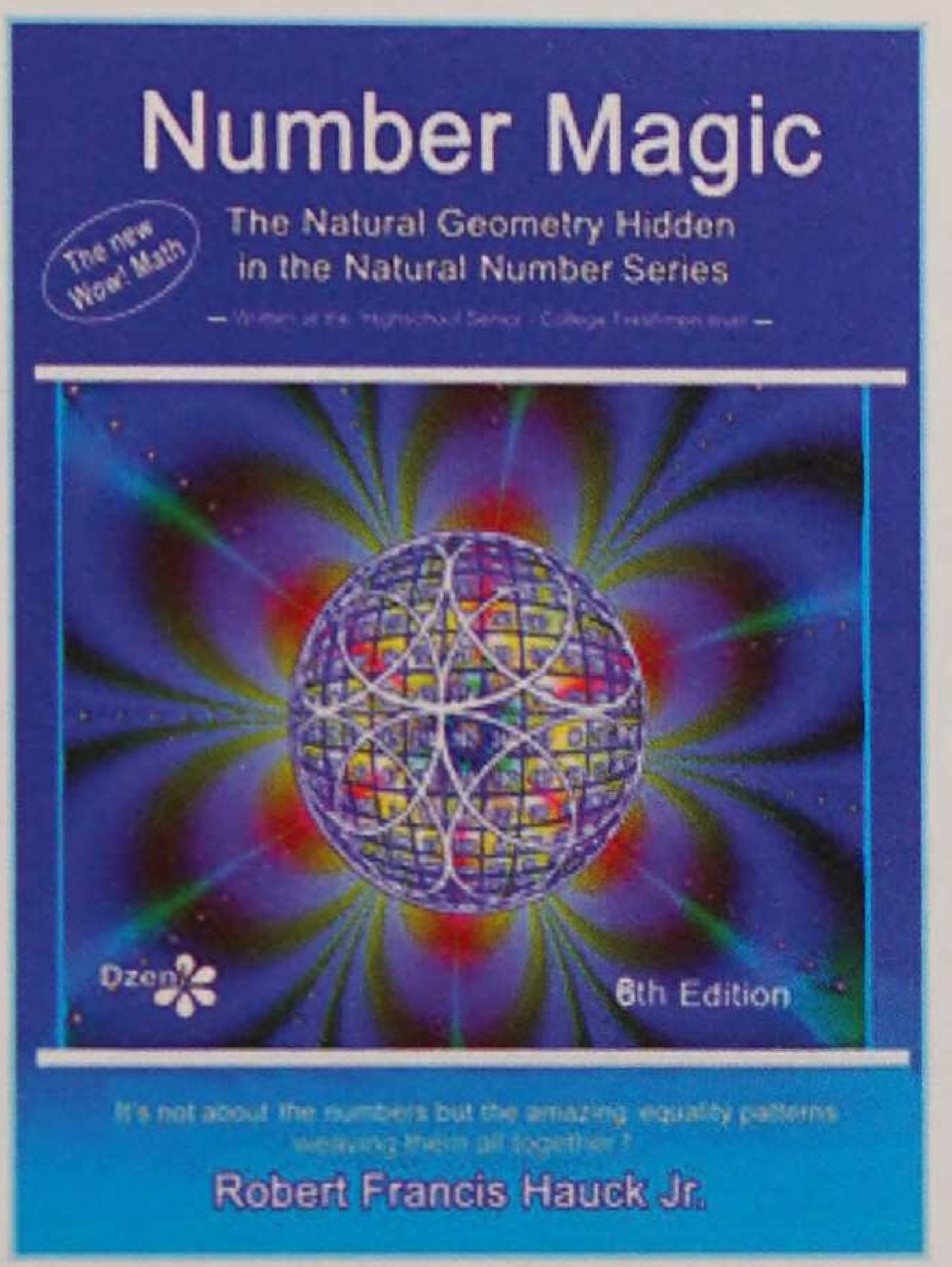
1. All but Class-2 squares have the anchor-dot distribution property.
2. One set of anchor-dot patterns in Class-1 and Class-5 squares are centered around the points of equality between the modulus and integer functions on their size squares.
3. Another two sets of dot patterns in Class-1 and Class-5 squares arise from the centers of transposed complementary tiling patterns. Beyond size-7, the location of these dots are not related to the points of modulus and integer functions' equalities.
4. All other Classes have off-set rectangular or tilt-tile patterns which are unrelated to the points of modulus and integer functions' equalities.
5. Each equal-summing anchor-dot distribution pattern adds **n** additional summations which sum to the square's characteristic number.
6. This brings the total number of all characteristic equal summations to at least:
   1) $\mathbf{2n^2 + 8n}$ for Class-1 squares greater than size-7:
      - i. $\mathbf{2n^2}$ from **C5** unique complementary tile patterns
      - ii. **4n** from directional string summations
      - iii. **4n** from anchor-dot distribution patterns

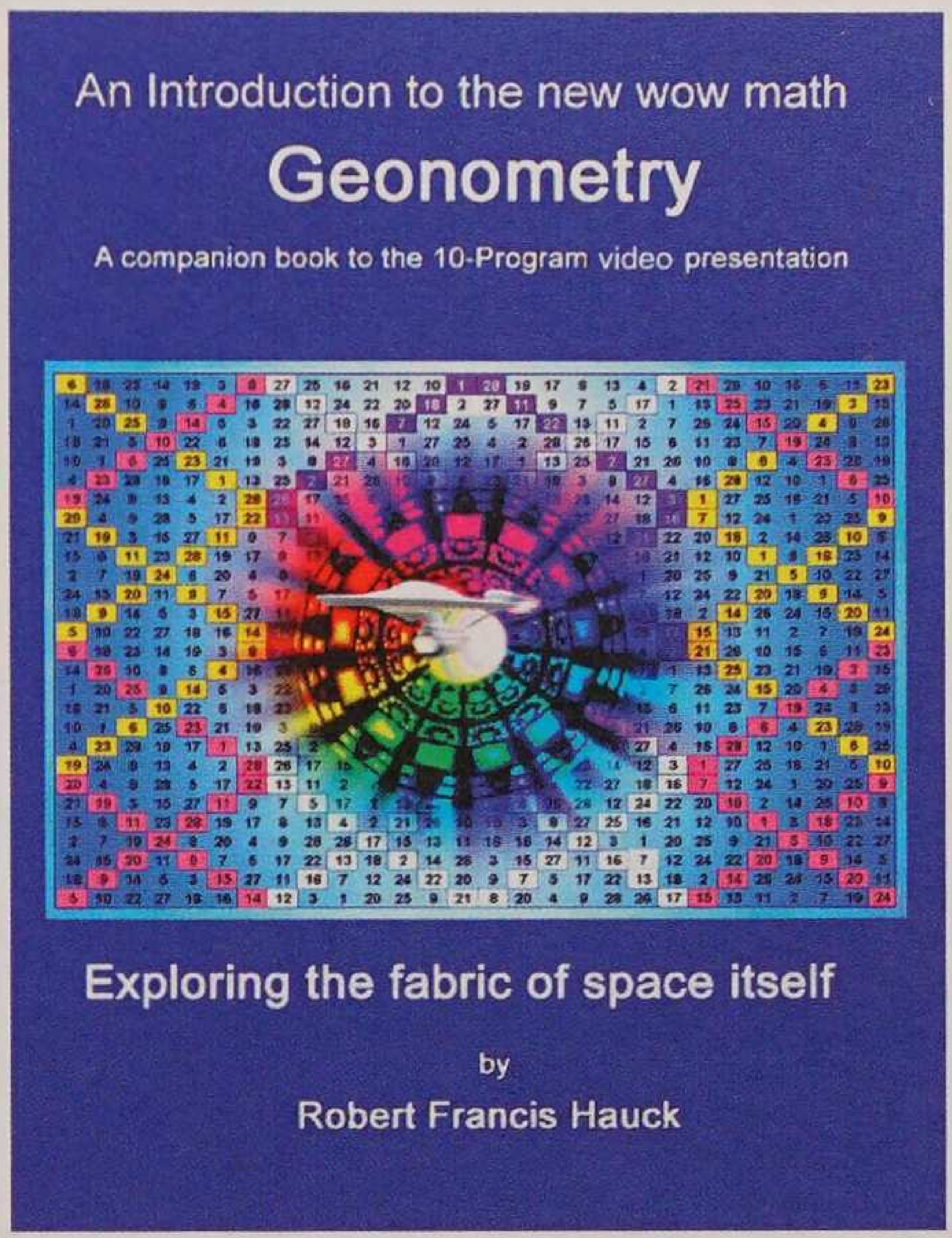
   2) $\mathbf{2n^2 + 4n + 1}$ for Class-4 squares greater than size-12:
      - i. $\mathbf{2n^2}$ from **C5** ◊ & **X** tile patterns
      - ii. **4n** from directional string summations
      - iii. **1** from characteristic circles

   3) $\mathbf{n^2 + 8n}$ for Classes 3 & 5 and Class-6 squares greater than size-9:
      - i. $\mathbf{n^2}$ from continuous **bx** block-square modularity
      - ii. **4n** from directional string summations
      - iii. **4n** from anchor-dot distribution patterns

7. The Class-6 squares of sizes $\mathbf{n = b^2}$ equal to **9** and **25** which are **bx**-modular are only those which were derived from collapsing a 5-dimensional quintacube of the same size **n** along its B-axis to a size **n** quadracube. Quintacubes are not dealt with in this program series because their numeric descriptions are too large to be legible in the companion book to this program series. It was these projections of quadracubes onto the 2-dimensional landscape that were used for generating the perfect Class-6 squares with continuous modularity in this program series. Due to their lack of legibility on a movie screen, these squares are only depicted in the alternate book, **Number Magic**.

We have come to the end of Program 2. The companion book to this series is the only source for anchor-dot patterns depicted in Part III as these were just recently discovered.





Number Magic – The Natural Geometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension.

Eighth Edition
Black & white print (350+ pages)

An Introduction to the new wow math Geonometry

ISBN 978-1-479-23823-1

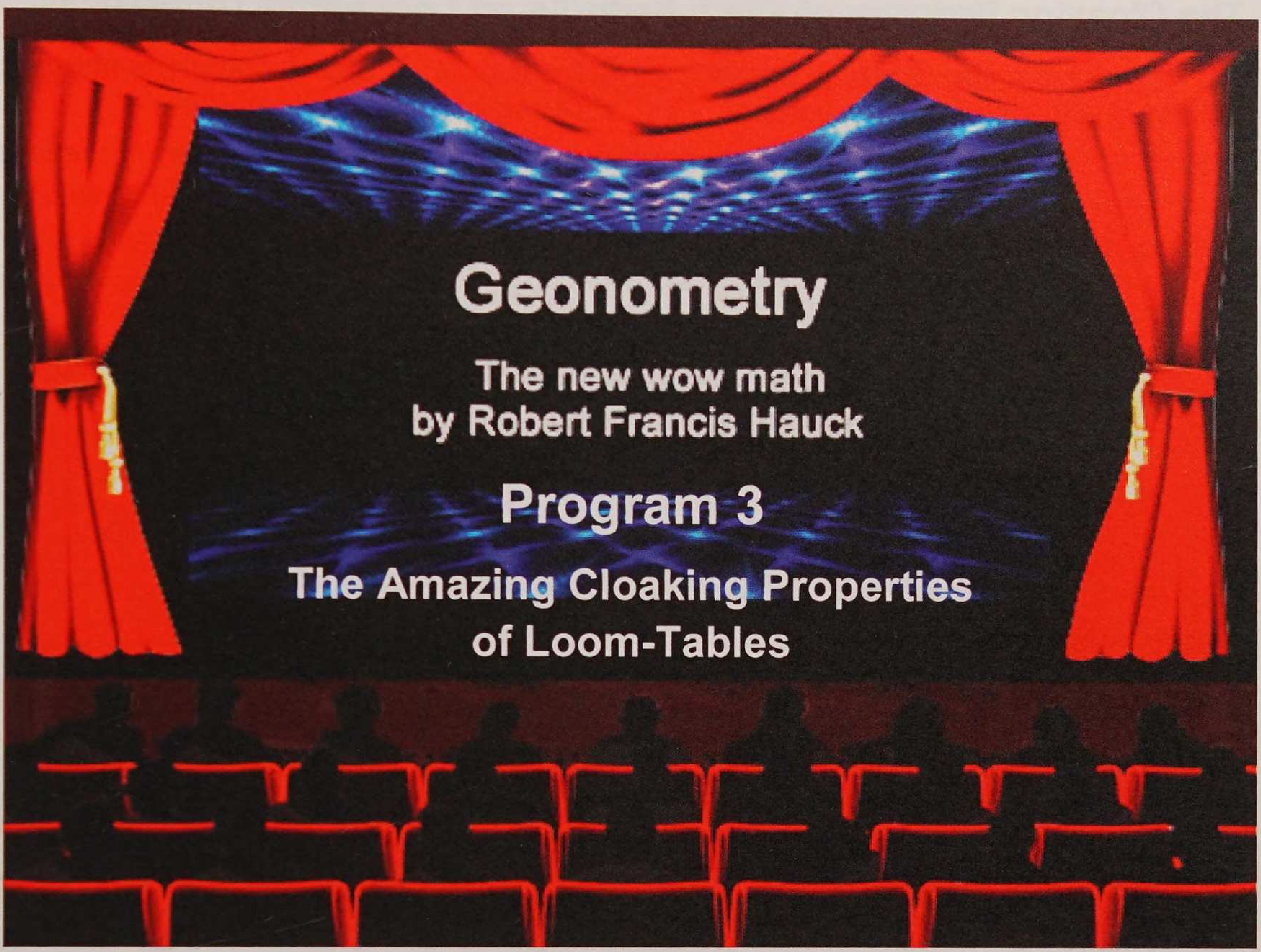
Contains all the slides and narration in this 10-program video series.
Selected examples.

Fourth Edition
Printed in color. (345+ pages)

The next program is devoted to demonstrating the cloaking properties of complementary loom-tables. We have already seen how the fabric of space is interwoven into quilt patterns; next it will be seen that these quilt patterns jointly cloak each other no matter where their centers are located relative to each other. This property provides a profound reason of just why these patterns haven't yet been uncovered by the Cosmologists with their convoluted 11-dimensional String Theory.

Further, it will also provide the reason for why Atomic Scientists will never directly see the electron shells around the nucleus of atoms – the real reason behind the Schrödinger Effect. Everything they proclaim has been deduced from thousands of experiments in atom smashers but never observed directly.

Program 3



Geonometry

The new wow math
by Robert Francis Hauck

Program 3

The Amazing Cloaking Properties
of Loom-Tables

This program is devoted to demonstrating the cloaking properties of complementary loom-tables. We have already seen how space is woven from equal-summing 1-dimensional numerical “strings” in Class-1 loom tables. Now, it will be seen that the word “strings” has a more fundamental meaning here than just *linear summations*.

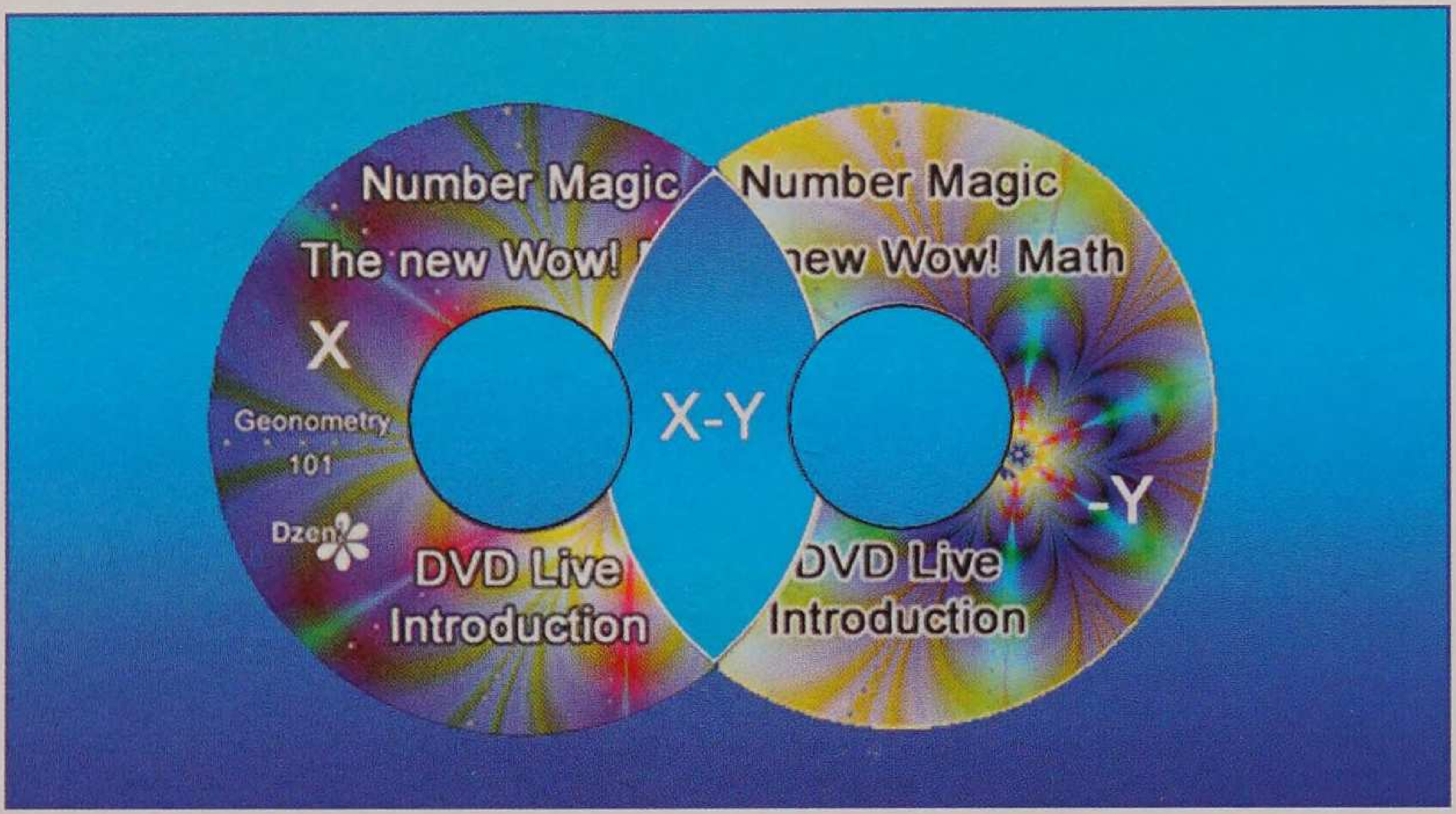
These cloaking properties will come into play later in Program 5 where Geonometry is used to describe the orbital distribution of electrons in atoms. So take note.

Toward the end of the program, the 1st application of Geonometry shows how the double helix could arise from the 2-dimensional spatial fabric to form the basis for DNA and variations of it to arise naturally.

Program 3

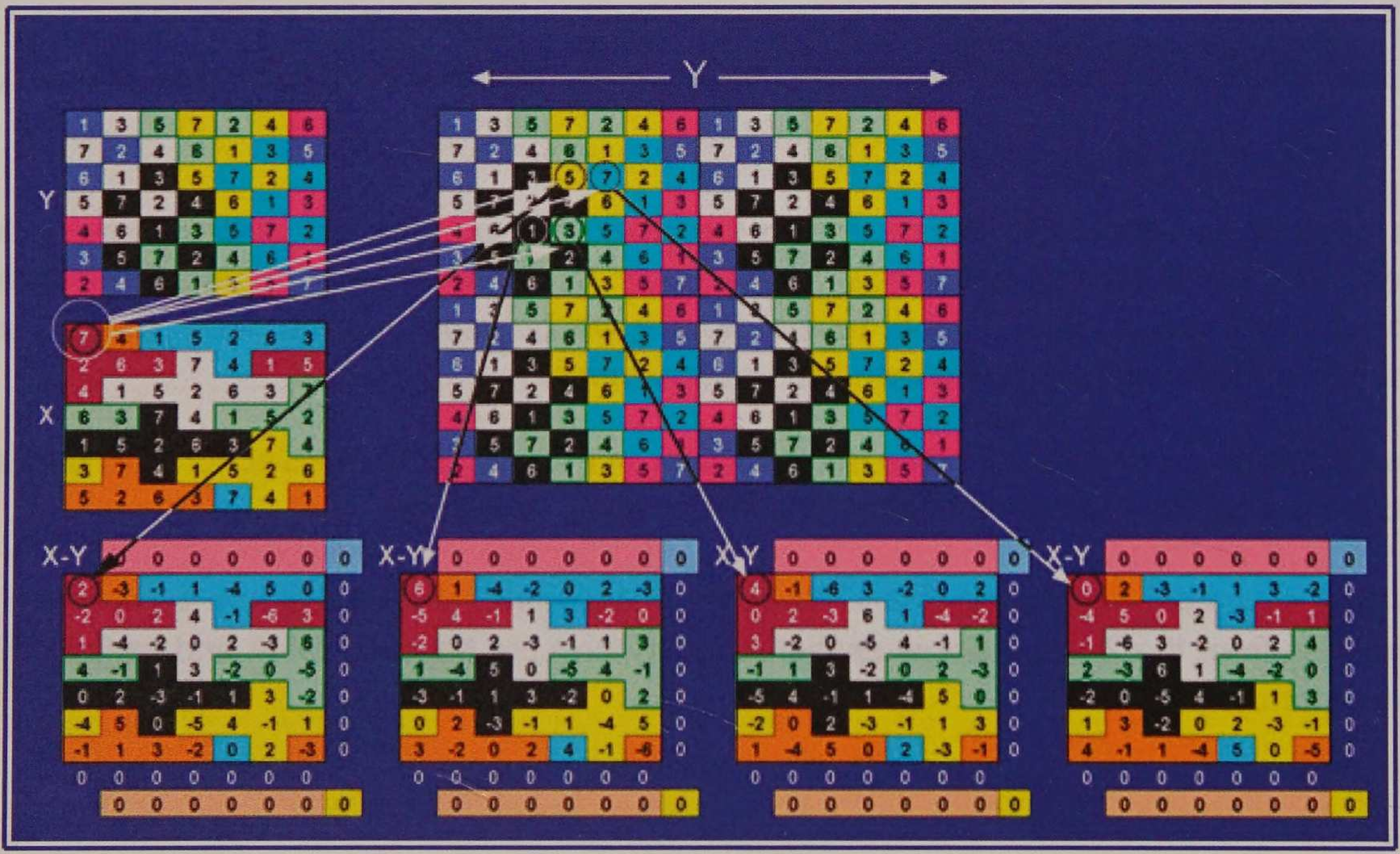
The Cloaking Property of Tiling Patterns on Loom-table Differences

Next we will observe the cloaking property of Classes-1, 4, 5 and 6 complementary loom tables covered with identical tiling patterns. First, we'll begin with squares of Class-1.



Class-1 Squares

Here the size-7 loom table has been duplicated four times to make it easier to see what is meant by cloaking pattern. The tables at the bottom use the loom table **X** at left center to take various roaming differences with table **Y** starting with the upper corner of **X**, number **7**.



Here is the absolutely amazing result: All the rows, columns, diagonals and tiles sum equally to **0** everywhere the differences are taken. It's as if neither loom table exists; one loom table cloaks the other.

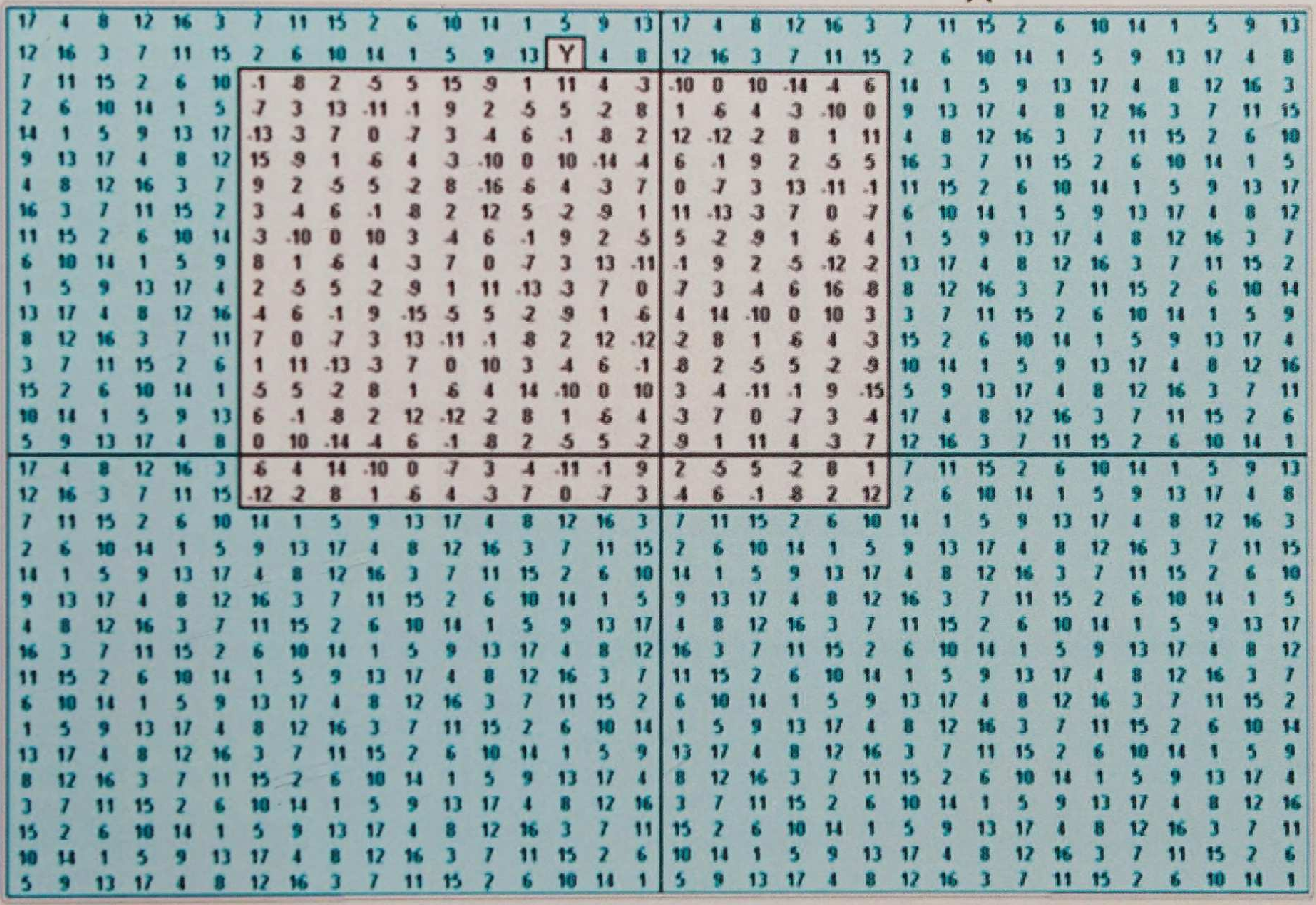
Program 3

Roaming size-17 Loom Difference Table

V(n) = X(n) – Y(n)

This slide shows a composite table formed from the loom table **X** duplicated four times. The loom table **Y** is then subtracted from the composite table as shown in white. Now this loom table can be subtracted from the composite table anywhere and what is shown on the next slide will occur.

X X



17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	Y	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	-1	-8	2	-5	5	15	-9	1	11	4	-3	-10	0	10	-14	-4	6	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	-7	3	13	-11	-1	9	2	-5	5	-2	8	1	-6	4	-3	-10	0	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	-13	-3	7	0	-7	3	-4	6	-1	-8	2	12	-12	-2	8	1	11	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	15	-9	1	-6	4	-3	-10	0	10	-14	-4	6	-1	9	2	-5	5	16	3	7	11	15	2	6	10	14	1	5
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13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
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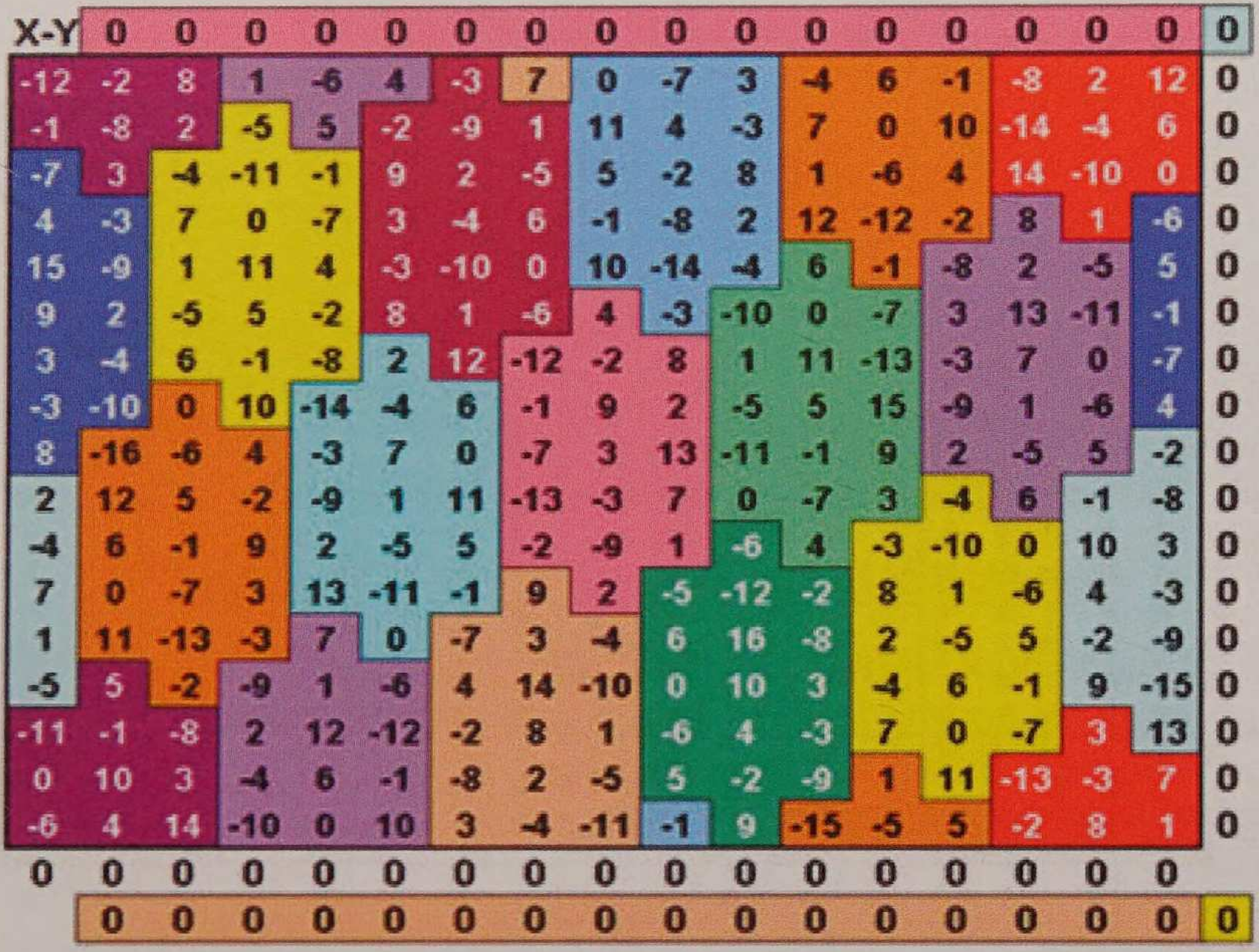
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

X X

Program 3

Same size-17 loom difference table with tile patterns A & B

Again, every row, column, diagonal and tile sums to **0** everywhere. And not only that but the tiles can be located anywhere in the table and each will still sum to **0**. It's as if the loom membranes didn't even exist because they would be undetectable.



X-Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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-1	-8	2	-5	5	-2	-9	1	11	4	-3	7	0	10	-14	-4	6	0
-7	3	-4	-11	-1	9	2	-5	5	-2	8	1	-6	4	14	-10	0	0
4	-3	7	0	-7	3	-4	6	-1	-8	2	12	-12	-2	8	1	-6	0
15	-9	1	11	4	-3	-10	0	10	-14	-4	6	-1	-8	2	-5	5	0
9	2	-5	5	-2	8	1	-6	4	-3	-10	0	-7	3	13	-11	-1	0
3	-4	6	-1	-8	2	12	-12	-2	8	1	11	-13	-3	7	0	-7	0
-3	-10	0	10	-14	-4	6	-1	9	2	-5	5	15	-9	1	-6	4	0
8	-16	-6	4	-3	7	0	-7	3	13	-11	-1	9	2	-5	5	-2	0
2	12	5	-2	-9	1	11	-13	-3	7	0	-7	3	-4	6	-1	-8	0
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1	11	-13	-3	7	0	-7	3	-4	6	16	-8	2	-5	5	-2	-9	0
-5	5	-2	-9	1	-6	4	14	-10	0	10	3	-4	6	-1	9	-15	0
-11	-1	-8	2	12	-12	-2	8	1	-6	4	-3	7	0	-7	3	13	0
0	10	3	-4	6	-1	-8	2	-5	5	-2	-9	1	11	-13	-3	7	0
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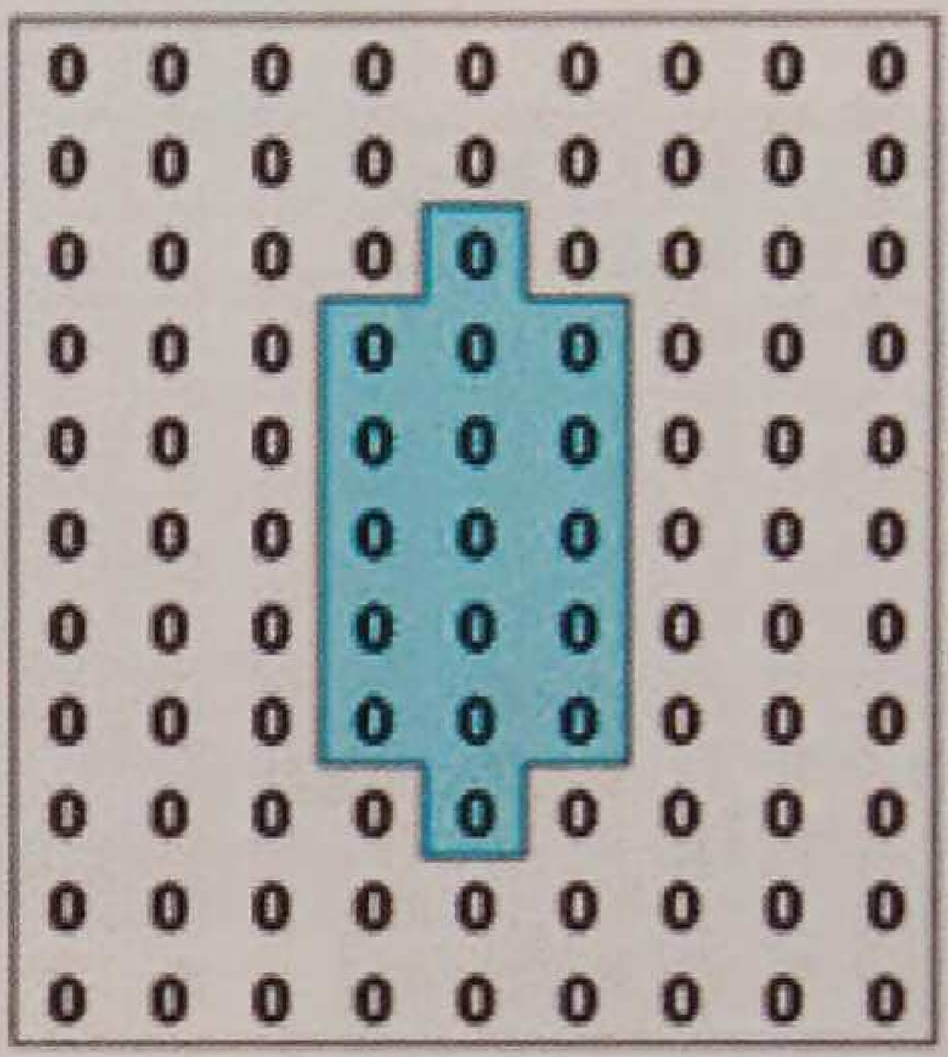
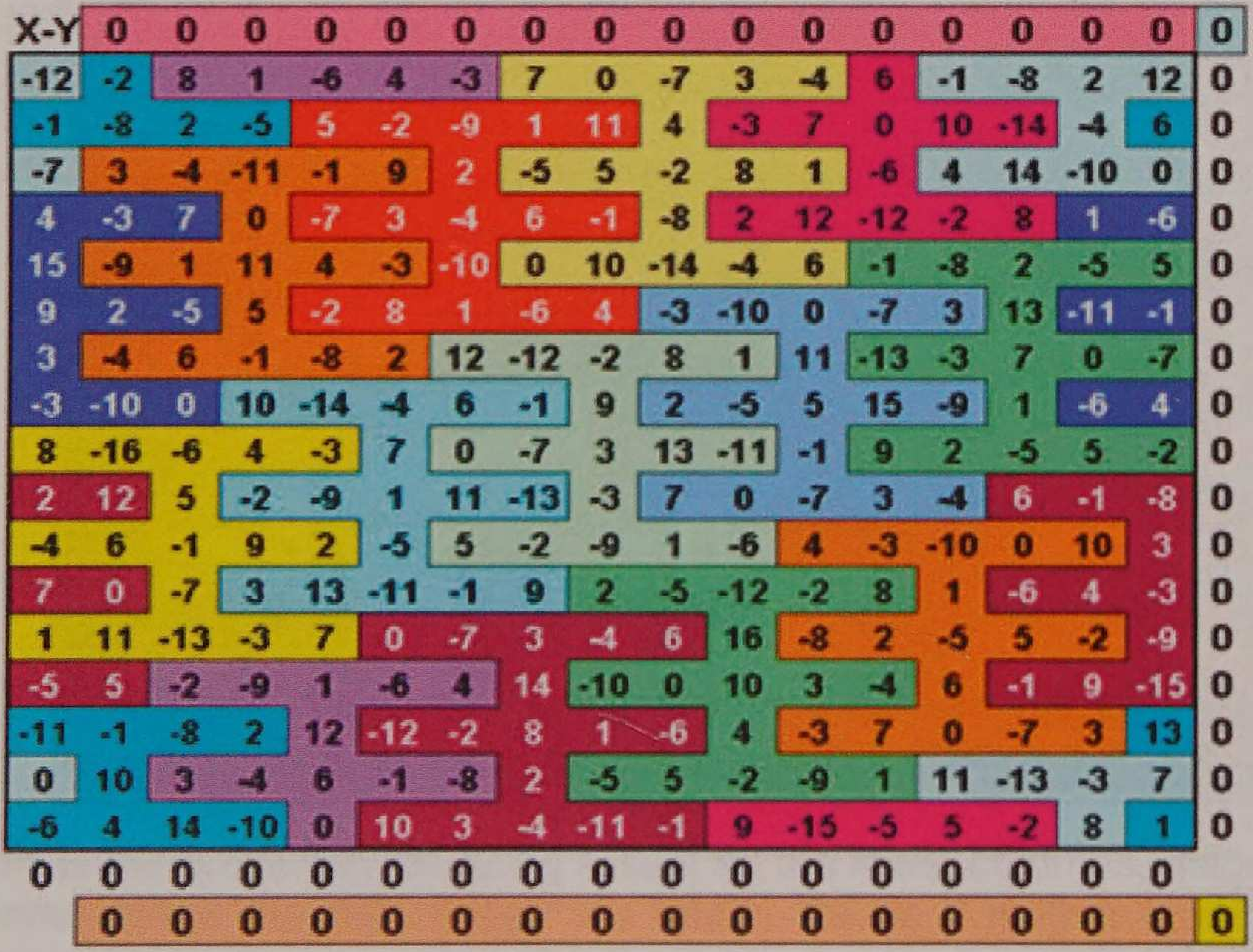
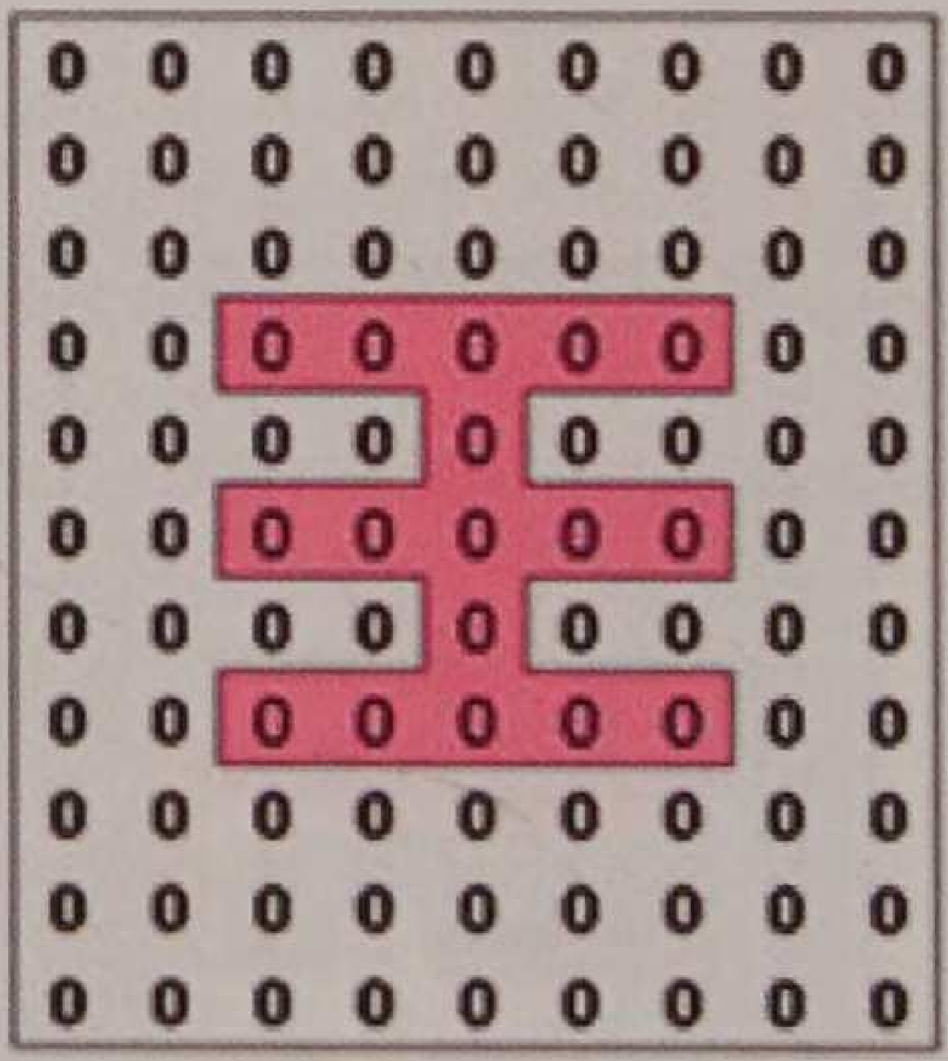


0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Individual tiles equal 0 anywhere throughout the difference table.



X-Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-12	-2	8	1	-6	4	-3	7	0	-7	3	-4	6	-1	-8	2	12	0
-1	-8	2	-5	5	-2	-9	1	11	4	-3	7	0	10	-14	-4	6	0
-7	3	-4	-11	-1	9	2	-5	5	-2	8	1	-6	4	14	-10	0	0
4	-3	7	0	-7	3	-4	6	-1	-8	2	12	-12	-2	8	1	-6	0
15	-9	1	11	4	-3	-10	0	10	-14	-4	6	-1	-8	2	-5	5	0
9	2	-5	5	-2	8	1	-6	4	-3	-10	0	-7	3	13	-11	-1	0
3	-4	6	-1	-8	2	12	-12	-2	8	1	11	-13	-3	7	0	-7	0
-3	-10	0	10	-14	-4	6	-1	9	2	-5	5	15	-9	1	-6	4	0
8	-16	-6	4	-3	7	0	-7	3	13	-11	-1	9	2	-5	5	-2	0
2	12	5	-2	-9	1	11	-13	-3	7	0	-7	3	-4	6	-1	-8	0
-4	6	-1	9	2	-5	5	-2	-9	1	-6	4	-3	-10	0	10	3	0
7	0	-7	3	13	-11	-1	9	2	-5	-12	-2	8	1	-6	4	-3	0
1	11	-13	-3	7	0	-7	3	-4	6	16	-8	2	-5	5	-2	-9	0
-5	5	-2	-9	1	-6	4	14	-10	0	10	3	-4	6	-1	9	-15	0
-11	-1	-8	2	12	-12	-2	8	1	-6	4	-3	7	0	-7	3	13	0
0	10	3	-4	6	-1	-8	2	-5	5	-2	-9	1	11	-13	-3	7	0
-6	4	14	-10	0	10	3	-4	-11	-1	9	-15	-5	5	-2	8	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

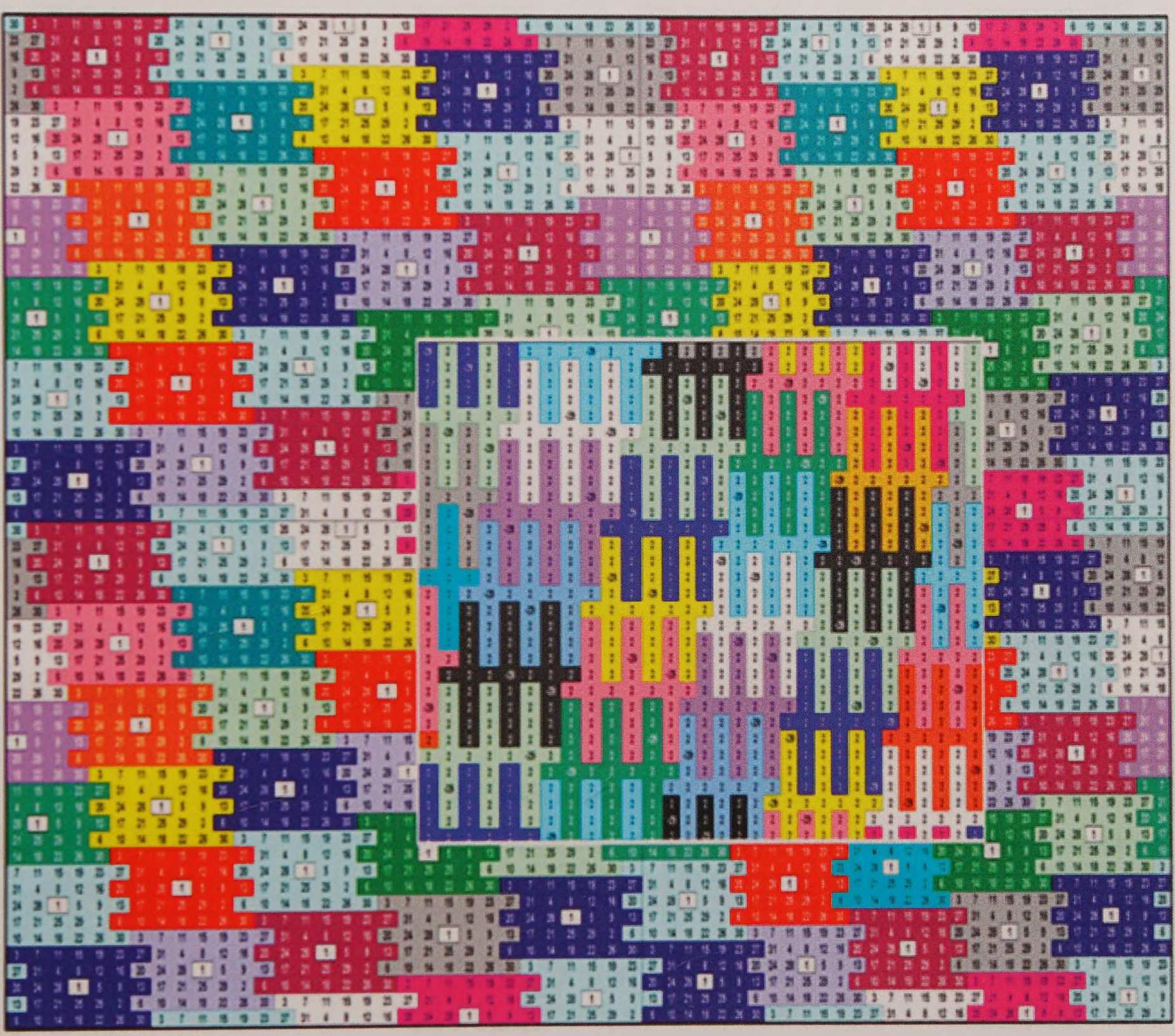


0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Program 3

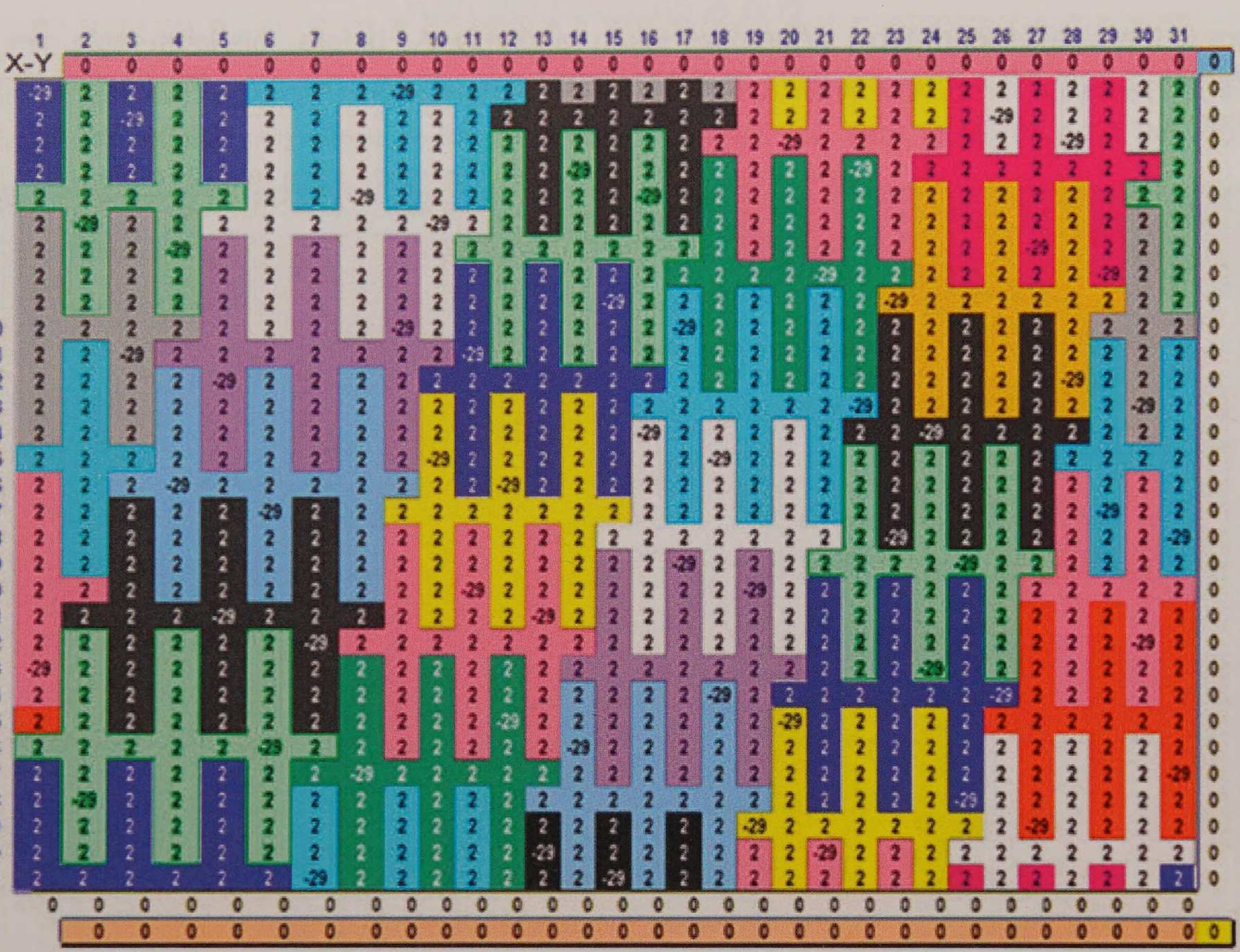
Continuous Loom Table Differences of the Size-31 Square

Here is the size-31 loom table differences taken randomly. As before, this loom table can be subtracted from the composite table anywhere and what will be shown on the next slide will occur.



Program 3

Size-31 loom difference table



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
X-Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	0
3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	0
4	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	0
5	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
6	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
7	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	0
8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	0
9	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	0
10	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
11	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
12	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	0
13	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	0
14	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	0
15	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	0
16	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
17	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	0
18	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	0
19	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	0
20	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	0
21	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
22	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	0
23	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	0
24	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	0
25	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	0
26	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
27	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	0
28	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	0
29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	0
30	2	2	2	2	2	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	0
31	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	-29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In each tile here there are **29** **2**'s and **2** negative **29**'s, all summing to **0**.

In general for looms of size-31, depending on where the off-center differences are taken, there will be **(31-j)** values of **+j** and **j** values of negative values **(j-31)**, together all summing to **0.**

This holds for **j = 1** thru **30**.

When **j = 31**, the centers of **Y** are aligned with **X** and their differences are as was originally depicted earlier in program 2. Recall that they summed to **0** there too.

Program 3

For Class-4 Squares

Here is the cloaking property of these Class-4 tiling patterns which are very different from Class-1 tiling patterns. Here, every diamond and X-pattern sums to **0**.

Just as for Class-1 squares, the loom tables do not need to be aligned. This property is continuous for Class-4 squares too. Each loom table cloaks the other.

Size-12

Here is the size-12 square with segregated diamond and X patterns (re:p.1-47)

	1	2	3	4	5	6	7	8	9	10	11	12	
12x12		870	870	870	870	870	870	870	870	870	870	870	870
1	67	84	49	24	133	6	139	12	121	96	61	78	870
2	107	41	128	89	32	110	35	113	56	17	104	38	870
3	69	100	51	22	117	4	141	28	123	94	45	76	870
4	75	46	93	124	27	142	3	118	21	52	99	70	870
5	44	98	59	14	119	29	116	26	131	86	47	101	870
6	73	66	91	126	7	144	1	138	19	54	79	72	870
7	31	120	13	60	97	42	103	48	85	132	25	114	870
8	140	2	95	122	71	77	68	74	23	50	143	5	870
9	33	136	15	58	81	40	105	64	87	130	9	112	870
10	111	10	129	88	63	106	39	82	57	16	135	34	870
11	11	137	20	53	80	62	83	65	92	125	8	134	870
12	109	30	127	90	43	108	37	102	55	18	115	36	870
	870	870	870	870	870	870	870	870	870	870	870	870	
		870	870	870	870	870	870	870	870	870	870	870	870

Here are the loom-tables derived in the base **24** instead of **12**. That produced complementary loom tables with larger numbers.

X(24)	150	150	150	150	150	150	150	150	150	150	150	150
19	12	1	24	13	6	19	12	1	24	13	6	150
11	17	8	17	8	14	11	17	8	17	8	14	150
21	4	3	22	21	4	21	4	3	22	21	4	150
3	22	21	4	3	22	3	22	21	4	3	22	150
20	2	11	14	23	5	20	2	11	14	23	5	150
1	18	19	6	7	24	1	18	19	6	7	24	150
7	24	13	12	1	18	7	24	13	12	1	18	150
20	2	23	2	23	5	20	2	23	2	23	5	150
9	16	15	10	9	16	9	16	15	10	9	16	150
15	10	9	16	15	10	15	10	9	16	15	10	150
11	17	20	5	8	14	11	17	20	5	8	14	150
13	6	7	18	19	12	13	6	7	18	19	12	150
150	150	150	150	150	150	150	150	150	150	150	150	
	150	150	150	150	150	150	150	150	150	150	150	150

Y(24)	42	42	42	42	42	42	42	42	42	42	42	42
3	4	3	1	6	1	6	1	6	4	3	4	42
5	2	6	4	2	5	2	5	3	1	5	2	42
3	5	3	1	5	1	6	2	6	4	2	4	42
4	2	4	6	2	6	1	5	1	3	5	3	42
2	5	3	1	5	2	5	2	6	4	2	5	42
4	3	4	6	1	6	1	6	1	3	4	3	42
2	5	1	3	5	2	5	2	4	6	2	5	42
6	1	4	6	3	4	3	4	1	3	6	1	42
2	6	1	3	4	2	5	3	4	6	1	5	42
5	1	6	4	3	5	2	4	3	1	6	2	42
1	6	1	3	4	3	4	3	4	6	1	6	42
5	2	6	4	2	5	2	5	3	1	5	2	42
42	42	42	42	42	42	42	42	42	42	42	42	
	42	42	42	42	42	42	42	42	42	42	42	42

Then when their differences were obtained, the numbers were also larger but closer together in value. And since this size-12 square is pairwise row-symmetric, the difference table is perfectly row-wise anti-symmetric as seen here. Here is the resulting difference table **V(12)**:

Now it's quite easy to see that both the X and diamond tiles sum to **0** everywhere.

V(12) = 7xX(24)-25xY(24)

	0	0	0	0	0	0	0	0	0	0	0	0
58	-16	-68	143	-59	17	-17	59	-143	68	16	-58	0
-48	69	-94	19	6	-27	27	-6	-19	94	-69	48	0
72	-97	-54	129	22	3	-3	-22	-129	54	97	-72	0
-79	104	47	-122	-29	4	-4	29	122	-47	-104	79	0
90	-111	2	73	36	-15	15	-36	-73	-2	111	-90	0
-93	51	33	-108	24	18	-18	-24	108	-33	-51	93	0
-1	43	66	9	-118	76	-76	118	-9	-66	-43	1	0
-10	-11	61	-136	86	-65	65	-86	136	-61	11	10	0
13	-38	80	-5	-37	62	-62	37	5	-80	38	-13	0
-20	45	-87	12	30	-55	55	-30	-12	87	-45	20	0
52	-31	115	-40	-44	23	-23	44	40	-115	31	-52	0
-34	-8	-101	26	83	-41	41	-83	-26	101	8	34	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

This size square only has a cloaking pattern which is continuous vertically in just four places: Those highlighted patterns straddling the centerline, those straddling the left and right edges, and those that are exactly inbetween. All the rest, that is, those tiles not contiguously highlighted, do not cancel out.

This segregated continuity in the tiling pattern will be referenced as *quasi-continuous*. This quasi-continuous tiling pattern only arises for the size-12 square. After this, all larger Class-4 squares are totally continuous in their cloaking property.

Program 3

Size-16

Here are the size-16 loom-tables and their difference table.

Modulus(16)

X		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136
	8	9	1	16	8	9	1	16	8	9	1	16	8	9	1	16	136
	10	7	15	2	10	7	15	2	10	7	15	2	10	7	15	2	136
	14	3	11	6	14	3	11	6	14	3	11	6	14	3	11	6	136
	4	13	5	12	4	13	5	12	4	13	5	12	4	13	5	12	136
	5	12	4	13	5	12	4	13	5	12	4	13	5	12	4	13	136
	11	6	14	3	11	6	14	3	11	6	14	3	11	6	14	3	136
	15	2	10	7	15	2	10	7	15	2	10	7	15	2	10	7	136
	1	16	8	9	1	16	8	9	1	16	8	9	1	16	8	9	136
	8	9	1	16	8	9	1	16	8	9	1	16	8	9	1	16	136
	10	7	15	2	10	7	15	2	10	7	15	2	10	7	15	2	136
	14	3	11	6	14	3	11	6	14	3	11	6	14	3	11	6	136
	4	13	5	12	4	13	5	12	4	13	5	12	4	13	5	12	136
	5	12	4	13	5	12	4	13	5	12	4	13	5	12	4	13	136
	11	6	14	3	11	6	14	3	11	6	14	3	11	6	14	3	136
	15	2	10	7	15	2	10	7	15	2	10	7	15	2	10	7	136
	1	16	8	9	1	16	8	9	1	16	8	9	1	16	8	9	136
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136

Integer (16)

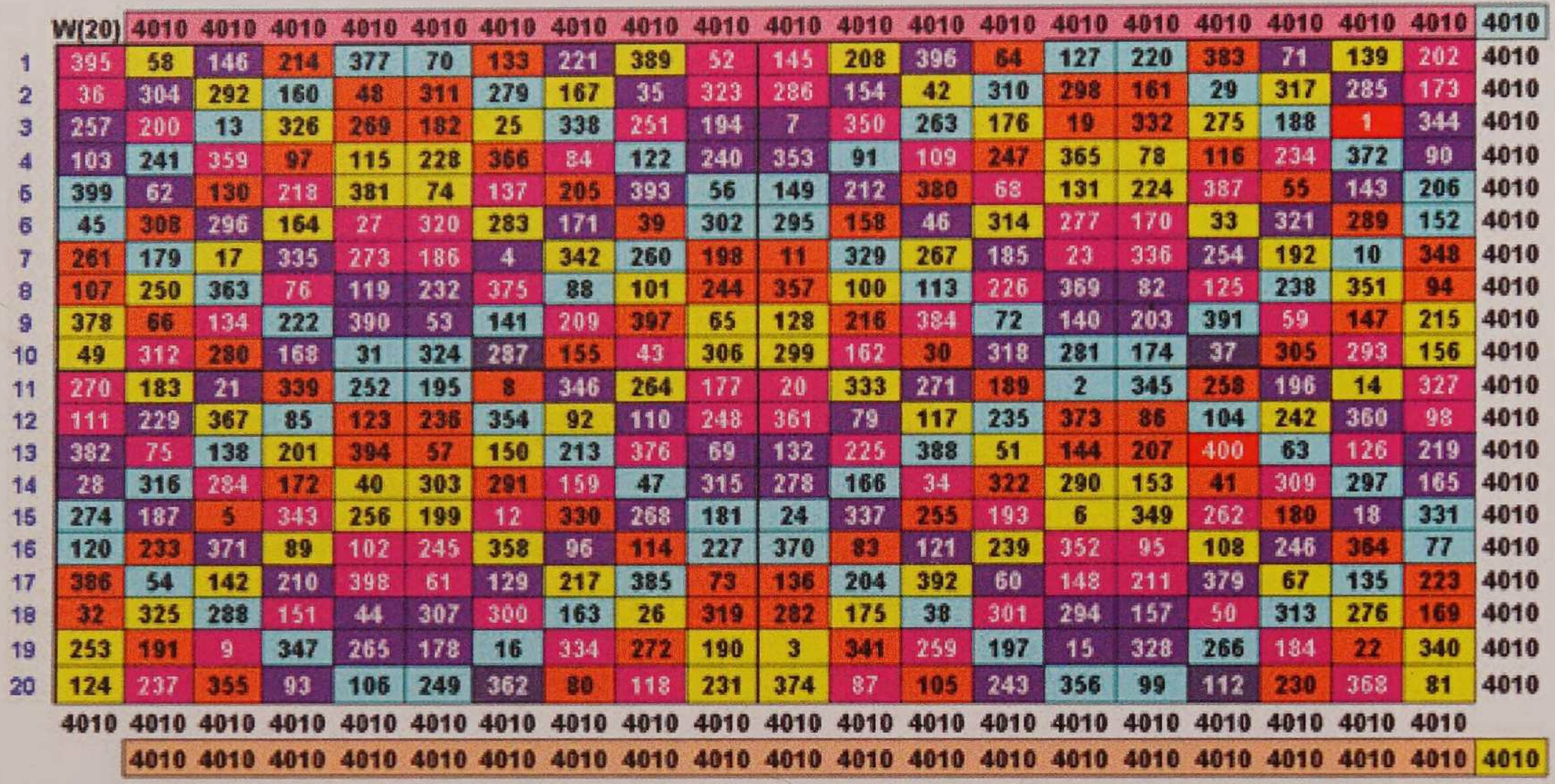
Y		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136
	1	7	3	5	12	14	10	16	9	15	11	13	4	6	2	8	136
	12	14	10	16	1	7	3	5	4	6	2	8	9	15	11	13	136
	4	6	2	8	9	15	11	13	12	14	10	16	1	7	3	5	136
	9	15	11	13	4	6	2	8	1	7	3	5	12	14	10	16	136
	5	3	7	1	16	10	14	12	13	11	15	9	8	2	6	4	136
	16	10	14	12	5	3	7	1	8	2	6	4	13	11	15	9	136
	8	2	6	4	13	11	15	9	16	10	14	12	5	3	7	1	136
	13	11	15	9	8	2	6	4	5	3	7	1	16	10	14	12	136
	5	3	7	1	16	10	14	12	13	11	15	9	8	2	6	4	136
	16	10	14	12	5	3	7	1	8	2	6	4	13	11	15	9	136
	8	2	6	4	13	11	15	9	16	10	14	12	5	3	7	1	136
	13	11	15	9	8	2	6	4	5	3	7	1	16	10	14	12	136
	1	7	3	5	12	14	10	16	9	15	11	13	4	6	2	8	136
	12	14	10	16	1	7	3	5	4	6	2	8	9	15	11	13	136
	4	6	2	8	9	15	11	13	12	14	10	16	1	7	3	5	136
	9	15	11	13	4	6	2	8	1	7	3	5	12	14	10	16	136
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
X-Y		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	7	2	-2	11	-4	-5	-9	0	-1	-6	-10	3	4	3	-1	8	0
2	-2	-7	5	-14	9	0	12	-3	6	1	13	-6	1	-8	4	-11	0
3	10	-3	9	-2	5	-12	0	-7	2	-11	1	-10	13	-4	8	1	0
4	-5	-2	-6	-1	0	7	3	4	3	6	2	7	-8	-1	-5	-4	0
5	0	9	-3	12	-11	2	-10	1	-8	1	-11	4	-3	10	-2	9	0
6	-5	-4	0	-9	6	3	7	2	3	4	8	-1	-2	-5	-1	-6	0
7	7	0	4	3	2	-9	-5	-2	-1	-8	-4	-5	10	-1	3	6	0
8	-12	5	-7	0	-7	14	2	5	-4	13	1	8	-15	6	-6	-3	0
9	3	6	-6	15	-8	-1	-13	4	-5	-2	-14	7	0	7	-5	12	0
10	-6	-3	1	-10	5	4	8	1	2	5	9	-2	-3	-4	0	-7	0
11	6	1	5	2	1	-8	-4	-3	-2	-7	-3	-6	9	0	4	5	0
12	-9	2	-10	3	-4	11	-1	8	-1	10	-2	11	-12	3	-9	0	0
13	4	5	1	8	-7	-2	-6	-3	-4	-3	-7	0	1	6	2	5	0
14	-1	-8	4	-13	10	-1	11	-2	7	0	12	-5	2	-9	3	-10	0
15	11	-4	8	-1	6	-13	-1	-6	3	-12	0	-9	14	-5	7	2	0
16	-8	1	-3	-4	-3	10	6	1	0	9	5	4	-11	2	-2	-7	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

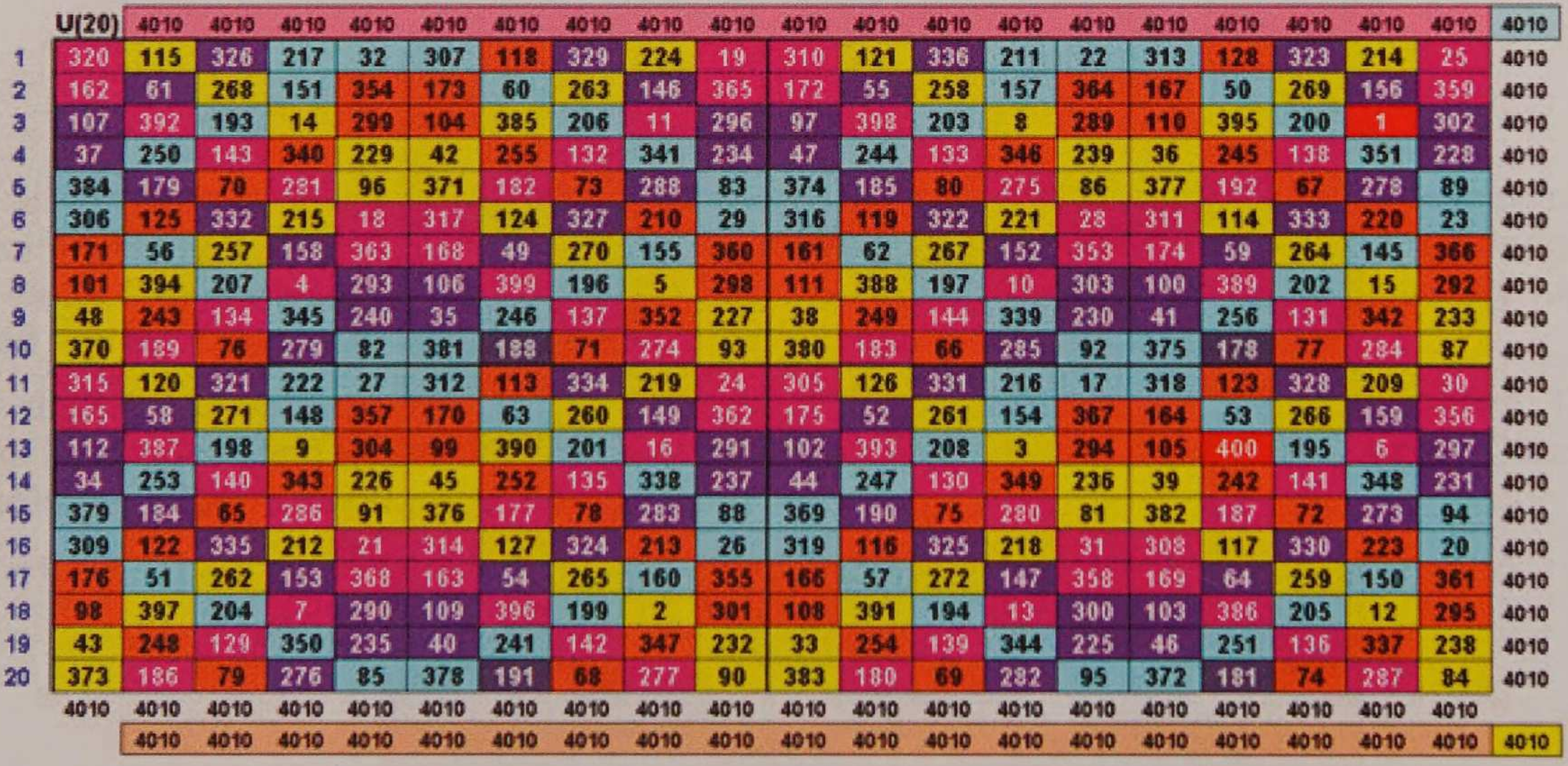
Program 3

Size-20

Here is the size-20 primal square and its dual. Neither is pairwise symmetric, yet both are still pangenic.



W(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	395	58	146	214	377	70	133	221	389	52	145	208	396	64	127	220	383	71	139	202	4010
2	36	304	292	160	48	311	279	167	35	323	286	154	42	310	298	161	29	317	285	173	4010
3	257	200	13	326	269	182	25	338	251	194	7	350	263	176	19	332	275	188	1	344	4010
4	103	241	359	97	115	228	366	84	122	240	353	91	109	247	365	78	116	234	372	90	4010
5	399	62	130	218	381	74	137	205	393	56	149	212	380	68	131	224	387	55	143	206	4010
6	45	308	296	164	27	320	283	171	39	302	295	158	46	314	277	170	33	321	289	152	4010
7	261	179	17	335	273	186	4	342	260	198	11	329	267	185	23	336	254	192	10	348	4010
8	107	250	363	76	119	232	375	88	101	244	357	100	113	226	369	82	125	238	351	94	4010
9	378	66	134	222	390	53	141	209	397	65	128	216	384	72	140	203	391	59	147	215	4010
10	49	312	280	168	31	324	287	155	43	306	299	162	30	318	281	174	37	305	293	156	4010
11	270	183	21	339	252	195	8	346	264	177	20	333	271	189	2	345	258	196	14	327	4010
12	111	229	367	85	123	236	354	92	110	248	361	79	117	235	373	86	104	242	360	98	4010
13	382	75	138	201	394	57	150	213	376	69	132	225	388	51	144	207	400	63	126	219	4010
14	28	316	284	172	40	303	291	159	47	315	278	166	34	322	290	153	41	309	297	165	4010
15	274	187	5	343	256	199	12	330	268	181	24	337	255	193	6	349	262	180	18	331	4010
16	120	233	371	89	102	245	358	96	114	227	370	83	121	239	352	95	108	246	364	77	4010
17	386	54	142	210	398	61	129	217	385	73	136	204	392	60	148	211	379	67	135	223	4010
18	32	325	288	151	44	307	300	163	26	319	282	175	38	301	294	157	50	313	276	169	4010
19	253	191	9	347	265	178	16	334	272	190	3	341	259	197	15	328	266	184	22	340	4010
20	124	237	355	93	106	249	362	80	118	231	374	87	105	243	356	99	112	230	368	81	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

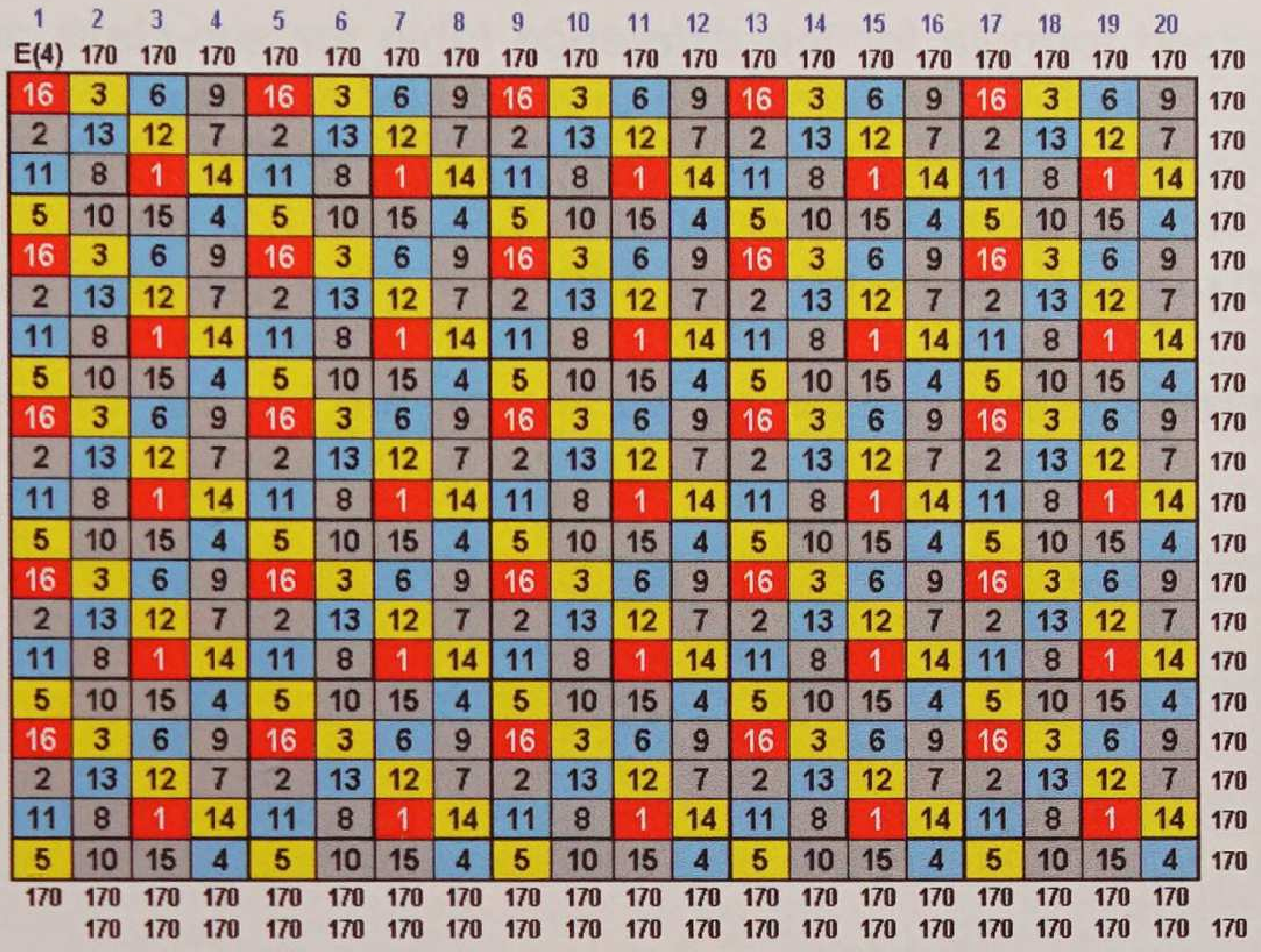


U(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	320	115	326	217	32	307	118	329	224	19	310	121	336	211	22	313	128	323	214	25	4010
2	162	61	268	151	354	173	60	263	146	365	172	55	258	157	364	167	50	269	156	359	4010
3	107	392	193	14	299	104	385	206	11	296	97	398	203	8	289	110	395	200	1	302	4010
4	37	250	143	340	229	42	255	132	341	234	47	244	133	346	239	36	245	138	351	228	4010
5	384	179	70	281	96	371	182	73	288	83	374	185	80	275	86	377	192	67	278	89	4010
6	306	125	332	215	18	317	124	327	210	29	316	119	322	221	28	311	114	333	220	23	4010
7	171	56	257	158	363	168	49	270	155	360	161	62	267	152	353	174	59	264	145	366	4010
8	101	394	207	4	293	106	399	196	5	298	111	388	197	10	303	100	389	202	15	292	4010
9	48	243	134	345	240	35	246	137	352	227	38	249	144	339	230	41	256	131	342	233	4010
10	370	189	76	279	82	381	188	71	274	93	380	183	66	285	92	375	178	77	284	87	4010
11	315	120	321	222	27	312	113	334	219	24	305	126	331	216	17	318	123	328	209	30	4010
12	165	58	271	148	357	170	63	260	149	362	175	52	261	154	367	164	53	266	159	356	4010
13	112	387	198	9	304	99	390	201	16	291	102	393	208	3	294	105	400	195	6	297	4010
14	34	253	140	343	226	45	252	135	338	237	44	247	130	349	236	39	242	141	348	231	4010
15	379	184	65	286	91	376	177	78	283	88	369	190	75	280	81	382	187	72	273	94	4010
16	309	122	335	212	21	314	127	324	213	26	319	116	325	218	31	308	117	330	223	20	4010
17	176	51	262	153	368	163	54	265	160	355	166	57	272	147	358	169	64	259	150	361	4010
18	98	397	204	7	290	109	396	199	2	301	108	391	194	13	300	103	386	205	12	295	4010
19	43	248	129	350	235	40	241	142	347	232	33	254	139	344	225	46	251	136	337	238	4010
20	373	186	79	276	85	378	191	68	277	90	383	180	69	282	95	372	181	74	287	84	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

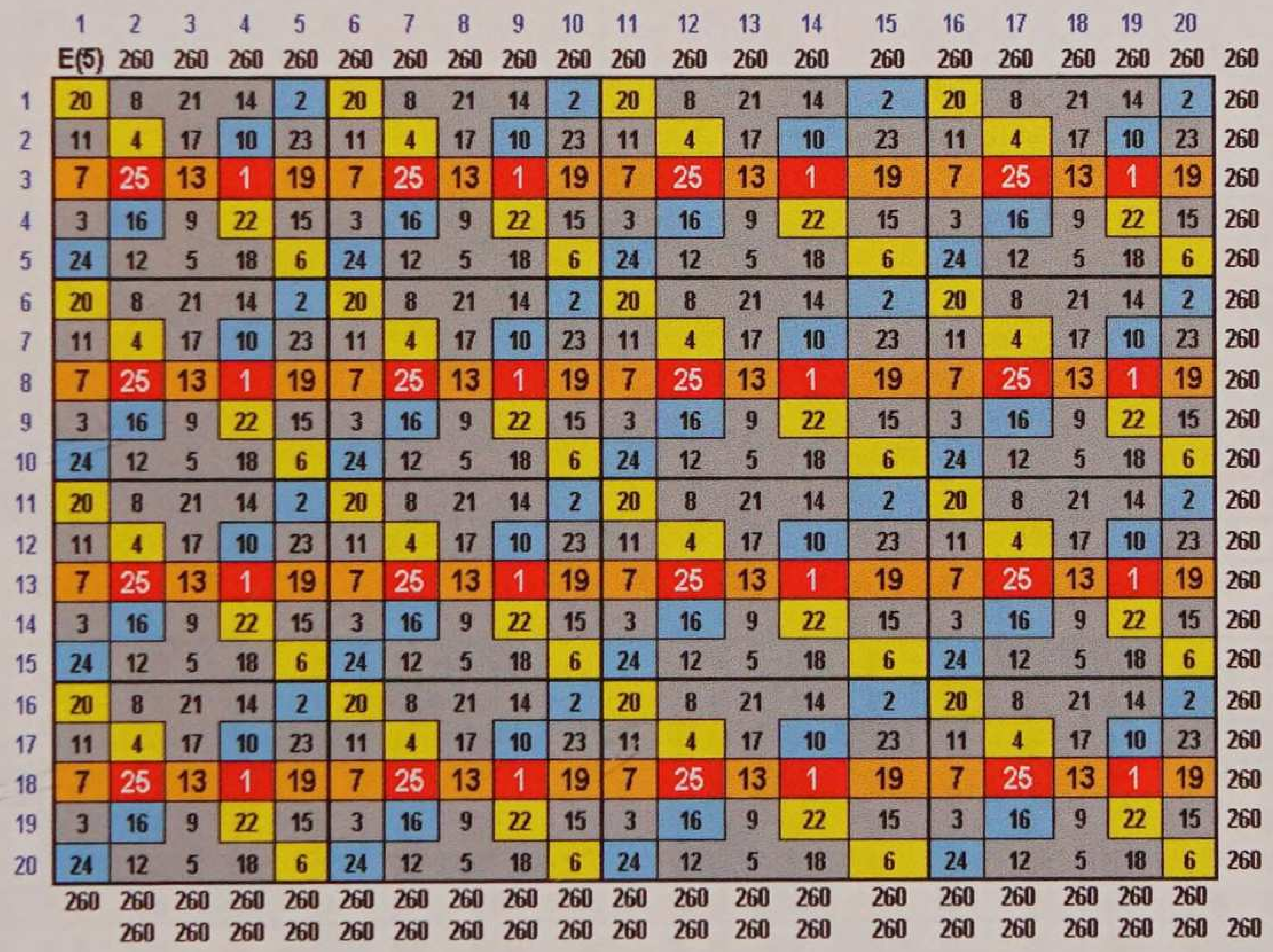
Now, it needs to be pointed out here that loom tables of a sub-class of Class-4 squares where **n = 4b** and **b** is an odd number, which are derived in the base **n** using the modulus and integer functions (**2.3**) and (**2.4**) back in Program 2, do not yield squares in which both the primal and dual have any tiling patterns at all. For that to happen, smaller squares involved in their composition must be expanded into block-square tiling patterns as shown on the next page.

Program 3

Here are the Expansion tables **E(4)** and **E(5)** used to generate the size **20** primary squares.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
E(4)	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	
	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	E(5)	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260
1	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
2	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
3	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
4	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
5	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
6	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
7	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
8	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
9	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
10	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
11	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
12	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
13	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
14	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
15	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
16	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
17	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
18	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
19	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
20	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260

And here is how the primal and dual are derived:

(3.10) $n = m_1 \times m_2$ where $m_1 \neq m_2$ and at least one of m_1 or m_2 is an odd number > **3**.

(3.11) $W(n) = (m_2)^2 (E(m_1) - |1|) + E(m_2)$ Generation of the Primal square

(3.12) $U(n) = (m_1)^2 (E(m_2) - |1|) + E(m_1)$ Generation of the Dual square

Both **W** and **U** will be perfect and have identical properties. Both will be pangenic and will have their extreme numbers in the same location.

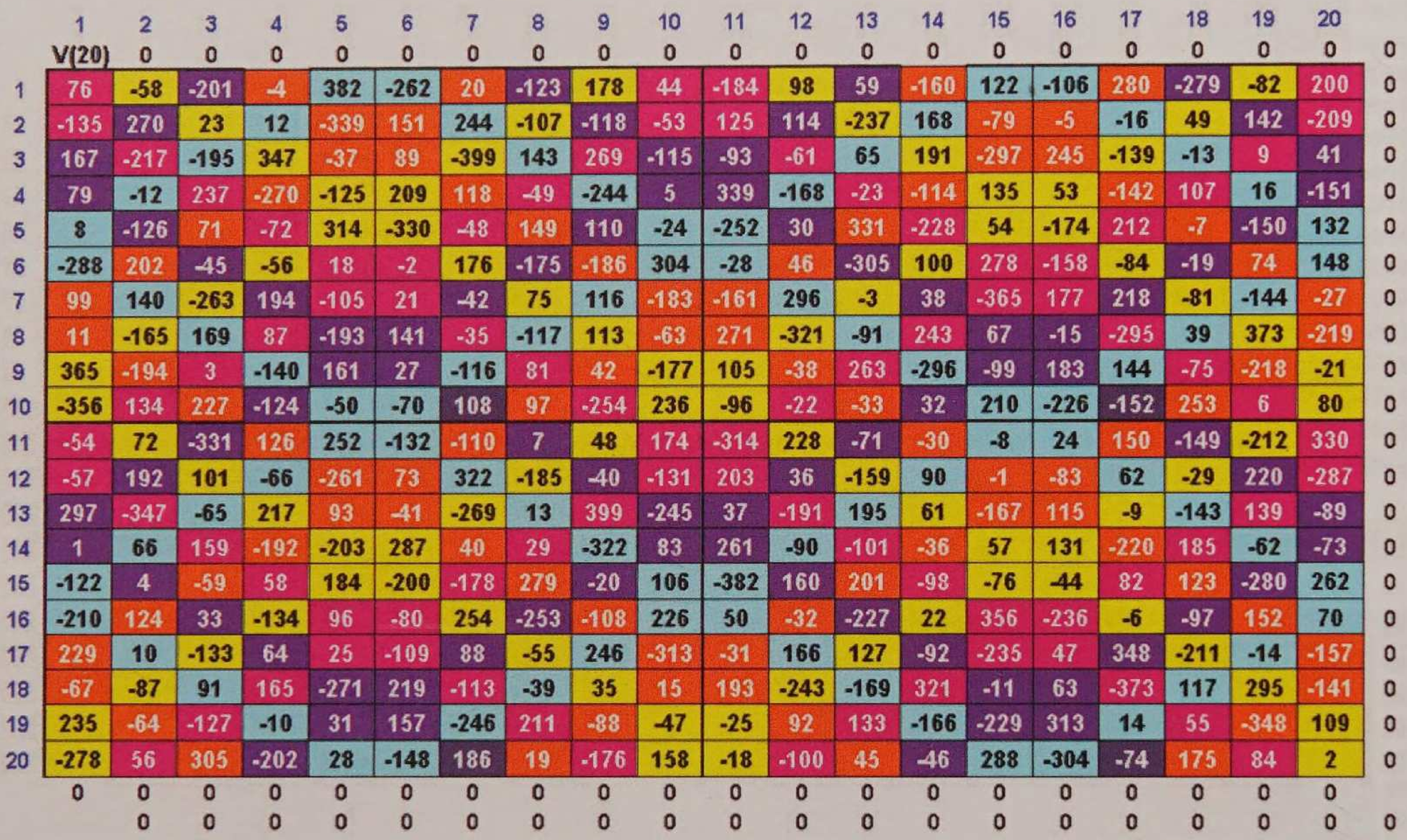
Their loom tables are the expansion tables **E(m)** and **E(n)**. These tables can be interchanged between (**3.11**) and (**3.12**). Consequently **W** and **U** are clearly the dual of each other. The method is called the **ATE** method for *Added Tiled Expansions* and will be detailed more in Program 8.

Size-20 Loom Difference table

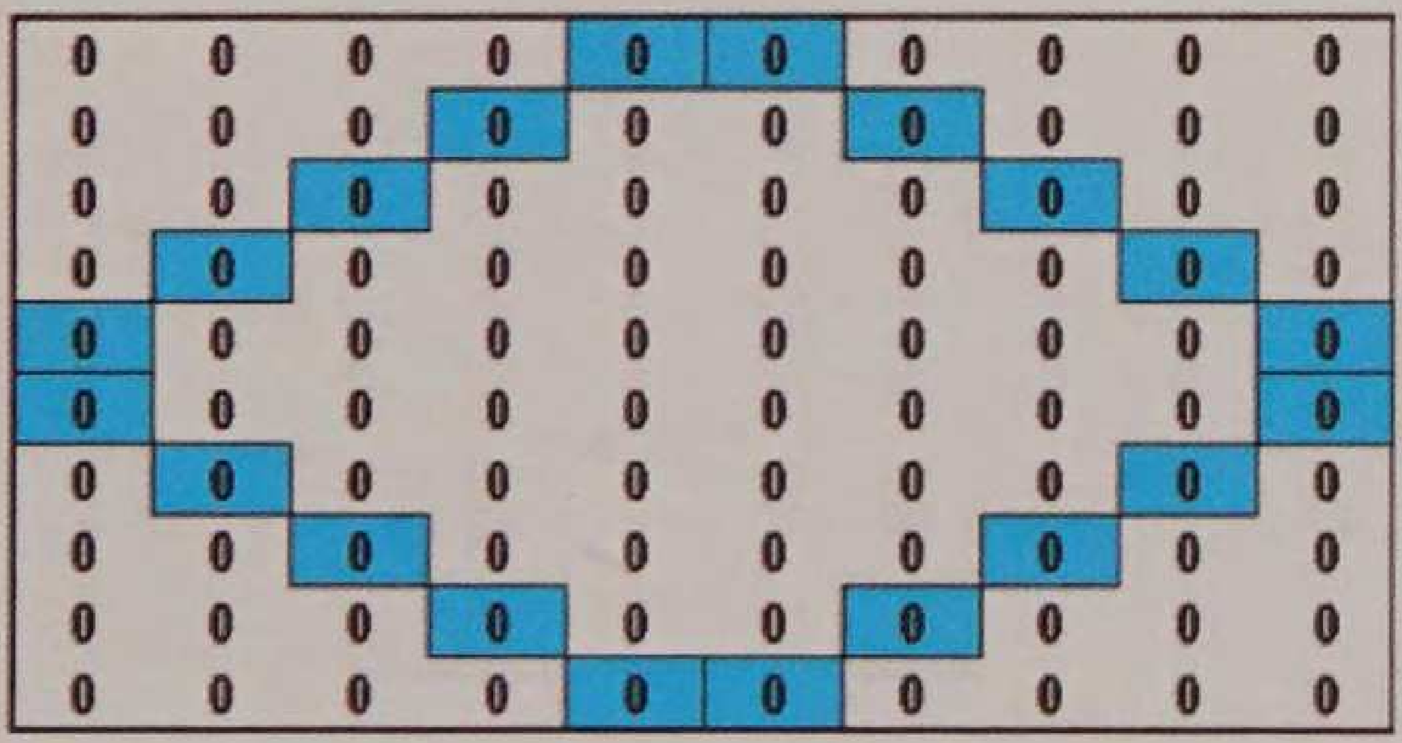
Here is the generalized formula for the difference table for sub-class size **mn** Class-4 loom tables, that are expanded from primal size **n** and size **m** squares, **W(m)** and **W(n)**. The characteristic numbers of these squares are denoted by C_m and C_n, respectively:

$$(3.13)\quad V(n) = m_1 C_{m2} E(m_2) - m_2 C_{m1} E(m_1)$$

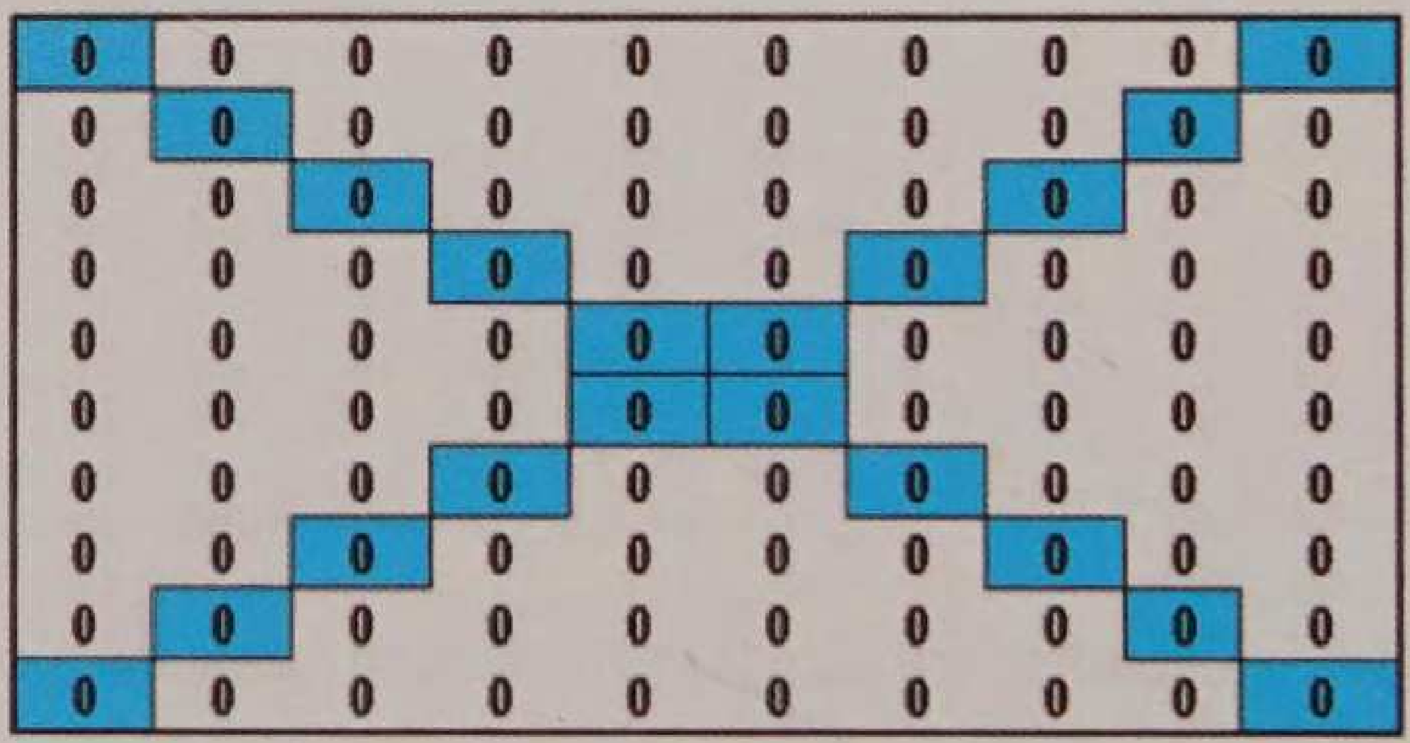
This yields the difference table here for $m_1 = 4$ and $m_2 = 5$.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	V(20)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	76	-58	-201	-4	382	-262	20	-123	178	44	-184	98	59	-160	122	-106	280	-279	-82	200	0
2	-135	270	23	12	-339	151	244	-107	-118	-53	125	114	-237	168	-79	-5	-16	49	142	-209	0
3	167	-217	-195	347	-37	89	-399	143	269	-115	-93	-61	65	191	-297	245	-139	-13	9	41	0
4	79	-12	237	-270	-125	209	118	-49	-244	5	339	-168	-23	-114	135	53	-142	107	16	-151	0
5	8	-126	71	-72	314	-330	-48	149	110	-24	-252	30	331	-228	54	-174	212	-7	-150	132	0
6	-288	202	-45	-56	18	-2	176	-175	-186	304	-28	46	-305	100	278	-158	-84	-19	74	148	0
7	99	140	-263	194	-105	21	-42	75	116	-183	-161	296	-3	38	-365	177	218	-81	-144	-27	0
8	11	-165	169	87	-193	141	-35	-117	113	-63	271	-321	-91	243	67	-15	-295	39	373	-219	0
9	365	-194	3	-140	161	27	-116	81	42	-177	105	-38	263	-296	-99	183	144	-75	-218	-21	0
10	-356	134	227	-124	-50	-70	108	97	-254	236	-96	-22	-33	32	210	-226	-152	253	6	80	0
11	-54	72	-331	126	252	-132	-110	7	48	174	-314	228	-71	-30	-8	24	150	-149	-212	330	0
12	-57	192	101	-66	-261	73	322	-185	-40	-131	203	36	-159	90	-1	-83	62	-29	220	-287	0
13	297	-347	-65	217	93	-41	-269	13	399	-245	37	-191	195	61	-167	115	-9	-143	139	-89	0
14	1	66	159	-192	-203	287	40	29	-322	83	261	-90	-101	-36	57	131	-220	185	-62	-73	0
15	-122	4	-59	58	184	-200	-178	279	-20	106	-382	160	201	-98	-76	-44	82	123	-280	262	0
16	-210	124	33	-134	96	-80	254	-253	-108	226	50	-32	-227	22	356	-236	-6	-97	152	70	0
17	229	10	-133	64	25	-109	88	-55	246	-313	-31	166	127	-92	-235	47	348	-211	-14	-157	0
18	-67	-87	91	165	-271	219	-113	-39	35	15	193	-243	-169	321	-11	63	-373	117	295	-141	0
19	235	-64	-127	-10	31	157	-246	211	-88	-47	-25	92	133	-166	-229	313	14	55	-348	109	0
20	-278	56	305	-202	28	-148	186	19	-176	158	-18	-100	45	-46	288	-304	-74	175	84	2	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Not only is it **0** geonomically but all of its diamond and X tiles sum continuously to **0** too. Consequently, the size-20 square possesses the cloaking property.

Common divisors may be factored out of formula (**3.13**). As it stands here, $m_1 \mathrm{x} C_{m2} = 4 \times 65 = 260$ and $m_2 \mathrm{x} C_{m1} = 5 \times 34 = 170$. The number **10** can be factored out of both coefficients without effect, yielding **17** and **26** as coefficients of **E(5)** and **E(4)**, respectively. That was done here to minimize the absolute values of the differences.

So now, for size **n = 4b** where $m_1 = 4$ and $m_2 = b$, an odd number **> 3**, the size **n** Class-4 square will have at least one ultra-perfect version of both primal and dual square that possesses complementary loom-tables with tiling patterns that have the continuous cloaking property.

Program 3

Loom-tables derived in the base 20

Now having derived a size-20 square **W(20)** using the size-4 and size-5 expansion tables, **E(4)** and **E(5)**, the following loom tables **X(20)** and **Y(20)** were then derived from the standard modulus and integer functions (**2.1**) and (**2.2**), respectively, from generated **W(20)**. Doing so, the following loom tables were derived:

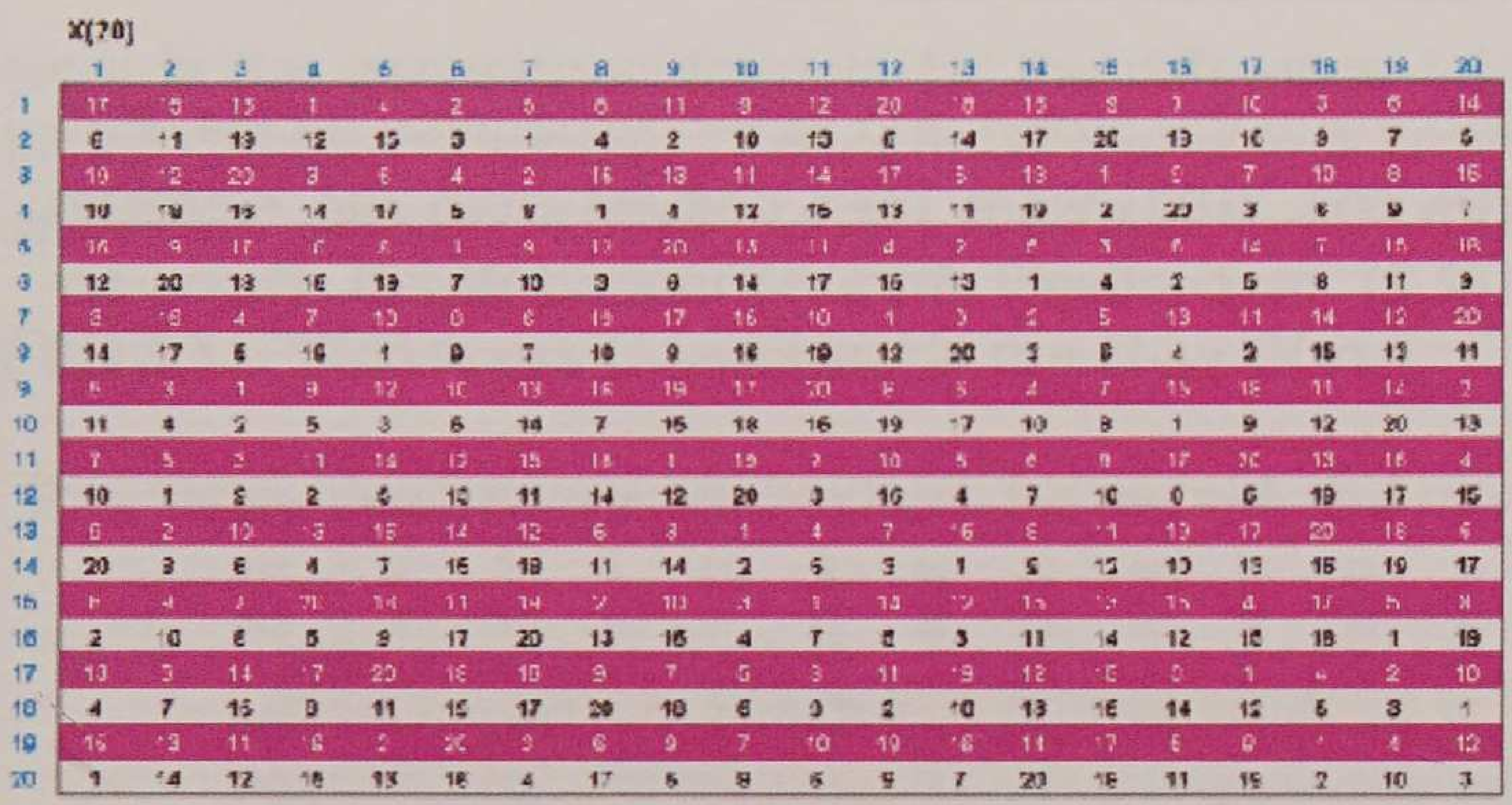
X(20)	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210
17	15	13	1	4	2	5	8	11	9	12	20	18	16	19	7	10	3	6	14	210
8	11	19	12	15	3	1	4	2	10	13	6	14	17	20	18	16	9	7	5	210
19	12	20	3	6	4	2	15	13	11	14	17	5	18	1	9	7	10	8	16	210
10	18	16	14	17	5	8	1	4	12	15	13	11	19	2	20	3	6	9	7	210
16	19	17	10	8	1	9	12	20	13	11	4	2	5	3	6	14	7	15	18	210
12	20	18	16	19	7	10	3	6	14	17	15	13	1	4	2	5	8	11	9	210
3	16	4	7	10	8	6	19	17	15	18	1	9	2	5	13	11	14	12	20	210
14	17	5	18	1	9	7	10	8	16	19	12	20	3	6	4	2	15	13	11	210
5	3	1	9	12	10	13	16	19	17	20	8	6	4	7	15	18	11	14	2	210
11	4	2	5	3	6	14	7	15	18	16	19	17	10	8	1	9	12	20	13	210
7	5	3	11	14	12	15	18	1	19	2	10	8	6	9	17	20	13	16	4	210
18	1	9	2	5	13	11	14	12	20	3	16	4	7	10	8	6	19	17	15	210
9	2	10	13	16	14	12	5	3	1	4	7	15	8	11	19	17	20	18	6	210
20	8	6	4	7	15	18	11	14	2	5	3	1	9	12	10	13	16	19	17	210
6	9	7	20	18	11	19	2	10	3	1	14	12	15	13	16	4	17	5	8	210
2	10	8	6	9	17	20	13	16	4	7	5	3	11	14	12	15	18	1	19	210
13	6	14	17	20	18	16	9	7	5	8	11	19	12	15	3	1	4	2	10	210
4	7	15	8	11	19	17	20	18	6	9	2	10	13	16	14	12	5	3	1	210
15	13	11	19	2	20	3	6	9	7	10	18	16	14	17	5	8	1	4	12	210
1	14	12	15	13	16	4	17	5	8	6	9	7	20	18	11	19	2	10	3	210
210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	
	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210

Y(20)	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210
20	11	8	3	20	12	9	2	20	11	9	2	19	11	8	3	20	11	9	2	210
4	7	15	16	3	8	15	17	4	7	15	16	4	7	14	16	3	8	15	16	210
12	18	1	10	13	19	1	10	12	19	1	9	13	18	2	10	12	19	1	10	210
6	5	17	13	6	5	18	14	6	5	17	14	6	4	18	13	7	5	17	14	210
19	11	8	3	20	11	9	2	20	11	8	3	20	12	9	2	20	11	9	2	210
4	7	14	16	3	8	15	16	4	7	15	16	3	8	15	17	4	7	15	16	210
13	18	2	10	12	19	1	10	12	18	1	10	13	19	1	10	12	19	1	9	210
6	4	18	13	7	5	17	14	6	5	17	13	6	5	18	14	6	5	17	14	210
20	12	9	2	20	11	9	2	19	11	8	3	20	11	9	2	20	11	8	3	210
3	8	15	17	4	7	15	16	4	7	14	16	3	8	15	16	4	7	15	16	210
13	19	1	10	12	19	1	9	13	18	2	10	12	19	1	10	12	18	1	10	210
6	5	18	14	6	5	17	14	6	4	18	13	7	5	17	14	6	5	17	13	210
20	11	9	2	20	11	8	3	20	12	9	2	20	11	9	2	19	11	8	3	210
3	8	15	16	4	7	15	16	3	8	15	17	4	7	15	16	4	7	14	16	210
12	19	1	10	12	18	1	10	13	19	1	10	12	19	1	9	13	18	2	10	210
7	5	17	14	6	5	17	13	6	5	18	14	6	5	17	14	6	4	18	13	210
20	11	9	2	19	11	8	3	20	11	9	2	20	11	8	3	20	12	9	2	210
4	7	15	16	4	7	14	16	3	8	15	16	4	7	15	16	3	8	15	17	210
12	19	1	9	13	18	2	10	12	19	1	10	12	18	1	10	13	19	1	10	210
6	5	17	14	6	4	18	13	7	5	17	14	6	5	17	13	6	5	18	14	210
210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	
	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210	210

Not much to look at, but amazingly the modulus loom-table **X(20)** possesses the same number-sequence patterns found in Class-1 squares; this is shown on the next page.

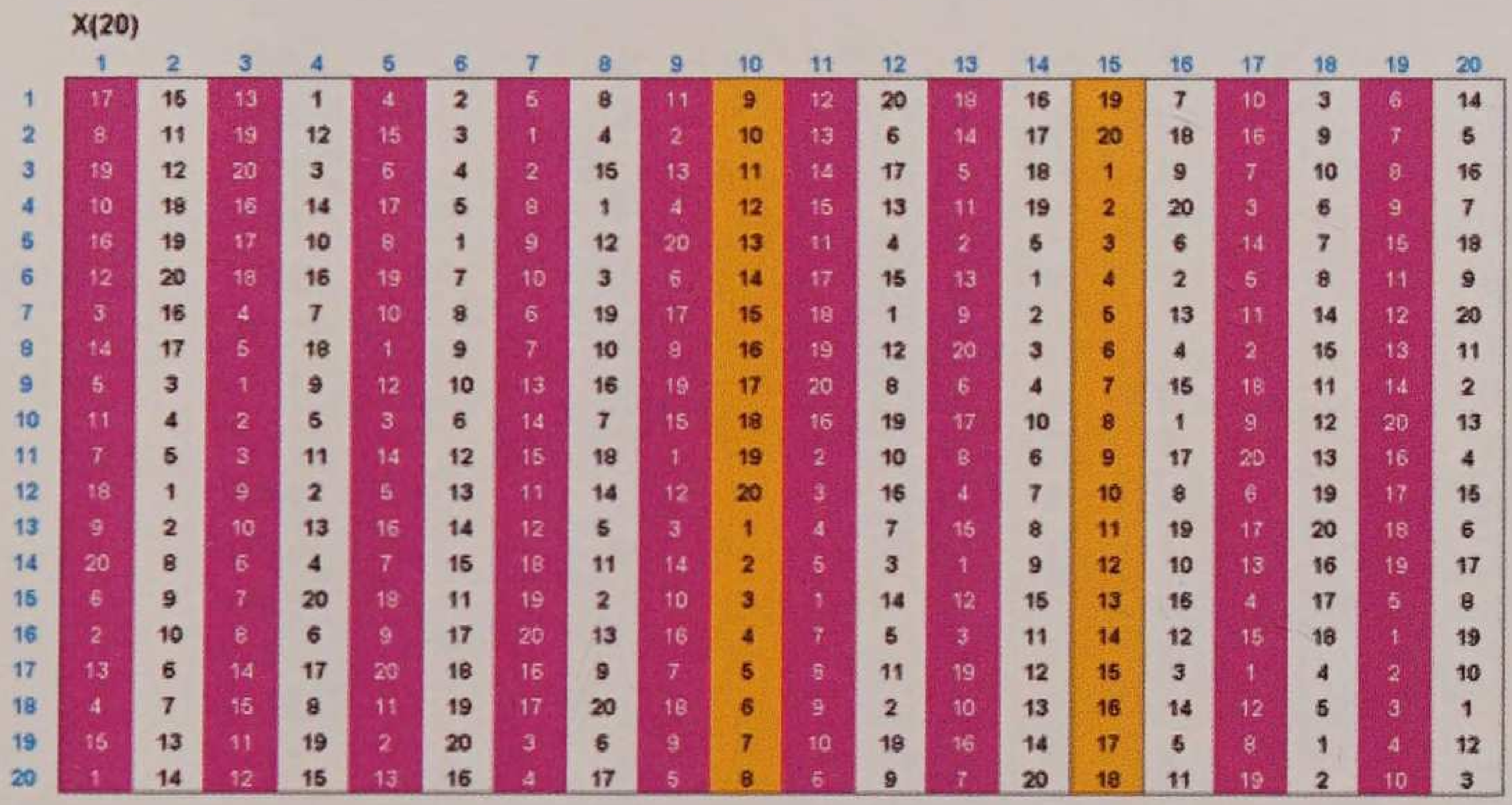
Program 3

Class-1 string summations found in a Class-4 loom-table



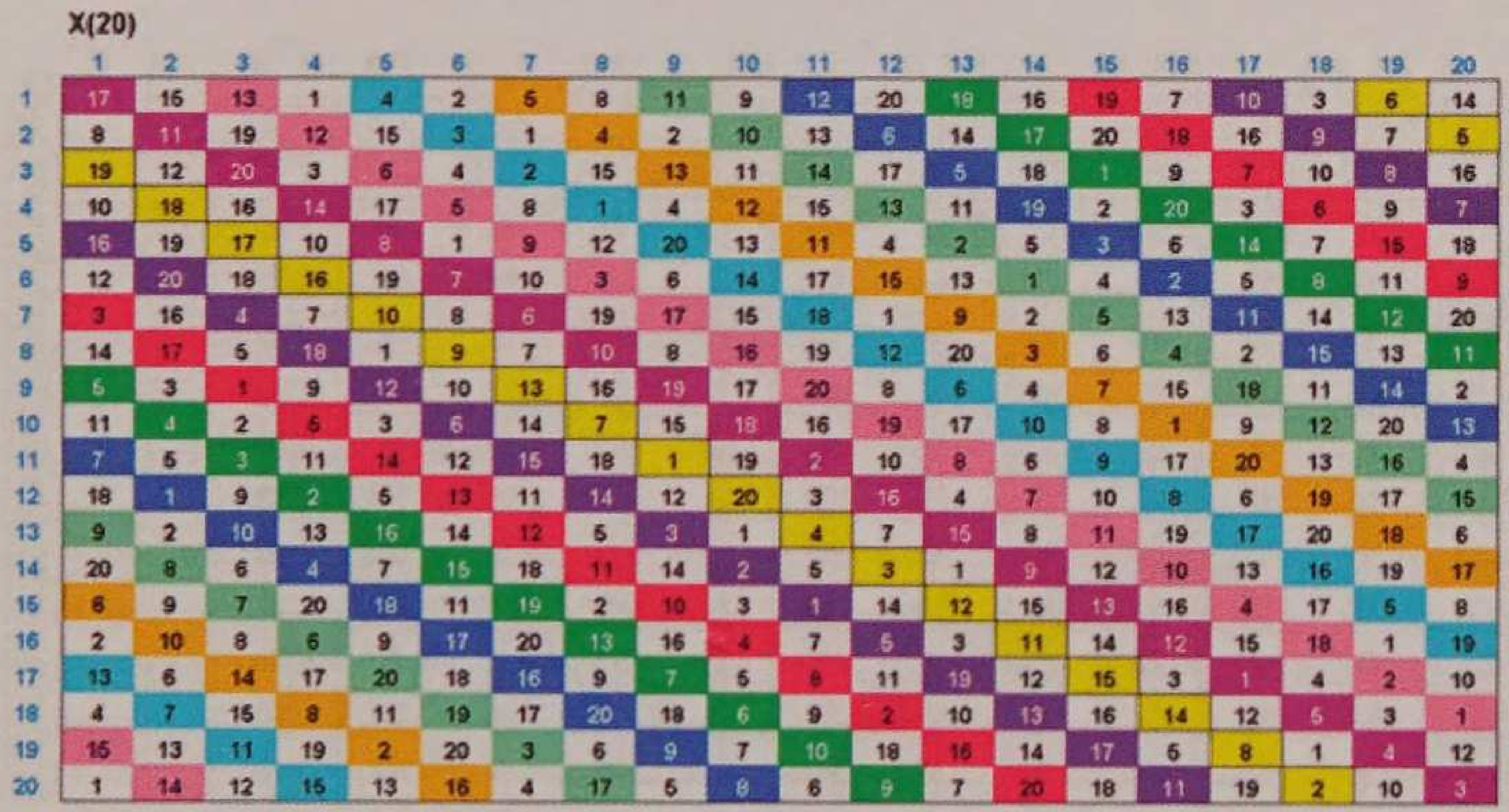
X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17	15	13	1	4	2	5	8	11	9	12	20	18	16	19	7	10	3	6	14
2	8	11	19	12	15	3	1	4	2	10	13	6	14	17	20	18	16	9	7	5
3	19	12	20	3	6	4	2	15	13	11	14	17	5	18	1	9	7	10	8	16
4	10	18	16	14	17	5	8	1	4	12	15	13	11	19	2	20	3	6	9	7
5	16	19	17	10	8	1	9	12	20	13	11	4	2	5	3	6	14	7	15	18
6	12	20	18	16	19	7	10	3	6	14	17	15	13	1	4	2	5	8	11	9
7	3	16	4	7	10	8	6	19	17	15	18	1	9	2	5	13	11	14	12	20
8	14	17	5	18	1	9	7	10	8	16	19	12	20	3	6	4	2	15	13	11
9	5	3	1	9	12	10	13	16	19	17	20	8	6	4	7	15	18	11	14	2
10	11	4	2	5	3	6	14	7	15	18	16	19	17	10	8	1	9	12	20	13
11	7	5	3	11	14	12	15	18	1	19	2	10	8	6	9	17	20	13	16	4
12	18	1	9	2	5	13	11	14	12	20	3	16	4	7	10	8	6	19	17	15
13	9	2	10	13	16	14	12	5	3	1	4	7	15	8	11	19	17	20	18	6
14	20	8	6	4	7	15	18	11	14	2	5	3	1	9	12	10	13	16	19	17
15	6	9	7	20	18	11	19	2	10	3	1	14	12	15	13	16	4	17	5	8
16	2	10	8	6	9	17	20	13	16	4	7	5	3	11	14	12	15	18	1	19
17	13	6	14	17	20	18	16	9	7	5	8	11	19	12	15	3	1	4	2	10
18	4	7	15	8	11	19	17	20	18	6	9	2	10	13	16	14	12	5	3	1
19	15	13	11	19	2	20	3	6	9	7	10	18	16	14	17	5	8	1	4	12
20	1	14	12	15	13	16	4	17	5	8	6	9	7	20	18	11	19	2	10	3



X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17	15	13	1	4	2	5	8	11	9	12	20	18	16	19	7	10	3	6	14
2	8	11	19	12	15	3	1	4	2	10	13	6	14	17	20	18	16	9	7	5
3	19	12	20	3	6	4	2	15	13	11	14	17	5	18	1	9	7	10	8	16
4	10	18	16	14	17	5	8	1	4	12	15	13	11	19	2	20	3	6	9	7
5	16	19	17	10	8	1	9	12	20	13	11	4	2	5	3	6	14	7	15	18
6	12	20	18	16	19	7	10	3	6	14	17	15	13	1	4	2	5	8	11	9
7	3	16	4	7	10	8	6	19	17	15	18	1	9	2	5	13	11	14	12	20
8	14	17	5	18	1	9	7	10	8	16	19	12	20	3	6	4	2	15	13	11
9	5	3	1	9	12	10	13	16	19	17	20	8	6	4	7	15	18	11	14	2
10	11	4	2	5	3	6	14	7	15	18	16	19	17	10	8	1	9	12	20	13
11	7	5	3	11	14	12	15	18	1	19	2	10	8	6	9	17	20	13	16	4
12	18	1	9	2	5	13	11	14	12	20	3	16	4	7	10	8	6	19	17	15
13	9	2	10	13	16	14	12	5	3	1	4	7	15	8	11	19	17	20	18	6
14	20	8	6	4	7	15	18	11	14	2	5	3	1	9	12	10	13	16	19	17
15	6	9	7	20	18	11	19	2	10	3	1	14	12	15	13	16	4	17	5	8
16	2	10	8	6	9	17	20	13	16	4	7	5	3	11	14	12	15	18	1	19
17	13	6	14	17	20	18	16	9	7	5	8	11	19	12	15	3	1	4	2	10
18	4	7	15	8	11	19	17	20	18	6	9	2	10	13	16	14	12	5	3	1
19	15	13	11	19	2	20	3	6	9	7	10	18	16	14	17	5	8	1	4	12
20	1	14	12	15	13	16	4	17	5	8	6	9	7	20	18	11	19	2	10	3



X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17	15	13	1	4	2	5	8	11	9	12	20	18	16	19	7	10	3	6	14
2	8	11	19	12	15	3	1	4	2	10	13	6	14	17	20	18	16	9	7	5
3	19	12	20	3	6	4	2	15	13	11	14	17	5	18	1	9	7	10	8	16
4	10	18	16	14	17	5	8	1	4	12	15	13	11	19	2	20	3	6	9	7
5	16	19	17	10	8	1	9	12	20	13	11	4	2	5	3	6	14	7	15	18
6	12	20	18	16	19	7	10	3	6	14	17	15	13	1	4	2	5	8	11	9
7	3	16	4	7	10	8	6	19	17	15	18	1	9	2	5	13	11	14	12	20
8	14	17	5	18	1	9	7	10	8	16	19	12	20	3	6	4	2	15	13	11
9	5	3	1	9	12	10	13	16	19	17	20	8	6	4	7	15	18	11	14	2
10	11	4	2	5	3	6	14	7	15	18	16	19	17	10	8	1	9	12	20	13
11	7	5	3	11	14	12	15	18	1	19	2	10	8	6	9	17	20	13	16	4
12	18	1	9	2	5	13	11	14	12	20	3	16	4	7	10	8	6	19	17	15
13	9	2	10	13	16	14	12	5	3	1	4	7	15	8	11	19	17	20	18	6
14	20	8	6	4	7	15	18	11	14	2	5	3	1	9	12	10	13	16	19	17
15	6	9	7	20	18	11	19	2	10	3	1	14	12	15	13	16	4	17	5	8
16	2	10	8	6	9	17	20	13	16	4	7	5	3	11	14	12	15	18	1	19
17	13	6	14	17	20	18	16	9	7	5	8	11	19	12	15	3	1	4	2	10
18	4	7	15	8	11	19	17	20	18	6	9	2	10	13	16	14	12	5	3	1
19	15	13	11	19	2	20	3	6	9	7	10	18	16	14	17	5	8	1	4	12
20	1	14	12	15	13	16	4	17	5	8	6	9	7	20	18	11	19	2	10	3

X(20)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17	15	13	1	4	2	5	8	11	9	12	20	18	16	19	7	10	3	6	14
2	8	11	19	12	15	3	1	4	2	10	13	6	14	17	20	18	16	9	7	5
3	19	12	20	3	6	4	2	15	13	11	14	17	5	18	1	9	7	10	8	16
4	10	18	16	14	17	5	8	1	4	12	15	13	11	19	2	20	3	6	9	7
5	16	19	17	10	8	1	9	12	20	13	11	4	2	5	3	6	14	7	15	18
6	12	20	18	16	19	7	10	3	6	14	17	15	13	1	4	2	5	8	11	9
7	3	16	4	7	10	8	6	19	17	15	18	1	9	2	5	13	11	14	12	20
8	14	17	5	18	1	9	7	10	8	16	19	12	20	3	6	4	2	15	13	11
9	5	3	1	9	12	10	13	16	19	17	20	8	6	4	7	15	18	11	14	2
10	11	4	2	5	3	6	14	7	15	18	16	19	17	10	8	1	9	12	20	13
11	7	5	3	11	14	12	15	18	1	19	2	10	8	6	9	17	20	13	16	4
12	18	1	9	2	5	13	11	14	12	20	3	16	4	7	10	8	6	19	17	15
13	9	2	10	13	16	14	12	5	3	1	4	7	15	8	11	19	17	20	18	6
14	20	8	6	4	7	15	18	11	14	2	5	3	1	9	12	10	13	16	19	17
15	6	9	7	20	18	11	19	2	10	3	1	14	12	15	13	16	4	17	5	8
16	2	10	8	6	9	17	20	13	16	4	7	5	3	11	14	12	15	18	1	19
17	13	6	14	17	20	18	16	9	7	5	8	11	19	12	15	3	1	4	2	10
18	4	7	15	8	11	19	17	20	18	6	9	2	10	13	16	14	12	5	3	1
19	15	13	11	19	2	20	3	6	9	7	10	18	16	14	17	5	8	1	4	12
20	1	14	12	15	13	16	4	17	5	8	6	9	7	20	18	11	19	2	10	3

The modulus loom table derived directly from **W(20)** in the base **20** has the Class-1 string-summing patterns. Observe that every row, column and diagonal in both major and minor directions contains the numbers **1** thru **20** exactly once.

Further, note the two vertical columns which contain these numbers in their natural sequence. Again it's another integrated naturally-woven numeric fabric. However, the corresponding integer loom-table has none of these properties.

Neither loom-table derived from **W(20)** in the base **20** possesses the size-20 characteristic tiling patterns either. For the primary squares to possess these tiling patterns jointly, it requires both loom-tables to possess them.

Although not shown in this program, the size **28** square has all of these properties too, or lack thereof, as cited here for the size **20** square. Check it out for yourself.

Note that this sub-class of **n = 4b** applies only to odd **b**.

Now for **b = 3**, going in reverse order by **8**, neither size **12**'s loom-tables have these characteristic Class-1 string-summing properties. The size-12 is embryonic and has an insufficient quantity of numbers to possess this property. So they begin at size **20** for **b = 5** and continue on from there in size increments of **8**.

Program 3

Here is the size **20** dual **U2(20)** derived from the loom-tables **X(20)** and **Y(20)** in the base **20** according to formula **(2.4)** of Program 2. Note that it does not possess continuous tiling patterns like the primal version **W(20)** does.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
U2(20)	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
	340	291	248	3	80	32	89	142	220	171	229	382	359	311	368	123	200	51	109	262	4010
	144	207	375	236	283	48	15	77	24	187	255	116	264	327	394	356	303	168	135	96	4010
	372	238	381	50	113	79	21	290	252	219	261	329	93	358	2	170	132	199	141	310	4010
	186	345	317	273	326	85	158	14	66	225	297	254	206	364	38	393	47	105	177	134	4010
	319	371	328	183	160	11	169	222	400	251	208	63	40	92	49	102	280	131	289	342	4010
	224	387	354	316	363	128	195	56	104	267	335	296	243	8	75	37	84	147	215	176	4010
	53	318	62	130	192	159	101	370	332	298	341	10	173	39	81	250	212	279	221	389	4010
	266	324	98	353	7	165	137	194	146	305	377	233	386	45	118	74	26	285	257	214	4010
	100	52	9	162	240	191	249	302	379	331	388	143	120	71	129	282	360	211	268	23	4010
	203	68	35	97	44	107	275	136	284	347	314	376	323	188	155	16	164	227	395	256	4010
	133	99	41	210	272	239	281	349	13	378	22	190	152	119	161	330	392	258	301	70	4010
	346	5	178	34	86	245	217	274	226	384	58	313	67	125	197	154	106	365	337	293	4010
	180	31	189	242	320	271	228	83	60	12	69	122	300	151	209	362	339	391	348	103	4010
	383	148	115	76	124	287	355	216	263	28	95	57	4	167	235	196	244	307	374	336	4010
	112	179	121	390	352	218	361	30	193	59	1	270	232	299	241	309	73	338	82	150	4010
	27	185	157	114	166	325	397	253	306	65	138	94	46	205	277	234	286	344	18	373	4010
	260	111	269	322	399	351	308	163	140	91	149	202	380	231	288	43	20	72	29	182	4010
	64	127	295	156	204	367	334	396	343	108	175	36	184	247	315	276	223	88	55	17	4010
	292	259	201	369	33	398	42	110	172	139	181	350	312	278	321	90	153	19	61	230	4010
	6	265	237	294	246	304	78	333	87	145	117	174	126	385	357	213	366	25	198	54	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

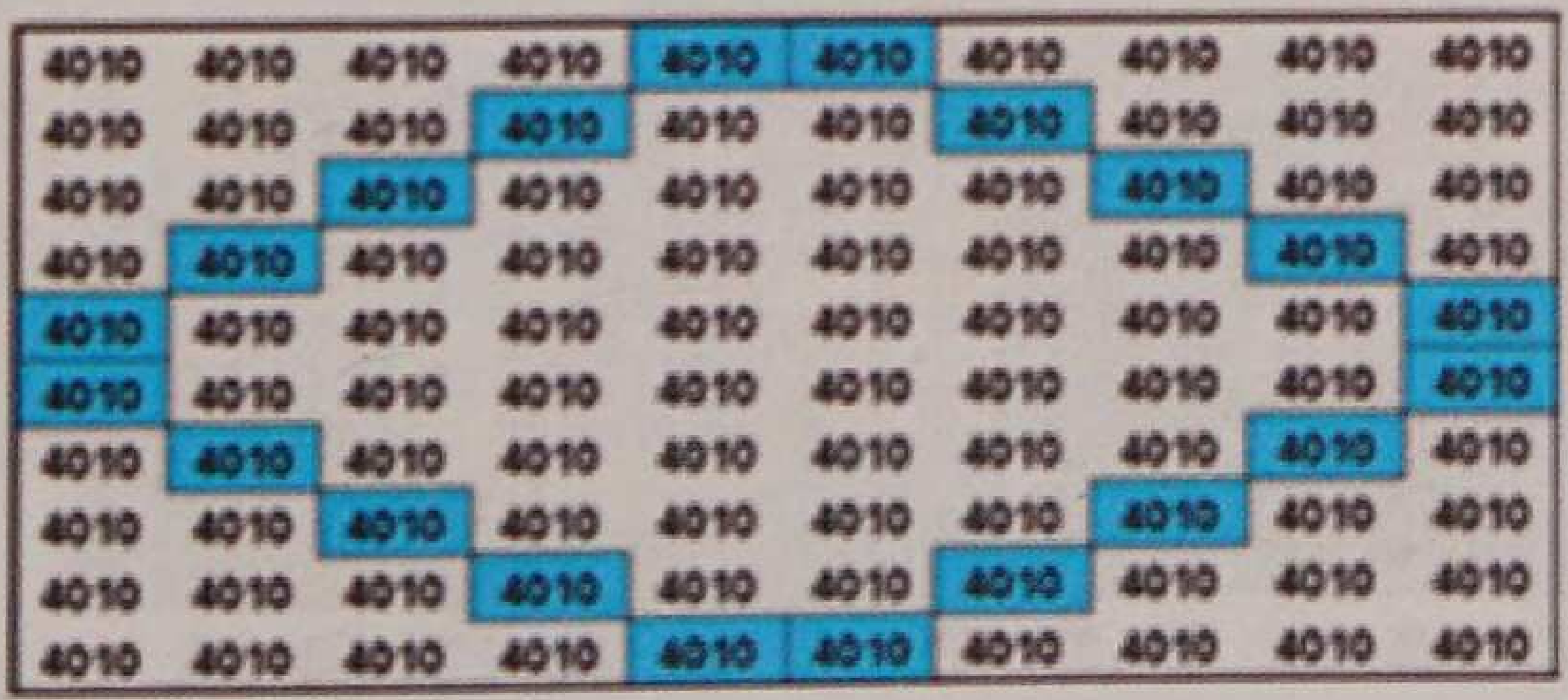
However the dual derived from (**3.12**) using the expansion tables **E(4)** and **E(5)** does have the same continuous tiling patterns as does the primal square as seen below.

This is just one demonstration which shows how a sub-class of Class-4 squares can be generated which will possess complementary tiling patterns for both primal and dual squares simultaneously.

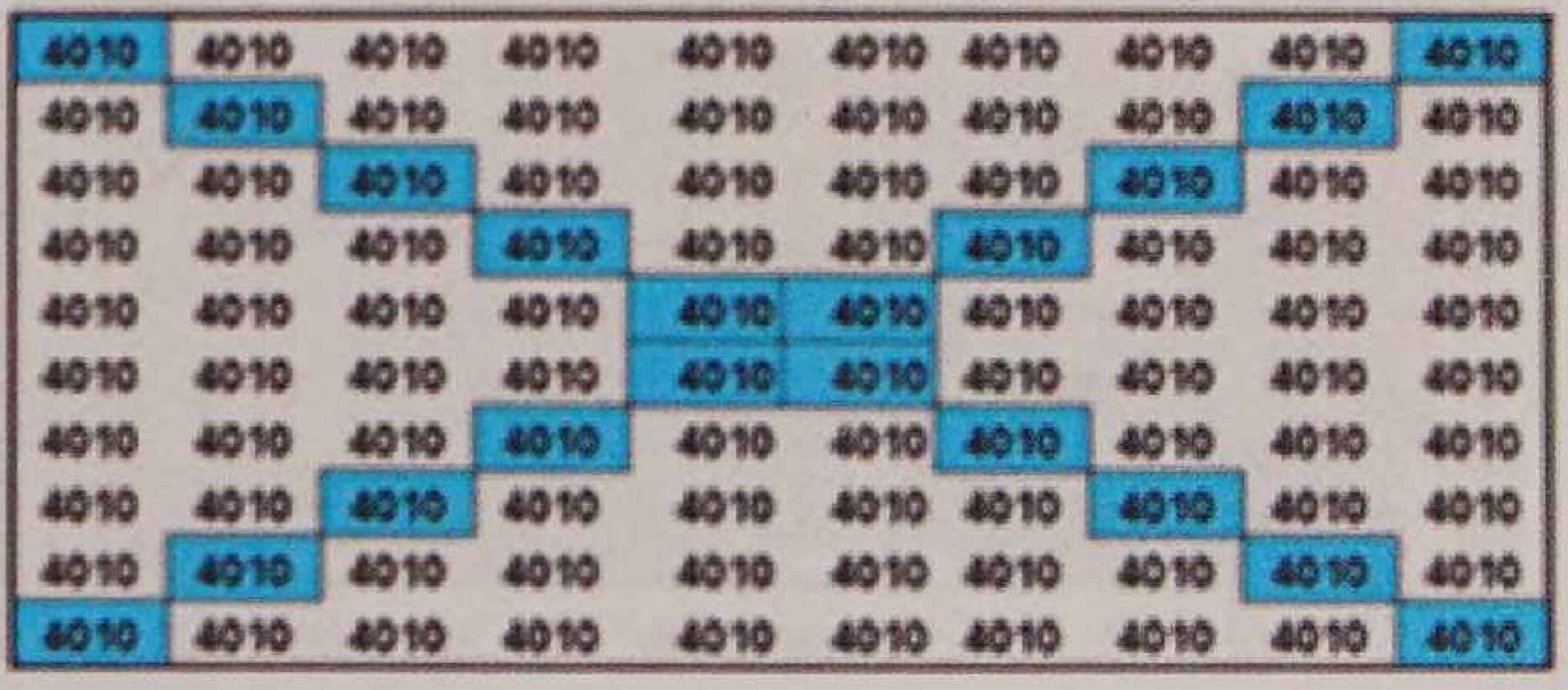
4409	4409	4409	4409	3611	3611	4010	3611	4409	4409	4409	4409
4010	4409	4010	4010	4010	3611	4010	4808	4409	4808	4409	4010
4010	4409	4409	3611	3611	3611	3611	4010	4808	4409	4808	4409
4010	4808	3611	4808	3611	3212	4010	4010	4010	4808	4409	3212
4409	4010	4010	3611	3212	3611	3212	4010	4409	4409	4409	4409
4010	4010	4010	4010	3611	3212	3611	3212	4010	4409	4010	4010
4010	4010	3611	3611	3611	3611	3611	4409	4010	4409	4409	3611
4409	3212	4010	4409	4010	4010	4010	3611	4010	4808	3611	4808
3611	3611	4010	3611	4409	4409	4409	4409	4409	4010	4010	3611
4010	3611	4010	4808	4409	4808	4409	4010	4010	4010	4010	4010
3611	3611	3611	4010	4808	4409	4808	4409	4010	4010	3611	3611
3611	3212	4010	4010	4010	4808	4409	3212	4409	3212	4010	4409

Two other successful methods for generating such Class-4 squares whicxh use a spread-pattern distribution of a smaller size square, the pattern computed from very elementary binary patterns of **1**'s and **0**'s, are shown in Program 8,

U1(20)																					
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	320	115	326	217	32	307	118	329	224	19	310	121	336	211	22	313	128	323	214	25	4010
2	162	61	268	151	354	173	60	263	146	365	172	55	258	157	364	167	50	269	156	359	4010
3	107	392	193	14	299	104	385	206	11	296	97	398	203	8	289	110	395	200	1	302	4010
4	37	250	143	340	229	42	255	132	341	234	47	244	133	346	239	36	245	138	351	228	4010
5	384	179	70	281	96	371	182	73	288	83	374	185	80	275	86	377	192	67	278	89	4010
6	306	125	332	215	18	317	124	327	210	29	316	119	322	221	28	311	114	333	220	23	4010
7	171	56	257	158	363	168	49	270	155	360	161	62	267	152	353	174	59	264	145	366	4010
8	101	394	207	4	293	106	399	196	5	298	111	388	197	10	303	100	389	202	15	292	4010
9	48	243	134	345	240	35	246	137	352	227	38	249	144	339	230	41	256	131	342	233	4010
10	370	189	76	279	82	381	188	71	274	93	380	183	66	285	92	375	178	77	284	87	4010
11	315	120	321	222	27	312	113	334	219	24	305	126	331	216	17	318	123	328	209	30	4010
12	165	58	271	148	357	170	63	260	149	362	175	52	261	154	367	164	53	266	159	356	4010
13	112	387	198	9	304	99	390	201	16	291	102	393	208	3	294	105	400	195	6	297	4010
14	34	253	140	343	226	45	252	135	338	237	44	247	130	349	236	39	242	141	348	231	4010
15	379	184	65	286	91	376	177	78	283	88	369	190	75	280	81	382	187	72	273	94	4010
16	309	122	335	212	21	314	127	324	213	26	319	116	325	218	31	308	117	330	223	20	4010
17	176	51	262	153	368	163	54	265	160	355	166	57	272	147	358	169	64	259	150	361	4010
18	98	397	204	7	290	109	396	199	2	301	108	391	194	13	300	103	386	205	12	295	4010
19	43	248	129	350	235	40	241	142	347	232	33	254	139	344	225	46	251	136	337	238	4010
20	373	186	79	276	85	378	191	68	277	90	383	180	69	282	95	372	181	74	287	84	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010



4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010



4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Program 3

Size-24

n = 4b where b = 6, an even number

This size-24 square here was derived by the **SPD** method described in Program 8 using the size **12** square in a **24x24** spread-pattern expansion. Applying the **ATE** method using the size **3** and **8** squares produced a perfect size **24** square but it lacked the characteristic diamond and X tiling patterns shown here. This same negative result was also obtained by using the **TAP** method. Only the **SPD** method was successful but it took normalization of the square to symmetrically align the numbers **1** and n^2 plus the swapping of three column-pairs to get the square to be totally pairwise row-symmetric.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	217	66	235	414	439	432	145	138	163	342	511	360	505	354	523	126	151	144	6924
2	103	192	85	564	313	546	319	552	301	204	97	186	391	480	373	276	25	258	31	264	13	492	385	474	6924
3	356	506	311	194	143	149	140	146	95	554	359	509	68	218	23	482	431	437	428	434	383	266	71	221	6924
4	105	208	87	562	297	544	321	568	303	202	81	184	393	496	375	274	9	256	33	280	15	490	369	472	6924
5	327	514	345	160	135	178	111	154	129	520	351	538	39	226	57	448	423	466	399	442	417	232	63	250	6924
6	83	209	92	557	296	566	299	569	308	197	80	206	371	497	380	269	8	278	11	281	20	485	368	494	6924
7	325	534	343	162	115	180	109	174	127	522	331	540	37	246	55	450	403	468	397	462	415	234	43	252	6924
8	139	156	121	528	349	510	355	516	337	168	133	150	427	444	409	240	61	222	67	228	49	456	421	438	6924
9	323	545	344	161	104	182	107	185	128	521	320	542	35	257	56	449	392	470	395	473	416	233	32	254	6924
10	141	172	123	526	333	508	357	532	339	166	117	148	429	460	411	238	45	220	69	244	51	454	405	436	6924
11	291	550	309	196	99	214	75	190	93	556	315	574	3	262	21	484	387	502	363	478	381	268	27	286	6924
12	116	170	131	518	335	533	332	530	347	158	119	173	404	458	419	230	47	245	44	242	59	446	407	461	6924
13	289	570	307	198	79	216	73	210	91	558	295	576	1	282	19	486	367	504	361	498	379	270	7	288	6924
14	247	48	229	420	457	402	463	408	445	60	241	42	535	336	517	132	169	114	175	120	157	348	529	330	6924
15	500	362	455	50	287	5	284	2	239	410	503	365	212	74	167	338	575	293	572	290	527	122	215	77	6924
16	249	64	231	418	441	400	465	424	447	58	225	40	537	352	519	130	153	112	177	136	159	346	513	328	6924
17	471	370	489	16	279	34	255	10	273	376	495	394	183	82	201	304	567	322	543	298	561	88	207	106	6924
18	227	65	236	413	440	422	443	425	452	53	224	62	515	353	524	125	152	134	155	137	164	341	512	350	6924
19	469	390	487	18	259	36	253	30	271	378	475	396	181	102	199	306	547	324	541	318	559	90	187	108	6924
20	283	12	265	384	493	366	499	372	481	24	277	6	571	300	553	96	205	78	211	84	193	312	565	294	6924
21	467	401	488	17	248	38	251	41	272	377	464	398	179	113	200	305	536	326	539	329	560	89	176	110	6924
22	285	28	267	382	477	364	501	388	483	22	261	4	573	316	555	94	189	76	213	100	195	310	549	292	6924
23	435	406	453	52	243	70	219	46	237	412	459	430	147	118	165	340	531	358	507	334	525	124	171	142	6924
24	260	26	275	374	479	389	476	386	491	14	263	29	548	314	563	86	191	101	188	98	203	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Program 3

Size-24 Loom Tables

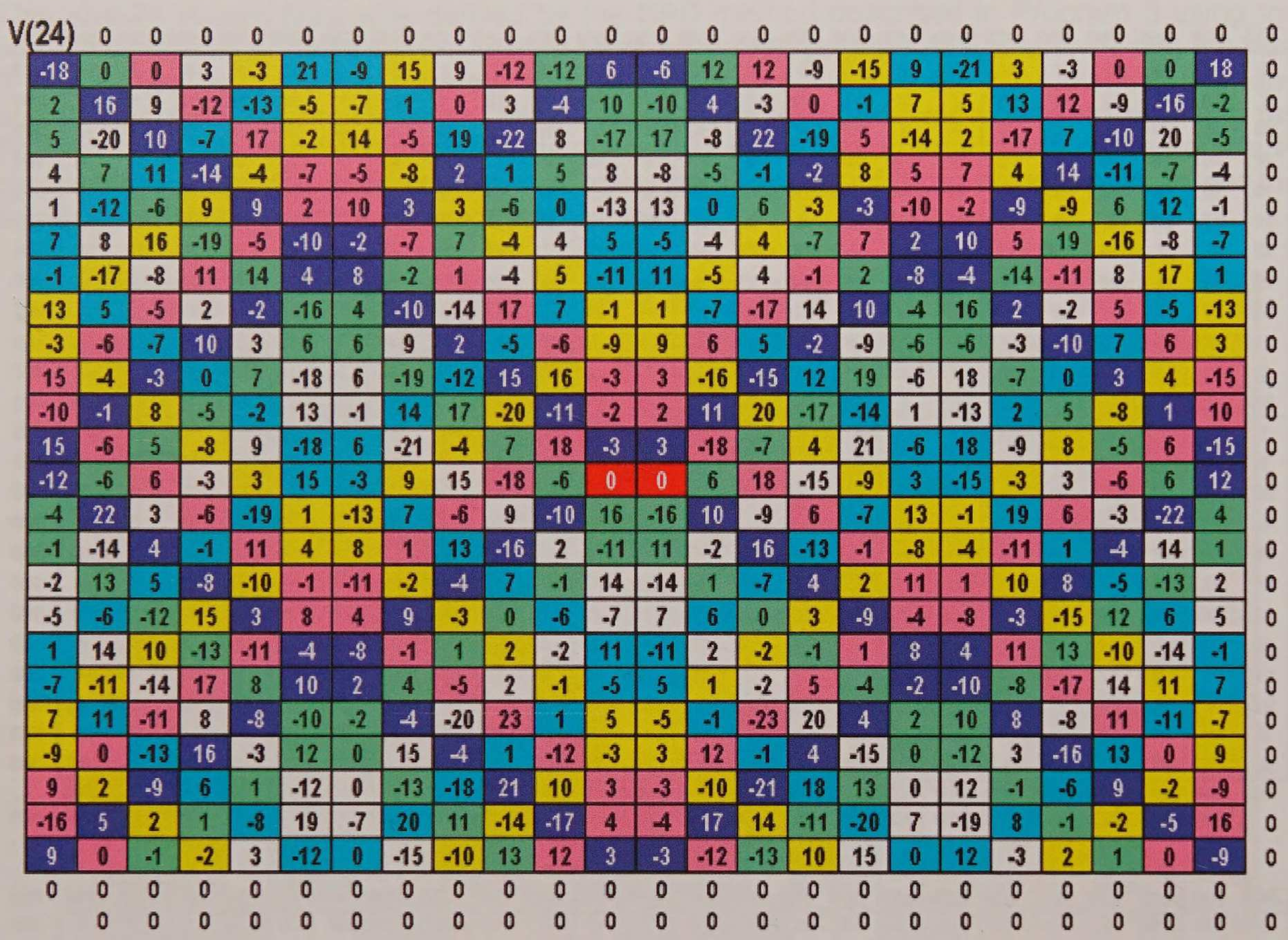
Here are the loom tables for the size-24 square seen in Program 2 taken in the base **24**.

X(24)	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300
1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	300
7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	300
20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	300
9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	300
15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	300
11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	300
13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	300
19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	300
11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	300
21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	300
3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	300
20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	300
1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	1	18	19	6	7	24	300
7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	7	24	13	12	1	18	300
20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	20	2	23	2	23	5	300
9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	9	16	15	10	9	16	300
15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	15	10	9	16	15	10	300
11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	11	17	20	5	8	14	300
13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	13	6	7	18	19	12	300
19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	19	12	1	24	13	6	300
11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	11	17	8	17	8	14	300
21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	21	4	3	22	21	4	300
3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	3	22	21	4	3	22	300
20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	20	2	11	14	23	5	300
300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	
	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300

Y(24)	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300
19	18	19	3	10	3	10	3	10	18	19	18	7	6	7	15	22	15	22	15	22	6	7	6	300
5	8	4	24	14	23	14	23	13	9	5	8	17	20	16	12	2	11	2	11	1	21	17	20	300
15	22	13	9	6	7	6	7	4	24	15	22	3	10	1	21	18	19	18	19	16	12	3	10	300
5	9	4	24	13	23	14	24	13	9	4	8	17	21	16	12	1	11	2	12	1	21	16	20	300
14	22	15	7	6	8	5	7	6	22	15	23	2	10	3	19	18	20	17	19	18	10	3	11	300
4	9	4	24	13	24	13	24	13	9	4	9	16	21	16	12	1	12	1	12	1	21	16	21	300
14	23	15	7	5	8	5	8	6	22	14	23	2	11	3	19	17	20	17	20	18	10	2	11	300
6	7	6	22	15	22	15	22	15	7	6	7	18	19	18	10	3	10	3	10	3	19	18	19	300
14	23	15	7	5	8	5	8	6	22	14	23	2	11	3	19	17	20	17	20	18	10	2	11	300
6	8	6	22	14	22	15	23	15	7	5	7	18	20	18	10	2	10	3	11	3	19	17	19	300
13	23	13	9	5	9	4	8	4	24	14	24	1	11	1	21	17	21	16	20	16	12	2	12	300
5	8	6	22	14	23	14	23	15	7	5	8	17	20	18	10	2	11	2	11	3	19	17	20	300
13	24	13	9	4	9	4	9	4	24	13	24	1	12	1	21	16	21	16	21	16	12	1	12	300
11	2	10	18	20	17	20	17	19	3	11	2	23	14	22	6	8	5	8	5	7	15	23	14	300
21	16	19	3	12	1	12	1	10	18	21	16	9	4	7	15	24	13	24	13	22	6	9	4	300
11	3	10	18	19	17	20	18	19	3	10	2	23	15	22	6	7	5	8	6	7	15	22	14	300
20	16	21	1	12	2	11	1	12	16	21	17	8	4	9	13	24	14	23	13	24	4	9	5	300
10	3	10	18	19	18	19	18	19	3	10	3	22	15	22	6	7	6	7	6	7	15	22	15	300
20	17	21	1	11	2	11	2	12	16	20	17	8	5	9	13	23	14	23	14	24	4	8	5	300
12	1	12	16	21	16	21	16	21	1	12	1	24	13	24	4	9	4	9	4	9	13	24	13	300
20	17	21	1	11	2	11	2	12	16	20	17	8	5	9	13	23	14	23	14	24	4	8	5	300
12	2	12	16	20	16	21	17	21	1	11	1	24	14	24	4	8	4	9	5	9	13	23	13	300
19	17	19	3	11	3	10	2	10	18	20	18	7	5	7	15	23	15	22	14	22	6	8	6	300
11	2	12	16	20	17	20	17	21	1	11	2	23	14	24	4	8	5	8	5	9	13	23	14	300
300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	
	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300

Program 3

Here is the loom difference table **V(24)**. Because the size-24 is pairwise row-symmetric, so is **V.** It is then quite obvious that all the diamond and X-patterns on **V** sum to **0** everywhere too. Since **W(24)** is **6x**-modular, these **6x6** block-squares sum to **0** everywhere too.



V(24)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-18	0	0	3	-3	21	-9	15	9	-12	-12	6	-6	12	12	-9	-15	9	-21	3	-3	0	0	18	0
2	16	9	-12	-13	-5	-7	1	0	3	-4	10	-10	4	-3	0	-1	7	5	13	12	-9	-16	-2	0
5	-20	10	-7	17	-2	14	-5	19	-22	8	-17	17	-8	22	-19	5	-14	2	-17	7	-10	20	-5	0
4	7	11	-14	-4	-7	-5	-8	2	1	5	8	-8	-5	-1	-2	8	5	7	4	14	-11	-7	-4	0
1	-12	-6	9	9	2	10	3	3	-6	0	-13	13	0	6	-3	-3	-10	-2	-9	-9	6	12	-1	0
7	8	16	-19	-5	-10	-2	-7	7	-4	4	5	-5	-4	4	-7	7	2	10	5	19	-16	-8	-7	0
-1	-17	-8	11	14	4	8	-2	1	-4	5	-11	11	-5	4	-1	2	-8	-4	-14	-11	8	17	1	0
13	5	-5	2	-2	-16	4	-10	-14	17	7	-1	1	-7	-17	14	10	-4	16	2	-2	5	-5	-13	0
-3	-6	-7	10	3	6	6	9	2	-5	-6	-9	9	6	5	-2	-9	-6	-6	-3	-10	7	6	3	0
15	-4	-3	0	7	-18	6	-19	-12	15	16	-3	3	-16	-15	12	19	-6	18	-7	0	3	4	-15	0
-10	-1	8	-5	-2	13	-1	14	17	-20	-11	-2	2	11	20	-17	-14	1	-13	2	5	-8	1	10	0
15	-6	5	-8	9	-18	6	-21	-4	7	18	-3	3	-18	-7	4	21	-6	18	-9	8	-5	6	-15	0
-12	-6	6	-3	3	15	-3	9	15	-18	-6	0	0	6	18	-15	-9	3	-15	-3	3	-6	6	12	0
-4	22	3	-6	-19	1	-13	7	-6	9	-10	16	-16	10	-9	6	-7	13	-1	19	6	-3	-22	4	0
-1	-14	4	-1	11	4	8	1	13	-16	2	-11	11	-2	16	-13	-1	-8	-4	-11	1	-4	14	1	0
-2	13	5	-8	-10	-1	-11	-2	-4	7	-1	14	-14	1	-7	4	2	11	1	10	8	-5	-13	2	0
-5	-6	-12	15	3	8	4	9	-3	0	-6	-7	7	6	0	3	-9	-4	-8	-3	-15	12	6	5	0
1	14	10	-13	-11	-4	-8	-1	1	2	-2	11	-11	2	-2	-1	1	8	4	11	13	-10	-14	-1	0
-7	-11	-14	17	8	10	2	4	-5	2	-1	-5	5	1	-2	5	-4	-2	-10	-8	-17	14	11	7	0
7	11	-11	8	-8	-10	-2	-4	-20	23	1	5	-5	-1	-23	20	4	2	10	8	-8	11	-11	-7	0
-9	0	-13	16	-3	12	0	15	-4	1	-12	-3	3	12	-1	4	-15	0	-12	3	-16	13	0	9	0
9	2	-9	6	1	-12	0	-13	-18	21	10	3	-3	-10	-21	18	13	0	12	-1	-6	9	-2	-9	0
-16	5	2	1	-8	19	-7	20	11	-14	-17	4	-4	17	14	-11	-20	7	-19	8	-1	-2	-5	16	0
9	0	-1	-2	3	-12	0	-15	-10	13	12	3	-3	-12	-13	10	15	0	12	-3	2	1	0	-9	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Program 3

Size-28 and it's loom tables' Difference table

Here's the size-28 square again. The square is neither pair-wise row nor centrally symmetric. Yet each diamond and X tile-pattern sums to $\mathbf{C_{28}}$ continuously.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
1	203	459	715	194	205	468	724	154	214	470	733	156	223	479	693	165	225	488	695	174	234	448	704	176	243	450	713	185	10990
2	779	111	269	434	739	113	278	436	748	122	287	396	750	131	289	405	759	140	249	407	768	142	258	416	777	102	260	425	10990
3	88	589	558	324	97	598	567	333	50	607	569	342	59	616	578	295	68	618	587	304	77	627	540	313	79	636	549	322	10990
4	524	388	14	662	526	348	16	671	535	357	25	673	495	359	34	682	504	368	36	642	506	377	45	651	515	379	5	653	10990
5	218	481	688	167	227	490	697	169	236	443	706	178	245	452	708	187	198	461	717	196	207	463	726	149	216	472	735	158	10990
6	752	133	291	400	761	135	251	409	770	144	253	418	772	104	262	427	781	106	271	429	741	115	280	438	743	124	282	398	10990
7	61	611	580	297	70	620	582	306	72	629	542	315	81	631	551	317	90	591	560	326	92	600	562	335	52	609	571	337	10990
8	497	361	29	684	499	370	38	644	508	372	47	646	517	381	7	655	519	390	9	664	528	350	18	666	537	352	27	675	10990
9	240	454	710	189	200	456	719	191	209	465	728	151	211	474	730	160	220	483	690	162	229	485	699	171	238	445	701	180	10990
10	774	99	264	422	783	108	273	431	736	117	275	440	745	126	284	393	754	128	293	402	763	137	246	411	765	146	255	420	10990
11	83	633	553	319	85	593	555	328	94	602	564	330	54	604	573	339	63	613	575	299	65	622	584	308	74	624	544	310	10990
12	512	383	2	657	521	392	11	659	530	345	20	668	539	354	22	677	492	363	31	686	501	365	40	639	510	374	49	648	10990
13	213	476	732	155	222	478	692	164	231	487	694	173	233	447	703	182	242	449	712	184	202	458	721	193	204	467	723	153	10990
14	747	121	286	395	756	130	288	404	758	139	248	413	767	141	257	415	776	101	266	424	778	110	268	433	738	119	277	435	10990
15	56	606	568	341	58	615	577	301	67	617	586	303	76	626	546	312	78	635	548	321	87	595	557	323	96	597	566	332	10990
16	534	356	24	679	494	358	33	681	503	367	42	641	505	376	44	650	514	385	4	652	523	387	13	661	532	347	15	670	10990
17	235	442	705	177	244	451	714	186	197	460	716	195	206	469	725	148	215	471	734	157	224	480	687	166	226	489	696	175	10990
18	769	143	259	417	771	103	261	426	780	112	270	428	740	114	279	437	749	123	281	397	751	132	290	406	760	134	250	408	10990
19	71	628	541	314	80	637	550	316	89	590	559	325	98	599	561	334	51	608	570	343	60	610	579	296	69	619	588	305	10990
20	507	378	46	645	516	380	6	654	525	389	8	663	527	349	17	672	536	351	26	674	496	360	35	683	498	369	37	643	10990
21	208	464	727	150	217	473	729	159	219	482	689	168	228	484	698	170	237	444	707	179	239	453	709	188	199	462	718	190	10990
22	742	116	274	439	744	125	283	399	753	127	292	401	762	136	252	410	764	145	254	419	773	105	263	421	782	107	272	430	10990
23	93	601	563	336	53	603	572	338	62	612	581	298	64	621	583	307	73	630	543	309	82	632	552	318	91	592	554	327	10990
24	529	344	19	667	538	353	28	676	491	362	30	685	500	371	39	638	509	373	48	647	518	382	1	656	520	391	10	665	10990
25	230	486	700	172	232	446	702	181	241	455	711	183	201	457	720	192	210	466	722	152	212	475	731	161	221	477	691	163	10990
26	757	138	247	412	766	147	256	414	775	100	265	423	784	109	267	432	737	118	276	441	746	120	285	394	755	129	294	403	10990
27	66	623	585	302	75	625	545	311	84	634	547	320	86	594	556	329	95	596	565	331	55	605	574	340	57	614	576	300	10990
28	502	366	41	640	511	375	43	649	513	384	3	658	522	386	12	660	531	346	21	669	533	355	23	678	493	364	32	680	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	
		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990

Here is the size-28 difference-table between its two expansion tables **E(7)** and **E(4)** (neither of which is shown here) as expressed in terms of their minimum integral ratio. It is geonomically equal to **0** – not as obvious as were the Class-4 squares when **b** was an even number. Further, all of its diamond and X tiling patterns sum equally to **0**. So the size-28 square has the cloaking property too. It just takes some calculating in software program Excel™ to certify all that.

V(28) = 50 E(7) - 17 E(4)

	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
131	194	257	-599	97	41	104	81	-56	7	-49	47	-209	-146	631	-106	-243	-299	597	-259	-396	381	444	-293	-549	347	291	-446	0
52	-71	-108	-264	732	-105	-261	-298	579	-258	-414	382	545	-411	-448	229	392	-564	232	195	239	-598	79	42	86	82	45	-111	0
-563	633	277	-160	-716	480	124	-313	83	327	90	-466	-70	174	-63	333	-223	140	-216	180	-376	-13	583	27	-410	-166	430	-126	0
-28	-365	-188	275	-62	315	-222	122	-215	162	-375	88	465	128	-528	-65	312	-25	-562	615	278	-178	-715	462	125	-212	-35	428	0
-124	-180	716	-140	-277	-333	563	-174	-430	466	410	-327	-583	313	376	-480	216	160	223	-633	63	126	70	166	-90	-27	-83	13	0
511	-445	-482	314	358	-479	198	161	205	-632	164	8	171	48	11	-145	18	14	-142	-179	698	-139	-295	-332	664	-292	-329	348	0
-104	259	-97	299	-257	106	-131	146	-291	-47	549	-7	-444	-81	396	-41	-597	599	243	-194	-631	446	209	-347	49	293	56	-381	0
431	94	-443	-99	397	-59	-596	581	244	-93	-749	547	91	-246	-69	394	57	-399	-103	241	-96	281	-256	207	-249	247	-409	54	0
-498	279	342	-514	182	245	189	-548	29	92	36	132	-5	-61	2	-21	-158	-214	682	-55	-311	-248	529	-208	-464	432	495	-361	0
137	133	-23	-60	-16	-20	-176	-213	783	-173	-210	-366	630	-326	-363	433	477	-360	-516	280	324	-513	283	127	290	-666	130	-26	0
-478	-115	362	-75	-512	565	328	-228	-665	412	175	-262	15	378	22	-415	-138	225	-12	265	-172	72	-165	112	-325	38	515	78	0
176	-280	16	360	23	-433	-137	326	-130	366	-290	173	-283	213	-324	20	516	60	-477	-133	363	26	-630	666	210	-127	-783	513	0
-39	-95	-32	64	-192	-129	648	-89	-345	-282	614	-242	-379	398	461	-395	-532	364	308	-429	148	211	155	-582	114	58	121	98	0
596	-241	-397	399	443	-394	-431	246	409	-547	249	93	256	-581	96	59	103	99	-57	-94	69	-54	-91	-247	749	-207	-244	-281	0
-19	344	107	-449	-53	191	-46	231	-206	157	-199	197	-359	4	481	44	-393	-149	447	-109	-546	531	294	-143	-699	497	141	-296	0
-198	179	-358	-14	482	145	-511	-48	329	-8	-664	632	295	-161	-698	479	142	-314	-18	445	-11	-348	-171	292	-164	332	-205	139	0
-413	483	427	-310	-566	330	274	-463	233	177	240	-616	80	24	87	183	-73	-10	-66	30	-226	-163	733	-123	-260	-316	580	-276	0
222	-615	62	25	188	65	28	-128	35	-88	-125	-162	715	-122	-278	-315	562	-275	-312	365	528	-428	-465	212	375	-462	215	178	0
-274	-30	566	10	-427	-183	413	-24	-580	616	260	-177	-733	463	226	-330	66	310	73	-483	-87	276	-80	316	-240	123	-233	163	0
261	-195	-732	564	108	-229	-52	411	-45	-382	-86	258	-79	298	-239	105	-232	264	-392	71	448	111	-545	-82	414	-42	-579	598	0
46	109	53	149	-107	-44	19	-4	-141	-197	699	-157	-294	-231	546	-191	-447	449	393	-344	-481	296	359	-497	199	143	206	-531	0
681	-156	-193	-349	647	-309	-346	331	494	-343	-499	297	341	-496	181	144	307	-649	147	-9	154	31	-6	-43	1	-3	-159	-196	0
-648	429	192	-364	32	395	39	-398	-121	242	-114	282	-155	89	-148	129	-308	-64	532	95	-461	-98	379	-58	-614	582	345	-211	0
-113	383	-273	190	-266	230	-426	37	533	77	-460	-116	380	-76	-613	683	227	-110	-766	530	74	-263	33	377	40	-416	-120	224	0
-328	-265	512	-225	-362	415	478	-378	-515	262	325	-412	165	228	172	-565	12	75	138	115	-22	-78	-15	-38	-175	-112	665	-72	0
426	-530	266	110	273	-683	113	76	120	116	-40	-77	-33	-37	-74	-230	766	-190	-227	-383	613	-224	-380	416	460	-377	-533	263	0
-189	55	-182	214	-342	21	498	61	-495	-132	464	-92	-529	548	311	-245	-682	514	158	-279	-2	361	5	-432	-36	208	-29	248	0
346	9	-647	649	193	-144	-681	496	159	-297	-1	343	6	-331	-154	309	-147	349	-307	156	-181	196	-341	3	499	43	-494	-31	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

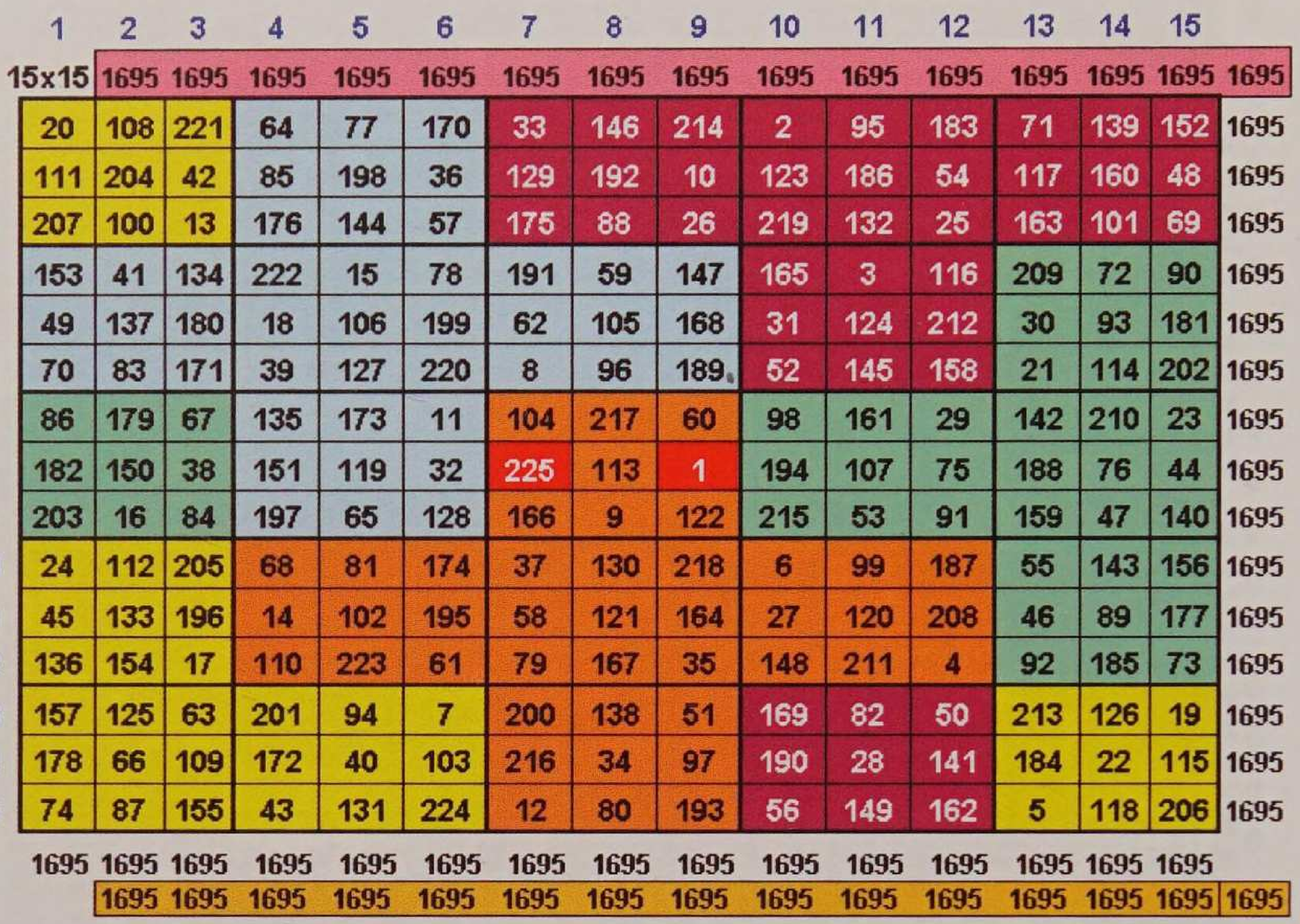
Program 3

For Class-3 Squares

Size **n** = **3b** where **b** is an odd-number **> 3**

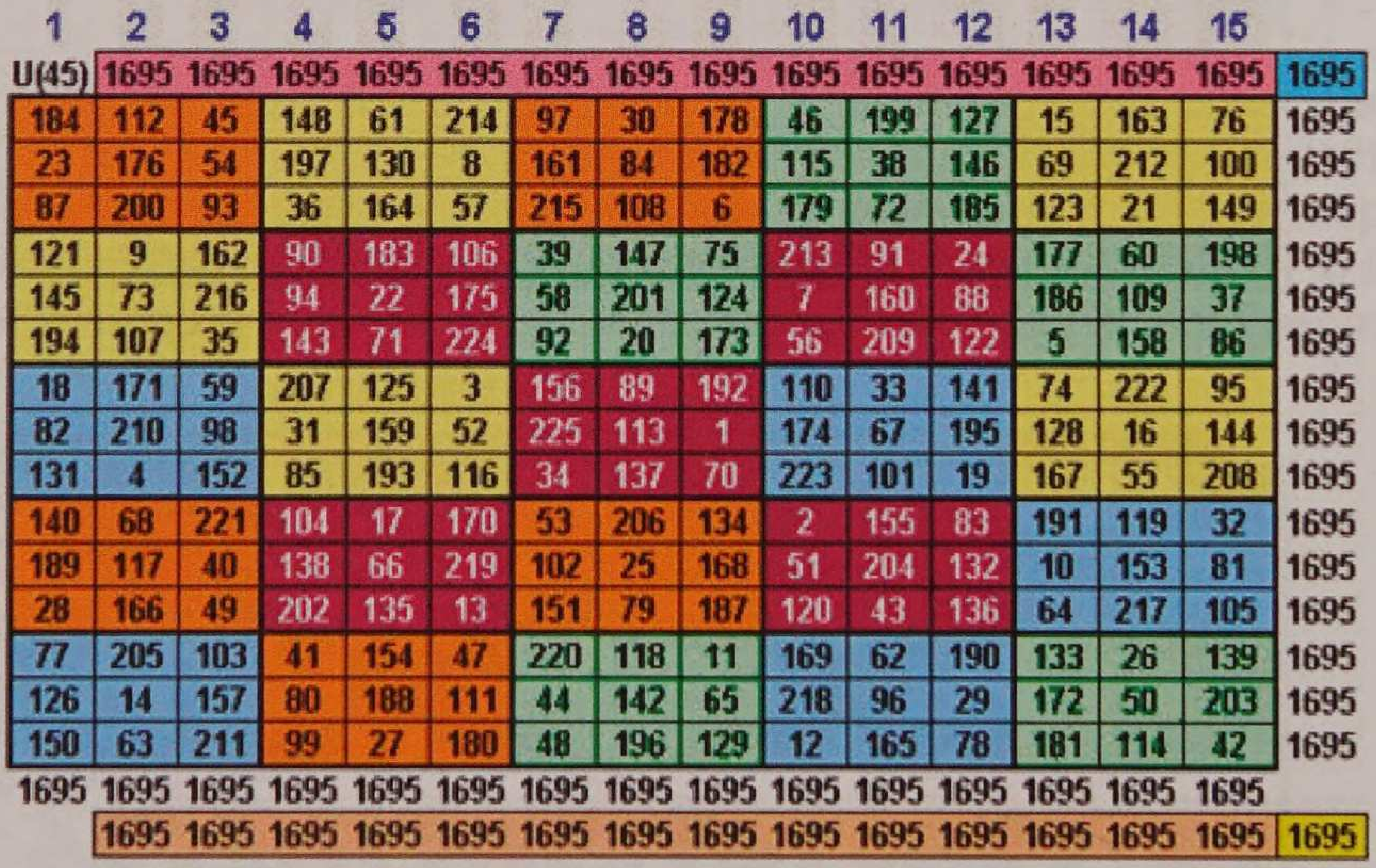
Size-15 block-patterns

Here is the size-15 square. It has **5** **3x3** block-square tiling patterns which sum to **3** times the square's characteristic number. Note that the tiling pattern is identical with the "**+**" pattern of size-5 square but at the **3x3** block-square level.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	15x15	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
1	20	108	221	64	77	170	33	146	214	2	95	183	71	139	152	1695
2	111	204	42	85	198	36	129	192	10	123	186	54	117	160	48	1695
3	207	100	13	176	144	57	175	88	26	219	132	25	163	101	69	1695
4	153	41	134	222	15	78	191	59	147	165	3	116	209	72	90	1695
5	49	137	180	18	106	199	62	105	168	31	124	212	30	93	181	1695
6	70	83	171	39	127	220	8	96	189	52	145	158	21	114	202	1695
7	86	179	67	135	173	11	104	217	60	98	161	29	142	210	23	1695
8	182	150	38	151	119	32	225	113	1	194	107	75	188	76	44	1695
9	203	16	84	197	65	128	166	9	122	215	53	91	159	47	140	1695
10	24	112	205	68	81	174	37	130	218	6	99	187	55	143	156	1695
11	45	133	196	14	102	195	58	121	164	27	120	208	46	89	177	1695
12	136	154	17	110	223	61	79	167	35	148	211	4	92	185	73	1695
13	157	125	63	201	94	7	200	138	51	169	82	50	213	126	19	1695
14	178	66	109	172	40	103	216	34	97	190	28	141	184	22	115	1695
15	74	87	155	43	131	224	12	80	193	56	149	162	5	118	206	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
		1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

The dual square has the same properties. Note that the tiling pattern is identical with the "X" pattern of size-5 square but at the **3x3** block-square level.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
U(45)	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
184	112	45	148	61	214	97	30	178	46	199	127	15	163	76	1695
23	176	54	197	130	8	161	84	182	115	38	146	69	212	100	1695
87	200	93	36	164	57	215	108	6	179	72	185	123	21	149	1695
121	9	162	90	183	106	39	147	75	213	91	24	177	60	198	1695
145	73	216	94	22	175	58	201	124	7	160	88	186	109	37	1695
194	107	35	143	71	224	92	20	173	56	209	122	5	158	86	1695
18	171	59	207	125	3	156	89	192	110	33	141	74	222	95	1695
82	210	98	31	159	52	225	113	1	174	67	195	128	16	144	1695
131	4	152	85	193	116	34	137	70	223	101	19	167	55	208	1695
140	68	221	104	17	170	53	206	134	2	155	83	191	119	32	1695
189	117	40	138	66	219	102	25	168	51	204	132	10	153	81	1695
28	166	49	202	135	13	151	79	187	120	43	136	64	217	105	1695
77	205	103	41	154	47	220	118	11	169	62	190	133	26	139	1695
126	14	157	80	188	111	44	142	65	218	96	29	172	50	203	1695
150	63	211	99	27	180	48	196	129	12	165	78	181	114	42	1695
1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

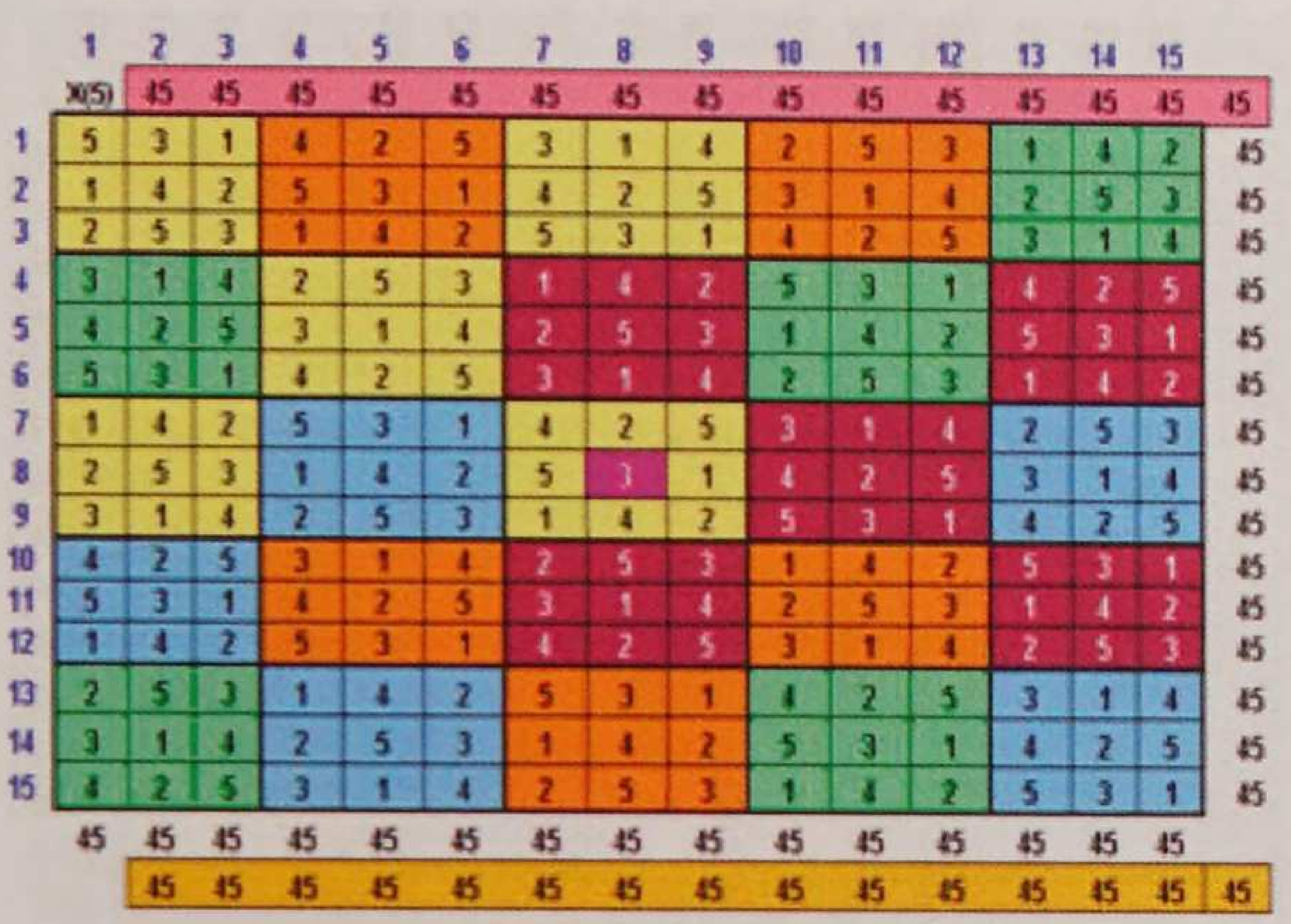
The loom tables were computed using the base **5** instead of **15**. In fact, all Class-3 loom tables must be derived in the base **b** where **n = 3b**, otherwise they will not be geonomic.

(3.14) $W(15) = 5 \cdot (Y(5)-|1|) + X(5)$ for generation of the Primal square

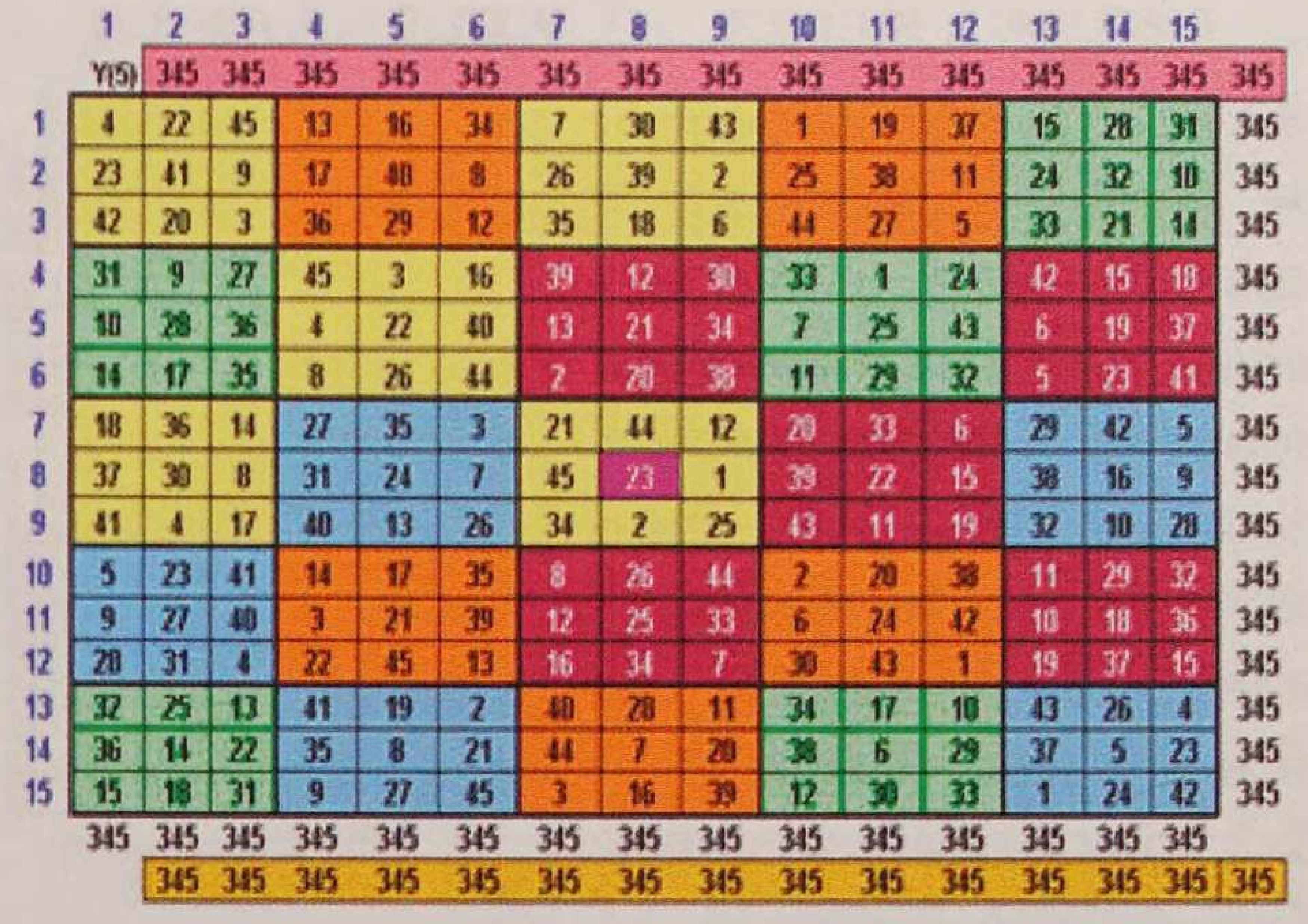
(3.15) $U(15) = 3 \cdot 15 \cdot (X(5)-|1|) + Y(5)$ for generation of the Dual square

Program 3

This slide shows loom tables **X** & **Y** for the size-15 square derived in the base 5.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	X(5)	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
1	5	3	1	4	2	5	3	1	4	2	5	3	1	4	2	45
2	1	4	2	5	3	1	4	2	5	3	1	4	2	5	3	45
3	2	5	3	1	4	2	5	3	1	4	2	5	3	1	4	45
4	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5	45
5	4	2	5	3	1	4	2	5	3	1	4	2	5	3	1	45
6	5	3	1	4	2	5	3	1	4	2	5	3	1	4	2	45
7	1	4	2	5	3	1	4	2	5	3	1	4	2	5	3	45
8	2	5	3	1	4	2	5	3	1	4	2	5	3	1	4	45
9	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5	45
10	4	2	5	3	1	4	2	5	3	1	4	2	5	3	1	45
11	5	3	1	4	2	5	3	1	4	2	5	3	1	4	2	45
12	1	4	2	5	3	1	4	2	5	3	1	4	2	5	3	45
13	2	5	3	1	4	2	5	3	1	4	2	5	3	1	4	45
14	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5	45
15	4	2	5	3	1	4	2	5	3	1	4	2	5	3	1	45
	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	
		45	45	45	45	45	45	45	45	45	45	45	45	45	45	45



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	Y(5)	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345
1	4	22	45	13	16	34	7	30	43	1	19	37	15	28	31	345
2	23	41	9	17	40	8	26	39	2	25	38	11	24	32	10	345
3	42	20	3	36	29	12	35	18	6	44	27	5	33	21	14	345
4	31	9	27	45	3	16	39	12	30	33	1	24	42	15	18	345
5	10	28	36	4	22	40	13	21	34	7	25	43	6	19	37	345
6	14	17	35	8	26	44	2	20	38	11	29	32	5	23	41	345
7	18	36	14	27	35	3	21	44	12	20	33	6	29	42	5	345
8	37	30	8	31	24	7	45	23	1	39	22	15	38	16	9	345
9	41	4	17	40	13	26	34	2	25	43	11	19	32	10	28	345
10	5	23	41	14	17	35	8	26	44	2	20	38	11	29	32	345
11	9	27	40	3	21	39	12	25	33	6	24	42	10	18	36	345
12	28	31	4	22	45	13	16	34	7	30	43	1	19	37	15	345
13	32	25	13	41	19	2	40	28	11	34	17	10	43	26	4	345
14	36	14	22	35	8	21	44	7	20	38	6	29	37	5	23	345
15	15	18	31	9	27	45	3	16	39	12	30	33	1	24	42	345
	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345	
		345	345	345	345	345	345	345	345	345	345	345	345	345	345	345

Here is a version of a difference-table that is geonomically **0.** It was derived here by the formula:

(3.16) $V = \overline{y}X - \overline{x}Y$

where $\overline{x}$ is the average value of **X** and $\overline{y}$ is the average value of **Y** that have been derived in the lowest base number consistent with their size.

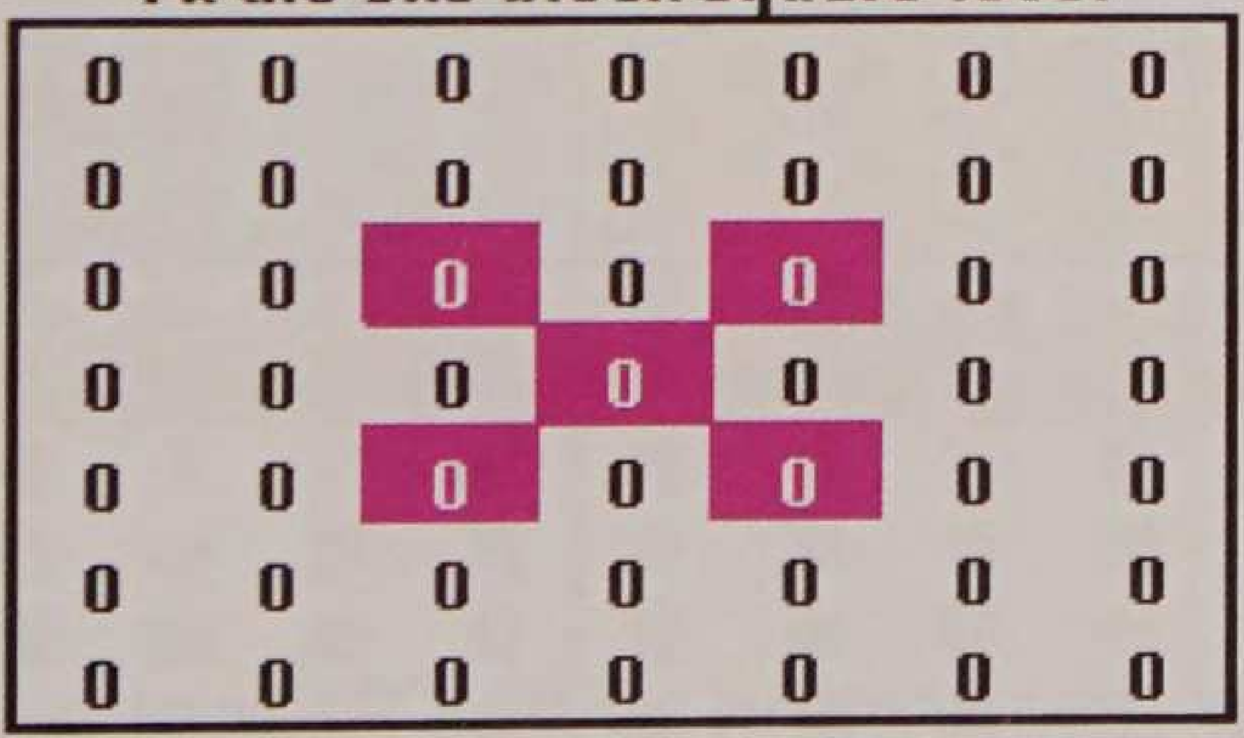
This formula is a more generalized version than the simple difference table **V = X – Y**.

V = 23X-3Y

V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
103	3	-112	53	-2	13	48	-67	-37	43	58	-42	-22	8	-47	0
-46	-31	19	64	-51	-1	14	-71	109	-6	-91	59	-26	19	39	0
-80	55	60	-85	5	10	10	15	5	-40	-35	100	-30	-40	50	0
-24	-4	11	-89	106	21	-94	56	-44	16	66	-49	-34	1	61	0
62	-38	7	57	-43	-28	7	52	-33	2	17	-83	97	12	-88	0
73	18	-82	68	-32	-17	63	-37	-22	13	28	-27	8	23	-77	0
-31	-16	4	34	-36	14	29	-86	79	9	-76	74	-41	-11	54	0
-65	25	45	-70	20	25	-20	0	20	-25	-20	70	-45	-25	65	0
-54	11	41	-74	76	-9	-79	86	-29	-14	36	-34	-4	16	31	0
77	-23	-8	27	-28	-13	22	37	-63	17	32	-68	82	-18	-73	0
88	-12	-97	83	-17	-2	33	-52	-7	28	43	-57	-7	38	-62	0
-61	-1	34	49	-66	-16	44	-56	94	-21	-106	89	-11	4	24	0
-50	40	30	-100	35	40	-5	-15	-10	-10	-5	85	-60	-55	80	0
-39	-19	26	-59	91	6	-109	71	-14	1	51	-64	-19	31	46	0
47	-8	22	42	-58	-43	37	67	-48	-13	2	-53	112	-3	-103	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

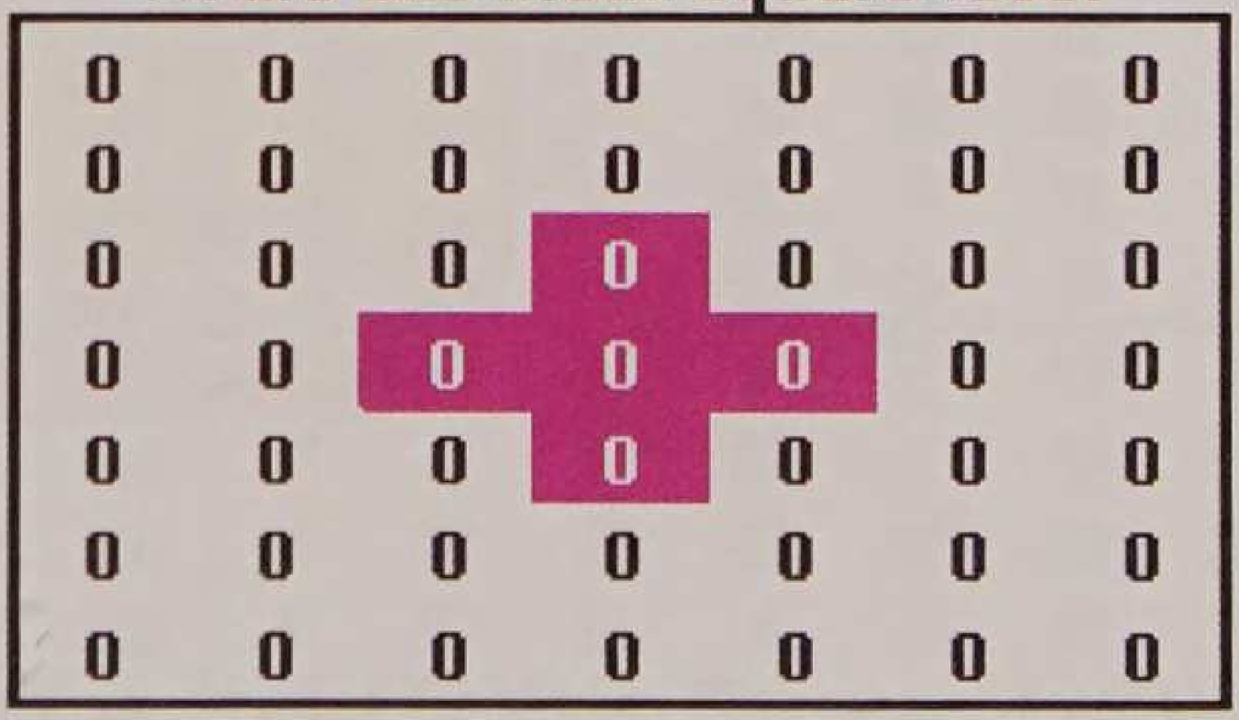
It just so happens that the loom tables of squares whose size is not an arithmetic product of two odd prime numbers have equal loom-table averages. So the averages in formula (**3.16**) are the same and can be factored out, thereby yielding just **V = X – Y**. This is possible because the characteristic number of **V** is always **0** and can be treated as such algebraically when in geonomic form.

At the 3x3 block-square level



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

At the 3x3 block-square level



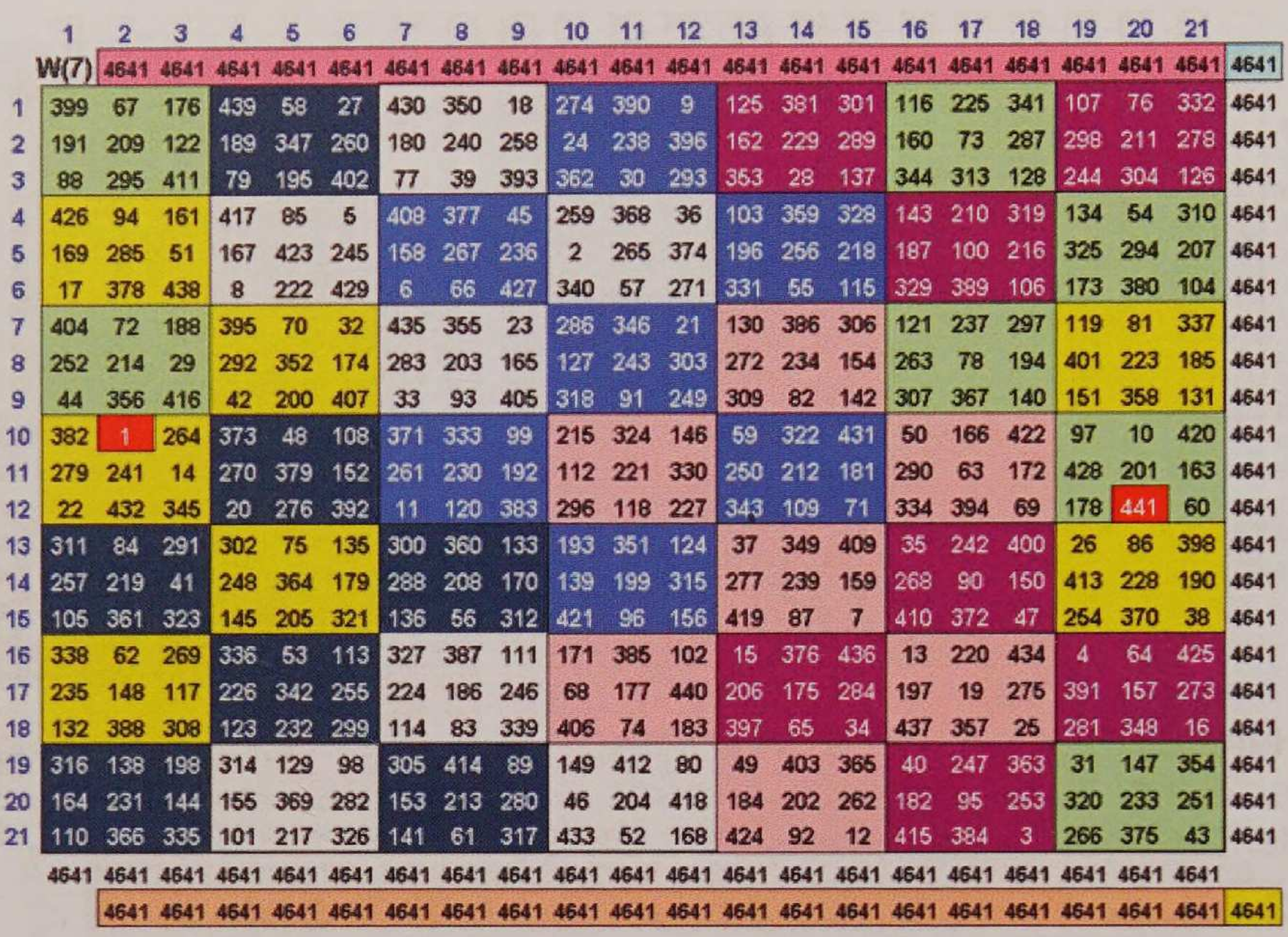
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Although only shown here, this cloaking property holds for all Class-3 squares to follow. Thus, all Class-3 squares possess the cloaking property. You can check them out in Excel™.

Program 3

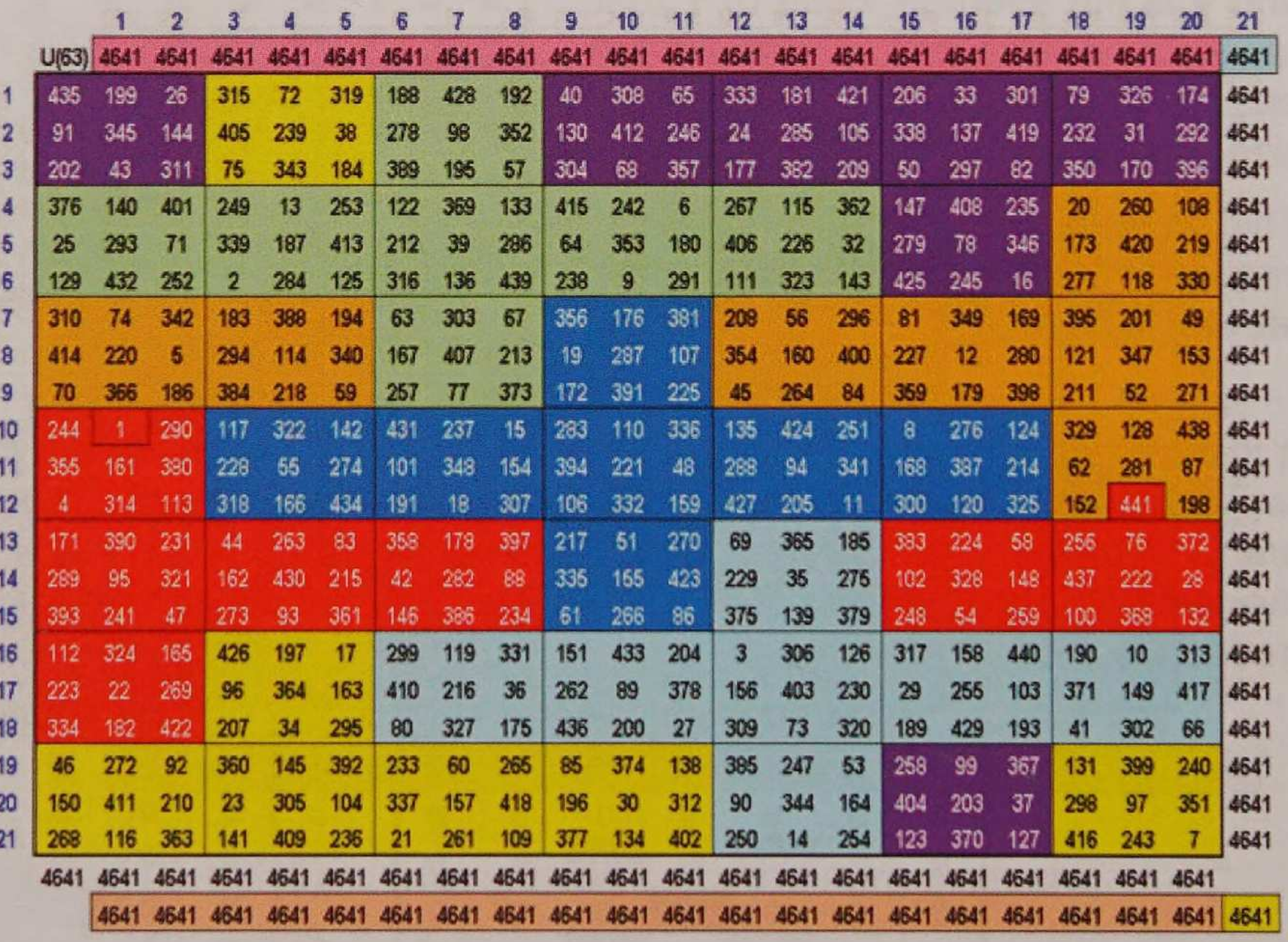
Size-21 block-patterns

Here is the size-21 square. It has **7** 3x3 block-square tiling patterns which sum to **3** times the square's characteristic number. Note that the tiling pattern is identical with the A-pattern of size-7 square but at the 3x3 block-square level. This pattern sums to **3** times the size-21 square's characteristic number continuously.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
	W(7)	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
1	399	67	176	439	58	27	430	350	18	274	390	9	125	381	301	116	225	341	107	76	332	4641
2	191	209	122	189	347	260	180	240	258	24	238	396	162	229	289	160	73	287	298	211	278	4641
3	88	295	411	79	195	402	77	39	393	362	30	293	353	28	137	344	313	128	244	304	126	4641
4	426	94	161	417	85	5	408	377	45	259	368	36	103	359	328	143	210	319	134	54	310	4641
5	169	285	51	167	423	245	158	267	236	2	265	374	196	256	218	187	100	216	325	294	207	4641
6	17	378	438	8	222	429	6	66	427	340	57	271	331	55	115	329	389	106	173	380	104	4641
7	404	72	188	395	70	32	435	355	23	286	346	21	130	386	306	121	237	297	119	81	337	4641
8	252	214	29	292	352	174	283	203	165	127	243	303	272	234	154	263	78	194	401	223	185	4641
9	44	356	416	42	200	407	33	93	405	318	91	249	309	82	142	307	367	140	151	358	131	4641
10	382	1	264	373	48	108	371	333	99	215	324	146	59	322	431	50	166	422	97	10	420	4641
11	279	241	14	270	379	152	261	230	192	112	221	330	250	212	181	290	63	172	428	201	163	4641
12	22	432	345	20	276	392	11	120	383	296	118	227	343	109	71	334	394	69	178	441	60	4641
13	311	84	291	302	75	135	300	360	133	193	351	124	37	349	409	35	242	400	26	86	398	4641
14	257	219	41	248	364	179	288	208	170	139	199	315	277	239	159	268	90	150	413	228	190	4641
15	105	361	323	145	205	321	136	56	312	421	96	156	419	87	7	410	372	47	254	370	38	4641
16	338	62	269	336	53	113	327	387	111	171	385	102	15	376	436	13	220	434	4	64	425	4641
17	235	148	117	226	342	255	224	186	246	68	177	440	206	175	284	197	19	275	391	157	273	4641
18	132	388	308	123	232	299	114	83	339	406	74	183	397	65	34	437	357	25	281	348	16	4641
19	316	138	198	314	129	98	305	414	89	149	412	80	49	403	365	40	247	363	31	147	354	4641
20	164	231	144	155	369	282	153	213	280	46	204	418	184	202	262	182	95	253	320	233	251	4641
21	110	366	335	101	217	326	141	61	317	433	52	168	424	92	12	415	384	3	266	375	43	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	
		4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

Here is the size-21 dual square. The dual version has the same properties as the primal. It is shown with the characteristic B-pattern tiling of the size-7 square in **7** **3x3** block-squares. This pattern sums to **3** times the size-21 square's characteristic number continuously.



		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	U(63)	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
1	435	199	26	315	72	319	188	428	192	40	308	65	333	181	421	206	33	301	79	326	174	4641
2	91	345	144	405	239	38	278	98	352	130	412	246	24	285	105	338	137	419	232	31	292	4641
3	202	43	311	75	343	184	389	195	57	304	68	357	177	382	209	50	297	82	350	170	396	4641
4	376	140	401	249	13	253	122	369	133	415	242	6	267	115	362	147	408	235	20	260	108	4641
5	25	293	71	339	187	413	212	39	286	64	353	180	406	226	32	279	78	346	173	420	219	4641
6	129	432	252	2	284	125	316	136	439	238	9	291	111	323	143	425	245	16	277	118	330	4641
7	310	74	342	183	388	194	63	303	67	356	176	381	208	56	296	81	349	169	395	201	49	4641
8	414	220	5	294	114	340	167	407	213	19	287	107	354	160	400	227	12	280	121	347	153	4641
9	70	366	186	384	218	59	257	77	373	172	391	225	45	264	84	359	179	398	211	52	271	4641
10	244	1	290	117	322	142	431	237	15	283	110	336	135	424	251	8	276	124	329	128	438	4641
11	355	161	380	228	55	274	101	348	154	394	221	48	288	94	341	168	387	214	62	281	87	4641
12	4	314	113	318	166	434	191	18	307	106	332	159	427	205	11	300	120	325	152	441	198	4641
13	171	390	231	44	263	83	358	178	397	217	51	270	69	365	185	383	224	58	256	76	372	4641
14	289	95	321	162	430	215	42	282	88	335	165	423	229	35	276	102	328	148	437	222	28	4641
15	393	241	47	273	93	361	146	386	234	61	266	86	375	139	379	248	54	259	100	368	132	4641
16	112	324	165	426	197	17	299	119	331	151	433	204	3	306	126	317	158	440	190	10	313	4641
17	223	22	269	96	364	163	410	216	36	262	89	378	156	403	230	29	255	103	371	149	417	4641
18	334	182	422	207	34	295	80	327	175	436	200	27	309	73	320	189	429	193	41	302	66	4641
19	46	272	92	360	145	392	233	60	265	85	374	138	385	247	53	258	99	367	131	399	240	4641
20	150	411	210	23	305	104	337	157	418	196	30	312	90	344	164	404	203	37	298	97	351	4641
21	268	116	363	141	409	236	21	261	109	377	134	402	250	14	254	123	370	127	416	243	7	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	
		4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

(3.17) W(21) = 7 x (Y(7)-|1|) + X(7) for generation of the Primal square

(3.18) U(21) = 3 x 21 x (X(7)-|1|) + Y(7) for generation of the Dual square

Program 3

Size-21 Loom Tables

Here are the size-21 loom tables derived from the primal in the base **7**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
X(7)	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84
1	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	84
2	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	84
3	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	84
4	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	84
5	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	84
6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	84
7	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	84
8	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	84
9	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	84
10	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	84
11	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	84
12	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	84
13	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	84
14	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	84
15	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	84
16	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	84
17	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	84
18	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	84
19	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	84
20	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	84
21	5	2	6	3	7	4	1	5	2	6	3	7	4	1	5	2	6	3	7	4	1	84
	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	
	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84

Note that the ratio of the loom tables' centers is **32 / 4= 8**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
Y(7)	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672
1	57	10	26	63	9	4	62	50	3	40	56	2	18	55	43	17	33	49	16	11	48	672
2	28	30	18	27	50	38	26	35	37	4	34	57	24	33	42	23	11	41	43	31	40	672
3	13	43	59	12	28	58	11	6	57	52	5	42	51	4	20	50	45	19	35	44	18	672
4	61	14	23	60	13	1	59	54	7	37	53	6	15	52	47	21	30	46	20	8	45	672
5	25	41	8	24	61	35	23	39	34	1	38	54	28	37	32	27	15	31	47	42	30	672
6	3	54	63	2	32	62	1	10	61	49	9	39	48	8	17	47	56	16	25	55	15	672
7	58	11	27	57	10	5	63	51	4	41	50	3	19	56	44	18	34	43	17	12	49	672
8	36	31	5	42	51	25	41	29	24	19	35	44	39	34	22	38	12	28	58	32	27	672
9	7	51	60	6	29	59	5	14	58	46	13	36	45	12	21	44	53	20	22	52	19	672
10	55	1	38	54	7	16	53	48	15	31	47	21	9	46	62	8	24	61	14	2	60	672
11	40	35	2	39	55	22	38	33	28	16	32	48	36	31	26	42	9	25	62	29	24	672
12	4	62	50	3	40	56	2	18	55	43	17	33	49	16	11	48	57	10	26	63	9	672
13	45	12	42	44	11	20	43	52	19	28	51	18	6	50	59	5	35	58	4	13	57	672
14	37	32	6	36	52	26	42	30	25	20	29	45	40	35	23	39	13	22	59	33	28	672
15	15	52	47	21	30	46	20	8	45	61	14	23	60	13	1	59	54	7	37	53	6	672
16	49	9	39	48	8	17	47	56	16	25	55	15	3	54	63	2	32	62	1	10	61	672
17	34	22	17	33	49	37	32	27	36	10	26	63	30	25	41	29	3	40	56	23	39	672
18	19	56	44	18	34	43	17	12	49	58	11	27	57	10	5	63	51	4	41	50	3	672
19	46	20	29	45	19	14	44	60	13	22	59	12	7	58	53	6	36	52	5	21	51	672
20	24	33	21	23	53	41	22	31	40	7	30	60	27	29	38	26	14	37	46	34	36	672
21	16	53	48	15	31	47	21	9	46	62	8	24	61	14	2	60	55	1	38	54	7	672
	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	
	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672	672

Program 3

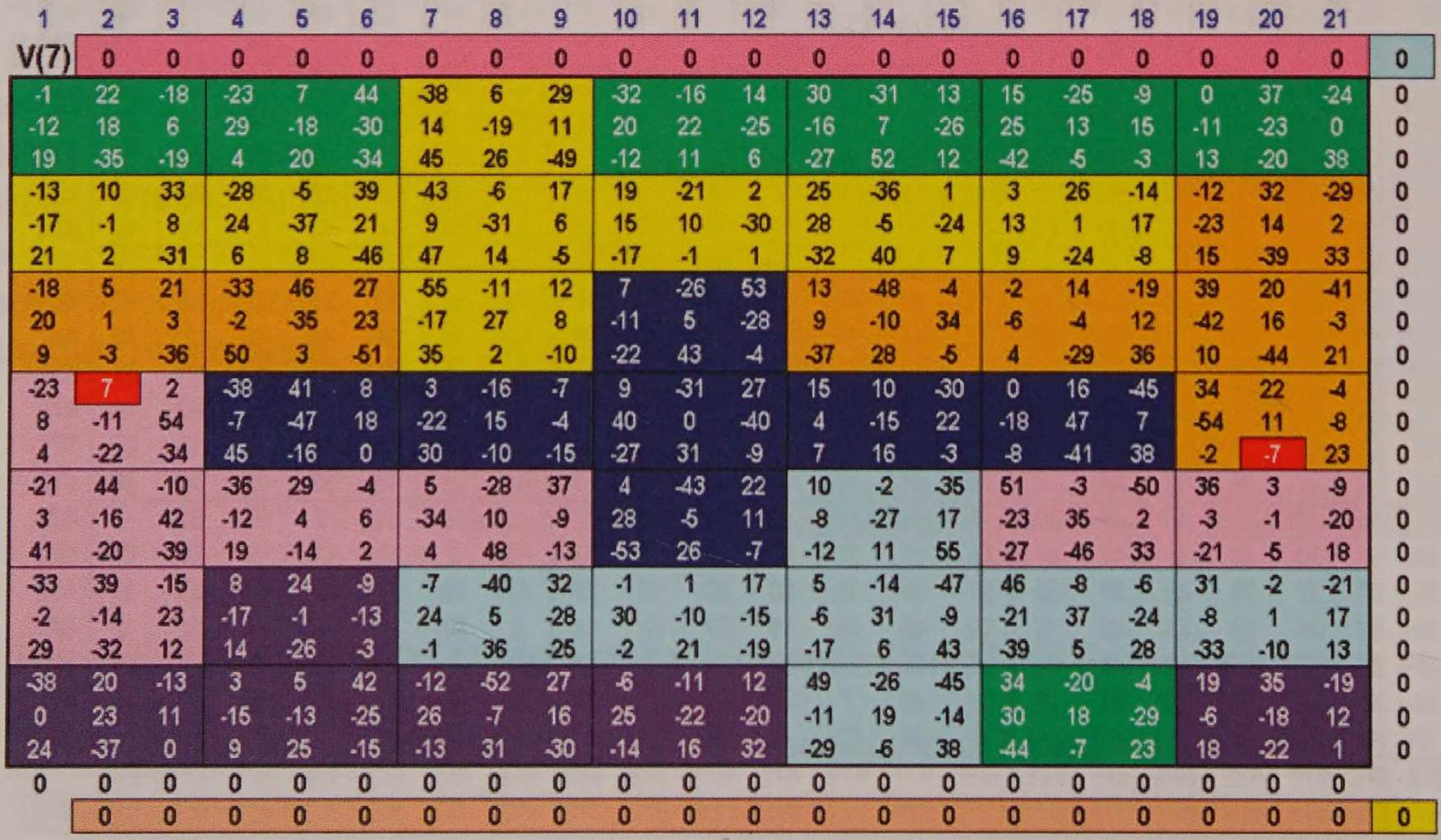
Size-21 Loom Table Difference Table

V(7) = 8X(7) - Y(7)

Here is the loom-table difference table. The number **8** in the formula comes from **the ratio of the loom tables' centers.**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
V(7)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	-1	22	-18	-23	7	44	-38	6	29	-32	-16	14	30	-31	13	15	-25	-9	0	37	-24	0
	-12	18	6	29	-18	-30	14	-19	11	20	22	-25	-16	7	-26	25	13	15	-11	-23	0	0
	19	-35	-19	4	20	-34	45	26	-49	-12	11	6	-27	52	12	-42	-5	-3	13	-20	38	0
	-13	10	33	-28	-5	39	-43	-6	17	19	-21	2	25	-36	1	3	26	-14	-12	32	-29	0
	-17	-1	8	24	-37	21	9	-31	6	15	10	-30	28	-5	-24	13	1	17	-23	14	2	0
	21	2	-31	6	8	-46	47	14	-5	-17	-1	1	-32	40	7	9	-24	-8	15	-39	33	0
	-18	5	21	-33	46	27	-55	-11	12	7	-26	53	13	-48	-4	-2	14	-19	39	20	-41	0
	20	1	3	-2	-35	23	-17	27	8	-11	5	-28	9	-10	34	-6	-4	12	-42	16	-3	0
	9	-3	-36	50	3	-51	35	2	-10	-22	43	-4	-37	28	-5	4	-29	36	10	-44	21	0
	-23	7	2	-38	41	8	3	-16	-7	9	-31	27	15	10	-30	0	16	-45	34	22	-4	0
	8	-11	54	-7	-47	18	-22	15	-4	40	0	-40	4	-15	22	-18	47	7	-54	11	-8	0
	4	-22	-34	45	-16	0	30	-10	-15	-27	31	-9	7	16	-3	-8	-41	38	-2	-7	23	0
	-21	44	-10	-36	29	-4	5	-28	37	4	-43	22	10	-2	-35	51	-3	-50	36	3	-9	0
	3	-16	42	-12	4	6	-34	10	-9	28	-5	11	-8	-27	17	-23	35	2	-3	-1	-20	0
	41	-20	-39	19	-14	2	4	48	-13	-53	26	-7	-12	11	55	-27	-46	33	-21	-5	18	0
	-33	39	-15	8	24	-9	-7	-40	32	-1	1	17	5	-14	-47	46	-8	-6	31	-2	-21	0
	-2	-14	23	-17	-1	-13	24	5	-28	30	-10	-15	-6	31	-9	-21	37	-24	-8	1	17	0
	29	-32	12	14	-26	-3	-1	36	-25	-2	21	-19	-17	6	43	-39	5	28	-33	-10	13	0
	-38	20	-13	3	5	42	-12	-52	27	-6	-11	12	49	-26	-45	34	-20	-4	19	35	-19	0
	0	23	11	-15	-13	-25	26	-7	16	25	-22	-20	-11	19	-14	30	18	-29	-6	-18	12	0
	24	-37	0	9	25	-15	-13	31	-30	-14	16	32	-29	-6	38	-44	-7	23	18	-22	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The difference table is continuously **7x**-modular at **0**. It is also totally anti-symmetric. The **3x3** size-7 tile patterns A and B sum to **0** continuously.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
V(7)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	-1	22	-18	-23	7	44	-38	6	29	-32	-16	14	30	-31	13	15	-25	-9	0	37	-24	0
	-12	18	6	29	-18	-30	14	-19	11	20	22	-25	-16	7	-26	25	13	15	-11	-23	0	0
	19	-35	-19	4	20	-34	45	26	-49	-12	11	6	-27	52	12	-42	-5	-3	13	-20	38	0
	-13	10	33	-28	-5	39	-43	-6	17	19	-21	2	25	-36	1	3	26	-14	-12	32	-29	0
	-17	-1	8	24	-37	21	9	-31	6	15	10	-30	28	-5	-24	13	1	17	-23	14	2	0
	21	2	-31	6	8	-46	47	14	-5	-17	-1	1	-32	40	7	9	-24	-8	15	-39	33	0
	-18	5	21	-33	46	27	-55	-11	12	7	-26	53	13	-48	-4	-2	14	-19	39	20	-41	0
	20	1	3	-2	-35	23	-17	27	8	-11	5	-28	9	-10	34	-6	-4	12	-42	16	-3	0
	9	-3	-36	50	3	-51	35	2	-10	-22	43	-4	-37	28	-5	4	-29	36	10	-44	21	0
	-23	7	2	-38	41	8	3	-16	-7	9	-31	27	15	10	-30	0	16	-45	34	22	-4	0
	8	-11	54	-7	-47	18	-22	15	-4	40	0	-40	4	-15	22	-18	47	7	-54	11	-8	0
	4	-22	-34	45	-16	0	30	-10	-15	-27	31	-9	7	16	-3	-8	-41	38	-2	-7	23	0
	-21	44	-10	-36	29	-4	5	-28	37	4	-43	22	10	-2	-35	51	-3	-50	36	3	-9	0
	3	-16	42	-12	4	6	-34	10	-9	28	-5	11	-8	-27	17	-23	35	2	-3	-1	-20	0
	41	-20	-39	19	-14	2	4	48	-13	-53	26	-7	-12	11	55	-27	-46	33	-21	-5	18	0
	-33	39	-15	8	24	-9	-7	-40	32	-1	1	17	5	-14	-47	46	-8	-6	31	-2	-21	0
	-2	-14	23	-17	-1	-13	24	5	-28	30	-10	-15	-6	31	-9	-21	37	-24	-8	1	17	0
	29	-32	12	14	-26	-3	-1	36	-25	-2	21	-19	-17	6	43	-39	5	28	-33	-10	13	0
	-38	20	-13	3	5	42	-12	-52	27	-6	-11	12	49	-26	-45	34	-20	-4	19	35	-19	0
	0	23	11	-15	-13	-25	26	-7	16	25	-22	-20	-11	19	-14	30	18	-29	-6	-18	12	0
	24	-37	0	9	25	-15	-13	31	-30	-14	16	32	-29	-6	38	-44	-7	23	18	-22	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

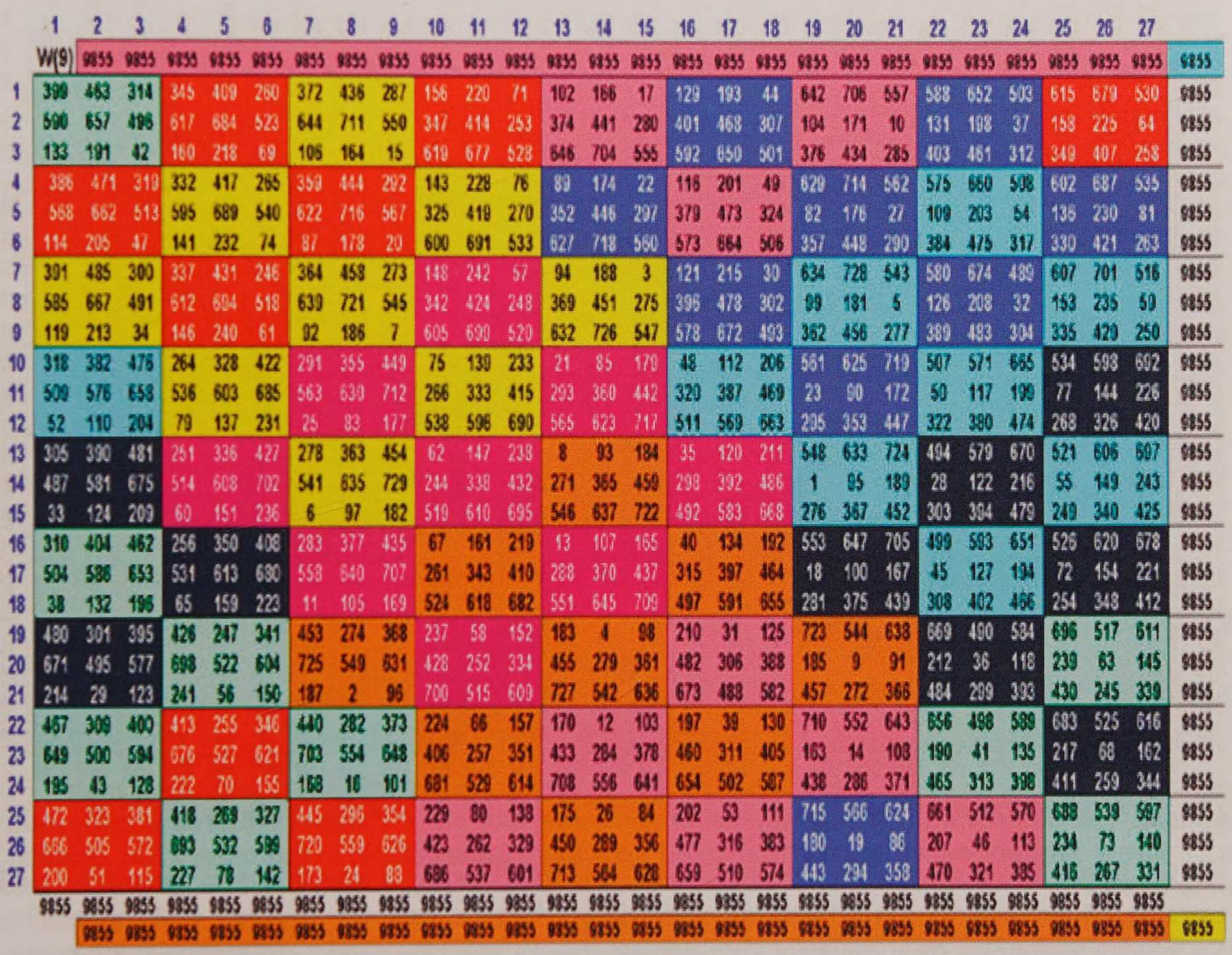
Program 3

Size-27 block-patterns

Here is the size-27 square. It has **9 3x3** block-square tiling patterns which sum to **3** times the square's characteristic number.

Note that the tiling pattern here is new because the size-9 square is not a Class-1 square like the size-5 and size-7 squares which have characteristic tiling patterns.

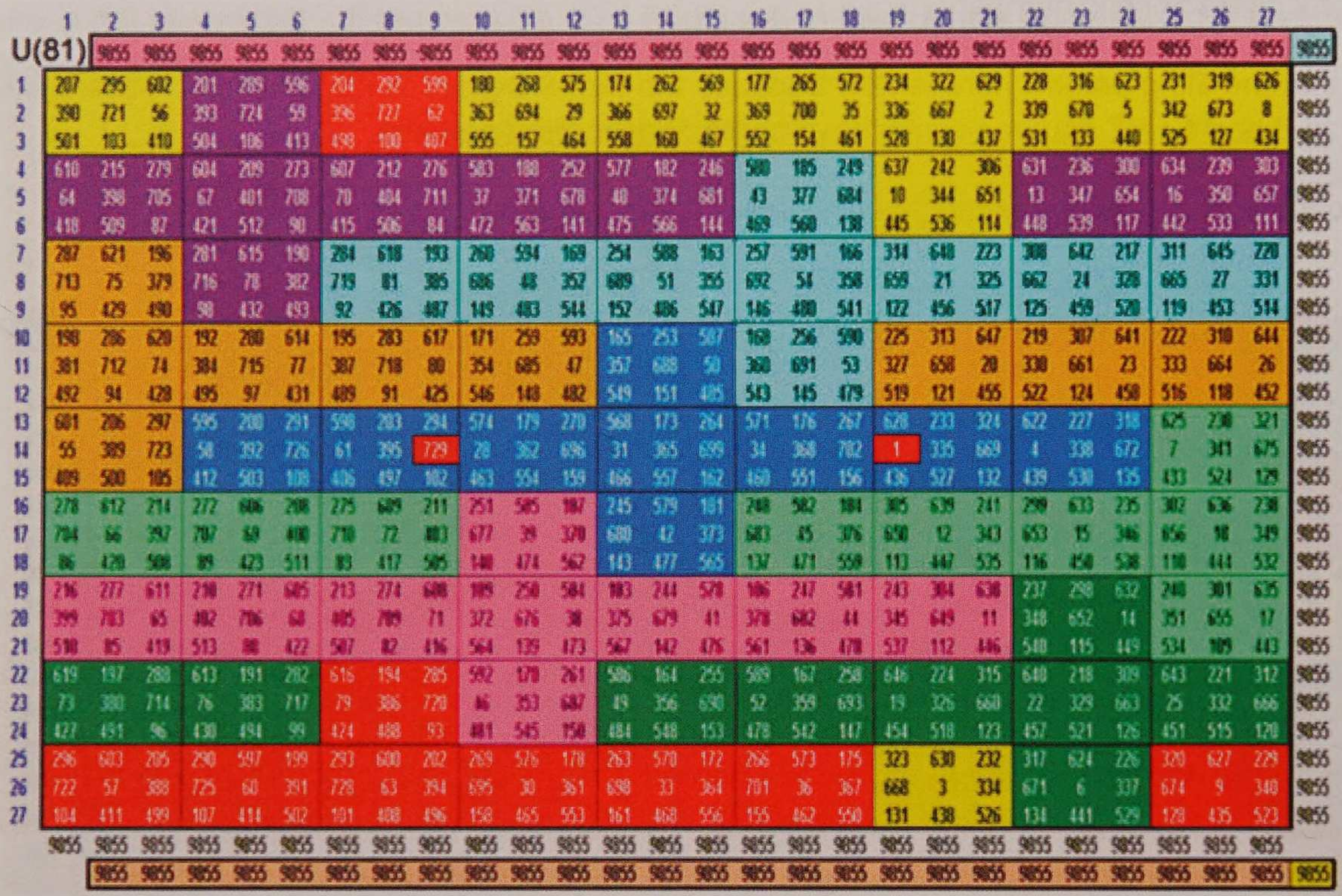
The loom tables were computed using the base **9** instead of **27**.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
W(9)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	399	463	314	345	409	260	372	436	287	156	220	71	102	166	17	129	193	44	642	706	557	588	652	503	615	679	530	9855
2	590	657	496	617	684	523	644	711	550	347	414	253	374	441	280	401	468	307	104	171	10	131	198	37	158	225	64	9855
3	133	191	42	160	218	69	106	164	15	619	677	528	646	704	555	592	650	501	376	434	285	403	461	312	349	407	258	9855
4	386	471	319	332	417	265	359	444	292	143	228	76	89	174	22	116	201	49	629	714	562	575	660	508	602	687	535	9855
5	568	662	513	595	689	540	622	716	567	325	419	270	352	446	297	379	473	324	82	176	27	109	203	54	136	230	81	9855
6	114	205	47	141	232	74	87	178	20	600	691	533	627	718	560	573	664	506	357	448	290	384	475	317	330	421	263	9855
7	391	485	300	337	431	246	364	458	273	148	242	57	94	188	3	121	215	30	634	728	543	580	674	489	607	701	516	9855
8	585	667	491	612	694	518	639	721	545	342	424	248	369	451	275	396	478	302	99	181	5	126	208	32	153	235	59	9855
9	119	213	34	146	240	61	92	186	7	605	699	520	632	726	547	578	672	493	362	456	277	389	483	304	335	429	250	9855
10	318	382	476	264	328	422	291	355	449	75	139	233	21	85	179	48	112	206	561	625	719	507	571	665	534	598	692	9855
11	509	576	658	536	603	685	563	630	712	266	333	415	293	360	442	320	387	469	23	90	172	50	117	199	77	144	226	9855
12	52	110	204	79	137	231	25	83	177	538	596	690	565	623	717	511	569	663	295	353	447	322	380	474	268	326	420	9855
13	305	390	481	251	336	427	278	363	454	62	147	238	8	93	184	35	120	211	548	633	724	494	579	670	521	606	697	9855
14	487	581	675	514	608	702	541	635	729	244	338	432	271	365	459	298	392	486	1	95	189	28	122	216	55	149	243	9855
15	33	124	209	60	151	236	6	97	182	519	610	695	546	637	722	492	583	668	276	367	452	303	394	479	249	340	425	9855
16	310	404	462	256	350	408	283	377	435	67	161	219	13	107	165	40	134	192	553	647	705	499	593	651	526	620	678	9855
17	504	586	653	531	613	680	558	640	707	261	343	410	288	370	437	315	397	464	18	100	167	45	127	194	72	154	221	9855
18	38	132	196	65	159	223	11	105	169	524	618	682	551	645	709	497	591	655	281	375	439	308	402	466	254	348	412	9855
19	480	301	395	426	247	341	453	274	368	237	58	152	183	4	98	210	31	125	723	544	638	669	490	584	696	517	611	9855
20	671	495	577	698	522	604	725	549	631	428	252	334	455	279	361	482	306	388	185	9	91	212	36	118	239	63	145	9855
21	214	29	123	241	56	150	187	2	96	700	515	609	727	542	636	673	488	582	457	272	366	484	299	393	430	245	339	9855
22	467	309	400	413	255	346	440	282	373	224	66	157	170	12	103	197	39	130	710	552	643	656	498	589	683	525	616	9855
23	649	500	594	676	527	621	703	554	648	406	257	351	433	284	378	460	311	405	163	14	108	190	41	135	217	68	162	9855
24	195	43	128	222	70	155	168	16	101	681	529	614	708	556	641	654	502	587	438	286	371	465	313	398	411	259	344	9855
25	472	323	381	418	269	327	445	296	354	229	80	138	175	26	84	202	53	111	715	566	624	661	512	570	688	539	597	9855
26	666	505	572	693	532	599	720	559	626	423	262	329	450	289	356	477	316	383	180	19	86	207	46	113	234	73	140	9855
27	200	51	115	227	78	142	173	24	88	686	537	601	713	564	628	659	510	574	443	294	358	470	321	385	416	267	331	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Program 3

Here is the dual of the size-27 square. It has the same tiling properties as the primal version. It is shown with another new characteristic tiling pattern at the **3x3** block-square level. This pattern too sums continuously to **3** times the size-27 square's characteristic number.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
U(81)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	207	295	602	201	289	596	204	292	599	180	268	575	174	262	569	177	265	572	234	322	629	228	316	623	231	319	626	9855
2	390	721	56	393	724	59	396	727	62	363	694	29	366	697	32	369	700	35	336	667	2	339	670	5	342	673	8	9855
3	501	103	410	504	106	413	498	100	407	555	157	464	558	160	467	552	154	461	528	130	437	531	133	440	525	127	434	9855
4	610	215	279	604	209	273	607	212	276	583	188	252	577	182	246	580	185	249	637	242	306	631	236	300	634	239	303	9855
5	64	398	705	67	401	708	70	404	711	37	371	678	40	374	681	43	377	684	10	344	651	13	347	654	16	350	657	9855
6	418	509	87	421	512	90	415	506	84	472	563	141	475	566	144	469	560	138	445	536	114	448	539	117	442	533	111	9855
7	287	621	196	281	615	190	284	618	193	260	594	169	254	588	163	257	591	166	314	648	223	308	642	217	311	645	220	9855
8	713	75	379	716	78	382	719	81	385	686	48	352	689	51	355	692	54	358	659	21	325	662	24	328	665	27	331	9855
9	95	429	490	98	432	493	92	426	487	149	483	544	152	486	547	146	480	541	122	456	517	125	459	520	119	453	514	9855
10	198	286	620	192	280	614	195	283	617	171	259	593	165	253	587	168	256	590	225	313	647	219	307	641	222	310	644	9855
11	381	712	74	384	715	77	387	718	80	354	685	47	357	688	50	360	691	53	327	658	20	330	661	23	333	664	26	9855
12	492	94	428	495	97	431	489	91	425	546	148	482	549	151	485	543	145	479	519	121	455	522	124	458	516	118	452	9855
13	601	206	297	595	200	291	598	203	294	574	179	270	568	173	264	571	176	267	628	233	324	622	227	318	625	230	321	9855
14	55	389	723	58	392	726	61	395	729	28	362	696	31	365	699	34	368	702	1	335	669	4	338	672	7	341	675	9855
15	409	500	105	412	503	108	406	497	102	463	554	159	466	557	162	460	551	156	436	527	132	439	530	135	433	524	129	9855
16	278	612	214	272	606	208	275	609	211	251	585	187	245	579	181	248	582	184	305	639	241	299	633	235	302	636	238	9855
17	704	66	397	707	69	400	710	72	403	677	39	370	680	42	373	683	45	376	650	12	343	653	15	346	656	18	349	9855
18	86	420	508	89	423	511	83	417	505	140	474	562	143	477	565	137	471	559	113	447	535	116	450	538	110	444	532	9855
19	216	277	611	210	271	605	213	274	608	189	250	584	183	244	578	186	247	581	243	304	638	237	298	632	240	301	635	9855
20	399	703	65	402	706	68	405	709	71	372	676	38	375	679	41	378	682	44	345	649	11	348	652	14	351	655	17	9855
21	510	85	419	513	88	422	507	82	416	564	139	473	567	142	476	561	136	470	537	112	446	540	115	449	534	109	443	9855
22	619	197	288	613	191	282	616	194	285	592	170	261	586	164	255	589	167	258	646	224	315	640	218	309	643	221	312	9855
23	73	380	714	76	383	717	79	386	720	46	353	687	49	356	690	52	359	693	19	326	660	22	329	663	25	332	666	9855
24	427	491	96	430	494	99	424	488	93	481	545	150	484	548	153	478	542	147	454	518	123	457	521	126	451	515	120	9855
25	296	603	205	290	597	199	293	600	202	269	576	178	263	570	172	266	573	175	323	630	232	317	624	226	320	627	229	9855
26	722	57	388	725	60	391	728	63	394	695	30	361	698	33	364	701	36	367	668	3	334	671	6	337	674	9	340	9855
27	104	411	499	107	414	502	101	408	496	158	465	553	161	468	556	155	462	550	131	438	526	134	441	529	128	435	523	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

(3.19) **W(27) = 9 x (Y(9)-|1|) + X(9)** for generation of the Primal square

(3.20) **U(27) = 3 x 27 x (X(9)-|1|) + Y(9)** for generation of the Dual square

Remark: The size-27 square is a special case among the other Class-3 squares shown thus far. It has a different dual that can be derived from loom tables taken to the base **27** in addition to the base **9**.

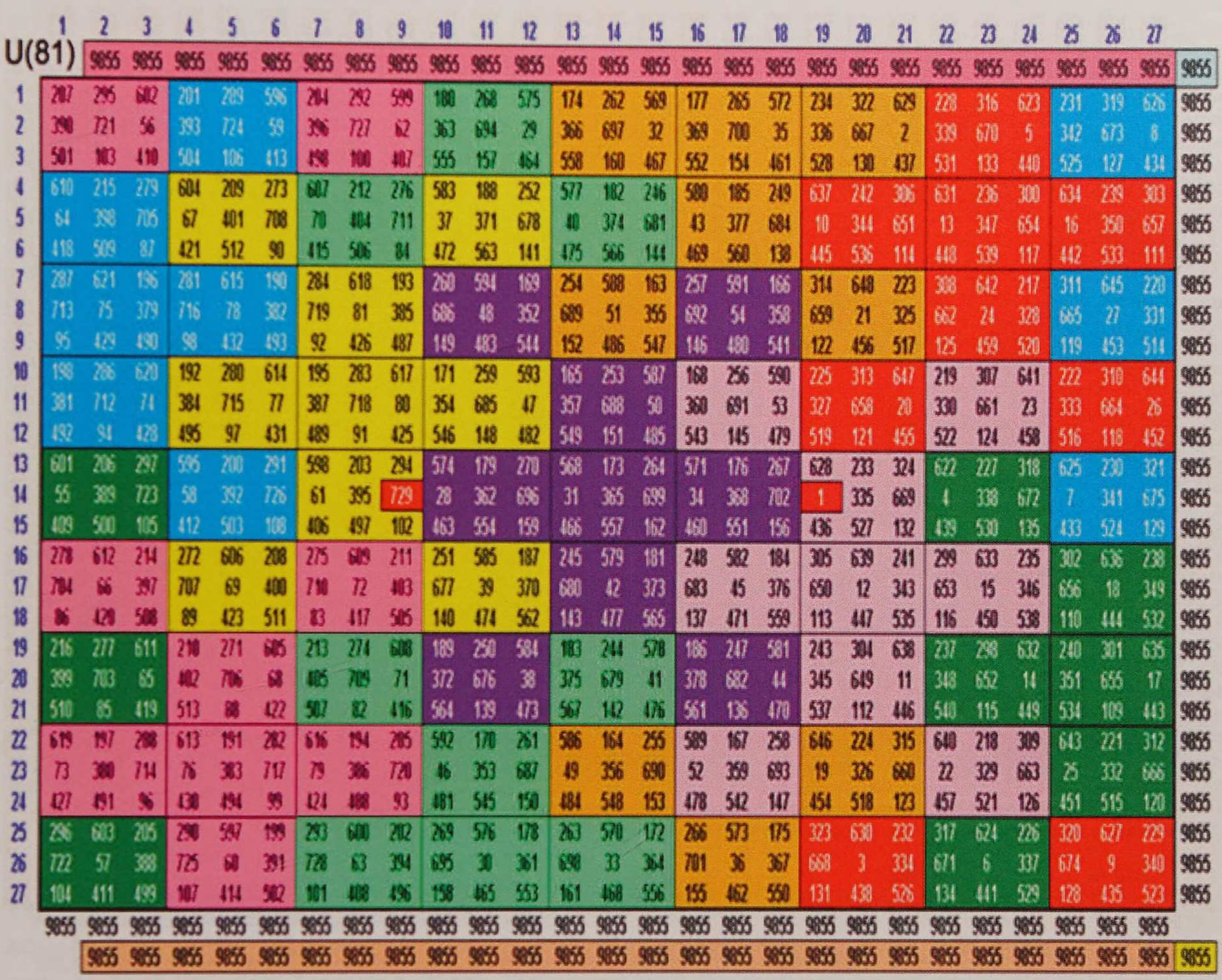
Those loom tables are geonomically ultra-perfect too. Further their loom difference table sums to **0** everywhere. So do all the block-tile patterns shown here. Consequently it possesses the cloaking property.

Since this was the only exception found in the range of Class-3 squares investigated, the size **27** square would be better treated as a square from Class-6, which it will be henceforth, where $\mathbf{n = b^a}$, **b = 3** and **a = 3**.

Program 3

The third tiling pattern on the size-27 dual square

Here is yet a third characteristic tiling pattern for the size-27 Class-3 square shown here on the dual square. Again, the **3x3** block-square tiles in this tiling pattern also sum to 3 times the square's characteristic number.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
U(81)	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
1	207	295	602	201	289	596	204	292	599	180	268	575	174	262	569	177	265	572	234	322	629	228	316	623	231	319	626	9855
2	390	721	56	393	724	59	396	727	62	363	694	29	366	697	32	369	700	35	336	667	2	339	670	5	342	673	8	9855
3	501	103	410	504	106	413	498	100	407	555	157	464	558	160	467	552	154	461	528	130	437	531	133	440	525	127	434	9855
4	610	215	279	604	209	273	607	212	276	583	188	252	577	182	246	580	185	249	637	242	306	631	236	300	634	239	303	9855
5	64	398	705	67	401	708	70	404	711	37	371	678	40	374	681	43	377	684	10	344	651	13	347	654	16	350	657	9855
6	418	509	87	421	512	90	415	506	84	472	563	141	475	566	144	469	560	138	445	536	114	448	539	117	442	533	111	9855
7	287	621	196	281	615	190	284	618	193	260	594	169	254	588	163	257	591	166	314	648	223	308	642	217	311	645	220	9855
8	713	75	379	716	78	382	719	81	385	686	48	352	689	51	355	692	54	358	659	21	325	662	24	328	665	27	331	9855
9	95	429	490	98	432	493	92	426	487	149	483	544	152	486	547	146	480	541	122	456	517	125	459	520	119	453	514	9855
10	198	286	620	192	280	614	195	283	617	171	259	593	165	253	587	168	256	590	225	313	647	219	307	641	222	310	644	9855
11	381	712	74	384	715	77	387	718	80	354	685	47	357	688	50	360	691	53	327	658	20	330	661	23	333	664	26	9855
12	492	94	428	495	97	431	489	91	425	546	148	482	549	151	485	543	145	479	519	121	455	522	124	458	516	118	452	9855
13	601	206	297	595	200	291	598	203	294	574	179	270	568	173	264	571	176	267	628	233	324	622	227	318	625	230	321	9855
14	55	389	723	58	392	726	61	395	729	28	362	696	31	365	699	34	368	702	1	335	669	4	338	672	7	341	675	9855
15	409	500	105	412	503	108	406	497	102	463	554	159	466	557	162	460	551	156	436	527	132	439	530	135	433	524	129	9855
16	278	612	214	272	606	208	275	609	211	251	585	187	245	579	181	248	582	184	305	639	241	299	633	235	302	636	238	9855
17	704	66	397	707	69	400	710	72	403	677	39	370	680	42	373	683	45	376	650	12	343	653	15	346	656	18	349	9855
18	86	420	508	89	423	511	83	417	505	140	474	562	143	477	565	137	471	559	113	447	535	116	450	538	110	444	532	9855
19	216	277	611	210	271	605	213	274	608	189	250	584	183	244	578	186	247	581	243	304	638	237	298	632	240	301	635	9855
20	399	703	65	402	706	68	405	709	71	372	676	38	375	679	41	378	682	44	345	649	11	348	652	14	351	655	17	9855
21	510	85	419	513	88	422	507	82	416	564	139	473	567	142	476	561	136	470	537	112	446	540	115	449	534	109	443	9855
22	619	197	288	613	191	282	616	194	285	592	170	261	586	164	255	589	167	258	646	224	315	640	218	309	643	221	312	9855
23	73	380	714	76	383	717	79	386	720	46	353	687	49	356	690	52	359	693	19	326	660	22	329	663	25	332	666	9855
24	427	491	96	430	494	99	424	488	93	481	545	150	484	548	153	478	542	147	454	518	123	457	521	126	451	515	120	9855
25	296	603	205	290	597	199	293	600	202	269	576	178	263	570	172	266	573	175	323	630	232	317	624	226	320	627	229	9855
26	722	57	388	725	60	391	728	63	394	695	30	361	698	33	364	701	36	367	668	3	334	671	6	337	674	9	340	9855
27	104	411	499	107	414	502	101	408	496	158	465	553	161	468	556	155	462	550	131	438	526	134	441	529	128	435	523	9855
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

This instance where there is more than just two distinct simultaneously-occurring complementary tiling patterns for one size of square and its dual is not antithetical to squares' pairwise complementary tiling patterns because the size-27 square is **3x**-modular while all the other Class-3 squares shown here of size **3b** are not **bx**-modular. This means that you could pick out any **nine 3x3** block-squares from anywhere in the size-27 table and that pattern would sum to **3** times the square's characteristic number continuously.

This pattern is not true-to-form because the size-9 square possesses no inherent unique tiling patterns. So actually, all tiling patterns for squares of any class are only pair-complementary, if they possess tiling patterns at all.

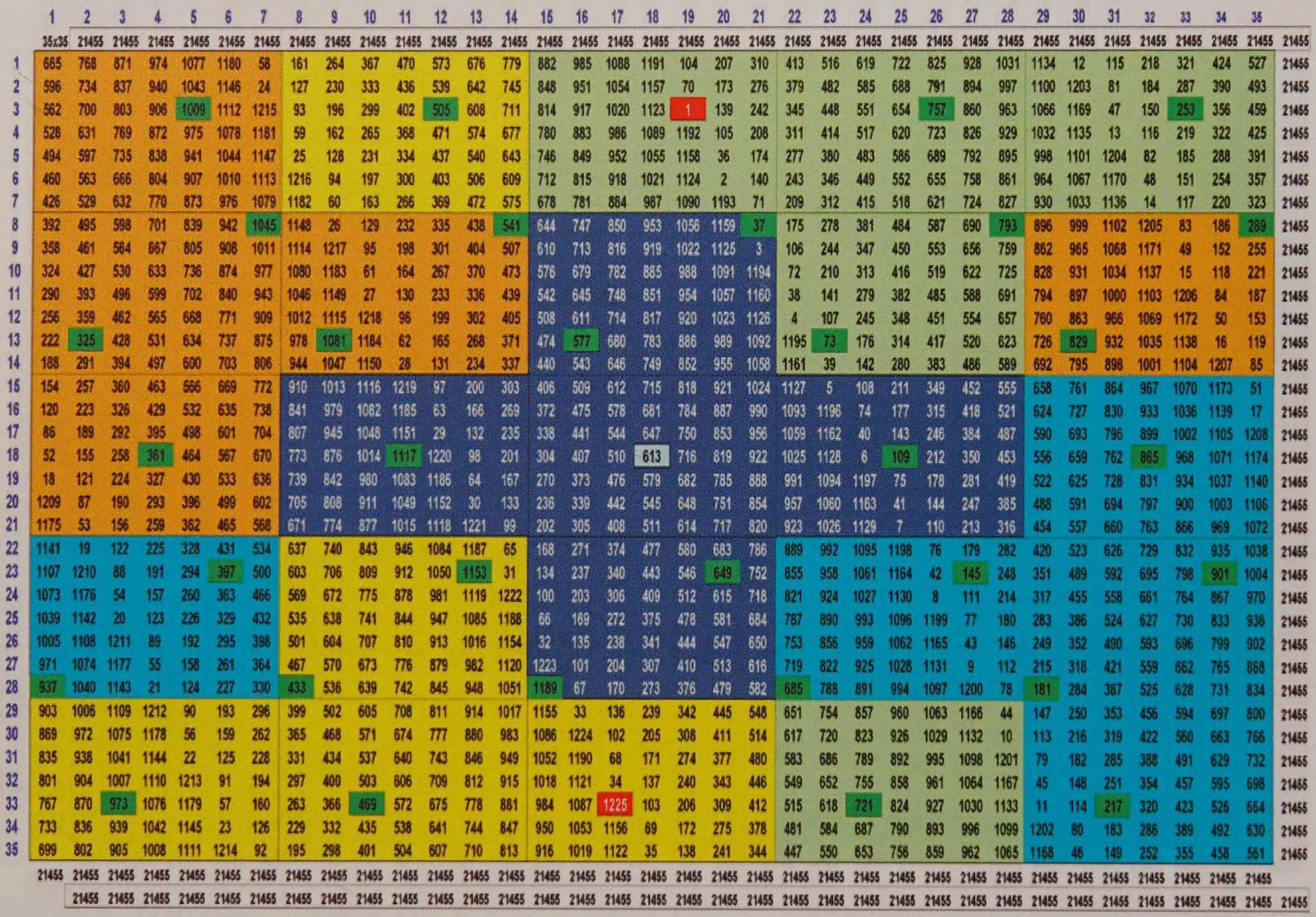
Program 3

For Class-5 Squares

Size **n = ab**, **a & b** odd numbers **a > b > 3**

Next we'll investigate the tiling patterns for Class-5 squares. Only one size is small enough to be barely legible and that is the size-35 square where **n = 5x7.**

Size-35 primal square with size-7 block-tile patterns



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	35x35	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	665	768	871	974	1077	1180	58	161	264	367	470	573	676	779	882	985	1088	1191	104	207	310	413	516	619	722	825	928	1031	1134	12	115	218	321	424	527	21455
2	596	734	837	940	1043	1146	24	127	230	333	436	539	642	745	848	951	1054	1157	70	173	276	379	482	585	688	791	894	997	1100	1203	81	184	287	390	493	21455
3	562	700	803	906	1009	1112	1215	93	196	299	402	505	608	711	814	917	1020	1123	1	139	242	345	448	551	654	757	860	963	1066	1169	47	150	253	356	459	21455
4	528	631	769	872	975	1078	1181	59	162	265	368	471	574	677	780	883	986	1089	1192	105	208	311	414	517	620	723	826	929	1032	1135	13	116	219	322	425	21455
5	494	597	735	838	941	1044	1147	25	128	231	334	437	540	643	746	849	952	1055	1158	36	174	277	380	483	586	689	792	895	998	1101	1204	82	185	288	391	21455
6	460	563	666	804	907	1010	1113	1216	94	197	300	403	506	609	712	815	918	1021	1124	2	140	243	346	449	552	655	758	861	964	1067	1170	48	151	254	357	21455
7	426	529	632	770	873	976	1079	1182	60	163	266	369	472	575	678	781	884	987	1090	1193	71	209	312	415	518	621	724	827	930	1033	1136	14	117	220	323	21455
8	392	495	598	701	839	942	1045	1148	26	129	232	335	438	541	644	747	850	953	1056	1159	37	175	278	381	484	587	690	793	896	999	1102	1205	83	186	289	21455
9	358	461	564	667	805	908	1011	1114	1217	95	198	301	404	507	610	713	816	919	1022	1125	3	106	244	347	450	553	656	759	862	965	1068	1171	49	152	255	21455
10	324	427	530	633	736	874	977	1080	1183	61	164	267	370	473	576	679	782	885	988	1091	1194	72	210	313	416	519	622	725	828	931	1034	1137	15	118	221	21455
11	290	393	496	599	702	840	943	1046	1149	27	130	233	336	439	542	645	748	851	954	1057	1160	38	141	279	382	485	588	691	794	897	1000	1103	1206	84	187	21455
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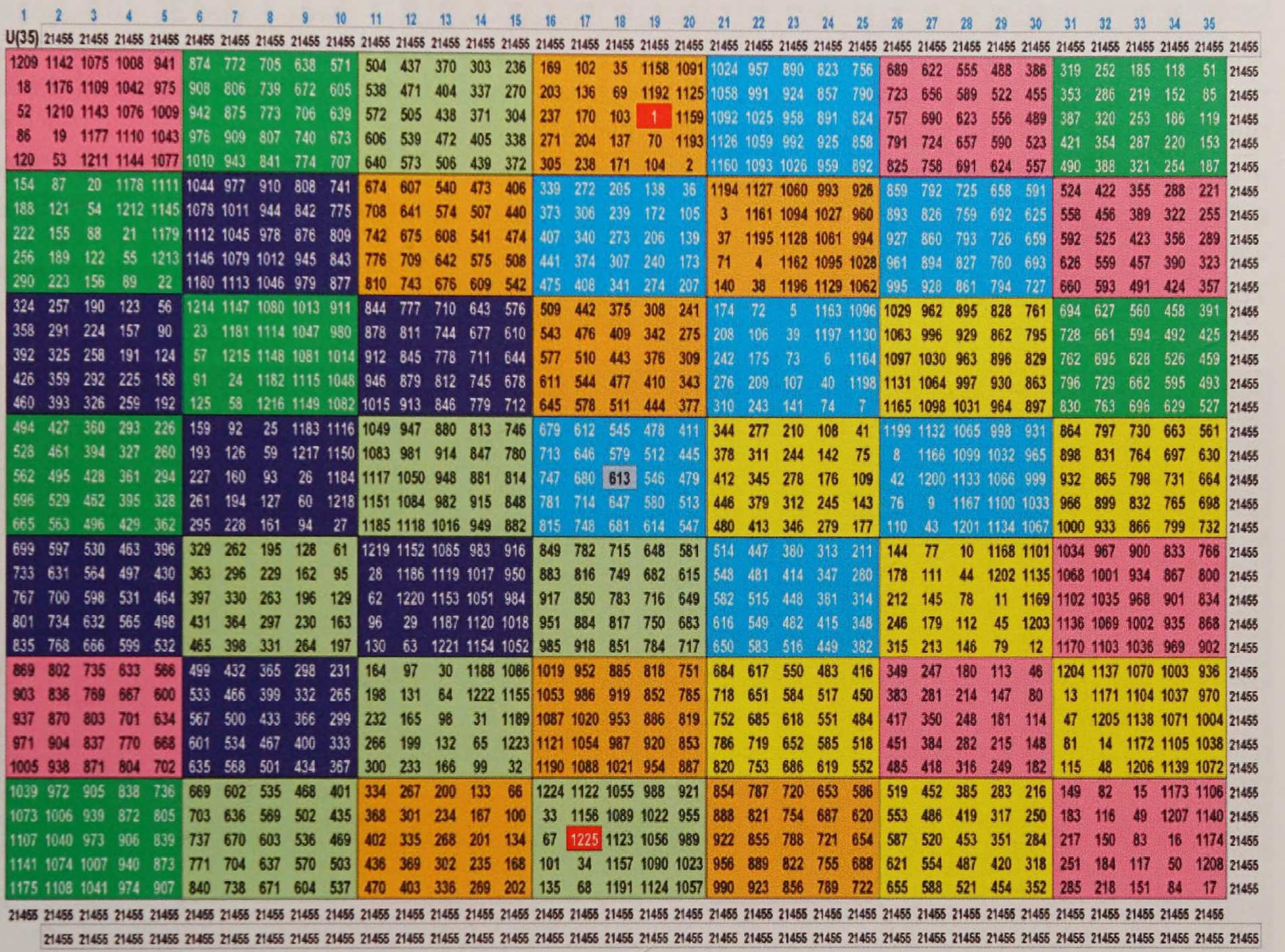
13	222	325	428	531	634	737	875	978	1081	1184	62	165	268	371	474	577	680	783	886	989	1092	1195	73	176	314	417	520	623	726	829	932	1035	1138	16	119	21455
14	188	291	394	497	600	703	806	944	1047	1150	28	131	234	337	440	543	646	749	852	955	1058	1161	39	142	280	383	486	589	692	795	898	1001	1104	1207	85	21455
15	154	257	360	463	566	669	772	910	1013	1116	1219	97	200	303	406	509	612	715	818	921	1024	1127	5	108	211	349	452	555	658	761	864	967	1070	1173	51	21455
16	120	223	326	429	532	635	738	841	979	1082	1185	63	166	269	372	475	578	681	784	887	990	1093	1196	74	177	315	418	521	624	727	830	933	1036	1139	17	21455
17	86	189	292	395	498	601	704	807	945	1048	1151	29	132	235	338	441	544	647	750	853	956	1059	1162	40	143	246	384	487	590	693	796	899	1002	1105	1208	21455
18	52	155	258	361	464	567	670	773	876	1014	1117	1220	98	201	304	407	510	613	716	819	922	1025	1128	6	109	212	350	453	556	659	762	865	968	1071	1174	21455
19	18	121	224	327	430	533	636	739	842	980	1083	1186	64	167	270	373	476	579	682	785	888	991	1094	1197	75	178	281	419	522	625	728	831	934	1037	1140	21455
20	1209	87	190	293	396	499	602	705	808	911	1049	1152	30	133	236	339	442	545	648	751	854	957	1060	1163	41	144	247	385	488	591	694	797	900	1003	1106	21455
21	1175	53	156	259	362	465	568	671	774	877	1015	1118	1221	99	202	305	408	511	614	717	820	923	1026	1129	7	110	213	316	454	557	660	763	866	969	1072	21455
22	1141	19	122	225	328	431	534	637	740	843	946	1084	1187	65	168	271	374	477	580	683	786	889	992	1095	1198	76	179	282	420	523	626	729	832	935	1038	21455
23	1107	1210	88	191	294	397	500	603	706	809	912	1050	1153	31	134	237	340	443	546	649	752	855	958	1061	1164	42	145	248	351	489	592	695	798	901	1004	21455
24	1073	1176	54	157	260	363	466	569	672	775	878	981	1119	1222	100	203	306	409	512	615	718	821	924	1027	1130	8	111	214	317	455	558	661	764	867	970	21455
25	1039	1142	20	123	226	329	432	535	638	741	844	947	1085	1188	66	169	272	375	478	581	684	787	890	993	1096	1199	77	180	283	386	524	627	730	833	936	21455
26	1005	1108	1211	89	192	295	398	501	604	707	810	913	1016	1154	32	135	238	341	444	547	650	753	856	959	1062	1165	43	146	249	352	490	593	696	799	902	21455
27	971	1074	1177	55	158	261	364	467	570	673	776	879	982	1120	1223	101	204	307	410	513	616	719	822	925	1028	1131	9	112	215	318	421	559	662	765	868	21455
28	937	1040	1143	21	124	227	330	433	536	639	742	845	948	1051	1189	67	170	273	376	479	582	685	788	891	994	1097	1200	78	181	284	387	525	628	731	834	21455
29	903	1006	1109	1212	90	193	296	399	502	605	708	811	914	1017	1155	33	136	239	342	445	548	651	754	857	960	1063	1166	44	147	250	353	456	594	697	800	21455
30	869	972	1075	1178	56	159	262	365	468	571	674	777	880	983	1086	1224	102	205	308	411	514	617	720	823	926	1029	1132	10	113	216	319	422	560	663	766	21455
31	835	938	1041	1144	22	125	228	331	434	537	640	743	846	949	1052	1190	68	171	274	377	480	583	686	789	892	995	1098	1201	79	182	285	388	491	629	732	21455
32	801	904	1007	1110	1213	91	194	297	400	503	606	709	812	915	1018	1121	34	137	240	343	446	549	652	755	858	961	1064	1167	45	148	251	354	457	595	698	21455
33	767	870	973	1076	1179	57	160	263	366	469	572	675	778	881	984	1087	1225	103	206	309	412	515	618	721	824	927	1030	1133	11	114	217	320	423	526	664	21455
34	733	836	939	1042	1145	23	126	229	332	435	538	641	744	847	950	1053	1156	69	172	275	378	481	584	687	790	893	996	1099	1202	80	183	286	389	492	630	21455
35	699	802	905	1008	1111	1214	92	195	298	401	504	607	710	813	916	1019	1122	35	138	241	344	447	550	653	756	859	962	1065	1168	46	149	252	355	458	561	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

The shaded patterns are the continuously equal-summing size-5 characteristic tile pattern B at the **7x7** block-square level. Each tile sums to **7**-times the size-35 square's characteristic number. Although not shown here, the same also holds for the size-7 tile pattern A too.

Program 3

Size-35 dual square with size-5 block-tile patterns

Here is the size-35 square's dual square.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	1209	1142	1075	1008	941	874	772	705	638	571	504	437	370	303	236	169	102	35	1158	1091	1024	957	890	823	756	689	622	555	488	386	319	252	185	118	51	21455
2	18	1176	1109	1042	975	908	806	739	672	605	538	471	404	337	270	203	136	69	1192	1125	1058	991	924	857	790	723	656	589	522	455	353	286	219	152	85	21455
3	52	1210	1143	1076	1009	942	875	773	706	639	572	505	438	371	304	237	170	103	1	1159	1092	1025	958	891	824	757	690	623	556	489	387	320	253	186	119	21455
4	86	19	1177	1110	1043	976	909	807	740	673	606	539	472	405	338	271	204	137	70	1193	1126	1059	992	925	858	791	724	657	590	523	421	354	287	220	153	21455
5	120	53	1211	1144	1077	1010	943	841	774	707	640	573	506	439	372	305	238	171	104	2	1160	1093	1026	959	892	825	758	691	624	557	490	388	321	254	187	21455
6	154	87	20	1178	1111	1044	977	910	808	741	674	607	540	473	406	339	272	205	138	36	1194	1127	1060	993	926	859	792	725	658	591	524	422	355	288	221	21455
7	188	121	54	1212	1145	1078	1011	944	842	775	708	641	574	507	440	373	306	239	172	105	3	1161	1094	1027	960	893	826	759	692	625	558	456	389	322	255	21455
8	222	155	88	21	1179	1112	1045	978	876	809	742	675	608	541	474	407	340	273	206	139	37	1195	1128	1061	994	927	860	793	726	659	592	525	423	356	289	21455
9	256	189	122	55	1213	1146	1079	1012	945	843	776	709	642	575	508	441	374	307	240	173	71	4	1162	1095	1028	961	894	827	760	693	626	559	457	390	323	21455
10	290	223	156	89	22	1180	1113	1046	979	877	810	743	676	609	542	475	408	341	274	207	140	38	1196	1129	1062	995	928	861	794	727	660	593	491	424	357	21455
11	324	257	190	123	56	1214	1147	1080	1013	911	844	777	710	643	576	509	442	375	308	241	174	72	5	1163	1096	1029	962	895	828	761	694	627	560	458	391	21455
12	358	291	224	157	90	23	1181	1114	1047	980	878	811	744	677	610	543	476	409	342	275	208	106	39	1197	1130	1063	996	929	862	795	728	661	594	492	425	21455
13	392	325	258	191	124	57	1215	1148	1081	1014	912	845	778	711	644	577	510	443	376	309	242	175	73	6	1164	1097	1030	963	896	829	762	695	628	526	459	21455
14	426	359	292	225	158	91	24	1182	1115	1048	946	879	812	745	678	611	544	477	410	343	276	209	107	40	1198	1131	1064	997	930	863	796	729	662	595	493	21455
15	460	393	326	259	192	125	58	1216	1149	1082	1015	913	846	779	712	645	578	511	444	377	310	243	141	74	7	1165	1098	1031	964	897	830	763	696	629	527	21455
16	494	427	360	293	226	159	92	25	1183	1116	1049	947	880	813	746	679	612	545	478	411	344	277	210	108	41	1199	1132	1065	998	931	864	797	730	663	561	21455
17	528	461	394	327	260	193	126	59	1217	1150	1083	981	914	847	780	713	646	579	512	445	378	311	244	142	75	8	1166	1099	1032	965	898	831	764	697	630	21455
18	562	495	428	361	294	227	160	93	26	1184	1117	1050	948	881	814	747	680	613	546	479	412	345	278	176	109	42	1200	1133	1066	999	932	865	798	731	664	21455
19	596	529	462	395	328	261	194	127	60	1218	1151	1084	982	915	848	781	714	647	580	513	446	379	312	245	143	76	9	1167	1100	1033	966	899	832	765	698	21455
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21	699	597	530	463	396	329	262	195	128	61	1219	1152	1085	983	916	849	782	715	648	581	514	447	380	313	211	144	77	10	1168	1101	1034	967	900	833	766	21455
22	733	631	564	497	430	363	296	229	162	95	28	1186	1119	1017	950	883	816	749	682	615	548	481	414	347	280	178	111	44	1202	1135	1068	1001	934	867	800	21455
23	767	700	598	531	464	397	330	263	196	129	62	1220	1153	1051	984	917	850	783	716	649	582	515	448	381	314	212	145	78	11	1169	1102	1035	968	901	834	21455
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25	835	768	666	599	532	465	398	331	264	197	130	63	1221	1154	1052	985	918	851	784	717	650	583	516	449	382	315	213	146	79	12	1170	1103	1036	969	902	21455
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27	903	836	769	667	600	533	466	399	332	265	198	131	64	1222	1155	1053	986	919	852	785	718	651	584	517	450	383	281	214	147	80	13	1171	1104	1037	970	21455
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30	1005	938	871	804	702	635	568	501	434	367	300	233	166	99	32	1190	1088	1021	954	887	820	753	686	619	552	485	418	316	249	182	115	48	1206	1139	1072	21455
31	1039	972	905	838	736	669	602	535	468	401	334	267	200	133	66	1224	1122	1055	988	921	854	787	720	653	586	519	452	385	283	216	149	82	15	1173	1106	21455
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34	1141	1074	1007	940	873	771	704	637	570	503	436	369	302	235	168	101	34	1157	1090	1023	956	889	822	755	688	621	554	487	420	318	251	184	117	50	1208	21455
35	1175	1108	1041	974	907	840	738	671	604	537	470	403	336	269	202	135	68	1191	1124	1057	990	923	856	789	722	655	588	521	454	352	285	218	151	84	17	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

The shaded patterns are the continuously equal-summing tile patterns, again at the block-square level. This time they are each composed of **5x5** block-squares in the characteristic tile pattern A of the size-7 square. Each tile sums to **5** times the size-35 square's characteristic number. Although not shown here, the same also holds for the size-5 tile pattern B too.

Now, the principle here is this: tiles composed of **bxb** block-squares will each sum equally to **b** times the characteristic number of the square of size **ab**. Likewise, tiles composed of **axa** block-squares will each sum equally to **a** times the characteristic number of the square of size **ab**.

The tables shown here were derived in the base **35**. There are **3** different versions of each Class-5 square of size **n = ab**: each one derived with loom tables in one of the bases a^2, **ab** and b^2. Their generation and relationship among each other will be addressed in Program 8.

Program 3

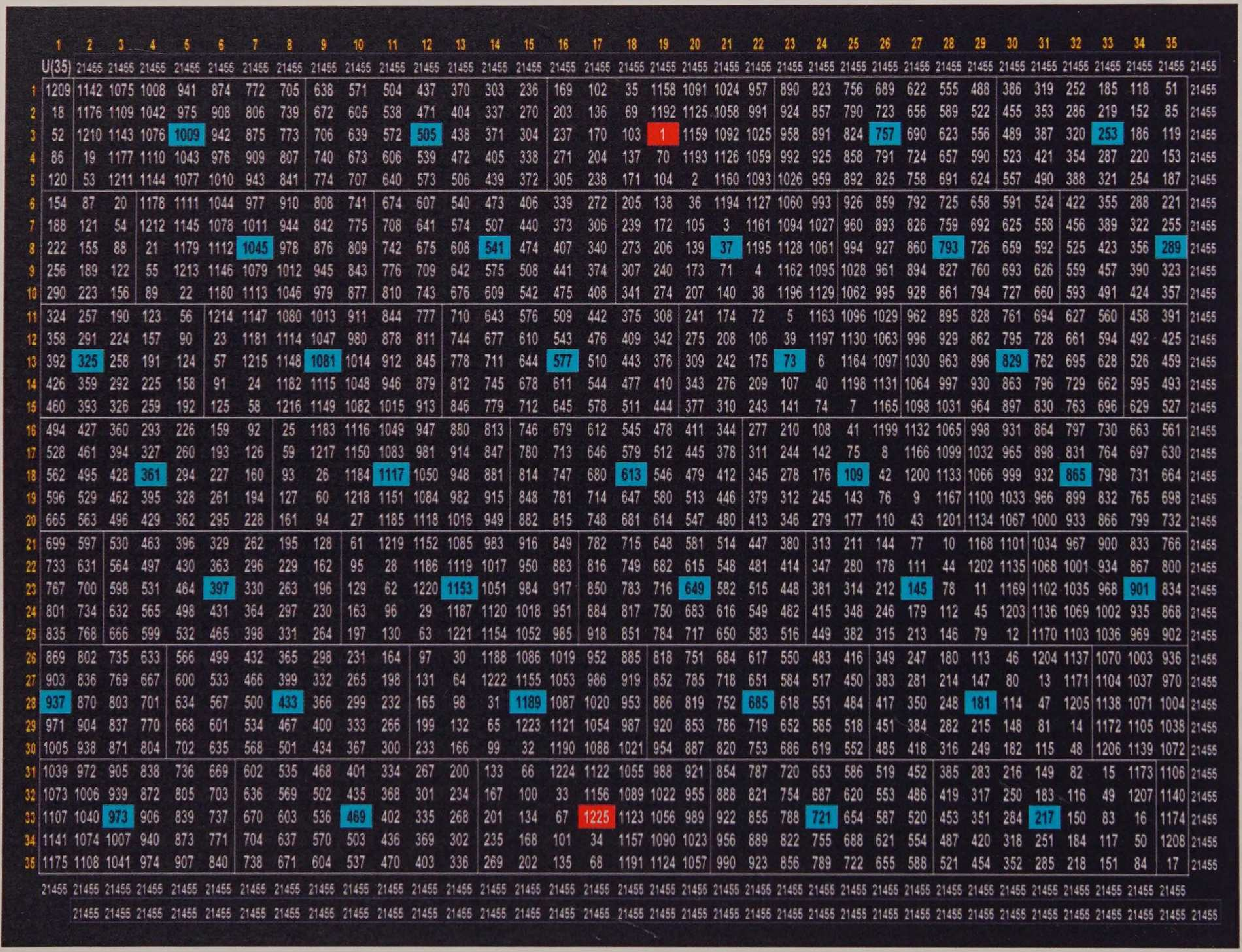
The Class-5 uniform-summation property

The size-35 dual square here has been blocked-out with **5x7** rectangles with these numbers located in their centers.

The numbers in the square with a highlighted background are those values for which the modulus and integer functions are equal. It has already been stated in this program that these numbers all sum to the square's characteristic number.

It was seen in Part III of Program 2 that all the numbers in the same relative location within each rectangle also sum to the square's characteristic number. And the numbers in these same rectangles in the original primal square do so too.

This property occurs uniformly as shown here for any perfect square.

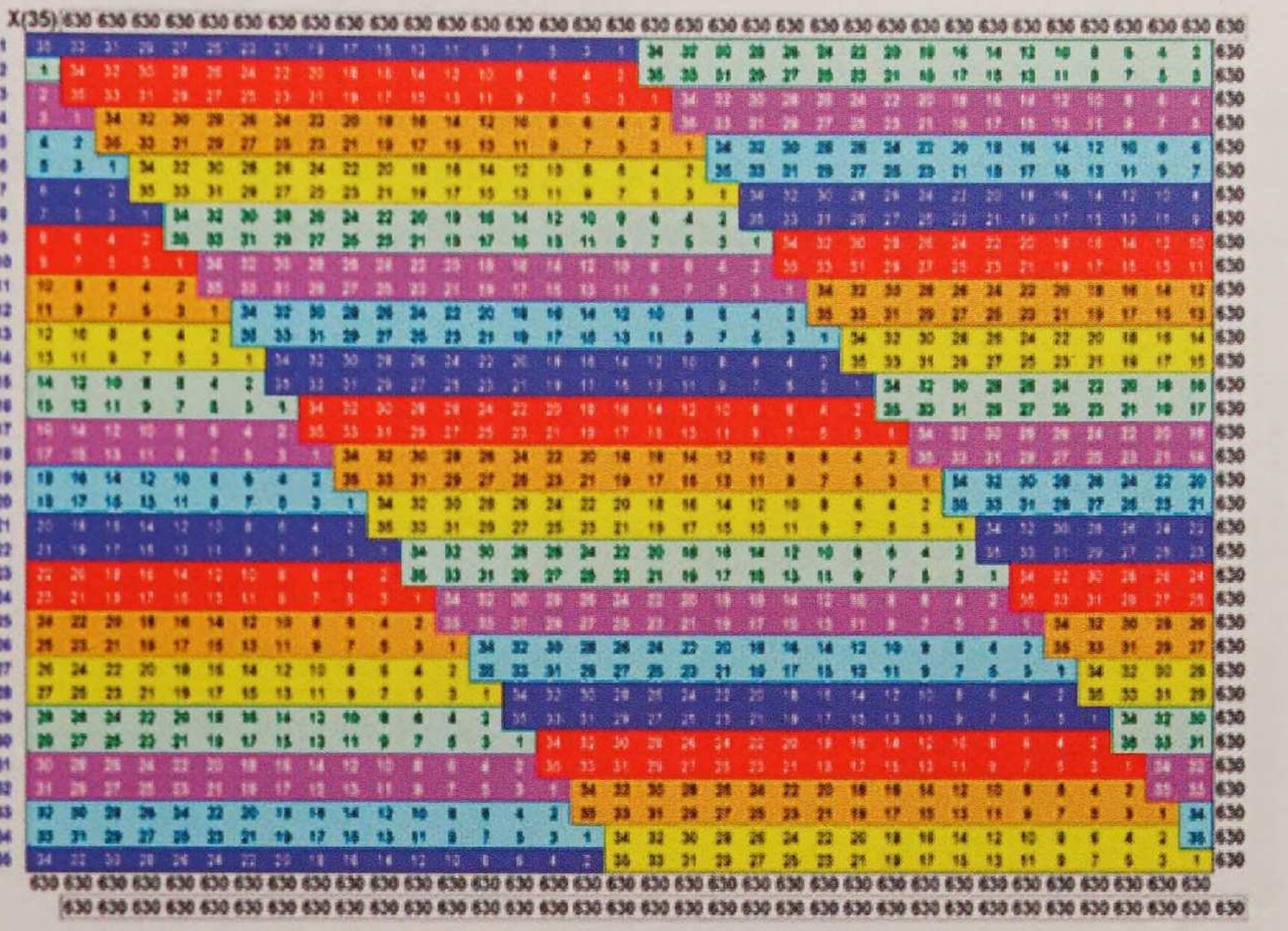


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	1209	1142	1075	1008	941	874	772	705	638	571	504	437	370	303	236	169	102	35	1158	1091	1024	957	890	823	756	689	622	555	488	386	319	252	185	118	51	21455
2	18	1176	1109	1042	975	908	806	739	672	605	538	471	404	337	270	203	136	69	1192	1125	1058	991	924	857	790	723	656	589	522	455	353	286	219	152	85	21455
3	52	1210	1143	1076	1009	942	875	773	706	639	572	505	438	371	304	237	170	103	1	1159	1092	1025	958	891	824	757	690	623	556	489	387	320	253	186	119	21455
4	86	19	1177	1110	1043	976	909	807	740	673	606	539	472	405	338	271	204	137	70	1193	1126	1059	992	925	858	791	724	657	590	523	421	354	287	220	153	21455
5	120	53	1211	1144	1077	1010	943	841	774	707	640	573	506	439	372	305	238	171	104	2	1160	1093	1026	959	892	825	758	691	624	557	490	388	321	254	187	21455
6	154	87	20	1178	1111	1044	977	910	808	741	674	607	540	473	406	339	272	205	138	36	1194	1127	1060	993	926	859	792	725	658	591	524	422	355	288	221	21455
7	188	121	54	1212	1145	1078	1011	944	842	775	708	641	574	507	440	373	306	239	172	105	3	1161	1094	1027	960	893	826	759	692	625	558	456	389	322	255	21455
8	222	155	88	21	1179	1112	1045	978	876	809	742	675	608	541	474	407	340	273	206	139	37	1195	1128	1061	994	927	860	793	726	659	592	525	423	356	289	21455
9	256	189	122	55	1213	1146	1079	1012	945	843	776	709	642	575	508	441	374	307	240	173	71	4	1162	1095	1028	961	894	827	760	693	626	559	457	390	323	21455
10	290	223	156	89	22	1180	1113	1046	979	877	810	743	676	609	542	475	408	341	274	207	140	38	1196	1129	1062	995	928	861	794	727	660	593	491	424	357	21455
11	324	257	190	123	56	1214	1147	1080	1013	911	844	777	710	643	576	509	442	375	308	241	174	72	5	1163	1096	1029	962	895	828	761	694	627	560	458	391	21455
12	358	291	224	157	90	23	1181	1114	1047	980	878	811	744	677	610	543	476	409	342	275	208	106	39	1197	1130	1063	996	929	862	795	728	661	594	492	425	21455
13	392	325	258	191	124	57	1215	1148	1081	1014	912	845	778	711	644	577	510	443	376	309	242	175	73	6	1164	1097	1030	963	896	829	762	695	628	526	459	21455
14	426	359	292	225	158	91	24	1182	1115	1048	946	879	812	745	678	611	544	477	410	343	276	209	107	40	1198	1131	1064	997	930	863	796	729	662	595	493	21455
15	460	393	326	259	192	125	58	1216	1149	1082	1015	913	846	779	712	645	578	511	444	377	310	243	141	74	7	1165	1098	1031	964	897	830	763	696	629	527	21455
16	494	427	360	293	226	159	92	25	1183	1116	1049	947	880	813	746	679	612	545	478	411	344	277	210	108	41	1199	1132	1065	998	931	864	797	730	663	561	21455
17	528	461	394	327	260	193	126	59	1217	1150	1083	981	914	847	780	713	646	579	512	445	378	311	244	142	75	8	1166	1099	1032	965	898	831	764	697	630	21455
18	562	495	428	361	294	227	160	93	26	1184	1117	1050	948	881	814	747	680	613	546	479	412	345	278	176	109	42	1200	1133	1066	999	932	865	798	731	664	21455
19	596	529	462	395	328	261	194	127	60	1218	1151	1084	982	915	848	781	714	647	580	513	446	379	312	245	143	76	9	1167	1100	1033	966	899	832	765	698	21455
20	665	563	496	429	362	295	228	161	94	27	1185	1118	1016	949	882	815	748	681	614	547	480	413	346	279	177	110	43	1201	1134	1067	1000	933	866	799	732	21455
21	699	597	530	463	396	329	262	195	128	61	1219	1152	1085	983	916	849	782	715	648	581	514	447	380	313	211	144	77	10	1168	1101	1034	967	900	833	766	21455
22	733	631	564	497	430	363	296	229	162	95	28	1186	1119	1017	950	883	816	749	682	615	548	481	414	347	280	178	111	44	1202	1135	1068	1001	934	867	800	21455
23	767	700	598	531	464	397	330	263	196	129	62	1220	1153	1051	984	917	850	783	716	649	582	515	448	381	314	212	145	78	11	1169	1102	1035	968	901	834	21455
24	801	734	632	565	498	431	364	297	230	163	96	29	1187	1120	1018	951	884	817	750	683	616	549	482	415	348	246	179	112	45	1203	1136	1069	1002	935	868	21455
25	835	768	666	599	532	465	398	331	264	197	130	63	1221	1154	1052	985	918	851	784	717	650	583	516	449	382	315	213	146	79	12	1170	1103	1036	969	902	21455
26	869	802	735	633	566	499	432	365	298	231	164	97	30	1188	1086	1019	952	885	818	751	684	617	550	483	416	349	247	180	113	46	1204	1137	1070	1003	936	21455
27	903	836	769	667	600	533	466	399	332	265	198	131	64	1222	1155	1053	986	919	852	785	718	651	584	517	450	383	281	214	147	80	13	1171	1104	1037	970	21455
28	937	870	803	701	634	567	500	433	366	299	232	165	98	31	1189	1087	1020	953	886	819	752	685	618	551	484	417	350	248	181	114	47	1205	1138	1071	1004	21455
29	971	904	837	770	668	601	534	467	400	333	266	199	132	65	1223	1121	1054	987	920	853	786	719	652	585	518	451	384	282	215	148	81	14	1172	1105	1038	21455
30	1005	938	871	804	702	635	568	501	434	367	300	233	166	99	32	1190	1088	1021	954	887	820	753	686	619	552	485	418	316	249	182	115	48	1206	1139	1072	21455
31	1039	972	905	838	736	669	602	535	468	401	334	267	200	133	66	1224	1122	1055	988	921	854	787	720	653	586	519	452	385	283	216	149	82	15	1173	1106	21455
32	1073	1006	939	872	805	703	636	569	502	435	368	301	234	167	100	33	1156	1089	1022	955	888	821	754	687	620	553	486	419	317	250	183	116	49	1207	1140	21455
33	1107	1040	973	906	839	737	670	603	536	469	402	335	268	201	134	67	1225	1123	1056	989	922	855	788	721	654	587	520	453	351	284	217	150	83	16	1174	21455
34	1141	1074	1007	940	873	771	704	637	570	503	436	369	302	235	168	101	34	1157	1090	1023	956	889	822	755	688	621	554	487	420	318	251	184	117	50	1208	21455
35	1175	1108	1041	974	907	840	738	671	604	537	470	403	336	269	202	135	68	1191	1124	1057	990	923	856	789	722	655	588	521	454	352	285	218	151	84	17	21455
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
		21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 3

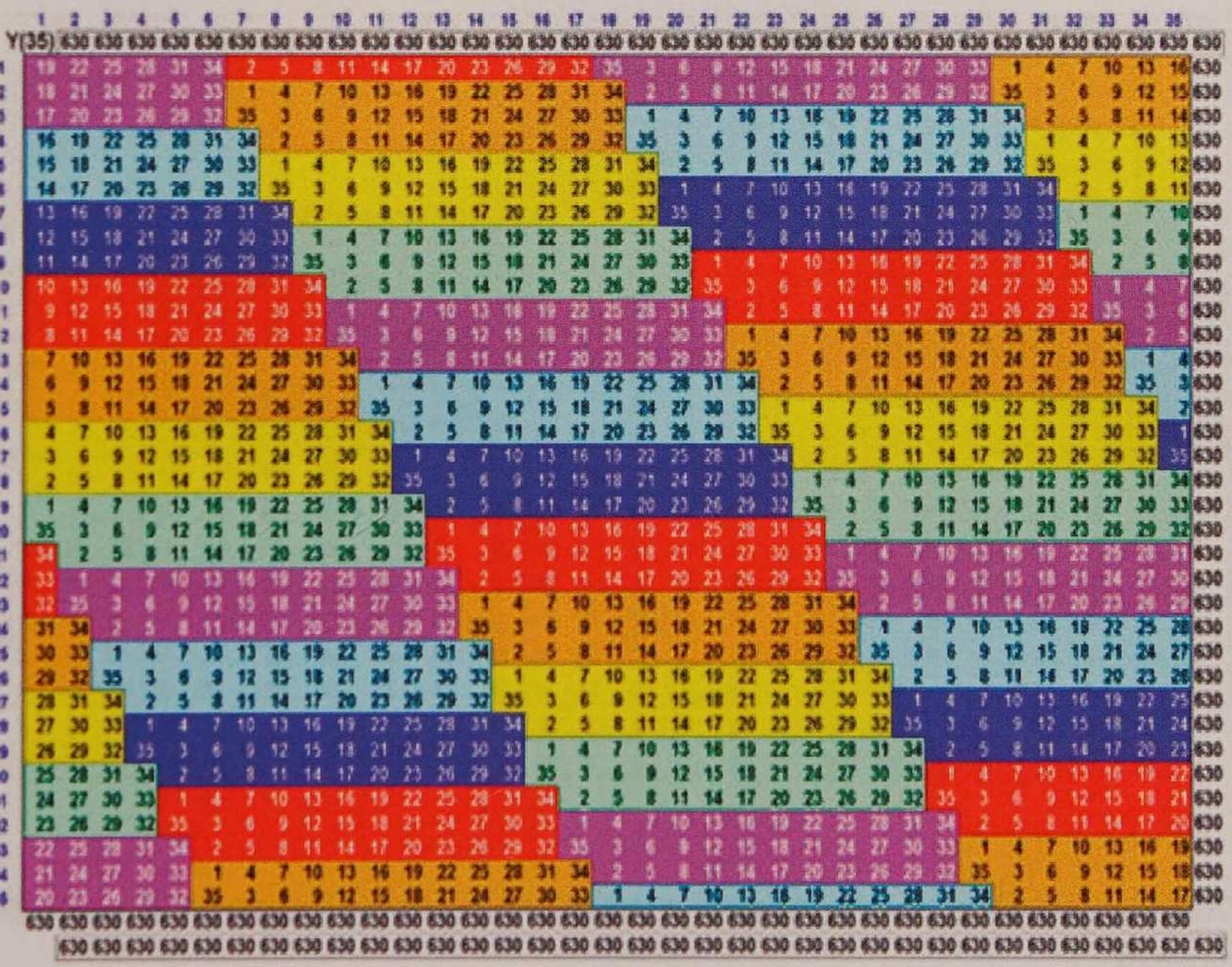
Size-35 Modulus and Integer Loom Table Patterns

Here is the size-35 loom table **X(35)** taken in the base **35**. Every row, column and diagonal contains the numbers **1** thru **35.**



Each colored section contains all the numbers **1** thru **35**. Note that all the even numbers are in the top row of each section. Dub this tile pattern P, for the size-35 modulus loom table **X(35)**.

Here is the size-35 loom table **Y(35)** taken in the base **35.** Again, every row, column and diagonal contains the numbers **1** thru **35** too.



Each colored section also contains all the numbers **1** thru **35**. These sections highlight a tiling pattern, dub it Q, for the size-35 integer loom table **Y(35)**.

Note that tile pattern P above contains **2** rows while pattern Q here contains **3** rows

Further, tiling pattern P contains exactly **2** tiles horizontally per **3** rows whereas here tiling pattern Q contains exactly **3** tiles horizontally per **5** rows.

Now here is what is so surprising: Tile pattern P doesn't work on **Y(35)** and tile pattern Q doesn't work on **X(35)** ! And moreover, neither works on the primary dual squares **W(35)** and **U(35)** because they are not shared in common between the loom tables.

Program 3

The Cloaking property of size-35 loom tables

V1(35) = X1(35) – Y1(35)

Here is the difference table of loom tables derived in the base **35**. It is totally pairwise anti-symmetric. All the rows, columns and main and wrap diagonals sum to **0**. All **7x7** block-squares in both A and B tile patterns of the size-5 square sum continuously to **0**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
X(35)-Y(35)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-16	-11	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	34	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	-1	4	9	14	0
2	17	-13	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	32	-33	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	0
3	15	-15	-10	-5	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	5	10	0
4	13	18	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	33	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	-7	-2	3	8	0
5	11	16	-14	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	0
6	9	14	19	-11	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	-1	4	0
7	7	12	17	-13	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	32	2	-28	-23	-18	-13	-8	-3	2	7	12	17	-13	-8	-3	2	0
8	5	10	15	20	-10	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	0
9	3	8	13	18	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	-7	-2	0
10	1	6	11	16	21	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	-14	-9	-4	0
11	-1	4	9	14	19	-11	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	0
12	-3	2	7	12	17	22	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	-13	-8	0
13	-5	0	5	10	15	20	-10	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	0	5	-25	-20	-15	-10	-5	0	5	10	15	-15	-10	0
14	-7	-2	3	8	13	18	23	-7	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	0
15	-9	-4	1	6	11	16	21	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	-14	0
16	-11	-6	-1	4	9	14	19	24	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	-16	0
17	-13	-8	-3	2	7	12	17	22	-8	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	0
18	-15	-10	-5	0	5	10	15	20	25	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	0
19	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	8	-22	-17	-12	-7	-2	3	8	13	0
20	16	-14	-9	-4	1	6	11	16	21	26	-4	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	-24	-19	-14	-9	-4	1	6	11	0
21	14	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	0
22	12	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	7	-23	-18	-13	-8	-3	2	7	0
23	10	15	-15	-10	-5	0	5	10	15	20	25	-5	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	10	-20	-15	-10	-5	0	5	0
24	8	13	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	-22	-17	-12	-7	-2	3	0
25	6	11	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	11	-19	-14	-9	-4	1	0
26	4	9	14	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	-21	-16	-11	-6	-1	0
27	2	7	12	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	-18	-13	-8	-3	0
28	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	10	-20	-15	-10	-5	0
29	-2	3	8	13	-17	-12	-7	-2	3	8	13	18	23	28	-2	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	13	-17	-12	-7	0
30	-4	1	6	11	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	11	-19	-14	-9	0
31	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	14	-16	-11	0
32	-8	-3	2	7	12	-18	-13	-8	-3	2	7	12	17	22	27	32	-33	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	-18	-13	0
33	-10	-5	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	5	10	15	-15	0
34	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	33	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	13	-17	0
	-14	-9	-4	1	6	11	-19	-14	-9	-4	1	6	11	16	21	26	31	-34	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	11	16	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

However, neither loom-table tile patterns P nor Q sum to 0 here because they are not complementary.

Program 3

Simultaneously, all 5x5 block-squares in both A and B tile patterns of the size-7 square sum continuously to 0 too.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
X(35)-Y(35)		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-16	-11	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	34	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	-1	4	9	14	0
2	17	-13	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	32	-33	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	0
3	15	-15	-10	-5	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	5	10	0
4	13	18	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	33	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	-7	-2	3	8	0
5	11	16	-14	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	0
6	9	14	19	-11	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	-1	4	0
7	7	12	17	-13	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	32	2	-28	-23	-18	-13	-8	-3	2	7	12	17	-13	-8	-3	2	0
8	5	10	15	20	-10	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	0
9	3	8	13	18	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	-7	-2	0
10	1	6	11	16	21	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	-14	-9	-4	0
11	-1	4	9	14	19	-11	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	0
12	-3	2	7	12	17	22	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	-13	-8	0
13	-5	0	5	10	15	20	-10	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	0	5	-25	-20	-15	-10	-5	0	5	10	15	-15	-10	0
14	-7	-2	3	8	13	18	23	-7	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	0
15	-9	-4	1	6	11	16	21	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	-14	0
16	-11	-6	-1	4	9	14	19	24	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	-16	0
17	-13	-8	-3	2	7	12	17	22	-8	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	0
18	-15	-10	-5	0	5	10	15	20	25	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	0
19	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	8	-22	-17	-12	-7	-2	3	8	13	0
20	16	-14	-9	-4	1	6	11	16	21	26	-4	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	-24	-19	-14	-9	-4	1	6	11	0
21	14	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	0
22	12	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	7	-23	-18	-13	-8	-3	2	7	0
23	10	15	-15	-10	-5	0	5	10	15	20	25	-5	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	10	-20	-15	-10	-5	0	5	0
24	8	13	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	-22	-17	-12	-7	-2	3	0
25	6	11	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	11	-19	-14	-9	-4	1	0
26	4	9	14	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	-21	-16	-11	-6	-1	0
27	2	7	12	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	-18	-13	-8	-3	0
28	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	10	-20	-15	-10	-5	0
29	-2	3	8	13	-17	-12	-7	-2	3	8	13	18	23	28	-2	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	13	-17	-12	-7	0
30	-4	1	6	11	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	11	-19	-14	-9	0
31	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	14	-16	-11	0
32	-8	-3	2	7	12	-18	-13	-8	-3	2	7	12	17	22	27	32	-33	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	-18	-13	0
33	-10	-5	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	5	10	15	-15	0
34	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	33	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	13	-17	0
	-14	-9	-4	1	6	11	-19	-14	-9	-4	1	6	11	16	21	26	31	-34	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	11	16	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Program 3

There's even more. Those **5x7** segments that we just observed also sum to **0** on the difference table **V(35)** too. That result stems from the total anti-symmetry of each **5x7** segment itself.

V(35)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-16	-11	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	34	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	-1	4	9	14	0
17	-13	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	32	-33	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	0
15	-15	-10	-5	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	5	10	0
13	18	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	33	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	-7	-2	3	8	0
11	16	-14	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	0
9	14	19	-11	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	-1	4	0
7	12	17	-13	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	32	2	-28	-23	-18	-13	-8	-3	2	7	12	17	-13	-8	-3	2	0
5	10	15	20	-10	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	0
3	8	13	18	-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	-7	-2	0
1	6	11	16	21	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	-14	-9	-4	0
-1	4	9	14	19	-11	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	-11	-6	0
-3	2	7	12	17	22	-8	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	-13	-8	0
-5	0	5	10	15	20	-10	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	0	5	-25	-20	-15	-10	-5	0	5	10	15	-15	-10	0
-7	-2	3	8	13	18	23	-7	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	-12	0
-9	-4	1	6	11	16	21	-9	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	-24	-19	-14	-9	-4	1	6	11	16	-14	0
-11	-6	-1	4	9	14	19	24	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	-16	0
-13	-8	-3	2	7	12	17	22	-8	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	7	-23	-18	-13	-8	-3	2	7	12	17	0
-15	-10	-5	0	5	10	15	20	25	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	-25	-20	-15	-10	-5	0	5	10	15	0
-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	8	-22	-17	-12	-7	-2	3	8	13	0
16	-14	-9	-4	1	6	11	16	21	26	-4	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	-24	-19	-14	-9	-4	1	6	11	0
14	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	-21	-16	-11	-6	-1	4	9	0
12	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	7	-23	-18	-13	-8	-3	2	7	0
10	15	-15	-10	-5	0	5	10	15	20	25	-5	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	10	-20	-15	-10	-5	0	5	0
8	13	-17	-12	-7	-2	3	8	13	18	23	28	-2	3	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	-22	-17	-12	-7	-2	3	0
6	11	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	26	-4	1	6	11	-19	-14	-9	-4	1	0
4	9	14	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	-21	-16	-11	-6	-1	0
2	7	12	-18	-13	-8	-3	2	7	12	17	22	27	-3	2	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	-18	-13	-8	-3	0
0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	-5	0	5	10	-20	-15	-10	-5	0
-2	3	8	13	-17	-12	-7	-2	3	8	13	18	23	28	-2	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	13	-17	-12	-7	0
-4	1	6	11	-19	-14	-9	-4	1	6	11	16	21	26	31	1	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	11	-19	-14	-9	0
-6	-1	4	9	-21	-16	-11	-6	-1	4	9	14	19	24	29	-1	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	-6	-1	4	9	14	-16	-11	0
-8	-3	2	7	12	-18	-13	-8	-3	2	7	12	17	22	27	32	-33	-28	-23	-18	-13	-8	-3	2	7	12	17	22	-8	-3	2	7	12	-18	-13	0
-10	-5	0	5	10	-20	-15	-10	-5	0	5	10	15	20	25	30	0	-30	-25	-20	-15	-10	-5	0	5	10	15	20	-10	-5	0	5	10	15	-15	0
-12	-7	-2	3	8	-22	-17	-12	-7	-2	3	8	13	18	23	28	33	-32	-27	-22	-17	-12	-7	-2	3	8	13	18	23	-7	-2	3	8	13	-17	0
-14	-9	-4	1	6	11	-19	-14	-9	-4	1	6	11	16	21	26	31	-34	-29	-24	-19	-14	-9	-4	1	6	11	16	21	-9	-4	1	6	11	16	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Wherever these polka-dot patterns are found, they will sum to **0** on the loom-table difference table too.

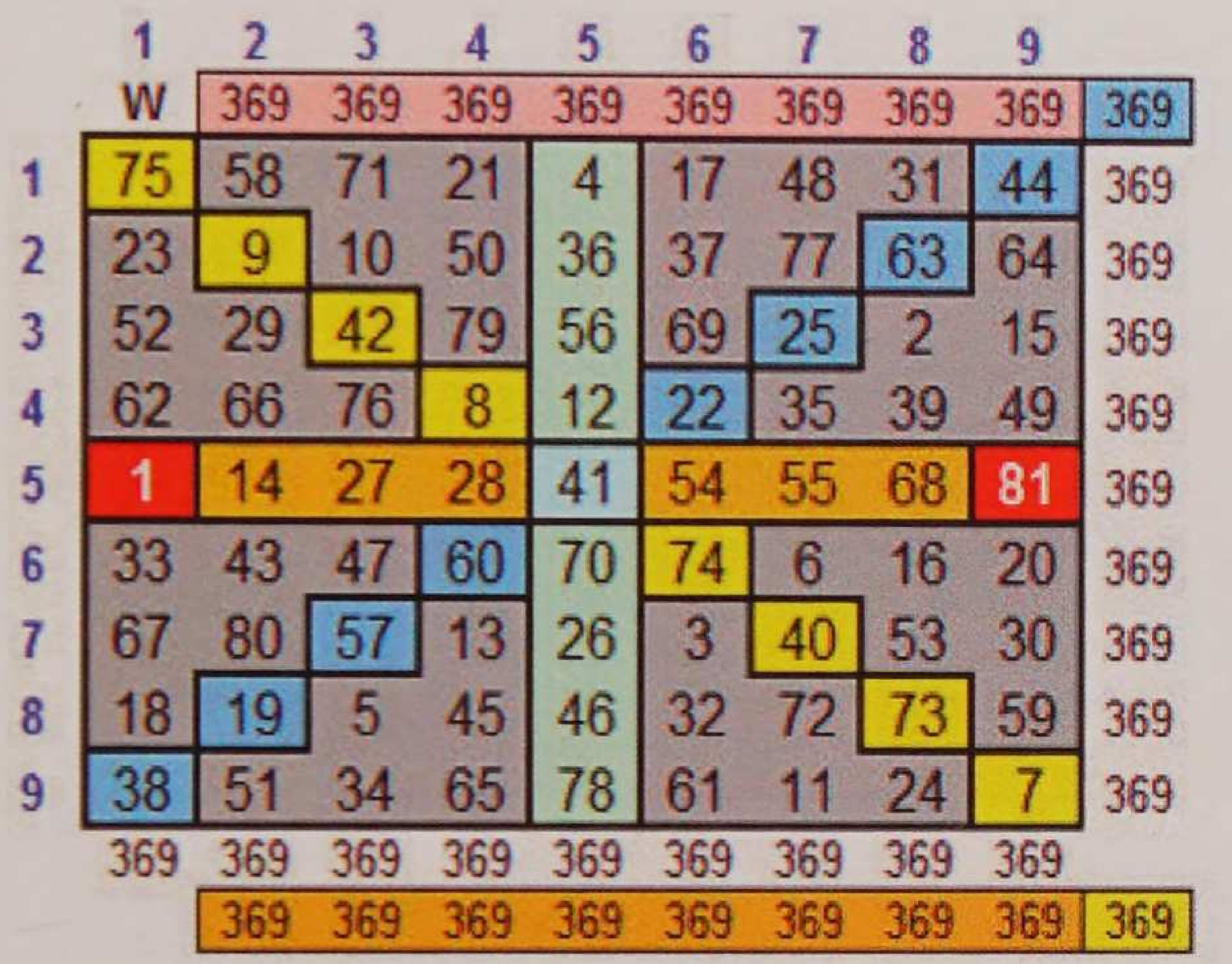
Program 3

For Class-6 Squares

Size **n = b^{a}**, **b** is an odd number ≥ **3**; **a** ≥ **2**

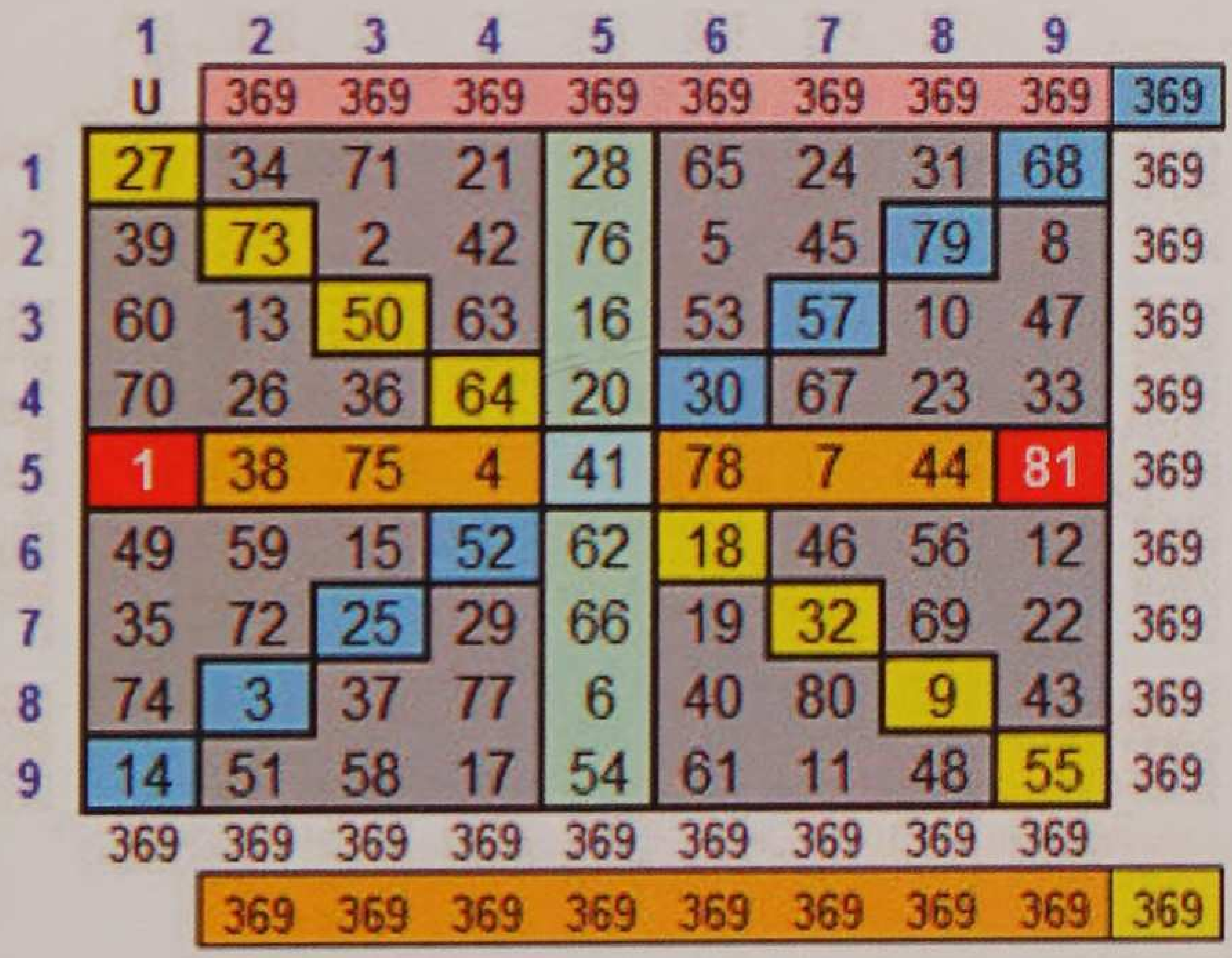
Size-9 Square

Here is the Class-6 size-9 square and its dual with their shared complementary loom tables. The loom tables were derived in the base **9**.

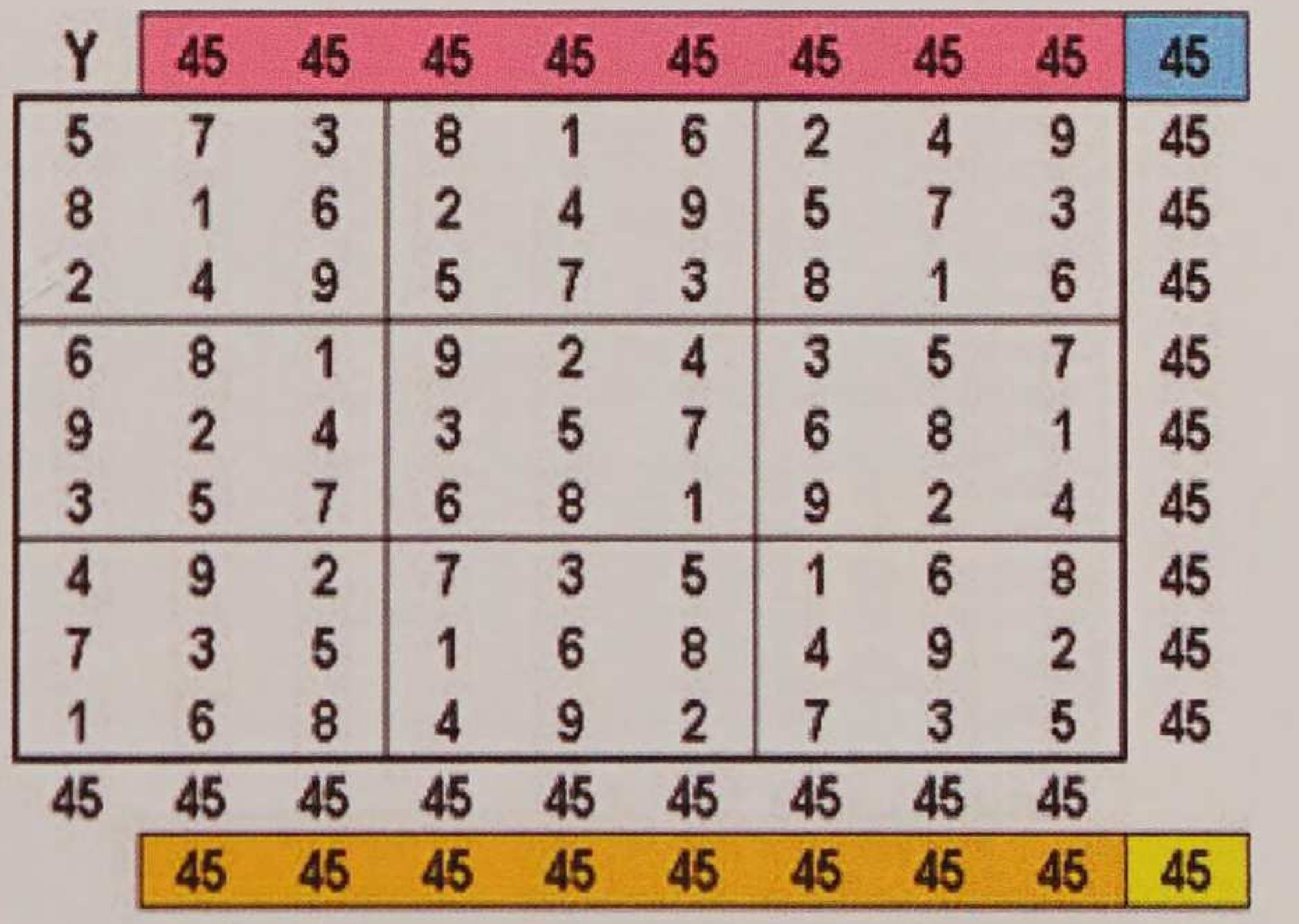


	1	2	3	4	5	6	7	8	9	
W	369	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

X	45	45	45	45	45	45	45	45	45
8	4	3	8	4	3	8	4	3	45
1	9	5	1	9	5	1	9	5	45
6	2	7	6	2	7	6	2	7	45
4	3	8	4	3	8	4	3	8	45
9	5	1	9	5	1	9	5	1	45
2	7	6	2	7	6	2	7	6	45
3	8	4	3	8	4	3	8	4	45
5	1	9	5	1	9	5	1	9	45
7	6	2	7	6	2	7	6	2	45
45	45	45	45	45	45	45	45	45	
	45	45	45	45	45	45	45	45	45



	1	2	3	4	5	6	7	8	9	
U	369	369	369	369	369	369	369	369	369	369
1	27	34	71	21	28	65	24	31	68	369
2	39	73	2	42	76	5	45	79	8	369
3	60	13	50	63	16	53	57	10	47	369
4	70	26	36	64	20	30	67	23	33	369
5	1	38	75	4	41	78	7	44	81	369
6	49	59	15	52	62	18	46	56	12	369
7	35	72	25	29	66	19	32	69	22	369
8	74	3	37	77	6	40	80	9	43	369
9	14	51	58	17	54	61	11	48	55	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369



Y	45	45	45	45	45	45	45	45	45
5	7	3	8	1	6	2	4	9	45
8	1	6	2	4	9	5	7	3	45
2	4	9	5	7	3	8	1	6	45
6	8	1	9	2	4	3	5	7	45
9	2	4	3	5	7	6	8	1	45
3	5	7	6	8	1	9	2	4	45
4	9	2	7	3	5	1	6	8	45
7	3	5	1	6	8	4	9	2	45
1	6	8	4	9	2	7	3	5	45
45	45	45	45	45	45	45	45	45	
	45	45	45	45	45	45	45	45	45

Both the primal and its dual square are continuously **3x** modular with **3x3** block-squares each summing to **369**, the square's characteristic number. The size-9 square has no tiling pattern outside of this **3x-modularity**.

Cloaking Property of Size-9 Loom Tables

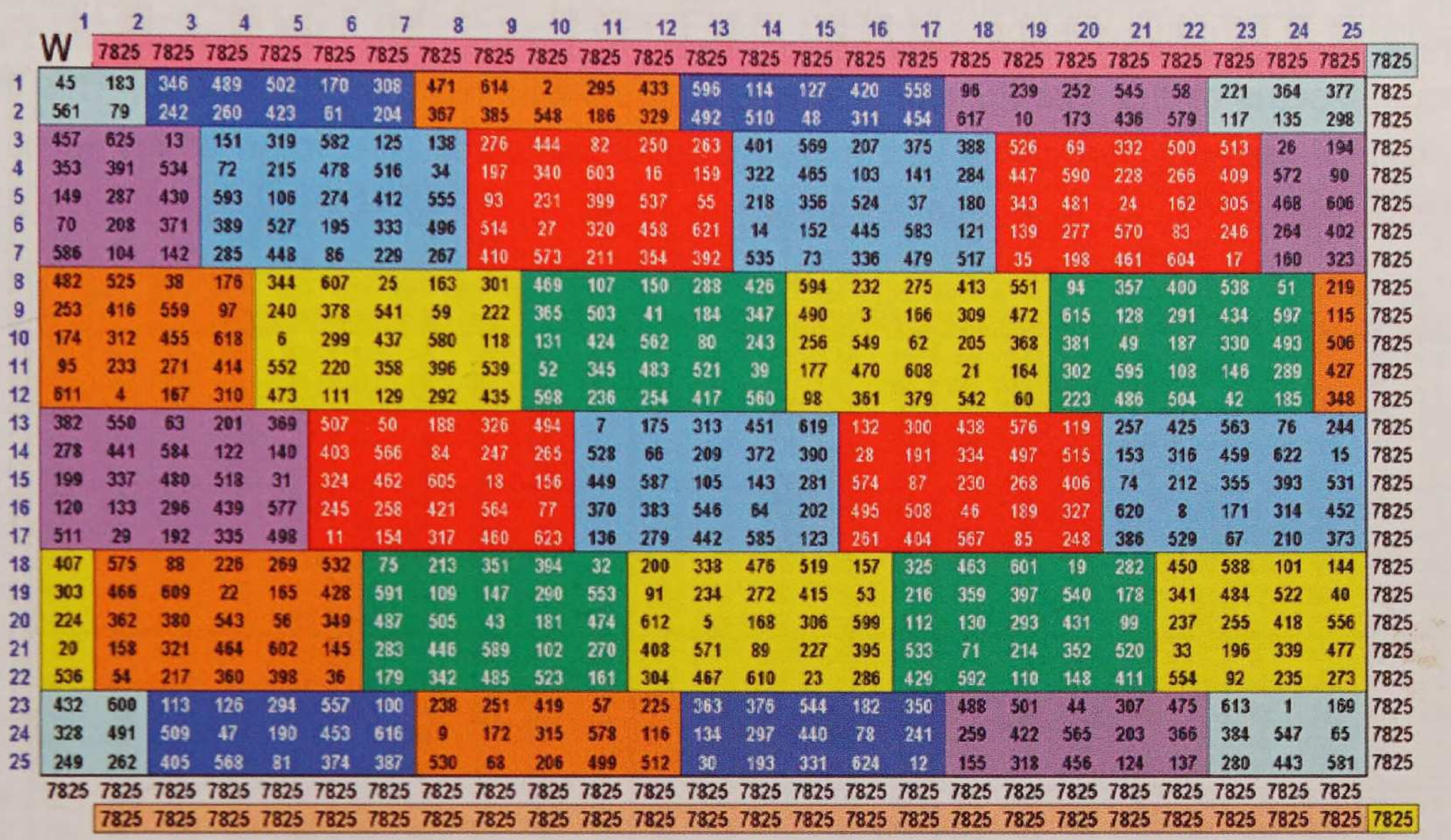
	1	2	3	4	5	6	7	8	9	
	X-Y	0	0	0	0	0	0	0	0	0
1	-6	-3	0	0	3	6	-3	0	3	0
2	2	8	-1	-1	5	-4	-4	2	-7	0
3	1	-2	1	-2	-5	-2	4	1	4	0
4	1	-5	-5	7	1	1	4	-2	-2	0
5	0	3	6	-3	0	3	-6	-3	0	0
6	2	2	-4	-1	-1	-7	5	5	-1	0
7	-4	-1	-4	2	5	2	-1	2	-1	0
8	7	-2	4	4	-5	1	1	-8	-2	0
9	-3	0	3	-6	-3	0	0	3	6	0
	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	
	X-Y	0	0	0	0	0	0	0	0	0
1	0	-6	-3	6	0	3	3	-3	0	0
2	-1	5	5	-4	2	2	-7	-1	-1	0
3	-3	0	3	-6	-3	0	0	3	6	0
4	-2	1	-8	4	7	-2	1	4	-5	0
5	6	0	3	3	-3	0	0	-6	-3	0
6	4	-2	-2	1	-5	-5	7	1	1	0
7	-7	-4	2	-1	2	8	-4	-1	5	0
8	4	4	1	1	1	-2	-2	-2	-5	0
9	-1	2	-1	-4	-1	-4	2	5	2	0
	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0

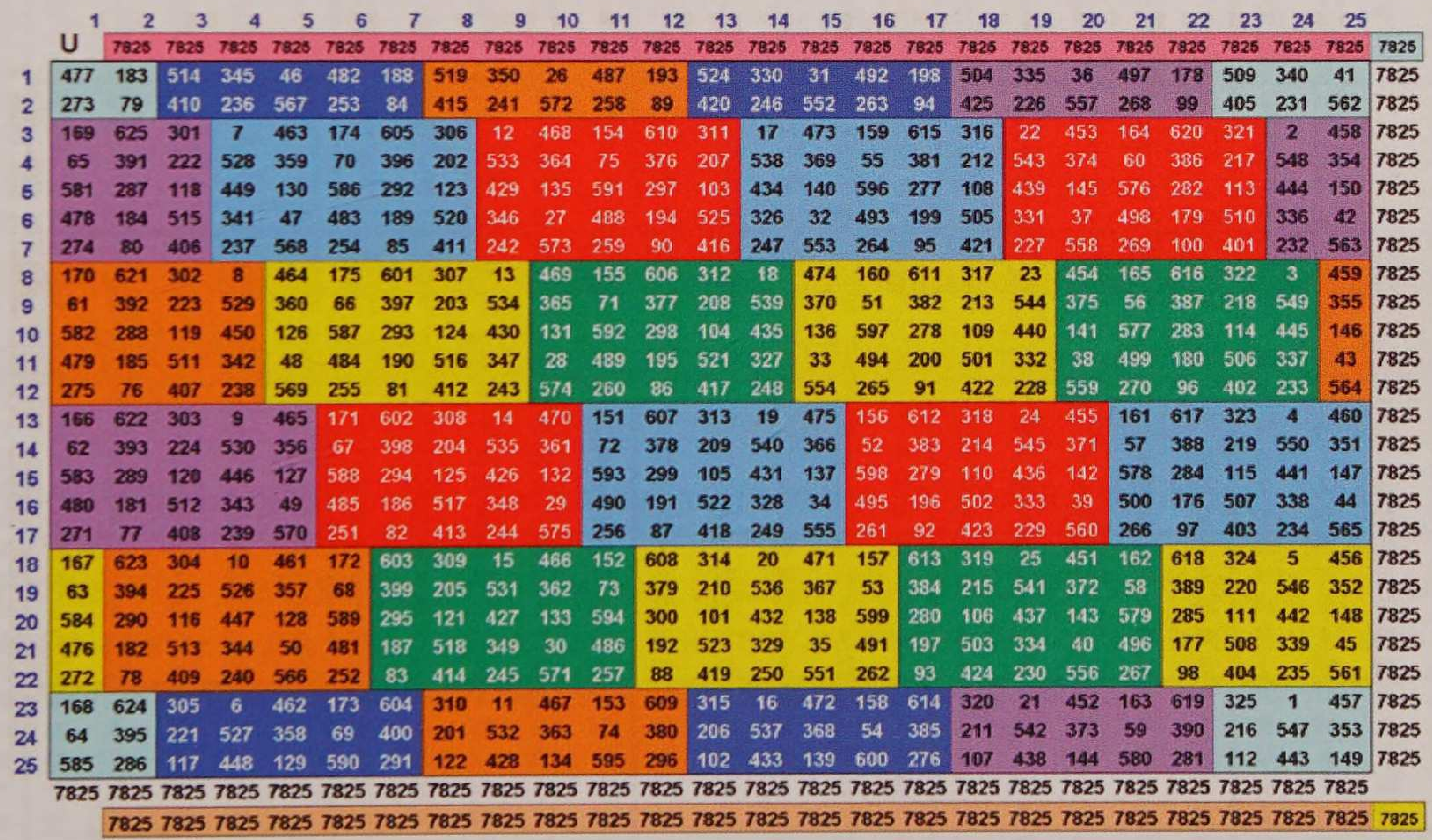
Shown here are loom table differences taken aligned one-on-one (top) and at random (bottom). In addition to all the rows, columns and diagonals summing to **0** in the loom difference tables, all the 3x3 block squares sum continuously to **0** too. So the Size-9 square possesses the cloaking property.

Program 3

Size-25 Primal and Dual Squares



W	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
6	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
7	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
8	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
9	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
10	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
11	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
12	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
13	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
14	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
15	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
16	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
17	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
18	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
19	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
20	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
21	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
22	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
23	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
24	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
25	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825



U	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	477	183	514	345	46	482	188	519	350	26	487	193	524	330	31	492	198	504	335	36	497	178	509	340	41	7825
2	273	79	410	236	567	253	84	415	241	572	258	89	420	246	552	263	94	425	226	557	268	99	405	231	562	7825
3	169	625	301	7	463	174	605	306	12	468	154	610	311	17	473	159	615	316	22	453	164	620	321	2	458	7825
4	65	391	222	528	359	70	396	202	533	364	75	376	207	538	369	55	381	212	543	374	60	386	217	548	354	7825
5	581	287	118	449	130	586	292	123	429	135	591	297	103	434	140	596	277	108	439	145	576	282	113	444	150	7825
6	478	184	515	341	47	483	189	520	346	27	488	194	525	326	32	493	199	505	331	37	498	179	510	336	42	7825
7	274	80	406	237	568	254	85	411	242	573	259	90	416	247	553	264	95	421	227	558	269	100	401	232	563	7825
8	170	621	302	8	464	175	601	307	13	469	155	606	312	18	474	160	611	317	23	454	165	616	322	3	459	7825
9	61	392	223	529	360	66	397	203	534	365	71	377	208	539	370	51	382	213	544	375	56	387	218	549	355	7825
10	582	288	119	450	126	587	293	124	430	131	592	298	104	435	136	597	278	109	440	141	577	283	114	445	146	7825
11	479	185	511	342	48	484	190	516	347	28	489	195	521	327	33	494	200	501	332	38	499	180	506	337	43	7825
12	275	76	407	238	569	255	81	412	243	574	260	86	417	248	554	265	91	422	228	559	270	96	402	233	564	7825
13	166	622	303	9	465	171	602	308	14	470	151	607	313	19	475	156	612	318	24	455	161	617	323	4	460	7825
14	62	393	224	530	356	67	398	204	535	361	72	378	209	540	366	52	383	214	545	371	57	388	219	550	351	7825
15	583	289	120	446	127	588	294	125	426	132	593	299	105	431	137	598	279	110	436	142	578	284	115	441	147	7825
16	480	181	512	343	49	485	186	517	348	29	490	191	522	328	34	495	196	502	333	39	500	176	507	338	44	7825
17	271	77	408	239	570	251	82	413	244	575	256	87	418	249	555	261	92	423	229	560	266	97	403	234	565	7825
18	167	623	304	10	461	172	603	309	15	466	152	608	314	20	471	157	613	319	25	451	162	618	324	5	456	7825
19	63	394	225	526	357	68	399	205	531	362	73	379	210	536	367	53	384	215	541	372	58	389	220	546	352	7825
20	584	290	116	447	128	589	295	121	427	133	594	300	101	432	138	599	280	106	437	143	579	285	111	442	148	7825
21	476	182	513	344	50	481	187	518	349	30	486	192	523	329	35	491	197	503	334	40	496	177	508	339	45	7825
22	272	78	409	240	566	252	83	414	245	571	257	88	419	250	551	262	93	424	230	556	267	98	404	235	561	7825
23	168	624	305	6	462	173	604	310	11	467	153	609	315	16	472	158	614	320	21	452	163	619	325	1	457	7825
24	64	395	221	527	358	69	400	201	532	363	74	380	206	537	368	54	385	211	542	373	59	390	216	547	353	7825
25	585	286	117	448	129	590	291	122	428	134	595	296	102	433	139	600	276	107	438	144	580	281	112	443	149	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Here is another Class-6 square, this time of size-25. Both the primal and dual versions possess **5x5** block-squares which continuously sum to the size-25 square's characteristic number anywhere and everywhere. It is 5x-modular. Due to the nature of their shape, these block squares can be shifted both horizontally and vertically to create many different complementary tiling patterns.

Class-6 squares use the same generation formulas as Class-1 squares.

Program 3

Size-25 Loom Tables X(25) & Y(25)

Here are the size-25 complementary loom tables. Each **5-by-5** block-square contains the numbers **1** thru **25** exactly once. Size-25 squares are continuously **5x5** modular and each block square sums to the size-25 loom-table's characteristic number, L_{25} **= 325**. This is the source for block-square modularity in the primal and dual tables. The loom tables were both derived in the base **25**.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
X	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
1	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
2	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
3	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
4	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
5	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
6	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
7	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
8	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
9	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
10	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
11	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
12	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
13	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
14	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
15	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
16	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
17	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
18	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
19	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
20	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
21	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
22	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
23	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
24	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
25	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
Y	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
1	2	8	14	20	21	7	13	19	25	1	12	18	24	5	6	17	23	4	10	11	22	3	9	15	16	325
2	23	4	10	11	17	3	9	15	16	22	8	14	20	21	2	13	19	25	1	7	18	24	5	6	12	325
3	19	25	1	7	13	24	5	6	12	18	4	10	11	17	23	9	15	16	22	3	14	20	21	2	8	325
4	15	16	22	3	9	20	21	2	8	14	25	1	7	13	19	5	6	12	18	24	10	11	17	23	4	325
5	6	12	18	24	5	11	17	23	4	10	16	22	3	9	15	21	2	8	14	20	1	7	13	19	25	325
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8	20	21	2	8	14	25	1	7	13	19	5	6	12	18	24	10	11	17	23	4	15	16	22	3	9	325
9	11	17	23	4	10	16	22	3	9	15	21	2	8	14	20	1	7	13	19	25	6	12	18	24	5	325
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11	4	10	11	17	23	9	15	16	22	3	14	20	21	2	8	19	25	1	7	13	24	5	6	12	18	325
12	25	1	7	13	19	5	6	12	18	24	10	11	17	23	4	15	16	22	3	9	20	21	2	8	14	325
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14	12	18	24	5	6	17	23	4	10	11	22	3	9	15	16	2	8	14	20	21	7	13	19	25	1	325
15	8	14	20	21	2	13	19	25	1	7	18	24	5	6	12	23	4	10	11	17	3	9	15	16	22	325
16	5	6	12	18	24	10	11	17	23	4	15	16	22	3	9	20	21	2	8	14	25	1	7	13	19	325
17	21	2	8	14	20	1	7	13	19	25	6	12	18	24	5	11	17	23	4	10	16	22	3	9	15	325
18	17	23	4	10	11	22	3	9	15	16	2	8	14	20	21	7	13	19	25	1	12	18	24	5	6	325
19	13	19	25	1	7	18	24	5	6	12	23	4	10	11	17	3	9	15	16	22	8	14	20	21	2	325
20	9	15	16	22	3	14	20	21	2	8	19	25	1	7	13	24	5	6	12	18	4	10	11	17	23	325
21	1	7	13	19	25	6	12	18	24	5	11	17	23	4	10	16	22	3	9	15	21	2	8	14	20	325
22	22	3	9	15	16	2	8	14	20	21	7	13	19	25	1	12	18	24	5	6	17	23	4	10	11	325
23	18	24	5	6	12	23	4	10	11	17	3	9	15	16	22	8	14	20	21	2	13	19	25	1	7	325
24	14	20	21	2	8	19	25	1	7	13	24	5	6	12	18	4	10	11	17	23	9	15	16	22	3	325
25	10	11	17	23	4	15	16	22	3	9	20	21	2	8	14	25	1	7	13	19	5	6	12	18	24	325
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

Program 3

Size-25 Loom Difference Table

Here is the size-25 loom difference table. Every **5x5** block-square sums to **0** continuously. Further, randomly aligned differences do the same.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	X(25)-Y(25)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	18	0	7	-6	-19	13	-5	2	-11	1	8	-10	-3	9	-4	3	-15	17	4	-9	-2	5	12	-1	-14	0
	-12	0	7	-1	6	8	-5	2	-6	1	3	-10	-3	-11	21	-2	-15	-8	9	16	-7	-20	12	4	11	0
	-12	0	12	-6	6	-17	20	7	-11	1	3	15	2	-16	-4	-2	10	-3	-21	16	-7	5	-8	-1	11	0
	-12	0	-13	19	6	-17	-5	7	14	1	-22	15	2	9	-4	-2	10	-3	4	-9	-7	5	-8	-1	11	0
	18	0	-13	-6	1	13	-5	-18	14	-4	8	-10	2	9	-9	3	10	-3	4	-14	23	5	-8	-1	-19	0
	17	-1	6	-2	-20	12	-6	1	-7	0	7	-11	-4	13	-5	2	-16	16	8	-10	-3	4	11	3	-15	0
	-13	-1	11	-2	5	7	-6	6	-7	0	2	-11	1	-12	20	-3	-16	-4	8	15	-8	-21	16	3	10	0
	-13	4	11	-7	5	-18	24	6	-12	0	2	19	1	-17	-5	-3	14	-4	-22	15	-8	9	-9	-2	10	0
	-8	-1	-14	18	5	-13	-6	6	13	0	-18	14	1	8	-5	2	9	-4	3	-10	-3	4	-9	-2	10	0
	17	-1	-14	-7	5	12	-6	-19	13	0	7	-11	1	8	-5	2	9	-4	3	-10	22	4	-9	-2	-15	0
	16	-2	10	-3	-21	11	-7	5	-8	-1	6	-12	0	12	-6	1	-17	20	7	-11	-4	3	15	2	-16	0
	-14	3	10	-3	4	6	-2	5	-8	-1	1	-7	0	-13	19	-4	-12	-5	7	14	-9	-17	15	2	9	0
	-9	3	10	-8	4	-14	23	5	-13	-1	6	18	0	-18	-6	1	13	-5	-23	14	-4	8	-10	-3	9	0
	-9	-2	-15	17	9	-14	-7	5	12	4	-19	13	0	7	-1	1	8	-5	2	-6	-4	3	-10	-3	14	0
	16	-2	-15	-3	4	11	-7	-20	17	-1	6	-12	0	12	-6	1	8	-5	7	-11	21	3	-10	2	-16	0
	15	2	9	-4	-22	10	-3	4	-9	-2	5	-8	-1	11	-7	0	-13	19	6	-12	-5	7	14	1	-17	0
	-10	2	9	-4	3	10	-3	4	-9	-2	5	-8	-1	-14	18	0	-13	-6	6	13	-5	-18	14	1	8	0
	-10	2	9	-9	8	-15	22	4	-14	3	5	17	-1	-19	-2	0	12	-6	-24	18	-5	7	-11	-4	13	0
	-10	-3	-16	21	8	-15	-8	4	16	3	-20	12	-1	11	-2	0	7	-6	6	-7	-5	2	-11	1	13	0
	15	-3	-11	-4	3	10	-8	-16	16	-2	5	-13	4	11	-7	0	7	-1	6	-12	20	2	-6	1	-17	0
	19	1	8	-5	-23	14	-4	3	-10	-3	9	-9	-2	10	-8	4	-14	18	5	-13	-1	6	13	0	-18	0
	-11	1	8	-5	7	9	-4	3	-10	2	4	-9	-2	-15	22	-1	-14	-7	5	17	-6	-19	13	0	12	0
	-11	1	8	-5	7	-16	21	3	-10	2	4	16	-2	-15	-3	-1	11	-7	-20	17	-6	6	-12	0	12	0
	-11	-4	-12	20	7	-16	-9	8	15	2	-21	11	3	10	-3	-1	6	-2	5	-8	-6	1	-7	0	12	0
	14	1	-12	-5	2	9	-4	-17	15	-3	4	-9	3	10	-8	-1	11	-2	5	-13	19	6	-7	0	-18	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The yellow cells highlight points of equality between the complementary loom tables.

Make note: The loom tables of all Class-6 squares also possess the cloaking property. This cloaking property is an amazing property that may someday find security applications – that's yet another potential supplication that this math would be good for.

Program 3

Summary Table of Cloaking Property by Class

	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6
Cloaking Complementary looms	**Yes**	**No**	**Yes, but only at the bxb block-square level**	**Yes**	**Yes, but only at the bxb block-square level**	**Yes**

All but Class-2 squares possess the cloaking property.

Note that Class-3 & Class-5 squares possess the cloaking property but only at the block-square pattern level.

Unification of all the Square-generation Formulas

Complementary loom tables **X** and **Y** exist for Classes 3, 4 & 5 squares only if derived in the base **b** where **c** is the class of square of size **n**:

(3.21) $b = n/c$

(3.22) $W(n) = n/c\,(\,Y(b) - |1|\,) + X(b)$ for generation of the Primal square

(3.23) $U(n) = c \cdot n\,(\,X(b) - |1|\,) + Y(b)$ for generation of the Dual square

Class-2 squares do not possess complementary loom tables that are geonomic and so are excluded from these formulas.

Recall that for Class-1 squares **b = n/c = n** and **c▪n = n** since **c = 1**. So class was not visibly part of the generation formulas.

It was then determined later that Class-4 squares whose size was a multiple of **8** and Classes-5 and 6 squares could not only be treated the same as Class-1 squares, but also could be treated like Class-3 squares here too.

For Class-5 squares, these more generalized formulas only hold if **a** replaces **c**, where **a>b>3** and **n = ab**.

However, this generalized form completely eliminates the loom-cloaking property for all but Class-1 squares.

Remark: The size-27 square is a special case among the Class-3 squares shown thus far. It has a different dual that can be derived from loom tables taken to the base **27** as well as the base **9**. Those loom tables are also geonomically ultra-perfect.

Further, their loom difference table sums to **0** everywhere. So do all the block-tile patterns. Consequently it possesses the cloaking property.

Since this square was the only exception found in the range of Class-3 squares investigated, the size-27 square would be better treated as a square from Class-6 and has therefore been included in that class too where **b = 3** and **a = 3**; $n = b^a$.

Program 3

Application #1 of Geonometry

The Membrane of Life

In this segment of the program we will explore the helical patterns in Class-5 squares and see just how Geonometry can be applied to the formation of DNA molecules.

To verify what is presented next, you will need to refer to the size-35 square and its loom tables shown earlier in this program.

Program 3

The Double and Triple Helixes in Class-5 Squares
Size-35 Modulus Loom Table X(35)

The size-35 modulus loom table is seen to contain a double helical pattern when the distribution of even numbers is highlighted against the distribution of odd numbers. Note the zipper-like pattern where the rows change from even numbers to odd numbers.

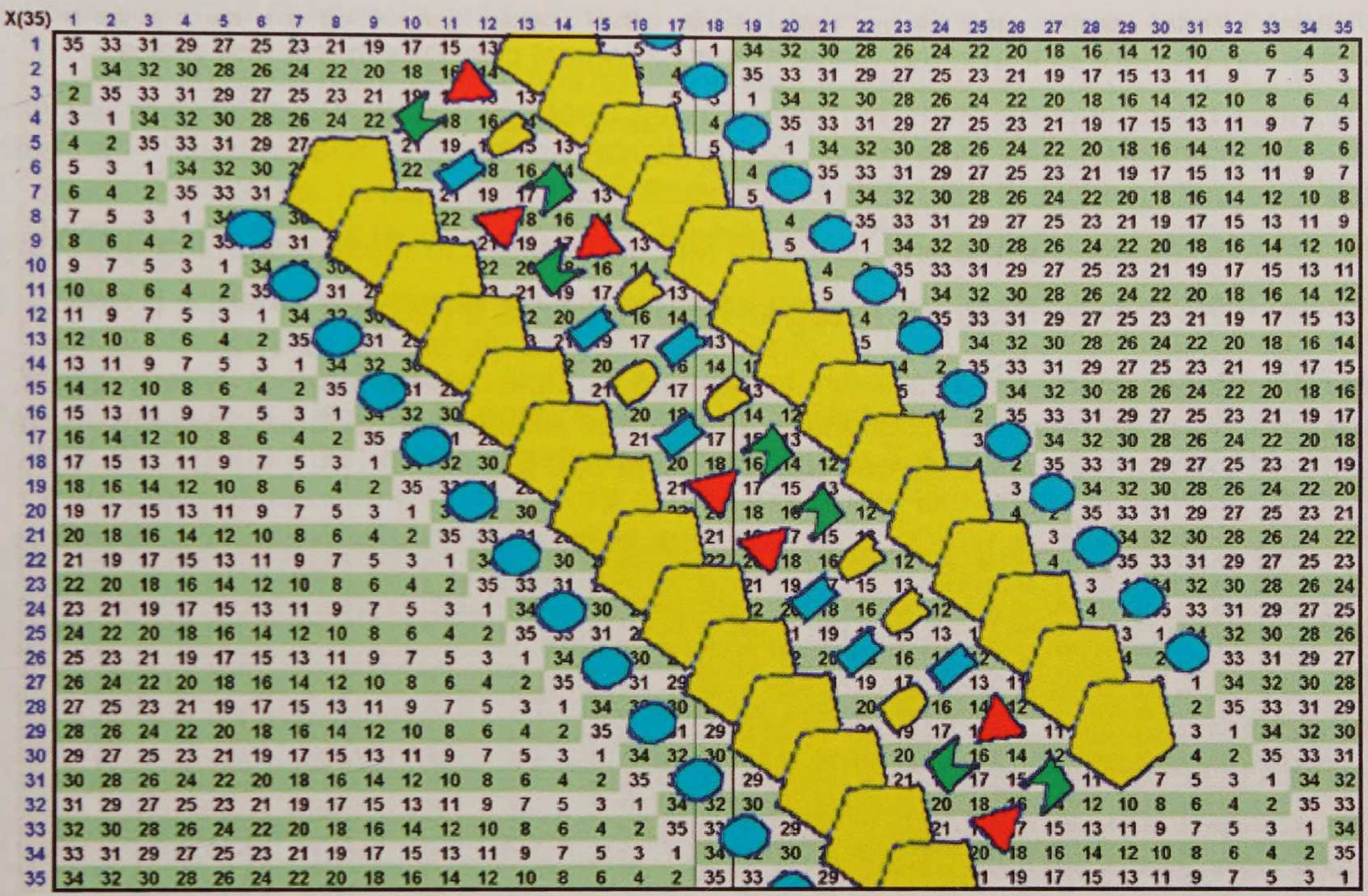
Down 2 over 1

X(35)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2
2	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3
3	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4
4	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5
5	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6
6	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7
7	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8
8	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9
9	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10
10	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11
11	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12
12	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13
13	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14
14	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15
15	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16
16	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17
17	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18
18	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19
19	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20
20	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21
21	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22
22	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23
23	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24
24	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25
25	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26
26	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27
27	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28
28	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29
29	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30
30	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31
31	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32
32	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33
33	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34
34	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35
35	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1

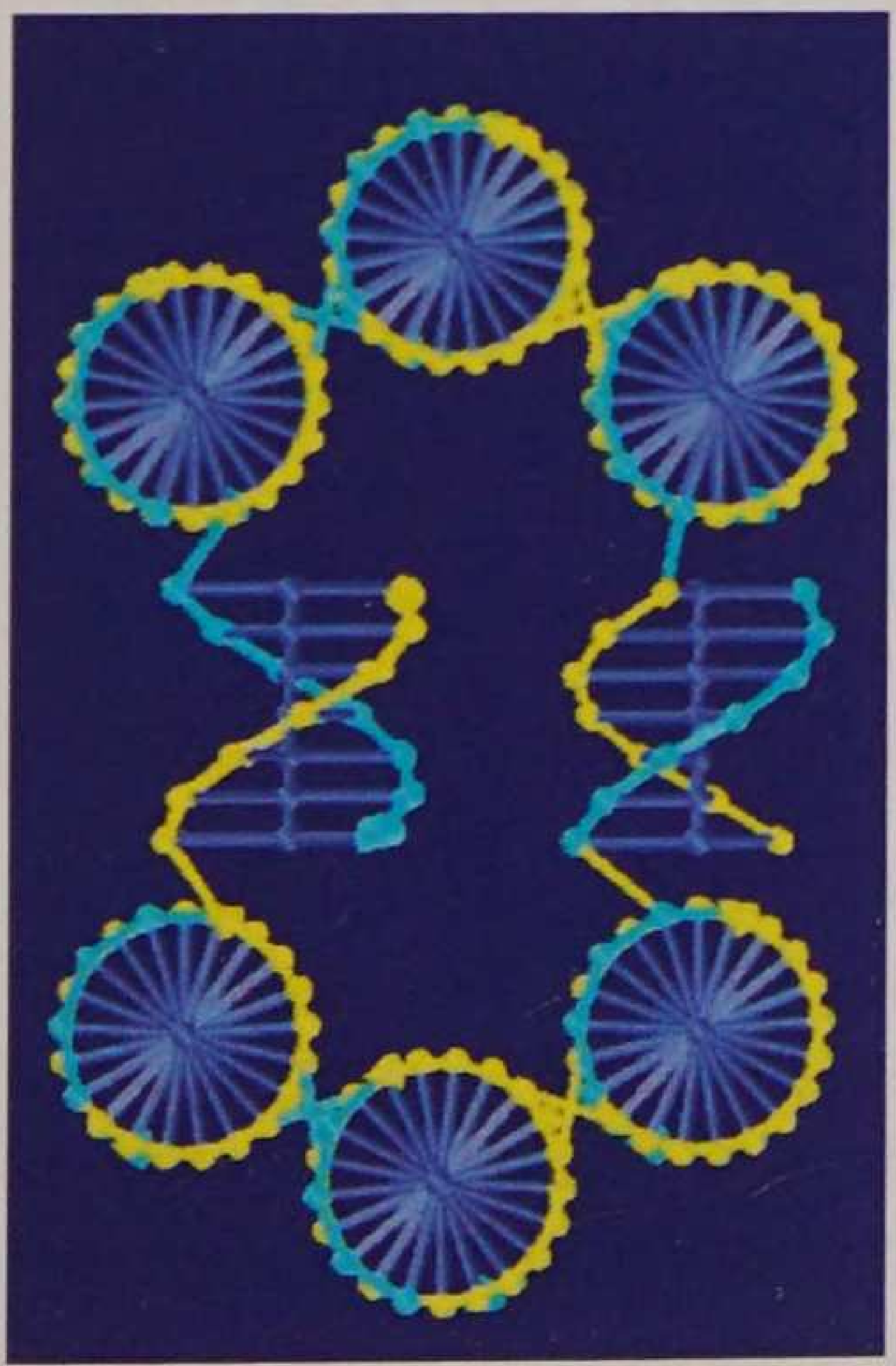
Program 3

The membrane of Life

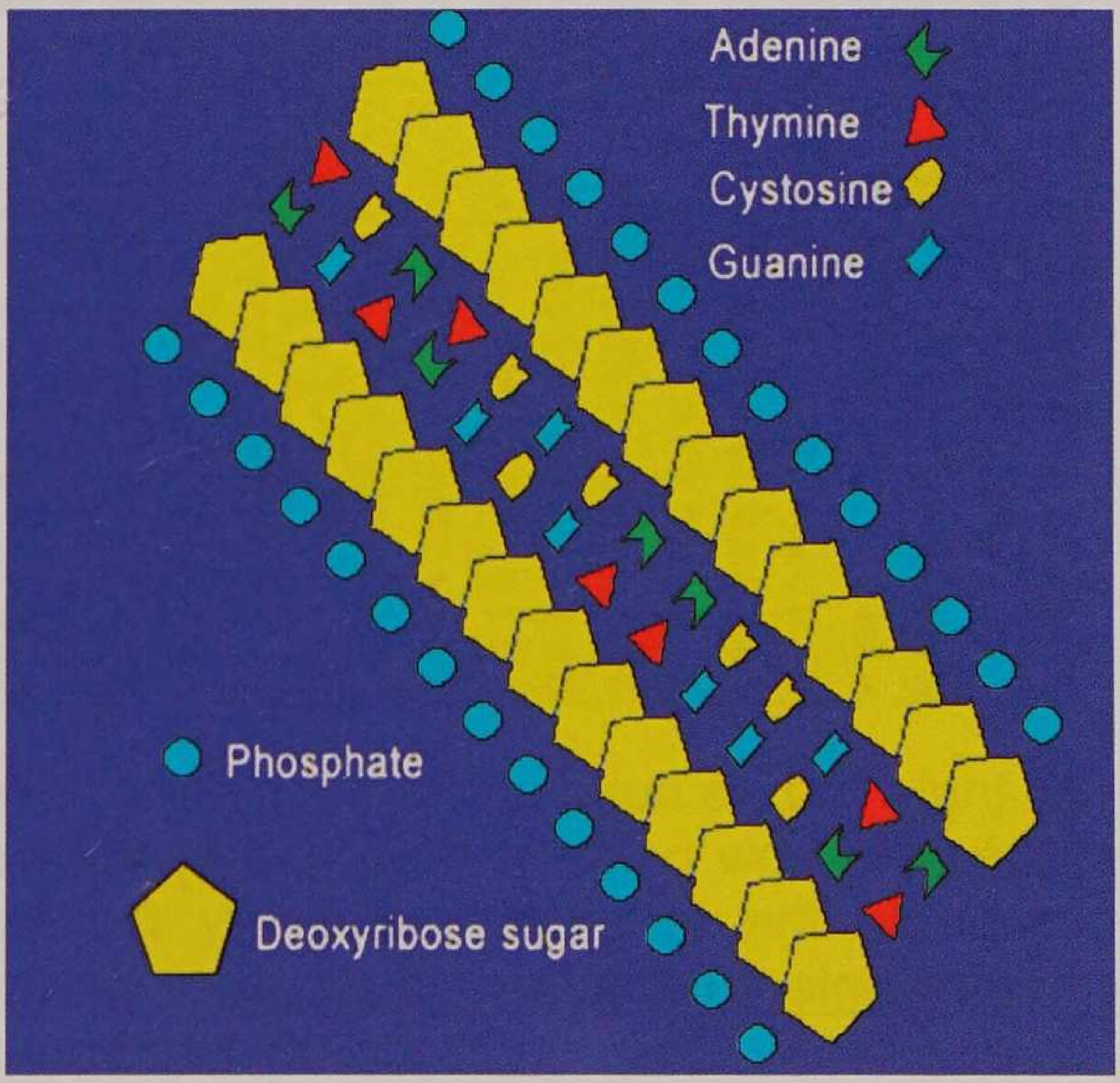
The double helical pattern may explain how phosphate atoms connected with sugar molecules collect along the zipper-like seams in the planar frequencies of a spatial membrane from a primordial soup of various nucleotides. Once these sugar molecules capture nucleotides, these nucleotides are attracted to their counterpart by a non-covalent weak-bond. This configuration then automatically contorts itself into a 3-dimensional double helix. The two seams connect in a spiraling ladder structure due to the dissymmetry of adjacent sugar molecules.



At left is the double helical closed loop DNA structure looking from the top down.



At right is shown the inherent bonds between nucleotides and how they would attract one another in this curling-up process. Note how Adenine is only attractted to Thymine and Cytosine is only attracted to Guanine.



The implication here is that there definitely is more to evolution than just natural selection.

For instances: What caused the big jumps such as going from amphibians to mammals? Or from cold-blooded creatures to warm-blooded ones? Or from 4-legged animals to ones with flapping wings for flight? How did that just evolve from something less complex to something functionally more complex with no track record inbetween?

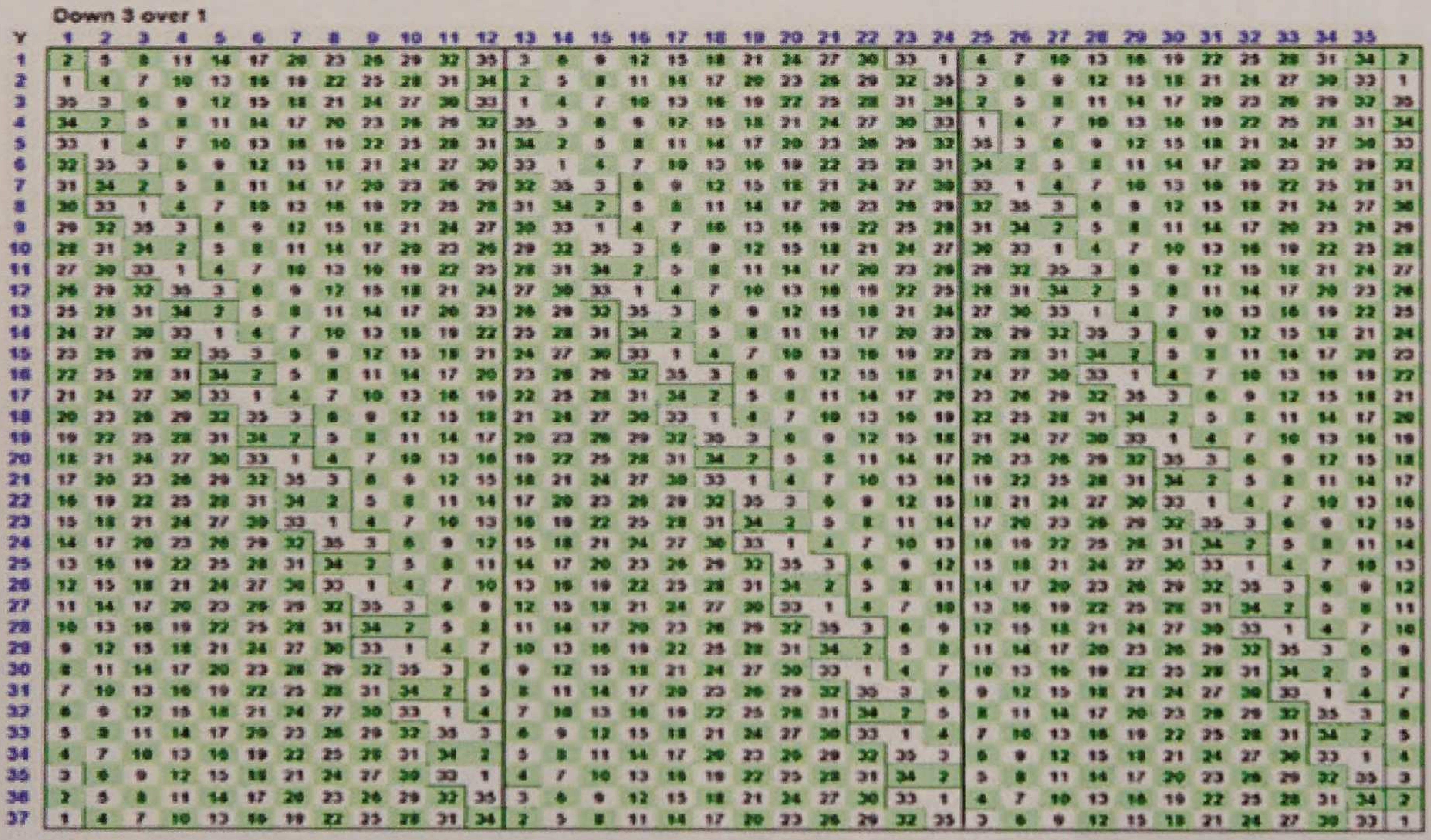
How did the first mammary gland evolve in mammals? How could a female's milk-producing mammary glands evolve through natural selection over time? Wouldn't they need to be fully functional right from the get-go? Wouldn't the mother require to be fed herself upon her birth yet without an initial source of milk? There had to be something more than natural selection of the fittest going on here to introduce these major biological changes in the first place.

Might all these major sudden shifts in function have been the result of a sudden major shift in the DNA of offspring caused by a ripple in the structured fabric of space itself? – something like a transient rolling wave in the fabric of space akin to a tsunami?

Don't be afraid to demand some answers from Micro-biologists to these disturbing questions.

Size 35 Integer Loom Table Y(35)

The size-35 integer loom table is seen to contain a triple helical pattern when, again, the distribution of even numbers is highlighted against the distribution of odd numbers. Note the checkerboard pattern this time between the even and odd numbers.



Down 3 over 1

Y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
1	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2
2	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1
3	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35
4	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34
5	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33
6	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32
7	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31
8	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30
9	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29
10	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28
11	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27
12	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26
13	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25
14	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24
15	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23
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20	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18
21	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17
22	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16
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31	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7
32	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6
33	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5
34	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4
35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3
36	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1	4	7	10	13	16	19	22	25	28	31	34	2
37	1	4	7	10	13	16	19	22	25	28	31	34	2	5	8	11	14	17	20	23	26	29	32	35	3	6	9	12	15	18	21	24	27	30	33	1

The triple helix is not a result of a natural assembly process as is the double helix, but this very structure is now being studied and synthesized by molecular biologists for creating other life forms. So the template necessary for this to take place may also be the underlying complementary harmonic vibrations in Class-5 planar membranes. We will see in Program 8 just how these Class-5 vibrational membranes can be expanded to any multiple of its size by three different expansion methods. So these flattened helical spatial patterns can exist at any multiple of its basic size.

Notes

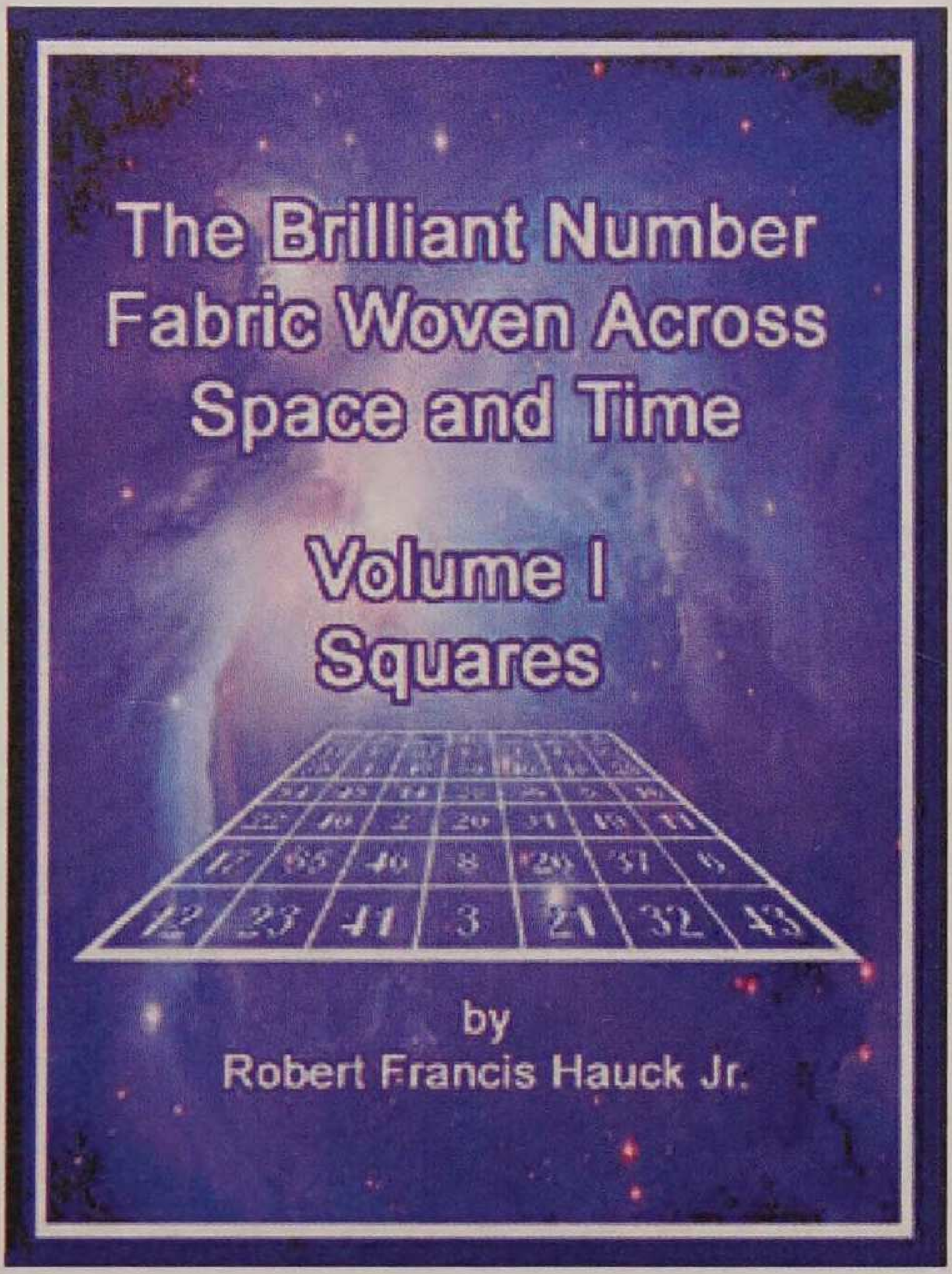
1. All classes of squares, except those of Class-2, possess the loom-cloaking property.
2. Class-3 squares have continuous complementary tiling patterns at the **bxb** block-square level where **n = 3b**.
3. Class-4 squares have continuous diamond and X complementary tiling patterns that are confined to **2bx2b** block-squares where **n = 4b**.
4. Class-5 squares have continuous tiling patterns simultaneously at the block-square level at both of their factor's sizes **a** & **b** where **n = ab**.
5. Class-6 squares are simply continuously **bx-**modular at the **b** block-square level where $\mathbf{n = b^a}$. So its tiles are simply **bxb** block-squares.
6. The version of the size-35 square shown in this program was derived in the base **35** by the **ATE** generation method described in Program 8. Only Class-5 squares generated from loom-tables in the base **n = ab** exhibit the zipper-like patterns between their strings of even and odd numbers.
7. There are three subclasses of Class-4 squares, where **n = 4b**:
    a. **n** is a multiple of **8**.
    b. **b** is an odd-number.
    c. **b** is an even number not divisible by **4**,. (This sub-class is the only one to make use of near-perfect Class-2 squares such as **6**, **10** and **14** in their manifestation and that involves the **TAP** expansion method in Program 8.)

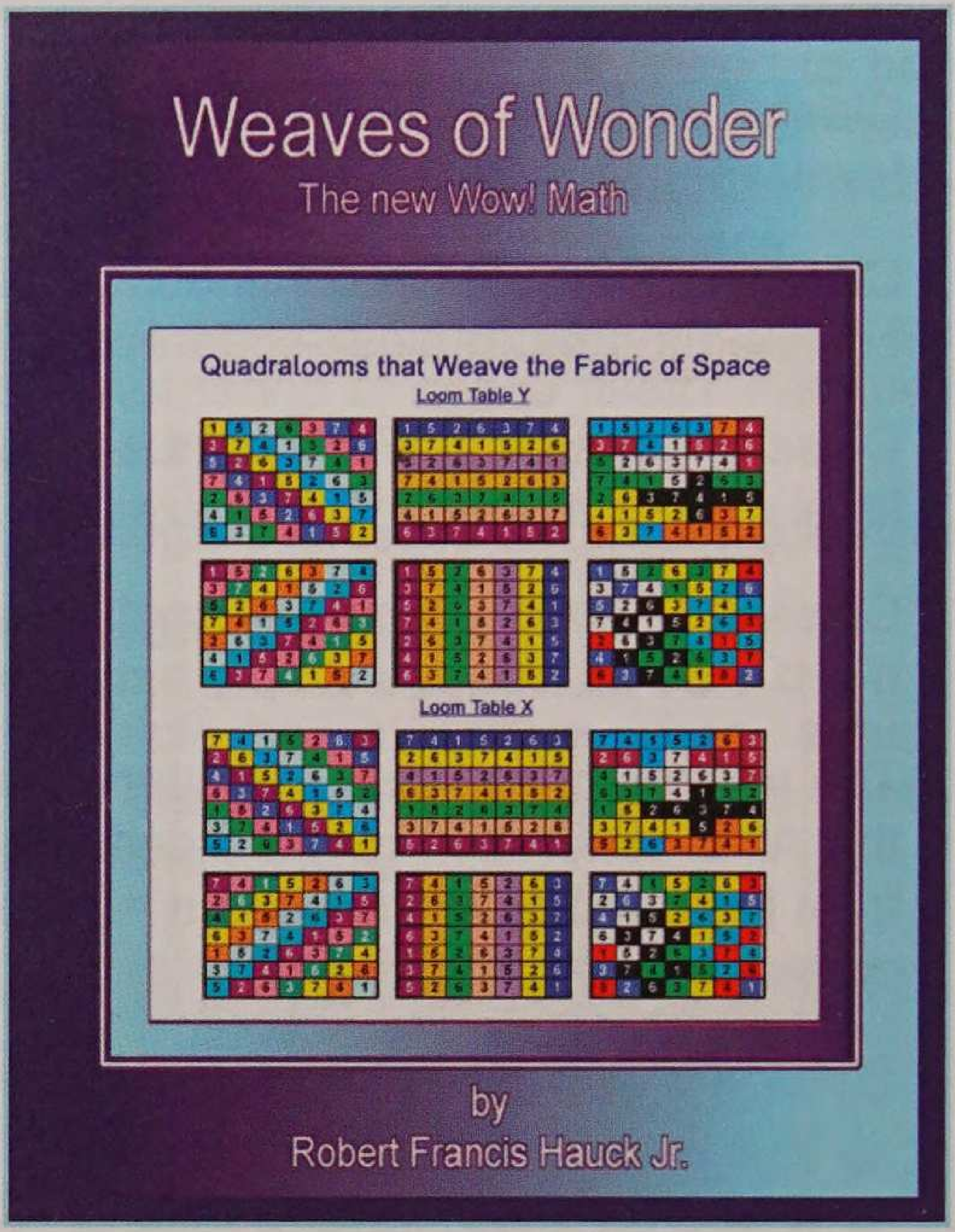
* * *

Program 3

We have come to the end of program 3 of the new **math**, **Geonometry**. I hope that this program has been as awe-inspiring for you as it was for me in its development.

Here are the two books upon related to this program.





The Brilliant Number Fabric Woven across Space and Time
Volume I - Squares

ISBN: 978-1-461-06984-3

Third Edition (140 pages)

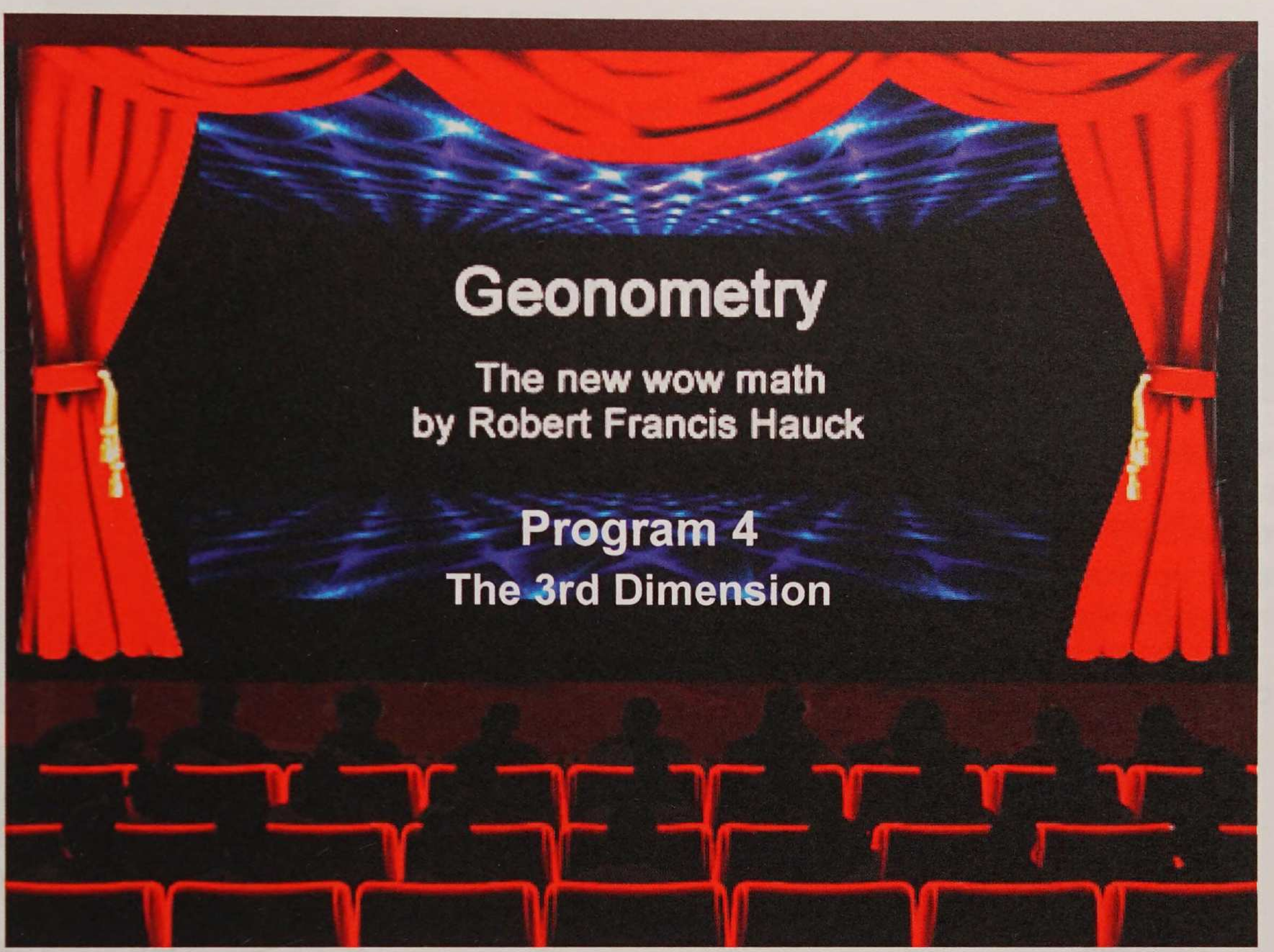
Weaves of Wonder –The New Wow Math

ISBN: 978-1-469-93296-5

Shows how to construct geonomic squares from loom tables;
First Edition (128 pages)

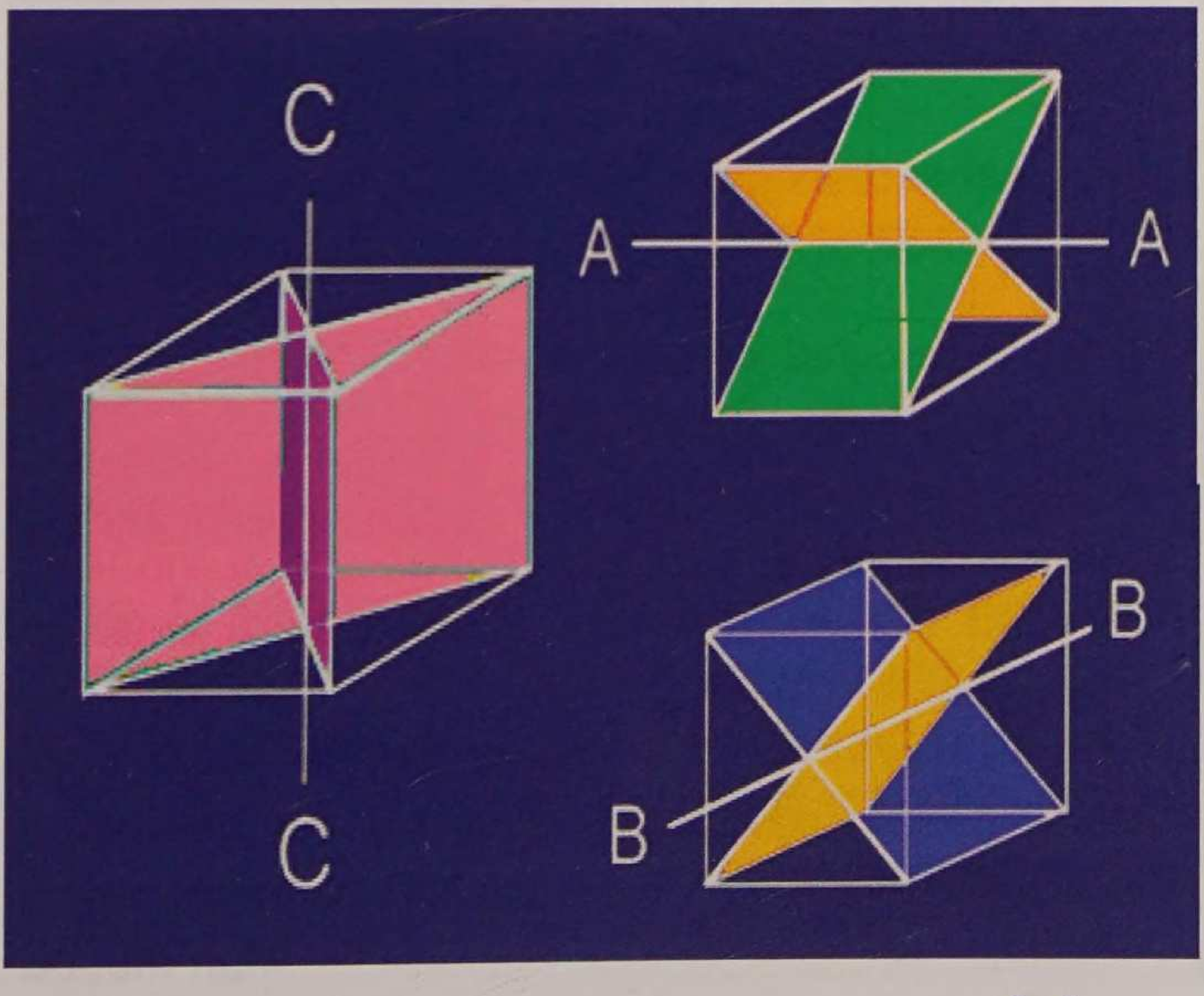
In the next program we will explore the properties of the 3rd dimension and the amazing equal-summing 1-dimensional and 2-dimensional patterns discovered in geonomic cubes. We will accomplish this through the compact view of yet another type of table, called the **depth-sum table**. There we will begin the incredible mathematical journey up through higher dimensions and down to lower sub-dimensions as only Geonometry can provide.

Program 4



Next we will address cubic tables which are shown to actually tap into, measure and map 3-dimensional space.

Introduction



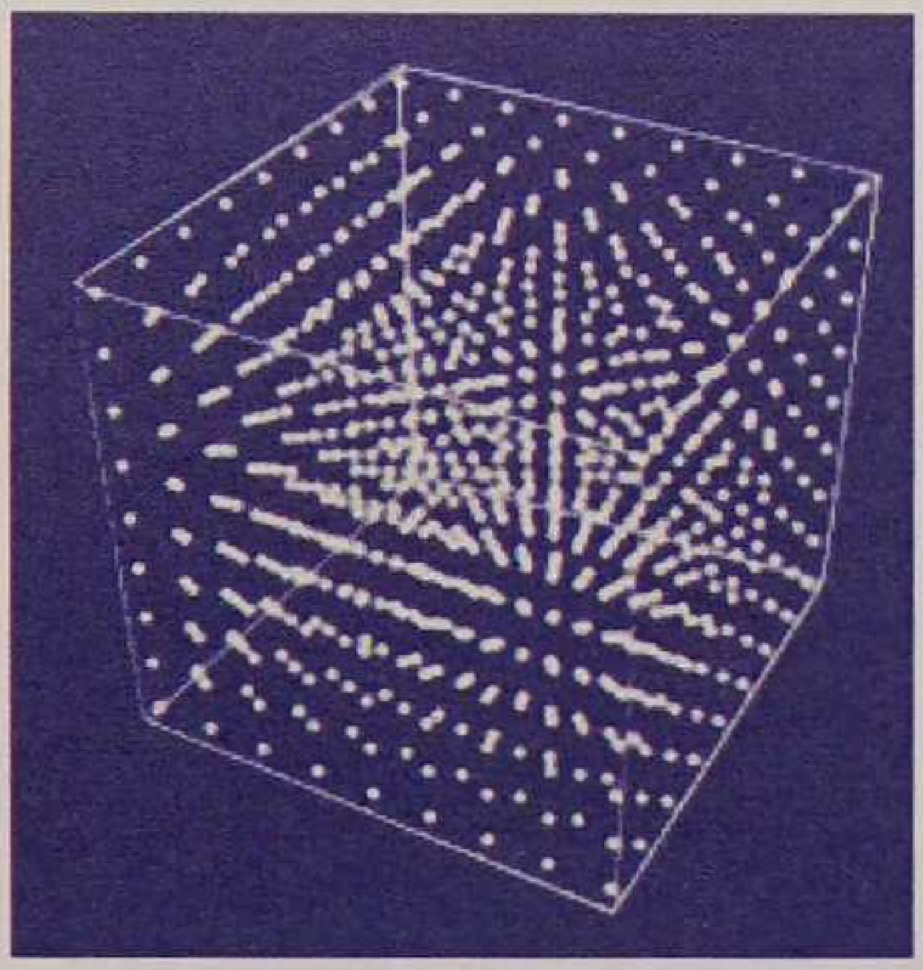
Here are the **3** directions that will be referred to by letters **A B** & **C** in identifying the depth-sum tables taken along their designated axis.

All cubes are described by rectangular tables of embedded squares along the B-axis; akin to the cards in a deck of cards.

A cube of size **n** has **n** block-squares of size **n-by-n** embedded within it. Bottom-to-top in the table correlates with going from front-to-back in the cube along the B-axis.

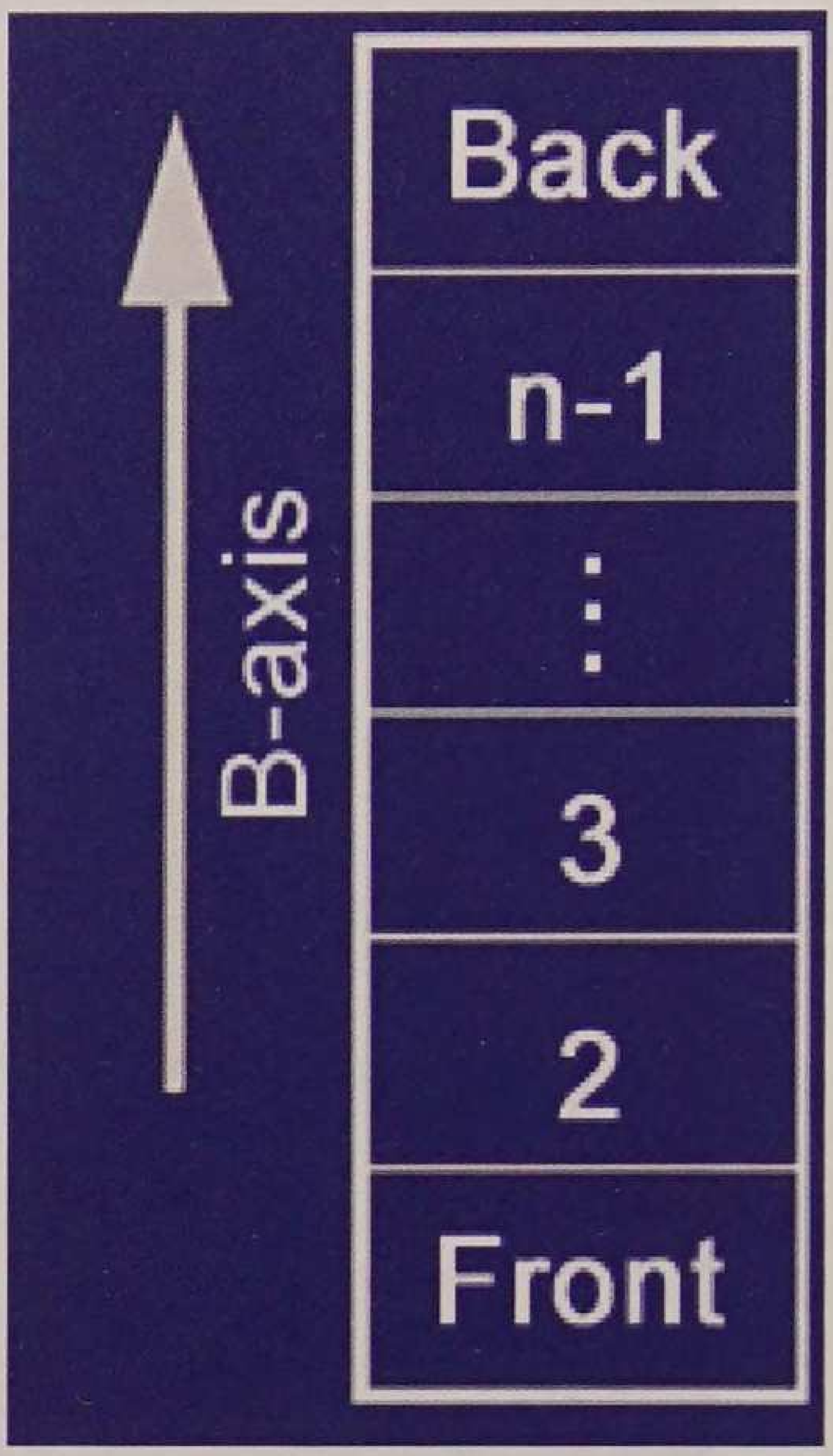
Program 4

Cubic Definitions

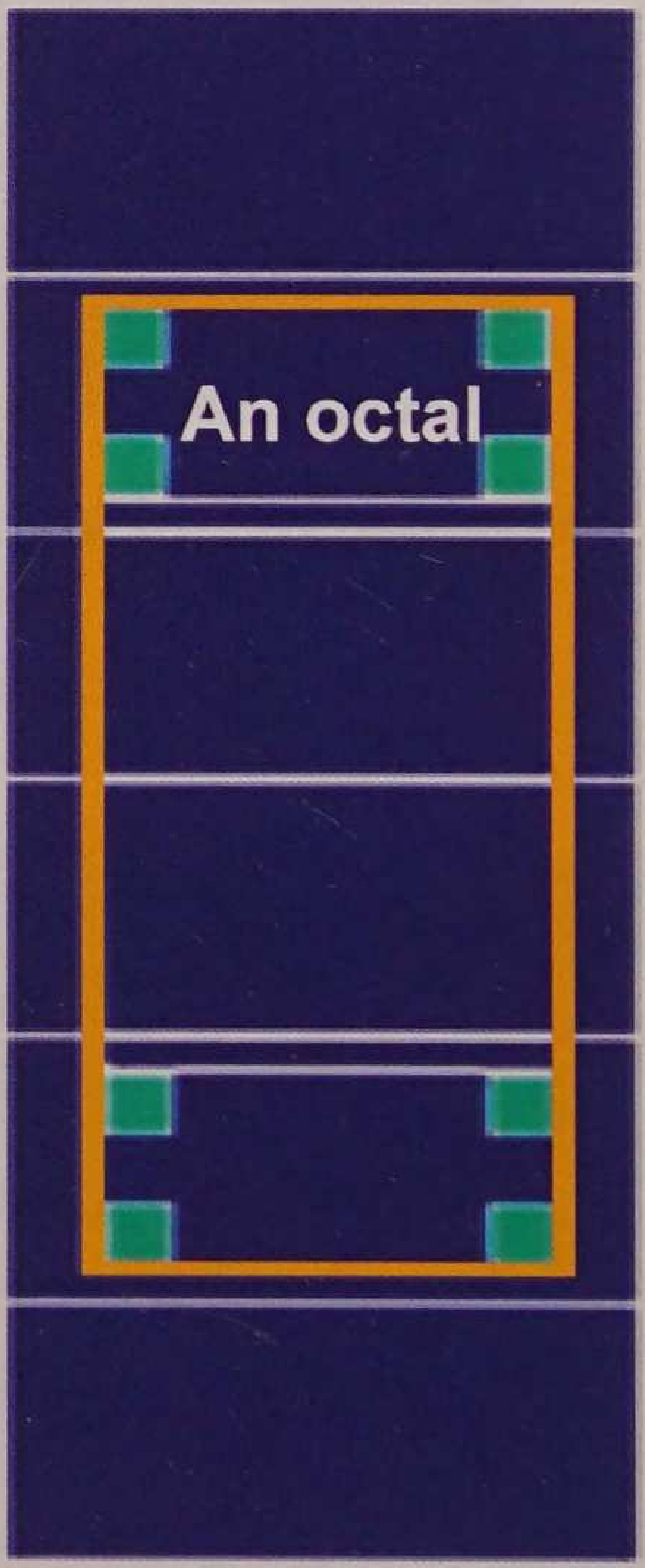


As shown at right, the bottom embedded block-square lies at the front of the cube along the B-axis and the top block-square lies at the back.

Here are some basic definitions for cubes:



The **octal** is the 3-dimensional counter-part of the quadral in squares. These are the sum of the numbers at the **8** corners of a symmetrically-centered box within the cube. In a cubic table, they are the numbers in identical quadrals of two equidistant centrally-symmetric embedded block-squares as shown below at right.



A cube has the status of being **perfect** if:

1. All the linear channels along the A-axis and pillars along the C-axis each sum equally.
2. All the main planar diagonals sum equally.
3. All octals sum equally: that is, the cube is 3-dimensionally pangenic.

A perfect cube is **absolutely-perfect** when, in addition:

1. All the diagonal planes sum equally.
2. The depth-sum table along the **B** axis collapses to a perfect square.

A cube is also **ultra-perfect** when it contains equal-summing dual tile patterns in each of its embedded squares.

Note: the tile patterns are confined to squares and do not cross the borders of embedded squares but do wrap around the individual squares themselves, both horizontally and vertically.

All cubes that are associated with Class-1 squares are ultra-perfect. On the otherhand, cubes whose corresponding squares are from Class-4 which have characteristic equal-summing diamond and X-tile patterns have not been found to be also ultra-perfect in that these equal-summing patterns were lost in the construction method for cubes.

Program 4

Class-1 Cubes

Size-5 Cube

Block-square minor diagonal sums

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

		1	2	3	4	5		
			1575	1575	1575	1575	1575	
	1	95	33	121	64	2	315	
	2	111	54	17	85	48	315	
5	3	7	100	38	101	69	315	1575
	4	28	116	59	22	90	315	
	5	74	12	80	43	106	315	
	1	45	108	71	14	77	315	
	2	61	4	92	35	123	315	
4	3	82	50	113	51	19	315	1575
	4	103	66	9	97	40	315	
	5	24	87	30	118	56	315	
	1	120	58	21	89	27	315	
	2	11	79	42	110	73	315	
3	3	32	125	63	1	94	315	1575
	4	53	16	84	47	115	315	
	5	99	37	105	68	6	315	
	1	70	8	96	39	102	315	
	2	86	29	117	60	23	315	
2	3	107	75	13	76	44	315	1575
	4	3	91	34	122	65	315	
	5	49	112	55	18	81	315	
	1	20	83	46	114	52	315	
	2	36	104	67	10	98	315	
1	3	57	25	88	26	119	315	1575
	4	78	41	109	72	15	315	
	5	124	62	5	93	31	315	
		1575	1575	1575	1575	1575		
			1575	1575	1575	1575	1575	

Quadrals

277	152	277
277	277	277

Octals

504	504	504
504	504	504

Quadrals

202	327	202
327	202	202

Octals

504	504	504
504	504	504

Quadrals

252	252	252
252	252	252

Octals

504	504	504
504	504	
504	504	

Quadrals

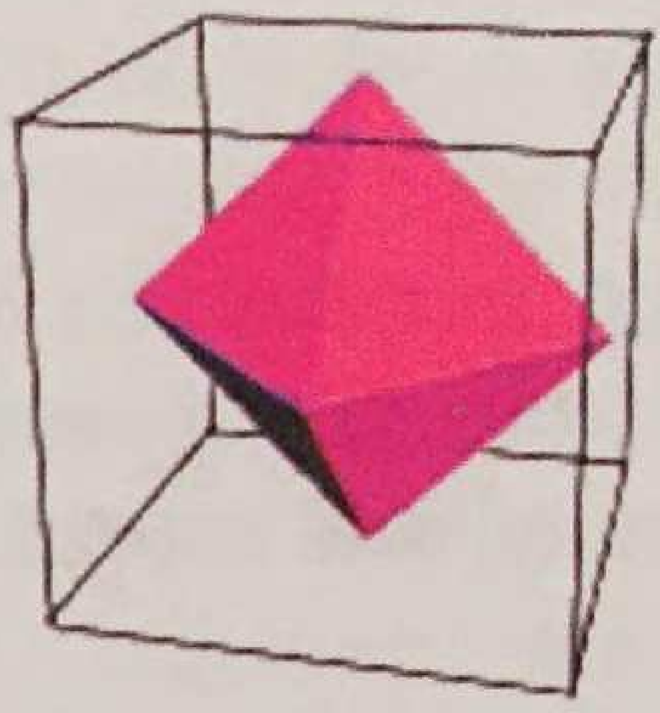
302	177	302
177	302	302

Octahedrons

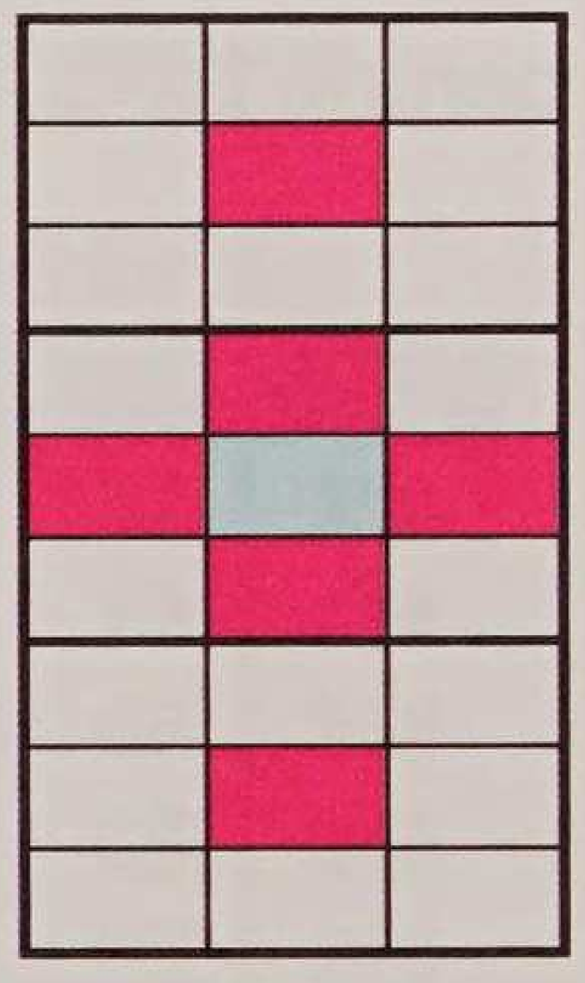
378
378

Quadrals

227	352	227
227	227	227



Octahedron in 3D



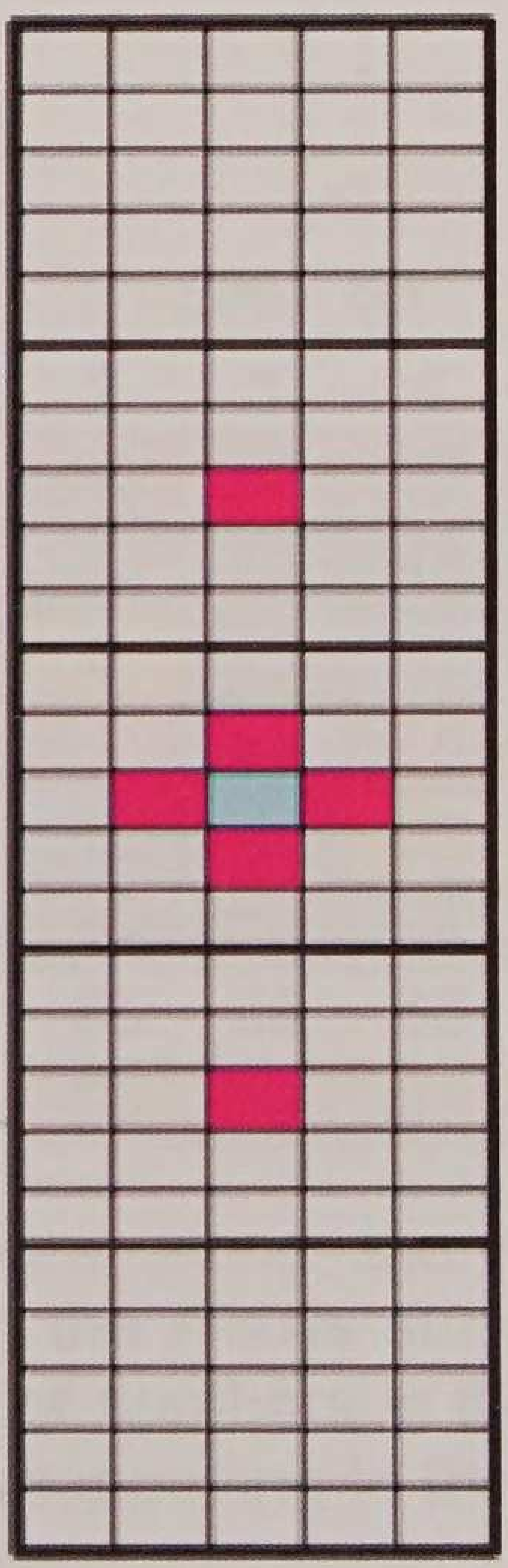
Block-square major diagonal sums

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

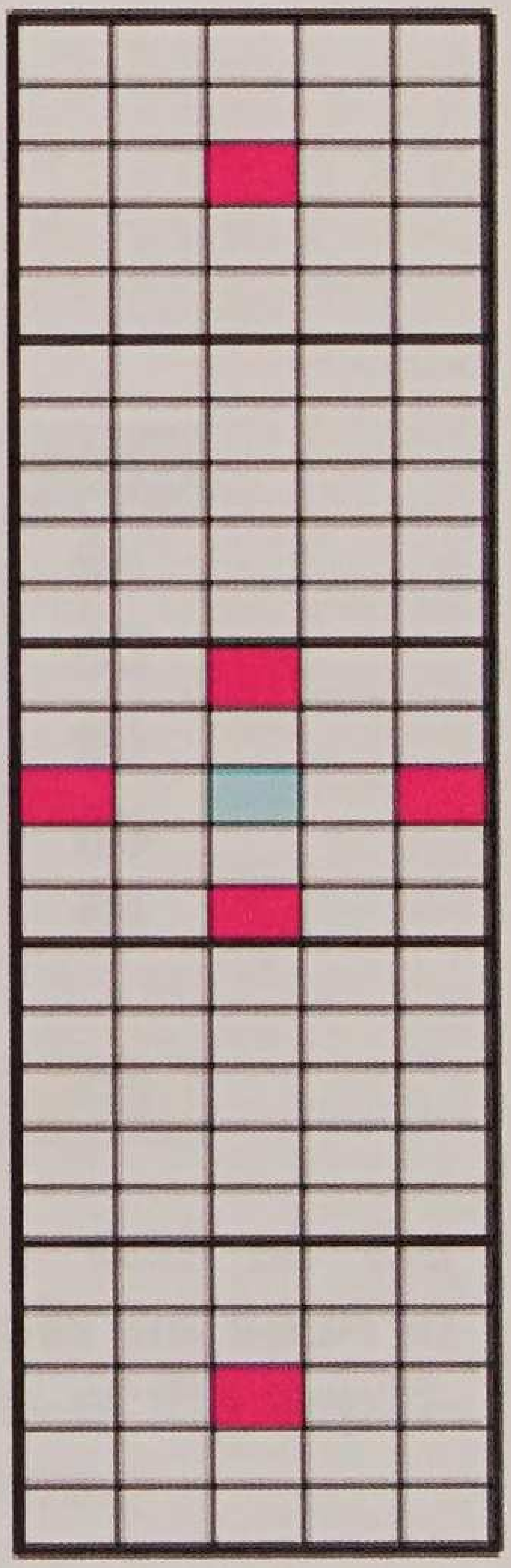
Here is an absolutely perfect cubic table of size-5 from class-1. Its octals sum equally to **504**.

There are two octahedrons in this cube, one nested within the other, whose **6** corners sum equally to **378**. This octahedral value is always 3-quarters of the kernel number. These octahedral patterns are depicted in the rectangular table of the cube on the right, by crimson-colored cells. Their shape and relationship to the cube is depicted in the picture at top right. Their associated cells in the cubic table are shown at bottom right.

Octahedron #1



Octahedron #2



Program 4

The colored cells in the quadral and octal sums at center are those which associated with the central embedded square. The top row in the last octal summation box is the octals formed from two symmetrically positioned quadrals in the central square. Those octal sums were obtained from adding together the top and bottom rows of the central quadral summation box.

The other octal sums were obtained from adding together the numbers in the same location from two symmetrically located quadral sum boxes. Throughout this program, this is the typical representation.

Every row, column and diagonal in both minor and major directions, including wrapping in every block-square, sums to **315**. This qualifies the cube to be also *absolutely* - perfect.

Only the central block-square #3 is pangenic as a square, with quadrals equal to **252**.

Program 4

5x3D Depth-sum Tables

Depth-sums along A-axis

A	1575	1575	1575	1575	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
1575	1575	1575	1575	1575	
	1575	1575	1575	1575	1575

Depth-sum quadrals

1260	1260	1260
1260	1260	1260

Depth-sums along B-axis

B	1575	1575	1575	1575	1575
350	290	355	320	260	1575
305	270	335	300	365	1575
285	375	315	255	345	1575
265	330	295	360	325	1575
370	310	275	340	280	1575
1575	1575	1575	1575	1575	
	1575	1575	1575	1575	1575

Depth-sum quadrals

1260	1260	1260
1260	1260	1260

Depth-sums along C-axis

C	1575	1575	1575	1575	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
315	315	315	315	315	1575
1575	1575	1575	1575	1575	
	1575	1575	1575	1575	1575

Depth-sum quadrals

1260	1260	1260
1260	1260	1260

The depth-sum table **B** collapses to an ultra-perfect size-5 square by first dividing every number in the depth-sum table by **5** and then subtracting **50** from each number in the resulting table. That number is **1** less than the minimum number in the table resulting from the first operation. But make note that that number is always the difference between the pivot numbers of the same size square and cube.

Reduction of Depth-sum Table B to a perfect size-5 square

Depths-sums B divided by 5

B/5	1575	1575	1575	1575	1575
70	58	71	64	52	315
61	54	67	60	73	315
57	75	63	51	69	315
53	66	59	72	65	315
74	62	55	68	56	315
315	315	315	315	315	
	315	315	315	315	315

B/5 - 50

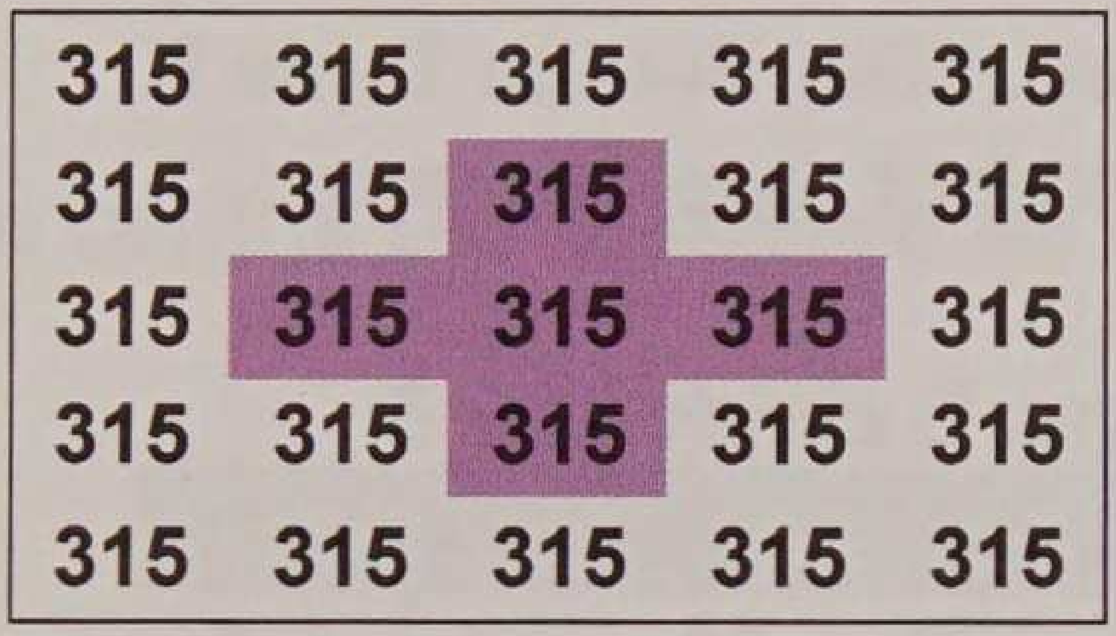
	1	2	3	4	5	
	5x5	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

All of the pillars along the C-axis and all the channels along the A-axis sum equally as seen in the duplication of numbers in the cells of the depth-sum tables corresponding to those axes.

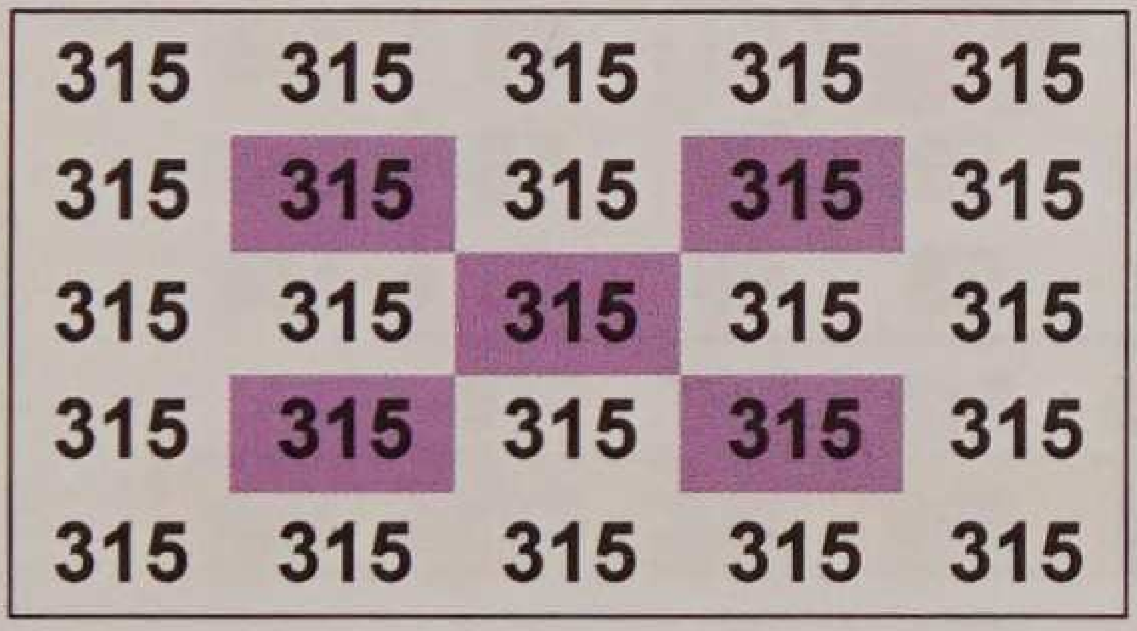
Program 4

5x3D Cubic Tiling Patterns

At right are the five size-5 embedded squares with one of two continuously complementary tiling patterns high-lighted. The other complementary tile pattern consisting of an X-pattern does the same. Both sum to **315** in every tile. The tiling can be relocated anywhere across each embedded square and still sum to **315**. This qualifies the cube to be *ultra*-perfect.



315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

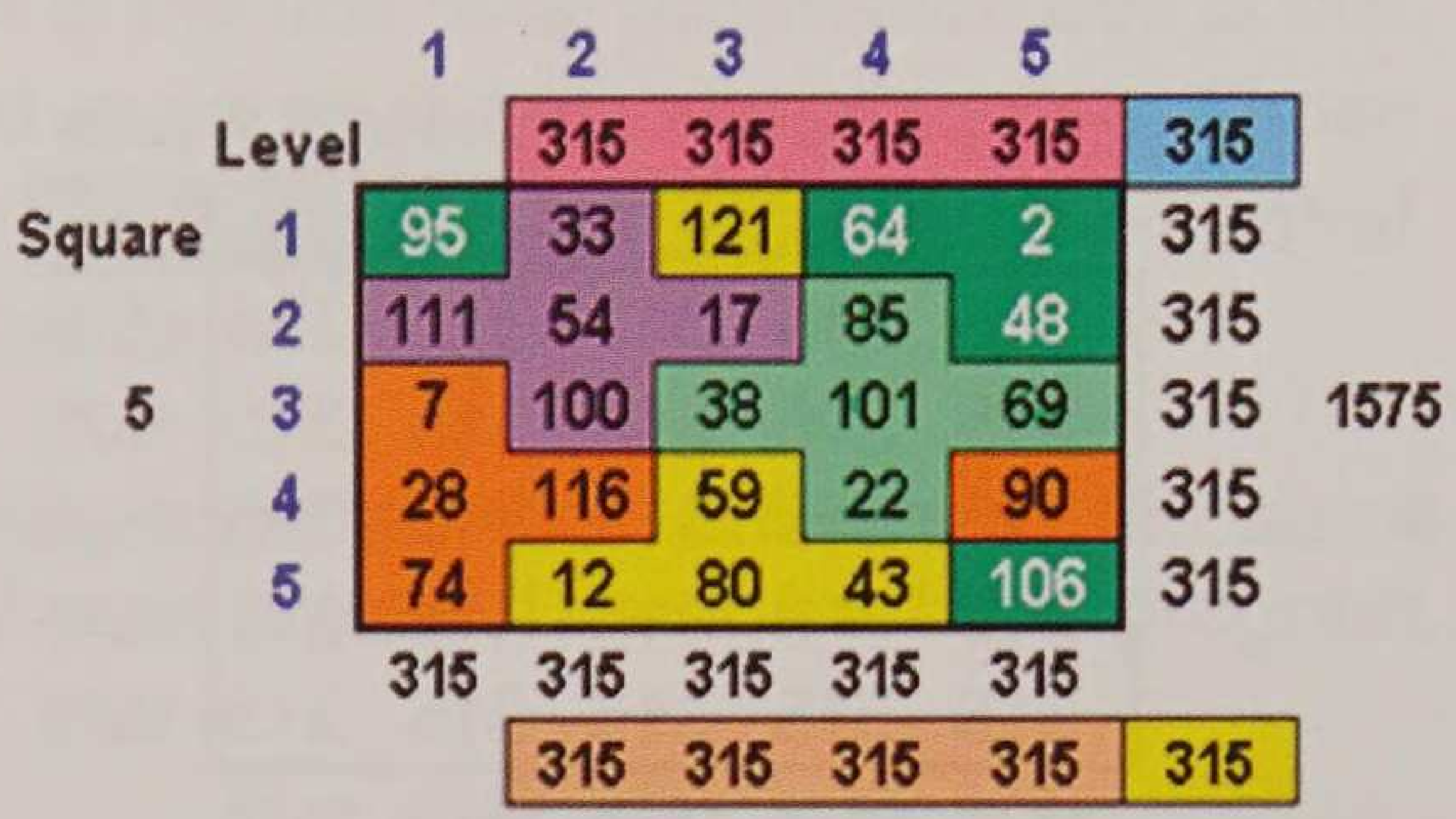


315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

Every row, column and diagonal in both minor and major directions, including wrapping in every block-square, sums to **315**. This qualifies the cube to be also *absolutely-perfect*.

Only the central block-square #3 is pangenic as a square, with quadrals equal to **252**.

Note: The tile patterns do not extend across their square's borders to adjacent squares; keep in mind that we're looking at three dimensions here, 2-dimensional slices at a time.



			1	2	3	4	5		
Level				315	315	315	315	315	
Square		1	95	33	121	64	2	315	
		2	111	54	17	85	48	315	
	5	3	7	100	38	101	69	315	1575
		4	28	116	59	22	90	315	
		5	74	12	80	43	106	315	
			315	315	315	315	315		
				315	315	315	315	315	

Level				315	315	315	315	315	
Square		1	45	108	71	14	77	315	
		2	61	4	92	35	123	315	
	4	3	82	50	113	51	19	315	1575
		4	103	66	9	97	40	315	
		5	24	87	30	118	56	315	
			315	315	315	315	315		
				315	315	315	315	315	

Level				315	315	315	315	315	
Square		1	120	58	21	89	27	315	
		2	11	79	42	110	73	315	
	3	3	32	125	63	1	94	315	1575
		4	53	16	84	47	115	315	
		5	99	37	105	68	6	315	
			315	315	315	315	315		
				315	315	315	315	315	

Level				315	315	315	315	315	
Square		1	70	8	96	39	102	315	
		2	86	29	117	60	23	315	
	2	3	107	75	13	76	44	315	1575
		4	3	91	34	122	65	315	
		5	49	112	55	18	81	315	
			315	315	315	315	315		
				315	315	315	315	315	

Level				315	315	315	315	315	
Square		1	20	83	46	114	52	315	
		2	36	104	67	10	98	315	
	1	3	57	25	88	26	119	315	1575
		4	78	41	109	72	15	315	
		5	124	62	5	93	31	315	
			315	315	315	315	315		
				315	315	315	315	315	

Program 4

Class-2 Cube

Size-6

Here is the Class-2 size-6 cube. It is geonomically perfect but not absolutely-perfect because not all of its planar diagonals sum equally parallel to its B-axis. But all of its octals sum equally.

Square		1	2	3	4	5	6		
			3888	3906	3906	3924	3906	3906	
	1	204	97	42	175	120	13	651	
	2	89	68	149	143	2	200	651	
6	3	58	153	112	33	208	87	651	3906
	4	160	135	34	183	82	57	651	
	5	122	11	206	80	65	167	651	
	6	18	187	108	37	174	127	651	
	1	168	133	6	211	84	49	651	
	2	125	32	185	107	38	164	651	
5	3	22	189	76	69	172	123	651	3906
	4	196	99	70	147	118	21	651	
	5	86	47	170	116	29	203	651	
	6	54	151	144	1	210	91	651	
	1	60	169	114	31	192	85	651	
	2	161	140	5	215	74	56	651	
4	3	130	9	184	105	64	159	651	3906
	4	16	207	106	39	154	129	651	
	5	194	83	62	152	137	23	651	
	6	90	43	180	109	30	199	651	
	1	24	205	78	67	156	121	651	
	2	197	104	41	179	110	20	651	
	3	94	45	148	141	28	195	651	3906
3	4	52	171	142	3	190	93	651	
	5	158	119	26	188	101	59	651	
	6	126	7	216	73	66	163	651	
	1	132	25	186	103	48	157	651	
	2	17	212	77	71	146	128	651	
2	3	202	81	40	177	136	15	651	3906
	4	88	63	178	111	10	201	651	
	5	50	155	134	8	209	95	651	
	6	162	115	36	181	102	55	651	
	1	96	61	150	139	12	193	651	
	2	53	176	113	35	182	92	651	
1	3	166	117	4	213	100	51	651	3906
	4	124	27	214	75	46	165	651	
	5	14	191	98	44	173	131	651	
	6	198	79	72	145	138	19	651	
		3906	3906	3906	3906	3906	3906		
			3924	3906	3906	3888	3906	3906	

Quadrals

6

362	578	362
578	146	578
362	578	362

5

362	578	362
578	146	578
362	578	362

4

434	434	434
434	434	434
434	434	434

3

434	434	434
434	434	434
434	434	434

2

506	290	506
290	722	290
506	290	506

1

506	290	506
290	722	290
506	290	506

Octals

6 + 1

868	868	868
868	868	868
868	868	868

5 + 2

868	868	868
868	868	868
868	868	868

4 + 3

868	868	868
868	868	868
868	868	868

Depth-sums along A-axis

A	3906	3906	3906	3906	3906	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
3906	3906	3906	3906	3906	3906	
	3906	3906	3906	3906	3906	3906

Depth-sum Quadrals

2604	2604	2604
2604	2604	2604
2604	2604	2604

Depth-sums along B-axis

B	3888	3906	3906	3924	3906	3906
684	690	576	726	612	618	3906
642	732	570	750	552	660	3906
672	594	564	738	708	630	3906
636	702	744	558	600	666	3906
624	606	696	588	714	678	3906
648	582	756	546	720	654	3906
3906	3906	3906	3906	3906	3906	
	3924	3906	3906	3888	3906	3906

Depth-sum Quadrals

2604	2604	2604
2604	2604	2604
2604	2604	2604

Depth-sums along C-axis

C	3906	3906	3906	3906	3906	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
651	651	651	651	651	651	3906
3906	3906	3906	3906	3906	3906	
	3906	3906	3906	3906	3906	3906

Depth-sum Quadrals

2604	2604	2604
2604	2604	2604
2604	2604	2604

There are no octahedrons in even size cubes because their existence requires a distinct central row and column which only odd-size squares have.

It is evident from the depth-sum tables that all of the pillars along the C-axis and all the channels along the A-axis sum equally from the duplication of numbers in the cells of the depth-sum tables corresponding to axes A and C.

Depth-sums along the B-axis

B	3888	3906	3906	3924	3906	3906
684	690	576	726	612	618	3906
642	732	570	750	552	660	3906
672	594	564	738	708	630	3906
636	702	744	558	600	666	3906
624	606	696	588	714	678	3906
648	582	756	546	720	654	3906
3906	3906	3906	3906	3906	3906	
	3924	3906	3906	3888	3906	3906

B/6	648	651	651	654	651	651
114	115	96	121	102	103	651
107	122	95	125	92	110	651
112	99	94	123	118	105	651
106	117	124	93	100	111	651
104	101	116	98	119	113	651
108	97	126	91	120	109	651
651	651	651	651	651	651	
	654	651	651	648	651	651

B/6-90	108	111	111	114	111	111
24	25	6	31	12	13	111
17	32	5	35	2	20	111
22	9	4	33	28	15	111
16	27	34	3	10	21	111
14	11	26	8	29	23	111
18	7	36	1	30	19	111
111	111	111	111	111	111	
	114	111	111	108	111	111

6x6	108	111	111	114	111	111
16	27	34	3	10	21	111
14	11	26	8	29	23	111
18	7	36	1	30	19	111
24	25	6	31	12	13	111
17	32	5	35	2	20	111
22	9	4	33	28	15	111
111	111	111	111	111	111	
	114	111	111	108	111	111

Collapsing the size-6 cube to a size-6 punctuated-perfect square

Its depth sum table **B** will collapse to a punctuated-perfect size-6 square by:

1. Dividing **B** by **6**
2. Subtracting **90**
3. Wrapping the rows/columns to center the beginning and ending numbers.

The resulting version can then be converted to a near-perfect square by interchanging two numbers in one of its columns.

Note that the number **90** subtracted in step #2 is always **1** less than the minimum number in the table resulting from step #1. But it is also the difference between the characteristic numbers of the cube and square of size-6.

This, of course, generalizes to the differences between the characteristic numbers of squares and cubes of the same size **n** in this collapsation process.

Program 4

Class-3 Cube

Size-3

3x3x3

		1	2	3		
Squares			126	126	126	
	1	17	4	21	42	
3	2	10	9	23	42	**126**
	3	15	2	25	42	
	1	22	12	8	42	
2	2	27	14	1	42	**126**
	3	20	16	6	42	
	1	3	26	13	42	
1	2	5	19	18	42	**126**
	3	7	24	11	42	
		126	126	126		
			126	126	126	

Quadrals

78	39
56	56
34	73

Octals

112	112
112	

Octahedron

84

Depth sums along A axis

	A	126	126	126
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
		126	126	126

Depth-sum quadrals

168	168

Depth sums along B axis

	B	126	126	126
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
		126	126	126

Depth-sum quadrals

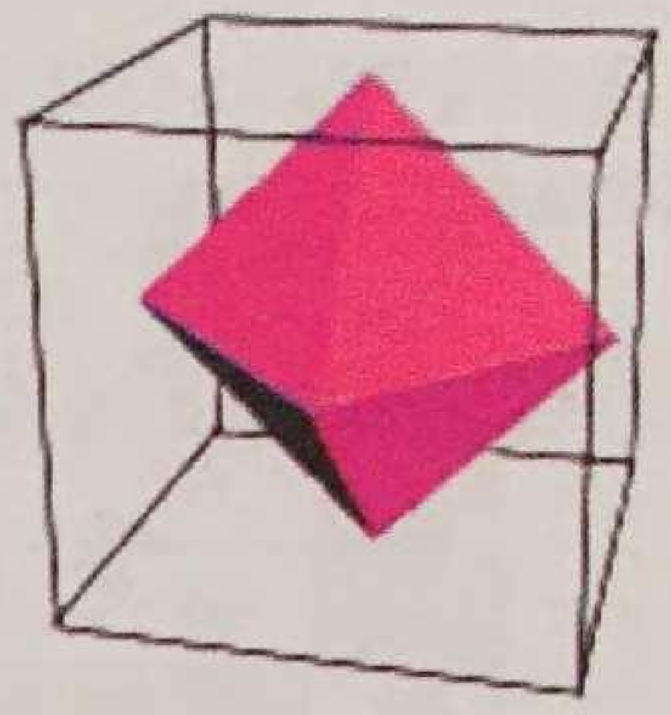
168	168

Depth sums along C axis

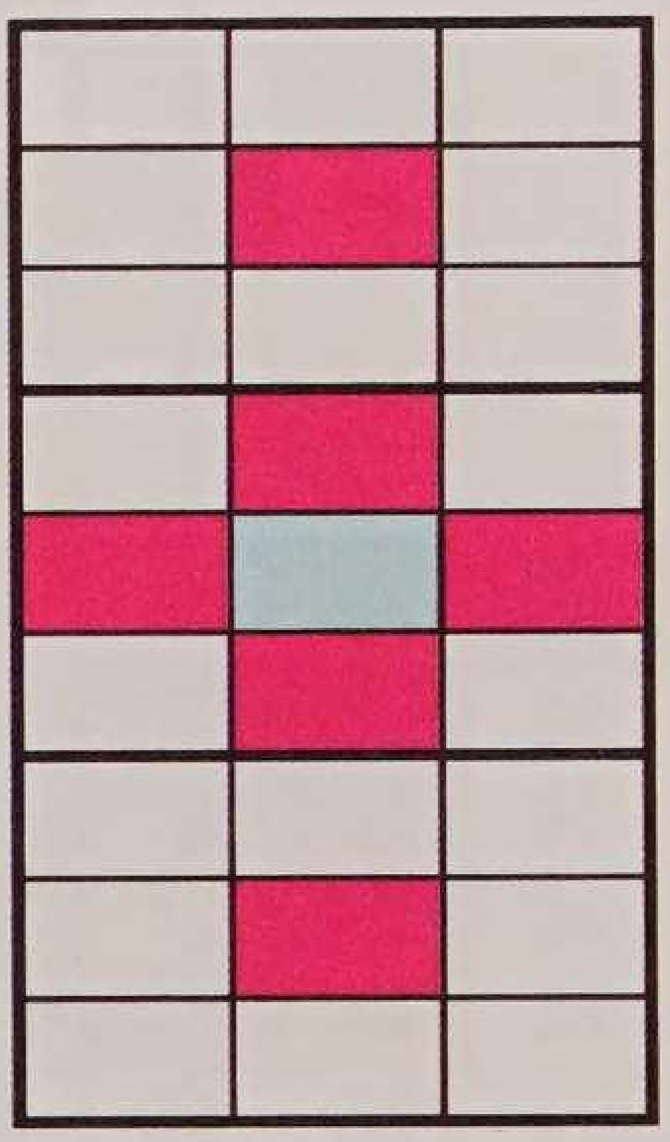
	C	126	126	126
1	42	15	69	126
2	69	42	15	126
3	15	69	42	126
	126	126	126	
		45	207	126

Depth-sum quadrals

168	168



Octahedron in 3D



Here is a size-3 cube. It is perfect but is not absolutely perfect as seen by two unequal planar diagonals in its depth-sum table **C**. Notice how this would be missed if it were not for depth-sum tables.

Further, its depth-sum table **B** will not collapse to a size-3 square because it has a duplication of numbers. The principle is -- that eventhough a geonomic cube is perfect, it will not collapse to a square if its associated size square is not perfect. That's how true-to-form Geonometry actually is!

Its octals all sum equally to **112**. Its sole octahedron sums to **84 = ¾ x 112**.

Program 4

Class-4

Absolutely- Perfect Size-4 Cube

Here is the size-4 cube from Class-4. It is absolutely perfect in that the planar summations in all 9 directions sum equally to the cube's characteristic number. It is pangenic because all its octals sum equally; and surprisingly so do the quadrals in the embedded squares.

The duplication of numbers in **A** & **C** shows the equality among channels and pillars.

4x3D

Square							
		520	520	520	520		
	1	37	58	15	20	130	
4	2	16	19	38	57	130	520
	3	50	45	28	7	130	
	4	27	8	49	46	130	
	1	21	42	63	4	130	
3	2	64	3	22	41	130	520
	3	2	61	44	23	130	
	4	43	24	1	62	130	
	1	5	26	47	52	130	
2	2	48	51	6	25	130	520
	3	18	13	60	39	130	
	4	59	40	17	14	130	
	1	53	10	31	36	130	
1	2	32	35	54	9	130	520
	3	34	29	12	55	130	
	4	11	56	33	30	130	
		520	520	520	520		
		520	520	520	520		

Quadrals

130	130	130	130
130	130	130	130
130	130	130	130
130	130	130	130

Octals

260	260
260	260
260	260
260	260

Depth-sum along A-axis

A	520	520	520	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
520	520	520	520	
	520	520	520	520

Depth-sum along B-axis

B	520	520	520	520
116	136	156	112	520
160	108	120	132	520
104	148	144	124	520
140	128	100	152	520
520	520	520	520	
	520	520	520	520

Depth-sum along C-axis

C	520	520	520	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
130	130	130	130	520
520	520	520	520	
	520	520	520	520

Program 4

Segregated into squares

4	130	130	130	130
37	58	15	20	130
16	19	38	57	130
50	45	28	7	130
27	8	49	46	130
	130	130	130	130

Quadrals

130	130
130	130

3	130	130	130	130
21	42	63	4	130
64	3	22	41	130
2	61	44	23	130
43	24	1	62	130
	130	130	130	130

Quadrals

130	130
130	130

1	130	130	130	130
53	10	31	36	130
32	35	54	9	130
34	29	12	55	130
11	56	33	30	130
	130	130	130	130

Quadrals

130	130
130	130

2	130	130	130	130
5	26	47	52	130
48	51	6	25	130
18	13	60	39	130
59	40	17	14	130
	130	130	130	130

Quadrals

130	130
130	130

Here are the four embedded squares. They are all independently pangenic. They sum equally in both their rows and columns.

On the right, the series of operations on the depth-sum table B collapses it to a perfect square.

Depth-sums along B-axis

B	520	520	520	520
116	136	156	112	520
160	108	120	132	520
104	148	144	124	520
140	128	100	152	520
520	520	520	520	
	520	520	520	520

B/4

29	34	39	28
40	27	30	33
26	37	36	31
35	32	25	38

B/4-24

5	10	15	4
16	3	6	9
2	13	12	7
11	8	1	14

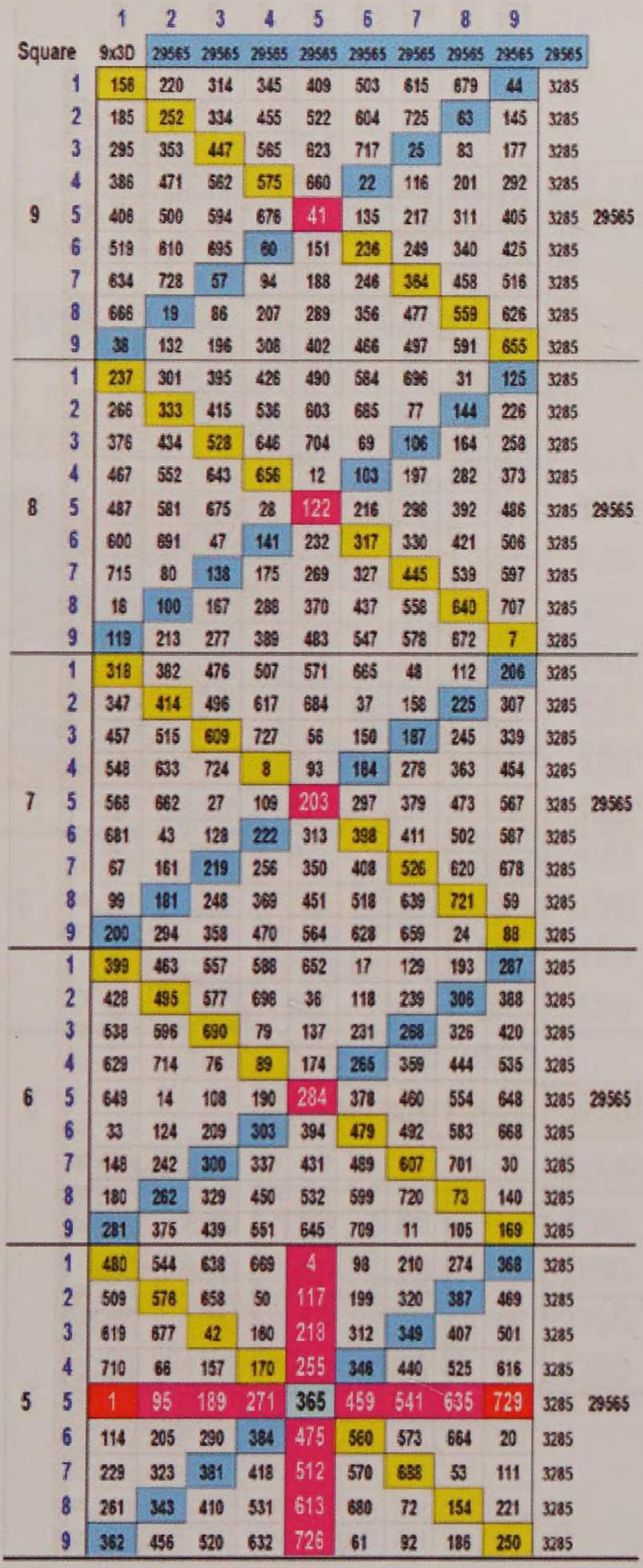
4x4	34	34	34	34
5	10	15	4	34
16	3	6	9	34
2	13	12	7	34
11	8	1	14	34
34	34	34	34	
	34	34	34	34

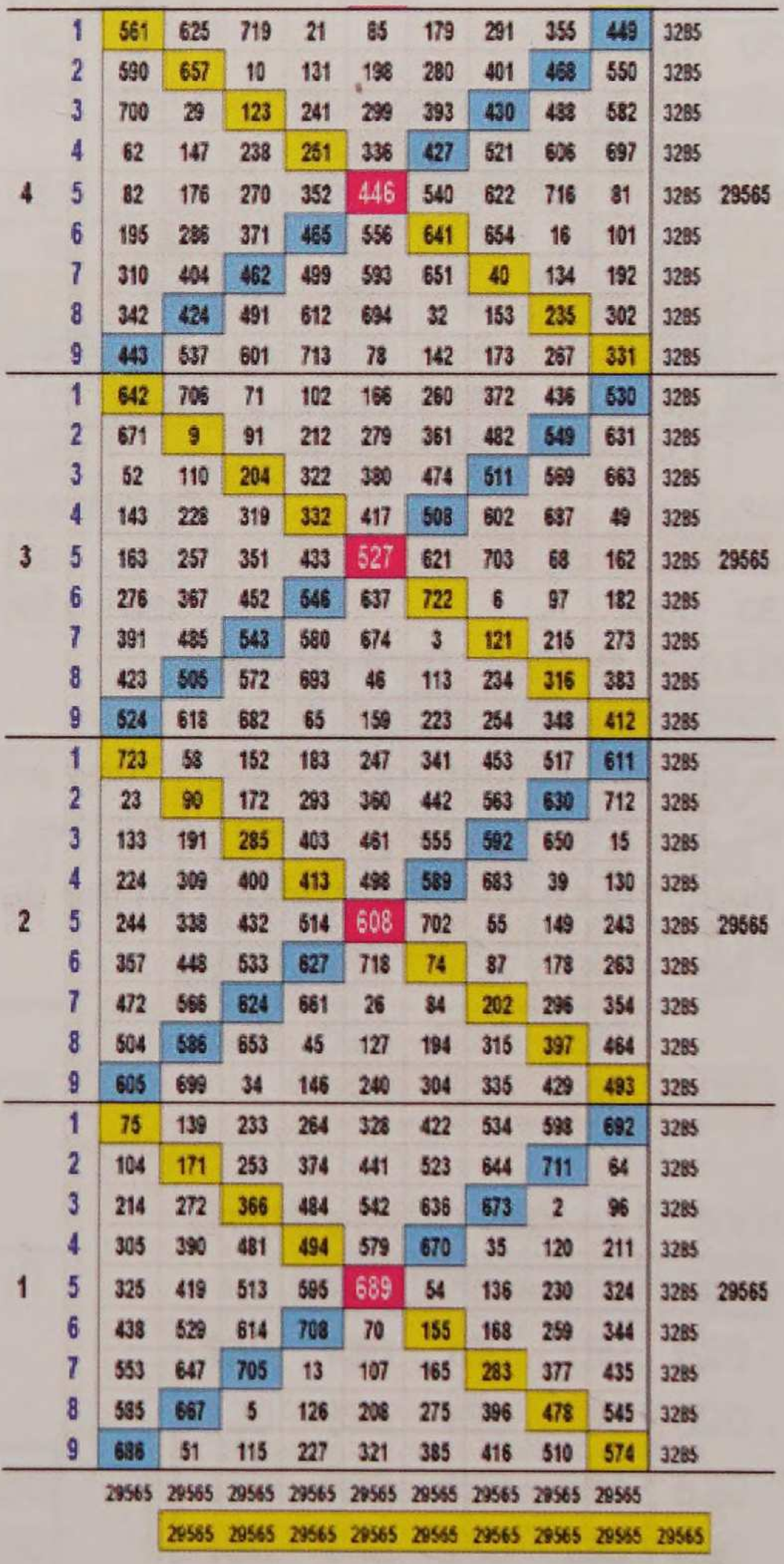
Program 4

Class-6 Cube

Size 9

Here is the Class-3 cube of size-9. It is **absolutely perfect** – all of its diagonal planes in **9** directions, **2** in parallel with each of its **3** axes, and **3** in alignment with each axis-pair all sum to the cube's characteristic number, as seen in all **3** of its depth-sum tables. As seen in the colored table at the bottom, all of the main and wrap major diagonals in each embedded square sum equally too.





		1	2	3	4	5	6	7	8	9		
Square		9x3D	29565	29565	29565	29565	29565	29565	29565	29565	29565	
	1	158	220	314	345	409	503	615	679	44	3285	
	2	185	252	334	455	522	604	725	63	145	3285	
	3	295	353	447	565	623	717	25	83	177	3285	
	4	386	471	562	575	660	22	116	201	292	3285	
9	5	406	500	594	676	41	135	217	311	405	3285	29565
	6	519	610	695	60	151	236	249	340	425	3285	
	7	634	728	57	94	188	246	364	458	516	3285	
	8	666	19	86	207	289	356	477	559	626	3285	
	9	38	132	196	308	402	466	497	591	655	3285	
	1	237	301	395	426	490	584	696	31	125	3285	
	2	266	333	415	536	603	685	77	144	226	3285	
	3	376	434	528	646	704	69	106	164	258	3285	
	4	467	552	643	656	12	103	197	282	373	3285	
8	5	487	581	675	28	122	216	298	392	486	3285	29565
	6	600	691	47	141	232	317	330	421	506	3285	
	7	715	80	138	175	269	327	445	539	597	3285	
	8	18	100	167	288	370	437	558	640	707	3285	
	9	119	213	277	389	483	547	578	672	7	3285	
	1	318	382	476	507	571	665	48	112	206	3285	
	2	347	414	496	617	684	37	158	225	307	3285	
	3	457	515	609	727	56	150	187	245	339	3285	
	4	548	633	724	8	93	184	278	363	454	3285	
7	5	568	662	27	109	203	297	379	473	567	3285	29565
	6	681	43	128	222	313	398	411	502	587	3285	
	7	67	161	219	256	350	408	526	620	678	3285	
	8	99	181	248	369	451	518	639	721	59	3285	
	9	200	294	358	470	564	628	659	24	88	3285	
	1	399	463	557	588	652	17	129	193	287	3285	
	2	428	495	577	698	36	118	239	306	388	3285	
	3	538	596	690	79	137	231	268	326	420	3285	
	4	629	714	76	89	174	265	359	444	535	3285	
6	5	649	14	108	190	284	378	460	554	648	3285	29565
	6	33	124	209	303	394	479	492	583	668	3285	
	7	148	242	300	337	431	489	607	701	30	3285	
	8	180	262	329	450	532	599	720	73	140	3285	
	9	281	375	439	551	645	709	11	105	169	3285	
	1	480	544	638	669	4	98	210	274	368	3285	
	2	509	578	658	50	117	199	320	387	469	3285	
	3	619	677	42	160	213	312	349	407	501	3285	
	4	710	66	157	170	255	346	440	525	616	3285	
5	5	1	95	189	271	365	459	541	635	729	3285	29565
	6	114	205	290	384	475	560	573	664	20	3285	
	7	229	323	381	418	512	570	688	53	111	3285	
	8	261	343	410	531	613	680	72	154	221	3285	
	9	362	456	520	632	726	61	92	186	250	3285	
	1	561	625	719	21	85	179	291	355	449	3285	
	2	590	657	10	131	198	280	401	468	550	3285	
	3	700	29	123	241	299	393	430	488	582	3285	
	4	62	147	238	251	336	427	521	606	697	3285	
4	5	82	176	270	352	446	540	622	716	81	3285	29565
	6	195	286	371	465	556	641	654	16	101	3285	
	7	310	404	462	499	593	651	40	134	192	3285	
	8	342	424	491	612	694	32	153	235	302	3285	
	9	443	537	601	713	78	142	173	267	331	3285	
	1	642	706	71	102	166	260	372	436	530	3285	
	2	671	9	91	212	279	361	482	549	631	3285	
	3	52	110	204	322	380	474	511	569	663	3285	
	4	143	228	319	332	417	508	602	687	49	3285	
3	5	163	257	351	433	527	621	703	68	162	3285	29565
	6	276	367	452	546	637	722	6	97	182	3285	
	7	391	485	543	580	674	3	121	215	273	3285	
	8	423	505	572	693	46	113	234	316	383	3285	
	9	524	618	682	65	159	223	254	348	412	3285	
	1	723	58	152	183	247	341	453	517	611	3285	
	2	23	90	172	293	360	442	563	630	712	3285	
	3	133	191	285	403	461	555	592	650	15	3285	
	4	224	309	400	413	498	589	683	39	130	3285	
2	5	244	338	432	514	608	702	55	149	243	3285	29565
	6	357	448	533	627	718	74	87	178	263	3285	
	7	472	566	624	661	26	84	202	296	354	3285	
	8	504	586	653	45	127	194	315	397	464	3285	
	9	605	699	34	146	240	304	335	429	493	3285	
	1	75	139	233	264	328	422	534	598	692	3285	
	2	104	171	253	374	441	523	644	711	64	3285	
	3	214	272	366	484	542	636	673	2	96	3285	
	4	305	390	481	494	579	670	35	120	211	3285	
1	5	325	419	513	595	689	54	136	230	324	3285	29565
	6	438	529	614	708	70	155	168	259	344	3285	
	7	553	647	705	13	107	165	283	377	435	3285	
	8	585	667	5	126	208	275	396	478	545	3285	
	9	686	51	115	227	321	385	416	510	574	3285	
		29565	29565	29565	29565	29565	29565	29565	29565	29565		
			29565	29565	29565	29565	29565	29565	29565	29565	29565	

3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285

Program 4

Size-9 Cube's Octal and Octahedral sums

Quadrals

9

893	1622	1622	1622	1622
1622	893	1622	1622	1622
1622	1622	893	1622	1622
1622	1622	1622	893	1622

8

488	1217	1946	1946	1946
1217	1217	1217	1946	1946
1946	1217	1217	1217	1946
1946	1946	1217	1217	488

7

812	812	1541	2270	2270
812	1541	1541	1541	2270
1541	1541	1541	1541	812
2270	1541	1541	812	812

6

1136	1136	1136	1865	2594
1136	1136	1865	1865	1136
1136	1865	1865	1136	1136
1865	1865	1136	1136	1136

5

1460	1460	1460	1460	1460
1460	1460	1460	1460	1460
1460	1460	1460	1460	1460
1460	1460	1460	1460	1460

4

1784	1784	1784	1055	326
1784	1784	1055	1055	1784
1784	1055	1055	1784	1784
1055	1055	1784	1784	1784

3

2108	2108	1379	650	650
2108	1379	1379	1379	650
1379	1379	1379	1379	2108
650	1379	1379	2108	2108

2

2432	1703	974	974	974
1703	1703	1703	974	974
974	1703	1703	1703	974
974	974	1703	1703	2432

1

2027	1298	1298	1298	1298
1298	2027	1298	1298	1298
1298	1298	2027	1298	1298
1298	1298	1298	2027	1298

Octals

1 + 9

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

2 + 8

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

3 + 7

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

4 + 6

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920
2920	2920	2920	2920	2920

Octahedrons

2190
2190
2190
2190

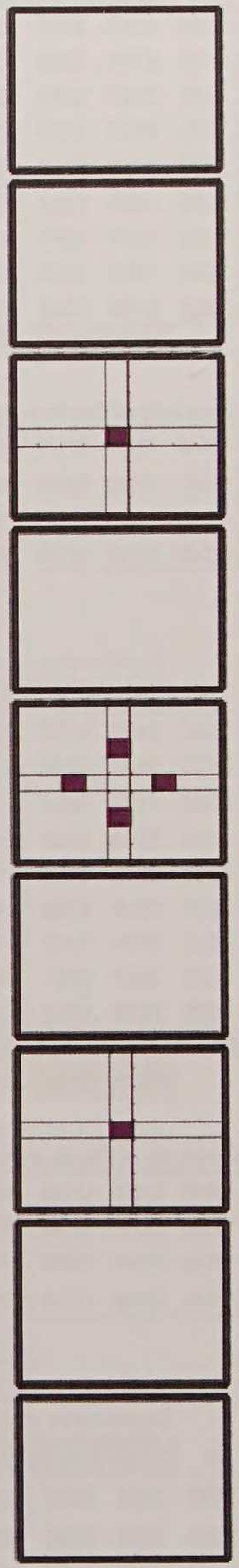
Here are the quadral sums per each embedded block-square and their combination into octal sums. They are all equal to

$2920 = q = 8/9^2 \times Cn$.

The octahedrals all sum equally to

2190 = ¾ q.

Octahedral sums are the sum of one diamond quadral in the central block-square and the central numbers of two opposing block-squares that are as equidistant from the center of the cube as the chosen diamond is from the center of the central block square. **Schema at right**



Program 4

Collapsing the depth-sum table B to a perfect square

Depth-sum tables **A** and **C** show that all the channels along the A-axis and all the pillars along the C-axis sum equally because their cells are all equal. The equal summing rows and columns in depth-sum table **B** also confirm this.

The differences in numbers in depth-sum table **B** show that the channels along the **B** axis are <u>unequal</u>; but this provides for its collapse to a perfect size-9 square as a result of these differences. Again, the reduction number is the difference between the averages of the size-9 square and cube.

Depth-sums along axis-A

A	29565	29565	29565	29565	29565	29565	29565	29565	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
	29565	29565	29565	29565	29565	29565	29565	29565	29565

Quadrals of Depth-sum Table A

13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140

Depth-sums along axis-B

B	29565	29565	29565	29565	29565	29565	29565	29565	29565
3591	3438	3555	3105	2952	3069	3348	3195	3312	29565
3123	2997	3006	3366	3240	3249	3609	3483	3492	29565
3384	3177	3294	3627	3420	3537	3141	2934	3051	29565
3474	3510	3600	2988	3024	3114	3231	3267	3357	29565
2925	3042	3159	3168	3285	3402	3411	3528	3645	29565
3213	3303	3339	3456	3546	3582	2970	3060	3096	29565
3519	3636	3429	3033	3150	2943	3276	3393	3186	29565
3078	3087	2961	3321	3330	3204	3564	3573	3447	29565
3258	3375	3222	3501	3618	3465	3015	3132	2979	29565
29565	29565	29565	29565	29565	29565	29565	29565	29565	
	29565	29565	29565	29565	29565	29565	29565	29565	29565

Depth-sums divided by 9

B/9	3285	3285	3285	3285	3285	3285	3285	3285	3285
399	382	395	345	328	341	372	355	368	3285
347	333	334	374	360	361	401	387	388	3285
376	353	366	403	380	393	349	326	339	3285
386	390	400	332	336	346	359	363	373	3285
325	338	351	352	365	378	379	392	405	3285
357	367	371	384	394	398	330	340	344	3285
391	404	381	337	350	327	364	377	354	3285
342	343	329	369	370	356	396	397	383	3285
362	375	358	389	402	385	335	348	331	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	
	3285	3285	3285	3285	3285	3285	3285	3285	3285

Quadrals of Depth-sum Table B

13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140
13140	13140	13140	13140	13140

B/9 reduced by 324

9x9	369	369	369	369	369	369	369	369	369
75	58	71	21	4	17	48	31	44	369
23	9	10	50	36	37	77	63	64	369
52	29	42	79	56	69	25	2	15	369
62	66	76	8	12	22	35	39	49	369
1	14	27	28	41	54	55	68	81	369
33	43	47	60	70	74	6	16	20	369
67	80	57	13	26	3	40	53	30	369
18	19	5	45	46	32	72	73	59	369
38	51	34	65	78	61	11	24	7	369
369	369	369	369	369	369	369	369	369	
	369	369	369	369	369	369	369	369	369

Depth-sums along axis-C

C	29565	29565	29565	29565	29565	29565	29565	29565	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	29565
3285	3285	3285	3285	3285	3285	3285	3285	3285	
29565	29565	29565	29565	29565	29565	29565	29565	29565	
	29565	29565	29565	29565	29565	29565	29565	29565	29565

The size-9 square has no tiling pattern so it cannot be ultra-perfect. Nonetheless it is absolutely perfect because all **9** planar directional sums are equal to its characteristic number.

Program 4

Embedded Characteristic Spheres in Class-4 Cubes

Just as for squares, the cube's kernel number divides the cube's characteristic number exactly without a fractional remnant only for Class-4 cubes. However, the size-2 cube does so too because the whole cube is its only octal.

Determining which size cubes have Characteristic spheres

Size Cube	Characteristic number	Kernel Number	# of Octals = characteristic no.
1	1	8	0.125
2	18	18	1
3	126	112	1.125
4	520	260	2
5	1,575	504	3.125
6	3,906	868	4.5
7	8,428	1,376	6.125
8	16,416	2,052	8
9	29,565	2,920	10.125
10	50,050	4,004	12.5
11	80,586	5,328	15.125
12	124,488	6,916	18
13	185,731	8,792	21.125
14	269,010	10,980	24.5
15	379,800	13,504	28.125
16	524,416	16,388	32
17	710,073	19,656	36.125
18	944,946	23,332	40.5
19	1,238,230	27,440	45.125
20	1,600,200	32,004	50
21	2,042,271	37,048	55.125
22	2,577,058	42,596	60.5
23	3,218,436	48,672	66.125
24	3,981,600	55,300	72

The formula for the right-hand column is

(4.1) $$C_n/q = n^2/8$$

where **q** = octal sum (3D kernel number).

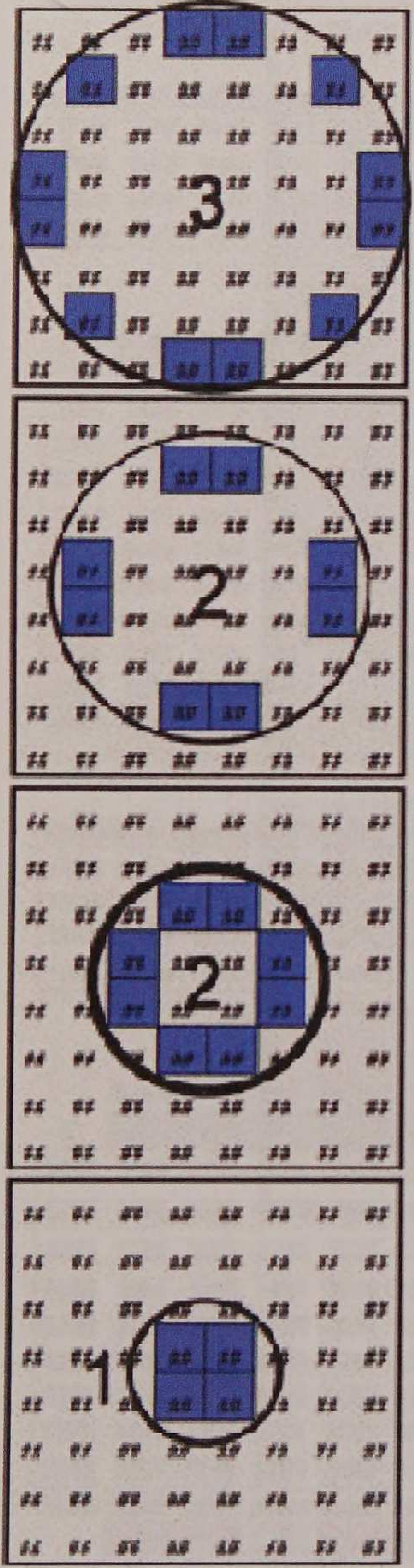
Program 4

A characteristic sphere is composed of impinging octals which sum to the cube's characteristic number.

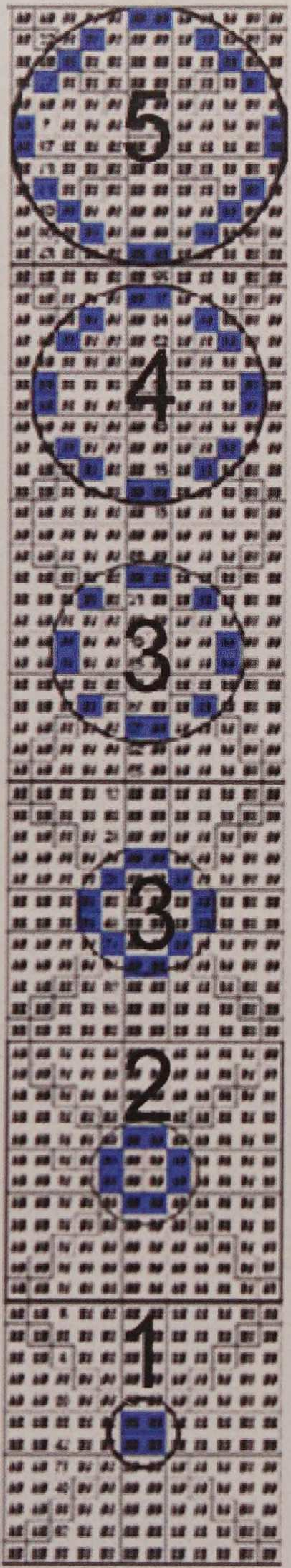
Here are the cells participating in impinging the interior of characteristic spheres for size cubes **8, 12** and **16**.Their octals are **8**, **18** and **32**, respectively.

The size-4 cube (not shown) has 2 octals.

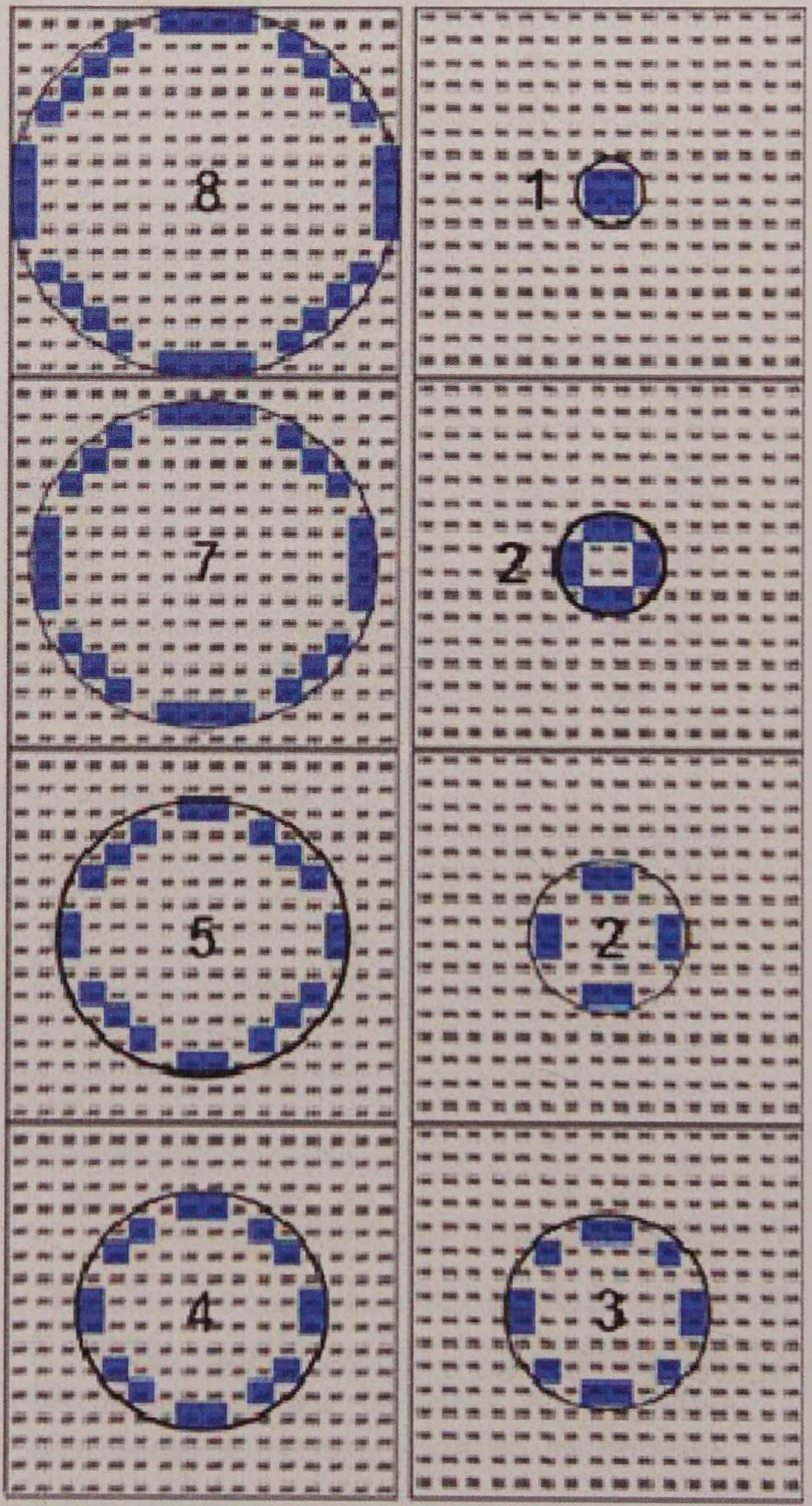
Size-8 Cube
Bottom half



12x Cube
Bottom Half



Size-16 Cube
Bottom half **(cont'd)**

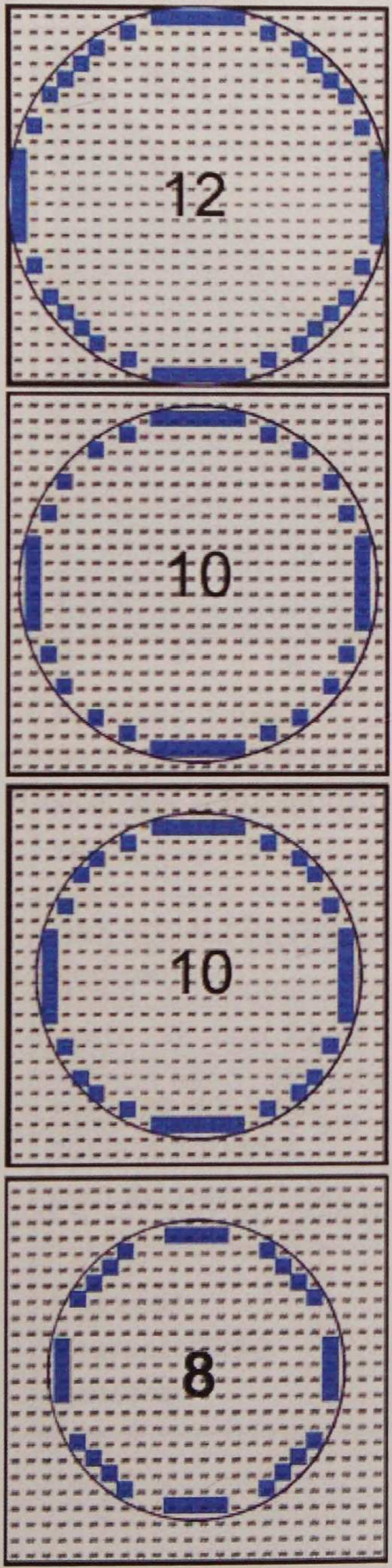


Program 4

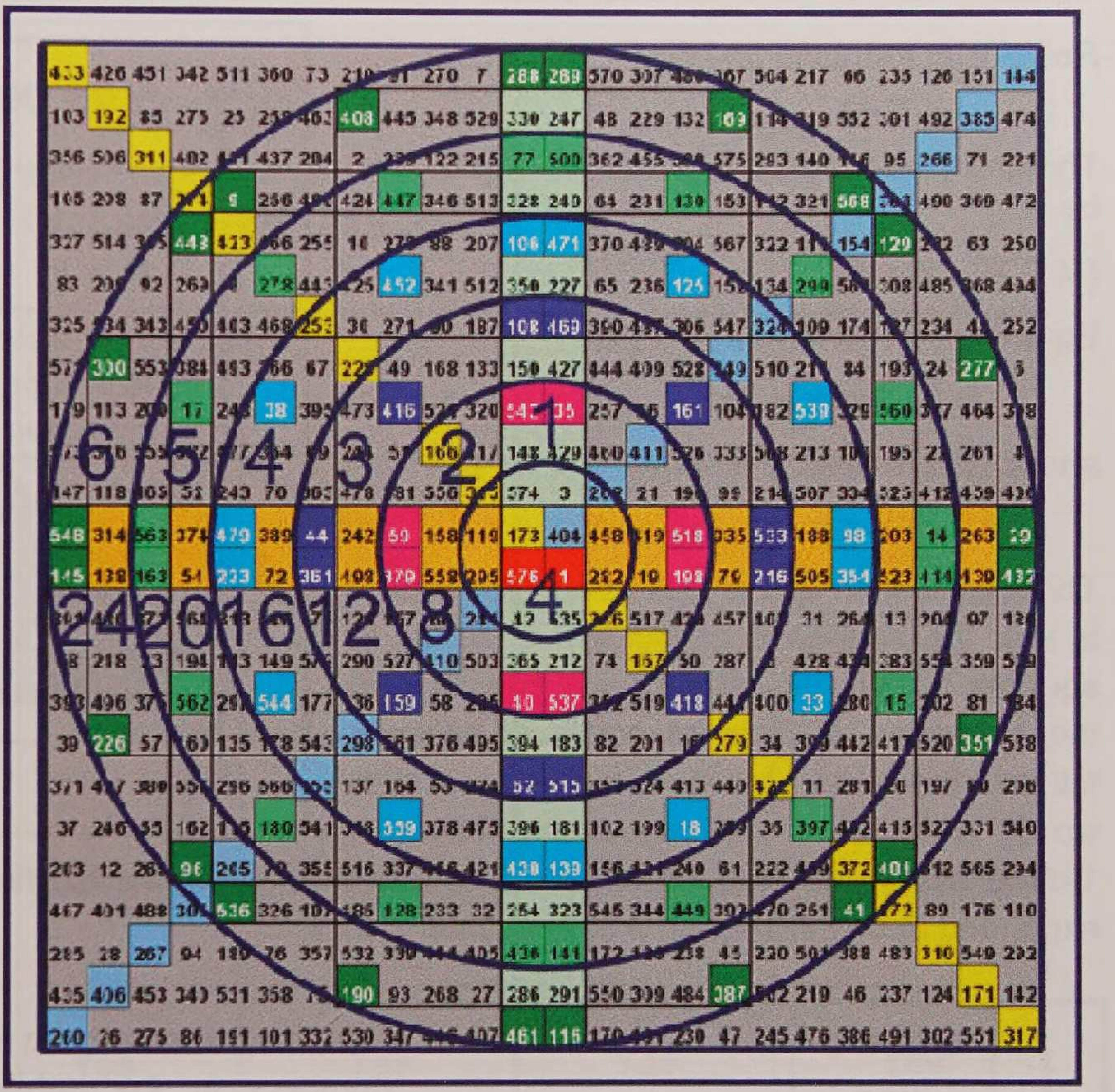
Octal and Quadral Differences between Circles and Cubes

This slide explains the difference in the number of octals participating in impinging the series of nested circles of spheres with the number of quadrals involved in impinging the nested circles of squares.

24x Cube



24x Square



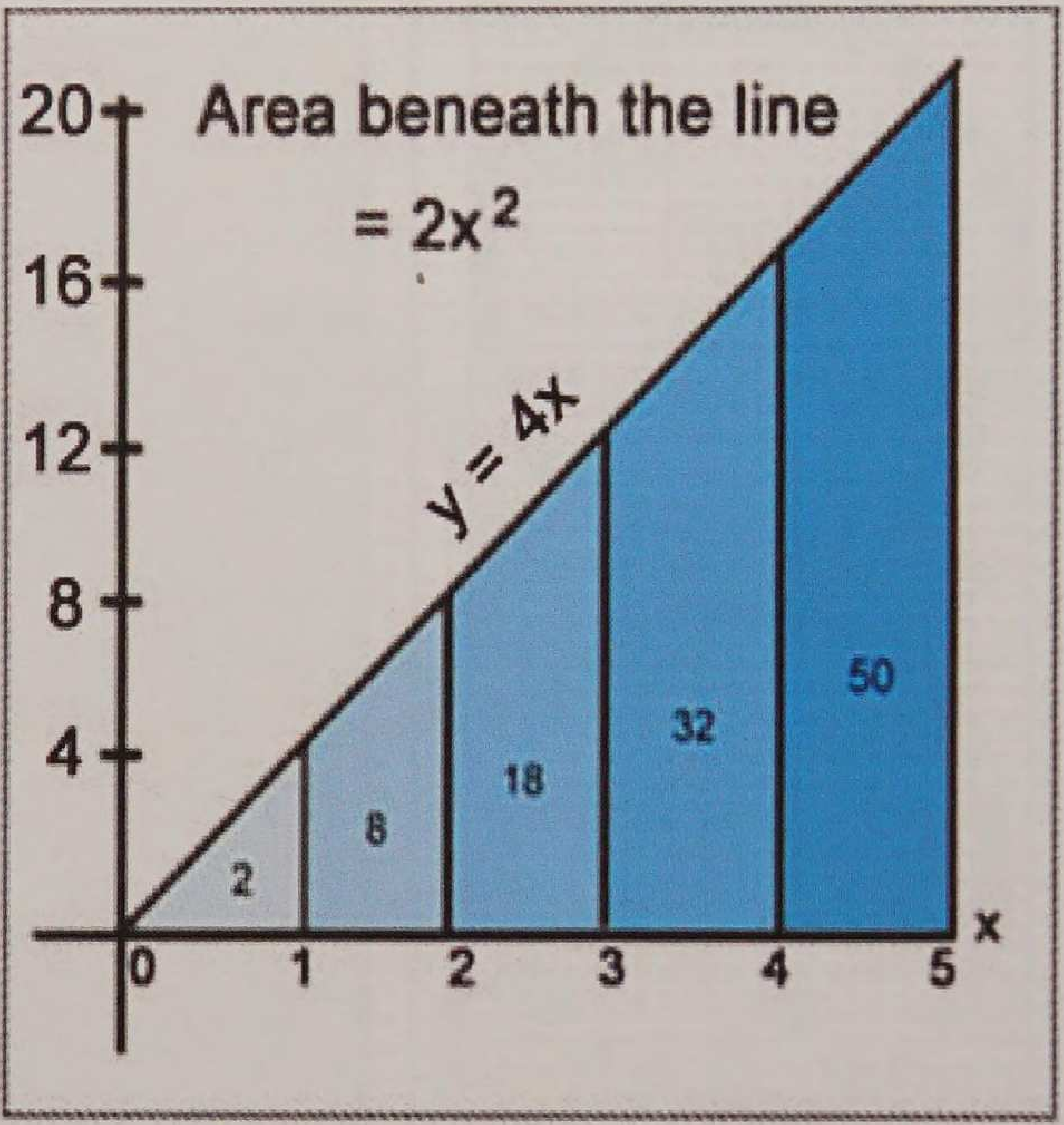
When characteristic circles were depicted for a series of independent squares, the difference between sizes of consecutive squares was **4**, where here for the nested circles within a single sphere, that size difference at each level is merely **2**. Whence the reason for the larger count for octals in cubes than for quadrals in squares of the same size.

Program 4

Summary of Spheres

Formula (**4.1**) is the formula for this series in terms of size and kernel number that we saw three slides prior.

(4.1) $C_n / q = (n^2 / 8) \times q$ where **q** = octal sum (i.e. 3D kernel number).



Recall that formula (**1.1**) from Program 1?

$$y = 4x$$

The area under this straight line is measured by **z**:

(4.2) $z = 2x^2$

Equating (**4.1**) and (**4.2**) we get

$$n^2 / 8 = z = 2x^2$$

and solving this for **n** in terms of **x** we get

$$n = 4x = y$$

The significance of this one-to-one correlation is that the sequence of incident quadrals in squares follows the straight line **y = 4x** and the sequence of incident octals in spheres follows the area under this line. In other words, a 2-dimensional relationship follows a 1-dimensional line, and a 3-dimensional relationship follows a 2-dimensional area. This will be expanded upon in Program 9 to any dimension **k**.

x	Cube's size n	C_n	z
1	4	2q	**2**
2	8	8q	**8**
3	12	18q	**18**
4	16	32q	**32**
5	20	50q	50
6	24	72q	72

The thing to note here is the series of unshadded numbers in the rightmost column for class-4 squares. They number **2, 8, 18** and **32**. This is much more than just coincidental! These numbers will be demonstrated to be the very key to cracking the hidden code underlying the distribution of electrons in the atoms, coming up next in Program 5.

Program 4

Summary of Cubic Properties

This table summarizes the properties of geonomic cubes by class. It is not to be studied here but may serve as a reference guide to be returned to as needed for comprehending the broader property-distribution pattern. All this detail is printed in your companion book to this program series.

Properties of Cubes	Odd		Even	
Class	**Classes 1 & 6**	**Classes 3 & 5**	**Class 2**	**Class 4**
Size factors	n is a prime or square of an odd number	n = ab both odd	n = 2a odd a > 1	n = 2b b even
Perfection Level	Both Absolute & Ultra-perfect	Absolutely Perfect	Perfect	Absolutely perfect
Pangenic Octal sums equal	Yes	Yes	Yes	Yes
Possess equal block-square row and column sums	Yes	Yes	Yes	Yes
Has equal channels and pillars	Yes	Yes	Yes	Yes
Embedded block-squares are ultra-perfect squares with 3D pangenicity	Yes	No	No	No
Possess characteristic spheres	No	No	No	Yes

Program 4

Notes

1. Class-5 cubes have not been explored and consequently their properties are at this time unknown. So all the notes exclude Class-5 cubes. “All” means “All except Class-5”.

2. In all cubes, all the numbers in each embedded block-square sum to the cube’s characteristic number.

3. All cubes are pangenic in that the corners of all their centrally located octals sum equally.

4. In Class1 and 3 cubes, all their centrally located octagons sum equally too.

5. All classes of cubes are perfect in that all their **nxn** horizontal, vertical and main diagonal planes sum to the cube’s characteristic number.

6. Class-1 cubes have equal-summing block squares which sum to the cube’s characteristic number.

7. Each embedded block-square in a Class-1 cube possesses the continuous complementary tiling patterns of the comparable size-n square.

8. Only the size-9 Class-6 cube has been explored and was found to have all the basic properties of Class-1 cubes including equal summing centrally symmetric octals and octagons, except having contiguous tiling patterns in its embedded block squares.

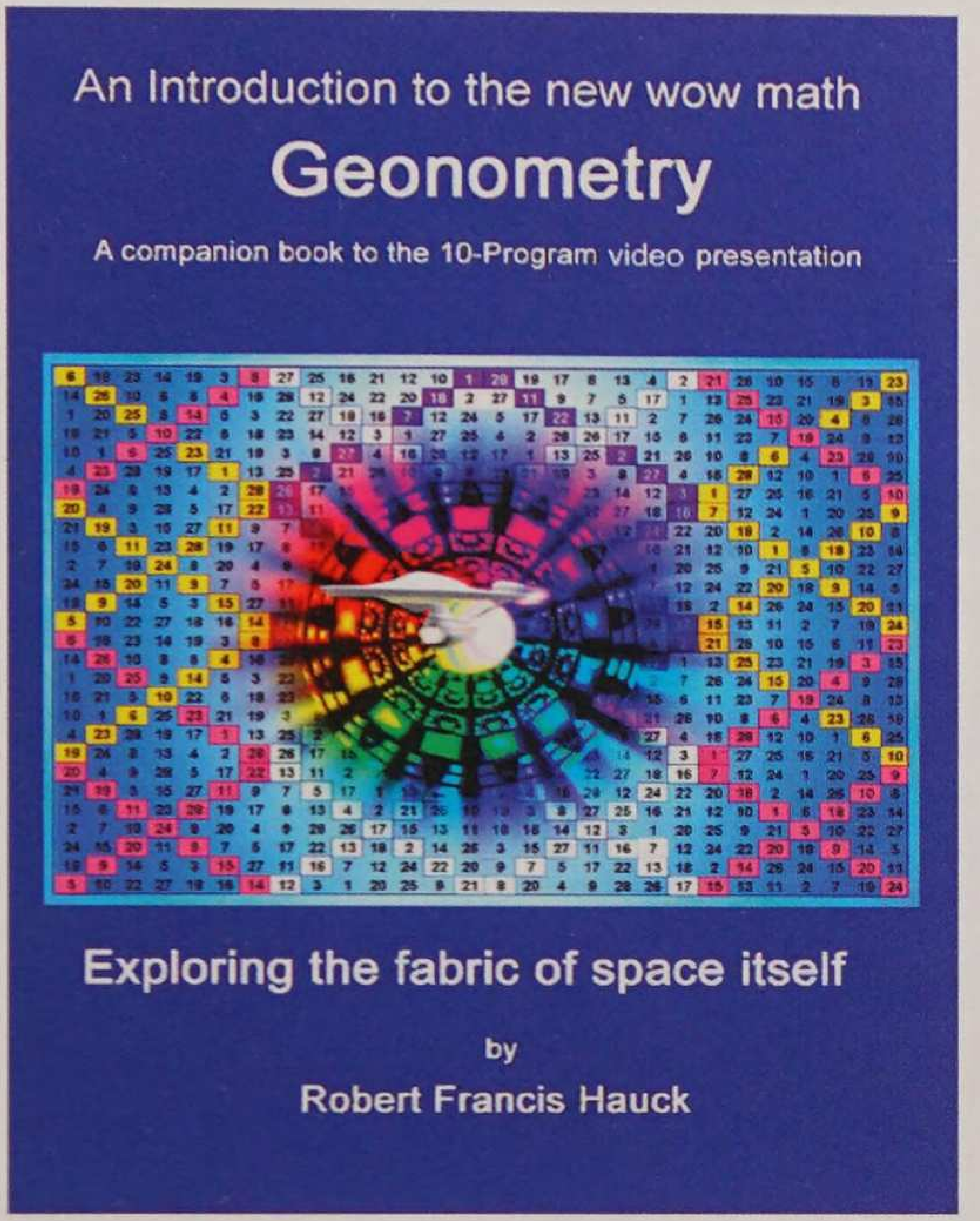
9. All classes of cubes except those of Class-2 collapse to a perfect square.

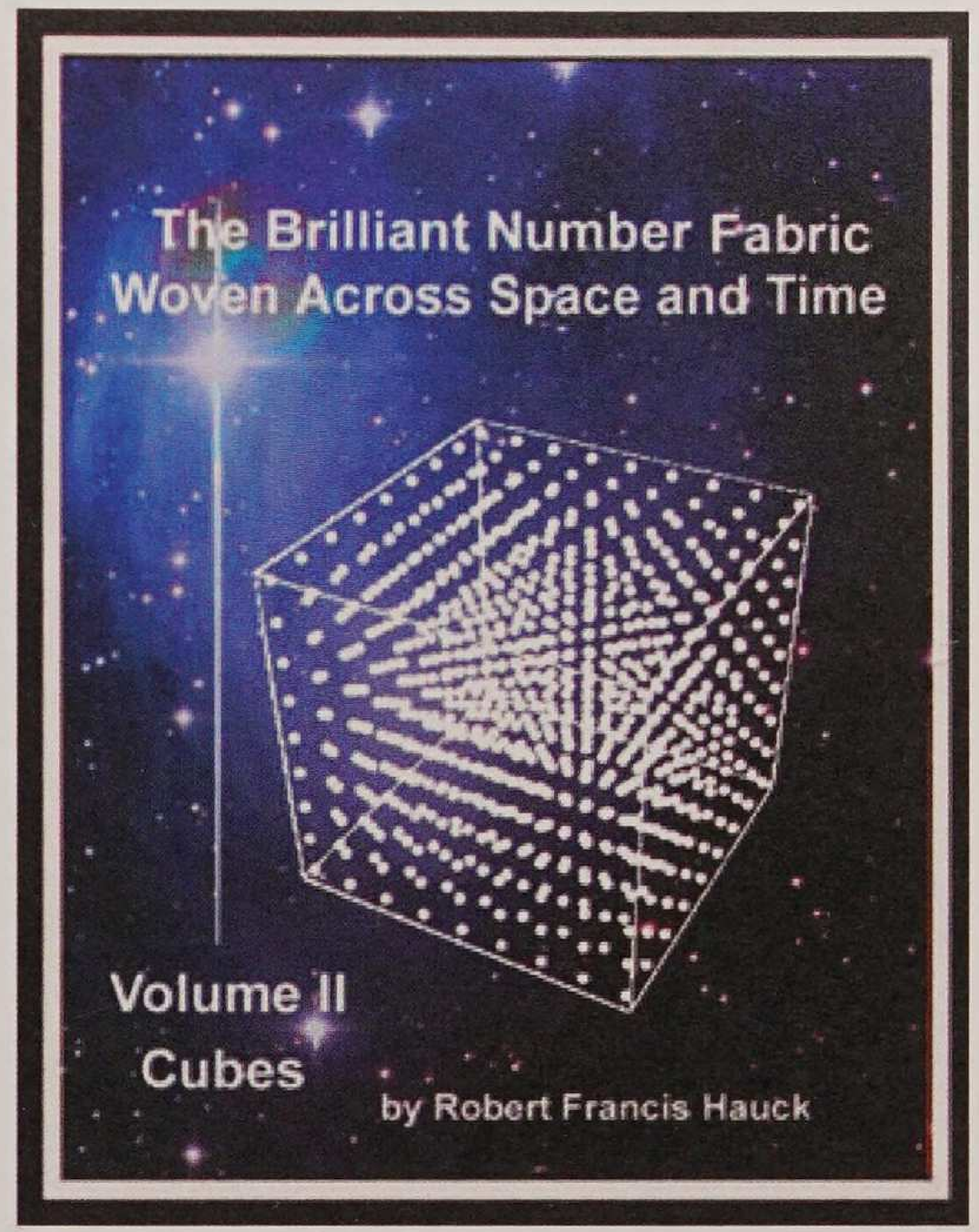
10. All Class-2 cubes collapse to a punctuated-perfect square with only 4 unequal wrap diagonals.

11. All Class-4 cubes possess a characteristic sphere.

Program 4

Here are the two books upon which this program was based.





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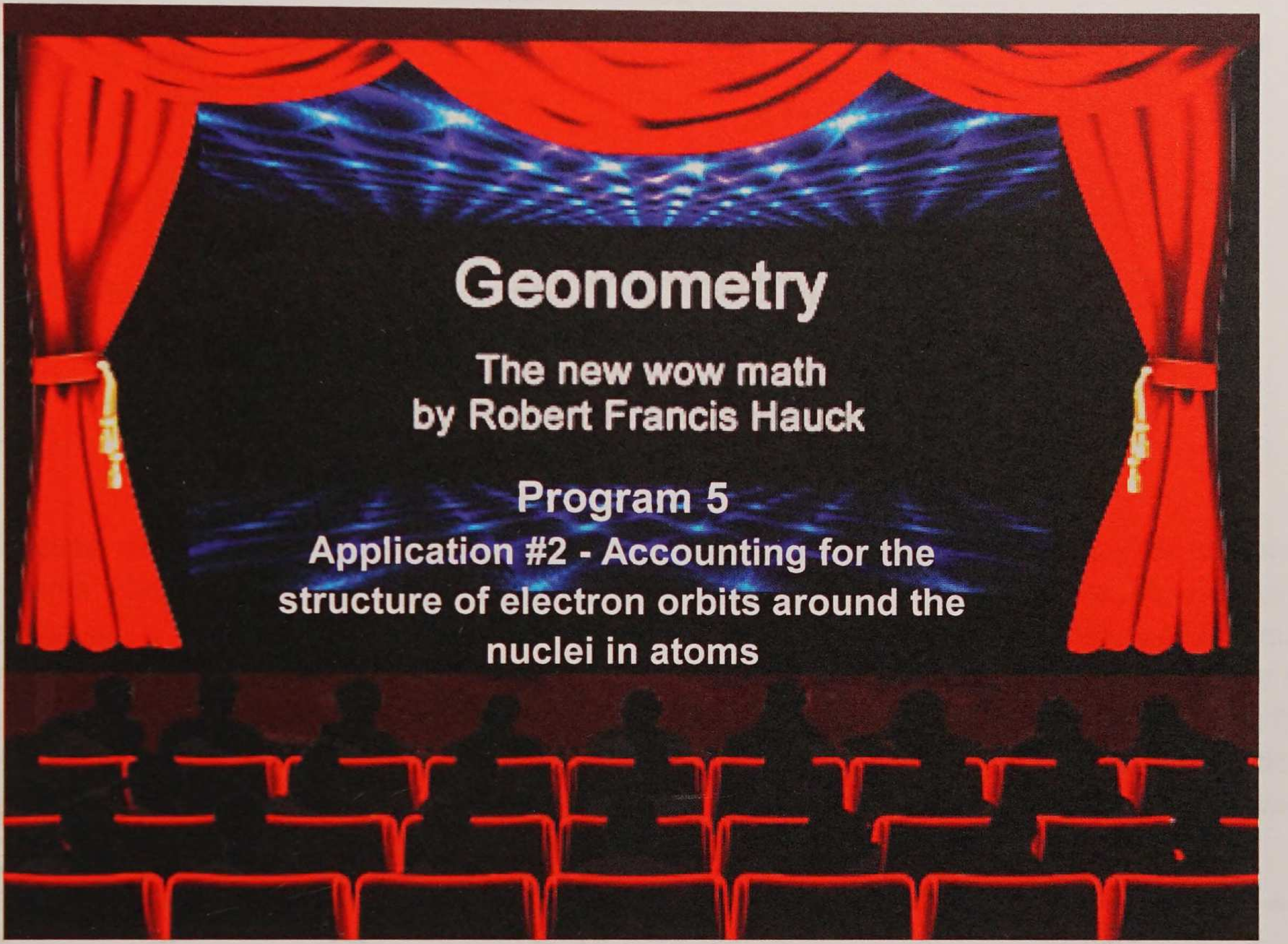
In the next program we will get to witness just how the mathematics of 2- and 3-dimensional Geonometry can explain and account for the distribution and navigation of the electrons around the nuclei of atoms.

Program 4

Program 5

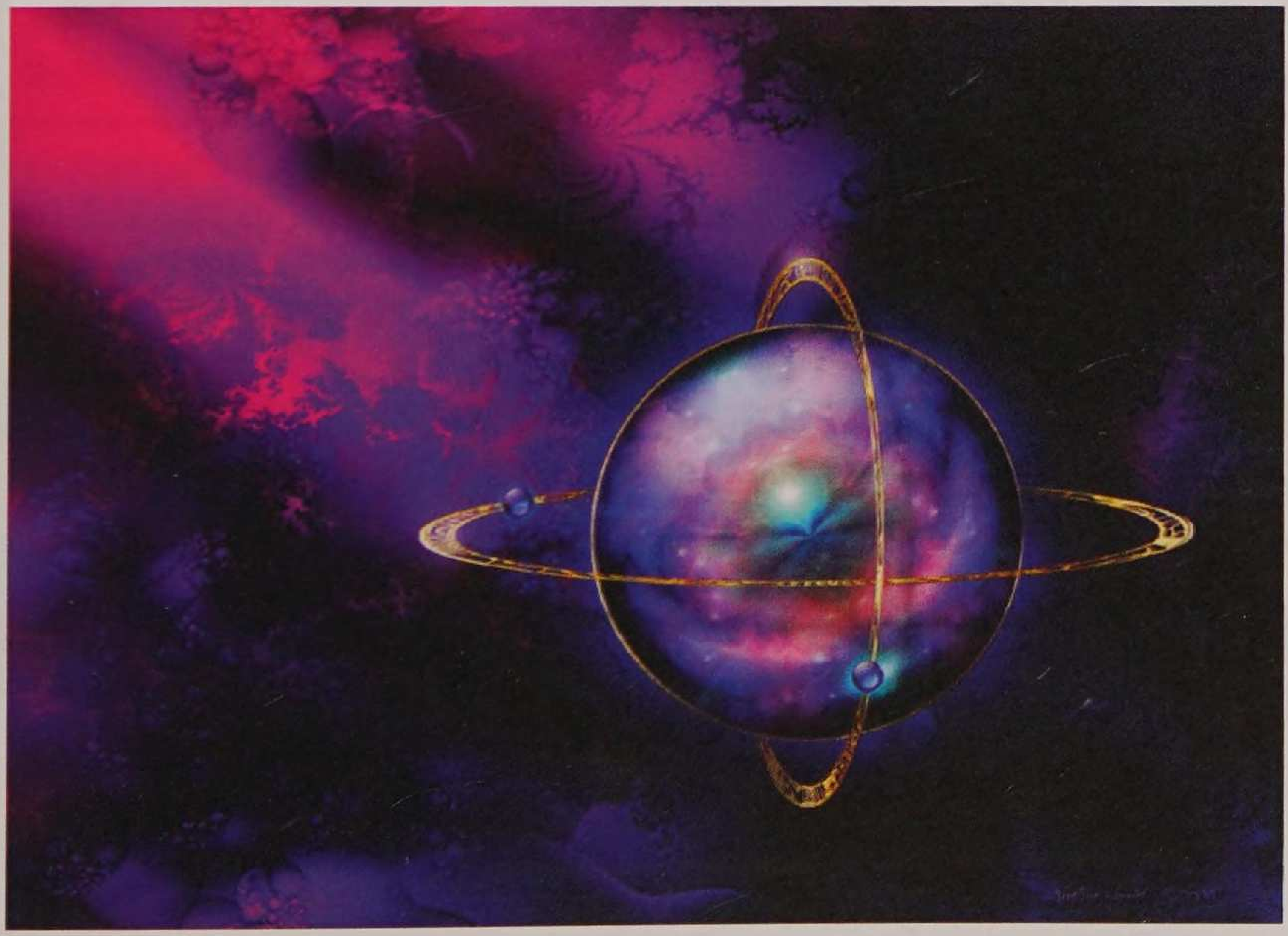
Application #2 of Geonometry



Geonometry

The new wow math
by Robert Francis Hauck

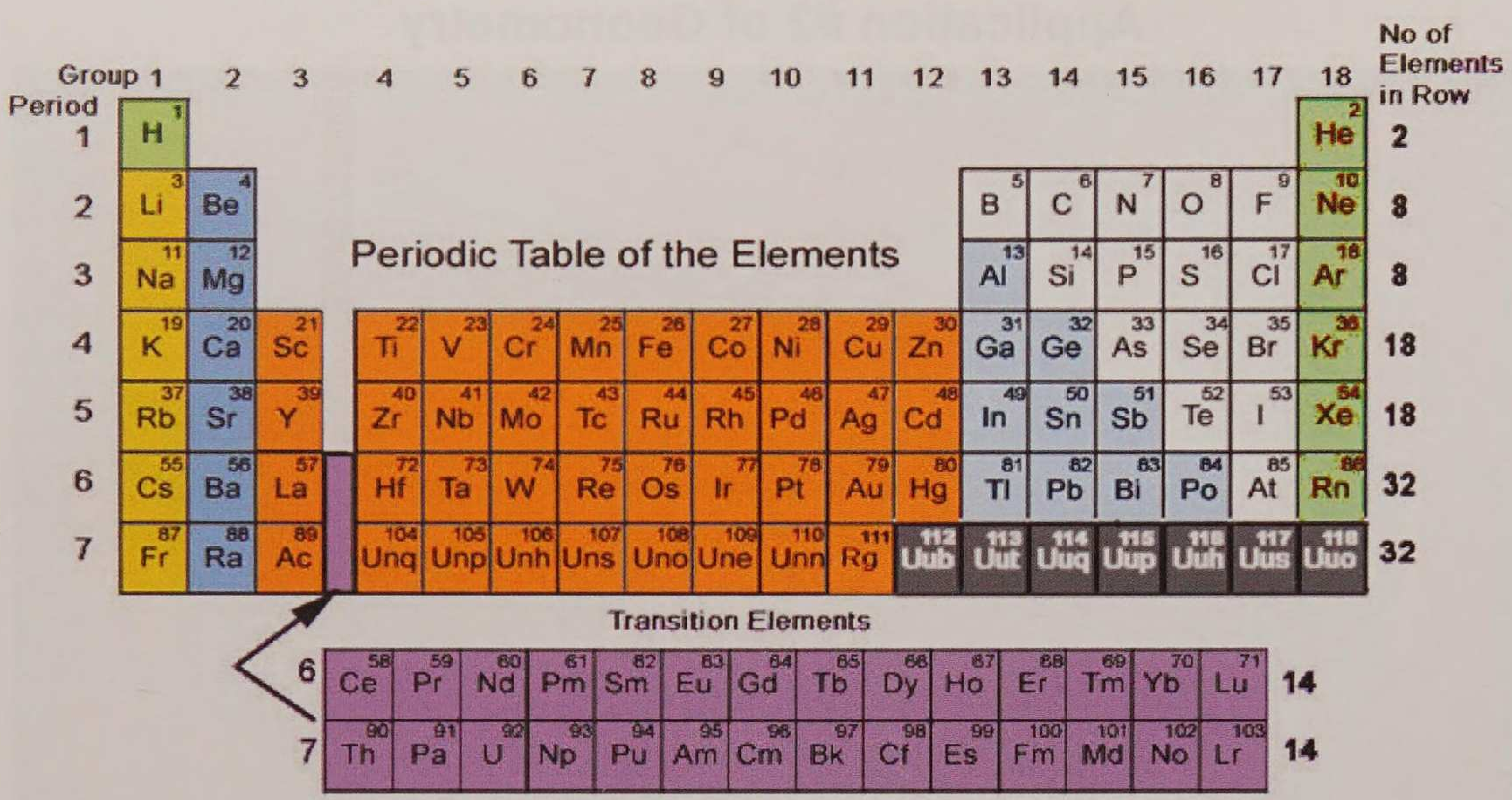
Program 5
Application #2 - Accounting for the structure of electron orbits around the nuclei in atoms



Next we will observe the first major application of Geonometry when we interpret the distribution and orbital patterns of electrons at the atomic level **1/3D**. In this program we shall explore the electron pattern in each of the noble elements and see just how Geonometry has been applied to gain a better understanding of what has already been found in Atomic Physics.

Program 5

Accounting for the Distribution of Electrons in the Atoms



Periodic Table of the Elements

Period	Group 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	No of Elements in Row
1	1 H																	2 He	2
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	8
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	8
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	18
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	18
6	55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn	32
7	87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn	111 Rg	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo	32

Transition Elements

6	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	14
7	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	14

Antimatter Matter.

[32, 32, 18, 18, 8, 8, 2, 2, 8, 8, 18, 18, 32, 32]

[64, 36, 16, 4, 16, 36, 64]

[8, 6, 4, 2, 4, 6, 8]

[4, 3, 2, 1, 2, 3, 4] ⟸ **a perfect harmonic wave-form pattern**

Note the sequence of the number series **2, 8, 18** and **32** at the far right. These were the number of octals incident to cubes of sizes **4, 8, 12** and **16** whose kernel numbers of those sizes summed to the characteristic number. This correlation is profound because it correlates one hundred percent to the number of elements in consecutive row-pairs of the Periodic Table of Elements.

The series at bottom shows what this series would look-like if it were extended to the left to include the atoms composed of anti-matter. These are atoms whose constituents are positrons instead of electrons and anti-protons with the opposite charge of protons. Neutrons of matter and antimatter are theoretically the same.

This expanded series can be reduced by simple arithmetic operations to a series of consecutive numbers stretching in opposite directions with the number **1** at its center.

First, the duplicated numbers in the series are combined into separate sums.

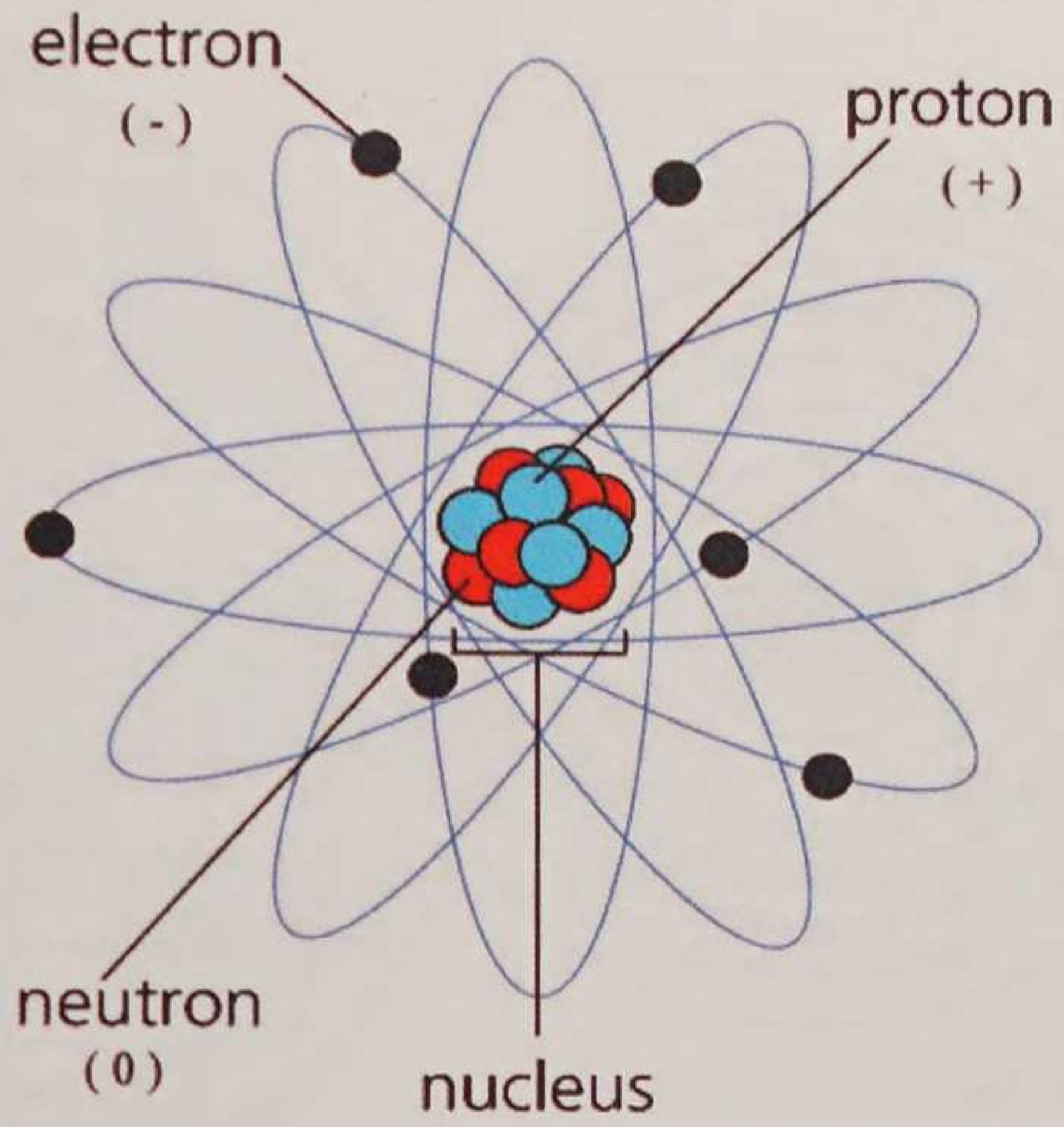
Next, these sums are converted to their square root.

And finally, those square-root numbers are divided by **2**.

This is what is defined mathematically as a perfect harmonic waveform pattern.

Program 5

The Niels-Bohr Model of the Atom



This slide depicts the Niels-Bohr model of the atoms of the noble elements, that is, those elements whose electron shells are full as in column **18** of the Periodic Table. Its electrons, each having a negative charge, are depicted as orbiting the nucleus which consists of an equal number of protons with a positive charge. These protons are separated from each other by intervening neutrons in close proximity, having no charge, which serve as insulation among the positively-charged protons, which in-turn are attracting the electrons which have orbital motion. That description has been used by chemists for over a hundred years but has been improved upon since its acceptance back a century ago as shown in the next slide.

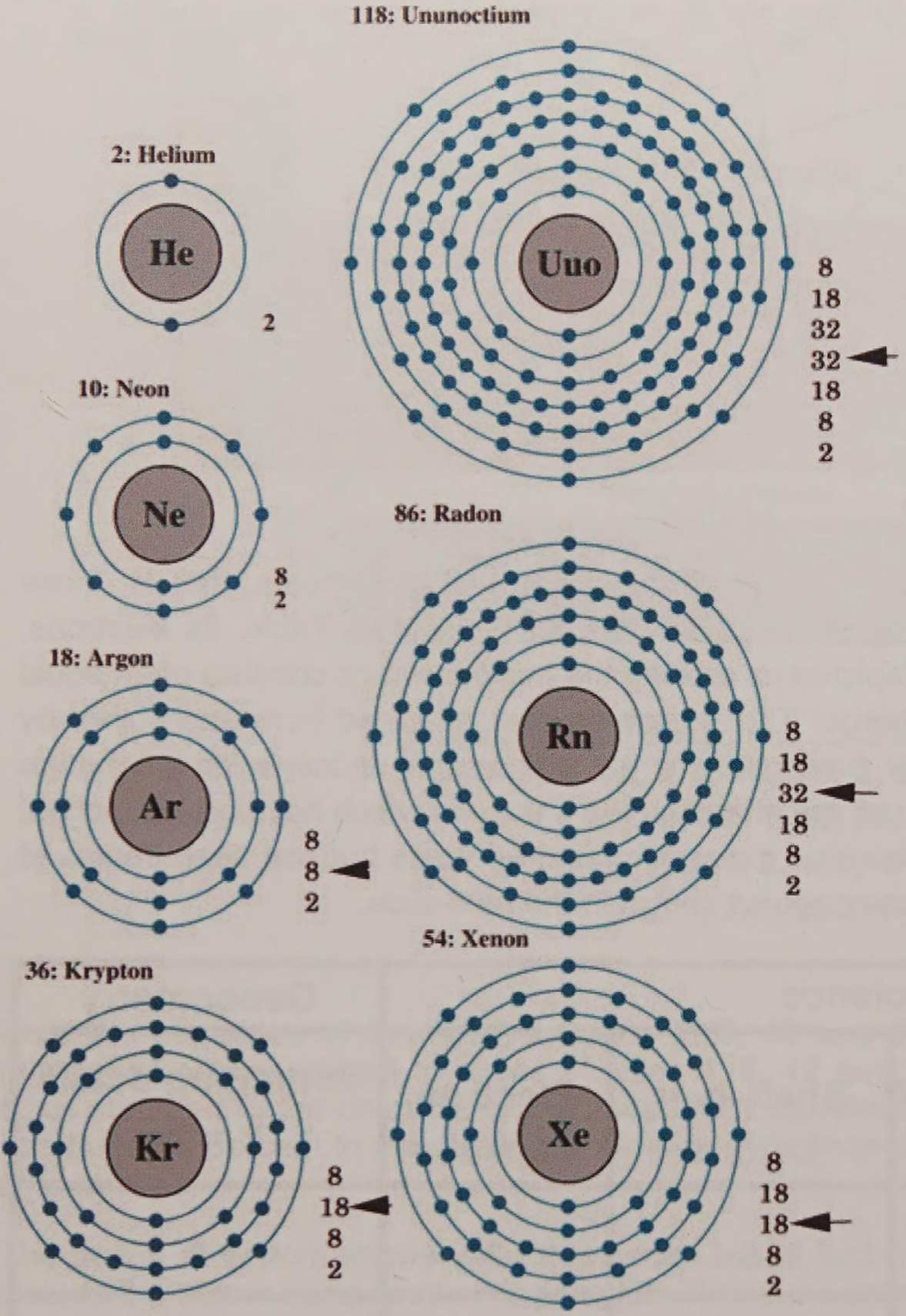
Atomic Science				Geonometry
Principal Quantum Number x	**Energy level**	**Shell–pair Letters**	**Electron Capacity**	**Impinging octals** $Y = 2x^2$
1	**s**	**K & L**	**2**	**2**
2	**p**	**M & N**	**8**	**8**
3	**d**	**O & P**	**18**	**18**
4	**f**	**Q & R**	**32**	**32**

The variable **x** represents the Principal Quantum Number, the number of the energy level in the associated electron shell-pair.

Note that **y =** the multiple of the kernel number **q** which equals the size-4x cube's characterristic number = the number of octals impinging the cube's characteristic sphere.

Program 5

Distribution pattern of electrons in the Noble elements



The distribution of electrons has been determined by atomic physicists to consist of nested shells of electrons as shown for the noble elements here. Note that each new completed shell originates within the middle of all the previous nested shells. That's a big correction to the Niels-Bohr model.

However, don't accept this model as fact; Geonometry has a different view yet as shown at bottom.

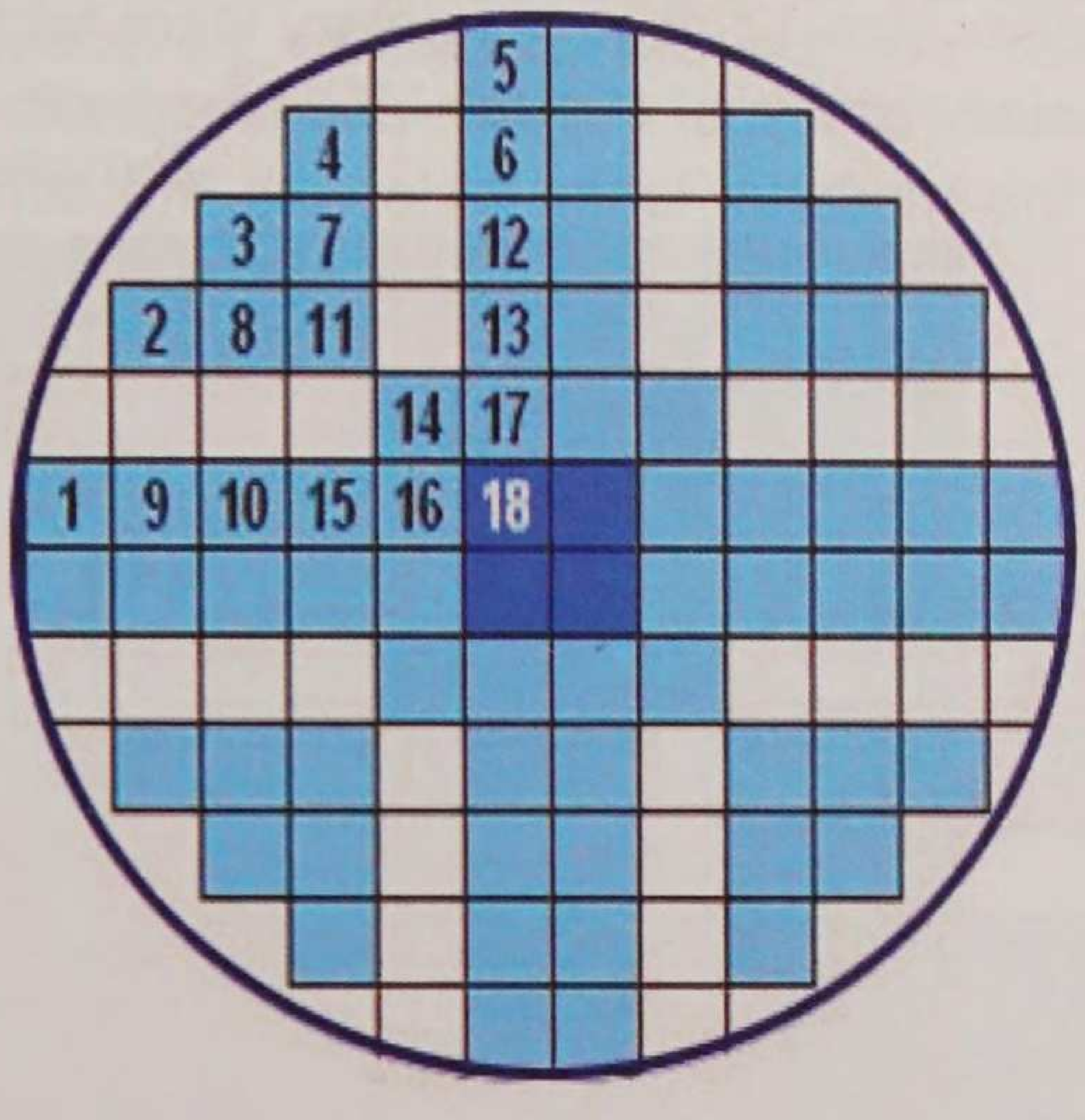
Now we have observed in Program 4 that the number of octals in the characteristic spheres for cubes of sizes **4, 8, 12** and **16** are equal to the numbers **2, 8, 18** and **32**, respectively. Note that each of these numbers minus **1** are the numbers **1, 7, 17** and **31** respectively, the latter of which are sequential prime-numbers.

We have already seen in Program 2 that geonomic squares for these sizes greater than **1** are Class-1 squares that have dual tiling patterns which cover the entire square exactly.

Electrons	Exterior Interior	Total
Helium	2	2
Neon	2, 8	10
Argon	2, 8, 8	18
Krypton	2, 8, 8, 18	36
Xenon	2, 8,8, 18, 18	54
Radon	2, 8, 8, 18, 18, 32	86
Ununoctium	2, 8 ,8, 18, 18, 32, 32	118

Program 5

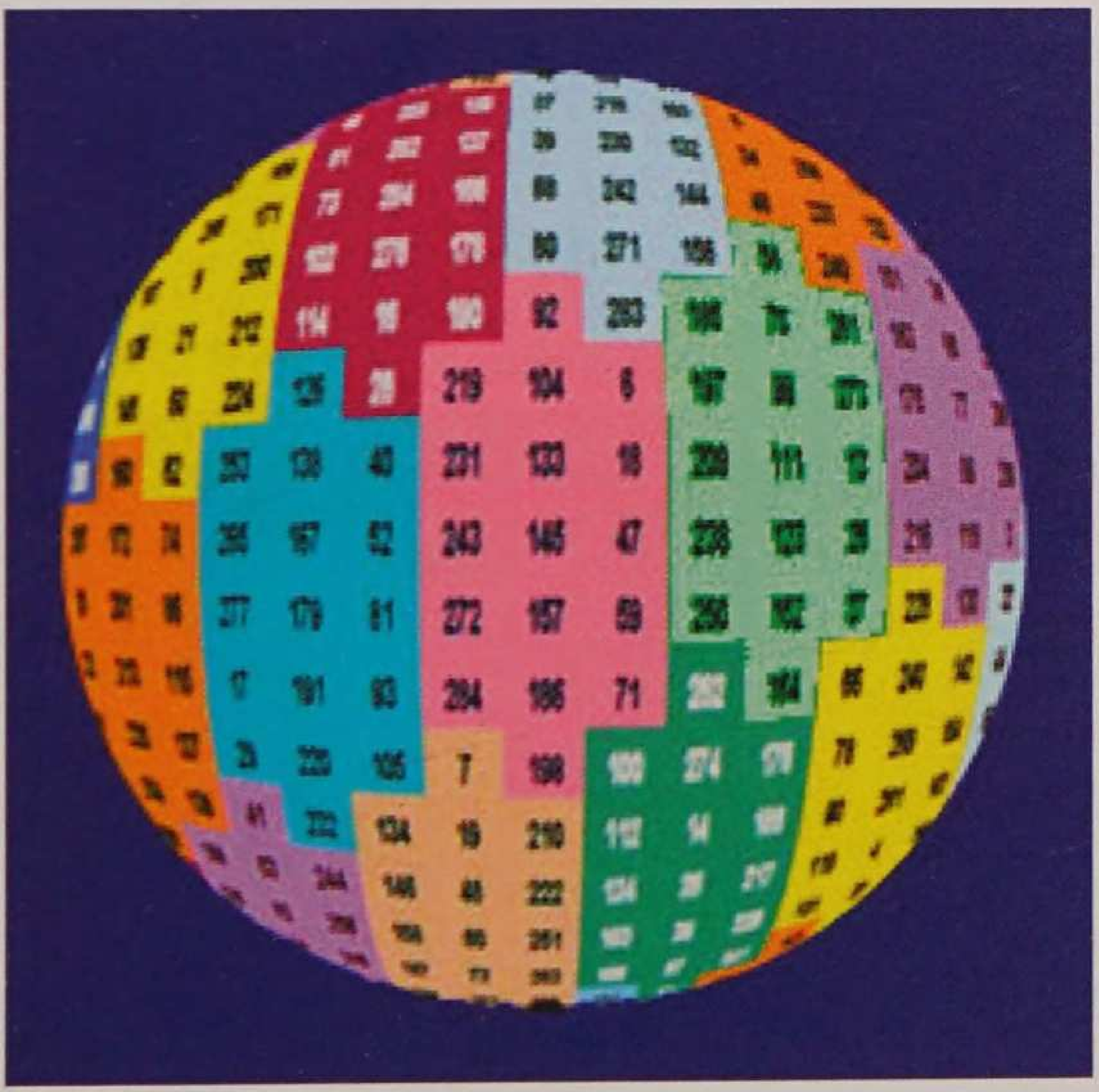
The Distribution of Octals in the Size-12 Characteristic Sphere

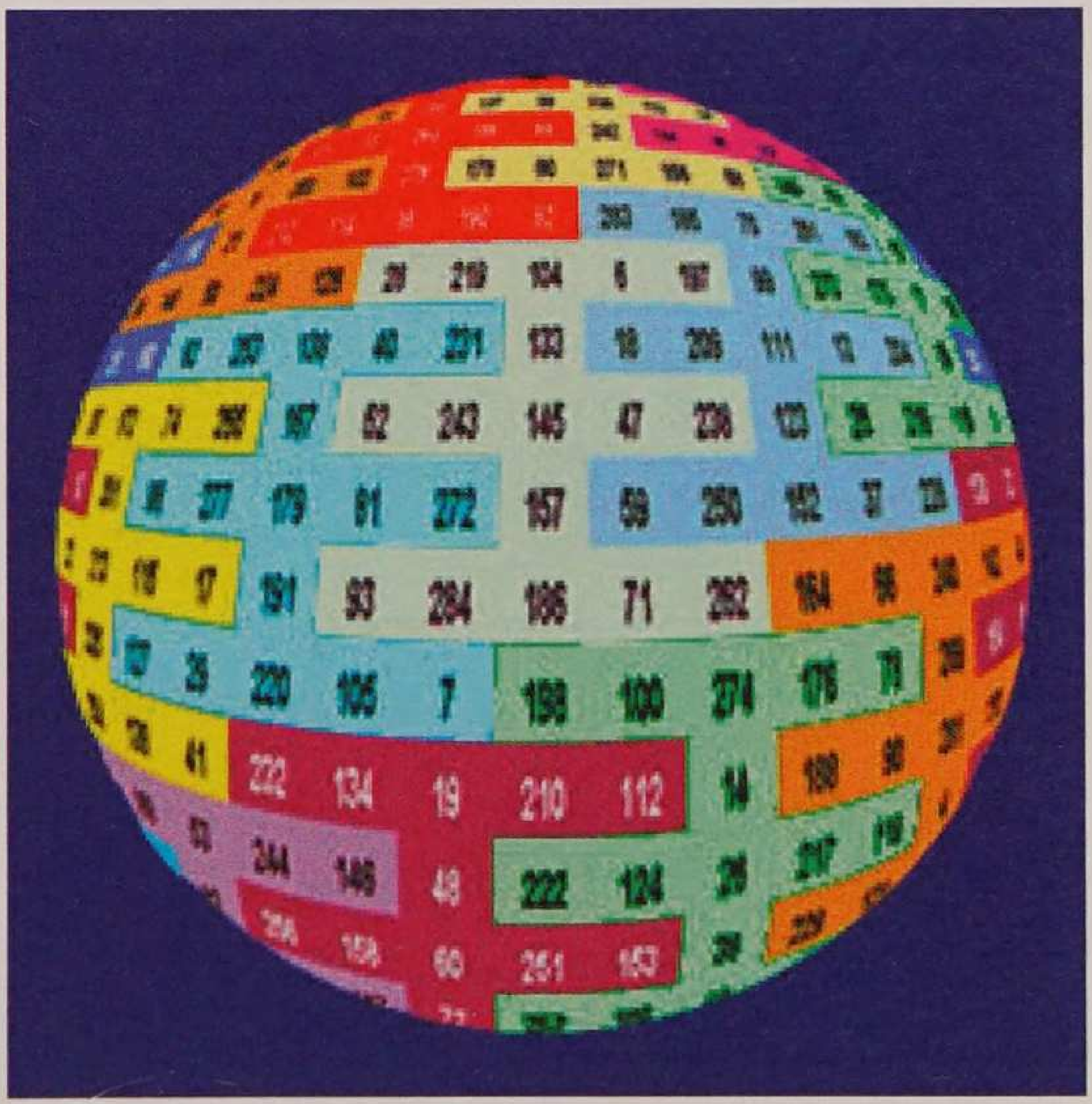


The size-12 cube's inscribed sphere showing the location of its 18 octals looking from top-down

The **18** octals of the size-12 cube's inscribed sphere indicates that it has the capacity for **18** electrons per spherical electron shell.

Tiling Patterns on the Surface of Characteristic Spheres





Picture the sze-17 tile patterns **A** & **B** wrapped around the size-12 characteristic sphere. This sphere would have the capacity for **18** electrons on its surface. At any one instant one of the **18** electrons passes through one of the non-tiled polar regions.

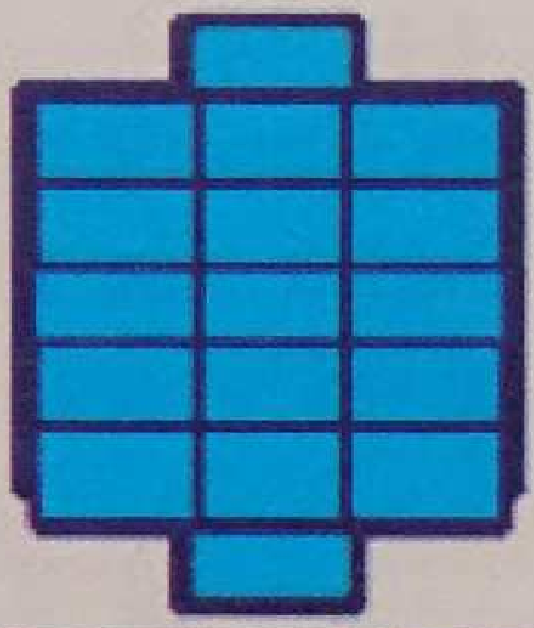
If these two tiling patterns were converted to their shared complementary loom tables, we would get two complementary spheres each with a capacity of **18** electrons.

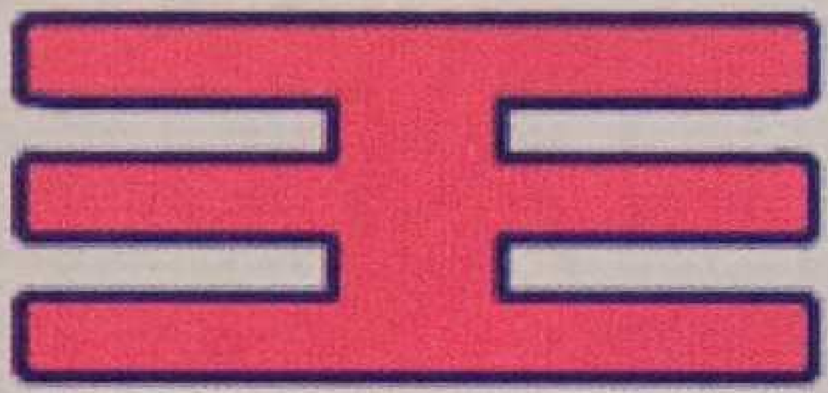
One can picture one tiling pattern guiding the electrons in one electron shell and the other tiling pattern orchestrating the activity of electrons in the other shell within each associated shell-pair. Further, the vibrating spherical-surface membrane of one shell could operate according to the modulus loom table and the other according to the integer loom table.

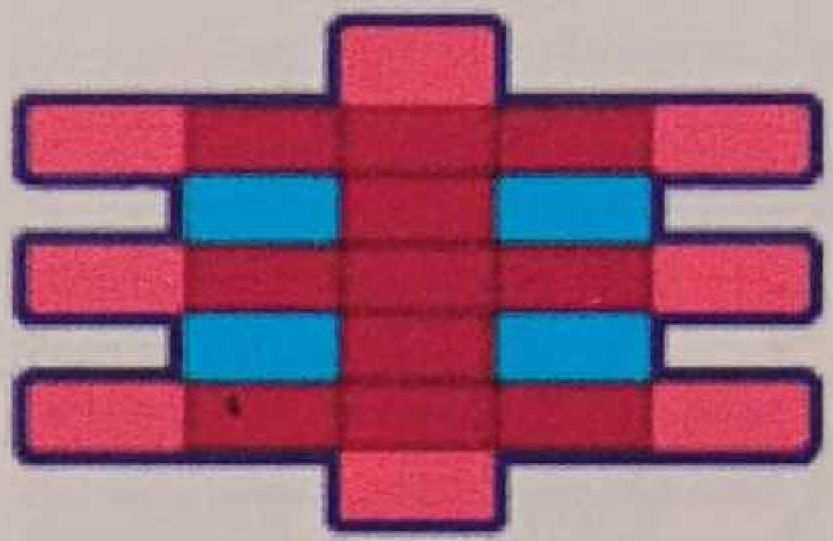
Program 5

Tiling Patterns of Loom Tables on the Surface of Characteristic Spheres

The complementary tiling patterns of prime-number size-17 complementary loom tables serve as templates for the location of electrons in adjacent shells of Krypton and Neon with electron counts of **18** electrons, each shown here on the modulus loom table with the different tile centers aligned over the number **9**.







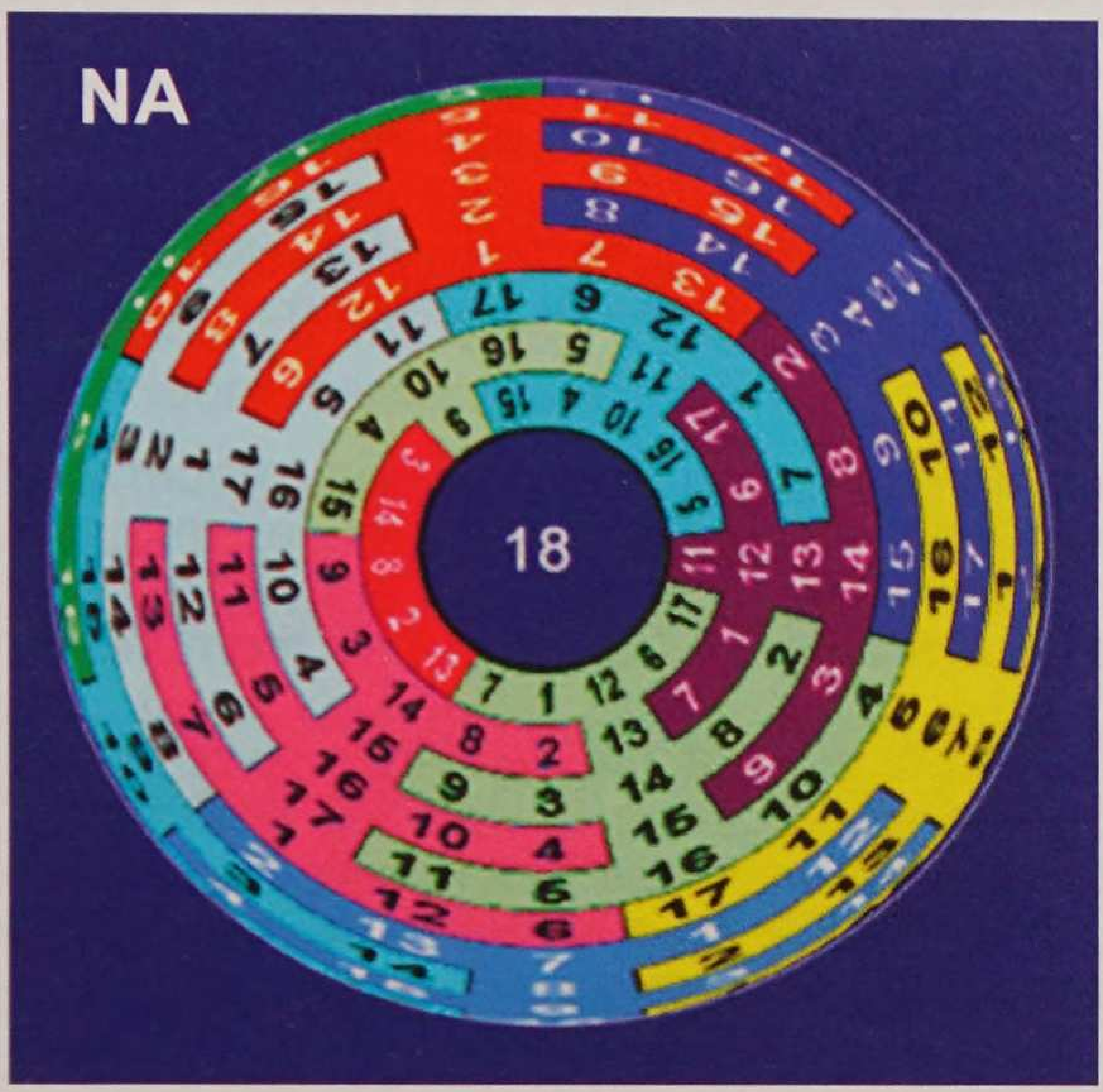
17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

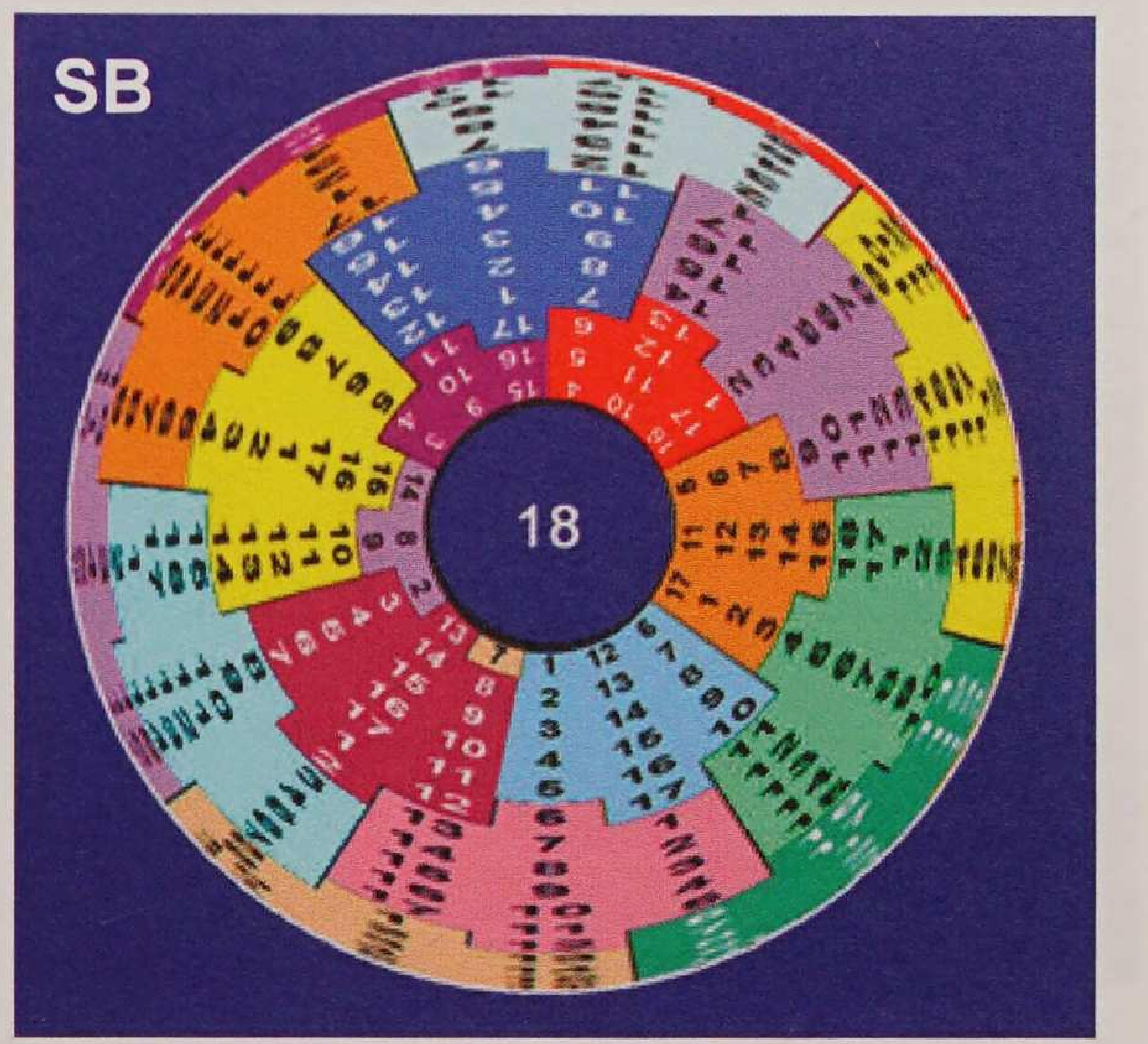
17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

Program 5

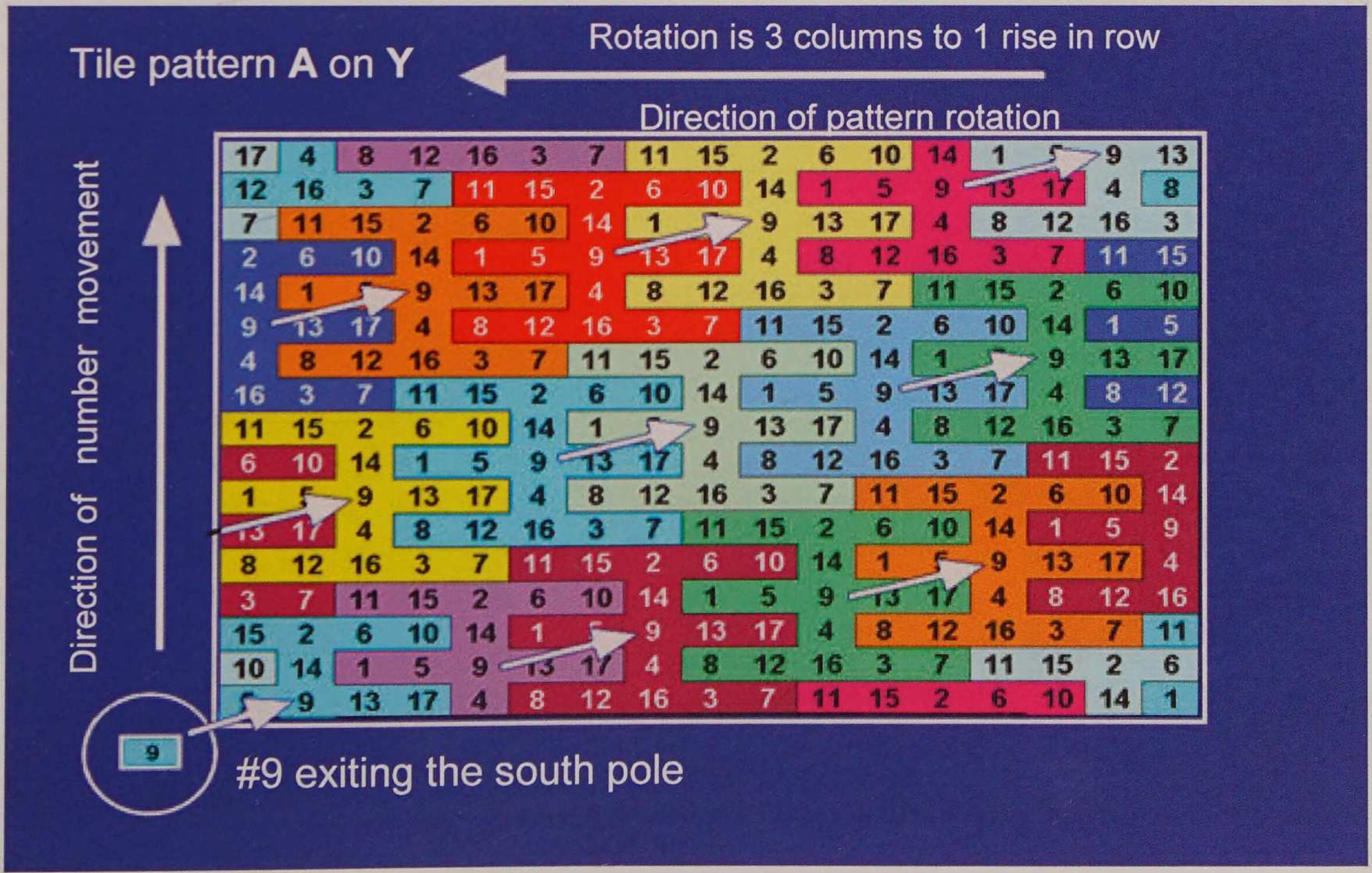
Orbital motion of Electrons according to Geonometry

Here are the tiling patterns **A** and **B** for the size-17 square centered at the North and South poles respectively, wrapped around the surface of the size-12 cube's loom tables' characteristic sphere. **NA** stands for **North** pole and tile pattern **A**. **SB** stands for **South** pole and tile pattern **B.**





Each pattern has room for **18** electrons on its surface. At any one instant one of the **18** electrons in each shell would pass through a non-tiled polar region, both north and south of each of the paired spherical shells. These regions would be glide areas without any propelling vibrations. Further, due to the density of electrons passing through the opposing poles, these poles would function as the polar axis for the alignment of the shells in the shell-pair.



Tile pattern **A** on **Y**

Rotation is 3 columns to 1 rise in row

Direction of pattern rotation

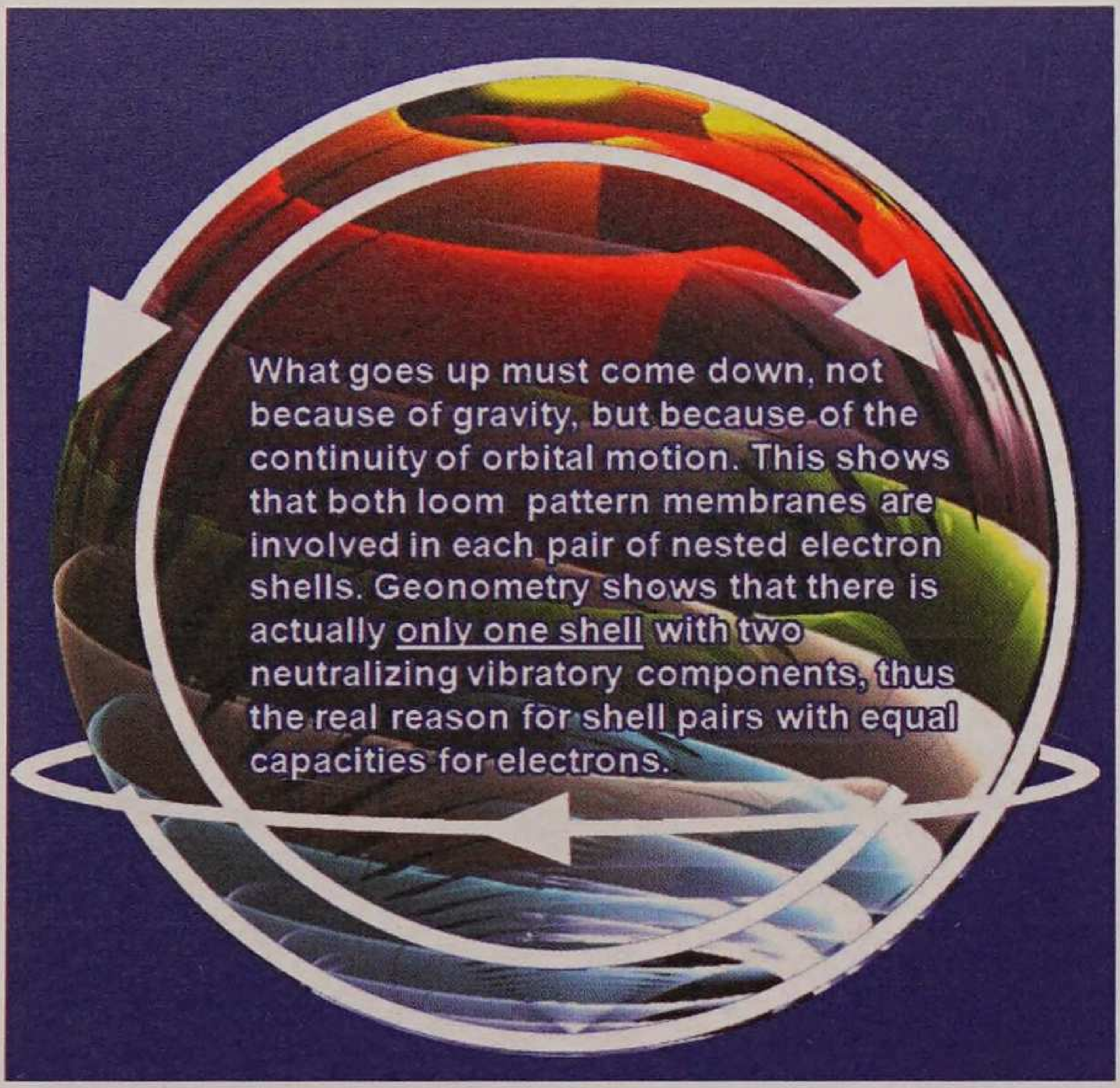
Direction of number movement

17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13
12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8
7	11	15	2	6	10	14	1	[illegible]	9	13	17	4	8	12	16	3
2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15
14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10
9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5
4	8	12	16	3	7	11	15	2	6	10	14	1	[illegible]	9	13	17
16	3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12
11	15	2	6	10	14	1	[illegible]	9	13	17	4	8	12	16	3	7
6	10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2
1	5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14
13	17	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9
8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	17	4
3	7	11	15	2	6	10	14	1	5	9	13	17	4	8	12	16
15	2	6	10	14	1	5	9	13	17	4	8	12	16	3	7	11
10	14	1	5	9	13	17	4	8	12	16	3	7	11	15	2	6
5	9	13	17	4	8	12	16	3	7	11	15	2	6	10	14	1

#9 exiting the south pole

Program 5

Now, picture the tiling pattern as rotating from right to left. As it does, the center of the next tile jumps 1 row upwards in 3 paces of rotation leftward toward the north pole of the sphere. Now picture the electron riding these vibrations straight upward and crossing over the north pole of the sphere.

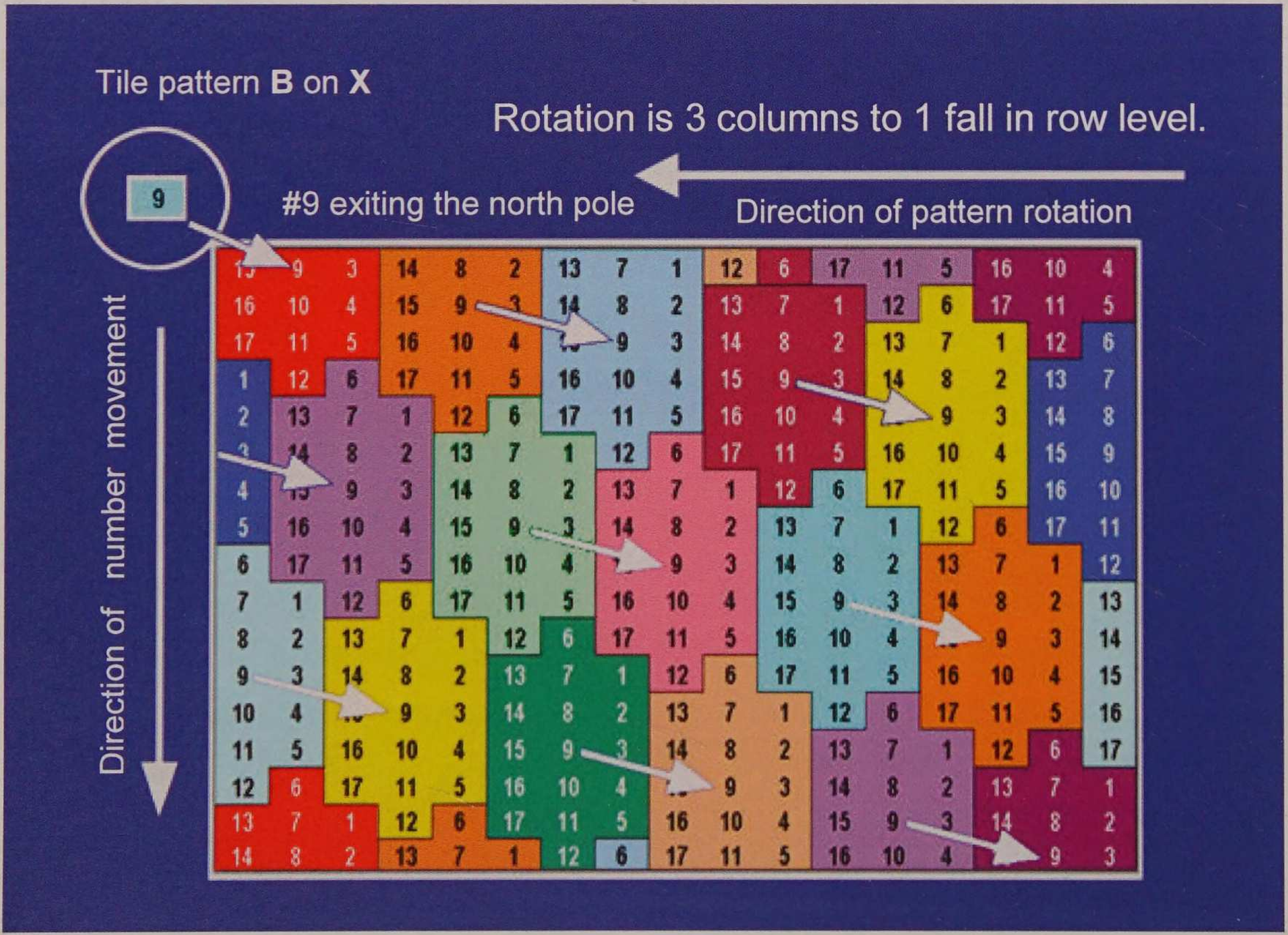


What goes up must come down, not because of gravity, but because of the continuity of orbital motion. This shows that both loom pattern membranes are involved in each pair of nested electron shells. Geonometry shows that there is actually only one shell with two neutralizing vibratory components, thus the real reason for shell pairs with equal capacities for electrons.

What goes up must come down, not because of gravity, but because of the continuity of orbital motion. This shows that both loom pattern membranes are involved by necessity in each pair of nested electron shells. Geonometry shows that actually there is **one shell** with two neutralizing components, thus the real reason for **shell pairs** with equal capacities for electrons.

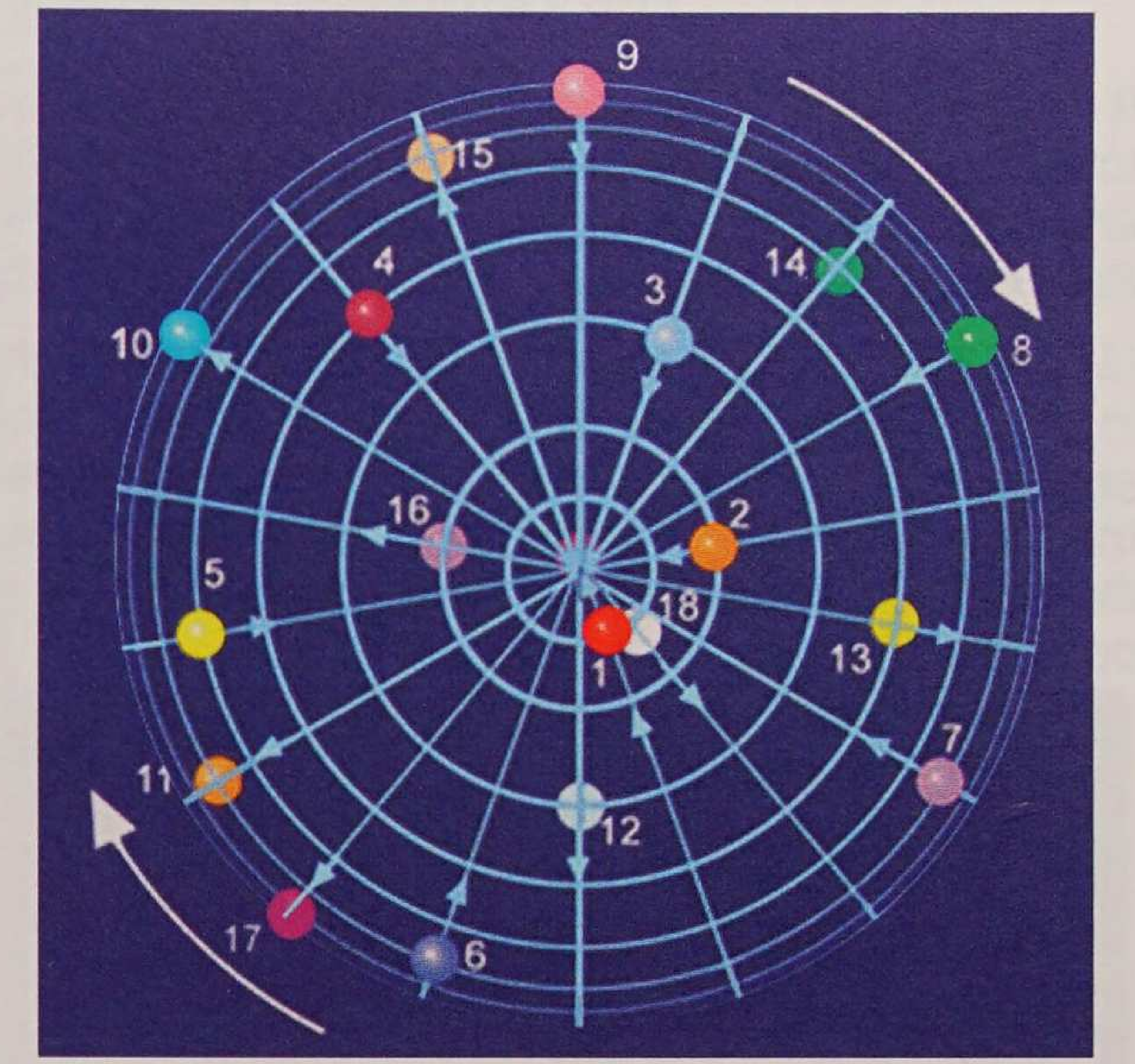
We've already seen the cloaking property of tiling patterns in Program 4. So these laminated tiling patterns need not even be aligned in order to be neutral outside their shell-pair membrane.

On the other half of the shell-pair, picture the descention of electrons toward the south pole. For every 3 paces of rotation leftward, the electron riding the wave membrane within the tile descends one row straight downward.



15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4
16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11	5
17	11	5	16	10	4	[illegible]	9	3	14	8	2	13	7	1	12	6
1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13	7
2	13	7	1	12	6	17	11	5	16	10	4	[illegible]	9	3	14	8
3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15	9
4	15	9	3	14	8	2	13	7	1	12	6	17	11	5	16	10
5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17	11
6	17	11	5	16	10	4	[illegible]	9	3	14	8	2	13	7	1	12
7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2	13
8	2	13	7	1	12	6	17	11	5	16	10	4	[illegible]	9	3	14
9	3	14	8	2	13	7	1	12	6	17	11	5	16	10	4	15
10	4	[illegible]	9	3	14	8	2	13	7	1	12	6	17	11	5	16
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6	17
12	6	17	11	5	16	10	4	[illegible]	9	3	14	8	2	13	7	1
13	7	1	12	6	17	11	5	16	10	4	15	9	3	14	8	2
14	8	2	13	7	1	12	6	17	11	5	16	10	4	[illegible]	9	3

Program 5



This shows the only orbital pattern that can be inferred from the spinning tiling patterns.

The electrons are numbered consecutively from the top downward.

As the dual tile patterns are rotating clockwise each electron advances one more level straight up toward the north pole or straight down toward the south pole depending upon which loom membrane it is riding.

Half of the **18** electrons are heading upward toward the north pole and half are heading downward toward the south pole.

Both loom membranes are involved as are both tiling patterns: **A** on **Y** and **B** on **X**.
The double membrane actually has the capacity for **36** electrons before it is completely filled. Hence the reason for atomic physicists' classification of the electron distribution into combined shell-pairs. Geonometry shows how this shell-pair is actually constructed as two laminated complementary joint-neutralizing vibrating membranes.

Further, the vibrations at the poles are zero, so it's a glide area for crossing electrons. All the tiled areas have powering vibrations proportional to the average of the numbers in each tile. So in the case of **18** electrons here, that number is **9,** which is the central number in each tile.

The same construct explains the electron shell pairs having **8** and **32** electrons per shell. In these cases the shell pair consists of two tiling patterns on two laminated vibrating loom membranes of prime-number sizes **7** and **31**, respectively. This was the motivation for showing these size loom tables and tiling patterns in detail back in Programs 2 and 3.

The electron shell containing just **2** electrons is just a simplification of the larger shells where here there is one shell-pair with just two shells, one for the "up" portion and one for the "down" portion of the orbit. Its capacity then is just 2 electrons, each shepparding the other thru mutual repelling electro-parity.

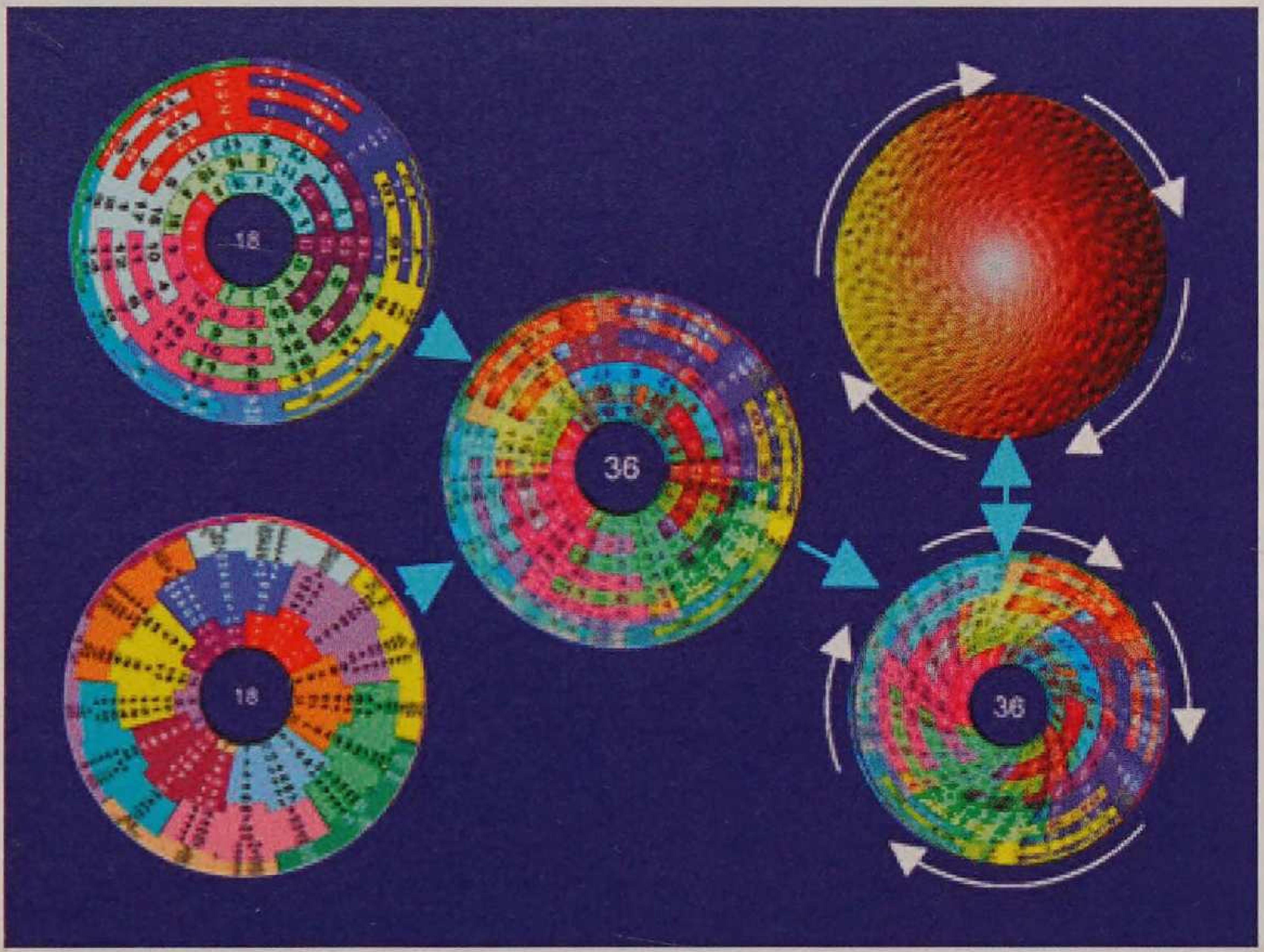
Now, we have already seen back in Program 3 that the tiling patterns on the loom tables had central numbers that ranged the gamut of **1** thru **n** for all Class-1 squares. Thus, we may interpret the central numbers in the A and B patterns on complementary loom tables **X** and **Y**, respectively, to be the energy level of all $\mathbf{2n^2}$ electrons in that shell-pair, thus accounting for the series of consecutive elements which have the same electron energy level in their shell-pairs **(2, 16, 36, 64}**. Thus, we have accounted for all the electrons in terms of size-n patterns A and B wrapped around the associated characteristic sphere of geonomic cubes of sizes **4**, **8**, **12** & **16**.

Program 5

The Double-Shell Cloaking of Electron Shells

It is physically impossible to isolate one individual electron to take its temperature, so to speak, to determine its energy level. Atomic physicists have derived their categorization of electron energy levels by averaging thousands upon thousands of atom-smasher measurements. So their categorizations are only the averages of what is detectable.

Geonometry tells a different story. It indicates that there must be **18** energy levels of electrons in the innermost shell of Krypton and Xenon of which only **17** are non-zero. And that these energy levels derive from the spherical membrane vibrating at **17** identical energies in each of the size-12 characteristic spherical shells in their shared electron shell-pair. And further, it is these vibrations that drive the electrons in their orbits with equal velocities around the common compact nucleus.



These distinct electron shells can never be observed in pairs directly by atomic physicists because the pairs of same-size loom tables involved cloak one another regardless of the location of the dual tiling patterns relative to one another – and that was proven mathematically in Program 4. The only alignment that is necessary in this interpretation is that over the poles.

But such alignment is guaranteed if the vibrating membranes spin on a common axis and that was demonstrated quite vividly here in Program 5.

Atomic Physics has determined that there must be two equal but opposite spins for the electrons in any shell-pair as $+\frac{1}{2}$ and $-\frac{1}{2}$. In the case for Krypton and Xenon here, that spin sign is the opposite up-down directions of the electrons in the complementary tiling patterns on complementary loom-tables spinning in the same direction. The centers of the tiles in each spinning complementary tiling pattern were seen to be the average of all the numbers in each tile, **9**, accounting for the value $\frac{1}{2}$ for the size-18 shells of the shell-pair:

$$(5.1)\quad \bar{x} = \bar{y} = \tfrac{1}{2}n$$

Extension of electron distribution pattern to other shell-pairs

The same construct explains the electron shell-pairs having **8** and **32** electrons per shell. In these cases the shell-pair consists of **2** tiling patterns on **2** laminated vibrating loom membranes of prime-number sizes **7** and **31,** respectively.

The electron shell containing just **2** electrons is just a simplification of the larger shells. Here there is one shell-pair with just two shells, one for the up and one for the down portion of the orbit. Its capacity then is just **2** electrons with spins of $+\frac{1}{2}$ and $-\frac{1}{2}$.

The location of poles may differ among different shell-pairs. That is, the spin detected in one shell-pair may be different from the spin of any other shell-pair. We're talking about shell-pairs here.

Program 5

Electron Fill Patterns

Electron Count		Electron-fill Sequence			
#	Element	1	2	3	4
1	Hydrogen	1			
2	Helium	1	1		
3	Lithium	1	1	1	
4	Berylium	1	1	2	
5	Boron	1	1	3	
6	Carbon	1	1	4	
7	Nitrogen	1	1	5	
8	Oxygen	1	1	6	
9	Fluorine	1	1	7	
10	Neon	1	1	8	

Electron Count		Electron-fill Sequence			
#	Element	1	2	3	4
11	Sodium	1	1	8	1
12	Magnesium	1	1	8	2
13	Aluminum	1	1	8	3
14	Silicon	1	1	8	4
15	Phosphorus	1	1	8	5
16	Sulfur	1	1	8	6
17	Chlorine	1	1	8	7
18	Argon	1	1	8	8

Electron Count		Electron-fill Sequence					
#	Element	1	2	3	4	5	6
19	Potassium	1	1	8	8	1	
20	Calcium	1	1	8	8	2	
21	Scandium	1	1	8	9	2	
22	Titanium	1	1	8	10	2	
23	Vanadium	1	1	8	11	2	
24	Chromium	1	1	8	13	1	
25	Manganese	1	1	8	13	2	
26	Iron	1	1	8	14	2	
27	Cobalt	1	1	8	15	2	
28	Nickel	1	1	8	17	1	
29	Copper	1	1	8	18	1	
30	Zinc	1	1	8	18	2	
31	Gallium	1	1	8	18	3	
32	Germanium	1	1	8	18	4	
33	Arsenic	1	1	8	18	5	
34	Selenium	1	1	8	18	6	
35	Bromine	1	1	8	18	7	
36	Krypton	1	1	8	18	8	
		1	2	3	4	5	6

Electron Count		Electron-fill Sequence					
#	Element	1	2	3	4	5	6
37	Rubidium	1	1	8	18	8	1
38	Strontium	1	1	8	18	8	2
39	Yttrium	1	1	8	18	9	2
40	Zirconium	1	1	8	18	10	2
41	Niobium	1	1	8	18	12	1
42	Molybdenum	1	1	8	18	13	1
43	Technetium	1	1	8	18	13	2
44	Ruthenium	1	1	8	18	15	2
45	Rhodium	1	1	8	18	16	1
46	Palladium	1	1	8	18	18	0
47	Silver	1	1	8	18	18	1
48	Cadmium	1	1	8	18	18	2
49	Indium	1	1	8	18	18	3
50	Tin	1	1	8	18	18	4
51	Antimony	1	1	8	18	18	5
52	Tellurium	1	1	8	18	18	6
53	Iodine	1	1	8	18	18	7
54	Xenon	1	1	8	18	18	8
		1	2	3	4	5	6

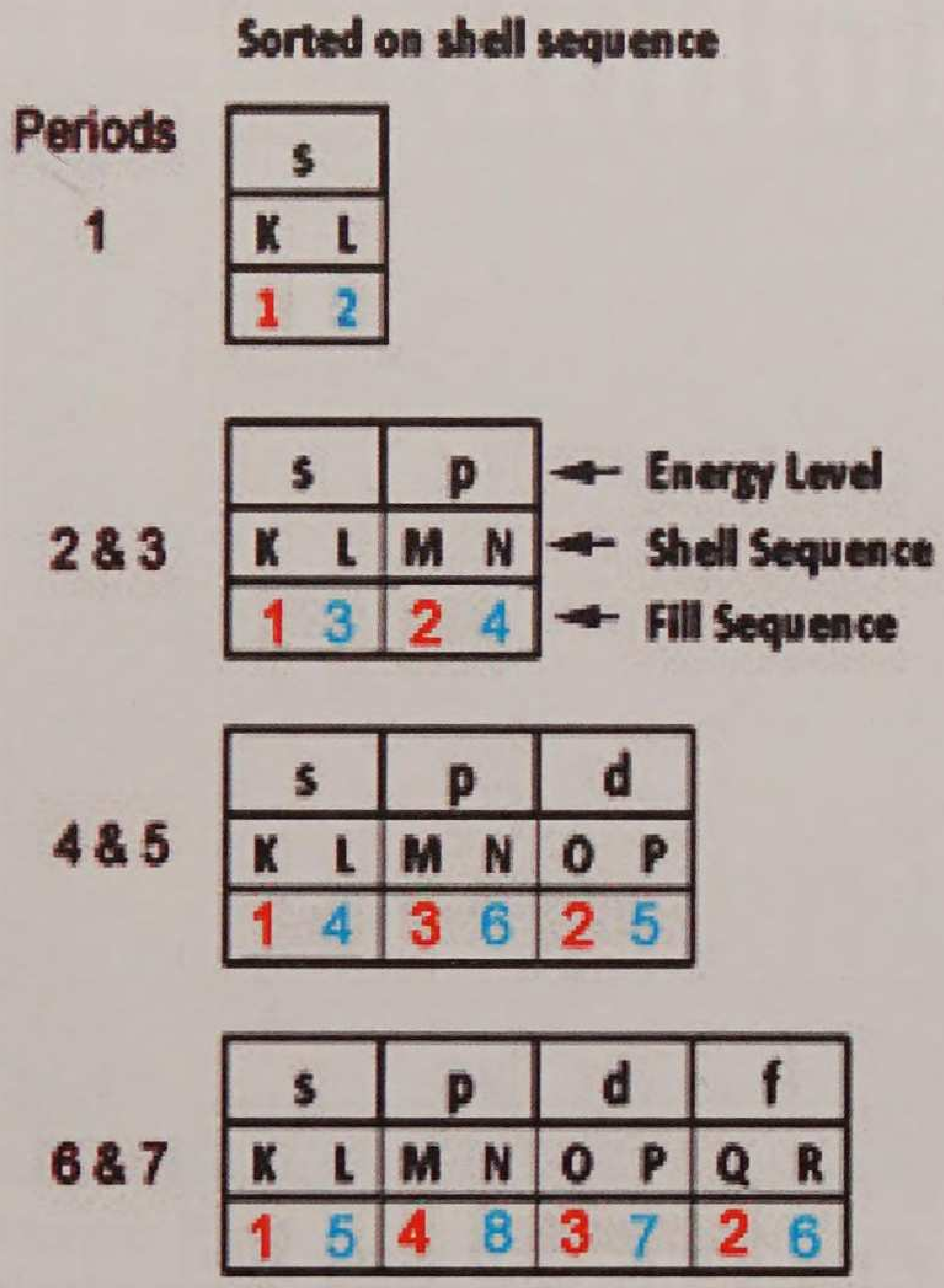
Electron Count		Electron-fill Sequence							
#	Element	1	2	3	4	5	6	7	8
55	Cesium	1	1	8	18	18	8	1	
56	Barium	1	1	8	18	18	8	2	
57	Lanthanum	1	1	8	18	18	9	2	
58	Cerium	1	1	8	18	19	9	2	
59	Praseodymium	1	1	8	18	21	8	2	
60	Neodymium	1	1	8	18	22	8	2	
61	Promethium	1	1	8	18	23	8	2	
62	Samarium	1	1	8	18	24	8	2	
63	Europium	1	1	8	18	25	8	2	
64	Gadolinium	1	1	8	18	26	8	2	
65	Terbium	1	1	8	18	27	8	2	
66	Dysprosium	1	1	8	18	28	8	2	
67	Holmium	1	1	8	18	29	8	2	
68	Erbium	1	1	8	18	30	8	2	
69	Thulium	1	1	8	18	31	8	2	
70	Ytterbium	1	1	8	18	32	8	2	
71	Lutetium	1	1	8	18	32	9	2	
72	Hafnium	1	1	8	18	32	10	2	
73	Tantalum	1	1	8	18	32	11	2	
74	Tungsten	1	1	8	18	32	12	2	
75	Rhenium	1	1	8	18	32	13	2	
76	Osmium	1	1	8	18	32	14	2	
77	Iridium	1	1	8	18	32	15	2	
78	Platinum	1	1	8	18	32	17	1	
79	Gold	1	1	8	18	32	18	1	
80	Mercury	1	1	8	18	32	18	2	
81	Thallium	1	1	8	18	32	18	3	
82	Lead	1	1	8	18	32	18	4	
83	Bismuth	1	1	8	18	32	18	5	
84	Polonium	1	1	8	18	32	18	6	
85	Astatine	1	1	8	18	32	18	7	
86	Radon	1	1	8	18	32	18	8	
		1	2	3	4	5	6	7	8

Electron Count		Electron-fill Sequence							
#	Element	1	2	3	4	5	6	7	8
87	Francium	1	1	8	18	32	18	8	1
88	Radium	1	1	8	18	32	18	8	2
89	Actinium	1	1	8	18	32	18	9	2
90	Thorium	1	1	8	18	32	18	10	2
91	Protactinium	1	1	8	18	32	20	9	2
92	Uranium	1	1	8	18	32	21	9	2
93	Neptunium	1	1	8	18	32	22	9	2
94	Plutonium	1	1	8	18	32	24	8	2
95	Americium	1	1	8	18	32	25	8	2
96	Curium	1	1	8	18	32	25	9	2
97	Berkelium	1	1	8	18	32	27	8	2
98	Californium	1	1	8	18	32	28	8	2
99	Einsteinium	1	1	8	18	32	29	8	2
100	Fermium	1	1	8	18	32	30	8	2
101	Mendelevium	1	1	8	18	32	31	8	2
102	Nobelium	1	1	8	18	32	32	8	2
103	Lawrencium	1	1	8	18	32	32	8	3
104	Rutherfordium	1	1	8	18	32	32	10	2
105	Dubnium	1	1	8	18	32	32	11	2
106	Seaborgium	1	1	8	18	32	32	13	1
107	Bohrium	1	1	8	18	32	32	13	2
108	Hassium	1	1	8	18	32	32	14	2
109	Meitnerium	1	1	8	18	32	32	15	2
110	Darmstadtium	1	1	8	18	32	32	18	0
111	Roentgenium	1	1	8	18	32	32	18	1
112	Copernicium	1	1	8	18	32	32	18	2
113	Ununtrium	1	1	8	18	32	32	18	3
114	Flerovium	1	1	8	18	32	32	18	4
115	Ununpentium	1	1	8	18	32	32	18	5
116	Livermorium	1	1	8	18	32	32	18	6
117	Ununseptium	1	1	8	18	32	32	18	7
118	Ununoctium	1	1	8	18	32	32	18	8
		1	2	3	4	5	6	7	8

Electron Distribution among Shells

The next slide helps to comprehend this somewhat complex electron tumbling process as electrons are added.

On the prior page is the sequence table for which shell gets an electron as atoms are derived from the preceding one in the electron shell filling process. The colored numbers indicate where unusual rearrangements occur.



Sorted on shell sequence

Periods

1

s	
K	L
1	2

2&3

s		p	
K	L	M	N
1	3	2	4

4&5

s		p		d	
K	L	M	N	O	P
1	4	3	6	2	5

6&7

s		p		d		f	
K	L	M	N	O	P	Q	R
1	5	4	8	3	7	2	6

The elements with all applicable shells filled to capacity are called “noble” elements and are highlighted in yellow. These are the elements in the rightmost column of the Periodic Table.

Note that this chart has **8** columns, not **7** as seen in all current textbooks on Chemistry. It appears rather redundant to have all **1**’s in **2** columns as opposed to all **2**’s in **1** column, but Geonometry indicates that there must be **8** electron shells, not **7**. Having shell-pairs for all the electrons is more consistent with the double counting of additional electrons in every other noble element beginning with Helium.

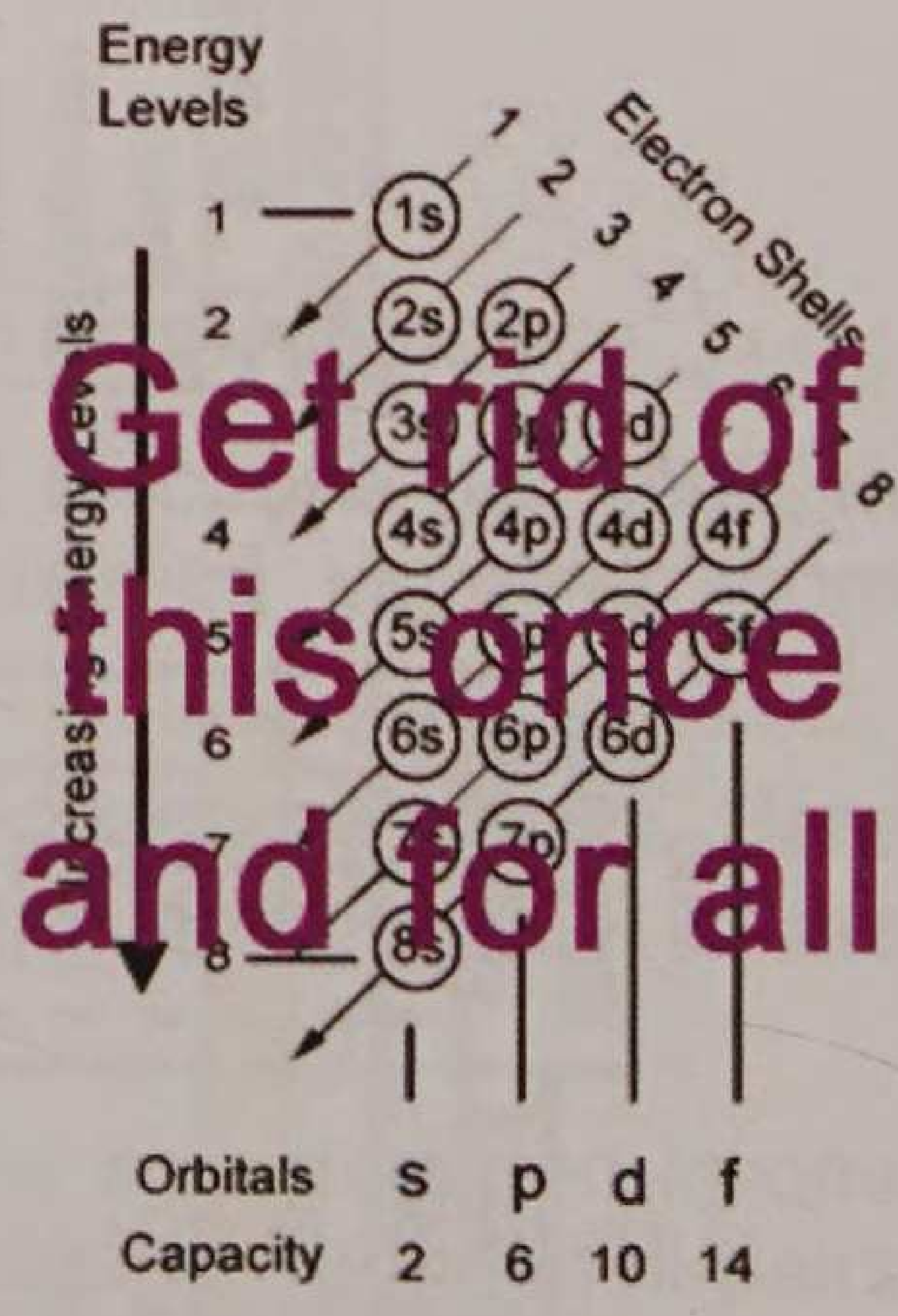
It is important to note that the numbered sequence relates strictly to the sequence in which electron are added to the atoms.

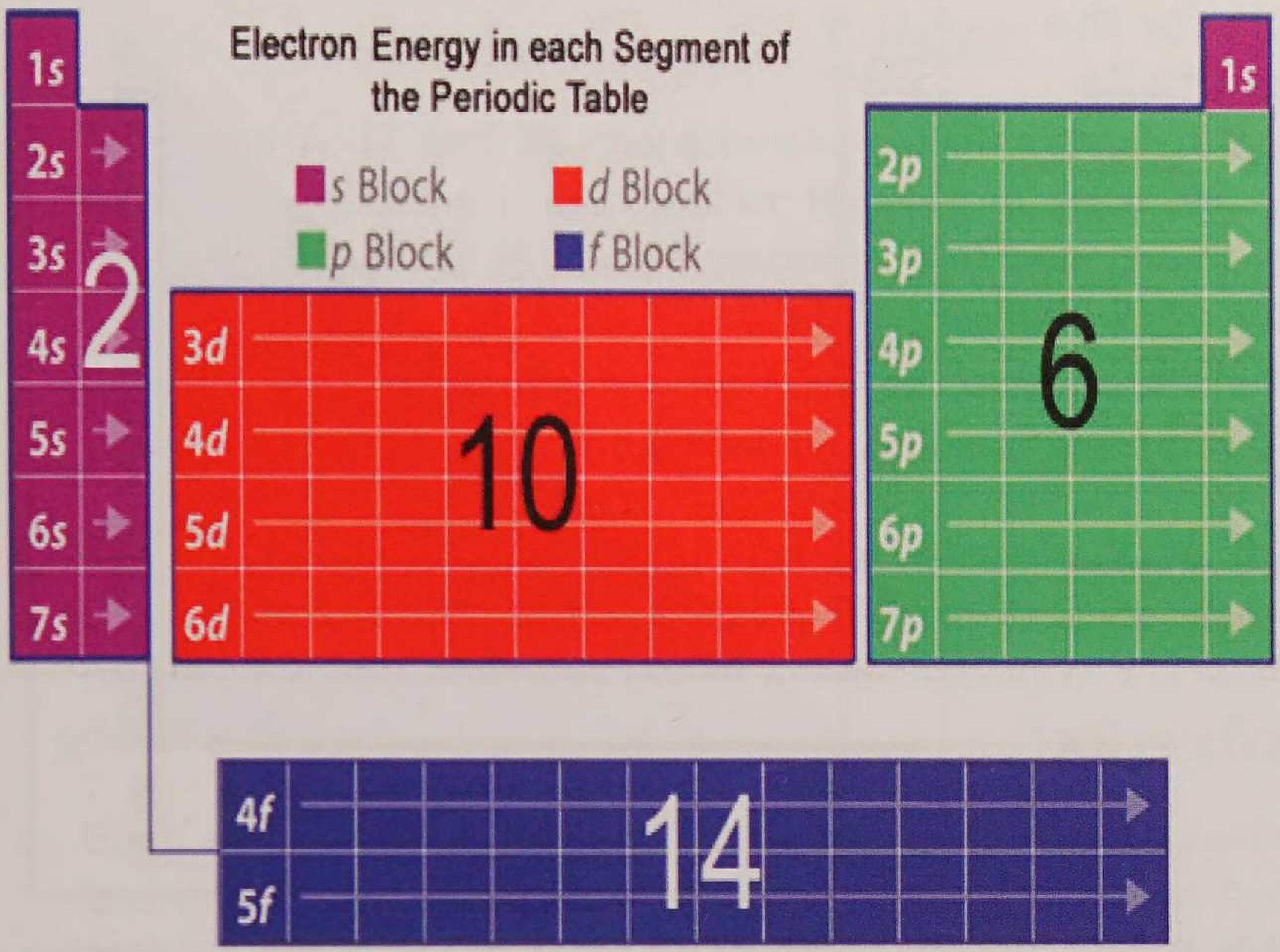
That sequence indicated by Geonometry is shown here sorted on the energy level. Geonometry indicates all of these shell-pairs must be adjacent for them to function together properly; they should not be separated as some scientists picture them being split as was seen earlier.

Observe that the **shell-pairs** are half-filled with the 1st **2** electrons in the outermost shell-pair and then the balance from the innermost to outermost vacant half-shell.

On the next page is the distribution of these orbitals throughout the Periodic Table. Observe that every shell has all of its electrons at one distinct energy level.

At right is the atomic scientists’ contorted way of looking at these orbitals from an abbreviated perspective. This may be more convenient for scientists but this just makes understanding more obtuse for students. It took me two days to see that this chart matched up to the Periodic Table on the next page There is pictured the easy way to view these average electron energy levels without this confusing diagonally-oriented graph. If you want all the energies for a particular atom, just trace your way back up through the table to Hydrogen. All this chart is on the right is a shorthand notation for recovering all the energies for consecutive orbitals. You don’t need it.





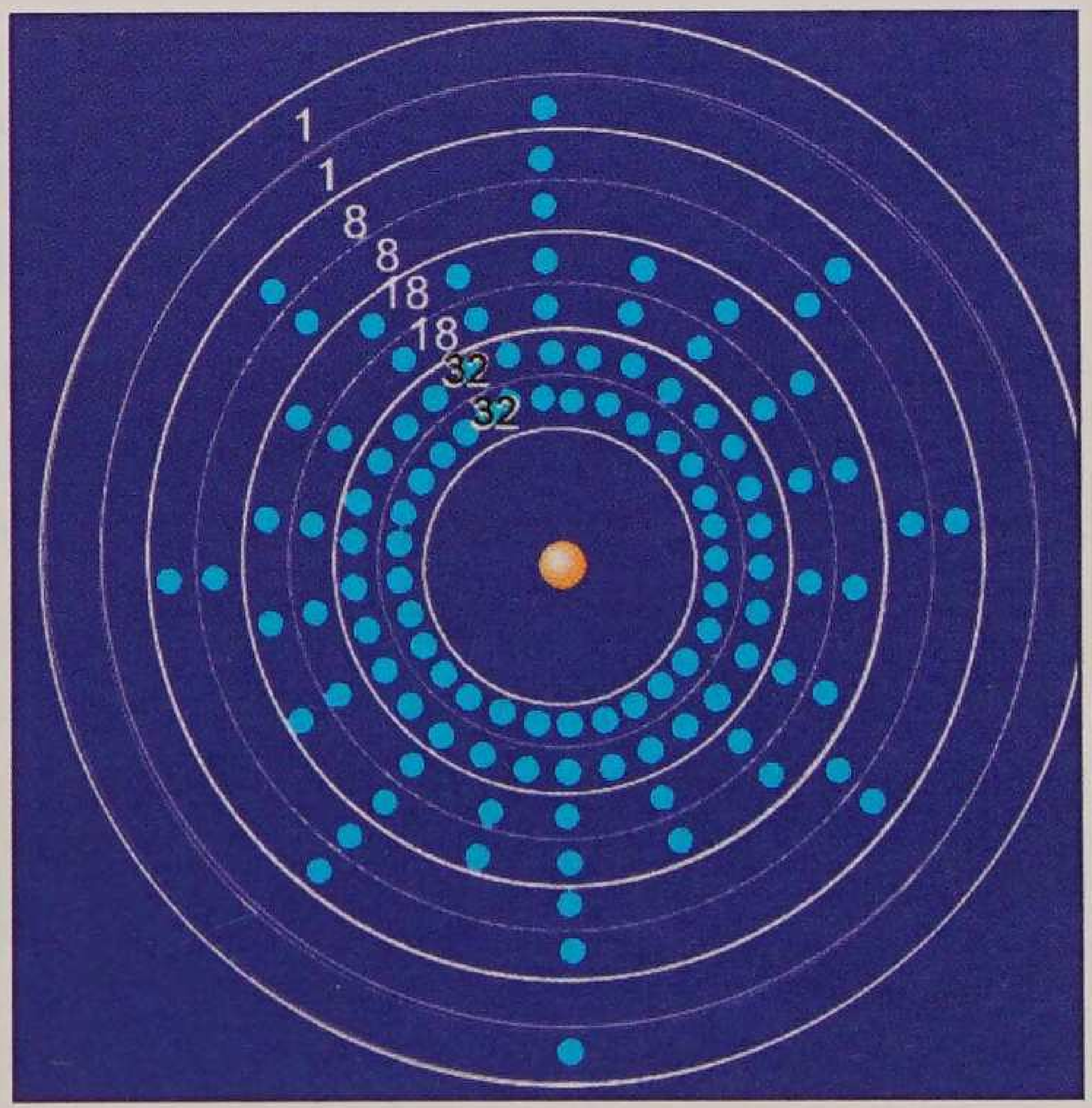
Orbital Capacities

Here is the distribution of these orbitals throughout the Periodic Table. Observe that every shell has all its electrons at its own distinct energy level. Observe that the orbital capacity of each section **s, p, d, f**, is **2, 6, 10** & **14,** respectively.

And note that these capacities are **2** times the series of odd-prime numbers **1, 3, 5, 7.**

So once again, odd-prime-numbers are playing a prominent cohesive role in the organization of atoms!.

Geonometry's view of the electron distribution pattern



Atomic Scientists have determined that the outer **2** electrons tumble inward when the 3rd electron is added to the outer shell. The inward movement transfers **3** electrons to the nearest shell-pair with vacant capacity. From there on, the outer shell-pair of capacity **2** remains vacant until the newly receptive shell-pair reaches its capacity. Then the process repeats itself.

Laminated shell-pairs have the joint capacities of **2**, **16**, **36** and **64**.

All of this makes a whole lot more sense and is a more organized framework than the helter-skelter motion envisioned by today's atomic physicists, which is somewhat reminiscent of the old Niels-Bohr model shown at the beginning of this program.

Of course, the physics had to come first before the math could be applied, but Geonometry depicts the electron properties quite vividly for better explaining the inherent source of the physics.

So now it should be evident just how the natural fundamental geometric patterns inherent in Geonometry can be applied to explain naturally occurring phenomenon at the sub-dimensional space of the atom!

That is why Geonometry has been dubbed the "new wow math".

Program 7 will explain the structure of the nucleus of atoms in terms of Geonometry from discoveries made in 4-dimensional space coming up next in Program 6.

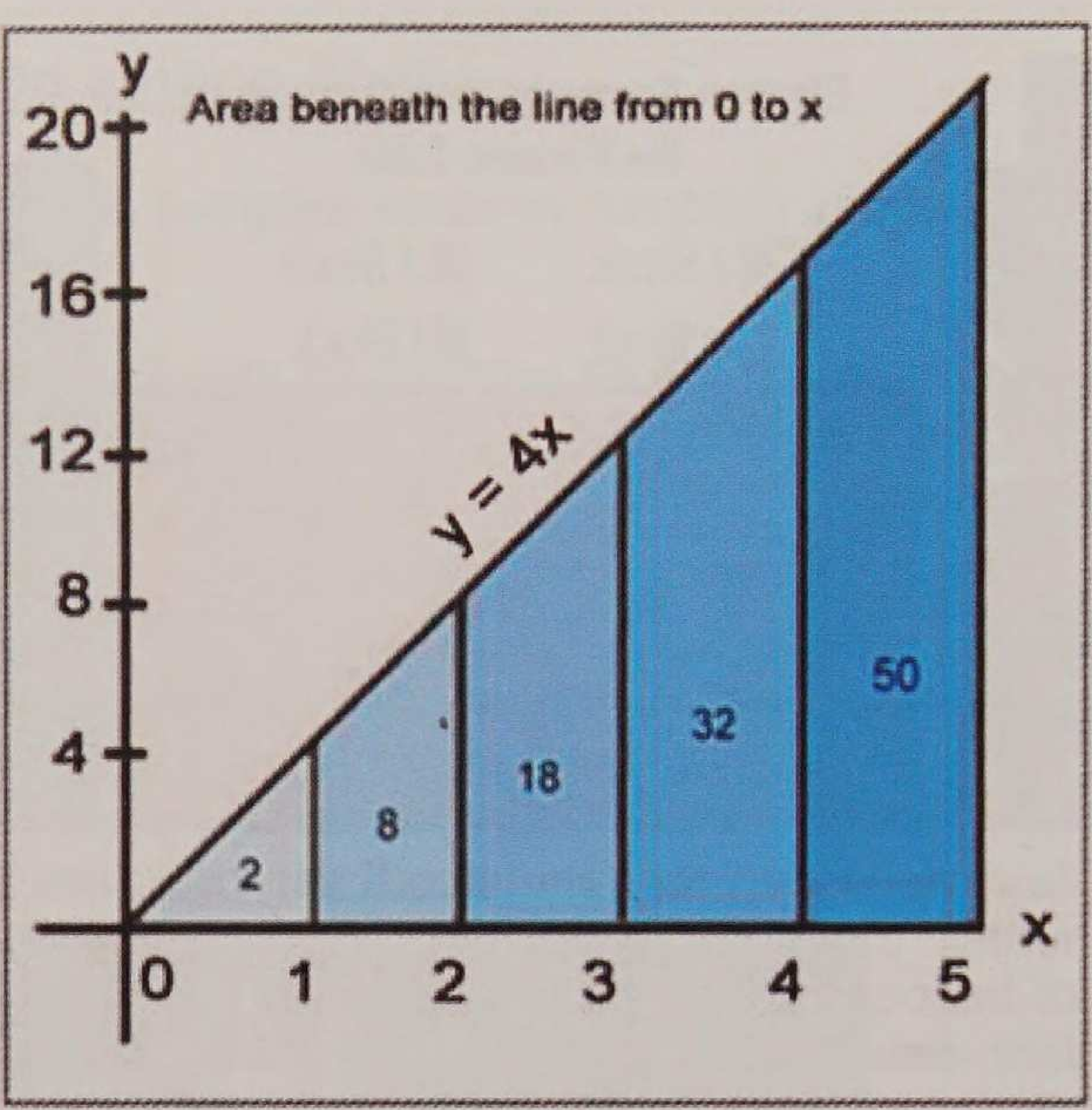
Program 5

Derivation of the Formula for the Number of Electrons in Shells

For those who have had at least an introduction to Integral Calculus 101, here is the derivation of the formula for the number of electrons per level of electron shell. The count of electrons in the series-level for variable **x** is the area under the line **y = 4x**, from **0** out to its numbered level.

Integral calculus shows that the number of electrons per level **x** is $\mathbf{2x^2}$:

$$(5.2)\qquad \int_0^y y\,\partial y = \int_0^x 4x\,\partial x = 2x^2$$



Note that **2-0=2**; **8-2=6**; **18-8=10**; and **32-18=14**. These are the capacities of the successive shells, **s, p, d, f**, respectively.

Derivation of the number series for the number of elements in consecutive periods of the Periodic Table

Now the series of numbers for the count of electrons or positrons in full electron shells among both matter and antimatter atoms can be constructed from simple operations of basic arithmetic on the basic harmonic series **1** thru **4** as indicated by the operations below. And this produces the numerical solution to the calculus formula but now built upon a perfectly harmonic pattern:

	Anti-matter / Matter	
↑ $2x^2$	**[32, 32, 18, 18, 8, 8, 2, 2, 8, 8, 18, 18, 32, 32]**	Split numbers into halves & separate.
$4x^2$	**[64, 36, 16, 4, 16, 36, 64]**	Square the numbers.
2x	**[8, 6, 4, 2, 4, 6, 8]**	Multiply series by **2**.
x start	**[4, 3, 2, 1, 2, 3, 4]**	Basic harmonic number pattern

And this produces the numerical solution to the calculus formula (**5.2**)

Everyone can now contemplate about what was just presented. It was shown here in Program 5 that Geonometry explains the number and the distribution pattern of electrons plus the existence of neutral complementary pairs of electron shells in the atoms of the noble elements, including Helium. That was a totally new mathematical explanation. In this slide, the formula is confirmed by the classical mathematics of integral calculus as well as the simple 1-dimensional arithmetical operations on a perfect harmonic string.

The distribution of anti-matter will be addressed next in Program 6.

Program 5

Notes

1. The sizes **7**, **17** and **31** characteristic complementary tiling patterns furnish the vibrating membrane patterns to account for the four electron shell pairs. These shell pairs have capacities for **2**, **16**, **36** and **64** electrons, and the individual shells have electron capacities of (**1** & **1**) (**8** & **8**) (**18** & **18**) (**32** & **32**) respectively in that order. Characteristic spheres of Geonomic cubes of size $\mathbf{n = 4x}$ have $\mathbf{2x^2}$ octals which sum to the cube's characteristic number. This accounts for the shell capacities of electrons: **2**, **8**, **18**, **32**

2. The shell pairs account for the total pair capacity, **2**, **16**, **36**, **64** which is $\mathbf{4x^2}$ for $\mathbf{x>1}$. For $\mathbf{x = 1}$, what is known for certain in Atomic Science stems from the possibility that the unseen complementary shell also with a capacity of **2** is related to antimatter and cannot be detected among ordinary matter.

3. Each electron shell-pair has **2** glide regions with no pulsating vibrations, one at the north and one at the south pole regions where electrons go from up to down and vice versa within their rotating shell-pair, thus accounting for the **±½** spins among electrons.

4. The cloaking property of Class-1 complementary tiling patterns prevent atomic scientists from directly observing these electron shell pairs. Everything claimed by Atomic Science regarding electron distribution around the nucleus has been inferred from energy measurements but never observed directly.

5. Atomic Science claims that the additional electron added in going from one atom to the next enters an orbit associated with the lowest vacant energy level.

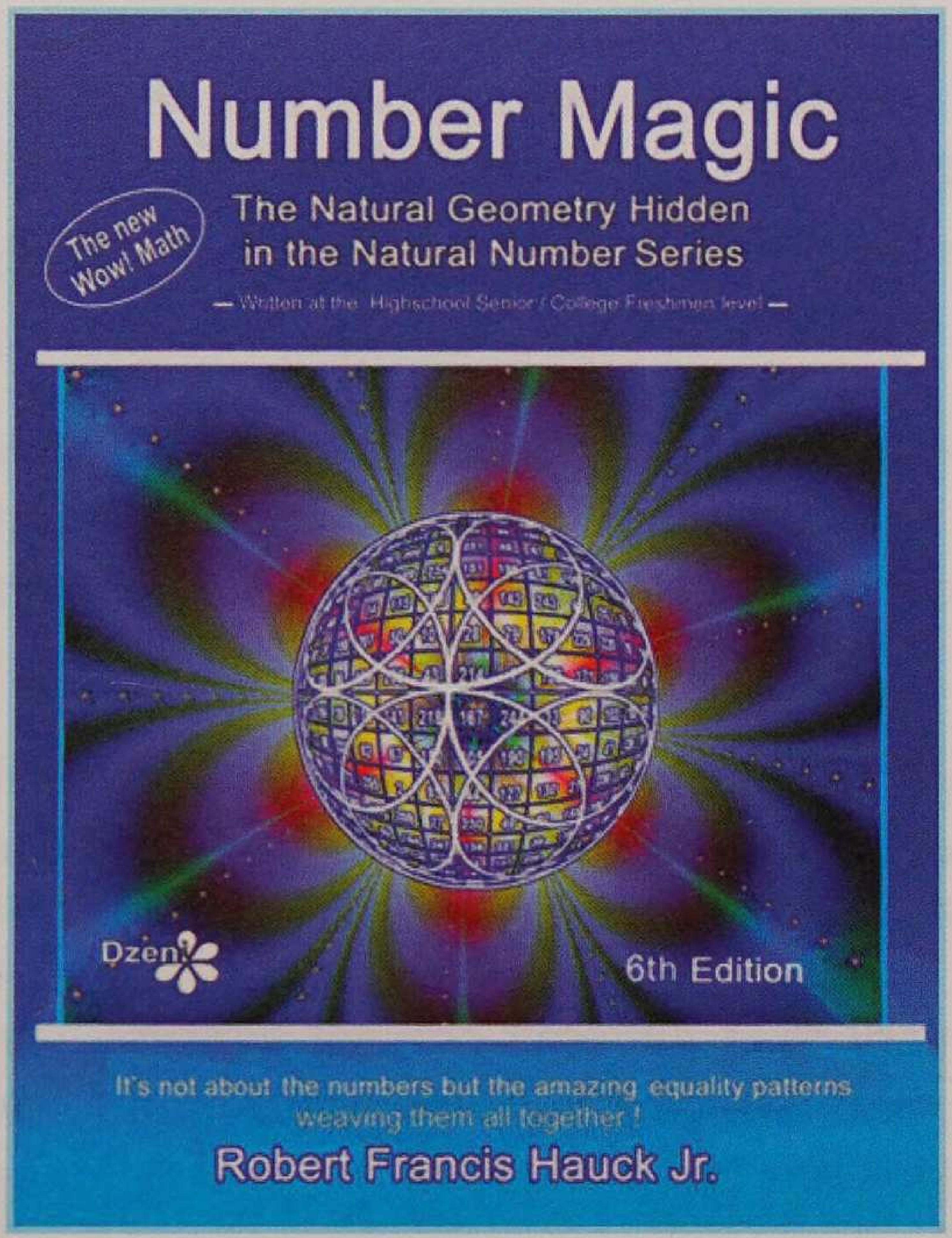
6. Geonometry claims that each shell-pair fills to only half its capacity before additional electrons begin filling in the next level closer to the nucleus. After the electron shell nearest the nucleus gets half full, it continues accepting electrons until its laminated other half gets completely filled. Then the next lesser distant shell-pair starts getting filled and so on until the outermost shell-pair with capacity of **2** electrons gets completely filled in <u>again</u>. This sequence is necessary to explain why the shells function together in pairs. Shell-pairs are necessary to complete the north-to-south pole and south-to-north pole transitions in the orbital pattern. These electrons' motions are powered by the rotational multi-layered pulsating fabric of space surrounding the nucleus.

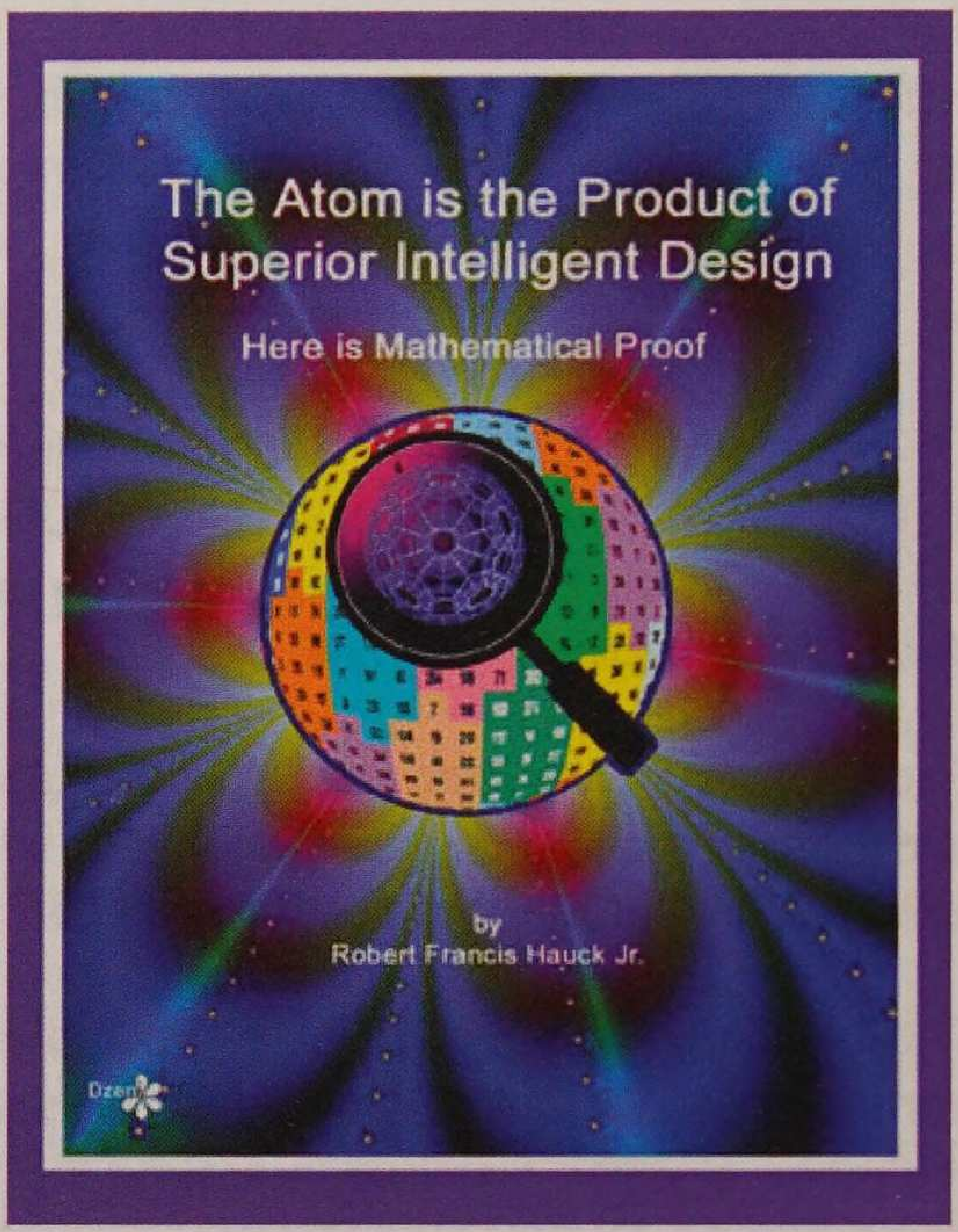
7. The higher the number of the shell-pair, the higher is the energy of its orbiting electrons. All electrons within the same shell-possess the same energy. Electrons within a single shell-pair differ in spin energy by ±½ between the two shells in the shell-pair to account for the upward and downward velocity described by Geonometry.

Program 5

We have come to the end of Program 5 of the new math, Geonometry.

Here are the author's books upon which this program was based:





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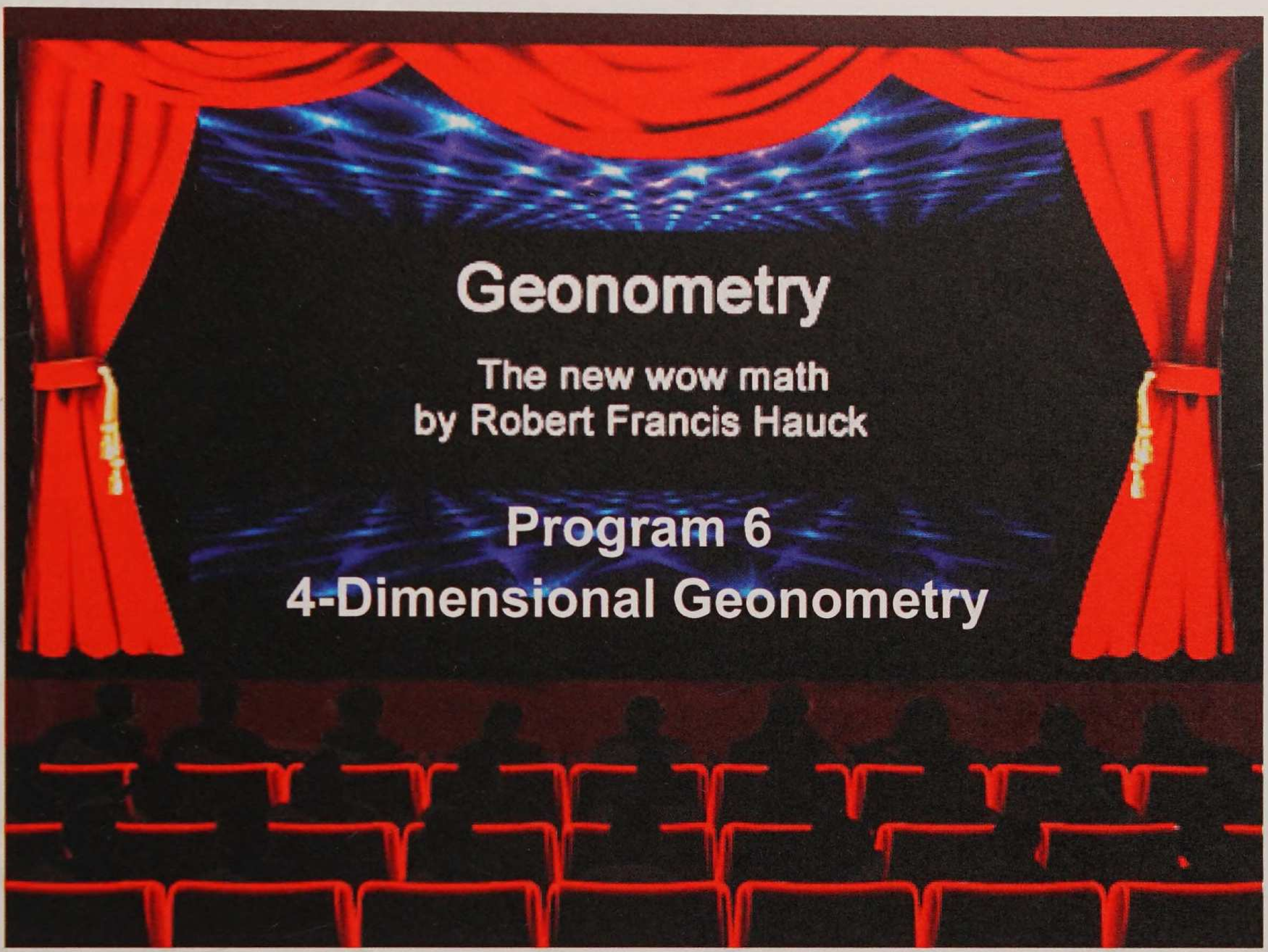
Program 6

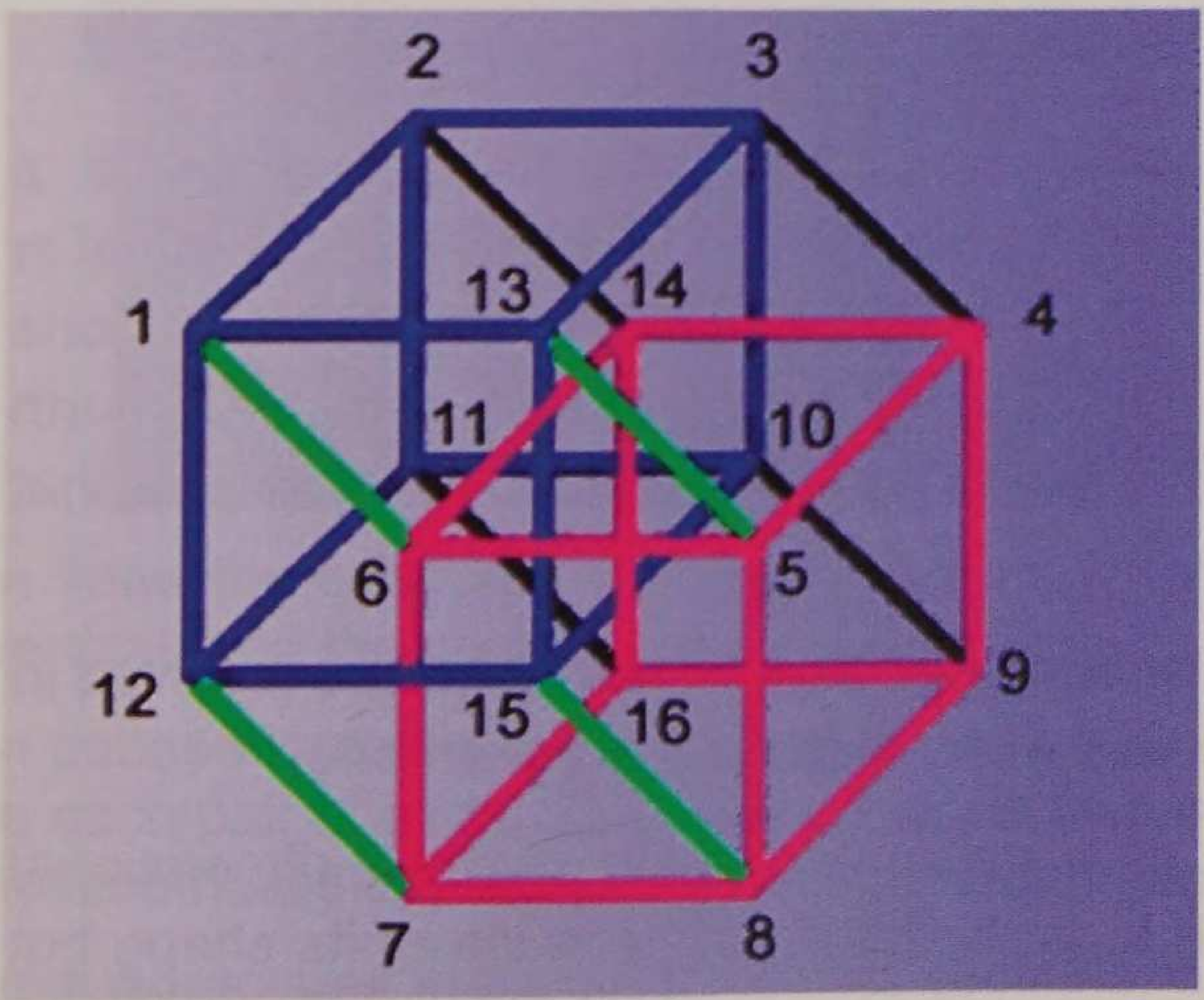


Geonometry

The new wow math
by Robert Francis Hauck

Program 6
4-Dimensional Geonometry

In this program, we shall explore the amazing spatial properties of the fourth dimension.

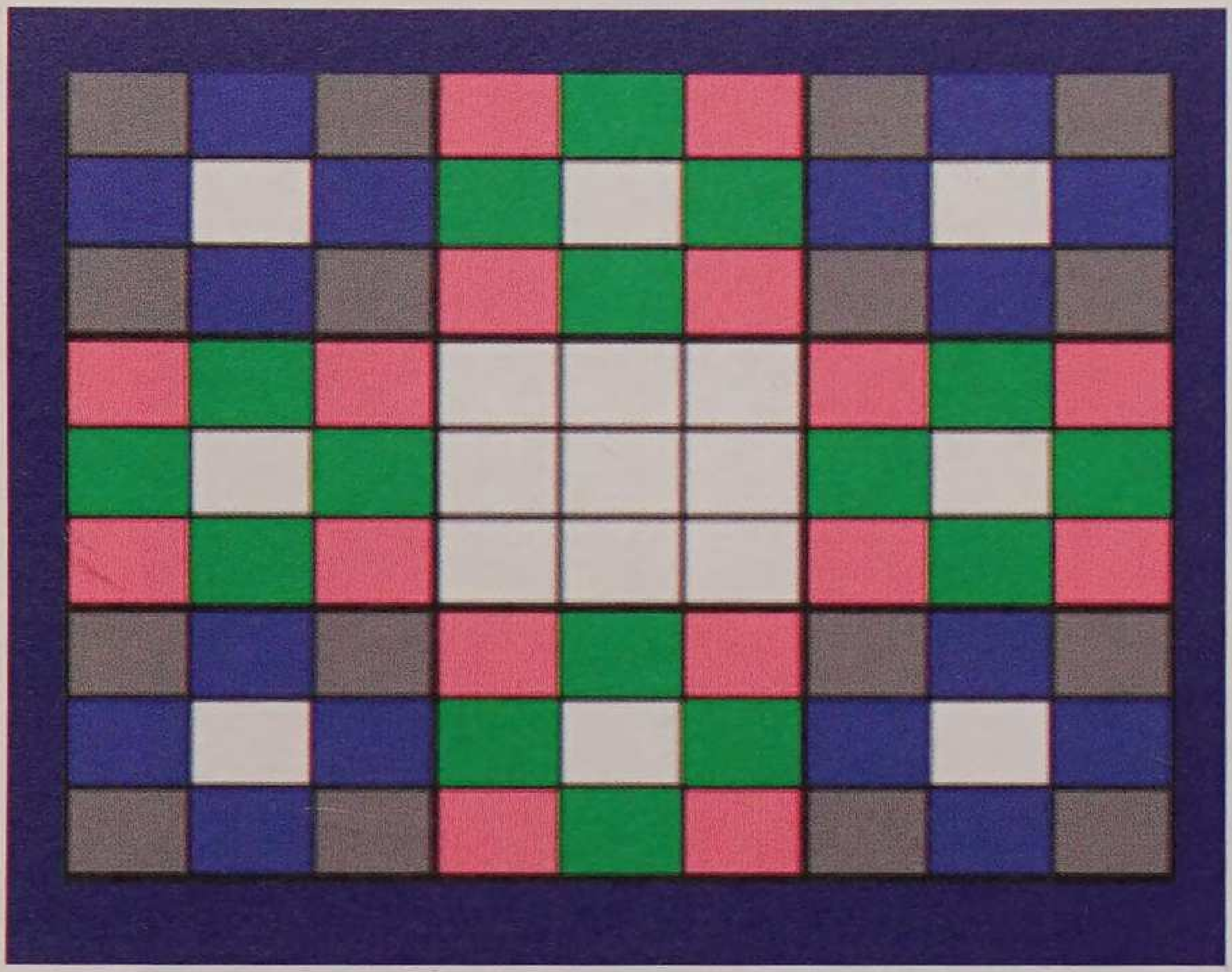


Here is a 3-dimensional view of a 4-dimensional hypercube. Like the square and cube earlier, this figure shows the basic rectangular structure of 4-dimensional space. Note that now the quadracubic enclosure possesses **16** 90° corners or points. This structure need only have equal length sides parallel to each of the 4 axes; they need not to be equal between different axes. As there is no description of this structure in classical math, it will be called a **hexadectal**, which means a 16-corner "boxahedron".

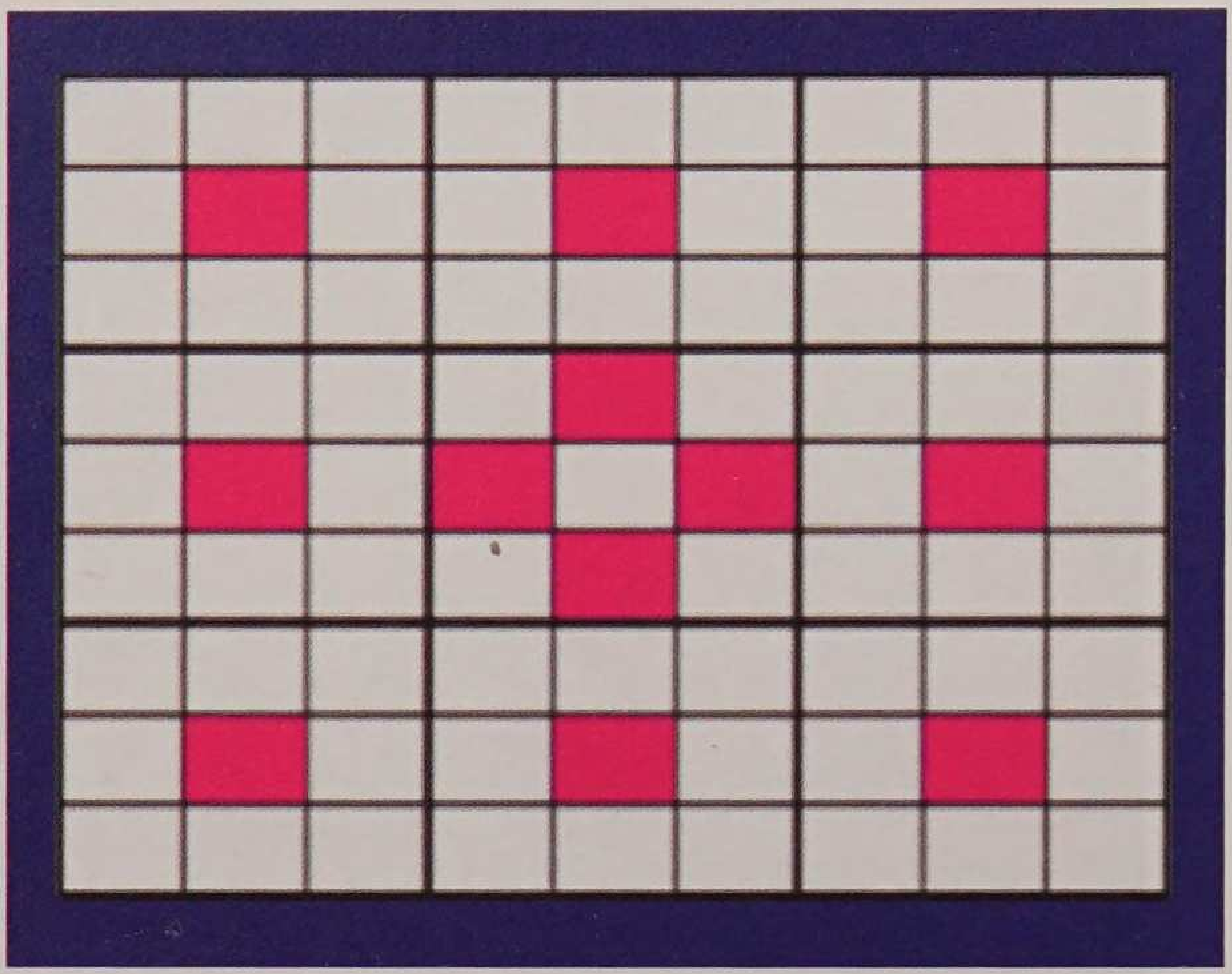
Program 6

Definitions

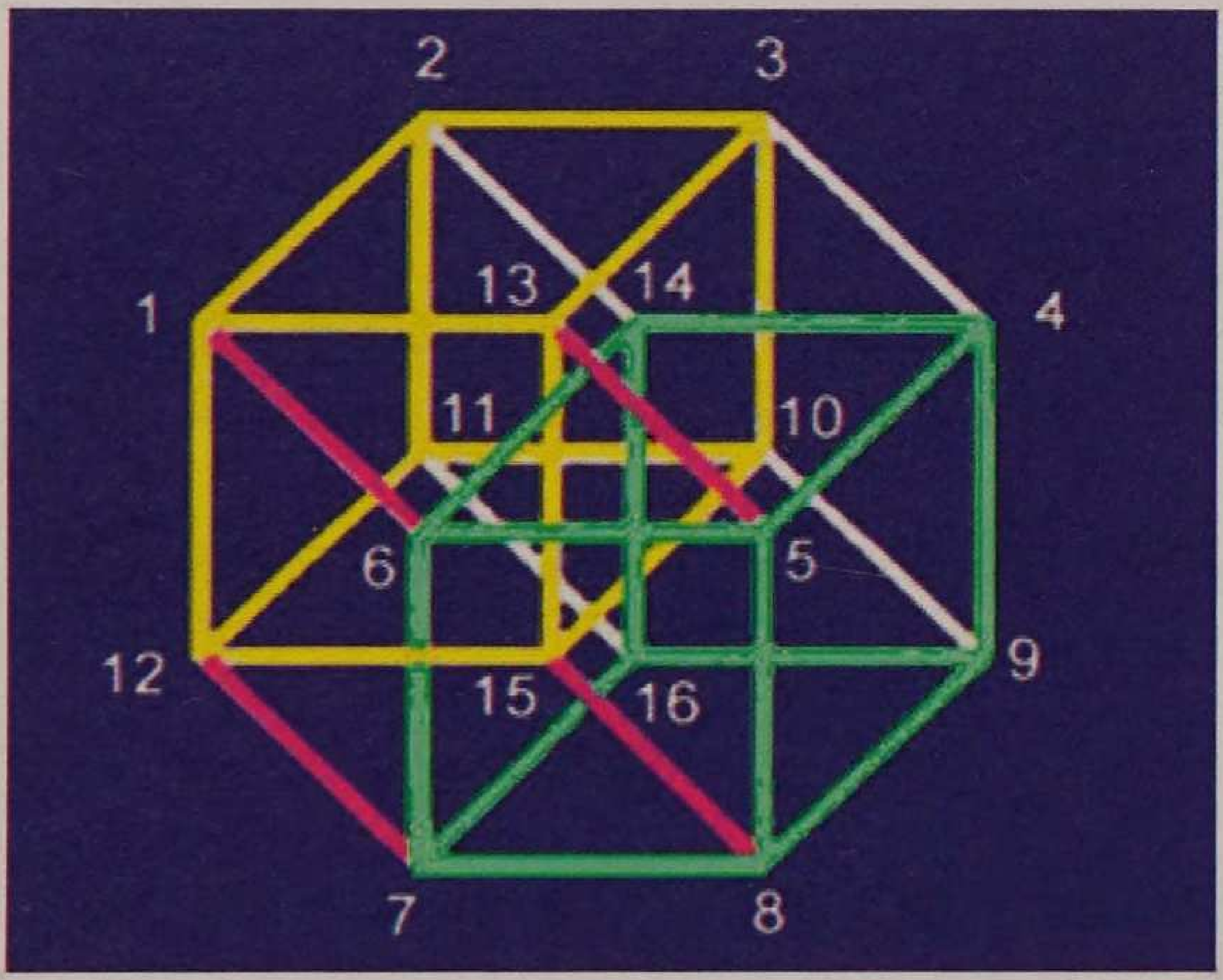
Hexadectals

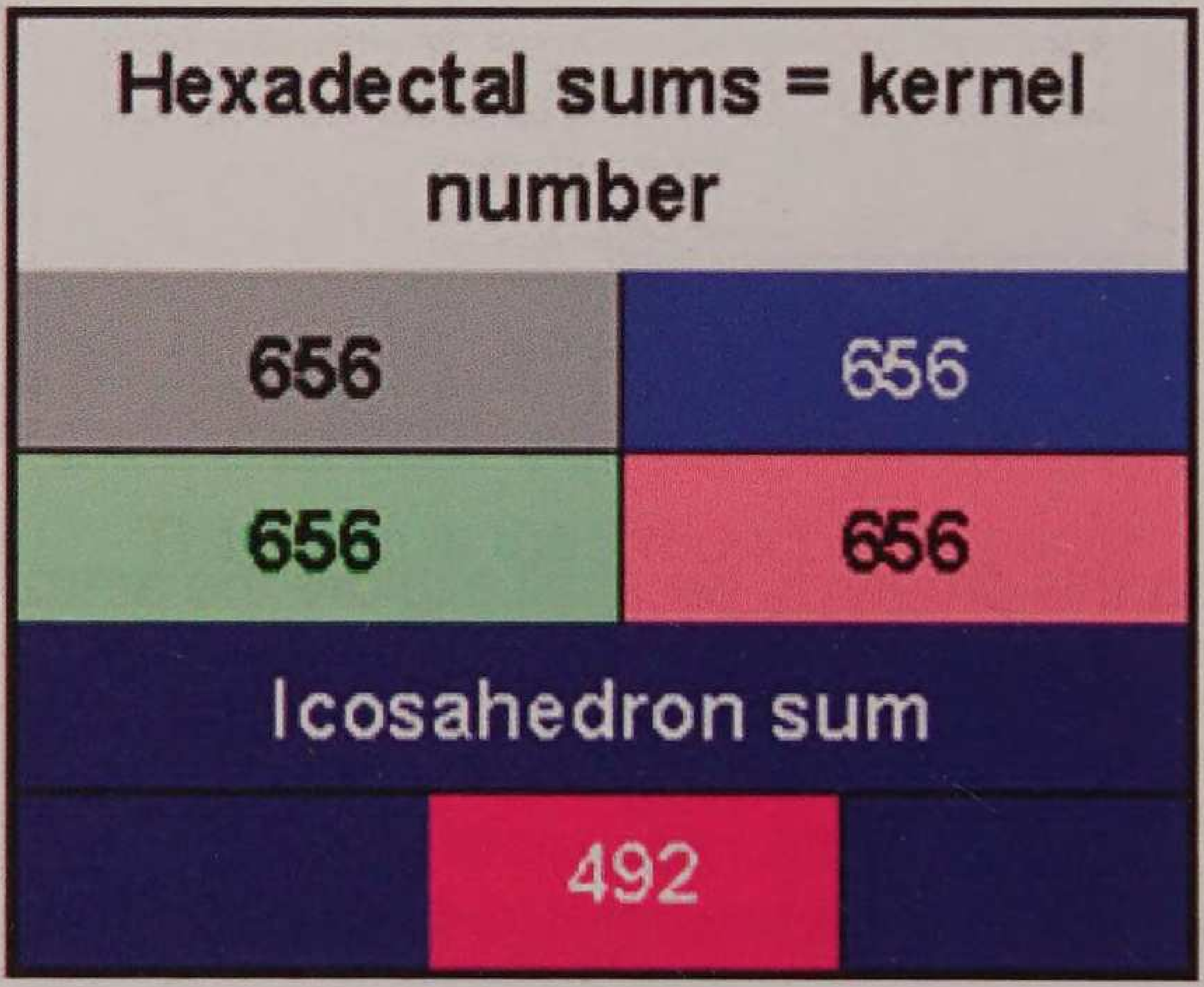


Icosahedron



Icosahedral sums = ¾ hexadectal sums





Hexadectal sums = kernel number	
656	656
656	656
Icosahedron sum	
492	

Each square 4-dimensional geonomic table represents a 4-dimensional cube, called a **quadracube**. **The hexadectal** is the 16-corner version in 4-dimensions of the 8-corner octal in 3-dimensions. In all the quadracubes to follow, each has all of its hexadectal summations equal. So consequently, all the quadracubes are pangenic with respect to the fourth dimension. The hexadectals' equal value in 4-dimensional space is again referred to as that size table's kernel number.

The figure at upper-left highlights with different color shadings those cells which are involved in computing the sums of the various nested hexadectals when there is more than one.

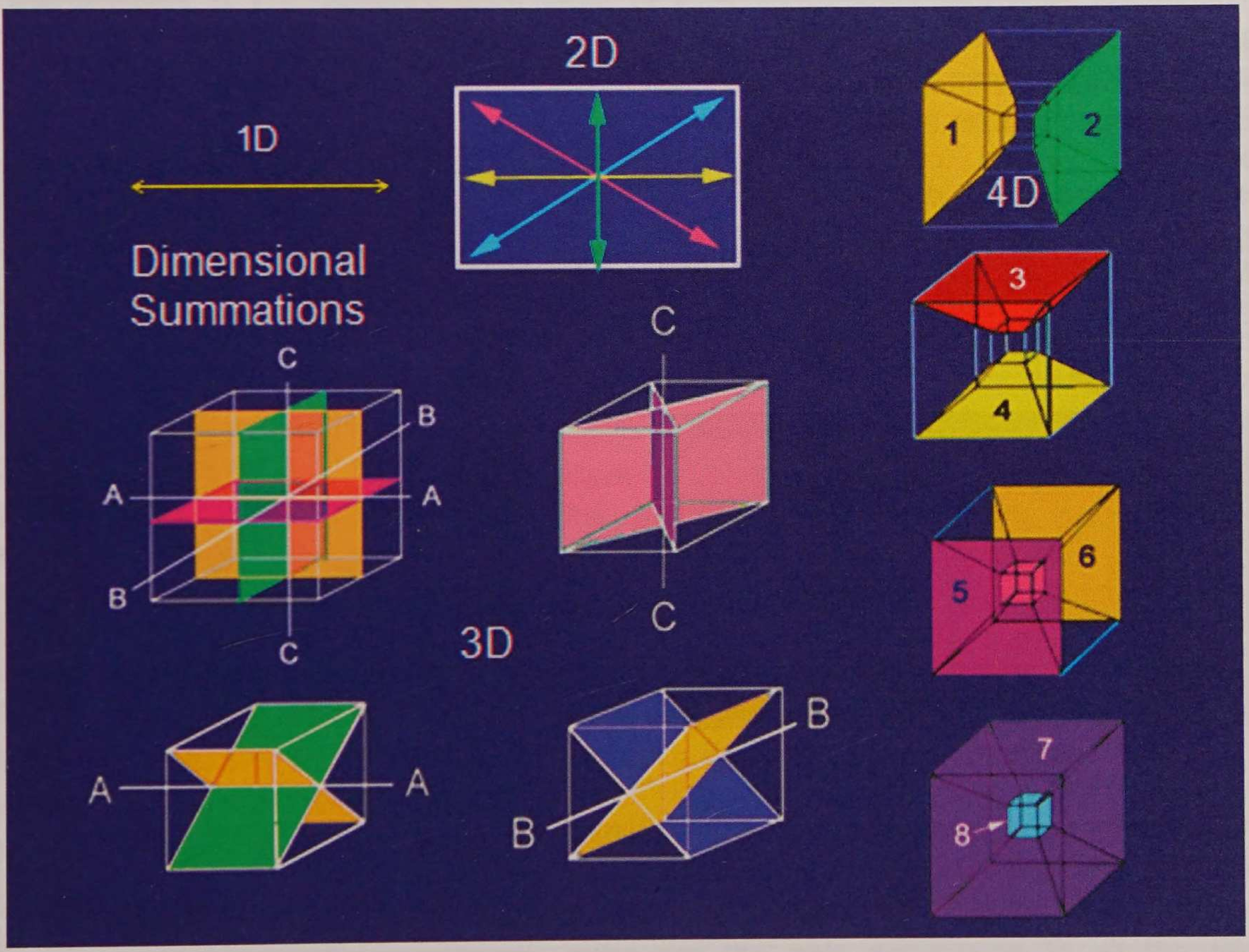
The figure at upper-right highlights the corners of a geometry which is the 4-dimensional counterpart of the octahedron in cubes, called the **icosahedron** not because of its shape but merely because it has **12** corners. Each icosahedron always sums to 3/4 of the kernel number.

Program 6

Here are some more needed definitions.

We say that a quadracube is **perfect** whenever all of its cubical summations are equal to its characteristic number. It is **absolutely perfect** when each of its embedded squares of size **n** sum equally. A quadracube of size **n** has continuous **nx-block-square modularity** when every size **n** block-square also sums to the same number continuously across the table, including wrapping across the edges. A quadracube of size **n** is said to be **ultra-perfect** when all the equal tiling patterns for the square of size **n** persist in each embedded square.

A quadracube of size **n** is said to collapse to a cube when either of its depth-sum tables taken along the **B** or **D** axis reduces to a rectangular cubic table by first dividing all the numbers in the depth-sum table of size **n**, by **n** itself, and then subtracting from each number in the table the minimum number less 1. For odd-size squares, this difference is actually the difference between the central numbers (dimensional averages) in the square and cube of size **n**.



In lines there is only **1** linear direction. In squares there are **4** linear directions: horizontal, vertical, and the corner-to-corner diagonals.

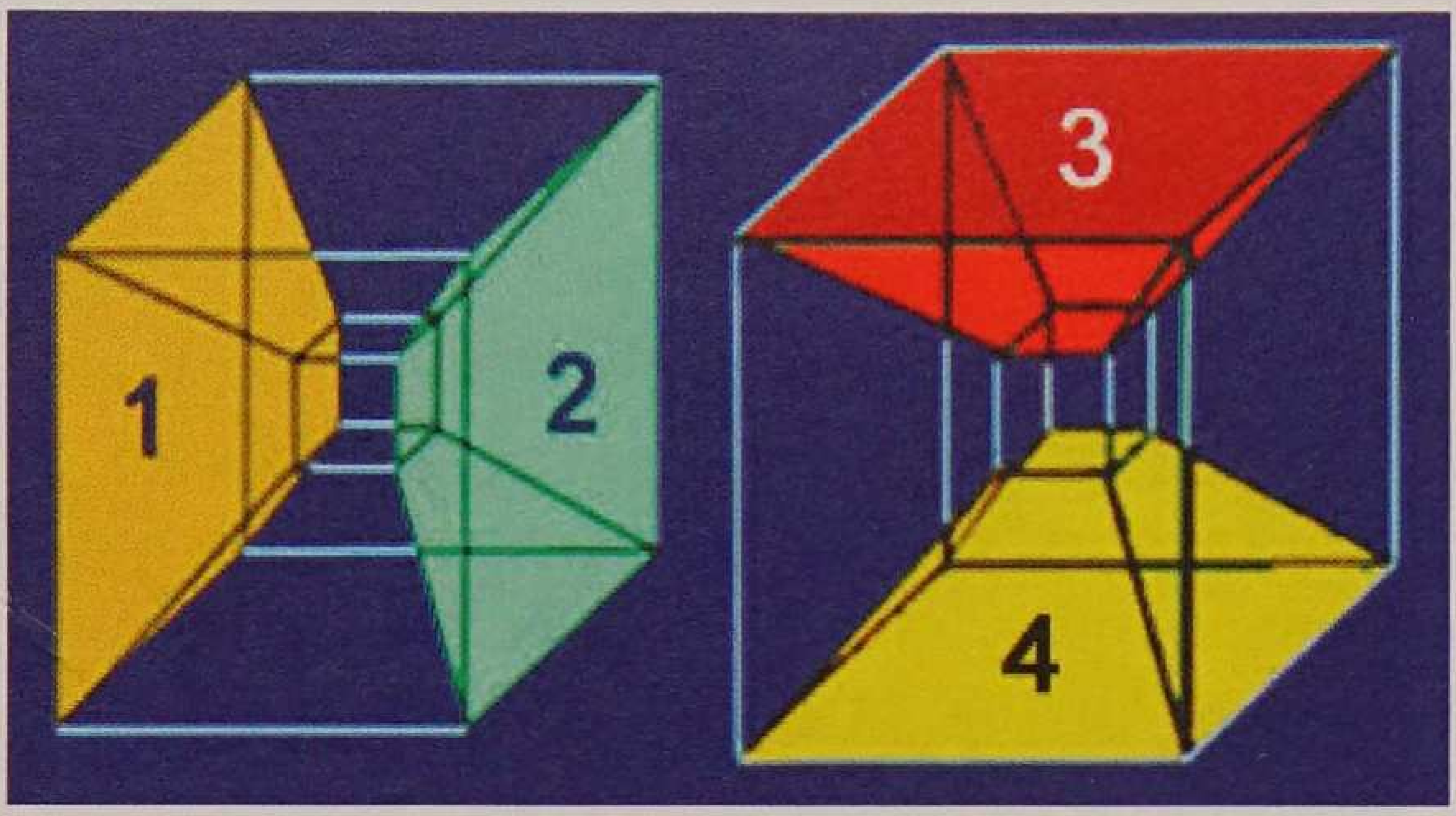
In cubes there are **9** planar directions: **2** intersecting along each of their **3** axes; and **3** parallel to each pair of axes. Each plane has its own combination of directions. There are no corner-to-symmetrically-opposite-corner cubic summations because these would be linear, not planar.

In quadracubes there are **8** cubical sections, each with its own combination of directions.

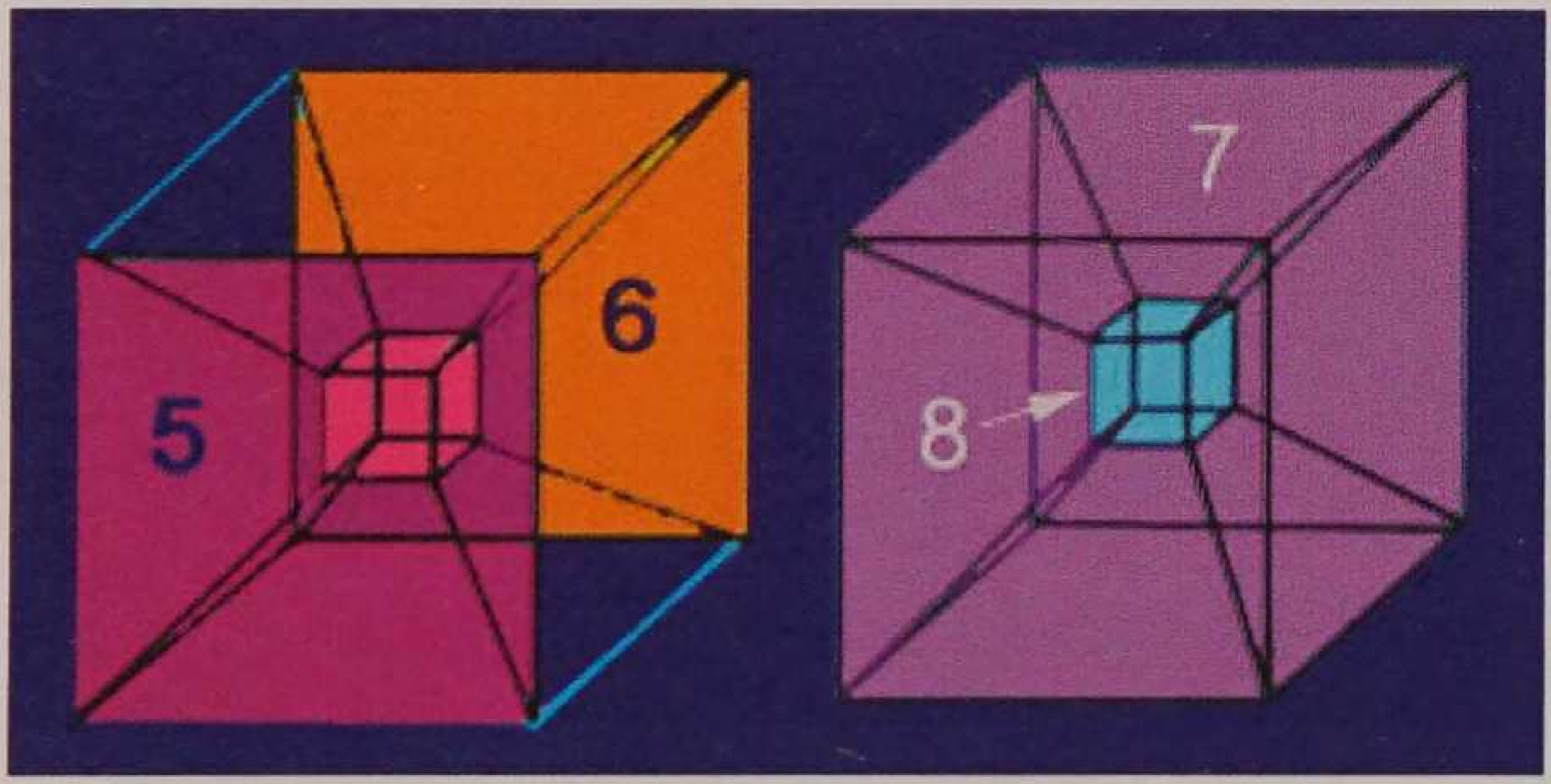
Program 6

Quadracubes by Class

Class-2 2x4D Quadracube



Here is the smallest quadracube. It is of size-2. It contains **4** intersecting embedded size-2 cubes as seen in the upper two tables where two vertical cubes intersect 2 horizontal cubes. Its dual quadracube is depicted below and is represented by the lower two tables. So between the primal and dual quadracubes together, the number of embedded cubes totals **8**. In general, that count is always **4n**.



Cubes

3	16	9	7	2
	4	5	11	14
4	1	8	10	15
	13	12	6	3

Quadrals

34	34	68
34	34	68

Cubes	1		2	
	16	9	7	2
	4	5	11	14
	1	8	10	15
	13	12	6	3

Quadrals

34	34
34	34
68	68

Cubes	5		6	
5	16	3	10	5
	13	2	11	8
6	1	14	7	12
	4	15	6	9

Quadrals

34	34
34	34
68	68

Cubes

7	16	3	10	5
	13	2	11	8
8	1	14	7	12
	4	15	6	9

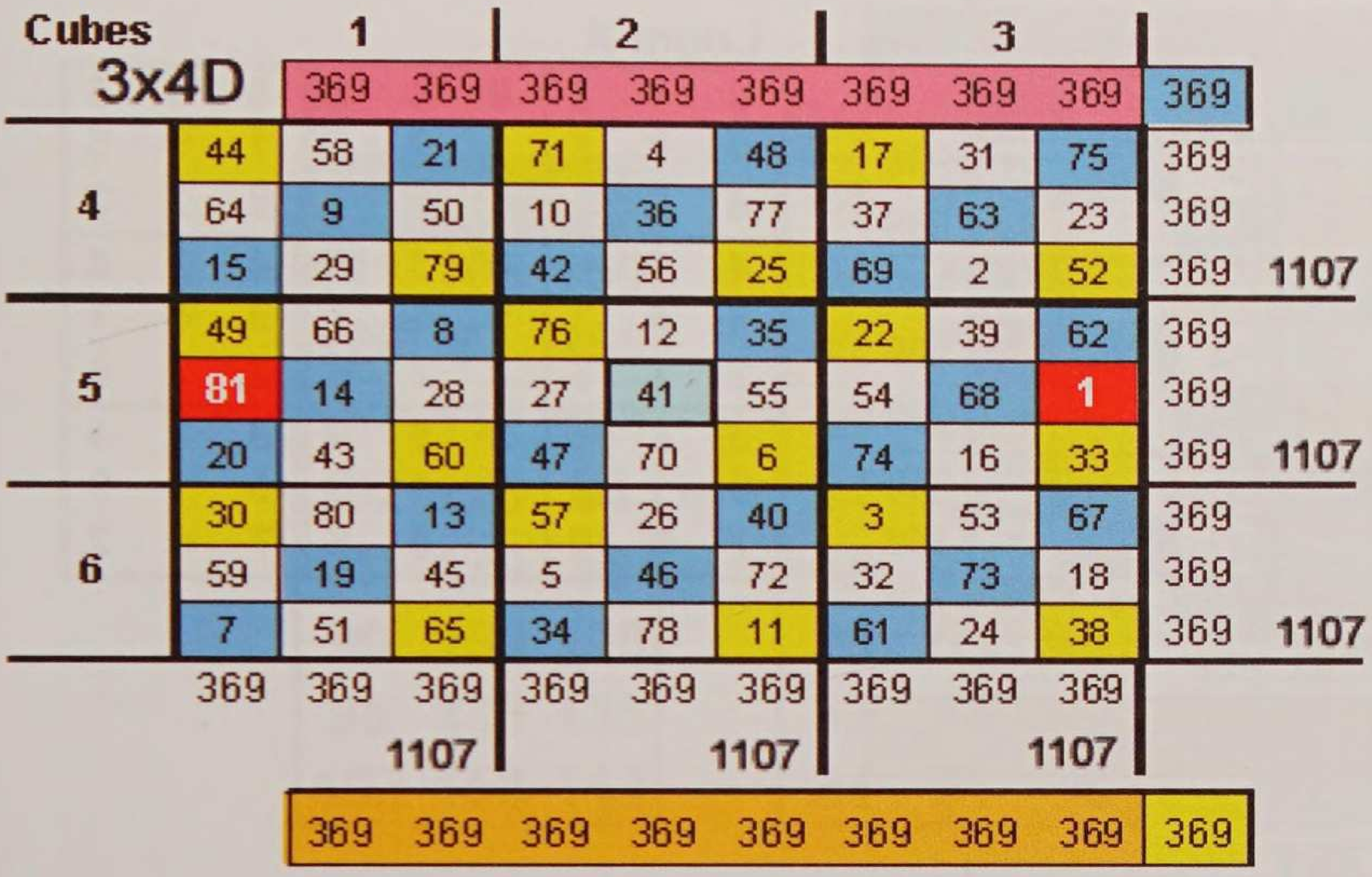
Quadrals

34	34	68
34	34	68

All the embedded cubes sum equally to **68**, so the quadracube is perfect. Every **2x2** embedded square quadrant sums to **34**. So the size-2 quadracube is also absolutely perfect.

Program 6

A Class-3 Quadracube



Cubes	1			2			3				
3x4D	369	369	369	369	369	369	369	369	369		
4	44	58	21	71	4	48	17	31	75	369	
	64	9	50	10	36	77	37	63	23	369	
	15	29	79	42	56	25	69	2	52	369	1107
5	49	66	8	76	12	35	22	39	62	369	
	81	14	28	27	41	55	54	68	1	369	
	20	43	60	47	70	6	74	16	33	369	1107
6	30	80	13	57	26	40	3	53	67	369	
	59	19	45	5	46	72	32	73	18	369	
	7	51	65	34	78	11	61	24	38	369	1107
	369	369	369	369	369	369	369	369	369		
			1107			1107			1107		
	369	369	369	369	369	369	369	369	369		

Here is the size-3 quadracube. It is a Class-3 hypercube by definition.

It consists of **9** embedded 3x3 block-squares.
Its characteristic number is the sum of all the numbers in each embedded cube. That number here is **3** times **369** which equals **1107**,

Each embedded block-square sums to **369**. Consequently, all the main and wrap diagonal sums at the block-square level sum equally. Again that number here is **3** times **369** which equals the quadracube's characteristic number, **1107**. So the quadracube is absolutely-perfect.

1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107

1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107
1107	1107	1107	1107	1107	1107	1107	1107	1107

Here is a tiling pattern which continuously sums to the quadracube's characteristic number. Each tiling pattern contains **27** number cells and equivalently **9** **1x3** or **3x1** blocks which altogether sum to the quadracube's characteristic number.

Now, here is a surprising discovery: Each tiling pattern is its complement's transpose.

Similar to the complementary tiling patterns of the size-5 square that are the 45° rotation of the other, these tiling patterns here for the size-3 quadracube are the only occurrence in all the tiling patterns for cubes discovered so far where the tile and its transpose constitute the totality of complementary patterns.

Program 6

Dual Size-3 Quadracubes and their Common Loom Tables

Cubes		1			2			3		
	W		369	369	369	369	369	369	369	369
4	44	58	21	71	4	48	17	31	75	369
	64	9	50	10	36	77	37	63	23	369
	15	29	79	42	56	25	69	2	52	369
5	49	66	8	76	12	35	22	39	62	369
	81	14	28	27	41	55	54	68	1	369
	20	43	60	47	70	6	74	16	33	369
6	30	80	13	57	26	40	3	53	67	369
	59	19	45	5	46	72	32	73	18	369
	7	51	65	34	78	11	61	24	38	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

Loom X

8	4	3	8	4	3	8	4	3
1	9	5	1	9	5	1	9	5
6	2	7	6	2	7	6	2	7
4	3	8	4	3	8	4	3	8
9	5	1	9	5	1	9	5	1
2	7	6	2	7	6	2	7	6
3	8	4	3	8	4	3	8	4
5	1	9	5	1	9	5	1	9
7	6	2	7	6	2	7	6	2

Cubes		7			8			9		
	U		369	369	369	369	369	369	369	369
10	68	34	21	71	28	24	65	31	27	369
	8	73	42	2	76	45	5	79	39	369
	47	13	63	50	16	57	53	10	60	369
11	33	26	64	36	20	67	30	23	70	369
	81	38	4	75	41	7	78	44	1	369
	12	59	52	15	62	46	18	56	49	369
12	22	72	29	25	66	32	19	69	35	369
	43	3	77	37	6	80	40	9	74	369
	55	51	17	58	54	11	61	48	14	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

Loom Y

5	7	3	8	1	6	2	4	9
8	1	6	2	4	9	5	7	3
2	4	9	5	7	3	8	1	6
6	8	1	9	2	4	3	5	7
9	2	4	3	5	7	6	8	1
3	5	7	6	8	1	9	2	4
4	9	2	7	3	5	1	6	8
7	3	5	1	6	8	4	9	2
1	6	8	4	9	2	7	3	5

Here is the second dual in the series of three distinct dual quadracubes of size-3.

The complementary loom tables for both are depicted at right.

In **X**, every column and all diagonals, main and wrap, contain the numbers from **1** through **9** exactly once.

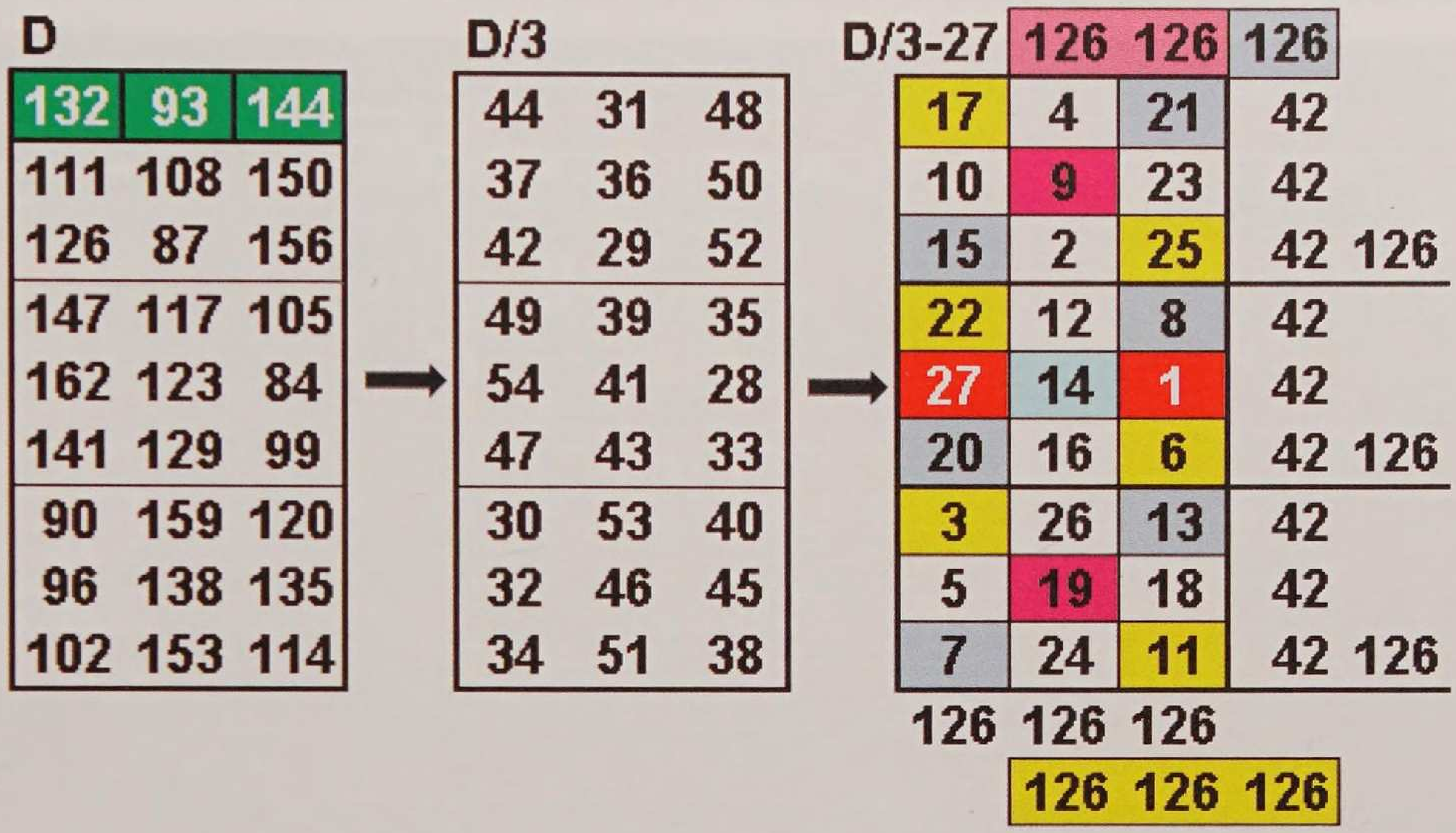
In **Y**, every row, column and all minor diagonals contains the numbers from **1** through **9** exactly once.

Further, every embedded size-3 square in either loom table contains the numbers from **1** through **9** exactly once.

Because there are numerical differences in the row, column or diagonal properties between **X** and **Y**, the dual quadracubes are always pairwise centrally symmetric and can never be pairwise row or column symmetric.

Program 6

Reduction of depth-sum table along the D-axis



D

132	93	144
111	108	150
126	87	156
147	117	105
162	123	84
141	129	99
90	159	120
96	138	135
102	153	114

→

D/3

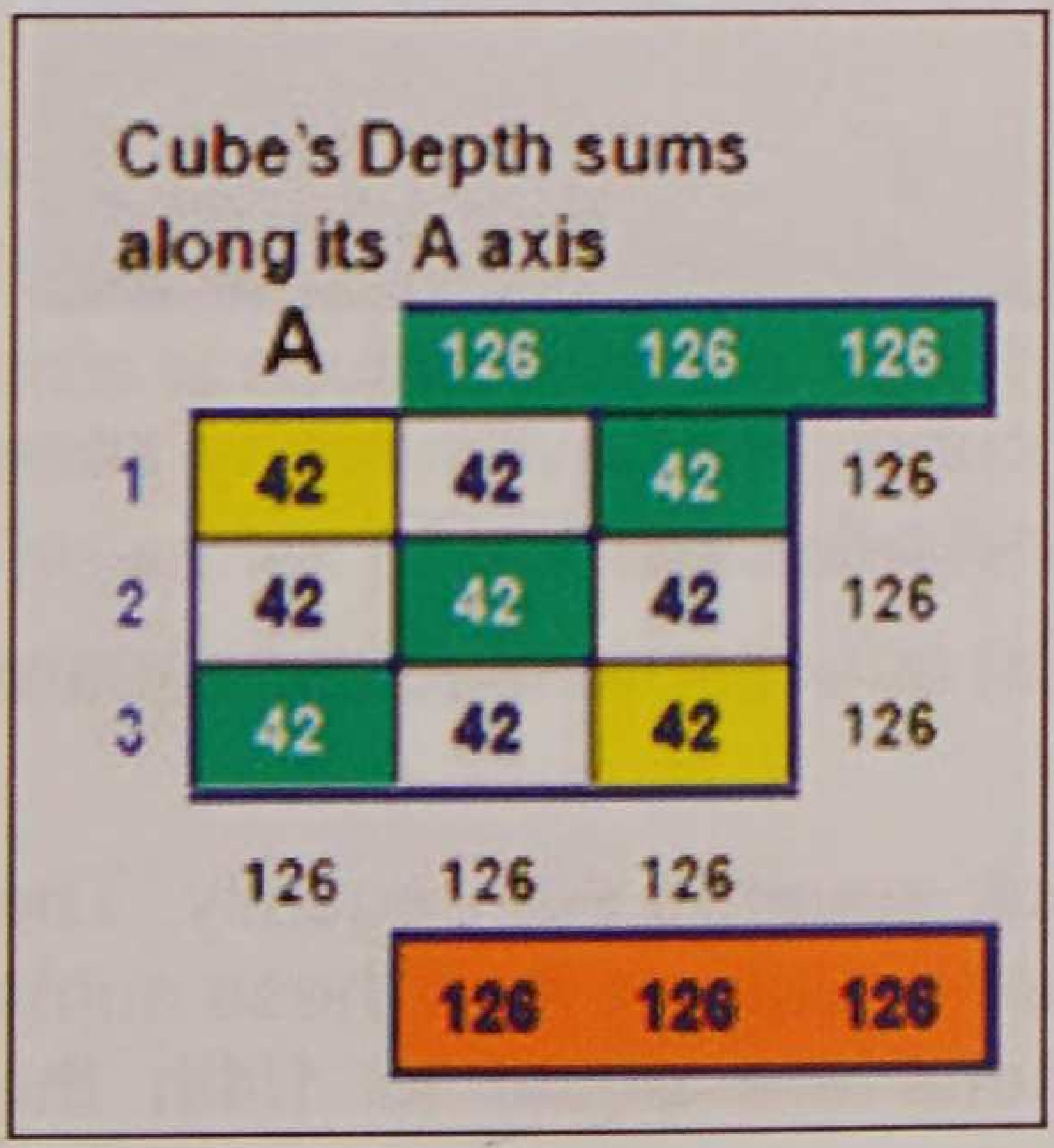
44	31	48
37	36	50
42	29	52
49	39	35
54	41	28
47	43	33
30	53	40
32	46	45
34	51	38

→

D/3-27	126	126	126	
17	4	21	42	
10	9	23	42	
15	2	25	42	126
22	12	8	42	
27	14	1	42	
20	16	6	42	126
3	26	13	42	
5	19	18	42	
7	24	11	42	126
126	126	126		
	126	126	126	

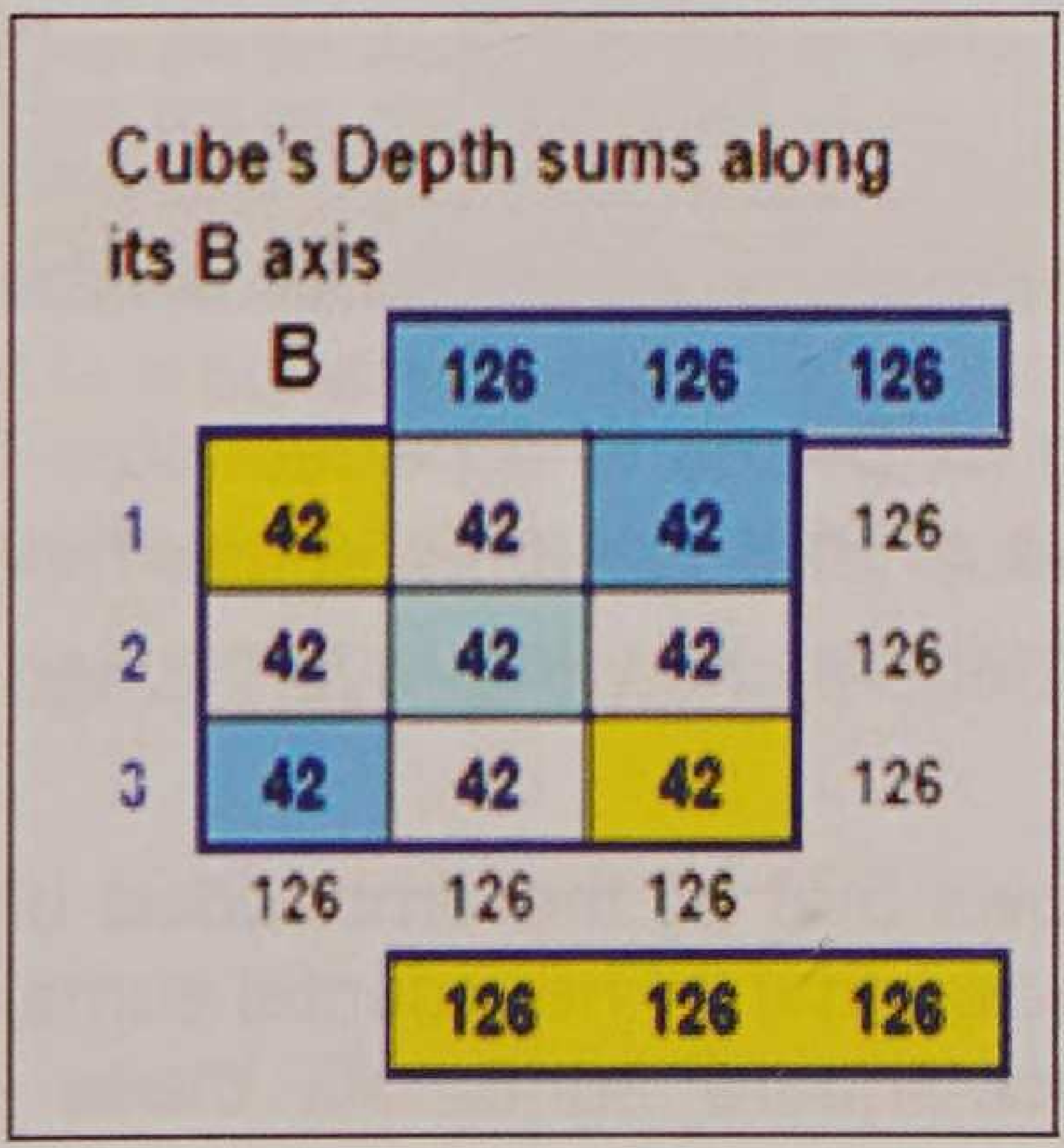
The size-3 quadracube will collapse along its D-axis to a perfect size-3 cube. However, the resulting cube will not collapse further to a size-3 square along any of its axes because the size-3 square is imperfect. Such a collapse is automatically prevented by the duplication of numbers in all 3 of the cubes depth-sum tables. Again, this shows how true-to-form Geonometry actually is!

The dual version does not collapse to a cube. We'll see shortly that this is a fundamental property of higher dimensions.



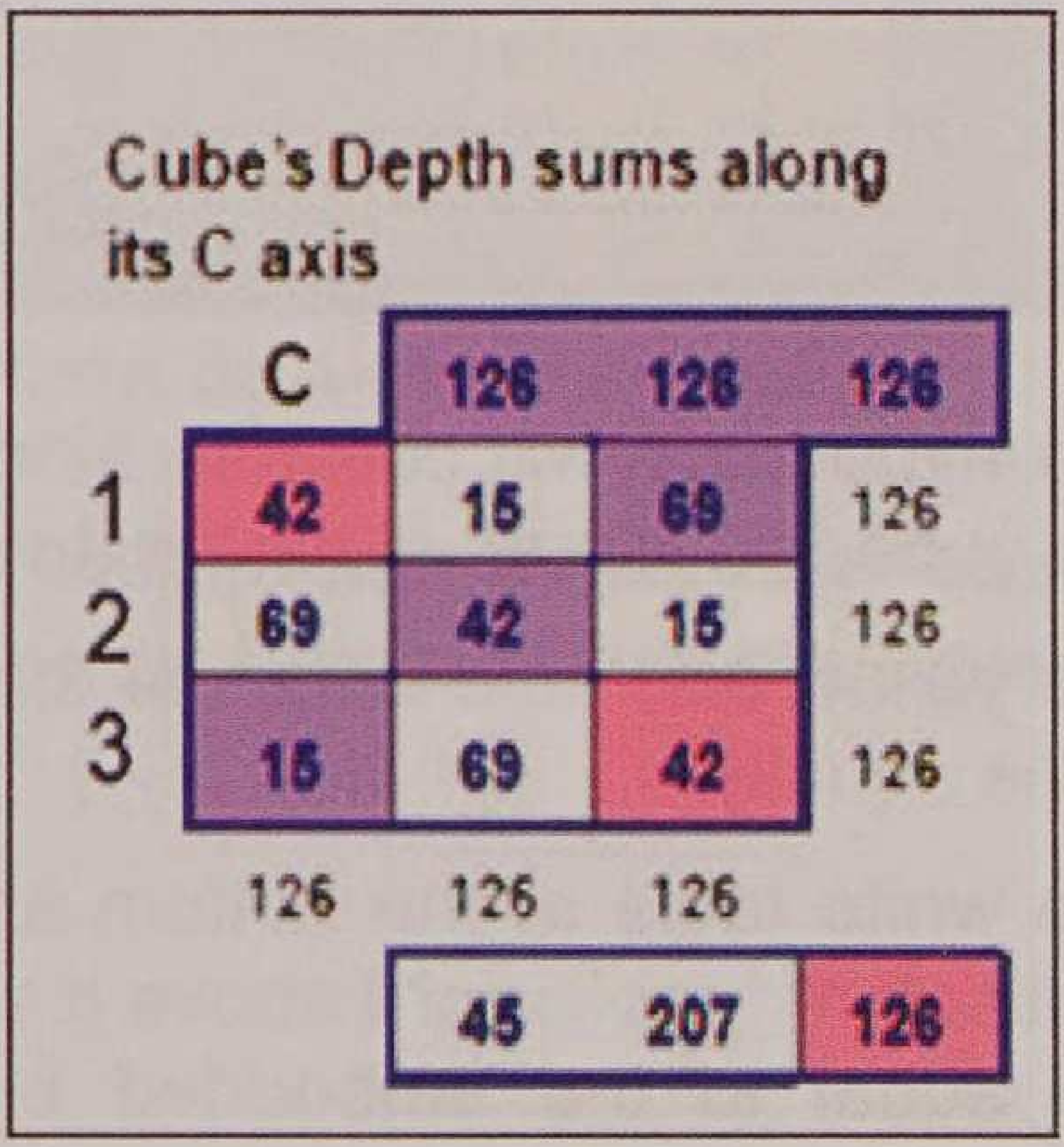
Cube's Depth sums along its A axis

A	126	126	126	
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
		126	126	126



Cube's Depth sums along its B axis

B	126	126	126	
1	42	42	42	126
2	42	42	42	126
3	42	42	42	126
	126	126	126	
		126	126	126



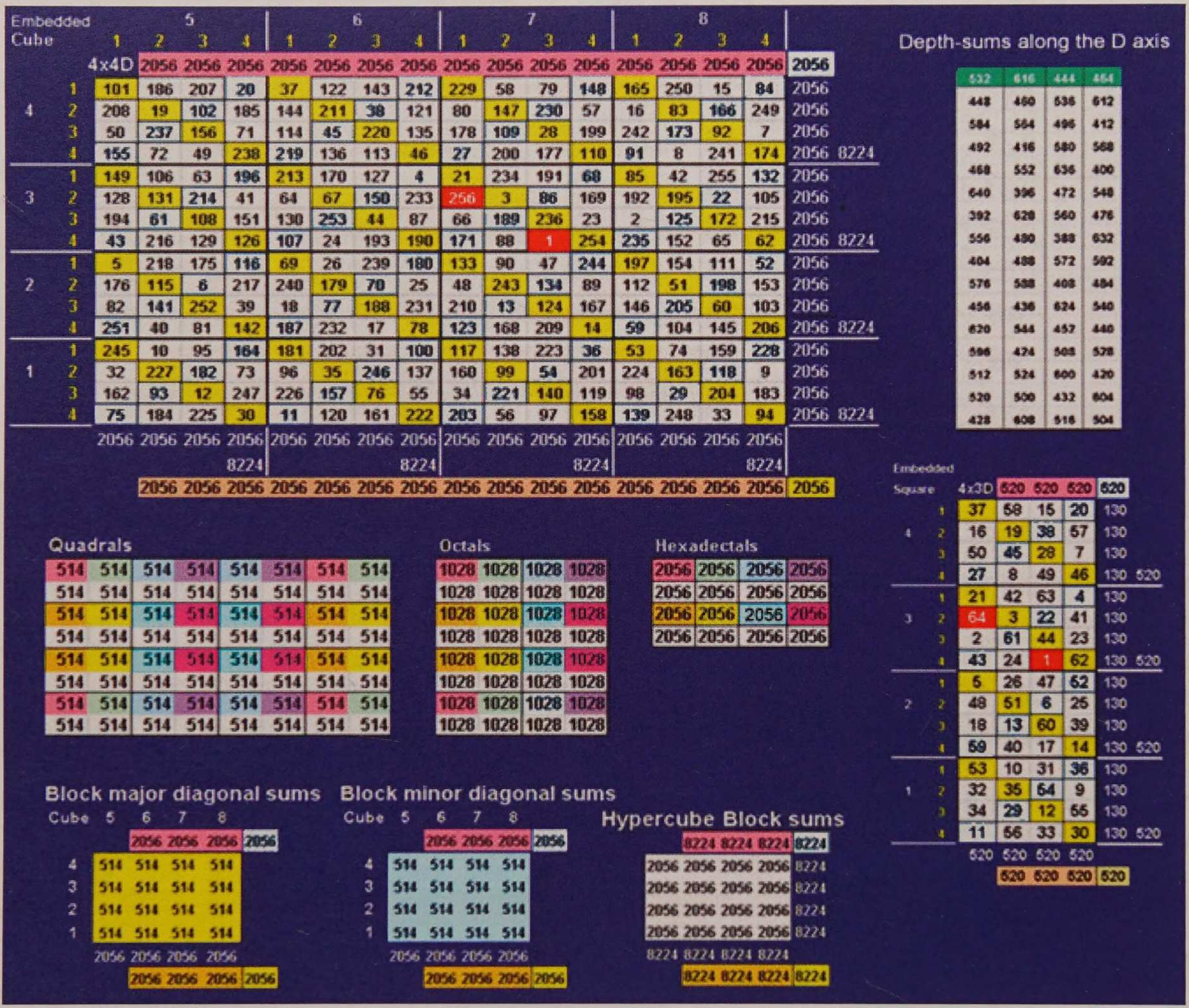
Cube's Depth sums along its C axis

C	126	126	126	
1	42	15	69	126
2	69	42	15	126
3	15	69	42	126
	126	126	126	
		45	207	126

Program 6

Class-4 Quadracube

Here is the absolutely-perfect Class-4 quadracube of size-4. All the numbers in each embedded cube sum to the quadracube's characteristic number. Consequently all of the horizontal and vertical cubic sections sum equally to the quadracubes characteristic number **8224**.



Embedded Cube		5				6				7				8					
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4		
	4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
4	1	101	186	207	20	37	122	143	212	229	58	79	148	165	250	15	84	2056	
	2	208	19	102	185	144	211	38	121	80	147	230	57	16	83	166	249	2056	
	3	50	237	156	71	114	45	220	135	178	109	28	199	242	173	92	7	2056	
	4	155	72	49	238	219	136	113	46	27	200	177	110	91	8	241	174	2056	8224
3	1	149	106	63	196	213	170	127	4	21	234	191	68	85	42	255	132	2056	
	2	128	131	214	41	64	67	150	233	256	3	86	169	192	195	22	105	2056	
	3	194	61	108	151	130	253	44	87	66	189	236	23	2	125	172	215	2056	
	4	43	216	129	126	107	24	193	190	171	88	1	254	235	152	65	62	2056	8224
2	1	5	218	175	116	69	26	239	180	133	90	47	244	197	154	111	52	2056	
	2	176	115	6	217	240	179	70	25	48	243	134	89	112	51	198	153	2056	
	3	82	141	252	39	18	77	188	231	210	13	124	167	146	205	60	103	2056	
	4	251	40	81	142	187	232	17	78	123	168	209	14	59	104	145	206	2056	8224
1	1	245	10	95	164	181	202	31	100	117	138	223	36	53	74	159	228	2056	
	2	32	227	182	73	96	35	246	137	160	99	54	201	224	163	118	9	2056	
	3	162	93	12	247	226	157	76	55	34	221	140	119	98	29	204	183	2056	
	4	75	184	225	30	11	120	161	222	203	56	97	158	139	248	33	94	2056	8224
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
					8224				8224				8224				8224		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	

Depth-sums along the D axis

532	616	444	464
448	460	536	612
584	564	496	412
492	416	580	568
468	552	636	400
640	396	472	548
392	628	560	476
556	480	388	632
404	488	572	592
576	588	408	484
456	436	624	540
620	544	452	440
596	424	508	528
512	524	600	420
520	500	432	604
428	608	516	504

Quadrals

514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514
514	514	514	514	514	514	514	514

Octals

1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028
1028	1028	1028	1028

Hexadectals

2056	2056	2056	2056
2056	2056	2056	2056
2056	2056	2056	2056
2056	2056	2056	2056

Embedded Square							
	4x3D	520	520	520	520		
4	1	37	58	15	20	130	
	2	16	19	38	57	130	
	3	50	45	28	7	130	
	4	27	8	49	46	130	520
3	1	21	42	63	4	130	
	2	64	3	22	41	130	
	3	2	61	44	23	130	
	4	43	24	1	62	130	520
2	1	5	26	47	52	130	
	2	48	51	6	25	130	
	3	18	13	60	39	130	
	4	59	40	17	14	130	520
1	1	53	10	31	36	130	
	2	32	35	54	9	130	
	3	34	29	12	55	130	
	4	11	56	33	30	130	520
		520	520	520	520		
			520	520	520	520	

Block major diagonal sums

Cube	5	6	7	8	
		2056	2056	2056	2056
4	514	514	514	514	
3	514	514	514	514	
2	514	514	514	514	
1	514	514	514	514	
	2056	2056	2056	2056	
		2056	2056	2056	2056

Block minor diagonal sums

Cube	5	6	7	8	
		2056	2056	2056	2056
4	514	514	514	514	
3	514	514	514	514	
2	514	514	514	514	
1	514	514	514	514	
	2056	2056	2056	2056	
		2056	2056	2056	2056

Hypercube Block sums

	8224	8224	8224	8224
2056	2056	2056	2056	8224
2056	2056	2056	2056	8224
2056	2056	2056	2056	8224
2056	2056	2056	2056	8224
8224	8224	8224	8224	
	8224	8224	8224	8224

The hypercube will collapse along its D-axis to a perfect size-4 cube. We have already seen this cube in Program 4 where it was seen to collapse to a perfect size-4 square.

The yellow and blue tables at the bottom show that every diagonal of every embedded square sums equally.

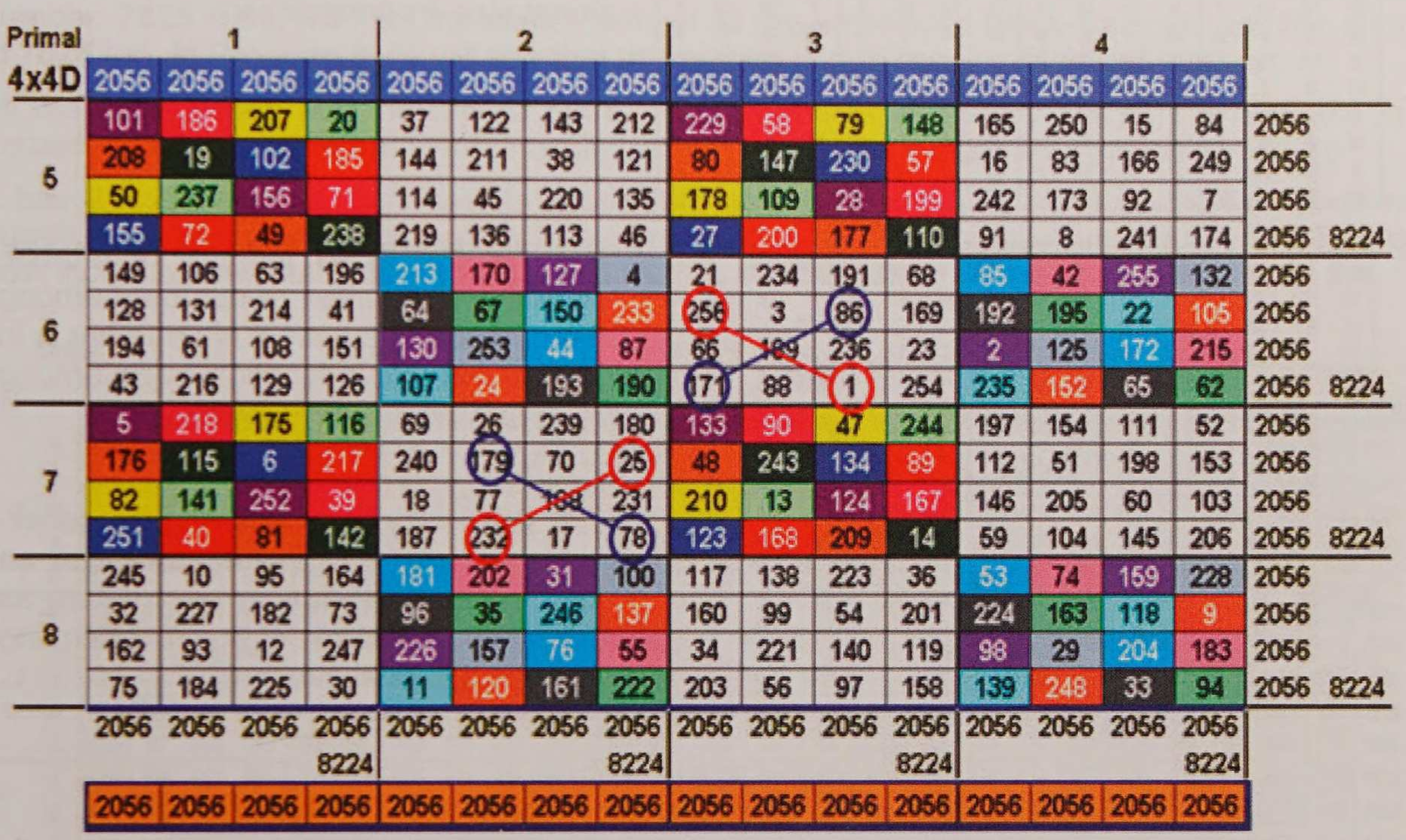
The white table at the bottom shows that all the embedded block squares sum equally. The multicolored table right above it shows that all hexadectal sums are equal too. And these sums are equal to the embedded block-square sums. All these sums are equal to **1/4**th the quadracube's characteristic number.

Note that the quadracube sums geonomically at the block-square level.

Program 6

The size-4 quadracube's equal-summing row-pairs are now identified by matching colors within the embedded squares.

Note that the pattern is distinctly different from the equal-summing pair-patterns already identified for the size-16 square which has symmetric row-pairs or centrally symmetric pairs. The pattern here has neither.



Primal	1				2				3				4					
4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
5	101	186	207	20	37	122	143	212	229	58	79	148	165	250	15	84	2056	
	208	19	102	185	144	211	38	121	80	147	230	57	16	83	166	249	2056	
	50	237	156	71	114	45	220	135	178	109	28	199	242	173	92	7	2056	
	155	72	49	238	219	136	113	46	27	200	177	110	91	8	241	174	2056	8224
6	149	106	63	196	213	170	127	4	21	234	191	68	85	42	255	132	2056	
	128	131	214	41	64	67	150	233	256	3	86	169	192	195	22	105	2056	
	194	61	108	151	130	253	44	87	66	189	236	23	2	125	172	215	2056	
	43	216	129	126	107	24	193	190	171	88	1	254	235	152	65	62	2056	8224
7	5	218	175	116	69	26	239	180	133	90	47	244	197	154	111	52	2056	
	176	115	6	217	240	179	70	25	48	243	134	89	112	51	198	153	2056	
	82	141	252	39	18	77	188	231	210	13	124	167	146	205	60	103	2056	
	251	40	81	142	187	232	17	78	123	168	209	14	59	104	145	206	2056	8224
8	245	10	95	164	181	202	31	100	117	138	223	36	53	74	159	228	2056	
	32	227	182	73	96	35	246	137	160	99	54	201	224	163	118	9	2056	
	162	93	12	247	226	157	76	55	34	221	140	119	98	29	204	183	2056	
	75	184	225	30	11	120	161	222	203	56	97	158	139	248	33	94	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
				8224				8224				8224				8224		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

The equal-summing pairs are distributed within each embedded block-square and are related to each other as depicted by the colors and connected circles.

A square geonomic table is either 2- or 4-dimensional. The size-16 2-dimensional square is either row pair-wise or centrally pair-wise equal summing. This square table is neither. This demonstrates the dimensionality of the table as not being one of 2-dimensions eventhough the table is square and of a size that is a square number. That confirms its 4-dimensionality.

The series of complementary loom tables generate dual quadracubes with an identical equal-summing pair pattern as the primal quadracube, again confirming its 4-dimensional structure.

Program 6

X(16)

5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14
5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4
16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9
2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7
11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14

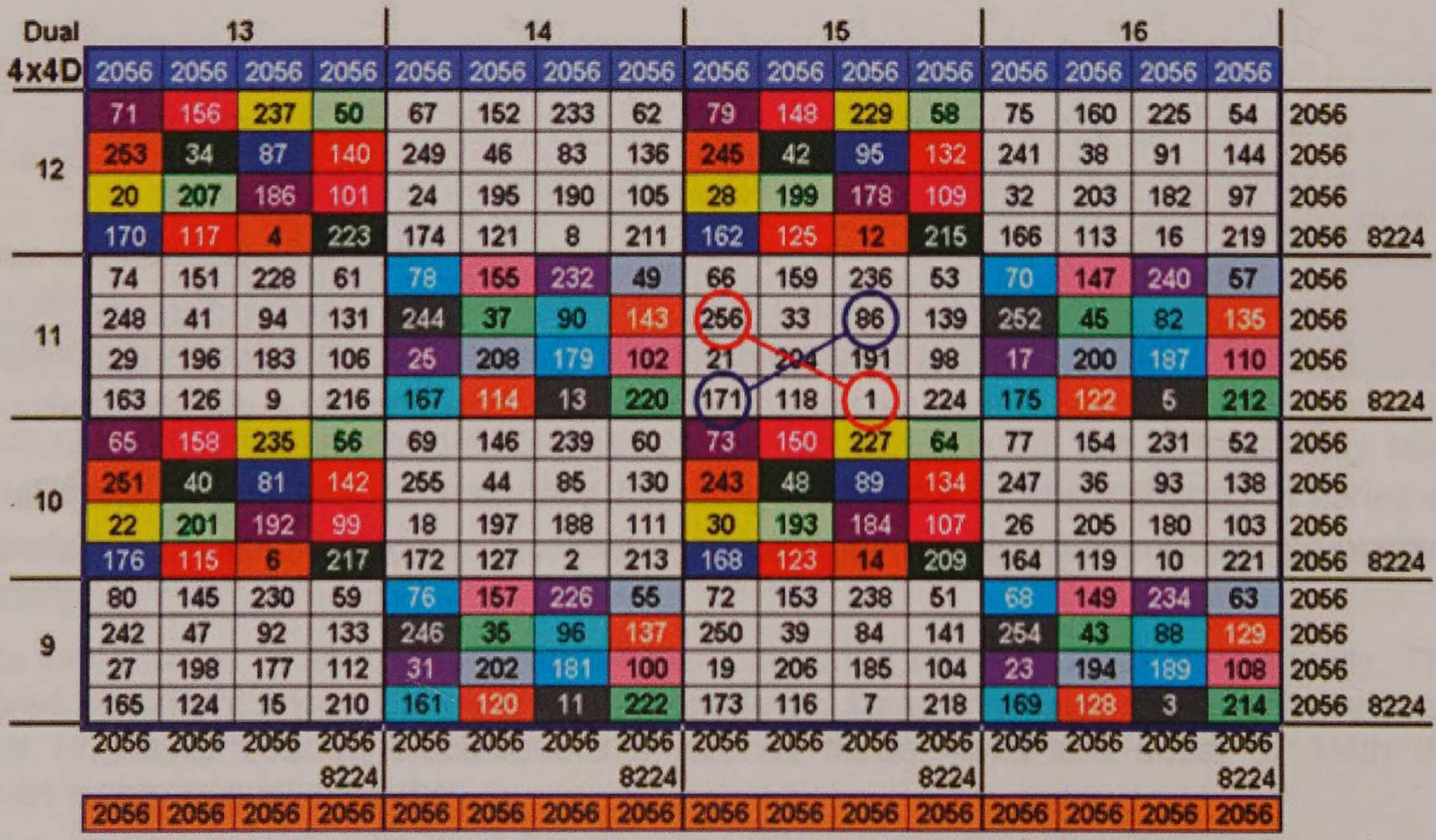
Here is the **4x4D** modulus loom-table. Note the repetition of embedded squares. The distribution of numbers within each block-square is the same as that for the modulus of the 2-dimensional size-4 primal square.

Y(16)

7	12	13	2	3	8	9	14	15	4	5	10	11	16	1	6
13	2	7	12	9	14	3	8	5	10	15	4	1	6	11	16
4	15	10	5	8	3	14	9	12	7	2	13	16	11	6	1
10	5	4	15	14	9	8	3	2	13	12	7	6	1	16	11
10	7	4	13	14	11	8	1	2	15	12	5	6	3	16	9
8	9	14	3	4	5	10	15	16	1	6	11	12	13	2	7
13	4	7	10	9	16	3	6	5	12	15	2	1	8	11	14
3	14	9	8	7	2	13	12	11	6	1	16	15	10	5	4
1	14	11	8	5	2	15	12	9	6	3	16	13	10	7	4
11	8	1	14	15	12	5	2	3	16	9	6	7	4	13	10
6	9	16	3	2	5	12	15	14	1	8	11	10	13	4	7
16	3	6	9	12	15	2	5	8	11	14	1	4	7	10	13
16	1	6	11	12	13	2	7	8	9	14	3	4	5	10	15
2	15	12	5	6	3	16	9	10	7	4	13	14	11	8	1
11	6	1	16	15	10	5	4	3	14	9	8	7	2	13	12
5	12	15	2	1	8	11	14	13	4	7	10	9	16	3	6

Here is the **4x4D** integer loom-table. Note the distribution of numbers among the embedded squares. No number is in the same relative location twice among all the block-squares.

The complementary loom-tables generate a perfect dual quadracube with an identical equal-summing pair pattern as the primal quadracube, again confirming its 4-dimensional structure.



Dual	13				14				15				16					
4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
12	71	156	237	50	67	152	233	62	79	148	229	58	75	160	225	54	2056	
	253	34	87	140	249	46	83	136	245	42	95	132	241	38	91	144	2056	
	20	207	186	101	24	195	190	105	28	199	178	109	32	203	182	97	2056	
	170	117	4	223	174	121	8	211	162	125	12	215	166	113	16	219	2056	8224
11	74	151	228	61	78	155	232	49	66	159	236	53	70	147	240	57	2056	
	248	41	94	131	244	37	90	143	256	33	86	139	252	45	82	135	2056	
	29	196	183	106	25	208	179	102	21	204	191	98	17	200	187	110	2056	
	163	126	9	216	167	114	13	220	171	118	1	224	175	122	5	212	2056	8224
10	65	158	235	56	69	146	239	60	73	150	227	64	77	154	231	52	2056	
	251	40	81	142	255	44	85	130	243	48	89	134	247	36	93	138	2056	
	22	201	192	99	18	197	188	111	30	193	184	107	26	205	180	103	2056	
	176	115	6	217	172	127	2	213	168	123	14	209	164	119	10	221	2056	8224
9	80	145	230	59	76	157	226	55	72	153	238	51	68	149	234	63	2056	
	242	47	92	133	246	35	96	137	250	39	84	141	254	43	88	129	2056	
	27	198	177	112	31	202	181	100	19	206	185	104	23	194	189	108	2056	
	165	124	15	210	161	120	11	222	173	116	7	218	169	128	3	214	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
				8224				8224				8224				8224		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

Program 6

Class-1 Quadracubes

Size-5

Here is the size-5 quadracube of Class-1. All of its **10** embedded cubical sections each sum equally to the quadracube's characteristic number **39,125**.

All of its **25** embedded squares sum equally to **7825, 1/5** of the quadracube's characteristic number: **7825 = 39,125 / 5**. The center cells of all the embedded block-squares together sum to **7825** too. In Program 8 we will see that this pattern is only the *tip of the iceberg*!

All the hexadectals sum equally to **5008**. The hexadectal sums are just any **4** quadrals in embedded squares that are symmetrically positioned around the central embedded square.

It has **2** icosahedrons. One has its associated cells highlighted in crimson and the other in purple. Note that pairwise the centrally-symmetrical same-colored cells sum equally to **626**. So both icosahedrons sum equally to **6 x 626 = 3756 = 3/4 x 5008**.

Cubes				1					2					3					4					5			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
		5x4D	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
	1	470	158	621	314	2	220	533	371	64	377	595	283	121	439	127	345	33	496	189	502	95	408	246	564	252	7825
	2	611	304	17	460	173	361	54	392	210	548	111	429	142	585	298	486	179	517	335	48	236	554	267	85	423	7825
6	3	7	475	163	601	319	382	225	538	351	69	132	600	288	101	444	507	350	38	476	194	257	100	413	226	569	7825
	4	153	616	309	22	465	528	366	59	397	215	278	116	434	147	590	28	491	184	522	340	403	241	559	272	90	7825
	5	324	12	455	168	606	74	387	205	543	356	449	137	580	293	106	199	512	330	43	481	574	262	80	418	231	7825
	1	420	233	571	264	77	170	608	321	14	452	545	358	71	389	202	295	108	446	139	577	45	483	196	514	327	7825
	2	561	254	92	410	248	311	4	467	160	623	61	379	217	535	373	436	129	592	285	123	186	504	342	35	498	7825
7	3	82	425	238	551	269	457	175	613	301	19	207	550	363	51	394	582	300	113	426	144	332	50	488	176	519	7825
	4	228	566	259	97	415	603	316	9	472	165	353	66	384	222	540	103	441	134	597	290	478	191	509	347	40	7825
	5	274	87	405	243	556	24	462	155	618	306	399	212	530	368	56	149	587	280	118	431	524	337	30	493	181	7825
	1	495	183	521	339	27	245	558	271	89	402	620	308	21	464	152	370	58	396	214	527	120	433	146	589	277	7825
	2	511	329	42	485	198	261	79	417	235	573	11	454	167	610	323	386	204	542	360	73	136	579	292	110	448	7825
8	3	32	500	188	501	344	407	250	563	251	94	157	625	313	1	469	532	375	63	376	219	282	125	438	126	594	7825
	4	178	516	334	47	490	553	266	84	422	240	303	16	459	172	615	53	391	209	547	365	428	141	584	297	115	7825
	5	349	37	480	193	506	99	412	230	568	256	474	162	605	318	6	224	537	355	68	381	599	287	105	443	131	7825
	1	445	133	596	289	102	195	508	346	39	477	570	258	96	414	227	320	8	471	164	602	70	383	221	539	352	7825
	2	586	279	117	435	148	336	29	492	185	523	86	404	242	560	273	461	154	617	310	23	211	529	367	60	398	7825
9	3	107	450	138	576	294	482	200	513	326	44	232	575	263	76	419	607	325	13	451	169	357	75	388	201	544	7825
	4	128	591	284	122	440	503	341	34	497	190	253	91	409	247	565	3	466	159	622	315	378	216	534	372	65	7825
	5	299	112	430	143	581	49	487	180	518	331	424	237	555	268	81	174	612	305	18	456	549	362	55	393	206	7825
	1	395	208	546	364	52	145	583	296	114	427	520	333	46	489	177	270	83	421	239	552	20	458	171	614	302	7825
	2	536	354	67	385	223	286	104	442	135	598	36	479	192	510	348	411	229	567	260	98	161	604	317	10	473	7825
10	3	57	400	213	526	369	432	150	588	276	119	182	525	338	26	494	557	275	88	401	244	307	25	463	151	619	7825
	4	203	541	359	72	390	578	291	109	447	140	328	41	484	197	515	78	416	234	572	265	453	166	609	322	15	7825
	5	374	62	380	218	531	124	437	130	593	281	499	187	505	343	31	249	562	255	93	406	624	312	5	468	156	7825
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
			7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Block major diagonal sums

Cube	1	2	3	4	5	
		7825	7825	7825	7825	7825
6	1565	1565	1565	1565	1565	7825
7	1565	1565	1565	1565	1565	7825
8	1565	1565	1565	1565	1565	7825
9	1565	1565	1565	1565	1565	7825
10	1565	1565	1565	1565	1565	7825
		7825	7825	7825	7825	7825

Hypercube 5x5 block sums

Cube	1	2	3	4	5
6	7825	7825	7825	7825	7825
7	7825	7825	7825	7825	7825
8	7825	7825	7825	7825	7825
9	7825	7825	7825	7825	7825
10	7825	7825	7825	7825	7825

Block minor diagonal sums

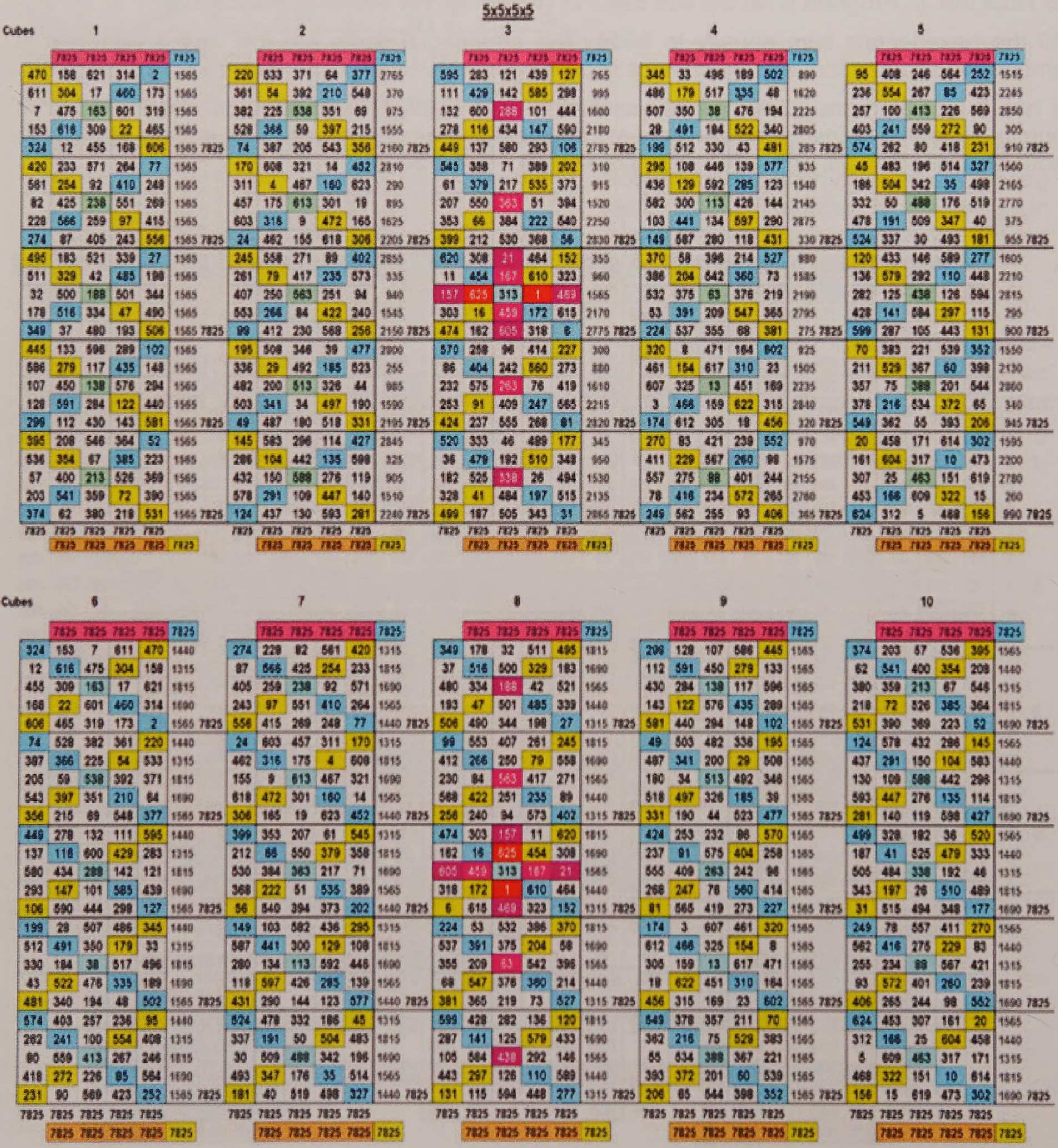
Cube	1	2	3	4	5	
		7825	7825	7825	7825	7825
6	1565	1565	1565	1565	1565	7825
7	1565	1565	1565	1565	1565	7825
8	1565	1565	1565	1565	1565	7825
9	1565	1565	1565	1565	1565	7825
10	1565	1565	1565	1565	1565	7825
		7825	7825	7825	7825	7825

Program 6

Size-5 Quadracube's 10 Embedded Cubes

Here all of the **10** cubes embedded within the size-5 quadracube are shown separately. Note that each is in itself perfect because all of its planar block-square sums equal **7825**.

All the octals in the central cubes **3** & **8** sum to **2504**. So the hexadectals formed from their octals sum to **5008**. Of all the hexadectals in the quadracube, this is the only case where they are generated by only two embedded cubes, not four.



5x5x5x5

Cubes 1

	7825	7825	7825	7825	7825	
470	158	621	314	2	1565	
611	304	17	460	173	1565	
7	475	163	601	319	1565	
153	616	309	22	465	1565	
324	12	455	168	606	1565	7825
420	233	571	264	77	1565	
561	254	92	410	248	1565	
82	425	238	551	269	1565	
228	566	259	97	415	1565	
274	87	405	243	556	1565	7825
495	183	521	339	27	1565	
511	329	42	485	198	1565	
32	500	188	501	344	1565	
178	516	334	47	490	1565	
349	37	480	193	506	1565	7825
445	133	596	289	102	1565	
586	279	117	435	148	1565	
107	450	138	576	294	1565	
128	591	284	122	440	1565	
299	112	430	143	581	1565	7825
395	208	546	364	52	1565	
536	354	67	385	223	1565	
57	400	213	526	369	1565	
203	541	359	72	390	1565	
374	62	380	218	531	1565	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

2

	7825	7825	7825	7825	7825	
220	533	371	64	377	2765	
361	54	392	210	548	370	
382	225	538	351	69	975	
528	366	59	397	215	1555	
74	387	205	543	356	2160	7825
170	608	321	14	452	2810	
311	4	467	160	623	290	
457	175	613	301	19	895	
603	316	9	472	165	1625	
24	462	155	618	306	2205	7825
245	558	271	89	402	2855	
261	79	417	235	573	335	
407	250	563	251	94	940	
553	266	84	422	240	1545	
99	412	230	568	256	2150	7825
195	508	346	39	477	2800	
336	29	492	185	523	255	
482	200	513	326	44	985	
503	341	34	497	190	1590	
49	487	180	518	331	2195	7825
145	583	296	114	427	2845	
286	104	442	135	598	325	
432	150	588	276	119	905	
578	291	109	447	140	1510	
124	437	130	593	281	2240	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

3

	7825	7825	7825	7825	7825	
595	283	121	439	127	265	
111	429	142	585	298	995	
132	600	288	101	444	1600	
278	116	434	147	590	2180	
449	137	580	293	106	2785	7825
545	358	71	389	202	310	
61	379	217	535	373	915	
207	550	363	51	394	1520	
353	66	384	222	540	2250	
399	212	530	368	56	2830	7825
620	308	21	464	152	355	
11	454	167	610	323	960	
157	625	313	1	469	1565	
303	16	459	172	615	2170	
474	162	605	318	6	2775	7825
570	258	96	414	227	300	
86	404	242	560	273	880	
232	575	263	76	419	1610	
253	91	409	247	565	2215	
424	237	555	268	81	2820	7825
520	333	46	489	177	345	
36	479	192	510	348	950	
182	525	338	26	494	1530	
328	41	484	197	515	2135	
499	187	505	343	31	2865	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

4

	7825	7825	7825	7825	7825	
345	33	496	189	502	890	
486	179	517	335	48	1620	
507	350	38	476	194	2225	
28	491	184	522	340	2805	
199	512	330	43	481	285	7825
295	108	446	139	577	935	
436	129	592	285	123	1540	
582	300	113	426	144	2145	
103	441	134	597	290	2875	
149	587	280	118	431	330	7825
370	58	396	214	527	980	
386	204	542	360	73	1585	
532	375	63	376	219	2190	
53	391	209	547	365	2795	
224	537	355	68	381	275	7825
320	8	471	164	602	925	
461	154	617	310	23	1505	
607	325	13	451	169	2235	
3	466	159	622	315	2840	
174	612	305	18	456	320	7825
270	83	421	239	552	970	
411	229	567	260	98	1575	
557	275	88	401	244	2155	
78	416	234	572	265	2760	
249	562	255	93	406	365	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

5

	7825	7825	7825	7825	7825	
95	408	246	564	252	1515	
236	554	267	85	423	2245	
257	100	413	226	569	2850	
403	241	559	272	90	305	
574	262	80	418	231	910	7825
45	483	196	514	327	1560	
186	504	342	35	498	2165	
332	50	488	176	519	2770	
478	191	509	347	40	375	
524	337	30	493	181	955	7825
120	433	146	589	277	1605	
136	579	292	110	448	2210	
282	125	438	126	594	2815	
428	141	584	297	115	295	
599	287	105	443	131	900	7825
70	383	221	539	352	1550	
211	529	367	60	398	2130	
357	75	388	201	544	2860	
378	216	534	372	65	340	
549	362	55	393	206	945	7825
20	458	171	614	302	1595	
161	604	317	10	473	2200	
307	25	463	151	619	2780	
453	166	609	322	15	260	
624	312	5	468	156	990	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

Cubes 6

	7825	7825	7825	7825	7825	
324	153	7	611	470	1440	
12	616	475	304	158	1315	
455	309	163	17	621	1815	
168	22	601	460	314	1690	
606	465	319	173	2	1565	7825
74	528	382	361	220	1440	
387	366	225	54	533	1315	
205	59	538	392	371	1815	
543	397	351	210	64	1690	
356	215	69	548	377	1565	7825
449	278	132	111	595	1440	
137	116	600	429	283	1315	
580	434	288	142	121	1815	
293	147	101	585	439	1690	
106	590	444	298	127	1565	7825
199	28	507	486	345	1440	
512	491	350	179	33	1315	
330	184	38	517	496	1815	
43	522	476	335	189	1690	
481	340	194	48	502	1565	7825
574	403	257	236	95	1440	
262	241	100	554	408	1315	
80	559	413	267	246	1815	
418	272	226	85	564	1690	
231	90	569	423	252	1565	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

7

	7825	7825	7825	7825	7825	
274	228	82	561	420	1315	
87	566	425	254	233	1815	
405	259	238	92	571	1690	
243	97	551	410	264	1565	
556	415	269	248	77	1440	7825
24	603	457	311	170	1315	
462	316	175	4	608	1815	
155	9	613	467	321	1690	
618	472	301	160	14	1565	
306	165	19	623	452	1440	7825
399	353	207	61	545	1315	
212	66	550	379	358	1815	
530	384	363	217	71	1690	
368	222	51	535	389	1565	
56	540	394	373	202	1440	7825
149	103	582	436	295	1315	
587	441	300	129	108	1815	
280	134	113	592	446	1690	
118	597	426	285	139	1565	
431	290	144	123	577	1440	7825
524	478	332	186	45	1315	
337	191	50	504	483	1815	
30	509	488	342	196	1690	
493	347	176	35	514	1565	
181	40	519	498	327	1440	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

8

	7825	7825	7825	7825	7825	
349	178	32	511	495	1815	
37	516	500	329	183	1690	
480	334	188	42	521	1565	
193	47	501	485	339	1440	
506	490	344	198	27	1315	7825
99	553	407	261	245	1815	
412	266	250	79	558	1690	
230	84	563	417	271	1565	
568	422	251	235	89	1440	
256	240	94	573	402	1315	7825
474	303	157	11	620	1815	
162	16	625	454	308	1690	
605	459	313	167	21	1565	
318	172	1	610	464	1440	
6	615	469	323	152	1315	7825
224	53	532	386	370	1815	
537	391	375	204	58	1690	
355	209	63	542	396	1565	
68	547	376	360	214	1440	
381	365	219	73	527	1315	7825
599	428	282	136	120	1815	
287	141	125	579	433	1690	
105	584	438	292	146	1565	
443	297	126	110	589	1440	
131	115	594	448	277	1315	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

9

	7825	7825	7825	7825	7825	
299	128	107	586	445	1565	
112	591	450	279	133	1565	
430	284	138	117	596	1565	
143	122	576	435	289	1565	
581	440	294	148	102	1565	7825
49	503	482	336	195	1565	
487	341	200	29	508	1565	
180	34	513	492	346	1565	
518	497	326	185	39	1565	
331	190	44	523	477	1565	7825
424	253	232	86	570	1565	
237	91	575	404	258	1565	
555	409	263	242	96	1565	
268	247	76	560	414	1565	
81	565	419	273	227	1565	7825
174	3	607	461	320	1565	
612	466	325	154	8	1565	
305	159	13	617	471	1565	
18	622	451	310	164	1565	
456	315	169	23	602	1565	7825
549	378	357	211	70	1565	
362	216	75	529	383	1565	
55	534	388	367	221	1565	
393	372	201	60	539	1565	
206	65	544	398	352	1565	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

10

	7825	7825	7825	7825	7825	
374	203	57	536	395	1565	
62	541	400	354	208	1440	
380	359	213	67	546	1315	
218	72	526	385	364	1815	
531	390	369	223	52	1690	7825
124	578	432	286	145	1565	
437	291	150	104	583	1440	
130	109	588	442	296	1315	
593	447	276	135	114	1815	
281	140	119	598	427	1690	7825
499	328	182	36	520	1565	
187	41	525	479	333	1440	
505	484	338	192	46	1315	
343	197	26	510	489	1815	
31	515	494	348	177	1690	7825
249	78	557	411	270	1565	
562	416	275	229	83	1440	
255	234	88	567	421	1315	
93	572	401	260	239	1815	
406	265	244	98	552	1690	7825
624	453	307	161	20	1565	
312	166	25	604	458	1440	
5	609	463	317	171	1315	
468	322	151	10	614	1815	
156	15	619	473	302	1690	7825
7825	7825	7825	7825	7825		
	7825	7825	7825	7825	7825	

Program 6

Size-5 Quadracube's Depth-sum Tables

This slide depicts the four depth-sum tables of the size-5 quadracube. Note that both depth-sums taken along the A and C axes have all their numbers equal. This shows that all the channels and pillars in each embedded cube sum equally.

A

1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565

B

2850	1540	355	2195	885
305	2145	960	2800	1615
910	2875	1565	255	2220
1515	330	2170	985	2825
2245	935	2775	1590	280
1600	290	2230	945	2760
2180	895	2835	1550	365
2785	1625	315	2130	970
265	2205	920	2860	1575
995	2810	1525	340	2155
350	2165	980	2820	1510
930	2770	1585	300	2240
1535	375	2190	880	2845
2140	955	2795	1610	325
2870	1560	275	2215	905
2225	915	2855	1570	260
2805	1520	335	2175	990
285	2250	940	2755	1595
890	2830	1545	360	2200
1620	310	2150	965	2780
975	2790	1605	320	2135
1555	270	2210	925	2865
2160	1000	2815	1505	345
2765	1580	295	2235	950
370	2185	900	2840	1530

C

1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565
1565	1565	1565	1565	1565

D

1725	1415	1855	1570	1260
1805	1520	1335	1675	1490
1285	1750	1440	1755	1595
1390	1830	1545	1360	1700
1620	1310	1650	1465	1780
1475	1790	1605	1320	1635
1555	1270	1710	1425	1865
1660	1500	1815	1505	1345
1765	1580	1295	1735	1450
1370	1685	1400	1840	1530
1850	1540	1355	1695	1385
1305	1645	1460	1800	1615
1410	1875	1565	1255	1720
1515	1330	1670	1485	1825
1745	1435	1775	1590	1280
1600	1290	1730	1445	1760
1680	1395	1835	1550	1365
1785	1625	1315	1630	1470
1265	1705	1420	1860	1575
1495	1810	1525	1340	1655
1350	1665	1480	1820	1510
1430	1770	1585	1300	1740
1535	1375	1690	1380	1845
1640	1455	1795	1610	1325
1870	1560	1275	1715	1405

This slide shows the quadracube's depth-sum table **D** being collapsed along its D-axis to a perfect size-5 cube.

D

1725	1415	1855	1570	1260
1805	1520	1335	1675	1490
1285	1750	1440	1755	1595
1390	1830	1545	1360	1700
1620	1310	1650	1465	1780
1475	1790	1605	1320	1635
1555	1270	1710	1425	1865
1660	1500	1815	1505	1345
1765	1580	1295	1735	1450
1370	1685	1400	1840	1530
1850	1540	1355	1695	1385
1305	1645	1460	1800	1615
1410	1875	1565	1255	1720
1515	1330	1670	1485	1825
1745	1435	1775	1590	1280
1600	1290	1730	1445	1760
1680	1395	1835	1550	1365
1785	1625	1315	1630	1470
1265	1705	1420	1860	1575
1495	1810	1525	1340	1655
1350	1665	1480	1820	1510
1430	1770	1585	1300	1740
1535	1375	1690	1380	1845
1640	1455	1795	1610	1325
1870	1560	1275	1715	1405

→

D/5

345	283	371	314	252
361	304	267	335	298
257	350	288	351	319
278	366	309	272	340
324	262	330	293	356
295	358	321	264	327
311	254	342	285	373
332	300	363	301	269
353	316	259	347	290
274	337	280	368	306
370	308	271	339	277
261	329	292	360	323
282	375	313	251	344
303	266	334	297	365
349	287	355	318	256
320	258	346	289	352
336	279	367	310	273
357	325	263	326	294
253	341	284	372	315
299	362	305	268	331
270	333	296	364	302
286	354	317	260	348
307	275	338	276	369
328	291	359	322	265
374	312	255	343	281

→

E = D/5-250

95	33	121	64	2
111	54	17	85	48
7	100	38	101	69
28	116	59	22	90
74	12	80	43	106
45	108	71	14	77
61	4	92	35	123
82	50	113	51	19
103	66	9	97	40
24	87	30	118	56
120	58	21	89	27
11	79	42	110	73
32	125	63	1	94
53	16	84	47	115
99	37	105	68	6
70	8	96	39	102
86	29	117	60	23
107	75	13	76	44
3	91	34	122	65
49	112	55	18	81
20	83	46	114	52
36	104	67	10	98
57	25	88	26	119
78	41	109	72	15
124	62	5	93	31

Program 6

Derived perfect cube collapses further to a perfect square

Block-square minor diagonal sums

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

		1575	1575	1575	1575	1575	
	95	33	121	64	2	315	
	111	54	17	85	48	315	
5	7	100	38	101	69	315	1575
	28	116	59	22	90	315	
	74	12	80	43	106	315	
	45	108	71	14	77	315	
	61	4	92	35	123	315	
4	82	50	113	51	19	315	1575
	103	66	9	97	40	315	
	24	87	30	118	56	315	
	120	58	21	80	27	315	
	11	79	42	110	73	315	
3	32	125	63	1	94	315	1575
	53	16	84	47	115	315	
	99	37	105	68	6	315	
	70	8	96	39	102	315	
	86	29	117	60	23	315	
2	107	75	13	76	44	315	1575
	3	91	34	122	65	315	
	49	112	55	18	81	315	
	20	83	46	114	52	315	
	36	104	67	10	98	315	
1	57	25	88	26	119	315	1575
	78	41	109	72	15	315	
	124	62	5	93	31	315	
	1575	1575	1575	1575	1575		
		1575	1575	1575	1575	1575	

Block-square major diagonal sums

315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315
315	315	315	315	315

BD Depth-sums

350	290	355	320	260
305	270	335	300	365
285	375	315	255	345
265	330	295	360	325
370	310	275	340	280

BD/5

70	58	71	64	52
61	54	67	60	73
57	75	63	51	69
53	66	59	72	65
74	62	55	68	56

BD/5 - 50

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

	1	2	3	4	5	
	5x5	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

Quadrals

52	52	52
52	52	52

This slide shows that the derived cube is absolutely perfect.

It also shows that this cube will collapse further to a perfect size-5 square.

Program 6

Depth-sum tables B and D collapse along their B-axes to identical perfect squares

This slide shows that the size-5 quadracube will collapse along either its B or D-axis directly to a perfect size-5 square.

B

2850	1540	355	2195	885
305	2145	960	2800	1615
910	2875	1565	255	2220
1515	330	2170	985	2825
2245	935	2775	1590	280
1600	290	2230	945	2760
2180	895	2835	1550	365
2785	1625	315	2130	970
265	2205	920	2860	1575
995	2810	1525	340	2155
350	2165	980	2820	1510
930	2770	1585	300	2240
1535	375	2190	880	2845
2140	955	2795	1610	325
2870	1560	275	2215	905
2225	915	2855	1570	260
2805	1520	335	2175	990
285	2250	940	2755	1595
890	2830	1545	360	2200
1620	310	2150	965	2780
975	2790	1605	320	2135
1555	270	2210	925	2865
2160	1000	2815	1505	345
2765	1580	295	2235	950
370	2185	900	2840	1530

D

1725	1415	1855	1570	1260
1805	1520	1335	1675	1490
1285	1750	1440	1755	1595
1390	1830	1545	1360	1700
1620	1310	1650	1465	1780
1475	1790	1605	1320	1635
1555	1270	1710	1425	1865
1660	1500	1815	1505	1345
1765	1580	1295	1735	1450
1370	1685	1400	1840	1530
1850	1540	1355	1695	1385
1305	1645	1460	1800	1615
1410	1875	1565	1255	1720
1515	1330	1670	1485	1825
1745	1435	1775	1590	1280
1600	1290	1730	1445	1760
1680	1395	1835	1550	1365
1785	1625	1315	1630	1470
1265	1705	1420	1860	1575
1495	1810	1525	1340	1655
1350	1665	1480	1820	1510
1430	1770	1585	1300	1740
1535	1375	1690	1380	1845
1640	1455	1795	1610	1325
1870	1560	1275	1715	1405

Depth-sums

BB

8000	7700	8025	7850	7550
7775	7600	7925	7750	8075
7675	8125	7825	7525	7975
7575	7900	7725	8050	7875
8100	7800	7625	7950	7650

DB

8000	7700	8025	7850	7550
7775	7600	7925	7750	8075
7675	8125	7825	7525	7975
7575	7900	7725	8050	7875
8100	7800	7625	7950	7650

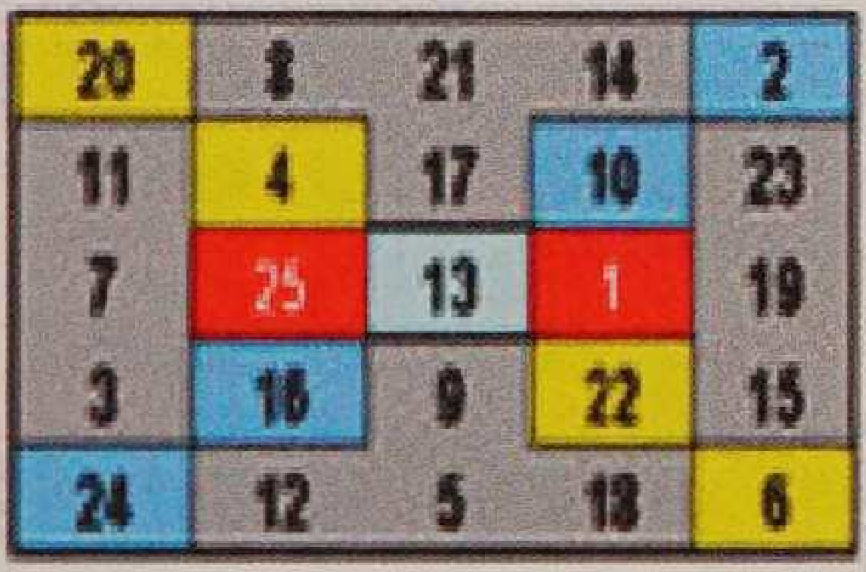
BB/25

320	308	321	314	302
311	304	317	310	323
307	325	313	301	319
303	316	309	322	315
324	312	305	318	306

DB/25

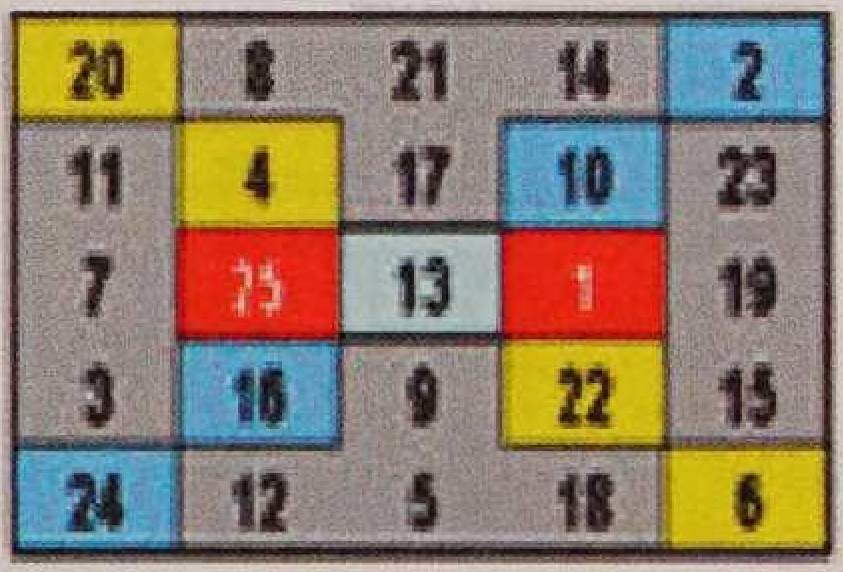
320	308	321	314	302
311	304	317	310	323
307	325	313	301	319
303	316	309	322	315
324	312	305	318	306

BB/25 - 300



20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

DB/25 - 300



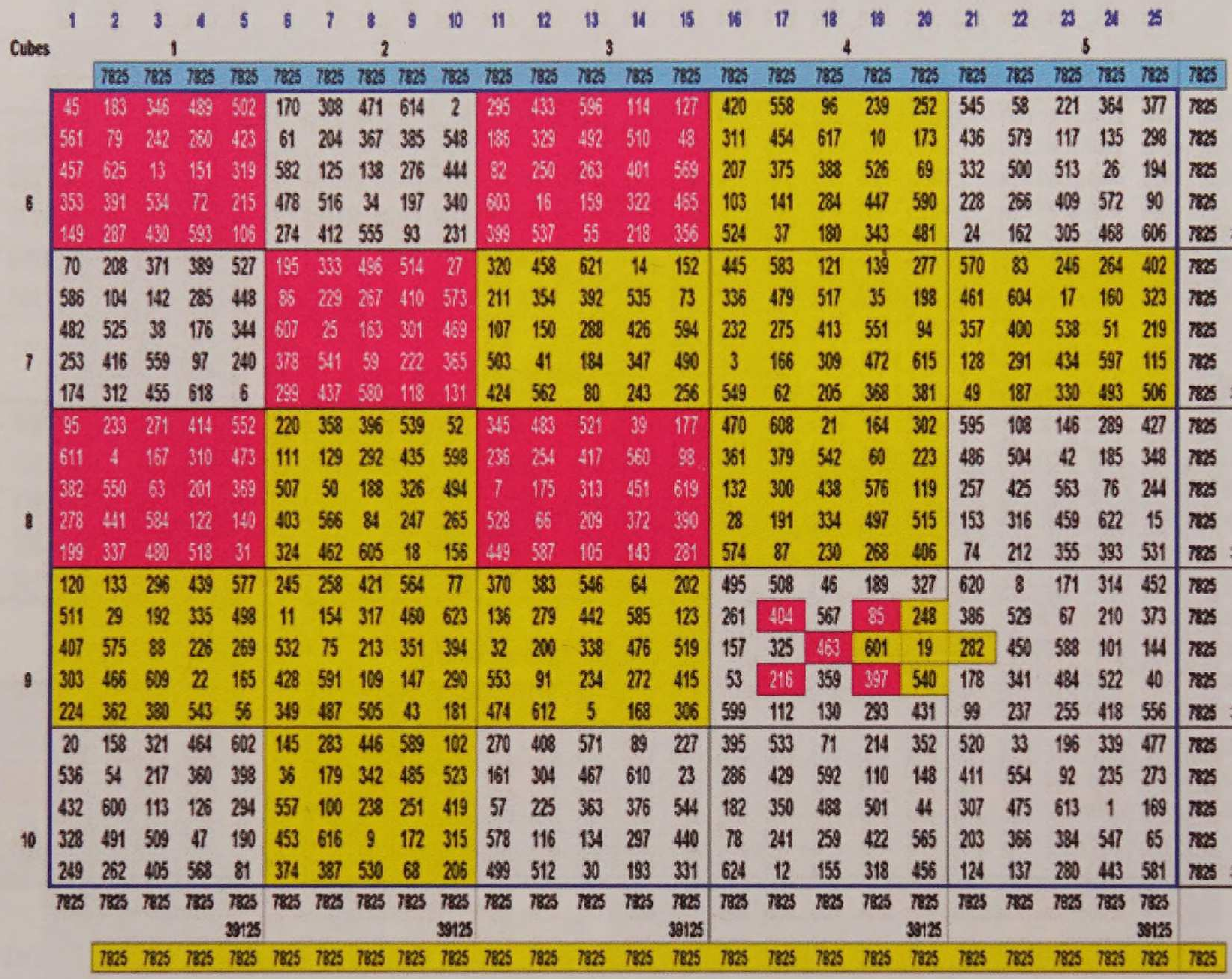
20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

Although the Depth-sum tables **B** and **D** are different, note that the depth-sums **BB** and **DB** of depth-sum tables **B** and **D**, respectively, taken along their own B-axis are identical.

Program 6

Size-5 quadracube's characteristic tiling patterns at the block-square Level

This picture shows that those size-5 **+** and **X** tiling patterns no longer hold at the cellular level in the quadracube but now hold at the **5x5** block-square level.



		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25		
	Cubes			1					2					3					4					5				
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
1		45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825	
2		561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825	
3		457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825	
4	6	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825	
5		149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825	39125
6		70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825	
7		586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825	
8		482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825	
9	7	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825	
10		174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825	39125
11		95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825	
12		611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825	
13		382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825	
14	8	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825	
15		199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825	39125
16		120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825	
17		511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825	
18		407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825	
19	9	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825	
20		224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825	39125
21		20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825	
22		536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825	
23		432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825	
24	10	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825	
25		249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825	39125
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825		
						39125					39125					39125					39125					39125		
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	

All **5x5** block-square tile patterns A & B each sum to the quadracube's characteristic number **39,125** continuously – the same as all the numbers in each horizontal and vertical cube.

The tiling patterns involve the embedded block-squares of **n+1 = 6** intersecting cubes. That is an amazing newly-discovered general tiling-pattern property for quadracubes. However, that doesn't grant the quadracube the status of being ultra-perfect because the tiling pattern is not confined to each block-square by itself.

Nonetheless, the tiling pattern is continuous throughout the quadracube.

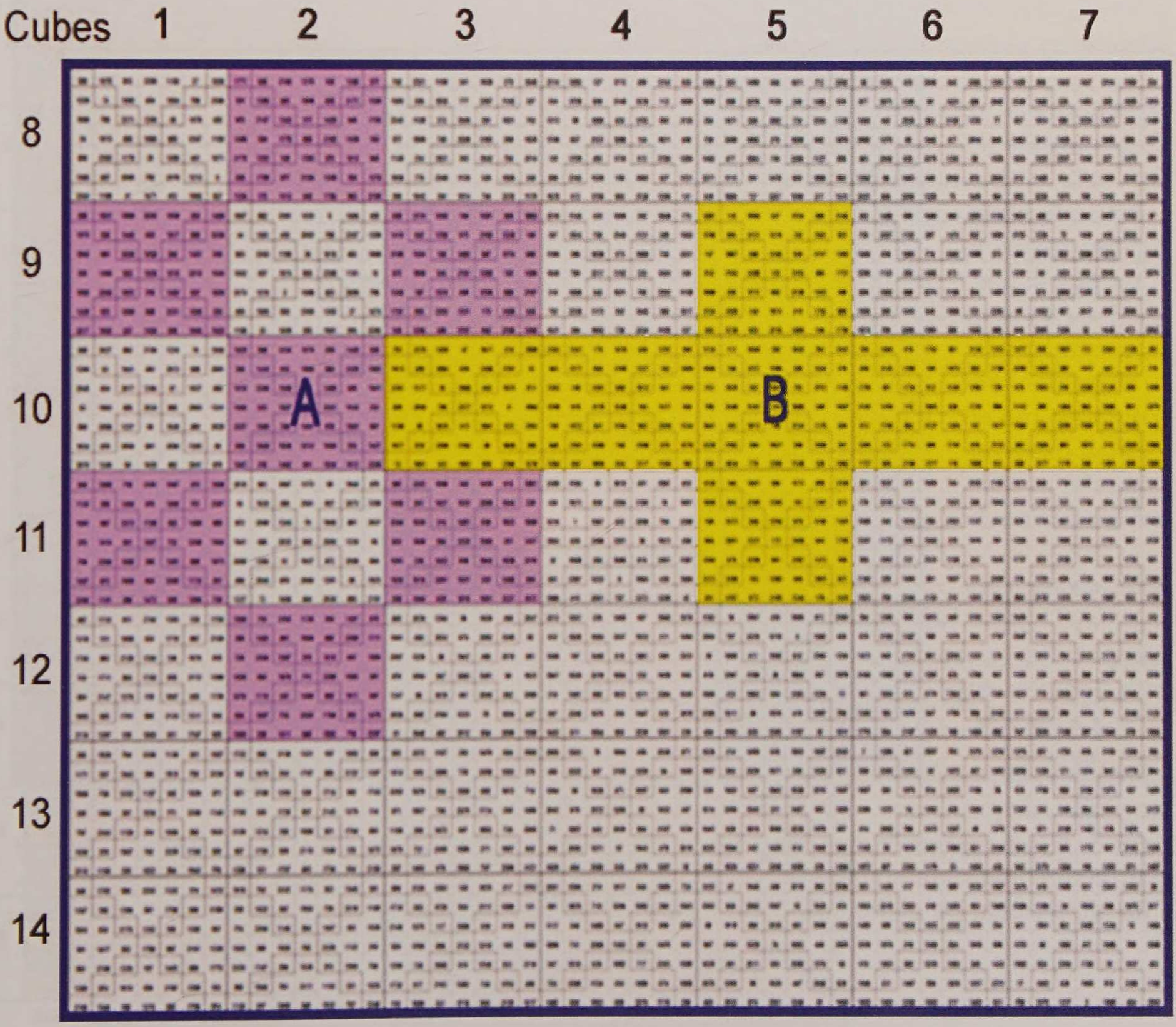
Program 6

Size-7

The size-7 quadracube is too large to show the numbers legibly. Nonetheless it's the patterns here that are important.

This same property applies to all quadracubes in Class-1 as demonstrated again here for the size-7 quadracube. Both size-7 characteristic tiling patterns are at the size-7 block-square level as highlighted. The tiling patterns mimic those for the size-7 square. All the numbers in each pattern sum exactly to the quadracube's characteristic number continuously.

The tiling pattern involves the embedded block-squares of **n+1 = 8** intersecting cubes.



Program 6

Categorization of Quadracubes by Class

Quadracubes can also be categorized by class. The table lists the properties common among all the sizes of quadracube in each class.

Class 1	Class 2	Class 3	Class 4	Class 5
n is an odd prime >3 n = 1b (3, 5 & 7)	n = 2b b is odd (2 & 6)	n = 3b b is odd ≥ 1 (3)	n = 2b b is even (4)	n = ab a & b odd numbers ≥ 3 (9)
Absolutely perfect Continuously nx modular All embedded cubes are ultra-perfect in that each sums to the quadracube's characteristic number and all tiling patterns persist in each embedded square Possesses a dual Primal collapses to both a perfect cube and a perfect square	Absolutely perfect Continuously nx modular for b>1 Possesses a dual Dual doesn't collapse to a lower dimension; only the primal does Primal collapses to perfect cube	Absolutely perfect Continuously 3x modular Possesses a dual Dual doesn't collapse to a lower dimension; only the primal does Primal collapses to a perfect cube	Absolutely perfect Continuously nx modular Possesses a dual Dual doesn't collapse to a lower dimension; only the primal does Primal collapses to a perfect cube and a perfect square	Unknown Smallest size is 9 Largest size quadracube investigated is size-7 and is not of this class

Class-5 quadracubes' properties are unknown as of this compilation; the largest size of quadracube investigated is of size-7 and is not of this class. The smallest such size is of size-9. Its table would be of the size **81x81**; way too large to be legible for print.

Program 6

Exploration of an alternative Size-4 Quadracube

In the course of navigating the 4th dimension, I happened upon this size-4 quadracube. What is different between this one and the previous one of the same size is this: eventhough this version will collapse to a perfect cube too, it will not collapse to a perfect square as the first one did. Consequently it has no relationship with the perfect size-4 square. It was generated from an imperfect square with the same ± deviations in its wrap diagonals, which canceled out when they were incorporated into a size-4 cube making the resulting cube also perfect. That cube was used to generate this quadracube. This version of the size-4 quadracube has equal-summing row-pair summations and some amazing properties that lend insight into hyper- and sub-dimensional spaces.

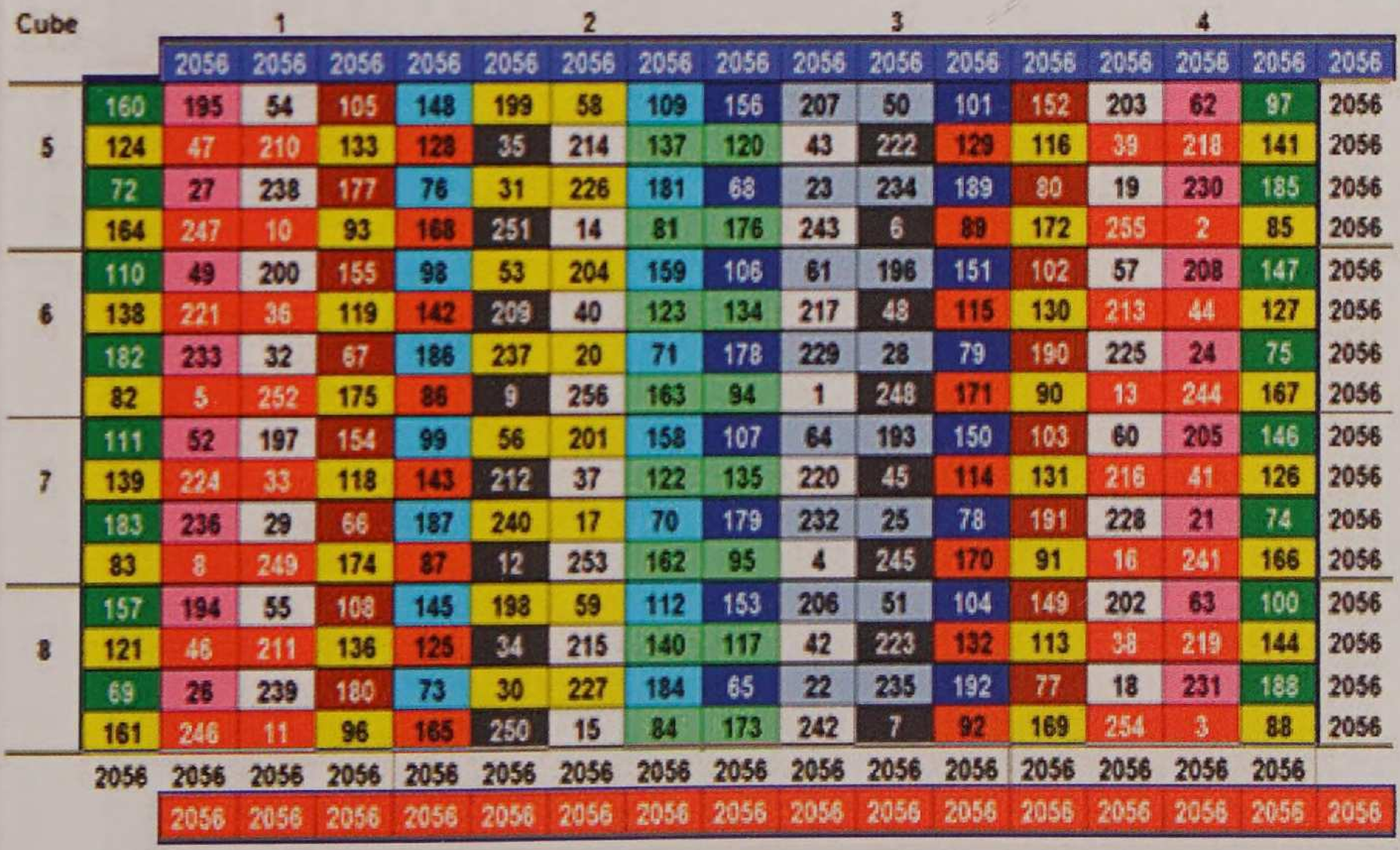
Size 4 Primal Quadracube

Cube			1				2				3				4		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056
	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056
6	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056
	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056
7	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056
	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056
8	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056
	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Its equal row-pair summations are those with the same color in each row.

Complementary loom tables generate a dual quadracube with an identical equal-summing pair pattern as the primal quadracube, again confirming its 4-dimensionality.

Size 4 Dual Quadracube

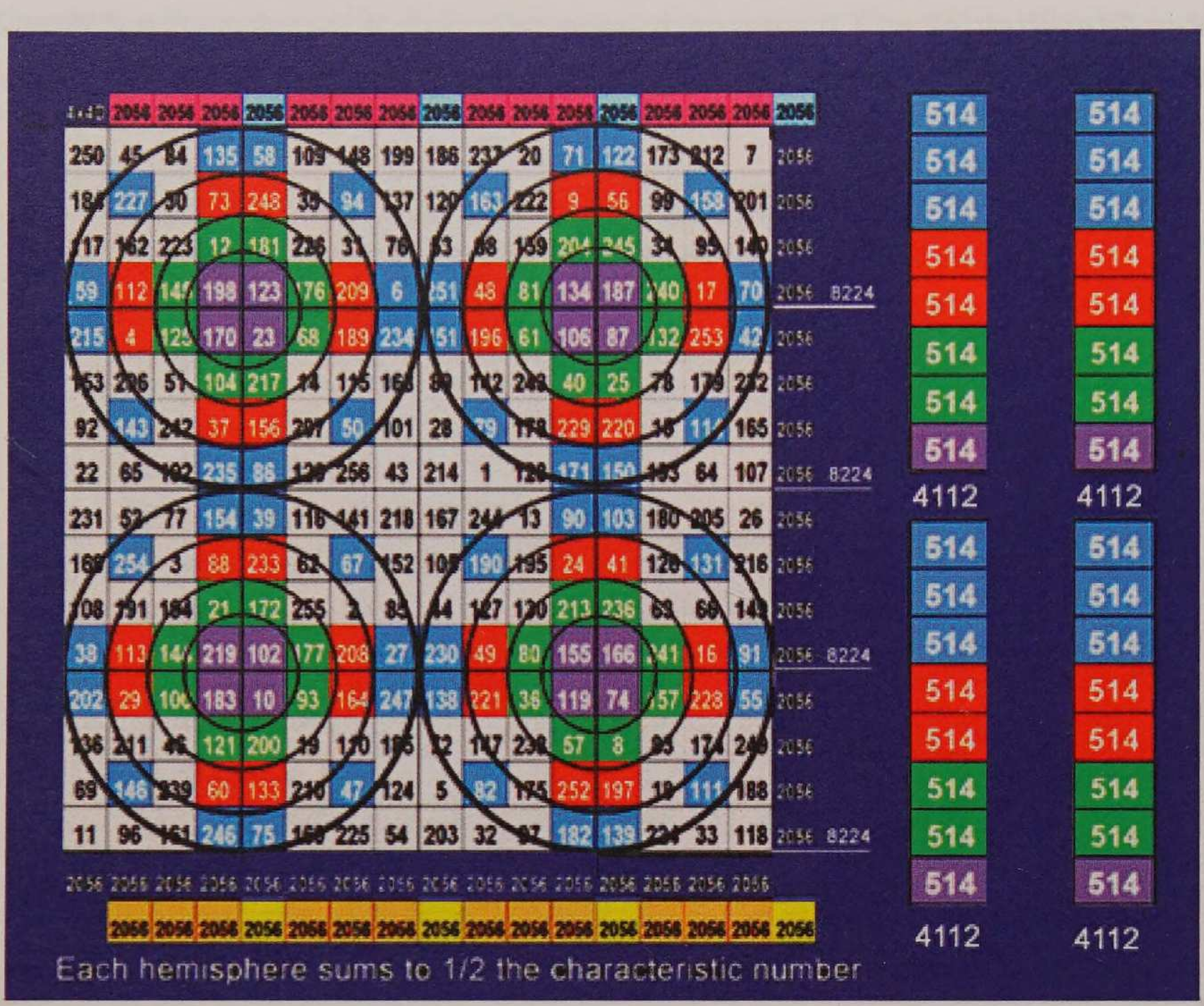


Cube			1				2				3				4		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
5	160	195	54	105	148	199	58	109	156	207	50	101	152	203	62	97	2056
	124	47	210	133	128	35	214	137	120	43	222	129	116	39	218	141	2056
	72	27	238	177	76	31	226	181	68	23	234	189	80	19	230	185	2056
	164	247	10	93	168	251	14	81	176	243	6	89	172	255	2	85	2056
6	110	49	200	155	98	53	204	159	106	61	196	151	102	57	208	147	2056
	138	221	36	119	142	209	40	123	134	217	48	115	130	213	44	127	2056
	182	233	32	67	186	237	20	71	178	229	28	79	190	225	24	75	2056
	82	5	252	175	86	9	256	163	94	1	248	171	90	13	244	167	2056
7	111	52	197	154	99	56	201	158	107	64	193	150	103	60	205	146	2056
	139	224	33	118	143	212	37	122	135	220	45	114	131	216	41	126	2056
	183	236	29	66	187	240	17	70	179	232	25	78	191	228	21	74	2056
	83	8	249	174	87	12	253	162	95	4	245	170	91	16	241	166	2056
8	157	194	55	108	145	198	59	112	153	206	51	104	149	202	63	100	2056
	121	46	211	136	125	34	215	140	117	42	223	132	113	38	219	144	2056
	69	26	239	180	73	30	227	184	65	22	235	192	77	18	231	188	2056
	161	246	11	96	165	250	15	84	173	242	7	92	169	254	3	88	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

The discovery of these two primary quadracubes here provided a picture of the 4th dimension that is absolutely astounding as you will now behold.

Program 6

Embedded Hemispheres in the Primal Quadracube

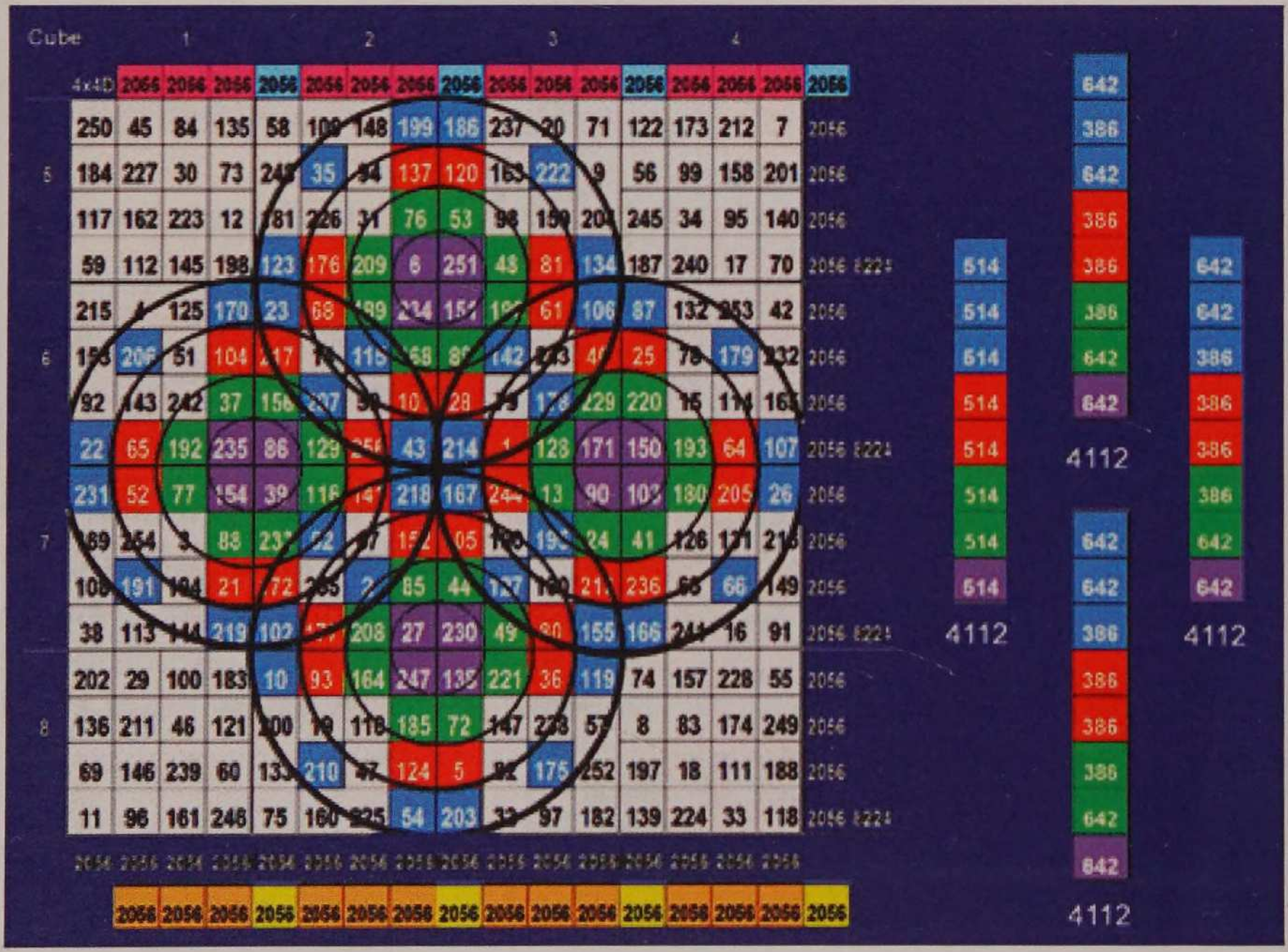


4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056	
	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056	
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056	
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056	8224
	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056	
	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056	
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056	
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056	8224
	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056	
	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056	
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056	
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056	8224
	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056	
	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056	
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056	
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

514	514
514	514
514	514
514	514
514	514
514	514
514	514
514	514
4112	4112
514	514
514	514
514	514
514	514
514	514
514	514
514	514
514	514
4112	4112

Each hemisphere sums to 1/2 the characteristic number

The quadracube possesses **4 non-intersecting** inscribed hemispheres each of which sums to **1/2** the hypercube's characteristic number.



Cube			1				2				3				4			
	4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056	
5	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056	
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056	
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056	8224
	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056	
6	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056	
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056	
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056	8224
	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056	
7	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056	
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056	
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056	8224
	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056	
8	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056	
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056	
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	

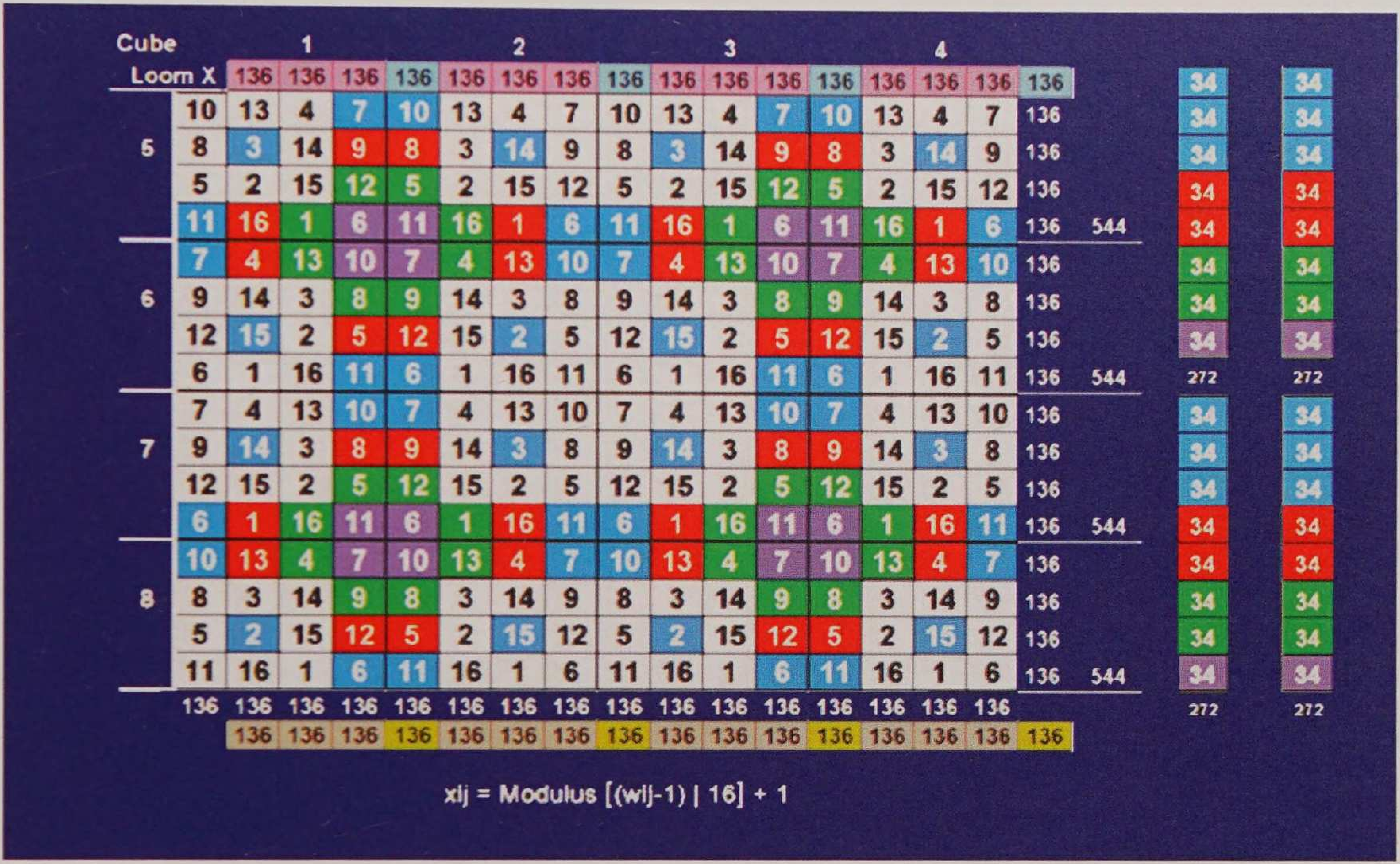
	642	
	386	
	642	
	386	
514	386	642
514	386	642
514	642	386
514	642	386
514	4112	386
514		386
514	642	642
514	642	642
4112	386	4112
	386	
	386	
	386	
	642	
	642	
	4112	

It also simultaneously possesses **4** inscribed hemispheres that are **intersecting,** each of which again sums to **1/2** the hypercube's characteristic number.

Program 6

Uncovering the Size-4 Characteristic Torus via Loom Tables

Its modulus loom table shows the same equal-summing hemispherical patterns. Note that number **34** for each of the quadrals in the embedded squares. That's the characteristic number of the size-4 square!

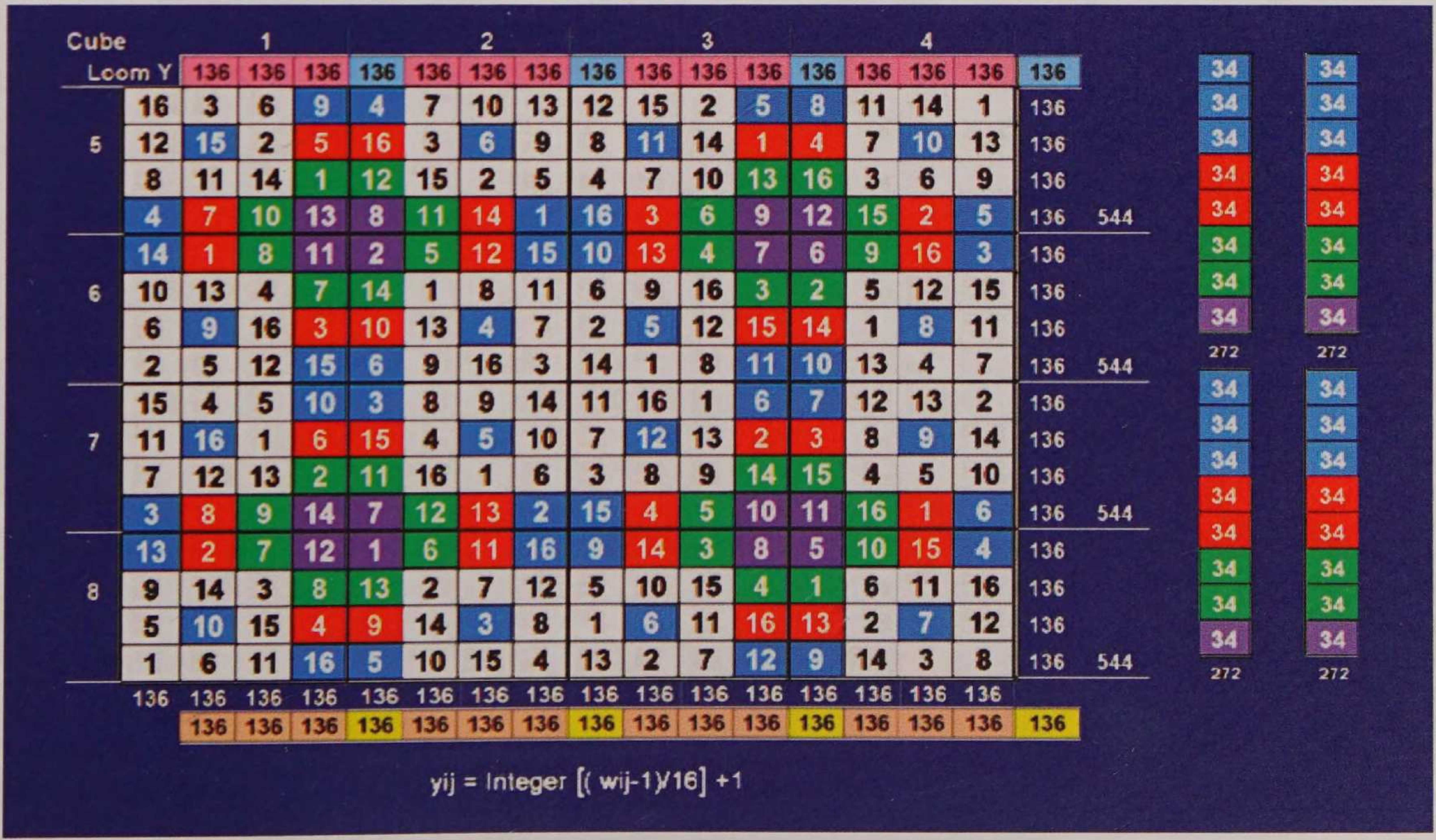


Cube			1				2				3				4			
Loom X		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
5	10	13	4	7	10	13	4	7	10	13	4	7	10	13	4	7	136	
	8	3	14	9	8	3	14	9	8	3	14	9	8	3	14	9	136	
	5	2	15	12	5	2	15	12	5	2	15	12	5	2	15	12	136	
	11	16	1	6	11	16	1	6	11	16	1	6	11	16	1	6	136	544
6	7	4	13	10	7	4	13	10	7	4	13	10	7	4	13	10	136	
	9	14	3	8	9	14	3	8	9	14	3	8	9	14	3	8	136	
	12	15	2	5	12	15	2	5	12	15	2	5	12	15	2	5	136	
	6	1	16	11	6	1	16	11	6	1	16	11	6	1	16	11	136	544
7	7	4	13	10	7	4	13	10	7	4	13	10	7	4	13	10	136	
	9	14	3	8	9	14	3	8	9	14	3	8	9	14	3	8	136	
	12	15	2	5	12	15	2	5	12	15	2	5	12	15	2	5	136	
	6	1	16	11	6	1	16	11	6	1	16	11	6	1	16	11	136	544
8	10	13	4	7	10	13	4	7	10	13	4	7	10	13	4	7	136	
	8	3	14	9	8	3	14	9	8	3	14	9	8	3	14	9	136	
	5	2	15	12	5	2	15	12	5	2	15	12	5	2	15	12	136	
	11	16	1	6	11	16	1	6	11	16	1	6	11	16	1	6	136	544
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136		
		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	

34	34
34	34
34	34
34	34
34	34
34	34
34	34
34	34
272	272
34	34
34	34
34	34
34	34
34	34
34	34
34	34
34	34
272	272

xlj = Modulus [(wlj-1) | 16] + 1

So does its integer loom table. Again every quadral in each embedded square sums to **34**.



Cube			1				2				3				4			
Loom Y		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
5	16	3	6	9	4	7	10	13	12	15	2	5	8	11	14	1	136	
	12	15	2	5	16	3	6	9	8	11	14	1	4	7	10	13	136	
	8	11	14	1	12	15	2	5	4	7	10	13	16	3	6	9	136	
	4	7	10	13	8	11	14	1	16	3	6	9	12	15	2	5	136	544
6	14	1	8	11	2	5	12	15	10	13	4	7	6	9	16	3	136	
	10	13	4	7	14	1	8	11	6	9	16	3	2	5	12	15	136	
	6	9	16	3	10	13	4	7	2	5	12	15	14	1	8	11	136	
	2	5	12	15	6	9	16	3	14	1	8	11	10	13	4	7	136	544
7	15	4	5	10	3	8	9	14	11	16	1	6	7	12	13	2	136	
	11	16	1	6	15	4	5	10	7	12	13	2	3	8	9	14	136	
	7	12	13	2	11	16	1	6	3	8	9	14	15	4	5	10	136	
	3	8	9	14	7	12	13	2	15	4	5	10	11	16	1	6	136	544
8	13	2	7	12	1	6	11	16	9	14	3	8	5	10	15	4	136	
	9	14	3	8	13	2	7	12	5	10	15	4	1	6	11	16	136	
	5	10	15	4	9	14	3	8	1	6	11	16	13	2	7	12	136	
	1	6	11	16	5	10	15	4	13	2	7	12	9	14	3	8	136	544
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136		
		136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	

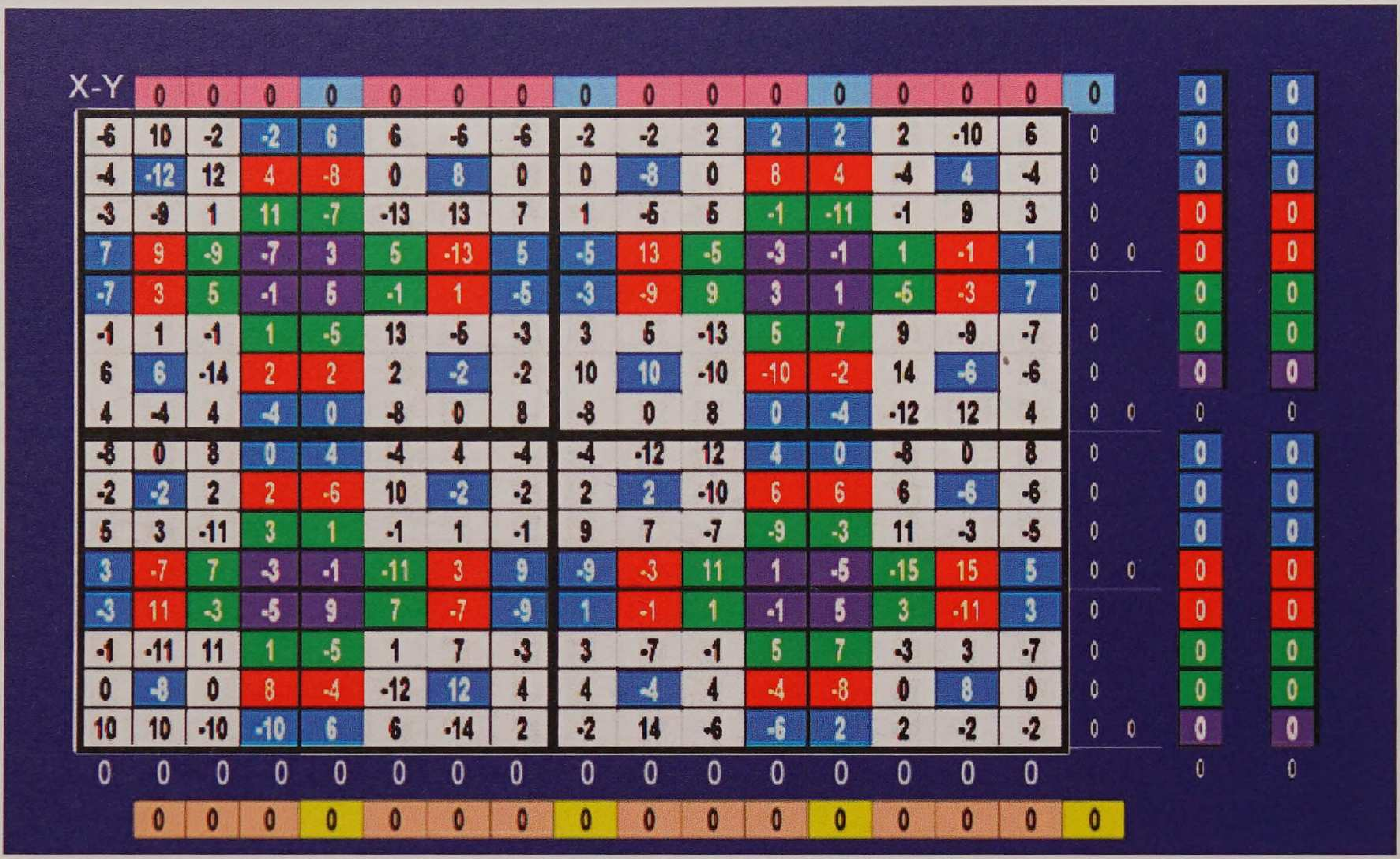
34	34
34	34
34	34
34	34
34	34
34	34
34	34
34	34
272	272
34	34
34	34
34	34
34	34
34	34
34	34
34	34
34	34
272	272

yij = Integer [(wij-1)/16] +1

Program 6

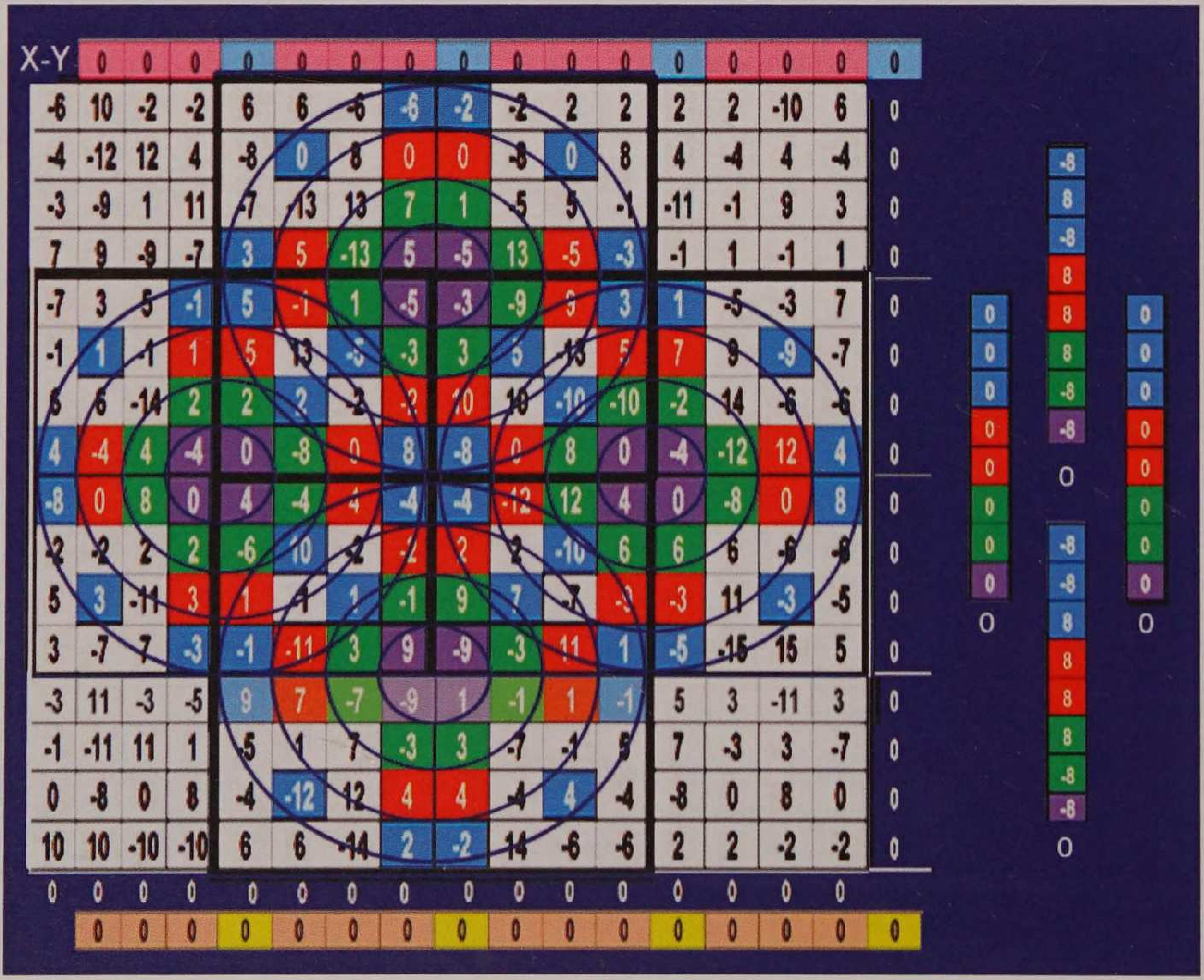
Loom-table Differences

Here is the loom difference table. All four hemispherical patterns sum to **0**.



X-Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0
	-6	10	-2	-2	6	6	-6	-6	-2	-2	2	2	2	2	-10	6	0		0	0
	-4	-12	12	4	-8	0	8	0	0	-8	0	8	4	-4	4	-4	0		0	0
	-3	-9	1	11	-7	-13	13	7	1	-5	5	-1	-11	-1	9	3	0		0	0
	7	9	-9	-7	3	5	-13	5	-5	13	-5	-3	-1	1	-1	1	0	0	0	0
	-7	3	5	-1	5	-1	1	-5	-3	-9	9	3	1	-5	-3	7	0		0	0
	-1	1	-1	1	-5	13	-5	-3	3	5	-13	5	7	9	-9	-7	0		0	0
	6	6	-14	2	2	2	-2	-2	10	10	-10	-10	-2	14	-6	-6	0		0	0
	4	-4	4	-4	0	-8	0	8	-8	0	8	0	-4	-12	12	4	0	0	0	0
	-8	0	8	0	4	-4	4	-4	-4	-12	12	4	0	-8	0	8	0		0	0
	-2	-2	2	2	-6	10	-2	-2	2	2	-10	6	6	6	-6	-6	0		0	0
	5	3	-11	3	1	-1	1	-1	9	7	-7	-9	-3	11	-3	-5	0		0	0
	3	-7	7	-3	-1	-11	3	9	-9	-3	11	1	-5	-15	15	5	0	0	0	0
	-3	11	-3	-5	9	7	-7	-9	1	-1	1	-1	5	3	-11	3	0		0	0
	-1	-11	11	1	-5	1	7	-3	3	-7	-1	5	7	-3	3	-7	0		0	0
	0	-8	0	8	-4	-12	12	4	4	-4	4	-4	-8	0	8	0	0		0	0
	10	10	-10	-10	6	6	-14	2	-2	14	-6	-6	2	2	-2	-2	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			

And so do the **intersecting** hemispheres.



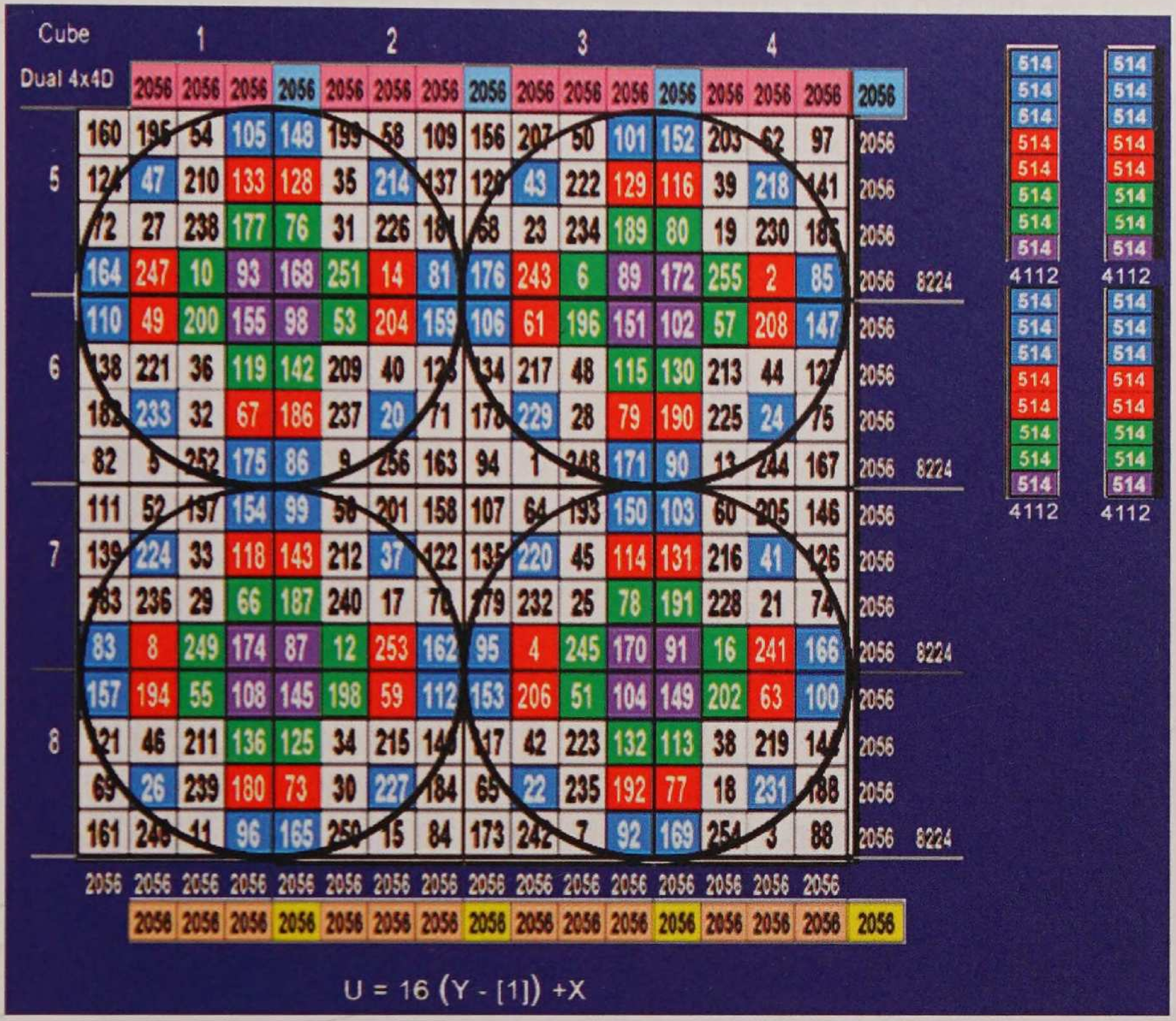
X-Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	-6	10	-2	-2	6	6	-6	-6	-2	-2	2	2	2	2	-10	6	0
	-4	-12	12	4	-8	0	8	0	0	-8	0	8	4	-4	4	-4	0
	-3	-9	1	11	-7	-13	13	7	1	-5	5	-1	-11	-1	9	3	0
	7	9	-9	-7	3	5	-13	5	-5	13	-5	-3	-1	1	-1	1	0
	-7	3	5	-1	5	-1	1	-5	-3	-9	9	3	1	-5	-3	7	0
	-1	1	-1	1	-5	13	-5	-3	3	5	-13	5	7	9	-9	-7	0
	6	6	-14	2	2	2	-2	-2	10	10	-10	-10	-2	14	-6	-6	0
	4	-4	4	-4	0	-8	0	8	-8	0	8	0	-4	-12	12	4	0
	-8	0	8	0	4	-4	4	-4	-4	-12	12	4	0	-8	0	8	0
	-2	-2	2	2	-6	10	-2	-2	2	2	-10	6	6	6	-6	-6	0
	5	3	-11	3	1	-1	1	-1	9	7	-7	-9	-3	11	-3	-5	0
	3	-7	7	-3	-1	-11	3	9	-9	-3	11	1	-5	-15	15	5	0
	-3	11	-3	-5	9	7	-7	-9	1	-1	1	-1	5	3	-11	3	0
	-1	-11	11	1	-5	1	7	-3	3	-7	-1	5	7	-3	3	-7	0
	0	-8	0	8	-4	-12	12	4	4	-4	4	-4	-8	0	8	0	0
	10	10	-10	-10	6	6	-14	2	-2	14	-6	-6	2	2	-2	-2	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	-8	-8	0
0	8	-8	0
0	-8	8	0
0	8	8	0
0	8	8	0
0	8	8	0
0	-8	-8	0
0	-8	-8	0
0	0	0	0

Program 6

Embedded Hemispheres in the Dual Quadracube

This slide depicts the non-intersecting embedded hemispheres of the dual quadracube.

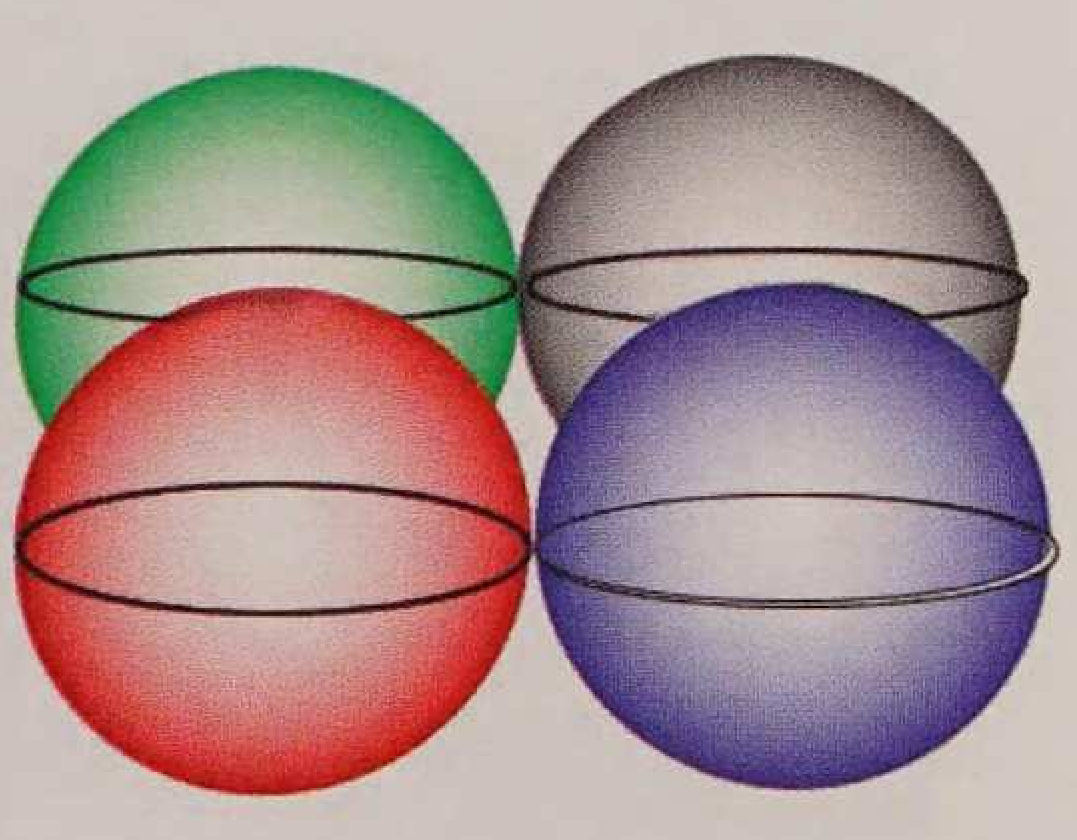


Cube	1				2				3				4					
Dual 4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	160	195	54	105	148	199	58	109	156	207	50	101	152	203	62	97	2056	
5	124	47	210	133	128	35	214	137	120	43	222	129	116	39	218	141	2056	
	72	27	238	177	76	31	226	181	68	23	234	189	80	19	230	185	2056	
	164	247	10	93	168	251	14	81	176	243	6	89	172	255	2	85	2056	8224
	110	49	200	155	98	53	204	159	106	61	196	151	102	57	208	147	2056	
6	138	221	36	119	142	209	40	123	134	217	48	115	130	213	44	127	2056	
	182	233	32	67	186	237	20	71	178	229	28	79	190	225	24	75	2056	
	82	5	252	175	86	9	256	163	94	1	248	171	90	13	244	167	2056	8224
	111	52	197	154	99	56	201	158	107	64	193	150	103	60	205	146	2056	
7	139	224	33	118	143	212	37	122	135	220	45	114	131	216	41	126	2056	
	183	236	29	66	187	240	17	70	179	232	25	78	191	228	21	74	2056	
	83	8	249	174	87	12	253	162	95	4	245	170	91	16	241	166	2056	8224
	157	194	55	108	145	198	59	112	153	206	51	104	149	202	63	100	2056	
8	121	46	211	136	125	34	215	140	117	42	223	132	113	38	219	144	2056	
	69	26	239	180	73	30	227	184	65	22	235	192	77	18	231	188	2056	
	161	246	11	96	165	250	15	84	173	242	7	92	169	254	3	88	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	

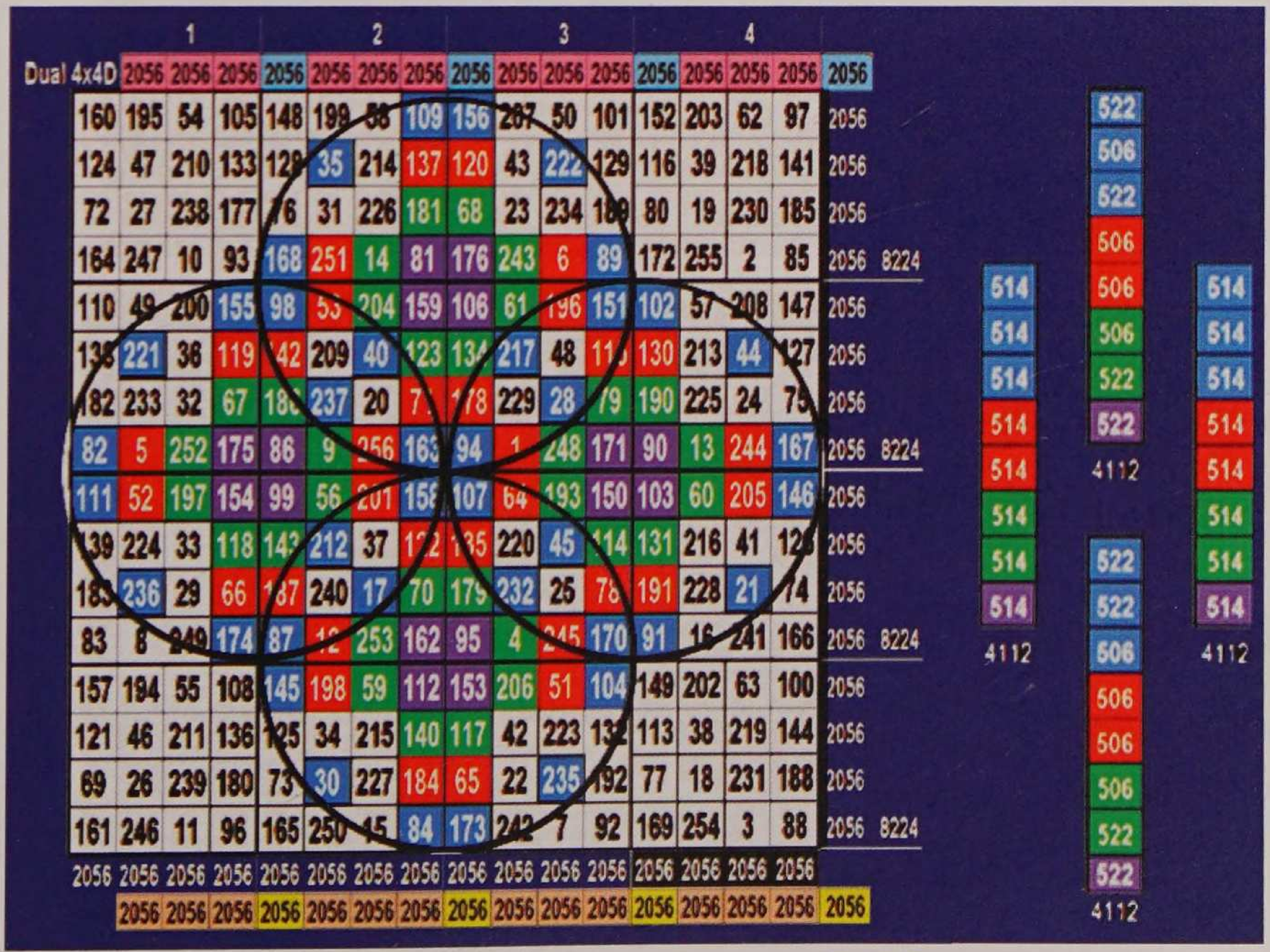
514	514
514	514
514	514
514	514
514	514
514	514
514	514
514	514
4112	4112
514	514
514	514
514	514
514	514
514	514
514	514
514	514
514	514
4112	4112

U = 16 (Y - [1]) +X

Note that every quadral in the four quadrants sums equally to **514**. So all the hemispheres sum equally to **4112**.



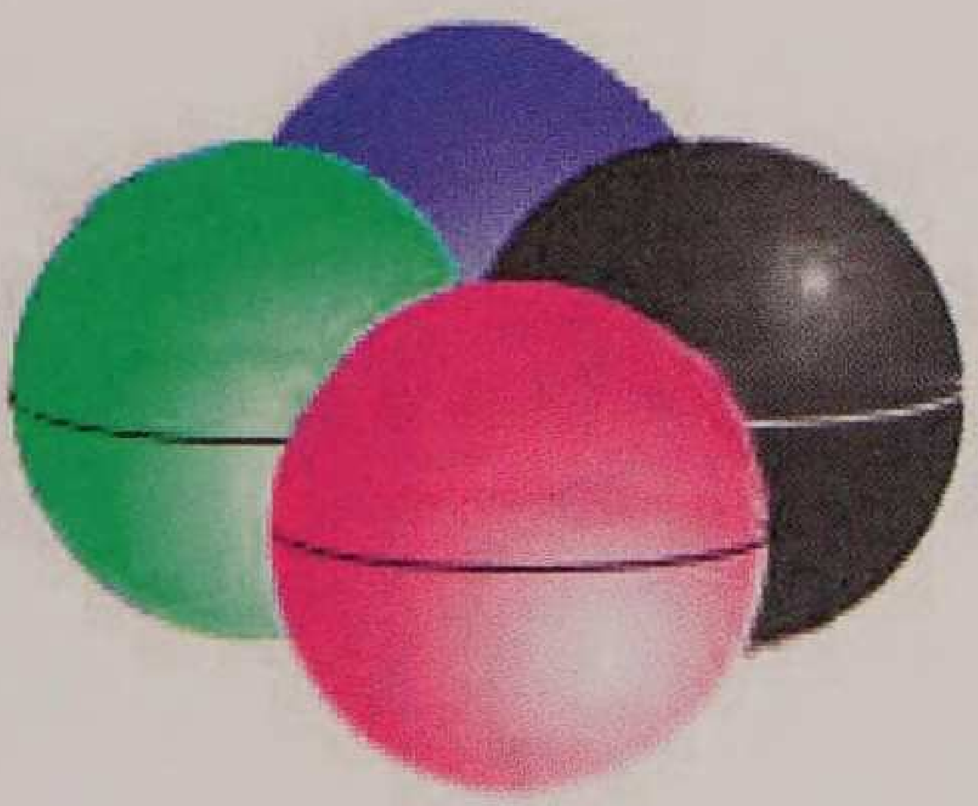
This slide depicts the intersecting embedded hemispheres of the dual quadracube.



	1				2				3				4					
Dual 4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	160	195	54	105	148	199	58	109	156	207	50	101	152	203	62	97	2056	
	124	47	210	133	128	35	214	137	120	43	222	129	116	39	218	141	2056	
	72	27	238	177	76	31	226	181	68	23	234	189	80	19	230	185	2056	
	164	247	10	93	168	251	14	81	176	243	6	89	172	255	2	85	2056	8224
	110	49	200	155	98	53	204	159	106	61	196	151	102	57	208	147	2056	
	138	221	36	119	142	209	40	123	134	217	48	115	130	213	44	127	2056	
	182	233	32	67	186	237	20	71	178	229	28	79	190	225	24	75	2056	
	82	5	252	175	86	9	256	163	94	1	248	171	90	13	244	167	2056	8224
	111	52	197	154	99	56	201	158	107	64	193	150	103	60	205	146	2056	
	139	224	33	118	143	212	37	122	135	220	45	114	131	216	41	126	2056	
	183	236	29	66	187	240	17	70	179	232	25	78	191	228	21	74	2056	
	83	8	249	174	87	12	253	162	95	4	245	170	91	16	241	166	2056	8224
	157	194	55	108	145	198	59	112	153	206	51	104	149	202	63	100	2056	
	121	46	211	136	125	34	215	140	117	42	223	132	113	38	219	144	2056	
	69	26	239	180	73	30	227	184	65	22	235	192	77	18	231	188	2056	
	161	246	11	96	165	250	15	84	173	242	7	92	169	254	3	88	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	

	522	
	506	
	522	
514	506	514
514	506	514
514	506	514
514	522	514
514	522	514
514	4112	514
514		514
514	522	514
4112	522	4112
	506	
	506	
	506	
	506	
	522	
	522	
	4112	

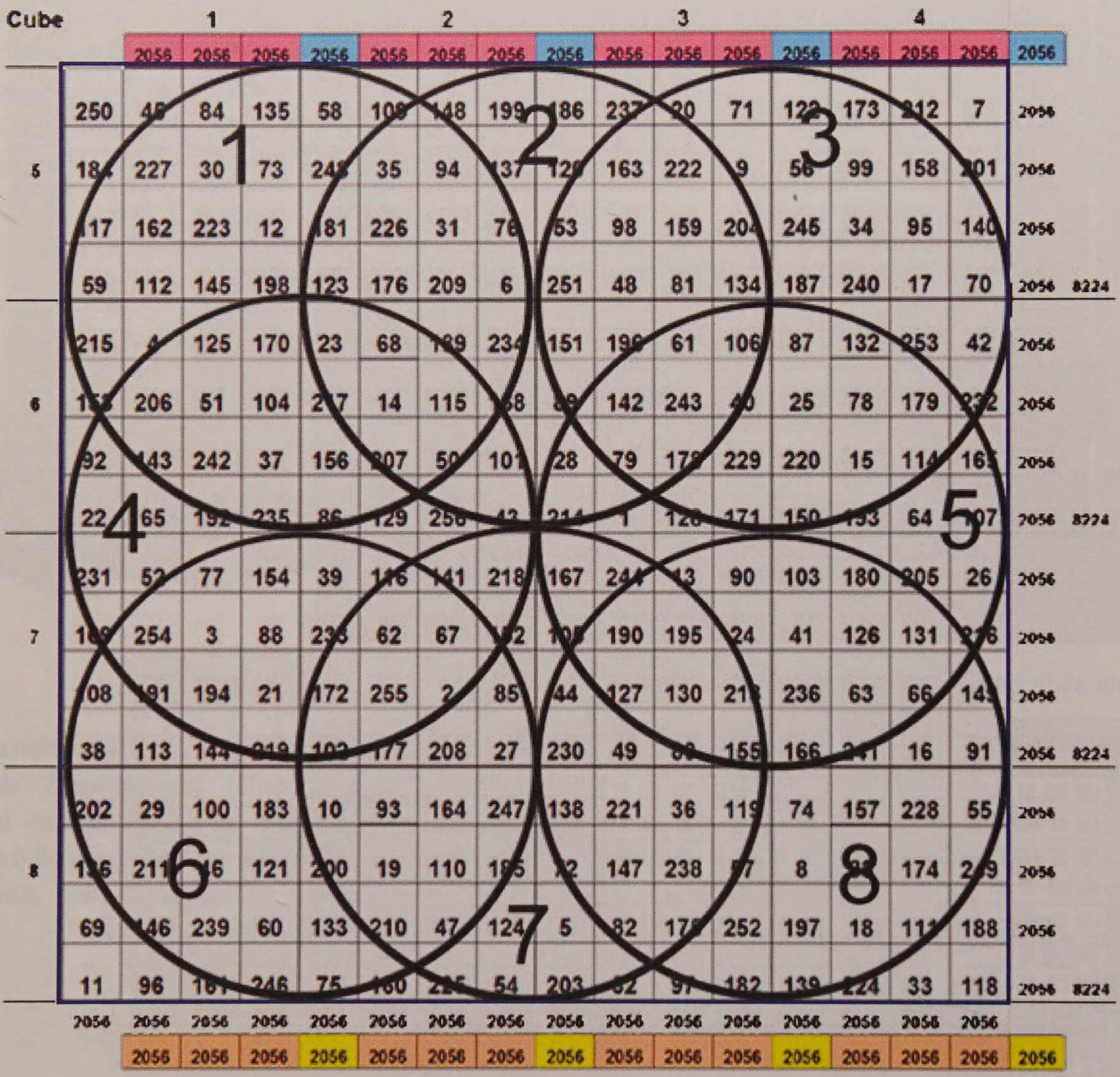
Only in the two horizontal hemispheres do all the quadrals sum to **514**. Nonetheless all the hemispheres do sum equally to **4112**.



Program 6

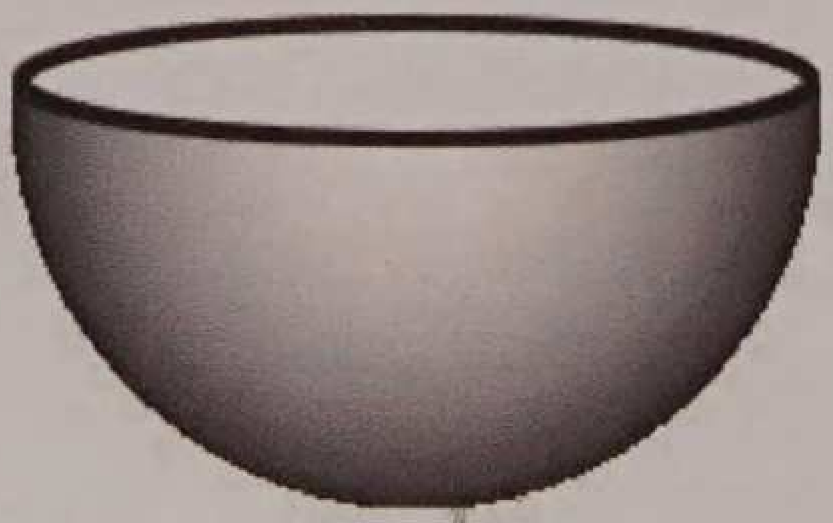
All the Hidden Hemispheres in Both the Primal and Dual

Here are all **8** hemispheres that lay hidden in the primal quadracube. The same pattern exists in its dual quadracube too.



Cube			1				2				3				4			
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	
	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056	
5	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056	
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056	
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056	8224
	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056	
6	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056	
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056	
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056	8224
	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056	
7	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056	
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056	
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056	8224
	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056	
8	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056	
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056	
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	

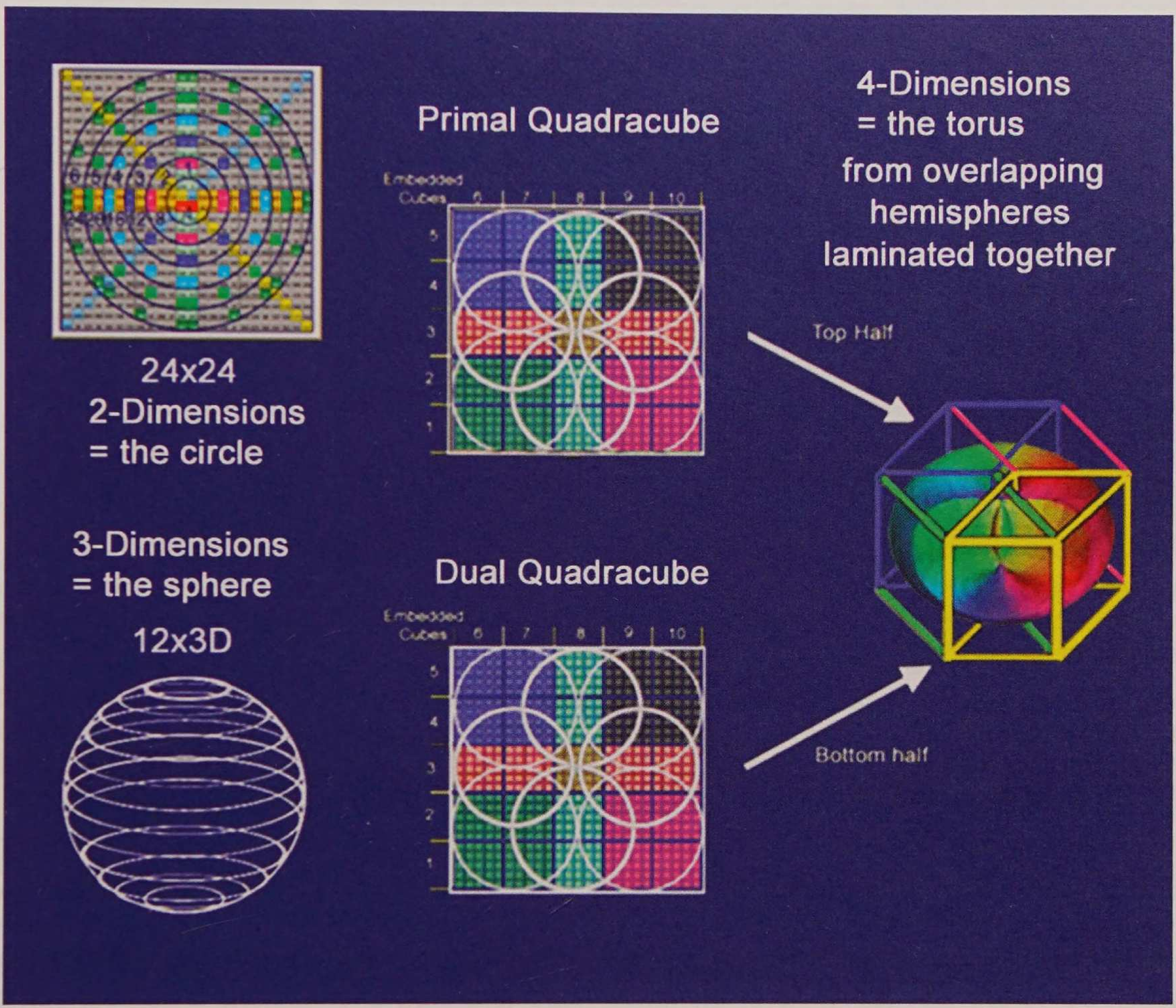
Each circle represents a 3D characteristic hemisphere.



Program 6

The Characteristic Torus

This slide depicts the complete pattern when the table of the size-4 quadracube's primal version is laminated to its dual version. We get a torus in 4-dimensional space!



24x24
2-Dimensions
= the circle

Primal Quadracube

4-Dimensions
= the torus
from overlapping
hemispheres
laminated together

3-Dimensions
= the sphere

12x3D

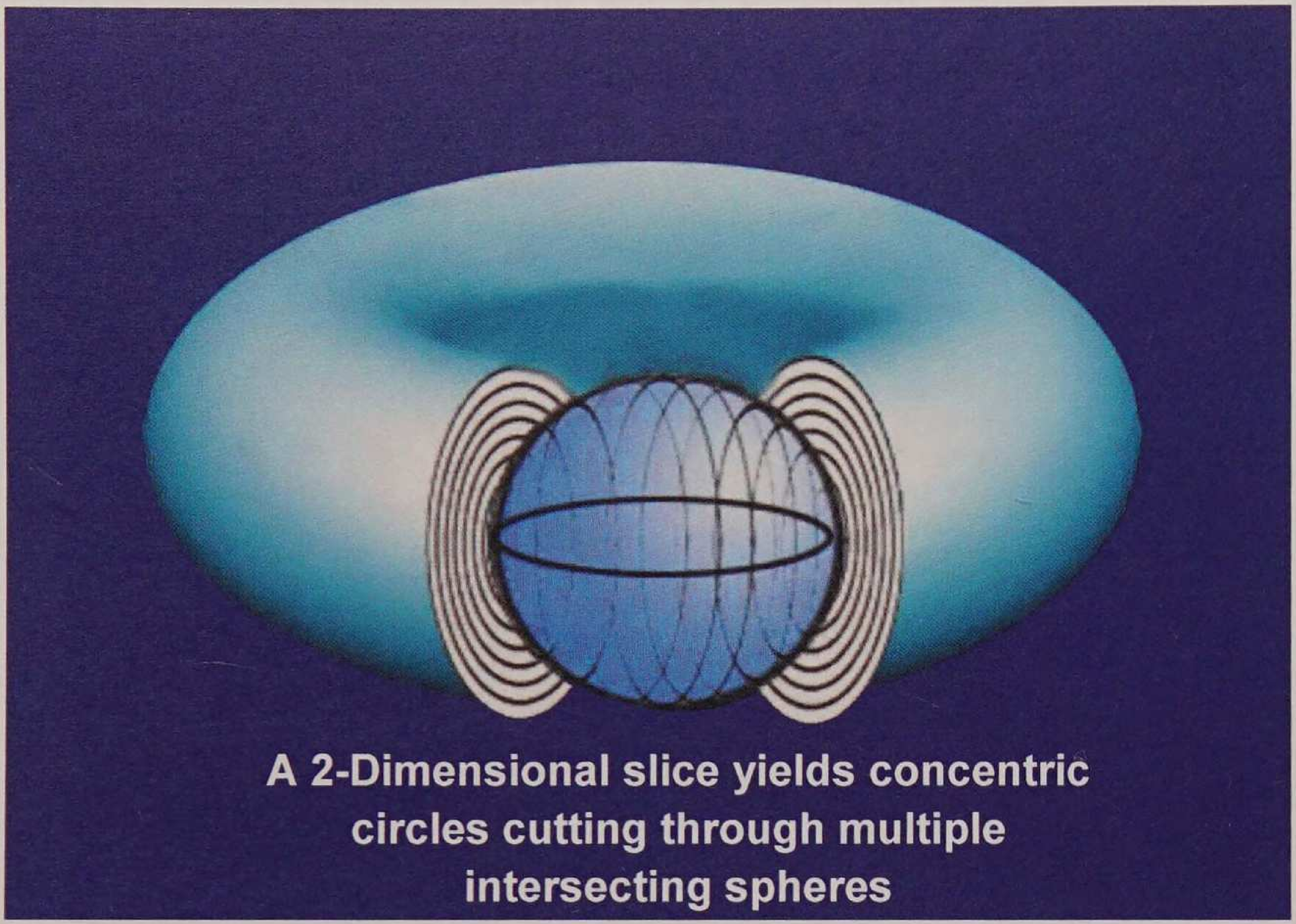
Dual Quadracube

In squares we had the characteristic circle; in cubes we had the characteristic sphere; and now in quadracubes we have the characteristic torus.

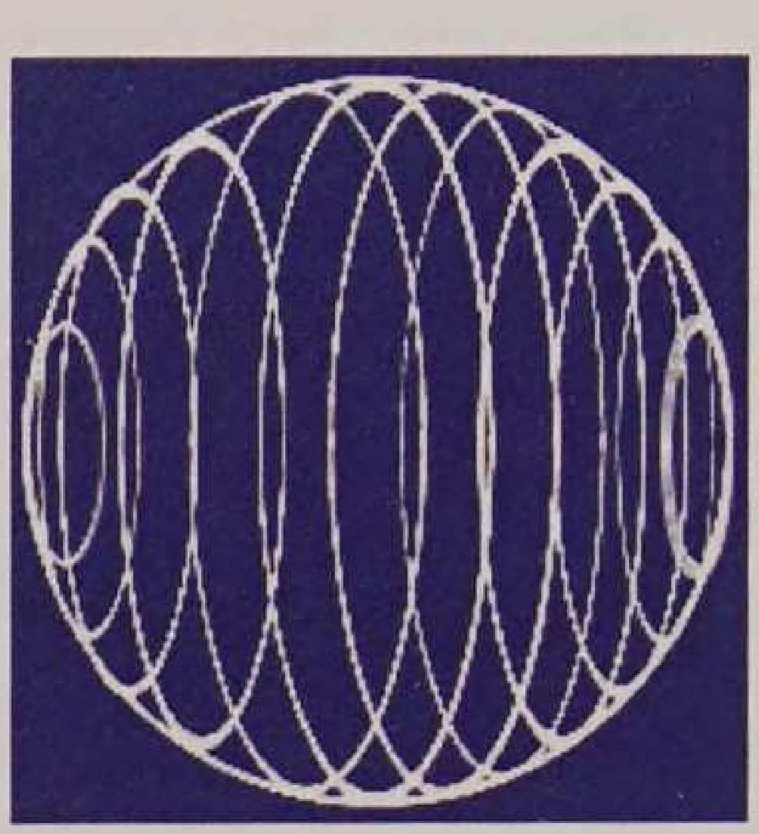
This slide shows the torus resulting from laminating together the complementary loom tables which lie at the heart of the torus embedded in the two dual quadracubes.

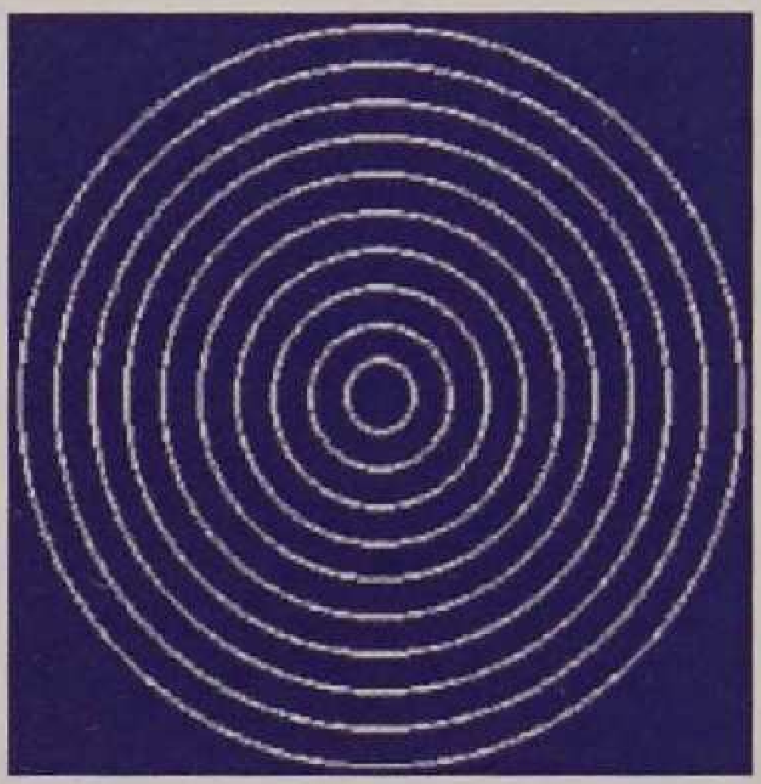
Program 6

A 3-Dimensional sectional cutout yields a sphere



A 2-Dimensional slice yields concentric circles cutting through multiple intersecting spheres



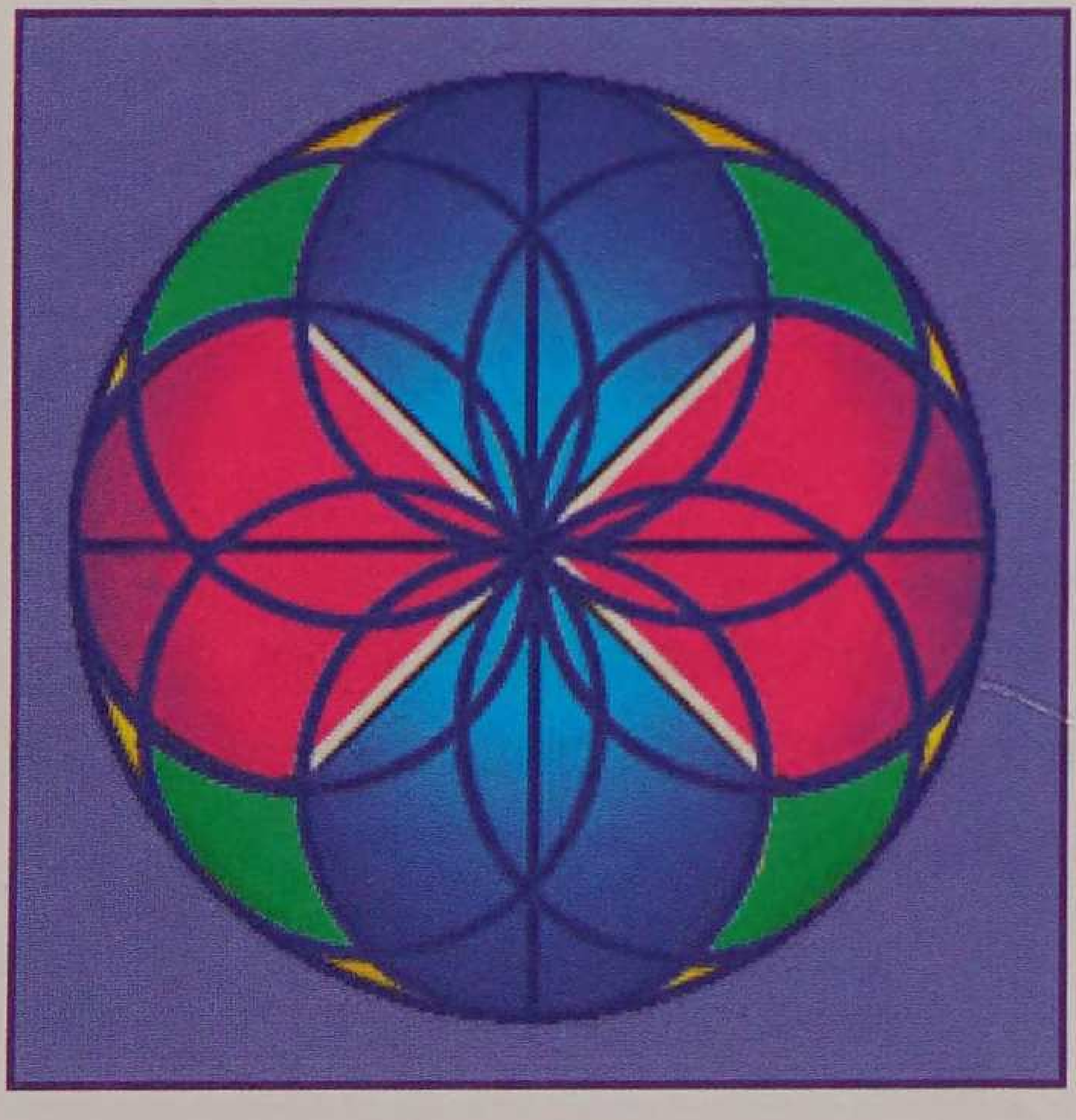


This slide shows that a cubic section cut from a torus is a sphere, composed of 2 hemispheres. The planar slice is a series of nested concentric circles, each one from different intersecting embedded spheres.

So we have now seen how we first discovered the characteristic circle in squares, then characteristic spheres in cubes, and now characteristic toruses in quadracubes. This establishes that just as 2-dimensional space is circular and 3-dimensional space is spherical, 4-dimensional space is toroidal !

As such, only two of those figures are **isotropic** – that is, the spaces measure out equally in all directions. The torus clearly does not measure out equally; its outer rim is way longer than its inner one. So a single torus is not the whole picture.

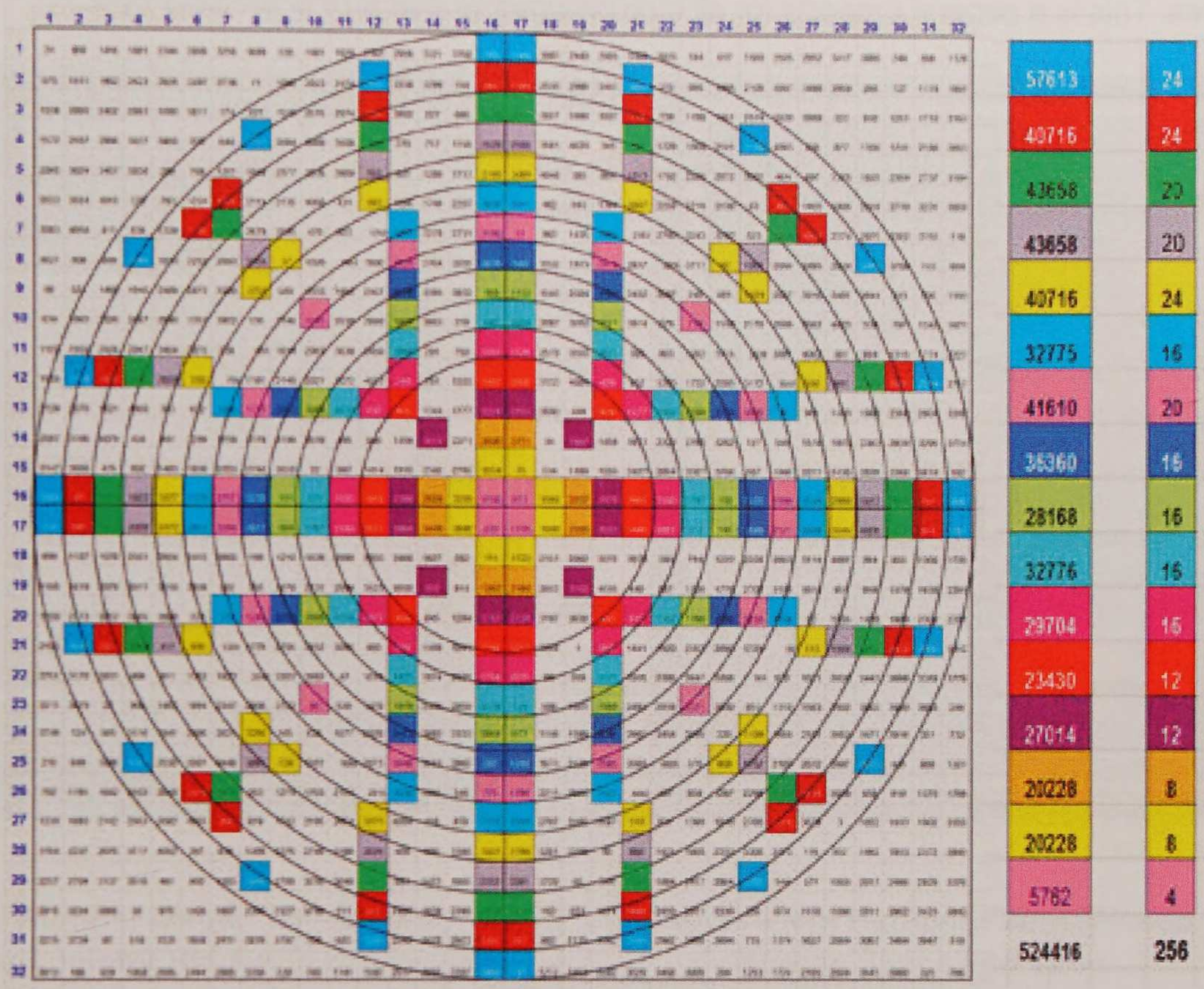
A picture of 4-dimensional space from perspective of 5D



It will be seen in Program 9 that in dimensions greater than **2D** that there is more than one dual. In **4D**, specifically, there are 4 quadracubes that are related to each other through a circular series of **4** connections. The **4** quadracubes taken **2** at a time in adjacent circular series to create complete spheres via two-fold laminations yields **4** toroidal spaces in sequence. Each toroidal space intersects the other three. It amounts to four donuts with very small center holes intersecting each other at uniformly-spaced 3-dimensional angles as seen here.

Here is what the entire 4-dimensional space looks like from the perspective of 5-dimensional space as determined by Geonometry.

Size-8 Quadracube's Hemispherical Pattern



57613	24
40716	24
43658	20
43658	20
40716	24
32775	16
41610	20
35360	16
28168	16
32776	16
29704	16
23430	12
27014	12
20228	8
20228	8
5762	4
524416	256

Shown above is the pattern which sums to half the characteristic number for each 3-dimensional quadrant of the size-8 quadracube. The size-8 quadracubic table is way too large to depict all the numbers. It would take **eight** pages of **32x32** tables to demonstrate the differently located **8** hemispheric patterns, each summing to **524,416**, half the characteristic number, **1,048,832**. The pattern involves **256** cells as depicted by the count strip on the right of the table.

Some fundamental calculations will confirm the practicality of this pattern of **256** numbers thus avoiding the display of all the **4000+** numbers.

The total sum of the numbers in the entire **64x64** quadracubic table is the sum of the series of from **1** through **4096** consecutive numbers; that equals **8,390,656**. So the dimensional average is **8,390,656 / 4096 = 2048.5**.

Observe that **524,416 / 256 = 2048.5** too. Consequently, a pattern of **256** select cells summing to half the characteristic number in each selected quadrant or 32x32-midsection of the quadracubic table is clearly doable.

Program 6

So just as for the **4x4D** quadracube, the table of the quadracube's dual is necessary to complete the 4-dimensional characteristic torus composed again of **8** complete intersecting spheres. This is a general property for all quadracubes whose size is divisible by **4**.

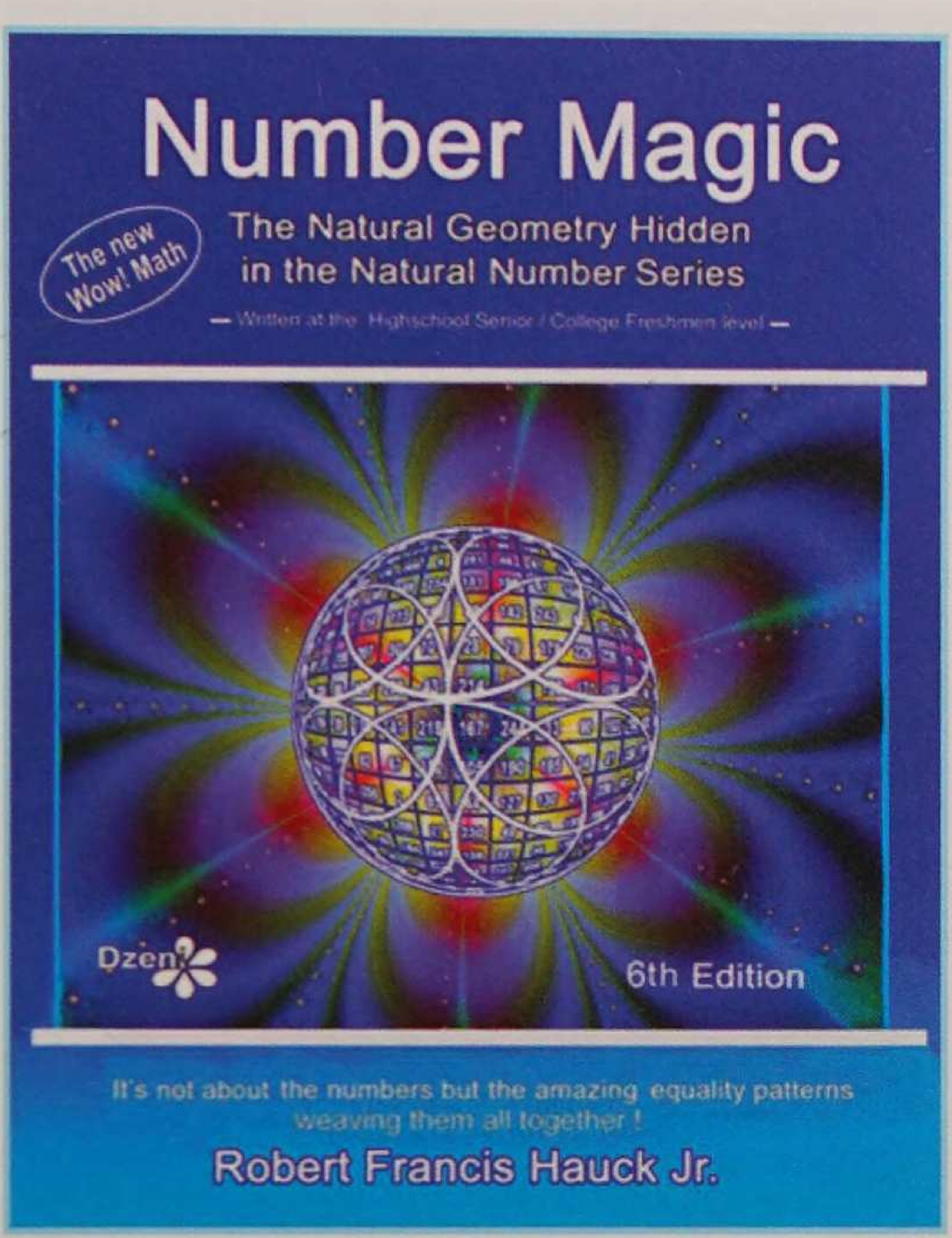
Notes

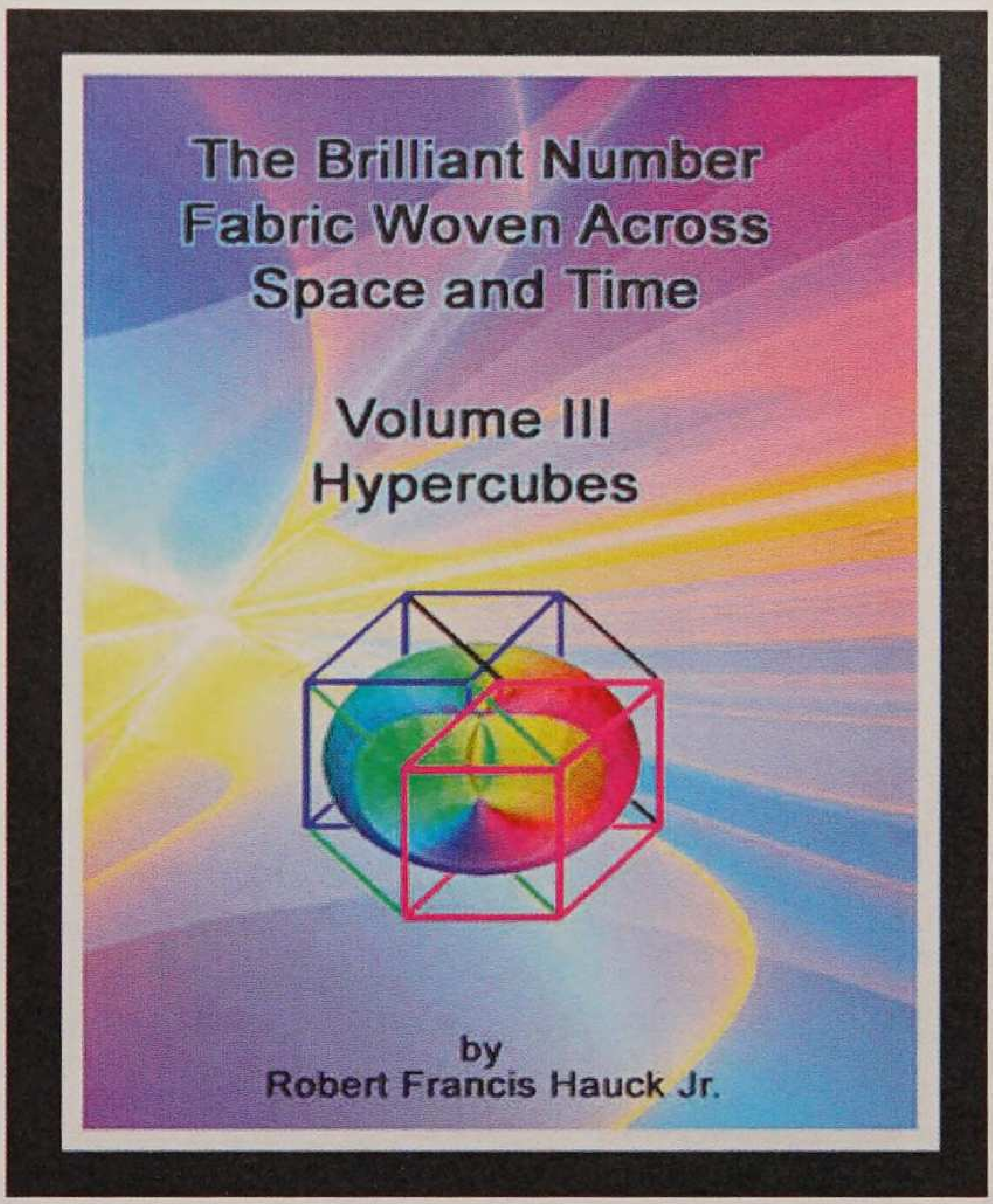
1. A quadracube of size **n** is composed of **2n** embedded cubes, half of which are vertically oriented and half which are oriented horizontally.
2. Each embedded cube sums to the quadracube's characteristic number.
3. A quadracube of size **n** is composed of $\mathbf{n}^2$ embedded block squares, each of which sums equally to **1/n**-th of the quadracube's characteristic number.
4. The centers of all embedded cubes together sum to **1/n**-th of the quadracube's characteristic number.
5. A quadracube will collapse to a perfect geonomic cube along both its **B** and **D** axes.
6. Quadracubes of size **4x** with equal-summing pairwise row patterns possess **8** characteristic hemispheres. Laminated together with one of their **3** dual quadracubes produces a 4-dimensional characteristic torus. These special quadracubes can lend insight into structures within sub-dimensional spaces **1/3D** and **1/4D**.

Program 6

We have come to the end of Program 6.

Here are the books upon which this program was based.





Number Magic The Natural Geonometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension.

Eighth Edition (350+ pages)

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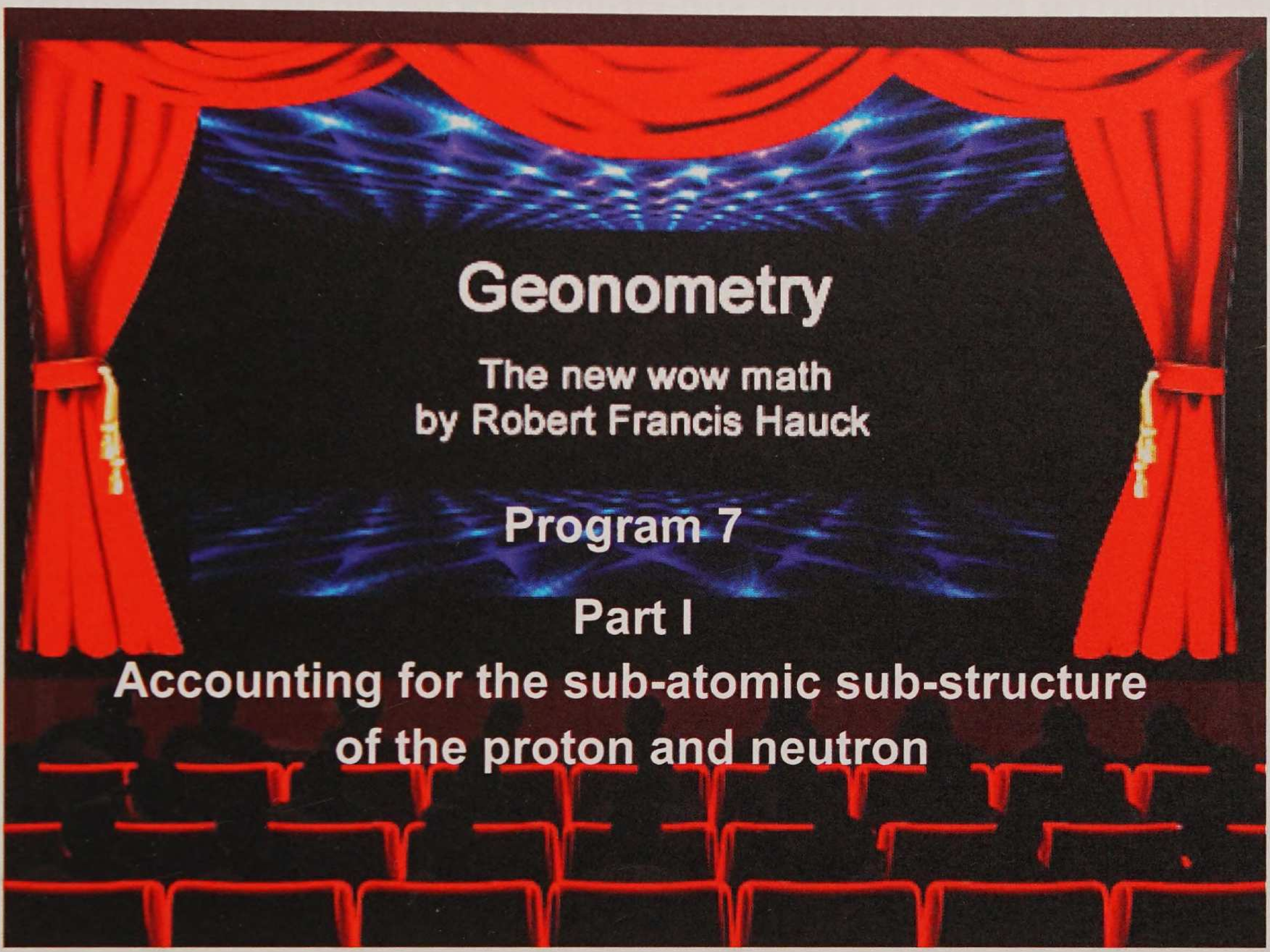
ISBN: 978-1-461-08073-2

Deals only with 4- and 5-dimensional hypercube that are legible in print.

Third Edition (74 pages)

In the next program we're going sub-dimensional once again, but this time all the way down to the basement **1/5D** were we will encounter the particles that make up **quarks** in **1/4D**!

Program 7



Geonometry

The new wow math
by Robert Francis Hauck

Program 7

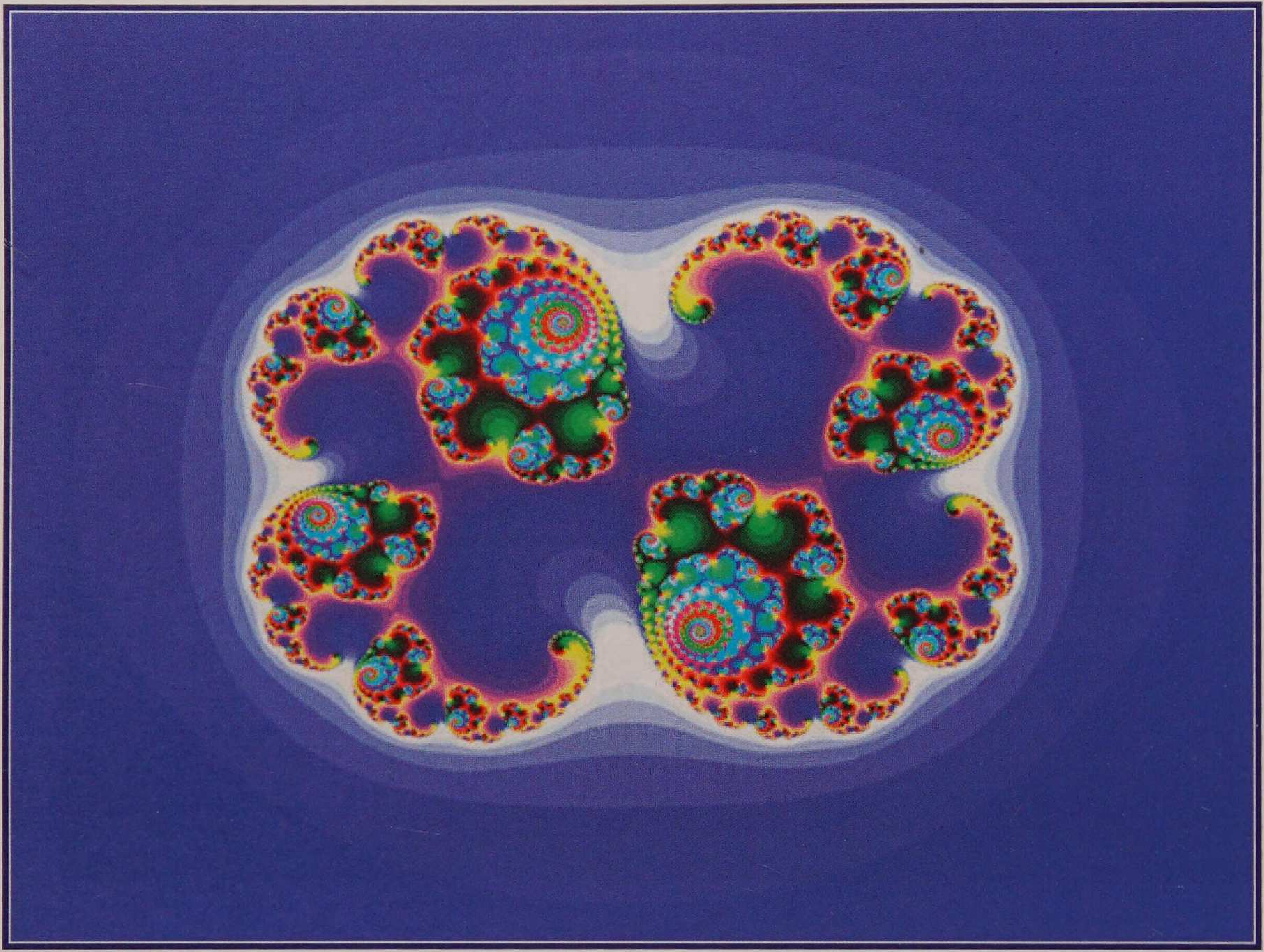
Part I
Accounting for the sub-atomic sub-structure
of the proton and neutron

In this first Part we will be demonstrating how the fundamental properties of 4-dimensional space that we've seen in the prior program can be applied in the exploration of sub-dimensional space **1/4D**. It's quite a mind-trip!

Program 7

Application #3

Accounting for the sub-atomic sub-structure of the proton and neutron



We have seen that the pattern properties observed in the 2nd dimension led to an interpretation of DNA molecular formation in the sub-dimension of **1/2D**, and the 3rd dimension led to a mathematical interpretation of the electron distribution in atoms at the sub-dimension **1/3D**. Having now observed the properties of the 4th dimension, we'll be investigating the quadracube's properties at the sub-dimensional level, **1/4D**.

Program 7

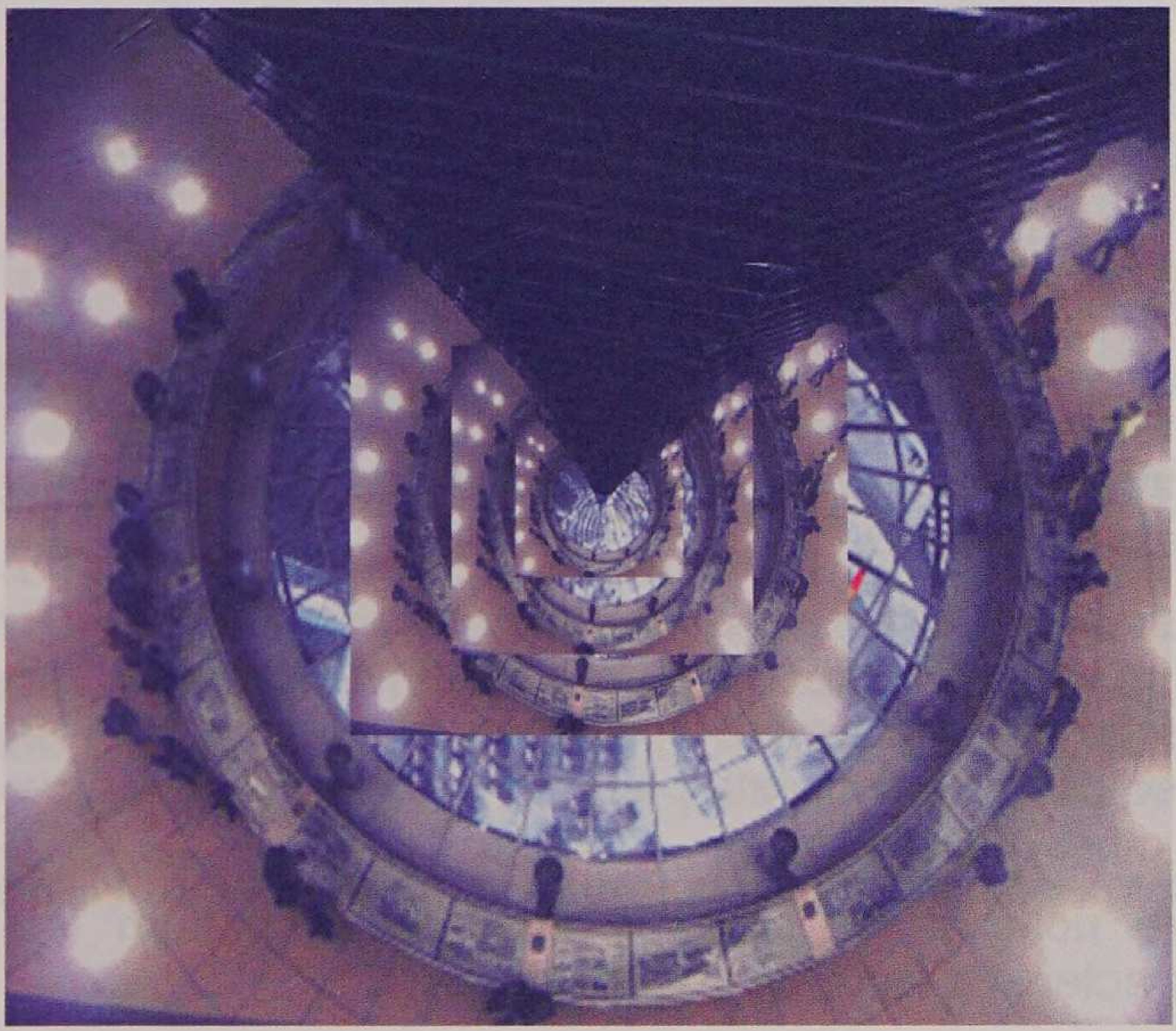
Dimensional Levels

The table here lists descriptive examples of the objects observed at the various levels of multi-dimensional and sub-dimensional space. Of course, with the current state of Man's knowledge, objects in 4-dimensional space can only be conjectured about -- such as has been done here.

Table of Dimensional Levels

Dimensions	Description (examples)
4	Hyperspace (multiple 3D universes)
3	Cubic space
2	Planar space
1	Linear space
1/2	Molecular level
1/3	Atomic level
1/4	Particle level (electrons, protons, neutrons, baryons)
1/5	Sub-particle level (quarks, leptons, hadrons)

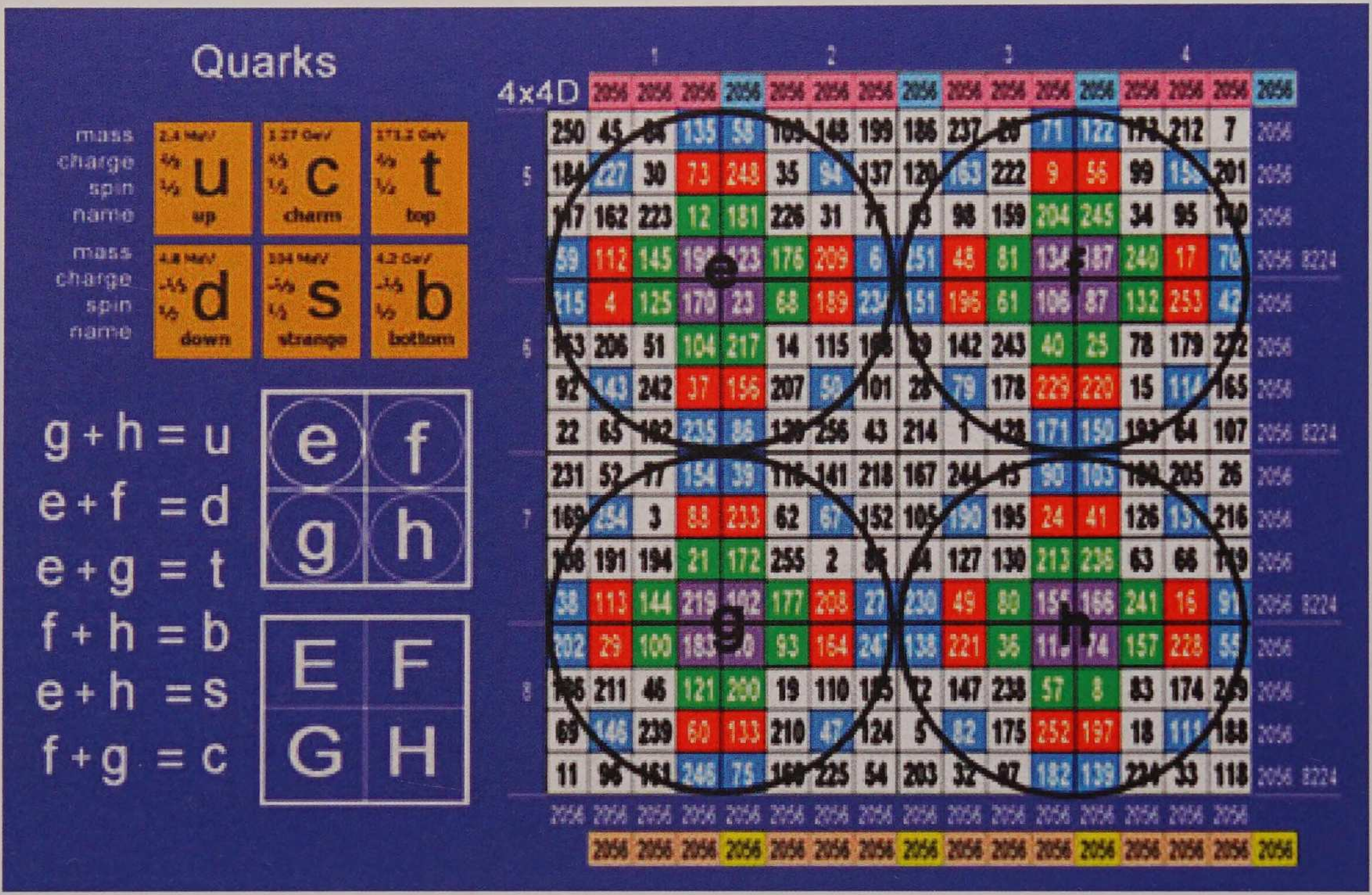
Next we're going down the "wellevator" to the basement of the universe at level **1/5D** and see what we can fathom by applying what we just learned from 4-dimensional Geonometry.



Program 7

Quarks

The particles found at this subdimensional level are called quarks. A quark is an elementary particle and a fundamental constituent of matter. Quarks combine to form composite particles called baryons, the most stable of which are protons and neutrons, the basic components of nuclei of the atoms that we have already encountered at the sub-dimensional level **1/3D**. It has been determined that there are six types of quarks, labeled **u & d, s & c,** and **t & b**, standing for **up & down / strange & charm / top & bottom**, respectively. We'll see shortly just how the physicists arrived at these names.



g + h = u
e + f = d
e + g = t
f + h = b
e + h = s
f + g = c

4x4D	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056	
5	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056	
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056	
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056	8224
	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056	
6	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056	
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056	
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056	8224
	231	52	77	154	39	116	141	218	167	244	45	90	103	180	205	26	2056	
7	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056	
	108	191	194	21	172	255	2	85	84	127	130	213	236	63	66	149	2056	
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056	8224
	202	29	100	183	[illegible]	93	164	247	138	221	36	11	74	157	228	55	2056	
8	[illegible]	211	46	121	200	19	110	[illegible]	[illegible]	147	238	57	8	83	174	249	2056	
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056	
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

The heavier quarks rapidly change into **up** and **down** quarks through a process of particle decay in an automatic transformation from a higher mass state to a lower mass state. Because of this, **up** and **down** quarks are generally stable and consequently are the only ones found in the nuclei of atoms; the other four quarks are transient and only encountered in particle colliders.

We have already seen that the size-4 quadracube had **4** independent hemispheres embedded within its table. These hemispheres have been labeled with the small letters **e f g & h.** These **4** quadrants, each consisting of just one hemisphere, are identified with corresponding capital letters **E F G & H**.

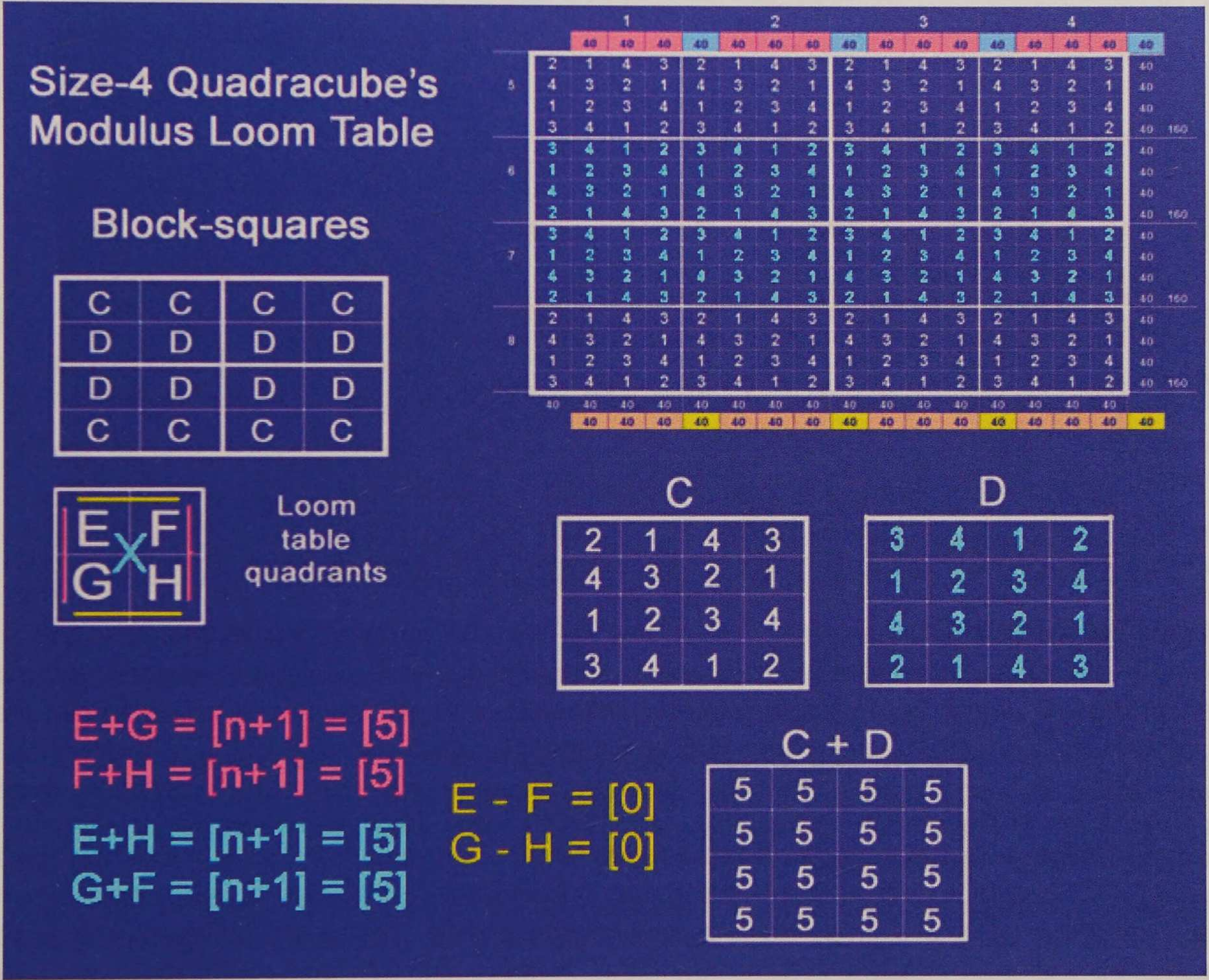
These hemispheres may be combined in **6** different combinations to form complete spheres in which all their impinging octals sum to the quadracube's characteristic number, as shown. Whichever hemisphere correlates to which quark is immaterial at the moment. Nonetheless these patterns correlate perfectly to the stability of the neutron and proton and the instability of the other 4 baryons.

Take note of the six combinations at bottom left of the four hemispheres which form characteristic spheres in 4-dimensions.

Program 7

Size-4 Quadracube's Modulus Loom table

This is the **modulus loom table** for the size-4 quadracube. Note that it has only **2** distinct embedded size-4 squares, labeled here as **C** & **D.** This modulus table may be represented in terms of these block-squares as depicted at left. The corresponding quadrants of the loom table, which contain the characteristic hemispheres, are denoted again here by the capital letters **E F G** & **H**.



Size-4 Quadracube's Modulus Loom Table

	1				2				3				4					
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	
	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40	
5	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40	
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40	
	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40	160
	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40	
6	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40	
	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40	
	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40	160
	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40	
7	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40	
	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40	
	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40	160
	2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3	40	
8	4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1	40	
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	40	
	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	40	160
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40		
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	

Block-squares

C	C	C	C
D	D	D	D
D	D	D	D
C	C	C	C

Loom table quadrants

C

2	1	4	3
4	3	2	1
1	2	3	4
3	4	1	2

D

3	4	1	2
1	2	3	4
4	3	2	1
2	1	4	3

E+G = [n+1] = [5]
F+H = [n+1] = [5]

E+H = [n+1] = [5]
G+F = [n+1] = [5]

E - F = [0]
G - H = [0]

C + D

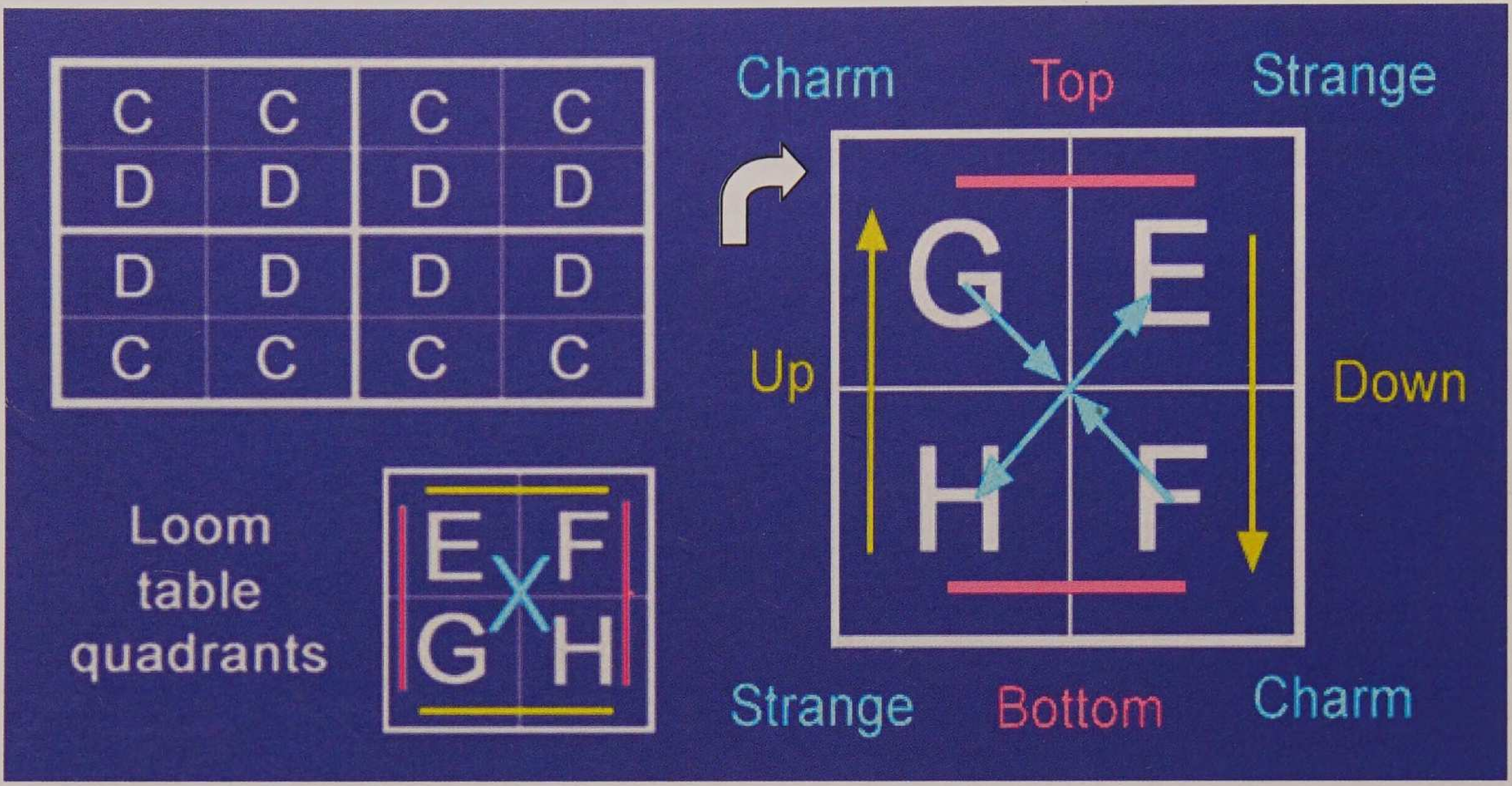
5	5	5	5
5	5	5	5
5	5	5	5
5	5	5	5

Now observe that the four **addition** combinations of these quadrants, as expressed in the lower left corner, yield matrices of all **5**'s. And 2 **difference** combinations next to them yield matrices of all **0**'s. Those combinations which correspond to the matrix of all **5**'s are those with different **complementary** loom table patterns. Those combinations which correspond to the matrix of all **0**'s are those with **identical** loom table patterns. This shows that of the **6** possible combinations of hemispheres, **2** are **identical** and **4** are **complementary.**

These patterns correlate perfectly to the stability of the neutron and proton and the instability of the other 4 baryons.

Program 7

100% correlation with the atomic physicists' model and descriptions



By rotating the diagram 90° clockwise, the model derived here exhibits a100% correlation with the atomic physicists' model and descriptions.

Note that **G & H** and **E & F** are the only two combinations that are composed from quadracubic quadrants with identical numerical patterns. They now correspond to the "**up**" and "**down**" quarks. And it is these two quarks that make up the two baryons with the least mass and the most stability: the proton and neutron.

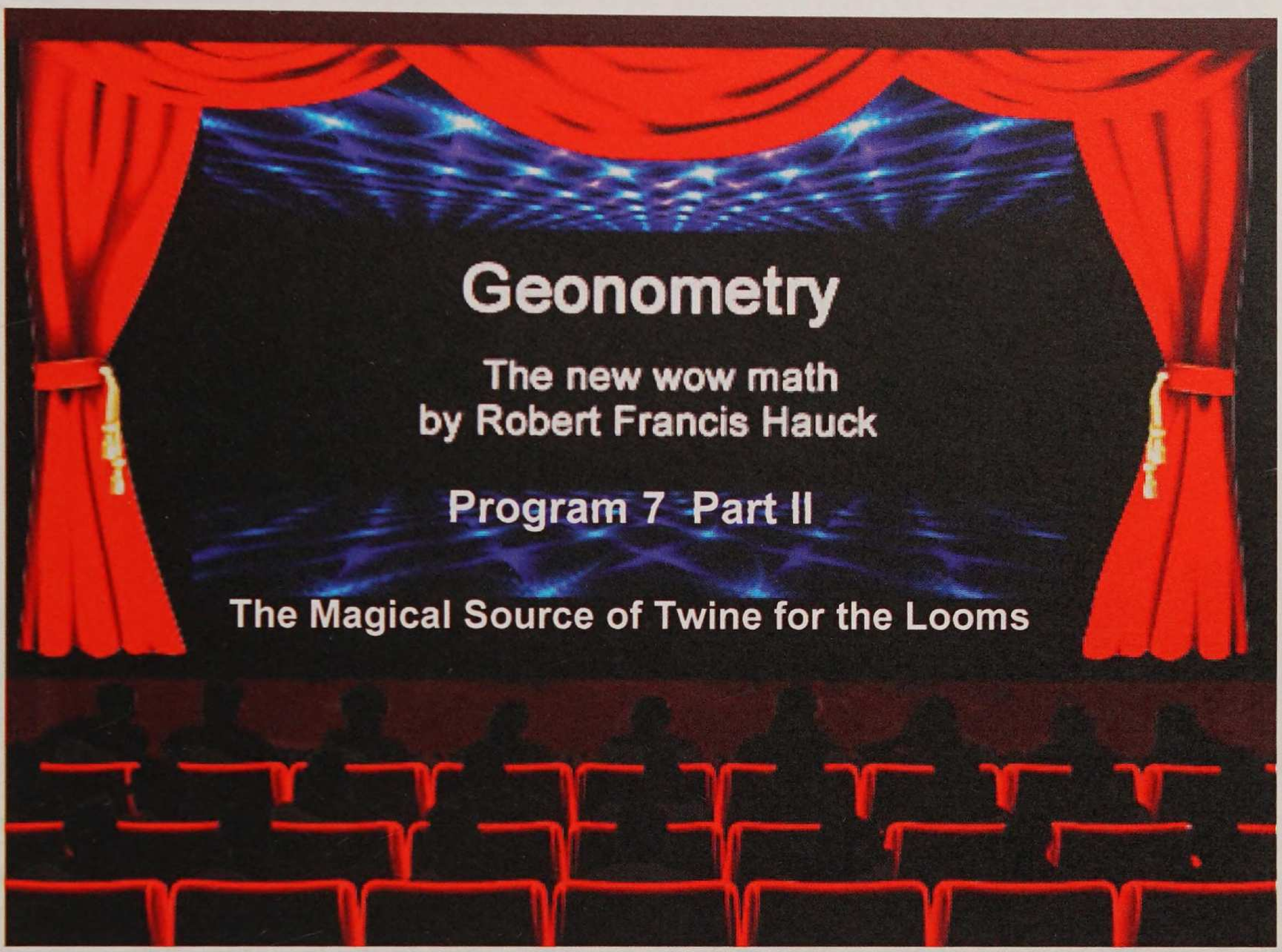
The combinations of **E** & **H** repel each other and are strangers. The combinations of **F** & **G** are attracted to each other because they possess charm.

The combinations **G** & **E** just happen to be at the top of the relationship box and **H** & **F** just happen to be at the bottom of the box when the particle physicists were organizing the properties into a 4-quadrant relationship box. That's how they got their quirky names: **up down / strange charm / top bottom**.

Recall that these quadrants correspond to characteristic hemispheres in **4D** space for both loom tables **X** & **Y** and for both primary quadracubes **W** & **U** simultaneously. All this cannot be just coincidental! This demonstrates that the properties uncovered in **4D space** have fundamental application for comprehending the basic properties at the **1/4D** sub-dimensional level!

So once again Geonometry has supplied the math with which to explain the properties of basic structures, from one of a higher dimensional level to one of a lower level corresponding to the inverse sub-dimension.

Program 7



Geonometry

The new wow math
by Robert Francis Hauck

Program 7 Part II

The Magical Source of Twine for the Looms

In this segment of the program, we will explore a completely new type of square whose properties are truly magical. This type of square allows for duplication of numbers but in a very specific normalized manner. They are called **Matchmaker squares**. What you will discover in this program is a fundamental relationship unknown in Matrix Theory prior to the release of this program series.

Caution: The matchmaker square is what is categorized in mathematics as recreational math. It is shown here to exhibit its fundamental connection to Geonomical math. This correspondence between these two distinct categories of math should not be used to lump Geonometry in with recreational math to taint Geonometry as being merely a topic of curiosity.

The Source of Origin of the Matchmaker's Square

Of all the squares dubbed "magic", the matchmaker's square is truly magic. It wasn't invented as such; it was discovered by the author quite by accident while trying to contrive an example to demonstrate the effectiveness of a scheduling algorithm that he developed for National Semiconductor Corporation to maximize production priorities. The matrix you will see here is the actual table of priorities used in that intended demonstration. It was never used because to his stunning surprise, he kept getting the same priority answer in the quest of maximizing the total priorities no matter in which order the items were to be scheduled.

Geonometry is the mathematics of the **fabric of space**. The matchmaker square is the essential basic source of all the Class-1 squares and, whether you believe it or not, is the **twine** used by the *quadralooms* of Lucasfilm *Stuntworks Division* to create the **fabric of space** as you will now see for yourself.

Program 7

The Five Basic Properties of the Matchmaker's Magic Square

9x9	1	2	3	4	5	6	7	8	9
1	9	4	11	6	3	8	5	10	7
2	14	9	16	11	8	13	10	15	12
3	11	6	13	8	5	10	7	12	9
4	12	7	14	9	6	11	8	13	10
5	7	2	9	4	1	6	3	8	5
6	10	5	12	7	4	9	6	11	8
7	15	10	17	12	9	14	11	16	13
8	8	3	10	5	2	7	4	9	6
9	13	8	15	10	7	12	9	14	11

A matching in a matchmaker's square is a selection of a different column for every distinct row and summing the numbers in the cells at their intersections.

Here is an example of a size-9 numeric table of what is called the matchmaker's magic square.

It has these **5** properties:

1. **The Dimensional Average Property:** All the numbers average to its size **n**, here **9**.
2. **The Matchings Property**: All *matchings* sum to the same number, $\mathbf{n^2}$; here **81**.
3. **The Totality Property**: All the numbers in the square sum to $\mathbf{n^3}$**,** here **729**.
4. **The Subset Property**: Any smaller square, called an **abbreviated square**, taken from anywhere in the table has all possible *matching*s summing to the same number. This number is different for different abbreviated squares. Moreover, these squares need not be composed of numbers from adjacent rows and columns -- any number of **m** independent rows and **m** independent columns may be selected at random and used to make an abbreviated **m-by-m** square from the cells common among their intersections -- of course keeping their places in the abbreviated square relative to the bigger square. And every *matching* within the abbreviated square will sum to the same number! This is just an amazing property that derives from both the numbers and the space in which they are strewn, that is, their underlying organized pattern.
5. **The Invariance-under-permutation Property**: The rows and columns may be swapped one pair at a time, that is, rearranged into whatever order you like, by interchanging the position of any two rows or any two columns, any number of times in series, like spinning a Rubric's Cube, and the square will still have the same properties #1 thru #4 ! Formally, the matchmaker's square is said to be **invariant under column and row permutation**.

Note: Properties #4 & #5 are proven as *theorems* in the Appendix of the book, **Number Magic**.

Program 7

Characteristic number for the abbreviated square = **41.**
All **24** (4x3x2) possible matchings sum to **41**

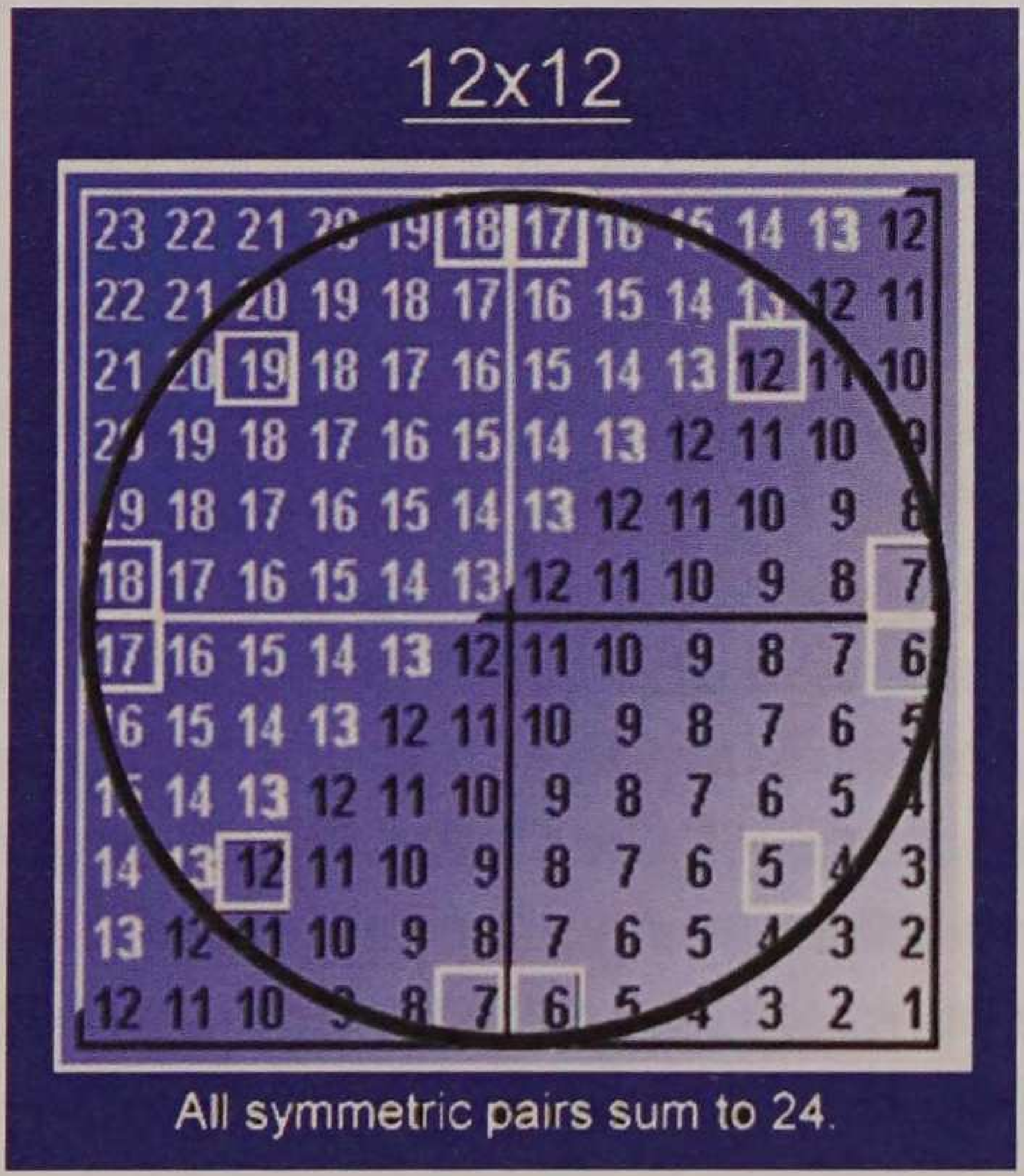
	Columns			
Rows	3	5	7	9
1	11	3	5	7
4	14	6	8	10
7	17	9	11	13
9	15	7	9	11

The Size-9 mm-Square Normalized

9x9	81	81	81	81	81	81	81	81	81	
	17	16	15	14	13	12	11	10	9	
	16	15	14	13	12	11	10	9	8	
	15	14	13	12	11	10	9	8	7	
	14	13	12	11	10	9	8	7	6	
	13	12	11	10	9	8	7	6	5	81
	12	11	10	9	8	7	6	5	4	
	11	10	9	8	7	6	5	4	3	
	10	9	8	7	6	5	4	3	2	
	9	8	7	6	5	4	3	2	1	
					81					
	81	81	81	81	81	81	81	81	81	

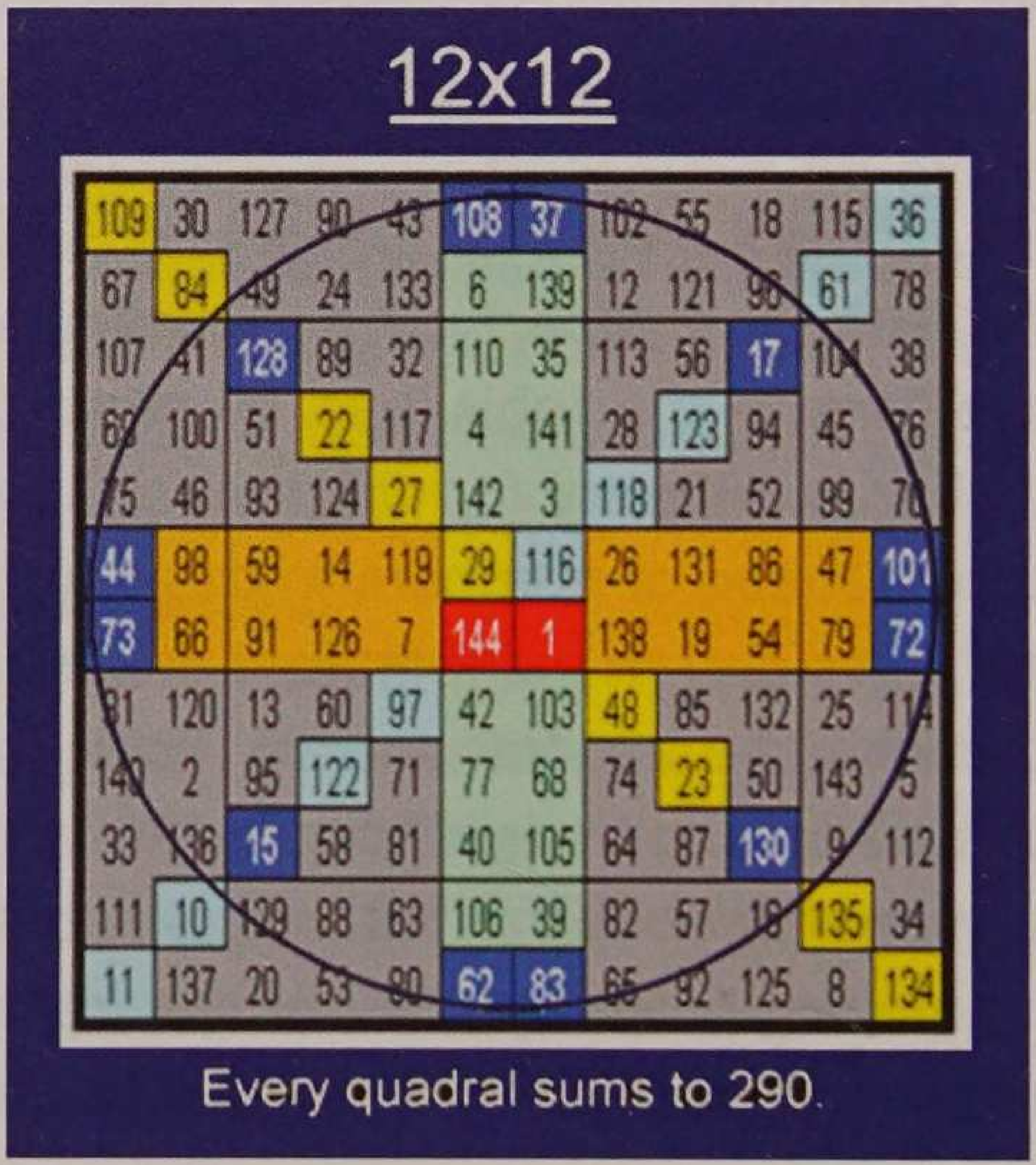
Here is the size-9 matchmaker's square unscrambled. Note the obvious number pattern. When the numbers are laid out in this fashion, the square is said to be **normalized**.

Comparison of Characteristic Circles between Class-4 Regular Squares and Matchmaker Squares of the same size


12x12

23	22	21	20	19	18	17	16	15	14	13	12
22	21	20	19	18	17	16	15	14	13	12	11
21	20	19	18	17	16	15	14	13	12	11	10
20	19	18	17	16	15	14	13	12	11	10	9
19	18	17	16	15	14	13	12	11	10	9	8
18	17	16	15	14	13	12	11	10	9	8	7
17	16	15	14	13	12	11	10	9	8	7	6
16	15	14	13	12	11	10	9	8	7	6	5
15	14	13	12	11	10	9	8	7	6	5	4
14	13	12	11	10	9	8	7	6	5	4	3
13	12	11	10	9	8	7	6	5	4	3	2
12	11	10	9	8	7	6	5	4	3	2	1

All symmetric pairs sum to 24.


12x12

109	30	127	90	43	108	37	102	55	18	115	36
67	84	49	24	133	6	139	12	121	96	61	78
107	41	128	89	32	110	35	113	56	17	104	38
69	100	51	22	117	4	141	28	123	94	45	76
75	46	93	124	27	142	3	118	21	52	99	70
44	98	59	14	119	29	116	26	131	86	47	101
73	66	91	126	7	144	1	138	19	54	79	72
31	120	13	60	97	42	103	48	85	132	25	114
140	2	95	122	71	77	68	74	23	50	143	5
33	136	15	58	81	40	105	64	87	130	9	112
111	10	129	88	63	106	39	82	57	16	135	34
11	137	20	53	80	62	83	65	92	125	8	134

Every quadral sums to 290.

Here are the characteristic circles of both the matchmaker's square of size-12 and the regular size-12 square. They are shown side-by-side for direct comparison.

Observe that in both cases, eventhough both of these tables and their characteristic numbers are different, the **12** impinging cells of the largest inscribed circle in each table sum to each square's distinct characteristic number. This one-to-one correspondence between matchmaker squares and regular geonomic squares holds for every size of square in Class-4. This is another amazing discovery of Geonometry.

Matchmakers Magic Cube

Here is the size-5 matchmaker's cube.

Its three depth-sum tables depicted at left are all identical.

Observe that the cube appears as being numerically identical along each of the three axial directions as viewed horizontally from the left and behind, and vertically from the top; then again horizontally from the right and front, and vertically from the bottom.

5x3D mm-cube

A	175	175	175	175	175
55	50	45	40	35	
50	45	40	35	30	
45	40	35	30	25	175
40	35	30	25	20	
35	30	25	20	15	
		175			
	175	175	175	175	175

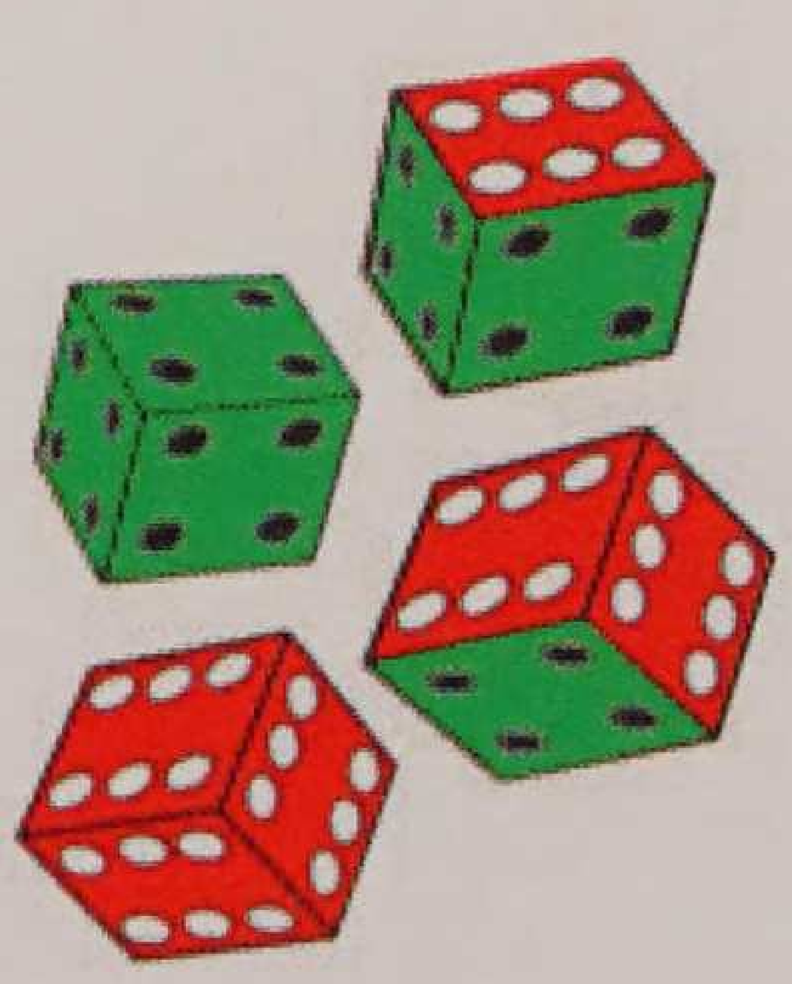
B	175	175	175	175	175
55	50	45	40	35	
50	45	40	35	30	
45	40	35	30	25	175
40	35	30	25	20	
35	30	25	20	15	
		175			
	175	175	175	175	175

C	175	175	175	175	175
55	50	45	40	35	
50	45	40	35	30	
45	40	35	30	25	175
40	35	30	25	20	
35	30	25	20	15	
		175			
	175	175	175	175	175

View along B-axis
Front to Back

	13	12	11	10	9	5
	12	11	10	9	8	4
5	11	10	9	8	7	3
	10	9	8	7	6	2
	9	8	7	6	5	1
	12	11	10	9	8	5
	11	10	9	8	7	4
4	10	9	8	7	6	3
	9	8	7	6	5	2
	8	7	6	5	4	1
	11	10	9	8	7	5
	10	9	8	7	6	4
3	9	8	7	6	5	3
	8	7	6	5	4	2
	7	6	5	4	3	1
	10	9	8	7	6	5
	9	8	7	6	5	4
2	8	7	6	5	4	3
	7	6	5	4	3	2
	6	5	4	3	2	1
	9	8	7	6	5	5
	8	7	6	5	4	4
1	7	6	5	4	3	3
	6	5	4	3	2	2
	5	4	3	2	1	1
	5	4	3	2	1	

The three axial perspectives depicted below show that the matchmaker's cube is like a 6-sided dice with three adjoining faces dotted with one number and their three opposing faces with another.



A-axis

Last column in each block-square

13	12	11	10	9	5
12	11	10	9	8	4
11	10	9	8	7	3
10	9	8	7	6	2
9	8	7	6	5	1
5	4	3	2	1	

First column in each block-square

9	8	7	6	5	5
8	7	6	5	4	4
7	6	5	4	3	3
6	5	4	3	2	2
5	4	3	2	1	1
5	4	3	2	1	

B-axis

Back
Last block-square

	13	12	11	10	9
	12	11	10	9	8
5	11	10	9	8	7
	10	9	8	7	6
	9	8	7	6	5

Front
First block-square

	9	8	7	6	5
	8	7	6	5	4
1	7	6	5	4	3
	6	5	4	3	2
	5	4	3	2	1

C-axis

Last row in each block-square

5	13	12	11	10	9
4	12	11	10	9	8
3	11	10	9	8	7
2	10	9	8	7	6
1	9	8	7	6	5
	5	4	3	2	1

First row in each block-square

5	9	8	7	6	5
4	8	7	6	5	4
3	7	6	5	4	3
2	6	5	4	3	2
1	5	4	3	2	1
	5	4	3	2	1

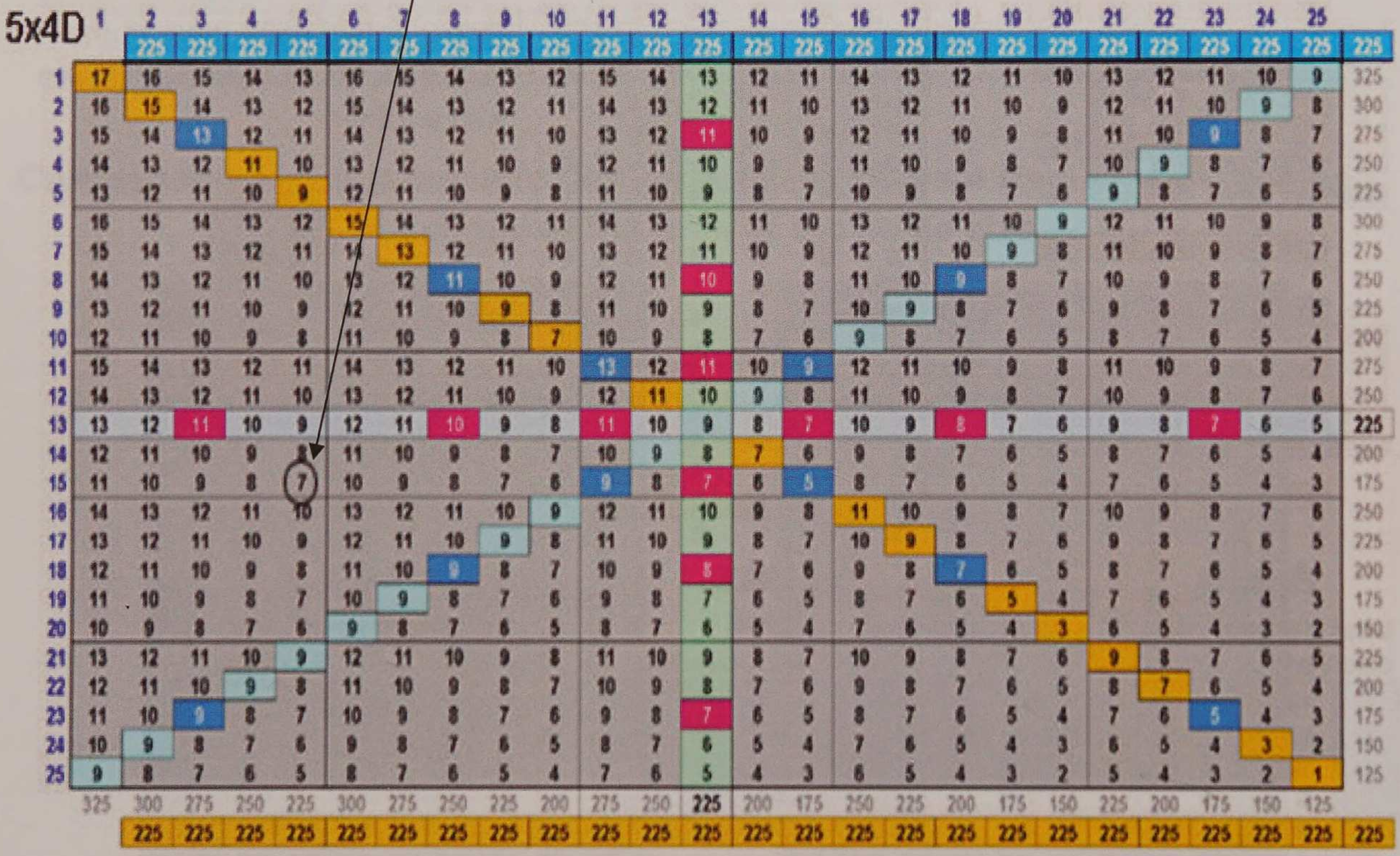
Program 7

Matchmakers Magic Quadracube

Here is the size-4 matchmakers quadracube. Its embedded hexadectals and its two icosahedrons each sum equally within their distinct geometries. The cells involved in the two icosahedral summations are highlighted by crimson and blue.

9	8	7	6	5
8	7	6	5	4
⑦	6	5	4	3
6	5	4	3	2
5	4	3	2	1

The numbers in lower right-hand corner of each **5x5** block-square are just the expansion of each number into an mm block -square of the same size.



5x4D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
		225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225
1	17	16	15	14	13	16	15	14	13	12	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	325
2	16	15	14	13	12	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	300
3	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	275
4	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
5	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
6	16	15	14	13	12	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	300
7	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	275
8	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
9	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
10	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
11	15	14	13	12	11	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	275
12	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
13	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
14	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
15	11	10	9	8	⑦	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	175
16	14	13	12	11	10	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	250
17	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
18	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
19	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	175
20	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	6	5	4	3	2	150
21	13	12	11	10	9	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	225
22	12	11	10	9	8	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	200
23	11	10	9	8	7	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	175
24	10	9	8	7	6	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	6	5	4	3	2	150
25	9	8	7	6	5	8	7	6	5	4	7	6	5	4	3	6	5	4	3	2	5	4	3	2	1	125
	325	300	275	250	225	300	275	250	225	200	275	250	225	200	175	250	225	200	175	150	225	200	175	150	125	
		225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225

The hexadectal sums are just any **4** quadrals in four embedded squares symmetrically positioned around the central embedded square.

The **36** hexadectal summations are each equal to **144**.

The cells involved in the **2** icosahedral summations are highlighted by crimson and blue. Each sums to **108** = ¾ of **144**.

Program 7

A matching in 4-dimensions in a size-5 **quadracube** selects **5** numbers such that no two numbers are selected from neither the same cube nor from the same location in all the other embedded squares among all the embedded cubes. All of its 4-dimensional matchings of **5** selected cells will sum equally everywhere to **45**. That's **5 x 9** where the number **9** is the dimensional average of the table for 4-dimensions.

Then the table can be treated at the **cubic** block-square level. Every embedded horizontal cube is matched with one vertical cube. Then a complete matching is made within their intersecting block-square. That's **25** matchings in all. All of these matchings will sum to the same number every time. In this 4D table of size-5 it will sum to **225**. That's equal to **9 x 25** where the number **9** is again the dimensional average of the table for 3-dimensions.

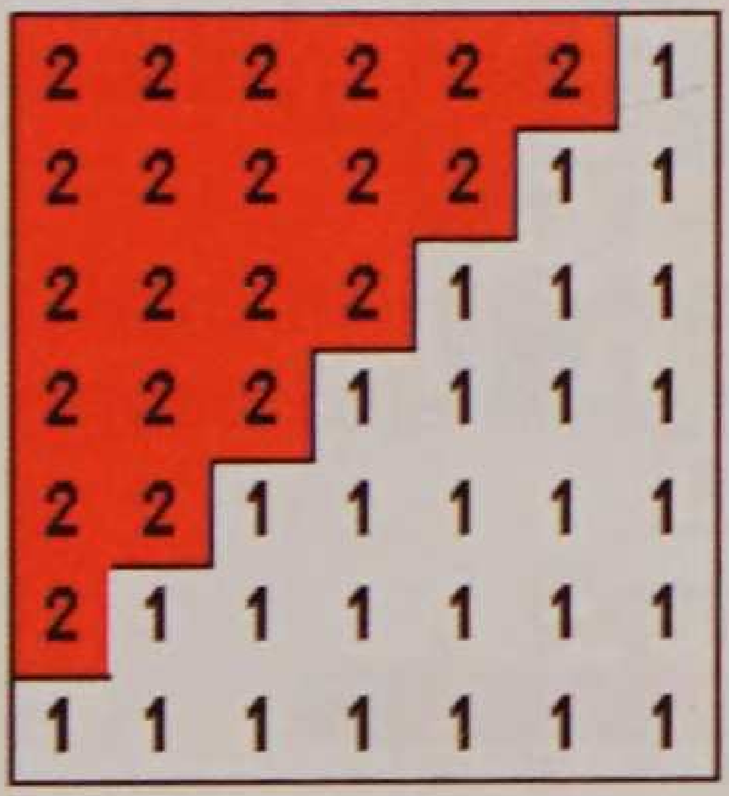
Further, you can just do a row-to-column matching as if the table were an ordinary matchmaker square of size **25**. Again you will have **25** matchings, total. All of these matchings will also sum to **225**.

Note that for matchmaker squares of odd-size **n**, the dimensional average, i.e. pivot number, **p = n**.

Loom Tables of Matchmaker Squares

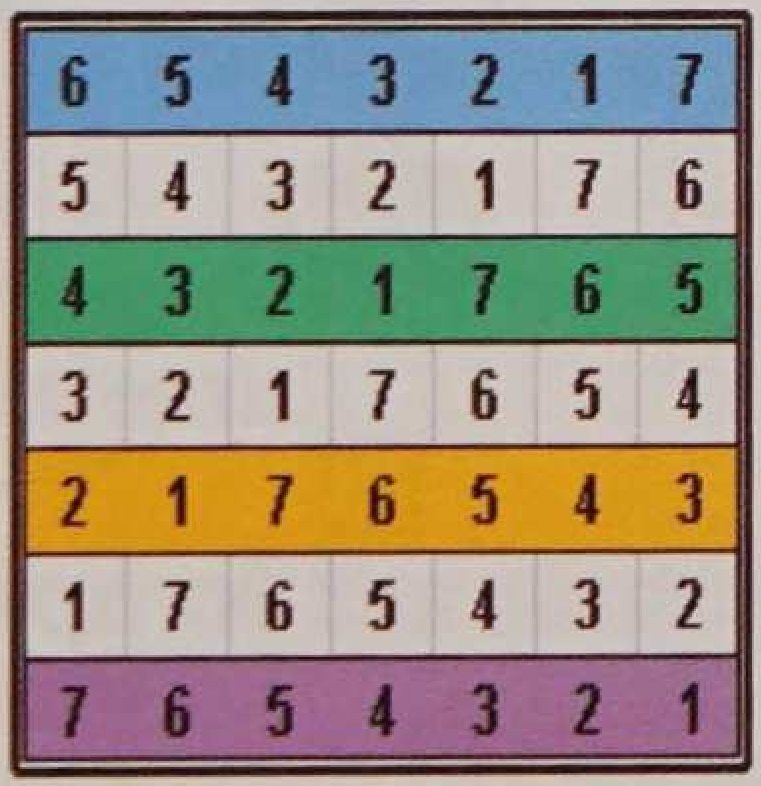
Now here is where we encounter the fundamental relationship between matchmaker squares and regular Class-1 geonomic squares that was promised earlier in this program.

Y



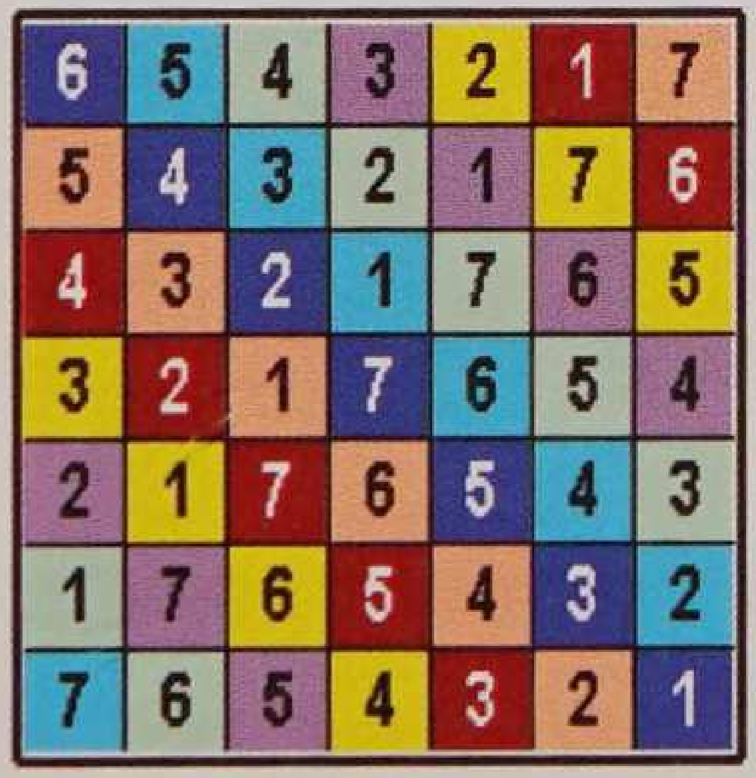
2	2	2	2	2	2	1
2	2	2	2	2	1	1
2	2	2	2	1	1	1
2	2	2	1	1	1	1
2	2	1	1	1	1	1
2	1	1	1	1	1	1
1	1	1	1	1	1	1

X

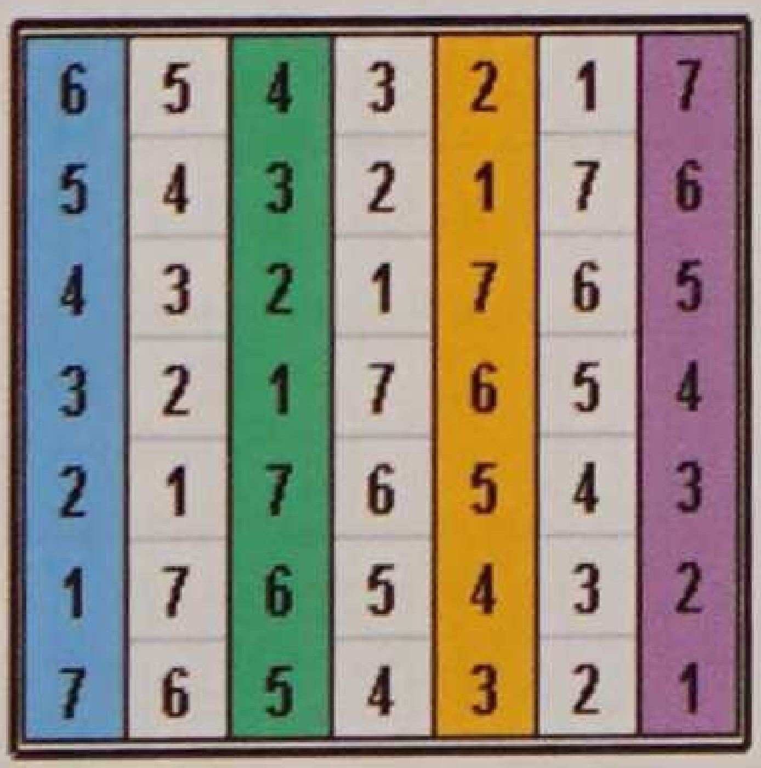


6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

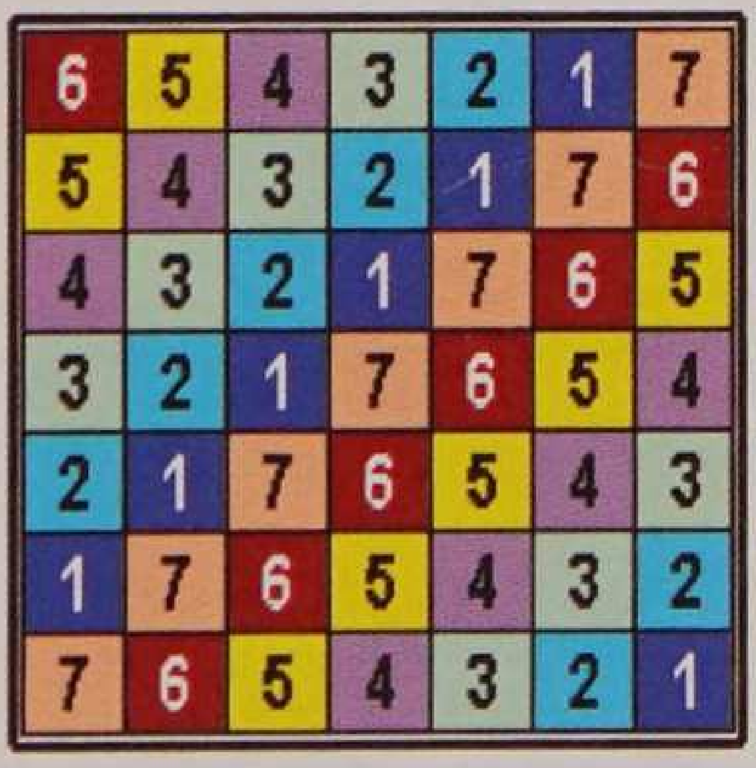
X



6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1



6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1



6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

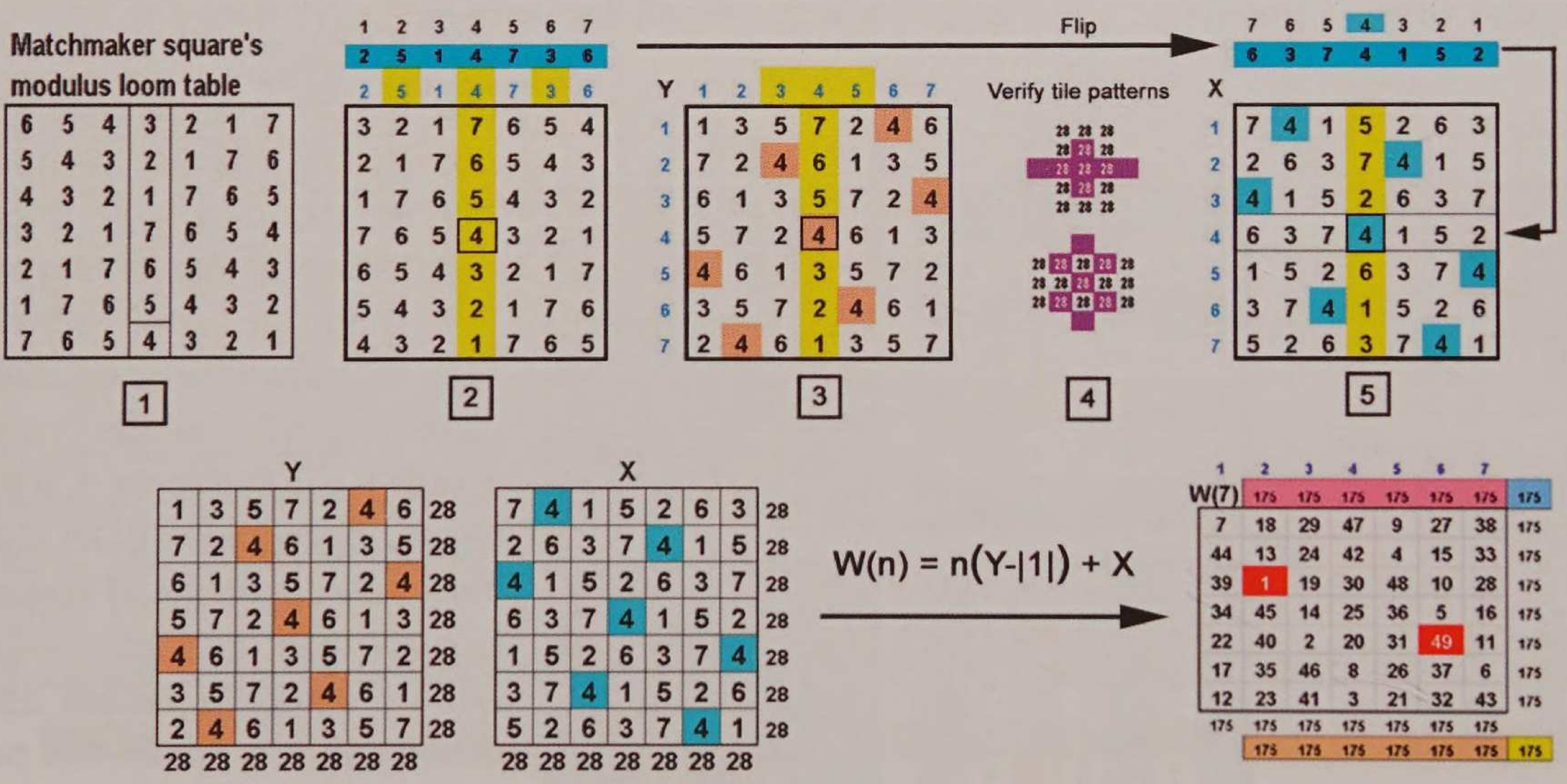
The matchmaker's square of size-7 here, is decomposed into its modulus and integer loom tables as depicted at left. The modulus loom table is seen to have row and column patterns in which each contains the numbers **1** thru **7** exactly once. Each of its minor diagonals contains **7** repetitions of one distinct number, with no diagonal pattern being duplicated. So it only has partially the basic number distribution of a loom table because it lacks the complete pattern of having the numbers from **1** thru **7** exactly once in all four directional summations.

On the next slide we'll convert this matchmaker loom-table to a modulus loom-table for a size-7 regular geonomic square in just **5** steps.

Program 7

Generation of Ultra-perfect Class-1 Squares from Matchmakers Squares of the same size

1. First, the rows of the matchmaker's modulus loom-table are sorted on its central column from large to small. Note that this brings the table's average **4=½(n+1)** into the center.
2. Next, the columns of that result are sorted on the resequencing pattern along the top (see algorithm below).
3. This yields a candidate for the integer loom-table **Y**. Verify / test for characteristic complementary tiling patterns. Transpose the table if not all are equal. If neither the derived table nor its transpose yields all equal-summing tiling patterns, increment the spacing parameter Δ**h** by **1** and continue from step #2. When all tiles sum equally, you will have the integer loom-table **Y**. Go to step #4.
4. **X** is derived as follows:
   - i. Highlight the cells with numbers equal to **½(n+1)** in **Y**.
   - ii. Copy the highlighted pattern to **X**. Flip the dot pattern in **X** horizontally and recolor.
   - iii. Flip the column-resequencing pattern for **Y** and use for the sequence of values in **X** for all the rows. If **Y** was derived from a transposed table, use the column-resequencing pattern for the columns in **X** instead.
   - iv. Insert the row/column values such that their centers align with the highlighted cell and wrap the excess numbers.
5. Compute the primal square from the standard generation formula: **W = n(Y-|1|) + X**.



6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1

1	2	3	4	5	6	7
2	5	1	4	7	3	6
2	5	1	4	7	3	6
3	2	1	7	6	5	4
2	1	7	6	5	4	3
1	7	6	5	4	3	2
7	6	5	4	3	2	1
6	5	4	3	2	1	7
5	4	3	2	1	7	6
4	3	2	1	7	6	5

Y	1	2	3	4	5	6	7
1	1	3	5	7	2	4	6
2	7	2	4	6	1	3	5
3	6	1	3	5	7	2	4
4	5	7	2	4	6	1	3
5	4	6	1	3	5	7	2
6	3	5	7	2	4	6	1
7	2	4	6	1	3	5	7

	7	6	5	4	3	2	1
	6	3	7	4	1	5	2
X							
1	7	4	1	5	2	6	3
2	2	6	3	7	4	1	5
3	4	1	5	2	6	3	7
4	6	3	7	4	1	5	2
5	1	5	2	6	3	7	4
6	3	7	4	1	5	2	6
7	5	2	6	3	7	4	1

Y

1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

X

7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

1	2	3	4	5	6	7	
W(7)	175	175	175	175	175	175	175
7	18	29	47	9	27	38	175
44	13	24	42	4	15	33	175
39	1	19	30	48	10	28	175
34	45	14	25	36	5	16	175
22	40	2	20	31	49	11	175
17	35	46	8	26	37	6	175
12	23	41	3	21	32	43	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

Constructing the resequencing pattern

Starting with the sorted matchmaker loom-table (again), place the number **(n+1)/2** above the center column. Initially with Δ**h** set at the last successful value, place the next number in increasing sequence Δ**h** columns away to the left. Continue this process wrapping back to the right side as many times as is necessary until some column is renumbered **#n**. Then going back to the center column again, place the next number in decreasing sequence Δ**h** columns away to the right. Continue this process wrapping back to the left side as many times as necessary until the last column is renumbered **#1**. In repeat operations increment Δ**h** by **1**

Program 7

Resequencing Table	
Size square	**Column pattern**
5	2
7	2
11	2
13	3
17	3T
19	5
23	5T
29	5
31	6T

In the resequencing table at right, a T indicates that subsequent to the sort on columns, the resulting square needs to be transposed to exhibit the characteristic complementary tile patterns.

This method has been called the **Double-Quark method** to signify its fundamental significance; what the double-quark is to the fundamental existence of the neutron and proton as shown earlier in Part I, this method is fundamental to the existence of Class-1 odd-prime-number size squares generated from the complementary loom-tables **X** and **Y**.

Now, these resequencing patterns were determined by trial and error. The complementary tiling patterns were discovered by trying out different patterns alongside the simultaneous generation of the geonomic square based on what was already discovered for squares of lesser sizes. The table here lists the results of these discoveries up thru size-31.

Observe the sequence of increasing Δ**h** in the Resequencing Table. It was found that no viable loom table was derived from Δ**h** being any less than all the prior sizes of successful loom tables.

This is the best that has been observed so far. Obviously there is plenty of opportunity for further development with respect to uncovering additional ultra-perfect Class-1 squares. But their discovery will remain on an enlightened trial-and-error basis for the foreseeable future.

Nonetheless, this table along with the 5-step **Double-Quark** algorithm on the prior page allows anyone to generate ultra-perfect Class-1 squares up to and beyond size-**31** very directly using Microsoft's Excel™ program. That computer program can construct the initial table, sort its columns, and perform the transpose with just a few clicks on the computer screen.

* * *

Next, the generation of the size-31 square will be shown to exemplify the fundamental nature of these column sorting patterns.

Conversion of the matchmaker square to the ultra-perfect size-31 geonomic square

Here is an example of the redistribution sequences for the size **31** square which is the largest print-legible size of Class-1 squares:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
29	24	19	14	9	4	30	25	20	15	10	5	31	26	21	16	11	6	1	27	22	17	12	7	2	28	23	18	13	8	3

Resequence columns from black to blue: Here is the pre-sorted size-31 matchmaker modulus loom-table depicting its resequencing patterns where Δ**h** = **6**. It makes no difference whether the sequence incrementation initially proceeds from the left or right; it just needs to be consistent in its application. Here the incrementation is reversed from the previous example.

On the next page is the derived loom-table **Y** after resequencing its columns. Below it is its transpose with all the cells containing **#16** highlighted in blue.

Program 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	29	24	19	14	9	4	30	25	20	15	10	5	31	26	21	16	11	6	1	27	22	17	12	7	2	28	23	18	13	8	3
1	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17
2	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18
3	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19
4	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20
5	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21
6	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22
7	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23
8	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24
9	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25
10	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26
11	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27
12	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28
13	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29
14	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30
15	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31
16	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
17	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
18	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
19	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4
20	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5
21	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6
22	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7
23	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8
24	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9
25	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10
26	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11
27	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
28	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13
29	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14
30	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15
31	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16

Transposed

Y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
3	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10
5	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4
6	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
7	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
8	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
9	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11
10	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5
11	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
12	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
13	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
14	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12
15	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6
16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
17	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
18	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
19	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13
20	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7
21	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1
22	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
23	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
24	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14
25	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8
26	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2
27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
28	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
29	16	17	18	19	20	21		23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
30	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9
31	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
29	24	19	14	9	4	30	25	20	15	10	5	31	26	21	16	11	6	1	27	22	17	12	7	2	28	23	18	13	8	3

Above is the resequencing pattern again. It is next transposed and used for the columns instead of rows in **X** because **Y** needed to be transposed to possess the standard characteristic tile patterns. The dot pattern of **Y** was flipped horizontally and colored pink for use in **X**. The resequencing number-pattern is then used for the columns by placing it in each column so that the #16 appears in each pink dot. Below is the derived modulus loom-table **X**. Note that downwardly **#21** always precedes **#16**, except of course when **#16** is at the very top.

X	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
31	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	496
30	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	496
29	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	496
28	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	496
27	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	496
26	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	496
25	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	496
24	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	496
23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	496
22	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	496
21	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	496
20	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	496
19	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	496
18	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	496
17	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	496
16	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	496
15	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	496
14	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	496
13	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	496
12	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	496
11	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	496
10	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	496
9	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	496
8	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	496
7	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	496
6	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	496
5	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	496
4	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	496
3	10	4	29	23	17	11	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	496
2	5	30	24	18	12	6	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	496
1	31	25	19	13	7	1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	496
	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	496	

Here is proof that that the double-quark method will always yield a Class-1 square that is geonomically ultra-perfect:

Because every row, column and diagonal in **Y** contains the numbers from **1** thru **n** exactly once, the sum of y_{ij} over **j=1** thru **n** equals L_n, the loom-tables characteristic number. But so does the resequence series and these form all the rows or columns of **X**.

Now, the resequence series was determined such that **Y** possessed complementary tiling patterns, so **Y** qualifies as a Class-1 loom-table. The anchor-dot pattern in **X** was chosen to correspond to be the horizontal flip of that of **Y** and this provided the distribution of numbers in **X** such that every row, column and diagonal contained the numbers from **1** thru **n** exactly once too.

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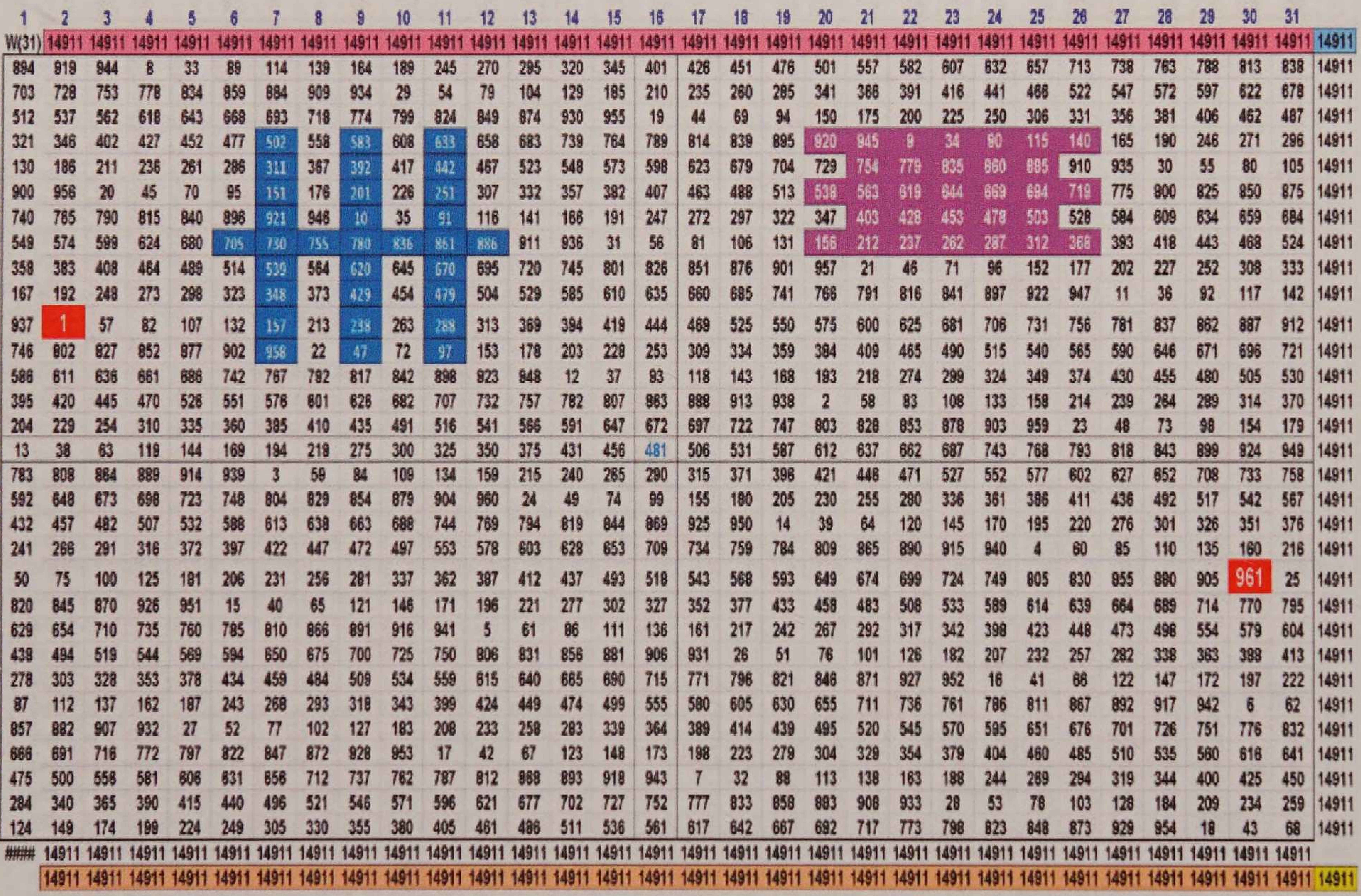
Hence, **X** too qualifies as a loom table for a Class-1 square. The fact that **X** also possessed identical tiling patterns is due to the property that the tiling patterns identified during the derivation of **Y** are characteristic of all loom tables of size **n**. Consequently, **X** could only have tiling patterns that were simultaneously complementary and be identical to those of **Y**. **QED**

Both loom-tables must possess a given property for the primary squares **W** and **U** to have them too. Here is the size-31 square derived from **X** and **Y** by the standard regeneration formula.

It has the complementary tiling patterns characteristic of all size-31 squares.

Observe its pairwise central symmetry. This guarantees its pangenicity.

Again, it's another ultra-perfect square.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
W(31)	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911
1	894	919	944	8	33	89	114	139	164	189	245	270	295	320	345	401	426	451	476	501	557	582	607	632	657	713	738	763	788	813	838	14911
2	703	728	753	778	834	859	884	909	934	29	54	79	104	129	185	210	235	260	285	341	366	391	416	441	466	522	547	572	597	622	678	14911
3	512	537	562	618	643	668	693	718	774	799	824	849	874	930	955	19	44	69	94	150	175	200	225	250	306	331	356	381	406	462	487	14911
4	321	346	402	427	452	477	502	558	583	608	633	658	683	739	764	789	814	839	895	920	945	9	34	90	115	140	165	190	246	271	296	14911
5	130	186	211	236	261	286	311	367	392	417	442	467	523	548	573	598	623	679	704	729	754	779	835	860	885	910	935	30	55	80	105	14911
6	900	956	20	45	70	95	151	176	201	226	251	307	332	357	382	407	463	488	513	538	563	619	644	669	694	719	775	800	825	850	875	14911
7	740	765	790	815	840	896	921	946	10	35	91	116	141	166	191	247	272	297	322	347	403	428	453	478	503	528	584	609	634	659	684	14911
8	549	574	599	624	680	705	730	755	780	836	861	886	911	936	31	56	81	106	131	156	212	237	262	287	312	368	393	418	443	468	524	14911
9	358	383	408	464	489	514	539	564	620	645	670	695	720	745	801	826	851	876	901	957	21	46	71	96	152	177	202	227	252	308	333	14911
10	167	192	248	273	298	323	348	373	429	454	479	504	529	585	610	635	660	685	741	766	791	816	841	897	922	947	11	36	92	117	142	14911
11	937	1	57	82	107	132	157	213	238	263	288	313	369	394	419	444	469	525	550	575	600	625	681	706	731	756	781	837	862	887	912	14911
12	746	802	827	852	877	902	958	22	47	72	97	153	178	203	228	253	309	334	359	384	409	465	490	515	540	565	590	646	671	696	721	14911
13	586	611	636	661	686	742	767	792	817	842	898	923	948	12	37	93	118	143	168	193	218	274	299	324	349	374	430	455	480	505	530	14911
14	395	420	445	470	526	551	576	601	626	682	707	732	757	782	807	863	888	913	938	2	58	83	108	133	158	214	239	264	289	314	370	14911
15	204	229	254	310	335	360	385	410	435	491	516	541	566	591	647	672	697	722	747	803	828	853	878	903	959	23	48	73	98	154	179	14911
16	13	38	63	119	144	169	194	219	275	300	325	350	375	431	456	481	506	531	587	612	637	662	687	743	768	793	818	843	899	924	949	14911
17	783	808	864	889	914	939	3	59	84	109	134	159	215	240	265	290	315	371	396	421	446	471	527	552	577	602	627	652	708	733	758	14911
18	592	648	673	698	723	748	804	829	854	879	904	960	24	49	74	99	155	180	205	230	255	280	336	361	386	411	436	492	517	542	567	14911
19	432	457	482	507	532	588	613	638	663	688	744	769	794	819	844	869	925	950	14	39	64	120	145	170	195	220	276	301	326	351	376	14911
20	241	266	291	316	372	397	422	447	472	497	553	578	603	628	653	709	734	759	784	809	865	890	915	940	4	60	85	110	135	160	216	14911
21	50	75	100	125	181	206	231	256	281	337	362	387	412	437	493	518	543	568	593	649	674	699	724	749	805	830	855	880	905	961	25	14911
22	820	845	870	926	951	15	40	65	121	146	171	196	221	277	302	327	352	377	433	458	483	508	533	589	614	639	664	689	714	770	795	14911
23	629	654	710	735	760	785	810	866	891	916	941	5	61	86	111	136	161	217	242	267	292	317	342	398	423	448	473	498	554	579	604	14911
24	438	494	519	544	569	594	650	675	700	725	750	806	831	856	881	906	931	26	51	76	101	126	182	207	232	257	282	338	363	388	413	14911
25	278	303	328	353	378	434	459	484	509	534	559	615	640	665	690	715	771	796	821	846	871	927	952	16	41	66	122	147	172	197	222	14911
26	87	112	137	162	187	243	268	293	318	343	399	424	449	474	499	555	580	605	630	655	711	736	761	786	811	867	892	917	942	6	62	14911
27	857	882	907	932	27	52	77	102	127	183	208	233	258	283	339	364	389	414	439	495	520	545	570	595	651	676	701	726	751	776	832	14911
28	666	691	716	772	797	822	847	872	928	953	17	42	67	123	148	173	198	223	279	304	329	354	379	404	460	485	510	535	560	616	641	14911
29	475	500	556	581	606	631	656	712	737	762	787	812	868	893	918	943	7	32	88	113	138	163	188	244	269	294	319	344	400	425	450	14911
30	284	340	365	390	415	440	496	521	546	571	596	621	677	702	727	752	777	833	858	883	908	933	28	53	78	103	128	184	209	234	259	14911
31	124	149	174	199	224	249	305	330	355	380	405	461	486	511	536	561	617	642	667	692	717	773	798	823	848	873	929	954	18	43	68	14911
####	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	
	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911	14911

And now that you know just how all odd-prime-number size squares can be manifested to be ultra-perfect, you'll get to see just how all composite-size squares can be manifested from these ultra-perfect squares in the next program. Then you will be able to generate all perfect geonomic squares of any desired size on your own and explore still larger squares.

Further, you'll get to see how the near-perfect Class-2 squares are generated too.

You are situated just at the beginning of this new math, Geonometry, and there is a lot more opportunity for the discovery of additional properties and patterns now that the foundation has been laid.

Program 7

Deriving an alternative perfect square from the anchor-dot pattern

			X							Y			
3	6	2	5	1	4	7	6	4	2	7	5	3	1
5	1	4	7	3	6	2	5	3	1	6	4	2	7
7	3	6	2	5	1	4	4	2	7	5	3	1	6
2	5	1	4	7	3	6	3	1	6	4	2	7	5
4	7	3	6	2	5	1	2	7	5	3	1	6	4
6	2	5	1	4	7	3	1	6	4	2	7	5	3
1	4	7	3	6	2	5	7	5	3	1	6	4	2
3	6	2	5	1	4	7	6	4	2	7	5	3	1
5	1	4	7	3	6	2	5	3	1	6	4	2	7
7	3	6	2	5	1	4	4	2	7	5	3	1	6
2	5	1	4	7	3	6	3	1	6	4	2	7	5
4	7	3	6	2	5	1	2	7	5	3	1	6	4
6	2	5	1	4	7	3	1	6	4	2	7	5	3
1	4	7	3	6	2	5	7	5	3	1	6	4	2

At left are the loom tables **X** and **Y** derived from the primal size-7 square **W** on the prior page. Each has been duplicated once below them to facilitate the derivation of loom tables from the anchor-dot pattern.

The highlighted numbers are the **7** points of equality between them with the numbers ranging from **1** thru **7**. Below these are the loom tables **X*** and **Y*** derived by merely taking the numbers highlighted in each row in the upper block and making a row from their values as encountered sequentially going down the rows in each table.

Then this pattern is dragged down vertically recording the values as subsequent rows in **X*** and **Y*** until the last row in each has been completed. Compare the first row of both **X*** and **Y*** with the numbers highlighted above.

X*

	28	28	28	28	28	28	28
2	5	1	4	7	3	6	28
4	7	3	6	2	5	1	28
6	2	5	1	4	7	3	28
1	4	7	3	6	2	5	28
3	6	2	5	1	4	7	28
5	1	4	7	3	6	2	28
7	3	6	2	5	1	4	28
28	28	28	28	28	28	28	
	28	28	28	28	28	28	28

Y*

2	5	1	4	7	3	6
1	4	7	3	6	2	5
7	3	6	2	5	1	4
6	2	5	1	4	7	3
5	1	4	7	3	6	2
4	7	3	6	2	5	1
3	6	2	5	1	4	7

Y^{*T}

	28	28	28	28	28	28	28
2	1	7	6	5	4	3	28
5	4	3	2	1	7	6	28
1	7	6	5	4	3	2	28
4	3	2	1	7	6	5	28
7	6	5	4	3	2	1	28
3	2	1	7	6	5	4	28
6	5	4	3	2	1	7	28
28	28	28	28	28	28	28	
	28	28	28	28	28	28	28

Next **Y*** is transposed to get **Y***^{T}. The new square is derived from the standard generation formula using the reconstructed modulus loom-table **X*** and the transpose of the reconstructed integer loom-table **Y***^{T} to get **W***. Then **W*** may be normalized and summed geonomically to produce another perfect size-7 square **W*(7)** as shown here. Check it out.

$W^* = 7*(Y^* - | 1 |) + X^*$

9	5	43	39	35	24	20
32	28	17	13	2	47	36
6	44	40	29	25	21	10
22	18	14	3	48	37	33
45	41	30	26	15	11	7
19	8	4	49	38	34	23
42	31	27	16	12	1	46

W*(7) After normalization

	175	175	175	175	175	175	175
31	27	16	12	1	46	42	175
5	43	39	35	24	20	9	175
28	17	13	2	47	36	32	175
44	40	29	25	21	10	6	175
18	14	3	48	37	33	22	175
41	30	26	15	11	7	45	175
8	4	49	38	34	23	19	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

This method is a general method that can be applied to any size Class-1 square, even those points of equality between its own loom-tables linearly distributed, viz. along a main diagonal or across a row or column. Observe this on the next page.

Program 7

The perpetuity property of the anchor-dot derivative method

Dot Patterns	
W1	(1; 2)
W2	(1;-2)
W3	(2;-1)
W4	(1; 2)
W5	(1; 1)
W6	(0; 1)
W7	(1;-1)
W8	(1; 1)
W9	(2; 1)
W10	(2; 1)
W11	(1;-2)
W12	(1;-1)

Here are 11 consecutive applications of the anchor-dot derivative method to the original size-7 square. It shows how the points of equality get redistributed. This method will produce all **56** distinct normalized perfect versions of the size-7 square.

In the process, either **X*** or **Y*, but not both,** can become non-geonomic by having all numbers equal in each of its same-directional diagonals. In that case, the transpose of the loom-table that is geonomic is used along with its non-transposed version to continue on. Such occurred here for **X*** in deriving **W11** from **W10**. Then **Y*** and its transpose were used to generate **W11**. Normalization had no effect on the result. The last square in the process needs normalization just to be pangenic; that's all.

Another glitch occurs when the anchor-dots all get distributed along a single row or column. That is resolved by setting **X*=X** and **Y*=Y**. The transpose of **Y*** will break this logjam as seen in the transition from **W6** to **W7**.

Here, the centers of the tiling patterns for **W1** are aligned with the anchor-dot patterns. Note however that characteristic tiling patterns **A** & **B** or **hA** & **hB** may be temporarely lost at some points in the sequence. They will reappear and vanish again and again. That's a topic for further exploration.

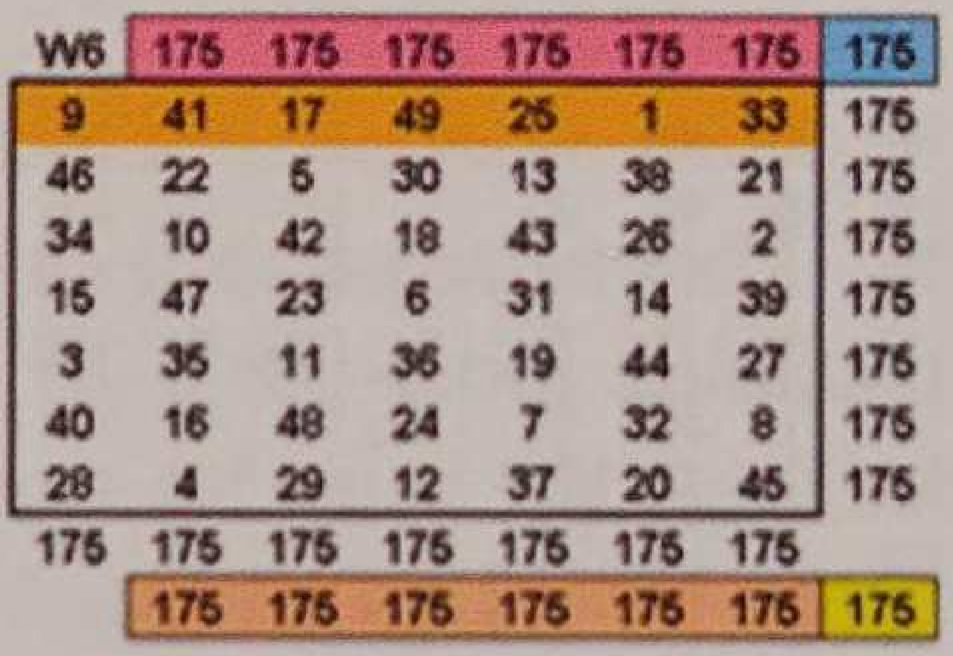
W1	175	175	175	175	175	175	175
38	27	9	47	29	18	7	175
33	15	4	42	24	13	44	175
28	10	48	30	19	1	39	175
16	5	36	25	14	45	34	175
11	49	31	20	2	40	22	175
6	37	26	8	46	35	17	175
43	32	21	3	41	23	12	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

W2	175	175	175	175	175	175	175
9	5	43	39	35	24	20	175
32	28	17	13	2	47	36	175
6	44	40	29	25	21	10	175
22	18	14	3	48	37	33	175
45	41	30	26	15	11	7	175
19	8	4	49	38	34	23	175
42	31	27	16	12	1	46	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

W3	175	175	175	175	175	175	175
9	31	4	26	48	21	36	175
18	40	13	35	1	23	45	175
27	49	15	37	10	32	5	175
29	2	24	46	19	41	14	175
38	11	33	6	28	43	16	175
47	20	42	8	30	3	25	175
7	22	44	17	39	12	34	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

W4	175	175	175	175	175	175	175
9	15	28	34	40	46	3	175
4	10	16	22	35	41	47	175
48	5	11	17	23	29	42	175
36	49	6	12	18	24	30	175
31	37	43	7	13	19	25	175
26	32	38	44	1	14	20	175
21	27	33	39	45	2	8	175
175	175	175	175	175	175	175	
	126	175	224	273	322	28	77

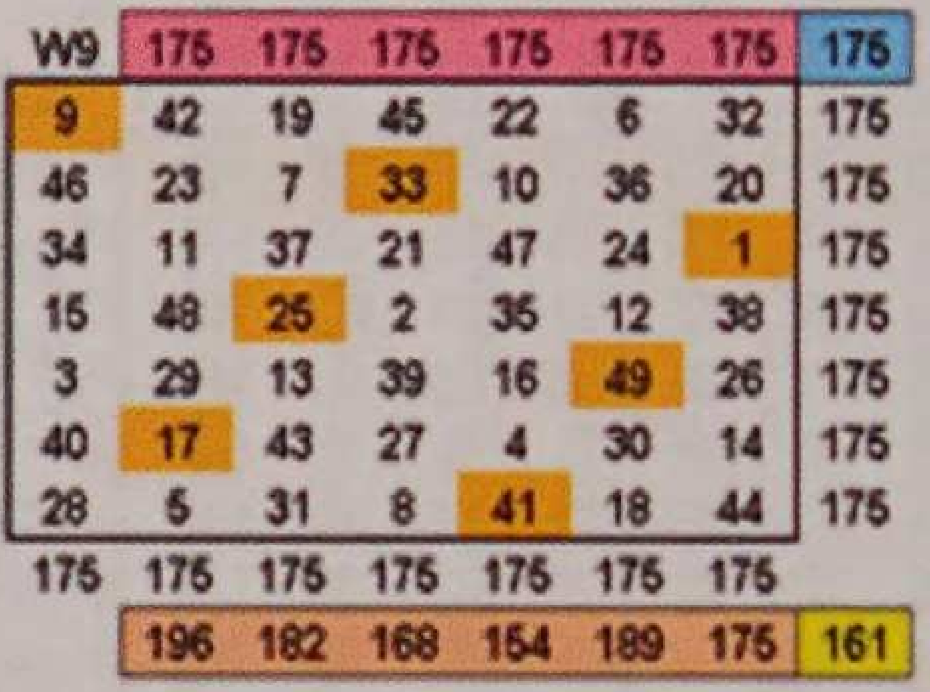
W5	175	175	175	175	175	175	175
9	6	45	42	32	22	19	175
39	29	26	16	13	3	49	175
20	10	7	46	36	33	23	175
43	40	30	27	17	14	4	175
24	21	11	1	47	37	34	175
5	44	41	31	28	18	8	175
35	25	15	12	2	48	38	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175



W6	175	175	175	175	175	175	175
9	41	17	49	25	1	33	175
46	22	5	30	13	38	21	175
34	10	42	18	43	26	2	175
15	47	23	6	31	14	39	175
3	35	11	36	19	44	27	175
40	16	48	24	7	32	8	175
28	4	29	12	37	20	45	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

W7	161	175	189	154	168	182	196
9	46	34	15	3	40	28	175
4	41	22	10	47	35	16	175
48	29	17	5	42	23	11	175
36	24	12	49	30	18	6	175
31	19	7	37	25	13	43	175
26	14	44	32	20	1	38	175
21	2	39	27	8	45	33	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

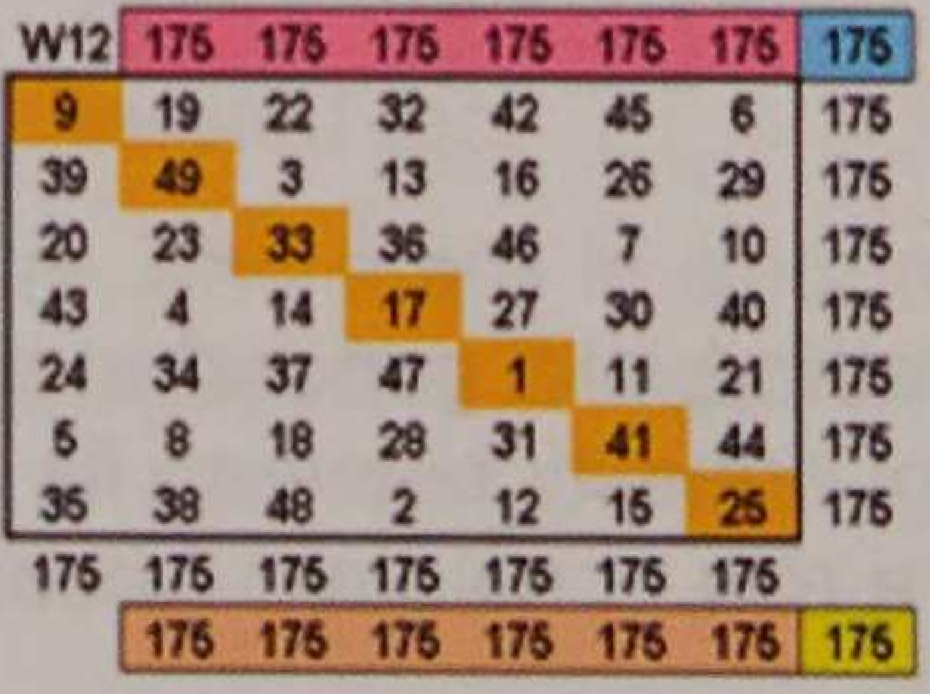
W8	175	175	175	175	175	175	175
9	6	45	42	32	22	19	175
39	29	26	16	13	3	49	175
20	10	7	46	36	33	23	175
43	40	30	27	17	14	4	175
24	21	11	1	47	37	34	175
5	44	41	31	28	18	8	175
35	25	15	12	2	48	38	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175



W9	175	175	175	175	175	175	175
9	42	19	45	22	6	32	175
46	23	7	33	10	36	20	175
34	11	37	21	47	24	1	175
15	48	25	2	35	12	38	175
3	29	13	39	16	49	26	175
40	17	43	27	4	30	14	175
28	5	31	8	41	18	44	175
175	175	175	175	175	175	175	
	196	182	168	154	189	175	161

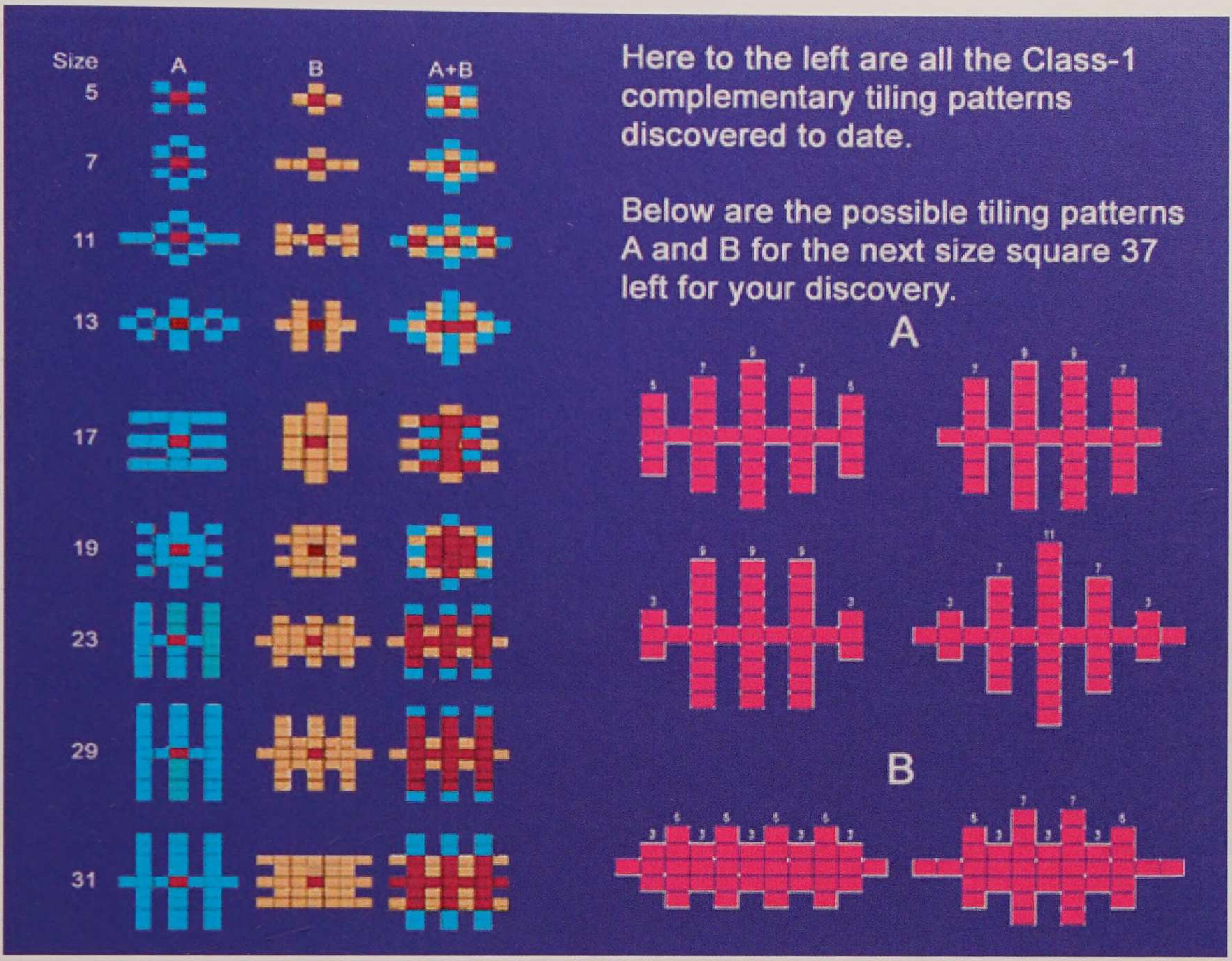
W10	175	175	175	175	175	175	175
9	47	29	18	7	38	27	175
32	21	3	41	23	12	43	175
6	37	26	8	46	35	17	175
22	11	49	31	20	2	40	175
45	34	16	5	36	25	14	175
19	1	39	28	10	48	30	175
42	24	13	44	33	15	4	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

W11	175	175	175	175	175	175	175
9	27	38	7	18	29	47	175
39	1	19	30	48	10	28	175
20	31	49	11	22	40	2	175
43	12	23	41	3	21	32	175
24	42	4	15	33	44	13	175
5	16	34	45	14	25	36	175
35	46	8	26	37	6	17	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175



W12	175	175	175	175	175	175	175
9	19	22	32	42	45	6	175
39	49	3	13	16	26	29	175
20	23	33	36	46	7	10	175
43	4	14	17	27	30	40	175
24	34	37	47	1	11	21	175
5	8	18	28	31	41	44	175
35	38	48	2	12	15	25	175
175	175	175	175	175	175	175	
	175	175	175	175	175	175	175

Program 7



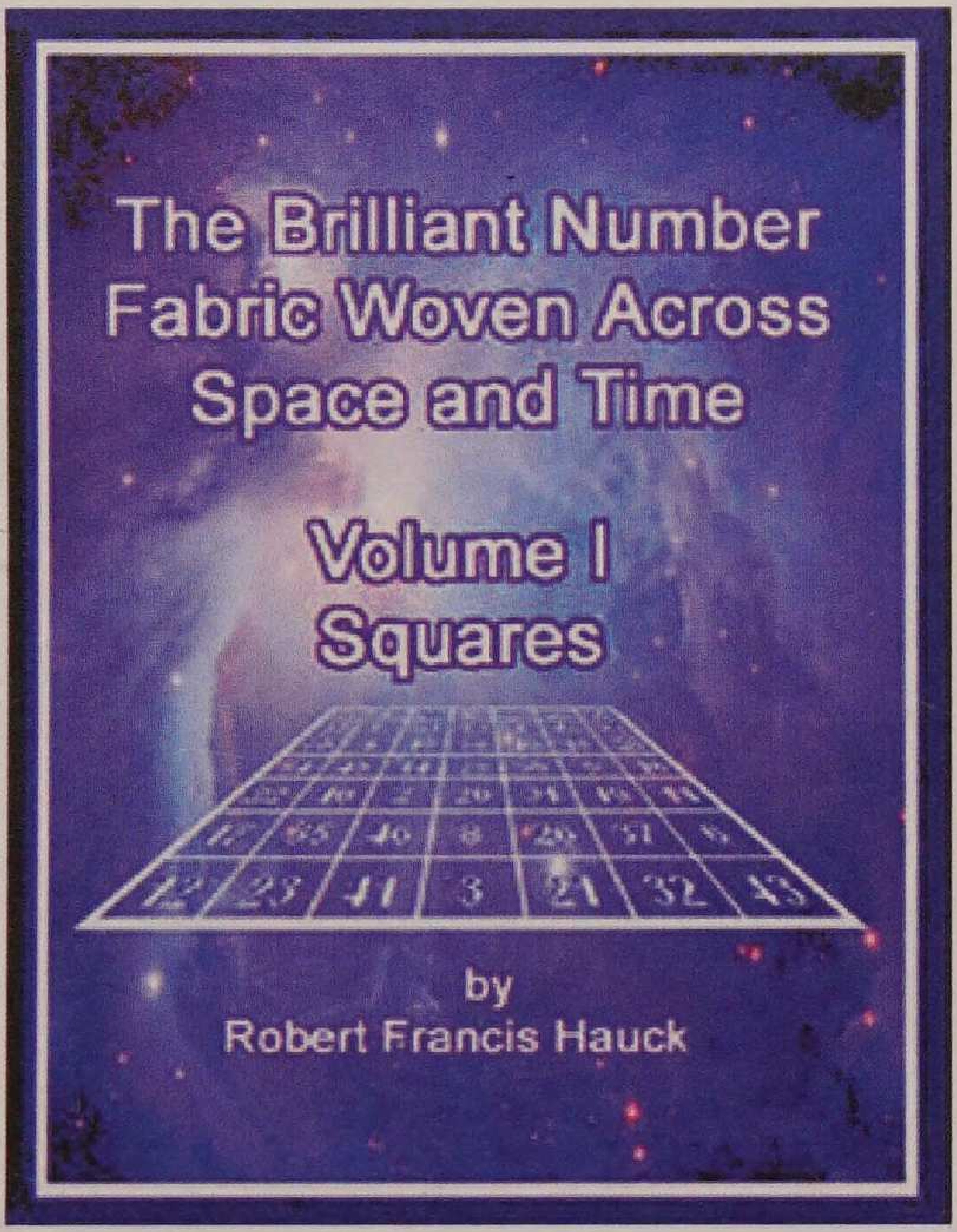
Here to the left are all the Class-1 complementary tiling patterns discovered to date.

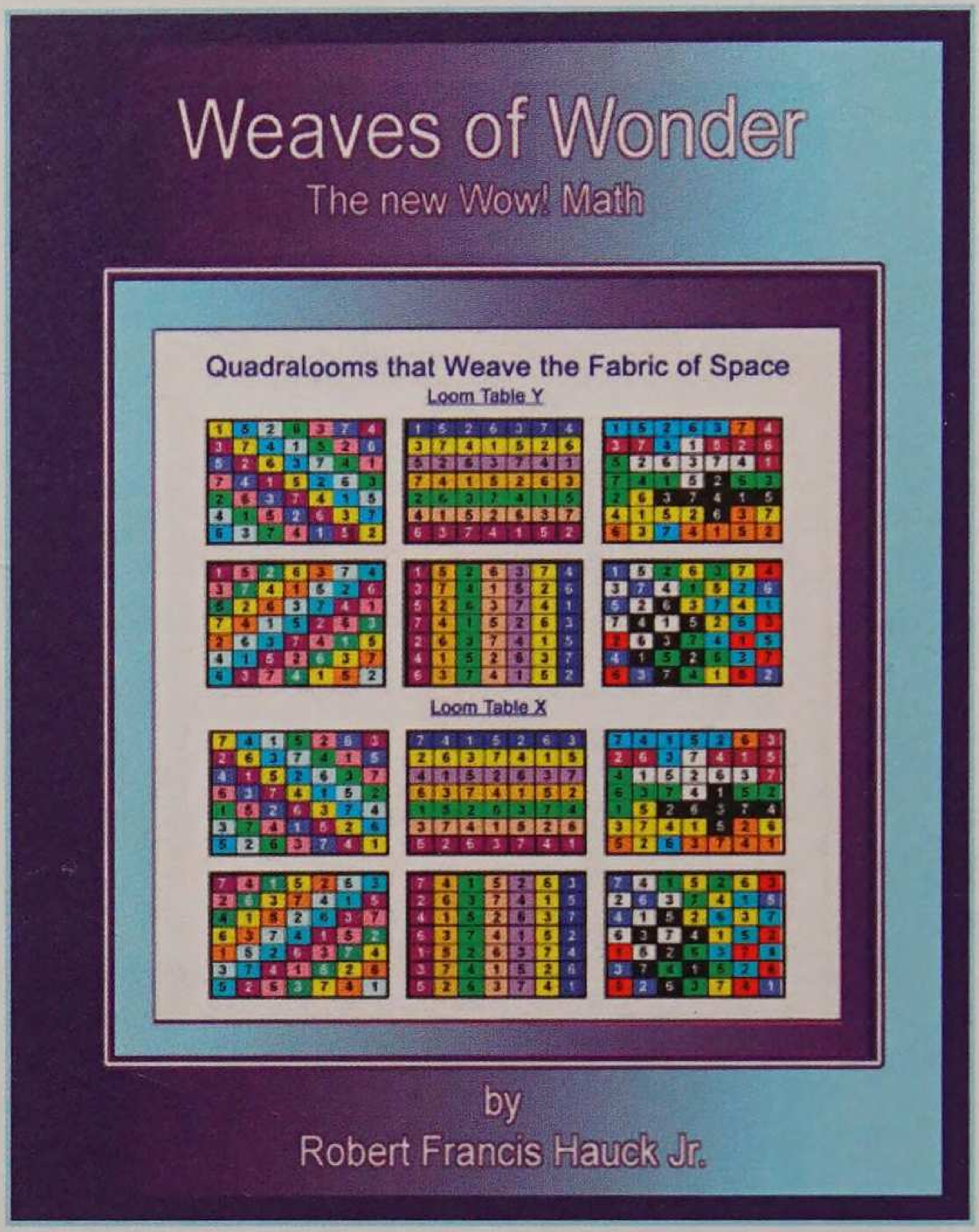
Below are the possible tiling patterns A and B for the next size square 37 left for your discovery.

Notes

1. The modulus loom table of a size **4x** quadracube correlates 100% with the relationship among the **6** quarks in sub-dimension **1/4D**.
2. The modulus loom table of the matchmaker's square derives the complementary loom tables of any ultra-perfect square without any knowledge of the Class-1 square beforehand. That method, called the **Double-Quark** method, was described in this program. It always resulted in a non-linear distribution of the numbers for which the modulus and integer functions are equal. As such, the squares are not only ultra-perfect but possess characteristic anchor-dot patterns distinctly different from those formed from, and in addition to, the centers of their tiling patterns.
3. Geonometry is the mathematics of the **fabric of space**. The matchmaker square is the essential basic source of all the Class-1 squares and is the very **twine** used by the **looms** to generate the fabric of space. It is the mathematical equivalent in its significance to the **up** and **down** quarks that are so fundamental to the existence of the neutron and proton. That is why the resequencing method was called **The Double Quark algorithm**.

We have come to the end of Program 7. I hope that this program has been as awe-inspiring for you as it was for me in its discovery. Here are the books upon which this program was based.





The Brilliant Number Fabric Woven across Space and Time Volume I -- Squares

ISBN: 978-1-461-06984-3

Fifth Edition (128 pages)

Weaves of Wonder – The New Wow Math

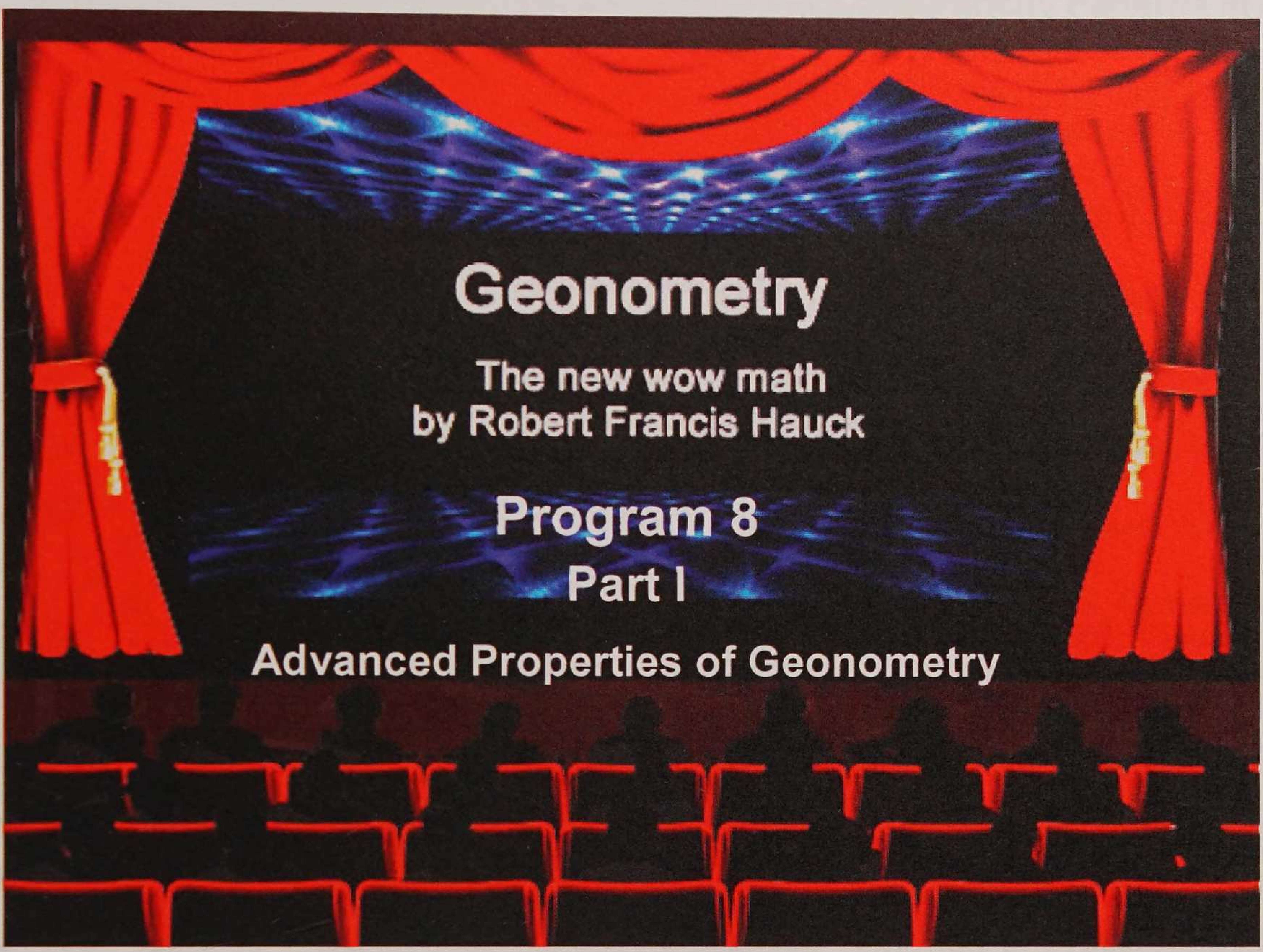
ISBN: 978-1-469-93296-5

Shows how to construct geonomic squares from loom tables.

Second Edition (120 pages)

In the next program we'll look at some advanced properties of Geonometry. We have been referencing Program 8 all throughout this program series with respect to various methods of manifestation for composite-size squares. From these methods and additional fundamental properties yet to be described, we will get an even broader picture of the spatial fabric and its pervasiveness. Come back again for yet some more mind-blowing “Wows!”

Program 8



Geonometry

The new wow math
by Robert Francis Hauck

Program 8
Part I

Advanced Properties of Geonometry

This program demonstrates some remarkable properties in Geonometry. Here we will get an even broader picture of the spatial fabric and its pervasiveness. These membrane patterns detected in Geonometry through complementary loom tables may well extend across vast distances on a cosmological level too. This program looks at multi-dimensional space from the point of view of expansion properties and not-so-obvious hidden patterns.

Program 8

The Corner Triangle Property

Here is an example of the corner triangle property. This property is such that every one of the triangles, which include one corner and its related main diagonal and all number cells in-between, sum equally. It is just coincidental that the triangular sum here is exactly 4 times the size-7 square's characteristic number. The multiple is always ½(**n+1**).

Size-7 perfect square

	1	2	3	4	5	6	7	
7x7		175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

Quadral sums

100	100	100	100
100	100	100	100
100	100	100	100

Quarter triangle sums

700	700
700	700

Every square that's perfect has this property.Class-2 squares are not perfect and so do not have this property.

Now since each quarter triangle shares one of the main diagonals which sum equally to **175**, eliminating the main diagonal from each quarter square will yield four independent triangles of size **n-1** which sum equally to **525 = 700 - 175 = 3 x 175 = 7 x 25 = ½(n-1)C_n.**

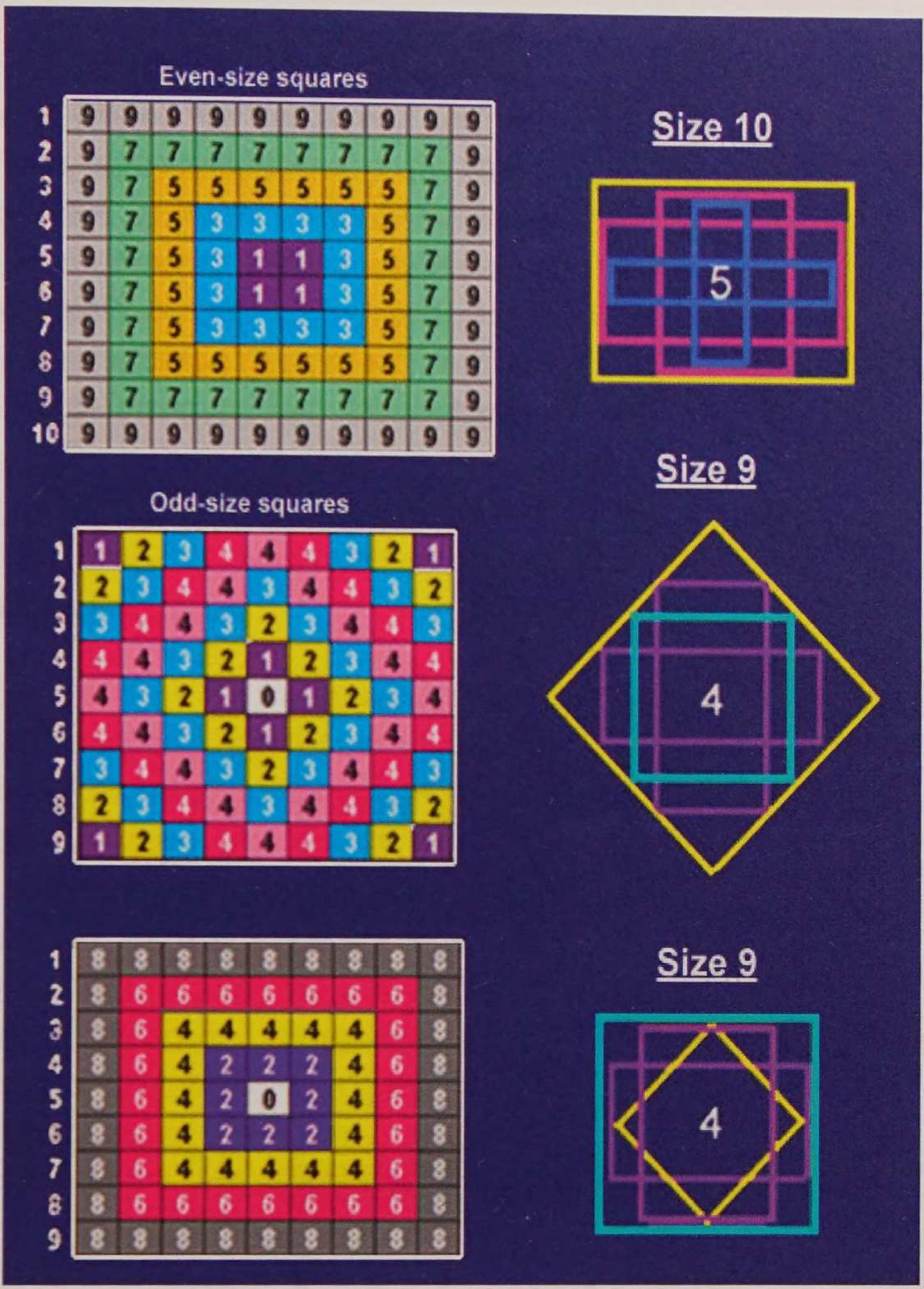
This leads to the 4th-degree polynomial expression for each of the **4** separate equal-summing triangles of size **n-1** as:

$$(8.1)\qquad \Lambda_n = \tfrac{1}{2}(n-1)\,C_n = \tfrac{1}{4}\,n(n^3 - n^2 + n^1 - n^0)$$

where the expression in parenthesis is a 3rd-degree monomial with alternating signs of unity which in turn can be factored as:

$$(8.2)\qquad (n^3 - n^2 + n^1 - n^0) = (n-1)(n^2+1).$$

Program 8



1	9	9	9	9	9	9	9	9	9	9
2	9	7	7	7	7	7	7	7	7	9
3	9	7	5	5	5	5	5	5	7	9
4	9	7	5	3	3	3	3	5	7	9
5	9	7	5	3	1	1	3	5	7	9
6	9	7	5	3	1	1	3	5	7	9
7	9	7	5	3	3	3	3	5	7	9
8	9	7	5	5	5	5	5	5	7	9
9	9	7	7	7	7	7	7	7	7	9
10	9	9	9	9	9	9	9	9	9	9

1	1	2	3	4	4	4	3	2	1
2	2	3	4	4	3	4	4	3	2
3	3	4	4	3	2	3	4	4	3
4	4	4	3	2	1	2	3	4	4
5	4	3	2	1	0	1	2	3	4
6	4	4	3	2	1	2	3	4	4
7	3	4	4	3	2	3	4	4	3
8	2	3	4	4	3	4	4	3	2
9	1	2	3	4	4	4	3	2	1

1	8	8	8	8	8	8	8	8	8
2	8	6	6	6	6	6	6	6	8
3	8	6	4	4	4	4	4	6	8
4	8	6	4	2	2	2	4	6	8
5	8	6	4	2	0	2	4	6	8
6	8	6	4	2	2	2	4	6	8
7	8	6	4	4	4	4	4	6	8
8	8	6	6	6	6	6	6	6	8
9	8	8	8	8	8	8	8	8	8

Concentricity Patterns in all Pangenic Tables

All pangenic squares have the following *concentricity* patterns:

Even-size squares as depicted at the top have frames that sum consecutively to the series:

1, 3, 5, 7 ... (n-1)

The numbers in concentric frames indicate the number of quadrals in that respective frame. The diagram to its right shows the source of the **5** quadrals in the 3rd frame.

Similar patterns are found in odd-size squares too:

Two odd-size square representations are shown here at left. The diagrams at their right depict the concentric frame with **4** quadrals and show where these quadrals are located in that frame.

Now observe the patterns of the quantity of quadrals in concentric frames emanating from the center: In odd-size squares, the pattern is both:

0, 1, 2, 3 ... (n-1)/2, (n-1)/2 ... 3, 2, 1, 0

and **0, 2, 4, 6 ... (n-1).**

The first series is a perfect harmonic waveform. Clearly, the latter series for odd-size squares can be converted to an intelligent pattern by adding **1** to each number to get: **1, 3, 5, 7 ... n.**

Now each of these series is either an *intelligent pattern* or a perfect *harmonic waveform as* defined at the beginning of this program series. Since all tables in Geonometry of any dimension are pangenic, all squares of any class possess these nested concentric frames whose total numbers in each frame are multiples of its kernel number which sum sequentially along either an intelligent-pattern or a harmonic waveform.

Since octals in cubes and hexadectals in quadracubes are composed of symmetrically located quadrals, these patterns extend to higher dimensions too. In odd-size tables of higher dimensions, the central embedded square is not involved in octal or hexadectal summations, so it doesn't come into play there.

In quadracubes, not only does the intelligent pattern appear in the quadrals among 4 symmetrically located embedded block squares, but it appears at the block-square level too, all simultaneously!

Here we see that intelligent patterns and harmonic waveforms arise naturally in Geonometry. So now, having observed all the amazing equality patterns in Class-1 and Class-4 tables plus having seen the roles that Class-1 and Class-4 tables played in interpreting the structure of the atom, and to find yet even more, can you deny the underlying intelligent patterning inherent in the fabric of space?

Program 8

The Zero-differential Property

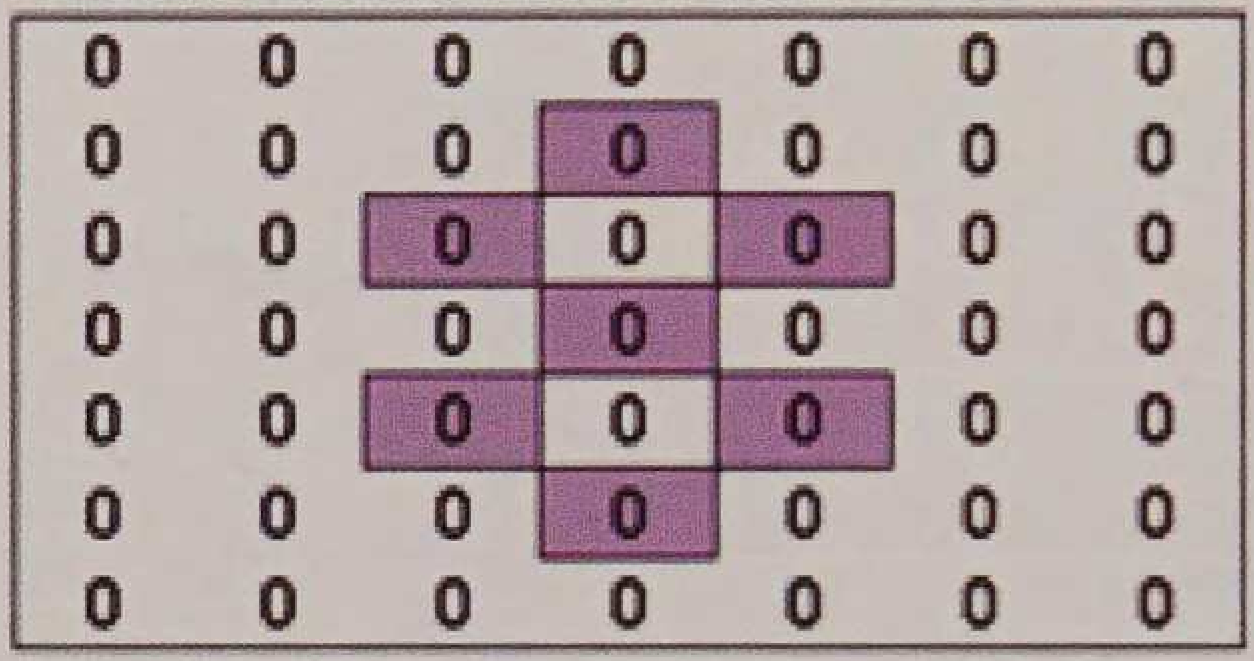
	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175
1	7	18	29	47	9	27	38	175
2	44	13	24	42	4	15	33	175
3	39	1	19	30	48	10	28	175
4	34	45	14	25	36	5	16	175
5	22	40	2	20	31	49	11	175
6	17	35	46	8	26	37	6	175
7	12	23	41	3	21	32	43	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

Here is shown column-difference and row-difference tables for the size-7 square. Observe that the characteristic number of these difference-tables is zero. Further, the size-7 complementary characteristic tile patterns both sum to zero everywhere in the difference table. This property holds for all ultra-perfect squares.

Column differences

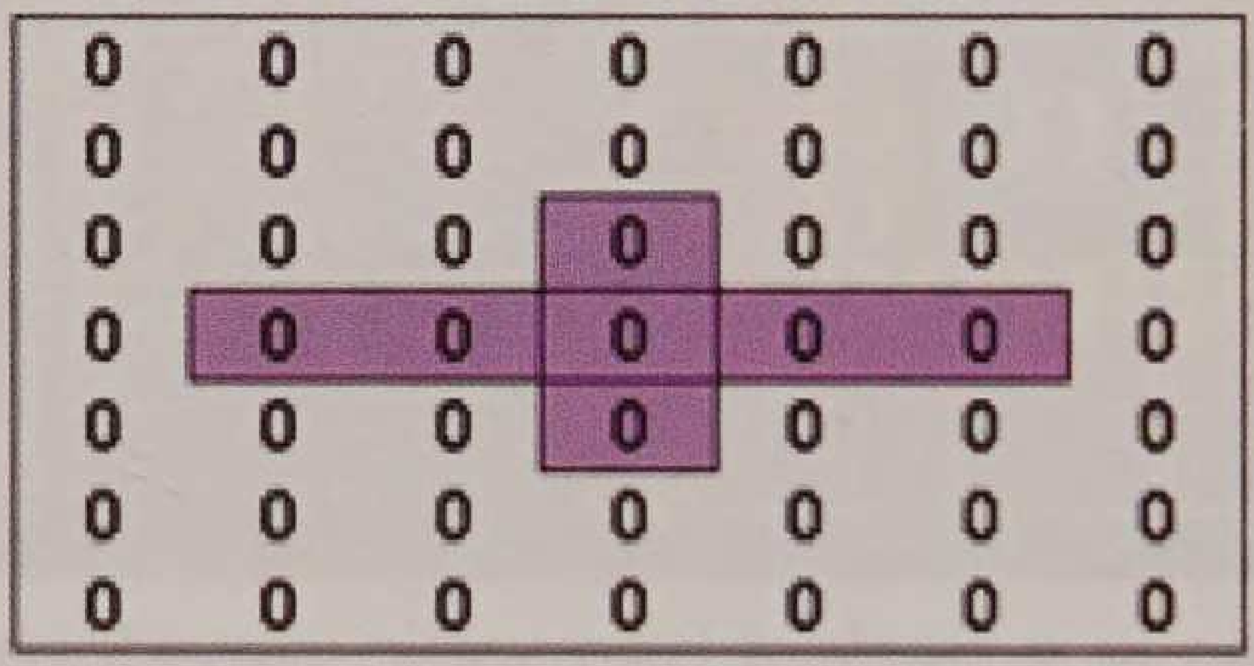
	0	0	0	0	0	0	0
-11	-11	-18	38	-18	-11	31	0
31	-11	-18	38	-11	-18	-11	0
38	-18	-11	-18	38	-18	-11	0
-11	31	-11	-11	31	-11	-18	0
-18	38	-18	-11	-18	38	-11	0
-18	-11	38	-18	-11	31	-11	0
-11	-18	38	-18	-11	-11	31	0
0	0	0	0	0	0	0	
	0	0	0	0	0	0	0

Pattern A sum of col. differences



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of col. differences

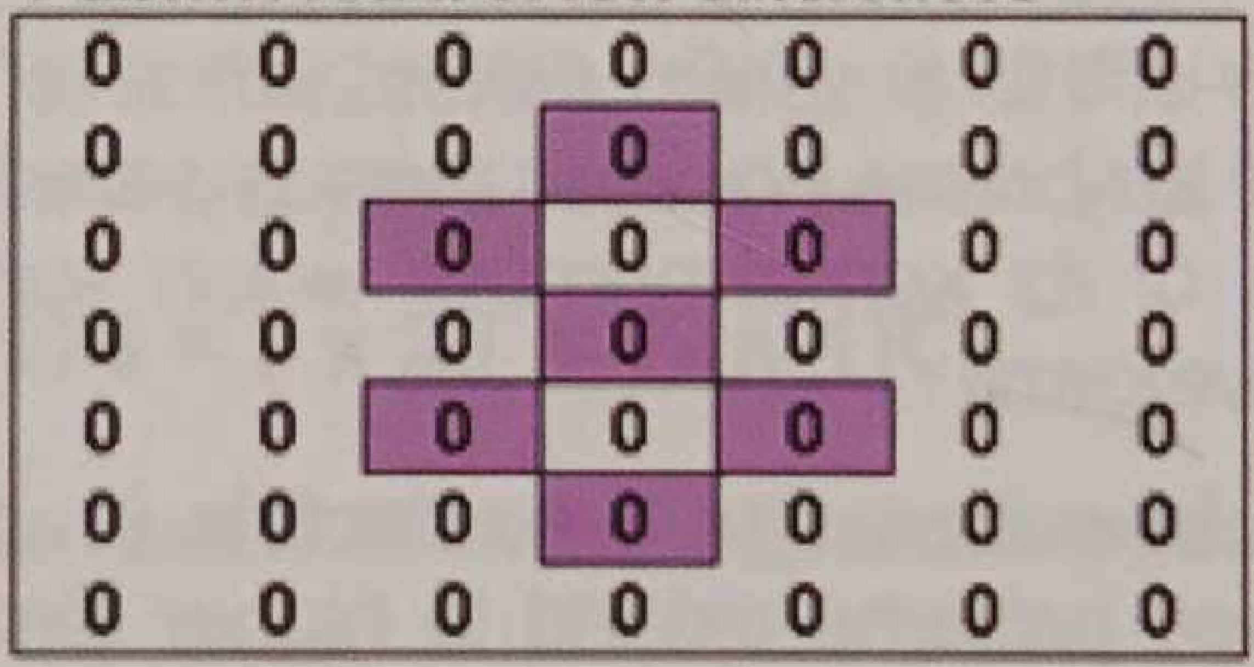


0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Row differences

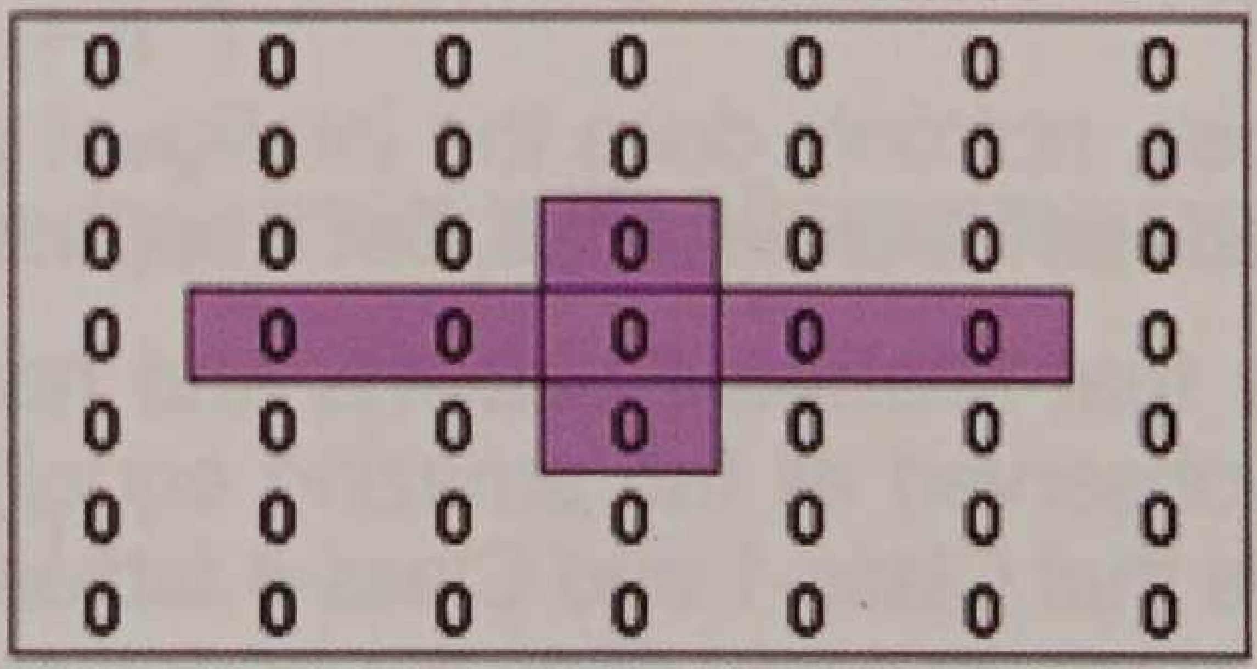
	0	0	0	0	0	0	0
-37	5	5	5	5	12	5	0
5	12	5	12	-44	5	5	0
5	-44	5	5	12	5	12	0
12	5	12	5	5	-44	5	0
5	5	-44	12	5	12	5	0
5	12	5	5	5	5	-37	0
5	5	12	-44	12	5	5	0
0	0	0	0	0	0	0	
	0	0	0	0	0	0	0

Pattern A sum of row differences



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of row differences



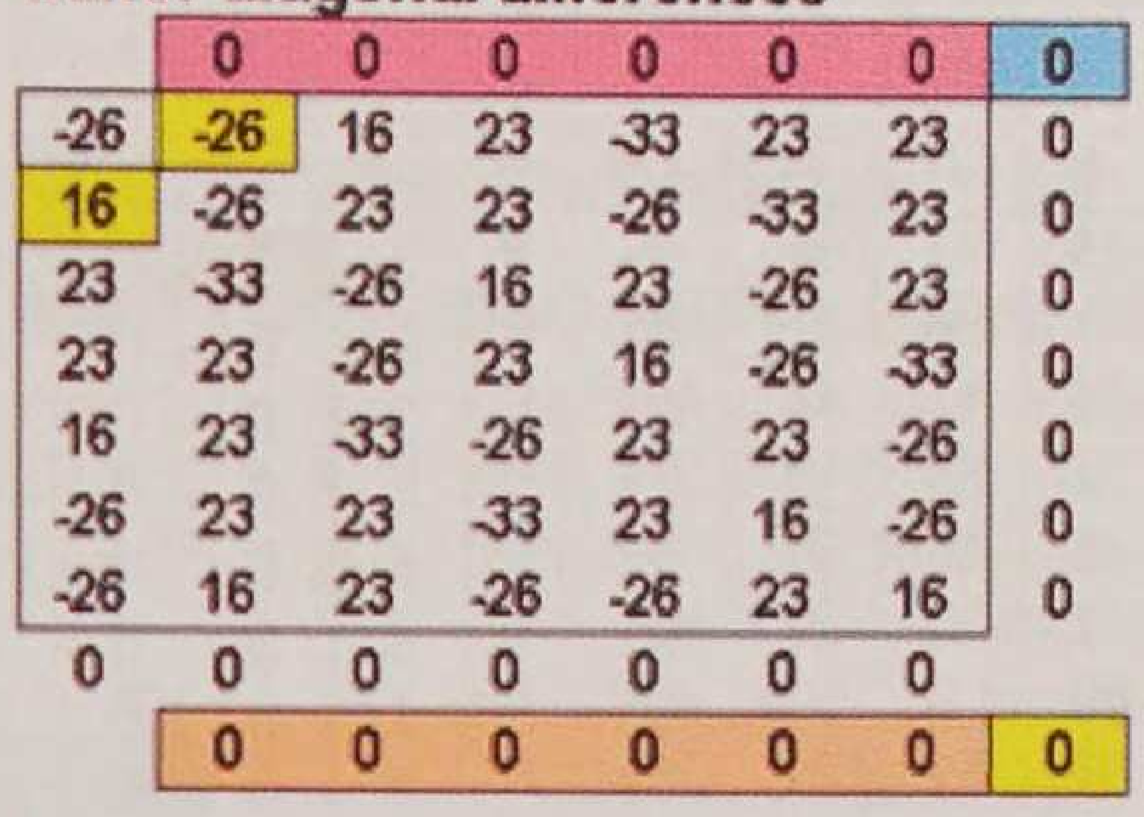
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

This property also holds for the characteristic block-square modularity common to all class-3 squares too.

Program 8

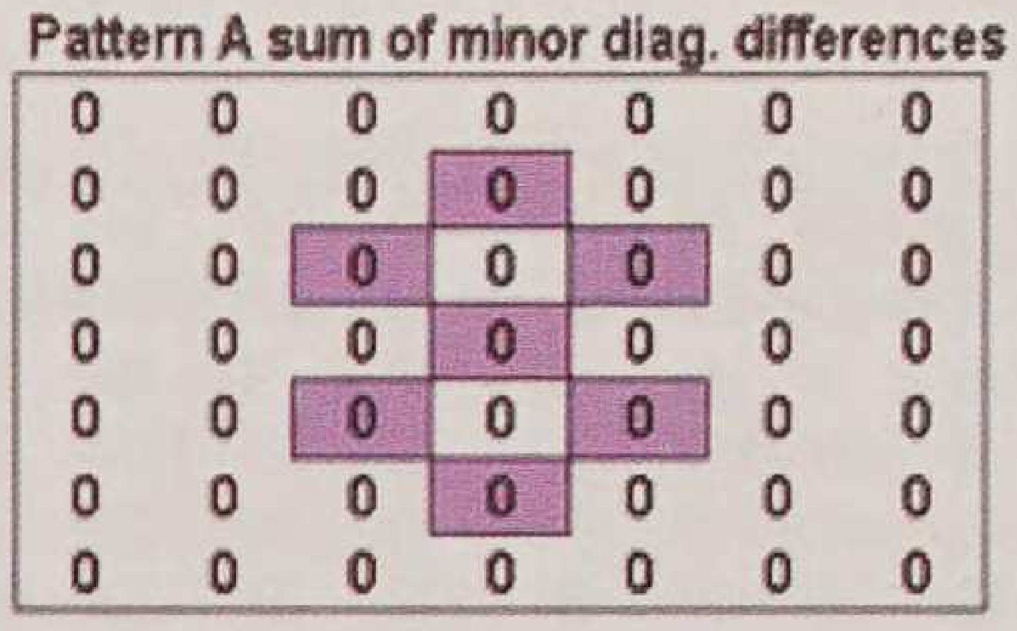
Here is shown two more difference tables for the size-7 square. They now are differences along the two different diagonal directions. Observe that the characteristic number of these difference-tables is zero too. Further, the size-7 characteristic tile patterns also both sum to zero in these difference tables everywhere. This property holds for all ultra-perfect squares.

Minor diagonal differences



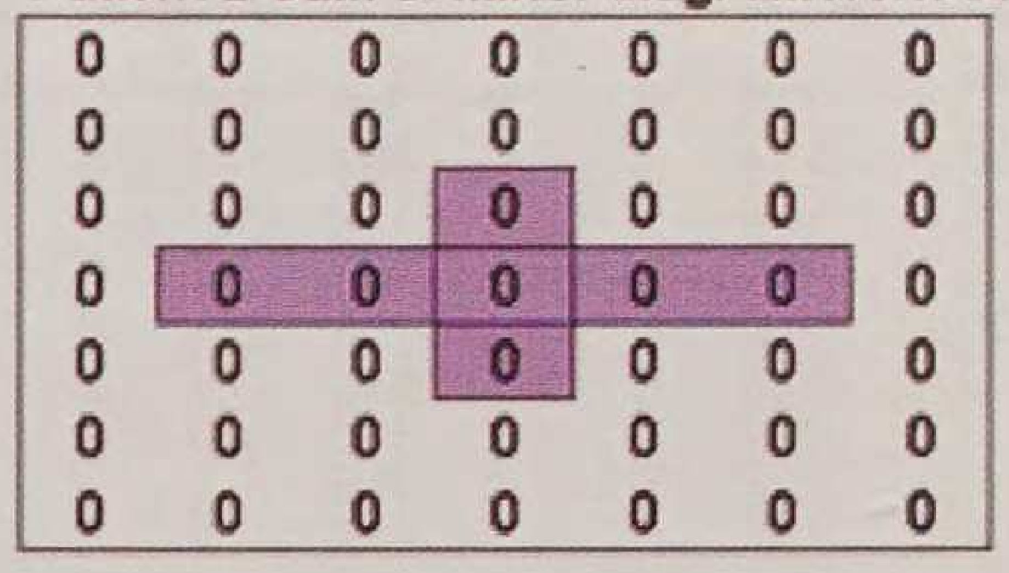
	0	0	0	0	0	0	0
-26	-26	16	23	-33	23	23	0
16	-26	23	23	-26	-33	23	0
23	-33	-26	16	23	-26	23	0
23	23	-26	23	16	-26	-33	0
16	23	-33	-26	23	23	-26	0
-26	23	23	-33	23	16	-26	0
-26	16	23	-26	-26	23	16	0
0	0	0	0	0	0	0	
	0	0	0	0	0	0	0

Pattern A sum of minor diag. differences



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

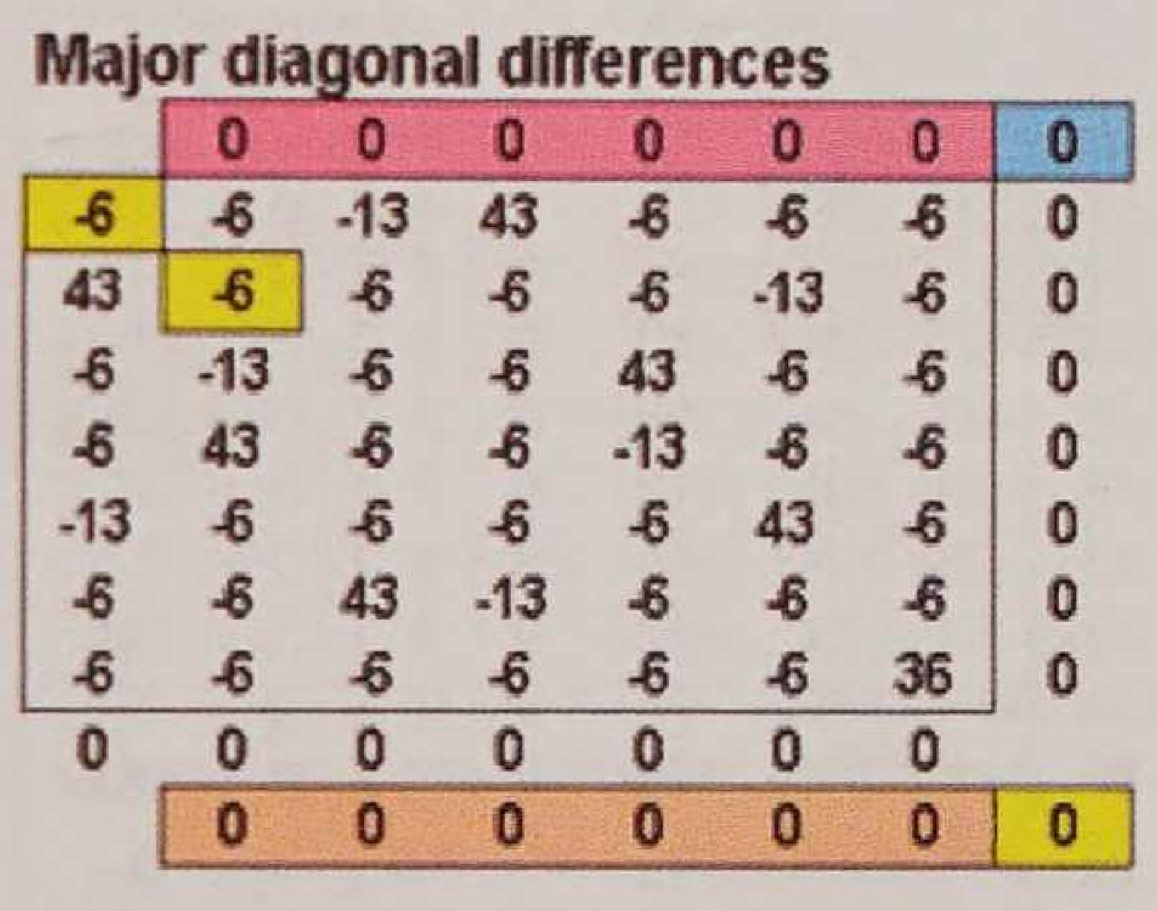
Pattern B sum of minor diag. differences



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

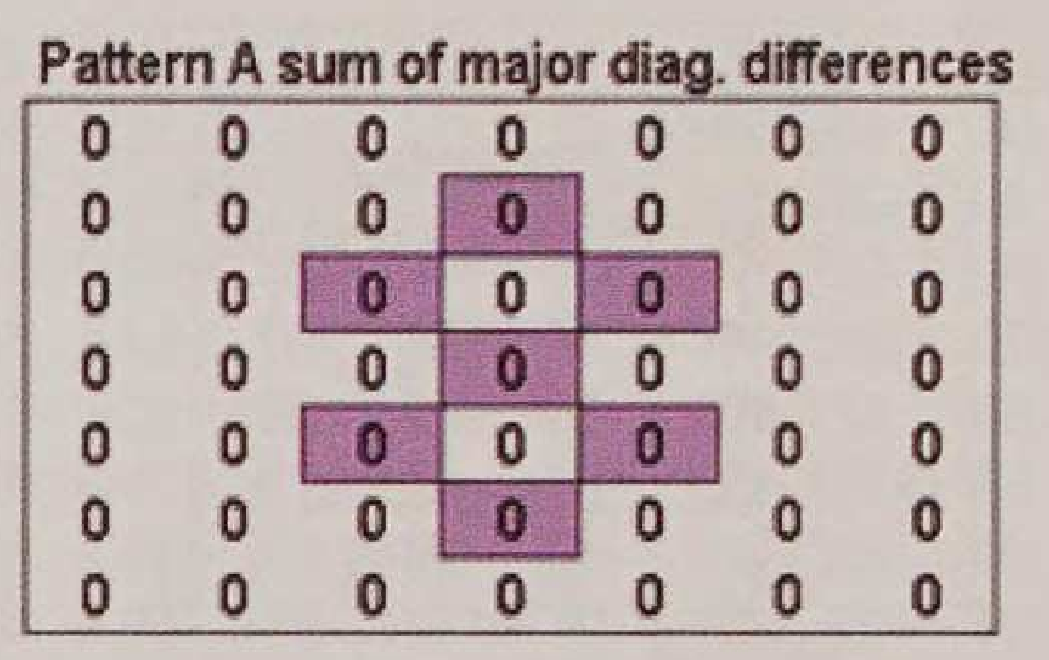
This property might seem obvious for sums taken in the same direction as were the differences because these differences wrap around in that direction. But to also hold for both tile patterns and for all the other directional differences all simultaneously in all instances is absolutely astounding!

Major diagonal differences



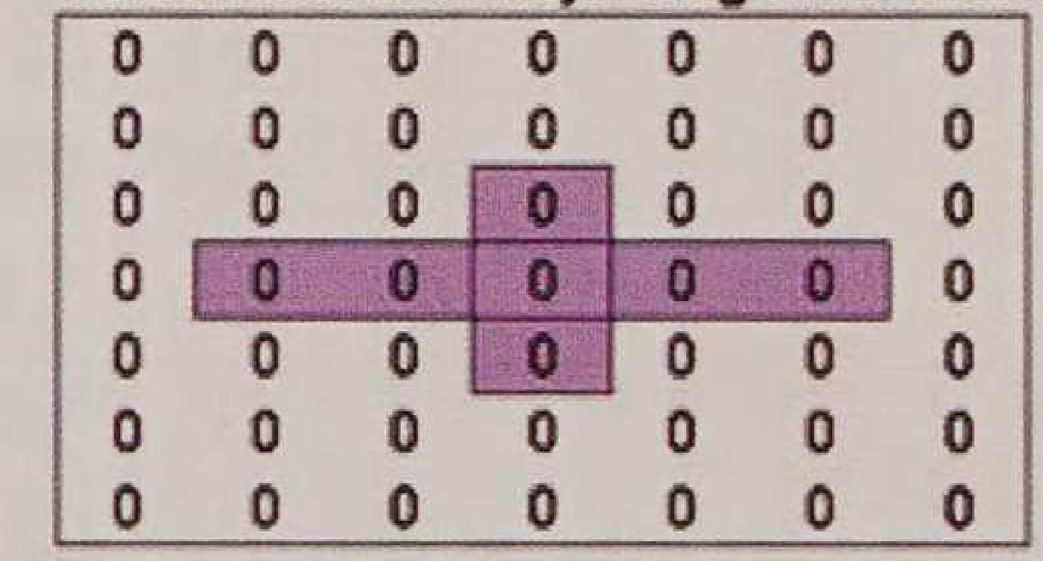
	0	0	0	0	0	0	0
-6	-6	-13	43	-6	-6	-6	0
43	-6	-6	-6	-6	-13	-6	0
-6	-13	-6	-6	43	-6	-6	0
-6	43	-6	-6	-13	-6	-6	0
-13	-6	-6	-6	-6	43	-6	0
-6	-6	43	-13	-6	-6	-6	0
-6	-6	-6	-6	-6	-6	36	0
0	0	0	0	0	0	0	
	0	0	0	0	0	0	0

Pattern A sum of major diag. differences



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Pattern B sum of major diag. differences



0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

In differential calculus, a *zero-differential* in two directions (2 variables) indicates a plane of balance. Here it means that the numeric table is perfectly balanced with respect to all 4 directions simultaneously! When, in addition, both of its inherent tile patterns sum to zero throughout all the difference tables, each tile is balanced throughout the numeric table in all 4 directions at every point.

If a square composed of multiple same-size ultra-perfect squares were to be laid atop a placid pool of water and a stone were pitched onto it anywhere within and clear of its edges, no ripples would emanate from the square.

In classical mathematics this property is called **isotropism**. In this new numeric geometry, the tables are said to be ***differentially isotropic***, i.e. their difference sums are invariant with respect to all the directions regardless of the direction in which the adjacent number-pair differences are taken.

Among all the possible square tables of sequential numbers, only ultra-perfect geonomic squares exhibit this property. For Classes 1 & 4 squares it's their tile patterns which sums to **0** anywhere and everywhere. For Classes 3, 5 & 6 squares, it's their block-square modularity and/or block-tile patterns which sum to **0** anywhere. And that's for every class of square except those of Class-2.

Program 8

The Zero-Differential property on the Size-12 Class-4 Square

	0	0	0	0	0	0	0	0	0	0	0	0
-50	-44	99	-10	-40	-45	94	-8	-45	-46	104	-9	0
121	-23	-27	-67	113	-27	-23	-59	117	-31	-31	-63	0
-98	4	45	50	-100	9	46	40	-99	14	44	45	0
28	58	-117	32	26	63	-116	22	27	68	-118	27	0
-41	-41	99	-13	-49	-45	103	-5	-45	-49	95	-9	0
112	-26	-27	-64	122	-27	-32	-62	117	-28	-22	-63	0
-104	10	45	44	-94	9	40	46	-99	8	50	45	0
31	67	-117	23	23	63	-113	31	27	59	-121	27	0
-44	-50	99	-4	-46	-45	100	-14	-45	-40	98	-9	0
118	-32	-27	-58	116	-27	-26	-68	117	-22	-28	-63	0
-95	13	45	41	-103	9	49	49	-99	5	41	45	0
22	64	-117	26	32	63	-122	28	27	62	-112	27	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0
-106	65	86	-40	-97	56	74	-43	-94	68	83	-52	0
131	-88	-61	11	128	-85	-49	20	119	-97	-52	23	0
-81	45	99	-63	-81	45	99	-63	-81	45	99	-63	0
49	-20	-119	97	52	-23	-131	88	61	-11	-128	85	0
-92	61	76	-50	-101	70	88	-47	-104	58	79	-38	0
126	-90	-54	18	126	-90	-54	18	126	-90	-54	18	0
-88	47	104	-58	-79	38	92	-61	-76	50	101	-70	0
59	-16	-133	83	56	-13	-121	92	47	-25	-124	95	0
-99	63	81	-45	-99	63	81	-45	-99	63	81	-45	0
121	-92	-47	25	124	-95	-59	16	133	-83	-56	13	0
-74	43	94	-68	-83	52	106	-65	-86	40	97	-56	0
54	-18	-126	90	54	-18	-126	90	54	-18	-126	90	0
0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
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3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
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11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0
15	42	59	-107	16	29	51	-102	23	37	52	-115	0
33	-84	-16	61	28	-76	-3	60	20	-83	-8	68	0
-53	103	-18	-31	-55	108	-17	-41	-54	113	-19	-36	0
8	-61	-20	84	3	-68	-28	83	16	-60	-33	76	0
20	35	49	-114	21	43	56	-109	13	30	57	-101	0
22	-80	-9	62	32	-81	-14	64	27	-82	-4	63	0
-57	114	-13	-35	-56	101	-21	-30	-49	109	-20	-43	0
15	-66	-34	79	10	-58	-21	78	2	-65	-26	86	0
19	31	54	-103	17	36	55	-113	18	41	53	-108	0
26	-79	-2	66	21	-86	-10	65	34	-78	-15	58	0
-52	107	-23	-42	-51	115	-16	-37	-59	102	-15	-29	0
4	-62	-27	80	14	-63	-32	82	9	-64	-22	81	0
0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
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9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0
56	-109	13	30	57	-101	20	35	49	-114	21	43	0
-10	65	34	-78	-15	58	26	-79	-2	66	21	-86	0
-17	-41	-54	113	-19	-36	-53	103	-18	-31	-55	108	0
-21	78	2	-65	-26	86	15	-66	-34	79	10	-58	0
51	-102	23	37	52	-115	15	42	59	-107	16	29	0
-14	64	27	-82	-4	63	22	-80	-9	62	32	-81	0
-16	-37	-59	102	-15	-29	-52	107	-23	-42	-51	115	0
-28	83	16	-60	-33	76	8	-61	-20	84	3	-68	0
55	-113	18	41	53	-108	19	31	54	-103	17	36	0
-3	60	20	-83	-8	68	33	-84	-16	61	28	-76	0
-21	-30	-49	109	-20	-43	-57	114	-13	-35	-56	101	0
-32	82	9	-64	-22	81	4	-62	-27	80	14	-63	0
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	1	2	3	4	5	6	7	8	9	10	11	12
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2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

Program 8

Class-2 squares lack the zero-differential property

This is typical of near-perfect Class-2 squares and shows that they lack the zero-differential property that all perfect squares share. They also don't have tile patterns or any modularity property that would sum to **0** either.

	1	2	3	4	5	6		
W(6)		111	111	111	111	111	111	
1	6	33	12	25	4	31	111	
2	17	16	21	15	22	19	110	(-1)
3	30	3	36	1	34	7	111	
4	32	9	26	11	28	5	111	
5	18	23	14	24	13	20	112	(+1)
6	8	27	2	35	10	29	111	
	111	111	111	111	111	111		
		111	111	111	111	111	111	

	1	2	3	4	5	6	
1	6	33	12	25	4	31	6
2	17	16	21	15	22	19	17
3	30	3	36	1	34	7	30
4	32	9	26	11	28	5	32
5	18	23	14	24	13	20	18
6	8	27	2	35	10	29	8
	6	33	12	25	4	31	6

	0	0	0	0	0	0
-27	21	-13	21	-27	25	0
1	-5	6	-7	3	2	0
27	-33	35	-33	27	-23	0
23	-17	15	-17	23	-27	0
-5	9	-10	11	-7	2	0
-19	25	-33	25	-19	21	0
0	0	0	0	0	0	
	0	0	0	0	0	0

	0	0	0	0	0	0
-11	17	-9	10	-18	12	1
-13	13	-15	14	-12	12	-1
-2	-6	10	-10	6	2	0
14	-14	12	-13	15	-15	-1
10	-4	12	-11	3	-9	1
2	-6	-10	10	6	-2	0
0	0	0	0	0	0	
	0	0	0	0	0	0

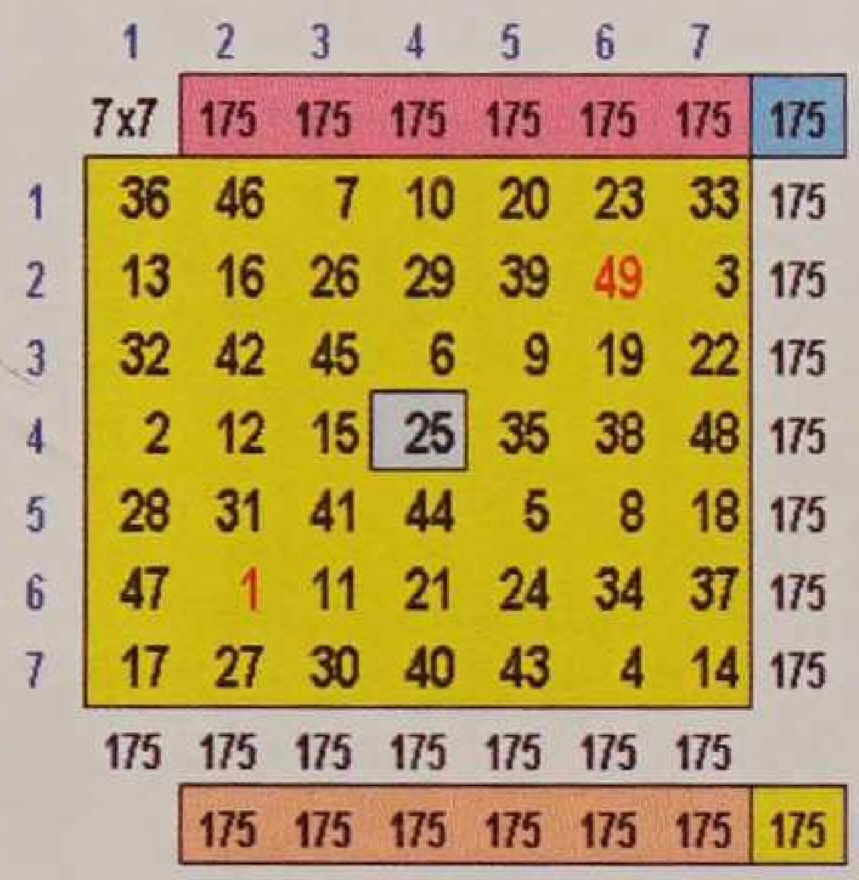
	0	0	0	0	0	0
-10	12	-3	3	-15	14	1
14	-20	20	-19	15	-11	-1
21	-23	25	-27	29	-25	0
9	-5	2	-2	8	-13	-1
-9	21	-21	14	-16	12	1
-25	15	-23	31	-21	23	0
0	0	0	0	0	0	
	0	0	0	0	0	0

	0	0	0	0	0	0
-16	4	-4	11	-9	13	-1
14	-18	21	-21	15	-10	1
29	-27	25	-23	21	-25	0
9	-3	3	-4	8	-12	1
-15	13	-22	22	-10	11	-1
-21	31	-23	15	-25	23	0
0	0	0	0	0	0	
	0	0	0	0	0	0

Program 8

The Sub-additive Property of all perfect Squares

The sub-additive property of perfect squares is that, given a square **W** of size **n**, a square **R** of the same size whose numbers are the **(n-1)x(n-1)** continuous summations within **W**, with wrapping imposed in the summations, will reduce back to **W** upon collapse and normalization. Here it is demonstrated for the size 7 square Class-1 square. The extreme values are depicted in **red**.



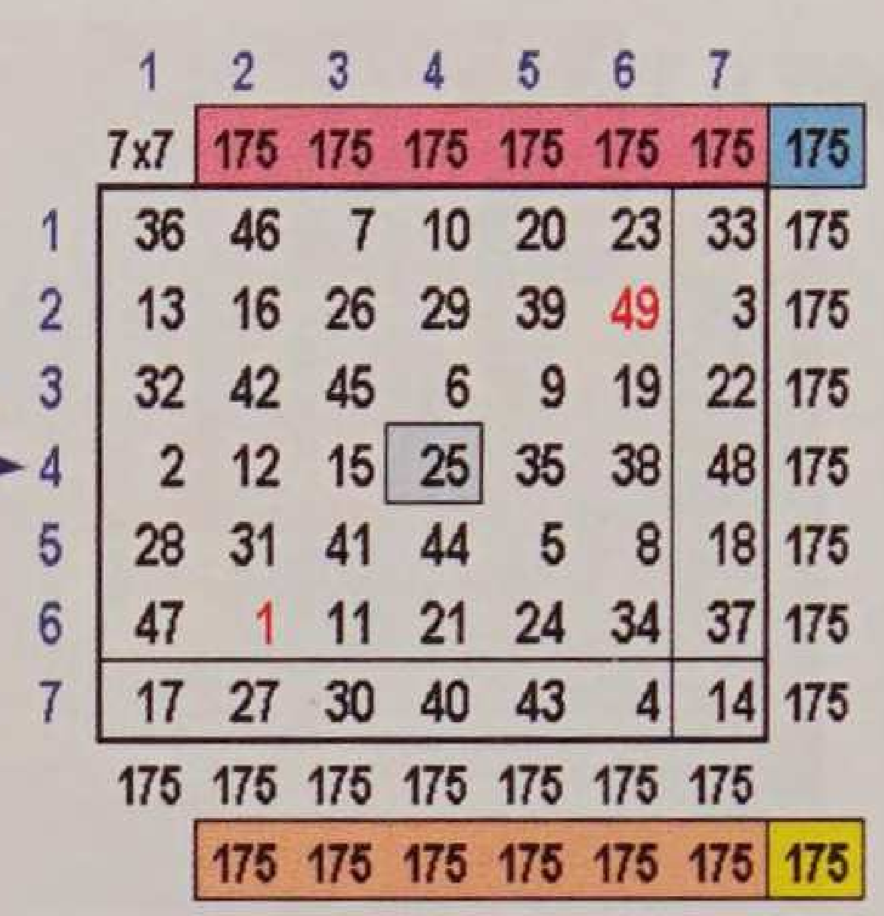
	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175
1	36	46	7	10	20	23	33	175
2	13	16	26	29	39	49	3	175
3	32	42	45	6	9	19	22	175
4	2	12	15	25	35	38	48	175
5	28	31	41	44	5	8	18	175
6	47	1	11	21	24	34	37	175
7	17	27	30	40	43	4	14	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

R(6x6) Minimum = 876

889	892	902	905	915	918	879
908	911	921	882	885	895	898
878	888	891	901	904	914	924
897	907	917	920	881	884	894
923	877	887	890	900	910	913
893	903	906	916	919	880	883
912	922	876	886	896	899	909

R-875

14	17	27	30	40	43	4
33	36	46	7	10	20	23
3	13	16	26	29	39	49
22	32	42	45	6	9	19
48	2	12	15	25	35	38
18	28	31	41	44	5	8
37	47	1	11	21	24	34



	1	2	3	4	5	6	7	
	7x7	175	175	175	175	175	175	175
1	36	46	7	10	20	23	33	175
2	13	16	26	29	39	49	3	175
3	32	42	45	6	9	19	22	175
4	2	12	15	25	35	38	48	175
5	28	31	41	44	5	8	18	175
6	47	1	11	21	24	34	37	175
7	17	27	30	40	43	4	14	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

Next, this property is demonstrated for the size-11 square from Class-1.

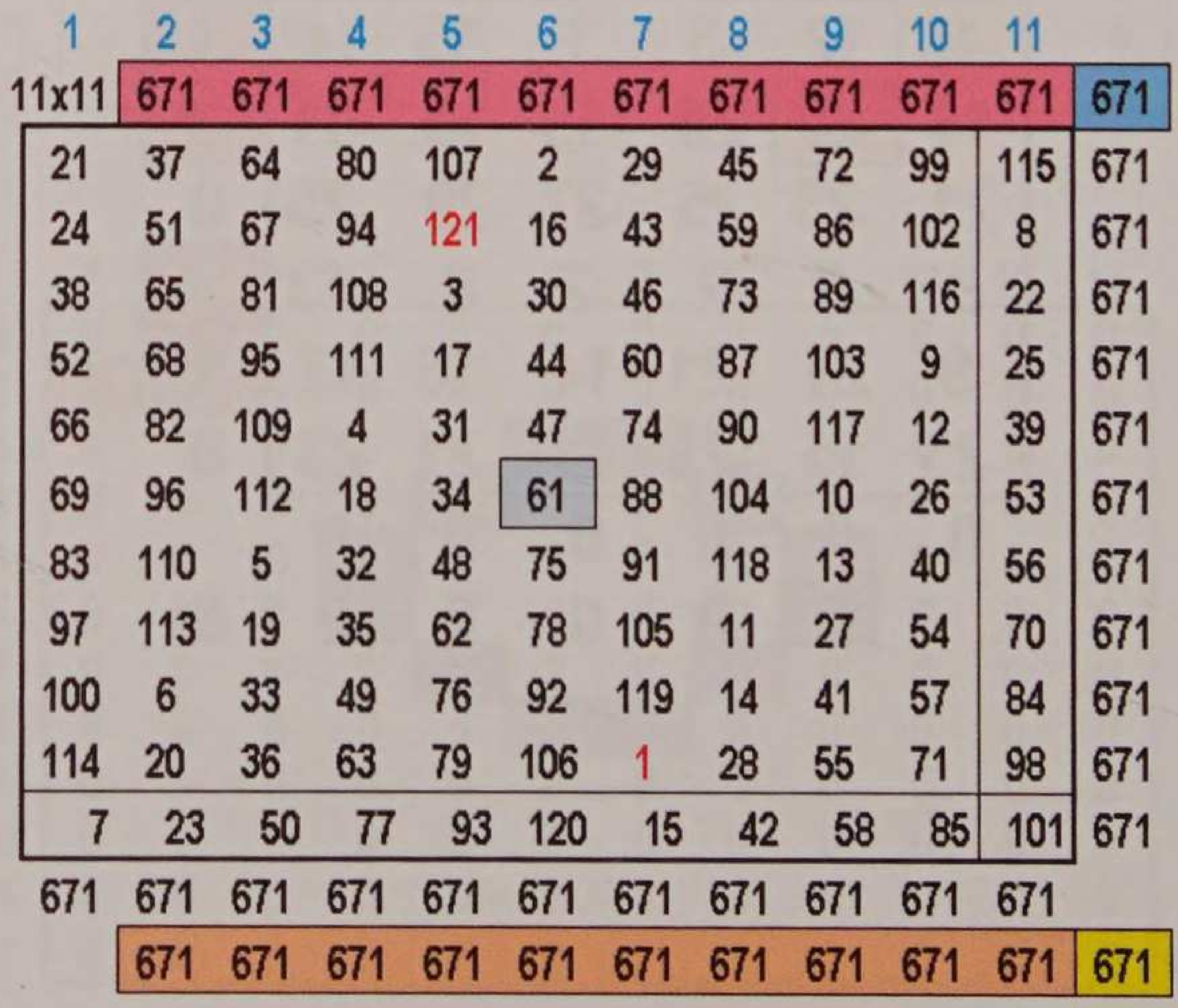
11x11	671	671	671	671	671	671	671	671	671	671	671
21	37	64	80	107	2	29	45	72	99	115	671
24	51	67	94	121	16	43	59	86	102	8	671
38	65	81	108	3	30	46	73	89	116	22	671
52	68	95	111	17	44	60	87	103	9	25	671
66	82	109	4	31	47	74	90	117	12	39	671
69	96	112	18	34	61	88	104	10	26	53	671
83	110	5	32	48	75	91	118	13	40	56	671
97	113	19	35	62	78	105	11	27	54	70	671
100	6	33	49	76	92	119	14	41	57	84	671
114	20	36	63	79	106	1	28	55	71	98	671
7	23	50	77	93	120	15	42	58	85	101	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671

R(11x11) Minimum = 6040

1	2	3	4	5	6	7	8	9	10	11
6140	6046	6062	6089	6116	6132	6159	6054	6081	6097	6124
6154	6060	6076	6103	6119	6146	6041	6068	6084	6111	6138
6047	6063	6090	6106	6133	6160	6055	6082	6098	6125	6141
6061	6077	6104	6120	6147	6042	6069	6085	6112	6128	6155
6064	6091	6107	6134	6150	6056	6083	6099	6126	6142	6048
6078	6105	6121	6148	6043	6070	6086	6113	6129	6156	6051
6092	6108	6135	6151	6057	6073	6100	6127	6143	6049	6065
6095	6122	6149	6044	6071	6087	6114	6130	6157	6052	6079
6109	6136	6152	6058	6074	6101	6117	6144	6050	6066	6093
6123	6139	6045	6072	6088	6115	6131	6158	6053	6080	6096
6137	6153	6059	6075	6102	6118	6145	6040	6067	6094	6110

R-6039

1	2	3	4	5	6	7	8	9	10	11
101	7	23	50	77	93	120	15	42	58	85
115	21	37	64	80	107	2	29	45	72	99
8	24	51	67	94	121	16	43	59	86	102
22	38	65	81	108	3	30	46	73	89	116
25	52	68	95	111	17	44	60	87	103	9
39	66	82	109	4	31	47	74	90	117	12
53	69	96	112	18	34	61	88	104	10	26
56	83	110	5	32	48	75	91	118	13	40
70	97	113	19	35	62	78	105	11	27	54
84	100	6	33	49	76	92	119	14	41	57
98	114	20	36	63	79	106	1	28	55	71



1	2	3	4	5	6	7	8	9	10	11	
11x11	671	671	671	671	671	671	671	671	671	671	671
21	37	64	80	107	2	29	45	72	99	115	671
24	51	67	94	121	16	43	59	86	102	8	671
38	65	81	108	3	30	46	73	89	116	22	671
52	68	95	111	17	44	60	87	103	9	25	671
66	82	109	4	31	47	74	90	117	12	39	671
69	96	112	18	34	61	88	104	10	26	53	671
83	110	5	32	48	75	91	118	13	40	56	671
97	113	19	35	62	78	105	11	27	54	70	671
100	6	33	49	76	92	119	14	41	57	84	671
114	20	36	63	79	106	1	28	55	71	98	671
7	23	50	77	93	120	15	42	58	85	101	671
671	671	671	671	671	671	671	671	671	671	671	
	671	671	671	671	671	671	671	671	671	671	671

Program 8

Size-5 Square

Here is the size-5 Class-1 square. The table below is composed of quadrants of the size-5. This construct makes it easy to calculate those block-sums that wrap across the edges of the square.

20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6

P Sums of 2x2s

43	50	62	49	56
47	59	41	53	60
51	63	45	57	44
55	42	54	61	48
64	46	58	40	52

Q Sums of 3x3s

126	113	120	107	119
105	117	129	111	123
114	121	108	115	127
118	125	112	124	106
122	109	116	128	110

R Sums of 4x4s

201	219	207	200	213
197	215	203	216	209
218	206	199	212	205
214	202	220	208	196
210	198	211	204	217

P−39	65	65	65	65	65
4	11	23	10	17	65
8	20	2	14	21	65
12	24	6	18	5	65
16	3	15	22	9	65
25	7	19	1	13	65
65	65	65	65	65	
	65	65	65	65	65

Q−104	65	65	65	65	65
22	9	16	3	15	65
1	13	25	7	19	65
10	17	4	11	23	65
14	21	8	20	2	65
18	5	12	24	6	65
65	65	65	65	65	
	65	65	65	65	65

R−195	65	65	65	65	65
6	24	12	5	18	65
2	20	8	21	14	65
23	11	4	17	10	65
19	7	25	13	1	65
15	3	16	9	22	65
65	65	65	65	65	
	65	65	65	65	65

Normalized

	65	65	65	65	65
6	18	5	12	24	65
15	22	9	16	3	65
19	1	13	25	7	65
23	10	17	4	11	65
2	14	21	8	20	65
65	65	65	65	65	
	65	65	65	65	65

Normalized

	65	65	65	65	65
6	18	5	12	24	65
15	22	9	16	3	65
19	1	13	25	7	65
23	10	17	4	11	65
2	14	21	8	20	65
65	65	65	65	65	
	65	65	65	65	65

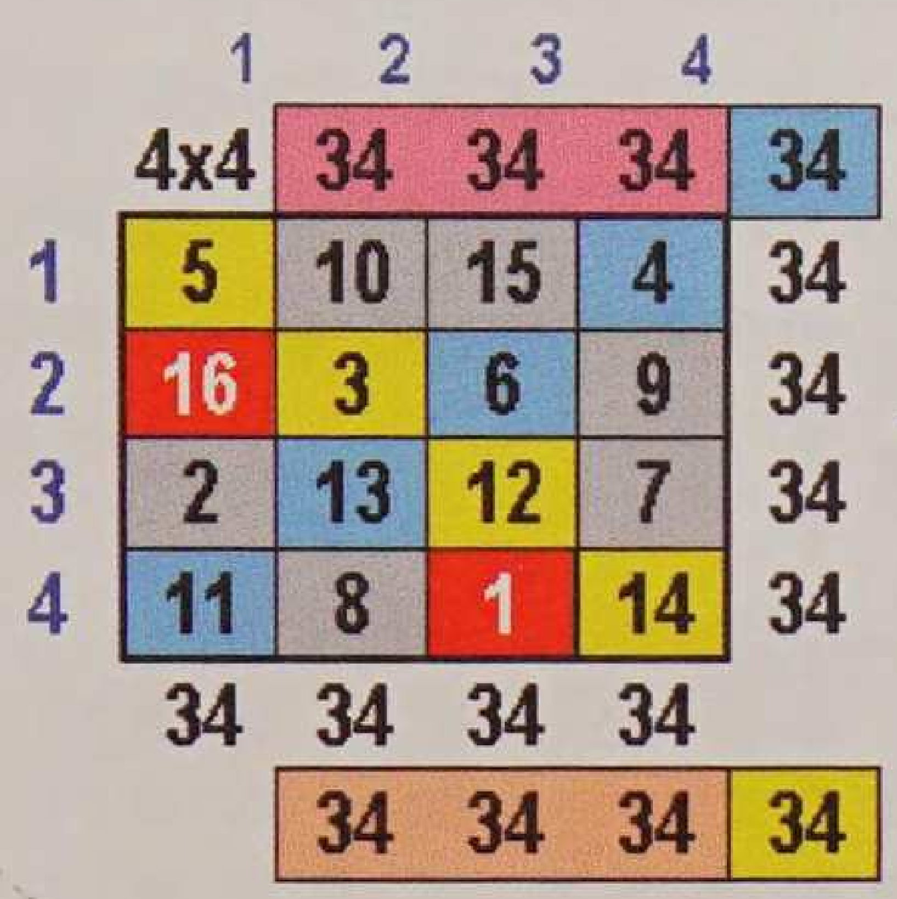
Normalized

	65	65	65	65	65
20	8	21	14	2	65
11	4	17	10	23	65
7	25	13	1	19	65
3	16	9	22	15	65
24	12	5	18	6	65
65	65	65	65	65	
	65	65	65	65	65

Block-square sums of size **2x2**, **3x3** and **4x4** are taken with wrapping in the top table. Upon reduction and normalization which places the pivot number at its center, all such derived **5x5** tables reduce to either the original perfect square or its reflection. This property extended to every sub-size block-square and was found to apply only to the size-5 square.

Program 8

Here is the perfect size-4 square from Class-4. The sub-additive property yields its identical twin.



	1	2	3	4	
	4x4	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
		34	34	34	34

→

R(8x8) Min = 69

82	79	76	69
72	73	78	83
77	84	71	74
75	70	81	80

→

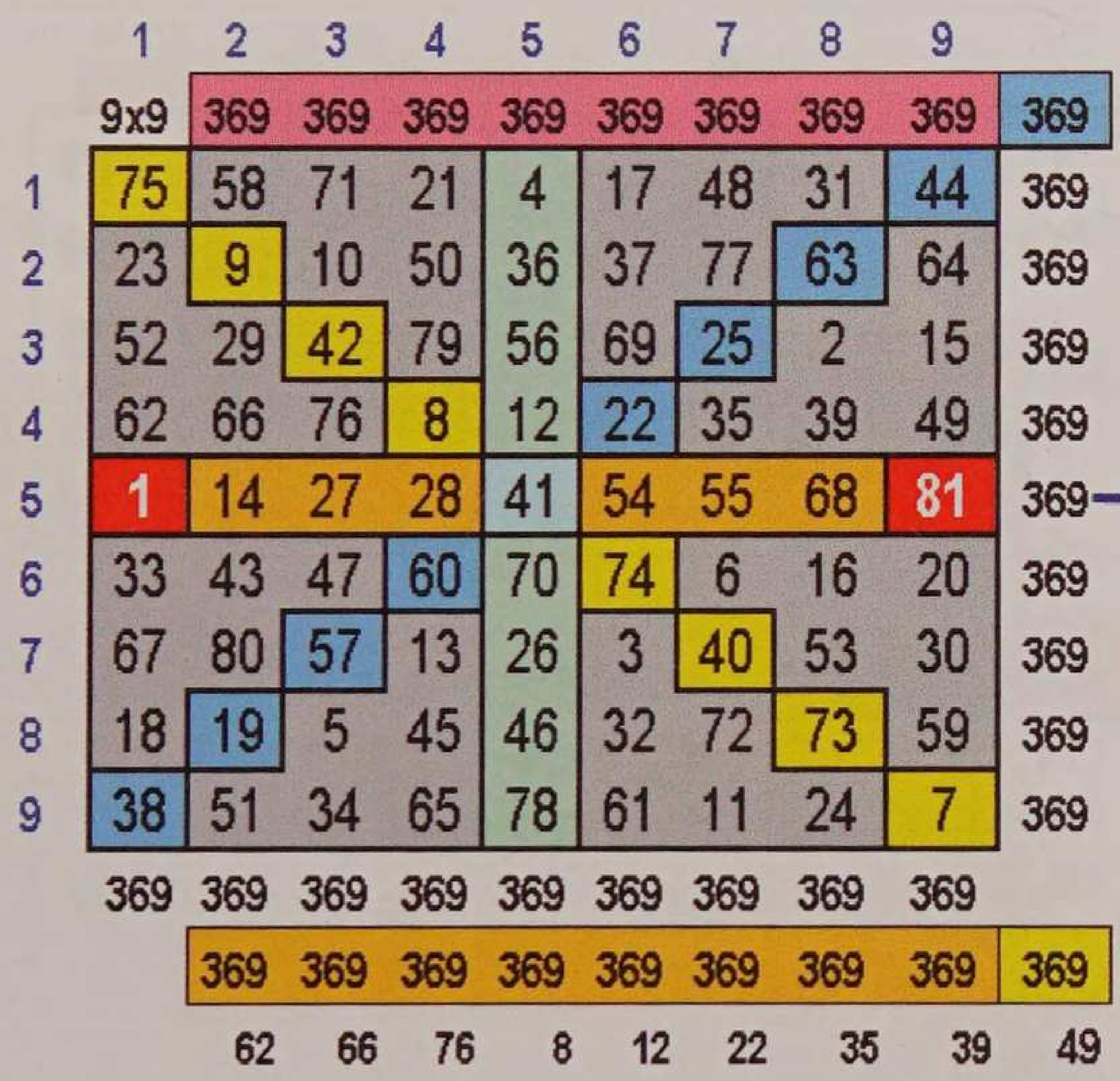
R-68

14	11	8	1
4	5	10	15
9	16	3	6
7	2	13	12

→

	1	2	3	4	
	W(4)	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
		34	34	34	34

The same thing results for the size-9 square from Class-6.



	1	2	3	4	5	6	7	8	9	
	9x9	369	369	369	369	369	369	369	369	369
1	75	58	71	21	4	17	48	31	44	369
2	23	9	10	50	36	37	77	63	64	369
3	52	29	42	79	56	69	25	2	15	369
4	62	66	76	8	12	22	35	39	49	369
5	1	14	27	28	41	54	55	68	81	369
6	33	43	47	60	70	74	6	16	20	369
7	67	80	57	13	26	3	40	53	30	369
8	18	19	5	45	46	32	72	73	59	369
9	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

62 66 76 8 12 22 35 39 49

→

R(8x8) Minimum = 2584

2590	2621	2634	2617	2648	2661	2644	2594	2607
2627	2658	2641	2654	2604	2587	2600	2631	2614
2647	2606	2592	2593	2633	2619	2620	2660	2646
2598	2635	2612	2625	2662	2639	2652	2608	2585
2632	2645	2649	2659	2591	2595	2605	2618	2622
2664	2584	2597	2610	2611	2624	2637	2638	2651
2603	2616	2626	2630	2643	2653	2657	2589	2599
2613	2650	2663	2640	2596	2609	2586	2623	2636
2642	2601	2602	2588	2628	2629	2615	2655	2656

→

R-2583

7	38	51	34	65	78	61	11	24
44	75	58	71	21	4	17	48	31
64	23	9	10	50	36	37	77	63
15	52	29	42	79	56	69	25	2
49	62	66	76	8	12	22	35	39
81	1	14	27	28	41	54	55	68
20	33	43	47	60	70	74	6	16
30	67	80	57	13	26	3	40	53
59	18	19	5	45	46	32	72	73

→

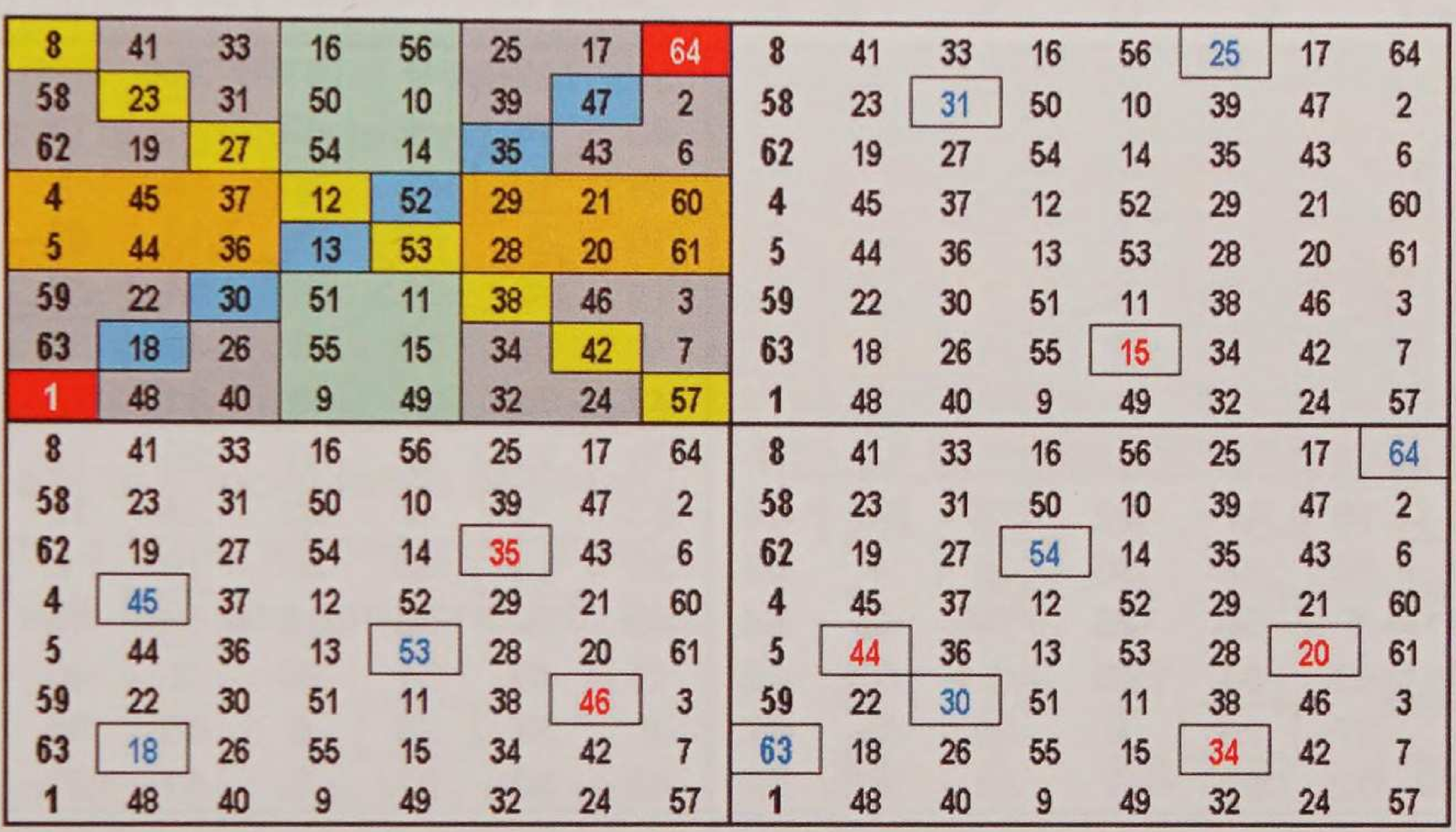
	1	2	3	4	5	6	7	8	9	
	W(9)	369	369	369	369	369	369	369	369	369
	75	58	71	21	4	17	48	31	44	369
	23	9	10	50	36	37	77	63	64	369
	52	29	42	79	56	69	25	2	15	369
	62	66	76	8	12	22	35	39	49	369
	1	14	27	28	41	54	55	68	81	369
	33	43	47	60	70	74	6	16	20	369
	67	80	57	13	26	3	40	53	30	369
	18	19	5	45	46	32	72	73	59	369
	38	51	34	65	78	61	11	24	7	369
	369	369	369	369	369	369	369	369	369	
		369	369	369	369	369	369	369	369	369

Program 8

The Random Difference Property for all Perfect Geonomic Squares

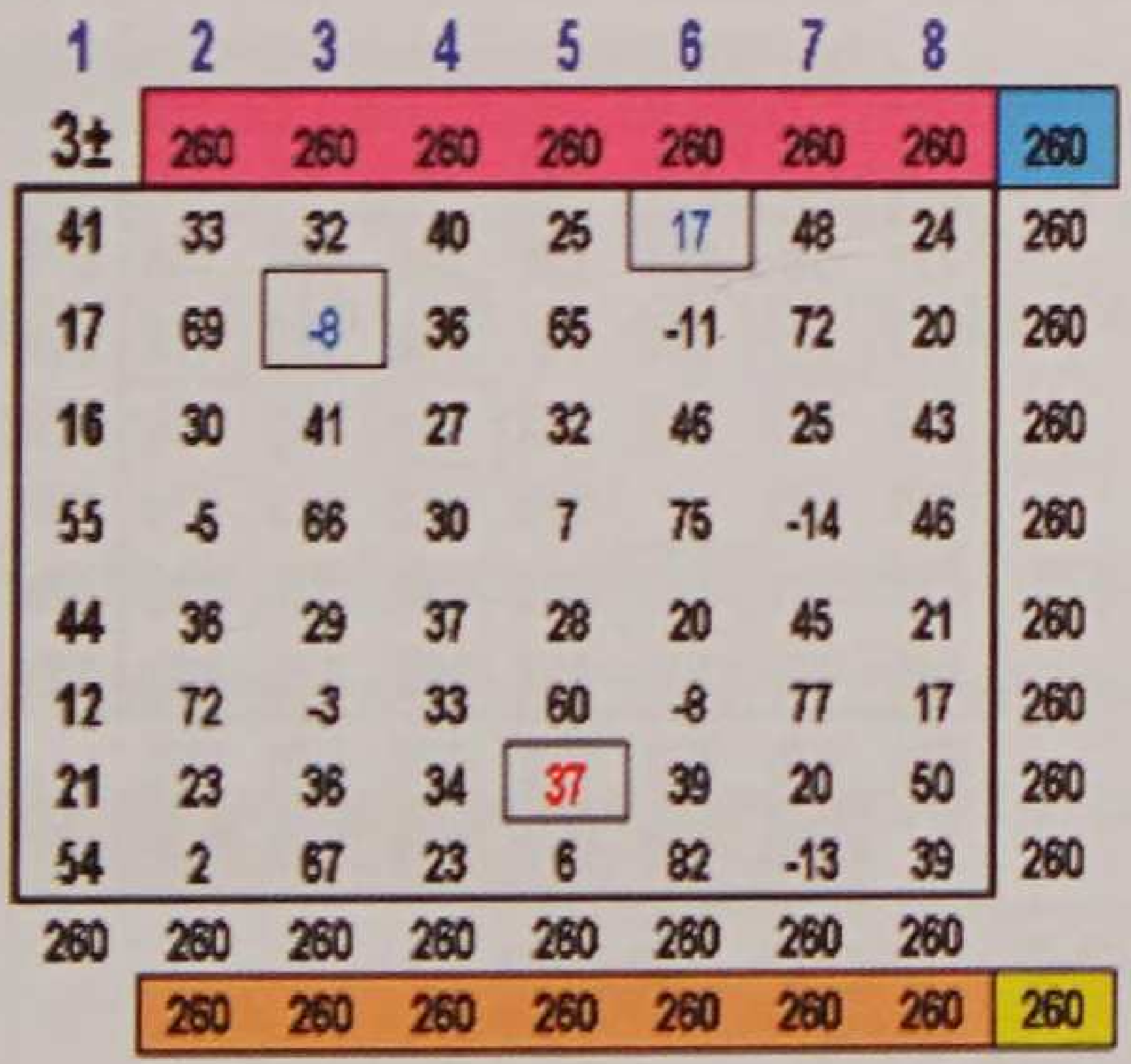
Here is an interesting property. In any perfect size **n** geonomic square, a random selection of an odd quantity of **m** distinct cells such that **(m-1)/2** of them are subtracted from **(m+1)/2** of them and the result is put into a square of size **n** in such a manner which distributes the pattern of alternating differences continuously across the new square, will have a characteristic number that sums geonomically everywhere to that of that of the original square.

Here is the size-8 square duplicated four times to make the calculations easier when the patterns are wrapped in the continuous summation process. The pattern of the numbers in **blue** are the cells from which the numbers in **red** are subtracted relatively.

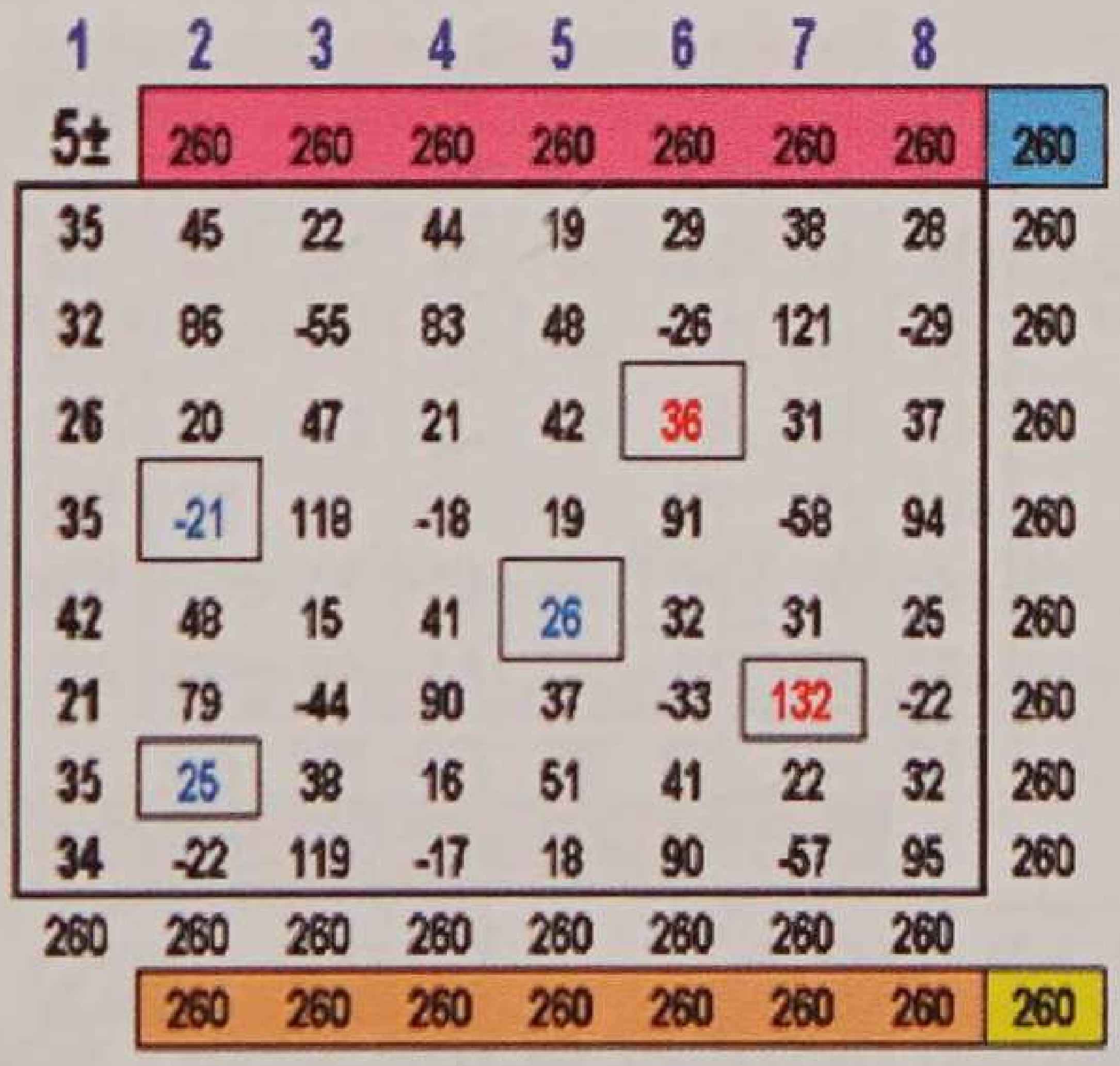


8	41	33	16	56	25	17	64	8	41	33	16	56	25	17	64
58	23	31	50	10	39	47	2	58	23	31	50	10	39	47	2
62	19	27	54	14	35	43	6	62	19	27	54	14	35	43	6
4	45	37	12	52	29	21	60	4	45	37	12	52	29	21	60
5	44	36	13	53	28	20	61	5	44	36	13	53	28	20	61
59	22	30	51	11	38	46	3	59	22	30	51	11	38	46	3
63	18	26	55	15	34	42	7	63	18	26	55	15	34	42	7
1	48	40	9	49	32	24	57	1	48	40	9	49	32	24	57
8	41	33	16	56	25	17	64	8	41	33	16	56	25	17	64
58	23	31	50	10	39	47	2	58	23	31	50	10	39	47	2
62	19	27	54	14	35	43	6	62	19	27	54	14	35	43	6
4	45	37	12	52	29	21	60	4	45	37	12	52	29	21	60
5	44	36	13	53	28	20	61	5	44	36	13	53	28	20	61
59	22	30	51	11	38	46	3	59	22	30	51	11	38	46	3
63	18	26	55	15	34	42	7	63	18	26	55	15	34	42	7
1	48	40	9	49	32	24	57	1	48	40	9	49	32	24	57

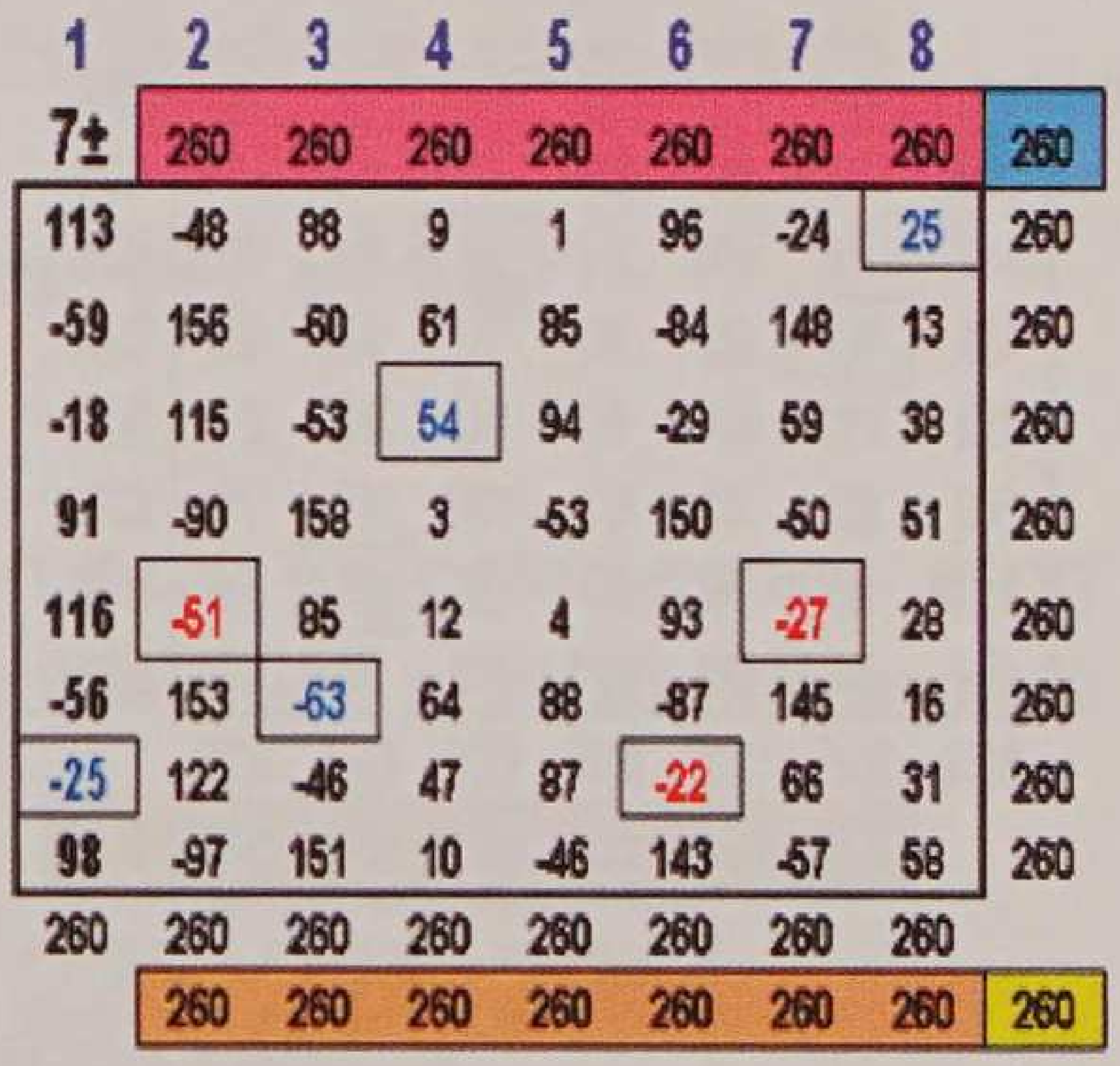
Here is the result of that continuous summation process showing the difference pattern for **m = 3**, **5** & **7**. It makes no difference whether **m** is less than or greater than **n**.



1	2	3	4	5	6	7	8	
3±	260	260	260	260	260	260	260	260
41	33	32	40	25	17	48	24	260
17	69	-8	36	65	-11	72	20	260
16	30	41	27	32	46	25	43	260
55	-5	66	30	7	75	-14	46	260
44	36	29	37	28	20	45	21	260
12	72	-3	33	60	-8	77	17	260
21	23	36	34	37	39	20	50	260
54	2	67	23	6	82	-13	39	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260



1	2	3	4	5	6	7	8	
5±	260	260	260	260	260	260	260	260
35	45	22	44	19	29	38	28	260
32	86	-55	83	48	-26	121	-29	260
26	20	47	21	42	36	31	37	260
35	-21	118	-18	19	91	-58	94	260
42	48	15	41	26	32	31	25	260
21	79	-44	90	37	-33	132	-22	260
35	25	38	16	51	41	22	32	260
34	-22	119	-17	18	90	-57	95	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260



1	2	3	4	5	6	7	8	
7±	260	260	260	260	260	260	260	260
113	-48	88	9	1	96	-24	25	260
-59	156	-60	61	85	-84	148	13	260
-18	115	-53	54	94	-29	59	38	260
91	-90	158	3	-53	150	-50	51	260
116	-51	85	12	4	93	-27	28	260
-56	153	-63	64	88	-87	145	16	260
-25	122	-46	47	87	-22	66	31	260
98	-97	151	10	-46	143	-57	58	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260

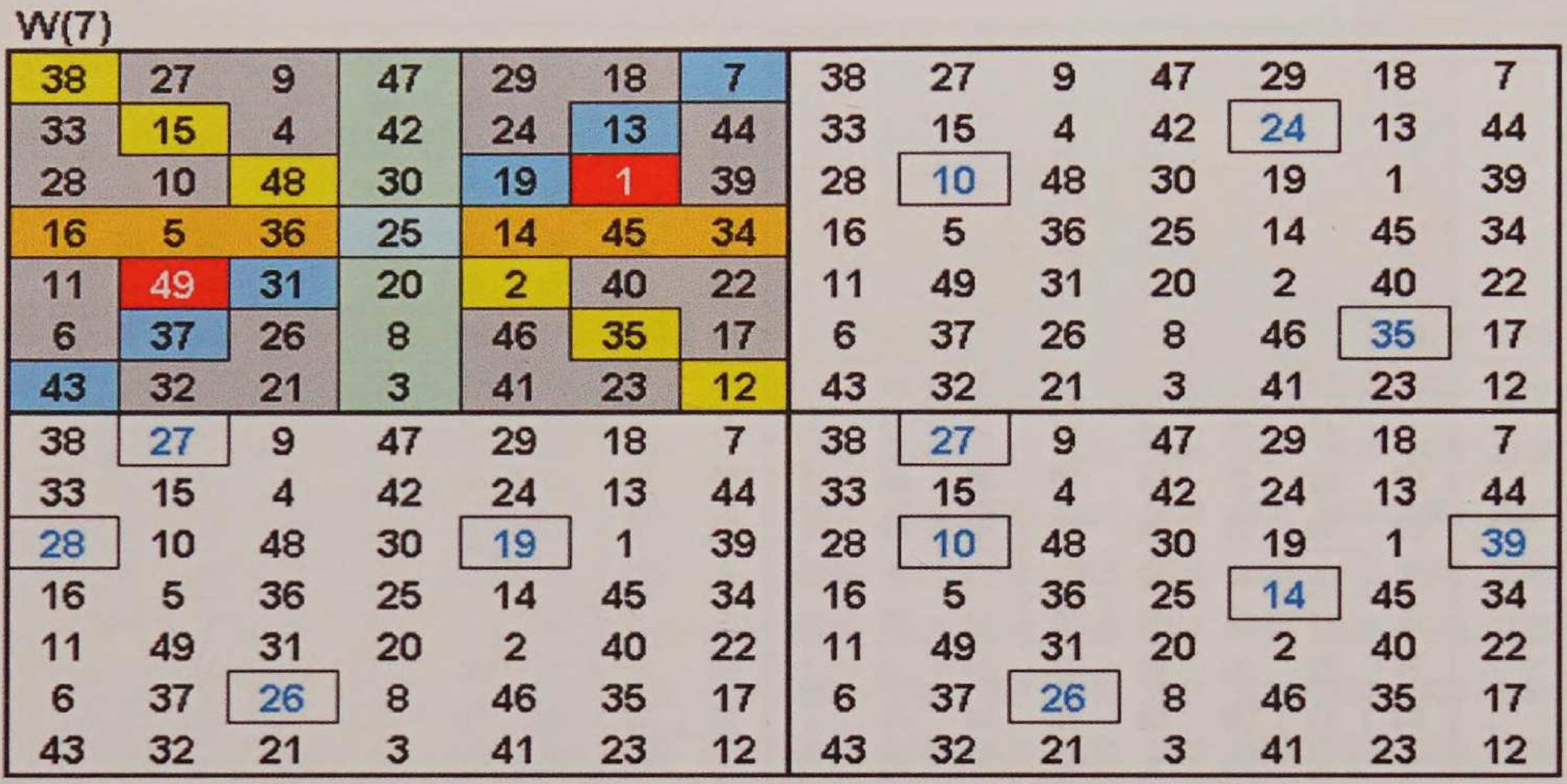
Class-2 squares are only near-perfect and so do not possess this property.

Program 8

The Random Addition Property for all Perfect Geonomic Squares

Here is another interesting property along the same lines. In any perfect size **n** geonomic square, a random selection of a quantity of **m** distinct cells are added together and the result is put into a square of size **n** in such a manner which distributes the pattern of selected cells continuously across a size **n** square, will have a characteristic number that sums geonomically everywhere to **m** times that of that of the original **square**.

Here is the size-7 square duplicated four times to make the calculations easier when the patterns are wrapped in the continuous summation process. The cells with the numbers in **blue** form the pattern which has been selected at random.



W(7)

38	27	9	47	29	18	7	38	27	9	47	29	18	7
33	15	4	42	24	13	44	33	15	4	42	24	13	44
28	10	48	30	19	1	39	28	10	48	30	19	1	39
16	5	36	25	14	45	34	16	5	36	25	14	45	34
11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	37	26	8	46	35	17	6	37	26	8	46	35	17
43	32	21	3	41	23	12	43	32	21	3	41	23	12
38	27	9	47	29	18	7	38	27	9	47	29	18	7
33	15	4	42	24	13	44	33	15	4	42	24	13	44
28	10	48	30	19	1	39	28	10	48	30	19	1	39
16	5	36	25	14	45	34	16	5	36	25	14	45	34
11	49	31	20	2	40	22	11	49	31	20	2	40	22
6	37	26	8	46	35	17	6	37	26	8	46	35	17
43	32	21	3	41	23	12	43	32	21	3	41	23	12

Here is the result of that continuous summation process showing the addition pattern for **m = 3**, **4** & **5**. It makes no difference whether **m** is less than or greater than **n**.

+Random 3	525	525	525	525	525	525	525
69	78	80	89	42	51	116	525
47	49	107	74	76	85	87	525
81	83	92	45	54	105	65	525
52	110	63	72	88	90	50	525
79	95	48	57	108	68	70	525
113	66	75	77	86	53	55	525
84	44	60	111	71	73	82	525
525	525	525	525	525	525	525	
	525	525	525	525	525	525	525

+Random 4	700	700	700	700	700	700	700
100	28	180	122	64	62	144	700
66	57	153	88	44	168	124	700
32	184	112	68	59	148	97	700
61	150	92	41	172	128	56	700
181	116	72	49	152	94	36	700
140	96	38	176	125	60	65	700
120	69	53	156	84	40	178	700
700	700	700	700	700	700	700	
	700	700	700	700	700	700	700

+Random 5	875	875	875	875	875	875	875
116	138	167	147	71	107	129	875
77	99	135	108	144	166	146	875
136	172	145	76	105	127	114	875
104	133	106	142	164	151	75	875
170	143	81	103	132	112	134	875
131	111	140	162	149	73	109	875
141	79	101	137	110	139	168	875
875	875	875	875	875	875	875	
	875	875	875	875	875	875	875

The characteristic number of the size-7 square is **175.**

525 = 3x175; 700 = 4x175 and 875 = 5x175.

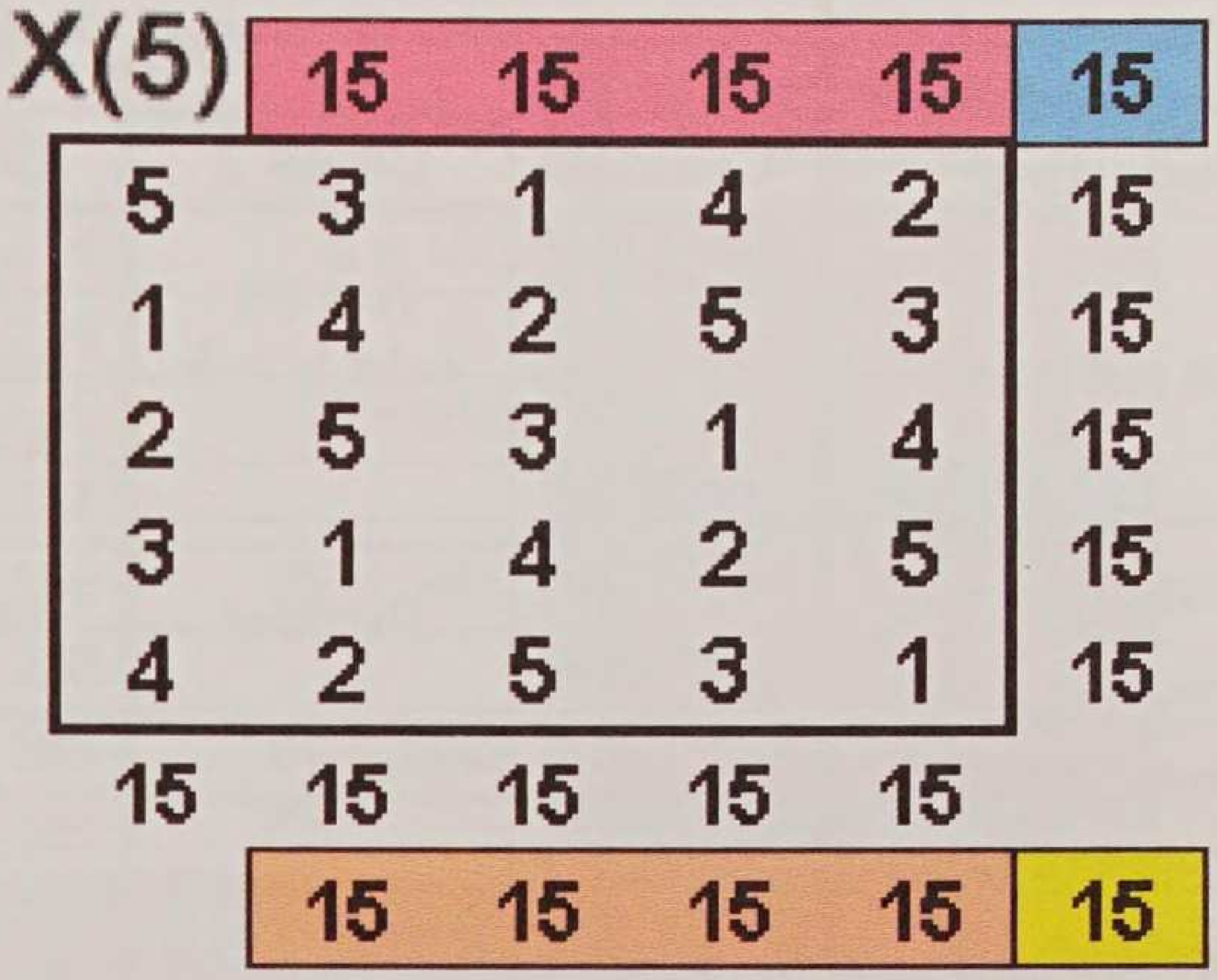
Program 8

The Cubic Conversion Property

Unknown to most engineers and even math teachers worldwide, there is a little-known theorem published circa 1950 by the late George Pólya, Professor Emeritus of Mathematics at Stanford University which states that:

The sum of the cubes of consecutive numbers from 1 thru n equals the sum of the numbers 1 thru n, quantity squared.

$$\sum_{j=1}^{n} j^{3} = \{ \sum_{j=1}^{n} j \}^{2} \tag{8.4}$$



X(5)						
		15	15	15	15	15
	5	3	1	4	2	15
	1	4	2	5	3	15
	2	5	3	1	4	15
	3	1	4	2	5	15
	4	2	5	3	1	15
	15	15	15	15	15	
		15	15	15	15	15

$$z_{ij} = x_{ij}^{3}$$

Z(5)						
		225	225	225	225	225
	125	27	1	64	8	225
	1	64	8	125	27	225
	8	125	27	1	64	225
	27	1	64	8	125	225
	64	8	125	27	1	225
	225	225	225	225	225	
		225	225	225	225	225

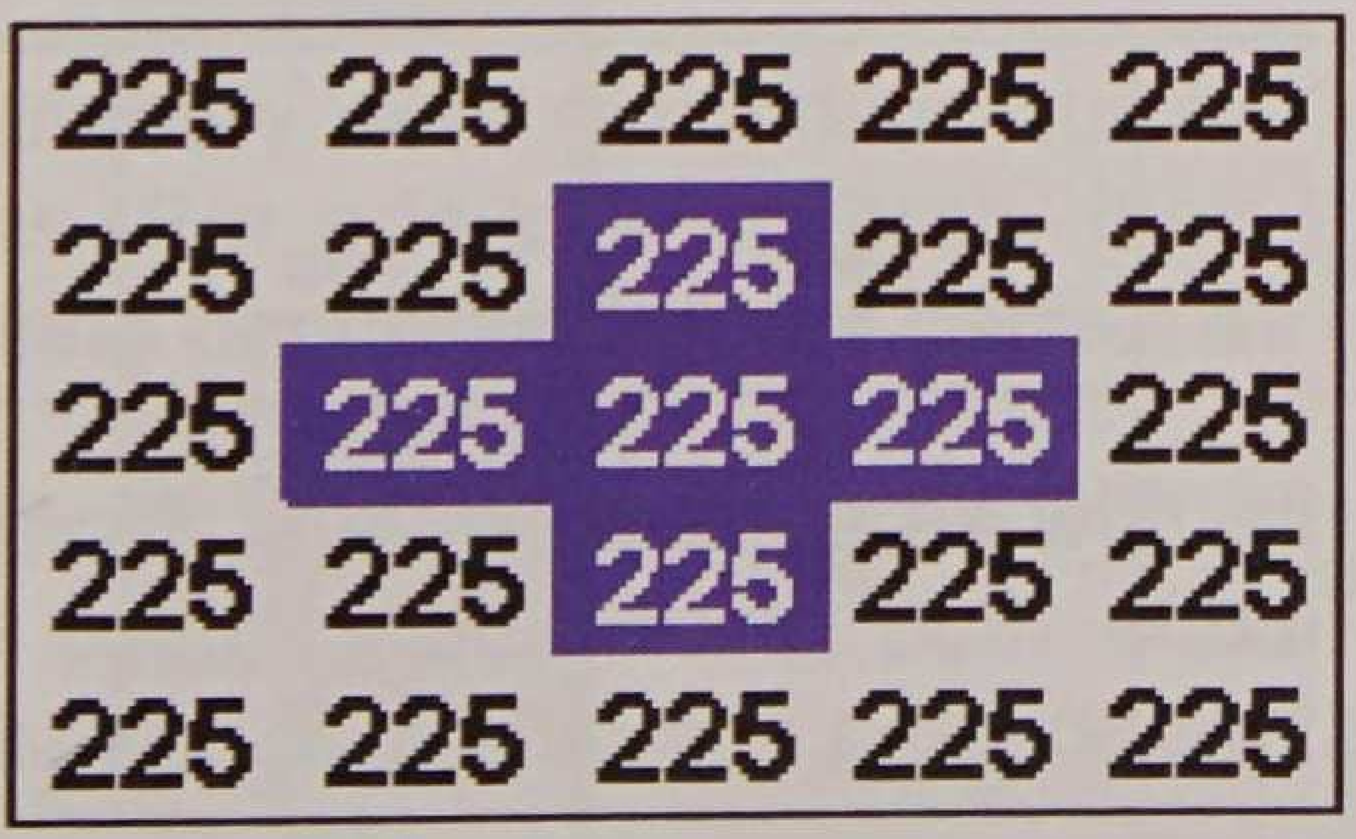
225 = 15 x 15

In Geonometry that translates to: **The cubing of all the numbers in a loom table of Class-1 squares yields a table whose characteristic number will be the square of its former self!**

At right is the modulus loom table of the size-5 square. Its cubic conversion is shown below it.

Since all loom tables of Class-1 squares contain the numbers from **1** thru **n** exactly once in all 4 directions, Polya's theorem applies to all 4 directional sums simultaneously.

Further, since all Class-1 squares are ultra-perfect, the tiles in both complementary tiling patterns on the cubic conversion of both complementary loom tables each sum to the square of the original characteristic number too!



225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225

225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225
225	225	225	225	225

There is future discovery potential here applying functions to the number sequences in loom-tables.

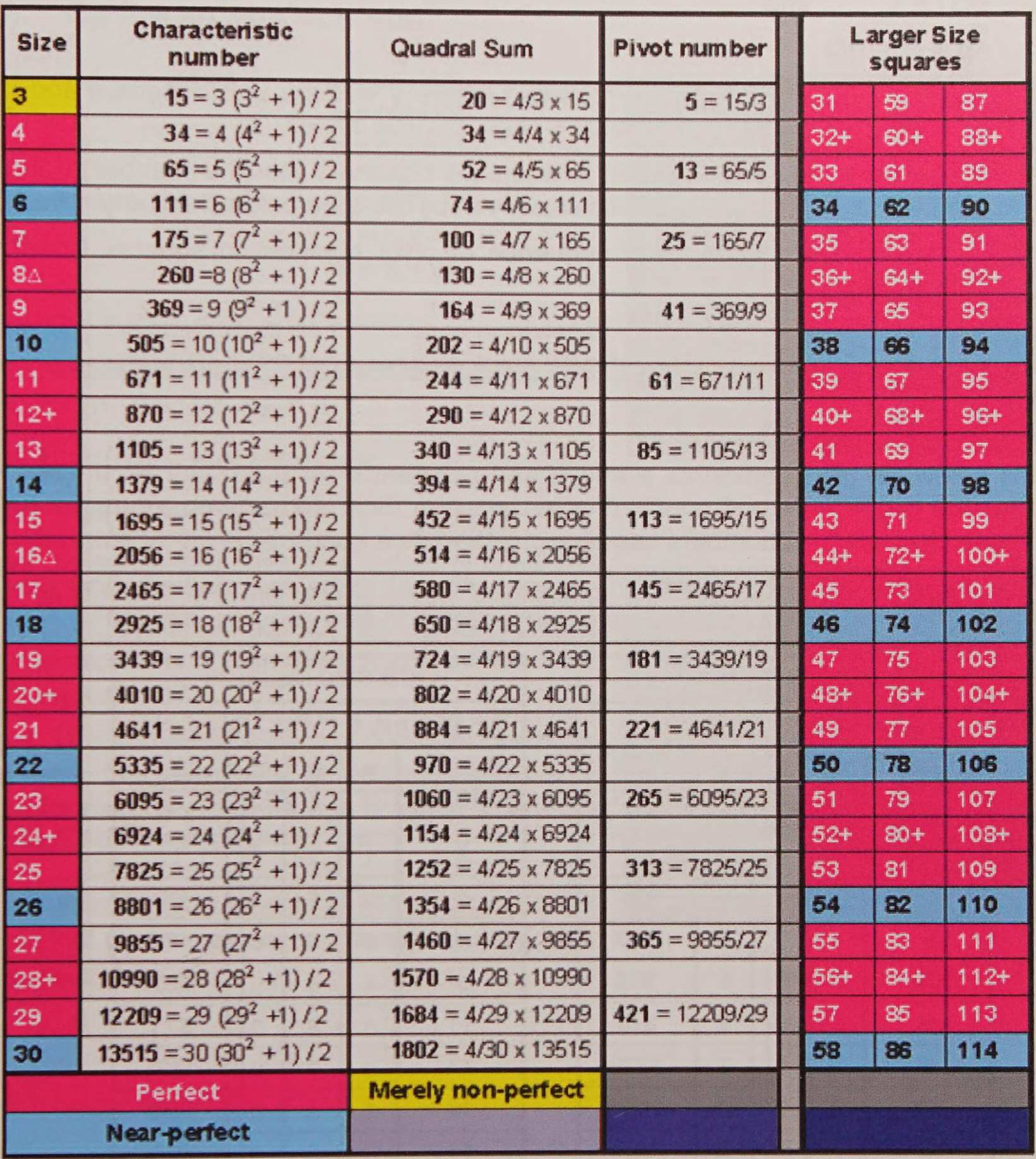
Program 8

Summary of Properties of Squares by Class

Summary of Squares' Properties	Odd Size				Even Size	
	Class 1	Class 3	Class 5	Class 6	Class 2	Class 4
Size factors	n is an odd prime number	$n = 3b$ odd $b > 3$	$n = ab$ both odd $a > b > 3$	$n = b^2$ odd prime $b \geq 3$	$n = 2b$ b is odd $b \geq 3$	$n = 4b$ b is any positive number
Perfection Level Program 1	Ultra-perfect	Perfect	Perfect	Perfect	Near-perfect	Ultra-Perfect
Pangenicity Program 1	Yes	Yes	Yes	Yes	Yes	Yes
Characteristic circles Program 1	No	No	No	No	No	Yes
Continuous Modularity Program 1	None	Always **3x** modular	None	Always **bx** modular	None	Always **2bx** modular
Characteristic tiling patterns Programs 2 & 3	Yes	Yes at 3x block-square level	Yes at both ax & bx block-square level	Yes at bx block-square level	No	Yes
Anchor-dot Patterns **Program 2**	Yes	Yes	Yes	Yes	No	Yes
Anchor-dot perpetuity property **Program 7**	Yes	No	No	No	No	No
Corner Triangle property Program 8	Yes	Yes	Yes	Yes	No	Yes
Sub-additive property **Program 8**	Yes	Yes	Yes	Yes	No	Yes
Concentricity property **Program 8**	Yes	Yes	Yes	Yes	Yes	Yes
Zero-differential property Program 8	Yes	Yes	Yes	Yes	No	Yes
Cubic–conversion property Program 8	Yes	No	No	No	No	No

Program 8

Table of Characteristic numbers



Size	Characteristic number	Quadral Sum	Pivot number	Larger Size squares		
3	**15** = 3 (3^2 + 1) / 2	**20** = 4/3 x 15	**5** = 15/3	31	59	87
4	**34** = 4 (4^2 + 1) / 2	**34** = 4/4 x 34		32+	60+	88+
5	**65** = 5 (5^2 + 1) / 2	**52** = 4/5 x 65	**13** = 65/5	33	61	89
6	**111** = 6 (6^2 + 1) / 2	**74** = 4/6 x 111		**34**	**62**	**90**
7	**175** = 7 (7^2 + 1) / 2	**100** = 4/7 x 165	**25** = 165/7	35	63	91
8△	**260** =8 (8^2 + 1) / 2	**130** = 4/8 x 260		36+	64+	92+
9	**369** = 9 (9^2 + 1) / 2	**164** = 4/9 x 369	**41** = 369/9	37	65	93
10	**505** = 10 (10^2 + 1) / 2	**202** = 4/10 x 505		**38**	**66**	**94**
11	**671** = 11 (11^2 + 1) / 2	**244** = 4/11 x 671	**61** = 671/11	39	67	95
12+	**870** = 12 (12^2 + 1) / 2	**290** = 4/12 x 870		40+	68+	96+
13	**1105** = 13 (13^2 + 1) / 2	**340** = 4/13 x 1105	**85** = 1105/13	41	69	97
14	**1379** = 14 (14^2 + 1) / 2	**394** = 4/14 x 1379		**42**	**70**	**98**
15	**1695** = 15 (15^2 + 1) / 2	**452** = 4/15 x 1695	**113** = 1695/15	43	71	99
16△	**2056** = 16 (16^2 + 1) / 2	**514** = 4/16 x 2056		44+	72+	100+
17	**2465** = 17 (17^2 + 1) / 2	**580** = 4/17 x 2465	**145** = 2465/17	45	73	101
18	**2925** = 18 (18^2 + 1) / 2	**650** = 4/18 x 2925		**46**	**74**	**102**
19	**3439** = 19 (19^2 + 1) / 2	**724** = 4/19 x 3439	**181** = 3439/19	47	75	103
20+	**4010** = 20 (20^2 + 1) / 2	**802** = 4/20 x 4010		48+	76+	104+
21	**4641** = 21 (21^2 + 1) / 2	**884** = 4/21 x 4641	**221** = 4641/21	49	77	105
22	**5335** = 22 (22^2 + 1) / 2	**970** = 4/22 x 5335		**50**	**78**	**106**
23	**6095** = 23 (23^2 + 1) / 2	**1060** = 4/23 x 6095	**265** = 6095/23	51	79	107
24+	**6924** = 24 (24^2 + 1) / 2	**1154** = 4/24 x 6924		52+	80+	108+
25	**7825** = 25 (25^2 + 1) / 2	**1252** = 4/25 x 7825	**313** = 7825/25	53	81	109
26	**8801** = 26 (26^2 + 1) / 2	**1354** = 4/26 x 8801		**54**	**82**	**110**
27	**9855** = 27 (27^2 + 1) / 2	**1460** = 4/27 x 9855	**365** = 9855/27	55	83	111
28+	**10990** = 28 (28^2 + 1) / 2	**1570** = 4/28 x 10990		56+	84+	112+
29	**12209** = 29 (29^2 +1) / 2	**1684** = 4/29 x 12209	**421** = 12209/29	57	85	113
30	**13515** = 30 (30^2 + 1) / 2	**1802** = 4/30 x 13515		**58**	**86**	**114**
Perfect		**Merely non-perfect**				
Near-perfect						

Here is a summary of characteristic numbers for squares up through size-30 along with their quadral sums. For odd-size squares, their pivot numbers are also listed. It cites the size-3 square as being geonomic but not perfect as a consequence of its embryonic size.

It identifies those even-size squares with all equal row-pair summations and cites the size-8 and size-16 squares as having both equal row-pair and centrally-symmetric pair sums. It shows the pattern of perfect and near-perfect squares up through size **114.**

The right-hand column lists the pivot umber of each odd-size square. Note the incremental pattern: **(4, 8, 12, 16, ... 4x)**. This may be expressed in formulas as:

(8.5 a) $p(n) = p(n-2) + \Delta p$ where

(8.5 b) $p(1) \equiv 1$ and

(8.5 c) $\Delta p = 4(n-1)/2$

Remember that formula **y = 4x** back in Program 1 where **y** was the size of square and **x** was the number of quadrals incident to the square's characteristic circle?

And again, that formula back in Program 4 when we observed that the number of incident octals in the characteristic spheres of cubes was given by the area below the line **y = 4x** ?

Well here it is again in the incremental sequence of pivot numbers in squares where now **y = Δp** and **x = (n-1)/2**.

Characteristic numbers of squares expressed as a quadratic

(8.6) $C_n = an^2 + bn + c$

The tables below list the coefficients **a** and **b** and constant **c** for the quadratic in (**8.6**). The formulas for the coefficients and constant differ between even and odd-size squares but the fact that these characteristic numbers can be expressed as a quadratic is pretty amazing.

Just the visible patterns in each table here are sufficient to be the mathematical proof by induction, one for the even-size squares and one for the odd-size squares, separately.

Even-size squares				
a = c= n/2, b = 0				
Size n	**a**	**b**	**c**	**C_n**
2	1	0	1	5
4	2	0	2	34
6	3	0	3	111
8	4	0	4	260
12	5	0	5	505
14	6	0	6	870
16	7	0	7	1379
18	8	0	8	2056
20	9	0	9	2925
22	10	0	10	4010
24	11	0	11	5335
26	12	0	12	6924
28	13	0	13	8801
30	14	0	14	10990
32	15	0	15	13515
34	16	0	16	16400

Odd-size squares				
a = (n-1)/2, b = (n+1)/2, c = 0				
Size n	**a**	**b**	**c**	**C_n**
1	0	1	0	1
3	1	2	0	11
5	2	3	0	65
7	3	4	0	175
9	4	5	0	369
11	5	6	0	671
13	6	7	0	1105
15	7	8	0	1695
17	8	9	0	2465
19	9	10	0	3439
21	10	11	0	4641
23	11	12	0	6095
25	12	13	0	7825
27	13	14	0	9855
29	14	15	0	12209
31	15	16	0	14911

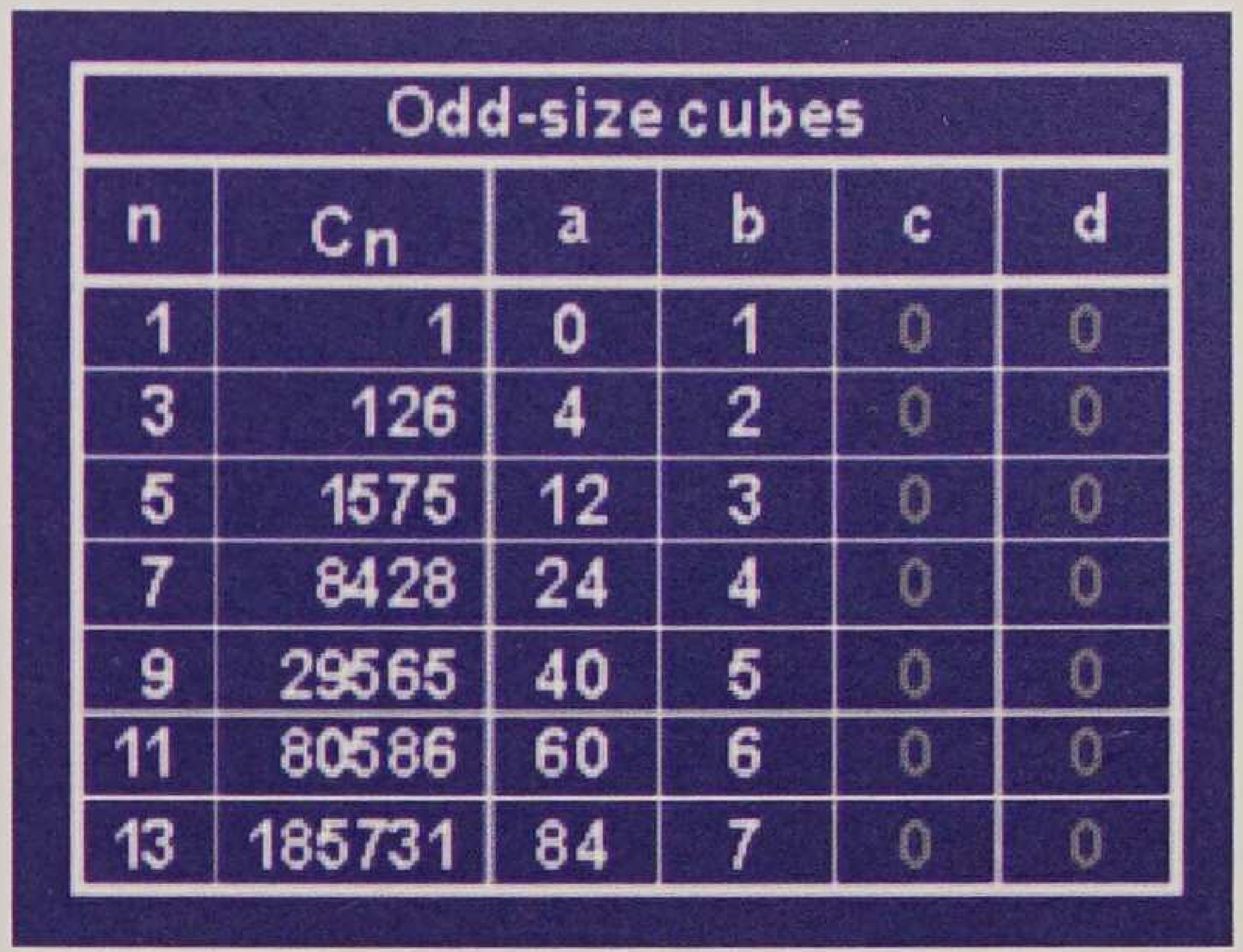
Program 8

Characteristic numbers of cubes expressed as a 3rd-degree polynomial

(8.7) $C_n^{(3)} = an^3+bn^2+cn^1+dn^0$

where for **odd**-size cubes: $a = (n^2-1)/2$
$b = (n+1)/2$
$c = d = 0$

and for **even**-size cubes: $a = n^2/2$
$c = n/2$
$b = d = 0$



Odd-size cubes					
n	Cn	a	b	c	d
1	1	0	1	0	0
3	126	4	2	0	0
5	1575	12	3	0	0
7	8428	24	4	0	0
9	29565	40	5	0	0
11	80586	60	6	0	0
13	185731	84	7	0	0

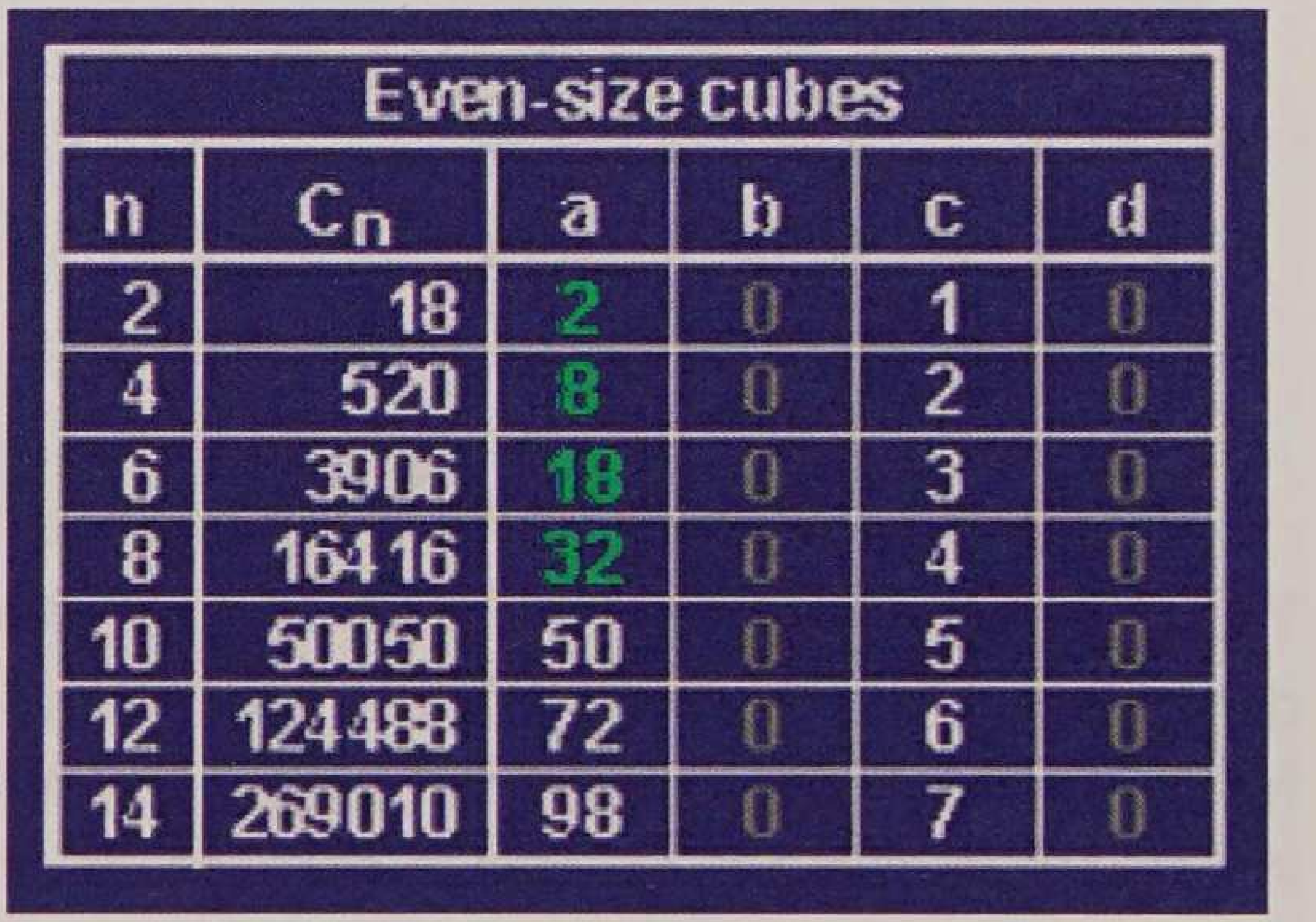
Representing the coefficients as functions of **n**, observe that **a(n_2)** and **c(n_2)** for even sizes **n_2** and **a(n_1)** and **b(n_1)** for odd sizes **n_1**, then for **$n_2 = n_1 + 1$**:

(8.8.1) $a(n_2) - a(n_1) = n_2$ and

(8.8.2) $b(n_1) - c(n_2) = 0$

Observe the series of the first four values of coefficient **a** for even-size cubes. Is this not the series of numbers, namely {**2, 8, 8, 18, 18, 32, 32**}, for elements in rows of the Periodic Table of Elements? And aren't these the first four values for **a**?

And 4, 8, 12 & 16 were the sizes of cubes for characteristic spheres that were used in determining the shell capacity for electrons seen back in Program-5. Isn't this just **y = 4c**? Again we see **y = 4x**.

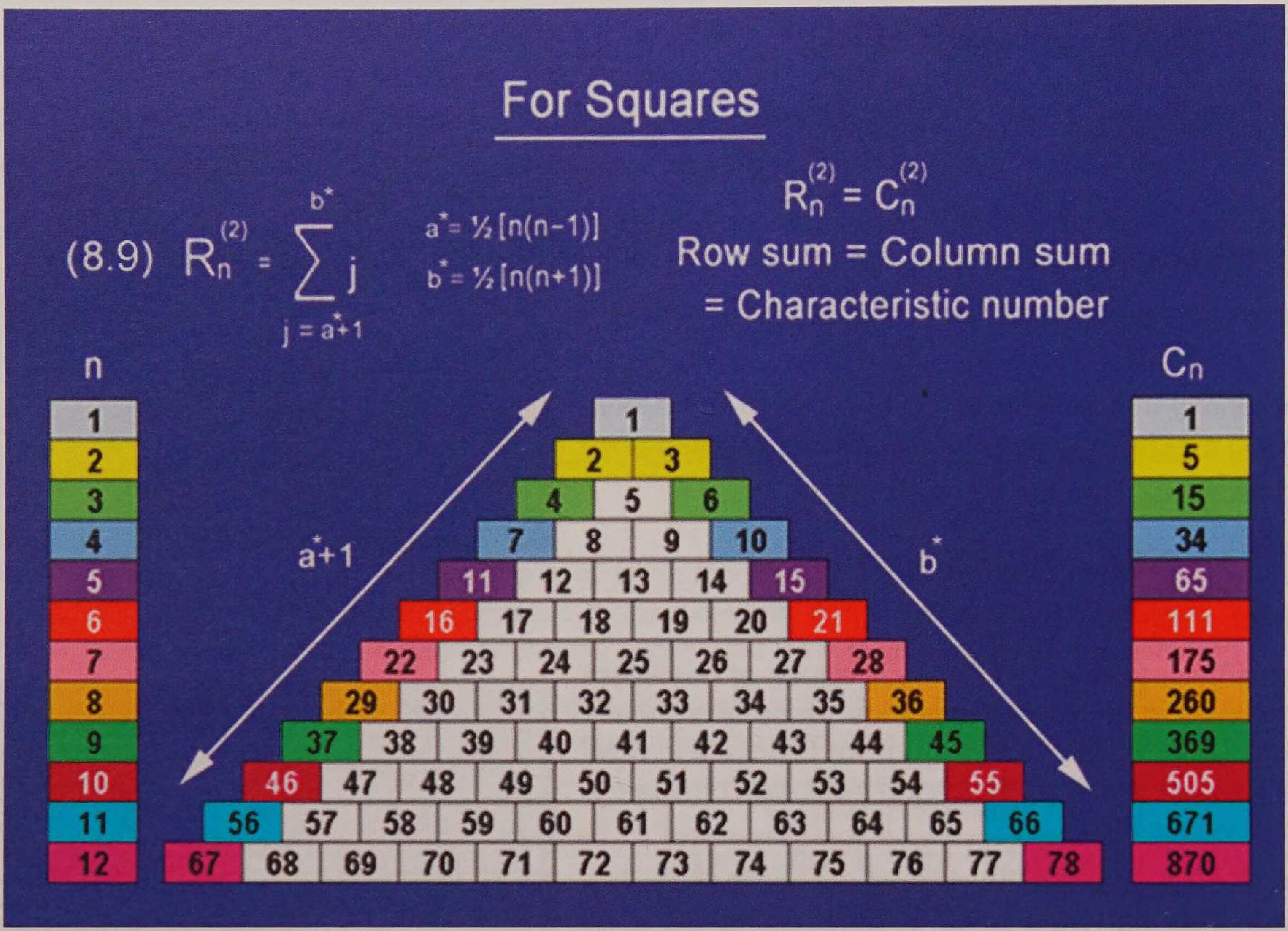


Even-size cubes					
n	Cn	a	b	c	d
2	18	2	0	1	0
4	520	8	0	2	0
6	3906	18	0	3	0
8	16416	32	0	4	0
10	50050	50	0	5	0
12	124488	72	0	6	0
14	269010	98	0	7	0

Now, all this is not just coincidental – It is further evidence that Geonometry is uncovering the very fabric of space itself !!!

Program 8

The Index Triangle's Correlation with Characteristic Numbers of Squares



For Squares

$$(8.9)\quad R_n^{(2)} = \sum_{j=a^*+1}^{b^*} j \qquad a^* = \tfrac{1}{2}[n(n-1)] \quad b^* = \tfrac{1}{2}[n(n+1)]$$

$$R_n^{(2)} = C_n^{(2)}$$

Row sum = Column sum
= Characteristic number

n		C_n
1	1	1
2	2 3	5
3	4 5 6	15
4	7 8 9 10	34
5	11 12 13 14 15	65
6	16 17 18 19 20 21	111
7	22 23 24 25 26 27 28	175
8	29 30 31 32 33 34 35 36	260
9	37 38 39 40 41 42 43 44 45	369
10	46 47 48 49 50 51 52 53 54 55	505
11	56 57 58 59 60 61 62 63 64 65 66	671
12	67 68 69 70 71 72 73 74 75 76 77 78	870

This picture shows a triangular index table where the cells are numbered sequentially from top-down by row and within each row from left-to-right. Note the relationship between **a** and **b** in formula (**8.9**) and on the left and right edges of the index triangle. The amazing feature of this index table is that all the numbers in the **n**-th row sum to the characteristic number for all geonomic squares of size **n**. The largest number in an index triangle of size **n** is:

(8.10) $\quad \mathbf{N_n = n(n+1)/2}$

Sum of the numbers in the n-th row = the characteristic number of square of size n:

(8.11) $\quad \mathbf{C_n = n(n^2+1)/2}$

Note the correspondence between formula (**8.10**) for the count of numbers in the triangular table of size **n** and formula (**8.11**) for the characteristic numbers of squares of size **n**. The only difference between them is simply **n** verses $\mathbf{n^2}$ as highlighted in **blue**.

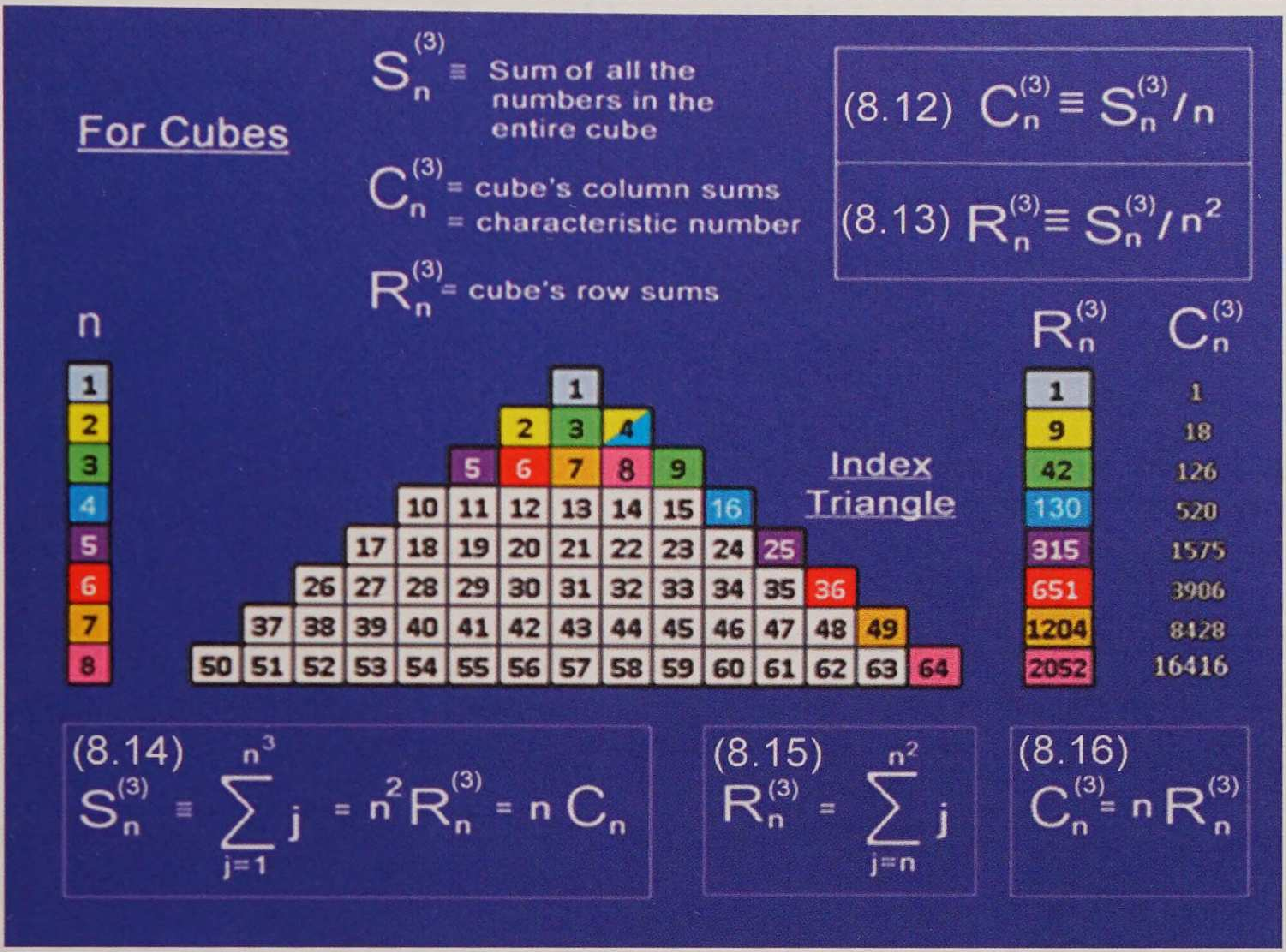
The formula for the common row sums **Rn** for squares of size **n** is given in terms of the index triangle row's starting and ending numbers. In geonomic squares, all of the row sums and the column sums are equal. So $\mathbf{R_n = C_n}$ here.

Note that the values of the cells on the right side of the triangle are simply the sum of all the numbers in the vector to the left of the triangle from the top down to the cell of the same color.

Observe that the superscript number **2** of **Cn** in (**8.9**) stands for the characteristic number for geonomic tables of size **n** in dimension-2.

Program 8

Characteristic Numbers of Cubes and Triangular Index Table Sums



For Cubes

$S_n^{(3)} \equiv$ Sum of all the numbers in the entire cube

$C_n^{(3)}$ = cube's column sums = characteristic number

$R_n^{(3)}$ = cube's row sums

(8.12) $C_n^{(3)} \equiv S_n^{(3)}/n$

(8.13) $R_n^{(3)} \equiv S_n^{(3)}/n^2$

Index Triangle

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16
17 18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33 34 35 36
37 38 39 40 41 42 43 44 45 46 47 48 49
50 51 52 53 54 55 56 57 58 59 60 61 62 63 64

n	$R_n^{(3)}$	$C_n^{(3)}$
1	1	1
2	9	18
3	42	126
4	130	520
5	315	1575
6	651	3906
7	1204	8428
8	2052	16416

(8.14) $S_n^{(3)} \equiv \sum_{j=1}^{n^3} j = n^2 R_n^{(3)} = n\,C_n$

(8.15) $R_n^{(3)} = \sum_{j=n}^{n^2} j$

(8.16) $C_n^{(3)} = n\,R_n^{(3)}$

Like the correspondence of triangular index sums with characteristic numbers of squares, here is yet another amazing correspondence now between consecutive numbers on an index triangle and characteristic numbers of cubes.

The index triangle here is different from the one for squares. There, each row contained only one more number than the preceding row; here it contains two more numbers per row.

It's quite amazing that all the ending numbers in each row of this index triangle are consecutive **square numbers**.

Here's how the process works in this index triangle: On the left, **n** is selected and the color noted. The same colored cell in the triangle with the least value greater than **n** is determined. Next, these numbers are summed in sequence from number **n** thru $\mathbf{n^2}$ also of the same color on the right side of the triangle. That sum is the value $\mathbf{R_n}$ of all the cube's rows (**8.15**).

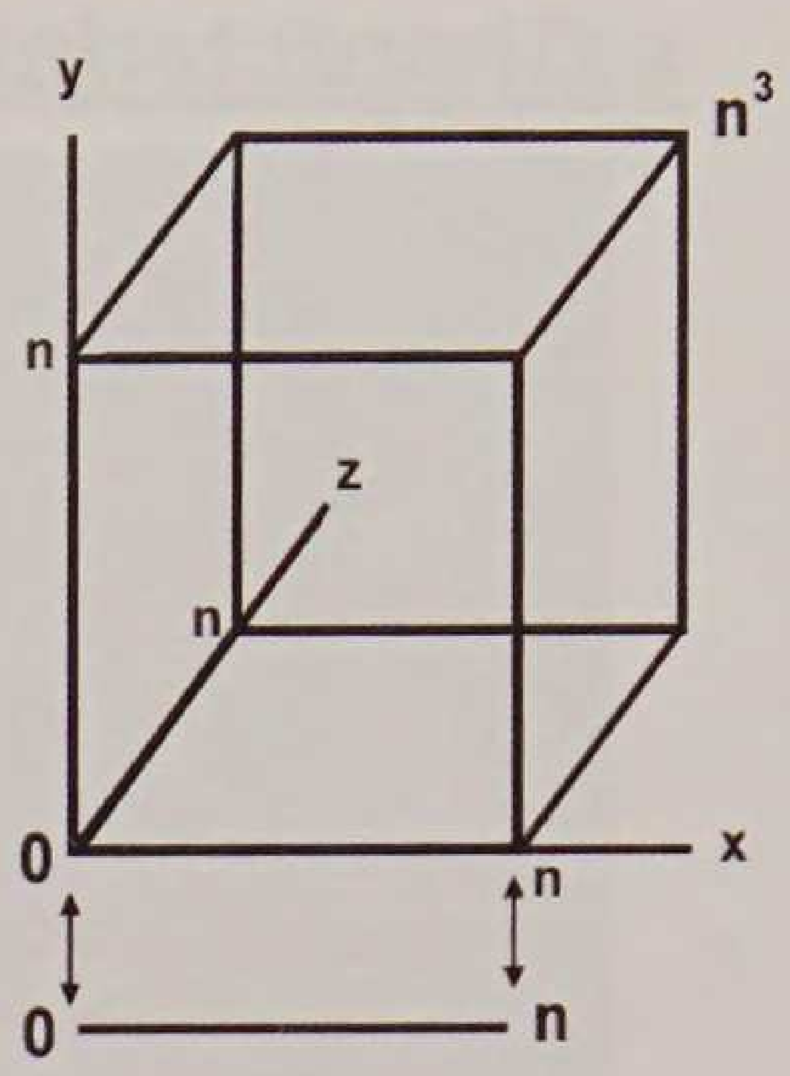
Then $\mathbf{R_n}$ is multiplied by **n** and that number is the value of all the cube's columns (**8.16**). This is also the characteristic number $\mathbf{C_n}$ for the cube of size **n**. Those numbers are listed at far right for cubes of sizes **1** thru **8**.

Note in the triangle that the number **4's** background is doubly colored. That's because it is the ending number for **n = 2** and the starting number for **n = 4**. In larger index tables this will occur on the right hand side of the triangle for every number that is both the starting number and an ending number for a lesser number in the triangle.

Although $\mathbf{S_n}$ is defined as the sum of all the numbers in the cube starting from **1** and ending at $\mathbf{n^3}$, it is quite amazing that it is directly related to $\mathbf{R_n}$ which is the sum of numbers starting at **n** an ending at $\mathbf{n^2}$ for a total of only **n+1** numbers in all when compared to the $\mathbf{n^3}$ numbers otherwise involved! Further averaging $\mathbf{S_n}$ by **n** yields the column sums as per formula (**8.12**) and averaging $\mathbf{S_n}$ by $\mathbf{n^2}$ yields the row sums per formula (**8.13**). Yet both $\mathbf{R_n}$ and $\mathbf{C_n}$ can be obtained with less computation from the shorter-length sums in the index triangle.

The Pattern of Characteristic Numbers as Volume Averages between basic geometries in higher and lower Dimensions

The picture at right shows that the characteristic number for a geonomic **square** (2-dimensions) is the dimensional average of a **cube** (3-dimensions) and a **string** (1-dimension) all of the same size **n**. The formula for C_n below the picture yields the characteristic number for any square of size **n**.



$C_n = ½ (n^3+n)$

The table below shows that the distance between the two dimensions involved for characteristic numbers in **k**-dimensions expands as the dimension increases. That expansion difference Δ between the powers of **n** in the rightmost column is equal to **k** itself.

In general, for any dimension **k**, the characteristic number for a geonomic table of size **n** may be expressed as:

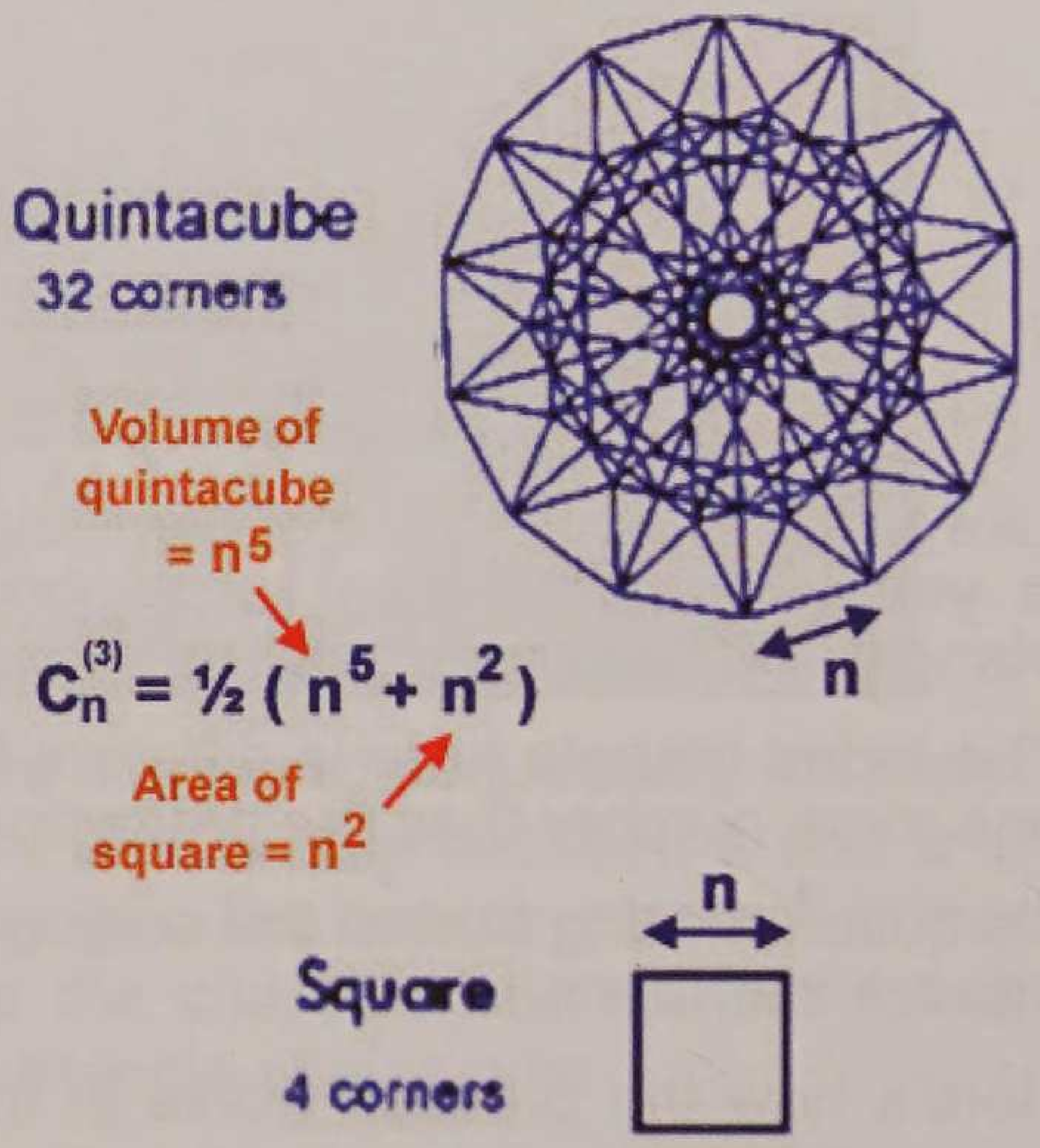
(8.17) $$C_n^{(k)} = S_n^{(k)}/n = n^k(n^k+1)/2n = ½\,(n^{(2k-1)} + n^{(k-1)})$$

Note that for strings where **k=1**, which involves **0D,** (**8.17**) yields $C_n^{(1)}$ **= (n+1)/2** as expected,.

k	(2k-1)	(k-1)	Δ
1	1	0	1
2	3	1	2
3	5	2	3
4	7	3	4
5	9	4	5
6	11	5	6
7	13	6	7

Geonometry is going to give this simple formula a very profound interpretation. This is exemplified by the pictorial to the right.

Here, the characteristic number of any size **cube** is the just the average of the **quintacube's** volume and the area of a **square of** the same size. This averaging of measures between two different dimensions, one lower and one larger, yielding the fundamental characteristic number of any size-**n** geonomic table in an intervening dimension is the profound interpretation.



$C_n^{(3)} = ½\,(n^5 + n^2)$

This has implication for the rate of the passage of time among different dimensions as follows:

If time and spatial volume are correlated in any way, this would imply that the rate of the passage of time in dimension **k+1** is somehow correlated with the average of the rates of time-passage between dimension **2k** and **k**. If so, given that the rate of time passage in the lower dimension was faster than the dimension above it would imply that the rate of time-passage in the higher dimension is slower than that in the **intervening** dimension.

It is well known in Atomic Science as well as in Micro-biology that events take place in nano-seconds and micro-seconds, respectively. So time is in fact faster in dimensions less than 3D. That implies that the passage of time in the 4th dimension is slower than the passage of time in the 3rd dimension.

This observation is the basis upon which Geonometry predicts that such is the case in its big picture of the multi-dimensional universe shown at the end of Program 9.

Program 8

The loom table hidden in the tiling patterns of Class-1 Cubes

Here is a size-7 Class-1 Cube. It is absolutely ultra-perfect. That is, all of its embedded squares sum equally in all four directions and it has continuously equal-summing complementary tiling patterns, A and B. (Shown on the next 2 pages.)

The 7x- Cube

1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
8428	8428	8428	8428	8428	8428	8428

← Every minor diagonal in each block-square sums to **1204**.

Square										
	1	56	263	127	341	156	27	234	1204	
	2	191	13	220	91	249	113	327	1204	
	3	284	99	313	177	48	206	77	1204	
7	4	34	241	63	270	134	299	163	1204	8428
	5	120	334	149	20	227	98	256	1204	
	6	213	84	291	106	320	184	6	1204	
	7	306	170	41	199	70	277	141	1204	
	1	252	116	323	194	9	223	87	1204	
	2	44	209	73	287	102	309	180	1204	
	3	137	295	166	30	244	59	273	1204	
6	4	230	94	259	123	330	152	16	1204	8428
	5	316	187	2	216	80	294	109	1204	
	6	66	280	144	302	173	37	202	1204	
	7	159	23	237	52	266	130	337	1204	
	1	105	312	176	47	205	76	283	1204	
	2	240	62	269	140	298	162	33	1204	
	3	333	148	19	226	97	255	126	1204	
5	4	83	290	112	319	183	5	212	1204	8428
	5	169	40	198	69	276	147	305	1204	
	6	262	133	340	155	26	233	55	1204	
	7	12	219	90	248	119	326	190	1204	
	1	301	165	29	243	58	272	136	1204	
	2	93	258	122	336	151	15	229	1204	
	3	186	1	215	79	293	108	322	1204	
4	4	279	143	308	172	36	201	65	1204	8428
	5	22	236	51	265	129	343	158	1204	
	6	115	329	193	8	222	86	251	1204	
	7	208	72	286	101	315	179	43	1204	
	1	154	18	225	96	254	125	332	1204	
	2	289	111	318	189	4	211	82	1204	
	3	39	197	68	275	146	304	175	1204	
3	4	132	339	161	25	232	54	261	1204	8428
	5	218	89	247	118	325	196	11	1204	
	6	311	182	46	204	75	282	104	1204	
	7	61	268	139	297	168	32	239	1204	
	1	7	214	78	292	107	321	185	1204	
	2	142	307	171	42	200	64	278	1204	
	3	235	50	264	128	342	157	28	1204	
2	4	328	192	14	221	85	250	114	1204	8428
	5	71	285	100	314	178	49	207	1204	
	6	164	35	242	57	271	135	300	1204	
	7	257	121	335	150	21	228	92	1204	
	1	203	67	274	145	303	174	38	1204	
	2	338	160	24	238	53	260	131	1204	
	3	88	246	117	324	195	10	224	1204	
1	4	181	45	210	74	281	103	310	1204	8428
	5	267	138	296	167	31	245	60	1204	
	6	17	231	95	253	124	331	153	1204	
	7	110	317	188	3	217	81	288	1204	
		8428	8428	8428	8428	8428	8428	8428		

Quadrals

737	737	394	737
737	394	1080	737
737	737	737	394

835	492	835	492
492	835	492	835
835	835	492	835

590	933	590	590
590	590	933	590
933	590	590	590

688	688	688	688
688	688	688	688
688	688	688	688

786	443	786	786
786	786	443	786
443	786	786	786

541	884	541	884
884	541	884	541
541	541	884	541

639	639	982	639
639	982	296	639
639	639	639	982

Octals

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

1376	1376	1376	1376
1376	1376	1376	1376
1376	1376	1376	1376

Octahedrons

1032
1032
1032

8428	8428	8428	8428	8428	8428	8428
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204
1204	1204	1204	1204	1204	1204	1204

← Every major diagonal in each block-square sums to **1204**.

Program 8

Tiling pattern A on embedded squares

Each embedded square contains complementary tiling patterns in which each tile sums to **1204**. (Only one of which is shown here; the other is shown on the next page)

The number strips on the right are lists of tile centers in the respective tiling pattern, sorted in order of increasing value. They are put into a square according to their sequence in the cube. This square is shown at the next-to-last square on the right. Left-to-right in the square corresponds to front-to-back in the cube. The last table on the right is the modulus table derived from the square of center-numbers above it. The same thing is shown on the next page for the other tiling pattern.

Embedded		1	2	3	4	5	6	7	
Square 7			1204	1204	1204	1204	1204	1204	1204
		56	263	127	341	156	27	234	1204
	2	191	13	220	91	249	113	327	1204
	3	284	99	313	177	48	206	77	1204
7	4	34	241	63	270	134	299	163	1204
	5	120	334	149	20	227	98	256	1204
	6	213	84	291	106	320	184	6	1204
	7	306	170	41	199	70	277	141	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

Tile centers
6
56
99
149
199
249
299

Embedded									
Square 6			1204	1204	1204	1204	1204	1204	1204
	1	252	116	323	194	9	223	87	1204
	2	44	209	73	287	102	309	180	1204
	3	137	295	166	30	244	59	273	1204
6	4	230	94	259	123	330	152	16	1204
	5	316	187	2	216	80	294	109	1204
	6	66	280	144	302	173	37	202	1204
	7	159	23	237	52	266	130	337	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

2
52
102
152
202
252
295

Embedded									
Square 5			1204	1204	1204	1204	1204	1204	1204
	1	105	312	176	47	205	76	283	1204
	2	240	62	269	140	298	162	33	1204
	3	333	148	19	226	97	255	126	1204
5	4	83	290	112	319	183	5	212	1204
	5	169	40	198	69	276	147	305	1204
	6	262	133	340	155	26	233	55	1204
	7	12	219	90	248	119	326	190	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

5
55
105
148
198
248
298

Embedded									
Square 4			1204	1204	1204	1204	1204	1204	1204
	1	301	165	29	243	58	272	136	1204
	2	93	258	122	336	151	15	229	1204
	3	186	1	215	79	293	108	322	1204
4	4	279	143	308	172	36	201	65	1204
	5	22	236	51	265	129	343	158	1204
	6	115	329	193	8	222	86	251	1204
	7	208	72	286	101	315	179	43	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

1
51
101
151
201
251
301

Embedded									
Square 3			1204	1204	1204	1204	1204	1204	1204
	1	154	18	225	96	254	125	332	1204
	2	289	111	318	189	4	211	82	1204
	3	39	197	68	275	146	304	175	1204
3	4	132	339	161	25	232	54	261	1204
	5	218	89	247	118	325	196	11	1204
	6	311	182	46	204	75	282	104	1204
	7	61	268	139	297	168	32	239	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

4
54
104
154
197
247
297

Embedded									
Square 2			1204	1204	1204	1204	1204	1204	1204
	1	7	214	78	292	107	321	185	1204
	2	142	307	171	42	200	64	278	1204
	3	235	50	264	128	342	157	28	1204
2	4	328	192	14	221	85	250	114	1204
	5	71	285	100	314	178	49	207	1204
	6	164	35	242	57	271	135	300	1204
	7	257	121	335	150	21	228	92	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

7
50
100
150
200
250
300

Embedded									
Square 1			1204	1204	1204	1204	1204	1204	1204
	1	203	67	274	145	303	174	38	1204
	2	338	160	24	238	53	260	131	1204
	3	88	246	117	324	195	10	224	1204
1	4	181	45	210	74	281	103	310	1204
	5	267	138	296	167	31	245	60	1204
	6	17	231	95	253	124	331	153	1204
	7	110	317	188	3	217	81	288	1204
		1204	1204	1204	1204	1204	1204	1204	
			1204	1204	1204	1204	1204	1204	1204

3
53
103
153
203
246
296

1	2	3	4	5	6	7
3	7	4	1	5	2	6
53	50	54	51	55	52	56
103	100	104	101	105	102	99
153	150	154	151	148	152	149
203	200	197	201	198	202	199
246	250	247	251	248	252	249
296	300	297	301	298	295	299

Table composed of external columns above

3	7	4	1	5	2	6
4	1	5	2	6	3	7
5	2	6	3	7	4	1
6	3	7	4	1	5	2
7	4	1	5	2	6	3
1	5	2	6	3	7	4
2	6	3	7	4	1	5

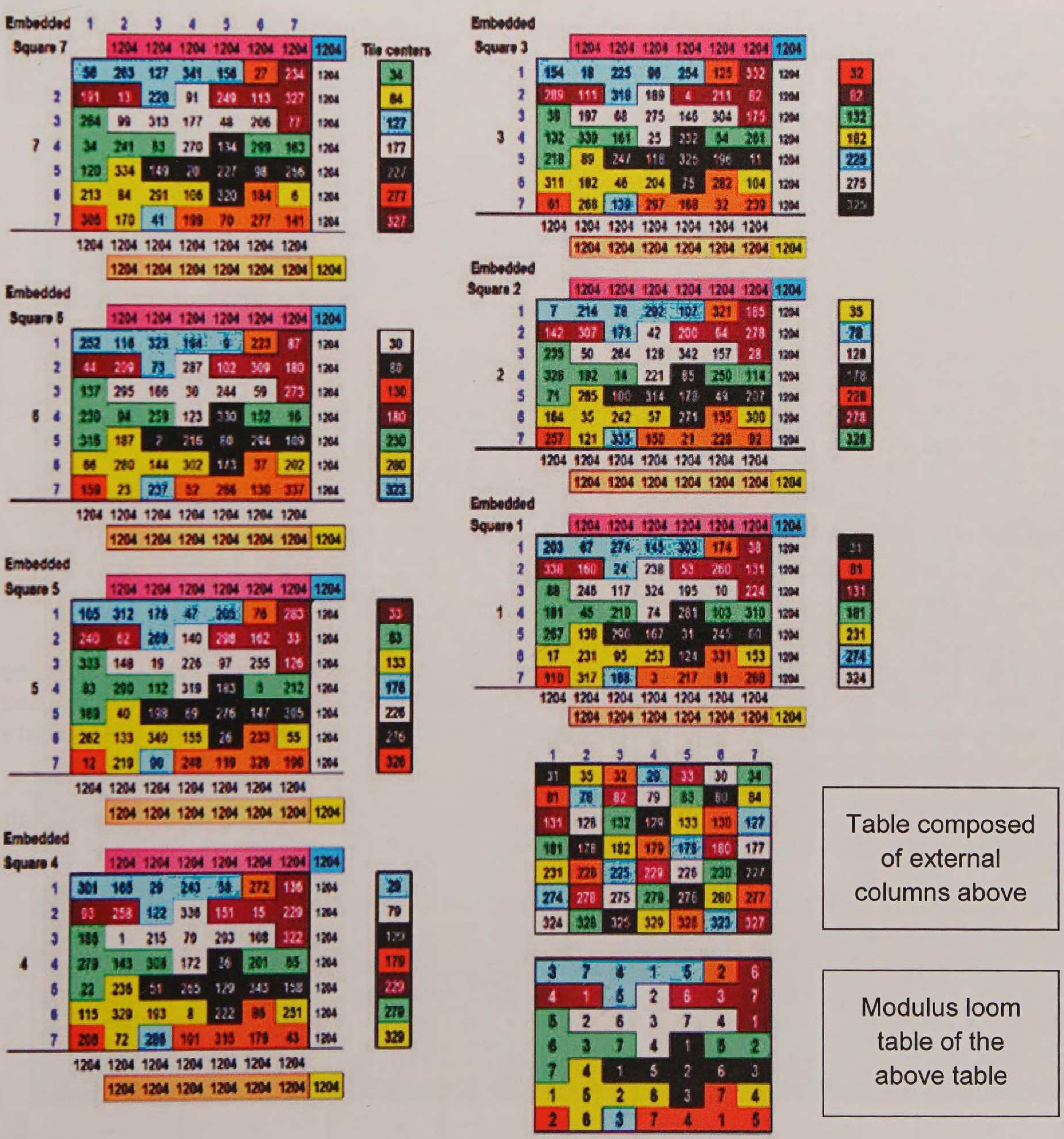
Modulus loom table of the above table

Program 8

Tiling pattern B on embedded squares

Although the tile centers between the two tiling patterns were deliberately made non-co-incidental, Lo and behold, both derived modulus tables are numerically identical!

And moreover, the table possesses all the characteristics of a loom table because every row, column and diagonal, including wrap diagonals in both directions, contains all the numbers **1** through **7** exactly once. We shall identify this loom table in what follows as **X***.



Embedded Square 7		1	2	3	4	5	6	7		Tile centers
			1204	1204	1204	1204	1204	1204	1204	
		56	263	127	341	156	27	234	1204	34
	2	191	13	220	91	249	113	327	1204	84
	3	284	99	313	177	48	206	77	1204	127
7	4	34	241	63	270	134	299	163	1204	177
	5	120	334	149	20	227	98	256	1204	227
	6	213	84	291	106	320	184	6	1204	277
	7	306	170	41	199	70	277	141	1204	327
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

Embedded Square 6										
			1204	1204	1204	1204	1204	1204	1204	
	1	252	116	323	194	9	223	87	1204	30
	2	44	209	73	287	102	309	180	1204	80
	3	137	295	166	30	244	59	273	1204	130
6	4	230	94	259	123	330	152	16	1204	180
	5	316	187	2	216	80	294	109	1204	230
	6	66	280	144	302	173	37	202	1204	280
	7	159	23	237	52	266	130	337	1204	323
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

Embedded Square 5										
			1204	1204	1204	1204	1204	1204	1204	
	1	105	312	176	47	205	76	283	1204	33
	2	240	62	269	140	298	162	33	1204	83
	3	333	148	19	226	97	255	126	1204	133
5	4	83	290	112	319	183	5	212	1204	176
	5	169	40	198	69	276	147	305	1204	226
	6	262	133	340	155	26	233	55	1204	276
	7	12	219	90	248	119	326	190	1204	326
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

Embedded Square 4										
			1204	1204	1204	1204	1204	1204	1204	
	1	301	165	29	243	58	272	136	1204	29
	2	93	258	122	336	151	15	229	1204	79
	3	186	1	215	79	293	108	322	1204	129
4	4	279	143	308	172	36	201	65	1204	179
	5	22	236	51	265	129	343	158	1204	229
	6	115	329	193	8	222	86	251	1204	279
	7	208	72	286	101	315	179	43	1204	329
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

Embedded Square 3										
			1204	1204	1204	1204	1204	1204	1204	
	1	154	18	225	96	254	125	332	1204	32
	2	289	111	318	189	4	211	82	1204	82
	3	39	197	68	275	146	304	175	1204	132
3	4	132	339	161	25	232	54	261	1204	182
	5	218	89	247	118	325	196	11	1204	225
	6	311	182	46	204	75	282	104	1204	275
	7	61	268	139	297	168	32	239	1204	325
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

Embedded Square 2										
			1204	1204	1204	1204	1204	1204	1204	
	1	7	214	78	292	107	321	185	1204	35
	2	142	307	171	42	200	64	278	1204	78
	3	235	50	264	128	342	157	28	1204	128
2	4	328	192	14	221	85	250	114	1204	178
	5	71	285	100	314	178	49	207	1204	228
	6	164	35	242	57	271	135	300	1204	278
	7	257	121	335	150	21	228	92	1204	328
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

Embedded Square 1										
			1204	1204	1204	1204	1204	1204	1204	
	1	203	67	274	145	303	174	38	1204	31
	2	338	160	24	238	53	260	131	1204	81
	3	88	246	117	324	195	10	224	1204	131
1	4	181	45	210	74	281	103	310	1204	181
	5	267	138	296	167	31	245	60	1204	231
	6	17	231	95	253	124	331	153	1204	274
	7	110	317	188	3	217	81	288	1204	324
		1204	1204	1204	1204	1204	1204	1204		
			1204	1204	1204	1204	1204	1204	1204	

1	2	3	4	5	6	7
31	35	32	29	33	30	34
81	78	82	79	83	80	84
131	128	132	129	133	130	127
181	178	182	179	176	180	177
231	228	225	229	226	230	227
274	278	275	279	276	280	277
324	328	325	329	326	323	327

Table composed of external columns above

3	7	4	1	5	2	6
4	1	5	2	6	3	7
5	2	6	3	7	4	1
6	3	7	4	1	5	2
7	4	1	5	2	6	3
1	5	2	6	3	7	4
2	6	3	7	4	1	5

Modulus loom table of the above table

Program 8

The geonomic Square Hidden in the Cube's Tiling Patterns

The table at upper left is the transpose of **X*** that we just derived from the modulus table of the tile center numbers.

To its right is its complementary loom-table denoted by Y^{*H} that was obtained by horizontally flipping X^{*T}.

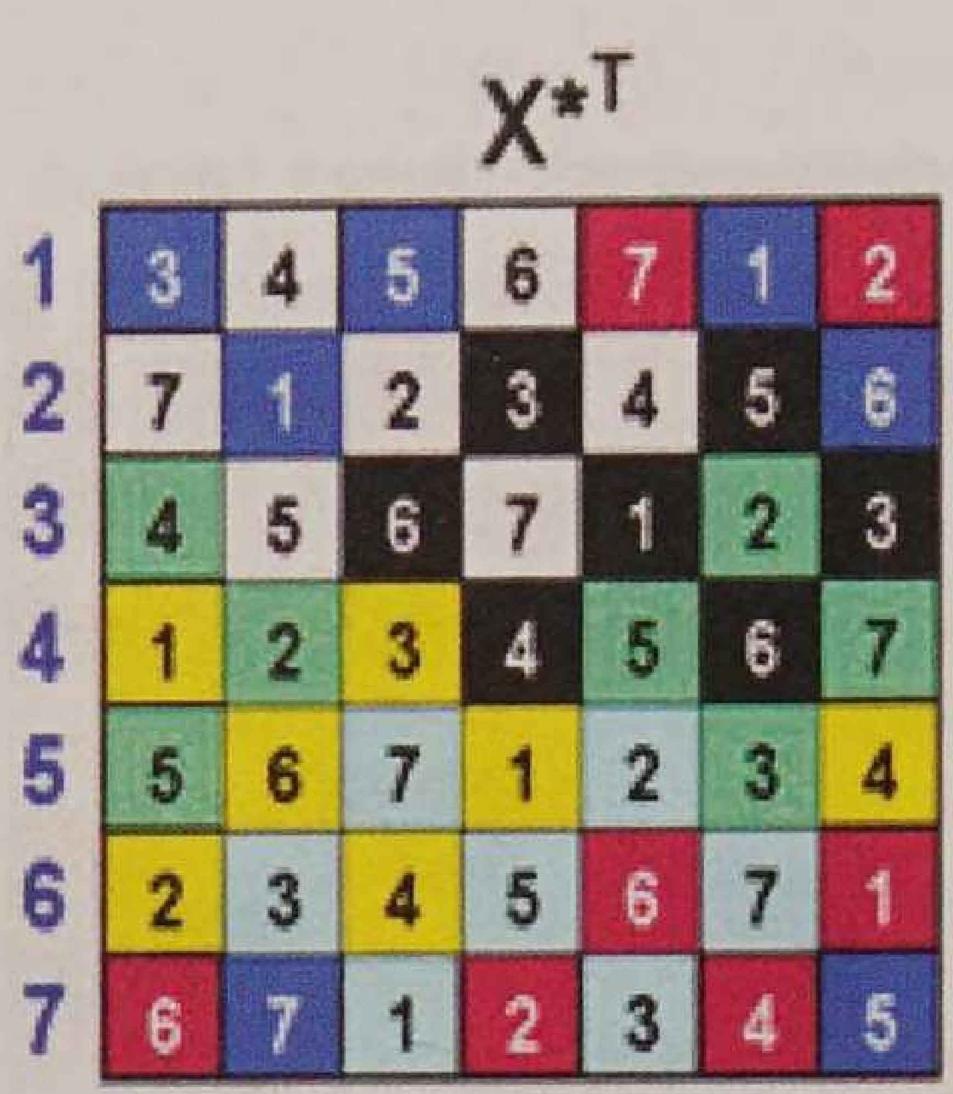
Then applying X^{*T} and Y^{*H} to the standard **2D** formula for generation of the primal:

$$W^* = n(Y^{*H} - |1|) + X^{*T}$$

we get **W*** as seen at bottom left.

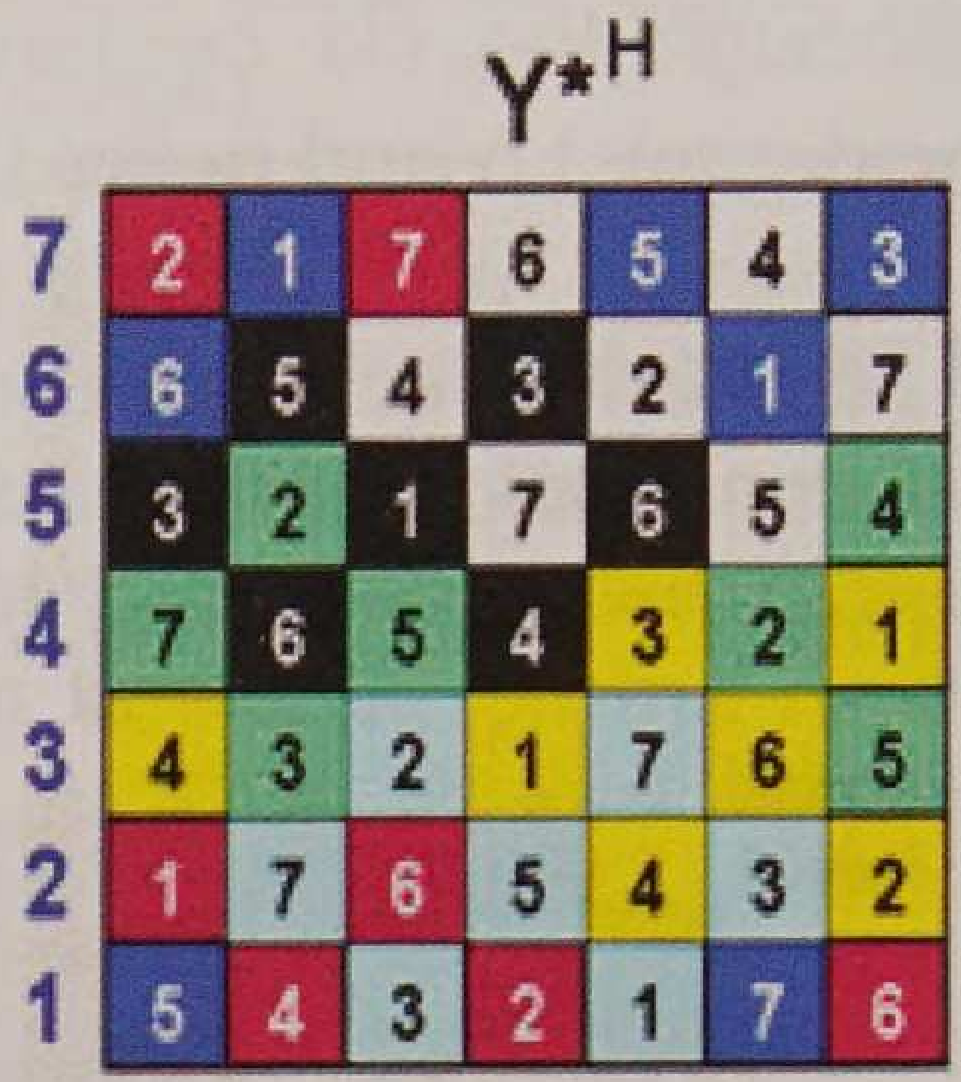
This derived square is seen to be an <u>ultra-perfect</u> size-7 square. Now that's one big surprise. Who could have guessed that?

Note that all the values that the modulus and integer functions have in common are located in the center column of **W***. This is the result of flipping X^{*T} horizontally to get Y^{*H}.



X^{*T}

1	3	4	5	6	7	1	2
2	7	1	2	3	4	5	6
3	4	5	6	7	1	2	3
4	1	2	3	4	5	6	7
5	5	6	7	1	2	3	4
6	2	3	4	5	6	7	1
7	6	7	1	2	3	4	5

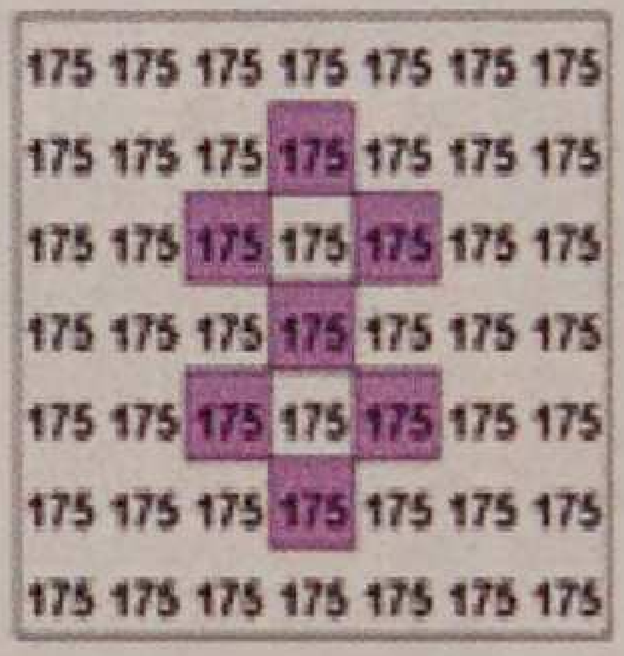


Y^{*H}

7	2	1	7	6	5	4	3
6	6	5	4	3	2	1	7
5	3	2	1	7	6	5	4
4	7	6	5	4	3	2	1
3	4	3	2	1	7	6	5
2	1	7	6	5	4	3	2
1	5	4	3	2	1	7	6

W*								
		175	175	175	175	175	175	175
1	10	4	47	41	35	22	16	175
2	42	29	23	17	11	5	48	175
3	18	12	6	49	36	30	24	175
4	43	37	31	25	19	13	7	175
5	26	20	14	1	44	38	32	175
6	2	45	39	33	27	21	8	175
7	34	28	15	9	3	46	40	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175



175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

Now here's the second big surprise: Since these tiles can be moved continuously in the cube by moving the entire cubical tiling pattern, this implies that there are **7** distinctly different tables of sorted tile-center numbers. And surprisingly, all **7** yield loom tables identical to **X***. That's another amazing discovery!

This same result can be obtained from the tiling patterns in the cube's first and second duals too. (Not shown)

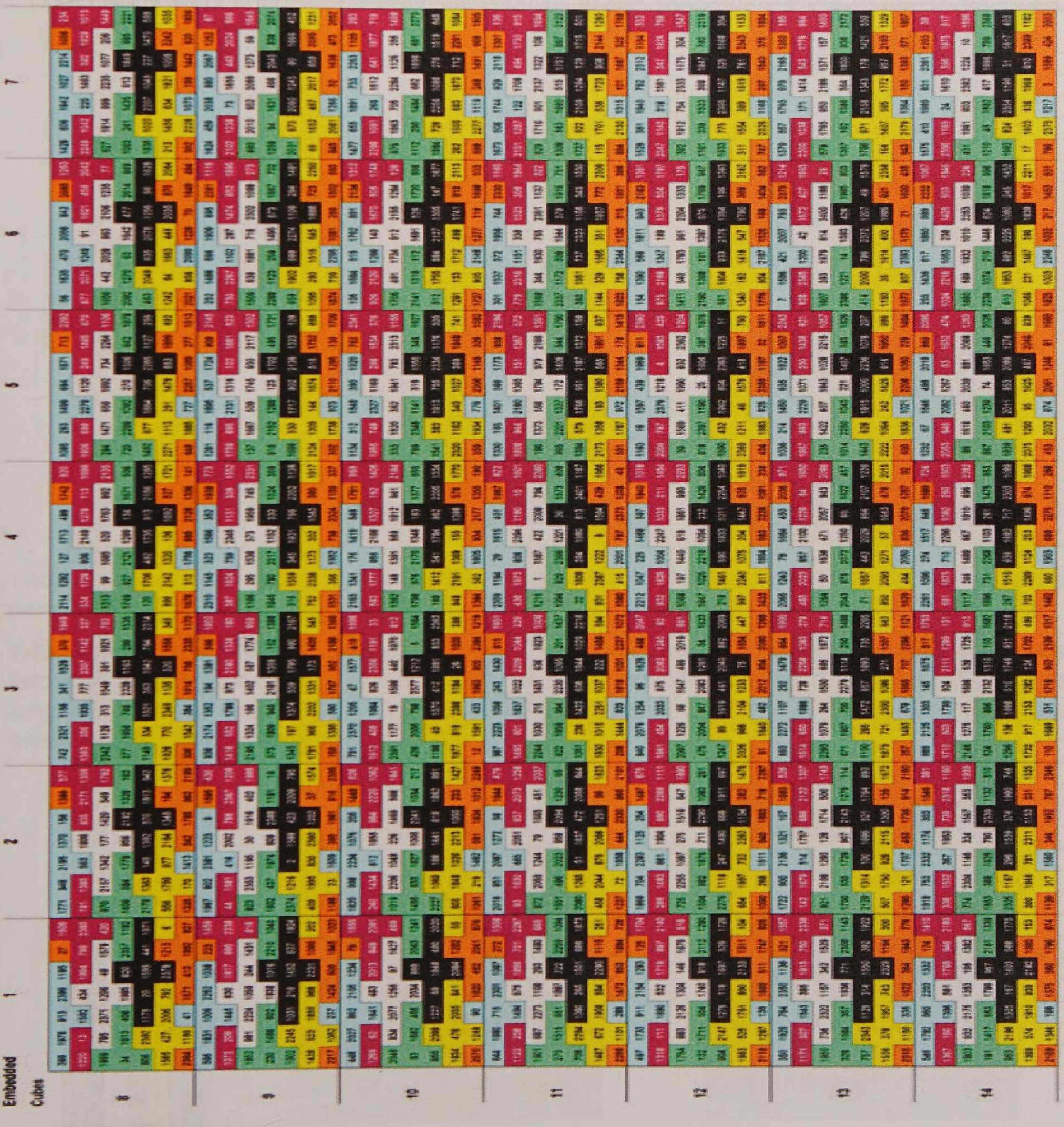
The first dual **U1*** of **W*** is obtained by interchanging the derived loom-tables in the standard generation formula.

Program 8

The loom table hidden in the tiling patterns of Class-1 Quadracubes

Next, we'll examine the patterns inherent in the tiling pattern of a 4-dimensional quadracube of size-7. Although hardly legible, the larger size-7 quadracube was chosen over the size-5 quadracube because it was seen earlier that the size-5 table has some peculiar additional properties that stem from its characteristic tile patterns' transposable symmetry.

We've already seen the acquisition of the hidden modulus loom table from sorted tile centers in 3-dimensions. What is depicted here holds in general for all prime-number-size tables in 4-dimensions and above.



Program 8

Follow these steps: First the center numbers are obtained from each tile's center. In each column in the first table below are the tile centers in the corresponding embedded square in a horizontally embedded cube.

Then separately within each block of its own, each column is sorted on its increasing values to arrive at the second table.

Then the modulus function is applied to each value of the second table to get the third table. What results are 7-duplicates of the modulus loom table, one for each horizontal cube.

The transpose of this loom-table can be derived from using the horizontally flipped version of pattern hB on the vertically embedded cubes. (That's not interesting enough to be shown too.)

L = center numbers of tiles

34	1406	377	1749	720	2092	1063
427	1799	770	2142	1113	84	1456
813	2185	1156	127	1499	470	1842
1206	177	1549	520	1892	863	2235
1992	963	2335	1306	277	1649	620
2385	1356	327	1699	670	2042	1013
1599	570	1942	913	2285	1256	227
230	1602	573	1945	916	2288	1259
623	1995	966	2338	1309	280	1652
1009	2381	1352	323	1695	666	2038
1059	30	1402	373	1745	716	2088
1845	816	2188	1159	130	1502	473
2238	1209	180	1552	523	1895	866
1452	423	1795	766	2138	1109	80
83	1455	426	1798	769	2141	1112
476	1848	819	2191	1162	133	1505
862	2234	1205	176	1548	519	1891
1255	226	1598	569	1941	912	2284
2041	1012	2384	1355	326	1698	669
2091	1062	33	1405	376	1748	719
1648	619	1991	962	2334	1305	276
279	1651	622	1994	965	2337	1308
672	2044	1015	2387	1358	329	1701
715	2087	1058	29	1401	372	1744
1108	79	1451	422	1794	765	2137
1894	866	2237	1208	179	1551	522
2287	1258	229	1601	572	1944	915
1501	472	1844	815	2187	1158	129
132	1504	475	1847	818	2190	1161
525	1897	868	2240	1211	182	1554
911	2283	1254	225	1597	568	1940
1304	275	1647	618	1990	961	2333
1747	718	2090	1061	32	1404	375
2140	1111	82	1454	425	1797	768
1697	668	2040	1011	2383	1354	325
328	1700	671	2043	1014	2386	1357
378	1750	721	2093	1064	35	1407
764	2136	1107	78	1450	421	1793
1157	128	1500	471	1843	814	2186
1943	914	2286	1257	228	1600	571
2336	1307	278	1650	621	1993	964
1550	521	1893	864	2236	1207	178
181	1553	524	1896	867	2239	1210
574	1946	917	2289	1260	231	1603
960	2332	1303	274	1646	617	1989
1353	324	1696	667	2039	1010	2382
1796	767	2139	1110	81	1453	424
2189	1160	131	1503	474	1846	817
1403	374	1746	717	2089	1060	31

L centers sorted within each block-row

34	177	327	127	277	84	227
427	570	377	520	670	470	620
813	963	770	913	720	863	1013
1206	1356	1156	1306	1113	1256	1063
1599	1406	1549	1699	1499	1649	1456
1992	1799	1942	1749	1892	2042	1842
2385	2185	2335	2142	2285	2092	2235
230	30	180	323	130	280	80
623	423	573	373	523	666	473
1009	816	966	766	916	716	866
1059	1209	1352	1159	1309	1109	1259
1452	1602	1402	1552	1695	1502	1652
1845	1995	1795	1945	1745	1895	2038
2238	2381	2188	2338	2138	2288	2088
83	226	33	176	326	133	276
476	619	426	569	376	519	669
862	1012	819	962	769	912	719
1255	1062	1205	1355	1162	1305	1112
1648	1455	1598	1405	1548	1698	1505
2041	1848	1991	1798	1941	1748	1891
2091	2234	2384	2191	2334	2141	2284
279	79	229	29	179	329	129
672	472	622	422	572	372	522
715	865	1015	815	965	765	915
1108	1258	1058	1208	1358	1158	1308
1501	1651	1451	1601	1401	1551	1701
1894	2044	1844	1994	1794	1944	1744
2287	2087	2237	2387	2187	2337	2137
132	275	82	225	32	182	325
525	668	475	618	425	568	375
911	718	868	1011	818	961	768
1304	1111	1254	1061	1211	1354	1161
1697	1504	1647	1454	1597	1404	1554
1747	1897	2040	1847	1990	1797	1940
2140	2283	2090	2240	2383	2190	2333
328	128	278	78	228	35	178
378	521	671	471	621	421	571
764	914	721	864	1014	814	964
1157	1307	1107	1257	1064	1207	1357
1550	1700	1500	1650	1450	1600	1407
1943	1750	1893	2043	1843	1993	1793
2336	2136	2286	2093	2236	2386	2186
181	324	131	274	81	231	31
574	374	524	667	474	617	424
960	767	917	717	867	1010	817
1353	1160	1303	1110	1260	1060	1210
1403	1553	1696	1503	1646	1453	1603
1796	1946	1746	1896	2039	1846	1989
1314	1308	1323	1311	1331	1313	1302

X = Modulus 7 of L

6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2
6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2

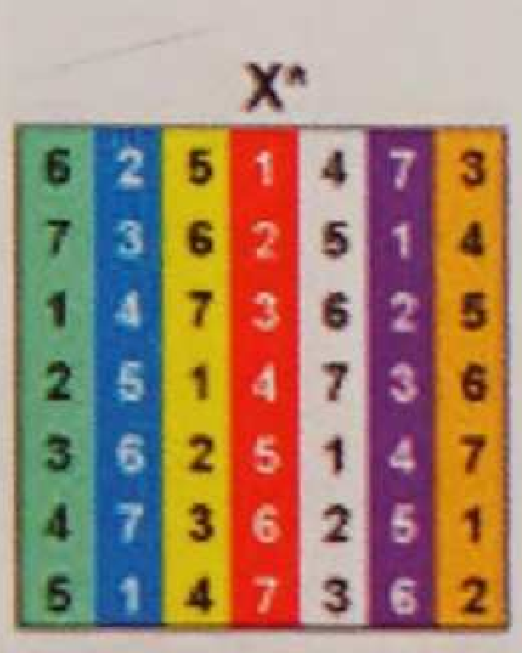
Program 8

Generating Ultra-perfect Dual Squares from the Derived Modulus Loom Table

An ultra-perfect square can be derived from this loom table **X*** just as was shown for 3-dimensions. Equivalently, the derived modulus loom-table can be vertically flipped instead of transposed to get a viable complementary integer loom-table. And to preserve the orientation of the tile patterns, both loom tables need to be transposed.

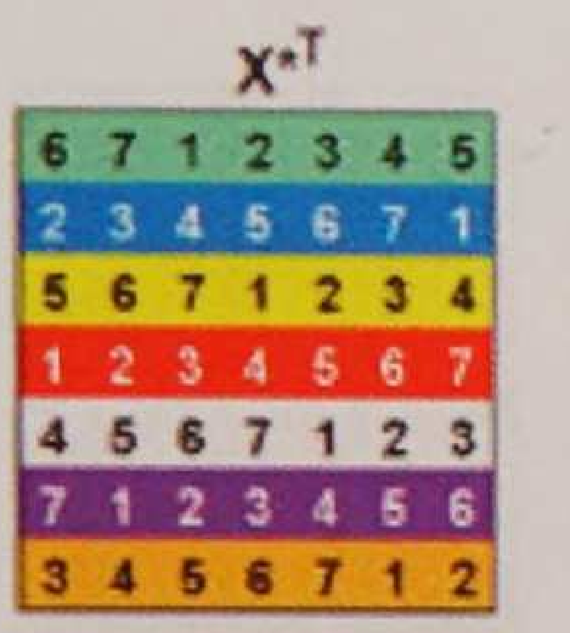
It is these versions that produce the hidden geonomic squares shown here. Square **U1*** can produce two more additional dual squares (not shown) with all the same properties.

X^*



6	2	5	1	4	7	3
7	3	6	2	5	1	4
1	4	7	3	6	2	5
2	5	1	4	7	3	6
3	6	2	5	1	4	7
4	7	3	6	2	5	1
5	1	4	7	3	6	2

X^{*T}

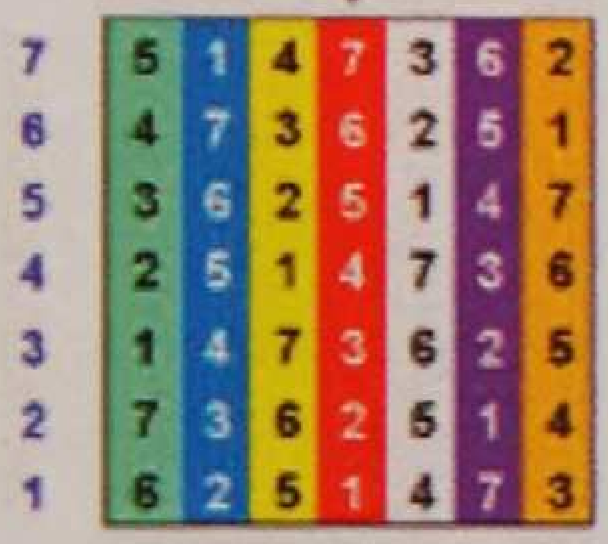


6	7	1	2	3	4	5
2	3	4	5	6	7	1
5	6	7	1	2	3	4
1	2	3	4	5	6	7
4	5	6	7	1	2	3
7	1	2	3	4	5	6
3	4	5	6	7	1	2

	1	2	3	4	5	6	7	
	W*	175	175	175	175	175	175	175
1	34	28	15	9	3	46	40	175
2	2	45	39	33	27	21	8	175
3	26	20	14	1	44	38	32	175
4	43	37	31	25	19	13	7	175
5	18	12	6	49	36	30	24	175
6	42	29	23	17	11	5	48	175
7	10	4	47	41	35	22	16	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

	1	2	3	4	5	6	7	
	U1*	175	175	175	175	175	175	175
1	40	46	3	9	15	28	34	175
2	8	21	27	33	39	45	2	175
3	32	38	44	1	14	20	26	175
4	7	13	19	25	31	37	43	175
5	24	30	36	49	6	12	18	175
6	48	5	11	17	23	29	42	175
7	16	22	35	41	47	4	10	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

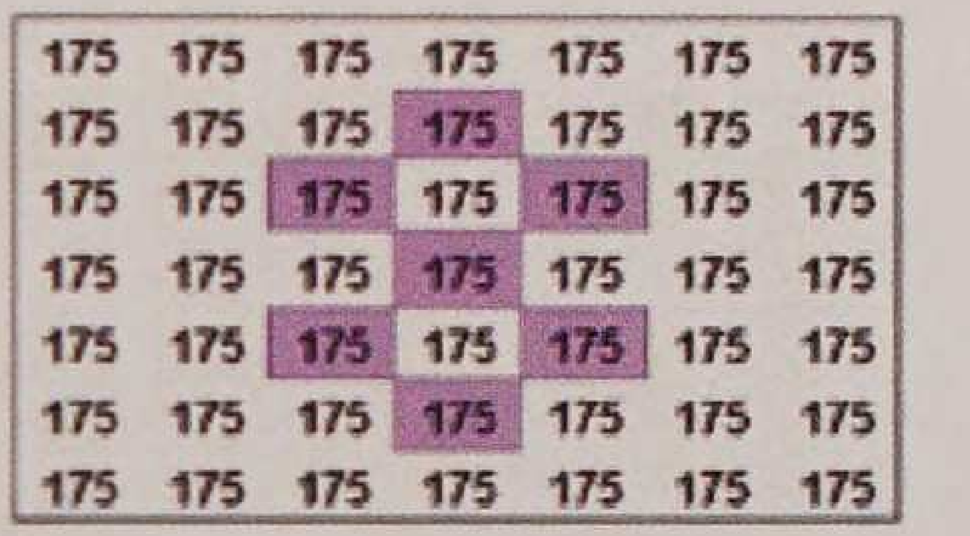
Y^*



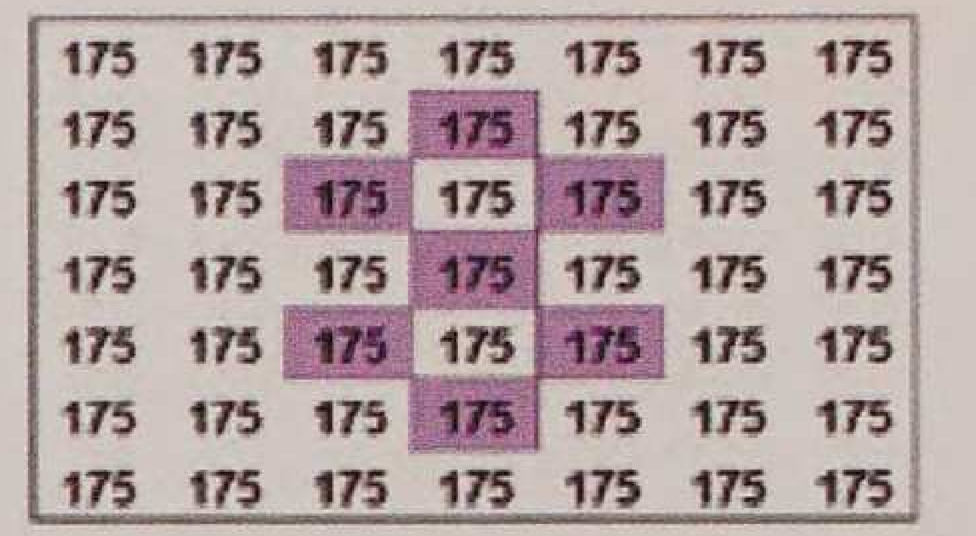
7	5	1	4	7	3	6	2
6	4	7	3	6	2	5	1
5	3	6	2	5	1	4	7
4	2	5	1	4	7	3	6
3	1	4	7	3	6	2	5
2	7	3	6	2	5	1	4
1	6	2	5	1	4	7	3

Y^{*T}

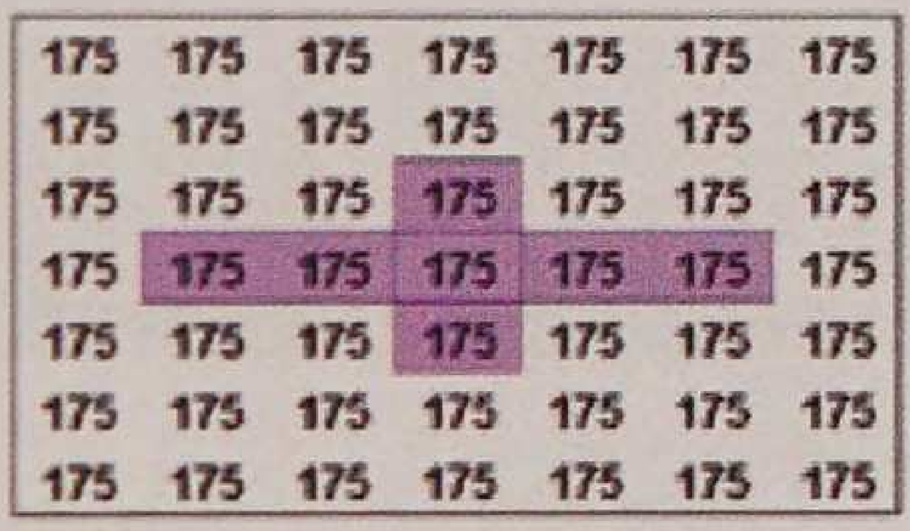
5	4	3	2	1	7	6
1	7	6	5	4	3	2
4	3	2	1	7	6	5
7	6	5	4	3	2	1
3	2	1	7	6	5	4
6	5	4	3	2	1	7
2	1	7	6	5	4	3



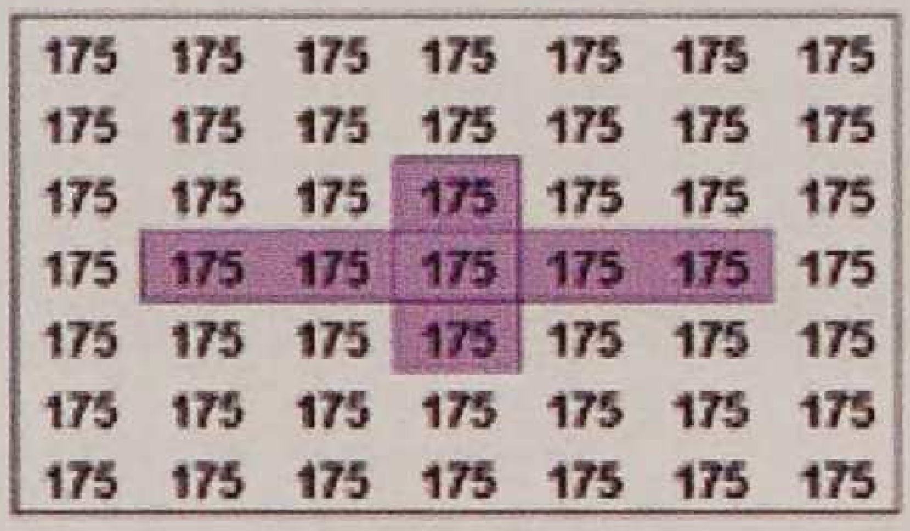
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175



175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175



175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

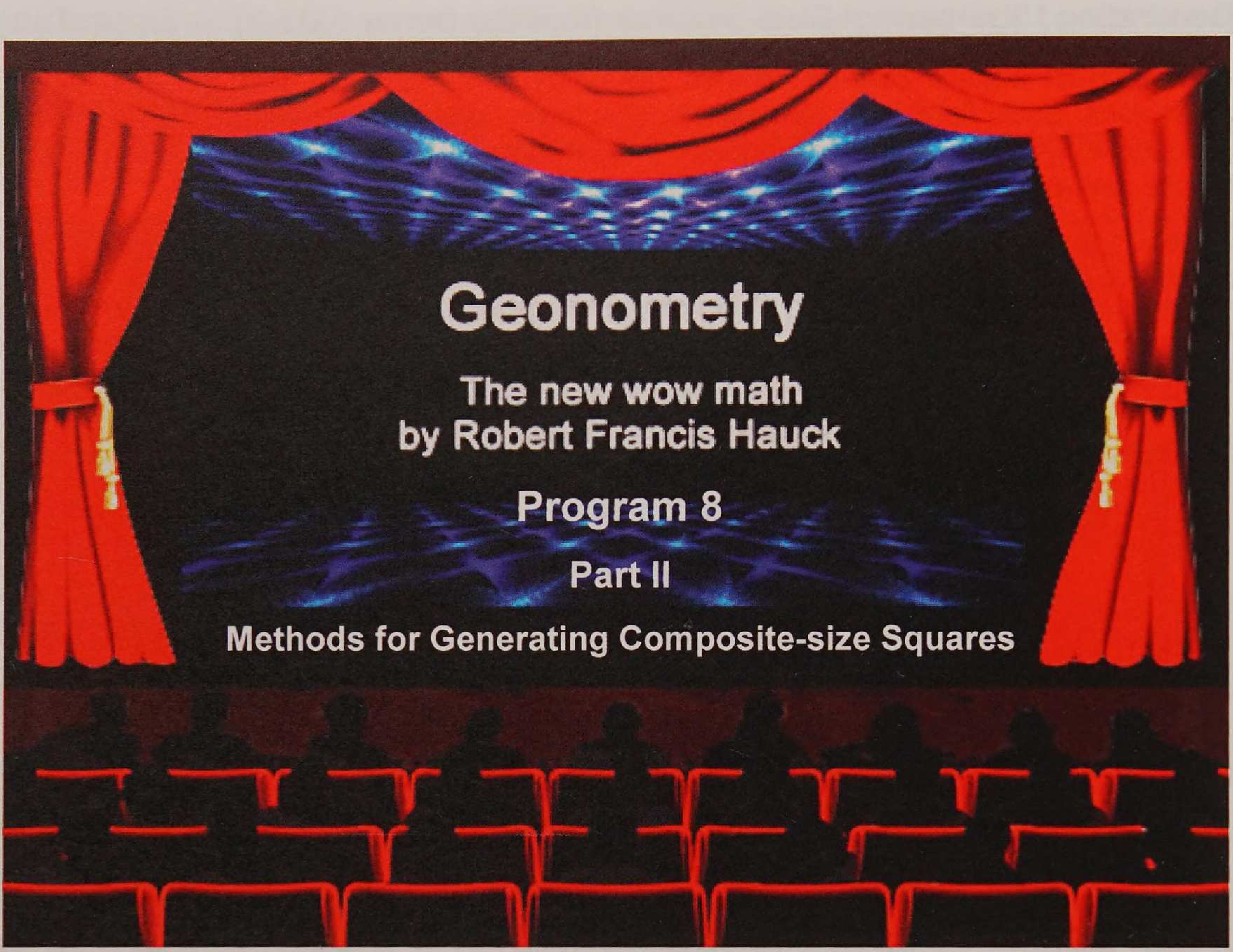


175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175
175	175	175	175	175	175	175

These hidden loom tables in 3-dimensions and now in 4-dimensions lend credence to the pervasive undulating harmonic fabric of space that Geonometry has inadvertently tapped into!

What this implies is this: There are basic fundamental harmonic patterns running everywhere throughout any multi-dimensional space. This is an indication of the existence of the *ether* long sought-after since the 15th century. Here is another mathematical confirmation of the existence of the very fabric of space itself.

* * *



Geonometry

The new wow math
by Robert Francis Hauck

Program 8

Part II

Methods for Generating Composite-size Squares

Here in Part II we will see just how the squares beyond Class-1 are generated. There are five distinct methods for doing this; there is one method that is used in one of other methods; and there is one method for co-mingling regular geonomic squares with the matchmaker square of the same size, for a total of **7** more methods in addition to the Double Quark algorithm seen back in Program 7.

Program 8

#1: Expansion by the Balloon Method

The demonstration here shows that the size-5 perfect square can be expanded by enlarging its cells to any size block-square uniformly and the resulting square will still be perfect. This property holds for any size square that is perfect or near-perfect. Considering that the wrap diagonals cut across the block-squares in different proportions from multiplicity **3** onward, this is just an amazing property! Although it does have a mathematical basis, it's not worth the time and effort to explain it all.

The characteristic number $\mathbf{C_{mn}}$ of each square of size **n** of multiplicity **m**, where **p** is the dimensional average of the square, is given by the general formula:

$$(8.18) \qquad C_{mn} = pmn$$

Here **n = 5**, **m** ranges from **1** thru **3** and **p = 13**. The characteristic numbers are **65**, **130** and **195**, which are **1**, **2** and **3** times **pn = 65**, respectively. Observe that the number **p** is the same for every multiplicity.

W(5)		65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

W(5,2)		130	130	130	130	130	130	130	130	130	130
1	20	20	8	8	21	21	14	14	2	2	130
2	20	20	8	8	21	21	14	14	2	2	130
3	11	11	4	4	17	17	10	10	23	23	130
4	11	11	4	4	17	17	10	10	23	23	130
5	7	7	25	25	13	13	1	1	19	19	130
6	7	7	25	25	13	13	1	1	19	19	130
7	3	3	16	16	9	9	22	22	15	15	130
8	3	3	16	16	9	9	22	22	15	15	130
9	24	24	12	12	5	5	18	18	6	6	130
10	24	24	12	12	5	5	18	18	6	6	130
	130	130	130	130	130	130	130	130	130	130	
		130	130	130	130	130	130	130	130	130	130

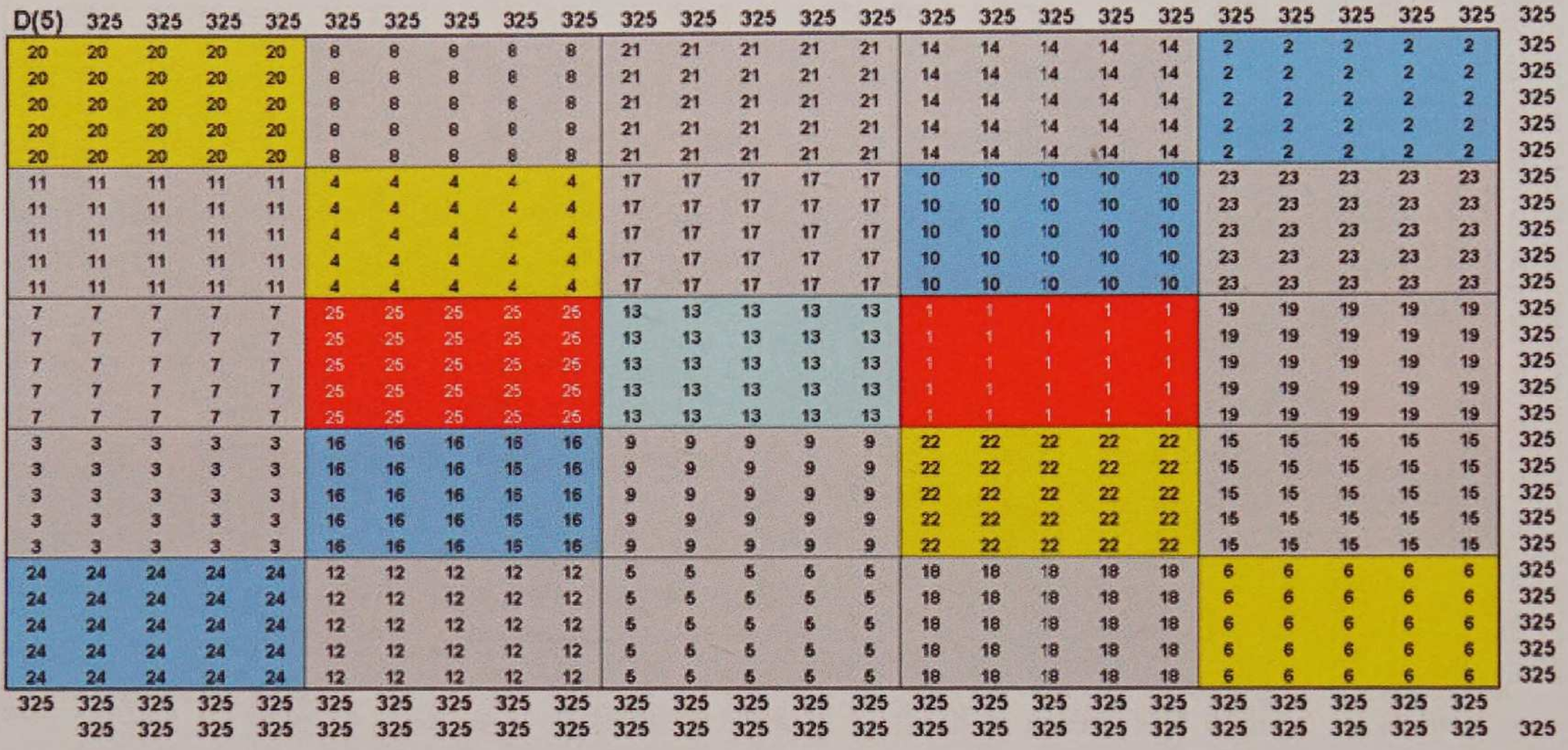
W(5,3)		195	195	195	195	195	195	195	195	195	195	195	195	195	195	195
1	20	20	20	8	8	8	21	21	21	14	14	14	2	2	2	195
2	20	20	20	8	8	8	21	21	21	14	14	14	2	2	2	195
3	20	20	20	8	8	8	21	21	21	14	14	14	2	2	2	195
4	11	11	11	4	4	4	17	17	17	10	10	10	23	23	23	195
5	11	11	11	4	4	4	17	17	17	10	10	10	23	23	23	195
6	11	11	11	4	4	4	17	17	17	10	10	10	23	23	23	195
7	7	7	7	25	25	25	13	13	13	1	1	1	19	19	19	195
8	7	7	7	25	25	25	13	13	13	1	1	1	19	19	19	195
9	7	7	7	25	25	25	13	13	13	1	1	1	19	19	19	195
10	3	3	3	16	16	16	9	9	9	22	22	22	15	15	15	195
11	3	3	3	16	16	16	9	9	9	22	22	22	15	15	15	195
12	3	3	3	16	16	16	9	9	9	22	22	22	15	15	15	195
13	24	24	24	12	12	12	5	5	5	18	18	18	6	6	6	195
14	24	24	24	12	12	12	5	5	5	18	18	18	6	6	6	195
15	24	24	24	12	12	12	5	5	5	18	18	18	6	6	6	195
	195	195	195	195	195	195	195	195	195	195	195	195	195	195	195	
		195	195	195	195	195	195	195	195	195	195	195	195	195	195	195

Program 8

#2: Expansion by the T-Ball method

(The merging of a Tiled-expansion with a Balloon-expansion)

This method is applicable to generating Class-6 squares from Class-1 squares. The top table is a ballooned expansion of the size-5 square to one of size-25, denoted by **D(5)**. Below is a size-25 expansion by tiling with the size-5 square, denoted by **E(5)**. Both are generated from the same size-5 perfect square **W(5)**.



D(5)	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
20	20	20	20	20	8	8	8	8	8	21	21	21	21	21	14	14	14	14	14	2	2	2	2	2	325
11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
11	11	11	11	11	4	4	4	4	4	17	17	17	17	17	10	10	10	10	10	23	23	23	23	23	325
7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
7	7	7	7	7	25	25	25	25	25	13	13	13	13	13	1	1	1	1	1	19	19	19	19	19	325
3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
3	3	3	3	3	16	16	16	16	16	9	9	9	9	9	22	22	22	22	22	15	15	15	15	15	325
24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
24	24	24	24	24	12	12	12	12	12	5	5	5	5	5	18	18	18	18	18	6	6	6	6	6	325
325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
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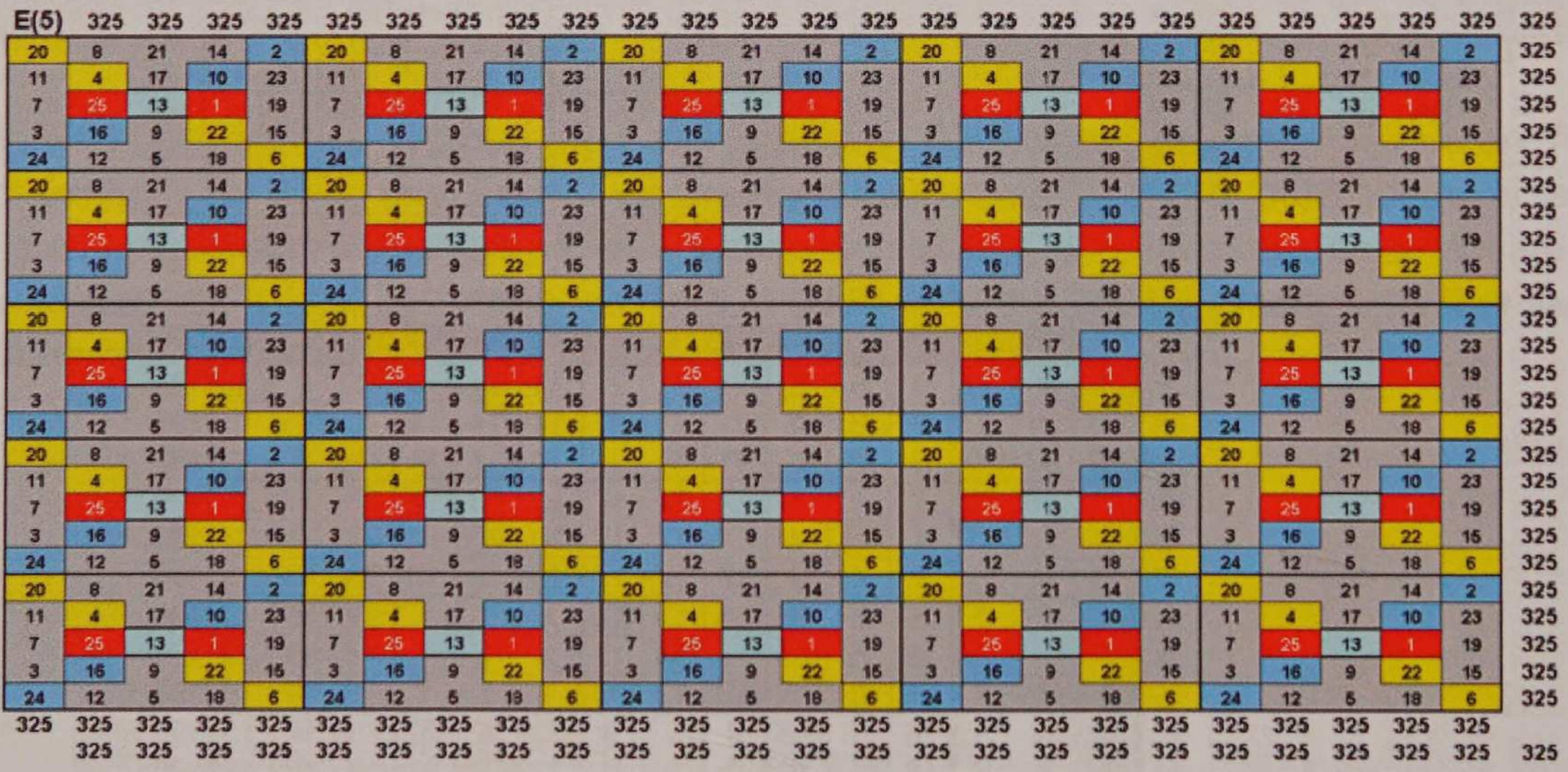


E(5)	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	325
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	325
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	325
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	325
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	325
325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	
	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325

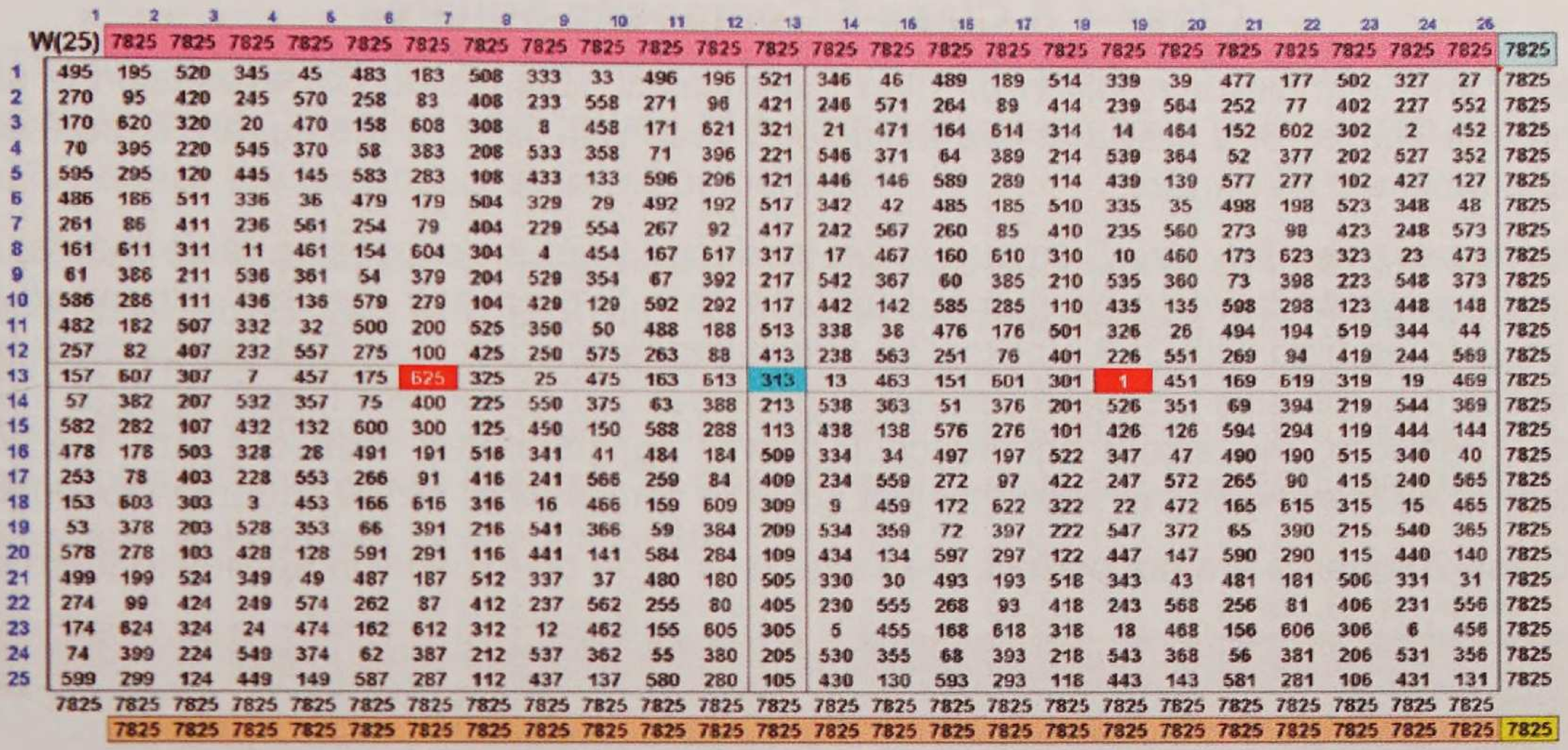
Two distinct perfect size-25 squares, each the dual of the other, are next generated from the two formulas for **n = 5**:

$$(8.19)\quad W(n^2) = n^2\,(E(n)-|1|) + D(n)$$

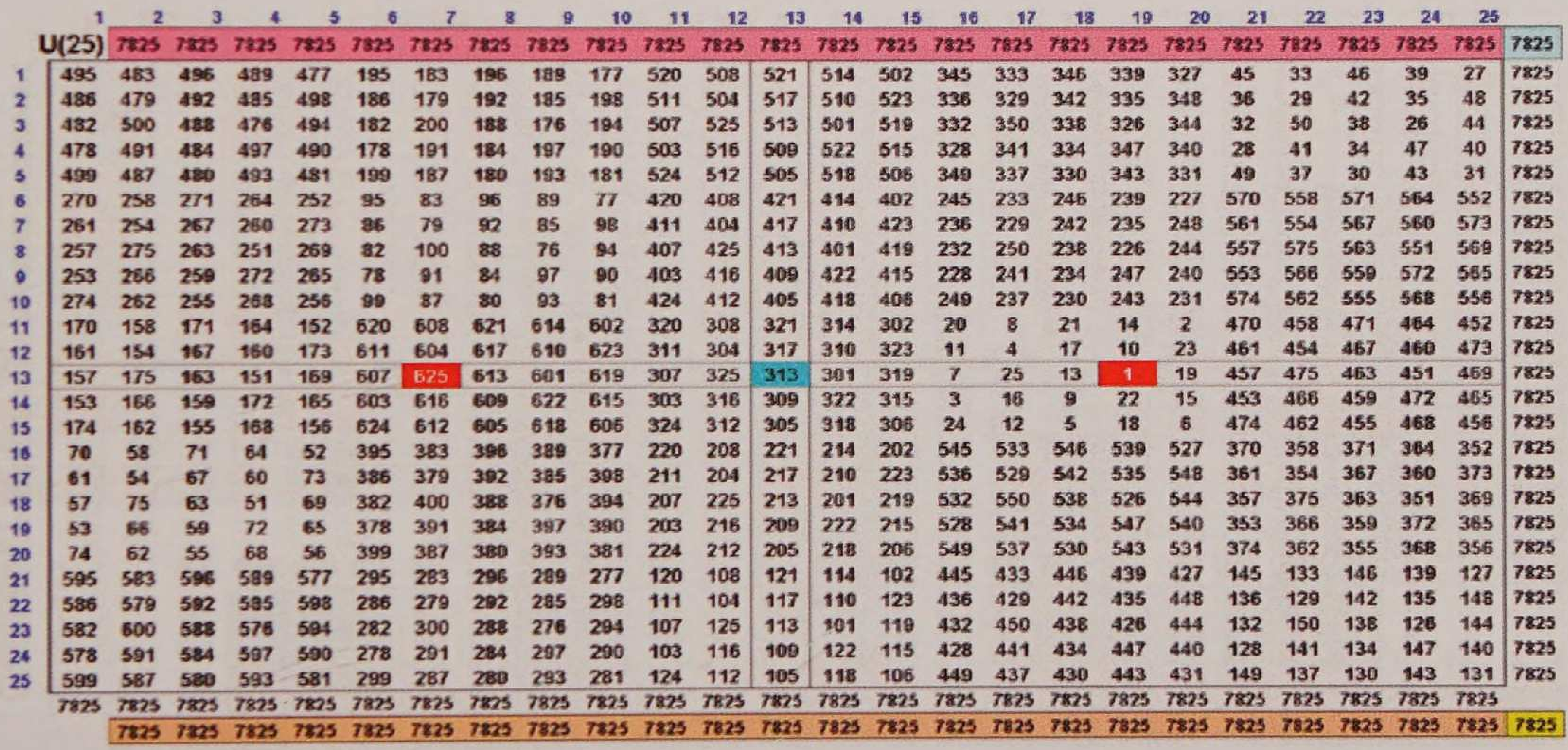
$$(8.20)\quad U(n^2) = n^2\,(D(n)-|1|) + E(n)$$

Program 8

Here are the generated dual squares **W(25)** and **U(25)**.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	495	195	520	345	45	483	183	508	333	33	496	196	521	346	46	489	189	514	339	39	477	177	502	327	27	7825
2	270	95	420	245	570	258	83	408	233	558	271	96	421	246	571	264	89	414	239	564	252	77	402	227	552	7825
3	170	620	320	20	470	158	608	308	8	458	171	621	321	21	471	164	614	314	14	464	152	602	302	2	452	7825
4	70	395	220	545	370	58	383	208	533	358	71	396	221	546	371	64	389	214	539	364	52	377	202	527	352	7825
5	595	295	120	445	145	583	283	108	433	133	596	296	121	446	146	589	289	114	439	139	577	277	102	427	127	7825
6	486	186	511	336	36	479	179	504	329	29	492	192	517	342	42	485	185	510	335	35	498	198	523	348	48	7825
7	261	86	411	236	561	254	79	404	229	554	267	92	417	242	567	260	85	410	235	560	273	98	423	248	573	7825
8	161	611	311	11	461	154	604	304	4	454	167	617	317	17	467	160	610	310	10	460	173	623	323	23	473	7825
9	61	386	211	536	361	54	379	204	529	354	67	392	217	542	367	60	385	210	535	360	73	398	223	548	373	7825
10	586	286	111	436	136	579	279	104	429	129	592	292	117	442	142	585	285	110	435	135	598	298	123	448	148	7825
11	482	182	507	332	32	500	200	525	350	50	488	188	513	338	38	476	176	501	326	26	494	194	519	344	44	7825
12	257	82	407	232	557	275	100	425	250	575	263	88	413	238	563	251	76	401	226	551	269	94	419	244	569	7825
13	157	607	307	7	457	175	625	325	25	475	163	613	313	13	463	151	601	301	1	451	169	619	319	19	469	7825
14	57	382	207	532	357	75	400	225	550	375	63	388	213	538	363	51	376	201	526	351	69	394	219	544	369	7825
15	582	282	107	432	132	600	300	125	450	150	588	288	113	438	138	576	276	101	426	126	594	294	119	444	144	7825
16	478	178	503	328	28	491	191	516	341	41	484	184	509	334	34	497	197	522	347	47	490	190	515	340	40	7825
17	253	78	403	228	553	266	91	416	241	566	259	84	409	234	559	272	97	422	247	572	265	90	415	240	565	7825
18	153	603	303	3	453	166	616	316	16	466	159	609	309	9	459	172	622	322	22	472	165	615	315	15	465	7825
19	53	378	203	528	353	66	391	216	541	366	59	384	209	534	359	72	397	222	547	372	65	390	215	540	365	7825
20	578	278	103	428	128	591	291	116	441	141	584	284	109	434	134	597	297	122	447	147	590	290	115	440	140	7825
21	499	199	524	349	49	487	187	512	337	37	480	180	505	330	30	493	193	518	343	43	481	181	506	331	31	7825
22	274	99	424	249	574	262	87	412	237	562	255	80	405	230	555	268	93	418	243	568	256	81	406	231	556	7825
23	174	624	324	24	474	162	612	312	12	462	155	605	305	5	455	168	618	318	18	468	156	606	306	6	456	7825
24	74	399	224	549	374	62	387	212	537	362	55	380	205	530	355	68	393	218	543	368	56	381	206	531	356	7825
25	599	299	124	449	149	587	287	112	437	137	580	280	105	430	130	593	293	118	443	143	581	281	106	431	131	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
U(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	495	483	496	489	477	195	183	196	189	177	520	508	521	514	502	345	333	346	339	327	45	33	46	39	27	7825
2	486	479	492	485	498	186	179	192	185	198	511	504	517	510	523	336	329	342	335	348	36	29	42	35	48	7825
3	482	500	488	476	494	182	200	188	176	194	507	525	513	501	519	332	350	338	326	344	32	50	38	26	44	7825
4	478	491	484	497	490	178	191	184	197	190	503	516	509	522	515	328	341	334	347	340	28	41	34	47	40	7825
5	499	487	480	493	481	199	187	180	193	181	524	512	505	518	506	349	337	330	343	331	49	37	30	43	31	7825
6	270	258	271	264	252	95	83	96	89	77	420	408	421	414	402	245	233	246	239	227	570	558	571	564	552	7825
7	261	254	267	260	273	86	79	92	85	98	411	404	417	410	423	236	229	242	235	248	561	554	567	560	573	7825
8	257	275	263	251	269	82	100	88	76	94	407	425	413	401	419	232	250	238	226	244	557	575	563	551	569	7825
9	253	266	259	272	265	78	91	84	97	90	403	416	409	422	415	228	241	234	247	240	553	566	559	572	565	7825
10	274	262	255	268	256	99	87	80	93	81	424	412	405	418	406	249	237	230	243	231	574	562	555	568	556	7825
11	170	158	171	164	152	620	608	621	614	602	320	308	321	314	302	20	8	21	14	2	470	458	471	464	452	7825
12	161	154	167	160	173	611	604	617	610	623	311	304	317	310	323	11	4	17	10	23	461	454	467	460	473	7825
13	157	175	163	151	169	607	625	613	601	619	307	325	313	301	319	7	25	13	1	19	457	475	463	451	469	7825
14	153	166	159	172	165	603	616	609	622	615	303	316	309	322	315	3	16	9	22	15	453	466	459	472	465	7825
15	174	162	155	168	156	624	612	605	618	606	324	312	305	318	306	24	12	5	18	6	474	462	455	468	456	7825
16	70	58	71	64	52	395	383	396	389	377	220	208	221	214	202	545	533	546	539	527	370	358	371	364	352	7825
17	61	54	67	60	73	386	379	392	385	398	211	204	217	210	223	536	529	542	535	548	361	354	367	360	373	7825
18	57	75	63	51	69	382	400	388	376	394	207	225	213	201	219	532	550	538	526	544	357	375	363	351	369	7825
19	53	66	59	72	65	378	391	384	397	390	203	216	209	222	215	528	541	534	547	540	353	366	359	372	365	7825
20	74	62	55	68	56	399	387	380	393	381	224	212	205	218	206	549	537	530	543	531	374	362	355	368	356	7825
21	595	583	596	589	577	295	283	296	289	277	120	108	121	114	102	445	433	446	439	427	145	133	146	139	127	7825
22	586	579	592	585	598	286	279	292	285	298	111	104	117	110	123	436	429	442	435	448	136	129	142	135	148	7825
23	582	600	588	576	594	282	300	288	276	294	107	125	113	101	119	432	450	438	426	444	132	150	138	126	144	7825
24	578	591	584	597	590	278	291	284	297	290	103	116	109	122	115	428	441	434	447	440	128	141	134	147	140	7825
25	599	587	580	593	581	299	287	280	293	281	124	112	105	118	106	449	437	430	443	431	149	137	130	143	131	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Now, here is what's so amazing: **D(5) = Y(25)** is the integer loom table and **E(5) = X(25)** is the modulus loom table that can both be derived from **W(25)** in the base **25**.

And further, **Y(25)** is the modulus table and **X(25)** is the integer table that can both be derived from the dual **U(25)** in the base **25**. So clearly, **D(5)** and **E(5)** are complementary loom tables for the size-25 dual squares.

Note that both **W(25)** and **U(25)** are pairwise centrally symmetric just like the size-5 square **W(5)** that was involved in their manifestation.

It is pretty obvious from **U(25)** that the resulting squares possess no modularity.

* * *

This ends the description of the **T-Ball** expansion method.

Program 8

#3: Expansion by the ATE Method for generating Class-4 & Class-5 Composite Squares

The name **ATE** stands for Addition of Tiled Expansions. This method of expansion is simply the tiling of a square of a size that is a multiple of itself with itself. If the square is perfect, the expanded table will be perfect.

Note, however, since the size-3 square is not perfect, to get an expansion table that is, each size-3 square would have to be married up with its reflection and the **9x9** expansion table would have to be tiled with **3x6** blocks. But this is impossible because **9** is not divisible by **2** ! That implies that the size-3 square can never be expanded geonomically with an odd multiple of itself. Therefore, the size-9 square could never be generated from the size-3 square's expansion. It will be shown in Program 9 just how the ultra-perfect size-9 square was found.

Since Class-2 squares are not perfect, the same restriction on expansion applies to them too.

E(3)	60	60	60	60	60	60	60	60	60	60	60	60
2	7	6	6	7	2	2	7	6	6	7	2	60
9	5	1	1	5	9	9	5	1	1	5	9	60
4	3	8	8	3	4	4	3	8	8	3	4	60
4	3	8	8	3	4	4	3	8	8	3	4	60
9	5	1	1	5	9	9	5	1	1	5	9	60
2	7	6	6	7	2	2	7	6	6	7	2	60
2	7	6	6	7	2	2	7	6	6	7	2	60
9	5	1	1	5	9	9	5	1	1	5	9	60
4	3	8	8	3	4	4	3	8	8	3	4	60
4	3	8	8	3	4	4	3	8	8	3	4	60
9	5	1	1	5	9	9	5	1	1	5	9	60
2	7	6	6	7	2	2	7	6	6	7	2	60
60	60	60	60	60	60	60	60	60	60	60	60	
	60	60	60	60	60	60	60	60	60	60	60	60

E(4)	102	102	102	102	102	102	102	102	102	102	102	102
5	10	15	4	5	10	15	4	5	10	15	4	102
16	3	6	9	16	3	6	9	16	3	6	9	102
2	13	12	7	2	13	12	7	2	13	12	7	102
11	8	1	14	11	8	1	14	11	8	1	14	102
5	10	15	4	5	10	15	4	5	10	15	4	102
16	3	6	9	16	3	6	9	16	3	6	9	102
2	13	12	7	2	13	12	7	2	13	12	7	102
11	8	1	14	11	8	1	14	11	8	1	14	102
5	10	15	4	5	10	15	4	5	10	15	4	102
16	3	6	9	16	3	6	9	16	3	6	9	102
2	13	12	7	2	13	12	7	2	13	12	7	102
11	8	1	14	11	8	1	14	11	8	1	14	102
102	102	102	102	102	102	102	102	102	102	102	102	
	102	102	102	102	102	102	102	102	102	102	102	102

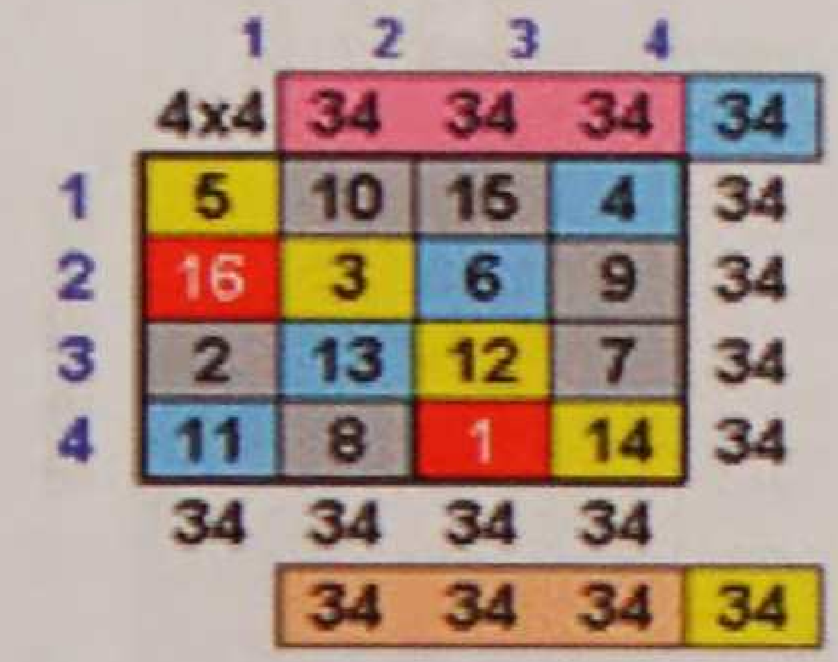
	E(5)	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260
1	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
2	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
3	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
4	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
5	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
6	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
7	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
8	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
9	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
10	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
11	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
12	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
13	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
14	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
15	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
16	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
17	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
18	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
19	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
20	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260

Program 8

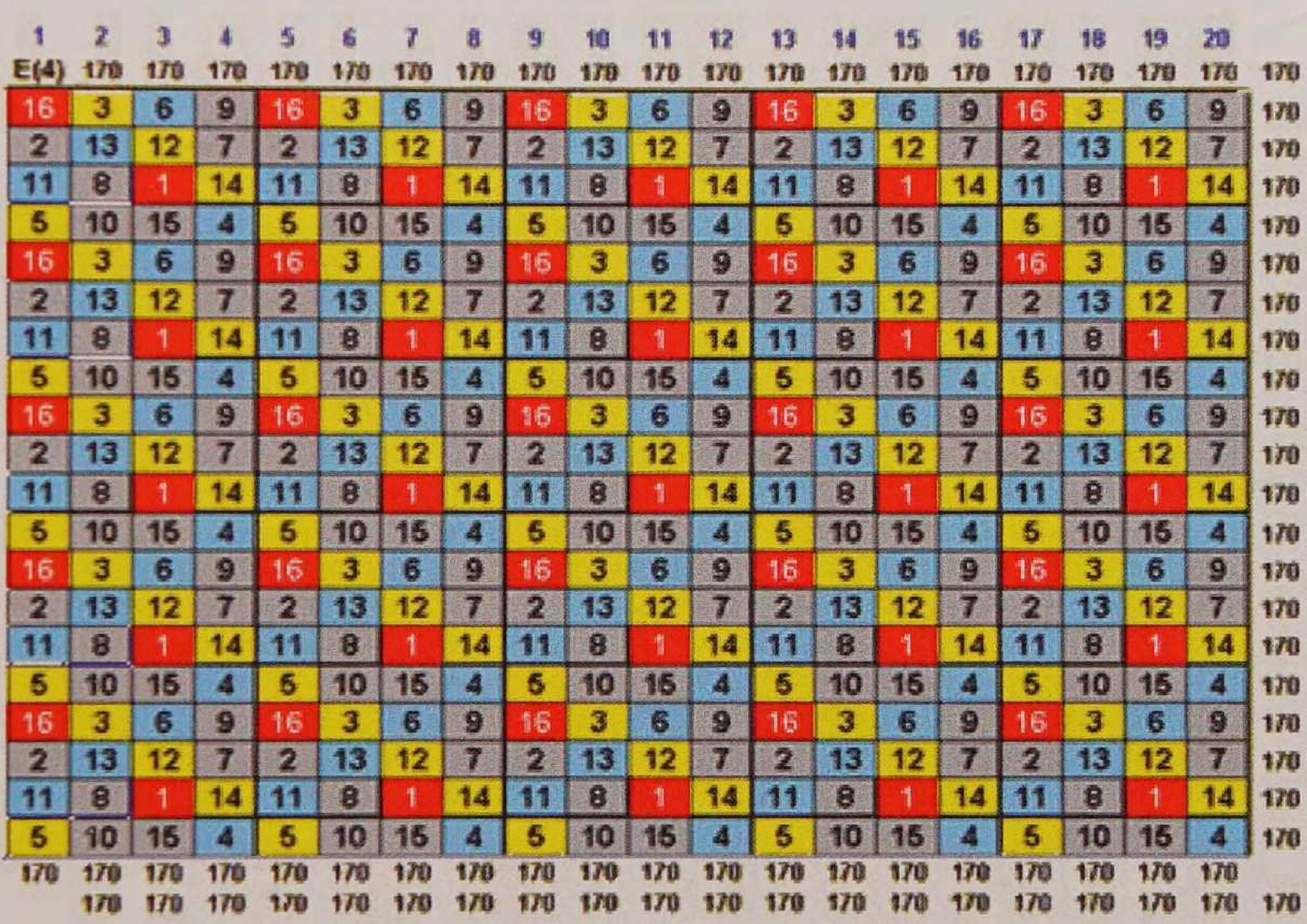
Application to composing Class-4 Squares

The **ATE** method applies to the generation of Class-4 and Class-5 squares. This method of expansion is simply the tiling of a square of a size that is a multiple of itself with itself. If the square is perfect, the expanded table will be perfect.

First we'll see an application to the generation of the size-20 Class-4 square. Shown here are the expansion tables **E(4)** and **E(5)** of size-20 tiled with size-4 and size-5 perfect squares, respectively. These two expanded squares shall be merged together by addition to get a size-20 square **W(20)** without any duplication of numbers. In order to do that, the tilings need to be exact multiples of each other: **E(4)** has **25** tiles of the size-4 square and **E(5)** has **16** tiles of the size-5 square, yielding two tables of size **20**.

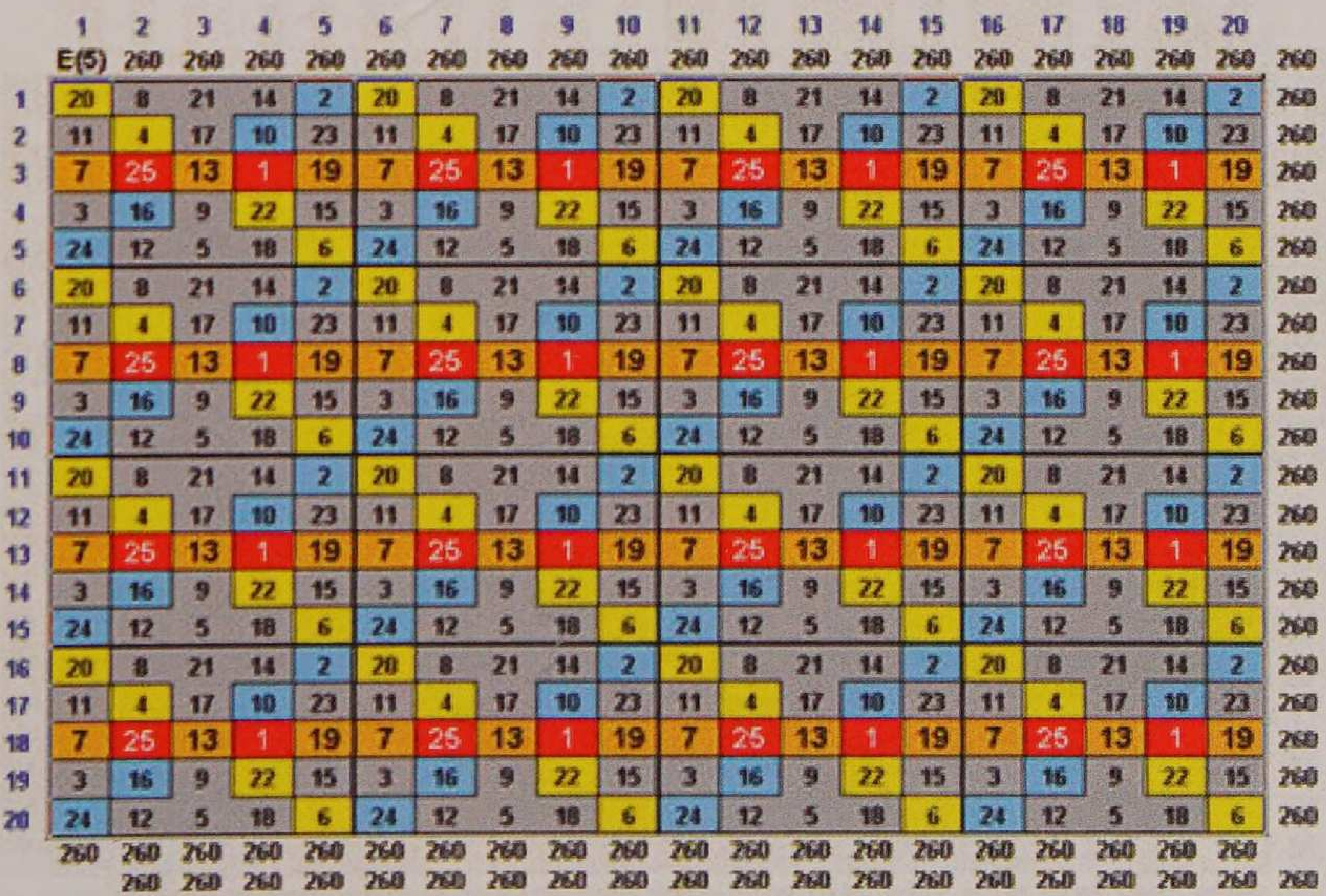


	1	2	3	4	
4x4	34	34	34	34	34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
		34	34	34	34



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
E(4)	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	16	3	6	9	170
	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	2	13	12	7	170
	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	11	8	1	14	170
	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	5	10	15	4	170
	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	
		170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170	170

	1	2	3	4	5	
5x5	65	65	65	65	65	65
	20	8	21	14	2	65
	11	4	17	10	23	65
	7	25	13	1	19	65
	3	16	9	22	15	65
	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
E(5)	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260
1	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
2	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
3	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
4	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
5	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
6	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
7	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
8	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
9	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
10	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
11	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
12	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
13	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
14	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
15	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
16	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	260
17	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	260
18	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	260
19	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	260
20	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	260
	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260

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Here is the result of that addition according to the formula:

$$(8.21) \quad W1(20) = 5^2(E(4) - |1|) + E(5)$$

Note the location of the extreme numbers **1** and **400** highlighted in red.

W20	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	395	58	146	214	377	70	133	221	389	52	145	208	396	64	127	220	383	71	139	202	4010
2	36	304	292	160	48	311	279	167	35	323	266	154	42	310	298	161	29	317	285	173	4010
3	257	200	13	326	269	182	25	338	251	194	7	350	263	176	19	332	275	188	1	344	4010
4	103	241	359	97	115	228	366	84	122	240	353	91	109	247	365	78	116	234	372	90	4010
5	399	62	130	218	381	74	137	205	393	56	149	212	380	68	131	224	387	55	143	206	4010
6	45	308	296	164	27	320	283	171	39	302	295	158	46	314	277	170	33	321	289	152	4010
7	261	179	17	335	273	186	4	342	260	198	11	329	267	185	23	336	254	192	10	348	4010
8	107	250	363	76	119	232	375	88	101	244	357	100	113	226	369	82	125	238	351	94	4010
9	378	66	134	222	390	53	141	209	397	65	128	216	384	72	140	203	391	59	147	215	4010
10	49	312	280	168	31	324	287	155	43	306	299	162	30	318	281	174	37	305	293	156	4010
11	270	183	21	339	252	195	8	346	264	177	20	333	271	189	2	345	258	196	14	327	4010
12	111	229	367	85	123	236	354	92	110	248	361	79	117	235	373	86	104	242	360	98	4010
13	382	75	138	201	394	57	150	213	376	69	132	225	388	51	144	207	400	63	126	219	4010
14	28	316	284	172	40	303	291	159	47	315	278	166	34	322	290	153	41	309	297	165	4010
15	274	187	5	343	256	199	12	330	268	181	24	337	255	193	6	349	262	180	18	331	4010
16	120	233	371	89	102	245	358	96	114	227	370	83	121	239	352	95	108	246	364	77	4010
17	386	54	142	210	398	61	129	217	385	73	136	204	392	60	148	211	379	67	135	223	4010
18	32	325	288	151	44	307	300	163	26	319	282	175	38	301	294	157	50	313	276	169	4010
19	253	191	9	347	265	178	16	334	272	190	3	341	259	197	15	328	266	184	22	340	4010
20	124	237	355	93	106	249	362	80	118	231	374	87	105	243	356	99	112	230	368	81	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

Next the roles of **E(4)** and **E(5)** are interchanged as in:

$$(8.22) \quad W2(20) = 4^2(E(5) - |1|) + E(4)$$

And we get a completely different perfect size-20 square. Yet each has the same properties as the other. Both have **diamond** and **X** characteristic tiling patterns and both are pangenic. However, neither is row nor centrally pairwise symmetric.

Again, note the location of the extreme numbers **1** and **400** highlighted in red. They are in the same location in both squares **W1** and **W2**.

U20	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	300	343	108	271	339	184	247	12	180	223	88	151	320	64	127	391	60	204	367	31	4010
2	302	76	235	388	143	216	374	129	282	57	115	268	23	196	355	9	162	336	95	249	4010
3	333	390	241	117	174	30	82	357	213	270	121	198	54	309	361	237	294	150	1	78	4010
4	46	13	378	325	286	152	119	65	27	392	258	205	166	133	99	344	306	272	239	185	4010
5	380	24	187	351	20	264	327	91	260	303	168	231	399	144	207	72	140	283	48	111	4010
6	83	156	315	69	122	396	55	209	362	36	295	348	103	276	334	189	242	17	175	228	4010
7	14	369	321	297	254	110	61	38	393	350	201	177	134	90	42	317	273	230	181	158	4010
8	126	193	59	304	366	232	299	145	6	73	338	385	246	112	179	25	87	352	218	265	4010
9	359	104	267	32	200	243	8	171	340	84	147	311	80	224	387	51	220	363	128	291	4010
10	163	236	394	149	202	77	135	288	43	116	375	29	182	356	15	269	322	96	255	308	4010
11	194	50	2	377	233	290	141	118	74	329	381	257	214	170	21	98	353	310	261	137	4010
12	206	172	139	85	47	312	278	225	186	153	19	364	326	292	259	105	66	33	398	345	4010
13	40	284	347	11	280	323	188	251	319	164	227	92	160	203	68	131	400	44	107	371	4010
14	142	316	75	229	382	56	215	368	123	296	354	109	262	37	195	248	3	176	335	89	4010
15	274	130	81	58	313	370	221	197	154	10	62	337	293	250	101	178	34	389	341	217	4010
16	386	252	219	165	26	93	358	305	266	132	199	45	7	372	238	285	146	113	79	324	4010
17	120	263	28	191	360	4	167	331	100	244	307	71	240	383	148	211	379	124	287	52	4010
18	222	97	155	208	63	136	395	49	102	376	35	289	342	16	275	328	183	256	314	169	4010
19	253	210	161	138	94	349	301	277	234	190	41	18	373	330	281	157	114	70	22	397	4010
20	67	332	298	245	106	173	39	384	346	212	279	125	86	53	318	365	226	192	159	5	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

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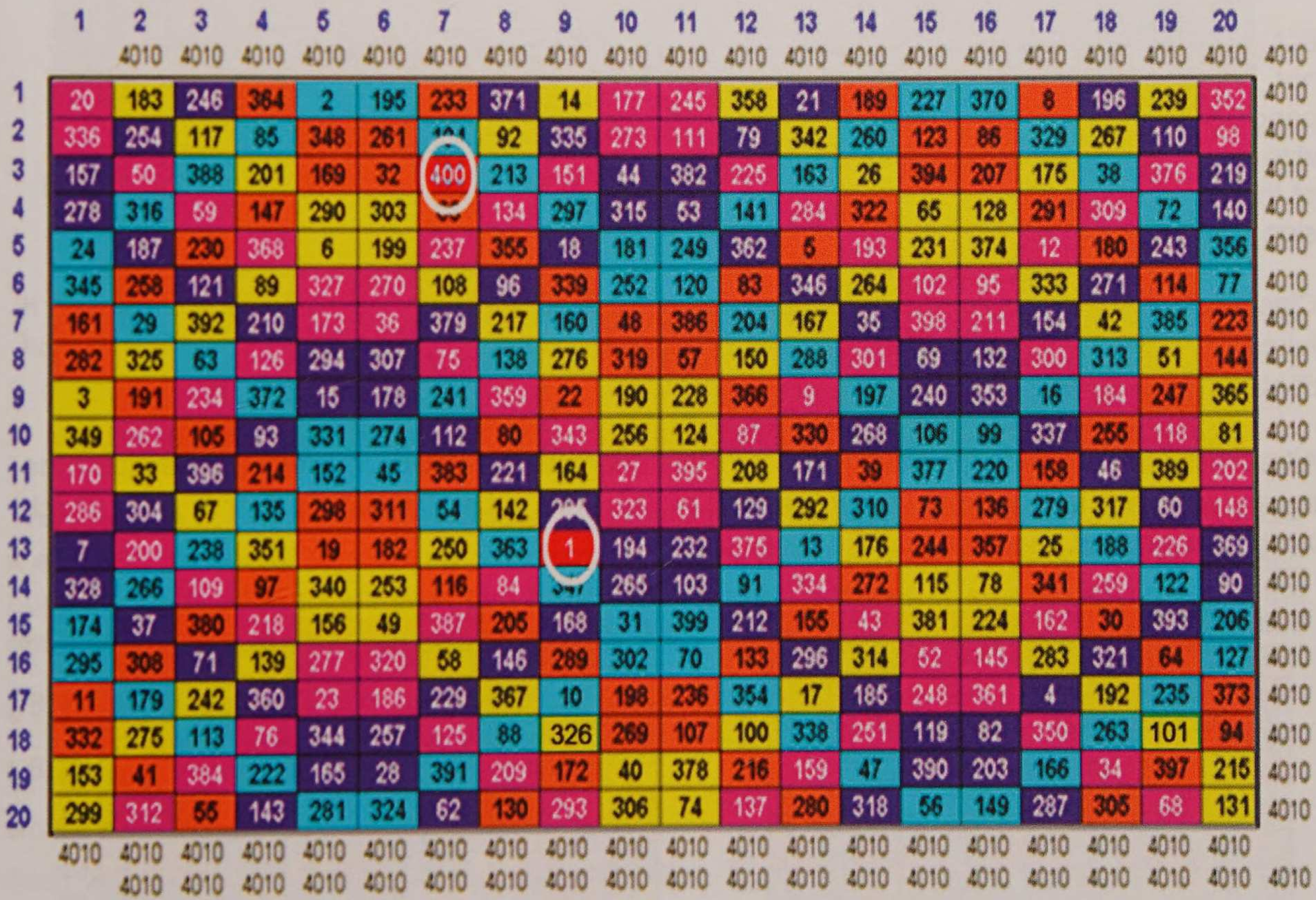
Further, the size-20 square is a matrix of even size, yet the maximum and minimum numbers are separated by an odd number of columns and an odd number of rows. So the two extreme numbers can never be realigned to be equal-summing pairwise symmetric, neither centrally nor row-wise.

That size-20 square we were evaluating for tiling patterns back in Program 2 is shown here again. It is a completely different table than **W1** and **W2** and was obtained by the **TAP** expansion method **#5**. Yet it too didn't have any pairwise symmetry either.

Squares of sizes **20**, **28** and **35** were derived in this manner. The series of size-35 squares depicted later in this book are the result of this **ATE** method. Of these three different size squares, only the size **35** was pairwise symmetric. In general, odd-size matrices can be made to be pairwise symmetric by this **ATE** method of expansion, but even-size ones cannot.

Note where the extreme numbers highlighted in red are located here compared with the ones in the two preceding tables.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
1	20	183	246	364	2	195	233	371	14	177	245	358	21	189	227	370	8	196	239	352	4010
2	336	254	117	85	348	261	104	92	335	273	111	79	342	260	123	86	329	267	110	98	4010
3	157	50	388	201	169	32	400	213	151	44	382	225	163	26	394	207	176	38	376	219	4010
4	278	316	59	147	290	303	56	134	297	315	63	141	284	322	65	128	291	309	72	140	4010
5	24	187	230	368	6	199	237	355	18	181	249	362	5	193	231	374	12	180	243	356	4010
6	345	258	121	89	327	270	108	96	339	252	120	83	346	264	102	95	333	271	114	77	4010
7	161	29	392	210	173	36	379	217	160	48	386	204	167	35	398	211	154	42	385	223	4010
8	282	325	63	126	294	307	75	138	276	319	57	150	288	301	69	132	300	313	51	144	4010
9	3	191	234	372	15	178	241	359	22	190	228	366	9	197	240	353	16	184	247	365	4010
10	349	262	105	93	331	274	112	80	343	256	124	87	330	268	106	99	337	255	118	81	4010
11	170	33	396	214	152	45	383	221	164	27	395	208	171	39	377	220	158	46	389	202	4010
12	286	304	67	135	298	311	54	142	285	323	61	129	292	310	73	136	279	317	60	148	4010
13	7	200	238	351	19	182	250	363	1	194	232	375	13	176	244	357	25	188	226	369	4010
14	328	266	109	97	340	253	116	84	347	265	103	91	334	272	115	78	341	259	122	90	4010
15	174	37	380	218	156	49	387	205	168	31	399	212	155	43	381	224	162	30	393	206	4010
16	295	308	71	139	277	320	58	146	289	302	70	133	296	314	52	145	283	321	64	127	4010
17	11	179	242	360	23	186	229	367	10	198	236	354	17	185	248	361	4	192	235	373	4010
18	332	275	113	76	344	257	125	88	326	269	107	100	338	251	119	82	350	263	101	94	4010
19	153	41	384	222	165	28	391	209	172	40	378	216	159	47	390	203	166	34	397	215	4010
20	299	312	55	143	281	324	62	130	293	306	74	137	280	318	56	149	287	305	68	131	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	
		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

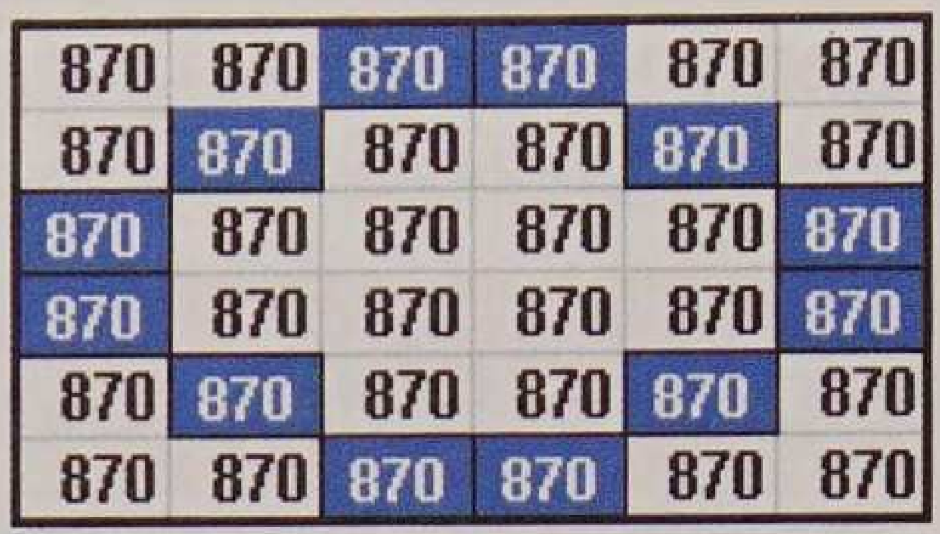
Expansion and merging of the size-3 and size-4 squares to get a size-12 Class-4 square

Here is one configuration for size-12 that was found to have continuous diamond and X-pattern tilings. But it is not pangenic.

It was obtained from the two tables below.

Note that **E(4)** is composed of duplicates of the perfect size-4 square while **E(3)** is composed of four variations of the imperfect size-3 square in order to make the expanded table be geonomic.

E(4) is obviously pangenic, but **E(3)** is not. Therein lies the source of the loss of pangenicity in the derived dual squares of size-12. This loss only occurs when one of the participating squares in the expansion is imperfect.



870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870

870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870
870	870	870	870	870	870

	1	2	3	4	5	6	7	8	9	10	11	12	
W12		870	870	870	870	870	870	870	870	870	870	870	870
9	38	88	132	33	43	83	128	34	42	87	133	29	870
10	144	23	46	73	140	27	54	77	136	19	50	81	870
11	13	111	107	62	12	112	103	57	17	116	102	58	870
12	94	66	8	125	93	67	4	120	98	71	3	121	870
13	45	86	127	28	41	90	135	32	37	82	131	36	870
14	137	25	51	78	142	20	47	79	141	24	52	74	870
15	11	115	105	60	16	110	101	61	15	114	106	56	870
16	99	68	1	118	95	72	9	122	91	64	5	126	870
17	40	84	134	35	39	85	130	30	44	89	129	31	870
18	139	21	53	80	138	22	49	75	143	26	48	76	870
19	18	113	100	55	14	117	108	59	10	109	104	63	870
20	92	70	6	123	97	65	2	124	96	69	7	119	870
	870	870	870	870	870	870	870	870	870	870	870	870	
		870	870	870	870	870	870	870	870	870	870	870	870

Class-4 squares of sizes **n = 4b** where **b** is an odd number can be composed in this manner. It takes some rearranging of the rows and columns to get the resulting square to exhibit continuous tiling patterns.

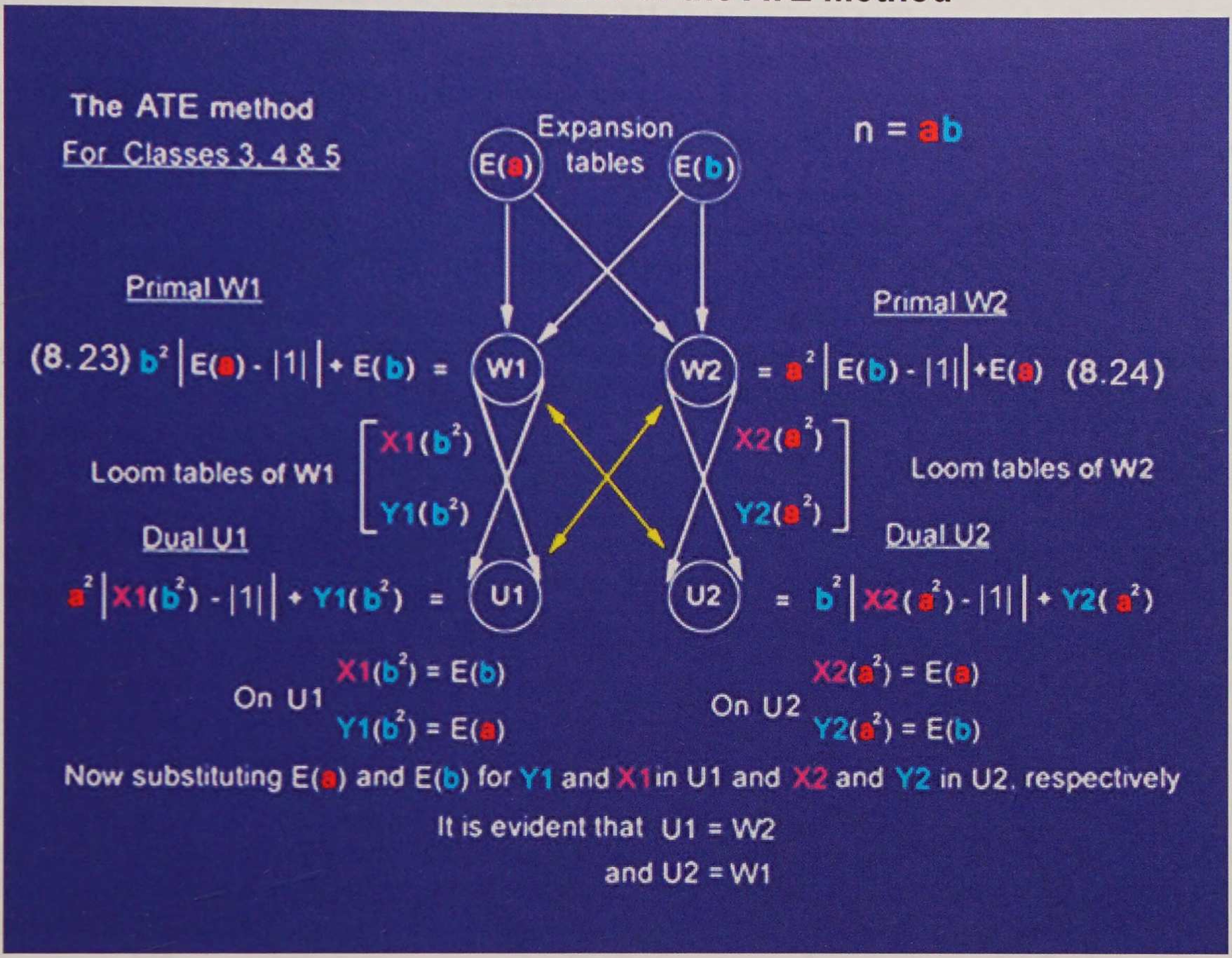
All such squares of size greater than **12** in this sub-class will also be pangenic because the square of size **b** laid out repetitively like **E(4)** will be from either Class-1, 3, 5 or 6, all of which will be perfect and pangenic just like W(4).

E(3)												
	60	60	60	60	60	60	60	60	60	60	60	60
2	7	6	6	7	2	2	7	6	6	7	2	60
9	5	1	1	5	9	9	5	1	1	5	9	60
4	3	8	8	3	4	4	3	8	8	3	4	60
4	3	8	8	3	4	4	3	8	8	3	4	60
9	5	1	1	5	9	9	5	1	1	5	9	60
2	7	6	6	7	2	2	7	6	6	7	2	60
2	7	6	6	7	2	2	7	6	6	7	2	60
9	5	1	1	5	9	9	5	1	1	5	9	60
4	3	8	8	3	4	4	3	8	8	3	4	60
4	3	8	8	3	4	4	3	8	8	3	4	60
9	5	1	1	5	9	9	5	1	1	5	9	60
2	7	6	6	7	2	2	7	6	6	7	2	60
60	60	60	60	60	60	60	60	60	60	60	60	
	60	60	60	60	60	60	60	60	60	60	60	60

E(4)												
	102	102	102	102	102	102	102	102	102	102	102	102
5	10	15	4	5	10	15	4	5	10	15	4	102
16	3	6	9	16	3	6	9	16	3	6	9	102
2	13	12	7	2	13	12	7	2	13	12	7	102
11	8	1	14	11	8	1	14	11	8	1	14	102
5	10	15	4	5	10	15	4	5	10	15	4	102
16	3	6	9	16	3	6	9	16	3	6	9	102
2	13	12	7	2	13	12	7	2	13	12	7	102
11	8	1	14	11	8	1	14	11	8	1	14	102
5	10	15	4	5	10	15	4	5	10	15	4	102
16	3	6	9	16	3	6	9	16	3	6	9	102
2	13	12	7	2	13	12	7	2	13	12	7	102
11	8	1	14	11	8	1	14	11	8	1	14	102
102	102	102	102	102	102	102	102	102	102	102	102	
	102	102	102	102	102	102	102	102	102	102	102	102

Program 8

Mathematical Proof of the ATE Method



The ATE method

For Classes 3, 4 & 5

Expansion tables

E(a) E(b)

n = ab

Primal W1

(8.23) $b^2\left|E(a) - |1|\right| + E(b) = W1$

Primal W2

$W2 = a^2\left|E(b) - |1|\right| + E(a)$ (8.24)

Loom tables of W1 $X1(b^2)$, $Y1(b^2)$

Loom tables of W2 $X2(a^2)$, $Y2(a^2)$

Dual U1

$a^2\left|X1(b^2) - |1|\right| + Y1(b^2) = U1$

Dual U2

$U2 = b^2\left|X2(a^2) - |1|\right| + Y2(a^2)$

On U1 $X1(b^2) = E(b)$, $Y1(b^2) = E(a)$

On U2 $X2(a^2) = E(a)$, $Y2(a^2) = E(b)$

Now substituting E(a) and E(b) for Y1 and X1 in U1 and X2 and Y2 in U2, respectively

It is evident that U1 = W2

and U2 = W1

The merging of two different expanded tiled matrices deals with two squares of different sizes **a** and **b** to get one of size **n = ab**. The factors must be unequal and one factor must be an odd-number. Their addition will yield two completely different squares of size **ab** according to the order in which their expansion tables are merged.

(8.23) $\mathbf{W1(n)} = a^2(E(b) - |1|) + E(a)$

(8.24) $\mathbf{W2(n)} = b^2(E(a) - |1|) + E(b)$

where **n = ab, a ≠ b, a** or **b** an odd-number

Both will be perfect and have identical properties. Both will be pangenic and will have their extreme numbers in the same location.

Their loom tables are the expansion tables **E(a)** and **E(b)**. The schematic above proves that **W1** and **W2** are truly the dual of each other.

Suppose we have two primal squares **W1** and **W2**, each of size **n** generated by merging two tiled expansion tables **E(a)** and **E(b)** according to the two addition formulas(**8.23**) and (**8.24**).

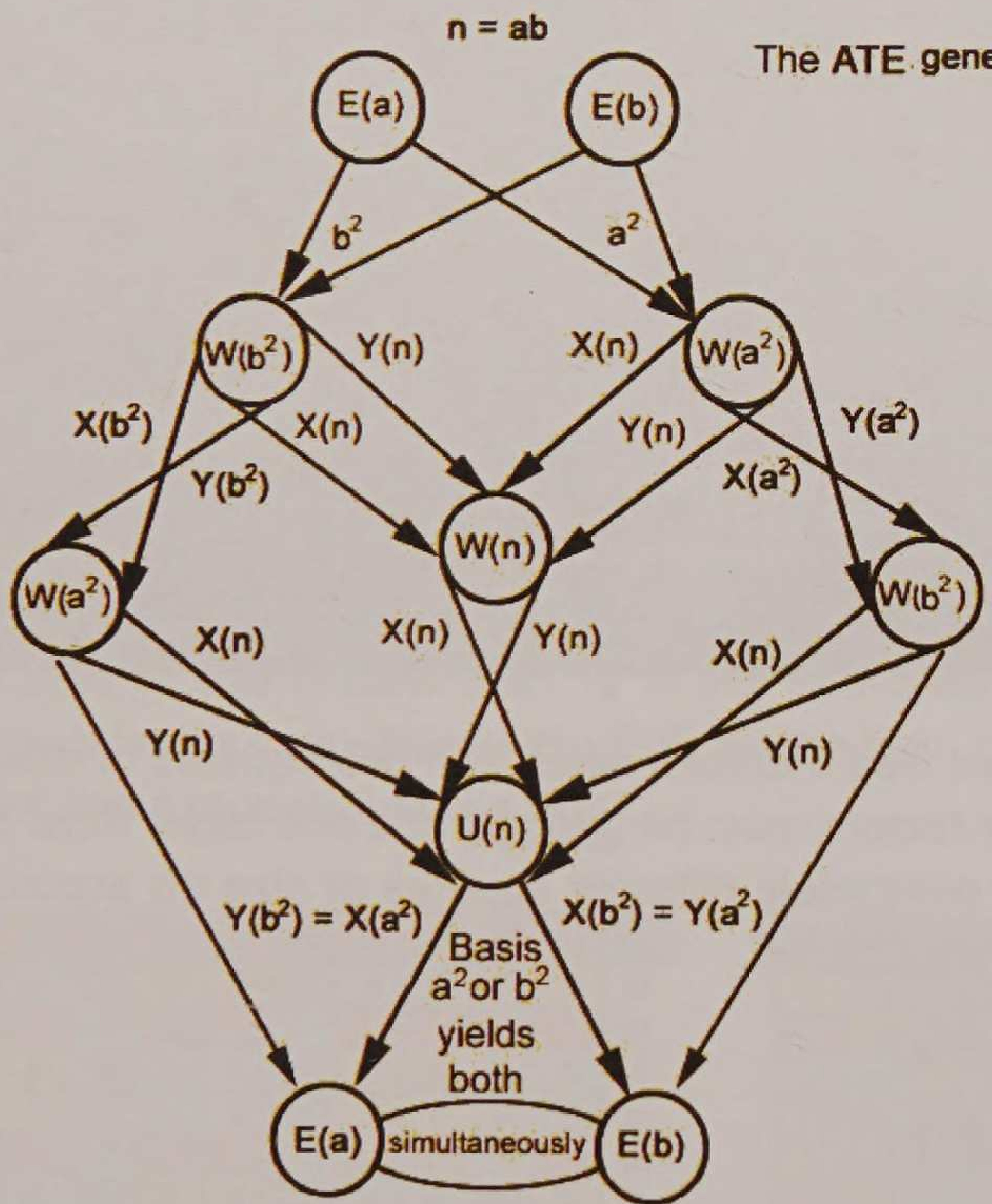
Next complementary loom tables derived from **W1** and **W2** are interchanged in their generation formulas for their respective primal versions. This yields two distinct duals, **U1** and **U2**.

At this point, the two sets of complementary loom tables have no relationship to one another, and neither do the two resulting dual squares **U1** and **U2** because they have been constructed and derived in two different bases, **a** and **b.**

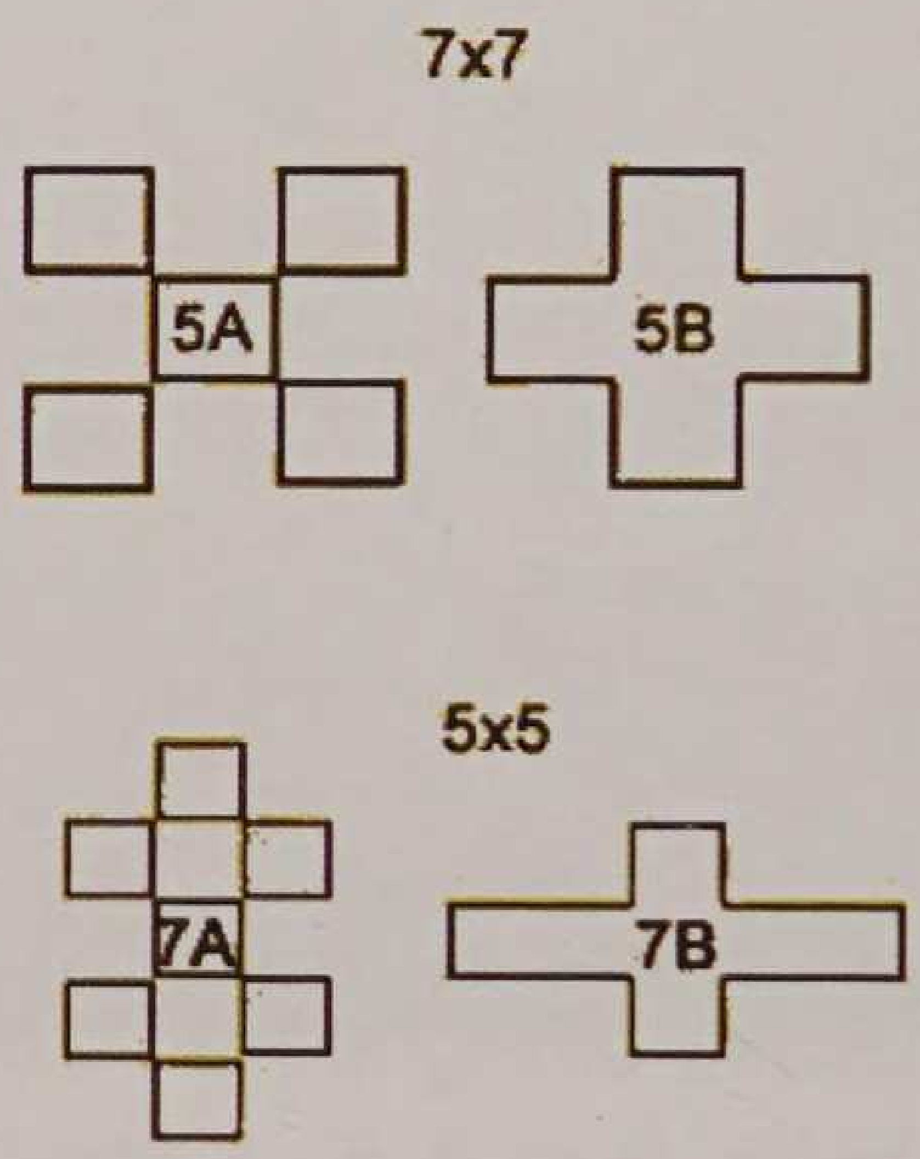
Yet, here's the amazing part: the primal squares initially derived from the expansion tables are related because their modulus and integer loom-tables **X1** and **Y1** taken from **W1** in the base a^2 yield both original expansion tables **E(a)** and **E(b)** in that order. Additionally **X2** and **Y2** taken in that order from **W2** in the other base b^2 also yield both original expansion tables **E(b)** and **E(a)** but now in reverse order. For instance, where the modulus table **X1** yields **E(b)**, the modulus table **X2** yields **E(a)**.

Now substituting **E(a)** for **Y1** in **U1** and for **X2 in U2** and **E(b)** for **X1 in U1** and for **Y2** in **U2**, we readily observe that **W1** is identically **U2** and **W2** is identically **U1. QED**

Application to Class-5 Squares



The ATE generation method for Class-5 squares



For Class-4 squares there were four different versions of each square of size **n = ab**: each pair derived with loom tables in one of the bases: a^2 or b^2. For Class-5 squares there are two more, now in the base **n = ab**.

For Class-5 squares there are two styles of tiling pattern: one which is composed of **bxb** block-squares in the size <u>a</u> characteristic tiling patterns; the other which is composed of **axa** block-squares in the size <u>b</u> characteristic tiling patterns.

All Class-5 squares, derived in any one of the three <u>bases</u>, will have both pairs of tiling patterns and consequently will have **4** distinct equal-summing tiling patterns simultaneously. This only occurs for Class-5 squares derived by the **ATE** method.

Program 8

Once any primal or dual square is derived in either base $\mathbf{a}^2$ or $\mathbf{b}^2$, all the other versions can be derived in any of the other bases by deriving the modulus and integer loom tables from it in the desired base. Then its dual can be generated by interchanging the roles of the loom tables in the generating formula. This doesn't work for Class-4 squares because their loom tables derived in the composite base **n** are not geonomic.

Further, the complementary expansion tables **E(a)** and **E(b)** can be recovered by deriving the loom tables in either base $\mathbf{a}^2$ or $\mathbf{b}^2$ from any of the three primal versions or the one dual version.

Note 1: This only applies to **Class-4 squares**. The modulus and integer functions taken in the base **n** for Class-4 squares derived by the ATE method here will often fail to yield loom tables that are geonomic. Even if they don't, the squares will lack continuous tiling patterns. So the expansion tables for Class-4 squares actually need to be produced directly by the patterned expansion of the **TAP** (*Tiled And Pattern*) method #5 or a closely-related **SPD** (Spread-Pattern Distribution) method #6, both to be described later. When using the **ATE** method here, the loom-tables can be subsequently derived in either base $\mathbf{a}^2$ or $\mathbf{b}^2$ from the primary squares generated from them.

Note 2: This only applies to **Class-5 squares**. The demonstrated helical patterns shown earlier in Program 3 of the size-35 Class-5 square was only found in loom tables that were subsequently derived in the **base n = ab** from either square $\mathbf{W(a^2)}$ or $\mathbf{W(b^2)}$ originally generated in the base $\mathbf{a}^2$ or $\mathbf{b}^2$, respectively.

Note 3: Squares of sizes **20**, **28** and **35** were derived using the **ATE** method here. Of these three squares of different sizes, only the size **35** was pairwise centrally symmetric. In general, only odd-size matrices can be made to be pairwise symmetric by this method of expansion; the even-size ones cannot.

* * *

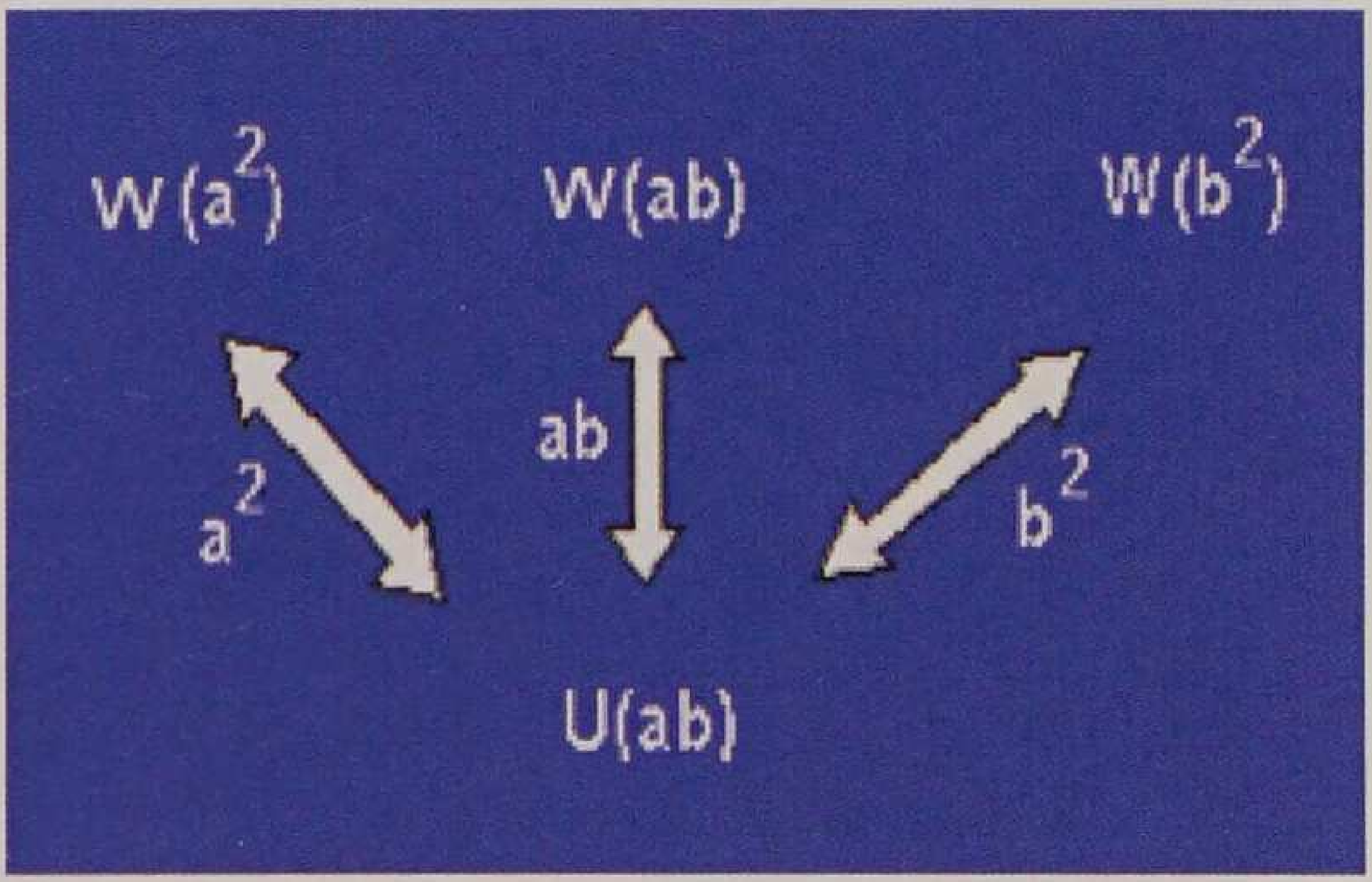
The 6 versions of the size-35 square

Of the 6 derived squares: W(49) & **U(49)**; **W(25)** & **U(25)**; and **W(35)** & **U(35)**, these reduce to only three because

(8.25a) $\mathbf{W(35) = U(25) = W(49)}$

(8.25b) $\mathbf{U(49) = W(25)}$

(8.25c) $\mathbf{U(35) \neq}$ **any other version**



Only **U(35)** is unique. Nonetheless it can be derived from any of **W(35)**, **W(49)** or **U(25)** in the base **35**, **49** or **25**, respectively!
Now what is so amazing is that, while three different versions can derive an identical table using three different number bases, the converse is true: a single square can produce three different perfect geonomic squares of the same size with the same characteristic number using three different bases. That property only exists among Class-5 squares.

And all three versions of the size-35 squares have all four equal-summing block-square tiling patterns simultaneously. Now all that's wow math !!

Program 8

Size-35 Square derived in the base 49 with size-7 tiling patterns

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
W5(49)	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465
1	969	370	989	684	78	949	350	1010	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	980	361	987	675	76	940	390	1009	655	56	21465
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21465
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21465
4	114	748	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21465
5	1138	580	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21465
6	937	308	1006	645	95	966	340	986	674	75	939	389	1015	654	55	960	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21465
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21465
8	332	1203	597	47	911	312	1183	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21465
9	131	758	396	1071	710	111	729	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21465
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21465
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21465
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21465
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21465
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	708	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21465
15	1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21465
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21465
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	430	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21465
18	310	1181	624	25	890	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21465
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1059	708	21465
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	201	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21465
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21465
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21465
23	327	1109	602	42	896	307	1220	621	16	906	336	1200	601	44	915	309	1180	630	24	895	338	1209	683	4	924	318	1189	632	33	897	298	1218	612	13	926	21465
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21465
25	1143	544	232	858	290	1172	573	212	838	281	1152	553	241	857	261	1132	575	221	847	298	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21465
26	942	392	1011	657	51	971	365	991	688	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21465
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21465
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21465
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21465
30	1160	554	200	875	259	1140	563	229	848	249	1169	583	209	877	278	1142	543	238	867	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	840	289	21465
31	959	353	1028	667	48	932	362	1008	647	97	961	382	981	676	77	941	391	1018	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21465
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21465
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21465
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21465
35	1170	571	217	836	286	1159	551	239	865	266	1130	580	219	845	288	1150	560	199	874	268	1139	582	228	854	248	1160	562	208	876	277	1148	542	237	856	257	21465
	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	
	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465

Size-35 Dual Square derived in the base 49 with size-5 tiling patterns

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U(49)	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465	21465
1	945	658	221	1164	702	445	158	946	664	202	1170	708	446	164	927	670	208	1171	714	427	170	933	671	214	1152	720	433	171	939	652	220	1158	721	439	152	21465
2	811	354	92	1035	598	311	1079	817	360	98	1036	579	317	1085	823	361	79	1042	585	323	1086	804	367	85	1048	586	304	1092	810	373	86	1029	592	310	1098	21465
3	682	250	1188	726	469	7	975	688	226	1194	732	475	13	951	694	232	1200	738	451	19	957	700	238	1176	744	457	25	963	676	244	1182	750	463	1	969	21465
4	378	116	884	622	340	1103	841	384	122	890	603	341	1109	847	390	103	891	609	347	1115	828	391	109	897	615	328	1116	834	397	115	878	616	334	1122	840	21465
5	274	1212	755	493	31	999	537	255	1218	756	499	37	980	543	256	1224	762	480	43	981	549	262	1205	768	481	49	987	530	268	1206	774	487	30	993	531	21465
6	145	908	646	189	1127	870	408	146	914	627	195	1133	871	414	127	920	633	196	1139	852	420	133	921	639	177	1145	858	421	139	902	645	183	1146	864	402	21465
7	1061	779	517	60	1023	561	279	1067	785	523	61	1004	567	285	1073	786	504	67	1010	573	286	1054	792	510	73	1011	554	292	1060	798	511	54	1017	560	298	21465
8	932	675	213	1151	719	432	175	938	651	219	1157	725	438	151	944	657	225	1163	701	444	157	950	663	201	1169	707	450	163	926	669	207	1175	713	426	169	21465
9	803	366	84	1047	590	303	1091	809	372	90	1028	591	309	1097	815	353	91	1034	597	315	1078	816	359	97	1040	578	316	1084	822	365	78	1041	584	322	1090	21465
10	699	237	1180	743	456	24	962	680	243	1181	749	462	5	968	681	249	1187	730	468	6	974	687	230	1193	731	474	12	955	693	231	1199	737	455	18	956	21465
11	395	108	896	614	327	1120	833	396	114	877	620	333	1121	839	377	120	883	621	339	1102	845	383	121	889	602	345	1108	846	389	102	895	608	346	1114	827	21465
12	261	1204	767	485	48	986	529	267	1210	773	486	29	992	535	273	1211	754	492	35	998	536	254	1217	760	498	36	979	542	260	1223	761	479	42	985	548	21465
13	132	925	638	176	1144	857	425	138	901	644	182	1150	863	401	144	907	650	188	1126	869	407	150	913	626	194	1132	875	413	126	919	632	200	1138	851	419	21465
14	1053	791	509	72	1015	553	291	1059	797	515	53	1016	559	297	1065	778	516	59	1022	565	278	1066	784	522	65	1003	566	284	1072	790	503	66	1009	572	290	21465
15	949	662	205	1168	736	449	162	930	668	206	1174	712	430	168	931	674	212	1155	718	431	174	937	655	218	1156	724	437	155	943	656	224	1162	705	443	156	21465
16	820	358	96	1039	577	320	1083	821	364	77	1045	583	321	1089	802	370	83	1046	589	302	1095	808	371	89	1027	595	308	1096	814	352	95	1033	596	314	1077	21465
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18	382	125	888	601	344	1107	850	388	101	894	607	350	1113	826	394	107	900	613	326	1119	832	400	113	876	619	332	1125	838	376	119	882	625	338	1101	844	21465
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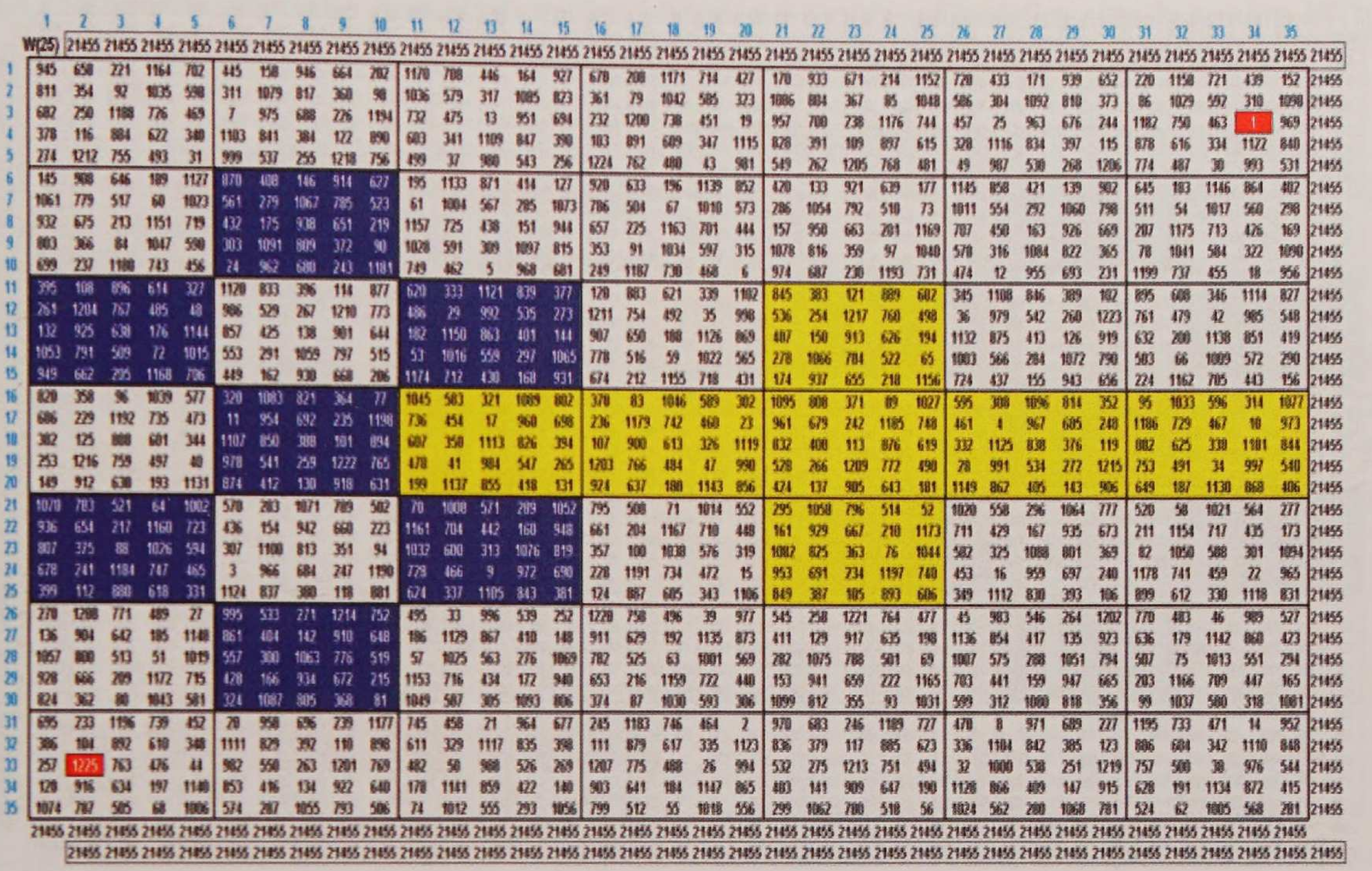
Program 8

Size-35 Square derived in the base 25 with size-7 tiling patterns



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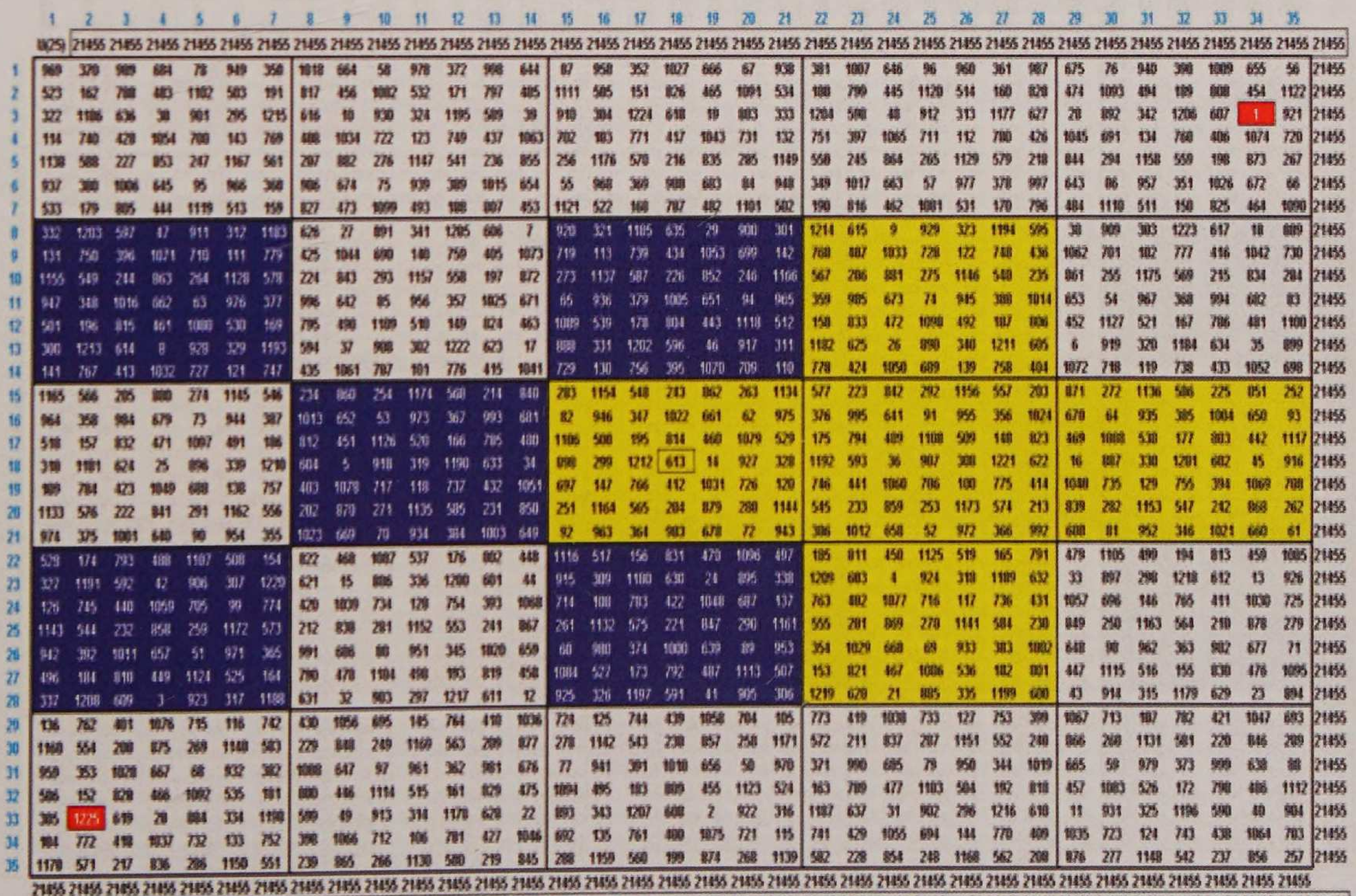
Size-35 Dual Square derived in the base 25 with size-5 tiling patterns



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28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	628	21	885	335	1199	608	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	385	1225	619	28	884	334	1190	599	49	913	314	1170	628	22	893	343	1207	600	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1056	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1178	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 8

Size-35 Square derived in the base 35 with size-7 tiling patterns

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
W1(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	969	370	989	684	78	949	350	1018	664	58	978	372	998	644	87	958	352	1027	666	67	938	381	1007	646	96	960	361	987	675	76	940	390	1009	655	56	21455
2	523	162	788	483	1102	503	191	817	456	1082	532	171	797	485	1111	505	151	826	465	1091	534	180	799	445	1120	514	160	828	474	1093	494	189	808	454	1122	21455
3	322	1186	636	30	901	295	1215	616	10	930	324	1195	589	39	910	304	1224	618	19	883	333	1204	598	48	912	313	1177	627	28	892	342	1206	607	1	921	21455
4	114	740	428	1054	700	143	769	408	1034	722	123	749	437	1063	702	103	771	417	1043	731	132	751	397	1065	711	112	780	426	1045	691	134	760	406	1074	720	21455
5	1138	588	227	853	247	1167	561	207	882	276	1147	541	236	855	256	1176	570	216	835	285	1149	550	245	864	265	1129	579	218	844	294	1158	559	198	873	267	21455
6	937	380	1006	645	95	966	360	986	674	75	939	389	1015	654	55	968	369	988	683	84	948	349	1017	663	57	977	378	997	643	86	957	351	1026	672	66	21455
7	533	179	805	444	1119	513	159	827	473	1099	493	188	807	453	1121	522	168	787	482	1101	502	190	816	462	1081	531	170	796	484	1110	511	150	825	464	1090	21455
8	332	1203	597	47	911	312	1163	626	27	891	341	1205	606	7	920	321	1185	635	29	900	301	1214	615	9	929	323	1194	595	38	909	303	1223	617	18	889	21455
9	131	750	396	1071	710	111	779	425	1044	690	140	759	405	1073	719	113	739	434	1053	699	142	768	407	1033	728	122	748	436	1062	701	102	777	416	1042	730	21455
10	1155	549	244	863	264	1128	578	224	843	293	1157	558	197	872	273	1137	587	226	852	246	1166	567	206	881	275	1146	540	235	861	255	1175	569	215	834	284	21455
11	947	348	1016	662	63	976	377	996	642	85	956	357	1025	671	65	936	379	1005	651	94	965	359	985	673	74	945	388	1014	653	54	967	368	994	682	83	21455
12	501	196	815	461	1080	530	169	795	490	1109	510	149	824	463	1089	539	178	804	443	1118	512	158	833	472	1098	492	187	806	452	1127	521	167	786	481	1100	21455
13	300	1213	614	8	928	329	1193	594	37	908	302	1222	623	17	888	331	1202	596	46	917	311	1182	625	26	890	340	1211	605	6	919	320	1184	634	35	899	21455
14	141	767	413	1032	727	121	747	435	1061	707	101	776	415	1041	729	130	756	395	1070	709	110	778	424	1050	689	139	758	404	1072	718	119	738	433	1052	698	21455
15	1165	566	205	880	274	1145	546	234	860	254	1174	568	214	840	283	1154	548	243	862	263	1134	577	223	842	292	1156	557	203	871	272	1136	586	225	851	252	21455
16	964	358	984	679	73	944	387	1013	652	53	973	367	993	681	82	946	347	1022	661	62	975	376	995	641	91	955	356	1024	670	64	935	385	1004	650	93	21455
17	518	157	832	471	1097	491	186	812	451	1126	520	166	785	480	1106	500	195	814	460	1079	529	175	794	489	1108	509	148	823	469	1088	538	177	803	442	1117	21455
18	310	1181	624	25	896	339	1210	604	5	918	319	1190	633	34	898	299	1212	613	14	927	328	1192	593	36	907	308	1221	622	16	887	330	1201	602	45	916	21455
19	109	784	423	1049	688	138	757	403	1078	717	118	737	432	1051	697	147	766	412	1031	726	120	746	441	1060	706	100	775	414	1040	735	129	755	394	1069	708	21455
20	1133	576	222	841	291	1162	556	202	870	271	1135	585	231	850	251	1164	565	204	879	280	1144	545	233	859	253	1173	574	213	839	282	1153	547	242	868	262	21455
21	974	375	1001	640	90	954	355	1023	669	70	934	384	1003	649	92	963	364	983	678	72	943	386	1012	658	52	972	366	992	680	81	952	346	1021	660	61	21455
22	528	174	793	488	1107	508	154	822	468	1087	537	176	802	448	1116	517	156	831	470	1096	497	185	811	450	1125	519	165	791	479	1105	499	194	813	459	1085	21455
23	327	1191	592	42	906	307	1220	621	15	886	336	1200	601	44	915	309	1180	630	24	895	338	1209	603	4	924	318	1189	632	33	897	298	1218	612	13	926	21455
24	126	745	440	1059	705	99	774	420	1039	734	128	754	393	1068	714	108	783	422	1048	687	137	763	402	1077	716	117	736	431	1057	696	146	765	411	1030	725	21455
25	1143	544	232	858	259	1172	573	212	838	281	1152	553	241	867	261	1132	575	221	847	290	1161	555	201	869	270	1141	584	230	849	250	1163	564	210	878	279	21455
26	942	392	1011	657	51	971	365	991	686	80	951	345	1020	659	60	980	374	1000	639	89	953	354	1029	668	69	933	383	1002	648	98	962	363	982	677	71	21455
27	496	184	810	449	1124	525	164	790	478	1104	498	193	819	458	1084	527	173	792	487	1113	507	153	821	467	1086	536	182	801	447	1115	516	155	830	476	1095	21455
28	337	1208	609	3	923	317	1188	631	32	903	297	1217	611	12	925	326	1197	591	41	905	306	1219	620	21	885	335	1199	600	43	914	315	1179	629	23	894	21455
29	136	762	401	1076	715	116	742	430	1056	695	145	764	410	1036	724	125	744	439	1058	704	105	773	419	1038	733	127	753	399	1067	713	107	782	421	1047	693	21455
30	1160	554	200	875	269	1140	583	229	848	249	1169	563	209	877	278	1142	543	238	857	258	1171	572	211	837	287	1151	552	240	866	260	1131	581	220	846	289	21455
31	959	353	1028	667	68	932	382	1008	647	97	961	362	981	676	77	941	391	1010	656	50	970	371	990	685	79	950	344	1019	665	59	979	373	999	638	88	21455
32	506	152	820	466	1092	535	181	800	446	1114	515	161	829	475	1094	495	183	809	455	1123	524	163	789	477	1103	504	192	818	457	1083	526	172	798	486	1112	21455
33	305	1225	619	20	884	334	1198	599	49	913	314	1178	628	22	893	343	1207	608	2	922	316	1187	637	31	902	296	1216	610	11	931	325	1196	590	40	904	21455
34	104	772	418	1037	732	133	752	398	1066	712	106	781	427	1046	692	135	761	400	1075	721	115	741	429	1055	694	144	770	409	1035	723	124	743	438	1064	703	21455
35	1170	571	217	836	286	1150	551	239	865	266	1130	580	219	845	288	1159	560	199	874	268	1139	582	228	854	248	1168	562	208	876	277	1148	542	237	856	257	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Size-35 Dual derived in the base 35 with size-5 tiling patterns

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
U1(35)	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455
1	833	676	309	650	248	133	1200	100	1174	772	1148	746	624	474	563	448	46	415	20	1087	972	1061	939	544	878	518	361	239	335	178	1042	152	1009	859	702	21455
2	1135	740	618	959	592	435	531	409	14	1116	226	1055	933	1029	907	505	355	724	329	207	296	146	1003	853	1222	820	670	794	644	277	120	461	94	1168	68	21455
3	220	1084	194	1016	901	499	875	718	316	692	290	175	997	107	1216	814	1190	788	631	271	605	490	88	422	62	1129	769	1103	946	586	920	560	403	1	377	21455
4	284	162	258	136	1210	75	1177	782	660	756	599	477	573	451	56	1123	23	1097	975	1071	914	547	397	521	371	214	338	188	1045	895	984	862	712	836	686	21455
5	628	962	567	445	43	419	17	1091	236	1058	943	541	882	515	358	734	332	182	1039	149	1013	856	1197	830	673	313	647	252	130	464	104	1171	776	1145	743	21455
6	937	1026	904	509	843	728	326	204	300	143	1007	117	1219	824	667	798	641	274	615	458	98	1165	65	1139	737	1113	956	589	439	528	413	11	380	230	1052	21455
7	261	111	1213	818	1187	785	635	759	689	487	85	426	59	1133	33	1100	958	583	924	557	400	496	374	224	1081	191	1020	890	994	872	715	320	689	294	172	21455
8	570	455	53	387	27	1094	979	1068	911	551	885	525	368	211	342	185	1049	159	581	866	709	840	683	281	657	255	140	1207	72	1181	779	1155	753	596	481	21455
9	879	512	362	731	336	179	303	153	1010	860	1194	827	677	801	651	249	127	468	101	1175	40	1142	747	625	966	564	442	538	416	21	1088	233	1062	940	1036	21455
10	1223	821	1162	795	638	278	612	462	95	429	69	1136	741	1110	953	593	927	532	410	8	384	227	1056	201	1023	908	506	847	725	323	699	297	147	1004	114	21455
11	63	1130	30	1104	947	1078	921	554	404	493	378	221	345	195	1017	902	991	869	719	808	693	291	169	265	108	1217	82	1184	789	632	763	606	484	580	423	21455
12	365	706	339	189	1046	156	985	863	1204	837	680	285	654	259	137	471	76	1178	783	1152	750	600	969	574	452	50	391	24	1098	243	1065	915	548	889	522	21455
13	674	805	648	246	622	465	105	1172	37	1146	744	1120	963	561	446	535	420	18	352	237	1059	944	1033	876	516	850	735	333	176	307	150	1014	124	1191	831	21455
14	5	1107	957	590	931	529	407	583	381	231	1053	198	1027	905	1001	844	722	327	696	301	144	268	118	1220	825	1159	792	642	766	616	459	92	433	66	1140	21455
15	349	192	1021	166	988	873	716	812	690	288	664	262	112	1214	79	1180	786	1127	760	603	408	577	427	60	394	34	1101	951	1075	918	558	892	497	375	218	21455
16	658	256	134	475	73	1182	47	1149	754	597	973	571	449	545	380	78	1095	240	1069	912	1043	806	519	369	703	343	186	310	160	982	867	1201	834	684	773	21455
17	960	565	934	539	417	15	356	234	1063	208	1038	880	513	854	732	330	671	304	154	1011	121	1195	820	1169	802	645	250	619	469	102	436	41	1143	748	1117	21455
18	1024	909	998	841	726	815	700	298	141	272	115	1224	89	1156	796	639	770	613	456	587	430	70	1137	2	1111	954	1085	920	526	411	500	385	228	317	202	21455
19	109	478	83	1185	790	1124	757	607	976	581	424	57	398	31	1105	215	1072	922	555	896	494	372	713	346	196	1018	163	992	870	1211	809	687	292	661	266	21455
20	453	542	392	25	359	244	1066	916	1040	883	523	857	707	340	183	314	157	906	131	1198	830	681	777	655	253	629	472	77	1179	44	1153	751	1092	970	568	21455
21	1008	851	729	334	668	308	151	275	125	1192	832	1166	799	649	738	623	466	99	440	38	1147	12	1114	964	562	938	536	414	510	353	238	1060	205	1034	877	21455
22	86	1160	793	1134	767	610	460	584	434	67	401	6	1108	958	1082	925	530	899	504	382	225	321	199	1028	173	995	845	723	819	697	295	636	269	119	1221	21455
23	395	35	1102	212	1076	919	1050	893	491	376	710	350	193	282	167	989	874	1208	806	691	780	665	263	106	482	80	1189	54	1121	761	604	980	578	421	552	21455
24	704	337	678	311	161	983	128	1202	835	1176	774	652	257	626	476	74	443	48	1150	755	1089	967	572	941	546	389	22	363	241	1070	180	1037	887	520	861	21455
25	803	646	742	620	463	594	437	42	1144	9	1118	961	1057	935	533	418	507	357	235	324	209	1031	881	1005	848	733	822	672	305	148	279	122	1196	96	1163	21455
26	1112	222	1079	929	527	903	501	379	720	318	203	1025	170	999	842	1218	816	694	299	633	273	116	485	90	1157	797	1131	764	614	948	588	431	64	405	3	21455
27	190	286	164	993	138	1205	810	688	784	662	260	601	479	84	1186	51	1125	758	1099	977	575	425	549	399	32	366	216	1073	923	1047	890	495	864	714	347	21455
28	745	630	473	71	447	45	1154	19	1086	971	569	945	543	386	517	360	245	1067	177	1041	884	1015	858	701	341	675	315	158	247	132	1199	839	1173	771	656	21455
29	1054	932	537	906	511	354	232	328	206	1035	145	1002	852	730	826	669	302	643	276	126	1193	93	1167	800	1141	739	617	467	591	441	39	408	13	1115	965	21455
30	174	996	846	1215	813	698	787	637	270	113	489	87	1161	61	1128	768	611	952	585	428	559	402	7	1109	219	1083	926	1022	900	498	383	717	322	200	289	21455
31	483	81	450	55	1122	762	1096	974	579	913	553	396	29	370	213	1077	187	1044	894	492	868	711	344	685	283	168	990	135	1209	807	1183	781	659	264	598	21455
32	540	390	514	364	242	331	181	1038	888	1012	855	705	829	679	312	155	251	129	1203	103	1170	775	653	749	627	470	566	444	49	1151	16	1090	968	1064	942	21455
33	849	1225	823	666	306	640	280	123	457	97	1164	804	1138	736	621	955	595	438	36	412	10	1119	229	1051	936	534	910	508	351	727	325	210	1032	142	1006	21455
34	1158	58	1132	765	1106	949	582	432	556	406	4	373	223	1080	930	1019	897	502	871	721	319	197	293	171	1000	110	1212	817	695	791	634	267	608	486	91	21455
35	524	367	217	1074	184	1048	891	987	865	708	348	682	287	165	254	139	1206	811	1180	778	663	752	602	480	78	454	52	1126	26	1093	978	576	917	550	393	21455
21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	
	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455	21455

Program 8

#4: Expansion by the Bootstrap method

This method is called the *bootstrap* method because it only utilizes the size-**b** square **W(b)** by itself. It applies in general to the composition of Class-6 squares.

	1	2	3	4	5	
	W(5)	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

This method starts with the scaling up of the **W(b)** square by formula (**8.26**) to get **Z(b)** as shown here

Then **Z** is permuted first by one column per each block-column **n-1** times in series to form the first block-row of an expansion table **D** as shown below. Next the first block-row in **D** is permuted by one row per each block-row below it **n-1** times to form the size-b^2 table $\mathbf{D(b^2)}$.

$$(8.26) \qquad Z(b) = b^2(W(b) - |1|)$$

	1	2	3	4	5	
	Z(5)	1500	1500	1500	1500	1500
1	475	175	500	325	25	1500
2	250	75	400	225	550	1500
3	150	600	300	0	450	1500
4	50	375	200	525	350	1500
5	575	275	100	425	125	1500
	1500	1500	1500	1500	1500	
		1500	1500	1500	1500	1500

Observe that the rows and columns of the block-square in the upper left corner are permuted across block-rows and block-columns as shown here so that each number never appears in the same place twice among all the **25 5x5** block-squares.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	D(25)	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500
1	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
2	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
3	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
4	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
5	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
6	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
7	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
8	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
9	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
10	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
11	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
12	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
13	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
14	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
15	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
16	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
17	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
18	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
19	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
20	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
21	250	75	400	225	550	550	250	75	400	225	225	550	250	75	400	400	225	550	250	75	75	400	225	550	250	7500
22	150	600	300	0	450	450	150	600	300	0	0	450	150	600	300	300	0	450	150	600	600	300	0	450	150	7500
23	50	375	200	525	350	350	50	375	200	525	525	350	50	375	200	200	525	350	50	375	375	200	525	350	50	7500
24	575	275	100	425	125	125	575	275	100	425	425	125	575	275	100	100	425	125	575	275	275	100	425	125	575	7500
25	475	175	500	325	25	25	475	175	500	325	325	25	475	175	500	500	325	25	475	175	175	500	325	25	475	7500
	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	
		7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500

Program 8

Next the tiled expansion table **E(b^2)** is added to **D(b^2)** to get **W*(b^2)**.

E(25)

20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6
20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2	20	8	21	14	2
11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23	11	4	17	10	23
7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19	7	25	13	1	19
3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15	3	16	9	22	15
24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6	24	12	5	18	6

Then **W*(25)** is generated from

(8.27) $$W^*(b^2) = D(b^2) + E(b^2)$$

After this, **W*(b^2)** is normalized by moving the upper-left block-square to the central position by permuting block-rows and block-columns to get **W(b^2) as** shown here.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
W(25)	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
1	320	8	471	164	602	620	308	21	464	152	170	608	321	14	452	470	158	621	314	2	20	458	171	614	302	7825
2	211	529	367	60	398	386	204	542	360	73	61	379	217	535	373	361	54	392	210	548	536	354	67	385	223	7825
3	107	450	138	576	294	282	125	438	126	594	582	300	113	426	144	132	600	288	101	444	432	150	588	276	119	7825
4	503	341	34	497	190	178	516	334	47	490	478	191	509	347	40	28	491	184	522	340	328	41	484	197	515	7825
5	424	237	555	268	81	99	412	230	568	256	274	87	405	243	556	574	262	80	418	231	249	562	255	93	406	7825
6	420	233	571	264	77	95	408	246	564	252	270	83	421	239	552	570	258	96	414	227	245	558	271	89	402	7825
7	311	4	467	160	623	611	304	17	460	173	161	604	317	10	473	461	154	617	310	23	11	454	167	610	323	7825
8	207	550	363	51	394	382	225	538	351	69	57	400	213	526	369	357	75	388	201	544	532	375	63	376	219	7825
9	103	441	134	597	290	278	116	434	147	590	578	291	109	447	140	128	591	284	122	440	428	141	584	297	115	7825
10	524	337	30	493	181	199	512	330	43	481	499	187	505	343	31	49	487	180	518	331	349	37	480	193	506	7825
11	520	333	46	489	177	195	508	346	39	477	495	183	521	339	27	45	483	196	514	327	345	33	496	189	502	7825
12	411	229	567	260	98	86	404	242	560	273	261	79	417	235	573	561	254	92	410	248	236	554	267	85	423	7825
13	307	25	463	151	619	607	325	13	451	169	157	625	313	1	469	457	175	613	301	19	7	475	163	601	319	7825
14	203	541	359	72	390	378	216	534	372	65	53	391	209	547	365	353	66	384	222	540	528	366	59	397	215	7825
15	124	437	130	593	281	299	112	430	143	581	599	287	105	443	131	149	587	280	118	431	449	137	580	293	106	7825
16	120	433	146	589	277	295	108	446	139	577	595	283	121	439	127	145	583	296	114	427	445	133	596	289	102	7825
17	511	329	42	485	198	186	504	342	35	498	486	179	517	335	48	36	479	192	510	348	336	29	492	185	523	7825
18	407	250	563	251	94	82	425	238	551	269	257	100	413	226	569	557	275	88	401	244	232	575	263	76	419	7825
19	303	16	459	172	615	603	316	9	472	165	153	616	309	22	465	453	166	609	322	15	3	466	159	622	315	7825
20	224	537	355	68	381	399	212	530	368	56	74	387	205	543	356	374	62	380	218	531	549	362	55	393	206	7825
21	220	533	371	64	377	395	208	546	364	52	70	383	221	539	352	370	58	396	214	527	545	358	71	389	202	7825
22	111	429	142	585	298	286	104	442	135	598	586	279	117	435	148	136	579	292	110	448	436	129	592	285	123	7825
23	507	350	38	476	194	182	525	338	26	494	482	200	513	326	44	32	500	188	501	344	332	50	488	176	519	7825
24	403	241	559	272	90	78	416	234	572	265	253	91	409	247	565	553	266	84	422	240	228	566	259	97	415	7825
25	324	12	455	168	606	624	312	5	468	156	174	612	305	18	456	474	162	605	318	6	24	462	155	618	306	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

This pattern is unique among all the geonomic squares investigated to date: here all the numbers for which the modulus and integer functions are equal lie in the central embedded **5x5** block-square! This centralizing property is the result of the block-normalization of **W***.

Program 8

The central block-square sums totally to the square's characteristic number, but is the only one to do so. The central numbers of all the **5x5** block-squares sum to the square's characteristic number too.

Each **5x5** block-square is geonomic in that all its 4-directional sums are equal to **1565**.

Only the central block-square is pangenic in that all its quadrals sum to **1252 = 4 x 313**.

Pangenic

	1	2	3	4	5	
	5x5	1565	1565	1565	1565	1565
1	495	183	521	339	27	1565
2	261	79	417	235	573	1565
3	157	625	313	1	469	1565
4	53	391	209	547	365	1565
5	599	287	105	443	131	1565
	1565	1565	1565	1565	1565	
		1565	1565	1565	1565	1565

Not pangenic

	1	2	3	4	5	
	5x5	1565	1565	1565	1565	1565
1	145	583	296	114	427	1565
2	36	479	192	510	348	1565
3	557	275	88	401	244	1565
4	453	166	609	322	15	1565
5	374	62	380	218	531	1565
	1565	1565	1565	1565	1565	
		1565	1565	1565	1565	1565

The size-25 square is the smallest Class-6 square that can be bootstrapped and the largest such size still legible. So it's all you will get to see regarding this centralizing property.

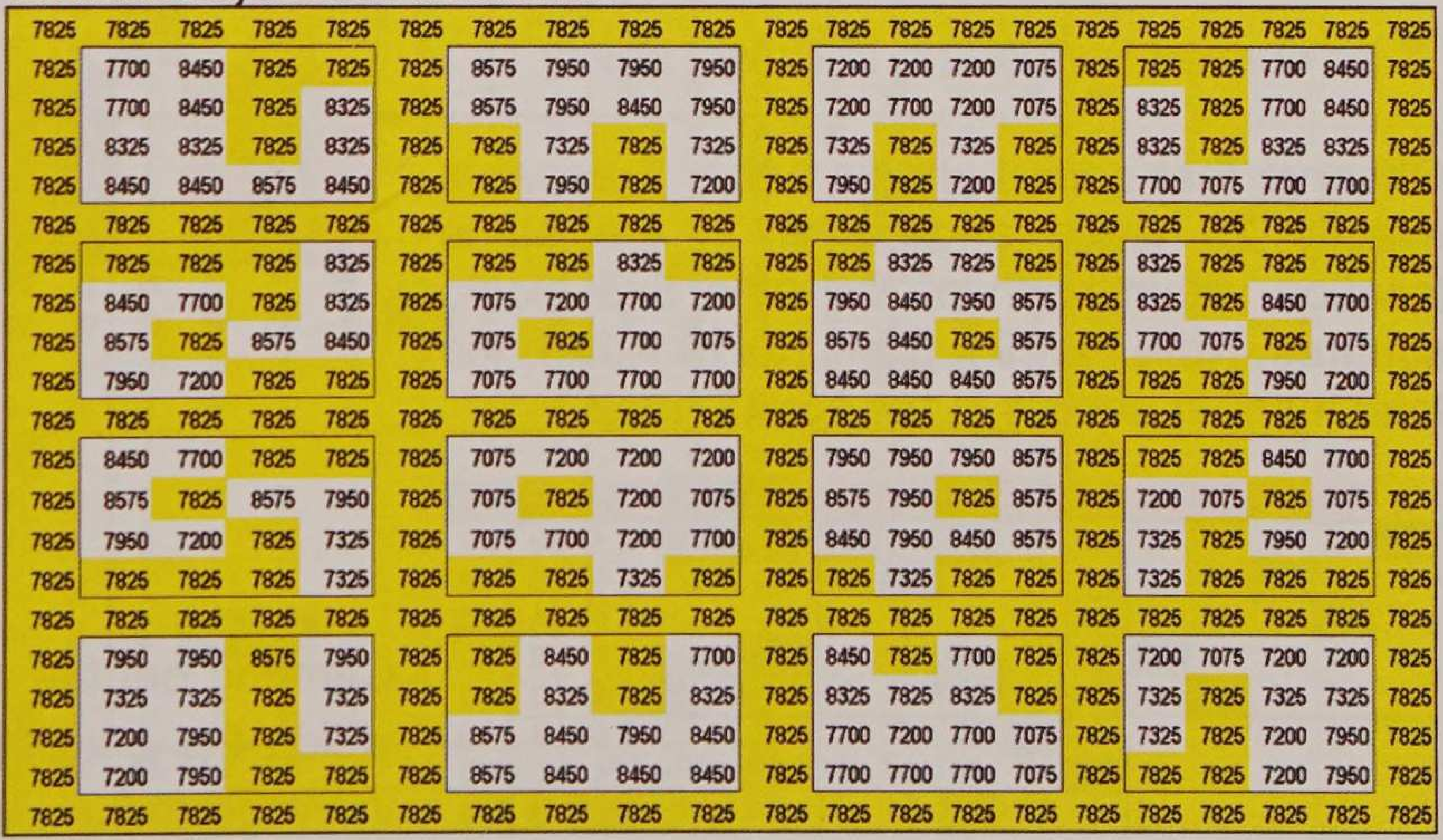
From the centralizing property stems the following quasi-continuous modularity property as shown below. The **5x5** block-squares which are certain to sum to the square's characteristic number reside only in locations where no more than one boundary between adjacent embedded block-squares is crossed.

This completes discussion of the Bootstrap expansion method.

* * *

Note: None of the expansion methods, the **T-Ball** method #2, the **ATE** method #3, nor the **Bootstrap** method #4 will produce a Class-6 square that possesses continuous modularity.

Square-number sizes $\mathbf{b}^2$ that are continuously **bx** modular are only the result of a 2-dimensional fractal projection of a 4-dimensional size-**b** quadracube onto a 2-dimensional square of size $\mathbf{b}^2$.



Not continuously 5x modular in derived form

7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	7700	8450	7825	7825	7825	8575	7950	7950	7950	7825	7200	7200	7200	7075	7825	7825	7825	7700	8450	7825
7825	7700	8450	7825	8325	7825	8575	7950	8450	7950	7825	7200	7700	7200	7075	7825	8325	7825	7700	8450	7825
7825	8325	8325	7825	8325	7825	7825	7325	7825	7325	7825	7325	7825	7325	7825	7825	8325	7825	8325	8325	7825
7825	8450	8450	8575	8450	7825	7825	7950	7825	7200	7825	7950	7825	7200	7825	7825	7700	7075	7700	7700	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	7825	7825	7825	8325	7825	7825	7825	8325	7825	7825	7825	8325	7825	7825	7825	8325	7825	7825	7825	7825
7825	8450	7700	7825	8325	7825	7075	7200	7700	7200	7825	7950	8450	7950	8575	7825	8325	7825	8450	7700	7825
7825	8575	7825	8575	8450	7825	7075	7825	7700	7075	7825	8575	8450	7825	8575	7825	7700	7075	7825	7075	7825
7825	7950	7200	7825	7825	7825	7075	7700	7700	7700	7825	8450	8450	8450	8575	7825	7825	7825	7950	7200	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	8450	7700	7825	7825	7825	7075	7200	7200	7200	7825	7950	7950	7950	8575	7825	7825	7825	8450	7700	7825
7825	8575	7825	8575	7950	7825	7075	7825	7200	7075	7825	8575	7950	7825	8575	7825	7200	7075	7825	7075	7825
7825	7950	7200	7825	7325	7825	7075	7700	7200	7700	7825	8450	7950	8450	8575	7825	7325	7825	7950	7200	7825
7825	7825	7825	7825	7325	7825	7825	7825	7325	7825	7825	7825	7325	7825	7825	7825	7325	7825	7825	7825	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
7825	7950	7950	8575	7950	7825	7825	8450	7825	7700	7825	8450	7825	7700	7825	7825	7200	7075	7200	7200	7825
7825	7325	7325	7825	7325	7825	7825	8325	7825	8325	7825	8325	7825	8325	7825	7825	7325	7825	7325	7325	7825
7825	7200	7950	7825	7325	7825	8575	8450	7950	8450	7825	7700	7200	7700	7075	7825	7325	7825	7200	7950	7825
7825	7200	7950	7825	7825	7825	8575	8450	8450	8450	7825	7700	7700	7700	7075	7825	7825	7825	7200	7950	7825
7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Further, only those quadracubes that were obtained from collapsing a 5-dimensional quintacube of the same size along its B-axis would yield such modularity. Collapsing 5-dimensional quintacubes to 4-dimensional quadracubes is only demonstrated in the book **Number Magic** as 5-dimensional squares are too large to be legible or comprehendible even on a large movie screen. Dimensional fracticality is addressed in Program 9.

Program 8

#5: Expansion by the TAP method

The merging of a Tiled-expansion And a Pattern table

This method applies in general to the composition of Classes 2, 3 & 4 squares. Classes 3 & 4 use two loom tables, one leads to the derivation of the other according to formula (**8.29**) for Class-3 squares or (**8.31**, later) for Class-4 squares. Together they form a composite loom table upon which the composite square can be readily manifested as follows.

Application to Class-3 Squares

Let **Y** denote the basic loom table and $\mathbf{Y}^T$ its transpose. The composite loom table **X** for expanded squares is obtained from:

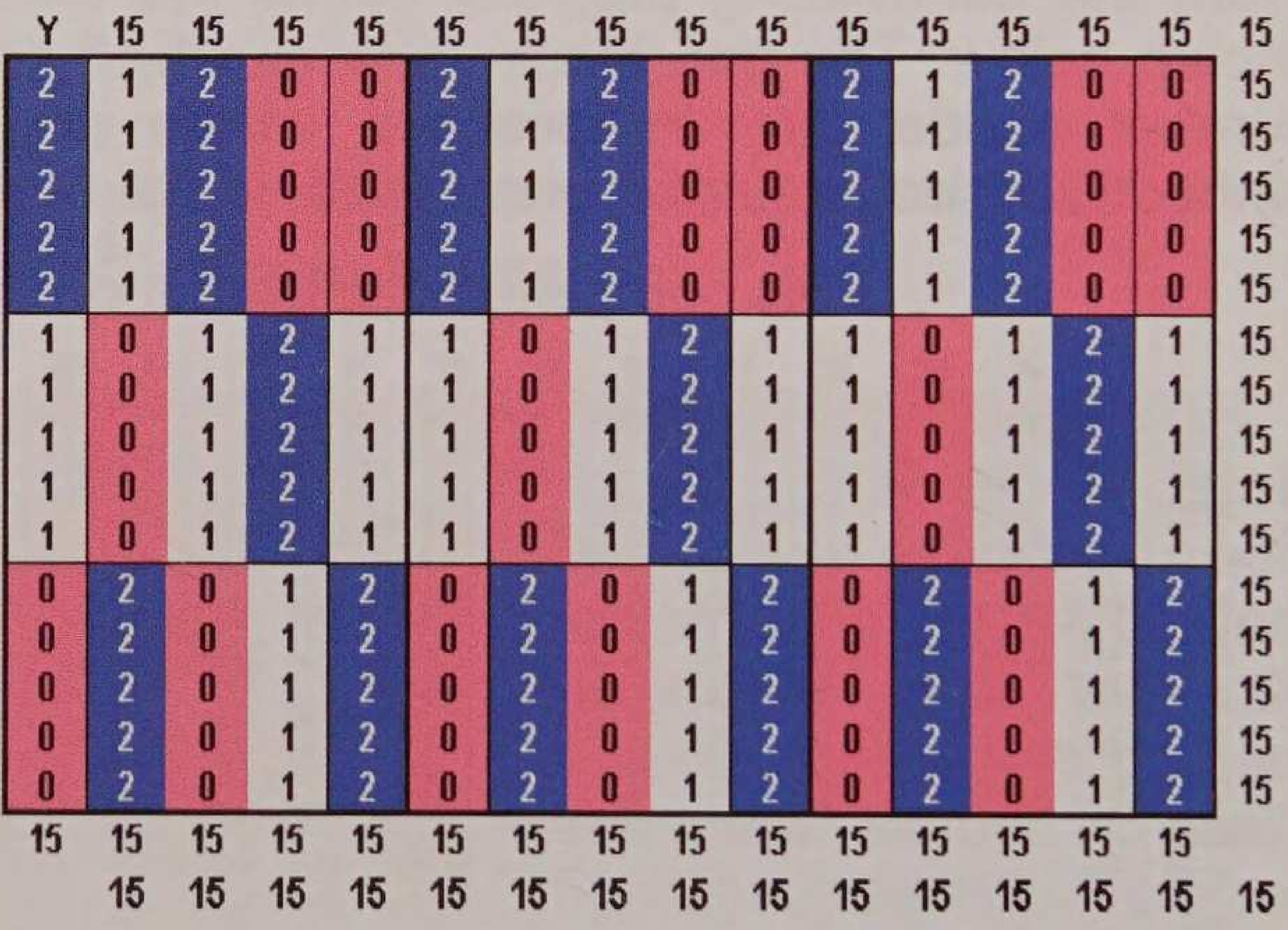
$$(\,8.28\,)\qquad X = Y + 3Y^T$$

Then the Class-3 square **W(3n)** is obtained from the size **3n** table tiled with $\mathbf{W_n}$ according to:

$$(\,8.29\,)\qquad W(3n) = n^2 X + E(n) = n^2 \begin{vmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{vmatrix} + \begin{vmatrix} W_n & W_n & W_n \\ W_n & W_n & W_n \\ W_n & W_n & W_n \end{vmatrix}$$

The rightmost matrix in the formula above is the tiled expansion table **E** to size **3n** of the size-n square $\mathbf{W_n}$.

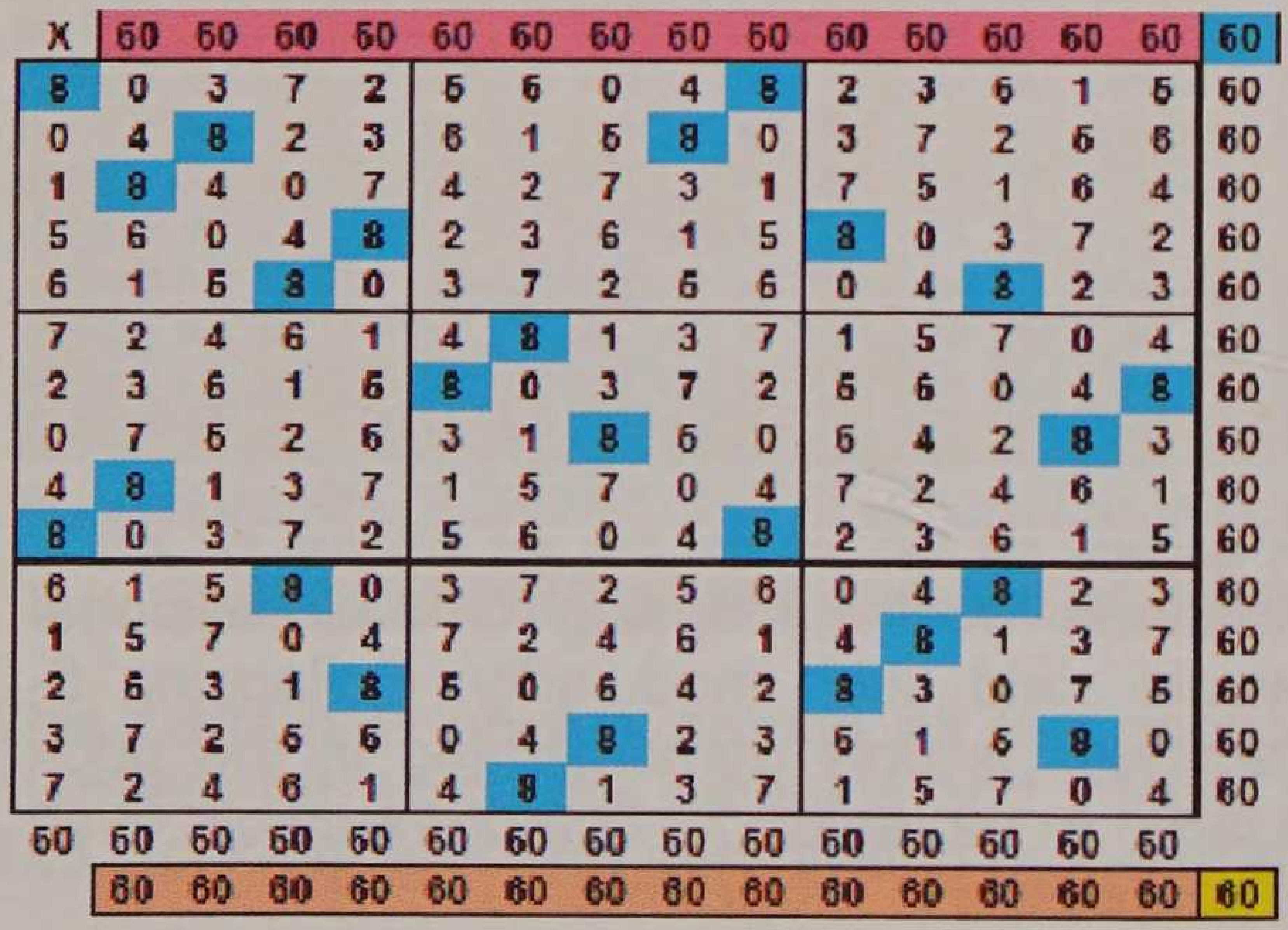
The pattern table **Y** of size **3n** follows the basic pattern shown here for the size**15** square.



Y	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
2	1	2	0	0	2	1	2	0	0	2	1	2	0	0	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
1	0	1	2	1	1	0	1	2	1	1	0	1	2	1	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
0	2	0	1	2	0	2	0	1	2	0	2	0	1	2	15
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15

X as derived from (**8.28**) is shown below.

Observe that each number **0** thru **8** appears exactly **25** times and never in the same location within each of the nine **5x5** block-squares. Note that **9 x 25 = 15x15 = 225**, the total number of cells in the **15x15** table.



X	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
8	0	3	7	2	5	6	0	4	8	2	3	6	1	5	60
0	4	8	2	3	6	1	5	8	0	3	7	2	5	6	60
1	8	4	0	7	4	2	7	3	1	7	5	1	6	4	60
5	6	0	4	8	2	3	6	1	5	8	0	3	7	2	60
6	1	5	8	0	3	7	2	5	6	0	4	8	2	3	60
7	2	4	6	1	4	8	1	3	7	1	5	7	0	4	60
2	3	6	1	5	8	0	3	7	2	5	6	0	4	8	60
0	7	5	2	6	3	1	8	5	0	6	4	2	8	3	60
4	8	1	3	7	1	5	7	0	4	7	2	4	6	1	60
8	0	3	7	2	5	6	0	4	8	2	3	6	1	5	60
6	1	5	8	0	3	7	2	5	6	0	4	8	2	3	60
1	5	7	0	4	7	2	4	6	1	4	8	1	3	7	60
2	6	3	1	8	5	0	6	4	2	8	3	0	7	5	60
3	7	2	5	6	0	4	8	2	3	6	1	5	8	0	60
7	2	4	6	1	4	8	1	3	7	1	5	7	0	4	60
60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	
	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60

Carrying out the calculations of formula (**8.29**) yields the perfect size-15 square seen earlier in Program 3.

Program 8

Here is the pattern **Y** for the size **21** square.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
Y	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
1	1	0	1	2	1	1	1	1	0	1	2	1	1	1	1	0	1	2	1	1	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
0	2	2	0	1	2	0	0	2	2	0	1	2	0	0	2	2	0	1	2	0	21
21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21

And here is the **X** loom- table derived from formula (**8.28**):

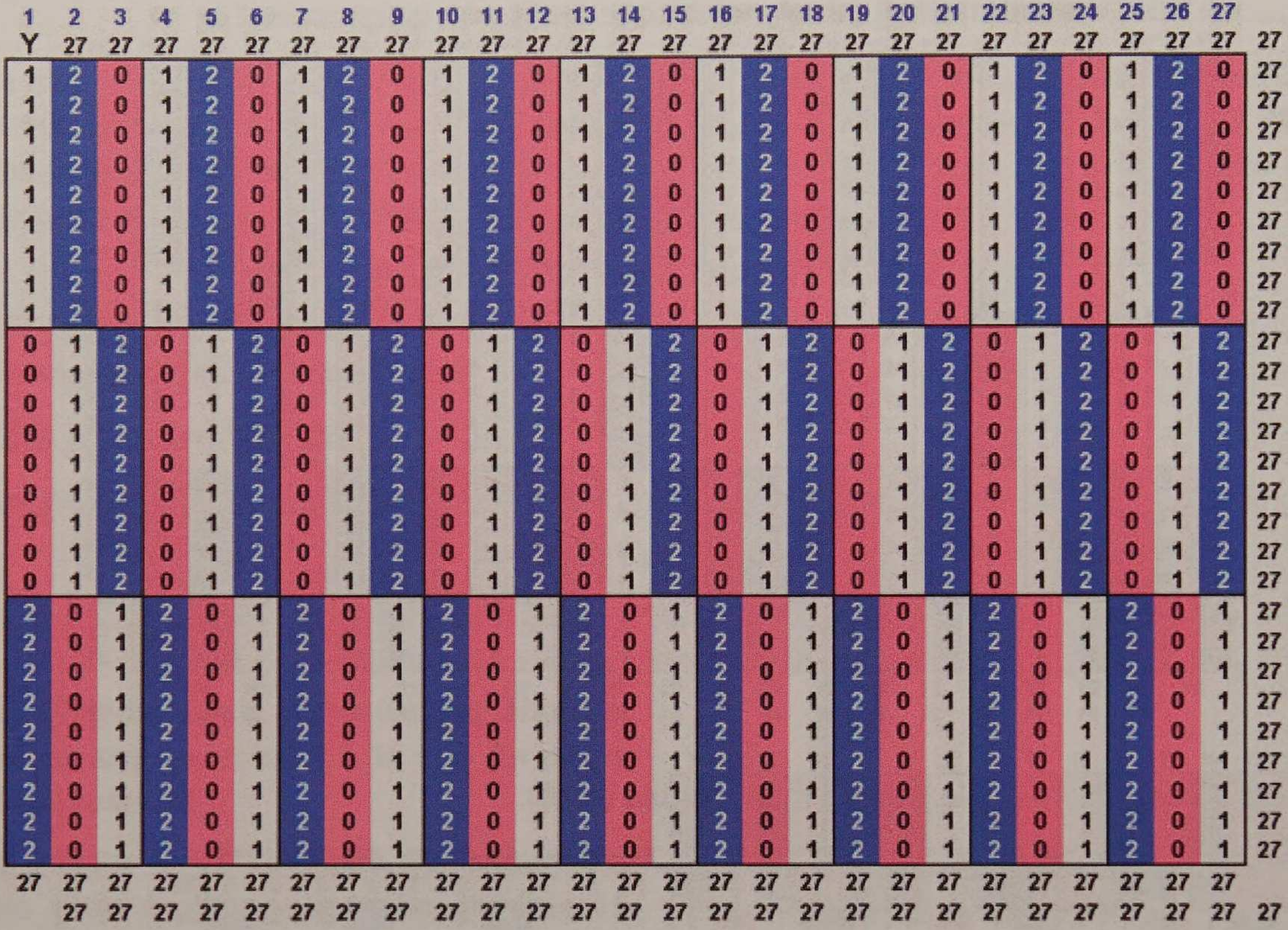
X	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84
8	2	5	8	2	2	8	7	1	4	7	1	1	7	6	0	3	6	0	0	6	84
6	0	3	6	0	0	6	7	1	4	7	1	1	7	8	2	5	8	2	2	8	84
7	1	4	7	1	1	7	6	0	3	6	0	0	6	8	2	5	8	2	2	8	84
8	2	5	8	2	2	8	7	1	4	7	1	1	7	6	0	3	6	0	0	6	84
6	0	3	6	0	0	6	8	2	5	8	2	2	8	7	1	4	7	1	1	7	84
6	0	3	6	0	0	6	7	1	4	7	1	1	7	8	2	5	8	2	2	8	84
8	2	5	8	2	2	8	7	1	4	7	1	1	7	6	0	3	6	0	0	6	84
5	5	2	5	8	5	5	4	4	1	4	7	4	4	3	3	0	3	6	3	3	84
3	3	0	3	6	3	3	4	4	1	4	7	4	4	5	5	2	5	8	5	5	84
4	4	1	4	7	4	4	3	3	0	3	6	3	3	5	5	2	5	8	5	5	84
5	5	2	5	8	5	5	4	4	1	4	7	4	4	3	3	0	3	6	3	3	84
3	3	0	3	6	3	3	5	5	2	5	8	5	5	4	4	1	4	7	4	4	84
3	3	0	3	6	3	3	4	4	1	4	7	4	4	5	5	2	5	8	5	5	84
5	5	2	5	8	5	5	4	4	1	4	7	4	4	3	3	0	3	6	3	3	84
2	8	8	2	5	8	2	1	7	7	1	4	7	1	0	6	6	0	3	6	0	84
0	6	6	0	3	6	0	1	7	7	1	4	7	1	2	8	8	2	5	8	2	84
1	7	7	1	4	7	1	0	6	6	0	3	6	0	2	8	8	2	5	8	2	84
2	8	8	2	5	8	2	1	7	7	1	4	7	1	0	6	6	0	3	6	0	84
0	6	6	0	3	6	0	2	8	8	2	5	8	2	1	7	7	1	4	7	1	84
0	6	6	0	3	6	0	1	7	7	1	4	7	1	2	8	8	2	5	8	2	84
2	8	8	2	5	8	2	1	7	7	1	4	7	1	0	6	6	0	3	6	0	84
84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	
	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84

Observe that each number **0** thru **8** now appears exactly **49** times in **X** and never in the same location within each of the nine **7x7** block-squares. Note that **9 x 49 = 21x21 = 441**, the total number of cells in the **21x21** table.

Carrying out formula (**8.29**) yields the perfect size-21 square also seen earlier in Program 3.

Program 8

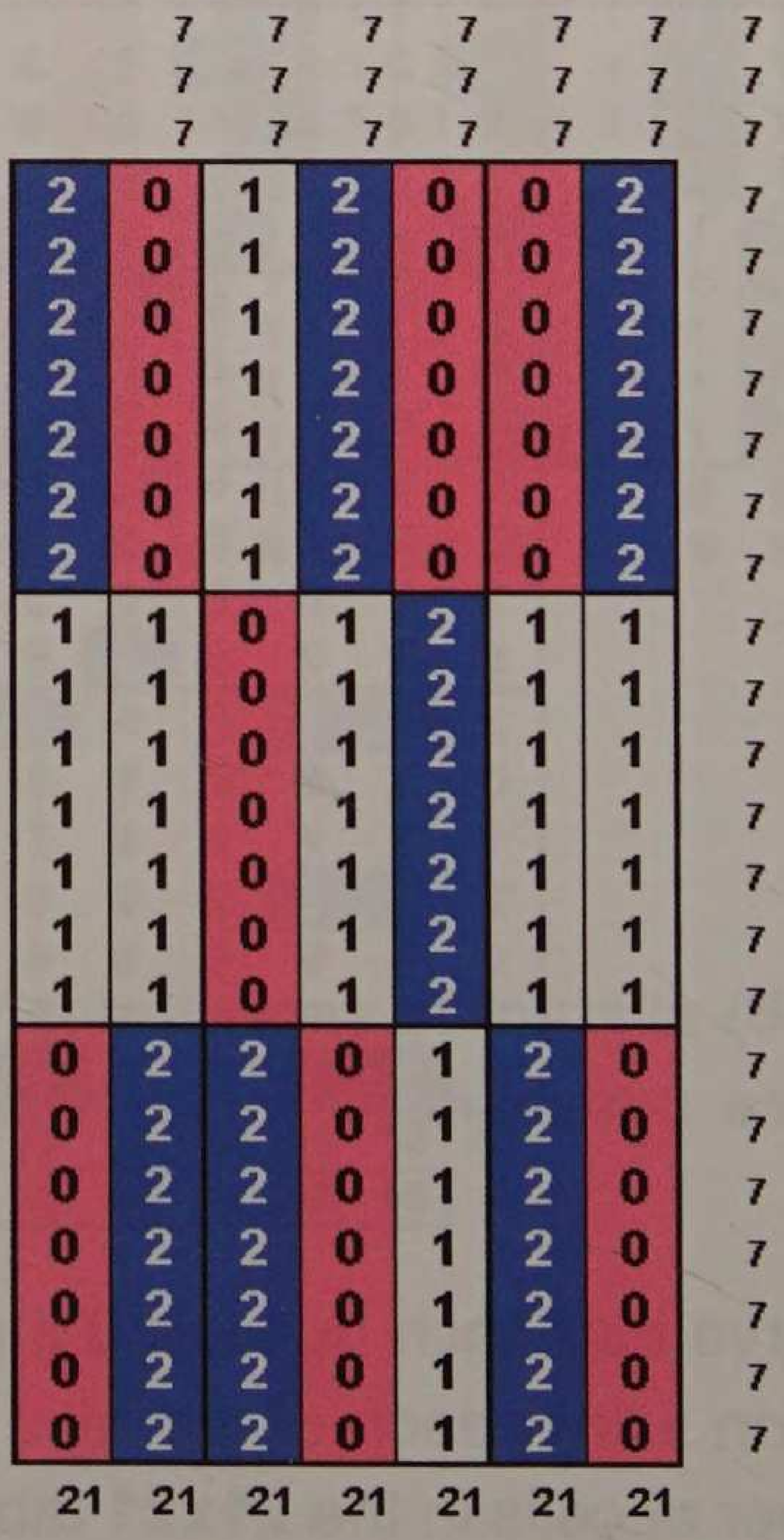
Pattern **Y** can be expanded further just by repeating the basic pattern.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
Y	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	27
27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27

This method was used to generate the size **27** perfect square seen in Program 3.

Different sizes involve different arrangements of the columns shown here. It is best to first get a rectangular table of size **3n x n** which is geonomic as shown here at right and then triplicate it to get a **3n x 3n** size square table as shown on the prior page for the size-21 square.



	7	7	7	7	7	7	7
	7	7	7	7	7	7	7
	7	7	7	7	7	7	7
2	0	1	2	0	0	2	7
2	0	1	2	0	0	2	7
2	0	1	2	0	0	2	7
2	0	1	2	0	0	2	7
2	0	1	2	0	0	2	7
2	0	1	2	0	0	2	7
2	0	1	2	0	0	2	7
1	1	0	1	2	1	1	7
1	1	0	1	2	1	1	7
1	1	0	1	2	1	1	7
1	1	0	1	2	1	1	7
1	1	0	1	2	1	1	7
1	1	0	1	2	1	1	7
1	1	0	1	2	1	1	7
0	2	2	0	1	2	0	7
0	2	2	0	1	2	0	7
0	2	2	0	1	2	0	7
0	2	2	0	1	2	0	7
0	2	2	0	1	2	0	7
0	2	2	0	1	2	0	7
0	2	2	0	1	2	0	7
21	21	21	21	21	21	21	

Program 8

Application of the TAP method to Class-4 Squares

A similar expansion method can be applied to get Class-4 squares.

The formulas (**8.28**) and (**8.29**) are specialized to handle expansions to double the size of the square. To formulate this process, the vertically and/or horizontally flipped version of $\mathbf{W_n}$ shall be denoted by the symbol $\overline{\mathbf{W}}_\mathbf{n}$.

$$(8.30)\qquad X = Y_1 + 2Y_2$$

$$(8.31)\quad W(2n) = n^2 \begin{vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{vmatrix} + \begin{vmatrix} W_n & \overline{W}_n \\ W_n & \overline{W}_n \end{vmatrix}$$

		-6		6	
	4x4	28	34	40	34
1	8	2	4	12	26
2	10	16	14	6	46
3	11	3	1	7	22
4	5	13	15	9	42
	34	34	34	34	
		28	34	40	34
		-6		6	

Here is a size-4 square that is purposely imperfect and it will be used to show how squares of size **2n** can be obtained from Class-2 squares of size **n**. Here $\mathbf{W_n}$ is the size-4 square shown above. This size-4 square here has its wrap diagonals merely differing in sign. So it suffices to flip the square horizontally for the variances to cancel out.

Because the square also had unequal rows, it also has to be flipped vertically too.

So there was a double flip involved here to get $\overline{\mathbf{W}}\mathbf{n}$.

	1	2	3	4	5	6	7	8	
	Y1	4	4	4	4	4	4	4	4
1	1	0	1	0	1	0	1	0	4
2	1	0	1	0	1	0	1	0	4
3	0	1	0	1	0	1	0	1	4
4	0	1	0	1	0	1	0	1	4
5	0	1	0	1	0	1	0	1	4
6	0	1	0	1	0	1	0	1	4
7	1	0	1	0	1	0	1	0	4
8	1	0	1	0	1	0	1	0	4
	4	4	4	4	4	4	4	4	
		4	4	4	4	4	4	4	4

X	12	12	12	12	12	12	12	12
3	0	3	0	3	0	3	0	12
3	0	3	0	3	0	3	0	12
2	1	2	1	2	1	2	1	12
2	1	2	1	2	1	2	1	12
0	3	0	3	0	3	0	3	12
0	3	0	3	0	3	0	3	12
1	2	1	2	1	2	1	2	12
1	2	1	2	1	2	1	2	12
12	12	12	12	12	12	12	12	
	12	12	12	12	12	12	12	12

Y2	4	4	4	4	4	4	4	4
1	0	1	0	1	0	1	0	4
1	0	1	0	1	0	1	0	4
1	0	1	0	1	0	1	0	4
1	0	1	0	1	0	1	0	4
0	1	0	1	0	1	0	1	4
0	1	0	1	0	1	0	1	4
0	1	0	1	0	1	0	1	4
0	1	0	1	0	1	0	1	4
4	4	4	4	4	4	4	4	
	4	4	4	4	4	4	4	4

8x8	260	260	260	260	260	260	260	260
24	9	8	25	40	57	56	41	260
58	39	42	55	10	23	26	7	260
14	19	30	3	62	35	46	51	260
36	61	52	45	20	13	4	29	260
17	16	1	32	33	64	49	48	260
63	34	47	50	15	18	31	2	260
11	22	27	6	59	38	43	54	260
37	60	53	44	21	12	5	28	260
260	260	260	260	260	260	260	260	
	260	260	260	260	260	260	260	260

Here is the resulting perfect size-8 square from (**8.31**). It is **4x2** modular.

Program 8

Next, we'll see the same method for deriving a size-12 perfect square from a near-perfect size-6 square. Here are the patterns for Y_1 and Y_2

Y_1	1	2	3	4	5	6	7	8	9	10	11	12	
		6	6	6	6	6	6	6	6	6	6	6	6
1	1	0	1	0	1	0	1	0	1	0	1	0	6
2	1	0	1	0	1	0	1	0	1	0	1	0	6
3	1	0	1	0	1	0	1	0	1	0	1	0	6
4	0	1	0	1	0	1	0	1	0	1	0	1	6
5	0	1	0	1	0	1	0	1	0	1	0	1	6
6	0	1	0	1	0	1	0	1	0	1	0	1	6
7	0	1	0	1	0	1	0	1	0	1	0	1	6
8	0	1	0	1	0	1	0	1	0	1	0	1	6
9	0	1	0	1	0	1	0	1	0	1	0	1	6
10	1	0	1	0	1	0	1	0	1	0	1	0	6
11	1	0	1	0	1	0	1	0	1	0	1	0	6
12	1	0	1	0	1	0	1	0	1	0	1	0	6
	6	6	6	6	6	6	6	6	6	6	6	6	
		6	6	6	6	6	6	6	6	6	6	6	6

Y_2	1	2	3	4	5	6	7	8	9	10	11	12	
		6	6	6	6	6	6	6	6	6	6	6	6
1	1	0	1	0	1	0	1	0	1	0	1	0	6
2	1	0	1	0	1	0	1	0	1	0	1	0	6
3	1	0	1	0	1	0	1	0	1	0	1	0	6
4	1	0	1	0	1	0	1	0	1	0	1	0	6
5	1	0	1	0	1	0	1	0	1	0	1	0	6
6	1	0	1	0	1	0	1	0	1	0	1	0	6
7	0	1	0	1	0	1	0	1	0	1	0	1	6
8	0	1	0	1	0	1	0	1	0	1	0	1	6
9	0	1	0	1	0	1	0	1	0	1	0	1	6
10	0	1	0	1	0	1	0	1	0	1	0	1	6
11	0	1	0	1	0	1	0	1	0	1	0	1	6
12	0	1	0	1	0	1	0	1	0	1	0	1	6
	6	6	6	6	6	6	6	6	6	6	6	6	
		6	6	6	6	6	6	6	6	6	6	6	6

Here is the compound loom table **X** derived from (**8.30**).

X	1	2	3	4	5	6	7	8	9	10	11	12	
		18	18	18	18	18	18	18	18	18	18	18	18
1	3	0	3	0	3	0	3	0	3	0	3	0	18
2	3	0	3	0	3	0	3	0	3	0	3	0	18
3	3	0	3	0	3	0	3	0	3	0	3	0	18
4	2	1	2	1	2	1	2	1	2	1	2	1	18
5	2	1	2	1	2	1	2	1	2	1	2	1	18
6	2	1	2	1	2	1	2	1	2	1	2	1	18
7	0	3	0	3	0	3	0	3	0	3	0	3	18
8	0	3	0	3	0	3	0	3	0	3	0	3	18
9	0	3	0	3	0	3	0	3	0	3	0	3	18
10	1	2	1	2	1	2	1	2	1	2	1	2	18
11	1	2	1	2	1	2	1	2	1	2	1	2	18
12	1	2	1	2	1	2	1	2	1	2	1	2	18
	18	18	18	18	18	18	18	18	18	18	18	18	
		18	18	18	18	18	18	18	18	18	18	18	18

Then W_{12} is obtained from (**8.31**), using the size-6 near-perfect squares shown below.

W_6	111	111	111	111	111	111		
1	24	25	6	31	12	13	111	
2	17	32	5	35	2	23	114	3
3	22	9	4	33	28	15	111	
4	16	27	34	3	10	21	111	
5	14	11	26	8	29	20	108	-3
6	18	7	36	1	30	19	111	
	111	111	111	111	111	111		
	111	111	111	111	111	111		

$\bar{W}_6$	111	111	111	111	111	111		
1	18	7	36	1	30	19	111	
2	14	11	26	8	29	20	108	-3
3	16	27	34	3	10	21	111	
4	22	9	4	33	28	15	111	
5	17	32	5	35	2	23	114	3
6	24	25	6	31	12	13	111	
	111	111	111	111	111	111		
	111	111	111	111	111	111		

And here is the resulting size-12 perfect square. So now we see just how useful those near-perfect Class-2 squares are for generating larger size squares of Class-4.

	1	2	3	4	5	6	7	8	9	10	11	12	
	12x12	870	870	870	870	870	870	870	870	870	870	870	870
1	109	30	127	90	43	108	37	102	55	18	115	36	870
2	67	84	49	24	133	6	139	12	121	96	61	78	870
3	107	41	128	89	32	110	35	113	56	17	104	38	870
4	69	100	51	22	117	4	141	28	123	94	45	76	870
5	75	46	93	124	27	142	3	118	21	52	99	70	870
6	44	98	59	14	119	29	116	26	131	86	47	101	870
7	73	66	91	126	7	144	1	138	19	54	79	72	870
8	31	120	13	60	97	42	103	48	85	132	25	114	870
9	140	2	95	122	71	77	68	74	23	50	143	5	870
10	33	136	15	58	81	40	105	64	87	130	9	112	870
11	111	10	129	88	63	106	39	82	57	16	135	34	870
12	11	137	20	53	80	62	83	65	92	125	8	134	870
	870	870	870	870	870	870	870	870	870	870	870	870	
		870	870	870	870	870	870	870	870	870	870	870	870

This size-12 square has the quasi-continuous diamond and X-tile patterns as shown back in Program 2. Both the **TAP** method here and the **ATE** method shown earlier yielded quasi-continuous tiling patterns for the size-12 square. However, only the **TAP** method here furnished a square that was also pairwise row-symmetric. Consequently it is the better method of the two for generating Class-4 squares. There is a related method, #6 **SPD**, which is just as good.

Generating the Size-16 square

Here are the pattern tables **Y1** and **Y2** for generating the size-16 square from the size-8 square.

Y_2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	
	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8

Y_1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	8
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	
	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8

X	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	24
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	24
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	24
3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	24
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	24
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	24
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	24
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	24
0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	24
0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	24
0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	24
0	3	0	3	0	3	0	3	0	3	0	3	0	3	0	3	24
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	24
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	24
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	24
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	24
24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	
	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24

Since the size-8 square is perfect to begin with, no flipping is required and $\mathbf{W_8}$ and $\mathbf{\overline{W}_8}$ are identical.

Here is the complementary loom table $\mathbf{X_{16}}$. The expansion pattern should by now be quite obvious.

Whichever size-8 square is used for $\mathbf{W_8}$, its pairwise symmetry will be transferred to the resulting size-16 square.

Program 8

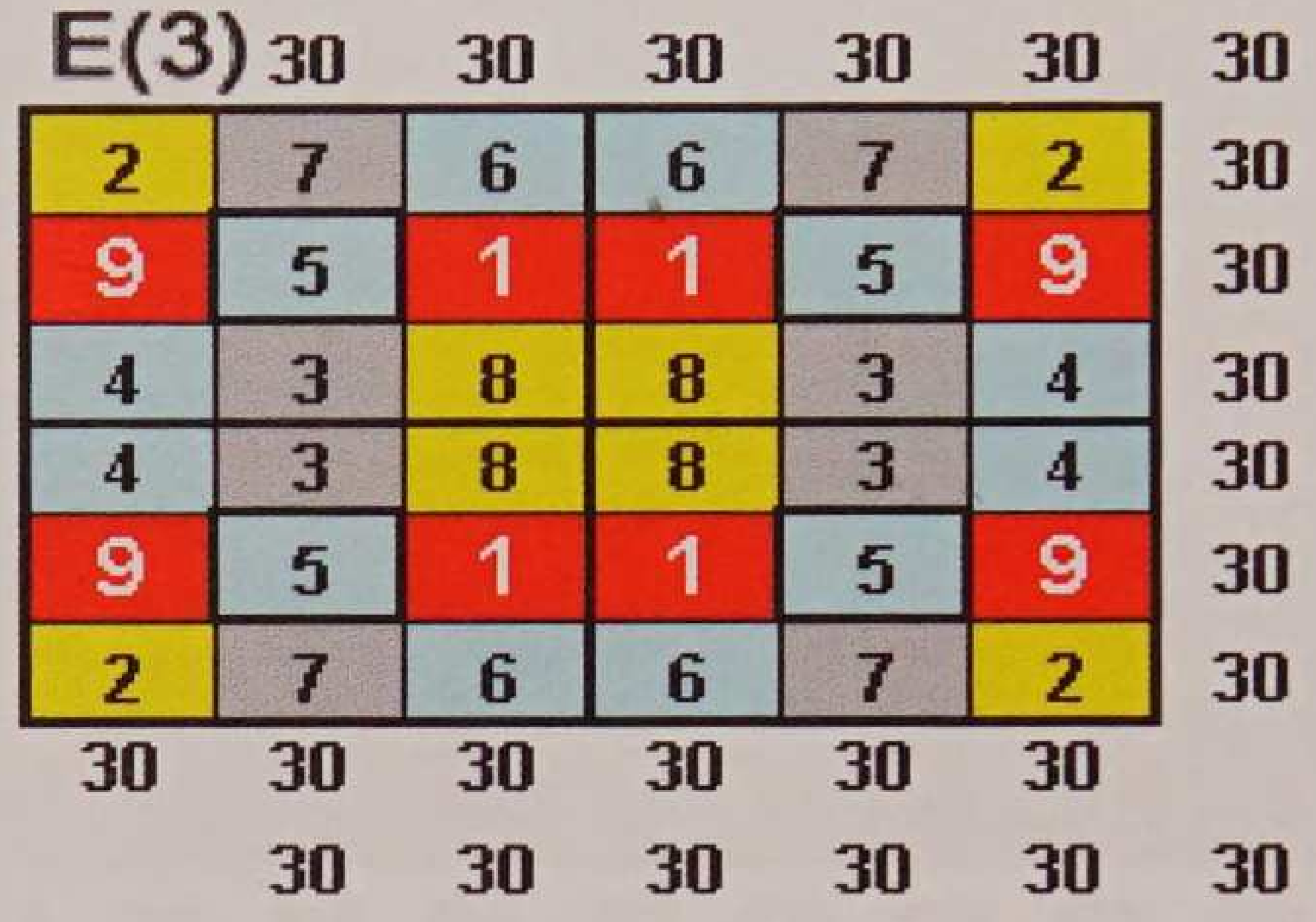
Application of the TAP method to generating Class-2 Squares

Here are three patterns to demonstrate the general pattern that can be applied as **X** in formula (**8.31**), shown here for **n = 3**, **5** and **7**.

$$(8.32)\quad W(2n) = n^2 X_{2n} + E(n) = \begin{vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{vmatrix} + \begin{vmatrix} W(n)_{11} & W(n)_{12} \\ W(n)_{21} & W(n)_{22} \end{vmatrix}$$

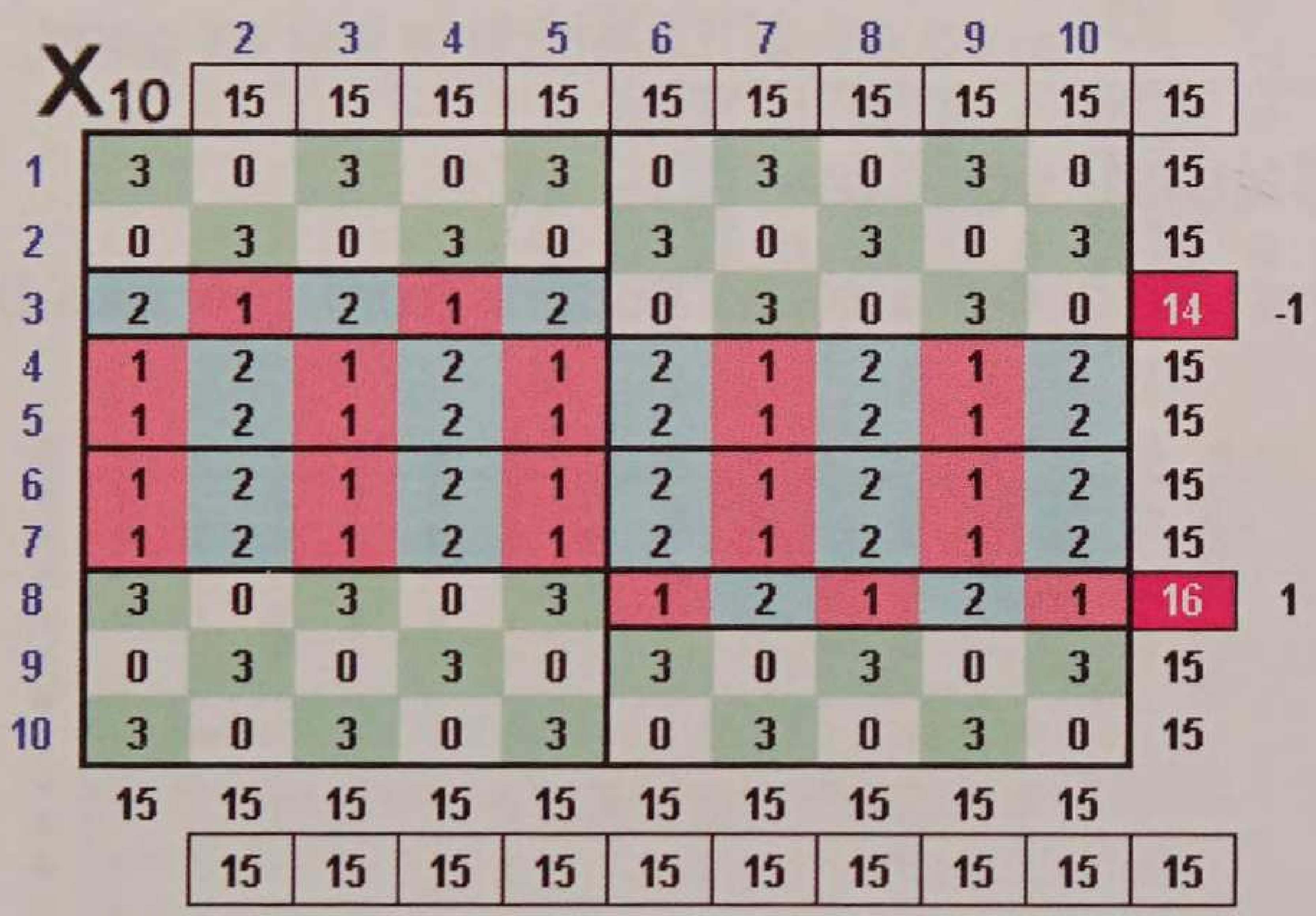
X_6

		2	3	4	5	6		
		9	9	9	9	9	9	
1	3	0	3	0	3	0	9	
2	0	3	0	2	1	2	8	-1
3	1	2	1	2	1	2	9	
4	1	2	1	2	1	2	9	
5	1	2	1	3	0	3	10	1
6	3	0	3	0	3	0	9	
	9	9	9	9	9	9		
		9	9	9	9	9	9	



E(3)

	30	30	30	30	30	30
2	7	6	6	7	2	30
9	5	1	1	5	9	30
4	3	8	8	3	4	30
4	3	8	8	3	4	30
9	5	1	1	5	9	30
2	7	6	6	7	2	30
30	30	30	30	30	30	
	30	30	30	30	30	30

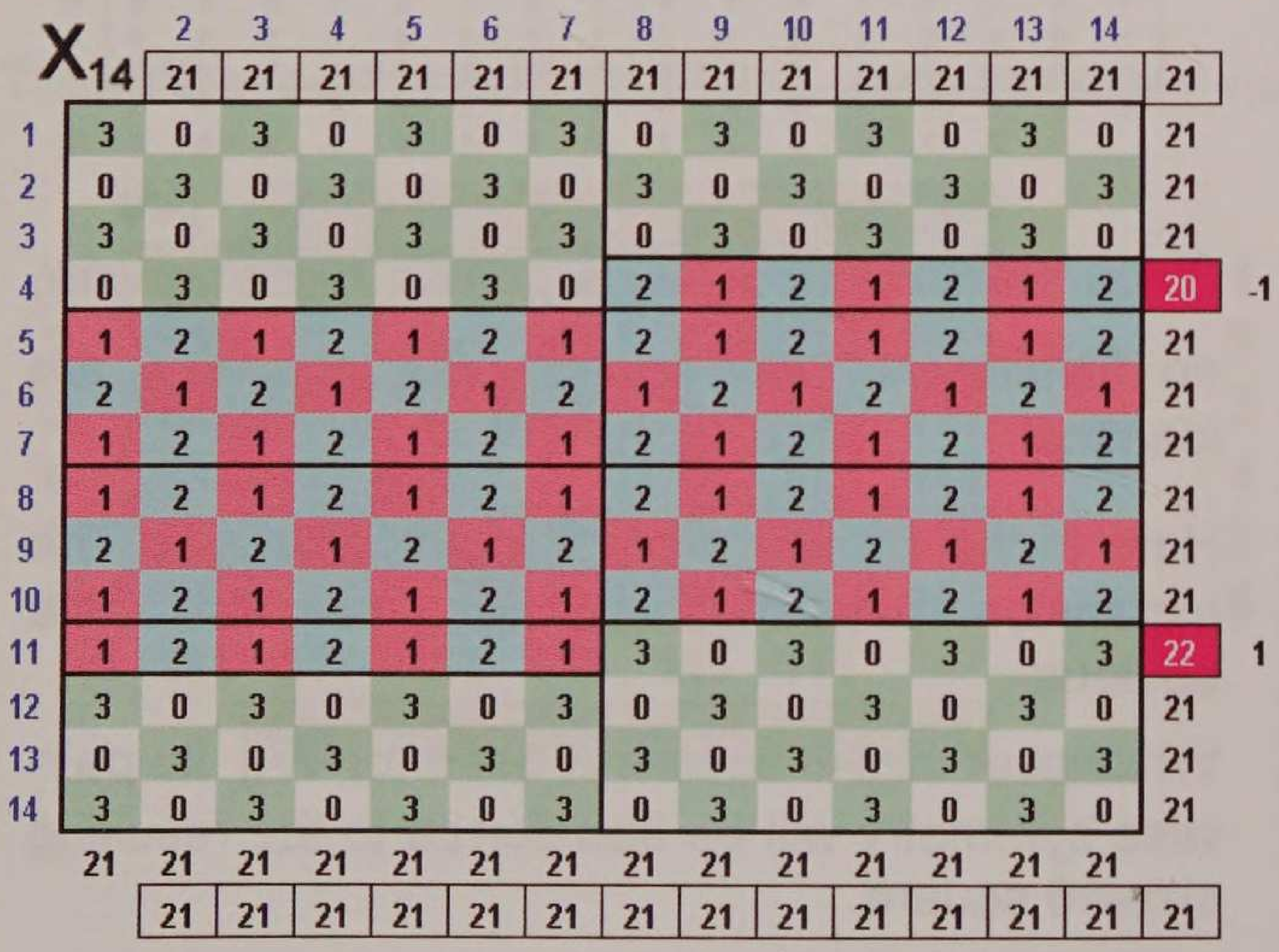


X_{10}

		2	3	4	5	6	7	8	9	10		
		15	15	15	15	15	15	15	15	15	15	
1	3	0	3	0	3	0	3	0	3	0	15	
2	0	3	0	3	0	3	0	3	0	3	15	
3	2	1	2	1	2	0	3	0	3	0	14	-1
4	1	2	1	2	1	2	1	2	1	2	15	
5	1	2	1	2	1	2	1	2	1	2	15	
6	1	2	1	2	1	2	1	2	1	2	15	
7	1	2	1	2	1	2	1	2	1	2	15	
8	3	0	3	0	3	1	2	1	2	1	16	1
9	0	3	0	3	0	3	0	3	0	3	15	
10	3	0	3	0	3	0	3	0	3	0	15	
	15	15	15	15	15	15	15	15	15	15		
		15	15	15	15	15	15	15	15	15	15	

E(5)

		2	3	4	5	6	7	8	9	10
		130	130	130	130	130	130	130	130	130	130
1	20	8	21	14	2	20	8	21	14	2	130
2	11	4	17	10	23	11	4	17	10	23	130
3	7	25	13	1	19	7	25	13	1	19	130
4	3	16	9	22	15	3	16	9	22	15	130
5	24	12	5	18	6	24	12	5	18	6	130
6	20	8	21	14	2	20	8	21	14	2	130
7	11	4	17	10	23	11	4	17	10	23	130
8	7	25	13	1	19	7	25	13	1	19	130
9	3	16	9	22	15	3	16	9	22	15	130
10	24	12	5	18	6	24	12	5	18	6	130
	130	130	130	130	130	130	130	130	130	130	
		130	130	130	130	130	130	130	130	130	130



X_{14}

		2	3	4	5	6	7	8	9	10	11	12	13	14		
		21	21	21	21	21	21	21	21	21	21	21	21	21	21	
1	3	0	3	0	3	0	3	0	3	0	3	0	3	0	21	
2	0	3	0	3	0	3	0	3	0	3	0	3	0	3	21	
3	3	0	3	0	3	0	3	0	3	0	3	0	3	0	21	
4	0	3	0	3	0	3	0	2	1	2	1	2	1	2	20	-1
5	1	2	1	2	1	2	1	2	1	2	1	2	1	2	21	
6	2	1	2	1	2	1	2	1	2	1	2	1	2	1	21	
7	1	2	1	2	1	2	1	2	1	2	1	2	1	2	21	
8	1	2	1	2	1	2	1	2	1	2	1	2	1	2	21	
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	21	
10	1	2	1	2	1	2	1	2	1	2	1	2	1	2	21	
11	1	2	1	2	1	2	1	3	0	3	0	3	0	3	22	1
12	3	0	3	0	3	0	3	0	3	0	3	0	3	0	21	
13	0	3	0	3	0	3	0	3	0	3	0	3	0	3	21	
14	3	0	3	0	3	0	3	0	3	0	3	0	3	0	21	
	21	21	21	21	21	21	21	21	21	21	21	21	21	21		
		21	21	21	21	21	21	21	21	21	21	21	21	21	21	

There are four different versions of **W(3)** in (**8.32**) in the tiled expansion **E(3)** to make it geonomic (top right). That is the only case where this is necessary because all the other expansion tables are composed of Class-1 tiles which are all perfect.

The tiled expansion of the **W(5)** and **W(7)** squares is obvious and has therefore been omitted.

The tiling pattern for all larger squares should now be obvious based on the three examples for X_{2n} shown at left here.

* * *

This concludes the demonstration of the **TAP** expansion method.

Program 8

#6 Expansion by the SPD method

The abbreviation **SPD** stands for *Spread Pattern Distribution*. This is an amazing method of expansion using a simple binary pattern for generating Class-4 squares of size **n = 4b** from a perfect size **2b** square. Here is the binary pattern for generating the spread expansion of the size **12** square to one of size **24**. Observe that $\mathbf{Y_2}$ here is just the transpose of $\mathbf{Y_1}$: $\mathbf{Y_2} = \mathbf{Y_1}^T$.

From this point on, formula (**8.30**) of the **TAP** method is applied to generate the spread pattern **X** for **W(12)**, which is then used in formula (**8.32**) to generate **W(24)** (next page).

Y_1

		12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
1	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
2	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
3	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
4	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
5	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
6	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
7	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
8	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
9	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
10	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
11	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
12	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
13	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
14	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
15	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
16	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
17	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
18	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	12
19	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
20	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
21	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
22	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
23	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
24	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	12
	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	
		12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12

Y_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
		12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
2	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
3	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
4	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
6	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
7	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
8	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
9	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
10	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
11	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
12	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
13	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
14	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
15	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
16	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
17	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
18	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
19	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
20	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
21	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
22	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
23	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	12
24	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	12
	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	
		12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12

Program 8

X		36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
1	3	2	3	2	3	2	1	0	1	0	1	0	2	3	2	3	2	3	0	1	0	1	0	1	36
2	1	0	1	0	1	0	3	2	3	2	3	2	0	1	0	1	0	1	2	3	2	3	2	3	36
3	3	2	3	2	3	2	1	0	1	0	1	0	2	3	2	3	2	3	0	1	0	1	0	1	36
4	1	0	1	0	1	0	3	2	3	2	3	2	0	1	0	1	0	1	2	3	2	3	2	3	36
5	3	2	3	2	3	2	1	0	1	0	1	0	2	3	2	3	2	3	0	1	0	1	0	1	36
6	1	0	1	0	1	0	3	2	3	2	3	2	0	1	0	1	0	1	2	3	2	3	2	3	36
7	2	3	2	3	2	3	0	1	0	1	0	1	3	2	3	2	3	2	1	0	1	0	1	0	36
8	0	1	0	1	0	1	2	3	2	3	2	3	1	0	1	0	1	0	3	2	3	2	3	2	36
9	2	3	2	3	2	3	0	1	0	1	0	1	3	2	3	2	3	2	1	0	1	0	1	0	36
10	0	1	0	1	0	1	2	3	2	3	2	3	1	0	1	0	1	0	3	2	3	2	3	2	36
11	2	3	2	3	2	3	0	1	0	1	0	1	3	2	3	2	3	2	1	0	1	0	1	0	36
12	0	1	0	1	0	1	2	3	2	3	2	3	1	0	1	0	1	0	3	2	3	2	3	2	36
13	1	0	1	0	1	0	3	2	3	2	3	2	0	1	0	1	0	1	2	3	2	3	2	3	36
14	3	2	3	2	3	2	1	0	1	0	1	0	2	3	2	3	2	3	0	1	0	1	0	1	36
15	1	0	1	0	1	0	3	2	3	2	3	2	0	1	0	1	0	1	2	3	2	3	2	3	36
16	3	2	3	2	3	2	1	0	1	0	1	0	2	3	2	3	2	3	0	1	0	1	0	1	36
17	1	0	1	0	1	0	3	2	3	2	3	2	0	1	0	1	0	1	2	3	2	3	2	3	36
18	3	2	3	2	3	2	1	0	1	0	1	0	2	3	2	3	2	3	0	1	0	1	0	1	36
19	0	1	0	1	0	1	2	3	2	3	2	3	1	0	1	0	1	0	3	2	3	2	3	2	36
20	2	3	2	3	2	3	0	1	0	1	0	1	3	2	3	2	3	2	1	0	1	0	1	0	36
21	0	1	0	1	0	1	2	3	2	3	2	3	1	0	1	0	1	0	3	2	3	2	3	2	36
22	2	3	2	3	2	3	0	1	0	1	0	1	3	2	3	2	3	2	1	0	1	0	1	0	36
23	0	1	0	1	0	1	2	3	2	3	2	3	1	0	1	0	1	0	3	2	3	2	3	2	36
24	2	3	2	3	2	3	0	1	0	1	0	1	3	2	3	2	3	2	1	0	1	0	1	0	36
	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
		36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36

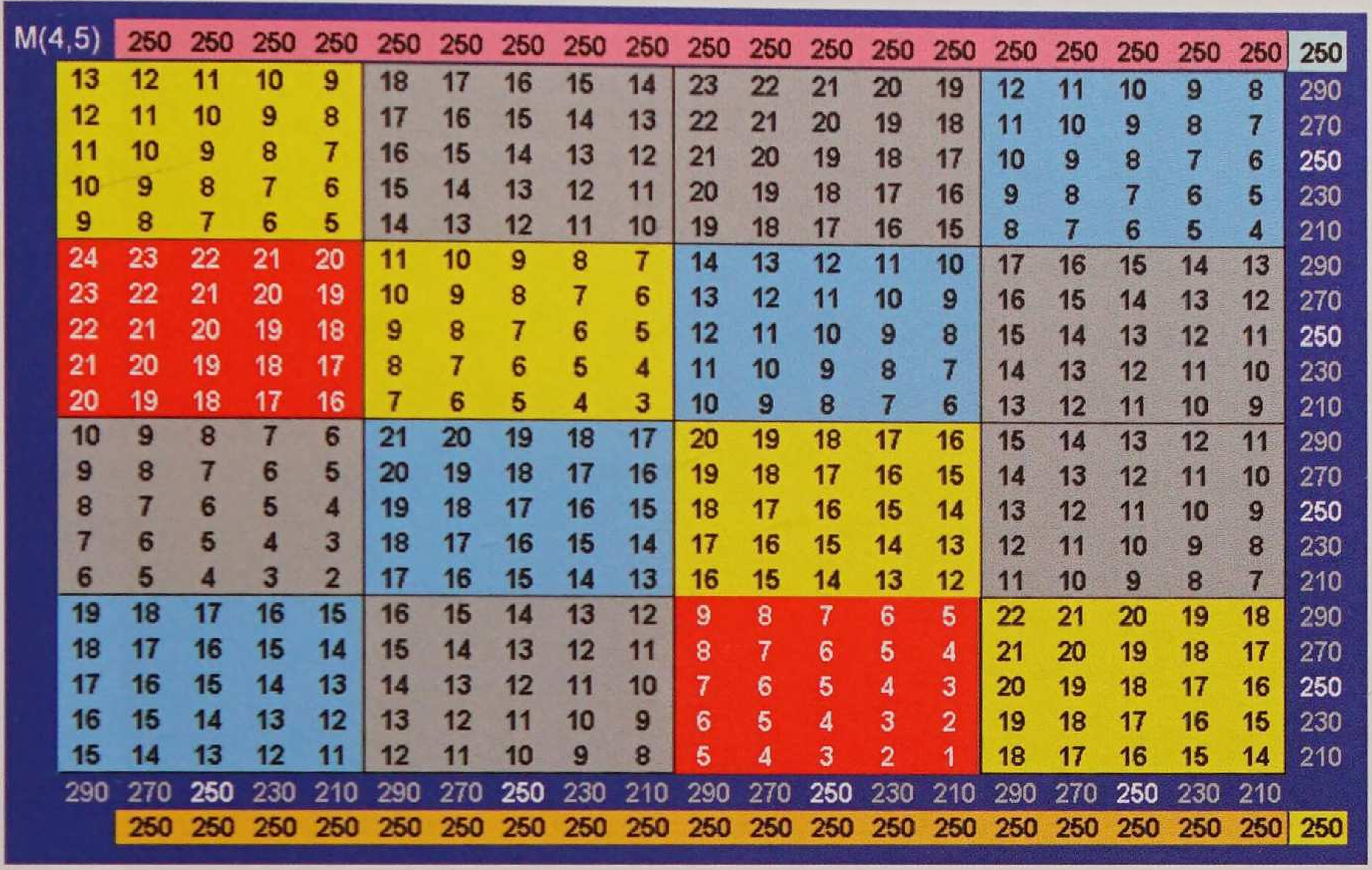
At top is the spread-pattern **X** derived from (**8.30**).

Below is the resulting size **24** square derived from (**8.32**) after centering the starting and ending numbers. The red bars indicate those columns which are among themselves pairwise row-symmetric. The size **24** square shown back in Program 3 that was totally pairwise row-symmetric was obtained after columns 7 thru 9 here were swapped intact with columns 19 thru 21. Surprisingly, this exchange preserved the square's perfect geonomicity and induced continuity of its characteristic tiling patterns which are segregated in the size **12** square!

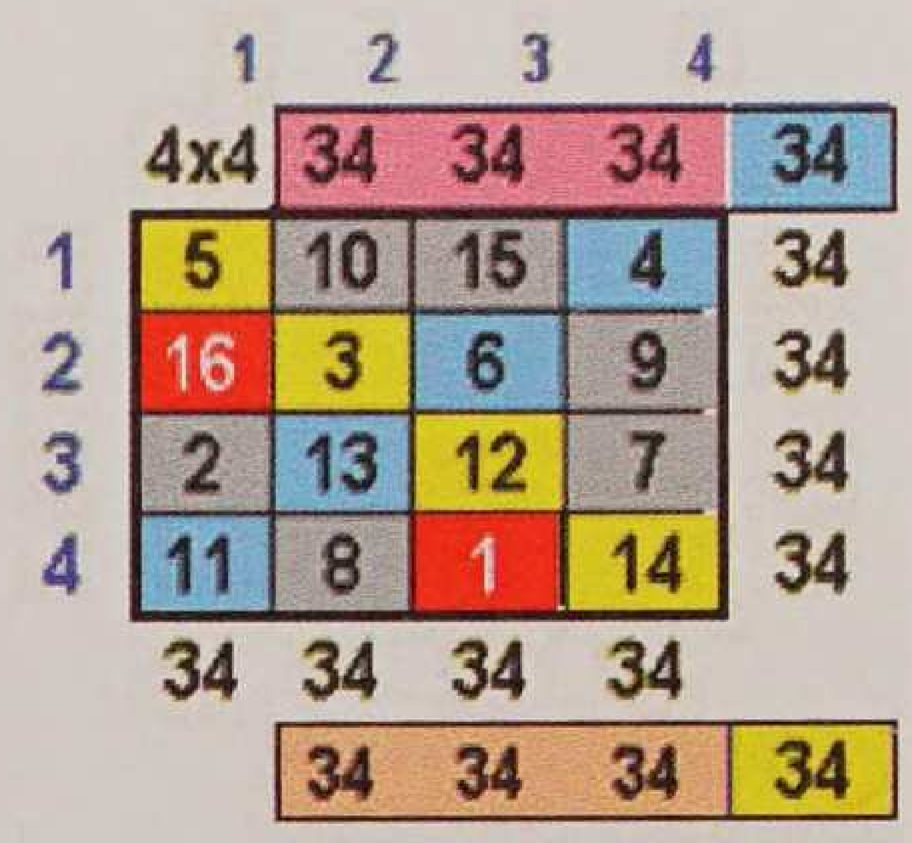
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
	W(24)	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
1	433	426	451	54	223	72	505	354	523	414	439	432	145	138	163	342	511	360	217	66	235	126	151	144	6924
2	103	192	85	564	313	546	31	264	13	204	97	186	391	480	373	276	25	258	319	552	301	492	385	474	6924
3	356	506	311	194	143	149	428	434	383	554	359	509	68	218	23	482	431	437	140	146	95	266	71	221	6924
4	105	208	87	562	297	544	33	280	15	202	81	184	393	496	375	274	9	256	321	568	303	490	369	472	6924
5	327	514	345	160	135	178	399	442	417	520	351	538	39	226	57	448	423	466	111	154	129	232	63	250	6924
6	83	209	92	557	296	566	11	281	20	197	80	206	371	497	380	269	8	278	299	569	308	485	368	494	6924
7	325	534	343	162	115	180	397	462	415	522	331	540	37	246	55	450	403	468	109	174	127	234	43	252	6924
8	139	156	121	528	349	510	67	228	49	168	133	150	427	444	409	240	61	222	355	516	337	456	421	438	6924
9	323	545	344	161	104	182	395	473	416	521	320	542	35	257	56	449	392	470	107	185	128	233	32	254	6924
10	141	172	123	526	333	508	69	244	51	166	117	148	429	460	411	238	45	220	357	532	339	454	405	436	6924
11	291	550	309	196	99	214	363	478	381	556	315	574	3	262	21	484	387	502	75	190	93	268	27	286	6924
12	116	170	131	518	335	533	44	242	59	158	119	173	404	458	419	230	47	245	332	530	347	446	407	461	6924
13	289	570	307	198	79	216	361	498	379	558	295	576	1	282	19	486	367	504	73	210	91	270	7	288	6924
14	247	48	229	420	457	402	175	120	157	60	241	42	535	336	517	132	169	114	463	408	445	348	529	330	6924
15	500	362	455	50	287	5	572	290	527	410	503	365	212	74	167	338	575	293	284	2	239	122	215	77	6924
16	249	64	231	418	441	400	177	136	159	58	225	40	537	352	519	130	153	112	465	424	447	346	513	328	6924
17	471	370	489	16	279	34	543	298	561	376	495	394	183	82	201	304	567	322	255	10	273	88	207	106	6924
18	227	65	236	413	440	422	155	137	164	53	224	62	515	353	524	125	152	134	443	425	452	341	512	350	6924
19	469	390	487	18	259	36	541	318	559	378	475	396	181	102	199	306	547	324	253	30	271	90	187	108	6924
20	283	12	265	384	493	366	211	84	193	24	277	6	571	300	553	96	205	78	499	372	481	312	565	294	6924
21	467	401	488	17	248	38	539	329	560	377	464	398	179	113	200	305	536	326	251	41	272	89	176	110	6924
22	285	28	267	382	477	364	213	100	195	22	261	4	573	316	555	94	189	76	501	388	483	310	549	292	6924
23	435	406	453	52	243	70	507	334	525	412	459	430	147	118	165	340	531	358	219	46	237	124	171	142	6924
24	260	26	275	374	479	389	188	98	203	14	263	29	548	314	563	86	191	101	476	386	491	302	551	317	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	
		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

#7: Expansion by the Matchmaker Pattern

Here is another amazing expansion property: When the size-4 square is expanded to a size-20 square with the size-5 matchmaker pattern, all the equal string summations that are characteristic of matchmaker squares are found in the expanded table.



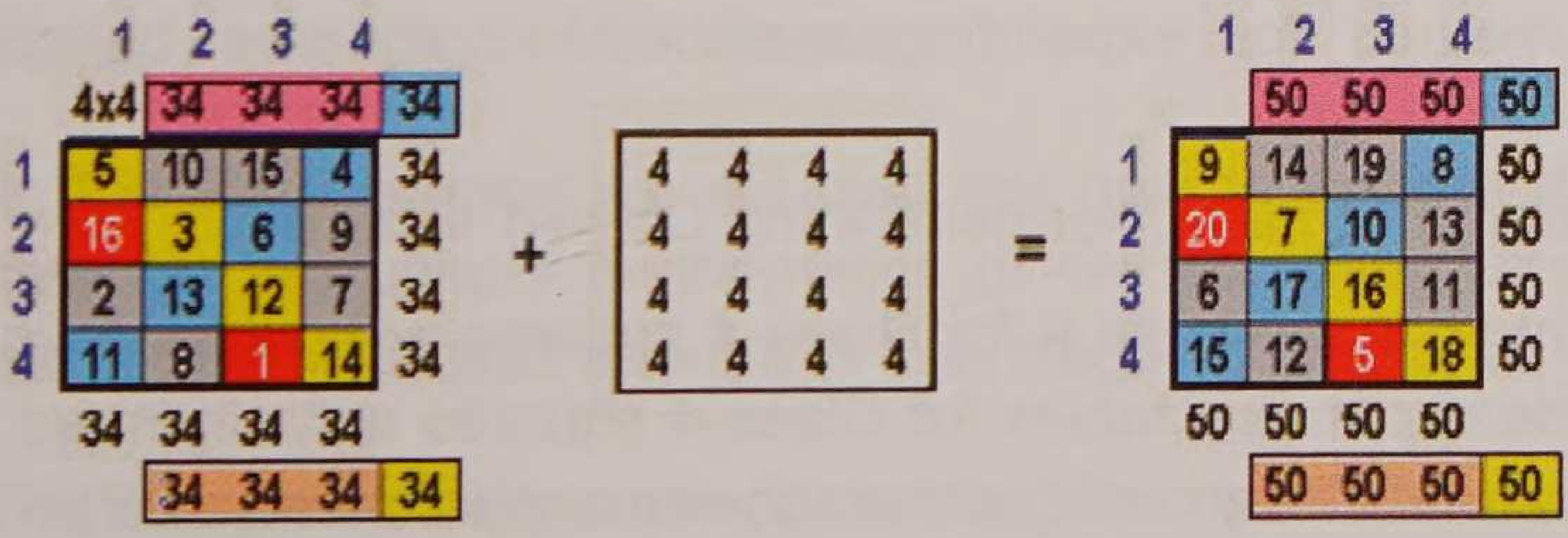
M(4,5)	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250
	13	12	11	10	9	18	17	16	15	14	23	22	21	20	19	12	11	10	9	8	290
	12	11	10	9	8	17	16	15	14	13	22	21	20	19	18	11	10	9	8	7	270
	11	10	9	8	7	16	15	14	13	12	21	20	19	18	17	10	9	8	7	6	250
	10	9	8	7	6	15	14	13	12	11	20	19	18	17	16	9	8	7	6	5	230
	9	8	7	6	5	14	13	12	11	10	19	18	17	16	15	8	7	6	5	4	210
	24	23	22	21	20	11	10	9	8	7	14	13	12	11	10	17	16	15	14	13	290
	23	22	21	20	19	10	9	8	7	6	13	12	11	10	9	16	15	14	13	12	270
	22	21	20	19	18	9	8	7	6	5	12	11	10	9	8	15	14	13	12	11	250
	21	20	19	18	17	8	7	6	5	4	11	10	9	8	7	14	13	12	11	10	230
	20	19	18	17	16	7	6	5	4	3	10	9	8	7	6	13	12	11	10	9	210
	10	9	8	7	6	21	20	19	18	17	20	19	18	17	16	15	14	13	12	11	290
	9	8	7	6	5	20	19	18	17	16	19	18	17	16	15	14	13	12	11	10	270
	8	7	6	5	4	19	18	17	16	15	18	17	16	15	14	13	12	11	10	9	250
	7	6	5	4	3	18	17	16	15	14	17	16	15	14	13	12	11	10	9	8	230
	6	5	4	3	2	17	16	15	14	13	16	15	14	13	12	11	10	9	8	7	210
	19	18	17	16	15	16	15	14	13	12	9	8	7	6	5	22	21	20	19	18	290
	18	17	16	15	14	15	14	13	12	11	8	7	6	5	4	21	20	19	18	17	270
	17	16	15	14	13	14	13	12	11	10	7	6	5	4	3	20	19	18	17	16	250
	16	15	14	13	12	13	12	11	10	9	6	5	4	3	2	19	18	17	16	15	230
	15	14	13	12	11	12	11	10	9	8	5	4	3	2	1	18	17	16	15	14	210
	290	270	250	230	210	290	270	250	230	210	290	270	250	230	210	290	270	250	230	210	
	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250



	1	2	3	4	
4x4	34	34	34		34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34		34

Observe how the expansion is done by comparing the numbers in yellow cells in the square with the lower right-hand corner in the yellow block squares of the expanded table.

A matching in this expanded square is equal to the sum of matchings in each block square. A matching here within a block-square always equals **5** times its center number. The center numbers can be put into a size-4 table as shown here just by adding **4** to each number in the regular square of size-4 and then summing geonomically.



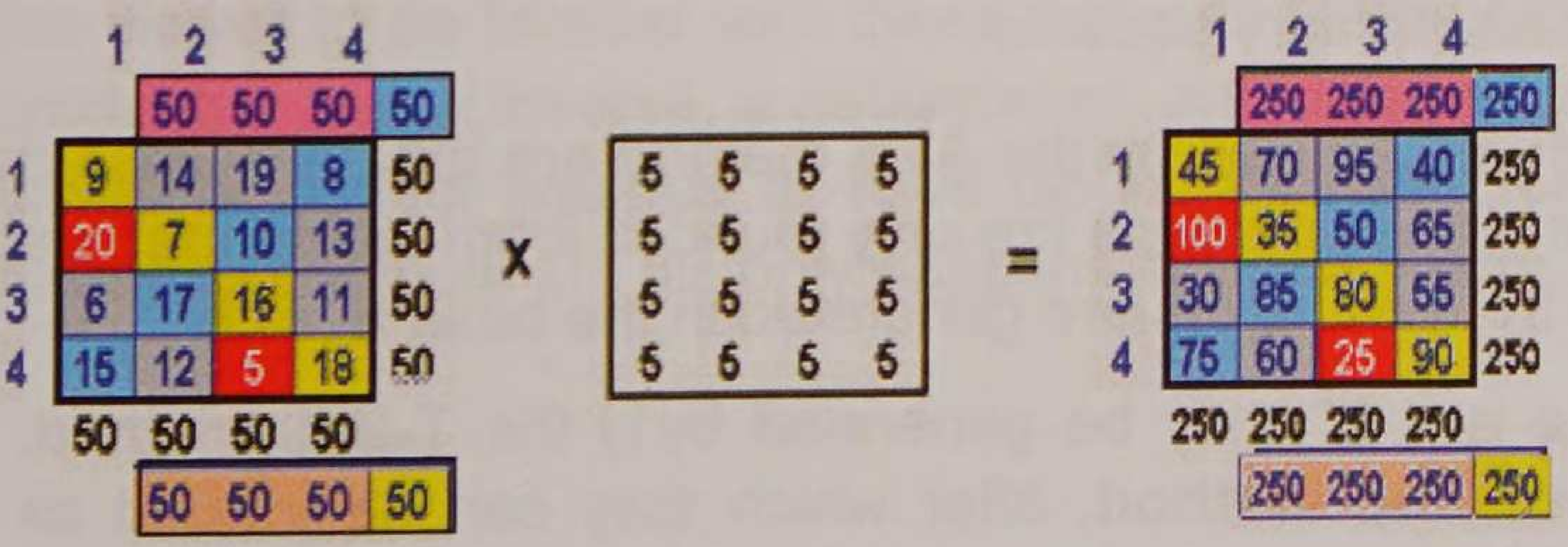
	1	2	3	4	
4x4	34	34	34		34
1	5	10	15	4	34
2	16	3	6	9	34
3	2	13	12	7	34
4	11	8	1	14	34
	34	34	34	34	
	34	34	34		34

\+

4	4	4	4
4	4	4	4
4	4	4	4
4	4	4	4

=

	1	2	3	4	
	50	50	50		50
1	9	14	19	8	50
2	20	7	10	13	50
3	6	17	16	11	50
4	15	12	5	18	50
	50	50	50	50	
	50	50	50		50



	1	2	3	4	
	50	50	50		50
1	9	14	19	8	50
2	20	7	10	13	50
3	6	17	16	11	50
4	15	12	5	18	50
	50	50	50	50	
	50	50	50		50

x

5	5	5	5
5	5	5	5
5	5	5	5
5	5	5	5

=

	1	2	3	4	
	250	250	250		250
1	45	70	95	40	250
2	100	35	50	65	250
3	30	85	80	55	250
4	75	60	25	90	250
	250	250	250	250	
	250	250	250		250

Then that table is multiplied by **5**. It is now seen that the sum of all the block-square matchings equals all the overall matchings in the expanded square! So the matching property is retained in this expansion too.

Like the other expansions for regular squares, this matchmaker expansion property holds for all perfect squares too.

Note, however, the **invariance under row and column permutation** property now holds only at the block-square level.

* * *

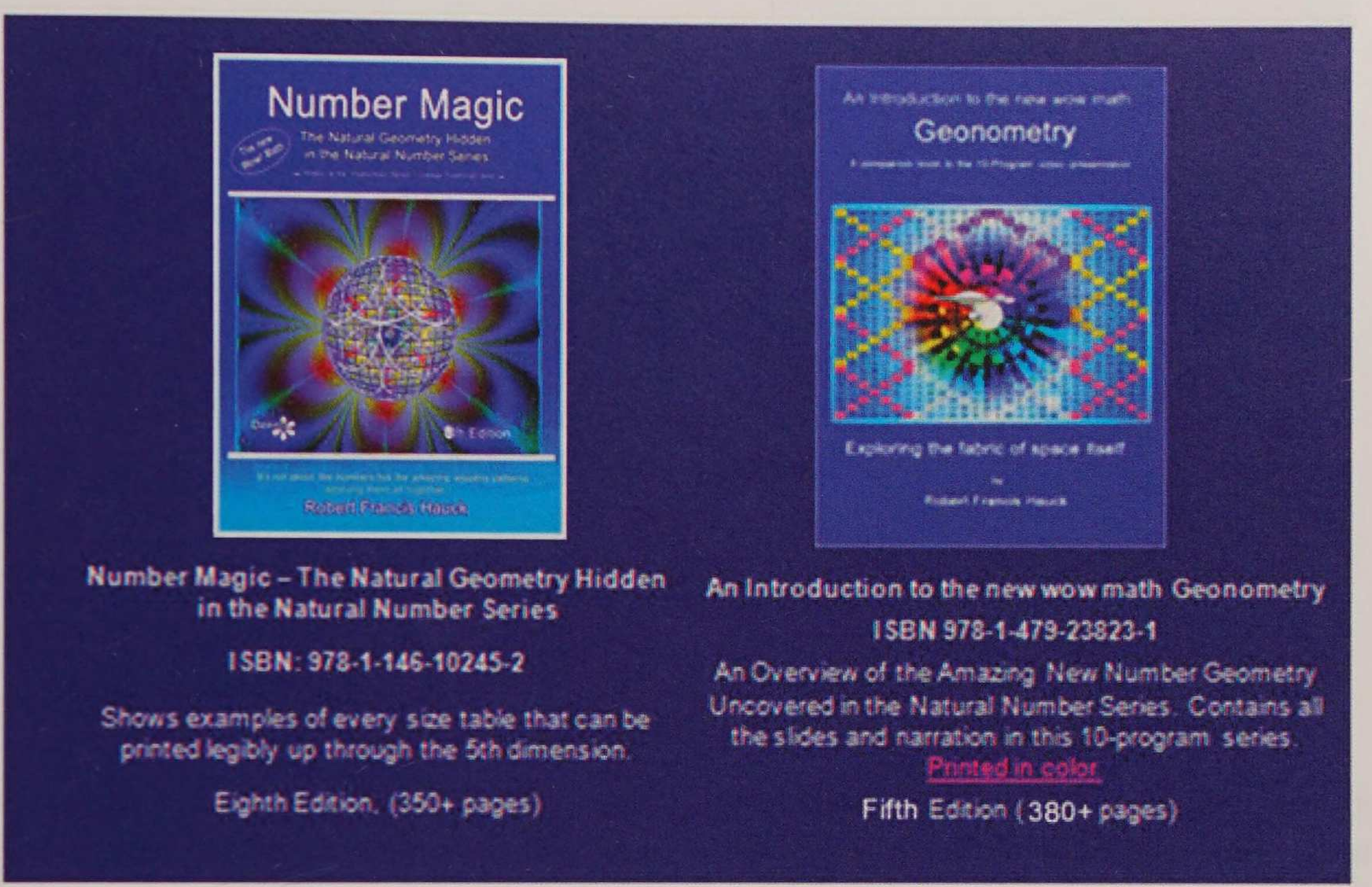
This ends the discussion on table-expansions.

Notes

1. The zero-differential property holds for all ultra-perfect squares of all classes except for those of Class-2.
2. The sub-additive property holds for every size sub-additive block only for the size-5 square.
3. The increment in the values of pivot numbers for consecutive odd-size squares follows the formula **$y = \Delta p = 2(n-1) = 4x$**; another instance of **$y = 4x$**.
4. The characteristic number of a <u>square</u> of size **n** is directly related to the **n**-th row of an index triangle.
5. The characteristic number of a <u>cube</u> of size **n** is directly related to the sum of consecutive numbers from **n** to **n^2** in an index triangle.
6. The Polya formula (**8.4**) applied to loom-tables' cells in the cubic-conversion method results in a matrix whose characteristic number is the square of the loom-tables' original characteristic number.
7. There are really a total of 8 expansion methods for squares:
    1) The **Balloon** method for expanding any class of square except Class-2.
    2) The **T-Ball** method for generating Class-6 squares.
    3) The **ATE** method for generating Class-4 and Class-5 squares.
    4) The **Bootstrap** method for generating Class-6 squares.
    5) The **TAP** method for generating Class-2, Class-3 and Class-4 squares.
    6) The **SPD** method for generating Class-4 squares.
    7) The **Matchmaker-pattern** method for expanding matchmaker squares using Class-1 squares.
    8) The **Double-Quark** method from Program 7 for generating Class-1 squares.
8. The modulus and integer functions taken in the base **n** for Class-4 squares will fail to yield loom tables that are geonomic. So the expansion tables for Class-4 squares actually need to be produced directly by the ATE method. They can then be subsequently derived as **E(a)** and **E(b)** in either base **a^2** or **b^2** from the primary squares.
9. Class-5 squares of a size **$n = ab$** that are derived by the **ATE** method are the only Class-5 squares that possess helical patterns. These patterns are only found in loom tables that are subsequently derived in the base **ab** from either square generated in the base **a^2** or **b^2**.
10. Class-6 squares which are of a size **$n = b^2$** may be generated by1) the **T-Ball** method, 2) the **TAP** method and 3) the **Bootstrap method**, after which they can be treated as Class-1 squares in generating their dual from the resulting primal square. Even so, Class-6 squares cannot be derived from matchmaker squares as all Class-1 squares can. Further, Class-6 squares derived by any of these methods will lack any continuous modularity. Only the **2D** projection of a **4D** quadracube that was initially obtained from the collapse of a **5D** quintacube will possess this modularity property.

Program 8

We have come to the end of Program 8 of the new math, **Geonometry**. Here are the two books upon which this program was based.



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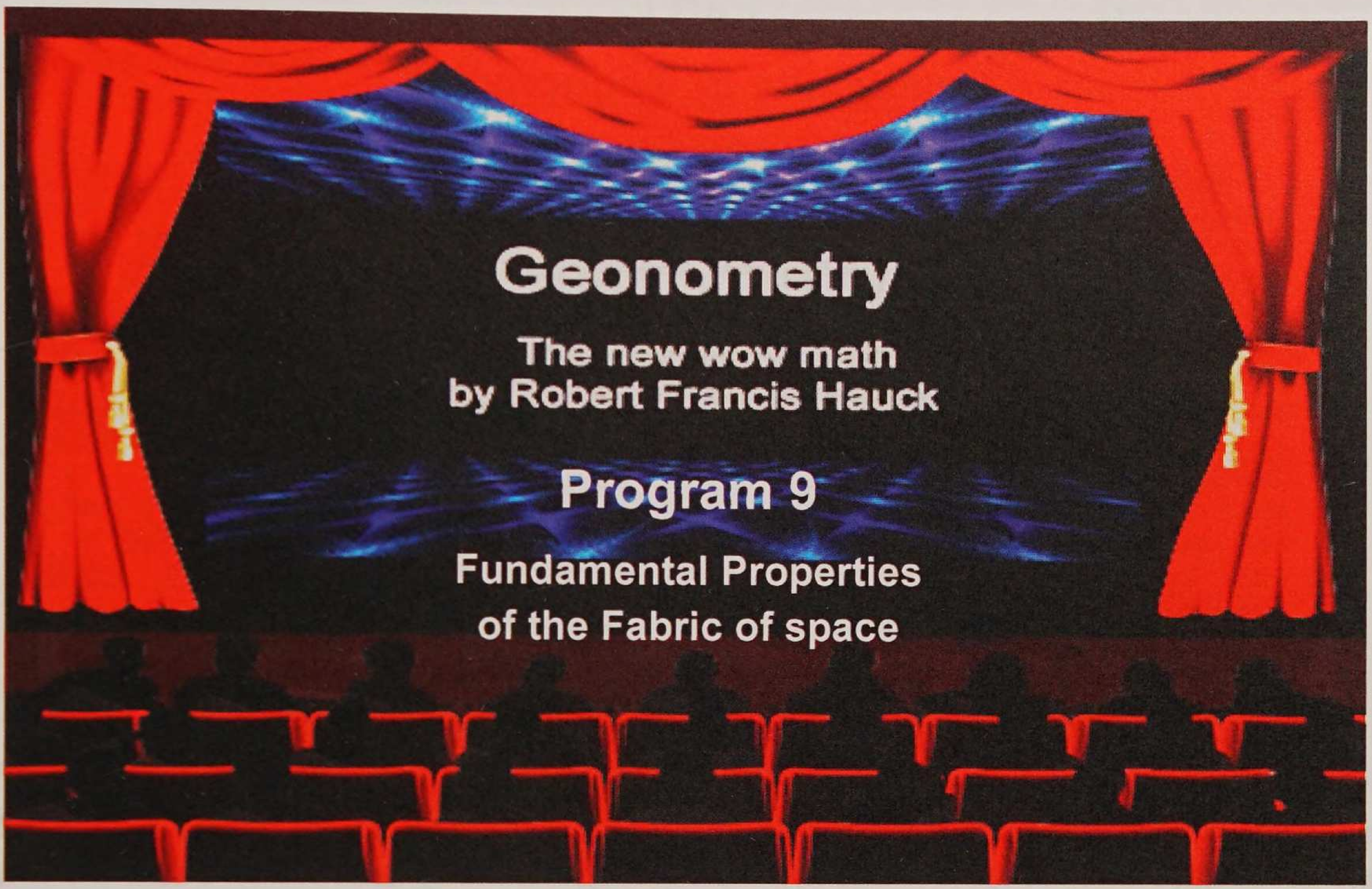
In the next program, we will see some amazing mathematics from classical math that is already embodied in Geonometry.

In the first segment we will discover how to explore the properties of any higher dimension through the unique mathematics of Geonometry. This section does not require any higher math than what was needed in the foregoing programs.

For comprehending and appreciating the math in the second segment, it will be essential for the viewer to be familiar with these topics in Matrix Algebra: a matrix product, a matrix inner-product, a matrix inverse, a determinant, and eigenvalues. All of these will be shown for loom-tables of perfect squares to have closed formulas for predicting the result without actually having to compute it! This only happens in Geonometry and rarely, if ever, in Classical Math.

Program 9



Geonometry

The new wow math
by Robert Francis Hauck

Program 9

Fundamental Properties
of the Fabric of space

There are still yet a number of amazing general properties of perfect dimensional geonomic tables. These we'll look at next. It will provide an even more fundamental picture of the natural fabric of space.

Here, we'll see some amazing mathematics from classical math that is already embodied in Geonometry.

This program is segregated into two sections.

In the first segment we will discover how to explore the properties of any higher dimension through the unique mathematics of Geonometry. Here we'll prove mathematically that space is fractal – what happens in higher dimensions is repeated in lower dimensions. This section doesn't require any higher math that what was needed in the foregoing programs.

For comprehending and appreciating the math in the second segment, it would be helpful if the viewer was familiar with these topics in Matrix Algebra: a matrix inner product and matrix multiplication, a determinant, a matrix inverse, and eigenvalues. All of these will be shown for loom tables of perfect squares.

Program 9

Part I

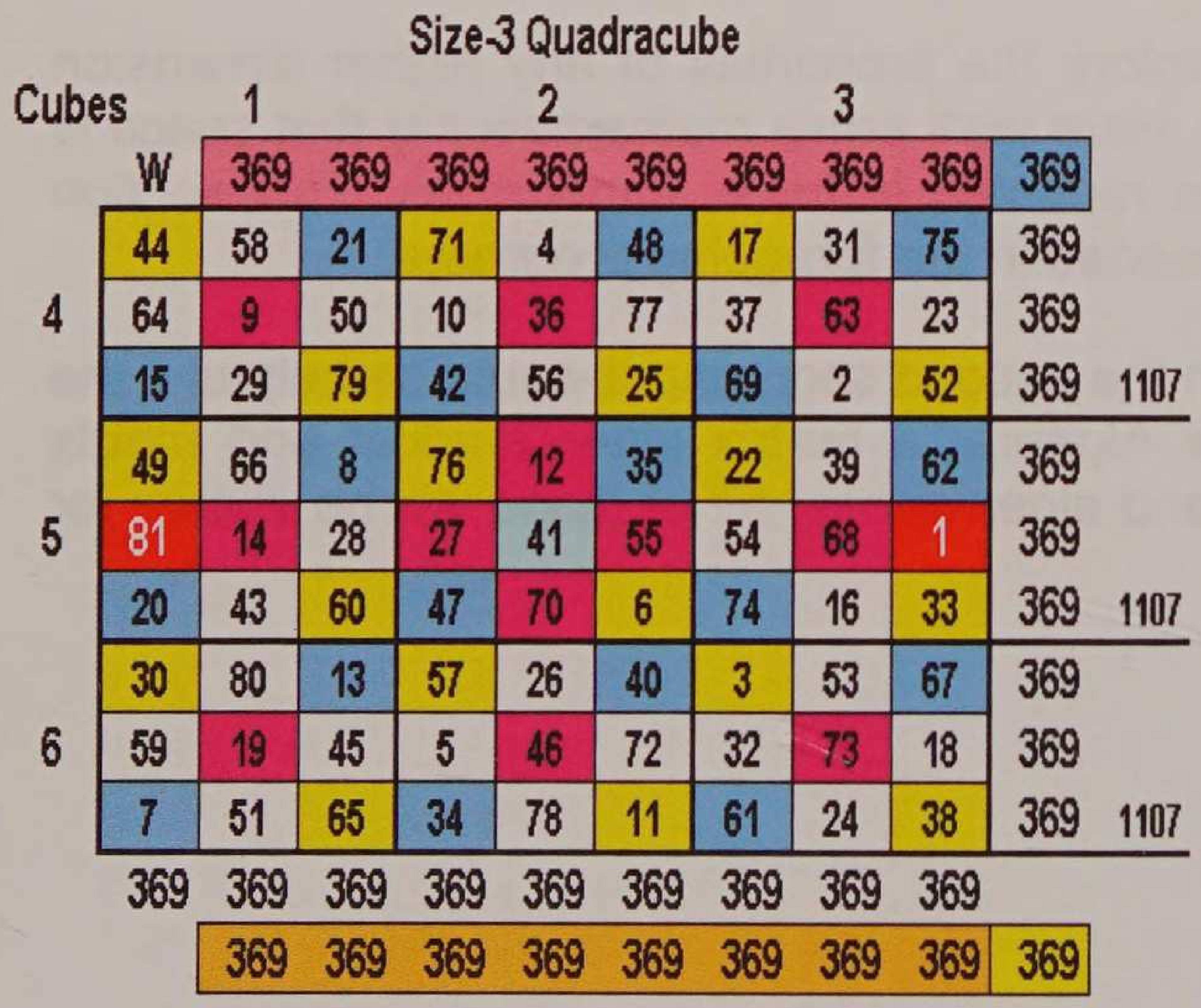
Fracticality in Geonometry

The Mandelbrot Fractal Pattern

Next, we'll look at the property of Fracticality in geonomic squares. Being "fractal" means that a pattern is composed of parts that are repetitions of itself. Here is the Mandelbrot fractal pattern. Note those yellow patterns all throughout the picture. Those are replications of the big pattern. Well, Geonometry is also fractal.

Remember those tiling patterns for Class-3 squares of sizes **15** and **21** that were shown back in Program 3? Those complementary tiling patterns of sizes **5** and **7** respectively were at the block-square level. Those patterns qualify as being **fractal**.

Fracticality in 4-Dimensions



Size-3 Quadracube

Cubes		1			2			3			
	W	369	369	369	369	369	369	369	369	369	
	44	58	21	71	4	48	17	31	75	369	
4	64	9	50	10	36	77	37	63	23	369	
	15	29	79	42	56	25	69	2	52	369	1107
	49	66	8	76	12	35	22	39	62	369	
5	81	14	28	27	41	55	54	68	1	369	
	20	43	60	47	70	6	74	16	33	369	1107
	30	80	13	57	26	40	3	53	67	369	
6	59	19	45	5	46	72	32	73	18	369	
	7	51	65	34	78	11	61	24	38	369	1107
	369	369	369	369	369	369	369	369	369		
		369	369	369	369	369	369	369	369	369	

Here is a size-3 quadracube. It contains **9** embedded size-3 squares each of which sums to the size-9's characteristic number **369**. In fact, the size-3 quadracube projects onto 2-dimensions as a square of size-9. Consequently, the quadracube is fractal because the summation patterns hold for two different sizes of table in two different dimensions.

The size-3 quadracube is continuously **3x** modular, so any configuration of **3x3** block-squares in a **3** block-square tiling pattern will sum to the quadracube's characteristic number, **1107**, such as in the letter **V**.

Program 9

Here is a size-4 quadracube. It projects onto 2-dimensions as a perfect size-16 square with continuous **4x**-modularity. As such, any **4x4** block-square tile configuration of **4** block-squares will sum to the quadracube's characteristic number.

4x4D

Cube	1				2				3				4					
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
5	250	45	84	135	58	109	148	199	186	237	20	71	122	173	212	7	2056	
	184	227	30	73	248	35	94	137	120	163	222	9	56	99	158	201	2056	
	117	162	223	12	181	226	31	76	53	98	159	204	245	34	95	140	2056	
	59	112	145	198	123	176	209	6	251	48	81	134	187	240	17	70	2056	8224
6	215	4	125	170	23	68	189	234	151	196	61	106	87	132	253	42	2056	
	153	206	51	104	217	14	115	168	89	142	243	40	25	78	179	232	2056	
	92	143	242	37	156	207	50	101	28	79	178	229	220	15	114	165	2056	
	22	65	192	235	86	129	256	43	214	1	128	171	150	193	64	107	2056	8224
7	231	52	77	154	39	116	141	218	167	244	13	90	103	180	205	26	2056	
	169	254	3	88	233	62	67	152	105	190	195	24	41	126	131	216	2056	
	108	191	194	21	172	255	2	85	44	127	130	213	236	63	66	149	2056	
	38	113	144	219	102	177	208	27	230	49	80	155	166	241	16	91	2056	8224
8	202	29	100	183	10	93	164	247	138	221	36	119	74	157	228	55	2056	
	136	211	46	121	200	19	110	185	72	147	238	57	8	83	174	249	2056	
	69	146	239	60	133	210	47	124	5	82	175	252	197	18	111	188	2056	
	11	96	161	246	75	160	225	54	203	32	97	182	139	224	33	118	2056	8224
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		
				8224				8224				8224				8224		
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056		

Here is the size-5 quadracube generated directly from the size-5 cube. All of its embedded block-squares sum to **1/5**-th of the quadracube's characteristic number. As such, any **5x5** block-square tile configuration of **5** block-squares will sum to the quadracube's characteristic number.

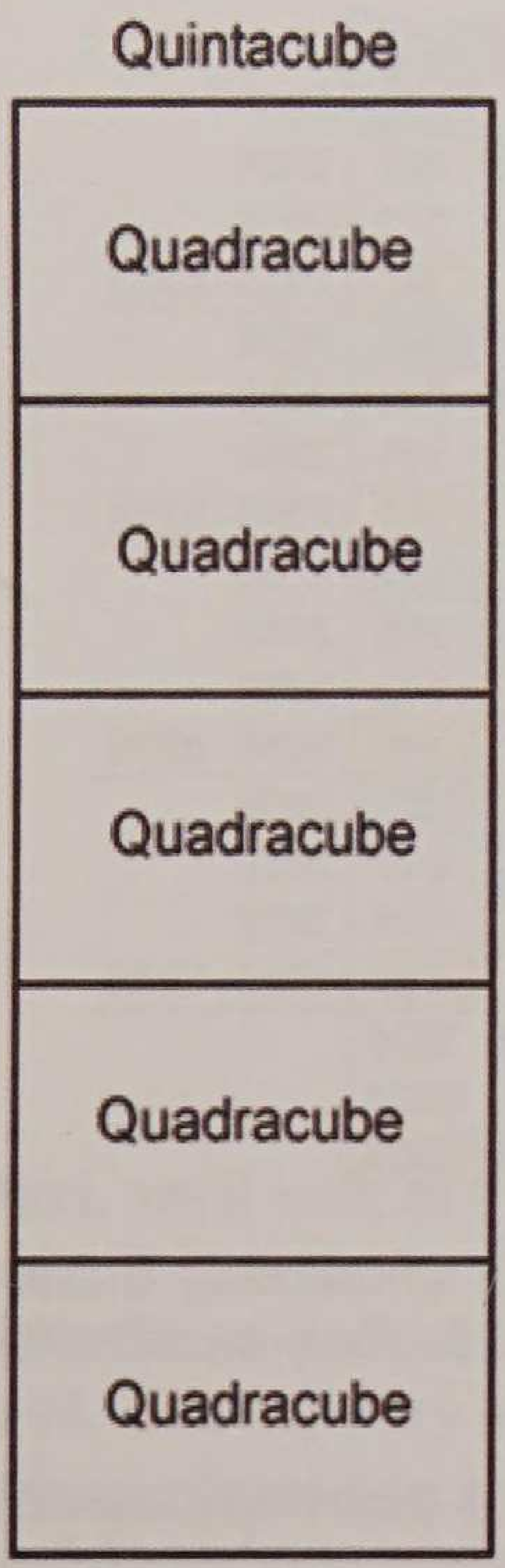
Further, it is also simultaneously equivalent to a size **25** Class-6 square. As such it is also continuously **5x** modular there where any size **5** block-square will sum equally to the characteristic number of the size-25 square.

Cubes		1					2					3					4					5					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
	5x4D	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
	1	45	183	346	489	502	170	308	471	614	2	295	433	596	114	127	420	558	96	239	252	545	58	221	364	377	7825
	2	561	79	242	260	423	61	204	367	385	548	186	329	492	510	48	311	454	617	10	173	436	579	117	135	298	7825
6	3	457	625	13	151	319	582	125	138	276	444	82	250	263	401	569	207	375	388	526	69	332	500	513	26	194	7825
	4	353	391	534	72	215	478	516	34	197	340	603	16	159	322	465	103	141	284	447	590	228	266	409	572	90	7825
	5	149	287	430	593	106	274	412	555	93	231	399	537	55	218	356	524	37	180	343	481	24	162	305	468	606	7825
	1	70	208	371	389	527	195	333	496	514	27	320	458	621	14	152	445	583	121	139	277	570	83	246	264	402	7825
	2	586	104	142	285	448	86	229	267	410	573	211	354	392	535	73	336	479	517	35	198	461	604	17	160	323	7825
7	3	482	525	38	176	344	607	25	163	301	469	107	150	288	426	594	232	275	413	551	94	357	400	538	51	219	7825
	4	253	416	559	97	240	378	541	59	222	365	503	41	184	347	490	3	166	309	472	615	128	291	434	597	115	7825
	5	174	312	455	618	6	299	437	580	118	131	424	562	80	243	256	549	62	205	368	381	49	187	330	493	506	7825
	1	95	233	271	414	552	220	358	396	539	52	345	483	521	39	177	470	608	21	164	302	595	108	146	289	427	7825
	2	611	4	167	310	473	111	129	292	435	598	236	254	417	560	98	361	379	542	60	223	486	504	42	185	348	7825
8	3	382	550	63	201	369	507	50	188	326	494	7	175	313	451	619	132	300	438	576	119	257	425	563	76	244	7825
	4	278	441	584	122	140	403	566	84	247	265	528	66	209	372	390	28	191	334	497	515	153	316	459	622	15	7825
	5	199	337	480	518	31	324	462	605	18	156	449	587	105	143	281	574	87	230	268	406	74	212	355	393	531	7825
	1	120	133	296	439	577	245	258	421	564	77	370	383	546	64	202	495	508	46	189	327	620	8	171	314	452	7825
	2	511	29	192	335	498	11	154	317	460	623	136	279	442	585	123	261	404	567	85	248	386	529	67	210	373	7825
9	3	407	575	88	226	269	532	75	213	351	394	32	200	338	476	519	157	325	463	601	19	282	450	588	101	144	7825
	4	303	466	609	22	165	428	591	109	147	290	553	91	234	272	415	53	216	359	397	540	178	341	484	522	40	7825
	5	224	362	380	543	56	349	487	505	43	181	474	612	5	168	306	599	112	130	293	431	99	237	255	418	556	7825
	1	20	158	321	464	602	145	283	446	589	102	270	408	571	89	227	395	533	71	214	352	520	33	196	339	477	7825
	2	536	54	217	360	398	36	179	342	485	523	161	304	467	610	23	286	429	592	110	148	411	554	92	235	273	7825
10	3	432	600	113	126	294	557	100	238	251	419	57	225	363	376	544	182	350	488	501	44	307	475	613	1	169	7825
	4	328	491	509	47	190	453	616	9	172	315	578	116	134	297	440	78	241	259	422	565	203	366	384	547	65	7825
	5	249	262	405	568	81	374	387	530	68	206	499	512	30	193	331	624	12	155	318	456	124	137	280	443	581	7825
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	
		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

Program 9

Fracticality in 5-Dimensions

The size-5 quintacube in 5-dimensions is way too large to show, so we'll deal with a facsimile of it here. It is depicted in detail over **5** consecutive pages in the book, **Number Magic**.



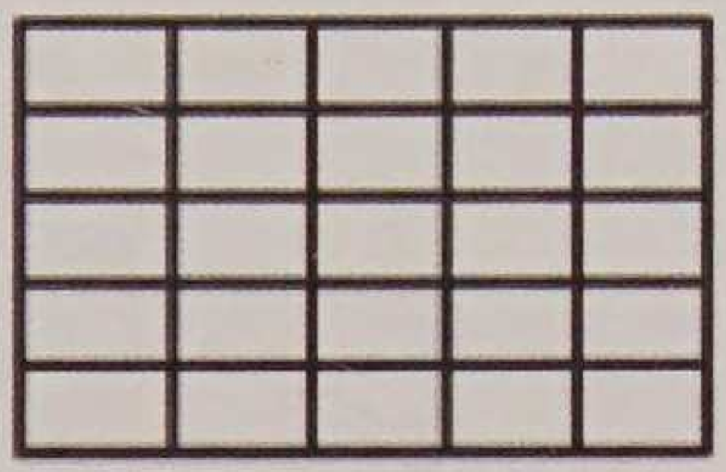


Table is too large to show the numbers.

A size-5 quintacube is composed of 125 5x5 embedded squares that are perfect in that they each have all the equal summations that perfect squares have.

This quintacube can be viewed as a size-5 cube of 5x5 block-squares.

Therefore the quintacube is fractal too but at the block-square level.

Prima-fascia proof of the fractal nature of Space

The perfect size-9 square was first discovered as the result of collapsing a 5-dimen-sional size-3 quintacube to a 4-dimensional size-3 quadra-cube.

An absolutely perfect size-9 square could not be derived itself because the size-3 square cannot be expanded to a size-9 square due to the fact that it is imperfect.

Further, constructing a size-9 loom table with the property of having the numbers **1** thru **9** strung equally in all 4 directions, is impossible.

The perfect size-3 quadracube could not be derived from the expansion of the size-3 cube either, as were the size-4 and size-5 quadracubes, because although the size-3 cube is perfect, it is not ultra-perfect and that was seen as such back in Program 4. There would always be some planar wrap diagonals that would not sum equally.

	1	2	3	4	5	6	7	8	9	
	Y(9)	45	45	45	45	45	45	45	45	45
1	9	7	8	3	1	2	6	4	5	45
2	3	1	2	6	4	5	9	7	8	45
3	6	4	5	9	7	8	3	1	2	45
4	7	8	9	1	2	3	4	5	6	45
5	1	2	3	4	5	6	7	8	9	45
6	4	5	6	7	8	9	1	2	3	45
7	8	9	7	2	3	1	5	6	4	45
8	2	3	1	5	6	4	8	9	7	45
9	5	6	4	8	9	7	2	3	1	45
	45	45	45	45	45	45	45	45	45	
		45	45	45	45	45	45	45	45	45

Here is the derived integer loom table. Observe that both main diagonals each contain only **3** of the **9** numbers with the number **5** shared between them. Yet the loom table is still geonomic. That's pretty amazing!

Consequently, if geonomic space were not fractal throughout, there would be no absolutely-perfect size-9 square. That's prima-fascia proof of the fractal nature of the fabric of space.

Program 9

Generalized Formulas for the Cubic Dual Series

Here are the generalized formulas for generating a series of dual cubes from the original one. It takes three generations to arrive back at the starting cube. The diagram shows that only the original cube will collapse to a square, if Geonometry allows it.

The crossing of directional arrows in the diagram represents the continued exchange of the roles between derived loom tables at each step when navigating from one version of the cube to the next. It takes three generations to arrive back at the starting cube.

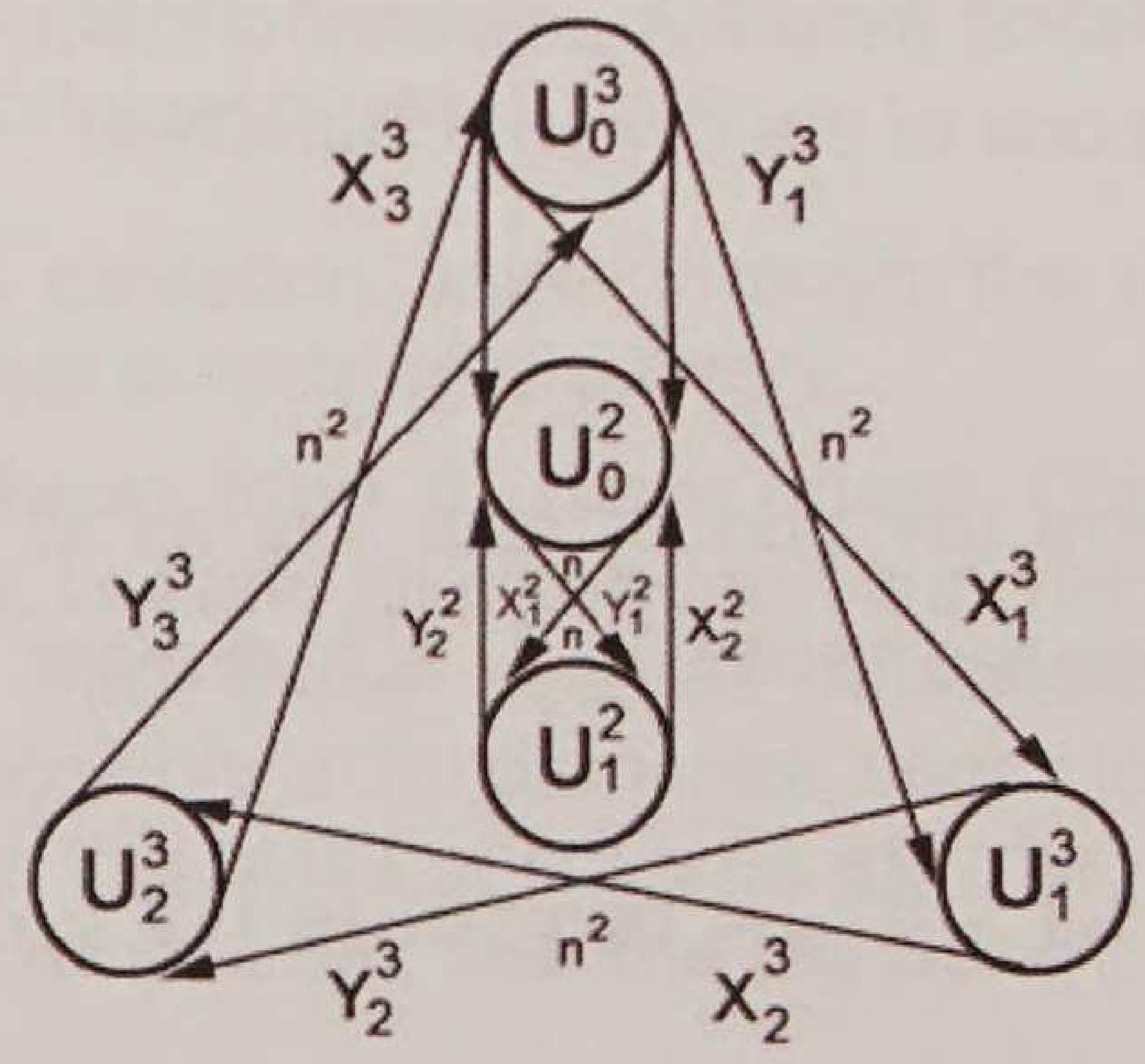
The formulas for obtaining the cubic dual $\mathbf{U_1}$ from the primal cube $\mathbf{W}$ are as follows:

(9.1) $X_1 = \text{modulus } |W - |1|\,|\,n| + 1$

(9.2) $Y_1 = \text{integer } |(W - |1|) / n| + 1$ where $|\mathbf{1}|$ is a cubic matrix of all **1**'s,

from which **W** may be reconstructed from

(9.3) $W = n\,(Y_1 - |1|) + X_1$



Then the dual $\mathbf{U_1}$ of $\mathbf{W}$ may be constructed by interchanging the roles of **X** and **Y** and using the base $\mathbf{n^2}$ instead of **n**:

(9.4) $U_1 = n^2\,(X_1 - |1|) + Y_1$

Next, $\mathbf{U_1}$ can be decomposed into its own distinct complementary loom tables as follows:

(9.5) $X_2 = \text{modulus } |U_1 - |1|\,|\,n| + 1$

(9.6) $Y_2 = \text{integer } |(U_1 - |1|) / n| + 1$

Now the **dual of the dual**, $\mathbf{U_2}$, can be obtained from the first dual cube $\mathbf{U_1}$ following the same procedure:

(9.7) $U_2 = n^2\,(X_2 - |1|) + Y_2$

Neither $\mathbf{U_1}$ nor $\mathbf{U_2}$ equals **W**, viz. neither the dual nor the dual of its dual equals the primal cubic version as is otherwise common for squares, where there, the dual of its dual does regenerate the primal square directly. The dual of the dual of the dual of **W** is yet a third absolutely perfect cube, $\mathbf{U_3}$, in the series of duals and this finally does equal **W**.

(9.8) $X_3 = \text{modulus } |U_2 - |1|\,|\,n| + 1$

(9.9) $Y_3 = \text{integer } |(U_2 - |1|) / n| + 1$

(9.10) $U_3 = n^2\,(X_3 - |1|) + Y_3 = W_3$

The navigation path is pictured here at right. The diagram shows that only the original cube will collapse to a square, if Geonometry allows it.

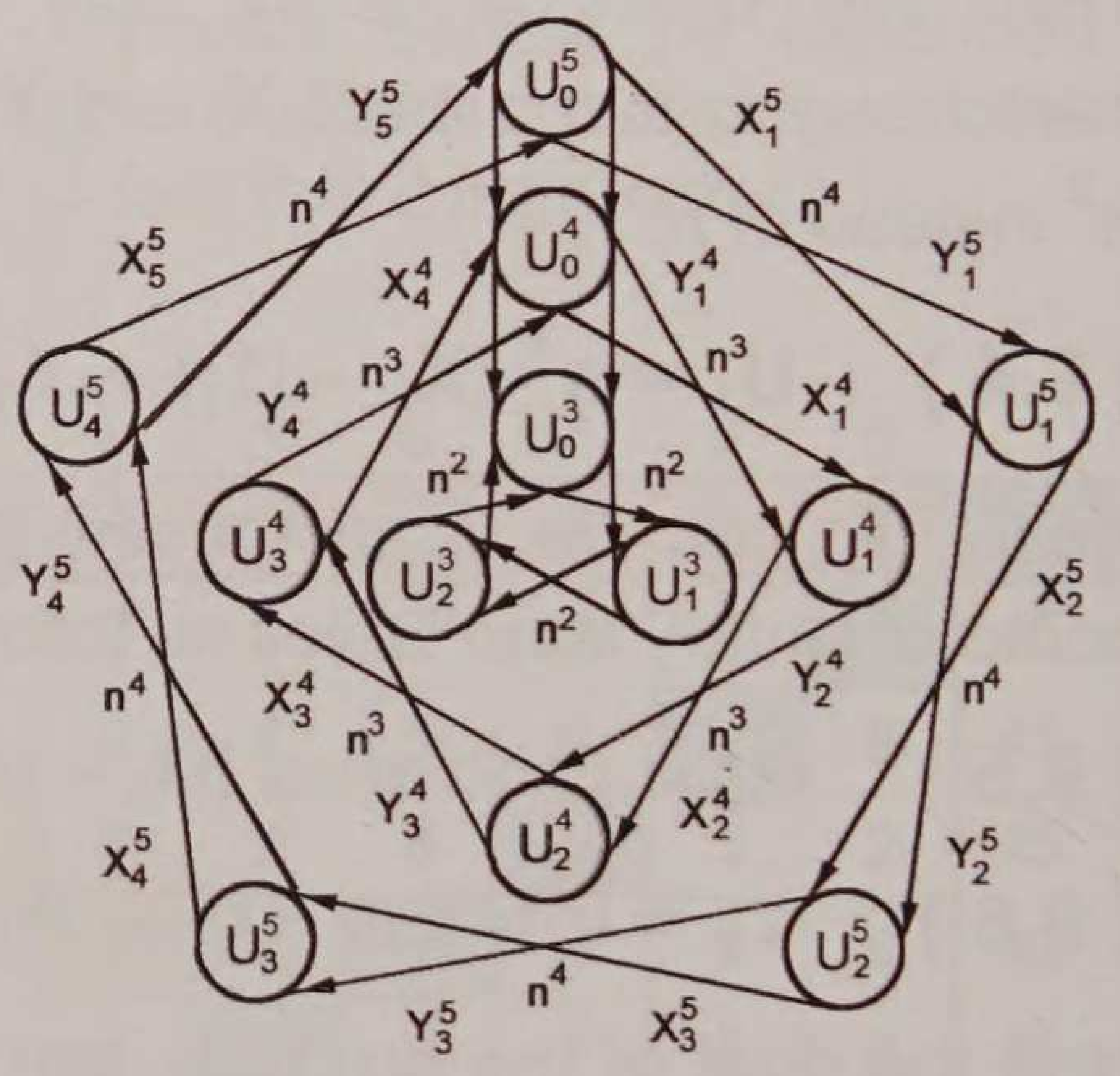
Program 9

Navigating Multidimensional Space via Class-1 Geonomic Tables

Here is the mathematical generalization of duality theory in Geonometry as it applies to the series of dual geonomic tables of dimension **k**. In dimension **k**, there is a series of successive dual tables that lead back to the original version in **k** generations. This is described by generalized formulas (**9.11**) through (**9.14**). However only the **k-th** dual will collapse down to the next lower dimension of **k -1** through one or more of its depth-sum tables. And that **k**-th dual table is identical to the original geonomic table of dimension **k** back where it all started.

The diagrams at right show this navigational network among dimensions **5** down through **2**.

The crossovers of the links represent the interchange of the roles between the modulus and integer loom tables in each transition to the next successive dual. The downward vertical transitions between adjacent dimensional levels represent the collapse of the depth-sum table along one of the dimensional axes.



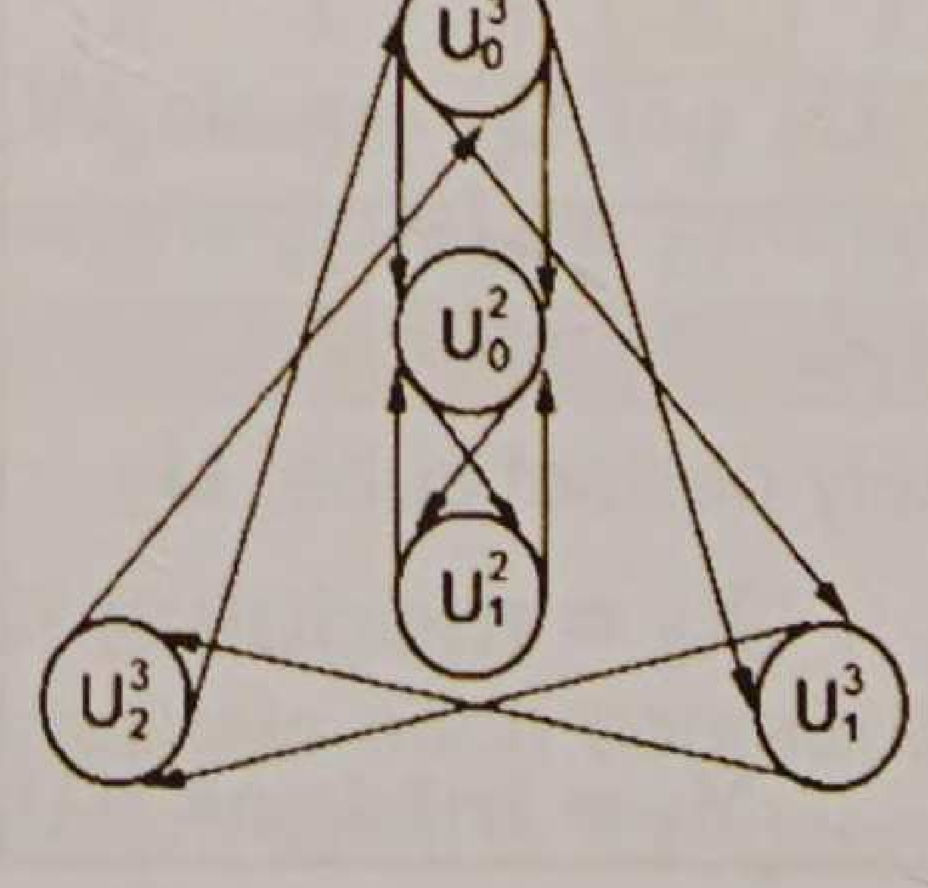
Defining the primal table $\mathbf{W_K}$ as $\mathbf{U^k{}_0}$, and letting <u>**h**</u> run through the series **1, 2 … k,** we get:

$$(9.11)\quad X^k{}_h = \text{modulus}\,[U^k{}_{h-1} - |1|\;|\;n] + |1|$$

$$(9.12)\quad Y^k{}_h = \text{integer}\,[(U^k{}_{h-1} - |1|)\,/\,n] + |1|$$

$$(9.13)\quad U^k{}_h = n^{k-1}\,(X^k{}_h - |1|) + Y^k{}_h$$

$$(9.14)\quad U^k{}_k = U^k{}_0,\quad \text{in dimension } \mathbf{k}.$$

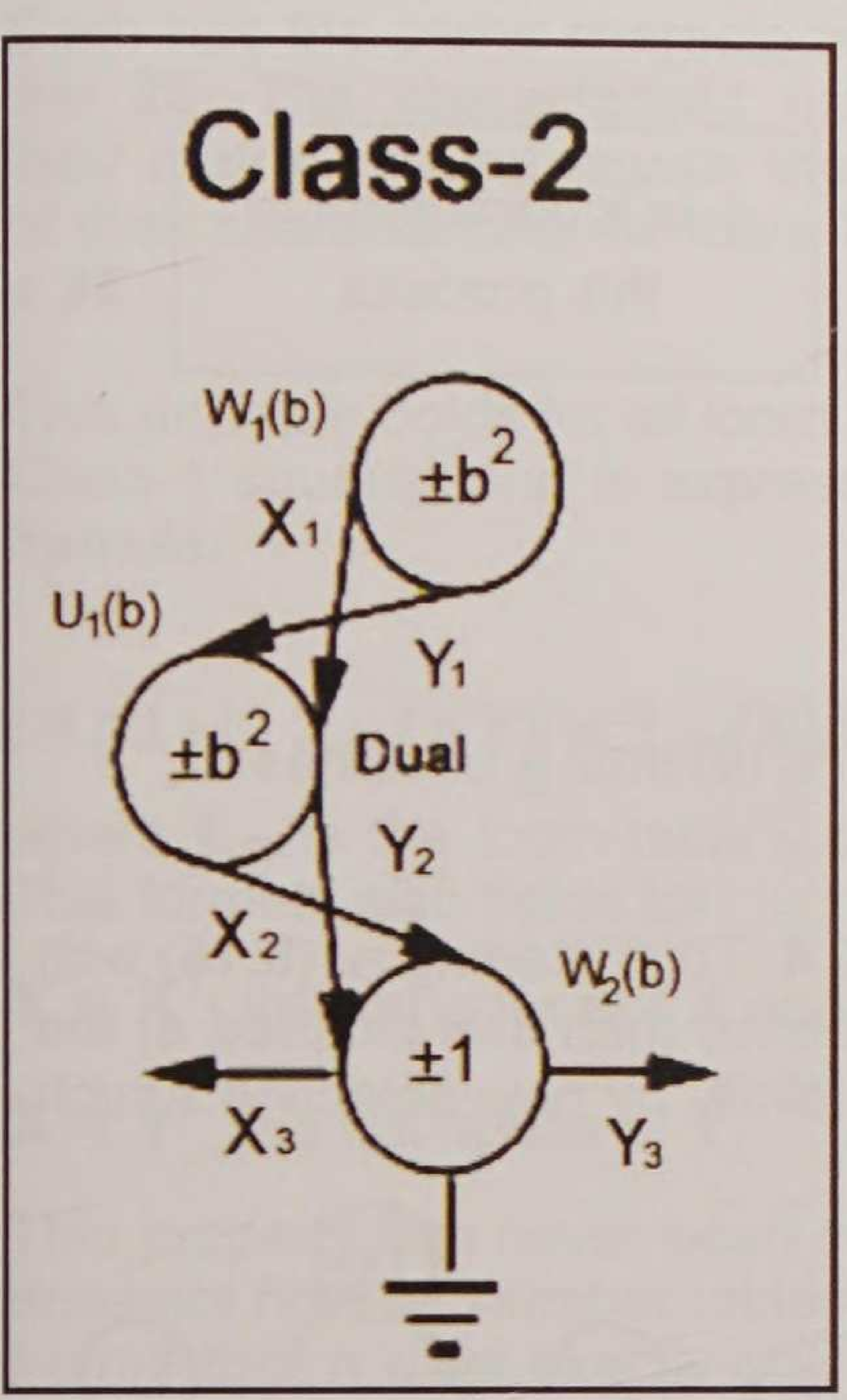


All dual tables derived by this serial dualization process are absolutely ultra-perfect, if their corresponding size square is also, in that they still retain all the equal-summing dual tiling patterns of $\mathbf{W_K}$ within each of their embedded block-squares. However, only $\mathbf{U^k{}_0}$ will collapse to the corresponding geometry $\mathbf{U^{k-1}{}_0}$ in the next lower dimension **k-1** by one or more of its depth-sum tables.

Program 9

The Crash and Burn of the dual for Class-2 squares

Here is what happens for a Class-2 square of size **$n = 2b$** when the generation formula for Class-1 squares is used in deriving the dual of the dual in an attempt to return to the primal square; it ends in a primal square where the deviation between the two unequal rows diminishes from **$\pm b^2$** to **± 1.** At this point the loom tables derived from the dual become non-geonometric in that their columns do not all sum equally. That is, the generation process becomes grounded; it cannot return to the version of the original primal square.



It was this last version that was used for the primal table in this program because it was that table which had the minimum possible deviation from the characteristic number. That's why the Class-2 squares have no dual: the grounded version was used as the original primal version.

This minimization of the deviation value through the dualization process was another surprising discovery.

Various methods have been tried to make these Class-2 squares function normally, but that all proved to be in vain. So it has been concluded that these squares represent an unavoidable wrinkle or possibly a fundamental seam in the fabric of space. Someday, this class of squares may be used to prove that something cannot exist, like "the Big Bang".

k-1 Different navigational paths in the series of duals in prime-number dimension k

Generalizing (**9.11**) through (**9.14**) even further we get for the next dual **#h** in the series of duals

$$X_h \equiv \text{modulus}\,[\,U_h - |1| \mid n^t\,] + |1| \tag{9.15}$$

$$Y_h \equiv \text{integer}\,[\,(U_h - |1|)\,/\,n^t\,] + |1| \tag{9.16}$$

$$s = \text{modulus}\,[\,h+t \mid k\,] \tag{9.17}$$

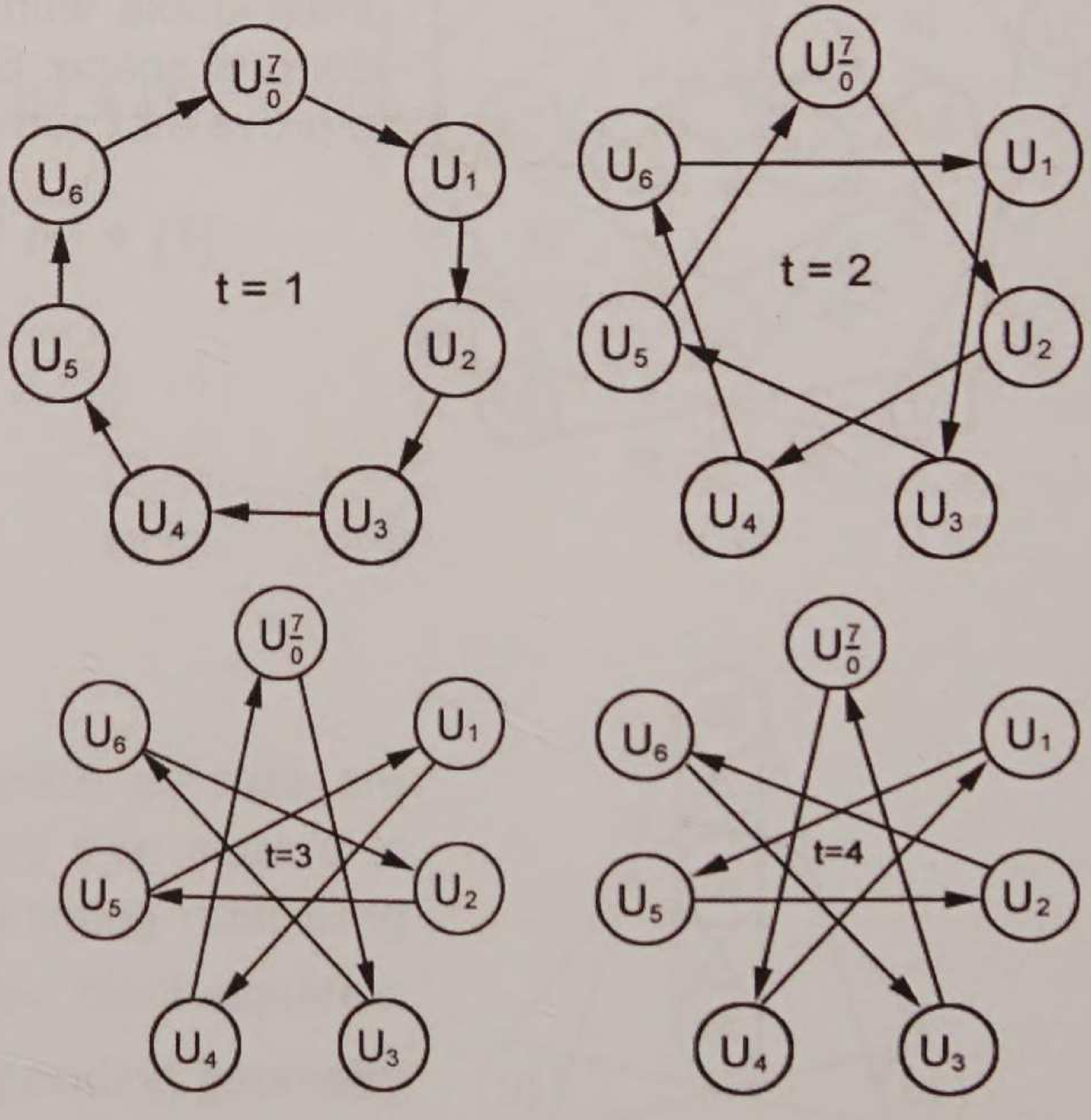
$$U_s = n^{k-t}\,(X_h - |1|) + Y_h \tag{9.18}$$

$$h \longleftarrow s \tag{9.19}$$

t is a parameter in this process.

Starting at (**9.15**) with **h** = **0**, repeat the process from (**9.15**) thru (**9.19**) until **h** becomes **0** again in (**9.19**).

The diagrams depict the series of duals for **k=7** and **t = 1, 2, 3** & **4**. Thus starting at (**9.15**) with **h** = **0**, one can navigate k-dimensional space in a different sequence than that pictured at the upper left diagram for **t=1**, which was described in the previous slide, by now selecting **t** such that **k > t > 1**.



When **k** is an odd prime-number, regardless of the value of **t** in the range of **k > t ≥ 1**, it will always still take **k** steps to get back to $\mathbf{U_0}$.

Note that the path for **t = 4** is just the reverse of the sequence when **t = 3**. When **t = 6**, the path is the reverse of **t=1**. The same holds for when **t = 5**; it is the reverse of **t = 2**. This is always the case between the paths for **t** and **k–t, t < k**.

Keep in mind that this process only works for Class-1 and Class-6 cubes and hypercubes of odd prime-number dimensions.

This formulation implies the existence of quantum harmonics throughout prime-number dimensions of space. It's just amazing how both odd-prime-number sizes and odd-prime-number dimensions are so definitive of the properties of multi-dimensional space.

* * *

We are next entering that segment of the program where we'll be dealing with some higher math concepts from classical mathematics.

Program 9

Part II

Complementary Loom-tables' Matrix Product

Here is the matrix product of complementary loom tables **X** and **Y** of size-7. Each has the same characteristic number **28**. The characteristic number of their matrix product equals the product of their characteristic numbers **784 = 28 x 28.**

X

7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

Y

1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

This principle holds for all loom tables of Class-1 squares and is expressed in general by the formula:

$$L_n(XY) = L_n(X)\,L_n(Y) = L_n^2 \qquad (9.20)$$

where L_n is the loom-table's characteristic number. This formula also holds for

$Y = X^T$, the transpose of **X** and

$X = Y^T$, the transpose of **Y**.

Matrix product

XY		784	784	784	784	784	784	784
	98	119	126	119	98	112	112	784
	126	119	98	112	112	98	119	784
	98	112	112	98	119	126	119	784
	112	98	119	126	119	98	112	784
	119	126	119	98	112	112	98	784
	119	98	112	112	98	119	126	784
	112	112	98	119	126	119	98	784
	784	784	784	784	784	784	784	
		784	784	784	784	784	784	784

This property has never been observed in Matrix Theory before. That is because this property does not hold for number tables that are not Class-1 loom-tables, i.e. the sequence of numbers from **1** thru **n** exist exactly once in every row, column and diagonal in both directions. So this property of matrix products of Class-1 complementary loom-tables is only found in tables that are geonomically ultra-perfect.

Here's another property that follows from George Polya's formula (**8.3**) back in Program 8:

(**9.20**) becomes

$$L_n(XY) = L_n^2 = \Big[\sum_{j=1}^{n} j\Big]^2 = \sum_{j=1}^{n} j^3 \qquad (9.21)$$

That's new to Matrix Algebra too!

Here's another surprise: while both tile patterns A and B sum continuously to the loom tables' characteristic number on both **X** and **Y** individually, only the **transpose** of the tile patterns now do so on the matrix product **XY** for all Class-1 tables of size **n > 5**.

A^T

784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784

B^T

784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784
784	784	784	784	784	784	784

Program 9

For Class-4, 5 & 6 loom tables of size **n = ab**:

$$X(b^2)\,Y(b^2) \qquad\qquad X(a^2)\,Y(a^2)$$

$$= E(b)\,E(a) \qquad\qquad = E(a)\,E(b)$$

In fact

Identical !!

$$= \left| L_n(E(b))/a \times L_n\,(E(a))/b \right|$$

$$= \left| C_b \times C_a \right|$$

Note that the last expression is a table of size-n of constant values equal to the product of the tiling square's characteristic number $\mathbf{C_b}$ of **W(b)** and $\mathbf{C_a}$ of **W(a)**.

For Class-6 square's loom tables used in the **T-Ball** method, set **a = b** and substitute **D(b)** for **E(a)** in the formulation here. Then the constant in the matrix product is $(\mathbf{C_b})^2$.

This property applies to matrix products of compatible tiled expansion tables used in the **ATE** and **T-Ball** methods and no others.

Both matrix products **X(n)Y(n)** and **Y(n)X(n)** in the base **n = ab** however will fail to yield a matrix of all constants for loom tables for:

1. Class-4 squares because their loom tables to the base **n** are not pangenic and are therefore incompatible.
2. Class-5 squares just because they just don't.

For Class-6 squares generated by the **T-Ball** method, contrary to that for Classes 4 & 5 squares generated by loom tables with the **ATE** method, both **X(n)** and **Y(n)** can indeed be derived in the base **n**.

The smallest Class-4 square in which this can be demonstrated is of size-20 and the smallest Class-5 square is of size-35. In this book are shown the size-20 square to the bases **16** and **25** and three versions of the size-35 square to the bases **25**, **35** and **49**. From these you can derive the various loom tables in Excel™ and use Excel multiplication to compute their matrix products in their respective bases and verify what was just presented as formulas here.

E(4) x E(5)

2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210
2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210	2210

The size-25 Class-6 square, its dual and complementary loom tables were shown in their entirety in Programs 3 & 8.

Example

$$\begin{aligned} &2210 \\ &= (170 \times 260)/20 \\ &= (170)/5 \times (260)/4 \\ &= 34 \times 65 \\ &= C_4 \times C_5 \end{aligned}$$

Program 9

Determinants of loom tables

Here is another interesting property: Every loom table – both the modulus loom and the integer loom simultaneously – derived from any geonomic square which has a non-zero determinant, has a non-zero determinant too. That might be anticipated; however, what is so amazing is its **absolute value** is always the same among all different loom tables of all perfect squares of that size! That is, for that size of loom table, it is the **Loom-table's Characteristic Determinant** (**LCD**).

Here are closed formulas for the non-zero determinants of loom tables. Closed formulas for determinants are extremely rare in Matrix Algebra because a determinant is primarily a long drawn-out systematic matrix calculation that yields only a single number. Yet formula (**9.22**) here gives the formula for the determinant solely in terms of the loom table's characteristic number.

The **LCD** for size **n** loom tables were all found to be expressible as

(9.22) $$\text{Det } |X_n| = \pm L_n n^{n-2} = \text{Det } |Y_n|$$

where L_n is the loom table's characteristic number. Note that L_n here is just the sum of numbers **1** through **n**.

So (**9.22**) becomes

(9.23) $$\text{Det } |X_n| = \pm 1/2\ (n+1)\ n^{n-1} = -\text{ Det } |Y_n|$$

The determinant for X_n is always the negative of the determinant for Y_n. That is yet again another amazing property!

Five examples whose determinants were obtained from computations made on internet *Quick Math* are given below as verification of formula (**9.23**):

Examples:

- The size **3** loom tables' determinants are **±18** which equals **$\pm 1/2\ (4)\ 3^2$**.
- The size **5** loom tables' determinants are **±1875** which equals **$\pm 1/2\ (6)\ 5^4$**.
- The size **7** loom tables' determinants are **±470,596** which equals **$\pm 1/2\ (8)\ 7^6$**.
- The size **11** loom tables' determinants are **±155,624,547,606** which equals **$\pm 1/2\ (12)\ 11^9$**.

Observe how rapidly these determinants increase. Consequently these formulas are essential because the value of the determinant quickly outpaces the capacity for computer accuracy.

Program 9

Determinants for Geonomic Squares

Here is a summary of determinants for geonomic squares up through size-35. Notice how rapidly these values expand!

Size	Determinant
3	-360
4	0
5	-4680000
6	0
7	-34602923880
8	0
9	0
10	0
11	4103774968930397760660
12	0
13	4613321938573815398620416 x 10^4
14	0
15	0
16	0
17	-343347181240267750640339540289566234937600
18	0
19	-1960777328288893166110031542996078227727835490624
20	0
21	0
22	0
23	218409064520903617186499340425741355982940132563581674197645060
24	0
25	3937983554928570896236323828343302021489868164062500000000 x 10^8
26	0
27	0
28	0
29	-3300425835451301007376277661222514481517264760581329043482170674435540767116884280576
30	0
31	-1385831137266546108389738098552594583748517804154657576998604543332646496673871863773855900
32	0
33	0
34	0
35	607264854990001244217328738660095747790077104674354315699626139615073234935683471679687500 x 10^{19}
36	0

Perfect & non-singular	Perfect & singular	Near-perfect & singular	Imperfect & non-singular

Only squares which have loom tables with a non-zero determinant have a non-zero determinant themselves and we'll attempt to formulate those for squares next.

Program 9

Determinants of Class-1 Squares

Determinant of W(5)

4680000

Remainder	Factors
7488	5^4
117	2^6
13	3^2
1	$13 = L_5 - 2$

Determinant of W(7)

34602923880

Remainder	Factors	
294120	7^6	
49020	6	
9804	5	
2451	4	
817	3	
19	43	$= (3 L_7 - 2)/2$
1	19	$= (2L_7 + 1)/3$

Determinant of W(11)

4103774968930397766066O

Remainder	Factors	
13075891740	11^{12}	
2615178348	5	
871726116	3	
435863058	3	
217931529	2	
72643843	2	
48787	**1489**	**Dead-end**

Both factors are primes
without Geonomic rhymes

Here are the determinants and their factors for the three smallest consecutive Class-1 squares: sizes **5**, **7** and **11**.

The factors are shown to demonstrate that there is no systematic formulation of the determinants of primal squares. Observe the two large primes at the bottom of Size-11's factors which preclude making any further progress. These large prime numbers have no interpretation in terms of the size-11 square's properties and are therefore useless.

However, note that the loom tables' common determinant can be factored out and hence always wholly divides the primal square's determinant:

$$\text{Det } |X(5)| = 5^4 \times 3$$
$$\text{Det } |X(7)| = 7^6 \times 4$$
$$\text{Det } |X(11)| = 11^{10} \times 6$$

Loom tables' Eigenvalues

Here are two fundamental theorems from Matrix Algebra regarding *eigenvalues*:

I. The determinant of a matrix is equal to the product of its eigenvalues,

$$\textbf{Det } |X_n| = \prod \lambda j = \lambda 1\, \lambda 2\, \lambda 3 \ldots \lambda n.$$

II. The sum of a matrix's eigenvalues is equal to the sum of the numbers along its major diagonal, called the **trace** of the matrix.

$$\textbf{tr } |X_n| = \sum \lambda j = \lambda 1 + \lambda 2 + \lambda 3 + \ldots + \lambda n$$

We saw that the formula (**9.23**) gave the determinant for a Class-1 loom table of prime-number size **n** as

$$(9.23) \qquad \textbf{Det } |X_n| = \tfrac{1}{2}(n+1)\, n^{n-1} = L_n \times n_1 \times n_2 \times \ldots \times n_{n-2}$$

And since every prime-number size loom table contains the numbers from **1** thru **n** in every diagonal, the ***trace*** of the loom table X_n of size **n** is simply the sum of numbers **1** thru **n**:

$$(9.24) \qquad \textbf{tr } |X_n| = \sum j = 1 + 2 + 3 + \ldots + n\text{-}1 + n = L_n$$

Website **www.bluebit.gr/matrix-calculator/calculate.aspx** gives these imaginary numbers for the size-7 loom table X_7. Just bear with me a moment; we're not getting into imaginary numbers for all but a moment.

The trace of loom table **tr** $|X_7| = 28 = L_7$. It is obvious that when these imaginary numbers are summed at right that all but the number **28** cancels out in both the real and imaginary components.

```
Eigenvalues

(28.000, 0.000i)
( 5.061, 0.000i)
( 2.531, 4.383i)
( 2.531,-4.383i)
(-5.061, 0.000i)
(-2.531, 4.383i)
(-2.531,-4.383i)
```

And we have already seen that **Det**$|X_7|$ = **470596**. Excel gives **470625.41** for the product. Clearly the value determined from formula (**9.23**) given by Geonometry is the more accurate one. That's the benefit of having generalized closed formulas.

Now, let me make this perfectly clear: Decimal fractions, real and imaginary numbers have no place in Geonometry. Geonometry is about whole integral numbers and their fundamental equal-summing patterns. The same point is made for matrix inverses of loom tables: the numerators can be expressed in whole numbers and the denominator can be expressed as an inverse of a product of two whole numbers.

Here's where this new math butts-up against classical math. Since we already have a closed formula for the determinant and the loom table's characteristic number is the dominant eigenvalue, that's all that is needed in engineering applications. The rest lies at the noise level for all the other eigenvalues and their associated eigenvectors. The only vital point in applied Geonometry is that the dominant eigenvalue for Class-1 loom tables, dominant by an order of magnitude, is λ_1 where:

$$(9.25) \qquad \lambda_1(n) = L_n$$

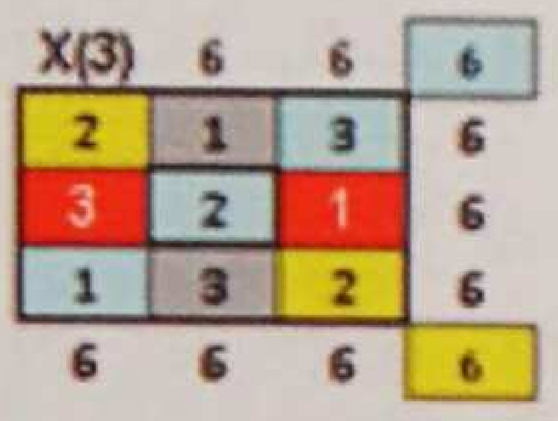
The rest of the eigenvalues and their associated eigenvectors are superfluous.

Program 9

Loom table's Matrix Inverse

A third useful theorem is that a matrix has an inverse only if all of its eigenvalues are all non-zero.

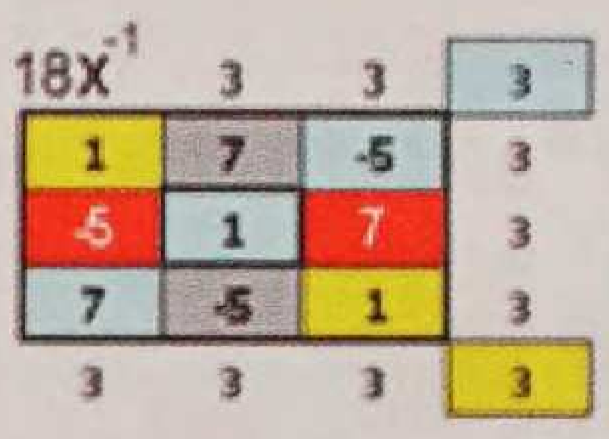
Here are the modulus loom tables of sizes **3** thru **7** and their matrix inverses. These are sufficient to generalize formulas for all Class-1 squares.



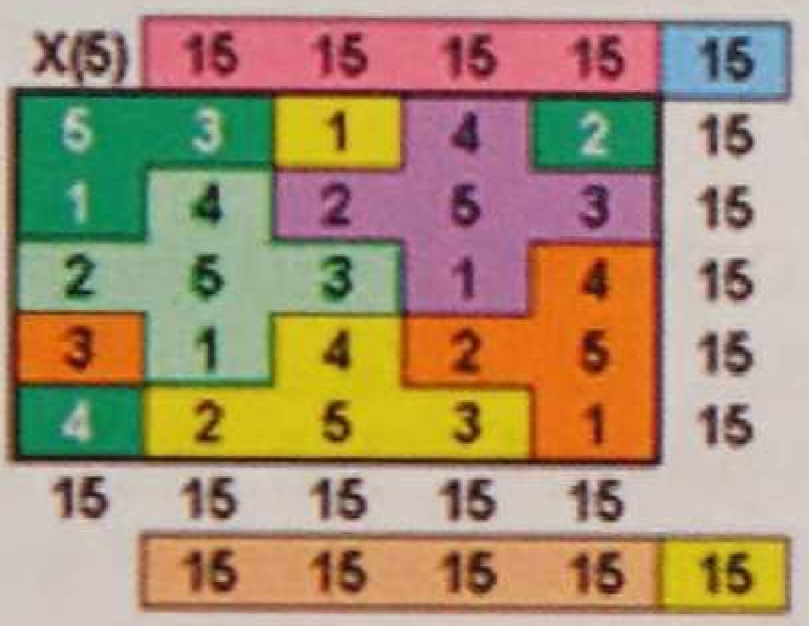
X(3) 6	6	6	6
2	1	3	6
3	2	1	6
1	3	2	6
6	6	6	6

Inverse

$$\frac{1}{18}\begin{bmatrix} 1 & 7 & -5 \\ -5 & 1 & 7 \\ 7 & -5 & 1 \end{bmatrix}$$



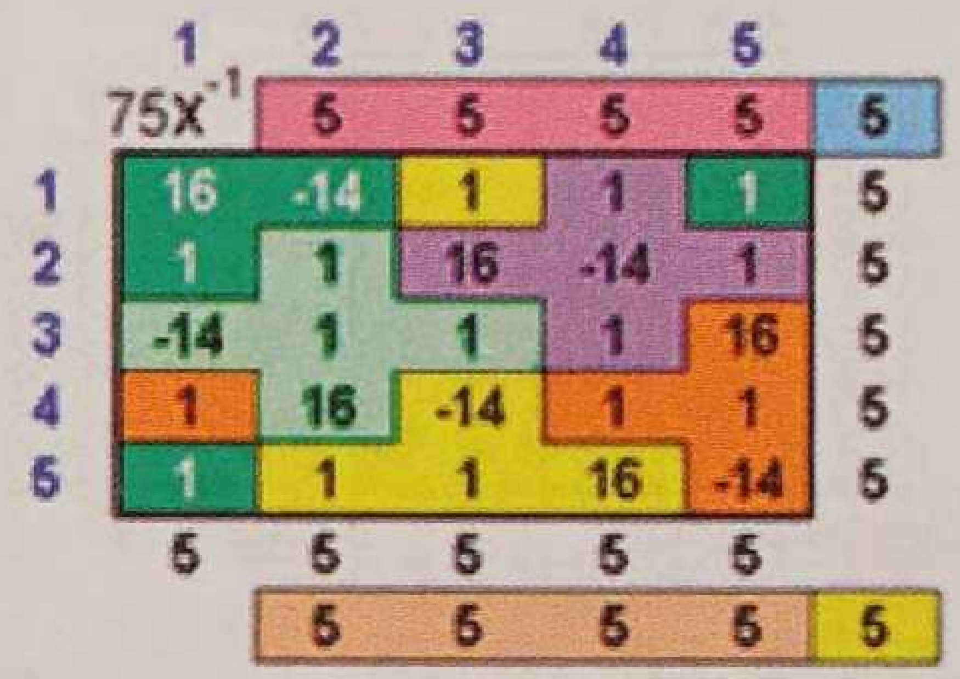
$18X^{-1}$ 3	3	3	3
1	7	-5	3
-5	1	7	3
7	-5	1	3
3	3	3	3



X(5)	15	15	15	15	15
5	3	1	4	2	15
1	4	2	5	3	15
2	5	3	1	4	15
3	1	4	2	5	15
4	2	5	3	1	15
15	15	15	15	15	
	15	15	15	15	15

Inverse

$$\frac{1}{75}\begin{bmatrix} 16 & -14 & 1 & 1 & 1 \\ 1 & 1 & 16 & -14 & 1 \\ -14 & 1 & 1 & 1 & 16 \\ 1 & 16 & -14 & 1 & 1 \\ 1 & 1 & 1 & 16 & -14 \end{bmatrix}$$

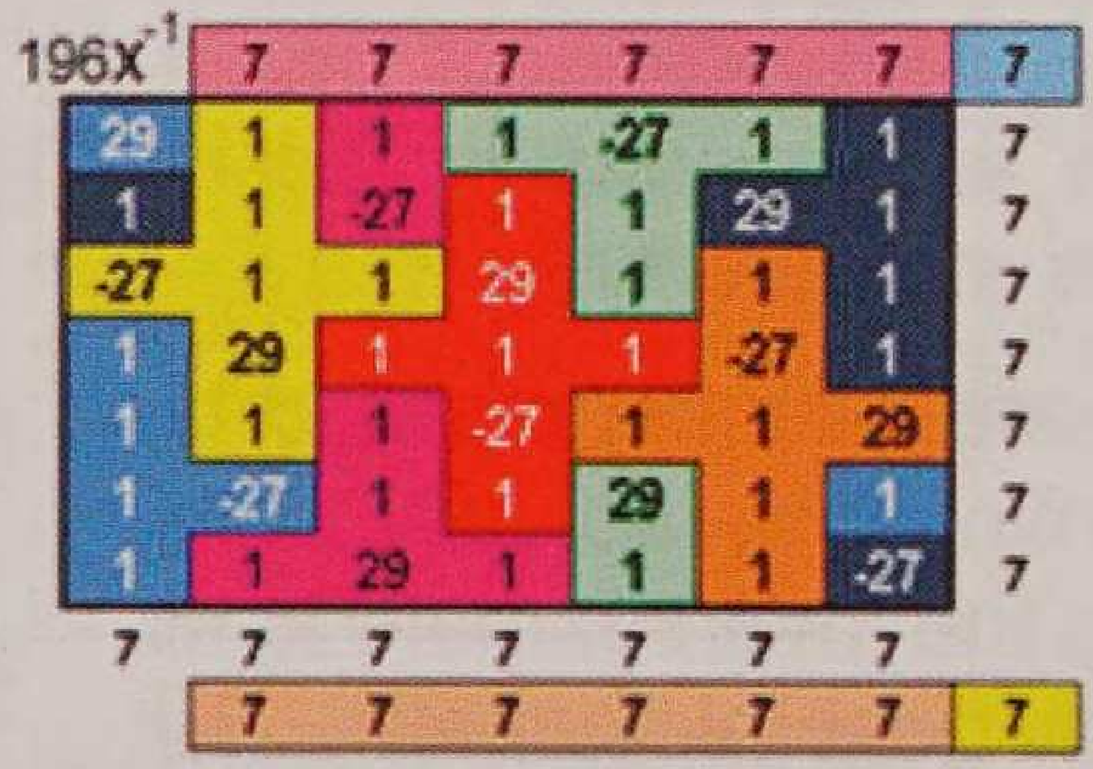


	1	2	3	4	5	
	$75X^{-1}$	5	5	5	5	5
1	16	-14	1	1	1	5
2	1	1	16	-14	1	5
3	-14	1	1	1	16	5
4	1	16	-14	1	1	5
5	1	1	1	16	-14	5
	5	5	5	5	5	
		5	5	5	5	5

X(7)	28	28	28	28	28	28	28
7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	
	28	28	28	28	28	28	28

Inverse

$$\frac{1}{196}\begin{bmatrix} 29 & 1 & 1 & 1 & -27 & 1 & 1 \\ 1 & 1 & -27 & 1 & 1 & 29 & 1 \\ -27 & 1 & 1 & 29 & 1 & 1 & 1 \\ 1 & 29 & 1 & 1 & 1 & -27 & 1 \\ 1 & 1 & 1 & -27 & 1 & 1 & 29 \\ 1 & -27 & 1 & 1 & 29 & 1 & 1 \\ 1 & 1 & 29 & 1 & 1 & 1 & -27 \end{bmatrix}$$



$196X^{-1}$	7	7	7	7	7	7	7
29	1	1	1	-27	1	1	7
1	1	-27	1	1	29	1	7
-27	1	1	29	1	1	1	7
1	29	1	1	1	-27	1	7
1	1	1	-27	1	1	29	7
1	-27	1	1	29	1	1	7
1	1	29	1	1	1	-27	7
7	7	7	7	7	7	7	
	7	7	7	7	7	7	7

1. First, we observe that the common denominator d_n in the inverse for size **n** is given by

   (9.26) $\quad d_n = n L_n$

2. Secondly, the matrix of numerators always contain the numbers $(1 \pm L_n)$ exactly once and **n-2** number **1**'s in every row, column, diagonal and the transpose of characteristic tiles. These each sum to **n** as seen in the calculation below:

   $(1+L_n) + (1-L_n) + (n-2) = (L_n - L_n) + (2 - 2) + n = n.$

3. Thirdly, the matrix of numerators placed in a geonomic framework has a characteristic number equal to its own size **n**.

The size-3 square has no tiling pattern and the size-5 square has totally symmetric tiling patterns. But size-7 demonstrates point #2. Further, although not shown, what holds for the transpose of tile pattern B also holds for the transpose of its complementary tile pattern A.

Program 9

X

7	4	1	5	2	6	3	28
2	6	3	7	4	1	5	28
4	1	5	2	6	3	7	28
6	3	7	4	1	5	2	28
1	5	2	6	3	7	4	28
3	7	4	1	5	2	6	28
5	2	6	3	7	4	1	28
28	28	28	28	28	28	28	

$$X^{-1} = 1/196 \times \begin{pmatrix} 29 & 1 & 1 & 1 & -27 & 1 & 1 \\ 1 & 1 & -27 & 1 & 1 & 29 & 1 \\ -27 & 1 & 1 & 29 & 1 & 1 & 1 \\ 1 & 29 & 1 & 1 & 1 & -27 & 1 \\ 1 & 1 & 1 & -27 & 1 & 1 & 29 \\ 1 & -27 & 1 & 1 & 29 & 1 & 1 \\ 1 & 1 & 29 & 1 & 1 & 1 & -27 \end{pmatrix}$$

Y

1	3	5	7	2	4	6	28
7	2	4	6	1	3	5	28
6	1	3	5	7	2	4	28
5	7	2	4	6	1	3	28
4	6	1	3	5	7	2	28
3	5	7	2	4	6	1	28
2	4	6	1	3	5	7	28
28	28	28	28	28	28	28	

$$Y^{-1} = 1/196 \times \begin{pmatrix} -27 & 29 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -27 & 29 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -27 & 29 & 1 \\ 29 & 1 & 1 & 1 & 1 & 1 & -27 \\ 1 & -27 & 29 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -27 & 29 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -27 & 29 \end{pmatrix}$$

The inverse's table of numerators can be generated by **$1 \pm L_7$** in a continuous step pattern as shown here in yellow and **1**'s everywhere else. Similarly for the pattern in the inverse of the Integer loom table.

Note that the geonomic characteristic number of the numerator matrix is exactly its own size, **7**.

The next two loom-table inverses are of size-17. Note that their patterns are distinctly different from the size-7 inverses.

Even the direction of propagation and overlap in the inverses are different.

So these patterns need to be determined by actually computing the inverse; there is no apriori general pattern. But once having uncovered the patterns, they can be applied directly without error henceforth.

$$X(17)^{-1} = \frac{1}{2601} \times \begin{pmatrix} 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 \\ 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 \\ 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 \\ 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 \\ -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 154 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -152 \end{pmatrix}$$

Observe that the absolute value of each of the larger two numbers in the size-17 inverse average **153**. And on the prior slide, the absolute value of each of the larger two numbers in the size-7 inverse averaged **28**. These are the loom-tables' characteristic numbers respectively.

$n = 17$

$L_{17} = 153$

$d_{17} = 2601$

$= 17 \times 153$

$= n L_n$

$$Y(17)^{-1} = \frac{1}{2601} \times \begin{pmatrix} 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 \\ 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 \\ 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 \\ -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 \\ 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 \\ 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 \\ 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 154 & -152 & 1 & 1 \end{pmatrix}$$

Program 9

The Matrix Product of a Loom-table's Inverse with its Transpose

In regular Matrix Algebra, the inverse of a matrix times itself equals the Identity matrix, i.e. . $Q^{-1}Q = I$. In Geonometry, there is a property resulting from the product of a loom-table's **inverse** with its **transpose** as shown here. This does not hold for any flipped version of **X**.

X

1	2	1	7	6	5	4	3
2	6	5	4	3	2	1	7
3	3	2	1	7	6	5	4
4	**7**	**6**	**5**	**4**	**3**	**2**	**1**
5	4	3	2	1	7	6	5
6	1	7	6	5	4	3	2
7	5	4	3	2	1	7	6

X^T

2	6	3	**7**	4	1	5
1	5	2	**6**	3	7	4
7	4	1	**5**	2	6	3
6	3	7	**4**	1	5	2
5	2	6	**3**	7	4	1
4	1	5	**2**	6	3	7
3	7	4	**1**	5	2	6
1	2	3	4	5	6	7

X^{-1}

0.005	0.005	0.005	0.148	0.005	-0.138	0.005
-0.138	0.005	0.005	0.005	0.005	0.148	0.005
0.148	0.005	-0.138	0.005	0.005	0.005	0.005
0.005	0.005	0.148	0.005	-0.138	0.005	0.005
0.005	0.005	0.005	0.005	0.148	0.005	-0.138
0.005	-0.138	0.005	0.005	0.005	0.005	0.148
0.005	0.148	0.005	-0.138	0.005	0.005	0.005

$X^{-1}X^T$

	1	1	1	1	1	1	1
0.429	0.429	0.429	0.429	-0.571	0.429	-0.571	1
0.429	-0.571	0.429	-0.571	0.429	0.429	0.429	1
-0.571	0.429	0.429	0.429	0.429	-0.571	0.429	1
0.429	0.429	-0.571	0.429	-0.571	0.429	0.429	1
0.429	-0.571	0.429	0.429	0.429	0.429	-0.571	1
0.429	0.429	0.429	-0.571	0.429	-0.571	0.429	1
-0.571	0.429	-0.571	0.429	0.429	0.429	0.429	1
1	1	1	1	1	1	1	
	1	1	1	1	1	1	1

A

1	1	1	1	1	1	1
1	1	1	**1**	1	1	1
1	1	**1**	1	**1**	1	1
1	1	1	**1**	1	1	1
1	1	**1**	1	**1**	1	1
1	1	1	**1**	1	1	1
1	1	1	1	1	1	1

B

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	**1**	1	1	1
1	**1**	**1**	**1**	**1**	**1**	1
1	1	1	**1**	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

The product $X^{-1}X^T$ is filled with fractions, yet its geonomic summations are all equal to **1**. Wow!

Here is the complementary loom-table **Y**, its transpose Y^T, its inverse Y^{-1} and product $Y^{-1}Y^T$.

Y

1	3	4	5	**6**	7	1	2
2	7	1	2	**3**	4	5	6
3	4	5	6	**7**	1	2	3
4	1	2	3	**4**	5	6	7
5	5	6	7	**1**	2	3	4
6	2	3	4	**5**	6	7	1
7	6	7	1	**2**	3	4	5

Y^T

3	7	4	1	5	2	6
4	1	5	2	6	3	7
5	2	6	3	7	4	1
6	**3**	**7**	**4**	**1**	**5**	**2**
7	4	1	5	2	6	3
1	5	2	6	3	7	4
2	6	3	7	4	1	5
1	2	3	4	5	6	7

Y^{-1}

0.005	0.148	0.005	-0.138	0.005	0.005	0.005
0.005	-0.138	0.005	0.005	0.005	0.005	0.148
0.005	0.005	0.005	0.005	0.148	0.005	-0.138
0.005	0.005	0.148	0.005	-0.138	0.005	0.005
0.148	0.005	-0.138	0.005	0.005	0.005	0.005
-0.138	0.005	0.005	0.005	0.005	0.148	0.005
0.005	0.005	0.005	0.148	0.005	-0.138	0.005

$Y^{-1}Y^T$

	1	1	1	1	1	1	1
-0.143	-0.143	-0.143	-0.143	0.857	-0.143	0.857	1
-0.143	0.857	-0.143	0.857	-0.143	-0.143	-0.143	1
0.857	-0.143	-0.143	-0.143	-0.143	0.857	-0.143	1
-0.143	-0.143	0.857	-0.143	0.857	-0.143	-0.143	1
-0.143	0.857	-0.143	-0.143	-0.143	-0.143	0.857	1
-0.143	-0.143	-0.143	0.857	-0.143	0.857	-0.143	1
0.857	-0.143	0.857	-0.143	-0.143	-0.143	-0.143	1
1	1	1	1	1	1	1	
	1	1	1	1	1	1	1

A

1	1	1	1	1	1	1
1	1	1	**1**	1	1	1
1	1	**1**	1	**1**	1	1
1	1	1	**1**	1	1	1
1	1	**1**	1	**1**	1	1
1	1	1	**1**	1	1	1
1	1	1	1	1	1	1

B

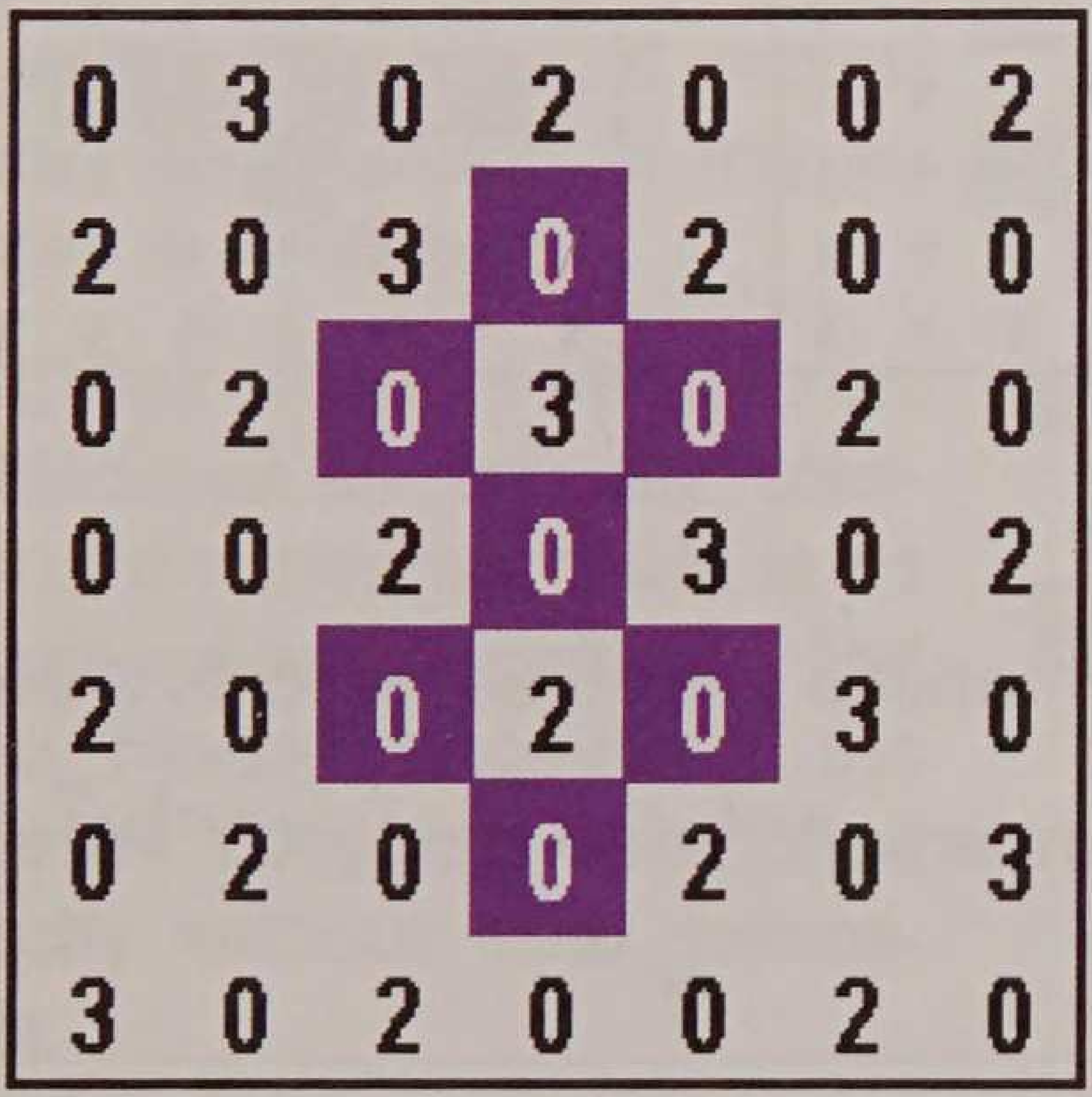
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	**1**	1	1	1
1	**1**	**1**	**1**	**1**	**1**	1
1	1	1	**1**	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

The same result holds for the product $Y^{-1}Y^T$.

Geonomic Properties of the standard Identity Matrix

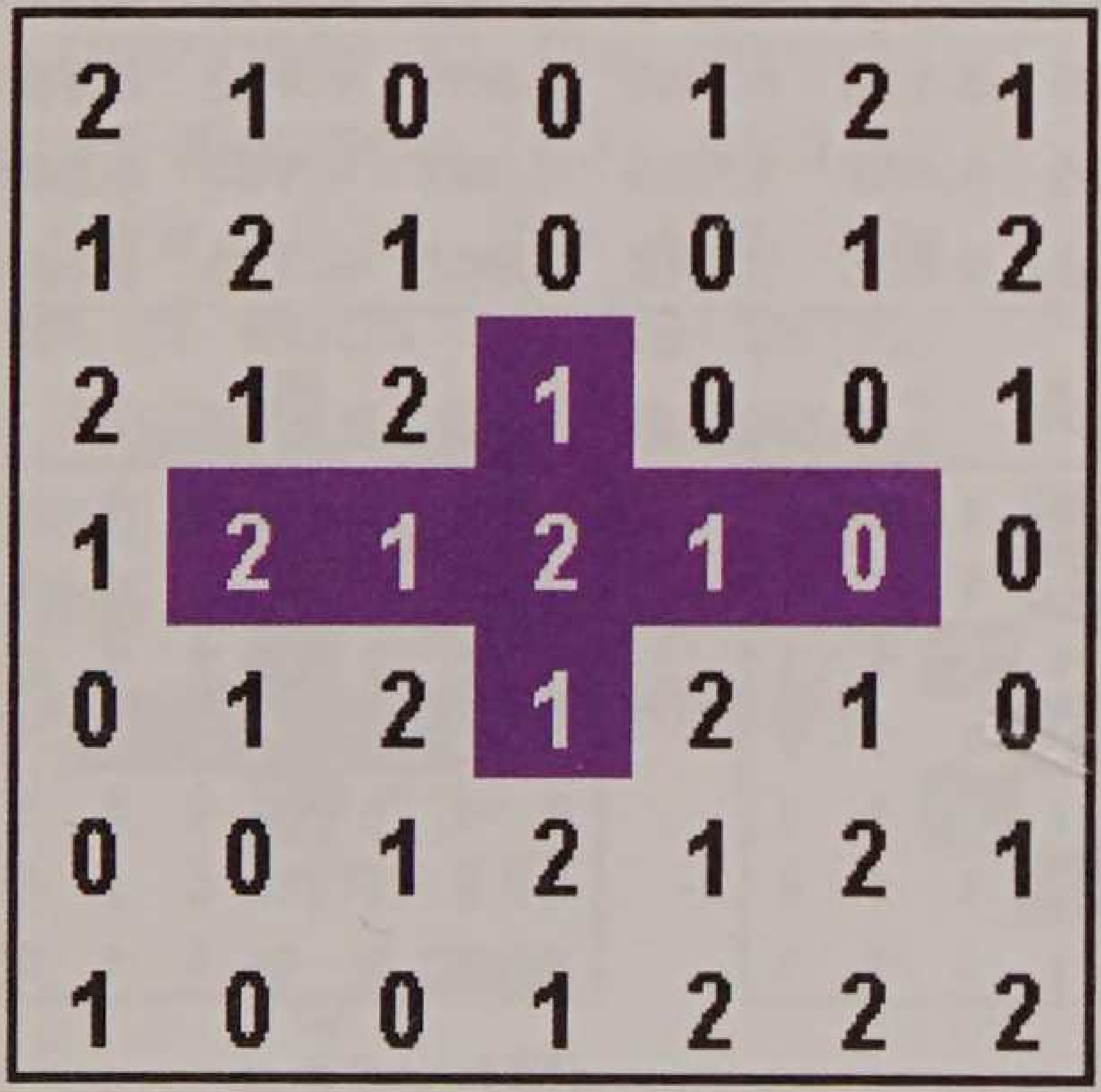
I_7	1	1	1	1	1	1	1
1	0	0	0	0	0	0	1
0	1	0	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	0	1	0	0	0	1
0	0	0	0	1	0	0	1
0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	1
1	1	1	1	1	1	1	
	0	0	0	0	0	0	7

A



0	3	0	2	0	0	2
2	0	3	0	2	0	0
0	2	0	3	0	2	0
0	0	2	0	3	0	2
2	0	0	2	0	3	0
0	2	0	0	2	0	3
3	0	2	0	0	2	0

B



2	1	0	0	1	2	1
1	2	1	0	0	1	2
2	1	2	1	0	0	1
1	2	1	2	1	0	0
0	1	2	1	2	1	0
0	0	1	2	1	2	1
1	0	0	1	2	2	2

Here is the standard identity matrix of Matrix Algebra. This is the result one gets when pre-multiplying any non-singular matrix by its inverse.

Put in geonomic form, it displays no continuously equal summations along its major diagonals. Further, it has no continuously equal summations in either of the size-7 characteristic complementary tiling patterns.

This demonstrates that Geonometry is distinct from Matrix Algebra.

What is shown here for the size-7 identity matrix also holds for the size-11 identity matrix of the same class.

Now, cosmologists rely on Matrix Theory to compute their elements of String Theory in **11** contorted dimensions. Yet Geonometry has been demonstrated to yield a more consistent and perfect property for this matrix product of table and its inverse.

This is proof that Geonometry is operating on a level never seen before – and in my opinion, is tapping, measuring and mapping the harmonic fabric of space itself.

Consequently, those concepts put forth in the latter part of this program regarding the properties of multi-dimensional space at various levels shouldn't be dismissed as fantasy. Of course they are only conjectures at this point in time, but they have as much credibility as the cosmological notions so prevalent among scientists today.

For you students watching this program, it is an encouragement to question what you are being taught as theory and to learn to think outside of the academic box. There are amazing discoveries to be made about the characteristics of multi-dimensional space by your generation. So keep your mind open and come on board with this new math. Learn its amazing properties so you're able to apply them in your chosen field of endeavor, whatever that may be.

Program 9

Kernel Value Multiples equal to the Characteristic Number for any Dimension

Now, lets turn our attention to getting a grasp on multi-dimensional space.

Here is a table listing the dimensions and the count of kernel numbers that equal the characteristic number in tables of size **4x** in different dimensions. Recall that this multiple determined circles in squares, spheres in cubes and toruses in quadracubes. $\mathbf{V}^k$ represents the list of kernel numbers corresponding to dimension **k**.

Kernel Value Multiples equal to the Characteristic Number in dimensions 2 thru 5

Table Size n	n/2	V_2	V_3	V_4	V_5
4	2	1	2	4	8
8	4	2	8	32	128
12	6	3	18	108	648
16	8	4	32	256	2048
20	10	5	50	500	5000
24	12	6	72	864	10,368
28	14	7	98	1372	19,208
32	16	8	128	2048	32,768

Noting that:

$$V_3 = (n/2)^1 V_2 \qquad V_4 = (n/2)^2 V_2 \qquad V_5 = (n/2)^3 V_2$$

$$V_4 = (n/2)^1 V_3 \qquad V_5 = (n/2)^2 V_3$$

$$V_5 = (n/2)^1 V_4$$

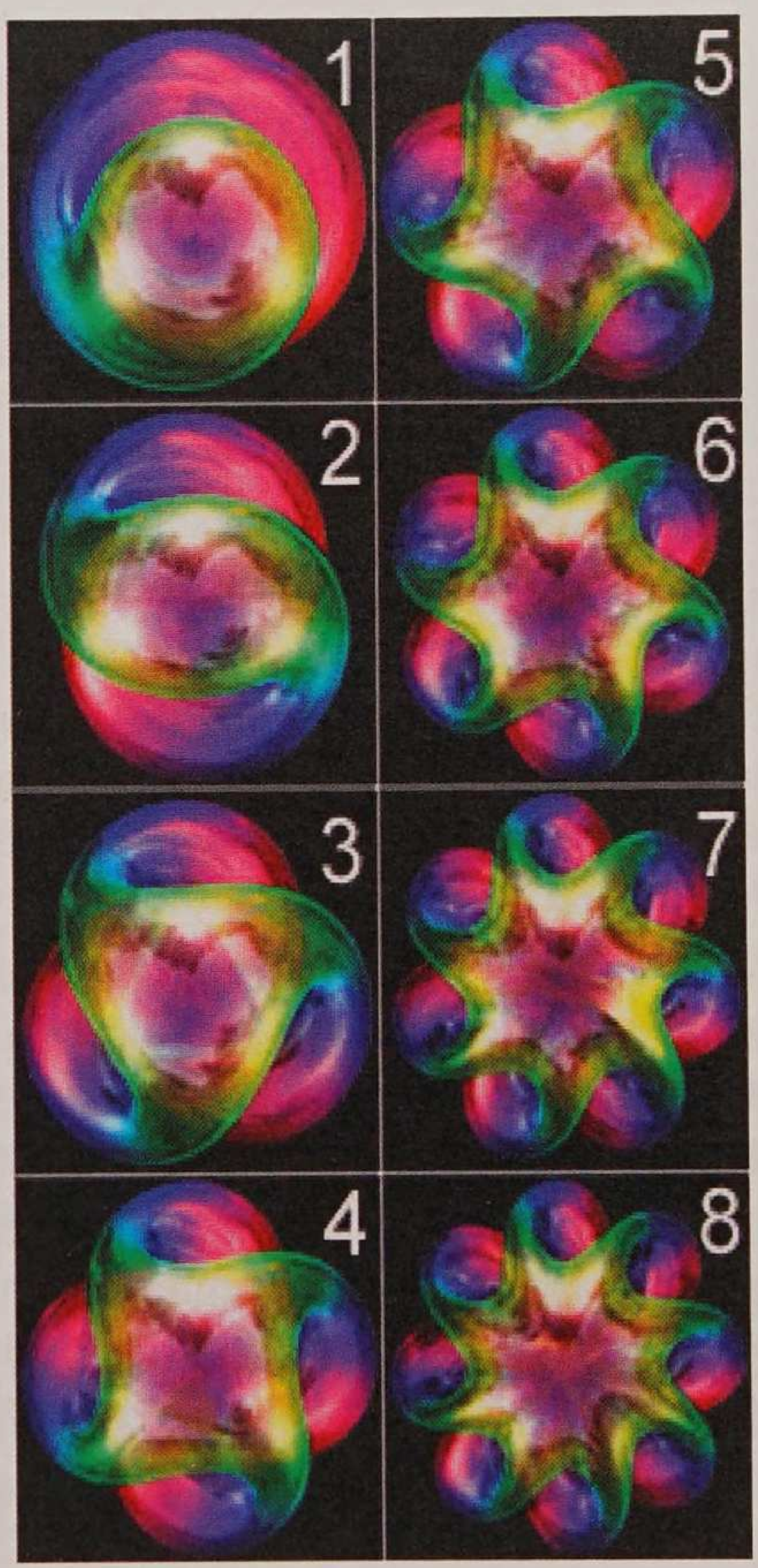
and that

$$(\ 9.27\ ) \qquad V_2 = n/4$$

Putting everything in terms of $\mathbf{V_2}$, we see that the foregoing relationships can be generalized as:

$$(\ 9.28\ ) \qquad V_k = (n/4)\,(n/2)^{k-2} = n^{k-1}/2^{\,k}$$

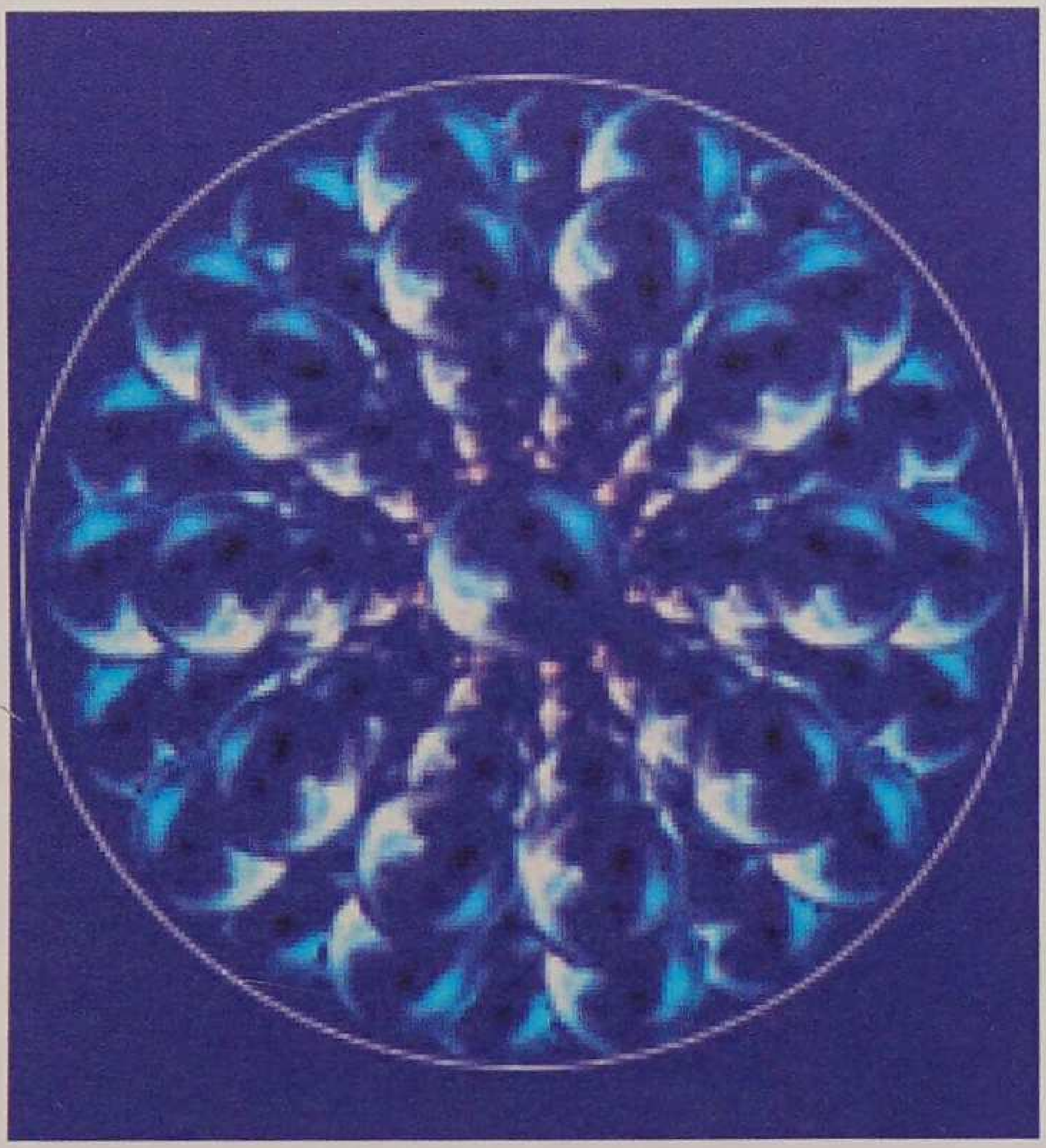
What this demonstrates is that as the dimensions increase and the sub-dimensions decrease away from the point of singularity, the convolution of space from contorted spheres increases as shown at right.

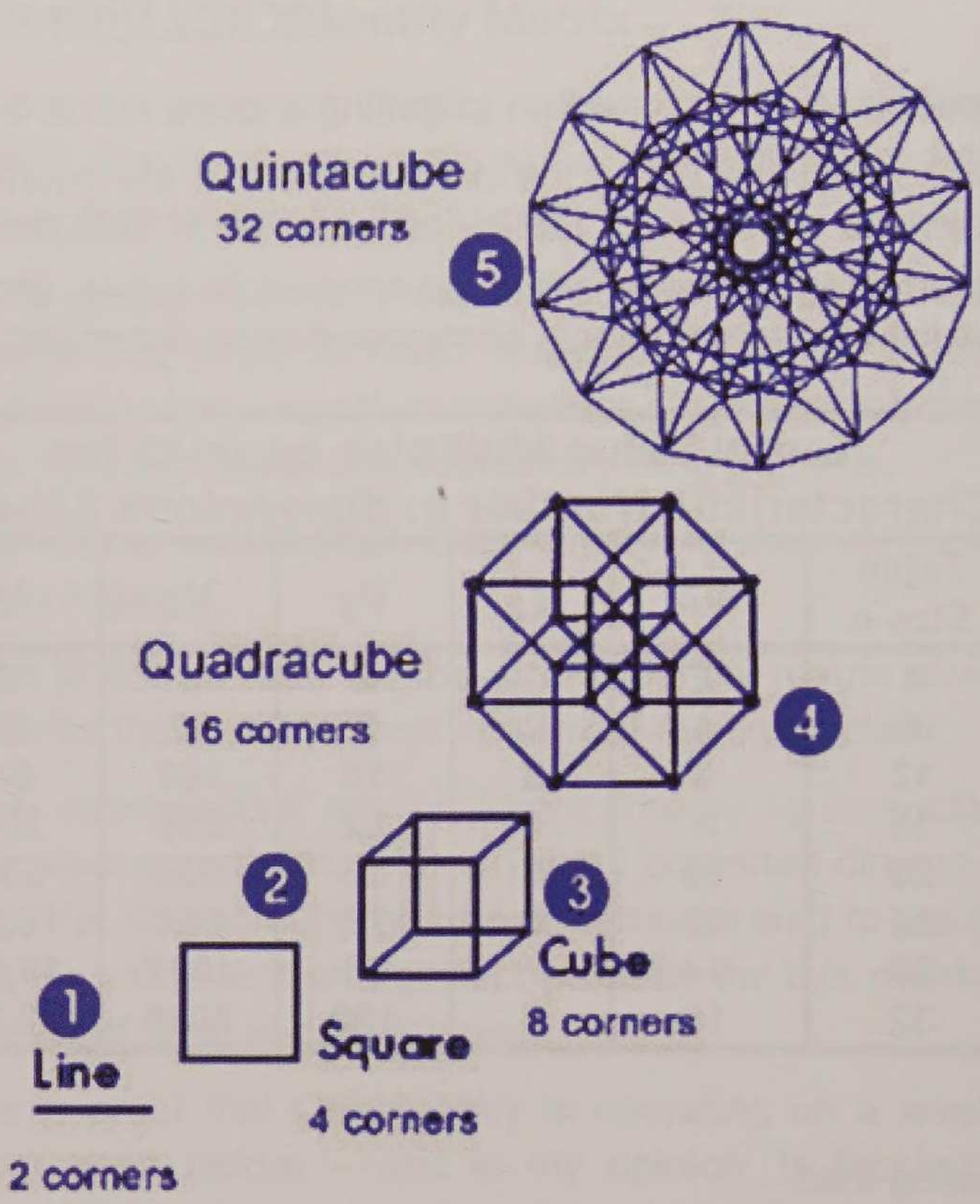


Multi-dimensional Exotic Spheres in increasing hyper- and sub-dimensional Spaces

Methods for constructing these exotic colored spheres, may be found on the internet at www.bugman123.com/Math/index.html

The Increasing Complexity of Space





The **32** cornered figure at upper right is called a duohexadectal. Remember how the kernel values were linked with circles in squares, spheres in cubes and toruses in quadracubes. This figure corresponds to the quintacube in 5-dimensions.

What curved figure would that determine in 5-dimensions among the 4 integrated quadracubes in a 5-dimensional quadracubic projection we saw earlier in this program when 5-dimensions was shown to be fractal? Could it again be another sphere generated from multiple 3D spheres as pictured here at left?

It's pretty obvious to me that 5-dimensions takes the shape of a sphere made up of many spheres like atoms are made of many bundled neutrons and protons. Could this structure go on in both higher and lower dimensions indefinitely? Why would it have to end on some higher or lower dimensional level? Is it just only the 2nd and 3rd dimensions that can support life?

The Big Picture of the Multi-dimensional Universe

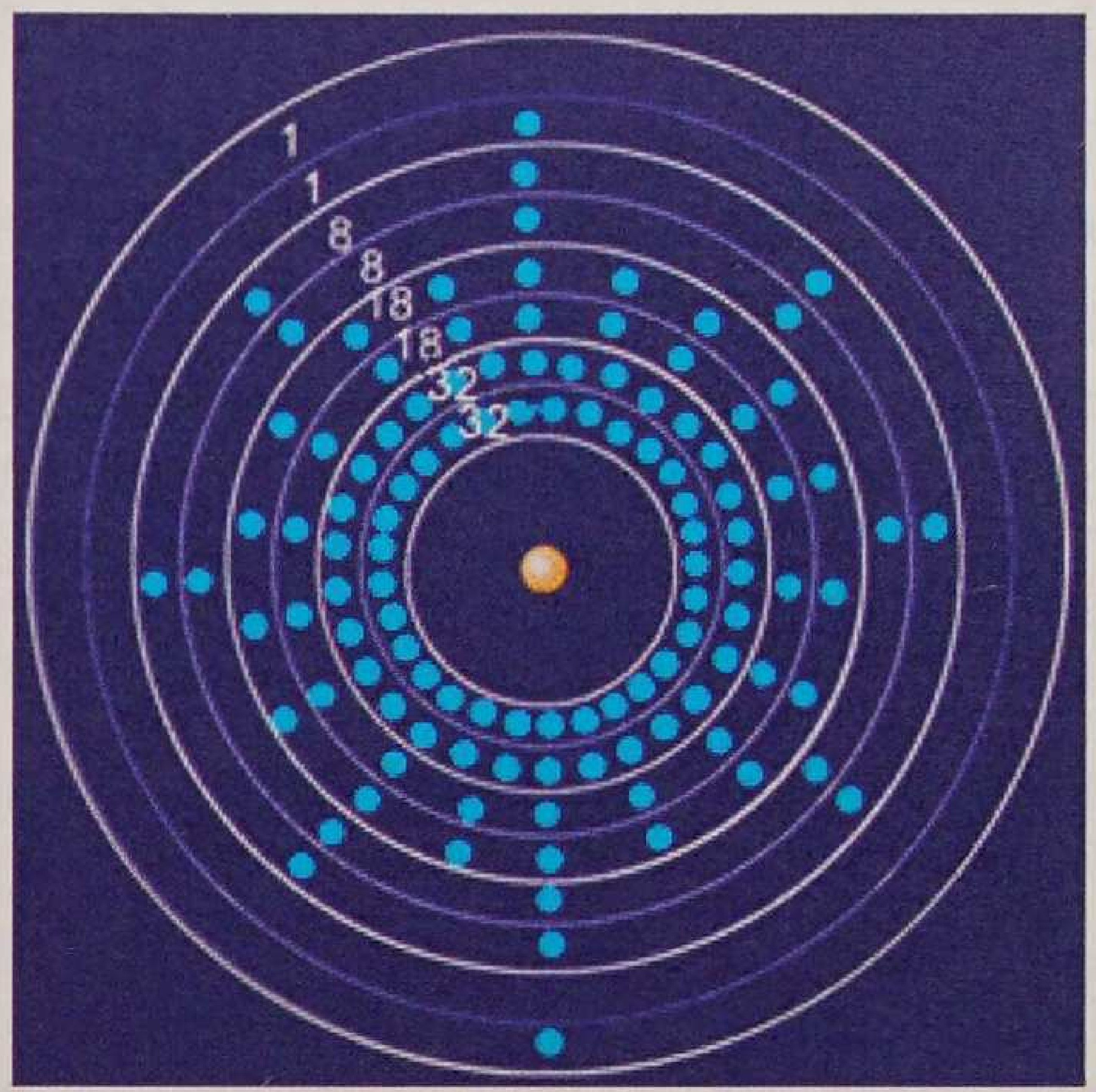
With geonometry we are able to evaluate the general nature of any dimensional level. However, it will take much more powerful computers than a PC or laptop to do it and it will also take specially written softwarewith computational capabilities beyond Microsoft's Excel program to do it. The foundation for future research has now been laid by Geonometry.

Next, we'll take a look-see at just what the big-picture of the multi-dimensional universe might look like based on notions that are not yet confirmed by either Mathematics or Science. These notions grant the next generation of technologists a vast arena of exciting exploration that could well lead to numerous Nobel prizes among them.

Program 9

First, the Penetrating Picture of Sub-dimensional space

We know that particle accelerators and atom smashers measure their observations in **milliseconds** in **1/3D** and **microseconds** in **1/4D**, so there should be no argument against accepting that time passes at an accelerated pace when descending different sub-dimensional levels.



Given our midpoint perspective of time in 3-dimensions as the basis upon which to relate our tick-tocks to the passage of time in sub-dimensions, which are orders of magnitude faster than our "seconds", the passage of time in the 4th dimension should be at least an order of magnitude slower than the passage of time in our 3rd dimension, just based merely on the progression just cited. So time may not be a dimension in and of itself as the Theory of Relativity presumes, but may be a property of each dimensional level. What happens as an object accelerates to light-speed is just its entry into the 4-th dimension where time is relatively much slower than in 3-dimensional space. And just as Einstein's theory concludes, spacemen would return younger than their Earth-bound counterparts. It's just that the properties of 4-dimensional space remained unknown to Einstein and to this day still remain unknown to Science. Geonometry has provided that missing link here, so now its exploration and even exploitation can proceed.

We have seen just how the basic patterns that are found in Geonometry of the 2nd, 3rd & 4th dimensions can be applied to provide the basic mathematics for interpreting the supposedly scientifically-known atomic structures in sub-dimensions **1/2D**, **1/3D** and **1/4D**, respectively.

It was seen back in Program 5 that Geonometry provided a new reasonable view of how the nested electron-shell pairs were positioned around the nucleus. What that indicates about dimensions below the point of inversion is that sub-dimensional space expands inward, not outward as they do in dimensional spaces above the point-of-inversion.

The sequence of electron addition in going from one atom to the next in increasing atomic number need not follow the sequence of the acquiring shell's location relative to its distance from the nucleus. Atomic-scientists might consider revising their perspective in their viewing of sub-dimensional space and not view the preceding shell-pairs as being saturated and then being cleaved in half by newly added shells. This will never be known for certain because the electron shells are cloaked by the very vibrational fabric of space itself anyway. This current view by Science was conjured up from trying to make sense of the thousands upon thousands of atom-smasher measurements taken only a half century ago.

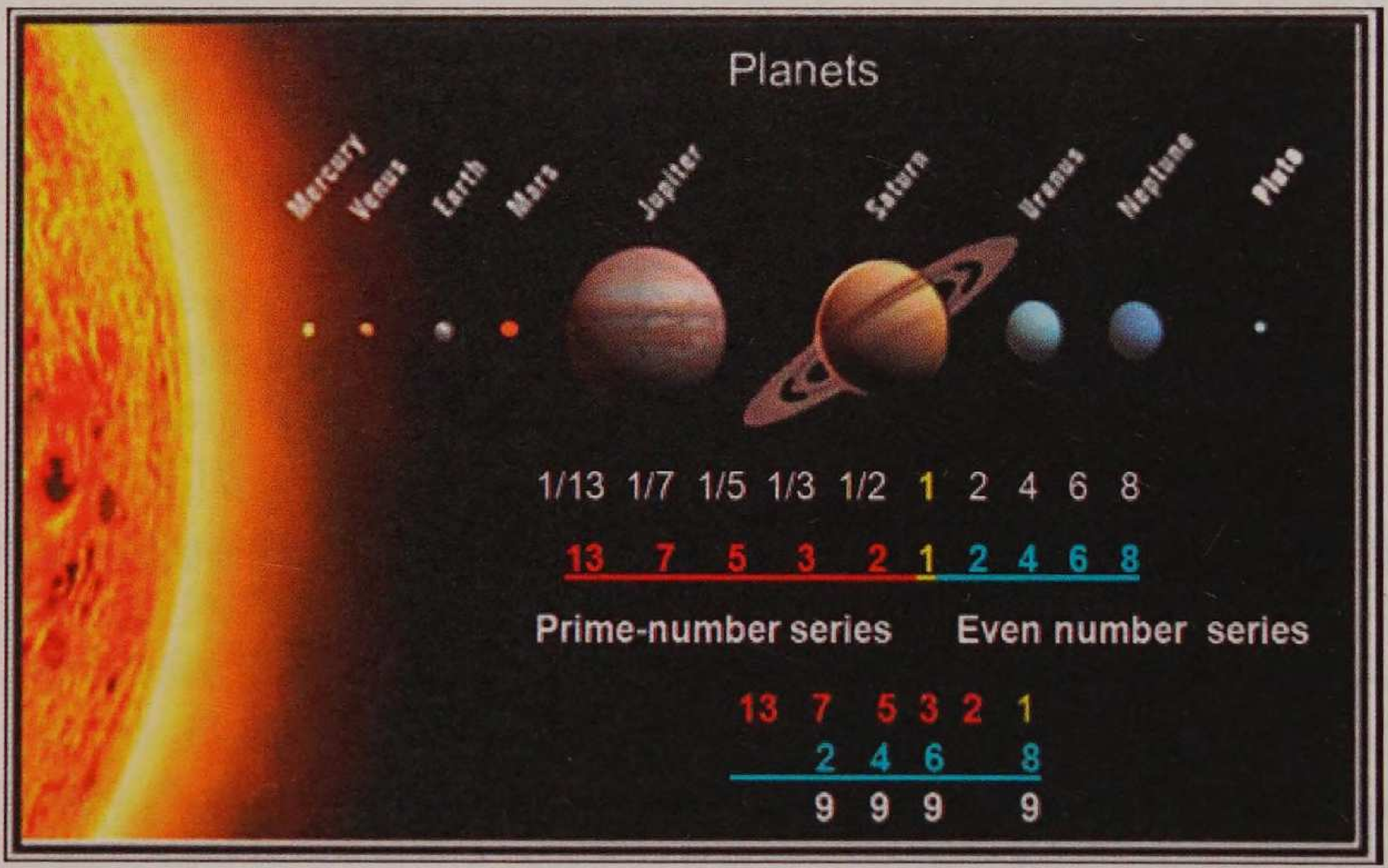
Geonometry explains how shells work in pairs, which Atomic Physics fails to do. Considering the lightest shell-pair as a single electron shell which then gets buried by the filling in of other shells of more capacity is just nonsensical. All throughout the addition of electrons, it was seen right here in your companion book to this series from the data provided by Atomic Science itself that there were always **1**, **2** and sometimes even **3** fringe electrons in the outermost shell at the points of transition to the start of the filling-in of the next lower shell with the next **3** electrons.

I say that it makes more sense if the shell-pairs, the emphasis is on "pairs", not "shells", to become only half full before the next lower shell begins accepting electrons, on and on, shell to next lower shell, until the lowest shell pair gets completely full. Then the remaining half-pairs begin filling up in succession progressing away from the nucleus, until the outermost shell-pair with only the capacity of 2 electrons becomes filled-in once again. Then the process ends with Ununoctium (#118) having all of its shell-pairs saturated.

Further, Geonometry predicts that there should be two more elements beyond element #118 because there may be another shell-pair #5 residing further inside the known shell-pair #4 having the capacity of **100** electrons (**$4x^2$** = **100** for **x** = **5**) and that shell-pair could tolerate receiving a mere **2** more electrons and protons. It follows from the stability of all elements in Groups I & II (columns) of the Periodic Table that these new elements should be stable too. The hurdle for nuclear scientists to clear is merging the nuclei of two stable elements together to get one with the electron capacity of **119** or **120**.

Next, the Picture of 3-Dimensional Space

Harmonics in the Spacing of Solar System Planets



Solar System Planets	Distance from Sun in a.u.	Orbit relative to Jupiter's
Mercury	0.39	1/13
Venus	0.72	1/7
Earth	1	1/5
Mars	1.5	1/3
Asteroids	2.7	1/2

Solar System Planets	Distance from Sun in a.u.	Orbit relative to Jupiter's
Jupiter	5.2	1
Saturn	9.5	2
Uranus	19.2	4
Neptune	30.1	6
(Pluto)	39.5	8

Program 9

Here is our solar system. We encountered this slide way back in Program 1. It is shown again here because the next slide will need to refer to the ratios of a few planetary orbital diameters.

The orbital distances from the Sun are listed in the table. This distribution pattern of orbiting material (planets and the belt of asteroids) is most dense midway from the sun. It becomes less dense closer to the sun according to the inverses of a series of ascending odd prime-numbers, and less dense out away from the nebular center according to a series of ascending even numbers.

The series of red numbers are the inverses of the numbers above them. Those are the ratios of the inner planets' distance from the Sun relative to Jupiter's.

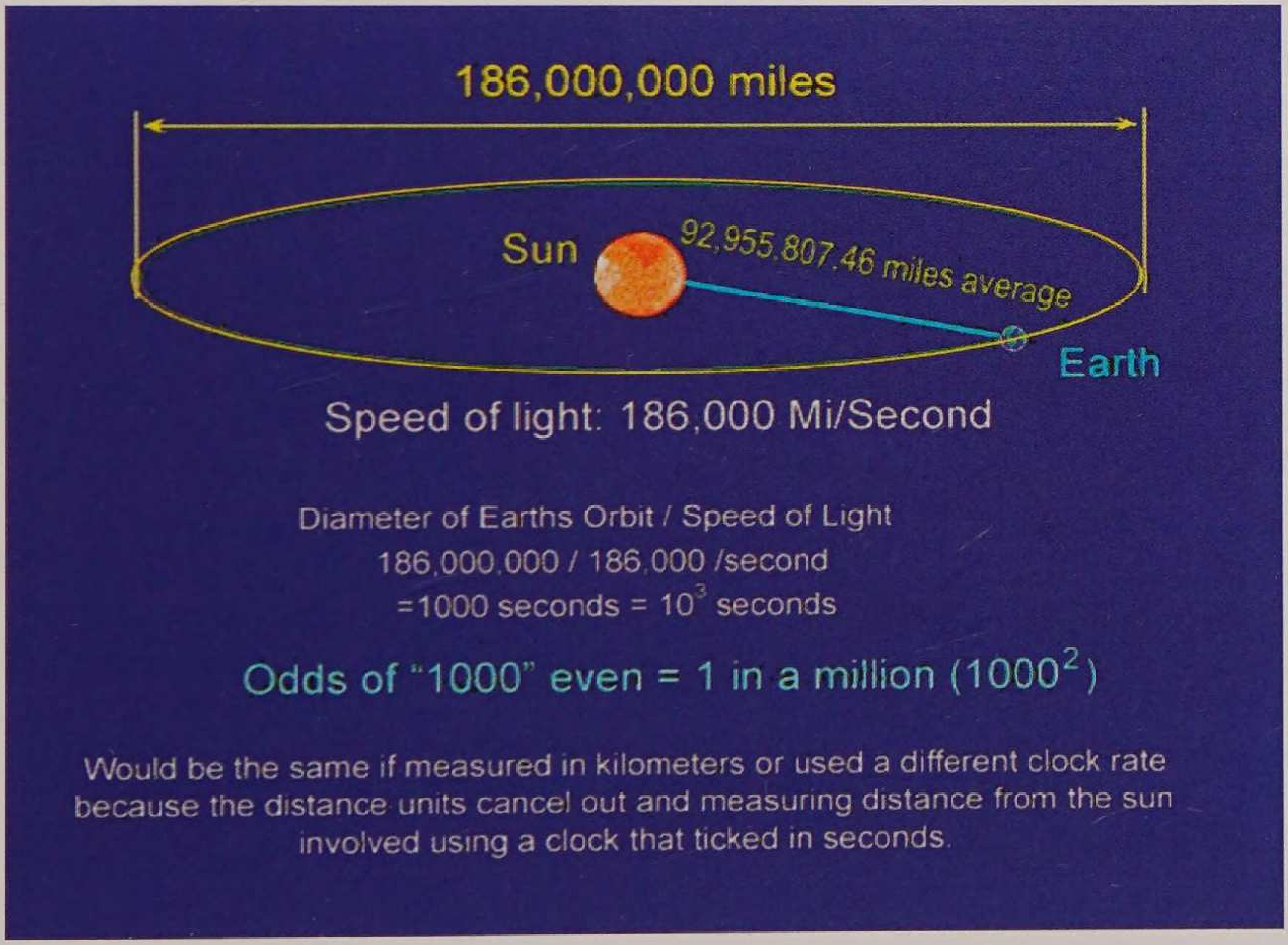
The series of blue numbers are the multiples of the distance from the Sun of outer planets relative to Jupiter's.

Notice how the numbers in red follow a series of increasing prime numbers to the left and how the series of blue numbers follow a series to the right that increases by **2**, all of this starting from a **1** corresponding to the orbital distance of Jupiter.

Light-Speeds in 3-Dimensional Space

Let's address something never talked about in schools because everyone believes that light speed is a constant.

The Michelson-Morley experiment back in 1887 established the speed of light to be **186,000** miles per second. Then all the astrophysicists took that speed to apply evenly throughout all of space so they would have a means for calibrating cosmic distances. All that which that experiment established is that such is the speed of light on Earth at the distance of the Earth from the Sun; nothing more.



Speed of light: 186,000 Mi/Second

Diameter of Earths Orbit / Speed of Light
186,000,000 / 186,000 /second
=1000 seconds = 10^3 seconds

Odds of "1000" even = 1 in a million (1000^2)

Would be the same if measured in kilometers or used a different clock rate because the distance units cancel out and measuring distance from the sun involved using a clock that ticked in seconds.

Let's take a closer look at this Michelson-Morley measurement: The center of the Earth is approximately **93** million miles (one astronomical unit) from the center of the Sun, so the diameter of Earth's orbit is **186 million miles**. Now do the math: the measured speed of light is exactly **1000 times the diameter of Earth's orbit**!

186.000,000 mi / 186,000 mi/sec = 1000 sec = 10^3 sec = 1 kilosecond

Or is it an indication of the pace of 3-dimensional time as well as one of the speed of light could be pegged to the distance from a star?

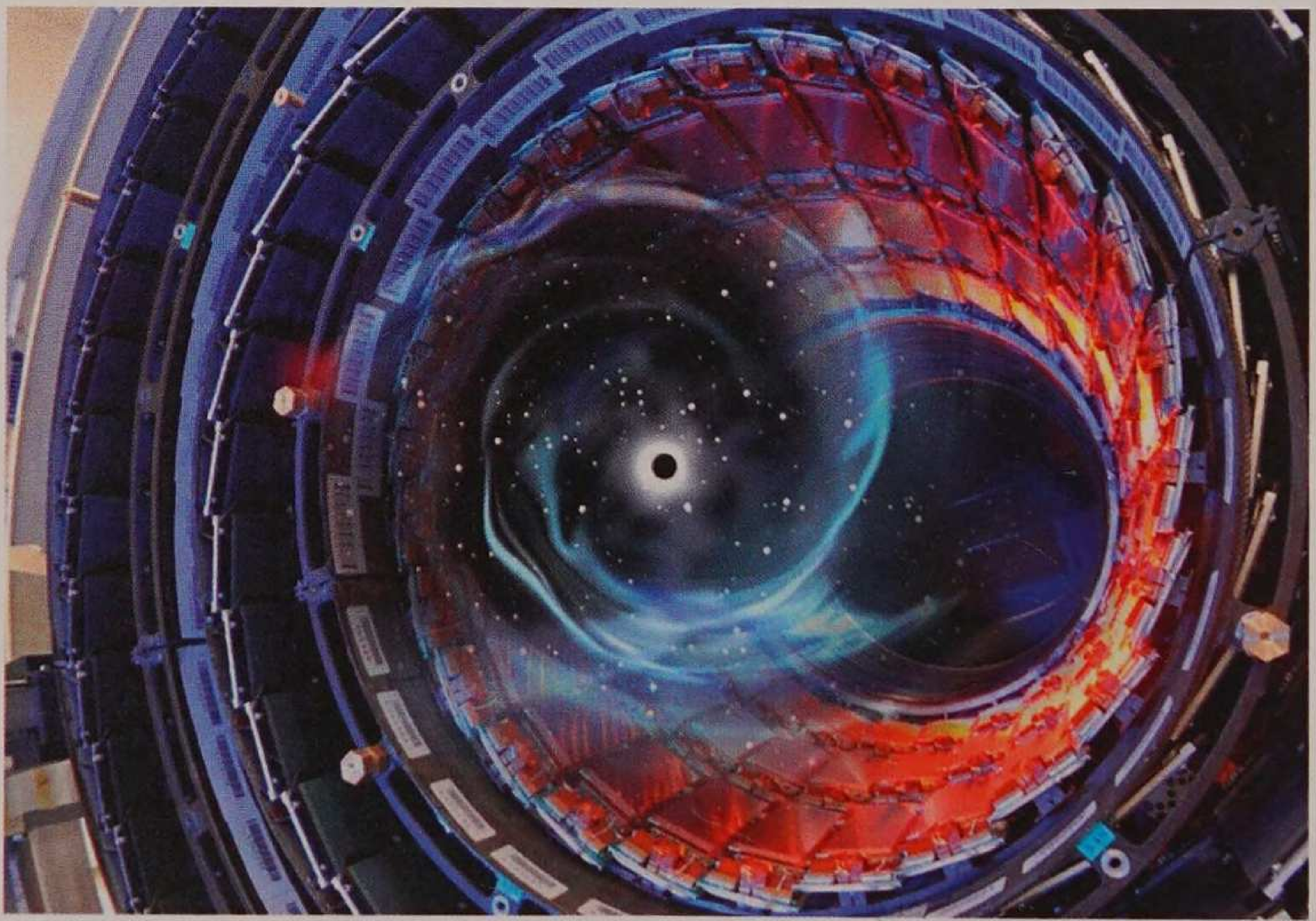
Why is the light speed limit so amazingly correlated with the Earth's distance from the sun? It appears to me that the only thing that is constant is the time of **1 kilo-second**. I consider that number to be **the** 3-dimensional constant, not the light-speed limit. In Geonometry's perspective, that's the unit of time relative to the 3rd dimension.

Given the measured distances of the planetary orbits, could the speed of light on Mars be **5/3**rds that on Earth? Could the speed of light on Jupiter be **5** times that on Earth? That's the ratios of the diameters of their orbits relative to Earth's. We won't know until we at least perform the Michelson-Morley experiment on Mars with distances measured-out physically. That experiment is awaiting the next batch of astronauts, space engineers and astrophysicists.

Could the velocity of time be different on different dimensional levels too?

We know that particle accelerators and atom smashers measure their observations in **milliseconds** in **1/3D** and **nanoseconds** in **1/4D**, so there should be no argument against accepting that time passes at an accelerated pace when descending sequential sub-dimensional levels. So the passage of time in the 4th dimension should be at least an order of magnitude slower than the passage of time in the 3rd dimension, just based merely on the inverse of the progression just cited.

The particle accelerator CERN in Switzerland may someday send particles speeding at 3-dimensional light speed. I conjecture that these particles will never be detected because they will disappear into the 4th dimension before their capture.



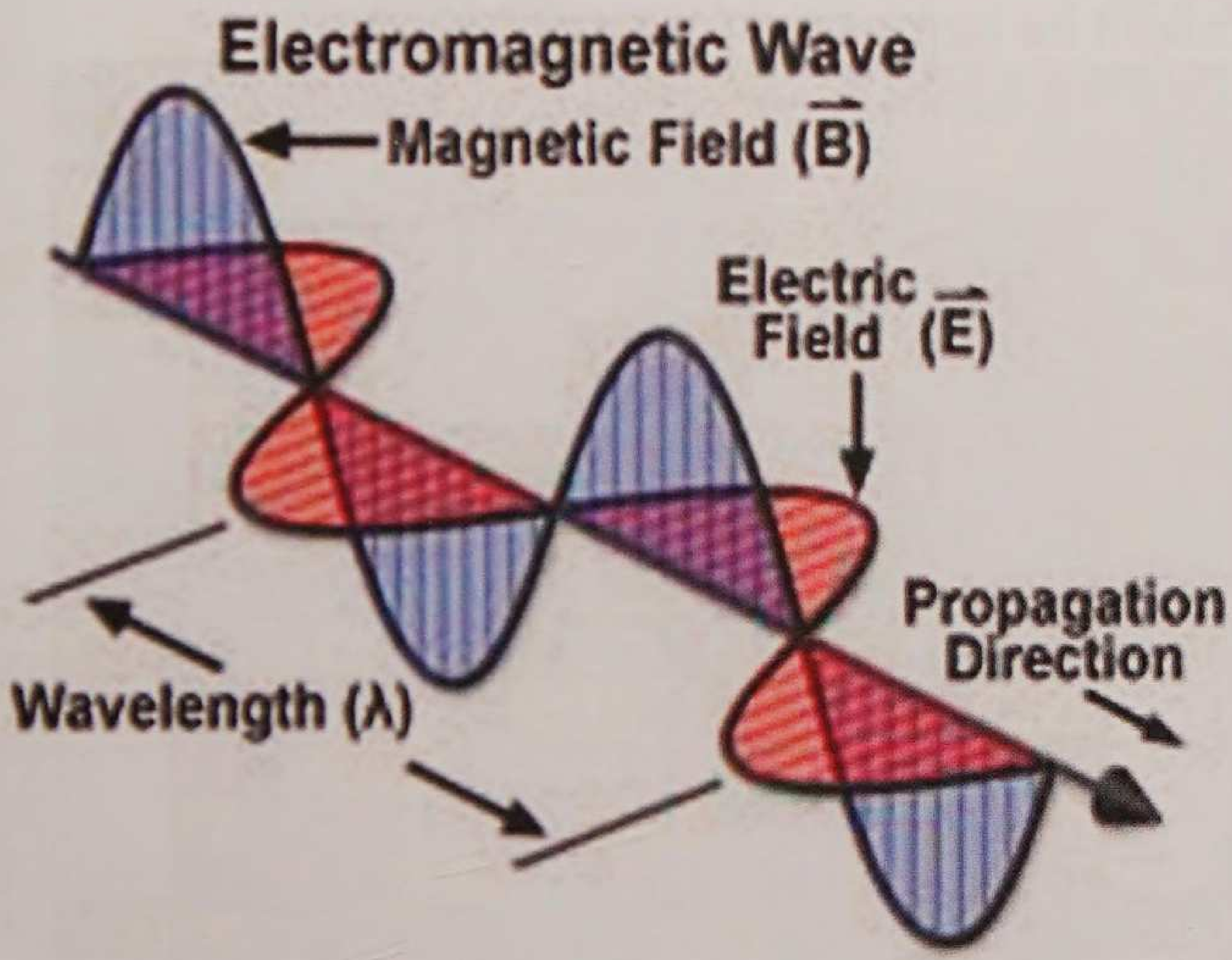
I consider black-holes to be the intersection of other universes that are connected via the overlapping of spheres throughout the four 4-dimensional toruses as seen earlier in this program series. And what falls into these black holes emerges in other universes as decomposed elementary matter only to later become embryonic substances for star formations there. And likewise, matter is constantly being fed into our universe as cosmic elementary decomposed matter, the so-called *dark matter*. Thus the multi-dimensional universe is interconnected and self renewing. That is why I don't buy the Big-Bang theory. That theory is an *assumption*, neither provable nor disprovable. It's merely a hopeful **belief**, not a fact.

And that could explain the detected continual expansion of our own universe. These embryonic stars in our universe are filling in the voids left by the universe's expansion. It's just the way the multi-dimensional universe interacts among its many intersecting dimensions.

Mankind is just now beginning to poke its collective head out the window on space. Science needs to determine the light-speed limit on Mars and make a determined effort to detect the existence of 4th dimension.

The Fabric of Space Itself

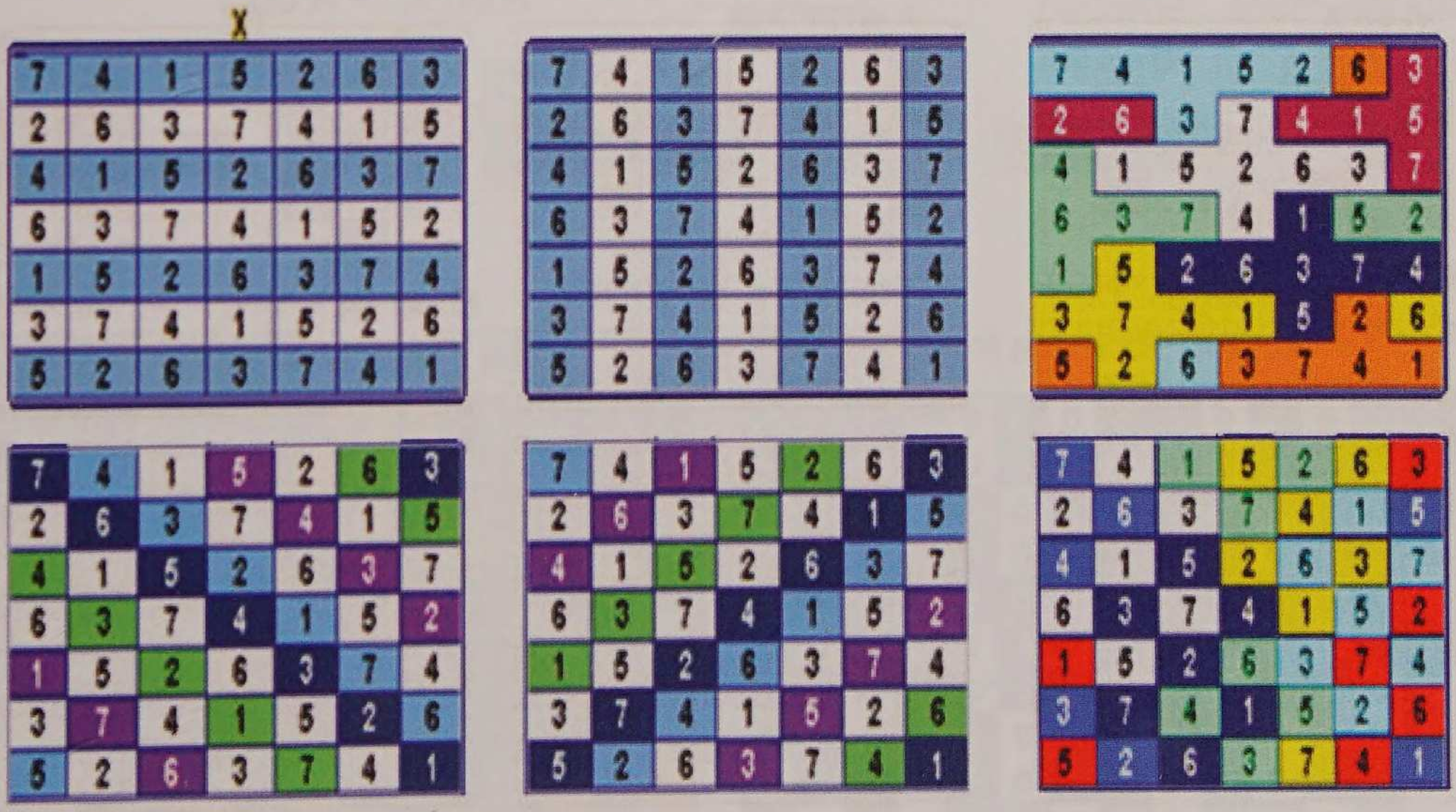
Here is the complementary relationship between electric and magnetic fields in an electromagnetic wave. This fundamental complementarity was fathomed long after electricity and magnetism were discovered independently.



Before electromagnetism was harnessed, this relationship between magnetic and electric fields was unknown. Only by trial-and-error research was it discovered. And once discovered, electric motors and generators were soon developed right afterwards.

The point being made here is that just as a 3-dimensional electromagnetic wave lies at the heart of modern-day electric motors, the complementarity and interwoven cross-patterns of loom tables that were shown throughout **2D** potentially lays at the heart of the vibrational **3D** spatial fabric.

The linear number distribution patterns interwoven in a single loom table



7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

7	4	1	5	2	6	3
2	6	3	7	4	1	5
4	1	5	2	6	3	7
6	3	7	4	1	5	2
1	5	2	6	3	7	4
3	7	4	1	5	2	6
5	2	6	3	7	4	1

Program 9

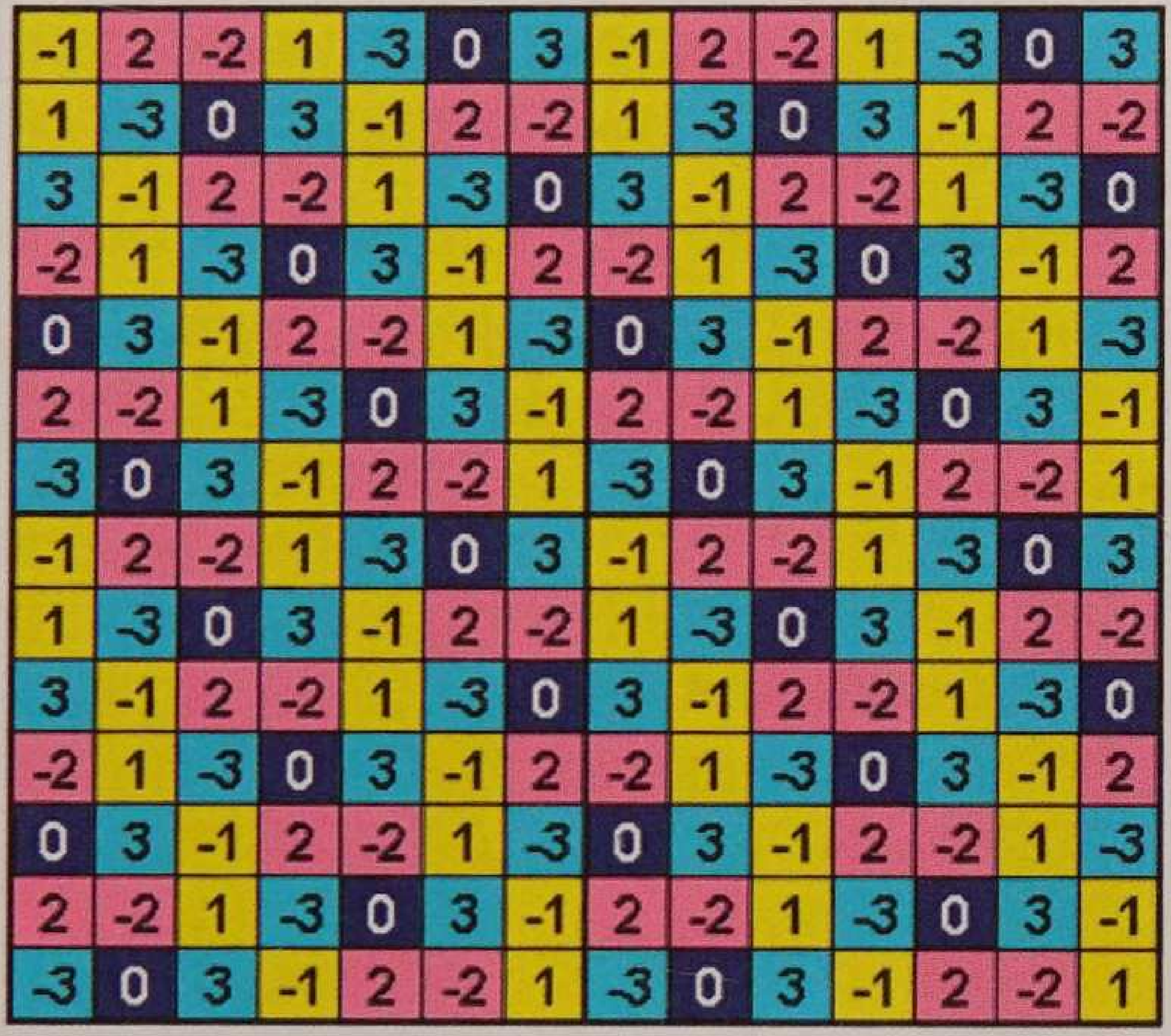
The size-7 loom tables as have been portrayed do not expose the inherent embedded waveform. To do that, they need be balanced anti-symmetrically. This is done by reducing each loom table by its dimensional average $\bar{x} = \bar{y} = (n+1)/2$. The results for **X** and **Y** are shown here where the loom-tables are duplicated four times each to better observe the patterns **F** and **G.** The absolute values of the same numbers are identically colored.

Below these are the size-7 characteristic tile patterns **A** and **B** extrapolated from colored patterns **F** and **G**. To the far right, the tile patterns are added together to get a combined value for both tile patterns **A** on **X+Y** and **B** on **X+Y**. Then these tile patterns are integrated to observe their continuous patterns across 2-dimensional space, shown at the bottom.

Now observe that the continuous pattern **B** cancels itself out by always having adjacent equal ± values, while pattern **A** possesses a continuous **±5** wave pattern. It is this undulating wave pattern that rolls across the spatial fabric that might be harnessed for propulsion.

$X^* = X-|4|$

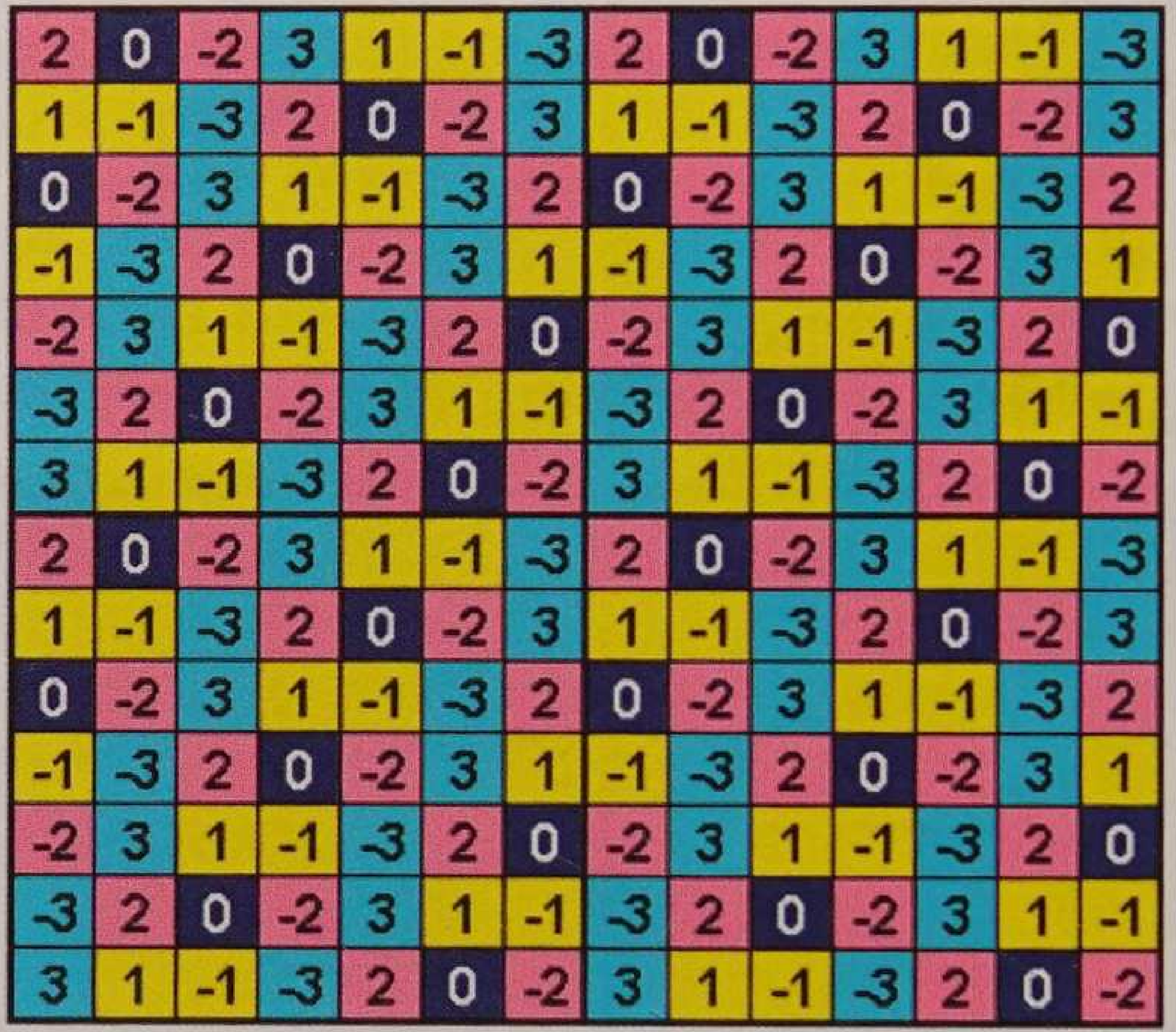
Pattern F



-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3
1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2
3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0
-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2
0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3
2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1
-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1
-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3
1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2
3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3	0
-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1	2
0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1	-3
2	-2	1	-3	0	3	-1	2	-2	1	-3	0	3	-1
-3	0	3	-1	2	-2	1	-3	0	3	-1	2	-2	1

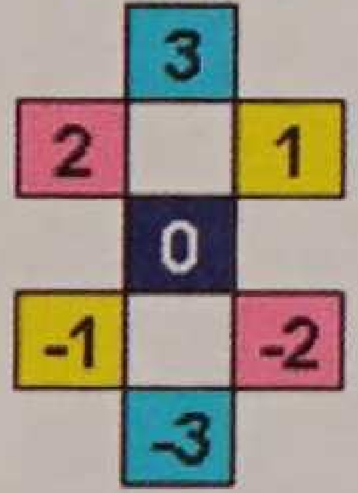
$Y^* = Y+|4|$

Pattern G

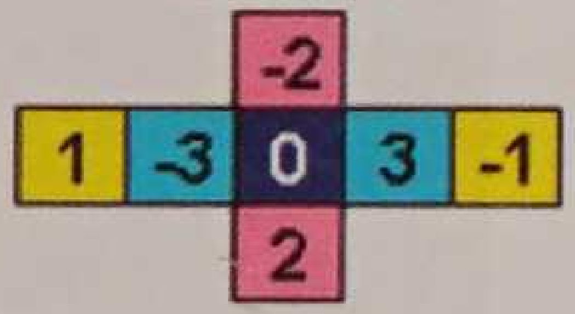


2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3
1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3
0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2
-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1
-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0
-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1
3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2
2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3
1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3
0	-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2
-1	-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1
-2	3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0
-3	2	0	-2	3	1	-1	-3	2	0	-2	3	1	-1
3	1	-1	-3	2	0	-2	3	1	-1	-3	2	0	-2

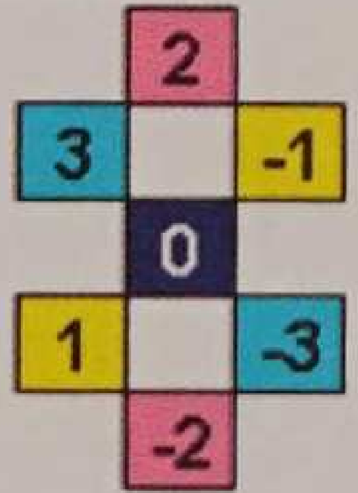
A on X^*



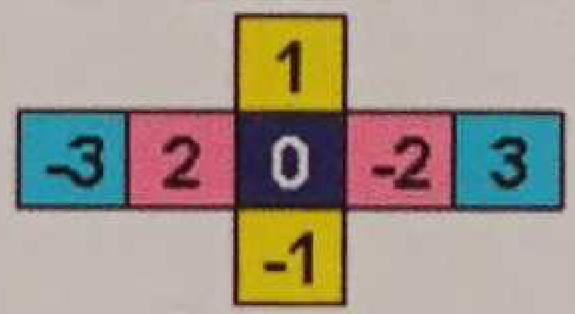
B on X^*



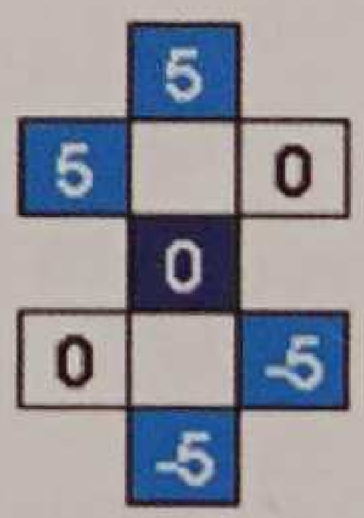
A on Y^*



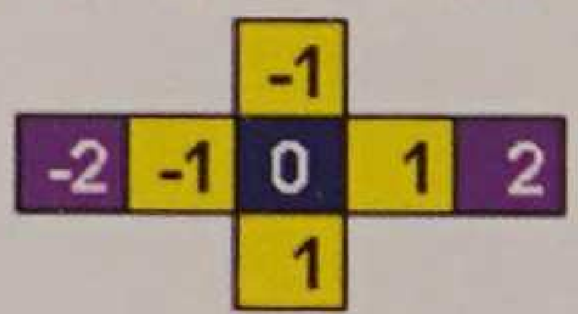
B on Y^*



A on X^* + A on Y^*

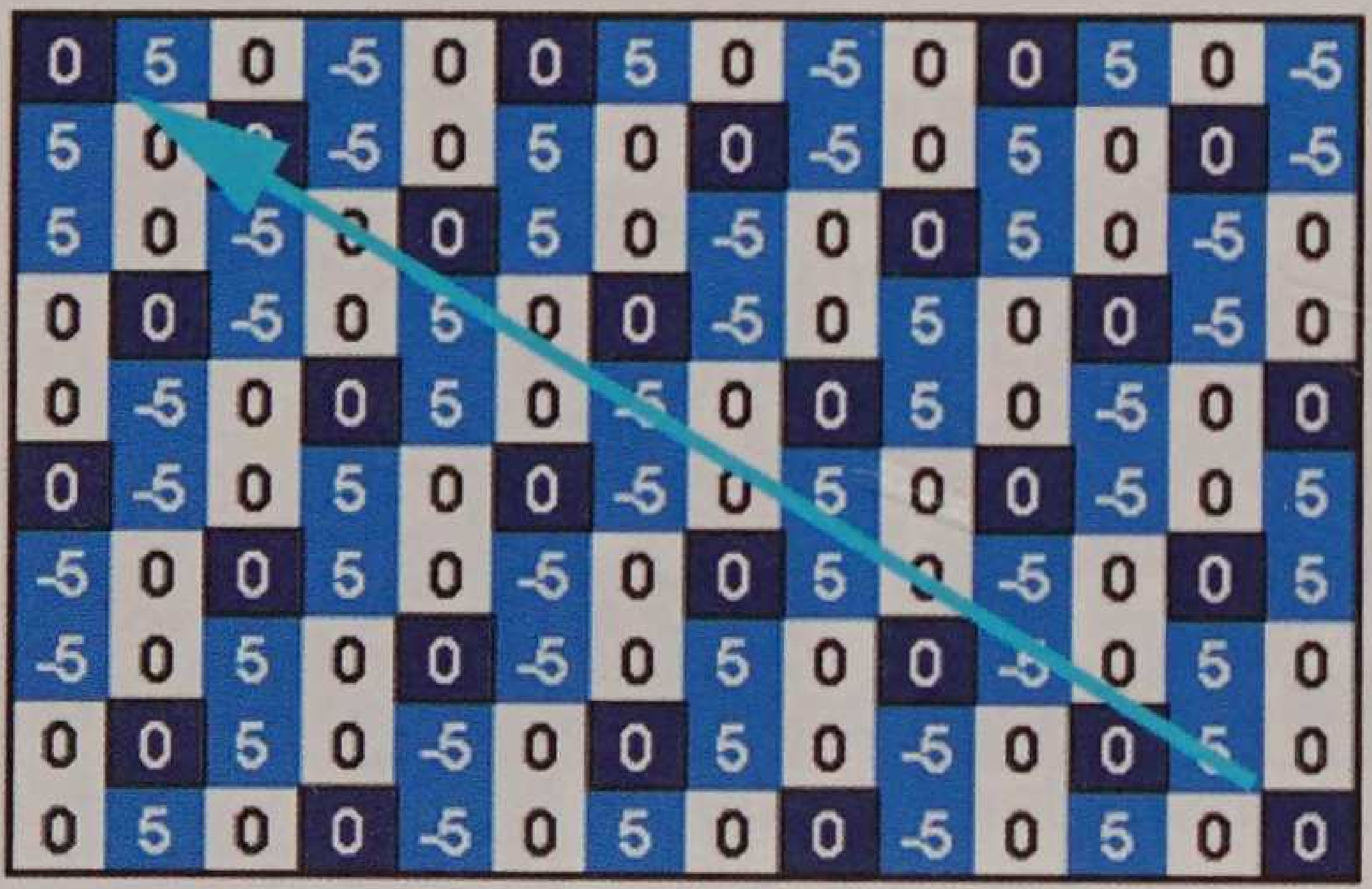


B on X^* + B on Y^*



A on (X^*+Y^*)

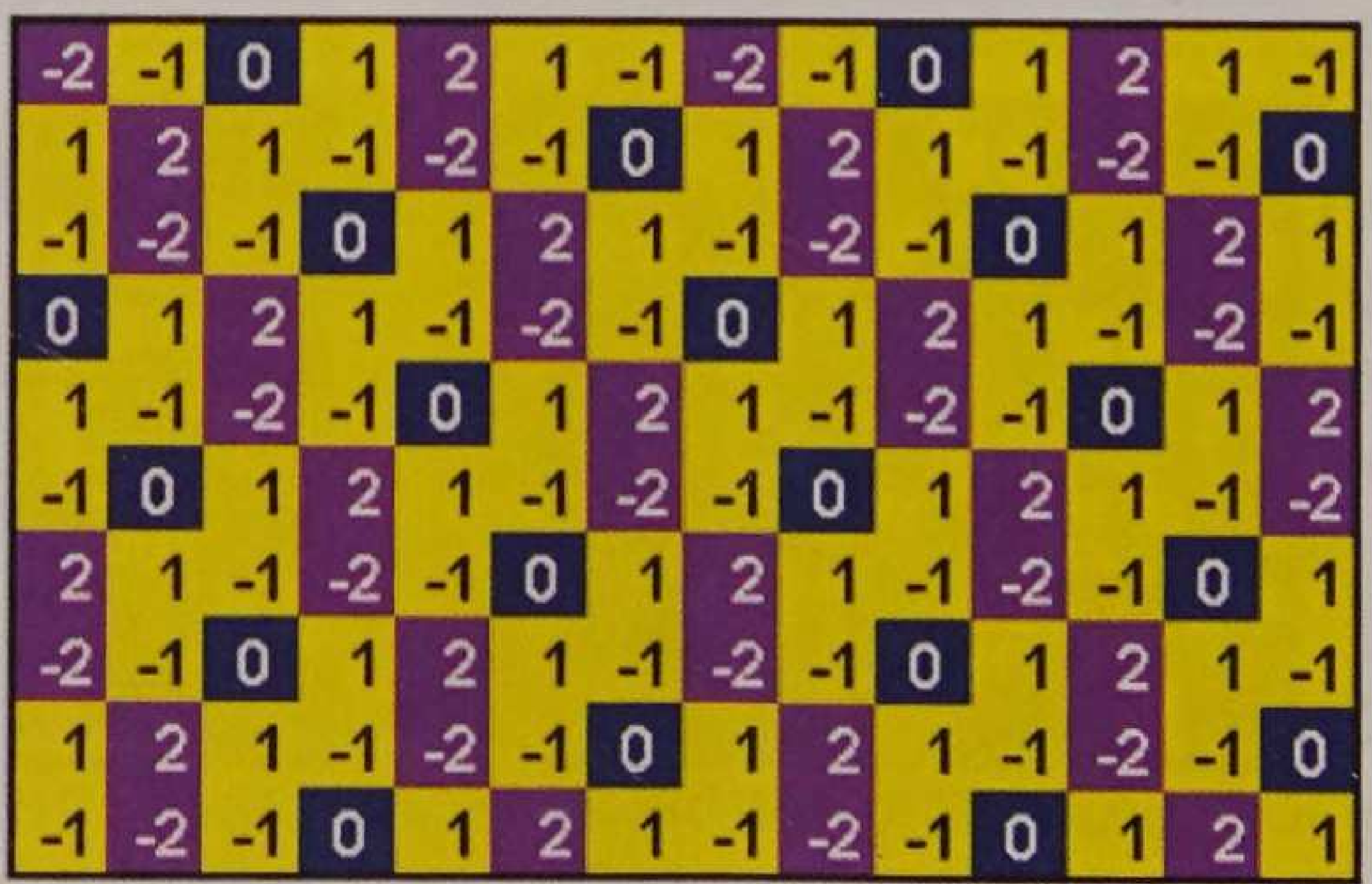
Rolling ±5 wave form



0	5	0	-5	0	0	5	0	-5	0	0	5	0	-5
5	0	0	-5	0	5	0	0	-5	0	5	0	0	-5
5	0	-5	0	0	5	0	-5	0	0	5	0	-5	0
0	0	-5	0	5	0	0	-5	0	5	0	0	-5	0
0	-5	0	0	5	0	-5	0	0	5	0	-5	0	0
0	-5	0	5	0	0	-5	0	5	0	0	-5	0	5
-5	0	0	5	0	-5	0	0	5	0	-5	0	0	5
-5	0	5	0	0	-5	0	5	0	0	-5	0	5	0
0	0	5	0	-5	0	0	5	0	-5	0	0	5	0
0	5	0	0	-5	0	5	0	0	-5	0	5	0	0

B on (X^*+Y^*)

Neutral



-2	-1	0	1	2	1	-1	-2	-1	0	1	2	1	-1
1	2	1	-1	-2	-1	0	1	2	1	-1	-2	-1	0
-1	-2	-1	0	1	2	1	-1	-2	-1	0	1	2	1
0	1	2	1	-1	-2	-1	0	1	2	1	-1	-2	-1
1	-1	-2	-1	0	1	2	1	-1	-2	-1	0	1	2
-1	0	1	2	1	-1	-2	-1	0	1	2	1	-1	-2
2	1	-1	-2	-1	0	1	2	1	-1	-2	-1	0	1
-2	-1	0	1	2	1	-1	-2	-1	0	1	2	1	-1
1	2	1	-1	-2	-1	0	1	2	1	-1	-2	-1	0
-1	-2	-1	0	1	2	1	-1	-2	-1	0	1	2	1

Program 9

X

10	4	9	3	8	2	7	1	6	11	5
2	7	1	6	11	5	10	4	9	3	8
5	10	4	9	3	8	2	7	1	6	11
8	2	7	1	6	11	5	10	4	9	3
11	5	10	4	9	3	8	2	7	1	6
3	8	2	7	1	6	11	5	10	4	9
6	11	5	10	4	9	3	8	2	7	1
9	3	8	2	7	1	6	11	5	10	4
1	6	11	5	10	4	9	3	8	2	7
4	9	3	8	2	7	1	6	11	5	10
7	1	6	11	5	10	4	9	3	8	2

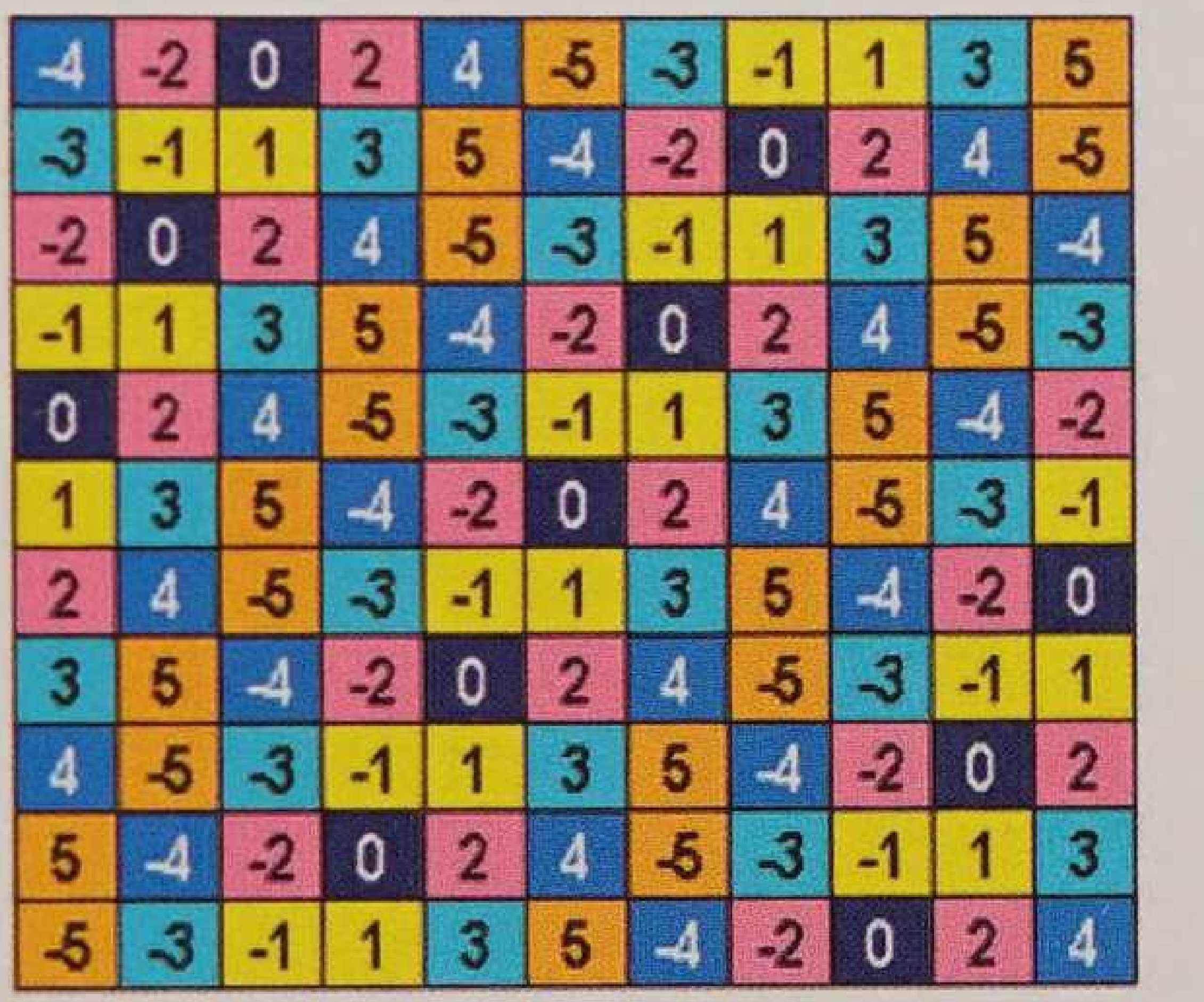
Y

2	4	6	8	10	1	3	5	7	9	11
3	5	7	9	11	2	4	6	8	10	1
4	6	8	10	1	3	5	7	9	11	2
5	7	9	11	2	4	6	8	10	1	3
6	8	10	1	3	5	7	9	11	2	4
7	9	11	2	4	6	8	10	1	3	5
8	10	1	3	5	7	9	11	2	4	6
9	11	2	4	6	8	10	1	3	5	7
10	1	3	5	7	9	11	2	4	6	8
11	2	4	6	8	10	1	3	5	7	9
1	3	5	7	9	11	2	4	6	8	10

X* =X-|6|

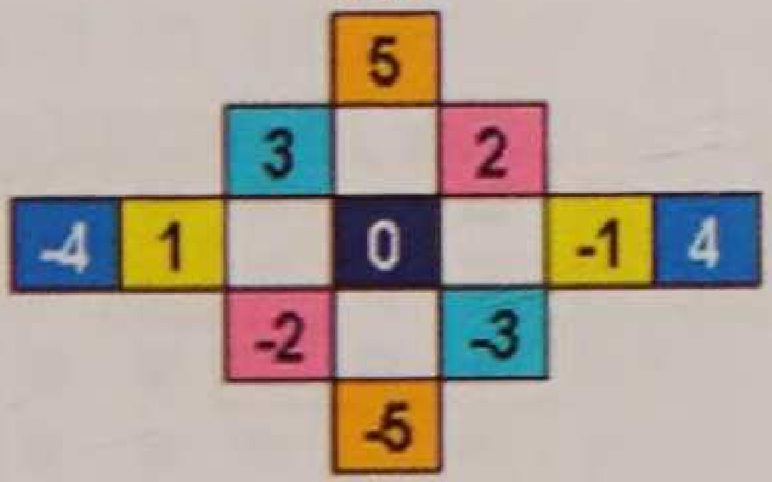
4	-2	3	-3	2	-4	1	-5	0	5	-1
-4	1	-5	0	5	-1	4	-2	3	-3	2
-1	4	-2	3	-3	2	-4	1	-5	0	5
2	-4	1	-5	0	5	-1	4	-2	3	-3
5	-1	4	-2	3	-3	2	-4	1	-5	0
-3	2	-4	1	-5	0	5	-1	4	-2	3
0	5	-1	4	-2	3	-3	2	-4	1	-5
3	-3	2	-4	1	-5	0	5	-1	4	-2
-5	0	5	-1	4	-2	3	-3	2	-4	1
-2	3	-3	2	-4	1	-5	0	5	-1	4
1	-5	0	5	-1	4	-2	3	-3	2	-4

Y* = Y-|6|

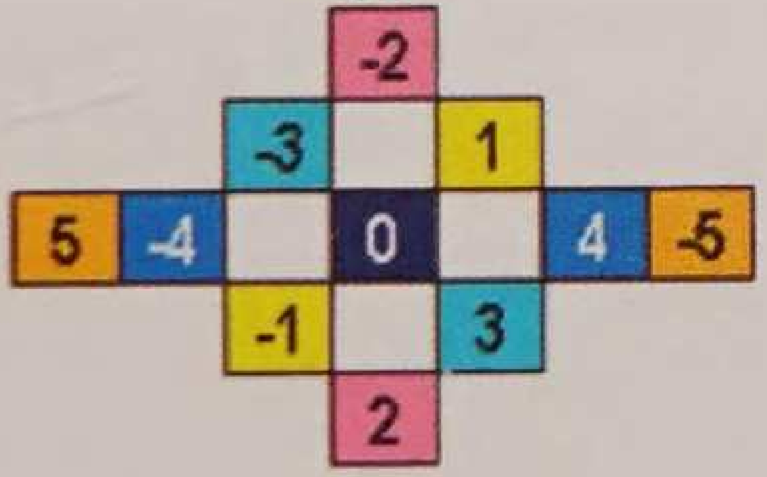


-4	-2	0	2	4	-5	-3	-1	1	3	5
-3	-1	1	3	5	-4	-2	0	2	4	-5
-2	0	2	4	-5	-3	-1	1	3	5	-4
-1	1	3	5	-4	-2	0	2	4	-5	-3
0	2	4	-5	-3	-1	1	3	5	-4	-2
1	3	5	-4	-2	0	2	4	-5	-3	-1
2	4	-5	-3	-1	1	3	5	-4	-2	0
3	5	-4	-2	0	2	4	-5	-3	-1	1
4	-5	-3	-1	1	3	5	-4	-2	0	2
5	-4	-2	0	2	4	-5	-3	-1	1	3
-5	-3	-1	1	3	5	-4	-2	0	2	4

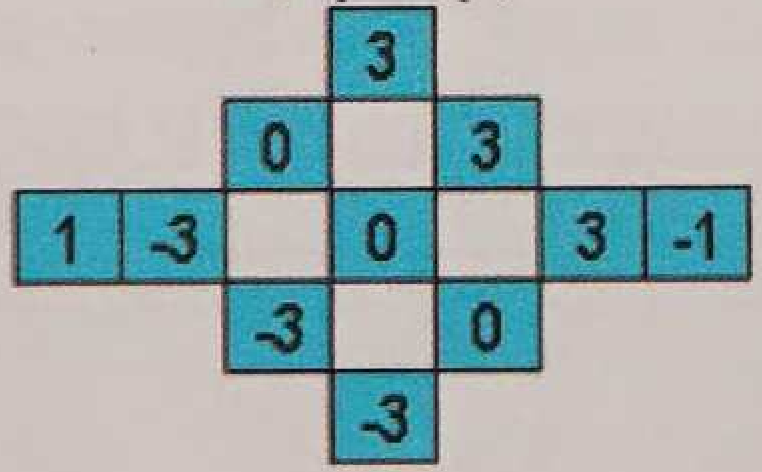
AX*



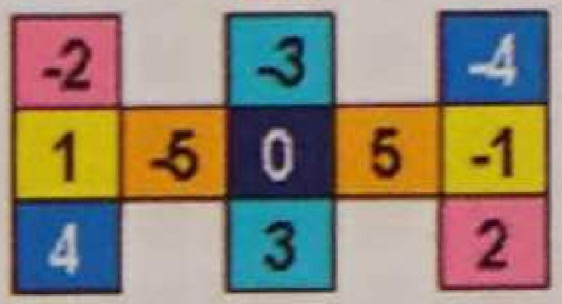
AY*



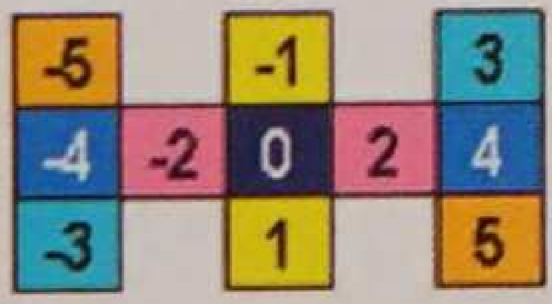
A(X*+Y*)



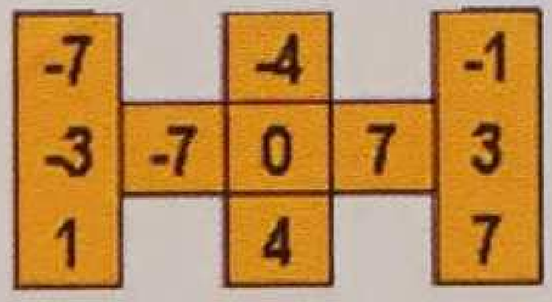
BX*



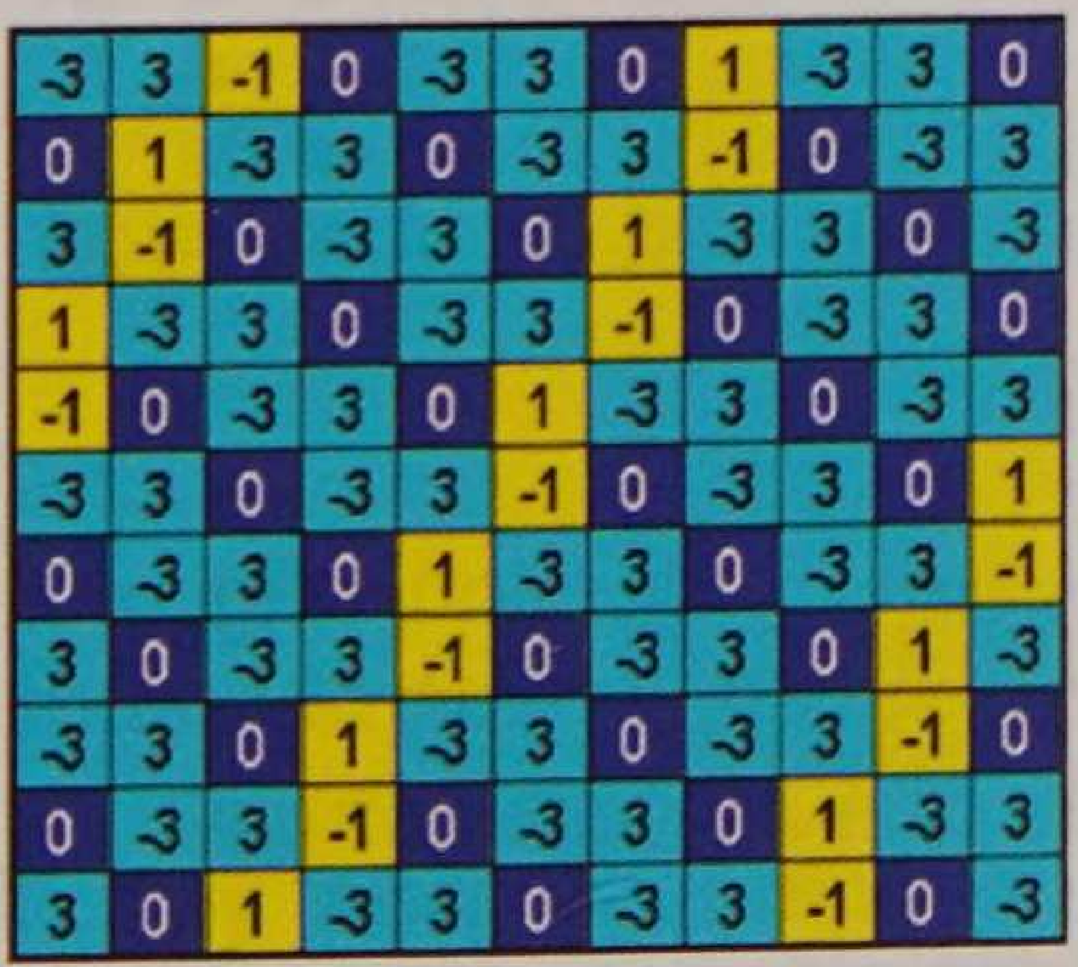
BY*



B(X*+Y*)

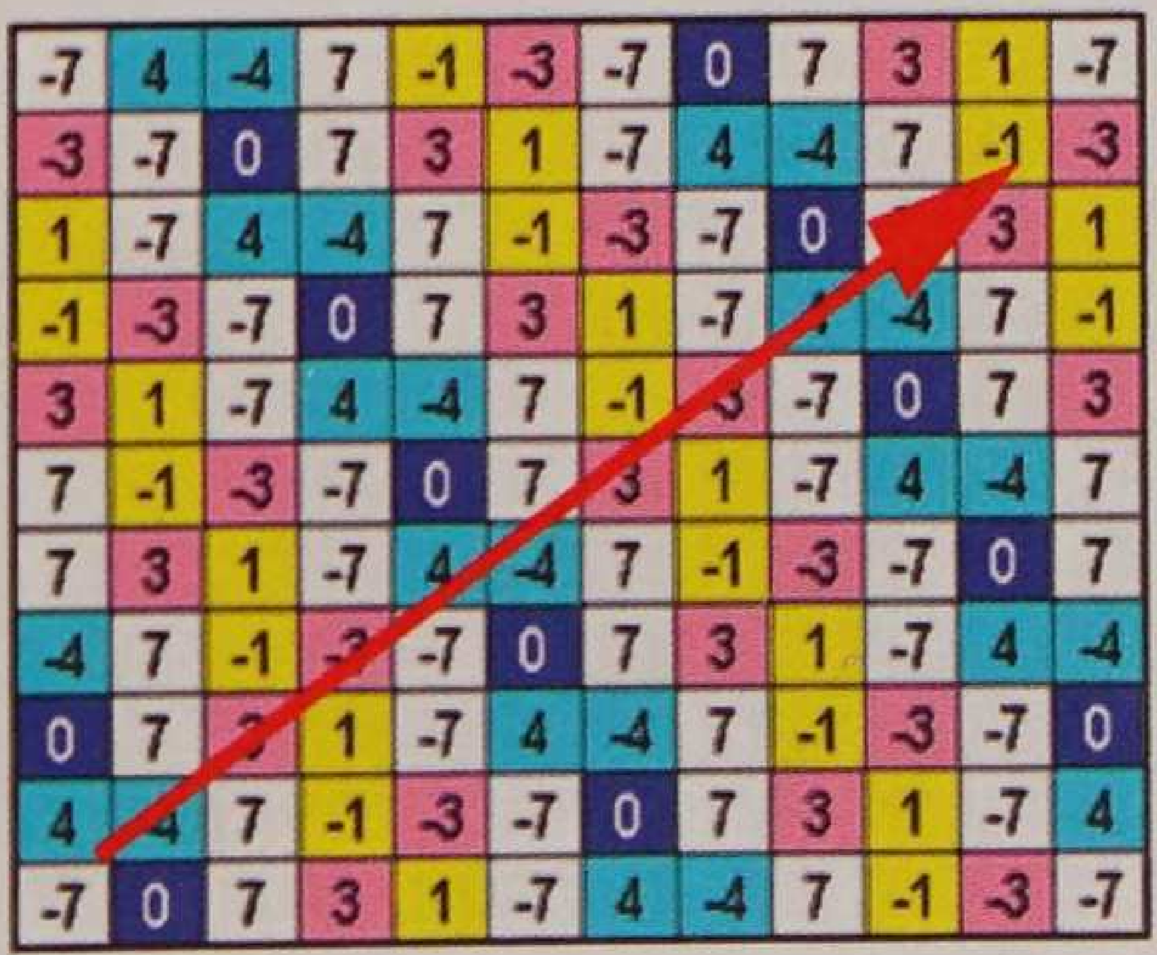


A on X*+Y*
Neutral



-3	3	-1	0	-3	3	0	1	-3	3	0
0	1	-3	3	0	-3	3	-1	0	-3	3
3	-1	0	-3	3	0	1	-3	3	0	-3
1	-3	3	0	-3	3	-1	0	-3	3	0
-1	0	-3	3	0	1	-3	3	0	-3	3
-3	3	0	-3	3	-1	0	-3	3	0	1
0	-3	3	0	1	-3	3	0	-3	3	-1
3	0	-3	3	-1	0	-3	3	0	1	-3
-3	3	0	1	-3	3	0	-3	3	-1	0
0	-3	3	-1	0	-3	3	0	1	-3	3
3	0	1	-3	3	0	-3	3	-1	0	-3

B on X*+Y*
Rolling ±7 wave form



-7	4	-4	7	-1	-3	-7	0	7	3	1	-7
-3	-7	0	7	3	1	-7	4	-4	7	-1	-3
1	-7	4	-4	7	-1	-3	-7	0	[illegible]	3	1
-1	-3	-7	0	7	3	1	-7	4	-4	7	-1
3	1	-7	4	-4	7	-1	-3	-7	0	7	3
7	-1	-3	-7	0	7	-3	1	-7	4	-4	7
7	3	1	-7	4	-4	7	-1	-3	-7	0	7
-4	7	-1	-3	-7	0	7	3	1	-7	4	-4
0	7	-3	1	-7	4	-4	7	-1	-3	-7	0
4	-4	7	-1	-3	-7	0	7	3	1	-7	4
-7	0	7	3	1	-7	4	-4	7	-1	-3	-7

Here are the size-11 loom-tables. Each has been balanced anti-symmetrically by subtracting **6** from each number. The equal absolute values have been highlighted by identical coloring.

From these patterns, **AX, AY, BX** and **BY** are derived. Then tile pattern **A** is integrated to cover **X+Y** and likewise for tile pattern **B**.

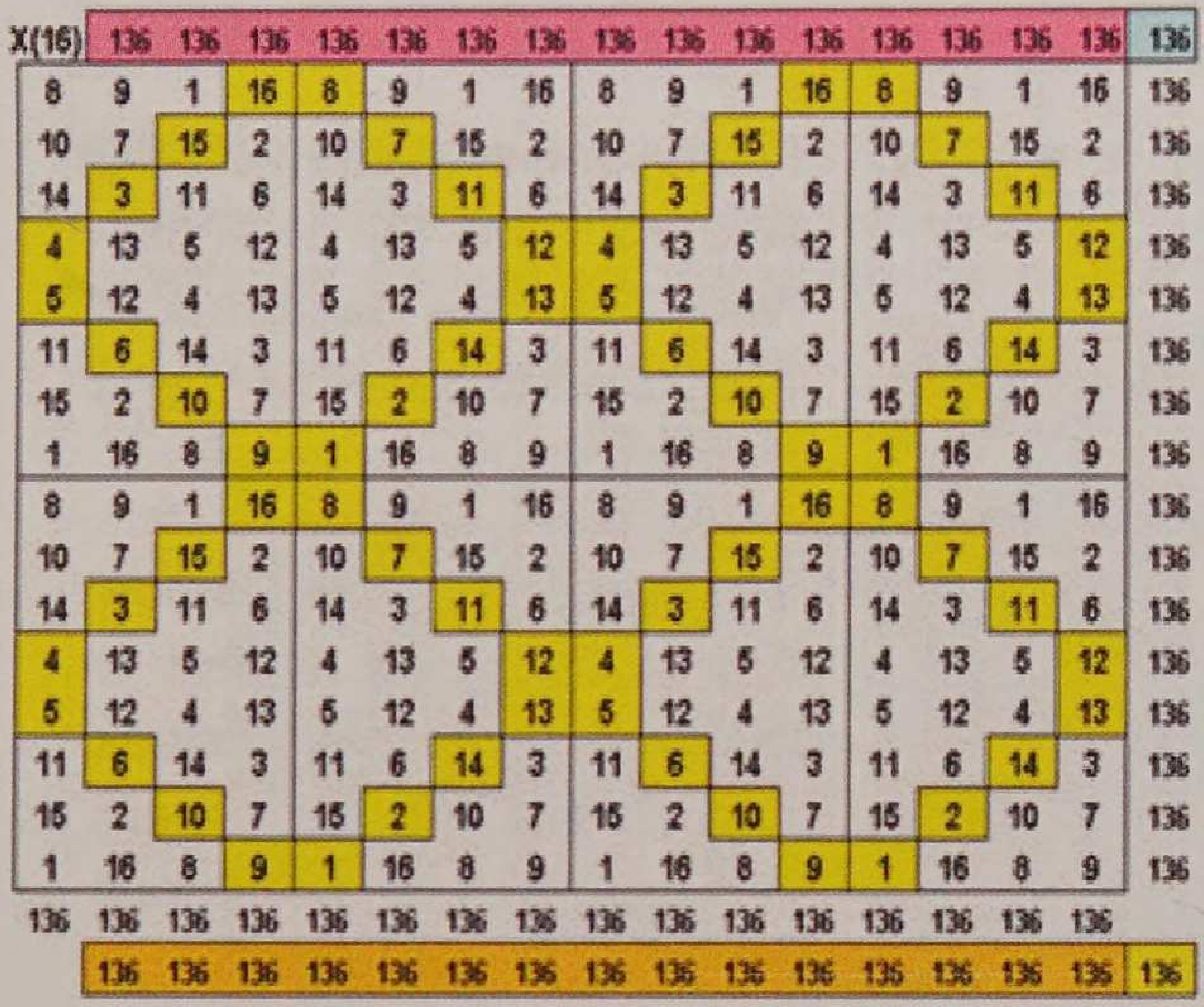
The patterns on **X+Y** show that **A** becomes neutral in that all adjacent values cancel out. Only tile pattern **B** possesses a continuous **±7** rolling wave pattern.

This procedure will fathom a continuous wave pattern for every Class-1 square. Consequently, it is these patterns that should be explored for propulsion and levitation.

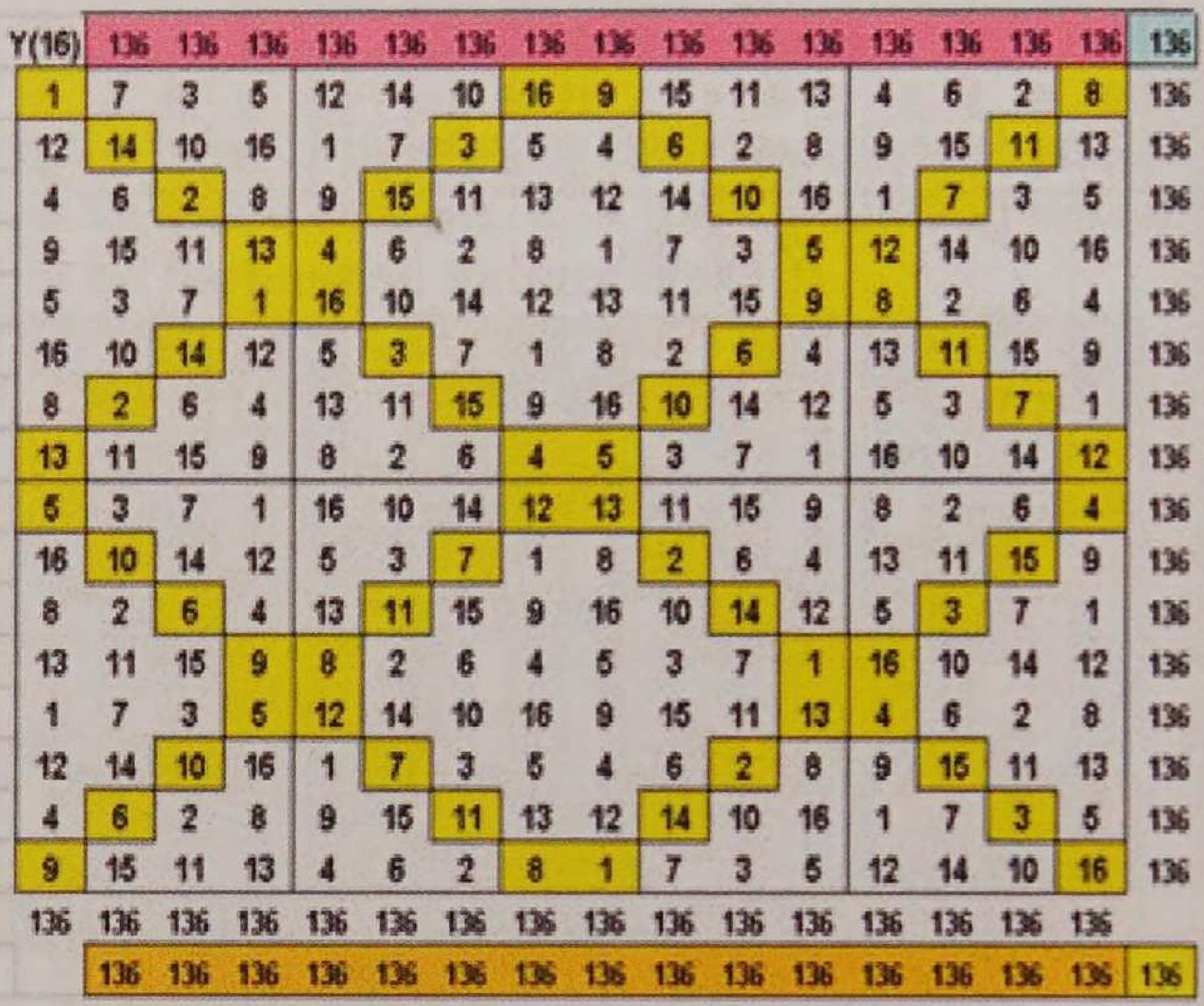
Note that the tiling patterns on **X** and **Y** contain all the number from **0** thru **(n-1)/2**. This only occurs for an ultra-perfect rendition of the Class-1 square; viz. it possess both complementary characteristic tiling patterns.

Program 9

Next, we'll look at Class-4 squares which also have continuous tiling patterns. The smallest size Class-4 square whose loom-tables independently possess both tiling patterns ◊ and **X** is the size-16. Here are the loom-tables **X** and **Y**. We have observed that both are pairwise centrally symmetrical. If each loom-table is flipped both vertically and horizontally and added back to itself, we get a table of all constants Z in both cases. If this pattern were to be projected onto a confined fabric of space of size-16, a wall of positive constants would emerge – and this is the sought after propulsion force needed to surf "empty" space.



X(16)	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136
8	9	1	16	8	9	1	16	8	9	1	16	8	9	1	16	136
10	7	15	2	10	7	15	2	10	7	15	2	10	7	15	2	136
14	3	11	6	14	3	11	6	14	3	11	6	14	3	11	6	136
4	13	5	12	4	13	5	12	4	13	5	12	4	13	5	12	136
5	12	4	13	5	12	4	13	5	12	4	13	5	12	4	13	136
11	6	14	3	11	6	14	3	11	6	14	3	11	6	14	3	136
15	2	10	7	15	2	10	7	15	2	10	7	15	2	10	7	136
1	16	8	9	1	16	8	9	1	16	8	9	1	16	8	9	136
8	9	1	16	8	9	1	16	8	9	1	16	8	9	1	16	136
10	7	15	2	10	7	15	2	10	7	15	2	10	7	15	2	136
14	3	11	6	14	3	11	6	14	3	11	6	14	3	11	6	136
4	13	5	12	4	13	5	12	4	13	5	12	4	13	5	12	136
5	12	4	13	5	12	4	13	5	12	4	13	5	12	4	13	136
11	6	14	3	11	6	14	3	11	6	14	3	11	6	14	3	136
15	2	10	7	15	2	10	7	15	2	10	7	15	2	10	7	136
1	16	8	9	1	16	8	9	1	16	8	9	1	16	8	9	136
136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	
	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136



Y(16)	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136	136
1	7	3	5	12	14	10	16	9	15	11	13	4	6	2	8	136
12	14	10	16	1	7	3	5	4	6	2	8	9	15	11	13	136
4	6	2	8	9	15	11	13	12	14	10	16	1	7	3	5	136
9	15	11	13	4	6	2	8	1	7	3	5	12	14	10	16	136
5	3	7	1	16	10	14	12	13	11	15	9	8	2	6	4	136
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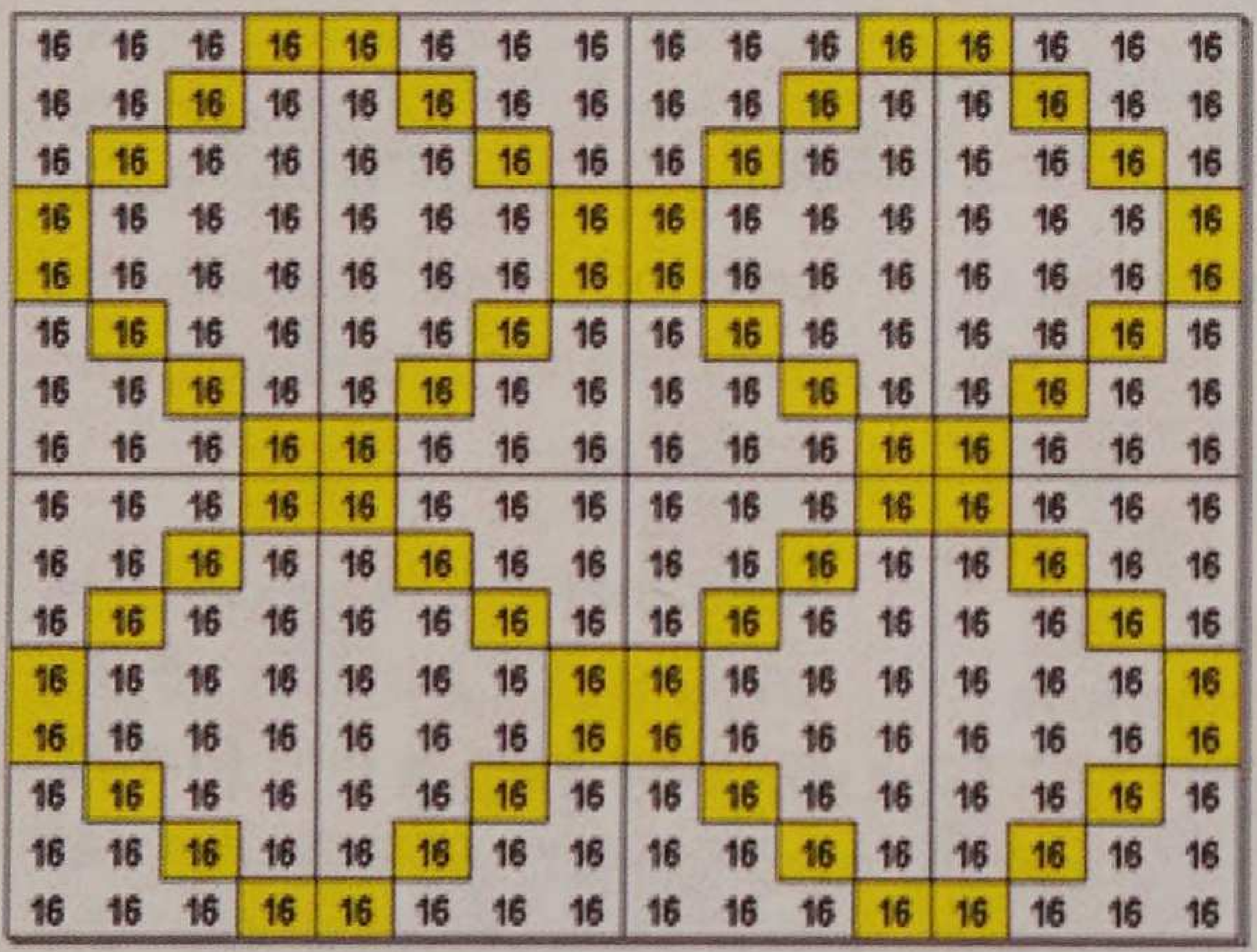
Z1 = X + hvX - |1|



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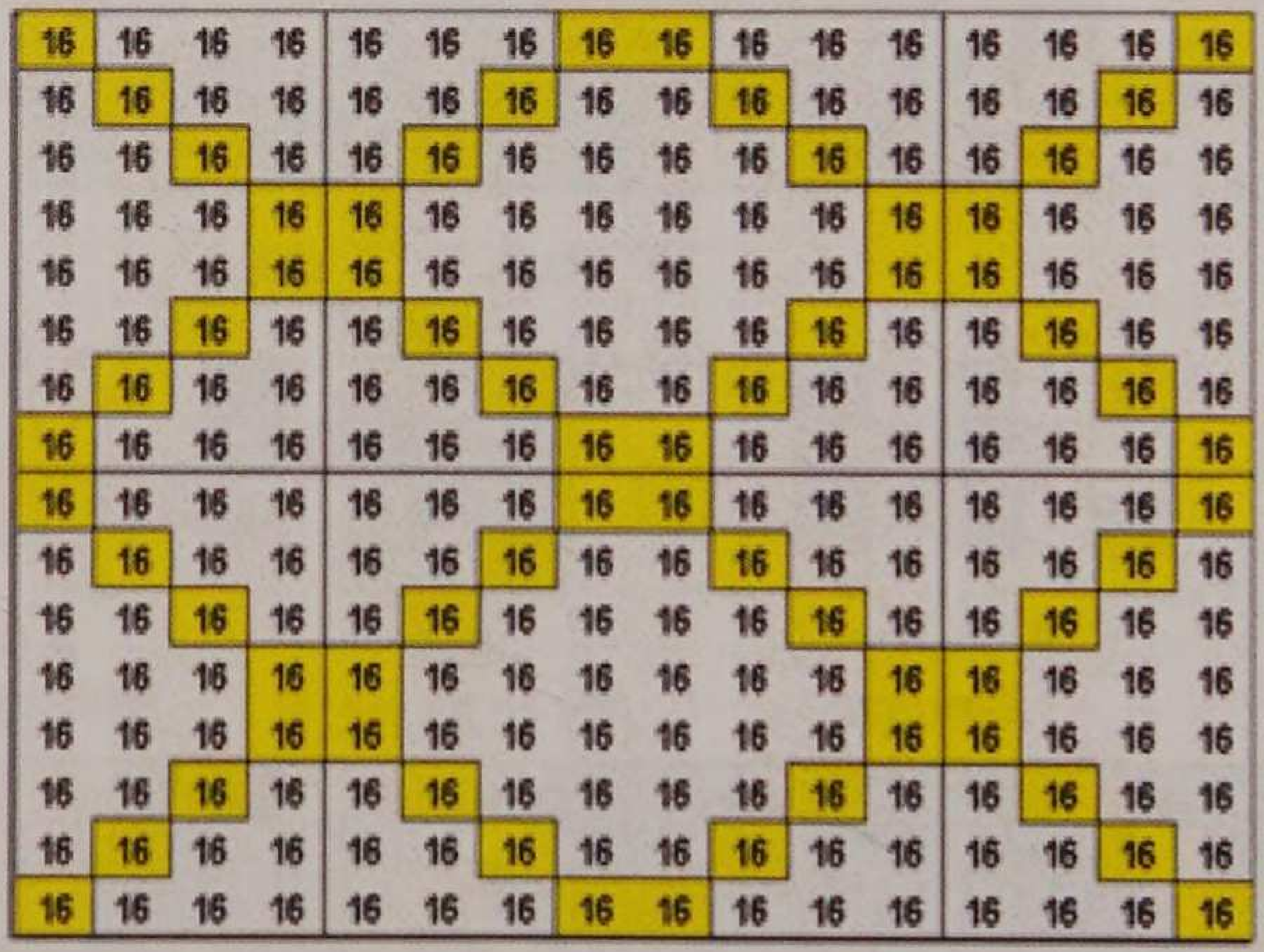
Z2 = Y + hvY - |1|



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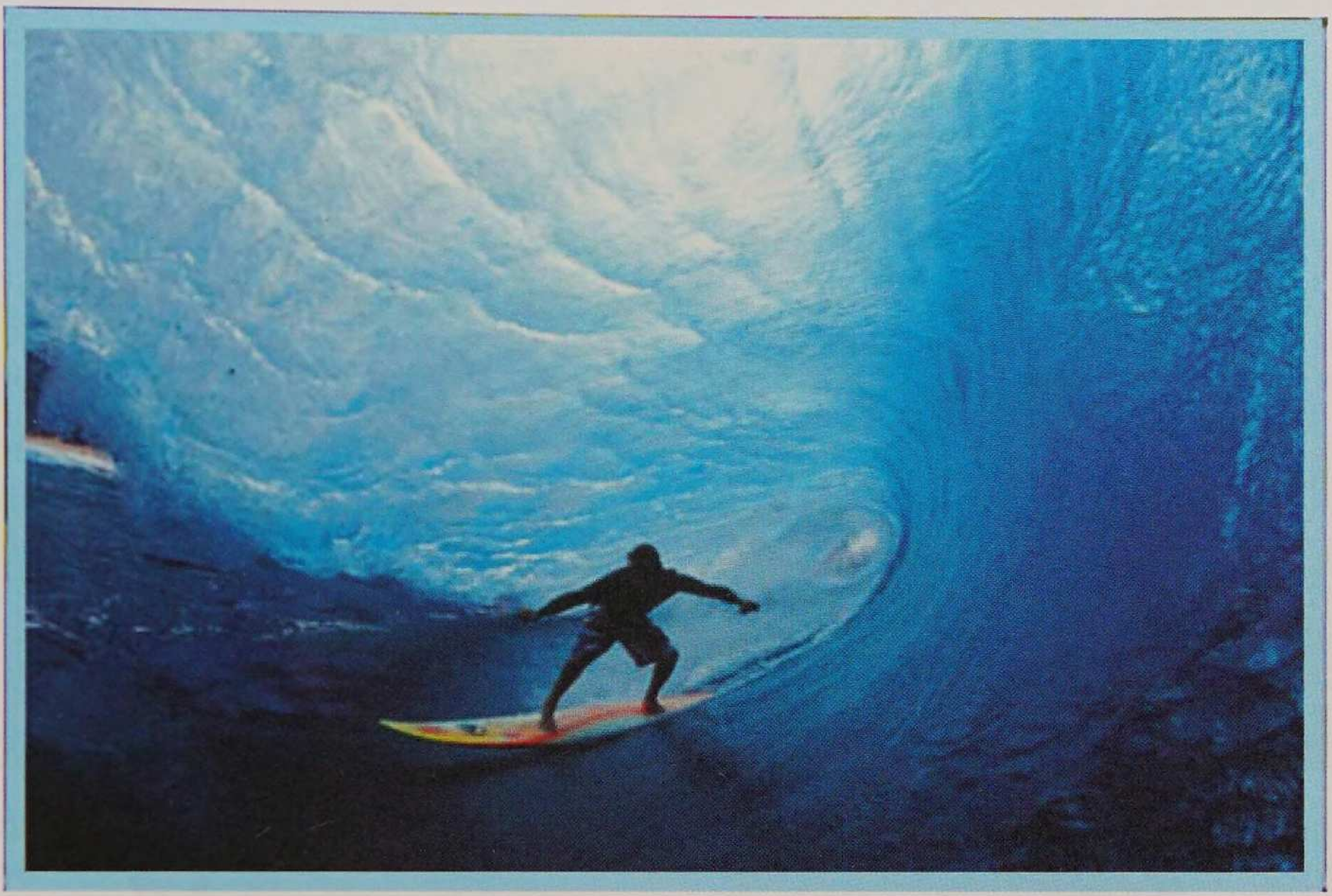
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

So now it should be clear as to why the Starship Enterprise has two propulsion engines: they're just not only for balance, but each one is for generating the double flip version of one of the dual loom patterns simultaneously !!! And that provides **$2n^2$** points of **n**-powered thrust.

Now that these spatial patterns have been identified and measured mathematically, the trial-and-error research can begin looking for them with detectors using phasers (sound), lasers or nano-pulse generators. And once discovered, analyzed and categorized, the spatial fabric could be developed for levitation and spatial propulsion in relatively short order.

Just as we have used the undulating patterns in the oceans to surf its waves,



and in times of civilizations past where we used the sails on boats to capture the circular patterns of wind to cross the ocean-divides and discover new continents,

it may now be possible to identify the spatial harmonic vibrations among the vibrating membranes within the fabric of space itself that could well be harnessed to surf the universe throughout 3- and 4-dimensional space. We just have to tap into the fabric of 3-dimensional space to get into hyperspace.

And once this can be done with a yet to be developed spatial surfing technology, Mankind will be able to ride the continuously pulsating spatial fabric and discover new worlds!

The challenge for the next generation of scientists is to find ways to begin the exploration of the 4th dimension. As you have already seen, Geonometry can provide some fundamental guidance in such an undertaking.

Then a whole new world would open up just as it has in the modern era of today with all the electronics and electrical mechanics unknown only 1½ centuries ago. Then the jet engine, long airport runways and long-distance haulage by truck and rail will become a thing of the past.

Star Trek Enterprise breaking through the 3rd dimensional light–speed barrier while going into the 4th dimension

This picture is a virtual representation of how the spaceship such as the famous **Starship Enterprise** would appear upon breaking through the light-speed barrier of the 3rd dimension as seen from within 4-dimensional space as it entered the 4th dimension. Note the harmonic patterned trail it leaves behind it as it surfs the harmonic vibrations of 4-dimensional space.

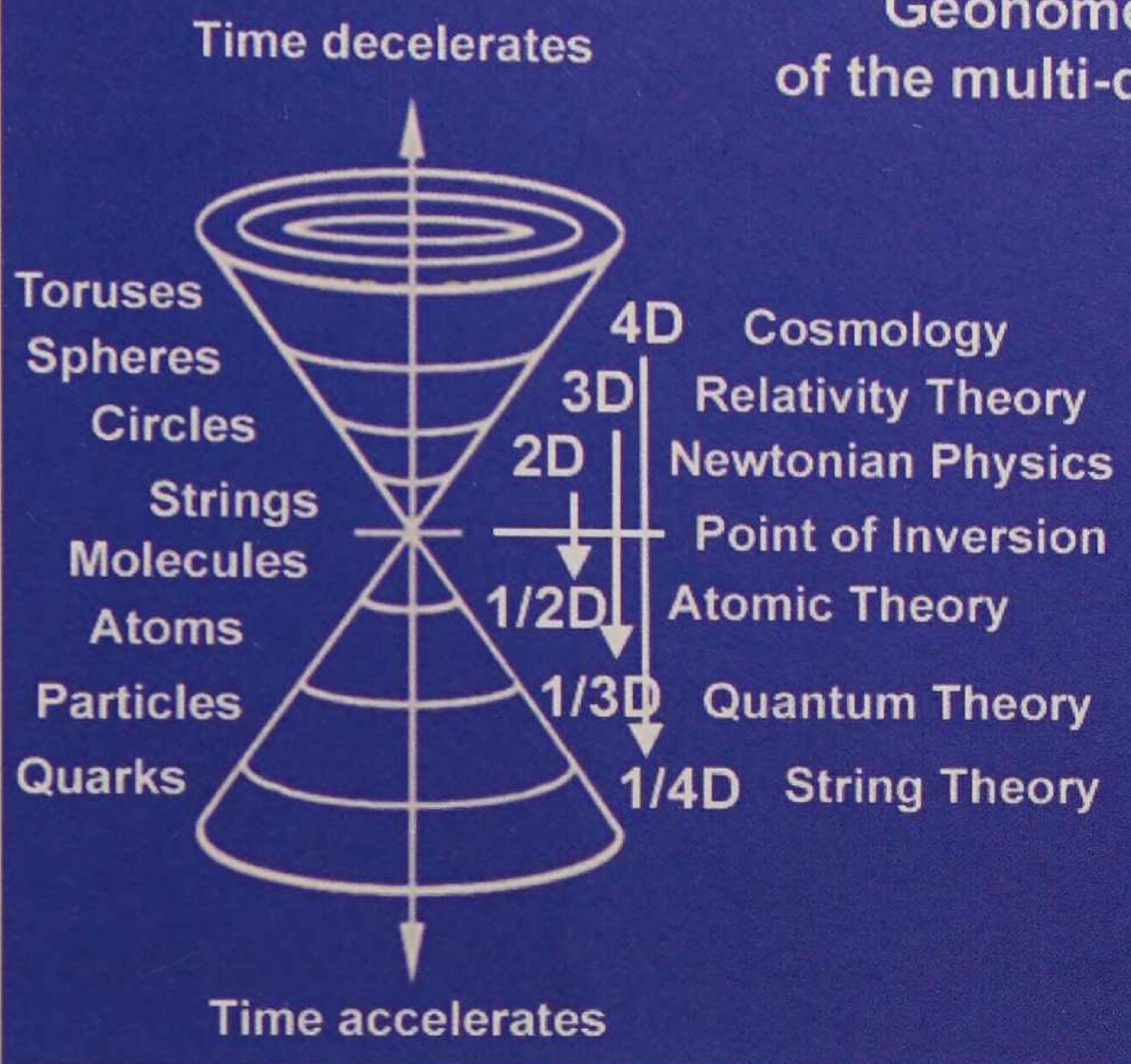
This program series has shown mathematically just why these spatial harmonics haven't yet been observed by Science. Program 3 demonstrated that there are two complementary fundamental vibrational patterns which must coexist together simultaneously and that these dual patterns continuously cloak one another! The spatial *ether* is there, but to detect it, it needs to be exposed by removing, replacing or mimicking one of the complementary vibrational layers in the natural geonomic pattern with one of our own fabrication.

If you desire to participate in this very next major development, you will need to know the principles and properties of Geonometry and seek a job in fundamental research discovering and harnessing them.

Program 9

Finally, Geonometry vs. Cosmology

Geonometry's Big Picture of the multi-dimensional universe



Dimension	Time Constant	
4	Mega-second	10^4
3	Kilosecond	10^3
2	Decasecond	10^2
1	Second	10^1
1/2	Decisecond	10^{-1}
1/3	Microsecond	10^{-2}
1/4	Millisecond	10^{-3}
1/5	Nanosecond	10^{-4}

Below the point of inversion into the sub-dimensional realm, space expands inwardly (gets more dense and complex) and time accelerates proportional to the dimensional distance from the point of inversion. (Observe the time table to the far right.)

Above the point-of inversion, space expands outwardly (gets spread apart and rarified) and time decelerates proportional to the dimensional distance from the point of inversion.

To the right of the figure is listed the various scientific theories that have emerged for explaining the structure of space at the various dimensional levels. The big effort today in Cosmology is to find some mathematical principle to unite all these formulations into one grand theory of the universe. It's an attempt by Science to discover the superior intelligence underlying the interwoven structure of the spatial fabric.

Even today, String Theory is graduating from the mathematics involving string vibrations to one involving vibrational patterns of membranes, called M-Theory. That is, Cosmology is evolving from a 1-dimensional viewpoint to a 2-dimensional view; one from a linear perspective to one that's planar – supposedly a sign of progress. Yet still Cosmology can't tie all the different dimensional mathematics into a single coherent theory.

Geonometry is the expansion of linear arithmetic to the arithmetic of 2-dimensions and beyond. In this same quest, Geonometry is the math that does have the potential to uncover that hidden fundamental design without recourse to all that far-out convoluted 11-dimensional math of Cosmology.

Whereas Cosmology extends its math out to 11 dimensions of space, the accomplishments of Geonometry thus far in this same context were made possible by simply confining the exploration of harmonic interwoven numeric patterns to fixed prime-number size spaces and in the process uncovering the very structure of space itself. You be your own judge if this new math has succeeded.

Program 9

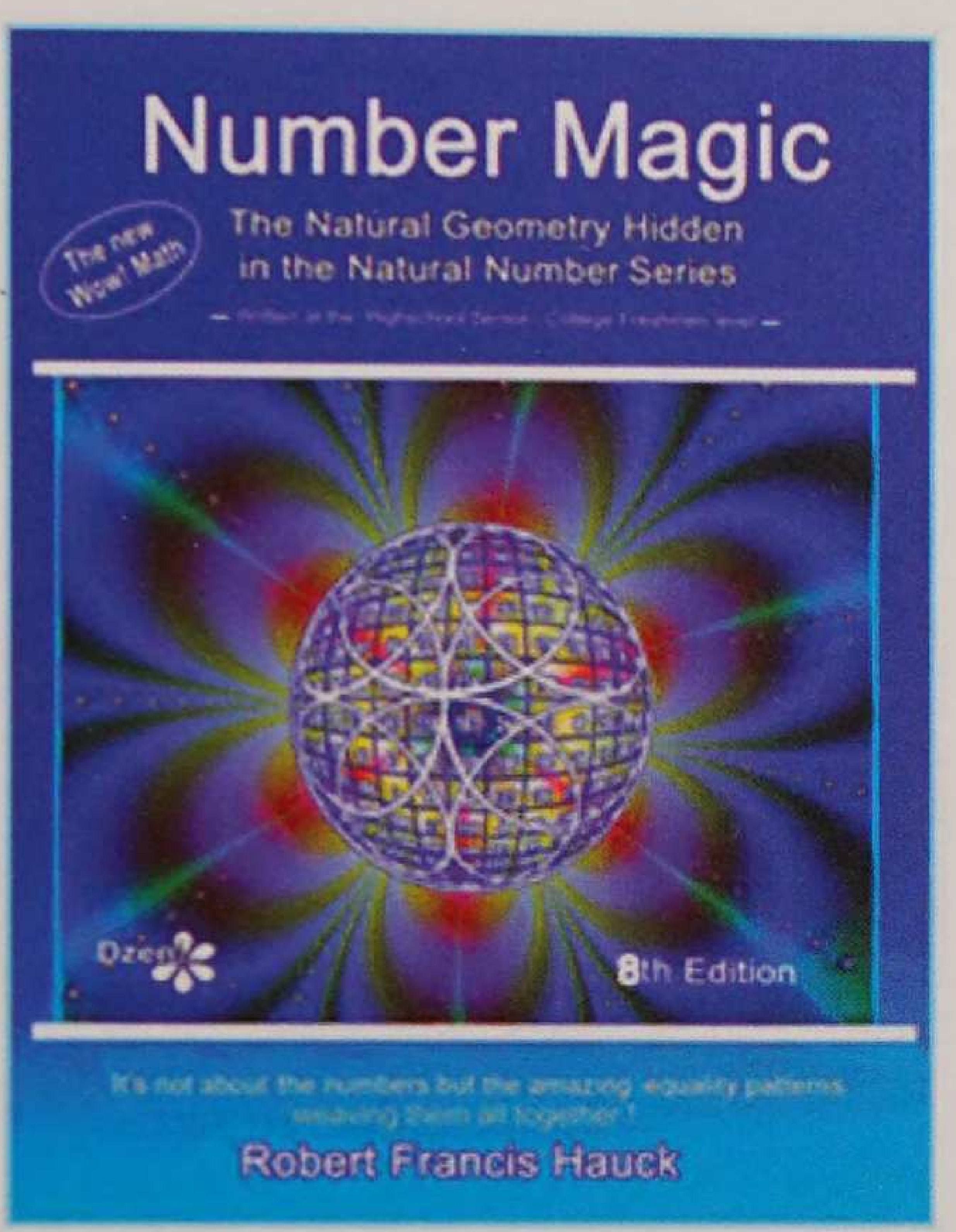
Notes

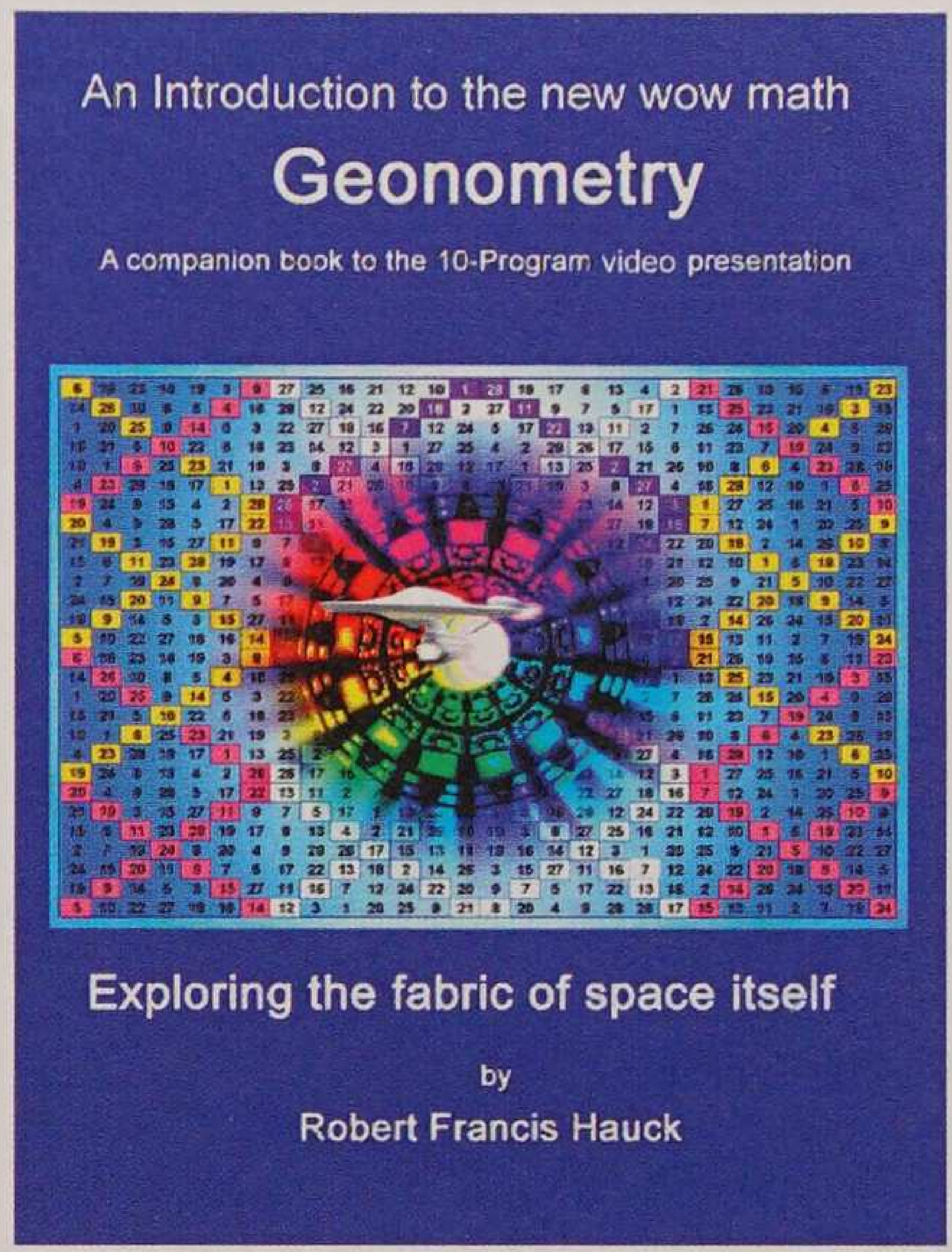
1. Geonomic space can be navigated by a series of successive Class-1 duals in 3-dimensions and higher.
2. The complete 4-dimensional space consists of four intersecting toruses forming a hyper-sphere as seen from the 5th dimension.
3. Dimensions 4 and 5 are fractal in that Class-1 size **n** quadracubes and quintacubes project onto to 2-dimensional space as perfect size $\mathbf{n}^2$ Class-6 squares.
4. The characteristic number of a matrix product of complementary loom tables is the square of the loom tables' common characteristic number.
5. A matrix product of compatible tiled expansion tables results in a matrix of all constant numbers equal to the product of the characteristic numbers of their constituent tiling squares.
6. The series of kernel numbers which equal the characteristic number for hypercubes of any dimension can be expressed as a function solely of its size **n**.
7. The complementary loom-tables of all Class-1 squares have a matrix inverse whose common denominator is an integer that can be factored out leaving all the numerators expressed as integers in matrix form. Further, the pattern of numbers in the matrix of numerators follows a consistent numeric pattern throughout each size which geonomically has as its characteristic number its own size **n**.
8. Loom-tables have a dominant eigenvalue equal to their characteristic number which can be expressed solely in terms of its size **n**.
9. The determinants of loom-tables all have a very simple closed form as a function solely in terms of their size **n**.
10. Determinants of perfect squares cannot be expressed as a closed formula like its loom-tables can.

Program 9

Here is the master book **Number Magic** which contains much of the material presented in this program series plus containing numerical tables for all legible sizes of square, cube and hypercubes of 4 & 5-dimensions. The only book which contains all of the material presented in this program series is **An Introduction to the New Wow Math -- Geonometry**. You should have had in it your possession throughout this entire program series.

It is available only from the publisher's internet website **www.CreateSpace.com**





Number Magic – The Natural Geometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension.

Eighth Edition
Black & white print (350+ pages)

An Introduction to the new wow math Geonometry

ISBN 978-1-479-23823-1

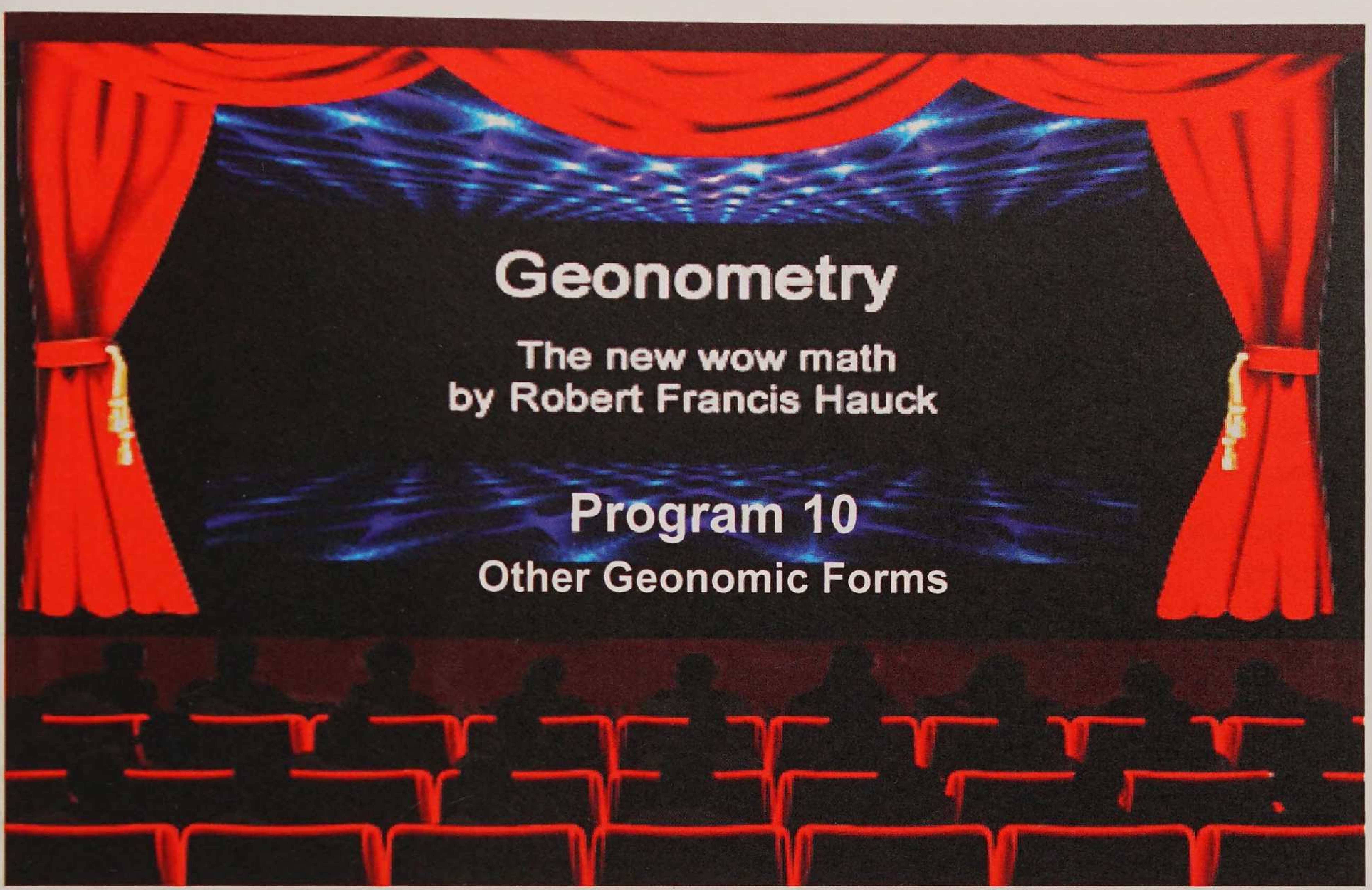
Contains all the slides and narration in this 10-program video series.
Selected examples.

Fifth Edition
Printed in color (380+ pages)

In the next program, we will explore other geonomic forms.

There we will discover through the application of Geonometry to hexagons just what makes snowflakes so hexagonally symmetrically perfect. Scientists are to this day unable to systematically explain this phenomenon by the chemical makeup of water alone. That program will provide yet the final fundamental proof that Geonometry is truly uncovering the real hidden fabric of space.

Program 10



Other Geonomic Forms

First well see how easy it is to derive the loom-tables for generating geonomic triangles.

Then we'll look at generating geonomic diamonds from perfect adjacent-sized geonomic squares. And in the process, we'll discover a second method that when it is applicable that will yield an identical result as the brute-force method does but in a very obtuse manner.

Then we'll look at generating geonomic hexagons from hexagonal loom-tables that employ some of the same basic principles we discovered in triangles. Here we will observe the inherent vibratory membrane for snowflake formation.

Geonometry shows definitively just what makes snowflakes so hexagonally symmetrically perfect. Scientists still to this day are unable to systematically explain this phenomenon by the chemical makeup of water and the static electricity of vapor. In otherwords, Science cannot use its own knowledge resources to provide even a near plausible explanation that doesn't rely on some undetectable perfectly symmetrical force. It demonstrates once again that Geonometry is uncovering the real hidden fabric of space.

Triangles

Here are three of the smallest triangles with equal-summing sides. The center triangle with sides of length **2** is not a viable frame for equal-summing sides because it is impossible to have **3** distinct numbers to sum equally together in pairs. So all summations within triangular frames will exempt the central triangle of size-2.

On the otherhand, all triangles will have equal-summing corner triangles consisting of just **3** cells each. Their common sum is indicated by the external numbers in blue at the three corners.

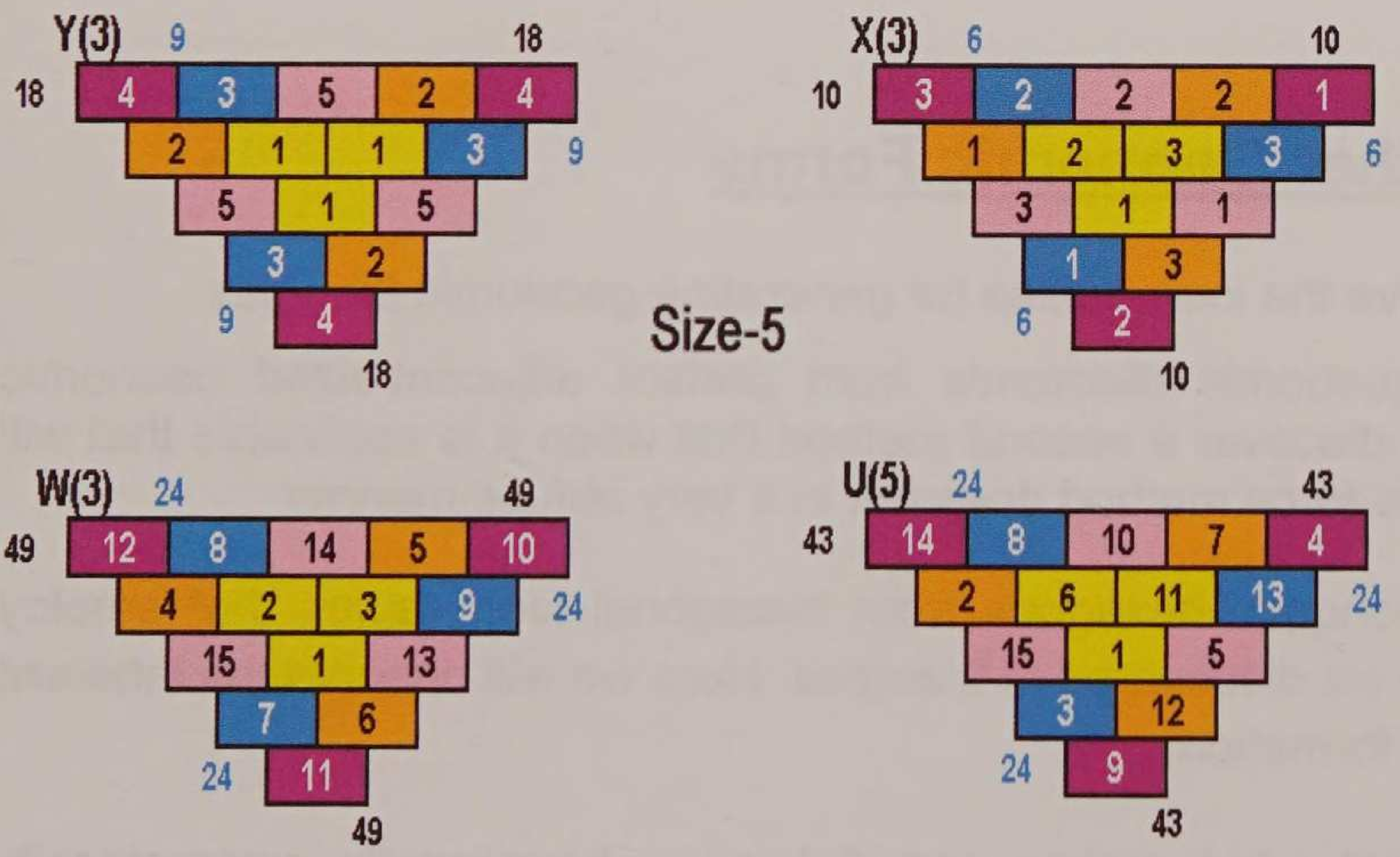
The size-5 triangle has only a single viable frame; the size-7 triangle has two viable frames; and the size-9 has three.

Both loom tables are shown along with the dual triangles generated from them according to the formulas:

(10.1) $$W_n(3) = 3(Y_n(3) - |1|) + X_n(3)$$

(10.2) $$U_n(m) = m(X_n(3) - |1|) + Y_n(3)$$

(10.3) $m \equiv \mathrm{Int}[N_n/3]$ where N_n is the number of cells in a triangle of size n.

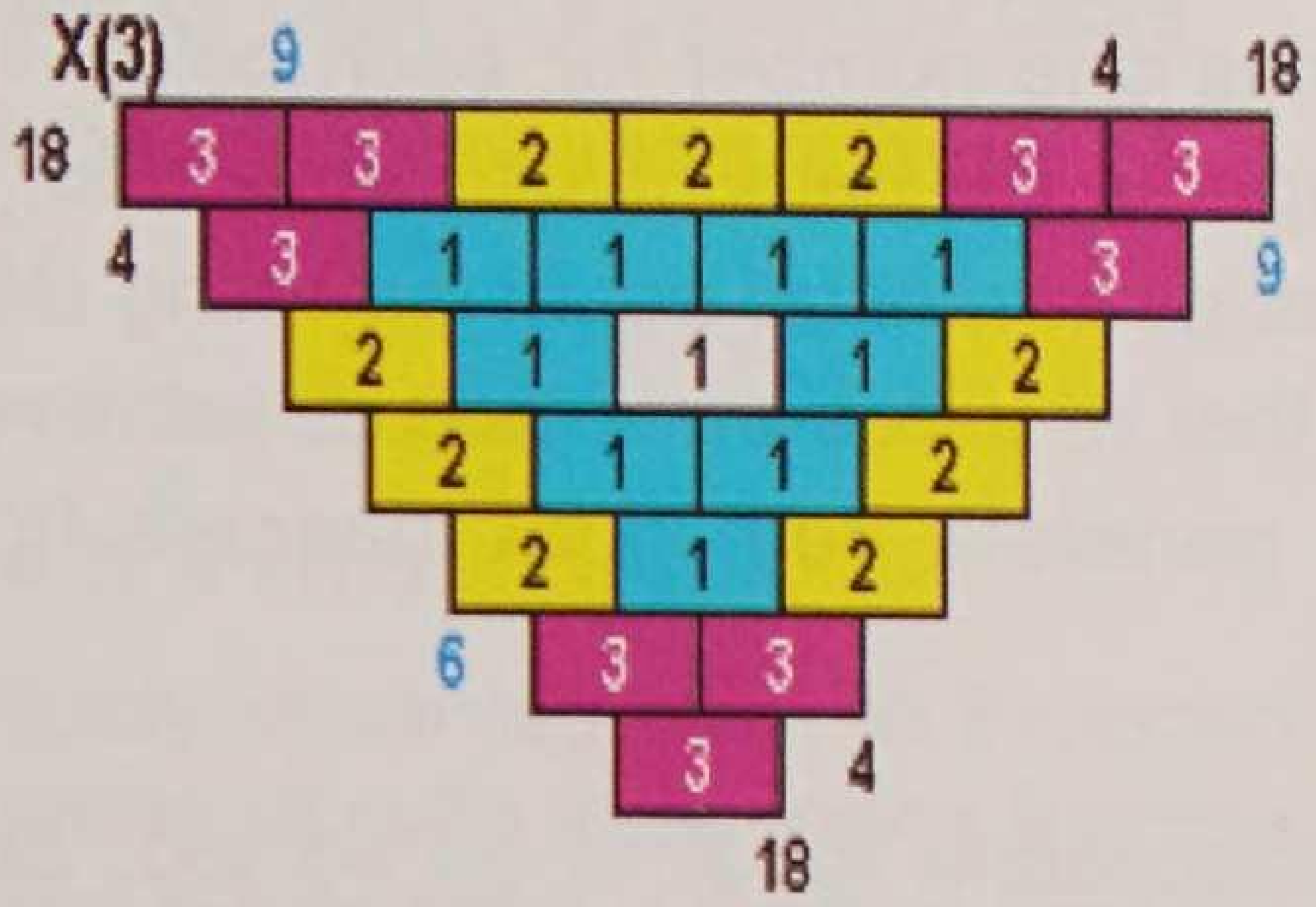


Geonomic triangles are fairly easy to construct from loom tables. In the case of the size-5 there are **5** distinct colors of **3** cells each which cover all **15** cells symmetrically.

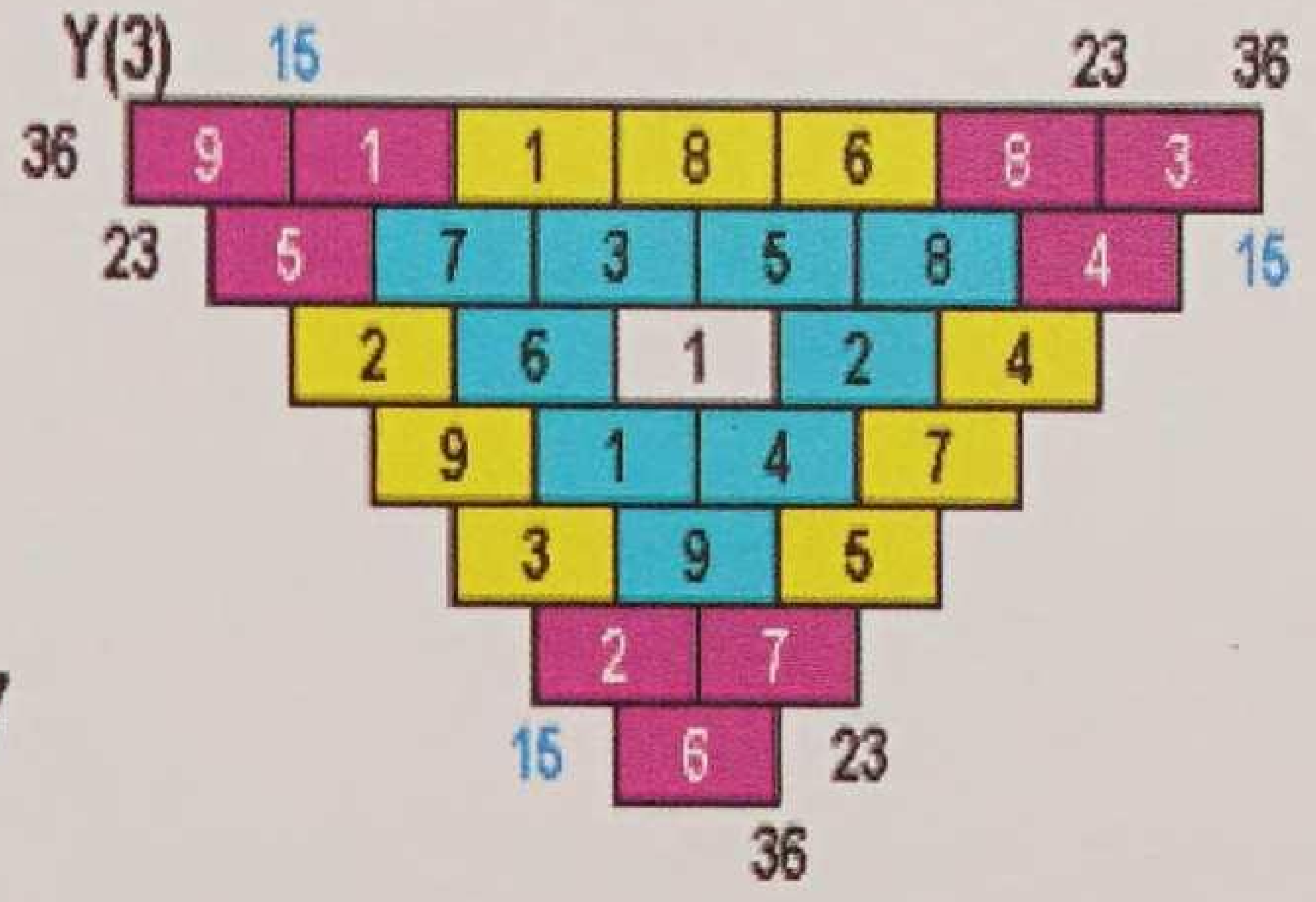
In the size-7 triangle with **28** cells, there are **3** colors of **9** cells each and which together symmetrically cover all the cells except the central one. In these cases, the central cell is left alone uncolored and filled with the number **1** in all of its tables. Further all frames are included in having equal-summing sides.

In the size-9 triangle with **45** cells, there are **5** colors of **9** cells each, which altogether symmetrically cover all the cells. All of its frames have equal-summing sides. These three examples cover the three basic situations that arise in triangles.

Program 10

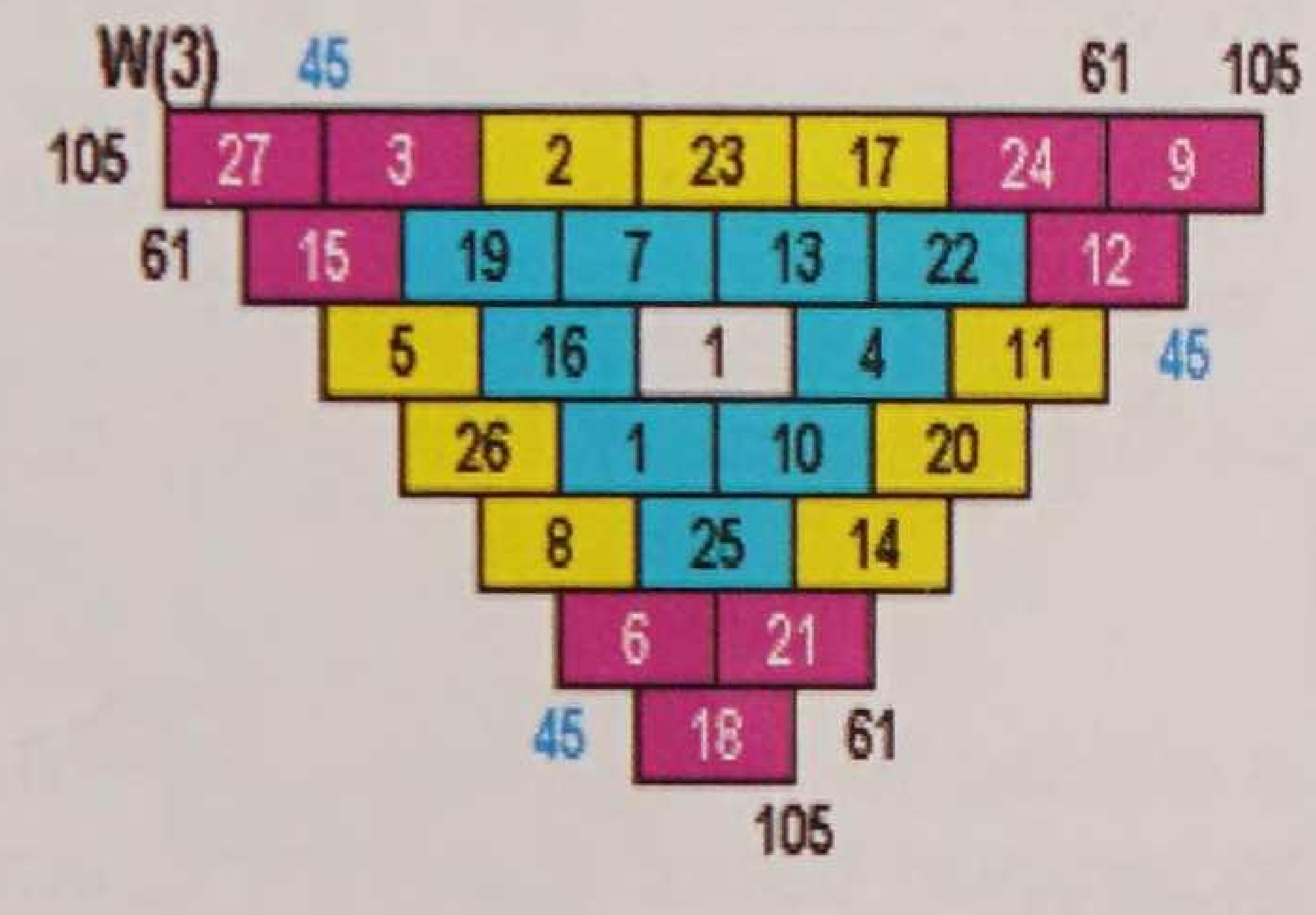


18 3 3 2 2 2 3 3
4 3 1 1 1 1 3 9
2 1 1 1 2
2 1 1 2
2 1 2
6 3 3
3 4
18

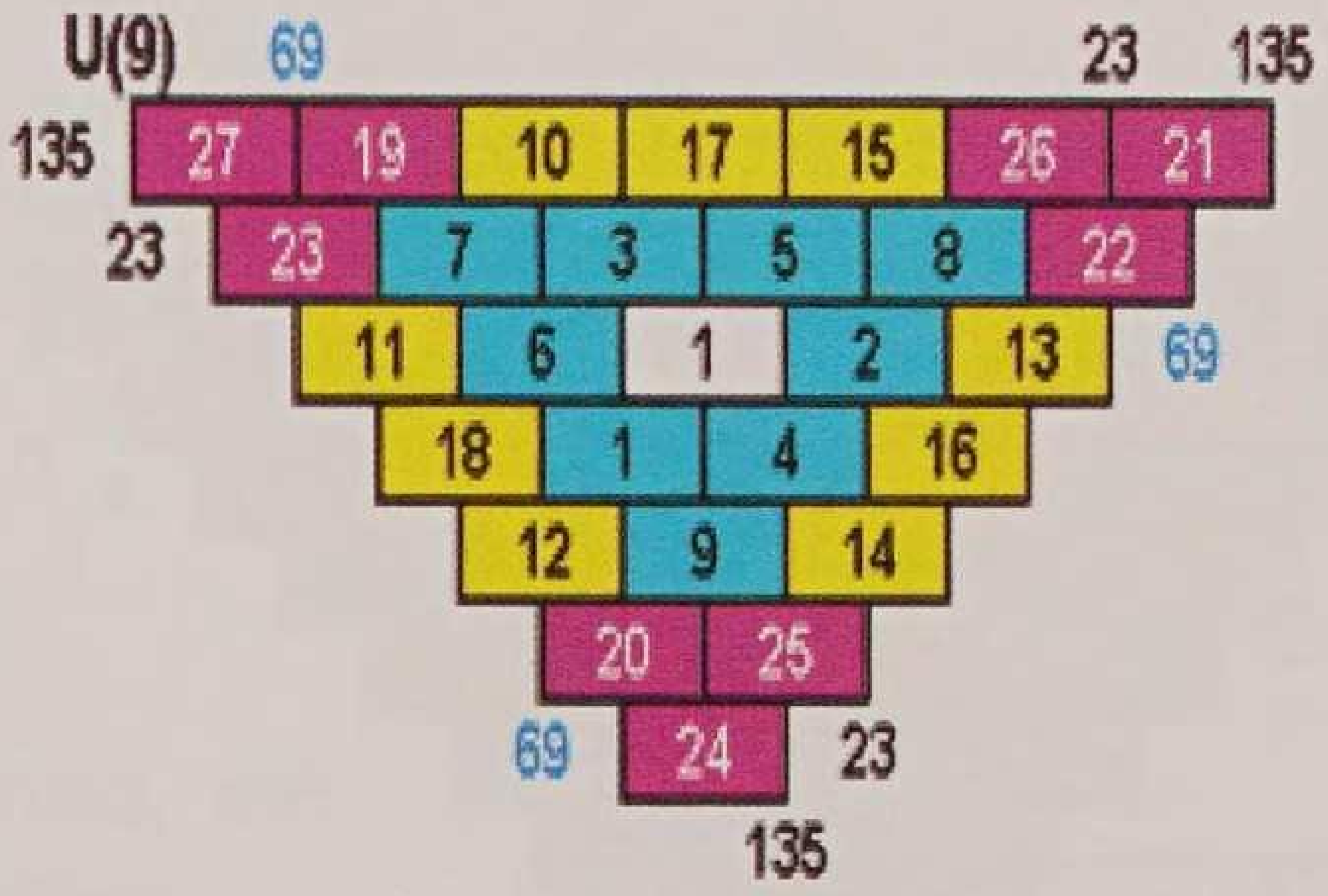


36 9 1 1 8 6 8 3
23 5 7 3 5 8 4 15
2 6 1 2 4
9 1 4 7
3 9 5
2 7
15 6 23
36

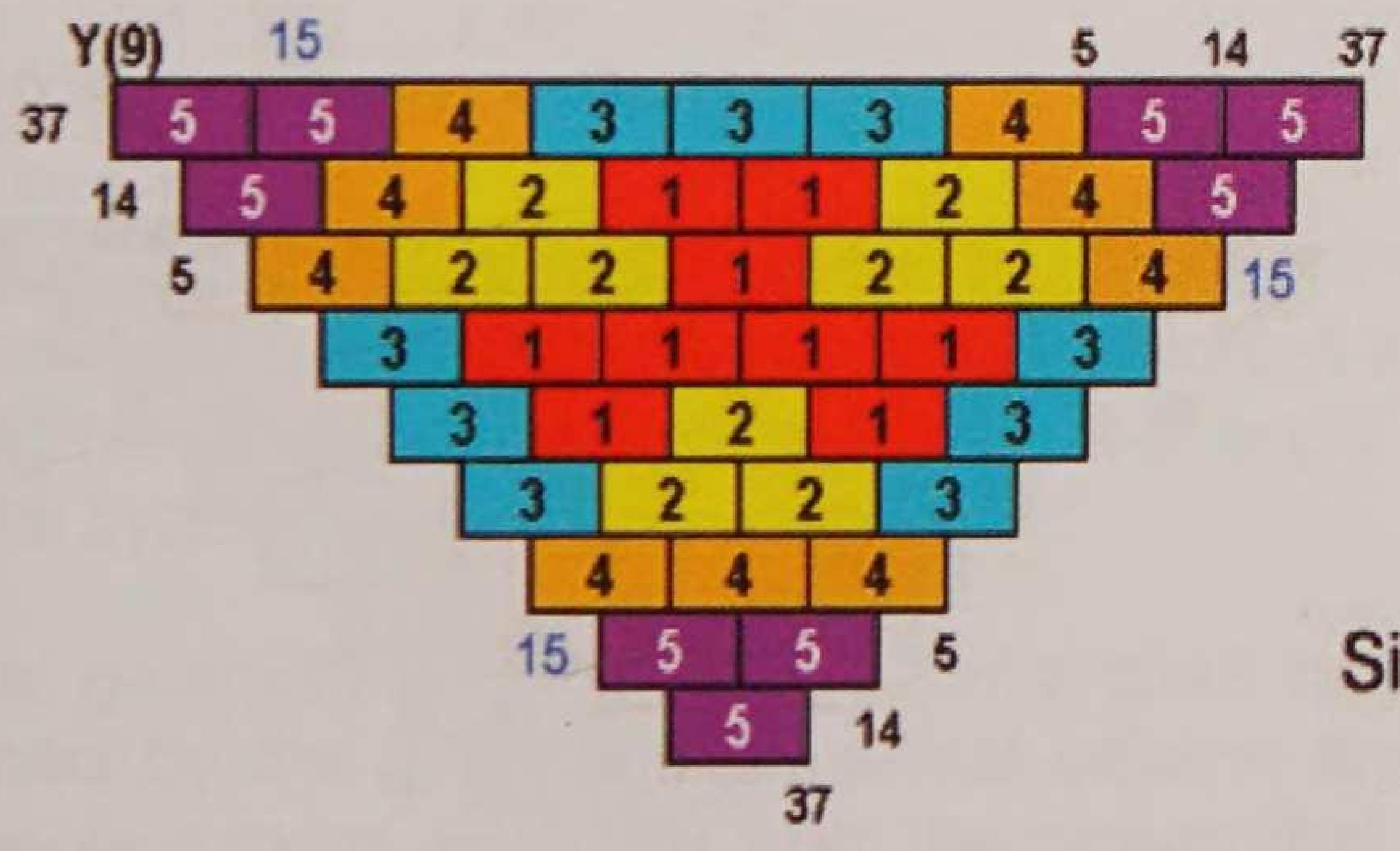
Size-7



105 27 3 2 23 17 24 9
61 15 19 7 13 22 12
5 16 1 4 11 45
26 1 10 20
8 25 14
6 21
45 18 61
105

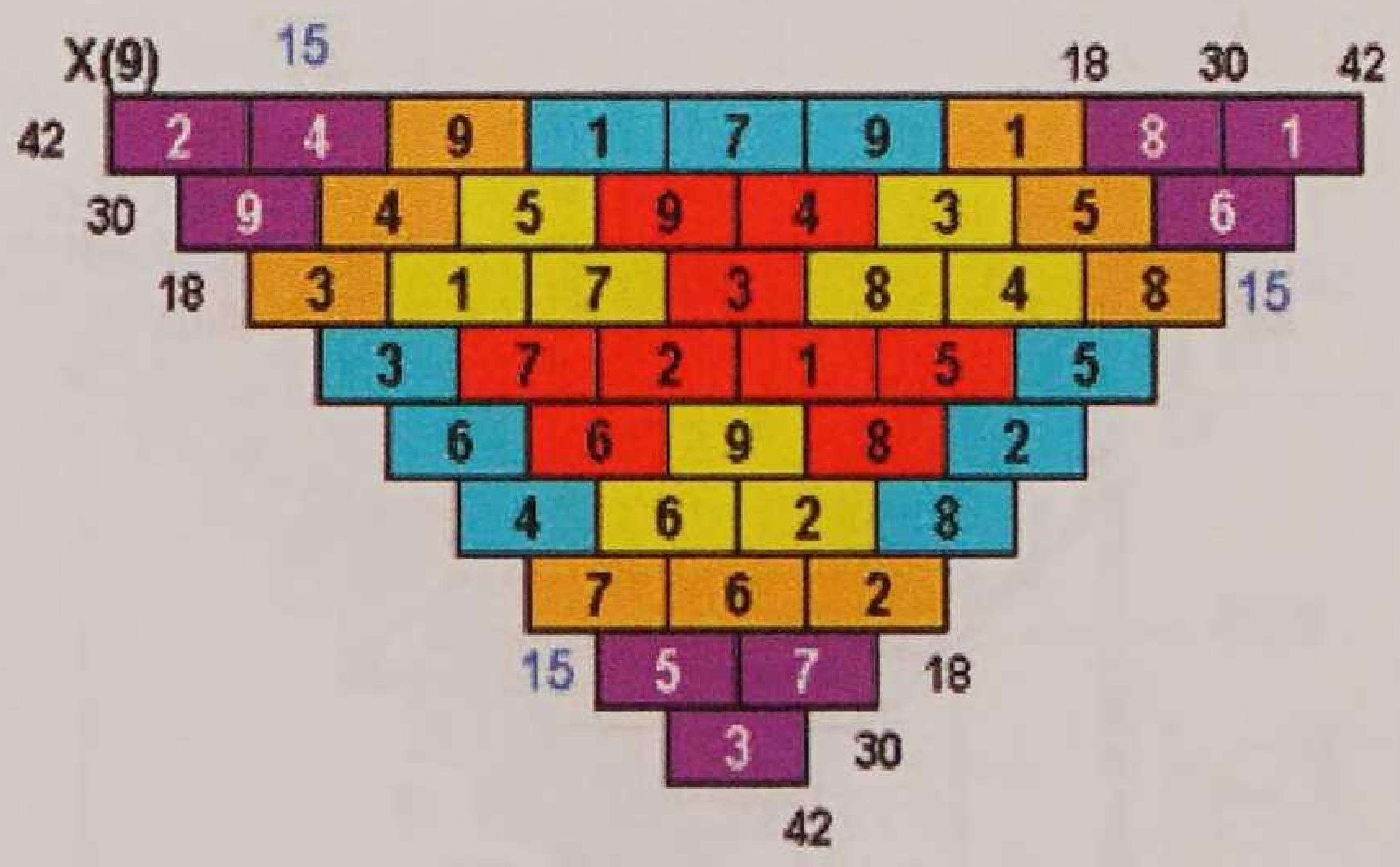


135 27 19 10 17 15 26 21
23 23 7 3 5 8 22
11 6 1 2 13 69
18 1 4 16
12 9 14
20 25
69 24 23
135

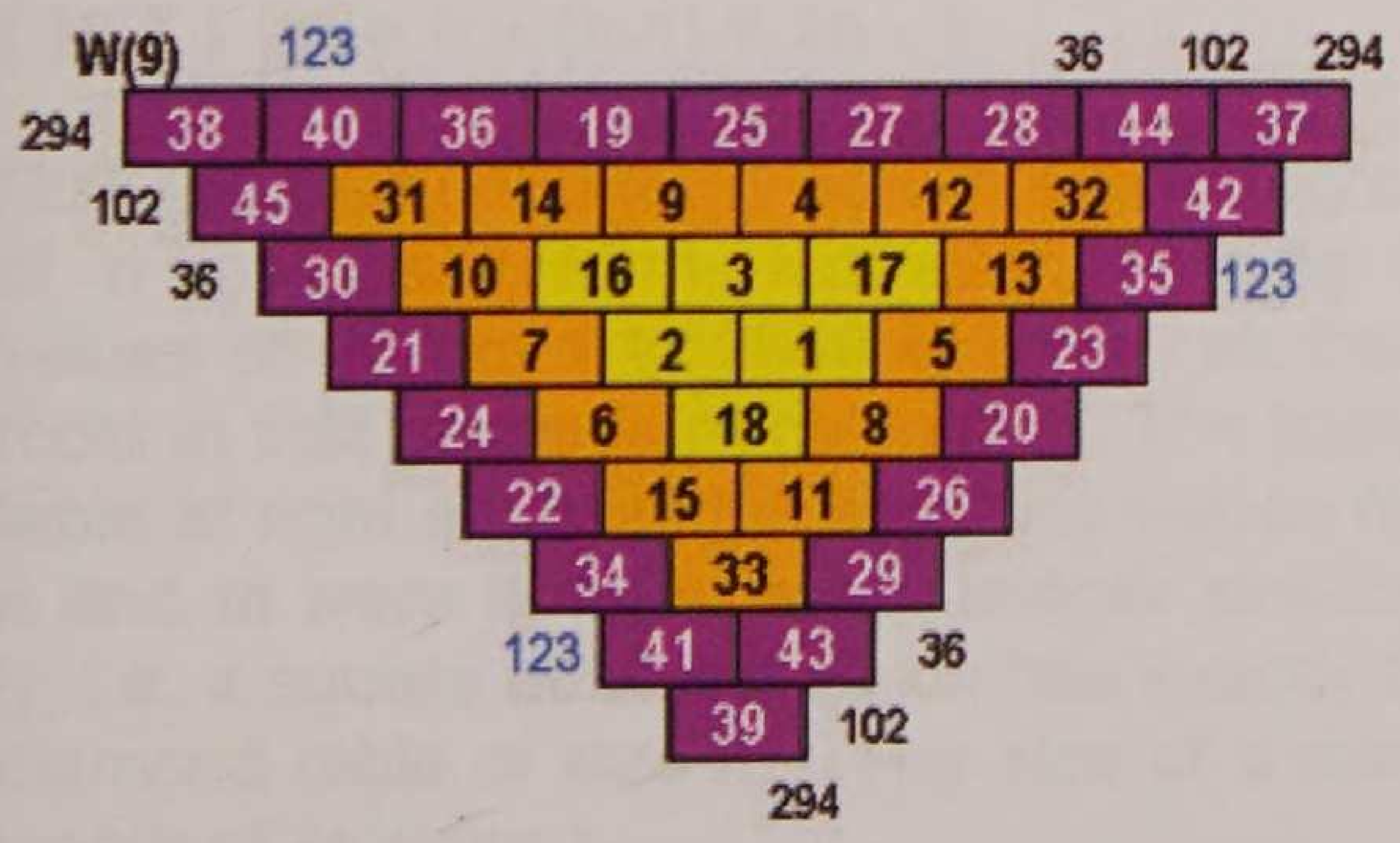


37 5 5 4 3 3 3 4 5 5
14 5 4 2 1 1 2 4 5
5 4 2 2 1 2 2 4 15
3 1 1 1 1 3
3 1 2 1 3
3 2 2 3
4 4 4
15 5 5 5
5 14
37

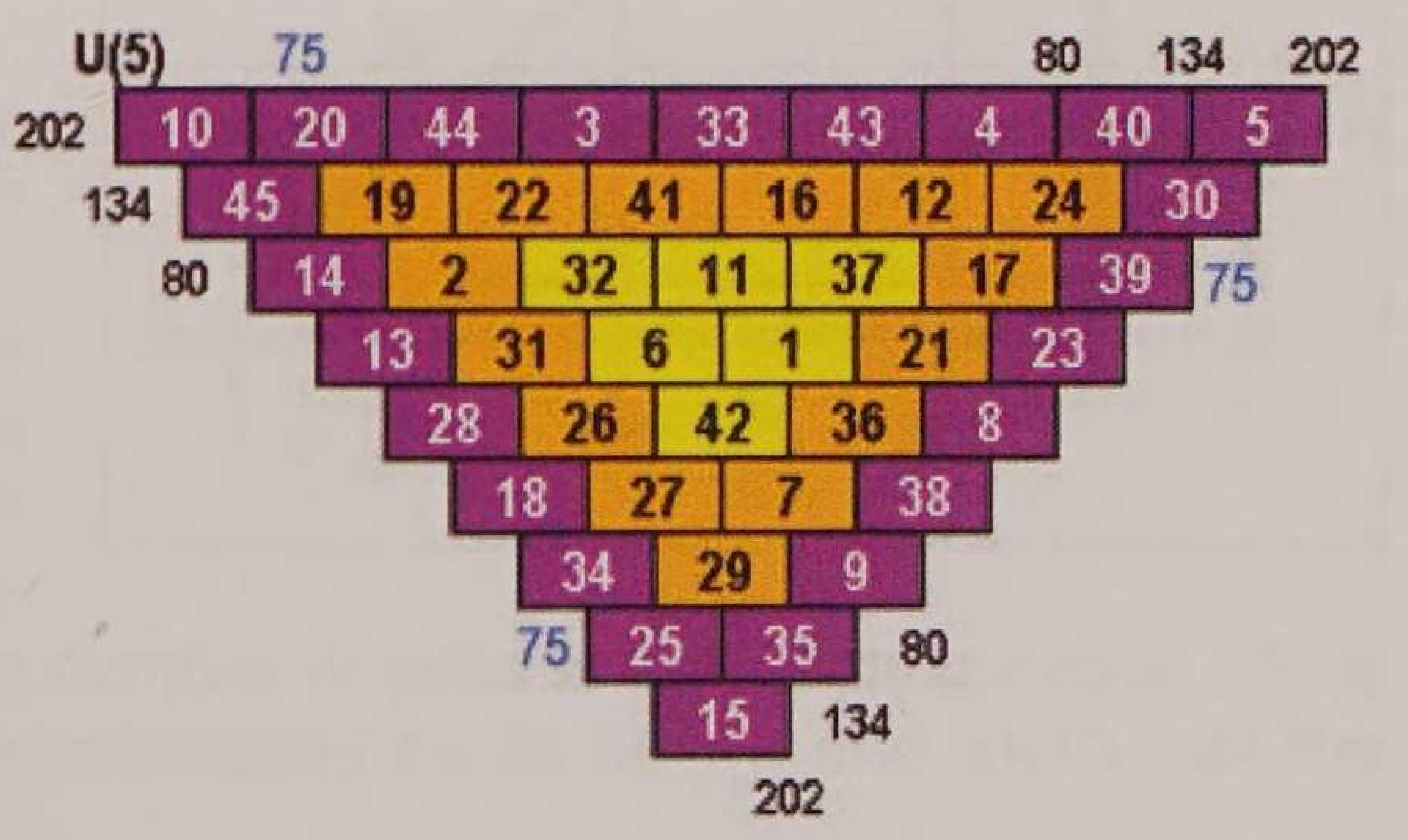
Size-9



42 2 4 9 1 7 9 1 8 1
30 9 4 5 9 4 3 5 6
18 3 1 7 3 8 4 8 15
3 7 2 1 5 5
6 6 9 8 2
4 6 2 8
7 6 2
15 5 7 18
3 30
42



294 38 40 36 19 25 27 28 44 37
102 45 31 14 9 4 12 32 42
36 30 10 16 3 17 13 35 123
21 7 2 1 5 23
24 6 18 8 20
22 15 11 26
34 33 29
123 41 43 36
39 102
294

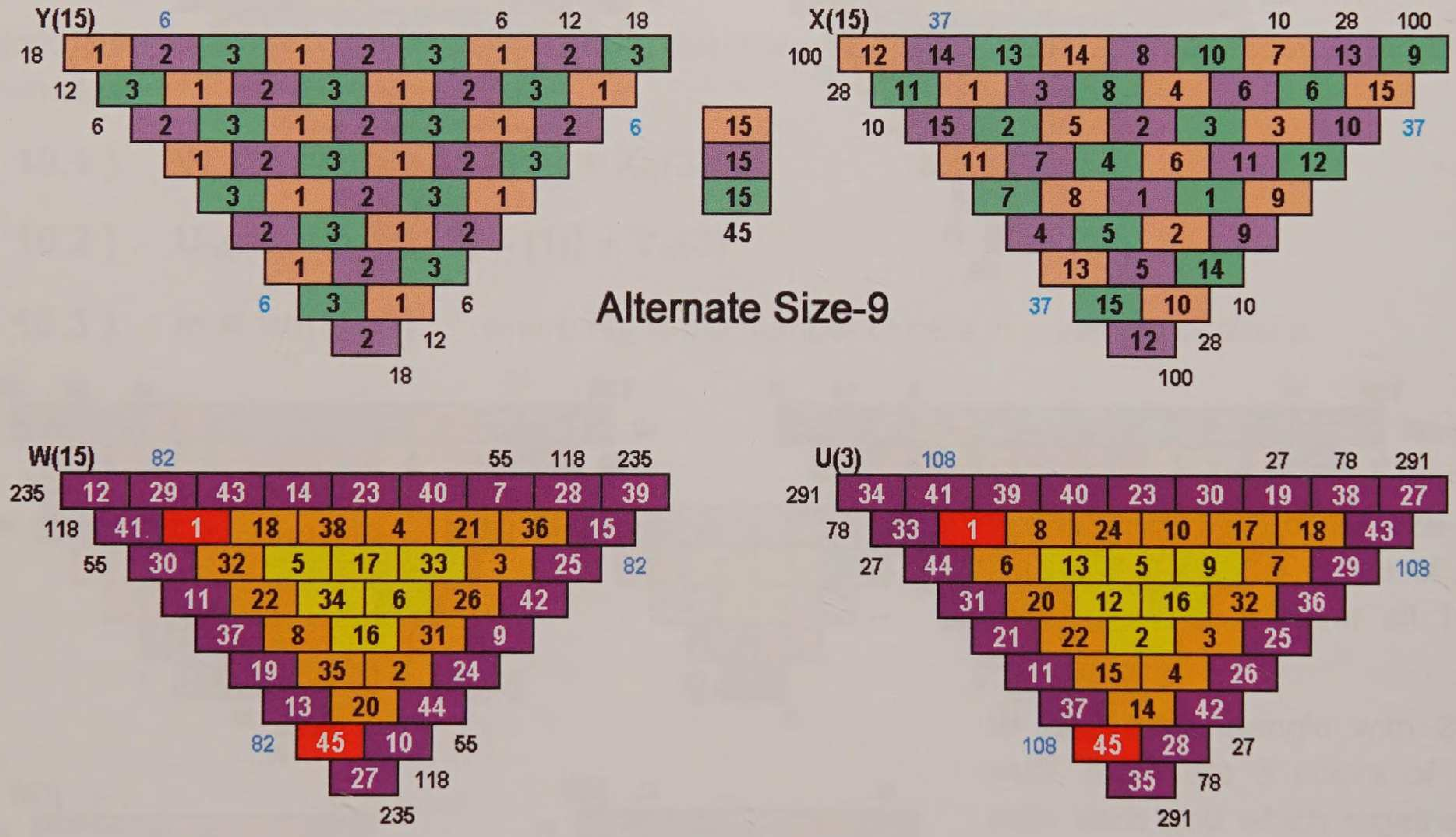


202 10 20 44 3 33 43 4 40 5
134 45 19 22 41 16 12 24 30
80 14 2 32 11 37 17 39 75
13 31 6 1 21 23
28 26 42 36 8
18 27 7 38
34 29 9
75 25 35 80
15 134
202

Program 10

Here is an alternative pattern for deriving the size-9 triangle. All triangles of odd-size can be derived with this 3-color checker-board pattern. There are **15** cells of each color all separated from cells of like color. The integer loom-table **Y** has one number for each color. The modulus loom table **X** has the numbers from **1** thru **15** throughout the cells of the same color. These cells cover the table of **45** cells completely. Clearly, **Y** has all frames of equal summing sides because of its symmetry of numbers. It's the numbers in **X** that requires arranging to achieve that result. Once achieved, the loom-tables can be used to generate both dual triangles from formulas (10.1) and (**10.2**) where **m = 15** from (**10.3**).

Note that only **Y** has triangular modularity. It is impossible for **X** to do so too. So the primal tables do not have this property.



Alternate Size-9

Y(15) 6 6 12 18
18 1 2 3 1 2 3 1 2 3
12 3 1 2 3 1 2 3 1
6 2 3 1 2 3 1 2 6
1 2 3 1 2 3
3 1 2 3 1
2 3 1 2
1 2 3
6 3 1 6
2 12
18

15
15
15
45

X(15) 37 10 28 100
100 12 14 13 14 8 10 7 13 9
28 11 1 3 8 4 6 6 15
10 15 2 5 2 3 3 10 37
11 7 4 6 11 12
7 8 1 1 9
4 5 2 9
13 5 14
37 15 10 10
12 28
100

W(15) 82 55 118 235
235 12 29 43 14 23 40 7 28 39
118 41 1 18 38 4 21 36 15
55 30 32 5 17 33 3 25 82
11 22 34 6 26 42
37 8 16 31 9
19 35 2 24
13 20 44
82 45 10 55
27 118
235

U(3) 108 27 78 291
291 34 41 39 40 23 30 19 38 27
78 33 1 8 24 10 17 18 43
27 44 6 13 5 9 7 29 108
31 20 12 16 32 36
21 22 2 3 25
11 15 4 26
37 14 42
108 45 28 27
35 78
291

At this time no further equal-summation properties have been found for triangles.

Relationship between Geonomic Squares and Diamonds

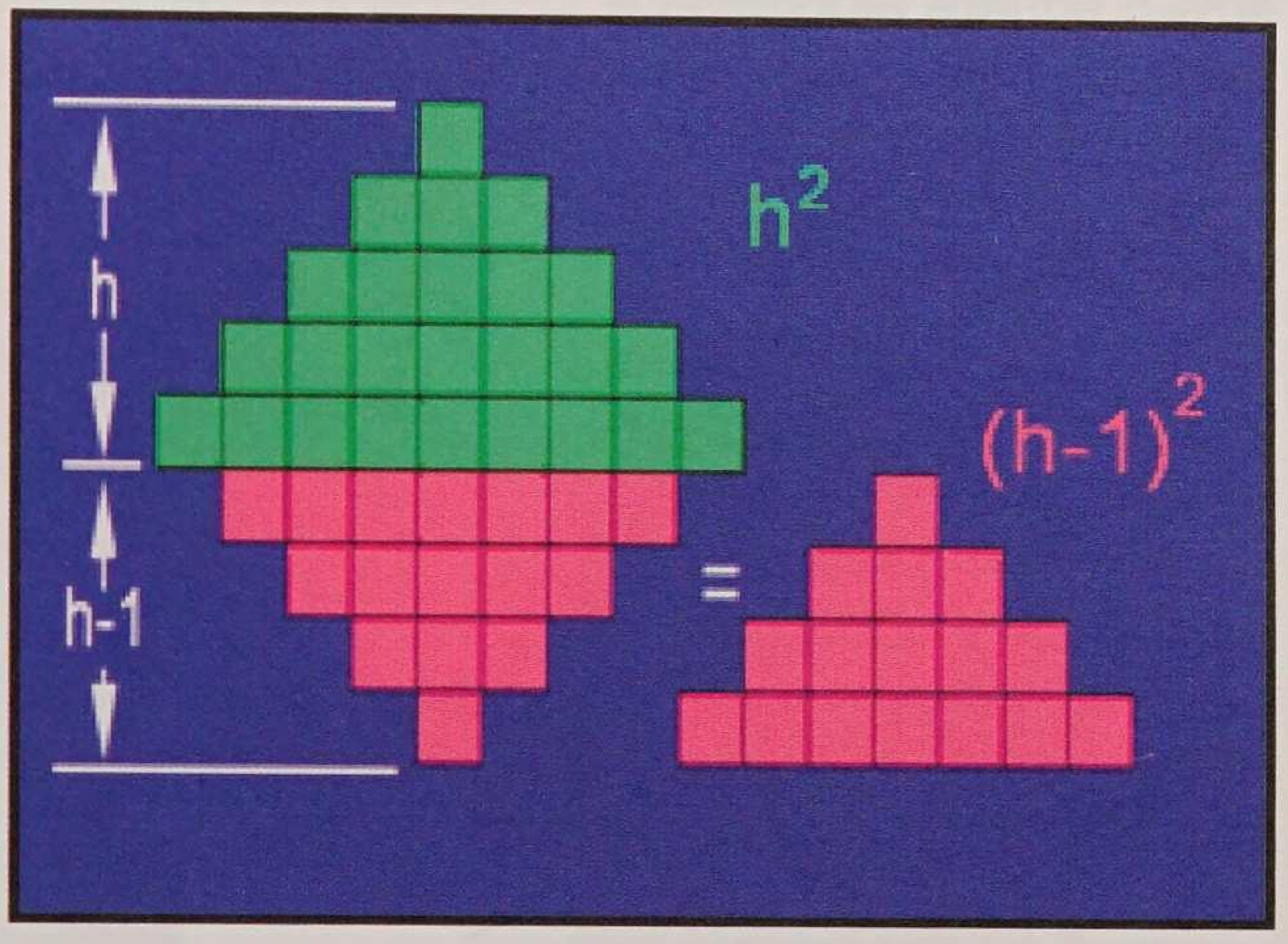
N ≡ the number of cells in a diamond table

h ≡ height of the triangle by the number of cells vertically

n ≡ the size of the square

(10.4) $N = h^2 + (h-1)^2$

We have already observed back in Program 6 that the number of cells in an index triangle of height **h** was **h-squared**. Observe that the number of cells **N** in any diamond may be expressed as in formula (**10.4**) as the sum of two index triangles, one of height **h** and the other of height **h-1**.



Since the number of cells in any square of size-n is n^2,

(10.5) $n^2 = N$

Equating this with the number of cells in a diamond, we get the formula for the required number of cells they must have in common:

(10.6) $n^2 = h^2 + (h-1)^2$

Note that (**10.6**) is a special case of the *Pythagorean Theorem* for right triangles where **h** and **h-1** are the lengths of orthogonal sides of a right-triangle and **n** is its hypotenuse.

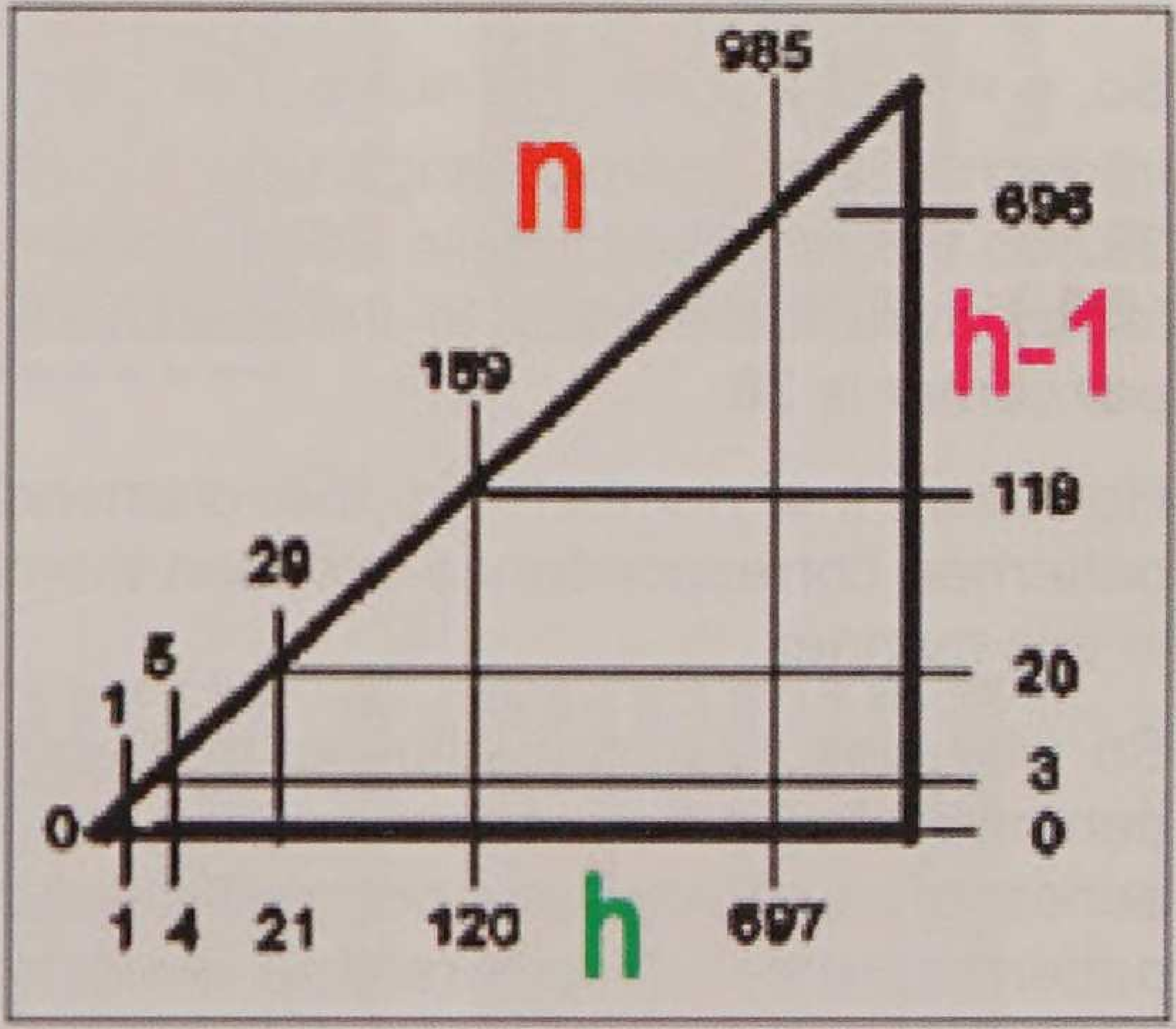
Diamond size ↓ | Square size ↓

m	h	N	n
1	1	1	1
7	4	25	5
41	21	841	29
239	120	28561	169
1393	697	970225	985

The geometric size of a diamond shall be determined by the length of its central column and row. Denote it by **m**. It is clear from the picture that:

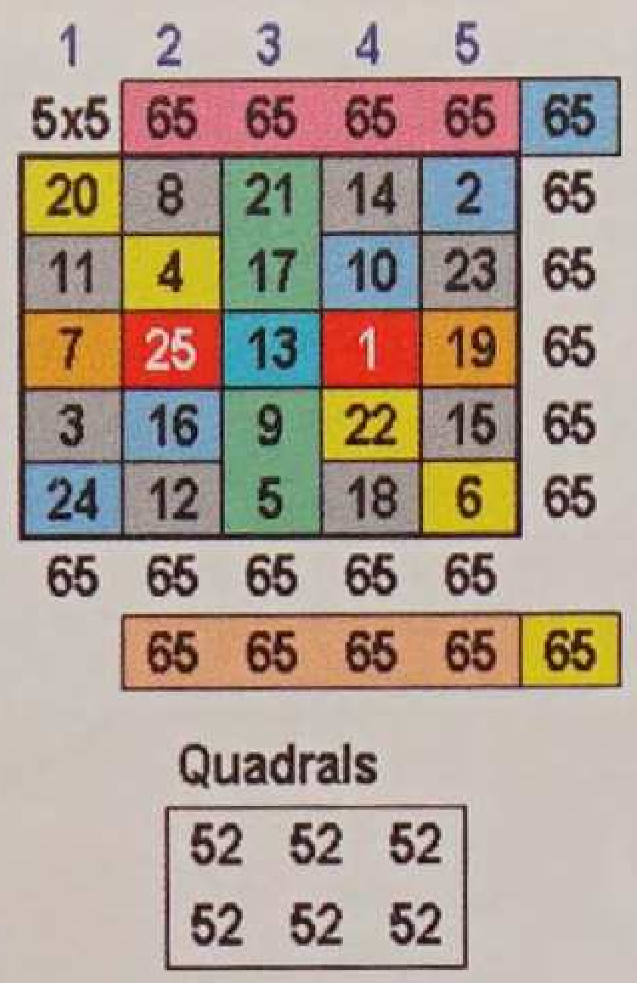
(10.7) $m = h + (h-1) = 2h - 1.$

Now (**10.6**) is not directly solvable for integral values of **n** in closed form. However, running through values of **n** from **1** to **2600** in Excel, all the square roots in that range were calculated. The triangle and table at right show all the resulting values for which **n** and **m** were both an exact integer simultaneously, i.e. a square of size **n** which has exactly the right number of cells **N** for reconfiguration into a diamond table of size **m**. (The size of a diamond is measured by its height and width, not the length of its sides.)



So to get a diamond from a size-**29** square, we will need to construct a size **41** diamond.

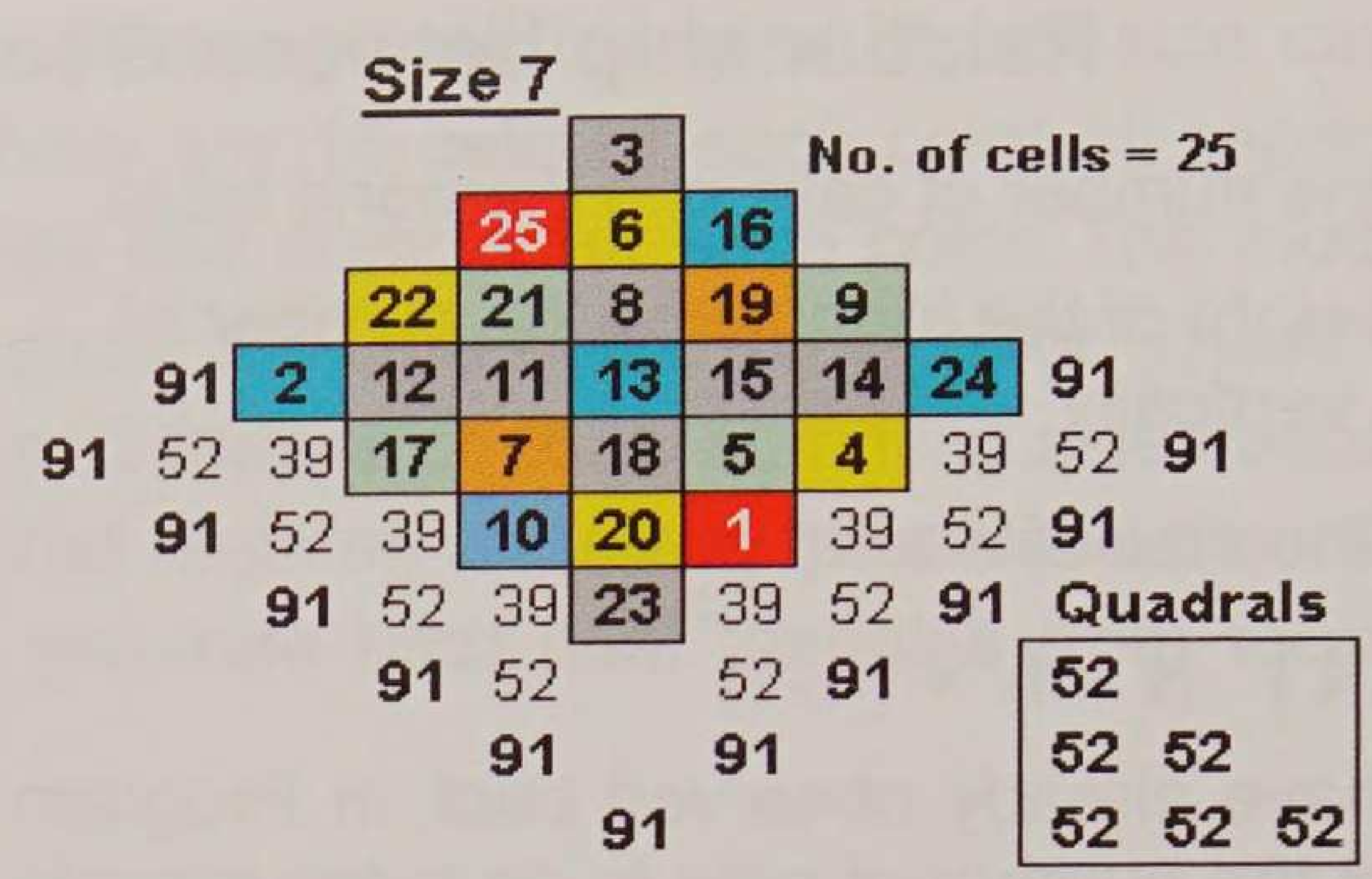
Here we see the perfect size-5 square alongside a perfect size-7 diamond. Observe that the characteristic number for the diamond is **91**.



	1	2	3	4	5	
	5x5	65	65	65	65	65
1	20	8	21	14	2	65
2	11	4	17	10	23	65
3	7	25	13	1	19	65
4	3	16	9	22	15	65
5	24	12	5	18	6	65
	65	65	65	65	65	
		65	65	65	65	65

Quadrals

52	52	52
52	52	52



Size 7

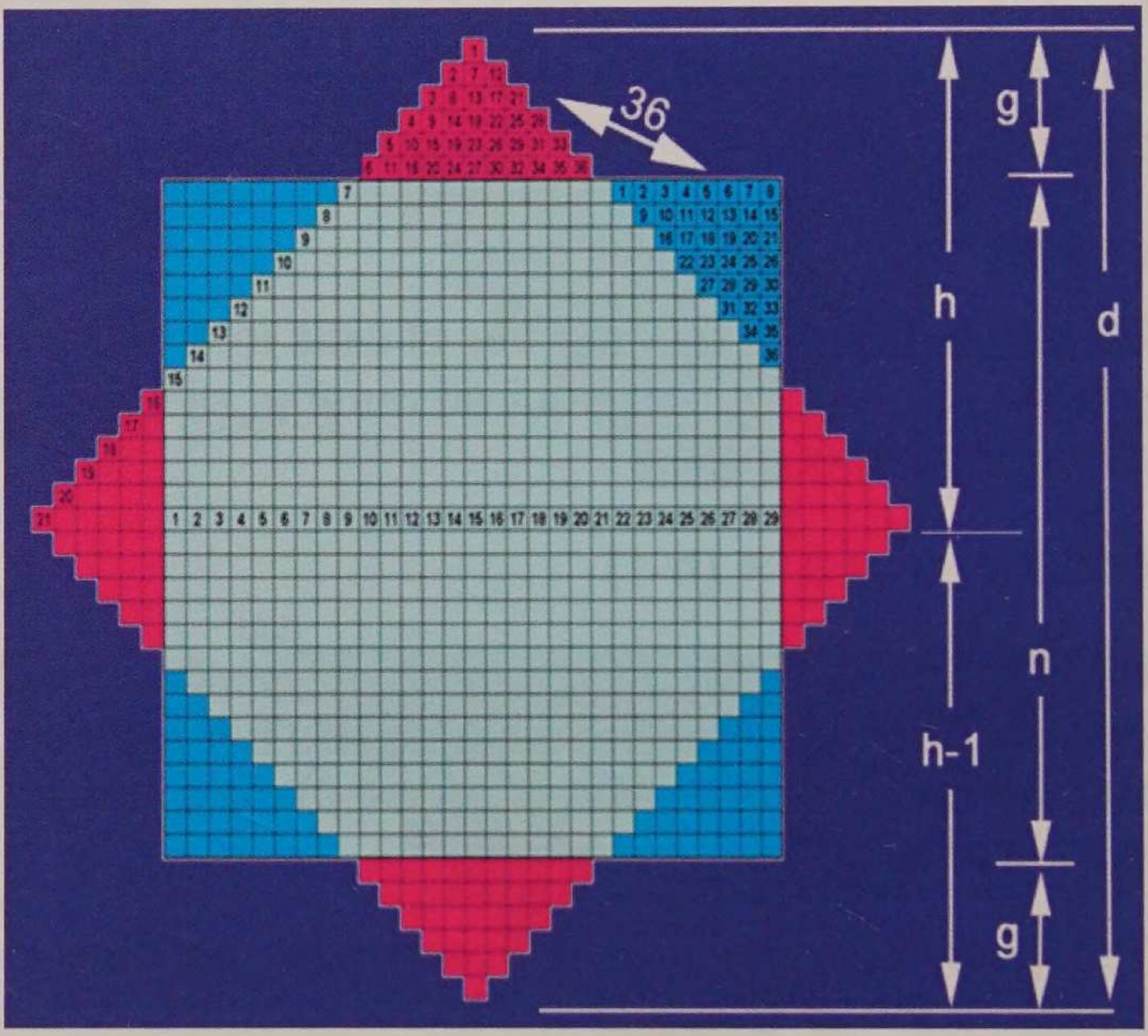
No. of cells = 25

					3					
				25	6	16				
			22	21	8	19	9			
	91	2	12	11	13	15	14	24	91	
91	52	39	17	7	18	5	4	39	52	91
	91	52	39	10	20	1	39	52	91	
		91	52	39	23	39	52	91		
			91	52		52	91			
				91		91				
					91					

Quadrals

52		
52	52	
52	52	52

Note that the quadral sums are the same between the square and diamond. That will always be the case. Quadrals in both are equal to **52**. Both the square and diamond are pairwise centrally symmetric. Note that in the diamond the short and long diagonal sums together equal **91**, the same value to which the full-length horizontal and vertical cells sum.



Here is that size-**29** square converted into a size-**21** diamond. Let **g** ≡ the height of each corner triangle. Then

$$(10.8)\quad 2g = d - n$$
$$= h + (h-1) - n$$
$$= 2h - (n+1)$$

The count of numbers in an index triangle of height **g** is g^2. For **h = 21** and **n = 29,**

$$2g = 42 - (29+1) = 12$$

So, **g = 12/2 = 6** and the number of cells in each of the corner triangles is $6^2 = 36$. So the number of cells to be transferred from the square to the diamond per corner is **36**.

However, the numbers in the diamond and the square that are both geonomic have no patterned correspondence between them. So the size-**21** geonomic diamond cannot be derived in this manner.

So why was this correlation of squares with diamonds made here in the first place? What it demonstrates is that in many occasions, although with less and less frequency, the exact same series of numbers can possess two different numerical geometries simultaneously whose patterns are not directly related to each other on a one-to-one basis. I find that just amazing! It's like having a ghost and a goblin in the same body.

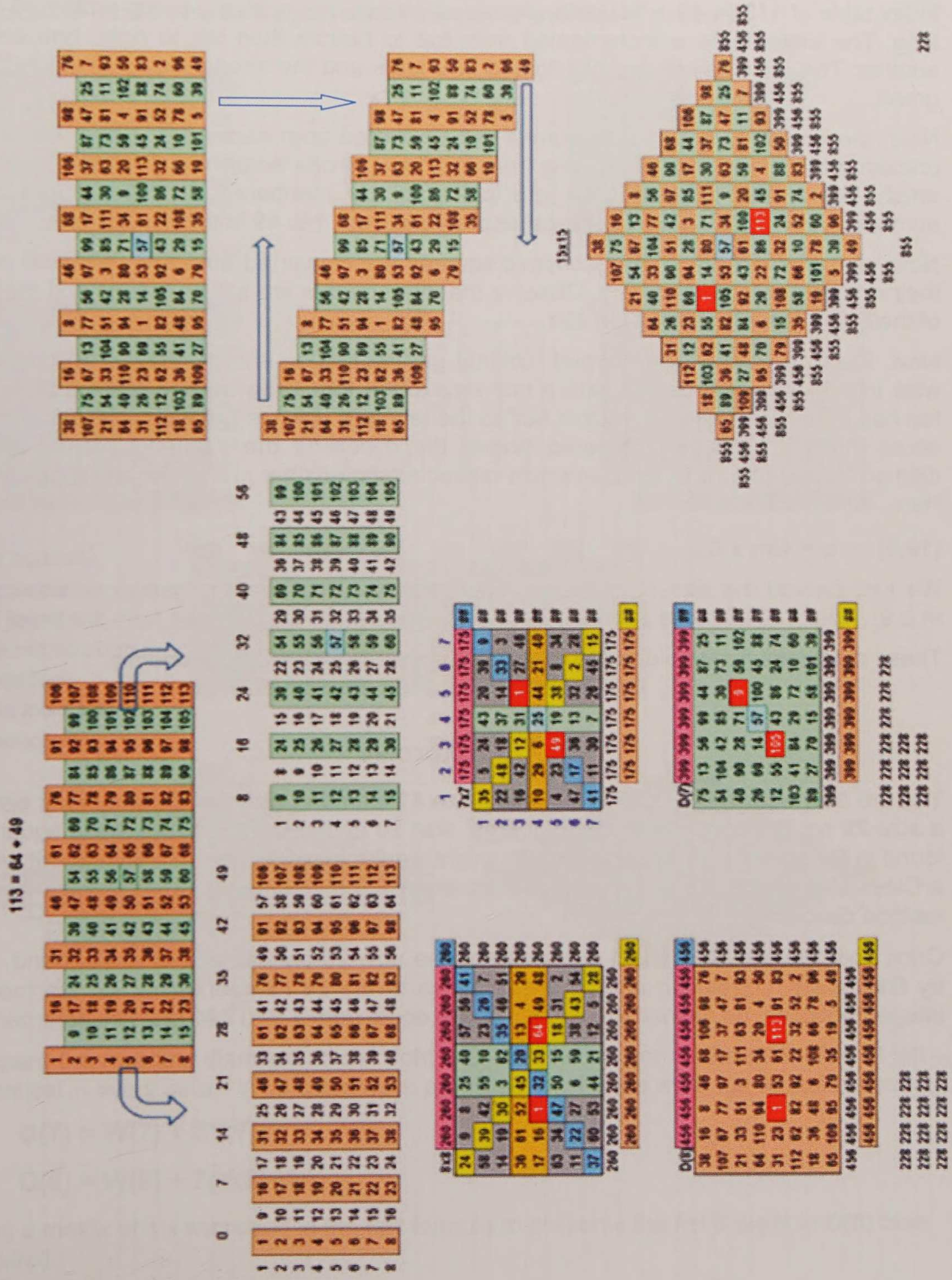
In the next slide, we'll see how geonomic diamonds can be derived by two successive geonomic squares. With this very simple technique, you can readily derive the perfect pangenic size **21** diamond to match the diamond format depicted here and compare it with the perfect size **29** square for any correspondences. The size-**29** square is depicted in the book, **Number Magic**, that would otherwise require a large matchmaker's square in Excel to manifest on your own.

Program 10

Generating Geonomic Diamonds from Geonomic Squares

The Laminated Indexing Method LIM

Deriving a Geonomic Diamond from two Perfect Squares



The Laminated Indexing Method LIM

We're going to generate a perfect diamond from merging two perfect consecutive squares. We'll choose the size-7 & size-8 squares to generate a size-15 diamond. First we start with a double-index table of **113** (= 49 + 64) cells aligned as shown in a bumpety-bump pattern, **8** high by **15** long. The index table is incremented from top to bottom then left to right, one column after another. The tall columns are highlighted in orange and the shorter columns are highlighted in green.

Next, these differently-colored columns are separated from each other. Then the columns are preceded by consecutive numbering from top down in one empty column then right to the next empty column until all columns are preceded by numbered columns. There should be numbering from **1** to **64** in the leftmost section and from **1** to **49** in the second one.

Now, all the numbers in both geonomic squares are converted from the uncolored numbers to their adjacent colored numbers. Observe that the squares are still geonomic and that both sets of their quadrals are all equal to **228**.

Next, the two squares are merged, column per column as shown. Then we tilt-wrap column-wise into a stair-case pattern with a half-step at the top. Then we tilt-wrap the table row-wise, top half to the right and the bottom half to the left. and now we have a perfect diamond with all equal rhomboid quadrals centered across the middle of the diamond. All the quadrals for diamonds sum to **4/m** of the diamond's characteristic number just as for rectangles in squares. Here, **4/15 x 855 = 228**.

(10.9) $q = 4/m \times C_m$

We just derived the size-15 diamond. This method will derive any geonomic diamond of size **m ≥ 9**. Note that all of the diamonds with the cell-structure as depicted here will be of odd size.

These diamonds have no duals because their loom tables are not geonomic.

Exercise

You are now equipped to derive a perfect size **41** diamond with the same number sequence as a size **29** square from two perfect squares, size **20** and size **21**. Both of these squares can be found in Program 3. If you succeed, you might want a size **29** square to compare it with. That is a Class-1 square that can be derived from a size-29 matchmaker square by the double-quark method described in Program 7.

Once both the size **41** perfect diamond and the size **29** perfect square are at hand, you might try finding any correlation among the locations of the **29** numbers in which the modulus and integer functions, as defined in Program 3, are equal from **1** thru **841** **(29^2)** in increments of **30**.

Program 10

The Bump and Grind BAG Method

Here is another method for creating geonomic diamonds. This one only works using a Class-1 and an adjacent size Class-4 square because it requires viable geonomic **integer** loom-tables for both. To demonstrate this, we'll again select the size-7 and size-8 squares.

	1	2	3	4	5	6	7	
W(7)		175	175	175	175	175	175	175
1	35	5	24	43	20	39	9	175
2	22	48	18	37	14	33	3	175
3	16	42	12	31	1	27	46	175
4	10	29	6	25	44	21	40	175
5	4	23	49	19	38	8	34	175
6	47	17	36	13	32	2	28	175
7	41	11	30	7	26	45	15	175
	175	175	175	175	175	175	175	
		175	175	175	175	175	175	175

\+ 8*

Y(7)

5	1	4	7	3	6	2
4	7	3	6	2	5	1
3	6	2	5	1	4	7
2	5	1	4	7	3	6
1	4	7	3	6	2	5
7	3	6	2	5	1	4
6	2	5	1	4	7	3

=

	1	2	3	4	5	6	7	
D(7)		399	399	399	399	399	399	399
1	75	13	56	99	44	87	25	399
2	54	104	42	85	30	73	11	399
3	40	90	28	71	9	59	102	399
4	26	69	14	57	100	45	88	399
5	12	55	105	43	86	24	74	399
6	103	41	84	29	72	10	60	399
7	89	27	70	15	58	101	39	399
	399	399	399	399	399	399	399	
		399	399	399	399	399	399	399

	1	2	3	4	5	6	7	8	
W(8)		260	260	260	260	260	260	260	260
1	24	9	8	25	40	57	56	41	260
2	58	39	42	55	10	23	26	7	260
3	14	19	30	3	62	35	46	51	260
4	36	61	52	45	20	13	4	29	260
5	17	16	1	32	33	64	49	48	260
6	63	34	47	50	15	18	31	2	260
7	11	22	27	6	59	38	43	54	260
8	37	60	53	44	21	12	5	28	260
	260	260	260	260	260	260	260	260	
		260	260	260	260	260	260	260	260

\+ 7*

Y(8)

3	2	1	4	5	8	7	6
8	5	6	7	2	3	4	1
2	3	4	1	8	5	6	7
5	8	7	6	3	2	1	4
3	2	1	4	5	8	7	6
8	5	6	7	2	3	4	1
2	3	4	1	8	5	6	7
5	8	7	6	3	2	1	4

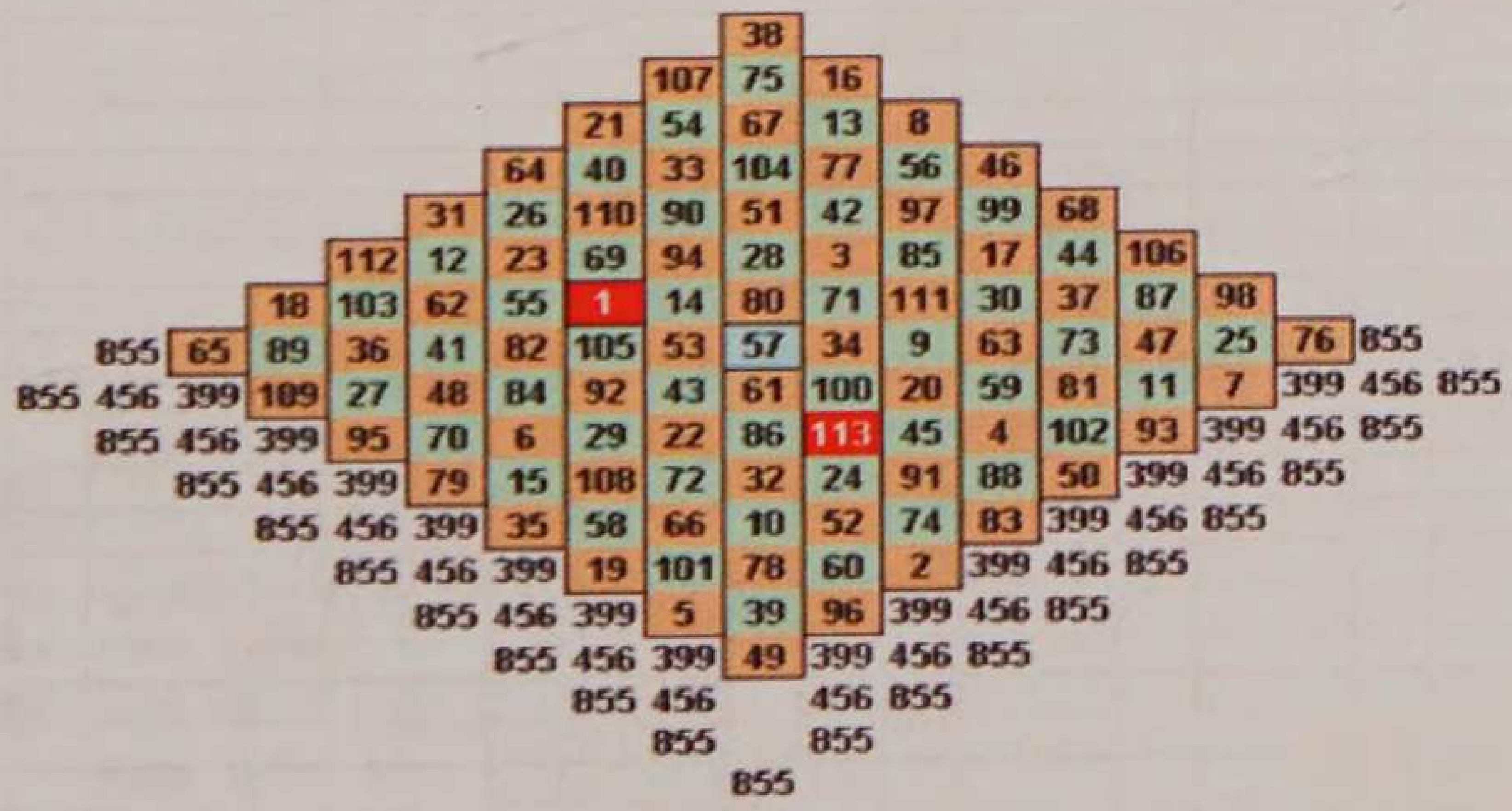
=

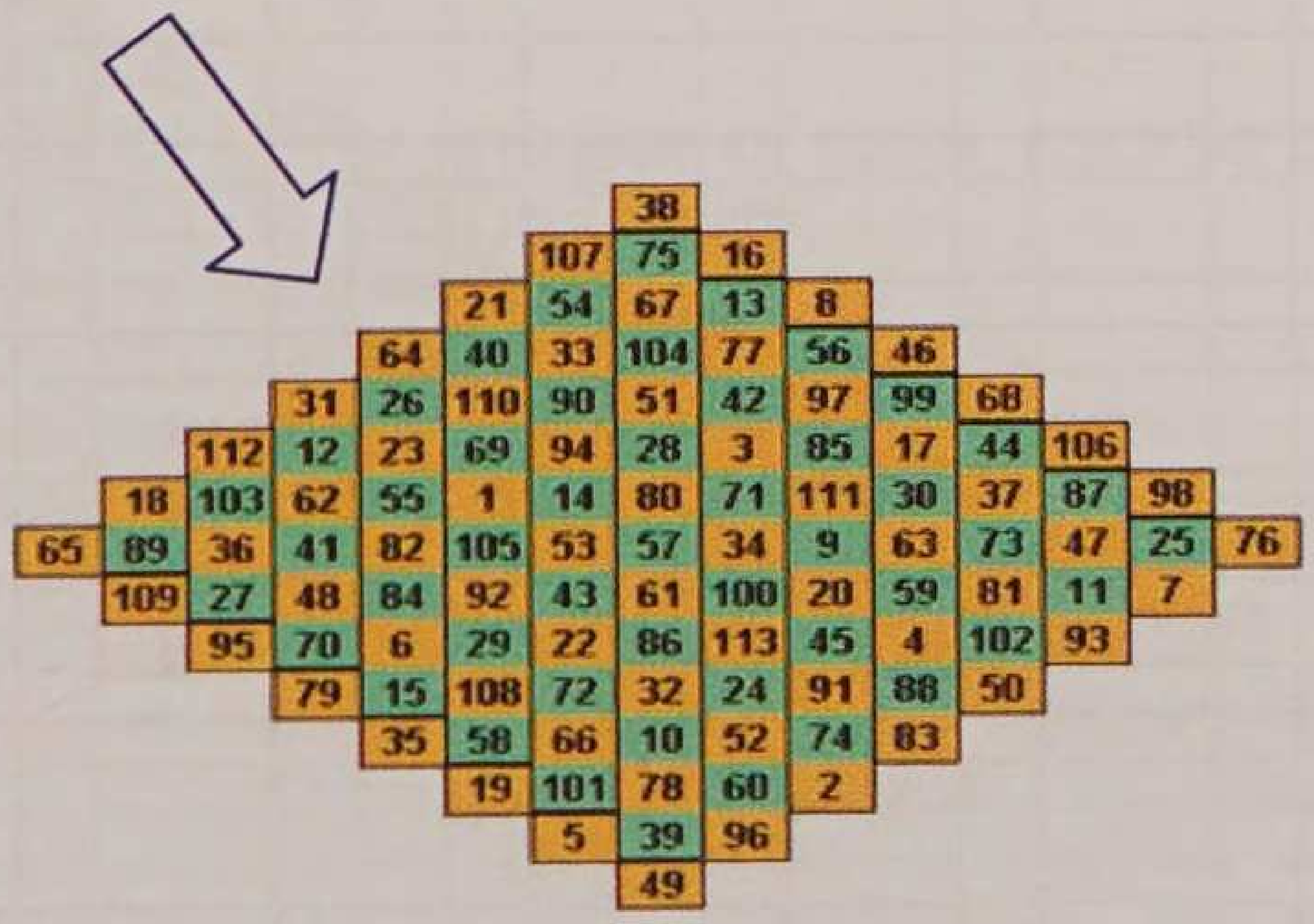
D(8)								
	456	456	456	456	456	456	456	456
38	16	8	46	68	106	98	76	456
107	67	77	97	17	37	47	7	456
21	33	51	3	111	63	81	93	456
64	110	94	80	34	20	4	50	456
31	23	1	53	61	113	91	83	456
112	62	82	92	22	32	52	2	456
18	36	48	6	108	66	78	96	456
65	109	95	79	35	19	5	49	456
456	456	456	456	456	456	456	456	
	456	456	456	456	456	456	456	456

Note: The centrally symmetric pairwise version of the size-8 square will not conform to this procedure because its loom tables are not geonomic.

38		16		8		46		68		106		98		76
107	75	67	13	77	56	97	99	17	44	37	87	47	25	7
21	54	33	104	51	42	3	85	111	30	63	73	81	11	93
64	40	110	90	94	28	80	71	34	9	20	59	4	102	50
31	26	23	69	1	14	53	57	61	100	113	45	91	88	83
112	12	62	55	82	105	92	43	22	86	32	24	52	74	2
18	103	36	41	48	84	6	29	108	72	66	10	78	60	96
65	89	109	27	95	70	79	15	35	58	19	101	5	39	49

15x15





Now, compare the resulting diamond on the right with the one we got on the prior slide at left – they're identical in every detail! Who could have predicted that? Here were the formulas used:

(10.10) $D(7) = W(7) + 8Y(7)$

(10.11) $D(8) = W(8) + 7(Y(8)-|1|)$

Subtracting a matrix of 1's was used in the last formula to preserve the fist **8** digits in **D(8)** from being modified.

Program 10

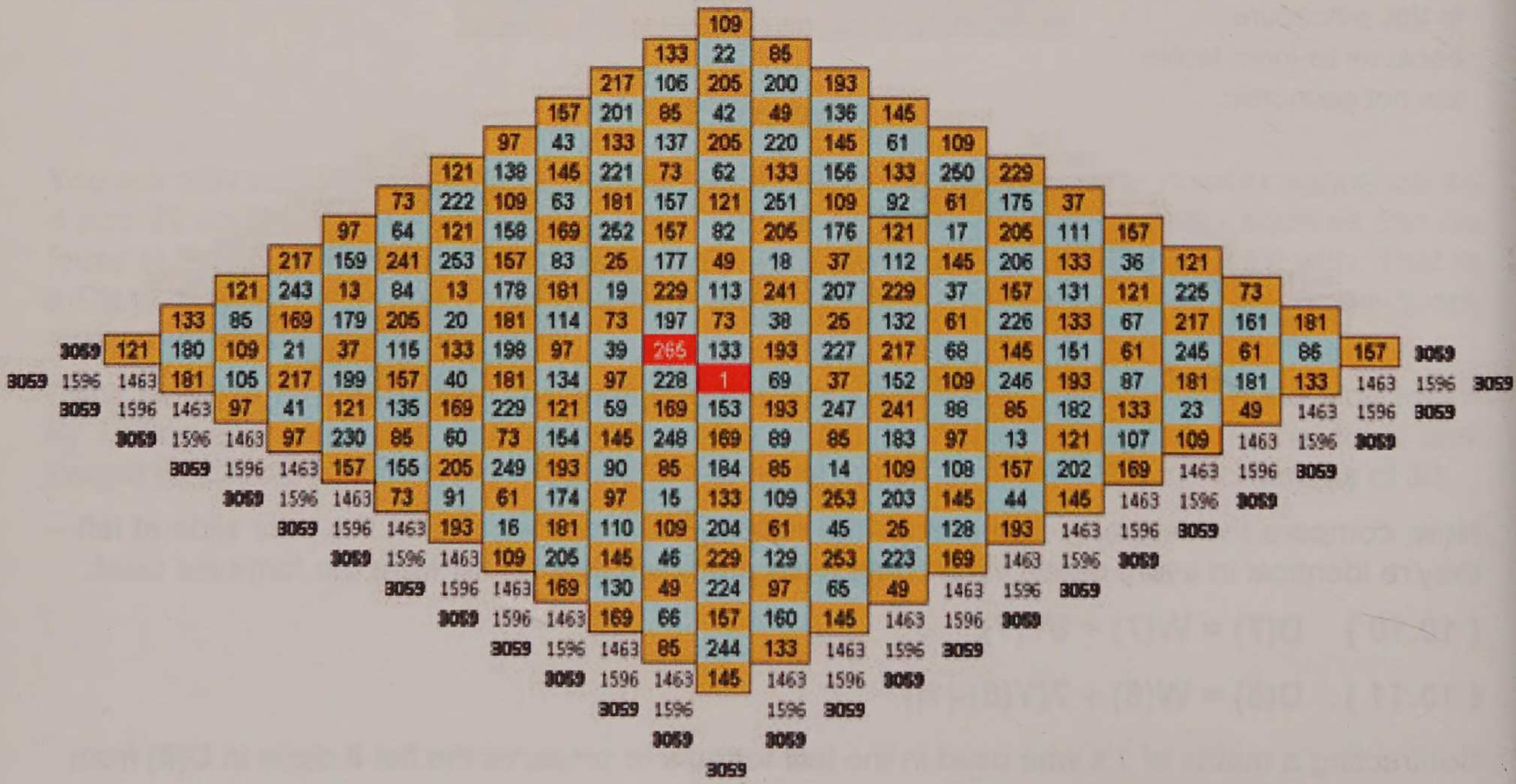
The Size-23 Diamond

Here is the size-23 diamond composed from the size-11 and size-12perfect squares using the **BAG** method.

	W(11)	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463
1	22	200	136	61	250	175	111	36	225	161	86	1463
2	106	42	220	156	92	17	206	131	67	245	181	1463
3	201	137	62	251	176	112	37	226	151	87	23	1463
4	43	221	157	82	18	207	132	68	246	182	107	1463
5	138	63	252	177	113	38	227	152	88	13	202	1463
6	222	158	83	19	197	133	69	247	183	108	44	1463
7	64	253	178	114	39	228	153	89	14	203	128	1463
8	159	84	20	198	134	59	248	184	109	45	223	1463
9	243	179	115	40	229	154	90	15	204	129	65	1463
10	85	21	199	135	60	249	174	110	46	224	160	1463
11	180	105	41	230	155	91	16	205	130	66	244	1463
	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	
		1463	1463	1463	1463	1463	1463	1463	1463	1463	1463	1463

	1	2	3	4	5	6	7	8	9	10	11	12	
		1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596
1	109	85	193	145	109	229	37	157	121	73	181	157	1596
2	133	205	49	145	133	61	205	133	121	217	61	133	1596
3	217	85	205	133	109	121	145	157	133	61	181	49	1596
4	157	133	73	121	205	37	229	61	145	193	133	109	1596
5	97	145	181	157	49	241	25	217	109	85	121	169	1596
6	121	109	169	25	229	73	193	37	241	97	157	145	1596
7	73	121	157	181	73	265	1	193	85	109	145	193	1596
8	97	241	13	181	97	97	169	169	85	253	25	169	1596
9	217	13	205	133	181	121	145	85	133	61	253	49	1596
10	121	169	37	157	169	73	193	97	109	229	97	145	1596
11	133	109	217	121	85	205	61	181	145	49	157	133	1596
12	121	181	97	97	157	73	193	109	169	169	85	145	1596
	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	
		1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596	1596

109		85		193		145		109		229		37		157		121		73		181		157
133	22	205	200	49	136	145	61	133	250	61	175	205	111	133	36	121	225	217	161	61	86	133
217	106	85	42	205	220	133	156	109	92	121	17	145	206	157	131	133	67	61	245	181	181	49
157	201	133	137	73	62	121	251	205	176	37	112	229	37	61	226	145	151	193	87	133	23	109
97	43	145	221	181	157	157	82	49	18	241	207	25	132	217	68	109	246	85	182	121	107	169
121	138	109	63	169	252	25	177	229	113	73	38	193	227	37	152	241	88	97	13	157	202	145
73	222	121	158	157	83	181	19	73	197	265	133	1	69	193	247	85	183	109	108	145	44	193
97	64	241	253	13	178	181	114	97	39	97	228	169	153	169	89	85	14	253	203	25	128	169
217	159	13	84	205	20	133	198	181	134	121	59	145	248	85	184	133	109	61	45	253	223	49
121	243	169	179	37	115	157	40	169	229	73	154	193	90	97	15	109	204	229	129	97	65	145
133	85	109	21	217	199	121	135	85	60	205	249	61	174	181	110	145	46	49	224	157	160	133
121	180	181	105	97	41	97	230	157	155	73	91	193	16	109	205	169	130	169	66	85	244	145



Program 10

Geonomic Hexagons

The properties necessary to qualify for a numeric hexagonal table to be a *geonomic* hexagon are:

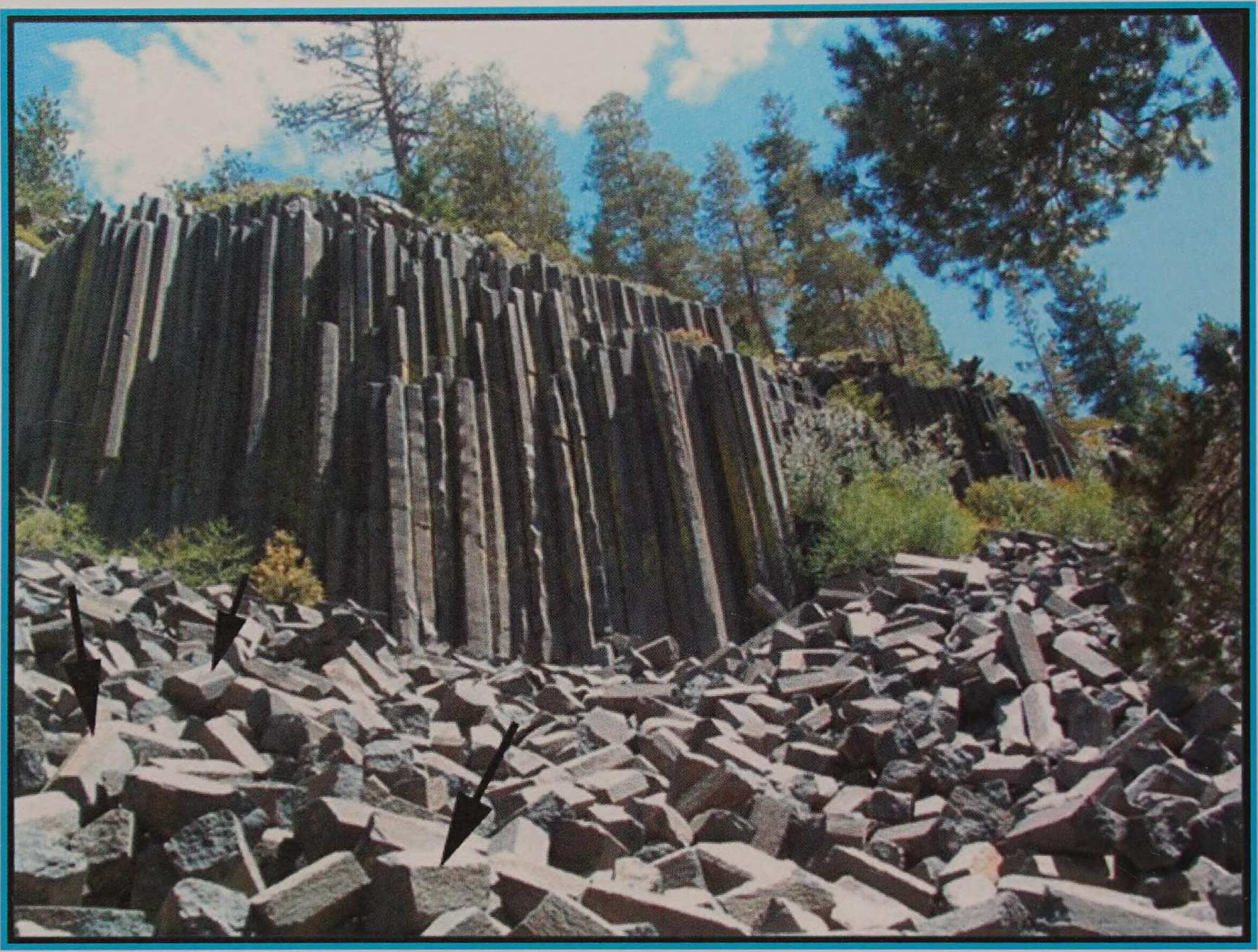
1. The numbers range in series starting from **1** to the total number of cells, middle cell excluded.
2. There are **3** summing directions.
3. Each frame's sides sum equally, with **1** minor variance permitted in the next to the innermost frame.
4. The innermost frame is exempt because is impossible to have all adjacent pairs equal by sharing any **6** distinct numbers in a pairwise round-robin sequence.

By definition, numerical hexagons determined by others thus far that are to be found on the internet at Wikipedia.com do not satisfy these pre-conditions and are consequently not *geonomic*. Further, those numeric patterns yield no sensical loom tables. In fact, their hexagonal tables cannot even be regenerated by the loom tables derived from them. As a consequence they have no dual counterparts like those of the hexagonal tables shown in this program. And further, none of them possess any inherent patterns.

So, in this program, you're in for something very new.

Application #4 – Inherent Vibratory Membrane for Snowflake Formation

Devils Post-pile Rock Formation



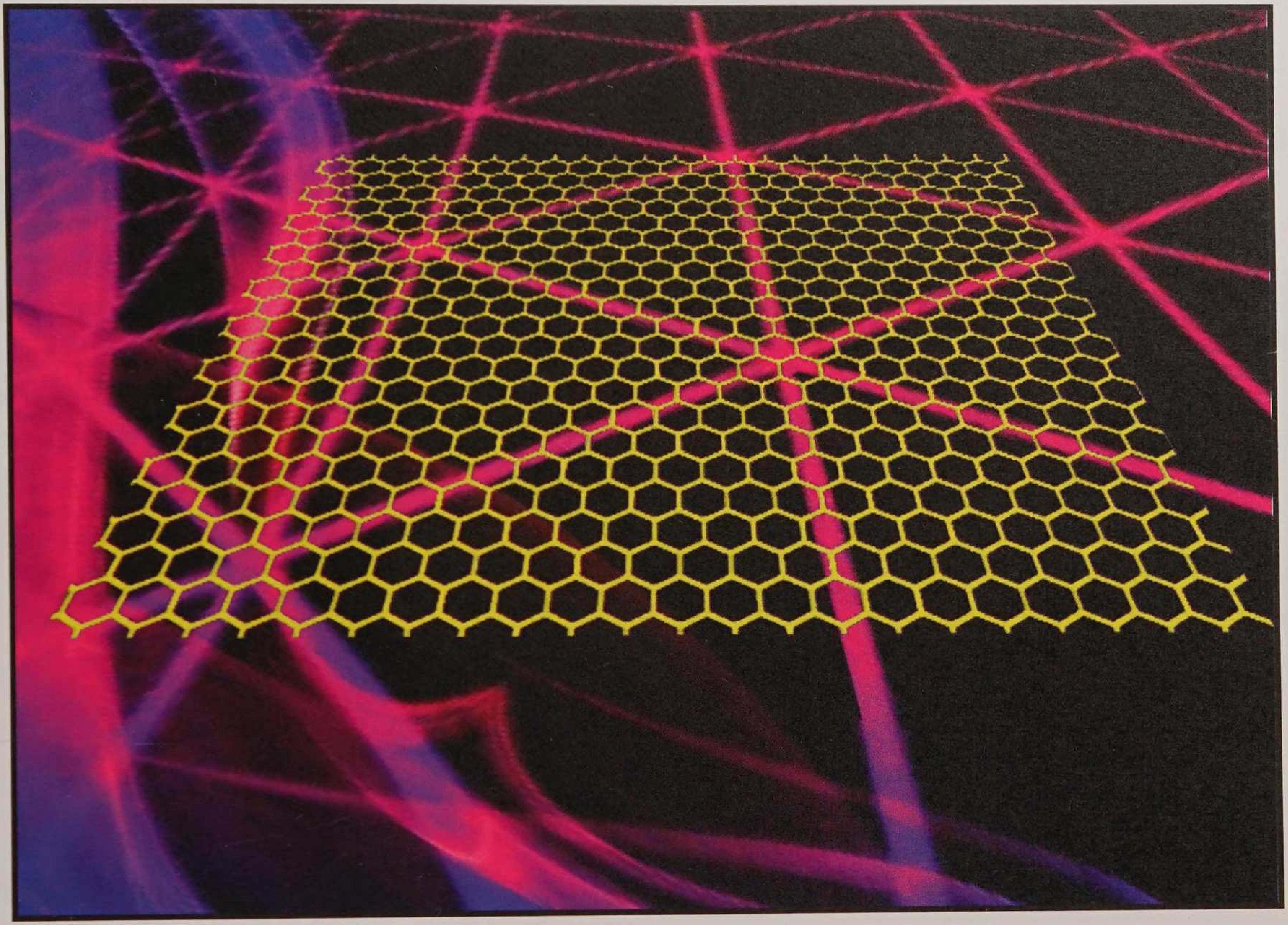
Before exploring hexagonal tables, we need to pictorially explore the rock formations at Devils Post Pile in the Sierra Nevada mountain range just west of the Sierra Nevada crest. The formation is a rare sight in the geological world and ranks as one of the world's finest examples of columnar basalt. Its columns tower **60** feet high and display an unusual hexagonal symmetry from top to bottom.

The post-pile was created by a lava flow which reached a thickness of **500** feet. Because of its unusual thickness, much of the mass of pooled lava cooled very slowly and evenly over time, which is why the columns are so long and so symmetrical, forming almost perfect vertical hexagonal columns.

The picture of Devils Post-pile is shown here to demonstrate that when liquid material is subject to uniform cooling, it shrinks into hexagonal segments.

Program 10

The Conversion of Vapor-laden air into Frozen Hexagonal Segments

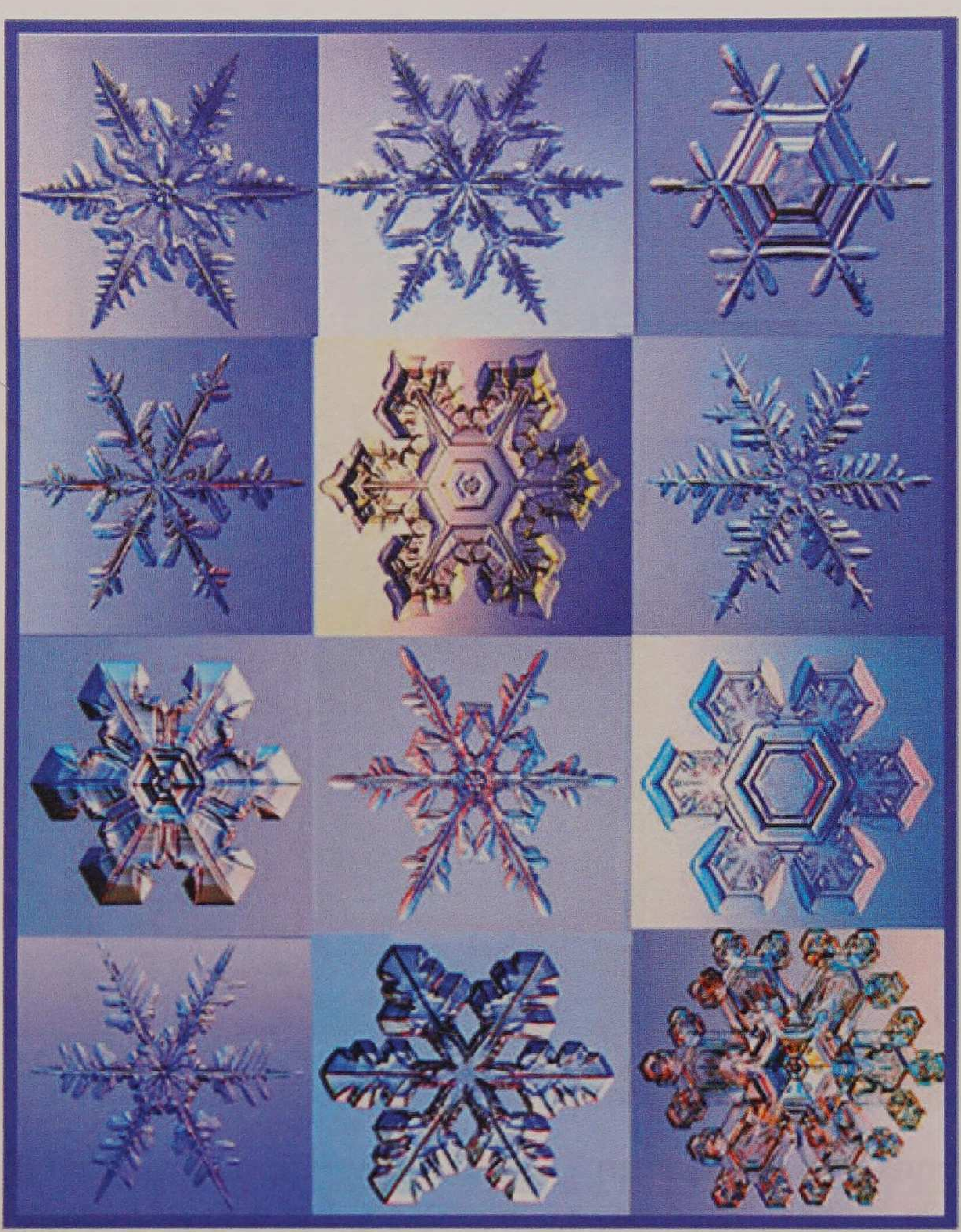


Even though ice expands when it forms from water, when it converts directly to a frozen blanket from moisture-laden vapor, it actually shrinks. When snow condenses out of a moisture-laden vapor, it first is a continuous sheet sandwiched between two contacting layers of air, one cold enough to make ice crystals form and the other layer containing the moisture-laden vapor. This thin blanket of frozen homogenous vapor then naturally separates uniformly into detached hexagonal segments.

From this point onward, these hexagonal membranes of ice configure their icy content into snowflakes according to the harmonic vibrational membranes naturally inherent in their local space. Here is where Geonometry picks up the story of the snowflake-formation process that Science can’t explain.

Program 10

Snowflakes are Numerical Weaves of Wonder

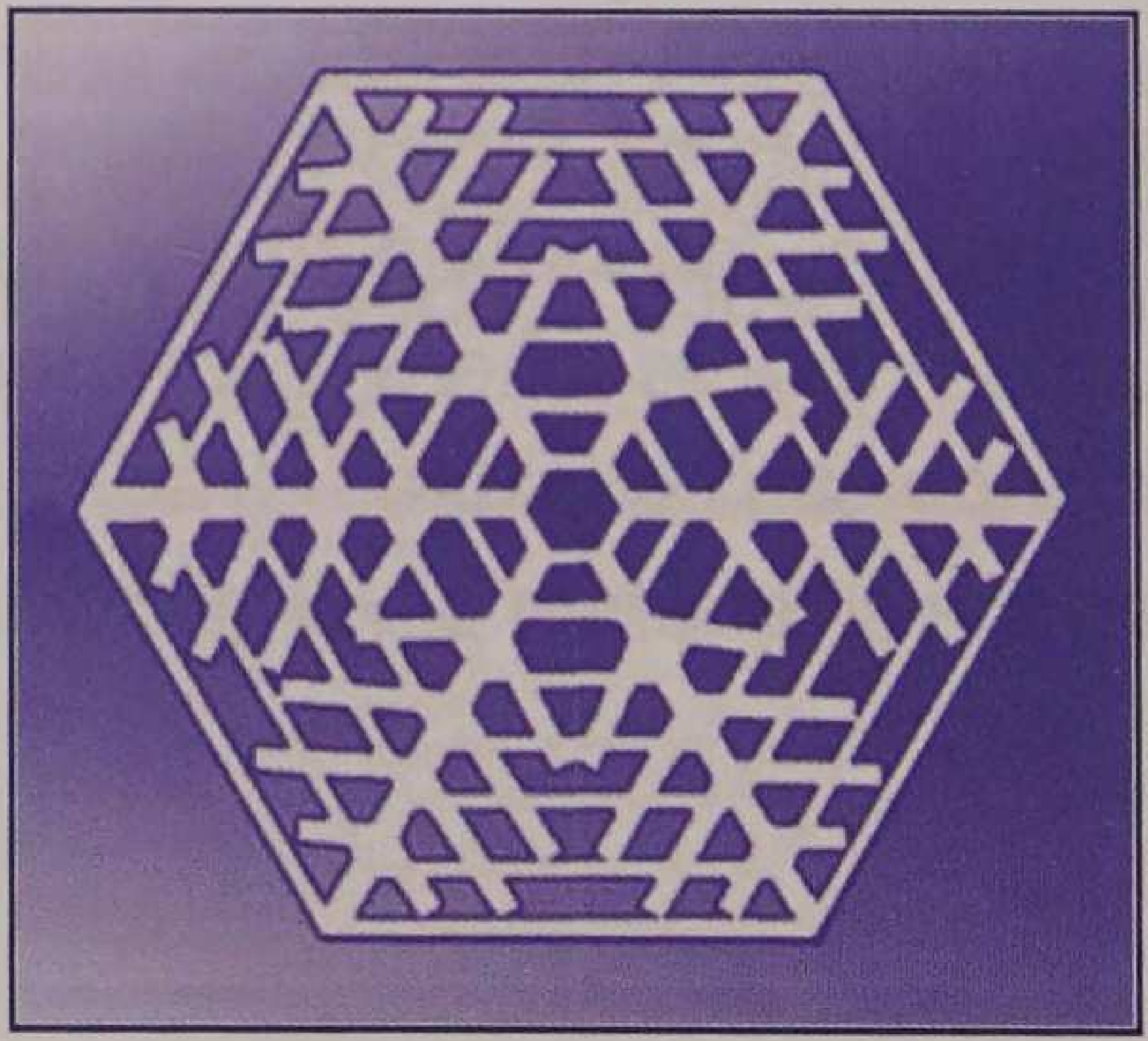


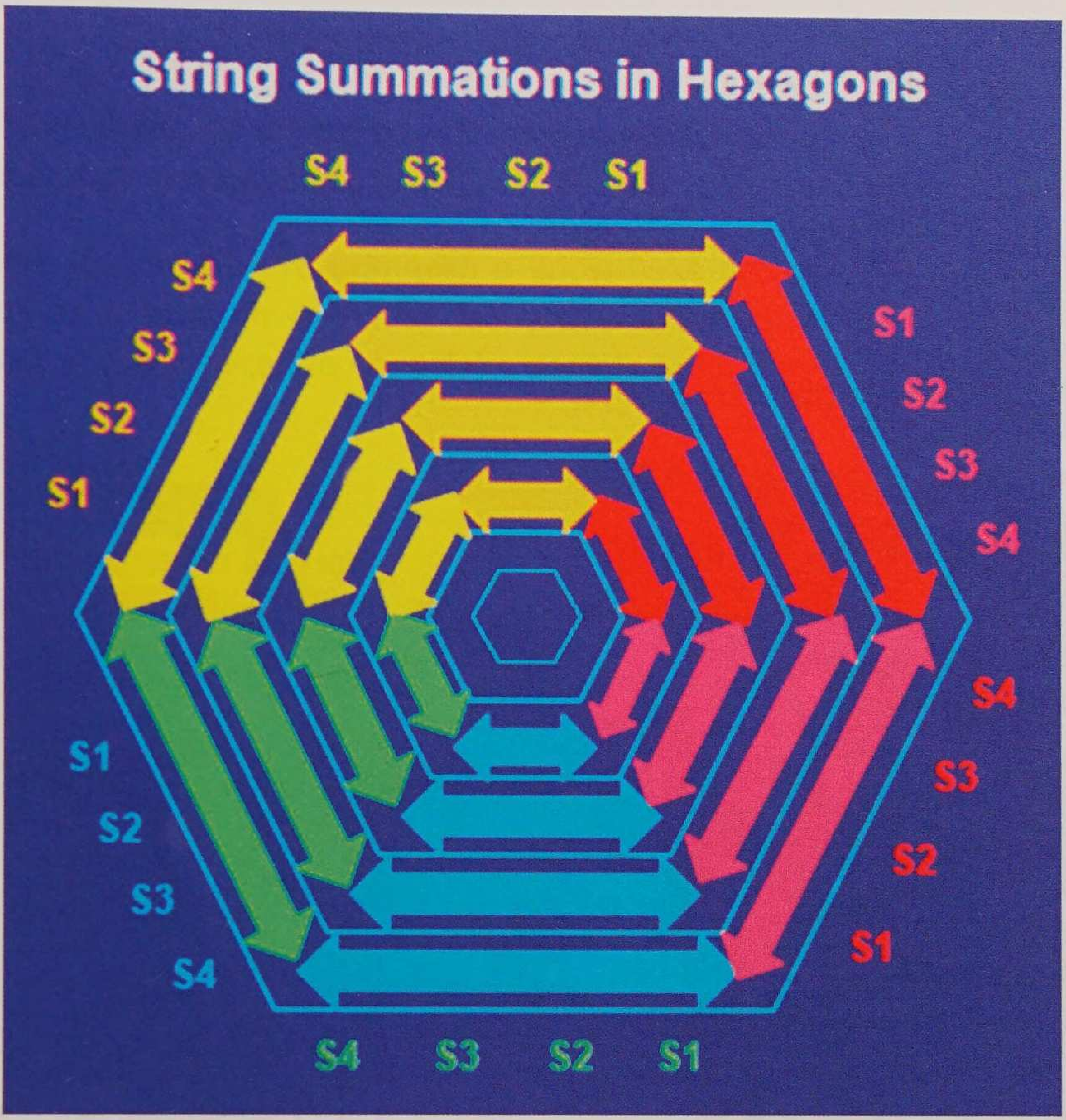
Snowflakes are really Numerical Weaves of Wonder.

This composite picture shows the perfect 3-directional symmetry of various snowflakes. This perfect symmetry cannot be explained by the molecular structure of water. So it must be coming from somewhere else. That is what will be demonstrated here mathematically.

Photos Courtesy of Kenneth G. Libbrecht PhD.
at Caltech University re:
www.its.caltech.edu/~atomic/snowcrystals/photos

The diagram to the right shows the snowflake pattern spread across a series of nested hexagonal frames of decreasing size. This is the form of the equal summation patterns that will be shown for various size hexagonal tables to follow.





This diagram shows the relationship of numbers outside the hexagon to the summations taken within it. The lettered points correspond to summations taken along their identically colored arrows, with the number corresponding to the length of the arrow: the larger its value, the longer the summation string and conversely, the smaller its value, the shorter the summation string.

All but the next to last smallest frame will have **6** equal-summing sides. By the nature of space itself, the modulus table may have one side that is greater or lesser by **±1.**

Each hexagonal table contains all the numbers in the series from **1** thru capital **N** given by formulas (**10.12**) and (**10.13**) for a hexagon whose size is **n**, as measured by the number of cells per hexagonal side in the outermost frame.

(10.12) $m = 3(n-1)$

(10.13) $N = nm$

Derivations of the two dual hexagons and their complementary loom tables will use both numbers **n** and **m** as the bases involved.

The central numbers of all hexagonal tables will be excluded, because they cause more complications in the formulations than their accommodation would resolve. To make the formulas all work together, the center number in all tables will be made a **1**.

The size-5 hexagon

Here are the size-5 dual geonomic hexagons. The primal hexagon **W** has all its calculations done in the base **m=12**; the dual **U** has all its calculations done in the base **n=5**. Here are the formulas for those calculations:

(10.14) $X(m) = \mathrm{Mod}\,[W(m)-1 \mid m] + |1|$

(10.15) $Y(m) = \mathrm{Int}\,[(W(m)-1) / m] + |1|$

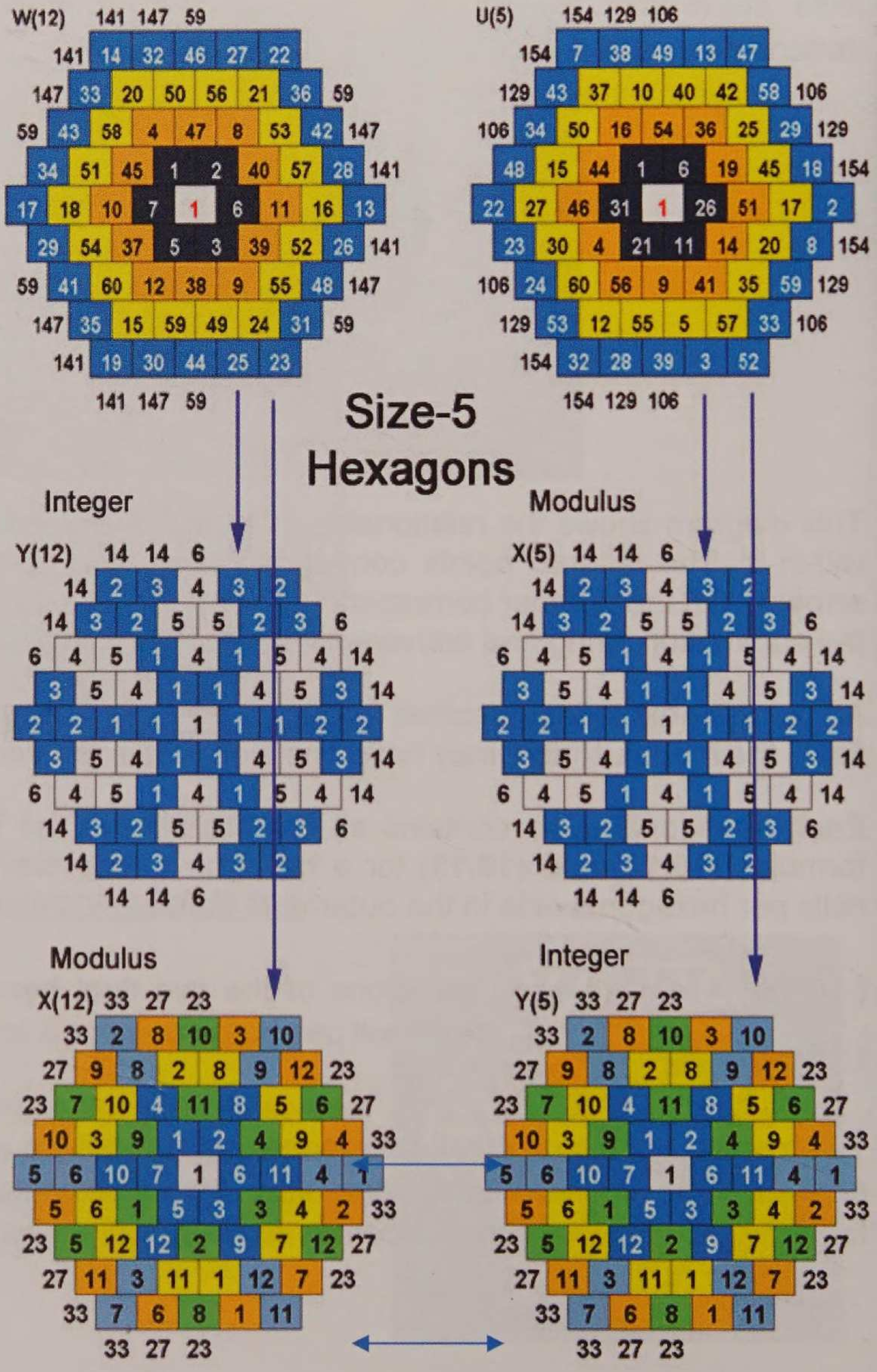
(10.16) $X(n) = \mathrm{Int}\,[(U(n)-1) / n] + |1|$

(10.17) $Y(n) = \mathrm{Mod}\,[U(n)-1 \mid n] + |1|$

Now here's the amazing part:
just like for squares.

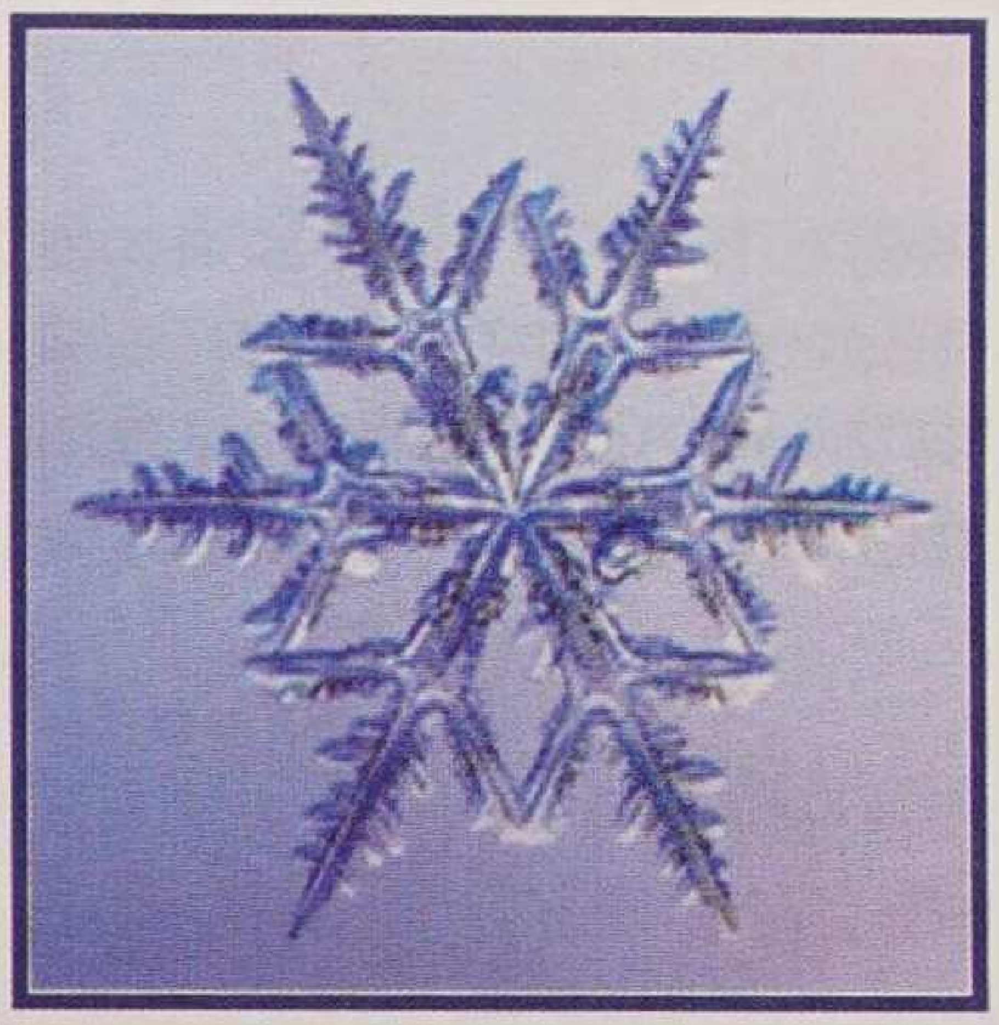
(10.18) $\begin{cases} Y(m) = X(n) \\ X(m) = Y(n) \end{cases}$

In what follows, tables **Y(n)** and **Y(m)** will be shown along with the primal hexagon **W(m)**. Derivation of their duals **U(n)** will be left as an exercise if you want to explore and verify them.



Program 10

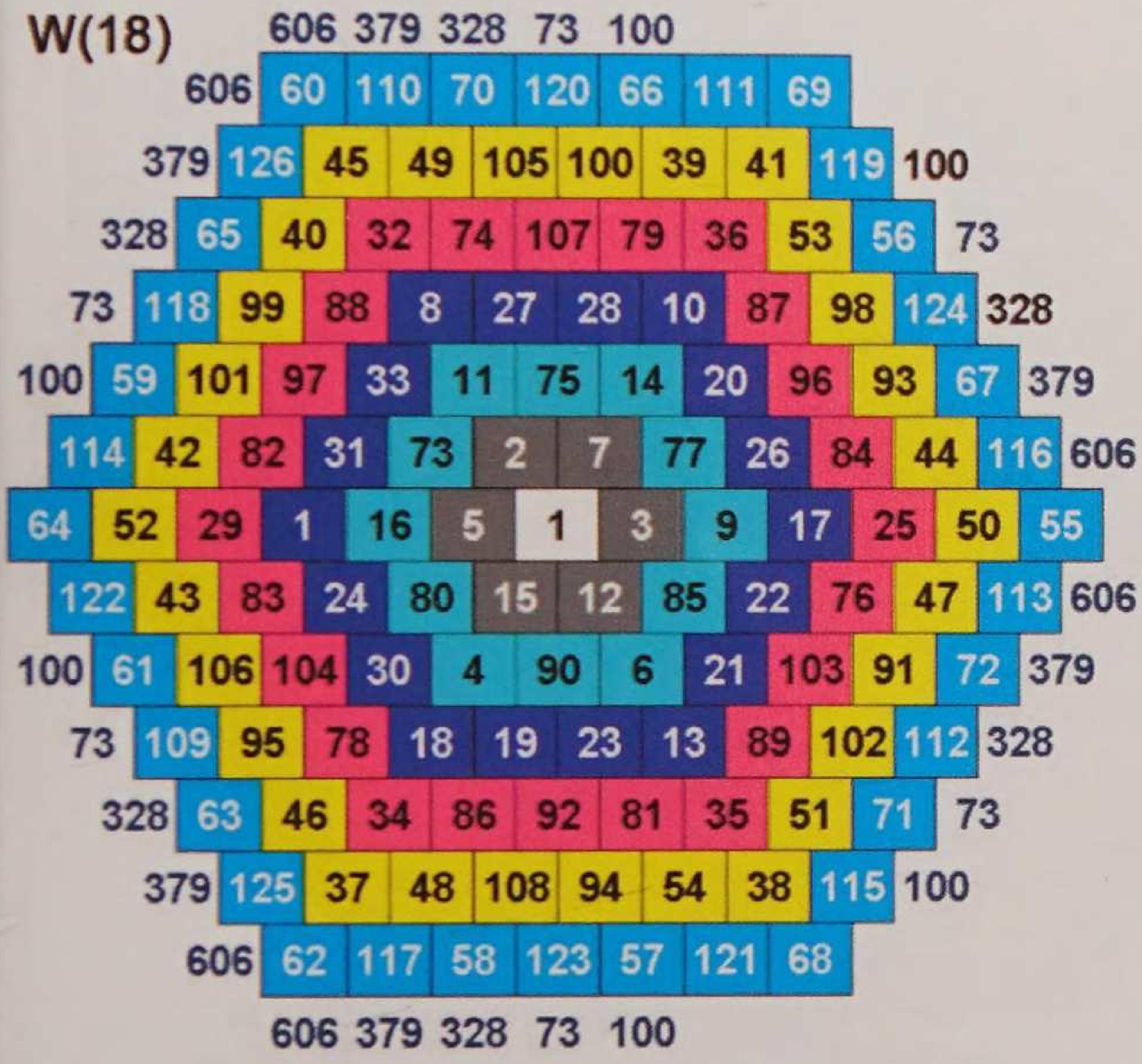
Size-7 Hexagon



Here is the size-7 hexagon. All of its nested frames have equal-summing sides. It is geonomically perfect.

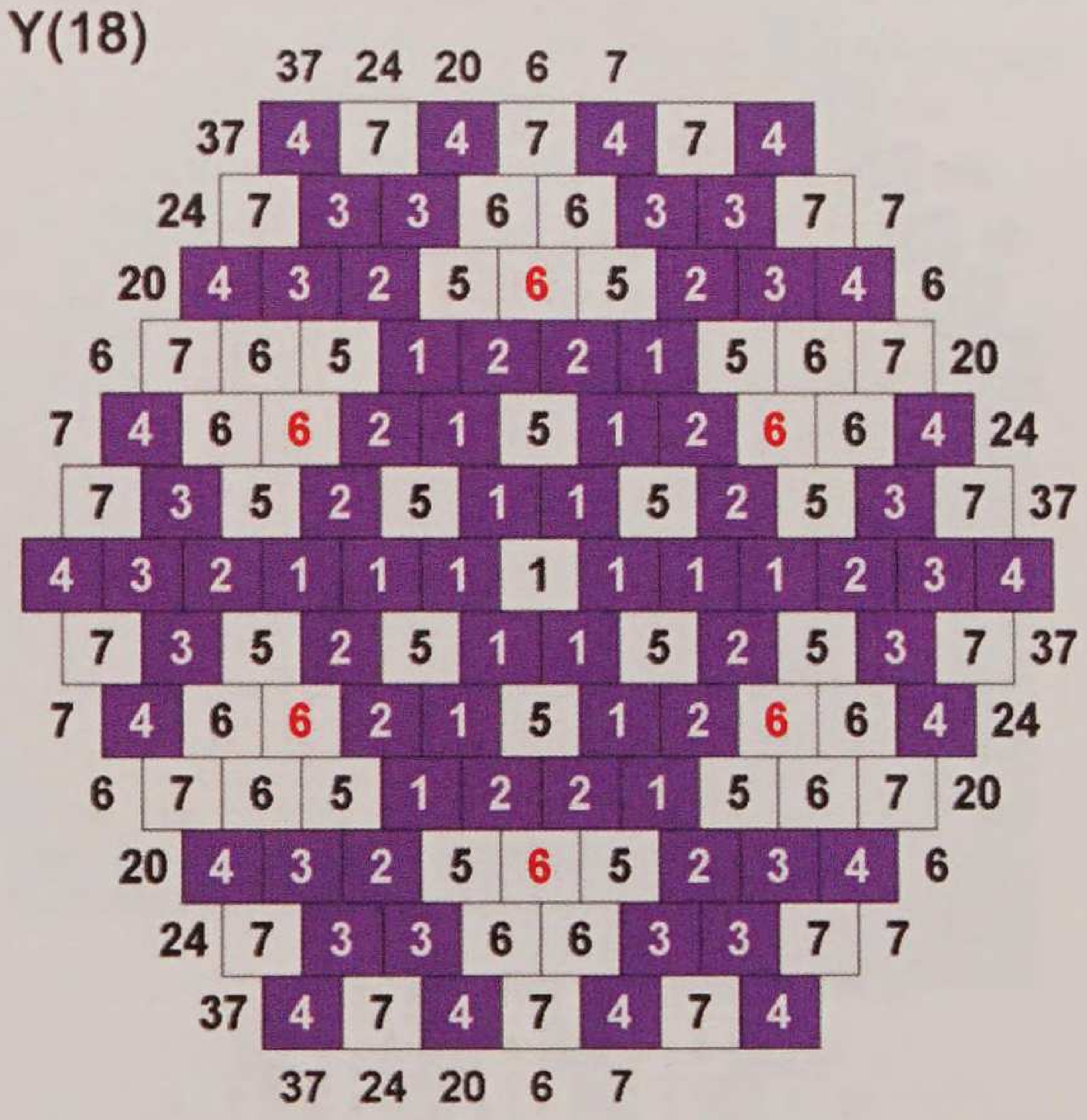
Its integer loom table in the base **18** unmasks a snow-flake pattern upon shading numbers **1** through **4**. All of its frames have equal-summing sides. The pattern mimics the snowflake depicted at right.

W(18)



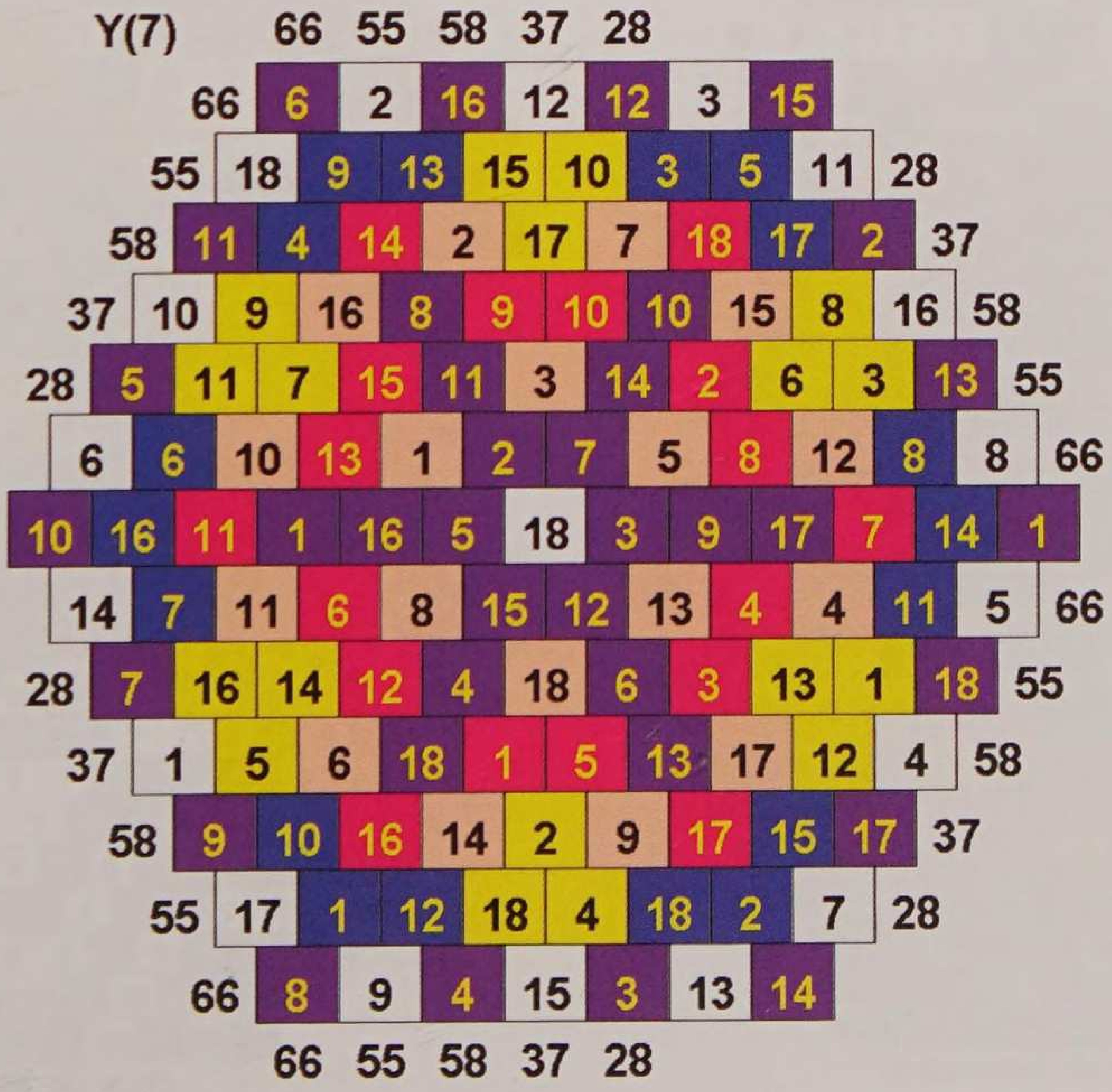
606 379 328 73 100
606 60 110 70 120 66 111 69
379 126 45 49 105 100 39 41 119 100
328 65 40 32 74 107 79 36 53 56 73
73 118 99 88 8 27 28 10 87 98 124 328
100 59 101 97 33 11 75 14 20 96 93 67 379
114 42 82 31 73 2 7 77 26 84 44 116 606
64 52 29 1 16 5 1 3 9 17 25 50 55
122 43 83 24 80 15 12 85 22 76 47 113 606
100 61 106 104 30 4 90 6 21 103 91 72 379
73 109 95 78 18 19 23 13 89 102 112 328
328 63 46 34 86 92 81 35 51 71 73
379 125 37 48 108 94 54 38 115 100
606 62 117 58 123 57 121 68
606 379 328 73 100

Y(18)



37 24 20 6 7
37 4 7 4 7 4 7 4
24 7 3 3 6 6 3 3 7 7
20 4 3 2 5 6 5 2 3 4 6
6 7 6 5 1 2 2 1 5 6 7 20
7 4 6 6 2 1 5 1 2 6 6 4 24
7 3 5 2 5 1 1 5 2 5 3 7 37
4 3 2 1 1 1 1 1 1 1 2 3 4
7 3 5 2 5 1 1 5 2 5 3 7 37
7 4 6 6 2 1 5 1 2 6 6 4 24
6 7 6 5 1 2 2 1 5 6 7 20
20 4 3 2 5 6 5 2 3 4 6
24 7 3 3 6 6 3 3 7 7
37 4 7 4 7 4 7 4
37 24 20 6 7

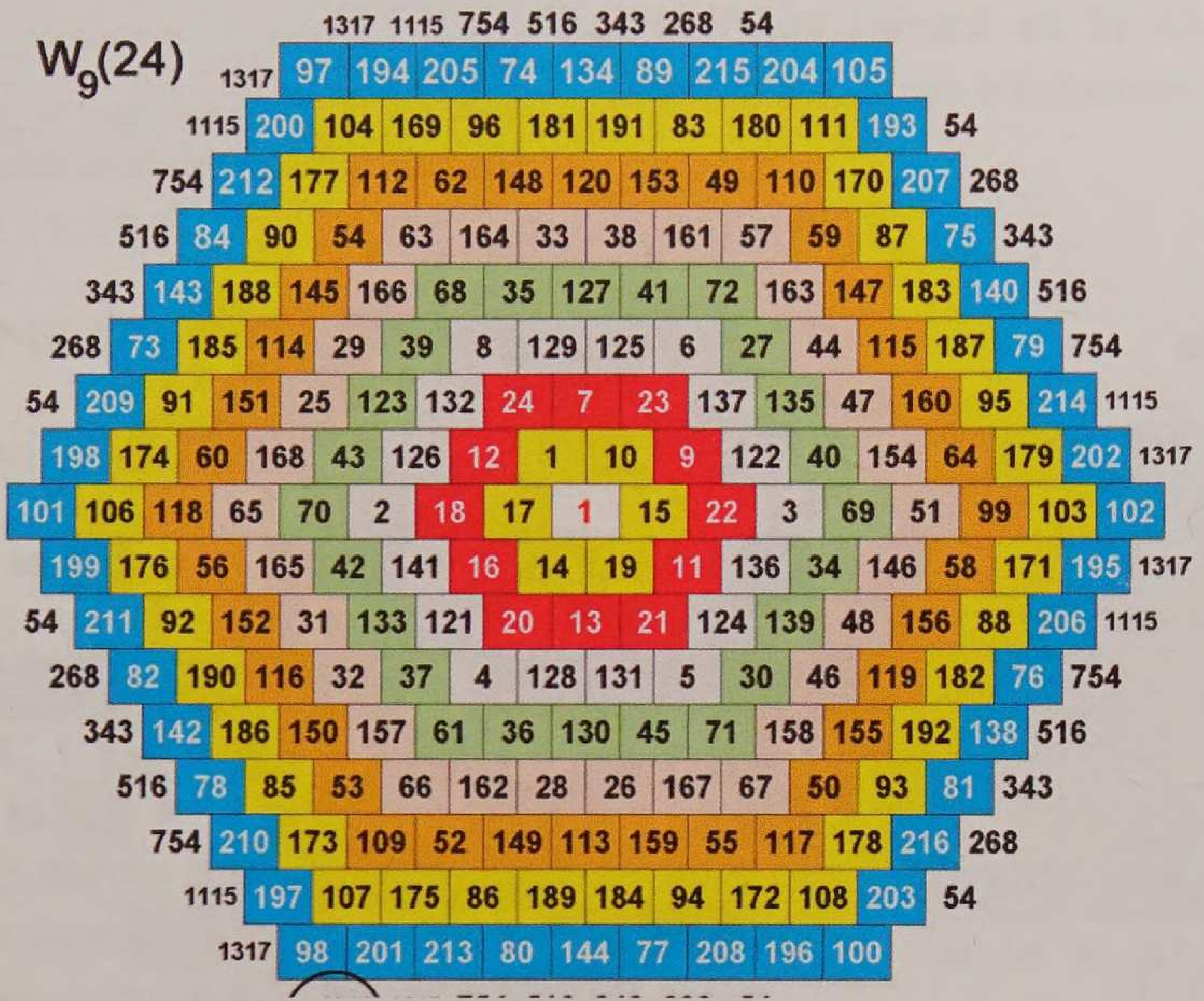
Y(7)



66 55 58 37 28
66 6 2 16 12 12 3 15
55 18 9 13 15 10 3 5 11 28
58 11 4 14 2 17 7 18 17 2 37
37 10 9 16 8 9 10 10 15 8 16 58
28 5 11 7 15 11 3 14 2 6 3 13 55
6 6 10 13 1 2 7 5 8 12 8 8 66
10 16 11 1 16 5 18 3 9 17 7 14 1
14 7 11 6 8 15 12 13 4 4 11 5 66
28 7 16 14 12 4 18 6 3 13 1 18 55
37 1 5 6 18 1 5 13 17 12 4 58
58 9 10 16 14 2 9 17 15 17 37
55 17 1 12 18 4 18 2 7 28
66 8 9 4 15 3 13 14
66 55 58 37 28

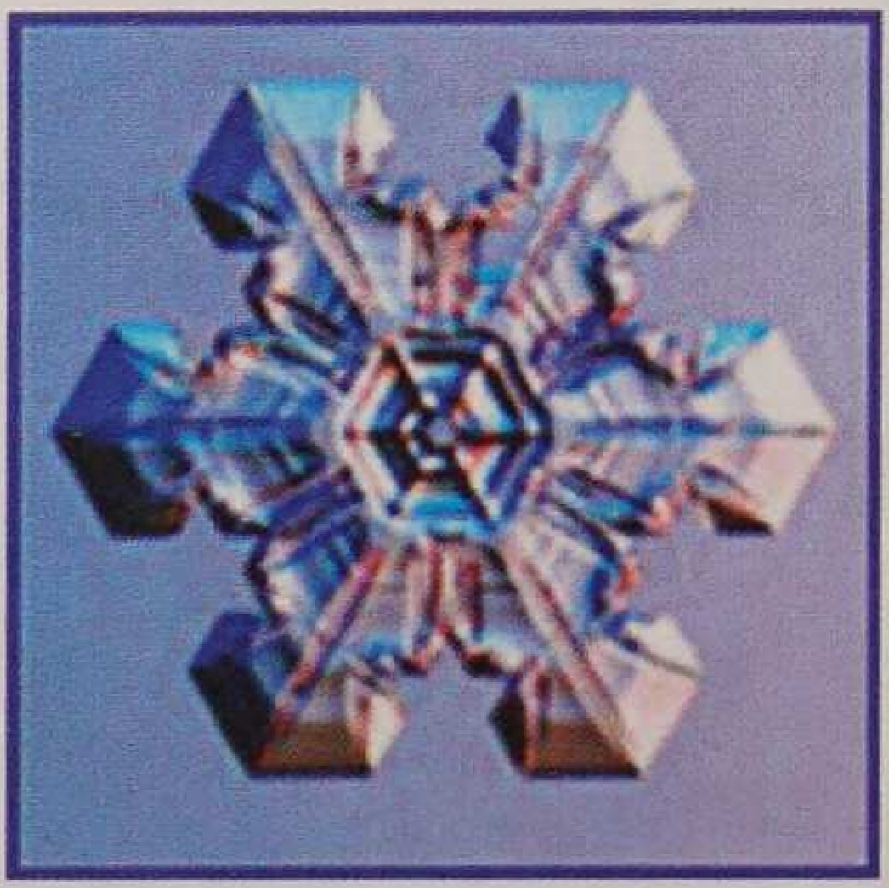
Size-9 Hexagon

Here is the size-9 hexagon. It contains all of the numbers from **1** thru **216**. All but one of its nested frames have equal-summing sides. Its outermost frame has one side that is off by **+1**. This minor variation in the outermost frame is a spatial wrinkle that cannot be eliminated but merely shifted to another frame.

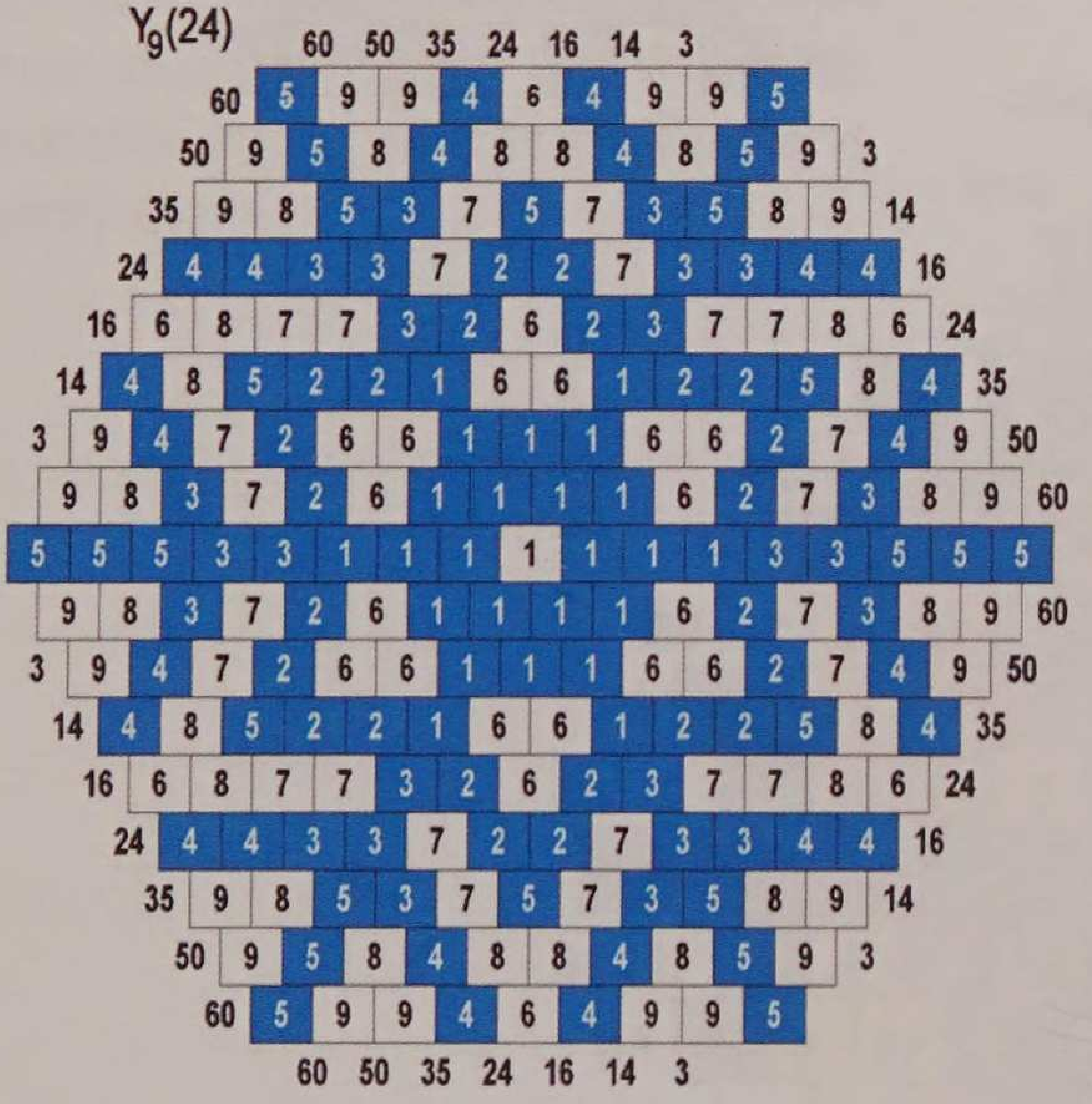


$W_9(24)$

```
          1317 1115 754 516 343 268 54
1317 97 194 205 74 134 89 215 204 105
1115 200 104 169 96 181 191 83 180 111 193 54
754 212 177 112 62 148 120 153 49 110 170 207 268
516 84 90 54 63 164 33 38 161 57 59 87 75 343
343 143 188 145 166 68 35 127 41 72 163 147 183 140 516
268 73 185 114 29 39 8 129 125 6 27 44 115 187 79 754
54 209 91 151 25 123 132 24 7 23 137 135 47 160 95 214 1115
198 174 60 168 43 126 12 1 10 9 122 40 154 64 179 202 1317
101 106 118 65 70 2 18 17 1 15 22 3 69 51 99 103 102
199 176 56 165 42 141 16 14 19 11 136 34 146 58 171 195 1317
54 211 92 152 31 133 121 20 13 21 124 139 48 156 88 206 1115
268 82 190 116 32 37 4 128 131 5 30 46 119 182 76 754
343 142 186 150 157 61 36 130 45 71 158 155 192 138 516
516 78 85 53 66 162 28 26 167 67 50 93 81 343
754 210 173 109 52 149 113 159 55 117 178 216 268
1115 197 107 175 86 189 184 94 172 108 203 54
1317 98 201 213 80 144 77 208 196 100
```



The integer loom-table in the base **m=24** reveals the hidden snowflake by highlighting the numbers **1** thru **5**. All of its frames have equal-summing sides. It mimics the snow-flake pattern above.



$Y_9(24)$

```
      60 50 35 24 16 14 3
60 5 9 9 4 6 4 9 9 5
50 9 5 8 4 8 8 4 8 5 9 3
35 9 8 5 3 7 5 7 3 5 8 9 14
24 4 4 3 3 7 2 2 7 3 3 4 4 16
16 6 8 7 7 3 2 6 2 3 7 7 8 6 24
14 4 8 5 2 2 1 6 6 1 2 2 5 8 4 35
3 9 4 7 2 6 6 1 1 1 6 6 2 7 4 9 50
9 8 3 7 2 6 1 1 1 1 6 2 7 3 8 9 60
5 5 5 3 3 1 1 1 1 1 1 1 3 3 5 5 5
9 8 3 7 2 6 1 1 1 1 6 2 7 3 8 9 60
3 9 4 7 2 6 6 1 1 1 6 6 2 7 4 9 50
14 4 8 5 2 2 1 6 6 1 2 2 5 8 4 35
16 6 8 7 7 3 2 6 2 3 7 7 8 6 24
24 4 4 3 3 7 2 2 7 3 3 4 4 16
35 9 8 5 3 7 5 7 3 5 8 9 14
50 9 5 8 4 8 8 4 8 5 9 3
60 5 9 9 4 6 4 9 9 5
      60 50 35 24 16 14 3
```

Y(9)

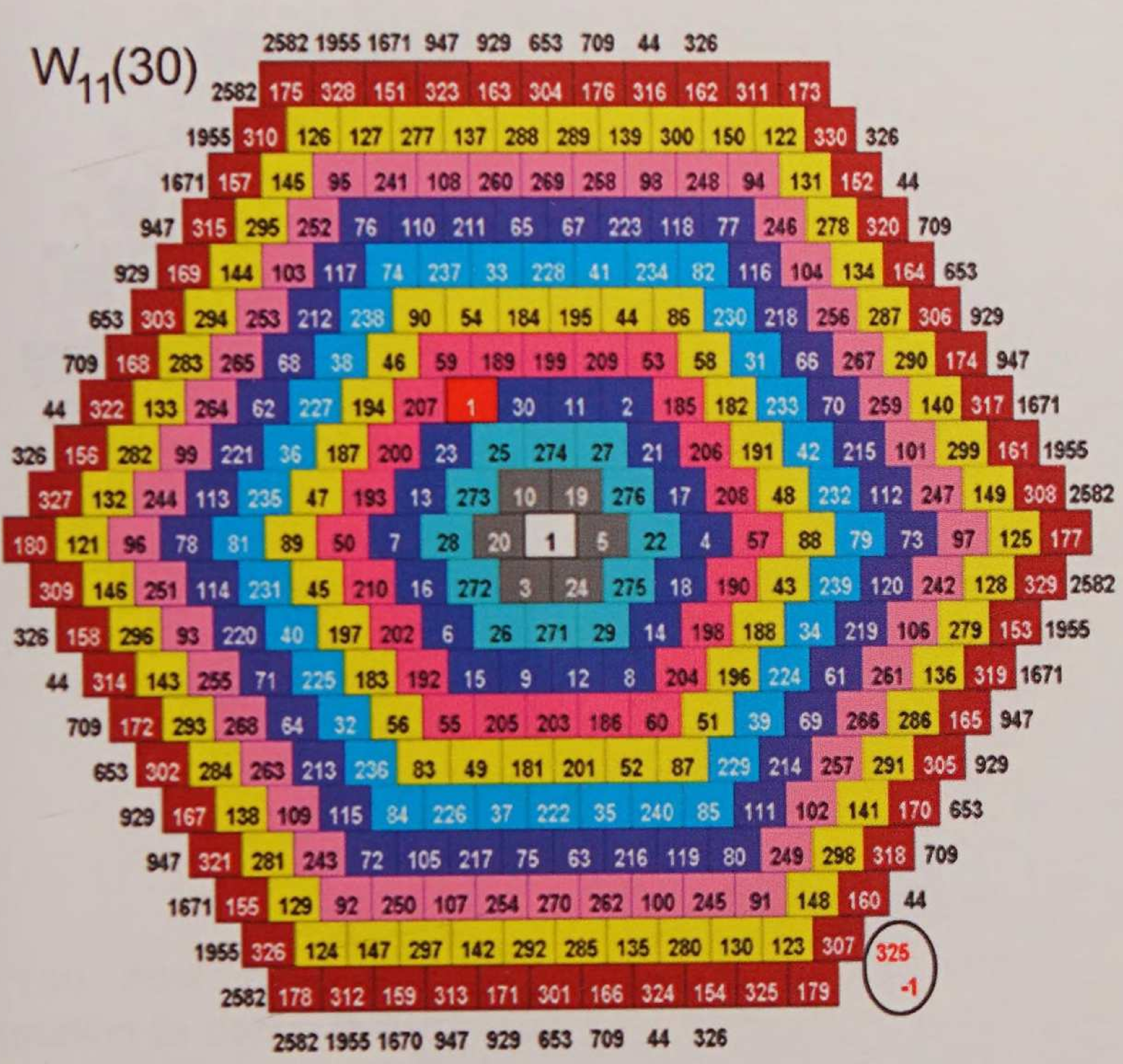
```
      93 107 82 84 79 28 54
93 1 2 13 2 14 17 23 12 9
107 8 8 1 24 13 23 11 12 15 1 54
82 20 9 16 14 4 24 9 1 14 2 15 28
84 12 18 6 15 20 9 14 17 9 11 15 3 79
79 23 20 1 22 20 11 7 17 24 19 3 15 20 84
28 1 17 18 5 15 8 9 5 6 3 20 19 19 7 82
54 17 19 7 1 3 12 24 7 23 17 15 23 16 23 22 107
6 6 12 24 19 6 12 1 10 9 2 16 10 16 11 10 93
5 10 22 17 22 2 18 17 1 15 22 3 21 3 3 7 6
7 8 8 21 18 21 16 14 19 11 16 10 2 10 3 3 93
54 19 20 8 7 13 1 20 13 21 4 19 24 12 16 14 107
28 10 22 20 8 13 4 8 11 5 6 22 23 14 4 82
79 22 18 6 13 13 12 10 21 23 14 11 24 18 84
84 6 13 5 18 18 4 2 23 19 2 21 9 79
82 18 5 13 4 5 17 15 7 21 10 24 28
107 5 11 7 14 21 16 22 4 12 11 54
93 2 9 21 8 24 5 16 4 4
      94 107 82 84 79 28 54
      +1
```

Program 10

Size-11 Hexagon

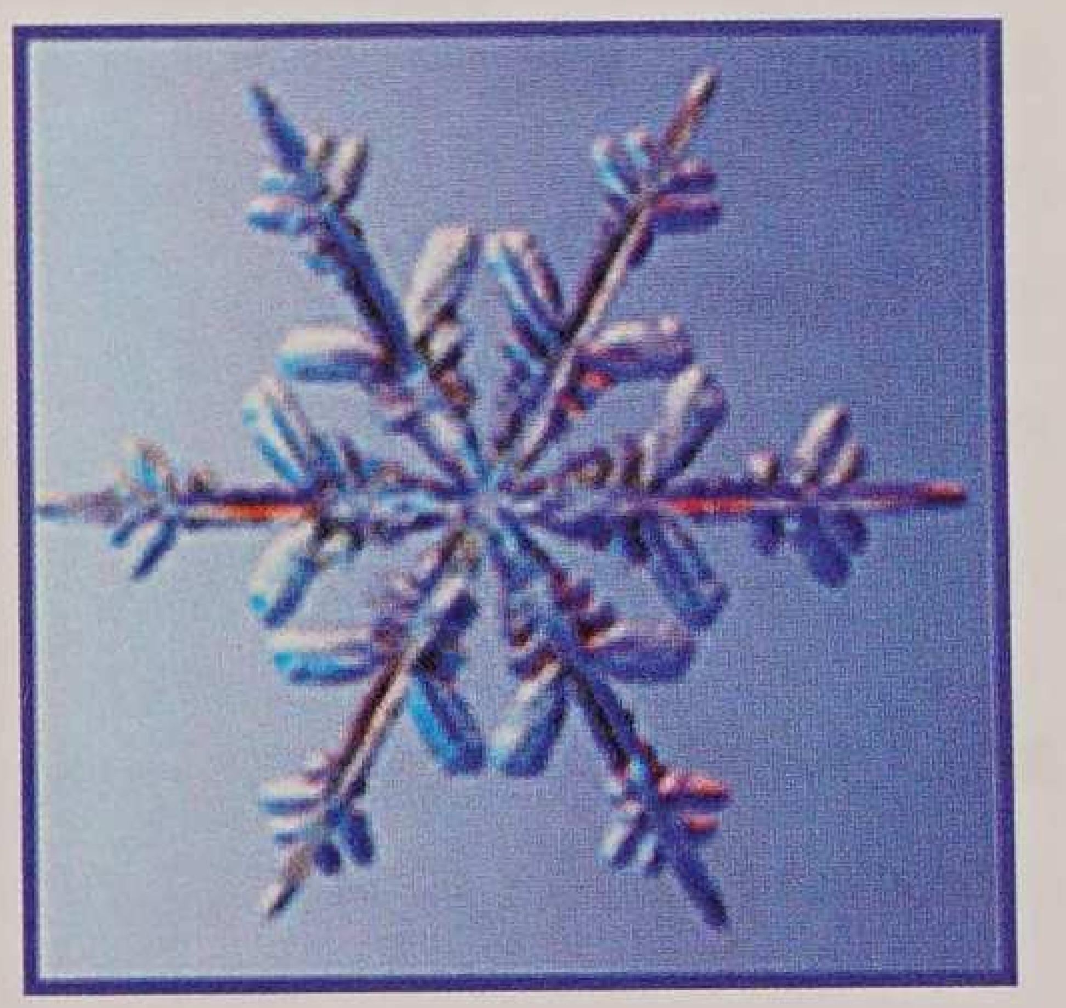
Here is the size-11 hexagon. It contains all the numbers from **1** to **330**. All but one of its nested frames have equal-summing sides. Its innermost viable frame has one side that is off by **1**. This minor variation in the innermost viable frame cannot be eliminated, only shifted to another frame.

$W_{11}(30)$

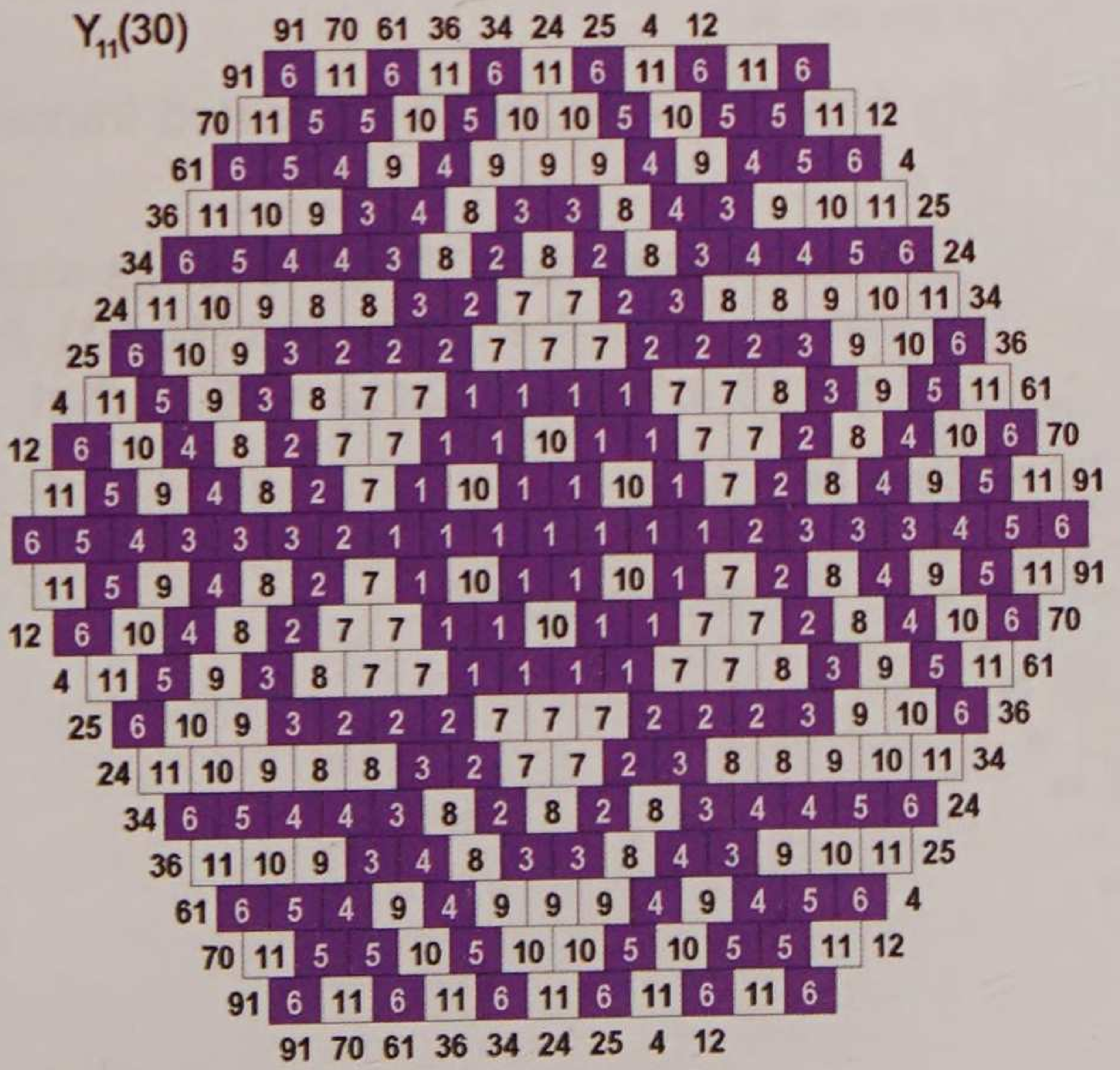


2582 1955 1671 947 929 653 709 44 326
2582 175 328 151 323 163 304 176 316 162 311 173
1955 310 126 127 277 137 288 289 139 300 150 122 330 326
1671 157 145 95 241 108 260 269 258 98 248 94 131 152 44
947 315 295 252 76 110 211 65 67 223 118 77 246 278 320 709
929 169 144 103 117 74 237 33 228 41 234 82 116 104 134 164 653
653 303 294 253 212 238 90 54 184 195 44 86 230 218 256 287 306 929
709 168 283 265 68 38 46 59 189 199 209 53 58 31 66 267 290 174 947
44 322 133 264 62 227 194 207 1 30 11 2 185 182 233 70 259 140 317 1671
326 156 282 99 221 36 187 200 23 25 274 27 21 206 191 42 215 101 299 161 1955
327 132 244 113 235 47 193 13 273 10 19 276 17 208 48 232 112 247 149 308 2582
180 121 96 78 81 89 50 7 28 20 1 5 22 4 57 88 79 73 97 125 177
309 146 251 114 231 45 210 16 272 3 24 275 18 190 43 239 120 242 128 329 2582
326 158 296 93 220 40 197 202 6 26 271 29 14 198 188 34 219 106 279 153 1955
44 314 143 255 71 225 183 192 15 9 12 8 204 196 224 61 261 136 319 1671
709 172 293 268 64 32 56 55 205 203 186 60 51 39 69 266 286 165 947
653 302 284 263 213 236 83 49 181 201 52 87 229 214 257 291 305 929
929 167 138 109 115 84 226 37 222 35 240 85 111 102 141 170 653
947 321 281 243 72 105 217 75 63 216 119 80 249 298 318 709
1671 155 129 92 250 107 254 270 262 100 245 91 148 160 44
1955 326 124 147 297 142 292 285 135 280 130 123 307 325 -1
2582 178 312 159 313 171 301 166 324 154 325 179
2582 1955 1670 947 929 653 709 44 326

Its Integer loom-table reveals the hidden snowflake pattern by highlighting the numbers **1** thru **6**. It mimics the snowflake shown here. All of its sides

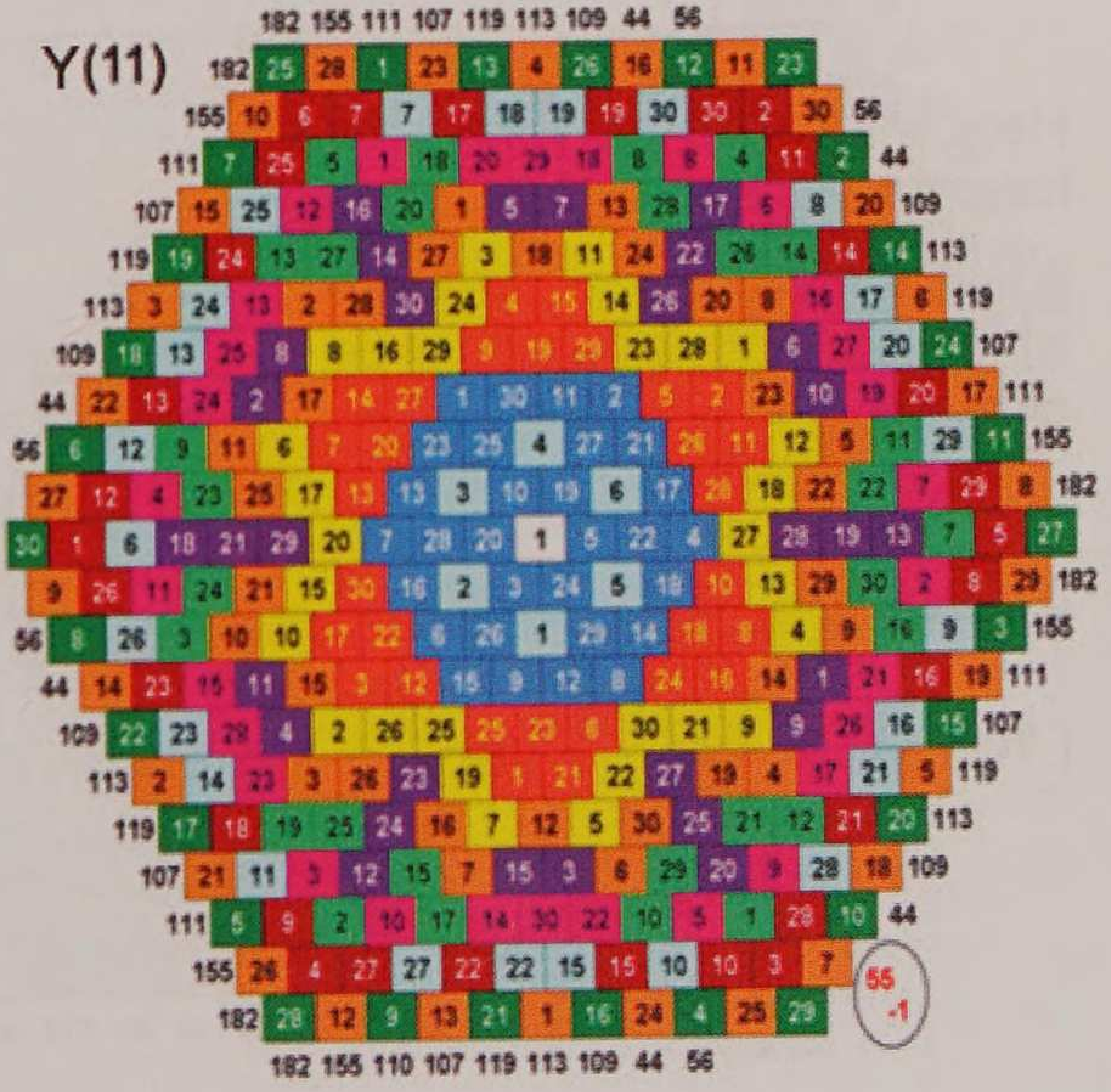


$Y_{11}(30)$



91 70 61 36 34 24 25 4 12
91 6 11 6 11 6 11 6 11 6 11 6
70 11 5 5 10 5 10 10 5 10 5 5 11 12
61 6 5 4 9 4 9 9 9 4 9 4 5 6 4
36 11 10 9 3 4 8 3 3 8 4 3 9 10 11 25
34 6 5 4 4 3 8 2 8 2 8 3 4 4 5 6 24
24 11 10 9 8 8 3 2 7 7 2 3 8 8 9 10 11 34
25 6 10 9 3 2 2 2 7 7 7 2 2 2 3 9 10 6 36
4 11 5 9 3 8 7 7 1 1 1 1 7 7 8 3 9 5 11 61
12 6 10 4 8 2 7 7 1 1 10 1 1 7 7 2 8 4 10 6 70
11 5 9 4 8 2 7 1 10 1 1 10 1 7 2 8 4 9 5 11 91
6 5 4 3 3 3 2 1 1 1 1 1 1 1 2 3 3 3 4 5 6
11 5 9 4 8 2 7 1 10 1 1 10 1 7 2 8 4 9 5 11 91
12 6 10 4 8 2 7 7 1 1 10 1 1 7 7 2 8 4 10 6 70
4 11 5 9 3 8 7 7 1 1 1 1 7 7 8 3 9 5 11 61
25 6 10 9 3 2 2 2 7 7 7 2 2 2 3 9 10 6 36
24 11 10 9 8 8 3 2 7 7 2 3 8 8 9 10 11 34
34 6 5 4 4 3 8 2 8 2 8 3 4 4 5 6 24
36 11 10 9 3 4 8 3 3 8 4 3 9 10 11 25
61 6 5 4 9 4 9 9 9 4 9 4 5 6 4
70 11 5 5 10 5 10 10 5 10 5 5 11 12
91 6 11 6 11 6 11 6 11 6 11 6
91 70 61 36 34 24 25 4 12

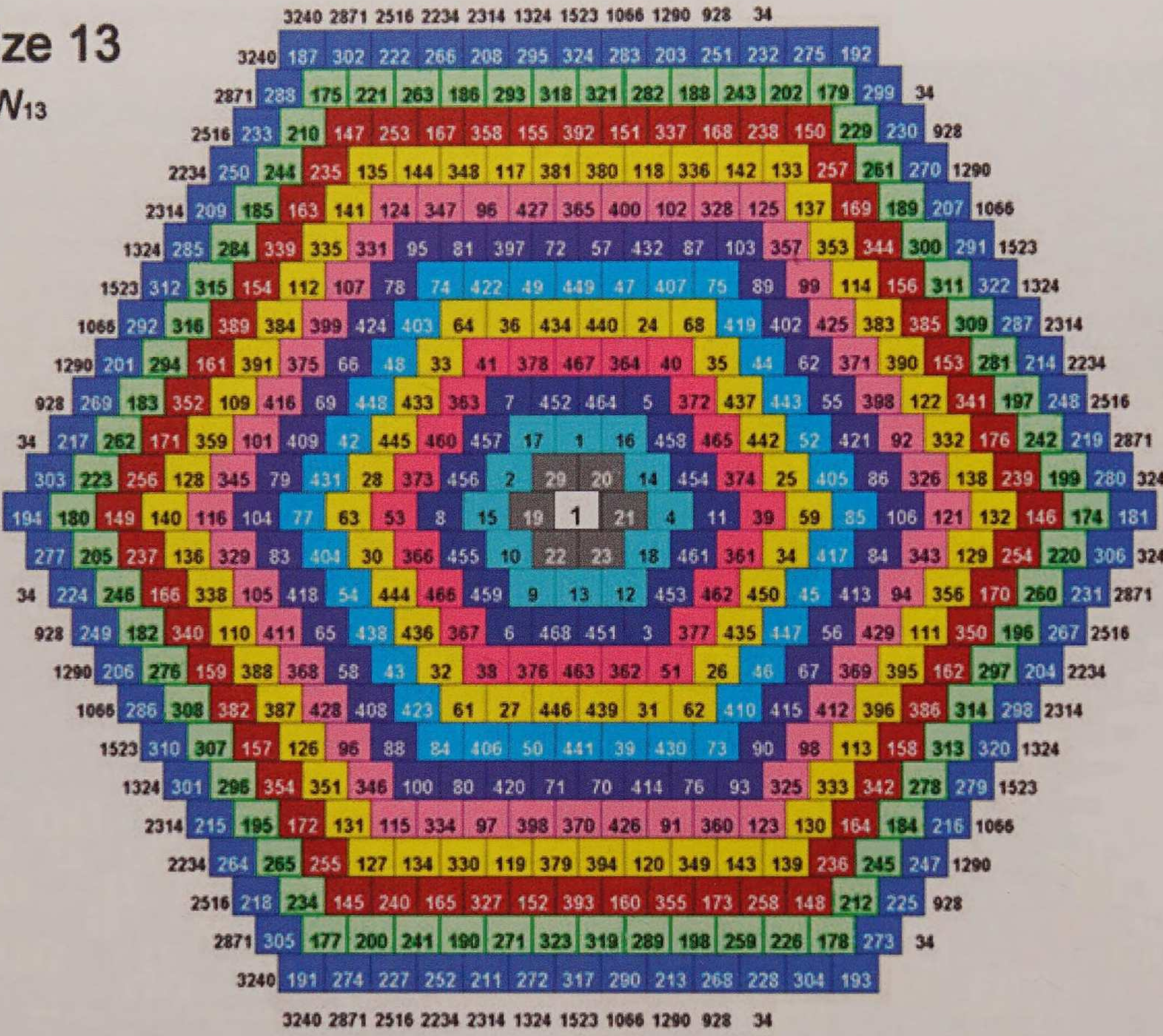
Y(11)



182 155 111 107 119 113 109 44 56
182 25 28 1 23 13 4 26 16 12 11 23
155 10 6 7 7 17 18 19 19 30 30 2 30 56
111 7 25 5 1 18 20 29 18 8 8 4 11 2 44
107 15 25 12 16 20 1 5 7 13 28 17 6 8 20 109
119 19 24 13 27 14 27 3 18 11 24 22 26 14 14 14 113
113 3 24 13 2 28 30 24 4 15 14 26 20 8 16 17 6 119
109 18 13 25 8 8 16 29 9 19 29 23 28 1 6 27 20 24 107
44 22 13 24 2 17 14 27 1 30 11 2 5 2 23 10 19 20 17 111
56 6 12 9 11 6 7 20 23 25 4 27 21 26 11 12 5 11 29 11 155
27 12 4 23 25 17 13 13 3 10 19 6 17 28 18 22 22 7 29 8 182
30 1 6 18 21 29 20 7 28 20 1 5 22 4 27 28 19 13 7 5 27
9 26 11 24 21 15 30 16 2 3 24 5 18 10 13 29 30 2 8 29 182
56 8 26 3 10 10 17 22 6 26 1 29 14 18 8 4 9 16 9 3 155
44 14 23 15 11 15 3 12 15 9 12 8 24 16 14 1 21 16 19 111
109 22 23 28 4 2 26 25 25 23 6 30 21 9 9 26 16 15 107
113 2 14 23 3 26 23 19 1 21 22 27 19 4 17 21 5 119
119 17 18 19 25 24 16 7 12 5 30 25 21 12 21 20 113
107 21 11 3 12 15 7 15 3 6 29 20 9 28 18 109
111 5 9 2 10 17 14 30 22 10 5 1 28 10 44
155 26 4 27 27 22 22 15 15 10 10 3 7 55 -1
182 28 12 9 13 21 1 16 24 4 25 29
182 155 110 107 119 113 109 44 56

Size-13 Primal Hexagon

Size 13
W$_{13}$



3240 2871 2516 2234 2314 1324 1523 1066 1290 928 34
3240 187 302 222 266 208 295 324 283 203 251 232 275 192
2871 288 175 221 263 186 293 318 321 282 188 243 202 179 299 34
2516 233 210 147 253 167 358 155 392 151 337 168 238 150 229 230 928
2234 250 244 235 135 144 348 117 381 380 118 336 142 133 257 261 270 1290
2314 209 185 163 141 124 347 96 427 365 400 102 328 125 137 169 189 207 1066
1324 285 284 339 335 331 95 81 397 72 57 432 87 103 357 353 344 300 291 1523
1523 312 315 154 112 107 78 74 422 49 449 47 407 75 89 99 114 156 311 322 1324
1066 292 316 389 384 399 424 403 64 36 434 440 24 68 419 402 425 383 385 309 287 2314
1290 201 294 161 391 375 66 48 33 41 378 467 364 40 35 44 62 371 390 153 281 214 2234
928 269 183 352 109 416 69 448 433 363 7 452 464 5 372 437 443 55 398 122 341 197 248 2516
34 217 262 171 359 101 409 42 445 460 457 17 1 16 458 465 442 52 421 92 332 176 242 219 2871
303 223 256 128 345 79 431 28 373 456 2 29 20 14 454 374 25 405 86 326 138 239 199 280 3240
194 180 149 140 116 104 77 63 53 8 15 19 1 21 4 11 39 59 85 106 121 132 146 174 181
277 205 237 136 329 83 404 30 366 455 10 22 23 18 461 361 34 417 84 343 129 254 220 306 3240
34 224 246 166 338 105 418 54 444 466 459 9 13 12 453 462 450 45 413 94 356 170 260 231 2871
928 249 182 340 110 411 65 438 436 367 6 468 451 3 377 435 447 56 429 111 350 196 267 2516
1290 206 276 159 388 368 58 43 32 38 376 463 362 51 26 46 67 369 395 162 297 204 2234
1066 286 308 382 387 428 408 423 61 27 446 439 31 62 410 415 412 396 386 314 298 2314
1523 310 307 157 126 96 88 84 406 50 441 39 430 73 90 98 113 158 313 320 1324
1324 301 296 354 351 346 100 80 420 71 70 414 76 93 325 333 342 278 279 1523
2314 215 195 172 131 115 334 97 398 370 426 91 360 123 130 164 184 216 1066
2234 264 265 255 127 134 330 119 379 394 120 349 143 139 236 245 247 1290
2516 218 234 145 240 165 327 152 393 160 355 173 258 148 212 225 928
2871 305 177 200 241 190 271 323 319 289 198 259 226 178 273 34
3240 191 274 227 252 211 272 317 290 213 268 228 304 193
3240 2871 2516 2234 2314 1324 1523 1066 1290 928 34

Here is the size-13 hexagon. It contains all the numbers from **1** to **468**. All of its nested frames have equal-summing sides.

It was derived in the base **m’** = **18** = **½m** = **½36**, **half** the usual base **m = 36** for **W** for its size, which became unwieldy. To be in conformance with the relationship of the two bases (**10.13**), its dual hexagon should be derived in **26**, double the base of **n = 13**: **n’ = 2n = 2x13 = 26** so that

$$m'n' = m/2 \times \tfrac{1}{2}n = mn = N.$$

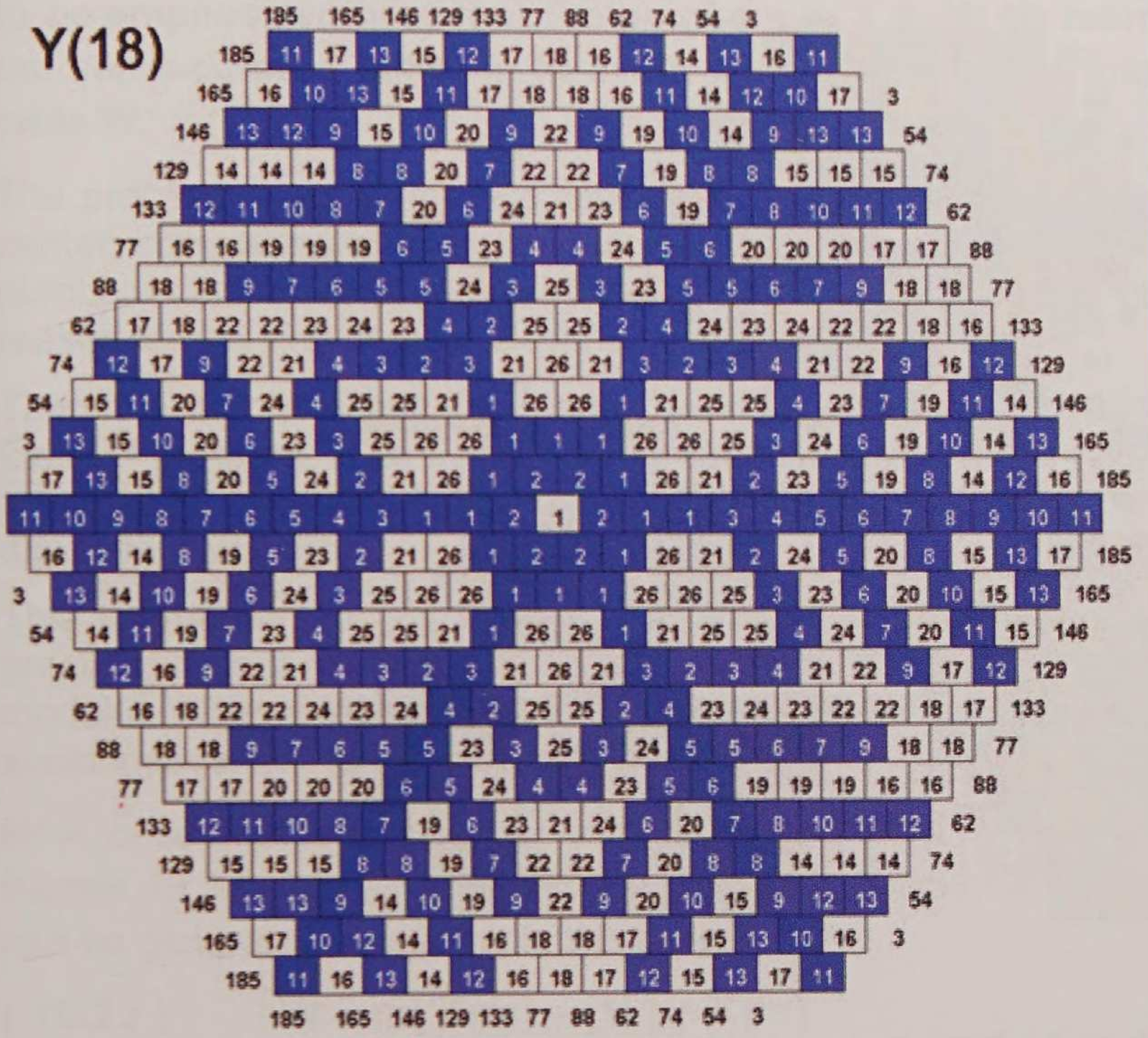
In general, using the fraction **1/g** works in all cases where **g** divides **m** exactly.

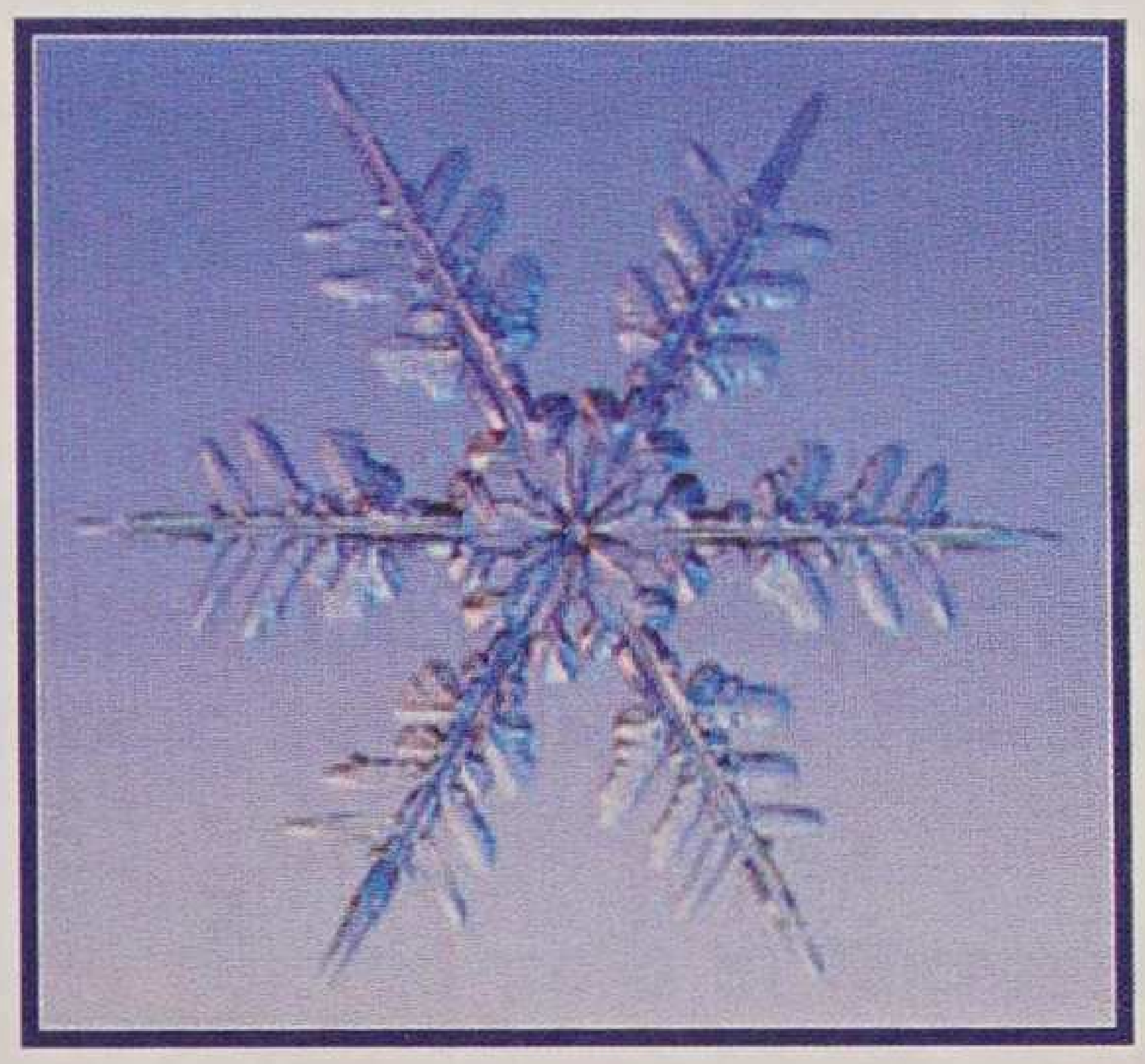
(10.19) N = m/g x gn

Program 10

The Size-13 Integer Loom Table

Y(18)


185 165 146 129 133 77 88 62 74 54 3
185 11 17 13 15 12 17 18 16 12 14 13 16 11
165 16 10 13 15 11 17 18 18 16 11 14 12 10 17 3
146 13 12 9 15 10 20 9 22 9 19 10 14 9 13 13 54
129 14 14 14 8 8 20 7 22 22 7 19 8 8 15 15 15 74
133 12 11 10 8 7 20 6 24 21 23 6 19 7 8 10 11 12 62
77 16 16 19 19 19 6 5 23 4 4 24 5 6 20 20 20 17 17 88
88 18 18 9 7 6 5 5 24 3 25 3 23 5 5 6 7 9 18 18 77
62 17 18 22 22 23 24 23 4 2 25 25 2 4 24 23 24 22 22 18 16 133
74 12 17 9 22 21 4 3 2 3 21 26 21 3 2 3 4 21 22 9 16 12 129
54 15 11 20 7 24 4 25 25 21 1 26 26 1 21 25 25 4 23 7 19 11 14 146
3 13 15 10 20 6 23 3 25 26 26 1 1 1 26 26 25 3 24 6 19 10 14 13 165
17 13 15 8 20 5 24 2 21 26 1 2 2 1 26 21 2 23 5 19 8 14 12 16 185
11 10 9 8 7 6 5 4 3 1 1 2 1 2 1 1 3 4 5 6 7 8 9 10 11
16 12 14 8 19 5 23 2 21 26 1 2 2 1 26 21 2 24 5 20 8 15 13 17 185
3 13 14 10 19 6 24 3 25 26 26 1 1 1 26 26 25 3 23 6 20 10 15 13 165
54 14 11 19 7 23 4 25 25 21 1 26 26 1 21 25 25 4 24 7 20 11 15 146
74 12 16 9 22 21 4 3 2 3 21 26 21 3 2 3 4 21 22 9 17 12 129
62 16 18 22 22 24 23 24 4 2 25 25 2 4 23 24 23 22 22 18 17 133
88 18 18 9 7 6 5 5 23 3 25 3 24 5 5 6 7 9 18 18 77
77 17 17 20 20 20 6 5 24 4 4 23 5 6 19 19 19 16 16 88
133 12 11 10 8 7 19 6 23 21 24 6 20 7 8 10 11 12 62
129 15 15 15 8 8 19 7 22 22 7 20 8 8 14 14 14 74
146 13 13 9 14 10 19 9 22 9 20 10 15 9 12 13 54
165 17 10 12 14 11 16 18 18 17 11 15 13 10 16 3
185 11 16 13 14 12 16 18 17 12 15 13 17 11
185 165 146 129 133 77 88 62 74 54 3



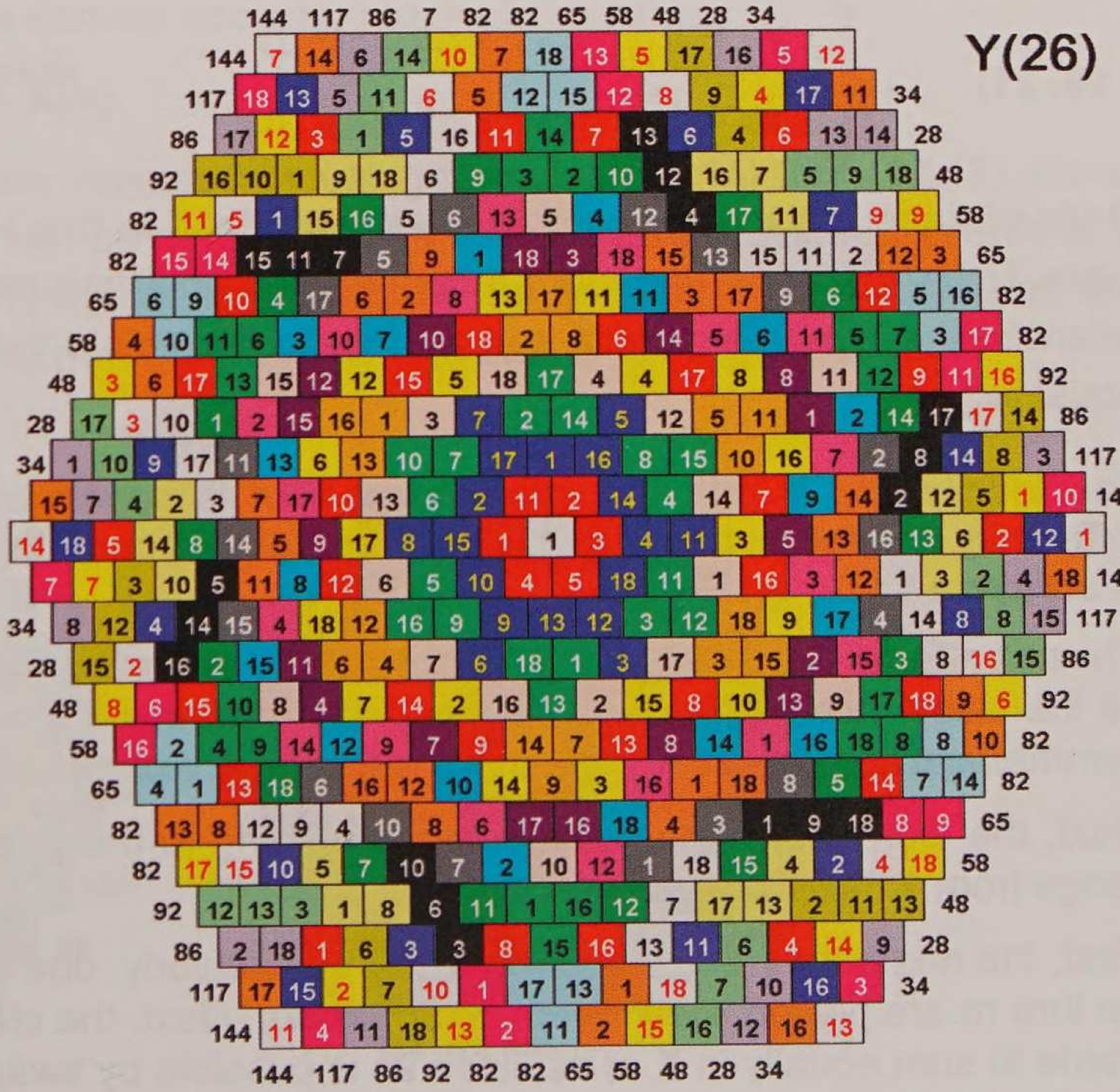
The size-13 hexagons integer loom table derived using the base **m' = 18 = 36/2** unmasks a snowflake pattern upon shading numbers **1** thru **13 =** **½n'**. The pattern mimics the depicted snowflake.

The integer loom-table to the base **n' = 26** is shown below.

You now have enough information to derive the dual in light of **(10.18)** to the base **26** from

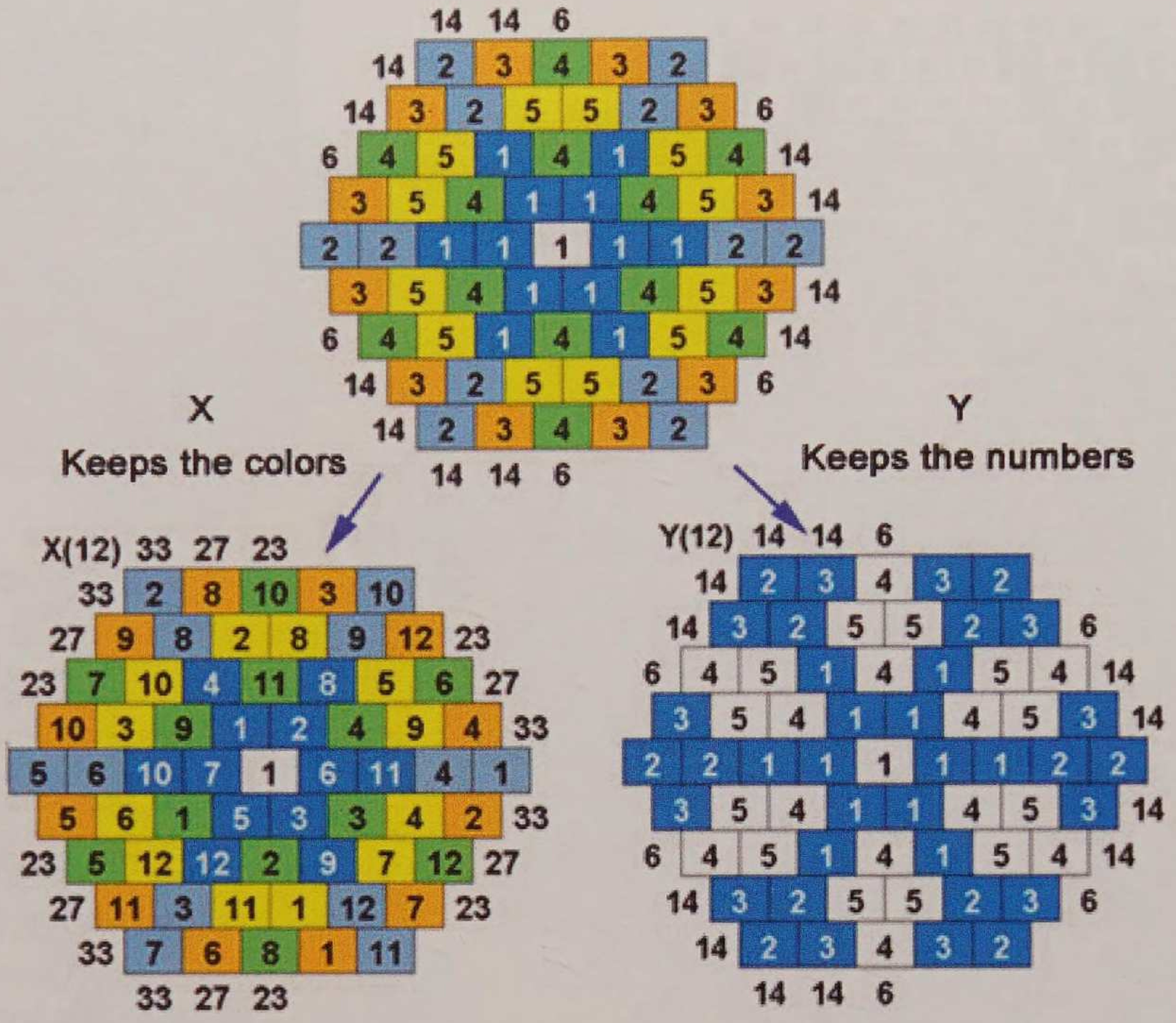
(10.20) $U = n'(Y(n') - |1|) + X(n')$

Y(26)


144 117 86 7 82 82 65 58 48 28 34
144 7 14 6 14 10 7 18 13 5 17 16 5 12
117 18 13 5 11 6 5 12 15 12 8 9 4 17 11 34
86 17 12 3 1 5 16 11 14 7 13 6 4 6 13 14 28
92 16 10 1 9 18 6 9 3 2 10 12 16 7 5 9 18 48
82 11 5 1 15 16 5 6 13 5 4 12 4 17 11 7 9 9 58
82 15 14 15 11 7 5 9 1 18 3 18 15 13 15 11 2 12 3 65
65 6 9 10 4 17 6 2 8 13 17 11 11 3 17 9 6 12 5 16 82
58 4 10 11 6 3 10 7 10 18 2 8 6 14 5 6 11 5 7 3 17 82
48 3 6 17 13 15 12 12 15 5 18 17 4 4 17 8 8 11 12 9 11 16 92
28 17 3 10 1 2 15 16 1 3 7 2 14 5 12 5 11 1 2 14 17 17 14 86
34 1 10 9 17 11 13 6 13 10 7 17 1 16 8 15 10 16 7 2 8 14 8 3 117
15 7 4 2 3 7 17 10 13 6 2 11 2 14 4 14 7 9 14 2 12 5 1 10 144
14 18 5 14 8 14 5 9 17 8 15 1 1 3 4 11 3 5 13 16 13 6 2 12 1
7 7 3 10 5 11 8 12 6 5 10 4 5 18 11 1 16 3 12 1 3 2 4 18 144
34 8 12 4 14 15 4 18 12 16 9 9 13 12 3 12 18 9 17 4 14 8 8 15 117
28 15 2 16 2 15 11 6 4 7 6 18 1 3 17 3 15 2 15 3 8 16 15 86
48 8 6 15 10 8 4 7 14 2 16 13 2 15 8 10 13 9 17 18 9 6 92
58 16 2 4 9 14 12 9 7 9 14 7 13 8 14 1 16 18 8 8 10 82
65 4 1 13 18 6 16 12 10 14 9 3 16 1 18 8 5 14 7 14 82
82 13 8 12 9 4 10 8 6 17 16 18 4 3 1 9 18 8 9 65
82 17 15 10 5 7 10 7 2 10 12 1 18 15 4 2 4 18 58
92 12 13 3 1 8 6 11 1 16 12 7 17 13 2 11 13 48
86 2 18 1 6 3 3 8 15 16 13 11 6 4 14 9 28
117 17 15 2 7 10 1 17 13 1 18 7 10 16 3 34
144 11 4 11 18 13 2 11 2 15 16 12 16 13
144 117 86 92 82 82 65 58 48 28 34

Program 10

Constructing Dual Snowflake-Patterned Complementary Loom-Tables



Denoting the hexagonal loom-tables by **X ≡ |** x_{ij} **|** and **Y ≡ |** y_{ij} **|**:

$$(10.\ 21)\qquad \begin{cases} y_{ij} = i & \text{for } 1 \le i \le n \\ x_{ij} = j & \text{for } 1 \le j \le m \end{cases} \qquad \text{where } m = 3(n-1)$$

Ordinarily the subscripts **ij** would denote the cell in the **i**-th row and **j**-th column of a square table. Here, in view of the loom table patterns and their numbering within them, the subscript **i** refers to the **i**-th pattern of **Y** and **j** refers to the cell in the same **i**-th pattern but in **X** which is equal to **j**.

The loom table **Y** with the intended pattern is constructed first. The top hexagon has five differently colored patterns numbered separately from **1** thru **n,** here **n = 5**. The central number is always a **1**.

These patterns are arranged in a snowflake design for numbers **1** thru **3**, i.e. **(n + 1)/2**, so that all the viable nested frames in **Y** have equal-summing sides. These patterns are laid out to be symmetrical across each of the three diametrical axes as shown in the hexagon on the right.

Next, the loom table **X** is constructed according to the colored patterns at top. These numbers range from **1** thru **m**; here **m = 12**.

First, the numbers **1** thru **n** in **X** are entered circularly, one number per side until all the numbers up thru **m** are placed within each color pattern. Next, the challenge is to get all the sides of each frame to sum equally in **X** or as close as is possible by swapping the locations of the numbers in **X** within their same-colored patterns.

The frame-side equalizing process involves rearranging numbers in **X** in one or more frames simultaneously, all the while keeping each number within its corresponding color pattern. **It has to be emphasized here that the numbers in X must be rearranged within X to stay within the same-colored patterns to avoid any duplication of numbers in the resulting master table W.**

The process starts with the outermost frame and proceeds frame by frame inward toward the center, completing each frame before going on to the next. Frequently the need arises to swap numbers across multiple frames, some already completed, in order to be successful. That's the reason for working at least 2 frames simultaneously.

Then the numbers can be readily re-arranged among identically colored cells in Microsoft's Excel™ program. (The Excel™ program contains a limited number of discernible colors so only a subset of consecutive frames on larger hexagonal tables can be worked on at any one time due to this limitation.)

This rearrangement process is a real mental challenge but still surmountable after some practice. It took me 20 hours of swapping numbers to get the numerical layout of the size-13 modulus table **X** in the base **18**. Much time must also be spent in color formatting in order to avoid inadvertent duplication of numbers.

Now, once both loom tables **X** and **Y** are completed by having all equal sides in all their nested frames, or as near as possible in the smallest viable frame, then the master hexagon table **W** can be generated according to the formula:

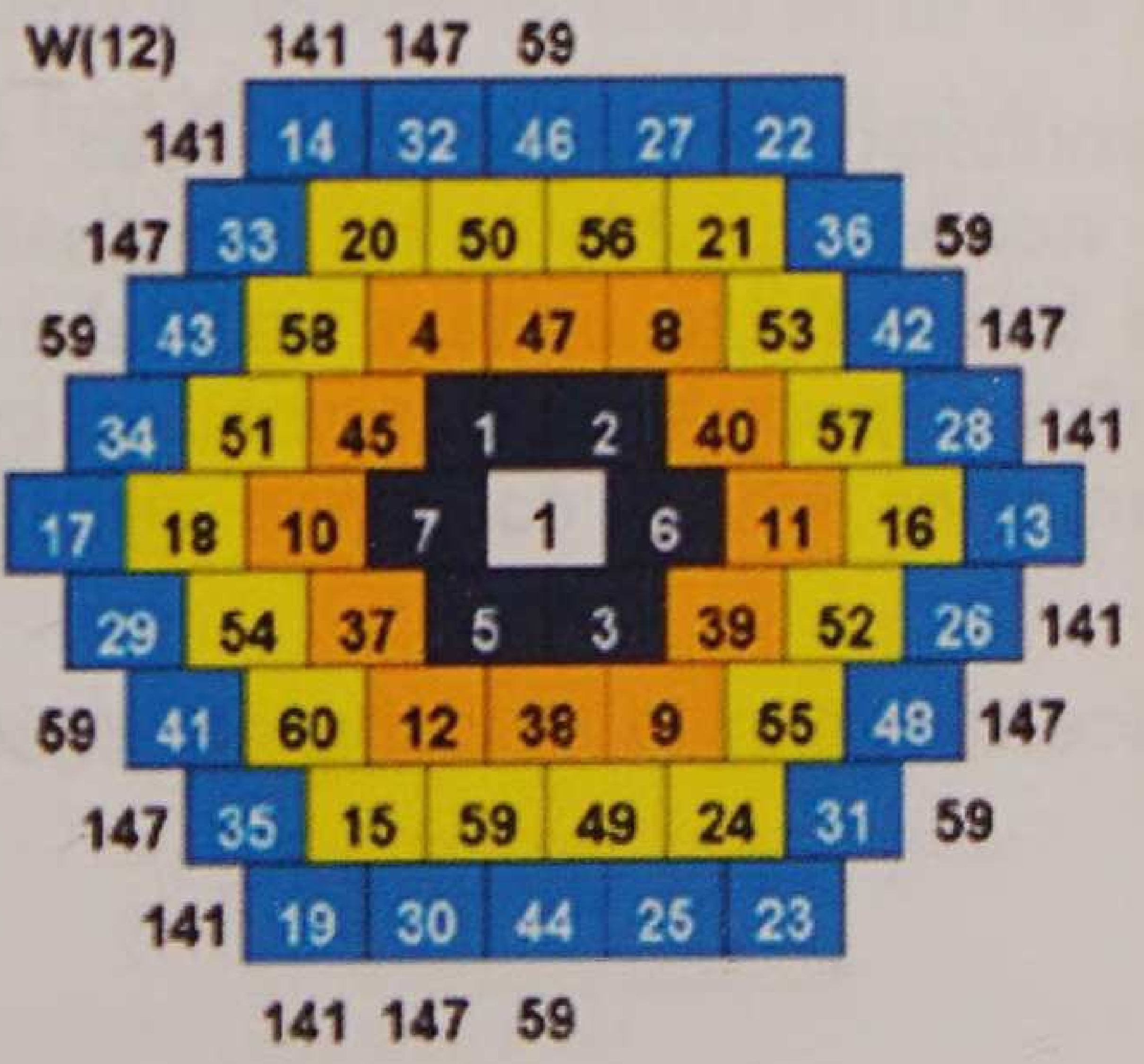
$$W = m(\, Y(m) - |1|\,) + X(m) \qquad (10.22)$$

And the dual hexagon table **U** can be derived according to the formula:

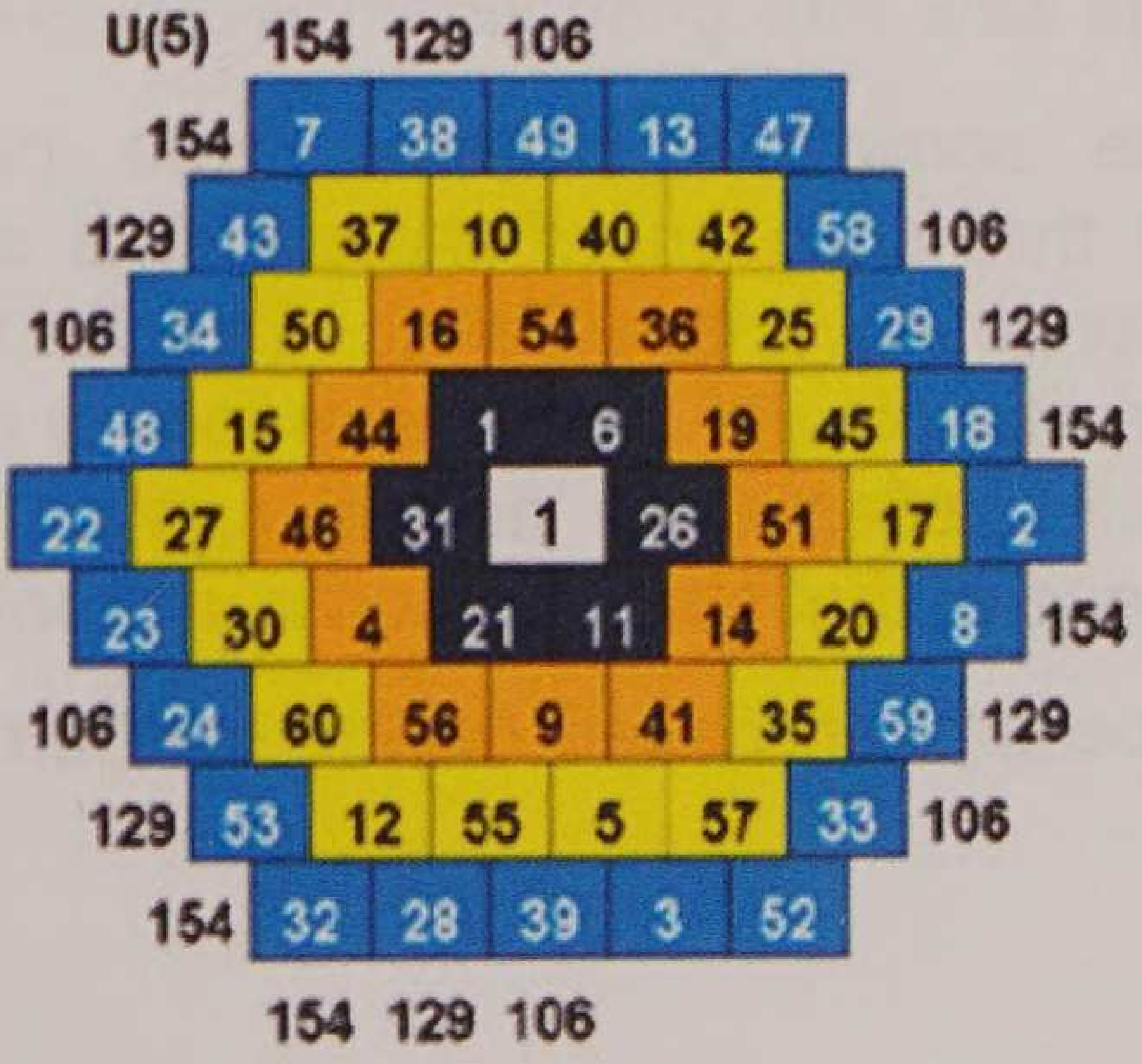
$$U = n(\, Y(n) - |1|\,) + X(n) \qquad (10.23)$$

Now, this whole process can be easily reversed by using formulas **(10.14)** thru (**10.17**) shown earlier so that both **X(m)** and **Y(m)** can be derived from **W** anew. This provides the means to regenerate the snowflake pattern in **Y(m)** that has now been buried within **W** in the base **m**, and to generate the snowflake pattern **X(n)** now buried within the dual **U** in the base **n**.

Size 5



W(12) 141 147 59
141 14 32 46 27 22
147 33 20 50 56 21 36 59
59 43 58 4 47 8 53 42 147
34 51 45 1 2 40 57 28 141
17 18 10 7 1 6 11 16 13
29 54 37 5 3 39 52 26 141
59 41 60 12 38 9 55 48 147
147 35 15 59 49 24 31 59
141 19 30 44 25 23
141 147 59



U(5) 154 129 106
154 7 38 49 13 47
129 43 37 10 40 42 58 106
106 34 50 16 54 36 25 29 129
48 15 44 1 6 19 45 18 154
22 27 46 31 1 26 51 17 2
23 30 4 21 11 14 20 8 154
106 24 60 56 9 41 35 59 129
129 53 12 55 5 57 33 106
154 32 28 39 3 52
154 129 106

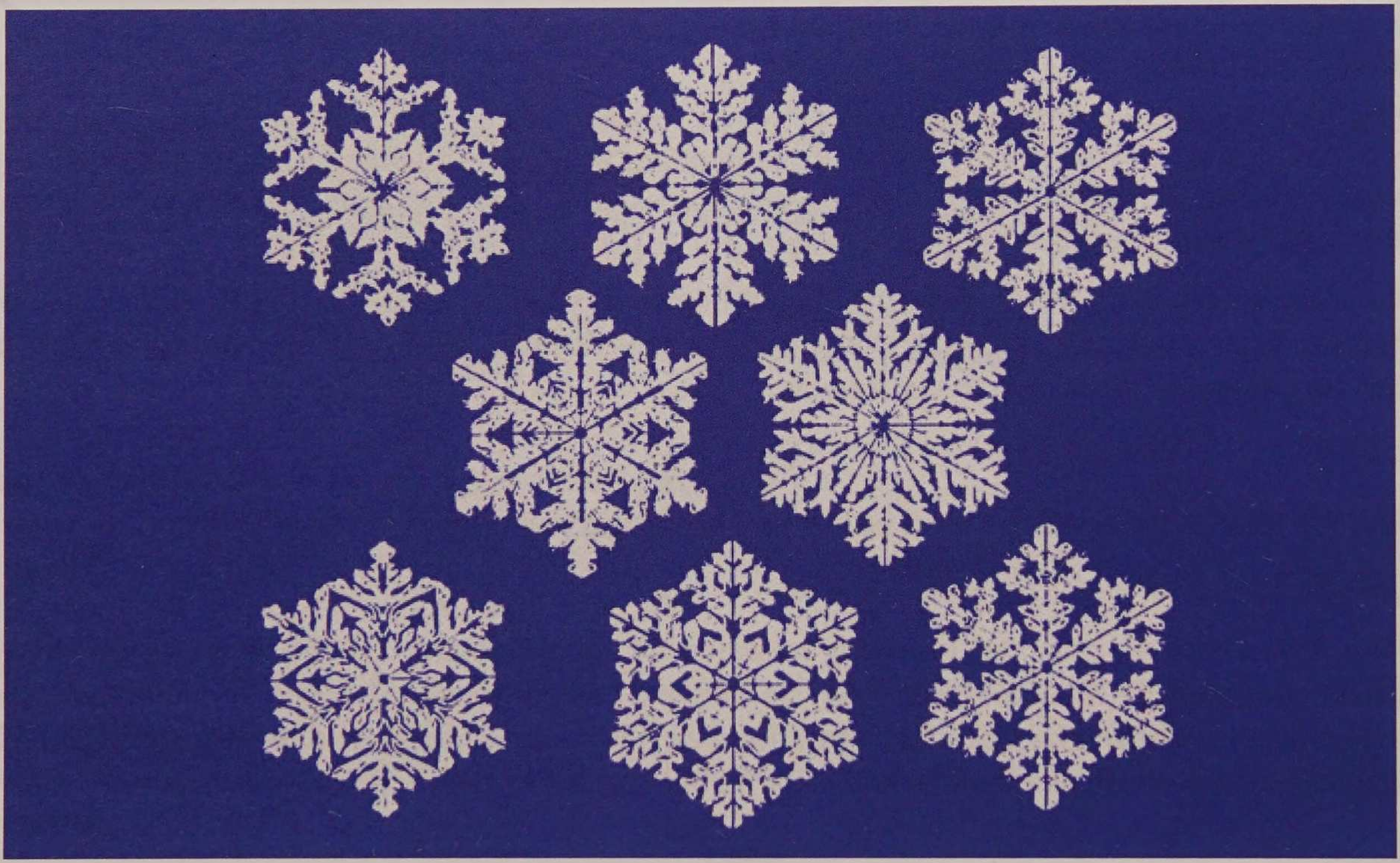
Both the original size-5 hexagonal table and its dual possess the uninterrupted number series **1** thru **60**.

Keep in mind that the central number is just an artifact to facilitate the natural duality possessed by the **N** off-center cells in hexagons. Hexagons always possess 3**n(n-1)+1** cells.

Note that **N =nm =3n(n-1)** is always wholly divisible by **6**:

(10.24) $N/6 = 3n(n-1)/6 = n(n-1)/2$

and either **n** or **n-1** is an even number.



Although these snowflake patterns were prefabricated, this process does demonstrate mathematically that such snowflake patterns are indeed possible to be manifested from the numerically interwoven spatial fabric. Despite the fact that the pattern was fabricated, these tables are still a mapping of sized, hexagonally confined space itself as embodied by the natural number series. It reflects the natural spatial harmonics wholly contained within a confined geometrical space of that size and configuration!

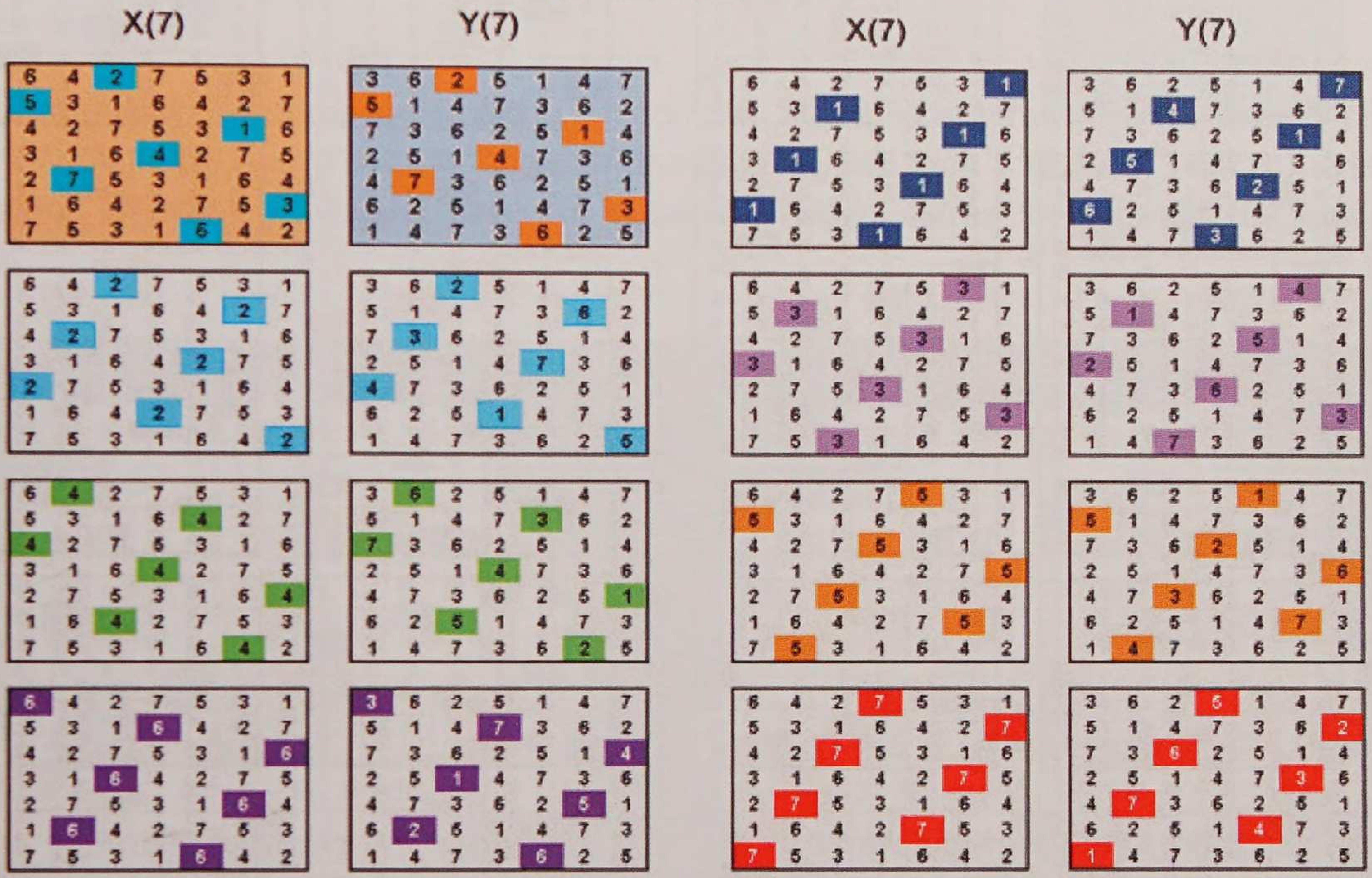
This will now be proven based on the mathematics common to all complementary geonomic loom-tables.

Program 10

Correspondences of values between complementary loom-tables of squares

Here are two size-7 complementary loom-tables **X(7)** and **Y(7)** from Class-1 squares.

Complementary Loom-tables



X(7)

6	4	**2**	7	5	3	1
5	3	1	6	4	2	7
4	2	7	5	3	**1**	6
3	1	6	**4**	2	7	5
2	**7**	5	3	1	6	4
1	6	4	2	7	5	**3**
7	5	3	1	**6**	4	2

Y(7)

3	6	**2**	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	**1**	4
2	5	1	**4**	7	3	6
4	**7**	3	6	2	5	1
6	2	5	1	4	7	**3**
1	4	7	3	**6**	2	5

X(7)

6	4	2	7	5	3	**1**
5	3	**1**	6	4	2	7
4	2	7	5	3	**1**	6
3	**1**	6	4	2	7	5
2	7	5	3	**1**	6	4
1	6	4	2	7	5	3
7	5	3	**1**	6	4	2

Y(7)

3	6	2	5	1	4	**7**
5	1	**4**	7	3	6	2
7	3	6	2	5	**1**	4
2	**5**	1	4	7	3	6
4	7	3	6	**2**	5	1
6	2	5	1	4	7	3
1	4	7	**3**	6	2	5

6	4	**2**	7	5	3	1
5	3	1	6	4	**2**	7
4	**2**	7	5	3	1	6
3	1	6	4	**2**	7	5
2	7	5	3	1	6	4
1	6	4	**2**	7	5	3
7	5	3	1	6	4	**2**

3	6	**2**	5	1	4	7
5	1	4	7	3	**6**	2
7	**3**	6	2	5	1	4
2	5	1	4	**7**	3	6
4	7	3	6	2	5	1
6	2	5	**1**	4	7	3
1	4	7	3	6	2	**5**

6	4	2	7	5	**3**	1
5	**3**	1	6	4	2	7
4	2	7	5	**3**	1	6
3	1	6	4	2	7	5
2	7	5	**3**	1	6	4
1	6	4	2	7	5	**3**
7	5	**3**	1	6	4	2

3	6	2	5	1	**4**	7
5	**1**	4	7	3	6	2
7	3	6	2	**5**	1	4
2	5	1	4	7	3	6
4	7	3	**6**	2	5	1
6	2	5	1	4	7	**3**
1	4	**7**	3	6	2	5

6	**4**	2	7	5	3	1
5	3	1	6	**4**	2	7
4	2	7	5	3	1	6
3	1	6	**4**	2	7	5
2	7	5	3	1	6	**4**
1	6	**4**	2	7	5	3
7	5	3	1	6	**4**	2

3	**6**	2	5	1	4	7
5	1	4	7	**3**	6	2
7	3	6	2	5	1	4
2	5	1	**4**	7	3	6
4	7	3	6	2	5	**1**
6	2	**5**	1	4	7	3
1	4	7	3	6	**2**	5

6	4	2	7	**5**	3	1
5	3	1	6	4	2	7
4	2	7	**5**	3	1	6
3	1	6	4	2	7	**5**
2	7	**5**	3	1	6	4
1	6	4	2	7	**5**	3
7	**5**	3	1	6	4	2

3	6	2	5	**1**	4	7
5	1	4	7	3	6	2
7	3	6	**2**	5	1	4
2	5	1	4	7	3	**6**
4	7	**3**	6	2	5	1
6	2	5	1	4	**7**	3
1	**4**	7	3	6	2	5

6	4	2	7	5	3	1
5	3	1	**6**	4	2	7
4	2	7	5	3	1	**6**
3	1	**6**	4	2	7	5
2	7	5	3	1	**6**	4
1	**6**	4	2	7	5	3
7	5	3	1	**6**	4	2

3	6	2	5	1	4	7
5	1	4	**7**	3	6	2
7	3	6	2	5	1	**4**
2	5	**1**	4	7	3	6
4	7	3	6	2	**5**	1
6	**2**	5	1	4	7	3
1	4	7	3	**6**	2	5

6	4	2	**7**	5	3	1
5	3	1	6	4	2	**7**
4	2	**7**	5	3	1	6
3	1	6	4	2	**7**	5
2	**7**	5	3	1	6	4
1	6	4	2	**7**	5	3
7	5	3	1	6	4	2

3	6	2	**5**	1	4	7
5	1	4	7	3	6	**2**
7	3	**6**	2	5	1	4
2	5	1	4	7	**3**	6
4	**7**	3	6	2	5	1
6	2	5	1	**4**	7	3
1	4	7	3	6	2	5

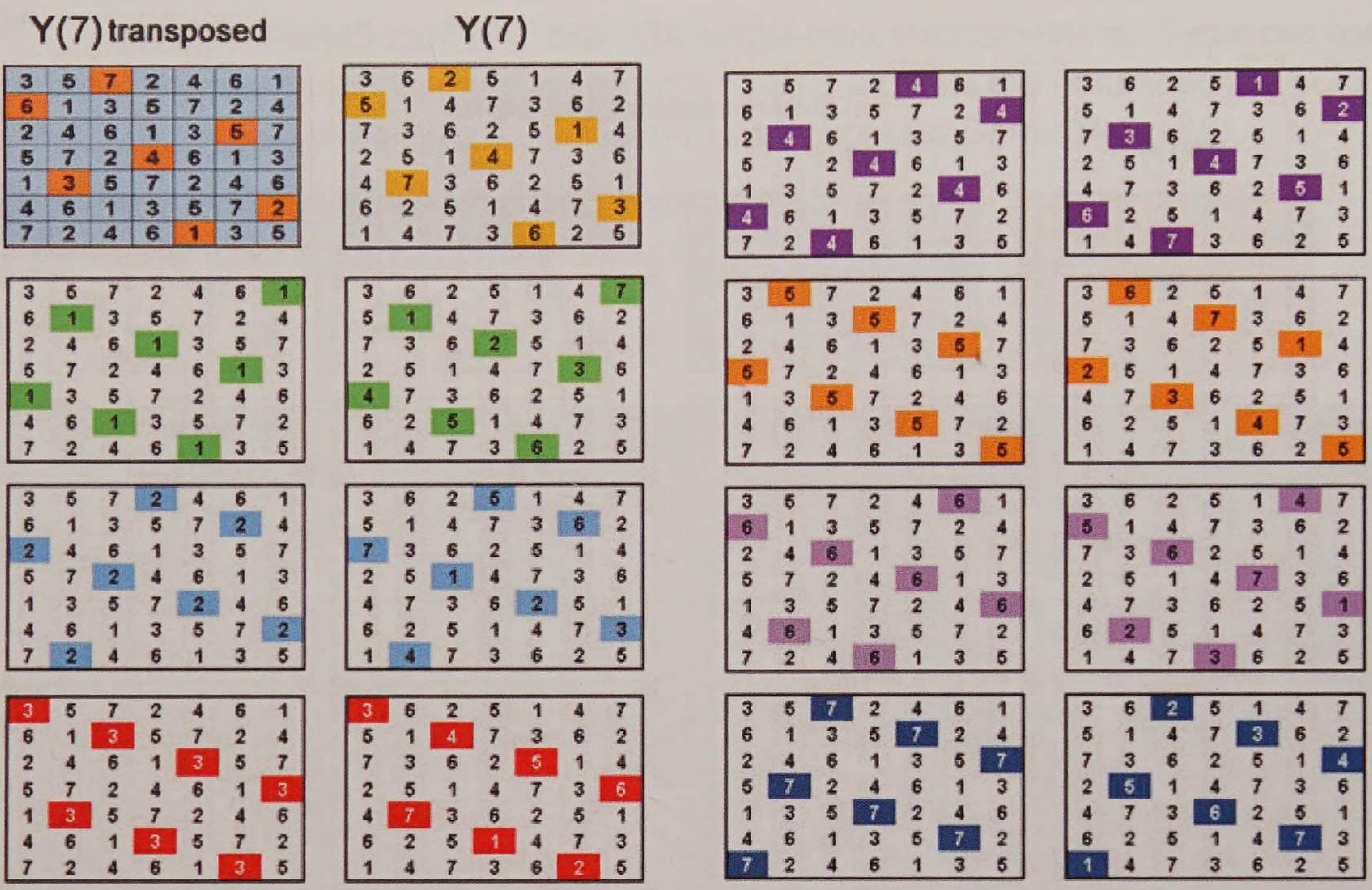
At top-left are the loom-tables highlighting the numbers that they have in a common location. Note that they have numbers **1** thru **7** in common locations.

Then what follows is the highlighting of the locations of just one number throughout **X**, from **1** thru **7**, exactly once and then transferring the highlight pattern over to **Y**. Note that the highlighted numbers in **Y** range from **1** thru **7** exactly once in each and every instance.

The same correspondences arise when the like-number highlighting that was applied to **X** are applied on **Y** first and then transferred to **X**.

Program 10

Here is the size-7 integer loom-table and its transpose; both highlighted in the same manner between them as were the integer loom-table with its complementary modulus loom-table.



Y(7) transposed

3	5	**7**	2	4	6	1
6	1	3	5	7	2	4
2	4	6	1	3	**5**	7
5	7	2	**4**	6	1	3
1	**3**	5	7	2	4	6
4	6	1	3	5	7	**2**
7	2	4	6	**1**	3	5

Y(7)

3	6	**2**	5	1	4	7
5	1	4	7	3	6	2
7	3	6	2	5	**1**	4
2	5	1	**4**	7	3	6
4	**7**	3	6	2	5	1
6	2	5	1	4	7	**3**
1	4	7	3	**6**	2	5

3	5	7	2	**4**	6	1
6	1	3	5	7	2	**4**
2	**4**	6	1	3	5	7
5	7	2	**4**	6	1	3
1	3	5	7	2	**4**	6
4	6	1	3	5	7	2
7	2	**4**	6	1	3	5

3	6	2	5	**1**	4	7
5	1	4	7	3	6	**2**
7	**3**	6	2	5	1	4
2	5	1	**4**	7	3	6
4	7	3	6	2	**5**	1
6	2	5	1	4	7	3
1	4	**7**	3	6	2	5

3	5	7	2	4	6	**1**
6	**1**	3	5	7	2	4
2	4	6	**1**	3	5	7
5	7	2	4	6	**1**	3
1	3	5	7	2	4	6
4	6	**1**	3	5	7	2
7	2	4	6	**1**	3	5

3	6	2	5	1	4	**7**
5	**1**	4	7	3	6	2
7	3	6	**2**	5	1	4
2	5	1	4	7	**3**	6
4	7	3	6	2	5	1
6	2	**5**	1	4	7	3
1	4	7	3	**6**	2	5

3	**5**	7	2	4	6	1
6	1	3	**5**	7	2	4
2	4	6	1	3	**5**	7
5	7	2	4	6	1	3
1	3	**5**	7	2	4	6
4	6	1	3	**5**	7	2
7	2	4	6	1	3	**5**

3	**6**	2	5	1	4	7
5	1	4	**7**	3	6	2
7	3	6	2	5	**1**	4
2	5	1	4	7	3	6
4	7	**3**	6	2	5	1
6	2	5	1	**4**	7	3
1	4	7	3	6	2	**5**

3	5	7	**2**	4	6	1
6	1	3	5	7	**2**	4
2	4	6	1	3	5	7
5	7	**2**	4	6	1	3
1	3	5	7	**2**	4	6
4	6	1	3	5	7	**2**
7	**2**	4	6	1	3	5

3	6	2	**5**	1	4	7
5	1	4	7	3	**6**	2
7	3	6	2	5	1	4
2	5	**1**	4	7	3	6
4	7	3	6	**2**	5	1
6	2	5	1	4	7	**3**
1	**4**	7	3	6	2	5

3	5	7	2	4	**6**	1
6	1	3	5	7	2	4
2	4	**6**	1	3	5	7
5	7	2	4	**6**	1	3
1	3	5	7	2	4	**6**
4	**6**	1	3	5	7	2
7	2	4	**6**	1	3	5

3	6	2	5	1	**4**	7
5	1	4	7	3	6	2
7	3	**6**	2	5	1	4
2	5	1	4	**7**	3	6
4	7	3	6	2	5	**1**
6	**2**	5	1	4	7	3
1	4	7	**3**	6	2	5

3	5	7	2	4	6	1
6	1	**3**	5	7	2	4
2	4	6	1	**3**	5	7
5	7	2	4	6	1	**3**
1	**3**	5	7	2	4	6
4	6	1	**3**	5	7	2
7	2	4	6	1	**3**	5

3	6	2	5	1	4	7
5	1	**4**	7	3	6	2
7	3	6	2	**5**	1	4
2	5	1	4	7	3	**6**
4	**7**	3	6	2	5	1
6	2	5	**1**	4	7	3
1	4	7	3	6	**2**	5

3	5	**7**	2	4	6	1
6	1	3	5	**7**	2	4
2	4	6	1	3	5	**7**
5	**7**	2	4	6	1	3
1	3	5	**7**	2	4	6
4	6	1	3	5	**7**	2
7	2	4	6	1	3	5

3	6	**2**	5	1	4	7
5	1	4	7	**3**	6	2
7	3	6	2	5	1	**4**
2	**5**	1	4	7	3	6
4	7	3	**6**	2	5	1
6	2	5	1	4	**7**	3
1	4	7	3	6	2	5

Again as before, the loom-table and its transpose possess the numbers **1** thru **7** among the same locations exactly once, but not identically number for number.

As the pattern is shifted, its transpose has all one number highlighted while those same locations within itself run the gamut from **1** thru **7** exactly once.

Well this is exactly what was done in hexagons but using different number bases which were related by the formula:

(10.25) $\mathbf{N_n = mn}$, $\mathbf{N_n}$ divisible by **6**.

As long as the number of cells in the hexagon excluding the center could be expressed as formula (**10.24**), the bases **m** and **n** could be used between the complementary loom tables.

For instance, we used the bases **7** and **18** for the size-7 hexagonal loom-tables which contain **126** off-center cells. However we could just as well used dual bases **6** and **21**.

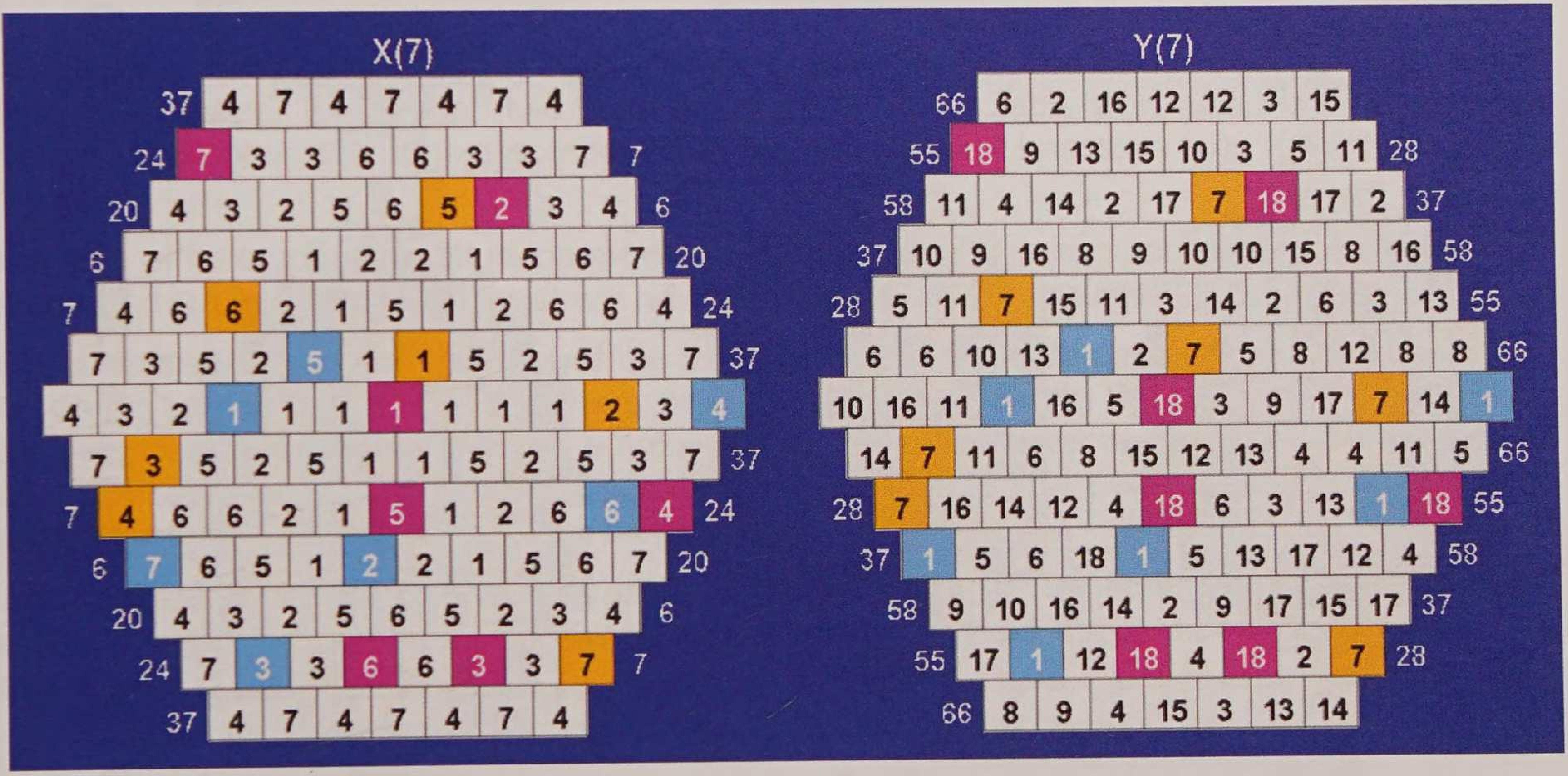
When constructing the complementary loom-tables for the size-7 hexagon, we used the bases **5** and **12**, **5x12 = 60**. We could have just as well as used the bases **6** and **10**.

You might try out this alternative combination in which the bases are closer together.

Correspondences of values between complementary loom-tables of geonomic hexagons

Here again are the complementary loom-tables of the size-7 **geonomic** hexagon seen earlier. On the right, the three numbers **1**, **7** and **18** have been highlighted in three distinct colors. These highlights were then transferred from the integer loom-table on the right to the modulus loom-table on the left.

Observe that each color in the table on the left contains the numbers from **1** thru **7** exactly once. In fact, this occurs for every number between the loom-tables from **1** thru **18**. This is merely a generalization of what we've just seen for squares. This confirms the loom-tables' complementarity.



X(7)

37 4 7 4 7 4 7 4
24 7 3 3 6 6 3 3 7 7
20 4 3 2 5 6 5 2 3 4 6
6 7 6 5 1 2 2 1 5 6 7 20
7 4 6 6 2 1 5 1 2 6 6 4 24
7 3 5 2 5 1 1 5 2 5 3 7 37
4 3 2 1 1 1 1 1 1 1 2 3 4
7 3 5 2 5 1 1 5 2 5 3 7 37
7 4 6 6 2 1 5 1 2 6 6 4 24
6 7 6 5 1 2 2 1 5 6 7 20
20 4 3 2 5 6 5 2 3 4 6
24 7 3 3 6 6 3 3 7 7
37 4 7 4 7 4 7 4

Y(7)

66 6 2 16 12 12 3 15
55 18 9 13 15 10 3 5 11 28
58 11 4 14 2 17 7 18 17 2 37
37 10 9 16 8 9 10 10 15 8 16 58
28 5 11 7 15 11 3 14 2 6 3 13 55
6 6 10 13 1 2 7 5 8 12 8 8 66
10 16 11 1 16 5 18 3 9 17 7 14 1
14 7 11 6 8 15 12 13 4 4 11 5 66
28 7 16 14 12 4 18 6 3 13 1 18 55
37 1 5 6 18 1 5 13 17 12 4 58
58 9 10 16 14 2 9 17 15 17 37
55 17 1 12 18 4 18 2 7 28
66 8 9 4 15 3 13 14

In squares, $N_2 = n^2$ and the bases were common. In hexagons, the bases were different but still related similarly as **in** formula (**10.24**).

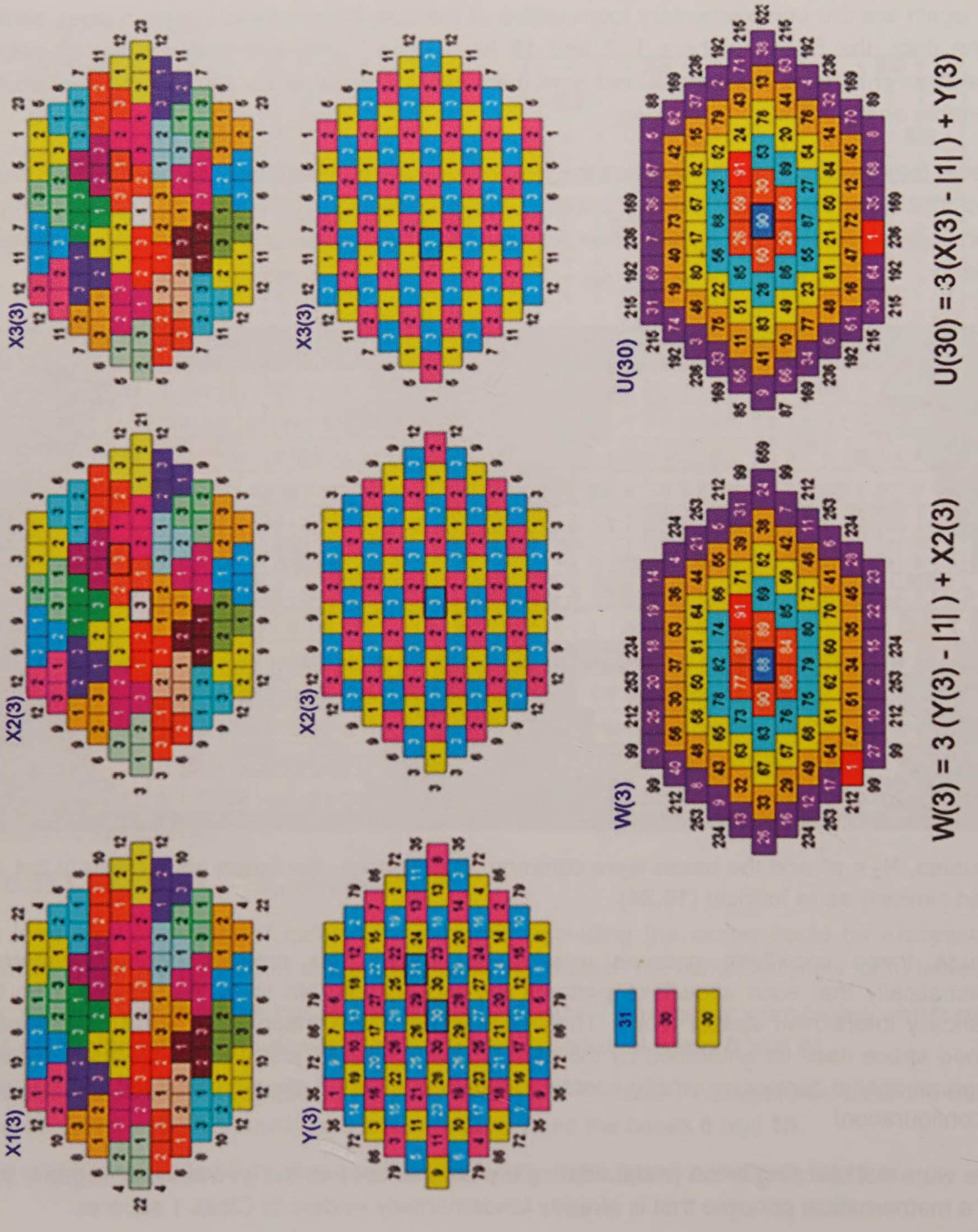
Although these snowflake patterns were prefabricated, this process does demonstrate mathematically that such snowflake patterns are indeed possible to be manifested from the numerically interwoven spatial fabric. These tables are still a mapping of sized, hexagonally confined space itself as embodied by the natural number series just as for squares. It reflects the natural spatial harmonics wholly contained within a confined geometrical space of that size and configuration!

So we were not cheating when prefabricating the snowflakes into the geonomic hexagons; they obey a mathematical principle that is already fundamentally evident in Class-1 squares.

We'll take this one step further on the next page.

Program 10

Alternate Size-7 Hexagons



$U(30) = 3(X(3) - |1|) + Y(3)$

$W(3) = 3(Y(3) - |1|) + X2(3)$

Program 10

The Triangle Modular Method – Tri-Mod

Here's yet another method for generating geonomic hexagons.

This slide shows a size-6 hexagonal loom-table with just the numbers **1** thru **3**. Note the triangular patterns in the tables at the top. These three tables have been given identities as **X1**, **X2** and **X3** corresponding to the number in their upper left-hand corner. **X2** and **X3** are easily obtained from **X1** just by roll-wrapping its rows left **1** or **2** cells, respectively. Note that all the **X**'s are triangularly 2x-modular: any triangle of **3** adjacent cells anywhere sums to **6**.

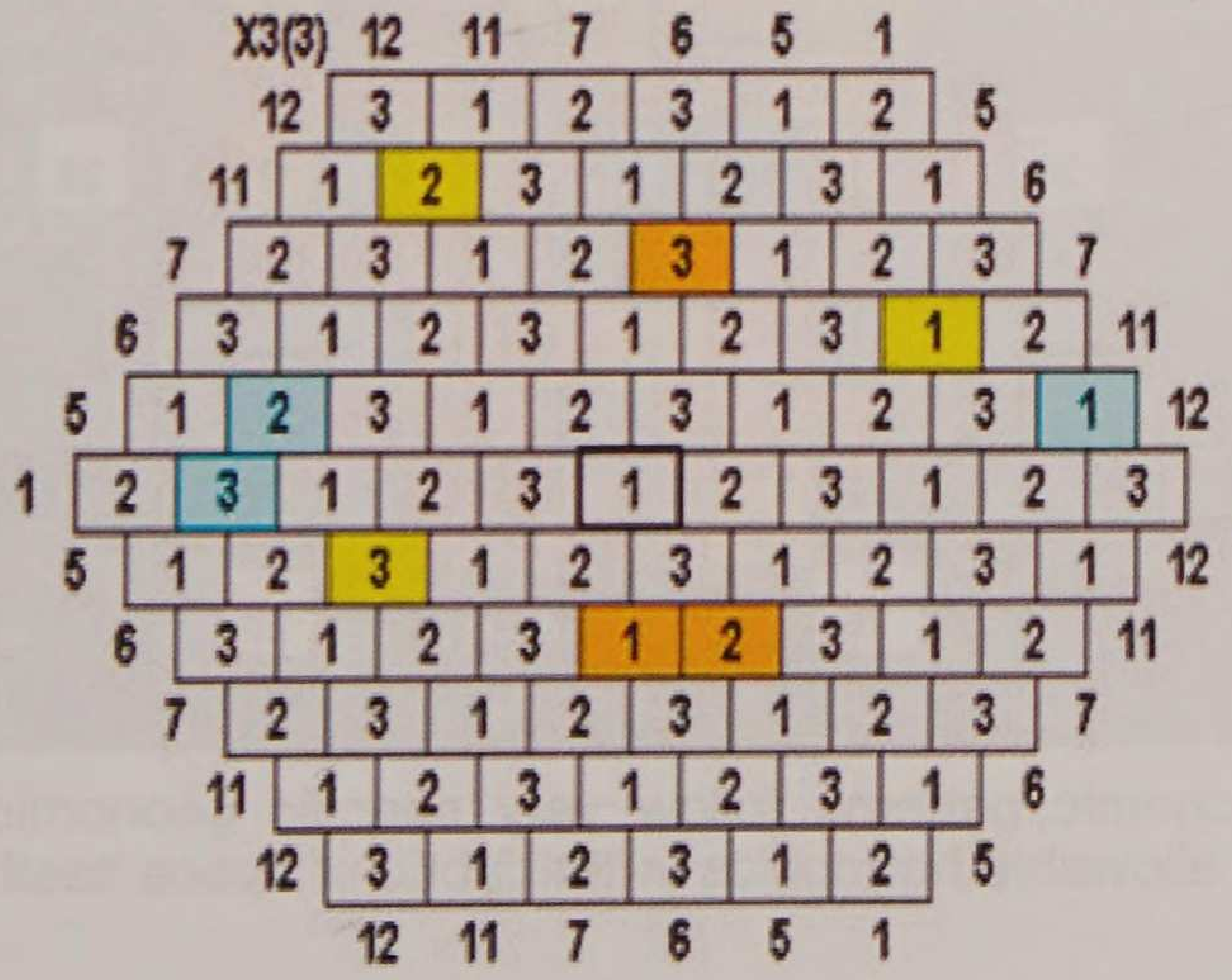
The **X2** and **X3** hexagons in the middle row now have their cells colored differently for each identical number in **X1**. This color pattern is carried over to the integer loom table **Y** on the left where the numbers **1** thru **30** are used to fill in the cells of the same color so that the sides of their distinct nested viable frames are equal within the frame. The number **31** is needed for the blue cells as there are **31** of those.

The hexagonal tables in the bottom row were obtained according to the formulas by using **Y** in both cases and **X2** for generating **W** and **X3** for generating **U** in the same fashion as we have all along for squares. The reason two versions of the modulus table were used is that each was necessary to obtain the largest number **91** in the two different bases. In fact, **X1** will not yield a **91** with **Y**.

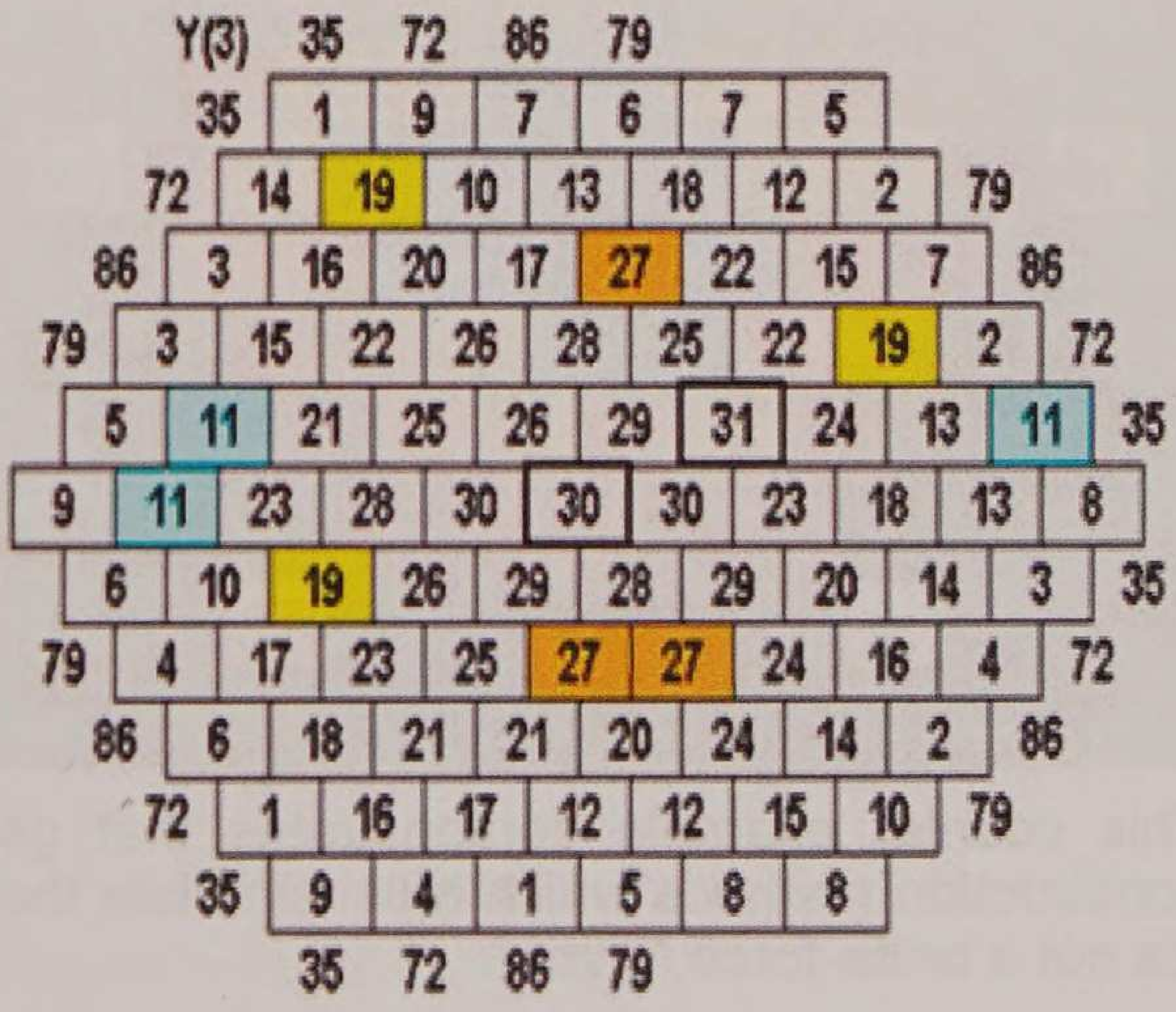
This type of hexagonal modulus loom-table with the pangenic triangular 2x-modularity property can be derived in this manner for every hexagon whose size is a multiple of **3**.

In your spare time, make note that of the three diametrical diagonals in each of **W** and **U**, only one doesn't sum equally with the others: in **W(3)** it's off by **1** and in **U(30)** it's off by **3**. Once the size gets to be **9** and greater when employing the triangular checker-board pattern, they might have a better chance of all summing equally. Then you will have a perfectly balanced geonomic hexagon with the pangenic triangular property.

And just as was shown on the slide prior to last, coloring all identical numbers in **Y** with the same color and transferring the coloring pattern to **X** will yield the numbers from **1** thru **3** for each distinct color transferred.



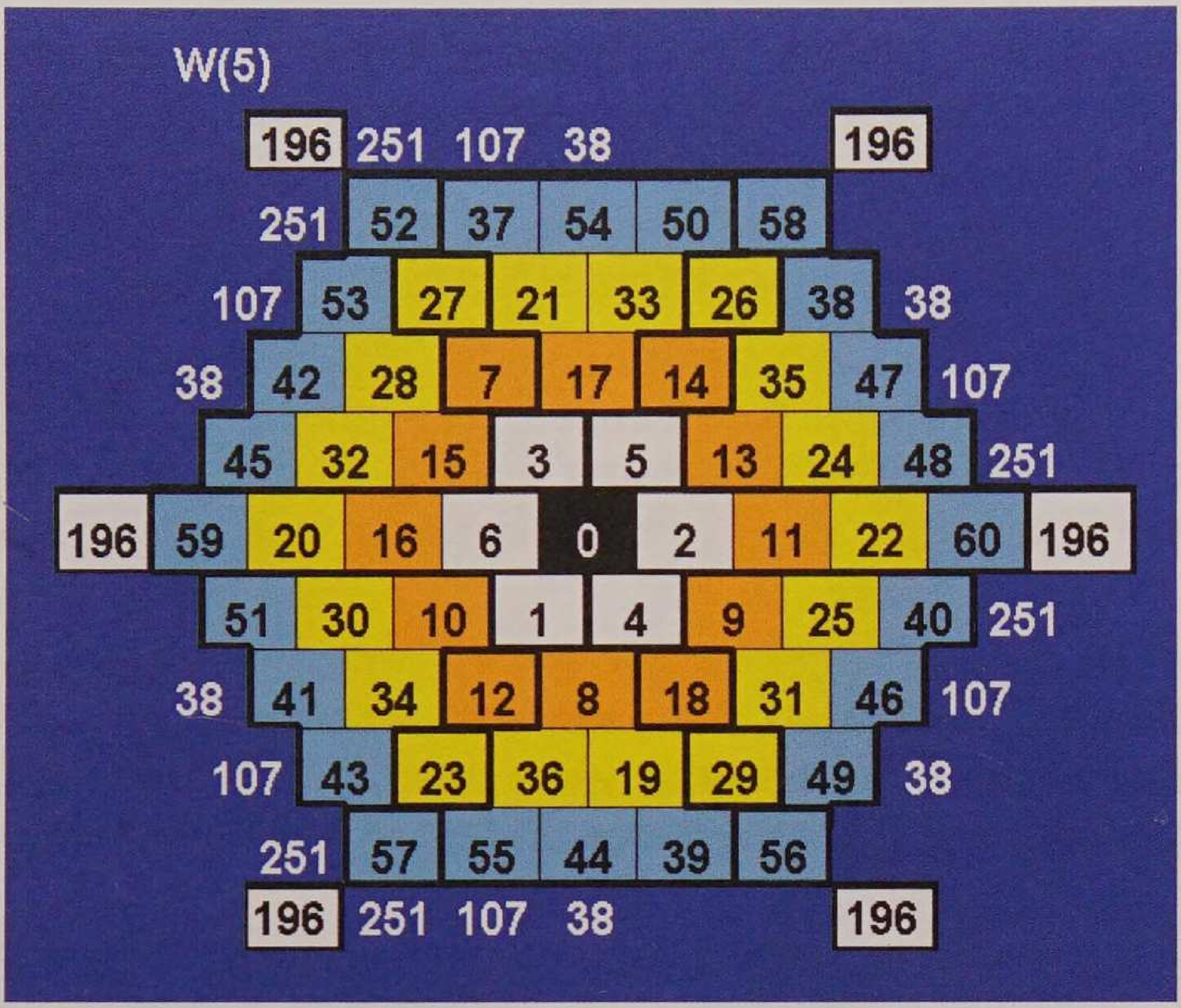
X3(3)	12	11	7	6	5	1						
12	3	1	2	3	1	2	5					
11	1	2	3	1	2	3	1	6				
7	2	3	1	2	3	1	2	3	7			
6	3	1	2	3	1	2	3	1	2	11		
5	1	2	3	1	2	3	1	2	3	1	12	
1	2	3	1	2	3	1	2	3	1	2	3	
5	1	2	3	1	2	3	1	2	3	1	12	
6	3	1	2	3	1	2	3	1	2	11		
7	2	3	1	2	3	1	2	3	7			
11	1	2	3	1	2	3	1	6				
12	3	1	2	3	1	2	5					
	12	11	7	6	5	1						



Y(3)	35	72	86	79								
35	1	9	7	6	7	5						
72	14	19	10	13	18	12	2	79				
86	3	16	20	17	27	22	15	7	86			
79	3	15	22	26	28	25	22	19	2	72		
	5	11	21	25	26	29	31	24	13	11	35	
	9	11	23	28	30	30	30	23	18	13	8	
	6	10	19	26	29	28	29	20	14	3	35	
79	4	17	23	25	27	27	24	16	4	72		
86	6	18	21	21	20	24	14	2	86			
72	1	16	17	12	12	15	10	79				
35	9	4	1	5	8	8						
	35	72	86	79								

As of the latest revision of this companion book, even-sized hexagons have not been systematically explored, so there is plenty of opportunity for discovery of new properties there.

What may seem as a geonomic hexagon many times isn't



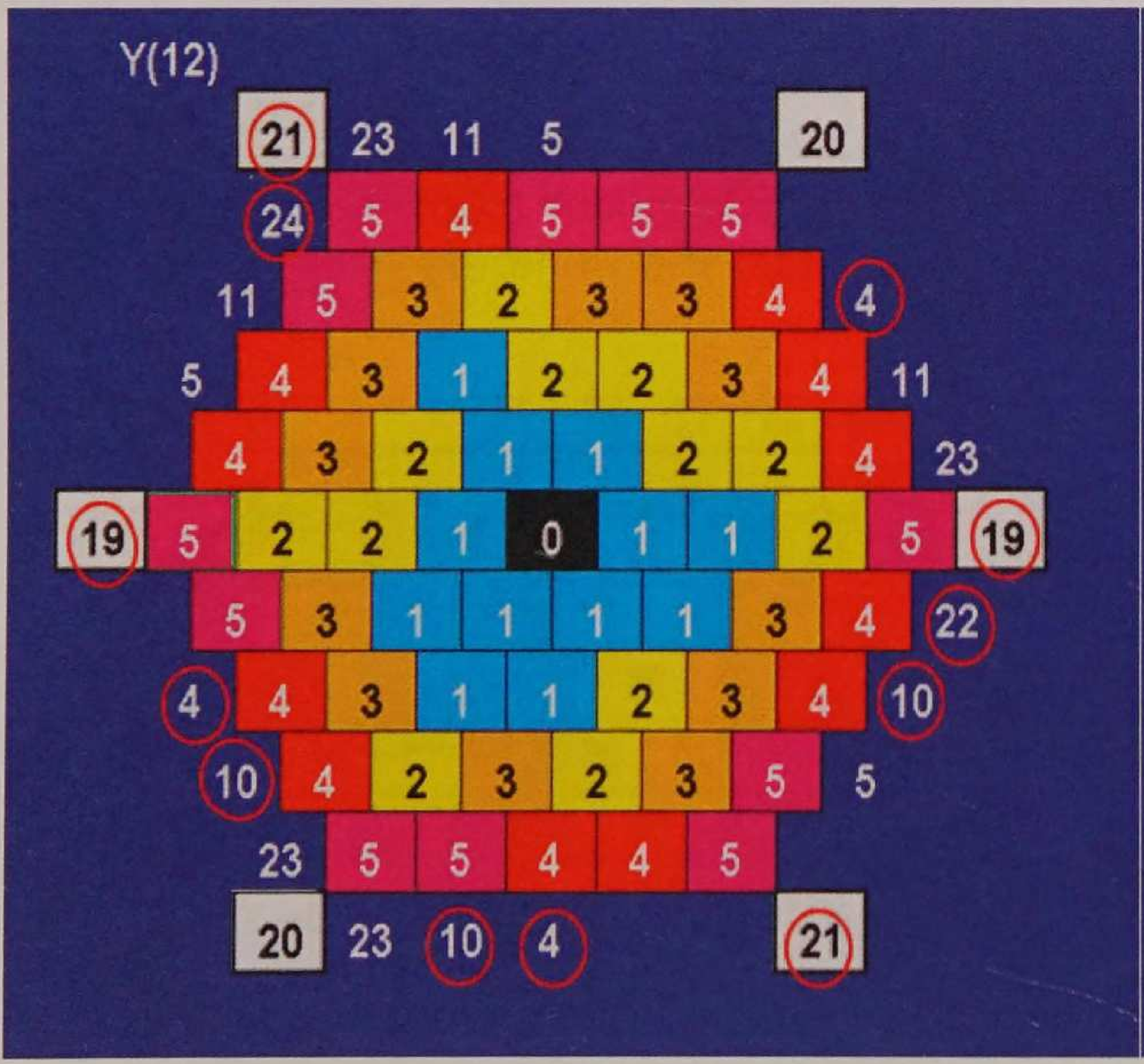
Here is a size-5 hexagon that is seemingly perfect:

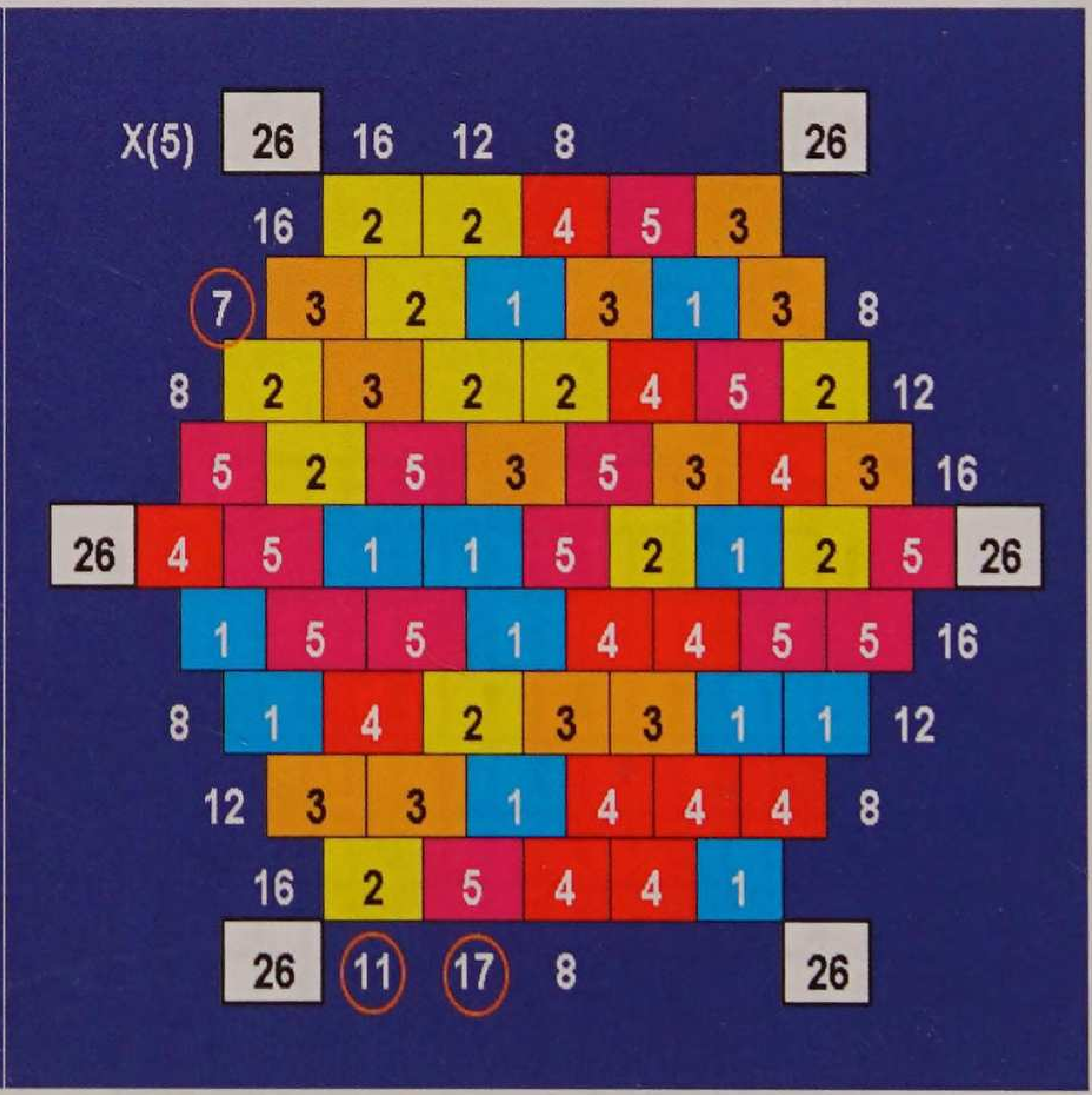
All of its nested frames have equal-summing sides; and its 3 diametrical sums are equal too.

It was constructed by manipulation of numbers **1** thru **60** as follows: the numbers **1** thru **6** were used in the smallest frame; the numbers **7** thru **18** in the second smallest; the numbers **19** thru **36** in the next to largest frame; and the numbers **37** thru **60** in the largest.

What would seem to have some interesting properties when converted to its complementary loom-tables, i.e. **X(5)** and **Y(12)**, actually has <u>none</u> as seen below.

Observe that the equal summations were not preserved throughout either loom-table. Further, there are no natural harmonic patterns either – It's all scattered confusion.



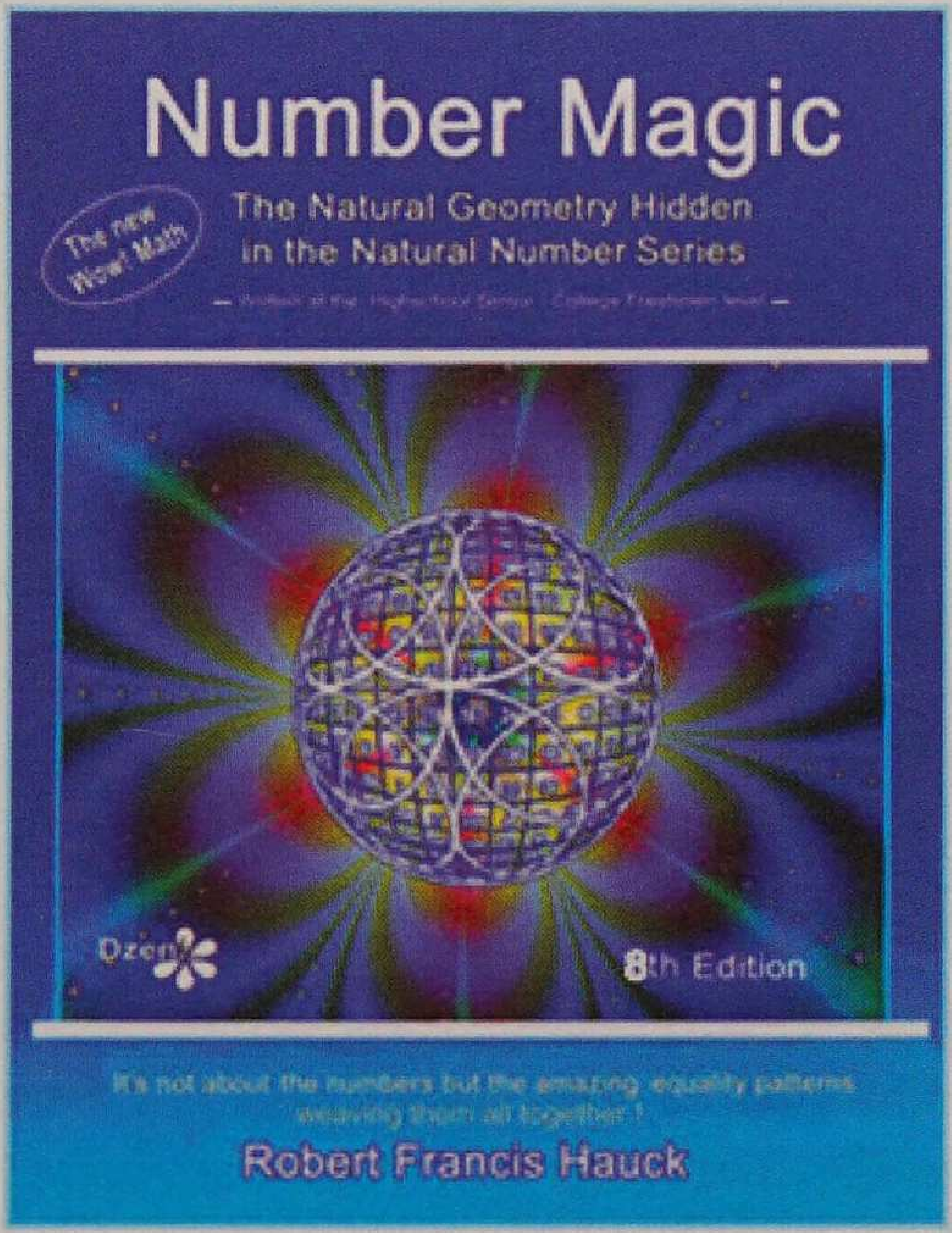


This counter example demonstrates that geonomic patterns follow very specific geonomic construction methods which must simulate the allowable harmonics in the fabric of space itself. It's not a brute-force "given".

Notes

1. Perfect geonomic diamonds can be generated by merging 2 consecutive size squares.
    a) The *laminated indexing method* **LIM** will always work with any two adjacent size squares. It will even work with a Class-2 square involved.
    b) The method that will match the laminated indexing method is the **BAG** method in which a multiple of the square's integer loom table was added to it and then merged with an adjacent size square having undergone the same procedure. This method will only work between adjacent Class-1 and Class-4 squares. Further, it will only work with Class-4 squares that are pairwise row-symmetric.
    c) The Tri-Mod method uses modular triangles on hexagons whose size is a multiple of **3** to generate its loom-tables. This method may not yield hidden snowflake patterns if such patterns are not consciously embedded in the integer loom-table upon its construction.
2. The size **5** hexagon is the smallest odd-size hexagon that is geonomically perfect in both its pair of complementary loom tables and its pair of dual hexagons.
3. The size **7** is the next size to be perfect, followed by size **13**. The sizes **9** and **11** inbetween may be imperfect merely due to the manner in which the modulus loom-table was constructed; it may possibly be improved upon.
4. Mathematically, geonomic hexagons are unrelated to hexagons appearing on the internet which have every linear slice in all 3 directions summing equally to a single number. Those constructs have no relationship or application to measuring and mapping the fabric of space.
5. Geonomic hexagons show how snowflakes form so symmetrically perfect along three distinct equally spaced polarized directions and do so with such branching complexity.

We have come to the end of Program 10. Here are the two books upon which this program was based.



Number Magic – The Natural Geometry Hidden in the Natural Number Series

ISBN: 978-1-146-10245-2

Shows examples of every size table that can be printed legibly up through the 5th dimension

Eighth Edition,
(350+ pages)

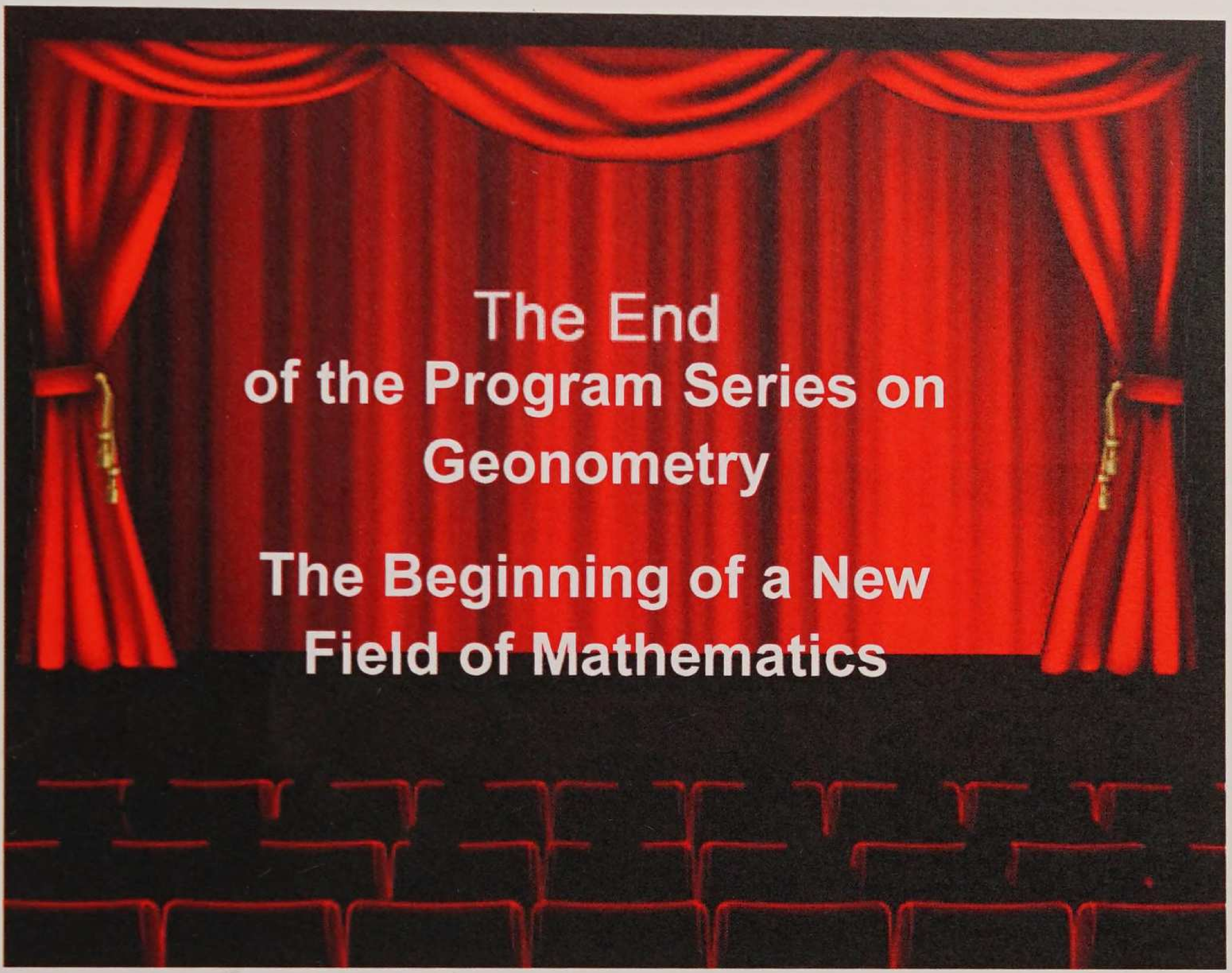
Snowflakes are a "Weave of Wonder"

ISBN: 978-1-469-94348-0

Describes how these hexagonal tables are constructed through creating loom tables with embedded patterns

Second Edition
(30 pages)

Only a few selected math students from the various highschools were selected by their math teachers to be in attendance for this demonstration. So now consider yourselves privileged to have seen the mathematics that very few others will ever get to see, intertwined throughout the fabric of space itself.



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The Beginning of a New
Field of Mathematics

We have come to the end of the exploration of perfectly harmonic patterns in the basic fabric of space through the new **wow math** of **Geonometry**. This new math demonstrates that there are perfectly harmonic quantizations of space ranging from the infinitesimal to the cosmological. Thus Geonometry offers a new simple mathematical tool to search for these amazing and very enlightening hidden patterns and to understand and interpret them.

This has been a real mind-trip for me over the course of the past 8 years of exploration as each amazing and sometimes profoundly stunning discovery was made, one after another after another, without any impedance whatsoever, almost on a daily basis. It was all there just awaiting discovery, like the New World was for Columbus back at the close of the 14th century. He had his ships; I had my math and science background at the ready.

May your success in its future applications be as exciting and rewarding for you as this exploration was for me. The rest is now up to you.

Should you succeed at constructing sizes of tables that were listed in blue back in Program 1, or construct snowflake patterns from even sizes of hexagons, please contact me and provide the solutions so that such may be included in future programs and book revisions. You will be given full recognition in the credits section. Contact information is on the cover-page of this book.

Glossary
Of
Terms and Definitions

Glossary

Prime numbers

When the term "**prime**" or "**prime numbers**" is used in this book, it is to denote an **odd-prime** or **all odd primes**, respectively. It's ridiculously burdensome to have to use the prefix "**odd**" every time when referencing all primes other than **2** since the number **2 is the only even prime** in the entire number series while the number of odd-primes is infinite. In my view, the number **2** should be dismissed from its inclusion in prime numbers altogether since, other than it has no other divisor than itself in the entire number sequence, it possesses none of the other attributes in Geonometry of the odd primes and vice versa. The number **2** is really a "**sub-prime**" as this book demonstrates over and over again and begs demotion like the planet Pluto (circa 2007) so we can talk about just odd primes without the need for all the preliminary qualifiers every time we cite them. The mention of "even prime" in this book is made the exception.

Averages

Dimensional average

The sum $\mathbf{S_k(n)}$ of all the numbers in a table of size **n** and dimension **k** divided by the number of numbers (n^k) in the table: $\mathbf{S_k(n) / n^k}$. Applies to both regular and mm-tables of dimension **k**.

Linear average: The dimensional average for 1-dimension ($\mathbf{k = 1}$).

Planar average: The dimensional average for 2-dimensions ($\mathbf{k = 2}$).

Cubic average: The dimensional average for 3-dimensions ($\mathbf{k = 3}$).

Quartic average: The dimensional average for 4-dimensions ($\mathbf{k = 4}$).

Quintic average: The dimensional average for 5-dimensions ($\mathbf{k = 5}$).

Conceptual Notions

Size vs. Order

Classical math uses the term "order" to describe the size of the matrix. This is a remnant of the days when the properties in Matrix Algebra applied to consecutive sizes and as such could be considered ordered. In Geonometry, "order" has no meaning because the fundamental properties are scattered across 6 distinct classes of square. Here there is no consecutive ordering of size with property. Further, "order" has 2 syllables while "size" has only 1, yet they have the same connotation. So "size" is not only justified but preferred.

Intelligent pattern

The existence of a numeric pattern among serial elements that is conceptual in its design such that only a being of at least human intelligence could comprehend it as being a pattern.

Numerically, it is any number sequence whose elements can be reduced by one or more algebraic operations to one of the following sequences:

a) all elements equal a single constant,
b) the sequence of natural numbers; **1, 2, 3, 4,5, 6 ...**
c) the odd-number sequence **1, 3, 5 ,7, 9, 11...**
d) the sequence of odd prime numbers **1, 3, 5, 7, 11, 13, 19 ,23, ...**

Note that a consecutive even number sequence can be reduced to the natural number sequence merely by dividing all the numbers by **2** or by subtracting **1**.

Harmonic waveform

A series of consecutive numbers whose values follow a pattern:

n, n-1,n-2, ... , 3, 2, 1, 2, 3, ... , n-2, n-1, n, n-1,n-2, ...

Measure

The summation across a table in any of the four directions: top-to-bottom, left-to-right, upper-left to lower-right, upper-right to lower-left.

Mapping

The uncovering of natural numeric sequences in the 4 directional strings and tilings in loom-tables.

Glossary

The 4th dimension

Here it's simply the **fourth spatial dimension**, **4D**, and should not be confused with a time dimension. Since the Theory of Relativity was conceived in a 3-dimensional framework, it was expedient back then to assign time to a fourth dimension.

According to the General Theory of Relativity, traveling matter reaching the speed of light trades one dimension of space for one of time and consequently time comes to a halt in the direction traveled. This notion has no place here in the context of this treatise. The perspective here is that time is a property of the dimensional level in mega-space and is not itself a dimension.

According to The American Mathematical Institute, mega-space is comprised of at least **26** purely spatial dimensions. It is my own conjecture that the higher the dimensional level in mega-space, the slower the clock-rate; the lower the dimensional level, the faster the clock-rate, viz. the faster atomic clock verses the standard clock in 3-dimensions. The geonomic basis for this conjecture is explained in Program 8 (re: The Pattern of Characteristic Numbers as Volume Averages between basic geometries in higher and lower Dimensions, (p. **8-20**).

Geometries

Quadral

A set of **4** numbers corresponding to the corners of a rectangle or diamond (rhombus) that is centered in the table.

Octal

A set of **8** numbers corresponding to the corners of a rectangular box that is centered in the table, usually composed of two matching quadrals from separate blocks, each block being directionally opposite from each other and equidistant from the center of the table.

Hexadectal

A set of **16** numbers corresponding to the corners of a 4D hyper-dimensional box that is centered in the table, composed of two matching octals from separately embedded cubes, each cube being directionally opposite and equidistant across the center of the table or equivalently four matching quadrals from four separate block-squares, each block-square being equidistant from the center of the table forming a symmetric pattern. (See pattern p.**6-2**)

Duohexadectal

A set of **32** numbers corresponding to the corners of a 5-dimensional box that is centered in the table, composed of two matching hexadectals from separate quadracubes, each quadracube being directionally opposite from each other and equidistant from the center of the table.

Octahedron

A set of **6** numbers corresponding to the corners of two pyramids joined together at their bases. Only defined for a 3D table of odd size. It consists of the center numbers of block-pairs equidistant above and below the center block along with the set of **4** numbers of a corresponding diamond in the central block. (See pattern p.**4-13**) There is one octahedron associated with each nested cubical lamination within the boundary of the cube.

Icosahedron

Defined only for a 3D table of odd size. A set of **12** numbers corresponding to the centers of **8** distinct blocks equidistant about the center of a 4D table of odd size and the corresponding **4** numbers of a diamond within the central block. (See pattern p.**6-2**)

Duocosahedron

A set of **24** numbers corresponding to the two icosahedrons from two opposing embedded quadracubes equidistant from the central quadracube in a 5D table of odd-size, or two segregated icosahedrons from the central embedded quadracube.

Quadracube

A 4-dimensional hypercube.

Glossary

Table Terminology

Channel

The string summation along any **horizontal** linear path as viewed along the cited axis; an individual depth sum along the A, B or D-axis.

Pillar

The string summation along any **vertical** linear path as viewed along the C-axis; relates to the individual depth-sums in depth-sum table C.

Block

A square array corresponding to a 2-dimensional group of distinct, contiguous numbers.

Major diagonal

A diagonal that goes from upper-left to lower-right.

Minor diagonal

A diagonal that goes from upper-right to lower-left.

Wrap diagonal

A diagonal which does not originate at a corner of the table and, where its downward path meets the edge, resumes on the next row at the opposite side of the table and continues until it either reaches the other opposite edge again and continues or reaches the bottom of the table and terminates its decent.

Pivot number

The central number in odd-size tables; equals the **dimensional average** of all the numbers in the table.

Kernel number

The number **q**, characteristic of the table's size **n** and the number of dimensions **k** in which the table is contained that is the value to which the $\mathbf{2^k}$ corners of the characteristic dimensionally-geometric figures all sum.

Density

The proportion of **geometric equalities** equal to the kernel number, characteristic for the size and dimensions of the table, compared with the number of all such possible geometric summations. Any geonomic square, cube, diamond or hypercube of any size which exhibits **100% density**, **(**i.e. maximum density) is said to be ***pangenic*** (see below).

Pangenic

The term applied to any table with its dimensional geometry summing equally throughout the table. All geonomic tables in this book are pangenic.

Pangenicity

The property of being pangenic.

Concentricity

The property of nested concentric frames which parallel the perimeter shape of a table which exhibit an intelligent number pattern as defined on page G-2 in going from the center outward. See Program 8, page 8-3 for examples. Property follows from pangenicity.

Glossary

Table Operations

Table collapsing

The process of summing all the numbers in each cubic or hypercubic table along one axial direction to get a table called a **depth-sum**.

Collapsing the depth-sum table itself to get a geonomic table of equal size in a lower dimension.

Table expanding

The process of expanding a square table to a size-multiple of itself.

Roll-wrapping

The sliding of rows horizontally or columns vertically and then repositioning the excess beyond the original border to the vacated cells in their original order.

Tilt-wrapping

The sliding of rows horizontally or columns vertically in proportion to their distance from a lower left corner and then repositioning the excess beyond the original border to the vacated cells in their original order.

Equalities

Planar equality

The property that the sum of all the numbers in every directional 2-dimensional slice through a cube -- horizontally, vertically from each of the two orthogonal directions, and diagonally along the main diagonals from each of the six orthogonal inclinations with wrapping included equals the cube's characteristic number.

Pairwise central-symmetry

The property of equality among all pair sums of two distinct numbers equidistant from and diametrically opposite the table's center.

Pairwise row-symmetry

The property of equality among all pair sums of two distinct numbers in the same row equidistant from the table's vertical centerline.

Symmetric-pair equality

The property of equality among all pair sums of numbers which are symmetrically opposite the center of the table.

Row-pair equality

The property that has every pair of numbers symmetrically located on each side of the row's center adds to the same number.

Cross-directional equalities

Equal summations in a table of numbers formed from every linear path in all four directions: top-to-bottom, left-to-right, upper-left to lower-right, and upper-right to lower-left, with all wrap diagonals included in the diagonal summations.

Complementary equalities

Two distinct geometrical patterns of numbers in a table that always occur simultaneously and sum equally. This includes loom-tables in their entirety.

Spatial Terms

Hyperspace

Any dimension higher than 3D.

Hypercube

A table of numbers of any dimension greater than three.

Hyperplane

A 2-dimensional (planar) slice through any hypercube.

Glossary

Characterizations

Squares

A square of consecutive numbers may have any of the following summation properties:

1. Equal column sums
2. Equal row sums
3. Equal main diagonal sums
4. Equal quadrals
5. Equal major wrap diagonal sums
6. Equal minor wrap diagonal sums

Geonomic square

Pangenic square with properties #1 thru #4.

Perfect square

Pangenic square with properties #1 thru #6. They are "perfect" because they add up equally from four distinct directions, plus have all their quadrals equal, the maximum possible number of linear equalities in 2-dimensions in ordinary circumstances. (See **complementary patterns**.)

Near-perfect square

A square whose summation equalities bring it very close to attaining its named status but is short from achieving it because two of their row summations differ from equality by **±1**. Still considered a geonomic square because its rows could all be made to sum equally by a simple interchange of two numbers in the same column without affecting equal column and main diagonal sums, but will affect four wrap-diagonal sums.

Punctuated perfect square

A punctuated-perfect square is one that is otherwise perfect except for some of its wrap diagonal sums being periodically unequal. A near-perfect square always converts to a punctuated perfect square, but not conversely.

Ultra-perfect square

A perfect square that also exhibits two complementary tile patterns whose elements sum to the square's characteristic number anywhere throughout the table, in addition to all the other equalities ascribed to perfect status, i.e. an order of magnitude beyond perfection, as defined in **continuous complementary tiling patterns**.

Cubes

A cube may have

1. its two main planar diagonals equal to its characteristic number
2. all of its linear sums (channels) equal along the A-axis, specifically, all depth-sums equal in table **A**,
3. all of its linear sums (pillars) equal along the C-axis, specifically, all depth-sums equal in table **C**,
4. all of its octals equal.
5. all of its wrap and main planar diagonals equal for all six possible diagonal directions,

Perfect cube

A cube which has properties #1 thru #4 for cubes (above) except the C-axis depth-sum wrap diagonals are not all equal. Nonetheless, the rectangular cubic table has all equal planar diagonal sums as indicated by a row of equal diagonal sums at both the top and bottom of the table. It gives the appearance of being absolutely perfect but is deficient in property #5.

Cubes whose size is even but not divisible by **4** can be manifested as perfect but not absolutely perfect. These size cubes only have merely a near-perfect square counterpart.

Glossary

Absolutely perfect cube

A cube which has all of the cubic equalities #1 thru #5 above is absolutely perfect in that there are no more linear and planar equalities possible. Cubes whose size is divisible by **4** and all odd-size cubes greater than **3** are absolutely perfect.

Absolutely ultra-perfect cube

An absolutely perfect cube whose embedded squares are ultra-perfect in that they possess dual simultaneous tiling patterns characteristic of the same-size square; they only lack 2-dimensional pangenicity because they participate in a cubic pangenicity instead. They have even more summations equaling the cube's characteristic number than just absolutely-perfect cubes – they have equal-summing complementary tiling patterns too.

Hypercubes

Perfect hypercube

A quadracube which has embedded cubes which are all perfect.

Absolutely perfect hypercube

A quadracube which has embedded vertical and horizontal cubes which are all absolutely perfect.

Geometrical Properties

Continuous modularity

The property that all **nxn (or mxn, m<n)** block-squares anywhere in a table all sum to the same number. For perfect cubes and quadracubes of size **n**, that number equals the table's characteristic number.

Complementary tile patterns

The property unique to *ultra-perfect* Class-1 and Class-4 squares and cubes where each and every number in the table serves as the center of two distinct but partially overlapping geometrical configurations involving numbers that within each pattern sum to the characteristic number of the square.

Continuous complementary tiling patterns

For all perfect squares of size **n**, there are **n** such non-overlapping repetitions of a single pattern of **n**-connected numbers which blanket the entire table without voids (wrapping imposed). Moreover, these integrated composite "tiling" patterns may be dragged across the table with wrapping imposed and all the equalities persist.

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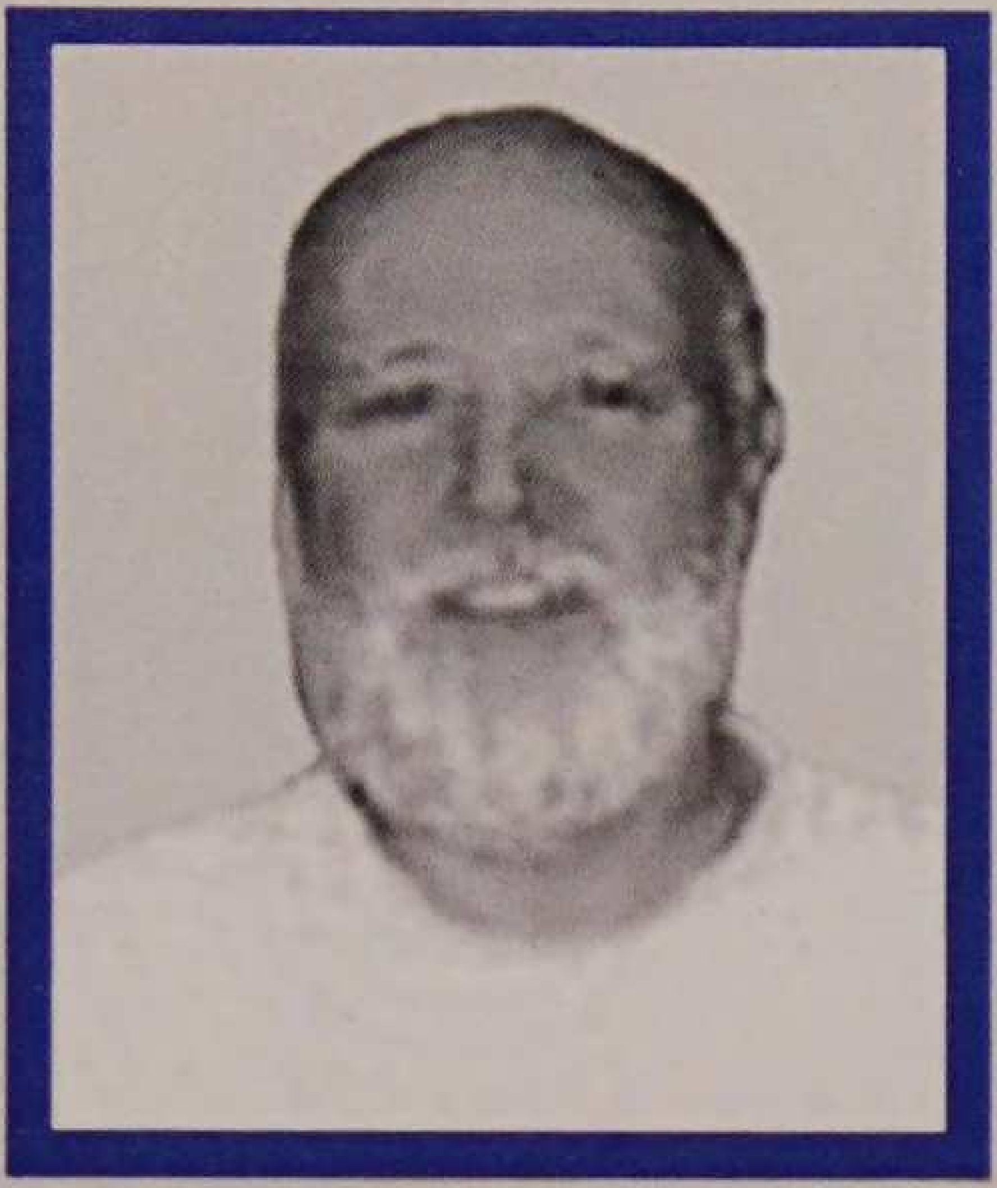

The author has degrees in mathematics from Penn State University (BS '64) and Stanford University (MS '71).

This book is the result of 8 years of exploration and discovery employing MicroSoft's Excel software.

The book is a companion treatise to the 10-Program Series on Geonometry. Every slide shown there is copied here along with the narration in print.

Geonometry is a whole new mathematics on a very basic level. It mostly involves simple arithmatic of addition, subtraction, multiplication and division of whole numbers in tables. There are no fractions involved.

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It accounts for the distribution and orbital layers of electrons in the atom.

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It describes just why snowflakes are perfectly hexagonally symmetric.

It identifies a harmonic pattern in the orbits of the planets around the Sun.

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