STUDIES IN PHILOSOPHY

XV

STRICT FINITISM

AN EXAMINATION OF LUDWIG WITTGENSTEIN'S *REMARKS ON THE FOUNDATIONS OF MATHEMATICS*

by

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PREFACE

In an essay entitled "Wittgenstein's Remarks on the Foundation of Mathematics", in the *British Journal for the Philosophy of Science*, IX, No. 34 (August, 1958), G. Kreisel reviewed the book which I shall examine in this essay. He closed this review by commenting: "now it seems to me to be a surprisingly insignificant product of a sparkling mind". Although one can sympathize with this comment, it is not entirely accurate. It is not surprising that the book as a whole seems insignificant because it is a posthumous selection of remarks from his note-books. These remarks had been written over a period of at least seven years, 1937 to 1944. In the text they are organized according to their date of composition, rather than according to a connecting theme, though I must confess that in my essay I treat his work as a treatise. Furthermore, many of the remarks are very obscure. They are like the expressions of an insight which one jots down in a note-book and then needs several pages to *unpack* it. Unfortunately, Wittgenstein did not *unpack* his insights. I submit that my theory of strict finitism which is presented in Chapters II and III is an *unpacking* of several of his remarks, and of those remarks which express his deepest insight on mathematics. There is no doubt that if one thought that the only significant remarks on the foundations of mathematics occurred in mathematical journals, such as the *Journal of Symbolic Logic,* then Wittgenstein's *Remarks* would seem surprisingly insignificant. There is no discussion of the technical problems of axiom systems except to say that they are philosophically unimportant. Only an elementary knowledge of symbolic logic is required to read his attacks on formal proofs in mathematics and the attempt to show that mathematics is logic in *Principia Mathematica*. Although Wittgenstein's *Remarks on the Foundations of Mathematics* is insignificant as a mathematical or metamathematical treatise, it is not fair to expect it to be one. A brief glance through it immediately shows that it is not at all like the technical articles on that branch of mathematics called "the foundations of mathematics". I hope that my essay is no more technical than Wittgenstein's *Remarks*.

However, many of Wittgenstein's remarks have surprising philosophical significance. In my opinion the most significant points are the following three. Wittgenstein accounts for the necessary truth of simple geometry and the arithmetic of small numbers without any trace of platonism; this is what I call his strict finitism. He finds the proper place for self-evidence; in other words, he recognizes the importance of not being able to conceive of the denial of a proposition without using inconceivability of the denial as a criterion for or definition of necessary truth. Thirdly, he shows, by non-mathematical means, the mathematical and philosophic irrelevance of attempts such as *Principia Mathematica*, to reduce mathematics to logic. I do believe that these *Remarks* contain a conclusive refutation of Logicism.

This essay discusses the same topic as my dissertation: *An Examination of Ludwig Wittgenstein's Remarks on the Foundations of Mathematics*, viz., Wittgenstein's *Remarks*. The only similarity between the two, however, lies in the presentation of strict finitism. In my dissertation, I tried to uncover an implicit metaphysics in Wittgenstein's strict finitistic philosophy of mathematics and then tried to refute Wittgenstein by refuting this metaphysics which I attributed to him. In this essay my aim is not to criticize Wittgenstein. My aim has been primarily to develop some lines of thought suggested in Wittgenstein's *Remarks* in hope that such developments may make reading these remarks of Wittgenstein's easier and more profitable.

Throughout my essay I abbreviate Wittgenstein's *Remarks on the Foundation of Mathematics* as *RFM*. *RFM* is broken up into five books: I, II, III, IV, and V. Each book is divided into remarks numbered by arabic numerals. Thus I shall refer to specific passages, such as remark 22 in Book II by citing book number and remark number, eg., II-22. I abbreviate Wittgenstein's *Philosophical Investigations* as *PI*. Most of the papers on *RFM* which I consider are reprinted in: *Philosophy of Mathematics,* Selected Readings, edited by Paul Benacerraf and Hilary Putnam (Prentice Hall, 1964). I abbreviate this book of papers as *BP*. I hope that the absence of footnotes facilitates reading.

TABLE OF CONTENTS

I

PHILOSOPHICAL PROBLEMS AT THE FOUNDATIONS OF MATHEMATICS

> I have not yet made the role of miscalculating clear. The role of the proposition: "I must have miscalculated". It is really the key to an understanding of the 'foundations' of mathematics. (II-90)

1. It is not obvious that the role of an admission of miscalculating is the key to an understanding of the foundations of mathematics. What is the role of a proposition? What is meant by "foundations of mathematics"? How could an understanding of such an apparently minute point, namely an admission of miscalculation, be so helpful in such an apparently large topic, namely the foundations of mathematics? Yet, Wittgenstein is basically correct. To show that he is correct, I shall first describe what I believe Wittgenstein meant in this passage by "foundations of mathematics". Then, I shall explain how understanding something which you could call the role of "I must have miscalculated" provides answers to some of the crucial questions about the foundations of mathematics. However, my main goal in this chapter is not to explain this passage about miscalculating. Rather, I hope, while explaining this passage about miscalculating, to reach the main goal of this chapter – to explain what I consider to be an investigation into the foundations of mathematics.

In this essay, by "foundations of mathematics" I mean, and I believe that in *RFM* Wittgenstein meant, philosophic foundations of mathematics as opposed to mathematical foundations of mathematics. In IV-52 Wittgenstein wrote:

The philosopher must twist and turn about so as to pass by the mathematical problems, and not run up against one, – which would have to be solved before he could go further.

His labour in philosophy is as it were an idleness in mathematics.

In V-13 he wrote:

The *mathematical* problems of what is called foundations are no more the foundations of mathematics for us than the painted rock is the support of a painted tower.

But what is the philosophy of mathematics or an investigation into the philosophic foundations of mathematics? And what, if anything, are mathematical foundations of mathematics? In my opinion, someone is investigating the philosophic foundations of mathematics when he tries to answer one or more of the following questions. (I have obtained the idea of the philosophy of mathematics being based on questions like these from the Introduction of Stephen Körner's *The Philosophy of Mathematics* (London, 1960).

Group I: Questions on the ontology of non-applied mathematics.

(i) When we do non-applied mathematics are we talking about some objects?

(ii) When we do non-applied mathematics are we talking about non-ordinary objects?

(iii) If we are talking about non-ordinary objects when we do non-applied mathematics, what are they like and how do we get knowledge of them?

Group II: Questions on the epistemology of non-applied mathematics.

(i) Are there truths in non-applied mathematics?

(ii) If there are truths in non-applied mathematics, are they necessary truths?

(iii) If the truths of non-applied mathematics are necessary truths, what does it mean to say that they are necessarily true in addition to their being merely true and how do we come to know that they must be true?

Group III: Questions designed to check whether or not answers to the preceding questions on the ontology and epistemology of non-applied mathematics enable the philosopher to account for applied mathematics.

(i) Does a bit of non-applied mathematics become applied mathematics by making it be about objects if it were about no objects as non-applied mathematics? Or, if it were about objects when non-applied, does it become applied by making it be about a different kind of object? If non-applied mathematics is about the same objects as applied mathematics, what is the difference between applied and non-applied mathematics?

(ii) Does a proposition of non-applied mathematics become applied by making it be a truth about something if it were not true when non-applied? Are propositions of applied mathematics necessarily true? If propositions of applied mathematics and non-applied mathematics are both necessarily true, what differentiates applied from non-applied mathematics?

2. Before I state what I mean by an investigation into the mathematical foundations of mathematics, I shall comment extensively on these questions leading to a philosophy of mathematics. What is meant by "applied" and "non-applied"? I cannot now say precisely what is the difference between non-applied and applied mathematics. To state precisely the difference between non-applied and applied mathematics is, as is clear from Group III, one of the problems of the philosophy of mathematics. However, the following distinction will suffice for our present purpose of illustrating these groups of questions. A person has done applied mathematics if he makes or accepts a non-mathematical statement and he has obtained it by doing mathematics. The mathematics that he used to conclude his non-mathematical proposition is applied mathematics.

For instance, a person may say "This box of soap weighs three pounds." The statement that a box of soap weighs three pounds is not a mathematical statement. Assume that we ask this person how he arrived at this conclusion because we know that he did not weigh the box. If he answers that he figured it out because he knew that this box and a smaller one, which weighs one third what it weighs, together weigh four pounds, he has done applied mathematics.

Of course, when doing his applied mathematics, the person gives mathematical expression to facts about his environment. In our example, the man would have expressed the relevant facts in the system of equations: $b_1 + b_2 = 4$ and $b_2 = b_1/3$. In this system b_1 stands for the weight of big box and b_2 for the weight of the smaller box. Similarly, conclusions about the pressures of gases, accelerations of objects towards one another, and the amount of interest earned in four months are drawn by having a mathematical expression for some facts of nature and then computing the non-mathematical conclusion from them. (I include statements of initial conditions such as $T = 17°C$ as mathematical expressions of natural facts.) Such conclusions about pressures, etc., we would normally say are obtained as a result of doing applied mathematics. According to my characterization of applied mathematics, the drawing of such conclusions would be typical applications of mathematics. So I take my characterization of applied mathematics as justified. Note, however, that I do not consider giving mathematical expression to natural facts applied mathematics, although it may be called applying concepts used in mathematics. In this I also agree with common sense because to state laws of nature mathematically is not considered to be a function of mathematics; it is considered to be a function of natural science.

An advantage of characterizing applied mathematics as the mathematics that a person uses to arrive at a non-mathematical conclusion is that I do not beg the philosophic question of whether the subject matter and modality of mathematical propositions are the same or different in applied and non-applied mathematics. (By modality of a proposition I mean whether it is necessarily true (or false) or contingently true (or false). Very likely the person in our example concluded that the big box weighed three pounds by solving the system of equations: $b_1 + b_2 = 4$ and $b_2 = b_1/3$. His solving of this system was doing applied mathematics. However, in solving this system we do not know whether or not he has done the same as he would have done if he were solving this system as a problem in non-applied mathematics. For instance, when he solved this system he may have been doing the same as he did as a student in an algebra class or he may have done something quite different.

3. A person has done non-applied mathematics if he has done mathematics but has not done it for the purpose of drawing a non-mathematical conclusion. By doing mathematics I have in mind proving a theorem of mathematics or computing the value of a function. Since computing the value of a function, eg., computing that the cube root of 27 is 3, can be regarded as proving a low level theorem, eg., $\sqrt[3]{27} = 3$; I would say that doing non-applied mathematics is proving mathematical theorems. What, then, is mathematics? Mathematics is what is considered to be mathematics by university departments of mathematics and editors of mathematical journals.

4. My characterization of mathematics requires comment on two points. The first is that I admit that what we ordinarily call computing or calculating can be non-applied (pure) mathematics as much as proving the fundamental theorem of arithmetic is doing pure mathematics. (Establishing that $15 \times 13 = 195$ is a typical example of what we ordinarily call computing or calculating.) In this I agree with Wittgenstein. The opening quotation of this chapter hints that he regarded calculating as an important part of non-applied mathematics. Also, a reading of *RFM* will show that he is trying to gain a philosophic understanding of mathematics, to answer the three groups of questions listed above, primarily by asking these questions about that level of mathematics we call calculating. Indeed in his "Mathematics and the 'Language Game'" (*BP* 481-490), Alan R. Anderson went so far as to criticize Wittgenstein for identifying mathematics with calculating procedures. But Wittgenstein did not make the factual error of identifying mathematics with calculating procedures. For instance, in IV-46 he admitted: "'Mathematics' is *not* a sharply delimited concept." Thus he would not believe that he could identify mathematics, which he cannot clearly identify, with calculating. Also in II-46 he explicitly claimed: "mathematics is a MOTLEY of techniques of proof".

5. If Wittgenstein made any kind of error of identifying mathematics with calculating, it was the error of holding that a philosophic understanding of calculating both applied and non-applied, gives a philosophic understanding of mathematics on any level. I do believe that Wittgenstein thought that a philosophic understanding of calculating would provide a philosophic understanding of all of mathematics. To support this, I again refer to the opening quotation and his practice in *RFM*. I would also call attention to IV-16 in which he wrote:

> Strangely, it can be said that there is so to speak a solid core to all these glistening concept-formations. And I should like to say that that is what makes them into mathematical productions.

In our development of his philosophy, we shall see that Wittgenstein regards mathematics as a way of forming concepts. However, an important point to make here is that the assertion of there being this solid core should not be read as contradicting the claim that there is a motley of mathematical techniques of proof. It should, I submit, be taken as a suggestion that there is one facet of all mathematical thinking such that, if properly understood in any one of the manifold branches of mathematics, this understanding would provide a philosophic understanding of all other branches of mathematics. Now it is a philosophic error to hold that a philosophic understanding of calculating suffices for a philosophic understanding of mathematics only if there is no such solid core. So whether or not Wittgenstein made a philosophic error by focusing his attention primarily on calculating can be judged only after we have his philosophic examination of calculating.

6. The second point of comment on my characterization of non-applied mathematics is the loose characterization of mathematics as that which mathematicians call mathematics. Such a characterization is certainly compatible with Wittgenstein's admission that "mathematics" is a vague, – not sharply delimited –, concept. I interpret "vague" or "not sharply delimited "as meaning that there are no necessary conditions for applying "mathematics".