Prime Evolution

Interview with Matthew Watkins

Matthew Watkins' Web-based Number Theory and Physics Archive *and its speculative twin* Inexplicable Secrets of Creation[1] *– hosted by the mathematics department of Exeter University in the UK where Watkins is an honorary research fellow – have grown into a unique resource. The archive brings together work from the plurality of disciplines contributing to an as yet unnamed field of research concerned with the startling connections between number theory – particularly the Riemann Hypothesis on the distribution of the prime numbers – and the physical sciences. Watkins talks to* **COLLAPSE** *about his rôle in, and motivations for, catalysing and disseminating the field, about the latest developments in the search for the hypothetical 'Riemann dynamics', about the nature of discovery in mathematics and its academic and cultural status.*

1. *At* http://www.maths.ex.ac.uk/~mwatkins/. Dr. Watkins has kindly assembled a 'primer' for the mathematical concepts discussed in this interview: at http://www.maths.ex.ac.uk/~mwatkins/zeta/collapseglossary.htm

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Collapse: The primes have perennially been hailed as 'mysterious'. In modern mathematics this mystery has condensed around the problem which Riemann's Hypothesis concerns. We can find primes as we count along the number line, but we have no way of predicting in general where and how densely they will occur. A lack of determinable global order, then.

Matthew Watkins: But first there's a major question concerning what is meant by 'order'. I'm often asked, is there a pattern in the primes, is there an order, but what does that mean? If you try to reformulate these questions very precisely, you're forced to consider what it would it mean for there to be order, or a pattern. I mean, there are patterns like wallpaper patterns, where you have a block of something repeating. Well, almost by definition the primes can't do that. But what sort of pattern could there be, what sort of order could there be? The idea that there might be a pattern, the importance of there being a pattern in the primes – these aren't things you can rigorously pin down.

C: Couldn't you use an information-theoretical definition of pattern?

MW: You could come up with a definition, one of an endless number of possible definitions, from information theory or some related discipline, of what a pattern is, and then apply it. But I think there's still the basic fact that when people who aren't familiar with any of those definitions are asking 'is there a pattern?' they don't mean anything that such a definition could capture, they mean something else – they don't really know quite what, but it seems important to them that there should be; whether or not there is a pattern in the primes they see as an important question. And it struck me when I was thinking about this that it's more *feeling-based*, it's not a rational question they're posing. You can try to construct rational questions around it. People have, and such questions have given rise to a large part of that body of work we call number theory.

C: But initially it's more like the expression of an instinct for pattern recognition?

MW: Perhaps. As Jung said – almost as the culmination of his work on archetypes – the set of positive integers, taken as a whole, corresponds to the *archetype of order*. So, in a sense, all notions of order, of something coming before something else, of things being in a sequence, all of that ultimately can be linked back to our instinctive grasping of there being a number system underlying our experience. Now that number system turns out to have embedded within it an enigma, a problem bordering on the paradoxical: is there order in the way this thing's put together or not? We feel there should be, but we aren't entirely sure how to ask the question – basically, we don't really know. So we start by asking whether there's order in the number system, and the unintended result of our probing into this matter is that what we ultimately mean by order *in any sense* gets indirectly thrown into question.

People also frequently ask about the existence of a *formula* – is there a formula for the prime numbers? Well, again, that's difficult because, yes there is, there's the Riemann-von Mangoldt explicit formula, which effectively generates exactly the distribution of prime numbers as 'output' – but you need the complete set of *Riemann zeros* as input. This is an infinite set, and to produce it you effectively need the complete set of prime numbers, so there's a circularity. So it's a formula, but not the kind of formula which people who ask this question have in mind. There are also algorithms – rigorous procedures – which can systematically generate the primes. One could arguably call these 'formulas', but they're basically methods of computation, and the computations quickly become intractably huge…so we're not talking about anything that can systematically spit out primes one after another in the sense that people might have in mind when they ask about the existence of a formula.

C: As the years have gone on, mathematicians' ingenuity and the employment of new technologies have seen an acceleration in the conquest of the critical line of Riemann zeta zeros. But does the fact we've got, say, one billion of the zeros make it any less mysterious than when we had a hundred? Does the apparent success of the Riemann Hypothesis (RH) militate against the conception of the primes as mysterious?

MW: First of all, you can't really talk about RH being 'successful', it's still a hypothesis. RH doesn't predict the primes as such, but the theory of Riemann's zeta function, from which it emerges, allows us to understand the distribution of primes much more deeply. At the heart of this theory is the peculiar sequence of 'zeros' now known as 'Riemann zeros', 'Riemann zeta zeros' or sometimes just 'zeta zeros' – these are what RH directly concerns.

What's happened really is that RH has displaced the mystery. The primes are no longer mysterious, you could argue – we now know that they are exactly governed. Initially, it was found that they're governed by a logarithmic distribution, a sort of gradual thinning out, in an almost statistical sense – that provides reliable but approximate information about the primes. Riemann later found that the logarithmic distribution is 'modulated' by an infinite set of waves, where each wavelength corresponds to one of the Riemann zeros. We're in the realm of proven mathematical results here, and these *precisely* pin down the primes, so in that sense, all mystery is gone; but in actuality the mystery has been pushed back, or displaced. The mystery now is, *where the hell did these Riemann zeros come from*? We can calculate hundreds of billions of them, we've got a vast, intricate body of precise mathematical results concerning them which ultimately brings us to a big, important, question about whether they'll all lie on the 'critical line' – that question is RH. But ultimately, what *are* they?

Since the seventies, this idea that they might be *vibrations of something* has taken root and has now been more-or-less universally accepted, on the basis of a lot of computational evidence together with a mysterious, suggestive mathematical 'coincidence' involving something called the Selberg Trace Formula – and that ties in with certain unexpected connections with physics.

So if we've got vibrations of a mysterious 'something' underlying the number system, in a sense the primes are no longer the mystery, the primes have been taken care of, the mystery has been displaced. The primes are our obvious way into the mystery, but ultimately it's a mystery about the system of positive integers, about 'order', and arguably even about *time*.

C: To return to the question of order, are the zeros any more ordered than the primes?

MW: The set of primes and the set of Riemann zeros are in some sense 'dual' structures. There's a variant of what's called a 'Fourier duality' between them. To put it simply, you can use the zeros to generate the set of primes: if you have just the zeros and the explicit formula, you can effectively 'put the zeros in and get the primes out'. And it also works in the opposite direction. So the two generate each other. In a sense the primes are more well-behaved in that they're all integers, they all fall on this nice 'grid' of positive integers. The primes can be explained to a schoolchild, a five-year-old is capable of understanding the idea of prime numbers. They are there among the familiar positive integers, the usual counting numbers, and counting is a ubiquitous part of our everyday experience.

They're dual, so in some sense the two could be seen as equally important, two sides of the same coin. However, the Riemann zeros are very different – they're not integers, they're what we call 'transcendental', irrational numbers; you need a degree in mathematics before you can even begin to understand the definition of them, and relative to the total population, only the tiniest handful of people have any real understanding of what is currently known about them. And they appear to have absolutely nothing to do with ordinary everyday experience.

C: We could say that the zeros are not a solution to the problem, but the problem itself, expressed in a domain that's more difficult for us to access; the exact nature of this domain then becomes the real focus of interest.

MW: Yes, the zeros are the problem, and thus the problem's been displaced to somewhere we're much less familiar with. Counting, you know...Ancient Greeks and earlier people could count pebbles out on the ground, subdivide them into piles and contemplate different types of numbers – 'perfect numbers', 'triangular numbers', prime numbers – and they were able to develop a certain amount of theory. But that's just one side of the coin. On the other side, there was no way they could have contemplated the Riemann zeros: (a) you need a theory of 'functions of a complex variable', and (b) in order to calculate more than the first handful of them you need a pretty powerful computer.

It reminds me of the central image in the film *2001*: It's as if we've dug this monolithic thing up, it's been there for aeons, as a structure it's overwhelmingly impressive, and everyone concerned is flabbergasted, asking themselves how did *that* get there, you know: it comes from *somewhere* else, somewhere beyond, and it induces a sense of almost religious awe.

One suspects that if a mathematical structure underlying or 'explaining' the Riemann zeros were to emerge – that is, if in fifty or a hundred years someone comes up with something new which 'explains' the zeros in the way the zeros 'explain' the primes – then that new structure is just going to open up another even deeper mystery. Paul Erdös, who published more mathematics papers than anyone else ever, and who was primarily a number theorist, said that it's going to be at least a million years before we understand the primes, and even then we won't really understand them.

C: Is it a properly transcendental problem, relating to the limits of our thought: the more that we think, the further the problem moves away from us?

MW: Well, again, we don't know that yet: it may be, but then who knows – maybe it'll all neatly tie up somehow. But it *feels* to me that the problem has a quest-like quality. The fact that the metaphorical image of the Holy Grail has been invoked a few times in the literature, as well as a lot of language poetically invoking the feminine and generally suggesting an 'otherness', suggests that I'm not the only one thinking like this. I've had an interesting dialogue with some Jungians about this aspect of RH.

The problem of the primes isn't just different from other mathematical problems, it *precedes* them. All other mathematical problems rely on the fact that there are positive integers. Without the set of positive integers, those other mathematical problems couldn't exist. So the problem of the primes is *the* problem in a sense, it's beyond the most basic, it's there before all the others are there. As soon as you've got counting, as soon as you've got any notion of repetition, then the problem of the primes is there waiting to be discovered.

If we don't understand the prime numbers, we don't understand the positive integers. And if we don't understand the positive integers, then I don't know if we understand anything at all, because all science is entirely built on measurement, and you can't measure anything until you can count. All our rational scientific thought relies on these very basic ideas of order and counting.

One of the most important quotations that I've reproduced on my website is this, from Gerald Tenenbaum (Institut Élie Cartan):

> As archetypes of our representation of the world, numbers form, in the strongest sense, part of ourselves, to such an extent that it can legitimately be asked whether the subject of study of arithmetic is not the human mind itself. From this a strange fascination arises: how can it be that these numbers, which lie so deeply within ourselves, also give rise to such formidable enigmas? Among all these mysteries, that of the prime numbers is undoubtedly the most ancient and most resistant.[2]

So, in probing the mystery of the prime numbers we're effectively on a sort of journey to the centre of the mind, or of the collective human psyche, and ultimately to the point where that interfaces with the physical world which it finds itself inhabiting. That quote perhaps best conveys some feeling as to why I'm so gripped by this stuff.

C: The story of the modern theory of primes begins

2. G. Tenenbaum and M. Mendès France, *The Prime Numbers and Their Distribution* (AMS, 2000) p.1

with Gauss's initial success in predicting approximately the distribution of primes. How do we get from there to the pioneering interdisciplinary work that your web-archive charts?

MW: Gauss – although he didn't publish, he supposedly got there first – Gauss and Legendre noticed that there was at least a 'statistical' thinning out of the primes that you could quantify. Riemann later uncovered the zeros of his zeta function – the Riemann zeros – and so was able to pin it down much more rigorously. But there's a fifty-year gap where...actually, I don't know what mathematicians felt during that time. Practically, they were trying to refine the approximations; Chebyshev and others improved the approximation of how many primes you'll find in any given chunk of the number line. But whether there was an expectation that eventually someone would find a way to make this exact, or whether there was a general feeling that 'this is the best we'll ever do', I don't know, and I can't recall seeing anything in the literature of that period where feelings about this matter were expressed. Once Riemann's work came along then no-one was really interested in what people *used* to think. The history of mathematical ignorance isn't as well documented as the history of mathematical discovery.

There are some parallels with the situation we're in now, where there's a mystery about this proposed 'Riemann dynamics', this hypothetical dynamical system underlying the Riemann zeros.

C: The complex plane is the most important mathematical support of RH itself. And here already a transformation takes place – de Sautoy[3] talks about it as a sort of magic mirror we step through – which seems to unfold things we thought we knew, in a completely different space – as it turns out, very fruitfully for mathematics and the physical sciences alike.

There's obviously something very powerful about the complex plane itself which, at the very least, corresponds in some way to physical reality, and so the fact that it was also the complex plane which facilitated Riemann's insight into the prime distribution is itself suggestive.

MW: The complex plane appears to have a life of its own. Complex numbers are absolutely necessary to describe quantum-mechanical phenomena. Electricians use the complex unit *i* just to work with AC electricity, so something as 'nuts-and-bolts' as the National Electricity Grid depends on the complex plane. And yet it is this supremely mysterious thing. I mean, all those fractals that started to circulate in the 1980's – a lot of people don't realise what they're looking at, but those are things that naturally inhabit the complex plane. Without the complex plane you wouldn't be able to see such objects, that's their natural domain. And then the Riemann zeta

3. De Sautoy, M. *The Music of the Primes*. London: HarperCollins, 2004

function, with all its strange properties; Riemann's big step was to take a function which Euler had looked at and ask, what would that do if we extended it into the complex plane? And what it was found to do then spawned the great mystery of the Riemann zeros.

Another strange thing worth mentioning: One tends to think of temperature as existing on a linear scale, a one-dimensional scale. But in statistical mechanics, by constructing a function of temperature, the 'partition function', and extending it out to the complex plane, you find that it has a set of 'singularities', off the familiar real number line, in this other two-dimensional region that doesn't seem to have anything to do with temperature or any other aspect of practical measurable physical reality. Yet these singularities correspond to *phase transitions* of the system. Without the complex plane you'd never have known they were there. The same thing happens with the zeta function, it's got a set of singular points in the complex plane, the Riemann zeros off the real line. From the behaviour of the zeta function *on* the real line, you would never have guessed they were there.

Various people have put forward models of two-dimensional time – imaginary time certainly gets used, complex time. Such models can be used in attempts to explain otherwise inexplicable phenomena, but none of this can be applied to our normal experience of reality, you can't really do anything with it. I would say that the complex plane is still deeply mysterious. It's 'behind the scenes' of reality as we experience it.

C: And historically, complex numbers had been discovered long before there was any sense of their ultimate utility. Only later did it became evident that something which seemed to have been a mathematical fiction, was hugely important to work in these fields.

MW: Absolutely, the word imaginary, you know – you've got the 'real' numbers and the 'imaginary' numbers – it's a very unfortunate name, but it's simply because of the history of the thing. For quite a while, no-one thought these things had any 'reality' to them, primarily because they didn't correspond to anything experiential in the way 'real' numbers were seen to.

C: It's difficult to ignore this experimental evidence that complex numbers relate to something in reality: we have to take account of these things which just impress themselves upon us. The traits of the complex plane are obviously real, but they don't correspond to any actual object, any actual thing we can get hold of. They're distributed through reality itself.

MW: Yes, the system of complex numbers is there, I don't know 'where' it is, but it's not just something we *invented*. And, interestingly, it's most directly evident at the subatomic level. As I said, the theory of AC electricity relies on it, but then ultimately that's a quantum-mechanical phenomenon, scaled up to the level where we can, say, run a toaster on it. Functions of a complex variable get used in statistical mechanics, aerodynamics, *etc.*, but those are fairly indirect manifestations of something very deep, I feel. The fact that the complex plane relates so closely to quantum mechanics means that in macroscopic reality, it permeates everything, as you say, and yet nobody had a clue it was there until relatively recently. Even after it had been mathematically brought into consciousness it was still seen as just a fiction.

As for the primes, you can't understand the distribution of primes until you've grasped the Riemann zeros. And the Riemann zeros live on the complex plane, inarguably. The 'nontrivial' zeros, the ones RH concerns, inhabit a narrow vertical strip in the complex plane. The RH simply says that they all – the entire infinite set of Riemann zeros – lie on the 'critical line' which runs up the middle of this 'critical strip'.

Now, to prove RH would be an exact mathematical task, so RH gets a lot of press – there's the whole fame-and-fortune thing, literally a million-dollar prize, this idea of something like winning the ultimate intellectual gold medal, you know – but you've either done it or you haven't, it's very clear-cut. But I'm more interested in the less clear-cut questions – what *are* the Riemann zeros, from where do they originate?

To answer this we may need something else as new

4. See http://www.claymath.org/millenium/

and unexpected as the complex plane was when it was first introduced, something we haven't thought of yet, a new mathematical 'environment' in which these things will become perfectly clear. But that may well lead to another body of questions which are even more baffling.

But "from where do the zeros originate" – what does that mean? They're seemingly vibrations of something, but what? What is that thing going to be – is it going to be a mathematical model of a dynamic system that people may or may not be able to physically manifest? If it is possible to physically manifest it and someone does...what then are we confronted with?

One gets a very strong feeling that until we understand the what the zeros 'are', we won't be in a position to prove RH. These two issues are tied together. But the former isn't yet a precise question, whereas 'is the RH true' *is*.

C: It is said that in mathematics a question isn't even a question if you can't formulate it precisely: mathematics is the art of formalising problems, so if you can't do that then in a sense it falls outside of mathematics.

MW: Yes, and so something with this kind of quasi-mathematical character is generally regarded with a certain suspicion; it's neither one thing nor the other.

C: A mystery rather than a problem, then.

MW: Yes, and I suppose I tend to be attracted to the mysteries.

C: Practically speaking, how does the hypothetical positing of a Riemann dynamics change the nature of the search for a proof of RH?

MW: It brings other people in, it brings the physicists in. Before, you had analytic number theorists hammering away at this problem. And now probability theorists, geometers and physicists are all contenders, and they all have pieces of the puzzle. It's broadened the scene, if you like, of people concerned with the problem. But it also has given a deeper sense of what's at stake; again, if there is a dynamic system underlying the Riemann zeta function, well then it underlies the number system; if it underlies the number system then it underlies everything, or at least everything that rational scientific thought concerns itself with. And so, again, we're force to ask what is it, where does it 'live', what does it 'do'? And perhaps the most important question is, *what is the time parameter*? Because a dynamical system always has a time parameter according to which it 'evolves' – so what kind of time are we talking about in this case? So it basically opens a whole new can of philosophical worms. It makes me think of what Hilbert said, when he was asked about RH, he said that it isn't just the most important problem in mathematics, it's the most important problem. And I think a lot of people might just think, yes, that's because he was a mathematician, he was biased...but I think he knew what he was talking about. He and Pólya first proposed that there might be a 'Riemann operator', that the zeros might be a spectrum of something. They didn't suggest a dynamical system as such, but they could be said to have laid the groundwork for that. So I think Hilbert may have sensed something very big going on there, which he was trying to express in that pronouncement.

C: The first steps towards elaborating the nature of the Riemann dynamics comes with Julia's interpretation of the zeta function as a thermodynamic partition function. What is a partition function, and in what sense can one speak of the primes as a numerical gas – Julia's 'free Riemann gas'? Is it simply a useful metaphor taken from thermodynamics, or is there a more substantial link?

MW: Well, firstly, Julia's work doesn't directly address the issue of the Riemann dynamics, although there may well be a deep connection there.

Your last question is difficult to answer, but it would be hard to deny that there's a sort of a metaphor here, in that there's a strong resemblance between certain aspects of the zeta function and the theory of thermodynamic partition functions. But it goes deeper than a superficial resemblance. There are enough corresponding elements, that Julia included what he called a 'dictionary' in the paper he first published about this.[4] It consists of two columns, with number theoretical structures on one side and corresponding thermodynamic structures on the other. And the correspondences are such that, if you're sufficiently familiar with number theory and statistical mechanics, you can't deny there's something...there's a very strong link there. So you could call this a metaphor, but I would maintain that it's more than just a metaphor in the familiar sense, *i.e.* a useful way of explaining what something is by means of something else which isn't directly related to it.

Now what is a partition function, in statistical physics, or statistical mechanics? Well, in classical mechanics, a billiard table is often used as an example: you've got a finite number of billiard balls bouncing off each other, bouncing off the sides, they're colliding, energy is being transferred between them, there are various angles, positions and momenta involved. And the idea is that you've got a sufficiently simple system that you can keep track of each individual object and what it's doing. But a problem arises when you've got something like a box of gas: that's effectively like a giant three-dimensional billiard table, but there are too many components to keep track of what each one is doing. You're not actually going to be able to do anything in that way, so you're going to have to study it in the sort of way sociologists study society – they can't possibly consider all the specifics of

4. Julia, B.L. 'Statistical theory of numbers', in M. Waldschmidt, et. al. (eds.) *Number Theory and Physics*. Springer Proceedings in Physics 47 (Springer, 1989)

each individual person, so they must look at overall statistical trends in the population.

Suppose you had a quantity of gas particles in this room, and they were all roaming freely. It would be very surprising to find them all clustered up in one corner. One expects a more uniform spread. But, *there's no real reason they can't do that.* It's like if I toss a coin fifty times, I'd be very surprised if I got fifty heads or fifty tails, but there's no reason why that can't happen. That would be no more unreasonable than any other outcome of fifty coin-tosses, it's just that it's extremely *improbable* because, unlike any other outcome, *there's only one way of arriving at it.* Similarly, there are proportionally few possible configurations of those gas particles where they're all squashed in one corner, compared to the vast proportion of configurations where they're more-or-less uniformly distributed.

Now suppose you have a box of gas, and the gas consists of particles which can jump between different energy levels in an effectively random way. This time you're concerned, rather than with the spatial distribution, with the *total energy* of the system – that's simply what you get when you add together all the individual particle energies. You can ask about the probability of the system having a particular total energy, and it turns out to be rather like the situation with the spatial distribution. That is, the system tends towards a mid-range total energy on the whole, while the highest and lowest ranges of *possible* total energy are much more improbable – because their occurrence requires something akin to a huge number of coin-tosses producing almost all heads or almost all tails.

So what you're looking at with thermodynamics is the probability that you'll find a box of gas or some similarly complex system in one state or another. And the partition function takes a unit of probability and 'partitions' or subdivides it, so that you end up with a curve describing in precise terms the relative probability of finding the total energy of the system at any particular level. So the partition function will basically return probabilities that a system is in one of any number of possible states. The partition functions Julia refers to are functions of temperature – as the temperature of the system varies, the probabilities also vary, and the partition function is able to provide a precise probabilistic distribution of possible total energies at any given temperature.

Now partition functions, it turns out, are the key to understanding statistical mechanical systems; they 'encapsulate' such systems. The partition function in this context is a function of temperature, and temperature would naturally be seen as a variable which varied on the real line – on the *positive* real line, if you're working with absolute temperature. Well, nineteenth century mathematics suddenly allowed the possibility of extending such a variable to the complex plane, regardless of what a complex-valued temperature might actually refer to. You take your partition function which is supposed to be returning probabilities of a system being in some energy state or other based on a real-valued temperature variable, extend it to the complex plane, and you find there are singularities hidden out there which tell you about the possible existence of phase transitions in your system. These are very important for understanding the system, but, as I said earlier, you wouldn't see them if you didn't have access to the complex plane.

Now Julia wasn't the first – George Mackey got there first, although it wasn't widely noticed. Julia discovered it independently and then Donald Spector, a couple of years later – they all noticed that if you treat the primes as your basic particles, and each prime p is thought of as having as its 'energy' the natural logarithm of p – that logarithm turns out to be very important, logarithms show up everywhere in analytic number theory – then the Riemann zeta function very naturally falls into the rôle of being the partition function of an abstract numerical 'gas' which is made of this set of particles – what Julia calls the 'free Riemann gas'. Imagine a fluctuating integer, where prime factors are coming and going all the time, joining and leaving, so the energy of that integer is going up and down, the more prime factors there are the higher the energy, and the less prime factors the lower the energy. The zeta function naturally becomes the partition function of such a system. The 'pole' of the zeta function – this unique singularity of the zeta function at the point 1 in the complex plane where the zeta function isn't defined, where it effectively becomes infinite – corresponds very naturally to something in thermodynamics called a *Hagedorn catastrophe*, a phenomenon involving the energy levels crowding together so the system hits a critical state and shifts into an altogether different mode. So the pole of the zeta function is associated with this 'catastrophe', and based on what I was just saying, the Riemann zeros also become linked to phase transitions, in a way that no-one entirely understands. And there's more...those are just the basic points, there are further subtleties which suggest that, in some sense, thinking of the zeta function as a partition function goes beyond mere metaphor. It's a metaphor, but it's a metaphor that goes deep enough to suggest to me that *the number system has some sort of quasi-physical quality*.

C: How are we to interpret this? There's a perplexing quality to these propositions, one is never sure whether what's being revealed is a progression, or simply a restatement of the same problem in different terms.

MW: Possibly, but you see, the mathematics that's come out of studying things like boxes of gas, *that that should be applicable at all* to studying something as fundamental as the positive integers, to me comes across as sort of *uncanny*. I think that's a good word to capture how a lot of people have reacted to these discoveries. It's hard to see how it's simply a reformulation of the problem.

You'd never have got there if you hadn't studied the boxes of gas in the first place. When you ask how we can best interpret this, the only answer I can come up with is I honestly don't know. To me it points to something fascinating which we haven't yet entirely understood or taken into account.

Now, interestingly, Alain Connes' (College de France, IHES, Vanderbilt) model involving what's called a *C*-dynamical system* – his attempt to try and describe the Riemann dynamics, which hasn't yet fully succeeded, although it's certainly opened up some new vistas – was inspired by Julia's paper, but Connes uses the partition function in a somewhat different sense. The partition functions I've been describing, the ones associated with boxes of gas, *etc.*, could be called 'classical partition functions' as they belong to 'classical statistical mechanics'. But there are also partition functions used in *quantum* statistical mechanics, which take some of the same concepts down to the quantum level.

Connes takes certain elements of quantum statistical mechanics and applies them to the zeta function, treating it as a partition function, and this reveals certain things which again push the metaphor, in my mind, so far that it can't be regarded as *just* a metaphor.

C: So there is a direct link between the quantum-mechanical interpretation and the thermodynamic?

MW: I think there must be, although it's not yet entirely clear what it would be. Of the two most extensive pages in my web-archive, one deals with the spectral interpretation – Hilbert and Pólya's suggestion that the Riemann zeros might be vibrational frequencies of something and Michael Berry's (Bristol University) physics-inspired work concerning what that 'something' might be. Berry and his colleague Jon Keating have outlined a whole set of dynamical properties characterising this hypothetical Riemann dynamics. And the other page deals with the thermodynamic or statistical mechanics side of things – you've got Julia, Spector, Mackey, who all put forward the idea that the zeta function is a partition function, which would suggest that the zeros are in fact phase transitions of something. So these two currents of research are seemingly different approaches, not obviously compatible. Alain Connes has begun to bridge the gap, though. He has taken Julia's suggestion about zeta as a partition function, shifted it into the realm of quantum statistical mechanics, and then brought in p-adic and adelic number systems, and a lot of other very deep mathematics including something called noncommutative geometry, which is about as difficult as current mathematics gets. He's managed to describe a dynamical system, or at least sketch out the beginnings of one, which produces the Riemann zeros as vibrational frequencies, but where the zeta function is also playing the rôle of a partition function, so there is a link there.

C: Connes' *adele* is an infinite-dimensional space in which each dimension is folded, so to speak, with the frequency of each prime.

MW: Yes, that's almost it. An adele is a generalised kind of number which contains an infinite number of co-ordinates, one associated with each prime number, effectively, and then an extra one, which corresponds to the continuum of real numbers.

The adelic number system embraces all of the different *p*-adic number systems – 2-adic, 3-adic, 5-adic, 7-adic, etc. *p*-adics and adeles constitute yet another aspect of number theory finding its way into physics, thereby suggesting that things aren't the way we thought they were.

The Archimedean principle, the basic principle of all measurement, is based on rational numbers, on ratios. If you have a line segment and a longer line segment, by taking the shorter line segment and joining it end to end a finite number of times, you will always be able to exceed the longer line segment. That seems obvious – it's the basis on which I can take a ruler and measure this room. If I kept joining it end to end and I never got to the end of the room, then measurement wouldn't work very well! So, the universe at the macroscopic scale is Archimedean: the Archimedean principle applies. And the number system we generally use, the continuum of real numbers, is an Archimedean system.

Now, the real number continuum is based on a particular arbitrary choice of how we 'close' the system of rational numbers. The rational numbers are fairly simple, well-determined, or given, if you like – canonical. You've got your integers, and then you start taking ordinary fractions and that fills in the gaps – it doesn't fill in *all* the gaps, but it densely fills in the number line. The 'holes' that remain are the irrational numbers, which can't be expressed as ratios of integers, $\sqrt{2}$ being the one that, it's widely believed, was first discovered, and π being undoubtedly the most famous. But there's not just a handful of exceptions, these irrational numbers are in some sense *more common* than the rational numbers.

The question is, given the system of rational numbers, how do you fill in the holes, how do you seal the whole thing up? Well, the method we've ended up adopting produces the system of real numbers, which is a system in which the Archimedean principle applies. And that's based on defining the holes, the irrational numbers, as the 'limits' of sequences of rational numbers. But to define the limits, you have to have a sense of distance; put simply, a sequence *converges* when its elements get closer and closer to something, and the notion of 'closer' requires some sense of distance. The sense of distance we use to define the real numbers is the obvious one: the distance between any two rational numbers on the real number line is what you get when you subtract the smaller from the larger. But that's an arbitrary way of defining distance. It turns out that, within the logical constraints which apply, there are an infinite number of other meaningful, consistent ways you can define what distance is, and each leads to a different notion of 'closure' and hence to a different number system. So you're still starting with the rationals, but the way you 'fill in the holes' is completely different, and you end up with a different kind of mathematics. Now this was discovered by Hensel in the late 1890's, and very quickly the possible ways of closing the rationals were classified. It turns out that there are infinitely many of them, and that they correspond to prime numbers: there's the 2-adic system, the 3-adic system, the 5-adic system, the 7-adic system, all the way up, and then finally there's the ∞-adic system, which corresponds to the usual system of real numbers, and which suggests the existence of what's called the 'prime at infinity', a deeply mysterious thing, which an Israeli mathematician called Shai Haran has written a whole book about[5].

But the point is, in a 2-adic, 3-adic or 5-adic number system, the distance between two rational numbers has nothing to do with the traditional distance between two points on a ruler anymore, rather it's about arithmetic relationships involving divisibility of numerators and denominators by the prime p which characterises the p-adic system in question. So things that would look very close together on a ruler could be huge distances apart, and vice versa, things that are vast distances apart in a

5. M.J. Shai Haran *The Mysteries of the Real Prime*. London Mathematical Society Monographs, Oxford: OUP, 2001.

normal Euclidean sense could be very close together in a *p*-adic sense.

C: And the adelic system is built up of all these?

MW: An adele is a generalised number which has an infinite number of co-ordinates. One's a 2-adic number, one's a 3-adic number, one's a 5-adic number: one for each prime. They're usually written as:

(2-adic number, 3-adic number, 5-adic number...; real number)

so you get one of each. When, at the end of the nineteenth century, these *p*-adic number systems were discovered, it was realised that we've been doing all our physics on the basis that time and space are like the real number continuum. That's the assumption; all the Einsteinian, Riemannian, Minkowskian manifolds, space-time manifolds, were based on real numbers extending in different dimensions. But why should we assume the universe is 'real', in that sense? You could formulate a 17-adic manifold and do space-time physics in it, or a 37-adic manifold; but then, why pick one prime rather than another? Hence the idea arose, why not chuck them all in, create a system which involves all of them at once – this is the adelic approach, described in very crude terms. Hence *p*-adic and adelic physics – there are people developing models of *p*-adic physics where the *p* is just left as an arbitrary *p*, where it would work for any prime, basically re-building physics according to these new number systems. So you've got *p*-adic models of time, *p*-adic models of probability. A lot of it really turns your ideas of the world on their head.

Now Connes has come up with a dynamical system on a space of adeles, which generates the spectrum of Riemann zeros. The problem is that the system he's starting with has already got the prime numbers built in to it, so some people would say, well, he's really only reformulated the problem. But I suspect there's a lot more to it than that. It's not quite the dynamical system that is being sought in connection with RH, but it is widely seen as a valuable step in the right direction.

Even more interesting than Connes' work, from my point-of-view, is that of the lesser-known Michel Lapidus (University of California-Riverside), another Frenchman with a staggeringly broad view of mathematics and physics. I recently had the privilege of proofreading his latest book – I hope it will come out this year, it's been a long time in the pipeline. It's called *In Search of the Riemann Zeros* and it brings all of these ideas together. And he's taken Connes' idea even further. He's got a set of ideas involving quantum statistical mechanics, *p*-adics and adeles, dynamical systems, vibrational frequencies, partition functions, it's all in there, but also fractals, string theory...

C: The adele already intuitively brings to mind string theory, because of the way everything seems to be bound up with the nature of these peculiarly convoluted spaces.

MW: There's been a lot of work done on p-adic and adelic string theory, but that's not quite what you mean. Lapidus has actually come up with a fascinating connection. He was working on something he called 'fractal strings', but these didn't have anything to do with the 'string theory' physicists study, it was just the name that he had given to these particular mathematical objects. And then he generalised them to something called 'fractal membranes'. But since he came up with that, oddly enough, he's found that aspects of string theory relate directly and unexpectedly to the mathematics. His model involves a dynamical system, a *non-commutative flow of fractal membranes in a moduli space*...

C: Which sounds wonderful!

MW: Yes, at a very naïve level, I just enjoy all the extraordinary language. But, more seriously, I have a certain emotional investment in Lapidus actually being onto something, because if he's correct, it turns out that his 'flow', this very strange, highly counterintuitive, non-commutative geometrical 'flow' projects down into a simpler realm, into the number system, as a flow of 'generalised prime numbers' on a line. This is very close to some strange speculative ideas I made public back in

1999[6]. Lapidus contacted me a few years ago to say this, as it had come to his attention when I first put it up on the Web. Now, it's not that I influenced him, it's almost as if I caught a glimpse of some future mathematics which will follow from his current work. I don't know how I can explain what happened… it's as if I caught a glimpse of something which was coming, but I didn't have the language to describe it accurately, so I just described it as well as I could in this rather naïve way. And so in a way I now feel somewhat vindicated concerning my slightly crackpot idea, because of Lapidus' work.

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In some ways, I think all this intellectualising and mathematics isn't really that good for me, and isn't really what I 'should' be doing. But part of me can't entirely detach myself from it. The speculation I just mentioned, which now appears to be at least partly vindicated, gripped me in a profound way. This event had a precedent a few years earlier when I became *convinced* there was some connection between the Gaussian probability distribution and the prime numbers: that was driven by a sort of compulsion that was, looking back, was quite...not psychotic – it didn't lead to any sort of negative behaviour – but it did rather take over my psyche.

I was one of those kids who, it was obvious fairly early on, could excel at mathematics, and being a fairly

6. *See* http://www.maths.ex.ac.uk/~mwatkins/isoc/evolutionnotes.htm

scrawny, unattractive young person, one latches on to anything one is good at – it provides a sense of importance. It was as simple as that, it wasn't any sort of noble motive for seeking the truth or anything. I was living in the States as a teenager, starting to get interested in things like radical art movements and philosophy. If circumstances had been different I'd have probably studied something else, but I wanted to get out of the States, that was quite a big thing for me then, so to get into a British university my best bet was to apply to do a maths degree, which I did. And then I sailed through that, and got offered a place on a PhD programme, which seemed like a great idea – effectively being paid to explore ideas which I found quite interesting and which I seemed to have an aptitude for exploring. So that was all fairly accidental, and there was no real motive behind it, if you like. It was just the way my life unfolded. But by the time I was doing the PhD I was starting to engage with a lot of other non-mathematical ideas and people, and there was a real sense that, hang on, where is this going, is this really what I want to be doing? At that point I was more interested in 'seeking the truth' – it sounds a bit grandiose, but I wasn't interested in a stable career, and the idea of deriving some sort of self-esteem from being an accomplished mathematician, that no longer seemed to be of any importance. So I started thinking, if I'm seeking the truth, is the truth to be found here, is this really what I should be doing? And then the disillusionment set in. After a year of being on a Royal

Society European fellowship, there was a distinct sense that modern mathematics was becoming irreparably fragmented, and I felt like I was being made very comfortable in an ivory tower, in a vast field of other ivory towers, between which there was relatively little communication. And then there were all sorts of personal factors, just the way my life was going, people I knew, a sense of imminent global catastrophe...

This was 1995, so perhaps there was a touch of millenarian hysteria involved! There was a sense that, as a mathematician, I was part of the problem rather than part of the solution. A lot of my friends were involved in ecological activism and things like that, and I started to formulate a worldview wherein science had become the new, unacknowledged, religion of industrialised society, and mathematics was the inner priesthood of science. To put it in very simple terms, Western culture runs on science, and science runs on maths. So I saw myself as being trained up for this priesthood which was unconsciously steering the world to complete destruction and meaninglessness. And so there was a sense of guilt, almost, that I was involved in this. So I just broke out and floated around doing all sorts of interesting things for a few years, had a great time – I don't regret that at all. I never imagined that I'd get involved in mathematics again.

But then certain ideas about prime numbers started to percolate in my mind. I'd never really looked at number theory in any detail, had just a very basic number theory course as an undergraduate. But shortly after I 'dropped out', I started thinking about prime numbers and the fact that they have a sort of 'random' quality…and at the same time thinking about the Gaussian distribution, the bell curve, and the ubiquity of that, the fact that almost anything that you can name, count, measure, and gather data on tends to scatter along this particular ideal exponential curve. I remember posting a question on an Internet newsgroup back in 1995, trying to get somebody to explain to me why this thing shows up everywhere: not just in the biological realm, but in much more convoluted 'cultural' realms – I expect that you could count the number of appearances of a letter of the alphabet on the front page of a newspaper over so many years or months, and you'd find the same thing. And the purely mathematical explanations put forward made sense to some extent, but I still felt there was some huge mystery lurking behind the Gaussian distribution, the fact that it shows up everywhere. I scribbled all sorts of half-baked ideas down, some of which seem ridiculous now, some still of great interest. But I became convinced – and I still don't know where this came from – I became *utterly* convinced that the distribution of prime numbers in some sense was very deeply linked to this, to the ubiquity of the Gaussian distribution, that they were two sides of something.

And what's strange is that almost seven years later, I discovered there was something called the *Erdös-Kac theorem*, which was proved in 1940, and which I'd never even heard mentioned before. This was the beginning of *probabilistic number theory*, and it basically states that the distribution of prime factors of large integers follows a Gaussian distribution. Obviously, the larger the integer, the more prime factors it is likely to have, but you rescale in a way that takes that into account, so you're dealing purely with the seeming randomness in the fact that some numbers have got lots of prime factors and some numbers have only got one – and you end up with a bell curve. And not just an approximate one, this is what really struck me: if I was to measure the population over time of sparrows in the garden out there, or the way that those sunflower seeds fall on the ground [*pointing to bird-feeder hanging in a tree*], if I had large enough numbers I may well get very nice approximations of the bell curve. A high-resolution computer image might even match the ideal mathematical bell curve in every detail. But they're *always* approximate; in fact all use of statistical inference in science is based on finite amounts of data, which give rise to approximate bell-curves or other distributions. With the Erdös-Kac theorem on the other hand, the *n*, the number of elements in your data set, *actually* tends to infinity. This is what really struck me about all this: *n* can tend to infinity only when you have an infinite amount of whatever it is you're dealing with. And integers are the only thing, effectively, which we have – at least theoretically – access to an infinite amount of.

So, I haven't fully delved into this, but there's a problem with the use of infinity in statistics and probability theory. It's fine in some sort of abstract Platonic sense, but when you start applying it to the world, *there is no infinity*. But it does apply absolutely, precisely – and this is the theorem which Erdös and Kac proved – that as *n* tends to infinity, the distribution of prime factors tends to this distribution. So, in some sense, that's the only 'true' Gaussian distribution there really is, the 'oldest' one, the most primordial. As soon as you've got positive integers, that's hidden there within them. Any other instances of the Gaussian distribution, you know, bird populations or currency fluctuations or anything else like that, not only are these approximate, but they require all sorts of complicated categories and definitions. So, anyway, I still can't quite explain why I was so gripped by this idea of the prime numbers and the Gaussian distribution being linked, but I was, and it's as if I was somehow unconsciously aware of something and couldn't manage to pin it down, you know. I tried endlessly to find some way of relating these things and failed. Had this been 2005 rather than 1995 I probably would have quickly found out about the Erdös-Kac theorem using websearches.

So as a result of this unresolved compulsion, I had a certain amount of prime number-related activity going on in my mind. Then, in the winter of 1998 I went back to the States to visit my parents who were still out there, and I had a lot of free time. I found a long thin piece of cardboard and drew a number line, circled all the prime numbers, and then started drawing arcs between the prime numbers and their multiples. So every number was connected to all of its prime factors by arcs emanating out from that number to the left. The number fifteen would have two arcs emerging from it, one going to number three, the other to five. And a prime number would have no arcs going to the left, only arcs going to the right. Now obviously you can never draw the complete thing, but I drew enough of it that you could get a sense of there being something, a connectedness, a 'messy' connectedness, like a nervous system, or mycelium, or...I don't know, I can't quite describe it, but I just spent a long time looking at this, I had it up on the bedroom wall. And as a result of internalising that image, I started to think that it was perhaps the gaps *between the primes* that were most important…but I was somehow naïve enough to think that possibly no-one else had thought of that, whereas in fact quite a lot of work has been done on the gaps between the primes and yes, they are important. But I started thinking that maybe the gaps, suitably rescaled, are the things which distribute in a Gaussian way. I tried to run some computer models, to calculate the gaps and analyse their distribution – but not having access to the necessary computational power, that wasn't really going anywhere.

And then this image of the interconnectedness of the primes, the whole number system as a single connected entity, with each prime as a sort of 'nexus', the whole thing exploded in my mind – it was something very sudden, and the initial impression I got was that the primes themselves were imbued with a sort of 'charge'...I think I'd read somewhere that average gaps between consecutive primes are logarithmic, that is the average gap between a prime p and the next prime is log *p*, the natural logarithm of *p*. Obviously the gaps can vary wildly from this average, but the average is a precise mathematical result, becoming increasingly precise as we allow *p* to tend to infinity. I was suddenly gripped by this idea that the primes themselves were imbued with a kind of charge, something like an electrical charge, and that that log *p* was the clue, that was the charge of the prime *p*. At the time I was unaware of Julia's thermodynamic approach which associates with each prime *p* the energy log *p*, and also that certain proposed dynamical schemes involve 'orbits' with period log *p* associated with each prime *p*.

C: So the magnitude of the gap before the prime would be its charge?

MW: Well, for sufficiently large primes *p*, the gap before and the gap after would both be approximately log *p*. And I had the idea that these primes were in some sense repelling each other and that the bigger the prime, the greater the charge and the stronger the repulsion, hence the bigger the gap. This all came tumbling in as a single thought, really – the account I'm giving now is an attempt to reconstruct and coherently describe it. But rapidly following this initial impression was the idea was that, well, if there's that kind of repulsion involved then what I'm looking at is a *frozen image of something which was previously in motion* – this is what I got a very strong inner-visual sense of. I try to describe it to people like this: imagine attaching a wire to a wall and then stretching it away from the wall, effectively off to infinity, and then marking out with tiny white dots equal spaces representing the integers, and then imagine little tiny magnetic beads, mutually repulsive particles, positioned along the wire at positions 2,3,5,7,11, *etc.*, that is, at the positions of what we call the prime numbers. Now set up a camera, and then subject the whole area to a huge fluctuating magnetic field, causing the beads to move up and down the wire, driven not just by the field, but by their mutual repulsion. Film that, and then run the film backwards. What you'd see is all these particles moving around on the wire and repelling each other, responding to each other, and then eventually coming to rest at the positions we associate with the primes. That's the image.

Now I was well aware of the obvious question: how do we interpret the time parameter here? This is a huge problem – we're not talking about time in the familiar clock sense, not in the historical sense. I certainly wasn't under any illusion that anything like this had 'happened' at any point in the past. I was suggesting that the system had a 'past', but that it wasn't part of the *historical* past, rather of some other time-like dimension. And rather than thinking, that's ridiculous, I won't think that, I tried to suspend disbelief and see where it would take me. So the basic thought then was, okay, if what we're looking at is a frozen image of something which was previously in motion, the motion must have subsided for some reason – so what we're looking at must be something in a state of equilibrium. So, what kind of equilibrium? Well, I came up with a crude notion of 'arithmetic equilibrium': Why have the magnetic beads come to rest where they are? Well, if we freeze the motion at any moment, so you've got an infinite sequence of tiny beads whose positions don't necessarily correspond to positive integers – they could be any real numbers – and then generate all possible finite multiplicative combinations of those numbers, that would produce something analogous to the positive integers. The positive integers, recall, can be generated as the set of all finite multiplicative combinations of the primes. But these new 'integers' would not be anything like the familiar integers, they'd generally be all over the place. They wouldn't be nicely arranged, equally-spaced. But if the particles ever happened to reach the point where they collectively inhabited the positions associated with what we now call the primes, the 'integers' they'd generate would be equally spaced. So, I thought, it's *equal-spacedness* which is a key to this 'arithmetic equilibrium' which, according to my scheme, has been achieved in the number system.

C: Something like an entropic sequence, heading towards an attractor.

MW: Something like that, I was thinking in terms of all sorts of ideas I had partial understanding of – my understanding of physics is very piecemeal, it was even more so then. So many ideas were feeding in. I started to think, how would it begin? Maybe something like a big bang, where you've got all the particles squeezed together at the wall, at the end of the wire, but with something like an infinite magnetic field produced by the wall, and then you let go, and they all explode outwards. At any moment you could freeze the image and generate all the finite multiplicative combinations, the set of 'integers' that they generate: I called these 'generalised primes' and 'generalised integers'. Well, it turns out that Arne Beurling, a relatively obscure Norwegian mathematician, had come up with this idea of generalised primes and generalised integers many decades previously. To better understand the familiar primes he'd started looking at the question, suppose we 'change' the primes, what can we then say about the associated integers and their asymptotic distribution? Martin Huxley (Cardiff University), who's quite an eminent number theorist, got in touch with me as a result of my original website, to say, oh yes, there is actually a name for those, they're called 'Beurling generalized primes'.

C: The distinction being between the primes as we know them and, as it were, a generalised function of 'priming' by which a number system is generated.

MW: Yes, it's a bit like that, taking the idea of the primes not as indivisible integers, but as a set of generators. But the idea of them flowing or moving, no-one as far as I knew had ever put that idea forward. And so I came up with what I decided was almost a 'creation story', some sort of strange mythological mathematics – the creation story behind the number system. Whether there was this 'big bang' thing at the beginning or not, I wasn't sure...but the idea was that, okay, these generalised primes were somehow set in motion. Remember, there are these generalised prime particles, and then there's a kind of invisible set of generalised integers that they're embedded in, that they're generating, which are also in motion. And, at any moment, the 'heterogeneity' of these generalised integers, their lack of equal-spacedness, is creating some kind of 'tension' which is affecting the particles' charges. The idea of fixed log p charges gave way to the idea of fluctuating charges, governed by the spacing within the generalised integers at any given moment. So you can almost think of the distribution of these generalised integers trying to space itself out by 'influencing' the generalised primes and their charges so that their mutual repulsion eventually leads them to a stable configuration, an attractor point – that would be the arithmetic equilibrium. Having reached that – the familiar configuration of primes – the generalised integers would be nothing but the familiar positive integers 1,2,3,... The perfect equal-spacedness of these would result in all forces on the generalised primes dropping away, and the number system has then 'come into being'.

That was the 'story' I came up with, that all came a bit later, trying to make sense of this image that I originally had of the primes being charged, mutually repulsive, and in motion – or having *been in motion.* At the time, it had felt like, this is profoundly important and I have to act on it, I was being somehow compelled to act on it. It felt like the most important...certainly the *strangest* idea ever to enter my mind. And, insofar as I can grasp what is meant by 'numinous', it was charged with a numinous quality.

I was hoping to be able to actually describe the scheme in serious mathematical terms, to reveal that there was some mathematical integrity behind it, but that never happened...So all I had was this nebulous idea about an evolutionary dynamical system underlying the primes. And it was an idea which seemed very strange, I can't emphasise that enough – I couldn't really justify it using any sort of logical or mathematical reasoning, and yet it gripped me psychologically with such force that I couldn't let go of it, I was driven to try and make sense of it. And that led me to create a website...you know, this is what you do in 1998, you create a website, and then you start emailing various eminent mathematicians and physicists to try and get them to look at what you're doing. And as a result of that, a few people were quite helpful and responsive, I was sent some relevant literature, and I started to realise that actually, there are a lot of strange, unexplained connections between number theory and physics. These things seemed to me to be circumstantial evidence supporting my strange insight, whatever it might have been, or whatever value it might have had. They too suggested the number system had some mysterious 'quasi-physical' character. This may have been wishful thinking on my part, but the material was undeniably fascinating in its own right, so I started compiling it into a web-archive, intended to, at least indirectly, back up my idea. Eventually, though, my original idea began to become a bit of an embarrassment to me – it seemed quite nave and ill-informed. So, as the archiving took on a life of its own, and I became fascinated with all this serious maths and physics that I had become aware of, I gradually buried the original idea inside a vast web-archive. But I never entirely removed it, somehow still sensing, or hoping, that there was something of value there.

All my attempts to come up with a mathematical model, a dynamical system that would correspond to that image, had failed. I had struggled because I didn't have anything like the mathematical abilities that would be required for that. And in fact, I now feel vindicated in that it's not that I wasn't capable enough to do it; in order to describe anything like a flow in this space of Beurling prime configurations wherein what's called the classical prime configuration, the usual primes, constitutes some kind of dynamical equilibrium – in order to describe anything like that you need to do what Michel Lapidus has done, and introduce a noncommutative flow on a moduli space of fractal membranes. And there was no way in 1998-9 that I could have had access to those ideas. So – and again, this isn't a serious proposition, but the only way I can make sense of this for myself – it was as if I'd caught a little precognitive glimpse of some future mathematics, sensed the importance of it, tried to get it down, but didn't have the language to get it down, did the best I could, and put it out on the Web. This then led on to me putting a lot of time and effort into what was effectively public service web-archiving for a few years, which has been quite fulfilling, but it was initially just a consequence of the original 'flash', and the compulsion it induced in me. Now I'm feeling somewhat vindicated that someone appropriately qualified has shown that there does appear to be something like this underlying the number system.

C: Is there an analogy between what you're describing and what happened historically with non-Euclidean space – could it be seen as an arithmetical version of that, with the unknown time parameter as something as unanticipated as the curvature of space?

MW: Yeah, in the sense that you're breaking out of what is considered to be the only possible version of something, into a whole range of possible versions, and that initially seems 'mad' to many onlookers.

C: At the time, the idea that space could be folded or that space could be curved seemed insane. Nevertheless, such new generalisations are arguably the very movement of science itself.

MW: I think it was Gauss, Boylai and Lobachevsky who simultaneously came up with the same basic idea of parabolic geometry, and at least one of them was afraid to even mention it to anyone. If I had still been involved in serious mathematical research in 1998-9, if there had been a career at stake, my guess is that, having had the same experience, I may well have thought twice about going public with these ideas. Whereas as it was, it didn't really matter.

C: An interesting example of how being embedded in a discipline, having a reputation, and no doubt having funding depending on it, would actually stop you from saying something – there wouldn't be any channel through which to get it out.

MW: In a way, I was in a perfect position to just have a go, to push it out there.

I've read accounts of mathematicians trying to describe how they made certain great conceptual leaps. The big difference is that the leaps they made were into something that could actually be mathematically described, and ultimately, you know, were incorporated into legitimate mathematics. Whereas I just had a sort of mad flash, a glimpse of something which, as yet, is not legitimate mathematics, it's just a vague impression.

C: Yet the structural detail in which you described it makes it something more than simply a vague idea.

MW: Well I'm not sure that the *detail* of what I've described adds any validity. Had it not been for Lapidus' work coming along, I probably would have entirely disowned it by now. But at the time, there was a conviction that there was *something* in it, but it was hard to know what to call it. There was an awkwardness because, it falls between the usual categories...I suppose it could be called phenomenology or something, there's probably a legitimate-sounding name that someone could come up with. But when I put it out on the Web I was quite careful, because I was well aware of all sorts of cranks on the Web ranting about how they've discovered this or that revolutionary idea, or proved Einstein wrong, or whatever. And I so I tried to be very understated in how I presented it – you know, I've had this idea, and I don't know what it means, it may well be meaningless, but I invite people to either show me why it's meaningless, or else indicate what it might lead to. And gradually it began to happen. But I don't know if it really contributed to anything. I think Michel Lapidus would probably have reached the same conclusions regardless. Perhaps it did influence him, I don't know, but I don't think so. So in a way, if I did catch a glimpse of some sort of future mathematical discovery, it would have occurred anyway, so what's the value of what I did?

C: At least, it does lead one to think about mathematics not in terms of the points at which people draw everything together, make it into a formal system, but rather these discontinuous moments when, inexplicably, things move, things split apart and something new opens up?

MW: A crack opens up and something doesn't quite make sense.

C: From what you know of the mathematical community, is it the case that the sort of research you are pursuing is not accepted, that they're not interested in it?

MW: There's a small enclave of perhaps slightly more open-minded, more unusual mathematicians, who are prepared to discuss these sorts of things privately. The vast majority are slightly bemused or just not interested, they're too busy with their own work to stop and think about what it all might mean. Mathematicians aren't generally encouraged to think about 'meaning'. They don't really need to, they've got a very exact discipline, they've got theorems to prove and things like that. Basically, what I'm doing, I couldn't call it mathematical research. You've called it fundamental research...you could call it that, I know what you mean. The way I see it I'm just trying to raise certain questions and generate discussion, and I'd say the vast majority of the mathematical community just isn't going to engage with that, which is okay. Because I'm not actually doing mathematics, I'm not engaged in mathematics research in the way they are; I'm playing a different game, asking questions about what mathematics means, what it is, how we relate to it. But at the same time I'm not part of the philosophy-of-mathematics community either, which is involved in something much more rigorous and disciplined than what I'm doing.

I suppose because I've got more time I'm in a better position to just stop and think: what's the point, why are we looking at this stuff anyway, what does it mean? Professional mathematicians these days tend to be extremely busy, they've got to theorems to prove, papers to publish, conferences to attend. They need to keep their careers afloat, and so they've got a lot less time to think about what this stuff might *mean*.

But the thing about the Web – and this is quite an important factor in what I'm doing – is that it's possible for me to say what I think and to discuss it with large numbers of other people in the academic world, without having any formal academic status and without having to get anything published. And I can change it as I go along – there's no final document, that's the other thing. I don't publish articles, I can just put together vague rambling webpages and then keep changing them as my ideas change.

C: This is a striking aspect of your research – the presentation of it is very open: no need to hold back until you've got an completely solid hypothesis and then put it online tentatively as a preprint. The site is continually updated, and you're creating this network which connects together all these scientists who it seems are working on related problems but don't always know of each other: in some cases you're actually notifying them of each other's work.

MW: I've spent a lot of time emailing relevant researchers and alerting them to the existence of new articles or preprints which they may well be interested in. And it's difficult to quantify, but I do seem to have stimulated a certain amount of interdisciplinary work. I've created a rôle for myself which hasn't really got a name yet, and as far as I know, no-one's prepared to fund me, but I'm doing my bit to weave together these threads of research.

Part of what caused my disillusionment with mathematics, which caused me to drop out in the first place, was...well, the overriding impression was the biblical image of the Tower of Babel. It occurred to me that if you were to put the names of all professional research mathematicians in the world into a hat and pick out two, the chance of there being any real overlap in their research interests would be quite small, and this continues to get smaller. It was as if mathematical research was getting so fragmented that there was no longer any effective communication possible. So in a way, I suppose what's needed, if one wants to try and fix this, is people who are not specialising, but rather trying to get an overall picture and to weave it all together by creating lines of communication. I didn't come into this with that intention, but that seems to have been the rôle I've created for myself. I haven't got any answers at all. I just feel that there are questions that are important and which aren't being asked – possibly because there just isn't the language in which to ask them coherently yet. But at the same time, because there are no real constraints on me, I don't have to prove myself to anyone, publish anything, or stay within any particular boundaries, I can just throw out certain ideas, get people thinking about things, suggest connections between things in such a way as to indicate the existence of something which we can't yet pin down perhaps, but which will come into focus the more we look at it.

In the mathematical community, at least the proportionally small number of people I've communicated with, I do get a sense that there's a sense of wonder there which is something unquantifiable, something that you couldn't prove a theorem about, but which is nonetheless there. It's something to do with these individuals' emotional, psychological or even spiritual orientations, I suppose. But a lot of mathematicians, I'm afraid, do tend towards the familiar stereotype of socially inept, almost mildly autistic people who have very little time for the unquantifiable aspects of life. And so there is an almost scathing disregard from some quarters. I think – I feel – that anything that's vague or a little bit ephemeral, they see that as worse than useless, perhaps because their own self-esteem and status is tied up in a self-image of being the guardians of some sort of absolute inarguable exactitude and truth.

C: Your guiding thread is a fascination with how mathematics relates to reality, rather than with mathematics *per se*.

This seems to be related to the fundamental problematic which appears right at the very origin, you could say the co-origin, of mathematics, philosophy and natural science: with the Pythagoreans, who realised that operations carried out on numbers applied – rigorously, but for them somewhat magically – to natural phenomena, and so put forward the idea that reality was actually nothing but numbers, reality was structured by number. In a sense they put forward a type of mathematical empiricism, *i.e.* the idea that you could go out and explore the world, and what you would expect to find was relationships between numbers, and you could understand the natural world like that. Now this came to a catastrophic end with the discovery of irrational numbers...

MW: Yeah, the legendary drowning at sea of Hippasus of Metapontum – it's fascinating stuff, a pivotal event in human history...

C: Certain aspects of the natural world were shown to exceed number – or number as it was conceived then. Certain quantities which can be mathematically described (the diagonal of a square with side length 1, the area of a circle with radius 1, the golden ratio) cannot be expressed as ratios of integers, they are 'alogos' or, as we now say, irrational.

After a long period under the influence of Aristotle's instrumentalism, for which every sublunary physical phenomena was subject to an inevitable degradation, meaning that exact mathematics was applicable only to astronomy, the celestial and sublunary worlds were (blasphemously) reunified, most of all by Kepler, under a single mathematical physics, reinvigorating the Pythagorean dream of a mathematical natural science.

Then in the nineteenth-century mathematics seemed to exceed its reference to the real world, to claim its own autonomous consistency, and any necessary link with the natural sciences was removed, mathematics asserted its independence from any application; its applicability to the physical world even seemed to become a sort of mathematical ghetto.

Now, in the work you're looking at, it seems that we return once again to a Pythagoreanism but with a strange twist...

MW: Yes, something's been turned on its head. I've been fascinated by Pythagoras and the Pythagoreans for a long time. Sometimes I think, you know, in a way I'm acting a bit like a 'neo-Pythagorean'…but as you say, there's a strange twist there. I think a lot of people forget, when Pythagoras is discussed as 'the first mathematician', that he had one foot in mathematics and another one in a sort of shamanic, mystical-type reality.

C: Whereas the Pythagoreans discovered in numbers the semi-divine property of rigorously elucidating nature, we have this experimentally and theoretically-vindicated body of method and knowledge taken from natural science, with whose aid we're trying to illuminate what now seems like a somewhat opaque and mysterious numerical realm; and there are these things within number which still don't really make sense. Mathematicians such as Chaitin [*see article in the current volume–ed.*] have said that mathematics must now become a quasi-empirical practice – this is in relation to his own work, but it might perhaps equally be applied here.

MW: Some of the quotes I have on the site agree: Martin Gardner said something about how some problems of number theory might be undecidable and might need a sort of mathematical 'Uncertainty Principle'. Timothy Gowers wrote that the primes somehow *feel like experimental data*, but at the same time he's well aware that they are rigidly determined.

We find ourselves in a situation where Michael Berry, studying spectra of quantum mechanical systems, can take techniques he's developed to classify or better understand certain types of physical systems and apply them to the Riemann zeros, in order to produce a hypothesis that we will get a particular 'number variance' in the far reaches of the spectrum of Riemann zeros – then years later, you know, computer power reaches the point where zeros can be calculated at that scale, the 'number variance' computed...and the graphs match up perfectly. It's the first time I'm aware of when a physicist was able to tell pure mathematicians something new based entirely on his familiarity with physical systems.

C: Does the field then become *de facto* an experimental one? You have the a hypothetical physical system which will produce the system of vibrations which the Riemann zeros seem to correspond to. And the only way to find out whether there's really any system which is adequate to that would be by experimentation – in the same way that the Higgs Boson hypothesised to glue together the results of quantum physics must now be sought experimentally – hence the construction of CERN's much-anticipated Large Hadron Collider. Does someone have to *build* the Riemann dynamical system?

MW: Michael Berry has said he's absolutely convinced that, if such a thing is physically possible, someone will make one of these things in a lab, and then the Riemann zeros will actually come out on the instrument readings. But at the moment there's no-one actually conducting any experiments which are getting anywhere near that, or even attempting to. You do have physicists taking certain ideas – largely mathematical models intended for physical systems – and applying them to aspects of the zeta function. There is an experimental branch of study of course, you've got people looking at the Riemann zeros themselves, which contain a wealth of data – we've got, I believe, hundreds of billions of them calculated now – this is being done with grid computing[8]. The gaps between them and all kinds of other things you can measure when you've got a set of seemingly random real numbers, are being analysed using a variety of statistical methods, random matrix theory is being applied. So these are, to some extent, experimental studies. Marek Wolf (Institute of Theoretical Physics, Wroclaw) experimentally detected a widespread physical phenomenon called '1/*f* noise' in the distribution of prime numbers.

The prime numbers continue out to infinity, we've known they go on forever since Euclid, but we can only calculate them up to a point. We tend to think our current computers are 'powerful' , and we think we can find 'big' prime numbers – you know, now and again one will even make it into the news. But *there's no such thing as a 'big' number*, this is what I always try to get across to laypeople – because the number system goes on forever, however far we look, proportionally it's still an infinitesimal step into an infinite unknown .

C: And, of course, in consequence, no matter how many zeros are found, one never comes any closer to a *proof* of RH.

MW: Yes, exactly. There's the duality between Riemann zeros and primes, and so the same idea applies with the zeros. We can never calculate more than an infinitesimal proportion of them. Sometimes I use the analogy of large telescopes: you're looking out into space, and the more you can see, the more you can deduce about the nature of the universe you live in. Analogously, we can 'see into' the number line a certain distance, what we think is a 'long way' – but again, it's meaningless, really, to say a 'long way' or a 'big number'. Of course we can see further than we've ever seen before, so we can detect certain apparent patterns which can give rise to hypotheses that we can then attempt to prove. Similarly we can look further than ever up the critical line now, and with hundreds of billions of Riemann zeros we can test certain hypotheses and generate new ones. So there's an experimental element in that. But as far as the hypothesised Riemann dynamics goes, the quest to try and pin down something like a Riemann dynamics isn't really being furthered by experimental science as such, rather the progress seems to be coming from mathematicians like Connes, Lapidus and Christopher Deninger (University of Münster). But these people – well, certainly Connes and Lapidus – do have a very broad interest in large areas of both mathematics and physics, which is what makes their work so interesting.

It would be misleading to suggest that mathematics has become an empirical science, since exact formulations are still possible – even in these more hazy areas – at least we can't rule out the possibility of exact formulations. But an empirical approach has become potentially useful. In connection with this, I should mention the emergence of probabilistic number theory, which in itself raises huge questions. Probabilistic number theory effectively started in 1940 with the Erdös-Kac theorem which I mentioned earlier, the discovery that the number of prime factors in 'large' integers has a kind of random distribution which follows the Gaussian distribution or bell curve. That discovery led to a whole outpouring of theorems and conjectures which have collectively become known as probabilistic number theory, where you apply the methods of probability theory, and make use of the key idea that divisibility by a prime *p* and divisibility by a different prime *q* are 'statistically independent events', one has absolutely no bearing on the other. When you deal with probability you deal with this idea of independent events – *well, these are arguably the most independent 'events' there can ever be*. Physical events in any well-prepared experiment, you might think they are independent; but ultimately every particle of the universe is gravitationally pulling on every other particle, everything is linked, although the effects are generally negligible and impossible to quantify. The only place where things are *totally* independent is in the number system – the divisibility of an integer by two different prime numbers. So here is a place where you can apply probability theory, where everything is *entirely* exact, where you can let your *n* tend to infinity and that actually refers to something. Probabilistic number theory allows you to *prove* things about prime numbers and about the number system generally, using the techniques of probability theory, and that seems highly counterintuitive. The fact that it works at all raises questions which are more like 'mysteries' than formal mathematical problems.

There are three separate areas worth mentioning here: the emergence of probabilistic number theory, the effectiveness of the analogy with statistical mechanics – partition functions, *etc.* which I described earlier – and then the rôle of random matrix theory, which was developed for modelling subatomic phenomena, but then was accidentally found in the 70's to apply directly to the theory of the Riemann zeros. So you've got three separate areas of randomness-based thinking, stochastic disciplines if you like. They deal with large systems which have too many components to keep track of individually – these components must be treated almost sociologically, as populations, and subjected to probabilistic or statistical thinking. All three areas have been effective in furthering our understanding of the number system. Now, again, mathematicians would tend to focus on at most one of these things, see what could be achieved and perhaps make a few sober remarks on what it all might mean. But to me, the fact that you've got these three areas, all of a stochastic nature, shedding light on the primes and the Riemann zeros, points to something very strange. We've got primes, the most basic things in the universe as we experience it – the sequence of prime numbers is the most basic non-trivial information there is, it's the one thing you can't argue with anyone about, it's the one thing all lifeforms in the universe could potentially relate to. And yet in some ways they seem to be best understood using a type of analysis more appropriate to weather systems, roulette wheels, boxes of gas, *etc.*

I've always thought of probability theory as a slightly 'tainted' branch of mathematics for three reasons: Firstly, it's origins are not entirely honourable – I seem to recall that it has its roots in an historical accumulation of gambling techniques which got distilled into a formal theory by Pascal. Secondly, it deals with 'events', repeatable 'events', which are categories of physical phenomena, 'occurrences' of one type or another which can be quantified, measured, counted, numerically analysed, *etc.* whereas truly 'pure' mathematics doesn't rely on anything in physical reality in quite this way. Finally, by its very nature, probability theory tends to deliver imprecise information – there's always a margin of error. And yet this system of thought, which has been developed in order to deal in an approximate way with large, complicated physical systems, seems so perfectly applicable to something which is so fundamental, which is characterised by an absolute precision, and which underlies everything else – the distribution of primes! It's as if we've got something back-to-front. It's similarly interesting that probability should have such a fundamental rôle in quantum mechanics: an ultra-simplified account of what QM tells us is that, insofar as it can be understood as being made of particles, the universe can also be understood as being made of 'fields of probability' . Probability theory in a casino, yes; or in a meterology lab... But prime numbers? The fundamental level of matter? These are things we instinctively feel should be totally deterministic and rigid. And to me, this suggests we're looking at something the wrong way 'round – something's been turned on its head. It's as if 'randomness', or some essential, almost esoteric quality associated with randomness – that quality evidenced in our failure to really understand what we mean by 'randomness' – is emanating up from these fundamental realms. We've been dragging it down from the macroscopic scale, the casino scale, down to this micro-level, in a numerical and physical sense, and finding that it helps us understand something. But I feel something's back-to-front there.

A mathematician called R.C. Vaughan states in one of my archived quotations that it's obvious that the prime numbers are random, but we don't know what randomness is. And there is a real problem with defining randomness. There are several definitions, information theorists, probability theorists, have put forward definitions of what it means for something to be random. The definitions overlap to a large extent, but ultimately, when is a string of digits random? If I give you a block of a thousand 0's and 1's, it might *look* completely random, it might even pass numerous tests run on it for randomness…but then I could reveal, well actually, no, it's a thousand digits of π starting from the two-millionth digit. And then it's not random anymore. So there's the whole question of what randomness is. This is one of the central themes that fascinates me: where does this notion come from, where does it lead us in our understanding of the reality we inhabit, and why does it tie in so closely with both the fundamentals of the number system and of particle physics?

And then there's the difficulty of talking about having two of anything, that in order to have two of anything you have to have a category which those two objects both belong to. But the categories are always imprecise. We have to partition spacetime into blocks with 'fuzzy' boundaries, and then attempt to match aspects of these blocks up with some ideal which exists in a sort of mental hyperspace, a Platonic realm of sorts. So we're projecting these categories onto the universe which actually aren't intrinsic in the universe; we're setting out these boundaries, but the boundaries are blurry. Yet, despite the possible problems this blurriness might cause, on a practical level we're able to then extract data which fits remarkably well against certain probability distributions. The most ubiquitous and I think the most important one is the Gaussian or bell curve – and this, as we can see from the Erdös-Kac theorem, has a mysterious and fundamental relationship with the number system we're using to count members of our fuzzy-boundaried categories in the first place.

The effectiveness of statistical inference in the hard sciences and the social sciences – I'm sure this would be widely disputed, but I feel there is a mystery there which isn't really being acknowledged, and it has to do with how we can name and count anything, and how, when we do name, count and measure things they seem to collectively accord with these ideal mathematical blueprints or templates. That says more about the way our mental hyperspace is being mapped onto the physical universe than about anything *intrinsic* in the physical universe.

C: When you look at the local you expect to find precision, whereas with the global you're happy with statistical data. Here we're looking at these local, precise conditions and there seems to be randomness 'built into' them in a way that's not immediately comprehensible: After all, they're not statistical aggregates in any obvious sense.

MW: Yes, the set of positive integers is in a category of its own, *there's just one number system.* Yet, it's as if this entity – if we take the positive integers, the primes and the zeta function as aspects of a single thing, different aspects of the same entity – rather than being a carved-in-stone, unique thing, is actually just one example of a class of things, and we're able to apply statistical analysis because of that. This is why, when I started finding out about these things, I felt my 'prime evolution' thing might have something in it, this idea of the number system being a frozen state of something which had previously inhabited many different states. I've had certain quite critical, serious-minded people react to some of my more sensational suggestions by saying, well all this number theory and physics, there's nothing mysterious at all – the universe follows mathematical laws, so of course we'd expect certain aspects of number theory to show up in the physical world. If they'd look a bit deeper into this, they'd see what I meant: yes, it's not surprising, given that maths underlies all of physics, that we might get, say, particular values of the zeta function showing up in string theory, or the theory of integer partitions relating to Bose-Einstein condensates or whatever: you get these odd little instances of number theory/physics correspondence; I've catalogued a lot of these in my web-archive. But that's not the really interesting stuff. What's much more surprising is the way physics seems to be pointing the way for understanding the zeta function, and often this is statistical or stochastic physics, as if the zeta function – and in some sense, then, the number system – is just one example of a more general phenomenon. And I don't think anyone disputes the spectral nature of the Riemann zeros now. But it's not one archetypal ubiquitous spectrum we see showing up all over physics. If we saw 'the zeta spectrum' – as it might be called – everywhere, then it would somehow feel a lot less mysterious. We'd probably feel quite comfortable with such an affirmation of the old idea that the number system directly underlies the structure of the physical universe. But the Riemann zeros take the form of an almost disconcertingly arbitrary-looking spectrum, never known of by humans prior to the late 1850's. In the very recent past we've been confronted with the fact that it has all the fingerprints of membership in certain classes, very wide classes, of very specific physical systems, as if it's just one element of a whole class, a population of things. So it's a bit like the way you might be able to, based on the postcode of a UK resident, predict certain things about his or her attitudes, abilities, tastes, whatever – because you've got statistical information about the population, you can make plausible hypotheses about this specific individual.

And it's as if the primes-zeta entity, whatever you want to call it, despite its seemingly fundamental, unique status, is just one individual in a wider class of things. But the space in which that class exists is something we haven't even begun to imagine might exist, or we haven't got any access to.

So we have this image of a frozen system, something congealing into a state, and then…it's as if you walked into a concert hall and caught the last note of a symphony, and everybody's applauding ecstatically and you're wondering, what's all the fuss about? You didn't witness the process that led up to that last note, and it's like, with the prime numbers, we're just walking in on the last moment, the culmination of something. As if there was a whole 'symphony' that led up to that, and humanity may be on the verge of revealing it.

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C: All of the foregoing seems to suggest that what we think of as simple and elegant foundations may in fact be the eventual product of something which is rather complex, even beyond our comprehension. So we'd have to separate out what seems simple and elegant to us, from what is actually fundamental in the universe, and this is another sense in which mathematics mirrors the condition of theoretical physics, in which, characteristically, the further we go towards the fundamental, the stranger things become (string theory being a case in point).

Rather than defining the primes on the basis of the supposedly fundamental and simple number line, in fact it seems that, when we look through this complex theoretical-mathematical prism you have described, there's actually something more fundamental about the primes. The primes themselves produce...

MW: ...the number line, yes, you can see it that way. I came up with this naïve idea, before I really learned any of the more serious stuff, this was after I had been thinking about the Erdös-Kac theorem, the primes and the Gaussian distribution, but before I 'experienced' the dynamical aspect of the primes. I was thinking about how we tend to construct the primes. We're taught to construct the number line starting with one and then using the Peano axioms, you know, there's an axiom that basically says, whatever number you arrive at you can always add another one to it. And I thought, hold on, where does this come from, this idea that you can always add another one, and I started to question that as something that might not be as obvious as it first seems. There's some hidden assumption there about order, time or something, I felt.

And I thought, well, there's an alternate approach we could adopt here, we could start with an infinite alphabet of meaningless symbols, an infinite alphabet of meaningless yet distinct symbols, and then create the dictionary of all possible words of finite length out of that alphabet.

This alphabet of symbols would correspond to the prime numbers. By combining the symbols in all finite possible combinations, you generate the set of words in your infinitely-long dictionary – this corresponds to the fact that if you combine the primes in all finite multiplicative combinations, you get the set of positive integers. Except now *there's no sense of order*: Because we're not starting with the positive integers, we don't need to think of one prime number as being 'greater than' another. The primes are not embedded in the positive integers yet, they're just these free-floating abstract symbols. So I used to try and conjure up this image of bubbles floating in an imaginary space, each with an exotic glyph, a symbol from our 'alphabet of primes' on it. The idea is that you can then join any number of these bubbles in any combination, including repeats. All possible such bubble-clusters are to be found floating somewhere in this space. Some are larger than others in the sense that there are more bubbles in the cluster – that is, more prime factors – but there's no sense of a cluster coming 'before' or 'after' another cluster. It's only when you cross the Rubicon of deciding which alphabetic symbol is going to be your '2' that you start to create some sense of order.

So I had these hints and intuitions – I couldn't really pin them down to anything very rigorous – that we've been thinking about randomness and the fundamentals of reality in a back-to-front way. We've got ourselves into a kind of confusion where everything seems immensely complicated when we delve down to the fundamentals of either the number system – which seems at least partly to inhabit the realm of psyche – or of the physical world, the world of matter – just open a textbook on analytic number theory or quantum mechanics and you'll see what I mean. I felt this issue could be addressed if we examined some of our 'obvious' assumptions. We think we've taken the obvious construction – that is, you start with one, then you add one, and then you add another one, this idea you can always add another one. Rather, what if we start with the primes, and build the number system up that way? The whole 'order' thing then becomes more of a 'phenomenon' than something axiomatic...

C: Coincidentally, the 'legendary' Dr. Daniel Barker also devised a notation system for the positive integers based upon prime factorisation, which is very close to what you're talking about here.[7] You have these inseparable lexicographical units from which numbers are composed, and they could be in any order. He was interested in place value as a culturally-repressive numerical practice, and this was a way of doing away with place value completely. Each number would just be like a collection of boulders or something.

MW: The lexicographical approach, yes. I've tried to get this across to some lay-people I've talked to. There's the

7. *See* http://abstractdynamics.org/005047.html

COLLAPSE VOLUME I Erratum:
p162 Note 7 should read http://hyperstition.abstractdynamics.org/archives/005047.html

fundamental theorem of arithmetic – literally *the most important thing we know about the number system.* And no more than 0.1% the population have even heard of it, I'd guess. It basically says that every integer breaks down uniquely into prime factors. And we've got this strange situation where almost nobody knows this, this simple fact, the most important thing we know about the number system. This is straying into other territory, but to me, humanity's relationship with number is rather unhealthy, because we've built this entire civilisation around the mathematical sciences, and yet the ordinary population knows nothing of the basics, and often finds mathematics a source of fear and unpleasantness. I try and conjure up this image of these bubbles, the fact that the clusters can be as large as you want, you can have huge 'planets' of prime factor bubbles joined together – *there's no upper size limit.* And so something like the greatest common divisor can then be explained very simply, it's just the intersection, literally where the two clusters intersect. The least common multiple can be similarly explained. Prime numbers distinguish themselves from non-prime integers because they are individual bubbles. The integer 1 is *the absence of any bubble*, the empty background space, the blank page in the "dictionary" I mentioned earlier…

And then you imagine stringing the entire set of clusters out in a line according to this 'order' thing, and you start to see that there's a counterintuitive variation in the sequence – you get small clusters, huge clusters and single bubbles all intermingling according to no sensible scheme. And this is the sort of thing that I'd eventually like to push further out into the public domain just to see what sort of effect it would have, when people start looking at their supposedly familiar number system in this new light. Because people tend to think of the number system like a row of boxes of cereal in a supermarket, just identical units stacked together, a sort of homogeneous featureless thing that just goes on: each number is just the previous one plus one, there's nothing much there, nothing of interest. And it was Frank Sommen, a really remarkable, imaginative Flemish mathematician who I worked with during my PhD studies, who once said to me, every positive integer is a different animal. I came to see exactly what he meant: each one's got its own 'anatomy', every one's a different story, and that starts to become apparent as soon as you realise that each integer factors in a unique way into prime numbers.

C: This is a basic intuition that one finds in 'primitive' numerological systems.

MW: Yes, and in children as well, with their favourite numbers, and feelings about each of the first few positive integers – ethnomathematics and children.

C: Something that gets beaten out of people by mathematics: when people start learning mathematics, it's as if the first task is to extirpate any idea that numbers have quality. Mathematics is in fact often seen as constitutively opposed to any such intuition.

MW: Yes. Marie-Louise von Franz, one of my favourite writers, who studied under Jung and wrote a lot about number archetypes, she talked about number having both quantitative and qualitative aspects. The quantitative is obvious, we all use numbers to count. Cultures who revere certain numbers and have mystical beliefs about them which we might laugh at, they still use them to count with and to trade, they recognise that they have a quantitative aspect. This is the aspect of number that has given rise to economics and technology; but equally, perhaps even more importantly, there's the qualitative aspect that only survives in our culture in children having favourite numbers, some adults having lucky numbers, not wanting to sleep on the thirteenth floor of an hotel, the way they might choose lottery numbers, that sort of thing. But, you know, in 'serious' society numbers are supposed to be *entirely* quantitative. Von Franz wrote about a traditional Chinese story involving eleven generals who, faced with some very difficult military situation, took a vote as to whether they should attack or retreat. Three voted to attack and eight voted to retreat. So what did they do? They attacked, because three was a more favourable number – it wasn't a *bigger* number, but it was a number associated with unanimity, or some other favorable quality like that. And the attack was a success.

So it's interesting that they could build a civilisation that was able to have a functioning economy and military and to govern millions of people – clearly they were intensely aware that number had a quantitative aspect – but there was also a serious engagement with the *qualitative* aspect which is dismissed in our present culture as entirely superstitious. Now I'm not encouraging people to engage in completely arbitrary numerology, I mean, I've looked at a lot of that new age numerology literature, and the problem is, nothing can be verified: someone can write a book saying a particular number means something, and someone else can write another one saying it means the complete opposite. It just confuses matters, as there's never any consensus or certainty in these interpretations. That's why professional mathematicians would almost unanimously just react against it and say it's all rubbish.

C: But is there any way to talk about it which doesn't get into that morass of mysticism?

MW: There are two approaches: one is the serious attempt by Jung and his followers to catalogue all of the ethnomathematical systems, undertaking a serious study and survey of various cultures and their relationship with number, trying to find common threads, and through psychoanalytical work and dream studies, trying to find extract essential patterns to build up a body of material from which we could possibly deduce something about how number interfaces with the psyche at a fundamental level. The other approach is to seriously study number theory, because as far as I'm concerned, that *is* numerology, really – you're looking at the properties of integers, and if you study it to a certain depth it takes you into the realms of what you could only call the mystical or the uncanny, where cracks seem to open up in your normal understanding of reality.

C: Is that perhaps what characterises number theory as opposed to mathematics, what makes it a very different discipline?

MW: Well, number theory is universally acknowledged as a branch of mathematics. It can't really be separated from it like that. But it arguably has a unique status at the very heart of mathematics. You're working at the very root of it all, dealing with the simplest objects, the positive integers. And yet you come across these counterintuitively complicated structures and results. You can separate mathematics into branches and disciplines but they all ultimately overlap and interrelate. Gauss (who himself was called the 'prince of mathematicians') called mathematics 'the queen of the sciences', and number theory 'the queen of mathematics'. The idea is that number theory is generally seen as the pinnacle, in that it contains the most difficult problems; also it's concerned with the integers, and all of the rest of mathematics ultimately relies on integers. Hence it's not surprising that problems of number theory do seep into other areas of mathematics, and even physics. What *is* surprising is that physics is beginning to shed light on number theoretical structures like the zeta function, as if it were just one of a class of objects, whereas it's meant to be this fundamental object underlying everything.

What I'm trying to describe with my clusters of bubbles isn't intended as any sort of serious mathematical proposition, it's just a picturesque visualisation – trying to look at the number system from another angle, if you like. But there's a hidden assumption within the Peano axioms, I think, which needs to be addressed – although I don't think I'm the one to address it. It concerns the axiom which allows you to always add one. Even in the proof of the infinitude of primes, I sense some sort of subtle circularity there – the idea is that, if the number of primes were finite, you could multiply them all together and then add one. And that rapidly leads to a contradiction concerning primeness and divisibility…hence there must be infinitely many primes. So that takes you back to the Peano axioms, the idea that you can always add one. But in my visualisation, multiplying them all together would correspond to building one mighty cluster using one of each type of bubble. And in that visualisation 'adding 1' is a far less obvious operation. This ties in with problems of time, the idea of time, repetition, even basic physical questions: you know, this 'adding 1' business presupposes that you've got a physical space, something like the space we're familiar with, in which you can make a sequence of marks, or a time continuum in which you can make a sequence of utterances or beats. And I feel there may be subtle assumptions concerning the homogeneity of time and space involved in this, too.

C: These questions of time and space must fall out from the primes' intimate connection to the relationship between multiplication and addition.

MW: Brian Conrey, who's President of the American Institute of Mathematics, and Alain Connes have both been quoted as saying that RH is ultimately concerned with the basic intertwining of addition and multiplication. And if we haven't really got a clue how to prove RH – which we don't – we're going to have to own up, we don't even understand how addition and multiplication interrelate. A more succinct, precise way of describing these two possible constructions of the primes that I have outlined – the conventional 'just add 1' approach, and my 'lexicographical' approach with its equivalent clusters-of-bubbles visualisation – is given by Grald Tenenbaum, who certainly knows what he's talking about:

> Addition and multiplication equip the set of positive natural numbers with the double structure of an Abelian semigroup. The first [addition] is associated

> with a total order relation as it is generated by the single number one.

So if you've got addition and you've got this single number 1, you can generate the postive integers just by adding 1 plus 1, 1 plus 1 plus 1, etc. If you take 1 as your 'additive generator', the universe generated is the set of positive integers.

> The second [multiplication], reflecting the partial order of divisibility,

This probably isn't the time to get into the subtle issues of 'order' in mathematics – you've got 'total order' and 'partial order': addition relates to total order, where something definitively comes before or after something else; and divisibility relates to partial order, a less distinctive type of order, although I won't get into the details of that…

> [Multiplication], reflecting the partial order of divisibility, has an infinite number of generators, the prime numbers.

So, now, rather than starting with just the number 1 and combining it with itself in every possible way using addition, we start with this infinite set of primes and then take all possible *multiplicative* combinations.

> Defined since antiquity, this key concept has yet to deliver up all of its secrets, and there are plenty of them.[8]

8. Tenenbaum and France, *op. cit.*

It has the quality of a square peg in a round hole, this tension between addition and multiplication. It's almost like, despite the inarguable perfection of the number system, they don't really fit together very well, and they generate what I feel is something like friction, and this produces the sprawling mass of definitions, theorems, lemmas and conjectures that we call analytic number theory. There's a novel by Apostolos Doxiadis called *Uncle Petros and Goldbach's Conjecture* – it's written as fiction, but he gets some key ideas across through an elderly mathematician character. This is very well put, I feel:

> Multiplication is unnatural in the same sense that addition is natural. It's a contrived second order concept, no more really than a series of additions of equal elements.[9]

So that's the point, that 3x5, you can see that as 0+3+3+3+3+3 – you start with nothing, zero, and add three five times. So in a sense you can build multiplication out of addition, whereas it doesn't work the other way around. So addition is a first order operation, and multiplication is, as he's saying, unnatural, in that it's 'second order'. The thing that struck me about it when I was dwelling on this for a while was that it has to do with *time*, it has to do with repetition. And it also relates to the very deep issues concerning the whole idea of where number comes from and how we define number. As I hinted earlier, I've spent a lot of time thinking about how you could

9. Doxiadis, A. *Uncle Petros and Goldbach's Conjecture*. NY: Bloomsbury, 2000.

ever have two of anything. You know, there are two people sitting here in this room right now, but that relies on the definition of what a 'person' is. We define the category linguistically, and we think we know what a 'person' is, but you can imagine some sort of genetically-engineered mutant that may or may not be a 'person' depending on how the definition was formulated, and the definition's made of words and each word is imprecise, is subject to interpretation. So any type of category you define is going to have a 'fuzzy' boundary, so...although it works quite well for day-to-day affairs, counting things works fairly well, you've got fifteen sheep in your paddock. But you can always contrive some convoluted situation where, maybe it's fourteen sheep or maybe it's fifteen – is that odd looking creature really a 'sheep' or is it something else?

So, it comes down to issues of language and definition. We consider chunks of spacetime, we recognise patterns and say, yes, that chunk of spacetime falls into such-and-such a category. As I said, I started to wonder how you can really have two of anything. Every entity ultimately distinguishes itself from every other, these categories are not mathematically precise, there's an arbitrary element involved in deciding whether things get included – "where do you draw the line?" as they say. And yet these categories are the essence of counting, and if there's a problem with applying the concept '2' to our experience then there's going to be a problem with all of the other positive integers.

Exceptionally, when you get down to the subatomic level you can have two of something, because each individual electron is absolutely indistinguishable from the others. So that's interesting, that this concept makes sense at the subatomic level but then 'fuzzes out' at macroscopic scales.

But the thing is, when you say '3x7', you're effectively saying 'three sevens'. So, seven pebbles in a row – you count out seven by adding one plus one plus one, etc. That feels quite 'natural'. But then, to make the leap to 'three lots of seven'…you can have three giraffes or three potatos, the fuzzy boundaries mean that's a difficult enough issue as it is, but 'three sevens' presupposes that a 'seven' is *something that there can be more than one of* in some sense...

C: One would have to say that the multiplier and the multiplicand are somehow of a different order, two different types of numbers are involved in the operation.

MW: Yes, one is operating on the other. If you add, it doesn't matter...I mean, it's true to say that 3x7 is the same as 7x3, you've got this basic 'commutative' property applying to the positive integers. But when you consider the 'act' of 3x7, the three is how many times you're doing something, whether it's laying out a row of seven beans or playing seven drumbeats, and the seven is some kind of an extension in space or time. Whereas in adding

3+7 or 7+3, both numbers play the same rôle. So there's something there, not easy to pin down, which we don't understand, and I have a very deep sense that we won't really understand it until we really understand *time*. It has something to do with time. Our inability to understand the primes, our inability to prove RH is a symptom of our inability to understand the relationship between addition and multiplication, and that is related to our relationship with time.

C: On your site you quote J.J. Sylvester:

> I have sometimes thought that the profound mystery which envelops our conceptions relative to prime numbers depends upon the limitations of our faculties in regard to time, which like space may be in essence poly-dimensional and that this and other such sort sort of truths would become self-evident to a being whose mode of perception is according to superficially as opposed to our own limitation to linearly extended time.[10]

MW: I think he must have been thinking about the relationship of multiplication and addition in terms of time. This was 1888, so RH had been posed, but mathematicians long before RH understood that the enigma of the prime numbers was rooted in the uneasy relationship of addition and multiplication. So possibly he had a

10. Sylvester, J.J. 'On certain inequalities relating to prime numbers', *Nature* 38 (1888) pp259-262, reproduced in *Collected Mathematical Papers*, Volume 4. NY: Chelsea,1973 p.600

sense that the relationship had something to do with time. But he says 'the profound mystery which envelopes our conceptions relative to prime numbers' – in other words, the puzzling interface of addition and multiplication – 'depends upon the limitations of our faculties in regard to time'. So if there were a higher dimensional, a two-dimensional 'time surface' or something like it – the word 'superficially' is being used by Sylvester in the original sense meaning 'relating to surfaces' – our minds, normally constrained to a 'timeline', could perhaps 'spread out across it' in some sense. It's perhaps a bit like being able to come up off the surface of the earth and look down from a third dimension to get a sense of how things are laid out, whereas when you're stuck on the ground, certain things are not at all apparent...but these are all very vague and intuitive ideas.

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C: In one of the papers you link to in the archive[11], Volovich suggests a most extreme and startling explanation for the concurrence of physics and mathematics.

MW: Yes, and you may have noticed that he quotes Pythagoras at the beginning, a slightly amusing Greek-to-Russian-to-English compound translation of "all is number" – "the whole thing is a number". I got very excited when I first found that paper, because he's suggesting that number theory is the ultimate physical theory. That

11. I.V. Volovich, "Number theory as the ultimate physical theory", Preprint CERN-TH **87** 4781-4786 (1987)

came out in 1987 as a preprint at CERN – he's an accomplished physicist – but it was never published in a journal. The fact that it never got published and the fact that he hasn't responded to my questions about it could suggest that he's backed away from it somewhat. I can't speak for him, but I wonder if he's slightly embarrassed by its more grandiose claims, in the way I was suggesting earlier that physicists and mathematicians can be.

But the thing is, he has done this vast body of work on *p*-adic physics, which I referred to earlier. And the rise of *p*-adic physics is a very interesting thing in itself because, you see, even though the universe at the scale of this room is Archimedean – I can lay my ruler end to end and will eventually reach the end of the room – the universe is *not* Archimedean at all scales. Below the Planck scale, it's no longer Archimedean. Below this 10^{-35}m or so – which to some people sounds too small to worry about, but you just take a metre, then a tenth, then a tenth, not that many times, really…It's not that our instruments aren't precise enough to measure below that scale, it's that the whole idea of measurement as we've formulated it ceases to make consistent sense. And effectively, space becomes non-Archimedean below that scale. There's a similar scale with time and other fundamental quantities, below which they become non-Archimedean. You can theoretically join some unit of measurement end-to-end and never achieve a given, finite extension.

This has led people to think that maybe *p*-adic physics, where you're dealing with a non-Archimedean number system, would be more appropriate for application at the sub-Planck scale. And Volovich seems to be suggesting that different non-Archimedean number systems could apply to different regions of space and time at different scales. Again, I'm not entirely sure: large parts of the paper are beyond the scope of my present understanding. I'm intrigued by his referenced to 'fluctuating number systems', but I don't know whether he means fluctuating with time, or in some other more generalised sense.

People are now starting to think about applying *p*-adic mathematics to the physical world. Each *p*-adic number system provides a different sense of 'distance' between two rational numbers, and that notion of distance then allows you to define all the other numbers which aren't rational via precise mathematical concepts involving 'limits'. I mentioned this earlier. This distance or 'metric' is defined in terms of divisibility of primes. It has to do with highest powers: for instance, in a 7-adic metric, finding the distance between two rationals involves basically looking for the highest power of 7 that divides into the numerator of their difference – that difference of course is also a rational number – when it is expressed as a fraction in lowest terms. As a result of that, number theory comes flooding into your *p*-adic physics: if you start looking at *p*-adic or adelic space and time, issues associated with the prime numbers become directly relevant. Of all of this number theory/physics material I'm archiving this is the area I'm least familiar with.

C: Saying that the means of measurement, that the possibility of measurement has changed is one thing, but saying that numbers are actually the 'atoms' themselves, so to speak, is something else: that means that there is no longer some *thing* you're measuring. The measurement itself takes on a sort of substantiality.

MW: Yes, these are very difficult notions to grasp, in so far as I understand what's being proposed. I think, perhaps like myself, Volovich caught a glimpse of something, got quite excited about it and wrote it down; he's quoted Pythagoras – it's as if there's some mystical quality to his insight.

C: There might be thousands of these papers hidden everywhere that people haven't published.

MW: I'm not sure it would be in the thousands, but who knows…There's a general hesitance to stick one's neck out. If I'm helping to encourage that sort of thing, then I suppose that's a useful contribution.

C: Exeter University has granted you an honorary fellowship and hosts the web- archive, but there is no funding available for your work. Apart from your own fascination with the subject, what drives you to continue this labour of archiving and making your own speculative connections public?

MW: Over the years after I'd dropped out of formal academia, I spent a lot of time thinking through and honing these ideas about mathematics being some sort of inner priesthood of our scientistic culture that's in the process of destroying the ecosystem, and wondering what could be done about it, how do we change this, you know? I felt that campaigning to stop the destruction of this or that rainforest isn't going to be enough, you've got to go right to the core, to the root of the problem, the fulcrum. And, reading von Franz, with her ideas about ethnomathematics, and quantity and quality, and reading René Guenon, who – although I don't embrace his traditionalist fundamentalism – wrote a fascinating book called *The Reign of Quantity and the Signs of the Times*, I started having this idea that only when Western Culture re-evaluates its relationship with number can there be any real change in the way we relate to the world, because we've got stuck in a 'quantocentric' view of the world. And so I have felt at times that what I was trying to bring forth – whether it was in my strange 1998 'evolutionary' notion or just in my networking of various people's work via my web-archive – was an acceleration towards an imminent transformation in our relationship with the number system. I was quite driven for a while, but I've become considerably more cautious and sober in my approach to this since. I saw what I perceived to be clues...felt that it *had* to be coming, and only through that sort of transformation will the Western project ever be able to steer itself in a less destructive direction. At times I've felt that I had an important rôle to play – not that I was 'chosen' to do it or anything, but that my work was cut out for me, and it was an important mission. Other times, I've been much less certain, and wondered, you know, why am I sitting in front of this computer editing HTML, when I could be spending the same time and effort campaigning for, say, the rights of an indigenous tribe having its land ravaged by a multinational corporation. I had to justify this to myself when people I knew were involved in things like that, by telling myself, well actually they're just dealing with the symptoms, whereas I'm trying to deal with the root of the problem. So it verged on an idealism, almost an activism.

C: The point being that rather than lamenting the destructive rôle of number and of science, one tries to recognise that there's something else within number, and as you said, to re-evaluate our relationship to it, which is not to say to reject it, but to become *more* numerate...

MW: Yeah, which is what I saw around me, people being very suspicious towards mathematics, hating it, seeing it as controlling and evil, and I thought, no, we need to get inside it, try to understand where it comes from and how it works.

But then I started to question whether I was just creating a whole set of complex and noble motivations for myself when in fact it was just my ego or desire to be acknowledged for what I'd achieved, or, you know, just wanting some sort of recognition or status. I was continually wondering what it was that was motivating me, and trying to rein myself in and consider the worst possible motivations as well as the best.

I had a kind of motivational collapse in early 2005, when I was struck by a very deep sense of there being insufficient time; you know, I had this grandiose hope of helping to effect some sort of long term change in culture and the way in which we deal with the number system. I started to think, maybe what I'm contributing to *would* have that effect if there were a few more centuries left of relatively leisurely culture and well-funded academia to take these ideas on and develop them, but, you know, we're facing multiple global crises, and this sort of thing is never really going to have time to take root.

I've since drifted in and out of this activity periodically, found what I think is a healthy level of interest in these matters. But I don't strongly believe that I'm part of some current of cultural change anymore, I'm just...I suppose you just can't know what effect you're having, particularly with the Web, when you're pushing ideas out. You don't know who's reading them and what they're going to do with them – a bright teenager who reads my website might be inspired to study mathematics and, influenced by some of the hints, clues, suggestions, *etc.* I've assembled, go on to make amazing discoveries...who knows?

There's also the whole relationship between psyche and matter which seems to have been at the centre of all my interests over the years. I got involved in parapsychology for a while, online psychokinesis research in 1996, wondering whether there really was something in that, and what it would imply concerning the psyche-matter interface. There's also a very exciting interdisciplinary field of 'consciousness studies' emerging, and which I've been following, people trying to understand the physics of consciousness, looking at microtubules in brain cells and how quantum mechanical phenomena at that scale might help to explain the origins of consciousness – physicists, neurologists, philosophers, psychologists, anthropologists, psychopharmacologists, *etc.* are all contributing to this field. Then there's all the Jungian theory concerning myth, archetype, synchronicity and the 'psychoid' level of reality – a kind of psycho-physical interface. The simple fact that mathematics is able to describe the world at all, that's a mystery involving mental constructs being mapped mapping onto material reality. There's the 'mind-brain problem' which philosophers debate. And then dreaming, shamanism, schizophrenia, quantum-mechanical paradoxes, these are all things I've spent a lot of time thinking about, reading about – generally wondering how it all fits together. And it had occurred to me that these topic cluster around the central mystery of how matter and psyche interface. But I'd been thinking about prime numbers, etc. for a few years before it occurred to me that this is very much part of the same picture. I'd been exploring the interface of physics – which concerns matter, obviously – and number theory, which, as that Tenenbaum quote suggested, is really an exploration of 'the mind itself'. And the research I've been interested in archiving displays a two-way traffic: Number theorists have been providing concepts and structures which physicists have used to better understand the world of matter. Physicists have been able to, using their understanding of matter, shed light on the internal workings of the number system. Even number theory *without* the physics is implicated: although number is widely considered as a mental construct, at the same time it manifests directly in the world of matter: when you consider a quartz crystal or a five-petalled wildflower, it's hard to deny there's an essential 'sixness' or 'fiveness' there. So, number itself is a bridge of sorts between psyche and matter.

This last idea, that number is a bridge between psyche and matter, comes quite close to something Jung was exploring in his later career. He left a lot of incomplete work when he died, and I believe he left von Franz to look at number archetypes. He'd looked at individual integers, the first few integers and their various associations. But later, more importantly, he'd come up with the idea that, not individual numbers with their associations, but the *set of positive integers as a single entity* is in itself an archetype, the archetype of order.

Now what has distinguished Western culture from the rest of humanity, what characterises the Sumerian-to-Babylonian-to-Greek-to-Roman-to-Western-European cultural current that dominates the planet with its measurement and science and so on, is the way we've dealt with this archetype which normally inhabits the collective unconscious. I picture it as a sort of mysterious sea creature – we've hooked it and we've hauled it out from the dark depths into the daylight of consciousness. We've taken something that was primarily unconscious, and which would naturally manifest primarily via the number archetypes and number associations in other cultures. We've dragged this thing out of the sea and onto the land, cut it up and studied it, studied its anatomy in great detail in order to obtain a new kind of magic, if you like, and that, I came to believe, was the root of all the world's problems.

But then we have this emergence into consciousness of the set of the prime numbers buried within the set of positive integers, a hidden archetype within an archetype, a kind of chaos within order, the black dot in the yin half of the yin-yang symbol; the emergence of *that* archetype – the prime numbers, the zeta function and everything they entail – into mass consciousness, is just starting now, really. The first four 'popular' books on RH have all come out in the last couple of years...it's strange that this should all be happening so suddenly. Thinking along the quasi-Jungian lines I've sketched out, the integration of these ideas into consciousness, the idea of the Riemann zeros having their origins in some 'older' or 'deeper' numerical reality, something more 'primordial', *etc.* may turn out to be of profound historical significance. According to the insanely optimistic wishful thinking which I've since distanced myself from, this could be the event that would start to alleviate the effects of rampant 'quantocentrism' and put things back into balance.

C: I wonder whether the growth of 'popular science' could play a rôle here – thinking in particular of the many books which have been published on RH.

MW: The fact that you've got four books on RH out suddenly – why is this, why hadn't this happened before? I'm sure a few years ago most people involved would have said that it's impossible to explain RH to laypeople. But four authors have done their best, with varying degrees of success. The books have all been well-received, have sold fairly well. So why is this happening? The mystically inclined might invoke an unseen force that's trying to bring these ideas into consciousness. Jungians might talk about 'compensation' and the collective unconscious. But more simplistically, more materialistically, it's market forces, it's capitalism, and it's because people are looking for meaning. Many are turning to New Age cultism, some are turning to born-again Christianity, Scientology, fundamentalist Islam, whatever. But there are a lot of people who are aware that the real 'guardians of truth' these days are not priests and monks, but scientists and mathematicians, and yet, they find themselves in a position where they don't know anything about the essential subject matter. So they want someone to explain, say, the mysteries of quantum physics to them. I get this all the time, people really wanting me to explain quantum physics, fractals, relativity, the golden mean, chaos theory, p; there's a handful of things that people get really excited and obsessed about, you know. And of course the market system rises to meet a demand, a growing demand for meaning. The problem is that capitalism doesn't care whether a book is accurate or well-written, it just cares about sales figures. So as a result you get gross oversimplifications hitting the market and sometimes selling quite well. Because the market has expanded, there is more competition, and ideally, if you believe in the effectiveness of capitalism, then the 'best' stuff will float to the top – but 'best' in this sense doesn't necessarily correlate with truthfulness or accuracy, rather with how successfully the book quenches readers' thirst for meaning. There does seem to have been a certain amount of progress, though. I don't really watch much TV, but it does now appear that with the computer graphics available, it's possible to make some things a lot more visually accessible, so viewers can at least get a flavour of the problem, or of what's at stake.

But the really deep stuff, the major philosophical problems underlying maths and physics…it's hard to imagine that there really is a shortcut to years and years of disciplined study. I mean, you might be able to get the basics of something across to a few, a small section of the population who are already interested and whose minds are structured in a certain way – it's not to do with levels of intelligence, just a certain kind of intelligence. You've got committees for the popular understanding of science and things of that nature, but they're very marginal. Unless there were a major cultural shift, unless you had major government funding, and the top layer of mathematicians and scientists committing themselves full-time to bringing this stuff through into popular culture...but there's no motivation for that to happen – governments aren't interested in educating their populations except in ways which will further economic growth. They want a certain proportion of young people to be trained up to be economists, accountants, engineers, *etc.* 'Truth' doesn't really come into it. So I doubt it…but, again, you never know, some major cultural shift could occur where the demand for this sort of knowledge reaches the point where the best people would feel obliged to provide it. Or, possibly, there could be some sci-fi type breakthrough involving direct brain-to-brain knowledge transfers, you know, you can't rule these things out, but I'm not holding my breath!

You've probably noticed, part of my website is very formal-academic, the web-archive aspect; and part of it is just about getting fundamental ideas across to people who are open to them and just want to understand their reality a bit better. I have felt in the past, with my 'activist' hat on, that it's important to bring some of these issues to widespread public attention – the basic issues of the number system. At this stage I don't know if it *is* 'important' or not, but I'd be very interested to know what the overall effect of that kind of exposure would be. Again, I suppose I am still gripped by the idea that, if we transform humanity's relation with number, that could have a positive transformative effect. I suspect I'm still partially motivated by that belief at an almost subconscious level.

The only thing I can really say with any confidence at all is that I think we're on the verge – and again, the timescale is very indefinite here – but Western Civilisation is on the verge of collectively realising that the number system is something very different from what it had previously thought it to be. I haven't got a particular theory about what it is, I just know it isn't what we think it is.