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TRAJECTORY DESIGN STRATEGIES FOR THE NGST L_2 LIBRATION POINT MISSION

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and

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Abstract

The Origins' Next Generation Space Telescope (NGST) trajectory design is addressed in light of improved methods for attaining constrained orbit parameters and their control at the exterior collinear libration point, L_2. The use of a dynamical systems approach, state-space equations for initial libration orbit control, and optimization to achieve constrained orbit parameters are emphasized. The NGST trajectory design encompasses a direct transfer and orbit maintenance under a constant acceleration. A dynamical systems approach can be used to provide a biased orbit and stationkeeping maintenance method that incorporates the constraint of a single axis correction scheme.

INTRODUCTION

Sun-Earth libration point orbits serve as excellent locations for scientific investigations of stellar and galactic physics. These orbits are often selected to minimize environmental impacts and disturbances and to maximize observing efficiency. Trajectory design in support of such missions are increasingly challenging as more complex mission designs are envisioned. To meet these challenges trajectory design software must be further updated to incorporate better understanding of the libration orbit solution space. Thus improving the efficiency and expanding the capabilities of current approaches. Recently applied to trajectory design, dynamical systems approach now offers new insights into the natural dynamics associated with the multi-body problem.[1,2] This approach allows a more rapid and robust methodology to libration orbit and transfer orbit design.

Even though libration orbits have become more mainstream and many missions to the exterior Sun-Earth collinear libration points, L_1 and L_2, are now proposed, the number of operational missions flown have been few in number totaling only four. There are currently three missions awaiting launch over the next two years and one L_2 mission in design, the Next Generation Space Telescope (NGST).[3] Of these eight missions, all but one was designed and

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supported by the Goddard Space Flight Center (GSFC). While similar in dynamical properties, the diversity of the libration orbits is revealed by their orientations and amplitudes in a Sun-Earth rotating coordinate frame centered at the libration point. For the International Sun-Earth Explorer (ISEE-3) the complexity of mission design was handled through a combination of analytical and numerical methods to predetermine the reference libration orbit, along with proven operational numerical techniques for targeting and optimization.[4,5] The International Solar Terrestrial Program Solar Heliospheric Observatory (SOHO) mission was the next true libration orbiter, with orbit amplitudes equal to that of the ISEE-3 mission.[6,7] While the transfer and mission orbit is similar to ISEE-3, the stationkeeping control method does not follow that of re-targeting back to a predetermined reference path. SOHO stationkeeping is performed to ensure that the orbit completes another revolution which has an added benefit of minimizing the ΔV required. ACE, the most recent mission, was designed differently in that the transfer orbit was adjusted to allow a capture into a small L_1 Lissajous orbit.[8] GSFC support over the next several years will include the Microwave Anisotropy Probe (MAP), Triana, and NGST missions. Unique to these missions are the transfer trajectories and libration orbits designed to meet linear and non-linear orbit goals.

NGST Mission

The GSFC NGST concept design shown in Figure 1 is part of the NASA Origins Program. It is designed to be the successor to the Hubble Space Telescope. The majority of the NGST observations will be in the infrared part of the spectrum. To accommodate this spectrum, it is important that the telescope be kept at low temperatures and that observations be free of sunlight entering the telescope either by reflection or directly. An orbit at a suitable distance from the Earth and moon and their reflected sunlight is desirable. While there are several orbits that are satisfactory from a thermal point of view, an orbit in the vicinity of the L_2 point is chosen. The overall trajectory is similar to that used for SOHO or ACE, but at L_2. Based on mission constraints, two orbit amplitudes of 800,000 km and 400,000 km in the +y-axis direction perpendicular to the Sun-Earth line are investigated. Amplitudes in the other directions are allowed to float as long as they meet the NGST requirements listed in table 1. The NGST launch is planned for 2008 on either an expendable launch vehicle (ELV) or Shuttle. In either case, launch will be into a low circular parking orbit (~185km) with a 28.5° inclination. Unique to the concept design is the use of a 300m^2 solar shade.

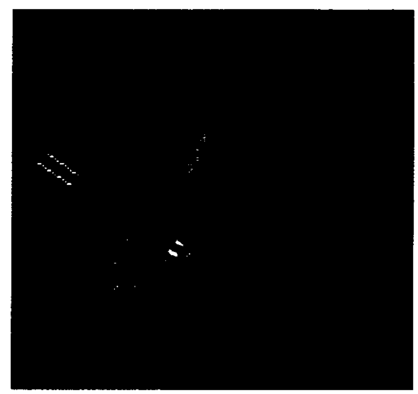


Figure 1 NGST Concept

Table 1
NGST Orbit Requirements

Requirement Type	Requirement Parameter	Impact
Shadow	None Allowed	Opening lissajous or quasi-halo
Transfer Type	Direct	Minimize Lunar Encounters
Mission Duration	10 years	Orbit Stability
Maneuver Direction	Along SRC +X-axis only	Stationkeeping
Spacecraft Design	Propulsion System	Single SRC Axis Control
Spacecraft Design	Solar Shade ~300m^2	Constant Accelerations
y-axis Orbit Amplitude	8e5 km > A_y < 4e5 km	Transfer Design and Maintenance

MISSION DESIGN

GSFC libration point mission design capabilities have significantly improved over the last decade. The success of GSFC support is based in an accurate numerical computational regime. Before 1990, mainframe computers were the only resource to compute trajectories for libration orbits. The software of choice at that time was the Goddard Mission Analysis System (GMAS). This software had complete optimization functionality as well as the capability to model all the required perturbing forces. The software was unique at the time since it allowed object modules to be linked into the run sequence as a way to allow the user access to data for trajectory analysis. During the early 1990's, the GSFC operational PC program called Swingby was developed.[9] Swingby was developed as a replacement for GMAS but with an interactive graphical user interface to provide instantaneous feedback of the trajectory design in multiple coordinate systems. It was designed to be a generic tool to support a variety of missions including, lunar, planetary, libration, and deep space and of course gravity assisted trajectory designs. Swingby provided complete mission analysis and operations for the WIND, SOHO, ACE, and is currently being used for Triana analysis and as an independent check for MAP. Additionally, Lunar Prospector and Clementine also used Swingby for mission design and maneuver planning. With the unprecedented success of Swingby, GSFC invested in a COTS program called Astrogator, produced by Analytical Graphics Inc. that is based on Swingby design and mathematical specifications.

It is important that libration trajectories be modeled accurately. The software must integrate spacecraft trajectories very accurately and model both impulsive and finite maneuvers. Swingby and Astrogator allow this by incorporating various high order numerical integrators. Precise force modeling includes up to 100x100 Earth and lunar gravity potentials, solar radiation pressure, multiple 3rd-body perturbation effects and an atmospheric drag model. Trajectory targeting and optimization is performed by varying user-selected parameters to achieve the required goals. A differential corrector is routinely used as the method of choice for targeting. Both programs use B-plane and libration coordinate targets. These software tools are also excellent for prelaunch analysis including error analysis, launch window calculations, finite engine modeling, and ephemeris generation.

Shooting Methods

The trajectory design can be computed using GSFC's Swingby or Astrogator software. Currently, both of these programs use a direct shooting approach (forward or backward) for targeting and meeting mission goals. A shooting method is generally used to achieve orbit goals for both programs described above. The usual sequence of this method is to vary the initial conditions though predefined perturbations. The initial conditions include the orbital initial conditions, an applied ΔV, or spacecraft design parameter to meet goals that include orbital parameters such as period, position, velocity, amplitude, etc. The general targeting procedures used in developing a baseline transfer trajectory are:

- Target a trajectory that yields an escape trajectory towards the L_2 libration point with the Moon at the appropriate geometry.
- Target a solar rotating libration point coordinate system goal that either achieves an x-z plane crossing velocity near zero which yields a second x-z plane crossing or a predefined state and time. Then target to a multiple period revolution at L_2 using this target method.
- Incorporate conditions to achieve the correct orientation of the Lissajous pattern.

- For stationkeeping, target multiple x-z plane crossings that ensure an orbit that meets requirements.

This procedure is duplicated for significant changes in launch date or to include lunar phasing loop strategies. Targeting to an opening Lissajous pattern assures that the spacecraft will not pass through the shadow for multiple revolutions assuming control of the unstable mode. While this procedure will achieve the required orbit, it is not robust for rapidly changing requirements. In order to decrease the difficulty in meeting mission orbit parameters and constraints in a direct targeting approach, the application of a dynamical system approach is investigated and incorporated into the overall trajectory design technique.

New Strategies: Libration Mission Design Improvements

As mission concepts become more ambitious, increasing innovation is necessary in the design of the trajectory. Design capabilities for libration point missions have significantly improved in recent years. The success of Swingby for construction of trajectories in this regime is evidence of the improvement in computational capabilities. Nevertheless, conventional tools do not currently incorporate any theoretical understanding of the multi-body problem and do not exploit dynamical relationships. An in depth discussion of the versatility of dynamical systems as they apply to libration trajectory design were previously presented and is summarized below with permission from the prime author.[2]

Dynamical Systems Approach

Nonlinear dynamical systems theory (DST) offers new insights in multi-body regimes, where qualitative information is necessary concerning sets of solutions and their evolution.[1] DST is, of course, a broad subject area. For application to spacecraft trajectory design, it is helpful to first consider special solutions and invariant manifolds, since this aspect of DST offers immediate insights. Under a GSFC grant, Purdue University investigated various dynamical systems methodologies that now are included in software called Generator. In Generator, different types of solution arcs, some based on dynamical systems theory, are input to a process that differentially corrects the trajectory segments to produce a complete path in a complex dynamical model. A two level iteration scheme is utilized whenever differential corrections are required. This approach produces position continuity and then a velocity continuity for a given trajectory. An understanding of the solution space then forms a basis for computation of a preliminary libration and transfer orbit solution and the end-to-end approximation can then be transferred to a direct targeting methods like Swingby for final adjustments for launch window, launch vehicle error analysis, maneuver planning, or higher order modeling. Our current goal is to blend dynamical systems theory, which employs the dynamical relationships to construct the solution arcs into Swingby or Astrogator with strength in numerical analysis.

The geometrical theory of dynamical systems is based in phase space and begins with special solutions that include equilibrium points, periodic orbits, and quasi-periodic motions. Differential manifolds are introduced as the geometrical model for the phase space of dependent variables. An invariant manifold is defined as an n-dimensional surface such that an orbit starting on the surface remains on the surface throughout its dynamical evolution. So, an invariant manifold is a set of orbits that form a surface. Invariant manifolds, in particular stable, unstable, and center manifolds, are key components in the analysis of the phase space. Bounded motions which include periodic orbits such as halo orbits exist in the center manifold, as well as transitions from one type of bounded motion to another. Sets of orbits that approach or depart an invariant manifold asymptotically are also invariant manifolds (under certain conditions) and these are the stable and unstable manifolds, respectively, associated with the linear stable and unstable modes.

The periodic halo orbits, as defined in the circular restricted problem, are used as a reference solution for investigating the phase space in this analysis. It is possible to exploit the hyperbolic nature of these orbits by using the associated stable and unstable manifolds to generate transfer trajectories as well as general trajectory arcs in this L_2 region of space.

Lissajous-Manifold-Transfer Generation

The computation process of the stable and unstable manifolds, shown in Table 2, is associated with particular halo orbit design parameters and is accomplished numerically in a straightforward manner. The procedure is based on the availability of the monodromy matrix (the variational or state transition matrix after one period of motion) associated with the lissajous orbit. A similar state transition matrix of this sort can be computed using the state equations of motion based on circular three-body restricted motion.[10] This matrix essentially serves to define a discrete linear map of a fixed point in some arbitrary Poincare section. As with any discrete mapping of a fixed point, the characteristics of the local geometry of the phase space can be determined from the eigenvalues and eigenvectors of the monodromy matrix. These are characteristics not only of the fixed point, but of the lissajous orbit. The local approximation of the stable and unstable manifolds involves calculating the eigenvectors of the monodromy matrix that are associated with the stable and unstable eigenvalues. This approximation can be propagated to any point along the halo orbit using the state transition matrix.

The first step is to generate the lissajous orbit of interest. This is indicated in Table 2 by "Lissajous". With this information, the monodromy matrix can then be computed (assuming periodic motion). Also, in the Monodromy"block, the eigenvalues/eigenvectors associated with the nominal orbit are computed and near the fixed point, the half-manifold is determined to first order, by the stable eigenvector.

The next step in Manifold is then to globalize the stable manifold. This can be accomplished by numerically integrating backwards in time. It also requires an initial state that is near but not necessarily on the halo orbit. A linear approximation is utilized to get this initial state displaced along the stable eigenvector. Higher order expressions are available but not necessary. A displacement is selected that avoids violating the linear estimate, yet the displacement is not so small that the time of flight becomes too large due to the asymptotic nature of the stable manifold. Note that a similar procedure can be used to approximate and generate the unstable manifold. The stable and unstable manifolds for any fixed point along a halo orbit are one-dimensional and this fact implies that the stable/unstable manifolds for the entire halo orbit are two-dimensional. This is an important concept when considering design options.

With the manifold as an initial guess, one can then perform differential corrections in the Transfer block that meet all the trajectory constraints while achieving an Earth access region. This final step provides the necessary conditions that are used in the numerical shooting process.

Table 2
Dynamical System Approach Segments

Utility	*Input*	*Output*
Phase (Generic Orbit)	**User Data**	**Control Angles For Lissajous**
Lissajous	**Universe And User Data**	**Patch Point And Lissajous Orbit**
Monodromy (Periodic Orbit)	**Universe And Lissajous Output**	**Fixed Points And Stable And Unstable Manifold Approximations**
Manifold	**Universe And Monodromy Output**	**1-Dimensional Manifold**
Transfer	**Universe, User Selected Patch Points, Manifold Output**	**Transfer Trajectory From Earth To L_1 Or L_2**

APPLICATIONS TO NGST LIBRATION ORBIT DESIGN

The above dynamical system approach is now applied to the NGST mission orbit parameters for generation of the libration orbit, the transfer orbit, and used in stationkeeping considerations.

NGST Trajectory Design: Libration Orbit

The design of the NGST libration orbit begins with the generator dynamical system approach. The required orbit parameters of y-axis amplitude of 800,000km or 400,000km for two cases are input into the generation of a lissajous orbit. The resulting output as shown in Figures 2 and 3 are a result of the lissajous segment. This orbit reflects the use of multiple bodies, elliptical approximation of the orbit, and solar radiation pressure (SRP). The algorithms used include parameters of a Richardson-Cary model. The orbits as shown meets all the NGST requirements because this is the starting point versus the end conditions of a shooting method. Figure 2a shows the NGST orbit in an SRC frame with a y-axis amplitude of 800K km. It is a class I orbit that has an opening z-axis component. Figure 2b shows the compliment of the Sun-Earth-Vehicle (SEV) angle. A maximum of 30^0 and minimum of 4^0 is achieved to meet all lighting constraints. Similar to Figure 2, Figure 3 shows the results for a 400K y-axis amplitude km. The SEV angle is maintained between 15^0 and 4^0, roughly the size of the lunar orbit radius

NGST Trajectory Design: Direct Transfer

Given a libration orbit with the above NGST requirements, a transfer trajectory is sought that will also minimize fuel requirements and incorporate possible NGST constraints. While a trajectory design approach similar to that used for SOHO or ACE can be pursued, the application of a dynamical system approach is investigated and is incorporated into the overall trajectory design technique. Using invariant manifolds and the NSGT orbit parameters, libration orbits and transfer paths can be computed; a surface is projected onto configuration space and the three-dimensional plots appears in Figures 4 and 5 upside-down to show detail. This particular section of the surface is associated with the "Earth Access region" along the L_2 libration point orbit. An interesting observation is apparent as motion proceeds along the center of the surface. The smoothness of the surface is interrupted because a few of the trajectories pass close to the Moon upon Earth departure.

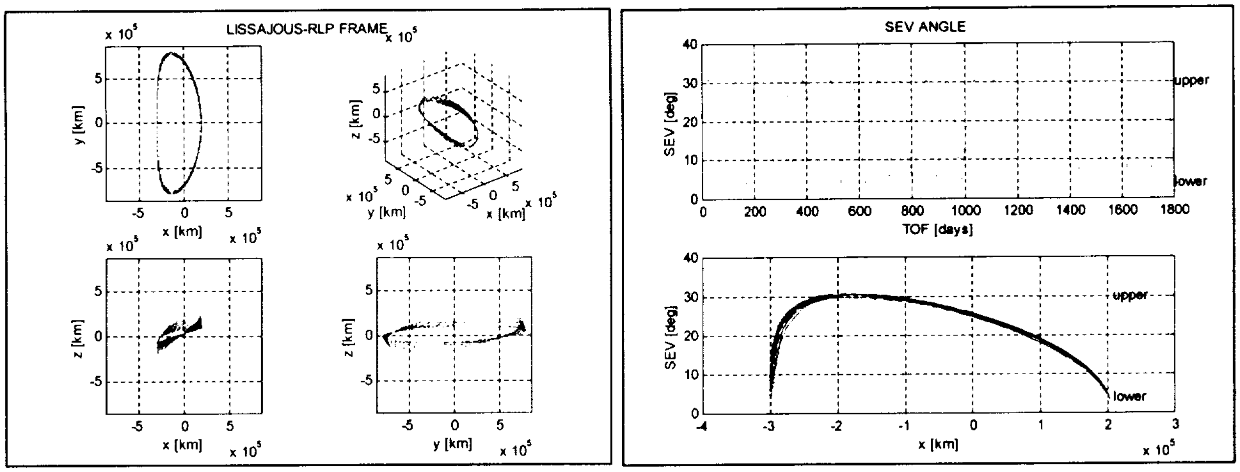


Figure 2 - Lissajous Pattern and Sun-Earth-Angles for 800K Orbit

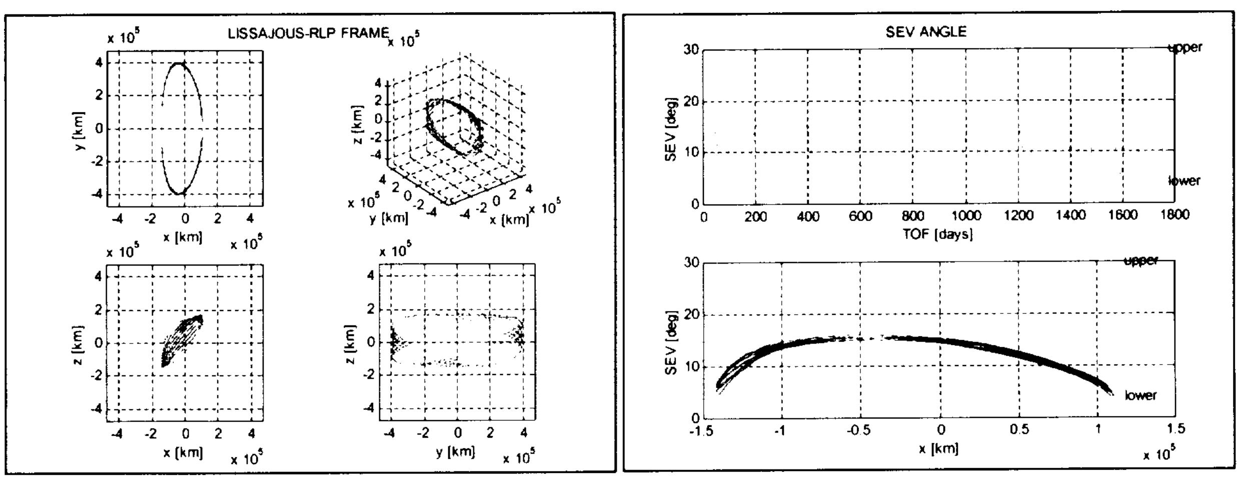


Figure 3 - Lissajous Pattern and Sun-Earth-Angles for 400K Orbit

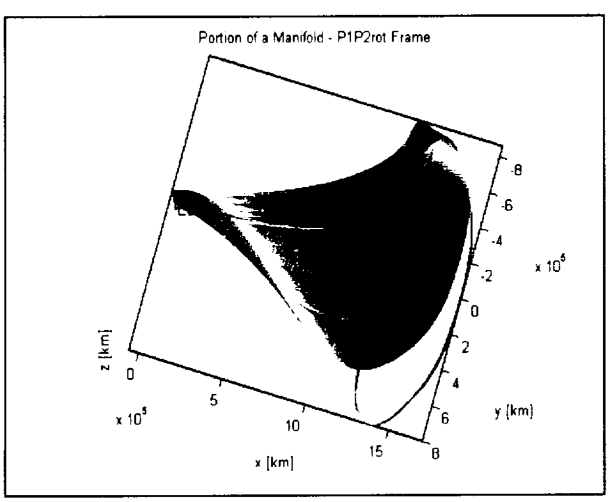


Figure 4 NGST 800km Y-axis Amplitude

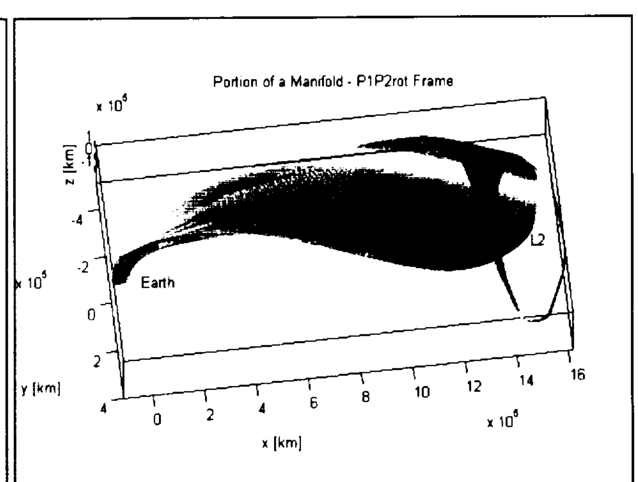
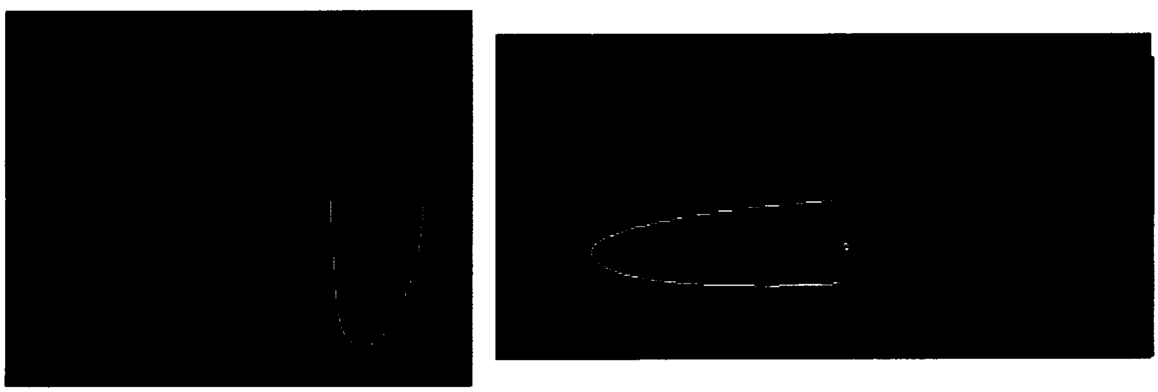


Figure 5 NGST 400km Y-axis Amplitude

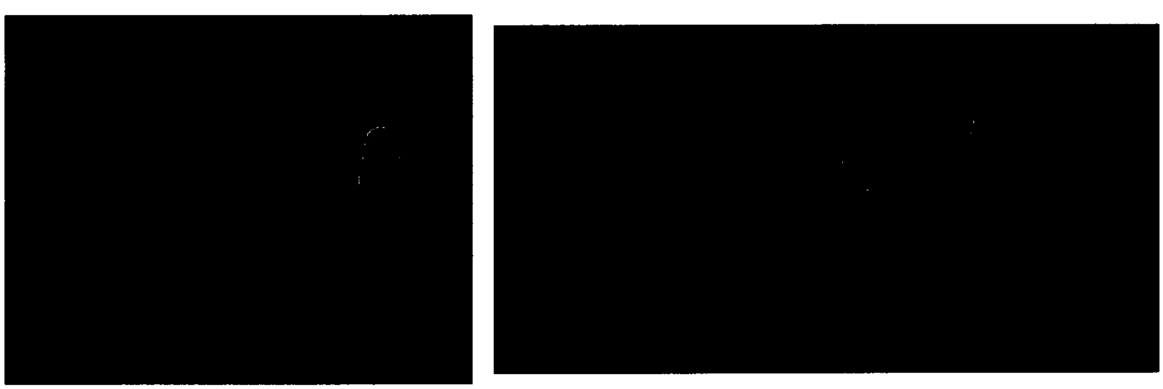
Lunar gravity assists were not incorporated into the approximation for the manifolds, but no special consideration was involved to avoid the Moon either. Using information available in Figure 4 and 5, the one trajectory that passes closest to the Earth is identified and used as the initial guess for the transfer path. The larger size of the Lissajous orbit reduces the Earth passage distance and minimizes any insertion ΔV. Given the initial guess, the transfer is differentially corrected to meet the requirements of achieving both the lissajous orbit and an Earth parking orbit. From this point, the solution is input directly into numerical tools and appears in Figure 6. Swingby and other tools are used for further visualization, analysis of launch vehicle and maneuver errors, midcourse corrections, and other design considerations. The ΔV requirements for each transfer into the NGST libration orbit are presented in Table 3.

Table 3
Required ΔVs for Transfers to Lissajous Orbits

	800K km Y - amplitude	400K km Y - amplitude
Injection ΔV	**3.17 km/s**	**3.18 km/s**
Libration Orbit Insertion ΔV	**12.0 m/s**	**123.0 m/s**



a) 800K y-amplitude, Ecliptic Plane and y-z Plane View in Solar Rotating Coordinates



b) 400K y-amplitude, Ecliptic Plane and y-z Plane View in Solar Rotating Coordinates

Figure 6 Numerical Application of Dynamical Initial Conditions

Biased Libration Orbits

The NGST design characteristics include a large solar shade of approximately $300m^2$. This in turn yields a constant acceleration that must be taken into account for both the libration orbit and transfer orbit design. Additionally, a design to accommodate thermal and optical constraints requires that no thrusts are applied such that the exhaust is near the telescope optics. This in combination with the observation pointing requirements that are generally in the anti-sun direction imposes a stationkeeping control strategy which allows ΔVs only in the solar rotating coordinate (SRC) +x-axis direction. For NGST, a biased orbit is therefore needed that allows a stationkeeping strategy that not only incorporates the constant acceleration but necessitates the control of an unstable orbit in a constrained direction. SOHO, ACE and ISEE-3/ICE also were required to follow constraints placed on the maneuver by the orbit and spacecraft design.[11,12]

Stability and Single-Axis Stationkeeping

As with all libration orbits about the unstable L_1 and L_2 points, stationkeeping is required. Typically, without the NGST requirements of a biased orbit requirement, a ΔV budget of approximately 4 m/s per year is used as a preliminary number. For the NGST design, only thrust in the SRC +x-axis is allowed. Fortunately, this direction also offers the minimal fuel and ΔV required to maintain the orbit. For this paper, two strategies are presented. A biased orbit achieved through inclusions of accelerations and a biased orbit achieved through the use of deterministic ΔVs and accelerations. Our model assumes solar radiation pressure (SRP) is perpendicular to the solar shade surface and directed along Sun-spacecraft line, so the direction varies a little. Operational attitude changes for science is not included for this analysis but must be included once a sequence of attitudes is established by the science team before launch. This would have an effect of introducing accelerations in the velocity vector direction.

Constant Acceleration Strategy

In our first strategy, a biased orbit is computed for the 800K km y-axis amplitude that incorporates a solar radiation pressure acceleration which is either double that of the expected acceleration of the conceptual spacecraft design or compared to a traditional orbit that does not account for SRP accelerations. This acceleration is also used to compute a transfer trajectory that places NGST on a libration orbit biased in the SRC -x-axis towards the Earth. The difference in this orbit in the direction of the Earth is relatively small, less than 15000km, as compared to the x-axis amplitudes in rotating libration center coordinate system of 1e5 to 3e5 km. Once the transfer is completed and an orbit insertion state is achieved, stationkeeping will be required as the orbit will deviate as usual. Figure 7 presents the x-axis difference for an expected acceleration versus no accelerations modeled, and the difference in the expected acceleration and a doubled acceleration.

For the case that inflates the acceleration to twice the expected amount, the orbit is biased so that the first orbit maneuver, or the libration orbit insertion maneuver is in the required SRC + x-axis direction. The maneuver is then allowed to underburn by a few percent to always under-achieve the required energy for maintaining a complete orbit revolution. The energy is such that all the trajectories are homoclinic in nature. The subsequent maneuver, then, is also performed in the SRC +x-axis direction. Again this maneuver performance is purposely under-achieved requiring yet another maneuver that is in the required direction. Table 4 presents results of this strategy with the maneuver magnitude in meters per sec. Each maneuver was performed, arbitrarily, at the x-z plane crossing with a target of achieving a zero velocity in the x axis direction at the following x-z plane crossing. A period of two years with four orbit revolutions was obtained. While the total magnitude per year is approximately 3.6 m/s, the strategy can be used to reduce the ΔV required to less than 1 m/s.

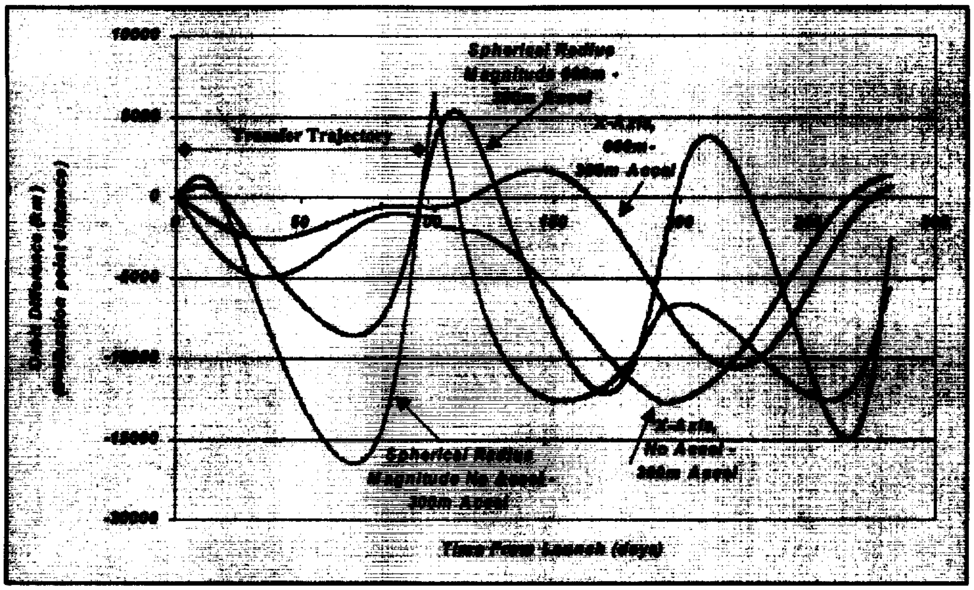


Figure 7 – Biased Libration Orbit Differences

Table 4
ΔVs for Constant Acceleration Strategy

Maneuver Required (m/s)	Maneuver Performed (m/s)	Percent Underburn
2.6025800	2.4984768	4.0
1.2919410	1.1309081	12.0
1.3298389	1.1702582	12.0
2.6928000	2.5850880	4.0
0.8238900	0.7901664	4.0

In Figure 8, all the post maneuver trajectories of the libration orbit from the above strategy are shown for the following two crossings after each maneuver. For this case, the accelerations can optionally be included in the dynamical approach that computes a baseline libration orbit or included in the targeting procedure afterwards. Note that the orbital C3 energy is maintained below zero that the orbits are homoclinic and therefore return to the Earth region.

Deterministic Maneuver Strategy

Another single axis maneuver accommodation can also be found by including deterministic ΔVs in the nominal libration point orbit that is determined using the two-level differential scheme that is part of an overall dynamical systems approach.[13,14] The expected acceleration due to the SRP is also modeled in the differential equations of motion. This is implemented in the generator utility that computes the required libration orbit as discussed above. The deterministic maneuvers are pre-specified to be any appropriate value that accommodates the

SRP acceleration and maintains the corrective maneuvers (i.e., stationkeeping) in a positive x direction.

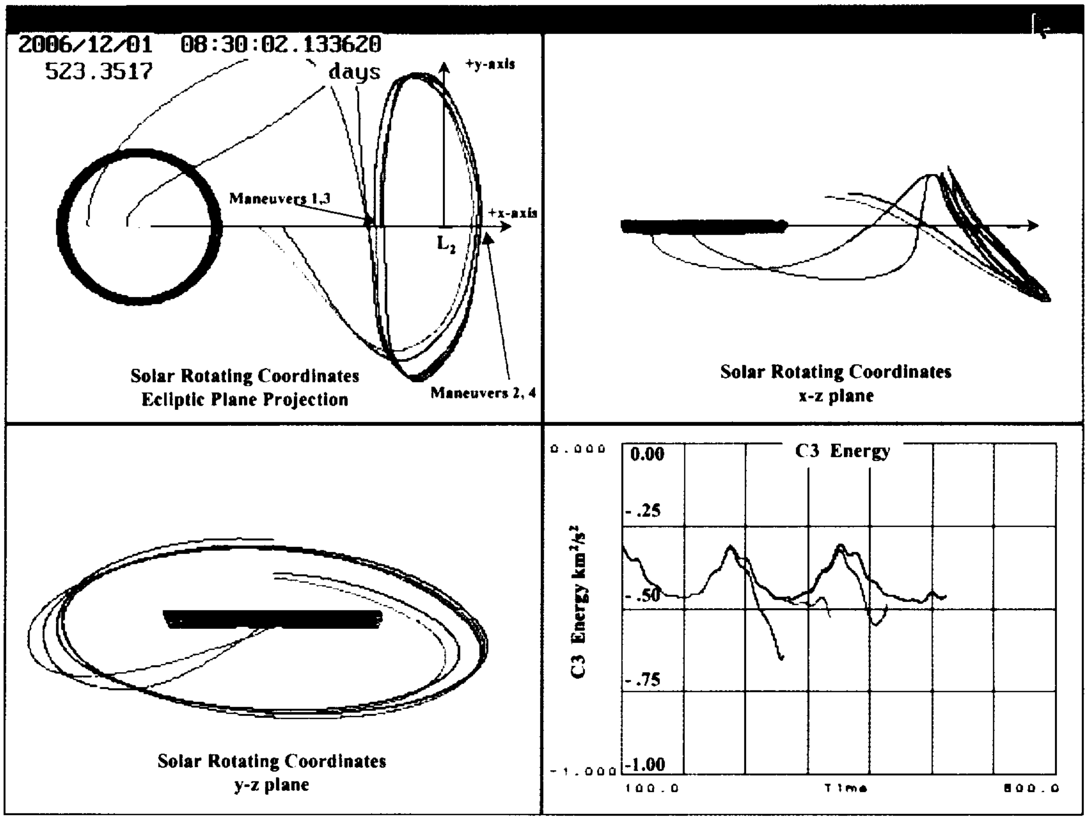


Figure 8 Earth Return Trajectories

As an example, 1 m/s deterministic maneuvers, directed entirely along +x-axis and applied at four locations about the libration orbit, can be incorporated in the baseline design of the libration point orbit. This can be seen in Figure 9 where the blue dots represent the maneuver locations. The Lissajous used in this design had amplitudes with z-excursion of 250,000 km and amplitude in the direction of the y-axis of 400,000 km. From a modeled traditional Lissajous, the SRP acceleration will shift the solution about 11,500 km toward the Earth as shown previously in Figure 8. Adding both SRP and the 1m/s bias maneuvers shifted the nominal solution about 12,500 km toward the Earth. This result is borne out by the data. The relatively large SRP force contributes a dynamically significant term in the model for NGST, so it is reasonable to incorporate it in the analysis. If SRP is not included in the baseline NGST design, it can be offset with corrective maneuvers, but the maneuvers would be required in the opposite (-x) direction. However, we can compute an acceptable trajectory by incorporating this force. Then, only parameter uncertainty in the SRP model is necessary in any subsequent error analysis.

The visual difference in the libration orbits for the above strategies is shown in Figure 10. The red indicates the deterministic maneuver trajectory, the green the SRP accelerations only, and the magenta neither strategy applied. Obviously, all the solutions look the same in the scale of the figures. In both figures, the blue dots indicate the locations of the bias maneuvers. The locations were arbitrary spread out evenly over each revolution. There is no reason to think this is the best distribution though.

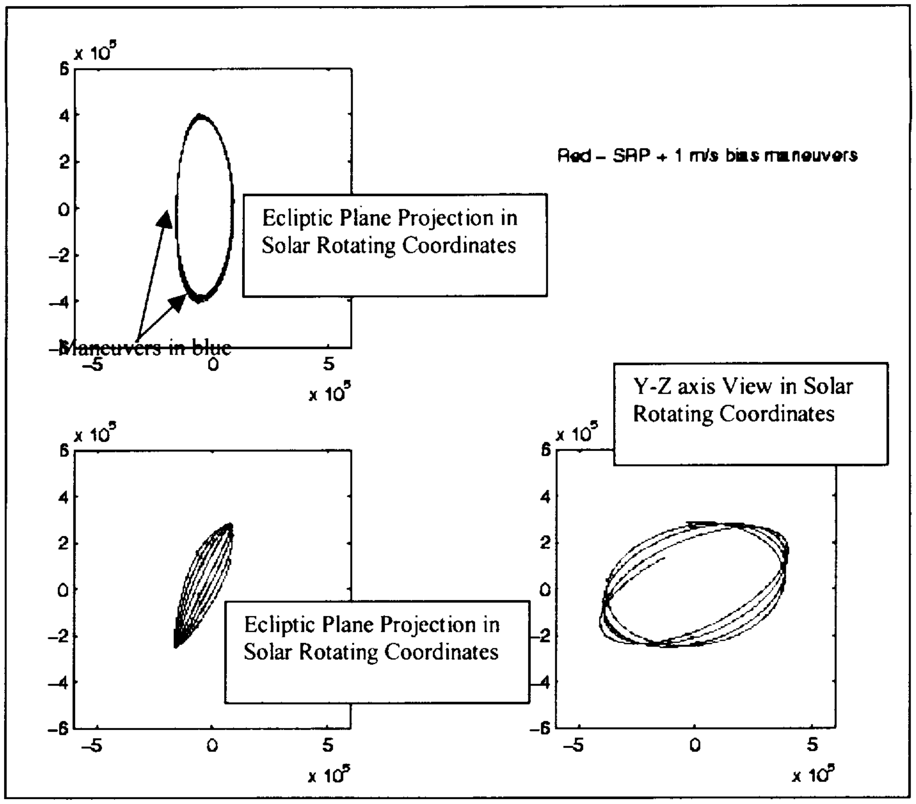


Figure 9 Biased Orbit with Deterministic Maneuvers

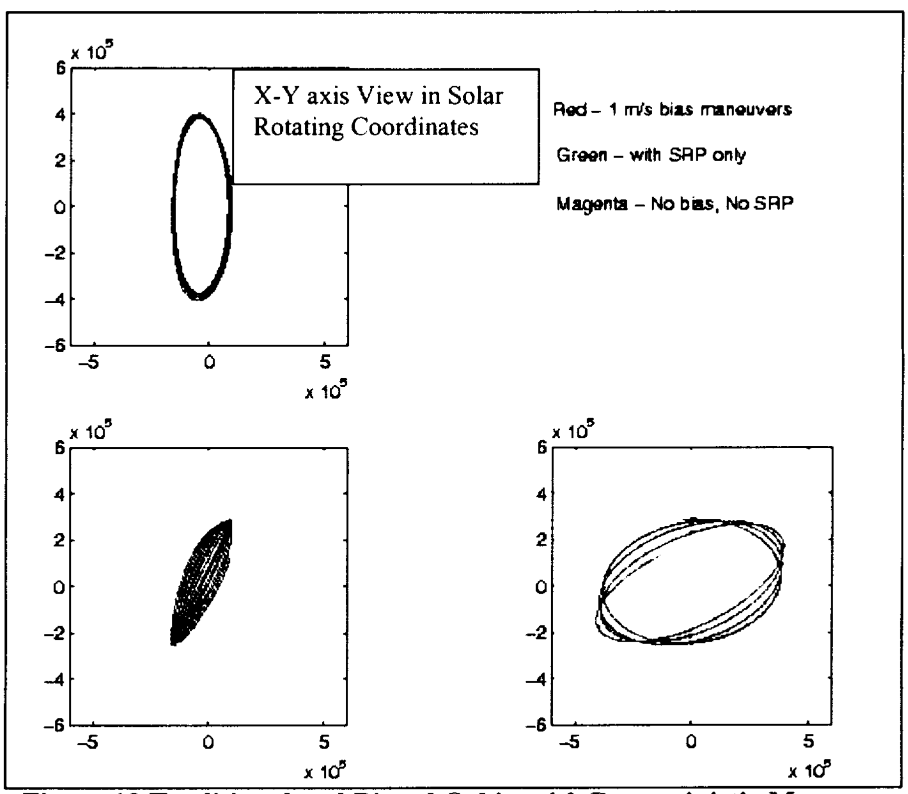


Figure 10 Traditional and Biased Orbit with Deterministic Maneuvers or SRP Accelerations

Considering the strategy that employs biased deterministic maneuvers in the baseline orbit, it is useful to complete a small stationkeeping error analysis to demonstrate that the resulting stationkeeping maneuvers can, in fact, be parallel to x-axis and positive, as required throughout the mission.[15] The problem that was formulated can be stated simply as follows: given a large solar shade, how can a strategy be developed so that *all* stationkeeping maneuvers are directed away from the Sun (i.e., directed in the $+x$ direction)? It is assumed that the thrusters do not have a minimum bound in the size of the maneuver. (The methodology can accommodate both lower and upper bounds as necessary.) The strategy must be sufficiently robust that the times of the maneuvers can be predetermined. The nominal trajectory for the stationkeeping analysis is then biased with some specified number and magnitude of deterministic maneuvers in the direction of the Sun, and the stationkeeping maneuvers are placed on top of these deterministic maneuvers. The magnitude of the deterministic maneuvers is specified so that any computed stationkeeping maneuver (potentially, in conjunction with momentum dumps and/or other corrective actions) that is added to the deterministic maneuver. This will result in a net maneuver that is directed away from the Sun. (If appropriate, the magnitude of the bias maneuver can also be specified to maintain a resultant maneuver that is larger than some lower bound on the thrusters.) This magnitude is primarily a function of the number of the number of maneuvers per revolution. For example, if there are four biasing maneuvers per revolution, then the size of a stationkeeping maneuver will generally be less than 0.3 m/s and most often close to .2 m/s.

To demonstrate this approach, consider the performance of one potential stationkeeping strategy. From a number of different possible controllers, one is selected to maintain the nominal biased trajectory with uncertainties. Orbit determination errors are included in every simulation and the nominal orbit includes solar radiation pressure determined for a solar shade that is 300 square meters in size. The nominal also includes four bias maneuvers per revolution of magnitude .5 m/s, all in the $+x$ direction. (With .5 m/s biased maneuvers, the trajectory appears very similar to that in Figure 9.) The controller is a simple targeter, that is, a differential corrector that changes the velocity *only* in the rotating x direction in order to target the rotating x position component at the next maneuver. Targeting in this manner results in a one step computation of the ΔV that is designed to reduce the position errors, at the target time, between the spacecraft trajectory and the nominal to zero. Note that this *computation* is based on a linear propagation of the current state errors to the target time via the state transition matrix associated with the nominal solution. So, a very small residual error will remain at the target. The target time is defined here as the time specified for the next maneuver and, in this case, that target time is pre-specified as the time of the next bias maneuver. The inputs for this set of trials appears in Table 5. The maneuver execution errors are included as proportional and fixed. The proportional errors are implemented so that they will be added downtrack, that is, parallel to the direction of the ΔV vector, and crosstrack, or perpendicular to the direction of the ΔV vector.

The results of the error analysis appear in Table 6. Note that this is the result for the *stationkeeping*; the bias maneuvers are already included in the nominal. Thus, for 30 maneuvers over 3.3 years (6.5 revolutions), the cost for the deterministic trajectory is 15 m/s plus the estimated stationkeeping cost in the table. However, all maneuvers are totally in the positive x direction.

If the libration point trajectory is increased in size such that A_y is increased to 800,000 km, the orbit can still be maintained with .5 m/s bias maneuvers. The average expected stationkeeping cost, over 1000 trails, increases slightly but is still within the stationkeeping budget and all maneuvers are in the specified direction for the error models incorporated here.

Further analysis can examine these options in more detail. The particular stationkeeping methodology employed here for the orbit maintenance was somewhat arbitrary; the targeter was selected for simplicity and was not tuned to this application. Alternative strategies may serve to reduce the cost and four bias maneuvers per revolution may not be the best choice for cost and/or operational reasons. Also, the locations of the bias Δ Vs are arbitrary in this study. Nevertheless, the strategy to bias the nominal successfully achieves the objective and such a libration point orbit can be maintained for reasonable costs.

TABLE 5
Inputs for the Stationkeeping Analysis

L_2 trajectory	A_y = 400,000 km; A_z = 250,000 km
Trajectory Duration	730 (~2 years), 1200 days (~3.3 years)
Bias Maneuvers	.5 m/s each
Tracking Interval	10 days
$\sigma_x, \sigma_y, \sigma_z$ tracking errors	6.13, 3.26, 9.50 km
$\sigma_{\dot{x}}, \sigma_{\dot{y}}, \sigma_{\dot{z}}$ tracking errors	6.34, 1.75, 5.90 mm/s
$\sigma_{\dot{x}}, \sigma_{\dot{y}}, \sigma_{\dot{z}}$ for mnvrs (prop)	2.0%, 1.33%, 1.33% magnitude
$\sigma_{\dot{x}}, \sigma_{\dot{y}}, \sigma_{\dot{z}}$ for mnvrs (fixed)	3.33, 1.66, 1.66 mm/s (3 σ)
Injection error	Same σ s as tracking errors
1000 monte carlo simulations for each test Four maneuvers per revolution at specified times	

TABLE 6
Resulting Stationkeeping Costs Over 730 Days, 1200 Days

L_2 Trajectory	A_y = 400,000 km	A_z =250,000km
Stationkeeping duration	730 days	1200 days
Average total cost for 1000 Trials	1.149 m/s	2.113 m/s
Number of Maneuvers (all trials)	18	30
Average Value: Max Stationkeep Δ V	.17168 m/s	.205 m/s
Average Value: Min Stationkeep Δ V	.00500 m/s	.003 m/s

SUMMARY

A key factor in designing missions in the vicinity of libration points is to understand the natural dynamics of the region. For instance, non-linearities may restrict the range of applicability of linear theory. Application of a dynamical approach to libration orbit design results in providing a rapid numerical design process by allowing the mission specific orbit to be designed first and then used in a continuous process for the transfer trajectory design. With this information, non-linear, invariant manifolds associated with the special solution can be computed for a number of cases. Special solutions such as periodic halo orbits and their transfer trajectories can then be studied in more detail. A biased orbit can also be constructed using a dynamical approach whereby accelerations and deterministic maneuvers can be incorporated into the state equations.

CONCLUSIONS

Trajectory design in support of L_2 and L_1 missions is increasingly challenging as more complex missions are envisioned. Software tools for trajectory design in this regime must be further developed to incorporate better understanding of the solution space, improving the efficiency, and expanding the capabilities of current approaches. A dynamical systems approach offers insights into the natural dynamics associated with the multi-body problem. The goal of this effort is the blending of analysis from dynamical systems theory with the well-established NASA Goddard software programs to enhance and expand the capabilities for mission design. A dynamical approach can be used to provide a biased orbit and stationkeeping maintenance method that incorporates the constraint of a single axis correction scheme.

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REFERENCES

1. K. Howell, B. Barden, and M. Lo, " Application of Dynamical System Theory to Trajectory Design for a Libration Point Mission", *Journal of the Astronautical Sciences*, Vol. 45 No. 2, 1997, pp. 161-178.

2. J.J Guzman, D. S. Cooley, K.C. Howell, and D.C. Folta, "Trajectory Design strategies that incorporate Invariant Manifolds and Swingby", AAS 98-349 AIAA/AAS Astrodynamics Conference, Boston, August 10-12, 1998.

3 "NGST Website," http://ngst.gsfc.nasa.gov/. Goddard Space Flight Center

4 R. W. Farquhar, D. Muhonen, and D. Richardson, "Mission Design for a Halo Orbiter of the Earth", *Journal of Spacecraft and Rockets*, Vol 14, No. 3 , 1977, pp. 170-177.

5. R. W. Farquhar, "The Flight of ISEE-3/ICE:Origins, Mission History, and a Legacy", AIAA/AAS Astrodynamics Conference, Boston, AIAA-98-4464, August 10-12, 1998.

6. D. Dunham, S. Jen, C. Roberts, A.Seacord, P.Shearer, D. Folta, and D. Muhonen, "Transfer Trajectory Design for the SOHO Libration Point Mission", 43rd Congress of the International Astronautical Federation, Washington D.C. 1992.

7. S. Stalos, D. Folta, B. Short, J. Jen, and A. Seacord, "Optimum Transfer to a Large-Amplitude Halo Orbit for the Solar and Heliospheric Observatory (SOHO) Spacecraft, AAS 93-294, Flight Mechanics Symposium, GSFC, 1993.

8. P. Shearer and T. Harrington, "Trajectory Optimization for the ACE Halo Orbit Mission", AIAA/AAS Astrodynamics Specialist Conference, July 1996, San Diego, CA.

9. J. Carrico, C. Schiff, L.Roszman, H.Hooper, D. Folta, and K. Richon, "An Interactive Tool for Design and Support of Lunar, Gravity Assist, and Libration Point Trajectories", AIAA 93-1126, AIAA/AHS/ASEE Aerospace Design Conference, CA., 1993.

10. D. Folta, R. Carpenter, and C. Wagner, "Formation Flying with Decentralized Control in Libration Point Orbits", International Space Symposium, Biarritz, France. June 2000.

11. D. Dunham and C. Roberts, "Stationkeeping Techniques for libration-Point Satellites", AAS 98-4466 AIAA/AAS Astrodynamics Conference, Boston, August 10-12, 1998.

12. D. Muhonen and D. Folta, "Accelerometer Enhanced Trajectory Control for the ISEE-3 Halo Orbit", Journal of Astronautical Sciences, 33 , 1985, pp-289-300

13. K.C. Howell, and B.T. Barden, "Biasing Maneuvers for Directional Control of Stationkeeping Delta V's in Libration Point Orbits," Purdue IOM AAE-9840-007, June 1998.

14. K.C. Howell, and B.T. Barden "Investigation of the Effects of Maneuver Uncertainties on a Finite Set of Different Controllers and Maneuver Strategies," Purdue IOM AAE-9840-009, July 1998.

15. K.C. Howell, and H.J. Pernicka, "Station-Keeping Method for Libration Point Trajectories," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, January-February 1993, pp. 151-15