

Introduction

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It falls to me to introduce the various texts assembled in this issue of *Io*. I have decided that the most expeditious way of suggesting the thread of connectivity that runs through them is to begin with a somewhat personal narration of the circumstances which, in fact, have brought them together.

In the summer of 1982 I participated in a seminar at The Massachusetts Institute of Technology convened by the linguist Haj Ross. Ross gathered twelve artists from various disciplines to discuss what he took to be certain fundamental formal structures underlying the aesthetics not only of art, but of all creativity including mathematics and to some degree science. Occasionally attending the seminar were linguists, physicists and mathematicians; and, during the course of the summer, connections between mathematical structures, aesthetic notions of form, and eastern meditative practices were frequently commented upon. In my case, an interest in mathematical studies that had lain dormant since high school was awakened and I began to entertain the possibility of exploring the relationships between mathematical abstraction, artistic abstraction, and the abstract structures implicit in such practices as Zen meditation and other contemplative disciplines.

Two things particularly struck me at MIT: first, I became aware that certain mathematicians had, in fact, explicitly associated the states of mind involved in mathematical abstraction with contemplative concerns; second, that though most traditional contemplative practices encourage a *cessation* of cognitive activity, the very structures of

the practices leading to such cessation are frequently themselves expressed or at least expressible in eminently abstract terms. One performs logical operations upon the continuum of awareness in order to curtail the proliferation and differentiation of events within it. Furthermore, upon developing their powers of concentration and continuous attention to a sufficient degree, meditators discover that cognitive operations are involved in the most concrete and least discursive aspects of awareness: spontaneous acts of identification and differentiation occur unremittingly and the imputation of existence to objects seems to subtend all conscious life. But such acts of identification, differentiation and imputation form the basis for mathematical abstraction as well.

When I returned from MIT I began informing myself about various branches of modern mathematical thought and philosophy with a view primarily to exploring these interconnections. By the middle of the fall of 1982 I had been introduced to Christer Hennix, and something of an apprenticeship in the intricacies of his mathematical work began. Hennix is a Swedish-born musician, "concept" artist and poet, as well as the holder of an advanced degree in philosophy. Since 1980 Hennix has held various positions as a visiting professor of logic at MIT AI laboratory, Cambridge, Mass., and as an assistant professor of mathematics and computer science at State University of New York at New Platz, N.Y. He is the principal student of the founder of the Ultra-Intuitionist School of the Foundations of Mathematics, A.S. Yessenin-Volpin.

Inspired by this apprenticeship, in the winter of 1984 I succeeded in interesting two of my poet-associates, Don Byrd and George Quasha, in joining with me in a more or less formal series of seminars to center around Hennix's work, but to involve—in addition to mathematics and logic—poetic and other artistic practices, various facets of contemporary philosophy, Pre-Socratic thought and the Greek Mysteries, visionary poetics and experience, oriental, psychedelic and other forms of contemplative explorations. The seminars ran through the summer of 1984 under the rubric of "The Rhinebeck Institute." (Hennix lives in Rhinebeck New York and most of the sessions were held at his studio there.) Some of the poems by George Quasha included in this issue were read and discussed by us at some of the seminar sessions. The subject matter of Don Byrd's essay (which is a chapter from his current prose project, *The Poetics of Common Knowledge*), namely, the relationship between Cartesian mathematics and modern poetics, formed an

important area of discussion.

In addition to the above-mentioned participants, the seminar was occasionally attended by Susan Quasha, Dawn Felicioni, psychologist Anne McClellan, composer Niel Rolnick, and curator (of the Blum Gallery at Bard College) Linda Weintraub.

Though never attending the seminar, Henry Flynt and A.S. Yessenin-Volpin were each in different ways important "presences." Most of us were studying their writings and their work was frequently alluded to during the sessions.

The remainder of this introduction will attempt to summarize a fraction of the material we discussed during those seminars under the following topics: the Parmenides Poem (a founding text of Western rational philosophy that also happens to be a visionary journey); the work of L.E.J. Brouwer (the Dutch mathematician and mystic who founded the Intuitionist School of foundations of mathematics); the work of A.S. Yessenin-Volpin (founder of Ultra-Intuitionism); and the work of Christer Hennix. I will leave the texts of Flynt, Quasha, Byrd—as well as my own poems—to speak for themselves.

The Parmenides Poem

A consistent concern throughout the seminar was the study, interpretation and translation of Parmenides. The seminar culminated with a public lecture given by Hennix on "Eleatic Set Theory," which I introduced with an earlier version of the translation printed at the end of this issue.

Parmenides is usually recognized as the beginning of the "rational" tradition in Western Philosophy. He is the first to leave a text behind in which the assertions are supported by definite arguments, and where belief demands proof.

Yet the Parmenides that appears in his fragmentary poem is not simply a rationalist in any familiar modern sense. He writes, for one thing, in verse, not prose. The occasion for his setting forth his arguments is a visionary journey; his mouth piece, a goddess; and the object of his vision, an enigmatic and mysterious totality that, though unnamed, is referred to by the locution "esti" (it is) or "on" (Being). The Goddess who speaks of Being is Dike or Justice, and thus though the technical philosophy is thoroughly abstract in content, its domain seems to be the whole of human action, and its attitude, ethical.

The Parmenides poem in fact opens a rather impressively large range of philosophical issues which remain unresolved to this day: the

questions of the unity of knowledge, the unity of being, the unity of knowledge *and* being, the nature of proof, the nature of thought itself, the nature of justice, the relations between thought, proof, knowledge and justice, the possibility of rational cosmology and the relationship between logic and cosmology, the existence and nature of abstract entities, the privileged status of philosophical knowledge—all these questions *became* explicitly formulatable questions immediately following the production of Parmenides' poem, none of them are definitely resolved today, and the future of science, culture and the human spirit all depend to a far from trivial degree on how these questions come to be reflected upon.

But Parmenides was a visionary and a poet as well as a philosopher and mathematician, and as we were a group of poets concerned with philosophy, mathematics and visionary matters, we took the task of discussing, interpreting and translating Parmenides as an appropriate collective concern.

The text of the poem comes to us in fragments recorded by later philosophers: Aristotle and Simplicius, principally. It is generally assumed that it appeared in two parts: one delineating a "true" path, the other a representation of a path of mere opinion. The reason for this division is not clear from a study of the existing fragments. Numerous analyses have been suggested. It is also uncertain what proportion of the entire poem we are in possession of, and, in the case of certain passages, to which section they belong.

The poem is written in dactylic hexameters, the meter of the Homeric Poems and the Delphic oracular responses; however, the diction is not derived from the Homeric dialect and if the formulaic theory of the technique of Homeric poetry is correct, the Parmenidean poem does not employ it. Exactly why Parmenides wrote in hexameters when there was already precedent for philosophers to write without them is conjecture.

Though the thought of the poem is supported by argumentation, this thought manifests in the context of an ecstatic vision. Parmenides is taken on a journey in a chariot, and with the assistance of the daughters of the sun god, he ascends into a super-celestial region normally forbidden to mortal inspection. Parmenides' personal qualifications for the journey combine with a certain good fortune to create the conditions for his welcome by the goddess.

Parmenides' chariot journey resonates with similar voyages occurring in the texts of cultures geographically proximate to and contem-

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porary with the Classical Greeks, cultures whose interrelation with the Hellenic world is speculative but by no means impossible. I am thinking, in particular, of the Vision of the Chariot in *Ezekiel* and the metaphor of the chariot in the *Upanishads*. A variant of the image of the chariot as a metaphor for a spiritual journey is taken up, of course, in Plato's *Phaedrus*.

Parmenides tells us that the horses that drive his chariot have the capacity to take him as far as he wishes to travel. I read the horses as images for his own mental capacities, the chariot as the vehicle of thought *cum* contemplation upon which he journeys. Parmenides has mastered his own intelligence and his own mind as far as that is possible, and this qualifies him for contact with a transcendent source of wisdom, the goddess herself.

The goddess herself is unique to Parmenides. I speculate that the reverence paid to the goddess justifies the use of Heroic verse. In any case the hexameters do not appear to be wielded ironically: Parmenides is not mocking the poetic tradition by using its style. Though his thought is a new vision of reality and a radical critique of ancestral myth and recent philosophy alike, something of the sense of the sacred as manifested in the poetic tradition is being honored and carried forward here. Again, though Parmenides radically decomposes previous thought, the sacred awe of the truth that such predecessor ontologies radiated in their poetries is being transmitted in a new mode.

The symbolism of the poem is a species of mystical allegory wherein persons and objects, though figurative, cannot be replaced by the abstractions for which in fact they stand. Dike is more than abstract "Justice," yet she is also more than an anthropomorphic goddess: Parmenides' conversation with her and the instruction he receives from her constitute a communication and instruction from Justice itself; and Justice is taken as somehow identical with the transcendent object, the It Is, of which the goddess speaks. That Dike speaks in arguments and proofs bespeaks the nature of Justice: Justice is inseparable from correct reasoning, and the rationality of Being conjoins its own theodicy.

The restrictions on reasoning that Justice demands are stringent. In particular, a certain form of negative reasoning or negative existence is radically forbidden. That which Is Not cannot be asserted To Be, and thus no creature made up partly of Being and partly of Non-Being can be granted any share in existence at all. Being is whole, self-identical,

bounded, indivisible, and eternal. It is like a "well-rounded sphere."

It has often been wondered how Parmenides could have permitted himself numerous negative expressions in the delineation of Being, if the existence of non-being is to be so severely interdicted. But a recent study by Scot Austin (*Parmenides, Being, Bounds and Logic*, 1986, New Haven, Yale University Press) demonstrates that the ban on negative language is itself limited to a ban on the positive assertion of negative subjects: such constructions as "the non-existence of X exists." It does not disallow such assertions as "X does not have property y." Austin argues that Parmenides' language is consistently disciplined in this way, so that the charge of self-contradiction or verbal unconsciousness will not hold.

The image of the "well-rounded" sphere is extraordinary. It has often been taken literally, as if Parmenides meant to project a spherical cosmology of a material sort. In fact, the image is clearly a simile—it is meant to express the concept that measurable distances, (such as the distances measured by geometers) have no application within Being. There is no distance as far as things of the mind are concerned. The aspect of the sphere that is intended in the simile is that which is contained in its formal definition: that all points on its surface are equidistant from its center. Similarly, all "points" or thoughts within Being are correlated without subdivision or hierarchical organization of any kind.

Parmenides' "sphere of being" is often, in recent literature, taken as the prototype of the kind of systematizing, totalizing cosmologies and philosophies which, imposing themselves on the world, justify hierarchically-organized social structures, mind-stunning super-systems, and all the political, social and theological tyrannies of western history. But it seems to me it is important to notice that whatever the aspirations of systematic thinkers in the west from Plato through Hegel and Marx to Carnap and Goodman may be, none of them in fact takes up the challenge of the restrictions upon intellectual assertion leveled by the goddess. Plato (who is most often taken to be Parmenides' heir) after explicitly merging the concept of Being and Non-being to form the hybrid Becoming, throws up his hands in despair of ever understanding what the Master really meant. The cosmological sphere of Being that Plato presents in the *Timaeus*, and which is generally thought to stem from Parmenides, is in fact a version of the sphere that Parmenides describes in the second part of his poem—the part presumably dedicated to a presentation of the way of "opinion," namely

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—a “likely construction”, an “eokos logos.” But whereas for Plato such constructions are things one proposes when one has a hint of the truth but cannot prove it, precisely such hints and propositions are, by Parmenides, interdicted.

Parmenides, then, far from being the progenitor of the totalizations and hierarchies of subsequent western thought, and thus the figure most to be assailed at a time in history when western thought is tottering, stands outside of the whole tradition as a monstrosity and a castigation against it, out of its origin. Parmenides was a hope that has not yet been fulfilled because his actual proposition has never been seriously entertained or analyzed with any particular depth.

The connection between the Parmenides poem and the cultures of India has been remarked upon. Something like Parmenides’ sphere of Being exists as a symbol for the Buddhist’s principle of the Dharma-kaya; and, the expression of the absolute object of contemplation through the denial of a series of predicates is a common feature of Indian absolutism. Ingenious analyses have been offered likening Parmenides’ chariot journey—with its axle that gives off musical tones and radiates heat and light—to the processes of Kundalini Yoga. That Parmenides was educated as a Pythagorean is part of the Parmenides tradition. That Pythagoras’ doctrine, with its vegetarianism, metempsychosis and general soteriological scheme owes *its* similarity to Indian metaphysics to Pythagoras’ personal journeys to the east is part of the Pythagorean tradition. Thus the possibility exists that the Parmenides poem reflects a hidden dimension of cross-cultural fertilization between India and the Ancient Mediterranean.

Another aspect of the Parmenides poem that looks suspiciously like typically Indian approaches to epistemology is its bipartite structure: the poem contains a doctrine of “two-truths,” one pointing towards an absolute that cannot be expressed properly in speech but can be acceded to through direct realization, the other a relative, conventional truth—a matter of historical convenience, common sense, or heuristic practice. The doctrine of two-truths is prominent in both Buddhist and Hindu metaphysics. Whether or not Parmenides’ poem owes anything to the east in this regard, the division of the poem into two contradictory doctrines deserves comment. A text that argues elaborately for the connection with India and which we discussed in Rhinebeck is Oscar Marcel Hinze’s *Tantra Vidya* (Motilal Banarsidass, Delhi, 1979). I will reserve my own remarks on this topic for the supplement to this issue of *Io* where there will be further material

dealing with Parmenidean questions.

My translation of the Parmenides poem provides a few new readings but I was mostly concerned to take the text seriously rhetorically as a poem intended to celebrate the goddess Dike. Once again, I reserve comment on specific interpretative aspects of the translation for a supplement to this issue of *Io*.

Translation of course *implies* interpretation, and the question of interpretation/translation was an important aspect of the entire Rhinebeck enterprise. Quasha's poems to some extent "translate" the states of cognitivity internal to their own composition, as well as interact in concrete and subversive ways with the "interpretative" machinery and habits of their readers. Hennix's text "translates/interprets" texts from the logico-mathematical tradition—Frege, Wittgenstein, Brouwer—in part by inserting them in a series which includes Parmenides on the one hand and the Japanese *Hekigan Roku* on the other.

L.E.J. Brouwer

After Parmenides, a second personage studied by us in Rhinebeck was the modern Dutch mathematician, L.E.J. Brouwer. Brouwer is the founder of the Intuitionist School of Foundations of Mathematics. He was himself a contemplative and a mystic, and the inspiration and motivation for his mathematical work remained throughout his career inseparable from his spiritual concerns. Brouwer is a major influence on modern mathematical theory, both behind the scenes and center stage, though his work is not frequently studied today in academic circles even where intuitionism of a certain type is in favor. He is, however, a major influence on both A.S. Yessenin-Volpin and Christer Hennix. The appendix to this introduction includes several of the key passages from Brouwer's work that express his views on the contemplative life. I will attempt, in due course, a conjecture as to the connection between mathematics and contemplation his work suggests.

In order to see Brouwer's position in mathematical thought, it is necessary to grasp how the intuitionist school compares in outlook with other mathematical philosophies. I will sketch these views very briefly, and refer the reader to the introduction and opening selections in the anthology of essays edited by Paul Benacerraf and Hilary Putnam, *Philosophy of Mathematics, Selected readings*, (2d edition, Cambridge University Press, 1983) for further discussion.

It is often remarked that there have been three distinct approaches to the question of the ontological status of mathematical objects in

modern times:

1. classicism (or platonism)
2. formalism (or conventionalism)
3. intuitionism (or constructivism)

with constructivism playing a part in formalism.

For the classicist or platonist, mathematical objects are objectively-existing entities. The mathematician discovers these entities and studies their properties in a manner analogous to the way a natural scientist discovers and studies natural phenomena. Thus, the intellectual activities involved in developing mathematical systems are extrinsic to the systems themselves. Natural numbers, ideal geometrical objects, axiomatic structures—in short, mathematical entities in general—are considered to be existences independent of the thinking mathematician. Mathematical language expresses but does not determine mathematical truths, and the entire mathematical universe is conceived to be a hierarchy with logic and set theory at the apex.

In contrast, for the formalist, mathematical objects are purely conventional. They consist solely of written marks and the rules devised by persons for the manipulation of those marks. Mathematical systems have no ontological significance apart from the ontology implied in their being concrete inscriptions of signs. Whereas for the classicist a proof is a demonstration that certain objects exist in an ideal mathematical cosmos, for the formalist a proof is simply an algorithm: a strict procedure for deriving strings of symbols from other strings according to perfectly explicit rules. Any relationship to “reality” that such systems may possess derives from interpretations or “models” of those systems: ways of associating certain strings of symbols with physical or other realities. The intellectual activities involved in developing mathematical systems are perfectly extrinsic to the systems themselves, except in the sense that the activity of devising the rules of procedure is an intellectual activity. The rules once devised, however, can be executed perfectly well by a computing device without reference to human intervention. Mathematical language (the marks and the rules for use) constitutes the whole of mathematics.

For the intuitionist, mathematical objects are constructions of the human intellect: the “creating” or “creative” subject, to use a phrase coined by Brouwer. A mathematical object is intrinsic to the intellectual activity involved in thinking it. It is neither true by convention, nor is it a feature of the objective universe. In fact, in its most radical formulations, both the objective universe and the world of human

convention are themselves derivative of the mathematical activity of thinking subjects. A mathematical object becomes real by its being thought, by its being constructed concretely in the mind of a thinking mathematician.

While classicism is in a sense derived from platonic philosophy with its emphasis on the reality of abstract ideas, its modern developments are due more to the mathematicians of the Renaissance and finally to Descartes than to the notions of the Greeks. (For an excellent account of the development of Renaissance mathematics and how it differs from that of the properly Classical period, see Jacob Klein's *Greek Mathematics and The Rise of Modern Algebra* and Don Byrd's essay in this issue.)

Modern formalism is associated mainly with David Hilbert. Intuitionism derives ultimately from Kant's apriorism, but gets its major impetus from Brouwer. Brouwer's approach to mathematics was based as we have noted, upon a perspective that was decidedly "mystical" or "contemplative" in origin. Brouwer's Ph.D. thesis, *Over de Grondslagen der Wiskunde*, in which many of his major mathematical notions receive their initial formulations, was composed simultaneously with his *Life, Art, and Mysticism*. The two texts were intended to complement each other, though his thesis advisor forced him to remove the passages from his thesis which would have provided the links between them. Brouwer never retracted his mystical views, however embarrassing they proved to his tutors and subsequently to his disciples.

Brouwer's intuitionism begins with a severe critique of classical mathematics on numerous points — points which were later taken up by Hilbert and his followers and incorporated in the conventionalist program. Later too, Brouwer's intuitionistic reconstruction of mathematics was rewritten by his disciples without grounding the system upon the fundamental intuitionistic acts that had been their origin. In other words, much of what is presently called "intuitionism," though derived indeed from Brouwer's alternative methods for constructing mathematics, is intuitionism in name alone in that it does not seriously depend upon the concrete acts of intuition in the mathematical thinker's mind.

Brouwer viewed mathematics as being built up from certain profoundly private internal acts of the thinking psyche. These acts take place in the loneliness of the thinker's internal life and constitute the inner gestures performed between the self and its deity. Mathematical

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intuition is thus fundamentally independent of language —independent even of the disambiguated language of mathematical thought —and profoundly removed from the language of everyday. Mathematical language functions to help the thinker remember his thoughts or to aid in the stimulation of such thoughts in others, but is finally inessential to the nature of mathematical truth itself.

The controversy between the three main schools of mathematical thought of course touches many philosophical topics, but let me confine myself to the following two:

1. the question of the ontological validity of natural science;
2. the link between intuitionistic thought and contemplative modes of enlightenment.

Natural science aspires to validation through a certain hierarchical organization. The social sciences and “sciences of man” must depend upon the biological facts pertaining to the human species as discovered by biologists. Biology in turn grounds its analyses of biological phenomena upon the chemical substrate of those phenomena; and chemistry has achieved “certainty” through its dependence upon quantum physics. Physics is the ideal and basis for all natural science in that its findings are expressed in precise mathematical language. Indeed, the truth claims of the physicist are really limited to the formulae and equations which depict the regularities discovered in the experimental data. Thus, if physics possesses determinate ontological content, that content both determines the ontology of the other sciences and rests upon the ontological import (or lack thereof) of the mathematical language in which it is expressed.

Of course this view of things is over-simplified and not without its challengers, and would need much qualification to be made precise —there are numerous biological and sociological concepts which are not derived from the position in the above hierarchy: concepts of structure, function, pattern etc. But all such concepts apply to physical systems which in turn have their analyses on the next hierarchical level. If the independent concepts are to be given ontological significance without recourse to this founding level, the entire edifice of physical science loses its coherence, and ontological principles outside of physical science must be sought to ground these structures. Again, certain phenomenologists argue that the hierarchy sketched above itself grows as “regional ontologies” or as derivative structures from more primitive domains. But in such cases the ontological significance of science is sought outside of the domain of science. The point here is simply that if

the edifice of science as a whole demands recognition as positive knowledge, then it must be able to provide its own ontology. In any case the ontological significance attached to the mathematical language in which the base of the system is expressed will determine just what the ontological significance of the sciences are. It will be argued by some that, presently, science proceeds not as an organized hierarchy of disciplines, but as a multiplicity of heterogeneous activities with various relations to each other and to the world. But this condition of incoherent multiplicity follows from the abandonment of ontological finality as a desideratum, a consequence of precisely the formalist philosophy. At the very least what can be said is that the current state of things rests upon a philosophical outlook which itself is far from unanimsously defended.

In any case, the ontology (or lack thereof) of natural science depends upon the nature of the "truth" inherent in mathematical language, or the nature of mathematical validity itself. If there is NO CONSENSUS on the nature of mathematical language, any consensus the physicists arrive at regarding their own theories must be considered indecisive. The many assertions that the ontological meaning of the physicists' theories can well remain indeterminate (since it is the pragmatic, i.e. technological, efficacy of these theories that really backs up their truth claims) are all well and good IF one is willing to admit the reduction of all science, including theoretical physics, to the province of technology. This admission would entail that there is no "pure" science at all, and that to look to science for anything more than technical means for attaining practical ends, must be renounced.

We turn now to a discussion of the link between intuitionistic philosophy and contemplative concerns.

We will see the intuitionist in two quite distinct though related attitudes. One of these is in fact not incompatible with at least certain formalist-constructivist possibilities and has a reasonable number of adherents (including, in a sense, Yessenin-Volpin, though Yessenin-Volpin goes much further than other formal-intuitionists in emphasizing the concrete realization of construction methods. Also, Yessenin-Volpin's work is ontologically *anarchic* rather than, as is the case with formalism, *indifferent*.) The second attitude, though clearly the attitude of Brouwer himself, has few vocal adherents in the mathematical community. Let us call the first attitude *formal intuitionism* and the second, *intuitionism proper*. Formal intuitionism and intuitionism proper do not differ necessarily in their propositional content, but

rather in their ontological stance. Thus, I will sketch first formal intuitionism, and what I say here can be assumed to hold for intuition proper as well, though not conversely.

Formal intuitionism coincides with classical mathematics when dealing with finite mathematical objects. But the main developments of modern mathematics have explored the rich domain of infinite or "transfinite" magnitudes and orderings, so the coincidence is in fact relatively trivial. Formal intuitionism denies the law of the excluded middle for infinitary systems. And this denial does not refer in particular to the demand that the value "undecidable" be admitted as an alternative. In classical mathematics, a proposition is *undecidable* or *unsolvable* if the application of the procedures of the system yields NEITHER the values "true" nor "false", when carried out. Such propositions are said to be *independent* of the formal system chosen and they may therefore serve as *axioms*. Hence, the Russell Paradox in naive set theory can be considered undecidable, and in certain interpretations of the "Liar Paradox" the liar-statement is judged to be undecidable. But for the intuitionist, since an object cannot be said to exist until it has been constructed, nor can it be denied that it might exist until it has been shown that it cannot be constructed — there will be objects whose existence can neither be affirmed nor denied. These objects will also not necessarily lead to paradoxes. The point is not that the existence of such objects cannot be decided formally, but rather that their existence HAS NOT YET been determined. Time distinctions thus become an important part of Intuitionist thinking, especially, in the system of Yessenin-Volpin.

A frequently cited example:

Consider the decimal expansion of Π . A procedure has long been known for indefinitely extending the sequence of digits defining the value of this irrational number. Today, this procedure is being carried out to ever-further decimal places by computers, so that progressively the value of Π will grow more and more lengthy and more and more "precise". Now consider some arbitrary sequence of digits (a,b,c, . . . n) that has not as yet turned up as a particular segment of the sequence of digits constituting the decimal expansion of Π . We ask whether the proposition:

P.1. "(a,b,c, . . . n) belongs to the decimal expansion of Π "

is true or false? According to a classicist, this question has a unique answer that already exists: it just hasn't been discovered yet. But the

intuitionist points out that, although as soon as $(a, b, c, \dots n)$ has been found, the value “true” (or not false) can be assigned to the proposition, since the expansion of Π does not terminate, the procedure could be applied indefinitely without our having a proof that $P.I.$ is either true or false. There never will come a time when we will have generated a sufficiently long enough decimal expansion to decide the question negatively. The intuitionist therefore argues that the answer to our question does not at the present time exist and that the classical position has been refuted. We cannot know anything about the properties of such a sequence until we actually construct it.

The example of the decimal expansion of Π is not trivial. The question it raises infects the entire question of the “existence” of irrational numbers (which are understood as non-repeating non-terminating decimals) and through these to the understanding of the structure of the mathematical continuum, i.e. the points laid out on the “real line”. For Brouwer, the continuum is not a unique concept with eternal properties to be established once and for all. Its nature varies with both the specific details of its construction and the precise degree to which procedures of its construction, at a given point in time, have been carried out.

Formal intuitionism thus amounts to a restriction on logical rules when applied to infinite objects. As such, it can be taken over whole cloth and given a formalist interpretation. Such in fact was carried out by students of Brouwer; and, today, when logic itself has been generalized within Category Theory, Intuitionistic Logic and Intuitionistic Set Theory appear as formal variants of mathematical structures, as legitimate within logic as non-Euclidean geometries are within modern geometry.

But what is the relationship between formal intuitionism and intuitionism proper — intuitionism as invented and presumably practiced by Brouwer? And how does this relationship bear on the possibility of a rapprochement between mathematical thought and contemplative consciousness?

Brouwer’s intuitionist mathematics cuts down severely on the universes (species) of mathematical objects. The motivation for doing this was to provide a foundation for mathematics. In prosecution of this aim, Intuitionism is in competition with the project initiated by Frege, watered down by Russell and Whitehead, and brought to a head by Hilbert and his school (including the young Gödel), to reduce all of mathematics to a *finitary* structure such as, for example, a finite set of

axioms. Frege had actually proceeded along intuitionistic or semi-intuitionistic lines, identifying logical norms with general principles which can be *effectively carried out* (though he is commonly thought to belong to the classical camp); Hilbert, working formalistically, also viewed logic with a constructive interpretation where, in particular, the concept of *decidability* was emphasized: you stipulate the rules of logic explicitly so that they can appear as algorithms. But the goal of Hilbert's project was to reduce all of mathematics to a finitary system called *metamathematics* (or *proof-theory*) and, as a by-product, to obtain a *consistency proof* for classical arithmetic by the use of logical principles even more restricted than Brouwer's! In fact, this is precisely Hilbert's "second problem." In contrast to both of these, Brouwer thought all intellectual activities carried out by means of notational systems (whether those systems are derived from ordinary language as in philosophy or inscribed in the rarified notation of the formal logic of Russell, Whitehead et. al.) — that all intellectual disciplines of this sort are in fact initiated by human conventions for human convenience. Mathematics, however, is based on fundamental and inalienable intuitions that precede all logical or linguistic norms. The foundations of mathematics is not to be sought in the logic of formal systems at all. Mathematics is to be founded by linking it to fundamental intuitions of thought-processes of the thinking subject. Moreover, these intuitions of thought-processes are at the basis not only of mathematical thinking, but of consciousness itself. Brouwer's speculation founds mathematics upon specific modes of human intuition, while at the same time finds human consciousness itself to be a form of mathematics (mathematics being understood precisely in the intuitionistic sense).

The fundamental intuition that grounds both human consciousness and mathematics, Brouwer calls the awareness of "a move of time." A move of time is a minimal element of consciousness. It is similar in nature to the notion of a "point-instant," (familiar from occasionalist thought in its various forms—Islamic, Jewish, Buddhist, Cartesian, Whiteheadian, Russellian, etc.) i.e., the notion that time (and the material objects, or moments of awareness within time) consists of a linear continuum of point-like moments of infinitesimal duration, which vanish as they arise and are replaced spontaneously by successor moments. This concept of time, however, abstracts from dynamic temporal experience and reduces the enigmas of temporality to the problem of the linear continuum. For Brouwer, the linear continuum itself is to be built up from the experience of time, so such a

reduction is impossible. What is required is that certain fundamental properties of the continuum (for instance, that neither temporal continuity nor discreteness are derived from each other) must already be present in the temporal experience from which they are to be extrapolated. Brouwer accomplishes this by asserting that the fundamental element of temporal consciousness already has an irreducibly complex structure: it is not a point-instant, an indivisible, durationless unity, but a "twoity"—a dyad with the following characteristic: each moment of consciousness spontaneously splits into two parts—1, the trace of itself surviving as a spontaneous memory; 2, the succeeding moment. This new moment repeats the process, similarly splitting into the trace of itself and its own successor. Both the dynamic character of time and the way consciousness is bound up with temporality are thus built into the fundamental concept. The duality, continuity/discreteness, is also established thus at the foundation.

The discovery of the twoity as the fundamental entity is to be an act the thinking subject must perform for itself. It involves a deliberate concentration of consciousness on its own streaming, and a concrete wresting of this fundamental abstract structural core. The twoity and the succession of twoities are to be discerned as the "empty" substratum for all experience by an act of self-intervention that the reader may feel, as I do, is profoundly connected to the act of "turning into oneself" that Brouwer recommends in *Life, Art, and Mysticism* (see Appendix II). Brouwer does not explicitly draw this connection, but whether or not his mystical speculation will be experienced as relevant to his mathematical thinking depends upon our providing that link. Indeed, whether or not his demand for the restriction of the use of the law of the excluded middle can be connected to his founding of mathematics on radical temporal experience may depend entirely upon whether or not one undergoes that radical temporal experience. In any case, from the procession of empty twoities, one proceeds to develop the elementary series of mathematic thought.

Now the procession of twoities when it becomes explicit for consciousness finds the construction of the conscious intellect, yet at the same time, the procession of twoities itself is inherent in the subject. It is through processes directly analogous to the development of explicit mathematical thinking that consciousness builds up a world. The link between mathematical thinking and the development of consciousness (what Hennix will call attention to as the link between *semeiosis* and *cosmosis*) is the notion of a series. Consciousness expe-

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riences two species of series: one, it discovers as causally-related temporal sequences; the other, it constructs into things—objects with durative identity. A causal sequence requires that the order relationship among the members of the series be preserved, while an object is coalesced from a series of impressions whose members may be considered independently of their position in that series. Brouwer writes in “Consciousness, Philosophy and Mathematics” (1948; *Collected Works*, p. 1235):

An iterative complex of sensations, whose elements have an invariable order of succession in time, whilst if one of its elements occurs, all following elements are expected to occur likewise, in the right order of succession, is called a *causal sequence*.

On the other hand there are iterative complexes of sensations whose elements are permutable in point of time. Some of them are completely estranged from the subject. They are called *things*.

Our belief that event X is the cause of a subsequent Y which in turn is the cause of a third event Z, depends upon the temporal order of X, Y and Z. If Z precedes X, it normally cannot be the cause of it. But the sequence of momentary impressions of an object O can be recalled in any order we choose without disturbing our belief that the object O is the same entity, perduring throughout its various momentary appearances. Causality and thinghood are thus both brought under the single concept of a series, and the latter concept is derived from a fundamental intuition of time. The world is built up by the soul as a complex concatenation of both kinds of such series.

At this point I refer the reader to the passages from Brouwer’s work included as an appendix to this introduction. These passages bear upon the notion of the sequences of twoities, and the relation between Brouwer’s intuitionism and his mysticism. First, Brouwer’s statement of “The Two Acts of Intuitionism” (Appendix I). These passages were repeated in subsequent works and stand as the founding statements of intuitionist thought. They will be followed by excerpts from *Life, Art, and Mysticism*. Both sets are presented as selected by Hennix and published as excerpts in his text *Brouwer’s Lattice*. *Brouwer’s Lattice* was “required reading” at the seminars and the excerpts represent what Hennix wished to present to us of Brouwer’s mystical thought.

Twoity

There have been very few commentators, besides Hennix, on the connection between Brouwer's contemplative and mathematical views. It has seemed difficult to many to see how Brouwer on the one hand derives arithmetical systems from his twoities, and how, on the other hand, the system derived from twoities and series of twoities link to his mystical ideas.

If the serious business of the soul involves a turning into the self and a turning away from "the sad world" of constructed, willful, public reality, as Brouwer maintains in *Art, Mysticism and Life*, how can the creation of a mathematical universe facilitate this "turning into the self"?

Tentatively we can conjecture that Brouwer's construction of mathematics provides him with the possibility of exhausting the activity of his own intellect in a way that liberates him from the cunning and deceit of the "Sad World." Brouwer's intuitionism provides, at every step, links back to "the Creative Subject's" own activity in constructing its cosmos and provides a system of procedures which makes it possible for anyone whose motivations are congruent with his own to retrace the path of humanity's intellectual catastrophe (as outlined in the excerpts given in our appendix), and, at the same time, to avoid both the absolutism of classical mathematics and the nihilism of the formalist school.

I will develop this conjecture by considering how Brouwer escapes from the deceptive consequences of the concatenation of the Sad World. (I use Brouwer's phrase "the Sad World" to refer to the world of human experience and culture as built up from serial concatenations.) This will involve first of all an interpretation of how both arithmetic and consciousness are derived from the series of twoities.

The generation of the twoities in Brouwer's system can be interpreted as developing along a pair of axes: the reiterated series of empty twoities proceed concretely in time along a horizontal axis; processes of intellectually constructing further series of twoities (or multiplicities of series of twoities) occur along the vertical axis.

The horizontal series of twoities, developing in time, constitute a concrete experience of emptiness—a fundamental experience of time to which the Creative Subject returns *ad libidum*, as a way of "emptying out" its consciousness, at any point in the process of construction. The series of empty twoities forms a fixed series, a bottom line to which

consciousness is always free to return. It is the basis of all constructed existence, the reiterable fundamental intuition grounding and re-grounding both thought and the world grasped by perception and thought. At every moment it is independent of all concrete content, yet, because it is wedded to a specifically subjective experience of time itself, it is the most concrete of things.

The vertical series of constructions represents successive generations of indices for chains of positive cognitive acts, perceptions, and mathematical constructions. The complexity of our world and of our thought, and our bewilderment before both—all derive from the interplay of these vertical constructions proceeding in forgetfulness of their purely derivative character.

The vertical axis is built up as follows:

Arising from any twoitly along the horizontal axis, conceive a series of abstract constructions. At the position immediately above any selected twoitly, find a conceptual copy of that twoitly together with a copy of the two unities of which the original twoitly is composed.

These two unities, it will be remembered, represent a pure moment of sentience (divested of all content) which has spontaneously split into a successor moment and a trace of itself. The abstract copy of the original twoitly together with the copy of the two unities constitutes a new twoitly: a new bi-unity, an abstract representation of the original pair of unities of which the original twoitly was composed, and the conceptual combination of these two unities into a new bi-unity. This process can now be iterated, each new abstract twoitly being composed of the two prior unities together with their unification into a new unity, constituting once again a new bi-unity. Each successive iteration of this process grows more complex, in the sense that the content of each consists of larger and larger complexes of twoities and unities. And the series of ever greater complexes of twoities and unities will be able to serve as an index for ordering other sequences of objects, perceptions, or constructions. The series of twoities, ever growing in complexity, serves as a construction of a natural number series or even the continuum of real numbers itself.

While the horizontal series takes place in the concrete real time of the Creative Subject, with each twoitly existing only in its own moment and surviving only as its own trace in the successor twoitly, the vertical series generates an atemporal conceptual space, in which each successive abstract twoitly enjoys a kind of hallucinatory pseudo-existence.

Each successive stage in the development of the "vertical" series

must take place within the real-time existence of the creative subject, yet it is precisely the property of this "ascending" series of abstractions to neglect this temporal character, and to generate a symbolic space in which predecessor twoities continue to exist as successor twoities are formulated.

Though we have been referring to the original succession of horizontal twoities as a series, as a matter of fact, the possibility of anything existing in series only emerges with the vertical series. The horizontally-extending emergings of twoities perpetually perishes, leaving only its traces in successor twoities. It only becomes possible to identify the horizontally emerging twoities *as* a series after we have indices for them. And this becomes possible through the fact that the ordering of the vertical series can serve as a system of indices. (The possibility of the construction of the vertical series, however, derives from the fact that the horizontally-extending twoities have a dyadic structure: the individual point-moments of temporal consciousness are in fact bi-unitary, for it is only by abstracting from the possibility of a trace of a predecessor moment that an abstract series of ascending twoities can be formed.)

The vertical series of twoities can, as we mentioned, be treated as a natural number series: a set of indices whereby other series of otherwise absolutely transitory phenomena in the experience of the Creative Subject can be linked to a conceptual space which enjoys quasi-independence from the uncompromised evanescence of the temporal domain. Thus, the two species of series (which Brouwer identifies with objects and causal sequences) are concatenated by consciousness by linking them to members in the vertical series of twoities which, in the form of natural number series, serve as indices for their members.

The existence of both objects and causal sequences are capable of being cognized by the Creative Subject because they can be indexed by the members of the vertical series of twoities.

But because the entire vertical series of twoities has no concrete existence, the Creative Subject is at every point of its existence free to "drop down", as it were, to the horizontal axis and recover the emptiness of its own actual condition. The vertical series of twoities (and the causal and objective series for which it serves as an index) only exists as long as a concrete Creative Subject sustains it and no longer. The entire construction of the Sad World is thus seen as being at every point capable of being collapsed back into the empty but concrete continuum of the creative subject's only authentic life.

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This possibility of “dropping down” to the horizontal axis at any point in its life allows the Creative Subject to remain free from ensnarement by the construction of intellect and consciousness, free at every moment to resume its silent communion with its spiritual ground.

In Hennix’s work, the concept of the Creative Subject, taken over from Brouwer, together with this bi-axial construction, gets explicit development. In his “Theory of The Creative Subject” (page 383) the atomic conceptual acts called “noemas” correspond to Brouwer’s series of temporal twoities, while the vertically-rising series of denotational connections correspond to Brouwer’s vertical axis of intellectually constructed twoities. But before looking at Hennix’s work we need to sketch some of the main ideas of Yessenin-Volpin. We will also look at two further topics of general interest that play key roles in Hennix’s thought: Category Theory and The Procedure of Diagonal Substitution.

A.S. Yessenin-Volpin

Alexander Sergevich Yessenin-Volpin is a Russian refugee who has recently become a naturalized American citizen and who presently lives in Boston. He is perhaps best known as a co-founder of the Soviet Human Rights Movement (together with Sakharov), but he is also the founder of “The Ultra-Intuitionist School in the Foundations of Mathematics.” An outline of the program for this school is contained in an article published in Kino, Myhill, Velsey: *Intuitionism & Proof Theory* (Amsterdam, 1970). Though it is far too subtle and intricate for me to summarize fully in these pages, I strongly recommend it to the readers of *Io*. It makes few demands upon the reader in terms of formal preparation, develops its own symbolism quite lucidly, and is thus a text which can be read by the interested novice.

To give a sense of who Yessenin-Volpin is as a human-rights activist, I quote here from a biographical, editorial introduction to an article by Yessenin-Volpin from *The Humanist*, January/February 1973. “After many attempts to leave the Soviet Union failed, Aleksandr Yessenin-Volpin, distinguished mathematical logician, was finally granted an exit visa in May of 1972. . . .

“Dr. Volpin had been a key figure in the democratic movement in the Soviet Union, and, as a result, he had been constantly harassed and intimidated by the government. During Stalin’s era, he was committed to a mental institution for writing two anti-Stalinist poems. . . . After Stalin’s death he received a pardon. He edited the section on Mathematical Logic of the journal *Mathematica*. . . . In 1959, he was invited

to participate in an international symposium in Poland but was denied a visa. He was imprisoned in 1959 and put in a special mental hospital in Leningrad, but was released in 1961. . . .

“On December 5, 1965 (Constitution Day) he participated in a demonstration on behalf of Siniavski and Daniel in Pushkin Plaza, Moscow, carrying a sign reading “RESPECT THE CONSTITUTION! He was arrested, but released. . . . In February, 1968, he applied for a visa to travel abroad. He was, for the fifth time, incarcerated in a mental institution. A letter of protest from 95 mathematicians and scientists helped to secure his release.”

Yessenin-Volpin calls his program “an anti-traditional program” and it is based upon a putting into doubt of at least eleven principle assumptions usually left unexamined in classical mathematics. Rigor in mathematics generally develops by making explicit assumptions which hitherto had been either concealed or taken for granted. Making such assumptions explicit may have the destructive but salutary effect of pointing out contradictions hidden in what had seemed to be a secure system. But such destructive discoveries most frequently lead to a deepening of mathematical understanding and the opening of fresh ground.

Ground-breaking criticism of this order, however, frequently meets with a two-fold resistance: first, the desire not to shake unnecessarily the foundations of systems and practices apparently in good use; second, the seeming self-evidence, triviality and harmlessness of the assumptions exposed. Novices in mathematics complain that much discussion in foundations amounts to quibbling over what seem to be pointless differences. Yet it is precisely at the level of seemingly unquestionable assertions that the structure of our unconscious cognitive commitments reveal themselves. For this reason, Yessenin-Volpin operates on the principle that everything should be open to question.

I will not be able to examine all the points that Yessenin-Volpin challenges, but will sketch out the implications of a few of them. First of all, he rejects the uniqueness of the system of natural numbers. The natural numbers are not a system of entities existing in a single form for all time. They must be constructed concretely for each context of use. And different constructions can be brought into comparison. Yessenin-Volpin shows that it is possible, for instance, to disagree about the nature of an infinite series of natural numbers: the classical concept of infinity holds that the natural numbers form a completed totality. Yet it is possible to hold that such a totality does not exist, i.e. that the natural

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numbers only extend as far as it is effectively possible to compute them. To give an *informal* example: a number such as $[10^{12}]$ to which no one has ever counted, since it is not a *humanly* (non-computer-aided) feasible number, can be defined as "playing the role" of an infinite number in a humanly-feasible substructure of arithmetic. Since it can be shown that it is possible to construct number systems according to these two different concepts of the meaning of the term "infinity," it is thereby informally demonstrated that the natural numbers do not form a unique system, i.e. that axioms of limitation of size in arithmetic are *independent* from each other. Yessenin-Volpin generalizes this idea presented as systems in which two (or more) number systems are employed concurrently, and specific rules for embedding one inside the other are established. (The uniqueness of the natural numbers is a main feature of the classical belief that mathematical entities are objective and independent of our constructions. The recognition of the plurality of number-systems actually is a return to a way of thinking about numbers which was current among the Greeks. See Jacob Klein's *Greek Mathematics and the Rise of Modern Algebra*.)

Yessenin-Volpin also doubts the principle of mathematical induction and the assumption, which he calls the "locality principle," that, "If the axioms of a formal system are true and the rules of inference conserve the truth, then each theorem is true." Induction is the principle which allows the classical mathematician to assume that all the natural numbers "already" exist without having to be constructed or to prove that every natural number is finite. (0 is finite and if n is finite so is $n+1$. Then all natural numbers are finite.) Induction allows the mathematician to assume that properties that hold for numbers hold for their successors. Volpin shows that the locality principle and induction require each other: to derive the theorems from an axiomatic system, you must assume that induction holds; but, to demonstrate the truth of induction, you make use of an axiomatic system and in particular the locality principle. The two principles form a vicious circle and therefore ought not to be used at the *foundations* of mathematics.

The above considerations do not mean that there cannot be explicit and well-defined employments of axiomatic systems. They only imply that such systems do not exist as absolute constructions, guaranteed to formalize truth without any prior considerations as to the specific aims and conditions of their being set up. In other words, Yessenin-Volpin requires that whatever mathematical system one employs, the aims and rules for its constructions must be specified explic-

itly beforehand. The terms, demonstrations, and deductions of a classical system are only what he calls "termoids," "demonstroids" and "deductoids": i.e. they are demonstrations and proofs which require further comment showing that their construction has been carried out in conformation with the explicitly stated "proto-theories": the definitions, rules and aims that are established before the axioms and rules of inference are laid down. In particular, in order for a formal proof to be "convincing" all termoids which occur in such a proof must be shown to be *terms*, i.e. signs which demonstrably carry denotations.

The fact that not only definitions but AIMS must be stipulated before a mathematical system gets under way, radically relativizes and concretizes the entire practice of mathematics. It effectively destroys the insularity from social, personal, and ethical contexts that mathematical thinking has traditionally enjoyed. For if it is required that aims for mathematical thinking must be made explicit, then very general considerations about the nature of aims and means become directly connected to mathematics.

The Logic of the Moral Sciences is a treatise stating precisely the ethical principles which he recommends. As an ethical theory it is distinguished from other formal ethical theories by the fact that it does not reduce general ethical principles to a formal language assumed to be valid independently of ethics. Rather, it attempts to think through a set of principles which, though far broader than those involved in specific mathematical formalisms, nonetheless are required in order to specify the conditions under which mathematical systems are to be set up. These principles are concerned with the nature of aims and rules, and thus find applications in other areas of human endeavor where aims and rules are relevant. In particular, Yessenin-Volpin develops his ethical theory in relationship to matters of jurisprudence. But it is clear that an extension of these ideas could be developed, say, in aesthetics, or religious practice.

Two aspects of Yessenin-Volpin's system deserve special comment. First, Yessenin-Volpin defines logic itself as *the science of avoiding errors*. Errors are understood relative to aims. Logic then is a discipline that must be developed locally in every case relative to specified goals. It is not an aprioristic science that legislates demands about what we should or should not think without regard to the purposes for which we ourselves in each case employ logic. The consequences of logic are thus only binding relative to our own intentions.

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Second, relative to situations in which we cannot afford to make errors—where, in other words, logic is indispensable—it is propitious that we should wish to banish from our considerations acts of “faith.” Faith here means trust in assertions adopted without “sufficient reason,” i.e. without proof. But proof does not mean in general the algorithms by which theorems are generated from axioms according to inference rules. A proof is rather “any fair means of establishing incontestability.” “Incontestability” is not defined independently of ourselves as concrete users of logic; and, even in the face of a proof which we are unable to contest, we retain the freedom to neglect it. Though Yessenin-Volpin’s system demands a level of rigor unprecedented in any prior mathematical philosophy, the point of the rigor is not to establish an absolutely binding universe of truths, but to secure explicitly our freedom in regard to the employment of our reason.

Another interesting feature of Yessenin-Volpin’s ethical system is the concept of a “logic of confidence”. Occasions arise where it is not possible to banish faith utterly from our considerations, but where it is appropriate to trust in the assertions of others. The logic of confidence thus involves the syllogism, I have confidence in X; X asserts Y; therefore, I am confident in asserting Y. Acts of confidence are capable of being made explicit, and rules for accepting and denying confidence can be stated.

The relativization of logic also involves calling attention to what Yessenin-Volpin calls “collations” and “tactics of attention and neglect.” Collations are connections between things. They are of two kinds: identifications and distinctions. These are taken as the atomic actions of our reason. They are things we do. Thus “identity” and “difference” are not fundamental ontological categories: they are consequences of our activities undertaken in concrete contexts. Similarly, the rules by which we follow a given rational procedure depend upon “tactics” by which we follow the rules and other tactics by which we give ourselves the permission to neglect certain aspects of the situations in which we are operating. Both these species of activities must be made explicit as a pre-requisite for *any* reasoning about them.

Finally, the relativization of logic and the insistence that all steps in a mathematical activity be explicitly justifiable (in the last instance by the law of sufficient reason) imply that the temporal ordering of our logical activities is non-trivial to their results. It matters whether or not a given assertion has as yet been demonstrated. Thus a “tense-logic” must be developed making all these relations explicit.

All of these matters are sketched out in the paper included in these pages.

Category Theory

Category theory is among the most abstract systems of mathematical thought that exist to date. The “categories” of category theory are themselves whole mathematical systems; thus, the sets in Zermelo-Frankel set-theory are objects in a category, topological spaces form a category, linear algebra is a category, group theory is a category (category theory generalizes the well-known concept of a semi-group with unity). In other words, category theory generalizes over and draws its concepts by abstracting from lower-order mathematical disciplines, much as traditional algebra generalizes over arithmetic or topology generalizes over geometry.

Category theory, however, reaches such a level of abstraction that even logical constants and most of the logical rules applicable in lower branches of mathematics are themselves susceptible to analyses and thus to variations.

Though category theory generalizes over lower disciplines, it remains in its concepts and terms quite free of the disciplines it “categorizes.” It may more fruitfully be thought of as an independent body of concepts from which other disciplines can be *derived*, than as an abstractive generalization from those disciplines.

Initially, the inventors of category theory hoped to provide a foundation for mathematics alternative to that provided by set theory, but that hope was disappointed because of the theory’s extremely high level of generalization. Eventually, Lawvere and Tierney elaborated the notion of a *topos*: a topos adds additional structure to the notion of a category and makes it appear to be suitable for the founding of mathematics. The topos concept, however, turned out to involve intuitionistic principles, (formally intuitionistic principles, that is), so one had the rather paradoxical situation of being in possession of an intuitionistic theory of the foundations of classical mathematics. It is this circumstance upon which Hennix seizes in his work.

The entities with which category theory concerns itself are called “objects” and “morphisms.” An object may be anything at all *conceived of as an object*. Thus, the objects of a category might be sets or natural numbers or topological spaces; but, they might also be states of consciousness or ordinary items of the every day world. The “morphisms”, which are generally indicated by arrows connecting the symbols for

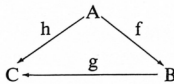
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objects, stand for operations which are performed on, among, or between the objects. If the category is set-theory, for example, the objects would be the sets and the morphisms, the set-functions such as membership, intersection, union, and the like.

In addition to possessing objects and morphisms, in order to qualify as a category, a system must enforce certain rules. Morphisms must be capable of "composition": that is to say, a transitive relation between morphisms must obtain, such that the morphisms can be performed in sequence: if an arrow f connects an object A to an object B

$A \xrightarrow{f} B$, and another arrow g connects object B to object

$B \xrightarrow{g} C$, there will always be a third arrow h called the *composition* of f with g connecting A to C . The diagram



is called a "commutative" diagram and is used in category theory to render, in a compact and intuitively obvious notation, the relations between the objects and the morphisms. The composition of arrows must be transitive except in the case of the "empty category": the category which has no objects and no arrows. If a mathematical system does not exhibit transitivity, then it cannot be generated within category theory, i.e. an intransitive system is represented by the empty category.

In standard developments of category theory the empty category has little application. As most mathematical systems affirm transitivity, the empty category is developed within category theory mostly to complete the symbolism. Yet in Hennix's use of category theory, it is precisely the empty or "almost empty" category that proves to be the richest and most rewarding of study, since his concern is frequently with *intransitive* structures—structures which thus occupy what he calls "the interior" of the empty category (interpreted as a set).

Since category theory deals with principles held in common by different systems, special processes are required to establish connections, transformations, and relations between different categories. Such processes are called "functors" and, like morphisms, are indicated

by arrows, pointing now from category to category rather from object to object.

Among the morphisms that get particular attention in Hennix's work are the "pushouts" and "pullbacks." Pushouts may be viewed as arrows that indicate the generation of theorems from axioms within a category, while pullbacks may be viewed as indicating the reduction of the theorems to the axioms from which they have been generated. (These ideas come from algebraic geometry, in particular, sheaf-theory, and are expressed structurally as "Grothendiech topologies".) A functor that gets special attention is the universal "forgetful" functor which eliminates levels of specification between categories. By a judicious use of pullbacks, forgetful functors and the empty category, Hennix is able to indicate structures which obtain for aesthetic and ascetic processes, usually thought to be incapable of logical presentation.

A topos, as we mentioned, is a category with certain important additional structural features. It was through the elaboration of the topos concept that, as stated above, Lawvere and Tierney were able to initiate the employment of category theory as a theory of the foundations of mathematics. Rather surprisingly, even the Environment E discussed extensively by Hennix below is a topos, or rather contains an unlimited "universe" of toposes.

Toposes contain methods for classifying and indexing the objects of the category, so that it becomes possible in a topos to symbolize a collection of objects, relations and operations together with a system of indices distinguishing and classifying those objects, relationships and operations. It is this feature of the topos that makes it an appropriate structure for the study of the logics implicit in cosmologies, psychologies, and all disciplines where objects must be considered together with their modes of representation.

Diagonal Arguments

The Yellow Book deals throughout with constructions that are known in mathematical language as "diagonal" constructions. These constructions extend formal systems where the application of the operations of such systems to certain values leads to contradiction. A diagonal construction often becomes necessary, for instance, where systems encounter self-reference: if a function is taken as an argument for itself, the value of the function is either un-defined (and the system is incomplete), or the system becomes notationally inconsistent (and the system is over-complete), or else the system must be expanded in some

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way to accommodate the self-referential case (making use, for instance, of fixed points in metamathematical extensions). The expansion of the system is obtained by first adjoining to it the operation of "diagonalization" and then picking any diagonal formula as a "new axiom". A situation where diagonalization is exemplified usually occurs whenever a formal system is construed as taking arguments for ALL its values (unrestricted quantification). If a free-variable formula is construed with its *free-variable* formulae as ranging over all its values, then there will occur a case where the system will have to refer to itself. In general, where unrestricted self-reference is allowed, paradoxes arise.

Diagonal constructions occur in mathematics where the limits of a given notation are reached, yet where one insists on the possibility of continuing to apply the operations already defined in particular, iterated compositions of operations with themselves. Thus, the creation of fractions to indicate the ratios between whole numbers, the creation of irrationals to indicate roots of numbers not perfect squares, and the recursive development of multiplication from addition or exponentiation from multiplication are all instances of diagonalization.

In modern mathematics, diagonalization plays an important role in the concept of the "transfinite" numbers of Cantor, and the battle over the legitimacy of the transfinite numbers rages over the interpretation of the diagonal procedure that produces them. Again, it is the "diagonal" case in Frege's foundations of arithmetic, the existence of sets which can take themselves as members, that generates the paradox discovered by Russell which constitutes the central crisis in modern mathematical thought; and a very elaborate diagonal procedure is at the heart of Gödel's famous incompleteness theorems. Finally, the entire issue of what is "unsayable" in Wittgenstein's *Tractatus Logico-Philosophicus* hinges upon the diagonal construction which would have to be provided were the process of "picturing" allowed to picture itself.

There is a singular interest attachable to diagonal and self-referential constructions which has been frequently commented upon in both popular and serious mathematical literature, for self-referential situations do not only show up under formal conditions: they may appear wherever a symbolic system is in use, whether such a system is an ordinary language, a philosophical or technical discipline, a biological system of cognition or perception, a work of art, or a system of spiritual exercises. That is, in any system of representation where the free-variable terms of the system have been allowed to range over the

totality of their domain, there will occur a diagonal case, a term which refers to the system itself. In each such system, the diagonal or self-referential case will have the appearance of singularity, a moment of confusion, an occasion for doubt, or else will seem a justification for the expansion of the system, for a mystical flight into transcendent realms—depending on the context of the system and the perspicacity of its devisor.

A few examples: the diagonal case in a psychological theory occurs when that theory attempts to account for how the psychology of the theorist affects the psychological theory being propounded (one thinks of C.G. Jung's famous remark, "we do nothing but dream the myth onward" or the problem of transference and counter-transference in psychoanalysis); the diagonal case occurs in anthropology, where anthropologists inquire into the cultural conditions of their own research (and of course, the Lévi-Strauss of *Triste-Tropiques* and *La Pensée Sauvage* comes to mind); the diagonal case occurs in literature in a great variety of forms and wherever authors refer to their own texts (numerous instances of this occur in the poems of George Quasha in this issue); in physics, where experimentalists must give a physical account of their experimental apparatus; and in contemplative exercises, where an instruction to attend to phenomena in a certain way must be applied to the phenomena of obeying that very instruction.

It is generally not observed that the consequences and relevance of each of these cases depend totally upon the concrete circumstances under which they arise. Yet, most frequently, the diagonal case forces a re-examination of the terms of the particular system, often causing doubt, dismay, or grandiose mystification as the case may be. Wittgenstein believed that many paradoxes and puzzles for thought, being bewitchments foisted upon us by this general exigency of our notational systems, could be banished simply by noticing and disallowing diagonal situations from our languages. This observation led him to dismiss the significance of both the Russell Paradox, which could be handled by simply eliminating self-referential sets from set theory, and the Gödel incompleteness theorems, which appeared to him to involve "a puffed up" notion of proof. (p. 132., *Remarks on the Foundations of Mathematics*, 2nd ed.)

It is well-known that historically there have been three typical responses to diagonal situations: rejection of the diagonal case as meaningless; rejection of the system as incoherent; or expansion of the system into a new dimension. In the first instance, one rejects the

diagonal case as meaningless according to definition: psychology disqualifies itself from studying the psychology of its own theorizing, a judge disqualifies himself from sitting in trials wherein he may be a party, transfinite numbers are simply denied existence within the mathematical cosmos. But the problem with such a blanket solution is that, though in some cases self-elimination may be appropriate (the case of the judge for instance), in others, the refusal to confront the diagonal case simply begs the question. It may be quite important to know how the psychology of a psychologist affects his psychological theory, or how the physics of a physicist's apparatus influences his experiments. Even transfinite numbers, given appropriate constructions, have their uses.

But if the diagonal case is naively accepted, contradictions in the system arise and the system loses its epistemological coherence altogether: some solution to the Russell Paradox must be accomplished, or else the resulting inconsistencies in set theory render the latter useless for founding arithmetic; if the psychological condition of the psychological theorist determines the content of his theory, the theory is deprived of convincingness; and, if the meditator applies the instruction "refrain from intentional activities" to the activity of meditating itself, he will abandon his session.

Where diagonalization is not abandoned, often a new dimension of the discipline in question comes into view. Diagonalization thus seems to be a moment of creative fertility, wherein the limitations of a notational system are reached and new vistas of conceptualization may be opened. But here the dangers are that of grandiosity and mystification, where irrelevant "acts of faith" (see Yessenin-Volpin below) may be involved. If diagonalization is permitted without restriction, as, for instance, in classical mathematics, we may find ourselves committed to an over-populated cosmos of abstract entities, actually created by the excesses of the notational system.

The later Wittgenstein's relentless assault on Platonic idealism has its roots in his objection to diagonalization procedures, for the belief in the existence of platonic forms has its source in the diagonal situation that arises whenever one inquires about the *existence* not of the entities that fall under a given concept, but of the *concept itself*.

Particularly poignant and anxiety-provoking cases of diagonal substitution occur for the Buddhist meditator, and I will try to show how the example of the Zen student confronting his Koan is analogous to the style of treating diagonal arguments recommended by the ultra-

intuitionist program.

Numerous exercises require Zen practitioners to refrain from conceptual thought, to refrain from activity, or to perceive all phenomena and all thought as "empty" or void. But the thought by which the meditation itself is set in motion is itself a concept, and meditation is itself a kind of activity. Meditators must thus wrestle with the diagonal case propounded by the instruction they are attempting to follow, until a break-through, wherein the experience allowing them to differentiate the appropriate levels of application of the instruction, occurs. Diagonalization, in this way, is at the heart of many Zen "Koans" or "Model Subjects" (as they are termed in the *Hekigan Roku*, quoted by Hennix below.) The meditator is forced to address a vivid contradiction which has the structure of a diagonal argument.

Now, Zen meditators notoriously face the irascibility of Zen masters, and Zen masters may refuse the student's solution to the koan for two contrasting reasons: either the student fails to experience the bite of the koan—refuses, that is, to take the contradiction seriously, never actually confronting the diagonal case (such a student is like the formalist theorist who saves the system by rejecting the diagonal case as simply undefined within it—the specific contradiction is resolved, but that is all that has been gained); or else, the student allows himself to grow dizzy with the koan: he experiences a kind of psychic vertigo or ecstasy, allowing the diagonal dilemma to generate hallucinations and grandiose (even transfinite) constructions. His answers to the koan are up in the clouds; his state of mind is inflated and out of touch with the task of confronting the nature of his own mind—the task that is at the heart of the meditative process. The Zen master cuts him down with some direct reminder of his immediate and earthbound condition. Such a meditator resembles the classical theorist, carried away by the platonistic possibilities of his diagonal constructions.

But successful Zen students are able to see the diagonal construction for what it is and pass beyond it. They attain a vantage point that is not confined or defined by the language in which the meditational instruction is couched. They discover in their own experience the very mind that is struggling with the koan, and they do so not by blanking out all cognitive activity or refusing to confront the contradiction, but by exercising their faculties to the limit. They cover the entire space of their cognitive language and step beyond it—not into an ever more elaborate and mystifying hyper-reality of thought construction, but onto the immediate ground of their own activity. They are thereby able

to grasp the condition and the structure of the exercise which brought them to their insight.

The acceptable solution to the koan is eminently practical, but by no means reductive: the concrete, down-to-earth mind of the meditator is by no means the mundane confusion of the unenlightened: it is ordinary mind brought into contact with its experiential basis, capable of saturating, at a single stroke, the space of its own activity.

The analogy with the ultra-intuitionist position is this: in dealing with the problem of diagonalization, the ultra-intuitionist steers a middle way between reductive formalism and classical ideational realism. Approaches to diagonalization, like approaches to the construction of number systems in general, are multiple, and tailored to meet specific applications. Because the steps in the procedures of construction must be monitored and ordered, and proofs for the actual completion of the steps must accompany their being carried out, the exact range of application of the propositions of the system can be monitored, specified, and varied according to aim and need. It is unnecessary to banish diagonalization altogether, and it is unnecessary to grant unlimited license to diagonal constructions. Rather, the actual procedure of construction is always kept firmly in view, without arbitrarily and obtusely restricting any constructions which might prove to be of use.

The Yellow Book

Christer Hennix's *The Yellow Book* is a work in progress (Part I of which appeared in *WCH WAY # 5* edited by Don Byrd and Jed Rasula), and is composed in such a way that between any two pages a new page may be inserted (dense order pagination). It is my ambition to provide a commentary for this text (and its dense ordering) at some point in the future, but here I will confine myself to a general description of the work and a somewhat random discussion of some of its salient features.

The Yellow Book consists of excerpts from material written over the last fifteen years, some of which appeared previously as part of Hennix's *Brouwer's Lattice, Notes on Toposes and Adjoints* (1976), and *17 Points on Intensional Logics* (1979). *Notes on Toposes and Adjoints* itself was composed from numerous notebooks to accompany an installation of the (E-) environment, *TOPOSES AND ADJOINTS*, at Moderna Museet, Stockholm, 4 IX-17 X, 1976. The text was published as an adjunction to *Brouwer's Lattice* and was at once a catalogue for

the installation, a specification of its intellectual parameters, and an element in the installation itself. The installation was a work of "Concept Art" more or less as understood and defined by Henry Flynt (cf. his texts included below).

Hennix is presently working towards a re-installation of his environment, which an extended version of the present text will accompany. A description of the non-textual aspects of the installation will enhance our understanding of the present text.

The environment TOPOSES AND ADJOINTS (referred to as \hat{E}^* in the text) consisted of a body of works (1-3) in different media elaborating intertransmutable and even parallel aesthetic and epistemological processes.

1. A TEXT, exhibited on a wall outside the exhibition room of the environment and sold as the catalogue.

2. The installation itself, consisting of a museum room containing acrylic diagrams, monochromatic computer graphics displays, a pair of ultra-black paintings; the diagrams are large-scale wall paintings of presentations of figures implicitly given by the Text, the ultra-black paintings painted by the author and executed with pine-soot as pigment for creating an ultra-black color and applied on highly polished (mirror-like) aluminum square-surfaces, the monochromatic computer graphics ultimately reducible to finite texts defining image-specific computer programs.

3. A continuous, composite sound wave form electronically composed in 1970 and left ongoing from that time to the present.

To participate in TOPOSES AND ADJOINTS actually required spending many hours, not to say days, in attendance at the installation. In attending this programmed environment, one might devote oneself to contemplating the composite sound wave form and its formal relationship with the many ways of reading the graphic material as well as the spatial presentation itself while epistemologically guided by a study of the TEXT.

As mentioned above, the various media of the work were conceived in parallel and are systematically reflected in each other. Each medium—painting, wave form, space, or mathematical text—proposes a certain process of composition and follows that process to one or more limit conditions, for example: a surface of a light-metal mirror with particles of pine soot as the ultra-black paintings are executed until they become totally saturated. Though *discrete* (infinitesimal) particles of soot are applied, this application is continued until an

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impression of continuous density is reached. At this point, in a sense, an entire world of possible images has been traversed, and yet the final effect—that of a continuously covered surface—repeats and returns to the initial condition, a blankness or mirror stage in which there is no differentiation of parts (so-called chaotic topological spaces). Sustained contemplation of the black paintings in conjunction with the TEXT, however, recovers all stages of the process: the individual particles of soot become visible again, when perceived as “dissolved” into the printing ink of the text of the formal theory chosen to capture all theorems about a continuum of densely ordered ultra-black soot-particles and for which the paintings act as “models”. That is, the “visibility” of the individual soot-particles becomes a semiotical process accompanying acts of self-illumination.

The process of contemplating the composite sound-wave form takes one on a parallel journey through another continuum. The sound wave is composed of three continuous frequencies (sine-waves), selected and structurally organized by Hennix by a technique of eliminating unwanted frequencies from the range of combination tones generated by a selected fundamental tone and the three concurrent sine-waves. The composite sound-wave form is thus generated recursively and continuously; each infinitesimal moment of the composite sound-wave is generated as a recursion performed upon the moment just gone by. Compositional processes have been reduced to a minimum, yet a most stringent format of just intonation is performed uninterrupted, for which a precise grammar is established. What one hears upon entering the environment E sounds, at first, like a “single note composition”. But upon allowing one’s attention to be absorbed by the ongoing process of the perception of the single note, the various excluded combination tones and their relationships begin to emerge. If one develops what the TEXT calls a “sustained feeling of awareness” in relation to the sine-wave triad, an infinitely rich world of sound figures begins to emerge in one’s consciousness. Further, if one has mastered the conceptual machinery of the TOPOSES AND ADJOINT TEXT, or at least is familiar with the rudiments of Harmonic Analysis (Fourier series etc.), one will find in the contemplation of this composite sound wave form a single SEMIOTIC OBJECT, a philosophical or poetic SIGN whose meanings are generated in the listener’s consciousness through the interaction of his/her attention, the structural properties of the composite sound-wave form itself, and the concepts grasped from the study of the TEXT. The indefinite construction of this process is

referred to by Hennix as a relation of "homosemeiosis." In particular, the general concept of a "form of thought," given as a space defined by the variables determining its geometry, can be expressed by the same notation as that used to specify the three sine waves of which the sound is composed. As the sound is received as an interpretation of the sound wave in the listener's experience, so a form of thought requires models or interpretations in the Creative Subject's mind in order to be understood. The interpretations for both sound form and form of thought are possible worlds which can meet or even coalesce as a result of a *homotopy operation* (mutual deformations of abstract conceptual spaces).

Now for the Text itself. *The Yellow Book* sketches out domains of abstract thought from various regions of advanced mathematics, in particular topology, topos theory (from category theory), an ultra-intuitionistic version of transfinite arithmetic, and Montague Grammar, interpreted as a generalization of the language of *Tractatus Logico-Philosophicus*. Though the language of these disciplines are employed with formal rigor, the bases and the outcomes of Hennix's speculations by no means leave these theories in place. Just as the soot paintings begin with a blank mirror space and end with a return to the original condition of blankness (by a route that takes the viewer through a domain infinitely rich in specific content and detail), so the theoretical structures of the TEXT build up structures from fundamentally "empty" concepts and collapse all the structures generated back into a recapitulated emptiness at the end, (while at the same time allowing a phantasmagorically rich array of mathematical structures and possibilities to shimmer before the contemplating intellect in passing).

Though the languages of mathematics form the medium of the text's expression, the positivity of mathematical thought is by no means the goal of the enterprise. Quite the contrary: TOPOSES AND ADJOINTS rather diminishes confidence even in intuitionistic mathematical reasoning while at the same time exhausting the mathematical intellect and satisfying its demands. And it does this for ends which are at once mathematical, philosophical, aesthetic, spiritual, political and ethical.

We can see how these ends are accomplished through a brief look at a feature of the TEXT designated as the Theory of The Creative Subject together with its models. Both the notion of the Creative

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Subject and the structure of its models stem from Brouwer. The Creative Subject is a difficult and somewhat ambiguous conception. It is perhaps best understood as an ideal figure for the individual insofar as he/she engages in mathematical thought. Insofar as the whole of one's experience is involved in the scope of its cognitivity and insofar as cognitivity in general is to be treated according to intuitionistic concepts, the Creative Subject may serve as a first approximation of a model for one's entire cognitive life.

Hennix develops several Diagrams for the Creative Subject and uses them to generalize Brouwer's concept of the fundamental twofolds which found all cognitive experience. In one of those diagrams (page 383) Hennix does so along lines similar to those I developed above. This diagram consists of a pair of axes. The horizontal axis represents the series of fundamental intuitions or cognitions called *noemas*. The vertical axis represents chains of cognitions derived from these noemas by means of *arrows* or set theoretical operations. Arrows symbolize *denotational connections* between cognitions. Denotational connections are connections established according to the law of sufficient reason.

There would be numerous applications of the diagrams of the Creative Subject, but the aesthetic principles under which Hennix developed his constructions includes the preference that these applications include a context for working out a foundation for one's personal enlightenment, on the one hand, and a paradigm for the solution of culturally relevant theoretical problems on the other. Both of these demands are fulfillable. In applying the Diagram of the Creative Subject (the diagram on page 383 mentioned above) to one's individual cognitive life, one selects as noemas whatever one finds to be one's fundamental cognitive commitments. These commitments are treated as "languageless" perceptions. The Diagram is applied as a function of those perceptions. It is required that all derivative cognitive commitments be connected by means of denotational connections, i.e. cognitions rising from the initial noemas are guided principally by the *law of sufficient reason*. The Diagram thus serves as a means for testing the grounding of one's fundamental ideas. For the most part, this *initial application* of the Diagram acts as a technique for emptying one's mind of its commitments, since the denotational connections between one's associated thoughts can rarely be provided on account of a lack of sufficient reason. The horizontal axis along which the noemas appear may be interpreted as a version of Brouwer's horizontal axis—namely,

as the concrete procession of temporal moments conceived of as empty twoities. The emptying out of the vertical dimension of the diagram (due to the failure to discover denotational connections) will then appear as a process of continually returning from the incoherence of one's mental life to the empty substratum of the concrete stream of one's temporal existence—a return which can be experienced as facilitating the subjective exploration of one's authentic inner life.

The second aesthetic preference mentioned above is fulfilled by the consideration that the application of the theory of the Creative Subject to certain philosophical texts problematizes those texts, in the precise sense that every fundamental statement presented by such a text will ultimately be analyzed as some set of noemas, while every line of thinking derived from that set will become subjected to the law of sufficient reason. If the set of noemas is empty, the meaning of any derivative statement will be empty. If the set of noemas is non-empty, its members will form the foundation of the statement's meaning. As a matter of fact, Hennix is able to produce, by means of the application of The Theory of the Creative Subject, solutions to several outstanding problems such as the cosmological *Night Sky Paradox* (why is the night sky dark?) and the metamathematical *Consistency Problem of Arithmetic* (why is $0=1$ not an arithmetical theorem?) These solutions are possible because, in a manner analogous to the way Brouwer's twoities are capable of generating arithmetical series (or even the continuum of real numbers) the chain of noemas can be interpreted as a generalization of the generation of number series in general, so that number series of different lengths and distinct closure properties can be constructed or generated for different uses. Since the unveiling of the fact that different number concepts are frequently applied indiscriminately, as if they were the same number concept, the flexibility of the Theory of the Creative Subject and its models provides a cogent tool for various analytical tasks demanded by the deepest problems of philosophy.

Again, just as Brouwer's First Act of Intuitionism finds both culturally-relevant mathematical constructions and spiritually-motivated private worlds, so the models for the Creative Subject allow for a reconciliation between private enlightenment and public problematic.

A complete exposition of Hennix's thought is impossible in this introduction. I have therefore selected, somewhat arbitrarily, two topics for discussion which I hope may prove helpful in orienting the reader to *The Yellow Book*. The topics are : *Motivations for Post-*

Wittgensteinian Formal Philosophy; From a Theory of Meaning to a Theory of Self-Illumination.

**Motivations for
Post-Wittgensteinian Formal Philosophy**

The Yellow Book contains an interpretation/translation of certain key passages from Wittgenstein's *Tractatus*. I therefore wish to present a discussion of Hennix's concern with "formal philosophy"—a concern which may seem incongruous in what we might call the post-Wittgensteinian age.

It is well-known that Wittgenstein, beginning with his return to philosophy in 1929 (after hearing Brouwer lecture in Vienna), progressively abandoned the formal aspects of the enterprise undertaken in the *Tractatus Logico-Philosophicus*. This process of abandonment culminated, perhaps, in the powerful passages of *Philosophical Investigations* wherein Wittgenstein renounces many of the aims of formal philosophy: he renounces the ambition to purge natural language of its ambiguities and metaphysical obscurities; he renounces the hope of establishing, once-and-for-all, the logical structure underpinning language, material reality, and human thought; he renounces the pursuit of the chimerical and fascinating aspects of formal, logical thought—the appearance of depth that logical enigmas display, the appearance of adamant, crystalline fixedness that logic promises to depict.

Given what I take to be Hennix's acceptance of the late Wittgenstein's renunciations, it is necessary to make clear what motivates Hennix's use of formal vocabulary and logical apparatus, and what justifies his apparent valorization of the *Tractatus*. (It may be remarked at the outset that in fact the later Wittgenstein by no means abandoned formal philosophy—see, for instance, *Philosophical Grammar*.)

In order to approach Hennix's conception of formal philosophy, it is necessary first to look at several possible post-Wittgensteinian motivations for the development of formal philosophies—motivations which certainly do *not* underlie Hennix's concerns.

Accepting Wittgenstein's reasons for renouncing the project of discovering a unique and universal formal philosophical system, one might still wish to make use of formal thinking as a sort of personal "therapy." Disciplining one's language, defining terms and issues, and working out systems of inference might appeal as a reasonable procedure for making one's way through the morass of intellectual life and the chaos of ordinary language. A private individual might choose to

formalize his/her thought without making any claim upon public consensus and having no ambition to compete with other systems for universal truth.

Though not at all precluded, (during the Rhinebeck seminars Hennix often talked about the Socratic sense of philosophy as a kind of therapy for curing the confusions of the ordinary mind, and Wittgenstein himself often spoke of the work of philosophy being a kind of therapy aimed at eliminating conceptual maladies), such uses of formal philosophy are betrayed by the rhetorical attitude of Hennix's texts which seem at any rate to aim at a tone of ontological assurance that would make no sense at all for purely private utterance: Hennix's pronouncements are punctuated by moments that rise to an almost oratorical state of publicity, yet which at the same time limit their absolute claims by opening large questions about the very existence of the texts in which they are enunciated. Hennix states flatly that "reality" is a result of authority. But ontological authority is confined to conditions of textuality, since "Being" remains only formalizable as a term in a text, while the ontological status of the text itself, being unformalizable, is undetermined or incomplete. "Being" is also identified with "discernings" of boundaries occurring within "spaces" which are expressions of modalities of consciousness and guided/directed with the aid of collections of textual signs. "Thought" is inseparable from acts of attention and attention is directed by acts of thought, represented, say, by the shadows of the objects in the ontology of algebraic geometry, again understood as textual structures. But textual structures are inseparable from human aims and situations, so that most of the ontologies in the environment É documented in *The Yellow Book* are upon reflection both thoroughly constructivist and intuitionistic in that they are not asserted independently of their conditions of construction and are at every point dependent upon subjective acts of thought. They intentionally fail to guarantee the unique logical cosmos to which the rhetorical gestures in which they are developed suggest they might provide access.

The rhetorical appeals to what may appear to be a "vatic," oracular or authoritarian logical universality are never what the texts aim at conceptually. This actually should be obvious since Hennix operates throughout within Yessenin-Volpin's ethical position which at every point bestows upon the reader maximum freedom to reject even *proven* propositions: proofs are intelligible only in terms of situations where aims are defined; however, the apparent epistemological anarchy is

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subjected to the restriction that not all propositions are asserted and not all procedures recommended, because they are invariably *subject to individual choices and purposes*. (cf. Cage, Fahlström, Johns: The very *choice* suffices to make them one's own.) Proofs are not obedient to absolute principles but are defined as "fair means of establishing incontestable propositions convincingly,"—convincingness being a matter of the logic of individual acceptance. *Correctness* is relative to aims. Logic itself is defined as *the science of avoiding errors* and is demanded only where errors are 1. definable and 2. intolerable.

The requirement that logic specify principles of correctness regarding procedures for attaining aims where error is intolerable makes this format of logic not only useful, but an ultimate court of appeal for technological theories. And, of course, the use of formal theories within specific sciences is in Wittgenstein precisely the appropriate place for them. What is interdicted is the metaphysical extension of theoretical truths beyond the concrete sphere of their application. Something like Yessenin-Volpin's approach is in practice appealed to in every technological situation where other formal theories fail to accommodate themselves to practical exigencies, and, as developed by Hennix, Yessenin-Volpin may be seen as the true inheritor of Wittgenstein's late work.

But, again, the refinements and improvements of the logic of technology are not the principal aim of Hennix's work. (One might notice that Yessenin-Volpin's "eleutheric" ethics not only specifies the logic for technological projects, but also spells out and attempts to justify a hierarchy of ethical aims that would limit the probability of undesirable side-effects attendant upon the accomplishment of technological aims. Thus, the very theory that shows how technological projects might be facilitated actually establishes severe limitations upon their pursuit. Not all sufficient means are justified by an aim, although, necessarily, all *necessary* such means are.)

There is one more use that is commonly found for formal theories these days—though I don't think that the reader is likely to mistake this use for Hennix's own. Formalizations of psychological, social, aesthetic, and many other situations are frequently attempted where these situations are assumed to be understood intuitively well enough so that "common sense" can provide a criterion guiding the formalization. Such formalizations "explain" nothing that is not already contained in common sense, and, though "harmless" in the sense that they seem to pretend little regarding metaphysical universes beyond the familiar

world, they can be quite pernicious in that they tend to reify and render fixed precisely the unquestioned assumptions of the social epoch under which they are undertaken).

What then can be said about Hennix's use of formal philosophy, given his disengagement from the above motives? To answer this question we pass to our second topic.

***From a Theory of Meaning
to a Theory of Self-Illumination***

At a fundamental level of analysis, Hennix conceives the meaning of a proposition to be one of the main subjects of ethics (since the "reality" of meaning is also a function of authority), and, insofar as uncertainty of the meaning-relation is intolerable, the notion of proof will interfere with the notions of semantics. Thus, for Hennix, the meaning of a proposition is neither the concatenation of the meanings of its terms, the truth or falsity of such concatenations when measured against objective states of affairs, nor again the use to which the proposition is put in ordinary life. The logical meaning of a proposition is a function of the associated propositions which establish it.

This may seem to burden the propositions of everyday life to such an extent that they would for the most part become meaningless. This consequence is accepted by Hennix and points to an important second role that proofs play in Hennix's thought (beyond grounding propositions) which is, in fact, Hennix's main motive for doing formal philosophy: to allow the thinking consciousness to grasp the extreme degree to which its own perceptions and cognitions are vitiated with uncertainty. We live in a world saturated with cognitions for which we have no demonstrations, and to the degree that we allow this situation to go unexamined, we live in a world that is meaningless, when grasped from a certain ethical viewpoint.

This rather severe picture of the state of our cognitive universe, however, is not asserted as an authoritarian dogma (in the style of the Positivism of the thirties) to be imposed by a logical elite upon an ignorant populace; it is solely presented as an experience which can be turned into an instrument to be developed by those with sufficient experience to find a use for it. Because the concept of proof that Hennix adopts comes under the requirements of Yessenin-Volpin's "eleutheric" ethics, what counts as an acceptable proof for any proposition depends upon the aims inherent in the concrete situation of which the proposition is a part. A proposition is not true or false independently of its use

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context—yet, with formalization, this context is specifiable within parameters defined as narrowly as one requires. Thus, if one's concerns were in fact the trustworthiness of perceptions and cognitions in the everyday world, the proof-theory of meaning would be a particularly powerful tool for *displaying the extreme degree of unreliability inherent in our ordinary mundane beliefs*: one might thus be moved to examine the bases upon which the most elementary of our sensations, perceptions and assertions are constructed, and thus to find how frequently such constructions are, in fact, vacuous. The collection of meaningful assertions about the world turns out in general to be empty. Yet, the world, evacuated by the plethora of meaningless assumptions, cognitions, and assertions which normally cloud and clutter it, does not by any means vanish from view: in fact the empty space—the space vacated by the meaningless propositions—turns out to have a populous interior: an interior surveyable without interference from propositional assertions of any kind. (It is in this context that Hennix has formulated a particularly striking interpretation of Parmenides which to some extent also has been brought forward by S. Austin in the work referred to above, and which will be developed in the supplement to this issue of *Io*.)

I will conclude these remarks with a look at the specific sense in which the notion of “enlightenment” or “self-illumination” is developed in Hennix's work. I have, on the authority of the text itself, mentioned analogies with Zen meditation, and *The Yellow Book* is indeed organized around a series of “Model Subjects,” a term which translates “Koan” and which Hennix borrows from the Shaw translation of *The Blue Cliff Records (Hekigan Roku)*. But the analogy is, to some extent, misleading, in that the disciplines or concatenation of disciplines, or forms of mental activity described by Hennix are not dependent upon a reference to Buddhist metaphysics as traditional or textual authority, nor upon participation in Buddhist institutions for authorizations of practice. (The parallels, however, between Hennix's soteriological epistemology and that of the Indian Logician Dharmakirti and the development of his thought within the Madhyamika Prasangika school go beyond those drawn in our introduction. I will certainly take this topic up in some detail in the future.) Though the enlightenment sought suggests Buddhistic forms of illumination, the methods and sociology, as it were, remain western, in the sense that they are conducted on the ground of the individual intellect and by the individual—responsible to his/her own intellectual and ethical life alone.

The substance of enlightenment or self-illumination for Hennix begins with an ascesis that aims at the establishment of the Creative Subject's independence from "mundane" forms of thought and from the ordinary language which saturates and controls ordinary cognitive life. One attempts with unremitting assiduity to identify the forms of thought which rule one's conscious and unconscious cognitivity, (the ultimate such forms are one's "noemas"), and through the application of analytical techniques one withdraws and reduces the streaming of associative thought to which they give rise.

But for Hennix it is insufficient simply to identify and suppress these noemas: the drift of linguistically saturated and alienated consciousness cannot be neutralized by simply observing the concrete content and activity of the movement of one's mind, for the general structures which determine this drift will remain operative unless they are pacified by constructions aimed precisely at consuming, replacing or reducing them. The general structural form of their production must be discerned. Such discernings will provide the preconditions for the reduction down to and ultimately of the noemas. Reduction of the noemas is possible, I repeat, through the comprehension of the general structure of thought and the assiduous application of this comprehension to one's own cognitive activity. Now, the discernment of the general form of thought involves an analysis of the linguistic forms in which thought is ordinarily encountered, and in this way Hennix's work has its roots in a tradition of poetics beginning perhaps with Hölderlin and extending through Mallarmé to Blanchot, Lacan, and Foucault. These writers delineate with great subtlety the problematic of the Cartesian subject with regard to its alienation and annihilation through the evolution of an awareness of language as such: the subject loses itself in its reproduction as the "I-speak" of a beginningless and interminable verbal stream, whose origin is inaccessible but whose existence as an ungraspable totality prefigures the entire of the space of individual linguistic consciousness.

But whereas the latter three of these writers base their linguistic thinking on the tradition of Saussure, Hennix works with concepts found in Montague Grammar that are in radical competition with specifically the Chomsky school. It is Montague that is, in any case, important for Hennix's work—in particular for his theories of abstract grammars and higher-order programming languages (logic programs). Montague worked from the premise that all language can be treated as a species of abstract mathematical structure, with the grammars of

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concrete languages falling out as special cases of purely formal possibilities constructed a priori from set-theoretical considerations. Hennix "lifts" Montague grammar and locates it within category theory, and thus brings it into contact with fundamental logical structures in an atmosphere where the possibility of freely choosing the basic logical laws underlying the linguistic structures becomes available for individually-determined conceptual use. By working with the "forgetful functors" "pullbacks" and the "empty category" in ways consonant with but not intended by the originators of category theory, Hennix, prescribes a method for eliminating cognitive commitments at a single stroke. Since the possibilities for this elimination are capable of a priori development, criteria can be envisaged by which the individual can test his/her advancement along the path. The application of such criteria are what make it possible for the individual to travel this path without guidance of tradition or teacher.

The condition of enlightenment aimed at, then, is a condition where the Creative Subject has rigorously liberated him/herself from the drift of ordinary cognition. The interest of such a condition actually *begins* at this point: one wants to imagine how the entire domain of human activity might be re-visioned and the entire domain of technology and culture revitalized under such conditions. Unfortunately, all that has been accomplished thus far is the determination of the "way", though it should be obvious that this has been a formidable task; nonetheless, it is only the beginning of what appears to be an inexhaustible journey. Though the path of *The Yellow Book* is thoroughly destructive (as is the initiation of Intuitionism through Brouwer's "first act") the consequence of this destruction is to introduce profoundly hopeful perspectives into a cultural and historical milieu for the most part dominated by intellectual triviality, myopia and fatigue.