BARBARA CUBED

The Manual of PURE LOGIC

Written and Illustrated
by
C. F. RUSSELL

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FOR
THE PRACTICUS

CONTENTS

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BARBARA CUBED

ALL ARGUMENTS ARE A FORTIORI

I. DEFINITION—The word, Logic, comes from the Greek, **Logos**, which means reasoning &/or its immediate expression in rational language. The term, Science, stands for a body of systematised knowledge—not guesswork—about some one subject. An Art is a code of rules in practice. As descriptive of the Science of the Art of Reasoning, Logic deals with the **What** & the **How** of Exact Inference or Accurate Deduction.

Pure, or Formal, Logic is not concerned with Etymological, Grammatical, Rhetorical, Dialectical, Psychological, Philosophical, Epistemological & Metaphysical problems, except where it is necessary to approach the borders of these fields while brushing out from its own domain the debris of the centuries. The discussion of Fallacies, even, is out of place here; for such are but violations of the rules of correct reasoning. To determine the general form of all reasoning we must make an exhaustive study of all possible examples; this automatically provides a basis for classifying all fallacies.

Although an essential aid in the pursuit of Truth, Pure Logic does not occupy itself with gathering data or testing them except where such is involved in the process of transforming them into **logical objects**, but it begins to operate only after all pertinent data are assembled. Then it deals only with antecedents & consequents; something is given & something necessarily follows. It is the duty of our science to understand the whole nature of illative force so as to weigh the validity of an argument. It exposes the essence of argument per se & classifies all possible arguments. It lays bare the proper form of all inference & explains in detail the necessary & sufficient process of all deduction.

In short, we may say that our subject is limited in scope strictly to that with which we shall be concerned in this book & all other matters are entirely extraneous.

II. LOGICAL OBJECTS—Ordinary, rational thinking & its communication as in common discourse is the rough material which Logic takes, transforms & refines into that specific sort of thing which it can handle, called a **Logical Object**, of which there are three kinds, Names, Equations & Syllogisms. A **Name** is a noun, or substantive word, phrase or clause, which is properly qualified & quantified.

An **Equation** is a sentence or assertion in which two Names are conjoined or copulated & whose import or function is to declare or indicate that these two Names stand for identically the same thing. The fundamental law of logic, the primary expression of illative force is the definition of an Equation as above, or briefly:—AN EQUATION IS THE COMBINATION OF TWO NAMES OF ONE & THE SAME THING.

A **Syllogism** is a group of three Equations containing in all but three different classes of Names, viz—Major Term, Minor Term & Middle Term.

Formally, the syllogism consists of Major Premise, Minor Premise & Conclusion. The major term is the predicate of the major premise & of the conclusion; the minor term is the subject of the minor premise & of the conclusion; the middle term is the subject of the major premise, the predicate of the minor premise & does not appear in the conclusion. This arrangement of the terms & sequence of the premises is the "first figure" of Aristotle. In pure logic "figure" has no illative significance & no use except as a device of convenience. Thus we put all arguments into the first figure for the sake of uniformity thereby making comparison more easy & clear. Similarly with "subject" & "predicate"; since both terms of the equation are both qualified & quantified they become proper logical objects & are equated, hence equal to each other & can be substituted for each other, so they may be exchanged or transposed without altering the meaning of the equation or disturbing its illative import.

In Pure Logic there is no room for ambiguity or equivocation; the meaning of every statement & every component thereof must always be definite & adequately & precisely explicit; viz—we must deal only with **logical objects.**

III. QUALITY—This is that attribute of a Name which tells whether it refers to something in or not in a given class. The two species of quality, **Affirmative** & **Negative**, are mutually exclusive or contradictory.

In logic, each name has three essential parts: (1) the **character**, the part which expresses a concept & gives it denotation enabling it to represent a class or have meaning; (2) the **quality**, the part or mark which decides whether we speak of a subject affirmatively or negatively; & (3) the **quantity.**

Names are expressed in **Notation** by sigils or symbols or, in print, letters. A Capital is used for an Affirmative & a small, or lower-case letter, for a Negative, as, e.g. **M** for "men" & **m** for "not-men."

The negative particle (not) is always attached to the name & never to the **copula**, which is always an **equals sign** (=), its equivalent "is" or "are," or it may be left out & simply understood. Thus the logical import of "A slave is not a free person," is "All slaves are some not-free-persons."

An equation is made by the simple adjunction of two names as **S P** for **S** = **P**, or **S p′** for **S** = some not-**P**. The terms on each side of the equation are different expressions for one & the same thing. Consequently we cannot combine two contradictory names to make a **valid** equation. If **S P** be valid, then **S p** is invalid.

The fundamental law of logic, the basis of all illative force, viz—the definition of an equation, involves what are termed **the three laws of thought.**

(1) THE LAW OF IDENTITY, expressed as **S** = **S**, or **SS**, or **P** = **P** (**PP**), or **MM**, or **m′m′**—i.e. the equation of any two identical terms, signifies that a thing is itself. This is simply adopting a canon of self-consistency; having once named a thing we keep the same signification throughout our argument.

(2) THE LAW NON-CONTRADICTION, expressed by **S** = not-**s**, or **S** ¢ **s**—i.e. the equation of any term with the contradictory of its contradictory or else the inequation of two contradictory terms, asserts that a thing is not anything not-itself. Just as (1) says that the terms of an equation must mean the same thing; so (2) says that if they do not mean the same thing they cannot constitute an equation.

(3) THE LAW OF EXCLUDED MIDDLE (or Third), expressed in, either **S** or **s** = **S**, or, either **S P** or **S p**; i.e. any name can be coupled with any other name or else with that other name's contradictory to make an equation. In short, **S** + **s** = all there is; together with its contradictory any term includes a conceptional scope which is total or universal. Anything at all, if it can be spoken-of as **S** or as **s**, then it must be either **S** or **s**, there is no other alternative.

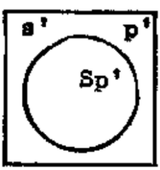
IV. QUANTITY—This is that attribute or mark of a Name which tells whether it refers to the whole or else only a part of what its character denotes or connotes. The two kinds of quantity are Universal & Particular & are mutually exclusive or contradictory.

In Notation, a prime (′), is attached to the letter which stands for a part only & not the whole of a class or thing; thus **S′** is a **particular** term & means "Some, not all, **S**"; it never can mean "Some, perhaps all, **S**."

The **Universal** term merely omits the prime; thus **S** means "All **S**" or "Every **S**", whereas **s** (a small **s**) stands for "All not-**S**," when no prime is attached to the letter.

The equation, **S p′**, means "All **S** is some not-**P**." **S p′** = **p′ S**. Thus, if we fix on **S** as the subject & **p′** as the predicate, since both are proper logical objects, we can transpose them without changing the logical force of the equation.

The sanction for the mutual contradiction between the two species of quantity is easily supported. Note that if "All **S** is some not-**P**" (**S p′**), then there must be some not-**P** (**p′**) which is not-**S** (**s′**).



The validity of this deduction is demonstrated after the Eulerian fashion by the diagram here. Let the square contain everything which is not-**P** (**p**). Let the circle contain everything which is **S**, then whatever is not-**S** (**s**) will be outside of the circle, whether or not such **s** remains within the confines of the square. The original equation asserts that All **S** is some not-**P** (**Sp′**), but does not say that **S** is all **p**, in fact, by definition of particularity, it affirms categorically that **S** is not-all not-**P**. We see then with our eyes &/or mind that there remains some **p** (**p′**) which positively is not-**S** (**s′**). From **S p′** we can then always deduce **s′p′**. Thus if we make a statement about **s′** or **p′** we are making an assertion about some, not all; "some" is not "all." Again, suppose **p′S** as illustrated in the diagram, i.e. some not-**P** is all of **S**. Here we particularise or prime a term (**p′**) to indicate that we have made a division or sexion or partition of its extension. We make two parts of **p**, what lies within the circle, **S** & what is within the square of **p** but outside of **S**. What is outside of **S** belongs to **s**, but is not all **s**, but only some **s** (**s′**). Hence there is some not-**P** (**p′**) which is some not-**S** (**s′**), or, the equation **s′p′** is valid. **To reason correctly one must concentrate the attention.** In fine, reasoning is nothing else but a development of the faculty of attention.

V. TYPES OF NAMES—Since each name has two attributes & each attribute has two species, there must be $2 \times 2 = 4$ types of names. In notation these 4 are **S**, **S′** **s** & **s′**, tabulated as follows:

	Affirmative	Negative
Universal	(All S) S	(All not-S) s
Particular	(Some S) S'	(Some not-S) s'

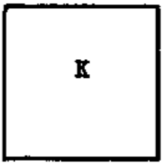
VI. TYPES OF EQUATIONS—Since there are 4 types of names, in an equation there are 4 sorts of subjects to be attached to the same 4 sorts of predicates; hence there are $4 \times 4 = 16$ types of equations, tabulated as follows:

	Predicates			
	P	p	P'	p'
Subjects				
S - - -	SP	Sp	SP'	Sp'
s - - -	sP	sp	sP'	sp'
S' - - -	S'P	S'p	S'P'	S'p'
s' - - -	s'P	s'p	s'P'	s'p'

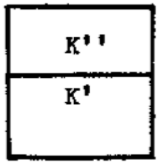
This table is exhaustive; there is no possible assertion which cannot be reduced to one of these 16 types.

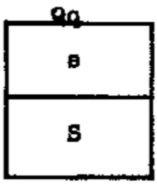
VII. DEMONSTRATION—Clear thinking will inevitably discover that the true nature of illative force is wholly empirical. To prove is to demonstrate; the process is always sensible or perceptual, either as sensual or imaginative. It may be refined & abbreviated, as in the higher mathematics, for we often grasp an intricate conception by means of a single stenographic symbol. But soar as high as we may into the realm of the abstract, yet at least some symbol must be present in the mind, else we are not reasoning at all, but merely muddling in chaos not yet become cosmos. The sign may refer to a group of things, or an organised unity; a part may stand for a whole; there may be all sorts of relations between the thing & its symbol; the symbol may be metaphorical or it may be purely arbitrary, nevertheless the symbol itself must be a percept of some kind. Whatever cannot be displayed, illustrated, or exemplified in some way cannot enter the realm of discourse or reasoning. To reason is simply to discern something, as in a picture; to reason logically is nothing else than to render obvious or evident what that picture contains, that is, call it to our attention. It is a matter of naming & equating names. Here, practice makes perfect, the logician acquires skill & becomes an Artist. The artist aspires to reach exactitude & certitude & studies to become a Scientist.

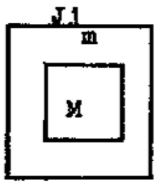
The primitive of logic is the Name, which, as said, possesses character, quality & quantity—no more & no less. Let us draw an enclosed space, as a square—or rectangle, the shape is not important—to represent everything which can be named, to denote the whole universe, to contain everything that exists, that is, or is real, every thing or being. Let us call this geometrical figure

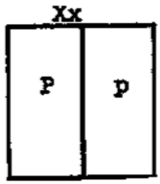


K, i.e. all **K**. Whatever is not-**K** is not a thing, nothing. Nothing exists outside of the universe. **K′** is some **K**, **K″** is the rest of **K**, some (other) **K**.



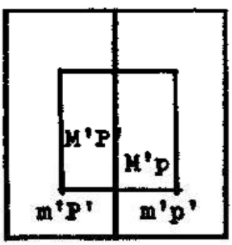




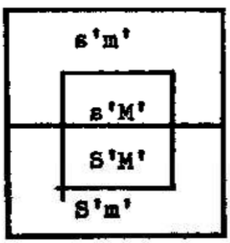


Now let **K** be divided in two parts, **K′** & **K″**, let the lower part be named **S**, then the upper part is not-**S** or **s**, i.e. everything which is not **S**. Similarly, let there be another dichotomy of **K**, as inner & outer, or as we shall see later, back & front, called respectively, **M** & **m**. Finally, divide **K** into **P** & **p**, as shown above.

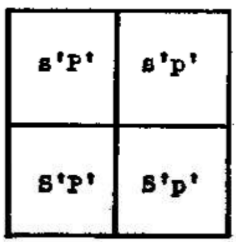
Let these three letters represent the three terms of a syllogism, **S** & **s** for affirmative & negative minor term; **M** & **m** for the middle term & **P** & **p** for affirmative & negative major term. Let us call these three dichotomies of the universe, respectively, **Qq** or Salt; **Jj** or Quick-



(Major Premise)



(Minor Premise)

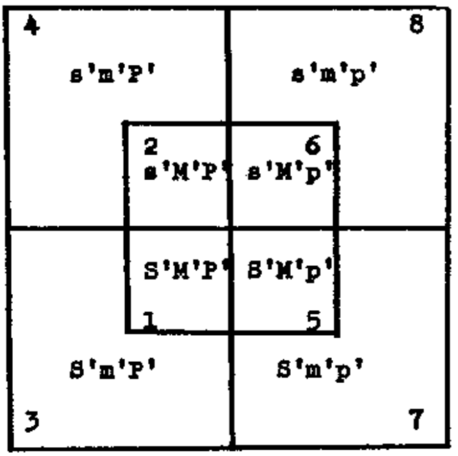


(Conclusion)

silver & **Xx** or Sulphur. In short, we have a representation in two dimensions, as a **logical frame**, of a cube or three-dimensional figure, in which the salt dimension or **principal** is **Q** for the male pole or bottom of the cube = **S** & **q** for the female or top = **s**; the back & the front of the cube, **M** & **m** = **J** & **j**, the forward & backward principal, Quicksilver & the Sulphur or left & right dimension = **X** & **x** = **P** & **p**.

A combination of the **Jj** & **Xx** principal dichotomies gives us the major premise; the **Qq** & **Jj** combination yields the minor premise & the **Qq** & **Xx** combination is the conclusion. See the illustrations & read or interpret the **frames** thus:—Major Premise—**M′P′** = Some **M** is some **P**; **M′p′** = Some not-**M** is some not-**P**; **m′P′** = Some not-**M** is some **P**; and **m′p′** = Some not-**M** is some not-**P**; & so on, similarly with the other premise & the conclusion.

VIII. THE LOGICAL FRAME—Now, combine all three dichotomies into one logical frame containing 2 x 2 x 2 = 8 sexions called points (#s) or numbers, from **1** to 8, respectively, as shown in the diagram here. Thus, in #1 we put or find everything which is at the same time **S**, & **M** & **P**. In #2 we see whatever is **s**, **M** & **P**; in #3 are all things which are **S**, **m**, & **P**, and so on, with the other #s.



The logical frame corresponds in two dimensions to the logical cube in three & for logical purposes is its exact equivalent. Divide the cube into bottom & top, or **S** & not-**S(s)** into back & front, **M** & **m**; & into left & right (**P** & **p**).

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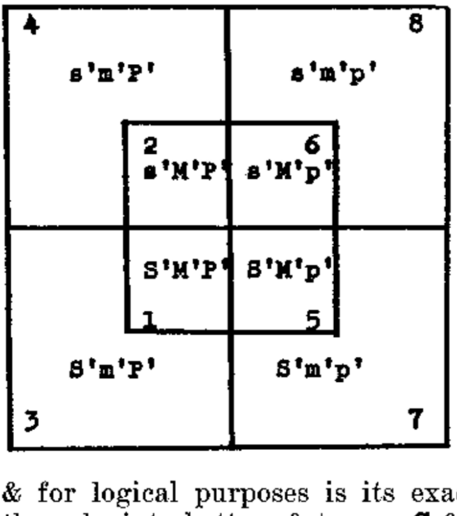

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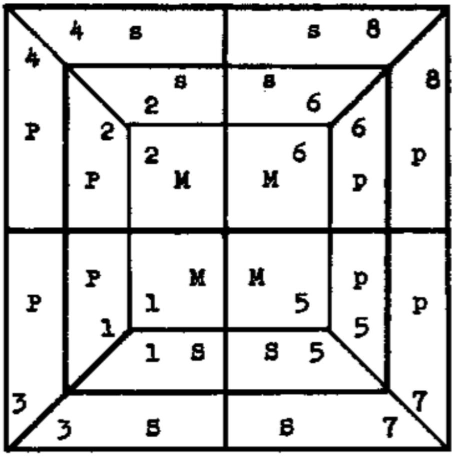
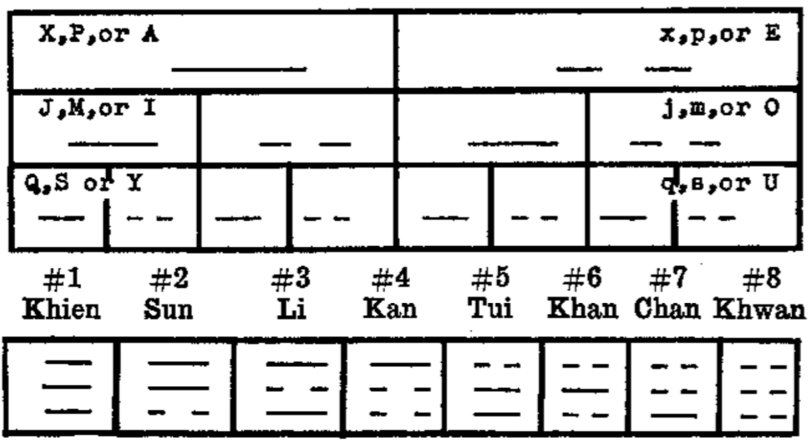
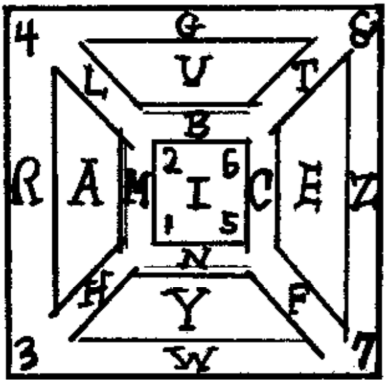
#1 = bottom, back & left; #2 = top, back, left; #3 = bottom, front, left; #6 = top, back, right; #8 = top, front, right, & so on. The three dimensions, or principals, are Salt or **Q**, Quicksilver or **J**, & Sulphur or **X**; thus #4, e.g. = **qjX** = **smP**, for female or negative salt & quicksilver & male or affirmative sulphur. A pole or half the cube in any principal is also called a Metal. Thus, **Q** = silver, **q** = lead; **J** = mercury, j = tin; **X** = copper, **x** = iron. The Alchemical nomenclature is given here for reference; see Book CHAMELEON for further ideas.



IX. TRIGRAMMIC ANALYSIS—One half (metal) of the cube in any principal is expressed by a **gram**, or **Yao**, which stands for one term of an equation. If the term is affirmative, or the pole male, the line is unbroken, ———, & called a **Yang**; if it is female or negative, the line is broken & called a **Yin**.



The trigram is made (erected) & read from the bottom up; bottom gram = salt, middle = quicksilver, top = sulphur. The cube has twelve edges = **crystals**, each composed of two **#s** & ascribed severally to the signs of the zodiac & lettered; the 4 in each principal correspond to the 4 elements—see table below.



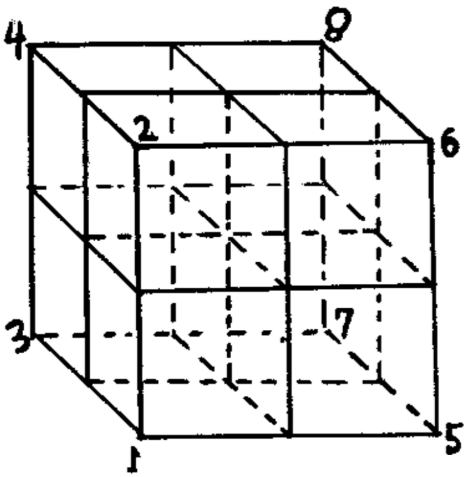
Fire	Air	Water	Earth
Sulphur			
N	W	G	B
Pisces	Aries	Sagittary	Virgo
1-5	3-7	4-8	2-6
Quicksilver			
M	L	T	F
Gemini	Capricorn	Leo	Cancer
1-3	2-4	6-8	5-7
Salt			
M	C	Z	R
Aquarius	Libra	Scorpio	Taurus
1-2	5-6	7-8	3-4

One combination of terms, as **SP**, **Sm**, **mP**, etc., viz—an equation, is represented by a **Digram**, or two grams or Yaos, one above the other,

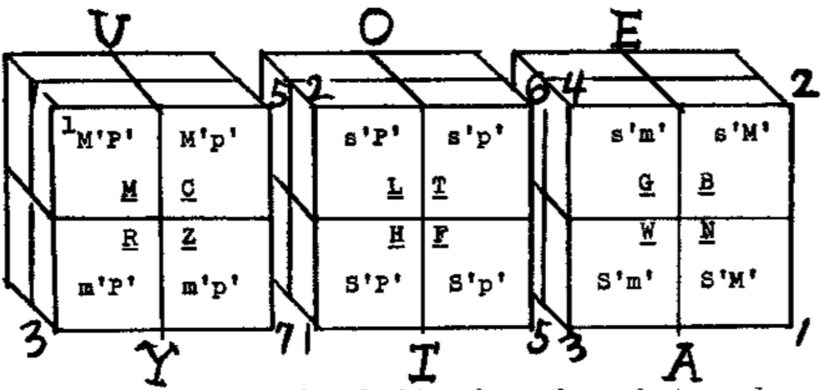
as ——— – – ———

(Fire) (Air) (Earth)

& – – = (water).



When it is desired to show the principals of the Yaos, the extra or unsexed place is filled simply with a dot (.). There are twelve principalled digrams for the twelve crystals, counted



four in each dimension, looking from the male towards the female pole, beginning with #1 of the Cube & counting **Deosil** (=clockwise), ascribing elements to them in the sequence, **Fire, Air, Water & Earth**. Thus, the edge, which contains #1 + #2 = Fire of Salt; #5 + #6 = Air of Salt; #7 + #8 = Water of Salt & #3 + #4 = Earth of Salt; similarly, Fire of Sulphur = #1 + #5; Fire of Quicksilver = #1 + #3 & so on. The letters, (**S**), (**P**), (**V**), (**D**), in parentheses stand for the Vitals: Fire, Air, Water, Earth. Thus #4 = **(V)X**, **(p)j**, **(d)q** = **Male Water** of **Sulphur**; **Female Air** of **Quicksilver** & **Female Earth** of **Salt.**

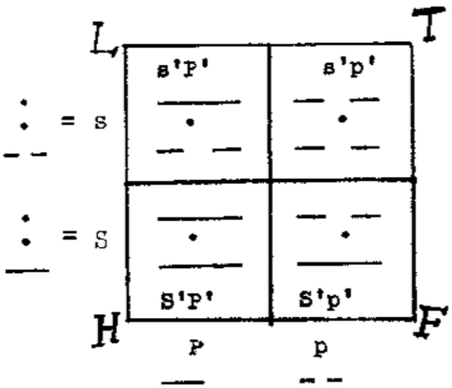
X. CONVERSION or IMMEDIATE INFERENCE—In order to give the universal quantity to any term in an equation it is necessary to cross-off or eliminate one or more crystals from the frame. There must be at least one universal term in a premise if that premise is to have any logical force = power to eliminate a part of the logical frame.

CRYSTAL ELIMINATIONS

Major premises eliminate Salt Crystals.

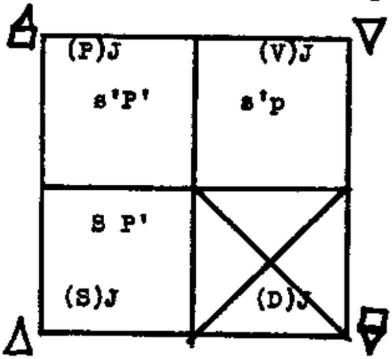
Minor premises eliminate Sulphur Crystals.

Conclusions eliminate Quicksilver Crystals.



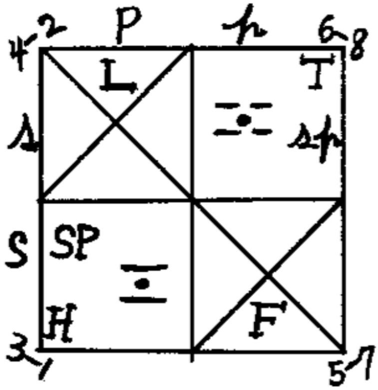
Suppose we take the statement, "All **S** is some **P′**", or, in notation, **S P′**. Draw the frame, as above, from the "I" or male quicksilver viewpoint; each compartment represents a quicksilver crystal of which we see here the male pole; we are looking at Mercury, the back of the Cube. **S** is the bottom, **P** is the left; so when we say, ALL **S** IS SOME **P** = **S P′**, we interpret this in the frame as "All of the bottom is some of the left"; consequently there can be no bottom on the right, which means that we must cross-off or eliminate that crystal where the bottom & the right come together, viz—the points, 5 & 7, or the **F** crystal, which we proceed to do—see diagram below. In short, if all **S** is **P′**, there can be no **S** which is not-**P** (**p**), so we eliminate the fourth or **Earth** corner of **Quicksilver.**

Now from the same diagram from which we read **SP′**, we can also read **s′p** = ALL NOT-**P** IS SOME NOT-**S**. Now, take this equation, as the original & see what crystal it eliminates. Thus if **ps′** then there can be no **p** which is **S**, hence we must cross-off the crystal which combines copper with silver, viz, the **F** crystal, just as before. **ps′** = **s′p** is the **OBVERSE** of **SP′**. The Obverse & its obvertend are of the same value, i.e. have the same logical force, which means **they eliminate the same crystals.**



XI. OBVERSION WITH BOTH TERMS UNIVERSAL —Take, e.g., the equation, **S P** = All **S** is all **P**. **The Formula of Elimination requires that we change the**

quality of the term other than universal, then eliminate that crystal where this new combination is found. Thus, with **SP′**, we change the **P** to **p** & eliminate the combination, **Sp.** But when both terms are universal we must do this twice. Hence we must eliminate the two combinations, **Sp** & **sP**. Big **S** & little **p** are found together in the **F** crystal &



little **s** & big **P** occur together in the **L** crystal, so we eliminate both **F** & **L**, leaving **H** = **SP** & **T** = **sp**, the 1-3 & the 6-8 crystals. The result then is that All **S** is all **P** & All not-**S** is all not-**P**. These two equations are obverses of each other, or related, either way, as obvertend & obverse & they are precisely, logically equivalent.

XII. TYPES OF CONVERSION—The first type, Obversion, can always be performed whenever at least one term of the equation is universal. In the case where one term is universal & the other particular, one only crystal is eliminated; where both are universal, two crystals are deleted. In the latter case the obverse is the only converse. In the former instance there is also another converse, called the SUBVERSE, which can be found by changing the quality of the universal term & also its quantity, leaving the other term as it was; thus **S P′** has **s′P′** for its subverse. Note that the subverse of the obverse is identical with the subverse of the obvertend. Thus **S P′** has **s′p** for its obverse & the latter also has **s′P′** for its own subverse. The rule for Obversion in general is TO CHANGE THE QUALITY OF BOTH TERMS & EXCHANGE THEIR QUANTITIES. In the case of **SP** we get **sp** by the first part of the rule & since both terms are universal an exchange of their respective quantities leaves both, as before, universal. The obverse can be reconverted into the obvertend but the subverse cannot be reconverted into the subvertend, since both terms are particular & this kind of an equation yields only what is called a COVERSE, which is found by changing the quality of one of the terms at a time, thus **S′P′** has **s′P′** or **S′p′** for a coverse & either of the last two has **s′p′**, so that, given any one of these 4 types, the other 3 may be deduced therefrom either directly or indirectly. However, one must always be careful to work only with what can be deduced from a diagrammatic representation of a premise or of an argument, to use only those converses which can be read from the same diagram which shows the convertend. **Always demonstrate an inference.**

XIII. DIMINUTION OF EQUATIONAL TYPES.

Since obversion gives another equation which has the very same logical force as the obvertend, as representing different species of logical validity there is no need, in a practical list, to include both the obverse & its obvertend, since both will cross-off the same crystals from the frame & equally well.

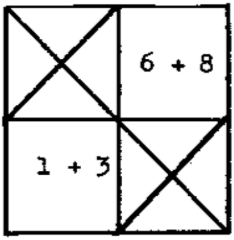
The complete list of 16 types of equations includes 12 in which at least one term is universal, & consequently, which can be obverted. These 12, then, must represent 6 pairs, for each of the 12 can be obverted & the obverses of all 12 must include the same 12 types listed in the beginning, though differently placed in the table. We can then discard 6 of the 12, leaving the other 6 to serve the purpose, making the rejection according to some convenient principle of classification, such, e.g., as giving preference to affirmative & universal subject over negative & particular. Thus we save **SP** & discard **sp**; we keep **sP′** & do away with **S′p**; we prefer **sp′** to **S′P**, & so on. At the same time we can further reduce the list by discarding all except one of the types where both terms are particular, since by coversion these 4 are equivalent; of these latter let us keep **S′P′**. Now we have reduced the 16, first to 10, then to 7 types:—

SP **Sp** **SP′** **Sp′** **sP′** **sp′** **S′P′**

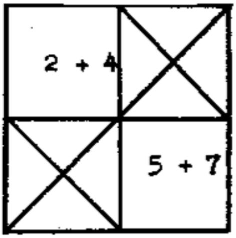
which include all equations which respectively have different crystal elimination pictures.

XIV. THE SEVEN INDEPENDENT TYPES DEMONSTRATED—Taking each equation by itself, drawing a frame & then eliminating the crystals which are inconsistent with the premise, we get the following table showing the logical force of each of these types.

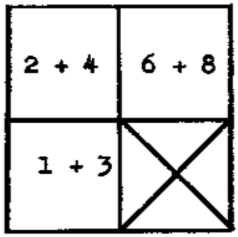
(1)
SP ¢ Sp, sP



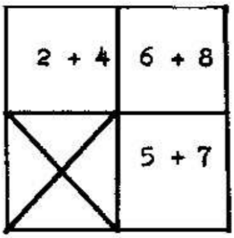
(2)
Sp ¢ SP, sp



(3)
SP′ ¢ Sp

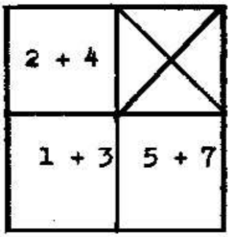


(4)



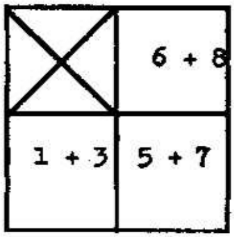
Sp′ ¢ SP

(5)

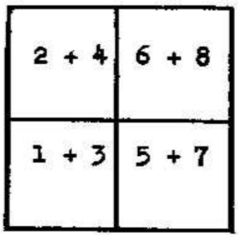


sP′ ¢ sp

(6)



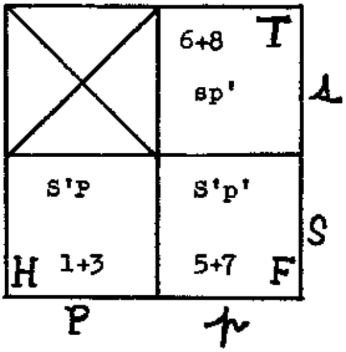
sp′ ¢ sP



(7)
S′P′ ¢ O

Since the premises show the combinations of salt & sulphur, the conclusions are quicksilver crystals, some of which are eliminated except in the case of (7), where both terms are particular. The crystals that remain, two #s to each, show the conclusions to be deduced from the premises. These can be read or interpreted directly from each frame. Take, e.g., (6) = **sp′ ¢ sP**, eliminating the **L** crystal, 2 + 4, which is where **s** & **P** occur together, leaving crystals **T**, **H** & **F**, or water, fire & earth of quicksilver, respectively.

To read the conclusion take one whole metal as the subject of the equation & another metal for the predicate. If a portion of a metal is crossed-off then the corresponding term is universal. Thus in the picture, no part of the bottom, **S**, has been eliminated, consequently with respect to **P** or **p** we can make no assertion concerning **S** universally, but only in particular. That is to say, some **S** (**S′**) is on the **P** side & **S′** is on the not-**P** (**p**) side, but it is not possible to say that **All S** is **P′** or **All S** is **p′**. There are four possible combinations or equations into which the two letters can enter, thus the subject may be affirmative or negative & similarly with the predicate.



Take (1) the affirmative subject with the affirmative predicate, **S** with **P**; here we see some **S** but all **P**, hence **H** is read as **S′P** = Some **S** is all **P**, or All **P** is some **S**. (2) Take the affirmative subject with the negative predicate, **S** with **p**; here in **F** we have some **S** & also some **p**, for there is also **p** in **T** & **S** in **H**; hence **F** is read as **S′p′** = Some **S** is some not-**P**. (3) Take the negative subject with the affirmative predicate, **s** with **P**; this crystal (**L**) is crossed-off, so we cannot read anything from this combination. (4) Take the negative subject with the negative predicate, **s** with **p**; **T** must be read as all **s**, for the rest of **s** is in **L** & crossed-off & **p** is **p′** for in **F** there is more **p**; hence **T** is read as **sp′** = All **s** is some not-**P**. RULE—**If both crystals of one metal remain INTACT then that metal must be primed in any conclusion, but if one crystal of a metal has been eliminated then that same metal is universal.** Read the whole systematically so as to exhaust the possibilities. Thus **S** goes with both **P** & **p** so **S** must be primed; **s** goes only with **P** so **s** is not primed; **P** goes only with **S** hence **P** is not primed, while **p** goes with both **S** & **s**, hence **p** is primed. This analysis & tabulation shows the crystal picture of the conclusion; a further treatment, which makes a punctual or exhaustive analysis is explained next.

H 1+3	F 5+7	T 6+8
P	p'	p'
Ṡ'	Ṡ'	ṡ
S'P	S'p'	sp'

XV. QUANTIFICATION TECHNIQUE—Write down in consecutive sequence the points which remain in the frame, as, e.g., with (6) = **sp′** —

#1	3	5	6	7	8
P	P	p	p	p	p
Ṡ	Ṡ	Ṡ	ṡ	Ṡ	ṡ

Here we put a dot (.) in place of the middle term, **M** or **m** & indicate the principal, sexed components of each #. Now, since a conclusion can be read only with respect to the whole picture of one metal, viz, one sex of a principal, if the metal is primed in one converse it must be primed in every other; if universal in one instance it is universal throughout the picture, viz, wherever that same quality occurs—**this is a rule.**

Now, the first step in this technique, after obtaining from the picture the above punctual table, is to segregate the crystals which are sufficient for the purpose of the operation of priming by disregarding those which make the same qualitative equation. Thus, we see that both #1 & #3 give the combination **SP**, so we encircle #3 or put parentheses around its components; similarly with #7 which = #5 & #8 which = #6. The systematic way to do this is to begin at the left with **SP** & look across to see if big **S** & big **P** occur together again; if so, then encircle them to show that we disregard them in the next step & so on with all the others in rotation. This leaves #s 1, 5 & 6 as shown here & we are now ready to attach the primes as follows.

1	(#3)	5	6	(#7)	(#8)
P	(P)	p	p	(p)	(p)
Ṡ	(Ṡ)	Ṡ	ṡ	(Ṡ)	(ṡ)

	#1	#5	#6
X=	P	p	p
J=	•	•	•
Q=	S	S	s

Look across, confining your

attention this time to a single principal: where the same quality is repeated in the same principal attach a prime to the same quality in all cases; thus in **X** we see two negative **p's**, hence **p** is primed to **p′**; in **Q**, both **S's** must be primed; but not the **s** or the **P**.

'1	'5'	6'
P	p'	p'
S'	S'	s

The resulting combinations are the conclusions which can be read respectively from each point. Thus #1 = **S′P**; #5 = **S′p′** & #6 = **sp′**; note that 1 & 6 are related as obvertend & obverse, while 5 is the subverse of either 1 or 6. The student should work out in a similar fashion the quantification from each of the types as illustrated in XIV.

XVI. SYZYGIES—We have considered an equation & its conversion, or inference in two dimensions; now we can extend the study to three principals to exhaust the subject. A combination of **premises** in which three different principals are exhibited is called a **Syzygy**. Thus, each premise shows two different principals, one for the subject & another for the predicate & since there are but three in all, if three are shown in two premises one of the three must be duplicated & this one is the **middle** term.

You see three principals, **Q**, **J** & **X** in two equations. The combinations possible are **QJ**, **QX** & **JX**. Two of these combinations must be since these are the only types of syzygy we can have. In practice we adopt (II) since

(I)	or	(II)	or	(III)
Q J		Q J		Q X
Q X		J X		J X

here the middle term is J, but we reverse the order of the premises, putting the major first, thus—J X, then it is in the "first" figure & the conclusion is Q J Q X & the three equations together constitute the standard **syllogism**. Now, since there are seven unique or independent (not-further reducible) types of equations & a syzygy has two equations, there must be $7 \times 7 = 49$ types of syzygies, which we put in the first figure as

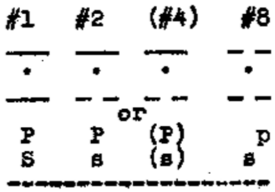
M P = major premise
S M = minor premise

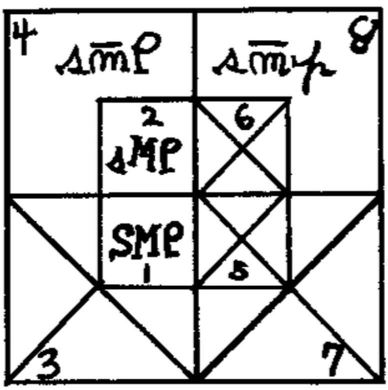
When both premises are properly qualified & quantified in both terms thereof the conclusion (**S P**) follows as a matter of deduction from a frame from which each premise is allowed separately to eliminate its own inconsistent crystals. The conclusion must follow from the premises; it cannot be written down arbitrarily; hence there are also but 49 types of syllogisms, all but one of which have a valid conclusion. In fine, we work out each premise by itself in the same frame & add the results, reading the conclusion according to the #s not eliminated by the several premises. Usually the premises will have crossed-off certain points in common. The process is essentially the same as conversion, nothing new entering into it.

XVII. SYLLOGISMS—Briefly, a syllogism is a syzygy with the conclusion added; a true conclusion consists of the logical frame with or without crystals eliminated. To work out an example, first, draw the frame with the 8 #s, as illustrated here. Take, say,

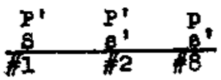
MP¢ ; the major premise, **MP′** ¢ **Mp**,
SM′
SP′ the C crystal = 5 + 6.

The minor premise, **SM′** ¢ **Sm**, the W crystal, 3 + 7. This leaves #s 1, 2, 4, & 8, from which to array the conclusion.





The duplicate is #4 which repeats #2, so we encircle #4 to disregard it in the priming. In **X** we have **P** twice, hence **P′**. In **Q s** occurs twice, hence **s′**. Both **S** & **p** remain universal. The conclusion to be preferred is #1, **SP′**, whose obverse is



#8, **s′p** & #2 is the subverse of either #1 or #8. We can read all this directly from the frame at a glance, viz —All **S** is some **P**; all not-**P** is some not-**S** & some not-**S** is some **P**. Inspection yields also the premises with their own converses respectively, **MP′**, **m′p, m′P′** & **SM′**, **s′m, s′M.** #4 is the same as #2, for the conclusion, but for the premises make their own combinations to work the quantification technique. The student should practice demonstrating all the syllogisms in the table of XIX.

XVIII. THE SHORT CUT—In contrast with the above which may be termed the Geometrical method there is an Algebraic or purely Notational process of solving syllogisms which gives the main conclusion without using the logical frame & the quantification technique; then from this main answer all the converses may be deduced legitimately. The rules for the **Short Cut** are as follows.

First, put the syzygy into what is called the **Working Form** which is such that at least one Middle is universal & both middles are of the same quality. This can be done if at least one of the four terms is universal, otherwise not, viz. in the case where all four are particular & which does not yield a valid conclusion.

If the given syzygy is not already in this working form convert one or both premises, trying first obversion, then subversion & if necessary use coversion. Then, with the syzygy in working form **simply set down the terms of the conclusion (S & P), retaining their quality as in the premises & for quantity attach a prime only if there is at least one in the same column.** This means that if the term is universal in the premises it remains universal in the conclusion unless the opposite **middle** is particular; if the term is particular in the premises, or if the opposite middle is particular, then the term is particular in the conclusion.

Take, e.g. M P', not in working form for the middles
are not S m' of the same quality, although one of
them is universal. Obvert the minor premise = **s′M**, then
M P' fulfils the requirements, so that we simply write
s'M down the conclusion, **s′P′**.

Take another example— m'P'; here it is necessary
to obvert the minor S m' = s'M & then covert
the major to **M′P′** then ——— M'P' the requirements are fulfilled & we conclude s'P'. Test this by

the geometrical method as follows. Both terms of the major premise are particular both in the covertend & coverse, hence it eliminates no crystal. The minor will eliminate the same crystal either as obvertend or obverse, viz ¢ **SM** = **N** = 1 + 5, leaving 2, 3, 4, 6, 7, & 8. Of these (4) & (8) duplicate 2 & 6, so from 2, 3, 6, 7, we derive respectively, **s′P′**, **S′P′**, **s′p′** & **S′p′** which are the converses of **s′P′** the answer. The student should work out all cases this way.

XIX. TABLE OF SOLVED SYLLOGISMS

Minor	(Major Premises) MP	Mp	MP'	Mp'	mP'	mp'	M'P'
SM-	SP	Sp	SP'	Sp'	sP'	sp'	S'P'
Sm-	Sp	SP	sP'	sp'	SP'	Sp'	S'P'
SM'-	SP'	Sp'	SP'	Sp'	S'P'	S'P'	S'P'
Sm'-	SP'	SP'	S'P'	S'P'	SP'	Sp'	S'P'
sM'-	Sp'	sp'	Sp'	S'P'	S'P'	S'P'	S'P'
sm'-	sP'	sP'	S'P'	SP'	Sp'	sp'	S'P'
S'M'-	S'P'	S'P'	S'P'	S'P'	S'P'	S'P'	----

XX. TABLE OF SYLLOGISMS IN PUNCTUAL NOTATION

	(Major Premises & what they eliminate) M P / 3456	M p / 1278	M P' / 56	Mp' / 12	m P' / 78	m p' / 34	M'P' / 0
SM ≠ 2367	1	5	1'	5'	4'	8'	'1'
Sm ≠ 1458	7	3	2'	6'	3'	7'	'3'
SM' ≠ 37	1'	5'	1'	5'	'1'	'1'	'1'
Sm' ≠ 15	1'	3'	'1'	'3'	3'	7'	'3'
sM' ≠ 48	7'	6'	5'	'3'	'1'	'1'	'1'
sm' ≠ 26	2'	4'	'1'	1'	7'	8'	'1'
S'M' ≠ 0	'1'	'3'	'1'	'3'	'1'	'1'	---

To interpret, simply write the conclusion with subject & predicate of same quality as in the indicated point & put the prime on the subject if at the left, if at the right it goes with the predicate, thus, e.g. 7′ = **Sp′**; ′6′ = **s′p′**, ′3 = **S′P**.

XXI. TABLE OF SYLLOGISMS AS PUNCTUAL REMAINDERS

Minor Premises Eliminate Sulphur Crystals	Major Premises: MP	Mp	MP'	Mp'	mP'	mp'	M'P'
	(Eliminate Salt Crystals)						
	C-R	M-Z	C	M	Z	R	O
S M ≠ W-B-	1,8	4,5	1,4,8	4,5,8	1,4,5	1,5,8	1,4,5,8
s M ≠ G-N-	2,7	3,6	2,3,7	3,6,7	2,3,6	2,6,7	2,3,6,7
S'M ≠ B-	1,7,8	3,4,5	1,3,4,7,8	3,4,5,7,8	1,3,4,5	1,5,7,8	1,3,4,5,7,8
s'M ≠ N-	2,7,8	3,4,6	2,3,4,7,8	3,4,6,7,8	2,3,4,6	2,6,7,8	2,3,4,6,7,8
S'm ≠ G-	1,2,7	3,5,6	1,2,3,7	3,5,6,7	1,2,3,5,6	1,2,5,6,7	1,2,3,5,6,7
s'm ≠ W-	1,2,8	4,5,6	1,2,4,8	4,5,6,8	1,2,4,5,6	1,2,5,6,8	1,2,4,5,6,8
S'M' ≠ O-	1,2,7,8	3,4,5,6	1,2,3,4,7,8	3,4,5,6,7,8	1,2,3,4,5,6	1,2,5,6,7,8	1,2,3,4,5,6,7,8

XXII. CONCERNING THE A FORTIORI ARGUMENT—The basic law of Logic is the Definition of the Equation—**An Equation is a combination of two names of the same thing.** All proof is empirical. While granting that all reasoning is either deduction, as immediate inference, or conversion, viz, from one premise, or else as mediate inference, or syllogism, viz, from two premises, it must be clearly understood that the validity of such is on **a posteriori** not on **a priori** grounds. The algorithms of reasoning, whether geometrical or algebraic, pictorial or symbolic, demonstrative or notational, merely express in different ways what is always empirical. It is true that all reasoning can be reduced to these prescribed forms, but their strength does not manifest some innate principle of the mind, but, rather, a property of truth which is independent of the mind; the mind merely perceives it. The ancient claim that all arguments can be reduced to syllogisms, if we count also the immediate inference as included in this form, cannot be disproved. Mathematical reasoning is obviously an aggregate of conversion & syllogism wherever actual reasoning is involved, for the rest it is either introductory, explanatory, critical or technical. The only possible doubt to be cast on the thesis concerns the so-called **a fortiori** argument & here, as a matter of cold blooded fact it should be emphasised that every syllogism can & must be reduced to the **a fortiori** argument before its illative validity can be demonstrated; thus for practical purposes they are one & the same thing. It is absurd then to suppose that the **a fortiori** argument cannot be reduced-to or identified-with the syllogism, although all pseudo-logicians have hitherto failed in this performance, most of them merely begging the question or else introducing extraneous matter into the process, which really should be very simple—as follows.

Take the usual example as given—**A** is greater than **B**, but **B** is greater than **C**; hence **A** is greater than **C**. Obviously "**B**" & "greater than **B**" are terms differing more than simply in quantity &/or quality, hence the appearance of "four" instead of the legitimate "three" terms of the syllogism, presenting the snag which has tripped would-be logical feet for centuries. But if we put the argument in the following perfectly correct form, the difficulty vanishes at once.

ALL **the greatness of B** is SOME OF **the greatness of A**,
M **P′**
All **the greatness of C** is some of **the greatness of B**,
S **M′**
Hence,
All **the greatness of C** is some of **the greatness of A**.
S **P′**

This is the classic type called "**Barbara**". Any other **a fortiori** can be similarly reduced, even as the above, merely by discerning what the argument is truly about. Take, e.g. the following: John is older than James, but James is older than George; hence John is older than George. Here the question is about the ages of three boys. Put the argument in this form—

All **the age of James** is some of **the age of John**,
M **P′**
All **the age of George** is some of **the age of James**,
S **M′**
Hence,
All **the age of George** is some of **the age of John.**
S **P′**

Simple & neat! Indeed, all problems about equality, identity, etc., are **a fortiori** arguments, as, e.g., **A** = **B**, here the question is of "equality"; the equality of **A** means "whatever is equal to **A** (in size, number, or whatever the argument calls-for); in the case of **A** is **B**, it is the "identity" of **A** & of **B**, which is to be considered either in whole or in part. No further discussion is necessary.

XXIII. BARBARA CUBED