COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

VOLUME I

PRINCIPLES OF PHILOSOPHY

AND

VOLUME II

ELEMENTS OF LOGIC

EDITED BY

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AND

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VOLUME I

PRINCIPLES OF PHILOSOPHY

Errata

1.1, line 11 *for* Stagyrite *read* Stagirite
1.20n* *for* vol. 9 *read* 8.7ff
1.133–134n† *for* c.1894 *read* 1901
1.283n‡ *for* vols. 7 and 8 *read* vol. 7
I, p. 178n* *for* From an unidentified fragment. *read* From *Basis of Pragmatism*, pp. 65–67.
1.545n* *for* c.1893 *read* 1893
1.553, line 11 *for* _he *read* the
1.573n* *for* Pragmatism *read* Pragmaticism
1.575n† *for* 2.8n. *read* 2.8.
1.584n*, line 5 *for* ch.11 *read* ch.12
1.585n† *for* vol. 9 *read* 8.132–152, except 136n3 and 138n4
I, P. 365: Index s.n. Boethius, *add* 659
I, p. 365: Index, *add* Eckius, 405n
I, p. 366: Index s.n. Spencer, *add* 174 *delete* 533
I, p. 377: Index, *add* Hemilogical, 567

INTRODUCTION

Charles Sanders Peirce plays a unique rôle in the history of American philosophy. During his own lifetime he published no book on philosophy, and except for a relatively short period he held no university chair from which to impress his influence upon students; yet he has come to be recognized as the founder of the one distinctive movement which this country has produced.

Pragmatism, as it developed, followed the pattern of William James' thought and that of John Dewey rather than the conceptions of Peirce; but it was Peirce, as James and Dewey magnanimously insisted, who defined the principle of the movement and gave it the first impetus. Never indeed a leader of movements, Peirce was an originator of ideas. He clearly formulated in his writings many conceptions which are only today beginning to find recognition, and there are implications in his thought which have not yet been fully developed.

Articles on pragmatism represent only one phase of his work. Some of his best thought was devoted to logical problems: to the logic of classes and relations, the theory of signs, scientific method, to probability and induction, and to the logical analysis of mathematics. In the development of exact or mathematical logic his papers represent the most important and considerable contributions in the period between Boole's *Laws of Thought* and Schröder's *Vorlesungen*. His writings on logic touch almost every point of theoretical interest in the subject.

His published papers, about seventy-five in number, include the series of articles on pragmatism, the logical papers, and important discussions of metaphysical problems. There are about twice as many book reviews. From these published works one may gather some suggestion of the versatility of his interests and the wide range of his studies, which included subjects as remote and unexpected as geodesy and astronomy, telepathy, criminology, and optics. But perhaps because carefully edited for publication, these papers and reviews fail to

reveal as they might another side of Peirce — his humor, freshness, pithiness of phrase, his exuberance of idea, erratic self-consciousness and self-confidence, his endless projection of vast systematic constructions, the gleams of genius described by James in his famous phrase as "flashes of brilliant light relieved against Cimmerian darkness." Only in the less formal writings does Peirce emerge as his friends at Harvard knew him in the great period of philosophy there at the turn of the century.

After Peirce's death in 1914, his unpublished manuscripts came into the care of the Department of Philosophy at Harvard University. They number several hundreds, not including fragments, the fruit of a long life devoted almost exclusively to philosophy and to science in a great variety of forms. These manuscripts represent all stages of incompleteness. Frequently there is no date or title, and many leaves are out of place or altogether missing. Some of them were rewritten as many as a dozen times: it is often evident that Peirce himself was not able to select the final form. Some are clearly identifiable as earlier drafts of his published papers; others one may assume to have been such drafts, although they differ from the published papers so much as to make this a matter of doubt. Often these unpublished studies contain passages, or longer portions, which impress those who have examined them as being of greater worth or clarity than those in the published articles. There are, likewise, a number of studies, often completed and of considerable length, and yet plainly unrelated to any which were printed. Sometimes they can be identified, through contemporary correspondence, as definite projects for publication which for one or another reason, never came to fruition. Often, however, there is no indication of such definite intent; he seems to have written merely from the impulse to formulate what was in his mind. Nevertheless, Peirce's studies of this kind are usually fairly continuous and systematic. If their merely private or preliminary nature is at all betrayed, this is because in them Peirce allows himself to follow out the ramifications of his topic, so that digressions appear which are inadmissible in print, but which show vividly the interconnectedness of his thought and the unsystematic character of his writings.

INTRODUCTION

Peirce possessed the system-making mind. That the merely external exigencies of his life and the indifference of publishers prevented any full-length presentation of his philosophy is a tragedy. And it is a tragedy which cannot now be set right. His system cannot be completely reconstructed; even the attempt would mean taking indefensible liberties with the manuscripts. The most that can be done is to select, with such judgment as one can command, the most important of these u .published papers and to compare them with his published writings on the same topic. Such selection is always difficult. Illuminating passages of great interest must be passed by because inextricably connected with other material the inclusion of which is not justified. On the other hand, because the doctrines they present are too important to be omitted, papers and fragments must often be included although one is sure that the author would not have printed them in their present condition. Often there are alternative drafts of the same study, one distinctly superior in some portion or respect; the other, in some other portion or respect. In such cases a choice is necessary, although any choice is a matter of regret.

In general, when Peirce's thought is at its best, he writes least well. For relatively superficial and transient topics he commanded a facile style, as in the many engaging contributions to *The Nation*. And in his more serious published work, he never allowed anything to leave his hand until it had attained a certain clarity and continuity. But when he is most in earnest (the manuscripts make this evident), the systematic and detailed character of his thought impedes his pen: he is likely to fall into some harsh jargon of his own, adopted in the interests of precision. The neatly turned phrase or brief and striking statement must often be rejected, in favor of one more technically accurate, or more complicated in the interest of adequacy. It is only just, however, to recognize that there are infelicities of style which occur in some of the papers included in these volumes which Peirce himself would never have allowed to remain in the final published form.

The more important of these manuscripts of Peirce, as well as his published papers, have now been brought together in some ten volumes which will appear in rapid succession. The first volume contains in outline his system, so far as it can be

presented, his writings on scientific method and the classification of the sciences, his doctrine of the categories, and his work on ethics. The next volume deals with the theory of signs and meaning, traditional logic, induction, the science of discovery and probability; and the third volume reprints his published work on modern logic. The fourth includes his unpublished original contributions to the foundations of mathematics, logic and graphs. The fifth volume contains his papers on pragmatism. The sixth is concerned with metaphysics. It is expected that the remaining volumes will contain his writings on physics and psychology, as well as his reviews, letters and biography.

Nearly all the members of the Department during the last fifteen years, as well as many others who were interested in Peirce, have devoted much time to the often very intractable material of the manuscripts. But the final and laborious work of selecting, arranging and preparing the papers for the press has been done by Dr. Charles Hartshorne, formerly Instructor in Philosophy at Harvard and by Dr. Paul Weiss, who is at present an Instructor in Philosophy at this university. The Department desires to express its gratitude to the many friends who have contributed generously towards the expense of printing the volumes.

* * *

Wherever possible Peirce's punctuation and spelling have been retained. Titles supplied by the editors for papers previously published are marked with an ^{E}, while Peirce's titles for unpublished papers are marked with a ^{P}. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not marked. Remarks and additions by the editors are inclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout each volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the paragraphs.

HARVARD UNIVERSITY
AUGUST, 1931.

PREFACE*

1. To erect a philosophical edifice that shall outlast the vicissitudes of time, my care must be, not so much to set each brick with nicest accuracy, as to lay the foundations deep and massive. Aristotle builded upon a few deliberately chosen concepts — such as matter and form, act and power — very broad, and in their outlines vague and rough, but solid, unshakable, and not easily undermined; and thence it has come to pass that Aristotelianism is babbled in every nursery, that "English Common Sense," for example, is thoroughly peripatetic, and that ordinary men live so completely within the house of the Stagyrite that whatever they see out of the windows appears to them incomprehensible and metaphysical. Long it has been only too manifest that, fondly habituated though we be to it, the old structure will not do for modern needs; and accordingly, under Descartes, Hobbes, Kant, and others, repairs, alterations, and partial demolitions have been carried on for the last three centuries. One system, also, stands upon its own ground; I mean the new Schelling-Hegel mansion, lately run up in the German taste, but with such oversights in its construction that, although brand new, it is already pronounced uninhabitable. The undertaking which this volume inaugurates is to make a philosophy like that of Aristotle, that is to say, to outline a theory so comprehensive that, for a long time to come, the entire work of human reason, in philosophy of every school and kind, in mathematics, in psychology, in physical science, in history, in sociology, and in whatever other department there may be, shall appear as the filling up of its details. The first step toward this is to find simple concepts applicable to every subject.†

2. But before all else, let me make the acquaintance of my reader, and express my sincere esteem for him and the deep pleasure it is to me to address one so wise and so patient. I

* 1 and 2 are from "A Guess at the Riddle" (c. 1898), see bk. III, ch. 3, §1 note. 3–7 and 8–14 are two fragments, c. 1897.

† See 23ff. and book III.

know his character pretty well, for both the subject and the style of this book ensure his being one out of millions. He will comprehend that it has not been written for the purpose of confirming him in his preconceived opinions, and he would not take the trouble to read it if it had. He is prepared to meet with propositions that he is inclined at first to dissent from; and he looks to being convinced that some of them are true, after all. He will reflect, too, that the thinking and writing of this book has taken, I won't say how long, quite certainly more than a quarter of an hour, and consequently fundamental objections of so obvious a nature that they must strike everyone instantaneously will have occurred to the author, although the replies to them may not be of that kind whose full force can be instantly apprehended.

3. The reader has a right to know how the author's opinions were formed. Not, of course, that he is expected to accept any conclusions which are not borne out by argument. But in discussions of extreme difficulty, like these, when good judgment is a factor, and pure ratiocination is not everything, it is prudent to take every element into consideration. From the moment when I could think at all, until now, about forty years, I have been diligently and incessantly occupied with the study of methods [of] inquiry, both those which have been and are pursued and those which ought to be pursued. For ten years before this study began, I had been in training in the chemical laboratory. I was thoroughly grounded not only in all that was then known of physics and chemistry, but also in the way in which those who were successfully advancing knowledge proceeded. I have paid the most attention to the methods of the most exact sciences, have intimately communed with some of the greatest minds of our times in physical science, and have myself made positive contributions — none of them of any very great importance, perhaps — in mathematics, gravitation, optics, chemistry, astronomy, etc. I am saturated, through and through, with the spirit of the physical sciences. I have been a great student of logic, having read everything of any importance on the subject, devoting a great deal of time to medieval thought, without neglecting the works of the Greeks, the English, the Germans, the French, etc., and have produced systems of my own both in deductive and in inductive logic.

In metaphysics, my training has been less systematic; yet I have read and deeply pondered upon all the main systems, never being satisfied until I was able to think about them as their own advocates thought.

4. The first strictly philosophical books that I read were of the classical German schools; and I became so deeply imbued with many of their ways of thinking that I have never been able to disabuse myself of them. Yet my attitude was always that of a dweller in a laboratory, eager only to learn what I did not yet know, and not that of philosophers bred in theological seminaries, whose ruling impulse is to teach what they hold to be infallibly true. I devoted two hours a day to the study of Kant's *Critic of the Pure Reason* for more than three years, until I almost knew the whole book by heart, and had critically examined every section of it. For about two years, I had long and almost daily discussions with Chauncey Wright, one of the most acute of the followers of J. S. Mill.

5. The effect of these studies was that I came to hold the classical German philosophy to be, upon its argumentative side, of little weight; although I esteem it, perhaps am too partial to it, as a rich mine of philosophical suggestions. The English philosophy, meagre and crude, as it is, in its conceptions, proceeds by surer methods and more accurate logic. The doctrine of the association of ideas is, to my thinking, the finest piece of philosophical work of the prescientific ages. Yet I can but pronounce English sensationalism to be entirely destitute of any solid bottom. From the evolutionary philosophers, I have learned little; although I admit that, however hurriedly their theories have been knocked together, and however antiquated and ignorant Spencer's *First Principles* and general doctrines, yet they are under the guidance of a great and true idea, and are developing it by methods that are in their main features sound and scientific.

6. The works of Duns Scotus have strongly influenced me. If his logic and metaphysics, not slavishly worshipped, but torn away from its medievalism, be adapted to modern culture, under continual wholesome reminders of nominalistic criticisms, I am convinced that it will go far toward supplying the philosophy which is best to harmonize with physical science. But other conceptions have to be drawn from the history of science and from mathematics.

7. Thus, in brief, my philosophy may be described as the attempt of a physicist to make such conjecture as to the constitution of the universe as the methods of science may permit, with the aid of all that has been done by previous philosophers. I shall support my propositions by such arguments as I can. Demonstrative proof is not to be thought of. The demonstrations of the metaphysicians are all moonshine. The best that can be done is to supply a hypothesis, not devoid of all likelihood, in the general line of growth of scientific ideas, and capable of being verified or refuted by future observers.

8. Religious infallibilism, caught in the current of the times, shows symptoms of declaring itself to be only practically speaking infallible; and when it has thus once confessed itself subject to gradations, there will remain over no relic of the good old tenth-century infallibilism, except that of the infallible scientists, under which head I include, not merely the kind of characters that manufacture scientific catechisms and homilies, churches and creeds, and who are indeed "born missionaries," but all those respectable and cultivated persons who, having acquired their notions of science from reading, and not from research, have the idea that "science" means knowledge, while the truth is, it is a misnomer applied to the pursuit of those who are devoured by a desire to find things out. . . .

9. Though infallibility in scientific matters seems to me irresistibly comical, I should be in a sad way if I could not retain a high respect for those who lay claim to it, for they comprise the greater part of the people who have any conversation at all. When I say they lay claim to it, I mean they assume the functions of it quite naturally and unconsciously. The full meaning of the adage *Humanum est errare*, they have never waked up to. In those sciences of measurement which are the least subject to error — metrology, geodesy, and metrica[1] astronomy — no man of self-respect ever now states his result, without affixing to it its *probable error*; and if this practice is not followed in other sciences it is because in those the probable errors are too vast to be estimated.

10. I am a man of whom critics have never found anything good to say. When they could see no opportunity to injure me, they have held their peace. The little laudation I have had has

come from such sources, that the only satisfaction I have derived from it, has been from such slices of bread and butter as it might waft my way. Only once, as far as I remember, in all my lifetime have I experienced the pleasure of praise — not for what it might bring but in itself. That pleasure was beatific; and the praise that conferred it was meant for blame. It was that a critic said of me that I did not seem to be *absolutely sure of my own conclusions*. Never, if I can help it, shall that critic's eye ever rest on what I am now writing; for I owe a great pleasure to him; and, such was his evident animus, that should he find that out, I fear the fires of hell would be fed with new fuel in his breast.

11. My book will have no instruction to impart to anybody. Like a mathematical treatise, it will suggest certain ideas and certain reasons for holding them true; but then, if you accept them, it must be because you like my reasons, and the responsibility lies with you. Man is essentially a social animal: but to be socia' one thing, to be gregarious is another: I decline to serve as bellwether. My book is meant for people who *want to find out;* and people who want philosophy ladled out to them can go elsewhere. There are philosophical soup shops at every corner, thank God!

12. The development of my ideas has been the industry of thirty years. I did not know as I ever should get to publish them, their ripening seemed so slow. But the harvest time has come, at last, and to me that harvest seems a wild one, but of course it is not I who have to pass judgment. It is not quite you, either, individual reader; it is experience and history.

13. For years in the course of this ripening process, I used for myself to collect my ideas under the designation *fallibilism;* and indeed the first step toward *finding out* is to acknowledge you do not satisfactorily know already; so that no blight can so surely arrest all intellectual growth as the blight of cocksureness; and ninety-nine out of every hundred good heads are reduced to impotence by that malady — of whose inroads they are most strangely unaware!

14. Indeed, out of a contrite fallibilism, combined with a high faith in the reality of knowledge, and an intense desire to find things out, all my philosophy has always seemed to me to grow. . . .

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GENERAL HISTORICAL ORIENTATION

PRINCIPLES OF PHILOSOPHY[P]

CHAPTER 1

LESSONS FROM THE HISTORY OF PHILOSOPHY

§1. NOMINALISM*

15. Very early in my studies of logic, before I had really been devoting myself to it more than four or five years, it became quite manifest to me that this science was in a bad condition, entirely unworthy of the general state of intellectual development of our age; and in consequence of this, every other branch of philosophy except ethics — for it was already clear that psychology was a special science and no part of philosophy — was in a similar disgraceful state. About that time — say the date of Mansel's *Prolegomena Logica*† — Logic touched bottom. There was no room for it to become more degraded. It had been sinking steadily, and relatively to the advance of physical science, by no means slowly from the time of the revival of learning — say from the date of the last fall of Constantinople.‡ One important addition to the subject had been made early in the eighteenth century, the Doctrine of Chances. But this had not come from the professed logicians, who knew nothing about it. Whewell, it is true, had been doing some fine work; but it was not of a fundamental character. De Morgan and Boole had laid the foundations for modern exact logic, but they can hardly be said to have begun the erection of the edifice itself. Under these circumstances, I naturally opened the dusty folios of the scholastic doctors. Thought generally was, of course, in a somewhat low condition under the Plantagenets. You can appraise it very well by the impression that Dante, Chaucer, Marco Polo, Froissart, and the great cathedrals make upon us. But [their] logic, relatively to the general condition of

* From the "Lowell Lectures of 1903," Lecture IIIa.

† 1851.

‡ 1453.

thought, was marvellously exact and critical. They can tell us nothing concerning methods of reasoning since their own reasoning was puerile; but their analyses of thought and their discussions of all those questions of logic that almost trench upon metaphysics are very instructive as well as very good discipline in that subtle kind of thinking that is required in logic.

16. In the days of which I am speaking, the age of Robert of Lincoln, Roger Bacon, St. Thomas Aquinas, and Duns Scotus, the question of nominalism and realism was regarded as definitively and conclusively settled in favor of realism. You know what the question was. It was whether *laws* and general *types* are figments of the mind or are real. If this be understood to mean whether there really are any laws and types, it is strictly speaking a question of metaphysics and not of logic. But as a first step toward its solution, it is proper to ask whether, granting that our common-sense beliefs are true, the analysis of the meaning of those beliefs shows that, according to those beliefs, laws and types are objective or subjective. This is a question of logic rather than of metaphysics — and as soon as this is answered the reply to the other question immediately follows after.

17. Notwithstanding a great outburst of nominalism in the fourteenth century which was connected with politics, the nominalists being generally opposed to the excessive powers of the pope and in favor of civil government, a connection that lent to the philosophical doctrine a factitious following, the Scotists, who were realists, were in most places the predominant party, and retained possession of the universities. At the revival of learning they stubbornly opposed the new studies; and thus the word *Duns*, the proper name of their master, came to mean an adversary of learning. The word originally further implied that the person so called was a master of subtle thought with which the humanists were unable to cope. But in another generation the disputations by which that power of thought was kept in training had lost their liveliness; and the consequence was that Scotism died out when the strong Scotists died. It was a mere change of fashion.

18. The humanists were weak thinkers. Some of them no doubt might have been trained to be strong thinkers; but they had no severe training in thought. All their energies went to

writing a classical language and an artistic style of expression. They went to the ancients for their philosophy; and mostly took up the three easiest of the ancient sects of philosophy, Epicureanism, Stoicism, and Scepticism. Epicureanism was a doctrine extremely like that of John Stuart Mill. The Epicureans alone of the later ancient schools believed in inductive reasoning, which they grounded upon the uniformity of nature, although they made the uniformity of nature to consist in somewhat different characters from those Stuart Mill emphasizes. Like Mill, the Epicureans were extreme nominalists. The Stoics advocated the flattest materialism, which nobody any longer has any need of doing since the new invention of Monism enables a man to be perfectly materialist in substance, and as idealistic as he likes in words. Of course the Stoics could not but be nominalists. They took no stock in inductive reasoning. They held it to be a transparent fallacy. The Sceptics of the *Renaissance* were something like the agnostics of the generation now passing away, except that they went much further. Our agnostics contented themselves with declaring everything beyond ordinary generalizations of experience to be unknowable, while the Sceptics did not think any scientific knowledge of any description to be possible. If you turn over the pages, for example, of Cornelius Agrippa's book *De* [*incertitudine et*] *vanitate scientiarum* [*et artium*] [1531], you will find he takes up every science in succession, arithmetic, geometry, mechanics, optics, and after examination pronounces each to be altogether beyond the power of the human mind. Of course, therefore, as far as they believed in anything at all, the Sceptics were nominalists.

19. In short, there was a tidal wave of nominalism. Descartes was a nominalist. Locke and all his following, Berkeley, Hartley, Hume, and even Reid, were nominalists. Leibniz was an extreme nominalist, and Rémusat [C. F. M.?] who has lately made an attempt to repair the edifice of Leibnizian monadology, does so by cutting away every part which leans at all toward realism. Kant was a nominalist; although his philosophy would have been rendered compacter, more consistent, and stronger if its author had taken up realism, as he certainly would have done if he had read Scotus. Hegel was a nominalist of realistic yearnings. I might continue the list

much further. Thus, in one word, all modern philosophy of every sect has been nominalistic.

20. In a long notice of Frazer's *Berkeley*, in the *North American Review* for October, 1871,* I declared for realism. I have since very carefully and thoroughly revised my philosophical opinions more than half a dozen times, and have modified them more or less on most topics; but I have never been able to think differently on that question of nominalism and realism. In that paper I acknowledged that the tendency of science has been toward nominalism; but the late Dr. Francis Ellingwood Abbot in the very remarkable introduction to his book entitled "Scientific Theism" [1885], showed on the contrary, quite conclusively, that science has always been at heart realistic, and always must be so; and upon comparing his writings with mine, it is easily seen that these features of nominalism which I pointed out in science are merely superficial and transient.

21. The heart of the dispute lies in this. The modern philosophers — one and all, unless Schelling be an exception — recognize but one mode of being, the being of an individual thing or fact, the being which consists in the object's crowding out a place for itself in the universe, so to speak, and reacting by brute force of fact, against all other things. I call that existence.

22. Aristotle, on the other hand, whose system, like all the greatest systems, was evolutionary, recognized besides an embryonic kind of being, like the being of a tree in its seed, or like the being of a future contingent event, depending on how a man shall decide to act. In a few passages Aristotle seems to have a dim *aperçue* of a third mode of being in the *entelechy*. The embryonic being for Aristotle was the being he called matter, which is alike in all things, and which in the course of its development took on form. Form is an element having a different mode of being. The whole philosophy of the scholastic doctors is an attempt to mould this doctrine of Aristotle into harmony with christian truth. This harmony the different doctors attempted to bring about in different ways. But all the realists agree in reversing the order of Aristotle's evolution by making the form come first, and the individuation of that

* See vol. 9.

form come later. Thus, they too recognized two modes of being; but they were not the two modes of being of Aristotle.

23. My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future.

24. Let us begin with considering actuality, and try to make out just what it consists in. If I ask you what the actuality of an event consists in, you will tell me that it consists in its happening *then* and *there*. The specifications *then* and *there* involve all its relations to other existents. The actuality of the event seems to lie in its relations to the universe of existents. A court may issue *injunctions* and *judgments* against me and I not care a snap of my finger for them. I may think them idle vapor. But when I feel the sheriff's hand on my shoulder, I shall begin to have a sense of actuality. Actuality is something *brute*. There is no reason in it. I instance putting your shoulder against a door and trying to force it open against an unseen, silent, and unknown resistance. We have a two-sided consciousness of effort and resistance, which seems to me to come tolerably near to a pure sense of actuality. On the whole, I think we have here a mode of being of one thing which consists in how a second object is. I call that Secondness.

25. Besides this, there are two modes of being that I call Firstness and Thirdness. Firstness is the mode of being which consists in its subject's being positively such as it is regardless of aught else. That can only be a possibility. For as long as things do not act upon one another there is no sense or meaning in saying that they have any being, unless it be that they are such in themselves that they may perhaps come into relation with others. The mode of being a *redness*, before anything in the universe was yet red, was nevertheless a positive qualitative possibility. And redness in itself, even if it be embodied, is something positive and *sui generis*. That I call Firstness. We naturally attribute Firstness to outward objects, that is we suppose they have capacities in themselves which may or may not be already actualized, which may or may not ever be actualized, although we can know nothing of such possibilities [except] so far as they are actualized.

26. Now for Thirdness. Five minutes of our waking life will hardly pass without our making some kind of prediction; and in the majority of cases these predictions are fulfilled in the event. Yet a prediction is essentially of a general nature, and cannot ever be completely fulfilled. To say that a prediction has a decided tendency to be fulfilled, is to say that the future events are in a measure really governed by a law. If a pair of dice turns up sixes five times running, that is a mere uniformity. The dice might happen fortuitously to turn up sixes a thousand times running. But that would not afford the slightest security for a prediction that they would turn up sixes the next time. If the prediction has a tendency to be fulfilled, it must be that future events have a tendency to conform to a general rule. "Oh," but say the nominalists, "this general rule is nothing but a mere word or couple of words!" I reply, "Nobody ever dreamed of denying that what is general is of the nature of a general sign; but the question is whether future events will conform to it or not. If they will, your adjective 'mere' seems to be ill-placed." A rule to which future events have a tendency to conform is *ipso facto* an important thing, an important element in the happening of those events. This mode of being which *consists*, mind my word if you please, the mode of being which *consists* in the fact that future facts of Secondness will take on a determinate general character, I call a Thirdness.

§2. CONCEPTUALISM*

27. Many philosophers call their variety of nominalism, "conceptualism"; but it is essentially the same thing; and their not seeing that it is so is but another example of that loose and slapdash style of thinking that has made it possible for them to remain nominalists. Their calling their "conceptualism" a middle term between realism and nominalism is itself an example in the very matter to which nominalism relates. For while the question between nominalism and realism is, in its nature, susceptible of but two answers: yes and no, they make an idle and irrelevant point which had been thoroughly considered by all the great realists; and instead of

* From "Essays on Meaning," June, 1909.

drawing a valid distinction, as they suppose, only repeat the very same confusion of thought which made them nominalists. The question was whether all properties, laws of nature, and predicates of more than an actually existent subject are, without exception, mere figments or not.[1] The conceptualists seek to wedge in a third position conflicting with the principle of excluded middle. They say, "Those universals are real, indeed; but they are only real thoughts." So much may be said of the philosopher's stone. To give that answer constitutes a man a nominalist. Are the laws of nature, and that property of gold by which it will yield the purple of Cassius, no more real than the philosopher's stone? No, the conceptualists admit that there is a difference; but they say that the laws of nature and the properties of chemical species are results of thinking. The great realists had brought out all the truth there is in that much more distinctly long before modern conceptualism appeared in the world. They showed that the general is not capable of full actualization in the world of action and reaction but is of the nature of what is thought, but that our thinking only apprehends and does not create thought, and that that thought may and does as much govern outward things as it does our thinking. But those realists did not fall into any confusion between the real fact of having a dream and the illusory object dreamed. The conceptualist doctrine is an undisputed truism about *thinking*, while the question between nominalists and realists relates to *thoughts*, that is, to the objects which thinking enables us to know.

[1] It must not be imagined that any notable realist of the thirteenth or fourteenth century took the ground that any "universal" was what we in English should call a "thing," as it seems that, in an earlier age, some realists and some nominalists, too, had done; though perhaps it is not quite certain that they did so, their writings being lost. Their very definition of a "universal" admits that it is of the same generic nature as a word, namely, is: "Quod natum optum est *praedicari* de pluribus." Neither was it their doctrine that any "universal" *itself* is real. They might, indeed, some of them, think so; but their realism did not consist in *that* opinion, but in holding that what the word *signifies*, in contradistinction to what it can be truly said of, is real. Anybody may happen to opine that "the" is a real English word; but that will not constitute him a realist. But if he thinks that, whether the word "hard" itself be real or not, the property, the character, the predicate, *hardness*, is not invented by men, as the word is, but is really and truly in the hard things and is one in them all, as a description of habit, disposition, or behavior, *then* he *is* a realist.

§3. THE SPIRIT OF SCHOLASTICISM*

28. . . . [The] history of logic is not altogether without an interest as a branch of history. For so far as the logic of an age adequately represents the methods of thought of that age, its history is a history of the human mind in its most essential relation — that is to say with reference to its power of investigating truth. But the chief value of the study of historical philosophy is that it disciplines the mind to regard philosophy with a cold and scientific eye and not with passion as though philosophers were contestants.

29. British logic is a subject of some particular interest inasmuch as some peculiar lines of thought have always been predominant in those islands, giving their logicians a certain family resemblance, which already begins to appear in very early times. The most striking characteristic of British thinkers is their nominalistic tendency. This has always been and is now very marked. So much so that in England and in England alone are there many thinkers more distinguished at this day as being nominalistic than as holding any other doctrines. William Ockham or Oakum, an Englishman, was beyond question the greatest nominalist that ever lived; while Duns Scotus, another British name, it is equally certain is the subtilest advocate of the opposite opinion. These two men, Duns Scotus and William Ockham, are decidedly the greatest speculative minds of the middle ages, as well as two of the profoundest metaphysicians that ever lived. Another circumstance which makes [the] logic of the British Islands interesting is that there more than elsewhere have the studies of the logic of the natural sciences been made. Already we find some evidences of English thought running in that direction, when we meet with that singular phenomenon Roger Bacon — a man who was scientific before science began. At the first dawn of the age [of] science, Francis Bacon wrote that professedly and really logical treatise, the *Novum Organum*, a work the celebrity of which perhaps exceeds its real merits. In our own day, the writings of Whewell, Mill, and Herschel afford some of the finest accounts of the methods of thought in science. Another direction in which logical thought has gone farther in England than

* From Lecture I, "Early Nominalism and Realism" of the "Lectures on British Logicians," delivered at Harvard in 1869.

elsewhere is in mathematico-formal logic — the chief writers on which are Boole, DeMorgan, and the Scotch Sir William Hamilton — for although Hamilton was so bitter against mathematics, that his own doctrine of the quantified predicate is essentially mathematical is beyond intelligent dispute. This fondness for the formal part of logic had already appeared in the middle ages, when the nominalistic school of Ockham—the most extremely scholastic of the scholastics — and next to them the school of Scotus carried to the utmost the doctrines of the *Parva Logicalia* which were the contribution of those ages to this branch of the science. And those *Parva Logicalia* may themselves have had an English origin, for the earliest known writer upon the subject — unless the Synopsis Αριστοτελους Οργανον be attributed to Psellus — was an Englishman, William Shirwood. . . .*

30. The most striking characteristic of medieval thought is the importance attributed to authority. It was held that authority and reason were two coördinate methods of arriving at truth, and far from holding that authority was secondary to reason, the scholastics were much more apt to place it quite above reason. When Berengarius in his dispute with Lanfranc remarked that the whole of an affirmation does not stand after a part is subverted, his adversary replied: "The sacred authorities being relinquished, you take refuge in dialectic, and when I am to hear and to answer concerning the ministry of the Faith, I prefer to hear and to answer the sacred authorities which are supposed to relate to the subject rather than dialectical reasons." To this Berengarius replied that St. Augustine in his book *De doctrina christiana* says that what he said concerning an affirmation is bound up indissolubly with that very eternity of truth which is God. But added: "Maximi plane cordis est, per omnia ad dialecticum confugere, quia confugere ad eam ad rationem est confugere, quo qui non confugit, cum secundum rationem sit factus ad imaginem Dei, suum honorem reliquit, nec potest renovari de die in diem ad imaginem Dei."† Next to sacred authorities — the Bible, the church and the fathers — that of Aristotle of course ranked the highest. It could be denied, but the presumption was immense against his being wrong on any particular point.

* Cf. Prantl's *Geschichte der Logik*, 2 Aufl. Bd. 2, S. 266; Bd. 3, S. 10ff.

† Ibid., vol. 2, p. 72.

31. Such a weight being attached to authority — a weight which would be excessive were not the human mind at that time in so uneducated a state that it could not do better than follow masters, since it was totally incompetent to solve metaphysical problems for itself — it follows naturally that originality of thought was not greatly admired, but that on the contrary the admirable mind was his who succeeded in interpreting consistently the dicta of Aristotle, Porphyry, and Boethius. Vanity, therefore, the vanity of cleverness, was a vice from which the schoolmen were remarkably free. They were minute and thorough in their knowledge of such authorities as they had, and they were equally minute and thorough in their treatment of every question which came up.

32. All these characters remind us less of the philosophers of our day than of the men of science. I do not hesitate to say that scientific men now think much more of authority than do metaphysicians; for in science a question is not regarded as settled or its solution as certain until all intelligent and informed doubt has ceased and all competent persons have come to a catholic agreement, whereas fifty metaphysicians, each holding opinions that no one of the other forty-nine can admit, will nevertheless generally regard their fifty opposite opinions as more certain than that the sun will rise tomorrow. This is to have what seems an absurd disregard for others' opinions. The man of science attaches positive value to the opinion of every man as competent as himself, so that he cannot but have a doubt of a conclusion which he would adopt were it not that a competent man opposes it; but on the other hand, he will regard a sufficient divergence from the convictions of the great body of scientific men as tending of itself to argue incompetence, and he will generally attach little weight to the opinions of men who have long been dead and were ignorant of much that has been since discovered which bears upon the question in hand. The schoolmen, however, attached the greatest authority to men long since dead, and there they were right, for in the dark ages it was not true that the later state of human knowledge was the most perfect, but on the contrary. I think it may be said then that the schoolmen did not attach too much weight to authority, although they attached much more to it than we ought to do or than ought or could be

attached to it in any age in which science is pursuing a successful and onward course — and of course infinitely more than is attached to it by those intellectual nomads, the modern metaphysicians, including the positivists.

33. In the slight importance they attached to a brilliant theory, the schoolmen also resembled modern scientific men, who cannot be comprehended in this respect at all by men not scientific. The followers of Herbert Spencer, for example, cannot comprehend why scientific men place Darwin so infinitely above Spencer, since the theories of the latter are so much grander and more comprehensive. They cannot understand that it is not the sublimity of Darwin's theories which makes him admired by men of science, but that it is rather his minute, systematic, extensive, strict, scientific researches which have given his theories a more favorable reception — theories which in themselves would barely command scientific respect. And this misunderstanding belongs to all those metaphysicians who fancy themselves men of science on account of their metaphysics. This same scientific spirit has been equally misunderstood as it is found in the schoolmen. They have been above all things found fault with because they do not write a literary style and do not "study in a literary spirit." The men who make this objection cannot possibly comprehend the real merits of modern science. If the words *quidditas*, *entitas*, and *haecceitas* are to excite our disgust, what shall we say of the Latin of the botanists, and the style of any technically scientific work? As for that phrase "studying in a literary spirit" it is impossible to express how nauseating it is to any scientific man, yes even to the scientific linguist. But above all things it is the searching thoroughness of the schoolmen which affiliates them with men of science and separates them, world-wide, from modern so-called philosophers. The thoroughness I allude to consists in this, that in adopting any theory, they go about everywhere, they devote their whole energies and lives in putting it to tests *bona fide* — not such as shall merely add a new spangle to the glitter of their proofs but such as shall really go toward satisfying their restless insatiable impulse to put their opinions to the *test*. Having a theory, they must apply it to every subject and to every branch of every subject to see whether it produces a result in accordance with the only cri-

teria they were able to apply—the truth of the Catholic faith and the teaching of the Prince of Philosophers.

34. Mr. George Henry Lewes in his work on Aristotle* seems to me to have come pretty near to stating the true cause of the success of modern science when he has said that it was *verification*. I should express it in this way: modern students of science have been successful because they have spent their lives not in their libraries and museums but in their laboratories and in the field; and while in their laboratories and in the field they have been not gazing on nature with a vacant eye, that is, in passive perception unassisted by thought, but have been *observing* — that is, perceiving by the aid of analysis — and testing suggestions of theories. The cause of their success has been that the motive which has carried them to the laboratory and the field has been a craving to know how things really were, and an interest in finding out whether or not general propositions actually held good — which has overbalanced all prejudice, all vanity, and all passion. Now it is plainly not an essential part of this method in general that the tests were made by the observation of natural objects. For the immense progress which modern mathematics has made is also to be explained by the same intense interest in testing general propositions by particular cases — only the tests were applied by means of particular demonstrations. This is observation, still, for as the great mathematician Gauss has declared — algebra is a science of the eye,† only it is observation of artificial objects and of a highly recondite character. Now this same unwearied interest in testing general propositions is what produced those long rows of folios of the schoolmen, and if the test which they employed is of only limited validity so that they could not unhampered go on indefinitely to further discoveries, yet the *spirit*, which is the most essential thing — the *motive*, was nearly the same. And how different this spirit is from that of the major part, though not all, of modern philosophers — even of those who have called themselves empirical, no man who is actuated by it can fail to perceive.

* *Aristotle: A Chapter from the History of Science*, London (1864).

† Quoted by Sylvester in his Presidential Address to the British Assn. in 1868. See Sylvester's *Mathematical Papers*, vol. 2, p. 654.

§4. KANT AND HIS REFUTATION OF IDEALISM*

35. Kant's whole philosophy turns upon his logic. He gives the name of logic to the greater part of his *Critic of the Pure Reason*, and it is a result of the great fault of his logical theory that he does not extend that name to the whole work. This greatest fault was at the same [time] the greatest merit of his doctrine: it lay in his sharp discrimination of the intuitive and the discursive processes of the mind. The distinction itself is not only familiar to everybody but it had long played a part in philosophy. Nevertheless, it is on such obvious distinctions that the greater systems have been founded, and [Kant] saw far more clearly than any predecessor had done the whole philosophical import of this distinction. This was what emancipated him from Leibnizianism, and at the same time turned him against sensationalism. It was also what enabled him to see that no general description of existence is possible, which is perhaps the most valuable proposition that the *Critic* contains. But he drew too hard a line between the operations of observation and of ratiocination. He allows himself to fall into the habit of thinking that the latter only begins after the former is complete; and wholly fails to see that even the simplest syllogistic conclusion can only be drawn by observing the relations of the terms in the premisses and conclusion. His doctrine of the *schemata* can only have been an afterthought, an addition to his system after it was substantially complete. For if the *schemata* had been considered early enough, they would have overgrown his whole work.

36. Kant's refutation of idealism in the second edition of the *Critic of the Pure Reason* has been often held to be inconsistent with his main position or even to be knowingly sophistical. It appears to me to be one of the numerous passages in that work which betray an elaborated and vigorous analysis, marred in the exposition by the attempt to state the argument more abstractly and demonstratively than the thought would warrant.

In "Note 1," Kant says that his argument beats idealism

* 35 is an unpublished, uncompleted review of T. K. Abbott's translation of Kant's *Introduction to Logic*, etc. Longmans Green & Co., 1885. 37–38 is "Notes on the Question of the Existence of an External World." c. 1890. 36 and 39 are from fragmentary alternative mss. of that same date.

at its own game. How is that? The idealist says that all that we know immediately, that is, otherwise than inferentially, is what is *present* in the mind; and things out of the mind are not so present. The whole idealist position turns upon this conception of the *present.*

37. The idealistic argument turns upon the assumption that certain things are absolutely "present," namely what we have in mind at the moment, and that nothing else can be immediately, that is, otherwise than inferentially known. When this is once granted, the idealist has no difficulty in showing that that external existence which we cannot know immediately we cannot know, at all. Some of the arguments used for this purpose are of little value, because they only go to show that our knowledge of an external world is fallible; now there is a world of difference between fallible knowledge and no knowledge. However, I think it would have to be admitted as a matter of logic that if we have no immediate perception of a non-ego, we can have no reason to admit the supposition of an existence so contrary to all experience as that would in that case be.

38. But what evidence is there that we can immediately know only what is "present" to the mind? The idealists generally treat this as self-evident; but, as Clifford jestingly says, "it is evident" is a phrase which only means "we do not know how to prove." The proposition that we can immediately perceive only what is present seems to me parallel to that other vulgar prejudice that "a thing cannot act where it is not." An opinion which can only defend itself by such a sounding phrase is pretty sure to be wrong. That a thing cannot act where it is not is plainly an induction from ordinary experience, which shows no forces except such as act through the resistance of materials, with the exception of gravity which, owing to its being the same for all bodies, does not appear in ordinary experience like a force. But further experience shows that attractions and repulsions are the universal types of forces. A thing may be said to be wherever it acts; but the notion that a particle is absolutely present in one part of space and absolutely absent from all the rest of space is devoid of all foundation. In like manner, the idea that we can immediately perceive only what is present seems to be founded on our ordinary experi-

ence that we cannot recall and reëxamine the events of yesterday nor know otherwise than by inference what is to happen tomorrow. Obviously, then, the first move toward beating idealism at its own game is to remark that we apprehend our own ideas only as flowing in time, and since neither the future nor the past, however near they may be, is *present*, there is as much difficulty in conceiving our perception of what passes within us as in conceiving external perception. If so, replies the idealist, instead of giving up idealism we must go still further to nihilism. Kant does not notice this retort; but it is clear from his footnote that he would have said: Not so; for it is impossible we should so much as think we think in time unless we do think in time; or rather, dismissing blind impossibility, the mere imagination of time is a clear perception of the past. Hamilton* stupidly objects to Reid's phrase "immediate memory"; but an immediate, intuitive consciousness of time clearly exists wherever time exists. But once grant immediate knowledge in time, and what becomes of the idealist theory that we immediately know only the *present?* For the present can contain no time.

39. But Kant does not pursue this line of thought along the straight road to its natural result; because he is a sort of idealist himself. Namely, though not idealistic as to the substance of things, he is partially so in regard to their accidents. Accordingly, he introduces his distinction of the variable and the persistent (*beharrlich*), and seeks to show that the only way we can apprehend our own flow of ideas, binding them together as a connected flow, is by attaching them to an immediately perceived persistent externality. He refuses to inquire how that immediate external consciousness is possible, though such an inquiry might have probed the foundations of his system.

§5. HEGELISM†

40. The critical logicians have been much affiliated to the theological seminaries. About the thinking that goes on in laboratories they have known nothing. Now the seminarists

* Sir William Hamilton's *Discussions on Philosophy and Literature*, ch. 2, p. 55. What Hamilton objects to is "*immediate* knowledge of the *past*" as a definition of memory.

† 40 and 41–2 are from separate unidentified fragments, c. 1892.

and religionists generally have at all times and places set their faces against the idea of continuous growth. That disposition of intellect is the most catholic element of religion. Religious truth having been once defined is never to be altered in the most minute particular; and theology being held as queen of the sciences, the religionists have bitterly fought by fire and tortures all great advances in the true sciences; and if there be no true continuous growth in men's ideas where else in the world should it be looked for? Thence, we find this folk setting up hard lines of demarcation, or great gulfs, contrary to all observation, between good men and bad, between the wise and foolish, between the spirit and the flesh, between all the different kinds of objects, between one quantity and the next. So shut up are they in this conception of the world that when the seminarist Hegel discovered that the universe is everywhere permeated with continuous growth (for that, and nothing else, is the "Secret of Hegel") it was supposed to be an entirely new idea, a century and a half after the differential calculus had been in working order.

41. Hegel, while regarding scientific men with disdain, has for his chief topic the importance of continuity, which was the very idea the mathematicians and physicists had been chiefly engaged in following out for three centuries. This made Hegel's work less correct and excellent in itself than it might have been; and at the same time hid its true mode of affinity with the scientific thought into which the life of the race had been chiefly laid up. It was a misfortune for Hegelism, a misfortune for "philosophy," and a misfortune (in lesser degree) for science.

42. My philosophy resuscitates Hegel, though in a strange costume.

CHAPTER 2

*LESSONS FROM THE HISTORY OF SCIENCE**

§1. THE SCIENTIFIC ATTITUDE

43. If we endeavor to form our conceptions upon history and life, we remark three classes of men. The first consists of those for whom the chief thing is the qualities of feelings. These men create art. The second consists of the practical men, who carry on the business of the world. They respect nothing but power, and respect power only so far as it [is] exercized. The third class consists of men to whom nothing seems great but reason. If force interests them, it is not in its exertion, but in that it has a reason and a law. For men of the first class, nature is a picture; for men of the second class, it is an opportunity; for men of the third class, it is a cosmos, so admirable, that to penetrate to its ways seems to them the only thing that makes life worth living. These are the men whom we see possessed by a passion to learn, just as other men have a passion to teach and to disseminate their influence. If they do not give themselves over completely to their passion to learn, it is because they exercise self-control. Those are the natural scientific men; and they are the only men that have any real success in scientific research.

44. If we are to define science, not in the sense of stuffing it into an artificial pigeon-hole where it may be found again by some insignificant mark, but in the sense of characterizing it as a living historic entity, we must conceive it as that about which such men as I have described busy themselves. As such, it does not consist so much in *knowing*, nor even in "organized knowledge," as it does in diligent inquiry into truth for truth's sake, without any sort of axe to grind, nor for the sake of the delight of contemplating it, but from an impulse to penetrate into the reason of things. This is the sense in which

* A manuscript of notes for a projected, but never completed, *History of Science*, c. 1896.

this book is entitled a History of *Science*. Science and philosophy seem to have been changed in their cradles. For it is not knowing, but the love of learning, that characterizes the scientific man; while the "philosopher" is a man with a system which he thinks embodies all that is best worth knowing. If a man burns to learn and sets himself to comparing his ideas with experimental results in order that he may correct those ideas, every scientific man will recognize him as a brother, no matter how small his knowledge may be.

45. But if a man occupies himself with investigating the truth of some question for some ulterior purpose, such as to make money, or to amend his life, or to benefit his fellows, he may be ever so much better than a scientific man, if you will — to discuss that would be aside from the question — but he is not a scientific man. For example, there are numbers of chemists who occupy themselves exclusively with the study of dyestuffs. They discover facts that are useful to scientific chemistry; but they do not rank as genuine scientific men. The genuine scientific chemist cares just as much to learn about erbium — the extreme rarity of which renders it commercially unimportant — as he does about iron. He is more eager to learn about erbium if the knowledge of it would do more to complete his conception of the Periodic Law, which expresses the mutual relations of the elements.

§2. THE SCIENTIFIC IMAGINATION

46. When a man desires ardently to know the truth, his first effort will be to imagine what that truth can be. He cannot prosecute his pursuit long without finding that imagination unbridled is sure to carry him off the track. Yet nevertheless, it remains true that there is, after all, nothing but imagination that can ever supply him an inkling of the truth. He can stare stupidly at phenomena; but in the absence of imagination they will not connect themselves together in any rational way. Just as for Peter Bell a cowslip was nothing but a cowslip, so for thousands of men a falling apple was nothing but a falling apple; and to compare it to the moon would by them be deemed "fanciful."

47. It is not too much to say that next after the passion to

learn there is no quality so indispensable to the successful prosecution of science as imagination. Find me a people whose early medicine is not mixed up with magic and incantations, and I will find you a people devoid of all scientific ability. There is no magic in the medical Papyrus Ebers. The stolid Egyptian saw nothing in disease but derangement of the affected organ. There never was any true Egyptian science.

48. There are, no doubt, kinds of imagination of no value in science, mere artistic imagination, mere dreaming of opportunities for gain. The scientific imagination dreams of explanations and laws.

§3. SCIENCE AND MORALITY

49. A scientific man must be single-minded and sincere with himself. Otherwise, his love of truth will melt away, at once. He can, therefore, hardly be otherwise than an honest, fair-minded man. True, a few naturalists have been accused of purloining specimens; and some men have been far from judicial in advocating their theories. Both of these faults must be exceedingly deleterious to their scientific ability. But on the whole, scientific men have heen the best of men. It is quite natural, therefore, that a young man who might develope into a scientific man should be a well-conducted person.

50. Yet in more ways than one an exaggerated regard for morality is unfavorable to scientific progress. I shall present only one of those ways. It will no doubt shock some persons that I should speak of morality as involving an element which can become bad. To them good conduct and moral conduct are one and the same — and they will accuse me of hostility to morality. I regard morality as highly necessary; but it is a means to good life, not necessarily coextensive with good conduct. Morality consists in the folklore of right conduct. A man is brought up to think he ought to behave in certain ways. If he behaves otherwise, he is uncomfortable. His conscience pricks him. That system of morals is the traditional wisdom of ages of experience. If a man cuts loose from it, he will become the victim of his passions. It is not safe for him even to reason about it, except in a purely speculative way. Hence, morality is essentially conservative. Good morals and good manners are identical, except that tradition

attaches less importance to the latter. The gentleman is imbued with conservatism. This conservatism is a habit, and it is the law of habit that it tends to spread and extend itself over more and more of the life. In this way, conservatism about morals leads to conservatism about manners and finally conservatism about opinions of a speculative kind. Besides, to distinguish between speculative and practical opinions is the mark of the most cultivated intellects. Go down below this level and you come across reformers and rationalists at every turn — people who propose to remodel the ten commandments on modern science. Hence it is that morality leads to a conservatism which any new view, or even any free inquiry, no matter how purely speculative, shocks. The whole moral weight of such a community will be cast against science. To inquire into nature is for a Turk very unbecoming to a good Moslem; just as the family of Tycho Brahe regarded his pursuit of astronomy as unbecoming to a nobleman. (See Thomas Nash in *Pierce Pennilesse* for the character of a Danish nobleman.)

51. This tendency is necessarily greatly exaggerated in a country when the "gentleman," or recognized exponent of good manners, is appointed to that place as the most learned man. For then the inquiring spirit cannot say the gentlemen are a lot of ignorant fools. To the moral weight cast against progress in science is added the weight of superior learning. Wherever there is a large class of academic professors who are provided with good incomes and looked up to as gentlemen, scientific inquiry must languish. Wherever the bureaucrats are the more learned class, the case will be still worse.

§4. MATHEMATICS

52. The first questions which men ask about the universe are naturally the most general and abstract ones. Nor is it true, as has so often been asserted, that these are the most difficult questions to answer. Francis Bacon is largely responsible for this error, he having represented — having nothing but his imagination and no acquaintance with actual science to draw upon — that the most general inductions must be reached by successive steps. History does not at all bear out

that theory. The errors about very general questions have been due to a circumstance which I proceed to set forth.

53. The most abstract of all the sciences is mathematics. That this is so, has been made manifest in our day; because all mathematicians now see clearly that mathematics is only busied about *purely hypothetical questions*. As for what the truth of existence may be the mathematician does not (*qua* mathematician) care a straw. It is true that early mathematicians could not clearly see that this was so. But for all their not seeing it, it was just as true of the mathematics of early days as of our own. The early mathematician might perhaps be more inclined to assert roundly that two straight lines in a plane cut by a third so as to make the sum of the internal angles on one side less than two right angles would meet at some finite distance on that side if sufficiently produced; although, as a matter of fact, we observe no such tendency in Euclid. But however that may have been, the early mathematician had certainly no more tendency than the modern to *inquire into the truth of that postulate;* but quite the reverse. What he really did, therefore, was merely to deduce consequences of unsupported assumptions, whether he recognized that this was the nature of his business or not. Mathematics, then, really was, for him as for us, the most abstract of the sciences, cut off from all inquiry into existential truth. Consequently, the tendency to attack the most abstract problems first, not because they were *recognized* as such, but because such they *were*, led to mathematics being the earliest field of inquiry.

54. We find some peoples drawn more toward arithmetic; others more toward geometry. But in either case, a correct method of reasoning was sure to be reached before many centuries of real inquiry had elapsed. The reasoning would be at first awkward, and one case would be needlessly split up into several. But still all influences were pressing the reasoner to make use of a diagram, and as soon as he did that he was pursuing the correct method. For mathematical reasoning consists in constructing a diagram according to a general precept, in observing certain relations between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms. All valid necessary reasoning is in

fact thus diagrammatic.* This, however, is far from being obviously true. There was nothing to draw the attention of the early reasoners to the need of a diagram in such reasoning. Finding that by their inward meditations they could deduce the truth concerning, for example, the height of an inaccessible pillar, they naturally concluded the same method could be applied to positive inquiries.

In this way, early success in mathematics would naturally lead to bad methods in the positive sciences, and especially in metaphysics.

§5. SCIENCE AS A GUIDE TO CONDUCT

55. We have seen how success in mathematics would necessarily create a confidence altogether unfounded in man's power of eliciting truth by inward meditation without any aid from experience. Both its confidence in what is within and the absolute certainty of its conclusions lead to the confusion of *a priori* reason with conscience. For conscience, also, refuses to submit its dicta to experiment, and makes an absolute dual distinction between right and wrong. One result of this is that men begin to rationalize about questions of purity and integrity, which in the long run, through moral decay, is unfavorable to science. But what is worse, from our point of view, they begin to look upon science as a guide to conduct, that is, no longer as pure science but as an instrument for a practical end. One result of this is that all probable reasoning is despised. If a proposition is to be applied to action, it has to be embraced, or believed without reservation. There is no room for doubt, which can only paralyze action. But the scientific spirit requires a man to be at all times ready to dump his whole cartload of beliefs, the moment experience is against them. The desire to learn forbids him to be perfectly cocksure that he knows already. Besides positive science can only rest on experience; and experience can never result in absolute certainty, exactitude, necessity, or universality. But it is precisely with the universal and necessary, that is, with Law, that [con]science concerns itself. Thus the real character of science is destroyed as soon as it is made an adjunct to conduct; and especially all progress in the inductive sciences is brought to a standstill.

* See 66, 240, 369 and vol. 4, bk. II.

§6. MORALITY AND SHAM REASONING

56. The effect of mixing speculative inquiry with questions of conduct results finally in a sort of half make-believe reasoning which deceives itself in regard to its real character. Conscience really belongs to the subconscious man, to that part of the soul which is hardly distinct in different individuals, a sort of community-consciousness, or public spirit, not absolutely one and the same in different citizens, and yet not by any means independent in them. Conscience has been created by experience just as any knowledge is; but it is modified by further experience only with secular* slowness.

57. When men begin to rationalize about their conduct, the first effect is to deliver them over to their passions and produce the most frightful demoralization, especially in sexual matters. Thus, among the Greeks, it brought about pæderasty and a precedence of public women over private wives. But ultimately the subconscious part of the soul, being stronger, regains its predominance and insists on setting matters right. Men, then, continue to tell themselves they regulate their conduct by reason; but they learn to look forward and see what conclusions a given method will lead to before they give their adhesion to it. In short, it is no longer the reasoning which determines what the conclusion shall be, but it is the conclusion which determines what the reasoning shall be. This is sham reasoning. In short, as morality supposes self-control, men learn that they must not surrender themselves unreservedly to any method, without considering to what conclusions it will lead them. But this is utterly contrary to the single-mindedness that is requisite in science. In order that science may be successful, its votaries must hasten to surrender themselves at discretion to experimental inquiry, in advance of knowing what its decisions may be. There must be no reservations.

58. The effect of this shamming is that men come to look upon reasoning as mainly decorative, or at most, as a secondary aid in minor matters — a view not altogether unjust, if questions of conduct are alone to interest us. They, therefore, demand that it shall be plain and facile. If, in special cases, complicated reasoning is indispensable, they hire a specialist to perform it. The result of this state of things is, of course, a rapid

* On this use of "secular" see 176.

deterioration of intellectual vigor, very perceptible from one generation to the next. This is just what is taking place among us before our eyes; and to judge from the history of Constantinople, it is likely to go on until the race comes to a despicable end.

§7. THE METHOD OF AUTHORITY

59. When society is broken into bands, now warring, now allied, now for a time subordinated one to another, man loses his conceptions of truth and of reason. If he sees one man assert what another denies, he will, if he is concerned, choose his side and set to work by all means in his power to silence his adversaries. The truth for him is that for which he fights.

60. The next step which is to be expected in a logical development not interrupted by accidental occurrences will consist in the recognition that a central authority ought to determine the beliefs of the entire community. As far as morals and religion go, this plan admirably fulfills its purpose of producing uniformity. But in order that it may do this, it is desirable that there should be another less absolute authority which shall declare, not infallibly but yet with a weight of collective learning, the propositions which science from time to time puts out of reasonable doubt, and which shall aid the researches of competent investigators. The value of such services in the development of science is immense; though they are accompanied by very serious disadvantages in not allowing to unofficial studies the weight which ought to be accorded to them. The history of science is full of examples of this sort.

§8. SCIENCE AND CONTINUITY

61. One of the worst effects of the influence of moral and religious reasonings upon science lies in this, that the distinctions upon which both insist as fundamental are dual distinctions, and that their tendency is toward an ignoring of all distinctions that are not dual and especially of the conception of continuity. Religion recognizes the saints and the damned. It will not readily admit any third fate. Morality insists that a motive is either good or bad. That the gulf between them is

bridged over and that most motives are somewhere near the middle of the bridge, is quite contrary to the teachings of any moral system which ever lived in the hearts and consciences of a people.

62. It is not necessary to read far in almost any work of philosophy written by a man whose training is that of a theologian, in order to see how helpless such minds are in attempting to deal with continuity. Now continuity, it is not too much to say, is the leading conception of science. The complexity of the conception of continuity is so great as to render it important wherever it occurs. Now it enters into every fundamental and exact law of physics or of psychics that is known. The few laws of chemistry which do not involve continuity seem for the most part to be very roughly true. It seems not unlikely that if the veritable laws were known continuity would be found to be involved in them. . . .*

§9. THE ANALYTIC METHOD

63. The first problems to suggest themselves to the inquirer into nature are far too complex and difficult for any early solution, even if any satisfactorily secure conclusion can ever be drawn concerning them. What ought to be done, therefore, and what in fact is done, is at first to substitute for those problems others much simpler, much more abstract, of which there is a good prospect of finding probable solutions. Then, the reasonably certain solutions of these last problems will throw a light more or less clear upon more concrete problems which are in certain respects more interesting.

64. This method of procedure is that Analytic Method to which modern physics owes all its triumphs. It has been applied with great success in psychical sciences also. (Thus, the classical political economists, especially Ricardo, pursued this method.)† It is reprobated by the whole Hegelian army, who think it ought to be replaced by the "Historic Method," which studies complex problems in all their complexity, but which cannot boast any distinguished successes.

* See vol. 6, bk. I.

† Cf. 4.115.

§10. KINDS OF REASONING*

65. There are in science three fundamentally different kinds of reasoning, Deduction (called by Aristotle συναγωγή or αναγωγή), Induction (Aristotle's and Plato's ἐπαγωγή) and Retroduction (Aristotle's ἀπαγωγή, but misunderstood because of corrupt text, and as misunderstood usually translated *abduction*).† Besides these three, Analogy (Aristotle's παραδειγμα) combines the characters of Induction and Retroduction.

66. *Deduction* is that mode of reasoning which examines the state of things asserted in the premisses, forms a diagram of that state of things, perceives in the parts of that diagram relations not explicitly mentioned in the premisses, satisfies itself by mental experiments upon the diagram that these relations would always subsist, or at least would do so in a certain proportion of cases, and concludes their necessary, or probable, truth. For example, let the premiss be that there are four marked points upon a line which has neither extremity nor furcation. Then, by means of a diagram,

we may conclude that there are two pairs of points such that in passing along the line in any way from one to the other point of either pair, one point of the second pair will be passed an odd number of times and the other point an even (or zero) number of times. This is *deduction*.

67. *Induction* is that mode of reasoning which adopts a conclusion as approximate, because it results from a method of inference which must generally lead to the truth in the long run. For example, a ship enters port laden with coffee. I go aboard and sample the coffee. Perhaps I do not examine over a hundred beans, but they have been taken from the middle, top, and bottom of bags in every part of the hold. I conclude by *induction* that the whole cargo has approximately the same value per bean as the hundred beans of my sample. All that induction can do is to ascertain the value of a ratio.

* Cf. vol. 2, bk. III.

† Peirce usually calls it abduction; sometimes hypothesis.

68. *Retroduction* is the provisional adoption of a hypothesis, because every possible consequence of it is capable of experimental verification, so that the persevering application of the same method may be expected to reveal its disagreement with facts, if it does so disagree. For example, all the operations of chemistry fail to decompose hydrogen, lithium, glucinum, boron, carbon, nitrogen, oxygen, fluorine, sodium, . . . gold, mercury, thallium, lead, bismuth, thorium, and uranium. We provisionally suppose these bodies to be simple; for if not, similar experimentation will detect their compound nature, if it can be detected at all. That I term *retroduction*.

69. *Analogy* is the inference that a not very large collection of objects which agree in various respects may very likely agree in another respect. For instance, the earth and Mars agree in so many respects that it seems not unlikely they may agree in being inhabited.

70. The methods of reasoning of science have been studied in various ways and with results which disagree in important particulars. The followers of Laplace treat the subject from the point of view of the theory of probabilities. After corrections due to Boole* and others,† that method yields substantially the results stated above. Whewell‡ described the reasoning just as it appeared to a man deeply conversant with several branches of science as only a genuine researcher can know them, and adding to that knowledge a full acquaintance with the history of science. These results, as might be expected, are of the highest value, although there are important distinctions and reasons which he overlooked. John Stuart Mill endeavored to explain the reasonings of science by the nominalistic metaphysics of his father. The superficial perspicuity of that kind of metaphysics rendered his logic extremely popular with those who think, but do not think profoundly; who know something of science, but more from the outside than the inside, and who for one reason or another delight in the simplest theories even if they fail to cover the facts.

71. Mill denies that there was any reasoning in Kepler's

* *Laws of Thought*, chs. 16–21.

† Including C. S. Peirce. See Paper No. 1, vol. 3.

‡ *The Philosophy of the Inductive Sciences*, 1840.

procedure. He says it is merely a description of the facts.* He seems to imagine that Kepler had all the places of Mars in space given him by Tycho's observations; and that all he did was to generalize and so obtain a general expression for them. Even had that been all, it would certainly have been inference. Had Mill had even so much practical acquaintance with astronomy as to have practised discussions of the motions of double stars, he would have seen that. But so to characterize Kepler's work is to betray total ignorance of it. Mill certainly never read the *De Motu* [*Motibus*] *Stellae Martis*, which is not easy reading. The reason it is not easy is that it calls for the most vigorous exercise of all the powers of reasoning from beginning to end.

72. What Kepler had given was a large collection of observations of the apparent places of Mars at different times. He also knew that, in a general way, the Ptolemaic theory agrees with the appearances, although there were various difficulties in making it fit exactly. He was furthermore convinced that the hypothesis of Copernicus ought to be accepted. Now this hypothesis, as Copernicus himself understood its first outline, merely modifies the theory of Ptolemy so far as [to] impart to all the bodies of the solar system one common motion, just what is required to annul the mean motion of the sun. It would seem, therefore, at first sight, that it ought not to affect the appearances at all. If Mill had called the work of Copernicus mere description he would not have been *so very far* from the truth as he was. But Kepler did not understand the matter quite as Copernicus did. Because the sun was so near the centre of the system, and was of vast size (even Kepler knew its diameter must be at least fifteen times that of the earth), Kepler, looking at the matter dynamically, thought it must have something to do with causing the planets to move in their orbits. This retroduction, vague as it was, cost great intellectual labor, and was most important in its bearings upon all Kepler's work. Now Kepler remarked that the lines of apsides of the orbits of Mars and of the earth are not parallel; and he utilized various observations most ingeniously to infer that they probably intersected in the sun. Consequently, it must be supposed that a general description of the motion would be

* *Ibid.*, bk. III, ch. 2, §3.

simpler when referred to the sun as a fixed point of reference than when referred to any other point. Thence it followed that the proper times at which to take the observations of Mars for determining its orbit were when it appeared just opposite the sun — the true sun — instead of when it was opposite the *mean* sun, as had been the practice. Carrying out this idea, he obtained a theory of Mars which satisfied the longitudes at all the oppositions observed by Tycho and himself, thirteen in number, to perfection. But unfortunately, it did not satisfy the latitudes at all and was totally irreconcilable with observations of Mars when far from opposition.

73. At each stage of his long investigation, Kepler has a theory which is approximately true, since it approximately satisfies the observations (that is, within $8'$, which is less than any but Tycho's observations could decisively pronounce an error), and he proceeds to modify this theory, after the most careful and judicious reflection, in such a way as to render it more rational or closer to the observed fact. Thus, having found that the centre of the orbit bisects the eccentricity, he finds in this an indication of the falsity of the theory of the equant and substitutes, for this artificial device, the principle of the equable description of areas. Subsequently, finding that the planet moves faster at ninety degrees from its apsides than it ought to do, the question is whether this is owing to an error in the law of areas or to a compression of the orbit. He ingeniously proves that the latter is the case.

74. Thus, never modifying his theory capriciously, but always with a sound and rational motive for just the modification he makes, it follows that when he finally reaches a modification — of most striking simplicity and rationality — which exactly satisfies the observations, it stands upon a totally different logical footing from what it would if it had been struck out at random, or the reader knows not how, and had been found to satisfy the observation. Kepler shows his keen logical sense in detailing the whole process by which he finally arrived at the true orbit. This is the greatest piece of Retroductive reasoning ever performed.

§11. THE STUDY OF THE USELESS

75. . . . The old-fashioned political economist adored, as alone capable of redeeming the human race, the glorious principle of individual greed, although, as this principle requires for its action hypocrisy and fraud, he generally threw in some dash of inconsistent concessions to virtue, as a sop to the vulgar Cerberus. But it is easy to see that the only kind of science this principle would favor would be such as is immediately remunerative with a great preference for such as can be kept secret, like the modern sciences of dyeing and perfumery. Kepler's discovery rendered Newton possible, and Newton rendered modern physics possible, with the steam engine, electricity, and all the other sources of the stupendous fortunes of our age. But Kepler's discovery would not have been possible without the doctrine of conics. Now contemporaries of Kepler — such penetrating minds as Descartes and Pascal — were abandoning the study of geometry (in which they included what we now call the differential calculus, so far as that had at that time any existence) because they said it was so UTTERLY USELESS. There was the future of the human race almost trembling in the balance; for had not the geometry of conic sections already been worked out in large measure, and had their opinion that only sciences apparently useful ought to be pursued, [prevailed] the nineteenth century would have had none of those characters which distinguish it from the *ancien régime*.

76. True science is distinctively the study of useless things. For the useful things will get studied without the aid of scientific men. To employ these rare minds on such work is like running a steam engine by burning diamonds.

77. The University of Paris encouraged useless studies in the most effective way possible, by training so many men as to be almost sure of getting a large proportion of all the minds that could be very serviceable in such studies. At the same time, it provided a sure living not only for such as were really successful, but even for those whose talents were of a somewhat inferior kind. On the other hand, like all universities, it set up an official standard of truth, and frowned on all who questioned it. Just so, the German universities for a whole generation turned the cold shoulder to every man who did not extol their stale Hegelianism, until it became a stench in the nostrils of

every man of common sense. Then the official fashion shifted, and a Hegelian is today treated in Germany with the same arrogant stupidity with which an anti-Hegelian formerly was. Of course, so-called "universities," whose purpose is not the solution of great problems, but merely the fitting of a selection of young men to earn more money than their fellow citizens not so favored, have for the interests of science none of the value of the medieval and German universities, although they exercise the same baleful influence to about the same degree.

78. The small academies of continental Europe are reasonably free from the gravest fault of the universities. Their defect is that while they indirectly do much for their few members they extend little aid to the younger men, except that of giving a general tone of respectability to pure science.

79. The larger bodies give much less aid to individuals; but they begin to aid them sooner. They have a distinct though limited use when they are specialized, like the Union of German chemists. But whether the Royal Society has been as serviceable to science as the French *Académie des Sciences* may be doubted.

§12. *IL LUME NATURALE*

80. In examining the reasonings of those physicists who gave to modern science the initial propulsion which has insured its healthful life ever since, we are struck with the great, though not absolutely decisive, weight they allowed to instinctive judgments. Galileo appeals to *il lume naturale* at the most critical stages of his reasoning. Kepler, Gilbert, and Harvey — not to speak of Copernicus — substantially rely upon an inward power, not sufficient to reach the truth by itself, but yet supplying an essential factor to the influences carrying their minds to the truth.

81. It is certain that the only hope of retroductive reasoning ever reaching the truth is that there may be some natural tendency toward an agreement between the ideas which suggest themselves to the human mind and those which are concerned in the laws of nature.

§13. GENERALIZATION AND ABSTRACTION

82. The most important operation of the mind is that of generalization. There are some exceedingly difficult questions of theoretical logic connected with generalization. On the other hand, there are some valuable lessons which evade those puzzles. If we look at any earlier work upon mathematics as compared with a later one upon the same subject, that which most astonishes us is to see the difficulty men had in first seizing upon general conceptions which after we become a little familiarized to them are quite matters of course. That an Egyptian should have been able to think of adding one-fifth and one-fifth, and yet should not have been content to call the sum two-fifths, but must call it one-third plus one-fifteenth, as if he could not conceive of a sum of fractions unless their denominators were different, seems perverse stupidity. That decimals should have been so slow in coming in, and that, when they did come, the so-called decimal point should be written as if the relation of units to tenths were somehow peculiar, while what was logically called for was simply some mark attached to the units place, so that instead of 3.14159 [what] should have been written [was] $\hat{3}14159$, seems very surprising. That Descartes should have thought it necessary to work problems in analytical geometry four times over, according to the different quadrants between the axes of coördinates in which the point to be determined might occur, is astonishing. That which the early mathematicians failed to see in all these cases was that some feature which they were accustomed to insert into their theorems was quite irrelevant and could perfectly well be omitted without affecting in the slightest degree the cogency of any step of the demonstrations.

83. Another operation closely allied to generalization is abstraction; and the use of it is perhaps even more characteristic of mathematical reasoning than is generalization. This consists of seizing upon something which has been conceived as a ἔπος πτερόεν, a meaning not dwelt upon but through which something else is discerned, and converting it into an ἔπος ἀπτερόεν, a meaning upon which we rest as the principal subject of discourse. Thus, the mathematician conceives an operation as something itself to be operated upon. He conceives the collection of places of a moving particle as itself a

place which can at one instant be totally occupied by a filament, which can again move, and the aggregate of all its places, considered as possibly occupied in one instant, is a surface, and so forth.

84. The intimate connection between generalization and continuity is to be pointed out.*

§14. THE EVALUATION OF EXACTITUDE

85. For every line of scientific research there is in any given stage of its development, an appropriate standard of certitude and exactitude, such that it is useless to require more, and unsatisfactory to have less. This is a part of the doctrine of the Economy of Research. When Phœnix† made his celebrated survey of the route from San Francisco to the Mission of Dolores, the distance required was the sum of two parts, one of them resting on the guess of a driver, while the other was determined at great expense to a transcendental precision. As long as one part of the distance was extremely uncertain, there was no use in spending much money in ascertaining the other part precisely. For there is a relation between the value of an increased certainty of an item of knowledge and the cost of such increase of certainty, which enables us to determine whether it is better to expend our genius, energy, time, and money upon one investigation or upon another.

86. If a result is to be used merely to confirm the result of an independent investigation, it may have a high value even though its probability is not very high. But if it is only to be used in combination with other results, very little will be gained by increasing its probability far beyond the probabilities of those others. Of course, knowledge that is to be put to special purposes may need to be more precise than other knowledge. Thus, it pays to determine the places of a thousand stars with the utmost accuracy, leaving hundreds of thousands only roughly located, and others only recorded upon photographs. But where a high degree of exactitude and probability is unattainable, that is no reason for refusing to accept such knowledge as we can attain. Because we cannot reach great certainty about the life and teachings of Pythagoras is no reason

* See vol. 6, bk. I, ch. 7.

† In his *Phoenixiana*, "Official Report."

for sulkily dismissing the subject as one we know nothing about, as Dr. Ed. Zeller* would have us do.

§15. SCIENCE AND EXTRAORDINARY PHENOMENA

87. Science is from the nature of its procedure confined to the investigation of the ordinary course of nature. I do not mean that it cannot investigate individual objects, such as the earth. But all its explanations of such objects must be limited to the supposition that they have come about in the ordinary course of nature. A statistical result may be obtained.

88. We may find that such and such a proportion of calves have five legs. But we never can conclude with any probability that the ratio is strictly zero; and even if we knew that the proportion of men with golden thighs is exactly zero, that would be no argument at all against Pythagoras having had a golden thigh. For something might be true of one man, or any number of men, and yet might occur in the long run in a finite number of cases out of an infinite series. Now a finite number divided by infinity is exactly zero. That Pythagoras had a golden thigh is the testimony of history. It is asserted by Aristotle, of all possible authorities the highest, by both Porphyry and Jamblichus after Nicomachus, by Herodotus, by Plutarch, Diogenes Laertius, Aelian, Apollonius,† etc. This is far stronger testimony than we have for the resurrection of Jesus. Are we then to admit as a part of the science of history that Pythagoras had a golden thigh?

89. To do so would be to make a retroductive inference. Now a retroductive conclusion is only justified by its *explaining* an observed fact. An explanation is a syllogism of which the major premiss, or rule, is a known law or rule of nature, or other general truth; the minor premiss, or case, is the hypothesis or retroductive conclusion, and the conclusion, or result, is the observed (or otherwise established) fact. Such an explanation, in this case, would be like this:

* *Der Philosophie der Griechen*, S. 279.

† Peirce seems to have secured his authorities from Zeller's *A History of Greek Philosophy*, 1881, vol. 1, p. 328, n. 4. Zeller's references are not all accurate, and the authorities quoted are not independent. Peirce's annotated copy of this book is now, through the gift of his wife, the property of the Harvard College Library.

Every fact about Pythagoras (unless kept secret or insignificant) would be reported by his ancient biographers.

That Pythagoras had a golden thigh was a fact about Pythagoras neither secret nor insignificant.

∴ That Pythagoras had a golden thigh would be reported by all his ancient biographers.

90. But this syllogism may be condemned at once on the ground that it supposes we have statistical knowledge about such kinds of facts as are quite contrary to the usual course of nature. If the reply be made that it could make in regard to the reporting of the fact no difference whether it were a natural one or not, I rejoin, that granting that, it is not to the purpose. It only goes to show that there is no difference between natural and supernatural facts in this respect; from which the only just inference is that no such proposition can be known even in respect to natural facts. This, indeed, is the case. We cannot say that every remarkable public fact about Pythagoras would be reported, but only that every phenomenon would be told as it appeared to people in an almost primitive state of civilization. Nobody can think that the golden thigh was treated as a modern assayer would treat a gold brick. It was probably flexible and therefore its golden appearance was superficial. One of these days, we may find out something about the ancient Persians, Chorasmians, or Brahmins which may make this story significant. At present, it only illustrates the impossibility of science making any assertion about a fact out of the course of nature. Pythagoras was certainly a wonderful man. We have no right, at all, to say that supernal powers had not put a physical mark upon him as extraordinary as his personality. Science can no more deny a miracle than it can assert one.

91. But although science cannot infer any particular violation of the ordinary course of nature, it may very well be that it should find evidence that such violations are so frequent and usual that this fact is itself a part of the ordinary course of nature. For that reason, it is perfectly proper that science should inquire, for example, into the evidences of the fulfillment of prayers, etc. That is something open to experimental inquiry; and until such inquiry has been instituted nobody is entitled to any opinion whatever, or any bias, as to its result.

§16. REASONING FROM SAMPLES

92. Many persons seem to suppose that the state of things asserted in the premisses of an induction renders the state of things asserted in the conclusion probable. The fact that Macaulay's essay on Bacon was admired in its day shows how little the absurdity of such a position was perceived. Even John Stuart Mill holds that the uniformity of nature makes the one state of things follow from the other. He overlooks the circumstance that if so it ought to follow necessarily, while in truth no definite probability can be assigned to it without absurd consequences. He also overlooks the fact that inductive reasoning does not invariably infer a uniformity; it may infer a diversity. I watch the throws of a die, I notice that about half are odd and half are even, and that they follow one another with the utmost irregularity. I conclude that about half of all the throws of that die are odd and that the odd and even follow one another with great irregularity. How can any principle of uniformity account for the truth of such an induction? Mill never made up his mind in what sense he took the phrase "uniformity of nature" when he spoke of it as the basis of induction. In some passages he clearly means any special uniformity by which a given character is likely to belong to the whole of a species, a genus, a family, or a class if it belongs to any members of that group. In this sense, as well as in others, overlooked by Mill, there is no doubt the knowledge of a uniformity strengthens an inductive conclusion; but it is equally free from doubt that such knowledge is not essential to induction. But in other passages Mill holds that it is not the knowledge of the uniformity, but the uniformity itself that supports induction, and furthermore that it is no special uniformity but a general uniformity in nature. Mill's mind was certainly acute and vigorous, but it was not mathematically accurate; and it is by that trait that I am forced to explain his not seeing that this general uniformity could not be so defined as not on the one hand to appear manifestly false or on the other hand to render no support to induction, or both. He says it means that under similar circumstances similar events will occur. But this is vague. Does he mean that objects alike in all respects but one are alike in that one? But plainly no two different real objects are alike in all respects

but one. Does he mean that objects *sufficiently* alike in other respects are alike in any given respect? But that would be but another way of saying that no two different objects are alike in all respects but one. It is obviously true; but it has no bearing on induction, where we deal with objects which we well know are, like all existing things, alike in numberless respects and unlike in numberless other respects.*

93. The truth is that induction is reasoning from a sample taken at random to the whole lot sampled. A sample is a *random* one, provided it is drawn by such machinery, artificial or physiological, that in the long run any one individual of the whole lot would get taken as often as any other. Therefore, judging of the statistical composition of a whole lot from a sample is judging by a method which will be right on the average in the long run, and, by the reasoning of the doctrine of chances, will be nearly right oftener than it will be far from right.

94. That this does justify induction is a mathematical proposition beyond dispute. It has been objected that the sampling cannot be random in this sense. But this is an idea which flies far away from the plain facts. Thirty throws of a die constitute an approximately random sample of all the throws of that die; and that the randomness should be approximate is all that is required.

95. This account of the rationale of induction is distinguished from others in that it has as its consequences two rules of inductive inference which are very frequently violated, although they have sometimes been insisted upon. The first of these is that the sample must be a random one. Upon that I shall not dwell here. The other rule is that the character, toward the ascertainment of the proportionate frequency of which in the lot sampled [the sampling is done], must not be determined by the character of the particular sample taken. For example, we must not take a sample of eminent men, and studying over them, find that they have certain characters and conclude that all eminent men will have those characters. We must first decide for what character we propose to examine the sample, and only after that decision examine the sample. The reason is that any sample will be peculiar and unlike the aver-

* Mill's views on induction are examined in more detail in vol. 2, bk. III, ch. 9.

age of the lot sampled in innumerable respects. At the same time it will be approximately like the average of the whole lot in the great majority of respects.

96. In order to illustrate the necessity of this rule I take a random sample of eminent persons. It is quite a random one, for it consists of the first names on pages 100, 300, 500, 700, 900, of Phillips's *Great Index of Biography* [*Biographical Reference*, second edition, 1881]. The names are as follows:

	Born	*Died*
Francis Baring	1740	1810 Sept. 12
Vicomte de Custine	1760	1794 Jan. 3
Hippostrates (of uncertain age)		
Marquis d' O.	1535	1594 Oct. 24
Theocrenes	1480	1536 Oct. 18

Now I might, in violation of the above rule of predesignation, draw the following inductions:

1. Three-fourths of these men were born in a year whose date ends in a cipher. Hence about three-fourths of all eminent men are probably so born. But, in fact, only one in ten is so born.

2. Three eminent men out of four die in autumn. In fact, only one out of four.

3. All eminent men die on a day of the month divisible by three. In fact, one out of three.

4. All eminent men die in years whose date doubled and increased by one gives a number whose last figure is the same as that in the tens' place of the date itself. In fact, only one in ten.

5. All eminent men who were living in any year ending in forty-four died at an age which after subtracting four becomes divisible by eleven. All others die at an age which increased by ten is divisible by eleven.

97. This rule is recognized in the requirement of physicists that a theory shall furnish predictions which shall be verified before any particular weight is accorded to it. The medical men, too, who deserve special mention for the reason that they have had since Galen a logical tradition of their own, recognize this rule, however dimly, in their working against reasoning "*post hoc, ergo propter hoc.*". . .

§17 THE METHOD OF RESIDUAL PHENOMENA

98. The so-called "method of residual phenomena" is so simple that it hardly calls for any remark. At any early stage of science when there are few observations of a given matter, and those rough ones, a law is made out which, when the observations come to be increased in number and made more accurate, is found not to hold exactly. The departures from this law are found themselves to follow a law which may now be shown to be true. But at a still later date it is found that this law again is interfered with, that there are still more minute departures from it, and these departures are again found to follow a law. All the successive laws so found may be real, or they may be merely empirical formulae. . . .

§18. OBSERVATION

99. I have already remarked that a definition of science in general which shall express a really intelligent conception of it as a living historic entity must regard it as the occupation of that peculiar class of men, the scientific men. The same remark may be extended to definitions of the different branches of science. The men who pursue a given branch herd together. They understand one another; they live in the same world, while those who pursue another branch are for them foreigners.

100. It will be found upon close examination that that which renders the modes of thought of the students of a special branch of science peculiar is that their experience lies in a peculiar region. And the cause of this is that they are trained and equipped to make a peculiar kind of observations. The man who is continually making chemical analyses lives in a different region of nature from other men. The same thing is even more true of men who are constantly using a microscope.

101. It comes to this, that sciences must be classified according to the peculiar means of observation they employ.

102. So too the great landmarks in the history of science are to be placed at the points where new instruments, or other means of observation, are introduced. Astronomy before the telescope and astronomy after the telescope. Prephotographic astronomy and photographic astronomy. Chemistry before the exact analytic balance, and after.

§19. EVOLUTION

103. The evolutionary theory in general throws great light upon history and especially upon the history of science — both its public history and the account of its development in an individual intellect. As great a light is thrown upon the theory of evolution in general by the evolution of history, especially that of science — whether public or private.

104. The main theories of the evolution of organic species are three. First, the theory of Darwin, according to which the entire interval from Moner to Man has been traversed by successive purely fortuitous and insensible variations *in reproduction*. The changes on the whole follow a determinate course simply because a certain amount of change in certain directions destroys the species altogether, as the final result of successive weakenings of its reproductive power. Second, the theory of Lamarck, according to which the whole interval has been traversed by a succession of very minute changes. But these have not taken place in reproduction, which has absolutely nothing to do with the business, except to keep the average individuals plastic by their youth. The changes have not been fortuitous but wholly the result of strivings of the individuals. Third, the theory of cataclysmal evolution, according to which the changes have not been small and have not been fortuitous; but they have taken place chiefly in reproduction. According to this view, sudden changes of the environment have taken place from time to time. These changes have put certain organs at a disadvantage, and there has been an effort to use them in new ways. Such organs are particularly apt to sport in reproduction and to change in the way which adapts them better to their recent mode of exercise.

105. Notwithstanding the teachings of Weismann, it seems altogether probable that all three of these modes of evolution have acted. It is probable that the last has been the most efficient. These three modes of organic evolution have their parallels in other departments of evolution.

106. Let us consider, for example, the evolution of standards of weights and measures. In order to define the word "pound" in the *Century Dictionary*,* I made a list of about

* See 209. Peirce wrote the definitions of terms in mechanics, mathematics astronomy, astrology, weights and measures, logic, metaphysics, all those relating to universities, and many on psychology for the *Century Dictionary*, edition of 1889.

four hundred pounds which had been in use in different parts of Europe — undoubtedly a very incomplete list, for it was confined in great measure to certain provinces concerning which I was able to obtain information. Each individual pound or measuring stick is from time to time copied; and at length the old one becomes destroyed. The measure of each copy is imperceptibly larger or smaller than its immediate prototype. If then these variations cannot, by gradual summation, produce a standard much smaller without that standard being destroyed as inconvenient while no such destruction would follow upon an increase of the standard, the average of the standards will slowly grow larger by Darwinian evolution. If there were a disposition on the part of owners of pounds to file them down, so as to make them lighter, though not enough to be noticed, then these filed pounds being copied, and the copies filed, there would be a gradual lightening of the pound by Lamarckian evolution. But it is very unlikely that either of these two modes has been a considerable factor in the actual evolution of weights and measures. As long as their circumstances are unchanged, human communities are exceedingly conservative. Nothing short of the despotism of a modern government with a modern police can cause a change in weights and measures. But from time to time changes occur which cause trade to take new routes. Business has to be adapted to new conditions; and under such influences we find all those habits of communities which are rendered unsuitable by the change become plastic enough. Then it is that a new pound or a new yard may be made which is a compromise between a desire to retain old ways and a desire to please new-comers.

107. In the evolution of science, a Darwinian mode of evolution might, for example, consist in this, that at every recall of a judgment to the mind — say, for example, a judgment in regard to some such delicate question as the marriage of the clergy — a slight fortuitous modification of the judgment might take place; the modified judgment would cause a corresponding modification of the belief-habit, so that the next recall would be influenced by this fortuitous modification, though it would depart more or less from it by a new fortuitous modification. If, however, by such summation of modifications an

opinion quite untenable were reached, it would either be violently changed or would be associationally weak and not apt to be recalled. The effect of this would be in the long run that belief would move away from such untenable positions. It is possible that such a mode of influence may affect our instinctive feelings; but there can be nothing of this sort in science, which is controlled and exact. But another sort of Darwinian evolution undoubtedly does take place. We are studying over phenomena of which we have been unable to acquire any satisfactory account. Various tentative explanations recur to our minds from time to time, and at each occurrence are modified by omission, insertion, or change in the point of view, in an almost fortuitous way. Finally, one of these takes such an aspect that we are led to dismiss it as impossible. Then, all the energy of thought which had previously gone to the consideration of that becomes distributed among the other explanations, until finally one of them becomes greatly strengthened in our minds.

108. Lamarckian evolution might, for example, take the form of perpetually modifying our opinion in the effort to make that opinion represent the known facts as more and more observations came to be collected. This is all the time going on in regard, for example, to our estimate of the danger of infection of phthisis. Yet, after all, it does not play a prominent part in the evolution of science. The physical journals — say, for example, Poggendorff's [*Annalen der Physik*] and *Beiblätter* — publish each month a great number of new researches. Each of these is a distinct contribution to science. It represents some good, solid, well-trained labor of observation and inference. But as modifying what is already known, the average effect of the ordinary research may be said to be insignificant. Nevertheless, as these modifications are not fortuitous but are for the most part movements toward the truth — could they be rightly understood, all of them would be so — there is no doubt that from decade to decade, even without any splendid discoveries or great studies, science would advance very perceptibly. We see that it is so in branches of physics which remain for a long time without any decisive conquests. It was so, for example, in regard to the classification of the chemical elements in the lapse of time from Berzelius to

Mendeléeff, as the valuable history of Venable* shows. This is an evolution of the Lamarckian type.

109. But this is not the way in which science mainly progresses. It advances by leaps; and the impulse for each leap is either some new observational resource, or some novel way of reasoning about the observations. Such novel way of reasoning might, perhaps, be considered as a new observational means, since it draws attention to relations between facts which would previously have been passed by unperceived.

[I] illustrate by the discoveries of Pasteur,† who began by applying the microscope to chemistry. He picked out the right- and left-handed crystals of tartaric acid. The two kinds have absolutely the same properties except in regard to direction of rotation of the plane of polarization and in their chemical relations to other "optically active" bodies. Since this method of picking out individual crystals was so slow, Pasteur looked for other means. Ferments of appropriate kinds were found to have the same effect. The microscope showed these were due to living organisms, which Pasteur began studying. At that time the medical world was dominated by Claude Bernard's dictum that a disease is not an entity but merely a sum of symptoms.‡ This was pure metaphysics which only barricaded inquiry in that direction. But that was a generation which attached great value to nominalistic metaphysics. Pasteur began with the phylloxera. He found it influenced the "optical activity" of the sugar. This pointed to a ferment and therefore to an entity. He began to extend the doctrine to other diseases. The medical men, dominated by the metaphysics of Claude Bernard, raised all sorts of sophistical objections. But the method of cultures and inoculation proved the thing, and here we see new ideas connected with new observational methods and a fine example of the usual process of scientific evolution. It is not by insensible steps.

§20. SOME A PRIORI DICTA

110. The last fifty years have taught the lesson of not trifling with facts and not trusting to principles and methods which

* *The Development of the Periodic Law*, Easton, Pa., 1896.

† See *Oeuvres de Pasteur*, vol. 1, p. 83, Paris, 1922.

‡ *Leçons de Pathologie expérimental*, 2^{me} leçon, Paris, 1872.

are not logically founded upon facts and which serve only to exclude testimony from consideration.

111. Such, for example, was the dictum of Claude Bernard that a disease is not an entity — a purely metaphysical doctrine. But the observation of facts has taught us that a disease is in many, if not most, serious cases, just as much an entity as a human family consisting of father, mother, and children.

112. Such was the dictum of the old psychology which identified the soul with the ego, declared its absolute simplicity, and held that its faculties were mere names for logical divisions of human activity. This was all unadulterated fancy. The observation of facts has now taught us that the ego is a mere wave in the soul, a superficial and small feature, that the soul may contain several personalities and is as complex as the brain itself, and that the faculties, while not exactly definable and not absolutely fixed, are as real as are the different convolutions of the cortex.

113. Such were the dicta by means of which the internal criticism of historical documents was carried to such a height that it often amounted to the rejection of all the testimony that has come down to us, and the substitution for it of a dream spun out of the critic's brain. But archeological researches have shown that ancient testimony ought to be trusted in the main, with a small allowance for the changes in the meanings of words. When we are told that Pythagoras had a golden thigh, we are to remember that to the ancients gold did not mean a chemical element of atomic weight 197.5 and specific gravity 19.3, melting at 1045° C. and forming saline compounds of the types AuX and AuX_3. It meant something of metallic lustre, warmer in color than electrum and cooler than copper. Dr. Schliemann's discoveries were the first socdolager that "higher criticism" received. It has since got many others.

114. Such was the dictum of Laplace that stones do not come from heaven.

115. Such were the dicta by which everything of the nature of extraordinary powers connected with psychological states of which the hypnotic trance is an example were set down as tricks. At present, while the existence of telepathy cannot be said to be established, all scientific men are obliged by observed facts to admit that it presents at least a very serious problem requiring respectful treatment.

§21. THE PAUCITY OF SCIENTIFIC KNOWLEDGE

116. Persons who know science chiefly by its results — that is to say, have no acquaintance with it at all as a living inquiry — are apt to acquire the notion that the universe is now entirely explained in all its leading features; and that it is only here and there that the fabric of scientific knowledge betrays any rents.

117. But in point of fact, notwithstanding all that has been discovered since Newton's time, his saying that we are little children picking up pretty pebbles on the beach while the whole ocean lies before us unexplored remains substantially as true as ever, and will do so though we shovel up the pebbles by steam shovels and carry them off in carloads. An infinitesimal ratio may be multiplied indefinitely and remain infinitesimal still.

118. In the first place all that science has done is to study those relations between objects which were brought into prominence and conceiving which we had been endowed with some original knowledge in two instincts — the instinct of *feeding*, which brought with it elementary knowledge of mechanical forces, space, etc., and the instinct of *breeding*, which brought with it elementary knowledge of psychical motives, of time, etc. All the other relations of things concerning which we must suppose there is vast store of truth are for us merely the object of such false sciences as judicial astrology, palmistry, the doctrine of signatures, the doctrine of correspondences, magic, and the like.

119. In the next place, even within the very bounds to which our science is confined, it is altogether superficial and fragmentary. Want of knowledge of the constitution of matter and of electricity. The conservation of forces, as Helmholtz first enunciated it, untenable; whether it can be universally true in any sense is a difficult problem. To strengthen it Helmholtz greatly insisted on discontinuities — a most objectionable theory from every point of view. Mind quite as little understood as matter, and the relations between the two an enigma. The forces we know can be but a small part of all those that are operative. Our ignorance of small things and great, of distant times and of very slow operations. We are equally ignorant of very rapid performances which neverthe-

less we know to take place. Our science is altogether middle-sized and mediocre. Its insignificance compared with the universe cannot be exaggerated.

§22. THE UNCERTAINTY OF SCIENTIFIC RESULTS

120. It is a great mistake to suppose that the mind of the active scientist is filled with propositions which, if not proved beyond all reasonable cavil, are at least extremely probable. On the contrary, he entertains hypotheses which are almost wildly incredible, and treats them with respect for the time being. Why does he do this? Simply because any scientific proposition whatever is always liable to be refuted and dropped at short notice. A hypothesis is something which looks as if it might be true and were true, and which is capable of verification or refutation by comparison with facts. The best hypothesis, in the sense of the one most recommending itself to the inquirer, is the one which can be the most readily refuted if it is false. This far outweighs the trifling merit of being likely. For after all, what is a *likely* hypothesis? It is one which falls in with our preconceived ideas. But these may be wrong. Their errors are just what the scientific man is out gunning for more particularly. But if a hypothesis can quickly and easily be cleared away so as to go toward leaving the field free for the main struggle, this is an immense advantage.

121. Retroduction goes upon the hope that there is sufficient affinity between the reasoner's mind and nature's to render guessing not altogether hopeless, provided each guess is checked by comparison with observation. It is true that agreement does not show the guess is right; but if it is wrong it must ultimately get found out. The effort should therefore be to make each hypothesis, which is practically no more than a question, as near an even bet as possible.

§23. THE ECONOMY OF RESEARCH

122. Dr. Ernst Mach, who has one of the best faults a philosopher can have, that of riding his horse to death, does just this with his principle of Economy in science.* But of

* See, e.g., the lecture on the "Economical Nature of Physical Inquiry" in the *Popular Scientific Lectures* (1895).

course there is a doctrine of the Economies of Research. One or two of its principles are easily made out. The value of knowledge is, for the purposes of science, in one sense absolute. It is not to be measured, it may be said, in money; in one sense that is true. But knowledge that leads to other knowledge is more valuable in proportion to the trouble it saves in the way of expenditure to get that other knowledge. Having a certain fund of energy, time, money, etc., all of which are merchantable articles to spend upon research, the question is how much is to be allowed to each investigation; and *for us* the value of that investigation is the amount of money it will pay us to spend upon it. *Relatively*, therefore, knowledge, even of a purely scientific kind, has a money value.

This value increases with the fullness and precision of the information, but plainly it increases slower and slower as the knowledge becomes fuller and more precise. The cost of the information also increases with its fullness and accuracy, and increases faster and faster the more accurate and full it is. It therefore *may* be the case that it does not pay to get *any* information on a given subject; but, at any rate, it *must* be true that it does not pay (in any given state of science) to push the investigation beyond a certain point in fullness or precision.

123. If we have a number of studies in which we are interested, we should commence with the most remunerative and carry that forward until it becomes no more than equally remunerative with the commencement of another; carry both forward at such rates that they are equally remunerative until each is no more remunerative than a third, and so on.

124. If two or more kinds of knowledge are so related that one can replace the other so that the possession of one renders the other less profitable, this will diminish the investigation of either while increasing the investigation of all.

125. If two or more kinds of information are of use only as supplementing one another, that is, only when combined together, this will increase the investigations until there is little or no profit from the least profitable kind of research.

CHAPTER 3

NOTES ON SCIENTIFIC PHILOSOPHY

§1. LABORATORY AND SEMINARY PHILOSOPHIES*

126.... The kind of philosophy which interests me and must, I think, interest everybody is that philosophy, which uses the most rational methods it can devise, for finding out the little that can as yet be found out about the universe of mind and matter from those observations which every person can make in every hour of his waking life. It will not include matters which are more conveniently studied by students of special sciences, such as psychology. Thus, everybody has remarked that there are four prominent qualities of the sense of taste, sweet, sour, salt, and bitter. But there may be other tastes, not so readily made out without special study; and in any case tastes are conveniently studied in connexion with flavors and odors, which make a difficult experimental inquiry. Besides, the four tastes are altogether special and throw no light on the problems which, on account of their extreme generality, will naturally be examined by a class of researchers of entirely different aptitudes from those which adapt men to the discovery of recondite facts.

127. If anybody asks what there is in the study of obvious phenomena to make it particularly interesting, I will give two answers. The first is the one which seems to me the strongest; the other is that which nobody can fail to feel the force of. The first answer is that the spirit in which, as it seems to me, philosophy ought to be studied is the spirit in which every branch of science ought to be studied; namely, the spirit of joy in learning ourselves and in making others acquainted with the glories of God. Each person will feel this joy most in the

* From "Introduction showing the point of view from which Philosophy appears to the author to be an interesting subject to a man of common-sense," in the Notebook, "Sketch of Some Proposed Chapters on the Sect of Philosophy Called Pragmatism." c. 1905.

particular branch of science to which his faculties are best adapted. It is not a sin to have no taste for philosophy as I define philosophy. As a matter of fact, however, almost everybody does feel an interest in philosophical problems, especially at that time of life at which he is spoiling for an intellectual tussle.

128. It is true that philosophy is in a lamentably crude condition at present; that very little is really established about it; while most philosophers set up a pretension of knowing all there is to know — a pretension calculated to disgust anybody who is at home in any real science. But all we have to do is to turn our backs upon all such truly vicious conduct, and we shall find ourselves enjoying the advantages of having an almost virgin soil to till, where a given amount of really scientific work will bring in an extraordinary harvest, and that a harvest of very fundamental truth of exceptional value from every point of view.

129. This consideration touches upon the second reason for studying laboratory-philosophy (as contradistinguished from seminary-philosophy). It is that the special sciences are obliged to take for granted a number of most important propositions, because their ways of working afford no means of bringing these propositions to the test. In short, they always rest upon metaphysics. At one time, for example, we find physicists, Kelvin, Maxwell and others, assuming that a body cannot act where it is not, meaning by "where it is not" where its lines of force do not centre. At another time, we find them assuming that the laws of mechanics (including the principles of metric geometry) hold good for the smallest corpuscles. Now it is one thing to infer from the laws of little things how great things, that consist of little things, will act; but it is quite a different thing to infer from the phenomena presented by great things how single things billions of times smaller will act. It is like inferring that because in any country one man in so many will commit suicide, therefore every individual, once in such a period of time, will make an attempt at suicide. The psychical sciences, especially psychology, are, if possible, even more necessitated to assume general principles that cannot be proved or disproved by their ordinary methods of work. The philosopher alone is equipped with the facilities for examining

such "axioms" and for determining the degree to which confidence may safely be reposed in them. Find a scientific man who proposes to get along without any metaphysics — not by any means every man who holds the ordinary reasonings of metaphysicians in scorn — and you have found one whose doctrines are thoroughly vitiated by the crude and uncriticized metaphysics with which they are packed. We must philosophize, said the great naturalist Aristotle*— if only to avoid philosophizing. Every man of us has a metaphysics, and has to have one; and it will influence his life greatly. Far better, then, that that metaphysics should be criticized and not be allowed to run loose. A man may say "I will content myself with common sense." I, for one, am with him there, in the main. I shall show why I do not think there can be any *direct* profit in going behind common sense — meaning by common sense those ideas and beliefs that man's situation absolutely forces upon him. We shall later see more definitely what is meant.† I agree, for example, that it is better to recognize that some things are red and some others blue, in the teeth of what optical philosophers say, that it is merely that some things are resonant to shorter ether waves and some to longer ones. But the difficulty is to determine what really is and what is not the authoritative decision of common sense and what is merely *obiter dictum*. In short, there is no escape from the need of a critical examination of "first principles."

§2. AXIOMS‡

130. The science which, next after logic, may be expected to throw the most light upon philosophy, is mathematics. It is historical fact, I believe, that it was the mathematicians Thales, Pythagoras, and Plato who created metaphysics, and that metaphysics has always been the ape of mathematics. Seeing how the propositions of geometry flowed demonstratively from a few postulates, men got the notion that the same must be true in philosophy. But of late mathematicians have fully agreed that the axioms of geometry (as they are wrongly called) are not by any means evidently true. Euclid, be it

* *Metaphysics*, bk. I, 982b–3a.

† See vol. 5, bk. II, ch. 7 and bk. III, chs. 2 and 3.

‡ Unpaginated fragment, c. 1893.

observed, never pretended they were evident; he does not reckon them among his κοιναὶ ἔννοιαι, or things everybody knows,[1] but among the αἰτήματα, postulates, or things the author must beg you to admit, because he is unable to prove them. At any rate, it is now agreed that there is no reason whatever to think the sum of the three angles of a triangle precisely equal to 180 degrees. It is generally admitted that the evidence is that the departure from 180 degrees (if there is any) will be greater the larger the triangle, and in the case of a triangle having for its base the diameter of the earth's orbit and for its apex the furthest star, the sum hardly can differ, according to observation, so much as 0.1″. It is probable the discrepancy is far less. Nevertheless, there is an infinite number of different possible values, of which precisely 180 degrees is only one; so that the probability is as 1 to ∞ or 0 to 1, that the value is just 180 degrees. In other words, it seems for the present impossible to suppose the postulates of geometry precisely true. The matter is reduced to one of evidence; and as absolute precision [is] beyond the reach of direct observation, so it can never be rendered probable by evidence, which is indirect observation.

131. Thus, the postulates of geometry must go into the number of things approximately true. It may be thousands of years before men find out whether the sum of the three angles of a triangle is greater or less than 180 degrees; but the presumption is, it is one or the other.

132. Now what is metaphysics, which has always formed itself after the model of mathematics, to say to this state of things? The mathematical axioms being discredited, are the metaphysical ones to remain unquestioned? I trow not. There is one proposition, now held to be very certain, though denied throughout antiquity, namely that every event is precisely determined by general laws, which evidently never can be rendered probable by observation, and which, if admitted, must, therefore, stand as self-evident. This is a metaphysical postulate closely analogous to the postulates of geometry. Its fate is

[1] Except the proposition that two lines cannot enclose a space, though only one of the three best manuscripts places even this in the list. But what Euclid meant was that two straight lines can have but one intersection, which *is* evident.

sealed. The geometrical axioms being exploded, this is for the future untenable. Whenever we attempt to verify a physical law, we find discrepancies between observation and theory, which we rightly set down as errors of observation. But now it appears we have no reason to deny that there are similar, though no doubt far smaller, discrepancies between the law and the real facts. As Lucretius says,* the atoms swerve from the paths to which the laws of mechanics would confine them. I do not now inquire whether there is or not any positive evidence that this is so. What I am at present urging is that this arbitrariness is a conception occurring in logic, encouraged by mathematics, and ought to be regarded as a possible material to be used in the construction of a philosophical theory, should we find that it would suit the facts. We observe that phenomena approach very closely to satisfying general laws; but we have not the smallest reason for supposing that they satisfy them precisely.

§3. THE OBSERVATIONAL PART OF PHILOSOPHY†

133. Every science has a mathematical part, a branch of work that the mathematician is called in to do. We say, "Here, mathematician, suppose such and such to be the case. Never you mind whether it is really so or not; but tell us, supposing it to be so, what will be the consequence." Thus arise mathematical psychology, mathematical stylometry, mathematical economics, mathematical physics, mathematical chemistry, mathematical meteorology, mathematical biology, mathematical geology, mathematical astronomy, etc., etc., etc. But there is none of these mathematical offices which constitutes quite so large a proportion of the whole science to which it is annexed as mathematical philosophy, for the obvious reason that the observational part of philosophy is a simple business, compared, for example, with that of anatomy or biography, or any other special science.

134. To assume, however, that the observational part of philosophy, because it is not particularly laborious, is therefore

* *De Rerum Natura*, bk. II, l. 216ff.

† From "The Idea of a Law of Nature among the Contemporaries of David Hume and among Advanced Thinkers of the Present Day," c. 1894.

easy, is a dreadful mistake, into which the student is very apt to fall, and which gives the death-blow to any possibility of his success in this study. It is, on the contrary, extremely difficult to bring our attention to elements of experience which are continually present. For we have nothing in experience with which to contrast them; and without contrast, they cannot excite our attention. We can only contrast them with imaginary states of things; but even what we imagine is but a crazy-quilt of bits snipped off from actual experiences. The result is that roundabout devices have to be resorted to, in order to enable us to perceive what stares us in the face with a glare that, once noticed, becomes almost oppressive with its insistency. This circumstance alone would be sufficient to render philosophical observation difficult — much more difficult, for example, than the kind of observation which the painter has to exercise. Yet this is the least of the difficulties of philosophy. Of the various hindrances more serious still, I may mention once more the notion that it is an extremely easy thing to perceive what is before us every day and hour. But quite the worst is, that every man becomes more or less imbued with philosophical opinions, without being clearly aware of it. Some of these, it is true, may be right opinions; if he is a quite uneducated man, they doubtless will be so. But even if they are right, or nearly right, they prevent true observation as much as a pair of blue spectacles will prevent a man from observing the blue of the sky. The man will hold the right opinion, but not knowing that it might be founded upon direct observation, he will class it among articles of faith of a pretty dubious character. The more a man is educated in other branches, but not trained in philosophy, the more certain it is that two-thirds of his stock of half-conscious philosophical opinions will be utterly wrong, and will completely blind him to the truth, which he will gradually become unable so much as to conceive. I remember a really eminent French *savant*, who had sojourned for very many months in America, but who must have imbibed in his childhood the notion, then common in France, that Englishmen and Americans interject into every second sentence a certain word which the French imagine to be English. He belonged to one of the most observant of races; he was naturally a keen observer; and he was trained in an observational science; and

yet, in order to assimilate himself as much as possible to American ways, he used to think it necessary to greet one every morning with a "How do you do, goddam?" and to keep it up all day. He actually believed that he had observed that such was the American style. The educated man who is a beginner in philosophy is just like that man, who (be it remembered) had been moving about in America for years; — and by a beginner in philosophy I wish to be understood as meaning, in the case of an educated man, one who has not been seriously, earnestly, and single-mindedly devoted to the study of it for more than six or eight years. For there is no other science for which the preparatory training requires to be nearly so severe and so long, no matter how great the natural genius of the student may be. For a plain man or a boy who should be early taken in hand by an instructor capable of making him comprehend both sides of every question, the time, without doubt, can be greatly reduced, with untiring industry and energy on the pupil's part.

§4. THE FIRST RULE OF REASON*

135. Upon this first, and in one sense this sole, rule of reason, that in order to learn you must desire to learn, and in so desiring not be satisfied with what you already incline to think, there follows one corollary which itself deserves to be inscribed upon every wall of the city of philosophy:

Do not block the way of inquiry.

136. Although it is better to be methodical in our investigations, and to consider the economics of research, yet there is no positive sin against logic in *trying* any theory which may come into our heads, so long as it is adopted in such a sense as to permit the investigation to go on unimpeded and undiscouraged. On the other hand, to set up a philosophy which barricades the road of further advance toward the truth is the one unpardonable offence in reasoning, as it is also the one to which metaphysicians have in all ages shown themselves the most addicted.

Let me call your attention to four familiar shapes in which this venomous error assails our knowledge:

* From unpaginated ms. "F. R. L.," c. 1899.

137. The first is the shape of absolute assertion. That we can be sure of nothing in science is an ancient truth. The Academy taught it. Yet science has been infested with over-confident assertion, especially on the part of the third-rate and fourth-rate men, who have been more concerned with teaching than with learning, at all times. No doubt some of the geometries still teach as a self-evident truth the proposition that if two straight lines in one plane meet a third straight line so as to make the sum of the internal angles on one side less than two right angles those two lines will meet on that side if sufficiently prolonged. Euclid, whose logic was more careful, only reckoned this proposition as a *Postulate*, or arbitrary Hypothesis. Yet even he places among his axioms the proposition that a part is less than its whole, and falls into several conflicts with our most modern geometry in consequence. But why need we stop to consider cases where some subtilty of thought is required to see that the assertion is not warranted when every book which applies philosophy to the conduct of life lays down as positive certainty propositions which it is quite as easy to doubt as to believe?

138. The second bar which philosophers often set up across the roadway of inquiry lies in maintaining that this, that, and the other never can be known. When Auguste Comte was pressed to specify any matter of positive fact to the knowledge of which no man could by any possibility attain, he instanced the knowledge of the chemical composition of the fixed stars; and you may see his answer set down in the *Philosophie positive*.* But the ink was scarcely dry upon the printed page before the spectroscope was discovered and that which he had deemed absolutely unknowable was well on the way of getting ascertained. It is easy enough to mention a question the answer to which is not known to me today. But to aver that that answer will not be known tomorrow is somewhat risky; for oftentimes it is precisely the least expected truth which is turned up under the ploughshare of research. And when it comes to positive assertion that the truth never will be found out, that, in the light of the history of our time, seems to me more hazardous than the venture of Andrée.†

* 19^{me} leçon.

† In 1897 Salomon August Andrée attempted to fly over the polar regions in a balloon. He died in the attempt.

139. The third philosophical stratagem for cutting off inquiry consists in maintaining that this, that, or the other element of science is basic, ultimate, independent of aught else, and utterly inexplicable — not so much from any defect in our knowing as because there is nothing beneath it to know. The only type of reasoning by which such a conclusion could possibly be reached is *retroduction*. Now nothing justifies a retroductive inference except its affording an explanation of the facts. It is, however, no explanation at all of a fact to pronounce it *inexplicable*. That, therefore, is a conclusion which no reasoning can ever justify or excuse.

140. The last philosophical obstacle to the advance of knowledge which I intend to mention is the holding that this or that law or truth has found its last and perfect formulation — and especially that the ordinary and usual course of nature never can be broken through. "Stones do not fall from heaven," said Laplace, although they had been falling upon inhabited ground every day from the earliest times. But there is no kind of inference which can lend the slightest probability to any such absolute denial of an unusual phenomenon.

§5. FALLIBILISM, CONTINUITY, AND EVOLUTION*

141. All positive reasoning is of the nature of judging the proportion of something in a whole collection by the proportion found in a sample. Accordingly, there are three things to which we can never hope to attain by reasoning, namely, absolute certainty, absolute exactitude, absolute universality. We cannot be absolutely certain that our conclusions are even approximately true; for the sample may be utterly unlike the unsampled part of the collection. We cannot pretend to be even probably exact; because the sample consists of but a finite number of instances and only admits special values of the proportion sought. Finally, even if we could ascertain with absolute certainty and exactness that the ratio of sinful men to all men was as 1 to 1; still among the infinite generations of men there would be room for any finite number of sinless men with-

* From unpaginated, untitled ms. (or mss.) which to judge from 159 was intended as part of a lecture. c. 1897.

out violating the proportion. The case is the same with a seven legged calf.

142. Now if exactitude, certitude, and universality are not to be attained by reasoning, there is certainly no other means by which they can be reached.

143. Somebody will suggest *revelation*. There are scientists and people influenced by science who laugh at revelation; and certainly science has taught us to look at testimony in such a light that the whole theological doctrine of the "Evidences" seems pretty weak. However, I do not think it is philosophical to reject the possibility of a revelation. Still, granting that, I declare as a logician that revealed truths — that is, truths which have nothing in their favor but revelations made to a few individuals — constitute by far the most uncertain class of truths there are. There is here no question of universality; for revelation is itself sporadic and miraculous. There is no question of mathematical exactitude; for no revelation makes any pretension to that character. But it does pretend to be *certain;* and against that there are three conclusive objections. First, we never can be absolutely certain that any given deliverance really is inspired; for that can only be established by reasoning. We cannot even prove it with any very high degree of probability. Second, even if it is inspired, we cannot be sure, or nearly sure, that the statement is true. We know that one of the commandments was in one of the Bibles printed with[out] a *not* in it.* All inspired matter has been subject to human distortion or coloring. Besides we cannot penetrate the counsels of the most High, or lay down anything as a principle that would govern his conduct. We do not know his inscrutable purposes, nor can we comprehend his plans. We cannot tell but he might see fit to inspire his servants with errors. In the third place, a truth which rests on the authority of inspiration only is of a somewhat incomprehensible nature; and we never can be sure that we rightly comprehend it. As there is no way of evading these difficulties, I say that revelation, far from affording us any certainty, gives results less certain than other sources of information. This would be so even if revelation were much plainer than it is.

144. But, it will be said, you forget the laws which are

*The "Wicked Bible" of 1631 omitted "not" from the Seventh Commandment.

known to us *a priori*, the axioms of geometry, the principles of logic, the maxims of *causality*, and the like. Those are absolutely certain, without exception and exact. To this I reply that it seems to me there is the most positive historic proof that innate truths are particularly uncertain and mixed up with error, and therefore *a fortiori* not without exception. This historical proof is, of course, not infallible; but it is very strong. Therefore, I ask *how do you know* that *a priori* truth is certain, exceptionless, and exact? You cannot know it by *reasoning*. For that would be subject to uncertainty and inexactitude. Then, it must amount to this that you know it *a priori;* that is, you take *a priori* judgments at their own valuation, without criticism or credentials. That is barring the gate of inquiry.

145. Ah! but it will be said, you forget direct experience. Direct experience is neither certain nor uncertain, because it affirms nothing — it just *is*. There are delusions, hallucinations, dreams. But there is no mistake that such things really do appear, and direct experience means simply the appearance. It involves no error, because it testifies to nothing but its own appearance. For the same reason, it affords no certainty. It is not *exact*, because it leaves much vague; though it is not *inexact* either; that is, it has no false exactitude.

146. All this is true of direct experience at its first presentation. But when it comes up to be criticized it is past, itself, and is represented by *memory*. Now the deceptions and inexactitude of memory are proverbial.

147. . . . On the whole, then, we cannot in any way reach perfect certitude nor exactitude. We never can be absolutely sure of anything, nor can we with any probability ascertain the exact value of any measure or general ratio.

This is my conclusion, after many years study of the logic of science; and it is the conclusion which others, of very different cast of mind, have come to, likewise. I believe I may say there is no tenable opinion regarding human knowledge which does not legitimately lead to this corollary. Certainly there is nothing new in it; and many of the greatest minds of all time have held it for true.

148. Indeed, most everybody will admit it until he begins to see what is involved in the admission — and then most people will draw back. It will not be admitted by persons

utterly incapable of philosophical reflection. It will not be fully admitted by masterful minds developed exclusively in the direction of action and accustomed to claim practical infallibility in matters of business. These men will admit the incurable fallibility of all opinions readily enough; only, they will always make exception of their own. The doctrine of fallibilism will also be denied by those who fear its consequences for science, for religion, and for morality. But I will take leave to say to these highly conservative gentlemen that however competent they may be to direct the affairs of a church or other corporation, they had better not try to manage science in that way. Conservatism — in the sense of a dread of consequences — is altogether out of place in science — which has on the contrary always been forwarded by radicals and radicalism, in the sense of the eagerness to carry consequences to their extremes. Not the radicalism that is cocksure, however, but the *radicalism that tries experiments*. Indeed, it is precisely among men animated by the spirit of science that the doctrine of fallibilism will find supporters.

149. Still, even such a man as that may well ask whether I propose to say that it is not quite certain that twice two are four — and that it is even not probably quite exact! But it would be quite misunderstanding the doctrine of fallibilism to suppose that it means that twice two is probably not exactly four. As I have already remarked, it is not my purpose to doubt that people can usually *count* with accuracy. Nor does fallibilism say that men cannot attain a sure knowledge of the creations of their own minds. It neither affirms nor denies that. It only says that people cannot attain absolute certainty concerning questions of fact. Numbers are merely a system of names devised by men for the purpose of counting.* It is a matter of real fact to say that in a certain room there are two persons. It is a matter of fact to say that each person has two eyes. It is a matter of fact to say that there are four eyes in the room. But to say that *if* there are two persons and each person has two eyes there *will be* four eyes is not a statement of fact, but a statement about the system of numbers which is our own creation.

150. Still, if the matter is pressed, let me ask whether any

* See 4.155ff.

individual here present thinks there is no room for possible doubt that twice two is four?

What do *you* think? You have heard of hypnotism. You know how common it is. You know that about one man in *twenty* is capable of being put into a condition in which he holds the most ridiculous nonsense for unquestionable truth. How does any individual here know but that I am a hypnotist and that when he comes out of my influence he may see that twice two is four is merely his distorted idea; that in fact everybody knows it isn't so? Suppose the individual I am addressing to be enormously wealthy. Then I ask: "Would you, in view of this possibility — or with the possibility that you are seized with a temporary insanity, risk your entire fortune this minute against one cent, on the truth of twice two being four?" You certainly ought not to do so; for you could not go on making very many millions of such bets before you would *lose!* Why, according to my estimate of probabilities there is not a single truth of science upon which we ought to bet more than about a million of millions to one — and that truth will be a general one and not a special fact. People say "Such a thing is as certain as that the sun will rise tomorrow!" I like that phrase for its great moderation because it is infinitely far from certain that the sun will rise tomorrow.

151. To return to our friends the Conservatives; these ladies and gentlemen will tell me this doctrine of fallibilism can never be admitted because the consequences from it would undermine Religion. I can only say I am very sorry. The doctrine is true; — without claiming absolute certainty for it, it is *substantially* unassailable. And if its consequences are antagonistic to religion, so much the worse for religion. At the same time, I do not believe they are so antagonistic. The dogmas of a church may be infallible — infallible in the sense in which it is infallibly true that it is wrong to murder and steal — practically and substantially infallible. But what use a church could make of a mathematical infallibility, I fail to see. *Messieurs et mesdames les conservateurs* have generally taken the lead in determining what the church should say to the novelties of science; and I don't think they have managed the business with very distinguished success so far. They have begun by recoiling with horror from the alleged heresies —

about the rotundity of the earth, about its rotation, about geology, about Egyptian history, and so forth — and they have ended by declaring that the church never breathed a single word against any of these truths of science. Perhaps, it be just so with fallibility. For the present those knowing in divine things insist that infallibility is the prerogative of the church, but maybe bye and bye we shall be told that this infallibility had always been taken in an *ecclesiastical sense.* And that will be *true*, too. I should not wonder if the churches were to be quite agile in reformed teachings during the coming thirty years. Even one that mainly gathers in the very ignorant and the very rich may feel young blood in its veins.

152. But doubtless many of you will say, as many most intelligent people have said, Oh, we grant your *fallibilism* to the extent you insist upon it. It is nothing new. Franklin said a century ago that nothing was certain. We will grant it would be foolish to bet ten years' expenditure of the United States Government against one cent upon any fact whatever. But practically speaking many things are substantially certain. So, after all, of what importance is your *fallibilism?*

We come then to this question: of what importance is it? Let us see.

153. How *can* such a little thing be of importance, you will ask? I answer: after all there is a difference between something and nothing. If a metaphysical theory has come into general vogue, which can rest on nothing in the world but the assumption that absolute exactitude and certitude are to be attained, and if that metaphysics leaves us unprovided with pigeonholes in which to file important facts so that they have to be thrown in the fire — or to resume our previous figure if that metaphysical theory seriously blocks the road of inquiry — then it is comprehensible that the little difference between a degree of evidence extremely high and absolute certainty should after all be of great importance as removing a mote from our eye.

154. Let us look then at two or three of the grandest results of science and see whether they appear any different from a fallibilist standpoint from what they would to an infallibilist.

Three of the leading conceptions of science may be glanced at — I mean the ideas of force, of continuity, and of evolution.

155. . . . The fourth law of motion was developed about forty years ago* by Helmholtz and others. It is called the law of the conservation of energy; but in my opinion that is a very misleading name, implying a peculiar aspect of the law under which the real fact at the bottom of it is not clearly brought out. It is therefore not suitable for an abstract and general statement, although it is a point of view which is very serviceable for many practical applications. But the law generally stated is that the changes in the velocities of particles depend exclusively on their relative positions.

It is not necessary now to examine these laws with technical accuracy. It is sufficient to notice that they leave the poor little particle no option at all. Under given circumstances his motion is precisely laid out for him.

We can from the nature of things have no evidence at all tending to show that these laws are absolutely exact. But in some single cases we can see that the approximation to exactitude is quite wonderful.

These laws have had a very wonderful effect upon physical sciences, because they have shown the very high degree of exactitude with which nature acts — at least, in simple configurations. But, as I said before, the logic of the case affords us not one scintilla of reason to think that this exactitude is perfect.

156. The illustrious Phœnix [G. H. Derby], you remember, wrote a series of lectures on astronomy to be delivered at the Lowell Institute in Boston.† But owing to the unexpected circumstance of his not being invited to give any lectures at that Institution, they were ultimately published in *The San Diego Herald*. In those lectures in treating of the sun he mentions how it once stood still at the command of Joshua. But, says he, I never could help thinking that it might have wiggled a very little when Joshua was not looking directly at it. The question is whether particles may not spontaneously swerve by a very little — less than we can perceive — from the exact

* To judge from this, the ms. should be dated ten years earlier. But the absence of the terms and the handwriting in earlier mss., and their presence in mss. dated 1897–8 seem to indicate that the editorial dating is correct.

† *Phœnixiana*, "Lectures on Astronomy."

requirements of the laws of mechanics. We cannot possibly have a right to deny this. For such a denial would be a claim to absolute exactitude of knowledge. On the other hand, we never can have any right to suppose that any observed phenomenon is simply a sporadic spontaneous irregularity. For the only justification we can have for supposing anything we don't see is that it would explain how an observed fact could result from the ordinary course of things. Now to suppose a thing sporadic, spontaneous, irregular, is to suppose it departs from the ordinary course of things. That is blocking the road of inquiry; it is supposing the thing inexplicable, when a supposition can only be justified by its affording an explanation.

157. But we may find a general class of phenomena, forming a part of the general course of things, which are explicable not as *an irregularity*, but as the resultant effect of a whole class of irregularities.

Physicists often resort to this kind of explanation to account for phenomena which appear to violate the law of the conservation of energy. The general properties of gases are explained by supposing the molecules are moving about in every direction in the most diverse possible ways. Here, it is true, it is supposed that there is only so much irregularity as the laws of mechanics permit — but the principle is there of explaining a general phenomenon by the statistical regularities that exist among irregularities.

158. As there is nothing to show that there is not a certain amount of absolute spontaneity in nature, despite all laws, our metaphysical pigeon-holes should not be so limited as to exclude this hypothesis, provided any general phenomena should appear which might be explained by such spontaneity.

159. Now in my opinion there are several such general phenomena. Of these I will at this moment instance but one.

It is *the* most obtrusive character of nature. It is so obvious, that you will hardly know at first what it is I mean. It is curious how certain facts escape us because they are so pervading and ubiquitous; just as the ancients imagined the music of the spheres was not heard because it was heard all the time. But will not somebody kindly tell the rest of the audience what is the most marked and obtrusive character of nature? Of course, I mean the variety of nature.

160. Now I don't know that it is logically accurate to say that this marvellous and infinite diversity and manifoldness of things is a sign of spontaneity. I am a logical analyst by long training, you know, and to say this is a manifestation of spontaneity seems to me faulty analysis. I would rather say it *is* spontaneity. I don't know what you can make out of the meaning of spontaneity but newness, freshness, and diversity.

161. Let me ask you a little question? Can the operation of *law* create diversity where there was no diversity before? Obviously not; under given circumstances mechanical law prescribes *one* determinate result.

I could easily prove this by the principles of analytical mechanics. But that is needless. You can see for yourselves that law prescribes like results under like circumstances. That is what the word *law* implies. So then, all this exuberant diversity of nature cannot be the result of law. Now what is spontaneity? It is the character of not resulting by law from something antecedent.

162. Thus, the universe is *not* a mere mechanical result of the operation of blind law.* The most obvious of all its characters cannot be so explained. It is the multitudinous facts of all experience that show us this; but that which has opened our eyes to these facts is the principle of fallibilism. Those who fail to appreciate the importance of fallibilism reason: we see these laws of mechanics; we see how extremely closely they have been verified in some cases. We suppose that what we haven't examined is like what we have examined, and that these laws are absolute, and the whole universe is a boundless machine working by the blind laws of mechanics. This is a philosophy which leaves no room for a God! No, indeed! It leaves even human consciousness, which cannot well be denied to exist, as a perfectly idle and functionless *flâneur* in the world, with no possible influence upon anything — not even upon itself. Now will you tell me that this fallibilism amounts to nothing?

163. But in order really to see all there is in the doctrine of fallibilism, it is necessary to introduce the idea of continuity, or unbrokenness. This is the leading idea of the differential calculus and of all the useful branches of mathematics; it plays

* See vol. 6, bk. I.

a great part in all scientific thought, and the greater the more scientific that thought is; and it is the master key which adepts tell us unlocks the arcana of philosophy.

164. We all have some idea of continuity. Continuity is fluidity, the merging of part into part. But to achieve a really distinct and adequate conception of it is a difficult task, which with all the aids possible must for the most acute and most logically trained intellect require days of severe thought. If I were to attempt to give you any logical conception of it, I should only make you dizzy to no purpose. I may say this, however. I draw a line. Now the points on that line form a continuous series. If I take any two points on that line, however close together, other points there are lying between them. If that were not so,the series of points would not be continuous. It might be so, even if the series of points were not continuous. . . .

165. You will readily see that the idea of continuity involves the idea of infinity. Now, the nominalists tell us that we cannot reason about infinity, or that we cannot reason about it *mathematically*. Nothing can be more false. Nominalists cannot reason about infinity, because they do not reason logically about anything. Their reasoning consists of performing certain processes which they have found worked well — without having any insight into the conditions of their working well. This is not logical reasoning. It naturally fails when infinity is involved; because they reason about infinity as if it were finite. But to a logical reasoner, reasoning about infinity is decidedly simpler than reasoning about finite quantity.

166. There is one property of a continuous expanse that I must mention, though I cannot venture to trouble you with the demonstration of it. It is that in a continuous expanse, say a continuous line, there are continuous lines infinitely short. In fact, the whole line is made up of such infinitesimal parts. The property of these infinitely small spaces is — I regret the abstruseness of what I am going to say, but I cannot help it — the property which distinguishes these infinitesimal distances is that a certain mode of reasoning which holds good of all finite quantities and of some that are not finite does not hold good of them. Namely, mark any point on the line A. Suppose that point to have any character; suppose, for instance,

it is *blue*. Now suppose we lay down the rule that every point within an inch of a blue point shall be painted blue. Obviously, the consequence will be that the whole line will have to be blue. But this reasoning does not hold good of infinitesimal distances. After the point A has been painted blue, the rule that every point infinitesimally near to a blue point shall be painted blue will not necessarily result in making the whole blue. Continuity involves infinity in the strictest sense, and infinity even in a less strict sense goes beyond the possibility of direct experience.

167. Can we, then, ever be sure that anything in the real world is continuous? Of course, I am not asking for an absolute certainty; but can we ever say that it is so with any ordinary degree of security? This is a vitally important question. I think that we have one positive direct evidence of continuity and on the first line but one. It is this. We are immediately aware only of our present feelings — not of the future, nor of the past. The past is known to us by present memory, the future by present suggestion. But before we can interpret the memory or the suggestion, they are past; before we can interpret the present feeling which means memory, or the present feeling that means suggestion, since that interpretation takes time, that feeling has ceased to be present and is now past. So we can reach no conclusion from the present but only from the past.

168. How do we know then on the whole that the past ever existed, that the future ever will exist? How do we know there ever was or ever will be anything but the present instant? Or stop: I must not say *we*. How do I know that anybody but myself ever existed or even I myself exist except for one single instant, the present, and that all this business is not an illusion from top to bottom? Answer: I don't know. But I am trying the hypothesis that it is real, which seems to work excellently so far. Now if this is real, the past is really known to the present. How can it be known? Not by inference; because as we have just seen we can make no inference from the present, since it will be past before the inference gets drawn.

169. Then we must have an immediate consciousness of the past. But if we have an immediate consciousness of a state of consciousness past by one unit of time and if that past state

involved an immediate consciousness of a state then past by one unit, we now have an immediate consciousness of a state past by two units; and as this is equally true of all states, we have an immediate consciousness of a state past by four units, by eight units, by sixteen units, etc.; in short we must have an immediate consciousness of every state of mind that is past by any finite number of units of time. But we certainly have not an immediate consciousness of our state of mind a year ago. So a year is more than any finite number of units of time in this system of measurement; or, in other words, there is a measure of time infinitely less than a year. Now, this is only true if the series be continuous. Here, then, it seems to me, we have positive and tremendously strong reason for believing that time really is continuous.

170. Equally conclusive and direct reason for thinking that space and degrees of quality and other things are continuous is to be found as for believing time to be so. Yet, the reality of continuity once admitted, reasons are there, divers reasons, some positive, others only formal, yet not contemptible, for admitting the continuity of all things. I am making a bore of myself and won't bother you with any full statement of these reasons, but will just indicate the nature of a few of them. Among formal reasons, there are such as these, that it is easier to reason about continuity than about discontinuity, so that it is a convenient assumption. Also, in case of ignorance it is best to adopt the hypothesis which leaves open the greatest field of possibility; now a continuum is merely a discontinuous series with additional possibilities. Among positive reasons, we have that apparent analogy between time and space, between time and degree, and so on. There are various other positive reasons, but the weightiest consideration appears to me to be this: How can one mind act upon another mind? How can one particle of matter act upon another at a distance from it? The nominalists tell us this is an ultimate fact — it cannot be explained. Now, if this were meant in [a] merely practical sense, if it were only meant that we know that one thing does act on another but that how it takes place we cannot very well tell, up to date, I should have nothing to say, except to applaud the moderation and good logic of the statement. But this is not what is meant; what is meant is that we

come up, bump against actions absolutely unintelligible and inexplicable, where human inquiries have to stop. Now that is a mere *theory*, and nothing can justify a theory except its explaining observed facts. It is a poor kind of theory which in place of performing this, the sole legitimate function of a theory, merely supposes the facts to be inexplicable. It is one of the peculiarities of nominalism that it is continually supposing things to be absolutely inexplicable. That blocks the road of inquiry. But if we adopt the theory of continuity we escape this illogical situation. We may then say that one portion of mind acts upon another, because it is in a measure immediately present to that other; just as we suppose that the infinitesimally past is in a measure present. And in like manner we may suppose that one portion of matter acts upon another because it is in a measure in the same place.

171. If I were to attempt to describe to you in full all the scientific beauty and truth that I find in the principle of continuity, I might say in the simple language of Matilda the Engaged, "the tomb would close over me e'er the entrancing topic were exhausted"— but not before my audience was exhausted. So I will just drop it here. Only, in doing so, let me call your attention to the natural affinity of this principle to the doctrine of fallibilism. The principle of continuity is the idea of fallibilism objectified. For fallibilism is the doctrine that our knowledge is never absolute but always swims, as it were, in a continuum of uncertainty and of indeterminacy. Now the doctrine of continuity is that *all things* so swim in continua.

172. The doctrine of continuity rests upon observed fact as we have seen. But what opens our eyes to the significance of that fact is fallibilism. The ordinary scientific infallibilist — of which sect Büchner in his *Kraft und Stoff* affords a fine example — cannot accept *synechism*, or the doctrine that all that exists is continuous — because he is committed to discontinuity in regard to all those things which he fancies he has exactly ascertained, and especially in regard to that part of his knowledge which he fancies he has exactly ascertained to be *certain*. For where there is continuity, the exact ascertainment of real quantities is too obviously impossible. No sane man can dream that the ratio of the circumference to the

diameter could be exactly ascertained by measurement. As to the quantities he has not yet exactly ascertained, the Büchnerite is naturally led to separate them into two distinct classes, those which may be ascertained hereafter (and there, as before, continuity must be excluded), and those absolutely unascertainable — and these in their utter and everlasting severance from the other class present a new breach of continuity. Thus scientific infallibilism draws down a veil before the eyes which prevents the evidences of continuity from being discerned.

But as soon as a man is fully impressed with the fact that absolute exactitude never can be known, he naturally asks whether there are any facts to show that hard discrete exactitude really exists. That suggestion lifts the edge of that curtain and he begins to see the clear daylight shining in from behind it.

173. But fallibilism cannot be appreciated in anything like its true significancy until evolution has been considered. This is what the world has been most thinking of for the last forty years — though old enough is the general idea itself. Aristotle's philosophy, that dominated the world for so many ages and still in great measure tyrannizes over the thoughts of butchers and bakers that never heard of him — is but a metaphysical evolutionism.

174. Evolution means nothing but *growth* in the widest sense of that word. Reproduction, of course, is merely one of the incidents of growth. And what is growth? Not mere increase. Spencer says it is the passage from the homogeneous to the heterogeneous — or, if we prefer English to Spencerese — *diversification*. That is certainly an important factor of it. Spencer further says that it is a passage from the unorganized to the organized; but that part of the definition is so obscure that I will leave it aside for the present. But think what an astonishing idea this of *diversification* is! Is there such thing in nature as increase of variety? Were things simpler, was variety less in the original nebula from which the solar system is supposed to have grown than it is now when the land and sea swarms with animal and vegetable forms with their intricate anatomies and still more wonderful economies? It would seem as if there were an increase in variety, would it not? And yet mechanical law, which the scientific infallibilist

tells us is the only agency of nature, mechanical law can never produce diversification. That is a mathematical truth — a proposition of analytical mechanics; and anybody can see without any algebraical apparatus that mechanical law out of like antecedents car nly produce like consequents. It is the very idea of law. So if observed facts point to real growth, they point to another agency, to spontaneity for which infallibilism provides no pigeon-hole. And what is meant by this passage from the less organized to the more organized? Does it mean a passage from the less bound together to the more bound together, the less connected to the more connected, the less regular to the more regular? How can the regularity of the world increase, if it has been absolutely perfect all the time?

175. . . . Once you have embraced the principle of continuity no kind of explanation of things will satisfy you except that they *grew*. The infallibilist naturally thinks that everything always was substantially as it is now. Laws at any rate being absolute could not grow. They either always were, or they sprang instantaneously into being by a sudden fiat like the drill of a company of soldiers. This makes the laws of nature absolutely blind and inexplicable. Their why and wherefore can't be asked. This absolutely blocks the road of inquiry. The fallibilist won't do this. He asks may these *forces* of nature not be somehow amenable to reason? May they not have naturally grown up? After all, there is no reason to think they are absolute. If all things are continuous, the universe must be undergoing a continuous growth from non-existence to existence. There is no difficulty in conceiving existence as a matter of degree. The reality of things consists in their persistent forcing themselves upon our recognition. If a thing has no such persistence, it is a mere dream. Reality, then, is persistence, is regularity. In the original chaos, where there was no regularity, there was no existence. It was all a confused dream. This we may suppose was in the infinitely distant past. But as things are getting more regular, more persistent, they are getting less dreamy and more real.

Fallibilism will at least provide a big pigeon-hole for facts bearing on that theory.

Book II

THE CLASSIFICATION OF THE SCIENCES

PROEM: THE ARCHITECTONIC CHARACTER OF PHILOSOPHY*

176. The universally and justly lauded parallel which Kant draws between a philosophical doctrine and a piece of architecture has excellencies which the beginner in philosophy might easily overlook; and not the least of these is its recognition of the cosmic character of philosophy. I use the word "cosmic" because *cosmicus* is Kant's own choice; but I must say I think *secular* or *public* would have approached nearer to the expression of his meaning. Works of sculpture and painting can be executed for a single patron and must be by a single artist. A painting always represents a fragment of a larger whole. It is broken at its edges. It is to be shut up in a room and admired by a few. In such a work individuality of thought and feeling is an element of beauty. But a great building, such as alone can call out the depths of the architect's soul, is meant for the whole people, and is erected by the exertions of an army representative of the whole people. It is the message with which an age is charged, and which it delivers to posterity. Consequently, thought characteristic of an individual — the piquant, the nice, the clever — is too little to play any but the most subordinate *rôle* in architecture. If anybody can doubt whether this be equally true of philosophy, I can but recommend to him that splendid third chapter of the Methodology, in the *Critic of the Pure Reason.*

177. To the cosmological or secular character of philosophy (to which, as closely connected, Kant with his unfailing discernment joins the circumstance that philosophy is a thing that has to grow by the fission of minute parts and not by accretion) is due the necessity of planning it out from the beginning. Of course, every painting likewise has its composition; but composition is not a very weighty problem, except in that kind of painting which is accessory to architecture, or is, at any rate, very public in its appeal. Indeed historical painting is one of those exceptions which go to prove the rule that in

* Apparently a foreword to a volume of the *Principles of Philosophy*, c. 1896.

works which aim at being secular, rather than individualistic, the preliminary business of planning is particularly important and onerous.

178. And the reason is very plain and simple. The instincts of the lower animals answer their purposes much more unerringly than a discursive understanding could do. But for man discourse of reason is requisite, because men are so intensively individualistic and original that the instincts, which are racial ideas, become smothered in them. A deliberate logical faculty, therefore, has in man to take their place; and the sole function of this logical deliberation is to grind off the arbitrary and the individualistic character of thought. Hence, wherever the arbitrary and the individualistic is particularly prejudicial, there logical deliberation, or discourse of reason, must be allowed as much play as possible.

179. That is why philosophy ought to be deliberate and planned out; and that is why, though pitchforking articles into a volume is a favorite and easy method of bookmaking, it is not the one which Mr. Peirce has deemed to be the most appropriate to the exposition of the principles of philosophy; so that, instead of making up this book by a collection of his old papers with additions, as he was urged to do, he has preferred to write it entirely anew, as if he had never before set pen to paper.*

* However, for the only philosophical work Peirce ever completed, *The Grand Logic*, the "pitchfork" method was used. The editors, of course, were compelled to "pitchfork," though they have tried to do it according to a plan, suggested by the classifications contained in the present book.

CHAPTER 1

AN OUTLINE CLASSIFICATION OF THE SCIENCES*

180. This classification, which aims to base itself on the principal affinities of the objects classified, is concerned not with all possible sciences, nor with so many branches of knowledge, but with sciences in their present condition, as so many businesses of groups of living men. It borrows its idea from Comte's classification; namely, the idea that one science depends upon another for fundamental principles, but does not furnish such principles to that other. It turns out that in most cases the divisions are trichotomic; the First of the three members relating to universal elements or laws, the Second arranging classes of forms and seeking to bring them under universal laws, the Third going into the utmost detail, describing individual phenomena and endeavoring to explain them. But not all the divisions are of this character.

The classification has been carried into great detail;† but only its broader divisions are here given.

181. All science is either, A. Science of Discovery; B. Science of Review; or C. Practical Science.

182. By "science of review" is meant the business of those who occupy themselves with arranging the results of discovery, beginning with digests, and going on to endeavor to form a philosophy of science. Such is the nature of Humboldt's *Cosmos*, of Comte's *Philosophie positive*, and of Spencer's

* Pp. 5–9 of *A Syllabus of Certain Topics of Logic*, 1903, Alfred Mudge & Son, Boston, bearing the following preface: "This syllabus has for its object to supplement a course of eight lectures to be delivered at the Lowell Institute, by some statements for which there will not be time in the lectures, and by some others not easily carried away from one hearing. It is intended to be a help to those who wish seriously to study the subject, and to show others what the style of thought is that is required in such study. Like the lectures themselves, this syllabus is intended chiefly to convey results that have never appeared in print; and much is omitted because it can be found elsewhere."

† See 203ff. where, also, some modifications of the present scheme are to be found. Cf. e.g. 181 and 239.

Synthetic Philosophy. The classification of the sciences belongs to this department.

183. Science of Discovery is either, I. Mathematics; II. Philosophy; or III. Idioscopy.*

184. Mathematics studies what is and what is not logically possible, without making itself responsible for its actual existence. Philosophy is *positive science*, in the sense of discovering what really is true; but it limits itself to so much of truth as can be inferred from common experience. Idioscopy embraces all the special sciences, which are principally occupied with the accumulation of new facts.

185. Mathematics may be divided into *a*. the Mathematics of Logic; *b*. the Mathematics of Discrete Series; *c*. the Mathematics of Continua and Pseudo-continua.

I shall not carry this division further. Branch *b* has recourse to branch *a*, and branch *c* to branch *b*.

186. Philosophy is divided into *a*. Phenomenology; *b*. Normative Science; *c*. Metaphysics.

Phenomenology ascertains and studies the kinds of elements universally present in the phenomenon; meaning by the *phenomenon*, whatever is present at any time to the mind in any way. Normative science distinguishes what ought to be from what ought not to be, and makes many other divisions and arrangements subservient to its primary dualistic distinction. Metaphysics seeks to give an account of the universe of mind and matter. Normative science rests largely on phenomenology and on mathematics; metaphysics on phenomenology and on normative science.

187. Idioscopy has two wings: α. the Physical Sciences; and β. the Psychical, or Human Sciences.

Psychical science borrows principles continually from the physical sciences; the latter very little from the former.

188. The physical sciences are: *a*. Nomological, or General, Physics; *b*. Classificatory Physics; *c*. Descriptive Physics.

Nomological physics discovers the ubiquitous phenomena of the physical universe, formulates their laws, and measures their constants. It draws upon metaphysics and upon mathematics for principles. Classificatory physics describes and classifies physical forms and seeks to explain them by the laws

* See 242n for Bentham's definition of this term.

discovered by nomological physics with which it ultimately tends to coalesce. Descriptive physics describes individual objects — the earth and the heavens — endeavors to explain their phenomena by the principles of nomological and classificatory physics, and tends ultimately itself to become classificatory.

189. The Psychical Sciences are: *a*. Nomological Psychics or Psychology; *b*. Classificatory Psychics, or Ethnology; *c*. Descriptive Psychics, or History.

Nomological psychics discovers the general elements and laws of mental phenomena. It is greatly influenced by phenomenology, by logic, by metaphysics, and by biology (a branch of classificatory physics). Classificatory psychics classifies products of mind and endeavors to explain them on psychological principles. At present it is far too much in its infancy (except linguistics, to which reference will be made below) to approach very closely to psychology. It borrows from psychology and from physics. Descriptive psychics endeavors in the first place to describe individual manifestations of mind, whether they be permanent works or actions; and to that task it joins that of endeavoring to explain them on the principles of psychology and ethnology. It borrows from geography (a branch of descriptive physics), from astronomy (another branch) and from other branches of physical and psychical science.

I now consider the subdivisions of these sciences, so far as they are so widely separated as quite to sunder the groups of investigators who today study them.

190. Phenomenology is, at present, a single study.

191. Normative science has three widely separated divisions: i. Esthetics; ii. Ethics; iii. Logic.

Esthetics is the science of ideals, or of that which is objectively admirable without any ulterior reason. I am not well acquainted with this science; but it ought to repose on phenomenology. Ethics, or the science of right and wrong, must appeal to Esthetics for aid in determining the *summum bonum*. It is the theory of self-controlled, or deliberate, conduct. Logic is the theory of self-controlled, or deliberate, thought; and as such, must appeal to ethics for its principles. It also depends upon phenomenology and upon mathematics. All thought

being performed by means of signs, logic may be regarded as the science of the general laws of signs. It has three branches: 1, Speculative Grammar, or the general theory of the nature and meanings of signs, whether they be icons, indices, or symbols; 2, Critic, which classifies arguments and determines the validity and degree of force of each kind; 3, Methodeutic, which studies the methods that ought to be pursued in the investigation, in the exposition, and in the application of truth. Each division depends on that which precedes it.

192. Metaphysics may be divided into, i, General Metaphysics, or Ontology; ii, Psychical, or Religious, Metaphysics, concerned chiefly with the questions of 1, God, 2, Freedom, 3, Immortality; and iii, Physical Metaphysics, which discusses the real nature of time, space, laws of nature, matter, etc. The second and third branches appear at present to look upon one another with supreme contempt.

193. Nomological physics is divided into, i, Molar Physics, Dynamics and Gravitation; ii, Molecular Physics, Elaterics and Thermodynamics; iii, Etherial Physics, Optics and Electrics. Each division has two subdivisions. The dependence of the divisions is well marked.

194. Classificatory physics seems, at present, as a matter of fact, to be divided, quite irrationally and most unequally, into i, Crystallography; ii, Chemistry; iii, Biology.

195. But crystallography is rather an offshoot from chemistry, to which it furnishes a few facts, but hardly a principle. It is highly mathematical and depends also on elaterics. Biology might be regarded (although, as a matter of fact, no such view is taken) as the chemistry of the albumoids and of the forms they assume. It is probable that all the differences of races, individuals, and tissues are chemical, at bottom. At any rate, the possible varieties of albuminoids are amply sufficient to account for all the diversity of organic forms.

196. Pure chemistry seems, at present, to consist of, 1, Physical Chemistry, consisting of the old chemical physics and the modern chemical dynamics; 2, Organic Chemistry, Aliphatic and Aromatic; 3, Inorganic Chemistry, consisting of the doctrine of the elements, their atomic weights, periodicity, etc., and the doctrine of compounds.

197. Biology is divided into, 1, Physiology; and 2, Anat-

omy. Physiology is closely allied to chemistry and physics. Anatomy is divided into many distinct fields, according to the nature of the forms studied.

198. Descriptive physics is divided into, 1, Geognosy, and, 2, Astronomy. Both have various well-known subdivisions.

199. Psychology is most naturally divided, according to the methods it follows, into, i, Introspectional Psychology; ii, Experimental Psychology; iii, Physiological Psychology; iv, Child Psychology.

This division only admits those parts of psychology which investigate the general phenomena of mind. Special psychology belongs to classificatory psychics. Both experimental and physiological psychology are dependent upon introspective psychology. But it is hard to say which of them derives most from the other. Child psychology depends on all the others. Psychology is too young a science to have any further living divisions than such as are here admitted.

200. Classificatory psychics is divided into, i, Special Psychology, itself consisting of, 1, Individual Psychology; 2, Psychical Heredity; 3, Abnormal Psychology; 4, Mob Psychology; 5, Race Psychology; 6, Animal Psychology; ii, Linguistics, a vast science, divided according to the families of speech, and cross-divided into, 1, Word Linguistics; 2, Grammar; and there should be a comparative science of forms of composition; iii, Ethnology, divided into, 1, the Ethnology of Social Developments, customs, laws, religion, and traditions; and, 2, the Ethnology of Technology.

201. Descriptive psychics is divided into, i, History proper, itself divided according to the nature of its data into, 1, Monumental History; 2, Ancient History with all other history that is drawn from few and general testimonies; 3, History drawn from a wealth of documents, as Modern History, generally. History has, beside, two cross-divisions; the one into, 1, Political History; 2, History of the Different Sciences; 3, History of Social Developments, religion, law, slavery, manners, etc.; the other according to the different parts of the world and the different peoples whose history is studied; ii, Biography, which at present is rather a mass of lies than a science; iii, Criticism, the study of individual works of mind, itself divided into, 1, Literary Criticism; 2, Art Criticism, of which

the latter is divided into many departments, as Criticism of Military Operations, Criticism of Architecture, etc.

202. The classification of practical sciences has been elaborated by the author, but will not here be touched upon.* No classification of the science of review has been attempted.

* See 243.

CHAPTER 2

*A DETAILED CLASSIFICATION OF THE SCIENCES**

§1. NATURAL CLASSES

203. Many have been the attempts at a general classification of the sciences. Dr. Richardson's little book upon the subject[1] is quite incomplete, only enumerating one hundred and forty-six systems. They are naturally many, because not only are their purposes various, but their conceptions of a science are divergent, and their notions of what classification is are still more so. Many of these schemes introduce sciences which nobody ever heard of; so that they seem to aim at classifying, not actually existent sciences, but possible sciences. A somewhat presumptuous undertaking is that of classifying the science of the remote future. On the other hand, if classifications are to be restricted to sciences actually existing at the time the classifications are made, the classifications certainly ought to differ from age to age. If Plato's classification was satisfactory in his day, it cannot be good today; and if it be good now, the inference will be that it was bad when he proposed it.

This business of classifying sciences is not one to be undertaken precipitately or off-hand. That is plain. We should not begin the execution of the task until we have well considered, first, what classification is; and secondly, what science is. . . .

204. The first question then, that it seems well to consider (remembering that classification is one of the topics of logic to be dealt with more scientifically in its proper place, and that I can here only skim the surface of it) is, What is meant by a true and natural class? A great many logicians say there is no such thing; and, what is strange, even many students of

* Section 1, ch. 2, of the "Minute Logic," 1902.

[1] *Classification, Theoretical and Practical*, by Ernest Cushing Richardson, C. Scribner's Sons, N. Y., 1901.

taxonomic sciences not only follow this opinion, but allow it a great part in determining the conclusions of botany and zoölogy. The cause of their holding this opinion has two factors; first, that they attach a metaphysical signification to the term *natural* or *real class*, and secondly, that they have embraced a system of metaphysics which allows them to believe in no such thing as that which they have defined a real or natural class to be. Far be it from me to wish to close any avenue by which truth may be arrived at; and if botanists and zoölogists come to the conclusion that botany and zoölogy must rest upon metaphysics, I have not a word of objection to make. Only I can tell them that metaphysics is a most difficult science, presenting more pitfalls for the uninformed than almost any, which a mere amateur at it would be foolish to fancy that he could escape. Therefore, if botany and zoölogy must perforce rest upon metaphysics, by all means let this metaphysics be recognized as an explicit branch of those sciences, and be treated in a thoroughgoing and scientific manner. Having devoted many years to it, I am entitled to my opinion upon a metaphysical question, although it may be a mistaken one; and my opinion is that it is a shallow and sciolistic metaphysics which declares a "real class," in the sense which those writers attach to the term, to be an impossible thing. At the same time, I am unable to see any need at all in positive science for considering such metaphysically real classes. To my apprehension the business of classification has no concern with them, but only with true and natural classes, in another and a purely experiential sense. For example, if I were to attempt to classify the arts, which I shall not do, I should have to recognize, as one of them, the art of illumination, and should have occasion to remark that lamps form a true, real, and natural class, because every lamp has been made and has come into being as a result of an aim common and peculiar to all lamps. A *class*, of course, is the total of whatever objects there may be in the universe which are of a certain description. What if we try taking the term "natural," or "real, class" to mean a class of which all the members owe their existence as members of the class to a common final cause? This is somewhat vague; but it is better to allow a term like this to remain vague, until we see our way to rational precision. In the case of lamps, we know what that cause is: that instinct

which enables us to distinguish human productions and to divine their purpose informs us of this with a degree of certainty which it were futile to hope that any science should surpass. But in the case of natural classes the final cause remains occult. Perhaps, since phrases retain their sway over men's minds long after their meaning has evaporated, it may be that some reader, even at this day, remains imbued with the old notion that there are no final causes in nature; in which case, natural selection, and every form of evolution, would be false. For evolution is nothing more nor less than the working out of a definite end. A final cause may be conceived to operate without having been the purpose of any mind: that supposed phenomenon goes by the name of *fate*. The doctrine of evolution refrains from pronouncing whether forms are simply fated or whether they are providential; but that definite ends are worked out none of us today any longer deny. Our eyes have been opened; and the evidence is too overwhelming. In regard to natural objects, however, it may be said, in general, that we do not know precisely what their final causes are. But need that prevent us from ascertaining whether or not there is a common cause by virtue of which those things that have the essential characters of the class are enabled to exist?

205. The manner of distribution of the class-character will show, with a high degree of certainty, whether or not it is determinative of existence. Take, for example, the class of animals that have legs. The use of legs is clear to us, having them ourselves. But if we pass the animal kingdom in review, we see that in the majority of branches there are no such organs of locomotion; while in the others they are present throughout some whole classes, and absent throughout others; and in still others are sometimes present, sometimes absent. With such a distribution, this mode of locomotion may be so connected with the possibility of a form, that two animals of the same order could not differ in respect to using legs; but it is evident that animals having legs do not form a natural group; for they are not separated from all others in any other important particular. We thus get a tolerably clear idea of what a natural class is: it will amply suffice for our present purpose; though we can hardly hope that it will turn out to be logically accurate. We also see that, when an object has been made with a pur-

pose, as is, of course, the case with the sciences, no classes can be more fundamental nor broader than those which are defined by the purpose. A purpose is an operative desire. Now a desire is always general; that is, it is always some *kind* of thing or event which is desired; at least, until the element of will, which is always exercised upon an individual object upon an individual occasion, becomes so predominant as to overrule the generalizing character of desire. Thus, desires create classes, and extremely broad classes. But desires become, in the pursuit of them, more specific. Let us revert, for example, to lamps. We desire, in the first instance, merely economical illumination. But we remark that that may be carried out by combustion, where there is a chemical process kindling itself, or heat may be supplied from without in electric lighting, or it may be stored up, as in phosphorescence. These three ways of carrying out our main purpose constitute subsidiary purposes.[1] So if we decide upon electric lighting, the question will be between incandescent and arc lighting. If we decide upon combustion, the burning matter may itself become incandescent, or its heat may serve to render another more suitable thing incandescent, as in the Welsbach burner. Here is a complication which will ordinarily be advantageous, since by not making the same thing fulfill the two functions of supplying heat to produce incandescence and of incandescing upon being heated, there is more freedom to choose things suitable to the two functions. This is a good example of that sort of natural class which Agassiz called an order; that is, a class created by a useful complication of a general plan.

206. Closely connected with the fact that every desire is general, are two other facts which must be taken into account in considering purposive classes. The first of these is that a desire is always more or less variable, or vague. For example, a man wants an economical lamp. Then if he burns oil in it, he will endeavor to burn that oil which gives him sufficient light at the lowest cost. But another man, who lives a little further from the source of supply of that oil and a little nearer the source of a different oil may find that different oil to be the

[1] I am here influenced by the *Essay on Classification* [1857] of L. Agassiz, whose pupil I was for a few months. This work appeared at a most inauspicious epoch.

better for him. So it is with the desires of one individual. The same man who prefers veal to pork as a general thing, may think that an occasional spare rib is better than having cold boiled veal every day of his life. In short, variety is the spice of life for the individual, and practically still more so for a large number of individuals; and as far as we can compare Nature's ways with ours, she seems to be even more given to variety than we. These three cases may be very different on their subjective side; but for purposes of classification they are equivalent.

207. But not only is desire *general* and *vague*, or indeterminate; it has besides a certain longitude, or *third* dimension. By this I mean that while a certain ideal state of things might most perfectly satisfy a desire, yet a situation somewhat differing from that will be far better than nothing; and in general, when a state is not too far from the ideal state, the nearer it approaches that state the better. Moreover, the situation of things most satisfactory to one desire is almost never the situation most satisfactory to another. A brighter lamp than that I use would perhaps be more agreeable to my eyes; but it would be less so to my pocket, to my lungs, and to my sense of heat. Accordingly, a compromise is struck; and since all the desires concerned are somewhat vague, the result is that the objects actually will cluster about certain middling qualities, some being removed this way, some that way, and at greater and greater removes fewer and fewer objects will be so determined. Thus, clustering distributions will characterize purposive classes.

208. One consequence of this deserves particular notice, since it will concern us a good deal in our classification of the sciences, and yet is quite usually overlooked and assumed not to be as it is. Namely, it follows that it may be quite impossible to draw a sharp line of demarcation between two classes, although they are real and natural classes in strictest truth. Namely, this will happen when the form about which the individuals of one class cluster is not so unlike the form about which individuals of another class cluster but that variations from each middling form may precisely agree. In such a case, we may know in regard to any intermediate form what proportion of the objects of that form had one purpose and what proportion the other; but unless we have some supplementary

information we cannot tell which ones had one purpose and which the other.

209. The reader may be disposed to suspect that this is merely a mathematician's fancy, and that no such case would be likely ever to occur. But he may be assured that such occurrences are far from being rare. In order to satisfy him that this state of things does occur, I will mention an incontestable instance of it;— incontestable, at least, by any fair mind competent to deal with the problem. Prof. [W. M.] Flinders Petrie, whose reasoning powers I had admired long before his other great scientific qualities had been proved, among which his great exactitude and circumspection as a metrologist concerns us here, exhumed, at the ancient trading town of Naucratis, no less than 158 balance-weights having the Egyptian *ket* as their unit.[1] The great majority of them are of basalt and syenite, material so unchangeable that the corrections needed to bring them to their original values are small. I shall deal only with 144 of them from each of which Mr. Petrie has calculated the value of the *ket* to a tenth of a Troy grain. Since these values range all the way from 137 to 152 grains, it is evident that the weights were intended to be copies of several different standards, probably four or five; for there would be no use of a balance, if one could detect the errors of the balance-weights by simply "hefting" them, and comparing them with one's memory of the standard weight. Considering that these weights are small, and were therefore used for weighing costly or even precious matter, our knowledge of the practice of weighing among the ancients gives us ground for thinking it likely that about half the weights would depart from their virtual standards by more, and about half by less, than, say, four or five tenths of one per cent, which, upon a *ket*, would be from half to two-thirds of a grain. Now the whole interval here is fourteen and one-half grains; and between 136.8 grains to 151.3 grains there is no case of an interval of more than a third of a grain not represented by any weight among the 144. To a person thoroughly familiar with the theory of errors this shows that there must be four or five different standards to which different ones aim to conform. . . . In order to represent these observations, I have adopted the following rough-and-ready

[1] *Egyptian Exploration Fund*, Third Memoir, 1886.

theory; for to make elaborate calculations would, from every point of view, be a waste of time. I have assumed that there were five different standards; that the weights depart from their standards according to the probability curve; and that the probable error of a single weight is five-eighths of a grain. I assume that of the 144 weights

36 were designed to conform to a standard of 139.2 grs.
25 were designed to conform to a standard of 142.2 grs.
26 were designed to conform to a standard of 144.7 grs.
23 were designed to conform to a standard of 146.95 grs.
34 were designed to conform to a standard of 149.7 grs.

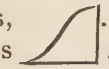
. . . I repeat that this theory has not been the subject of any but the simplest calculations. It is obvious that some such theory must be true; but to decide how near my theory probably comes to the true theory or how it ought to be modified, would be a very intricate problem for the solution of which the data are probably insufficient. It does not concern us here; our object being merely to make it clear that truly natural classes may, and undoubtedly often do merge into one another inextricably.

It is, I think, pretty certain that there were as many as five standards. Before the adoption of the metric system, every city throughout the greater part, if not all, the continent of Europe had its own pound, like its own *patois*. See the article "pound" in the *Century Dictionary*,* which was based on a list of some three hundred of such pounds whose values were known to me, a list now kept in manuscript in the Astor Library. That the same state of things must have been true in ancient Egypt may be inferred from the looseness of the tie which bound the different provinces of that empire together. Even their religions were different; so that *a fortiori* their *kets* would be so. Besides, none of the *kets* carry any authoritative mark; which is pretty conclusive proof that the central government did not intervene. It is, therefore, probable that the five standards were those of five towns with which Naucratis carried on trade. Yet virtual standards may be created in other ways. For example, where government does not insure uniformity in weights, it is usual for buyers to bring their own weights. It would thus naturally happen that some balance-

* p. 4657, edition of 1889.

weights would be manufactured for the use of buyers, and others for the use of sellers; and thus there would naturally be a tendency to the crystallization of a heavier and a lighter norm.

210. As for my assumption that the departures of the single weights from their virtual standards conform to the probability curve, it was only adopted as a ready way of imparting definiteness to the problem. Rich as is the store of data given by Petrie, it is insufficient, apparently, for determining the true law of those departures. If the workmen were sufficiently skillful (as I believe they would be) the departures would follow the probability curve. But if they were unskillful, it would be desirable to ascertain by what process the weights were made. The weights, being of stone, are not loaded; so that the adjustment was made by grinding, exclusively. Did the workman, then, have a balance by his side, or did he finish the weight by guesswork? In the latter case, inspection (and some sort of inspection there must, in this case, have been) would reject all weights outside a certain "tolerance," as it is called in coinage. Those that were too light would have to be thrown away. They would lie in a heap, until they reappeared to deceive a future archeologist. Petrie's weights, however, are somewhat heavier, not lighter, than independent evidence would lead us to believe the *ket* to have been. Those that were too heavy would be reground, but would for the most part still be rather heavier than the standard. The consequence would be that the [error] curve would be cut down vertically at two ordinates (equally distant, perhaps, from the standard), while the ordinate of its maximum would be at the right of that of the standard. If the workman had a balance at hand, and frequently used it during the process of adjustment, the form of the error-curve would depend upon the construction of the balance. If it were like a modern balance, so as to show, not only that one mass is greater than another, but also whether it is much or little greater, the workman would keep in one pan a weight of the maximum value that he proposed to himself as permissible for the weight he was making; and in all his successive grindings would be aiming at that. The consequence would be a curve  concave upwards and stopping abruptly at its maximum ordinate: a form easily

manageable by a slight modification of the method of least squares. But most of the balances shown upon the Egyptian monuments are provided with stops or other contrivances which would be needless if the balances were not top-heavy. Such balances, working automatically, are in use in all the mints of the civilized world, for throwing out light and heavy coins. Now a top-heavy balance will not show that two weights are equal, otherwise than by remaining with either end down which may be down. It only shows when, a weight being already in one pan, a decidedly heavier weight is placed in the other. The workman using such a balance would have no warning that he was approaching the limit, and would be unable to aim at any definite value, but (being, as we are supposing, devoid of skill), would have to grind away blindly, trying his weight every time he had ground off about as much as the whole range of variation which he proposed to allow himself. If he always ground off precisely the same amounts between successive tryings of his weight, he would be just as likely to grind below his maximum by any one fraction of the amount taken off at a grinding as by any other; so that his error curve would be a horizontal line cut off by vertical ordinates; thus,  . But since there would be a variability in the amount taken off between the trials, the curve would show a contrary flexure; thus,  . It must be admitted that the distribution of Petrie's *kets* is suggestive of this sort of curve, or rather of a modification of it due to a middling degree of skill.

211. I hope this long digression (which will be referred to with some interest when we come to study the theory of errors) will not have caused the reader to forget that we were engaged in tracing out some of the consequences of understanding the term "natural," or "real," class to mean a class the existence of whose members is due to a common and peculiar final cause. It is, as I was saying, a widespread error to think that a "final cause" is necessarily a purpose. A purpose is merely that form of final cause which is most familiar to our experience. The signification of the phrase "final cause" must be determined by its use in the statement of Aristotle* that all

* See *Meta.* 44 b 1 and 70 b 26.

causation divides into two grand branches, the efficient, or forceful; and the ideal, or final. If we are to conserve the truth of that statement, we must understand by final causation that mode of bringing facts about according to which a general description of result is made to come about, quite irrespective of any compulsion for it to come about in this or that particular way; although the means may be adapted to the end. The general result may be brought about at one time in one way, and at another time in another way. Final causation does not determine in what particular way it is to be brought about, but only that the result shall have a certain general character.

212. Efficient causation, on the other hand, is a compulsion determined by the particular condition of things, and is a compulsion acting to make that situation begin to change in a perfectly determinate way; and what the general character of the result may be in no way concerns the efficient causation. For example, I shoot at an eagle on the wing; and since my purpose — a special sort of final, or ideal, cause — is to hit the bird, I do not shoot directly at it, but a little ahead of it, making allowance for the change of place by the time the bullet gets to that distance. So far, it is an affair of final causation. But after the bullet leaves the rifle, the affair is turned over to the stupid efficient causation, and should the eagle make a swoop in another direction, the bullet does not swerve in the least, efficient causation having no regard whatsoever for results, but simply obeying orders blindly. It is true that the force of the bullet conforms to a law; and the law is something general. But for that very reason the law is not a force. For force is compulsion; and compulsion is *hic et nunc*. It is either that or it is no compulsion. Law, without force to carry it out, would be a court without a sheriff; and all its dicta would be vaporings. Thus, the relation of law, as a cause, to the action of force, as its effect, is final, or ideal, causation, not efficient causation. The relation is somewhat similar to that of my pulling the hair trigger of my rifle, when the cartridge explodes with a force of its own, and off goes the bullet in blind obedience to perform the special instantaneous beginning of an act that it is, each instant, compelled to commence. It is a vehicle of compulsion *hic et nunc*, receiving and transmitting it; while I receive and transmit ideal influence, of which I am a vehicle.

213. When we speak of an "idea," or "notion," or "conception of the mind," we are most usually thinking — or trying to think — of an idea abstracted from all efficiency. But a court without a sheriff, or the means of creating one, would not be a court at all; and did it ever occur to you, my reader, that an idea without efficiency is something equally absurd and unthinkable? Imagine such an idea if you can! Have you done so? Well, where did you get this idea? If it was communicated to you *viva voce* from another person, it must have had efficiency enough to get the particles of air vibrating. If you read it in a newspaper, it had set a monstrous printing press in motion. If you thought it out yourself, it had caused something to happen in your brain. And again, how do you know that you did have the idea when this discussion began a few lines above, unless it had efficiency to make some record on the brain? The court cannot be imagined without a sheriff. Final causality cannot be imagined without efficient causality; but no whit the less on that account are their modes of action polar contraries. The sheriff would still have his fist, even if there were no court; but an efficient cause, detached from a final cause in the form of a law, would not even possess efficiency: it might exert itself, and something might follow *post hoc*, but not *propter hoc;* for *propter* implies potential regularity. Now without law there is no regularity; and without the influence of ideas there is no potentiality.

214. The light of these reflections brings out into distinct view characters of our definition of a real class which we might otherwise have overlooked or misinterpreted. Every class has its definition, which is an idea; but it is not every class where the *existence*, that is, the occurrence in the universe of its members is due to the active causality of the defining idea of the class. That circumstance makes the epithet *natural* particularly appropriate to the class. The word *natura* evidently must originally have meant *birth;* although even in the oldest Latin it very seldom bears that meaning. There is, however, a certain sub-conscious memory of that meaning in many phrases; just as with words from φύσις, there is the idea of springing forth, or a more vegetable-like production, without so much reference to a progenitor. Things, it may be, φύεται spontaneously; but *nature* is an inheritance.

215. Heredity, of which so much has been said since 1860, is not a force but a law, although, like other laws, it doubtless avails itself of forces. But it is essentially that the offspring shall have a *general* resemblance to the parent, not that this general resemblance happens to result from this or that blind and particular action. No doubt, there is some blind efficient causation; but it is not that which constitutes the heredity, but, on the contrary, the general resemblance.

216. So, then, those naturalists are right who hold that the action of evolution in reproduction produces *real* classes, as by the very force of the words it produces *natural* classes. In considering the classification of sciences, however, we have no need of penetrating the mysteries of biological development; for the generation here is of ideas by ideas — unless one is to say, with many logicians, that ideas arise from the consideration of facts in which there are no such ideas, nor any ideas. That opinion is a superficial one, allied, on one side of it, to the notion that the only final cause is a purpose. So, those logicians imagine that an idea has to be connected with a brain, or has to inhere in a "soul." This is preposterous: the idea does not belong to the soul; it is the soul that belongs to the idea. The soul does for the idea just what the cellulose does for the beauty of the rose; that is to say, it affords it opportunity. It is the court-sheriff, the arm of the law.

217. I fear I may be producing the impression of talking at random. It is that I wish the reader to "catch on" to my conception, my point of view; and just as one cannot make a man see that a thing is red, or is beautiful, or is touching, by describing redness, beauty, or pathos, but can only point to something else that is red, beautiful, or pathetic, and say, "Look here too for something like that there," so if the reader has not been in the habit of conceiving ideas as I conceive them, I can only cast a sort of dragnet into his experience and hope that it may fish up some instance in which he shall have had a similar conception. Do you think, reader, that it is a positive fact that

"Truth, crushed to earth, shall rise again,"

or do you think that this, being poetry, is only a pretty fiction? Do you think that, notwithstanding the horrible wickedness of every mortal wight, the idea of right and wrong is nevertheless

the greatest power on this earth, to which every knee must sooner or later bow or be broken down; or do you think that this is another notion at which common sense should smile? Even if you are of the negative opinion, still you must acknowledge that the affirmative is intelligible. Here, then, are two instances of ideas which either have, or are believed to have, life, the power of bringing things to pass, here below. Perhaps you may object that right and wrong are only a power because there are, or will be, powerful men who are disposed to make them so; just as they might take it into their heads to make tulip-fancying, or freemasonry, or Volapük a power. But you must acknowledge that this is not the position of those on the affirmative side. On the contrary, they hold that it is the idea which will create its defenders, and render them powerful. They will say that if it be that freemasonry or its foe, the Papacy, ever pass away — as perhaps either may — it will be precisely because they are ideas devoid of inherent, incorruptible vitality, and not at all because they have been unsupplied with stalwart defenders. Thus, whether you accept the opinion or not, you must see that it is a perfectly intelligible opinion that ideas are not all mere creations of this or that mind, but on the contrary have a power of finding or creating their vehicles, and having found them, of conferring upon them the ability to transform the face of the earth.

218. If you ask what mode of being is supposed to belong to an idea that is in no mind, the reply will come that undoubtedly the idea must be embodied (or ensouled—it is all one) in order to attain complete being, and that if, at any moment, it should happen that an idea — say that of physical decency — was quite unconceived by any living being, then its mode of being (supposing that it was not altogether dead) would consist precisely in this, namely, that it was about to receive embodiment (or ensoulment) and to work in the world. This would be a mere potential being, a being *in futuro;* but it would not be the utter nothingness which would befall matter (or spirit) if it were to be deprived of the governance of ideas, and thus were to have no regularity in its action, so that throughout no fraction of a second could it steadily act in any general way. For matter would thus not only not actually exist, but it would not have even a potential existence, since potentiality

is an affair of ideas. It would be just downright Nothing.

219. It so happens that I myself believe in the eternal life of the ideas Truth and Right. I need not, however, insist upon that for my present purpose, and have only spoken of them in order to make my meaning clear. What I do insist upon is not now the infinite vitality of those particular ideas, but that every idea has in some measure, in the same sense that those are supposed to have it in unlimited measure, the power to work out physical and psychical results. They have life, generative life.

That it is so is a matter of experiential fact. But whether it is so or not is not a question to be settled by producing a microscope or telescope or any recondite observations of any kind. Its evidence stares us all in the face every hour of our lives. Nor is any ingenious reasoning needed to make it plain. If one does not see it, it is for the same reason that some men have not a sense of sin; and there is nothing for it but to be born again and become as a little child. If you do not see it, you have to look upon the world with new eyes.

220. I may be asked what I mean by the objects of [a] class *deriving their existence* from an idea. Do I mean that the idea calls new matter into existence? Certainly not. That would be pure intellectualism, which denies that blind force is an element of experience distinct from rationality, or logical force. I believe that to be a great error; but I need not stop to disprove it now, for those who entertain it will be on my side in regard to classification. But it will be urged that if that is not my meaning, then the idea merely confers upon the members of the class its character; and since every class has a defining character, any one class is as "natural" or "real" as another, if that term be taken in the sense I give to it. I cannot, however, quite admit that. Whether or not every class is or is not more or less a natural class is a question which may be worth consideration; but I do not think that the relation of the idea to the members of the natural class is simply that it is applicable to them as a predicate, as it is to every class equally. What I mean by the idea's conferring existence upon the individual members of the class is that it confers upon them the power of working out results in this world, that it confers upon them, that is to say, organic existence, or, in one word, life. The existence of an individual man is a totally different thing from

the existence of the matter which at any given instant happens to compose him, and which is incessantly passing in and out. A man is a wave, but not a vortex. Even the existence of the vortex, though it does happen to contain, while it lasts, always the same particles, is a very different thing from the existence of these particles. Neither does the existence of wave or vortex consist merely in the fact that something is true of whatever particles compose them; although it is inseparably bound up with that fact. Let me not be understood as proposing any new definitions of a vortex and a wave. What I mean is this. Take a corpse: dissect it, more perfectly than it ever was dissected. Take out the whole system of blood vessels entire, as we see them figured in the books. Treat the whole systems of spinal and sympathetic nerves, the alimentary canal with its adjuvants, the muscular system, the osseous system, in the same way. Hang these all in a cabinet so that from a certain point of view each appears superposed over the others in its proper place. That would be a singularly instructive specimen. But to call it a man would be what nobody would for an instant do or dream. Now the best definition that ever was framed is, at best, but a similar dissection. It will not really work in the world as the object defined will. It will enable us to see how the thing works, in so far as it shows the efficient causation. The final causation, which is what characterizes the *definitum*, it leaves out of account. We make smoke rings. We make one pass through another, and perform various experiments, which give us an imperfect idea, yet some idea, of what a vortex really is. *How* all these things happen can be traced out from the definition. But the *rôle* that vortices really play in the universe — no insignificant one, if all matter is built of them — the real life of them, depends upon the idea of them, which simply finds its opportunity in those circumstances that are enumerated in the definition. Efficient causation is that kind of causation whereby the parts compose the whole; final causation is that kind of causation whereby the whole calls out its parts. Final causation without efficient causation is helpless; mere calling for parts is what a Hotspur, or any man, may do; but they will not come without efficient causation. Efficient causation without final causation, however, is worse than helpless, by far; it is mere chaos; and

chaos is not even so much as chaos, without final causation; it is blank nothing.

221. The writer of a book can do nothing but set down the items of his thought. For the living thought, itself, in its entirety, the reader has to dig into his own soul. I think I have done my part, as well as I can. I am sorry to have left the reader an irksome chore before him. But he will find it worth the doing.

222. So then, a natural class being a family whose members are the sole offspring and vehicles of one idea, from which they derive their peculiar faculty, to classify by abstract definitions is simply a sure means of avoiding a natural classification. I am not decrying definitions. I have a lively sense of their great value in science. I only say that it should not be by means of definitions that one should seek to find natural classes. When the classes have been found, then it is proper to try to define them; and one may even, with great caution and reserve, allow the definitions to lead us to turn back and see whether our classes ought not to have their boundaries differently drawn. After all, boundary lines in some cases can only be artificial, although the classes are natural, as we saw in the case of the *kets*. When one can lay one's finger upon the purpose to which a class of things owes its origin, then indeed abstract definition may formulate that purpose. But when one cannot do that, but one can trace the genesis of a class and ascertain how several have been derived by different lines of descent from one less specialized form, this is the best route toward an understanding of what the natural classes are. This is true even in biology; it is much more clearly so when the objects generated are, like sciences, themselves of the nature of ideas.

223. There are cases where we are quite in the dark, alike concerning the creating purpose and concerning the genesis of things; but [there are cases] where we find a system of classes connected with a system of abstract ideas — most frequently numbers — and that in such a manner as to give us reason to guess that those ideas in some way, usually obscure, determine the possibilities of the things. For example, chemical compounds, generally — or at least the more decidedly characterized of them, including, it would seem, the so-called elements— seem to belong to types, so that, to take a single example,

chlorates $KClO_3$, manganates* $KMnO_3$, bromates $KBrO_3$, rutheniates† $KRuO_3$, iodates KIO_3, behave chemically in strikingly analogous ways. That this sort of argument for the existence of natural classes — I mean the argument drawn from types, that is, from a connection between the things and a system of formal ideas — may be much stronger and more direct than one might expect to find it, is shown by the circumstance that ideas themselves — and are they not the easiest of all things to classify naturally, with assured truth?— can be classified on no other grounds than this, except in a few exceptional cases. Even in these few cases, this method would seem to be the safest. For example, in pure mathematics, almost all the classification reposes on the relations of the forms classified to numbers or other multitudes. Thus, in topical geometry, figures are classified according to the whole numbers attached to their *choresis*, *cyclosis*, *periphraxis*, *apeiresis*, etc. As for the exceptions, such as the classes of hessians, jacobians, invariants, vectors, etc., they all depend upon types, too, although upon types of a different kind. It is plain that it must be so; and all the natural classes of logic will be found to have the same character.

§2. NATURAL CLASSIFICATIONS

224. There are two remarks more about natural classification which, though they are commonplace enough, cannot decently be passed by without recognition. They have both just been virtually said, but they had better be more explicitly expressed and put in a light in which their bearing upon the practice of classification shall be plain. The descriptive definition of a natural class, according to what I have been saying, is not the essence of it. It is only an enumeration of tests by which the class may be recognized in any one of its members. A description of a natural class must be founded upon samples of it or typical examples. Possibly a zoölogist or a botanist may have so definite a conception of what a species is that a single type-specimen may enable him to say whether a form of which he finds a specimen belongs to the same species or not. But it will be much safer to have a large number of individual

* K_2MnO_4.

† K_2RuO_4.

specimens before him, from which he may get an idea of the amount and kind of individual or geographical variation to which the given species is subject. In proportion as the category of the class is higher, the greater will be the need of a multiplicity of examples. True, a naturalist may be so familiar with what a genus is, what a family is, what an order is, what a class is, that if you were to show him a new specimen of a hitherto unknown class, he could, with that single specimen before him, sit down and write out definitions, not only of its class, but also of its order, of its family, and of its genus, as well as of its species. Such a feat would display marvellous familiarity with what those categories [mean] in botany and in zoology; but intellectually it would be a performance of no high order, and the less so the greater the certainty of the conclusion. Generalization broad, luminous, and solid must enter into an intellectual performance in order to command much admiration. Such generalization, which teaches a new and clear lesson upon the truth of which reliance can be placed, requires to be drawn from many specimens. We shall endeavor, in that way, to define each class, that is to enumerate characters which are absolutely decisive as to whether a given individual does or does not belong to the class. But it may be, as our *kets* show, that this is altogether out of the question; and the fact that two classes merge is no proof that they are not truly distinct natural classes.

225. For they may, nevertheless, be genealogically distinct, just as no degree of resemblance between two men is proof positive that they are brothers. Now genealogical classification, among those objects of which the genesis is genealogical, is the classification we can most certainly rely upon as being natural. No harm will be done if, in those cases, we *define* the natural classification as the genealogical classification; or, at least, [if] we make the genealogical character one of the essential characters of a natural classification. It can not be more; because if we had before us, ranged in ancestral order, all the intermediate forms through which the human stock has passed in developing from non-man into man, it is plain that other considerations would be necessary in determining (if it admitted of determination) at what point in the series the forms begin to merit the name of human.

226. The sciences are, in part, produced each from others. Thus, spectroscopic astronomy has for its parents, astronomy, chemistry, and optics. But this is not the whole genesis nor the principal part of the genesis of any broad and definite science. It has its own peculiar problem springing from an idea. That geometry derived its birth from land surveying is the tradition, which is borne out by the tradition that it took its origin in Egypt where the yearly floods must have rendered accurate surveying of special importance. Moreover, the wonderful accuracy of the dimensions of the great pyramid exhibit a degree of skill in laying out ground which could only have been attained by great intellectual activity; and this activity could hardly fail to lead to some beginnings of geometry. We may, therefore, accept with considerable confidence the tradition involved in the very name of geometry. Speaking in a broad, rough way, it may be said that the sciences have grown out of the useful arts, or out of arts supposed to be useful. Astronomy out of astrology; physiology, taking medicine as a halfway out of magic; chemistry out of alchemy; thermotics from the steam-engine, etc. Among the theoretical sciences, while some of the most abstract have sprung straight from the concretest arts, there is nevertheless a well-marked tendency for a science to be first descriptive, later classificatory, and lastly to embrace all classes in one law. The classificatory stage may be skipped. Yet in the truer order of development, the generation proceeds quite in the other direction. Men may and do begin to study the different kinds of animals and plants before they know anything of the general laws of physiology. But they cannot attain any true understanding of taxonomic biology until they can be guided by the discoveries of the physiologists. Till then the study of mollusks will be nothing but conchology. On the other hand the physiologist may be aided by a fact or two here and there drawn from taxonomic biology; but he asks but little and that little not very urgently of anything that the taxonomist can tell him and that he could not find out for himself.

227. All natural classification is then essentially, we may almost say, an attempt to find out the true genesis of the objects classified. But by genesis must be understood, not the efficient action which produces the whole by producing the

parts, but the final action which produces the parts because they are needed to make the whole. Genesis is production from ideas. It may be difficult to understand how this is true in the biological world, though there is proof enough that it is so. But in regard to science it is a proposition easily enough intelligible. A science is defined by its problem; and its problem is clearly formulated on the basis of abstracter science. This is all I intended to say here concerning classification, in general.

228. Having found the natural classes of the objects to be classified, we shall then use the same methods — probably, in most cases, the third — in order to discover the natural classes of those classes that we have found. Is this the whole business of classification? No serious student can hold it to be so. The classes found have to be defined, naturally if possible, but if not, then at least conveniently for the purposes of science. They have not only to be defined but described, a story without an end. This applies, of course, not merely to the species or immediate classes of the objects described, but to the higher orders of classes. There may also be between the different classes relations, each of which appertains just as much to the description of any one of the set of classes to which it belongs as to any other.

229. In regard to the higher orders of classes, so far as concerns animals, Louis Agassiz* thought that he was able to characterize in general terms the different categories of classes which zoölogists talk of. That is, he undertook to say what sort of characters distinguish *branches* from branches, *classes* from classes, *orders* from orders, *families* from families, *genera* from genera, and *species* from species. His general classification of animals has passed away; and few naturalists attach much importance to his characterizations of the categories. Yet they are the outcome of deep study, and it is a merit of them that they involve no attempt at hard abstract accuracy of statement. How can he have been so long immersed in the study of nature without some truth sticking to him? I will just set down his vague definitions and allow myself to be vaguely influenced by them, so far as I find anything in the facts that answers to his descriptions. Although I am an ignoramus in biology, I ought by this time to recognize meta-

* In the *Essay on Classification.*

physics when I meet with it; and it is apparent to me that those biologists whose views of classification are most opposite to those of Agassiz are saturated with metaphysics in its dangerous form — *i.e.* the unconscious form — to such an extent that what they say upon this subject is rather the expression of a traditionally absorbed fourteenth century metaphysics than of scientific observation.

230. It would be useless for our purpose to copy the definitions of Agassiz[1] had he not expressed them in the briefest terms, as follows:

Branches are characterized by the plan of structure;

Classes, by the manner in which that plan is executed, as far as ways and means are concerned; ("Structure is the watchword for the recognition of classes." Page 145.)

Orders, by the degrees of complication of that structure; ("The leading idea . . . is that of a definite rank among them." Page 151.)

Families, by their form, as determined by structure; ("When we see new animals, does not the first glance, that is, the first impression made upon us by their form, give us a very correct idea of their nearest relationship? . . . So form is characteristic of families; . . . I do not mean the mere outline, but form as determined by structure." Pages 159, 160.)

Genera, by the details of the execution in special parts;

Species, by the relations of individuals to one another and to the world in which they live, as well as by the proportions of their parts, their ornamentation, etc.

231. All classification, whether artificial or natural, is the arrangement of objects according to ideas. A natural classification is the arrangement of them according to those ideas from which their existence results. No greater merit can a taxonomist have than that of having his eyes open to the ideas in nature; no more deplorable blindness can afflict him than that of not seeing that there are ideas in nature which determine the existence of objects. The definitions of Agassiz will, at least, do us the service of directing our attention to the supreme importance of bearing in mind the final cause of objects in finding out their own natural classifications.

[1] Ib., quarto 1857, p. 170. The reader will perceive by the date that these ideas were put forth at a somewhat inauspicious moment.

§3. THE ESSENCE OF SCIENCE

232. So much in regard to classification. Now if we are to classify the sciences, it is highly desirable that we should begin with a definite notion of what we mean by a science; and in view of what has been said of natural classification, it is plainly important that our notion of science should be a notion of science as it lives and not a mere abstract definition. Let us remember that science is a pursuit of living men, and that its most marked characteristic is that when it is genuine, it is in an incessant state of metabolism and growth. If we resort to a dictionary, we shall be told that it is systematized knowledge. Most of the classifications of the sciences have been classifications of systematized and established knowledge — which is nothing but the exudation of living science;— as if plants were to be classified according to the characters of their gums. Some of the classifications do even worse than that, by taking science in the sense attached by the ancient Greeks, especially Aristotle, to the word ἐπιστήμη. A person can take no right view of the relation of ancient to modern science unless he clearly apprehends the difference between what the Greeks meant by ἐπιστήμη and what we mean by knowledge. The best translation of ἐπιστήμη is "comprehension." It is the ability to define a thing in such a manner that all its properties shall be corollaries from its definition. Now it may be that we shall ultimately be able to do that, say for light or electricity. On the other hand, it may equally turn out that it forever remains as impossible as it certainly is to define number in such a way that Fermat's and Wilson's theorems should be simple corollaries from the definition. I do not mean to deny that those theorems are deducible from the definition. All that is here being urged turns on the falsity of the old notion that all deduction is corollarial deduction. But, at any rate, the Greek conception of knowledge was all wrong in that they thought that one must advance in direct attack upon this ἐπιστήμη; and attached little value to any knowledge that did not manifestly tend to that. To look upon science in that point of view in one's classification is to throw modern science into confusion.

233. Another fault of many classifications — or if not a fault, it is at least a purpose very different from that which I

should be bold enough to attempt — is that they are classifications not of science as it exists, but of systematized knowledge such as the classifier hopes may some time exist. I do not believe it is possible to have that intimate acquaintance with the science of the indefinite future that the discovery of the real and natural classification of it would require. At any rate, I will make no such attempt, except in one department, and there only partially and timidly.

234. Let us look upon science — the science of today — as a living thing. What characterizes it generally, from this point of view, is that the thoroughly established truths are labelled and put upon the shelves of each scientist's mind, where they can be at hand when there is occasion to use things — arranged, therefore, to suit his special convenience — while science itself, the living process, is busied mainly with conjectures, which are either getting framed or getting tested. When that systematized knowledge on the shelves is used, it is used almost exactly as a manufacturer or practising physician might use it; that is to say, it is merely applied. If it ever becomes the object of science, it is because in the advance of science, the moment has come when it must undergo a process of purification or of transformation.

235. A scientific man is likely in the course of a long life to pick up a pretty extensive acquaintance with the results of science; but in many branches, this is so little necessary that one will meet with men of the most deserved renown in science who will tell you that, beyond their own little nooks, they hardly know anything of what others have done. Sylvester always used to say that he knew very little mathematics: true, he seemed to know more than he thought he did. In various branches of science, some of the most eminent men first took up those subjects as mere pastimes, knowing little or nothing of the accumulations of knowledge. So it was with the astronomer Lockyer: so it has been with many naturalists. Now, did those men gradually become men of science as their stores of knowledge increased, or was there an epoch in their lives, before which they were amateurs and after which they were scientists? I believe that the answer is that, like any other regeneration, the metamorphosis is commonly sudden, though sometimes slow. When it is sudden, what is it that constitutes

the transformation? It is their being seized with a great desire to learn the truth, and their going to work with all their might by a well-considered method to gratify that desire. The man who is working in the right way to learn something not already known is recognized by all men of science as one of themselves, no matter how little he is informed. It would be monstrous to say that Ptolemy, Archimedes, Eratosthenes and Posidonius were not scientific men because their knowledge was comparatively small. The life of science is in the desire to learn. If this desire is not pure, but is mingled with a desire to prove the truth of a definite opinion, or of a general mode of conceiving of things, it will almost inevitably lead to the adoption of a faulty method; and *in so far* such men, among whom many have been looked upon in their day as great lights, are not genuine men of science; though it would be foul injustice to exclude them absolutely from that class. So if a man pursues a futile method through neglect to inform himself of effective methods, he is no scientific man; he has not been moved by an intelligently sincere and effective desire to learn. But if a man simply fails to inform himself of previous work which would have facilitated his own, although he is to blame, it would be too harsh to say that he has violated the essential principles of science. If a man pursues a method which, though very bad, is the best that the state of intellectual development of his time, or the state of the particular science he pursues, would enable a man to take — I mean, for example, such men as Lavater, Paracelsus and the earlier alchemists, the author of the first chapter of Genesis, and the old metaphysicians — we perhaps cannot call them scientific men, while perhaps we ought to do so. Opinions would differ about this. They are, at any rate, entitled to an honorable place in the vestibule of science. A pretty wild play of the imagination is, it cannot be doubted, an inevitable and probably even a useful prelude to science proper. For my part, if these men really had an effective rage to learn the very truth, and did what they did as the best way they knew, or could know, to find it out, I could not bring myself to deny them the title. The difficulty is that one of the things that coheres to that undeveloped state of intelligence is precisely a very imperfect and impure thirst for truth. Paracelsus and the alchemists were rank charlatans seeking for gold

more than for truth. The metaphysicians were not only pedants and pretenders, but they were trying to establish foregone conclusions. These are the traits which deprive those men of the title scientist, although we ought to entertain a high respect for them as mortals go; because they could no more escape the corruptness of their aims than they could the deficiencies of their knowledge. Science consists in actually drawing the bow upon truth with intentness in the eye, with energy in the arm.

236. Such being the essence of science, it is obvious that its first offspring will be men — men whose whole lives are devoted to it. By such devotion each of them acquires a training in making some particular kind of observations and experiments. (Unfortunately, his acquisition of books, instruments, laboratory, etc., depends upon qualifications in which the man of science is usually rather wanting — as wealth, diplomacy, popularity as a teacher — so that he is less likely to be provided with them than are men less qualified to use them for the advancement of science.) He will thus live in quite a different world — quite a different aggregate of experience — from unscientific men and even from scientific men pursuing other lines of work than his. He naturally converses with and reads the writings of those who, having the same experience, have ideas interpretable into his own. This society develops conceptions of its own. Bring together two men from widely different departments — say a bacteriologist and astronomer — and they will hardly know what to say to one another; for neither has seen the world in which the other lives. True, both use optical instruments; but the qualities striven for in a telescopic objective are of no consequence in a microscopical objective; and all the subsidiary parts of telescope and microscope are constructed on principles utterly foreign to one another — except their stiffness.

237. Here, then, are natural classes of sciences all sorted out for us in nature itself, so long as we limit our classification to actually recognized sciences. We have only to look over the list of scientific periodicals and the list of scientific societies to find the families of science, ready named. I call such classes families because Agassiz tells us that it is the family which strikes the observer at first glance. To make out the genera

and especially the species, closer examination is requisite; while the knowledge of orders, classes, and branches calls for a broader acquaintance with science.

§4. THE DIVISIONS OF SCIENCE

238. The first great division of science will be according to its fundamental purpose, making what I shall term *branches* of science. A modification of a general purpose may constitute a *subbranch*. All knowledge whatever comes from observation; but different sciences are observational in such radically different ways that the kind of information derived from the observation of one department of science (say natural history) could not possibly afford the information required of observation by another branch (say mathematics). I call groups based on such considerations *classes*, and modifications of the same nature *subclasses*. Observation is, in Agassiz's phrase, the "ways and means" of attaining the purpose of science. Of two departments of science A and B, of the same class, A may derive special facts from B for further generalization while supplying B with principles which the latter, not aiming so high, is glad to find ready-made. A will rank higher than B, by virtue of the greater generality of its object, while B will be richer and more varied than A. I call groups based on these considerations *orders*, or if based on modifications of the same sort of idea, *suborders*. A given science with a special name, a special journal, a special society, studying one group of facts, whose students understand one another in a general way and naturally associate together, forms what I call a *family*. A subdivision of it on the same principle, but taken more minutely, I term a *subfamily*. I can give no such definitions of genera and species, not having carried my classification of the sciences to these minutiæ. For it is to be understood that I have not first fixed my definitions of *branch*, *class*, *order*, and *family*, and then adapted the classification to those definitions, but, on the contrary, the classification was first entirely formed (except that the categories of subbranches, subclasses, and suborders had, in some cases, not been interposed, and in others had been confounded with the classes above them) before any idea of employing the terms *branch*, *class*, *order*, and *family* entered my head, and it was not until this was done that first the appropri-

ateness of these terms struck me. I can, however, say with some confidence that I should not regard a family as constituted merely by the class of facts studied, were there no concomitant difference of procedure, giving an all-round peculiar character to the study of that subject; nor do I believe that a mere difference in the things studied could appear to me a sufficient foundation for a difference between genera. Since writing that sentence, I notice that I have made inorganic and organic chemistry *sub*genera. But, then, everybody knows that there is far more difference between inorganic and organic chemistry than that the latter studies the compounds of a somewhat peculiar element. Their whole aims and ways of thinking, as well as their manipulation, are in distinct contrast.

239. I recognize two branches of science: Theoretical, whose purpose is simply and solely knowledge of God's truth; and Practical, for the uses of life. In Branch I, I recognize two subbranches, of which, at present, I consider only the first, [the sciences of discovery]. Among the theoretical sciences [of discovery], I distinguish three classes, all resting upon observation, but being observational in very different senses.[1]

240. The first is mathematics, which does not undertake to ascertain any matter of fact whatever, but merely posits hypotheses, and traces out their consequences. It is observational, in so far as it makes constructions in the imagination according to abstract precepts, and then observes these imaginary objects, finding in them relations of parts not specified in the precept of construction. This is truly observation, yet certainly in a very peculiar sense; and no other kind of observation would at all answer the purpose of mathematics.[2]

[1] Some catholic writers recognize sciences resting upon authority. No doubt, everybody of good sense believes some things substantially because he has been brought up to do so; but according to my conception of what science is, *that* is not science. Indeed, belief proper has nothing to do with science. [Baldassare] Lablanca [*Dialettica*, vol. II, lib. IV, c. 1, 1875] admits a class of documentary sciences. This is more plausible; although, as that author admits, documentary evidence enters into every science, while nothing can have rested wholly on documentary evidence *to the original authors of the documents*. He reckons as documentary sciences, history, linguistics, political economy, statistics, and geography. But it is quite plain that these do not form a natural group; especially since this geography must include physical geography.

[2] Many writers of France (as Comte and Ribot), and of Germany (as Schopenhauer and Wundt), and a few in England (as Cave), have given mathematics

241. Class II is philosophy, which deals with positive truth, indeed, yet contents itself with observations such as come within the range of every man's normal experience, and for the most part in every waking hour of his life. Hence Bentham calls this class, *cœnoscopic*.* These observations escape the untrained eye precisely because they permeate our whole lives, just as a man who never takes off his blue spectacles soon ceases to see the blue tinge. Evidently, therefore, no microscope or sensitive film would be of the least use in this class. The observation is observation in a peculiar, yet perfectly legitimate, sense. If philosophy glances now and then at the results of special sciences, it is only as a sort of condiment to excite its own proper observation.

242. Class III is Bentham's *idioscopic*†; that is, the special sciences, depending upon special observation, which travel or other exploration, or some assistance to the senses, either instrumental or given by training, together with unusual diligence, has put within the power of its students. This class manifestly divides itself into two subclasses, the physical and the psychical sciences; or, as I will call them, physiognosy and psychognosy. Under the former is to be included physics, chemistry, biology, astronomy, geognosy, and whatever may be like these sciences; under the latter, psychology, linguistics, ethnology, sociology, history, etc. Physiognosy sets forth the workings of efficient causation, psychognosy of final causation. But the two things call for different eyes. A man will be no

the first place among the sciences, contrary to the doctrine of Plato and Aristotle, which has caused so many to place it below philosophy in point of abstractness. I mention this to show that I am taking no revolutionary position here: I am open to charges enough of heresy to answer to, to make me desire to avoid those that can be avoided.

* "*Cœnoscopic* . . . from two Greek words, one of which signifies *common* — things belonging to others in common; the other *looking to*. By *cœnoscopic ontology*, then, is designated that part of the science which takes for its subject those properties which are considered as possessed in common by all the individuals belonging to the class which the name *ontology* is employed to designate, *i.e.* by *all individuals*." *The Works of Jeremy Bentham*, Edinburgh, 1843, viii, 83, footnote.

† "*Idioscopic* . . . from two Greek words, the first of which signifies *peculiar*. In *Idioscopic ontology*, then, we have that branch of art and science which takes for its subject such properties as are considered as peculiar to different classes of beings, some to one such class, some to another." *Ibid.*

whit the worse physiognosist for being utterly blind to facts of mind; and if we sometimes find observation in a psychognosist, it will, unless by exception, be found not to be of a purely physical fact. Thus, a philologist may have a fine ear for language-sounds; but it is by no means pure physical resemblance which determines whether a given sound is or is not "the" Italian close *o*, for example, as it is naïvely called: it is psychical habit. In any simple physical sense the sounds not distinguished from that differ much more from one another than almost any of them do from sounds which would not be tolerated for "the" close *o*. So, this fine phonetic observation of the linguist is a knack of understanding a virtual convention. The two kinds of observation are different; but they do not seem to be quite so different as both alike are from the observation of the philosopher and the mathematician; and this is why, though I, at first, was inclined to give each of them equal rank with those classes, it has at length appeared certain that they should be placed a little lower.

243. I still persist in leaving unnoticed a certain subbranch of theoretical science [the sciences of review]; and as for the practical sciences,[1] I shall merely mention a few of them, just to give an idea of what I refer to under that name. I mean, then, all such well-recognized sciences now *in actu*, as pedagogics, gold-beating, etiquette, pigeon-fancying, vulgar arithmetic, horology, surveying, navigation, telegraphy, printing, bookbinding, paper-making, deciphering, ink-making, librarian's work, engraving, etc.* In short, this is by far the more various of the two branches of science. I must confess to being utterly bewildered by its motley crowd, but fortunately the natural classification of this branch will not concern us in logic — at least, will not do so as far as I can perceive.

244. Now let us consider the relations of the classes of science to one another. We have already remarked that rela-

[1] Some writers so little comprehend the motives of science that they imagine all sciences are directed to practical ends, as was the general understanding before the Nineteenth Century. Thus [Luigi] Ferrarese in 1828 [in his *Saggio di una nuova classificazione delle scienze*] divided all sciences into three groups according as their purpose is to maintain health, to further perfectionment, or to prevent degradation; the first and third being separated by a thin partition indeed.

* The editors have abbreviated a very long list.

tions of generation must always be of the highest concern to natural classification, which is, in fact, no more nor less than an account of the existential, or *natural*, birth concerning relations of things; meaning by birth the relations of a thing to its originating final causes.

245. Beginning with Class I, mathematics meddles with every other science without exception. There is no science whatever to which is not attached an application of mathematics. This is not true of any other science, since pure mathematics has not, as a part of it, any application of any other science, inasmuch as every other science is limited to finding out what is positively true, either as an individual fact, as a class, or as a law; while pure mathematics has no interest in whether a proposition is existentially true or not. In particular, mathematics has such a close intimacy with one of the classes of philosophy, that is, with logic, that no small acumen is required to find the joint between them.

246. Next, passing to Class II, philosophy, whose business it is to find out all that can be found out from those universal experiences which confront every man in every waking hour of his life, must necessarily have its application in every other science. For be this science of philosophy that is founded on those universal phenomena as small as you please, as long as it amounts to anything at all, it is evident that every special science ought to take that little into account before it begins work with its microscope, or telescope, or whatever special means of ascertaining truth it may be provided with.

247. It might, indeed, very easily be supposed that even pure mathematics itself would have need of one department of philosophy; that is to say, of logic. Yet a little reflection would show, what the history of science confirms, that that is not true. Logic will, indeed, like every other science, have its mathematical parts. There will be a mathematical logic just as there is a mathematical physics and a mathematical economics. If there is any part of logic of which mathematics stands in need — logic being a science of fact and mathematics only a science of the consequences of hypotheses — it can only be that very part of logic which consists merely in an application of mathematics, so that the appeal will be, not

of mathematics to a prior science of logic, but of mathematics to mathematics. Let us look at the rationale of this a little more closely. Mathematics is engaged solely in tracing out the consequences of hypotheses. As such, she never at all considers whether or not anything be existentially true, or not. But now suppose that mathematics strikes upon a snag; and that one mathematician says that it is evident that a consequence follows from a hypothesis, while another mathematician says it evidently does not. Here, then, the mathematicians find themselves suddenly abutting against brute fact; for certainly a dispute is not a rational consequence of anything. True, this fact, this dispute, is no part of mathematics. Yet it would seem to give occasion for an appeal to logic, which is generally a science of fact, being a science of truth; and whether or not there be any such thing as truth is a question of fact. However, because this dispute relates merely to the consequence of a hypothesis, the mere careful study of the hypothesis, which is pure mathematics, resolves it; and after all, it turns out that there was no occasion for the intervention of a science of reasoning.

248. It is often said that the truths of mathematics are infallible. So they are, if you mean practical infallibility, infallibility such as that of conscience. They appear even as theoretically infallible, if they are viewed through spectacles that cut off the rays of blunder. I never yet met with boy or man whose addition of a long column, of fifty to a hundred lines, was absolutely infallible, so that adding it a second time could in no degree increase one's confidence in the result, nor ought to do so. The addition of that column is, however, merely a repetition of $1+1=2$; so that, however improbable it may be, there is a certain finite probability that everybody who has ever performed this addition of 1 and 1 has blundered, except on those very occasions on which we are accustomed to suppose (on grounds of probability merely) that they *did* blunder. Looked at in this light, every mathematical inference is merely a matter of probability. At any rate, in the sense in which anything in mathematics is certain, it is most certain that the whole mathematical world has often fallen into error, and that, in some cases, such errors have stood undetected for a couple of millennia. But no case is adducible in which

the science of logic has availed to set mathematicians right or to save them from tripping. On the contrary, attention once having been called to a supposed inferential blunder in mathematics, short time has ever elapsed before the whole mathematical world has been in accord, either that the step was correct, or else that it was fallacious; and this without appeal to logic, but merely by the careful review of the mathematics as such. Thus, historically mathematics does not, as *a priori* it cannot, stand in need of any separate science of reasoning.

249. But mathematics is the only science which can be said to stand in no need of philosophy, excepting, of course, some branches of philosophy itself. It so happens that at this very moment the dependence of physics upon philosophy is illustrated by several questions now on the *tapis*. The question of non-Euclidean geometry may be said to be closed. It is apparent now that geometry, while in its main outlines, it must ever remain within the borders of philosophy, since it depends and must depend upon the scrutinizing of everyday experience, yet at certain special points it stretches over into the domain of physics. Thus, space, as far as we can see, has three dimensions; but are we quite sure that the corpuscles into which atoms are now minced have not room enough to wiggle a little in a fourth? Is physical space hyperbolic, that is, infinite and limited, or is it elliptic, that is, finite and unlimited? Only the exactest measurements upon the stars can decide. Yet even with them the question cannot be answered without recourse to philosophy. But a question at this moment under consideration by physicists is whether matter consists ultimately of minute solids, or whether it consists merely of vortices of an ultimate fluid. The third possibility, which there seems to be reason to suspect is the true one, that it may consist of vortices in a fluid which itself consists of far minuter solids, these, however, being themselves vortices of a fluid, itself consisting of ultimate solids, and so on in endless alternation, has hardly been broached. The question as it stands must evidently depend upon what we ought to conclude from everyday, unspecialized observations, and particularly upon a question of logic. Another still warmer controversy is whether or not it is proper to endeavor to find a mechanical explanation of electricity, or whether it is proper, on the contrary, to leave the

differential equations of electrodynamics as the last word of science. This is manifestly only to be decided by a scientific philosophy very different from the amateurish, superficial stuff in which the contestants are now entangling themselves. A third pretty well defended opinion, by the way, is that instead of explaining electricity by molar dynamics, molar dynamics ought to be explained as a special consequence of the laws of electricity. Another appeal to philosophy was not long ago virtually made by the eminent electrician, the lamented Hertz, who wished to explain force, in general, as a consequence of unseen constraints. Philosophy alone can pronounce for or against such a theory. I will not undertake to anticipate questions which have not yet emerged; otherwise, I might suggest that chemists must ere long be making appeal to philosophy to decide whether compounds are held together by force or by some other agency. In biology, besides the old logico-metaphysical dispute about the reality of classifications, the momentous question of evolution has unmistakable dependence on philosophy. Then again, caryocinesis has emboldened some naturalists, having certain philosophical leanings, to rebel against the empire of experimental physiology. The origin of life is another topic where philosophy asserts itself; and with this I close my list, not at all because I have mentioned all the points at which just now the physical sciences are influenced by a philosophy, such as it is, but simply because I have mentioned enough of them for my present purpose.

250. The dependence of the psychical sciences upon philosophy is no less manifest. A few years ago, indeed, regenerate psychology, in the flush of her first success, not very wisely proposed to do without metaphysics; but I think that today psychologists generally perceive the impossibility of such a thing. It is true that the psychical sciences are not quite so dependent upon metaphysics as are the physical sciences; but, by way of compensation, they must lean more upon logic. The mind works by final causation, and final causation is logical causation. Note, for example, the intimate bearing of logic upon grammatical syntax. Moreover, everything in the psychical sciences is inferential. Not the smallest fact about the mind can be directly perceived as psychical. An emotion is directly felt as a bodily state, or else it is only known inferen-

tially. That a thing is agreeable appears to direct observation as a character of an object, and it is only by inference that it is referred to the mind. If this statement be disputed (and some will dispute it), all the more need is there for the intervention of logic. Very difficult problems of inference are continually emerging in the psychical sciences. In psychology, there are such questions as free-will and innate ideas; in linguistics, there is the question of the origin of language, which must be settled before linguistics takes its final form. The whole business of deriving ancient history from documents that are always insufficient and, even when not conflicting, frequently pretty obviously false, must be carried on under the supervision of logic, or else be badly done.

251. The influence of philosophy upon the practical sciences is less direct. It is only here and there that it can be detected; and ethics is the division of philosophy which most concerns these sciences. Ethics is courteously invited to make a suggestion now and then in law, jurisprudence, and sociology. Its sedulous exclusion from diplomacy and economics is immense folly. We are unhappily debarred from calling this folly stupendous or egregious, because it is merely the ordinary blindness of those who profoundly believe that lies are the most wholesome of diet, who, as Edgar Poe sagaciously said, when they get home, have once locked themselves in their several chambers, have undressed, knelt down by the bedside and said their prayers, got into bed, and blown out the candle, then, at length, and not till then, indulge in one veracious wink — the only veracious act of the day — and lull themselves to sleep with an inward ditty that Right is a silly thing without wealth or vigor in this work-a-day world. One day man shall start up out of his slumber to see by broad daylight that that despised idea has all along been the one irresistible power. Then may begin an era when it is counted within the practical sciences, one and all — when, in a word, a man will not design a stove nor order a coat without stopping first and sifting out his real desire — and it is prophecy as simple as *Barbara*, that, when that comes to pass, those sciences will answer even their lower and nearest purposes far more perfectly than at present they do. So, at any rate, the student of minute logic will be forced to think.

252. The direct action of the special psychical and physical sciences — the two subclasses of Class III — upon one another seems to be slight. One cannot see how, except in some accidental or exterior way, the psychical sciences are to influence the physical sciences, unless it should be found advantageous to call upon psychology to aid the physical observer in avoiding illusions and in diminishing his errors of observation. This, no doubt, deserves careful consideration; but I believe that, if the proper distinctions are drawn, it will be seen that as for illusions, the far better way, when it is practicable, as it almost always will be, will be to make the observations so simple and positive that no illusions can occur often enough to make any special dealing desirable; and as for errors of observation, it is best to treat them as residual phenomena like any other residual phenomena. That they are entirely physical every physicist must insist, physics being sufficiently advanced to see that all phenomena, without exception, are physical, for the purposes of physics. Soon we may hope that all psychologists, on their side, may be equally at one that all phenomena without exception are purely psychical for the purposes of psychics.

253. How far then are the psychical sciences influenced by physiognosy, or how far ought they to be so influenced? The theory of psychophysical parallelism would seem to imply that there is and can be no influence at all. But I must confess myself to be of the party which thinks that no psychical fact, as such, can be observed. The direct percept, as it first appears, appears as forced upon us brutally. It has no generality; and without generality there can be no psychicality. Physicality consists in being under the governance of physical, *i.e.*, efficient, causes, psychicality in being under the governance of psychical, *i.e.*, of final, causes. The percept brutally forces itself upon us; thus it appears under a physical guise. It is quite ungeneral, even antigeneral — in its character as percept; and thus it does not appear as psychical. The psychical, then, is not contained in the percept.

254. "But what?" some one will say, "Does one not perceive redness? And is not redness purely a psychical matter to which nothing in the physical world corresponds?" If one must answer *yes* or *no*, in the rough, of course one must say *yes*. Yet as to there being nothing in the physical universe that corre-

sponds to a given psychical phenomenon, the doctrine of parallelism itself disavows that opinion. Better let us say that in the present state of physical theory the peculiarity of redness finds no definite explanation. It would be an illogical presumption to say that it never can be explained. Redness, though a sensation, does not in the percept proclaim itself as such. At any rate, whether the psychical can be directly observed or not, no linguist, ethnologist, nor historian — no psychologist, even, in an unguarded moment — but will agree that his science rests very largely, if not quite entirely, upon physical *facts*.

255. This does not amount to an acknowledgment of need of help from the physical *sciences*. Some amount of such need and such help there is. It is easier detected than the dubious help received by physiognosy from psychognosy. The historian certainly depends in a measure upon physical geography. Linguistics must in the future receive substantial assistance from acoustics, in more than one direction, and from the anatomy of the vocal organs and of the ear. Besides such supplies of information, (which are relatively unimportant) psychognosy has received instruction and encouragement from the example of physiognosy in the nineteenth century. It has been helped to minute accuracy, to objectivity, to genuine love of truth as against the professor's profession of infallibility. Yet summing up all the items, the total influence is trifling compared with that of mathematics on philosophy or of both on idioscopy. Physics has, after all, supplied no principle to psychics, nor any great conception. On the contrary, every attempt to import into psychics the conceptions proper to physics has only led those who made it astray. All this confirms the justice of our rating of these two departments as subclasses.

256. We can now no longer postpone the recognition of a second subbranch of theoretical science. It is a department perfectly well recognized. It belongs by virtue of its purpose to the branch of Theory; yet varies enough in its purpose from the active science to be erected into a subbranch. It is the subject of Humboldt's *Cosmos*, Comte's *Philosophie positive*, and Spencer's *Synthetic Philosophy*. It is science *en retraite*, *Wissenschaft a.D.* Its design is to sum up the results of all the theoretical sciences and to study them as forming one system. It

may be called *retrospective* [or science of review], in contradistinction to *active* science.

257. We now come to consider groups one grade lower. Here is a point where I must confess to have hesitated. Our branches of science are distinguished by their different purposes; our classes by the fundamentally different nature of their observations. Logic suggests that *orders*, to which we next come, should be distinguished by the difference in the intellectual part of the business of the sciences under them; so that among the Physical Sciences, for example, we should have: first those which investigate the laws common to all matter; second, those which study the relations between different classes of physical objects; third, those whose objects are the understanding of different individual objects; and it is plain that a similar classification could be made in psychics. Still, although this seems *a priori* plausible, a positive guarantee that this will be a natural division is perhaps lacking. At any rate, no ground of assurance is evident. It has occurred to me that we might distribute the physical sciences into those which study objects predominantly under the dominion of force and those predominantly under the influence of final causality; giving physics and natural history. This separation would well accord with the way in which the men naturally cluster. But for that very reason, a suspicion is created that the point has not yet been reached where that cleavage should be made. Before we come to groups of men thoroughly understanding one another's work, we ought to consider groups of which the one stands in the relation of teacher of principles to another; just as, in a school, the relation of master and pupil makes a broader natural division than that between different forms or classes. . . .

258. Comte . . . produced a useful scale, as every candid man now confesses. It ran thus: Mathematics, Astronomy, Physics, Chemistry, Biology, Sociology. But sociology stands distinctly aloof from the others, as a psychical science. Astronomy, for Comte, meant the astronomy of his day, which was confined almost entirely to explaining the motions of the stars, and was thus directly dependent on mathematics. But our astronomy depends largely on chemistry. Striking out mathematics and sociology, which are not physical sciences, and put-

ting astronomy where it now seems to belong, we get Physics, Chemistry, Biology, Astronomy, or perhaps

	Physics	
Chemistry		Biology
Astronomy		

Geognosy was intended by Comte to be a subdivision of physics. But this is every way unnatural. Geognosy applies physics as well [as] biology (especially paleontology); so that a still better scheme would be

	Physics	
Chemistry		Biology
Astronomy		Geognosy

259. In this scheme, we see a return to my first idea. For physics, here, must mean general physics, so called; that is, the study of the laws and forces of nature. Chemistry must here be understood as the science of the different kinds of matter (which is substantially the definition of Ostwald and of Mendeléef). Thus the second lines are sciences of classes, or, say for brief, Classificatory Sciences, which of course have much more to do than merely to make schemes of classification. In the third line we find sciences descriptive and explanatory of individual objects, or individual systems; the heavens, and the earth. We may name them, by way of abridgment, Descriptive Sciences.

260. We may take it as settled, then, that nomological physics forms naturally the first order of the subclass of the physical sciences. But whether the others ought primarily to be divided according to the rows of the last scheme, or according to its columns is a question upon which a little further consideration needs to be bestowed. In this connection we remark that the affinity of geognosy to biology is hardly as decided as the vertical division would represent it to be. One cannot even say that chemistry is more allied to astronomy than it is to biology. Light may be thrown upon the matter by asking where crystallography and mineralogy should be placed. Nobody, surely, would hold it to be a natural classification to rank crystallography as coördinate with chemistry and biology. Nor does it belong to general nomological physics; for it is eminently a study of kinds, not of general laws. A suggestion of uniting it to biology would provoke a smile. It would seem,

then, that nothing remains but to treat it as a division of chemistry, in the sense of the study of the different kinds of matter. Two great authorities, Ostwald and Mendeléef, do, in fact, so define chemistry; but I venture to assert that chemists generally do not so understand their science, and that chemical laboratories are not equipped for that study. Chemistry is, as a matter of fact, pretty closely limited to the study of reactions, to the structure of compounds, and to the behaviour of elements in combinations. A chemist, as such, does not feel himself called upon to inquire further into the properties of the different substances than is required to identify them and to make out their constitutional relations. He would, for example, think it quite beyond his province to arrest his work to determine the constants of elasticity of a substance. To ask that of him, he would say, would be a mischievous mixing of vocations. The descriptions that chemists give of crystals — in most cases confined to their habits — would be regarded by a crystallographer as superficial; and if they occasionally go further, it is with a view to the identification of the substances. The definition of Ostwald and Mendeléeff, then (which I myself independently gave), defines a department of science of which chemistry is but a part. Let us call the science of the kinds of matter by the name of *chemology*. This science will have to describe all the special properties of all kinds of matter, and among these properties, will have to describe the shapes into which matter of different kinds grows. So, then, we may reckon crystallography as a branch of chemology.

261. We cannot but remark, in passing, that a certain doubt arises here; because the study of the different kinds of crystalline form — with their geometrical, elaterical, and optical relations to one another — will look upon the facts of crystallization from quite another standpoint from that of the chemologist who is considering the relations of the different kinds of matter to one another. But I pass that by for the moment in order to make another remark. Suppose it were settled that that difference of the points of view of the crystallographer and the chemologer were of subsidiary importance, and that the latter's business includes the study [of] all the forms that different kinds of matter naturally assume. Then I remark that there is a certain group of chemical bodies, the

albuminoids or protoplasms, of which, down to this time, the chemist can only say that they contain carbon (51 or 52 per cent), oxygen (20 to 23 per cent), nitrogen (16 to 19 per cent), hydrogen (about 7 per cent), sulphur (about 1 per cent), and probably often phosphorus and many other elements, and that there are something like fifteen thousand atoms to the molecule. These substances assume forms far more fantastical than crystals — namely all the forms that the biologist describes; and the mathematician assures us that even if the number of atoms to the molecule is greatly less than the number which Sabanajeff* has determined by an approved method, there nevertheless can be no doubt that it is sufficient to afford, on general principles of chemistry, enough different kinds of protoplasm for each organ, or even cell of every individual animal or plant that ever existed on earth to have a unique kind of its own, without seriously encroaching upon the wealth of varieties of these substances. So, then, we may rationally conclude that all the variety of the biological world is due to the variety of the different kinds of chemical substances of this group, with their corresponding variety of properties and of natural figures. Thereupon in comes the logician, and for his contribution to the discussion, declares it to be absolutely impossible to frame any definite hypothesis — however gratuitous — which should assign any other origin to the forms of animals and plants than the chemical constitution of the protoplasm. Imagine, if you like, that separate corpuscles related to atoms as atoms are to billiard balls, are endowed with free will, so that their motions are determined by persuasion and not by the general laws of physics. That, if proved, would be a momentous discovery enough. I know not what could tend more toward the obliteration of all distinction between psychognosy and physiognosy. Nevertheless, under that state of things it would remain true that the chemical constitution of the protoplasm, about which we now make no pretension to knowing anything, although it would then turn out to be so mighty strange, containing chemical elements that would put radium to the blush — that chemical constitution, I say, would even then be the sole determining cause of the forms of all

* See Poggendorff's *Biographisch-literarisches Handwörterbuch* (1883–1904), bk. IV, p. 1293, for an account of Sabanajeff's work and life.

animals and plants. So it would remain, though we were to suppose a special creative act at the birth or budding of each biological individual — as long as there remained an approximate regularity in the action — although this would be still more revolutionary of all our chemological conceptions. We all know the type of naturalist — often a justly honored man of science — who, at any suggestion that experiment can ever be of real avail in biology, pours forth a torrent in which feeling is more easily detected than logic. Some minds there are who seem to think that if A and B are radically dissimilar, it is weak to admit that they can be fundamentally unlike; although could this not be, there would be an end of natural classification. Nobody can dispute the fact that the albuminoids are radically unlike all chemical substances whose constitution we understand.

262. If then we are to have an order or suborder of physiognosy consisting of the study of kinds of matter and their natural forms, it is a requirement of logic that biology should be reckoned as a family of that order or suborder. It must no doubt be admitted that the study of the kinds of matter, chemology, is one thing and the study of the kinds of forms that matter may take is another. These will be two suborders of the order of classificatory physiognosy, or the study of physical kinds. But into which of these suborders biology should be placed is not so clear. Perhaps it should constitute a third suborder.

263. In addition, we must recognize a third order, descriptive and explanatory of the accidents of individual systems, apart from the study of the classes to which they belong.

264. Is there a somewhat parallel division of psychognosy? "Anthropology" is a word sometimes used in so broad a sense that it covers all psychognosy, or would do so, were the study of animals and of non-biological manifestations of mind not arbitrarily excluded. Of anthropology, so understood, the late Dr. Brinton * proposed a classification, an outline of which may usefully be put before the reader. It includes, in the first place, four grand divisions of somatology, ethnology, ethnography, and archeology. The first of these is purely physical,

* *Anthropology: As a Science*, by D. G. Brinton, published in pamphlet form, Philadelphia, 1892.

except that it strangely includes psychology, so that it does not now concern us. The fourth is purely descriptive, and largely physical. It will afford no help. Ethnology is made to include five departments, as follows: 1. Sociology. 2. Technology, embracing the fine and useful arts. 3. Religion. 4. Linguistics. 5. Folklore. Ethnography treats of the different races of men, and is largely physical. I have no objection to admitting that zoölogy must perforce take some cognizance of the instincts of animals, just as on the other hand, it is quite evident that their minds can never be understood without taking some account of their anatomy and physiology. But for all that, if we are to admit that the study of animals' bodies is a study of efficiency, while the study of their minds is a study of finality, a distinction the truth and unescapableness [of which] will only be emphasized the more we study the different phases and facets of the subject, then we must acknowledge that those two studies of animals' minds and of animals' bodies are widely different, however much they may overlap. But in truth the overlapping is quite trifling. Very little psychology is needed by the biologist; and no very deep biology by the psychologist.

265. Dr. Brinton's classification is artificial. He would hardly himself have contested that judgment. Of almost any subdivision of it, it may be said that no man could judiciously devote himself to all those studies exclusively. Probably no man ever did, if we read Brinton's explanation of what each includes. But the classification has a fault even worse than that of being artificial. There can be no objection to a man's engaging at one time in tracing out final, or mental, causation, and at another time in tracing out material, or efficient, causation. But to confound these two things together is fatal. That circumstance constitutes a certain measure of justification for the warfare that has been waged, in many quarters, upon "final causes"; and it equally justifies the dislike often felt to physical explanations. Longfellow used to say that he hated sciences. I can sympathize with him. For he lived so entirely in the psychical world, that science to him meant an endeavor to turn finality into efficiency; or as he would say, to refute poetry. It is most narrow not to consider final causes in the study of nature; but it is nonsense and utter confusion to treat

them as forces in the material sense. Dr. Brinton, along with ethnologists generally, appears to be oblivious to this, throughout, and to look upon the study of the psychical from a psychical point of view as essentially inexact. To ask whether a given fact is due to psychical or physical causes is absurd. Every fact has a physical side; perhaps every fact has a psychical side. Its physical aspect — as a mere motion — is due exclusively to physical causes; its psychical aspect — as a deed — is due exclusively to psychical causes. This remains true, though you accept every doctrine of telepathy, table-turning, or what you will. If I can turn a table by the force of my will, this will simply establish the fact that something between me and the table acts just as a stick with which I should poke the table would act. It would be a physical connection purely and simply, however interesting it might be to a psychologist. But on the other hand, as my hand obeys, in a general way, my commands, clutching what I tell it to clutch, though I leave to its better judgment all the *menu* of just how my general order is to be carried out (and so I do with my rapier, directing its point to move so and so, but how it is done I never know), so the table-turning experiment would, I suppose, show that I could give similar general orders to the untouched table. That would be purely psychical, or final, causation, in which particulars are disregarded. Meantime, one may note that the table certainly *will* turn, if I really and truly *will* that it shall without being too meticulous about ways and means.

266. Three items only of Brinton's somatology belong to psychognosy. They are, first, prosopology, so far as that refers to the dubious science of physiognomy; second, psychology, and third, criminal-anthropology. A great part of his ethnography simply considers men as biological forms. So, too, the physical geography, geology, and paleontology that he includes. This latter I had not supposed belonged to the science of man. On the other hand much of psychognosy is omitted; such as the study of animal and vegetable instinct (both of which, especially the latter, throw much light on man's nature), theology as such (supposing there is such a science), economics, esthetics (so far as it is neither philosophy on the one hand, nor practical science on the other), and history in all its many branches (and it seems to me strange that Dr. Brinton, who

makes almost everything else pertain to the science of man, should think that history does not do so), and biography.

267. Let us now, with Dr. Brinton's list before us, endeavor to survey psychognosy and make out its orders. In the first place, final causality, which is the object of psychical science, appears in three guises; first, quite detached from any biological organism; second, in biological individuals as vehicles; third, in societies, ranging from the family to that public which includes our indefinite "posterity." These distinctions, when we thus consider them together, impress us with a certain grandeur. It may be that this explains what, at any rate, is a fact, that the question has often pressed itself upon me whether they ought not to form the basis of the first division of the class of psychical sciences. But this would be merely, or mainly, a division according to the nature of the objects of study. We ought to classify the sciences according to their own natures; and not according to the nature of their objects in the least, except so far as this affects the nature of the studies of these objects.[1] But before taking anything of that sort into account, we ought to look for a division based on the differences of the intellectual factor in the work of science, such as has been found to constitute the three orders of physiognosy; to wit, the nomological, the classificatory, and the descriptive. These orders appear more and more clear, the further the subject is examined. Mind has its universal laws, operative wherever it is manifested, although these may be modified according to the mode of its incarnation or other manifestation. In studying the universal properties of mind, the student will, no doubt, have occasion to remark some of the peculiarities of different modes of manifestation of mind. It may easily happen to a young student that this study of special kinds of productions of mind comes to fascinate and absorb him far more than the thinner and abstracter science of mind's universal truths. It may happen to another student that while he makes elaborate studies of a special form of psychical fruit, he will never cease to pursue those studies with a view to their affording some clue to the general secrets of mind. Just so, a man may study the systems [of] crystals for the sake of their teachings concerning

[1] The opposite opinion is argued explicitly by Shields in his *Philosophia Ultima*, with as much force as the position allows.

the nature of elasticity, as Rankine did, or in hopes of learning from them something about light, as Brewster did; or on the other hand, being interested in crystals and their classes, with a view to gaining a better comprehension of them, he may make studies of their cohesion, as Haüy did; and with either of these motives, he may produce a memoir which, in itself considered, might very well be classed either as a contribution to nomological physics or to crystallography. Take a larger view of his work, and there will be no possible doubt that Brewster and Rankine were physicists, while Haüy was a botanist turned crystallographer.

268. What shall we classifiers do with studies which might equally belong to either of two groups? Shall we, for the sake of convenience, allow a little artificiality to enter into our classification, so as to give such a study an undoubted place? That would be compromise. Now we ought always to be willing to compromise judiciously in practical matters, never in theoretical science. But if there be any such thing as natural classification, it is the truth, the theoretical truth, which is not to be sacrificed to convenience. It may be different with a classification of sciences designed to govern the alcove-arrangement of a library. That is a question for separate discussion. I will only remark here that the purposes of alcove-arrangement are so multifarious, that it will not in many cases happen that the integral convenience of any one artificial arrangement is markedly superior to that of the natural arrangement. The natural classification of science must be based on the study of the history of science; and it is upon this same foundation that the alcove-classification of a library must be based. The natural classification of science is to be a classification of men of science; and because each great man's works are published in collected form, the alcove-classification of a library must also be a classification of men. At any rate, for our convenience in this chapter the plain truth untampered with alone will answer. Where it happens that the truth is that the defining lines between natural classes are not absolutely definite, it is that truth which we want stated.

269. Mind has its universal mode of action, namely, by final causation. The microscopist looks to see whether the motions of a little creature show any purpose. If so, there is

mind there. Passing from the little to the large, natural selection is the theory of how forms come to be adaptive, that is, to be governed by a *quasi* purpose. It suggests a machinery of efficiency to bring about the end — a machinery inadequate perhaps — yet which must contribute some help toward the result. But the being governed by a purpose or other final cause is the very essence of the psychical phenomenon, in general. There ought, therefore, one would think, to be under the order of psychonomy, or nomological psychognosy, a suborder which should seek to formulate with exactitude the law of final causation and show how its workings are to be traced out.

270. But under this universal law of mind, there are other laws, it may be equally ubiquitous yet not so abstract. There is, first of all, the great law of association (including fusion), a principle strikingly analogous to gravitation, since it is an attraction between ideas. There are, besides, other general phenomena of mind not explicable by association. The laws of all these phenomena will be studied under a second suborder of special nomological psychology.

271. As a second order, we have psychotaxy, not a very good name for classificatory psychognosy or the study of kinds of mental manifestation. This order falls into two suborders, the one embracing studies of mental performances and products, the other of incarnations, or ensoulments of mind. To the latter suborder I would refer all studies of the minds of insects and (when there are any) of octopuses, of sexual characteristics, of the seven ages of human life, of professional and racial types, of temperaments and characters. To the former suborder, I would refer the vast and splendidly developed science of linguistics, of customs of all kinds, of Brinton's ethnology generally.

272. A third order of psychognosy is descriptive and explanatory, but not in any predominant degree inductive. Those sciences which are mainly descriptive, which tell, for example, what an explorer has found, which give accounts of systems, as metrology, chronology, numismatics, heraldry, or examine individual productions of man, will form a descriptive suborder; while those which narrate sequences of events and show how one leads to another — History in short, whether of individuals, or of communities, or of fields of activity, or of the

development of minds, or of forms of social institutions, will form a second suborder.

§5. THE DIVISIONS OF PHILOSOPHY*

273. It is plain that philosophy cannot, like idioscopy, be split from top to bottom into an efficient and a final wing. For, not to mention other reasons, to philosophy must fall the task of comparing the two stems of causation and of exhuming their common root. In another way, however, philosophy falls asunder into two groups of studies to which the appellation of subclasses is alone appropriate, if we are to understand by a subclass a modification of that class-making sense in which philosophy may be said to be observational. For besides what constitutes — in the present stage of the study, at least — the main body of philosophy, resting exclusively upon universal experience, and imparting to it a tinge of necessity, there is a department of science which, while it rests, and can only rest, as to the bulk of it, upon universal experience, yet for certain special yet obtrusive points is obliged to appeal to the most specialized and refined observations, in order to ascertain what minute modifications of everyday experience they may introduce. If in these departments the teachings of ordinary experience took on the true complexion of necessity, as they usually do, it would hardly be in our power to appeal to special experience to contradict them. But it is a remarkable fact that though inattentive minds do pronounce the dicta of ordinary experience in these cases to be necessary, they do not appear so to those who examine them more critically. For example, everyday experience is that events occur in time, and that time has but one dimension. So much appears necessary. For we should be utterly bewildered by the suggestion that two events were each anterior to the other or that, happening at different times, one was not anterior to the other. But a two-dimensional anteriority is easily shown to involve a self-contradiction. So, then, that time is one-dimensional is, for the present, necessary; and we know not how to appeal to special experience to disprove it. But that space is three-dimensional involves no such necessity. We can perfectly well suppose that atoms or their corpuscles move freely in four or more dimensions. So

* Cf. vol. 5, bk I, lecture V, §1.

everyday experience seems to teach us that time flows continuously. But that we are not sure that it really does so, appears from the fact that many men of powerful minds who have examined the question are of the opinion that it is not so. Why may there not be a succession of stationary states, say a milliasse or so of them or perhaps an infinite multitude per second, and why may states of things not break abruptly from one to the next? Here the teachings of ordinary experience are, at least, difficult of ascertainment. There are cases where they are decidedly indefinite. Thus, such experience shows that the events of one day or year are not exactly like those of another, although in part there is a cyclical repetition. Speculative minds have asked whether there may not be a complete cycle at the expiration of which all things will happen again as they did before. Such is said to have been the opinion of Pythagoras; and the stoics took it up as a necessary consequence of their philistine views. Yet in our day, certain experiences, especially the inspiring history of science and art during the nineteenth century, have inclined many to the theory that there is endless progress, a definite current of change on the whole of the whole universe. What treasures would we not sacrifice for the sake of knowing for certain whether it really be so, or not! It is nothing to you or me, to our children, or to our remoter posterity. What concern have we with the universe, or with the course of ages? No more than my dog has in the book I am writing. Yet I dare say he would defend the manuscript from harm with his life. However, to return to the matter of progress, universal experience is rather for the notion than against it, since there is a current in time, so far as we can see: the past influences our intellect, the future our spirit, with entire uniformity. Still universal experience merely favors a guess as to larger periods.

274. There are two distinct questions to be answered concerning time, even when we have accepted the doctrine that it is strictly continuous. The first is, whether or not it has any exceptional instants in which it is discontinuous,— any abrupt beginning and end. Philosophers there have been who have said that such a thing is inconceivable; but it is perfectly conceivable to a mind which takes up intelligently and seriously the task of forming the conception. Men who are ready to

pronounce a thing impossible before they have seriously studied out the proper way of doing it, and especially without having submitted to a course of training in making the requisite exertion of will, merit contempt. When a man tells us something is inconceivable, he ought to accompany the assertion with a full narrative of all he has done in these two ways to see if it could not be conceived. If he fails to do that, he may be set down as a trifler. There is no difficulty in imagining that at a certain moment, velocity was suddenly imparted to every atom and corpuscle of the universe; before which all was absolutely motionless and dead. To say that there was no motion nor acceleration is to say there was no time. To say there was no action is to say there was no actuality. However contrary to the evidence, then, such a hypothesis may be, it is perfectly conceivable. The other question is whether time is infinite in duration or not. If it has no flaw in its continuity, it must, as we shall see in chapter 4,* return into itself. This may happen after a finite time, as Pythagoras is said to have supposed, or in infinite time, which would be the doctrine of a consistent pessimism.

275. Measurement, as shall, in due course, be distinctly proved†, is a business fundamentally of the same nature as classification; and just as there are artificial classifications in profusion, but only one natural classification, so there are artificial measurements to answer every demand; but only one of them is the natural measurement. If time returns into itself, an oval line is an icon [or analytic picture] of it. Now an oval line may be so measured as to be finite, as when we measure positions on a circle by an angular quantity, θ, running up to 360 degrees, where it drops to 0 degree (which is the natural measure in the case of the circle); or it may be measured so that the measure shall once pass through infinity, in going round the circle, as when we project the positions on the circumference from one of them as a centre upon a straight line on which we measure the shadows by a rigid bar, as in the accompanying figure, here. This is measuring by tan $\frac{1}{2}(\theta—\Theta)$, instead of by θ; where Θ depends upon the position of

* That part of ch. 4 does not appear to have been written, see 584n. The nature of time is discussed at some length in vol. 6.

† See 4.142ff.

the centre of projection. Such a mode of measurement has the mathematical convenience of using every real number once and once only. It is quite possible, however, to measure so as to run over the whole gamut of numbers twice or more times. The single projection from a point within the circle gives one repetition.

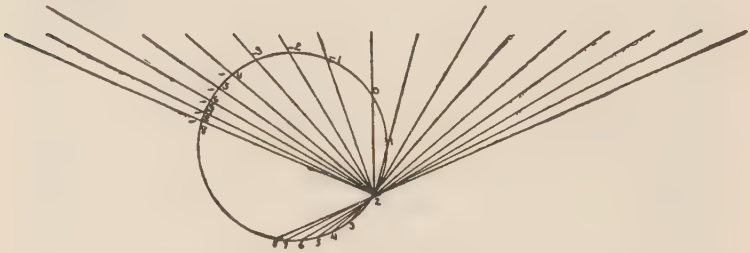


Figure 1

276. The question, however, is, What is the natural mode of measuring time? Has it absolute beginning and end, and does it reach or traverse infinity? Take time in the abstract and the question is merely mathematical. But we are considering a department of philosophy that wants to know how it is, not with pure mathematical time, but with the real time of history's evolution. This question concerns that evolution itself, not the abstract mathematical time. We observe the universe and discover some of its laws. Why, then, may we not discover the mode of its evolution? Is that mode of evolution, so far as we can discover, of such a nature that we must infer that it began and will end, whether this beginning and this end are distant from us by a finite number of days, hours, minutes, and seconds, or infinitely distant? In order to aid the reader in conceiving of a department of study which should make use of the discoveries of science to settle questions about the character of time as a whole, I have drawn three varieties of spirals.* The first of these has the equation $\theta = \frac{360^\circ}{\text{Log } 3} \log \frac{r - 1 \text{ inch}}{3 \text{ inches} - r}$. Imagine each revolution round the centre of the pencil point tracing the spirals, to represent the lapse of a year or any other cycle of time; and let r, the radius vector, represent the measure of the degree of evolution of the universe—it is not necessary to attach

* No such diagrams accompanied the ms.

any more definite idea to it. Then, if the universe obeys this law of evolution, it had an absolute beginning at a point of time in the past immeasurable in years. The degree of its stage of evolution was from the very first a positive quantity, 1; which constantly increases toward 3 which it will never surpass until its final destruction in the infinitely distant future. The second spiral is not strictly logarithmic. Its equation is $\theta = 360° \tan \frac{90° r}{1 \text{ inch}}$. Here again the universe is represented improving from a stage where $r = 1$ in the infinitely distant past to a stage where $r = 3$ in the infinitely distant future. But though this is infinitely distant when measured in years, evolution does not stop here, but continues uninterruptedly; and after another infinite series of years, $r = 5$; and so on endlessly. We must not allow ourselves to be drawn by the word "endless" into the fallacy of Achilles and the tortoise. Although, so long as r has not yet reached the value 3, another year will still leave it less than 3, yet if years do not *constitute* the flow of time, but only *measure* that flow, this in no wise prevents r from increasing in the flow of time beyond 3; so that it will be a question of fact whether or not, so far as we can make it out, the law of general evolution be such as to carry the universe beyond every fixed stage or not. It is very curious that in this case we can determine at exactly what season of the year in the infinitely distant future the value of r changes from being infinitesimally less to being infinitesimally more than 3. In the third spiral, of which the equation is $\frac{1}{r - 1/2 \text{ inch}} = 3 \log\left(1 + \text{anti-}\log \frac{90°}{\theta - 90°}\right)$, the universe was created a *finite* number of years ago in a stage of evolution represented by $r = 1/2$, and will go on for an infinite series of years approximating indefinitely to a state where $r = 2$, after which it will begin to advance again, and will advance until after another infinite lapse of years it will then in a finite time reach the stage when $r = 3 1/2$, when it will be suddenly destroyed. This last spiral is much the most instructive of the three; but all are useful. The reader will do well to study them.

277. Whether it is possible to make any scientific study of such questions and of the corresponding questions concerning physical geometry is a problem into which careful inquiry will have to be made in a subsequent chapter.* I must assume that my reader will desire to have this difficult problem cleared up; for if he is still in that stage of intellectual development in which he holds that he has already reached infallible conclusions on certain points, *e.g.*, that twice two makes four, that it is bad manners to marry one's grandmother, that he exists, that yesterday the sun set in the west, etc., so that to hear them seriously doubted fills him with disgust and anger (a little merriment could, perhaps, hardly be suppressed, and would not imply absolute infallibility), he cannot yet gain much from the perusal of this book, and had better lay it aside. Meantime, while it is still doubtful whether or not any knowledge of this kind is attainable, in view of the extreme interest of the questions, and in view of the fact that men of no small intellectual rank are endeavoring to illuminate them, we should by all means leave, for the present, a lodging for this group of studies in our scheme of classification.

278. One might well ask, however, whether their proper place is in philosophy or not rather in idioscopy, since they rest in part upon special observation. Every department of idioscopy builds upon philosophy, as we have seen. How then are these studies not idioscopic? Or, if they are not that, why not treat them as the zoölogists treat the tunicates, which, being neither strictly vertebrates nor by any means worms, are held to constitute a separate branch of the animal kingdom? As to that, I confess I am a little sceptical as to the decision of the zoölogists. But keeping to our proper question, every department of idioscopy is based upon special observation, and only resorts to philosophy in order that certain obstacles to its pursuing its proper special observational inquiries may be cleared out of the way. The sciences which we are now considering, on the contrary, are based upon the same sort of general experience upon which philosophy builds; and they only resort to special observation to settle some minute details, concerning which the testimony of general experience is possibly insufficient. It

* That chapter was not written. But see vol. 4, bk. I, ch. 4, and vols. 6 and 8, *passim*.

is true that they are thus of a nature intermediate between cœnoscopy and idioscopy; but in the main their character is philosophical. They form, therefore, a second subclass of philosophy, to which we may give the name of *theôrics*. As inquiry now stands, this subclass has but two divisions which can hardly rank as orders, but rather as families, *chronotheory* and *topotheôry*. This kind of study is in its first infancy. Few men so much as acknowledge that it is anything more than idle speculation. It may be that in the future the subclass will be filled up with other orders.

279. The first subclass, that of *necessary philosophy*, might be called *epistêmy*, since this alone among the sciences realizes the Platonic and generally Hellenic conception of επιστημη.[1] Under it, three orders stand out clearly.

280. The first of these is *Phenomenology*, or the Doctrine of Categories, whose business it is to unravel the tangled skein [of] all that in any sense appears and wind it into distinct forms; or in other words, to make the ultimate analysis of all experiences the first task to which philosophy has to apply itself. It is a most difficult, perhaps the most difficult, of its tasks, demanding very peculiar powers of thought, the ability to seize clouds, vast and intangible, to set them in orderly array, to put them through their exercises. The mere reading of this sort of philosophy, the mere understanding of it, is not easy. Anything like a just appreciation of it has not been performed by many of those who have written books. Original work in this department, if it is to be real and hitherto unformulated truth, is — not to speak of whether it is difficult or not — one of those functions of growth which every man, perhaps, in some fashion exercises once, some even twice, but which it would be next to a miracle to perform a third time.

281. Order II consists of the normative sciences. I wonder how many of those who make use of this term see any particular need of the word "normative." A normative science is one which studies what ought to be. How then does it differ from engineering, medicine, or any other practical science? If, however, logic, ethics, and esthetics, which are the families of normative science, are simply the arts of reasoning, of the conduct of life, and of fine art, they do not belong in the branch of

[1] It hardly need be said that *epistemology* is quite a different thing.

theoretic science which we are alone considering, at all. There is no doubt that they are closely related to three corresponding arts, or practical sciences. But that which renders the word normative needful (and not purely ornamental) is precisely the rather singular fact that, though these sciences do study what ought to be, *i.e.*, ideals, they are the very most purely theoretical of purely theoretical sciences. What was it that Pascal* said? "La vraie morale se moque de la morale." It is not worth while, in this corner of the book, to dwell upon so prominent a feature of our subject. The peculiar tinge of mind in these normative sciences has already been much insisted upon. It will come out in stronger and stronger colors as we go on.

282. Order III consists of metaphysics,† whose attitude toward the universe is nearly that of the special sciences (anciently, *physics* was its designation), from which it is mainly distinguished, by its confining itself to such parts of physics and of psychics as can be established without special means of observation. But these are very peculiar parts, extremely unlike the rest.

§6. THE DIVISIONS OF MATHEMATICS

283. Having now attained to a pretty clear apprehension of what a natural order of science is — deficient in distinctness though this apprehension be — we cannot, if we have any acquaintance with mathematics, consider that class of science, without seeing that none more manifestly falls into orders than this. The hypotheses of mathematics relate to systems which are either finite collections, infinite collections, or true continua; and the modes of reasoning about these three are quite distinct. These, then, constitute three orders. The last and highest kind of mathematics, consisting of topical geometry, has hitherto made very little progress; and the methods of demonstration in this order are, as yet, little understood. The study of finite collections divides into two suborders: first, that simplest kind of mathematics which is chiefly used in its application to logic, from which I find it almost impossible to

* Pensée 412, édition critique de *Pensées de Pascal* par G. Michaut, Fribourg, 1896.

† See 486ff. and vol. 6.

separate it*; and secondly, the general theory of finite groups. The study of infinite collections likewise divides into two suborders; first, arithmetic, or the study of the least multitudinous of infinite collections; and second, the calculus, or the study of collections of higher multitude. Hitherto, the calculus has been entirely confined to the study of collections of the lowest multitude above that of the collection of all integral numbers. This is studied either algebraically or geometrically, or, much more commonly, and perhaps more advantageously (though it is out of fashion to think so), by the two methods combined. The traditional division of mathematics, still much used, is into geometry and algebra — the division used by Jordanus Nemorarius† in the thirteenth century. It seems to me to be not only entirely artificial, but also extremely inconvenient from every point of view except the one of conforming to usage.‡

* This is the topic of ch. 3 of the "Minute Logic"; see vol. 4, bk. I, ch. 7.

† See M. Cantor's *Geschichte der Math.* II, Kap. 43–4.

‡ The remainder of this section of ch. 2 of the "Minute Logic" deals in considerable detail with the subdivisions of psychics and physics, and has been relegated to vols. 7 and 8.

Book III

PHENOMENOLOGY

CHAPTER 1

*INTRODUCTION**

§1. THE PHANERON†

284. Phaneroscopy is the description of the *phaneron;* and by the *phaneron* I mean the collective total of all that is in any way or in any sense present to the mind, quite regardless of whether it corresponds to any real thing or not. If you ask present *when*, and to *whose* mind, I reply that I leave these questions unanswered, never having entertained a doubt that those features of the phaneron that I have found in my mind are present at all times and to all minds. So far as I have developed this science of phaneroscopy, it is occupied with the formal elements of the phaneron. I know that there is another series of elements imperfectly represented by Hegel's Categories. But I have been unable to give any satisfactory account of them.

285. English philosophers have quite commonly used the word *idea* in a sense approaching to that which I give to *phaneron*. But in various ways they have restricted the meaning of it too much to cover my conception (if conception it can be called), besides giving a psychological connotation to their word which I am careful to exclude. The fact that they have the habit of saying that "there is no such idea" as this or that, in the very same breath in which they definitely describe the phaneron

* According to the scheme of classification given in the preceding book, phenomenology (or phaneroscopy) is the first division of philosophy, which is, in turn, the second of the sciences of discovery. The present book, to follow that scheme, should have been preceded by one on mathematics, the first of the sciences of discovery. Peirce's positive contributions to that science, however, are too technical for the general reader and his discussions of it are too closely interwoven with the discussions of other topics to make their inclusion in the present volume feasible. Most of the contributions to mathematics are to be found in vols. 3 and 4; the discussions regarding its nature are scattered throughout all the volumes; see *e.g.* 247ff.

† 284 is from the "Adirondack Lectures, 1905"; 285–287 are from "Logic viewed as Semeiotics, Introduction Number 2, Phaneroscopy," c. 1904.

in question, renders their term fatally inapt for my purpose.

286. There is nothing quite so directly open to observation as phanerons; and since I shall have no need of referring to any but those which (or the like of which) are perfectly familiar to everybody, every reader can control the accuracy of what I am going to say about them. Indeed, he must actually repeat my observations and experiments for himself, or else I shall more utterly fail to convey my meaning than if I were to discourse of effects of chromatic decoration to a man congenitally blind. What I term *phaneroscopy* is that study which, supported by the direct observation of phanerons and generalizing its observations, signalizes several very broad classes of phanerons; describes the features of each; shows that although they are so inextricably mixed together that no one can be isolated, yet it is manifest that their characters are quite disparate; then proves, beyond question, that a certain very short list comprises all of these broadest categories of phanerons there are; and finally proceeds to the laborious and difficult task of enumerating the principal subdivisions of those categories.

287. It will be plain from what has been said that phaneroscopy has nothing at all to do with the question of how far the phanerons it studies correspond to any realities. It religiously abstains from all speculation as to any relations between its categories and physiological facts, cerebral or other. It does not undertake, but sedulously avoids, hypothetical explanations of any sort. It simply scrutinizes the direct appearances, and endeavors to combine minute accuracy with the broadest possible generalization. The student's great effort is not to be influenced by any tradition, any authority, any reasons for supposing that such and such ought to be the facts, or any fancies of any kind, and to confine himself to honest, single-minded observation of the appearances. The reader, upon his side, must repeat the author's observations for himself, and decide from his own observations whether the author's account of the appearances is correct or not.

§2. VALENCIES*

288. There can be no psychological difficulty in determining whether anything belongs to the phaneron or not; for what-

* From "πλ," c. 1908.

ever seems to be before the mind *ipso facto* is so, in my sense of the phrase. I invite you to consider, not everything in the phaneron, but only its indecomposable elements, that is, those that are logically indecomposable, or indecomposable to direct inspection. I wish to make out a classification, or division, of these indecomposable elements; that is, I want to sort them into their different kinds according to their real characters. I have some acquaintance with two different such classifications, both quite true; and there may be others. Of these two I know of, one is a division according to the form or structure of the elements, the other according to their matter. The two most passionately laborious years of my life were exclusively devoted to trying to ascertain something for certain about the latter; but I abandoned the attempt as beyond my powers, or, at any rate, unsuited to my genius. I had not neglected to examine what others had done but could not persuade myself that they had been more successful than I. Fortunately, however, all taxonomists of every department have found classifications according to structure to be the most important.

289. A reader may very intelligently ask, How is it possible for an indecomposable element to have any differences of structure? Of internal logical structure it would be clearly impossible. But of external structure, that is to say, structure of its possible compounds, limited differences of structure are possible; witness the chemical elements, of which the "groups," or vertical columns of Mendeléeff's table, are universally and justly recognized as ever so much more important than the "series," or horizontal ranks in the same table. Those columns are characterized by their several valencies, thus:

He, Ne, A, Kr, X are medads ($\mu\eta\delta\acute{\epsilon}\nu$ none + the patronymic $= \iota\delta\eta\varsigma$).

H, L [Li], Na, K, Cu, Rb, Ag, Cs, –, –, Au, are monads;

G [Gl], Mg, Ca, Zn, Sr, Cd, Ba, –, –, Hg, Rd [Ra], are dyads;

B, Al, Sc, Ga, Y, In, La, –, Yb, Tc [Tl], Ac are triads;

C, Si, Ti, Ge, Zr, Sn, Co [Ce], –, –, Pc [Pb], Th, are tetrads;

N, P, V, As, Cb, Sb, Pr [Nd], –, Ta, Bi, Po [Pa], are properly pentads (as PCL_5, though owing to the junction of two pegs they often appear as triads. Their pentad character is particularly required to explain certain phenomena of albumins);

O, S, Cr, Se, Mo, Te, Nd [Sm], –, W, –, U, are properly

hexads (though by junction of bonds they usually appear as dyads);

F, Cl, Mn, Br, –, I, are properly heptads (usually appearing as monads);

Fe, Co, Ni, Ru, Rh, Pd, –, –, –, Os, Tr [Ir], Pt, are octads; (Sm, Eu, Gd, Er, Tb, Bz [?], Cl [Ct], are not yet placed in the table.)

290. So, then, since elements may have structure through valency, I invite the reader to join me in a direct inspection of the valency of elements of the phaneron. Why do I seem to see my reader draw back? Does he fear to be compromised by my bias, due to preconceived views? Oh, very well; yes, I do bring some convictions to the inquiry. But let us begin by subjecting these to criticism, postponing actual observation until all preconceptions are disposed of, one way or the other.

291. First, then, let us ask whether or not valency is the sole formal respect in which elements of the phaneron can possibly vary. But seeing that the possibility of such a ground of division is dependent upon the possibility of multivalence, while the possibility of a division according to valency can in nowise be regarded as a result of relations between bonds, it follows that any division by variations of such relations must be taken as secondary to the division according to valency, if such division there be. Now (my logic here may be puzzling, but it is correct), since my ten trichotomies of signs,* should they prove to be independent of one another (which is to be sure, highly improbable), would suffice to furnish us classes of signs to the number of

$$\begin{aligned} 3^{10} = (3^2)^5 = (10-1)^5 &= 10^5 - 5.10^4 \\ &\quad + 10.10^3 - 10.10^2 \\ &\quad + 5.10 - 1 \\ &= 50000 \\ &\quad + \ 9000 \\ &\quad + \ \ 49 \\ &= 59049 \end{aligned}$$

(*Voilà* a lesson in vulgar arithmetic thrown in to boot!), which

* See the letters to Lady Welby. These ten trichotomies are not to be confused with the ten not completely independent classes of signs given in vol. 2, bk. II. The latter originate from only three trichotomies; while the ten trichotomies yield sixty-six not completely independent classes of signs.

calculation threatens a multitude of classes too great to be conveniently carried in one's head, rather than a group inconveniently small, we shall, I think, do well to postpone preparations for further divisions until there be prospect of such a thing being wanted.

292. If, then, there be any formal division of elements of the phaneron, there must be a division according to valency; and we may expect medads, monads, dyads, triads, tetrads, etc. Some of these, however, can be antecedently excluded, as impossible; although it is important to remember that these divisions are not exactly like the corresponding divisions of Existential Graphs,* which have relation only to explicit indefinites. In the present application, a medad must mean an indecomposable idea altogether severed logically from every other; a monad will mean an element which, except that it is thought as applying to some subject, has no other characters than those which are complete in it without any reference to anything else; a dyad will be an elementary idea of something that would possess such characters as it does possess relatively to something else but regardless of any third object of any category; a triad would be an elementary idea of something which should be such as it were relatively to two others in different ways, but regardless of any fourth; and so on. Some of these, I repeat, are plainly impossible. A medad would be a flash of mental "heat-lightning" absolutely instantaneous, thunderless, unremembered, and altogether without effect. It can further be said in advance, not, indeed, purely *a priori* but with the degree of apriority that is proper to logic, namely, as a necessary deduction from the fact that there are signs, that there must be an elementary triad. For were every element of the phaneron a monad or a dyad, without the relative of teridentity† (which is, of course, a triad), it is evident that no triad could ever be built up. Now the relation of every sign to its object and interpretant is plainly a triad. A triad might be built up of pentads or of any higher perissad elements in many ways. But it can be proved — and really with extreme simplicity, though the statement of the general proof is confusing — that no element can have a higher valency than three.

* See vol. 4, bk. II.

† Cf. 346.

§3. MONADS, DYADS, AND TRIADS*

293. A thorough study of the logic of relatives confirms the conclusions which I had reached before going far in that study. It shows that logical terms are either monads, dyads, or polyads, and that these last do not introduce any radically different elements from those that are found in triads. I therefore divide all objects into monads, dyads, and triads; and the first step in the present inquiry is to ascertain what are the conceptions of the pure monad, free from all dyadic and triadic admixtures; of the dyad (which involves that of the monad) free from all triadic contamination, and what it is that is peculiar which the dyad adds to the monad; and of the triad (which involves those of the monad and dyad) and what it is that is characteristic of the triad.

§4. INDECOMPOSABLE ELEMENTS†

294. I doubt not that readers have been fretting over the ridiculous-seeming phrase "indecomposable element," which is as Hibernian as "necessary and sufficient condition" (as if "condition" meant no more than *concomitant* and as [if] *needful* were not the proper accompaniment of "sufficient"). But I have used it because I do not mean simply *element*. Logical analysis is not an analysis into existing elements. It is the tracing out of relations between concepts on the assumption that along with each given or found concept is given its negative, and every other relation resulting from a transposition of its correlates. The latter postulate amounts to merely identifying each correlate and distinguishing it from the others without recognizing any serial order among them. Thus to love and to be loved are regarded as the same concept, and not to love is also to be considered as the same concept. The combination of concepts is always by two at a time and consists in indefinitely identifying a subject of the one with a subject of the other, every correlate being regarded as a subject. Then if one concept can be accurately defined as a combination of others, and if these others are not of more complicated structure than the defined concept, then the defined concept is regarded as *ana-*

* From "The List of Categories: A Second Essay," c. 1894. 300 and 301 precede 293 in the ms.

† "The Basis of Pragmatism," Notebook I, c. 1905.

lyzed into these others. Thus A is grandparent of B, if and only if A is a parent of somebody who is a parent of B, therefore grandparent is analyzed into parent and parent. So stepparent, if taken as not excluding parentage, is analyzed into spouse and parent; and parent-in-law into parent and spouse.

295. These things being premised we may say in *primo*, there is no *a priori* reason why there should not be indecomposable elements of the phaneron which are what they are regardless of anything else, each complete in itself; provided, of course, that they be capable of composition. We will call these and all that particularly relates to them *Priman*. Indeed, it is almost inevitable that there should be such, since there will be compound concepts which do not refer to anything, and it will generally be possible to abstract from the internal construction that makes them compound, whereupon they become indecomposable elements.

296. In *secundo*, there is no *a priori* reason why there should not be indecomposable elements which are what they are relatively to a second but independently of any third. Such, for example, is the idea of otherness. We will call such ideas and all that is marked by them *Secundan* (*i.e.*, dependent on a second).

297. In *tertio* there is no *a priori* reason why there should not be indecomposable elements which are what they are relatively to a second and a third, regardless of any fourth. Such, for example, is the idea of *composition*. We will call everything marked by being a third or medium of connection, between a first and second anything, *tertian*.

298. It is *a priori* impossible that there should be an indecomposable element which is what it is relatively to a second, a third, and a fourth. The obvious reason is that that which combines two will by repetition combine any number.[1] Nothing could be simpler; nothing in philosophy is more important.

299. We find then *a priori* that there are three categories of undecomposable elements to be expected in the phaneron: those which are simply positive totals, those which involve dependence but not combination, those which involve combination.

Now let us turn to the phaneron and see what we find in fact.

[1] Thus stated, the principle does not seem to extend to abnumerable multitudes. Yet it must extend to them because, after all, the abnumerable is defined by means of combinations of two, and indeed must be so, since there is no form of combination not reducible to that.

CHAPTER 2

THE CATEGORIES IN DETAIL

A. FIRSTNESS

§1. THE SOURCE OF THE CATEGORIES*

300. The *list of categories*, or as Harris,† the author of *Hermes*, called them, the "philosophical arrangements," is a table of conceptions drawn from the logical analysis of thought and regarded as applicable to being. This description applies not merely to the list published by me in 1867,‡ and which I here endeavor to amplify, but also to the categories of Aristotle and to those of Kant. The latter have been more or less modified by different critics, as Renouvier, and still more profoundly by Hegel. My own list grew originally out of the study of the table of Kant.

301. I shall not here inquire how far it is justifiable to apply the conceptions of logic to metaphysics. For I hold the importance of that question, great as it is, to be perhaps secondary, and at any rate not paramount to that of the question what such conceptions would be. I may say, however, that in my own opinion, each category has to justify itself by an inductive examination which will result in assigning to it only a limited or approximate validity.

§2. THE MANIFESTATION OF FIRSTNESS§

302. The idea of First is predominant in the ideas of freshness, life, freedom. The free is that which has not another behind it, determining its actions; but so far as the idea of the negation of another enters, the idea of another enters; and

* From "The List of Categories: A Second Essay," c. 1894. 293 follows 301 in the ms.

† James Harris, in his *Philosophical Arrangements* (1775).

‡ See ch. 6.

§ From "The List of Categories: A Second Essay, X," c. 1894.

such negative idea must be put in the background, or else we cannot say that the Firstness is predominant. Freedom can only manifest itself in unlimited and uncontrolled variety and multiplicity; and thus the first becomes predominant in the ideas of measureless variety and multiplicity. It is the leading idea of Kant's "manifold of sense." But in Kant's synthetic unity the idea of Thirdness is predominant. It is an attained unity; and would better have been called totality; for that is the one of his categories in which it finds a home. In the idea of being, Firstness is predominant, not necessarily on account of the abstractness of that idea, but on account of its self-containedness. It is not in being separated from qualities that Firstness is most predominant, but in being something peculiar and idiosyncratic. The first is predominant in feeling, as distinct from objective perception, will, and thought.

§3. THE MONAD*

303. The pure idea of a *monad* is not that of an object. For an object is over against me. But it is much nearer an object than it is to a conception of self, which is still more complex. There must be some determination, or suchness, otherwise we shall think nothing at all. But it must not be an abstract suchness, for that has reference to a special suchness. It must be a special suchness with some degree of determination, not, however, thought as more or less. There is to be no comparison. So that it is a suchness *sui generis*. Imagine me to make and in a slumberous condition to have a vague, unobjectified, still less unsubjectified, sense of redness, or of salt taste, or of an ache, or of grief or joy, or of a prolonged musical note. That would be, as nearly as possible, a purely monadic state of feeling. Now in order to convert that psychological or logical conception into a metaphysical one, we must think of a metaphysical monad as a pure nature, or quality, in itself without parts or features, and without embodiment. Such is a pure monad. The meanings of names of "secondary" qualities are as good approximations to examples of monads as can be given.

* From "The List of Categories: A Second Essay," c. 1894. 303 follows 293 and is followed by 326 in the ms.

§4. QUALITIES OF FEELING*

304. . . . Among phanerons there are certain qualities of feeling, such as the color of magenta, the odor of attar, the sound of a railway whistle, the taste of quinine, the quality of the emotion upon contemplating a fine mathematical demonstration, the quality of feeling of love, etc. I do not mean the sense of actually experiencing these feelings, whether primarily or in any memory or imagination. That is something that involves these qualities as an element of it. But I mean the qualities themselves which, in themselves, are mere may-bes, not necessarily realized. The reader may be inclined to deny that. If so, he has not fully grasped the point that we are not considering what is true, not even what truly appears. I ask him to note that the word *red* means something when I say that the precession of the equinoxes is no more red than it is blue, and that it means just what it means when I say that aniline red is red. That mere *quality*, or suchness, is not in itself an occurrence, as seeing a red object is; it is a mere may-be. Its only being consists in the fact that there *might be* such a peculiar, positive, suchness in a phaneron. When I say it is a quality, I do not mean that it "inheres" in [a] subject. That is a phaneron peculiar to metaphysical thought, not involved in the sensation itself, and therefore not in the quality of feeling, which is entirely contained, or superseded, in the actual sensation. The Germans usually call these qualities feelings, feelings *of pleasure or pain*. To me this seems to be mere repetition of a tradition, never subjected to the test of observation. I can imagine a consciousness whose whole life, alike when wide awake and when drowsy or dreaming, should consist in nothing at all but a violet color or a stink of rotten cabbage. It is purely a question of what I can imagine and not of what psychological laws permit. The fact that I can imagine this, shows that such a feeling is not *general*, in the sense in which the law of gravitation is general. For nobody can imagine that law to have any being of any kind if it were impossible that there should exist any two masses of matter, or if there were no such things as motion. A true general cannot have any being unless there is to be some prospect of its sometime having occasion to be

* From "Logic viewed as Semeiotics, Introduction Number 2, Phaneroscopy," continuing 287.

embodied in a fact, which is itself not a law or anything like a law. A quality of feeling can be imagined to be without any occurrence, as it seems to me. Its mere may-being gets along without any realization at all.

§5. FEELING AS INDEPENDENT OF MIND AND CHANGE*

305. Suppose I begin by inquiring of you, Reader, in what particulars a feeling of redness or of purple without beginning, end, or change; or an eternally sounding and unvarying railway whistle; or a sempiterne thrill of joyous delight — or rather, such as would afford *us* delight, but supposed to be in that respect quite neutral — that should constitute the entire universe, would differ from a substance? I suppose you will tell me that no such thing could be alone in the universe because, firstly, it would require a mind to feel it, which would not be the feeling itself; secondly, the color or sound and probably also the thrill of delight would consist of vibrations; thirdly, none of them could last forever without a flow of time; fourthly, each would have a quality, which would be a determination in several respects, the color in hue, luminosity, chroma, and vividness; the sound in pitch, timbre (itself highly complex), loudness, and vividness; the delight more or less sensual, more or less emotional, more or less elevated, etc.; and fifthly, each would require a physical substratum altogether disparate to the feeling itself. But I point out to you that these things are only known to us by extraneous experience; none of them are either seen in the color, heard in the sound, or felt in the visceral sensation. Consequently, there can be no logical difficulty in supposing them to be absent, and for my part, I encounter not the slightest psychological difficulty in doing so, either. To suppose, for example, that there is a flow of time, or any degree of vividness, be it high or low, seems to me quite as uncalled for as to suppose that there is freedom of the press or a magnetic field.

* From "An Apology for Pragmaticism," intended for the January, 1907, *Monist*. See 4.540.

§6. A DEFINITION OF FEELING*

306. By a feeling, I mean an instance of that kind of consciousness which involves no analysis, comparison or any process whatsoever, nor consists in whole or in part of any act by which one stretch of consciousness is distinguished from another, which has its own positive quality which consists in nothing else, and which is of itself all that it is, however it may have been brought about; so that if this feeling is present during a lapse of time, it is wholly and equally present at every moment of that time. To reduce this description to a simple definition, I will say that by a feeling I mean an instance of that sort of element of consciousness which is all that it is positively, in itself, regardless of anything else.

307. A feeling, then, is not an event, a happening, a coming to pass, since a coming to pass cannot be such unless there was a time when it had not come to pass; and so it is not in itself all that it is, but is relative to a previous state. A feeling is a *state*, which is in its entirety in every moment of time as long as it endures. But a feeling is not a single state which is other than an exact reproduction of itself. For if that reproduction is in the same mind, it must be at a different time, and then the being of the feeling would be relative to the particular time in which it occurred, which would be something different from the feeling itself, violating the definition which makes the feeling to be all that it is regardless of anything else. Or, if the reproduction were simultaneous with the feeling, it must be in another mind, and thus the identity of the feeling would depend upon the mind in which it was, which is other than the feeling; and again the definition would be violated in the same way. Thus, any feeling must be identical with any exact duplicate of it, which is as much as to say that the feeling is simply a quality of immediate consciousness.

308. But it must be admitted that a feeling experienced in an outward sensation may be reproduced in memory. For to deny this would be idle nonsense. For instance, you experience, let us say, a certain color sensation due to red-lead. It has a definite hue, luminosity, and chroma. These [are] three elements — which are not separate in the feeling, it is true, and

* From "Phaneroscopy $\phi\alpha\nu$" intended for the January, 1907, *Monist*. See 4.540.

are not, therefore, in the feeling at all, but are said to be in it, as a way of expressing the results which would follow, according to the principles of chromatics, from certain experiments with a color disk, color-box, or other similar apparatus. In that sense, the color sensation which you derive from looking at the red-lead has a certain hue, luminosity, and chroma which completely define the quality of the color. The *vividness*, however, is independent of all three of these elements; and it is very different in the memory of the color a quarter of a second after the actual sensation from what it is in the sensation itself, although this memory is conceivably perfectly true as to hue, luminosity, and chroma, which truth constitutes it an exact reproduction of the entire quality of the feeling.

309. It follows that since the *vividness* of a feeling — which would be more accurately described as the vividness of a consciousness of the feeling — is independent of every component of the quality of that consciousness, and consequently is independent of the resultant of those components, which resultant quality is the feeling itself. We thus learn what vividness is not; and it only remains to ascertain what else it is.

310. To this end two remarks will be useful. The first is that of whatever is in the mind in any mode of consciousness there is necessarily an immediate consciousness and consequently a feeling. The proof of this proposition is very instructive as to the nature of feeling; for it shows that, if by *psychology* we mean the positive, or observational, science of the mind or of consciousness, then although the entire consciousness at any one instant is nothing but a feeling, yet psychology can teach us nothing of the nature of feeling, nor can we gain knowledge of any feeling by introspection, the feeling being completely veiled from introspection, for the very reason that it is our immediate consciousness. Possibly this curious truth was what Emerson was trying to grasp — but if so, pretty unsuccessfully — when he wrote the lines,

> The old Sphinx bit her thick lip —
> Said, "Who taught thee me to name?
> I am thy spirit, yoke-fellow,
> Of thine eye I am eyebeam.

"Thou art the unanswered question;
Couldst see thy proper eye,
Always it asketh, asketh;
And each answer is a lie."

But whatever he may have meant, it is plain enough that all that is immediately present to a man is what is in his mind in the present instant. His whole life is in the present. But when he asks what is the content of the present instant, his question always comes too late. The present has gone by, and what remains of it is greatly metamorphosed. He can, it is true, recognize that he was at that time, for example, looking at a specimen of red-lead, and must have seen that color, which, he perceives, is something positive and *sui generis*, of the nature of feeling. But nobody's immediate consciousness, unless when he was much more than half asleep, ever consisted wholly of a color-sensation; and since a feeling is absolutely simple and without parts — as it evidently is, since it is whatever it is regardless of anything else, and therefore regardless of any part, which would be something other than the whole — it follows that if the red color-sensation was not the whole feeling of the instant it has nothing in common with the feeling of the instant. Indeed, although a feeling is immediate consciousness, that is, is whatever of consciousness there may be that is immediately present, yet there is no consciousness in it because it is instantaneous. For we have seen already that feeling is nothing but a quality, and a quality is not conscious: it is a mere possibility. We can, it is true, see what a feeling in general is like; that, for example, this or that red is a feeling; and it is perfectly conceivable that a being should have that color for its entire consciousness, throughout a lapse of time, and therefore at every instant of that time. But such a being could never know anything about its own consciousness. It could not think anything that is expressible as a proposition. It could have no idea of such a thing. It would be confined to feeling that color. Thus, if you perceive that you must at the instant in question have been looking at a given specimen of red-lead, you know that that color has some resemblance to your feeling at that instant. But this only means that when the feeling gives place to comparison this resemblance appears.

But there is no resemblance at all in feeling, since feeling is whatever it is, positively and regardless of anything else, while the resemblance of anything lies in the comparison of that thing with something else. . . .

311. Every operation of the mind, however complex, has its absolutely simple feeling, the emotion of the *tout ensemble.* This is a secondary feeling or sensation excited from within the mind, just as the qualities of outward sense are excited by something psychic without us. It seems at first glance unaccountable that a mere slight difference in the speed of vibration should make such a difference of quality as that between deep vermillion and violet blue. But then it is to be remembered that it is doubtless our imperfect knowledge of those vibrations which has led us to represent them abstractly as differing only in quantity. There is already a hint in the behavior of electrons that a lower speed and a greater one have differences which we have not been aware of. People wonder, too, how dead matter can excite feelings in the mind. For my part, instead of wondering how it can be, I feel much disposed to deny downright that it is possible. These new discoveries have reminded us how very little we know of the constitution of matter; and I prefer to guess that it is a psychic feeling of red without us which arouses a sympathetic feeling of red in our senses.

§7. THE SIMILARITY OF FEELINGS OF DIFFERENT SENSORY MODES*

312. One of the old Scotch psychologists, whether it was Dugald Stewart or Reid† or which other matters naught, mentions, as strikingly exhibiting the disparateness of different senses, that a certain man blind from birth asked of a person of normal vision whether the color scarlet was not something like the blare of a trumpet; and the philosopher evidently expects his readers to laugh with him over the incongruity of the notion. But what he really illustrates much more strikingly is the dullness of apprehension of those who, like himself, had only the conventional education of the eighteenth century and remained wholly uncultivated in comparing ideas that in their matter

* From "Definition," 1910.

† Reid, *Inquiry into the Human Mind*, ch. 6, sec. II. But cf. Locke's *Essay*, bk. II, ch. 4, §5.

are very unlike. For everybody who has acquired the degree of susceptibility which is requisite in the more delicate branches of reasoning — those kinds of reasoning which our Scotch psychologist would have labelled "*Intuitions*" with a strong suspicion that they were delusions — will recognize at once so decided a likeness between a luminous and extremely chromatic scarlet, like that of the iodide of mercury as commonly sold under the name of scarlet [and the blare of a trumpet] that I would almost hazard a guess that the form of the chemical oscillations set up by this color in the observer will be found to resemble that of the acoustical waves of the trumpet's blare. I am only deterred from doing so by its being apparently true that our sense of hearing is entirely analytic; so that we are totally deaf to the wave of sound as it exists, and only hear the harmonic components regardless of the phases at which vibrations of commensurable lengths are combined.

§8. PRESENTMENTS AS SIGNS*

313. A mere presentment may be a sign. When the traditional blind man said he thought scarlet must be something like the sound of a trumpet, he had caught its blatancy very well; and the sound is certainly a presentment, whether the color[1] is so or not. Some colors are called gay, others sad. The sentiment of tones is even more familiar; that is, tones are signs of visceral qualities of feeling. But the best example is that of odors, for these are signs in more than one way. It is a common observation that odors bring back old memories. This I think must be due, in part at least, to the fact that, whether from the peculiar connection of the olfactory nerve with the brain or from some other cause, odors have a remarkable tendency to

* From "The Basis of Pragmatism," Notebook II, c. 1905.

[1] "As to colors, there is a somewhat serious difficulty in regarding them as presentments, because we cannot regard them as simple elements as long as they are contaminated with space-extension, which is something easily distinguishable and is also plainly not *priman*, since the space cannot of its nature be limited. Now the color not only cannot be dissociated from space, but it cannot even be prescinded from it. It can only be distinguished from it. We may, however, *neglect* the spatial element, and so reduce its emphasis indefinitely; and I am inclined to think that somehow colors may be regarded as presentments, though I cannot very clearly thread my way through the difficulty."— From "The Basis of Pragmatism," Notebook I, c. 1905.

presentmentate themselves, that is to occupy the entire field of consciousness, so that one almost lives for the moment in a world of odor. Now in the vacuity of this world, there is nothing to obstruct the suggestions of association. That is one way, namely by contiguous association, in which odors are particularly apt to act as signs. But they also have a remarkable power of calling to mind mental and spiritual qualities. This must be an effect of resemblance-association, if under resemblance-association we include all *natural* associations of different ideas. I certainly would do this; for I do not know what else resemblance can consist in.

A lady's favorite perfume seems to me somehow to agree with that of her spiritual being. If she uses none at all her nature will lack perfume. If she wears violet she herself will have the very same delicate fineness. Of the only two I have known to use rose, one was an artistic old virgin, a *grande dame;* the other a noisy young matron and very ignorant; but they were strangely alike. As for those who use heliotrope, frangipanni, etc., I know them as well as I desire to know them. Surely there must be some subtle resemblance between the odor and the impression I get of this or that woman's nature.

§9. THE COMMUNICABILITY OF FEELINGS*

314. Philosophers, who very properly call all things into question, have asked whether we have any reason to suppose that red looks to one eye as it does to another. I answer that slight differences there may be, but [consider the blind man imagining] red to resemble the blare of a trumpet. He had collected that notion from hearing ordinary people converse together about colors,and since I was not born to be one of those whom he had heard converse, the fact that I can see a certain analogy, shows me not only that my feeling of redness is something like the feelings of the persons whom he had heard talk, but also his feeling of a trumpet's blare was very much like mine. I am confident that a bull and I feel much alike at the sight of a red rag. As for the senses of my dog, I must confess that they seem very unlike my own, but when I reflect to how small a degree he thinks of visual images, and of how *smells*

* An undelivered (?) passage in Lecture IV of the "Lectures on Pragmatism," 1903.

play a part in his thoughts and imaginations analogous to the part played by *sights* in mine, I cease to be surprised that the perfume of roses or of orange flowers does not attract his attention at all and that the effluvia that interest him so much, when at all perceptible to me, are simply unpleasant. He does not think of smells as sources of pleasure and disgust but as sources of information, just as I do not think of blue as a nauseating color, nor of red as a maddening one. I know very well that my dog's musical feelings are quite similar to mine though they agitate him more than they do me. He has the same emotions of affection as I, though they are far more moving in his case. You would never persuade me that my horse and I do not sympathize, or that the canary bird that takes such delight in joking with me does not feel with me and I with him; and this instinctive confidence of mine that it is so, is to my mind evidence that it really is so. My metaphysical friend who asks whether we can ever enter into one another's feelings — and one particular sceptic whom I have in mind is a most exceptionally sympathetic person, whose doubts are born of her intense interest in her friends — might just as well ask me whether I am sure that red looked to me yesterday as it does today and that memory is not playing me false. I know experimentally that sensations do vary slightly even from hour to hour; but in the main the evidence is ample that they are common to all beings whose senses are sufficiently developed.

315. I hear you say: "All that is not *fact;* it is poetry." Nonsense! Bad poetry is false, I grant; but nothing is truer than true poetry. And let me tell the scientific men that the artists are much finer and more accurate observers than they are, except of the special minutiæ that the scientific man is looking for.

316. I hear you say: "This smacks too much of an anthropomorphic conception." I reply that every scientific explanation of a natural phenomenon is a hypothesis that there is something in nature to which the human reason is analogous; and that it really is so all the successes of science in its applications to human convenience are witnesses. They proclaim that truth over the length and breadth of the modern world. In the light of the successes of science to my mind there is a degree of baseness in denying our birthright as children of God and in

shamefacedly slinking away from anthropomorphic conceptions of the universe.

§10. TRANSITION TO SECONDNESS*

317. The whole content of consciousness is made up of qualities of feeling, as truly as the whole of space is made up of points or the whole of time of instants.

318. Contemplate anything by itself — anything whatever that can be so contemplated. Attend to the whole and drop the parts out of attention altogether. One can approximate nearly enough to the accomplishment of that to see that the result of its perfect accomplishment would be that one would have in his consciousness at the moment nothing but a quality of feeling. This quality of feeling would in itself, as so contemplated, have no parts. It would be unlike any other such quality of feeling. In itself, it would not even resemble any other; for resemblance has its being only in comparison. It would be a pure *priman*. Since this is true of whatever we contemplate, however complex may be the object, it follows that there is nothing else in immediate consciousness. To be conscious is nothing else than to feel.

319. What room, then, is there for *secundans* and *tertians?* Was there some mistake in our demonstration that they must also have their places in the phaneron? No, there was no mistake. I said that the phaneron is made up entirely of qualities of feeling as truly as space is entirely made up of points. There is a certain *protoidal* aspect — I coin the word for the need — under which space is truly made up of nothing but points. Yet it is certain that no collection of points — using the word collection to mean merely a plural, without the idea of the objects being brought together — no collection of points, no matter how abnumerable its multitude, can in itself constitute space. . . .

320. The phaneron does contain genuine *secundans*. Standing on the outside of a door that is slightly ajar, you put your hand upon the knob to open and enter it. You experience an unseen, silent resistance. You put your shoulder against the door and, gathering your forces, put forth a tremendous effort. Effort supposes resistance. Where there is no effort there is no resistance, where there is no resistance there is no effort either

* From "Pragmatism," Fragment 2, c. 1910.

in this world or any of the worlds of possibility. It follows that an effort is not a feeling nor anything *priman* or protoidal. There are feelings connected with it: they are the sum of consciousness during the effort. But it is conceivable that a man should have it in his power directly to summon up all those feelings, or any feelings. He could not, in any world, be endowed with the power of summoning up an effort to which there did not happen to be a resistance all ready to exist. For it is an absurdity to suppose that a man could directly will to oppose that very will. A very little thinking will show that this is what it comes to. According to such psychological analysis as I can make, effort is a phenomenon which only arises when one feeling abuts upon another in time, and which then always arises. But my psychological pretensions are little,if they exist at all, and I only mention my theory in order that contrast should impress the reader with the irrelevancy of psychology to our present problem, which is to say of what sort that is which is in our minds when we make an effort and which constitutes it an effort.

321. We live in two worlds, a world of fact and a world of fancy. Each of us is accustomed to think that he is the creator of his world of fancy; that he has but to pronounce his fiat, and the thing exists, with no resistance and no effort; and although this is so far from the truth that I doubt not that much the greater part of the reader's labor is expended on the world of fancy, yet it is near enough the truth for a first approximation. For this reason we call the world of fancy the internal world, the world of fact the external world. In this latter we are masters, each of us, of his own voluntary muscles, and of nothing more. But man is sly, and contrives to make this little more than he needs. Beyond that, he defends himself from the angles of hard fact by clothing himself with a garment of contentment and of habituation. Were it not for this garment, he would every now and then find his internal world rudely disturbed and his fiats set at naught by brutal inroads of ideas from without. I call such forcible modification of our ways of thinking the influence of the world of fact or *experience.* But he patches up his garment by guessing what those inroads are likely to be and carefully excluding from his internal world every idea which is likely to be so disturbed. Instead of wait-

ing for experience to come at untoward times, he provokes it when it can do no harm and changes the government of his internal world accordingly.

B. SECONDNESS

§1. FEELING AND STRUGGLE*

322. The second category that I find, the next simplest feature common to all that comes before the mind, is the element of struggle.

This is present even in such a rudimentary fragment of experience as a simple feeling. For such a feeling always has a degree of vividness, high or low; and this vividness is a sense of commotion, an action and reaction, between our soul and the stimulus. If, in the endeavor to find some idea which does not involve the element of struggle, we imagine a universe that consists of a single quality that never changes, still there must be some degree of steadiness in this imagination, or else we could not think about and ask whether there was an object having any positive suchness. Now this steadiness of the hypothesis that enables us to think about it — and to mentally manipulate it — which is a perfectly correct expression, because our thinking about the hypothesis really consists in making experiments upon it — this steadiness, I say, consists in this, that if our mental manipulation is delicate enough, the hypothesis will resist being changed. Now there can be no resistance where there is nothing of the nature of struggle or forceful action. By struggle I must explain that I mean mutual action between two things regardless of any sort of third or medium, and in particular regardless of any law of action.

323. I should not wonder if somebody were to suggest that perhaps the idea of a law is essential to the idea of one thing acting upon another. But surely that would be the most untenable suggestion in the world considering that there is no one of us who after lifelong discipline in looking at things from the necessitarian point of view† has ever been able to train himself to dismiss the idea that he can perform any specifiable act of

* From "Lectures on Pragmatism," II, First Draught, c. 1903.

† See vol. 6, bk. I, ch. 2.

the will. It is one of the most singular instances of how a preconceived theory will blind a man to facts that many necessitarians seem to think that nobody really believes in the freedom of the will, the fact being that he himself believes in it when he is not theorizing. However, I do not think it worth while to quarrel about that. Have your necessitarianism if you approve of it; but still I think you must admit that no law of nature makes a stone fall, or a Leyden jar to discharge, or a steam engine to work.

§2. ACTION AND PERCEPTION*

324. [There is a category] which the rough and tumble of life renders most familiarly prominent. We are continually bumping up against hard fact. We expected one thing, or passively took it for granted, and had the image of it in our minds, but experience forces that idea into the background, and compels us to think quite differently. You get this kind of consciousness in some approach to purity when you put your shoulder against a door and try to force it open. You have a sense of resistance and at the same time a sense of effort. There can be no resistance without effort; there can be no effort without resistance. They are only two ways of describing the same experience. It is a double consciousness. We become aware of ourself in becoming aware of the not-self. The waking state is a consciousness of reaction; and as the consciousness *itself* is two-sided, so it has also two varieties; namely, action, where our modification of other things is more prominent than their reaction on us, and perception, where their effect on us is overwhelmingly greater than our effect on them. And this notion, of being such as other things make us, is such a prominent part of our life that we conceive other things also to exist by virtue of their reactions against each other. The idea of other, of *not*, becomes a very pivot of thought. To this element I give the name of Secondness.

§3. THE VARIETIES OF SECONDNESS†

325. The idea of second is predominant in the ideas of causation and of statical force. For cause and effect are two;

* From "Lowell Lectures of 1903." Lecture III, vol. 1, 3d Draught. See 343.

† Unidentified fragment.

and statical forces always occur between pairs. Constraint is a Secondness. In the flow of time in the mind, the past appears to act directly upon the future, its effect being called memory, while the future only acts upon the past through the medium of thirds. Phenomena of this sort in the outward world shall be considered below. In sense and will, there are reactions of Secondness between the *ego* and the *non-ego* (which non-ego may be an object of direct consciousness). In will, the events leading up to the act are internal, and we say that we are agents more than patients. In sense, the antecedent events are not within us; and besides, the object of which we form a perception (though not that which immediately acts upon the nerves) remains unaffected. Consequently, we say that we are patients, not agents. In the idea of reality, Secondness is predominant; for the real is that which insists upon forcing its way to recognition as something *other* than the mind's creation. (Remember that before the French word, *second*, was adopted into our language, *other* was merely the ordinal numeral corresponding to *two*.) The real is active; we acknowledge it, in calling it the *actual*. (This word is due to Aristotle's use of ενεργεια, action, to mean existence, as opposed to a mere germinal state.) Again, the kind of thought of those dualistic philosophers who are fond of laying down propositions as if there were only two alternatives, and no gradual shading off between them, as when they say that in trying to find a law in a phenomenon I commit myself to the proposition that law bears absolute sway in nature, such thought is marked by Secondness.

§4. THE DYAD*

326. A *dyad* consists of two *subjects* brought into oneness. These subjects have their modes of being in themselves, and they also have their modes of being, as first and second, etc., in connection with each other. They are two, if not really, at least in aspect. There is also some sort of union of them. The dyad is not the subjects; it has the subjects as one element of it. It has, besides, a suchness of monoidal character; and it has suchness, or suchnesses, peculiar to it as a dyad. The dyad brings the subjects together, and in doing so imparts a char-

* From "The List of Categories: A Second Essay," continuing 303.

acter to each of them. Those characters are, in some sense, two. The dyad has also two sides according to which subject is considered as first. These two sides of the dyad form a second pair of subjects attached to the dyad; and they have their mode of union. Each of them also has a special character as a subject of the dyad.

This description shows that the dyad, in contrast to the monad, has a variety of features; and all these features present dyadic relations.

327. As an example of a dyad take this: God said, Let there be light, and there was light. We must not think of this as a verse of Genesis, for Genesis would be a third thing. Neither must we think of it as proposed for our acceptance, or as held for true; for we are third parties. We must simply think of God creating light by fiat. Not that the fiat and the coming into being of the light were two facts; but that it is in one indivisible fact. God and light are the subjects. The act of creation is to be regarded, not as any third object, but merely as the suchness of connection of God and light. The dyad is the fact. It determines the existence of the light, and the creatorship of God. The two aspects of the dyad are, first, that of God compelling the existence of the light, and that of the light as, by its coming into existence, making God a creator. This last is in the present example merely a mere point of view, without any reality corresponding to it. That is one of the special features of the particular example chosen. Of the two aspects of the dyad, then, one is in this instance, fundamental, real, and primary, while the other is merely derivative, formal, and secondary.

328. I chose this instance because it is represented as instantaneous. Had there been any process intervening between the causal act and the effect, this would have been a medial, or third, element. Thirdness, in the sense of the category, is the same as mediation. For that reason, pure dyadism is an act of arbitrary will or of blind force; for if there is any reason, or law, governing it, that mediates between the two subjects and brings about their connection. The dyad is an individual fact, as it existentially is; and it has no generality in it. The being of a monadic quality is a mere potentiality, without existence. Existence is purely dyadic.

329. It is to be noted that existence is an affair of blind force. "The very hyssop that grows on the wall exists in that chink because the whole universe could not prevent it." No law determines any atom to exist. Existence is presence in some experiential universe — whether the universe of material things now existing, or that of laws, or that of phenomena, or that of feelings — and this presence implies that each existing thing is in dynamical reaction with every other in that universe. Existence, therefore, is dyadic; though Being is monadic.

§5. POLAR DISTINCTIONS AND VOLITION*

330. Calling any distinction between two equally decided characters to which no third seems to be coördinate (although a neutrality separates them) a *polar* distinction, in the external world polar distinctions are few. That of past and future, with the resulting two ways of passing over a line (and consequent right-and left-handed spirals and helices, whence probably the magnetic and possibly the electric poles — supposing the latter to be truly "polar" in our sense), with the right and left sides of our bodies, and the two sexes, seems pretty much to exhaust the list of them. Yet for the much smaller universe of psychology, polar distinctions abound, most of them referring to volition. Thus, pleasure is any kind of sensation that one immediately seeks, pain any that one immediately shuns. Right and wrong are expressly volitional. Necessity and impossibility so obviously refer to volition that the words often need qualification to show that rational modifications of them are meant. The words reasonable and perverse imply that assent is as free as choice ever is, and so proclaim their volitional strain. Roget's *Thesaurus* illustrates the great aptitude of the psychical to *polar* distinction. Any very close examination of how far this is due to volition would cause us to wander quite away from the subject of this essay. It would show that dichotomy, meaning the fact that the elements that a distinction separates are just *two* in number, is strikingly often — perhaps that it is presumably always — due to volition. . . .

331. Although the mode of consciousness we call volition, or willing, contrasts decidedly with the mere perception that something has been done, yet it is not perfected, and perhaps

* Unidentified fragment.

does not take place at all, until something is actually effected. Trying to shove something too heavy for the man to stir nevertheless accomplishes, in considerable measure, the only thing that he directly willed to do — namely, to contract certain muscles. In the days of table-turning we used to be commanded to sit quite away from a table, and "*with all our might*" to will that the table should move; and since the whole weight of our outstretched arms soon made our finger-tips unconsciously numb (for things are not apt to be consciously unconscious; and there were other concurring physiological effects that we did not suspect), while we were possessed of no other "might" over the table than through our muscles, we used to be speedily rewarded, by a direct consciousness of willing that the table move, accompanied by the vision of its wondrous obedience. Until it moved, we were only *longing*, not *willing*. So when certain psychologists write, chiefly in French — a language abounding in exquisite distinctions, but one in which any analytical method of interpretation is so sure to lead to misunderstandings, that the language is not well adapted to psychology or philosophy — about "*involuntary attention*," they can only mean one of two things, either *unpremeditated* attention or attention influenced by conflicting *desires*. Though "desire" implies a tendency to volition, and though it is a natural hypothesis that a man cannot *will* to do that which he has no sort of desire to do, yet we all know conflicting desires but too well, and how treacherous they are apt to be; and a desire may perfectly well be discontented with volition, *i.e.*, with what the man *will* do. The consciousness of that truth seems to me to be the root of our consciousness of free will. "Involuntary attention" involves in correct English a contradiction *in adjecto*.

§6. EGO AND NON-EGO*

332. The triad, feeling, volition, cognition, is usually regarded as a purely psychological division. Long series of carefully planned self-experiments, persistent and much varied, though only qualitative, have left me little doubt, if any, that there are in those elements three quite disparate modes of awareness. That is a psychological proposition; but that which

* From "Phaneroscopy or the Natural History of Concepts," c. 1905.

now concerns us is not psychological, particularly; namely the differences between that of which we are aware in feeling, volition, and cognition. Feeling is a quality, but so far as there is mere feeling, the quality is not limited to any definite subject. We hear of a man whose mind is jaundiced. That phrase well expresses feeling without reason. Feeling also as such is unanalyzed. Volition is through and through dual. There is the duality of agent and patient, of effort and resistance, of active effort and inhibition, of acting on self and on external objects. Moreover, there is active volition and passive volition, or inertia, the volition of reform and the volition of conservatism. That shock which we experience when anything particularly unexpected forces itself upon our recognition (which has a cognitive utility as being a call for explanation of the presentment), is simply the sense of the volitional inertia of expectation, which strikes a blow like a water-hammer when it is checked; and the force of this blow, if one could measure it, would be the measure of the energy of the conservative volition that gets checked. Low grades of this shock doubtless accompany all unexpected perceptions; and every perception is more or less unexpected. Its lower grades are, as I opine, not without experimental tests of the hypothesis, that sense of externality, of the presence of a *non-ego*, which accompanies perception generally and helps to distinguish it from dreaming. This is present in all sensation, meaning by sensation the initiation of a state of feeling;— for by feeling I mean nothing but sensation *minus* the attribution of it to any particular subject. In my use of words, when an ear-splitting, soul-bursting locomotive whistle starts, there is a sensation, which ceases when the screech has been going on for any considerable fraction of a minute; and at the instant it stops there is a second sensation. Between them there is a state of feeling.

333. As for pleasure and pain, which Kant and others have represented to be of the essence of feeling, whether it be merely because they and the section of the psychological world for which at this moment I have the presumption to speak apply the word feeling to different modifications of awareness, or whether there be a faulty analysis on the one part or the other, we certainly do not think that unadulterated feeling, if that element could be isolated, would have any relation to pain or

to pleasure. For in our opinion if there be any quality of feeling common to all pleasurable experiences or components of experience, and another one quality of feeling common to all that is painful (which we are inclined to doubt, to say the least), then we hold the opinion that the one is the feeling of being attracted, the other that of being repelled, by the present state of experience. If there be two such feelings, they are feelings of states of volition. But perhaps pleasure and pain are nothing more than names for the state of being attracted and that of being repelled by present experience. Of course, feelings accompany them, but under the latter hypothesis no feeling would be common to all pleasures, and none to all pains. If we are right, the position of the hedonists is preposterous, in that they make mere feelings to be active agencies, instead of being merely conscious indications of real determinations of our subconscious volitional beings. [I may mention that their *talk* (however it may be with their thought) is further preposterous as seeming to make pain a mere privation of pleasure, although it is plain that it is pain that indicates an active, and pleasure only a passive, determination of our volitional being.]

334. As for volition, I would limit the term in one way and extend it in another. I would limit it to the momentary direct dyadic consciousness of an *ego* and a *non-ego* then and there present and reacting each upon the other. In one, the action is generally more active, in the other more passive; but precisely what this difference consists in I do not feel sure. I think, however, that the will to produce a change is active, the will to resist a change is passive. All sensation is essentially, by its very definition, active. The objection to this is that, according to it, the voluntary inhibition of a reflex should not give a sense of effort; and probably the definition of the distinction between the sense of externality in willing and in perception requires a supplement or other slight modification on this account. But the important point [is] that the sense of externality in perception consists in a sense of powerlessness before the overwhelming force of perception. Now the only way in which any force can be learned is by something like trying to oppose it. That we do something like this is shown by the shock we receive from any unexpected experience. It is the inertia of the mind, which tends to remain in the state in which it is. No doubt

there is a marked difference between the active and intentional volition of muscular contraction and the passive and unintentional volition that gives the shock of surprise and the sense of externality. But the two are to be classed together as alike modes of double consciousness, that is, of awareness, at once and in the same awareness, of an *ego* and a *non-ego*. . . .

§7. SHOCK AND THE SENSE OF CHANGE*

335. Some writers insist that all experience consists in sense-perception; and I think it is probably true that every element of experience is in the first instance applied to an external object. A man who gets up out of the wrong side of the bed, for example, attributes wrongness to almost every object he perceives. That is the way in which he experiences his bad temper. It cannot, however, be said that he *perceives* the perversity which he wrongly attributes to outward objects.

336. We perceive objects brought before us; but that which we especially experience — the kind of thing to which the word "experience" is more particularly applied — is an event. We cannot accurately be said to perceive events; for this requires what Kant called the "synthesis of apprehension," not however, by any means, making the needful discriminations. A whistling locomotive passes at high speed close beside me. As it passes the note of the whistle is suddenly lowered from a well-understood cause. I perceive the whistle, if you will. I have, at any rate, a sensation of it. But I cannot be said to have a sensation of the change of note. I have a sensation of the lower note. But the cognition of the change is of a more intellectual kind. That I experience rather than perceive. It is [the] special field of experience to acquaint us with events, with changes of perception. Now that which particularly characterizes sudden changes of perception is a *shock*. A shock is a volitional phenomenon. The long whistle of the approaching locomotive, however disagreeable it may be, has set up in me a certain inertia, so that the sudden lowering of the note meets with a certain resistance. That must be the fact; because if there were no such resistance there could be no shock when the change of note occurs. Now this shock is quite unmistakable. It is more particularly to changes and contrasts

* Ibid.

of perception that we apply the word "experience." We experience vicissitudes, especially. We cannot experience the vicissitude without experiencing the perception which undergoes the change; but the concept of *experience* is broader than that of *perception*, and includes much that is not, strictly speaking, an object of perception. It is the compulsion, the absolute constraint upon us to think otherwise than we have been thinking that constitutes experience. Now constraint and compulsion cannot exist without resistance, and resistance is effort opposing change. Therefore there must be an element of effort in experience; and it is this which gives it its peculiar character. But we are so disposed to yield to it as soon as we can detect it, that it is extremely difficult to convince ourselves that we have exerted any resistance at all. It may be said that we hardly know it except through the axiom that there can be no force where there is no resistance or inertia. Whoever may be dissatisfied with my statement will do well to sit down and cipher out the matter for himself. He may be able to formulate the nature of the oppositional element in experience, and its relation to ordinary volition better than I have done; but that there is an oppositional element in it, logically not easily distinguished from volition, will, I make no doubt at all, be his ultimate conclusion.

C. THIRDNESS

§1. EXAMPLES OF THIRDNESS*

337. By the third, I mean the medium or connecting bond between the absolute first and last. The beginning is first, the end second, the middle third. The end is second, the means third. The thread of life is a third; the fate that snips it, its second. A fork in a road is a third, it supposes three ways; a straight road, considered merely as a connection between two places is second, but so far as it implies passing through intermediate places it is third. Position is first, velocity or the relation of two successive positions second, acceleration or the relation of three successive positions third. But velocity in so far as it is continuous also involves a third. Continuity repre-

* Fragment, "Third," c. 1875.

sents Thirdness almost to perfection. Every process comes under that head. Moderation is a kind of Thirdness. The positive degree of an adjective is first, the superlative second, the comparative third. All exaggerated language, "supreme," "utter," "matchless," "root and branch," is the furniture of minds which think of seconds and forget thirds. Action is second, but conduct is third. Law as an active force is second, but order and legislation are third. Sympathy, flesh and blood, that by which I feel my neighbor's feelings, is third.

§2. REPRESENTATION AND GENERALITY*

338. The ideas in which Thirdness is predominant are, as might be expected, more complicated, and mostly require careful analysis to be clearly apprehended; for ordinary, unenergetic thought slurs over this element as too difficult. There is all the more need of examining some of these ideas.

339. The easiest of those which are of philosophical interest is the idea of a sign, or representation.† A sign stands *for* something *to* the idea which it produces, or modifies. Or, it is a vehicle conveying into the mind something from without. That for which it stands is called its *object;* that which it conveys, its *meaning;* and the idea to which it gives rise, its *interpretant.* The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object at its limit. The meaning of a representation can be nothing but a representation. In fact, it is nothing but the representation itself conceived as stripped of irrelevant clothing. But this clothing never can be completely stripped off; it is only changed for something more diaphanous. So there is an infinite regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handed along; and as representation, it has its interpretant again. Lo, another infinite series.

340. Some of the ideas of prominent Thirdness which, owing to their great importance in philosophy and in science,

* 338 and 9 are from an unidentified fragment; 340–2 are from a fragment, "Thirdness," c. 1895.

† Cf. vol. 2, bk. II.

require attentive study are generality, infinity, continuity, diffusion, growth, and intelligence.

341. Let us examine the idea of generality. Every cook has in her recipe-book a collection of rules, which she is accustomed to follow. An apple pie is desired. Now, observe that we seldom, probably never, desire a single individual thing. What we want is something which shall produce a certain pleasure of a certain kind. To speak of a single individual pleasure is to use words without meaning. We may have a single experience of pleasure; but the pleasure itself is a quality. Experiences are single; but qualities, however specialized, cannot be enumerated. There are some two dozen kinds of metals well known to me. I remember to have examined lumps of those qualities. But it is only the limitation of experience which attaches that number; there is simply no end to the metallic qualities I can imagine. I can imagine an infinite variety between tin and lead, or between copper and silver, or between iron and nickel, or between magnesium and aluminum. An apple pie, then, is desired — a good apple pie, made of fresh apples, with a crust moderately light and somewhat short, neither too sweet nor too sour, etc. But it is not any particular apple pie; for it is to be made for the occasion; and the only particularity about it is that it is to be made and eaten today. For that, apples are wanted; and remembering that there is a barrel of apples in the cellar, the cook goes to the cellar and takes the apples that are uppermost and handiest. That is an example of following a general rule. She is directed to take apples. Many times she has seen things which were called apples, and has noticed their common quality. She knows how to find such things now; and as long as they are sound and fine, any apples will do. What she desires is something of a given quality; what she has to take is this or that particular apple. From the nature of things, she cannot take the quality but must take the particular thing. Sensation and volition being affairs of action and reaction relate to particular things. She has seen only particular apples, and can take only particular apples. But desire has nothing to do with particulars; it relates to qualities. Desire is not a reaction with reference to a particular thing; it is an idea about an idea, namely, the idea of how delightful it would be for me, the cook's master,

to eat an apple pie. However, what is desired is not a mere unattached quality; what is desired is that the dream of eating an apple pie should be realized in Me; and this Me is an object of experience. So with the cook's desire. She has no particular apple pie she particularly prefers to serve; but she does desire and intend to serve an apple pie to a particular person. When she goes into the cellar for the apples, she takes whatever bowl or basket comes handy, without caring what one, so long as it has a certain size, is clean, and has other qualities, but having once selected it, into that particular bowl she intends to put some apples. She takes any apples that are handy and seem good; but having taken them she means to make a pie of those apples. If she chances to see some others in the kitchen, on her return from the cellar, she will not use them for the pie, unless for some reason she changes her mind. Throughout her whole proceedings she pursues an idea or dream without any particular thisness or thatness — or, as we say, *hecceity* — to it, but this dream she wishes to realize in connection with an object of experience, which as such, does possess hecceity; and since she has to act, and action only relates to this and that, she has to be perpetually making random selections, that is, taking whatever comes handiest.

342. The dream itself has no prominent thirdness; it is, on the contrary, utterly irresponsible; it is whatever it pleases. The object of experience as a reality is a second. But the desire in seeking to attach the one to the other is a third, or medium.

So it is with any law of nature. Were it but a mere idea unrealized — and it is of the nature of an idea — it would be a pure first. The cases to which it applies, are seconds.

§3. THE REALITY OF THIRDNESS*

343. . . . It is impossible to resolve everything in our thoughts into those two elements [of Firstness and Secondness]. We may say that the bulk of what is actually done consists of Secondness — or better, Secondness is the predominant character of what *has been* done. The immediate present, could we seize it, would have no character but its Firstness. Not that I mean to say that immediate consciousness (a pure fiction, by

* From the "Lowell Lectures of 1903," III, vol. 1, 3d Draught. See 324 and 521.

the way), would be Firstness, but that the *quality* of what we are immediately conscious of, which is no fiction, is Firstness. But we constantly predict what is to be. Now what is to be, according to our conception of it, can never become wholly past. In general, we may say that *meanings* are inexhaustible. We are too apt to think that what one *means* to do and the *meaning* of a word are quite unrelated meanings of the word "meaning," or that they are only connected by both referring to some actual operation of the mind. Professor Royce especially in his great work *The World and the Individual* has done much to break up this mistake. In truth the only difference is that when a person *means* to do anything he is in some state in consequence of which the brute reactions between things will be moulded [in] to conformity to the form to which the man's mind is itself moulded, while the meaning of a word really lies in the way in which it might, in a proper position in a proposition believed, tend to mould the conduct of a person into conformity to that to which it is itself moulded. Not only will meaning always, more or less, in the long run, mould reactions to itself, but it is only in doing so that its own being consists. For this reason I call this element of the phenomenon or object of thought the element of Thirdness. It is that which is what it is by virtue of imparting a quality to reactions in the future.

344. There is a strong tendency in us all to be sceptical about there being any real meaning or law in things. This scepticism is strongest in the most masculine thinkers. I applaud scepticism with all my heart, provided it have four qualities: first, that it be sincere and real doubt; second, that it be aggressive; third, that it push inquiry; and fourth, that it stand ready to acknowledge what it now doubts, as soon as the doubted element comes clearly to light. To be angry with sceptics, who, whether they are aware of it or not, are the best friends of spiritual truth, is a manifest sign that the angry person is himself infected with scepticism — not, however, of the innocent and wholesome kind that tries to bring truth to light, but of the mendacious, clandestine, disguised, and conservative variety that is afraid of truth, although truth merely means the way to attain one's purposes. If the sceptics think that any account can be given of the phenomena of the universe while

they leave Meaning out of account, by all means let them go ahead and try to do it. It is a most laudable and wholesome enterprise. But when they go so far as to say that there is no such idea in our minds, irreducible to anything else, I say to them, "Gentlemen, your strongest sentiment, to which I subscribe with all my heart, is that a man worthy of that name will not allow petty intellectual predilections to blind him to truth, which consists in the conformity of his thoughts to his purposes. But you know there is such a thing as a defect of candor of which one is not oneself aware. You perceive, no doubt, that if there be an element of thought irreducible to any other, it would be hard, on your principles, to account for man's having it, unless he derived it from environing Nature. But if, because of that, you were to turn your gaze away from an idea that shines out clearly in your mind, you would be violating your principles in a very much more radical way."

345. I will sketch a proof that the idea of meaning is irreducible to those of quality and reaction. It depends on two main premisses. The first is that every genuine triadic relation involves meaning, as meaning is obviously a triadic relation. The second is that a triadic relation is inexpressible by means of dyadic relations alone. Considerable reflexion may be required to convince yourself of the first of these premisses, that every triadic relation involves meaning. There will be two lines of inquiry. First, all physical forces appear to subsist between pairs of particles. This was assumed by Helmholtz in his original paper, *On the Conservation of Forces*.* Take any fact in physics of the triadic kind, by which I mean a fact which can only be defined by simultaneous reference to three things, and you will find there is ample evidence that it never was produced by the action of forces on mere dyadic conditions. Thus, your right hand is that hand which is toward the *east*, when you face the *north* with your head toward the *zenith*. Three things, east, west, and up, are required to define the difference between right and left. Consequently chemists find that those substances which rotate the plane of polarization to the right or left can only be produced from such [similar] active substances. They are all of such complex constitution

* *Über die Erhaltung der Kraft, Einleitung* (1847). See 1889 ed. in Ostwald's "Klassiker d.E.W." series.

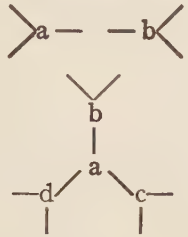
that they cannot have existed when the earth was very hot, and how the first one was produced is a puzzle. It cannot have been by the action of brute forces. For the second branch of the inquiry, you must train yourself to the analysis of relations, beginning with such as are very markedly triadic, gradually going on to others. In that way, you will convince yourself thoroughly that every genuine triadic relation involves thought or *meaning*. Take, for example, the relation of *giving*. A *gives* B to C. This does not consist in A's throwing B away and its accidentally hitting C, like the date-stone, which hit the Jinnee in the eye. If that were all, it would not be a genuine triadic relation, but merely one dyadic relation followed by another. There need be no motion of the thing given. Giving is a transfer of the right of property. Now right is a matter of law, and law is a matter of thought and meaning. I there leave the matter to your own reflection, merely adding that, though I have inserted the word "genuine," yet I do not really think that necessary. I think even degenerate triadic relations involve something like thought.

346. The other premiss of the argument that genuine triadic relations can never be built of dyadic relations and of qualities is easily shown. In existential graphs, a spot with one tail —X represents a quality, a spot with two tails — R — a dyadic relation.* Joining the ends of two tails is also a dyadic relation. But you can never by such joining make a graph with three tails. You may think that a node connecting three lines of identity Y is not a triadic idea. But analysis will show that it is so. I see a man on Monday. On Tuesday I see a man, and I exclaim, "Why, that is the *very* man I saw on Monday." We may say, with sufficient accuracy, that I directly experienced the identity. On Wednesday I see a man and I say, "That is the same man I saw on Tuesday, and consequently is the same I saw on Monday." There is a recognition of triadic identity; but it is only brought about as a conclusion from two premisses, which is itself a triadic relation. If I see two men at once, I cannot by any such direct experience identify both of them with a man I saw before. I can only identify them if I regard them, not as the *very* same, but as two different manifestations of the same man. But the idea of

* See vol. 4, bk. II.

manifestation is the idea of a sign. Now a sign is something, A, which denotes some fact or object, B, to some interpretant thought, C.

347. It is interesting to remark that while a graph with three tails cannot be made out of graphs each with two or one tail, yet combinations of graphs of three tails each will suffice to build graphs with every higher number of tails.



And analysis will show that every relation which is *tetradic*, *pentadic*, or of any greater number of correlates is nothing but a compound of triadic relations. It is therefore not surprising to find that beyond the three elements of Firstness, Secondness, and Thirdness, there is nothing else to be found in the phenomenon.

348. As to the common aversion to recognizing *thought* as an active factor in the real world, some of its causes are easily traced. In the first place, people are persuaded that everything that happens in the material universe is a motion completely determined by inviolable laws of dynamics; and that, they think, leaves no room for any other influence. But the laws of dynamics stand on quite a different footing from the laws of gravitation, elasticity, electricity, and the like. The laws of dynamics are very much like logical principles, if they are not precisely that. They only say how bodies will move after you have said what the forces are. They permit any forces, and therefore any motions. Only, the principle of the conservation of energy requires us to explain certain kinds of motions by special hypotheses about molecules and the like. Thus, in order that the viscosity of gases should not disobey that law we have to suppose that gases have a certain molecular constitution. Setting dynamical laws to one side, then, as hardly being positive laws, but rather mere formal principles, we have only the laws of gravitation, elasticity, electricity, and chem-

istry. Now who will deliberately say that our knowledge of these laws is sufficient to make us reasonably confident that they are absolutely eternal and immutable, and that they escape the great law of evolution? Each hereditary character is a law, but it is subject to developement and to decay. Each habit of an individual is a law; but these laws are modified so easily by the operation of self-control, that it is one of the most patent of facts that ideals and thought generally have a very great influence on human conduct. That truth and justice are great powers in the world is no figure of speech, but a plain fact to which theories must accommodate themselves.

349. The child, with his wonderful genius for *language*, naturally looks upon the world as chiefly governed by thought; for thought and expression are really one. As Wordsworth truly says, the child is quite right in this; he is an

"eye among the blind,
"On whom those truths do rest
"Which we are toiling all our lives to find."

But as he grows up, he loses this faculty; and all through his childhood he has been stuffed with such a pack of lies, which parents are accustomed to think are the most wholesome food for the child — because they do not think of his future — that he begins real life with the utmost contempt for all the ideas of his childhood; and the great truth of the immanent power of thought in the universe is flung away along with the lies. I offer this hypothetical explanation because, if the common aversion to regarding thought as a real power, or as anything but a fantastic figment, were really natural, it would make an argument of no little strength against its being acknowledged as a real power.

§4. PROTOPLASM AND THE CATEGORIES*

350. Thus mathematical considerations, by which I mean study as purely *a priori* and necessary as thought can be, have suggested and indeed insisted upon a classification of the elements of the phaneron, and so of the functions of the mind, and of the nervous system, of protoplasm itself, which empirical science will find very convenient. Instead of the familiar division of Tetens or Kant which makes pleasure-pain, cognition,

* From an unidentified fragment.

and volition the three categories of mental phenomena, we have feeling or quality, the action of opposition, and synthetic thought.

351. As to protoplasm, what the three *cenopythagorean categories*, as I call them, do, and what they are limited to doing, is to call attention to three very different characters of this chemical body. The first is a *posse* which it has in itself; for the *priman* stops at *can-bes* and never reaches to existence, which depends on interaction, or *secundanity*. This internal power which the category merely suggests, we recognize as that of feeling. Though it is priman, it is without any doubt dependent upon the extreme complexity of the protoplasmic molecule, if the word molecule can be applied to so intricate, unstable, and ununified a system. But it is the law of high numbers that extreme complication with a great multitude of independent similars results in a new simplicity. Next there is reactive force, a twoness, which is emphasized in the nerve cells together. It is the property by which any state of high cohesiveness tends to spread through the albuminoid matter. We usually call the property contractility. Thirdly, the categories suggest our looking for a synthetizing law; and this we find in the power of assimilation, incident to which is the habit-taking faculty. This is all the categories pretend to do. They suggest a way of thinking; and the possibility of science depends upon the fact that human thought necessarily partakes of whatever character is diffused through the whole universe, and that its natural modes have some tendency to be the modes of action of the universe.

352. In the study of logic I have found the cenopythagorean categories unlock many a secret.

§5. THE INTERDEPENDENCE OF THE CATEGORIES*

353. Perhaps it is not right to call these categories conceptions; they are so intangible that they are rather tones or tints upon conceptions. In my first attempt to deal with them,† I made use of three grades of separability of one idea from another. In the first place, two ideas may be so little allied

* From "One, Two, Three," c. 1880.

† See ch. 6.

that one of them may be present to the consciousness in an image which does not contain the other at all; in this way we can imagine *red* without imagining blue, and *vice versa;* we can also imagine sound without melody, but not melody without sound. I call this kind of separation *dissociation.* In the second place, even in cases where two conceptions cannot be separated in the imagination, we can often suppose one without the other, that is we can imagine data from which we should be led to believe in a state of things where one was separated from the other. Thus, we can suppose uncolored space, though we cannot dissociate space from color. I call this mode of separation *prescission.* In the third place, even when one element cannot even be supposed without another, they may oftimes be distinguished from one another. Thus we can neither imagine nor suppose a taller without a shorter, yet we can distinguish the taller from the shorter. I call this mode of separation *distinction.* Now, the categories cannot be dissociated in imagination from each other, nor from other ideas. The category of first can be prescinded from second and third, and second can be prescinded from third. But second cannot be prescinded from first, nor third from second. The categories may, I believe, be prescinded from any other one conception, but they cannot be prescinded from some one and indeed many elements. You cannot suppose a first unless that first be something definite and more or less definitely supposed. Finally, though it is easy to distinguish the three categories from one another, it is extremely difficult accurately and sharply to distinguish each from other conceptions so as to hold it in its purity and yet in its full meaning.

CHAPTER 3

A GUESS AT THE RIDDLE[*P]

PLAN OF THE WORK

354. Section 1. One, Two, Three. Already written.

Section 2. The triad in reasoning. Not touched. It is to be made as follows. 1. Three kinds of signs; as best shown in my last paper in the *Am. Jour. Math.*† 2. Term, proposition, and argument, mentioned in my paper on a new list of categories.‡ 3. Three kinds of argument, deduction, induction, hypothesis, as shown in my paper in *Studies in Logic.*§ Also three figures of syllogism, as shown there and in my paper on the Classification of Arguments.¶ 4. Three kinds of terms, absolute, relative, and conjugative, as shown in my first paper on Logic of Relatives.‖ There are various other triads which may be alluded to. The dual divisions of logic result from a false way of looking at things absolutely. Thus, besides affirmative and negative, there are really probable enunciations, which are intermediate. So besides universal and particular there are all sorts of propositions of numerical quantity. For example, the particular proposition: Some A is B, means "At least one A is B." But we can also say: At least 2 A's are B's. Also, All the A's but one are B's, etc., etc., *ad infinitum*. We pass from dual quantity, or a system of quantity such as that of Boolian algebra, where there are only two values, to plural quantity.

* c. 1890. One of the drafts of this work is headed: "Notes for a Book, to be entitled 'A Guess at the Riddle,' with a Vignette of the Sphynx below the Title." This caption is followed by the remark, "And this book, if ever written, as it soon will be if I am in a situation to do it, will be one of the births of time."

† 3. 359ff.

‡ Ch. 6 below.

§ Vol. 2, bk. III, ch. 8.

¶ Vol. 2, bk. III, ch. 2.

‖ Vol. 3, No. III.

Section 3. The triad in metaphysics. This chapter, one of the best, is to treat of the theory of cognition.

Section 4. The triad in psychology. The greater part is written.

Section 5. The triad in physiology. The greater part is written.

Section 6. The triad in biology. This is to show the true nature of the Darwinian hypothesis.

Section 7. The triad in physics. The germinal section. 1. The necessity of a natural history of the laws of nature, so that we may get some notion of what to expect. 2. The logical postulate for explanation forbids the assumption of any absolute. That is, it calls for the introduction of Thirdness. 3. Metaphysics is an imitation of geometry; and mathematicians having declared against axioms, the metaphysical axioms are destined to fall too. 4. Absolute chance. 5. The universality of the principle of habit. 6. The whole theory stated. 7. Consequences.

Section 8. The triad in sociology or, shall I say, pneumatology. That the consciousness is a sort of public spirit among the nerve-cells. Man as a community of cells; compound animals and composite plants; society; nature. Feeling implied in Firstness.

Section 9. The triad in theology. Faith requires us to be materialists without flinching.*

§1. TRICHOTOMY†[P]

355. Perhaps I might begin by noticing how different numbers have found their champions. Two was extolled by Peter Ramus, Four by Pythagoras, Five by Sir Thomas Browne, and so on. For my part, I am a determined foe of no innocent number; I respect and esteem them all in their several ways; but I am forced to confess to a leaning to the number Three in philosophy. In fact, I make so much use of threefold divisions in my speculations, that it seems best to commence by making a slight preliminary study of the conceptions upon which all such divisions must rest. I mean no more than the ideas of

* The last two sections do not seem to have been written.

† The sections of this book were originally called "chapters." There are a number of alternative versions of this section. 1 and 2 of the preface, are from one such alternative.

first, second, third — ideas so broad that they may be looked upon rather as moods or tones of thought, than as definite notions, but which have great significance for all that. Viewed as numerals, to be applied to what objects we like, they are indeed thin skeletons of thought, if not mere words. If we only wanted to make enumerations, it would be out of place to ask for the significations of the numbers we should have to use; but then the distinctions of philosophy are supposed to attempt something far more than that; they are intended to go down to the very essence of things, and if we are to make one single threefold philosophical distinction, it behooves us to ask beforehand what are the kinds of objects that are first, second, and third, not as being so counted, but in their own true characters. That there are such ideas of the really first, second, and third, we shall presently find reason to admit.

356. The first is that whose being is simply in itself, not referring to anything nor lying behind anything. The second is that which is what it is by force of something to which it is second. The third is that which is what it is owing to things between which it mediates and which it brings into relation to each other.

357. The idea of the absolutely first must be entirely separated from all conception of or reference to anything else; for what involves a second is itself a second to that second. The first must therefore be present and immediate, so as not to be second to a representation. It must be fresh and new, for if old it is second to its former state. It must be initiative, original, spontaneous, and free; otherwise it is second to a determining cause. It is also something vivid and conscious; so only it avoids being the object of some sensation. It precedes all synthesis and all differentiation; it has no unity and no parts. It cannot be articulately thought: assert it, and it has already lost its characteristic innocence; for assertion always implies a denial of something else. Stop to think of it, and it has flown! What the world was to Adam on the day he opened his eyes to it, before he had drawn any distinctions, or had become conscious of his own existence — that is first, present, immediate, fresh, new, initiative, original, spontaneous, free, vivid, conscious, and evanescent. Only, remember that every description of it must be false to it.

358. Just as the first is not absolutely first if thought along with a second, so likewise to think the second in its perfection we must banish every third. The second is therefore the absolute last. But we need not, and must not, banish the idea of the first from the second; on the contrary, the second is precisely that which cannot be without the first. It meets us in such facts as another, relation, compulsion, effect, dependence, independence, negation, occurrence, reality, result. A thing cannot be other, negative, or independent, without a first to or of which it shall be other, negative, or independent. Still, this is not a very deep kind of secondness; for the first might in these cases be destroyed yet leave the real character of the second absolutely unchanged. When the second suffers some change from the action of the first, and is dependent upon it, the secondness is more genuine. But the dependence must not go so far that the second is a mere accident or incident of the first; otherwise the secondness again degenerates. The genuine second suffers and yet resists, like dead matter, whose existence consists in its inertia. Note, too, that for the second to have the finality that we have seen belongs to it, it must be determined by the first immovably, and thenceforth be fixed; so that unalterable fixity becomes one of its attributes. We find secondness in occurrence, because an occurrence is something whose existence consists in our knocking up against it. A hard fact is of the same sort; that is to say, it is something which is there, and which I cannot think away, but am forced to acknowledge as an object or second beside myself, the subject or number one, and which forms material for the exercise of my will.

The idea of second must be reckoned as an easy one to comprehend. That of first is so tender that you cannot touch it without spoiling it; but that of second is eminently hard and tangible. It is very familiar, too; it is forced upon us daily; it is the main lesson of life. In youth, the world is fresh and we seem free; but limitation, conflict, constraint, and secondness generally, make up the teaching of experience. With what firstness

"The scarfed bark puts from her native bay;"

with what secondness

"doth she return,
With overweathered ribs and ragged sails."

But familiar as the notion is, and compelled as we are to acknowledge it at every turn, still we never can realize it; we never can be immediately conscious of finiteness, or of anything but a divine freedom that in its own original firstness knows no bounds.

359. First and second, agent and patient, yes and no, are categories which enable us roughly to describe the facts of experience, and they satisfy the mind for a very long time. But at last they are found inadequate, and the third is the conception which is then called for. The third is that which bridges over the chasm between the absolute first and last, and brings them into relationship. We are told that every science has its qualitative and its quantitative stage; now its qualitative stage is when dual distinctions — whether a given subject has a given predicate or not — suffice; the quantitative stage comes when, no longer content with such rough distinctions, we require to insert a possible halfway between every two possible conditions of the subject in regard to its possession of the quality indicated by the predicate. Ancient mechanics recognized forces as causes which produced motions as their immediate effects, looking no further than the essentially dual relation of cause and effect. That was why it could make no progress with dynamics. The work of Galileo and his successors lay in showing that forces are accelerations by which [a] state of velocity is gradually brought about. The words "cause" and "effect" still linger, but the old conceptions have been dropped from mechanical philosophy; for the fact now known is that in certain relative positions bodies undergo certain accelerations. Now an acceleration, instead of being like a velocity a relation between two successive positions, is a relation between three; so that the new doctrine has consisted in the suitable introduction of the conception of threeness. On this idea, the whole of modern physics is built. The superiority of modern geometry, too, has certainly been due to nothing so much as to the bridging over of the innumerable distinct cases with which the ancient science was encumbered; and we may go so far as to say that all the great steps in the method of science in every department have consisted in bringing into relation cases previously discrete.

360. We can easily recognize the man whose thought is

mainly in the dual stage by his unmeasured use of language. In former days, when he was natural, everything with him was unmitigated, absolute, ineffable, utter, matchless, supreme, unqualified, root and branch; but now that it is the fashion to be depreciatory, he is just as plainly marked by the ridiculous inadequacy of his expressions. The principle of contradiction is a shibboleth for such minds; to disprove a proposition they will always try to prove there lurks a contradiction in it, notwithstanding that it may be as clear and comprehensible as the day. Remark for your amusement the grand unconcern with which mathematics, since the invention of the calculus, has pursued its way, caring no more for the peppering of contradiction-mongers than an ironclad for an American fort.

361. We have seen that it is the immediate consciousness that is preëminently first, the external dead thing that is preëminently second. In like manner, it is evidently the representation mediating between these two that is preëminently third. Other examples, however, should not be neglected. The first is agent, the second patient, the third is the action by which the former influences the latter. Between the beginning as first, and the end as last, comes the process which leads from first to last.

362. According to the mathematicians, when we measure along a line, were our yardstick replaced by a yard marked off on an infinitely long rigid bar, then in all the shiftings of it which we make for the purpose of applying it to successive portions of the line to be measured, two points on that bar would remain fixed and unmoved. To that pair of points, the mathematicians accord the title of the absolute; they are the points that are at an infinite distance one way and the other as measured by that yard. These points are either really distinct, coincident, or imaginary (in which case there is but a finite distance completely round the line), according to the relation of the mode of measurement to the nature of the line upon which the measurement is made. These two points are the absolute first and the absolute last or second, while every measurable point on the line is of the nature of a third. We have seen that the conception of the absolute first eludes every attempt to grasp it; and so in another sense does that of the absolute second; but there is no absolute third, for the third is of its own nature relative, and this is what we are always think-

ing, even when we aim at the first or second. The starting-point of the universe, God the Creator, is the Absolute First; the terminus of the universe, God completely revealed, is the Absolute Second; every state of the universe at a measurable point of time is the third. If you think the measurable is all there is, and deny it any definite tendency whence or whither, then you are considering the pair of points that makes the absolute to be imaginary and are an Epicurean. If you hold that there is a definite drift to the course of nature as a whole, but yet believe its absolute end is nothing but the Nirvana from which it set out, you make the two points of the absolute to be coincident, and are a pessimist. But if your creed is that the whole universe is approaching in the infinitely distant future a state having a general character different from that toward which we look back in the infinitely distant past, you make the absolute to consist in two distinct real points and are an evolutionist.[1] This is one of the matters concerning which a man can only learn from his own reflections, but I believe that if my suggestions are followed out, the reader will grant that one, two, three, are more than mere count-words like "eeny, meeny, miny, mo," but carry vast, though vague ideas.

363. But it will be asked, why stop at three? Why not go on to find a new conception in four, five, and so on indefinitely? The reason is that while it is impossible to form a genuine three by any modification of the pair, without introducing something of a different nature from the unit and the pair, four, five, and every higher number can be formed by mere complications of threes. To make this clear, I will first show it in an example. The fact that A presents B with a gift C, is a triple relation, and as such cannot possibly be resolved into any combination of dual relations. Indeed, the very idea of a combination involves that of thirdness, for a combination is something which is what it is owing to the parts which it brings into mutual relationship. But we may waive that consideration, and still we cannot build up the fact that A presents C to B by

[1] The last view is essentially that of Christian theology, too. The theologians hold the physical universe to be finite, but considering that universe which they will admit to have existed from all time, it would appear to be in a different condition in the end from what it was in the beginning, the whole spiritual creation having been accomplished, and abiding.

any aggregate of dual relations between A and B, B and C, and C and A. A may enrich B, B may receive C, and A may part with C, and yet A need not necessarily give C to B. For that, it would be necessary that these three dual relations should not only coexist, but be welded into one fact. Thus we see that a triad cannot be analyzed into dyads. But now I will show by an example that a four can be analyzed into threes. Take the quadruple fact that A sells C to B for the price D. This is a compound of two facts: first, that A makes with C a certain transaction, which we may name E; and second, that this transaction E is a sale of B for the price D. Each of these two facts is a triple fact, and their combination makes up [as] genuine [a] quadruple fact as can be found. The explanation of this striking difference is not far to seek. A dual relative term, such as "lover" or "servant," is a sort of blank form, where there are two places left blank. I mean that in building a sentence round "lover," as the principal word of the predicate, we are at liberty to make anything we see fit the subject, and then, besides that, anything we please the object of the action of loving. But a triple relative term such as "giver" has two correlates, and is thus a blank form with three places left blank. Consequently, we can take two of these triple relatives and fill up one blank place in each with the same letter, X, which has only the force of a pronoun or identifying index, and then the two taken together will form a whole having four blank places; and from that we can go on in a similar way to any higher number. But when we attempt to imitate this proceeding with dual relatives, and combine two of them by means of an X, we find we only have two blank places in the combination, just as we had in either of the relatives taken by itself. A road with only three-way forkings may have any number of termini, but no number of straight roads put end on end will give more than two termini. Thus any number, however large, can be built out of triads; and consequently no idea can be involved in such a number, radically different from the idea of three. I do not mean to deny that the higher numbers may present interesting special configurations from which notions may be derived of more or less general applicability; but these cannot rise to the height of philosophical categories so fundamental as those that have been considered.

364. The argument of this book has been developed in the mind of the author, substantially as it is presented, as a following out of these three conceptions, in a sort of game of "follow-my-leader" from one field of thought into another. Their importance was originally brought home to me in the study of logic, where they play so remarkable a part that I was led to look for them in psychology. Finding them there again, I could not help asking myself whether they did not enter into the physiology of the nervous system. By drawing a little on hypothesis, I succeeded in detecting them there; and then the question naturally came how they would appear in the theory of protoplasm in general. Here I seemed to break into an interesting avenue of reflections giving instructive *aperçus* both into the nature of protoplasm and into the conceptions themselves; though it was not till later that I mapped out my thoughts on the subject as they are presented in Section 4. I had no difficulty in following the lead into the domain of natural selection; and once arrived at that point, I was irresistibly carried on to speculations concerning physics. One bold saltus landed me in a garden of fruitful and beautiful suggestions, the exploration of which long prevented my looking further. As soon, however, as I was induced to look further, and to examine the application of the three ideas to the deepest problems of the soul, nature, and God, I saw at once that they must carry me far into the heart of those primeval mysteries. That is the way the book has grown in my mind: it is also the order in which I have written it; and only this first chapter is more or less an afterthought, since at an earlier stage of my studies I should have looked upon the matter here set down as too vague to have any value. I should have discerned in it too strong a resemblance to many a crack-brained book that I had laughed over. A deeper study has taught me that even out of the mouths of babes and sucklings strength may be brought forth, and that weak metaphysical trash has sometimes contained the germs of conceptions capable of growing up into important and positive doctrines.

365. Thus, the whole book being nothing but a continual exemplification of the triad of ideas, we need linger no longer upon this preliminary exposition of them. There is, however, one feature of them upon which it is quite indispensable to

dwell. It is that there are two distinct grades of Secondness and three grades of Thirdness. There is a close analogy to this in geometry. Conic sections are either the curves usually so called, or they are pairs of straight lines. A pair of straight lines is called a degenerate conic. So plane cubic curves are either the genuine curves of the third order, or they are conics paired with straight lines, or they consist of three straight lines; so that there are the two orders of degenerate cubics. Nearly in this same way, besides genuine Secondness, there is a degenerate sort which does not exist as such, but is only so conceived. The medieval logicians (following a hint of Aristotle) distinguished between real relations and relations of reason. A real relation subsists in virtue of a fact which would be totally impossible were either of the related objects destroyed; while a relation of reason subsists in virtue of two facts, one only of which would disappear on the annihilation of either of the relates. Such are all resemblances: for any two objects in nature resemble each other, and indeed in themselves just as much as any other two; it is only with reference to our senses and needs that one resemblance counts for more than another. Rumford and Franklin resembled each other by virtue of being both Americans; but either would have been just as much an American if the other had never lived. On the other hand, the fact that Cain killed Abel cannot be stated as a mere aggregate of two facts, one concerning Cain and the other concerning Abel. Resemblances are not the only relations of reason, though they have that character in an eminent degree. Contrasts and comparisons are of the same sort. Resemblance is an identity of characters; and this is the same as to say that the mind gathers the resembling ideas together into one conception. Other relations of reason arise from ideas being connected by the mind in other ways; they consist in the relation between two parts of one complex concept, or, as we may say, in the relation of a complex concept to itself, in respect to two of its parts. This brings us to consider a sort of degenerate Secondness that does not fulfill the definition of a relation of reason. Identity is the relation that everything bears to itself: Lucullus dines with Lucullus. Again, we speak of allurements and motives in the language of forces, as though a man suffered compulsion from within. So with the voice of conscience: and

we observe our own feelings by a reflective sense. An echo is my own voice coming back to answer itself. So also, we speak of the abstract quality of a thing as if it were some second thing that the first thing possesses. But the relations of reason and these self-relations are alike in this, that they arise from the mind setting one part of a notion into relation to another. All degenerate seconds may be conveniently termed internal, in contrast to external seconds, which are constituted by external fact, and are true actions of one thing upon another.

366. Among thirds, there are two degrees of degeneracy. The first is where there is in the fact itself no Thirdness or mediation, but where there is true duality; the second degree is where there is not even true Secondness in the fact itself. Consider, first, the thirds degenerate in the first degree. A pin fastens two things together by sticking through one and also through the other: either might be annihilated, and the pin would continue to stick through the one which remained. A mixture brings its ingredients together by containing each. We may term these accidental thirds. "How did I slay thy son?" asked the merchant, and the jinnee replied, "When thou threwest away the date-stone, it smote my son, who was passing at the time, on the breast, and he died forthright." Here there were two independent facts, first that the merchant threw away the date-stone, and second that the date-stone struck and killed the jinnee's son. Had it been aimed at him, the case would have been different; for then there would have been a relation of aiming which would have connected together the aimer, the thing aimed, and the object aimed at, in one fact. What monstrous injustice and inhumanity on the part of that jinnee to hold that poor merchant responsible for such an accident! I remember how I wept at it, as I lay in my father's arms and he first told me the story. It is certainly just that a man, even though he had no evil intention, should be held responsible for the immediate effects of his actions; but not for such as might result from them in a sporadic case here and there, but only for such as might have been guarded against by a reasonable rule of prudence. Nature herself often supplies the place of the intention of a rational agent in making a Thirdness genuine and not merely accidental; as when a spark, as third, falling into a barrel of gunpowder, as first, causes an

explosion, as second. But how does nature do this? By virtue of an intelligible law according to which she acts. If two forces are combined according to the parallelogram of forces, their resultant is a real third. Yet any force may, by the parallelogram of forces, be mathematically resolved into the sum of two others, in an infinity of different ways. Such components, however, are mere creations of the mind. What is the difference? As far as one isolated event goes, there is none; the real forces are no more present in the resultant than any components that the mathematician may imagine. But what makes the real forces really there is the general law of nature which calls for them, and not for any other components of the resultant. Thus, intelligibility, or reason objectified, is what makes Thirdness genuine.

367. We now come to thirds degenerate in the second degree. The dramatist Marlowe had something of that character of diction in which Shakespeare and Bacon agree. This is a trivial example; but the mode of relation is important. In natural history, intermediate types serve to bring out the resemblance between forms whose similarity might otherwise escape attention, or not be duly appreciated. In portraiture, photographs mediate between the original and the likeness. In science, a diagram or analogue of the observed fact leads on to a further analogy. The relations of reason which go to the formation of such a triple relation need not be all resemblances. Washington was eminently free from the faults in which most great soldiers resemble one another. A centaur is a mixture of a man and a horse. Philadelphia lies between New York and Washington. Such thirds may be called intermediate thirds or thirds of comparison.

368. Nobody will suppose that I wish to claim any originality in reckoning the triad important in philosophy. Since Hegel, almost every fanciful thinker has done the same. Originality is the last of recommendations for fundamental conceptions. On the contrary, the fact that the minds of men have ever been inclined to threefold divisions is one of the considerations in favor of them. Other numbers have been objects of predilection to this philosopher and that, but three has been prominent at all times and with all schools. My whole method will be found to be in profound contrast with that of

Hegel; I reject his philosophy *in toto*. Nevertheless, I have a certain sympathy with it, and fancy that if its author had only noticed a very few circumstances he would himself have been led to revolutionize his system. One of these is the double division or dichotomy of the second idea of the triad. He has usually overlooked external Secondness, altogether. In other words, he has committed the trifling oversight of forgetting that there is a real world with real actions and reactions. Rather a serious oversight that. Then Hegel had the misfortune to be unusually deficient in mathematics. He shows this in the very elementary character of his reasoning. Worse still, while the whole burden of his song is that philosophers have neglected to take Thirdness into account, which is true enough of the theological kind, with whom alone he was acquainted (for I do not call it acquaintance to look into a book without comprehending it), he unfortunately did not know, what it would have been of the utmost consequence for him to know, that the mathematical analysts had in great measure escaped this great fault, and that the thorough-going pursuit of the ideas and methods of the differential calculus would be sure to cure it altogether. Hegel's dialectical method is only a feeble and rudimentary application of the principles of the calculus to metaphysics. Finally Hegel's plan of evolving everything out of the abstractest conception by a dialectical procedure, though far from being so absurd as the experientialists think, but on the contrary representing one of the indispensable parts of the course of science, overlooks the weakness of individual man, who wants the strength to wield such a weapon as that.

§2. THE TRIAD IN REASONING*

369. Kant, the King of modern thought, it was who first remarked the frequency in logical analytics of *trichotomics* or threefold distinctions. It really is so; I have tried hard and long to persuade myself that it is only fanciful, but the facts will not countenance that way of disposing of the phenomenon. Take any ordinary syllogism:

* From "One, Two, Three: Fundamental Categories of Thought and of Nature," c. 1885. This paper does not seem to form part of "A Guess at the Riddle," but is here inserted to take the place of the unwritten section 2 of the original work.

All men are mortal,
Elijah was a man;
Therefore, Elijah was mortal.

There are here three propositions, namely, two premisses and a conclusion; there are also three terms, *man*, *mortal*, and *Elijah*. If we transpose one of the premisses with the conclusion, denying both, we obtain what are called the indirect figures of syllogism; for example

All men are mortal,
But Elijah was not mortal;
Therefore, Elijah was not a man.

Elijah was not mortal,
But Elijah was a man;
Therefore, some men are not mortal.

Thus, there are three figures of ordinary syllogism. It is true there are other modes of inference which do not come under any of these heads; but that does not annul the fact that we have here a trichotomy. Indeed, if we examine by itself what is by some logicians called the fourth figure, we find that it also has three varieties related to one another as the three figures of ordinary syllogism. There is an entirely different way of conceiving the relations of the figures of syllogism; namely, by means of the conversion of propositions. But from that point of view also, the same classes are preserved. DeMorgan* has added a large number of new syllogistic moods which do not find places in this classification. The reasoning in these is of a peculiar character and introduces the principle of dilemma. Still, regarding these dilemmatic reasonings by themselves, they fall into three classes in a precisely similar manner. Again, I have shown† that the probable and approximate inferences of science must be classified on the very same principles, being either Deductions, Inductions, or Hypotheses. Other examples of threes in logic are statements of what is actual, what is possible, and what is necessary; the three kinds of forms, Names,‡ Propositions, and Inferences;§ affirmative, negative, and uncertain answers to a question. One very important triad is this: it has been found that there are three

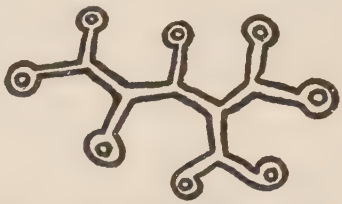
* *Formal Logic*, ch. 8. See also 2.568.
† See vol. 2, bk. III, chs. 2 and 5.
‡ Or Terms, but see 372.
§ Or Arguments.

kinds of signs which are all indispensable in all reasoning; the first is the diagrammatic sign or *icon*, which exhibits a similarity or analogy to the subject of discourse; the second is the *index*, which like a pronoun demonstrative or relative, forces the attention to the particular object intended without describing it; the third [or symbol] is the general name or description which signifies its object by means of an association of ideas or habitual connection between the name and the character signified.

370. But there is one triad in particular which throws a strong light on the nature of all the others. Namely, we find it necessary to recognize in logic three kinds of characters, three kinds of facts. First there are *singular* characters which are predicable of single objects, as when we say that anything is white, large, etc. Secondly, there are dual characters which appertain to pairs of objects; these are implied by all relative terms as "lover," "similar," "other," etc. Thirdly, there are plural characters, which can all be reduced to triple characters but not to dual characters. Thus, we cannot express the fact that A is a benefactor of B by any descriptions of A and B separately; we must introduce a relative term. This is requisite, not merely in English, but in every language which might be invented. This is true even of such a fact as A is taller than B. If we say, "A is tall, but B is short," the conjugation "but" has a relative force, and if we omit this word the mere collocation of the two sentences is a relative or dual mode of signifying. . . .

371. Let us now consider a triple character, say that A gives B to C. This is not a mere congeries of dual characters. It is not enough to say that A parts with C, and that B receives C. A synthesis of these two facts must be made to bring them into a single fact; we must express that C, in being parted with by A, is received by B. If, on the other hand, we take a quadruple fact, it is easy to express as a compound of two triple facts. . . . We are here able to express the synthesis of the two facts into one, because a triple character involves the conception of synthesis. Analysis involves the same relations as synthesis; so that we may explain the fact that all plural facts can be reduced to triple facts in this way. A road with a fork in it is the analogue of a triple fact, because it brings three termini into relation with one another. A dual fact is like a

road without a fork; it only connects two termini. Now, no combination of roads without forks can have more than two termini; but any number of termini can be connected by roads which nowhere have a knot of more than three ways. See the



figure, where I have drawn the termini as self-returning roads, in order to introduce nothing beyond the road itself. Thus, the three essential elements of a network of roads are *road about a terminus*, *roadway-connection*, and *branching;* and in like manner, the three fundamental categories of fact are, fact about an object, fact about two objects (relation), fact about several objects (synthetic fact).

372. We have seen that the mere coexistence of two singular facts constitutes a degenerate form of dual fact; and in like manner there are two orders of degeneracy in plural facts, for either they may consist in a mere synthesis of facts of which the highest is dual, or they may consist in a mere synthesis of singular facts. This explains why there should be three classes of *signs;* for there is a triple connection of *sign*, *thing signified*, *cognition produced in the mind*. There may be a mere relation of reason between the sign and the thing signified; in that case the sign is an *icon*. Or there may be a direct physical connection; in that case, the sign is an *index*. Or there may be a relation which consists in the fact that the mind associates the sign with its object; in that case the sign is a *name** [or *symbol*]. Now consider the difference between a logical *term*, a *proposition*, and an *inference*. A term is a mere general description, and as neither *icon* nor *index* possesses generality, it must be a name; and it is nothing more. A proposition is also a general description, but it differs from a term in that it purports to be in a real relation to the fact, to be really determined by it; thus, a proposition can only be formed of the conjunction of a name and an index. An inference, too, contains a general description. . . .

* Cf. 369.

§3. THE TRIAD IN METAPHYSICS[P]

373. I will run over all the conceptions that played an important part in the pre-Socratic philosophy and see how far they can be expressed in terms of one, two, three.

1. The first of all the conceptions of philosophy is that of a primal matter out of which the world is made. Thales and the early Ionian philosophers busied themselves mainly with this. They called it the ἀρχή, the beginning; so that the conception of first was the quintessence of it. Nature was a wonder to them, and they asked its explanation; from what did it come? That was a good question, but it was rather stupid to suppose that they were going to learn much even if they could find out from what sort of matter it was made. But to ask how it had been formed, as they doubtless did, was not an exhaustive question; it would only carry them back a little way. They wished to go to the very beginning at once, and in the beginning there must have been a homogeneous something, for where there was variety they supposed there must be always an explanation to be sought. The first must be indeterminate, and the indeterminate first of anything is the material of which it is formed. Besides, their idea was that they could not tell how the world was formed unless they knew from what to begin their account. The inductive [method] of explaining phenomena by tracing them back step by step to their causes was foreign not only to them but to all ancient and medieval philosophy; that is the Baconian idea. Indeterminacy is really a character of the first. But not the indeterminacy of homogeneity. The first is full of life and variety. Yet that variety is only potential; it is not definitely there. Still, the notion of explaining the variety of the world, which was what they mainly wondered at, by non-variety was quite absurd. How is variety to come out of the womb of homogeneity; only by a principle of spontaneity, which is just that virtual variety that is the first.*

§4. THE TRIAD IN PSYCHOLOGY†[P]

374. The line of reasoning which I propose to pursue is peculiar and will need some careful study to estimate the

* No more of this section seems to have been written; but see vol. 6.

† Cf. vol. 8.

strength of it. I shall review it critically in the last section, but meantime I desire to point out that the step I am about to take, which is analogous to others that will follow, is not so purely of the nature of a guess as might be supposed by persons expert in judging of scientific evidence. We have seen that the ideas of one, two, three, are forced upon us in logic, and really cannot be dispensed with. They meet us not once but at every turn. And we have found reason to think that they are equally important in metaphysics. How is the extraordinary prominence of these conceptions to be explained? Must it not be that they have their origin in the nature of the mind? This is the Kantian form of inference, which has been found so cogent in the hands of that hero of philosophy; and I do not know that modern studies have done anything to discredit it. It is true we no longer regard such a psychological explanation of a conception to be as final as Kant thought. It leaves further questions to be asked; but as far as it goes it seems to be satisfactory. We find the ideas of first, second, third, constant ingredients of our knowledge. It must then either be that they are continually given to us in the presentations of sense, or that it is the peculiar nature of the mind to mix them with our thoughts. Now we certainly cannot think that these ideas are given in sense. First, second, and third are not sensations. They can only be given in sense by things appearing labelled as first, second, and third, and such labels things do not usually bear. They ought therefore to have a psychological origin. A man must be a very uncompromising partisan of the theory of the *tabula rasa* to deny that the ideas of first, second, and third are due to congenital tendencies of the mind. So far there is nothing in my argument to distinguish it from that of many a Kantian. The noticeable thing is that I do not rest here, but seek to put the conclusion to the test by an independent examination of the facts of psychology, to see whether we can find any traces of the existence of three parts or faculties of the soul or modes of consciousness, which might confirm the result just reached.

375. Now, three departments of the mind have been generally recognized since Kant; they are: Feeling [of pleasure and pain], Knowing, and Willing. The unanimity with which this trisection of the mind has been accepted is, indeed, quite sur-

prising. The division did not have its genesis in the peculiar ideas of Kant. On the contrary, it was borrowed by him from dogmatic philosophers, and his acceptance of it was, as has been well remarked, a concession to dogmatism. It has been allowed even by psychologists to whose general doctrines it seems positively hostile.*

376. The ordinary doctrine is open to a variety of objections from the very point of view from which it was first delineated. First, desire certainly includes an element of pleasure quite as much as of will. Wishing is not willing; it is a speculative variation of willing mingled with a speculative and anticipatory feeling of pleasure. Desire should therefore be struck out of the definition of the third faculty, leaving it mere volition. But volition without desire is not voluntary; it is mere activity. Consequently, all activity, voluntary or not, should be brought under the third faculty. Thus attention is a kind of activity which is sometimes voluntary and sometimes not so. Second, pleasure and pain can only be recognized as such in a judgment; they are general predicates which are attached to feelings rather than true feelings. But mere passive feeling, which does not act and does not judge, which has all sorts of qualities but does not itself recognize these qualities, because it does not analyze nor compare — this is an element of all consciousness to which a distinct title ought to be given. Third, every phenomenon of our mental life is more or less like cognition. Every emotion, every burst of passion, every exercise of will, is like cognition. But modifications of consciousness which are alike have some element in common. Cognition, therefore, has nothing distinctive and cannot be regarded as a fundamental faculty. If, however, we ask whether there be not an element in cognition which is neither feeling, sense, nor activity, we do find something, the faculty of learning, acquisition, memory and inference, synthesis. Fourth, looking once more at activity, we observe that the only consciousness we have of it is the sense of resistance. We are conscious of hitting or of getting hit, of meeting with a *fact*. But whether the activity is within or without we know

* A number of manuscript pages seem to be missing here. 376–8 from "One, Two, Three: Fundamental Categories of Thought and Nature," have been substituted for them.

only by secondary signs and not by our original faculty of recognizing fact.

377. It seems, then, that the true categories of consciousness are: first, feeling, the consciousness which can be included with an instant of time, passive consciousness of quality, without recognition or analysis; second, consciousness of an interruption into the field of consciousness, sense of resistance, of an external fact, of another something; third, synthetic consciousness, binding time together, sense of learning, thought.

378. If we accept these [as] the fundamental elementary modes of consciousness, they afford a psychological explanation of the three logical conceptions of quality, relation, and synthesis or mediation. The conception of quality, which is absolutely simple in itself and yet viewed in its relations is seen to be full of variety, would arise whenever feeling or the singular consciousness becomes prominent. The conception of relation comes from the dual consciousness or sense of action and reaction. The conception of mediation springs out of the plural consciousness or sense of learning.

379. . . . We remember it [sensation]; that is to say, we have another cognition which professes to reproduce it; but we know that there is no resemblance between the memory and the sensation, because, in the first place, nothing can resemble an immediate feeling, for resemblance supposes a dismemberment and recomposition which is totally foreign to the immediate, and in the second place, memory is an articulated complex and worked-over product which differs infinitely and immeasurably from feeling. Look at a red surface, and try to feel what the sensation is, and then shut your eyes and remember it. No doubt different persons are different in this respect; to some the experiment will seem to yield an opposite result, but I have convinced myself that there is nothing in my memory that is in the least like the vision of the red. When red is not before my eyes, I do not see it at all. Some people tell me they see it faintly — a most inconvenient kind of memory, which would lead to remembering bright red as pale or dingy. I remember colors with unusual accuracy, because I have had much training in observing them; but my memory does not consist in any vision but in a habit by virtue of which I can recognize a newly presented color as like or unlike one I had

seen before. But even if the memory of some persons is of the nature of an hallucination, enough arguments remain to show that immediate consciousness or feeling is absolutely unlike anything else.

380. There are grave objections to making a whole third of the mind of the will alone. One great psychologist has said that the will is nothing but the strongest desire. I cannot grant that; it seems to me to overlook that fact which of all that we observe is quite the most obtrusive, namely, the difference between dreaming and doing. This is not a question of defining, but of noticing what we experience; and surely he who can confound desiring with doing must be a day-dreamer. The evidence, however, seems to be pretty strong that the consciousness of willing does not differ, at least not very much, from a sensation. The sense of hitting and of getting hit are nearly the same, and should be classed together. The common element is the sense of an actual occurrence, of actual action and reaction. There is an intense reality about this kind of experience, a sharp sundering of subject and object. While I am seated calmly in the dark, the lights are suddenly turned on, and at that instant I am conscious, not of a process of change, but yet of something more than can be contained in an instant. I have a sense of a saltus, of there being two sides to that instant. A consciousness of polarity would be a tolerably good phrase to describe what occurs. For will, then, as one of the great types of consciousness, we ought to substitute the polar sense.

381. But by far the most confused of the three members of the division, in its ordinary statement, is Cognition. In the first place every kind of consciousness enters into cognition. Feelings, in the sense in which alone they can be admitted as a great branch of mental phenomena, form the warp and woof of cognition, and even in the objectionable sense of pleasure and pain, they are constituents of cognition. The will, in the form of attention, constantly enters, and the sense of reality or objectivity, which is what we have found ought to take the place of will, in the division of consciousness, is even more essential yet, if possible. But that element of cognition which is neither feeling nor the polar sense, is the consciousness of a process, and this in the form of the sense of learning, of acquir-

ing, of mental growth is eminently characteristic of cognition. This is a kind of consciousness which cannot be immediate, because it covers a time, and that not merely because it continues through every instant of that time, but because it cannot be contracted into an instant. It differs from immediate consciousness, as a melody does from one prolonged note. Neither can the consciousness of the two sides of an instant, of a sudden occurrence, in its individual reality, possibly embrace the consciousness of a process. This is the consciousness that binds our life together. It is the consciousness of synthesis.

382. Here then, we have indubitably three radically different elements of consciousness, these and no more. And they are evidently connected with the ideas of one-two-three. Immediate feeling is the consciousness of the first; the polar sense is the consciousness of the second; and synthetical consciousness is the consciousness of a third or medium.

383. Note, too, that just as we have seen that there are two orders of Secondness, so the polar sense splits into two, and that in two ways, for first, there is an active and a passive kind, or will and sense, and second, there are external will and sense, in opposition to internal will (self-control, inhibitory will) and internal sense (introspection). In like manner, just as there are three orders of Thirdness, so there are three kinds of synthetical consciousness. The undegenerate and really typical form has not been made so familiar to us as the others, which have been more completely studied by psychologists; I shall therefore mention that last. Synthetical consciousness degenerate in the first degree, corresponding to accidental Thirdness, is where there is an external compulsion upon us to think things together. Association by contiguity is an instance of this; but a still better instance is that in our first apprehension of our experiences, we cannot choose how we will arrange our ideas in reference to time and space, but are compelled to think certain things as nearer together than others. It would be putting the cart before the horse to say that we are compelled to think certain things together because they are together in time and space; the true way of stating it is that there is an exterior compulsion upon us to put them together in our construction of time and space, in our perspective. Synthetical consciousness, degenerate in the second degree, corresponding

to intermediate thirds, is where we think different feelings to be alike or different, which, since feelings in themselves cannot be compared and therefore cannot be alike, so that to say they are alike is merely to say that the synthetical consciousness regards them so, comes to this, that we are internally compelled to synthesize them or to sunder them. This kind of synthesis appears in a secondary form in association by resemblance. But the highest kind of synthesis is what the mind is compelled to make neither by the inward attractions of the feelings or representations themselves, nor by a transcendental force of necessity, but in the interest of intelligibility that is, in the interest of the synthesizing "I think" itself; and this it does by introducing an idea not contained in the data, which gives connections which they would not otherwise have had. This kind of synthesis has not been sufficiently studied, and especially the intimate relationship of its different varieties has not been duly considered. The work of the poet or novelist is not so utterly different from that of the scientific man. The artist introduces a fiction; but it is not an arbitrary one; it exhibits affinities to which the mind accords a certain approval in pronouncing them beautiful, which if it is not exactly the same as saying that the synthesis is true, is something of the same general kind. The geometer draws a diagram, which if not exactly a fiction, is at least a creation, and by means of observation of that diagram he is able to synthesize and show relations between elements which before seemed to have no necessary connection. The realities compel us to put some things into very close relation and others less so, in a highly complicated, and in the [to?] sense itself unintelligible manner; but it is the genius of the mind, that takes up all these hints of sense, adds immensely to them, makes them precise, and shows them in intelligible form in the intuitions of space and time. Intuition is the regarding of the abstract in a concrete form, by the realistic hypostatization of relations; that is the one sole method of valuable thought. Very shallow is the prevalent notion that this is something to be avoided. You might as well say at once that reasoning is to be avoided because it has led to so much error; quite in the same philistine line of thought would that be; and so well in accord with the spirit of nominalism that I wonder some one does not put it forward. The true precept is

not to abstain from hypostatization, but to do it intelligently. . . .*

384. Kant gives the erroneous view that ideas are presented separated and then thought together by the mind. This is his doctrine that a mental synthesis precedes every analysis. What really happens is that something is presented which in itself has no parts, but which nevertheless is analyzed by the mind, that is to say, its having parts consists in this, that the mind afterward recognizes those parts in it. Those partial ideas are really not in the first idea, in itself, though they are separated out from it. It is a case of destructive distillation. When, having thus separated them, we think over them, we are carried in spite of ourselves from one thought to another, and therein lies the first real synthesis. An earlier synthesis than that is a fiction. The whole conception of time belongs to genuine synthesis and is not to be considered under this head.

§5. THE TRIAD IN PHYSIOLOGY[P]

385. Granted that there are three fundamentally different kinds of consciousness, it follows as a matter of course that there must be something threefold in the physiology of the nervous system to account for them. No materialism is implied in this, further than that intimate dependence of the action of the mind upon the body, which every student of the subject must and does now acknowledge. Once more a prediction, as it were, is made by the theory; that is to say, certain consequences, not contemplated in the construction thereof, necessarily result from it; and these are of such a character that their truth or falsehood can be independently investigated. Were we to find them strikingly and certainly true, a remarkable confirmation of the theory would be afforded. So much as this, however, I cannot promise; I can only say that they are not certainly false; and we must be content to trace out these consequences, and see what they are, and leave them to the future judgment of physiologists.

386. Two of the three kinds of consciousness, indeed, the simple and dual, receive an instant physiological explanation. We know that the protoplasmic content of every nerve-cell has

* Some manuscript pages seem to be missing here.

its active and passive conditions, and argument is unnecessary to show that feeling, or immediate consciousness, arises in an active state of nerve-cells. Experiments on the effects of cutting the nerves show that there is no feeling after communication with the central nerve-cells is severed, so that the phenomenon has certainly some connection with the nerve-cells; and feeling is excited by just such stimuli as would be likely to throw protoplasm into an active condition. Thus, though we cannot say that every nerve-cell in its active condition has feeling (which we cannot deny, however) there is scarce room to doubt that the activity of nerve-cells is the main physiological requisite for consciousness. On the other hand, the sense of action and reaction, or the polar sense, as we agreed to call it, is plainly connected with the discharge of nervous energy through the nerve-fibres. External volition, the most typical case of it, involves such a discharge into muscle cells. In external sensation, where the polar sense enters in a lower intensity, there is a discharge from the terminal nerve-cell through the afferent nerve upon a cell or cells in the brain. In internal volition, or self-control, there is some inhibitory action of the nerves, which is also known to involve the movement of nervous force; and in internal observation, or visceral sensation, there are doubtless transfers of energy from one central cell to another. Remembering that the polar sense is the sense of the difference between what was before and what is after a dividing instant, or the sense of an instant as having sides, we see clearly that the physiological concomitant of it must be some event which happens very quickly and leaves a more abiding effect, and this description suits the passage of a nervous discharge over a nerve-fibre so perfectly, that I do not think we need hesitate to set this phenomenon down as the condition of dual consciousness.

387. Synthetical consciousness offers a more difficult problem. Yet the explanation of the genuine form of that consciousness, the sense of learning, is easy enough; it is only the degenerate modes, the sense of similarity, and the sense of real connection, which oblige us to hesitate. With regard to these two degenerate forms, I am driven to make hypotheses.

388. When two ideas resemble one another, we say that they have something in common; part of the one is said to be

identical with a part of the other. In what does that identity consist? Having closed both eyes, I open first one and then shut it and open the other, and I say that the two sensations are alike. How can the impressions of two nerves be judged to be alike? It appears to me that in order that that should become possible, the two nerve-cells must probably discharge themselves into one common nerve-cell. In any case, it seems to me that the first supposition to make, for scientific observation to confirm or reject, is that two ideas are alike so far as the same nerve-cells have been concerned in the production of them. In short, the hypothesis is that resemblance consists in the identity of a common element, and that that identity lies in a part of the one idea and a part of the other idea being the feeling peculiar to the excitation of one or more nerve-cells.

389. When we find ourselves under a compulsion to think that two elements of experience which do not particularly resemble one another are, nevertheless, really connected, that connection must, I think, be due in some way to a discharge of nerve-energy; for the whole sense of reality is a determination of polar consciousness, which is itself due to such discharges. For example, I recognize that a certain surface on one side of a certain boundary is red, and on the other side is blue; or that any two qualities are immediately contiguous in space or time. If the contiguity is in time, it is by the polar sense directly that we are conscious of a dividing instant with its difference on the two sides. If the contiguity is in space, I think we have at first a completely confused feeling of the whole, as yet unanalyzed and unsynthesized, but afterward, when the analysis has been made, we find ourselves compelled, in recomposing the elements, to pass directly from what is on one side of the boundary to what is on the other. I suppose then that we are compelled to think the two feelings as contiguous because the nerve-cell whose excitation produces the feeling of one recalled sensation discharges itself into the nerve-cell whose excitation makes the feeling of the other recalled sensation.

390. The genuine synthetic consciousness, or the sense of the process of learning, which is the preëminent ingredient and quintessence of the reason, has its physiological basis quite evidently in the most characteristic property of the nervous

system, the power of taking habits. This depends on five principles, as follows. First, when a stimulus or irritation is continued for some time, the excitation spreads from the cells directly affected to those that are associated with it, and from those to others, and so on, and at the same time increases in intensity. Second, after a time fatigue begins to set in. Now besides the utter fatigue which consists in the cell's losing all excitability, and the nervous system refusing to react to the stimulus at all, there is a gentler fatigue, which plays a very important part in adapting the brain to serving as an organ of reason, this form of fatigue consisting in the reflex action or discharge of the nerve-cell ceasing to go on one path and either beginning on a path where there had been no discharge, or increasing the intensity of the discharge along a path on which there had been previously only a slight discharge. For example, one may sometimes see a frog whose cerebrum or brain has been removed, and whose hind leg has been irritated by putting a drop of acid upon it, after repeatedly rubbing the place with the other foot, as if to wipe off the acid, may at length be observed to give several hops, the first avenue of nervous discharge having become fatigued. Third, when, from any cause the stimulus to a nerve-cell is removed, the excitation quickly subsides. That it does not do so instantly is well known, and the phenomenon goes among physicists by the name of persistence of sensation. All noticeable feeling subsides in a fraction of a second, but a very small remnant continues for a much longer time. Fourth, if the same cell which was once excited, and which by some chance had happened to discharge itself along a certain path or paths, comes to get excited a second time, it is more likely to discharge itself the second time along some or all of those paths along which it had previously discharged itself than it would have been had it not so discharged itself before. This is the central principle of habit; and the striking contrast of its modality to that of any mechanical law is most significant. The laws of physics know nothing of tendencies or probabilities; whatever they require at all they require absolutely and without fail, and they are never disobeyed. Were the tendency to take habits replaced by an absolute requirement that the cell should discharge itself always in the same way, or according to any rigidly fixed con-

dition whatever, all possibility of habit developing into intelligence would be cut off at the outset; the virtue of Thirdness would be absent. It is essential that there should be an element of chance in some sense as to how the cell shall discharge itself; and then that this chance or uncertainty shall not be entirely obliterated by the principle of habit, but only somewhat affected. Fifth, when a considerable time has elapsed without a nerve having reacted in any particular way, there comes in a principle of forgetfulness or negative habit rendering it the less likely to react in that way. Now let us see what will be the result of these five principles taken in combination. When a nerve is stimulated, if the reflex activity is not at first of the right sort to remove the source of irritation, it will change its character again and again until the cause of irritation is removed, when the activity will quickly subside. When the nerve comes to be stimulated a second time in the same way, probably some of the other movements which had been made on the first occasion will be repeated; but, however this may be, one of them must ultimately be repeated, for the activity will continue until this does happen, I mean that movement which removes the source of irritation. On a third occasion, the process of forgetfulness will have been begun in regard to any tendency to repeat any of the actions of the first occasion which were not repeated on the second. Of those which were repeated, some will probably be repeated again, and some not; but always there remains that one which must be repeated before the activity comes to an end. The ultimate effect of this will inevitably be that a habit gets established of at once reacting in the way which removes the source of irritation; for this habit alone will be strengthened at each repetition of the experiment, while every other will tend to become weakened at an accelerated rate.

391. I have invented a little game or experiment with playing cards to illustrate the working of these principles; and I can promise the reader that if he will try it half a dozen times he will be better able to estimate the value of the account of habit here proposed. The rules of this game are as follows: take a good many cards of four suits, say a pack of fifty-two, though fewer will do. The four suits are supposed to represent four modes in which a cell may react. Let one suit, say

spades, represent that mode of reaction which removes the source of irritation and brings the activity to an end. In order readily to find a card of any suit as wanted, you had better lay all the cards down face up and distribute into four packets, each containing the cards of one suit only. Now take two spades, two diamonds, two clubs, and two hearts, to represent the original disposition of the nerve-cell, which is supposed to be equally likely to react in any of the four ways. You turn these eight cards face down and shuffle them with extreme thoroughness.[1] Then turn up cards from the top of this pack, one by one until a spade is reached. This process represents the reaction of the cell. Take up the cards just dealt off, and add to the pack held in the hand one card of each of those suits that have just been turned up (for habit) and remove from the pack one card of each suit not turned up (for forgetfulness). Shuffle, and go through with this operation thirteen times or until the spades are exhausted. It will then generally be found that you hold nothing but spades in your hand.

392. Thus we see how these principles not only lead to the establishment of habits, but to habits directed to definite ends, namely the removal of sources of irritation. Now it is precisely action according to final causes which distinguishes mental from mechanical action; and the general formula of all our desires may be taken as this: to remove a stimulus. Every man is busily working to bring to an end that state of things which now excites him to work.

393. But we are led yet deeper into physiology. The three fundamental functions of the nervous system, namely, first, the excitation of cells; second, the transfer of excitation over fibres; third, the fixing of definite tendencies under the influence of habit, are plainly due to three properties of the protoplasm or life-slime itself. Protoplasm has its active and its passive condition, its active state is transferred from one part of it to another, and it also exhibits the phenomena of habit.

[1] Cards are almost never shuffled enough to illustrate fairly the principles of probabilities; but if after being shuffled in any of the usual ways, they are dealt into three packs and taken up again, and then passed from one hand into the other one by one, every other one going to the top and every other to the bottom of the pack that thus accumulates in the second hand, and finally cut, the shuffling may be considered as sufficient for the purpose of this game. Whenever the direction is to shuffle, shuffling as thorough as this is meant.

But these three facts do not seem to sum up the main properties of protoplasm, as our theory would lead us to expect them to do. Still, this may be because the nature of this strange substance is so little understood; and if we had the true secret of its constitution we might see that qualities that now appear unrelated really group themselves into one, so that it may be after all that it accords with our theory better than it seems to do. There have been at least two attempts to explain the properties of protoplasm by means of chemical suppositions; but inasmuch as chemical forces are as far as possible themselves from being understood, such hypotheses, even if they were known to be correct, would be of little avail. As for what a physicist would understand by a molecular explanation of protoplasm, such a thing seems hardly to have been thought of; yet I cannot see that it is any more difficult than the constitution of inorganic matter. The properties of protoplasm are enumerated as follows: contractility, irritability, automatism, nutrition, metabolism, respiration, and reproduction; but these can all be summed up under the heads of sensibility, motion, and growth. These three properties are respectively first, second, and third. Let us, however, draw up a brief statement of the facts which a molecular theory of protoplasm would have to account for. In the first place, then, protoplasm is a definite chemical substance, or class of substances, recognizable by its characteristic relations. "We do not at present," says Dr. Michael Foster* (1879), "know anything definite about the molecular composition of active living protoplasm; but it is more than probable that its molecule is a large and complex one in which a proteid substance is peculiarly associated with a complex fat and with some representative of the carbohydrate group, *i.e.*, that each molecule of protoplasm contains residues of each of these three great classes. The whole animal body is modified protoplasm." The chemical complexity of the protoplasm molecule must be amazing. A proteid is only one of its constituents, and doubtless very much simpler. Yet chemists do not attempt to infer from their analyses the ultimate atomic constitution of any of the proteids, the number of atoms entering into them being so great as almost to nullify the law of multiple proportions. I do find

* The editors have been unable to locate this quotation.

in the book just quoted the following formula for nuclein, a substance allied to the proteids. It is $C_{29}H_{49}N_9P_3O_{22}$. But as the sum of the numbers of atoms of hydrogen, nitrogen, and phosphorus ought to be even, this formula must be multiplied by some even number; so that the number of atoms in nuclein must be two hundred and twenty-four at the very least. We can hardly imagine, then, that the number of atoms in protoplasm is much less than a thousand, and if one considers the very minute proportions of some necessary ingredients of animal and vegetable organisms, one is somewhat tempted to suspect that fifty thousand might do better, or even come to be looked upon in the future as a ridiculously small guess. Protoplasm combines with water in all proportions, the mode of combination being apparently intermediate between solution and mechanical mixture. According to the amount of water it contains, it passes from being brittle to being pliable, then gelatinous, then slimy, then liquid. Generally, it has the character of being elastico-viscous; that is to say, it springs back partially after a long strain, and wholly after a short one; but its viscosity is much more marked than its elasticity. It is generally full of granules, by which we can see slow streaming motions in it, continuing for some minutes in one way and then generally reversed. The effect of this streaming is to cause protuberances in the mass, often very long and slender. They occasionally stick up against gravity; and their various forms are characteristic of the different kinds of protoplasm. When a mass of it is disturbed by a jar, a poke, an electric shock, heat, etc., the streams are arrested and the whole contracts into a ball; or if it were very much elongated, sometimes breaks up into separate spheres. When the external excitation is removed, the mass sinks down into something like its former condition. Protoplasm also grows; it absorbs material and converts it into the like of its own substance; and in all its growth and reproduction, it preserves its specific characters.

394. Such are the properties that have to be accounted for. What first arrests our attention, as likely to afford the key to the problem, is the contraction of the mass of protoplasm on being disturbed. This is obviously due to a vast and sudden increase of what the physicists call "surface tension," or the pulling together of the outer parts, which phenomenon is always observed in liquids, and is the cause of their making

drops. This surface tension is due to the cohesion, or attraction between neighboring molecules. The question is, then, how can a body, on having its equilibrium deranged, suddenly increase the attractions between its neighboring molecules? These attractions must increase rapidly as the distance is diminished; and thus the answer suggests itself that the distance between neighboring molecules is diminished. True, the average distance must remain nearly the same, but if the distances which had previously been nearly equal are rendered unequal, the attractions between the molecules that are brought nearer to one another will be much more increased than those between those that are removed from one another will be diminished. We are thus led to the supposition that in the ordinary state of the substance, its particles are moving for the most part in complicated orbital or quasi-orbital systems, instead of in the chemical molecules or more definite systems of atoms of less complex substances, these particles thus moving in orbits not being, however, atoms, but chemical molecules. But we must suppose that the forces between these particles are just barely sufficient to hold them in their orbits, and that in fact, as long as the protoplasm is in an active condition, they are not all so held, but that one and another get occasionally thrown out of their orbits and wander about until they are drawn in to some other system. We must suppose that these systems have some approximate composition, about so many of one kind of particles and so many of another kind, etc., entering into them. This is necessary to account for the nearly constant chemical composition of the whole. On the other hand, we cannot suppose that the number of the different kinds is rigidly exact; for in that case we should not know how to account for the power of assimilation. We must suppose then that there is considerable range in the numbers of particles that go to form an orbital system, and that the somewhat exact chemical composition of the whole is the exactitude of a statistical average; just as there is a close equality between the proportions of the two sexes in any nation or province, though there is considerable inequality in each of the different households. Owing to the complexity of this arrangement, the moment that there is any molecular disturbance, producing perturbations, large numbers of the particles are thrown out

of their orbits, the systems are more or less deranged in the immediate neighborhood of the disturbance, and the harmonic relations between the different revolutions are somewhat broken up. In consequence of this, the distances between neighboring particles, which had presented a systematic regularity, now become extremely unequal, and their average attractions, upon which the cohesion depends, is increased. At the same time, the particles thrown out of their systems shoot into other systems and derange these in their turn, and so the disturbance is propagated throughout the entire mass. The source of disturbance, however, being removed, interchanges of energy take place, in which there is a tendency to equalize the *vis viva* of the different particles, and they consequently tend to sink down into orbital motions again, and gradually something very like the original state of things is reëstablished, the original orbital systems remaining, for the most part, and the wandering particles in large proportion finding places in these systems or forming new ones. Some of these particles will not find any places, and thus there will be a certain amount of wasting of the protoplasmic mass. If the same disturbance is repeated, so far as the orbital systems remain the same as they were before, there will be a repetition of almost exactly the same events. The same kinds of particles (the same I mean in mass, velocities, directions of movement, attractions, etc.) which were thrown out of the different systems before will generally get thrown out again, until, if the disturbance is repeated several times, there gets to be rather a deficiency of those kinds of particles in the different systems, when some new kinds will begin to be thrown out. These new kinds will differently perturb the systems into which they fly, tending to cause classes of particles like themselves to be thrown out, and, in that way, the direction of propagation of the disturbance, as well as its velocity and intensity, may be altered, and, in short, the phenomenon of fatigue will be manifested. Even when the protoplasmic mass is left to itself, there will be some wandering of particles, producing regions of slight disturbance, and so inequalities of tension; and thus, streams will be set up, movements of the mass will take place, and slender processes will be formed. If, however, the mass be left to itself for a very long time, all the particles that are

readily thrown out will, in all the changes that are rung on the combinations of situations and velocities in the orbital systems, get thrown out; while the others will constantly tend to settle down into more stable relations; and so the protoplasm will gradually take a passive state from which its orbital systems are not easily deranged. The food for those kinds of protoplasm that are capable of marked reaction has to be presented in chemically complex form. It must doubtless present particles just like those that revolve in the orbital systems of the protoplasm. In order to be drawn into an orbital system, a particle, whether of food matter or just thrown off from some other system, must have the right mass, must present itself at the right point, and move with the right velocity in the right direction and be subject to the right attractions. It will be right in all these respects, if it comes to take the place of a particle which has just been thrown off; and thus, particles taken in are particularly likely to be of the same material and masses and to take the same places in the orbits as those that have been shortly before thrown off. Now these particles being the exact representatives of those thrown off, will be likely to be thrown off by the same disturbances, in the same directions, and with the same results, as those which were thrown off before; and this accounts for the principle of habit. All the higher kinds of protoplasm, those for example which have any marked power of contraction, are fed with matter chemically highly complex.*

§6. THE TRIAD IN BIOLOGICAL DEVELOPMENT

395. Whether the part played by natural selection and the survival of the fittest in the production of species be large or small, there remains little doubt that the Darwinian theory indicates a real cause, which tends to adapt animal and vegetable forms to their environment. A very remarkable feature of it is that it shows how merely fortuitous variations of individuals together with merely fortuitous mishaps to them would, under the action of heredity, result, not in mere irregularity, nor even in a statistical constancy, but in continual and

* The following note is appended to this section: "Here the Chemical Idea." For this, see vol. 6, bk. I, ch. 8.

indefinite progress toward a better adaptation of means to ends. How can this be? What, abstractly stated, is the peculiar factor in the conditions of the problem which brings about this singular consequence?

396. Suppose a million persons, each provided with one dollar, to sit down to play a simple and fair game of chance, betting for example on whether a die turns up an odd or even number. The players are supposed to make their bets independently of one another, and each to bet on the result of each throw one dollar against a dollar on the part of the bank. Of course, at the very first bet, one-half of them would lose their only dollar and go out of the game, for it is supposed that no credit is allowed, while the other half would win each $1 and so come to be worth $2. Of these 500,000 players, after the second throw, 250,000 would have lost, and so be worth only $1 each, while the other 250,000 would have won, and so be worth $3. After the third throw, 125,000, or one-half of those who had had $1 each, would be ruined; 250,000 would be worth $2 (namely one-half the 250,000 who had had $1 each, and one-half the 250,000 who had had $3 each) and 125,000 would be worth $4 each. The further progress of the game is illustrated by the table on page 216, where the numbers of players are given having each possible sum after the first, second, third, etc., throws. It will be seen by the table that, at the end of the fourth throw, the most usual fortune is $3, at the end of the ninth $4, at the end of the sixteenth $5, and in like manner at the end of the twenty-fifth it would be $6, at the end of the thirty-sixth $7, and so forth. Here, then, would be a continual increase of wealth, which is a sort of "adaptation to one's environment," produced by a survival of the fittest, that is, by the elimination from the game of every player who has lost his last dollar. It is easy to see that the increase of average and usual wealth comes about by the subtraction of all those small fortunes which would be in the hands of men who had once been bankrupt had they been allowed to continue betting.

397. Now the adaptation of a species to its environment consists, for the purposes of natural selection, in a power of continuing to exist, that is to say, in the power of one generation to bring forth another; for as long as another generation

	First	*Second*	*Third*	*Fourth*	*Fifth*	*Sixth*	*Seventh*	*Eighth*	*Ninth*	*Tenth*	*Sixteenth*
$1	.....	250,000	.....	125,000	.....	76,875	.....	53,750	.....	40,312½	31,744
2	500,000	.....	250,000	.....	153,750	.....	107,500	.....	80,625	.....	.....
3	.....	250,000	.....	182,500	.....	138,125	.....	107,500	.....	86,406¼	52,292
4	.....	.....	125,000	.....	122,500	.....	107,500	.....	92,187½	.....	.....
5	.....	.....	.....	62,500	.....	76,875	.....	76,875	.....	77,070¼	55,542
6	.....	.....	.....	.....	31,250	.....	46,250	.....	61,952⅞*	.....	.....
7	.....	.....	.....	.....	.....	15,625	.....	27,031¼	.....	38,710¾	38,880
8	.....	.....	.....	.....	.....	.....	7,812½	.....	15,468½	.....	.....
9	.....	.....	.....	.....	.....	.....	.....	3,906¼	.....	8,710¾	19,226
10	.....	.....	.....	.....	.....	.....	.....	.....	1,953⅛	.....	.....
11	.....	.....	.....	.....	.....	.....	.....	.....	.....	976½	8,714
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
13	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1,587
14	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
15	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	229
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
17	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	15

* This figure should be 51,952⅞. Because of this error the fifth and seventh figures in the next column are each 5,000 in excess, and, with the exception of the two last items, the entire last column is wrong.

is brought forth the species will continue and as soon as this ceases it is doomed after one lifetime. This reproductive faculty, then, depending partly on direct fecundity, and partly on the animal's living through the age of procreation, is precisely what the Darwinian theory accounts for. This character plainly is one of those which has an absolute minimum, for no animal can produce fewer offspring than none at all and it has no apparent upper limit, so that it is quite analogous to the wealth of those players. It is to be remarked that the phrase "survival of the fittest" in the formula of the principle does not mean the survival of the fittest individuals, but the survival of the fittest types; for the theory does not at all require that individuals ill-adapted to their environment should die at an earlier age than others, so long only as they do not reproduce so many offspring as others; and indeed it is not necessary that this should go so far as to extinguish the line of descent, provided there be some reason why the offspring of ill-adapted parents are less likely than others to inherit those parents' characteristics. It seems likely that the process, as a general rule, is something as follows: A given individual is in some respect ill-adapted to his environment, that is to say, he has characters which are generally unfavorable to the production of numerous offspring. These characters will be apt to weaken the reproductive system of that individual, for various reasons, so that its offspring are not up to the average strength of the species. This second generation will couple with other individuals, but owing to their weakness, their offspring will be more apt to resemble the other parent, and so the unfavorable character will gradually be eliminated, not merely by diminished numbers of offspring, but also by the offspring more resembling the stronger parent. There are other ways in which the unfavorable characters will disappear. When the procreative power is weakened, there are many examples to show that the principle of heredity becomes relaxed, and the race shows more tendency to sporting. This sporting will go on until in the course of it the unfavorable character has become obliterated. The general power of reproduction thereupon becomes strengthened; with it the direct procreative force is reinforced, the hereditary transmission of characters again becomes more strict, and the improved type is hardened.

398. But all these different cases are but so many different modes of one and the same principle, which is the elimination of unfavorable characters. We see then that there are just three factors in the process of natural selection; to wit: first, the principle of individual variation or sporting; second, the principle of hereditary transmission, which wars against the first principle; and third, the principle of the elimination of unfavorable characters.

399. Let us see how far these principles correspond with the triads that we have already met with. The principle of sporting is the principle of irregularity, indeterminacy, chance. It corresponds with the irregular and manifold wandering of particles in the active state of the protoplasm. It is the bringing in of something fresh and first. The principle of heredity is the principle of the determination of something by what went before, the principle of compulsion, corresponding to will and sense. The principle of the elimination of unfavorable characters is the principle of generalization by casting out of sporadic cases, corresponding particularly to the principle of forgetfulness in the action of the nervous system. We have, then, here, a somewhat imperfect reproduction of the same triad as before. Its imperfection may be the imperfection of the theory of development.*

§7. THE TRIAD IN PHYSICS

400. Metaphysical philosophy may almost be called the child of geometry. Of the three schools of early Greek philosophers, two, the Ionic and the Pythagorean, were all geometers, and the interest of the Eleatics in geometry is often mentioned. Plato was a great figure in the history of both subjects; and Aristotle derived from the study of space some of his most potent conceptions. Metaphysics depends in great measure on the idea of rigid demonstration from first principles; and this idea, as well in regard to the process as the axioms from which it sets out, bears its paternity on its face. Moreover, the conviction that any metaphysical philosophy is possible has been upheld at all times, as Kant well says, by the example in geometry of a similar science.

* Cf. vol. 6, bk. I, ch. 10.

401. The unconditional surrender, then, by the mathematicians of our time of the absolute exactitude of the axioms of geometry cannot prove an insignificant event for the history of philosophy. Gauss, the greatest of geometers, declares that "there is no reason to think that the sum of the three angles of a triangle is exactly equal to two right angles."* It is true, experience shows that the deviation of that sum from that amount is so excessively small that language must be ingeniously used to express the degree of approximation: but experience never can show any truth to be exact, nor so much as give the least reason to think it to be so, unless it be supported by some other considerations. We can only say that the sum of the three angles of any given triangle cannot be much greater or less than two right angles; but that exact value is only one among an infinite number of others each of which is as possible as that. So say the mathematicians with unanimity.

402. The absolute exactitude of the geometrical axioms is exploded; and the corresponding belief in the metaphysical axioms, considering the dependence of metaphysics on geometry, must surely follow it to the tomb of extinct creeds. The first to go must be the proposition that every event in the universe is precisely determined by causes according to inviolable law. We have no reason to think that this is absolutely exact. Experience shows that it is so to a wonderful degree of approximation, and that is all. This degree of approximation will be a value for future scientific investigation to determine; but we have no more reason to think that the error of the ordinary statement is precisely zero, than any one of an infinity of values in that neighborhood. The odds are infinity to one that it is not zero; and we are bound to think of it as a quantity of which zero is only one possible value. Phœnix, in his *Lectures on Astronomy*,† referring to Joshua's commanding the sun to stand still, said that he could not help suspecting that it might have wiggled a very little when Joshua was not looking directly at it. We know that when we try to verify any law of nature by experiment, we always find discrepancies between the observations and the theory. These we rightly refer to errors

* See, *e.g.*, *General Investigations of Curved Surfaces*, art. 20.

† See 156.

of observation; but why may there not be similar aberrations due to the imperfect obedience of the facts to law?

403. Grant that this is conceivable and there can be nothing in experience to negative it. Strange to say, there are many people who will have a difficulty in conceiving of an element of lawlessness in the universe, and who may perhaps be tempted to reckon the doctrine of the perfect rule of causality as one of the original instinctive beliefs, like that of space having three dimensions. Far from that, it is historically altogether a modern notion, a loose inference from the discoveries of science. Aristotle* often lays it down that some things are determined by causes while others happen by chance. Lucretius,† following Democritus, supposes his primordial atoms to deviate from their rectilinear trajectories just fortuitously, and without any reason at all. To the ancients, there was nothing strange in such notions; they were matters of course; the strange thing would have been to have said that there was no chance. So we are under no inward necessity of believing in perfect causality if we do not find any facts to bear it out.

404. I am very far from holding that experience is our only light; Whewell's views of scientific method seem to me truer than Mill's; so much so that I should pronounce the known principles of physics to be but a development of original instinctive beliefs. Yet I cannot help acknowledging that the whole history of thought shows that our instinctive beliefs, in their original condition, are so mixed up with error that they can never be trusted till they have been corrected by experiment. Now the only thing that the inference from experience can ever teach us is the approximate value of a ratio. It all rests on the principle of sampling; we take a handful of coffee from a bag, and we judge that there is about the same proportion of sound beans in the whole bag that there is in that sample. At this rate, every proposition which we can be entitled to make about the real world must be an approximate one; we never can have the right to hold any truth to be exact. Approximation must be the fabric out of which our philosophy has to be built.

* *e.g.* in the *Physics* 195b, 31–198a, 13.

† Bk. II, l. 216–93.

405. I come now to another point. Most systems of philosophy maintain certain facts or principles as ultimate. In truth, any fact is in one sense ultimate — that is to say, in its isolated aggressive stubbornness and individual reality. What Scotus calls the hæcceities of things, the hereness and nowness of them, are indeed ultimate. Why this which is here is such as it is; how, for instance, if it happens to be a grain of sand, it came to be so small and so hard, we can ask; we can also ask how it got carried here; but the explanation in this case merely carries us back to the fact that it was once in some other place, where similar things might naturally be expected to be. Why IT, independently of its general characters, comes to have any definite place in the world, is not a question to be asked; it is simply an ultimate fact. There is also another class of facts of which it is not reasonable to expect an explanation, namely, facts of indeterminacy or variety. Why one definite kind of event is frequent and another rare, is a question to be asked, but a reason for the general fact that of events some kinds are common and some rare, it would be unfair to demand. If all births took place on a given day of the week, or if there were always more on Sundays than on Mondays, that would be a fact to be accounted for, but that they happen in about equal proportions on all the days requires no particular explanation. If we were to find that all the grains of sand on a certain beach separated themselves into two or more sharply discrete classes, as spherical and cubical ones, there would be something to be explained, but that they are of various sizes and shapes, of no definable character, can only be referred to the general manifoldness of nature. Indeterminacy, then, or pure firstness, and hæcceity, or pure secondness, are facts not calling for and not capable of explanation. Indeterminacy affords us nothing to ask a question about; hæcceity is the *ultima ratio*, the brutal fact that will not be questioned. But every fact of a general or orderly nature calls for an explanation; and logic forbids us to assume in regard to any given fact of that sort that it is of its own nature absolutely inexplicable. This is what Kant[1] calls a regulative principle, that is to say, an intellectual hope. The sole immediate purpose of thinking is to render things intelligible; and to think and yet in that very

[1] After the scholastics; See Eckius [?] in Petrus Hispanus 48b, nota 1.

act to think a thing unintelligible is a self-stultification. It is as though a man furnished with a pistol to defend himself against an enemy were, on finding that enemy very redoubtable, to use his pistol to blow his own brains out to escape being killed by his enemy. Despair is insanity. True, there may be facts that will never get explained; but that any given fact is of the number, is what experience can never give us reason to think; far less can it show that any fact is of its own nature unintelligible. We must therefore be guided by the rule of hope, and consequently we must reject every philosophy or general conception of the universe, which could ever lead to the conclusion that any given general fact is an ultimate one. We must look forward to the explanation, not of all things, but of any given thing whatever. There is no contradiction here, any more than there is in our holding each one of our opinions, while we are ready to admit that it is probable that not all are true; or any more than there is in saying that any future time will sometime be passed, though there never will be a time when all time is past.

406. Among other regular facts that have to be explained is law or regularity itself. We enormously exaggerate the part that law plays in the universe. It is by means of regularities that we understand what little we do understand of the world, and thus there is a sort of mental perspective which brings regular phenomena to the foreground. We say that every event is determined by causes according to law. But apart from the fact that this must not be regarded as absolutely true, it does not mean so much as it seems to do. We do not mean, for example, that if a man and his antipode both sneeze at the same instant, that that event comes under any general law. That is merely what we call a coincidence. But what we mean is there was a cause for the first man's sneezing, and another cause for the second man's sneezing; and the aggregate of these two events make up the first event about which we began by inquiring. The doctrine is that the events of the physical universe are merely motions of matter, and that these obey the laws of dynamics. But this only amounts to saying that among the countless systems of relationship existing among things we have found one that is universal and at the same time is subject to law. There is nothing except this singular char-

acter which makes this particular system of relationship any more important than the others. From this point of view, uniformity is seen to be really a highly exceptional phenomenon. But we pay no attention to irregular relationships, as having no interest for us.

407. We are brought, then, to this: conformity to law exists only within a limited range of events and even there is not perfect, for an element of pure spontaneity or lawless originality mingles, or at least must be supposed to mingle, with law everywhere. Moreover, conformity with law is a fact requiring to be explained; and since law in general cannot be explained by any law in particular, the explanation must consist in showing how law is developed out of pure chance, irregularity, and indeterminacy.

408. To this problem we are bound to address ourselves; and it is particularly needful to do so in the present state of science. The theory of the molecular constitution of matter has now been carried as far as there are clear indications to direct us, and we are now in the mists. To develope the mathematical consequences of any hypothesis as to the nature and laws of the minute parts of matter, and then to test it by physical experiment, will take fifty years; and out of the innumerable hypotheses that might be framed, there seems to be nothing to make one more antecedently probable than another. At this rate how long will it take to make any decided advance? We need some hint as to how molecules may be expected to behave; whether, for instance, they would be likely to attract or repel one another inversely as the fifth power of the distance, so that we may be saved from many false suppositions, if we are not at once shown the way to the true one. Tell us how the laws of nature came about, and we may distinguish in some measure between laws that might and laws that could not have resulted from such a process of development.

409. To find that out is our task. I will begin the work with this guess. Uniformities in the modes of action of things have come about by their taking habits. At present, the course of events is approximately determined by law. In the past that approximation was less perfect; in the future it will be more perfect. The tendency to obey laws has always been and always will be growing. We look back toward a point in the

infinitely distant past when there was no law but mere indeterminacy; we look forward to a point in the infinitely distant future when there will be no indeterminacy or chance but a complete reign of law. But at any assignable date in the past, however early, there was already some tendency toward uniformity; and at any assignable date in the future there will be some slight aberrancy from law. Moreover, all things have a tendency to take habits. For atoms and their parts, molecules and groups of molecules, and in short every conceivable real object, there is a greater probability of acting as on a former like occasion than otherwise. This tendency itself constitutes a regularity, and is continually on the increase. In looking back into the past we are looking toward periods when it was a less and less decided tendency. But its own essential nature is to grow. It is a generalizing tendency; it causes actions in the future to follow some generalization of past actions; and this tendency is itself something capable of similar generalizations; and thus, it is self-generative. We have therefore only to suppose the smallest spoor of it in the past, and that germ would have been bound to develop into a mighty and over-ruling principle, until it supersedes itself by strengthening habits into absolute laws regulating the action of all things in every respect in the indefinite future.

According to this, three elements are active in the world: first, chance; second, law; and third, habit-taking.

410. Such is our guess of the secret of the sphynx. To raise it from the rank of philosophical speculation to that of a scientific hypothesis, we must show that consequences can be deduced from it with more or less probability which can be compared with observation. We must show that there is some method of deducing the characters of the laws which could result in this way by the action of habit-taking on purely fortuitous occurrences, and a method of ascertaining whether such characters belong to the actual laws of nature.

411. The existence of things consists in their regular behavior. If an atom had no regular attractions and repulsions, if its mass was at one instant nothing, at another a ton, at another a negative quantity, if its motion instead of being continuous, consisted in a series of leaps from one place to another without passing through any intervening places, and

if there were no definite relations between its different positions, velocities and directions of displacement, if it were at one time in one place and at another time in a dozen, such a disjointed plurality of phenomena would not make up any existing thing. Not only substances, but events, too, are constituted by regularities. The flow of time, for example, in itself is a regularity. The original chaos, therefore, where there was no regularity, was in effect a state of mere indeterminacy, in which nothing existed or really happened.

412. Our conceptions of the first stages of the development, before time yet existed, must be as vague and figurative as the expressions of the first chapter of Genesis. Out of the womb of indeterminacy we must say that there would have come something, by the principle of Firstness, which we may call a flash. Then by the principle of habit there would have been a second flash. Though time would not yet have been, this second flash was in some sense after the first, because resulting from it. Then there would have come other successions ever more and more closely connected, the habits and the tendency to take them ever strengthening themselves, until the events would have been bound together into something like a continuous flow. We have no reason to think that even now time is quite perfectly continuous and uniform in its flow. The quasi-flow which would result would, however, differ essentially from time in this respect, that it would not necessarily be in a single stream. Different flashes might start different streams, between which there should be no relations of contemporaneity or succession. So one stream might branch into two, or two might coalesce. But the further result of habit would inevitably be to separate utterly those that were long separated, and to make those which presented frequent common points coalesce into perfect union. Those that were completely separated would be so many different worlds which would know nothing of one another; so that the effect would be just what we actually observe.

413. But Secondness is of two types. Consequently besides flashes genuinely second to others, so as to come after them, there will be pairs of flashes, or, since time is now supposed to be developed, we had better say pairs of states, which are reciprocally second, each member of the pair to the other. This

is the first germ of spatial extension. These states will undergo changes; and habits will be formed of passing from certain states to certain others, and of not passing from certain states to certain others. Those states to which a state will immediately pass will be adjacent to it; and thus habits will be formed which will constitute a spatial continuum, but differing from our space by being very irregular in its connections, having one number of dimensions in one place and another number in another place, and being different for one moving state from what it is for another.

414. Pairs of states will also begin to take habits, and thus each state having different habits with reference to the different other states will give rise to bundles of habits, which will be substances.[1] Some of these states will chance to take habits of persistency, and will get to be less and less liable to disappear; while those that fail to take such habits will fall out of existence. Thus, substances will get to be permanent.

415. In fact, habits, from the mode of their formation, necessarily consist in the permanence of some relation, and therefore, on this theory, each law of nature would consist in some permanence, such as the permanence of mass, momentum, and energy. In this respect, the theory suits the facts admirably.

416. The substances carrying their habits with them in their motions through space will tend to render the different parts of space alike. Thus, the dimensionality of space will tend gradually to uniformity; and multiple connections, except at infinity, where substances never go, will be obliterated. At the outset, the connections of space were probably different for one substance and part of a substance from what they were for another; that is to say, points adjacent or near one another for the motions of one body would not be so for another; and this may possibly have contributed to break substances into little pieces or atoms. But the mutual actions of bodies would have tended to reduce their habits to uniformity in this respect; and besides there must have arisen conflicts between the habits of bodies and the habits of parts of space, which would never have ceased till they were brought into conformity.

[1] I use substance, here, in the old sense of a thing, not in the modern chemical sense.

CHAPTER 4

THE LOGIC OF MATHEMATICS; AN ATTEMPT TO DEVELOP MY CATEGORIES FROM WITHIN[*P]

§1. THE THREE CATEGORIES

417. Although the present paper deals with mathematics, yet its problems are not mere mathematical problems. It is not proposed to inquire into the methods of reasoning of mathematics particularly, although this subject will incidentally be touched upon. But mathematics performs its reasonings by a *logica utens* which it develops for itself, and has no need of any appeal to a *logica docens;* for no disputes about reasoning arise in mathematics which need to be submitted to the principles of the philosophy of thought for decision. The questions which are here to be examined are, what are the different systems of hypotheses from which mathematical deduction can set out, what are their general characters, why are not other hypotheses possible, and the like. These are not problems which, like those of mathematics, repose upon clear and definite assumptions recognized at the outset; and yet, like mathematical problems, they are questions of possibility and necessity. What the nature of this necessity can be is one of the very matters to be discovered. This much, however, is indisputable: if there are really any such necessary characteristics of mathematical hypotheses as I have just declared in advance that we shall find that there [are], this necessity must spring from some truth so broad as to hold not only for the universe we know but for every world that poet could create. And this truth like every truth must come to us by the way of experience. No apriorist ever denied that. The first matters which it is pertinent to examine are the most universal categories of elements of all experience, natural or poetical.

* c. 1896. The first four pages of the manuscript are missing.

418. We remark among phenomena three categories of elements.

The first comprises the qualities of phenomena, such as red, bitter, tedious, hard, heartrending, noble; and there are doubtless manifold varieties utterly unknown to us. Beginners in philosophy may object that these are not qualities of things and are not in the world at all, but are mere sensations. Certainly, we only know such as the senses we are furnished with are adapted to reveal; and it can hardly be doubted that the specializing effect of the evolutionary process which has made us what we are has been to blot the greater part of the senses and sensations which were once dimly felt, and to render bright, clear, and separate the rest. But whether we ought to say that it is the senses that make the sense-qualities or the sense-qualities to which the senses are adapted, need not be determined in haste. It is sufficient that wherever there is a phenomenon there is a quality; so that it might almost seem that there is nothing else in phenomena. The qualities merge into one another. They have no perfect identities, but only likenesses, or partial identities. Some of them, as the colors and the musical sounds, form well-understood systems. Probably, were our experience of them not so fragmentary, there would be no abrupt demarcations between them, at all.* Still, each one is what it is in itself without help from the others. They are single but partial determinations.

419. The second category of elements of phenomena comprises the actual facts. The qualities, in so far as they are general, are somewhat vague and potential. But an occurrence is perfectly individual. It happens here and now. A permanent fact is less purely individual; yet so far as it is actual, its permanence and generality only consist in its being there at every individual instant. Qualities are concerned in facts but they do not make up facts. Facts also concern subjects which are material substances. We do not see them as we see qualities, that is, they are not in the very potentiality and essence of sense. But we feel facts resist our will. That is why facts are proverbially called brutal. Now mere qualities do not resist. It is the matter that resists. Even in actual sensation there is a reaction. Now mere qualities, unmaterialized, cannot

* Cf. 313; also vol. 6, bk. I, ch. 5.

actually react. So that, rightly understood, it is correct to say that we immediately, that is, directly perceive matter. To say that we only infer matter from its qualities is to say that we only know the actual through the potential. It would be a little less erroneous to say that we only know the potential through the actual, and only infer qualities by generalization from what we perceive in matter. All that I here insist upon is that quality is one element of phenomena, and fact, action, actuality is another. We shall undertake the analysis of their natures below.

420. The third category of elements of phenomena consists of what we call laws when we contemplate them from the outside only, but which when we see both sides of the shield we call thoughts. Thoughts are neither qualities nor facts. They are not qualities because they can be produced and grow, while a quality is eternal, independent of time and of any realization. Besides, thoughts may have reasons, and indeed, must have some reasons, good or bad. But to ask why a quality is as it is, why red is red and not green, would be lunacy. If red were green it would not be red; that is all. And any semblance of sanity the question may have is due to its being not exactly a question about quality, but about the relation between two qualities, though even this is absurd. A thought then is not a quality. No more is it a fact. For a thought is general. I had it. I imparted it to you. It is general on that side. It is also general in referring to all possible things, and not merely to those which happen to exist. No collection of facts can constitute a law; for the law goes beyond any accomplished facts and determines how facts that *may be*, but *all* of which never can have happened, shall be characterized. There is no objection to saying that a law is a general fact, provided it be understood that the general has an admixture of potentiality in it, so that no congeries of actions here and now can ever make a general fact. As *general*, the law, or general fact, concerns the potential world of quality, while as *fact*, it concerns the actual world of actuality. Just as action requires a peculiar kind of subject, matter, which is foreign to mere quality, so law requires a peculiar kind of subject, the thought, or, as the phrase in this connection is, the *mind*, as a peculiar kind of subject foreign to mere individual action. Law, then, is something as remote

from both quality and action as these are remote from one another.

421. Having thus by observation satisfied ourselves that there are these three categories of elements of phenomena, let us endeavor to analyze the nature of each, and try to find out why there should be these three categories and no others. This reason, when we find it, ought to be interesting to mathematicians; for it will be found to coincide with the most fundamental characteristic of the most universal of the mathematical hypotheses, I mean that of number.

§2. QUALITY

422. What, then, is a *quality?*

Before answering this, it will be well to say what it is not. It is not anything which is dependent, in its being, upon mind, whether in the form of sense or in that of thought. Nor is it dependent, in its being, upon the fact that some material thing possesses it. That quality is dependent upon sense is the great error of the conceptualists. That it is dependent upon the subject in which it is realized is the great error of all the nominalistic schools. A quality is a mere abstract potentiality; and the error of those schools lies in holding that the potential, or possible, is nothing but what the actual makes it to be. It is the error of maintaining that the whole alone is something, and its components, however essential to it, are nothing. The refutation of the position consists in showing that nobody does, or can, in the light of good sense, consistently retain it. The moment the fusillade of controversy ceases they repose on other conceptions. First, that the quality of red depends on anybody actually seeing it, so that red things are no longer red in the dark, is a denial of common sense. I ask the conceptualist, do you really mean to say that in the dark it is no longer true that red bodies are capable of transmitting the light at the lower end of the spectrum? Do you mean to say that a piece of iron not actually under pressure has lost its power of resisting pressure? If so, you must either hold that those bodies under the circumstances supposed assume the opposite properties, or you must hold that they become indeterminate in those respects. If you hold that the red body in the dark acquires a power of absorbing the long waves of the spectrum, and that

the iron acquires a power of condensation under small pressure, then, while you adopt an opinion without any facts to support it, you still admit that qualities exist while they are not actually perceived — only you transfer this belief to qualities which there is no ground for believing in. If, however, you hold that the bodies become indeterminate in regard to the qualities they are not actually perceived to possess, then, since this is the case at any moment in regard to the vast majority of the qualities of all bodies, you must hold that generals exist. In other words, it is concrete things you do not believe in; qualities, that is, generals — which is another word for the same thing — you not only believe in but believe that they alone compose the universe. Consistency, therefore, obliges you to say that the red body is red (or has some color) in the dark, and that the hard body has some degree of hardness when nothing is pressing upon it. If you attempt to escape the refutation by a distinction between qualities that are real, namely the mechanical qualities, and qualities that are not real, sensible qualities, you may be left there, because you have granted the essential point. At the same time, every modern psychologist will pronounce your distinction untenable. You forget perhaps that a realist fully admits that a sense-quality is only a possibility of sensation; but he thinks a possibility remains possible when it is not actual. The sensation is requisite for its apprehension; but no sensation nor sense-faculty is requisite for the possibility which is the being of the quality. Let us not put the cart before the horse, nor the evolved actuality before the possibility as if the latter *involved* what it only *evolves*. A similar answer may be made to the other nominalists. It is impossible to hold consistently that a quality only exists when it actually inheres in a body. If that were so, nothing but individual facts would be true. Laws would be fictions; and, in fact, the nominalist does object to the word "law," and prefers "uniformity" to express his conviction that so far as the law expresses what only *might* happen, but does not, it is nugatory. If, however, no law subsists other than an expression of actual facts, the future is entirely indeterminate and so is general to the highest degree. Indeed, nothing would exist but the instantaneous state; whereas it is easy to show that if we are going to be so free in calling elements fictions an instant is the

first thing to be called fictitious. But I confess I do not take pains accurately to answer a doctrine so monstrous, and just at present out of vogue.

423. So much for what quality is not. Now what *is* it? We do not care what meaning the usages of language may attach to the word. We have already seen clearly that the elements of phenomena are of three categories, quality, fact, and thought. The question we have to consider is how quality shall be defined so as to preserve the truth of that division. In order to ascertain this, we must consider how qualities are apprehended and from what point of view they become emphatic in thought, and note what it is that will and must be revealed in that mode of apprehension.

424. There is a point of view from which the whole universe of phenomena appears to be made up of nothing but sensible qualities. What is that point of view? It is that in which we attend to each part as it appears in itself, in its own suchness, while we disregard the connections. Red, sour, toothache are each *sui generis* and indescribable. In themselves, that is all there is to be said about them. Imagine at once a toothache, a splitting headache, a jammed finger, a corn on the foot, a burn, and a colic, not necessarily as existing at once — leave that vague — and attend not to the parts of the imagination but to the resultant impression. That will give an idea of a general quality of pain. We see that the idea of a quality is the idea of a phenomenon or partial phenomenon considered as a monad, without reference to its parts or components and without reference to anything else. We must not consider whether it exists, or is only imaginary, because existence depends on its subject having a place in the general system of the universe. An element separated from everything else and in no world but itself, may be said, when we come to reflect upon its isolation, to be merely potential. But we must not even attend to any determinate absence of other things; we are to consider the total as a unit. We may term this aspect of a phenomenon the *monadic* aspect of it. The quality is what presents itself in the *monadic* aspect.

425. The phenomenon may be ever so complex and heterogeneous. That circumstance will make no particular difference in the quality. It will make it more general. But one quality

is in itself, in its monadic aspect, no more general than another. The resultant effect has no parts. The quality in itself is indecomposable and *sui generis*. When we say that qualities are general, are partial determinations, are mere potentialities, etc., all that is true of qualities reflected upon; but these things do not belong to the quality-element of experience.

426. Experience is the course of life. The world is that which experience inculcates. Quality is the monadic element of the world. Anything whatever, however complex and heterogeneous, has its quality *sui generis*, its possibility of sensation, would our senses only respond to it. But in saying this, we are straying from the domain of the monad into that of the dyad; and such truths are best postponed until we come to discuss the dyad.

§3. FACT

427. Next, what is *fact?*

As before, it is not the usage of language which we seek to learn, but what must be the description of fact in order that our division of the elements of phenomena into the categories of quality, fact, and law may not only be true, but also have the utmost possible value, being governed by those same characteristics which really dominate the phenomenal world. It is first requisite to point out something which must be excluded from the category of fact. This is the general, and with it the permanent or eternal (for permanence is a species of generality), and the conditional (which equally involves generality). Generality is either of that negative sort which belongs to the merely potential, as such, and this is peculiar to the category of quality; or it is of that positive kind which belongs to conditional necessity, and this is peculiar to the category of law. These exclusions leave for the category of fact, first, that which the logicians call the *contingent*, that is, the accidentally actual, and second, whatever involves an unconditional necessity, that is, force without law or reason, *brute* force.

428. It may be said that there is no such phenomenon in the universe as brute force, or freedom of will, and nothing accidental. I do not assent to either opinion; but granting that both are correct, it still remains true that considering a single action by itself, apart from all others and, therefore,

apart from the governing uniformity, it is in itself brute, whether it show brute *force* or not. I shall presently point out a sense in which it does display force. That it is possible for a phenomenon in *some* sense to present force to our notice without emphasizing any element of law, is familiar to everybody. We often regard our own exertions of will in that way. In like manner, if we consider any state of an individual thing, putting aside other things, we have a phenomenon which is actual, but *in itself* is not necessitated. It is not pretended that what is here termed fact is the whole phenomenon, but only an element of the phenomenon — so much as belongs to a particular place and time. That when more is taken into account, the observer finds himself in the realm of law in every case, I fully admit. (Nor does that conflict with tychism.*)

429. On the other hand, if the view be limited to any part of the phenomenal world, however great, and this be looked upon as a monad, entirely regardless of its parts, nothing is presented to the observer but a quality. How much, then, must we attend to, in order to perceive the pure element of fact? There are certain occurrences which, when they come to our notice, we set down as "accidental." Now, although there is really no more of the factual element in these than in other facts, yet the circumstance that we call them *par excellence* contingent, or "accidental," would lead us to expect that which distinguishes the realm of fact from the realms of quality and of law, to be particularly prominent in them. We call such facts "coincidences," a name which implies that our attention is called in them to the coming together of two things. Two phenomena, and but two, are required to constitute a coincidence; and if there are more than two no new form of relationship appears further than a complication of pairs. Two phenomena, whose parts are not attended to, cannot display any law, or regularity. *Three* dots may be placed in a straight line, which is a kind of regularity; or they may be placed at the vertices of an equilateral triangle, which is another kind of regularity. But *two* dots cannot be placed in any particularly regular way, since there is but one way in which they can be placed, unless they were set together, when they would cease to be two. It is true that on the earth two dots may be placed

* See vol. 6, bk. I, ch. 2, and 6.102.

antipodally. But that is only one of the exceptions that prove the rule, because the earth is a third object there taken into account. So two straight lines in a plane can be set at right angles, which is a sort of regularity. But this is another rule proving exception, since $<AOB$ is made equal to $<BOC$. Now those angles are distinguished by being formed of two different parts of the line AC; so that really three things, OA, OB, and OC are considered. So much for accidental actuality. The type of brute force is the exertion of animal strength. Suppose I have long ago determined how and when I will act. It still remains to perform the act. That element of the whole operation is purely brute execution. Now observe that I cannot exert strength all alone. I can only exert my strength if there be something to resist me. Again duality is prominent, and this time in a [more] obtrusively dual way than before, because the two units are in two different relations the one to the other. In the coincidence the two phenomena are related in one way to one another. It is a monoidal dyad. But in the exertion of strength, although I act on the object and the object acts on me, which are two relations of one kind and joined in one reaction, yet in each of these two relations there is an agent and a patient, a doer and a sufferer, which are in contrary attitudes to one another. So that the action consists of two monoid dyads oppositely situated.

430. All this renders it quite certain that the nature of fact is in some way connected with the number two, and that of law with three or some higher number or numbers, just as we have already seen that quality is described by means of the number one. But although it is hardly more than might be expected to find that a particular category of the constituents of phenomena has a special capacity for relations of a certain form — that some are too complex to suit this matter, while others [are] too simple to call into action its distinctive powers — and that in that way that category comes to have an intimate affinity with a certain formal conception, yet it would certainly be astonishing if it should turn out that material constituents of phenomena were coextensive with formal ideas. We consequently wish to discover just what the connection of the dyad with fact is. We shall do well to postpone the consideration of those facts which seem to involve a triad, such as a process

with beginning, middle and end, until we have examined the nature of law. For we naturally suspect, after what has been pointed out above, that where there is a threeness in a fact, there an element of generality may lurk. Putting aside then, for the present, triadic facts, we may add to the properties of fact already noticed such others as may seem worth mention, and may then turn to the consideration of duality, its properties and different formal types, so as to compare these with what is to be remarked in regard to fact.

431. Whenever we come to know a fact, it is by its resisting us. A man may walk down Wall Street debating within himself the existence of an external world; but if in his brown study he jostles up against somebody who angrily draws off and knocks him down, the sceptic is unlikely to carry his scepticism so far as to doubt whether anything beside the ego was concerned in that phenomenon. The resistance shows him that something independent of him is there. When anything strikes upon the senses, the mind's train of thought is always interrupted; for if it were not, nothing would distinguish the new observation from a fancy. Now there is always a resistance to interruption; so that on the whole the difference between the operation of receiving a sensation and that of exerting the will is merely a difference of degree. We may, however, learn of a fact indirectly. Either the fact was experienced directly by some other person whose testimony comes to us, or else we know it by some physical effect of it. Thus we remark that the physical effects of a fact can take the place of experience of the fact by a witness. Hence, when we pass from the consideration of the appearance of a fact in experience to its existence in the world of fact, we pass from regarding the appearance as depending on opposition to our will to regarding the existence as depending on physical effects.

432. There can hardly be a doubt that the existence of a fact does consist in the existence of all its consequences. That is to say, if all the consequences of a supposed fact are real facts, that makes the supposed fact to be a real one. If, for example, something supposed to be a hard body acts in every respect like such a body, that constitutes the reality of that hard body; and if two seeming particles act in every respect as if they were attracting particles, that makes them really so.

This may be expressed by saying that the fact fights its way into existence; for it exists by virtue of the oppositions which it involves. It does not exist, like a quality, by anything essential, by anything that a mere definition could express. That does not help its mode of being. It might hinder it; because where there is not a unit there cannot be a pair; and where there is not a quality there cannot be a fact; or where there is not possibility there cannot be actuality. But that which gives actuality is opposition. The fact "takes place." It has its here and now; and into that place it must crowd its way. For just as we can only know facts by their acting upon us, and resisting our brute will (I say *brute* will, because after I have determined how and when I will exert my strength, the mere action itself is in itself brute and unreasoning), so we can only conceive a fact as gaining reality by actions against other realities. And further to say that something has a mode of being which lies not in itself but in its being over against a second thing, is to say that that mode of being is the *existence* which belongs to fact.

433. The same conclusion can be reached by another line of thought. There are different kinds of existence. There is the existence of physical actions, there is the existence of psychical volitions, there is the existence of all time, there is the existence of the present, there is the existence of material things, there is the existence of the creations of one of Shakespeare's plays, and, for aught we know, there may be another creation with a space and time of its own in which things may exist. Each kind of existence consists in having a place among the total collection of such a universe. It consists in being a second to any object in such universe taken as first. It is not time and space which produce this character. It is rather this character which for its realization calls for something like time and space.

434. When we speak of a fact as *individual*, or not general, we mean to attribute to it two characters each of which is altogether peculiar to facts. One of these is the character just described, the other having a mode of being independent of any qualities or determinations, or, as we may say, having brute fighting force, or self-assertion. The individual fact insists on being here irrespective of any reason, whether it be true or not

that when we take a broader view we are able to see that, without reason, it never could have been endowed with that insistency. This character makes a gulf between the *individual* fact and the *general* fact, or law, as well as between the individual *fact* and any quality, or mere possibility, which only mildly hopes it won't be intruding. But besides that character, individuality implies another, which is that the individual is determinate in regard to every possibility, or quality, either as possessing it or as not possessing it. This is the principle of excluded middle, which does not hold for anything general, because the general is partially indeterminate; and any philosophy which does not do full justice to the element of fact in the world (of which there are many, so remote is the philosopher's high walled garden from the market place of life, where fact holds sway), will be sure sooner or later to become entangled in a quarrel with this principle of excluded middle.

435. Thus far, in this section, attention has been called successively (but in no philosophical sequence) to six characteristic features of fact. In recollecting them, we may place at their head the circumstance that fact has distinct features, for this distinguishes it from quality although not from law. The others already examined have been as follows: second, facts are either accidentally actual or involve brute force; third, every fact has a here and now; fourth, fact is intimately associated with the dyad; fifth, every fact is the sum of its consequences; sixth, the existence of facts consists in fight; seventh, every fact is determinate in reference to every character. But in making our distribution of the elements of phenomena into quality, fact, and law, we were led to notice additional features of fact. I continue to take them up promiscuously.

436. The eighth feature of fact is that every fact has a subject, which is the grammatical subject of the sentence that asserts the existence of the fact. Indeed, in a logical sense, there are two subjects; for the fact concerns two things. One of these two subjects, at least, is a thing itself of the nature of fact, or we may express this in other words by saying that the existence of this subject is a fact. This subject is a *thing*. It has its here and now. It is the sum of all its characters, or consequences. Its existence does not depend upon any definition,

but consists in its reacting against the other things of the universe. Of it every quality whatever is either true or false. That this subject, whose actions all have single objects, is material, or physical substance, or *body*, not a psychical subject, we shall see when we come to consider psychical subjects in discussing the nature of law. This does not in the least contradict idealism, or the doctrine that material bodies, when the whole phenomenon is considered, are seen to have a psychical substratum.

437. The ninth feature of fact is that every fact is connected with a reciprocal fact, which may, or may not, be inextricably bound up with it. If one body strikes upon another, that second body reciprocally strikes upon it; and the two facts are inseparable. But if one body is hard, there must be a second body of some degree of hardness for the former to resist. Yet the annihilation of the second body would [not] destroy the hardness of the first. It would not affect it; for any other body that might grow hard at any time and the first body, remaining unaffected, would realize its hardness whenever the impact with the other should happen to occur. Here, therefore, the reciprocal fact is not so inseparable from the other. If a solid body suddenly melts, it will at once flow into the vacant parts of its vessel; and the beginning to any such consequent fact will be a change reciprocal to the first change. But there is no particular consequence which will be inseparable from the melting, perhaps. There may or may not be. So we see that the division between facts inseparable from reciprocal facts is not coincident with a division of facts into those whose reciprocal facts are separable and those whose reciprocal facts are inseparable.

438. The tenth feature of fact, which has just been illustrated is that its natural classification takes place by dichotomies.

439. The eleventh feature of dual fact is that if it involves any variation in time, this variation consists of a change in the qualities of its subjects, but never the annihilation or production of those subjects. We may, indeed, conceive of an action by which something is produced or destroyed. But either a third subject will be concerned, so that the fact is one of those the study of which we have expressly postponed, or that which

is produced or destroyed will be one of those facts whose reciprocal facts are separable. If a star suddenly bursts into view, when no external subject caused it to do so, then, just as the appearance will be irrefragable demonstration that something dark was there before, so the fact itself will constitute the previous existence of its subject. For this is the only method by which we can deduce metaphysical truths. Consequently, bodies, and the subjects of facts generally, are permanent and eternal.

440. The twelfth feature of fact is that it is accidental. That is to say, even if it involves brute force, and though that force be governed by a law which requires the acting body continually to exert this force, yet nevertheless the individual action is not involved in the existence of the fact, but on the contrary is something that can only happen by having a subject with an independent mode of being not dependent upon this nor upon any determination whatsoever. It is something which *happens*.

I have taken no pains to make this promiscuous list of properties of fact complete, having only cared that it should be sufficient to enable us to compare the characters of fact with those of duality and thus ultimately to attain an understanding of why all phenomena should be composed of quality, fact, and law.

§4. DYADS*

441. Let us now inquire what is involved in the conception of *two*, and in particular by what features a pair is distinguished from a single *one*, on the one hand, and from three, or any larger set, on the other.

442. A mathematician will be inclined to pronounce this the most ridiculously trifling question to be called a problem that could well be imagined. A pair, he may say, is just an object and an object, and that is all that is involved in this puffed-category of the *dyad*. But any logician will tell him that *that* statement, at any rate, is inaccurate. For the purposes of the logic of mathematics it is fatally inaccurate. A married couple is not a man. Neither is it a woman, and *a fortiori* it is not, at once, a man and a woman. Nor is it dis-

* Cf. vol. 3, No. XVIII.

junctively either a man or a woman. It is a third object, to whose constitution, which is its nature, and therefore to its existence, too, a man is requisite and a woman is requisite. A pair is an object to whose constitution a subject and another subject are necessary and sufficient. This corresponds to a part of feature number eight of fact.

443. But accepting this amendment, which to his customary way of thinking is microscopic, the mathematician will be inclined to say, here is a perfect definition; and excepting a few little corollaries, there is nothing more to be said of the dyad. It behooves me, then, to clearly state what the inquiry is which I propose to institute. It is not a mathematical inquiry; because the business of the mathematician is to frame an arbitrary hypothesis, which must be perfectly distinct at the outset, so far, at least, as concerns those features of it upon which mathematical reasoning can turn, and then to deduce from this hypothesis such necessary consequences as can be drawn by diagrammatical reasoning. The present problem is one of logical analysis. Instead of setting out with a distinct hypothesis of a diagrammatic kind, we have the confused fact that a dyad is a conception of the highest utility, though we are not prepared to say exactly what its nature is, nor even, in all cases, whether a given case should properly be reckoned as a duality or not. We are somewhat in the position of a naturalist who knows that whales are large swimming animals, which spout water, and yield blubber, spermaceti, and whalebone, but knows little else about them, and who proposes to himself to examine the anatomy and physiology of whales so as to assign them their place in the system of the animal kingdom. He does not intend to preserve the popular description nor delimitation of the class of whales. He will perhaps see reason to extend the name to some animals not popularly called whales and to refuse it to others that are so called. He will also subdivide the group, and classify it according to the facts. So far as our inquiry is a logical analysis, the greatest difference between it and that of a taxonomic biologist consists in the circumstance that we are not forced to institute special observations, because all the facts are either well known or can be ascertained by careful reflection upon those that are known.

444. But besides being logical in the sense of demanding

a logical analysis, our inquiry also relates to *two* as a conception of logic. The term "logic" is unscientifically by me employed in two distinct senses. In its narrower sense, it is the science of the necessary conditions of the attainment of truth. In its broader sense, it is the science of the necessary laws of thought, or, still better (thought always taking place by means of signs), it is general semeiotic, treating not merely of truth, but also of the general conditions of signs being signs (which Duns Scotus called *grammatica speculativa**), also of the laws of the evolution of thought, which since it coincides with the study of the necessary conditions of the transmission of meaning by signs from mind to mind, and from one state of mind to another, ought, for the sake of taking advantage of an old association of terms, be called *rhetorica speculativa*, but which I content myself with inaccurately calling *objective logic*, because that conveys the correct idea that it is like Hegel's logic. The present inquiry is a logical one in the broad sense. It is a study of dyads in the necessary forms of signs.

Our method must be to observe how logic requires us to think and especially to reason, and to attribute to the conception of the dyad those characters which it must have in order to answer the requirements of logic.

445. We can at once see that a pair, having a structure, must present a variety of features; and this is a character in which the dyad differs markedly from the monad, which having no structure nor parts in any sense, is bare of all features except that each *one* is something peculiar. This corresponds to feature number one of fact.

446. A monad has no units. This sounds paradoxical, and seems to the mathematician an *aperçu* from an arbitrary point of view; but we soon see that it is the suitable point of view for logical purposes. In the pair there are unit parts; and so there are in all higher sets. Let us inquire, then, what is the function of the units of a set in the constitution of that set. We must first remark that in logic a set cannot generally be adequately represented by a diagram of a promiscuous collection of dots. Of the multitudinous examples of this in mathematics it will be sufficient to call to mind the constituents of a determinant, and how they have to be arrayed in a square block. As a

* *Opera Omnia Collecta*, L. Durand, T. 1, pp. 45–76.

general rule, the form of connection (or a part of it, at least) must be considered in logic in case a set has to be considered as such. This form of connection belongs to the set and not to its units. Now reasoning is formal. That is to say, whatever inference is sound concerning one thing or one character is sound in regard to any other thing or character whose form of connection (so far as it need be considered) is strictly analogous to that of the former. All that has to be represented, then, for the purposes of logic, is the characters of the sets themselves; and the units need exhibit nothing except what is requisite to the exhibition of the characters belonging to sets. What, then, is the use of the units, at all? And how can they, when thus stripped of all qualities, contribute to the representation of characters of sets? The answer is that if all that were desired was to present for contemplation the character of a set, the statement of the mode of its connection in abstract terms, with no particular reference to the units, would be sufficient; and in point of fact, this is the general form which metaphysicians give to their statements, so far as the usages of speech render it convenient. But when, one set having been represented, it is desired to attach to it the representation of another set, and there is a unit or units which belong to both sets, then in order to show how the total set is composed of those two sets, it is necessary to take account of the identities of their common units. Now identity is a relation which cannot be implied by a general description of the identical things; and the descriptions of the sets, so far as they leave out the individual things, are general. Hence, it follows that the only purpose in indicating the units in the representation of the set, is in order that each of them may signify its identity with an individual of another set. The identity of different units of the same set might be similarly represented. Hence, passing from the representation of the set, to the set itself, as it is logically conceived, the only function of the units in it is to establish possible identities with the units of other sets. A unit, therefore, is something essential to a set whose existence consists in its possible identity with another unit of the same or another set. Now, identity is essentially a dual relation. That is, it requires two subjects and no more. If three objects are identical, this fact is entirely contained in the fact that the three

pairs of objects are identical. Hence a unit is something whose existence consists in a possible dyad of which it is the subject. Thus, there is an element of twoness in every set. So I was right in saying that the monad has no unit, since the monad in no wise involves the dyad.

447. There are certain truths about quality not considered in Section 2, for the reason that they were considered as belonging under the head of the dyad. They do not concern the monad in its aspect as one, but are dyads of monads. One of these is that whatever is a possible aspect irrespective of parts has possible parts. I mean that any object presenting a quality in its purity might be further determined. Every quality is, in itself, general. Given any possible determination, there is a possible further determination. In the beginning was nullity, or absolute indetermination, which, considered as the possibility of all determination, is being. A monad is a determination *per se*. Every determination gives a possibility of further determination. When we come to the dyad, we have the unit, which is, in itself, entirely without determination, and whose existence lies in the possibility of an identical opposite, or of being indeterminately over against itself alone, with a determinate opposition, or over-against-ness, besides.

It follows that a set considered apart from its units is a monad. In fact, in not considering the units, we allow all sets of the same general character to collect before us, and regard those sets as a monad without parts.

But a set considered as made up of units in a peculiar connection is a dyad if its units are two, a triad if they are three, etc. A part of the above corresponds to feature number eight of fact.

448. Let us further examine the dyads of monads just mentioned. But before doing so, this designation has to be defended. It may be objected, with apparent force, that the truth just mentioned about quality is general, and that, as general, it applies to innumerable qualities and not to a pair. This is perfectly true; but then, all that we are inquiring into concerning the monad, the dyad, and plural sets, is general, and therefore, from that point of view we are not studying the monad and the dyad at all, but polyads of monads and dyads.

This is true. Our thought is rational and, as such, general, or of the plural nature. But it relates to the monad and the dyad in part. So the truth just mentioned is general, but it relates to a single monad, any monad, and declares of it that a monad exists which is, for thought, equivalent to that monad further determined.

449. This is one of three regulative laws of logic of high importance which were enunciated by Kant in the *Critic of the Pure Reason*.* The other two are that there is a determination less than and included in any possible determination, and that between any two determinations, one included in the other, a third may be found. Besides these dyads, both whose subjects are monads, there are also certain dyads, one of whose subjects is a monad and the other a possible dyad, that is, a unit. And there are general laws connected with these.

450. The first of these is that any unit (or units) whatsoever contemplated in itself without conscious regard to its parts would, were our sense to respond to it, be seen to embody a monad. De Morgan propounded this law, so far as it is pertinent to formal logic, affirming that any collection of objects whatsoever possess universally some character which belongs to no other object at all. For, said he, they at least possess the character of being units of that collection. Considered as a *proof*, this begs the question; but considered as another way of formulating the same phenomenon, and as a way which throws some light upon it, it has its value. This coincides with the principle of excluded middle. Those objects of the universe which do not possess a given character possess another character which, in reference to that universe, is in the relation of negation to the first. Hence, it is impossible to form a single class of dyads; two classes of dyads must be formed at once. Hence, considering all the monads which can appear on the contemplation of sets of units of the universe in their monadic aspect, every single unit is determined to be one subject of a dyad which has any one of those monads as its second subject, namely it is either such a dyad as determines it to have the character of being one of the units which made up the object of the contemplation in which that monad appeared, or it is

* Appendix to the "Transcendental Dialectic."

such a dyad as determines the unit to have the character belonging to all the other units of the universe.

451. What has been here affirmed of collections of units is equally true of collections of monads. Namely, any monads may be contemplated together, and in their monadic aspect without regard to the single monads are seen to be one monad. There is thus a relation between monads similar to the relation of a unit to a monad. But there is this difference in the two cases: a monad thus embraced under another monad is so embraced in its very mode of being, while that a unit should be embraced under a monad has no concern with the mode of existence of the unit, which lies in its brute self-identity and otherness from all the rest. It is on the contrary an adventitious circumstance that this particular unit is embraced under this monad.

This corresponds to feature number seven and in part to feature number twelve of fact.

452. The metaphysical categories of quality, fact, and law, being categories of the matter of phenomena, do not precisely correspond with the logical categories of the monad, the dyad, and the polyad or higher set, since these are categories of the forms of experience. The dyads of monads, being dyads, belong to the category of the dyad. But since they are composed of monads as their sole matter, they belong materially to the category of quality, or the monad in its material mode of being. It cannot be regarded as a *fact* that scarlet is red. It is a *truth;* but it is only an essential truth. It is that in being which corresponds in thought to Kant's analytical judgment. It is a dyadism latent in monads.

453. I may pause here a moment to remark that when I say nullity consists of the possibility of the monad; that the unit consists of the possibility of the dyad, and the like, such statements have a Hegelian sound. Undoubtedly they are intrinsically of that nature. I follow an order of evolution in such phrases, the possibility evolves the actuality. So does Hegel. He reaches each category from the last preceding by virtually calling "next!" What his process [is] of making the next come and of recognizing it when it emerges is, however important it may be, yet, comparatively speaking a detail, wherein I sometimes agree with the great idealist and sometimes diverge

from his footsteps — for my own method has resulted from a more deliberate examination of the exact theory of logic (in which Hegel's age, and especially his own country, and more especially he himself were decidedly weak), and consequently has a broader form, capable of diversification to adapt itself to the special form of the germinal conception. It is not yet time to formulate it. I apply it; the reader follows it with approval if he can; and a later review will show what the laws of the procedure have unconsciously been.

454. The most important division of dyads is with reference to the character of their subjects. For subjects differ in regard to the nature of the dyads which they are capable of forming. They are either dyads formed merely from monads or they are dyads into which enter objects having a dyadic mode of being, that is, individual things, or units.

455. Dyads of the former kind are seen to subsist as soon as the two monads are regarded together; and arguing from knowledge to being (that is, merely abstracting from the imported idea of a knower), they do subsist in so far as the two monads are compossible, that is in so far as both are such monads as they are. When scarlet and red are contemplated together, the former as first, the latter as second, a certain aspect *sui generis* presents itself, like that which presents itself when toothache and ache are contemplated together, the former first, the latter second. This kind of dyadism or dyadic relation which is evolved from the very being of the subjects as soon as they are together, I call an *essential* dyadic relation, and the dyad so formed an *essential dyad*. This is the only kind of dyad that can be composed out of monads alone; because monads having no parts nor distinct features cannot, whether singly or collectively, have any characters except those which spring directly from their several beings *sui generis*.

456. Dyads that are *accidental*, that is, are collective characters of their subjects adventitious to their being, must therefore concern subjects (or one subject, at least) which is not a monad, and consequently having a mode of being over and above what its mere inward suchness involves. It must have a mode of being gained by its opposition to another, that suchness does not avail to confer. What is this mode of being in its most general terms? In order that our conception of it may

embrace every variety, let it begin as soon as the mode of being of the monad ends. Combine quality with quality after quality and what is the mode of being which such determinations approach indefinitely but altogether fail ever to attain? It is, as logicians have always taught, the *existence* of the individual. Individual existence whether of a thing or of a fact is the first mode of being that suchness fails to confer. Suchness, or the mode of being of the monad, is the mere possibility of an existent.

457. Existence is that mode of being which lies in opposition to another. To say that a table exists is to say that it is hard, heavy, opaque, resonant, that is, produces immediate effects upon the senses, and also that it produces purely physical effects, attracts the earth (that is, is heavy), dynamically reacts against other things (that is, has inertia), resists pressure (that is, is elastic), has a definite capacity for heat, etc. To say there is a phantom table by the side of it incapable of affecting any senses or of producing any physical effects whatever, is to speak of an imaginary table. A thing without oppositions *ipso facto* does not exist. Of course, the question arises, if everything that exists exists by its reactions, how does the total collection of things exist? This is a legitimate and valuable question, the answer to which brings out a new idea. But this is not the time to consider it. Our purpose of developing the complete scheme of philosophical ideas is defeated unless we take up the points one by one in their due order. That question about the totality of things throws no doubt upon the manifest truth that existence lies in opposition; and the very fact that the consideration of it would lead to a still more developed philosophy is the very reason why it must be postponed until we have mastered the conception of being through opposition.* Not only is this opposition essential to an individual thing or subject, but also to an individual fact. Its truth, or existence, is the sum of its effects.

458. *Hic et nunc* is the phrase perpetually in the mouth of Duns Scotus, who first elucidated individual existence. It is a forcible phrase if understood as Duns did understand it, not as describing individual existence, but as suggesting it by an example of the attributes found in this world to accompany it. Two drops of water retain each its identity and opposition to

* See 6.415.

the other no matter in what or in how many respects they are alike. Even could they interpenetrate one another like optical images (which are also individual), they would nevertheless react, though perhaps not at that moment, and by virtue of that reaction would retain their identities. The point to be remarked is that the qualities of the individual thing, however permanent they may be, neither help nor hinder its identical existence. However permanent and peculiar those qualities may be, they are but *accidents;* that is to say, they are not involved in the mode of being of the thing; for the mode of being of the individual thing is existence; and existence lies in opposition merely.

459. We observe no life in chemical atoms. They appear to have no organs by which they could act. Nor can any action proper gain actuality, that is, a place in the world of actions, for any subject. Yet the individual atom exists, not at all in obedience to any physical law which would be violated if it never had existed, nor by virtue of any qualities whatsoever, but simply by virtue of its arbitrarily interfering with other atoms, whether in the way of attraction or repulsion. We can hardly help saying that it blindly forces a place for itself in the universe, or willfully crowds its way in.

460. No reaction among individual things can create one of those things nor destroy it; for before its existence or after it there would not be anything to react. So that the fountain of existence must be sought elsewhere.

461. Existence, though brought about by dyadism, or opposition, as its proper determination, yet, when brought about, lies *abstractly and in itself considered*, within itself. It is numerical identity, which is a dyadic relation of a subject to itself of which nothing but an existent individual is capable. It is to be observed that numerical identity is not empty verbiage, as the identity of a quality with itself is, but is a positive fact. This is due to the possibility of the individual's assuming different accidents. Throughout all vicissitudes its oppositions to other things remain intact, although they may be accidentally modified; and therein is manifest the positive character of identity.

462. The only primary essential dyadism is that between a containing monadic quality and a contained monadic quality.

For qualities cannot resemble one another nor contrast with one another unless in respect to a third quality; so that the resemblance of qualities is triadic. This, however, is a point calling for reëxamination in a future revision of this analysis. If I am right, there is no further logical distinction between essential dyads.

463. But with regard to accidental dyads, the case is far otherwise. We must at once divide them into those of which one subject is a monad, and those of which neither subject is a monad. This division is closely allied to and immediately suggested by the last. Dyads of the former kind may be termed *inherential* — as, this thing possesses redness; those of the latter kind may be called *relational*.

464. An inherential dyad strongly resembles an essential dyad. Begin with any quality, as high-colored, and form an essential dyad, as *red is high-colored*. Form another with red as second subject; as, *scarlet is red*. Form another with scarlet as second subject; as, *mercuric-iodide-color is scarlet*. So we may conceive determination added to determination, and at the limit a color so specific that it can only belong to an individual object. This I say is the limit which lies just beyond the possible, but is indefinitely approachable. This limit is a dyad of inherence. It is, after all, however, radically different from the essential dyad, because the quality of the subject of inherence is a mere accident of that individual. Inherence may be regarded from another point of view. Namely, the individual subject may be conceived as brought into relation to itself by the possession of the attribute.

465. Relational dyads are not further divisible in regard to the metaphysical character of their subjects. But they are divisible in regard to the nature of the connection between their subjects. And, first of all, a division is suggested by the last remark concerning inherential dyads. Namely, every relational dyad is either a *dyad of identity*, in which the two subjects are existentially one and the same, or it is a *dyad of diversity*, in which the subjects are existentially two and distinct. This relational identity is not the identity of inherence, but the identity which is altogether independent of any accident or accidents. It will, however, involve such inherences as may belong to the individual and identical subject.

466. With this division another is closely connected; namely, a dyad of diversity may either be such that the connection between its units consists merely in their agreement or difference in respect to a monadic quality, or it may be such that the connection of the units depends upon their possessing some dyadic character or characters. This distinction is most deeply engraved into the natures of dyads. For what is a dyadic character? It is a character conferred upon one individual by another individual. It thus involves the idea of *action* or *force*, not in a narrow scientific sense, but in the sense in which we speak of the will as a force. We may say then that this division is into *qualitative* and *dynamical* diversities. Or, in place of qualitative diversity, it will perhaps be better to use the familiar phrase *partial agreements*.

467. Dynamic dyads are, in the first place, distinguished into those which, by virtue of the characters which they attribute to their subjects, put those two subjects into like relations each to the other, and into those which, so far as the characters they attribute to their subjects go, leave a distinction between the reciprocal relations. The former kind may be called *materially unordered*, the latter, *materially ordered*. Thus, A is one mile from B is a materially unordered relation, but A kills B is materially ordered, notwithstanding that it may happen that B also kills A.

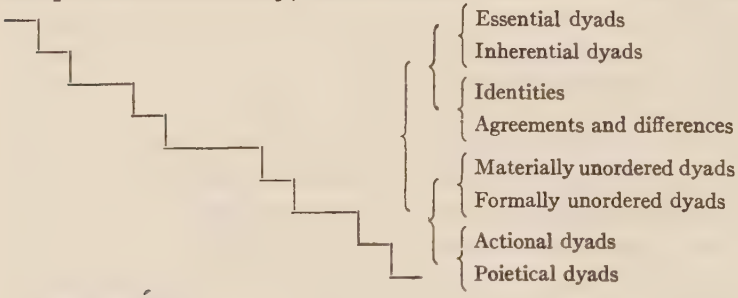
468. Closely connected with this distinction is another; namely, materially ordered dyads are divisible into those in which there is no existential or intrinsic distinction between the subjects as to which is first and which second, although in stating the fact language may require us to mark one as first and the other as second, and into those in which this distinction is existential. The former may be called *formally unordered* dyads, the latter *formally ordered*. Thus, when amber is rubbed against fur, one acquires resinous and the other vitreous electricity. The dyad is thus materially ordered. But, as far as we know, neither is to be regarded as distinctively agent or first in contradistinction to the patient, or second. When, however, of two oppositely electrified bodies one attracts the other, although the second equally attracts the first, yet the two attractions are distinct dyads and the attracting body is agent, or intrinsically first, while the attracted

body is intrinsically second. For one is determining and the other determined. Now the determining body is, in so far, left indeterminate; and indeterminacy, or possibility, as the character of the monad, is *first* relatively to determination, which, as essentially dyadic, is *second*.

469. There is no further room for distinction based upon the positions of the subjects; but the formally ordered dyads can still be divided with reference to the character of the dependence of one subject upon the other. Namely, this is either such that merely the monadic accidents of the second subject, or patient, are dependent upon the agent, or such that the dyadic existence of the patient is dependent upon the agent. The former may be termed *actional*, the latter *poietical*,* or *productive*, dyads.

No further distinctions seem to be relevant to the idea of the present analysis.

470. It will be remarked that the division is everywhere a dichotomy of the second of the two classes formed by the next preceding dichotomy. The result is that the ultimately undivided species form a staircase of successive steps. But the steps are not all equal. On the contrary, so thoroughly does twoness permeate the whole that the steps separate into successive pairs. There is also a marked distinction between the first pair of pairs and the second pair of pairs, which repeats the former with a variation. That is to say, the first pair of each of the two pairs of pairs arises from distinctions concerning the subjects, while the second pair of each pair of pairs arises from distinctions concerning the mode of connection of the subjects. The whole series of species of dyads are related like the phrases of a melody, as follows:



* From ποιέω.

The more this division is examined, the more clearly it will appear that it is not a fancifully imposed scheme, but springs inevitably from the evolution of the conceptions according to the general point of view adopted.

§5. TRIADS*

471. We come now to the *triad*. What is a triad? It is a three. But three what? If we say it is three subjects, we take at the outset an incomplete view of it. Let us see where we are, remembering that logic is to be our guide in this inquiry. The monad has no features but its suchness, which in logic is embodied in the signification of the verb. As such it is developed in the lowest of the three chief forms of which logic treats, the term, the proposition, and the syllogism. The dyad introduced a radically new sort of element, the subject, which first shows itself in the proposition. The dyad is the metaphysical correlative of the proposition, as the monad is of the term. Propositions are not all strictly and merely dyadic, although dyadism is their prominent feature. But strictly dyadic propositions have two subjects. One of these is active, or existentially prior, in its relation to the dyad, while the other is passive, or existentially posterior. A gambler stakes his whole fortune at an even game. What is the probability that he will gain the first risk? One half. What is the probability that he will gain the second risk? One fourth; for if he loses the first play, there will be no second. It is one alternative of the prior event which divides into two in the posterior event. So if A kills B, A first does something calculated to kill B, and then this subdivides into the case in which he does kill B and the case in which he does not. It is not B that does something calculated to make A kill him; or if he does, then he is an active agent and the dyad is a different one. Thus, there are in the dyad two subjects of different character, though in special cases the difference may disappear. These two subjects are the *units* of the dyad. Each is a *one*, though a dyadic one. Now the triad in like manner has not for its principal element merely a certain unanalyzable quality *sui generis*. It makes [to be sure] a certain feeling in us. [But] the formal rule governing the triad is that it remains equally true for all six permutations

* Cf. vol. 2, bk. II, ch. 2, §3 ff.

of A, B, C; and further, if D is in the same relation at once to A and B and to A and C, it is in the same relation to B and C; etc.

472. Each of the three subjects introduces a dyad into the triad, and so does each pair of subjects. The distinctive character or quality of the triad is a monadic element. The formal law of the triad is essentially triadic. It is in that, that the threeness inheres.

473. Every triad is either *monadically degenerate*, *dyadically degenerate*, or *genuine*. A monadically degenerate triad is one which results from the essence of three monads, its subjects. A dyadically degenerate triad is one which results from dyads. A genuine triad is one which cannot be resolved in any such way. That orange color is intermediate between red and yellow is a monoidally degenerate triad. So that one given quality is a compound of two others. So [that] red and green resemble violet more than they resemble each other. That red is a determination of color and scarlet of red involves a monadically degenerate triad and belongs to the class of essential triads; yet it is properly a dyadically degenerate triad where the component dyads are essential dyads. It is thus essential, but only indirectly essential. So that oranges and lemons smell alike, though it is properly only a dyad, yet may be considered as a triad, the common quality of smell being the third subject. That a citric taste and a perfume of a cologne water kind coexist in the lemon can only be regarded as a triad and not as a dyad. That A is father of B and B father of C is a genuinely dyadic degenerate triad. That A is as far north of B as B is east of C is a triad formed of two dyads of one kind and a dyad of another kind—(I mean the similarity of the other two, but this is accidental). This is an almost, but not quite, genuine triad. A is mother of B and B is wife of C. Here the two component dyads are more independent of one another. This is a purer case of the dyadic degenerate triad.

474. In considering the genuine triad, it is well to notice first that the last fact supposed involves the fact that A is mother-in-law of C, which is no triad, but a dyad. Indeed, every triad, as above remarked, involves a dyad; but it is the peculiarity of the dyadic triad that this dyad only differs from the triad in the lack of particularization of the mediating sub-

ject. So, reversing the process, every dyad by a particularization evolves a dyadic triad. Thus, A murders B is a generalization of A shoots that bullet, and the bullet fatally wounds B. This is true even in regard to the dyad, A winks, which evolves the triad, A experiences a nervous irritation and the nervous irritation causes winking of the eyelid. Such an evolution may be called an *explication* of the dyad. So the monad colored is explicated in the monadic dyad, red is colored, and red is explicated in scarlet is red. A triad may be explicated into a triadic tetrad. Thus, A gives B to C becomes A makes the covenant D with C and the covenant D gives B to C.

475. But if we compare the monad implicated in a genuine dyad, as *red* is in "this thing is red," with that dyad, we see that the latter is more than any mere explication of red. It is the truth of what Kant called a synthetic (that is, genuinely dyadic) judgment. It involves *existence*, while *red* or any mere explication of red is but a *possibility*. Even in "something is red," which leaves wholly indeterminate what it is that is red, and consequently does not really explicate red, at all, existence is just as positive as in, "this is red." Now let us consider the triad, A makes a contract with C. To say that A signs the document D and C signs the document D, no matter what the contents of that document, does not make a contract. The contract lies in the intent. And what is the intent? It is that certain conditional rules shall govern the conduct of A and of C. There is no positive fact in this; it is only conditional and intentional. Still less, if possible, is it any mere monadic quality. It has reference to conditions of experience, involving existence, involving dyadic fact. It may be said that it is a psychical fact. This is in so far true, that a psychical fact is involved; but there is no intent unless something be intended; and that which is intended cannot be covered by any facts; it goes beyond anything that can ever be done or have happened, because it extends over the whole breadth of a general condition; and a complete list of the possible cases is absurd. From its very nature, no matter how far specification has gone, it can be carried further; and the general condition covers all that incompletable possibility.

476. There, then, we have an example of a genuine triad and of a triadic conception. But what is the general descrip-

tion of a genuine triad? I am satisfied that no triad which does not involve generality, that is, the assertion of which does not imply something concerning *every possible* object of some description can be a genuine triad. The mere addition of one to two makes a triad; and therein is contained an idea entirely indecomposable into the ideas of one and two. For addition implies two subjects added, and something else as the result of the addition. Hence, it is wrong to define two as the sum of one and one; for according to such a definition, two would involve the idea of three. The idea characteristic of two is *other*. The corresponding idea characteristic of three is *third*. *Medium* is nearly as broad, and so is *uniter*.

477. The genuine triad contains no idea essentially different from those of *object*, *other*, *third*. But it involves the idea of a third not resoluble into a formless aggregation. In other words, it involves the idea of something more than *all that can* result from the successive addition of one to one. This "all that can" involves the idea of *every possible* something, and therefore of generality. The genuine triad, then, must involve generality.

478. The world of fact contains only what *is*, and not everything that is possible of any description. Hence, the world of fact cannot contain a genuine triad. But though it cannot contain a genuine triad, it may be governed by genuine triads.

So much for the division of triads into the monadic, dyadic, and triadic of [or?] genuine triads.

479. Dyadic triads are obviously of two kinds, first, those which have two monadic subjects, as a high perfume and a burning taste are united in many essential oils, and secondly, those which have all their subjects individuals.

480. Genuine triads are of three kinds. For while a triad if genuine cannot be in the world of quality nor in that of fact, yet it may be a mere law, or regularity, of quality or of fact. But a *thoroughly* genuine triad is separated entirely from those worlds and exists in the universe of *representations*. Indeed, representation necessarily involves a genuine triad. For it involves a sign, or representamen, of some kind, outward or inward, mediating between an object and an interpreting thought. Now this is neither a matter of fact, since thought

is general, nor is it a matter of law, since thought is living.

481. So much for the first order of subdivisions of the three classes of triads. Passing to the lower subdivisions, I find among those of the degenerate triads nothing of particular philosophical interest; though something may have been overlooked. But among the lower subdivisions of the genuine triads there is an abundance.

482. We first consider the first two of the three chief divisions of genuine triads, which are the laws of quality and the laws of fact. The laws of quality are all of one type. Namely, they all simply determine systems of qualities, of which Sir Isaac Newton's law of color-mixture with Dr. Thomas Young's supplement thereto, is the most perfect known example.

483. The laws of fact divide themselves at the outset into those which must be true if there be any true answer to every question that has a meaning, or, as we say, into laws *logically necessary* and laws *logically contingent.* To this division another is intimately connected. Namely, of laws logically contingent the most universal are of such a kind that they must be true provided every form which by logical necessity must be thought of a given subject is also a form of its real being. Calling this kind of necessity, metaphysical necessity, we may divide laws logically contingent into laws *metaphysically necessary* and laws *metaphysically contingent.*

484. The general law of quality, as distinct from the classificatory system of quality (of which we can have but a fragmentary knowledge), has three clauses, relating respectively to single qualities, to pairs of qualities, and to triads of qualities. The first clause is that every quality is perfect and in itself such as it is. The second more complex law is that two qualities have one or other of two sorts of relations to one another; namely, they may be, first, independent of one another, somewhat resembling and somewhat differing from one another, or secondly, one of them may be merely a further determination of the other, this latter being essentially the first of the pair in the order of evolution, or synthesis, while it is the second of the pair in the order of involution or analysis. The third clause relates to the respects, or third qualities, in which two compared qualities agree or differ. The first of these respects is the quality of the quality, or, as we may say, its

hue, in which respect the tastes of sugar and salt differ, or the pitch of sounds, or the respect in which red, blue, and green differ. The second respect is the absolute intensity of the quality, loudness in sounds, luminosity in color, strength in tastes and smells, etc. The third respect is purity, or the relative intensity of the strongest elements. It is great in high colors and in musical sounds. In some cases strength and weakness have peculiar hues. Bright colors tend toward yellow, dim colors toward violet. Very faint sounds tend toward a certain pitch. Purity and impurity may have their peculiar hues.

485. The general law of logic has likewise its three clauses. The monadic clause is that fact is in its existence perfectly definite. Inquiry properly carried on will reach some definite and fixed result or approximate indefinitely toward that limit. *Every subject is existentially determinate with respect to each predicate.* The dyadic clause is that there are two and but two possible determinations of each subject with reference to each predicate, the affirmative and the negative. Not only is the dyadic character manifest by the double determination, but also by the double prescription; first that the possibilities are two at least, and second that they are two at most. The determination is not both affirmative and negative, but it is either one or the other. A third limiting form of determination belongs to any subject [with regard] to [some other] one whose mode of existence is of a lower order, [the limiting case involving] a relative *zero*, related to the subjects of the affirmation and the negation as an inconsistent hypothesis is to a consistent one. The triadic clause of the law of logic recognizes three elements in truth, the idea, or predicate, the fact or subject, the thought which originally put them together and recognizes they are together; from whence many things result, especially a threefold inferential process which either first follows the order of involution from living thought or ruling law, and existential case under the condition of the law to the predication of the idea of the law in that case; or second, proceeds from the living law and the inherence of the idea of that law in an existential case, to the subsumption of that case and to the condition of the law; or third, proceeds from the subsumption of an existential case under the condition of a living

law, and the inherence of the idea of that law in that case to the living law itself.* Thus the law of logic governs the relations of different predicates of one subject.

486. The general law of metaphysics is little understood. The attention of thinkers has been so rivetted upon the question of its truth, that they have largely overlooked the importance of determining precisely what it is, even if it be not absolutely true, since it is certainly the product of natural thought and of reasoning which, however far it may be carried beyond the legitimate conclusion, is nevertheless true reasoning of a valid type. The difficulty of making here any brief statement of any value is great enough for that reason. But besides that, brief statements of a metaphysical kind can hardly be made intelligible. I can only notice some items of the law going to exhibit the threefold division of the law.

487. Metaphysics consists in the results of the absolute acceptance of logical principles not merely as regulatively valid, but as truths of being. Accordingly, it is to be assumed that the universe has an explanation, the function of which, like that of every logical explanation, is to unify its observed variety. It follows that the root of all being is One; and so far as different subjects have a common character they partake of an identical being. This, or something like this, is the monadic clause of the law. Second, drawing a general induction from all observed facts, we find all realization of existence lies in opposition, such as attractions, repulsions, visibilities, and centres of potentiality generally. "The very hyssop on the wall grows in that chink because the whole universe could not prevent its growing." This is, or is a part of, a dyadic clause of the law. Under the third clause, we have, as a deduction from the principle that thought is the mirror of being, the law that the end of being and highest reality is the living impersonation of the idea that evolution generates. Whatever is real is the law of something less real. Stuart Mill defined matter as a permanent possibility of sensation.† What is a permanent possibility but a law? Atom acts on atom, causing stress in the intervening matter. Thus force is the general fact of the states of atoms on the line. This is true of force in its widest

* Cf. 2.620 ff.

† In his *Examination of Sir Wm. Hamilton's Philosophy*, ch. 11.

sense, dyadism. That which corresponds to a general class of dyads is a representation of it, and the dyad is nothing but a conflux of representations. A general class of representations collected into one object is an organized thing, and the representation is that which many such things have in common. And so forth.

488. Passing to laws that are metaphysically contingent, that is, to such as are not necessarily involved in the literal extension to being of the necessary laws of logical truth, we may first divide these into those which impose upon the subjects of dyadic existence forms of reaction analogous to those of logic, that is, the *laws of time* (by which they evade the laws of logic in regard to contrary inherences) and those which have no relation to logic. And with this division another is closely connected, namely, the division of the latter class of laws into those which are imposed upon objects as reacting upon one another existentially, as merely coexistent, which are the laws of *space*, and into those which are only imposed upon objects in so far as their mode of existence is in its own metaphysical nature that of a subject, that is, laws of substantial things.

In regard to these two divisions a long and arduous philosophical discussion is quite ineluctable. It would be quite impracticable to summarize it in the present sketch of the shapes which are assumed by the three fundamental ideas of philosophy. All that can be done is to unfold in some measure the characteristics of the view here taken.

489. In the first place, then, it is plain enough that the law of time is not a metaphysical law. Our logical instinct tells us that. We took as the typical example of a metaphysical law, the law that whatever exists, although its existence is a matter of brute fact, irrespective of any qualities, must definitely possess or be without each monadic quality. Now we feel instinctively that the necessity of that is altogether higher than any necessity for the junctions, between the possessions by a subject of contrary attributes, to be related to one another like premisses and conclusions, as before and after. The one is the mere existential mirror of a law of logic. It is the requirement that that which is necessarily *true* (if there be any truth) shall be a part of the existential fact, and not merely of thought. But the other requires that the mere process of thought, which

logic regards as mental, and never insists upon predicating of the subject as true, shall itself be mirrored in existence. But while the law of time is not metaphysical, it is plainly, from that description of it, "next door to" a metaphysical law. This is the reason for making this division follow immediately after that into laws metaphysically necessary and contingent.

490. It will be very difficult for many minds — and for the very best and clearest minds, more difficult than for others — to comprehend the logical correctness of a view which does not put the assumption of time *before* either metaphysics or logic instead of *after* those kinds of necessity, as here arranged. But that is an objection, not to this particular item of the development, but to the general plan of it. To admit the force of the objection and carry it out to its consequences would simply result in reversing the whole order of development, making it begin with polyads, analyzing these into triads, and then finding dyads in triads, and monads in dyads. There is not only nothing erroneous in such an arrangement, but the conceptions cannot be thoroughly grasped until it has been carried out. But this is only one of two sides of the shield, both of which must be examined, and which have to be synthesized in the really philosophical view. The reason of this is, that although the view which takes the triad first is necessary to the understanding of any given point, yet it *cannot*, from the very nature of the case, be carried out in an entirely thoroughgoing manner. How, for instance, would you begin? By taking the triad *first*. You thus do, in spite of yourself, introduce the monadic idea of "first" at the very outset. To get at the idea of a monad, and especially to make it an accurate and clear conception, it is necessary to begin with the idea of a triad and find the monad-idea involved in it. But this is only a scaffolding necessary during the process of constructing the conception. When the conception has been constructed, the scaffolding may be removed, and the monad-idea will be there in all its abstract perfection. According to the path here pursued from monad to triad, from monadic triads to triadic triads, etc., we do not progress by logical involution — we do not say the monad *involves* a dyad — but we pursue a path of evolution. That is to say, we say that to carry out and perfect the monad, we need next a dyad. This seems to be a vague method when stated in

general terms; but in each case, it turns out that deep study of each conception in all its features brings a clear perception that precisely a given next conception is called for.

491. So far Hegel is quite right. But he formulates the general procedure in too narrow a way, making it use no higher method than dilemma, instead of giving it an observational essence. The real formula is this: a conception is framed according to a certain precept, [then] having so obtained it, we proceed to notice features of it which, though necessarily involved in the precept, did not need to be taken into account in order to construct the conception.* These features we perceive take radically different shapes; and these shapes, we find, must be particularized, or decided between, before we can gain a more perfect grasp of the original conception. It is thus that thought is urged on in a predestined path. This is the true evolution of thought, of which Hegel's dilemmatic method is only a special character which the evolution is sometimes found to assume. The great danger of the evolutionary procedure lies in forcing steps that are not inevitable, in consequence of not having a sufficiently distinct apprehension of the features of the conception in hand to see what it is that must immediately succeed it. The idea of time must be employed in arriving at the conception of logical consecution; but the idea once obtained, the time-element may be omitted, thus leaving the logical sequence free from time. That done, time appears as an existential analogue of the logical flow.

492. Time is said to be the form of inward intuition. But this is an error of the sort just considered. It confuses what is evolved from the time-idea with what is involved in it. The task of the analyst in making out the features of the time-law must begin by formulating precisely what it is which that law explicitly pretends to make subject to time. It is, in the first place, only real events that "take place," or have dates, in real time. Imaginary events, the course of a romance, are represented as having relations *like* those of time among one another, but they have no real places in time. A historical romance connects itself, more or less definitely, with real time; but that is because it "makes believe" they [the imaginary events] are real events. It is, then, only existentially real events which

* Cf. 6.302.

the law of time represents really to have places in real time. What, then, is a real event? It is an existential junction of incompossible facts. A pale yellowish iron solution mixed with a pale yellow solution of ferrocyanide of potassium suddenly turns deep blue. It is requisite that its being of a pale greenish or reddish yellow, and therefore not blue, should be a fact, and that the same thing's being blue should be a fact. Those two facts are contradictory. That is, that both should be true of precisely the same subject is absurd. But that they should be true of a subject existentially identical is not absurd, since they are mere accidents of an individual thing, which, as such, has no essence, its mode of being consisting in its forcing itself into a place in the world. Still, the two accidents could not be combined with one another. That *would* be absurd. For these accidents are monadic qualities, do have essences, and these essences are disparate. Their combination would have the form of a monadic triad but would not be a possible monadic triad; for it would violate a logical law. But though the two inherences cannot be *combined*, they can be *joined*. This junction is not a monadic triad, but it is of all forms of dyadic triad that one which most closely apes the monadic triad. Had we enumerated the divisions of dyadic triads, we should have been obliged to put this first of all. One kind of event, at least, then, is a dyadic triad of the very first kind, distinguished from the monadic triad in that it would, from the essence of the monadic qualities involved, have been contrary to a logical law, were it a monadic triad.

493. There are other sorts of events, somewhat more complex because the characters concerned are not simple monadic qualities. For example, A may make war upon B, that is, may pass from one sort of relation to B to another sort of relation to B. But they come to much the same thing. There is a repugnance between two monad elements. It is hardly for our present purposes worth while to undertake a long analysis in order to make the very slight correction of our definition of an event called for on this account. An event always involves a junction of contradictory inherences in the subjects existentially the same, whether there is a simple monadic quality inhering in a single subject, or whether they be inherences of contradictory monadic elements of dyads or polyads, in single sets of subjects. But there is a more important possible vari-

ation in the nature of events. In the kind of events so far considered, while it is not necessary that the subjects should be existentially of the nature of subjects — that is, that they should be substantial things — since it may be a mere wave, or an optical focus, or something else of like nature which is the subject of change, yet it is necessary that these subjects should be in some measure permanent, that is, should be capable of accidental determinations, and therefore should have dyadic existence. But the event may, on the other hand, consist in the coming into existence of something that did not exist, or the reverse. There is still a contradiction here; but instead of consisting in the material, or purely monadic, repugnance of two qualities, it is an incompatibility between two forms of triadic relation, as we shall better understand later. In general, however, we may say that for an event there is requisite: first, a contradiction; second, existential embodiments of these contradictory states; [third,] an immediate existential junction of these two contradictory existential embodiments or facts, so that the subjects are existentially identical; and fourth, in this existential junction a definite one of the two facts must be existentially first in the order of evolution and existentially second in the order of involution. We say the former is earlier, the latter later in time. That is, the past can in some measure work upon and influence (or flow into) the future, but the future cannot in the least work upon the past. On the other hand, the future can remember and know the past, but the past can only know the future so far as it can imagine the process by which the future is to be influenced.

494. Such, then, is the nature of an event. We can now go forward to an analysis of the substance of the law of time. It has three requirements, a monadic, a dyadic, and a triadic. The monadic clause in the law of time is that whatever fact or dyadic dyad exists, exists during a time, and in *this* time. The event is the existential junction of *states* (that is, of that which in existence corresponds to a *statement* about a given subject in representation) whose combination in one subject would violate the logical law of contradiction. The event, therefore, considered as a junction, is not a subject and does not inhere in a subject. What is it, then? Its mode of being is *existential quasi-existence*, or that approach to existence where

contraries can be united in one subject. Time is that diversity of existence whereby that which is existentially a subject is enabled to receive contrary determinations in existence. Phillip is drunk and Phillip is sober would be absurd, did not time make the Phillip of this morning another Phillip than the Phillip of last night. The law is that nothing dyadically exists as a subject without the diversification which permits it to receive contrary accidents. The instantaneous Phillip who can be drunk and sober at once has a potential being which does not quite amount to existence.

495. The dyadic requirement of the law of time is that if a subject existentially receives contrary attributes, of the two contrary states an existentially determinate one is first in the existential order of evolution and second in the existential order of involution, while the other is second in the existential order of evolution and first in the existential order of involution; and of any two events whatever, a determinate one is related to the other in this same way (although the two events are not joined, as the two states are joined in the event), unless they are independent of one another, or *contemporaneous*. Suppose I hold in my hand a leaden ball. I open my hand, the ball falls to the ground and rests there. There are three states of the ball: first, the ball is in my hand and is not on the ground; second, the ball is not in my hand and is not on the ground; third, the ball is not in my hand and is on the ground. Of the two events, the ball's leaving my hand and the ball's striking the ground, the former consists in the junction of the ball's being in my hand as first in evolution and the ball's being out of my hand as second in evolution. Hence, of the two states, the ball is in my hand but not on the ground and the ball is neither in my hand nor on the ground, the former is necessarily the first in evolution, being made so by the event. And of the two states, the ball is neither in my hand nor on the ground and the ball is not in my hand but is on the ground, the event of striking makes the former to be first in evolution. Thus, the order of the states is controlled by the nature of the events. But the events are nothing in themselves. But if the fall were instantaneous, if for example my hand intercepted at first a visual ray and were then removed, so that there were but two states — first, the hand visible, the ground invisible;

second, the hand invisible, the ground visible — then the two events are contemporaneous. If the two states, first "P and Q," second "not P and not Q" exist, then only one of the two states "P but not Q" and "Q but not P" can exist, for the reason that it is the dyadic character of the events that decides. Thus, supposing state "P and Q" and state "neither P nor Q" to both exist, and supposing that in the event "P – not P," P is first in evolution, then the state "P and Q" must antecede the state "neither P nor Q" in evolution, and consequently in the event "Q – not Q," Q must antecede not Q in evolution. These two events, "P becomes not P" and "Q becomes not Q," may then either antecede the other in evolution, and according as one or other antecedes, one or other of the two states, "P but not Q," and "not P but Q," becomes impossible. If the two events are contemporaneous, neither being existentially determined to be first in evolution, then these two states are both impossible.

496. The three possible temporal relations between two instantaneous events are naturally felt by us to mirror the three possible logical relations of two propositions which can be both true or both false, but are not logically equivalent (that is, have not by logical necessity the same value, as to being true or false). Namely of two such propositions, A and B, either, first, A can be false though B is true, but B must be true should A be true, or, second, either can be false though the other be true, so that they are independent of one another, or, third, A must be true should B be true, but B can be false though A is true. It is remarkable that we should instinctively connect the first case with the temporal succession of B after A, and the third case with the temporal succession of A after B, saying, in the former case, that B would *follow* from A and, in the latter, that A would *follow* from B. For superficial resemblances are the other way. We know what precedes in time from that which succeeds it much better than we know what is to come from that which goes before. This shows the instinct is not due to superficial resemblances. It is true that we know the conclusion later than we know the premisses; but we do not so much think of our *knowledge* as following as we do that one *fact* is logically sequent on the other. The instinct may, therefore, be presumed to be an obscure perception that temporal succession is a mirror of, or framework for, logical

sequence. Thus instinct with its almost unerring certainty favors this doctrine.

497. That of two events not contemporaneous one should happen before the other involves a *thisness* and thus a dyadism. For as it is impossible for us to indicate or ascertain one to be first by any general quality but only by a comparison with some standard experience, so it is impossible for a distinction of first and second to be except by a dyadic force of existence. That a determinate one shall be first and the other second requires reference to some kind of standard, since right and left are, as far as any monadic quality goes, just alike. There must be a standard first and second, and for any other pair there must be some way of bringing them into experiential connection one way and not the other way with this standard. This experiential reference to a standard in knowledge corresponds to an existential dyadic connection in fact. Otherwise there would be no truth in the knowledge.

498. So much for the dyadic clause of the law of time. The triadic clause is that time has no limit, and every portion of time is bounded by two instants which are of it, and between any two instants either way round, instants may be interposed such that taking any possible multitude of objects there is at least one interposed event for every unit of that multitude. This statement needs some explanation of its meaning. First what does it mean to say that time has no limit? This may be understood in a topical or a metrical sense. In a metrical sense it means there is no absolutely first and last of time. That is, while we must *adopt* a standard of first and last, there is nothing in its own nature the prototype of first and last. For were there any such prototype, that would consist of a pair of objects absolutely first and last. This, however, is more than is intended here. Whether that be true or not is a question concerning rather the events in time than time itself. What is here meant is that time has no instant from which there are more or less than two ways in which time is stretched out, whether they always be in their nature the foregoing and the coming after, or not. If that be so, since every portion of time is bounded by two instants, there must be a connection of time ring-wise. Events may be limited to a portion of this ring; but the time itself must extend round or else there will be

a portion of time, say future time and also past time, not bounded by two instants. The justification of this view is that it extends the properties we see belong to time to the whole of time without arbitrary exceptions not warranted by experience. Now, between any two events may be interposed not merely one event but a multitude of events greater than that statement would supply, a multitude of events as great as a multitude of objects describable. This may be really so or not, but this is the instinctive law of which we seem to be directly conscious.

499. By virtue of this, time is a continuum. For since the instants, or possible events, are as many as any collection whatever, and there is no maximum collection, it follows that they are more than any collections whatever. They must, therefore, be individually indistinguishable in their very existence — that is, are distinguishable and the parts distinguishable indefinitely, but yet not composed of individuals absolutely self-identical and distinct from one another — that is, they form a *continuum*. A continuum cannot be disarranged except to an insignificant extent. An instant cannot be removed. You can no more, by any decree, shorten a legal holiday by transferring its last instant to the work-day that follows that feast, than you can take away intensity from light, and keep the intensity on exhibition while the light is thrown into the ash-barrel. A limited line AB may be cut into two, AC and C′B, and its ends joined, C′ to A and C to B. That is to say, all this may be done in the imagination. We have a difficulty in imagining such a thing in regard to time. For in order that the time should flow continuously even in imagination from the end of one day into the beginning of a day that does historically come next, all the events must be prepared so that the states of things of these two instants, including states of gradual change, such as velocity, etc., shall be precisely the same. In the case of a line we do not think of this, although it is equally true, because we are unaccustomed to minutely dealing with the facts about single molecules and atoms upon which the cohesion of matter depends. We, therefore, see no particular difficulty in joining any end of a line to any other line's end continuously. This is as true a view as the other. As far as time itself goes, nothing prevents twenty-four hours being cut

out and the day before joining continuously to the day after, were there any power that could affect such a result. In such a case, the two instants brought together would be *identified*, or made one, which sufficiently shows their want of individual *self-identity* and repugnance to all others.

500. Intimately connected with the division of metaphysically contingent laws into laws which impose, upon inherences of different attributes in the same subject, forms analogous to forms of thought so that they may evade laws of logic and into those laws which have no reference to thought, there is a division of these latter laws into laws which impose, upon different subjects of precisely the same qualities, forms of relationship analogous to metaphysical forms so that they may evade the laws of metaphysics, that is, laws of space, and into laws which do not concern dyads of inherence but only dyads of reaction.

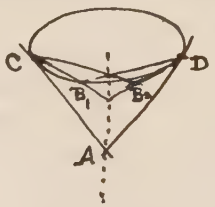
501. According to the metaphysical law of sufficient reason, alike in all respects two things cannot be. Space evades that law by providing places in which two things or any number, which are precisely alike, except that they are located in different places, themselves precisely alike in themselves, may exist. Thus, space does for different subjects of one predicate precisely what time does for different predicates of the same subject. And as time effects its evasion of the logical law by providing a form analogous to a logical form, so space effects its evasion of the metaphysical law by providing a form analogous to a metaphysical form. Namely, as metaphysics teaches that there is a succession of realities of higher and higher order, each a generalization of the last, and each the limit of a reality of the next higher order, so space presents points, lines, surfaces, and solids, each generated by the motion of a place of next lower dimensionality and the limit of a place of next higher dimensionality.

502. The last division of laws was a broad one. Now *a posteriori* laws are divided into those which are purely dynamical and those which are more or less intellectual, a division somewhat analogous to that of mental association into association by contiguity and by resemblance. The former are the nomological laws of physics. So far as our present science knows them, they are as follows:

503. First, every particle, or mathematically indivisible portion of matter, when not under a force, moves along a *ray*, or line belonging to a certain family of lines such that any four of them not all cut by each of an infinite multitude of rays is cut by just two rays.

504. Second, there is a *firmament*, or surface, severing space into worlds; and its properties are, first, that if (A), (B), (C), (D), (E), (F), are any points in a plane [1] section of it, the rays $\{AB\}$ and $\{DE\}$ will meet at a point $[\{AB\}\{DE\}]$ which is coradial with $[\{BC\}\{EF\}]$ and $[\{CD\}\{FA\}]$; secondly, no material particle ever comes to or leaves the firmament, nor does any plane fixedly connected with a particle ever move into or away from tangency with the firmament; and thirdly, if a body is *rigid*, that is, has only six degrees of freedom, so that all its radiform filaments are fixed when six of its particles are restricted to lying in fixed planes, or when six of its plane films are restricted to passing through fixed points, then, all its possible displacements are subject to the following conditions:

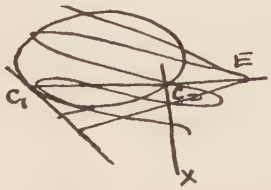
First, if two particles, A and B, of the rigid solid be situated in points $[A_1]$ and $[B_1]$ such that the ray $\{A_1 B_1\}$ has two points in the firmament, say $[C_1]$ and $[D_1]$, then A and B, however displaced, must lie in a ray that has two points in the firmament, and if any ray through $[A_1]$ has the two points $[C_2]$ and



$[D_2]$ in the firmament, then A remaining fixed in $[A_1]$, B can be displaced so as to occupy the point $[\{[\{C_1C_2\}\ \{D_1D_2\}]B_1\}\ \{C_2D_2\}]$ or the point $[\{[\{C_1D_2\}\{C_2D_1\}]B_1\}\{C_2D_2\}]$; but A and B can occupy simultaneously no pair of points which they are not necessarily able to occupy by virtue of this statement.

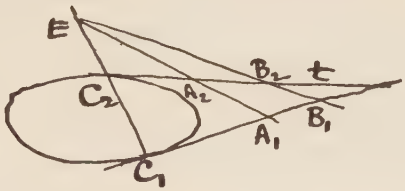
[1] A plane being the surface generated by a ray restricted to cutting two fixed rays which cut one another.

Second, if two particles, A and B, of the rigid solid are situated at points $[A_1]$ and $[B_1]$ such that the ray $\{A_1B_1\}$ has no point in the firmament, [then] in any plane containing $[A_1]$ and $[B_1]$ let [C] and [D] be the points of tangency of rays tangent to the firmament and passing through $[A_1]$. Then through



$[A_1]$ take any ray $\{r\}$ whatever, then $[\{[\{CB_1\}r]D\}\{[\{DB_1\}r]C\}]$ will be a point where the particle B may be while A is at $[A_1]$.

Third, if two particles, A and B, of the rigid solid are situated at points $[A_1]$ and $[B_1]$ such that the ray $\{A_1B_1\}$ has one



point in the firmament $[C_1]$, [then] in any plane through $\{A_1B_1\}$ take any other point $[C_2]$ on the firmament, and take any point [E] on the ray, $\{C_1C_2\}$. Then, if $\{t\}$ is the ray tangent to the firmament at $[C_2]$, A and B may be simultaneously at $[t\{EA_1\}]$ and $[t\{EB_1\}]$.

Every radial filament of a rigid body (supposed to fill all space) has its polar conjugate radial filament. Namely, one of these rays is the intersection of two planes tangent to the firmament, while the other passes through the two points of tangency. Every infinitesimal displacement of a rigid body is as if it were a part of a rigid body filling all space, and having two motions in one of which all the particles in one ray are fixed while all the plane films through its polar conjugate remain in the same plane, while in the other motion the reverse is the case.

505. Third, the effect of force upon a particle is to produce, while that force lasts, a component acceleration of the particle proportional to and in the ray of the force, and the resultant of such component accelerations is the same as if in each infinitesimal time, the different components acted successively, but each for a time equal to the whole of the infinitesimal time.

506. Fourth, the effect of a force between two particles is to give them opposite accelerations along the ray through them, these accelerations being inversely as certain quantities, called the *masses* of the accelerated particles, which masses are constant throughout all time.

507. Fifth, so far as force acts between pairs of particles regarded as mere occupiers of points, it depends upon the relative positions of the particles.

508. Sixth, it remains at present uncertain how the phenomena of elasticity, etc. are to be accounted for; but it is certain that all force cannot be positional attractions and repulsions. There is therefore some law additional to the last.

509. Seventh, all particles at a greater distance than a decimetre from one another attract one another nearly inversely as the square of the distance, the constant modulus being 6.658×10^{-8} (Boys).*

510. Eighth, particles closer together are known to attract one another more strongly, and it seems probable, although it is far from proved, that there are at least two kinds of particles attracting one another differently; but here our ignorance begins to be almost complete.

511. Laws which connect phenomena by a synthesis more or less intellectual, or inward, are divided somewhat broadly into laws of the inward relations, or resemblances, of bodies, and laws of mind.

512. The laws of resemblances and differences of bodies are classificatory, or chemical. We know little about them; but we may assert with some confidence that there are differences between substances — *i.e.*, differences in the smallest parts of bodies, and a classification based on that, and there are differences in the structure of bodies, and a classification based on that. Then of these latter we may distinguish dif-

* See "On the Newtonian Constant of Gravitation," *Philosophical Transactions*, London, 1895, 186A, p. 69.

ferences in the structure of the smallest pieces of bodies, depending on the shape and size of atomicules, and differences in the manner in which bodies are built up out of their smallest pieces. Here we have a distinction between that kind of structure which gives rise to forms without power of truth [true?] growth or inorganic structures, and the chemistry of protoplasms which develope [or] living organisms.

513. Finally laws of mind divide themselves into laws of the universal action of mind and laws of kinds of psychical manifestation.

514. Thus the general scheme of the division of laws is as follows:

- Logical law
- Metaphysical law
- Law of time
- Law of space
- Nomological laws of physics
- Classificatory laws of physics
- Nomological laws of psychics
- Classificatory laws of psychics

515. We now come to thoroughly genuine triads, the third class of the third class of triads; and at this stage of the inquiry it is well that we should take our bearings and note just where we are, in order that we may lay out our course for the next advances in the discussion. The monad has no features but its suchness, which appears in logic — let us remember that logic must be our guide throughout — as the signification of the verb. This already receives embodiment in the lowest of the chief forms of logic, the *term*. The dyad introduced a radically different sort of element, the subject, which first shows itself in the *proposition*. The dyadic proposition has two subjects, each a sort of mimic monad, but the two [are] of different kinds, one being active, the other passive. The triad brings a third sort of element, the expression of thought, or reasoning, consisting of a colligation of two propositions, not mere dyadic propositions, however, but general beliefs; and these two

propositions are connected by a common term and tend to produce a third belief. They not only tend to make the belief, but they also tend to render it true. This reason first emerges in the syllogism, which has three such colligations of premisses. Take the stock example,

All men die;
Enoch is a man;
Therefore, Enoch dies.

These propositions are not dyadic. The first is not so, because it is a rule, not a mere individual fact; the second is not so, because its second term is not a mere monadic quality, still less an individual unit, it is a class-term. The third is not so, because it is thought as a *result*. Each pair of these three propositions is a reason tending to render the third true. The first and third do so by means of their common monadic character. The first gives *dying* as a specimen character of all men; now the third declares as a consequence that Enoch dies. This gives Enoch one character of men, and so far as Enoch's dying is a consequence goes toward making him a man. The second and third propositions tend to make the first true by means of their common dyadic subject. The second proposition declares Enoch to be a specimen of a man. The third declares as a consequence that Enoch dies. This makes one man die; and so far as Enoch's dying is a consequence tends to render it true that all men die. Finally, the first two propositions bring about the truth of the third. In this particular case they do so absolutely. They generally tend to do so in a way which ought not to be more convincing, but is more in the way in which the objective truth is conceived to result than the other two. They do so by means of their community with respect to the middle term man, a term which as combining the characters of subject and predicate has a triadic element. For combination is triadism, and triadism is combination. Just as the logical verb with its signification reappears in metaphysics as a quality, an *ens* having a *nature* as its mode of being, and as a logical individual subject reappears in metaphysics as a thing, an *ens* having *existence* as its mode of being, so the logical reason, or premiss, reappears in metaphysics as a reason, an *ens* having a *reality*, consisting in a ruling both of the outward and of the

inward world, as its mode of being. The being of the quality lies wholly in itself, the being of the thing lies in opposition to other things, the being of the reason lies in its bringing qualities and things together.

516. In the degenerate dyad there is a metaphysical correspondent to a proposition; but it is a proposition whose two subjects are mere qualities. In the first degenerate triad there is a metaphysical correspondent to a syllogism; but it is a syllogism whose three reasons lie in mere qualities. Thus, orange color is intermediate between red and yellow. The syllogism is this:

Orange has in its own nature a certain
indescribable but felt relation to red;
Yellow has a similar relation to orange;
as a result, Yellow has a similar relation to red.

Now, if yellow has a relation to orange and as a result yellow has the same relation to red, this can only be because orange has that same relation to red.

517. In the second degenerate triad there is likewise a metaphysical correspondent to a syllogism; but it is a syllogism whose premisses lie in mere coexistences of dyadic facts. For example:

A is the mother of B;
B is the wife of C;
it results that A is the mother-in-law of C.

In the genuine triad, however, there is a real law, and a real case under the law; so that the reasons are not merely reasons in form, but they really govern the truth.

518. But though there be a real operation of law, yet one of the three reasons may be wanting in triadic reality:

All colors are compounds of so much red,
green, and blue;
Yellow is a color;
as a result, Yellow is compounded of proportions of
red, green, and blue.

The middle term here is little more than a disjunction of qualities, differing from that only in the separate colors not

being explicitly thought. Accordingly, that colors are compounded and that yellow is as a result so compounded only in form goes to make yellow a color; for in the very essence of color it is already given that yellow is a color. This triad is, therefore, only two-thirds genuine, one of its three reasons not being really operative.

519. A somewhat similar case arises when the middle term is a mere generalized dyadic existence.

All bodies are attracted toward one another proportionally to their masses and inversely as the square of the distance, multiplied by a fixed modulus;
The earth and moon have such and such masses and are at such a distance;
as a result, The earth and moon attract one another by so much.

But the last two propositions can hardly be said really to go toward making the truth of the first, since that law is nothing but the expression of the way bodies do move as facts. As far as it concerns the earth and moon it is so in the fact itself, and the earth and moon having such masses and distance as they have does not affect the brute fact, but only makes a certain proposition express that fact.

520. But there is a third kind of genuine triad in regard to which neither of the qualifications of their thorough genuineness applies for the reason that the result is of such a nature that it could not subsist were it not for the middle term which sustains it. A gives B to C. Say he does this by a formal legal act. Then, in this act A deprives himself of B; he also enters into an engagement with C and by virtue of these two sides of the act of gift, and of their unity, C acquires possession of B. But this is a remote result. The immediate result is that he acquires possession of B by the gift of A and without the action of A he could not acquire that possession.

CHAPTER 5

*DEGENERATE CASES**

§1. KINDS OF SECONDNESS

521. Very wretched is the notion of [the categories] that can be conveyed in one lecture. They must grow up in the mind, under the hot sunshine of hard thought, daily, bright, well-focussed, and well-aimed thought; and you must have patience, for long time is required to ripen the fruit. They are no inventions of mine. Were they so, that would be sufficient to condemn them. Confused notions of these elements appear in the first infancy of philosophy, and they have never entirely been forgotten. Their fundamental importance is noticed in the beginning of Aristotle's *De Caelo*, where it is said† that the Pythagoreans knew of them.

522. In Kant they come out with an approach to lucidity. For Kant possessed in a high degree all seven of the mental qualifications of a philosopher:

1. The ability to discern what is before one's consciousness.
2. Inventive originality.
3. Generalizing power.
4. Subtlety.
5. Critical severity and sense of fact.
6. Systematic procedure.
7. Energy, diligence, persistency, and exclusive devotion to philosophy.

523. But Kant had not the slightest suspicion of the inexhaustible intricacy of the fabric of conceptions, which is such that I do not flatter myself that I have ever analyzed a single idea into its constituent elements.

524. Hegel, in some respects the greatest philosopher that ever lived, had a somewhat juster notion of this complication,

* From the "Lowell Lectures of 1903," Lecture III, vol. 2, 3d Draught, following 349.

† 268a 11.

though an inadequate notion, too. For if he had seen what the state of the case was, he would not have attempted in one lifetime to cover the vast field that he attempted to clear. But Hegel was lamentably deficient in that fifth requisite of critical severity and sense of fact. He brought out the three elements much more clearly [than Kant did]; but the element of Secondness, of *hard fact*, is not accorded its due place in his system; and in a lesser degree the same is true of Firstness. After Hegel wrote, there came fifty years that were remarkably fruitful in all the means for attaining that fifth requisite. Yet Hegel's followers, instead of going to work to reform their master's system, and to render his statement of it obsolete, as every true philosopher must desire that his disciples should do, only proposed, at best, some superficial changes without replacing at all the rotten material with which the system was built up.

525. I shall not inflict upon you any account of my own labors. Suffice it to say that my results have afforded me great aid in the study of logic.

I will, however, make a few remarks on these categories. By way of preface, I must explain that in saying that the three, Firstness, Secondness, and Thirdness, complete the list, I by no means deny that there are other categories. On the contrary, at every step of every analysis, conceptions are met with which presumably do not belong to this series of ideas. Nor did an investigation of them occupying me for two years reveal any analysis of them into these as their constituents. I shall say nothing further about them, except incidentally.

526. As to the three universal categories, as I call them, perhaps with no very good reason for thinking that they are more universal than the others, we first notice that Secondness and Thirdness are conceptions of complexity. That is not, however, to say that they are complex conceptions. When we think of Secondness, we naturally think of two reacting objects, a first and a second. And along with these, as subjects, there is their reaction. But these are not constituents out of which the Secondness is built up. The truth is just reverse, [in] that the being a first or a second or the being a reaction each involves Secondness. An object cannot be a *second* of itself. If it is a second, it has an element of being what another makes

it to be. That is, the being a second involves Secondness. The reaction still more manifestly involves the being what another makes a subject to be. Thus, while Secondness is a fact of complexity, it is not a compound of two facts. It is a single fact about two objects. Similar remarks apply to Thirdness.

527. This remark at once leads to another. The Secondness of the second, whichever of the two objects be called the second, is different from the Secondness of the first. That is to say it *generally* is so. To kill and to be killed are different. In case there is one of the two which there is good reason for calling the first, while the other remains the second, it is that the Secondness is more accidental to the former than to the latter; that there is more or less approach to a state of things in which something, which is itself first, accidentally comes into a Secondness that does not really modify its Firstness, while its second in this Secondness is something whose *being* is of the nature of Secondness and which has no Firstness separate from this. It must be extremely difficult for those who are untrained to such analyses of conceptions to make any sense of all this. For that reason, I shall inflict very little of it upon you — just enough to show those who *can* carry what I say in their minds that it is by no means nonsense. The extreme kind of Secondness which I have just described is the relation of a *quality* to the *matter* in which that quality inheres. The mode of being of the quality is that of Firstness. That is to say, it is a possibility. It is related to the matter accidentally; and this relation does not change the quality at all, except that it imparts *existence*, that is to say, this very relation of inherence, to it. But the *matter*, on the other hand, has no being at all except the being a subject of qualities. This relation of really having qualities constitutes its *existence*. But if all its qualities were to be taken away, and it were to be left quality-less matter, it not only would not exist, but it would not have any positive definite possibility — such as an unembodied quality has. It would be nothing at all.

528. Thus we have a division of seconds into those whose very being, or Firstness, it is to be seconds, and those whose Secondness is only an accretion. This distinction springs out of the essential elements of Secondness. For Secondness involves Firstness. The concepts of the two kinds of Second-

ness are mixed concepts composed of Secondness and Firstness. One is the second whose very Firstness is Secondness. The other is a second whose Secondness is second to a Firstness. The idea of mingling Firstness and Secondness in this particular way is an idea distinct from the ideas of Firstness and Secondness that it combines. It appears to be a conception of an entirely different series of categories. At the same time, it is an idea of which Firstness, Secondness, and Thirdness are component parts, since the distinction depends on whether the two elements of Firstness and Secondness that are united are so united as to be one or whether they remain two. This distinction between two kinds of seconds, which is almost involved in the very idea of a second, makes a distinction between two kinds of Secondness; namely, the Secondness of genuine seconds, or matters, which I call genuine Secondness, and the Secondness in which one of the seconds is only a Firstness, which I call degenerate Secondness; so that this Secondness really amounts to nothing but this, that a subject, in its being a second, has a Firstness, or quality. It is to be remarked that this distinction arose from attending to extreme cases; and consequently subdivision will be attached to it according to the more or less essential or accidental nature of the genuine or the degenerate Secondness. With this distinction Thirdness has nothing to do, or at any rate has so little to do that a satisfactory account of the distinction need not mention Thirdness.

529. I will just mention that among Firstnesses there is no distinction of the genuine and the degenerate, while among Thirdnesses we find not only a genuine but two distinct grades of degeneracy.

§2. THE FIRSTNESS OF FIRSTNESS, SECONDNESS, AND THIRDNESS

530. But now I wish to call your attention to a kind of distinction which affects Firstness more than it does Secondness, and Secondness more than it does Thirdness. This distinction arises from the circumstance that where you have a triplet you have three pairs; and where you have a pair, you have two units. Thus, Secondness is an essential part of

Thirdness though not of Firstness, and Firstness is an essential element of both Secondness and Thirdness. Hence there is such a thing as the Firstness of Secondness and such a thing as the Firstness of Thirdness; and there is such a thing as the Secondness of Thirdness. But there is no Secondness of pure Firstness and no Thirdness of pure Firstness or Secondness. When you strive to get the purest conceptions you can of Firstness, Secondness, and Thirdness, thinking of quality, reaction, and mediation — what you are striving to apprehend is pure Firstness, the Firstness of Secondness — that is what Secondness is, of itself — and the Firstness of Thirdness. When you contrast the blind compulsion in an event of reaction considered as something which happens and which of its nature can never happen again, since you cannot cross the same river twice, when, I say, you contrast this compulsion with the logical necessitation of a *meaning* considered as something that has no being at all except so far as it actually gets embodied in an event of thought, and you regard this logical necessitation as a sort of actual compulsion, since the meaning must actually be embodied, what you are thinking of is a Secondness involved in Thirdness.

531. A Firstness is exemplified in every quality of a total feeling. It is perfectly simple and without parts; and everything has its quality. Thus the tragedy of King Lear has its Firstness, its flavor *sui generis*. That wherein all such qualities agree is universal Firstness, the very being of Firstness. The word *possibility* fits it, except that possibility implies a relation to what exists, while universal Firstness is the mode of being of itself. That is why a new word was required for it. Otherwise, "possibility" would have answered the purpose.

532. As to Secondness, I have said that our only direct knowledge of it is in willing and in the experience of a perception. It is in willing that the Secondness comes out most strongly. But it is not pure Secondness. For, in the first place, he who wills has a purpose; and that idea of purpose makes the act appear as a *means* to an end. Now the word *means* is almost an exact synonym to the word *third*. It certainly involves Thirdness. Moreover, he who wills is conscious of doing so, in the sense of *representing* to himself that he does so. But representation is precisely genuine Thirdness. You must

conceive an instantaneous consciousness that is instantly and totally forgotten and an effort without purpose. It is a hopeless undertaking to try to realize what consciousness would be without the element of representation. It would be like unexpectedly hearing a great explosion of nitroglycerine before one had recovered oneself and merely had the sense of the breaking off of the quiet. Perhaps it might not be far from what ordinary common sense conceives to take place when one billiard ball caroms on another. One ball "acts" on the other; that is, it makes an exertion *minus* the element of representation. We may say with some approach to accuracy that the general Firstness of all true Secondness is *existence*, though this term more particularly applies to Secondness in so far as it is an element of the reacting first and second. If we mean Secondness as it is an element of the occurrence, the Firstness of it is *actuality*. But actuality and existence are words expressing the same idea in different applications. Secondness, strictly speaking, is just when and where it takes place, and has no other being; and therefore different Secondnesses, strictly speaking, have in themselves no quality in common. Accordingly, existence, or the universal Firstness of all Secondness, is really not a quality at all. An actual dollar to your credit in the bank does not differ in any respect from a possible imaginary dollar. For if it did, the imaginary dollar could be imagined to be changed in that respect, so as to agree with the actual dollar. We thus see that actuality is not a *quality*, or mere mode of feeling. Hence Hegel, whose neglect of Secondness was due chiefly to his not recognizing any other mode of being than existence — and what he calls *existenz* is a special variety of it merely — regarded pure being as pretty much the same as nothing. It is true that the word "existence" names, as if it were an abstract possibility, that which is precisely the not having any being in abstract possibility; and this circumstance, when you look upon existence as the only being, seems to make existence all but the same as nothing.

533. To express the Firstness of Thirdness, the peculiar flavor or color of mediation, we have no really good word. *Mentality* is, perhaps, as good as any. poor and inadequate as it is. Here, then, are three kinds of Firstness, qualitative possibility, existence, mentality, resulting from applying Firstness

to the three categories. We might strike new words for them: primity, secundity, tertiality.

534. There are also three other kinds of Firstness which arise in a somewhat similar way; namely, the idea of a simple original quality, the idea of a quality essentially relative, such as that of being "an inch long"; and the idea of a quality that consists in the way something is thought or represented, such as the quality of being manifest.

535. I shall not enter into any exact analysis of these ideas. I only wished to give you such slight glimpse as I could of the sort of questions that busy the student of phenomenology, merely to lead up to Thirdness and to the particular kind and aspect of Thirdness which is the sole object of logical study. I want first to show you what genuine Thirdness is and what are its two degenerate forms. Now we found the genuine and degenerate forms of Secondness by considering the full ideas of first and second. Then the genuine Secondness was found to be reaction, where first and second are both true seconds and the Secondness is something distinct from them, while in degenerate Secondness, or mere reference, the first is a mere first never attaining full Secondness.

536. Let us proceed in the same way with Thirdness. We have here a first, a second, and a third. The first is a positive qualitative possibility, in itself nothing more. The second is an existent thing without any mode of being less than existence, but determined by that first. A *third* has a mode of being which consists in the Secondnesses that it determines, the mode of being of a law, or concept. Do not confound this with the ideal being of a quality in itself. A quality is something capable of being completely embodied. A law never can be embodied in its character as a law except by determining a habit. A quality is how something may or might have been. A law is how an endless future must continue to be.

537. Now in genuine Thirdness, the first, the second, and the third are all three of the nature of thirds, or thought, while in respect to one another they are first, second, and third. The first is thought in its capacity as mere possibility; that is, mere *mind* capable of thinking, or a mere vague idea. The *second* is thought playing the role of a Secondness, or event. That is, it is of the general nature of *experience* or *information*.

The third is thought in its role as governing Secondness. It brings the information into the mind, or determines the idea and gives it body. It is informing thought, or *cognition*. But take away the psychological or accidental human element, and in this genuine Thirdness we see the operation of a sign.

538. Every sign stands for an object independent of itself; but it can only be a sign of that object in so far as that object is itself of the nature of a sign or thought. For the sign does not affect the object but is affected by it; so that the object must be able to convey thought, that is, must be of the nature of thought or of a sign. Every thought is a sign. But in the first degree of degeneracy the Thirdness affects the object, so that this is not of the nature of a Thirdness — not so, at least, as far as this operation of degenerate Thirdness is concerned. It is that the third brings about a Secondness but does not regard that Secondness as anything more than a fact. In short it is the operation of executing an *intention*. In the last degree of degeneracy of Thirdness, there is thought, but no conveyance or embodiment of thought at all. It is merely that a fact of which there must be, I suppose, something like knowledge is *apprehended* according to a possible idea. There is an *instigation* without any *prompting*. For example, you look at something and say, "It is red." Well, I ask you what justification you have for such a judgment. You reply, "I *saw* it was red." Not at all. You saw nothing in the least like that. You saw an image. There was no subject or predicate in it. It was just one unseparated image, not resembling a proposition in the smallest particular. It instigated you to your judgment, owing to a possibility of thought; but it never told you so. Now in all imagination and perception there is such an operation by which thought springs up; and its only justification is that it subsequently turns out to be useful.

539. Now it may be that *logic* ought to be the science of Thirdness in general. But as I have studied it, it is simply the science of what must be and ought to be true representation, so far as representation can be known without any gathering of special facts beyond our ordinary daily life. It is, in short, the philosophy of representation.

540. The analysis which I have just used to give you some notion of genuine Thirdness and its two forms of degeneracy

is the merest rough blackboard sketch of the true state of things; and I must begin the examination of representation by defining representation a *little* more accurately. In the first place, as to my terminology, I confine the word *representation* to the operation of a sign or its *relation to* the object *for* the interpreter of the representation. The concrete subject that represents I call a *sign* or a *representamen*. I use these two words, *sign* and *representamen*, differently. By a *sign* I mean anything which conveys any definite notion of an object in any way, as such conveyers of thought are familiarly known to us. Now I start with this familiar idea and make the best analysis I can of what is essential to a sign, and I define a *representamen* as being whatever that analysis applies to. If therefore I have committed an error in my analysis, part of what I say about *signs* will be false. For in that case a *sign* may not be a *representamen*. The analysis is certainly true of the representamen, since that is all that word means. Even if my analysis is correct, something may happen to be true of all *signs*, that is of everything that, antecedently to any analysis, we should be willing to regard as conveying a notion of anything, while there might be something which my analysis describes of which the same thing is not true. In particular, all signs convey notions to *human minds;* but I know no reason why every representamen should do so.

541. My definition of a representamen is as follows:

A REPRESENTAMEN is a subject of a triadic relation TO a second, called its OBJECT, FOR a third, called its INTERPRETANT, this triadic relation being such that the REPRESENTAMEN determines its interpretant to stand in the same triadic relation to the same object for some interpretant.

542. It follows at once that this relation cannot consist in any actual event that ever can have occurred; for in that case there would be another actual event connecting the interpretant to an interpretant of its own of which the same would be true; and thus there would be an endless series of events which could have actually occurred, which is absurd. For the same reason the interpretant cannot be a *definite* individual object. The relation must therefore consist in a *power* of the representamen to determine *some* interpretant to being a representamen of the same object.

543. Here we make a new distinction. You see the principle of our procedure. We begin by asking what is the mode of being of the subject of inquiry, that is, what is its absolute and most universal Firstness? The answer comes, that it is either the Firstness of Firstness, the Firstness of Secondness, or the Firstness of Thirdness.

We then ask what is the universal Secondness, and what the universal Thirdness, of the subject in hand.

Next we say that Firstness of Firstness, that Firstness of Secondness and that Firstness of Thirdness, that have been described, have been the Firstness of the Firstness in each case. But what is the Secondness that is involved in it and what is the Thirdness?

So the Secondnesses as they have been first given are the Firstnesses of those Secondnesses. We ask what Secondness they involve and what Thirdness. And so we have endless questions, of which I have only given you small scraps.

The answers to these questions do not come of themselves. They require the most laborious study, the most careful and exact examination. The system of questions does not save that trouble in the least degree. It enormously increases it by multiplying the questions that are suggested. But it forces us along step by step to much clearer conceptions of the objects of logic than have ever been attained before. The *hard fact* that it has yielded such fruit is the principal argument in its favor.

544. The method has a general similarity to Hegel's. It would be historically false to call it a modification of Hegel's. It was brought into being by the study of Kant's categories and not Hegel's. Hegel's method has the defect of not working at all if you think with too great exactitude. Moreover, it presents no such definite question to the mind as this method does. This method works better the finer and more accurate the thought. The subtlest mind cannot get the best possible results from it; but a mind of very moderate skill can make better analyses by this method than the same mind could obtain without it, by far.

Analyses apparently conflicting may be obtained by this method by different minds, owing to the impossibility of conforming strictly to the requirements. But it does not follow that the results are utterly wrong. They will be two imperfect analyses, each getting a part of the truth.

CHAPTER 6

ON A NEW LIST OF CATEGORIES

§1. ORIGINAL STATEMENT*[E]

545. This paper is based upon the theory already established, that the function of conceptions is to reduce the manifold of sensuous impressions to unity and that the validity of a conception consists in the impossibility of reducing the content of consciousness to unity without the introduction of it.

546. This theory gives rise to a conception of gradation among those conceptions which are universal. For one such conception may unite the manifold of sense and yet another may be required to unite the conception and the manifold to which it is applied; and so on.

547. That universal conception which is nearest to sense is that of *the present, in general*. This is a conception, because it is universal. But as the act of *attention* has no connotation at all, but is the pure denotative power of the mind, that is to say, the power which directs the mind to an object, in contradistinction to the power of thinking any predicate of that object — so the conception of *what is present in general*, which is nothing but the general recognition of what is contained in attention, has no connotation, and therefore no proper unity. This conception of the present in general, of IT in general, is rendered in philosophical language by the word "substance" in one of its meanings. Before any comparison or discrimination can be made between what is present, what is present must have been recognized as such, as *it*, and subsequently the metaphysical parts which are recognized by abstraction are attributed to this *it*, but the *it* cannot itself be made a predicate. This *it* is thus neither predicated of a

* The first section of this chapter was published with this chapter heading in the *Proceedings of the American Academy of Arts and Sciences*, vol. 7, May 1867, pp. 287–298. It was intended as ch. 1 of the Grand Logic of 1893 and as Essay II of the Search for a Method, c. 1893.

subject, nor in a subject, and accordingly is identical with the conception of substance.

548. The unity to which the understanding reduces impressions is the unity of a proposition. This unity consists in the connection of the predicate with the subject; and, therefore, that which is implied in the copula, or the conception of *being*, is that which completes the work of conceptions of reducing the manifold to unity. The copula (or rather the verb which is copula in one of its senses) means either *actually is* or *would be*, as in the two propositions, "There *is* no griffin," and "A griffin *is* a winged quadruped." The conception of *being* contains only that junction of predicate to subject wherein these two verbs agree. The conception of being, therefore, plainly has no content.

If we say "The stove is black," the stove is the *substance*, from which its blackness has not been differentiated, and the *is*, while it leaves the substance just as it was seen, explains its confusedness, by the application to it of *blackness* as a predicate.

Though *being* does not affect the subject, it implies an indefinite determinability of the predicate. For if one could know the copula and predicate of any proposition, as ". . . . . is a tailed-man," he would know the predicate to be applicable to something supposable, at least. Accordingly, we have propositions whose subjects are entirely indefinite, as "There is a beautiful ellipse," where the subject is merely *something actual or potential;* but we have no propositions whose predicate is entirely indeterminate, for it would be quite senseless to say, "A has the common characters of all things," inasmuch as there are no such common characters.

Thus substance and being are the beginning and end of all conception. Substance is inapplicable to a predicate, and being is equally so to a subject.

549. The terms "precision"[1] and "abstraction," which

[1] *Precision.* (1) A high degree of approximation, only attainable by the thorough application of the most refined methods of science.

(2) Its earlier meaning, still more or less used by logicians, is derived from a meaning given to *praecisio* by Scotus and other scholastics: the act of supposing (whether with consciousness of fiction or not) something about one element of a percept, upon which the thought dwells, without paying any regard to other elements. Precision implies more than mere discrimination, which

were formerly applied to every kind of separation, are now limited, not merely to mental separation, but to that which arises from *attention to* one element and *neglect of* the other. Exclusive attention consists in a definite conception or *supposition* of one part of an object, without any supposition of the other. Abstraction or precision ought to be carefully distinguished from two other modes of mental separation, which may be termed *discrimination* and *dissociation*. Discrimination has to do merely with the senses of terms, and only draws a distinction in meaning. Dissociation is that separation which,

relates merely to the essence of a term. Thus I can, by an act of discrimination, separate color from extension; but I cannot do so by *precision*, since I cannot suppose that in any possible universe color (not color-sensation, but color as a quality of an object) exists without extension. So with *triangularity* and *trilaterality*. On the other hand, precision implies much less than dissociation, which, indeed, is not a term of logic, but of psychology. It is doubtful whether a person who is not devoid of the sense of sight can separate space from color by dissociation, or, at any rate, not without great difficulty; but he can, and, indeed, does do so, by *precision*, if he thinks a vacuum is uncolored. So it is, likewise, with space and tridimensionality.

Some writers called every description of abstraction by the name *precision*, dividing precision into the real and the mental, and the latter into the negative and the positive; but the better usage named these *abstraction* divided into *real* and *intentional*, and the latter into *negative* (in which character from which abstraction is made is imagined to be *deniable* of the subject prescinded) and into *precisive abstraction* or *precision*, where the subject prescinded is supposed (in some hypothetical state of things) without any supposition, whether affirmative or negative, in respect to the character abstracted. Hence, the *brocard: abstrahentium non est mendacium* (generally enunciated in connection with the *De Anima*, III, VII, 7). Scotus (in II *Physic.*, *Expositio* 20 *textus* 18) says: "Et si aliquis dicat, quod Mathematici tunc faciunt mendacium: quia considerant ista, quasi essent abstracta a motu, et materia; quae tamen sunt coniuncta materiae. Respondet, quod non faciunt mendacium: quia Mathematicus non considerat, utrum id, de quo demonstrat suas passiones, sit coniunctum materiae, vel abstractum a materia." This is not the place to treat of the many interesting logical, as well as psychological, discussions which have taken place concerning precision, which is one of the subjects which the scholastics treated in a comparatively modern way, although it leads directly to the question of nominalism and realism. It may, however, be mentioned that Scotus in many places draws a certain distinction variously designated by him and his followers (its nature and application is perhaps made as clear as anywhere in the *Opus Oxon.* III, xxii. qu. unica, "Utrum Christus fuerit homo in triduo," i. e. between the crucifixion and the resurrection), which the Thomists mostly dispute. There is some account of the matter in Chauvinus, *Lexicon* (2d ed.), under "Praecisio" . . . *Dictionary of Philosophy and Psychology*, vol. 2, pp. 323–4, Macmillan Co., New York, edition of 1911.

in the absence of a constant association, is permitted by the law of association of images. It is the consciousness of one thing, without the necessary simultaneous consciousness of the other. Abstraction or precision, therefore, supposes a greater separation than discrimination, but a less separation than dissociation. Thus I can discriminate red from blue, space from color, and color from space, but not red from color. I can prescind red from blue, and space from color (as is manifest from the fact that I actually believe there is an uncolored space between my face and the wall); but I cannot prescind color from space, nor red from color. I can dissociate red from blue, but not space from color, color from space, nor red from color.

Precision is not a reciprocal process. It is frequently the case, that, while A cannot be prescinded from B, B can be prescinded from A. This circumstance is accounted for as follows. Elementary conceptions only arise upon the occasion of experience; that is, they are produced for the first time according to a general law, the condition of which is the existence of certain impressions. Now if a conception does not reduce the impressions upon which it follows to unity, it is a mere arbitrary addition to these latter; and elementary conceptions do not arise thus arbitrarily. But if the impressions could be definitely comprehended without the conception, this latter would not reduce them to unity. Hence, the impressions (or more immediate conceptions) cannot be definitely conceived or attended to, to the neglect of an elementary conception which reduces them to unity. On the other hand, when such a conception has once been obtained, there is, in general, no reason why the premisses which have occasioned it should not be neglected, and therefore the explaining conception may frequently be prescinded from the more immediate ones and from the impressions.

550. The facts now collected afford the basis for a systematic method of searching out whatever universal elementary conceptions there may be intermediate between the manifold of substance and the unity of being. It has been shown that the occasion of the introduction of a universal elementary conception is either the reduction of the manifold of substance to unity, or else the conjunction to substance of another conception. And it has further been shown that the elements con-

joined cannot be supposed without the conception, whereas the conception can generally be supposed without these elements. Now, empirical psychology discovers the occasion of the introduction of a conception, and we have only to ascertain what conception already lies in the data which is united to that of substance by the first conception, but which cannot be supposed without this first conception, to have the next conception in order in passing from being to substance.

It may be noticed that, throughout this process, *introspection* is not resorted to. Nothing is assumed respecting the subjective elements of consciousness which cannot be securely inferred from the objective elements.

551. The conception of *being* arises upon the formation of a proposition. A proposition always has, besides a term to express the substance, another to express the quality of that substance; and the function of the conception of being is to unite the quality to the substance. Quality, therefore, in its very widest sense, is the first conception in order in passing from being to substance.

Quality seems at first sight to be given in the impression. Such results of introspection are untrustworthy. A proposition asserts the applicability of a mediate conception to a more immediate one. Since this is *asserted*, the more mediate conception is clearly regarded independently of this circumstance, for otherwise the two conceptions would not be distinguished, but one would be thought through the other, without this latter being an object of thought, at all. The mediate conception, then, in order to be *asserted* to be applicable to the other, must first be considered without regard to this circumstance, and taken immediately. But, taken immediately, it transcends what is given (the more immediate conception), and its applicability to the latter is hypothetical. Take, for example, the proposition, "This stove is black." Here the conception of *this stove* is the more immediate, that of *black* the more mediate, which latter, to be predicated of the former, must be discriminated from it and considered *in itself*, not as applied to an object, but simply as embodying a quality, *blackness*. Now this *blackness* is a pure species or abstraction, and its application to *this stove* is entirely hypothetical. The same thing is meant by "the stove is black," as by "there is blackness in the stove."

Embodying blackness is the equivalent of *black*.[1] The proof is this. These conceptions are applied indifferently to precisely the same facts. If, therefore, they were different, the one which was first applied would fulfil every function of the other; so that one of them would be superfluous. Now a superfluous conception is an arbitrary fiction, whereas elementary conceptions arise only upon the requirement of experience; so that a superfluous elementary conception is impossible. Moreover, the conception of a pure abstraction is indispensable, because we cannot comprehend an agreement of two things, except as an agreement in some *respect*, and this respect is such a pure abstraction as blackness. Such a pure abstraction, reference to which constitutes a *quality* or general attribute, may be termed a *ground*.

Reference to a ground cannot be prescinded from being, but being can be prescinded from it.

552. Empirical psychology has established the fact that we can know a quality only by means of its contrast with or similarity to another. By contrast and agreement a thing is referred to a correlate, if this term may be used in a wider sense than usual. The occasion of the introduction of the conception of reference to a ground is the reference to a correlate, and this is, therefore, the next conception in order.

Reference to a correlate cannot be prescinded from reference to a ground; but reference to a ground may be prescinded from reference to a correlate.

553. The occasion of reference to a correlate is obviously by comparison. This act has not been sufficiently studied by the psychologists, and it will, therefore, be necessary to adduce some examples to show in what it consists. Suppose we wish to compare the letters p and b. We may imagine one of them to be turned over on the line of writing as an axis, then laid upon the other, and finally to become transparent so that the other can be seen through it. In this way we shall form a new image which mediates between the images of the two letters, inasmuch as it represents one of them to be (when turned over) he likeness of the other. Again, suppose we think of a murderer as being in relation to a murdered person; in this case

[1] This agrees with the author of "De Generibus et Speciebus," *Ouvrages Inédits d'Abelard*, p. 528, [edited by V. Cousin, Paris, 1836].

we conceive the act of the murder, and in this conception it is represented that corresponding to every murderer (as well as to every murder) there is a murdered person; and thus we resort again to a mediating representation which represents the relate as standing for a correlate with which the mediating representation is itself in relation. Again, suppose we look up the word *homme* in a French dictionary; we shall find opposite to it the word *man*, which, so placed, represents *homme* as representing the same two-legged creature which *man* itself represents. By a further accumulation of instances, it would be found that every comparison requires, besides the related thing, the ground, and the correlate, also a *mediating representation which represents the relate to be a representation of the same correlate which this mediating representation itself represents*. Such a mediating representation may be termed an *interpretant*, because it fulfils the office of an interpreter, who says that a foreigner says the same thing which he himself says. The term representation is here to be understood in a very extended sense, which can be explained by instances better than by a definition. In this sense, a word represents a thing to the conception in the mind of the hearer, a portrait represents the person for whom it is intended to the conception of recognition, a weathercock represents the direction of the wind to the conception of him who understands it, a barrister represents his client to the judge and jury whom he influences.

Every reference to a correlate, then, conjoins to the substance the conception of a reference to an interpretant; and this is, therefore, the next conception in order in passing from being to substance.

Reference to an interpretant cannot be prescinded from reference to a correlate; but the latter can be prescinded from the former.

554. Reference to an interpretant is rendered possible and justified by that which renders possible and justifies comparison. But that is clearly the diversity of impressions. If we had but one impression, it would not require to be reduced to unity, and would therefore not need to be thought of as referred to an interpretant, and the conception of reference to an interpretant would not arise. But since there is a manifold of impressions, we have a feeling of complication or confusion,

which leads us to differentiate this impression from that, and then, having been differentiated, they require to be brought to unity. Now they are not brought to unity until we conceive them together as being *ours*, that is, until we refer them to a conception as their interpretant. Thus, the reference to an interpretant arises upon the holding together of diverse impressions, and therefore it does not join a conception to the substance, as the other two references do, but unites directly the manifold of the substance itself. It is, therefore, the last conception in order in passing from being to substance.

555. The five conceptions thus obtained, for reasons which will be sufficiently obvious, may be termed *categories*. That is,

Being
Quality (reference to a ground)
Relation (reference to a correlate)
Representation (reference to an interpretant)

Substance

The three intermediate conceptions may be termed accidents.

556. This passage from the many to the one is numerical. The conception of a *third* is that of an object which is so related to two others, that one of these must be related to the other in the same way in which the third is related to that other. Now this coincides with the conception of an interpretant. An *other* is plainly equivalent to a *correlate*. The conception of second differs from that of other, in implying the possibility of a third. In the same way, the conception of *self* implies the possibility of an *other*. The *ground* is the self abstracted from the concreteness which implies the possibility of another.

557. Since no one of the categories can be prescinded from those above it, the list of supposable objects which they afford is,

What is.
Quale (that which refers to a ground)
Relate (that which refers to ground and correlate)
Representamen (that which refers to ground, correlate, and interpretant)

It

558. A quality may have a special determination which prevents its being prescinded from reference to a correlate. Hence there are two kinds of relation.

First. That of relates whose reference to a ground is a prescindible or internal quality.

Second. That of relates whose reference to a ground is an unprescindible or relative quality.

In the former case, the relation is a mere *concurrence* of the correlates in one character, and the relate and correlate are not distinguished. In the latter case the correlate is set over against the relate, and there is in some sense an *opposition*.

Relates of the first kind are brought into relation simply by their agreement. But mere disagreement (unrecognized) does not constitute relation, and therefore relates of the second kind are only brought into relation by correspondence in fact.

A reference to a ground may also be such that it cannot be prescinded from a reference to an interpretant. In this case it may be termed an *imputed* quality. If the reference of a relate to its ground can be prescinded from reference to an interpretant, its relation to its correlate is a mere concurrence or community in the possession of a quality, and therefore the reference to a correlate can be prescinded from reference to an interpretant. It follows that there are three kinds of representations.

First. Those whose relation to their objects is a mere community in some quality, and these representations may be termed *likenesses*.*

Second. Those whose relation to their objects consists in a correspondence in fact, and these may be termed *indices* or *signs*.†

Third. Those the ground of whose relation to their objects is an imputed character, which are the same as *general signs*, and these may be termed *symbols*.

559. I shall now show how the three conceptions of reference to a ground, reference to an object, and reference to an interpretant are the fundamental ones of at least one universal science, that of logic. Logic is said to treat of second intentions

* In later writings called "icons."

† In later writings an index is always taken to be but one of many kinds of signs; a sign being understood in some sense similar to that given in 540.

as applied to first.* It would lead me too far away from the matter in hand to discuss the truth of this statement; I shall simply adopt it as one which seems to me to afford a good definition of the subject-genus of this science. Now, second intentions are the objects of the understanding considered as representations, and the first intentions to which they apply are the objects of those representations. The objects of the understanding, considered as representations, are symbols, that is, signs which are at least potentially general. But the rules of logic hold good of any symbols, of those which are written or spoken as well as of those which are thought. They have no immediate application to likenesses or indices, because no arguments can be constructed of these alone, but do apply to all symbols. All symbols, indeed, are in one sense relative to the understanding, but only in the sense in which also all things are relative to the understanding. On this account, therefore, the relation to the understanding need not be expressed in the definition of the sphere of logic, since it determines no limitation of that sphere. But a distinction can be made between concepts which are supposed to have no existence except so far as they are actually present to the understanding, and external symbols which still retain their character of symbols so long as they are only *capable* of being understood. And as the rules of logic apply to these latter as much as to the former (and though only through the former, yet this character, since it belongs to all things, is no limitation), it follows that logic has for its subject-genus all symbols and not merely concepts.[1] We come, therefore, to this, that logic treats of the reference of symbols in general to their objects. In this view it is one of a trivium of conceivable sciences. The first would treat of the formal conditions of

* See Peirce's definition in the *Century Dictionary* (1889) *Intention* 8; also Albertus Magnus, *Meta.* I, 1, 1, and Th. Aquinas, *Meta.* IV, 4, f. 43 v. A.

[1] Herbart says [*Lehrbuch*, 2 A., 1^te Kap., §34]: "Unsre sämmtlichen Gedanken lassen sich von zwei Seiten betrachten; theils als Thätigkeiten unseres Geistes, theils in Hinsicht dessen, *was* durch sie gedacht wird. In letzerer Beziehung heissen sie *Begriffe*, welches Wort, indem es das *Begriffene* bezeichnet, zu abstrahiren gebietet von der Art und Weise, wie wir den Gedanken empfangen, produciren oder reproduciren mögen." But the whole difference between a concept and an external sign lies in these respects which logic ought, according to Herbart, to abstract from.

symbols having meaning, that is of the reference of symbols in general to their grounds or imputed characters, and this might be called formal grammar;* the second, logic,† would treat of the formal conditions of the truth of symbols; and the third would treat of the formal conditions of the force of symbols, or their power of appealing to a mind, that is, of their reference in general to interpretants, and this might be called formal rhetoric.‡

There would be a general division of symbols, common to all these sciences; namely, into,

1°. Symbols which directly determine only their *grounds* or imputed qualities, and are thus but sums of marks or *terms;*

2°. Symbols which also independently determine their *objects* by means of other term or terms, and thus, expressing their own objective validity, become capable of truth or falsehood, that is, are *propositions;* and,

3°. Symbols which also independently determine their *interpretants*, and thus the minds to which they appeal, by premissing a proposition or propositions which such a mind is to admit. These are *arguments.*

And it is remarkable that, among all the definitions of the proposition, for example, as the *oratio indicativa*, as the subsumption of an object under a concept, as the expression of the relation of two concepts, and as the indication of the mutable ground of appearance, there is, perhaps, not one in which the conception of reference to an object or correlate is not the important one. In the same way, the conception of reference to an interpretant or third, is always prominent in the definitions of argument.

In a proposition, the term which separately indicates the object of the symbol is termed the subject, and that which indicates the ground is termed the predicate. The objects indicated by the subject (which are always potentially a plurality — at least, of phases or appearances) are therefore stated by the proposition to be related to one another on the

* Later called Speculative Grammar or Stechiology.

† Later called Critical Logic or Critic.

‡ Later called Speculative Rhetoric or Methodeutic.

ground of the character indicated by the predicate. Now this relation may be either a concurrence or an opposition. Propositions of concurrence are those which are usually considered in logic; but I have shown in a paper upon the classification of arguments* that it is also necessary to consider separately propositions of opposition, if we are to take account of such arguments as the following:

Whatever is the half of anything is less than that of which it is the half:

A is half of B;
A is less than B.

The subject of such a proposition is separated into two terms, a "subject nominative" and an "object accusative."

In an argument, the premisses form a representation of the conclusion, because they indicate the interpretant of the argument, or representation representing it to represent its object. The premisses may afford a likeness, index, or symbol of the conclusion. In deductive argument, the conclusion is represented by the premisses as by a general sign under which it is contained. In hypotheses, something *like* the conclusion is proved, that is, the premisses form a likeness of the conclusion. Take, for example, the following argument:

M is, for instance, P^{I}, P^{II}, P^{III}, and P^{IV};
S is P^{I}, P^{II}, P^{III}, and P^{IV}:
$\therefore$ S is M.

Here the first premiss amounts to this, that "P^{I}, P^{II}, P^{III}, and P^{IV}" is a likeness of M, and thus the premisses are or represent a likeness of the conclusion. That it is different with induction another example will show.

S^{I}, S^{II}, S^{III}, and S^{IV} are taken as samples of the collection M;
S^{I}, S^{II}, S^{III}, and S^{IV} are P:
$\therefore$ All M is P.

Hence the first premiss amounts to saying that "S^{I}, S^{II}, S^{III}, and S^{IV}" is an index of M. Hence the premisses are an index of the conclusion.

* See vol. 2, bk. III, ch. 2.

The other divisions of terms, propositions, and arguments arise from the distinction of extension and comprehension. I propose to treat this subject in a subsequent paper.* But I will so far anticipate that as to say that there is, first, the direct reference of a symbol to its objects, or its denotation; second, the reference of the symbol to its ground, through its object, that is, its reference to the common characters of its objects, or its connotation; and third, its reference to its interpretants through its object, that is, its reference to all the synthetical propositions in which its objects in common are subject or predicate, and this I term the information it embodies. And as every addition to what it denotes, or to what it connotes, is effected by means of a distinct proposition of this kind, it follows that the extension and comprehension of a term are in an inverse relation, as long as the information remains the same, and that every increase of information is accompanied by an increase of one or other of these two quantities. It may be observed that extension and comprehension are very often taken in other senses in which this last proposition is not true.

This is an imperfect view of the application which the conceptions which, according to our analysis, are the most fundamental ones find in the sphere of logic. It is believed, however, that it is sufficient to show that at least something may be usefully suggested by considering this science in this light.

§2. NOTES ON THE PRECEDING†‡

560. Before I came to man's estate, being greatly impressed with Kant's *Critic of the Pure Reason*, my father, who was an eminent mathematician, pointed out to me lacunæ in Kant's reasoning which I should probably not otherwise have discovered. From Kant, I was led to an admiring study of Locke, Berkeley, and Hume, and to that of Aristotle's *Organon*, *Metaphysics*, and psychological treatises, and somewhat later derived the greatest advantage from a deeply pondering perusal of some of the works of medieval thinkers, St. Augustine,

* Vol. 2, bk. II, ch. 5.

† 560–562 are from "Pragmatism," (Prag.[J]) c. 1905; 563 is from a fragment of a proposed "DI" Lecture c. 1898; 564–567 are from a fragment c. 1899.

‡ See also 2.340.

Abelard, and John of Salisbury, with related fragments from St. Thomas Aquinas, most especially from John of Duns, the Scot (Duns being the name of a then not unimportant place in East Lothian), and from William of Ockham. So far as a modern man of science can share the ideas of those medieval theologians, I ultimately came to approve the opinions of Duns, although I think he inclines too much toward nominalism. In my studies of Kant's great *Critic*, which I almost knew by heart, I was very much struck by the fact that, although, according to his own account of the matter, his whole philosophy rests upon his "functions of judgment," or logical divisions of propositions, and upon the relation of his "categories" to them, yet his examination of them is most hasty, superficial, trivial, and even trifling, while throughout his works, replete as they are with evidences of logical genius, there is manifest a most astounding ignorance of the traditional logic, even of the very *Summulæ Logicales*, the elementary schoolbook of the Plantagenet era. Now although a beastlike superficiality and lack of generalizing thought spreads like a pall over the writings of the scholastic masters of logic, yet the minute thoroughness with which they examined every problem that came within their ken renders it hard to conceive in this twentieth century how a really earnest student, goaded to the study of logic by the momentous importance that Kant attached to its details, could have reconciled himself to treating it in the debonnair and *dégagé* fashion that he did. I was thus stimulated to independent inquiry into the logical support of the fundamental concepts called categories.

561. The first question, and it was a question of supreme importance requiring not only utter abandonment of all bias, but also a most cautious yet vigorously active research, was whether or not the fundamental categories of thought really have that sort of dependence upon formal logic that Kant asserted. I became thoroughly convinced that such a relation really did and must exist. After a series of inquiries, I came to see that Kant ought not to have confined himself to divisions of propositions, or "judgments," as the Germans confuse the subject by calling them, but ought to have taken account of all elementary and significant differences of form among signs of all sorts, and that, above all, he ought not to have

left out of account fundamental forms of reasonings. At last, after the hardest two years' mental work that I have ever done in my life, I found myself with but a single assured result of any positive importance. This was that there are but three elementary forms of predication or signification, which as I originally named them (but with bracketed additions now made to render the terms more intelligible) were *qualities* (of feeling), (dyadic) *relations, and* (predications of) *representations.*

562. It must have been in 1866 that Professor De Morgan honored the unknown beginner in philosophy that I then was (for I had not earnestly studied it for more than ten years, which is a short apprenticeship in this most difficult of subjects), by sending me a copy of his memoir "On the Logic of Relations, etc."* I at once fell to upon it; and before many weeks had come to see in it, as De Morgan had already seen, a brilliant and astonishing illumination of every corner and every vista of logic. Let me pause to say that no decent semblance of justice has ever been done to De Morgan, owing to his not having brought anything to its final shape. Even his personal students, reverent as they perforce were, never sufficiently understood that his was the work of an exploring expedition, which every day comes upon new forms for the study of which leisure is, at the moment, lacking, because additional novelties are coming in and requiring note. He stood indeed like Aladdin (or whoever it was) gazing upon the overwhelming riches of Ali Baba's cave, scarce capable of making a rough inventory of them. But what De Morgan, with his strictly mathematical and indisputable method, actually accomplished in the way of examination of all the strange forms with which he had enriched the science of logic was not slight and was performed in a truly scientific spirit not unanimated by true genius. It was quite twenty-five years before my studies of it all reached what may be called a near approach toward a provisionally final result (absolute finality never being presumable in any universal science); but a short time sufficed to furnish me with mathematical demonstration that indecomposable predicates are of three classes: first, those which, like neuter verbs, apply but to a single subject; secondly, those which like

* "On the Syllogism IV, and the Logic of Relations," *Cambridge Philosophical Transactions*, vol. 10, pp. 331–358.

simple transitive verbs have two subjects each, called in the traditional nomenclature of grammar (generally less philosophical than that of logic) the "subject nominative" and the "object accusative," although the perfect equivalence of meaning between "A affects B" and "B is affected by A" plainly shows that the two things they denote are equally referred to in the assertion; and thirdly, those predicates which have three such subjects, or correlates. These last (though the purely formal, mathematical method of De Morgan does not, as far as I see, warrant this) never express mere brute fact, but always some relation of an intellectual nature, being either constituted by action of a mental kind or implying some general law.

563. As early as 1860, when I knew nothing of any German philosopher except Kant, who had been my revered master for three or four years, I was much struck with a certain indication that Kant's list of categories might be a part of a larger system of conceptions. For instance, the categories of relation — reaction, causality, and subsistence — are so many different modes of *necessity*, which is a category of modality; and in like manner, the categories of quality — negation, qualification, degree, and intrinsic attribution — are so many relations of inherence, which is a category of relation. Thus, as the categories of the third group are to those of the fourth, so are those of the second to those of the third; and I fancied, at least, that the categories of quantity, unity, plurality, totality, were, in like manner, different intrinsic attributions of quality. Moreover, if I asked myself what was the difference between the three categories of quality, the answer I gave was that negation was a merely *possible* inherence, quality in degree a *contingent* inherence, and intrinsic attribution a *necessary* inherence; so that the categories of the second group are distinguished by means of those of the fourth; and in like manner, it seemed to me that to the question how the categories of quantity — unity, plurality, totality — differ, the answer should be that *totality*, or system, is the intrinsic attribution which results from reactions, *plurality* that which results from causality, and *unity* that which results from inherence. This led me to ask, what are the conceptions which are distinguished by negative unity, qualitative unity, and intrinsic unity? I also asked, what are the different kinds of necessity by which

reaction, causality, and inherence are distinguished? I will not trouble the reader with my answers to these and similar questions. Suffice it to say that I seemed to myself to be blindly groping among a deranged system of conceptions; and after trying to solve the puzzle in a direct speculative, a physical, a historical, and a psychological manner, I finally concluded the only way was to attack it as Kant had done from the side of formal logic.

564. I must acknowledge some previous errors committed by me in expounding my division of signs into *icons*, *indices* and *symbols*. At the time I first published this division in 1867 I had been studying the logic of relatives for so short a time that it was not until three years later that I was ready to go to print with my first memoir on that subject. I had hardly commenced the cultivation of that land which De Morgan had cleared. I already, however, saw what had escaped that eminent master, that besides non-relative characters, and besides relations between pairs of objects, there was a third category of characters, and but this third. This third class really consists of plural relations, all of which may be regarded as compounds of triadic relations, that is, of relations between triads of objects. A very broad and important class of triadic characters [consists of] representations. A representation is that character of a thing by virtue of which, for the production of a certain mental effect, it may stand in place of another thing. The thing having this character I term a *representamen*, the mental effect, or thought, its *interpretant*, the thing for which it stands, its *object*.

565. In 1867, although I had proof (duly published)* that there was only a third category of characters besides non-relative characters and dual relations, yet I had not discovered that plural relations (which it had not occurred to me were sometimes not reducible to conjunctions of dual relations) constitute that third class. I saw that there must be a conception of which I could make out some features, but being unfamiliar with it in its generality, I quite naturally mistook it for that conception of *representation* which I obtained by generalizing for this very purpose the idea of a sign. I did not generalize enough, a form of error into which greater minds than mine

* 3.93ff.

might fall. I supposed the third class of characters was quite covered by the representative characters. Accordingly, I declared all characters to be divisible into *qualities* (non-relative characters), *relations*, and *representations*, instead of into non-relative characters, dual relations, and plural relations.

566. I observed in 1867* that dual relations are of two kinds according as they are or are not constituted by the relate and correlate possessing non-relative characters. This is correct. Two blue objects are *ipso facto* in relation to one another. It is important to remark that this is not true of characters so far as they are dissimilar. Thus, an orange and justice are not brought into relation to one another by the disparateness of their characters. Drag them into comparison, and then they stand in the relation of dissimilarity, a relation of a quite complex nature. But as the orange and justice exist, their qualities do not constitute a relation of dissimilarity. It must not be overlooked that dissimilarity is not simple otherness. Otherness belongs to hecceities. It is the inseparable spouse of identity: wherever there is identity there is necessarily otherness; and in whatever field there is true otherness there is necessarily identity. Since identity belongs exclusively to that which is *hic et nunc*, so likewise must otherness. It is, therefore, in a sense a dynamical relation, though only a relation of reason. It exists only so far as the objects concerned are, or are liable to be, forcibly brought together before the attention. Dissimilarity is a relation between characters consisting in otherness of all the subjects of those characters. Consequently, being an otherness, it is a dynamo-logical relation, existing only so far as the characters are, or are liable to be, brought into comparison by something besides those characters in themselves.

567. Similarity, on the other hand, is of quite a different nature. The forms of the words *similarity* and *dissimilarity* suggest that one is the negative of the other, which is absurd, since everything is both similar and dissimilar to everything else. Two characters, being of the nature of ideas, are, in a measure, the same. Their mere existence constitutes a unity of the two, or, in other words, pairs them. Things are similar and dissimilar so far as their characters are so. We see, then,

* 558.

that the first category of relations embraces only similarities; while the second, embracing all other relations, may be termed dynamical relations. At the same time, we see from the above remarks that the dynamical relations at once divide themselves into logical, hemilogical and non-logical relations. By logical relations, I mean those in respect to which all pairs [of] objects in the universe are alike; by hemilogical relations those in respect to which there is in reference to each object in the universe only one object (perhaps itself) or some definite multitude of objects which are different from others; while the alogical relations include all other cases. The logical and hemilogical relations belong to the old class of relations of reason, while relations *in re* are alogical. But there are a few not unimportant relations of reason which are likewise alogical. In my paper of 1867, I committed the error of identifying those relations constituted by non-relative characters with relations of equiparance, that is, with necessarily mutual relations, and the dynamical relations with relations of disquiparance, or possibly non-mutual relations. Subsequently, falling out of one error into another, I identified the two classes respectively with relations of reason and relations *in re*.

CHAPTER 7

*TRIADOMANY**

The author's response to the anticipated suspicion that he attaches a superstitious or fanciful importance to the number three, and forces divisions to a Procrustean bed of trichotomy.

568. I fully admit that there is a not uncommon craze for trichotomies. I do not know but the psychiatrists have provided a name for it. If not, they should. "Trichimania," [?] unfortunately, happens to be preëmpted for a totally different passion; but it might be called *triadomany*. I am not so afflicted; but I find myself obliged, for truth's sake, to make such a large number of trichotomies that I could not [but] wonder if my readers, especially those of them who are in the way of knowing how common the malady is, should suspect, or even opine, that I am a victim of it. But I am now and here going to convince those who are open to conviction, that it is not so, but that there is a good reason why a thorough student of the subject of this book† should be led to make trichotomies, that the nature of the science is such that not only is it to be expected that it should involve real trichotomies, but furthermore, that there is a cause that tends to give this form even to faulty divisions, such as a student, thirsting for thoroughness and full of anxiety lest he omit any branch of his subject, will be liable to fall into. Were it not for this cause, the trichotomic form would, as I shall show, be a strong argument in confirmation of the reasoning whose fruit should take this form.

569. My first argument in repelling the suspicion that the prevalence of trichotomies in my system is due purely to my predilection for that form, will be that were that predilection so potent, it would inevitably have made me equally given over to the trichotomic form of classification of whatever sub-

* 1910.

† Apparently "The Quest of Quest — An Inquiry into the Conditions of the Success in Inquiry (beyond the collection and observation of facts)," of which but a few pages were written.

ject I might work upon. But this is not at all the case. I once endeavored by going over the different classifications that I have made of subjects not of the special kind in which I find trichotomies to abound — a kind which I shall define below — to ascertain the relative frequency of different numbers of sub-classes in the divisions of classes generally, when the divisions were such as seemed to me undoubtedly to possess objective reality. I do not think my results of much value, on account of the great difference of the proportions in different kinds of subjects. Nevertheless, I will set them down. I found that among twenty-nine divisions of subjects not of the kind that specially abound in trichotomies, there would be eleven dichotomies, five trichotomies, and thirteen divisions into more than three parts. The fact that I got such a result, however rough it was, suffices to show that I have no marked predilection for trichotomies in general.

570. I come now to a second argument, or rather to a series of considerations not altogether foreign to what I have been saying. The warm friends who urged upon me these objections — and nothing can be more precious to a sincere student than frank and strongly put objections — were naturalists belonging to that family of minds to whom mathematics, even the simplest, seems a closed book. I would point out to them, or, to speak more accurately, I would tell them, that there is a world-wide difference between the divisions that one recognizes in classes whose essence one can comprehend, and the varieties that one observes from the outside, as one does those of objects of natural history, without being able to guess why they should be such as they seem to be, nor, except in the higher divisions, being at all sure that we have the full list of the parts, nor whether they result from a single division or from several, one succeeding another.

571. Agassiz, in his *Essay on Classification*, described well — I do not say perfectly, but relatively, well — what a classification of animals ought to be. But subsequent zoölogists seem to find that when he came to adjusting his idea to the facts of the animal kingdom, it did not seem to be a good fit. What wonder? It required the taxonomist to say what the idea of the Creator was, and the different manners in which the one idea was designed to be carried out. How can a

creature so place himself at the point of view of his Creator?

572. Soon the zoölogists began to classify according to the course of evolution. No doubt this had the advantage of turning their minds to problems within the scope of science. But I venture upon the observation that, granting the perfect success of their investigation, what they so ascertain is precisely the genealogy of species. Now genealogy is not at all the same thing as logical division. Nothing renders this clearer than the studies of Galton and others upon the phenomena of the inheritance of characters. I mean that this is shown even to those who have no definite idea of what logical division is; while for those who know what it is, the studies of Galton gave emphasis and illustration to what they must have fully realized already.

But when my critical friends counsel me to consider the marvellous multiplicity of sub-groups into which each group of the animal kingdom is divided at each division, I accept their suggestion, and turn to Huxley's famous volume on *The Anatomy of Vertebrate Animals*. I find that he first divides this branch into three provinces: the *Ichthyopsida*, the *Sauropsida*,[1] and the *Mammalia*. He divides each Province into Classes.*

[1] No ingenuity can make such manufactured words invariably suggest their meaning. A tortoise may perhaps "look like a lizard," but it is hard to see how a turkey or a heron does.

* The manuscript breaks off here.

Book IV

THE NORMATIVE SCIENCES

CHAPTER 1

*INTRODUCTION**

573. *Normative Science*† forms the mid-portion of cœnoscopy and its most characteristic part. . . . Logic, regarded from one instructive, though partial and narrow, point of view, is the theory of deliberate thinking. To say that any thinking is deliberate is to imply that it is controlled with a view to making it conform to a purpose or ideal. Thinking is universally acknowledged to be an active operation. Consequently, the control of thinking with a view to its conformity to a standard or ideal is a special case of the control of action to make it conform to a standard; and the theory of the former must be a special determination of the theory of the latter. Now special theories should always be made to rest upon the general theories of which they are amplifications. The present writer takes the theory of the control of conduct, and of action in general, so as to conform to an ideal, as being the mid-normative science; that is, as the second of the trio, and as that one of the three sciences in which the distinctive characters of normative science are most strongly marked. He will not undertake to pronounce any other distribution of the matter of normative science to be wrong; but, according to the dissection of that matter which seems to him to separate studies as they must be separated in research, such will be the mid-normative science. Since the normative sciences are usually held to be three, Logic, Ethics, and [Esthetics], and since he, too, makes them three, he would term the mid-

* From the "Basis of Pragmatism." 1906. See 5.549.

† Peirce came to recognize the nature of the Normative Sciences at a very late date (c. 1903). He wrote practically nothing on esthetics (see 2.197) and linked most of his discussions of practics and ethics with those on pragmatism and logic. Logic, the third of the Normative Sciences, being the subject on which Peirce spent about sixty years of intensive study and on which he left the most manuscripts, is the special topic of Volumes Two, Three, and Four. The present book, accordingly, dealing as it mainly does with but two subjects insufficiently studied, is unusually, but necessarily, short and unsatisfactory.

normative science ethics if this did not seem to be forbidden by the received acception of that term. He accordingly proposes to name the mid-normative science, as such (whatever its content may be) *antethics*, that is, that which is put in place of ethics, the usual second member of the trio. It is the writer's opinion that this *antethics* should be the theory of the conformity of action to an ideal. Its name, as such, will naturally be *practics*. Ethics is not practics; first, because ethics involves more than the theory of such conformity; namely, it involves the theory of the ideal itself, the nature of the *summum bonum;* and secondly, because, in so far as *ethics* studies the conformity of conduct to an ideal, it is limited to a particular ideal, which, whatever the professions of moralists may be, is in fact nothing but a sort of composite photograph of the conscience of the members of the community. In short, it is nothing but a traditional standard, accepted, very wisely, without radical criticism, but with a silly pretence of critical examination. The science of morality, virtuous conduct, right-living, can hardly claim a place among the heuretic sciences.

574. It has been a great, but frequent, error of writers on ethics to confound an ideal of conduct with a motive to action. The truth is that these two objects belong to different categories. Every action has a motive; but an ideal only belongs to a line [of] conduct which is deliberate. To say that conduct is deliberate implies that each action, or each important action, is reviewed by the actor and that his judgment is passed upon it, as to whether he wishes his future conduct to be like that or not. His ideal is the kind of conduct which attracts him upon review. His self-criticism, followed by a more or less conscious resolution that in its turn excites a determination of his habit, will, with the aid of the sequelæ, *modify* a future action; but it will not generally be a moving cause to action. It is an almost purely passive liking for a way of doing whatever he may be moved to do. Although it affects his own conduct, and nobody else's, yet the quality of feeling (for it is merely a quality of feeling) is just the same, whether his own conduct or that of another person, real or imaginary, is the object of the feeling; or whether it be connected with the thought of any action or not. If conduct is to be thoroughly deliberate, the ideal must be a habit of feeling which has grown up under the

influence of a course of self-criticisms and of hetero-criticisms; and the theory of the deliberate formation of such habits of feeling is what ought to be meant by *esthetics*.* It is true that the Germans, who invented the word, and have done the most toward developing the science, limit it to *taste*, that is, to the action of the *Spieltrieb* from which deep and earnest emotion would seem to be excluded. But in the writer's opinion the theory is the same, whether it be a question of forming a taste in bonnets or of a preference between electrocution and decapitation, or between supporting one's family by agriculture or by highway robbery. The difference of earnestness is of vast practical moment; but it has nothing to do with heuretic science.

According to this view, esthetics, practics, and logic form one distinctly marked whole, one separate department of heuretic science; and the question where precisely the lines of separation between them are to be drawn is quite secondary. It is clear, however, that esthetics relates to feeling, practics to action, logic to thought.

* Cf. 5.130, 5.553.

CHAPTER 2

*ULTIMATE GOODS**

575. It is pretty generally admitted that logic is a *normative* science, that is to say, it not only lays down rules which ought to be, but need not be followed; but it is the analysis of the conditions of attainment of something of which purpose is an essential ingredient. It is, therefore, closely related to an art; from which, however, it differs markedly in that its primary interest lies in understanding those conditions, and only secondarily in aiding the accomplishment of the purpose. Its business is analysis, or, as some writers prefer to say, definition.

The word *normative* was invented in the school of Schleiermacher.† The majority of writers who make use of it tell us that there are three normative sciences, logic, esthetics, and ethics, the doctrines of the true, the beautiful, and the good, a triad of ideals which has been recognized since antiquity. On the other hand, we quite commonly find the term "normative science" restricted to logic and ethics; and Schleiermacher himself states their purposes in a way that seems to give room for no third. The one, he says, relates to making thought conform to being, the other, to making being conform to thought. There seems to be much justice in this restriction. For that which renders logic and ethics peculiarly normative is that nothing can be either logically true or morally good without a purpose to be so. For a proposition, and especially the conclusion of an argument, which is only accidentally true is not logical. On the other hand, a thing is beautiful or ugly quite irrespective of any purpose to be so. It would seem, therefore, that esthetics is no more essentially normative than any nomological science. The science of optics, for example, might very well be regarded as the study of the conditions to be observed in making use of light. Under such a conception, nothing

* "Minute Logic," ch. 4, 1902–3.

† See 2.8n.

essential to optics would be omitted, nor anything foreign to it inserted. Those writers, however, who stand out for the trinity of normative sciences do so upon the ground that they correspond to three fundamental categories of objects of desire. As to that, the logician may be exempted from inquiring whether the beautiful is a distinct ideal or not; but he is bound to say how it may be with the true; and accordingly the intention of this chapter is to lay the foundation for the doctrine, which will appear more and more evident as we proceed, that that truth the conditions of which the logician endeavors to analyze, and which is the goal of the reasoner's aspirations, is nothing but a phase of the *summum bonum* which forms the subject of pure ethics, and that neither of those men can really understand himself until he perceives clearly that it is so.

576. I hope I shall not be thought to wander if I note one observation by the way, before formally settling down to the question. Were there nothing in reasoning more than the old traditional treatises set forth, then a rogue might be as good a reasoner as a man of honor; although a coward could not, even under such an idea of reasoning. But in induction a habit of probity is needed for success: a trickster is sure to play the confidence game upon himself. And in addition to probity, industry is essential. In the presumptive choice of hypotheses, still higher virtues are needed — a true elevation of soul. At the very lowest, a man must prefer the truth to his own interest and well-being and not merely to his bread and butter, and to his own vanity, too, if he is to do much in science. This will appear in the logical discussion; and it is thoroughly borne out by examining the characters of scientific men and of great heuretic students of all kinds. It is a remarkable fact that, excluding idle tales about pre-socratic philosophers, all history does not tell of a single man who has considerably increased human knowledge (unless theology be knowledge) having been proved a criminal. Of the four or five instances usually adduced, Seneca neither contributed to knowledge nor has been convicted of positive crime; Calvin was nothing but a theologian; the attacks upon Erasmus are beneath contempt; Bacon was no man of science, but only a grandiose writer, whose very style betrays him; Dr. Dodd was an ordinary commentator on the Bible; and nothing was proved against

Libri. The same may be said of whispers that this or that naturalist purloined specimens in the interest of science. The lofty character of the true man of science, physical or psychical, finds not one exception among a hundred. But it is needless to go to history for cases in which relatively small obliquities have prevented eminent scientists from achieving higher successes; for they abound in the experience of everybody who knows the scientific world from within. If it were true that every fallacy were a sin, logic would be reduced to a branch of moral philosophy. This is not true. But we can perceive that good reasoning and good morals are closely allied; and I suspect that with the further development of ethics this relation will be found to be even more intimate than we can, as yet, prove it to be.

577. There is room for doubt whether ethics is correctly described as a normative branch of philosophy. The doctrine of rights and duties is practical rather than normative; and if we are to use the word philosophy, as I intend to do, for that part of science which rests upon so much of experience as presses in upon every man during every hour of his waking life, then it is plain that the doctrine of rights and duties, which makes heavy drafts upon wisdom, or the knowledge which comes by reflection upon the total experience of a lifetime, as well as upon a learned acquaintance with the structure of the society in which one lives, stretches far beyond the familiar ground of philosophy. But the doctrine of rights and duties is a mere superstructure upon ethics proper. This groundwork philosophy will never disavow; for it is her pride and boast, the one branch of her work in which during the last three centuries an indisputable, steady progress has been made, not put to shame by the achievements of the special sciences. I wish as much could be said of logic. Concerning what, then, have all those writers whose subtle and beautiful discussions have built up the science of ethics been mainly occupying themselves? Surely not casuistry, or the determination of what under given circumstances ought to or may be done. They have been largely busied with the analysis of conscience, which as a psychological problem, mainly, belongs among the special psychical sciences. But the more important subject of their deliberations has been, What is good? Now this is hardly

a normative question: it is pre-normative. It does not ask for the conditions of fulfillment of a definitely accepted purpose, but asks what is to be sought, *not* for a reason, but back of every reason. Logic, as a true normative science, supposes the question of what is to be aimed at to be already answered before it could itself have been called into being. Pure ethics, philosophical ethics, is not normative, but pre-normative.

578. "If so, why this chapter?" I fear the reader will ask, and forthwith skip, as surplusage, the true life-germ of all the truths I have to unfold. "Never mind," you will say, "whether the aim which logic has in view is a good one, or not; as a matter of fact, we are interested in it. It is to learn the truth: no aim could be of more elementary simplicity. Let us turn to where we are told how to come to it." Well, if this aim is so readily comprehensible, suppose you tell me, to whom it does not seem so, what truth consists in. "Truth is the conformity of a representation to its object," says Kant.* One might make this statement more explicit; but for our present purpose it may pass. It is nearly correct, so far as it is intelligible. Only, what is that "object" which serves to define truth? Why it is the *reality:* it is of such a nature as to be independent of representations of it, so that, taking any individual sign or any individual collection of signs (such, for example, as all the ideas that ever enter into a given man's head) there is some character which that thing possesses, whether that sign or any of the signs of that collection represents the thing as possessing that character or not. Very good: now only tell me what it means to say that an object possesses a character, and I shall be satisfied. But even now, in advance of our study of definition, [we can] sufficiently see that we can only reach a conception of the less known through the more known, and that consequently the only meaning which we can attach to the phrase that a thing "has a character" is that something is *true* of it. So there we are, after threading the passages of this labyrinth, already thrown out at that very conception of truth at which we entered it. Indeed, when one comes to consider it, how futile it was to imagine that we were to clear up the idea of *truth* by the more occult idea of *reality!*

579. Yet the logician will never be scientifically or safely

* See, e.g., *C. d. R. V.* A58, 320, 462.

equipped for his explorations until he knows precisely what it is that he is seeking. The whole doctrine of logic depends upon that to a degree one could hardly foresee. The best way will be to go back to the beginning and inquire what it is that we can be content to wish for independently of any ulterior result. For the discussions of the moralists, who have not had logic in view, are not altogether adequate for our needs. In this inquiry we are not to look for any [discussion] leading to psychology; for the anatomy and physiology of the mind, or of the brain, though they may furnish a hint now and then, can after all not tell us that anything is desirable, except for some reason; while what we wish to know is what is desirable without any reason. Psychology might, it is true, discover that there is no way whatever in [which] certain things could become objects of desire; but it can only make such a discovery by relying upon direct self-questioning, as [to] what we do or do not desire, and such premisses of psychology are here precisely the conclusions of which we are in quest. So we must make up our minds to rely entirely upon self-questioning, with here and there perhaps some secondary aid from psychology.

580. Such self-questioning produces no infallible response. On the contrary, consciousness may be set down as one of the most mendacious witnesses that ever was questioned. But it is the only witness there is; and all we can do is to put it in the sweat-box and torture the truth out of it, with such judgment as we can command.

581. What I propose now to do is to pass in review every one of the general classes of objects which anybody could suppose to be an ultimate good, and to question consciousness, first, as to whether or not each of these in turn could content us as the sole ultimate good independently of any ulterior result, and if not, whether it can be considered to be in itself a good at all, irrespective of its effects. I shall arrange my list so as to commence with the most particular satisfactions and proceed step by step to the most general. But since there are in each grade several kinds of satisfactions, I shall begin in each grade of generality with the most immediate and selfish and go on by steps to the most subservient.

582. I begin, then, with simple satisfactions of the moment. The most immediate of these is the simple satisfaction of a

direct instinct. I am thirsty and I want a drink. Now our shifty witness, consciousness, is very ready with her answer that a drink is good but that momentary satisfactions are not the only good. Let us not be in haste to accept either answer. Men might easily argue — indeed, do argue — that there *can* be no other good than the satisfaction of the moment's desire. But the moment I hear that word *can* used, where nothing in the world is pertinent but observations of what is, I cast the judgment aside as worthless. For "cannot be" means "not in accordance with a hypothetical construction" intended, this time, to represent human nature. But I do not care about hypothetical constructions. I just want to know whether a man *does* ever find any other satisfaction than the simple satisfaction of the moment. If there is nothing good but the simple satisfaction of this moment, and all other moments are nothing, I must be in a state of perfect satisfaction or perfect dissatisfaction. Is that so? Obviously not: I may wish for something in spite of accompanying disadvantages. Therefore, the simple satisfaction of the moment is not all. There is at least complexity. Now can the simple satisfaction of the moment be, in itself, any good at all? Here consciousness is emphatic in her reply that the drink is good, however small a good. But there can be no harm in a little cross-examination of the witness. An absolutely simple satisfaction will involve no comparison, no measure, no degree. It will be perfect, if it exists at all. Now let be supposed that it could be proved to you that, I will not say for a moment only, but for the entire duration of a millionth of a second, you were to enjoy a simple satisfaction, say that of an agreeable color sensation, with no effects whatever of any kind, and of course no memory of it. Then, since this satisfaction would be perfect and immeasurable, and would be, O Consciousness, you say, a good, at how much would you value it? How many years of purgatory would you be willing to endure for the sake of it? Come, speak up. Would you endure five minutes of toothache? For the *knowledge* that you had, or were about to have, the strange experience, perhaps. But this would be an effect. You must suppose that you were to be utterly ignorant of whether you had, or were about to have, any such feeling. Would it not be precisely the same thing as if this had happened to some other

being, say to a mosquito, with this difference, that the mosquito is your neighbor, with whom you have some grain of sympathy while this isolated instant would really have no existence at all? I think I hear you murmur that an absolutely simple satisfaction would be an absurdity. Then such satisfaction is no part of the good. Still, it might be said that this result is owing to the absurd hypothesis of simplicity.

583. Let us, then, next consider whether the perfect and prompt satisfaction of every instinct is the only ultimate good. Though I cannot recall it at all, I think it very likely that in my childhood I read of a malevolent fairy who pronounced this curse upon an infant that during his entire life whatever wish he should conceive should be instantly gratified. If he wished for a drink, a drink should be instantly before him. If he wished it to taste differently, it should taste differently. If he was tired of sloth and wished he were working, he should be putting forth his strength. Only two things would be debarred. The first restriction should be that vague dissatisfaction, mere ennui at having his own way, should not be enough. He must form a definite wish. And secondly, wishing that his wishes should not be gratified should only be gratified until he made a positive wish. The instant he did that the satisfaction should come. I do not think that consciousness herself could have the face to pronounce this state of things good. The only alleviation of it would be the state of imbecility in which the person would be engulfed. Now I ask whether, in view of this, the mere satisfaction of an impulse can be said to be *per se* a good, at all. Under certain circumstances, the satisfaction may be good, no doubt; but is it so *per se* and *simpliciter?* Here again, we find ourselves contemplating an absurdity. A satisfaction cannot be divorced from its circumstances. It results then that the gratification of an instinct is not *per se* any part of the good. Still, it might be said that this is only because we have supposed an impossible situation in which there was no providing for future desires.

584. We will, therefore, next consider whether provision for satisfying future instinctive desires is the only ultimate good, and if not, whether it is, in itself, a good at all. It will here be pertinent to remark that although the state of things last imagined would not be good for a man, yet it does not

seem to differ much from the conditions under which my dog, and mare, and poultry seem to be enviably happy. Still, perhaps there may be a delusion here. The dog, I can perceive, considers that there is a heavy weight of responsibility upon him, and so do the mature poultry. Even the mare is not without this feeling; and perhaps she is not so entirely happy as the others. If there were an idiot about the place, could we regard it as an ultimate good that he should thus have all his instincts gratified? I think not. If it would produce a state of content in the poor fellow's mind, and if for any reason that were a good, then, for the sake of that effect, it is undeniable that the state of things supposed would be good: but that would not make it an ultimate good; on the contrary, it would furnish a *reason* for the sake of which it would be good. These considerations are extremely pertinent to the case we are now to consider, which is that of a person engaged busily in providing for his next day's wants, with just enough uncertainty as to his probable success to keep industrious. If, for any reason, or without an ulterior reason, it is desirable that he should be happy, and if his mental development is so low that those conditions would make him happy, as possibly they might make some creatures zoölogically human, then of course that would furnish an end as a means to which the state supposed would be good. But how is it with you, my Consciousness? Would you think it was reason enough for the creation of heaven and earth that it put you, or any other individual, into this condition of working for your living?*

* There are about five pages missing at this point. The manuscript then continues by repeating some of the foregoing, goes on to list a number of ultimate "ends" proposed by the early Greeks and concludes with a one-hundred-and-twenty-five-page discussion on the order, history, and contents of the Platonic Dialogues. Except for a short digression which will appear as §7–§8, ch. 11, bk. I, vol. 6, the rest of the manuscript will not be published.

CHAPTER 3

*AN ATTEMPTED CLASSIFICATION OF ENDS**

585. In the *Popular Science Monthly* for January, 1901, (LVIII *et seq.*)† I enumerated a number of ethical classes of motives, meaning by a motive, not a spring of action, but an aim or end appearing ultimate to the agent. Any such classification may be rendered more minute by subdivisions, or broader by aggregations of classes. My endeavor was to make my enumeration about evenly specific throughout. Upon a reëxamination of it, it appears to me to be sufficiently complete and systematic to afford a tolerable material to be cut up, worked over, and amplified into a satisfactory classification of ends. It is in the hope that others may be moved to interest themselves in this work and complete it, or help to complete it, that I now give an improved statement of it.

This statement will be facilitated and made clearer by a notation which is designed to show what the essential elements of the different ends appear to me to be.

586. A. A man may act in a quasi-hypnotic response to an instant command. I indicate this by the letter A.

B. A man may act from obedience, although not to a concrete command. I indicate this by B. In this case, he may still act as purely on the impulse of the moment as in case A. Only if he does so, while still acting from pure obedience, not from any impulse of his own, it must be a Mrs. Grundy, a *vague personification of the community*, which he obeys. I will indicate an end into which such personification enters as an element by a letter *z* following the capital letter.

Is there any way in which a man can act from pure obedience when there is no concrete command without the element *z*? Undoubtedly, provided he acts in obedience to a law. I will indicate that an end involves a conscious reference to a law, or

* c. 1903.

† See vol. 9.

general reason, by writing the figure 1 before the capital letter. We find, then, under B,

B*z*. Acting under dread of Mrs. Grundy, without generalizing her dictum.

1B. Acting under awe of a law, without criticizing its obligation.

But cannot the elements 1 and *z* be combined? Cannot a man act under the influence of a vague personification of the community and yet according to a general rule of conduct? Certainly: he so acts when he conforms to custom. Only if it is mere custom and not law, it is not a case of obedience, but of *conformity to norm*, or exemplar. (I never use the word *norm* in the sense of a precept, but only in that of a pattern which is copied, this being the original metaphor.) I indicate an end which presents a norm to be conformed to by the capital letter C.

Conformity to a norm may take place by an immediate impulse. It then becomes instinctive imitation. But here the man does not vaguely personify the community, but puts himself in the shoes of another person, as we say. I call this putting of oneself in another's place, *retroconsciousness*. I indicate that an end essentially involves retroconsciousness by writing the letter *y* after the capital.

Conformity to a norm may also take place without either the *y* or the *z* element. Only in this case the norm must be a definite ideal which is regarded as in itself καλὸς κ'ἀγαθός. I indicate an end which essentially involves the recognition of a definite ideal as universally and absolutely desirable by putting the figure 2 before the capital. Under C, then, we have the following cases:

C*y*. Instinctive imitation.

1C*z*. Conformity to custom.

2C. Conformity to the καλὸς κ'ἀγαθός, unanalyzed.

587. The elements 1 and *y* can be combined. That is to say, a man may act from putting himself in another's place and according to a general reason furnished by that retroconsciousness. That is, he acts for the sake of that other's welfare. The object need not be a person: an estate or a plant can be treated with the same affection. But this is no longer conformity to a norm; it is *devotion* to somebody or something.

588. In like manner, the elements 2 and *z* may be combined. That is to say, a man's ultimate end may lie in a vague personification of the community and at the same time may contemplate a definite general state of things as the *summum bonum*. That is, his heart may be set upon the welfare and safety of the community. But this again is *devotion*, not conformity to a norm. An end the adoption of which involves devotion shall be indicated by the capital letter D.

Devotion may operate in a momentary impulse. In that case, the agent does not put himself in the place of the object, — for that, without reflection, results merely*

589†. All these distinctions would be embraced by some such scheme as the following:

I. The end is to superinduce upon feeling a certain quality, pleasure.

II. The end is to extend the existence of a subject.
   1. Of something psychical, as a soul;
   2. Of something physical, as a race.

III. The end is to realize a general ideal.
   1. To bring about some general state of feeling, such as the greatest pleasure of the greatest number of persons;
   2. To impress a definite subject with a definite character.
      (a) This character being inward, such as altruistic sentiment;
      (b) This character being outward, such as the peace and prosperity of mankind.
   3. To further the realization of an ideal not definable in advance, otherwise than as that which tends to realize itself in the long run, or in some such way.
      (a) This ideal being supposed to be of the inward type;
      (b) This ideal being supposed to be of the outward type;
      (c) This ideal being purely methodical, and thus equally capable of inward and of outward realization.

* The manuscript breaks off here.

† 589–590 are from an untitled fragment, c. 1903.

590. The most serious defect of this classification lies in its subdivision of rationalistic theory into only two main branches splitting upon the insignificant question of whether the end is completely attainable or not. The truth is that there have been three grand classes of rationalistic moralists who have differed from one another upon the much more important question of the mode of being of the end. Namely, there have been those who have made the end purely subjective, a feeling of pleasure; there have been those who have made the end purely objective and material, the multiplication of the race; and finally there have been those who have attributed to the end the same kind of being that a law of nature has, making it lie in the rationalization of the universe.*

* See next chapter and 5.3, 5.433, where Peirce identifies himself with the last group.

CHAPTER 4

*IDEALS OF CONDUCT**

591. Every man has certain ideals of the general description of conduct that befits a rational animal in his particular station in life, what most accords with his total nature and relations. If you think this statement too vague, I will say, more specifically, that there are three ways in which these ideals usually recommend themselves and justly do so. In the first place certain kinds of conduct, when the man contemplates them, have an esthetic quality. He thinks that conduct fine; and though his notion may be coarse or sentimental, yet if so, it will alter in time and must tend to be brought into harmony with his nature. At any rate, his taste *is* his taste for the time being; that is all. In the second place, the man endeavors to shape his ideals into consistency with each other, for inconsistency is odious to him. In the third place, he imagines what the consequences of fully carrying out his ideals would be, and asks himself what the esthetic quality of those consequences would be.

592. These ideals, however, have in the main been imbibed in childhood. Still, they have gradually been shaped to his personal nature and to the ideas of his circle of society rather by a continuous process of growth than by any distinct acts of thought. Reflecting upon these ideals, he is led to *intend* to make his own conduct conform at least to a part of them — to that part in which he thoroughly believes. Next, he usually formulates, however vaguely, certain *rules of conduct.* He can hardly help doing so. Besides, such rules are convenient and serve to minimize the effects of future inadvertence and, what are well-named, the wiles of the devil within him. Reflection upon these rules, as well as upon the general ideals behind them, has a certain effect upon his disposition, so that what he naturally inclines to do becomes modified. Such being his condition,

* From the "Lowell Lectures of 1903," Lecture I, vol. 1, 3d Draught; 611–615 from vol. 2, 2d Draught, which is a continuation of vol. 1, 3d Draught.

he often foresees that a special occasion is going to arise; thereupon, a certain gathering of his forces will begin to work and this working of his being will cause him to consider how he will act, and in accordance with his disposition, such as it now is, he is led to form a *resolution* as to how he will act upon that occasion. This resolution is of the nature of a plan; or, as one might almost say, a *diagram*. It is a mental formula always more or less general. Being nothing more than an idea, this resolution does not necessarily influence his conduct. But now he sits down and goes through a process similar to that of impressing a lesson upon his memory, the result of which is that the *resolution*, or mental formula, is converted into a *determination*, by which I mean a really efficient agency, such that if one knows what its special character is, one can *forecast* the man's conduct on the special occasion. One cannot make forecasts that will come true in the majority of trials of them by means of any figment. It must be by means of something true and real.

593. We do not know by what machinery the conversion of a resolution into a determination is brought about. Several hypotheses have been proposed; but they do not much concern us just now. Suffice it to say that the determination, or efficient agency, is something hidden in the depths of our nature. A peculiar quality of feeling accompanies the first steps of the process of forming this impression; but later we have no direct consciousness of it. We may become aware of the disposition, especially if it is pent up. In that case, we shall recognize it by a feeling of *need*, of *desire*. I must notice that a man does not always have an opportunity to form a definite resolution beforehand. But in such cases there are less definite but still well-marked determinations of his nature growing out of the general rules of conduct that he has formulated; or in case no such appropriate rule has been formulated, his ideal of fitting conduct will have produced some disposition. At length, the anticipated occasion actually arises.

594. In order to fix our ideas, let us suppose a case. In the course of my reflexions, I am led to think that it would be well for me to talk to a certain person in a certain way. I resolve that I will do so when we meet. But considering how, in the heat of conversation, I might be led to take a different

tone, I proceed to impress the resolution upon my soul; with the result that when the interview takes place, although my thoughts are then occupied with the matter of the talk, and may never revert to my resolution, nevertheless the determination of my being does influence my conduct. All action in accordance with a determination is accompanied by a feeling that is pleasurable; but, whether the feeling at any instant is felt as pleasurable in that very instant or whether the recognition of it as pleasurable comes a little later is a question of fact difficult to make sure about.

595. The argument turns on the feeling of pleasure, and therefore it is necessary, in order to judge of it, to get at the facts about that feeling as accurately as we can. In beginning to perform any series of acts which had been determined upon beforehand, there is a certain sense of joy, an anticipation and commencement of a relaxation of the tension of need, which we now become more conscious of than we had been before. In the act itself taking place at any instant, it may be that we are conscious of pleasure; although that is doubtful. Before the series of acts are done, we already begin to review them, and in that review we recognize the pleasurable character of the feelings that accompanied those acts.

596. To return to my interview, as soon as it is over I begin to review it more carefully and I then ask myself whether my conduct accorded with my resolution. That resolution, as we agreed, was a mental formula. The memory of my action may be roughly described as an image. I contemplate that image and put the question to myself. Shall I say that that image satisfies the stipulations of my resolution, or not? The answer to this question, like the answer to any inward question, is necessarily of the nature of a mental formula. It is accompanied, however, by a certain quality of feeling which is related to the formula itself very much as the color of the ink in which anything is printed is related to the sense of what is printed. And just as we first become aware of the peculiar color of the ink and afterward ask ourselves whether it is agreeable or not, so in formulating the judgment that the image of our conduct does satisfy our previous resolution we are, in the very act of formulation, aware of a certain quality of *feeling*, the feeling of satisfaction — and directly afterward recognize that that feeling was pleasurable.

597. But now I may probe deeper into my conduct, and may ask myself whether it accorded with my general intentions. Here again there will be a judgment and a feeling accompanying it, and directly afterward a recognition that that feeling was pleasurable or painful. This judgment, if favorable, will probably afford less intense pleasure than the other; but the feeling of satisfaction which is pleasurable will be different and, as we say, a *deeper* feeling.

598. I may now go still further and ask how the image of my conduct accords with my ideals of conduct fitting to a man like me. Here will follow a new judgment with its accompanying feeling followed by a recognition of the pleasurable or painful character of that feeling. In any or all of these ways a man may criticize his own conduct; and it is essential to remark that it is not mere idle praise or blame such as writers who are not of the wisest often distribute among the personages of history. No indeed! It is approval or disapproval of the only respectable kind, that which will bear fruit in the future. Whether the man is satisfied with himself or dissatisfied, his nature will absorb the lesson like a sponge; and the next time he will tend to do better than he did before.

599. In addition to these three self-criticisms of single series of actions, a man will from time to time review his *ideals*. This process is not a job that a man sits down to do and has done with. The experience of life is continually contributing instances more or less illuminative. These are digested first, not in the man's consciousness, but in the depths of his reasonable being. The results come to consciousness later. But meditation seems to agitate a mass of tendencies and allow them more quickly to settle down so as to be really more conformed to what is fit for the man.

600. Finally, in addition to this personal meditation on the fitness of one's own ideals, which is of a practical nature, there are the purely theoretical studies of the student of ethics who seeks to ascertain, as a matter of curiosity, what the *fitness* of an ideal of conduct consists in, and to deduce from such definition of fitness what conduct ought to be. Opinions differ as to the wholesomeness of this study. It only concerns our present purpose to remark that it is in itself a purely theoretical inquiry, entirely distinct from the business of

shaping one's own conduct. Provided that feature of it be not lost sight of, I myself have no doubt that the study is more or less favorable to right living.

601. I have thus endeavored to describe fully the typical phenomena of controlled action. *They are not every one present in every case.* Thus, as I have already mentioned, there is not always an opportunity to form a resolution. I have specially emphasized the fact that conduct is determined by what precedes it in time, while the recognition of the pleasure it brings follows after the action. Some may opine that this is not true of what is called the pursuit of pleasure; and I admit that there is room for their opinion while I myself incline to think, for example, that the satisfaction of eating a good dinner is never a satisfaction in the present instantaneous state, but always follows after it. I insist, at any rate, that a *feeling*, as a mere appearance, can have no real power in itself to produce any effect whatever, however indirectly.

602. My account of the facts, you will observe, leaves a man at full liberty, no matter if we grant all that the necessitarians ask. That is, the man *can*, or if you please is *compelled*, to *make his life more reasonable.* What other distinct idea than that, I should be glad to know, can be attached to the word liberty?

603. Now let us compare the facts I have stated with the argument I am opposing. That argument rests on two main premisses; first, that it is unthinkable that a man should act from any other motive than pleasure, if his act be deliberate; and second, that action with reference to pleasure leaves no room for any distinction of right and wrong.

604. Let us consider whether this second premiss is really true. What would be requisite in order to destroy the difference between innocent and guilty conduct? The one thing that would do it would be to destroy the faculty of effective self-criticism. As long as that remained, as long as a man compared his conduct with a preconceived standard and that effectively, it need not make much difference if his only *real* motive were pleasure; for it would become disagreeable to him to incur the sting of conscience. But those who deluded themselves with that fallacy were so inattentive to the phenomena

that they confused the judgment, after the act, that that act satisfied or did not satisfy the requirements of a standard, with a pleasure or pain accompanying the act itself.

605. Let us now consider whether the other premiss is true, that it is unthinkable that a man should act deliberately except for the sake of pleasure. What is the element which it is in truth unthinkable that deliberate action should lack? It is simply and solely the determination. Let his determination remain, as it is certainly conceivable that it should remain, although the very nerve of pleasure were cut so that the man were perfectly insensible to pleasure and pain, and he will certainly pursue the line of conduct upon which he is intent. The only effect would be to render the man's intentions more inflexible — an effect, by the way, which we often have occasion to observe in men whose feelings are almost deadened by age or by some derangement of the brain. But those who have reasoned in this fallacious way have confounded together the determination of the man's nature, which is an efficient agency prepared previously to the act, with the comparison of conduct with a standard, which comparison is a general mental formula subsequent to the act, and, having identified these two utterly different things, placed them in the act itself as a mere quality of feeling.

606. Now if we recur to the defendant argument about reasoning, we shall find that it involves the same sort of tangle of ideas. The phenomena of reasoning are, in their general features, parallel to those of moral conduct. For reasoning is essentially thought that is under self-control, just as moral conduct is conduct under self-control. Indeed reasoning *is* a species of controlled conduct and as such necessarily partakes of the essential features of controlled conduct. If you attend to the phenomena of reasoning, although they are not quite so familiar to you as those of morals because there are no clergymen whose business it is to keep them before your minds, you will nevertheless remark, without difficulty, that a person who draws a rational conclusion, not only thinks it to be true, but thinks that similar reasoning would be just in every analogous case. If he fails to think this, the inference is not to be called reasoning. It is merely an idea suggested to his mind and which he cannot resist thinking is true. But not having been sub-

jected to any check or control, it is not deliberately approved and is not to be called reasoning. To call it so would be to ignore a distinction which it ill becomes a rational being to overlook. To be sure, every inference forces itself upon us irresistibly. That is to say, it is irresistible at the instant it first suggests itself. Nevertheless, we all have in our minds certain *norms*, or general patterns of right reasoning, and we can compare the inference with one of those and ask ourselves whether it satisfies that rule. I call it a rule, although the formulation may be somewhat vague; because it has the essential character of a rule of being a general formula applicable to particular cases. If we judge our norm of right reason to be satisfied, we get a feeling of approval, and the inference now not only appears as irresistible as it did before, but it will prove far more unshakable by any doubt.

607. You see at once that we have here all the main elements of moral conduct; the general standard mentally conceived beforehand, the efficient agency in the inward nature, the act, the subsequent comparison of the act with the standard. Examining the phenomena more closely we shall find that not a single element of moral conduct is unrepresented in reasoning. At the same time, the special case naturally has its peculiarities.

608. Thus, we have a general ideal of sound logic. But we should not naturally describe it as our idea of the kind of reasoning that befits men in our situation. How should we describe it? How if we were to say that sound reasoning is such reasoning that in every conceivable state of the universe in which the facts stated in the premisses are true, the fact stated in the conclusion will thereby and therein be true. The objection to this statement is that it only covers necessary reasoning, including reasoning about chances. There is other reasoning which is defensible as probable, in the sense that while the conclusion may be more or less erroneous, yet the same procedure diligently persisted in must, in every conceivable universe in which it leads to any result at all, lead to a result indefinitely approximating to the truth. When that is the case, we shall do right to pursue that method, provided we recognize its true character, since our relation to the universe does not permit us to have any necessary knowledge of

positive facts. You will observe that in such a case our ideal is shaped by the consideration of our situation relatively to the universe of existences. There are still other operations of the mind to which the name "reasoning" is especially appropriate, although it is not the prevailing habit of speech to call them so. They are conjectures, but rational conjectures; and the justification of them is that unless a man had a tendency to guess right, unless his guesses are better than tossing up a copper, no truth that he does not already virtually possess could ever be disclosed to him, so that he might as well give up all attempt to reason; while if he has any decided tendency to guess right, as he *may* have, then no matter how often he guesses wrong, he will get at the truth at last. These considerations certainly do take into account the man's inward nature as well as his outward relations; so that the ideals of good logic are truly of the same general nature as ideals of fine conduct. We saw that three kinds of considerations go to support ideals of conduct. They were, first, that certain conduct seems fine in itself. Just so certain conjectures seem likely and easy in themselves. Secondly, we wish our conduct to be consistent. Just so the ideal [of] necessary reasoning is consistency simply. Third, we consider what the general effect would be of thoroughly carrying out our ideals. Just so certain ways of reasoning recommend themselves because if persistently carried out they must lead to the truth. The parallelism, you perceive, is almost exact.

609. There is also such a thing as a general logical *intention*. But it is not emphasized for the reason that the will does not enter so violently into reasoning as it does into moral conduct. I have already mentioned the logical norms, which correspond to moral laws. In taking up any difficult problem of reasoning we formulate to ourselves a logical resolution; but here again, because the will is not at such high tension in reasoning as it often is in self-controlled conduct, these resolutions are not very prominent phenomena. Owing to this circumstance, the efficient determination of our nature, which causes us to reason in each case as we do, has less relation to resolutions than to logical norms. The act itself is, at the instant, irresistible in both cases. But immediately after, it is subjected to self-criticism by comparison with a previous

standard which is always the norm, or *rule*, in the case of reasoning, although in the case of outward conduct we are too often content to compare the act with the resolution. In the case of general conduct, the lesson of satisfaction or dissatisfaction is frequently not much taken to heart and little influences future conduct. But in the case of reasoning an inference which self-criticism disapproves is always instantly annulled, because there is no difficulty in doing this. Finally, all the different feelings which, as we noticed, accompanied the different operations of self-controlled conduct equally accompany those of reasoning, although they are not quite so vivid.

610. The parallelism is thus perfect. Nor, I repeat, could it fail to be so, if our description of the phenomena of controlled conduct was true, since reasoning is only a special kind of controlled conduct. . . .

611. What does right reasoning consist in? It consists in such reasoning as shall be conducive to our ultimate aim. What, then, is our ultimate aim? Perhaps it is not necessary that the logician should answer this question. Perhaps it might be possible to deduce the correct rules of reasoning from the mere assumption that we have some ultimate aim. But I cannot see how this could be done. If we had, for example, no other aim than the pleasure of the moment, we should fall back into the same absence of any logic that the fallacious argument would lead to. We should have no ideal of reasoning, and consequently no norm. It seems to me that the logician ought to recognize what our ultimate aim is. It would seem to be the business of the moralist to find this out, and that the logician has to accept the teaching of ethics in this regard. But the moralist, as far as I can make it out, merely tells us that we have a power of self-control, that no narrow or selfish aim can ever prove satisfactory, that the only satisfactory aim is the broadest, highest, and most general possible aim; and for any more definite information, as I conceive the matter, he has to refer us to the esthetician, whose business it is to say what is the state of things which is most admirable in itself regardless of any ulterior reason.

612. So, then, we appeal to the esthete to tell us what it is that is admirable without any reason for being admirable beyond its inherent character. Why, that, he replies, is the

beautiful. Yes, we urge, such is the name that you give to it, but what *is it?* What is this character? If he replies that it consists in a certain quality of feeling, a certain *bliss*, I for one decline altogether to accept the answer as sufficient. I should say to him, My dear Sir, if you can prove to me that this quality of feeling that you speak of does, as a fact, attach to what you call the beautiful, or that which would be admirable without any reason for being so, I am willing enough to believe you; but I cannot without strenuous proof admit that any particular quality of feeling is admirable without a reason. For it is too revolting to be believed unless one is forced to believe it.

613. A fundamental question like this, however practical the issues of it may be, differs entirely from any ordinary practical question, in that whatever is accepted as good in itself must be accepted without compromise. In deciding any special question of conduct it is often quite right to allow weight to different conflicting considerations and calculate their resultant. But it is quite different in regard to that which is to be the aim of all endeavor. The object admirable that is admirable *per se* must, no doubt, be general. Every ideal is more or less general. It may be a complicated state of things. But it must be a *single* ideal; it must have *unity*, because it is an idea, and unity is essential to every idea and every ideal. Objects of utterly disparate kinds may, no doubt, be admirable, because some special reason may make each one of them so. But when it comes to the ideal of the admirable, in itself, the very nature of its being is to be a precise idea; and if somebody tells me it is either this, or that, or that other, I say to him, It is clear you have no *idea* of what precisely it is. But an ideal must be capable of being embraced in a unitary idea, or it is no ideal at all. Therefore, there can be no compromises between different considerations here. The admirable ideal cannot be too extremely admirable. The more thoroughly it has whatever character is essential to it, the more admirable it must be.

614. Now what would the doctrine that that which is admirable in itself is a quality of feeling come to if taken in all its purity and carried to its furthest extreme — which should be the extreme of admirableness? It would amount to

saying that the one ultimately admirable object is the unrestrained gratification of a desire, regardless of what the nature of that desire may be. Now that is too shocking. It would be the doctrine that all the higher modes of consciousness with which we are acquainted in ourselves, such as love and reason, are good only so far as they subserve the lowest of all modes of consciousness. It would be the doctrine that this vast universe of Nature which we contemplate with such awe is good only to produce a certain quality of feeling. Certainly, I must be excused for not admitting that doctrine unless it be proved with the utmost evidence. So, then, what proof is there that it is true? The only reason for it that I have been able to learn is that *gratification*, *pleasure*, is the only conceivable result that is satisfied with itself; and therefore, since we are seeking for that which is *fine* and *admirable* without any reason beyond itself, *pleasure*, *bliss*, is the only object which can satisfy the conditions. This is a respectable argument. It deserves consideration. Its premiss, that pleasure is the only conceivable result that is perfectly self-satisfied, must be granted. Only, in these days of evolutionary ideas which are traceable to the French Revolution as their instigator, and still further back to Galileo's experiment at the leaning tower of Pisa, and still further back to all the stands that have been made by Luther and even by Robert of Lincoln against attempts to bind down human reason to any prescriptions fixed in advance — in these days, I say, when these ideas of progress and growth have themselves grown up so as to occupy our minds as they now do, how can we be expected to allow the assumption to pass that the admirable in itself is any stationary result? The explanation of the circumstance that the only result that is satisfied with itself is a quality of feeling is that reason always looks forward to an endless future and expects endlessly to improve its results.

615. Consider, for a moment, what Reason, as well as we can today conceive it, really is. I do not mean man's faculty which is so called from its embodying in some measure Reason, or Νοῦς, as a something manifesting itself in the mind, in the history of mind's development, and in nature. What is this Reason? In the first place, it is something that never can have been completely embodied. The most insignificant of general

ideas always involves conditional predictions or requires for its fulfillment that events should come to pass, and all that ever can have come to pass must fall short of completely fulfilling its requirements. A little example will serve to illustrate what I am saying. Take any general term whatever. I say of a stone that it is *hard*. That means that so long as the stone remains hard, every essay to scratch it by the moderate pressure of a knife will surely fail. To call the stone *hard* is to predict that no matter how often you try the experiment, it will fail every time. That innumerable series of conditional predictions is involved in the meaning of this lowly adjective. Whatever may have been done will not begin to exhaust its meaning. At the same time, the very being of the General, of Reason, is of such a mode that this being *consists* in the Reason's actually governing events. Suppose a piece of carborundum has been made and has subsequently been dissolved in aqua regia without anybody at any time, so far as I know, ever having tried to scratch it with a knife. Undoubtedly, I may have good reason, nevertheless, to call it hard; because some actual fact has occurred such that Reason compels me to call it so, and a general idea of all the facts of the case can only be formed if I do call it so. In this case, my calling it hard is an actual event which is governed by that law of hardness of the piece of carborundum. But if there were no actual fact whatsoever which was meant by saying that the piece of carborundum was hard, there would be not the slightest meaning in the word hard as applied to it. The very being of the General, of Reason, *consists* in its governing individual events. So, then, the essence of Reason is such that its being never can have been completely perfected. It always must be in a state of incipiency, of growth. It is like the character of a man which consists in the ideas that he will conceive and in the efforts that he will make, and which only develops as the occasions actually arise. Yet in all his life long no son of Adam has ever fully manifested what there was in him. So, then, the development of Reason requires as a part of it the occurrence of more individual events than ever can occur. It requires, too, all the coloring of all qualities of feeling, including pleasure in its proper place among the rest. This development of Reason consists, you will observe, in embodiment, that is, in manifestation. The creation of the

universe, which did not take place during a certain busy week, in the year 4004 B.C., but is going on today and never will be done, is this very developement of Reason. I do not see how one can have a more satisfying ideal of the admirable than the development of Reason so understood. The one thing whose admirableness is not due to an ulterior reason is Reason itself comprehended in all its fullness, so far as we can comprehend it. Under this conception, the ideal of conduct will be to execute our little function in the operation of the creation by giving a hand toward rendering the world more reasonable whenever, as the slang is, it is "up to us" to do so. In logic, it will be observed that knowledge is reasonableness; and the ideal of reasoning will be to follow such methods as must develope knowledge the most speedily. . . .

CHAPTER 5

VITALLY IMPORTANT TOPICS

§1. THEORY AND PRACTICE*

616. The early Greek philosopher, such as we read about in Diogenes Laertius, is certainly one of the most amusing curiosities of the whole human menagerie. It seems to have been demanded of him that his conduct should be in marked contrast with the dictates of ordinary common sense. Had he behaved as other men are supposed to do his fellow-citizens would have thought his philosophy had not taught him much. I know that historians possessed of "higher criticism" deny all the ridiculous anecdotes about the Hellenic sages. These scholars seem to think that logic is a question of literary taste, and their refined perceptions refuse to accept these narratives. But in truth even were taste carried to a point of delicacy exceeding that of the German professor — which he would think was pushing it quite into that realm of imaginary quantities which lies on the other side of infinity — it still would not weigh as logic, which is a matter of strict mathematical demonstration wherein opinion is of no weight at all.

617. Now scientific logic cannot approve that historical method which leads to the absolute and confident denial of all the positive testimony that is extant, the moment that testimony deviates from the preconceived ideas of the historian. The story about Thales falling into the ditch while pointing out the different stars to the old woman is told by Plato† about two centuries later. But Dr. Edouard Zeller‡ says he knows better, and pronounces the occurrence quite impossible. Were you to point out that the anecdote only attributes to Thales a character common to almost all mathematicians, this would afford him a new opportunity of applying his

* The first lecture on "Detached Ideas on Vitally Important Topics," of 1898. It is entitled "Philosophy and the Conduct of Life."

† Theaetetus 174A.

‡ *Die Philosophie der Griechen*, etc. 5 Auf. 1 Teil (1892) S. 183n.

favorite argument of objection, that the story is "too probable." So the assertion of half a dozen classical writers that Democritus was always laughing and Heraclitus always weeping "proclaims itself," says Zeller, "an idle fabrication,"* notwithstanding the supports it receives from the fragments. Even Zeller admits that Diogenes of Sinope was a trifle eccentric. Being a contemporary of Aristotle and one of the best-known men of Greece, his history cannot well be denied even by Zeller, who has to content himself with averring that the stories are "grossly exaggerated."† There was no other philosopher whose conduct according to all testimony was quite so extravagant as that of Pyrrho. The accounts of him seem to come direct from a writing of his devoted pupil, Timon of Phlius, and some of our authorities, of whom there are a dozen, profess to use this book. Yet Zeller and the critics do not believe them; and Brandis objects that the citizens of Elis would not have chosen a half-insane man high priest — as if symptoms of that kind would not have particularly recommended him for a divine office. That fashion of writing history is, I hope, now at last passing away.

618. However, disbelieve the stories if you will; you cannot refuse to admit that they show what kind of man the narrators expected a philosopher to be — if they were imaginary legends, all the more so. Now those narrators are a cloud of the sanest and soberest minds of antiquity — Plato, Aristotle, Cicero, Seneca, Pliny, Plutarch, Lucian, Ælian, and so forth. The Greeks expected philosophy to affect life — not by any slow process of percolation of forms, as *we* may expect that researches into differential equations, stellar photometry, the taxonomy of echinoderms, and the like will ultimately affect the conduct of life — but forthwith in the person and soul of the philosopher himself, rendering him different from ordinary men in his views of right conduct. So little did they separate philosophy from esthetic and moral culture that the *docti furor arduus Lucreti* could clothe an elaborate *cosmogony* in noble verse, for the express purpose of influencing men's lives; and Plato tells us in many places how inextricably he considers the study of Dialectic to be bound up with virtuous

* Ib. S. 626n and S. 845n.

† Ib. 4 Auf. 2 Teil 1 Abt. (1889) S. 283n.

living. Aristotle, on the other hand, set this matter right. Aristotle was not much of a Greek. That he was of full Greek blood is not likely. That he was not altogether a Greek-minded man is manifest. Though he belonged to the school of Plato, yet when he went there he was already a student, perhaps a personal pupil, of Democritus, himself another Thracian; and during his first years in Athens he cannot have had much intercourse with Plato, who was away at Syracuse a large part of the time. Above all Aristotle was an Asclepiades, that is to say, he belonged to a line every man of whom since the heroic age had, as a child, received a finished training in the dissecting-room. Aristotle was a thorough-paced scientific man such as we see nowadays, except for this, that he ranged over all knowledge. As a man of scientific instinct, he classed metaphysics, in which I doubt not he included logic, as a matter of course, among the sciences — sciences in *our* sense, I mean, what *he* called theoretical sciences — along with mathematics and natural science — natural science embracing what we call the physical sciences and the psychical sciences, generally. This theoretical science was for him one thing, animated by one spirit and having knowledge of theory as its ultimate end and aim. Esthetic studies were of a radically different kind; while morals, and all that relates to the conduct of life, formed a *third* department of intellectual activity, radically foreign in its nature and idea, from both the other two. Now, Gentlemen, it behooves me, at the outset of this course, to confess to you that in this respect I stand before you an Aristotelian and a scientific man, condemning with the whole strength of conviction the Hellenic tendency to mingle philosophy and practice.

619. There are sciences, of course, many of whose results are almost immediately applicable to human life, such as physiology and chemistry. But the true scientific investigator completely loses sight of the utility of what he is about. It never enters his mind. Do you think that the physiologist who cuts up a dog reflects, while doing so, that he may be saving a human life? Nonsense. If he did, it would spoil him for a scientific man; and then the vivisection would become a crime. However, in physiology and in chemistry, the man whose brain is occupied with utilities, though he will not do

much for science, may do a great deal for human life. But in philosophy, touching as it does upon matters which are, and ought to be, sacred to us, the investigator who does not stand aloof from all intent to make practical applications will not only obstruct the advance of the pure science, but, what is infinitely worse, he will endanger his own moral integrity and that of his readers.

620. In my opinion, the present infantile condition of philosophy — for as long as earnest and industrious students of it are able to come to agreement upon scarce a single principle, I do not see how it can be considered as otherwise than in its infancy — is due to the fact that during this century it has chiefly been pursued by men who have not been nurtured in dissecting-rooms and other laboratories, and who consequently have not been animated by the true scientific *Eros;* but who have on the contrary come from theological seminaries, and have consequently been inflamed with a desire to amend the lives of themselves and others, a spirit no doubt more important than the love of science, for men in average situations, but radically unfitting them for the task of scientific investigation. And it is precisely because of this utterly unsettled and uncertain condition of philosophy at present, that I regard any practical applications of it to religion and conduct as exceedingly dangerous. I have not one word to say against the philosophy of religion or of ethics in general or in particular. I only say that for the present it is all far too dubious to warrant risking any human life upon it. I do not say that philosophical science should not ultimately influence religion and morality; I only say that it should be allowed to do so only with secular slowness and the most conservative caution.

621. Now I may be utterly wrong in all this, and I do not propose to argue the question. I do not ask you to go with me. But to avoid any possible misapprehension, I am bound honestly to declare that I do not hold forth the slightest promise that I have any philosophical wares to offer you which will make you either better men or more successful men.

622. It is particularly needful that I should say this owing to a singular hybrid character which you will detect in these lectures. I was asked in December to prepare a course of lectures upon my views of philosophy. I accordingly set to work

to draw up in eight lectures an outline of one branch of philosophy, namely, Objective Logic.* But just as I was finishing one lecture word came that you would expect to be addressed on topics of vital importance, and that it would be as well to make the lectures detached. I thereupon threw aside what I had written and began again to prepare the same number of homilies on intellectual ethics and economics. They were wretched things; and I was glad enough to learn, when three-quarters of my task was done, that it would be desirable that as much as possible should be said of certain philosophical questions, other subjects being put in the background. At that time, however, it was too late to write a course which should set before you what I should have greatly desired to submit to your judgment. I could only patch up some fragments partly philosophical and partly practical. Thus, you will find me part of the time offering you detached ideas upon topics of vital importance, while part of the time I shall be presenting philosophical considerations, in which you will be able to feel an undercurrent toward that logic of things concerning which I shall have an opportunity to interject scarce one overt word.

623. I shall have a good deal to say about right reasoning; and in default of better I had reckoned *that* as a topic of vital importance. But I do not know that the theory of reasoning is quite vitally important. That it is absolutely essential in metaphysics, I am as sure as I am of any truth of philosophy. But in the conduct of life, we have to distinguish everyday affairs and great crises. In the great decisions, I do not believe it is safe to trust to individual reason. In everyday business, reasoning is tolerably successful; but I am inclined to think that it is done as well without the aid of theory as with it. A *logica utens*, like the analytical mechanics resident in the billiard player's nerves, best fulfills familiar uses.

624. In metaphysics, however, it is not so, at all; and the reason is obvious. The truths that the metaphysician infers can be brought to the test of experience, if at all, only in a department of experience quite foreign from that which furnishes his premisses. Thus a metaphysician who infers anything about a life beyond the grave can never find out for

* See vol. 6, bk. I, ch. 7, §2.

certain that his inference is false until he has gone out of the metaphysical business, at his present stand, at least. The consequence is that unless the metaphysician is a most thorough master of formal logic — and especially of the inductive side of the logic of relatives, immeasurably more important and difficult than all the rest of formal logic put together — he will inevitably fall into the practice of deciding upon the validity of reasonings in the same manner in which, for example, the practical politician decides as to the weight that ought to be allowed to different considerations, that is to say, by the impression those reasonings make upon the mind, only with this stupendous difference, that the one man's impressions are the resultant of long experiential training, while with such training the other man is altogether unacquainted. The metaphysician who adopts a metaphysical reasoning because he is impressed that it is sound, might just as well, or better, adopt his conclusions directly because he is impressed that they are true, in the good old style of Descartes and of Plato. To convince yourself of the extent to which this way of working actually vitiates philosophy, just look at the dealings of the metaphysicians with Zeno's objections to motion. They are simply at the mercy of the adroit Italian. For this reason, then, if for no other, the metaphysician who is not prepared to grapple with all the difficulties of modern exact logic had better put up his shutters and go out of the trade. Unless he will do one or the other, I tell him to his conscience that he is not the genuine, honest, earnest, resolute, energetic, industrious, and accomplished doubter that it is his duty to be.

625. But this is not all, nor half. For after all, metaphysical reasonings, such as they have hitherto been, have been simple enough for the most part. It is the metaphysical concepts which it is difficult to apprehend. Now the metaphysical conceptions, as I need not waste words to show, are merely adapted from those of formal logic, and therefore can only be apprehended in the light of a minutely accurate and thoroughgoing system of formal logic.

626. But in practical affairs, in matters of vital importance, it is very easy to exaggerate the importance of ratiocination. Man is so vain of his power of reason! It seems impossible for him to see himself in this respect, as he himself would see

himself if he could duplicate himself and observe himself with a critical eye. Those whom we are so fond of referring to as the "lower animals" reason very little. Now I beg you to observe that those beings very rarely commit a *mistake*, while we —— ! We employ twelve good men and true to decide a question, we lay the facts before them with the greatest care, the "perfection of human reason" presides over the presentment, they hear, they go out and deliberate, they come to a unanimous opinion, and it is generally admitted that the parties to the suit might almost as well have tossed up a penny to decide! Such is man's glory!

627. The mental qualities we most admire in all human beings except our several selves are the maiden's delicacy, the mother's devotion, manly courage, and other inheritances that have come to us from the biped who did not yet speak; while the characters that are most contemptible take their origin in reasoning. The very fact that everybody so ridiculously overrates his own reasoning is sufficient to show how superficial the faculty is. For you do not hear the courageous man vaunt his own courage, or the modest woman boast of her modesty, or the really loyal plume themselves on their honesty. What they are vain about is always some insignificant gift of beauty or of skill.

628. It is the instincts, the sentiments, that make the substance of the soul. Cognition is only its surface, its locus of contact with what is external to it.

629. Do you ask me to prove this? If so, you must be a rationalist, indeed. I can prove it — but only by assuming a logical principle of the demonstration of which I shall give a hint in the next lecture.* When people ask me to prove a proposition in philosophy I am often obliged to reply that it is a corollary from the logic of relatives. Then certain men say, "I should like exceedingly to look into this logic of relatives; you must write out an exposition of it." The next day I bring them a manuscript. But when they see that it is full of A, B, and C, they never look at it again. Such men — oh, well.

630. Reasoning is of three kinds. The first is necessary, but it only professes to give us information concerning the matter of our own hypotheses and distinctly declares that, if

* See "Introduction," vol. 4.

we want to know anything else, we must go elsewhere. The second depends upon probabilities. The only cases in which it pretends to be of value is where we have, like an insurance company, an endless multitude of insignificant risks. Wherever a vital interest is at stake, it clearly says, "Don't ask me." The third kind of reasoning tries what *il lume naturale*, which lit the footsteps of Galileo, can do. It is really an appeal to instinct. Thus reason, for all the frills it customarily wears, in vital crises, comes down upon its marrow-bones to beg the succour of instinct.

631. Reason is of its very essence egotistical. In many matters it acts the fly on the wheel. Do not doubt that the bee thinks it has a good reason for making the end of its cell as it does. But I should be very much surprised to learn that its reason had solved that problem of isoperimetry that its instinct has solved. Men many times fancy that they act from reason when, in point of fact, the reasons they attribute to themselves are nothing but excuses which unconscious instinct invents to satisfy the teasing "whys" of the *ego*. The extent of this self-delusion is such as to render philosophical rationalism a farce.

632. Reason, then, appeals to sentiment in the last resort. Sentiment on its side feels itself to be the man. That is my simple apology for philosophical sentimentalism.

633. Sentimentalism implies conservatism; and it is of the essence of conservatism to refuse to push any practical principle to its extreme limits — including the principle of conservatism itself. We do not say that sentiment is *never* to be influenced by reason, nor that under no circumstances would we advocate radical reforms. We only say that the man who would allow his religious life to be wounded by any sudden acceptance of a philosophy of religion or who would precipitately change his code of morals at the dictate of a philosophy of ethics — who would, let us say, hastily practice incest — is a man whom we should consider *unwise*. The regnant system of sexual rules is an instinctive or sentimental induction summarizing the experience of all our race. That it is abstractly and absolutely infallible we do not pretend; but that it is practically infallible for the individual — which is the only clear sense the word "infallibility" will bear — in that he ought to

obey it and not his individual reason, *that* we do maintain.

634. I would not allow to sentiment or instinct any weight whatsoever in theoretical matters, not the slightest. Right sentiment does not demand any such weight; and right reason would emphatically repudiate the claim if it were made. True, we are driven oftentimes in science to try the suggestions of instinct; but we only *try* them, we compare them with experience, we hold ourselves ready to throw them overboard at a moment's notice from experience. If I allow the supremacy of sentiment in human affairs, I do so at the dictation of reason itself; and equally at the dictation of sentiment, in theoretical matters I refuse to allow sentiment any weight whatever.

635. Hence, I hold that what is properly and usually called *belief*, that is, the adoption of a proposition as a κτῆμα ἐς ἀεί to use the energetic phrase of Doctor Carus,* has no place in science at all. We *believe* the proposition we are ready to act upon. *Full belief* is willingness to act upon the proposition in vital crises, *opinion* is willingness to act upon it in relatively insignificant affairs. But pure science has nothing at all to do with *action*. The propositions it accepts, it merely writes in the list of premisses it proposes to use. Nothing is *vital* for science; nothing can be. Its accepted propositions, therefore, are but opinions at most; and the whole list is provisional. The scientific man is not in the least wedded to his conclusions. He risks nothing upon them. He stands ready to abandon one or all as soon as experience opposes them. Some of them, I grant, he is in the habit of calling *established truths;* but that merely means propositions to which no competent man today demurs. It seems probable that any given proposition of that sort will remain for a long time upon the list of propositions to be admitted. Still, it may be refuted tomorrow; and if so, the scientific man will be glad to have got rid of an error. There is thus no proposition at all in science which answers to the conception of belief.

636. But in vital matters, it is quite otherwise. We must act in such matters; and the principle upon which we are willing to act is a *belief*.

637. Thus, pure theoretical knowledge, or science, has nothing directly to say concerning practical matters, and noth-

* *Fundamental Problems*, Open Court, Chicago (1891), p. 22.

ing even applicable at all to vital crises. Theory is applicable to minor practical affairs; but matters of vital importance must be left to sentiment, that is, to instinct.

638. Now there are two conceivable ways in which right sentiment might treat such terrible crises; on the one hand, it might be that while human instincts are not so detailed and featured as those of the dumb animals, yet they might be sufficient to guide us in the *greatest* concerns without any aid from reason, while on the other hand, sentiment might act to bring the vital crises under the domain of reason by rising under such circumstances to such a height of self-abnegation as to render the situation insignificant. In point of fact, we observe that a healthy natural human nature does act in both these ways.

639. The instincts of those animals whose instincts are remarkable present the character of being chiefly, if not altogether, directed to the preservation of the stock and of benefiting the individual very little, if at all, except so far as he may happen as a possible procreator to be a potential public functionary. Such, therefore, is the description of instinct that we ought to expect to find in man, in regard to vital matters; and so we do. It is not necessary to enumerate the facts of human life which show this, because it is too plain. It is to be remarked, however, that individuals who have passed the reproductive period, are more useful to the propagation of the human race than to [?] any other. For they amass wealth, and teach prudence, they keep the peace, they are friends of the little ones, and they inculcate all the sexual duties and virtues. Such instinct does, as a matter of course, prompt us, in all vital crises, to look upon our individual lives as small matters. It is no extraordinary pitch of virtue to do so; it is the character of every man or woman that is not despicable. Somebody during the Reign of Terror said: *Tout le monde croit qu'il est difficile de mourir. Je le crois comme les autres. Cependant je vois que quant on est là chacun s'en tire.* It is less characteristic of the woman because her life is more important to the stock, and her immolation less useful.

640. Having thus shown how much less vitally important reason is than instinct, I next desire to point out how exceedingly desirable, not to say indispensable, it is for the successful

march of discovery in philosophy and in science generally that practical utilities, whether low or high, should be *put out of sight* by the investigator.

641. The point of view of utility is always a narrow point of view. How much more we should know of chemistry today if the most practically important bodies had not received excessive attention; and how much *less* we should know, if the rare elements and the compounds which only exist at low temperatures had received only the *share* of attention to which their *utility* entitled them.

642. It is notoriously true that into whatever you do not put your whole heart and soul in that you will not have much success. Now, the two masters, *theory* and *practice*, you cannot serve. That perfect balance of attention which is requisite for observing the system of things is utterly lost if human desires intervene, and all the more so the higher and holier those desires may be.

643. In addition to that, in philosophy we have prejudices so potent that it is impossible to keep one's *sang-froid* if we allow ourselves to dwell upon them at all.

644. It is far better to let philosophy follow perfectly untrammeled a scientific method, *predetermined* in advance of knowing to what it will lead. If that course be honestly and scrupulously carried out, the results reached, even if they be not altogether true, even if they be grossly mistaken, can not but be highly serviceable for the ultimate discovery of truth. Meantime, sentiment can say "Oh well, philosophical science has not by any means said its last word yet; and meantime I will continue to believe *so and so*."

645. No doubt a large proportion of those who now busy themselves with philosophy will lose all interest in it as soon as it is forbidden to look upon it as susceptible of practical applications. We who continue to pursue the theory must bid *adieu* to them. But so we must in any department of pure science. And though we regret to lose their company, it is infinitely better that men devoid of genuine scientific curiosity should not barricade the road of science with empty books and embarrassing assumptions. . . .

646. The host of men who achieve the bulk of each year's new discoveries are mostly confined to narrow ranges. For

that reason you would expect the arbitrary hypotheses of the different mathematicians to shoot out in every direction into the boundless void of arbitrariness. But you do not find any such thing. On the contrary, what you find is that men working in fields as remote from one another as the African diamond fields are from the Klondike reproduce the same forms of novel hypothesis. Riemann had apparently never heard of his contemporary Listing. The latter was a naturalistic geometer, occupied with the shapes of leaves and birds' nests, while the former was working upon analytical functions. And yet that which seems the most arbitrary in the ideas created by the two men are one and the same form. This phenomenon is not an isolated one; it characterizes the mathematics of our times, as is, indeed, well known. All this crowd of creators of forms for which the real world affords no parallel, each man arbitrarily following his own sweet will, are, as we now begin to discern, gradually uncovering one great cosmos of forms, a world of potential being. The pure mathematician himself feels that this is so. He is not indeed in the habit of publishing any of his sentiments nor even his generalizations. The fashion in mathematics is to print nothing but demonstrations, and the reader is left to divine the workings of the man's mind from the sequence of those demonstrations. But if you enjoy the good fortune of talking with a number of mathematicians of a high order, you will find that the typical pure mathematician is a sort of Platonist. Only, he is [a] Platonist who corrects the Heraclitan error that the eternal is not continuous. The eternal is for him a world, a cosmos, in which the universe of actual existence is nothing but an arbitrary locus. The end that pure mathematics is pursuing is to discover that real potential world.

647. Once you become inflated with that idea, *vital importance* seems to be a very low kind of importance, indeed.

But such ideas are only suitable to regulate another life than this. Here we are in this workaday world, little creatures, mere cells in a social organism itself a poor and little thing enough, and we must look to see what little and definite task our circumstances have set before our little strength to do. The performance of that task will require us to draw upon all our powers, reason included. And in the doing of it we should

chiefly depend not upon that department of the soul which is most superficial and fallible — I mean our reason — but upon that department that is deep and sure — which is instinct.

648. Instinct is capable of developement and growth — though by a movement which is slow in the proportion in which it is vital; and this developement takes place upon lines which are altogether parallel to those of reasoning. And just as reasoning springs from experience, so the developement of sentiment arises from the soul's Inward and Outward Experiences. Not only is it of the same nature as the developement of cognition; but it chiefly takes place through the instrumentality of cognition. The soul's deeper parts can only be reached through its surface. In this way the eternal forms, that mathematics and philosophy and the other sciences make us acquainted with, will by slow percolation gradually reach the very core of one's being; and will come to influence our lives; and this they will do, not because they involve truths of merely vital importance, but because they are ideal and eternal verities.

§2. PRACTICAL CONCERNS AND THE WISDOM OF SENTIMENT*

649. Among the advantages which our humble cousins whom it pleases us to refer to as "the lower animals" enjoy over some of our own family is that they *never* reason about vitally important topics, and never have to lecture nor to listen to lectures about them. Docilely allowing themselves to be guided by their instincts into almost every detail of life, they live exactly as their Maker intended them to live. The result is, that they very rarely fall into error of any kind, and *never* into a vital one. What a contrast to our lives! Truly, that reason upon which we so plume ourselves, though it may answer for little things, yet for great decisions is hardly surer than a toss-up. . . .

650. Logic is computation, said Hobbes;† and those who have deepest delved in that dreary discipline testify that all

* The remainder of this chapter is from an alternate version of the first lecture on Detached Ideas entitled, "On Detached Ideas in General and on Vitally Important Topics," 1898. A number of duplicate passages have been omitted.

† *Logic or Computation*, part I, ch. 1.

reasoning whatever involves mathematics, and laugh over the fallacies of those who attempt to reason unmathematically. Now tell me, is mathematics an occupation for a gentleman and an athlete? Is not such drudgery fit only for the lower classes? One may well be struck with pity for the masses of population concentrated in New York and living under such unnatural conditions that they are forced to think mathematically. However, it is not as if they had the tender nurture of a cultured modern Harvard, that great eleemosynary institution that Massachusetts has established to the end that the *élite* of her youths may be aided to earning comfortable incomes and living softly cultured lives. The brains of those New York plebeians are coarse, strong, laboring brains, that don't know what it is to be free from mathematics. Their conceptions are crude and vulgar enough, but their vigor of reasoning would surprise you. I have seen my [private] scholars there wrestle with problems that I would no more venture to allow the exquisitely polished intellects of a modern university to attack than I would venture to toss a cannonball into an eggshell cup.

651. I intend to call upon you for no reasoning in these lectures more complicated than one of Hegel's dilemmas. For all reasoning is mathematical and requires effort; and I mean to shun the guilt of overstraining anybody's powers. That is why I have selected a subject for my lectures which is not at all in my line, but which I hope may prove to be to your taste.

652. On vitally important topics reasoning is out of place. . . . The very theory of reasoning, were we resolutely to attack it without any dread of mathematics, would furnish us conclusive reasons for limiting the applicability of reasoning to unimportant matters; so that, unless a problem is insignificant in importance compared with the aggregate of analogous problems, reasoning itself pronounces that there is a fallacy in submitting the question to reason, at all. That must remain merely an assertion, mathematics being *taboo*. . . .

653. In regard to the greatest affairs of life, the wise man follows his heart and does not trust his head. This should be the method of every man, no matter how powerful his intellect. More so still, perhaps, if mathematics is too difficult for him, that is to say, if he is unequal to any intricate reasoning whatsoever. Would not a man physically puny be a fool not to

recognize it, and to allow an insane megalomania to induce him to enter a match game of football? But the slightest of physical frames might as well attempt to force back a locomotive engine, as for the mightiest of mental giants to try to regulate his life advantageously by a purely reasoned-out theory.

654. Common sense, which is the resultant of the traditional experience of mankind, witnesses unequivocally that the heart is more than the head, and is in fact everything in our highest concerns, thus agreeing with my unproved logical theorem; and those persons who think that sentiment has no part in common sense forget that the dicta of common sense are objective facts, not the way some dyspeptic may feel, but what the healthy, natural, normal democracy thinks. And yet when you open the next new book on the philosophy of religion that comes out, the chances are that it will be written by an intellectualist who in his preface offers you his metaphysics as a guide for the soul, talking as if philosophy were one of our deepest concerns. How can the writer so deceive himself?

655. If, walking in a garden on a dark night, you were suddenly to hear the voice of your sister crying to you to rescue her from a villain, would you stop to reason out the metaphysical question of whether it were possible for one mind to cause material waves of sound and for another mind to perceive them? If you did, the problem might probably occupy the remainder of your days. In the same way, if a man undergoes any religious experience and hears the call of his Saviour, for him to halt till he has adjusted a philosophical difficulty would seem to be an analogous sort of thing, whether you call it stupid or whether you call it disgusting. If on the other hand, a man has had no religious experience, then any religion not an affectation is as yet impossible for him; and the only worthy course is to wait quietly till such experience comes. No amount of speculation can take the place of experience.

656. Pray pardon my hopping about from one branch of my discourse to another and back again with no more apparent purpose than a robin redbreast or a Charles Lamb. Because it would hardly be logically consistent for me to arrange my matter with scrupulously logical accuracy when the very thing I am driving at is that logic and reasoning are only of secondary importance. There are two psychological or anthropological

observations about our reasoning powers which it is convenient to insert here.

657. One is that powers of reasoning in any but the most rudimentary way are a somewhat uncommon gift, about as uncommon as a talent for music. Indeed, a much smaller number of persons actually attain to any proficiency in reasoning. But then the exercise of intricate ratiocination requires great energy and prolonged effort, while musical practice is nearly unmixed pleasure, I suppose, for those who do it well. Moreover, owing to several peculiar circumstances, good instruction in reasoning is exceedingly rare. As for what is taught in the colleges under the name of logic, oh dear, perhaps the less said the better. It is true that mathematics teaches one branch of reasoning. That is, indeed, its chief value in education. But how few teachers understand the logic of mathematics! And how few understand the psychology of the puzzled pupil! The pupil meets with a difficulty in Euclid. Two to one the reason is that there is a logical flaw. The boy, however, is conscious only of a mysterious hindrance. What his difficulty is he cannot tell the teacher; the teacher must teach him. Now the teacher probably never really saw the true logic of the passage. But he thinks he does because, owing to long familiarity, he has lost that sense of coming up against an invisible barrier that the boy feels. Had the teacher ever really conquered the logical difficulty himself, of course he would recognize just what it was, and thus would fulfill the first condition, at least, of being helpful. But not having conquered the difficulty, but only having worn out the sense of difficulty by familiarity, he simply cannot understand why the boy should feel any difficulty; and all he can do is to exclaim, "Oh, these stupid, stupid boys!" As if a physician should exclaim, "Oh, these horrid patients, they won't get well!" But suppose, by some extraordinary conjunction of the planets, a really good teacher of reasoning were to be appointed, what would be his first care? It would be to guard his scholars from that malady with which logic is usually infested, so that unless it runs off them like water from a duck, it is sure to make them the very worst of reasoners, namely, unfair reasoners, and what is worse unconsciously unfair, for the rest of their lives. The good teacher will therefore take the utmost pains to prevent the scholars

getting puffed up with their logical acquirements. He will wish to impregnate them with the right way of looking at reasoning before they shall be aware that they have learned anything; and he will not mind giving considerable time to that, for it is worth a great deal. But now come the examiner and the pupil himself. They want *results*, tangible to them. The teacher is dismissed as a failure, or, if he is allowed another chance, he will take good care to reverse the method of his teaching and give them *results* — especially, as that is the lazy way. These are some of the causes of there being so few strong reasoners in the world. But allowing for the influence of such causes as well as we can, the fact still remains that comparatively few persons are originally possessed of any but the feeblest modicum of this talent. What is the significance of that? Is it not a plain sign that the faculty of reasoning is not of the first importance to success in life? For were it so, its absence would cause the individual to postpone marriage and so affect his procreation; and thus natural selection would operate to breed the race for vigorous reasoning powers, and they would become common. And the study of characters confirms this conclusion. For though the men who are most extraordinarily successful evidently do reason deeply about the details of their business, yet no ordinary degrees of good success are influenced — otherwise than perhaps favorably — by any lack of great reasoning power. We all know highly successful men, lawyers, editors, scientific men — not to speak of artists — whose great deficiency in this regard is only revealed by some unforeseen accident.

658. The other observation I desired to make about the human reason is that we find people mostly modest enough about qualities which really go to making fine men and women — the courageous man not usually vaunting his courage, nor the modest woman boasting of her modesty, nor the loyal vain of their good faith: the things they are vain about are some insignificant gifts of beauty, or skill of some kind. But beyond all, with the exception of those who, being trained in logic, follow its rules and thus do not trust their direct reasoning powers at all, everybody else ridiculously overrates his own logic, and if he really has superior powers of reason is usually so consumed by conceit that it is far from rare to see a young

man completely ruined by it; so that one is sometimes tempted to think, and perhaps truly, that it conduces not only to a man's success from a worldly point of view but to his attaining any real elevation of character to be all but a fool in this regard, provided only he be perfectly aware of his own deficiency. . . .

659. All those modern books which offer new philosophies of religion, at the rate of one every fortnight on the average, are but symptoms of the temporary dissolution of the Christian faith. This appears as soon as we compare them with the works of religious philosophy of the ages of faith, such as the *Summa* of St. Thomas Aquinas or the *Opus Oxoniense* of Duns Scotus — the one reproducing without a shadow of mistrust every dogma of the Fathers of the Church, while the other displays a far stouter faith in maintaining that metaphysics has nothing to say either one way or the other concerning any question of religion, but leaves it to be decided by positive testimony or inspiration. The only old book which these modern philosophies of religion really resemble a good deal — except that they lack its terrible earnestness — is the *De consolatione philosophiae* and it is paying them a high compliment to say so. Boethius, you know, is utterly religionless, but he feels the need of religion and vainly tries to find a substitute for it in philosophy. His first two books are somewhat inspiring, because they breathe an unconscious religion. But as the work progresses, reasoning enters more and more into the thought, until the last book, which resembles a modern essay much more than all the rest, is a mere diet of bran for the hungered soul.

660. It is hardly necessary to insist here that the highly cultured classes of Christendom — excepting always those families which are so important as to be an object of solicitude on the part of the priests — are nowadays nearly destitute of any religion. It was made perfectly manifest five and twenty years ago or more — no matter for the exact date; it was at a date when men saturated with the mechanical philosophy were still hesitating to separate themselves from the church — when John Tyndall, in the innocence of his scientific heart, proposed to measure the efficacy of prayer by experimental statistics. Instantly, the clergy, one and all, instead of meeting the proposal with the candor with which Elijah met the priests of Baal — though by the way I notice some ingenious persons

think his barrels of water were really deodorized kerosene, which for a student of the history of chemistry, would, of itself, seem to be a good enough miracle — instead of thanking Tyndall for the idea, I say, the clergy to a man shrank back in terror, thus conclusively betraying to every eye their own utter disbelief in their own dogma. They pronounced it an impious proposition. But there was nothing more impious in it than in any other sort of inquiry into religion except this — that they feared it would bring all "talkee-talkee" to an end. Although it must be granted that in our country the clergy are by far the most sceptical class of the community, yet where the clergy stood a generation back, the bulk of the highly educated and cultured class stands now.

It is a thousand times better to have no faith at all in God or virtue than to have a hemi-hypocritical faith. . . .

§3. VITALLY IMPORTANT TRUTHS

661. Conservatism, true conservatism, which is sentimental conservatism, and by those who have no powers of observation to see what sort of men conservatives are, is often called stupid conservatism, an epithet far more applicable to the false conservatism that looks to see on which side bread is buttered — true conservatism, I say, means not trusting to reasonings about questions of vital importance but rather to hereditary instincts and traditional sentiments. Place before the conservative arguments to which he can find no adequate reply and which go, let us say, to demonstrate that wisdom and virtue call upon him to offer to marry his own sister, and though he be unable to answer the arguments, he will not act upon their conclusion, because he believes that tradition and the feelings that tradition and custom have developed in him are safer guides than his own feeble ratiocination. Thus, true conservatism is sentimentalism. Of course, sentiment lays no claim to infallibility, in the sense of *theoretical infallibility*, a phrase that logical analysis proves to be a mere jingle of words with a jangle of contradictory meanings. The conservative need not forget that he might have been born a Brahmin with a traditional sentiment in favor of *suttee* — a reflection that tempts him to become a radical. But still, on the whole,

he thinks his wisest plan is to reverence his deepest sentiments as his highest and ultimate authority, which is regarding them as *for him practically infallible* — that is, to say infallible in the only sense of the word in which *infallible* has any consistent meaning.

662. The opinion prevalent among radicals that conservatives, and sentimentalists generally, are fools is only a cropping-out of the tendency of men to conceited exaggeration of their reasoning powers. Uncompromising radical though I be upon some questions, inhabiting all my life an atmosphere of science, and not reckoned as particularly credulous, I must confess that the conservative sentimentalism I have defined recommends itself to my mind as eminently sane and wholesome. Commendable as it undoubtedly is to reason out matters of detail, yet to allow mere reasonings and reason's self-conceit to overslaw [over-slaugh? over-awe?] the normal and manly sentimentalism which ought to lie at the cornerstone of all our conduct seems to me to be foolish and despicable.

663. Philosophy after all is, at its highest valuation, nothing more than a branch of science, and as such is not a matter of vital importance; and those who represent it as being so are simply offering us a stone when we ask for bread. Mind, I do not deny that a philosophical or other scientific error may be fraught with disastrous consequences for the whole people. It might conceivably bring about the extirpation of the human race. Importance in that sense it might have in any degree. Nevertheless, in no case is it of *vital* importance.

664. A great calamity the error may be, *qua* event, in the sense in which an earthquake, or the impact of a comet, or the extinction of the sun would be an important event, and consequently, if it happens to lie in the line of my duty or of yours to investigate any philosophical question and to publish the more or less erroneous results of our investigations, I hope we shall not fail to do so, if we can. Certainly, any task which lies before us to be done has its importance. But there our responsibility ends. Nor is it the philosophy itself, *qua* cognition, that is vital, so much as it is our playing the part that is allotted to us.

665. You will observe that I have not said a single word in disparagement of the philosophy of religion, in general, which

seems to me a most interesting study, at any rate, and possibly likely to lead to some useful result. Nor have I attacked any sect of that philosophy. It is not the philosophy which I hold to be baleful, but the representing it to be of vital importance, as if any genuine religion could come from the head instead of from the heart.

666. Somewhat allied to the philosophy of religion is the science of ethics. It is equally useless. Now books of casuistry, indeed, using the word "casuistry" not in any technical sense, but merely to signify discussions of what ought to be done in various difficult situations, might be made at once extremely entertaining and positively useful. But casuistry is just what the ordinary treatises upon ethics do not touch, at least not seriously. They chiefly occupy themselves with reasoning out the basis of morality and other questions secondary to that. Now what's the *use* of prying into the philosophical basis of morality? We all know what morality is: it is behaving as you were brought up to behave, that is, to think you ought to be punished for not behaving. But to believe in thinking as you have been brought up to think defines *conservatism*. It needs no reasoning to perceive that morality is conservatism. But conservatism again means, as you will surely agree, not trusting to one's reasoning powers. To be a moral man is to obey the traditional maxims of your community without hesitation or discussion. Hence, ethics, which is reasoning out an explanation of morality is — I will not say immoral, [for] that would be going too far — composed of the very substance of immorality. If you ever happen to be thrown in with an unprofessional thief, the only very bad kind of thief, so as to be able to study his psychological peculiarities, you will find that two things characterize him; first, an even more immense conceit in his own reasoning powers than is common, and second, a disposition to reason about the basis of morals.

667. Ethics, then, even if not a positively dangerous study, as it sometimes proves, is as useless a science as can be conceived. But it must be said, in favor of ethical writers, that they are commonly free from the nauseating custom of boasting of the utility of their science.

668. Far be it from me to decry. Though I do hail from

New York,* I shall hardly be mistaken for a Wall Street Philistine. A useless inquiry, provided it is a systematic one, is pretty much the same thing as a scientific inquiry. Or at any rate if a scientific inquiry becomes by any mischance useful, that aspect of it has to be kept sedulously out of sight during the investigation or else, as I shall try to show you another evening, its hopes of success are fatally cursed.

669. As long as ethics is recognized as not being a matter of vital importance or in any way touching the student's conscience, it is, to a normal and healthy mind, a civilizing and valuable study — somewhat more so than the theory of whist, much more so than the question of the landing of Columbus, which things are insignificant not at all because they are useless, nor even because they are little in themselves, but simply and solely because they are detached from the great continuum of ideas.

670. It would be useless to enumerate the other sciences, since it would only be to reiterate the same declaration. As long as they are not looked at as practical, and so degraded to pot-boiling arts — as our modern writers degrade the philosophy of religion, in claiming that it is practical — for what difference does it make whether the pot to be boiled is today's or the hereafter's? They are all such that it would be far too little to say that they are valuable to us. Rather let our hearts murmur "blessed are we" if the immolation of our being can weld together the smallest part of the great cosmos of ideas to which the sciences belong.

671. Even if a science be useful — like engineering or surgery — yet if it is useful only in an insignificant degree as those sciences are, it still has a divine spark in which its petty practicality must be forgotten and forgiven. But as soon as a proposition becomes vitally important — then in the first place, it is sunk to the condition of a mere utensil; and in the second place, it ceases altogether to be scientific, because concerning matters of vital importance reasoning is at once an impertinence toward its subject matter and a treason against itself.

* Peirce was born in Cambridge, Mass., September 10, 1839, and lived there and in Milford, Pa., most of his life. For a time, however, he gave private instruction in logic at New York.

672. Were I willing to make a single exception to the principle I thus enunciate, and to admit that there was one study which was at once scientific and yet vitally important, I should make that exception in favor of logic; for the reason that if we fall into the error of believing that vitally important questions are to be decided by reasoning, the only hope of salvation lies in formal logic, which demonstrates in the clearest manner that reasoning itself testifies to its own ultimate subordination to sentiment. It is like a Pope who should declare *ex cathedra* and call upon all the faithful to implicitly believe on pain of damnation by the power of the keys that he was *not* the supreme authority.

673. Among vitally important truths there is one which I verily believe — and which men of infinitely deeper insight than mine have believed — to be solely supremely important. It is that vitally important facts are of all truths the veriest trifles. For the only vitally important matter is *my* concern, business, and duty — or yours. Now you and I — what are we? Mere cells of the social organism. Our deepest sentiment pronounces the verdict of our own insignificance. Psychological analysis shows that there is nothing which distinguishes my personal identity except my faults and my limitations — or if you please, my blind will, which it is my highest endeavor to annihilate. Not in the contemplation of "topics of vital importance" but in those universal things with which philosophy deals, the factors of the universe, is man to find his highest occupation. To pursue "topics of vital importance" as the first and best can lead only to one or other of two terminations — either on the one hand what is called, I hope not justly, Americanism, the worship of business, the life in which the fertilizing stream of genial sentiment dries up or shrinks to a rill of comic tit-bits, or else on the other hand, to monasticism, sleepwalking in this world with no eye nor heart except for the other. Take for the lantern of your footsteps the cold light of reason and regard your business, your duty, as the highest thing, and you can only rest in one of those goals or the other. But suppose you embrace, on the contrary, a conservative sentimentalism, modestly rate your own reasoning powers at the very mediocre price they would fetch if put up at auction, and then what do you come to? Why, *then*, the very

first command that is laid upon you, your quite highest business and duty, becomes, as everybody knows, to recognize a higher business than your business, *not* merely an avocation after the daily task of your vocation is performed, but a generalized conception of duty which completes your personality by melting it into the neighboring parts of the universal cosmos. If this sounds unintelligible, just take for comparison the first good mother of a family that meets your eye, and ask whether she is not a sentimentalist, whether you would wish her to be otherwise, and lastly whether you can find a better formula in which to outline the universal features of her portrait than that I have just given. I dare say you can improve upon that; but you will find one element of it is correct — especially if your understanding is aided by the logic of relatives — and that is that the supreme commandment of the Buddhistochristian religion is, to generalize, to complete the whole system even until continuity results and the distinct individuals weld together. Thus it is, that while reasoning and the science of reasoning strenuously proclaim the subordination of reasoning to sentiment, the very supreme commandment of sentiment is that man should generalize, or what the logic of relatives shows to be the same thing, should become welded into the universal continuum, which is what true reasoning consists in. But this does not reinstate reasoning, for this generalization should come about, not merely in man's cognitions, which are but the superficial film of his being, but objectively in the deepest emotional springs of his life. In fulfilling this command, man prepares himself for transmutation into a new form of life, the joyful Nirvana in which the discontinuities of his will shall have all but disappeared.

674. Do you know what it was that was at the root of the barbarism of the Plantagenet period and paralyzed the awakening of science from the days of Roger Bacon to those of Francis Bacon? We plainly trace it in the history, the writings, the monuments, of that age. It was the exaggerated interest men took in matters of vital importance.

675. Do you know what it is in Christianity that when recognized makes our religion an agent of reform and progress? It is its marking duty at its proper finite figure. Not that it diminishes in any degree its vital importance, but that behind

the outline of that huge mountain it enables us to descry a silvery peak rising into the calm air of eternity.

676. The generalization of sentiment can take place on different sides. Poetry is one sort of generalization of sentiment, and in so far is the regenerative metamorphosis of sentiment. But poetry remains on one side ungeneralized, and to that is due its emptiness. The complete generalization, the complete regeneration of sentiment is religion, which is poetry, but poetry completed.

677. That is about what I had to say to you about topics of vital importance. To sum it up, all sensible talk about vitally important topics must be commonplace, all reasoning about them unsound, and all study of them narrow and sordid.

INDEX OF PROPER NAMES*

* The numbers refer to paragraphs, not to pages.

INDEX OF SUBJECTS*

* The numbers refer to paragraphs, not to pages.

VOLUME II

ELEMENTS OF LOGIC

Errata

2.22n* *insert* 34
2.32 (p. 22, line 5) *for* ἀε *read* ἀει
2.45n* *for* vol. 8 *read* vols. 7 and 8
2.56n* *for* vol. 8 *read* 7.263f
2.147, line 3 *for* stimulation *read* simulation
2.196n* *for* VII *read* 7
2.219n* insert *A* before *Syllabus of Certain Topics of Logic*
2.243n* *for* vol. 9 *read* vol. 8
2.274n* *delete* (cf. note to ch. 1)
2.367, line 2 *for* not *read* now
2.391n1 (p. 238) *delete* [Ch. 1.]
2.391n2 (p. 238) *for* diffinitione *read* definitione
2.392n3 The second sentence is not in the original.
2.396 *for* it [an idea] *read* an idea
2.398n‡ for *Grundress* read *Grundriss*
2.431–433 A portion of this was not by Peirce at all, though accepted by him.
2.550n† *for* p. 424 *read* pp. 424–425
2.584n† *for* pp. 87–88 *read* pp. 87–89
2.713, line 14 *for* predictio *read* prediction
2.772n* *for* vols. 7 and 8 *read* 7.397f, 547, 555
II, p. 522: Index s.v. Abduction, validity of *for* 799 *read* 779
II, p. 526: Index s.v. Hemilogical *delete* 567

INTRODUCTION

Peirce liked to call himself a logician. For over fifty years he gave his best efforts to the development of logic, and there is justice in his contention that his point of view cannot be truly grasped without a mastery of his logical theories. His pragmatism, metaphysics, scientific analyses, historical studies, and even his experimental work depend in large part on these theories.

In the beginning Peirce planned to publish one original paper on logic every month, but he abandoned the attempt after his early papers were not well received. He continued, however, to write persistently on logical problems. His published papers and manuscripts cover almost the whole range of the subject. He seems to have read almost every available logical work, and to have paid particular attention to the logical writings of the Schoolmen and the English. Most of the manuscripts deal with logic. One is a complete book, his Grand Logic of 1893, for which he could not find a publisher. This is a detailed study of Aristotelian and Symbolic Logic, based largely on his own printed papers, some of which are included in the present volume. About ten years later he began to write his Minute Logic in twenty-three chapters, of which he unfortunately completed only three and a half. The present volume contains the first chapter and half of the second. The remainder may be found in Volume I. Volumes II, III, and IV contain the bulk of those logical papers which will be published. Volume III will include the published papers on Symbolic Logic; Volume IV, the unpublished writings on the foundations of mathematics and logic.

The theory of signs given in the present volume was subsequently expanded in the letters to Lady Welby, and is there shown to involve sixty-six classes of signs. Not all of these were analyzed, however. The present volume treats only of the ten classes of signs which Peirce analyzed in detail. His divisions, which reveal not only how his signs are classified,

but why, for example, there is only one kind of abduction, two kinds of deduction, and three kinds of induction, are determined in part by the following principle: that which is a Second is divisible into two parts, of which one is itself divisible into two parts, etc., and that which is a Third is divisible into three parts, one of which is a First, another of which is a Second, (and thus divisible into two parts), and the last a Third, which is itself divisible into threes, and so on, apparently without end. This theory of signs is a new discipline; its application in detail he left for others. How closely he thought it to be connected with metaphysics and pragmatism can be seen from Volume I, book iii, chapter 6, and from Volume IV, book i, chapter 6.

The aspect of logic which seems to have interested him longest and most deeply, and which makes his studies significant even today, is scientific methodology, particularly the logic of discovery. This includes his development of the "frequency theory" of probability, his original theory of abduction, or the method of obtaining new ideas, and his novel treatment of induction which is shown to be closely related to the other two methods.

The papers in this volume are in some cases forty years apart, yet they do not differ as widely as might be expected, largely because of Peirce's practice of expanding, clarifying, and working over the theories he early developed. The later papers differ from the earlier in detail and in clarity rather than in point of view; and where they do so differ, Peirce has usually indicated what the difference is.

It has been particularly difficult to select representative papers for this volume, because of the necessity of choosing from a great number of manuscripts covering in divergent ways the same ground from the same point of view. For the theory of signs an interweaving of many papers was necessary; for the logic of discovery the published papers have been reprinted, and only those manuscripts which were directly relevant to them have been chosen. Even then the volume is disproportionately large. To have included all of his significant writings on these subjects would have made necessary a number of volumes and increased unduly the size of this edition.

* * *

Wherever possible Peirce's punctuation and spelling have been retained. Titles supplied by the editors for papers previously published are marked with an ^{E}, while Peirce's titles for unpublished papers are marked with a ^{P}. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not marked. Remarks and additions by the editors are enclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout the volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the paragraphs.

HARVARD UNIVERSITY
FEBRUARY, 1932.

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Book I

GENERAL AND HISTORICAL SURVEY OF LOGIC

. . . All that you can find in print of my work on logic are simply scattered outcroppings here and there of a rich vein which remains unpublished. Most of it I suppose has been written down; but no human being could ever put together the fragments. I could not myself do so. — 1903

ELEMENTS OF LOGIC

CHAPTER 1

*CRITICAL ANALYSIS OF LOGICAL THEORIES**

§1. LOGIC'S PROMISES[P]

1. Begin, if you will, by calling logic the theory of the conditions which determine reasonings to be secure. A conception at once more exact and more extended may be looked for in the sequel.† Meantime, this description will serve for our preparatory studies of the yet unvisited country into which we propose to push our explorations. Logic, then, is a theory. The end of any theory is to furnish a rational account of its object. . . . A theory directly aims at nothing but knowing. Maybe, if it be sound, it is likely, some day, to prove useful. Still, fairness forbids our making utility the criterion of the excellence of the theory. One must acknowledge that such a way of thinking would not abase logic, as it does the generality of the sciences. To judge of logic by its applications would only be to judge of it by how far it does, or does not, in a certain manner, help us to understand things. Clearly, it is the proper aim of any theory to do that. Looking at the matter on the other side, this very consideration tends to show that the touchstone of utility is superfluous and impertinent for logic. Suppose the science supports the test; what then? That would merely show that it aids the understanding. Grant that it is a sound theory, and it must have been prepared to do that, application or no application.

2. From this point of view, we can see that logic may be useful, apart from any direct application, by supplying us with modes of conception which are useful. There are few theories, if any, of which this is equally true.

3. For the most part, theories do little or nothing for every-

* "Intended Characters of this Treatise," ch. 1 of the "Minute Logic" (1902).

† See 200.

day business. Nobody fit to be at large would recommend a carpenter who had to put up a pigsty or an ordinary cottage to make an engineer's statical diagram of the structure. In particular, applications of theories would be worse than useless where they would interfere with the operation of trained instincts. Who could play billiards by analytic mechanics? We all have a natural instinct for right reasoning, which, within the special business of each of us, has received a severe training by its conclusions being constantly brought into comparison with experiential results. Nay, we not only have a reasoning instinct, but as I shall early show,* we have an instinctive *theory* of reasoning, which gets corrected in the course of our experience. So, it would be most unreasonable to demand that the study of logic should supply an artificial method of doing the thinking that his regular business requires every man daily to do. . . .

4. Still, I repeat it, it is not in questions closely concerning a man's business that he can with any semblance of fairness look to finding the theory of reasoning helpful.† Such help is rather to be expected in extraordinary and unusual problems — especially in those of a speculative character, where conclusions are not readily checked by experience, and where our instinctive reasoning power begins to lose its self-confidence; as when we question what we ought to think about psychical research,‡ about the Gospels, about difficult questions of political economy,§ about the constitution of matter;¶ or when we inquire by what methods we can most speedily advance our knowledge of such matters.||

5. But, as I said before, were direct applications of logic, such as these, never useful, instead of being frequently so, as they are, yet its indirect utility, through the useful conceptions with which it supplies us, would be immense.

6. Meantime, its highest and greatest value is that it affords us an understanding of the processes of reasoning. That the

* See 186ff.
† Cf. vol. 1, bk. IV, ch. 5.
‡ See vol. 6, bk. II, ch. 7.
§ See 4.114ff.
¶ See 6.304ff.
|| See 101ff. and bk. III *passim*.

Platos are thoroughly right in that estimate will be more and more impressed upon our convictions as our acquaintance with the science grows.

7. On the other hand, we shall find reason to maintain, with Auguste Comte, that a theory cannot be sound unless it be susceptible of applications, immediate or remote, whether it be good economy so to apply it or not.* This is perhaps no more true of logic than of other theories; simply because it is perfectly true of all. Yet there is a special reason why it is more important to bear this point in mind in logic. Namely, logic is the theory of *right* reasoning, of what reasoning ought to be, not of what it is. On that account, it used to be called a *directive* science, but of late years Überweg's adjective *normative*[1] has been generally substituted. It might be that a normative science, in view of the economies of the case, should be quite useless for any practical application. Still, whatever fact had no bearing upon a conceivable application to practice would be entirely impertinent to such a science. It would be easy enough — much too easy — to marshal a goodly squadron of treatises on logic, each of them swelled out with matter foreign to any conceivable applicability until, like a corpulent man, it can no longer see on what it is standing, and the reader loses all clear view of the true problems of the science. But since the relation of the theory of logic to conceivable applications of it will, by and by, come up for closer examination, we need not now consider it further.

§2. OF MINUTE ACCURACY[P]

8. How shall the theory of right reasoning be investigated? The nature of the subject must be an important factor in

* For an extended discussion of this "Pragmatic" doctrine see vol. 5.

[1] The latter word is not, at least to one individual whom I wot of, particularly pleasing. The verb, *normo*, to square, is in the dictionary, but what ordinary reader of Latin can remember having met with it? Yet if the presumable motive for the substitution of the new adjective, namely, its avoidance of an apparent implication in "*directive*" that logic is a mere art, or practical science, approves itself to us, the twentieth century would laugh at us if we were too squeamish about the word's legitimacy of birth. [The *Vocabulaire de la Philosophie*, ed. by A. Lalande, vol. 2, p. 521, attributes the creation of this word, or at least its introduction into common speech, to Wundt. The Century Dictionary gets normative from *normo*, but not all the other standard works agree on this.]

determining the method. Before touching that, however, suppose we ask how, in the roughest sense, *any* theory ought to be investigated. Am I wrong in thinking I catch a whisper from good sense, that, for one item of the reply, a theory should be investigated *carefully and minutely?* Yet, strange to say, such a recommendation would be in flat contradiction to prevailing opinion. A month does not pass, scarce a fortnight will pass, without my attention being drawn to some new discussion by a man of strength relating to some broad, far-reaching question of science or philosophy. Every such dissertation will be sure to refer to principles of reasoning which are more or less contested. Upon the correctness of these the whole question hinges. How, then, do I find these logical principles are sought to be established? By the same severe and minute examination which the same author would approve in regard to a question of physics? Never: I am very sure he would condemn such piddling minuteness as inappropriate to so broad a question. He proceeds slap-dash, depicting the logical situation as in a blackboard diagram rather than as in a critically accurate anatomical plate. For the most part, he has but the vaguest notion of how he has come by his principles. He has gathered them casually, after the custom of *amateurs*. It might seem to behoove every man who has occasion to lay down principles of reasoning in a grave scientific discussion to be more than an *amateur* in logic. Voluminous writers, however, on logic there are who deliberately adopt vague substitutes for any definite method of establishing principles of reasoning.

9. When I was beginning my philosophical reading, my father, Benjamin Peirce,* forced me to recognize the extremely loose reasoning common to the philosophers. It was a matter open to the remark of every mathematician even before Weierstrass, when mathematical reasoning was far less strict than it has since become. The more recent philosophers certainly show an improvement in this respect. The metaphysics of our best contemporaries lacks but little of the rank of a science; but logical criticism has also grown more searching, and you may search the whole library of modern metaphysics from Descartes

* Interesting papers on Benjamin Peirce are to be found in *Benjamin Peirce, Biographical Sketch and Bibliography*, Mathematical Association of America, (1925), Oberlin, Ohio.

to the most accurate metaphysical reasoners* of today and hardly find a vital argument of an elaborate and apodictic kind that does not leave room to drive a coach and four through it.†

10. The effect upon the minds of those who have been nourished on such food mainly, becomes deplorably patent to everybody who finds himself in contact with them. Their natural sense of logic is enfeebled and diseased. I am confident I detect something of this even in the majority of those young men who become known to me as only having paid particular attention to philosophy in the universities; although there are some whose logical instincts have been too robust to be so easily debilitated.

11. The Greek philosophers could not be persuaded that minute analysis was proper in physical science. Born Hegelian sensualists, they could not divest themselves of their belief that no worse way of getting at any comprehension of a flower could be devised than beginning by picking it to pieces, and so spoiling the flower. What was the result? Manifold have been the theories that have been successively offered, considered, and rejected, to account for the non-success of the Greeks in physics. That the vast intellect of an Aristotle, so great in zoölogy, in the science of politics, in rhetoric, in the history of philosophy; so gigantic in ethics, logic, metaphysics, and psychology, should, in physics, have sunk into abject inferiority to the cranks of modern times, the refuters of Newton, the proposers of perpetual motions, has hitherto not been adequately explained. What better account of the matter could one desire than that in physics the Hellenic element of Aristotle's nature — that Greek estheticism which forbade analysis and required that the phenomenon should be contemplated in its concreteness — here governed him? That this was the cause is shown by the fact that all the other Greeks who shared the same prejudice were equally unsuccessful; while the few who did not share it, Hipparchus, Eratosthenes, Posidonius, Ptol-

* Royce was originally specified, his name being deleted, apparently, at the suggestion of William James. A number of similar changes seem to have been made throughout this chapter.

† See Preface to vol. 6 for Peirce's views regarding "scientific" metaphysics.

emy, Archimedes, were eminently successful in the physical sciences. In zoölogy, Asclepiad Aristotle, scion of a family whose every member, from the further prehistoric times, had been trained in medicine from childhood up, shared no Hellenic repugnance to dissection.[1] Nor did that repugnance ever extend to the non-sensuous objects with which Aristotle dealt in the sciences in which he most excelled.

12. In the northern Europe of the scholastic ages, the nullity of physics was due to a different cause, explicitly set forth by Roger Bacon.* Logic and metaphysics were studied with a considerable degree of minuteness and accuracy; so that, in spite of a barbaric civilization and other unfavorable influences, sufficiently obvious, they reached an excellence which our own generation has not been able to appreciate.

13. Our physical science, whatever extravagant historicists may say, seems to have sprung up uncaused except by man's intelligence and nature's intelligibility, which never could before be operative because it was not studied minutely. But modern philosophy had no such divine birth. On the contrary, it pays the usual tax upon inheritances from revolutions. It was the product of a double and triple revolution — the Renaissance, the Reformation, the Copernican revolution. It was, from the first, committed to a vendetta against all exact thinking and all punctual discussion. It has been a derelict on a vast ocean of surmise, drifting hither and thither, driven by storms, wrecked, shattered, its pieces dispersed even to opposite poles. Jetsam only is now occasionally cast up on the beaches of solid science, fraught with no cargo of wisdom except the one sad lesson of the issues of loose thinking.

14. Broad generalization is glorious when it is the inevitable outpressed juice of painfully matured little details of knowledge; but when it is not that, it is a crude spirit inciting only broils between a hundred little dogmas, each most justly condemning all the others. It is the usual fruit of sloth. A reader who is not disposed to work upon logic as slowly, as minutely, as laboriously as he would upon any other subject

[1] "*Mem.* Before this goes to press I have to go over three books: 1. Barthélemy St. Hilaire's Ed. of Aristotle's *Historia Animal;* 2. Littré's *Hippocrates;* 3. The best German history of medicine"— marginal note.

* See the *Opus Majus*, Parts I and VI.

whatever — at the very least — simply will have to go without learning much about the theory of reasoning from any source.

15. The present volume, at any rate, is written by one who does not look upon the subject as an amusement. He will endeavor to make the doctrine present an exterior as little odious as possible, to correspond with its inward divine beauty, and to harmonize with the deep happiness the study brings. But he will not attempt to prostitute the science to the purposes of the purposeless. In order to enjoy it, it will be needful to have one's heart set on something remote from enjoyment.

16. The reader will often think that the writer makes far too much of microscopic distinctions. But in this science errors become enormously magnified in their effects; and stricter accuracy is for that reason rendered obligatory. No distinction — the reader may assure himself — will here be insisted upon without an adequate motive, even if it be a mistaken one.

17. The reader may find the matter so dry, husky, and innutritious to the spirit that he cannot believe there is any human good in it. Stuff which offers no images more enlivening than that "Socrates is a man and therefore mortal" or that of "a person who stands to some woman in the relation of a lover of every benefactor of hers," may be too desperately inhuman to be accounted by him anything but diabolic idiocy, the product of hell's bedlam. But the fault is his. It shall not be more tedious than the multiplication table, that may be promised; and as the multiplication table is worth the pains of learning, unless one is a prince, so shall this be, even if one be a prince.

§3. DIFFERENT METHODS IN LOGIC[P]

18. To the melancholy disintegration of philosophic thought due to loose reasoning and the aversion to minute analysis may be plausibly attributed the springing up of a swarm of different methods of establishing the truths of logic. A conspectus of the principal of these may usefully be given. The reasons for each can only be indicated in the most abridged form, and such criticisms may be added as can be made instructive in the present nascent stage of our inquiry.

19. First: Some writers maintain that the goodness and badness of reasonings is not merely indicated by, but is constituted and composed of the satisfaction and dissatisfaction, respectively, of a certain logical feeling, or taste, within us. This opinion is held by one of the acutest and most in fashion of the logicians of today, Christopher Sigwart, an authorized translation of whose logic is published in Swan Sonnenschein's "Library of Philosophy" in two octavos of something like three and three-fourths millions of words.* Before a man sits down to read them he will in prudence calculate how many words he is laying his plans to read during his lifetime in all, how many of these he will appropriate to logic, how many to any one of a dozen schools of logic, how many to any one author. Schröder, the leader of exact logic in Germany, the fourth volume of whose treatise we are impatient to receive,† follows Sigwart in the opinion mentioned — possibly even goes beyond him. Some readers of the English version may object that I misinterpret Sigwart, and that he holds the satisfaction of the feeling of logicality not to constitute true logicality, as I say, but merely to be the sole indication of it, in the last analysis. His phrase is "Der Glaube an das Recht dieses Gefühls ist der letzte Ankergrund aller Gewissheit überhaupt."‡ By "Recht," I here understand "rightful authority to give the decision which it claims to give"; and "Gefühl," not precisely what *feeling* means in English, but a purely subjective feeling of pleasure or pain, which renders no decision except concerning itself. The translator (with the authorization of Dr. Sigwart, who possibly does not appreciate the exact sense of the English) seems to take "Recht" in the sense of "Richtigkeit," and "Gefühl" in the vague English sense, and thus weakens the proposition. Yet even so, so long as this feeling is made the sole evidence of logicality, underlying every other, it is, to all intents and purposes, made the essence of logicality.

20. It may be remarked that nobody would care to study logic unless he had already made up his mind that men were

* *Logic* (1895), translated by H. Dendy.

† *Algebra der Logik*. There is no fourth volume. Volume 2.2, which perhaps was what was meant, however, was published posthumously in 1905.

‡ Cf. *Logik*, §3, 1.

so apt to err in their sense of logicality as often to reason ill, and unless he held the distinction between reasoning well and reasoning ill was that the former is conducive to the knowledge of the *truth*, and the latter not so, and that by truth is meant something not dependent upon how we feel or think it to be. Upon Sigwart's principle the distinction would be a mere distinction of taste, or the satisfaction of a subjective feeling. This harmonizes only too well with the practice of German university professors, whose opinions are mainly founded on subjective feeling and upon fashion. In chapter 3, §6* we shall consider the argument by which this opinion is supported; and the reader will then be led clearly to understand how, without denying the existence of the logical sense, nor its intervention in all thought, I can maintain that it is extremely fallible, and that no appeal need or ought to be made to it in establishing the truths of logic. Judged by English standards and those which the present work aims to establish, Sigwart's teaching is calculated to undermine the vigor of reasoning, by a sort of phagedenic ulceration. So it would seem *a priori;* and *a posteriori* the impression made upon me by young reasoners who have been the most diligent students of Sigwart is that of debility and helplessness in thought.

21. Second: Since it must be nearly forty years since I read *La Logique* of the Abbé Gratry, a writer of subtlety and exactitude of thought as well as of elevation of reason, my account of his doctrine may not be accurate in its details. I insert it here because after feeling it seems natural to place any proposed method of basing logical principles upon direct individual experience. Now since those principles are general, only a mystical experience could give them. I cannot now assure myself that Gratry did so base any logical principles; but I remember that he considers every act of inductive reasoning in which one passes from the finite to the infinite — particularly every inference which from observation concludes that there is in certain objects of observation a true continuity which cannot be directly observed — to be due to a direct inspiration of the Holy Spirit.† Were that granted, however, consistency would require us to grant that the admission of a

* See also 5.85ff.

† Cf., e.g., *La Logique*, Paris (1855), vol. 2, pp. 196–7.

logical principle, which covers an infinity of possible inferences, is also a mystical experience.

22. Were I merely asked to grant that the anticipations of experience involved with (more accurately than "in") inductions cannot be accounted for except by the ancient hypothesis that man has been made in the image of his Maker, so far as his Reason goes, I should be compelled to admit this, for reasons which will be developed in good time.* But Gratry's doctrine is essentially different from that. Today, while I was putting my mare into her stable, in the dusk of the evening, I noticed a black streak upon the floor, which I at first took for a shadow. But upon closer inspection (for my eyes are not as good as they once were) I saw that it was a large black snake. I experienced a certain shock strong enough to enable me to perceive what that shock consisted in, namely, in a sense that the snake was there in spite of me. Now, even if I had anticipated seeing the snake, and even if, anticipating it, I had wished to see it, still, when I did come to see it, I should have experienced something of that same sense of being compelled to see it. Such a sense of compulsion, of a struggle between something within and something without, accompanies every experience whatever. How else can I distinguish between an experience and a play of fancy of extreme vividness, than by the sense of compulsion in the former case? And how can there be compulsion without resistance? Were Gratry right, then, every inductive reasoning which passes from observation of the finite and the discrete to belief in the infinite or the continuous ought to be accompanied by the sense that that belief was forced upon me, whether I will or no. That result, however, is, I believe, contradicted by observation. Instead of experiencing any such compulsion and struggle, I feel rather a sort of sympathy with nature which makes me sure that the continuity or the generality is there, somewhat as I felt sure I understood the particular state of mind of my mare at the time I was putting her up.

23. Thirdly: The opinion just now referred to, that logical principles are known by an inward light of reason, called the "light of nature" to distinguish it from the "light of grace"

* See, e.g., 769, 1.118 and 6.487ff.

which comes by revelation, has been the opinion entertained by the majority of careful logicians.

24. The phrase "light of reason," or its near equivalent, may probably be found in every literature. The "old philosopher" of China, Lao-Tze, who lived in the sixth century B. C. says for example,[1] "Whoso useth reason's light, and turneth back, and goeth home to its enlightenment, surrendereth not his person to perdition. This is called practising the eternal." The doctrine of a light of reason seems to be inwrapped in the old Babylonian philosophy of the first chapter of Genesis, where the Godhead says, "Let us make man in our image, after our likeness." It may, no doubt, justly be said that this is only an explanation to account for the resemblances of the images of the gods to men, a difficulty which the Second Commandment meets in another way. But does not this remark simply carry the doctrine back to the days when the gods were first made in man's image? To believe in a god at all, is not that to believe that man's reason is allied to the originating principle of the universe?

25. The reasonings of the present treatise will, I expect, make it appear that the history of science, as well as other facts, prove that there is a natural light of reason; that is, that man's guesses at the course of nature are more often correct than could be otherwise accounted for, while the same facts equally prove that this light is extremely uncertain and deceptive, and consequently unfit to strengthen the principles of logic in any sensible degree.

26. But the Aristotelians, who compose the majority of the more minute logicians, appeal directly to the light of reason, or to self-evidence, as the support of the principles of logic. Grote* and other empiricists think that they have proved that Aristotle did not do this, inasmuch as he considered the first principles to owe their origin to induction from sensible experiences. No doubt, Aristotle did hold that to be the case, and held moreover, that the general in the particular was directly perceived, an extraordinarily crude opinion. But that process of induction by which he held that first principles became known, was according to Aristotle not to be recovered and

[1] *Lao-Tze's Tao-Teh-King*, Paul Carus, Chicago (1898), ch. 52, sec. 3.

* In his *Aristotle*, 2d ed. (1880), p. 259, and p. 562.

criticized. It was not even voluntary. Consequently, if Aristotle had been asked how he knew that the same proposition could not be at once true and false, he could have given no other proof of it than its self-evidence.[1] Grote and those who agree with him, as well as some other schools of thinkers, quite overlook the important distinction between thought that can be controlled and thought which cannot be controlled. It is idle to criticize the latter. You cannot criticize what you do not doubt; although very many philosophers deceive themselves and others into the belief that they are criticizing what they hardly pretend to doubt, and so "argue" for foregone conclusions. A proof or genuine argument is a mental process which is open to logical criticism. If, therefore, a philosopher holds that a judgment, C, has been derived from an antecedent judgment, B, by a process of thought subject to the mind's self-control, and that the judgment B has been derived from a still earlier state of mind, A, by a process of thought not controllable, he may represent the process by which B has yielded C by a logical form of argument or proof; but the other process, by which B has been derived from A, has to be accepted as beyond criticism; and it cannot properly be represented by an argument, though it may be closely analogous to a logical argument in other respects. The truth is, that Aristotle, like all subsequent Greeks, unless we except the Epicureans, looked upon induction as logically indefensible. Consequently, where it was sub-conscious, it was treated as infallible, while it was not admitted into a scientific argument at all.

27. The conception of an argument or inference as a process only entitled to those designations by virtue of its being a subject of logical criticism is one which it is so important to grasp at the outset that I will here advert to another error of Aristotle's reasoning concerning first principles, which is unimportant in itself, but which leads to some further considerations on the nature of argument. Namely, Aristotle* argues that there must be certain first principles of science, because every scientific demonstration reposes upon a general principle as a premiss. If this premiss be scientifically demonstrated in its

[1] Indeed this is precisely the position he takes in the *Metaphysics* Γ iii, 1005b, 19.

* In *Posterior Analytics* I, ii.

turn, that demonstration must again have been based upon a general principle as its premiss. Now there must have been a beginning of the process, and therefore a first demonstration reposing upon an indemonstrable premiss. This is an argument like the Achilles and Tortoise argument of Zeno, except that, instead of going forward in time, it goes backward.* If we were to admit that the process of thought in the mind is really composed of distinct parts, corresponding to the arguments of the logical representation of it, each requiring a distinct effort of thought, then, indeed, we should have to admit Aristotle's reasoning, unless we were prepared to admit that an endless series of distinct efforts could actually be performed in a finite time; just as we should have to admit that Achilles could never overtake the tortoise if he had to resolve to run to where the tortoise then was, and having arrived there, to form a new resolution to run to the point which the tortoise had then attained. This would involve the assumption that Achilles could not run unless he saw the tortoise ahead of him. In like manner, the assumption that the reasoning process, as it is in the mind, consists of a succession of distinct arguments, each having a previously thought premiss, involves the assumption that reasoning cannot begin with the very perceptions of sense, since in these perceptions the process of thought has not yet begun; so that they do not contain any judgments capable of being exactly represented by propositions, or assertions. If that be so, there must clearly be a first premiss. But there is no necessity for supposing that the process of thought, as it takes place in the mind, is always cut up into distinct arguments. A man goes through a process of thought. Who shall say what the nature of that process was? He cannot; for during the process he was occupied with the object about which he was thinking, not with himself nor with his motions. Had he been thinking of those things his current of thought would have been broken up, and altogether modified; for he must then have alternated from one subject of thought to another. Shall he endeavor, after the course of thought is done, to recover it by repeating it, on this occasion interrupting it, and noting what he had last in mind? Then it will be extremely

* Cf. Lewis Carroll's "What the Tortoise said to Achilles," *Mind*, N. S. vol. 4, p. 278.

likely that he will be unable to interrupt it at times when the movement of thought is considerable; he will most likely be able to do so only at times when that movement was so slowed down that, in endeavoring to tell himself what he had in mind, he loses sight of that movement altogether; especially with language at hand to represent attitudes of thought, but not movements of thought. Practically, when a man endeavors to state what the process of his thought has been, after the process has come to an end, he first asks himself to what conclusion he has come. That result he formulates in an assertion, which, we will assume, has some sort of likeness — I am inclined to think only a conventionalized one — with the attitude of his thought at the cessation of the motion. That having been ascertained, he next asks himself how he is justified in being so confident of it; and he proceeds to cast about for a sentence expressed in words which shall strike him as resembling some previous attitude of his thought, and which at the same time shall be logically related to the sentence representing his conclusion, in such a way that if the premiss-proposition be true, the conclusion-proposition necessarily or naturally would be true. That argument is a representation of the *last part* of his thought, so far as its logic goes, that is, that the conclusion would be true supposing the premiss is so. But the self-observer has absolutely no warrant whatever for assuming that that premiss represented an attitude in which thought remained stock-still, even for an instant. If that is to be ascertained at all, it must be in some other way, probably by the physiologist. At present, such physiological surmise as we can make leans, I should say, toward the hypothesis that thought was in continual mutation. At any rate, physiology is not decisively against that hypothesis. Adopting that idea, the logical argument only represents the last part of thought, for the reason that it supposes a premiss which represents some attitude of thought which can only have resulted from thinking. Now if you only break off the last part of a time, you leave a previous time. If you break off the last part of this, you still leave a previous time; and there is no possibility of your breaking off so many last pieces that from what remains no last piece can be broken off. Hence, there is no necessity for a series of arguments representing a course of thought to

have a first argument, before which there was no argument in the thought, in the only sense in which there was any argument at all, in the process of thinking. For there is no fact in our possession to forbid our supposing that the thinking-process was one continuous (though undoubtedly varied) process. At any rate, it is only the self-defence of the process that is clearly broken up into arguments. It is more than doubtful whether what we can state as an argument or inference represents any part of the thinking except in the logical relation of the truth of the premiss to the truth of the conclusion. And, moreover, the argument so considered consists in the statements in words. How nearly they represent anything really in the thought is very doubtful, and is quite immaterial. The real thinking-process presumably begins at the very percepts. But a percept cannot be represented in words, and consequently, the first part of the thinking cannot be represented by any logical form of argument. Our logical account of the matter has to start from a *perceptual fact*, or proposition resulting from thought about a percept — thinking in its own movement presumably of the same nature as that which we represent by arguments and inferences, but not so representable in consequence of a defect in that method of representation.

28. Descartes and others have endeavored to bolster up the light of reason by make-believe arguments from the "veracity of God," and the like. They had better not have pretended to call that in question which they intended to prove, since the proofs, themselves, call for the same light to make them evident. Besides, reason left to itself at least believes in its own pronouncements, while it refuses to sanction the pretensions to infallibility made in its behalf on the ground of sundry "veracities." The celebrated criterion of clearness and distinctness, proposed by Descartes,* and amended to little purpose by Leibniz,† was, as Hamilton says, "nothing new," since it was no more than an utterly unsuccessful attempt to define the old "self-evidence" of the axioms of reason.

29. The "Criterion of Inconceivability" is a touch-stone

* *Discourse on Method*, pts. 2 and 4; *Principles of Philosophy*, I, 45, 46.

† "Meditationes de Cognitione, Veritate et Ideis" ed. Gerhardt, vol. 4, p. 422; transl. by G. M. Duncan in *Philosophical Works of Leibniz*, article III. Cf. also *Discours de la Métaphysique*, sec. 24 and *Nouveaux Essais*, II, ch. 29.

proposed for ascertaining whether or not a proposition is necessarily true, consisting in trying whether or not its denial is inconceivable.* It is taken for granted that a proposition offered as necessarily true will itself be necessary, that is, it will not only be true under all circumstances [which actually do occur] but will be an assertion that something would be true under all circumstances whatsoever. Its denial, therefore, will assert merely that under some circumstances the proposition would be false. By this denial being inconceivable is meant that it is quite impossible to definitely realize in imagination a state of things in which that would be false which the denial asserts to be false. Those who believe in the criterion innocently think that in order to ascertain whether the denial is thus inconceivable, all they have to do is to try an easy experiment and so find out, at once, whether they can imagine the state of things in question, or not. But as J. S. Mill puts it, "the history of science teems with inconceivabilities which have been conquered."† What is required, therefore, is that "inconceivable" should mean not merely unrealizable in imagination today but unrealizable after indefinite training and education. It certainly must be admitted by every reasonable disputant, that every proposition whose denial is in that sense absolutely and eternally inconceivable is false.‡ This I will prove upon the spot. I say, then, that a reasonable disputant disputes because he hopes, or at least, goes upon the assumption that the dispute will come to something; that is to say, that both parties will at length find themselves forced to a common belief which will be definitive and final. For otherwise, why dispute? To reach a final and compulsory belief is, therefore, what the reasonable disputant aims at. But what he aims at is the truth. Therefore, by the truth he means nothing more than a finally compulsory belief. If, then, you can prove to him that a necessary proposition is such that there will be a final, unshakable compulsion preventing him from imagining it to be false, you have proved to him that it has those characters

* See, e.g., Herbert Spencer, *First Principles*, Appleton & Co., N.Y., 4th ed. (1882), pp. 34–36.

† Cf. *Logic*, bk. II, ch. 5, sec. 6.

‡ Though the text is unmistakable, this should be "true" and not "false." See below and 48.

which he expresses by saying that the proposition is true. Now to say that a necessary proposition is true is to say that it is necessarily true. The proposition is therefore true by definition, if by inconceivable is meant eternally inconceivable. But a definition is not a criterion. For a criterion is a method of experiment by which something is ascertained which is a sure indication of whether or not something different, and less easy otherwise to find out, is true.

30. This criterion is specially deceptive for the reason that, in ordinary cases, if we ever can do a thing, we can already at least imagine some way in which it might be done. But if we cannot already imagine any way in which a proposition would be false, neither can we yet imagine any way of imagining it to be false. But for all that, the simplest little suggestion or information may at any moment put it into our power to imagine what no effort of thought could before enable us to imagine. It is like the egg of Columbus.

I will now give a single example of the inconceivabilities conquered with which, as Mill correctly says, the history of science teems. Euclid, some twenty-two centuries ago, laid it down as a "common notion," or axiom, evident to all men, that "a whole is greater than its part." For two millennia and more, this axiom was held to fulfill the ideal of an axiom better than any other, and when men wanted an example of an indubitable axiom, they commonly chose this. It is plain, therefore, that they could not realize in thought the truth of the contrary, try as they might. This is curious; for since Euclid's time and earlier it had never ceased to be a familiar truth that a finite magnitude added to an infinite one did not increase the latter. So, if during near 2200 years, among the millions of men who were continually declaring it inconceivable that a part should be as great as a whole, it had ever occurred to a single one to think how it would be if the part were infinite, it would have been all up with the immaculate fame of the axiom from that moment. At length, in the last quarter of the nineteenth century, somebody, I think it was Dr. Georg Cantor,* said: The even numbers are a part only of

* The fact that part of all the integers can be put in one-to-one correspondence with all the integers was known long before Cantor, though not the same use was made of the information. See, e.g., Galileo's *Mathematical Discourses*, Weston's translation, p. 46; Renouvier, *Année Philosophique* (1868), p. 37.

all the integer numbers. But every whole number has a double which is the double of no other integer number. So, for every integer number there is a separate and distinct double; and thus the doubles are as many as the integer numbers. But these doubles are all even numbers; and so, the partial collection is as great as the whole collection.

31. . . . I may as well notice here a style of reasoning much used for the support of instinctive beliefs. I mean the transcendental method. . . . The method was the invention of Kant, and in his hands it consists in showing, by some ingenious argument — different in every case — that the logical analysis of the process which the mind must go through shows that the proposition which is to be defended is involved in the *a priori* conditions of the possibility of practical everyday experience. If Kant had performed all the work which a thorough, scientific application of his method demanded, he would have had to postpone the publication of his *Critic of the Pure Reason* for another century, at least, which would have been regrettable. It would be radically contrary to Kant's principles to base logic (in the sense in which the word is used in this present book) upon the transcendental method. On the contrary, his whole critic of the understanding is deliberately based upon a scientific logic supposed to be already established. It is singular that, notwithstanding the gigantic logical strength of the *Critic of the Pure Reason*, and notwithstanding Kant's explicit teaching that this hinges upon the scientific perfection of the underlying formal, or ordinary, logic, yet he never touches this last doctrine without betraying unmistakable marks of hasty, superficial study.

32. What has been said of the utter impracticability of any one man's actually executing the design of the *Critic of the Pure Reason* is a hundredfold more true of Hegel's *Logik* (which, it must be understood, little more than incidentally treats of what we call logic). That work cannot justly be regarded as anything more than a sketch, notwithstanding a depth of thought in details which I believe in more on the testimony of other readers than because I have felt it myself. Hegel is also said to pursue the transcendental method; but his method has, it appears to me, only a very general and

slight resemblance to Kant's. The striking features of Hegel's procedure are only just discernible here and there in Kant. I will first describe his method generically, without adverting to certain most characteristic special features; because in this generic sense it has been adopted by many minds who would not accept it in its special form. Hegel begins, then, by assuming whatever appears most evident to an utterly unreflecting person, and sets it down. The only difference between the unreflecting person and Hegel, as he is in this mood, is that the former would consider the subject exhausted, and would pass to something else; while Hegel insists upon harping on that string until certain inevitable difficulties are met with. Hegel at once embraces these objections with the same good faith (for it is good faith, notwithstanding his being able, if he chooses, to see further) with which he assumed the original position. He pushes his objection for all it is worth — for more than it is worth, since the original position has something to say for itself, in reply. Hegel is anxious not to allow any "foreign considerations" to intervene in the struggle which ensues — that is to say, no suggestions from a more advanced stage of philosophical development. I cannot see that it would conflict with the spirit of the general method to allow suggestions from experience, provided they are such as would be inevitable, and such as would be within the grasp of the thought which for the moment occupies the theatre.

From this dispute there will ultimately emerge a final, definitive opinion, due to a new way of conceiving the facts. So, at least, the disputants hope and trust; and so we find it. The first unreflective view, so far as it is not the mere accident of individual caprice or of a singular individual ill-luck, is allowed to have a certain legitimacy; but the new conception alone puts the matter in a scientific light. It must be acknowledged that if this method of thought can be carried out, it is the best possible. It must represent the history of thought, too, as far as that history is not merely accidental. Now, considering the immense multitude of minds, and that it is the normal minds who are influential, the history of thought can hardly be very seriously accidental. It is true that there will be, in history, perturbations, due to the fact that all minds are not at exactly the same stage of development. It would certainly be a most

valuable method of thinking which not only should bring a man to a true thought, but to that thought which ought to be dominant with his contemporaries, and will become so in the main. But will this second conception, finality though it be, from its own a point of view, a κτῆμα ἐξ ἀεί, as my friend Dr. Paul Carus* says, remain a fixity without growth or change, such as the eternal hills are not so devoid of life as to be? No, indeed, says Hegel, and every man will echo his answer. It is of the nature of thought to grow; and now, not caring to adhere strictly to Hegel's text (as nobody animated by his spirit does, any longer), but guided rather by the history of thought, I ought to indicate in such rough way as I am prepared to do, some of the directions that growth will take.

33. The new conception will not be content to be restricted to the particular phenomena it was devised to explain: it will insist upon applying itself to analogous phenomena, and to others analogous to these again, without stint. For that purpose it must be widened and probably simplified and rendered more agreeable to reason. It will not be content with explaining the history of thought, but will aim to explain history in general. It will not be content with accounting for man, but will wish to grasp all the forms in the universe, which is greater than man. It will not be content with an accidental universe, but will wish to assimilate every possible universe that the mathematician can suggest. It will not be content with allowing to the unreflective view a sort of subordinate legitimacy, but will insist upon elevating it to a truth in full harmony with its own.

34. These tendencies are irrepressible: in the long run they will cause that which they need to come into being. But much more than that, they are thoroughly reasonable; and that which they call for ought to be. Now that which they demand above all is the fact and the admission that the world is reasonable — reasonably susceptible to becoming reasonable, for that is what it is, and all that it is, to be reasonable†— or in other words, that man is made after his maker's image.

35. Such is my rude notion of what the method is to which

* 1852–1919. Editor of the *Open Court* and the *Monist*.

† And the highest of all possible aims is to further concrete reasonableness. See 1.602, 1.615, 5.121, 5.433.

Hegel endeavors to impart exactitude. Vague applications of it recommend themselves to my faith; but I have never met with an attempt to state a transcendental argument with precision which began to convince me. At any rate, when I reexamine the logics of more or less Hegelian tendency which have appeared in the last quarter of a century, I must decline to allow any weight to such flummery. I do not mention earlier German works, because they are still worse.

36. Fourthly: Some writers make it the boast of their systems of logic that they rest upon a philosophical basis; others really use the same method, though they rather keep the fact in the back-ground, despite the good ring of saying that one's logic is philosophical. Only, if logic is to be a pavillion on the roof of metaphysics, then metaphysics cannot conveniently be made an upper story of logic, as Aristotle and Kant, the two greatest of metaphysical systematizers, would have it to be. To me, it seems that a metaphysics not founded on the science of logic is of all branches of scientific inquiry the most shaky and insecure, and altogether unfit for the support of so important a subject as logic, which is, in its turn, to be used as the support of the exactest sciences in their deepest and nicest questions.*

37. Some of the most celebrated logics, however, are written from the points of view of metaphysical sects. The *Organon* of Aristotle, the title which the collection of his logical treatises received, probably at the hands of Andronicus, the Rhodian,† who put them together about 60 B.C.[1] opens with a metaphysical book, the Categories, or Predicaments, although in that same treatise Aristotle evidently bases the metaphysics upon a grammatico-logical analysis of the Greek sentence. To this book was usually prefixed the treatise of Porphyry on the

* See 1.487, 1.624, 1.625.

† The 11th Scholarch of the Peripatetics, circa 70 B.C.

[1] "Organon" was first the name of the science, given to it by the early Peripatetics, because logic did not satisfy Aristotle's definitions either of Science or of Art. I fully accept the usual story about Aristotle's writings lying some centuries *perdus* until they were rescued and put in order first by Apellicon [circa 90 B.C., a famous book collector of Athens] and later by Andronicus. I consider the rejection of this story and consequent partial refusal to admit the authenticity of Aristotle's works as we have them, one of the extravagancies of "higher criticism."

Predicables. About half the scholastic works on logic are commentaries on the collection of books so formed. These works, therefore, base logic on metaphysics to some extent.

38. Of logics which in modern times more or less take for granted special systems of metaphysics, the earliest were a series of Aristotelian treatises. Gassendi* wrote an Epicurean logic, but of course knew nothing of the book of Philodemus on induction which turned up, centuries later, in Herculaneum.† The celebrated Port Royal logic entitled *L'Art de Penser*, of Antoine Arnauld (published in 1662) and Clauberg's *Logica Vetus et Nova* [1654], were Cartesian works. Malebranche wrote his own *Recherche de la Vérité* (1674–5) which likewise professed to be Cartesian, but was in truth rather Malebranchian, like much else called Cartesian. The *Medicina Mentis* of Graf von Tschirnhaus auf Kesselwalde [1687], today a familiar name to algebraists, stole the thunder of Spinoza. Locke's philosophy was represented in the treatises of Crousaz‡ Isaac Watts,§ and perhaps we may add of Condillac.¶ Leibnitzianism, systematized by Wolff, numbered its logics by scores. Kantianism had its Krug|| and Esser,** to mention only those of whom English readers are likely to know something; and every subsequent German philosopher, Baader, Fichte, Schelling, Hegel, Schleiermacher, Schopenhauer, von Hartmann, Dühring, down to Häckel, has been followed by his train of logicians.†† Thus, a large proportion of all the logics that have ever been written have more or less pursued this vicious order of thought.

39. Fifthly: Considerable controversy has taken place as to whether scientific results of psychology ought or ought not to be admitted among the premisses from which logical principles are to be deduced. Upon the affirmative side are the

* *Syntagma philosophicum*, pt. I, (1658).

† See T. Gomperz, *Philodemi de ira Libra*, (1864).

‡ *La Logique*, (1712).

§ *Logick*, (1724).

¶ *La Logique*, (1805) and *Langue des Calculs*, (1798).

|| *Logik oder Denklehre*, (1806).

** *System der Logik* (1823).

†† E.g., F. Hoffmann was a follower of Baader; Reinhold, Forberg, Schad were followers of Fichte; while Klein and Troxler were followers of Schelling.

Wolffian logicians* and many others, older and newer. J. S. Mill (*Examination of Hamilton* xx. 5th ed. p. 461) pronounces that "its [logic's] theoretic grounds are wholly borrowed from Psychology." A smaller but more powerful force is arrayed against the method. Kant holds that psychology has no influence upon logic. But then we are to remember that, according to Kant, nothing worth mention had been contributed to logic since Aristotle.† Herbart, however, had studied the subject to the best of his high powers; and his declaration is, "In der Logik ist es nothwendig alles Psychologische zu ignorieren." Among the thoroughly unpsychological works of later date is to be mentioned Riehl's *Philosophische Kriticismus*, 2 vols., 1876–9. Venn,‡ though a follower of Mill, whose logic is representative of associationalism, may be classed among those who do not make use of scientific results of psychology. Nobody will do injustice to the present treatise by describing its position as extremely unfavorable to the use of psychology in logic.

40. Of the arguments upon the affirmative side, none is stronger than the following. It is said that not to utilize in logic the results which psychologists have obtained concerning abstraction and concerning association would be a wasteful proceeding. It is also maintained that precepts setting forth how we *ought* to reason can have no rational basis except the science of the laws which describe how we *must* reason; and that, in fact, if this latter part of logic, which is, at the same time, a part of psychology, be dropped out, the purely normative part, which remains, is nothing but an auxilliary practical art.

41. The general answer to all this is that it only illustrates the dire confusion brought into both sciences when they are not so understood as to be widely distinct from one another. The logicians of the period from Descartes to Kant cannot be much blamed for seeing little distinction between psychology and logic, inasmuch as the psychology of their days, whether rational or empirical, consisted in little else than a logical analysis of the products of thought, as every psychologist of our day will admit. Even the pupils of James Mill, for all the

* E.g. G. B. Bülffinger, L. P. Thümming.

† *Critique of Pure Reason.* B, viii.

‡ *Empirical Logic,* (1889).

power of his extraordinary *Analysis of the Human Mind* [1829] were far from understanding psychology as it is now understood.

42. Modern psychology has made an admirable beginning. Every student of exact science who is in a situation to judge of what has been accomplished in this department must applaud it with his whole heart. Yet that it is only a beginning is shown by its present tendency to turn upon its axis, without making any great advance. Matters of brain-physiology and matters of consciousness elbow one another in unsympathetic juxtaposition, in a way which can only be transitional, and is a sign for us, as well as we can look forward to conceptions not yet attained, that psychologists do not yet understand what mind is, nor what it does. I am not at all prepared to clear the matter up; but I dimly discern, I think, that the physiological view has not sufficiently affected the introspective aspect; and possibly the converse is true, also. Perhaps the introspection is mainly illusion due to quasi-logical interpretations.*

43. But let me not be understood as having much faith in the current Wundtian monism. At any rate, however, it is pretty clear that there must be an application of scientific logic in order to separate the precipitate of physiology from the filtrate of logic. In this state of things, or in the real state of things which I am perhaps very mistakenly endeavoring to peer into, it ought, I think, to be admitted that psychological conceptions are not sufficiently matured to afford a safe foundation for any part of logic; and it seems still more evident that it is very much in the interest of psychology itself to restrain it from flowing over into the region of logic.

44. As for abstraction, I shall endeavor, in the course of this volume, to put before the reader reasons for thinking that it is a wholly unpsychological matter, the doctrine of which has gone far astray in consequence of the admixture of psychology with it.†

45. Something like psychological association certainly appears in logic; but in order that the relation of logical association to psychological association, a relation more interesting

* See 5.244ff.

† See 227, 364, 422 and 428.

to the psychologist than to the logician, may become clearly understood, it is desirable that the two theories should be developed separately and side by side.*

46. The notion that a normative science is necessarily of the nature of a practical art, in having no independent value as a pure theory, is one which, no doubt, arises naturally enough from a superficial survey. But in another chapter† we shall have occasion to inquire somewhat closely into the nature of the different sciences; and we shall then find that, so far is it from being true that the normative character must necessarily be exclusively due to the branch of knowledge that possesses it being a mere concrete application to a practical need of a theory which, in its pure development, never considered that need, that, on the contrary, this character may equally have its origin in the circumstance that the science which presents it is so very abstract, so alien to any experiential lineage, that ideals alone, in place of positive facts of experience, can be its proper objects.

47. It is J. S. Mill‡ who insists that how we *ought* to think can be ascertained in no other way than by reflection upon those psychological laws which teach us how we *must needs* think. But here we have to distinguish the case in which the compulsion attaches to that subconscious thought over which we have no control, and the case in which it attaches to conscious reasoning. In the former case, there is no room for logical criticism, at all. But because there is nothing to be said against our thinking in a certain way, in subconscious thought, when we cannot do otherwise, it does not, at all, follow that we ought to think in that way when we have our choice between several ways of thinking. If, however, Mill refers to a compulsion attaching to conscious thought, what he no doubt has in mind is, that a person ought to think in the way in which he *would be* compelled to think, if he duly reflected, and made his thoughts clear, and brought his whole knowledge to bear. But when he asserts that in such a case there is *no other reason* to be given for thinking in a given way than barely that the thinker is under compulsion so to think,

* Association is treated in detail in vol. 8.

† Vol. 1, bk. II, ch. 2.

‡ In his *Examination of Hamilton*, ch. XXI.

is he not applying that Criterion of Inconceivability against which we have heard him fulminate in his finest style? It is true that Mill does not say that there is no other reason in support of the conclusion, but only that there is no other reason why the reasoner ought to accept the conclusion. But this makes no pertinent difference; the arguments against the Criterion apply in this case.

48. As before, we are to distinguish between an absolute definitive compulsion of thought and a limited compulsion. To say that the reasoner would, if he reflected sufficiently, be under an absolute definitive compulsion to hold a certain opinion is, as was shown in the discussion of the Criterion, neither more nor less than to assert that the opinion is true. Now to say that a reasoner ought to believe something *for no other reason* than that it is true, is to say that there is no reason at all why he ought to do so. Besides, it is nonsense to say that the science of psychology establishes such a proposition, unless the subject of the belief happens to be the operations of the mind.

49. Nor can it be maintained that there is a distinct logic for reasonings whose conclusions are psychological. The evident truth is, that psychology never does prove a compulsion of thought of an absolute definitive kind for conscious operations of the mind. It establishes only associational compulsions; and with conquests over these Mr. Mill tells us the history of science teems.

50. Let us, however, come a little closer to the concrete. I do not remember any treatise on logic which tells the reader that if Sortes is a man, and all men are mortal, then Sortes *ought to be thought* mortal. Mill's *Logic* certainly says no such thing. What they all say is that Sortes must *be* mortal. Logical treatises never say anything about what "ought to be thought" as long as there is any compulsion of thought or reflection. In those cases they only speak of how the facts are. It is where there is no such compulsion that the "ought" finds room. Thus, we are told that we ought to try simple hypotheses before complex ones. But how such a maxim can be supported upon associational principles alone — which is what Mill must mean by laws showing how we must think, which have been discovered by psychology — I confess I do not see;

nor do I find anything in Mill's *System of Logic* to help me to see it.

51. Psychology must depend in its beginnings upon logic, in order to be psychology and to avoid being largely logical analysis. If then logic is to depend upon psychology in its turn, the two sciences, left without any support whatever, are liable to roll in one slough of error and confusion.

52. Sixthly: Other authors, indeed, a large majority of logicians, without citing results of scientific psychology in support of the principles of logic, yet incessantly refer to *data* of psychology — or to what would ordinarily be so considered, apparent self-observations that we think so and so — as showing what the truths of logic are. All this is beside the purpose. Logic is not the science of how we *do* think; but, in such sense as it can be said to deal with thinking at all, it only determines how we *ought* to think; nor how we ought to think in conformity with usage, but how we ought to think in order to think what is *true*. That a premiss should be pertinent to such a conclusion, it is requisite that it should relate, not to how we think, but to the necessary connections of different sorts of *fact*.

53. Herbart made a statement very nearly correct when he said that a *thought* (Begriff), in the sense in which alone logic deals with thoughts, is not a thinking but that which a thinking brings before the mind. The emphasis here is in the denial contained in the first clause. The rest plays no part in his treatise; and my only objection to his statement is that what the logician calls a thought, if he uses that mode of expression, cannot with pertinence signify even what is brought before the mind in thinking. If it be so brought before the mind (which seems improbable), it is of no concern to the logician.

54. Logic deals with the relations of knowledge and with arguments, or inferences. It may be that knowledge cannot be realized without somebody's thinking something corresponding to it, in the same sense in which color cannot be realized unless somebody sees it. "Full many a flower is born to blush unseen," seems to me to be true, nevertheless — decidedly more accurate than, "Full many a flower is of such a nature that it would blush, if light were irregularly reflected from it to my eye; but it never does blush, because I am not at this moment looking at it, or if I am, sufficient light is not

shining upon it." On the same principle, I think it more accurate to say that there are stores of knowledge in a library than to say that there are books there which, if I were reading, having become acquainted with the languages they are printed in, would be conveying knowledge to my mind. Surely, knowledge is decidedly more objective than color. This, however, is, for our present purpose, a secondary point. What is more important is that it certainly seems that an inference cannot have any life unless somebody performs a thinking process. Still, the inference, or argument, so far as logic can take any cognizance of it, may be, not to say probably is, of an entirely different construction from the thinking process, as I have already pointed out,* in considering Aristotle's view that there must be first principles of science.

55. If the reader will look at any theorem of Euclid (it had better be from the first book, which alone is drawn up with great logical care), he will notice that the proposition is first enunciated in abstract terms. This is the form of embodiment of it which is suitable for storage. But having so stated it, Euclid constructs a figure, and reënunciates the proposition with reference to that figure. Since this figure may be *any* figure conforming to the conditions of the abstract statement, the new enunciation is precisely equivalent to the original one, in meaning. The difference is, that from the abstract enunciation, left abstract, no inferences could be drawn, except such trivial syllogistic ones as the traditional treatises on logic alone consider, together with certain inferences of the same trivial character which the logic of relatives sets forth. In order to reason to any good effect from or to the abstract enunciation, it is necessary to bring it into this second shape. The new enunciation, the *exposition*, as it is termed, having been obtained, certain changes are made in the figure, consisting either in moving certain parts of it, or in adding new lines, or both, whereby we are led to perceive that those same relations hold good which are expressed in the second enunciation. This pointed out in the words "which had to be demonstrated," Euclid breaks off, without having actually reached the abstract statement. But had anybody asked him what he had proved, he would have repeated that abstract enunciation. In like

* In 27.

manner when we have finished a process of thinking, and come to the logical criticism of it, the first question we ask ourselves is "What did I conclude?" To that we answer with some *form of words,* probably. Yet we had probably not been thinking in any such form — certainly not, if our thought amounted to anything. Our whole logical criticism consists in investigating whether or not to one portion of knowledge, expressed presumably in a very different form from that in which it was thought, we can, without serious danger of error, attach a certain addition. What the process of thinking may have been has nothing to do with this question. There may, for aught we know or care, be a hundred ways of thinking in passing from such a premiss to such a conclusion. But the question is, whether, granting that there be such a thing as truth, which can be ascertained at all, such a way of adding conclusion to premiss, as that under examination, would lead to the ascertainment of the truth by the speediest path, or not. The whole logical inquiry relates to the *truth;* now the very idea of truth is that it is quite independent of what you or I may think it to be. How we think, therefore, is utterly irrelevant to logical inquiry.

56. I must be excused for dwelling on this point, for no other in all logic, although it is a science of subtleties, is so hard to see. The confusion is embedded in language, leaving no words available to epigrammatize the error. Now it is not of fools exclusively, but of the greater part of the thinking world that words are the money. A celebrated treatise [by Hobbes] is entitled *Logic, or Computation,* and although not all reasoning is computation, it is certainly true that numerical computation is reasoning. But calculating machines are in everyday use; and Babbage's analytical engine would perform considerable feats in mathematics. Other logical machines have been constructed.* All those instruments perform inferences; and those inferences are subject to the rules of logic. If from true premisses they always yield true conclusions, what more could be desired? Yet those machines have no souls that

* Peirce seems to have considered the construction of a logical machine. He cut out and arranged a number of overlapping papers to represent specific arguments; but it does not appear that he ever completed it or devised a key. In vol. 8 Peirce gives a detailed criticism of logical machines.

we know of. They do not appear to think, at all, in any psychical sense; and even if we should discover that they do so, it would be a fact altogether without bearing upon the logical correctness of their operations, which we should still have to assure ourselves of in the same way we do now.

57. The idea I am endeavoring to convey is difficult to seize. It cannot be seized by a reader who, instead of trying to seize it, puts himself into a resolutely hostile posture of mind toward it. Does he wish to know what I mean? If so, let him postpone criticism until he clearly apprehends what it is that is to be criticized. There are many minds who will be so occupied with a certain objection that it will quite eclipse the sense of what I have been saying. They will urge that those machines do not perform any inference, at all; that it is, on the contrary, we, who, knowing how they are constructed (be it in detail or in a general way, by testimony), infer that the number which is exhibited at the end of the process must be in a certain arithmetical relation to the numbers which determined the setting and working of the machine.

58. It will further be urged that if those machines were to be regarded as reasoning, there are others which would reason in far higher ways. For the calculating machines only execute variations upon $1+1=2$, while there are machines which may, with as much justice, be said to resolve problems before which generations of able mathematicians have fallen back, repulsed. Such, for example, are the solids of different shapes which yacht-designers drag through the water, and thereby come to the knowledge of arcana of hydrodynamics. Blocks of wood should seem, then, on my principles to be better reasoners than the brains of Gauss and Stokes. And why stop here? Any apparatus whatever used for experimentation would be, on the same principle, a logical machine. A steam-engine would be working out, at every revolution, its problem in thermodynamics; a simple match, scratched on a box, a question that we are unequal to so much as the formulating of.

59. This sounds crushing. What have I to say to it all? Simply that it is absolutely just. A logical machine differs from any other machine merely in working upon an excessively simple principle which is applied in a manifold and complex way, instead of upon an occult principle applied in a monoto-

nous way. If anybody wishes me to acknowledge that a logical machine reasons no more than any other machine, I do not know why I should not gratify him. That seems to me a matter of words. The result which the logical machine turns out has a relation to the data with which it was fed, which relation may be considered from the point of view of whether the former could be false so long as the latter are true. That is all there is in the facts of the case; and whether it is called reasoning or not I do not care. All that I insist upon is, that, in like manner, a man may be regarded as a machine which turns out, let us say, a written sentence expressing a conclusion, the man-machine having been fed with a written statement of fact, as premiss. Since this performance is no more than a machine might go through, it has no essential relation to the circumstance that the machine happens to work by geared wheels, while a man happens to work by an ill-understood arrangement of brain-cells; and if there be room for less, still less to the circumstance that a man thinks. Say, if you like, that thinking has everything to do with the life of reasoning; I still insist that it has nothing to do with the logical criticism, which is equally applicable to the machine's performances and to the man's. This is simply the question of whether or not the conclusion can be false while the premiss is true. Were it a question of whether man *can* reason ill, it might be well to examine the process and mechanism of his thinking. But there is no question that he often does reason ill; and that is the reason why we criticize reasoning and why we inquire whether or not a given way of proceeding from premiss to conclusion is conducive to the ascertainment of truth or the reverse.

60. Seventhly: Many of the works which come under the last two heads are more particularly distinguished by regarding logic either as consisting in, or as deduced from, a branch of knowledge which the Germans call *Erkenntnisslehre*, the doctrine of cognition, or from a closely allied science which they call *Wissenschaftslehre*, the doctrine of science, or *epistemology*. . . .

61. Wundt at the conclusion of his *Logic* (for that work does, at last, come to an end) says that, in "the true sense of the word," *Wissenschaftslehre* takes as the proper subject of its investigations the methods and results of the special

sciences, with a view to the formation of a metaphysics (*Weltanschauung*). Perhaps the clearest explanation is given by Benno Erdmann in the following words:* "The science whose subject is the common postulate underlying all sciences, including itself, may be designated by an expression coined by Fichte, *Wissenschaftslehre*. It is contra-distinguished from all other sciences by their being special while it is universal. Its problem may be traced back to the demand of Socrates for conceptual knowledge, or what Aristotle termed ὁρίζεσθαι καθόλου. It arose when the 'ideal of knowledge' awoke."

62. The word *Erkenntnisstheorie* has no such vagueness. It is the investigation of the sense in which knowledge is possible. It is often called the theory of cognition; but I do not see why we should not retain Kant's term, *critic*, especially as he borrowed it from the English;† for in our language this word has been in use since Hobbes and earlier, for the science of criticism, and was admitted by Johnson into his dictionary. The peculiar turn of meaning given to it by Kant, which makes it the critic of knowledge, or, as he would have said, the critic of the cognitive faculties, is quite admissible. Besides, the immense importance of Kant's work upon this problem imposes upon us the duty of accepting his word, as long as it is so far from being a bad one. . . .

63. Surely logic must begin with a critic of knowledge. That cannot be denied; for what prudent man would embark on any enterprise without first considering whether and how it could possibly succeed? Nor can it be denied that the theory of cognition is today one of the pearls of scientific psychology. But I contend that that propedeutic that is wanted for logic has no more to do with the psychological theory of cognition than logic itself is concerned with the psychical process of thinking. Even less were there room for less; since the psychological theory substantially ends where consciously controlled thought begins, with which alone logic has even an indirect connection.

64. It is true that students of *Erkenntnisslehre* say emphatically that it is not psychology. Some call it psychologico-metaphysical. Others deny it has any psychology in it. Benno

* *Logik*, (1892), Bd. I, S.9.

† Cf. 205.

Erdmann, himself a psychologist, formulates the problem of *Erkenntnisslehre* as follows: "What right have we to assume that ideas refer to objects out of the mind?" (*Mit welchen Rechte nehmen wir an, dass sich das Vorgestellte überhaupt auf ein Transscendentes bezieht?*)* Certainly psychology has no dealings with objects out of the mind. Still less has logic. Whether or not there is, at all, any such thing as Reality, the logician need not decide. He cannot hide from himself, any more than another man can, that objects very nearly like real things there are; and he cannot pretend to doubt it. But he sees, perhaps more clearly than other men, that approximation to reality and absolute reality itself are two different things. The mathematicians' *i*, of which the square is negative unity, *approximates* to reality. All that it is incumbent upon the logician to learn is what inferential habits are conducive to knowledge, and to positive knowledge, in case there be any reality of which it is possible to have positive knowledge, and are conducive to such semblance of positive knowledge as we can have, in case there is no perfect reality or in case otherwise true positive knowledge is impossible. But in order to solve even that problem, he has first to ascertain, in case there be any successful quest for knowledge, what the nature of knowledge would be; and for his purpose, knowledge may be something written down in a book.

65. The science which Berkeley, Kant, and others have developed, and which goes by the name of the theory of cognition, is an experiential, or positive science. It learns and teaches that certain things exist. It even makes special observations. But the experiential element in logic is all but nil. No doubt it is an observational science, in some sense; every science is that. Even pure mathematics observes its diagrams. But logic contents itself almost entirely, like mathematics, with considering what would be the case in hypothetical states of things. Unlike the special sciences, it is not obliged to resort to experience for the support of the laws it discovers and enunciates, for the reason that those laws are merely conditional, not categorical. The normative character of the science consists, precisely, in that condition attached to its laws. The only

* *Ibid.*, S.10.

purpose for which it is obliged to resort to experience is to establish a few facts, without which there could be no motive for its inquiries; and these facts are so extremely universal and atmospheric that no little acumen is required to make sure that they are anything more than empty formulae or at most hypotheses.

66. Logic is obliged to suppose (it need not assert) that there is knowledge embodied in some form, and that there is inference, in the sense that one embodiment of knowledge affects another. It is not obliged even so much as to *suppose* that there is consciousness. Descartes was of opinion that animals were unconscious automata. He might as well have thought that all men but himself were unconscious. To suppose them so does not annul the rules of logic. It still remains true that such and such a habit of determining one virtual store of knowledge by another will result in the concentration of actions so as to bring about definite ends. The essence of rationality lies in the fact that the rational being *will* act so as to attain certain ends. Prevent his doing so in one way, and he will act in some utterly different way which will produce the same result. Rationality is being governed by final causes. Consciousness, the feeling of the passing instant, has, as such, no room for rationality. The notion that logic is in any way concerned with it is a fallacy closely allied to hedonism in ethics.*

67. Eighthly: One needs but to turn over the leaves of a few of the first logical treatises that come to hand, especially if they are English, nor will he have to search long, in order to meet with appeals to the ordinary usages of speech as determinative of logical doctrines. Some recent books are quite crowded with this type of argument. It seems usually to be employed unreflectively; but there are works in which it is deliberately laid down as the principal basis of logical science.

68. The greater number of those who regard this method as very efficient would seem to have in mind an extremely small group of closely similar and highly peculiar languages — say Latin and a few literary languages of modern Europe. For some of the necessities of thought which they profess to deduce in this way are violated by the Greek, the language

* See 1.333.

nearest, in every sense, to these I have mentioned. Even if they took into account the entire group of Aryan and Shemitic languages — although within that group they would find modes of thought that would somewhat embarrass them, such as the usage of the Gaelic and Old Irish of putting the subject of a sentence in the genitive — yet this would be like judging of botanical possibilities by phanerogamous plants, so small and so peculiar is that group as compared with the great world of languages, the negative resemblance between which may be very roughly described by saying that they make no use of abstract ideas, but which differ as much from one another in their ways of thinking as they do from the inflected languages.

69. I must acknowledge that in opposition to this we find that there are a number of eminent linguists who maintain this method in its extremest possible form. But I am obliged to say that the most distinguished of these, Steinthal and Sayce, are men of great genius but not distinguished for the caution of their positions, even within their own specialties. When Sayce says that "had Aristotle been a Mexican, his system of logic would have assumed a wholly different form,"* I am willing to admit that there is a good deal of truth in that. It is lucky that Aristotle's only language was one that led him into as few errors as did the Greek. But so far am I from finding in this remark any encouragement to trust to the indications of language as evidence of logical necessity, that it seems to me to go quite the other way. The mere name of Steinthal no doubt overawes many, especially among those who have not read his principal books, at least, not critically. Moreover, it must not be forgotten that it is the usual practice of these Europeans who write grammars of non-Aryan languages, violently to adapt them to the Procrustean bed of Latin grammar. Even if one fully knows how false a representation this gives, it is hard, in writing a grammar, to resist the temptation to make use of brief familiar phrases which are, after all, as nearly right as any idea one can convey without much trouble and labor both to the reader and to oneself. For that reason, it will not suffice to get one's idea of an uninflected language from any mere grammar. It is necessary to have some real, living acquaintance with it, in order to appreciate its modes

* A. H. Sayce, *Introduction to the Science of Language* (1880), vol. 2, p. 329.

of thought, especially since these will be most difficult for us to grasp.

70. It seems to me, then, that appeals to language can serve no other purpose than as most inadequate and deceptive evidence of psychological necessities or tendencies; and these psychological necessities and tendencies, after they are ascertained, are utterly useless for the investigation of logical questions.

71. Ninthly: A well-known logician,* whose contributions to one part of the science are valuable, but whose treatise, as a whole, represents the agnostic disease at its crisis, adopts as his definition of good and bad reasoning its tendency toward and against the stability of the existing order of society. This method is open to most of the objections that have been urged against the foregoing eight, as well as to some others.

72. Tenthly: A few of the medieval logicians modified the principles of logic professedly so as to make them square with the authority of the church. This procedure should not be confounded with the passion of those ardent souls who delight to believe, at the dictates of their religion, all that they know to be most degradingly absurd. This method is not that, but consists in arguing that what seems to be absurd cannot really be so, since the church declares it to be true.

73. It may be the part of good sense for a person to employ a method of reasoning that he or she does not understand — say, for example, the method of least squares — on the recommendation of a respectable teacher; only it is not a scientific proceeding. Authority, from the nature of things, cannot *advance* knowledge.

74. Eleventhly: Appeal is often made by logical writers to the history of science as supporting their views of right reasoning. Whewell placed his *Novum Organum Renovatum* [1858], a work of high logical value, almost exclusively upon the basis of this method. As a source of secondary evidence upon such general doctrines of logic as may need such help, and even of primary evidence as to the value of special modes of reasoning, this method has been (it would be disgraceful to deny it) occasionally of no little service. In making use of it, it will be necessary, of course, to take good care that the history is open to no doubt. But there are great numbers of facts of

* Karl Pearson, in his *Grammar of Science* (1893), pp. 67–74.

scientific history about which it is as impossible for any sane man to entertain any real doubt (unless, indeed, we include German "higher critics" among that number) as about any item of the multiplication table. We also have to ask whether the facts are sufficiently numerous to lend any great certainty to an induction. That depends on what facts they are, as well as on how broad the induction may be that one thinks to rest upon them. It must be admitted that, as things are at present, certain special kinds of reasoning which rest mainly upon scientific experience are open to the more danger, in that those who employ them will most likely quite forget how unsafe they really are.

75. Twelfthly: Everyday experience, such as presses in upon every man, at every hour of his life, is open to no other doubt than that it may not have been correctly formulated in general terms. This must be the main source of what little matter of fact logic has occasion to assert. There still live men who talk of experience not yielding absolute certainty, absolute universality, absolute necessity, absolute precision. No; but there is nothing at all in our knowledge which we have any warrant at all for regarding as absolute in any particular. Absolute infallibility may belong to the pope and the ecumenical councils: it is outside my province to discuss that question. But I am quite confident it does not belong to the multiplication table. If I must make any exception, let it be that the assertion that every assertion but this is fallible, is the only one that is absolutely infallible. But though nothing else is *absolutely* infallible, many propositions are *practically* infallible; such as the dicta of conscience. As for those things which are known by everyday experience, let him doubt them who can lay his hand on his heart and say that he does doubt them. For the rest of us, it would be mendacity to say that our degree of assurance of them is unsatisfactory.

76. Thirteenthly: The chief source of logical truth, though never recognized by logicians, always has been and always must be the same as the source of mathematical truth. This was well shown in a little book entitled *Logische Studien* by the historian of materialism, Friedrich Albert Lange, published in 1877, having been unfortunately left unfinished at the author's death in 1875. I have found few books on logic so

instructive. I do not mean that the reader will find Lange's views exactly reproduced in this work, by any means, upon a single topic. But it has influenced me considerably, and I can recommend it as one of the very few works on logic that I have found too short.

77. What then is the source of mathematical truth? For that has been one of the most vexed of questions. I intend to devote an early chapter of this book to it.* I will merely state here that my conclusion agrees substantially with Lange's, that mathematical truth is derived from observation of creations of our own visual imagination, which we may set down on paper in form of diagrams. When it comes to logical truth, I do not think the intuition is quite what Lange describes it. He holds that we imagine something like an Euler's diagram; but I do not think that necessary. There are other ways, as I shall show, among which we may take our choice.† Lange holds up as a model Aristotle's proof of the conversion of the universal negative proposition. As well as I can translate Aristotle's untranslatable language, his proof reads as follows:

"If to none of the B's the (designation) A belongs; neither will the (designation) B belong to any of the A's. For if to any, as, for example, to C, it will not be true that to none of the B's the (designation) A belongs. For C is one of the B's."‡

78. It seems to me it would be much simpler to say that if No B is A, but some A is B, then we should have the two premisses of a syllogism in *Ferio*, from which we could conclude: Some A is not A, which is absurd. Hence, if No B is A, it cannot be true that some A is B; that is, it must be true that No A is B. The syllogistic form *Ferio* is

No B is A,
Some C is B,
∴ Some C is not A.

If for C we put A, we get the premisses above used. Now in my view, observation comes in to assure us that when A is substituted for C we do get these premisses, and it also enters in other similar ways.

* Ch. 3 of the "Minute Logic" published in vol. 4, bk. I, as No. 7.

† See vol. 4, bk. II.

‡ *Prior Analytics*, 1, 2*a*.

Aristotle seems to reason that if we were to assume that some A is B, supposing C to represent an A that is B, we should be obliged to conclude

C is A,
C is B,
∴ Some B is A.

There are reasons why I think this somewhat objectionable; but it seems to me to depend upon the same kind of observation.

CHAPTER 2

*PARTIAL SYNOPSIS OF A PROPOSED WORK IN LOGIC**

§1. ORIGINALITY, OBSISTENCE, AND TRANSUASION

79. This chapter's main use is to give the reader an idea of what sort of book this is to be.† One can see that its conceptions are unusual. We find ourselves in the vestibule of the labyrinth. Yes, The Labyrinth — in the Vestibule only, but yet in that tremendous, only Labyrinth. Thirteen doors, not yet opened, are before us. We choose the narrowest, the least prominent, the seldomest opened of any. . . .

80. That a reader should deliberately seek instruction from a treatise on logic is a proof that he has already made certain observations and reflections, and has acquired certain conceptions. I propose, at the outset, to invite the reader to give one more reconsideration, perhaps a little more deliberate than he has hitherto given, to these Pre-logical Ideas, in order to see how far they are well grounded, and in order to develope them perhaps a little more and penetrate to their real significance, as far as this can be done at this stage of the inquiry.

81. Some mathematicians, eminent for their success in their science, and who have particularly attended to the philosophy of it, regard Mathematics as a branch of Logic.‡ This is the more worthy of notice because it might with much justice be contended that mathematics is almost, if not quite, the only science which stands in need of no aid from a science of logic. Moreover, according to the opinion defended in the present treatise, logical truth is grounded upon a sort of observation of the same kind as that upon which mathematics is grounded. For these reasons, it is desirable at once to examine the nature

* The remainder of ch. 1 of the "Minute Logic."

† Not only was this book never completed, but many of the proposed discussions here outlined were never begun.

‡ E.g. Dedekind and Whitehead.

of the mathematician's procedure pretty thoroughly. I have reason to be confident that this study will be of aid to some of those who have no natural turn for mathematics. At the same time, I am bound to say that mathematics requires a certain vigor of thought, the power of concentration of attention, so as to hold before the mind a highly complex image, and keep it steady enough to be observed; and though training can do wonders in a short time in enhancing this vigor, still it will not make a powerful thinker out of a naturally feeble mind, or one that has been utterly debilitated by intellectual sloth.

82. There is another normative science which has a vital connection with logic, which has been strangely overlooked by almost all logicians. I mean Ethics. It is not necessary to be an acute reasoner in order to develop the truest moral conceptions; but I do aver, and will prove beyond dispute, that in order to reason well, except in a mere mathematical way, it is absolutely necessary to possess, not merely such virtues as intellectual honesty and sincerity and a real love of truth, but the higher moral conceptions.* I will not claim that the study of ethics is more directly conducive to good morals than, say, the reading of good poetry is conducive to the writing of good prose. But I will say that it affords a quite indispensable help to the understanding of logic. It is, moreover, a subtle sort of study, such as a person who is fond of logic cannot but find to his taste. . . .

83. Only after these topics shall have been disposed of will it become needful to take up that propedeutic to logic proper, that unpsychological *Erkenntnisslehre* at which I have hinted. I name it *Speculative Grammar*, after the title of a work by Duns Scotus having the same aim.

84. In announcing what I am going to say in this part of the book, I have to choose between utter unintelligibility and a lengthy anticipation of what is about to be proved, but can here only be asserted. I unhesitatingly take the latter course, since the ideas are put into such strange forms that a double exposition will aid the reader. I begin by endeavoring to strike the dominant note of the book with such force and clearness as I can command; for it is not only the key-note, but the key, to all logic. I essay an analysis of what appears in the

* See ch. 4 of the "Minute Logic" published in vol. 1, bk. IV as ch. 2.

world. It is not metaphysics that we are dealing with: only logic. Therefore, we do not ask what really is, but only what appears to everyone of us in every minute of our lives. I analyze experience, which is the cognitive resultant of our past lives, and find in it three elements. I call them *Categories*.* Would I could render them to the reader as vivid, as undeniable, as rational as they are to me. They will become so, if he will give thought enough to them. They appear in myriad shapes, of which, for the purpose of introducing the reader to them, I take the first that offers itself. A definition of experience happened, just now, to flow from my pen. It was a pretty good definition, I think: suppose we set out from that. Laconically speaking, experience is *esse in praeterito*. Only, remember, once more and once for all, that we do not mean what the secret nature of the fact is, but simply what we think it to be. Some fact there is. All experience compels your acknowledgment. What, then, is the fact that is present to you? Ask yourself: it is past. A fact is a *fait accompli;* its *esse* is *in praeterito*. The past compels the present, in some measure, at least. If you complain to the Past that it is wrong and unreasonable, it laughs. It does not care a snap of the finger for Reason. Its force is brute force. So then, you are compelled, brutally compelled, to admit that there is such an element in the world of experience as brute force. What then is brute force, or what does it seem to be? We ought to find little difficulty in answering that, since we are directly conscious (or seem to be, which is all that concerns us here) of exerting it ourselves. For, no matter how good a reason we may have for an act of the will, yet when we come to the exertion itself, reason does no part of the work: it is brute action. We can make no effort where we experience no resistance, no reaction. The sense of effort is a two-sided sense, revealing at once a something within and another something without. There is binarity in the idea of brute force; it is its principal ingredient. For the idea of brute force is little more than that of reaction; and this is pure binarity. Imagine two objects which are not merely *thought* as two, but of which something is true such that neither could be removed without destroying the fact supposed true of the other. Take, for example, a husband and

* Vol. 1, bk. III contains a detailed study of the categories.

wife. Here there is nothing but a real twoness; but it constitutes a reaction, in the sense that the husband makes the wife a wife in fact (not merely in some comparing thought); while the wife makes the husband a husband. A brute force is only a complication of binarities. It supposes not only two related objects, but that in addition to this state of things there is a *second* subsequent state. It further supposes two tendencies, one, of the one relate, tending to change the first relation in one way in the second state; the other, of the other relate, tending to change the same relation in a second way. Both those changes are in some way combined, so that each tendency is to some degree followed, to some degree modified. This is what we mean by *force*. It is almost pure binarity. The *bruteness* will consist in the absence of any reason, regularity, or rule, which should take part in the action as a third or mediating element. Binarity is one of my categories. I do not call it a conception; for it can be given in direct perception antecedent to thought. It penetrates every part of our inner world, as every part of the universe. The emotion of it becomes more like that of brute force in proportion to the development of this element [of binarity]. Among the inner shapes which binarity assumes are those of the *doubts* that are forced upon our minds. The very word "doubt," or "dubito," is the frequentative of "duhibeo"— i.e., *duo habeo*, and thus exhibits its binarity. If we did not struggle against doubt, we should not seek the truth. Binarity equally appears in negation, and in ordinary relative terms, even in similarity, and in a more real way in identity. The text* will show why individual existence is a markedly dualistic conception. Meantime, it is easy to see that only existing individuals can react against one another.

85. Let us now consider what could appear as being in the present instant were it utterly cut off from past and future. We can only guess; for nothing is more occult than the absolute present. There plainly could be no action; and without the possibility of action, to talk of binarity would be to utter words without meaning. There might be a sort of consciousness, or feeling, with no self; and this feeling might have its tone. Notwithstanding what William James has said, I do

* Cf. 3.93n, 3.611ff, 6.6.

not think there could be any continuity like space, which, though it may perhaps appear in an instant in an educated mind, I cannot think could do so if it had no time at all; and without continuity parts of the feeling could not be synthetized; and therefore there would be no recognizable parts. There could not even be a degree of vividness of the feeling; for this [the degree of vividness] is the comparative amount of disturbance of general consciousness by a feeling.* At any rate, such shall be our hypothesis, and whether it is psychologically true or not is of no consequence. The world would be reduced to a quality of unanalyzed feeling. Here would be an utter absence of binarity. I cannot call it unity; for even unity supposes plurality. I may call its form Firstness, Orience, or Originality. It would be something *which is what it is without reference to anything else* within it or without it, regardless of all force and of all reason. Now the world is full of this element of irresponsible, free, Originality. Why should the middle part of the spectrum look green rather than violet? There is no conceivable reason for it nor compulsion in it. Why was I born in the nineteenth century on Earth rather than on Mars a thousand years ago? Why did I today sneeze just five hours forty-three minutes and twenty-one seconds after a certain man in China whistled (supposing this did happen)? We know perhaps why a meteorite should fall to the earth, if it gets in the Earth's way; but why should the arrangements of nature be such that this particular meteorite was in the Earth's way? All these are facts which are as they are, simply because they happen to be so. We mostly neglect them; but there are cases, as in qualities of feeling, self-consciousness, etc., in which such isolated flashes come to the front. Originality, or Firstness, is another of my Categories.

86. Let us now take up being *in futuro*. As in the other cases, this is merely an avenue leading to a purer apprehension of the element it contains. An absolutely pure conception of a Category is out of the question. Being *in futuro* appears in mental forms, intentions and expectations. Memory supplies us a knowledge of the past by a sort of brute force, a quite binary action, without any reasoning. But all our knowledge of the future is obtained through the medium of some-

* Cf. 1.322.

thing else. To say that the future does not influence the present is untenable doctrine. It is as much as to say that there are no final causes, or ends. The organic world is full of refutations of that position. Such action [by final causation] constitutes evolution. But it is true that the future does not influence the present in the direct, dualistic, way in which the past influences the present. A machinery, a medium, is required. Yet what kind of machinery can it be? Can the future affect the past by any machinery which does not again itself involve some action of the future on the past? All our knowledge of the laws of nature is analogous to knowledge of the future, inasmuch as there is no direct way in which the laws can become known to us. We here proceed by experimentation. That is to say, we guess out the laws bit by bit. We ask, What if we were to vary our procedure a little? Would the result be the same? We try it. If we are on the wrong track, an emphatic negative soon gets put upon the guess, and so our conceptions gradually get nearer and nearer right. The improvements of our inventions are made in the same manner. The theory of natural selection is that nature proceeds by similar experimentation to adapt a stock of animals or plants precisely to its environment, and to keep it in adaptation to the slowly changing environment. But every such procedure, whether it be that of the human mind or that of the organic species, supposes that effects will follow causes on a principle to which the guesses shall have some degree of analogy, and a principle not changing too rapidly. In the case of natural selection, if it takes a dozen generations to sufficiently adapt a stock to a given change of the environment, this change must not take place more rapidly, or the stock will be extirpated instead of being adapted. It is no light question how it is that a stock in some degree out of adjustment to its environment immediately begins to sport, and that not wildly but in ways having some sort of relation to the change needed. Still more remarkable is the fact that a man before whom a scientific problem is placed immediately begins to make guesses, not wildly remote from the true guess. The physicist who observes a strange phenomenon in his laboratory, for example, does not begin by wondering whether the particular aspect of the planets at that moment had something to do with it — as Ernst

Mach* practically guesses that it is the fixed stars that keep a body moving in a right line at uniform speed — he looks about for some circumstance near at hand that may explain it. How is this marked, though excessively imperfect, divinatory power of guessing right on the part of the man and on the part of the organic stock, to be explained? Two alternatives only are open. On the one hand, we may say that there is a direct power of Reason to know how Reason will act; and that Nature is ruled by a Reasonable Power. On the other hand, we may say that the tendency to guess nearly right is itself the result of a similar experimental procedure. This involves a deeply interesting difficulty (not the mere stumbling over a *regressus ad infinitum*) which shall be touched upon before the close of this volume. As to the other hypothesis, it only concerns me here to say that as those peoples who believe in prophets look for this gift particularly among the insane, so the power here supposed would be altogether different from the operation of reasoning. Take experimental reasoning, for example. Here we have pairednesses between the experiments and the results of the experiments, consisting in the fact that the results follow the experiments in accordance with a previous hypothesis; and the nature of these pairednesses is such that they could not have existed had not a third thing, the prediction, been made. Just as a real pairedness consists in a fact being true of A which would be nonsense if B were not there, so we now meet with a Rational Threeness which consists in A and B being really paired by virtue of a third object, C. I tell my dog to go upstairs and fetch me my book, which he does. Here is a fact about three things, myself, the dog, and the book, which is no mere sum of facts relating to pairs, nor even a pairing of such pairs. I speak to the dog. I mention the book. I do those things together. The dog fetches the book. He does it in consequence of what I did. That is not the whole story. I not only simultaneously spoke to the dog and mentioned the book, but I mentioned the book to the dog; that is, I caused him to think of the book and to bring it. My relation to the book was that I uttered certain sounds which were understood by the dog to have reference to the book. What I did to the dog, beyond exciting his auditory

* See, for example, *Die Mechanik*, ch. II, vi, 6 and 9.

nerve, was merely to induce him to fetch the book. The dog's relation to the book was more prominently dualistic; yet the whole significance and intention of his fetching it was to obey me. In all action governed by reason such genuine triplicity will be found; while purely mechanical actions take place between pairs of particles. A man gives a brooch to his wife. The merely mechanical part of the act consists in his laying the brooch down while uttering certain sounds, and her taking it up. There is no genuine triplicity here; but there is no giving, either. The giving consists in his agreeing that a certain intellectual principle shall govern the relations of the brooch to his wife. The merchant in the Arabian Nights threw away a datestone which struck the eye of a Jinnee. This was purely mechanical, and there was no genuine triplicity. The throwing and the striking were independent of one another. But had he aimed at the Jinnee's eye, there would have been more than merely throwing away the stone. There would have been genuine triplicity, the stone being not merely thrown, but thrown *at* the eye. Here, *intention*, the mind's action, would have come in. Intellectual triplicity, or Mediation, is my third category.

87. There is no fourth, as will be proved.* This list of categories may be distinguished from other lists as the *Ceno-Pythagorean Categories*, on account of their connection with numbers. They agree substantially with Hegel's three moments. Could they be attributed to any thinker in well-known history, that would be almost enough to refute their claims to primitivity. It has occurred to me that perhaps Pythagoras brought them from Media or Aria; but careful examination has convinced me that there was not among the Pythagoreans the smallest approach to anything resembling the categories.

88. It is desirable that there should be technical terms for the categories. They should be expressive and not liable to be used in special senses in philosophy. The simplicity and pervasiveness of the categories render metaphorical designations quite impossible, since such a term, if at all appropriate, would contain the very category. There can be no *resemblance* to a category. A metaphorical name would probably contain the category in the first syllable, and the rest of the word would

* See, e.g., 1.298, 1.347.

be padding. I prefer, therefore, to borrow a word, or still better, to compose one, which, etymologically, if it may be, but by similarity with familiar words, indispensably, shall suggest a number of shapes in which the category is prominent. I propose to take the following terms on probation:

89. *Originality* is being such as that being is, regardless of aught else.

Obsistence (suggesting *obviate*, *object*, *obstinate*, *obstacle*, *insistence*, *resistance*, etc.) is that wherein secondness differs from firstness; or, is that element which taken in connection with Originality, makes one thing such as another compels it to be.

Transuasion (suggesting *translation*, *transaction*, *transfusion*, *transcendental*, etc.) is mediation, or the modification of firstness and secondness by thirdness, taken apart from the secondness and firstness; or, is being in creating Obsistence.

90. Although Originality is the most primitive, simple, and original of the categories, it is not the most obvious and familiar. We have thus far considered the categories in their original aspect. We now pass to the easier study of their obsistential shapes.

91. In the Obsistential aspect, Originality presents itself as a Quality, which is something which is such as it is, and is so free from Obsistence as not even to be self-identical, or individual. Two Qualities which are alike, as all Qualities are, are, in so far, the same Quality. Obsistence presents itself as a Relation, which is a fact concerning a set of objects, the Relates. A Relation is either *Genuine* or *Degenerate*. A Degenerate Relation is a fact concerning a set of objects which consists merely in a partial aspect of the fact that each of the Relates has its Quality. It is a Relation of Qualities; such as that A is greater than B. Its relates may be qualities or objects possessing qualities. It may be a Similarity, which is a more Degenerate form, or a Difference, which is a less Degenerate form, or it may be mixed. A Genuine Relation is one which is not necessarily involved in its Relates having any Qualities regardless of each other. Each relate is necessarily individual, or self-identical. Various other divisions of relations will be made; and the nature of identity, otherness,

coexistence, and incompossibility will be specially considered.*

92. Transuasion in its obsistent aspect, or *Mediation*, will be shown to be subject to two degrees of degeneracy. Genuine mediation is the character of a *Sign*. A *Sign* is anything which is related to a Second thing, its *Object*, in respect to a Quality, in such a way as to bring a Third thing, its *Interpretant*, into relation to the same Object, and that in such a way as to bring a Fourth into relation to that Object in the same form, *ad infinitum*. If the series is broken off, the Sign, in so far, falls short of the perfect significant character. It is not necessary that the Interpretant should actually exist. A being *in futuro* will suffice. Signs have two degrees of Degeneracy. A Sign degenerate in the lesser degree, is an Obsistent Sign, or *Index*, which is a Sign whose significance of its Object is due to its having a genuine Relation to that Object, irrespective of the Interpretant. Such, for example, is the exclamation "Hi!" as *indicative* of present danger, or a rap at the door as indicative of a visitor. A Sign degenerate in the greater degree is an Originalian Sign, or *Icon*, which is a Sign whose significant virtue is due simply to its Quality. Such, for example, are imaginations of how I would act under certain circumstances, as showing me how another man would be likely to act. We say that the portrait of a person we have not seen is *convincing*. So far as, on the ground merely of what I see in it, I am led to form an idea of the person it represents, it is an Icon. But, in fact, it is not a pure Icon, because I am greatly influenced by knowing that it is an *effect*, through the artist, caused by the original's appearance, and is thus in a genuine Obsistent relation to that original. Besides, I know that portraits have but the slightest resemblance to their originals, except in certain conventional respects, and after a conventional scale of values, etc. A Genuine Sign is a Transuasional Sign, or *Symbol*, which is a sign which owes its significant virtue to a character which can only be realized by the aid of its Interpretant. Any utterance of speech is an example. If the sounds were originally in part iconic, in part indexical, those characters have long since lost their importance. The words only stand for the objects they do, and signify the qualities they do,

* See, e.g., *Nomenclature and Divisions of Dyadic Relations*, Paper XVIII, vol. 3 for an extended treatment of Dyads.

because they will determine, in the mind of the auditor, corresponding signs. The importance of the above divisions, although they are new, has been acknowledged by all logicians who have seriously considered them. . . .

93. Logic is the science of the general necessary laws of Signs and especially of Symbols. As such, it has three departments. Obsistent logic, logic in the narrow sense, or *Critical Logic*, is the theory of the general conditions of the reference of Symbols and other Signs to their professed Objects, that is, it is the theory of the conditions of truth. Originalian logic, or *Speculative Grammar*, is the doctrine of the general conditions of symbols and other signs having the significant character. It is this department of general logic with which we are, at this moment, occupying ourselves. Transuasional logic, which I term *Speculative Rhetoric*, is substantially what goes by the name of methodology, or better, of *methodeutic*. It is the doctrine of the general conditions of the reference of Symbols and other Signs to the Interpretants which they aim to determine. . . .

94. In consequence of every sign determining an Interpretant, which is itself a sign, we have sign overlying sign. The consequence of this, in its turn, is that a sign may, in its immediate exterior, be of one of the three classes, but may at once determine a sign of another class. But this in its turn determines a sign whose character has to be considered. This subject has to be carefully considered, and order brought into the relations of the strata of signs, if I may call them so, before what follows can be made clear.

§2. TERMS, PROPOSITIONS, AND ARGUMENTS

95. Symbols, and in some sort other Signs, are either *Terms*, *Propositions*, or *Arguments*.* A *Term* is a sign which leaves its Object, and *a fortiori* its Interpretant, to be what it may. A *Proposition* is a sign which distinctly indicates the Object which it denotes, called its *Subject*, but leaves its Interpretant to be what it may. An *Argument* is a sign which distinctly represents the Interpretant, called its *Conclusion*, which it is intended to determine. That which remains of a

* Icons can be only terms; indices can be only terms or propositions (dicisigns), while symbols can be all three.

Proposition after removal of its Subject is a Term (a rhema) called its Predicate.* That which remains of an Argument when its Conclusion is removed is a Proposition called its Premiss, or (since it is ordinarily copulative) more usually its Premisses. . . .

96. Argument is of three kinds: *Deduction*, *Induction*, and *Abduction* (usually called adopting a hypothesis). An Obsistent Argument, or *Deduction*, is an argument representing facts in the Premiss, such that when we come to represent them in a Diagram we find ourselves compelled to represent the fact stated in the Conclusion; so that the Conclusion is drawn to recognize that, quite independently of whether it be recognized or not, the facts stated in the premisses are such as could not be if the fact stated in the conclusion were not there; that is to say, the Conclusion is drawn in acknowledgment that the facts stated in the Premiss constitute an Index of the fact which it is thus compelled to acknowledge.[1] All the demonstrations of Euclid are of this kind. Deduction is Obsistent in respect to being the only kind of argument which is compulsive. An originary Argument, or *Abduction*, is an argument which presents facts in its Premiss which present a similarity to the fact stated in the Conclusion, but which could perfectly well be true without the latter being so, much more without its being recognized; so that we are not led to assert the Conclusion positively but are only inclined toward admitting it as representing a fact of which the facts of the Premiss constitute an *Icon*. For example, at a certain stage of Kepler's eternal exemplar of scientific reasoning, he found that the observed longitudes of Mars, which he had long tried in vain to get fitted with an orbit, were (within the possible limits of error of the observations) such as they would be if Mars moved in an ellipse. The facts were thus, in so far, a *likeness* of those of motion in an elliptic orbit. Kepler did not conclude from this that the orbit really was an ellipse; but it did incline him to that idea so much as to decide him to undertake to ascertain whether virtual predictions about the latitudes and parallaxes based on this hypothesis would be verified or not. This

* Today the rhema, or rheme, is conventionally symbolized as $\varphi\hat{x}$ and is called a propositional function.

[1] The reader should refer to the definitions of Index, Icon, and Symbol [in 92].

probational adoption of the hypothesis was an Abduction. An Abduction is Originary in respect to being the only kind of argument which starts a new idea. A Transuasive Argument, or *Induction*, is an Argument which sets out from a hypothesis, resulting from a previous Abduction, and from virtual predictions, drawn by Deduction, of the results of possible experiments, and having performed the experiments, concludes that the hypothesis is true in the measure in which those predictions are verified, this conclusion, however, being held subject to probable modification to suit future experiments. Since the significance of the facts stated in the premisses depends upon their predictive character, which they could not have had if the conclusion had not been hypothetically entertained, they satisfy the definition of a Symbol of the fact stated in the conclusion. This argument is Transuasive, also, in respect to its alone affording us a reasonable assurance of an ampliation of our positive knowledge. By the term "virtual prediction," I mean an experiential consequence deduced from the hypothesis, and selected from among possible consequences independently of whether it is known, or believed, to be true, or not; so that at the time it is selected as a test of the hypothesis, we are either ignorant of whether it will support or refute the hypothesis, or, at least, do not select a test which we should not have selected if we had been so ignorant.

97. When Kepler had found that the elliptic orbit placed the planet Mars in the right longitudes, he proceeded to test the hypothesis in two ways. In the first place it had always been comparatively easy to find hypotheses approximately representing the longitudes, although not to the point of accuracy of Tycho Brahe's observations. But when these hypotheses were applied to the latitudes, it had always been found that additional hypotheses, of librations, or tiltings of the orbit of a complicated kind, having little verisimilitude, were required to come near to a representation of the latitudes. Kepler undertook the calculation of the latitudes from his elliptic theory without knowing whether the calculation would agree with the observation or not; but it was found that it did so most admirably. He then went back to the longitudes, and applied another test, of the success of which he could know nothing beforehand. What he had so far found was that the

planet was at the time of observation always in the direction in which it ought to be. But was it at the right distance? This could not be quite positively ascertained. But he could take two times at which Mars had been observed, and, at which according to the elliptic theory (which in this respect could hardly be in error) it was at the same point of its orbit, but at which it was certain that the earth was at widely different points in its orbit. The orbit of the earth is so nearly circular that there could be no doubt where it was at these times. These two places and the place of Mars (supposed the same at the two times) gave a triangle of which two angles and the intermediate side (the distance between the two positions of the earth) were known (the mean distance of the sun from the earth being taken as unit of distance). From that he could calculate the distance of Mars from the sun, with no assumption except that Mars was really at the same point of his orbit, about which there could (for a reason too long to set forth here) hardly be the least doubt, whether the elliptic orbit were correct or not. By trying this at times when Mars was at the two extremes of his orbit, and when he was at intermediate places, Kepler could get a test of the severest character as to whether the elliptic theory really flattened the orbit by the right amount or not. In the cases of the few, but well situated, pairs of observations which could be found that were suitable to this test, the accord of observation and theory was all that could be desired, and clinched the argument in the mind of every thinking person. It will be observed that the argument was very different from what it would have been if Kepler had merely taken all the observations of longitude, latitude, and parallax and had constructed from them a theory that would suit them all. That might evince no more than Kepler's extraordinary ingenuity. Nor was the last test the same that it would have been if Kepler, looking over the observations, and hunting for features of them that should suit the theory, had found this. That might only show that out of many features of the observations, some suited the theory. But his course was very different. He did not select this test because it would give a favorable result. He did not know that it would do so. He selected it because it was the test which Reason demanded should be applied. Let this course be pur-

sued, and no theories will stand long but those which are true. But the discussion of the strength of the argument belongs to Critical Logic, and not to Speculative Grammar.

§3. CLEARNESS OF IDEAS[P]

98. The division of all inference into Abduction, Deduction, and Induction may almost be said to be the Key of Logic.

99. After the thorough and careful discussion of all the above matters, involving many nice questions, including the one concerning which logicians are today disputing more than any other, many volumes having been devoted to it, I mean that of the nature of the proposition, and after every opinion has received its respectful hearing, we will come at last to the problem of Clearness, than which none in logic is more practically vital. I treated this subject in 1877,* and enunciated a maxim, the acceptance of which constitutes the position called Pragmatism, a question which has of late years largely occupied philosophers. My opinion remains substantially the same now† as then; but all those years have not passed without my learning something. I can now define the proposition more accurately, so as to close the door against those who would push the doctrine much further than I ever intended; and I can state the reasons for the rule in a manner which must be allowed to be more scientific, more convincing, and more definitory than before.

§4. ABDUCTION, DEDUCTION, AND INDUCTION

100. Critical Logic is then to be taken up.‡ I begin with necessary Deduction, treating it with as much completeness as possible. Yet I shall avoid wasting pages upon mere formalisms, except so far as their great familiarity entitles them to mention. I endeavor to include every form of necessary reasoning known to me. The utility of Abstractions in reasoning, which I shall here bring out, will be a novelty. I need hardly

* "The Fixation of Belief" (1877) and "How to Make Our Ideas Clear" (1878), chs. 4 and 5 of bk. II, vol. 5. The maxim is first stated in the latter paper.

† i.e., in 1902–3.

‡ In bk. III.

say that ordinary syllogistic will be but a small fraction of my doctrine. The main substance of that need hardly fill a page.

101. I next take up* that immensely important branch of deductive logic, the doctrine of chances, which has been called, with little exaggeration, the logic of the exact sciences. This involves several difficult questions, of which the two chief are on the one hand, the foundation of the doctrine, together with the nature of probability, and on the other hand, the admissibility of inverse probabilities. Both of these are matters of practical importance to us all; for although few have occasion to make numerical computations of probabilities, the use of the ideas and propositions of the calculus is most widely extended, and to great advantage, while, at the same time, even the greatest mathematicians† have fallen into fatal practical errors both in the theory and in the application of it. The first of the two questions mentioned is by no means one to be settled at one blow. A whole nest of fallacies is hidden in it. This is why I cannot here in a few words approximately define my position so that a person acquainted with the state of discussion can get a general idea of where I stand. I may, however, say that I am one of those who maintain that a probability must be a matter of positive knowledge, or confess itself a nullity. Yet I do not go to such an extreme length of empiricism as Mr. Venn.‡ On the other hand, some very acute, but in my opinion quite untenable, positions of Mr. F. Y. Edgeworth§ will receive examination. It is of the extremest importance to distinguish entirely different qualities commonly confounded under the name of probability. One of these, which I term "likelihood" is the most deceptive thing in the world, being nothing but the degree of conformity of a proposition to our preconceived ideas. When this is dignified by the name of probability, as if it were something on which vast Insurance Companies could risk their hundreds of millions, it does more harm than the yellow fever ever did. The probability proper is also an essentially inaccurate idea, calling for every precaution of pragmatism in the use of it, in which its inductive

* Bk. III, B.

† E.g. Laplace and Quetelet.

‡ See his *Logic of Chance*, (1866) and *Empirical Logic* (1889).

§ See Keynes' *Treatise on Probability* for a bibliography of Edgeworth's writings.

origin must be steadily kept in view as the compass by which we are to steer our bark safely on this ocean of probability. Induction might be accurately defined as the virtual inference of a probability, if probability could be defined without the idea of induction. When the philosophy of probability has once been put upon a sure footing, the question of inverse probabilities gives no serious difficulty. Nobody can go further than I in condemnation of this way of using probability, which completely vitiates the theory and practice of Inductive and Abductive reasoning, has set back civilization, and has corrupted ideals, to an extent so far beyond what anybody would believe possible without careful examination of the facts, that I know I must be laughed at for what seems a most ridiculous judgment. The reader would perhaps at length go with me if I could in this work enter into the history of current beliefs.

102. The discussion of probability naturally brings us to the interesting question of the validity of induction. I undertake to demonstrate mathematically that the validity of Induction, in the proper sense of the term, that is to say, experimental reasoning, follows, through the lemmas of probabilities, from the rudiments of the doctrine of necessary consequences, without any assumption whatever about the future being like the past, or similar results following similar conditions, or the uniformity of nature, or any such vague principle.* I shall set forth the reasoning in strict accuracy of form; and I defy anybody to find a flaw in it. The importance of the question for every man is tremendous. Having fully set forth my doctrine of induction, with the very strict rules to bind it down which are necessitated by the demonstration mentioned, I pass by for the present the consideration of all other theories, and proceed at once to the study of Abduction. Upon this subject, my doctrine has been immensely improved since my essay "A Theory of Probable Inference"† was published in 1883. In what I there said about "Hypothetic Inference" I was an explorer upon untrodden ground. I committed, though I half corrected, a slight positive error, which is easily set right without essentially altering my position. But my capital error was a negative one, in not perceiving that, according to my own

* See, e.g., bk. III, ch. 9.

† Bk. III, ch. 8.

principles, the reasoning with which I was there dealing could not be the reasoning by which we are led to adopt a hypothesis, although I all but stated as much. But I was too much taken up in considering syllogistic forms and the doctrine of logical extension and comprehension, both of which I made more fundamental than they really are. As long as I held that opinion, my conceptions of Abduction necessarily confused two different kinds of reasoning. When, after repeated attempts, I finally succeeded in clearing the matter up, the fact shone out that probability proper had nothing to do with the validity of Abduction, unless in a doubly indirect manner. But now a number of considerations offered themselves as possibly connected with the solution of the problem, and owing to the extreme weakness of this form of inference, it was difficult to make sure that they were irrelevant. I seemed to be lost in a pathless forest, until by minute application of the first principles, I found that the categories, which I had been led to neglect from not seeing how they were to be applied, must and in fact did furnish the clue that guided me through the maze. I prefer not to make any prefacial sketch of this doctrine, but shall ask the reader to judge of it, if at all, from its full exposition. I believe it to be the most important part of the book,* whether it be estimated from the theoretical or the practical side.

103. Having considered the three fundamental modes of argument, I now pass to the consideration of arguments which mingle the characters of these. Here belongs in the front rank the argument from analogy, followed by four ways of supporting hypotheses by uniformities, arguments from likelihood, from the resemblance of the future to the past, etc.

104. I now go back to consider other theories of the validity of induction and hypothesis than my own. I give as complete an account of these theories as I can. I show that the arguments which their authors identify with inductions have in some cases no force at all, but lead to manifest absurdity, and that in no cases are they nearly as strong as the true inductive argument. I furthermore show that they invariably leave the doors open, theoretically and practically, to bad reasoning. In fact, there are no rules at all, for the most part, which

* That part of the "Minute Logic" was not written. But see vol. 5, bk. I, ch. 7.

follow as necessary corollaries from the theories. From the ordinary theories of hypotheses, on the other hand, rules do flow; but they are particularly pestiferous rules, much worse than none at all; and these unfortunately have become, through judges' charges to juries and otherwise, widely current among people who never opened a book of logic. . . .

§5. SPECULATIVE RHETORIC*

105. All this brings us close to Methodeutic, or Speculative Rhetoric. The practical want of a good treatment of this subject is acute. It is not expected that any general doctrine shall teach men much about methods of solving problems that are familiar to them. But in problems a little remote from those to which they are accustomed, it is remarkable how not merely common minds, but those of the very highest order, stumble about helplessly. No class of thinkers can by anybody be rated higher in heuretic genius than the mathematicians; and yet see how they have boggled over comparatively simple problems of unfamiliar kinds, such as Fermat's theorems, Steiner's theorems, the problem of map-coloring, the theory of knots.

106. Many persons will think that there are other ways of acquiring skill in the art of inquiry which will be more instructive than the logical study of the theory of inquiry. That may be; I shall not dispute it; for it would carry me far beyond the confines of my province. I only claim that however much one may learn in other ways of the method of attacking an unfamiliar problem, something may be added to that knowledge by considering the general theory of how research must be performed. At the same time, it is this theory itself, for itself, which will here be the principal object.

107. In coming to Speculative Rhetoric, after the main conceptions of logic have been well settled, there can be no serious objection to relaxing the severity of our rule of excluding psychological matter, observations of how we think, and the like. The regulation has served its end; why should it be allowed now to hamper our endeavors to make methodeutic practically useful? But while the justice of this must be

* There is no systematic treatment of this subject. Remarks on the conditions of research and the principles of discovery are scattered through the volumes.

admitted, it is also to be borne in mind that there is a purely logical doctrine of how discovery must take place, which, however great or little is its importance, it is my plain task and duty here to explore. In addition to this, there may be a psychological account of the matter, of the utmost importance and ever so extensive. With this, it is not my business here to meddle; although I may here and there make such use of it as I can in aid of my own doctrine.

108. Time was when a theorem could constitute a considerable contribution to mathematical science. But now new theorems are turned out wholesale. A single treatise will contain hundreds of them. Nowadays methods alone can arrest attention strongly; and these are coming in such flocks that the next step will surely be to find a *method of discovering methods*.* This can only come from a theory of the method of discovery. In order to cover every possibility, this should be founded on a general doctrine of methods of attaining purposes, in general; and this, in turn, should spring from a still more general doctrine of the nature of teleological action, in general.†

109. Although the number of works upon Methodeutic since Bacon's *Novum Organum* has been large, none has been greatly illuminative. Bacon's work was a total failure, eloquently pointing out some obvious sources of error, and to some minds stimulating, but affording no real help to an earnest inquirer. THE book on this subject remains to be written; and what I am chiefly concerned to do is to make the writing of it more possible.

110. I do not claim that the part of the present volume which deals with Speculative Rhetoric will approach that ideal. As to the other parts of my book, this prefatory chapter commits me to producing a work of great importance or to being set down a drawler of nonsense. But for the methodeutic part, I only say that since my youth I have associated with strong thinkers and have never ceased to make it a point to study their handling of their problems in all its details. When I was young, no remark was more frequent than that a given method, though excellent in one science, would be dis-

* Cf. 3.364.

† See vol. 1, bk. II, ch. 2, §1–2.

astrous in another. If a mere aping of the externals of a method were meant, the remark might pass. But it was, on the contrary, applied to extensions of methods in their true souls. I early convinced myself that, on the contrary, that was the way in which methods must be improved; and great things have been accomplished during my life-time by such extensions. I mention my early foreseeing that it would be so, because it led me, in studying the methods which I saw pursued by scientific men, mathematicians, and other thinkers, always to seek to *generalize* my conception of their methods, as far as it could be done without destroying the forcefulness of those methods. This statement will serve to show about how much is to be expected from this part of my work.

§6. OBJECTIVE LOGIC*

111. With Speculative Rhetoric, Logic, in the sense of Normative Semeotic, is brought to a close. But now we have to examine whether there be a doctrine of signs corresponding to Hegel's objective logic; that is to say, whether there be a life in Signs, so that — the requisite vehicle being present — they will go through a certain order of development, and if so, whether this development be merely of such a nature that the same round of changes of form is described over and over again whatever be the matter of the thought or whether, in addition to such a repetitive order, there be also a greater life-history that every symbol furnished with a vehicle of life goes through, and what is the nature of it. There are minds who will pooh-pooh an idea of this sort, much as they would pooh-pooh a theory involving fairies. I have no objection to the pooh-pooh-ing of fairies, provided it be critical pooh-pooh-ing; but I wish I had the leisure to place before those gentlemen a work to be entitled The History of Pooh-pooh-ing. I think it would do them good; and make room in their minds for an essay upon the Logic of Pooh-pooh-ing. Mind, that if some forenoon, while I was in the midst of one of the most valuable of the chapters of my "Minute Logic," a rap should come at my outer door, and if, upon going to the door, I were to find two men who proposed to come in and discuss with me the prin-

* The closest approach to this subject is to be found in vol. 6, bk. I, ch. 7. But see 4.80.

ciples of Mormonism or Christian Science, I should promptly recommend them to apply elsewhere. This I should do upon the same grounds upon which I declined to join the American Psychical Research Society when it was started; namely, that I thought that to do so would be to sanction a probable great waste of time, together with the placing of some men in a compromising position. In like manner, if a reader who has thought it worth while to listen to what I have had to say upon normative logic finds objective logic too remote from his interests to care to listen to any discussion of it, I shall fully approve of his allowing the leaves of my chapter upon this subject to remain uncut. But my own position is different. It lies directly in the path of my duty to consider the question critically.

112. The first question, then, which I have to ask is: Supposing such a thing to be true, what is the kind of proof which I ought to demand to satisfy me of its truth? Am I simply to go through the actual process of development of symbols with my own thoughts, which are symbols, and am I to find in the sense of necessity and evidence of the following of one thought upon another an adequate assurance that the course followed is the necessary line of thought's development? That is the way the question has usually been put, hitherto, both by Hegelians and by Anti-Hegelians. But even if I were to find that the sequence of conceptions in Hegel's logic carried my mind irresistibly along its current, that would not suffice to convince me of its universal validity. Nor, on the other hand, does the mere fact that I do not find a single step of Hegel's logic, or any substitute for it that I have met with, either convincing or persuasive, give me any assurance whatever that there is no such life-history. It seems to me natural to suppose that it would be far easier satisfactorily to answer the question of whether there is such a thing than to find out what particular form that life-history would take if it were a reality; and not only natural to suppose so, but made as certain by solid reasons as any such anticipation in regard to proofs could well be.

113. I am not one of those transcendental apothecaries, as I call them — they are so skillful in making up a bill — who call for a quantity of big admissions, as indispensible *Voraus-*

setzungen of logic. I am not so indulgent as Argan* to suppose that they can seriously expect as much as half their demands to be allowed. I reduce the indispensability of their postulates far more than that, namely, all the way from universality to the single case that happens to have come up; and even then, I do not admit that indispensability is any ground of belief. It may be indispensable that I should have $500 in the bank — because I have given checks to that amount. But I have never found that the indispensability directly affected my balance, in the least. When a hand at whist has reached the point at which each player has but three cards left, the one who has to lead often goes on the assumption that the cards are distributed in a certain way, because it is only on that assumption that the odd trick can be saved. This is indisputably logical; and on a more critical analogous occasion there might be some *psychological* excuse, or even warrant, for a "will to believe" that such was really the case. But all that logic warrants is a *hope*, and not a belief. It must be admitted, however, that such hopes play a considerable part in logic. For example, when we discuss a vexed question, we *hope* that there is some ascertainable truth about it, and that the discussion is not to go on forever and to no purpose. A transcendentalist would claim that it is an indispensable "presupposition" that there is an ascertainable true answer to every intelligible question. I used to talk like that, myself; for when I was a babe in philosophy my bottle was filled from the udders of Kant. But by this time I have come to want something more substantial.

114. But whatever be the kind and degree of our logical assurance that there is any real world, external or internal, that same kind and degree of assurance we certainly have that there not only may be a living symbol, realizing the full idea of a symbol, but even that there actually is one.

115. I examine the question from this point of view. It certainly seems as if the mere hypothesis of such a thing as a symbol sufficed to demonstrate such a life-history. Still, a fallacy is to be suspected. How can a mere hypothesis prove so much as this seems to prove, if it proves anything? I call in the data of experience, not exactly the every-minute experi-

* See Molière's *Le Malade imaginaire.*

ence which has hitherto been enough, but the experience of most men, together with the history of thought. The conclusion seems the same. Yet still, the evidence is unsatisfactory. The truth is that the hypothesis involves the idea of a different mode of being from that of existential fact. This mode of being seems to claim immediate recognition as evident in the mere idea of it. One asks whether there is not a fallacy in using the ordinary processes of logic either to support it or to refute it.

116. Aristotelianism admitted two modes of being. This position was attacked by William Ockham, on the ground that one kind sufficed to account for all the phenomena. The hosts of modern philosophers, to the very Hegels, have sided with Ockham in this matter. But now the question comes before us for reëxamination: What are the modes of being? One might antecedently expect that the cenopythagorean categories would require three modes of being. But a little examination will show us that they could be brought into fairly presentable accordance with the theory that there were only two, or even only one. The question cannot be decided in that way. Besides, it would be illogical to rely upon the categories to decide so fundamental a question. The only safe way is to make an entirely fresh investigation. But by what method are we to pursue it? In such abstract questions, as we shall have already found, the first step, often more than half the battle, is to ascertain what we mean by the question — what we possibly *can* mean by it. We know already how we must proceed in order to determine what the meaning of the question is. Our sole guide must be the consideration of the use to which the answer is to be put — not necessarily the practical application, but in what way it is to subserve the *summum bonum*. Within this principle is wrapped up the answer to the question, what being is, and what, therefore, its modes must be. It is absolutely impossible that the word "Being" should bear any meaning whatever except with reference to the *summum bonum*. This is true of any word. But that which is true of one word in one respect, of another in another, of every word in some or another respect, that is precisely what the word "being" aims to express. There are other ways of conceiving Being — that it is that which manifests itself, that it is that

which produces effects — which have to be considered, and their relations ascertained.

117. Having thus worked out a tolerable conception of Being, we turn to modes of being. But these are metaphysical conceptions. Let us first inquire how the validity of *any* metaphysical conception is to be determined. For this purpose we have only to apply the principles of Speculative Rhetoric. We sketch out the method and apply it to a few metaphysical conceptions, such as Reality, Necessity, etc. In process of doing this, we discover that all such metaphysical conceptions are but determinations of the categories, and consequently form a regular system. We also find that they can be held as valid only in approximative and imperfect senses.

118. But this seems to be in conflict with our conception of Being, particularly as derived from the notion of symbol; which, however, is solidly founded, too. We now begin to see the sense of talking of modes of being. They are elements of coöperation toward the *summum bonum*. The categories now come in to aid us materially, and we clearly make out three modes or factors of being, which we proceed to make clear to ourselves. Arrived at this point, we can construct a *Weltanschauung*. From this platform, ethics acquires a new significance, as will be shown. Logic, too, shines forth with all its native nobility.* Common men carry this *Weltanschauung* in their breasts; and perhaps the pimp, the looting missionary, the Jay Gould, may, through the shadows of their degradation, catch now and then a purer glimpse of it, than the most earnest of citizens, the Cartises, the Emersons, the Bishop Myriels. It is beautifully universal; and one must acknowledge that there is something healthy in the philosophy of faith, with its resentment at logic as an impertinence. Only it is very infantile. Our final view of logic will exhibit it (on one side of it) as faith come to years of discretion.

* Cf. vol. 1, bk. IV.

CHAPTER 3

WHY STUDY LOGIC?[P]*

§1. THE PRE-LOGICAL SCIENCES

119. The long discussion of the Classification of the Sciences to which the last section was devoted was intended to bring to light the mode of relationship of logic to other theoretical inquiries; or, at least, to make the author's opinion explicit; for as yet the truth of what has been said remains to be proved. It is, however, not a heresy but a doctrine very widely entertained, since Auguste Comte wrote, that the sciences form a sort of ladder descending into the well of truth, each one leading on to another, those which are more concrete and special drawing their principles from those which are more abstract and general.

120. If this be so, and if the scheme of classification of the sciences that has been proposed be correct, it will follow that there are but five theoretical sciences which do not more or less depend upon the science of logic. One of these five is Logic itself, which must contrive, by hook or by crook, to work out its own salvation without a full pre-acquaintance with its own discoveries, but which, like any other science, will lay one stone upon another in the erection of its doctrine. This is the last of the five. The first is Mathematics. Mathematics may itself be regarded as an art of reasoning. Perhaps this is not the highest conception of it. But at any rate, mathematics has no occasion to inquire into the theory of the validity of its own argumentations; for these are more evident than any such theory could be. The second of the five is that department of philosophy called Phenomenology, whose business it is simply to draw up an inventory of appearances without going into any investigation of their truth. The third is Esthetics, if I am to take the word of others that there is such a science, I myself

* Second section of the second chapter, "Pre-logical Notions" of the "Minute Logic." For the first section, "The Classification of the Sciences," see vol. 1, bk. II, ch. 2.

being lamentably ignorant of it, as I fear will too plainly appear. The fourth is Ethics; certainly, one of the very subtlest of studies. The whole course of it seems to consist in painfully extricating oneself from one pitfall only straightway to fall into another. It might seem that logic was desirable in this deliberation; but I fear that logic, as a definite theory, can be of no avail until one knows what it is that one is trying to do, which is precisely what ethics has to determine. On the contrary, that has to be settled before one can form any sound system of logic, as we shall see in due time.

121. All the other sciences but those five, according to the principles herein to be defended, depend upon Logic. I do not mean merely that they practice logical reasoning: they draw principles from the theory of logic. This dependence will be most direct and intimate for those sciences which stand nearest after Logic in the scheme of the sciences; yet even those which are most remote, such as History on the Psychical side, and Geology on the Physical side, have sometimes to make direct appeal to the theory of evidence. Besides that, all these Descriptive sciences must be founded upon Classificatory Sciences. Now that the classificatory sciences have to make appeal to the science of rationality, and always have done so, in order to determine what they are to think of the reality of their own classifications, will not be denied. Moreover, the Classificatory Sciences are and must be founded upon the Nomological Sciences. Here we find the psychologists, on the one side, together with Galileo, Kepler, Newton, Descartes, and all the founders of nomological physics, on the other, making direct appeal to the theory of logic. In addition to that, these Nomological Sciences cannot avoid depending upon Metaphysics. It is when they promise themselves that they will not make any metaphysical assumptions that they are most in danger of slipping too deep into the metaphysical slough for deliverance, precisely because one cannot exercise control and criticism of what one does unconsciously. At a later stage of our logical studies this dependence of nomology upon metaphysics will appear very evident. As to Metaphysics, if the theory of logic which is to be developed in this book has any truth, the position of the two greatest of all metaphysicians, Aristotle and Kant, will herein be supported by satis-

factory proof, that that science can only rest directly upon the theory of logic. Indeed, it may be said that there has hardly been a metaphysician of the first rank who has not made logic his stepping-stone to metaphysics.

122. Such is the place of logic among the sciences; and such is its utility. Yet the reader will find that the aggregate value of all such applications will not compare with the treasure of the pure theory itself. For when he has surveyed the whole subject, he will see that the theory of logic, in so far as we attain to it, is the vision and the attainment of that Reasonableness for the sake of which the Heavens and the Earth have been created.

§2. TEN PRE-LOGICAL OPINIONS

123. Meantime, O Reader, not yet seeing the truth of this, why is it that you have undertaken the study of logic? You may have some excellent reasons which are peculiar to your personal relations to science and to life. But in addition to these, there are certain reasons which you must have, since they attach to the very essence of the study. Presuming that, aside from personal reasons, you desire in singleness of heart to examine the theory of reasoning under the guidance of an older student, I remark that this very fact is evidence that you are already a much better logician than are the mass of mankind, who are thoroughly persuaded that they reason well enough already. I do not mean to say that they maintain that none of them ever reasons wrong. Far from that; though they trust to common sense as affording all the security that could be desired for reasoning, yet their adhesion is majestically unanimous to the proposition that of all the race there is but one single individual who never falls into fallacy; and their only point of difference is that each is quite sure that he himself is that man. Unfortunately, to be cocksure that one is an infallible reasoner is to furnish conclusive evidence either that one does not reason at all, or that one reasons very badly, since that deluded state of mind prevents the constant self-criticism which is, as we shall see, the very life of reasoning. Congratulations, then, from my heart go out to you, my dear Reader, whom I assume to have a sincere desire to learn, not merely the *dicta* of common sense, but what good reasoning,

scientifically examined, shall prove to be. You are already an unusually good logician.

124. But now from what I know of you, I am led to think that you entertain certain ten opinions upon which I should like to offer some thoughts. You must almost certainly entertain these opinions, or you would not be wanting to study logic. Perhaps you and I do not think differently on most of these points. Yet it might be well to turn them over, and see what you do think, and why. Most of them are more or less controverted today.

125. In the first place, you would not wish to study logic unless you intended to reason; and you doubtless hold the purpose of reasoning to be the ascertainment of the truth. So it appears that you belong to the sect that maintains that there is such a thing as truth. Ought you not, then, to settle with yourself what that opinion of yours precisely amounts to, and further, what reason there is for entertaining it?

126. In the second place, you not only seem to fancy that there is such a thing as truth, but also that it can be found out and known, in some measure. What reason is there for that?

127. In the third place, you seem to think not only that some knowledge can be attained, but that it can be attained by reasoning.

128. In the fourth place, you seem to think that not only may reasoning lead to the truth, but that a man may be deceived by reasoning badly. This conception seems to demand scrutiny.

129. In the fifth place, I strongly suspect that you hold reasoning to be superior to intuition or instinctive uncritical processes of settling your opinions. What basis of fact is there for this opinion?

130. In the sixth place, I believe that you opine that you yourself are conscious of reasoning. I do not doubt that you are so, in some sense; but what is it precisely that you are conscious of?

131. In the seventh place, I am quite sure that you are already in possession of a logic, or theory of reasoning; and I wish to call your particular attention to the circumstance that in the eighth place, it would seem that though you entertain this theory of reasoning, you also entertain the opinion that

your theory is wrong. How can you opine that your opinion is wrong?

132. In the ninth place, although you think you reason wrong — else why study logic?— you think that by this reasoning process which is wrong you can correct your method, and demonstrate by bad reasoning beyond all peradventure that your improved reasoning is perfect.

133. In the tenth place, you would appear to be of opinion that by improving your theory of reasoning your practice of reasoning will be improved; much as if a man subject to palpitations of the heart should expect to be cured by reading a book on physiology. Is this a sound view or not?

134. I think you will agree with me that these ten opinions should be revised, and if necessary, corrected, before going any further. In regard to each one, you first wish to consider precisely what the opinion is, and next what reason there may be for adhering to it. We need not carry the discussion into any deep metaphysics. Simple answers will do to begin with; only they must be distinct and explicit. Profounder insight may come later.

§3. THE OBJECTIVITY OF TRUTH

135. You certainly opine that there is such a thing as Truth. Otherwise, reasoning and thought would be without a purpose. What do you mean by there being such a thing as Truth? You mean that something is SO — is correct, or just — whether you, or I, or anybody thinks it is so or not. Most persons, no doubt, opine that for every question susceptible of being answered by *yes* or *no*, one of these answers is true and the other false. Perhaps that is carrying the doctrine to an extravagant pitch.* At any rate, the mere fact that you wish to learn logic would not prove that you go so far as that. It only shows that you think that *some* question — some interesting question, what one, perhaps, you are not just now prepared to say — has one answer which is decidedly right, whatever people may think about it. The essence of the opinion is that there is *something* that is SO, no matter if there be an overwhelming vote against it. So you plainly opine. For if

* See 1.172.

thinking otherwise is going to make it otherwise, there is no use in reasoning or in studying logic.

136. Such is the opinion: there have been, and are today, numbers of eminent thinkers, philosophers, and even professed logicians, who have flatly denied it. One might produce stacks of *a priori* arguments against it. One might urge, for example, that there can be nothing of which God is ignorant. Even if there be no God, there can be nothing of which an Omniscient God would be ignorant, if there were such a Being. But whatever the Omniscient may think is *ipso facto* so. Consequently, the idea of Truth, in the sense of that which is so whether the thinker thinks it so or not, must be foreign to the mind of God. Of such truth Omniscience must be ignorant, and since He is by hypothesis ignorant of nothing, there is no such thing. Another argument is that if there be anything which is *so*, in spite of what be thought, there may be something which cannot be thought. For it is conceivable that all who could think it were destroyed. But it is inconceivable that there should be anything that cannot be thought, for to conceive this would be to think the very thing supposed to be unthinkable. Hence it is inconceivable that there should be any Truth independent of opinions about it.

137. One might equally argue *a priori* in favor of the Truth. For suppose there is not any proposition which is correct independently of what is thought about it. Then if there be any proposition which nobody ever thinks incorrect, it is as correct as possible and has all the truth there is. Consider, then, the proposition: "This proposition is thought by somebody to be incorrect." Now if it is, in fact, thought by somebody to be incorrect, then it is true. For that is precisely the statement. But if it is not thought by anybody to be incorrect, it has all the truth possible, if there is no truth independent of opinion. Here, then, is a proposition which is correct whether it is thought to be so or not. Therefore, there is such a thing as a proposition correct whatever may be opinions about it. But when we come to study logic, we shall find that all such *a priori* arguments, whether *pro* or *con*, about positive fact are rubbish. This question is a question of fact, and experience alone can settle it.

138. Experience is that determination of belief and cogni-

tion generally which the course of life has forced upon a man. One may lie about it; but one cannot escape the fact that some things *are* forced upon his cognition. There is the element of brute force, existing whether you opine it exists or not. Somebody may object that if he did not think so, he would not be forced to think so; so that it is not an instance in point. But this is a double confusion of ideas. For in the first place, that something is, even if you think otherwise, is not disproved but demonstrated if you cannot think otherwise; and in the next place, what experience forces a man to think, of course he must think. But he is not therein forced to think that it is *force* that makes him think so. The very opinion entertained by those who deny that there is any Truth, in the sense defined, is that it is not force, but their inward freedom which determines their experiential cognition. But this opinion is flatly contradicted by their own experience. They insist upon shutting their eyes to the element of compulsion, although it is directly experienced by them. The very fact that they can and do so shut their eyes confirms the proof that fact is independent of opinion about it.

139. Deceive yourself as you may, you have a direct experience of something reacting against you. You may suppose that there is some substance in which *ego* and *non-ego* have alike the roots of their being; but that is beside the question. The fact of the reaction remains. There is the proposition which is so, whatever you may opine about it. The essence of truth lies in its resistance to being ignored.

§4. DIRECT KNOWLEDGE

140. You further opine that there is such a thing as knowledge. Your thinking there is any use in logic betrays that opinion. For you, the *non-ego* is not an unknowable thing in itself. Since the above argument for reality is that it is experienced, the same argument compels you to admit that there is knowledge; so that that branch of this second question needs no further attention. But it will be well to notice roughly in what sense this argument compels you to admit the existence of knowledge.

141. The knowledge which you are compelled to admit is that knowledge which is directly forced upon you, and which

there is no criticizing, because it is directly forced upon you. For example, here I sit at my table with my inkstand and paper before me, my pen in my hand, my lamp at my side. It may be that all this is a dream. But if so, that such dream there is, is knowledge. But hold: what I have written down is only an imperfect description of the percept that is forced upon me. I have endeavored to state it in words. In this there has been an endeavor, purpose — something not forced upon me but rather the product of reflection. I was not forced to this reflection. I could not hope to describe what I see, feel, and hear, just as I see, feel, and hear it. Not only could I not set it down on paper, but I could have no kind of thought adequate to it or any way like it.[1]

Hundreds of percepts have succeeded one another while I have been setting down these sentences. I recognize that there is a percept or flow of percepts very different from anything I can describe or think. What precisely that is I cannot even tell myself. It would be gone, long before I could tell myself many items; and those items would be quite unlike the percepts themselves. In this thought there would always be effort or endeavor. Whatever is the product of effort might be suppressed by effort, and therefore is subject to possible error. I am forced to content myself not with the fleeting percepts, but with the crude and possibly erroneous thoughts, or self-informations, of what the percepts were. The science of psychology assures me that the very percepts were mental constructions, not the first impressions of sense. But what the first impressions of sense may have been, I do not know except inferentially and most imperfectly. Practically, the knowledge with which I have to content myself, and have to call "the evidence of my senses," instead of being in truth the evidence of the senses, is only a sort of stenographic report of that evidence, possibly erroneous.[2] In place of the *percept*, which, although not the first impression of sense, is a construction with which my will has had nothing to do, and may, there-

[1] I speak here too much of myself personally. Many men and women have imaginations resembling percepts up to the point of being mistaken for them. In others imagination is less vivid, down to my point, who see no resemblance between an imagination and a percept except that somehow they can be compared.

[2] For some people, however, this stenographic report seems to be illustrated with photographs. I can only adequately describe my own experience.

fore, properly be called the "evidence of my senses," the only thing I carry away with me is the *perceptual facts*, or the intellect's description of the evidence of the senses, made by my endeavor. These perceptual facts are wholly unlike the percept, at best; and they may be downright untrue to the percept.[1] But I have no means whatever of criticizing, correcting or recomparing them, except that I can collect new perceptual facts relating to new percepts, and on that basis may infer that there must have been some error in the former reports, or on the other hand I may in this way persuade myself that the former reports were true. The perceptual facts are a very imperfect report of the percepts; but I cannot go behind that record. As for going back to the first impressions of sense, as some logicians recommend me to do, that would be the most chimerical of undertakings.

142. The percepts, could I make sure what they were, constitute experience proper, that which I am forced to accept. But whether they are experience of the real world, or only experience of a dream, is a question which I have no means of answering with *absolute* certainty. I have, however, three tests which, though none of them is infallible, answer very well in ordinary cases. The first test consists in trying to dismiss the percepts. A fancy, or day-dream, can commonly be dismissed by a direct effort of will.[2] If I find that the flow of percepts persists consistently in spite of my will, I am usually satisfied. Still, it may be a hallucination. If I have reason to suspect that it is so, I apply the second test, which consists in asking some other person whether he sees or hears the same thing. If he does, and if several people do, that will ordinarily be taken as conclusive. Yet it is an established fact that some hallucinations and illusions affect whole companies of people. There remains, however, a third test that can be applied; and it is far the surest of the three. Namely, I may make use of my knowledge of the laws of nature (very fallible knowledge, confessedly) to predict that if my percept has its cause in the real world, a certain experiment must have a certain result —

[1] This I suppose would be true even for the most vivid imaginations.

[2] Personally, until old age began to mark its effects, I hardly had any pictorial dreams; and even now they are mere scraps of images to which incongruous abstract ideas are attached.

a result which in the absence of that cause would be not a little surprising. I apply this test of experiment. If the result does not occur my percept is illusory; if it does, it receives strong confirmation. For example, if I and all the company are so excited that we think we see a ghost, I can try what an unimaginative kodak would say to it.[1] So Macbeth made the experiment of trying to clutch the dagger.

143. All these tests, however, depend upon inference. The data from which inference sets out and upon which all reasoning depends are the *perceptual facts*, which are the intellect's fallible record of the *percepts*, or "evidence of the senses." It is these percepts alone upon which we can absolutely rely, and that not as representative of any underlying reality other than themselves.

§5. REASONING AND EXPECTATION

144. But since you propose to study logic, you have more or less faith in reasoning, as affording knowledge of the truth. Now reasoning is a very different thing indeed from the percept, or even from perceptual facts. For reasoning is essentially a voluntary act, over which we exercise control. If it were not so, logic would be of no use at all. For logic is, in the main, criticism of reasoning as good or bad. Now it is idle so to criticize[2] an operation which is beyond all control, correction, or improvement.

145. You have, therefore, to inquire, first, in what sense you have any faith in reasoning, seeing that its conclusions cannot in the least resemble the percepts, upon which alone implicit reliance is warranted. Conclusions of reasoning can little resemble even the *perceptual facts*. For besides being involuntary, these latter are strictly memories of what has taken place in the recent past, while all conclusions of reasoning partake of the general nature of expectations of the future. What two things can be more disparate than a memory and an expectation?

146. The reader questions, perhaps, the assertion that

[1] Personally, I never had anything like a hallucination except in the delirium of fever.

[2] I here use the word "criticize" in the philosophical sense. Criticism proper, literary criticism, does not necessarily approve or disapprove.

conclusions of reasoning are always of the nature of expectations. "What!" he will exclaim, "can we not reason about the authorship of the Junius Letters or the identity of the Man in the Iron Mask?" In a sense we can, of course. Still, the conclusion will not be at all like remembering the historical event. In order to appreciate the difference, begin by going back to the percept to which the memory relates. This percept is a single event happening *hic et nunc*. It cannot be generalized without losing its essential character. For it is an actual passage at arms between the non-ego and the ego. A blow is passed, so to say. Generalize the fact that you get hit in the eye, and all that distinguishes the actual fact, the shock, the pain, the inflammation, is gone. It is anti-general. The memory preserves this character, only slightly modified. The actual shock, etc., are no longer there, the quality of the event has associated itself in the mind with similar past experiences. It is a little generalized in the *perceptual fact*. Still, it is referred to a special and unique occasion, and the flavor of anti-generality is the predominant one. Now, on the other hand, consider what an expectation is. Begin with something in the distant future; and dismiss from your mind mechanical and necessitarian ideas, which essentially modify the pure notion of futurity. Looking at an expectation referring to the distant future, something, for example, which you propose to do, it is very little distinguished from a mere "may be." There is a sort of picture in your imagination whose outlines are vague and fluid. You do not attach it to any definite occasion, but you think vaguely that some definite occasion there is, to which that picture does attach itself, and in which it is to become individualized. You think that at present its state of being consists in the fact that either in your will or in somebody else's, or somehow in something analogous thereto in the nature of things, some rule is determined, or nearly determined, which will probably govern the individual event when it occurs. As the time approaches, the individualizing element, which is nothing but a general notion that some individual event is to be governed by this idea, acquires more and more prominence. But the expectation ever remains a general idea which we describe to ourselves as being attached to some generally described actual occasion; whereas the memory is merely

the reverberation of the shock of perception, essentially anti-general, though worn down here and there into generality by rubbing against memories of other similar occurrences. A perceptual fact is a memory hardly yet separated from the very percept. What is a conclusion of reasoning? It is a general idea to which at the suggestion of certain facts a certain general habit of reason has induced us to believe that a realization belongs. How slight is the difference between this and the description of an expectation! Yet, if we look closer, we can discern that the resemblance is nearer yet. For when we reason, we deliberately approve our conclusion. We look upon it as subject to criticism. We say to it: "Good! You will stand the fire, and come out of it with honour." Thus more than half an expectation enters into it as of its essence.

147. The second branch of the question, when you have decided in what your faith in reasoning consists, will inquire just what it is that justifies that faith. The stimulation of doubt about things indubitable or not really doubted is no more wholesome than is any other humbug; yet the precise specification of the evidence for an undoubted truth often in logic throws a brilliant light in one direction or in another, now pointing to a corrected formulation of the proposition, now to a better comprehension of its relations to other truths, again to some valuable distinctions, etc.

148. As to the former branch of this question, it will be found upon consideration that it is precisely the analogy of an inferential conclusion to an expectation which furnishes the key to the matter. An expectation is a habit of imagining. A habit is not an affection of consciousness; it is a general law of action, such that on a certain general kind of occasion a man will be more or less apt to act in a certain general way. An imagination is an affection of consciousness which can be directly compared with a percept in some special feature, and be pronounced to accord or disaccord with it. Suppose for example that I slip a cent into a slot, and expect on pulling a knob to see a little cake of chocolate appear. My expectation consists in, or at least involves, such a habit that when I think of pulling the knob, I imagine I see a chocolate coming into view. When the perceptual chocolate comes into view, my imagination of it is a feeling of such a nature that the percept can be

compared with it as to size, shape, the nature of the wrapper, the color, taste, flavor, hardness and grain of what is within. Of course, every expectation is a matter of inference. What an inference is we shall soon see* more exactly than we need just now to consider. For our present purpose it is sufficient to say that the inferential process involves the formation of a habit. For it produces a belief, or opinion; and a genuine belief, or opinion, is something on which a man is prepared to act, and is therefore, in a general sense, a habit. A belief need not be conscious. When it is recognized, the act of recognition is called by logicians a judgment, although this is properly a term of psychology. A man may become aware of any habit, and may describe to himself the general way in which it will act. For every habit has, or is, a general law. Whatever is truly general refers to the indefinite future; for the past contains only a certain collection of such cases that have occurred. The past is actual fact. But a general (fact) cannot be fully realized. It is a potentiality; and its mode of being is *esse in futuro*. The future is potential, not actual. What particularly distinguishes a general belief, or opinion, such as is an inferential conclusion, from other habits, is that it is active in the imagination. If I have a habit of putting my left leg into my trouser before the right, when I imagine that I put on my trousers, I shall probably not definitely think of putting the left leg on first. But if I *believe* that fire is dangerous, and I imagine a fire bursting out close beside me, I shall also imagine that I jump back. Conversely — and this is the most important point — a belief-habit formed in the imagination simply, as when I consider how I ought to act under imaginary circumstances, will equally affect my real action should those circumstances be realized. Thus, when you say that you have faith in reasoning, what you mean is that the belief-habit formed in the imagination will determine your actions in the real case. This is looking upon the matter from the psychological point of view. Under a logical aspect your opinion in question is that general cognitions of potentialities *in futuro*, if duly constructed, will under imaginary conditions determine *schemata* or imaginary skeleton diagrams with which percepts will accord when the real conditions accord with those imagi-

* 442.

nary conditions; or, stating the essence of the matter in a nutshell, you opine that percepts follow certain general laws. Exactly how far you hold that the percepts are determined by law is a matter of individual opinion. The mere fact that you hold reasoning to be useful only supposes that you think that *to some useful extent* percepts are under the governance of law. The habit of all loose thinkers is immensely to exaggerate the universality of what they believe, beyond all warrant. Some would boldly affirm that in all respects percepts are governed by law, a proposition which, if not logically absurd, is, at least, contrary to overwhelming evidence. Neither the mass of mankind nor even the most mechanical school of scientific speculators, of whom Laplace is the type, go to any such length. Various opinions have more or less vogue. We shall have to examine them later.

149. Now, taking the opinion in a vague sense, what evidence is there that it is true? In investigating this question, the first circumstance which strikes us is that this opinion is itself of a general nature. If percepts were not subject to law, at all, it would be a matter of indifference what our general ideas were. It might be convenient to act and think according to rules; but one set of rules would be superior to another set merely as being more conveniently carried out. It is safe to say that nobody ever did believe that percepts are in no degree subject to law, whatever nominalists may have persuaded themselves that they believed. But there is no more striking characteristic of dark ages, when thought was little developed, than the prevalence of a sentiment that an opinion was a thing to be chosen because one liked it, and which, having once been adopted, was to be fought for by fire and sword, and made to prevail. Take any general doctrine you please, and it makes no difference what facts may turn up: an ingenious logician will find means to fit them into the doctrine. Ask the theologians if this is not true. As civilization and enlightenment advance, however, this style of thought tends to weaken. Natural selection is against it; and it breaks down. Whatever one's theory may be as to the invalidity of human reason, there are certain cases where the force of conviction practically cannot be resisted; and one of these is the experience that one opinion is so far from being as strong as another in the long

run, though it receives equally warm support, that on the contrary, ideas utterly despised and frowned upon have an inherent power of working their way to the governance of the world, at last. True, they cannot do this without machinery, without supporters, without facts; but the ideas somehow manage to grow their machinery, and their supporters, and their facts, and to render the machinery, the supporters, and the facts strong. As intellectual development proceeds, we all come to believe in this more or less. Most of us, such is the depravity of the human heart, look askance at the notion that ideas have any power; although that some power they have we cannot but admit. The present work, on the other hand, will maintain the extreme position that every general idea has more or less power of working itself out into fact; some more so, some less so. Some ideas, the harder and more mechanical ones, actualize themselves first in the macrocosm; and the mind of man receives them by submitting to the teachings of nature. Other ideas, the more spiritual and moral ones, actualize themselves first in the human heart, and pass to the material world through the agency of man. Whether all this be true or not, it must at any rate be admitted by every candid man that he does believe firmly and without doubt that to some extent phenomena are regular, that is, are governed by general ideas; and so far as they are so, they are capable of prediction by reasoning.

150. I ought not to dismiss the question of the possibility of attaining a knowledge of the truth by reasoning without noticing an opinion which has of late years attained some vogue among men of science, that we cannot expect any physical hypothesis to maintain its ground indefinitely even with modifications, but must expect that from time to time there will be a complete cataclysm that shall utterly sweep away old theories and replace them by new ones. As far as I know, this notion has no other basis than the history of science. Considering how very, very little science we have attained, and how infantile the history of science still is, it amazes me that anybody should propose to base a theory of knowledge upon the history of science alone. An emmet is far more competent to discourse upon the figure of the earth than we are to say what future millennia and millionennia may have in store for physical

theories, with the atomic theory and the theory of light not yet one sleepy century old. The only really scientific theory that can be called old is the Ptolemaic system; and that has only been improved in details, not revolutionized. The most unhappy of physical theories has been that of Phlogiston; and even that was not altogether false, since something *is* lost from a burnt body; namely, energy.

§6. THE FALLIBILITY OF REASONING AND THE FEELING OF RATIONALITY*

151. Your studying logic is probably connected with a desire to reason *well* and to avoid reasoning *ill*. So that you probably conceive that reasoning may be either good or bad. If this is true, this matter of reasoning is a very singular function. There is no such thing as bad sight or bad hearing. True, these phrases are used; but all that is meant by "*bad*" in such locutions is *imperfect*. If a man is color-blind, he fails to perceive certain distinctions. But if there were a man to whom everything which looks to us red were to appear green, and *vice versa*, there would be no reason for stigmatizing such vision as bad. I detest the taste prevalent in a certain nation. Doubtless people of that nation would detest my taste; I should be heartily sorry if they did not. If either party were to go further and pronounce the other's taste to be *bad*, that would clearly imply that they made use of some other criterion than taste itself. Again, those whose sentiments I share abhor certain doctrines of certain writers upon Ethics — say, for example, those who make action the ultimate end of man.† I acknowledge that those writers are as much in earnest as we are, and I am quite aware that they reciprocate all our abhorrence. However it may be about taste, in regard to morals we can see ground for hope that debate will ultimately cause one party or both to modify their sentiments up to complete accord. Should it turn out otherwise, what can be said except that some men have one aim and some another? It would be monstrous for either party to pronounce the moral judgments of the other to be BAD. That would imply an appeal to some other tribunal.

* Cf. 5.85ff.

† Cf. 5.3.

152. In reasoning, however, your opinion is that we have the singular phenomenon of a physiological function which is open to approval and disapproval. In this you are supported by universal common sense, by the traditional logic, and by English logicians* as a body. But you are in opposition to German logicians† generally, who seldom notice fallacy, conceiving human reason to be an ultimate tribunal which cannot err.

153. Men of the best heart and purest intentions in the world have been known to commit actions contrary to their own moral principles simply because nothing in their lives had ever called their attention to the moral bearings of the kind of actions in question. In the same way, even upon the German theory, a man might reason badly from negligence, as in casting up a column of figures. Casuistry is in ill-repute, and somewhat deservedly so; and yet there are many nice questions of morals which are often discussed among men of the world; and it is a pity that there is not more discussion of that kind. I wonder some of our magazines do not open such a department. It often happens, I am sure, that a man seriously considers how it is his duty to act in a certain case, and yet comes to a conclusion quite the reverse of that which he would reach if certain aspects of the case had not escaped him. Yet everybody agrees that in following the dictates of his conscience he acts right, and that he would, under the circumstances, have done wrong had he acted in the manner in which he would have thought it right to act if he had carried his self-discussion further. This is because right and wrong are held to be, in the ultimate analysis, subjective. It is every man's duty to enlighten his conscience as much as possible. Conscience itself requires him to do that. But still, when all is done that circumstances permit, it is his duty to act conscientiously. Does an analogous state of things hold with regard to reasoning, or does it not? Is that necessarily good reasoning which is deliberately judged by the reasoner to be good reasoning, even although further consideration would show him that it was bad reasoning? Common sense and the English logicians answer *no*. Conscience is like our Supreme

* E.g., Boole, De Morgan, Whewell, J. S. Mill, Jevons, Venn, Pearson, MacColl.

† E.g., Sigwart, Wundt, Schuppe, Erdmann, Bergmann, Glogau, Husserl.

Court, which intends to frame its decisions according to the principles of law. But when it has decided a point, its decision becomes law, whether the wisest counsels would have maintained it or not. For the actual law consists in that which the court's officers will sustain. But according to the English logicians it is otherwise with rationality. Every reasoning holds out some expectation. Either, for example, it professes to be such that if the premisses are true the conclusion will always be true, or to be such that the conclusion will usually be true if the premisses are true, or to be a method of procedure which must ultimately lead to the truth, or makes some other such promise. If the facts bear out that promise, then, say the English, the reasoning is good. But if the facts violate the promise, the reasoning is bad, no matter how deliberately human reason may have approved of it. For the sole purpose of reasoning is, not to gratify a sense of rationality analogous to taste or conscience, but to ascertain the Truth, in the sense of that which is SO, no matter what be thought about it. If there is no such thing as Truth in this sense, the English opinion must fall. But no mere phenomenalism or idealism which would make thought, or something analogous to thought, a factor of reality, would be in conflict with the English opinion, so long as the doctrine continued to be that there is such a thing as Truth which is independent of what you or I or any group or generation of men may opine upon the subject. We can, of course, conceive of a man who should admit that there was such Truth, but who should prefer to believe what should be agreeable to reason; but English-speaking people generally want to know the truth, and to avoid disappointment, whether the truth be agreeable to reason or not. Again, a man may very well hold that Truth, in the above sense, exists, and may desire above all things to acquire a knowledge of it; and yet he may be of the opinion that the judgments of human reason accord so accurately with that truth, that there is no surer way of reasoning right than that of reasoning as the human mind naturally approves. This very sensible opinion will call for careful scrutiny below; but it supposes our main problem here, namely, whether rationality is wholly a subjective matter, like taste, or has a subjective and an objective side, like morals, or is wholly objective, to have been already

settled according to the English views. The consideration of this special variety of the English doctrine must therefore be postponed for a few pages.

154. I will now state the principal arguments which favor the German position. After that I will more fully expound the English position. This will enable us to comprehend better how the different German arguments hang together and to appreciate the distinctive character of the German doctrine, which it is important to appreciate, since thought having the same characteristic appears throughout the German logics. I will then set forth the arguments in favor of the English doctrine, and I hope that, by that time, it will be plain enough what our decision should be. As a supplement to the discussion, I will consider the opinion described in the last paragraph. The reader will not deem that I am detaining him too long over so momentous a question, even although this, with the other topics of this section, must come up for more scientific treatment at a later stage of our studies.

155. The first German argument is that to presume to say that a physiological operation, such as is reasoning, is good or bad, is a pre-scientific conception for which there is no warrant. It belongs to the theological era of thought about nature, during the reign of which men drag the images of the saints into the street, blackguard them and threaten them if they do not cause rain to fall. Nowadays, such conduct is deemed childish. Men no longer become angry with the attraction of gravitation; and it is equally absurd to praise or blame any natural operation of man's psycho-physical organism. It certainly must be admitted that there is some force in that argument.

156. The second argument is that before the present question was distinctly formulated, and while men's judgments were still unbiased by it, it came to be widely, if not generally, perceived that there is a family likeness between Esthetics, Ethics, and Logic. All three of them are purely theoretical sciences which nevertheless set up norms, or rules which need not, but which ought, to be followed. Now in the case of taste, it is recognized that the excellence of the norm consists exclusively in its accordance with the deliberate and natural judgment of the cultured mind. The best opinion about morality likewise is that it has its root in the nature of the

human soul, whether as a decree of reason, or what constitutes man's happiness, or in some other department of human nature. It is true that there are a few moralists who divorce the source of morality from human nature, but they are forced into a double doctrine; for they are still obliged to say that a man ought to obey his conscience, unless they abandon the very idea of morality. Nothing but the most evident necessity ought to persuade us that rationality, which is allied so closely to taste and to morals, is altogether unlike these in not being the product and expression of human reason. I will remark, at once, concerning this argument that it seems to me devoid of all force, for the reason that morality is far more objective than taste; and taste, morality, rationality, form a true sequence in this order. So that rationality ought according to the true analogy to be purely objective; taste being purely subjective, and morals half subjective, half objective. That morality is far more objective than taste is certainly the primitive judgment of common sense, to which some weight ought to be attached by those who propose to judge of reasons by natural common sense. It is true that the majority of writers on ethics in the past have made the root of morals subjective; but the best opinion is very plainly moving in the opposite direction. On the whole, therefore, the analogy of the trio is rather in favor of the English than the German doctrine.

157. The third argument is that science does not advance by revolutions, warfare, and cataclysms, but by coöperation, by each researcher's taking advantage of his predecessors' achievements, and by his joining his own work in one continuous piece to that already done. Now to suppose that there is any such objective entity as Rationality would be to break away from all modern thought from Descartes down. The English theory does set up such a noumenal Rationality in being dissatisfied with what appears to be rational to the normal human mind. The German might very well say that if the present book is to be taken as an exponent of the opinions that are in the nature of things bound up with the objective view of rationality, as it professes to be, then the break of that doctrine with all modern philosophy is manifest. "If Peirce's exposition of the English doctrine is to be accepted," they might say, "and it is perhaps the only one which goes to the

bottom of its philosophy, then that doctrine requires us to go back to the Aristotelian nonsense of *esse in futuro*, a conception too metaphysical for Hegel himself, which only such clouded intellects as the James Harrises and Monboddos have put up with. Something smacking very strongly of the extravagances of Wilhelmus Campallensis, who endowed abstract ideas with life, will have to be resuscitated in order to hold the parts of this doctrine together. The English thinkers themselves, after the most searching and the most friendly examination of their own philosophical classics, are today coming over to the leadership of Kant and of Hegel. According to Kant, all forms of thought, even those of metaphysical application, have their sole origin in the constitution of the human mind. Hegel has made it clear that if there is to be any philosophy, everything is to be traced back to one single elementary principle, and that the only one principle which can explain everything is thought, intellectual thought. But it is obviously an essential factor of the English doctrine of rationality that there should be a radically irrational element in Fact, an element of brute force, which this new defender of the doctrine pronounces to be anti-general!" All this has its force, without doubt, I acknowledge, as a rhetorical argument from probabilities; but it does not attack the heart of the question.

158. The fourth argument, and the chief one, upon the German side, is that even if we adopt the English doctrine, still, after all, our last and only reliance has to be upon the accuracy of the natural judgments of mind as to what is rational. Practically we must assume that those judgments are infallible. For whatever criterion of rationality we may adopt, in applying that criterion it will be necessary to reason. We thus ultimately rely upon the immediate testimony of the mind as to what is good reasoning; and if we are to rely on that, it is superfluous to call in any other witness. If a given witness, A, testifies of his own knowledge to a fact, the testimony of a second witness, B, concerning the same occurrence, is of no account, if we only know his testimony at second hand, through the report which A makes of it. For the testimony of B has no value unless A tells the truth; and if A tells the truth, his own direct testimony is conclusive. This A represents human reason; B whatever other criterion of rationality may be proposed.

I shall criticize this fourth argument below. But I will at once call attention to some features of it. In the first place, there is a great difference between believing evidence of a given kind and believing it to be infallible; so that if we were obliged to "trust" to the natural judgment of reason (which it will be found below that I dispute), still this would not at all amount to holding that natural judgment to be infallible. In the second place, even if the natural judgments of the mind concerning good and bad reasonings were infallible, that would not prove that rationality *consists* in the fact that certain reasonings are approved by the mind. The Pope is supposed to be infallible in his *ex cathedra* decisions as to what is pleasing to God; but it does not necessarily follow that God's being pleased has no other reality than the Pope's *ex cathedra* pronouncement. In the third place, if witness, A, testifies that he witnessed a fight in which he saw a certain John Doe aim a gun at Richard Roe and pull the trigger and shoot said Richard dead, and if he adds that another person, B, stood at his side and that B feels sure that though John Doe pulled the trigger his gun did not go off; and that Richard Roe did not fall until, a moment later, the third party, William Penn, shot him, it cannot be denied that what A tells about B does weaken his direct testimony. Whence, it follows that if A had testified that B stood at his side and confirmed his account of what he saw in every particular, it would have strengthened his testimony. Consequently, the reasoning that because we are obliged to trust to the direct judgment of the mind concerning the validity of an argument (were this true), therefore no other criterion of that validity assured to us by another direct judgment of reasoning could have any weight whatever, is thoroughly mistaken reasoning.

159. A fifth argument which might be offered in support of the German doctrine, but which the defenders of that doctrine do not seem to think worth notice, appears to me to be fully as strong as any of the other four. This argument would run thus: The universe has a rational constitution. Now if all men, everywhere and always, have unhesitatingly pronounced something to be a necessity of reason, this is a very strong argument that not merely to the human mind, but to mind, as mind, it would be such a necessity. But this, if it be true, is

but another aspect of the proposition that it is inherently rational. Then from this, taken in connection with the rational constitution of the universe, it will follow that a reasoner, trusting to this, cannot find that the facts will disappoint the expectation that this rationality justifies. Accordingly, that accordance of the facts with the promises of an argument which the English regard as *constituting* the rationality of the argument is, in truth, merely a corollary from the argument's being agreeable to Reason, as Reason, of which its catholic acceptance by all human minds is the sufficient evidence. Were this argument urged, my reply to it would be that a reasoner, as such, does not care whether such be the metaphysical constitution of the universe or not. Precisely what he is intent upon is that the facts shall not disappoint the promises of his argumentations. Therefore, the rationality of a reasoning, in the sense of that character of a reasoning at which the reasoner aims, does consist precisely in that necessary accord of the facts with the professions of the argument, in which the English doctrine makes it to consist.

160. I will now endeavor to state the English doctrine as I understand it, with such psychological additions as seem to be desirable in the present somewhat psychological discussion. It would seem, then, that in the simplest cases, the natural judgments of the mind in regard to what is logically necessary are as nearly infallible as are our ready judgments upon the simplest parts of the multiplication-table. They seem to be formed, too, in much the same way. Namely, in regard to the multiplication-table, we form our little diagram, perform our little experiment, and generalize the result under a conviction that what is true of one diagram of skeletal simplicity will be equally true, however complicated the nature of the units counted may be. So in the logical case, we imagine a simple case in which we should be called upon to act upon one rule or its denial, and we perceive that one of these rules would cause us to act at cross-purposes with ourselves. This is generalized under the conviction that it will make no difference what the nature of the case may be, so long as the logical relation remains the same. In more difficult cases, our generalizing conviction is less decided. We become doubtful. Our diagrammatic representation does not so accurately fit the case. Nevertheless, such

judgment as we do form is formed precisely according to the English theory. That is to say, we consider as well as we can, how we ought to form our inference so as not to be disappointed in the result. It is simply a confused judgment of the same nature. In considering, for example, the case of a dweller upon the shores of an inland sea who should arrive at the Bay of Biscay and see the tide rise for several times successively, and asking what the nature of the reasoning should be, common sense finds itself confronted with a problem which is too difficult for it, just as it was too difficult for Laplace and his successors. Inappropriate conceptions will be applied by common sense just as they were applied by the Laplacians. The only advantage of common sense will be that not having its attention drawn away to numerical rules, it will be a little less likely to overlook some considerations that the mathematicians overlooked in consequence of their [the considerations] being beyond the somewhat arbitrary limits of their [the mathematicians'] scales of measurement. There is probably no special instinct — using this word in a sense in which it shall embrace traditional as well as inherited habits — for rationality, such as there is for morality. Or, if there be such an instinct, it is of the most rudimentary and imperfect character. If not in all cases, at least in all cases which present any difficulty, we see natural judgments falling into the same errors into which early logicians fell. The common sense judgment concerning valid inference, so far as it is correct, seems to be nothing but a confused thinking of precisely the same nature as the English doctrine recommends. Common sense notions of logicality are in remarkable contrast to instincts in their want of obstinacy. Nothing so characterizes instincts as their persistence when all the lights of reason are against them, and this whether they are true inherited instincts or merely traditional. Well-bred people, for example, are full of traditional habits—prejudices, we call them — about manners. They may be in situations in which reason warns them that these habits are distinctly injurious to them; and still they have a difficulty in overcoming them even with a serious effort. The absence of any such phenomenon in regard to common sense judgments of logicality is striking. As soon as it appears that facts are against a given habit of reasoning, it at once loses its hold, without any such

obstinate and irrational persistency as shows itself, not only in questions of morals, but even in mere matters of manners, such as *corporal pudor*. No matter how strong and well-rooted in habit any rational conviction of ours may be, we no sooner find that another equally well-informed person doubts it, than we begin to doubt it ourselves. This is plainly shown by the anger such doubt excites in us. If our own belief were not disturbed, there is no reason why we should care what others believe; and if we are quite sure that we are better acquainted with the subject than are the doubters, their doubts will in fact provoke nothing more than a good-natured smile. Contrast this with our sentiments if, in a parlor in some Pacific Island, one of the ladies should call for a dry gown and take off the only garment she had on before the company, preparatory to putting on the other, without the others appearing to see anything out of the common course in it. All sorts of shivers would run through us; but there would not be a trace of anger, for the reason that our own instincts — or rather, comparatively modern traditions — would remain absolutely undisturbed.

161. The English logician, therefore, not only maintains that the proper way of deciding whether a given argument is valid or not is to consider whether there is anything in the constitution of the universe and the nature of things which insures that the facts shall be such as the argument promises that they shall be, but he goes so far as to maintain that our ordinary common sense judgments respecting the validity of arguments are formed in the same way, and differ from the judgments of scientific logic only in resting upon vaguer and less distinct thought. To ask him, therefore, to abandon his method of estimating the validity of arguments in favor of the German method of appeal to natural judgments of good sense, is, in his apprehension, neither more nor less than to ask him to abandon exact thinking for a kind of thinking which differs from it in no respect except that of being loose and confused.

162. The Germans, as everybody knows, have put all the rest of the civilized world deeply in their debt by producing the completest treatises upon almost all departments of science. Logic is no exception. The number of deeply thought out and original systems of logic which were elaborated in Germany

during the nineteenth century greatly exceeds that of such works produced in the same age in all the rest of the world. These German treatises have been great aids to me in writing the present book. This circumstance intensifies my desire to explain why it is that I not only constantly dissent from their doctrine, but am excited against them as pernicious writings. It is not their fault that they are pernicious. Contributions to philosophy might, as a general rule, almost be said to be useful in proportion as they are in error. For philosophy is still far from having reached the status of a mature science progressing in an orderly manner. It is still in that stage in which fundamental principles are disputed; and as long as that is the case, dispute is the most important business of a philosopher. But the only satisfactory refutation of a false system is that which is effected unintentionally and unconsciously by the vigorous defender of that system. He alone can make the falsity of it perfectly clear. But great genius and a great logical power are requisite for this. Few or none of the German logics are as strong as it is desirable that they should be. Still, their influence were wholesome, if it were not for the extraordinary deference which is paid to them by university men in this country and others. Deference to a treatise on logic is altogether out of place. It is an injury and almost an insult to the author. In another science this is not so true, because the author's long experience in his study justly gives a weight to his opinions beyond what can be attached to any reasons which he can express. But logic is purely a science of reasoning. The author is entitled to just so much approval as what he writes warrants, and no more. Moreover, in the present unsettled state of his science, it is his business to dispute, to give blows and take them; and it is no compliment to him to treat him tenderly. This, however, is the most innocent side of any deferential treatment of a work on logic. It is a terrible wrong to young students, who ought to be encouraged, urged, and if possible driven, to examine such works carefully, boldly, and actively — I had almost said aggressively; but that would be going too far. It is especially injurious to treat those works on logic deferentially whose doctrine is radically unsound in respect to the modes of argumentation which they approve and disapprove. This in my opinion is the case with the German

treatises. At the same time, they are certainly very carefully written and well studied works. Certain general causes have operated to set their logic wrong. In the first place, the German mind is very subjective. A man must be a poor observer of men not to be able to see that. In the physical and natural sciences this tendency is held in check and prevented from doing much harm, while it has the effect of producing circumspection, care, attention to all those precautions upon which success in those sciences largely depends. There are some of the psychical sciences which are so positive and objective that a dash of subjectivity is ordinarily a wholesome corrective. But as a general rule German subjectivity appears to disadvantage in the psychical sciences. The German students of these sciences, as a class, are apt to be carried away by theories and to draw their teachings too much from the *Ichheit*, as has been commonly remarked. In mathematics subjectivity can apparently do no harm. In the present unsettled state of logic, the effect of subjectivity is lamentable. It causes appeal to be made to feeling, to prejudice, instead of to fact, and blocks any real advance. It renders logic perfectly nugatory as far as any helpfulness to science is concerned, until at length this nugatoriness crystallizes into a *laissez-faire* policy. The course of science evidently could not be placed under any intellectual constraint. Scientific men will reason as they see fit. But they would see fit to listen to the counsels of logicians, if those counsels seemed to have a truly scientific basis. If any department of science is allowed to go on without any logical deliberation other than such as its own special studies lead it into, the result will be that, after more or less blundering, it will ultimately get its methods adapted to its temporary condition, only to plunge into new bewilderment when that temporary condition is outgrown. For the science, under such a policy, is guided by natural logicality corrected by experience. Now this natural logicality is precisely like scientific logic, except that it is thought hazily and inaccurately. It will ultimately lead to the same result, only with great waste of time and means of all sorts. Provided the truly scientific logic were put before the world, the subjective treatises could do no harm except that of occupying men's time in the examination of them, provided they were carefully examined and judged upon

their merits. It is the deference paid to them because they are written by supposed-to-be renowned German professors that really does the harm.

163. The subjective tendency of the German mind consists either in a hyperaesthesia, or else in an excessive attention to feelings in their immediacy, or more probably to both causes, each stimulating the other. The energy of the soul, being carried in that direction, is taken away from reasoning. This naturally would produce an effect which observation shows is somehow produced, namely, that the average German is not logically a very bright mind. Perhaps this contributes toward causing him to ponder very deeply any subject which occupies him; and since every man naturally talks most about subjects on which he has thought most, the German's natural dullness is in large part concealed. Nevertheless, it cannot escape a close observer. All the world over, it may be remarked that men who write logics are not on the average very good reasoners. Some men, no doubt, are led into the study of logic because their minds are naturally subtile; but there are more of whom one would guess that they must have been drawn to pay attention to logic because reasonings puzzled them which would not have puzzled an ordinarily intelligent man. These faults of reasoning to which the ordinary logics draw most attention, confusions between *all* and *some*, would seem to be so glaring that no man who really thinks could commit them, except just as he might, now and then, hastily write down that 5 and 6 make 2. The only way in which an intelligent man could *deliberately* commit any such fallacy would be by the inaccurate application to the case of some rule of logic. Now a most singular phenomenon characterizes all the German logics of the nineteenth century which I have examined — certainly, considerably over fifty of them — and distinguishes them from those of the English. It is that every one of them somewhere falls into a logical fallacy. Such things are not matters of opinion: they are indisputable. Besides, each such fault on the part of one German logician is condemned — tacitly, at any rate — by the general body of German logicians; for I am not now speaking of errors of reasoning which have been deliberately defended by this or that school of logicians: I am speaking of indefensible faults. In order to prove my as-

sertion that German logicians are universally capable of such faults, I intend to draw up a list of some of them and append it to this volume.* I hope it will serve to counteract the silly deference paid in this country to German logic — an attitude which no logician worthy of the name would desire that any student should assume toward his work.

164. Let us now briefly reëxamine the five arguments in favor of the German position. For in the light of what has been said their general character can better be appreciated. The first is that it is absurd to approve or disapprove of a physiological operation. I heartily endorse that remark. Every man who has learned to look upon nature from the physiological point of view must share the feeling. For my part, it so burns in my heart that if I could, I would abolish almost all punishment of grown people, and all judicial approval or disapproval except of the court's own officers. Let public opinion have its approvals and disapprovals, until public opinion learns better. But as for public force, let it be restricted to doing what is necessary to the welfare of society. Punishment, severe punishment, the barbaric punishment of a prison cell, infinitely more cruel than death, is not in the least conducive to public or to private welfare. As for the criminal classes, I would extirpate them, not by the barbarous method which some of those monsters whom economics has evolved propose, but by keeping the criminals confined in relative luxury, making them useful, and preventing reproduction. It would be easy to convert them from a source of enormous expense, and perpetual injury to people, into self-supporting harmless wards of the state. The only expense would be that of losing our darling revenge upon them. As for sporadic criminals, defalcators, murderers, and the like, I would deport them to an island, and leave them to govern themselves, and deal with one another. For trifling violations of order, trifling punishments might be retained.

165. I grant, then, that the feeling that it is absurd to approve or disapprove a physiological operation is a good and sound one. But this very attitude implies that *something* is to be approved and something disapproved. This being recognized, we are led to endeavoring to define what it is that may

* No such list has been found.

properly be approved or disapproved. The answer is easy enough. We approve of means of bringing about purposes which we embrace, assuming it to be in our power to adopt or to reject those means. As to the purposes themselves, every man must decide for himself, though others may offer suggestions. A physiological operation takes place under nature's laws, and is beyond our control. It is, therefore, idle to approve or to disapprove of it. But in this essential respect reasoning is not a physiological operation; being a method, perfectly under our control, of attaining a definite end, that of ascertaining how future phenomena will appear. As to the purpose of a physiological operation, we know nothing, unless we may presume that it is designed to perform the function which it does in fact perform. The Germans, with their disposition to look upon everything subjectively, that is, in its immediacy as feeling, are willing that we should like or dislike reasoning, as we might a piece of music; but they look upon the natural judgment of rationality as a mere judgment of feeling, overlooking the fact that the natural man, as much as the English logician, desires to ascertain the true facts, looks upon reasoning as a means of doing this, and adopts the mode of reasoning that he does adopt, because, as well as the confused state of his ideas enables him to judge, it must bring him to the truth, as a rule.

I have already said enough of the second argument which seeks to assimilate Logic to Esthetics. Its subjective leaning is manifest.

166. To the third argument, that the English view of logic involves a rupture with all modern metaphysics, I have as yet made no reply. This argument is not directly connected with the subjective attitude of thought, but is motived by another German tendency which has a rational foundation, though it takes a particular turn in Germany chiefly from an accidental cause, yet influenced in some degree by German subjective ways of feeling. It is quite true that the success of modern science largely depends upon a certain solidarity among investigators. I trust that the time is not far distant when logic may enter upon that path. All my efforts have, from the beginning of my career, been directed to that desirable result; and the logic of relatives has already become, partly by my labors, an

accepted division of science.* But such harmonious coöperation supposes that fundamental principles are settled. Until they become so, dispute has to be the method in which a given science shall make its way to the light. Unfortunately, in logic, opinion upon fundamentals is all at sixes and sevens. The natural effect of a disposition to take subjective views is to stimulate a spirit by which men fall into agreement in their philosophical sentiments from sympathy and as a matter of fashion. There is a deal of fashion in the prevalence of ideas in the German universities. The fact that a professor's revenue depends upon his estimation in the minds of young men who are incompetent to form any rational opinion concerning the value of his ideas is probably the chief cause which operates to make a good deal of irrational uniformity of opinion throughout the German universities. This is aided, however, by the sympathetic spirit of the German, which is an effect of his subjectivity. For a whole generation, ideas that were not Hegelian were looked upon throughout the German universities with the same utter contempt with which ideas that are Hegelian are now regarded in those same halls. But one and the same spirit has always been prevalent there; namely, to settle first what metaphysical ideas are agreeable to reason, that is, in effect, to the spirit of the day, and to shape the science of logic to fit those ideas. That method is no less than preposterous. The only rational way would be to settle first the principles of reasoning, and, that done, to base one's metaphysics upon those principles. Modern notions of metaphysics are not rationally entitled to any respect, because they have not been determined in that way, but on the contrary by a purely accidental circumstance, or by a circumstance which, so far as it can be looked upon as a reason, is a reason against rather than in favor of modern metaphysics. Namely, it came about in this way. In the Middle Ages, the great study in France and England, the countries that were destined to intellectual predominance in the early modern centuries, was Theology. The metaphysics of Aquinas, a modified Aristotelianism, had been immensely elaborated and deeply transformed by the vast logical genius of the British Duns Scotus, who died in 1308.

* See, e.g., Schröder's *Algebra der Logik*, 3.1; and the *Principia Mathematica*, *23ff.

The extreme intricacy of this system was felt to be an objection to it, and various attempts were made to introduce Nominalism — the simplest possible of all Logico-Metaphysical theories, if it can be sustained. These efforts finally culminated in the system of another Englishman, William Ockham. Scotism permitted assent to almost any theological dogma; it was specially favorable to those which involved mystery; it was naturally liked by ultramontanes and extreme clericals. Ockhamism, on the other hand, was naturally unfavorable to anything mysterious, not to say, to anything religious. Ockham was, if possible, more renowned as a bold opponent of Papal encroachments than as the inceptor of a new way of thinking. The result of the struggle that ensued was that when the new learning came, and science began to awaken, it was the Scotists who were in possession of the Universities. This accidental position it was, more than anything else, though the theological tendency of the doctrine aided, that caused the new scholars to look upon the Dunces as the ignorant fogies, *par excellence*, the principal enemies of all learning, progress and good sense. It was the intense hostility of the new men to fogyism, chiefly, which pushed them into a metaphysics of an Ockhamistic color. At any rate, no mistake can be greater than to suppose that Ockhamistic thought is naturally allied to the conceptions of modern science: it is antiscientific in essence. A scientific man whose only metaphysics has been such as his own studies have suggested will be definitely adverse to the ideas of Ockham, and, so far as his simple conceptions go, will agree with Scotus. Of course, he will know nothing of the opinions that are distinctively Scotistic, for so far as these are not technically logical they have in view theology.

167. In the Middle Ages the question between Scotism and Ockhamism had been closely argued. Had the conceptions of modern science been present to the minds of the disputants, the victory of the Scotists would have been more overwhelming than it was. As matters went, Ockhamism derived its chief strength from its political alliance.

168. So this is the way in which modern philosophy became pushed into Ockhamism. The new thinkers were incapable of the subtle thought that would have been necessary for any

adequate discussion of the question. They accepted nominalistic views upon the most superficial grounds. The question soon became buried and put out of sight by new questions which overlaid it, like new papers on an encumbered study table. In that way it has happened that the question has never attracted general, acute attention among modern metaphysicians. What is there in that genealogy which entitles the prevalent metaphysics to one feather's weight of rational authority? Authority is a thing not to be lightly introduced into science. Moreover, metaphysics ought to be founded on logic. To found logic on metaphysics is a crazy scheme. It was on logical grounds, and in a treatise on logic, that Ockham himself supported his nominalism.* For though his system had its faults, he had a more cultivated comprehension of the architecture of philosophy than have those modern Germans who boast that their systems of logic are "philosophical," that is, founded on metaphysics.

169. The fourth German argument is that we must, in any case, trust to the veracity of the natural instinct for rationality, and that consequently rationality consists in that judgment. I have already completely refuted that argument; but there is one point to be made against it which I have reserved for this place, because it so well illustrates the modes of thinking of the German logicians. Suppose this case. In the course of a long country ramble, I meet a boy whom I never saw before, and inquire my way of him. I take the turn which he directs me to take, but have not gone many steps further before I am overtaken by a man who informs me that that boy's mother has been arrested upon a charge of perjury and that I am wanted to testify to her veracity. "But," I say, "I never so much as heard of the woman. I did not know she existed." "Oh," says the man, "you must have known the boy had had a mother; and since you took the turn he told you to take, you are bound to acknowledge that the mother tells the truth, for it was she who had told the boy where this road leads to." What should I say to the man? I should say, "My dear sir, if you had asked me to testify that the woman *cannot* lie, for the reason that truth *consists* in her say-so, then I should think that your demand was as unjustifiable as any ever made of

* In his *Logica*, e.g., in I, xiv.

me." That would be an exaggeration, since his demand would not be in truth open to all the objections to which the argument of the German logicians is open; but as far as it goes, the analogy would be perfect. Somebody shakes a pair of dice in a dice-box and asks me to guess whether his next throw will be doublets or not. Before replying I make a mental diagram of all possible throws, and relying on that, I reply that I guess the throw will not show doublets. This is unquestionably a probable inference. In making it, the only thing that I am conscious of relying upon is my mental diagram, as representing the probable course of experience. That diagram takes the place of the boy of whom I inquired my way. But now the German logician assures me that the advice to guess against the doublets really originated in an instinct for rationality, of the existence of which I had no assurance, and have not yet; except that, as he says, the authority of the diagram must have had some origin, just as the boy must have had some mother. Although the dice have not yet been cast, and all I know is that my guess seems reasonable, I am asked to testify to the world and to myself, not only that this unheard-of instinct spoke truly on that occasion, but that it does so invariably: nay, that the truth of reasonings *consists* in that instinct's saying that they are true. Outside of a German treatise of logic, I never met with so bald a fallacy as that. So this is the sort of reasoning that it is thought not decent to controvert, unless it be with hesitancy, with the utmost humility, and with an implied acknowledgment of the impropriety of a mere American's controverting the opinion of a German.

170. If I may be allowed to use the word "habit," without any implication as to the time or manner in which it took birth, so as to be equivalent to the corrected phrase "habit or disposition," that is, as some general principle working in a man's nature to determine how he will act, then an instinct, in the proper sense of the word, is an inherited habit, or in more accurate language, an inherited disposition. But since it is difficult to make sure whether a habit is inherited or is due to infantile training and tradition, I shall ask leave to employ the word "instinct" to cover both cases. Now we certainly have habits of reasoning; and our natural judgments as to what is good reasoning accord with those habits. I am willing to grant

that it is probable that some of our judgments of rationality of the very simplest kind have at the bottom instincts in the above broad sense. I am inclined to think that even these have been so often furbished up and painted over by reflection upon the nature of things that they are, in mature life, mostly ordinary habits. In more complicated cases, say for example, in that guess about the pair of dice, I believe that our natural judgments as to what is reasonable are due to thinking over, ordinarily in a more or less confused way, what would happen. We imagine cases, place mental diagrams before our mind's eye, and multiply these cases, until a habit is formed of expecting that always to turn out the case, which has been seen to be the result in all the diagrams. To appeal to such a habit is a very different thing from appealing to any immediate instinct of rationality. That the process of forming a habit of reasoning by the use of diagrams is often performed there is no room for doubt. It is perfectly open to consciousness. Why may not all our natural judgments as to what is good reasoning be founded on habits formed in some such ways? If it be so, the German doctrine falls to the ground; for to form a notion of right reasoning from diagrams showing what will happen, is to form that notion virtually according to the English doctrine of logic, by reasoning from the nature of things. That is to say, a habit is involuntarily formed from the consideration of diagrams, which process when deliberately approved becomes inductive reasoning. Unless there be, in addition, some immediate instinctive feeling of rationality, the German theory cannot be correct. Yet proof of the existence of such an additional instinctive feeling is not forthcoming. Not even so much as a pretended proof of it is offered, nor so much as any likelihood of it, so high do these great German logicians hold themselves above the usual obligations of scientific logic.

171. On the contrary, pretty strong evidence is at hand that no such instinctive feeling exists. In the first place, our natural judgments as to what is good reasoning are accompanied by a sense of evidence: one thinks one *sees* that the fact is so and *must be* so, not merely that *we* cannot help thinking so. In this respect, these judgments contrast strongly with those of conscience. You and I have a horror of incest. We have been told that there is a reason for it; but that is open to doubt.

Reason or no reason, however, our aversion for and horror at the idea is simply felt, without any accompanying sense of evidence. So it is with what offends our taste. "I do not like you, Dr. Fell." I have no accompanying sense of its being a well-founded feeling. In regard to a simple syllogism it is quite otherwise. It is no blind, unaccountable impulse to reason in that way that I feel. I seem to perceive that so the facts must be. This difference between judgments of taste and morals on the one hand and of *rationality* on the other can hardly be accounted for on the German theory.

172. If, however, as the English suppose, the feeling of rationality is the product of a sort of subconscious reasoning — by which I mean an operation which would be a reasoning if it were fully conscious and deliberate — the accompanying feeling of evidence may well be due to a dim recollection of the experimentation with diagrams. There are many other facts which point in the same way, of which I will only mention one which seems almost conclusive. This is that if we practise logical reflexion according to the English method and are thus led to see that a certain method of reasoning promises nothing more than facts must from the nature of things bear out, we do not find that we have two distinct judgments of what is rational. If I am persuaded that incest will have deplorable effects upon off-spring, I feel a distinctly duplex condemnation of the practice, the one of a cool, almost sceptical kind, the other peremptory and without apology. There are some questions about which I, and I suppose it is the same with every thinking man, find these two voices quite at odds, my reason temperately but decidedly asserting that I ought to act in one way, my instincts, whether hereditary or conventional I cannot tell, most emphatically and peremptorily, though with no pretence to rationality, giving reason the lie. That *is* just such a phenomenon as would naturally be anticipated. It is very surprising that I do not find any such discord in my judgments as to what is good reasoning. There are various points in which my present opinions of what is good reasoning differ diametrically from those which I entertained before I had analyzed the matter after the English doctrine. But instead of my old feeling continuing to assert itself beside my new rationalized opinion, to say that it submits with docility is not enough: I cannot detect

the least trace of it remaining. If it were an immediate feeling, as the Germans suppose, it certainly would persist. The only possible explanation of its not doing so is that it was merely itself a confused conclusion of subconscious reason which feels itself superseded by clearer analysis of the same kind and along the same line.

173. I have discussed this matter at some length, because it is a momentous question for logic; and it seemed proper to turn it over upon different sides. But in truth the essence of the matter lies in a nutshell. Facts are hard things which do not consist in my thinking so and so, but stand unmoved by whatever you or I or any man or generations of men may opine about them. It is those facts that I want to know, so that I may avoid disappointments and disasters. Since they are bound to press upon me at last, let me know them as soon as possible, and prepare for them. This is, in the last analysis, my whole motive in reasoning. Plainly, then, I wish to reason in such way that the facts shall not, and cannot, disappoint the promises of my reasoning. Whether such reasoning is agreeable to my intellectual impulses is a matter of no sort of consequence. I do reason not for the sake of my delight in reasoning, but solely to avoid disappointment and surprise. Consequently, I ought to plan out my reasoning so that I evidently shall avoid those surprises. That is the *rationale* of the English doctrine. It is as perfect as it is simple.

174. It is now scarcely worth while to notice that opinion which acknowledges the English doctrine, yet prefers the authority of natural judgments concerning reasoning to those of analytic reason. It would be all very well to prefer an immediate instinctive judgment if there were such a thing; but there is no such instinct. What is taken for such is nothing but confused thought precisely along the line of the scientific analysis. It would be a somewhat extreme position to prefer confused to distinct thought, especially when one has only to listen to what the latter has to urge to find the former ready to withdraw its contention in the mildest acquiescence.

§7. REASONING AND CONDUCT

175. But while there is no independent instinctive sense of logicality, it is impossible to deny that there are instinctive

ways of forming opinions, especially if we continue to take instinct in that broad sense in which it will include all habits of which we are not prepared to render an account, or in one word all that goes by the name of the rule of thumb. In applying such instincts, such habits of unknown parentage, we do reason a little. But that little reasoning is based on some axiom or impression of opinion which we adopt uncritically, without any assurance that it is rational.

176. The popular notion is that Reason is far superior to any instinctive way of reaching the truth; and from your desire to study logic, I am perhaps warranted in presuming that such is your opinion. If so, in what respect do you hold reasoning to be superior to instinct? Birds and bees decide rightly hundreds of times for every time that they err. That would suffice to explain their imperfect self-consciousness; for if *error* be not pressed upon the attention of a being, there remains little to mark the distinction between the outer and the inner worlds.* A bee or an ant cannot — could not, though he were able to indulge in the pastime of introspection — ever guess that he acted from instinct. Accused of it, he would say, "Not at all! I am guided entirely by reason." So he is, in fact, in the sense that whatever he does is determined by virtual reasoning. He uses reason to adapt means to ends — that is, to his inclinations — just as we do; except that probably he has not the same self-consciousness. The point at which instinct intervenes is precisely in giving him inclinations which to us seem so singular. Just so, we, in the affairs of everyday life, merely employ reason to adapt means to inclinations which to us appear no more bizarre than those of a bee appear to him.

An old friend of mine once remarked to me that if a being not human were to observe mankind, he would be struck with admiration at that instinct which leads a large number of men each to contribute an insignificant sum so as to make up a fortune in the aggregate, and present it to one person chosen by lot; although certainly men who buy tickets to a lottery refrain from taking credit for their highly altruistic conduct with a modesty which does credit to their hearts. In the ordinary conduct of everyday affairs, men really do act from

* Cf. 5.234.

instinct; and their opinions are founded on instinct in the broad sense in which I here take that term. A small dose of reasoning is necessary to connect the instinct with the occasion: but the gist and character of their conduct is due to the instinct. It is only a remarkable man or a man in a remarkable situation, who, in default of any applicable rule of thumb, is forced to reason out his plans from first principles. In at least nine such cases out of every ten, he blunders seriously, even if he manages to escape complete disaster. We shall therefore be well within bounds in pronouncing Reason to be more than a thousand times as fallible as Instinct.

177. Invariably follow the dictates of Instinct in preference to those of Reason when such conduct will answer your purpose: that is the prescription of Reason herself.* Do not harbor any expectation that the study of logic can improve your judgment in matters of business, family, or other departments of ordinary life. Clear as it seems to me that certain *dicta* of my conscience are unreasonable, and though I know it may very well be wrong, yet I trust to its authority emphatically rather than to any rationalistic morality. This is the only rational course.

178. But *fortunately* (I say it advisedly) man is *not so happy* as to be provided with a full stock of instincts to meet all occasions, and so is forced upon the adventurous business of reasoning, where the many meet shipwreck and the few find, not old-fashioned happiness, but its splendid substitute, success. When one's purpose lies in the line of novelty, invention, generalization, theory — in a word, improvement of the situation — by the side of which happiness appears a shabby old dud — instinct and the rule of thumb manifestly cease to be applicable. The best plan, then, on the whole, is to base our conduct as much as possible on Instinct, but when we do reason to reason with severely scientific logic. It has seemed to me proper to say this in order that I might not be understood as promising for logic what she could not perform. Where reasoning of any difficulty is to be done concerning positive facts, that is to say, not mere mathematical deduction, the aid that logic affords is most important.

* Cf. 1.672.

§8. REASONING AND CONSCIOUSNESS

179. You opine that you yourself reason and are conscious of doing so. Undoubtedly, this is true in the sense that would naturally attach to the words. Yet it will be well, in beginning the study of logic, to be upon one's guard against sliding unawares from holding this to be undeniable in the ordinary sense in which it certainly is so, to holding it to be equally indubitable in another psychological sense.

180. Unless you are an old fox at logic such as the world does not harbor very many of, if we are to judge by what gets into print, or unless you are an extraordinarily vigorous thinker, you probably over-estimate very considerably the amount of logical reasoning which you perform. I know men justly renowned all over the civilized world as vigorous thinkers of whom I have satisfied myself that they perform little really logical thinking. They pass from premiss to conclusion of an argument, contenting themselves with remarking that that argument has the general appearance of belonging to the same class as many arguments of which they have had experience and which turned out to lead to true conclusions, but which they have never so analyzed as to be quite sure what conditions have to be fulfilled to necessitate their validity. This may be called reasoning; and if the reasoner were to recognize its vagueness and fragility, it might even be called logical reasoning of an excessively feeble kind. An argument which professes no more than it performs is sound, however little its performance may be. The difficulty is that the men who reason so do not recognize what it is that they are doing, but take the vague likeness of their argument to others that have succeeded for a perception of its necessary rigor, consequently telling themselves that they are reasoning in a very weighty and exact manner. Now an argument which takes itself as having a sort of weight of which it is really devoid, flying false colors, is not a good argument.

181. Of excessively simple reasonings a great deal is done which is unexceptionable. But leaving them out of account, the amount of logical reasoning that men perform is small, much smaller than is commonly supposed. It is really instinct that procures the bulk of our knowledge; and those excessively simple reasonings which conform to the requirements of logic

are, as a matter of fact, mostly performed instinctively or irreflectively.

182. Reasoning, properly speaking, cannot be unconsciously performed. A mental operation may be precisely like reasoning in every other respect except that it is performed unconsciously. But that one circumstance will deprive it of the title of reasoning. For reasoning is deliberate, voluntary, critical, controlled, all of which it can only be if it is done consciously. An unconscious act is involuntary: an involuntary act is not subject to control; an uncontrollable act is not deliberate nor subject to criticism in the sense of approval or blame. A performance which cannot be called good or bad differs most essentially from reasoning.

183. This does not imply that we must be aware of the whole process of the mind in reasoning or, indeed, of any portion of it. It is very desirable to have a clear apprehension of this distinction. We are, so to speak, responsible for the correctness of our reasonings. That is to say, unless we deliberately approve of them as rational, they cannot properly be called reasonings. But for this purpose, all that is necessary is that we should, in each case, compare premisses and conclusion, and observe that the relation between the facts expressed in the premisses involves the relation between facts implied in our confidence in the conclusion. What we call a reasoning is something upon which we place a stamp of rational approval. In order to do that, we must know what the reasoning is. In that sense, it must be a conscious act, just as a man is not bound by a contract if it can be proved that he signed it in his sleep. It must be his conscious act and deed. But for that purpose he only needs to know the character of the relation between the premisses and the conclusion. He need not know precisely what operations the mind went through in passing from the one to the other. That is a matter of detail which is not essential to his responsibility. The mind is like the conveyancer who has drawn up a deed. What books he looked into in choosing his verbiage is no concern of the person who signs, provided he knows what the paper binds him to doing.

184. I do not believe that, in the present state of psychology, anybody knows much about the operations of the mind in reasoning. There is no easy way in which such knowledge

can be had. There are those who say that all we have to do for this purpose is simply to watch what passes through consciousness. This word "consciousness" deceives them. It is as if someone were to argue that because science undertakes only to infer what is phenomenal, and phenomena are simply appearances, therefore, in order to make out what the processes of digestion are, it is only necessary to strip a man and watch his belly. All that we can find out by directly watching consciousness are the qualities of feeling, and those, not as they are felt, but as, after being felt, they are grouped. I can tell myself what the steps of a reasoning were; that is, I can enumerate some premisses and conclusions that were successively introduced; just as another man might give me the same information about his reasoning. It is difficult to make sure that such propositions actually passed through the mind. If they did, they were probably resting-places where the mental action ceased, and results were recorded, so that they could be recovered when the thinking should recommence.

185. At any rate, a knowledge of the processes of thinking, even if it were at hand, would be entirely irrelevant to that sort of knowledge of the nature of our reasonings which it is incumbent upon us to have in order that we may give them our deliberate approval.

These are points of importance for understanding the English, objective conception of logic.

§9. *LOGICA UTENS*

186. The opinions which you bring to the study of logic comprise among them a system of logic all made, although it is probably a little vague, in places. You know that this is substantially so, presuming that you are a reflective person, as you doubtless are; but you may perhaps be surprised that I should be so confident that it is so. It is simple enough, however. You would certainly not be interested in logic unless you were somewhat given to reasoning; probably not without being more or less addicted to self-observation. Now a person cannot perform the least reasoning without some general ideal of good reasoning; for reasoning involves deliberate approval of one's reasoning; and approval cannot be deliberate unless it is based upon the comparison of the thing approved with some

idea of how such a thing ought to appear. Every reasoner, then, has some general idea of what good reasoning is. This constitutes a theory of logic: the scholastics called it the reasoner's *logica utens*. Every reasoner whose attention has been considerably drawn to his inner life must soon become aware of this.

187. He, therefore, comes to the study of logic handicapped by a conceit that he knows something about it already, but, at the same time, aided by his being able to handle questions of logic with some confidence and familiarity. He ought to endeavor to suppress his conceit while preserving his disposition to think independently.

§10. *LOGICA UTENS* AND *LOGICA DOCENS*

188. But the fact that you are sincerely desirous of studying logic shows that you are not altogether satisfied with your *logica utens* nor with your powers of estimating the values of arguments. Of course, there is no good of entering upon any undertaking unless one desires the sole rational purpose of that undertaking, and is consequently more or less dissatisfied with one's present condition in that respect.

189. It is foolish, therefore, to study logic unless one is persuaded that one's own reasonings are more or less bad. Yet a reasoning is essentially something which one is deliberately convinced is good. There is a slight appearance of contradiction here, which calls for a little logic to remove it. The substance of an opinion is not the whole opinion. It has a mode. That is to say, the opinion has been approved because it has been formed in a certain way, and of opinions formed in that way, we have the opinion that relatively few are much in error. It is for that reason that we have adopted the opinion in question. Still, we attach but a limited degree of confidence to it, being of the opinion that out of a considerable number of opinions formed in the same way, some would probably be grossly erroneous. In this way, it might happen that you should hold that a large minority of your reasonings were bad, although you were inclined to adhere to each one singly. This is the general principle. But logicians are too apt to content themselves with the statement of general principles, and to overlook peculiar effects which may arise from complications

of them. The real situation in this case is too complicated to be considered to advantage; but we can illustrate the general way in which complexity may modify the effect of our general principle. Your reasonings are determined by certain general habits of reasoning, each of which has been, in some sense, approved by you. But you may recognize that your habits of reasoning are of two distinct kinds, producing two kinds of reasoning which we may call A-reasonings and B-reasonings. You may think that of the A-reasonings very few are seriously in error, but that none of them much advance your knowledge of the truth. Of your B-reasonings, you may think that so many of them as are good are extremely valuable in teaching a great deal. Yet of these B-reasonings you may think that a large majority are worthless, their error being known by their being subsequently found to come in conflict with A-reasonings. It will be perceived from this description that the B-reasonings are a little more than guesses. You will then be justified in adhering to those habits of reasoning which produced B-reasonings, by the reflection that if you do adhere to them, the evil effects of the bad ones will be mainly eliminated in course of time by opposing A-reasonings, while you will gain the important knowledge brought by the few B-reasonings that are good; whereas, if you were to discard those habits of reasoning which produced B-reasonings you would have nothing left but A-reasonings, and these could never afford you much positive knowledge. This imaginary illustration will serve to show how it might be that you should, with perfect consistency, hold your existing *logica utens* to be excessively unsatisfactory, although you are perfectly justified in adhering to it until you are in possession of a better system. Without knowing anything of your individual case, my general observation of the manner in which men reason leads me to believe it most probable that the above illustration about the A-reasonings and the B-reasonings represents, in a general way, your condition, except that you greatly overrate the value of many of the B-reasonings, which are really little more than guesses at truth, but are, many of them, regarded by you as inductions. If this be the case, a study of logic, while making your whole thought more accurate, will enable you to rate your B-reasonings more accurately, and to substitute for about half of them

reasonings that will not often deceive, while greatly improving the quality of those that will still remain more or less conjectural. This improvement will, however, be limited to logical reasonings; and of such you perhaps do not perform a great many. Those acts of the mind which chiefly depend upon instinct will remain unaffected, except that their true character will be recognized.

190. I am, in all this, adapting my tone to your probable state of feeling, that the principal value of logic lies in its supplying an art of reasoning; but I trust that by the time you have read this book through, you will have come to feel, as I do, that the greatest value of the science is of another and higher kind.

§11. THE IMPROVEMENT OF REASONING

191. In studying logic, you hope to correct your present ideas of what reasoning is good, what bad. This, of course, must be done by reasoning; and you cannot imagine that it is to be done by your accepting reasonings of mine which do not seem to you to be rational. It must, therefore, be done by means of the bad system of logic which you at present use. Some writers fancy that they see some absurdity in this. They say, "Logic is to determine what is good reasoning. Until this is determined reasoning must not be ventured upon. (They say it would be a "*petitio principii*"; but I avoid the technicality which must be unmeaning at the present stage of our inquiry, even if it really has any meaning, beyond my rendering of it.)* Therefore, the principles of logic must be determined without reasoning, by simple instinctive feeling." All this is fallacious. It springs from the wide-spread tendency, derived ultimately perhaps from Platonic influence, to take philosophical propositions in an exaggerated way. Let us rather state the case thus. At present, you are in possession of a *logica utens* which seems to be unsatisfactory. The question is whether, using that somewhat unsatisfactory *logica utens*, you can make out wherein it must be modified, and can attain to a better system. This is a truer way of stating the question; and so stated, it appears to present no such insuperable difficulty as is pretended. Suppose, for example, that we should show that your present logic

* See 614.

leads to flat self-contradiction. It is true that we must, then, in a sense, trust to natural logic in its declaration that two propositions flatly contradictory, the one of the other, cannot both be true. But in doing this, we are not relying upon the exactitude of any general logical principle, in all its generality. We are only confident that the two particular propositions in question are not both true. No matter how perfect a system of logic we may ever attain, still in applying that system, we must make use of intelligence. It is a misuse of terms to say that in doing this we "trust" to anything. We simply perceive that two certain propositions in flat contradiction to one another cannot both be true. We need not commit ourselves to any general principle; we simply recognize our inability to believe both at once. We simply recognize a mathematical necessity. Mathematics is not subject to logic. Logic depends on mathematics. The recognition of mathematical necessity is performed in a perfectly satisfactory manner antecedent to any study of logic. Mathematical reasoning derives no warrant from logic. It needs no warrant. It is evident in itself. It does not relate to any matter of fact, but merely to whether one supposition excludes another. Since we ourselves create these suppositions, we are competent to answer them. But it is when we pass out of the realm of pure hypothesis into that of hard fact that logic is called for. We then find that certain modes of reasoning are sound, because they must, by mathematical necessity, be sound, in whatever universe there may be in which there is such a thing as experience.

192. Nothing is more irrational than false pretence. We may all set that down as Axiom I, whether we are Artists, Practical Moralists, or Philosophers — a classification which embraces all mankind except the great world of Hypnotists — that is to say, the Medicine Men, the Confidence Men, the Horse Traders, the Diplomats, and all that lot, with which this book has no relation except that of alieniety. They and the burglars are useful members of society; but they are not likely to be readers of this book. For us, who are non-hypnotists, nothing is more irrational than false pretence. Yet the Cartesian philosophy, which ruled Europe for so long, is founded upon it. It pretended to doubt what it did not doubt. Let us not fall into that vice. You think that your *logica utens* is

more or less unsatisfactory. But you do not doubt that there is *some* truth in it. Nor do I; nor does any man. Why cannot men see that what we do not doubt, we do not doubt; so that it is false pretence to pretend to call it in question? There are certain parts of your *logica utens* which nobody really doubts. Hegel and his have loyally endeavored to cast a doubt upon it. The effort has been praiseworthy; but it has not succeeded. The truth of it is too evident. Mathematical reasoning holds. Why should it not? It relates only to the creations of the mind, concerning which there is no obstacle to our learning whatever is true of them. The method of this book, therefore, is to accept the reasonings of pure mathematics as beyond all doubt. It is fallible, as everything human is fallible. Twice two may perhaps not be four. But there is no more satisfactory way of assuring ourselves of anything than the mathematical way of assuring ourselves of mathematical theorems. No aid from the science of logic is called for in that field. As a fact, I have not the slightest doubt that twice two is four; nor have you. Then let us not pretend to doubt mathematical demonstrations of mathematical propositions so long as they are not open to mathematical criticism and have been submitted to sufficient examination and revision. The only concern that logic has with this sort of reasoning is to describe it.

193. That being settled, I propose, by purely mathematical reasoning, to show that in any world in which there is such a thing as the course of experience — an element which is absent from the world of pure mathematics — in such world a certain kind of reasoning must be valid which is not valid in the world of pure mathematics.

194. I have already stated in the second chapter what the course of the discussion is to be. I intend to restate it hereafter. But the interposition here of a third statement will not, by my reckoning, put the reader to any loss of time or patience. A book that goes to the bottom of an abstruse and complicated subject in such a manner that it can with profit be currently read ought to contain repetitions. Such at any rate, is the theory on which I have embarked the fortunes of this book.

195. I wish to show the reader how I am going to establish the doctrine of reason in making use of such imperfect reasoning powers as every reader will bring to the subject, and yet

without any inadmissible assumption. What a proposition it is that those who pronounce this to be impossible virtually maintain! That imperfect reason cannot perfect itself! How then, I should be glad to be informed, did man ever pass from a state of monership, and lower, to that of lordship of this globe? Oh, but it was not by a logical process. How so? It was by unwarrantable assumptions. What pedantry! So a man is not to be allowed to see what is reasonable unless he do so by the rules of art! Τὸ σάββατον δια τὸν ἄνθροπον ἐγένετο, οὐχ ὁ ἄνθροπος διὰ τὸ σάββατον, which being translated means, "Logic came about for the sake of reasonableness, not reasonableness for the sake of logic." Let us never lose sight of that truth, forgotten though it is, every day, in every walk of life, especially in well-regulated America!

§12. ESTHETICS, ETHICS, AND LOGIC

196. I shall begin with the reasoning of mathematics.* I shall analyze it and examine its nature. I shall not attempt to justify it. It will be time to defend it, when it shall once be deliberately doubted. Defense against sham doubt is but a blank-cartridge action. It is of no use. On the contrary, humbug is always harmful in philosophy.

197. Logic can be of no avail to mathematics; but mathematics lays the foundation on which logic builds; and those mathematical chapters will be quite indispensable. After them, it is my purpose to invite the reader to take up the study of Phenomenology.† In the derivation of this word, "phenomenon" is to be understood in the broadest sense conceivable; so that phenomenology might rather be defined as the study of what seems than as the statement of what appears. It describes the essentially different elements which seem to present themselves in what seems. Its task requires and exercises a singular sort of thought, a sort of thought that will be found to be of the utmost service throughout the study of logic. It can hardly be said to involve reasoning; for reasoning reaches a conclusion, and asserts it to be true however matters may seem; while in Phenomenology there is no assertion except

* Mathematics is dealt with in the next chapter of the "Minute Logic," published in vol. 4, bk. I, as no. VII.

† See vol. 1, bk. III.

that there are certain seemings; and even these are not, and cannot be asserted, because they cannot be described. Phenomenology can only tell the reader which way to look and to see what he shall see. The question of how far Phenomenology does reason will receive special attention. We shall next take up the logic of the normative sciences, of which logic itself is only the third, being preceded by Esthetics and Ethics. It is now forty-seven years ago that I undertook to expound Schiller's *Aesthetische Briefe* to my dear friend, Horatio Paine.* We spent every afternoon for long months upon it, picking the matter to pieces as well as we boys knew how to do. In those days, I read various works on esthetics; but on the whole, I must confess that, like most logicians, I have pondered that subject far too little. The books do seem so feeble. That affords one excuse. And then esthetics and logic seem, at first blush, to belong to different universes. It is only very recently that I have become persuaded that that seeming is illusory, and that, on the contrary, logic needs the help of esthetics. The matter is not yet very clear to me; so unless some great light should fall upon me before I reach that chapter, it will be a short one filled with doubts and queries mainly.†

198. Ethics‡ is another subject which for many years seemed to me to be completely foreign to logic. Indeed I doubted very much whether it was anything more than a practical science, or Art; although I had always been interested in ethical systems. About twenty years ago, I began to be impressed with the importance of the theory; but it has only been within five or six years that all the intimacy of its relation to logic has been revealed to me. It may very easily happen that the over-development of a man's moral conception should interfere with his progress in philosophy. The protoplasm of philosophy has to be in a liquid state in order that the operations of metabolism may go on. Now morality is a hardening agent. It is astonishing how many abominable scoundrels there are among sincerely moral people. The difficulty is that moral-

* A classmate — "a noble-hearted, sterling-charactered young gentleman . . almost the only real companion I have ever had."

† Only three and a half chapters of the "Minute Logic" were written, of which none is devoted to esthetics.

‡ See vol. 1, bk. IV and vol. 5, bk. I.

ity chokes its own stream. Like any other field, more than any other, it needs improvement, advance. Moral ideas must be a rising tide, or with the ebb foulness will be cast up. But morality, doctrinaire conservatist that it is, destroys its own vitality by resisting change, and positively insisting, This is eternally right: That is eternally wrong. The tendency of philosophers has always been to make their assertions too absolute. Nothing stands more in the way of a comprehension of the universe and of the mind. But in morals this tendency acquires triple strength. The practical side of ethics is its most obviously important side; and in practical matters, the first maxim is that everything may be exaggerated. That is the substance of Aristotle's *Ethics*. The moral spirit may very easily be carried to excess: all the more so, that the essence of that spirit is to insist upon its own absolute autocracy. All that is one side of the shield; but the other side is quite equally important. We are too apt to define ethics to ourselves as the science of right and wrong. That cannot be correct, for the reason that right and wrong are ethical conceptions which it is the business of that science to develope and to justify. A science cannot have for its fundamental problem to distribute objects among categories of its own creation; for underlying that problem must be the task of establishing those categories. The fundamental problem of ethics is not, therefore, What is right, but, What am I prepared deliberately to accept as the statement of what I want to do, what am I to aim at, what am I after? To what is the force of my will to be directed? Now logic is a study of the means of attaining the end of thought. It cannot solve that problem until it clearly knows what that end is. Life can have but one end. It is Ethics which defines that end. It is, therefore, impossible to be thoroughly and rationally logical except upon an ethical basis. Had I fully comprehended this great principle early in life, I should undoubtedly have been over-influenced by the moral spirit, and my present understanding of logic which, although it is far in advance of anybody else's, is, as compared with what it ought to be, as the scrawl on an infant's slate to a cartoon of Raphael, would have been even narrower and poorer than it now is — probably almost worthless. Before my logic was brought under the guidance of ethics, it was already a window through which

much important truth could be seen, but dim with dust, distorting details by striæ. Under the guidance of ethics I took it and melted it down, reduced it to a fluid condition. I filtered it till it was clear. I cast it in the true mould; and when it had become solid, I spared no elbow-grease in polishing it. It is now a comparatively brilliant lens, showing much that was not discernible before. I believe that it will only remain to those who come after me to perfect the processes. I am as confident as I am of death that Logic will hereafter be infinitely superior to what it is as I leave it; but my labors will have done good work toward its improvement.

199. What I have found to be true of Ethics I am beginning to see is true of Esthetics likewise. That science has been handicapped by the definition of it as the theory of beauty. The conception of beauty is but the product of this science, and a very inadequate attempt it is to grasp what it is that esthetics seeks to make clear. Ethics asks to what end all effort shall be directed. That question obviously depends upon the question what it would be that, independently of the effort, we should like to experience. But in order to state the question of esthetics in its purity, we should eliminate from it, not merely all consideration of effort, but all consideration of action and reaction, including all consideration of our receiving pleasure, everything in short, belonging to the opposition of the *ego* and the *non-ego*. We have not in our language a word of the requisite generality. The Greek καλός, the French *beau*, only come near to it, without hitting it squarely on the head. "Fine" would be a wretched substitute. Beautiful is bad; because one mode of being καλός essentially depends upon the quality being unbeautiful. Perhaps, however, the phrase "the beauty of the unbeautiful" would not be shocking. Still "beauty" is too skin-deep. Using καλός, the question of esthetics is, What is the one quality that is, in its immediate presence, καλός? Upon this question ethics must depend, just as logic must depend upon ethics. Esthetics, therefore, although I have terribly neglected it, appears to be possibly the first indispensable propedeutic to logic, and the logic of esthetics to be a distinct part of the science of logic that ought not to be omitted. This is a point concerning which it is not desirable to be in haste to come to a decided opinion.

200. When our logic shall have paid its *devoirs* to Esthetics and the Ethics, it will be time for it to settle down to its regular business. That business is of a varied nature; but so far as I intend in this place to speak of it, it consists in ascertaining methods of sound reasoning, and of proving that they are sound, not by any instinctive guarantee, but because it can be shown by the kinds of reasoning already considered, especially the mathematical, of one class of reasonings that they follow methods which, persisted in, must eventually lead to the truth in regard to those problems to which they are applicable, or, if not to the absolute truth, to an indefinite approximation thereto, while in regard to another class of reasonings, although they are so insecure that no reliance can be placed upon them, it will be shown in a similar way that yet they afford the only means of attaining to a satisfactory knowledge of the truth, in case this knowledge is ever to be attained at all, doing so by putting problems into such form that the former class of reasonings become applicable to them. This prospectus of how I am to proceed is sufficient to show that there can be no ground of reasonable complaint that unwarranted assumptions are made in the course of the discussion. Nothing will be assumed beyond what every sincere and intelligent person will and must confess is perfectly evident and which, in point of fact, is not really doubted by any caviller.

§13. UTILITY OF LOGICAL THEORY

201. You anticipate from the study of logic important improvement in your practice of reasoning. I have already endeavored to state just how far such improvement can be promised. Nobody will expect of theory that it should furnish skill, or render practice needless. A man of experience would be somewhat surprised to find that a writer on esthetics was an artist of power, or that a writer on ethics was a moral hero. Logicians are, as a rule, far from being the greatest of reasoners. Nevertheless, I am inclined to believe that the study of esthetics will be of benefit to an artist; though I know too little either of esthetics or of art to speak with confidence. Some writers on ethics deny that it affords any aid toward a moral life. I cannot but think that this is an exaggeration. I do not see how

the study of Plato, or Shaftesbury, or Kant can fail to make men better. I doubt if the reading of Hobbes could do a man of much power of thought any harm. As for logic, I must admit that it has turned out numberless captious thinkers who do not reason nearly as well as the average woman who does not remember ever to have seen the inside of a treatise on logic. But at any rate there is no cure for such people, unless it be to teach them a rational system of logic. I am inclined to think that one reason why logicians, as a class, are such bad reasoners, as they are apt to be, is that they were interested in logic, in the first place, because they found they had great difficulty in reasoning; and while other men are repelled from the study because they find its rules so nugatory and foreign to the spirit of true science, these men have continued the study because its mechanical and piddling character fits their own calibre of mind. It is the old traditional logic which I am now alluding to, whose most stupid maxims we today so often hear appealed to in the market-place as of acknowledged authority. But it is not seriously studied. When it was so studied in the middle ages, although it naturally could not foster reasoning such as that of a modern physicist, for example, yet it seems to me clear that it did cause great numbers of men to reason much more exactly and acutely than they would have done without such study.

202. For scientific reasoning about matters of science, scientific logic is indispensable.

§14. LOGIC*

203. Logic is a science which has not yet completed the stage of disputes concerning its first principles, although it is probably about to do so. Nearly a hundred definitions of it have been given. It will, however, generally be conceded that its central problem is the classification of arguments, so that all those that are bad are thrown into one division, and those which are good into another, these divisions being defined by marks recognizable even if it be not known whether the arguments are good or bad. Furthermore, logic has to divide good

* *Dictionary of Philosophy and Psychology* (1901–2, 1911), edited by J. M. Baldwin, Macmillan, New York, vol. 2, pp. 20–23, by Peirce and Mrs. Ladd-Franklin.

arguments by recognizable marks into those which have different orders of validity, and has to afford means for measuring the strength of arguments.

204. An approach to such a classification is made by every man whenever he reasons, in the proper sense of that term. It is true that the contemplation of a state of things believed to be real may cause the contemplator to believe something additional, without making any classification of such sequences. But in that case he does not criticize the procedure, nor so much as distinctly reflect that it is just. He can, consequently, not exercise any control over it. Now, that which is uncontrollable is not subject to any normative laws at all; that is, it is neither good nor bad; it neither subserves an end nor fails to do so. But it is only the deliberate adoption of a belief in consequence of the admitted truth of some other proposition which is, properly speaking, reasoning. In that case the belief is adopted because the reasoner conceives that the method by which it has been determined would either in no analogous case lead to a false conclusion from true premisses, or, if steadily adhered to, would at length lead to an indefinite approximation to the truth, or, at least, would assure the reasoner of ultimately attaining as close an approach to the truth as he can, in any way, be assured of attaining. In all reasoning, therefore, there is a more or less conscious reference to a general method, implying some commencement of such a classification of arguments as the logician attempts. Such a classification of arguments, antecedent to any systematic study of the subject, is called the reasoner's *logica utens*, in contradistinction to the result of the scientific study, which is called *logica docens*. See Reasoning.*

205. That part of logic, that is, of *logica docens*, which, setting out with such assumptions as that every assertion is either true or false, and not both, and that some propositions may be recognized to be true, studies the constituent parts of arguments and produces a classification of arguments such as is above described, is often considered to embrace the whole of logic; but a more correct designation is Critic (Greek κριτική. According to Diogenes Laertius,† Aristotle divided logic into

* 773ff.

† *Life of Aristotle*, bk. V, ch. 13.

three parts, of which one was πρὸς κρίσιν). This word, used by Plato (who divides all knowledge into *epitactic* and *critic*),* was adopted into Latin by the Ramists, and into English by Hobbes and Locke. From the last it was taken into German by Kant, who always writes it Critik, the initial *c* being possibly a reminiscence of its English origin. At present it is written Kritik in German. Kant is emphatic in the expression of the wish that the word may not be confounded with critique, a critical essay (German *Kritik*).†

206. It is generally admitted that there is a doctrine which properly antecedes what we have called critic. It considers, for example, in what sense and how there can be any true proposition and false proposition, and what are the general conditions to which thought or signs of any kind must conform in order to assert anything. Kant, who first raised these questions to prominence, called this doctrine *transcendentale Elementarlehre*, and made it a large part of his *Critic of the Pure Reason*. But the *Grammatica Speculativa* of Scotus is an earlier and interesting attempt. The common German word is *Erkenntnisstheorie*, sometimes translated Epistemology.

207. It is further generally recognized that another doctrine follows after critic, and which belongs to, or is closely connected with, logic. Precisely what this should contain is not agreed; but it must contain the general conditions requisite for the attainment of truth. Since it may be held to contain more, one hesitates to call it heuristic. It is often called Method; but as this word is also used in the concrete, methodic or methodeutic would be better.

208. For deciding what is good logic and what bad, appeal is made by different writers to one or more, generally several, of these eight sources: to direct dicta of consciousness, to psychology, to the usages of language, to metaphysical philosophy, to history, to everyday observation, to mathematics, and to some process of dialectic. In the middle ages appeal was frequently made to authority.

209. The appeal to direct consciousness consists in pronouncing certain reasoning to be good or bad because it is felt to be so. This is a very common method. Sigwart, for example,

* Cf. *Politicus*, 260.

† Cf. *Critik der Reinen Vernunft*, 2te Auflage, Einleitung VII.

bases all logic upon our invincible mental repulsion against contradiction, or, as he calls it, "the immediate feeling of necessity" (*Logic*, §3, 2). Those who think it worth while to make any defence at all of this proceeding urge, in effect, that, however far the logician may push his criticisms of reasoning, still, in doing so, he must reason, and so must ultimately rely upon his instinctive recognition of good and bad reasoning. Whence it follows that, in Sigwart's words, "every system of logic must rest upon this principle." It is, however, to be noted that among the dicta of direct consciousness, many pronounce certain reasonings to be bad. If, therefore, such dicta are to be relied upon, man not only usually has a tendency to reason right, but also sometimes has a tendency to reason wrong; and if that be so, the validity of a reasoning cannot *consist* in a man's having a tendency to reason in that way. Some say that the validity of reasoning consists in the "definitive dictum" of consciousness; but it has been replied that certain propositions in Euclid were studied for two thousand years by countless keen minds, all of whom had an immediate feeling of evidence concerning their proofs, until at last flaws were detected in those proofs, and are now admitted by all competent persons; and it is claimed that this illustrates how far from possible it is to make direct appeal to a definitive pronouncement. Besides, say those who object to this method, all reasoning and inquiry expects that there is such a thing as the truth concerning whatever question may be under examination. Now, it is of the very essence of this "truth," the meaning of the expectation, that the "truth" in no wise depends upon what any man to whom direct appeal can be made may opine about that question. *A fortiori* it does not depend upon whether I am satisfied with it or not. It is further insisted that there can be no genuine criticism of a reasoning until that reasoning is actually doubted; and no sooner is it actually doubted than we find that consciousness has revoked her *dictum* in its favour, if she ever made any. It is, indeed, maintained that so far from true is it that every system of logic must be based upon any instinctive recognition of good and bad reasoning, that it is quite impossible for any reasoning to be based upon such recognition in respect to that same reasoning. In reasoning, a man may feel sure he is right; but to "rest" that

confidence on nothing but itself is to rest it on nothing at all. If the fact that we must use our reasoning instinct in criticizing reasoning proves that we must appeal to nothing else in such criticism, it equally proves that we ought to follow the lead of that instinct without any logical control at all, which would be as much as to say that we ought not to reason at all. A man cannot criticize every part of his reasoning, since he cannot criticize the act of reasoning he is performing in the criticism, it is true. But he can criticize steps whose validity he doubts; and in doing so, ought to consider in what characters the validity of reasoning consists, and whether the reasoning in question possesses those characters.

210. Under an appeal to psychology is not meant every appeal to any fact relating to the mind. For it is, for logical purposes, important to discriminate between facts of that description which are supposed to be ascertained by the systematic study of the mind, and facts the knowledge of which altogether antecedes such study, and is not in the least affected by it; such as the fact that there is such a state of mind as doubt, and the fact that the mind struggles to escape from doubt. Even facts like these require to be carefully examined by the logician before he uses them as the basis of his doctrine. But many logicians have gone much further, and have avowedly based their systems upon one or another theory of psychology. Another class of logicians have professed to base logic upon a psychological theory of cognition. Of course, if this is done, such psychological doctrine is placed above logical criticism, or, at any rate, above logical support. For if the truth of a conclusion is known only from certain premisses, it cannot be used to support those premisses. Now, it may be doubted whether psychology is not, of all the special sciences, the one which stands most in need of appeal to a scientific logic.

211. Appeals to the usages of language are extremely common. They are made even by those who use algebraical notation in logic "in order to free the mind from the trammels of speech" (Schröder, *Logik*, i. p. iii). It is difficult to see what can be hoped for from such a proceeding, unless it be to establish a psychological proposition valid for all minds. But to do this, it would be necessary to look beyond the small and

very peculiar class of Aryan languages, to which the linguistic knowledge of most of those writers is confined. The Semitic languages, with which some of them are acquainted, are too similar to the Aryan greatly to enlarge their horizon. Moreover, even if other languages are examined, the value of any logical inferences from them is much diminished by the custom of our grammarians of violently fitting them to the Procrustean bed of Aryan grammar.

212. The objection which has been suggested to appeals to psychological results applies with far greater force to appeals to metaphysical philosophy, which, it will generally be conceded, can hardly take a step with security unless it rests upon the science of logic. Nevertheless, a great many logical treatises of various colors make it their boast that they are built upon philosophical principles.

213. Logicians occasionally appeal to the history of science. Such and such a mode of reasoning, it is said, for example, was characteristic of mediævalism or of ancient science; such another produced the successes of modern science. If logic is to be based upon probable reasonings, as some logicians maintain that it must be, such arguments, if critically examined, must be admitted to have great weight. They will naturally be out of place in a system of logic which professes to demonstrate from certain initial assumptions that the kinds of reasoning it recommends must be accepted.

214. There is probably room for dispute as to whether logic need assert anything at all as an absolute matter of fact. If it does not, any appeal to experience would seem to be irrelevant. If it does, still the opinion may be that such assertions of logic are of so exceedingly broad and slight a nature that the universal experience of every man's every day and hour puts them beyond all doubt — such experiences as that the world presents appearances of variety, of law, and of the real action of one thing upon another. As appearances, these things do not seem likely ever to be doubted. If logic has need of any facts, and if such facts will suffice, no objection can well be made to an appeal to them.

215. The boundary between some parts of logic and pure mathematics in its modern treatment is almost evanescent, as may be seen in Dedekind's *Was sind und was sollen die Zahlen*

(1888, English translation 1901). There are, however, departments of logic, such as the logic of probable inference (if that be regarded a part of logic), in which appeal is sometimes made to mathematical results, such as Bernoulli's law of high numbers. It seems to be the general opinion that nothing so difficult as mathematics can be admitted into, or be appealed to by, the science of logic, which has the peculiarity of consisting chiefly of truisms.

216. In mathematical reasoning there is a sort of observation. For a geometrical diagram or array of algebraical symbols is constructed according to an abstractly stated precept, and between the parts of such diagram or array certain relations are observed to obtain, other than those which were expressed in the precept. These being abstractly stated, and being generalized, so as to apply to every diagram constructed according to the same precept, give the conclusion. Some logicians hold that an equally satisfactory method depends upon a kind of inward observation, which is not mathematical, since it is not diagrammatic, the development of a conception and its inevitable transformation being observed and generalized somewhat as in mathematics; and those logicians base their science upon such a method, which may conveniently be termed, and is sometimes termed, a Dialectic. Other logicians regard such a method as either extremely insecure or as altogether illusory.

217. The generally received opinion among professors of logic is that all the above methods may properly be used on occasion, the appeal to mathematics, however, being less generally recognized.

218. *Literature:* the history of logic in Western Europe, down to the revival of learning, is given by Prantl, *Geschichte der Logik im Abendlande.* Upon the points upon which this author touches, he always affords valuable information, though his judgments are peremptory and slashing. Unfortunately, he omits much which was regarded by the authors of whom he treats as most important, because he does not himself so regard it. He also omits much which would be interesting to a reader taking a broader conception of logic. It is hardly necessary to say that upon some large subjects his views are controverted. . . .

Book II

SPECULATIVE GRAMMAR[P]

CHAPTER 1

*THE ETHICS OF TERMINOLOGY**

219. In order that my use of terms, notations, etc., may be understood, I explain that my conscience imposes upon me the following rules. Were I to make the smallest pretension to dictate the conduct of others in this matter, I should be reproved by the first of these rules. Yet if I were to develope the reasons the force of which I feel myself, I presume they would have weight with others.

220. Those reasons would embrace, in the first place, the consideration that the woof and warp of all thought and all research is symbols, and the life of thought and science is the life inherent in symbols; so that it is wrong to say that a good language is *important* to good thought, merely; for it is of the essence of it. Next would come the consideration of the increasing value of precision of thought as it advances. Thirdly, the progress of science cannot go far except by collaboration; or, to speak more accurately, no mind can take one step without the aid of other minds. Fourthly, the health of the scientific communion requires the most absolute mental freedom. Yet the scientific and philosophical worlds are infested with pedants and pedagogues who are continually endeavoring to set up a sort of magistrature over thoughts and other symbols. It thus becomes one of the first duties of one who sees what the situation is, energetically to resist everything like arbitrary dictation in scientific affairs, and above all, as to the use of terms and notations. At the same time, a general agreement concerning the use of terms and of notations — not too rigid, yet prevailing, with most of the co-workers in regard to most of the symbols, to such a degree that there shall be some small number of different systems of expression that have to be mastered — is indispensable. Consequently, since this is not to be brought about by arbitrary dictation, it must be brought about by the power of rational principles over the conduct of men.

* *Syllabus of Certain Topics of Logic* (1903), pp. 10–14, Alfred Mudge & Son, Boston. Continuing 1.202. Cf. 5.413, 5.502, and vol. 5, Appendix §4.

221. Now what rational principle is there which will be perfectly determinative as to what terms and notations shall be used, and in what senses, and which at the same time possesses the requisite power to influence all right-feeling and thoughtful men?

In order to find the answer to that question, it is necessary to consider, first, what would be the character of an ideal philosophical terminology and system of logical symbols; and, secondly, to inquire what the experience of those branches of science has been that have encountered and conquered great difficulties of nomenclature, etc., in regard to the principles which have proved efficacious, and in regard to unsuccessful methods of attempting to produce uniformity.

222. As to the ideal to be aimed at, it is, in the first place, desirable for any branch of science that it should have a vocabulary furnishing a family of cognate words for each *scientific* conception, and that each word should have a single exact meaning, unless its different meanings apply to objects of different categories that can never be mistaken for one another. To be sure, this requisite might be understood in a sense which would make it utterly impossible. For every symbol is a living thing, in a very strict sense that is no mere figure of speech. The body of the symbol changes slowly, but its meaning inevitably grows, incorporates new elements and throws off old ones. But the effort of all should be to keep the *essence* of every scientific term unchanged and exact; although absolute exactitude is not so much as conceivable. Every symbol is, in its origin, either an image of the idea signified, or a reminiscence of some individual occurrence, person or thing, connected with its meaning, or is a metaphor. Terms of the first and third origins will inevitably be applied to different conceptions; but if the conceptions are strictly analogous in their principal suggestions, this is rather helpful than otherwise, provided always that the different meanings are remote from one another, both in themselves and in the occasions of their occurrence. Science is continually gaining new conceptions; and every new *scientific* conception should receive a new word, or better, a new family of cognate words. The duty of supplying this word naturally falls upon the person who introduces the new conception; but it is a duty not to be undertaken without a thor-

ough knowledge of the principles and a large acquaintance with the details and history of the special terminology in which it is to take a place, nor without a sufficient comprehension of the principles of word-formation of the national language, nor without a proper study of the laws of symbols in general. That there should be two different terms of identical scientific value may or may not be an inconvenience, according to circumstances. Different systems of expression are often of the greatest advantage.

223. The ideal terminology will differ somewhat for different sciences. The case of philosophy is very peculiar in that it has positive need of popular words in popular senses — not as its own language (as it has too usually used those words), but as objects of its study. It thus has a peculiar need of a language distinct and detached from common speech, such a language as Aristotle, the scholastics, and Kant endeavored to supply, while Hegel endeavored to destroy it. It is good economy for philosophy to provide itself with a vocabulary so outlandish that loose thinkers shall not be tempted to borrow its words. Kant's adjectives "objective" and "subjective" proved not to be barbarous enough, by half, long to retain their usefulness in philosophy, even if there had been no other objection to them. The first rule of good taste in writing is to use words whose meanings will not be misunderstood; and if a reader does not know the meaning of the words, it is infinitely better that he should know he does not know it. This is particularly true in logic, which wholly consists, one might almost say, in exactitude of thought.

224. The sciences which have had to face the most difficult problems of terminology have unquestionably been the classificatory sciences of physics, chemistry, and biology. The nomenclature of chemistry is, on the whole, good. In their dire need, the chemists assembled in congress, and adopted certain rules for forming names of substances. Those names are well-known, but they are hardly used. Why not? Because the chemists were not psychologists, and did not know that a congress is one of the most impotent of things, even less influential by far than a dictionary. The problem of the biological taxonomists has, however, been incomparably more difficult; and they have solved it (barring small exceptions)

with brilliant success. How did they accomplish this? Not by appealing to the power of congresses, but by appealing to the power of the idea of right and wrong. For only make a man *really see* that a certain line of conduct is wrong, and he *will* make a strong endeavor to do the right thing — be he thief, gambler, or even a logician or moral philosopher. The biologists simply talked to one another, and made one another see that when a man has introduced a conception into science, it naturally becomes both his privilege and his duty to assign to that conception suitable scientific expressions; and that when a name has been conferred upon a conception by him to whose labors science is indebted for that conception, it becomes the duty of all — a duty to the discoverer, and a duty to science — to accept his name, unless it should be of such a nature that the adoption of it would be unwholesome for science; that should the discoverer fail in his duty, either by giving no name or an utterly unsuitable one, then, after a reasonable interval, whoever first has occasion to employ a name for that conception must invent a suitable one; and others ought to follow him; but that whoever deliberately uses a word or other symbol in any other sense than that which was conferred upon it by its sole rightful creator commits a shameful offence against the inventor of the symbol and against science, and it becomes the duty of the others to treat the act with contempt and indignation.

225. As fast as the students of any branch of philosophy educate themselves to a genuine scientific love of truth to the degree to which the scholastic doctors were moved by it, suggestions similar to those above will suggest themselves; and they will consequently form a technical terminology. In logic, a terminology more than passably good has been inherited by us from the scholastics. This scholastic terminology has passed into English speech more than into any other modern tongue, rendering it the most logically exact of any. This has been accompanied by the inconvenience that a considerable number of words and phrases of scientific logic have come to be used with a laxity quite astounding. Who, for example, among the dealers in Quincy Hall who talks of "articles of *prime necessity*," would be able to say what that phrase "prime necessity" strictly means? He could not have sought out a more technical

phrase. There are dozens of other loose expressions of the same provenance.

Having thus given some idea of the nature of the reasons which weigh with me, I proceed to state the rules which I find to be binding upon me in this field.

226. First. To take pains to avoid following any recommendation of an arbitrary nature as to the use of philosophical terminology.

Second. To avoid using words and phrases of vernacular origin as technical terms of philosophy.

Third. To use the scholastic terms in their anglicised forms for philosophical conceptions, so far as they are strictly applicable; and never to use them in other than their proper senses.

Fourth. For ancient philosophical conceptions overlooked by the scholastics, to imitate, as well as I can, the ancient expression.

Fifth. For precise philosophical conceptions introduced into philosophy since the middle ages, to use the anglicised form of the original expression, if not positively unsuitable, but only in its precise original sense.

Sixth. For philosophical conceptions which vary by a hair's breadth from those for which suitable terms exist, to invent terms with a due regard for the usages of philosophical terminology and those of the English language but yet with a distinctly technical appearance. Before proposing a term, notation, or other symbol, to consider maturely whether it perfectly suits the conception and will lend itself to every occasion, whether it interferes with any existing term, and whether it may not create an inconvenience by interfering with the expression of some conception that may hereafter be introduced into philosophy. Having once introduced a symbol, to consider myself almost as much bound by it as if it had been introduced by somebody else; and after others have accepted it, to consider myself more bound to it than anybody else.

Seventh. To regard it as needful to introduce new systems of expression when new connections of importance between conceptions come to be made out, or when such systems can, in any way, positively subserve the purposes of philosophical study.

CHAPTER 2

DIVISION OF SIGNS

§1. GROUND, OBJECT, AND INTERPRETANT*

227. Logic, in its general sense, is, as I believe I have shown, only another name for *semiotic* (*σημειωτική*), the quasi-necessary, or formal, doctrine of signs. By describing the doctrine as "quasi-necessary," or formal, I mean that we observe the characters of such signs as we know, and from such an observation, by a process which I will not object to naming Abstraction, we are led to statements, eminently fallible, and therefore in one sense by no means necessary, as to what *must be* the characters of all signs used by a "scientific" intelligence, that is to say, by an intelligence capable of learning by experience. As to that process of abstraction, it is itself a sort of observation. The faculty which I call abstractive observation is one which ordinary people perfectly recognize, but for which the theories of philosophers sometimes hardly leave room. It is a familiar experience to every human being to wish for something quite beyond his present means, and to follow that wish by the question, "Should I wish for that thing just the same, if I had ample means to gratify it?" To answer that question, he searches his heart, and in doing so makes what I term an abstractive observation. He makes in his imagination a sort of skeleton diagram, or outline sketch, of himself, considers what modifications the hypothetical state of things would require to be made in that picture, and then examines it, that is, *observes* what he has imagined, to see whether the same ardent desire is there to be discerned. By such a process, which is at bottom very much like mathematical reasoning, we can reach conclusions as to what *would be* true of signs in all cases, so long as the intelligence using them was scientific. The modes of thought of a God, who should possess an intuitive omniscience superseding reason, are put out of the question. Now the whole process of development among the com-

* From an unidentified fragment, c. 1897.

munity of students of those formulations by abstractive observation and reasoning of the truths which *must* hold good of all signs used by a scientific intelligence is an observational science, like any other positive science, notwithstanding its strong contrast to all the special sciences which arises from its aiming to find out what *must be* and not merely what *is* in the actual world.

228. A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes* called the *ground* of the representamen. "Idea" is here to be understood in a sort of Platonic sense, very familiar in everyday talk; I mean in that sense in which we say that one man catches another man's idea, in which we say that when a man recalls what he was thinking of at some previous time, he recalls the same idea, and in which when a man continues to think anything, say for a tenth of a second, in so far as the thought continues to agree with itself during that time, that is to have a *like* content, it is the same idea, and is not at each instant of the interval a new idea.

229. In consequence of every representamen being thus connected with three things, the ground, the object, and the interpretant, the science of semiotic has three branches. The first is called by Duns Scotus *grammatica speculativa*. We may term it *pure grammar*. It has for its task to ascertain what must be true of the representamen used by every scientific intelligence in order that they may embody any *meaning*. The second is logic proper. It is the science of what is quasi-necessarily true of the representamina of any scientific intelligence in order that they may hold good of any *object*, that is, may be true. Or say, logic proper is the formal science of the conditions of the truth of representations. The third, in imitation of Kant's fashion of preserving old associations of words in finding nomenclature for new conceptions, I call *pure rhetoric*. Its task is to

* *E.g.*, in 1.551.

ascertain the laws by which in every scientific intelligence one sign gives birth to another, and especially one thought brings forth another.

§2. SIGNS AND THEIR OBJECTS*

230. The word Sign will be used to denote an Object perceptible, or only imaginable, or even unimaginable in one sense — for the word "*fast*," which is a Sign, is not imaginable, since it is not *this word itself* that can be set down on paper or pronounced, but only *an instance* of it, and since it is the very same word when it is written as it is when it is pronounced, but is one word when it means "rapidly" and quite another when it means "immovable," and a third when it refers to abstinence. But in order that anything should be a Sign, it must "represent," as we say, something else, called its *Object*, although the condition that a Sign must be other than its Object is perhaps arbitrary, since, if we insist upon it we must at least make an exception in the case of a Sign that is a part of a Sign. Thus nothing prevents the actor who acts a character in an historical drama from carrying as a theatrical "property" the very relic that that article is supposed merely to represent, such as the crucifix that Bulwer's Richelieu holds up with such effect in his defiance. On a map of an island laid down upon the soil of that island there must, under all ordinary circumstances, be some position, some point, marked or not, that represents *qua* place on the map, the very same point *qua* place on the island. A sign may have more than one Object. Thus, the sentence "Cain killed Abel," which is a Sign, refers at least as much to Abel as to Cain, even if it be not regarded as it should, as having "*a killing*" as a third Object. But the set of objects may be regarded as making up one complex Object. In what follows and often elsewhere Signs will be treated as having but one object each for the sake of dividing difficulties of the study. If a Sign is other than its Object, there must exist, either in thought or in expression, some explanation or argument or other context, showing how — upon what system or for what reason the Sign represents the Object or set of Objects that it does. Now the Sign and the Explanation together make up another Sign, and

* From "Meaning," 1910.

since the explanation will be a Sign, it will probably require an additional explanation, which taken together with the already enlarged Sign will make up a still larger Sign; and proceeding in the same way, we shall, or should, ultimately reach a Sign of itself, containing its own explanation and those of all its significant parts; and according to this explanation each such part has some other part as its Object. According to this every Sign has, actually or virtually, what we may call a *Precept* of explanation according to which it is to be understood as a sort of emanation, so to speak, of its Object. (If the Sign be an Icon, a scholastic might say that the "*species*" of the Object emanating from it found its matter in the Icon. If the Sign be an Index, we may think of it as a fragment torn away from the Object, the two in their Existence being one whole or a part of such whole. If the Sign is a Symbol, we may think of it as embodying the "*ratio*," or reason, of the Object that has emanated from it. These, of course, are mere figures of speech; but that does not render them useless.)

231. The Sign can only represent the Object and tell about it. It cannot furnish acquaintance with or recognition of that Object; for that is what is meant in this volume by the Object of a Sign; namely, that with which it presupposes an acquaintance in order to convey some further information concerning it. No doubt there will be readers who will say they cannot comprehend this. They think a Sign need not relate to anything otherwise known, and can make neither head nor tail of the statement that every Sign must relate to such an Object. But if there be anything that conveys information and yet has absolutely no relation nor reference to anything with which the person to whom it conveys the information has, when he comprehends that information, the slightest acquaintance, direct or indirect — and a very strange sort of information that would be — the vehicle of that sort of information is not, in this volume, called a Sign.

232. Two men are standing on the seashore looking out to sea. One of them says to the other, "That vessel there carries no freight at all, but only passengers." Now, if the other, himself, sees no vessel, the first information he derives from the remark has for its Object the part of the sea that he does see, and informs him that a person with sharper eyes than his, or

more trained in looking for such things, can see a vessel there; and then, that vessel having been thus introduced to his acquaintance, he is prepared to receive the information about it that it carries passengers exclusively. But the sentence as a whole has, for the person supposed, no other Object than that with which it finds him already acquainted. The Objects — for a Sign may have any number of them — may each be a single known existing thing or thing believed formerly to have existed or expected to exist, or a collection of such things, or a known quality or relation or fact, which single Object may be a collection, or whole of parts, or it may have some other mode of being, such as some act permitted whose being does not prevent its negation from being equally permitted, or something of a general nature desired, required, or invariably found under certain general circumstances.

§3. DIVISION OF TRIADIC RELATIONS*

233. The principles and analogies of Phenomenology enable us to describe, in a distant way, what the divisions of triadic relations must be. But until we have met with the different kinds *a posteriori*, and have in that way been led to recognize their importance, the *a priori* descriptions mean little; not nothing at all, but little. Even after we seem to identify the varieties called for *a priori* with varieties which the experience of reflexion leads us to think important, no slight labour is required to make sure that the divisions we have found *a posteriori* are precisely those that have been predicted *a priori*. In most cases, we find that they are not precisely identical, owing to the narrowness of our reflexional experience. It is only after much further arduous analysis that we are able finally to place in the system the conceptions to which experience has led us. In the case of triadic relations, no part of this work has, as yet, been satisfactorily performed, except in some measure for the most important class of triadic relations, those of signs, or representamens, to their objects and interpretants.

234. Provisionally, we may make a rude division of triadic relations, which, we need not doubt, contains important truth, however imperfectly apprehended, into —

* §§3–10 are from "Nomenclature and Divisions of Triadic Relations, as far as they are determined,"— a manuscript continuation of the "Syllabus," c. 1903. See note to vol. 1, Bk. II, ch. 1, and 3.608n.

Triadic relations of comparison,

Triadic relations of performance, and

Triadic relations of thought.

Triadic relations of Comparison are those which are of the nature of logical possibilities.

Triadic relations of Performance are those which are of the nature of actual facts.

Triadic relations of Thought are those which are of the nature of laws.

235. We must distinguish between the First, Second, and Third Correlate of any triadic relation.

The First Correlate is that one of the three which is regarded as of the simplest nature, being a mere possibility if any one of the three is of that nature, and not being a law unless all three are of that nature.*

236. The Third Correlate is that one of the three which is regarded as of the most complex nature, being a law if any one of the three is a law, and not being a mere possibility unless all three are of that nature.*

* On Peirce's principle that possibilities determine only possibilities and laws are determined only by laws, the terms "First Correlate" and "Third Correlate" should be interchanged in 235–38. In this way one secures, in harmony with other writings, the ten classes mentioned in 238. They are as follows:

If the Third Correlate is a possibility, then

		First	*Second*	*Third*
(I)	1.	Possibility	Possibility	Possibility
(II)	2.	Existent	Possibility	Possibility
(III)	3.	Existent	Existent	Possibility
(V)	4.	Law	Possibility	Possibility
(VI)	5.	Law	Existent	Possibility
(VIII)	6.	Law	Law	Possibility
If the Second is an existent, then also				
(IV)	7.	Existent	Existent	Existent
(VII)	8.	Law	Existent	Existent
If the First is a law, then also				
(IX)	9.	Law	Law	Existent
(X)	10.	Law	Law	Law

By 242 and 274 the Representamen, Object, and Interpretant are the first, second, and third correlate respectively, while by 243ff. the representamen in itself, in relation to its object, and as interpreted, is the first, second, and third correlate respectively. The former division yields ten trichotomies and sixty-six classes of signs, the latter three trichotomies and ten classes of signs.

The bracketed roman numerals in the above table give the order of discussion in §7 and the designations in the table in 264. See also 243n.

237. The Second Correlate is that one of the three which is regarded as of middling complexity, so that if any two are of the same nature, as to being either mere possibilities, actual existences, or laws, then the Second Correlate is of that same nature, while if the three are all of different natures, the Second Correlate is an actual existence.*

238. Triadic relations are in three ways† divisible by trichotomy, according as the First, the Second, or the Third Correlate, respectively, is a mere possibility, an actual existent, or a law. These three trichotomies, taken together, divide all triadic relations into ten classes [see footnote to 235]. These ten classes will have certain subdivisions according as the existent correlates are individual subjects or individual facts, and according as the correlates that are laws are general subjects, general modes of fact, or general modes of law.

239. There will be besides a second similar division of triadic relations into ten classes, according as the dyadic relations which they constitute between either the First and Second Correlates, or the First and Third, or the Second and Third are of the nature of possibilities, facts, or laws; and these ten classes will be subdivided in different ways.‡

* The truth of this last clause is to be seen from case numbered 5 above. The truth of the rest of the proposition is to be seen from cases numbered 1, 7, and 10.

† The three ways are given in the notes to 243.

‡ Though Peirce has laid down the condition that a dyadic relation to be an existent requires both its correlates to be existents (cf. 283), he does not seem ever to have given the conditions involved in determining a dyadic relation to be of the nature of a law. In fact, his usual view is that there are no such dyadic relations. However, what seems to be meant here is that a dyadic relation is of the nature of law if both its correlates are laws. If, in addition, we accept the unstated propositions that a dyadic relation is a possibility if one correlate is a possibility, while a dyadic relation is an existent if one correlate is an existent and the other a law, we should get the following table:

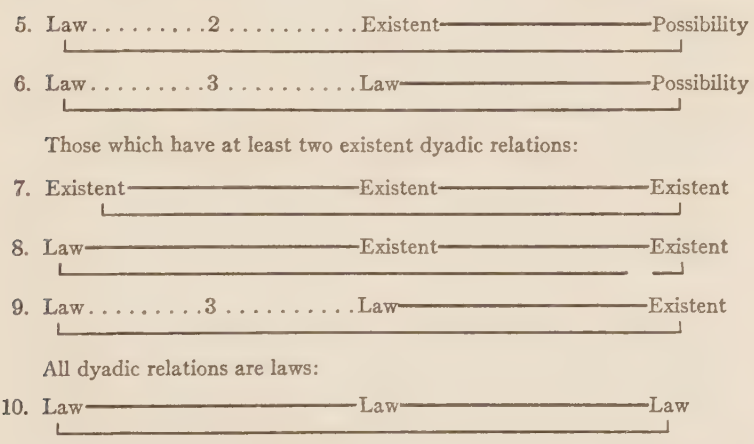
At least one dyadic relation of the nature of possibility:

	First	*Second*	*Third*
1.	Possibility	Possibility	Possibility
2.	Existent	Possibility	Possibility
3.	Existent	Existent	Possibility
4.	Law	Possibility	Possibility

240. It may be convenient to collect the ten classes of either set of ten into three groups according as all three of the correlates or dyadic relations, as the case may be, are of different natures, or all are of the same nature, or two are of one nature while the third is of a different nature.*

241. In every genuine Triadic Relation, the First Correlate may be regarded as determining the Third Correlate in some respect; and triadic relations may be divided according as that determination of the Third Correlate is to having some quality, or to being in some existential relation to the Second Correlate, or to being in some relation of thought to the Second for something†.

242. A *Representamen* is the First Correlate of a triadic relation, the Second Correlate being termed its *Object*, and the possible Third Correlate being termed its *Interpretant*, by which triadic relation the possible Interpretant is determined to be the First Correlate of the same triadic relation to the same



The black lines between the correlates are marks of the presence of the specified relation; the "....2...." and "....3...." stand for existential and rational dyadic relations respectively.

* I.e., 5 has its correlates all of different natures; 1, 7, 10 have their correlates all of the same nature; and the rest have two and only two correlates of the same nature; while 1, 2, 4, 7, 10, have all the dyadic relations of the same nature and 3, 5, 6, 8, 9, have only two of the same nature.

† In 1–6, the third correlate is determined by the first to have a quality; in 7–9, it is determined to have an existential relation to the second, and in 10 it is determined to have a relation of thought to the second for another correlate.

Object, and for some possible Interpretant. A *Sign* is a representamen of which some interpretant is a cognition of a mind. Signs are the only representamens that have been much studied.

§4. ONE TRICHOTOMY OF SIGNS

243. Signs are divisible by three trichotomies;* first, according as the sign in itself is a mere quality, is an actual existent, or is a general law;† secondly, according as the relation of the sign to its object consists in the sign's having some character in itself, or in some existential relation to that object, or in its relation to an interpretant;‡ thirdly, according as its Interpretant represents it as a sign of possibility or as a sign of fact or a sign of reason.§

244. According to the first division, a Sign may be termed a *Qualisign*, a *Sinsign*, or a *Legisign*.

A *Qualisign* is a quality which is a Sign. It cannot actually act as a sign until it is embodied; but the embodiment has nothing to do with its character as a sign.

245. A *Sinsign* (where the syllable *sin* is taken as meaning "being only once," as in *single*, *simple*, Latin *semel*, etc.) is an actual existent thing or event which is a sign. It can only be so through its qualities; so that it involves a qualisign, or rather, several qualisigns. But these qualisigns are of a peculiar kind and only form a sign through being actually embodied.

246. A *Legisign* is a law that is a Sign. This law is usually

* Peirce later (c. 1906, see e.g., 1.291, 4.530) discovered that there are ten trichotomies and sixty-six classes of signs. The analysis of the additional divisions was never satisfactorily completed; the best statement of them is to be found in the letters to Lady Welby, vol. 9. The present book, it is believed, contains most of Peirce's most thorough and authoritative work on signs.

The ten classes of signs derived from the three trichotomies here given are diagrammatically presented by Peirce in 264. If "Representamen," "Representamen as related to object," and "Interpreted Representamen" be substituted for first, second, and third correlate respectively, the tables of 235n and 239n should prove helpful schemata in §4–§7. The present section treats of the firstness, secondness, and thirdness of the *Representamen*.

† If we make the suggested substitutions we get the three groups consisting of: I; II, III, IV; and V–X.

‡ I.e., I, II, V; III, IV, VI, VII; VIII, IX, X.

§ I.e., The three groups of 241n — 1–6, 7–9; 10 i.e., I, II, III, V, VI, VIII; IV, VII, IX; X.

established by men. Every conventional sign is a legisign [but not conversely]. It is not a single object, but a general type which, it has been agreed, shall be significant. Every legisign signifies through an instance of its application, which may be termed a *Replica* of it. Thus, the word "the" will usually occur from fifteen to twenty-five times on a page. It is in all these occurrences one and the same word, the same legisign. Each single instance of it is a Replica. The Replica is a Sinsign. Thus, every Legisign requires Sinsigns. But these are not ordinary Sinsigns, such as are peculiar occurrences that are regarded as significant. Nor would the Replica be significant if it were not for the law which renders it so.

§5. A SECOND TRICHOTOMY OF SIGNS

247. According to the second trichotomy, a Sign may be termed an *Icon*, an *Index*, or a *Symbol*.

An *Icon* is a sign which refers to the Object that it denotes merely by virtue of characters of its own, and which it possesses, just the same, whether any such Object actually exists or not. It is true that unless there really is such an Object, the Icon does not act as a sign; but this has nothing to do with its character as a sign. Anything whatever, be it quality, existent individual, or law, is an Icon of anything, in so far as it is like that thing and used as a sign of it.

248. An *Index* is a sign which refers to the Object that it denotes by virtue of being really affected by that Object. It cannot, therefore, be a Qualisign, because qualities are whatever they are independently of anything else. In so far as the Index is affected by the Object, it necessarily has some Quality in common with the Object, and it is in respect to these that it refers to the Object. It does, therefore, involve a sort of Icon, although an Icon of a peculiar kind; and it is not the mere resemblance of its Object, even in these respects which makes it a sign, but it is the actual modification of it by the Object.

249. A *Symbol* is a sign which refers to the Object that it denotes by virtue of a law, usually an association of general ideas, which operates to cause the Symbol to be interpreted as referring to that Object. It is thus itself a general type or law, that is, is a Legisign. As such it acts through a Replica. Not only is it general itself, but the Object to which it refers

is of a general nature. Now that which is general has its being in the instances which it will determine. There must, therefore, be existent instances of what the Symbol denotes, although we must here understand by "existent," existent in the possibly imaginary universe to which the Symbol refers. The Symbol will indirectly, through the association or other law, be affected by those instances; and thus the Symbol will involve a sort of Index, although an Index of a peculiar kind. It will not, however, be by any means true that the slight effect upon the Symbol of those instances accounts for the significant character of the Symbol.

§6. A THIRD TRICHOTOMY OF SIGNS

250. According to the third trichotomy, a Sign may be termed a *Rheme*, a *Dicisign* or *Dicent Sign* (that is, a proposition or quasi-proposition), or an *Argument*.

A *Rheme** is a Sign which, for its Interpretant, is a Sign of qualitative Possibility, that is, is understood as representing such and such a kind of possible Object. Any Rheme, perhaps, will afford some information; but it is not interpreted as doing so.

251. A *Dicent Sign* is a Sign, which, for its Interpretant, is a Sign of actual existence. It cannot, therefore, be an Icon, which affords no ground for an interpretation of it as referring to actual existence. A Dicisign necessarily involves, as a part of it, a Rheme, to describe the fact which it is interpreted as indicating. But this is a peculiar kind of Rheme; and while it is essential to the Dicisign, it by no means constitutes it.

252. An *Argument* is a Sign which, for its Interpretant, is a Sign of law. Or we may say that a Rheme is a sign which is understood to represent its object in its characters merely; that a Dicisign is a sign which is understood to represent its object in respect to actual existence; and that an Argument is a Sign which is understood to represent its Object in its character as Sign. Since these definitions touch upon points at this time much in dispute, a word may be added in defence of them. A question often put is: What is the essence of a Judgment? A judgment is the mental act by which the judger seeks to impress upon himself the truth of a proposition. It is much

* See the second note to 95.

the same as an act of asserting the proposition, or going before a notary and assuming formal responsibility for its truth, except that those acts are intended to affect others, while the judgment is only intended to affect oneself. However, the logician, as such, cares not what the psychological nature of the act of judging may be. The question for him is: What is the nature of the sort of sign of which a principal variety is called a proposition, which is the matter upon which the act of judging is exercised? The proposition need not be asserted or judged. It may be contemplated as a sign capable of being asserted or denied. This sign itself retains its full meaning whether it be actually asserted or not.* The peculiarity of it, therefore, lies in its mode of meaning; and to say this is to say that its peculiarity lies in its relation to its interpretant. The proposition professes to be really affected by the actual existent or real law to which it refers. The argument makes the same pretension, but that is not the principal pretension of the argument. The rheme makes no such pretension.

253. The Interpretant of the Argument represents it as an instance of a general class of Arguments, which class on the whole will always tend to the truth. It is this law, in some shape, which the argument urges; and this "urging" is the mode of representation proper to Arguments. The Argument must, therefore, be a Symbol, or Sign whose Object is a General Law or Type. It must involve a Dicent Symbol, or Proposition, which is termed its *Premiss;* for the Argument can only urge the law by urging it in an instance. This Premiss is, however, quite different in force (*i.e.*, in its relation to its interpretant) from a similar proposition merely asserted; and besides, this is far from being the whole Argument. As for another proposition, called the Conclusion, often stated and perhaps required to complete the Argument, it plainly represents the Interpretant, and likewise has a peculiar force, or relation to the Interpretant. There is a difference of opinion among logicians as to whether it forms a part of the Argument or not; and although such opinions have not resulted from an exact analysis of the essence of Argument, they are entitled to weight. The present writer, without being absolutely confident, is strongly inclined to think that the Conclusion, although

* See 315.

it represents the Interpretant, is essential to the full expression of the Argument. It is usual with logicians to speak of the Premisses of an Argument, instead of the Premiss. But if there are more Premisses than one, the first step of the argumentation must be to colligate them into one Copulative Proposition: so that the only simple Argument of two Premisses is the Argument of Colligation. But even in this case, there are not properly two premisses. For whenever the mind is in a state ready to assert a proposition, P, it is already in a state of asserting a proposition, O, which the new proposition, P, only further determines; so that it is not P, merely, which comes to be asserted, but OP. In this view of the matter, there is no such thing as an Argument of Colligation. For to say that there is would make every judgment the conclusion of an argument. But if every judgment is to be regarded as the conclusion of an argument, which is, no doubt, an admissible conception, then it is the conclusion of a quite different kind of judgment from a mere Argument of Colligation. Thus, the Argument of Colligation is a form of Argument which is introduced into logic merely in order to avoid the necessity of considering the true nature of the Argument from which a Copulative Proposition has been derived. For that reason, it seems more proper in general to speak of the "Premiss" of an Argument than of its "Premisses." As to the word *Premiss* — in Latin of the thirteenth century *praemissa* — owing to its being so often used in the plural, it has become widely confounded with a totally different word of legal provenance, the "premisses," that is, the items of an inventory, etc., and hence buildings enumerated in a deed or lease. It is entirely contrary to good English usage to spell premiss, "premise," and this spelling (whose prevalence is due perhaps to Lord Brougham, or at least chiefly supported by his insistence), simply betrays ignorance of the history of logic, and even of such standard authors as Whateley, Watts, etc.*

§7. TEN CLASSES OF SIGNS

254. The three trichotomies of Signs result together in dividing Signs into *TEN CLASSES OF SIGNS*, of which numerous subdivisions have to be considered. The ten classes are as follows:

* See 582–3.

First: A Qualisign [*e.g.*, a feeling of "red"] is any quality in so far as it is a sign. Since a quality is whatever it is positively in itself, a quality can only denote an object by virtue of some common ingredient or similarity; so that a Qualisign is necessarily an Icon. Further, since a quality is a mere logical possibility, it can only be interpreted as a sign of essence, that is, as a Rheme.

255. Second: An Iconic Sinsign [*e.g.*, an individual diagram] is any object of experience in so far as some quality of it makes it determine the idea of an object. Being an Icon, and thus a sign by likeness purely, of whatever it may be like, it can only be interpreted as a sign of essence, or Rheme. It will embody a Qualisign.

256. Third: A Rhematic Indexical Sinsign [*e.g.*, a spontaneous cry] is any object of direct experience so far as it directs attention to an Object by which its presence is caused. It necessarily involves an Iconic Sinsign of a peculiar kind, yet is quite different since it brings the attention of the interpreter to the very Object denoted.

257. Fourth: A Dicent Sinsign [*e.g.*, a weathercock] is any object of direct experience, in so far as it is a sign, and, as such, affords information concerning its Object. This it can only do by being really affected by its Object; so that it is necessarily an Index. The only information it can afford is of actual fact. Such a Sign must involve an Iconic Sinsign to embody the information and a Rhematic Indexical Sinsign to indicate the Object to which the information refers. But the mode of combination, or *Syntax*, of these two must also be significant.

258. Fifth: An Iconic Legisign [*e.g.*, a diagram, apart from its factual individuality] is any general law or type, in so far as it requires each instance of it to embody a definite quality which renders it fit to call up in the mind the idea of a like object. Being an Icon, it must be a Rheme. Being a Legisign, its mode of being is that of governing single Replicas, each of which will be an Iconic Sinsign of a peculiar kind.

259. Sixth: A Rhematic Indexical Legisign [*e.g.*, a demonstrative pronoun] is any general type or law, however established, which requires each instance of it to be really affected by its Object in such a manner as merely to draw attention to that Object. Each Replica of it will be a Rhematic Indexical

Sinsign of a peculiar kind. The Interpretant of a Rhematic Indexical Legisign represents it as an Iconic Legisign; and so it is, in a measure — but in a very small measure.

260. Seventh: A Dicent Indexical Legisign [*e.g.*, a street cry] is any general type or law, however established, which requires each instance of it to be really affected by its Object in such a manner as to furnish definite information concerning that Object. It must involve an Iconic Legisign to signify the information and a Rhematic Indexical Legisign to denote the subject of that information. Each Replica of it will be a Dicent Sinsign of a peculiar kind.

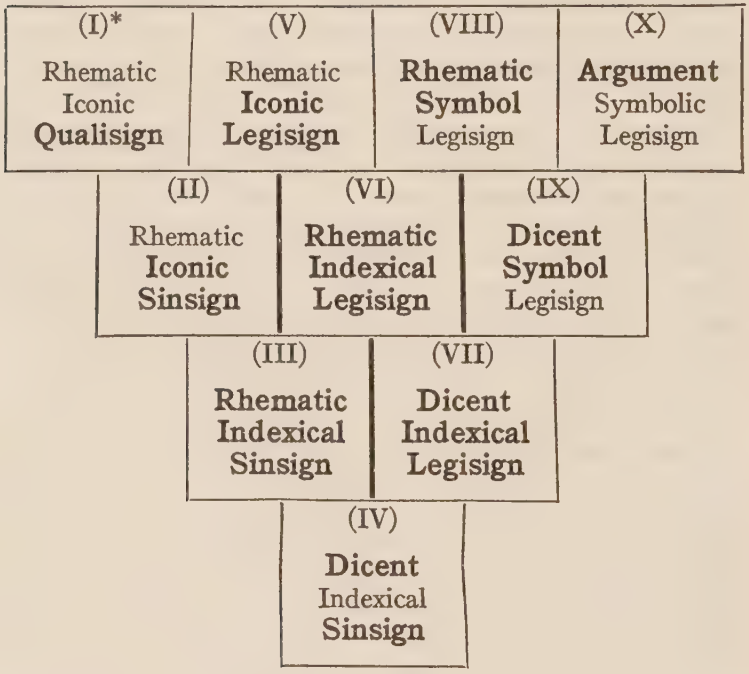
261. Eighth: A Rhematic Symbol or Symbolic Rheme [*e.g.*, a common noun] is a sign connected with its Object by an association of general ideas in such a way that its Replica calls up an image in the mind which image, owing to certain habits or dispositions of that mind, tends to produce a general concept, and the Replica is interpreted as a Sign of an Object that is an instance of that concept. Thus, the Rhematic Symbol either is, or is very like, what the logicians call a General Term. The Rhematic Symbol, like any Symbol, is necessarily itself of the nature of a general type, and is thus a Legisign. Its Replica, however, is a Rhematic Indexical Sinsign of a peculiar kind, in that the image it suggests to the mind acts upon a Symbol already in that mind to give rise to a General Concept. In this it differs from other Rhematic Indexical Sinsigns, including those which are Replicas of Rhematic Indexical Legisigns. Thus, the demonstrative pronoun "that" is a Legisign, being a general type; but it is not a Symbol, since it does not signify a general concept. Its Replica draws attention to a single Object, and is a Rhematic Indexical Sinsign. A Replica of the word "camel" is likewise a Rhematic Indexical Sinsign, being really affected, through the knowledge of camels, common to the speaker and auditor, by the real camel it denotes, even if this one is not individually known to the auditor; and it is through such real connection that the word "camel" calls up the idea of a camel. The same thing is true of the word "phoenix." For although no phoenix really exists, real descriptions of the phoenix are well known to the speaker and his auditor; and thus the word is really affected by the Object denoted. But not only are the Replicas of Rhematic Symbols very dif-

ferent from ordinary Rhematic Indexical Sinsigns, but so likewise are Replicas of Rhematic Indexical Legisigns. For the thing denoted by "that" has not affected the replica of the word in any such direct and simple manner as that in which, for example, the ring of a telephone-bell is affected by the person at the other end who wants to make a communication. The Interpretant of the Rhematic Symbol often represents it as a Rhematic Indexical Legisign; at other times as an Iconic Legisign; and it does in a small measure partake of the nature of both.

262. Ninth: A Dicent Symbol, or ordinary Proposition, is a sign connected with its object by an association of general ideas, and acting like a Rhematic Symbol, except that its intended interpretant represents the Dicent Symbol as being, in respect to what it signifies, really affected by its Object, so that the existence or law which it calls to mind must be actually connected with the indicated Object. Thus, the intended Interpretant looks upon the Dicent Symbol as a Dicent Indexical Legisign; and if it be true, it does partake of this nature, although this does not represent its whole nature. Like the Rhematic Symbol, it is necessarily a Legisign. Like the Dicent Sinsign it is composite inasmuch as it necessarily involves a Rhematic Symbol (and thus is for its Interpretant an Iconic Legisign) to express its information and a Rhematic Indexical Legisign to indicate the subject of that information. But its Syntax of these is significant. The Replica of the Dicent Symbol is a Dicent Sinsign of a peculiar kind. This is easily seen to be true when the information the Dicent Symbol conveys is of actual fact. When that information is of a real law, it is not true in the same fullness. For a Dicent Sinsign cannot convey information of law. It is, therefore, true of the Replica of such a Dicent Symbol only in so far as the law has its being in instances.

263. Tenth: An Argument is a sign whose interpretant represents its object as being an ulterior sign through a law, namely, the law that the passage from all such premisses to such conclusions tends to the truth. Manifestly, then, its object must be general; that is, the Argument must be a Symbol. As a Symbol it must, further, be a Legisign. Its Replica is a Dicent Sinsign.

264. The affinities of the ten classes are exhibited by arranging their designations in the triangular table here shown, which has heavy boundaries between adjacent squares that are appropriated to classes alike in only one respect. All other adjacent squares pertain to classes alike in two respects. Squares not adjacent pertain to classes alike in one respect only, except that each of the three squares of the vertices of the triangle pertains to a class differing in all three respects from the classes to which the squares along the opposite side of the triangle are appropriated. The lightly printed designations are superfluous.



§8. DEGENERATE SIGNS

265. In the course of the above descriptions of the classes, certain subdivisions of some of them have been directly or indirectly referred to. Namely, beside the normal varieties of Sinsigns, Indices, and Dicisigns, there are others which are

* See 235n and 243n for explanation of the roman numerals.

Replicas of Legisigns, Symbols, and Arguments, respectively. Beside the normal varieties of Qualisigns, Icons, and Rhemes, there are two series of others; to wit, those which are directly involved in Sinsigns, Indices, and Dicisigns, respectively, and also those which are indirectly involved in Legisigns, Symbols, and Arguments, respectively. Thus, the ordinary Dicent Sinsign is exemplified by a weathercock and its veering and by a photograph. The fact that the latter is known to be the effect of the radiations from the object renders it an index and highly informative. A second variety is a Replica of a Dicent Indexical Legisign. Thus any given street cry, since its tone and theme identifies the individual, is not a symbol, but an Indexical Legisign; and any individual instance of it is a Replica of it which is a Dicent Sinsign. A third variety is a Replica of a Proposition. A fourth variety is a Replica of an Argument. Beside the normal variety of the Dicent Indexical Legisign, of which a street cry is an example, there is a second variety, which is that sort of proposition which has the name of a well-known individual as its predicate; as if one is asked, "Whose statue is this?" the answer may be, "It is Farragut." The meaning of this answer is a Dicent Indexical Legisign. A third variety may be a premiss of an argument. A Dicent Symbol, or ordinary proposition, in so far as it is a premiss of an Argument, takes on a new force, and becomes a second variety of the Dicent Symbol. It would not be worth while to go through all the varieties; but it may be well to consider the varieties of one class more. We may take the Rhematic Indexical Legisign. *The* shout of "Hullo!" is an example of the ordinary variety — meaning, not an individual shout, but this shout "Hullo!" in general — this type of shout. A second variety is a constituent of a Dicent Indexical Legisign; as the word "that" in the reply, "that is Farragut." A third variety is a particular application of a Rhematic Symbol; as the exclamation "Hark!" A fourth and fifth variety are in the peculiar force a general word may have in a proposition or argument. It is not impossible that some varieties are here overlooked. It is a nice problem to say to what class a given sign belongs; since all the circumstances of the case have to be considered. But it is seldom requisite to be very accurate; for if one does not locate the sign precisely, one will easily

come near enough to its character for any ordinary purpose of logic.

§9. THE TRICHOTOMY OF ARGUMENTS

266. There are other subdivisions of some, at least, of the ten classes which are of greater logical importance. An Argument is always understood by its Interpretant to belong to a general class of analogous arguments, which class, as a whole, tends toward the truth. This may happen in three ways, giving rise to a trichotomy of all simple arguments into Deductions, Inductions, and Abductions.

267. A *Deduction* is an argument whose Interpretant represents that it belongs to a general class of possible arguments precisely analogous which are such that in the long run of experience the greater part of those whose premisses are true will have true conclusions. Deductions are either *Necessary* or *Probable*. Necessary Deductions are those which have nothing to do with any ratio of frequency, but profess (or their interpretants profess for them) that from true premisses they must invariably produce true conclusions. A Necessary Deduction is a method of producing Dicent Symbols by the study of a diagram. It is either *Corollarial* or *Theorematic*. A Corollarial Deduction is one which represents the conditions of the conclusion in a diagram and finds from the observation of this diagram, as it is, the truth of the conclusion. A Theorematic Deduction is one which, having represented the conditions of the conclusion in a diagram, performs an ingenious experiment upon the diagram, and by the observation of the diagram, so modified, ascertains the truth of the conclusion.

268. Probable Deductions, or more accurately, Deductions of Probability, are Deductions whose Interpretants represent them to be concerned with ratios of frequency. They are either *Statistical Deductions* or *Probable Deductions Proper*. A Statistical Deduction is a Deduction whose Interpretant represents it to reason concerning ratios of frequency, but to reason concerning them with absolute certainty. A Probable Deduction proper is a Deduction whose Interpretant does not represent that its conclusion is certain, but that precisely analogous reasonings would from true premisses produce true conclusions in the majority of cases, in the long run of experience.

269. An *Induction* is a method of forming Dicent Symbols concerning a definite question, of which method the Interpretant does not represent that from true premisses it will yield approximately true results in the majority of instances in the long run of experience, but does represent that if this method be persisted in, it will in the long run yield the truth, or an indefinite approximation to the truth, in regard to every question. An Induction is either a *Pooh-pooh Argument*, or an *Experimental Verification of a general Prediction*, or an Argument from a *Random Sample*. A Pooh-pooh Argument is a method which consists in denying that a general kind of event ever will occur on the ground that it never has occurred. Its justification is that if it be persistently applied on every occasion, it must ultimately be corrected in case it should be wrong, and thus will ultimately reach the true conclusion. A verification of a general prediction is a method which consists in finding or making the conditions of the prediction and in concluding that it will be verified about as often as it is experimentally found to be verified. Its justification is that if the Prediction does not tend in the long run to be verified in any approximately determinate proportion of cases, experiment must, in the long run, ascertain this; while if the Prediction will, in the long run, be verified in any determinate, or approximately determinate, proportion of cases, experiment must in the long run, approximately ascertain what that proportion is. An Argument from a Random Sample, is a method of ascertaining what proportion of the members of a finite class possess a predesignate, or virtually predesignate, quality, by selecting instances from that class according to a method which will, in the long run, present any instance as often as any other, and concluding that the ratio found for such a sample will hold in the long run. Its justification is evident.

270. An *Abduction* is a method of forming a general prediction without any positive assurance that it will succeed either in the special case or usually, its justification being that it is the only possible hope of regulating our future conduct rationally, and that Induction from past experience gives us strong encouragement to hope that it will be successful in the future.

§10. KINDS OF PROPOSITIONS

271. A Dicent Symbol, or general proposition, is either *Particular* or *Universal*. A Particular Dicent Symbol is represented by its Interpretant to indicate fact of existence; as, "Some swan is black," *i.e.*, there exists a black swan. A Universal Dicent Symbol is represented by its Interpretant to indicate a real law; as "No swan is black," *i.e.*, no amount of research *will ever* discover a black individual among swans. A Dicent Symbol is either *Non-relative* or *Relative*. A Non-relative Dicent Symbol is not concerned with the identity of more than one individual. But this must be understood in a particular way, the proposition being first expressed in an exemplar manner. Thus "No swan is black" seems to be concerned with the identity of all swans and all black objects. But it is to be understood that the proposition is to be considered under this form: Taking any one object in the universe you please, it is either not a swan or is not black. A Relative Dicent Symbol is concerned with the identity of more than one individual, or of what may be more than one, in an exemplar expression, as, "Take any individual, A, you please, and thereafter an individual, B, can be found, such that if A is a city of over a hundred thousand inhabitants, B will be a spot on this map corresponding to A." Whether a proposition is to be regarded as non-relative or relative depends on what use is to be made of it in argument. But it does not follow that the distinction is merely one of outward guise; for the force of the proposition is different according to the application that is to be made of it. It may here be noted as a matter of correct terminology (according to the views set forth in the second part [of the published portion] of this syllabus),* that a *Hypothetical Proposition* is any proposition compounded of propositions. The old doctrine is that a hypothetical proposition is either conditional, copulative, or disjunctive. But a conditional is properly a disjunctive proposition. Some propositions may equally well be regarded as copulative or disjunctive. Thus, at once, either Tully or not Cicero and either Cicero or not Tully, is the same as, either at once, Tully and Cicero or not Tully and not Cicero. Any definition may be regarded as a proposition of this sort; and for this reason such propositions

* See ch. 1.

might be termed *Definiform*, or *Definitory*. A copulative proposition is naturally allied to a particular proposition, a disjunctive proposition to a universal proposition.

272. If parts of a proposition be erased so as to leave *blanks* in their places, and if these blanks are of such a nature that if each of them be filled by a proper name the result will be a proposition, then the blank form of proposition which was first produced by the erasures is termed a *rheme*. According as the number of blanks in a rheme is 0, 1, 2, 3, etc., it may be termed a *medad* (from μηδέν, nothing), *monad*, *dyad*, *triad*, etc., rheme.

§11. REPRESENT*

273. To stand for, that is, to be in such a relation to another that for certain purposes it is treated by some mind as if it were that other.

Thus a spokesman, deputy, attorney, agent, vicar, diagram, symptom, counter, description, concept, premiss, testimony, all represent something else, in their several ways, to minds who consider them in that way. See Sign.† When it is desired to distinguish between that which represents and the act or relation of representing, the former may be termed the "representamen," the latter the "representation."

* *Dictionary of Philosophy and Psychology*, vol. 2, p. 464.
† 303–4.

CHAPTER 3

THE ICON, INDEX, AND SYMBOL

§1. ICONS AND HYPOICONS*

274. A *Sign*, or *Representamen*, is a First which stands in such a genuine triadic relation to a Second, called its *Object*, as to be capable of determining a Third, called its *Interpretant*, to assume the same triadic relation to its Object in which it stands itself to the same Object. The triadic relation is *genuine*, that is its three members are bound together by it in a way that does not consist in any complexus of dyadic relations. That is the reason the Interpretant, or Third, cannot stand in a mere dyadic relation to the Object, but must stand in such a relation to it as the Representamen itself does. Nor can the triadic relation in which the Third stands be merely similar to that in which the First stands, for this would make the relation of the Third to the First a degenerate Secondness merely. The Third must indeed stand in such a relation, and thus must be capable of determining a Third of its own; but besides that, it must have a second triadic relation in which the Representamen, or rather the relation thereof to its Object, shall be its own (the Third's) Object, and must be capable of determining a Third to this relation. All this must equally be true of the Third's Thirds and so on endlessly; and this, and more, is involved in the familiar idea of a Sign; and as the term Representamen is here used, nothing more is implied. A *Sign* is a Representamen with a mental Interpretant. Possibly there may be Representamens that are not Signs. Thus, if a sunflower, in turning towards the sun, becomes by that very act fully capable, without further condition, of reproducing a sunflower which turns in precisely corresponding ways toward the sun, and of doing so with the same reproductive power,

* 274–7, 283–4, 292–4 are from "Syllabus," c. 1902, no part of which was ever published (cf. note to ch. 1). 278–80 are from "That Categorical and Hypothetical Propositions are one in essence, with some connected matters," c. 1895, following 339; 281, 285, 297–302 are from chapter 2 of "The Art of Reasoning," c. 1895, while 282, 286–91 and 295–6 are from "The Short Logic," c. 1893.

the sunflower would become a Representamen of the sun. But *thought* is the chief, if not the only, mode of representation.

275. . . . The most fundamental [division of signs] is into *Icons*, *Indices*, and *Symbols*. Namely, while no Representamen actually functions as such until it actually determines an Interpretant, yet it becomes a Representamen as soon as it is fully capable of doing this; and its Representative Quality is not necessarily dependent upon its ever actually determining an Interpretant, nor even upon its actually having an Object.

276. An *Icon* is a Representamen whose Representative Quality is a Firstness of it as a First. That is, a quality that it has *qua* thing renders it fit to be a representamen. Thus, anything is fit to be a *Substitute* for anything that it is like. (The conception of "substitute" involves that of a purpose, and thus of genuine thirdness.) Whether there are other kinds of substitutes or not we shall see. A Representamen by Firstness alone can only have a similar Object. Thus, a Sign by Contrast denotes its object only by virtue of a contrast, or Secondness, between two qualities. A sign by Firstness is an image of its object and, more strictly speaking, can only be an *idea*. For it must produce an Interpretant idea; and an external object excites an idea by a reaction upon the brain. But most strictly speaking, even an idea, except in the sense of a possibility, or Firstness, cannot be an Icon. A possibility alone is an Icon purely by virtue of its quality; and its object can only be a Firstness. But a sign may be *iconic*, that is, may represent its object mainly by its similarity, no matter what its mode of being. If a substantive be wanted, an iconic representamen may be termed a *hypoicon*. Any material image, as a painting, is largely conventional in its mode of representation; but in itself, without legend or label it may be called a *hypoicon*.

277. Hypoicons may be roughly divided according to the mode of Firstness of which they partake. Those which partake of simple qualities, or First Firstnesses, are *images*; those which represent the relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts, are *diagrams;* those which represent the representative character of a representamen by representing a parallelism in something else, are *metaphors*.

278. The only way of directly communicating an idea is by means of an icon; and every indirect method of communicating an idea must depend for its establishment upon the use of an icon. Hence, every assertion must contain an icon or set of icons, or else must contain signs whose meaning is only explicable by icons. The idea which the set of icons (or the equivalent of a set of icons) contained in an assertion signifies may be termed the *predicate* of the assertion.

279. Turning now to the rhetorical evidence, it is a familiar fact that there are such representations as icons. Every picture (however conventional its method) is essentially a representation of that kind. So is every diagram, even although there be no sensuous resemblance between it and its object, but only an analogy between the relations of the parts of each. Particularly deserving of notice are icons in which the likeness is aided by conventional rules. Thus, an algebraic formula is an icon, rendered such by the rules of commutation, association, and distribution of the symbols. It may seem at first glance that it is an arbitrary classification to call an algebraic expression an icon; that it might as well, or better, be regarded as a compound conventional sign. But it is not so. For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction. Thus, by means of two photographs a map can be drawn, etc. Given a conventional or other general sign of an object, to deduce any other truth than that which it explicitly signifies, it is necessary, in all cases, to replace that sign by an icon. This capacity of revealing unexpected truth is precisely that wherein the utility of algebraical formulae consists, so that the iconic character is the prevailing one.

280. That icons of the algebraic kind, though usually very simple ones, exist in all ordinary grammatical propositions is one of the philosophic truths that the Boolean logic brings to light. In all primitive writing, such as the Egyptian hieroglyphics, there are icons of a non-logical kind, the ideographs. In the earliest form of speech, there probably was a large element of mimicry. But in all languages known, such representations have been replaced by conventional auditory signs. These, however, are such that they can only be explained by

icons. But in the syntax of every language there are logical icons of the kind that are aided by conventional rules. . . .

281. Photographs, especially instantaneous photographs, are very instructive, because we know that they are in certain respects exactly like the objects they represent. But this resemblance is due to the photographs having been produced under such circumstances that they were physically forced to correspond point by point to nature. In that aspect, then, they belong to the second class of signs, those by physical connection. The case is different if I surmise that zebras are likely to be obstinate, or otherwise disagreeable animals, because they seem to have a general resemblance to donkeys, and donkeys are self-willed. Here the donkey serves precisely as a probable likeness of the zebra. It is true we suppose that resemblance has a physical cause in heredity; but then, this hereditary affinity is itself only an inference from the likeness between the two animals, and we have not (as in the case of the photograph) any independent knowledge of the circumstances of the production of the two species. Another example of the use of a likeness is the design an artist draws of a statue, pictorial composition, architectural elevation, or piece of decoration, by the contemplation of which he can ascertain whether what he proposes will be beautiful and satisfactory. The question asked is thus answered almost with certainty because it relates to how the artist will himself be affected. The reasoning of mathematicians will be found to turn chiefly upon the use of likenesses, which are the very hinges of the gates of their science. The utility of likenesses to mathematicians consists in their suggesting in a very precise way, new aspects of supposed states of things. . . .

282. Many diagrams resemble their objects not at all in looks; it is only in respect to the relations of their parts that their likeness consists. Thus, we may show the relation between the different kinds of signs by a brace, thus:

$$\text{Signs: } \left\{ \begin{array}{l} \text{Icons,} \\ \text{Indices,} \\ \text{Symbols.} \end{array} \right.$$

This is an icon. But the only respect in which it resembles its object is that the brace shows the classes of *icons*, *indices*, and

symbols to be related to one another and to the general class of signs, as they really are, in a general way. When, in algebra, we write equations under one another in a regular array, especially when we put resembling letters for corresponding coefficients, the array is an icon. Here is an example:

$$a_1x+b_1y=n_1,$$
$$a_2x+b_2y=n_2.$$

This is an icon, in that it makes quantities look alike which are in analogous relations to the problem. In fact, every algebraical equation is an icon, in so far as it *exhibits*, by means of the algebraical signs (which are not themselves icons), the relations of the quantities concerned.

It may be questioned whether all icons are likenesses or not. For example, if a drunken man is exhibited in order to show, by contrast, the excellence of temperance, this is certainly an icon, but whether it is a likeness or not may be doubted. The question seems somewhat trivial.

§2. GENUINE AND DEGENERATE INDICES

283. An *Index* or *Seme** ($\sigma\tilde{\eta}\mu\alpha$) is a Representamen whose Representative character consists in its being an individual second. If the Secondness is an existential relation, the Index is *genuine*. If the Secondness is a reference, the Index is *degenerate*. A genuine Index and its Object must be existent individuals (whether things or facts), and its immediate Interpretant must be of the same character. But since every individual must have characters, it follows that a genuine Index may contain a Firstness, and so an Icon as a constituent part of it. Any individual is a degenerate Index of its own characters.

284. *Subindices* or *Hyposemes* are signs which are rendered such principally by an actual connection with their objects. Thus a proper name, personal demonstrative, or relative pronoun or the letter attached to a diagram, denotes what it does owing to a real connection with its object but none of these is an Index, since it is not an individual.

285. Let us examine some examples of indices. I see a man with a rolling gait. This is a probable indication that he is a sailor. I see a bowlegged man in corduroys, gaiters, and a jacket. These are probable indications that he is a jockey or

* "Seme" is usually reserved for indexical dicisigns which are only a subclass of the indices.

something of the sort. A sundial or a clock *indicates* the time of day. Geometricians mark letters against the different parts of their diagrams and then use these letters to indicate those parts. Letters are similarly used by lawyers and others. Thus, we may say: If A and B are married to one another and C is their child while D is brother of A, then D is uncle of C. Here A, B, C, and D fulfill the office of relative pronouns, but are more convenient since they require no special collocation of words. A rap on the door is an index. Anything which focusses the attention is an index. Anything which startles us is an index, in so far as it marks the junction between two portions of experience. Thus a tremendous thunderbolt indicates that *something* considerable happened, though we may not know precisely what the event was. But it may be expected to connect itself with some other experience.

286. . . . A low barometer with a moist air is an index of rain; that is we suppose that the forces of nature establish a probable connection between the low barometer with moist air and coming rain. A weathercock is an index of the direction of the wind; because in the first place it really takes the self-same direction as the wind, so that there is a real connection between them, and in the second place we are so constituted that when we see a weathercock pointing in a certain direction it draws our attention to that direction, and when we see the weathercock veering with the wind, we are forced by the law of mind to think that direction is connected with the wind. The pole star is an index, or pointing finger, to show us which way is north. A spirit-level, or a plumb bob, is an index of the vertical direction. A yard-stick might seem, at first sight, to be an icon of a yard; and so it would be, if it were merely intended to show a yard as near as it can be seen and estimated to be a yard. But the very purpose of a yard-stick is to show a yard nearer than it can be estimated by its appearance. This it does in consequence of an accurate mechanical comparison made with the bar in London called the yard. Thus it is a real connection which gives the yard-stick its value as a representamen; and thus it is an *index*, not a mere *icon*.

287. When a driver to attract the attention of a foot passenger and cause him to save himself, calls out "Hi!" so far as this is a significant word, it is, as will be seen below,

something more than an index; but so far as it is simply intended to act upon the hearer's nervous system and to rouse him to get out of the way, it is an index, because it is meant to put him in real connection with the object, which is his situation relative to the approaching horse. Suppose two men meet upon a country road and one of them says to the other, "The chimney of that house is on fire." The other looks about him and descries a house with green blinds and a verandah having a smoking chimney. He walks on a few miles and meets a second traveller. Like a Simple Simon he says, "The chimney of that house is on fire." "What house?" asks the other. "Oh, a house with green blinds and a verandah," replies the simpleton. "Where is the house?" asks the stranger. He desires some *index* which shall connect his apprehension with the house meant. Words alone cannot do this. The demonstrative pronouns, "this" and "that," are indices. For they call upon the hearer to use his powers of observation, and so establish a real connection between his mind and the object; and if the demonstrative pronoun does that — without which its meaning is not understood — it goes to establish such a connection; and so is an index. The relative pronouns, *who* and *which*, demand observational activity in much the same way, only with them the observation has to be directed to the words that have gone before. Lawyers use A, B, C, practically as very effective relative pronouns. To show how effective they are, we may note that Messrs. Allen and Greenough, in their admirable (though in the edition of 1877 [?], too small) Latin Grammar, declare that no conceivable syntax could wholly remove the ambiguity of the following sentence, "A replied to B that he thought C (his brother) more unjust to himself than to his own friend."* Now, any lawyer would state that with perfect clearness, by using A, B, C, as relatives, thus:

$$\text{A replied to B that he } \begin{matrix}(\text{A})\\(\text{B})\end{matrix}\text{, thought C}$$

$$\text{(his } \begin{matrix}(\text{A's})\\(\text{B's})\end{matrix}\text{, brother) more unjust to himself, } \begin{matrix}(\text{A})\\(\text{B})\\(\text{C})\end{matrix}\text{ than to his}$$

* *New Latin Grammar*, p. 131n (ed. 1884).

(A's)
(B's) own friend.[1] The terminations which in any inflected
(C's)
language are attached to words "governed" by other words, and which serve to show which the governing word is, by repeating what is elsewhere expressed in the same form, are likewise *indices* of the same relative pronoun character. Any bit of Latin poetry illustrates this, such as the twelve-line sentence beginning, "*Jam satis terris.*" Both in these terminations and in the A, B, C, a likeness is relied upon to carry the attention to the right object. But this does not make them icons, in any important way; for it is of no consequence how the letters A, B, C, are shaped or what the terminations are. It is not merely that one occurrence of an A is like a previous occurrence that is the important circumstance, but that *there is an understanding that like letters shall stand for the same thing*, and this acts as a force carrying the attention from one occurrence of A to the previous one. A possessive pronoun is two ways an index: first it indicates the possessor, and, second, it has a modification which syntactically carries the attention to the word denoting the thing possessed.

288. Some indices are more or less detailed directions for what the hearer is to do in order to place himself in direct experiential or other connection with the thing meant. Thus,

[1] Modern grammars define a pronoun as a word used in place of a noun. That is an ancient doctrine which, exploded early in the thirteenth century, disappeared from the grammars for several hundred years. But the substitute employed was not very clear; and when a barbarous rage against medieval thought broke out, it was swept away. Some recent grammars, as *Allen and Greenough's*, set the matter right again. There is no reason for saying that *I*, *thou*, *that*, *this*, stand in place of nouns; they indicate things in the directest possible way. It is impossible to express what an assertion refers to except by means of an index. A pronoun is an index. A noun, on the other hand, does not *indicate* the object it denotes; and when a noun is used to show what one is talking about, the experience of the hearer is relied upon to make up for the incapacity of the noun for doing what the pronoun does at once. Thus, a noun is an imperfect substitute for a pronoun. Nouns also serve to help out verbs. A pronoun ought to be defined as *a word which may indicate anything to which the first and second persons have suitable real connections, by calling the attention of the second person to it.* Allen and Greenough say, "pronouns indicate some person or thing without either naming or describing" [p. 128, edition of 1884]. This is correct — refreshingly correct; only it seems better to say what they *do*, and not merely what they don't.

the Coast Survey issues "Notices to Mariners," giving the latitude and longitude, four or five bearings of prominent objects, etc., and saying *there* is a rock, or shoal, or buoy, or lightship. Although there will be other elements in such directions, yet in the main they are indices.

289. Along with such indexical directions of what to do to find the object meant, ought to be classed those pronouns which should be entitled *selective* pronouns [or quantifiers] because they inform the hearer how he is to pick out one of the objects intended, but which grammarians call by the very indefinite designation of *indefinite* pronouns. Two varieties of these are particularly important in logic, the *universal selectives*, such as *quivis*, *quilibet*, *quisquam*, *ullus*, *nullus*, *nemo*, *quisque*, *uterque*, and in English, *any*, *every*, *all*, *no*, *none*, *whatever*, *whoever*, *everybody*, *anybody*, *nobody*. These mean that the hearer is at liberty to select any instance he likes within limits expressed or understood, and the assertion is intended to apply to that one. The other logically important variety consists of the *particular selectives*, *quis*, *quispiam*, *nescio quis*, *aliquis*, *quidam*, and in English, *some*, *something*, *somebody*, *a*, *a certain*, *some or other*, *a suitable*, *one*.

Allied to the above pronouns are such expressions as *all but one*, *one or two*, *a few*, *nearly all*, *every other one*, etc. Along with pronouns are to be classed adverbs of place and time, etc.

Not very unlike these are, *the first*, *the last*, *the seventh*, *two-thirds of*, *thousands of*, *etc.*

290. Other indexical words are prepositions, and prepositional phrases, such as, "on the right (or left) of." Right and left cannot be distinguished by any general description. Other prepositions signify relations which may, perhaps, be described; but when they refer, as they do oftener than would be supposed, to a situation relative to the observed, or assumed to be experientially known, place and attitude of the speaker relatively to that of the hearer, then the indexical element is the dominant element.[1]

[1] If a logician had to construct a language *de novo* — which he actually has almost to do — he would naturally say, I shall need prepositions to express the temporal relations of *before*, *after*, and *at the same time with*, I shall need prepositions to express the spatial relations of *adjoining*, *containing*, *touching*, of *in range with*, of *near to*, *far from*, of *to the right of*, *to the left of*, *above*, *below*, *before*,

291. Icons and indices assert nothing. If an icon could be interpreted by a sentence, that sentence must be in a "potential mood," that is, it would merely say, "Suppose a figure has three sides," etc. Were an index so interpreted, the mood must be imperative, or exclamatory, as "See there!" or "Look out!" But the kind of signs which we are now coming to consider are, by nature, in the "indicative," or, as it should be called, the *declarative* mood.[1] Of course, they can go to the expression of any other mood, since we may declare assertions to be doubtful, or mere interrogations, or imperatively requisite.

§3. THE NATURE OF SYMBOLS

292. A *Symbol* is a Representamen whose Representative character consists precisely in its being a rule that will determine its Interpretant. All words, sentences, books, and other conventional signs are Symbols. We speak of writing or pronouncing the word "man"; but it is only a *replica*, or embodiment of the word, that is pronounced or written. The word itself has no existence although it has a real being, *consisting*

behind, and I shall need prepositions to express motions into and out of these situations. For the rest, I can manage with metaphors. Only if my language is intended for use by people having some great geographical feature related the same way to all of them, as a mountain range, the sea, a great river, it will be desirable to have prepositions signifying situations relatively to that, as *across*, *seaward*, etc. But when we examine actual languages, it would seem as though they had supplied the place of many of these distinctions by gestures. The Egyptians had no preposition nor demonstrative having any apparent reference to the Nile. Only the Esquimos are so wrapped up in their bearskins that they have demonstratives distinguishing landward, seaward, north, south, east, and west. But examining the cases or prepositions of any actual language we find them a haphazard lot.

[1] The nomenclature of grammar, like that of logic, is derived chiefly from a late Latin,the words being transferred from the Greek, the Latin prefix translating the Greek prefix and the Latin stem the Greek stem. But while the logical words were chosen with fastidious care, the grammarians were excessively careless, and none more so than Priscian. The word *indicative* is one of Priscian's creations. It was evidently intended to translate Aristotle's ἀποφαντική. But this is precisely equivalent to *declarative* both in signification and according to the rules of transferrence, *de*, taking the place of ἀπο as is usual in these artificial formations (*de*monstration for ἀπόδειξις, etc.), and *clarare* representing φαίνειν, to make clear. Perhaps the reason Priscian did not choose the word *declaratiuus* was that Apuleius [see Prantl's *Geschichte der Logik*, I, 581], a great authority on words, had used this in a somewhat different sense.

in the fact that existents *will* conform to it. It is a general mode of succession of three sounds or representamens of sounds, which becomes a sign only in the fact that a habit, or acquired law, will cause replicas of it to be interpreted as meaning a man or men. The word and its meaning are both general rules; but the word alone of the two prescribes the qualities of its replicas in themselves. Otherwise the "word" and its "meaning" do not differ, unless some special sense be attached to "meaning."

293. A Symbol is a law, or regularity of the indefinite future. Its Interpretant must be of the same description; and so must be also the complete immediate Object, or meaning.[1] But a law necessarily governs, or "is embodied in" individuals, and prescribes some of their qualities. Consequently, a constituent of a Symbol may be an Index, and a constituent may be an Icon. A man walking with a child points his arm up into the air and says, "There is a balloon." The pointing arm is an essential part of the symbol without which the latter would convey no information. But if the child asks, "What is a balloon," and the man replies, "It is something like a great big soap bubble," he makes the image a part of the symbol. Thus, while the complete object of a symbol, that is to say, its meaning, is of the nature of a law, it must *denote* an individual, and must *signify* a character. A *genuine* symbol is a symbol that has a general meaning. There are two kinds of degenerate symbols, the *Singular Symbol* whose Object is an existent individual, and which signifies only such characters as that individual may realize; and the *Abstract Symbol*, whose only Object is a character.

294. Although the immediate Interpretant of an Index must be an Index, yet since its Object may be the Object of an Individual [Singular] Symbol, the Index may have such a Symbol for its indirect Interpretant. Even a genuine Symbol may be an imperfect Interpretant of it. So an *icon* may have a degenerate Index, or an Abstract Symbol, for an indirect

[1] There are two ways in which a Symbol may have a real Existential Thing as its real Object. First, the thing may conform to it, whether accidentally or by virtue of the Symbol having the virtue of a growing habit, and secondly, by the Symbol having an Index as a part of itself. But the immediate object of a symbol can only be a symbol and if it has in its own nature another kind of object, this must be by an *endless series*.

Interpretant, and a genuine Index or Symbol for an imperfect Interpretant.

295. A *Symbol* is a sign naturally fit to declare that the set of objects which is denoted by whatever set of indices may be in certain ways attached to it is represented by an icon associated with it. To show what this complicated definition means, let us take as an example of a symbol the word "loveth." Associated with this word is an idea, which is the mental icon of one person loving another. Now we are to understand that "loveth" occurs in a sentence; for what it may mean by itself, if it means anything, is not the question. Let the sentence, then, be "Ezekiel loveth Huldah." Ezekiel and Huldah must, then, be or contain indices; for without indices it is impossible to designate what one is talking about. Any mere description would leave it uncertain whether they were not mere characters in a ballad; but whether they be so or not, indices can designate them. Now the effect of the word "loveth" is that the pair of objects denoted by the pair of indices Ezekiel and Huldah is represented by the icon, or the image we have in our minds of a lover and his beloved.

296. The same thing is equally true of every verb in the declarative mood; and indeed of every verb, for the other moods are merely declarations of a fact somewhat different from that expressed by the declarative mood. As for a noun, considering the meaning which it has in the sentence, and not as standing by itself, it is most conveniently regarded as a portion of a symbol. Thus the sentence, "every man loves a woman" is equivalent to "whatever is a man loves something that is a woman." Here "whatever" is a universal selective index, "is a man" is a symbol, "loves" is a symbol, "something that" is a particular selective index, and "is a woman" is a symbol. . . .

297. The word *Symbol* has so many meanings that it would be an injury to the language to add a new one. I do not think that the signification I attach to it, that of a conventional sign, or one depending upon habit (acquired or inborn), is so much a new meaning as a return to the original meaning. Etymologically, it should mean a thing thrown together, just as ἔμβολον (embolum) is a thing thrown into something, a bolt, and παράβολον (parabolum) is a thing thrown besides, collateral

security, and ' *πόβολον* (hypobolum) is a thing thrown underneath, an antenuptial gift. It is usually said that in the word *symbol* the throwing together is to be understood in the sense of "to conjecture"; but were that the case, we ought to find that *sometimes* at least it meant a conjecture, a meaning for which literature may be searched in vain. But the Greeks used "throw together" (*συμβάλλειν*) very frequently to signify the making of a contract or convention. Now, we do find symbol (*σύμβολον*) early and often used to mean a convention or contract. Aristotle calls a noun a "symbol," that is, a conventional sign.* In Greek, watch-fire is a "symbol," that is, a signal agreed upon; a standard or ensign is a "symbol," a watchword is a "symbol," a badge is a "symbol"; a church creed is called a "symbol," because it serves as a badge or shibboleth; a theatre ticket is called a "symbol"; any ticket or check entitling one to receive anything is a "symbol." Moreover, any expression of sentiment was called a "symbol." Such were the principal meanings of the word in the original language. The reader will judge whether they suffice to establish my claim that I am not seriously wrenching the word in employing it as I propose to do.

298. Any ordinary word, as "give," "bird," "marriage," is an example of a symbol. It is *applicable to whatever may be found to realize the idea connected with the word;* it does not, in itself, identify those things. It does not show us a bird, nor enact before our eyes a giving or a marriage, but supposes that we are able to imagine those things, and have associated the word with them.

299. A regular progression of one, two, three may be remarked in the three orders of signs, Icon, Index, Symbol. The Icon has no dynamical connection with the object it represents; it simply happens that its qualities resemble those of that object, and excite analogous sensations in the mind for which it is a likeness. But it really stands unconnected with them. The index is physically connected with its object; they make an organic pair, but the interpreting mind has nothing to do with this connection, except remarking it, after it is established. The symbol is connected with its object by virtue of the idea

* *De Interpretatione*, II, 16a, 12.

of the symbol-using mind, without which no such connection would exist.

300. Every physical force reacts between a pair of particles, either of which may serve as an index of the other. On the other hand, we shall find that every intellectual operation involves a triad of symbols.

301. A symbol, as we have seen, cannot indicate any particular thing; it denotes a kind of thing. Not only that, but it is itself a kind and not a single thing. You can write down the word "star," but that does not make you the creator of the word, nor if you erase it have you destroyed the word. The word lives in the minds of those who use it. Even if they are all asleep, it exists in their memory. So we may admit, if there be reason to do so, that generals are mere words without at all saying, as Ockham* supposed, that they are really individuals.

302. Symbols grow. They come into being by development out of other signs, particularly from icons, or from mixed signs partaking of the nature of icons and symbols. We think only in signs. These mental signs are of mixed nature; the symbol-parts of them are called concepts. If a man makes a new symbol, it is by thoughts involving concepts. So it is only out of symbols that a new symbol can grow. *Omne symbolum de symbolo.* A symbol, once in being, spreads among the peoples. In use and in experience, its meaning grows. Such words as *force*, *law*, *wealth*, *marriage*, bear for us very different meanings from those they bore to our barbarous ancestors. The symbol may, with Emerson's sphynx, say to man,

Of thine eye I am eyebeam.

§4. SIGN†

303. Anything which determines something else (its *interpretant*) to refer to an object to which itself refers (its *object*) in the same way, the interpretant becoming in turn a sign, and so on *ad infinitum.*

No doubt, intelligent consciousness must enter into the series. If the series of successive interpretants comes to an end, the sign is thereby rendered imperfect, at least. If, an interpretant

* Cf. *Tractatus Logicæ*, I, xiv.

† *Dictionary of Philosophy & Psychology*, vol. 2, p. 527.

idea having been determined in an individual consciousness, it determines no outward sign, but that consciousness becomes annihilated, or otherwise loses all memory or other significant effect of the sign, it becomes absolutely undiscoverable that there ever was such an idea in that consciousness; and in that case it is difficult to see how it could have any meaning to say that that consciousness ever had the idea, since the saying so would be an interpretant of that idea.

304. A sign is either an *icon*, an *index*, or a *symbol*. An *icon* is a sign which would possess the character which renders it significant, even though its object had no existence; such as a lead-pencil streak as representing a geometrical line. An *index* is a sign which would, at once, lose the character which makes it a sign if its object were removed, but would not lose that character if there were no interpretant. Such, for instance, is a piece of mould with a bullet-hole in it as sign of a shot; for without the shot there would have been no hole; but there is a hole there, whether anybody has the sense to attribute it to a shot or not. A *symbol* is a sign which would lose the character which renders it a sign if there were no interpretant. Such is any utterance of speech which signifies what it does only by virtue of its being understood to have that signification.

§5. INDEX*

305. A sign, or representation, which refers to its object not so much because of any similarity or analogy with it, nor because it is associated with general characters which that object happens to possess, as because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other hand.

No matter of fact can be stated without the use of some sign serving as an index. If *A* says to *B*, "There is a fire," *B* will ask, "Where?" Thereupon *A* is forced to resort to an index, even if he only means somewhere in the real universe, past and future. Otherwise, he has only said that there is such an idea as fire, which would give no information, since unless it were known already, the word "fire" would be unin-

* *Ibid.*, vol. 1, pp. 531–2.

telligible. If *A* points his finger to the fire, his finger is dynamically connected with the fire, as much as if a self-acting fire-alarm had directly turned it in that direction; while it also forces the eyes of *B* to turn that way, his attention to be riveted upon it, and his understanding to recognize that his question is answered. If *A*'s reply is, "Within a thousand yards of here," the word "here" is an index; for it has precisely the same force as if he had pointed energetically to the ground between him and *B*. Moreover, the word "yard," though it stands for an object of a general class, is indirectly indexical, since the yard-sticks themselves are signs of the Parliamentary Standard, and that, not because they have similar qualities, for all the pertinent properties of a small bar are, as far as we can perceive, the same as those of a large one, but because each of them has been, actually or virtually, carried to the prototype and subjected to certain dynamical operations, while the associational compulsion calls up in our minds, when we see one of them, various experiences, and brings us to regard them as related to something fixed in length, though we may not have reflected that that standard is a material bar. The above considerations might lead the reader to suppose that indices have exclusive reference to objects of experience, and that there would be no use for them in pure mathematics, dealing, as it does, with ideal creations, without regard to whether they are anywhere realized or not. But the imaginary constructions of the mathematician, and even dreams, so far approximate to reality as to have a certain degree of fixity, in consequence of which they can be recognized and identified as individuals. In short, there is a degenerate form of observation which is directed to the creations of our own minds — using the word observation in its full sense as implying some degree of fixity and quasi-reality in the object to which it endeavours to conform. Accordingly, we find that indices are absolutely indispensable in mathematics; and until this truth was comprehended, all efforts to reduce to rule the logic of triadic and higher relations failed; while as soon as it was once grasped the problem was solved. The ordinary letters of algebra that present no peculiarities are indices. So also are the letters *A*, *B*, *C*, etc., attached to a geometrical figure. Lawyers and others who have to state a complicated affair with precision have recourse to

letters to distinguish individuals. Letters so used are merely improved relative pronouns. Thus, while demonstrative and personal pronouns are, as ordinarily used, "genuine indices," relative pronouns are "degenerate indices"; for though they may, accidentally and indirectly, refer to existing things, they directly refer, and need only refer, to the images in the mind which previous words have created.

306. Indices may be distinguished from other signs, or representations, by three characteristic marks: first, that they have no significant resemblance to their objects; second, that they refer to individuals, single units, single collections of units, or single continua; third, that they direct the attention to their objects by blind compulsion. But it would be difficult if not impossible, to instance an absolutely pure index, or to find any sign absolutely devoid of the indexical quality. Psychologically, the action of indices depends upon association by contiguity, and not upon association by resemblance or upon intellectual operations. See 1.558.

§6. SYMBOL*

307. A Sign (q.v.) which is constituted a sign merely or mainly by the fact that it is used and understood as such, whether the habit is natural or conventional, and without regard to the motives which originally governed its selection.

Σύμβολον is used in this sense by Aristotle several times in the *Peri hermeneias*, in the *Sophistici Elenchi*, and elsewhere.

308. THEMA:† A word proposed in 1635 by Burgersdicius [Burgersdyk] in his *Logic* (I., ii., §1), for that "quod intellectui cognoscendum proponi potest"; but what he seems to mean is what Aristotle sometimes vaguely expresses by λόγος, the immediate object of a thought, a meaning.

It is of the nature of a sign, and in particular of a sign which is rendered significant by a character which lies in the fact that it will be interpreted as a sign. Of course, nothing is a sign unless it is interpreted as a sign; but the character which causes it to be interpreted as referring to its object may be one which might belong to it irrespective of its object and though that object had never existed, or it may be in a relation

* *Ibid.*, vol. 2, p. 640.

† *Ibid.*, vol. 2, pp. 691–2.

to its object which it would have just the same whether it were interpreted as a sign or not. But the *thema* of Burgersdicius seems to be a sign which, like a word, is connected with its object by a convention that it shall be so understood, or else by a natural instinct or intellectual act which takes it as a representative of its object without any action necessarily taking place which could establish a factual connection between sign and object. If this was the meaning of Burgersdicius, his *thema* is the same as the present writer's "symbol." (See Sign.)

CHAPTER 4

*PROPOSITIONS**

§1. THE CHARACTERISTICS OF DICISIGNS

309. Of the three classes of the [third] trichotomy of representamens — the simple or substitutive signs, or *sumisigns* [*rhemes*]; the double or informational signs, quasi-propositions, or *dicisigns;* the triple or rationally persuasive signs, or *arguments*, or *suadisigns* — the one whose nature is, by all odds, the easiest to comprehend, is the second, that of quasi-propositions, despite the fact that the question of the essential nature of the "judgment" is today quite the most vexed of all questions of logic. The truth is that *all* these classes are of very intricate natures; but the problem of the day is needlessly complicated by the attention of most logicians, instead of extending to propositions in general, being confined to "judgments," or acts of mental acceptance of propositions, which not only involve characters, additional to those of propositions in general — characters required to differentiate them as propositions of a particular kind — but which further involve, beside the mental proposition itself, the peculiar act of assent. The problem is difficult enough, when we merely seek to analyze the essential nature of the *Dicisign*, in general, that is, the kind of sign that *conveys* information, in contradistinction to a sign [such as an icon] from which information may be derived.[1]

310. The readiest characteristic test showing whether a sign is a Dicisign or not is that a Dicisign is either true or false, but does not directly furnish reasons for its being so. This shows that a Dicisign must profess to refer or relate to something as having a real being independently of the repre-

* §§1–4 are from "Syllabus," c. 1902, continuing 294.

[1] To explain the judgment in terms of the "proposition" is to explain it by that which is essentially intelligible. To explain the proposition in terms of the "judgment" is to explain the self-intelligible in terms of a psychical act, which is the most obscure of phenomena or facts.

sentation of it as such, and further that this reference or relation must not be shown as rational, but must appear as a blind Secondness. But the only kind of sign whose object is necessarily existent is the genuine Index. This Index might, indeed, be a part of a Symbol; but in that case the relation would appear as rational. Consequently a Dicisign necessarily represents itself to be a genuine Index, and to be nothing more. At this point let us discard all other considerations, and see what sort of sign a sign must be that in any way represents itself to be a genuine Index of its Object, and nothing more. Substituting for "represents to be" a clearer interpretation, the statement is that the Dicisign's Interpretant represents an identity of the Dicisign with a genuine Index of the Dicisign's real Object. That is, the Interpretant represents a real existential relation or genuine Secondness, as subsisting between the Dicisign and its real Object. But the Interpretant of a Sign can represent no other Object than that of the Sign itself. Hence this same existential relation must be an Object of the Dicisign, if the latter have any real Object. This represented existential relation, in being an Object of the Dicisign, makes that real Object, which is correlate of this relation, also an Object of the Dicisign.

311. This latter Object may be distinguished as the *Primary Object,* the other being termed the *Secondary Object.* The Dicisign in so far as it is the relate of the existential relation which is the Secondary Object of the Dicisign, can evidently not be the entire Dicisign. It is at once a part of the Object and a part of the Interpretant of the Dicisign. Since the Dicisign is represented in its Interpretant to be an Index of a complexus as such, it must be represented in that same Interpretant to be composed of two parts, corresponding respectively to its Object and to itself [the Dicisign]. That is to say, in order to understand the Dicisign, it must be regarded as composed of two such parts whether it be in itself so composed or not. It is difficult to see how this can be, unless it really have two such parts; but perhaps this may be possible. Let us consider these two represented parts separately. The part which is represented to represent the Primary Object, since the Dicisign is represented to be an Index of its Object, must be represented as an Index, or some representamen of an Index, of the Pri-

mary Object. The part which is represented to represent a part of the Dicisign is represented as at once part of the Interpretant and part of the Object. It must, therefore, be represented as such a sort of Representamen (or to represent such a sort), as can have its Object and its Interpretant the same. Now, a Symbol cannot even have itself as its Object; for it is a law governing its Object. For example, if I say, "This proposition conveys information about itself," or "Let the term 'sphynx' be a general term to denote anything of the nature of a symbol that is applicable to every 'sphynx' and to nothing else," I shall talk unadulterated nonsense. But a Representamen mediates between its Interpretant and its Object, and that which cannot be the object of the Representamen cannot be the Object of the Interpretant. Hence, *a fortiori*, it is impossible that a Symbol should have its Object as its Interpretant. An Index can very well represent itself. Thus, every number has a double; and thus the entire collection of even numbers is an Index of the entire collection of numbers, and so this collection of even numbers contains an Index of itself. But it is impossible for an Index to be its own Interpretant, since an Index is nothing but an individual existence in a Secondness with something; and it only becomes an Index by being capable of being represented by some Representamen as being in that relation. Could this Interpretant be itself there would be no difference between an Index and a Second. An Icon, however, is strictly a possibility involving a possibility, and thus the possibility of its being represented as a possibility is the possibility of the involved possibility. In this kind of Representamen alone, then, the Interpretant may be the Object. Consequently, that constituent of the Dicisign, which is represented in the Interpretant as being a part of the Object, must be represented by an Icon or by a Representamen of an Icon. The Dicisign, as it must be understood in order to be understood at all, must contain those two parts. But the Dicisign is represented to be an Index of the Object, in that the latter involves something corresponding to these parts; and it is this Secondness that the Dicisign is represented to be the Index of. Hence the Dicisign must exhibit a connection between these parts of itself, and must represent this connection to correspond to a connection in the object between the

Secundal Primary Object [*i.e.*, the primary object so far as it is dyadic in structure] and the Firstness [or quality of the primary object] indicated by the part [of the Secundal Primary Object] corresponding to the Dicisign.

312. We conclude, then, that, if we have succeeded in threading our way through the maze of these abstractions, a Dicisign, defined as a Representamen whose Interpretant represents it as an Index of its Object, must have the following characters:

First: It must, in order to be understood, be considered as containing two parts. Of these, the one, *which may be called the Subject*, is or represents an Index of a Second existing independently of its being represented, while the other, *which may be called the Predicate*, is or represents an Icon of a Firstness [or quality or essence]. Second: These two parts must be represented as connected; and that in such a way that if the Dicisign has any Object, it [the Dicisign] must be an Index of a Secondness subsisting between the Real Object represented in one represented part of the Dicisign to be indicated and a Firstness represented in the other represented part of the Dicisign to be Iconized.

313. Let us now examine whether these conclusions, together with the assumption from which they proceed, hold good of all signs which profess to convey information without furnishing any rational persuasion of it; and whether they fail alike for all signs which do not convey information as well as for all those which furnish evidence of the truth of their information, or reasons for believing it. If our analysis sustains these tests, we may infer that the definition of the Dicisign on which they are founded holding, at least within the sphere of signs, is presumably sound beyond that sphere.

314. Our definition forbids an Icon to be a Dicisign, since the proper Interpretant of an Icon cannot represent it to be an Index, the Index being essentially more complicated than the Icon. There ought, therefore, to be no informational signs among Icons. We find that, in fact, Icons may be of the greatest service in obtaining information — in geometry, for example — but still, it is true that an Icon cannot, of itself, convey information, since its Object is whatever there may be which is like the Icon, and is its Object in the measure in which it is like the Icon.

§2. SUBJECTS AND PREDICATES

315. All propositions are informational Symbols. Our conclusions do not prevent Dicisigns from being Symbols; but let us begin by examining whether or not our definition and conclusions apply to ordinary propositions. To fix our ideas let us set down the proposition "Tully has a wart on his nose." That is a proposition whether it be true or not, whether anybody asserts it or not, and whether anybody assents to it or not. For an act of assertion supposes that, a proposition being formulated, a person performs an act which renders him liable to the penalties of the social law (or, at any rate, those of the moral law) in case it should not be true, unless he has a definite and sufficient excuse; and an act of assent is an act of the mind by which one endeavors to impress the meanings of the proposition upon his disposition, so that it shall govern his conduct, including thought under conduct, this habit being ready to be broken in case reasons should appear for breaking it. Now in performing either of these acts, the proposition is recognized as being a proposition whether the act be performed or not. Nor can a sound objection be grounded on the fact that a proposition is always understood as something that *might be* assented to and asserted.[1] For our definition of the Dicisign more than recognizes the truth of that in stating that (supposing the proposition to be a Dicisign) the Interpretant of it (that is, the mental representation, or thought, which it tends to determine), represents the proposition to be a genuine Index of a Real Object, independent of the representation. For an Index involves the existence of its Object. The definition [of the Dicisign] adds that this Object is a Secondness or real fact. That this is true of ordinary "ampliative" propositions, namely, that what they mean to represent is a fact, is beyond question. But as regards explicative propositions, and especially definitions, it may be doubted. If a definition is to be understood as introducing the definitum, so that it means "Let so and so — the definitum — mean so and so — the definition," then it is a proposition in the imperative mood,

[1] But if anybody prefers a form of analysis which gives more prominence to the unquestionable fact that a proposition is something capable of being assented to and asserted, it is not my intention to make any objection to that. I do not think my analysis does put quite the emphasis on that that it justly might.

and consequently, not a proposition; for a proposition is equivalent to a sentence in the indicative mood. The definition is thus only a proposition if the definitum be already known to the interpreter. But in that case it clearly conveys information as to the character of this definitum, which is matter of fact. But take an "analytical," *i.e.*, an explicative proposition; and to begin with, take the formula "A is A." If this be intended to state anything about real things, it is quite unintelligible. It must be understood to mean something about symbols; no doubt, that the substantive verb "is" expresses one of those relations that everything bears to itself, like "loves whatever may be loved by." So understood, it conveys information about a symbol. A symbol is not an individual, it is true. But any information about a symbol is information about every replica of it; and a replica is strictly an individual. What information, then, does the proposition "A is A" furnish concerning this replica? The information is that if the replica be modified so as to bring the same name before it and after it, then the result will be a replica of a proposition which will never be in conflict with any fact. To say that something *never* will be is not to state any real fact, and until some experience occurs — whether outward experience, or experience of fancies — which might be an occasion for a conflict with the proposition in question, it does not, to our knowledge, represent any actual Secondness. But as soon as such an occasion does arise, the proposition relates to the single replica that then occurs and to the single experience, and describes the relation between them. Similar remarks apply to every explicative proposition. The proposition "Every phoenix, in rising from its ashes, sings 'Yankee Doodle,'" will be, we may be confident, not in conflict with any experience. If so, it is perfectly true. "Every four-sided triangle is deep blue," is necessarily true, since it is impossible that any experience should conflict with it.* But both propositions are meaningless. Equally meaningless is any explicative proposition that is true, unless it be regarded as a proposition about a certain kind of symbol of which a replica

* If "some" be taken to involve the existence of what it quantifies, then I and O propositions of non-existents must both be false; by the square of opposition both E and A would then be true, so that all universals, whether affirmative or negative, are true of the non-existent. See also 324, 327, 369.

actually occurs. If "Man is a biped" be allowed to be an explicative proposition, it means nothing unless there be an occasion in which the name "man" may be applied. If there be such an occasion, in regard to that existential individual event, it is said that the term "biped" may be applied to it. That is, on an occasion on which the word "biped" is applied, the result will never be in conflict with any experience, real or imaginary. Thus every kind of proposition is either meaningless or has a real Secondness as its object. This is a fact that every reader of philosophy should constantly bear in mind, translating every abstractly expressed proposition into its precise meaning in reference to an individual experience. The system of existential graphs,* which is capable of expressing every proposition as analytically as may be desired, expresses an assertion by actually attaching an individual replica to the individual sheet, and such possible attachment is precisely what the Interpretant of a proposition represents before the proposition is asserted.

316. Let us now proceed to compare the conclusions from the abstract definition of a Dicisign with the facts about propositions. The first conclusion is that every proposition contains a *Subject* and a *Predicate*, the former representing (or being) an Index of the Primary Object, or Correlate of the relation represented, the latter representing (or being) an Icon of the Dicisign in some respect. Before inquiring whether every proposition has such parts, let us see whether the descriptions given of them are accurate, when there are such parts. The proposition "Cain kills Abel" has two subjects "Cain" and "Abel" and relates as much to the real Objects of one of these as to that of the other. But it may be regarded as primarily relating to the Dyad composed of Cain, as first, and of Abel, as second member. This Pair is a single individual object having this relation to Cain and to Abel, that its existence *consists* in the existence of Cain and in the existence of Abel and in nothing more. The Pair, though its existence thus depends on Cain's existence and on Abel's, is, nevertheless, just as truly existent as they severally are. The *Dyad* is not precisely the Pair. The Dyad is a mental Diagram consisting of two images of two objects, one existentially connected with one member of the pair, the other with the other; the one having attached to it,

* See vol. 4, bk. II.

as representing it, a Symbol whose meaning is "First," and the other a Symbol whose meaning is "Second." Thus, this diagram, the Dyad, represents Indices of Cain and Abel, respectively; and thus the subject conforms to our conclusion. Next consider the subject of this proposition, "Every man is the son of two parents." This supposes a mental diagram of a pair labelled "First" and "Second," as before (or rather by symbols equivalent to these for the special purpose) but instead of the two units of the Diagram being directly considered as Indices of two existent individuals, the Interpretant of the diagram represents, that if the interpreter of the whole proposition by an act of the mind actually attaches one of the units of the diagram to any individual man, there will be an existent relation attaching the other unit to a certain pair of individuals of which, if the interpreter of the whole proposition attaches one of them specially to that unit, then the predicate will be true of that individual Dyad in the order of its members. Of course, it is not meant that the person who sufficiently understands the diagram actually goes through this elaborate process of thought, but only that this is substantially what has to be done, completely and accurately to understand the proposition. The graph of the proposition will afford help in seeing that this is so. Here, as before, the Subject represents the individual Dyad, of which the proposition is the Symbol, to be represented by an Index. If the proposition has an abstract subject, as "Redness" or "Justice," it may either be treated, after the style of the scholastics, as an *exponible*, that is, as a proposition whose real construction is disguised by a grammatical trope; or, if this does not afford the true interpretation, the proposition discourses of a universe comprising one replica each of a collection of possible symbols, somewhat indefinite, but embracing all that need be considered. We cannot say "all that are pertinent," since no collection could exhaust the possible pertinent symbols. In the case of a *conditional* proposition,[1] "If it freezes tonight, your roses will be killed," the

[1] *Conditional* is the right appellation, and not *hypothetical*, if the rules of the author's Ethics of Philosophical Terminology [bk. II, ch. 1] are to be followed. The meaning of ὑποθετικός was quite unsettled with the Greeks; but the word seems ultimately to have come to be applied to any compound proposition; and so Apuleius, under Nero, uses the translation *conditionalis;* saying, "Propositionum igitur, perinde ut ipsarum conclusionum, duae species sunt: altera praedicativa,

meaning is that any replica of the proposition "It will freeze tonight" which may be true, coexists with a true replica of the proposition "your roses will be killed." This involves a representation of an Index just as much as does the subject of the proposition "Every rose will be killed."

317. Passing now to the consideration of the predicate, it is plain enough that the last proposition, or any at all like it, only conveys its signification by exciting in the mind some image or, as it were, a composite photograph of images, like the Firstness meant. This, however, does not squarely meet the question, which is not what our mental constitution causes to happen, but how the predicate represents the Firstness that it signifies.[1] The predicate is necessarily an *Iconic Sumisign* [Rheme] (which is not always true of the subject) and as such, as we should find by a full analysis of the Sumisign, essentially signifies what it does by representing itself to represent an Icon of it. Without an analysis of the Sumisign this point must remain a little obscure.

quae etiam simplex est; ut si dicamus, *qui regnat, beatus est:* altera substitutiva, vel conditionalis, quae etiam composita est; ut si aias: *qui regnat, si sapit, beatus est.* Substituis enim conditionem, qua, nisi sapiens est, non sit beatus." [See Prantl's *Geschichte der Logik*, I, 580–581.] But as early as Boëthius and Cassiodorus, that is, about A.D. 500, it was settled that *hypothetica* applies to any compound proposition, and *conditionalis* to a proposition asserting one thing only in case a condition set forth in a separate clause be fulfilled. This was the universally accepted use of the terms throughout the middle ages. Therefore, *hypotheticals* should have been divided into *disjunctives* and *copulatives*. They were usually divided into conditionals, disjunctives, and copulatives. But conditionals are really only a special kind of *disjunctives*. To say, "If it freezes tonight, your roses will be killed" is the same as to say, "It either will not freeze, or your roses will tonight be killed." A disjunctive does not exclude the truth of both alternatives, at once [cf. 345–7].

[1] Mill's term *connote* is not very accurate. Connote properly means to denote along with in a secondary way. Thus "killer" connotes a living thing killed. When the scholastics said that an adjective *connoted*, they meant it connoted the abstraction named by the corresponding abstract noun. But the ordinary use of an adjective involves no reference to any abstraction. The word *signify* has been the regular technical term since the twelfth century, when John of Salisbury (*Metalogicus*, II, xx) spoke of "quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa (i.e., adjectives) *significant*, et aliud esse quod *nominant*. Nominantur singularia (i.e., existent individual things and facts), sed universalia (i.e., Firstnesses) significantur." See my paper of Nov. 13, 1867 [next chapter], to which I might now [1902] add a multitude of instances in support of what is here said concerning *connote* and *signify*.

318. We next come to the question whether every proposition has a Subject and a Predicate. It has been shown above that this is true of a Conditional; and it is easily seen that it is equally so of any Disjunctive. Only, an ordinary Disjunctive has such a construction that one mode of analysis of it is as good as another. That is, to say, "Either A or B is true," may equally be regarded as saying, "A replica of a Symbol is true which is not true if no replica of A is true and no replica of B is true," or as saying, "If a replica of A is not true, a replica of B is true," or as saying, "If a replica of B is not true, a replica of A is true." These come to the same thing, just as "Some X is Y," "Some Y is X," and "Something is both X and Y" come to the same thing. The most perfectly thorough analysis throws the whole substance of the Dicisign into the Predicate. A copulative proposition even more obviously has a Subject and Predicate. It predicates the genuinely Triadic relation of *tricoexistence*, "P and Q and R coexist." For to say that both A and B is true is to say that something exists which *tricoexists* with true replicas of A and B. Some logical writers are so remarkably biassed or dense as to adduce the Latin sentences *fulget* and *lucet* as propositions without any subject. But who cannot see that these words convey no information at all without a reference (which will usually be Indexical, the Index being the common environment of the interlocutors) to the circumstances under which the Firstnesses they signify are asserted to take place?

319. The proposition should have an actual *Syntax*, which is represented to be the Index of those elements of the fact represented that correspond to the Subject and Predicate. This is apparent in all propositions. Since Abelard it has been usual to make this Syntax a third part of the proposition, under the name of the Copula. The historical cause of the emergence of this conception in the twelfth century was, of course, that the Latin of that day did not permit the omission of the verb *est*, which was familiarly, though not invariably, omitted in Greek, and not very uncommonly in classical Latin. In most languages there is no such verb. But it is plain that one does not escape the need of a Syntax by regarding the Copula as a third part of the proposition; and it is simpler to say that it is merely the accidental form that Syntax may take.

320. It has thus been sufficiently shown that all propositions conform to the definition of the Dicisign and to the corollaries drawn from that definition. A proposition is, in short, a Dicisign that is a Symbol. But an Index, likewise, may be a Dicisign. A man's portrait with a man's name written under it is strictly a proposition, although its syntax is not that of speech, and although the portrait itself not only represents, but is, a Hypoicon. But the proper name so nearly approximates to the nature of an Index, that this might suffice to give an idea of an informational Index. A better example is a photograph. The mere print does not, in itself, convey any information. But the fact, that it is virtually a section of rays projected from an object *otherwise known*, renders it a Dicisign. Every Dicisign, as the system of Existential Graphs fully recognizes, is a further determination of an already known sign of the same object. It is not, perhaps, sufficiently brought out in the present analysis. It will be remarked that this connection of the print, which is the quasi-predicate of the photograph, with the section of the rays, which is the quasi-subject, is the Syntax of the Dicisign; and like the Syntax of the proposition, it is a *fact* concerning the Dicisign considered as a First, that is, in itself, irrespective of its being a sign. Every informational sign thus involves a Fact, which is its Syntax. It is quite evident, then, that Indexical Dicisigns equally accord with the definition and the corollaries.

321. It will be remarked that this accord, both for propositions and for informational indices, is quite irrespective of their being asserted or assented to. Now in analyses hitherto proposed, it seems to have been thought that if assertion, or at any rate, assent, were omitted, the proposition would be indistinguishable from a compound general term — that "A man is tall" would then reduce to "A tall man." It therefore becomes important to inquire whether the definition of a Dicisign here found to be applicable to the former (even though it be not "judged"), may not be equally applicable to the latter. The answer, however, comes forthwith. Fully to understand and assimilate the symbol "a tall man," it is by no means requisite to understand it to relate, or to profess to relate, to a real Object. Its Interpretant, therefore, does not represent it as a genuine Index; so that the definition of the Dicisign

does not apply to it. It is impossible here fully to go into the examination of whether the analysis given does justice to the distinction between propositions and arguments. But it is easy to see that the proposition purports to intend to compel its Interpretant to refer to its real Object, that is represents itself as an Index, while the argument purports to intend not compulsion but action by means of comprehensible generals, that is, represents its character to be specially symbolic.

322. The above is the best analysis the author can, at present, make of the Dicisign. However satisfactory the main points of it may appear, it is not likely, on general principles, to stand without more or less amendment, though it would seem as if it could not but be pretty near to the truth. It is doubtful whether it applies fully to all kinds of propositions. This definition of the Dicisign will naturally lead one to guess that a Sumisign is any Representamen of which the Interpretant represents it as an Icon; and that the Argument or Suadisign is a Representamen of which the Interpretant represents it as a Symbol. Close examination encourages the student to believe that this is something like the truth, but so far as it has been carried, excites doubt whether this be the whole story. . . .

§3. DICHOTOMIES OF PROPOSITIONS

323. Indexical Dicisigns seem to have no important varieties; but propositions are divisible, generally by dichotomy primarily in various ways. In the first place, according to *Modality* or *Mode*, a proposition is either *de inesse* (the phrase used in the *Summulae*[1] [p. 71B]) or *modal*. A proposition *de inesse* contemplates only the existing state of things — existing,

[1] The *Summulae Logicales* of Petrus Hispanus, which Prantl [*Geschichte der Logik*, II, 266ff], a writer of little judgment and over-rated learning, whose useful history of Logic is full of blunders, misappreciations, and insensate theories, and whose own Billingsgate justifies almost any tone toward him, absurdly maintains that this book was substantially translated from a Greek book, which is manifestly from the Latin. The *Summulae* of Petrus Hispanus are nearly identical with some other contemporary works and evidently show a doctrine which had been taught in the schools from about A.D. 1200. After Boëthius, it is the highest authority for logical terminology, according to the present writer's ethical views.

that is, in the logical universe of discourse.* A modal proposition takes account of a whole range of possibility. According as it asserts something to be true or false throughout the whole range of possibility, it is *necessary* or *impossible*. According as it asserts something to be true or false within the range of possibility (not expressly including or excluding the existent state of things), it is *possible* or *contingent*. (The terms are all from Boëthius.)

324. A subject of a proposition is either *Singular*, *General*, or *Abstract*. It is singular if it indicates an otherwise known individual. It is general if it describes how an individual intended is to be selected. A general subject is (as commonly recognized) either *Universal* or *Particular* (and *Indefinite*). (These last three terms are found in Apuleius,† of Nero's time. But a senseless distinction between the indefinite and the particular is by the present writer unnoticed.) There is a complicated doctrine in the books as to the meaning of these terms, some kinds of universals [being taken as] asserting the existence of their subjects. The present writer makes all universals alike in not doing so. Then a *Universal* subject is one which indicates that the proposition applies to whatever individual there is in the universe or to whatever there *may be* of a general description without saying that there is any. A *Particular* subject is one which does not indicate what individual is intended, further than to give a general description of it, but does profess to indicate an existent individual at least. The order in which Universal and Particular subjects occur is material. Thus, "Some woman is adored by whatever Spaniard may exist," has its first subject "Some woman" particular, and its second "whatever Spaniard may exist" universal. But "Whatever Spaniard may exist adores some woman" has the same subjects in reverse order and so has a different meaning. It is quite conceivable that a subject should be so described as to be neither Universal nor Particular; as in *exceptives* (*Summulae*) as "Every man but one is a sinner." The same may be said of all kinds of numerical propositions, as "Any insect has an even number of legs." But these may

* I.e., it can be stated in terms of a material or Philonian implication. See 348n.

† Prantl, *op. cit.*, I, 581.

be regarded as Particular Collective Subjects. An example of a Universal Collective subject would be "Any two persons shut up together will quarrel." A collection is logically an individual. The distinction of Universal and Particular subjects is material, not merely formal; and it seems to be (and was regarded in the middle ages as being) of essentially the same nature as the distinction of Necessary and Possible propositions.

325. The distinction of *Hypothetical*, *Categorical*, and *Relative* propositions is also important. At any rate, the last has some important differences from the others.

326. The distinction between *Affirmative* and *Negative* propositions, as applied to ordinary categorical propositions, is purely a matter of form. A process called *infinitation* (used by Abelard, *Opera hactenus Inedita*, p. 225, and constantly ever since in all Western languages to this day), consisting in prefixing *non-* to a term, converts the proposition from a negative to an *affirmative* or so-called *Infinite* proposition. The difference between a negative and an infinite proposition is no more than that in Latin one may say *non est* or *est non*, without difference of meaning. "Socrates non est mortalis" is the usual form; but "Socrates est non mortalis" can equally be said. It must be remembered that logic has attracted to its study some of the most puerile of writers, and still continues to do so in some measure.

327. Finally, every proposition is either *true* or *false*. It is false if any proposition could be legitimately deduced from it, without any aid from false propositions, which would conflict with a direct perceptual judgment, could such be had. A proposition is true, if it is not false. Hence, an entirely meaningless form of proposition, if it be called a proposition, at all, is to be classed along with true propositions.

§4. A PRAGMATIC INTERPRETATION OF THE LOGICAL SUBJECT

328. Any symbol which may be a direct constituent of a proposition is called a *term* (*terminus*, Boëthius).* The logicians usually say that a categorical proposition has "two terms," its *subject* and its *predicate*, wherein, by a carelessness of expres-

* Prantl, *op. cit.*, I, 696.

sion, or by copying Aristotle,* they stumble upon the truth. Their usual *doctrine* is (though often not directly stated in one sentence), that such a proposition has three terms, the subject, predicate, and *copula* (Abelard).† The correct designation of the subject and predicate, in accord with their doctrine, is the *extremes*, which is translated from the same Greek word as *term* (ὅρος). The ordinary doctrine makes the copula the only verb, and all other terms to be either proper names or general class-names. The present author leaves the *is* as an inseparable part of the class-name; because this gives the simplest and most satisfactory account of the proposition.‡ It happens to be true that in the overwhelming majority of languages there are no general class names and adjectives that are not conceived as parts of some verb (even when there really is no such verb) and consequently nothing like a copula is required in forming sentences in such languages. The author (though with no pretension to being a linguist), has fumbled the grammars of many languages in the search for a language constructed at all in the way in which the logicians go out of their way to teach that all men think (for even if they do so, that has really nothing to do with logic). The only such tongue that he has succeeded in finding is the Basque, which seems to have but two or three verbs, all the other principal words being conceived as nouns. Every language must have proper names; and there is no verb wrapped up in a proper name. Therefore, there would seem to be a direct suggestion there of a true common noun or adjective. But, notwithstanding that suggestion, almost every family of man thinks of general words as parts of verbs. This seems to refute the logicians' psychology.

329. A proper name, when one meets with it for the first time, is existentially connected with some percept or other equivalent individual knowledge of the individual it names. It is *then*, and then only, a genuine Index. The next time one meets with it, one regards it as an Icon of that Index. The habitual acquaintance with it having been acquired, it becomes a Symbol whose Interpretant represents it as an Icon of an Index of the Individual named.

* "Ορον δὲ καλῶ εἰς ὃν διαλύεται ἡ πρότασις, οἷον τό τε κατηγορούμενον καὶ τὸ καθ' οὗ κατηγορεῖται, says Aristotle 24b.16.

† Prantl, *op. cit.*, II, 197.

‡ See also 3.459.

330. If you look into a textbook of chemistry for a definition of *lithium*, you may be told that it is that element whose atomic weight is 7 very nearly. But if the author has a more logical mind he will tell you that if you search among minerals that are vitreous, translucent, grey or white, very hard, brittle, and insoluble, for one which imparts a crimson tinge to an unluminous flame, this mineral being triturated with lime or witherite rats-bane, and then fused, can be partly dissolved in muriatic acid; and if this solution be evaporated, and the residue be extracted with sulphuric acid, and duly purified, it can be converted by ordinary methods into a chloride, which being obtained in the solid state, fused, and electrolyzed with half a dozen powerful cells, will yield a globule of a pinkish silvery metal that will float on gasolene; and the material of *that* is a specimen of lithium. The peculiarity of this definition —or rather this precept that is more serviceable than a definition — is that it tells you what the word lithium denotes by prescribing what you are to *do* in order to gain a perceptual acquaintance with the object of the word. Every subject of a proposition, unless it is either an Index (like the environment of the interlocutors, or something attracting attention in that environment, as the pointing finger of the speaker) or a Subindex (like a proper name, personal pronoun or demonstrative) must be a *Precept*, or Symbol, not only describing to the Interpreter what is to be done, by him or others or both, in order to obtain an Index of an individual (whether a unit or a single set of units) of which the proposition is represented as meant to be true, but also assigning a designation to that individual, or, if it is a set, to each single unit of the set. Until a better designation is found, such a term may be called a Precept. Thus, the Subject of the proposition, "Whatever Spaniard there may be adores some woman" may best be regarded as, "Take any individual, A, in the universe, and then there will be some individual, B, in the universe, such that A and B in this order form a dyad of which what follows is true," the Predicate being "—— is either not a Spaniard or else adores a woman that is ——."

331. Any term fit to be the subject of a proposition may be termed an *Onome*. A Categoreumatic term (Duns Scotus, but probably earlier) is any term fit to be the subject or predi-

cate of a proposition. A *Syncategoreumatic Term* or *Syncathegreuma* (*Summulae*)* is a Symbol going to make up a *Categoreumatic* Term. The Copula seems to fall between two stools, being neither categoreumatic nor syncategoreumatic. . . .

§5. THE NATURE OF ASSERTION†

332. Let us now consider in wnat the essential nature of *assertion* consists. I can here only restate, though in an improved form, a doctrine of *grammatica speculativa* which I first published in 1867.‡ Since that date, as my philosophical studies have progressed, I have been led half a dozen times and more to call the doctrine into serious question and to submit it to a rigid and thorough reëxamination. Each reëxamination, while leading to some modification more or less important, has reinstated the impeached doctrine in my estimation. I believe that I can now make a statement of it which shall leave little to be desired. At the same time, I will take occasion to acknowledge and explain the errors of my previous statements.

333. In such analysis of assertion there are two kinds of reasoning which we have to employ. On the one hand, we can directly observe what is familiar to our experience of assertions and seems to be inseparable from them. Professor Schröder calls this *rhetorical* evidence; and the designation is felicitous, because the reasoning in question has the characteristics of the inferences termed *rhetorical* by the old logicians. The term also harmonizes with my name of *speculative rhetoric* for the highest and most living branch of logic. To me personally, perhaps the designation gives that sort of satisfaction which so many schools have manifested in adopting appellations invented by their opponents as depreciative. For although Professor Schröder cannot but acknowledge the value and need of this kind of reasoning, a slight shade of disesteem seems to mingle with his approval on account of its undeniable formal imperfection. Now to me this very imperfection marks the reasoning as being drawn direct from those observational sources from whence all true reasoning must be drawn; and I have often remarked in the history of philosophy, that the reasonings

* See Prantl, *op. cit.*, II, 272.

† §5 and §6 are from "That Categorical and Hypothetical Propositions are one in essence, with some connected matters," c. 1895.

‡ "On a New List of Categories," vol. 1, bk. III, ch. 6.

which were somewhat dark and formally imperfect, often went the deepest. The other kind of reasoning which I employ in the analysis of assertion consists in deducing what the constituents of assertion must be from the theory, which I accept, that truth consists in the definitive compulsion of the investigating intelligence. This is systematical; but it is only half a method. For the deductions, or quasi-predictions, from theory having been made, it is requisite to turn to the rhetorical evidence and see whether or not they are verified by observation. If we find them to be so, not only does the analysis of assertion gain evidence of being completely rounded, but the theory of truth is rendered more probable.

334. In every assertion we may distinguish a speaker and a listener. The latter, it is true, need have only a problematical existence, as when during a shipwreck an account of the accident is sealed in a bottle and thrown upon the water. The problematical "listener" may be within the same person as the "speaker"; as when we mentally register a judgment, to be remembered later. If there be any act of judgment independent of any registry, and if it have any logical significance (which is disputable), we may say that in that case the listener becomes identical with the speaker.

335. The assertion consists in the furnishing of evidence by the speaker to the listener that the speaker believes something, that is, finds a certain idea to be definitively compulsory on a certain occasion. There ought, therefore, to be three parts in every assertion, a sign of the occasion of the compulsion, a sign of the enforced idea, and a sign evidential of the compulsion affecting the speaker in so far as he identifies himself with the scientific intelligence.

336. Because compulsion is essentially *hic et nunc*, the occasion of the compulsion can only be represented to the listener by compelling him to have experience of that same occasion. Hence it is requisite that there should be a kind of sign which shall act dynamically upon the hearer's attention and direct it to a special object or occasion. Such a sign I call an *Index*. It is true that there may, instead of a simple sign of this kind, be a precept describing how the listener is to act in order to gain the occasion of experience to which the assertion relates. But since this precept tells him how he is to act, and since

acting and being acted on are one and the same, and thus action is also *hic et nunc*, the precept must itself employ an Index or Indices. That to which the index directs attention may be called the subject of the assertion. . . .

337. The real world cannot be distinguished from a fictitious world by any description. It has often been disputed whether Hamlet was mad or not. This exemplifies the necessity of *indicating* that the real world is meant, if it be meant. Now reality is altogether dynamic, not qualitative. It consists in forcefulness. Nothing but a dynamic sign can distinguish it from fiction. It is true that no language (so far as I know) has any particular form of speech to show that the real world is spoken of. But that is not necessary, since tones and looks are sufficient to show when the speaker is in earnest. These tones and looks act dynamically upon the listener, and cause him to attend to realities. They are, therefore, the indices of the real world. Thus, there remains no class of assertions which involve no indices unless it be logical analyses and identical propositions. But the former will be misunderstood and the latter taken as nonsensical, unless they are interpreted as referring to the world of terms or concepts; and this world, like a fictitious world, requires an index to distinguish it. It is, therefore, a fact, as theory had pronounced, that one index, at least, must form a part of every assertion.

338. I term those occasions or objects which are denoted by the indices the *subjects* of the assertion. But these will not coincide with the objects denoted by the grammatical subjects. It has always been the habit of logicians to consider propositions only (or chiefly) after they have been expressed in certain standard, or canonical, forms. To treat them just as they are expressed in this or that language (as Hoppe and some others do) makes of logic a philological, not a philosophical, study. But the canonical forms chosen have been suggested by the usage of a narrow class of languages, and are calculated to lead philosophy astray. That which is called the *subject* is the noun which is in the nominative, although, even in our relatively small family of Indo-European languages, there are several in which that noun which in Latin, Greek, and the modern European languages is put in the nominative, is put in an oblique case. Witness the Irish and Gaelic. Often, too, the index is

not of the nature of a noun. It may be, as we have seen, a mere look or gesture. Then again it may be so disguised, that it is impossible to say with certainty whether it be an index, at all. It helps little to appeal to the meaning of the assertion; because it is in such cases difficult to say precisely what the meaning is. Thus, in the assertion, "All men are mortal," we may say that the subject is *every man*, or we may say that it is the collection *of men*, or that *every man and some mortal* are the two subjects, or that *everything* is the subject (the predicate being "is either not man or is mortal"), or that *everything* and *humanity* and *mortality* are the three subjects, or a hundred other dispositions. But if it is desired to adopt one constant canonical form, the best rule will be to use a separate index for everything which is indifferent from a logical point of view. That is, in this case to take *everything*, *humanity*, and *mortality* as the indices.

339. Every subject, when it is directly indicated, as *humanity* and *mortality* are, is singular. Otherwise, a precept, which may be called its *quantifier*, prescribes how it is to be chosen out of a collection, called its *universe*. In probable logic, the quantifiers — such as "nine out of ten," and the like — refer to an experiential course or "long run." But in necessary logic there is no reference to such a course of experience, and only two quantifiers are required; the *universal* quantifier, which allows any object, no matter what, to be chosen from the universe, and the *particular* quantifier, which prescribes that a suitable object must be chosen. When there are several quantified subjects, and when quantifications are different, the order in which they are chosen is material. It is the character of the quantifier of the *last chosen* subject which extends itself to the whole proposition. (In former statements, this last point was not clear to me.) While no other quantifiers than those two are indispensible, much more than mere brevity and convenience of writing is gained by using also two other "hemilogical"* quantifiers, the one permitting any object of the universe *but one* to be taken, the other restricting the liberty to one or other of a suitable two. The universe of a logical subject has always hitherto been assumed to be a discrete collection, so that the subject is an *individual* object or occasion. But in truth a

* See 1.567 for a definition of this term.

universe may be continuous, so that there is no part of it of which every thing must be either wholly true or wholly false. For example, it is impossible to find a part of a surface which must be all one color. Even a point of that surface may belong indifferently to three or more differently colored parts. But the logic of continuous universes awaits investigation. . . .

340. In 1867 I defined a symbol as any general representamen;* and so far I was right. But I immediately proceeded after the traditional manner, to divide symbols into *terms*, *propositions and argumentations*, with the meaning that "terms" have no assertoric element, and there I was wrong, although the division itself is not so much wrong as it is unimportant. Subsequently, noticing that I had classed natural symptoms both among indices and among symbols, I restricted symbols to conventional signs, which was another error. The truth is that my paper of 1867 was perhaps the least unsatisfactory, from a logical point of view, that I ever succeeded in producing; and for a long time most of the modifications I attempted of it only led me further wrong.

341. Every symbol, as involving an assertion, or rudimentary assertion, is general, in the sense in which we speak of a general sign. That is, the predicate is general. Even when we say "Boz was Charles Dickens," what we mean is that "Boz was the *same* as Charles Dickens," and *sameness* is a general, even a hemilogical, relation. For a predicate is of an ideal nature, and as such cannot be a mere hecceity. In fact in the proposition "Boz is Charles Dickens," the Subjects are Boz and Charles Dickens and the predicate is "*identical with.*" On the other hand, every general sign, even a "term," involves, at least, a rudimentary assertion. For what is a "term," or "class-name," supposed to be? It is something which signifies, or, to use J. S. Mills' objectionable terminology, "connotes" certain characters, and thereby denotes whatever possesses those characters. That is, it draws the attention to an idea, or mental construction, or diagram, of something possessing those characters, and the possession of those characters is kept in the foreground of consciousness. What does that mean unless that the listener says to himself, "that which is *here* (before the attention) possesses such and such characters"? That may not

* 1.559.

be quite a *proposition*, or fully an assertion, because the object of attention being in this case nothing but a mental creation, the listener does not tell himself what it is that is "here." It is, at least, not an assertion about the real world. But none the less it contains the assertoric element, the mental copula. When a listener hears the term "light," he proceeds to *create* in his mind an image thereof, and goes through the very same process of thought which is attributed to the Elohim in the first chapter of Genesis. "And God said, Let there be light; and there was light. And God saw the light, that it was good," — that is, that the light was, in fact, what was intended to be created. It amounted to saying "that is light"! Until this process is performed, the name excites no meaning in the mind of the listener. But I object to the triad, *term*, *proposition*, *inference*, if it be regarded as all-important in logic, on the ground that common nouns, which, with their equivalents, are what is meant by terms, are mere accidental grammatical forms which happen to be very prominent in the languages most familiar to us, but which hardly exist, or at least are far from prominent, in the vast majority of tongues, and are really not needed, at all, and ought to be unknown to the *Grammatica Speculativa*. It is absurd, indeed, to erect this unnecessary part of speech into a logical form and leave the indispensable prepositions unrepresented, merely because in Indo-European languages they often appear in the form of terminations.

342. At the same time, it must be admitted that the proposition, "Let l be light," or what is the same thing "l is light," where l is not otherwise defined, is only an assertion about a fleeting idea, far less developed than the proposition "Hamlet was mad," which relates to a great creation more enduring than bronze. Take away from any proposition its quantifying signs and such an expression is what remains. Remove the quantifier from the proposition "all men are mortal," or what is the same thing, "everything is either not a man or is mortal," and we have "x is either not a man or is mortal." Remove the quantifier from the proposition "Everything has some cause," or what is the same thing "Let A be anything; then there is something, B, such that B is the cause of A," and it becomes "B is the cause of A." Such rudimentary assertions — assertions in form with no substance — precisely express the mean-

ings of logical *terms*. In that sense, we may say that every proposition has as many terms as it has quantified subjects. Singular subjects are of a different nature. Every term is singular but indefinite. It may be affirmative or negative, according to the character of its predicate.

343. The copula differs from the subjects and predicate in being purely formal, and containing no special matter or complexity. No doubt, this is because we choose so to draw the lines between the different parts of the proposition as leave the copula no matter; but then there are sound reasons for so drawing those lines.

§6. RUDIMENTARY PROPOSITIONS AND ARGUMENTS

344. Having thus completed the analysis of the assertion, I now proceed to show, in one word, that in nearly the same sense in which a term is a rudimentary proposition, a proposition is, in its turn, a rudimentary argumentation. A term is a proposition with the subjects deprived of their forcefulness. Deprive the propositions of an argumentation of their assertiveness, and the result is an assertion. Thus, the argumentation,

Enoch was a man,
∴ Enoch was mortal,

becomes, on ceasing to assert the propositions,

If Enoch was a man, then Enoch was mortal.

Thus, the converse, at least, is true; and every so eviscerated argumentation is a proposition.

345. But now come almost the entire assemblage of German logicians, with Professor Schröder among them, and declare that hypothetical propositions and categorical propositions differ essentially from one another.* By a hypothetical proposition is meant, in that exact historical terminology which it is the great good fortune of logic to inherit, any proposition compounded of propositions. DeMorgan† has so completely treated the subject of logical combinations, that acquaintance with his work enables us to pronounce, at once, that there are

* Cf. Schröder, *Logik*, §28.

† E.g., in his *Formal Logic*, ch. 4, and his *Syllabus*, §21ff. See 366.

six species of simple hypotheticals ranged under two genera; and discussions by Mrs. Fabian [Ladd-] Franklin* and by her husband† show that complex hypotheticals having two members are numbered by tens of thousands. The simple species are as follows:

Genus. I. *Negative simple hypotheticals*
(not asserting nor denying either member of the hypothetical)

Species 1. *Conditional propositions.* If it thunders, it rains.
Species 2. *Disjunctive propositions.* It either thunders or it rains.
Species 3. *Repugnantial propositions.* It does not both thunder and rain.

Genus II. *Affirmative simple hypotheticals*
(either asserting or denying each member of the hypothetical)

Species 1. *Independential propositions.* It thunders without raining.
Species 2. *Conjunctive propositions.* It both thunders and rains.
Species 3. *Tertial propositions.* It neither thunders nor rains.

346. Most of these simple species were given as hypotheticals by medieval logicians. But Kant, who had arrived at the conviction that there must be three classes of propositions on every logical principle of division, having taken *categoricals* from one class, took the first two species of the negative genus of simple hypotheticals for his two other classes. But he called the conditionals, *hypothetical* propositions, so restricting that term, as, indeed, it had, by some logicians, already been restricted. Kant was poorly equipped to draw up a table of "Functions of Judgment." Even Lambert, the greatest formal logician of those days, could not have succeeded in the task; nor could a Kant and Lambert rolled into one gigantic intellect. Kant did not even give time enough to the task to form any idea of its magnitude. But subsequent German logicians, drifting about without any exact method, and at all periods and concerning all matters, far too gregarious, official,

* See *Studies in Logic*, edited by C. S. Peirce, Little, Brown and Co., Boston, 1883. "On the Algebra of Logic," by Christine Ladd, p. 61ff.

† Fabian Franklin, "A Point of Logical Notation," *Johns Hopkins University Circular*, p. 131, April 1881.

and party-adhering in their opinions, accepted the triad of *categorical*, *hypothetical*, and *disjunctive* propositions, partly because it seemed to be recommended on the metaphysical side, and partly because they had no method which could imperatively deny any view to which they were officially inclined. But Professor Schröder, being an exact logician, could not possibly accept that triad. Nevertheless, he holds categoricals to be essentially different from all hypotheticals in the broad sense of this word. The above analysis of the term, since it makes the term a proposition, makes categorical propositions compound, or hypothetical, propositions. But we cannot pass by the deliberate opinion of such a mind as Schröder without closer examination. . . .

347. The quantified subject of a hypothetical proposition is a *possibility*, or *possible case*, or *possible state of things*. In its primitive sense, that which is *possible* is a hypothesis which in a given state of information is not known, and cannot certainly be inferred, to be false. The assumed state of information may be the actual state of the speaker, or it may be a state of greater or less information. Thus arise various kinds of possibility. All these varieties of possibility are *ignorantial*, or *negative*. *Positive* possibility arises when our knowledge is such as is represented by a disjunctive proposition, that either A, or B, or C, or D, etc., is true. A, B, C, D, etc., are then the positively possible cases. Thus, in playing backgammon, there are twenty-one possible throws of the dice, at each play. The aggregate of the positively possible cases is the *range* or *universe* of possibility. The speaker of a hypothetical proposition does not necessarily possess a positive disjunctive knowledge; but he can make, at any rate, a *logical* disjunction which shall be necessarily true. The quantified subject may either be universal or particular. The particular negative and universal affirmative simple hypothetical propositions will be of a different and more simple character than those which are universal negative and particular affirmative. The following table shows this.

	Particular negative hypotheticals
Conditional.	It may not thunder or it may rain.
Disjunctive.	It may thunder or it may rain.
Repugnantial.	It may not thunder or it may not rain.

Universal affirmative hypotheticals

Independential. It must thunder and it cannot rain.
Conjunctive. It must thunder and it must rain.
Tertial. It cannot thunder and it cannot rain.

Universal negative hypotheticals

Conditional. In every possible case in which it should thunder it would rain.
Disjunctive. In every possible case it either thunders or rains.
Repugnantial. In no possible case will it both thunder and rain.

Particular affirmative hypotheticals

Independential. It may thunder without raining.
Conjunctive. It may thunder and rain, too.
Tertial. It may be that it will neither thunder nor rain.

348. In every fully developed hypothetical proposition, there is a range of possibilities. The proposition derives its characteristic from this. But the Philonians* maintain (and the Diodorans have usually admitted) that analysis ought to begin with the *consequentia simplex de inesse*, which is what a conditional proposition becomes for omnipotence. In other words, we are to begin by removing the quantification, and consider *singular* hypotheticals. That done, the conditional proposition becomes (according to Philonians) "In this case either it does not thunder or it rains." If we do not say what this case is, further than that it is some contemplated possibility, the singular hypothetical becomes a term. "In the case I am contemplating either it would not thunder or it would rain," amounts to, "Let us consider the case in which it either does not thunder or does rain," or to, "The case of its either raining or not thundering." The last two differ in the accidental syntax of familiar languages, but they do not differ in meaning.

349. In a paper which I published in 1880,† I gave an im-

* A Philonian is one who defines implication "materially" i.e., one who takes "P implies Q" to mean the same as "Not P or Q." There is a reference to the controversy between Philo the Megarian, Diodorus Cronus, and Chrysippus on this point in Cicero's *Acad. Quaesi.* II, 143; the issues between Philo and Diodorus are mentioned also in Sextus Empiricus, *Adv. Math.* VIII, 113–17. See also 3.441ff.

† "On the Algebra of Logic," vol. 3, no. VI.

perfect account of the algebra of the copula. I there expressly mentioned the necessity of quantifying the possible case to which a conditional or independential proposition refers. But having at that time no familiarity with the signs of quantification, the algebra of which I developed later,* the bulk of the chapter treated of simple consequences *de inesse*. Professor Schröder accepts this first essay as a satisfactory treatment of hypotheticals; and assumes, quite contrary to *my* doctrine, that the possible cases considered in hypotheticals have no multitudinous universe. This takes away from hypotheticals their most characteristic feature. It is the sole foundation of his section 45,† in which he notes various points of contrast, between hypotheticals and categoricals. According to this, hypotheticals are distinguished from categoricals in being more rudimentary and simple assertions; while the usual doctrine of those who maintain that there is a difference between the two forms of assertion is quite the reverse.

350. In several passages in Professor Schröder's first volume promises were held out that the opening section 28 of the next volume should clearly show the difference between hypotheticals and categoricals and should convict me of taking a narrow view of assertion. But when the second volume appeared that section seemed to me remarkably lame, considering the great force and exactitude of the writer's customary thought. There is so little in it that it would by itself convey quite a false idea of Professor Schröder's power as a logician.

351. Professor Schröder's main effort is to show that time has to be considered in the analysis of hypotheticals. But he gives no proof at all that time *must be* considered; he only shows how it may be considered. Nobody familiar with the logic of relatives needs to be told that it is easy to introduce the consideration of time, if it be desirable to do so. At any rate, when I maintain that categoricals are essentially the same as hypotheticals, I mean, essentially the same as *compound propositions*, without specially introducing the idea of time. It is evident, then, that all that discussion about time is quite beside the question in dispute; it contains a fault of logic.

352. The only other argument that I am able to extract

* Vol. 3, nos. XII and XIII, §3.

† In *Algebra der Logik*.

from Professor Schröder's section 28 is that any two terms could be taken as subject and predicate (in the old sense) of a categorical and the result will always be true or false, while in the case of the hypothetical the result will often be nonsensical, and neither true nor false. One cannot but remark that this argument seems to be at variance with the view of section 45. According to that section a hypothetical proposition does not essentially differ from an unquantified categorical. It is there even called by Professor Schröder a particular species of categorical. But here it appears as having properties which no categorical possesses. I am confident, however, that examination will convince the reader that it has no such properties. I first remark that a proposition does not cease to be true because it is nonsensical. A proposition is false if and only if something which it either expressly asserts or implies is false; and every proposition not false is true, by the principle of excluded middle. Hence, something not an assertion, considered as an assertion, is true. We may therefore put the question of truth and falsity aside; and ask whether it be true that a hypothetical can be nonsensical, but a categorical not. The truth is that nonsensical forms are so readily made in categoricals that the usage of language has taken them up and attached meanings to them. "What I am telling you is true," and "A man is a man," are frequently heard, although these are, in the strictest sense, nonsensical. One of Professor Schröder's examples of a nonsensical proposition is "This proposition is not true." But that is easily shown to involve contradiction, that is to imply two contradictory things.* It therefore does imply, or mean, something. A self-contradictory proposition is not meaningless; it means too much.† But if Professor Schröder intends to say that a categorical proposition cannot be self-contradictory, that is equally untenable. "A is not A" refutes that.

353. Common nouns are primitively used to denote "sense-percepts," while clauses of hypotheticals are commonly used to denote situations which sometimes occur. One denotes the object, the other the occasion of attention. There is a psychological distinction between them. But distinctions ought not

* See also 618.

† I.e., it means both p and not-p. See 383.

to be drawn in logic which can lead up to no discrimination between a good and a bad argumentation. For the purpose of logic, it makes no difference by what psychological action the attention is arrested. When an analysis shall have been made of a continuous logical universe, it may come to be shown that a logical distinction ought to be drawn between such a universe and a discrete universe; and it is, perhaps, a little more natural to connect the continuous universe with a hypothetical than with a categorical. Nevertheless, in very many cases the universe of hypothetical propositions is discrete; and in very many cases the universe of categorical propositions is continuous, like the example of the colored surface, above.

354. There are many languages in which the simplest assertions which we make in categorical form, take, as far as we can comprehend the psychical process, hypothetical forms. There is one of these tongues a smattering of which is not an uncommon accomplishment — a smattering sufficient to carry the student into the spirit of the language — I mean the Old Egyptian. There are few words in this language which are distinctively common nouns. Every general word excites a pictorical idea. Even to the modern student, the pictorial ideograph becomes a considerable part of the idea it excites; and the influence of the hieroglyphics, the modes of expression, etc., is to make "a composite of pictures" particularly expressive in describing the idea conveyed. Now our word "*is*," the copula, is commonly expressed in Old Egyptian by a demonstrative pronoun. It is evident that this demonstrative has in such sentences the force of a relative. Where is the verb? We feel that it is contained in the general words. In short, "man is mortal" is expressed in Old Egyptian in a form which expressed the following psychological process of thinking, "What is spoken of is man, *which* what is spoken of is mortal." This is precisely the way in which the same idea is conveyed in my general algebra of logic, where, putting h for man and d for mortal, I write

$$\pi_i \, h_i \prec d_i^*$$

This form equally serves for a universal categorical or a conditional proposition, and the fact that the mode of connec-

* I.e., for every individual it holds that if it is human, it is mortal; or, for all occasions it holds that what is human is mortal.

tion of the i with the h and d, appears a little different in the two cases from a psychological point of view, ought not to affect the logical classification.

355. But the reader will object that, even granting my contention that hypothetical propositions embrace all propositions, I am still far from having shown that endowing its members with assertiveness will convert it into a process of *argumentation*. I have only shown this, if I have shown it at all, in the case of universal conditional propositions. There is great force in this. The very idea of logic forces upon the logician the conception of inference, and inference involves the idea of necessary inference, and necessary inference involves the idea of the universal conditional proposition.

356. It remains to show in what manner I suppose the ideas of the other forms of propositions to be evolved; and this will be a chapter of what I have called "speculative rhetoric." I may begin by remarking that I use the sign $-\!\!<$ for the sign of inclusion. I believe I was the first to show, in 1867,* that Boole's algebra, as he left it, was unfit to express particular propositions. Following out that idea, I showed, in 1870,† before anybody else, that we needed in logic a sign corresponding to the sign $\leqq$, but that that sign is unsatisfactory because it implies that the relation is a combination of the relations expressed by $<$ and $=$, whereas in truth, as I demonstrated, it is more simple than either. I, therefore, proposed to replace the sign $\leqq$ by $-\!\!<$, at least in logic. The sign I proposed has the advantage that it can be easily made in the composing room, while the cursive form of it is rapidly written with two strokes. On account of my priority, the sign I proposed ought, in propriety, to be retained, unless it be open to very decided objections. I shall retain it. Accordingly

$$h_i -\!\!< d_i$$

means that on the occasion i, if the idea h is definitively forced upon the mind, then on the same occasion the idea d is definitively forced upon the mind. On the Philonian view this is the same as to say that on the occasion i, either the idea h is not definitively forced upon the mind or on the same occasion the idea d is definitively forced upon the mind. From that

* See 3.18.

† See 3.47n.

hypothesis, the rules of the sign $-<$ may be mathematically deduced. I do not give them here, because my ms. containing the development was many months ago borrowed by a friend; and I have not yet quite lost hope of recovering it, and thus sparing myself the labor of repeating the work.* It follows from the omitted development, that while this sign enables us, by taking letters to denote various propositions, to express many relations, yet unless we take a letter to denote a proposition known or assumed to be false, it never enables us to express that any assertion is false. There is a very good reason for adopting the convention that

$$a-<b-<c$$

shall mean $a-<(b-<c)$, and not $(a-<b)-<c$. We are thus led to inquire what must be the meaning of

$$a-<a-<a-<a-<a-<a-<a-<a$$

without end. This series of antecedents without a final consequent are seen to be equivalent to the denial of a.† Thus, without the introduction of any other sign, but merely by the idea of an endless sequence, after we already have the idea of successive sequence, we reach the idea of negation. Thus, the conceptions involved in argumentation produce the conception of the rejection of an argumentation. Hence, we are led to generalize our idea of argumentation, from the perception that one assertion has to be admitted because another is admitted, to embrace also that process of thought in which we think that though one assertion is true yet another is not thereby necessarily true. It is not the primitive conception of argumentation, but this generalized conception, which covers the entire field of hypotheticals. As soon as we have the idea of absurdity, we can conceive that a certain argument could logically lead to absurdity. Now an argument which can lead to absurdity is false; and an argument which is false can in some conceivable case lead to absurdity. Hence, as soon as we admit the idea of absurdity, we are bound to class the rejection of an argumentation among argumentations. Thus, as was said, a proposition is nothing more nor less than an

* A number of mss. on this topic have been found. They contain nothing not easily derived from the discussions in volume 3 and volume 4.

† I.e., it is equivalent to: not-a or not-a or not-a . . .

argumentation whose propositions have had their assertiveness removed, just as a term is a proposition whose subjects have had their denotative force removed.

§7. SUBJECT*

357. Whether or not every proposition has a principal subject, and, if so, whether it can or cannot have more than one, will be considered below. A proposition may be defined as a sign which separately indicates its object. For example, a portrait with the proper name of the original written below it is a proposition asserting that so that original looked. If this broad definition of a proposition be accepted, a proposition need not be a symbol. Thus a weathercock "tells" from which direction the wind blows by virtue of a real relation which it would still have to the wind, even if it were never intended or understood to indicate the wind. It separately indicates the wind because its *construction* is such that it must point to the quarter from which the wind blows; and this construction is distinct from its *position* at any particular time. But what we usually mean by a proposition or judgment is a symbolic proposition, or *symbol*, separately indicating its object. Every subject partakes of the nature of an index, in that its function is the characteristic function of an index, that of forcing the attention upon its object. Yet the subject of a symbolic proposition cannot strictly be an index. When a baby points at a flower and says, "Pretty," that is a symbolic proposition; for the word "pretty" being used, it represents its object only by virtue of a relation to it which it could not have if it were not intended and understood as a sign. The pointing arm, however, which is the subject of this proposition, usually indicates its object only by virtue of a relation to this object, which would still exist, though it were not intended or understood as a sign. But when it enters into the proposition as its subject, it indicates its object in another way. For it cannot be the subject of that symbolic proposition unless it is intended and understood to be so. Its merely being an index of the flower is not enough. It only becomes the subject of the proposition, because its being an index of the flower is evidence that it was *intended* to be. In like manner, all ordinary propositions refer

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 609–10.

to the real universe, and usually to the nearer environment. Thus, if somebody rushes into the room and says, "There is a great fire!" we know he is talking about the neighbourhood and not about the world of the *Arabian Nights' Entertainments*. It is the circumstances under which the proposition is uttered or written which indicate that environment as that which is referred to. But they do so not simply as index of the environment, but as evidence of an intentional relation of the speech to its object, which relation it could not have if it were not intended for a sign. The expressed subject of an ordinary proposition approaches most nearly to the nature of an index when it is a proper name which, although its connection with its object is purely intentional, yet has no reason (or, at least, none is thought of in using it) except the mere desirability of giving the familiar object a designation. Among, or along with, proper names we may put abstractions, which are the names of fictitious individual things, or, more accurately, of individuals whose being consists in the manner of being of something else. A kind of abstractions are individual collections, such as the "German people." When the subject is not a proper name, or other designation of an individual within the experience (proximate or remote) of both speaker and auditor, the place of such designation is taken by a virtual precept stating how the hearer is to proceed in order to find an object to which the proposition is intended to refer. If this process does not involve a regular course of experimentation, all cases may be reduced to two with their complications. These are the two cases: first, that in which the auditor is to take any object of a given description, and it is left to him to take any one he likes; and, secondly, the case in which it is stated that a suitable object can be found within a certain range of experience, or among the existent individuals of a certain class. The former gives the *distributed* subject of a *universal* proposition, as, "Any cockatrice lays eggs." It is not asserted that any cockatrice exists, but only that, if the hearer can find a cockatrice, to that it is intended that the predicate shall be applicable. The other case gives the *undistributed* subject of a *particular* proposition, as "Some negro albino is handsome." This implies that there is at least one negro albino. Among complications of these cases we may reckon such subjects as that of the proposition, "Every

fixed star but one is too distant to show a true disk," and, "There are at least two points common to all the circles osculating any given curve." The subject of a universal proposition may be taken to be, "Whatever object in the universe be taken"; thus the proposition about the cockatrice might be expressed: "Any object in the universe having been taken, it will either not be a cockatrice or it will lay eggs." So understood, the subject is not *asserted* to exist, but it is well known to exist; for the universe must be understood to be familiar to the speaker and hearer, or no communication about it would take place between them; for the universe is only known by experience. The particular proposition may still more naturally be expressed in this way, "There is something in the universe which is a negro albino that is handsome." No doubt there are grammatical differences between these ways of stating the fact; but formal logic does not undertake to provide for more than one way of expressing the same fact, unless a second way is requisite for the expression of inferences. The latter mode is, on the whole, preferable. A proposition may have several subjects. Thus the universe of projective geometry being understood, it is a true proposition that "Whatever individuals, A, B, C, and D may be, there are individuals E and F, such that whatever individual G may be, there is an individual H, and an individual I, such that, if A, B, C, and D are all straight lines, then E and F are straight lines, each intersecting A, B, C, and D, and E and F are not coincident; and if G is a straight line, not coincident with E, and not coincident with F, and if G intersects A, B, and C, it does not intersect D, unless H is a one-sheeted hyperboloid of which A, B, C, and D are generators, and J is a set of generators of H, to which A, B, C, and D all belong"; or, in our usual phraseology, any four straight lines in space are intersected by just two different straight lines, unless these four straight lines belong to one set of generators of a one-sheeted hyperboloid. Such a proposition is called a relative proposition. The order in which the selection of individuals is made is material when the selections are different in respect to distribution. The proposition may relate to the frequency with which, in the course of ordinary experience, a generic event is of a certain species. DeMorgan wishes to erect this into the general type

of propositions.* But this is to overlook a vital distinction between probability and that which a universal proposition asserts. To say that the probability that a calf will not have more than six legs is 1, is to say that in the long run, taking calves as they present themselves in experience, the ratio of the number of those with not more than six legs to the total number is 1. But this does not prevent there being any finite number of calves with more legs than six, provided that in the long run, that is, in an endless course of experience, their number remains finite, and does not increase indefinitely. A universal proposition, on the other hand, asserts, for example, that any calf which may exist, without exception, is a vertebrate animal. The universal proposition speaks of experience distributively; the probable, or statistical proposition, speaks of experience collectively.

§8. PREDICATE†

358. The view which pragmatic logic takes of the predicate, in consequence of its assuming that the entire purpose of deductive logic is to ascertain the necessary conditions of the truth of signs, without any regard to the accidents of Indo-European grammar, will be here briefly stated. Cf. Negation.‡

In any proposition, i.e., any statement which must be true or false, let some parts be struck out so that the remnant is not a proposition, but is such that it becomes a proposition when each blank is filled by a proper name. The erasures are not to be made in a mechanical way, but with such modifications as may be necessary to preserve the partial sense of the fragment. Such a residue is a *predicate*. The same proposition may be mutilated in various ways so that different fragments will appear as predicates. Thus, take the proposition "Every man reveres some woman." This contains the following predicates, among others:

". . . reveres some woman."

". . . is either not a man or reveres some woman."

"Any previously selected man reveres . . ."

"Any previously selected man is . . . "

* *Formal Logic*, ch. 8.

† *Dictionary of Philosophy and Psychology*, vol. 2, pp. 325–6.

‡ See 378–80.

§9. PREDICATION*

359. In logic: the joining of a predicate to a subject of a proposition so as to increase the logical breadth without diminishing the logical depth.

360. This still leaves room for understanding predication in various ways, according to the conception entertained of the dissection of a proposition into subject and predicate. It is a question under dispute today whether predication is the essential function of the proposition. Some maintain that the proposition "It rains" involves no predication. But if it is an assertion, it does not mean that it rains in fairyland, but the very act of saying anything with an appearance of seriously meaning it is an Index (q.v.)† that forces the person addressed to look about to see what it is to which what is being said refers. The "rains" recalls to his mind an image of fine up-and-down lines over the field of view; and he looks sharply out of the window, fully understanding that that visible environment is indicated as the subject where the lines of falling drops will be seen. In like manner, there is a predication in a conditional or other hypothetical proposition, in the same sense that some recognized range of experience or thought is referred to.

361. A few of the most frequently recurring scholastic phrases follow.

. . . *Analogical predication*; a rather favourite expression of Aquinas: predication in which the predicate is taken neither in its strict sense nor in an unrelated sense, but in a peculiar sense for which there is a good reason, as when a statue is said to be a man.

. . . *Denominative predication:* predication in which that whose nature it is to be a subject is taken as the subject, and something whose nature it is to be predicated is taken as the predicate; a predication of an accident of a substance. (It is well discussed by Scotus, *In univ. Porph.*, 9. 16, "Utrum haec sit vera, *Homo est animal*," where, as in the majority of scholastic disputations, the conclusion is foregone, and the interest lies in the formidable difficulties and how they are to be overcome.) Denominative predication, in its proper sense, is predi-

* *Ibid.*, vol. 2, pp. 326–9.

† 305–6.

cation of an accidental concrete term of its own subject; in a broad sense, it is the predication of any concrete of a suppositum, or of any subject of less breadth; in the widest sense, it is predication of any predicate of any subject. Denominative predication may be *a posteriori* or *a priori*, as *homo est albus*, *rationale est substantia*, *homo est animal.*

. . . *Dialectic predication*, as defined by Aristotle (I. *Top.*, x): the predication of a general term in a proposition which may result from an argument in a probable place, and not reducible to anything prior.

Direct predication: predication in the usual sense of representing that the breadth of the subject belongs to the predicate, and the depth of the predicate to the subject; or, in scholastic language, it is predication of a higher term of a lower one, of a passion of a subject, of an accident of a subject, of a mode of a quiddity, of a difference of a genus.

. . . *Essential predication:* in which the predicate is wholly contained in the essence of the subject. It is, therefore, in Kant's sense, an analytical judgment. But neither Kant nor the scholastics provide for the fact that an indefinitely complicated proposition, very far from obvious, may often be deduced by mathematical reasoning, or necessary deduction, by the logic of relatives, from a definition of the utmost simplicity, without assuming any hypothesis whatever (indeed, such assumption could only render the proposition deduced simpler); and this may contain many notions not explicit in the definition. This may be illustrated by the following: Man is a rational animal; hence, whatever is not a man is either, on the one hand, not rational, while either at the same time being an animal or else benefiting nothing except such objects as love nothing but fairies, or, on the other hand, is not an animal, while either being rational or standing to whatever fairy may exist in the relation of benefiting something that loves it. Now, if it be said that that is an analytical judgment, or essential predication, neither the definition of the scholastics nor that of Kant is adequate. But if it be said that it is not an essential predication, or analytical judgment, then the accidental predication and the synthetical judgment may be a necessary consequence, and a very recondite one, of a mere definition, quite contrary to what either Kant or the scholastics supposed and built upon.

Cf. Scotus (*In univ. Porph.*, 9. 12), who makes essential predication the predication of genus, species, or difference.

Exercised predication. The distinction between exercised and signate predication belongs to Scotus. (The passage which Prantl attributes to Antonius Andreas* is a quotation *verbatim* from Scotus, as often naturally happens in Prantl's *Geschichte.*) A *signate* predication is one which is *said to be* made, an *exercised* predication is one which *is* made; so that Scotus says: "A praedicari signato ad praedicari exercitum, [sive ad esse,] non tenet consequentia per se in eisdem terminis."† Scotus gives the following examples of the distinction, where the exercised predication is marked E, the signate S: S, *Genus praedicatur de specie;* E, *Homo est animal.* (The Lyons text here transposes the terms, which we give correctly.) S, *nego;* E, *non.* E, *tantum;* S, *excludo.* The abstract definition of Scotus is: "*Esse* in rebus primae intentionis, illud exercet, quod *praedicari* signat in secundis intentionibus.‡ Exercised predication is distinguished into *praedicatio de proprio supposito* and *praedicatio de subiecto;* the former is essential, the latter accidental.

Formal predication: predication where the predicate is in the concept of the subject, independent of any extrinsic cause or of any particular matter *in qua.* The difference between formal and essential predication is somewhat trivial and confused.

. . . *Natural predication:* when the subject and predicate ought to be so related according to their nature. This is substantially the definition given in many books; but it conveys little idea of how the expression is used. Natural predication is always divided into the identical and direct; non-natural predication is either *indirect,* i.e., *contra naturam,* or it is *praeter naturam,* i.e., *per accidens.* Examples of indirect predication, where the subject is related to the predicate as form to matter, are *alba est nix, animal est homo.* Examples of predication *praeter naturam,* where subject and predicate are related to some third term, as form to matter, are *album est dulce, dulce est album.* Examples of direct predication: *nix est alba, homo est animal.* Examples of identical predication: *gladius est ensis, Plato est Plato* (Conimbricenses *in Praef. Porph.*, q.i. art. 4). . . .

* See Prantl, *op. cit.*, III, 279.

† *Super Universalia Porphyrii,* qu. XIV.

‡ *Ibid.*

§10. QUANTITY*

362. (In logic and mathematics.) (1) Any Accident whereby a substance has part outside of part. Cf. Quantity (2).

This is the old definition; and it is true to the old meaning of the word in representing quantity as much more concrete than the modern conception. Quantity (see Aristotle's *Praedicamenta*, vi) is either discrete or continuous. Continuous quantity is either magnitude or time. The old definition of mathematics as the science of quantity is misunderstood, if quantity is here taken in the modern sense; it was only meant that mathematics treated of accidents having number, magnitude, or duration. There was therefore a mathematics of music.

363. (2)† In the general modern sense, quantity is a system of serial relationships.

Serial relationship differs from transitive relationship merely in the point of view, and (so closely connected are the two points of view) in hardly more than the mode of expression. Now, all transitive relation is traceable to inclusion. Hence, quantity might be defined as a system of inclusions looked upon as serial. It is very important to understand that quantity is a mere system of relative ordinal relations in a linear series. Each complete determination of quantity in a given system is a "value."

Quantity is either counted or measured. Counted quantity may have a finite multitude of values. Of systems of quantity of denumeral multitude, the simplest is that of the integer numbers. The system of rational fractions is the only other familiarly used. These fractions can, in several ways, be arranged in their order of quantity by mere counting.

364. (3) Concepts, or terms, are, in logic, conceived to have *subjective parts*, being the narrower terms into which they are divisible, and *definitive parts*, which are the higher terms of which their definitions or descriptions are composed: these relationships constitute "quantity."

This double way of regarding a class-term as a whole of parts is remarked by Aristotle in several places (e.g., *Met.*, Δ. xxv. 1023 b 22). It was familiar to logicians of every age.

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 410–12.

† Cf. vol. 4, bk. I, no. 4.

Thus Scotus Erigena calls logic "ars illa quae diuidit genera in species et species in genera resoluit."* John of Salisbury† refers to the distinction as "quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa [i.e., adjectives and the like] *significant*, et aliud esse quod *nominant*. Nominantur singularia, sed universalia significantur." For William of Auvergne, see Prantl, III, 77. The writer has a long list of similar passages before him. But the Aristotelians had their minds upon the discrimination of different kinds of predication, and insisted that the differences of different genera are different, thus forbidding cross-divisions. Arnauld, however, in *l'Art de penser*, conceives all predicates, or all essential predicates, as alike, without distinguishing *genus* and *differentia;* and was so led to devote a short chapter (vi) to *l'étendue* and *la compréhension* before taking up the predicables. But his services in the matter have been grossly exaggerated, and it really seems to have been Kant who made these ideas pervade logic and who first expressly called them quantities. But the idea was old. Archbishop Thomson,‡ W. D. Wilson,§ and C. S. Peirce¶ endeavor to make out a third quantity of terms. The last calls his third quantity "information," and defines it as the "sum of synthetical propositions in which the symbol is subject or predicate," antecedent or consequent. The word "symbol" is here employed because this logician regards the quantities as belonging to propositions and to arguments, as well as to terms. A distinction of *extensive* and *comprehensive distinctness* is due to Scotus (*Opus Oxon.*, I. ii. 3): namely, the usual effect upon a term of an increase of information will be either to increase its breadth without diminishing its depth, or to increase its depth without diminishing its breadth. But the effect may be to show that the subjects to which the term was already known to be applicable include the entire breadth of another term which had not been known to be so included. In that case, the first term has gained in *extensive distinctness*. Or the effect may be to teach that the marks already known to be

* *De divisione naturae* IV, 4.

† *Metalogicus* II, xx.

‡ *An Outline of the Necessary Laws of Thought* (1842), §§52, 54, 80.

§ *An Elementary Treatise on Logic*, (1856), I, ii, §5.

¶ 418.

predicable of the term include the entire depth of another term not previously known to be so included, thus increasing the *comprehensive distinctness* of the former term. The passage of thought from a broader to a narrower concept without change of information, and consequently with increase of depth, is called *descent;* the reverse passage, *ascent.* For various purposes, we often imagine our information to be less than it is. When this has the effect of diminishing the breadth of a term without increasing its depth, the change is called *restriction;* just as when, by an increase of real information, a term gains breadth without losing depth, it is said to gain extension. This is, for example, a common effect of *induction.* In such case, the effect is called generalization. A decrease of supposed information may have the effect of diminishing the depth of a term without increasing its information. This is often called *abstraction;* but it is far better to call it *prescission;* for the word *abstraction* is wanted as the designation of an even far more important procedure, whereby a transitive element of thought is made substantive, as in the grammatical change of an adjective into an abstract noun. This may be called the principal engine of mathematical thought. When an increase of real information has the effect of increasing the depth of a term without diminishing the breadth, the proper word for the process is *amplification.* In ordinary language, we are inaccurately said to *specify*, instead of to *amplify*, when we add to information in this way. The logical operation of forming a hypothesis often has this effect, which may, in such case, be called *supposition.* Almost any increase of depth may be called *determination.*

(4) Syllogistic is sometimes regarded as the mathematics of a system of quantities consisting of but two values, *truth* and *falsity.*

(5) The quantity of a proposition is that respect in which a universal proposition is regarded as asserting more than the corresponding particular proposition: the recognized quantities are Universal, Particular, Singular and — opposed to these as "definite"— Indefinite. *Quantitas* is used in this sense by Apuleius.*

365. *Quantification of the Predicate.* The attachment of signs

* See Prantl, *op. cit.*, I, 581.

of propositional quantity to the predicates of simple propositions is called by this name. The *dictum de omni* defines the relation of subject and predicate, so that "Any A is B" is to be understood as meaning "To whatever A is applicable, B is applicable." But this definition must be modified, in order to give any room for a quantification of the predicate. If then we are to take *all* and *some* in their proper distributive senses and not in collective senses, to say that "Every man is every animal" would, as Aristotle remarks, be absurd, unless it were meant that there was but one man and one animal, and that that one man was identical with that one animal. This system has never been proposed. But Hamilton,* with his followers, T. S. Baynes† and Calderwood, take the marks of quantity in a collective sense. They thus have, as one of the propositional forms, "Some man is not some animal," which precisely denies "Every man is every animal," in the distributive sense, and is entitled to an equal standing in logic. It does not deny "All man is all animal," in the collective sense of these logicians. This system had some vogue in its day.

366. *DeMorgan's System of Propositions.*‡ This permits the retention of the *dictum de omni*, merely applying propositional quality to the subject. We thus get the following eight forms of proposition:

)) To whatever A is applicable, B is applicable.
(·) To whatever A is inapplicable, B is applicable.
)·(To whatever A is applicable, B is inapplicable.
((To whatever A is inapplicable, B is inapplicable; i.e., To whatever B is applicable, A is applicable.
() To something to which A is applicable, B is applicable.
(·(To something to which A is applicable, B is inapplicable.
)·) To something to which A is inapplicable, B is applicable; i.e., To something to which B is applicable, A is inapplicable.
)(To something to which A is inapplicable, B is inapplicable.

The above is substantially one of DeMorgan's own forms of

* *Lectures on Logic*, XIII, pp. 243–48.
† *An Essay on the New Analytic of Logical Forms*, (1850).
‡ *Syllabus of a Proposed System of Logic* (1860). §21ff. See also 568.

statement, called by him onymatic.* There is no objection to this system; but it is an idle complication of forms which does not enable us to take account of any mode of inference that the old system does not cover. Still it does away with the figures of syllogism. But whatever the merits or demerits of the system, DeMorgan developed it with logical elegance.

§11. UNIVERSAL†

367. (1) This word was used in the middle ages where we should not use the word General. Another synonym was *praedicabile:* "Praedicabile est quod aptum natum est praedicari de pluribus," says Petrus Hispanus.‡ Albertus Magnus says, "Universale est quod cum sit in uno aptum natum est esse in pluribus."§ Burgersdicius, literally translating from Aristotle, says, "Universale (*τὸ καθ' ὅλου*) appello, quod de pluribus suapte natura praedicari aptum est," i.e., *ὃ ἐπὶ πλειόνων πέφυκε κατηγορεῖσθαι*. When the Scholastics talk of universals, they merely mean general terms (which are said to be *simple universals*), with the exception here following.

368. (2) The five terms of second intention, or more accurately the five classes of predicates, *genus*, *species*, *difference*, *property*, *accident*, were in the middle ages (as they still are) called "the predicables." But since predicable also means fit to be a predicate, in which sense it is almost an exact synonym of universal in the first sense, the five predicables came to be often referred to as "the universals."

369. (3) Predicated, or asserted, in a proposition *de omni;* said to be true, without exception, whatever there may be of which the subject term is predicable. See Quantity [§10].

Thus "any phoenix rises from its ashes" is a universal proposition. This is called the *complex* sense of universal. The subject must be taken in the distributive sense and not in the collective sense. Thus, "All man is all redeemed," which is Hamilton's "toto-total proposition,"¶ is not a universal proposition, or assertion *de omni*, in the sense defined by Aristotle

* *Ibid.*, §165.

† *Dictionary of Philosophy and Psychology*, vol. 2, pp. 737–40: 367–9 are by Peirce alone; 370–1, given in part only, are by Peirce and Mrs. C. Ladd-Franklin.

‡ *Summulae*, Tractatus II, p. 87C.

§ *De Praedicab.* II, 1, p. 11A.

¶ *Lectures on Logic*, App. V (d), (3).

in the *dictum de omni;* for it means that the collection of men is identical with the collection of the redeemed, and not that each man without exception is all redeemed. Leibnitz rightly insists that a universal proposition does not assert, or imply, the existence of its subject.* The first reason for this is that it accords with the definition, that is, the *dictum de omni*, which is that that is asserted universally of a subject which is said to be predicable of whatever that subject may be predicable. For this may be done without asserting that the subject is predicable of anything in the universe. The second reason is that the term *universal proposition* is a term of formal logic. Now the principal, or at least the most essential, business of formal logic is so to formulate direct syllogism as not to represent it as requiring more or less than it really does. Now the major premiss of a direct syllogism must be universal, but need not imply the existence of anything of which the subject should be predicable. Hence a form of universal proposition not asserting the existence of the subject is indispensable. Now that no second kind of universal proposition is needed will presently appear. The third reason is that it is necessary that formal logic should be provided with a form of proposition precisely denying every proposition coming under each of its simple forms. Now, if a universal proposition asserting the existence of its subject is regarded as a simple form of proposition — as, for example, "There are inhabitants of Mars and every one of them without exception has red hair"— its precise denial would be a particular proposition not asserting the existence of the subject, which would be a most singular form, hardly ever wanted, and manifestly complex, such as, "Either there is no inhabitant of Mars, or if there be, there is one at least who has not red hair." It is obviously far better to make the simple particular proposition assert the existence of its subject, "There is an inhabitant of Mars who has red hair," when the universal form will not make the same assertion, or imply it: "Whatever inhabitants of Mars there may be must, without exception, have red hair." If every particular proposition asserts the existence of its subject, then an affirmative particular proposition *implies* the existence of its predicate also. It would be a contradiction in terms to say that a propo-

* Cf. *Nouveaux Essais*, bk. IV, ch. 9.

sition *asserted* the existence of its predicate, since that of which a proposition *asserts* anything is its subject, not its predicate. But perhaps it is not quite accurate to say that the particular proposition *asserts* the existence of its subject. At any rate, this must not be understood as if, in such assertion, *existence* were a predicate not implied in a proposition which does not make this assertion (see Kant, *Krit. d. reinen Vernunft*, 1st ed., 599).

Every proposition refers to some index: universal propositions to the universe, through the environment common to speaker and auditor, which is an index of what the speaker is talking about. But the particular proposition asserts that, with sufficient means, in that universe would be found an object to which the subject term would be applicable, and to which further examination would prove that the image called up by the predicate was also applicable. That having been ascertained, it is an *immediate inference*, though not exactly *asserted* in the proposition, that there is some *indicable object* (that is, something *existent*) to which the predicate itself applies; so that the predicate also may be considered as referring to an index. Of course, it is perfectly legitimate, and in some aspects preferable, to formulate the particular proposition thus: "Something is, at once, an inhabitant of Mars and is red haired," and the universal proposition thus: "Everything that exists in the universe is, if an inhabitant of Mars, then also red haired." In this case, the universal proposition *asserts* nothing about existence; since it must already be well understood between speaker and auditor that the universe is *there*. The particular proposition in the new form asserts the existence of a vague something to which it pronounces "inhabitant of Mars" and "red haired" to be applicable.

The universal proposition must be understood as strictly excluding any single exception. It is thus distinguished from the proposition "The ratio of the number of A's to that of the A's that are B is as 1 : 1," not merely in being distributive in form instead of collective, but also in asserting much more. Thus the ratio of the multitude of all real numbers to those of them that are incommensurable is as 1 : 1, yet that does not prevent the commensurable numbers from existing, nor from being infinite in multitude. Were it proved that the ratio

of frequency of all events to such of them as were due to natural causation was 1 : 1, that would be no argument whatever against the existence of miracles; although it might (or might not, according to circumstances) be an argument against explaining any given event as miraculous, if such a hypothesis can be called an explanation. Now induction may conclude that the ratio of frequency of a specific to a generic event is 1 : 1, in the same approximate sense in which all inductive conclusions are to be accepted. Indeed, the ratios 1 : 1 and 0 : 1 may be inductively concluded with stronger confidence in their accuracy than any other ratio can be so concluded. But under no circumstances whatsoever can induction establish the accuracy or approximate accuracy of a strictly universal proposition, or that any given series of phenomenal events is, properly speaking, general (and therefore represents a possibly infinite class), or is even approximately general. Such propositions, outside of mathematics (taking this word so as to include all definitions and deductions from them), must either be entirely unwarranted, or must derive their warrant from some other source than observation and experiment. It might conceivably be established by testimony, as, for example, by a promise by a possibly immortal being to act in a certain way upon every occasion of a certain description; and thus it would not need to be an *a priori* judgment.

370. (4) . . . Descartes, Leibnitz, Kant, and others appeal to the universality of certain truths as proving that they are not derived from observation, either directly or by legitimate probable inference. There is only one such passage in Descartes; and even Leibnitz, though he frequently alleges the *necessity* of certain truths (that is, their being propositions of necessary mode) against Locke's opinion, yet in only one place (the *Avant-Propos* of the *Nouveaux Essais*) distinctly adds the criterion of universality. Descartes, Leibnitz, and Kant more or less explicitly state that that which they say cannot be derived from observation, or legitimate probable inference from observation, is a universal proposition in sense (3), that is, an assertion concerning every member of a general class without exception. Descartes (*Letter* xcix) argues that no legitimate inference can be made from external phenomena to the proposition that "Things equal to the same are equal to each

other," since that would be to infer a "universal" from a "particular." Leibnitz uses almost the same language: "D'où il nait une autre question, savoir, si toutes les vérités dépendent de l'expérience, c'est-à-dire de l'induction et des exemples, ou s'il y a un autre fondement . . . Or, tous les exemples qui confirment une vérité générale, de quelque nombre qu'ils soient, ne suffisent pas pour établir la nécessité universelle de cette même vérité: car il ne suit pas que ce qui est arrivé arrivera toujours de même."* Kant expresses himself still more unmistakably (*Krit. d. reinen Vernunft*, 2d ed., Einleitung, ii): "Erfahrung giebt niemals ihren Urtheilen wahre und strenge, sondern nur angenommene und comparative Allgemeinheit (durch Induction), so dass es eigentlich heissen muss: so viel wir bisher wahrgenommen haben, findet sich von dieser oder jener Regel keine Ausnahme. Wird also ein Urtheil in strenger Allgemeinheit gedacht, d. i. so, dass gar keine Ausnahme als möglich verstattet wird, so ist es nicht von der Erfahrung abgeleitet, sondern schlechterdings *a priori* gültig. Die empirische Allgemeinheit ist also nur eine willkührliche Steigerung der Gültigkeit, von der, welche in den meisten Fällen, zu der, die in allen gilt, wie z. B. in dem Satze: alle Körper sind schwer; wo dagegen strenge Allgemeinheit zu einem Urtheile wesentlich gehört, da zeigt diese auf einem besonderen Erkenntnissquell derselben, nämlich ein Vermögen des Erkenntnisses *a priori*. Nothwendigkeit und strenge Allgemeinheit sind also sichere Kennzeichen einer Erkenntniss *a priori*, und gehören auch unzertrennlich zu einander." But notwithstanding the fact that the whole logic of all these writers, especially Kant, requires the word universal to be understood in that sense, yet there are, in the works of all of them, some passages which lend a certain colour of excuse to the stupid blunder of some interpreters who teach that by necessity they mean the irresistible psychical force with which the proposition demands our assent, and that by universality they mean catholicity, i.e., the catholic acceptance of it *semper, ubique, et ab omnibus*. Descartes in particular, and Leibnitz in some measure, perhaps even Kant (though it would be very illogical for him to do so) did more or less attach weight to the irresistible apparent evidence, and to some degree to the

* Cf. *Nouveaux Essais, Avant-Propos.*

catholic acceptance, of propositions as tending to persuade us of their truth; but not as criteria of their origin. It is, however, to be noticed that false interpreters of Kant have used the word universal in the sense of being accepted by all men — the sense of κοινός in the phrase κοιναὶ ἔννοιαι.

371. The words universal and universality enter into various technical phrases:

. . . *Natural universal:* a natural sign predicable of a plurality of things, as smoke is a sign of fire. The nominalistic doctrine is that nothing out of the mind is universal in that sense. See Ockham, *Logica*, I. xiv *ad fin.*

. . . *Universal validity:* according to some logicians is the validity of such reasonings as are "calculated to operate conviction on all reasonable minds" (Hamilton, *Lect. on Logic*, xxvi). If he had omitted the word reasonable, and said "calculated to work conviction on all minds," this would not have proved they had any validity at all; for the validity of a reasoning depends upon whether it really will lead to the truth, and not upon whether it be believed that it will. Thus the word reasonable is the only pertinent word in the definition. But in fact there is no division of logical validity into universal and particular. . . .

§12. PARTICULAR*

372. In untechnical language, applied to single cases coming under general heads and occurring, or supposed to occur, in experience; in this sense it is also a substantive. The particulars are the experientially known circumstances of general nature, but as they appear in the individual case.

373. A particular proposition is one which gives a general description of an object and asserts that an object to which that description applies occurs in the universe of discourse, without asserting that it applies to the whole universe or to everything in the universe of a specified general description; as "Some dragons breathe fire." If we hold that the particular proposition asserts the existence of something, then the precise denial of it does not assert the existence of anything; as "No dragon breathing fire exists." It is, therefore, not true that from such precise denial any particular proposition fol-

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 265–6, by Peirce and Mrs. C. Ladd-Franklin.

lows, such as "Some dragon does not breathe fire." For if there is no dragon that does not breathe fire, this is false, although it may be true that there is no dragon that breathes fire.

For instance, from the particular proposition, "Some woman is adored by all Catholics," it follows that "Any Catholic that may exist adores a woman," i.e., "There is no Catholic that does not adore a woman," which is the precise denial of "Some Catholic non-adores all women," which is a particular proposition. From this, in turn, it follows that a woman adored by all Catholics does not exist, which is the precise denial of the first proposition, "Some woman is adored by all Catholics." Of every particular proposition the same thing is true. Thus, if "Some crow is white," it follows that "No inevitable consequence of whiteness is wanting to all crows," which is the precise denial of the particular proposition, "Some inevitable consequence of whiteness is wanting in all crows." Thus, from every particular proposition follows the precise denial of a particular proposition, but from no precise denial of a particular proposition can any particular proposition follow. But this does not extend to *a simple* particular proposition, such as "Something is white," since to say "Something is non-existent" (which the analogous treatment would yield) is an absurdity, and ought not to be considered as a proposition at all.

§13. QUALITY*

374. (In grammar and logic.) (1) Take a sentence in which a common noun or adjective is predicated of a proper noun, and imagine that there is something in the reality which corresponds to the form of the proposition. Then imagine that this form of fact consists in a relation of the objective subject, or substance, to one being, the same correlate for all cases where the same noun or adjective is predicated in the same sense, and that imaginary being, whether looked upon as real, or merely as a convenience of thought, is a *quality*. Thus, if anything is beautiful, white or incomprehensible, this consists in its possessing the quality of beauty, whiteness, or incomprehensibility.

375. (2) But in a more proper sense the term quality will

* *Ibid.*, vol. 2, pp. 408–9.

not be applied when the adjective, like *incomprehensible*, is conceived as signifying a relation. Thus, whiteness will be, in this narrow sense, a quality only so long as objects are thought as being white independently of anything else; but when this is conceived as a relation to the eye, "whiteness" is only a quality in a looser sense. Locke* defines quality as the power of producing an idea, which agrees with the above explanation tolerably.

Qualitas, having inevitably reached an excessively vague use, was in the Roman schools taken to designate almost any character or characters for which no other name was at hand. Thus arose a variety of special senses. Thus in grammar the difference between nouns which had a plural and those which had not was called a difference of quality; as was the difference between the personal pronouns and *qui*, *quis*, etc.

376. (3) In logic: the distinction between the affirmative and the negative proposition has been called the distinction of quality in propositions by all logicians, without interruption, from Apuleius, in the second century of our era, to our own contemporaries.

Kant, in order to round out a triad, added a third quality, called limitative, that of "Sortes est non homo," with a distinction from "Sortes non est homo." This will not bear criticism; but Kant's authority and the force of tradition have caused it to survive. As long as the universe of characters is unlimited, it is obvious that any collection of objects have some predicate common and peculiar to them. This being the case, as ordinary syllogistic tacitly assumes it is, the distinction between affirmative and negative propositions is purely relative to the particular predicate. No doubt many logicians have assumed that negative propositions are distinguished from ordinary affirmative propositions in not implying the reality of the subject. But what, then, does "Some patriarch does not die" mean? Besides, all admit that propositions *per se primo modo* do not imply the existence of the subject, although they be affirmative. At any rate, the resulting syllogistic, if consistent, is very objectionable. If, however, the universe of characters is limited, as it is in ordinary speech, where we say that logical inconsistency and mandarin oranges have nothing

* *Essay*, II, viii, 8.

in common, then the system of formal logic required will be a simple case of the logic of relatives (q.v.);* but the distinction of affirmative and negative propositions will become material or absolute, the forms of simple categorical propositions then being:

Any A possesses every character of the group β.
Any A wants every character of the group β.
Any A possesses some character of the group β.
Any A wants some character of the group β.
Some A possesses every character of the group β, etc.

377. (4) Quality, even in Aristotle, is especially employed to denote characters which constitute merits or demerits; and this word is remarkable for the number of specialized meanings that it bears. Since Kant it has been employed to designate the distinction of clear and obscure, or distinct and confused, etc. See the preceding topic.

Quality is distinguished as primary, secondary, secundo-primary, essential or substantial, accidental, manifest, occult, primitive, original, elementary, first, derived, real, intentional, imputed, passible, logical, propositional, active, alterant, affective, predicamental, etc.

§14. NEGATION†

378. Negation is used (1) logically, (2) metaphysically. In the logical sense it may be used (*a*) relatively, and (*b*) absolutely. Used relatively, when applied to a proposition, it may be understood (α) as denying the proposition, or (β) as denying the predicate.

379. (1) In its logical sense, negation is opposed to affirmation, although, when it is used relatively, this is perhaps not a convenient contrary term; in its metaphysical sense, negative is opposed to positive (fact, etc.)

The conception of negation, objectively considered, is one of the most important of logical relations; but subjectively considered, it is not a term of logic at all, but is prelogical. That is to say, it is one of those ideas which must have been fully developed and mastered before the idea of investigating

* See vol. 3, no. XX, §8.

† *Dictionary of Philosophy and Psychology*, vol. 2, pp. 146–7, by Peirce and Mrs. C. Ladd-Franklin.

the legitimacy of reasonings could have been carried to any extent.

The treatment of the doctrine of negation affords a good illustration of the effects of applying the principle of Pragmatism (q. v.)* in logic. The pragmatist has in view a definite purpose in investigating logical questions. He wishes to ascertain the general conditions of truth. Now, without of course undertaking to present here the whole development of thought, let it be said that it is found that the first step must be to define how two propositions can be so related that under all circumstances whatsoever,

The truth of the one entails the truth of the other,
The truth of the one entails the falsity of the other,
The falsity of the one entails the truth of the other,
The falsity of the one entails the falsity of the other.

This must be the first part of logic. It is deductive logic, or (to name it by its principal result) syllogistic. At all times this part of logic has been recognized as a necessary preliminary to further investigation. Deductive and inductive or methodological logic have always been distinguished; and the former has generally been called by that name.

In order to trace these relations between propositions, it is necessary to dissect the propositions to a certain extent. There are different ways in which propositions can be dissected. Some of them conduce in no measure to the solution of the present problem, and will be eschewed by the pragmatist at this stage of the investigation. Such, for example, is that which makes the copula a distinct part of the proposition. It may be that there are different ways of useful dissection; but the common one, which alone has been sufficiently studied, may be described as follows:

Taking any proposition whatever, as

"Every priest marries some woman to some man,"

we notice that certain parts may be struck out so as to leave a blank form, in which, if the blanks are filled by proper names (of individual objects known to exist), there will be a complete proposition (however silly and false). Such blank forms are, for example:

* See vol. 5, Introduction.

Every priest marries some woman to ——,
—— marries —— to some man,
—— marries —— to ——.

It may be that there is some language in which the blanks in such forms cannot be filled with proper names so as to make perfect propositions, because the syntax may be different for sentences involving proper names. But it does not matter what the rules of grammar may be.

The last of the above blank forms is distinguished by containing no selective word such as some, every, any, or any expression equivalent in force to such a word. It may be called a Predicate (q.v. [358]) or $\dot{\rho}\widetilde{\eta}\mu\alpha$. Corresponding to every such predicate there is another, such that if all the blanks in the two be filled with the same set of proper names (of individuals known to exist), one of the two resulting propositions will be true, while the other is false; as

Chrysostom marries Helena to Constantine;
Chrysostom non-marries Helena to Constantine.

It is true that the latter is not good grammar; but that is not of the smallest consequence. Two such propositions are said to be contradictories, and two such predicates to be negatives of one another, or each to result from the negation of the other. Two propositions involving selective expressions may be contradictories; but in order to be so, each selective has to be changed from indicating *a suitable selection* to indicating *any selection that may be made*, or vice versa. Thus the two following propositions are contradictories:

Every priest marries some woman to every man;
Some priest non-marries every woman to some man.

It is very convenient to express the negative of a predicate by simply attaching a *non* to it. If we adopt that plan, *non-non-marries* must be considered as equivalent to *marries*. It so happens that both in Latin and in English this convention agrees with the usage of the language. There is probably but a small minority of languages of the globe in which this very artificial rule prevails. Of two contradictory propositions each is said to result from the *negation* of the other.

The relation of negation may be regarded as defined by the

principles of contradiction and excluded middle. See Laws of Thought [bk. III, ch. 4, §15]. That is an admissible, but not a necessary, point of view. Out of the conceptions of non-relative deductive logic, such as consequence, coexistence or composition, aggregation, incompossibility, negation, etc., it is only necessary to select two, and almost any two at that, to have the material needed for defining the others. What ones are to be selected is a question the decision of which transcends the function of this branch of logic. Hence the indisputable merit of Mrs. Franklin's eight copula-signs, which are exhibited as of coördinate formal rank.* But, so regarded, they are not properly copulas or assertions of the relation between the several individual subjects and the predicate, but mere signs of the logical relations between different components of the predicate. The logical doctrine connected with those signs is of considerable importance to the theory of pragmatism.

. . . *Negant* or *negative negation* is the negation effected by attaching the negative particle to the copula in the usual Latin idiom, "Socrates non est stultus," in contradistinction to *infinite* (ἀόριστη), or *infinitant, negation*, which is effected by attaching the negative particle to the predicate, "Socrates est non stultus."

Kant revived this distinction in order to get a triad to make out the symmetry of his table of categories; and it has ever since been one of the deepest and dearest studies of German logicians. No idea is more essentially dualistic, and distinctly not triadic, than negation. *Not-*A = *other than* A = *a second thing to* A. Language preserves many traces of this. *Dubius* is between *two* alternatives, yea and nay.

380. (2) In the metaphysical sense, negation is the mere absence of a character or relation that is regarded as positive. It is distinguished from privation in not implying anything further.

Spinoza's celebrated saying, of which the Schellings have made so much, "omnis determinatio est negatio," has at least this foundation, that *determinatio* to one alternative excludes us from another. The same great truth is impressed upon youth in the utterance: "You cannot eat your cake and have it too."

* See *Dictionary of Philosophy and Psychology*, vol. 2, p. 369ff.

§15. LIMITATIVE*

381. (1) Applied to a third quality of judgments, additional to affirmative and negative. The idea of such a third quality originated among the Romans from the difference between "homo non est bonus" and "homo est non bonus," the latter being the limitative. . . .

It is one of the numerous cases in which accidents of language have affected accepted logical forms without any good reason. Boethius† and others applied the infinitation to the subject also, which DeMorgan‡ has shown makes a valuable addition to logic. Wolff,§ however, limited the modification to the predicate, without showing any serious reason for such application. Kant adopted it because it rounded out his triad of categories of quality. His defence, as reported by Jäsche, is that the negative excludes the subject from the sphere of the predicate, while the *unendliche*, limitative, or infinite judgment puts it into the infinite sphere outside the predicate. It is to be remarked that Kant regards a positive mark as differing *per se* from a negative one, and, in particular, as having a far narrower extension. Like most of the old logicians, he virtually limited the universe of marks to such as arrest our attention. If that had been explicitly and consistently done, it would have constituted an interesting particular logic, in which there would be a material and not merely formal difference between affirmative and negative facts. It is probable that Kant also understood the affirmative proposition to assert the existence of its subject, while the negative did not do so; so that "Some phoenixes do not rise from their ashes" would be true, and "All phoenixes do rise from their ashes" would be false. The limitative judgment would agree with the affirmative in this respect. This was probably his meaning, and he did not observe that his limitative judgment, "The human soul is immortal" (*nichtsterblich*), may be construed as equivalent to the conjunctive judgment, "The human soul is not mortal, and it is the human soul." No doubt Kant would

* *Ibid.*, vol. 2, pp. 6–7.
† See Prantl, *op. cit.*, I, 693.
‡ See e.g., *Formal Logic*, p. 37ff.
§ But cf. Wolff's *Logica*, §208ff.

have seen a world of difference between these two assertions. In that case he should have adopted a fourth quality, "The human soul is not immortal."

§16. MODALITY*

382. There is no agreement among logicians as to what modality consists in; but it is the logical qualification of a proposition or its copula, or the corresponding qualification of a fact or its form, in the ways expressed by the modes *possibile*, *impossibile*, *contingens*, *necessarium*.

Any qualification of a predication is a mode; and Hamilton says (*Lects. on Logic*, xiv) that "all logicians" call any proposition affected by a mode a modal proposition. This, however, is going much too far; for not only has the term usually been restricted in practice, from the age of Abelard, when it first appeared,† until now, to propositions qualified by the four modes "possible," "impossible," "necessary," and "contingent," with only occasional extension to any others, but positive testimonies to that effect might be cited in abundance.

The simplest account of modality is the scholastic, according to which the necessary (or impossible) proposition is a sort of universal proposition; the possible (or contingent, in the sense of not necessary) proposition, a sort of particular proposition. That is, to assert "*A* must be true" is to assert not only that *A* is true, but that *all* propositions analogous to *A* are true; and to assert "*A* may be true" is to assert only that *some* proposition analogous to *A* is true. If it be asked what is here meant by analogous propositions, the answer is — all those of a certain class which the conveniences of reasoning establish. Or we may say the propositions analogous to *A* are all those propositions which in some conceivable state of ignorance would be indistinguishable from *A*. Error is to be put out of the question; only ignorance is to be considered. This ignorance will consist in its subject being unable to reject certain potentially hypothetical states of the universe, each absolutely determinate in every respect, but all of which are, in fact, false. The aggregate of these unrejected falsities constitute the "range of possibility," or better, "of ignorance."

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 89–93.

† See Prantl, *op. cit.*, II, 158f.

Were there no ignorance, this aggregate would be reduced to zero. The state of knowledge supposed is, in necessary propositions, usually fictitious, in possible propositions more often the actual state of the speaker. The necessary proposition asserts that, in the assumed state of knowledge, there is no case in the whole range of ignorance in which the proposition is false. In this sense it may be said that an impossibility underlies every necessity. The possible proposition asserts that there is a case in which it is true.

Various subtleties are encountered in the study of modality. Thus, when the thinker's own state of knowledge is the one whose range of ignorance is in question, the judgments "*A* is true" and "*A* must be true," are not logically equivalent, the latter asserting a fact which the former does not assert, although the fact of its assertion affords direct and conclusive evidence of its truth. The two are analogous to "*A* is true" and "*A* is true, and I say so"; which are readily shown not to be logically equivalent by denying each, when we get "*A* is false" and "If *A* is true, I do not say so."

In the necessary particular proposition and the possible universal proposition there is sometimes a distinction between the "composite" and "divided" senses. "Some *S* must be *P*," taken in the composite sense, means that there is no case, in the whole range of ignorance, where some *S or other* is not *P;* but taken in the divided sense, it means that there is some *S* which same *S* remains *P* throughout the whole range of ignorance. So "Whatever *S* there may be may be *P*," taken in the composite sense, means that there is, in the range of ignorance, some hypothetic state of things (or it may be the unidentifiable true state, though this can hardly be the only such case) in which there either is no *S*, or every *S* there is is *P*; while in the divided sense, it means that there is no *S* at all in any hypothetic state but what in some hypothetic state or other is *P*. When there is any such distinction, the divided sense asserts more than the composite in necessary particular propositions, and less in possible universal. But in most cases the individuals do not remain identifiable throughout the range of possibility, when the distinction falls to the ground. It never applies to necessary universal propositions or to possible particular propositions.

383. Some logicians say that "*S* may be *P*" is not a proposition at all, for it asserts nothing. But if it asserted nothing, no state of facts could falsify it, and consequently the denial of it would be absurd. Now let *S* be "some self-contradictory proposition," and let *P* be "true." Then the possible proposition is "Some self-contradictory proposition may be true," and its denial is "No self-contradictory proposition can be true," which can hardly be pronounced absurd. It is true that those logicians usually take the form "*S* may be *P*" in the copulative sense "*S* may be *P*, and *S* may not be *P*," but this only makes it assert *more*, not *less*. The possible proposition, then, is a proposition. It not only must be admitted among logical forms, if they are to be adequate to represent all the facts of logic, but it plays a particularly important part in the theory of science. See Scientific Method, [vol. 7]. At the same time, according to the view of modality now under consideration, necessary and possible propositions are equipollent with certain assertory propositions; so that they do not differ from assertory propositions as universal and particular propositions differ from one another, but rather somewhat as hypothetical (i.e., conditional, copulative, and disjunctive), categorical, and relative propositions differ from one another — perhaps not quite so much.

According to this view, logically necessary and possible propositions relate to what might be known, without any knowledge whatever of the universe of discourse, but only with a perfectly distinct understanding of the meanings of words; geometrically necessary and possible propositions, to what a knowledge of the properties of space does or does not exclude; physical necessity, to what a knowledge of certain principles of physics does or does not exclude, etc. But when we say that of two collections one must be correspondentially greater than the other, but each cannot be correspondentially greater than the other, it has not been shown how this kind of necessity can be explained on the above principles.

384. The earliest theory of modality is Aristotle's, whose philosophy, indeed, consists mainly in a theory of modality. The student of Aristotle usually begins with the Categories; and the first thing that strikes him is the author's unconsciousness of any distinction between grammar and metaphysics,

between modes of signifying and modes of being. When he comes to the *metaphysical* books, he finds that this is not so much an oversight as an assumed axiom; and that the whole philosophy regards the existing universe as a performance which has taken its rise from an antecedent ability. It is only in special cases that Aristotle distinguishes between a possibility and an ability, between a necessity and a constraint. In this, he is perhaps nearer the truth than the system of equipollencies set forth above.

385. Kant seems to have been the first to throw any light upon the subject. To the old distinction between logical and real possibility and necessity he applied two new pairs of terms, analytic and synthetic, and subjective and objective. The following definitions (where every word is studied) certainly advanced the subject greatly:

"1. Was mit den formalen Bedingungen der Erfahrung (der Anschauung und den Begriffen nach) übereinkommt, ist *möglich.*

"2. Was mit den materialen Bedingungen der Erfahrung (der Empfindung) zusammenhängt, ist *wirklich.*

"3. Dessen Zusammenhang mit dem Wirklichen nach allgemeinen Bedingungen der Erfahrung bestimmt ist, ist (existirt) *nothwendig*" (*Krit. d. reinen Vernunft*, 1st ed., 219).

Kant holds that all the general metaphysical conceptions applicable to experience are capable of being represented as in a diagram, by means of the image of time. Such diagrams he calls "schemata." The schema of the possible he makes to be the figure of anything at any instant. The schema of necessity is the figure of anything lasting through all time (*ibid.*, 144, 145). He further states (*ibid.*, 74, footnote; Jäsche's *Logik*, Einl. ix, and elsewhere) that the possible proposition is merely conceived but not judged, and is a work of the apprehension (*Verstand*); that the assertory proposition is judged, and is, so far, a work of the judgment; and that the necessary proposition is represented as determined by law, and is thus the work of the reason (*Vernunft*). He maintains that his deduction of the categories shows that, and how, the conceptions originally applicable to propositions can be extended to modes of being — constitutively, to being having

reference to possible experience; regulatively, to being beyond the possibility of experience.

386. Hegel considers the syllogism to be the fundamental form of real being. He does not, however, undertake to work over, in the light of this idea, in any fundamental way, what is ordinarily called logic, but which, from his point of view, becomes merely subjective logic. He simply accepts Kant's table of functions of judgment, which is one of the most ill-considered performances in the whole history of philosophy. Consequently, what Hegel says upon this subject must not be considered as necessarily representing the legitimate outcome of his general position. His followers have been incompetent to do more. Rosenkranz (*Wissenschaft d. logischen Idee**) makes modality to represent the superseding of the form of the judgment and to be the preparation for that of the syllogism. In the *Encyclopädie*, Hegel's last statement, §§178–80, we are given to understand that the judgment of the *Begriff* has for its contents the totality (or, say, conformity to an ideal). In the first instance, the subject is singular, and the predicate is the reflection of the particular object upon the universal. That is, this or that object forced upon us by experience is judged to conform to something in the realm of ideas. But when this is doubted, since the subject does not, in itself, involve any such reference to the ideal world, we have the "possible" judgment, or judgment of doubt. But when the subject is referred to its genus, we get the apodictic judgment. But Hegel had already developed the ideas of possibility and necessity in the objective logic as categories of *Wesen*. In the *Encyclopädie* the development is somewhat as follows: *Wirklichkeit* is that whose mode of being consists in self-manifestation. As identity in general (the identity of *Sein* and *Existenz*) it is, in the first instance, possibility. That is to say, apparently, bare possibility, any fancy projected and regarded in the aspect of a fact. It is possible, for example, that the present Sultan may become the next Pope. But in the second movement arise the conceptions of the *Zufällig*, *Ausserlichkeit*, and "condition." The *Zufällig* is that which is recognized as merely possible: "*A* may be, but *A* may not be"; but it is also described by Hegel as that which has the *Grund*, or antecedent

* Bd. II, S. 127.

of its being, in something other than itself. The *Aeusserlichkeit* seems to be the having a being outside the ground of its being — an idea assimilated to caprice. That which such *Aeusserlichkeit* supposes outside of itself, as the antecedent of its being, is the presupposed condition. The third movement gives, in the first instance, "real possibility." In this we find the conceptions of "fact" (*Sache*), "activity" (*Thätigkeit*), and "necessity."

387. Lotze and Trendelenburg represent the first struggles of German thought to rise from Hegelianism. The most remarkable characteristic of Lotze's thought is, that he not only sees no urgency for unity of conception in philosophy, but holds that such unity would inevitably involve a falsity.* He represents a judgment as a means of apprehending becoming, in opposition to the concept, which apprehends being; but he says that the business of the judgment is to supply the cement for building up concepts. Accordingly, he has no doctrine of modality as a whole, but merely considers three cases, between which he traces no relation. Necessity may arise either out of the universal analytic judgment, the conditional judgment, or the disjunctive judgment. By the "judgment" is meant the meaning of a proposition. Lotze finds that the meaning of the analytical judgment is illogical, since it identifies contraries. However, the meaning of this meaning is justified by its not meaning to mean that the terms are identical, but only that the objects denoted by those terms are identical. The analytic proposition is, therefore, admissible, because it is practically meant to mean a particular proposition, that is, one in which the predicate is asserted of all the particulars. And the justification of the proposition, whose use was to be to connect elements of terms, is that, meant not as it is meant, but as it is meant to be meant, these elements are identical and do not need to be connected. In this way Lotze vindicates the necessity of the analytical categorical proposition. Coming next to conditionals, by thought of the same order, he finds that, assuming that the universe of real, intelligible objects is "coherent," we may be justified in asserting that the introduction of a condition X into a subject S gives rise to a predicate P as an analytical necessity; and for this purpose, when it is once

* Cf. *Logik* I, 1, §§33–35.

accomplished, it does not matter whether the ladder of the assumption of coherence remains or is taken away. Lotze treats the disjunctive proposition last, as if it were of a higher order, following Hegel in this respect. But what was excusable for Hegel is less so for Lotze, since he himself had signalized the significance of impersonal propositions, such as "it rains," "it thunders," "it lightens," whose only subject is the universe. Now, if there is any difference between "If it lightens, it thunders," and "Either it does not lighten or it thunders," it is that the latter considers the actual state of things alone, and the former a whole range of other possibilities. However, Lotze considers last the propositional form "S is P_1 or P_2 or P_3." Properly, this is not a disjunctive proposition, but only a proposition with a disjunctive predicate. Lotze considers it a peculiar form, because it cannot be represented by an Euler's diagram, which is simply a blunder. The necessity to which it gives rise must, therefore, either be the same as the conditional necessity, or else differ from it merely by greater simplicity. For other sound objections to Lotze's theory see Lange, *Logische Studien*, ii.

388. Trendelenburg (*Logische Untersuch.*, xiii) maintains that possibility and necessity can only be defined in terms of the antecedent (*Grund*), though he might, perhaps, object to the translation of *Grund* by so purely formal a word as "antecedent," notwithstanding its harmony with Aristotle. If all conditions are recognized, and the fact is understood from its entire *Grund*, so that thought quite permeates being — a sort of phrase which Trendelenburg always seeks — there is "necessity." If, on the other hand, only some conditions are recognized, but what is wanting in *Grund* is made up in thought, there is "possibility." In itself, an egg is nothing but an egg, but for thought it may become a bird. Trendelenburg will, therefore, neither admit, with Kant, that modality is originally a mere question of the attitude of the mind, nor with Hegel, whom he criticizes acutely, that it is originally objective.

389. Sigwart, who holds that logical questions must ultimately be decided by immediate feeling, and that the usages of the German language are the best evidence of what that feeling is, denies that the possible proposition is a proposition

at all, because it asserts nothing.* He forgets that if a proposition asserts nothing, the denial of it must be absurd, since it must exclude every possibility. Now, the denial of "I do not know but that *A* may be true" is "I know *A* is not true," which is hardly absurd. Sigwart, it is true, in accordance with usages of speech, takes "*A* may be true" in what the old logicians called the *sensus usualis*, that is, for the copulative proposition "A may be true, and further *A* may be not true." But this does not make it assert *less*, but *more*, than the technical form. In regard to the necessary proposition, Sigwart, following his guide, the usages of speech, finds that, "*A* must be true" asserts less than "*A* is true," so that from the latter the former follows, but not at all the latter from the former. This may be true for the usages of German speech, just as such phrases as "beyond every shadow of doubt," "out of all question," and the like, in our vernacular commonly betray the fact that there is somebody who not only doubts and questions, but flatly denies, the proposition to which they are attached. Bradley† accepts the sensational discovery of Sigwart.

390. Lange (loc. cit.) thinks the matter is put in the clearest light by the logical diagrams usually attributed to Euler, but really going back to Vives. "We, therefore, here again see," he says, "how spatial intuition, just as in geometry, verifies (*begründet*) a priority and necessity."

* *Logik*, §31.

† *Logic* (1883), ch. 7.

CHAPTER 5

TERMS[*E]

§1. THAT THESE CONCEPTIONS ARE NOT SO MODERN AS HAS BEEN REPRESENTED

391. The historical account usually given of comprehension and extension is this, "that the distinction, though taken in general terms by Aristotle,[1] and explicitly announced with scientific precision by one, at least, of his Greek commentators, had escaped the marvellous acuteness of the schoolmen, and remained totally overlooked and forgotten till the publication of the Port Royal Logic."[2] I would offer the following considerations to show that this interpretation of history is not exactly true. In the first place, it is said that a distinction was taken between these attributes, as though they were previously confounded. Now there is not the least evidence of this. A German logician,[3] has, indeed, by a subtle misconception, considered extension as a species of comprehension, but, to a mind beginning to reflect, no notions seem more unlike. The mental achievement has been the bringing of them into relation to one another, and the conception of them as factors of the import of a term, and not the separation of them. In the second place, it is correctly said that the doctrine taught by the Port Royalists is substantially contained in the work of a Greek commentator. That work is no other than Porphyry's

* §§1–6 are "Upon Logical Comprehension and Extension," *Proceedings of the American Academy of Arts and Sciences*, vol. 7, November 13, 1867, pp. 416–32, with additions and corrections c. 1870 and 1893; intended as Essay III of *Search for a Method* and as ch. 15 of the *Grand Logic*.

[1] Aristotle remarks in several places that genera and differences may be regarded as parts of species and species as equally parts of genera, e.g., in the 5th *Meta.* (1023b 22). The commentator referred to is perhaps Alexander Aphrodiensis . . . — 1893.

[2] This is quoted from Baines [Baynes'] (Port Royal Logic, 2d ed. p. xxxiii), who says that he is indebted to Sir William Hamilton for the information.

[3] Lotze.— 1893.

Isagoge;[1] and therefore it would be most surprising if the doctrine had been totally overlooked by the schoolmen, for whether their acuteness was as marvellous as Hamilton taught or not, they certainly studied the commentary in question as diligently as they did the Bible.[2] It would seem, indeed, that the tree of Porphyry involves the whole doctrine of extension and comprehension except the names. Nor were the scholastics without names for these quantities. The *partes subjectives* and *partes essentiales* are frequently opposed; and several other synonyms are mentioned by the Conimbricenses. It is admitted that Porphyry fully enunciates the doctrine; it must also be admitted that the passage in question is fully dealt with and correctly explained by the mediaeval commentators. The most that can be said, therefore, is that the doctrine of extension and comprehension was not a promiment one in the mediaeval logic.[3] [What the Port Royalists and later, still more decidedly, the Kantians, preached was the equivalence of the logical character of all essential predicates. They fused genera and differences while Porphyry, after Aristotle, is bent upon the discrimination of different kinds of predicates — 1893.]

392. A like degree of historical error is commonly committed in reference to another point which will come to be

[1] Porphyry appears to refer to the doctrine as an ancient one. [Ch. 1.]

[2] They were equally diligent in the study of Boëthius, who says: (*Opera*, p. 645) "Genus in divisione totum est, in diffinitione pars."

[3] The author of *De Generibus et Speciebus* opposes the *integral* and *diffinitive* wholes. John of Salisbury refers to the distinction of comprehension and extension, as something "quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa *significant*, et aliud esse quod *nominant*. Nominantur singularia, sed universalia significantur." (*Metalogicus*, lib. 2, cap. 20. Ed. of 1620, p. 111.) [By appellativa he means adjectives and the like. — 1893.]

Vincentius Bellovacensis (*Speculum Doctrinale*, Lib. III, cap. xi) has the following: "Si vero quæritur utrum hoc universale 'homo' sit in quolibet homine secundum se totum an secundum partem, dicendum est quod secundum se totum, id est secundum quamlibet sui partem diffinitivam. . . . , non autem secundum quamlibet partem subjectivam. . . ." William of Auvergne (Prantl's *Geschichte*, vol. 3, p. 77) speaks of "totalitatem istam, quæ est ex partibus rationis seu diffinitionis, et hae partes sunt genus et differentiæ; alio modo partes speciei individua sunt, quoniam ipsam speciem, cum de eis prædicatur, sibi invicem quodammodo partiunter." [See also Duns Scotus, *Opera* I, 137.] If we were to go to later authors, the examples would be endless. See any commentary in *Phys. Lib. I.*

treated of in this paper, allied, at least, as it is most intimately, with the subject of comprehension and extension, inasmuch as it also is founded on a conception of a term as a whole composed of parts — I mean the distinction of clear and distinct. Hamilton tells us: "We owe the discrimination to the acuteness of the great Leibniz. By the Cartesians the distinction had not been taken; though the authors of the Port Royal Logic came so near that we may well marvel how they failed explicitly to enounce it." (*Lectures on Logic;* Lecture IX.) Now, in fact, all that the Port Royalists say about this matter[1] is copied from Descartes,[2] and their variations from his wording serve only to confuse what in him is tolerably distinct. As for Leibniz, he himself expressly avows that the distinction drawn by Descartes is the same as his own.[3] Nevertheless, it is very much more clear with Leibniz than with Descartes. A philosophical distinction emerges gradually into consciousness; there is no moment in history before which it is altogether unrecognized, and after which it is perfectly luminous. Before Descartes, the distinction of confused and distinct had been thoroughly developed, but the difference between distinctness and clearness is uniformly overlooked. Scotus distinguishes between conceiving confusedly and conceiving the confused, and since any obscure concept necessarily includes more than its proper object, there is always in what is obscurely conceived a conception of something confused; but the schoolmen came no nearer than this to the distinction of Descartes and Leibniz.

§2. OF THE DIFFERENT TERMS APPLIED TO THE QUANTITIES OF EXTENSION AND COMPREHENSION

393. Extension and comprehension are the terms employed by the Port Royalists. Owing to the influence of Hamilton, *intension* is now frequently used for comprehension;[4] but it is liable to be confounded with intensity, and therefore is an objectionable word. It is derived from the use of cognate words by Cajetan and other early writers. *External* and

[1] Part I, ch. 9.

[2] *Principia*, Part I, §45 *et seq.*

[3] Eighth [ninth?] Letter to Burnet. Gerhardt's ed., vol. 3, p. 224.

[4] But *intension* was in use among the Leibnizians in the same sense.— 1893.

internal quantity are the terms used by many early Kantians. *Scope* and *force* are proposed by DeMorgan. *Scope* in ordinary language expresses extension, but *force* does not so much express comprehension as the power of creating a lively representation in the mind of the person to whom a word or speech is addressed. Mr. J. S. Mill has introduced the useful verbs *denote* and *connote*, which have become very familiar. It has been, indeed, the opinion of all* students of the logic of the fourteenth, fifteenth, and sixteenth centuries, that *connotation* was in those ages used exclusively for the reference to a second significate, that is (nearly) for the reference of a relative term (such as *father*, *brighter*, etc.) to the correlate of the object which it primarily denotes, and was never taken in Mill's sense of the reference of a term to the essential characters implied in its definition.[1] Mr. Mill has, however, considered himself entitled to deny this upon his simple dictum,† without the citation of a single passage from any writer of that time.[2] After explaining the sense in which he takes the term *connote*, Mill says: "The schoolmen, to whom we are indebted for the greater part of our logical language, gave us this also, and in this very sense. For though some of their general expressions countenance the use of the word in the more extensive and vague acceptation in which it is taken by Mr. [James] Mill, yet when they had to define it specifically as a technical term, and to fix its meaning as such, with that admirable precision which always characterized their definitions, they clearly explained that nothing was said to be connoted except *forms*, which word may generally, in their writings, be understood as synonymous with *attributes*."‡ As scholasticism is usually said to come to an end with Occam, this conveys the idea that *connote* was commonly employed by earlier writers. Yet the celebrated Prantl[3] considers it con-

* Originally, "the best."

[1] Cf. Morin, *Dictionnaire*, Tome I, col. 684 [685?]; Chauvin, *Lexicon*, both editions; Eustachius, *Summa*, Part I, Tr. I, qu. 6. [Aquinas, sentent 1, d. 8, q. 1, art. 1.]

† Originally, "authority."

[2] And such is the humility of his disciples, that not one has dared utter protest against this tax upon his credulity.— 1893.

‡ *Logic*, bk. I, ch. 2, §5, note.

[3] If I understand him, he expresses himself in his usual enigmatical style.—1870.

clusive proof that a passage in Occam's *Summa* is spurious, that *connotative* is there spoken of as a term in frequent use;[1] and remarks upon a passage of Scotus in which *connotatum* is found, that this conception is here met with for the first time.[2] The term occurs, however, in Alexander of Hales,[3] who makes *nomen connotans* the equivalent of *appellatio relativa*, and takes the relation itself as the object of *connotare*, speaking of creator as connoting the relation of creator to creature.[4] Occam's *Summa*[5] contains a chapter devoted to the distinction of absolute and connotative names. The whole deserves to be read, but I have only space to quote the following: "Nomen autem connotativum est illud quod significat aliquid primario et aliquid secundario; et tale nomen proprie habet diffinitionem exprimentem quid nominis et frequenter oportet ponere aliquid illius diffinitionis in recto et aliud in obliquo; sicut est de hoc nomine album, nam habet diffinitionem exprimentem quid nominis in qua una dictio ponitur in recto et alia in obliquo. Unde si queratur quid significat hoc nomen album, dices quod idem quod illa oratio tota 'aliquid informatum albedine' vel 'aliquid habens albedinem' et patet quod una pars orationis istius ponitur in recto et alia in obliquo. . . . Huiusmodi autem nomina connotativa sunt omnia nomina concreta primo modo dicta, et hoc *quia talia concreta significant unum in recto et aliud in obliquo*, hoc est dictu, in diffinitione exprimente quid nominis debet poni unus rectus significans unam rem et alius obliquus significans aliam rem, sicut patet de omnibus talibus, iustus, albus, animatus, et sic de aliis. Huiusmodi etiam nomina sunt omnia nomina relatiua, quia semper in eorum diffinitionibus ponuntur diversa idem diuersis modis vel diuersa significantia, sicut patet de hoc nomine simile. Mere autem absoluta sunt illa quae non significant aliquid principaliter et aliud vel idem secundario, sed quicquid significatur per tale nomen aeque primo significatur sicut patet de hoc nomine

[1] Prantl, *Geschichte*, vol. 3, p. 364.

[2] *Ibid.*, p. 134n. Scotus also uses the term. *Quodlib.* question 13, article 4.

[3] *Summa Theologica*, Part I, question 53. [This work was certainly written before 1280. Roger Bacon refers to it while saying that Albertus is still alive. — 1893.]

[4] The doctrine of *connotare* is part of the doctrine of *appellatio*, for which see Petrus Hispanus. — 1893.

[5] Part I, ch. 10. (Ed. of 1488, fol. 6, c.)

animal."* Eckius, in his comment on Petrus Hispanus, has also some extended remarks on the signification of the term *connote*, which agree in the main with those just quoted.[1] Mr. Mill's historical statement[2] cannot, therefore, be admitted.

394. Sir William Hamilton has borrowed from certain late Greek writers the terms *breadth* and *depth*, for extension and comprehension respectively.[3] These terms have great merits. They are brief; they are suited to go together; and they are very familiar. Thus, "wide" learning is, in ordinary parlance, learning of many things; "deep" learning, much knowledge of some things. I shall, therefore, give the preference to these terms. Extension is also called *sphere* and *circuit;* and comprehension, *matter* and *content.*

§3. OF THE DIFFERENT SENSES IN WHICH THE TERMS EXTENSION AND COMPREHENSION HAVE BEEN ACCEPTED

395. The terms *extension* and *comprehension*, and their synonyms, are taken in different senses by different writers. This is partly owing to the fact that while most writers speak only of the extension and comprehension of concepts, others apply these terms equally to concepts and judgments (Rösling)†, others to any mental representation (Überweg‡ and many French writers), others to cognition generally (Baumgarten§), others to "terms" (Fowler,¶ Spalding||), others to names (Shedden°); others to words (McGregor**), others to "mean-

* The last sentence in this quotation appears, in the original, nearly half a page before the rest of the quotation.

[1] Fol. 23 d. See also Tartareti Expositio in Petr. Hisp. towards the end. Ed. of 1509, fol. 91, b.

[2] . . . is simply rubbish. Civilization in England does not seem as yet to have reached the stage in which men feel shame in making positive assertions based on exceptional ignorance.— 1893.

[3] *Logic*, p. 100 [*i.e.*, Lect. viii, ¶24]. In the *Summa Logices* attributed to Aquinas, we read: "Omnis forma sub se habens multa, idest, quae universaliter sumitur, habet quamdam *latitudinem;* nam invenitur in pluribus, et dicitur de pluribus." (Tr. 1, c. 3.)

† *Logik* (1826), I, II; i, 1, iii, and ii, 1, iv.

‡ *System der Logik*, (1857), §§50, 53.

§ *Acroasis Logica*, ed. 2 (1773), §24.

¶ *The Elements of Deductive Logic* (1867), Pt. I, ch. 2.

|| *An Introduction to Logical Science* (1857), §§7, 30, 31.

° *Elements of Logic* (1864), p. 10.

** *A System of Logic* (1862), p. 191.

ings" (Jevons),* while one writer speaks only of the extension of *classes* and the comprehension of *attributes* (De Morgan in his Syllabus [§131]).

396. Comprehension is defined by the Port Royalists as "those attributes which it [an idea] involves in itself, and which cannot be taken away from it without destroying it."†

It will be remembered that the *marks* of a term are divided by logicians first into the necessary and the accidental, and that then the necessary marks are subdivided into such as are strictly essential, that is, contained in the definition, and such as are called proper. Thus, it is an essential mark of a triangle to have three sides; it is a proper mark to have its three angles equal to two right angles; and it is an accidental mark to be treated of by Euclid. The definition of the Port Royalists, therefore, makes comprehension include all necessary marks, whether essential or proper.

397. The Port Royalists attribute comprehension immediately to any ideas. Very many logicians attribute it immediately only to concepts. Now a concept, as defined by them, is strictly only the essence of an idea; they ought therefore to include in the comprehension only the essential marks of a term. These logicians, however, abstract so entirely from the real world, that it is difficult to see why these essential marks are not at the same time all the marks of the object as they suppose it.

398. There can, I think, be no doubt that such writers as Gerlach‡ and Sigwart§ make comprehension include all marks, necessary or accidental, which are universally predicable of the object of the concept.

Again, most German writers regard the comprehension as a sum either of concepts (Drobisch,¶ Bachmann,|| etc.) or of elements of intuition (Trendelenburg).° But many English writers regard it as the sum of real external attributes (Shedden,**

* *The Principles of Science* (1874), bk. I, ch. 2, p. 31.

† Baynes' translation, I, vi.

‡ *Grundress der Logik*, 2te Auf. (1822), §29.

§ *Logik*, §42.

¶ *Neue Darstellung der Logik*, 2te Auf. (1851), §23.

|| *System der Logik* (1828), Erster Theil, §48.

° *Logische Untersuchung*, 2te Auf. (1862), xv, 4.

** *Elements of Logic* (1864), pp. 10, 39ff.

Spalding,* Devey,† De Morgan,‡ Jevons,§ McGregor,¶ Fowler).|| According to most writers, comprehension consists of the (necessary) attributes *thought* as common to the objects. Shedden defines it as consisting of all the attributes common to the things denoted.

Again, most logicians consider as marks only such as are virtually[1] predicated; a few, perhaps, only such as are actually thought, and still fewer include those which are habitually thought. Here and there is found an author who makes comprehension include all true attributes, whether thought or not.

There is also a difference in the mode of reckoning up the marks. Most writers count all distinguishable marks, while a few consider coextensive marks as the same.

399. In the use of the term "extension" the want of a definite convention is still more marked. The Port Royalists define it as "those *subjects* to which the idea applies."° It would appear, therefore, that it might include mere fictions.

Others limit the term to *real* species, and at the same time extend it to single beings. This is the case with Watts,** and also with Friedrich Fischer.††

Others are most emphatic in declaring that they mean by it *things*, and not species, real or imaginary. This is the case with Bachmann,‡‡ Esser,§§ and Schulze.¶¶

* *An Introduction to Logical Science* (1857), §31.

† *Logic, or the Science of Inference* (1854), p. 42.

‡ Cf. *Syllabus*, §131.

§ *The Principles of Science* (1874), Bk. 1, ch. 2.

¶ *A System of Logic* (1862), p. 191.

|| *The Elements of Deductive Logic* (1867), Part I, ch. 2.

[1] I adopt the admirable distinction of Scotus between actual, habitual, and virtual cognition. [*Reportatei*, Ed. 1853, vol. 1, p. 147a. This distinction arose from mixed Aristotelean and Neoplatonic suggestions. Aristotle, as everybody knows, distinguished actual and potential thought. Alexander Aphrodisiensis distinguished material intellect (νοῦς ὑλικός), habitual intellect (νοῦς κατὰ ἕξιν) and *intellectus adeptus*. These two distinctions have little to do with one another. Still they were confounded by the Arabians, and the confused doctrine suggested to Scotus his brilliant and philosophical division. — 1893.]

° Baynes' translation I, vi.

** *Logick* (1725), Part I, ch. 3, §3.

†† *Lehrbuch der Logik* (1838), Drittes Kap., §37.

‡‡ *System der Logik* (1828), Erster Theil, §48.

§§ *System der Logik* 2te Auf. (1830), Erster Theil, §34.

¶¶ *Grundsätze der allgemeinen Logik* 5te Auf. (1831), §29.

Others make it include neither concepts nor things, but singular representations. This is the case with the strict Kantian. The following table exhibits this diversity:

Extension embraces

Individual representations, according to Kant,* E. Reinhold,† etc.
Representations, according to Fries,‡ Überweg,§ etc.
Real external things and species, according to Watts,¶ Shedden,|| etc.
Real external individual objects, according to Bachmann,° Devey,** etc.
Things, according to Schulze,†† Bowen,‡‡ etc.
Species, according to Drobisch,§§ De Morgan,¶¶ etc.
Objects (representations), according to Thomson,|| || etc.
Individuals, according to Mahan.°°
Concepts, according to Herbart,*** Vorländer,††† etc.
General terms, according to Spalding.‡‡‡
Psychical concepts, according to Strümpell.§§§
Variable marks, according to Ritter.¶¶¶

Again, logicians differ as to whether by extension they mean the concepts, species, things, or representations to which the term is habitually applied in the judgment, or all to which it is truly applicable. The latter position is held by Herbart, Kiesewetter, etc.; the former by Duncan, Spalding, Vorländer, Überweg, etc.

* Cf. *Logik*, her. v. G. B. Jäsche (1800), I, i, §§1–7.
† *Die Logik* (1827), S. 115.
‡ *System der Logik*, 3te Auf. (1837), §20.
§ *System der Logik* (1857), §53.
¶ *Logick* (1725), Part I, ch. 3, §3.
|| *Elements of Logic* (1864), pp. 39, 40.
° *System der Logik* (1828), Erster Theil, §48.
** *Logic, or the Science of Inference* (1854), p. 42.
†† *Grundsätze der allgemeinen Logik*, 5te Auf. (1831), §29.
‡‡ *A Treatise on Logic* (1864), p. 67.
§§ *Neue Darlellung der Logik*, 2te Auf. (1851), §23.
¶¶ *Formal Logic* (1847), p. 234.
|| || *Outline of the Laws of Thought*, 4 ed., pp. 99–102.
°° *Intellectual Philosophy*, 2d ed. (1847), ch. 7, 8.
*** *Lehrbuch zur Einleitung in die Philosophie* (1813), II, i, §40.
††† *Wissenschaft der Erkenntniss* (1847), II, i, 2, b.
‡‡‡ *An Introduction to Logical Science* (1857), §30.
§§§ *Entwurf der Logik* (1846), 4tes Kap.
¶¶¶ *Abriss der Philosophischen Logik* (1824), S.79.

Some logicians include only *actual* things, representations, etc., under extension (Bachmann, Fries, Herbart); others extend it to such as are merely possible (Esser, Ritter, Gerlach).

Finally, some few logicians speak of the two quantities as numerical, while most writers regard them as mere aggregates of diverse objects or marks.

[Dressler, following a hint from Beneke, distinguishes *real* and ideal extension and comprehension.— 1893.]

§4. DENIALS OF THE INVERSE PROPORTIONALITY OF THE TWO QUANTITIES, AND SUGGESTIONS OF A THIRD QUANTITY

400. Until lately the law of the inverse proportionality of extension and comprehension was universally admitted.[1] It is now questioned on various grounds.[2]

401. Drobisch* says that the comprehension varies arithmetically, while the extension varies geometrically. This is true, in one sense.

402. Lotze, after remarking that the only conception of a universal which we can have is the power of imagining singulars under it, urges that the possibility of determining a concept in a way corresponding to each particular under it is a mark of that concept, and that therefore the narrower concepts have as many marks as the wider ones. But, I reply,

[1] This law, algebraically stated, is that if a and b are logical terms so related that $a=bx$, then also $b=a+y$ and conversely. Numbers of German logicians are capable of denying this. — 1893.

[2] Hoppe reverses the law of Kant, and maintains that the wider the concept the greater its content. His idea, translated into Aristotelean phraseology, is this. He admits the second antepredicamental rule, that the differences of different genera are different (This, of itself, removes him widely from logicians for whom the distinction of comprehension and extension is the turning point of logic.) *Negro* is not a conception formed by the union of the two concepts *man* and *black*, but the peculiar differences of *negro* belong to negroes alone of all beings. This naturally carries him a step further, and he says the difference is of itself sufficient to constitute the pure concept, so that the genus is not an essential predicate. Thirdly, he finds that the characters of the narrower difference are less important (*wirkungsreich*) than those of the higher, and to have less important consequences is to have a smaller measure of predicates.— 1893.

* *Neue Darstellung der Logik*. 2te Auf. (1851), Anhang I.

these marks belong to the concept in its second intention, and are not common marks of those things to which it applies, and are therefore no part of the comprehension. They are, in fact, the very marks which constitute the extension. No one ever denied that extension is a mark of a concept; only it is a certain mark of second intention.

403. Vorländer's* objection is much more to the purpose. It is that, if from any determinate notion, as that of Napoleon, we abstract all marks, all determination, what remains is merely the conception *something*, which has no more extension than Napoleon. "Something" has an uncertain sphere, meaning either this thing or that or the other, but has no general extension, since it means one thing only. Thus, before a race, we can say that some horse will win, meaning this one, that one, or that one; but by some horse we mean but one, and it therefore has no more extension than would a term definitely indicating which — although this latter would be more determinate, that is, would have more comprehension. I am not aware that those who adhere to Kant's unmodified doctrine have succeeded in answering this objection.

404. Überweg has the following remarks:[1] "To the higher representation, since conformably to its definition it contains only the common elements of content of several lower representations, belongs in comparison to each of the lower a more limited content, but a wider circuit. The lower representation, on the contrary, has a richer content but narrower circuit. Yet by no means by every diminution or increase of a given content does the circuit increase or diminish, nor by every increase or diminution of a given circuit does the content diminish or increase." I am surprised that he does not explain himself further upon this point, which it is the principal object of this paper to develop.

405. De Morgan says:[2] "According to such statements as I have seen, 'man, residing in Europe, drawing breath north of the equator, seeing the sun rise before those in America,' would be a more intensively quantified notion than 'man residing in Europe'; but certainly not less *extensive*, for the third

* *Wissenschaft der Erkenntniss* (1847), S. 104–107.

[1] *System der Logik*, 2te Aufl., §54.

[2] *Formal Logic*, p. 234. His doctrine is different in the *Syllabus*.

and fourth elements of the notion must belong to those men to whom the first and second belong." Mr. De Morgan adopts the definitions of extension and comprehension given by the Port Royalists. According to those definitions, if the third and fourth elements necessarily belong to the notion to which the first and second belong, they are parts of the comprehension of that second notion which is composed of the first and second elements, and therefore the two notions are equal in comprehension; but if this is not the case, then the second notion can be predicated of subjects of which the first cannot, for example, of "man residing in Europe drawing breath south of the Equator"; for that there is really no such man will not affect the truth of the proposition, and therefore the second notion is more extensive than the first.

406. Two logicians only, as far as I remember, Archbishop Thomson[1] and Dr. W. D. Wilson,[2] while apparently admitting Kant's law, wish to establish a third quantity of concepts. Neither gentleman has defined his third quantity, nor has stated what its relations to the other two are. Thomson calls his Denomination. It seems to be the same as Extension regarded in a particular way. Dr. Wilson terms his new quantity Protension; it has something to do with time, and appears to be generally independent of the other two. It is plain, indeed, that as long as Kant's law holds, and as long as logical quantities can only be compared as being more or less and not directly measured, and as long as the different *kinds* of quantity cannot be compared at all, a third quantity must be directly proportional to one or other of the known quantities, and therefore must measure the same thing, or else must be independent of the other two, and be quite unconnected with them.

§5. THREE PRINCIPAL SENSES IN WHICH COMPREHENSION AND EXTENSION WILL BE TAKEN IN THIS PAPER*

407. I shall adopt Hamilton's terms, *breadth* and *depth*, for extension and comprehension respectively, and shall employ

[1] *Laws of Thought*, 4th ed., §§52, 80. [Cf. §54.]

[2] *Logic*, Part I, ch. 2, §5.

* This and the previous section were both numbered §4 in the original publication.

them in different senses, which I shall distinguish by different adjectives.

By the *informed breadth* of a term,[1] I shall mean all the real things of which it is predicable, with logical truth on the whole in a supposed state of information.[2] By the phrase "on the whole" I mean to indicate that all the information at hand must be taken into account, and that those things only of which there is on the whole reason to believe that a term is truly predicable are to be reckoned as part of its breadth.*

If T be a term which is predicable only of S′, S″, and S‴, then the S″s, the S‴s, and the S⁗s, will constitute the informed breadth of T. If at the same time, S′ and S″ are the subjects of which alone another term T′ can be predicated, and if it is not known that all S⁗s are either S′ or S″, then T is said to have a greater informed breadth than T′. If the S⁗s are known not to be all among the S″s and S‴s, this excess of breadth may be termed *certain*, and, if this is not known, it may be termed *doubtful*. If there are known to be S⁗s, not known to be S″s or S‴s, T is said to have a greater *actual* breadth than T′; but if no S⁗s are known except such are known to be S″s, and S‴s (though there may be others), T is said to have a greater *potential* breadth than T′. If T and T′ are conceptions in different minds, or in different states of the same mind, and it is known to the mind which conceives T that every S‴ is either S″ or S′, then T is said to be more *extensively distinct* than T′.[3]

[1] I restricted myself to *terms*, because at the time this chapter was first written (1867), I had not remarked that the whole doctrine of breadth and depth was equally applicable to *propositions* and to *arguments*. The breadth of a proposition is the aggregate of possible states of things in which it is true; the breadth of an argument is the aggregate of possible cases to which it applies. The depth of a proposition is the total of fact which it asserts of the state of things to which it is applied; the depth of an argument is the importance of the conclusions which it draws. In fact, every proposition and every argument can be regarded as a term.— 1893.

[2] It would seem needlessly to complicate the doctrine to introduce probabilities, and therefore it is understood that the information is *supposed to be* accepted absolutely.— 1893.

* Changed in 1870 and 1893 from "those things of which there is not . . . predicable are not . . ."

[3] For the distinction of extensive and comprehensive distinctness, see Scotus, i, dist. 2, qu. 3.

408. By the informed depth of a term, I mean all the real characters (in contradistinction to mere names) which can be predicated of it[1] (with logical truth, on the whole) in a supposed state of information, no character being counted twice over knowingly in the supposed state of information. The depth, like the breadth, may be certain or doubtful, actual or potential, and there is a comprehensive distinctness corresponding to extensive distinctness.

409. The informed breadth and depth suppose a state of information which lies somewhere between two imaginary extremes. These are, first, the state in which no fact would be known, but only the meaning of terms; and, second, the state in which the information would amount to an absolute intuition of all there is, so that the things we should know would be the very substances themselves, and the qualities we should know would be the very concrete forms themselves. This suggests two other sorts of breadth and depth corresponding to these two states of information, and which I shall term respectively the *essential*[2] and the *substantial* breadth and depth.

410. By the *essential depth* of a term, then, I mean the really conceivable qualities predicated of it in its definition.

411. The defined term will not perhaps be applicable to any real objects whatever. Let, for example, the definition of the term T be this,

Any T is both P′ and P″ and P‴,

then this sums up its whole meaning; and, as it may not be known that there is any such thing as P′, the meaning of T does not imply that it exists. On the other hand, we know that neither P′, P″, nor P‴ is coextensive with the whole sphere of being. For they are determinate qualities, and it is the very meaning of being that it is indeterminate, that is, is more extensive than any determinate term. In fact, P′, for example, is a real notion which we never could have except by means of its contrast to something else. Hence we must know that

[1] That is, of whatever things it is applicable to.

[2] The *essence* of a thing is the idea of it, the law of its being, which makes it the kind of thing it is, and which should be expressed in the definition of that kind.— 1893.

Whatever is not-P′ is not-T,
Whatever is not-P″ is not-T,
and Whatever is not-P‴ is not-T.

412. Thus, if we define the *essential breadth* of a term as those real things of which, according to its very meaning, a term is predicable, not-T has an essential breadth. We may therefore divide all terms into two classes, the essentially affirmative, or positive, and the essentially negative; of which the former have essential depth, but no essential breadth, and the latter essential breadth, but no essential depth.[1] It must be noted, however, that this division is not the same as the similar one which language makes. For example, *being*, according to this, is an essentially negative term, inasmuch as it means that which can be predicated of whatever you please, and so has an essential breadth; while *nothing* is an essentially positive term, inasmuch as it means that of which you are at liberty to predicate what you please, and therefore has an essential depth. The essential subjects of being cannot be enumerated, nor the essential predicates of nothing.

413. In essential breadth or depth, no two terms can be equal; for, were that the case, the two terms would have the same meaning, and therefore, for logical purposes, would be the same term. Two terms may have unknown relations in these quantities, on account of one or other of them not being distinctly conceived.

414. *Substantial breadth* is the aggregate of real substances of which alone a term is predicable with absolute truth. *Substantial depth* is the real concrete form which belongs to everything of which a term is predicable with absolute truth.

415. General terms denote several things. Each of these things has in itself no qualities, but only a certain concrete form which belongs to itself alone. This was one of the points brought out in the controversy in reference to the nature of universals.[2] As Sir William Hamilton [*Discussions*, Amer. Ed. p. 630] says, not even the humanity of Leibniz belongs to Newton, but a different humanity. It is only by abstraction,

[1] Negative terms are called by the logicians *infinite* (or, recently, *infinitated*). This is a translation of Aristotle's ἀόριστος [*De Interpr.* 3, 16b, 14], which really means "without definition" ὁρισμός.— 1893.

[2] See, for example, *De Generibus et Speciebus*, p. 548.

by an oversight, that two things can be said to have common characters. Hence, a general term has no *substantial depth.* On the other hand, particular terms, while they have *substantial depth,* inasmuch as each of the things, one or other of which is predicated of them, has a concrete form, yet have no *substantial breadth,* inasmuch as there is no aggregate of things to which alone they are applicable. In order to place this matter in a clearer light, I must remark, that I, in common with most logicians, take the copula in the sense of a sign of attribution, and not, like Hamilton, in the sense of a sign of equality in extension or comprehension. He exposes the proposition, "man is an animal," thus:

The extension of man	*Subject*
equals	*Copula*
a part or all of the extension of animal .	*Predicate*

And thus he makes the predicate particular. Others interpret it thus:

Every man	*Subject*
has all the attributes common to . .	*Copula*
every animal	*Predicate*

It is in this latter sense that the copula is considered in this paper. Now, a particular is, as has been said, an *alternative* subject. Thus, "Some S is M" means, if S′, S″, and S‴ are the singular S's, that "either S′, or else S″, or else S‴, has all the attributes belonging to M." A particular term, then, has a substantial depth, because it may have a predicate which is absolutely concrete, as in the proposition, "Some man is Napoleon." But if we put the particular into the predicate we have such a proposition as this: "M has all the attributes belonging to S′, or else all those belonging to S″, or else all those belonging to S‴." And this can never be true unless M is a single individual. Now a single individual substance is, I will not say an atom, but the smallest part of an atom, that is, nothing at all. So that a particular can have no *substantial breadth.* Now take the universal term "S." We can say, "Any S is M," but not if M is a real concrete quality. We cannot say, for instance, "Any man is Napoleon." On the other hand, we can say "Any M is S," even if M is a real substance or aggregate of substances. Hence a universal term has no *sub-*

stantial depth, but has *substantial breadth*. We may therefore divide all terms into substantial universals and substantial particulars.

416. Two terms may be equal in their substantial breadth and depth, and differ in their essential breadth and depth. But two terms cannot have relations of substantial breadth and depth which are unknown in the state of information supposed, because in that state of information everything is known.

417. In informed breadth and depth, two terms may be equal, and may have unknown relations. Any term, affirmative or negative, universal or particular, may have informed breadth or depth.

§6. THE CONCEPTIONS OF QUALITY, RELATION, AND REPRESENTATION, APPLIED TO THIS SUBJECT

418. In a paper presented to the Academy last May,* I endeavored to show that the three conceptions of reference to a ground, reference to a correlate, and references to an interpretant, are those of which logic must principally make use. I there also introduced the term "symbol," to include both concept and word. Logic treats of the reference of symbols in general to their objects. A symbol, in its reference to its object, has a triple reference:

First, Its direct reference to its object, or the real things which it represents;

Second, Its reference to its ground through its object, or the common characters of those objects;

Third, Its reference to its interpretant through its object, or all the facts known about its object.

What are thus referred to, so far as they are known, are:

First, The informed *breadth* of the symbol;

Second, The informed *depth* of the symbol;

Third, The sum of synthetical propositions in which the symbol is subject or predicate, or the *information* concerning the symbol.[1]

* *On a New List of Categories*, vol. 1, bk. III, ch. 6, §1.

[1] It will be seen that I depart widely from the ordinary use of this word to mean testimony given privately. As in metaphysics, information is the connection of form and matter, so it may in logic appropriately mean the measure of predication.— 1893.

419. By breadth and depth, without an adjective, I shall hereafter mean the informed breadth and depth.

It is plain that the breadth and depth of a symbol, so far as they are *not* essential, measure the *information* concerning it, that is, the synthetical propositions of which it is subject or predicate. This follows directly from the definitions of breadth, depth, and information. Hence it follows:

First, That, as long as the information remains constant, the greater the breadth, the less the depth;

Second, That every increase of information is accompanied by an increase in depth or breadth, independent of the other quantity;

Third, That, when there is no information, there is either no depth or no breadth, and conversely.

These are the true and obvious relations of breadth and depth. They will be naturally suggested if we term the information the *area*, and write —

$$\text{Breadth} \times \text{Depth} = \text{Area}.$$

[Analogous to increase of information in us, there is a phenomenon of nature — development — by which a multitude of things come to have a multitude of characters, which have been involved in few characters in few things.— 1893.]

420. If we learn that S is P, then, as a general rule, the depth of S is increased without any decrease of breadth, and the breadth of P is increased without any decrease of depth. Either increase may be *certain* or *doubtful*.

It may be the case that either or both of these increases does not take place. If P is a negative term, it may have no depth, and therefore adds nothing to the depth of S. If S is a particular term, it may have no breadth, and then adds nothing to the breadth of P. This latter case often occurs in metaphysics, and, on account of not-P as well as P being predicated of S, gives rise to an appearance of contradiction where there really is none; for, as a contradiction consists in giving to contradictory terms some breadth in common, it follows that, if the common subject of which they are predicated has no real breadth, there is only a verbal, and not a real contradiction. It is not really contradictory, for example, to say that a boundary is both within and without what it bounds. There is

also another important case in which we may learn that "S is P," without thereby adding to the depth of S or the breadth of P. This is when, in the very same act by which we learn that S is P, we also learn that P was covertly contained in the previous depth of S, and that consequently S was a part of the previous breadth of P. In this case, P gains in extensive distinctness and S in comprehensive distinctness.

421. We are now in condition to examine Vorländer's objection to the inverse proportionality of extension and comprehension. He requires us to think away from an object all its qualities, but not, of course, by thinking it to be without those qualities, that is, by denying those qualities of it in thought. How then? Only by supposing ourselves to be ignorant whether it has qualities or not, that is, by diminishing the supposed information; in which case, as we have seen, the depth can be diminished without increasing the breadth. In the same manner we can suppose ourselves to be ignorant whether any American but one exists, and so diminish the breadth without increasing the depth.

422. It is only by confusing a movement which is accompanied with a change of information with one which is not so, that people can confound generalization, induction, and abstraction. *Generalization* is an increase of breadth and a decrease of depth, without change of information. *Induction* is a certain increase of breadth without a change of depth, by an increase of believed information. *Abstraction* is a decrease of depth without any change of breadth, by a decrease of conceived information. *Specification* is commonly used (I should say unfortunately) for an increase of depth without any change of breadth, by an increase of asserted information. *Supposition* is used for the same process when there is only a conceived increase of information. *Determination*, for any increase of depth. *Restriction*, for any decrease of breadth; but more particularly without change of depth, by a supposed decrease of information. *Descent*, for a decrease of breadth and increase of depth, without change of information.[1]

[1] *Ascent* is the most unequivocal noun to denote the passage to a broader and less deep notion, without change of information; and other words of similar literal meaning are used in the same way. It is the decrease in depth, of course, which is directly expressed, the increase in breadth being implied. *Extension*,

423. Let us next consider the effect of the different kinds of reasoning upon the breadth, depth, and area of the two terms of the conclusion.

In the case of deductive reasoning it would be easy to show, were it necessary, that there is only an increase of the extensive distinctness of the major, and of the comprehensive distinctness of the minor, without any change in information. Of course, when the conclusion is negative or particular, even this may not be effected.

which directly expresses increase of breadth, has a somewhat different meaning. It is applied to the discovery (by increase of information) that a predicate applies — *mutatis mutandis* — to subjects to which it had not occurred to us to apply it. It involves no decrease of depth. Thus, Herbert Spencer says ["The Genesis of Science," *British Quarterly Review*, July, 1854] that the inversion of the barometer enabled us to *extend* the principles of mechanics to the atmosphere. Mathematicians frequently speak of the extension of a theorem. Thus, the modification of a theorem relating to plane curves so as to make it apply to all curves in space would be called an extension of that theorem. An extended theorem asserts all that the original theorem did, and more too. *Generalization* in its strict sense, means the discovery, by reflection upon a number of cases, of a general description applicable to them all. This is the kind of thought movement which I have elsewhere [509] called formal hypothesis, or reasoning from definition to definitum. So understood, it is not an increase in breadth but an increase in depth. For instance, I received today a number of English books printed by Hindoos in Calcutta. The manufacture is rude, yet peculiarly pleasing. Remembering other Indian manufactures I have seen, I now get a more definite conception of the characteristic of Indian taste. This, since it is an idea derived from the comparison of a number of objects, is called *generalization*. Yet it is not an extension of an idea already had, but, on the contrary, an increase of definiteness of the conceptions I apply to known things. Besides this, the proper meaning of the word generalization, there are two others which, though they are in good use, ought all the more for that to be severely frowned upon by all who have precision of philosophical terminology at heart. Namely, generalization is applied, secondly, to a particular kind of extension, namely to an extension in which the change of the predicate, in order to make it applicable to a new class of subjects, is so far from obvious, that it is the part of the mental process which chiefly attracts our notice. For example, what is usually called Fermat's theorem is that if ρ be a prime number, and a be any number not divisible by ρ, then $a^{\rho-1}$ leaves a remainder of 1 when divided by ρ. Now, what is called the generalized theorem of Fermat is that if κ is any integer number, and $\phi\kappa$ its totient, or the number of numbers as small as κ and prime to it, and if a be a number prime to κ, then $a^{\phi\kappa}$ leaves a remainder 1 when divided by κ. Instead of calling such process a generalization, it would be far better to call it a *generalizing extension*. — 1893.

424. Induction requires more attention. Let us take the following example:

S′, S″, S‴, and S^{IV} have been taken at random from among the M's;
S′, S″, S‴, and S^{IV} are P:
∴ any M is P.

We have here, usually, an increase of information. M receives an increase of depth, P of breadth. There is, however, a difference between these two increases. A new predicate is actually added to M; one which may, it is true, have been covertly predicated of it before, but which is now actually brought to light. On the other hand, P is not *yet* found to apply to anything but S′, S″, S‴, and S^{IV}, but only to apply to whatever else may hereafter be found to be contained under M. The induction itself does not make known any such thing.

425. Now take the following example of hypothesis:

M is, for instance, P′, P″, P‴, and P^{IV};
S is P′, P″, P‴, and P^{IV}:
∴ S is all that M is.

Here again there is an increase of information, if we suppose the premisses to represent the state of information before the inferences. S receives an addition to its depth; but only a potential one, since there is nothing to show that the M's have any common characters besides P′, P″, P‴, and P^{IV}. M, on the other hand, receives an actual increase of breadth in S, although, perhaps, only a *doubtful* one. There is, therefore, this important difference between induction and hypothesis, that the former potentially increases the breadth of one term, and actually increases the depth of another, while the latter potentially increases the depth of one term, and actually increases the breadth of another.

426. Let us now consider reasoning from definition to definitum, and also the argument from enumeration. A defining proposition has a meaning. It is not, therefore, a merely identical proposition, but there is a difference between the definition and the definitum. According to the received doctrine, this difference consists wholly in the fact that the definition is distinct, while the definitum is confused. But I think

that there is another difference. The definitum implies the character of being designated by a word, while the definition, previously to the formation of the word, does not. Thus, the definitum exceeds the definition in depth, although only *verbally*. In the same way, any unanalyzed notion carries with it a feeling — a constitutional word — which its analysis does not. If this be so, the definition is the predicate and the definitum the subject, of the defining proposition, and this last cannot be simply converted. In fact, the defining proposition affirms that whatever a certain name is applied to is supposed to have such and such characters; but it does not strictly follow from this, that whatever has such and such characters is actually called by that name, although it certainly *might* be so called. Hence, in reasoning from definition to definitum, there is a verbal increase of depth, and an actual increase of extensive distinctness (which is analogous to breadth). The increase of depth being merely verbal, there is no possibility of error in this procedure. Nevertheless, it seems to me proper, rather to consider this argument as a special modification of hypothesis than as a deduction, such as is reasoning from definitum to definition. A similar line of thought would show that, in the argument from enumeration, there is a verbal increase of breadth, and an actual increase of depth, or rather of comprehensive distinctness, and that therefore it is proper to consider this (as most logicians have done) as a kind of infallible induction. These species of hypothesis and induction are, in fact, merely hypotheses and inductions from the essential parts to the essential whole; this sort of reasoning from parts to whole being demonstrative. On the other hand, reasoning from the substantial parts to the substantial whole is not even a probable argument. No ultimate part of matter fills space, but it does not follow that no matter fills space.

SUPPLEMENT OF 1893*

427. The usefulness of the doctrine of logical quantity depends upon strict adherence to an exact terminology. To find the requisite terms, however, is not always easy.

An operation increasing the breadth of a term, whether with or without change of information, may be termed an *extension*

* "Terminology" a supplement to the foregoing.

of it. The word is most frequently employed, especially by mathematicians, to signify the application of a doctrine (perhaps with a slight modification) to a new sphere. This implies increased information. Still, the broader meaning here proposed is permitted by usage. In like manner, any diminution of breadth may be called *restriction*.

428. An operation increasing the depth of a term, whether with or without change of information, is known as a *determination*. The books generally give *abstraction* as the contrary of determination; but this is inadmissible. I would propose the word *depletion*. The adjective *abstract* was first used, in Latin, and in imitation of the Greek, of a geometrical form conceived as depleted of matter. Such a conception is intuitional, in the sense of being pictorial. In the seventh century, Isodorus Hispalensis defines *abstract number* in the same sense in which the phrase is still in vogue. But neither *abstract* nor any cognate word is found as a logical term until toward the close of the great dispute concerning realism and nominalism in the twelfth century, when the introduction of the term *abstraction* may be said to mark the cessation of that controversy, if not to be its most important fruit. The word is, with little doubt, a translation of the Greek ἀφαίρεσις, although no Greek text known at that time in the West, has been adduced, from which it could have been borrowed. The etymological meaning is, of course, drawing away from; this, however, does not mean, as is often supposed, drawing the attention away from an object, but, as all the early passages in both ancient languages fully demonstrate, drawing one element of thought (namely, the form) away from the other element (the matter), which last is then neglected. But even in the very first passage in which *abstraction* occurs as a term of logic, two distinct meanings of it are given, the one the contemplation of a form apart from matter, as when we think of *whiteness*, and the other the thinking of a nature *indifferenter*, or without regard to the differences of its individuals, as when we think of a *white* thing, generally.* The latter process is called, also, *precision* (or better, *prescission*): and it would greatly contribute to perspicuity of thought and expression if we were to return to the usage of the best scholastic doctors and designate it by that name exclusively,

* See Prantl, *op. cit.*, III, 94.

restricting abstraction to the former process by which we obtain notions corresponding to the "abstract nouns."* Modern logicians, especially the Germans, who have been lamentably sketchy in their studies of logic, have had the idea that these abstract nouns were mere affairs of grammar, with which the logician need not concern himself; but the truth is they are the very nerve of mathematical thinking. Thus, in the modern theory of equations, the action of changing the order of a number of quantities, is taken as itself a subject of mathematical operation, under the name of a substitution. So a straight line, which is nothing but a relation between points, is studied, and even *intuited*, as a distinct thing. It would be best to limit the word abstraction to this process; but if men cannot be persuaded to do this, the next best thing is to abolish the word abstraction altogether, and denominate this process *subjectifaction*. It is to be remarked that modern logicians and psychologists, though they have completely changed the breadth of abstraction, commonly applying the term not to subjectifaction but to prescission, nevertheless retain the mediaeval definition which was intended to apply not to the latter but to the former process. Namely, they define abstraction as attending to a part of an idea and neglecting the remainder. For attention is the pure denotative application, or breadth-function, of the thought-sign; it is the part that thought plays as an index. Of course, in saying this, I do not pretend to be giving a *psychological* account of attention, even if I can be persuaded that there is any such thing as psychology, apart from logic on the one hand and physiology on the other. Attention is a certain modification of the contents of consciousness with reference to a centre. This centre is where there is a strong sense-will reaction, which imparts to the idea the nature of an index (weathercock, sign post, or other blind forcible connection between thought and thing). Now, the subject of a proposition is just such an index. Hence, the real phenomenon of attending to a quality, say white, or making that the centre of thought, consists of thinking of it as the subject to which the other elements of thought are attributes. But *prescission*, if accurately analyzed, will be found not to be an affair of attention. We cannot prescind, but can only dis-

* Cf. 1. 549n.

tinguish, color from figure. But we can prescind the geometrical figure from color; and the operation consists in imagining it to be so illuminated that its hue cannot be made out (which we easily can imagine, by an exaggeration of the familiar experience of the indistinctness of hues in the dusk of twilight). In general, prescission is always accomplished by imagining ourselves in situations in which certain elements of fact cannot be ascertained. This is a different and more complicated operation than merely attending to one element and neglecting the rest. Thus, if the definition usually given of abstraction, that it is attention to a part of an idea with neglect of the rest, be accepted, the term must be applied, no longer to prescission but exclusively to subjectifaction.

429. So much for the terms expressive of increasing and diminishing logical breadth and depth, in general. In order to express an extension by depletion, and a determination by restriction, without change of information, we obviously stand in need of the words *generalization* and *specification*. Unfortunately, neither word is so used. By specification is invariably meant a determination by increased information. By generalization is sometimes meant, it is true, the extension of an idea by a considerable modification of its depth; but still there is commonly an increase of information. Even more frequently, generalization signifies a formal increase of depth, by a general idea being predicated of cases not before synthetized; and there may or may not be an increase of breadth. Finally, generalization, by a wanton abuse of language, is often used to mean simply an induction. It seems necessary, therefore, to abandon these words for the present purpose and to content ourselves with *ascent* and *descent*.

430. An increase of information, in general, is, in modern speech, called a *discovery*. The old word, *invention*, was much better, since this left *discovery* to be restricted to the finding of a new thing — as the discovery of America — while the finding out of a new character was specifically called a *detection*. Thus, Oldenburg, the Secretary of the Royal Society, writes in 1672, that the dispersion of light is "the oddest, if not the most considerable, detection which hath hitherto been made into the operations of nature." It is a pity these nice distinctions have been lost. We must now speak of the *discovery of an occurrence*

or *instance* and the *discovery of a property*. An imaginary increase of information is an *assumption* or *supposition;* but the former word is preferable. An increase of information by induction, hypothesis, or analogy, is a *presumption*. (A *legal presumption* is a presumption which follows an accepted rule of the courts, irrespective of the dictates of good sense.) A very weak presumption is a *guess*. A presumption opposed to direct testimony is a *conjecture*, or, if weak, a *surmise*.

SIGNIFICATION AND APPLICATION*

431. These are substitute terms for what are called by Mill and others connotation and denotation; for (1) the previously well-established use of connote was somewhat warped by Mill and his followers, and (2) these words may be applied to the corresponding properties of propositions as well as terms. The application of a term is the collection of objects which it refers to; of a proposition it is the instances of its holding good. The signification of a term is all the qualities which are indicated by it; of a proposition it is all its different implications.

432. Great confusion has arisen in logic from failing to distinguish between the different sorts of signification, or connotation, of a term: thus to the question, Are proper names connotative? "contradictory answers are given by ordinarily clear thinkers as being obviously correct," for the reason that they have not the same thing in mind under the term connotation. It is necessary to distinguish between; (1) the indispensable signification; (2) the banal signification; (3) the informational signification; and (4) the complete signification. (1) is so much as is contained in whatever may be fixed upon as the definition of the term — all those elements of the meaning in the absence of any one of which the name would not be applied; (2) is what "goes without saying," what is known to every one, and (3) is what there is occasion to give utterance to: these, of course, vary with the different individuals to whom the propposition is given out — that oxygen is exhilarating is informational to the student of chemistry, and banal to the teacher of chemistry (but false to those who are familiar with the latest

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 528–9; 431–3 are by Peirce and Mrs. C. Ladd-Franklin.

results of the science); (4) consists of all the valid predicates of the term in question. When I say, "The one I saw yesterday was John Peter," the indispensable signification of John Peter is simply an individual object of consciousness (usually a man, though it may be a dog, or a doll) whom it has been agreed to designate by that name; but the banal signification, to one who knows John Peter well, is very extensive.

433. The same characteristics apply to propositions as well as to terms: thus the complete signification (or implication) of *All x is y* is all its valid consequences, and its complete application (or range) is all those descriptions of circumstances under which it holds good — that is to say, all its sufficient antecedents.

434. A general term denotes whatever there may be which possesses the characters which it signifies; J. S. Mill uses, in place of signifies, the term connotes, a word which he or his father picked up in Ockham. But signify has been in uninterrupted use in this sense since the twelfth century, when John of Salisbury spoke of "quod fere in omnium ore celebre est aliud scilicet esse appellativa *significant*, et aliud esse quod, *nominant*. Nominantur singularia; sed universalia significantur."* Nothing can be clearer. There is no known occurrence of connote as early as this. Alexander of Hales (*Summa Theol.*, I. liii) makes *nomen connotans* the equivalent of *appellatio relativa*, and takes the relation itself as the accusative object of *connotare*, speaking of "creator" as *connoting* the relation of creator to creature. So Aquinas, *In sentent.*, I. dist. viii. q. 1, Art. 1. Subsequently, because adjectives were looked upon as relative terms, *white* being defined as "having whiteness," etc., the adjective was looked upon as connoting the abstraction, but never unless its supposed relative character was under consideration. Tataretus, for example, who wrote when the usage was fully established, will be found using such phraseology as the following: "Nulla relativa secundum se habent contrarium, cum non sint qualitates primae, sed solum relativa secundum dici, et hoc secundum esse absolutum et significatum principale eorum et non secundum esse respectivum et connotativum." Chauvin† (1st ed.) says: "Connotativum illud est cuius sig-

* *Metalogicus*, II, xx.

† *Lexicon Rationale.*

nificatum non sistit in se, sed necessario ad aliud refertur, vel aliud connotat. V. g. *Rex, magister, primus*."

It unfortunately happened, as the above quotations show, that the precise meaning recognized as proper to the word "signify" at the time of John of Salisbury (a younger contemporary of Abelard) was never strictly observed, either before or since; and, on the contrary, the meaning tended to slip towards that of "denote." Yet even now the propriety of John's remark must be recognized.

A number of works were written in the middle ages, *De modis significandi*, based upon Priscian (a contemporary of Boëthius), who in turn followed Apollonius the bad-tempered, "grammaticorum princeps," who lived in the time of Hadrian and Antoninus Pius. Cf. also Thurot, *Notices et Extraits des MSS.* xxii. Pt. II, and Duns Scotus, *Works*, Lyons edit. 1.

CHAPTER 6

*THE GRAMMATICAL THEORY OF JUDGMENT AND INFERENCE**

§1. JUDGMENTS

435. A judgment is an act of consciousness in which we recognize a belief, and a belief is an intelligent habit upon which we shall act when occasion presents itself. Of what nature is that recognition? It may come very near action. The muscles may twitch and we may restrain ourselves only by considering that the proper occasion has not arisen. But in general, we *virtually resolve* upon a certain occasion to act as if certain imagined circumstances were perceived. This act which amounts to such a resolve, is a peculiar act of the will whereby we cause an image, or *icon*, to be associated, in a peculiarly strenuous way, with an object represented to us by an *index*. This act itself is represented in the proposition by a *symbol*, and the consciousness of it fulfills the function of a symbol in the judgment. Suppose, for example, I detect a person with whom I have to deal in an act of dishonesty. I have in my mind something like a "composite photograph" of all the persons that I have known and read of that have had that character, and at the instant I make the discovery concerning that person, who is distinguished from others for me by certain indications, upon that index at that moment down goes the stamp of RASCAL, to remain indefinitely.

436. A proposition *asserts* something. That assertion is performed by the symbol which stands for the act of consciousness. That which accounts for *assertion* seeming so different from other sorts of signification is its *volitional* character.

437. Every assertion is an assertion that two different signs have the same object. If we ask why it should have that *dual* character, the answer is that volition involves an action and reaction. The consequences of this duality are found not only in the analysis of propositions, but also in their classification.

* From the "Short Logic," c. 1893, following 296.

438. It is impossible to find a proposition so simple as not to have reference to two signs. Take, for instance, "it rains." Here the icon is the mental composite photograph of all the rainy days the thinker has experienced. The index, is all whereby he distinguishes *that day*, as it is placed in his experience. The symbol is the mental act whereby [he] stamps that day as rainy. . . .

439. In order properly to exhibit the relation between premisses and conclusion of mathematical reasonings, it is necessary to recognize that in most cases the *subject-index* is compound, and consists of a *set* of indices. Thus, in the proposition, "A sells B to C for the price D," A, B, C, D form a set of four indices. The symbol "— sells — to — for the price —" refers to a mental icon, or idea of the act of sale, and declares that this image represents the *set* A, B, C, D, considered as attached to that icon, A as seller, C as buyer, B as object sold, and D as price. If we call A, B, C, D four *subjects* of the proposition and "— sells — to — for the price — " a *predicate*, we represent the logical relation well enough, but we abandon the Aryan syntax.

440. It may be asked, why may not an assertion identify the objects of any two signs whatever, as two indices? Why should it be limited to declaring the object of an *index* to be represented by an *icon*? The answer is that an assertion *may* identify the objects of any two signs whatever; yet that in every case this will amount to declaring that an *index*, or set of indices, is represented by an *icon*. For instance, let the proposition be, that William *Lamare*, the author of the book *Correctorium fratris Thomae* is really the William *Ware* who was the teacher of Duns Scotus. Here the objects of two indices are identified. But this is logically equivalent to the assertion that the icon of identity, that is, the mental composite image of two aspects of one and the same thing, represents the objects of the set of indices William *Mare* and William *Ware*.[1] We are not, indeed, absolutely forced to regard one of the signs as an *icon* in any case; but this is a very convenient way of taking account of certain properties of

[1] That *Marra* and *Warra* were really the same cannot be positively asserted; but the hypothesis suits the known facts remarkably well, except for the difference of names, which is perhaps not an insuperable obstacle.

inferences. It happens, too, to have some secondary advantages, such as that of agreeing with our natural metaphysics, and with our feeling in regard to subject and predicate.

441. As the index may be complex, so also may the icon. For instance, taking the universal selective index, *everything*, we may have an icon which is composed alternatively of two, a sort of composite of two icons, in the same way that any image is a "composite photograph" of innumerable particulars. Even what is called an "instantaneous photograph," taken with a camera, is a composite of the effects of intervals of exposure more numerous by far than the sands of the sea. Take an absolute instant during the exposure and the composite represents *this* among other conditions. Now, the two alternative icons are combined like that. We have an icon of this alternation, a composite of all the alternative cases we have thought of. The symbol asserts that one or other of those icons represents the universally selected index. Let one of the alternative icons be the idea of what is not a man, the other the idea of what is mortal. Then, the proposition will be: "Take anything you please, and it will either not be a man or will be mortal." Two signs so conjoined are said to be *aggregated*, or *disjunctively connected*, or *alternatively conjoined*. Take another example. Let the index be particularly selective. Let an icon be so compounded of two icons that in each variation of it both those icons are conjoined. For instance, let one be an *icon* of a Chinese, the other of a woman. Then, the combined *icon* will be an icon of a Chinese woman. Thus, the proposition will be: "Something can be so selected as to be at once a Chinese and a woman." Two signs so conjoined are said to be *combined*, or *conjunctively connected*, or *simultaneously conjoined*. . . .

§2. INFERENCE

442. It is now time to examine more carefully the nature of *inference*, or the conscious and controlled adoption of a belief as a consequence of other knowledge. The first step of inference usually consists in bringing together certain propositions which we believe to be true, but which, supposing the inference to be a new one, we have hitherto not considered together, or not as united in the same way. This step is called

colligation.* The compound assertion resulting from colligation is a *conjunctive proposition*, that is, it is a proposition with a composite icon, as well as usually with a composite index. Colligation is a very important part of reasoning, calling for genius perhaps more than any other part of the process. Many logicians refuse the name of reasoning to an inferential act of which colligation forms no part. Such an inferential act they all an *immediate inference*. This term may be accepted; but although colligation certainly gives a higher intellectuality to inference, yet its importance is exaggerated when it is represented to be of more account than the conscious control of the operation. The latter ought to determine the title of *reasoning*.

443. An inference, then, may have but a single premiss, or several premisses may be united by colligation. In the latter case, they form, when colligated, one conjunctive proposition. But even if there be but one premiss, the icon of that proposition is always more or less complex. The next step of inference to be considered consists in the contemplation of that complex icon, the fixation of the attention upon a certain feature of it, and the obliteration of the rest of it, so as to produce a new icon. . . .

444. Whenever one thing suggests another, both are together in the mind for an instant. In the present case, this conjunction is specially interesting, and in its turn suggests that the one necessarily involves the other. A few mental experiments — or even a single one, so expert do we become at this kind of experimental inquiry — satisfy the mind that the one icon would at all times involve the other, that is, suggest it in a special way, into which we shall soon inquire. Hence the mind is not only led from believing the premiss to judge the conclusion true, but it further attaches to this judgment another — that *every* proposition *like* the premiss, that is having an icon like it, *would* involve, and compel acceptance of, a proposition related to it as the conclusion then drawn is related to that premiss. [This is the third step of inference.] Thus we see, what is most important, that every inference is thought, at the time of drawing it, as one of a possible class of inferences. In the case of a rational inference, we *see*, in an icon which represents the dependence of the icon of the con-

* See 451n.

clusion upon the icon of the premiss, about what that class of inference is, although, as the outlines of icons are always more or less vague, there is always more or less of vagueness in our conception of that class of inferences. There is no other element of inference essentially different from those which have been mentioned. It is true that changes generally take place in the indices as well as in the icon of the premiss. Some indices may be dropped out. Some may be identified. The order of selections may sometimes be changed. But these all take place substantially in the same manner in which a feature of the icon attracts attention, and must be justified in the inference by experiments upon icons.

It thus appears that all knowledge comes to us by observation. A part is forced upon us from without and seems to result from Nature's mind; a part comes from the depths of the mind as seen from within, which by an egotistical anacoluthon we call *our* mind. The three essential elements of inference are, then, colligation, observation, and the judgment that what we observe in the colligated data follows a rule.[1]

[1] To speak summarily and use a symbol of abbreviation, rather than an analytical and iconical idea, we may say that the purpose of signs — which is the purpose of thought — is to bring truth to expression. The law under which a sign must be true is the law of inference; and the signs of a scientific intelligence must, above all other conditions, be such as to lend themselves to inference. Hence, the illative relation is the primary and paramount semiotic relation.

It might be objected that to say that the purpose of thought is to bring the truth to expression is to say that the production of *propositions*, rather than that of *inferences*, is the primary object. But the *production* of propositions is of the general nature of inference, so that inference is the essential function of the cognitive mind. — From the fragment used in ch. 2, §1.

Book III

CRITICAL LOGIC

A. EXPLICATIVE REASONING

B. AMPLIATIVE REASONING

A. EXPLICATIVE REASONING

CHAPTER 1

*THE ARISTOTELIAN SYLLOGISTIC**[1P]

§1. PRETENSIONS OF DEMONSTRATIVE REASONING

445. . . . It is easy to over-rate the importance of syllogism. Most of the older logics do that, in teaching that the substance of all reasoning lies in this. It is also easy to under-rate it; as many have done. The truth is, it is an essential element of almost all reasoning, perhaps of all.

A syllogism is a valid, *demonstrative*, *complete*, and *simply eliminative* argumentation.

446. To say that an argumentation is valid is to say that

* From chapter 9 of the *Grand Logic*, 1893. Cf. 3.162ff.

[1] To be well-read, or even fairly versed, in philosophy (no easy accomplishment) it is quite indispensible to have studied Aristotle; and the study of Aristotle may most conveniently begin with the two books of *Prior Analytics*, certainly the most elementary of all his writings. Two books precede these in the traditional arrangement (with which Aristotle himself probably had nothing to do). One of these, the *Predicaments* or *Categories*, is a metaphysico-logical treatise, of which only the outlines are important. The other, the *Peri hermeneias*, is purely logical, but difficult and confused; and the doctrine is not that of the *Analytics*. I should recommend every serious student of logic who can pick out easy Greek without much trouble to read the *Prior Analytics* at any rate, and the *Posterior Analytics* if he can find time. The *Posterior Analytics* is a splendid monument to the human intellect. Both treatises are in very easy Greek; and they have so much influenced medieval thought, and through that our own, that really a man does not understand what is said to him in the streets till he has read them. I would read them out of the Berlin edition; and if you want notes, there can be nothing better than the Greek scholia there given. Then by buying this edition, you have the advantage of having the index constantly at your hand; and it is of inestimable value, every day. Waitz's edition of the *Organon* is good; and Trendelenburg's *Beiträge*, *De Anima*, and little epitome [*Elementa Logices Aristoteleae*] are very valuable. There is a capital little epitome [*Outlines of the Philosophy of Aristotle*] by Wallace, and Grote's *Aristotle* has merit. But Grote is terribly one-sided. In fact, all modern commentators have strong leanings.

it is as truthful as it pretends to be. It is essential to reasoning, as already stated, that it should be accompanied by the reflection that it belongs to a class of reasonings, few or none of which lead from truth to falsity. All reasoning, therefore, makes a pretension; and if that pretension is true, the reasoning is *valid*.

447. *Demonstrative* reasoning pretends to be such that it is logically impossible for the premisses to be true while the conclusion is false. I think it would be just to add that demonstrative reasoning further pretends:

First, that its premisses are logically possible, or at least that the class of propositions to which they are considered as belonging contains possibly true propositions; and,

Second, that its conclusion is not logically necessary, or at least, that the class of propositions to which it is considered as belonging contains propositions not necessarily true.

This, at least, I think would have been Aristotle's way of conceiving the matter, had the question presented itself to him. For instance, if he had been asked what he would say of this reasoning:

> Chameleons assume the color of objects upon which they rest,
> ∴ Everything is what it is,

I think he would have said, this is no reasoning at all. Granted that the premiss cannot be true while the conclusion is false, because the conclusion cannot be false at all, reasoning pretends that there is some connection between premiss and conclusion, so that if the fact were the opposite of what is stated in the premiss, the conclusion would not necessarily be true. I think Aristotle would have made the same strictures upon an argument like this:

> Some parts are greater than their wholes;
> ∴ The eating of green fruit proves invariably fatal.

Such an argument cannot lead from true to false, because the premiss cannot be true. But if the reasoning pretends, as it seems to do, that from one thing being in a certain relation to another, it is safe to conclude a proposition about a totally different subject, in that sense it is false.

448. But although I think Aristotle or any other man of good sense would take this view, I propose to reject it, and to consider both the above reasonings as sound. My reason is, that such things are of no practical importance whatever — for as long as reasoning does not lead us astray, the whole purpose of logic is fulfilled — and to admit these reasonings as sound simplifies very considerably the whole doctrine of syllogism. In this I am by no means alone. Even in ancient times, many logicians took the same ground. . . .

449. A *complete* argument is one which professes to be not only necessary, but logically necessary.[1]

450. An *eliminative* argumentation is one which mentions something in the premisses in two opposite ways, so that it disappears from the conclusion. When we argue:

Washington was a high-minded politician;
∴ It is possible for a politician to be high-minded,

the argumentation is not eliminative. For what is dropped is mentioned but once. Again we may argue [that in:]

All men die;
∴ Holy men (if there are any) die,

we drop nothing, but insert something. Once more we may argue:

There are women whom all men reverence,
∴ Every man reverences some woman or another.

Here we neither drop nor insert. All these non-eliminative inferences are usually called *immediate inferences* by logicians; and have received very little attention. But Barbara and all eliminative inferences are said to be *mediate*. This terminology is ill-considered. Thus, the inference:

[1] An *incomplete* argumentation is properly called an *enthymeme*, which is often carelessly defined as a syllogism with a suppressed premiss, as if a sorites, or complex argumentation, could not equally give an enthymeme. The ancient definition of an *enthymeme* was "a rhetorical argumentation," and this is generally set down as a second meaning of the word. But it comes to the same thing. By a rhetorical argumentation was meant one not depending upon logical necessity, but upon common knowledge as defining a sphere of possibility. Such an argument is rendered logical by adding as a premiss that which it assumes as a leading principle.

Any two infinite planes have a line of intersection;
∴ Any three infinite planes have three lines of intersection,

is not eliminative, yet it plainly arises from steps of argumentation. We say of the three planes, A, B, C, that A and B, A and C, and B and C are pairs, each having an intersection, and we count three of these pairs. No logician has ever made out, or pretended to make out, that there is any sort of elimination between three premisses. Hence undecomposable eliminative arguments (not counting mere omissions) have two premisses each.

451. That the putting of the two premisses together is a distinct act of thought, so that the reasoning;

All men are mortal,
All patriarchs are men;
∴ All patriarchs are mortal,

really consists of these two steps:

All men are mortal,
All patriarchs are men;
∴ All men are mortal, and all patriarchs men;
∴ All patriarchs are mortal;

would be regarded by the ordinary logician as hair-splitting. Yet it is just in bringing the premisses together that all the difficulty lies! This preliminary uniting of the premisses is called *copulation*, or *colligation*.[1] Even between the copulate premiss and the conclusion another stage of thought might be inserted, which will appear if we vary one of the premisses thus:

All patriarchs are men, and all sinners are mortal;
∴ If all men are sinners, all patriarchs are mortal;
∴ All patriarchs are mortal.

[1] The latter term is more familiar to our generation, having been used by Whewell [*Novum Organum Renovatum*, II, iv]. But the former is the more legitimate historically. *Copulatum* with Aulus Gellius (XVI. viii. 10) translates the Stoical συμπεπλεγμένον in this sense. Conjunctions like *et* are called *copulative* by Priscian [*Institutiones Grammaticae*, lib. xvi, cap. 1]. Abelard uses *copulare*. We might use *colligation* where the propositions brought together are of one nature and function. But in syllogism, this is not the case. However, if the mood Darapti be admitted, it consists merely in compounding two premisses and dropping a term from the result. This will appear below.

This last step is enthymematic. It wants, to make it logical, the expression of the leading principle, "All men are sinners." But put *men* in place of sinners, and this becomes a logical principle, not needing to be set forth as a premiss. It seems, however, that logicians recognize no difference between saying, "If all men are men, A is true," and saying directly "A is true"; and I suppose they are right. We cannot recognize logical formulæ as, properly speaking, assertions.[1]

§2. RULES AND CASES

452. . . . In order to get at the real why and wherefore of Aristotle's choice of propositional forms, whatever he may himself have understood it to be, we must go back to this postulate:

We cannot know anything except it be a uniformity.

I am not pretending the uniformity or our knowledge need be perfect. Nor am I pretending we cannot become aware of a breach of uniformity. One uniformity may constitute a breach in another. I am simply suggesting, first, that an event altogether out of order and presenting no regularity could not come to our knowledge at all and second, that only in respect to its being orderly can we know it. I do not care to insist here upon the truth of this postulate. It would take me too far afield. I merely say that if it be granted, there seems to be a good reason for Aristotle's propositional forms; but if not, I am unable to defend the system.

A uniformity is a consequence. All we know is that from one thing another thing follows. Those two things themselves, when they come to be carefully thought out, are seen to be consequences, and so on indefinitely. I next ask that it be granted that there is an important class of inferences which have each of them more than one premiss. Of these, considering that each premiss judges or asserts that one thing follows

[1] What Kant calls an explicative, or analytical, judgment is either no judgment at all, because void of content (to use his phrase), or else it sets forth distinctly in the predicate what was only indistinctly thought (that is, not actually thought at all) in the subject. In that case, it is really synthetic, and rests on experience; only the experience on which it rests is mere internal experience — experience of our own imaginations. Association by resemblance, and association by contiguity: all lies in that great distinction.

from another, it is plain that the simplest is: from A follows B and from B follows C; and therefore from A follows C.

453. . . . [In the argument, "Men are sinners, and sinners are miserable; ∴ men are miserable,"] "Sinners are miserable," must be a *Rule* without exceptions. That is, it says in effect, if you take any sinner, you will find he is miserable. The second person appropriately expresses it, because there is a second premiss which draws attention to certain sinners, and virtually picks them out. If the rule has exceptions, all I can say is, that if you let *me* pick out the sinner he will turn out miserable. If I guarantee to find a miserable sinner, of course, I guarantee *there is* a sinner in the world. But if I turn the responsibility of picking out the sinner to you, I do not guarantee you can find one. I only say if you do find one, he will turn out miserable. This is the distinction between Universal and Particular propositions.*

The premiss

"Men are sinners,"

must refer to every character common to all sinners. No matter what the rule about sinners may be which the other premiss lays down, men are here said to be subject to that rule. If it were possible to find, in place of men, a race totally unlike sinners in every respect, the conclusion would hold that they were not miserable, if sinners were so. But that is an absurdity. For since sinners are things we can think about and speak of and reason correctly about sometimes, those other creatures would be creatures we could never think about, nor speak of, nor draw a single correct inference concerning them. For the same reason, while we can speak of angels as wanting *some* characters of sinners, it will not help us to assert they *possess* some characters of sinners, since every conceivable thing resembles every other in some respect — as for instance, in being conceivable or capable of being talked about, etc.[1]

* Cf. Hobbes, *Computation or Logic*, ch. II, §11.

[1] This does not hold in the case of a limited universe of marks. For if we are confining ourselves to a certain line of predicates, there will be nothing absurd in saying that things differ in every respect. In that case, there will be a lexis of predicates, distinct from the phasis. Certainly, if the nature of reasoning is to be explored, it is necessary to take account of cases in which we limit our thought to a particular order of predicates. Some logicians treat the subject as

When I speak of a common character of a class of objects, I must, for the purposes of the inference in Barbara, mean a general rule true of all that class. Now, if I say, you can take what rule you please applicable to all sinners, and it will be found applicable to men, I am not guaranteeing that there is any general rule true of all sinners. But when I say, I could find you a rule true of all sinners that does not hold good of all women (not, for instance, of the Blessed Virgin), I have committed myself to the proposition that there is such a rule. This is the distinction between *Affirmative* and *Negative* propositions. An affirmative proposition speaks of any general rule there may be, no matter what, while a negative says there is a rule and says that such a one can be elected so as to break down if applied to a certain subject (outside the class to which the rule refers).

454. We thus see how syllogistic theory calls for precisely the formal distinctions of propositions that Aristotle draws, and needs no others.

§3. THE QUADRANT

455. The distinction between Universal and Particular propositions is said to be the distinction in Quantity; that between Affirmative and Negative propositions the distinction in *Quality*. Such is the traditional terminology.[1] But this is a terrible abuse of the important words *quantity* and *quality*, the inconvenience of which is felt in studying the *Critic of the Pure Reason*. Therefore, notwithstanding their having a generation of occupancy for every card in the whist-pack, and one for the joker too, I for one shall vote to eject them. Let us say Universals and Particulars differ in *Lexis*, Affirmatives and

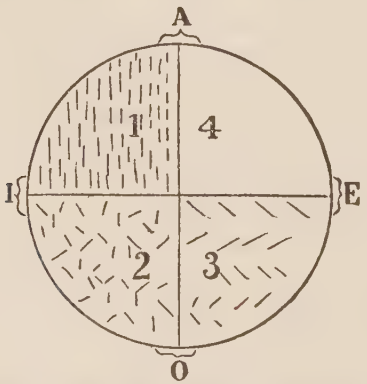
"extra-logical"; but that only means it is outside the scope of their own studies. If a mathematician should choose to characterize the differential calculus as "extra-mathematical," he would exhibit the same determination to keep his science small and simple that animates many of the logicians.

But although the limited universe of marks is not for me extra-logical, I think it is proper to exclude it from elementary syllogistic, for the reason that it is one of the simplest conceivable instances of the logic of relatives, and when that is treated this problem is virtually solved, even if it be not directly attended.

[1] It dates from Apuleius, and is more assified than golden. *Universal* and *Particular* have the same origin. *Affirmative* and *Negative* are words manufactured by Boethius. [See Prantl *op. cit.*, I, 691.]

Negatives in *Phasis*.[1] Lexis and Phasis are tell-way and say-way. Lexis is from λέγειν, to pick out, and also to tell; it is the mode of picking out, or of reckoning. Phasis is saying, in the sense of: "What do you say? Yes or No?"; being the base of κατάφασις, affirmation, and ἀπόφασις, negation. I really see no objection to them, except their novelty. For reversal of Lexis I shall use *metalexis*; for reversal of Phasis, *metaphasis*, though the meaning is nearly that of the Greek ἀντίφασις.

456. . . . Having taken the Diodoran, in opposition to the Philonian view of validity, Aristotle must for consistency hold the universal Affirmative implies the existence of its subject. . . . He must understand: "Some philosopher's stones are red" as not asserting the existence of any philosopher's stone. . . . As the distinction between Universal and Particular propositions concerns the subject, so the distinction between Affirmative and Negative ought, for the sake of symmetry, concern the predicate; so that the difference between asserting and not-asserting the existence of the subject ought to be the distinction between Universals and Particulars, not between Affirmatives and Negatives. Universal propositions do not, while particular propositions do, imply the existence of their subjects. The following figure illustrates the precise sense here assigned to the four forms A, E, I, O.



In the quadrant marked 1 there are lines which are all vertical; in the quadrant marked 2 some lines are vertical and some not; in quadrant 3 there are lines none of which are

[1] From φημί, not φαίνω; therefore nothing to do with phase.

vertical; and in quadrant 4 there are no lines. Now, taking *line* as subject and *vertical* as predicate,

A is true of quadrants 1 and 4 and false of 2 and 3.
E is true of quadrants 3 and 4 and false of 1 and 2.
I is true of quadrants 1 and 2 and false of 3 and 4.
O is true of quadrants 2 and 3 and false of 1 and 4.

Hence, A and O precisely deny each other, and so do E and I. But any other pair of propositions may be both true, or both false, or either true while the other is false.

457. Quadrant 1 includes the case in which the predicate covers the whole *universe* of discourse;[1] so that there is this intrinsic distinction between Affirmatives and Negatives, that the latter deny their predicates to be necessary, which the former permit; just as there is this intrinsic distinction between Universals and Particulars, that the latter assert the existence of their subjects, which the former do not insist upon.

458. There are some languages which take the negative particle in such a sense that a repetition of it is intensive; but I shall understand the negativing of a proposition to be a reversal of the above diagram across its sinister diagonal, interchanging quadrants 3 and 1, so that All S is not-not-P shall mean, All S is P. And in like manner, I shall use the word *some*, in such a sense that a repetition of it is not sinister, but to signify a reversal of the diagram across the dexter diagonal, interchanging quadrants 2 and 4, so that Some-some-S is P shall mean All S is P. This I do for the sake of symmetry; at the same time, it is easy to give an intelligible sense to it. To say: "Every S is P" is to say: "An S, even if one of the worst cases is selected, will be identical with a P, favorably chosen." To say: "Some S is P" is to say: "An S, if not one of the worst is chosen, will be identical with a P favorably chosen." But to say: "An S, if not other than one of the worst is chosen, will be identical with a P favorably chosen," reproduces the universal. By "favorably" is to be understood, favorably to the identity, but by the "worst cases" are to be understood those most calculated to overthrow the assertion. To say: "An S, if no one of the worst is selected, will be

[1] The term *universe*, now in general use, was introduced by De Morgan in 1846. *Cambridge Philosophical Transactions*, VIII, 380.

identical with a P unfavorably chosen," implies that every P is an S, just as "Any not-S is not P" implies the same thing. So to say: "An S, even if one of the worst cases is selected, is not identical with a P not favorably selected," is as much as to say that some P is not S, just as "Some not-S is P" implies the same thing. This meaning of the word "some" certainly departs very far indeed from the ordinary usage of speech. But that is nothing: it is perfectly intelligible, and is taken so as to give balance and symmetry to the logical system, which is a matter of the utmost importance, if that system is to fulfill a philosophical function. If the main object of the syllogistic forms were in actual application, to test reasonings as to whose validity or invalidity we found it difficult to decide, as some logicians seem naively to suppose, then their close connection with ordinary habits of thought might be a paramount consideration. But in reality, their main function is to give us an insight into the inward structure of reasoning in general; and for that purpose systematic perfection is indispensable. . . .

459. It is a blunder on Aristotle's part to call the propositions A and E contraries merely because they may both be false, but not both true. They ought to be called *incongruous* or *disparates*, and both these terms are somewhat in use. Subcontraries (a word of Boëthius,* imitating the ὑπεναντία of Ammonius) are propositions of opposite ecphasis but, being particular, both can be true, though both cannot be false. It would be well to follow the usage of those writers who call any two propositions which can logically both be true but not both false, subcontraries. *Contradictories* (Aristotle's ἀντικείμενα, the word *contradictoria* comes from Boëthius)† are two propositions which cannot both be true nor both false, but precisely deny one another. *Subaltern* (a word found in the translation of Porphyry's *Isagoge* by Marius Victorinus in the fourth century;‡ Porphyry's word is ὑπάλληλον, but in the present sense first found in Boëthius)§ is a particular proposition which follows by an immediate inference from its corresponding universal to which it is said to be subaltern.

460. But in my system none of the relations shown in the

* See Prantl, *op. cit.*, I, 687ff.

† *Ibid.*, 687.

‡ *Ibid.*, 661.

§ *Ibid.*, 684, 692.

diagram of Apuleius [the square of opposition] are preserved, except the two pairs of contradictories. All other pairs of propositions may be true together or false together.

A and E, All S is P, and No S is P, are true together when no S exists, and false together when part only of the S's are P. I and O, some S is P, some S is not P, are true and false together under precisely the opposite conditions.

A and I, Any S is P, Some S is P, are true together when there are S's all of which are P, and are false together when there are S's none of which are P. E and O, No S is P, and Some S is not P, are true and false together under precisely the opposite circumstances. . . .

CHAPTER 2

*ON THE NATURAL CLASSIFICATION OF ARGUMENTS**

Part I

§1. ESSENTIAL PARTS OF AN ARGUMENT

461. In this paper, the term "argument" will denote a body of premisses considered as such.[1] The term "premiss" will refer exclusively to something laid down (whether in any enduring and communicable form of expression, or only in some imagined sign), and not to anything only *virtually* contained in what is said or thought, and also exclusively to that part of what is laid down which is (or is supposed to be) relevant to the conclusion.[2]

462. Every inference involves the judgment that, if *such* propositions as the premisses are are true, then a proposition related to them, as the conclusion is, must be, or is likely to be, true. The principle implied in this judgment, respecting

* *Proceedings of the American Academy of Arts and Sciences*, vol. 7, April 9, 1867, pp. 261–87, with additions and corrections of 1893. Intended as Essay I of the *Search for a Method*.

[1] There can be little doubt that *argumentum* acquired its logical meaning in the Roman law courts; and Cicero not only uses it as above, but expressly defines it as "ratio rei dubiae faciens fidem." The definition of Boëthius, who intends to follow Cicero, makes it a *medium* proving a conclusion. *Medium* is here used in the sense of premiss; but since it usually means in logic a middle term, *argument* has been by many understood in that sense; and Hamilton is among those who go so far as to stigmatize the other and ancient use as improper; wishing to substitute for this the word *argumentation*. The substitution, however, seems to me historically wrong, contrary to common usage, and not particularly convenient. Still, to avoid reproach, I was inclined to replace *argument* in this essay by *inference* (for, as Locke well says, "to *infer* is nothing but, by virtue of one proposition laid down as true, to draw in another as true," and those who would restrict it to reasoning from effect to cause violate all good usage), until I reflected that to do so would have the air of admitting what I could never admit, that logic is primarily conversant with unexpressed thought and only secondarily with language.— 1893.

[2] So far as that is separable from the rest.— 1893.

a genus of argument, is termed the *leading principle* of the argument.

463. A *valid* argument is one whose leading principle is true.

464. In order that an argument should determine the necessary or probable truth of its conclusion, both the premisses and leading principle must be true.

§2. RELATIONS BETWEEN THE PREMISSES AND LEADING PRINCIPLE

465. The leading principle contains, by definition, whatever is considered requisite besides the premisses to determine the necessary or probable truth of the conclusion. And as it does not contain in itself the subsumption of anything under it, each premiss must, in fact, be equivalent to a subsumption under the leading principle.

The leading principle can contain nothing irrelevant or superfluous.

No fact, not superfluous, can be omitted from the premisses without being thereby added to the leading principle,[1] and nothing can be eliminated from the leading principle except by being expressed in the premisses. Matter may thus be transferred from the premisses to the leading principle, and *vice versa*.

There is no argument without premisses,[2] nor is there any without a leading principle.

466. It can be shown that there are arguments no part of whose leading principle can be transferred to the premisses, and that every argument can be reduced to such an argument by addition to its premisses. For, let the premisses of any argument be denoted by P, the conclusion by C, and the leading principle by L. Then, if the whole of the leading principle be expressed as a premiss, the argument will become

$$L \text{ and } P$$
$$\therefore C.$$

[1] Or else made a conditional antecedent to the conclusion. — 1893.

[2] To this it might be objected that if from premiss, P, we infer conclusion, C, then to infer "If P, then C," needs no premiss at all. But if the hypothetical judgment is *immediately* made, it is not inferential; and if not, it is requisite to begin with some premiss, though its modality be only problematical. — 1893.

But this new argument must also have its leading principle, which may be denoted by L'. Now, as L and P (supposing them to be true) contain all that is requisite to determine the probable or necessary truth of C, they contain L'. Thus L' must be contained in the leading principle, whether expressed in the premiss or not. Hence every argument has, as portion of its leading principle, a certain principle which cannot be eliminated from its leading principle. Such a principle may be termed a *logical principle.*

An argument whose leading principle contains nothing which can be eliminated is termed a *complete,* in opposition to an *incomplete, rhetorical,* or *enthymematic* argument.[1]

467. Since it can never be requisite that a fact stated should also be implied in order to justify a conclusion, every

[1] Neither of these terms is quite satisfactory. Enthymeme is usually defined as a syllogism with a premiss suppressed. This seems to determine the same sphere as the definition I have given; but the doctrine of a suppressed premiss is objectionable. The sense of a premiss which is said to be suppressed is either conveyed in some way, or it is not. If it is, the premiss is not suppressed in any sense which concerns the logician; if it is not, it ceases to be a premiss altogether. What I mean by the distinction is this. He who is convinced that Sortes is mortal because he is a man (the latter belief not only being the cause of the former, but also being felt to be so) necessarily says to himself that all *such* arguments are valid. This genus of argument is either clearly or obscurely recognized. In the former case, the judgment amounts to another premiss, because the proposition (for example), "All reasoning from humanity to mortality is certain," only says in other words that every man is mortal. But if the judgment amounts merely to this, that the argument in question belongs to some genus all under which are valid, then in one sense it does, and in another it does not, contain a premiss. It does in this sense, that by an act of attention such a proposition may be shown to have been virtually involved in it; it does not in this sense, that the person making the judgment did not *actually* understand this premiss to be contained in it. This I express by saying that this proposition is contained in the leading principle, but is not *laid down.* This manner of stating the matter frees us at once from all psychological perplexities; and at the same time we lose nothing, since all that we know of thought is but a reflection of what we know of its expression.

These vague arguments are just such as alone are suitable to oratory or popular discourse, and they are appropriate to no other; and this fact justifies the appellation, "rhetorical argument." There is also authority for this use of the term. "Complete" and "incomplete" are adjectives which I have preferred to "perfect" and "imperfect," as being less misleading when applied to argument, although the latter are the best when syllogism is the noun to be limited. [Perhaps it is necessary further to distinguish between a *complete* and a *logical* argument. [See 474n.]— 1893.]

logical principle considered as an assertion* will be found to be quite empty. The only thing it really enunciates is a rule of inference;† considered as expressing truth, it is nothing.[1] On this account that method of investigating logic which works upon syllogistic forms is preferable to another, too often confounded with it, which undertakes to enunciate logical principles.

§3. DECOMPOSITION OF ARGUMENT‡

468. Since a statement is not an argument for itself, no fact concluded can be stated in any one premiss. Thus it is no argument to say All *A* is *B*; *ergo* Some *A* is *B*.

469. If one fact has such a relation to another that, if the former is true, the latter is necessarily or probably true, this relation constitutes a determinate fact; and therefore, since the leading principle of a complete argument involves no matter of fact, every complete argument has at least two premisses.[2]

* Originally, "a proposition."

† Originally, "Considered as regulating the procedure of inference, it is determinate."

[1] Any assertion means merely how we would act under given circumstances, but a logical principle does not even mean this, but only what we would infer from certain premisses. — 1893.

‡ Originally numbered §4, though following immediately after §2.

[2] The view here taken appears inadmissible, unless it be understood to bear a greatly generalized meaning. For, according to this, from two premisses, "A" and "B," but one complete argument could be formed, namely, that which concludes the conjunctive proposition "A and B"; and so, all the arguments of which this paper treats would have to be excluded. But we must not use the word "argument" in a sense which completely annuls its utility. The mere "colligation of facts," to use Whewell's term, is a most important and difficult part of that whole operation which in its totality is called *reasoning*. But it is not the only part. Given the premiss that every man (living, dead, or unborn) is the son of a man, then by a process which is an important part of reasoning, we conclude that every man is a grandson (or a descendant of any order) of a man. Nothing is to be gained by excluding such an operation of thought from the number of arguments. To further illustrate this point, take any branch of pure mathematics — say the theory of numbers. What are its premisses? Some of them are mere definitions of terms, such as *product*, *sum*, and the like — terms which may be dispensed with entirely, their definitions always being substituted. The other premisses taken together define the relationship existing between integer numbers; and they may without difficulty be comprised in a single proposition, which will really be a definition of the subject-matter, number. Thus, the whole fabric of the theory of numbers, which in its *posse*, at least, may well be called vast, will be deduced from a single premiss. Nay, it would

470. Every conclusion may be regarded as a statement substituted for either of its premisses, the substitution being justified by the other premisses. Nothing is relevant to the other premisses, except what is requisite to justify this substitution. Either, therefore, these other premisses will by themselves yield a conclusion which, taken as a premiss along with the first premiss, justifies the final conclusion; or else some part of them, taken with the first premiss, will yield a conclusion which, taken as a premiss along with all the others, will again justify the final conclusion. In either case, it follows that every argument of more than two premisses can be resolved into a series of arguments of two premisses each. This justifies the distinction of *simple* and *complex* arguments.

§4. OF A GENERAL TYPE OF SYLLOGISTIC ARGUMENTS

471. A valid, complete, simple argument will be designated as a *syllogistic* argument.

472. Every proposition may, in at least one way, be put into the form,

$$S \text{ is } P;$$

the import of which is, that the objects to which S, or the *total subject*, applies have the characteristics attributed to every object to which P, or the *total predicate*, applies.

473. Every term has two powers or significations, according as it is subject or predicate. The former, which will here be termed its *breadth*, comprises the objects to which it is applied; while the latter, which will here be termed its *depth*, comprises the characters which are attributed to every one of the objects to which it can be applied. This breadth and depth must not be confounded with logical extension and comprehension, as these terms are usually taken.

474. Every substitution of one proposition for another must consist in the substitution of term for term. Such substitution can be justified only so far as the first term repre-

be very simple to make this single proposition the hypothetical antecedent of every conclusion, when it would be needed no more as a premiss. Nevertheless, it is impossible to regard reasoning otherwise than as a process; and as such, it must involve a substitution of a conclusion in place of premisses. The logic of relatives clearly shows that not all reasoning is eliminative; but so far as it is eliminative, it involves two premisses and is of the general type set forth in §4, below. — 1893.

sents what is represented by the second. Hence the only possible substitutions are —

First. The substitution for a term fulfilling the function of a subject of another whose breadth is included in that of the former; and

Second. The substitution for a term fulfilling the function of a predicate of another whose depth is included in that of the former.[1]

If, therefore, in either premiss a term appears as subject which does not appear in the conclusion as subject, then the other premiss must declare that the breadth of that term includes the breadth of the term which replaces it in the conclusion. But this is to declare that every object of the latter term has every character of the former. The eliminated term, therefore, if it does not fulfill the function of predicate in one premiss, does so in the other. But if the eliminated term fulfills the function of predicate in one premiss, the other premiss must declare that its depth includes that of the term which replaces it in the conclusion. Now, this is to declare that every character of the latter term belongs to every object of the former. Hence, in the other premiss, it must fulfill the function of a subject. Hence the general formula of all argument must be:

$$\begin{aligned} M &\text{ is } P, \\ S &\text{ is } M, \\ \therefore S &\text{ is } P; \end{aligned}$$

which is to be understood in this sense — that the terms of every syllogistic argument fulfill functions of subject and predicate as here indicated, but not that the argument can be grammatically expressed in this way.[2]

[1] Jevons is often referred to as the originator of the conception of reasoning as a substitution. He first set forth this idea in a little treatise called *The Substitution of Similars* [1869], which appeared after he had seen the present essay in print. But he never claimed the idea as original; nor do I. It was familiar to the Leibnizian logicians. In my opinion, the peculiar doctrine of the substitution of similars is utterly false and untenable. — 1893.

[2] This treatment of the subject has the effect of excluding the dilemma altogether from the scheme of classification. It was not until the Renaissance that this formal argument ever appeared in the logical treatises; though it had been given in the rhetorics since ancient times. The stock example of the dilemma is this from Aulus Gellius: a handsome wife would be unfaithful, which is bad; and an ugly wife would confer no pleasure, which is bad; therefore, to take any

Part II*

§1. OF APAGOGICAL FORMS

475. If C is true when P is, then P is false when C is. Hence, it is always possible to substitute for any premiss the denial of the conclusion, provided the denial of that premiss be at the same time substituted for the conclusion.[1] Hence, corresponding to every syllogistic argument in the general form,

$$S \text{ is } M,\ M \text{ is } P;$$
$$S \text{ is } P.$$

There are two others:

It is false that S is P, M is P;
It is false that S is M.

S is M, it is false that S is P;
It is false that M is P.

§2. OF CONTRADICTION

476. The apagogical forms make it necessary to consider in what way propositions deny one another.

If a proposition be put into the general form,

$$S \text{ is } P,$$

its contradictory has, first, as its subject, instead of S, "the S now meant"[2] or "some S"; and has, second, as its predicate, instead of P, that which differs from P or "not P."

477. From these relations of contradictories — from the necessities of the logic of apagogically related arguments, there-

wife would be bad. This is usually reduced to syllogism by assuming as a premiss a proposition which is nothing but a corollary from the principle of excluded middle; and this would be requisite to make the argument a *complete* one, according to the definition in the text. It would, however, have been proper to recognize, among incomplete arguments, those as *logical* which suppress no premiss except a mere logical principle, such as the principle of excluded middle. Such argument is a trifle less infantile than syllogism. By a dilemmatic argument I mean any argument whose validity depends upon the principle of excluded middle. Many of these are reckoned as syllogism by De Morgan [*Formal Logic*, p. 117ff]. I do not think anything should be a dilemma which does not depend on the same principle. — 1893.

* Peirce's first printed paper, of which Part II is substantially a restatement, is in the appendix to the present volume.

[1] This operation will be termed a *contraposition* of the premiss and conclusion.

[2] What S is meant being generally undetermined.

fore — arises the need of the two divisions of propositions into affirmative and negative on the one hand, and into universal and particular on the other. The contradictory of a universal proposition is particular, and the contradictory of an affirmative proposition is negative. Contradiction is a reciprocal relation, and therefore the contradictory of a particular proposition is universal, and that of a negative proposition is affirmative. The contradiction of particular and negative propositions could not be brought under the general formula, were the distinctions of affirmative and negative absolute and not merely relative; but, in fact, not-not-P is the same as P. And, if it is said that "what is now meant of the part of S meant at another time, is P," since the part of S meant at another time is left to be determined in whatever way the proposition made at another time may determine it, this can only be true if All S is P. Therefore, if one man says "some S is not P," and another replies, "some of that same S is P," this second person, since he allows the first man's some S, which has not been defined, to remain undefined, in effect says that All S is P.*

Whether contradictories differ in other respects than these well-known ones is an open question.

§3. OF BARBARA

478. Since some S means "the part now meant of S," a particular proposition is equivalent to a universal proposition with another subject; and in the same way, a negative proposition is equivalent to an affirmative proposition with another predicate.

The form, S is P,

therefore, as well as representing propositions in general, par-

* This sentence was deleted and the following substituted for it in 1893. "For example, suppose one man says 'Some S's are Q,' that is, 'I can so choose from the class of S's that those I choose shall be Q'; and suppose another replies, 'Some of that some are R,' that is, 'I can so choose from among those which you will choose that (whether they be Q or not) they shall be R.' Plainly he says in effect that all S's are R. It will be the same if instead of two men, it is but one man at two different times."

ticularly represents Universal Affirmative propositions; and thus the general form of syllogism:

$$M \text{ is } P,\ S \text{ is } M;$$
$$S \text{ is } P,$$

represents specially the syllogisms of the mood *Barbara*.

§4. OF THE FIRST FIGURE

479. Since, in the general form, S may be any subject and P any predicate, it is possible to modify Barbara by making the major premiss and conclusion negative, or by making the minor premiss and conclusion particular, or in both these ways at once. Thus we obtain all the modes of the first figure.

It is also possible to have such arguments as these:

Some M is P,

S has all the common characters of *that* part of M (whatever that part may be, and therefore of each and every M);

$\therefore$ S is P,

and

All not-M is P,

S is not M;

$\therefore S$ is P;

but as the theory of apagogical argument has not obliged us to take account of these peculiar modifications of subject and predicate, these arguments must be considered as belonging to Barbara. In this sense the major premiss must always be universal, and the minor affirmative.

Three propositions which are related to one another as though major premiss, minor premiss, and conclusion of a syllogism of the first figure will be termed respectively *Rule*, *Case*, and *Result*.

§5. SECOND AND THIRD FIGURES

480. Let the first figure be written thus:

Figure 1

Any **M** is / is not **P**
Any / Some **S** is **M**
Any / Some **S** is / is not **P**

481. Then its two apagogical modifications are the second and third figures.

Figure 2

Any **M** is / is not **P**
Some / Any **S** is not / is **P**
Some / Any **S** is not **M**

Figure 3

Some / Any **S** is not / is **P**
Any / Some **S** is **M**
Some **M** is not / is **P**

482.* It is customary to enumerate six moods of the third figure instead of four, and the moods Darapti and Felapton appear to be omitted. But a particular proposition is asserted (actually and not merely virtually) by the universal proposition which does not otherwise differ from it; and therefore Darapti is included both under Disamis and Datisi, and Felapton both under Bocardo and Ferison. (De Morgan.)

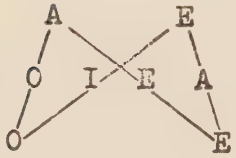
483. The second figure, from the assertion of the rule and the denial of the result, infers the denial of the case; the third figure, from the denial of the result and assertion of the case, infers the denial of the rule. Hence we write the moods as follows, by allowing inferences only on the straight lines:

* When Peirce incorporated this paper in the "Search for a Method" he deleted this paragraph and wrote instead: "It is customary to enumerate six moods of the third figure instead of four; but Darapti and Felapton are omitted, because when the universal premisses are not understood to assert the existence of their subjects these moods become invalid."

Figure 1

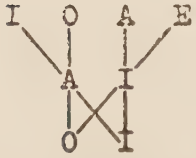
Assertion of Rule,
Assertion of Case;
Assertion of Result.

Figure 2



Assertion of Rule,
Denial of Result;
Denial of Case.

Figure 3



Denial of Result,
Assertion of Case;
Denial of Rule.

484. The symmetry of the system of moods of the three figures is further exhibited in the following table.

Enter at the top the proposition asserting or denying the rule; enter at the side the proposition asserting or denying the case; find in the body of the table the proposition asserting or denying the result. In the body of the table, propositions indicated by italics belong to the first figure, those by black-letter to the second figure, and those by script to the third figure.

	I	A	E	O
E		𝔈	𝔄	
A	*J*	*A*	*E*	*O*
I	*A*	*I*	*O*	*E*
O		𝔊	𝔍	

485. If, as the denial of the result in the second and third figures, we put the form "Any N is N," we have:

Figure 2	*Figure* 3
No M is N,	Any N is N,
Any N is N;	Some N is M;
$\therefore$ No N is M.	$\therefore$ Some M is N.

These are the formulae of the two simple conversions. Neither can be expressed syllogistically except in the figures in which they are here put (or in what is called the fourth figure, which we shall consider hereafter). If, for the denial of the result in the second figure, we put "No not-N is N" (where "not-N" has not as yet been defined) we obtain:

All M is N,
No not-N is N;
$\therefore$ No not-N is M.

In the same way, if we put "Some N is some-N" (where some-N has not been defined) for the denial of the result in the third figure, we have:

Some N is some-N,
All N is M;
$\therefore$ Some M is some-N.

These are the two ways of contraposing the Universal Affirmative.

486. There are two ostensive reductions of each mood of the second and third figures. I shall distinguish them as the short reduction and the long reduction. The short reduction is effected by converting or contraposing that premiss which is not the denial of the result. The long reduction is effected by transposing the premisses, contraposing or converting the denial of the result, and contraposing or converting the conclusion. The alteration thus produced in the order of the terms is shown in the following figure:

	Short Reduction	*Long Reduction*
N M	M N	M Ξ
Ξ M	Ξ M	N M
Ξ N	Ξ N	N Ξ
Σ Π	Σ Π	Σ P
Σ P	P Σ	Π Σ
P Π	P Π	Π P

The names bestowed by Shyreswood, or Petrus Hispanus,* upon the moods indicate the possibility of the short reduction in the case of Cesare and Festino of the second figure, and of Datisi and Ferison of the third figure; also the possibility of the long reduction of Camestres of the second figure and of Disamis of the third.

487. The short reduction of Camestres and Baroco is effected by introducing the term not-P, and defining it as that which S *is* when it is *not* P.[1] Hence for the second premiss (Any or some S is not P) we substitute "Any or some S is not-P"; and as the first premiss, Any M is P, gives by contraposition, Any not-P is not M, the moods:

Any M is P,
Any or some S is not P;
$\therefore$ Any or some S is not M,

are reduced to:

No not-P is M,
Any or some S is not-P;
$\therefore$ Any or some S is not M.

488. The short reduction of Disamis and Bocardo is effected by introducing the term some-S, defining it as that part of S which is or is not P when some S is or is not P.[2] We

* On the authorship of these mood names and other mnemonic devices, cf. Prantl, *op. cit.*, III, 11–50, passim.

[1] This inference,

S is not P,
$\therefore$ S is not-P,

being the conclusion of an affirmative from a negative premiss, can obviously not be ordinary syllogism. The suppressed premiss is,

Not-P is not P,

which is the definition of not-P. It will appear below (Part III, §1) that this is ordinary reasoning from definition to definitum, and is essentially of the second figure. — 1893.

[2] This is not reasoning from definition; for that is the substitution of a compact predicate for a complex one. The present inference is:

Some S is P,
$\therefore$ Some-S is P.

It draws a universal conclusion from a particular premiss, and is, therefore, not ordinary syllogism. The suppressed premiss is,

Some S is some-S;

and the inference is of the nature of reasoning from enumeration, as will be seen below. Such inference is essentially of the third figure. — 1893.

can therefore substitute for the first premiss, Some S is or is not P, All some-S is or is not P; while the second premiss, All S is M, can be contraposed into "Some M is some-S"; and thus the forms:

Some S is (*or* is not) P,
Any S is M;
$\therefore$ Some M is (*or* is not) P;

are reduced to the following:

Any some-S is (*or* is not) P,
Some M is some-S;
$\therefore$ Some M is (*or* is not) P.

489. To reduce Cesare, Festino, and Baroco in the long way, it is necessary to introduce the terms not-P and some-S. Not-P is defined as that class to which any M belongs which is not P. Hence for the first premiss of Cesare and Festino we can substitute "Any M is not-P." Some-S is defined as that class of S which is (or is not) P, when some S is (or is not) P. Hence for the second premisses of Festino and Baroco we can first substitute "Any some-S is (or is not) P"; and then, by contraposition or conversion, we obtain "Any P (or not-P) is not some-S." Then, by the transposition of the premisses, we obtain from Cesare, which is:

No M is P,
Any S is P;
($\therefore$ No S is M;)

Any not-P is not S,
Any M is not-P;
$\therefore$ Any M is not-S.

490. And from the conclusion of this reduced form we obtain the conclusion of Cesare by simple conversion. So Festino and its long reduction are:

Any M is not P,
Some S is P;
($\therefore$ Some S is not M;)

Any not-P is not some-S,
Any M is not-P;
$\therefore$ Any M is not some-S;

and the conclusion of Festino is obtained from that of the reduced form by a substitution which may be made syllogistically thus:

Any M is not some-S,
Some S is some-S;
$\therefore$ Some S is not M.

491. Baroco and its long reduction are:

Any M is P,	Any P is not some-S,
Some S is not P;	Any M is P;
($\therefore$ Some S is not M;)	$\therefore$ Any M is not some-S;

and the conclusion of Baroco is obtained from the conclusion of the reduction in the same way as that of Festino.

492. In order to reduce Datisi, Bocardo, and Ferison in the long way, we must define Some-S as that S which is M when some S is M, and Not-P as that which some (or any) S *is* when it is *not* P. Hence, for "Some S is M" we can substitute "Any some-S is M"; and for "Some (or any) S is not P," "Some (or any) S is not-P." "Some S is not-P" may be converted simply; and "Any S is not-P" may be contraposed so as to become "Some not-P is some-S." Then Datisi and its long reduction are:

Any S is P,	Any some-S is M,
Some S is M;	Some P is some-S;
($\therefore$ Some M is P;)	$\therefore$ Some P is M.

493. And from the conclusion of the reduction, the conclusion of Datisi is obtained by simple conversion. Ferison and its long reduction are:

Any S is not P,	Any some-S is M,
Some S is M;	Some not-P is some-S';
($\therefore$ Some M is not P;)	$\therefore$ Some not-P is M.

494. And from the conclusion of the reduction, the conclusion of Ferison may be obtained by a substitution whose possibility is expressed syllogistically thus:

Any not-P is not P,
Some not-P is M;
$\therefore$ Some M is not P.

495. Bocardo and its long reduction are

Some S is not P,	Any S is M,
Any S is M;	Some not-P is S;
($\therefore$ Some M is not P;)	$\therefore$ Some not-P is M.

And the conclusion of Bocardo is obtained from that of its reduction in the same way as the conclusion of Ferison.

496. The ostensive reduction of the indirect or apagogical figures may be considered as the exhibition of them under the general form of syllogism:

$$S \text{ is } M, \qquad M \text{ is } P;$$
$$\therefore S \text{ is } P.$$

But, in this sense, it is not truly a reduction if the substitutions made in the process are inferences. But although the possibility of the conversions and contrapositions can be expressed syllogistically, yet this can be done only by taking as one of the premisses:

"All N is N,"
"Any not-N is not N,"
or "Some N is some-N."

Now, these are properly not premisses, for they express no facts; they are merely forms of words without meaning.[1] Hence, as no complete argument has less than two premisses, the conversions and contrapositions are not inferences.[2] The only other substitutions which have been made have been of not-P and some-S for their definitions. These also can be put into syllogistic form; but a mere modification of language is not an inference. Hence no inferences have been employed in reducing the arguments of the second and third figures to such forms that they are readily perceived to come under the general form of syllogism.

There is, however, an intention in which these substitutions are inferential. For, although the passage from holding for true a fact expressed in the form "No A is B," to holding its converse, is not an inference, because, these facts being identical, the relation between them is not a fact; yet the passage from one of these forms taken merely as having *some* meaning, but not this or that meaning, to another, since these forms are not identical, and their logical relation is a fact, is an inference. This distinction may be expressed by saying that they are not inferences, but substitutions having the *form* of inferences.

Thus the reduction of the second and third figures, con-

[1] Or rather, they bear only the formal meaning that consists in showing certain forms of inference to be valid. — 1893.

[2] Or at most, they are but *formal* inferences in the sense in which this word has just been used. — 1893.

sidered as mere forms, is inferential; but when we consider only what is meant by any particular argument in an indirect figure, the reduction is a mere change of wording.

497. The substitutions made use of in the ostensive reductions are shown in the following table, where

e denotes simple conversion of E;
i denotes simple conversion of I;
a_2, contraposition of A into E;
a_3, contraposition of A into I;
o_2, the substitution of "Some S is not M" for "Any M is not some-S";
o_3, the substitution of "Some M is not P" for "Some not-P is M";
e'', introduction of not-P by definition;
i'', introduction of some-S by definition.

Reduction of Second Figure		
Name of Mood	Short Reduction	Long Reduction
Cesare	e	$e''\ a_2\ e$
Camestres	$a_2\ e''$	$e\ e$
Festino	e	$e''\ i''\ a_2\ o_2$
Baroco	$a_2\ e''$	$i''\ e\ o_2$

Reduction of Third Figure		
Name of Mood	Short Reduction	Long Reduction
Disamis	$a_3\ i''$	$i\ i$
Datisi	i	$i''\ a_3\ i$
Bocardo	$a_3\ i''$	$e''\ i\ o_3$
Ferison	i	$i''\ e''\ a_3\ o_3$

With the exception of the substitutions *i''* and *e''*, which will be considered hereafter, all those which are used in the reduction of the moods of either oblique figure have the form of inferences in the same figure.*

498. The so-called *reductio per impossibile* is the repetition or inversion of that contraposition of propositions by which the indirect figures have been obtained. Now, contradiction arises from a difference both in quantity and quality; but it is to be observed that, in the contraposition which gives the second figure, a change of the *quality* alone, and in that which gives the third figure a change of the *quantity* alone, of the contraposed propositions, is sufficient. This shows that the two contrapositions are of essentially different kinds, and that the reductions *per impossibile* of the second and third figures respectively involve the following formal inferences.[1]

Figure 2

The Result follows from the Case;
∴ The Negative of the Case follows from the Negative of the Result.

Figure 3

The Result follows from the Rule;
∴ The Rule changed in Quantity follows from the Result changed in Quantity.

But these inferences may also be expressed as follows:

Figure 2

Whatever (S) is M is $\overset{P}{\underset{\text{not}}{}}\ P$;
∴ Whatever (S) is $\overset{\text{not}}{P}\ P$ is not M.

Figure 3

Any $\overset{S}{\underset{\text{some}}{}}\ S$ is whatever (P or not-P) M is;
∴ Some M is whatever (P or not-P) $\overset{\text{some}}{S}\ S$ is.

* This paragraph was deleted in the "Search for a Method" and the following substituted: "Now, putting aside the long reductions of the moods concluding O, or a particular negative, which are needlessly complicated and involve irrelevant conversions, in every other case the forms of inference used belong to the very same figure as the syllogism to be reduced."

[1] A formal inference is a substitution having the form of an inference.

Now, the limitations in parentheses do not affect the essential nature of the inferences; and omitting them we have:

Figure 2

Any M is $\overset{P;}{\underset{\text{not } P;}{}}$
$\therefore$ Any $\overset{\text{not } P}{P}$ is not M.

Figure 3

Any $\overset{S}{\underset{\text{some } S}{}}$ is M;
$\therefore$ Some M is $\overset{\text{some } S}{\underset{S.}{}}$

We have already seen that the former of these is of the form of the second figure, and the latter of the form of the third figure of syllogism.

499. Hence, it appears that no syllogism of an indirect figure can be reduced to the first figure without a substitution which has the form of the very figure from which the syllogism is reduced. In other words, the indirect syllogisms are of an essentially different form from that of the first figure, although in a more general sense they come under that form.

§6. THE THEOPHRASTEAN MOODS

500. It is now necessary to consider the five moods of Theophrastus, viz., *Baralipton*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesomorum*. Baralipton is included in Dabitis, and Fapesmo in Frisesomorum, in the same way in which Darapti is included in Disamis and Datisi, and Felapton in Bocardo and Ferison. The Theophrastean moods are thus reduced to three, viz.:

Celantes	*Frisesomorum*	*Dabitis*
No X is Y,	No X is Y,	Some Y is Z,
All Z is X;	Some Y is Z;	All Z is X;
$\therefore$ Any Y is not Z.	$\therefore$ Some Z is not X.	$\therefore$ Some X is Y.

Suppose we have, first, a Rule; second, a Case under that rule, which is itself a Rule; and, third, a Case under this second rule, which conflicts with the first rule. Then it would be easy to prove that these three propositions must be of the form:

1. No X is Y.
2. All Z is X.
3. Some Y is Z.

These three propositions cannot all be true at once; if, then, any two are asserted, the third must be denied, which is what is done in the three Theophrastean moods.*

These moods are resolved into one another by the contraposition of propositions, and therefore should be considered as belonging to different figures.

501. They can be ostensively reduced to the first Aristotelian figure in two ways; thus,

		Short Reduction		*Long Reduction*	
B	A	B	A	B	Γ
Γ	B	Γ	B	A	B
A	Γ	Γ	A	A	Γ

502. The verses of Shyreswood† show how Celantes and Dabitis are to be reduced in the short way, and Frisesomorum in the long way. Celantes and its long reduction are as follows:

Any X is not Y,	Any not-X is not Z,
Any Z is X;	Any Y is not-X;
$\therefore$ Any Y is not Z.	$\therefore$ Any Y is not Z.

"Any X is not Y," becomes, by conversion, "Any Y is not X." The term "not-X" is then introduced, being defined as that which Y is when it is not X. Then "Z is X" becomes "Any not-X is not Z"; and, the premisses being transposed, the reduction is effected.

503. Dabitis and its long reduction are as follows:

Any Z is X,	Any some-Z is Y,
Some Y is Z;	Some X is some-Z;
$\therefore$ Some X is Y.	$\therefore$ Some X is Y.

"Some Y is Z" becomes, by conversion, "Some Z is Y." Then the term "some-Z" is introduced, being defined as that Z which is Y if "some Z is Y." Then "Any Z is X" becomes "Some X is some-Z," and, the premisses being transposed, the reduction is effected.

* Cf. Ladd-Franklin's "Antilogism" in "On the Algebra of Logic," *Studies in Logic*, p. 37ff.

† See Prantl, *op. cit.*, III, 15, 16.

504. Frisesomorum is:

Some Y is Z,
Any X is not Y;
$\therefore$ Some Z is not X.

Let some-Y be that Y which is Z when some Y is Z; and then we have:

Some Y is some-Y,
Any X is not Y;
$\therefore$ Some some-Y is not X.

Then let not-X be that which any Y is when some Y is not X, and we have,

Some some-Y is not-X,

which yields by conversion,

Some not-X is some-Y;

and we thus obtain the reduction:

Any some-Y is Z,
Some not-X is some-Y;
$\therefore$ Some not-X is Z.

From the conclusion of this reduction, the conclusion of Frisesomorum is justified as follows:

Some not-X is Z,
Any X is not not-X;
$\therefore$ Some Z is not X.

Another mode of effecting the short reduction of Frisesomorum is this: Let not-Y be that which any X is when no X is Y, and we have:

Some Y is Z,
Any not-Y is not Y;
$\therefore$ Some Z is not not-Y.

Let some-Z be that Z which is not not-Y when some Z is not-Y, and we have,

Any some-Z is not not-Y,

and by conversion,

Any not-Y is not some-Z.

Thus we obtain as the reduced form:

Any not-Y is not some-Z,
Any X is not-Y;
$\therefore$ Any X is some-Z.

From the conclusion of this reduction, we get that of Frisesomorum thus:

Some some-Z is Z,
Any X is not some-Z;
$\therefore$ Some Z is not X.

505. In either reduction of Celantes, if we neglect the substitution of terms for their definitions, the substitutions are all of the second syllogistic figure. This of itself shows that Celantes belongs to that figure, and this is confirmed by the fact that it concludes the denial of a case. In the same way, the reductions of Dabitis involve only substitutions in the third figure, and it concludes the denial of a rule. Frisesomorum concludes a proposition which is at once the denial of a rule and the denial of a case: its long reduction involves one conversion in the second figure and another in the third, and its short reductions involve conversions in Frisesomorum itself. It therefore belongs to a figure which unites the characters of the second and third, and which may be termed the second-third figure in Theophrastean syllogism.

506. There are, then, two kinds of syllogism — the Aristotelian and Theophrastean.[1] In the Aristotelian occur the first, second, and third figures, with four moods of each. In the Theophrastean occur the second, third, and second-third figures, with one mood of each. The first figure is the fundamental or typical one, and Barbara is the typical mood. There is a strong analogy between the figures of syllogism and the four forms of proposition. A is the fundamental form of proposition, just as the first figure is the fundamental form of syllogism. The second and third figures are derived from the first by the contraposition of propositions, and E and I are derived from A by the contraposition of terms, thus:

Any S is P.
Any not-P is not S. Some P is some-S.

[1] I leave the Theophrastean syllogism, as every logician has found it, almost entirely useless. — 1893.

O combines the modifications of *E* and *I*, just as the second-third figure combines the second and third. In the second-third figure, only *O* can be concluded, in the third only *I* and *O*, in the second only *E* and *O*, in the first either *A E I O*. Thus *A* is the first figure of proposition, *E* the second, *I* the third, *O* the second-third.[1]

§7. MATHEMATICAL SYLLOGISMS

507. A kind of argument very common in mathematics may be exemplified as follows:

Every part is less than that of which it is a part,
Boston is a part of the Universe;
∴ Boston is less than the Universe.

This may be reduced to syllogistic form thus:

Any relation of part to whole is a relation of less to greater,
The relation of Boston to the Universe is a relation of part to whole;
∴ The relation of Boston to the Universe is a relation of less to greater.

If logic is to take account of the peculiarities of such syllogisms, it would be necessary to consider some propositions as having three terms, subject, predicate, and object; and such propositions would be divided into *active* and *passive*. The varieties in them would be endless.

Part III

§1. INDUCTION AND HYPOTHESIS

508. In the syllogism:

$$\begin{aligned} &\text{Any } M \text{ is } P, \\ &\quad \Sigma' S' \text{ is } M; \\ \therefore\ &\Sigma' S' \text{ is } P; \end{aligned}$$

where $\Sigma' S'$ denotes the sum of all the classes which come under M, if the second premiss and conclusion are known to

[1] Hypotheticals have not been considered above, the well-known opinion having been adopted that, "If *A*, then *B*," means the same as "Every state of things in which *A* is true is a state of things in which *B* is (or will be) true."

be true, the first premiss is, by enumeration, true. Whence we have, as a valid demonstrative form of inference:

$$\begin{aligned} &\Sigma' \ S' \text{ is } P, \\ &\Sigma' \ S' \text{ is } M; \\ \therefore \quad &M \text{ is } P. \end{aligned}$$

This is called perfect induction. It would be better to call it formal induction.

509. In a similar way, from the syllogism:

$$\begin{aligned} &\text{Any } M \text{ is } \Pi' \ P', \\ &\text{Any } \ S \text{ is } \quad M; \\ \therefore &\text{Any } \ S \text{ is } \Pi' \ P'; \end{aligned}$$

where $\Pi' \ P'$ denotes the conjunction of all the characters of M, if the conclusion and first premiss are true, the second premiss is true by definition; so that we have the demonstrative form of argument:

$$\begin{aligned} &\text{Any } M \text{ is } \Pi' \ P', \\ &\text{Any } \ S \text{ is } \Pi' \ P'; \\ \therefore &\text{Any } \ S \text{ is } \quad M. \end{aligned}$$

This is reasoning from definition, or, as it may be termed, formal hypothesis.

510. One half of all possible propositions are true, because every proposition has its contradictory. Moreover, for every true particular proposition there is a true universal proposition, and for every true negative proposition there is a true affirmative proposition. This follows from the fact that the universal affirmative is the type of all propositions. Hence of all possible propositions in either of the forms,

$$\Sigma \ S' \text{ is } M, \text{ and } M \text{ is } \Pi' \ P',$$

one half are true. In an untrue proposition of either of these forms, some finite ratio of the S's or P's are not true subjects or predicates. Hence, of all propositions of either of these forms which are partly true, some finite ratio more than one half are wholly true. Hence, if in the above formulæ for formal induction or hypothesis, we substitute S' for $\Sigma' \ S'$ and P' for $\Pi' \ P'$ we obtain formulæ of probable inference. This reasoning gives no *determinate* probability to these modes of inference, but it is necessary to consider that, however weak

synthetic inference might have been at first, yet if it had the least positive tendency to produce truth, it would continually become stronger, owing to the establishment of more and more secure premisses.

511. The rules for valid induction and hypothesis deducible from this theory are as follows:

1. The explaining syllogism, that is to say, the deductive syllogism, one of whose premisses is inductively or hypothetically inferred from the other and from its conclusion, must be valid.

2. The conclusion is not to be held as absolutely true, but only until it can be shown that, in the case of induction, S' was taken from some narrower class than M, or, in the case of hypothesis, that P' was taken from some higher class than M.

3. From the last rule it follows as a corollary that in the case of induction the subject of the premisses must be a sum of subjects, and that in the case of hypothesis the predicate of the premisses must be a conjunction of predicates.

4. Also, that this aggregate must be of different objects or qualities and not of mere names.

5. Also, that the only principle upon which the instanced subjects or predicates can be selected is that of belonging to M.[1]

[1] Positivism, apart from its theory of history and of the relations between ths sciences, is distinguished from other doctrines by the manner in which it regarde hypotheses. Almost all men think that metaphysical theories are valueless, because metaphysicians differ so much among themselves; but the positivists give another reason, namely, that these theories violate the sole condition of all legitimate hypothesis. This condition is that every good hypothesis must be such as is certainly capable of subsequent verification with the degree of certainty proper to the conclusions of the branch of science to which it belongs. There is, it seems to me, a confusion here between the probability of an hypothesis in itself, and its admissibility into any one of those bodies of doctrine which have received distinct names, or have been admitted into a scheme of the sciences, and which admit only conclusions which have a very high probability indeed. I have here to deal with the rule only so far as it is a general canon of the *legitimacy* of hypotheses, and not so far as it determines their *relevancy* to a particular science; and I shall, therefore, consider only another common statement of it; namely, "that no hypothesis is admissible which is not capable of verification by direct observation." The positivist regards an hypothesis, not as an inference, but as a device for stimulating and directing observation. But I have shown above that certain premisses will render an hypothesis probable,

Hence the formulæ are:

Induction.

$S'\ S''\ S'''$, etc. are taken at random as M's,
$S'\ S''\ S'''$, etc. are P;
$\therefore$ Any M is probably P.

so that there is such a thing as legitimate hypothetic inference. It may be replied that such conclusions are not hypotheses, but inductions. That the sense in which I have used "hypothesis" is supported by good usage, I could prove by a hundred authorities. The following is from Kant: "An hypothesis is the holding for true of the judgment of the truth of a reason on account of the sufficiency of its consequents." Mill's definition (*Logic*, Book III, Ch. XIV §4) also nearly coincides with mine. Moreover, an hypothesis in every sense is an inference, because it is adopted for some reason, good or bad, and that reason, in being regarded as such, is regarded as lending the hypothesis some plausibility. The arguments which I term hypothetic are certainly not inductions, for induction is reasoning from particulars to generals, and this does not take place in these cases. The positivist canon for hypotheses is neither sufficient nor necessary. If it is granted that hypotheses are inferred, it will hardly be questioned that the observed facts must follow apodictically from the hypothesis without the aid of subsidiary hypotheses, and the characters of that which is predicated in the hypothesis, and from which the inference is drawn, must be taken as they occur, and not be picked out in order to make a plausible argument. That the maxim of the positivists is superfluous or worse, is shown; first, by the fact that it is not implied in the proof that hypothetic inference is valid; and next, by the absurdities to which it gives rise when strictly applied to history, which is entirely hypothetical, and is absolutely incapable of verification by direct observation. To this last argument I know of but two answers: first, that this pushes the rule further than was intended, it being considered that history has already been so verified; and second, that the positivist does not pretend to know the world as it absolutely exists, but only the world which appears to him. To the first answer, the rejoinder is that a rule must be pushed to its logical consequences in all cases, until it can be shown that some of these cases differ in some material respect from the others. To the second answer, the rejoinder is double: first, that I mean no more by "is" than the positivist by "appears" in the sense in which he uses it in saying that only what "appears" is known, so that the answer is irrelevant; second, that positivists, like the rest of the world, reject historic testimony sometimes, and in doing so distinguish hypothetically between what is and what in some other sense appears, and yet have no means of verifying the distinction by direct observation.

Another error in reference to hypothesis is that the antecedent probability of what is testified to cannot affect the probability of the testimony of a good witness. This is as much as to say that probable arguments can neither support nor weaken one another. Mr. Venn goes so far as to maintain the impossibility of a conflict of probabilities. The difficulty is instantly removed by admitting indeterminate probabilities.

Hypothesis

Any M is, for instance, $P'\ P''\ P'''$, etc.
S is $P'\ P''\ P'''$, etc.;
$\therefore$ S is probably M.

§2. MOODS AND FIGURES OF PROBABLE INFERENCE

512. It is obvious that the explaining syllogism of an induction or hypothesis may be of any mood or figure.

It would also seem that the conclusion of an induction or hypothesis may be contraposed with one of the premisses.

§3. ANALOGY

513. The formula of analogy is as follows:

S', S'', and S''' are taken at random from such a class that their characters at random are such as P', P'', P'''.

t is P', P'', and P''',
S', S'', and S''' are q;
$\therefore$ t is q.

Such an argument is double. It combines the two following:

1.

S', S'', S''' are taken as being P', P'', P''',
S', S'', S''' are q;
$\therefore$ (By induction) P', P'', P''' is q,

t is P', P'', P''';
$\therefore$ (Deductively) t is q.

2.

S', S'', S''' are, for instance, P', P'', P''',
t is P', P'', P''';
$\therefore$ (By hypothesis) t has the common characters of S', S'', S''',

S', S'', S''' are q;
$\therefore$ (Deductively) t is q.

Owing to its double character, analogy is very strong with only a moderate number of instances.

§4. FORMAL RELATIONS OF THE ABOVE FORMS OF ARGUMENT

514. If we take an identical proposition as the fact to be explained by induction and hypothesis, we obtain the following formulæ:

By Induction

S, S', S'' are taken at random as being M,
S, S', S'' have the characters common to S, S', S'';
$\therefore$ Any M has the characters common to S, S', S''.

By Hypothesis

M is, for instance, P, P', P'',
Whatever is at once P, P', and P'' is P, P', P'';
$\therefore$ Whatever is at once P, P', and P'' is M.

By means of the substitution thus justified, Induction and Hypothesis can be reduced to the general type of syllogism, thus:

Induction

S, S', S'' are taken as M,
S, S', S'' are P;
$\therefore$ Any M is P.

Reduction

S, S', S'' are P;
Almost any M has the common characters of S, S', S''.
$\therefore$ Almost any M is P.

Hypothesis

M is, for instance, P', P'', P''',
S is P', P'', P''';
$\therefore S$ is M.

Reduction

Whatever is, at once, P', P'', P''' is like M,
S is P', P'', P''';
$\therefore S$ is like M.

515. Induction may, therefore, be defined as argument which assumes that a whole collection, from which a number of instances have been taken at random, has all the common characters of those instances; hypothesis, as an argument

which assumes that a term which necessarily involves a certain number of characters, which have been lighted upon as they occurred, and have not been picked out, may be predicated of any object which has all these characters.

516. There is a resemblance between the transposition of propositions by which the forms of probable inference are derived and the contraposition by which the indirect figures are derived; in the latter case there is a *denial* or change of modal quality; while in the former there is reduction from certainty to probability, and from the sum of all results to some only, or a change in modal quantity. Thus probable inference is related to apagogical proof, somewhat as the third figure is to the second. Among probable inferences, it is obvious that hypothesis corresponds to the second figure, induction to the third, and analogy to the second-third.

CHAPTER 3

EXTENSION OF THE ARISTOTELIAN SYLLOGISTIC[P]

§1. ON A LIMITED UNIVERSE OF MARKS*

517. De Morgan and his followers frequently speak of a "limited universe of discourse" in logic.[1] An unlimited universe would comprise the whole realm of the logically possible. In such a universe, every universal proposition, not tautologous, is false; every particular proposition, not absurd, is true. Our discourse seldom relates to this universe: we are either thinking of the physically possible, or of the historically existent, or of the world of some romance, or of some other limited universe.

518. But besides its universe of objects, our discourse also refers to a universe of characters. Thus, we might naturally say that virtue and an orange have nothing in common. It is true that the English word for each is spelt with six letters, but this is not one of the marks of the universe of our discourse.

519. A universe of things is unlimited in which every combination of characters, short of the whole universe of characters, occurs in some object. In like manner, the universe of characters is unlimited in case every aggregate of things short of the whole universe of things possesses in common one of the characters of the universe of characters. Ordinary syllogistic as set forth [in 453] supposes an unlimited universe of characters.† The non-possession of any character is regarded as implying the possession of another character the negative

* §1 and §2 are "Note A," the Johns Hopkins *Studies in Logic*, edited by C. S. Peirce, Little, Brown and Co., Boston, (1883) pp. 182–6, as rewritten in 1893, for the *Grand Logic*, ch. 13. Cf. 3.345ff and 3.403F ff.

[1] The term was introduced by De Morgan in 1846. *Cambridge Philosophical Transactions*, VIII (1849), p. 380.

† Originally: "The conception of ordinary syllogism is so unclear that it would hardly be accurate to say that it supposes an unlimited universe of characters; but it comes nearer to that than to any other consistent view."

of the first. Nobody has more strenuously insisted upon this than De Morgan.*

520. In our ordinary discourse, on the other hand, not only are both universes limited, but, further than that, we have nothing to do with individual objects nor simple marks; so that we have simply the two distinct universes of things and marks related to one another, in general, in a perfectly indeterminate manner. The consequence is, that a proposition concerning the relations of two groups of marks is not necessarily equivalent to any proposition concerning classes of things; so that the distinction between propositions in extension and propositions in comprehension is a real one, separating two kinds of facts; whereas in the view of ordinary syllogistic the distinction only relates to two modes of considering any fact. To say that every object of the class S is included among the class of P's , of course must imply that every common character of the P's is a common character of the S's. But the converse implication is by no means necessary, except with an unlimited universe of marks. The reasonings in depth of which I have spoken, suppose, of course, the absence of any general regularity about the relations of marks and things.

521. I may mention here another respect in which this view differs from that of ordinary logic, although it is a point which has, so far as I am aware, no bearing upon the theory of probable inference. It is that under this view there are propositions of which the subject is a class of things, while the predicate is a group of marks. Of such propositions there are twelve species, distinct from one another in the sense that any fact capable of being expressed by a proposition of one of these species cannot be expressed by any proposition of another species.† The following are these forms. S means an object of the class S, while Π means a quality of the group Π; and "has" means possesses as a subject and attributed; "is had by" means inheres in; "wants" and "is wanted by" are negatives of "has."

1. Every S has every Π.
2. Some S has all Π.

* This sentence was added in 1893.

† The remainder of this section and the whole of the next differ in detail from the article as first published.

3. Every Π is had by some *S*.
4. Some Π is had by all *S*'s.
5. Every *S* has some Π.
6. Some *S* has some Π.
7. Every *S* wants every Π.
8. Some *S* wants all Π.
9. Every Π is wanted by some *S*.
10. Some Π is wanted by all *S*'s.
11. Every *S* wants some Π.
12. Some *S* wants some Π.*

These are the variations produced by the different connections of the verb without taking account of *not* applied to *S* and Π which would, of course, multiply them by four.

522. I term the above the forms of the *first order*, because they contain "has" or "is had by" "wants" or "is wanted by" once, and once only. The following is an example of a proposition of the *second order:*

"Every *S* wants some character possessed by every *P*." The following is of the *third order:*

"Some *S* wants some character had by every object that possesses all the Πs."

The following is of the *zero order:*

"Every *S* is a *P*."

523. It has been explained [in 458] that (speaking with sufficient accuracy for the purpose) "Some" means that the speaker is to select an instance, while "Every" or "Any" means that a second person is to perform the selection. Of course, it is easier to satisfy the conditions of a statement if one can select one's own examples, except for this, that he who undertakes to find an example guarantees that there *is* one, while if he leaves the selection to another, and there is none, his statement is not broken down. Consequently, from *Every* we can infer *Some*, provided only we have some other premiss which assures us that something exists. Thus, if and only if, we are assured some *S* exists we can infer 2 from 1,

* See 3.345 for the interpretation of some of these in terms of "relatives." They are perhaps clearer when expressed with quantifiers, thus: 1. (s) (Π) Πs. 2. (Σs) (Π) Πs. 3. (Π) (Σs, Πs. 4. (ΣΠ) (s, Πs. etc., where (Σs) means "some s" and (s) means "every s."

8 from 7, 6 from 5, and 12 from 11. In like manner, if, and only if, we are assured some Π exists we can infer 4 from 1, 6 from 3, 10 from 7, and 12 from 9.

524. If the first person is to select one object and a second person another, it will always be an advantage to the former to wait and see what selection the latter makes. Consequently, 3 always follows from 2. For 2, or

Some S has all Π's,

asserts that an S can be selected so that after it is selected, and the selection made known, no matter what Π be selected, that S will have that Π; while 3, or

Every Π is had by some S,

asserts that no matter what Π be selected, after it is selected, and the selection made known, an S can be so chosen that that S will have that Π.* In like manner, 5 always follows from 4, 9 from 8, and 11 from 10.

525. Form 1 is precisely denied by 12, 2 by 11, 3 by 10, 4 by 9, 5 by 8, and 6 by 7. Forms 2 and 10, 4 and 8 are, therefore, contraries: they cannot both be true but may both be false. Forms 3 and 11, 5 and 9 are sub-contraries: both cannot be false, but both may be true.

Other pairs of forms in the same terms may be both false, both true, or either false while the other is true.

526. In this system syllogisms hold between any two propositions having a middle term of different *phasis* in the two premisses. We should therefore expect in each figure 21 different species of syllogisms. But two of these infer no more than might be inferred from weaker premisses. I am indisposed to admit them as distinct species (for these species are more natural classes than are moods). Each species by metaphasis of the three terms and two verbs gives 32 moods, and if the premisses are of different form, 64. I enumerate the species in case the middle term is a group of qualities. I state them, when convenient, in the second figure, but regard figure little. I usually take the phasis of the verb differently in the two premisses.

* $(\Sigma s)\,(\Pi)\,\Pi s \prec (\Pi)\,(\Sigma s)\,\Pi s$.

First Species

Every P has no quality but μ's,
Every S has every μ;
$\therefore$ Every S has every quality in any P.

Second Species

Every P has μ's as its only qualities,
Some S has all μ's;
$\therefore$ Some S has every quality of any P.

Third Species

Every P has μ's for its only qualities,
Every μ is in some S or other;
$\therefore$ Qualities of S's are alone possessed by any P's.

This conclusion follows from the last.

Fourth Species

Every P has μ's for its only qualities,
A certain quality not μ is in all S's;
$\therefore$ There is a quality of all S's that is absent from all P's.

Hence, again, No S is a P. But this last inference would not hold if the phasis of the two premisses were not contrary.

Fifth Species

Every P has μ's for its sole qualities,
Every S has some quality or other beside the μ's;
$\therefore$ Every S possesses a quality unpossessed by any P.

Hence, again, No S is a P, an inference only holding on account of the antiphasis of the premisses. But the general conclusion of this species is much weaker than the last which asserts that there is a single quality which belongs to all S's and no P's; while the present inference is that each S has some quality or another absent from all P's.

Sixth Species

Every P has μ's for its sole qualities,
Some S has a quality not a μ;
$\therefore$ Some S has a quality unpossessed by any P.

Hence, again (on account of the antiphasis) Some S is not a P.

Seventh Species

Some P has μ's for its sole qualities,
Some S has all μ's;
$\therefore$ A certain S has all the qualities of a certain P.

Eighth Species

Every quality not a μ is absent from some P or other,
Some S has all μ's;
$\therefore$ Some S has all properties common to all P's.

This conclusion follows from the last.

Ninth Species

Every quality not a μ is absent from some P or other,
Every μ is in some S;
$\therefore$ Every property common to all Ps is in some S.

This conclusion follows from the last.

Tenth Species

According to the order of arrangement followed, I should here insert the syllogism from the premisses:

Some μ is absent from all P's,
Some S has all μ's;

But no more can be inferred from these premisses than if the last is weakened to

Every μ is in some S or other.

The conclusion in either case is:

Some S has a quality that no P has,

which is precisely the conclusion of the sixth species. As remarked, owing to the antiphasis, it further follows that

Some S is not a P.

Eleventh Species

Every P wants some μ,
Some S has every μ;
$\therefore$ Some S has some quality or other that each P wants.

Hence, further, owing to the antiphasis, some S is not P. The general conclusion here is much weaker than that of the last species, where there was said to be a quality possessed by an S but unpossessed by any P; while here it is only stated that there is a certain S which has one quality that one P wants

and perhaps a different one that others want, etc. But it is not so weak as if it only asserted that some S or other has each quality that any P wants. One of the premisses of the eleventh species follows from one of the tenth, and one of the tenth from one of the eleventh. But this does not balance matters, owing to one of these involving the middle term differently from the other.

Twelfth Species

Every P wants some μ or other,
Every μ is in some S or other;
$\therefore$ Every P wants some quality of some S.

In consequence of the antiphasis, we have the further inference, Every P is not some S. The general conclusion of the twelfth species follows from that of the eleventh.

Thirteenth Species

Some quality not a μ is absent from all P's,
Some μ is in all S's;
$\therefore$ Some quality of all S's is other than some quality absent from all P's.

We are now getting to premisses so weak that with one exception all further conclusions are spurious.* The present conclusion may be otherwise stated thus: The qualities absent from all P's together with those in all S's amount to two, at least.

Fourteenth Species

Every P wants some quality or other not μ,
Some μ is in all S's;
$\therefore$ Some quality of all S's is other than some quality or other absent from any given P.

This conclusion follows from the last. It does not speak of the quality of the S's as being other than one quality absent from all P's, but as being such that each P has some quality or other different from it.

* . . . "the conclusion is of a kind called *spurious* by De Morgan if, and only if, the middle term is affected by 'some' in both premisses"— in the original, p. 185. See 607; De Morgan, *Formal Logic*, pp. 153–4 and *Syllabus*, §76ff.; and B. Gilman, *Johns Hopkins University Circular*, August, 1882, on these "Spurious Propositions."

Fifteenth Species

Every P wants some quality or other not a μ,
Every S has some μ or other;

From these premisses two different conclusions follow, viz:

First, Every S has a quality other than some quality or other absent from each P.

Second, Every P wants a quality other than some quality or other of each S.

Each of these conclusions follows immediately from the conclusion of the Fourteenth Species.

Sixteenth Species

Next, according to the arrangement followed, I should put the premisses:

Some P wants all μ's,
Some S has some μ.

But no inference can be drawn from these which cannot equally be drawn when the first premiss is weakened, thus:

Every μ is absent from some P or other,
Some S has some μ;
$\therefore$ Some S has some quality not in some P.

Seventeenth Species

Some quality not μ is absent from all P's,
Some S has some μ;
$\therefore$ Some S has a quality different from some quality wanting to all P's.

This spurious conclusion follows from the conclusion of the Sixth Species.

Eighteenth Species

Every P wants some quality or other, other than μ,
Some S has some μ;
$\therefore$ Some S has a quality other than a quality absent from each P.

This conclusion follows from the last.

Nineteenth Species

Some P wants some quality not μ,
Some S has some μ;
$\therefore$ Some S possesses a quality other than some quality absent from a P.

527. Among the above nineteen propositions, I have introduced a misstatement or two in order that the reader may have the profitable pleasure of thinking the whole matter out *in his own way*, as an exercise in reasoning. If any logician does not think the above nineteen really different kinds of inference are as important as eighteen out of the nineteen traditional moods, then I do not envy him his perspicacity.

§2. GENERAL CANON OF SYLLOGISM[P]

528. It has for ages been customary with logicians, in addition to establishing valid forms of syllogism, to put forth certain rules independent of figure, or the special forms used, which were supposed to form of themselves sufficient criteria for the ascertainment of the validity or invalidity of a syllogistic argumentation.

529. Most of the rules so given fail at being universally true. The only such canon that really holds is that in order that there should be a syllogistic conclusion there must be two premisses containing a middle term "distributed" in one, but not in the other, and the conclusion will be drawn by compounding the two premisses in such a way that this middle term may be dropped upon the same principle upon which it is dropped in Barbara.

530. A term is said to be "distributed" when it substantially fulfills the function of an antecedent, and is said to be "undistributed" when it substantially fulfills the function of a consequent. When a proposition is made an antecedent, every possible case of its being true is spoken of; and when it is made a consequent every possible state of its being false is spoken of. For to say "If A is true, B is true," is to say that every possible state of things in which A is true is different from every possible state of things in which B is false. Thus, by considering in one premiss every possible state of things in which the middle is true and in the other every possible state of things in which the middle is false, between them every possible state of things is considered. The syllogism is thus conceived under the form

$$\text{If } M \text{ is true, } P \text{ is } \begin{cases} \text{true,} \\ \text{false,} \end{cases}$$

$$\text{If } M \text{ is false, } Q \text{ is } \begin{cases} \text{true,} \\ \text{false,} \end{cases}$$

$$\therefore \text{If } P \text{ is } \begin{cases} \text{false,} \\ \text{true} \end{cases} Q \text{ is } \begin{cases} \text{true,} \\ \text{false.} \end{cases}$$

This shows that every term not the middle term preserves in the conclusion the same distribution it had in the premiss.

531. The thirteenth species of syllogism suggests such forms as the following:

> The qualities of all S's together with those absent from all P's amount to two at least,
> The qualities absent from some S's amount to one at most;
> $\therefore$ The qualities of all S's amount to one at least.

This is plainly a case of the principle that the degree of particularity of the conclusion is the *algebraic sum* of those of the premisses.

§3. HAMILTON'S QUANTIFICATION OF THE PREDICATE*

532. Quite clearly, the Aristotelian syllogistic omits something. Many of the old logicians would dispute this; but it is because they forget altogether the purposes of logic, not so much its practical purpose, to teach men to reason well, as its theoretical purpose to analyze reasoning and see what it consists in, so far as it consists in what cannot be false, even if all unqualified opinions are mistaken. It may be, in fact I myself maintain, that it ought to be assumed that all our knowledge has some error in it — even our knowledge that there is something real. I do not believe that anything (unless it be God) quite fulfills the idea of the real. But then, nobody can sincerely doubt that things come pretty near to being real. Those facts that there is no room to question, logic ought (as I shall show) to take account of, so far as they concern reasoning. It is true, that logic does not go into the business of doing people's reasoning for them; so far as it can be relegated to anybody, mathematics undertakes that business. Hence it is that geometry, etc. are not parts of logic, but of mathematics. . . .

* From ch. 10, "Extension of the Aristotelian Syllogistic," of the *Grand Logic*, 1893.

Perhaps I am wrong about words; that it is not the logician *qua* logician, who has to analyze reasoning and explain the gist of it. Certainly, if a given man is usefully occupied, I do not mean to recommend him to do something else than he is inclined to undertake. But as for gentlemen who use the shibboleth of "*extralogical*"— and so offensively that shilaly was on my tongue — who seem to maintain, and many of them clearly say so, that the study of relations in general ought not be pursued by any man, I can only say that the sort of logic they are cultivating seems to me the barrenest and most trifling of things, unless it is destined to be developed into something more.

Until the time of Laurentius Valla, who died in 1457, no logic had ever mentioned the dilemma. The principle of excluded middle was "extralogical." This to my apprehension argues a certain narrowness on the part of the earlier logicians; and yet it is a fact admitting of no dispute that Aristotle and all the medieval logicians admitted far more into logic, than any but the most progressive party — and the party of progress is *ipso facto* perpetually in the minority — are willing to have the logical treatises of today consider. They, for example, were unanimous in thinking the general theory of relations a part of logic.

533. I propose in this chapter to consider one or two proposed enlargements of the Aristotelian syllogistic. Some of the proposals that have been made by the most eminent men seem to me utterly undeserving of mention, in a book like this which avoids altogether a criticism of recent contributions to logic. I barely except from these a proposal to which nothing induces me to refer except the singular celebrity it has attained, so that everybody has heard of it and expects to find something about it here, together with the fact that it will serve as a foil to bring out the merits of other proposals, and also as an instructive example of how not to do it — I allude to Sir William Hamilton's scheme of the quantified predicate. I am to a certain extent an admirer of Hamilton. I have learned much from his notes to Reid and some of his other writings. But formal logic is a mathematical subject; and Hamilton was exceptionally weak in mathematics. He wrote down eight propositions,

All S is all P,	Any S is not any-P,
All S is some P,	Any S is not some P,
etc.,	etc.,

and letting these propositions mean what the words seemed to suggest without any penetrating criticism — without a suspicion that anything of that sort was wanted — hastily drew up a silly table of inferences, which unmathematical logicians — the very ones who oppose any *rational* scheme of this sort as "extralogical"— extol to the skies. The reckless Hamilton flew like a dor-bug into the brilliant light of De Morgan's mind in a way which compelled the greatest formal logician that ever lived to examine and report upon the system. There was a considerable controversy; for Hamilton and several of his pupils were as able in controversy as they were impotent in inquiry — but De Morgan's final and unanswerable paper will be found in the *Cambridge Philosophical Transactions*, [1863] X, 428.*

534. Though Hamilton never wrote a book in exposition of his system, he did better. He drew up (in 1846) a list of sixteen very precisely expressed points which such an exposition ought to contain, and offered a prize for the performance. His most attentive scholar, Thomas Spencer Baynes, gained the prize by an essay most clear, most concise, and which convinces the reader he has the matter just as Hamilton at his best conceived it. Four years later, the essay was published;† and Hamilton again wrote to express his unqualified approval, except only that he offers a note "showing in what respect my [his] present view differs from those stated by you in the Essay." This note goes into the smallest minutiæ; so that there can be no possible mistake as to Hamilton's doctrine. Moreover, the appendices to Hamilton's *Discussions* and, much more important, the appendices to his *Lectures on Logic* contribute full verifications of every point; so that it has seldom happened in the history of philosophy that an author has left so fully satisfactory records of his thought behind him. Yet when, after Hamilton's death, De Morgan's attack came out, Baynes and all Hamilton's scholars were perfect know-noth-

* See also Appendix 1 to De Morgan's *Formal Logic*.

† *An Essay on the New Analytic of Logical Forms*, Edinburgh, (1850).

ings, could not say what he had taught at all, but thought it must have been something very different from what he and Baynes, for him, had most distinctly said, and which anybody with a head can see necessarily follows from the initial assumption.

The first of the sixteen requirements for the prize essay was that it should state "what logic *postulates* as a condition of its applicability." Note the tone of classroom question. No answer will gain the prize, one feels sure, unless it most precisely reproduces the author's opinion, without subtraction nor addition. Baynes' statement is admirably clear. The postulate is "that we be allowed to state in language what is contained in thought."*

That the doctrine of the paragraphs was really Hamilton's doctrine, is shown, not only by all the evidences above, but by the further testimony of Archbishop Thomson and Dean Mansel, two of his foremost pupils. The very style apes Hamilton. . . .

But though Hamilton was surer than sure that the true propositional forms were those eight, he never did succeed, nor have any of the repeaters of the shibboleth succeeded, in satisfying their own minds as to the meaning of those eight formulæ!! At one time, we find Hamilton holding, what the logic [of the paragraphs] would absolutely require, that each proposition is absolutely definite (*Lectures on Logic*, Appendix V, iv. (*a*)). He is not mathematician enough to see that in that case he would want either 16 or only 7 propositional forms. He himself only sees 4; he went over this statement to every class for twenty years, without seeing the absurdity of it; and it is repeated in Hamiltonian logics to this day! Every geometer knows there are only 3 nonmetrical situations which two conics can have relatively to one another (without contact). They are:

1. Coincidence,
2. Intersection,
3. Non-intersection.

Each conic may be regarded as representing the division of the universe into those things to which a term applies and those to which it does not apply. Then:

* See *Lectures on Logic*, VI, p. 114.

1. Coincidence gives two relations, between two terms, according as the two terms are on the same or opposite sides of the line. But Hamilton only recognizes the former.

2. Intersection gives only one relation.

3. Non-intersection gives four relations, two of which, however, are of the same form (the terms only being transposed). These cases Hamilton recognizes (considering them as one only), and also the case of S, P, and what is neither S nor P, making up the universe. But the case in which the different things in the universe are all either P or S exclusively or both at once, he overlooks.

535. Hamilton is thus shown (*a*) to be led by the nose by the words *some* and *all*, whose claim to admission into the predicate he admits without knowing what it means; and (*b*) to be utterly incapable of doing the simplest logical thinking, perfectly indispensable for the problem in hand. I will further say, (*c*) that the psychological analysis upon which the whole doctrine is built, according to which a proposition is a comparison of notions, is all wrong. It involves incidentally the absurdity that I cannot think one planet is inhabited without thinking whether they all are so, without exception. Further, (*d*) that his grand postulate is ridiculous. It is a mere trick to get us to subscribe to his humble petition to be allowed to say what he thinks, only to find that this implies that whatever a man has in thought is pertinent to his reasoning! (*e*) Moreover, De Morgan, in the memoir referred to, has shown beyond dispute that Hamilton's syllogistic involves paralogisms. Now, a man capable of a downright paralogism is an exceptionally weak reasoner; and this I think Hamilton's writings taken together show that he was. Yet in some respects his thinking powers were admirable.

§4. UNIVERSE OF DISCOURSE*[E]

536. In every proposition the circumstances of its enunciation show that it refers to some collection of individuals or of possibilities, which cannot be adequately described, but can only be indicated as something familiar to both speaker and auditor. At one time it may be the physical universe, at

* From "Universe," *Dictionary of Philosophy and Psychology*, vol. 2, p. 742, by Peirce and Mrs. C. Ladd-Franklin.

another it may be the imaginary "world" of some play or novel, at another a range of possibilities.

The term was introduced by De Morgan in 1846 (*Cambridge Philosophical Transactions*, viii, 380) but De Morgan never showed that he fully comprehended it. It does not seem to be absolutely necessary in all cases that there should be an index proper outside the symbolic terms of the proposition to show what it is that is referred to; but in general there is such an index in the environment common to speaker and auditor. This De Morgan has not remarked; but what he has remarked has likewise its importance, namely, that for the purposes of logic it makes no difference whether the universe be wide or narrow. The idea of a limited logical universe was adopted by Boole* and has been employed by all subsequent exact logicians. There is besides a universe of marks or characters, whenever marks are considered substantively, that is, as abstractions, as they commonly are in ordinary speech, even though the forms of language do not show it. Thus only, there comes to be a material difference between an affirmative and a negative proposition. For it will then alone be one thing to say that an object wants some character common to all men and another to say that it possesses every character common to all non-men. Only instead of giving *three* qualities it gives four, for the assertion may be that an object wants some character common to all non-men; a point made by ancient writers. . . .

* See *An Investigation of the Laws of Thought*, etc., p. 42.

CHAPTER 4

NOTES ON EXPLICATIVE REASONING

§1. LOGICAL*

537. Irrespective of any facts except those of which logic needs to take cognizance, such as the facts of doubt, truth, falsity, etc.

538. *Logical possibility* is, according to usage, freedom from all contradiction, explicit or implicit; and any attempt to reform the inaccuracy would only bring confusion.

539. *Logical necessity* is the necessity of that whose contrary is not logically possible.

540. *Logical induction* is an induction based on examination of every individual of the class to which the examination relates. Thus, conclusions from a census are logical inductions. While this mode of inference is a degenerate form of induction, it also comes into the class of dilemmatic reasoning.

541. *Logical truth* is a phrase used in three senses, rendering it almost useless.

(1) The harmony of a thought with itself. Most usually so defined, but seldom so employed. So far as this definition is distinct, it makes logical truth a synonym for logical possibility; but, no doubt, more is intended. (Hamilton, *Lectures on Logic*, xxvii.)

(2) The conformity of a thought to the laws of logic; in particular, in a concept, consistency; in an inference, validity; in a proposition, agreement with assumptions. This would better be called *mathematical truth*, since mathematics is the only science which aims at nothing more. (Kant, *Kritik der reinen Vernunft*, First ed., 294.)

(3) More properly, the conformity of a proposition with the reality, so far as the proposition asserts anything about the reality. Opposed, on the one hand, to metaphysical truth, which is an affection of the *ens*, and, on the other hand, to ethical *truth*, which is telling what a witness believes to be true. (Burgersdicius, *Inst. Met.*, chap. xviii.)

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 27–28.

542. . . . *Logical reasoning.* Reasoning in accordance with a Leading Principle [see 588f.] which thorough analysis, discussion, and experience have shown must lead to the truth, in so far as it is relied upon. But what Aristotle understood by a logical demonstration may be seen in his *De generatione animalium*, Lib. II, chap. viii.

543. . . . *Logical definition.* A strict definition by genus and specific difference. Ockham and his followers objected to the designation on the ground that the logician, as such, had no occasion to define any ordinary term, such as "man" (*Tractatus logicæ*, Pt. I, chap. xxvi).

§2. PURE*

544. . . . *Pure Logic*, a phrase often used, but to which no distinct conception can be attached. The following explanation by Hamilton (*Lectures on Logic*, App. I) is as good an explanation as can be given: "The doctrine which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the forms of thought, or in the conditions of the mind itself, which is the subject in which knowledge inheres — this science may be called *formal*, or *subjective*, or *abstract*, or *pure logic*. The science, again, which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the contents, materials, or objects about which logic is conversant — this science may be called *material*, or *objective*, or *concrete*, or *applied logic*." Perhaps we may say that pure logic is a logic deduced from hypotheses (which some will look upon as axioms) without any inquiry into the observational warrant for those hypotheses.

545. . . . *Pure probation*, or *proof*, is proof by deduction from hypotheses, or axioms, without any inquiry into the observational warrant for those premisses. Such is the usual reasoning of geometry.

546. *Pure proposition*, *enunciation*, or *judgment:* a proposition *de inesse*, a proposition not affected by modality. The pure proposition, as something merely proposed or contemplated, might be considered as a sort of problematic proposition. But, in fact, it is generally identified with the assertory proposition. . . .

* *Ibid.*, vol. 2, p. 402.

§3. *ORGANON**

547. Since neither the Aristotelian definition of a speculative science, nor of a practical science, nor of an art, seemed to suit logic very well, the early peripatetics and commentators denied that it was either a science or an art, and called it an instrument, ὄργανον; but they did not precisely define their meaning. It was negative chiefly. The collection of Aristotle's logical treatises, when it was made, thus came to be called the *Organon*. . . .

§4. INTENTION†

548. Aquinas (*Summa Theologica* I, 9.53); in classical writers an act of attention (and so Aquinas, *ibid.*, I. ii, 9.38, art. 2, and elsewhere); from *in*+*tendere*, to stretch. Aquinas seems sometimes to use the term for a mode of being (ibid., I. ii. 9.22) and sometimes for a relation (ibid., I, 9.29, art. 1; 9.76, art. 3, and esp. art. 4). A concept, as the result of attention.

First intentions are those concepts which are derived by comparing percepts, such as ordinary concepts of classes, relations, etc. *Second intentions* are those which are formed by observing and comparing first intentions. Thus the concept "class" is formed by observing and comparing class-concepts and other objects. The special class-concept, *ens*, or what is, in the sense of including figments as well as realities, can only have originated in that way. Of relative second intentions, four are prominent — identity, otherness, co-existence, and incompossibility.‡ Aquinas defined logic as the science of second intentions applied to first.

§5. MATERIAL LOGIC§

549. Formal logic classifies arguments by producing forms in which, the letters of the alphabet being replaced by any terms whatever, the result will be a valid, probable, or sophistic argument, as the case may be; material logic is a logic which does not produce such perfectly general forms, but considers a logical universe having peculiar properties.

* *Ibid.*, vol. 2, p. 219.

† *Ibid.*, vol. 1, p. 561.

‡ See vol. 4, bk. I, No. 3.

§ *Dictionary of Philosophy and Psychology*, vol. 2, pp. 44–45.

Such, for example, would be a logic in which every class was assumed to consist of a finite number of individuals; so that the syllogism of transposed quantity* would hold good. In most cases material logic is practically a synonym of applied logic. But a system like Hegel's may also properly be termed material logic. The term originated among the English Occamists of the fourteenth century, who declared Aristotle's logic to be material, in that it did not hold good of the doctrine of the Trinity.

§6. LOGICAL CONTRAPOSITION AND CONVERSION†

550. On page 148 of *Mind*, the Editor proposes to resolve the inference:

1. All S is P,
$\therefore$ No not-P is S.

into two steps, thus:

2. All S is P,
$\therefore$ No S is not-P,
$\therefore$ No not-P is S.

To this I object on the ground that both steps of the latter process depend on a property of the negative which is not essential to the validity of the inference proposed to be resolved. In the universal negative proposition, *homo non est animal*, the *non* qualifies the copula. The meaning of this qualification must, however, be defined to be such that the proposition is equivalent to *homo est non animal*, taken in such a sense that the existence of a man is not asserted. We may, therefore, substitute for the forms of inference [Nos. 1 and 2] in question,

3. All S is P,
$\therefore$ All not-P is not-S;

and

4. All S is P,
$\therefore$ All S is not-not-P,
$\therefore$ All not-P is not-S.

* See 579.

† *Mind*, vol. 1 (1876), p. 424.

The word *not* here has two properties. The first is that it is a relative term. To say that an animal is not a plant, is to say that it is *other than* every plant, just as we might say that it was *superior to every plant*. The second property is that the relative term *not*, like *cousin of*, *similar to*, etc. is its own converse [i.e., is symmetrical]. Now the first inference [Nos. 1 and 3] does not depend on this second property since it is of a form which holds good for all relative terms whatever. Thus we may reason,

All negroes are men,
∴ Every lover of all men is a lover of all negroes.

On the other hand both of the steps of the proposed resolution [No. 4] do depend on the convertible character of negation.

§7. OBVERSION*

551. Hamilton (*Lectures on Logic*, xiv, and especially Appendix V (c)) states that *Conversion* in logic is sometimes called obversion.

This is a surprising statement, which neither he nor his editors are able to support by citations. It is, therefore, not unlikely that Hamilton took it at second hand.

Bain (*Logic*, Pt. I, Bk. I, chap. iii, sec. 27) says: "In affirming one thing, we must be prepared to deny the opposite: 'the road is level,' 'it is not inclined,' are not two facts, but the same fact from its other side. This process is called obversion." Bain gives no reference. The regular scholastic name for the process he describes — a name given by Abelard (*Dialectica*, 225) — is *infinitatio*. This word is very common (see, for example, Albertus Magnus in III. *Peri hermeneias*, iii; Ockham, *Logica*, II. xii, xiii; and the index to Prantl, *Logik*, iv). But somebody may have got the notion that it was "barbarous," and have preferred to use a more classical-sounding designation.

§8. SYLLOGISM†

552. (1) According to Aristotle and the great body of logicians, a simple syllogism has two premisses and a conclusion,

* *Dictionary of Philosophy and Psychology*, vol. 2, p. 199.

† *Ibid.*, vol. 2, pp. 628–9, 633–9.

and three terms, one of which, the "middle," disappears from the conclusion. In the writer's opinion, the limitation to non-relative premisses is contrary to usage and to propriety.

553. (2) The sense laid down by Aristotle at the opening of his *Analytics*, 24 b 18, and repeated with verbal accuracy at the beginning of the *Topics* and at the beginning of the *Sophistici Elenchi*, and less formally in the logical part of the *Rhetoric* (A ii, §9), namely, Συλλογισμὸς δε ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι, "A *syllogism* is a symbol (λόγος) in which, some things having been posited, something different from the assumptions necessarily joins itself to them, by being involved in the being of the facts assumed."

It will be seen that, in this definition (1), no particular stress is placed upon the premisses being two in number; although Aristotle is afterwards emphatic in insisting that every syllogism has two premisses. The following inference is, on this definition, syllogistic: "*A* stands to a lover of *B* in the relation of benefiting everybody loved by him; hence *A* benefits *B*." The two essential characters are that the inference shall be necessary, and that the fact inferred shall be involved in the very being of the facts premissed, regardless of the manner in which those premissed facts may have become known. The operation called *colligation* by Whewell,* which consists in bringing the different premisses together and applying them, the one to another, or to a repetition of itself, in a particular way, wherein lies all that calls for sagacity in deductive reasoning, is then no part of the syllogism. Aristotle seems to say (1 *Anal. Pr.* xxxii) that some necessary inferences are not syllogisms; but the passage is not clear. However, if colligation is to be excluded from syllogism, then also such an inference as the following will be non-syllogistic: "Every man is an animal; hence whatever is in a given relation (as for example, 'other than') to every man is in that relation to every animal."

An argument consisting of a single syllogism is a monosyllogism, one of more than one a polysyllogism, called also *monosyllogistic* and *polysyllogistic proof*.

554. Trendelenburg, in his *Elementa Logices Aristoteleae*,†

* See *Novum Organum Renovatum*, II, iv.

† §20.

gives an account of the origin of the word, remarking that *συλλογίζεσθαι* is properly to conjoin in arithmetical computation or in reasoning. Thus, in the *Philebus*, 41 C, where Jowett translates "you may infer," the word means to put together two conclusions already reached and then deduce a further result. In the *Theaetetus*, 186 D, where it is said that *ἐπιστήμη* does not consist *ἐν τοῖς παθήμασι*, but *ἐν τῷ περὶ ἐκείνων συλλογισμῷ*, *συλλογισμός* is a synonym for what in the *Phaedrus*, 249 B, is called simply *λογισμός*, "*ἐκ πολλῶν ἰὸν αἰσθήσεων εἰς ἓν λογισμῷ ξυναιρούμενον.*" And Trendelenburg is of opinion that there are in Aristotle traces of the broader meaning, as when he speaks of *ὁ ἐξ ἐπαγωγῆς συλλογισμός*. Others, however, think that Aristotle's language shows that he drew a distinction between *ἐπαγωγή* and the syllogism from *ἐπαγωγή*, the latter being valid only in case of a complete enumeration. Bonitz, however, gives other instances of Aristotle's using the word in a broad sense. The chief of these is in *Rhetoric*, A xi. §23, where Cope's* note quoting a similar passage in the *Poetics* is interesting. Lutoslawski points out that one of Plato's very earliest dialogues, the *Charmides*, 161 A[1], contains a regular syllogism in *Cesare* introduced by the word *συλλογισάμενος* (160 E).† He also remarks‡ that in the *Philebus*, the term *μέσον* is used in the same technical sense as in Aristotle, and adds: "If we take into consideration that it would be entirely against Plato's view of literary composition to enumerate all possible figures of syllogism in a dialogue, as is done in Aristotle's treatise, it becomes quite possible and even probable that Aristotle's theory of syllogism was more than prepared by Plato." Of course this can be no more than a surmise, but it is a reasonable one, since Plato's mathematical mind would naturally have looked at reasoning in a formal light. At any rate it is almost incredible that any man should have struck out all that is in Aristotle's *Analytics* if the ground had not been prepared. Moreover,

* *Aristotle's Rhetoric with Cope's Commentary*, Sandys' edition, vol. 1, p. 217.

[1] Written, as there is strong unpublished ground for thinking, 394 B. C., while Aristotle only went to Athens 368 or 367 B. C. (Grote thinks not till 362 B. C.) All the other dialogues here mentioned are subsequent to Aristotle's joining the school.

† *The Origin and Growth of Plato's Logic* (1897), p. 203.

‡ *Ibid.*, p. 464.

Aristotle everywhere exhibits a mind quite unmathematical, so that one cannot but suspect that he received some hints towards a general outline of syllogistic from some source. Yet we cannot, for an instant, doubt Aristotle's veracity in the last chapter of the *Sophistici Elenchi*, where he says of syllogistic: "Of this subject, on the other hand, there has not been a part cultivated and a part neglected, but nothing whatsoever of it has hitherto existed. . . . About syllogizing I am aware of absolutely nothing having been previously said (*περὶ δὲ τοῦ συλλογίζεσθαι παντελῶς οὐδὲν εἴχομεν πρότερον ἄλλο λέγειν*)."[1]

555. (1) The syllogistic inference may be analysed into several distinct steps, each of which shall consist either in inserting or in omitting something. It does not evidently follow that such a method must afford a simpler representation of necessary inference than to take the inference of the *modus ponens* as elementary. But in point of fact it does so. The passage from premiss to conclusion may be regarded as a permissible transformation, and best, of a transformation of the nature of an insertion; that is to say, the conclusion is superadded to the premisses; since the premisses remain true.

556. Aristotle's verb *συμβαίνει* in the definition of the syllogism* takes this view. The relation between the protasis and apodosis of a hypothetical proposition differs formally from the relation between a premiss and conclusion in no essential respect, except that the protasis is not positively asserted. To regard the fact C as necessarily following from the fact A is to hold that, whenever facts analogous to A are true, a conclusion, related to them as C is to A, will always be true. In the proposition, "If A is true, C is true," we may have regard

[1] It has been argued that Aristotle may here, as it is said he often does, employ the first person plural to mean the students of Plato; and also that *πρότερον ἄλλο* would not exclude aid from contemporaries. The present writer, without making any pretension to philological learning, apprehends that it is quite clear that Aristotle is speaking of himself personally, and that he means that no doctrine of the syllogism, in which he now takes the first steps (*ὡς ἐκ τοιούτων ἐξ ἀρχῆς ὑπαρχόντων ἔχειν ἡ μέθοδος*) had existed before his *Analytics* and *Topics*. Such hints as he may have received from Plato cannot (the writer believes) have been in Aristotle's memory when he penned those words. But a man does not always know how he originally came by ideas which occupied him at first little, but afterwards more and more, up to almost complete absorption for many long years.

* *Prior Analytic*, I, 1, 24b.

only to the actual state of things; in any case the proposition is equivalent to "Either A is not true, or C is true." But more usually we refer to a range of possibilities, and mean that whenever any fact analogous to A is true, that which is related to it, as C is to A, is true. The same relation subsists between the subject and predicate of a universal proposition: to say that "All men are mortal" is to say that, taking any object X whatsoever, if X is a man, X is mortal. This agrees with the definition of universal predication given by Aristotle, and commonly known as the *dictum de omni*. It will be remarked that this definition does not make a universal proposition to assert the existence of its subject.

Now, since in any possible system of logical representation illative transformation must be performed and be recognized as permissible, it follows that any representation of a universal proposition which treats any other relation than that of the conclusion (with the premisses) to the premisses alone as the principal relation expressed by the proposition, leaves the logical analysis incomplete.

557. Three figures of syllogism were recognized by Aristotle, in the first of which the middle is subject of one premiss (the *major* premiss) and predicate of the other (the *minor* premiss); in the second the middle is the predicate of both premisses; while in the third the middle is subject of both premisses. Aristotle recognizes but four moods of syllogism in the first figure. Some early Peripatetic, Theophrastus it is said, added five indirect moods: *Baralipton*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesomorum*. It is rumoured that Galen first constituted a fourth figure by transposing the premisses of these. About the sixteenth century this figure began to be commonly admitted, and is now almost universally so. With this, the five moods have somewhat unnecessarily changed their names. Those now most usual are *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

558. The logic of Relatives (q. v.;* see also Symbolic Logic†) throws great light on syllogism. It shows that the copulated premisses are, as expressed in Peirce's algebra of dual relatives, in one of the three forms,

* 3.636.

† Vol. 4, bk. II, ch. 2.

$$(x\dagger\bar{y})(\check{y}\dagger z) \qquad (x\dagger\bar{y})\check{y}z \qquad (x\bar{y})\check{y}z.$$

These give respectively

$$x\dagger z \qquad xz \qquad x\top z^*.$$

The last is a so-called spurious conclusion, but such syllogisms are unscientifically excluded from consideration in almost all treatises. There remain, therefore, but two kinds of syllogism, the *universal* and the *particular*. Beginning with *Barbara*, it can be mathematically proved that every possible necessary inference from two premisses, both having the same form as the conclusion, must depend upon a relation of *inclusion* (see Schröder, *Algebra und Logik der Relative*, 337ff., where the "solution" given of transitiveness is the most accurate possible definition of *inclusion*, in that general sense in which a thing need not necessarily include itself). Thus:

The S's are included among the M's,
The M's are included among the P's;
$\therefore$ The S's are included among the P's.

So, for example, the pseudo-syllogism $S<M$, $M<P$, $\therefore S<P$, depends partly on the fact that $S<P$ implies that the units required to measure S are included among those required to measure P, and partly on the fact that $S<P$ implies that the units required to measure P are not included among those required to measure S. Putting, therefore, s, p, and m for the units required to measure S, P, and M, we have, on the one hand,

The s's are included among the m's,
The m's are included among the p's;
$\therefore$ The s's are included among the p's;

and, on the other hand,

The s's are included among the m's,
The p's are not included among the m's;
$\therefore$ The p's are not included among the s's.

If, in order to study the differences between the different moods of syllogism, we suppose that in the universal proposition S and P are modified by relatives, these must be the same for both, in order that *Barbara* should be valid. Calling

* $\dagger$ is the sign of relative addition, $\bar{y}$ is the negative of y, $\check{y}$ is the converse of y, xy means x and y, and $x\top z$ means x is other than z.

this common relative *h*, the form of the universal affirmative, A, can only be one of the two following:

Any *h* of an *s* is an *h* of a *p*;
Any *h* of every *s* is an *h* of every *p*.

These differ merely as being the converse of one another. For putting *k* for non-*h*, the latter is equivalent to

Any *k* of a *p* is a *k* of an *s*.

We may, therefore, assume the first of the two forms as the form of A. Then the validity of *Celarent* requires only that the universal negative E shall have for its subject, "Any *h* of an *s*" or "Something not an *h* of an *s*." The validity of *Darii* requires only that the predicate of the particular affirmative, I, shall be "is an *h* of a *p*." The validity of *Ferio* requires that the subject of the particular negative, O, should agree with that of I (in a sense not easy to explain without special notation), while its predicate should be the same as that of E, and further, that if either I or E expresses existence, so should O. The validity of *Baroko* and the moods of the second figure, generally, requires that the predicate of O (and of E) should be "is not an *h* of a *p*." The validity of *Bokardo* requires that the subject of O (and of I) should be either "Some *h* of an *s*" or else "Anything but an *h* of an *s*." The result is that there are three systems of propositions which give all the traditional moods, except *Darapti*, *Felapton*, *Baralipton*, and *Fapesmo*, which are invalid if universal propositions are understood to be the exact denials of particular propositions. These three systems are:

First System.

A. Any *h* of an *S* is an *h* of a *P*;
E. Any *h* of an *S* is not an *h* of a *P*;
I. Some *h* of an *S* is an *h* of a *P*;
O. Some *h* of an *S* is not an *h* of a *P*.

Second System.

A. Any *h* of an *S* is an *h* of a *P*;
E. Something not an *h* of an *S* is not an *h* of a *P*;
I. Whatever is not an *h* of an *S* is an *h* of a *P*;
O. Some *h* of an *S* is not an *h* of a *P*.

Third System.

A. Any h of an S is an h of a P;
E. Any h of an S is not an h of a P;
I. Whatever is not an h of an S is an h of a P;
O. Whatever is not an h of an S is not an h of a P.

The O of the third system might have been taken as A, and the same forms would have been reproduced in the same way with changed h. In the ordinary syllogistic the first system is used, and h is "identical with."

With a limited universe of marks, h will be a "character of."

559. It will be observed that *Celarent* and *Darii* introduce each an additional principle in slight modification of *Barbara*, and *Ferio* reunites those principles. The *second* and *third* figures introduce marked additional principles, which the so-called fourth figure, i.e., the Theophrastean syllogism, reunites. In the second figure there is a perceptible difference between the mode of inference of *Camestres* and *Baroko* on the one hand, *Cesare* and *Festino* on the other; although in *reasoning* itself it is morally impossible to distinguish *Camestres* from *Cesare*. Parallel remarks apply to the third figure. Of the Theophrastean moods, *Frisesomorum* alone is peculiar, *Celantes* and *Dabitis* depending merely on the principles of the second and third figures respectively.

The fact that the second and third figures involve peculiar principles is shown by the fact that the modes of reduction of any mood of each involve some peculiar immediate inference.

Thus, the reduction of the second figure involves the conversion of E. Now this conversion can be stated in syllogistic form; but it will be a syllogism in *Cesare*, which is the typical mood of the second figure, thus:

No M is N,
Any N is N;
$\therefore$ No N is M.

So the third figure involves the conversion of I, and when this is put into syllogistic form, the syllogism is in *Datisi*, the typical mood of the third figure, thus:

Any N is N,
Some N is M;
$\therefore$ Some M is N.

It may be said that the convertibility of I depends upon the equiparance* of the relation which I expresses. But even in that case, it must be noted that there is a certain difference between the *individual* and *definite* convertibility commonly understood by equiparance and the *indefinite* conversion, properly understood, of I, which is alone required in ordinary syllogistic. This will appear below.

560. The second and third figures can be reduced to the first apagogically, but the principles of the reduction are different. That of the second figure is that if the *negative* (not the denial) of the result of a hypothetical assumption under a known rule is found to be true, then the negative of the subsumption is true. Thus, referring to the usual syllogism, "All men are mortal, all patriarchs are men; hence, all patriarchs are mortal"; if the rule is correct, yet all patriarchs are immortal, it follows that all patriarchs are non-men. The statement of this principle is simply a form of the conversion of E. So the third figure may be apagogically reduced by the principle that if, a subsumption under a hypothetical rule being admitted, a partial result is found true, the partial truth of the rule follows. Thus, admitting that all patriarchs are men, if it be found that some patriarchs are mortal, it follows that at least some men are mortal. The second figure can always be reduced to the first by the simple conversion of the major premiss, this being first made negative, if necessary by infinitation. Thus,

$$\begin{aligned} &\text{Any } M \text{ is } P = \text{Any } M \text{ is not not-}P, \\ &\left.\begin{matrix}\text{Any} \\ \text{Some}\end{matrix}\right\} S \text{ is not } P; \\ \therefore &\left.\begin{matrix}\text{Any} \\ \text{Some}\end{matrix}\right\} S \text{ is not } M, \end{aligned}$$

is reduced by the conversion of the major to "No not-P is M." So the third figure is reduced to the first by the simple conversion of the minor premiss. But there is here a difficulty, owing to our not having in our ordinary languages a term to express the reversal of the quantity of a proposition; that is to say, a single operation which shall change Any to Some, and Some to Any.

* I.e., symmetry.

561. The difference between a particular and a universal proposition is that, in the former, the selection of the individual of which the predicate is asserted is transferred from the hearer to a person competent and friendly to the proposition. Let this transfer be repeated, and the choice goes back to the first party. Using "some" in the sense of prescribing such transfer, "Some-some-S is P" = Any S is P. In that way *Disamis* and *Bokardo* are reduced as follows. The moods are:

$$\begin{aligned} &\text{Some } S \left\{ \begin{matrix} \text{is} \\ \text{is not} \end{matrix} \right\} P, \\ &\text{Any } S \text{ is } M = \text{Some-some-}S \text{ is } M; \\ \therefore &\text{Some } M \left\{ \begin{matrix} \text{is} \\ \text{is not} \end{matrix} \right\} P. \end{aligned}$$

Converting "Some-some-S is M" to "Some M is Some-S," we have the syllogism of the first figure —

$$\begin{aligned} &\text{Any Some-}S \left\{ \begin{matrix} \text{is} \\ \text{is not} \end{matrix} \right\} P, \\ &\text{Some } M \text{ is Some-}S; \\ \therefore &\text{Some } M \left\{ \begin{matrix} \text{is} \\ \text{is not} \end{matrix} \right\} P. \end{aligned}$$

562. Any syllogism of the second or third figure can also be reduced by transposing the premisses, and converting the two propositions not converted in the method just described. But still the same kind of conversion continues to be required in the case of each figure, thus showing that the conversions of E and I are logically allied to the second and third figures respectively. *Camestres* and *Disamis* are the easiest so to reduce; *Festino* and *Ferison* the most difficult.

Festino.

$$\begin{aligned} &\text{Some } S \text{ is } P = \text{No Some-}S \text{ is not-}P, \\ &\text{No } M \text{ is } P = \text{Any } M \text{ is not-}P; \\ \therefore &\text{Some } S \text{ is not } M = \text{No Some-}S \text{ is } M. \end{aligned}$$

The conversion of E gives from the first premiss "No not-P is Some-S." This with the second premiss gives "No M is Some-S." Thence another conversion of E gives "No Some-S is M," which is the conclusion.

Ferison.

Some S is M = Any Some-S is M,
No S is P = Some-some-S is not-P;
$\therefore$ Some M is not P = Some M is not-P.

The conversion of I gives from the second premiss "Some not-P is Some-S." This with the first premiss gives "Some not-P is M," which by another conversion of I gives "Some M is not-P," the conclusion.

563. The conversion of E depends on the fact that otherness is an equiparance. If everybody loves everybody who loves him, it follows that if every man loves every woman, then every woman loves every man; and on the corresponding principle, if every S is other than every P, then every P is other than every S. The conversion of I, on the other hand, is justified by the fact that the propositional quantities of breadth and depth are inseparable. "Some S is P" = There is an embodiment of S conforming to the idea of P.

Now observe that we do not necessarily infer that that same P is S, but only that there is somewhere a P that is S. All that is needed for this inference is that whenever there is an embodiment of an idea conforming to a second idea, there is also an embodiment of the latter idea conforming to the idea first embodied. It is the principle that there cannot be a cork that fits a bottle unless there be a bottle that is fitted by a cork.

564. When we pass to probability the figures of syllogism become of an idea highly important. Thus, take the following reasoning, the nature of which we shall presently examine:

The probability of throwing doublets with a pair of dice is $\frac{1}{6}$,
Different throws of pairs of dice are independent;
$\therefore$ The probability of throwing precisely two doublets in six throws is just half that of throwing precisely one doublet in six throws.

In order to analyse this reasoning, it is necessary to note what the premisses and conclusion mean. The precise meaning of the first premiss is that we should square our actions on each single occasion to the fact that, taking throws of the dice as they occur in the course of experience, the ratio of the number

of throws of doublets hitherto to the total number of throws hitherto would, if the course of experience were endless, become, after a time, permanently remote from every other ratio than 1:6, but would never become so from this ratio. The meaning of the second premiss is that this is so not only for all the throws, but also for every endless portion of that totality of throws whose selection is determined by the ordinal relation of the throws composing it to previous throws of any given kind; so that, where such principle of selection allows, the throws are taken as they occur in the course of experience in endless succession.

The necessary conclusion is that we should square our action on any occasion to the principle that taking an endless succession and entirely different sets of six throws, just as they occur in the course of experience, the ratio of the number of sets containing two doublets to the number of sets containing one doublet will at length permanently depart from every other ratio than 1:2, but will not so depart from this ratio.

This inference is necessary; yet it is merely probable in this sense, that we cannot be sure that the number of sets of six throws containing just one doublet would be twice the number containing two doublets in a hundred trials, or in a million, or in any other fixed number. But what is certain is that any other ratio would eventually prove decidedly wrong, while 1:2 would not. This is a probable syllogism of the first figure, since it recognizes a case as coming under a general rule, the result of which it accepts as valid in that case. Of course the reader will not fail to observe that the sense in which the reasoning is said to be a syllogism is strictly defined. It is not meant that the above is a simple syllogism.

565. But if it were not known exactly how often with a given set of dice doublets would be thrown, since they may be similarly or dissimilarly loaded, and if we simply threw the dice and counted the whole number of throws and the number of throws of doublets, and if, finding that in twelve hundred throws there were about two hundred doublets, we were thence to conclude that in the long run about one-sixth of the throws would be doublets, this would be reasoning in the third figure. This would be in no sense necessary, except that if the experimentation were continued endlessly, it must approximate to

the true ratio at last. This form of reasoning differs from the probable syllogism of the first figure, in that there the precise value of the long-run ratio was stated in advance; while here the concluded ratio is subject to modification as experience is amplified.

It is inference in the third figure, since if the rule were that any ratio far from one-sixth would be satisfactory in the long run, the same ratio would probably be tolerably satisfactory in twelve hundred throws. Now, no ratio far from one-sixth is satisfactory for this set of twelve hundred throws. Hence, the rule is probably false. Now, the inference that a general rule is false because the result to which it would lead in a special case is not true is syllogism of the third figure. Although this mode of inference is not necessary, it is necessary that the method should ultimately yield an approximately correct conclusion.

566. A probable syllogism which appears, in consequence of a want of sharp insistence upon what is really the subject and what the predicate of propositions, to be in the second figure may be essentially of the same nature as a probable syllogism in the third figure; but it seems to be impossible that it should really be so, since there can be no strict objective probabilities of the kind required. If, for instance, we knew that in a considerable series of sets of six throws, one doublet precisely had occurred in a set twice as often as two doublets precisely, and we were thence to conclude that the two dice were probably normal, because, although other constitutions of dice might produce the same result, yet they are very unusual constitutions, we have here a probability of quite another kind; and instead of its being certain, as in the third figure, that the method of inference would in the long run lead to an indefinite approximation to the truth, it is here quite possible that the concluded constitution of the dice will always be entirely different from the true one. The three figures are therefore quite distinct; and we see that probability lends an importance to the three figures which modern logicians have not been willing to accord to them.

567. Aristotle pays no attention to hypothetical syllogisms; but it is said that Theophrastus studied them. We find them treated in considerable detail by Boëthius. During the middle

ages, purely formal syllogistic made no progress worth mention. About the middle of the nineteenth century some steps were taken. Sir W. Hamilton proposed a syllogistic system of no merit (see *An Essay on the New Analytic of Logical Forms*, by T. S. Baynes, Edinburgh, 1850) based on the following system of propositional forms, called the system of the thoroughgoing quantification of the predicate.

All A is all B, meaning that the whole collection of A's is identical with the whole collection of B's.

All A is some B, meaning that the whole collection of A's is identical with part of the collection of B's.

Some A is all B, meaning that a part of the collection of A's is identical with the whole collection of B's.

Some A is some B, meaning that a part of the collection of A's is identical with a part of the collection of B's.

Any A is not any B, meaning that the whole collection of A's is excluded from the whole collection of B's.

Some A is not any B, meaning that a part of the collection of A's is excluded from the whole collection of B's.

Any A is not some B, meaning that the whole collection of A's is excluded from a part of the collection of B's.

Some A is not some B, meaning that a part of the collection of A's is excluded from a part of the collection of B's.

There is also a pretence at a second series of meanings in depth, never clearly explained. Such a system is glaringly faulty; but it had some vogue in its day. There were also some other somewhat similar systems by Archbishop Thomson,* Spalding,† etc.

568. De Morgan constructed various systems of syllogistic of much greater merit, although, after all, they only complicate the subject to no purpose. His principal system, in which he postulates that no term is without breadth or is coextensive with the universe, is based on the following propositional forms:

* *Outline of the Laws of Thought*, Fourth ed., (1857), §§77ff.

† *An Introduction to Logical Science*, (1857), §§38ff.

$X))Y$. All X's are some Y's, i.e., Any X is a Y.

$X) \cdot (Y$. All X's are not (all) Y's, i.e., Any X is non-Y.

$X(\cdot)Y$. Everything is either some X or some Y (or both), i.e., Any non-X is Y.

$X((Y$. Some X's are all Y's, i.e., Any non-X is non-Y.

$X(\cdot (Y$. Some X's are not (all) Y's, i.e., Some X is non-Y.

$X()Y$. Some X's are some Y's, i.e., Some X is Y.

$X)(Y$. Some things are not either (all) X's or (all) Y's, i.e., Some non-X is non-Y.

$X) \cdot)Y$. All X's are not some Y, i.e., Some non-X is non-Y.

These forms arose, as one might guess, from the application of *not* to the subjects of the old four; so that it might be called the system of the thoroughgoing qualification of the subject. Whether or not this qualification of the subject can be said to be involved in the structure of any syllogism is the question upon the decision of which that of the acceptance of De Morgan's system must depend.

Under these conditions (figure of course disappearing) De Morgan gets eight universal syllogisms,

$$))))\quad (\cdot)))\quad (((\cdot)\quad ((((\quad (\cdot))\cdot(\quad)))\cdot(\quad)\cdot(((\quad)\cdot((\cdot);$$

eight *major-particular* syllogisms,

$$(((\quad)\cdot((\quad)))\cdot)\quad)))(\quad)\cdot((\cdot(\quad (((\cdot(\quad (\cdot))(\quad (\cdot))\cdot);$$

and eight *minor-particular* syllogisms,

$$())\quad)\cdot)))\quad)((\cdot)\quad)(((\quad)\cdot))\cdot(\quad ())\cdot(\quad (\cdot(((\quad (\cdot((\cdot).$$

De Morgan also takes account of complex premisses. Moreover, he developed the syllogistic of relative terms, and especially the highly important *syllogism of transposed quantity*; and he, Boole, and others studied numerically definite syllogisms.

569. The following technical phrases may be defined:

Arithmetical syllogism. Blundevile, in his *Arte of Logicke* (1599), in illustration of his position that "God hath prescribed certaine bounds of necessitie . . . which bounds are Syllogismes rightly made," instances this question: "If one pound of waxe be worth a groat, what is tenne pound of waxe worth? Marry tenne groates, which is prooved by a Syllogisme

in this manner: Every pound of waxe is worth a groat; but here is ten pound of waxe: *Ergo*, they are worth ten groats: and like as in these kinds of Syllogismes Arithmeticall, the proportion which is to bee judged by mans naturall knowledge, doth shew the Consequent to bee infallible, even so the Consequents in other Syllogismes are shewed to be infallible, by such demonstrations as are not farre fetched, or doubtfull, but are manifest, plaine and evident."

570. . . . *Compound syllogism:* a hypothetical syllogism, that is, a syllogism containing a disjunctive (or conditional) or copulative premiss.

571. *Conditional syllogism:* a syllogism containing a conditional premiss, especially the *Modus Ponens* and *Modus Tollens*, although some logicians refuse to these arguments the name of syllogism.

572. . . . *Demonstrative syllogism:* a syllogism which generates science.

573. *Dialectic syllogism:* a syllogism which generates opinion, being made of probable and credible propositions. The phrase is as old as Aristotle; but the use to which syllogism is put does not alter its nature. It is the argumentation or reasoning, not the logical form, which is dialectical.

574. . . . *Direct syllogism* (συλλογισμὸς δεικτικός): a syllogism proceeding from a *rule*, and the subsumption of a *case* under that rule, to the *result* of the rule in that case. One of the four moods, *Barbara*, *Celarent*, *Darii*, *Ferio*.

575. . . . *Hypothetical syllogism:* a syllogism either conditional (or disjunctive) or copulative. Often used for a syllogism conditional but not disjunctive, although there is no material difference between a conditional and a disjunctive proposition. A copulative proposition is the precise denial of a conditional or disjunctive proposition; and every conditional or disjunctive syllogism will by apagogical transformation give a copulative syllogism. But many logicians refuse to consider copulative syllogisms to be syllogisms on the ground that the conclusion contains nothing not stated in the premisses taken together; just as the same logicians exclude other arguments from the class of syllogisms because their conclusions contain matter not contained in the premisses taken together.

576. . . . *Perfect syllogism:* a syllogism of which no part of

the leading principle can be stated as a premiss and so eliminated from the leading principle; since if this is attempted it is still needed as leading principle.

577. . . . *Relative syllogism:* a syllogism involving relative terms. Such syllogisms have been recognized as proper subjects of logic by all logicians beginning with Aristotle.

578. . . . *Valid syllogism:* a syllogism whose premisses assert facts whose being is partly composed of the being of the fact asserted in the conclusion.

579. *Syllogism of transposed quantity:* a syllogism in which the whole quantity of one concluding term, or its contrary, is applied in a premiss to the other concluding term, or its contrary, by means of a relation of one-to-*N* correspondence. As in the following: Some *X*'s are not *Y*'s, *for* every *X* there is a *Y* which is *Z*; hence, some *Z*'s are not *X*'s.

580. *Negative syllogism:** any syllogism of the second figure, or the *Modus Tollens*, where the reasoning turns upon the change of quality. The canon of syllogism, that nothing can be concluded from two negatives, is inaccurate. What is requisite, in non-relative syllogism, is that the middle term should be once distributed and once undistributed. *Darapti* and *Felapton*, which appear to violate this rule, only do so because one of the premisses, so far as it is efficient, is virtually a particular. What is requisite is, that one of the interlocutors should select the individual denoted by the middle term in one premiss and the other in the other.

§9. MIDDLE TERM AND MIDDLE†

581. The adjective μέσος is applied in Greek to a third object additional to two others, when the idea of intervening can hardly be detected. It is, therefore, perhaps needless to seek further for Aristotle's intention in calling that term, by the consideration of which two others are illatively brought into one proposition as its subject and predicate, the middle term, or middle. It is the most important factor of Aristotle's theory of reasoning.

The same word means little more than third in the phrase "principle of excluded middle," which is, indeed, often called

* From article "Negative," *Dictionary of Philosophy and Psychology*, vol. 2, p. 148.

† *Ibid.*, vol. 2, p. 77, by Peirce and Mrs. C. Ladd-Franklin.

principium exclusi tertii. See Laws of Thought [§15]. On the other hand, something which partakes of each of two disparate natures, and renders them capable of influencing one another, is called a *tertium quid* (Aristotle's ἡ τρίτη οὐσία).

§10. PREMISE AND PREMISS*

582. (Two distinct words, recognized as such by older writers, but for the last century and more confounded. Premise is a legal word, derived from the French *premise*, which is a noun derived from the phrase *les choses premises*, used in inventories. Premiss is from the French noun *premisse*, and thence from the Low Latin *praemissa*, which goes back, as a substantive, to the early part of the thirteenth century. But it was hardly looked upon as a very good Latin at any time. *Propositio* replaced it, when elegance was preferred to technical accuracy): A proposition, the consideration of which has logically affected, or contributed to the determination of, a conclusion of reasoning.

An enthymeme is usually defined as a syllogism with a suppressed premise or premiss. Now, the expression of a train of thought may be elliptical, some thought being unexpressed in the confident anticipation that the reader, or hearer, will supply it. But in thought, a premise or premiss cannot be suppressed without ceasing to be either premise or premiss. If it be so suppressed, it enters into the leading principle of the inference. Every reasoning must proceed consciously upon some general principle, or it ceases to be a reasoning, and becomes a mere feeling of inability to think otherwise. On the other hand, when a principle of reasoning becomes by analysis distinctly apprehended and the precise effect which it has upon the conclusion understood, it becomes a *premise*, or, at least, a *premiss*. There is, therefore, no such thing as a suppressed *premiss*.

583. The word "premiss" became usual in the logical sense, in English, as early as Chaucer. In Wilson's *Rule of Reason* (1552) it does not occur, the phraseology there being like the following: "The double repeate, whiche is a woorde rehearsed in bothe Proposicions, must not entre into the conclusion." But in Blundevile's *Arte of Logicke* (1599) we read: "A Syl-

* *Ibid.*, vol. 2, pp. 330–31.

logisme is a kinde of argument contayning three Propositions, whereof the two first, commonly called the premisses," etc. In Watts's and other English logics it was spelt *premiss* and *premisses*. Johnson, however, in his *Dictionary*, gives *premises* in the plural and *premiss* in the singular, as distinct words, and remarks that the latter is little used in the plural outside of technical works.* In such works the word spelt with two s's continued to be employed.

§11. MNEMONIC VERSES AND WORDS†

584. (1) Instrumenta novem sunt, guttur, lingua, palatum
Quattuor et dentes, et duo labra simul.‡

The following mnemonic verses are contained in the *Summulae Logicales* of Petrus Hispanus, but were older, perhaps very much older.

(2) "Quae?" ca. vel hyp., "Qualis?" ne. vel aff., u. "Quanta?" univ. par. in. vel sing.§ (What is the substance of a proposition? categorical or hypothetical. What is its quality? negative or affirmative. What is its quantity? universal, particular, indefinite, or singular.)

(3) Simpliciter Fēcī, convertitur Ēvă per acci,
Āstō per contra: sic fit conversio tota.
Asserit $\overline{A}$, negat $\overline{E}$, sēd universaliter ambae;
Asserit $\overline{I}$, negat $\overline{O}$, sed particulariter ambo.¶

(*E* and *I* are converted simply; *E* and *A*, *per accidens; A* and *O*, *per contrapositionem.*)

(4) Prae, contradic.; post, contra.; prae postque, subalter.
Non omnis, quidam non; omnis non, quasi nullus;
Non nullus, quidam; sed "nullus non" valet "omnis";
Non aliquis, nullus; "non quidam non" valet "omnis";
Non alter, neuter; "neuter non" praestat "uterque."

* But see second edition (1755), London, under "Premiss" where it is said: "This word is rare in the singular."

† *Dictionary of Philosophy and Psychology*, vol. 2, pp. 87–88.

‡ Cf. Prantl, *op. cit.*, III, 41.

§ *Ibid.*, III, 27.

¶ *Ibid.*, III, 43.

(*Non* placed before *omnis* or *nullus* gives the contradictory proposition; placed after, the contrary; both before and after, the subalternate.)

(5) Primus, Ămābīmūs; Ēdēntūlī que, secundus;
Tertius, Īllĭăcē; Pūrpŭrĕā, reliquus.
Destruit ū totum, sēd ā confirmat utrumque;
Destruit ē dictum, destruit ī que modum.
Omne necessăriāt; impossibilē, quasi nullus;
Possibilē, quidam; quidam non, possībile non.
Ē dictum negat, ī que modum, nihil ā, sed ŭ totum.

(The first syllable of each of the four vocables *Amabīmus*, *Edentūli*, *Illiăce*, *Purpurĕa*, is for the possible mode; the second for the contingent; the third for the impossible; the fourth for the necessary. The vowel *a* signifies that both mode and "dictum" are to be taken assertorically; *e*, that the dictum is to be denied; *i*, that the mode is to be denied; *u*, that both mode and dictum are to be denied. Each word refers to a line or order of equipollent modal forms.)

(6) Tertius est quarto semper contrarius ordo.
Sit tibi linea subcontraria prima secundae.
Tertius est primo contradictorius ordo.
Pugnat cum quarto contradicendŏ secundus.
Prima subest quartae vice particularis habens se.
Hanc habet ad seriem se lege secunda sequentem.

(The relation of "Sortem impossibile est currere" and "Sortem necesse est currere" is that of contraries; they cannot be true at once. The relation "Sortem possibile est currere" and "Sortem possibile est non currere" is that of subcontraries; they cannot be false at once. The relation of "Sortem possibile est currere" and "Sortem impossibile est currere" is that of contradictories. The relation of "Sortem possibile est non currere" and "Sortem necesse est currere" is likewise that of contradictories. "Sortem possibile est currere" follows from "Sortem necesse est currere," as does "Sortem possibile est non currere" from "Sortem impossibile est currere.")

(7) Sub. prae. prima, secundā prae. bis, tertia sub. bis.

(The first figure contains the middle term as subject and predi-

cate; the second, the middle as predicated twice; the third, the middle twice as subject.)

(8) Bārbără, Cēlārēnt, Dărĭī, Fĕrĭō, Bărălīpton,
Cēlāntēs, Dăbĭtīs, Fāpēsmō, Frīsĕsŏmōrum.
Cēsărĕ, Cāmēstrēs, Fēstīnŏ, Bărōkŏ, Dărāpti,
Fēlāptōn, Dĭsămīs, Dātīsĭ, Bŏkārdŏ, Fĕrīson.*

(These are original names of the syllogistic moods, which there is no sufficient reason for abandoning. The direct moods of the first figure are recognizable by their containing no sign of conversion, *s*, *p*, or *k*; the indirect moods — or moods of the fourth figure — by their having those signs attached either to the third vowel or to the first two. In the second figure, one of the signs *s*, *p* is attached to the first vowel, or to the second and third, or *k* is attached to the second. In the names of the moods of the third figure, *s* or *p* is attached to the second vowel, or to the first and third, or *k* to the first. There are also names for syllogisms with weakened conclusions or strengthened premisses, as well as for indirect moods of the first figure considered as belonging to a fourth. But the above rules will enable a reader to identify them. Thus, *Bramantip* can be nothing but *Baralipton;* while *Barbari* is *Barbara* with a weakened conclusion. *Camenes* can be nothing but *Celantes; Dimaris* nothing but *Dabitis; Fesapo* nothing but *Fapesmo; Fresison* nothing but *Frisesomorum.* A writer who introduces an *m* into the name of a mood containing an *s* or *p* only after its third vowel, or who omits *m* from the name of a mood having *s* or *p* after the first and second vowels, uses the fourth figure.)

(9) Simpliciter vult *s*, verti *p* verŏ per acci.
M vult transponi, *k* per impossĭbile duci.
Servat maiorem variatque secunda minorem;
Tertia maiorem variat servatque minorem.

(*s*, in the name of a mood, shows that the proposition denoted by the preceding vowel is, in a preferred mode of reduction, to be converted simply; *p*, that it is to be converted *per accidens; m* shows that the premisses are to be transposed; *k*, that the preferred reduction is by reduction of the contradictory of the conclusion to an absurdity, this contradictory of the con-

* Cf. Prantl, *op. cit.*, III, 15–16.

clusion being, in the second figure, put in place of the minor premiss — the major being retained — and in the third figure in the place of the major — the minor being retained.)

A great number of other memorial words and verses have been proposed by logicians.

§12. REDUCTION*

585. Reduction has been recognized as necessary by Aristotle and almost all logicians; and before Kant it was generally recognized as proving that indirect syllogisms involved two or more steps of inference. But Kant from the same premisses inferred that reasoning in the first figure is the only reasoning; and this extraordinary conclusion is reached by simply not calling immediate inferences reasoning, because they have not two premisses. On that ground, nothing ought to be called reasoning except uniting two propositions into one copulative proposition. Cf. Syllogism [§8].

586. Reduction is either *ostensive* or *apagogical* (*per impossibile*). Ostensive reduction proceeds directly from the premisses of the syllogism to be reduced to its conclusion, by means of conversions and a direct syllogism. Apagogical reduction proceeds indirectly, by showing by direct syllogism, that from the denial of the conclusion of the syllogism to be reduced and one of the premisses, the denial of the other premiss would follow. Any syllogism of the second figure and any of the third except Darapti and Felapton (which are not valid if the universal be taken in the Philonian† sense) can be apagogically reduced.

Ostensive reduction is either short or long. Short reduction involves the conversion of one proposition only; long reduction the conversion of two propositions, and, except in the fourth figure, the transposition of the premisses.

587. In order to ascertain that which reduction is intended to ascertain, the nature of the different moods of syllogism, the first requisite is to recognize that we do not seek to trace out the process of reasoning; but what we seek to do is to analyse the precise logical conditions which render each mood valid. When this is recognized, it becomes easy to see and to gener-

* *Dictionary of Philosophy and Psychology*, vol. 2, p. 435.

† I.e., as not implying existence.

alize the relations expressed by the premisses and note just at what point they cease to be cogent. For this purpose, appeal must be made to the logic of Relatives (q. v.).* De Morgan has worked out the moods (*Cambridge Philosophical Transactions*, X, 350).

§13. LEADING PRINCIPLE†

588. It is of the essence of reasoning that the reasoner should proceed, and should be conscious of proceeding, according to a general habit, or method, which he holds would either (according to the kind of reasoning) always lead to the truth, provided the premisses were true; or, consistently adhered to, would eventually approximate indefinitely to the truth; or would be generally conducive to the ascertainment of truth, supposing there be any ascertainable truth. The effect of this habit or method could be stated in a proposition of which the antecedent should describe all possible premisses upon which it could operate, while the consequent should describe how the conclusion to which it would lead would be determinately related to those premisses. Such a proposition is called the "leading principle" of the reasoning.

589. Two different reasoners might infer the same conclusion from the same premisses; and yet their proceeding might be governed by habits which would be formulated in different, or even conflicting, leading principles. Only that man's reasoning would be good whose leading principle was true for all possible cases. It is not essential that the reasoner should have a distinct apprehension of the leading principle of the habit which governs his reasoning; it is sufficient that he should be conscious of proceeding according to a general method, and that he should hold that that method is generally apt to lead to the truth. He may even conceive himself to be following one leading principle when, in reality, he is following another, and may consequently blunder in his conclusion. From the effective leading principle, together with the premisses, the propriety of accepting the conclusion in such sense as it is accepted follows necessarily in every case. Suppose that the leading principle involves two propositions, L and L', and

* Vol. 3.

† *Dictionary of Philosophy and Psychology*, vol. 2, pp. 1–2.

suppose that there are three premisses, P, P', P''; and let C signify the acceptance of the conclusion, as it is accepted, either as true, or as a legitimate approximation to the truth, or as an assumption conducive to the ascertainment of the truth. Then, from the five premisses L, L', P, P', P'', the inference to C would be necessary; but it would not be so from L, L', P', P'' alone, for, if it were, P would not really act as a premiss at all. From P' and P'' as the sole premisses, C would follow, if the leading principle consisted of L, L', and P. Or from the four premisses L', P, P', P'', the same conclusion would follow if L alone were the leading principle. What, then, could be the leading principle of the inference of C from all five propositions L, L', P, P', P'', taken as premisses? It would be something already implied in those premisses; and it might be almost any general proposition so implied. Leading principles are, therefore, of two classes; and any leading principle whose truth is implied in the premisses of every inference which it governs is called a "logical" (or, less appropriately, a *formal*) leading principle; while a leading principle whose truth is not implied in the premisses is called a "factual" (or *material*) leading principle.

§14. *NOTA NOTÆ**

590. The logical principle *Nota notae est nota rei ipsius*, that is, the predicate of the predicate is the predicate of the subject, which is laid down in several places by Aristotle as the general principle of syllogism. The principal passages are as follows:

"When one thing is predicated of another as its subject, whatever is said of the predicate can also be said of the subject" (*Categ.*, iii. 1 b 10).

"Whatever is said of the predicate will hold also of the subject" (*Categ.*, v. 3 b 4).

"We say that something is predicated universally when nothing can be admitted as coming under the subject of which the predicate will not hold; and the same thing holds of negation" (1 *Anal. pr.*, i. 24 b 28). The term *nota notae* is from the first words of the original of this passage.

* *Ibid.*, vol. 2, p. 183.

591. "Of whatever the species is predicated, the genus is predicable" (*Topics*, Δ i. 121 a 25).

Some writers (as Hamilton, *Lectures on Logic*, Appendix VII, xxii, b) imagine a distinction between the *nota notae* and the *dictum de omni*. Some have been so extravagant as to attribute the former to Kant, in whose *Falsche Spitzfindigkeit* (1762, ii) it is very likely that the precise phrase "nota notae est nota rei ipsius" first occurs, though similar phrases, such as "cui conveniunt notae eidem quoque convenit nomen," are common in Wolff's and other logics of the eighteenth century. But it is clear that in Aristotle's mind it was one principle, essentially that which De Morgan well called the principle of the "transitiveness of the copula."

592. Aristotle, in the last but one of the above passages, seems to regard the *nota notae* as following from the definition of universal predication. To say that "Any S is P" is to say that of whatever S is true, P is true. This amounts to deriving the transitiveness of the copula from the transitiveness of illation. If from A follows B and from B follows C, then from A follows C. This, again, is equivalent to the principle that to say that from the truth of X follows the truth of the consequence that from Y follows Z, is the same as to say that from the joint truth of X and Y follows Z [i.e., $x \prec (y \prec z) = xy \prec z$].

§15. LAWS OF THOUGHT*

593. The three formulas of identity, contradiction, and excluded middle have been widely so known, though the doctrine that they are three coördinate and sufficient laws of all thought or of all reasoning has been held by a comparatively small party which hardly survives; and it is not too much to say that the doctrine is untenable. But the designation is so familiar and convenient that those formulas may very well be referred to as "the so-called three laws of thought." The formulas have usually been stated by those who upheld the doctrine as follows:

I. *The Principle of Identity:* A is A.

II. *The Principle of Contradiction:* A is not not-A.

III. *The Principle of Excluded Middle* or *Excluded Third:* everything is either A or not-A.

* *Ibid.*, vol. 1, pp. 641–44.

It is noticeable that two of these propositions are categorical and the third disjunctive, a circumstance demanding explanation for those who hold the distinction of categorical, conditional, and disjunctive propositions to be fundamental.

594. The meaning of the formula of identity presents only one small difficulty. If the copula "is" be taken in the sense of "is, if it exists," then the meaning of the formula is that no universal affirmative proposition having the same term as subject and predicate is false. If, however, the copula be understood to imply existence, the meaning is that no universal affirmative proposition is false in which the same term is subject and predicate, provided that term denotes any existing object. Or, the meaning may be that the same thing is true when the subject and predicate are the same proper name of an individual. In any case, it may properly be required that the precise meaning attached to the copula should be explained; and this explanation must in substance involve one or other of the above three statements; so that in any case the principle of identity is merely a part of the definition of the copula.

595. In like manner, if the word "not" is to be used in logical forms, its force should be explained with the utmost precision. Such an explanation will consist in showing that the relation it expresses belongs at once to certain classes of relations, probably not more than two, in view of the simplicity of the idea. Each of these two statements may be embodied in a formula similar, in a general way, to the formulas of contradiction and excluded middle. It has, therefore, seemed to Mill and to the "exact" logicians that these two formulas ought together to constitute a definition of the force of "not."

596. Other writers have regarded all three laws as "practical maxims." But practically nobody needs a maxim to remind him that a contradiction, for example, is an absurdity. It might be a useful injunction to tell him to beware of latent contradictions; but as soon as he clearly sees that a proposition is self-contradictory, he will have abandoned it before any maxim can be adduced. Seeing, then, that such formulas are required to define the relation expressed by "not," but are not required as maxims, it is in the former aspect that their true meanings are to be sought.

597. If it is admitted that they constitute a definition, they

must conform to the rules of definition. Considered as part of a definition, one of the commonest statements of the principle of contradiction, "*A* non est non-*A*," offends against the rule that the *definitum* must not be introduced into the definition. This is easily avoided by using the form "*A* est non non-*A*," "*A* is not not-*A*," or every term may be subsumed under the double negation of itself. If this form is adopted for the principle of contradiction, the principle of excluded middle ought to be "What is not not-*A* is *A*." If, however, we prefer to state the principle of excluded middle as "Everything is either *A* or not-*A*," then we should state the principle of contradiction as "What is, at once, *A* and not-*A* is nothing." There is no vicious circle here, since the term "nothing," or "non ens," may be formally defined without employing the particle "not" or any equivalent. Thus, we may express the principle of contradiction as follows:

> Whatever there may be which is both *A* and not-*A* is *X*, no matter what term *X* may be.

In either formula, *A* may be understood to be restricted to being an individual, or it may be allowed to be any term, individual or general. In the former case, in order to avoid conflict with the fundamental law that no true definition asserts existence, a special clause should be added, such as "if not-*A* there be." In the latter case, it should be stated that by "not-*A*" is not meant "not some *A*," but "not any *A*," or "other than whatever *A* there may be."

Bearing these points in mind, the formula "*A* is not-not-*A*," or "*A* is other than whatever is other than whatever is *A*," is seen to be a way of saying that the relation expressed by "not" is one of those which is its own converse, and is analogous to the following:

> Every rose is similar to whatever is similar to whatever is a rose;

which again is similar to the following:

Every man is loved by whatever loves whatever is a man. But if we turn to the corresponding formula of excluded middle, "Not-not-*A* is *A*," or "Whatever is not anything that is not

any A is A," we find that its meaning cannot be so simply expressed. Supposing that the relation r is such that it is true that

Whatever is r to whatever is r to whatever is A is A,

it can readily be proved that, whether the multitude of individuals in the universe be finite or infinite, each individual is either *non-r* to itself and to nothing else, or is one of a pair of individuals that are *non-r* to each other and to nothing else; and conversely, if the universe is so constituted, the above formula necessarily holds. But it is evident that if the universe is so constituted, the relation r is converse to itself; so that the formula corresponding to that of contradiction also holds. But this constitution of the universe does not determine r to be the relation expressed by "not." Hence, the pair of formulas,

A is not not-A,
Not not-A is A,

are inadequate to defining "not," and the former of them is mere surplusage. In fact, in a universe of monogamously married people, taking any class, the A's,

Every A is a non-spouse to whatever is non-spouse to every A,

and

Whatever is non-spouse to whatever is a non-spouse to every A is an A.

598. No such objection exists to the other pair of formulas:

Whatever is both A and not-A is nothing,
Everything is either A or not-A.

Their meaning is perfectly clear. Dividing all ordered pairs of individuals into those of the form $A : B$ and those of the form $A : A$,

The principle of contradiction excludes from the relation "not" all of the form $A : A$,*

The principle of excluded middle makes the relation of "not" to include all pairs of the form $A : B$.†

From this point of view, we see at once that there are three

* I.e., the principle of contradiction asserts that "not" is an alio-relative.

† I.e., the principle of excluded middle asserts that "not" is a concurrent.

other similar pairs of formulas defining the relations of identity, coexistence, and incompossibility,* as follows:

Whatever is A is identical with A; i.e,
Identity includes all pairs $A:A$.
Whatever is identical with A is A; i.e.,
Identity excludes all pairs $A:B$.
Whatever is A is coexistent with A;
i.e., Coexistence includes all pairs $A:A$.
Everything is either A or coexistent with A; i.e., Coexistence includes all pairs $A:B$.
Whatever is both A and incompossible with A is nothing;
i.e., Incompossibility excludes all pairs $A:A$.
Whatever there may be incompossible with A is A; i.e.,
Incompossibility excludes all pairs $A:B$.

599. Much has been written concerning the relations of the three principles to forms of syllogism. They have even been called *die Principien des Schliessens*, and have often been so regarded. Some points in reference to the meanings they have borne in such discussions require mention. Many writers have failed to distinguish sufficiently between reasoning and the logical forms of inference. The distinction may be brought out by comparing the moods Camestres and Cesare. Formally, these are essentially different. The form of Camestres is as follows:

Every P is an M,
Every S is other than every M;
$\therefore$ Every S is other than every P.

This form does not depend upon either clause of the definition of "not" or "other than." For if any other relative term, such as "lover of," be substituted for "other than," the inference will be equally valid. The form of Cesare is as follows:

Every P is other than every M,
Every S is an M;
$\therefore$ Every S is other than every P.

This depends upon the equiparance of "other than." For if we substitute an ordinary relative, such as *loves*, for "other than" in the premiss, the conclusion will be

Every S is loved by every P.

* Cf. 3.339.

(See De Morgan's fourth memoir on the syllogism, *Cambridge Philosophical Transactions*, X. (1860) 354.) The two forms are thus widely distinct in logic; and yet when a man actually performs an inference, it would be impossible to determine that he "reasons in" one of these moods rather than in the other. Either statement is incorrect. He does not, in strict accuracy, reason in any form of syllogism. For his reasoning moves in first intentions, while the forms of logic are constructions of second intentions. They are diagrammatic representations of the intellectual relation between the facts from which he reasons and the fact which he infers, this diagram necessarily making use of a particular system of symbols — a perfectly regular and very limited kind of language. It may be a part of a logician's duty to show how ordinary ways of speaking and of thinking are to be translated into that symbolism of formal logic; but it is no part of syllogistic itself. Logical principles of inference are merely rules for the illative transformation of the symbols of the particular system employed. If the system is essentially changed, they will be quite different. As the Booleans represent Cesare and Camestres, they appear, after literally translating the algebraic signs of those logicians into words, as follows:

A that is *B* is nothing,
C that is not *B* is nothing,
$\therefore$ *A* that is *C* is nothing.

The two moods are here absolutely indistinguishable.

600. From the time of Scotus down to Kant more and more was made of a principle agreeing in enunciation, often exactly, in other places approximately, with our principle of contradiction, and in the later of those ages usually called by that name, although earlier more often *principium primum*, *primum cognitum*, *principium identitatis*, *dignitas dignitatum*, etc. It would best be called the *Principle of Consistency*. Attention was called to it in the fourth book of Aristotle's *Metaphysics*. The meaning of this, which was altogether different, at least in post-scholastic times, from our principle of contradiction, is stated in the so-called *Monadologie* of Leibniz (§31) to be that principle by virtue of which we judge that to be false which involves a contradiction, and the denial of the contradiction to be true. The latter clause involves an appeal to the principle

of excluded middle as much as the former clause does to the formal principle of contradiction. And so the "principle of contradiction" was formerly frequently stated. But, in fact, neither is appealed to; for Leibnitz does not say that the contradiction is to be made explicit, but only that it is to be recognized as an inconsistency. Interpreted too strictly, the passage would seem to mean that all demonstrative reasoning is by the *reductio ad absurdum;* but this cannot be intended. All that is meant is that we draw that conclusion the denial of which would involve an absurdity — in short, that which consistency requires. This is a description, however imperfect, of the procedure of demonstrative Reasoning (q. v.)* and does not relate to logical forms. It deals with first, not second, intentions.

§16. REGULAR PROOF†

601. Proof which has the external form considered appropriate to making its cogency clear. The form of a regular demonstration is as follows: first, the proposition to be stated is precisely stated in general terms; second, the construction of a diagram is described conforming to the conditions of the proposition; third, the proposition is restated with reference to the construction; fourth, by means of additions to the diagram, parts of it are brought into comparison; from which it is made evident that the proposition is true of that construction. It is evident that to perfect the proof, it ought then to be shown that what is true of the particular construction will be true in every case.

§17. PERTINENT‡

602. In the doctrine of *obligationes*, in logic, "pertinent" is applied to a proposition whose truth or falsity would necessarily follow from the truth of the proposition to which it was said to be pertinent, and also of a term either necessarily true or necessarily false of another term to which it was said to be pertinent. Cf. *The Century Dictionary* [p. 4419, ed. of 1889].

* Ch. 10, §1.

† From article "Regular," *Dictionary of Philosophy and Psychology*, vol. 2, p. 439.

‡ *Ibid.*, vol. 2, p. 287.

§18. IMPLICIT*

603. Said of an element or character of a representation, whether verbal or mental, which is not contained in the representation itself, but which appears in the strictly logical (not merely in the psychological) analysis of that representation.

Thus, when we ordinarily think of something, say the Antarctic continent, as real, we do not stop to reflect that every intelligible question about it admits of a true answer; but when we logically analyse the meaning of reality, this result appears in the analysis. Consequently, only concepts, not percepts, can contain any implicit elements, since they alone are capable of logical analysis. An implicit contradiction, or contradiction *in adiecto*, is one which appears as soon as the terms are defined, irrespective of the properties of their objects. Thus there is, strictly speaking, no implicit contradiction in the notion of a quadrilateral triangle, although it is impossible. But, owing to exaggeration, this would currently be said to involve not merely an implicit, but an explicit contradiction, or contradiction in terms.

604. Any proposition which neither requires the exclusion from nor the inclusion in the universe of any state of facts or kind of object except such as a given second proposition so excludes or requires to be included, is implied in that second proposition in the logical sense of implication,† no matter how different it may be in its point of view, or otherwise. It is a part of the meaning of the copula "is" employed in logical forms of proposition, that it expresses a transitive relation, so that whatever inference from the proposition would be justified by the *dictum de omni* is implied in the meaning of the proposition. Nor could any rule be admitted as universally valid in formal logic, unless it were a part of the definition of one of the symbols used in formal logic. Accordingly, whatever can be logically deduced from any proposition is implied in it; and conversely. Whether what is implied will, or will not, be suggested by the contemplation of the proposition is a question of psychology. All that concerns logic is, whether all the facts

* *Ibid.*, vol. 1, pp. 525–26.

† This "logical sense of implication" corresponds to "entailment" and not to Philonian "material implication." It may be defined as the relation of a set of alternatives to the set which contains them.

excluded and required by the one proposition are among those so excluded or required by the other.

§19. OBSERVATION*

605. Attentive experience; especially, an act of voluntarily attentive experience, usually with some, often with great, effort.

More or less fixity in the object is requisite. Indeed, experience supposes that its object reacts upon us with some strength, much or little, so that it has a certain grade of reality or independence of our cognitive exertion. All reasoning whatever has observation as its most essential part. Whatever else there is in the act of reasoning is only preparatory to observation, like the manipulation of a physical experiment.

606. Much stress has been laid upon the distinction between "sciences of observation" and "sciences of experiment"; and undoubtedly there is a great contrast between the proceedings, let us say, of the anatomist and of the physiologist. Although the anatomist has to make many experiments (with stains, for example), yet the stress of his labour comes upon the act of observation; while the preparations for observation of the physiologist are far more elaborate, and the mere act of observation itself often very easy and coarse. The difference is, however, chiefly one of degree, and from a philosophical point of view is of quite secondary importance.

§20. SPURIOUS PROPOSITION†

607. (1) A proposition which from the constitution of the universe must be true (De Morgan, *Syllabus of Logic*, §76). (2) B. I. Gilman (*Johns Hopkins University Circular*, August, 1882), calls the conclusion from two particular premisses *spurious in the first degree*. Thus,

Some A is B,
Some C is not B;
$\therefore$ Some A is not some C.

This asserts the existence of an A and of a C, and further, that the number of the aggregate of the A's and C's is at least

* *Dictionary of Philosophy and Psychology*, vol. 2, p. 198, by Peirce and James Mark Baldwin, the editor.

† *Ibid.*, vol. 2, p. 588.

two. The conclusion from two premisses, of which one is particular and the other spurious in the first degree, gives a conclusion spurious in the second degree. There are also anti-spurious propositions, which are the precise denials of spurious propositions of the same degree.

§21. OPPOSITION*

608. One of Aristotle's *Postpredicaments*. There are said, in the book of *Categories* (cap. x), to be four kinds of opposites. Relative opposites are relate and correlate of a disquiparant relation.† Contrary opposites are the most unlike species of the same genus, as black and white, sickness and health. The third kind of opposition is between a habit and its privation, as sight and blindness. The fourth kind is between affirmation and negation. This passage has prevented the word opposite from taking any definite meaning in philosophy.

§22. INCONSISTENCY‡

609. The relation between two assertions which cannot be true at once, though it may not be a direct contradiction; as between a statement of items and a statement of their total.

610. A logical discrepancy, on the other hand, is a difference between two statements either difficult, or impossible to reconcile with the credibility of both. It is said to be negative if one assertion omits an inseparable part of the fact stated in another; as when one witness testifies that A pointed a pistol at B, and another that A shot at B. It is positive if one asserts what the other denies. But even then it may often be conciliable (verträglich); that is, may not prove that either statement is in other respects untrustworthy. See Bachmann, *Logik*, sec. 214 ff.

611. "Inconsistent" is applied to an assertion, or hypothesis, which either in itself, or in copulation with another proposition with which it is said to be inconsistent, might be known to be false by a man devoid of all information except the meanings of the words used and their syntax.

* *Ibid.*, vol. 2, p. 206.

† Or non-symmetrical. See 3.136c.

‡ *Ibid.*, vol. 1, p. 529.

Inconsistent differs from contradictory in being restricted usually to propositions, expressed or implied, and also in not implying that the falsity arises from a relation of negation. "That is John" and "It is Paul" are inconsistent, but hardly contradictory. Moreover, contradictory is also used in a peculiar sense in formal logic. Cf. Opposition.

§23. *REDUCTIO AD ABSURDUM**

612. (Latin translation of Aristotle's ἀπαγωγὴ εἰς τὸ ἀδύνατον.)† The disproof of a proposition by showing that among its consequences there is one which is impossible or simply false.

This mode of proof is generally considered to be unsatisfactory, as not showing on what general principle the proposition disproved is false. But it is very easy to convert any such proof into a direct proof. Take, for example, the seventh proposition of the first book of Euclid, that on the same side of the base AB two triangles ABC and ABD cannot exist having $AC=AD$ and $BC=BD$. Euclid proves this by showing that if there were two such triangles it would follow that the angles BDC and BCD were equal and also that they were unequal. But precisely the same steps of reasoning show that if there are two triangles ABC and ABD on the same side of AB, and if $AC=AD$, then BC is unequal to BD, which shows that there are not two triangles having $AC=AD$ and $BC=BD$, since things unequal are not equal.

§24. FALLACIES[E]

613. *Non sequitur*‡ (Latin for "it does not follow"). A name which belongs to the slang of the universities for the *fallacia consequentis* (called by Aristotle ὁπαρὰ τὸ ἑπόμενον ἔλεγχος, *De Sophist. Elen.*, 167 b 1), which is, strictly speaking, a fallacy which arises from a simple conversion of a universal affirmative, or transposing a protasis and apodosis, or ondition and consequent.

Thus, Aristotle§ tells us that the Eleatic Melissus argued that

* *Ibid.*, vol. 2, p. 434.

† *Anal. pr.* 25, 69a, 20.

‡ *Dictionary of Philosophy and Psychology*, vol. 2, p. 181.

§ *De Sophist. Elen.* I, 5, 167a, 36.

the universe is ungenerated, since nothing can be generated by what does not previously exist. The universe, then, not being generated, had no beginning; and having no beginning, it is infinite. But, as Aristotle remarks, although everything generated has a beginning, it does not follow (*non sequitur*, οὐκ ἀνάγκη δὲ τοῦτο συμβαίνειν) that everything that has a beginning is generated. A fever, for example, is not generated. Such fallacies are extremely common. De Morgan (*Formal Logic*, 268) gives this example: "Knowledge gives power, power is desirable, therefore knowledge is desirable." But though whatever is desirable has some desirable effect, it does not follow that whatever has any desirable effect is desirable. An attack of yellow fever has the desirable effect of rendering it unlikely the patient will for a long time have another; still, it is not itself desirable.

But the majority of logicians not only confound this fallacy with the *post hoc, ergo propter hoc*, which Aristotle considers immediately after, but even define it as "failure in the formal adequacy of the reason" (Sidgwick, *Fallacies*, II, ii, 4), or as "the introduction of new matter into the conclusion, which is not contained in the premisses" (Hyslop, *Logic*, xviii, 2), or as "the simple affirmation of a conclusion which does not follow from the premisses" (De Morgan, *loc. cit.*), or as "any argument which is of so loose and inconsequent a character that no one can discover any cogency in it" (Jevons, *Lessons in Logic*, xxi), or "to assume without warrant that a certain conclusion follows from premisses which have been stated" (Creighton, *Introductory Logic*, §46). Very many logicians omit it altogether, which is better.

Aristotle, however, could not express himself more precisely: Ὅπαρὰ τὸ ἑπόμενον ἔλεγχος διὰ τὸ οἴεσθαι ἀντιστρέφειν τὴν ἀκολούθησιν. That is, "from thinking that the *consequentia* can be converted." That is to say, thinking that because "If *A*, then *C*," therefore "If *C*, then *A*." Owing to the neglect of fallacies by the more scientific logicians, it is not easy to cite many who define the fallacy correctly. The Conimbricenses (than whom no authority is higher) do so (*Commentarii in Univ. Dialecticam* Arist. Stagir., *In lib. Elench.*, q. i. art. 4); also Eustachius (*Summa Philos.*, Tom. I, pars. III, tract. iii, disput. iii. 9.3); also Cope, an admirable student of Aristotle,

in his note on the *Rhetorics*, B. cap. xxiv. See also the *Century Dictionary* under "Fallacy." [Vol. 8, p. 2728, ed. 1889.]

614. *Petitio Principii** (Latin. This is a not very good translation of Aristotle's phrase τὸ ἐξ ἀρχῆς [or ἐν ἀρχῇ] αἰτεῖσθαι,† to beg what was proposed in the beginning). It is a Fallacy of a relatively high order, inasmuch as it cannot exist unless the conclusion truly follows from the premisses. To accuse a man of begging the question is in reality a plea which virtually admits that his reasoning is good. Its only fault is that it assumes as a premiss what no intelligent man who doubted the conclusion could know to be true.

A very necessary, though not always sufficient, precaution against this fallacy is to ask oneself whether the reasoning rests upon any observations, or inductions from observations, or even trustworthy hypothetic inferences from observations, which really involve the conclusion, relating to those matters of experience in reference to which the conclusion is important; and if relating to those things, whether in such a way and so closely that that conclusion really can have been implicitly asserted in those premisses. For example, to take an illustration partly fanciful, a man proposes to prove the reality or possibility of clairvoyance to me by proving to me that the sum of the angles of a triangle is two right angles. If, he says, you can sit in your study and know that this is true in the most distant parts of the universe, why may not an exceptionally gifted person know many facts about what happens only a hundred miles away? Upon that, I ask myself whether geometry rests upon any observations concerning clairvoyance or anything like clairvoyance. Nay, the consequence which my arguer has pointed out seems so cogent, and yet the line of reasoning so inadmissible, that I go up to the garret to exhume my old Euclid or Legendre [*Eléments de Géométrie*] to see how it is proved that sitting in my study I can know what the angles of the triangle whose vertices are at Sirius, Arcturus, and Fomalhaut, may sum up to. I find it is done by assuming that certain propositions about space are self-evident. Now, this may be safe enough so far as that sort of reasoning has been millions of times verified. But nothing of the sort has

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 287–88, by Peirce and Mrs. C. Ladd-Franklin.

† *Anal. Prior.*, II, 16.

been, or can be, verified exactly; and for such monstrous triangles a divergence from exactitude in the formula may be large, although for terrestrial triangles it be too minute for detection. In short, I am led to see that there must be a *petitio principii* in any argument which, resting merely on common sense, concludes the exact truth of any matter of fact.

615. *Philosopheme.** A demonstrative reasoning supposed to prove a truth from self-evident premisses.

It is necessarily, at best, a *Petitio Principii;* but it is the Aristotelian ideal of perfect reasoning (see Aristotle, *Top.* viii. 162 a 15; 279 a 30; 294 a 19).

616. *Saltus in concludendo:*† a paralogism which consists in proving something as an aid in proving something else and then supposing that something different has been proved. But by some writers a mere omission of an obvious step in a proof is called a saltus.

617. *Material Fallacy.*‡ This term originated with Whately (*Encyc. Metropolitana*, i. 218 b). Whately's material fallacies are those in which the conclusion does follow from the premisses. Therefore, excluding the multiple interrogation, which is no syllogism, of the rest of Aristotle's thirteen, only the *ignoratio elenchi* and the *petitio principii* are material.

Aldrich§ had modified Aristotle's division into fallacies *in dictione* and fallacies *extra dictionem;* making a division into *Sophismata in forma argumenti* (*sicubi conclusio non legitime consequatur ex praemissis*), and *Sophismata in materia argumenti* (*sicubi legitime non tamen vere concludere videatur syllogismus*). Under the latter head he placed the *ignoratio elenchi*, the *non causa pro causa*, the *non sequitur*, and the *petitio principii*. Whately's distinction is — whether from a theoretical or a practical point of view — by far the most important that can be drawn among fallacies; so that besides the reason of priority, which ought itself to be final, the needs of the logician forbid us to depart from Whately's definition. Some logicians do not admit material fallacies among the number of fallacies, but consider them to be faults of method

* *Dictionary of Philosophy and Psychology*, vol. 2, p. 290.
† *Ibid.*, from article "Saltus," vol. 2, p. 484.
‡ *Ibid.*, vol. 2, p. 44.
§ *Artis Logicae Rudimenta* (1862), p. 131ff.

(Hamilton, *Lectures on Logic*, xxvi; Ueberweg, *System der Logik*, §126, §137). E. E. Constance Jones (*Elements of Logic as a Science of Propositions*, §xxvi) reduces them to formal fallacies. Hyslop (*Elements of Logic*, chap. xvii) uses the term "material" fallacy, quite unjustifiably, to include all fallacies due to something in the matter of reasoning.

§25. *INSOLUBILIA**

618. A class of sophisms in which a question is put of such a nature that, whether it be answered affirmatively or negatively, an argument unimpeachable in form will prove the answer to be false.

The type is this. Given the following proposition:

This assertion is not true:

is that assertion, which proclaims its own falsity, and nothing else, true or false? Suppose it true. Then,

Whatever is asserted in it is true,
But that it is not true is asserted in it;
∴ By Barbara, That it is not true is true;
∴ It is not true.

Besides, if it is true, that it is true is true. Hence,

That it is not true is not true,
But that it is not true is asserted in the proposition;
∴ By Darapti, Something asserted in the proposition is not true:
∴ The proposition is not true.

On the other hand, suppose it is not true. In that case,

That it is not true is true,
But all that the proposition asserts is that it is not true;
∴ By Barbara, All that the proposition asserts is true;
∴ The proposition is true.

Besides, in this case,

Something the proposition asserts is not true,
But all that the proposition asserts is that it is not true;
∴ By Bokardo, That it is not true is not altogether true;
∴ That it is true is true;
∴ It is true.

* *Dictionary of Philosophy and Psychology*, vol. 1, p. 554.

Thus, whether it be true or not, it is both true and not. Now, it must be either true or not, hence it is both true and not, which is absurd.

Only two essentially distinct methods of solution have been proposed. One, which is supported by Ockham (*Summa totius logicae*, 3d div. of 3d part, cap. 38 and 45), admits the validity of the argumentation and its consequence, which is that there can be no such proposition, and attempts to show by other arguments that no proposition can assert anything of itself. Many logical writers follow Ockham in the first part of his solution, but fail to see the need of the second part. The other method of solution, supported by Paulus Venetus (*Sophismata Aurea*, sophisma 50), diametrically denies the principle of the former solution, and undertakes to show that every proposition virtually asserts its own truth. This method, therefore, denies the premiss of the antithesis that "all that the proposition asserts is that it is not true" since, like every other proposition, it also asserts its own truth, and is therefore contradictory and false, not in what it expressly asserts, but in what it implicitly asserts.* Some writers (as Fries)† hold that because every proposition asserts its own truth, therefore nothing is a proposition which asserts its own falsity. See Aristotle, *Sophisticae Elenchi*, cap. 25. Other proposed solutions of little importance are given by Paulus Venetus, *loc. cit.*‡

* For a further discussion of sophisms and insolubilia, see 5.33ff.

† *System der Logik.*

‡ See 3.446.

B. AMPLIATIVE REASONING

CHAPTER 5

*DEDUCTION, INDUCTION, AND HYPOTHESIS**

§1. RULE, CASE, AND RESULT[E]

619. The chief business of the logician is to classify arguments; for all testing clearly depends on classification. The classes of the logicians are defined by certain typical forms called syllogisms. For example, the syllogism called *Barbara* is as follows:

S is M, M is P;
Hence, S is P.

Or, to put words for letters —

Enoch and Elijah were men, all men die;
Hence, Enoch and Elijah must have died.

The "is P" of the logicians stands for any verb, active or neuter. It is capable of strict proof (with which, however, I will not trouble the reader) that all arguments whatever can be put into this form; but only under the condition that the *is* shall mean "*is* for the purposes of the argument" or "is represented by." Thus, an induction will appear in this form something like this:

These beans are two-thirds white,
But, the beans in this bag are (represented by) these beans;
∴ The beans in the bag are two-thirds white.

620. But, because all inference may be reduced in some way to *Barbara*, it does not follow that this is the most appropriate form in which to represent every kind of inference. On the contrary, to show the distinctive characters of different

* *Popular Science Monthly*, vol. 13, pp. 470–82 (1878); intended as Essay XIII of the *Search for a Method* (1893). It is the sixth and last of a series of papers on the "Illustrations of the Logic of Science," which appeared in the *Popular Science Monthly*. For the first and second papers, see vol. 5, bk. II, chs. 4 and 5; the third and fourth constitute chapters 6 and 7 of the present book; for the fifth paper, see vol. 6, bk. II, ch. 1.

sorts of inference, they must clearly be exhibited in different forms peculiar to each. *Barbara* particularly typifies deductive reasoning; and so long as the *is* is taken literally, no inductive reasoning can be put into this form. *Barbara* is, in fact, nothing but the application of a rule. The so-called major premiss lays down this rule; as, for example, *All men are mortal.* The other or minor premiss states a case under the rule; as, *Enoch was a man.* The conclusion applies the rule to the case and states the result: *Enoch is mortal.* All deduction is of this character; it is merely the application of general rules to particular cases. Sometimes this is not very evident, as in the following:

All quadrangles are figures,
But no triangle is a quadrangle;
Therefore, some figures are not triangles.

But here the reasoning is really this:
Rule. — Every quadrangle is other than a triangle.
Case. — Some figures are quadrangles.
Result. — Some figures are not triangles.

Inductive or synthetic reasoning, being something more than the mere application of a general rule to a particular case, can never be reduced to this form.

621. If, from a bag of beans of which we know that $\frac{2}{3}$ are white, we take one at random, it is a deductive inference that this bean is probably white, the probability being $\frac{2}{3}$. We have, in effect, the following syllogism:

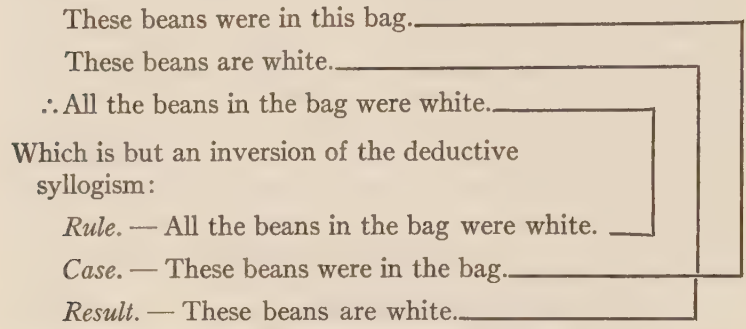
Rule. — The beans in this bag are $\frac{2}{3}$ white.

Case. — This bean has been drawn in such a way that in the long run the relative number of white beans so drawn would be equal to the relative number in the bag.

Result. — This bean has been drawn in such a way that in the long run it would turn out white $\frac{2}{3}$ of the time.

622. If instead of drawing one bean we draw a handful at random and conclude that about $\frac{2}{3}$ of the handful are probably white, the reasoning is of the same sort. If, however, not knowing what proportion of white beans there are in the bag, we draw a handful at random and, finding $\frac{2}{3}$ of the beans in the handful white, conclude that about $\frac{2}{3}$ of those in the bag are white, we are rowing up the current of deductive sequence, and are concluding a rule from the observation of a

result in a certain case. This is particularly clear when all the handful turn out one color. The induction then is:



So that induction is the inference of the *rule* from the *case* and *result*.

623. But this is not the only way of inverting a deductive syllogism so as to produce a synthetic inference. Suppose I enter a room and there find a number of bags, containing different kinds of beans. On the table there is a handful of white beans; and, after some searching, I find one of the bags contains white beans only. I at once infer as a probability, or as a fair guess, that this handful was taken out of that bag. This sort of inference is called *making an hypothesis*. It is the inference of a *case* from a *rule* and a *result*. We have, then —

DEDUCTION.

Rule. — All the beans from this bag are white.
Case. — These beans are from this bag.
∴ *Result.* — These beans are white.

INDUCTION.

Case. — These beans are from this bag.
Result. — These beans are white.
∴ *Rule.* — All the beans from this bag are white

HYPOTHESIS.

Rule. — All the beans from this bag are white.
Result. — These beans are white.
∴ *Case.* — These beans are from this bag.

We, accordingly, classify all inference as follows:

Inference.

Deductive or Analytic. Synthetic.

Induction. Hypothesis.

624. Induction is where we generalize from a number of cases of which something is true, and infer that the same thing is true of a whole class. Or, where we find a certain thing to be true of a certain proportion of cases and infer that it is true of the same proportion of the whole class. Hypothesis is where we find some very curious circumstance, which would be explained by the supposition that it was a case of a certain general rule, and thereupon adopt that supposition. Or, where we find that in certain respects two objects have a strong resemblance, and infer that they resemble one another strongly in other respects.

625. I once landed at a seaport in a Turkish province; and, as I was walking up to the house which I was to visit, I met a man upon horseback, surrounded by four horsemen holding a canopy over his head. As the governor of the province was the only personage I could think of who would be so greatly honored, I inferred that this was he. This was an hypothesis.

Fossils are found; say, remains like those of fishes, but far in the interior of the country. To explain the phenomenon, we suppose the sea once washed over this land. This is another hypothesis.

Numberless documents and monuments refer to a conqueror called Napoleon Bonaparte. Though we have not seen the man, yet we cannot explain what we have seen, namely, all these documents and monuments, without supposing that he really existed. Hypothesis again.

As a general rule, hypothesis is a weak kind of argument. It often inclines our judgment so slightly toward its conclusion that we cannot say that we believe the latter to be true; we only surmise that it may be so. But there is no difference except one of degree between such an inference and that by which we are led to believe that we remember the occurrences of yesterday from our feeling as if we did so.

§2. BAROCO AND BOCARDO; HYPOTHESIS AND INDUCTION[E]

626. Besides the way just pointed out of inverting a deductive syllogism to produce an induction or hypothesis, there is another. If from the truth of a certain premiss the truth of a certain conclusion would necessarily follow, then from the falsity of the conclusion the falsity of the premiss would follow. Thus, take the following syllogism in *Barbara:*

Rule. — All men are mortal,
Case. — Enoch and Elijah were men;
∴ *Result.* — Enoch and Elijah were mortal.

Now, a person who denies this result may admit the rule, and, in that case, he must deny the case. Thus:

Denial of Result. — Enoch and Elijah were not mortal,
Rule. — All men are mortal;
∴ *Denial of Case.* — Enoch and Elijah were not men.

This kind of syllogism is called *Baroco*, which is the typical mood of the second figure. On the other hand, the person who denies the result may admit the case, and in that case he must deny the rule. Thus:

Denial of the Result. — Enoch and Elijah were not mortal,
Case. — Enoch and Elijah were men;
∴ *Denial of the Rule.* — Some men are not mortal.

This kind of syllogism is called *Bocardo*, which is the typical mood of the third figure.

627. *Baroco* and *Bocardo* are, of course, deductive syllogisms; but of a very peculiar kind. They are called by logicians indirect moods, because they need some transformation to appear as the application of a rule to a particular case. But if, instead of setting out as we have here done with a necessary deduction in *Barbara*, we take a probable deduction of similar form, the indirect moods which we shall obtain will be —

Corresponding to *Baroco*, an hypothesis;
and, Corresponding to *Bocardo*, an induction.

For example, let us begin with this probable deduction in *Barbara:*

Rule. — Most of the beans in this bag are white,
Case. — This handful of beans are from this bag;
∴ *Result.* — Probably, most of this handful of beans are white.

Now, deny the result, but accept the rule:

Denial of Result. — Few beans of this handful are white,
Rule. — Most beans in this bag are white;
∴ *Denial of Case.* — Probably, these beans were taken from another bag.

This is an hypothetical inference. Next, deny the result, but accept the case:

Denial of Result. — Few beans of this handful are white.
Case. — These beans came from this bag.
∴ *Denial of Rule.* — Probably, few beans in the bag are white.

This is an induction.

628. The relation thus exhibited between synthetic and deductive reasoning is not without its importance. When we adopt a certain hypothesis, it is not alone because it will explain the observed facts, but also because the contrary hypothesis would probably lead to results contrary to those observed. So, when we make an induction, it is drawn not only because it explains the distribution of characters in the sample, but also because a different rule would probably have led to the sample being other than it is.

629. But the advantage of this way of considering the subject might easily be overrated. An induction is really the inference of a rule, and to consider it as the denial of a rule is an artificial conception, only admissible because, when statistical or proportional propositions are considered as rules, the denial of a rule is itself a rule. So, an hypothesis is really a subsumption of a case under a class and not the denial of it, except for this, that to deny a subsumption under one class is to admit a subsumption under another.

630. *Bocardo* may be considered as an induction, so timid as to lose its amplifiative character entirely. Enoch and Elijah are specimens of a certain kind of men. All that kind of men are shown by these instances to be immortal. But instead of boldly concluding that all very pious men, or all men favorites of the Almighty, etc., are immortal, we refrain from specifying the description of men, and rest in the merely explicative inference that *some* men are immortal. So *Baroco* might be considered as a very timid hypothesis. Enoch and Elijah are not mortal. Now, we might boldly suppose them to be gods or something of that sort, but instead of that we limit ourselves to the inference that they are of *some* nature different from that of man.

631. But, after all, there is an immense difference between the relation of *Baroco* and *Bocardo* to *Barbara* and that of Induction and Hypothesis to Deduction. *Baroco* and *Bocardo* are based upon the fact that if the truth of a conclusion necessarily follows from the truth of a premiss, then the falsity of the premiss follows from the falsity of the conclusion. This is always true. It is different when the inference is only probable. It by no means follows that, because the truth of a certain premiss would render the truth of a conclusion probable, therefore the falsity of the conclusion renders the falsity of the premiss probable. At least, this is only true, as we have seen in a former paper,* when the word "probable" is used in one sense in the antecedent and in another in the consequent.

§3. RULES FOR INDUCTION AND HYPOTHESES[E]

632. A certain anonymous writing is upon a torn piece of paper. It is suspected that the author is a certain person. His desk, to which only he has had access, is searched, and in it is found a piece of paper, the torn edge of which exactly fits, in all its irregularities, that of the paper in question. It is a fair hypothetic inference that the suspected man was actually the author. The ground of this inference evidently is that two torn pieces of paper are extremely unlikely to fit together by accident. Therefore, of a great number of inferences of this sort, but a very small proportion would be deceptive. The analogy of hypothesis with induction is so strong that some logicians have confounded them. Hypothesis has been called

* See, e.g., 515.

an induction of characters. A number of characters belonging to a certain class are found in a certain object; whence it is inferred that all the characters of that class belong to the object in question. This certainly involves the same principle as induction; yet in a modified form. In the first place, characters are not susceptible of simple enumeration like objects; in the next place, characters run in categories. When we make an hypothesis like that about the piece of paper, we only examine a single line of characters, or perhaps two or three, and we take no specimen at all of others. If the hypothesis were nothing but an induction, all that we should be justified in concluding, in the example above, would be that the two pieces of paper which matched in such irregularities as have been examined would be found to match in other, say slighter, irregularities. The inference from the shape of the paper to its ownership is precisely what distinguishes hypothesis from induction, and makes it a bolder and more perilous step.

633. The same warnings that have been given against imagining that induction rests upon the uniformity of Nature might be repeated in regard to hypothesis. Here, as there, such a theory not only utterly fails to account for the validity of the inference, but it also gives rise to methods of conducting it which are absolutely vicious. There are, no doubt, certain uniformities in Nature, the knowledge of which will fortify an hypothesis very much. For example, we suppose that iron, titanium, and other metals exist in the sun, because we find in the solar spectrum many lines coincident in position with those which these metals would produce; and this hypothesis is greatly strengthened by our knowledge of the remarkable distinctiveness of the particular line of characters observed. But such a fortification of hypothesis is of a deductive kind, and hypothesis may still be probable when such reënforcement is wanting.

634. There is no greater nor more frequent mistake in practical logic than to suppose that things which resemble one another strongly in some respects are any the more likely for that to be alike in others. That this is absolutely false, admits of rigid demonstration; but, inasmuch as the reasoning is somewhat severe and complicated (requiring, like all such reasoning, the use of A, B, C, etc., to set it forth), the reader would prob-

ably find it distasteful, and I omit it. An example, however, may illustrate the proposition: The comparative mythologists occupy themselves with finding points of resemblance between solar phenomena and the careers of the heroes of all sorts of traditional stories; and upon the basis of such resemblances they infer that these heroes are impersonations of the sun. If there be anything more in their reasonings, it has never been made clear to me. An ingenious logician, to show how futile all that is, wrote a little book, in which he pretended to prove, in the same manner, that Napoleon Bonaparte is only an impersonation of the sun. It was really wonderful to see how many points of resemblance he made out. The truth is, that any two things resemble one another just as strongly as any two others, if recondite resemblances are admitted. But, in order that the process of making an hypothesis should lead to a probable result, the following rules must be followed:

1. The hypothesis should be distinctly put as a question, before making the observations which are to test its truth. In other words, we must try to see what the result of predictions from the hypothesis will be.

2. The respect in regard to which the resemblances are noted must be taken at random. We must not take a particular kind of predictions for which the hypothesis is known to be good.

3. The failures as well as the successes of the predictions must be honestly noted. The whole proceeding must be fair and unbiased.

635. Some persons fancy that bias and counter-bias are favorable to the extraction of truth — that hot and partisan debate is the way to investigate. This is the theory of our atrocious legal procedure. But Logic puts its heel upon this suggestion. It irrefragably demonstrates that knowledge can only be furthered by the real desire for it, and that the methods of obstinacy, of authority, and every mode of trying to reach a foregone conclusion, are absolutely of no value.* These things are proved. The reader is at liberty to think so or not as long as the proof is not set forth, or as long as he refrains from examining it. Just so, he can preserve, if he likes, his freedom of opinion in regard to the propositions of geometry; only, in that

* See vol. 5, bk. II, ch. 4, §5.

case, if he takes a fancy to read Euclid, he will do well to skip whatever he finds with A, B, C, etc., for, if he reads attentively that disagreeable matter, the freedom of his opinion about geometry may unhappily be lost forever.

How many people there are who are incapable of putting to their own consciences this question, "Do I want to know how the fact stands, or not?"

The rules which have thus far been laid down for induction and hypothesis are such as are absolutely essential. There are many other maxims expressing particular contrivances for making synthetic inferences strong, which are extremely valuable and should not be neglected. Such are, for example, Mr. Mill's four methods. Nevertheless, in the total neglect of these, inductions and hypotheses may and sometimes do attain the greatest force.

§4. EMPIRICAL FORMULÆ AND THEORIES[E]

636. Classifications in all cases perfectly satisfactory hardly exist. Even in regard to the great distinction between explicative and ampliative inferences, examples could be found which seem to lie upon the border between the two classes, and to partake in some respects of the characters of either. The same thing is true of the distinction between induction and hypothesis. In the main, it is broad and decided. By induction, we conclude that facts, similar to observed facts, are true in cases not examined. By hypothesis, we conclude the existence of a fact quite different from anything observed, from which, according to known laws, something observed would necessarily result. The former, is reasoning from particulars to the general law; the latter, from effect to cause. The former classifies, the latter explains. It is only in some special cases that there can be more than a momentary doubt to which category a given inference belongs. One exception is where we observe, not facts similar under similar circumstances, but facts different under different circumstances — the difference of the former having, however, a definite relation to the difference of the latter. Such inferences, which are really inductions, sometimes present, nevertheless, some indubitable resemblances to hypotheses.

637. Knowing that water expands by heat, we make a number of observations of the volume of a constant mass of water at different temperatures. The scrutiny of a few of these suggests a form of algebraical formula which will approximately express the relation of the volume to the temperature. It may be, for instance, that v being the relative volume, and t the temperature, a few observations examined indicate a relation of the form —

$$v = 1 + at + bt^2 + ct^3.$$

Upon examining observations at other temperatures taken at random, this idea is confirmed; and we draw the inductive conclusion that all observations within the limits of temperature from which we have drawn our observations could equally be so satisfied. Having once ascertained that such a formula is possible, it is a mere affair of arithmetic to find the values of a, b, and c, which will make the formula satisfy the observations best. This is what physicists call an *empirical formula*, because it rests upon mere induction, and is not explained by any hypothesis.

Such formulæ, though very useful as means of describing in general terms the results of observations, do not take any high rank among scientific discoveries. The induction which they embody, that expansion by heat (or whatever other phenomenon is referred to) takes place in a perfectly gradual manner without sudden leaps or innumerable fluctuations, although really important, attracts no attention, because it is what we naturally anticipate. But the defects of such expressions are very serious. In the first place, as long as the observations are subject to error, as all observations are, the formula cannot be expected to satisfy the observations exactly. But the discrepancies cannot be due solely to the errors of the observations, but must be partly owing to the error of the formula which has been deduced from erroneous observations. Moreover, we have no right to suppose that the real facts, if they could be had free from error, could be expressed by such a formula at all. They might, perhaps, be expressed by a similar formula with an infinite number of terms; but of what use would that be to us, since it would require an infinite number of coefficients to be written down? When one quantity varies

with another, if the corresponding values are exactly known, it is a mere matter of mathematical ingenuity to find some way of expressing their relation in a simple manner. If one quantity is of one kind — say, a specific gravity — and the other of another kind — say, a temperature — we do not desire to find an expression for their relation which is wholly free from numerical constants, since if it were free from them when, say, specific gravity as compared with water, and temperature as expressed by the Centigrade thermometer, were in question, numbers would have to be introduced when the scales of measurement were changed. We may, however, and do desire to find formulæ expressing the relations of physical phenomena which shall contain no more arbitrary numbers than changes in the scales of measurement might require.

638. When a formula of this kind is discovered, it is no longer called an empirical formula, but a law of Nature; and is sooner or later made the basis of an hypothesis which is to explain it. These simple formulæ are not usually, if ever, exactly true, but they are none the less important for that; and the great triumph of the hypothesis comes when it explains not only the formula, but also the deviations from the formula. In the current language of the physicists, an hypothesis of this importance is called a theory, while the term hypothesis is restricted to suggestions which have little evidence in their favor. There is some justice in the contempt which clings to the word hypothesis. To think that we can strike out of our own minds a true preconception of how Nature acts, is a vain fancy. As Lord Bacon well says: "The subtlety of Nature far exceeds the subtlety of sense and intellect: so that these fine meditations, and speculations, and reasonings of men are a sort of insanity, only there is no one at hand to remark it."* The successful theories are not pure guesses, but are guided by reasons.

639. The kinetical theory of gases is a good example of this. This theory is intended to explain certain simple formulæ, the chief of which is called the law of Boyle. It is, that if air or any other gas be placed in a cylinder with a piston, and if its volume be measured under the pressure of the atmosphere, say fifteen pounds on the square inch, and if then another

* *Novum Organum*, bk. I, Aphorism X.

fifteen pounds per square inch be placed on the piston, the gas will be compressed to one-half its bulk, and in similar inverse ratio for other pressures. The hypothesis which has been adopted to account for this law is that the molecules of a gas are small, solid particles at great distances from each other (relatively to their dimensions), and moving with great velocity, without sensible attractions or repulsions, until they happen to approach one another very closely. Admit this, and it follows that when a gas is under pressure what prevents it from collapsing is not the incompressibility of the separate molecules, which are under no pressure at all, since they do not touch, but the pounding of the molecules against the piston. The more the piston falls, and the more the gas is compressed, the nearer together the molecules will be; the greater number there will be at any moment within a given distance of the piston, the shorter the distance which any one will go before its course is changed by the influence of another, the greater number of new courses of each in a given time, and the oftener each, within a given distance of the piston, will strike it. This explains Boyle's law. The law is not exact; but the hypothesis does not lead us to it exactly. For, in the first place, if the molecules are large, they will strike each other oftener when their mean distances are diminished, and will consequently strike the piston oftener, and will produce more pressure upon it. On the other hand, if the molecules have an attraction for one another, they will remain for a sensible time within one another's influence, and consequently they will not strike the wall so often as they otherwise would, and the pressure will be less increased by compression.

When the kinetical theory of gases was first proposed by Daniel Bernoulli,* in 1738, it rested only on the law of Boyle, and was therefore pure hypothesis. It was accordingly quite naturally and deservedly neglected. But, at present, the theory presents quite another aspect; for, not to speak of the considerable number of observed facts of different kinds with which it has been brought into relation, it is supported by the mechanical theory of heat. That bringing together bodies which attract one another, or separating bodies which repel one another, when sensible motion is not produced or destroyed,

* In his *Hydrodynamica.*

is always accompanied by the evolution of heat, is little more than an induction. Now, it has been shown by experiment that, when a gas is allowed to expand without doing work, a very small amount of heat disappears. This proves that the particles of the gas attract one another slightly, and but very slightly. It follows that, when a gas is under pressure, what prevents it from collapsing is not any repulsion between the particles, since there is none. Now, there are only two modes of force known to us, force of position or attractions and repulsions, and force of motion. Since, therefore, it is not the force of position which gives a gas its expansive force, it must be the force of motion. In this point of view, the kinetical theory of gases appears as a deduction from the mechanical theory of heat. It is to be observed, however, that it supposes the same law of mechanics (that there are only those two modes of force) which holds in regard to bodies such as we can see and examine, to hold also for what are very different, the molecules of bodies. Such a supposition has but a slender support from induction. Our belief in it is greatly strengthened by its connection with the law of Boyle, and it is, therefore, to be considered as an hypothetical inference. Yet it must be admitted that the kinetical theory of gases would deserve little credence if it had not been connected with the principles of mechanics.

640. The great difference between induction and hypothesis is, that the former infers the existence of phenomena such as we have observed in cases which are similar, while hypothesis supposes something of a different kind from what we have directly observed, and frequently something which it would be impossible for us to observe directly.* Accordingly, when we stretch an induction quite beyond the limits of our observation, the inference partakes of the nature of hypothesis. It would be absurd to say that we have no inductive warrant for a generalization extending a little beyond the limits of experience, and there is no line to be drawn beyond which we cannot push our inference; only it becomes weaker the further it is pushed. Yet, if an induction be pushed very far, we cannot give it much credence unless we find that such an extension explains some fact which we can and do observe. Here, then, we have a kind of mixture of induction and hypothesis sup-

* Cf. 511n.

porting one another; and of this kind are most of the theories of physics.

§5. ON THE DIFFERENCE BETWEEN INDUCTION AND HYPOTHESIS[E]

641. That synthetic inferences may be divided into induction and hypothesis in the manner here proposed,[1] admits of no question. The utility and value of the distinction are to be tested by their applications.

642. Induction is, plainly, a much stronger kind of inference than hypothesis; and this is the first reason for distinguishing between them. Hypotheses are sometimes regarded as provisional resorts, which in the progress of science are to be replaced by inductions. But this is a false view of the subject. Hypothetic reasoning infers very frequently a fact not capable of direct observation. It is an hypothesis that Napoleon Bonaparte once existed. How is that hypothesis ever to be replaced by an induction? It may be said that from the premiss that such facts as we have observed are as they would be if Napoleon existed, we are to infer by induction that *all* facts that are hereafter to be observed will be of the same character. There is no doubt that every hypothetic inference may be distorted into the appearance of an induction in this way. But the essence of an induction is that it infers from one set of facts another set of similar facts, whereas hypothesis infers from facts of one kind to facts of another. Now, the facts which serve as grounds for our belief in the historic reality of Napoleon are not by any means necessarily the only kind of facts which are explained by his existence. It may be that, at the time of his career, events were being recorded in some way not now dreamed of, that some ingenious creature on a neighboring planet was photographing the earth, and that these pictures on a sufficiently large scale may some time come into our possession, or that some mirror upon a distant star will, when the light reaches it, reflect the whole story back to earth. Never mind how improbable these suppositions are; every-

[1] This division was first made in a course of lectures by the author before the Lowell Institute, Boston, in 1866, and was printed in the *Proceedings of the American Academy of Arts and Sciences*, for April 9, 1867. [See 508–12.]

thing which happens is infinitely improbable. I am not saying that *these* things are likely to occur, but that *some* effect of Napoleon's existence which now seems impossible is certain nevertheless to be brought about. The hypothesis asserts that such facts, when they do occur, will be of a nature to confirm, and not to refute, the existence of the man. We have, in the impossibility of inductively inferring hypothetical conclusions, a second reason for distinguishing between the two kinds of inference.

643. A third merit of the distinction is, that it is associated with an important psychological or rather physiological difference in the mode of apprehending facts. Induction infers a rule. Now, the belief of a rule is a habit. That a habit is a rule active in us, is evident. That every belief is of the nature of a habit, in so far as it is of a general character, has been shown in the earlier papers of this series.* Induction, therefore, is the logical formula which expresses the physiological process of formation of a habit. Hypothesis substitutes, for a complicated tangle of predicates attached to one subject, a single conception. Now, there is a peculiar sensation belonging to the act of thinking that each of these predicates inheres in the subject. In hypothetic inference this complicated feeling so produced is replaced by a single feeling of greater intensity, that belonging to the act of thinking the hypothetic conclusion. Now, when our nervous system is excited in a complicated way, there being a relation between the elements of the excitation, the result is a single harmonious disturbance which I call an emotion. Thus, the various sounds made by the instruments of an orchestra strike upon the ear, and the result is a peculiar musical emotion, quite distinct from the sounds themselves. This emotion is essentially the same thing as an hypothetic inference, and every hypothetic inference involves the formation of such an emotion. We may say, therefore, that hypothesis produces the *sensuous* element of thought, and induction the *habitual* element. As for deduction, which adds nothing to the premisses, but only out of the various facts represented in the premisses selects one and brings the attention down to it, this may be considered as the logical formula for paying attention, which is the *volitional* element

* See, e.g., the first paper, vol. 5, bk. II, ch. 4.

of thought, and corresponds to nervous discharge in the sphere of physiology.*

644. Another merit of the distinction between induction and hypothesis is, that it leads to a very natural classification of the sciences and of the minds which prosecute them. What must separate different kinds of scientific men more than anything else are the differences of their *techniques*. We cannot expect men who work with books chiefly to have much in common with men whose lives are passed in laboratories. But, after differences of this kind, the next most important are differences in the modes of reasoning. Of the natural sciences, we have, first, the classificatory sciences, which are purely inductive — systematic botany and zoölogy, mineralogy, and chemistry. Then, we have the sciences of theory, as above explained — astronomy, pure physics, etc. Then, we have sciences of hypothesis — geology, biology, etc.†

There are many other advantages of the distinction in question which I shall leave the reader to find out by experience. If he will only take the custom of considering whether a given inference belongs to one or other of the two forms of synthetic inference given in 623, I can promise him that he will find his advantage in it, in various ways.

* Cf. 712.

† Cf. vol. 1, bk. II, ch. 2.

CHAPTER 6

*THE DOCTRINE OF CHANCES**

§1. CONTINUITY AND THE FORMATION OF CONCEPTS[E]

645. It is a common observation that a science first begins to be exact when it is quantitatively treated. What are called the exact sciences are no others than the mathematical ones. Chemists reasoned vaguely until Lavoisier showed them how to apply the balance to the verification of their theories, when chemistry leaped suddenly into the position of the most perfect of the classificatory sciences. It has thus become so precise and certain that we usually think of it along with optics, thermotics, and electrics. But these are studies of general laws, while chemistry considers merely the relations and classification of certain objects; and belongs, in reality, in the same category as systematic botany and zoölogy. Compare it with these last, however, and the advantage that it derives from its quantitative treatment is very evident.[1]

646. The rudest numerical scales, such as that by which the mineralogists distinguish the different degrees of hardness, are found useful. The mere counting of pistils and stamens sufficed to bring botany out of total chaos into some kind of form. It is not, however, so much from *counting* as from *measuring*, not so much from the conception of number as from

* *Popular Science Monthly*, vol. 12, pp. 604–15 (1878) with corrections of 1893 and a note of 1910; intended as ch. 18 of the *Grand Logic* (1893), and as Essay X of the *Search for a Method* (1893), the third of a series of papers on "Illustrations of the Logic of Science." See notes to ch. 5 and 6.410.

[1] This characterization of chemistry now sounds antiquated indeed; and yet it was justified by the general state of mind of chemists at that day, as is shown by the fact that only a few months before, van't Hoff had put forth a statement of the law of mass-action as something absolutely new to science. I am satisfied by considerable search after pertinent facts that no distinction between different allied sciences can represent any truth of fact other than a difference between what habitually passes in the minds, and moves the investigations of the two general bodies of the cultivators of those sciences at the time to which the distinction refers. — 1910.

that of continuous quantity, that the advantage of mathematical treatment comes. Number, after all, only serves to pin us down to a precision in our thoughts which, however beneficial, can seldom lead to lofty conceptions, and frequently descends to pettiness. Of those two faculties of which Bacon speaks,* that which marks differences and that which notes resemblances, the employment of number can only aid the lesser one; and the excessive use of it must tend to narrow the powers of the mind. But the conception of continuous quantity has a great office to fulfill, independently of any attempt at precision. Far from tending to the exaggeration of differences, it is the direct instrument of the finest generalizations. When a naturalist wishes to study a species, he collects a considerable number of specimens more or less similar. In contemplating them, he observes certain ones which are more or less alike in some particular respect. They all have, for instance, a certain S-shaped marking. He observes that they are not *precisely* alike, in this respect; the S has not precisely the same shape, but the differences are such as to lead him to believe that forms could be found intermediate between any two of those he possesses. He, now, finds other forms apparently quite dissimilar — say a marking in the form of a C — and the question is, whether he can find intermediate ones which will connect these latter with the others. This he often succeeds in doing in cases where it would at first be thought impossible; whereas, he sometimes finds those which differ, at first glance, much less, to be separated in Nature by the non-occurrence of intermediaries. In this way, he builds up from the study of Nature a new general conception of the character in question. He obtains, for example, an idea of a leaf which includes every part of the flower, and an idea of a vertebra which includes the skull. I surely need not say much to show what a logical engine is here. It is the essence of the method of the naturalist. How he applies it first to one character, and then to another, and finally obtains a notion of a species of animals, the differences between whose members, however great, are confined within limits, is a matter which does not here concern us. The whole method of classification must be considered later; but, at present, I only desire to point out that it is by taking advan-

* *Novum Organum*, bk. II, Aphorism XXVII.

tage of the idea of continuity, or the passage from one form to another by insensible degrees,[1] that the naturalist builds his conceptions. Now, the naturalists are the great builders of conceptions; there is no other branch of science where so much of this work is done as in theirs; and we must, in great measure, take them for our teachers in this important part of logic. And it will be found everywhere that the idea of continuity[2] is a powerful aid to the formation of true and fruitful conceptions. By means of it, the greatest differences are broken down and resolved into differences of degree, and the incessant application of it is of the greatest value in broadening our conceptions. I propose to make a great use of this idea[3] in the present series of papers; and the particular series of important fallacies, which, arising from a neglect of it,[4] have desolated philosophy, must further on be closely studied. At present, I simply call the reader's attention to the utility of this conception.

In studies of numbers, the idea of continuity is so indispensable, that it is perpetually introduced even where there is no continuity in fact, as where we say that there are in the United States 10.7 inhabitants per square mile, or that in New York 14.72 persons live in the average house.[5] Another example is that law of the distribution of errors which Quetelet, Galton, and others, have applied with so much success to the study of biological and social matters. This application of continuity to cases where it does not really exist illustrates, also, another point which will hereafter demand a separate study, namely, the great utility which fictions sometimes have in science.*

[1] "Or rather of an idea that continuity suggests — that of limitless intermediation; i.e., of a series between every two members of which there is another member of it"— to be substituted for the phrase "or . . . degrees."— 1893.

[2] For "continuity" substitute "limitless intermediation, the business of reasoning."— 1893.

[3] "And others that are involved in that of continuity."— 1893.

[4] For "neglect of" substitute "want of close study of these concepts."— 1893.

[5] This mode of thought is so familiarly associated with all exact numerical consideration, that the phrase appropriate to it is imitated by shallow writers in order to produce the appearance of exactitude where none exists. Certain newspapers, which affect a learned tone, talk of "the average man," when they simply mean *most men*, and have no idea of striking an average.

* See, e.g., 1.383.

§2. THE PROBLEM OF PROBABILITY[E]

647. The theory of probabilities is simply the science of logic quantitatively treated. There are two conceivable certainties with reference to any hypothesis, the certainty of its truth and the certainty of its falsity. The numbers *one* and *zero* are appropriated, in this calculus, to marking these extremes of knowledge; while fractions having values intermediate between them indicate, as we may vaguely say, the degrees in which the evidence leans toward one or the other. The general problem of probabilities is, from a given state of facts, to determine the numerical probability of a possible fact. This is the same as to inquire how much the given facts are worth, considered as evidence to prove the possible fact. Thus the problem of probabilities is simply the general problem of logic.

648. Probability is a continuous quantity, so that great advantages may be expected from this mode of studying logic. Some writers have gone so far as to maintain that, by means of the calculus of chances, every solid inference may be represented by legitimate arithmetical operations upon the numbers given in the premisses. If this be, indeed, true, the great problem of logic, how it is that the observation of one fact can give us knowledge of another independent fact, is reduced to a mere question of arithmetic. It seems proper to examine this pretension before undertaking any more recondite solution of the paradox.

But, unfortunately, writers on probabilities are not agreed in regard to this result. This branch of mathematics is the only one, I believe, in which good writers frequently get results entirely erroneous. In elementary geometry the reasoning is frequently fallacious, but erroneous conclusions are avoided; but it may be doubted if there is a single extensive treatise on probabilities in existence which does not contain solutions absolutely indefensible. This is partly owing to the want of any regular method of procedure; for the subject involves too many subtilities to make it easy to put its problems into equations without such an aid. But, beyond this, the fundamental principles of its calculus are more or less in dispute. In regard to that class of questions to which it is chiefly applied for practical purposes, there is comparatively little doubt; but in

regard to others to which it has been sought to extend it, opinion is somewhat unsettled.

This last class of difficulties can only be entirely overcome by making the idea of probability perfectly clear in our minds in the way set forth in our last paper.*

§3. ON DEGREES OF PROBABILITY[E]

649. To get a clear idea of what we mean by probability, we have to consider what real and sensible difference there is between one degree of probability and another.

The character of probability belongs primarily, without doubt, to certain inferences. Locke† explains it as follows: After remarking that the mathematician positively knows that the sum of the three angles of a triangle is equal to two right angles because he apprehends the geometrical proof, he thus continues: "But another man who never took the pains to observe the demonstration, hearing a mathematician, a man of credit, affirm the three angles of a triangle to be equal to two right ones, *assents* to it; *i.e.*, receives it for true. In which case the foundation of his assent is the probability of the thing, the proof being such as, for the most part, carries truth with it; the man on whose testimony he receives it not being wont to affirm anything contrary to, or besides his knowledge, especially in matters of this kind." The celebrated *Essay Concerning Humane Understanding* contains many passages which, like this one, make the first steps in profound analyses which are not further developed. It was shown‡ in the first of these papers that the validity of an inference does not depend on any tendency of the mind to accept it, however strong such tendency may be; but consists in the real fact that, when premisses like those of the argument in question are true, conclusions related to them like that of this argument are also true. It was remarked that in a logical mind an argument is always conceived as a member of a *genus* of arguments all constructed in the same way, and such that, when their premisses are real facts, their conclusions are so also. If the argument is demonstrative, then this is always so; if it is only

* See vol. 5, bk. II, ch. 5.

† *Essay*, bk. IV, ch. 15, §1.

‡ See vol. 5, bk. II, ch. 4, §2.

probable, then it is for the most part so. As Locke says, the probable argument is "*such as* for the most part carries truth with it."

650. According to this, that real and sensible difference between one degree of probability and another, in which the meaning of the distinction lies, is that in the frequent employment of two different modes of inference, one will carry truth with it oftener than the other. It is evident that this is the only difference there is in the existing fact. Having certain premisses, a man draws a certain conclusion, and as far as this inference alone is concerned the only possible practical question is whether that conclusion is true or not, and between existence and non-existence there is no middle term. "Being only is and nothing is altogether not," said Parmenides; and this is in strict accordance with the analysis of the conception of reality given in the last paper.* For we found that the distinction of reality and fiction depends on the supposition that sufficient investigation would cause one opinion to be universally received and all others to be rejected. That presupposition, involved in the very conceptions of reality and figment, involves a complete sundering of the two. It is the heaven-and-hell idea in the domain of thought. But, in the long run, there is a real fact which corresponds to the idea of probability, and it is that a given mode of inference sometimes proves successful and sometimes not, and that in a ratio ultimately fixed. As we go on drawing inference after inference of the given kind, during the first ten or hundred cases the ratio of successes may be expected to show considerable fluctuations; but when we come into the thousands and millions, these fluctuations become less and less; and if we continue long enough, the ratio will approximate toward a fixed limit. We may, therefore, define the probability of a mode of argument as the proportion of cases in which it carries truth with it.

651. The inference from the premiss, A, to the conclusion, B, depends, as we have seen, on the guiding principle, that if a fact of the class A is true, a fact of the class B is true. The probability consists of the fraction whose numerator is the number of times in which both A and B are true, and whose denominator is the total number of times in which A is true,

* See vol. 5, bk. II, ch. 5, §4.

whether B is so or not. Instead of speaking of this as the probability of the inference, there is not the slightest objection to calling it the probability that, if A happens, B happens. But to speak of the probability of the event B, without naming the condition, really has no meaning at all. It is true that when it is perfectly obvious what condition is meant, the ellipsis may be permitted. But we should avoid contracting the habit of using language in this way (universal as the habit is), because it gives rise to a vague way of thinking, as if the action of causation might either determine an event to happen or determine it not to happen, or leave it more or less free to happen or not, so as to give rise to an *inherent* chance in regard to its occurrence. It is quite clear to me that some of the worst and most persistent errors in the use of the doctrine of chances have arisen from this vicious mode of expression.[1]

§4. THREE LOGICAL SENTIMENTS[E]

652. But there remains an important point to be cleared up. According to what has been said, the idea of probability essentially belongs to a kind of inference which is repeated indefinitely. An individual inference must be either true or false, and can show no effect of probability; and, therefore, in reference to a single case considered in itself, probability can have no meaning. Yet if a man had to choose between drawing a card from a pack containing twenty-five red cards and a black one, or from a pack containing twenty-five black cards and a red one, and if the drawing of a red card were destined to transport him to eternal felicity, and that of a black one to consign him to everlasting woe, it would be folly to deny that he ought to prefer the pack containing the larger proportion of red cards, although, from the nature of the risk, it could not be repeated. It is not easy to reconcile this with our analysis of the conception of chance. But suppose he should choose the red pack, and should draw the wrong card, what consolation would he have? He might say that he had acted in accordance with reason, but that would only show that his reason was

[1] The conception of probability here set forth is substantially that first developed by Mr. Venn, in his *Logic of Chance*. Of course, a vague apprehension of the idea had always existed, but the problem was to make it perfectly clear, and to him belongs the credit of first doing this.

absolutely worthless. And if he should choose the right card, how could he regard it as anything but a happy accident? He could not say that if he had drawn from the other pack, he might have drawn the wrong one, because an hypothetical proposition such as, "if A, then B," means nothing with reference to a single case. Truth consists in the existence of a real fact corresponding to the true proposition. Corresponding to the proposition, "if A, then B," there may be the fact that *whenever* such an event as A happens such an event as B happens. But in the case supposed, which has no parallel as far as this man is concerned, there would be no real fact whose existence could give any truth to the statement that, if he had drawn from the other pack, he might have drawn a black card. Indeed, since the validity of an inference consists in the truth of the hypothetical proposition that *if* the premisses be true the conclusion will also be true, and since the only real fact which can correspond to such a proposition is that whenever the antecedent is true the consequent is so also, it follows that there can be no sense in reasoning in an isolated case, at all.

653. These considerations appear, at first sight, to dispose of the difficulty mentioned. Yet the case of the other side is not yet exhausted. Although probability will probably manifest its effect in, say, a thousand risks, by a certain proportion between the numbers of successes and failures, yet this, as we have seen, is only to say that it certainly will, at length, do so. Now the number of risks, the number of probable inferences, which a man draws in his whole life, is a finite one, and he cannot be absolutely *certain* that the mean result will accord with the probabilities at all. Taking all his risks collectively, then, it cannot be certain that they will not fail, and his case does not differ, except in degree, from the one last supposed. It is an indubitable result of the theory of probabilities that every gambler, if he continues long enough, must ultimately be ruined. Suppose he tries the martingale, which some believe infallible, and which is, as I am informed, disallowed in the gambling-houses. In this method of playing, he first bets say \$1; if he loses it he bets \$2; if he loses that he bets \$4; if he loses that he bets \$8; if he then gains he has lost $1+2+4=7$, and he has gained \$1 more; and no matter how many bets he loses, the first one he gains will make him \$1 richer than he

was in the beginning. In that way, he will probably gain at first; but, at last, the time will come when the run of luck is so against him that he will not have money enough to double, and must, therefore, let his bet go. This will *probably* happen before he has won as much as he had in the first place, so that this run against him will leave him poorer than he began; some time or other it will be sure to happen. It is true that there is always a possibility of his winning any sum the bank can pay, and we thus come upon a celebrated paradox that, though he is certain to be ruined, the value of his expectation calculated according to the usual rules (which omit this consideration) is large. But, whether a gambler plays in this way or any other, the same thing is true, namely, that if [he] plays long enough he will be sure some time to have such a run against him as to exhaust his entire fortune. The same thing is true of an insurance company. Let the directors take the utmost pains to be independent of great conflagrations and pestilences, their actuaries can tell them that, according to the doctrine of chances, the time must come, at last, when their losses will bring them to a stop. They may tide over such a crisis by extraordinary means, but then they will start again in a weakened state, and the same thing will happen again all the sooner. An actuary might be inclined to deny this, because he knows that the expectation of his company is large, or perhaps (neglecting the interest upon money) is infinite. But calculations of expectations leave out of account the circumstance now under consideration, which reverses the whole thing. However, I must not be understood as saying that insurance is on this account unsound, more than other kinds of business. All human affairs rest upon probabilities, and the same thing is true everywhere. If man were immortal he could be perfectly sure of seeing the day when everything in which he had trusted should betray his trust, and, in short, of coming eventually to hopeless misery. He would break down, at last, as every great fortune, as every dynasty, as every civilization does. In place of this we have death.

654. But what, without death, would happen to every man, with death must happen to some man. At the same time, death makes the number of our risks, of our inferences, finite, and so makes their mean result uncertain. The very idea of

probability and of reasoning rests on the assumption that this number is indefinitely great. We are thus landed in the same difficulty as before, and I can see but one solution of it. It seems to me that we are driven to this, that logicality inexorably requires that our interests shall *not* be limited. They must not stop at our own fate, but must embrace the whole community. This community, again, must not be limited, but must extend to all races of beings with whom we can come into immediate or mediate intellectual relation. It must reach, however vaguely, beyond this geological epoch, beyond all bounds. He who would not sacrifice his own soul to save the whole world, is, as it seems to me, illogical in all his inferences, collectively. Logic is rooted in the social principle.

To be logical men should not be selfish; and, in point of fact, they are not so selfish as they are thought. The willful prosecution of one's desires is a different thing from selfishness. The miser is not selfish; his money does him no good, and he cares for what shall become of it after his death. We are constantly speaking of *our* possessions on the Pacific, and of *our* destiny as a republic, where no personal interests are involved, in a way which shows that we have wider ones. We discuss with anxiety the possible exhaustion of coal in some hundreds of years, or the cooling-off of the sun in some millions, and show in the most popular of all religious tenets that we can conceive the possibility of a man's descending into hell for the salvation of his fellows.

Now, it is not necessary for logicality that a man should himself be capable of the heroism of self-sacrifice. It is sufficient that he should recognize the possibility of it, should perceive that only that man's inferences who has it are really logical, and should consequently regard his own as being only so far valid as they would be accepted by the hero. So far as he thus refers his inferences to that standard, he becomes identified with such a mind.

This makes logicality attainable enough. Sometimes we can personally attain to heroism. The soldier who runs to scale a wall knows that he will probably be shot, but that is not all he cares for. He also knows that if all the regiment, with whom in feeling he identifies himself, rush forward at once, the fort will be taken. In other cases we can only imitate the virtue.

The man whom we have supposed as having to draw from the two packs, who if he is not a logician will draw from the red pack from mere habit, will see, if he is logician enough, that he cannot be logical so long as he is concerned only with his own fate, but that that man who should care equally for what was to happen in all possible cases of the sort could act logically, and would draw from the pack with the most red cards, and thus, though incapable himself of such sublimity, our logician would imitate the effect of that man's courage in order to share his logicality.

But all this requires a conceived identification of one's interests with those of an unlimited community. Now, there exist no reasons, and a later discussion will show that there can be no reasons, for thinking that the human race, or any intellectual race, will exist forever. On the other hand, there can be no reason against it;[1] and, fortunately, as the whole requirement is that we should have certain sentiments, there is nothing in the facts to forbid our having a *hope*, or calm and cheerful wish, that the community may last beyond any assignable date.

655. It may seem strange that I should put forward three sentiments, namely, interest in an indefinite community, recognition of the possibility of this interest being made supreme, and hope in the unlimited continuance of intellectual activity, as indispensable requirements of logic. Yet, when we consider that logic depends on a mere struggle to escape doubt, which, as it terminates in action, must begin in emotion, and that, furthermore, the only cause of our planting ourselves on reason is that other methods of escaping doubt fail on account of the social impulse, why should we wonder to find social sentiment presupposed in reasoning? As for the other two sentiments which I find necessary, they are so only as supports and accessories of that. It interests me to notice that these three sentiments seem to be pretty much the same as that famous trio of Charity, Faith, and Hope, which, in the estimation of St. Paul, are the finest and greatest of spiritual gifts. Neither

[1] I do not here admit an absolutely unknowable. Evidence could show us what would probably be the case after any given lapse of time; and though a subsequent time might be assigned which that evidence might not cover, yet further evidence would cover it.

Old nor New Testament is a textbook of the logic of science, but the latter is certainly the highest existing authority in regard to the dispositions of heart which a man ought to have.

§5. FUNDAMENTAL RULES FOR THE CALCULATION OF CHANCES[E]

656. Such average statistical numbers as the number of inhabitants per square mile, the average number of deaths per week, the number of convictions per indictment, or, generally speaking, the numbers of x's per y, where the x's are a class of things some or all of which are connected with another class of things, their y's, I term *relative numbers*. Of the two classes of things to which a relative number refers, that one of which it is a number may be called its *relate*, and that one *per* which the numeration is made may be called its *correlate*.

657. Probability is a kind of relative number; namely, it is the ratio of the number of arguments of a certain genus which carry truth with them to the total number of arguments of that genus, and the rules for the calculation of probabilities are very easily derived from this consideration. They may all be given here, since they are extremely simple, and it is sometimes convenient to know something of the elementary rules of calculation of chances.

658. Rule I. *Direct Calculation.* — To calculate, directly, any relative number, say for instance the number of passengers in the average trip of a street-car, we must proceed as follows:

Count the number of passengers for each trip; add all these numbers, and divide by the number of trips. There are cases in which this rule may be simplified. Suppose we wish to know the number of inhabitants to a dwelling in New York. The same person cannot inhabit two dwellings. If he divide his time between two dwellings he ought to be counted a half-inhabitant of each. In this case we have only to divide the total number of the inhabitants of New York by the number of their dwellings, without the necessity of counting separately those which inhabit each one. A similar proceeding will apply wherever each individual relate belongs to one individual correlate exclusively. If we want the number of x's per y, and no x belongs to more than one y, we have only to divide the whole

number of x's of y's by the number of y's. Such a method would, of course, fail if applied to finding the average number of street-car passengers per trip. We could not divide the total number of travelers by the number of trips, since many of them would have made many passages.

To find the probability that from a given class of premisses, A, a given class of conclusions, B, follows, it is simply necessary to ascertain what proportion of the times in which premisses of that class are true, the appropriate conclusions are also true. In other words, it is the number of cases of the occurrence of both the events A and B, divided by the total number of cases of the occurrence of the event A.

659. Rule II. *Addition of Relative Numbers.* — Given two relative numbers having the same correlate, say the number of x's per y, and the number of z's per y, it is required to find the number of x's and z's together per y. If there is nothing which is at once an x and a z to the same y, the sum of the two given numbers would give the required number. Suppose, for example, that we had given the average number of friends that men have, and the average number of enemies, the sum of these two is the average number of persons interested in a man. On the other hand, it plainly would not do to add the average number of persons having constitutional diseases to the average number over military age, and to the average number exempted by each special cause from military service, in order to get the average number exempt in any way, since many are exempt in two or more ways at once.

This rule applies directly to probabilities, given the probability that two different and mutually exclusive events will happen under the same supposed set of circumstances. Given, for instance, the probability that if A then B, and also the probability that if A then C, then the sum of these two probabilities is the probability that if A then either B or C, so long as there is no event which belongs at once to the two classes B and C.

660. Rule III. *Multiplication of Relative Numbers.* — Suppose that we have given the relative number of x's per y; also the relative number of z's per x of y; or, to take a concrete example, suppose that we have given, first, the average number of children in families living in New York; and, second, the

average number of teeth in the head of a New York child — then the product of these two numbers would give the average number of children's teeth in a New York family. But this mode of reckoning will only apply in general under two restrictions. In the first place, it would not be true if the same child could belong to different families, for in that case those children who belonged to several different families might have an exceptionally large or small number of teeth, which would affect the average number of children's teeth in a family more than it would affect the average number of teeth in a child's head. In the second place, the rule would not be true if different children could share the same teeth, the average number of children's teeth being in that case evidently something different from the average number of teeth belonging to a child.

In order to apply this rule to probabilities, we must proceed as follows: Suppose that we have given the probability that the conclusion B follows from the premiss A, B and A representing as usual certain classes of propositions. Suppose that we also knew the probability of an inference in which B should be the premiss, and a proposition of a third kind, C, the conclusion. Here, then, we have the materials for the application of this rule. We have, first, the relative number of B's per A. We next should have the relative number of C's per B following from A. But the classes of propositions being so selected that the probability of C following from any B in general is just the same as the probability of C's following from one of those B's which is deducible from an A, the two probabilities may be multiplied together, in order to give the probability of C following from A. The same restrictions exist as before. It might happen that the probability that B follows from A was affected by certain propositions of the class B following from several different propositions of the class A. But, practically speaking, all these restrictions are of very little consequence, and it is usually recognized as a principle universally true that the probability that, if A is true, B is, multiplied by the probability that, if B is true, C is, gives the probability that, if A is true, C is.

There is a rule supplementary to this, of which great use is made. It is not universally valid, and the greatest caution has to be exercised in making use of it — a double care, first,

never to use it when it will involve serious error; and, second, never to fail to take advantage of it in cases in which it can be employed. This rule depends upon the fact that in very many cases the probability that C is true if B is, is substantially the same as the probability that C is true if A is. Suppose, for example, we have the average number of males among the children born in New York; suppose that we also have the average number of children born in the winter months among those born in New York. Now, we may assume without doubt, at least as a closely approximate proposition (and no very nice calculation would be in place in regard to probabilities), that the proportion of males among all the children born in New York is the same as the proportion of males born in summer in New York; and, therefore, if the names of all the children born during a year were put into an urn, we might multiply the probability that any name drawn would be the name of a male child by the probability that it would be the name of a child born in summer, in order to obtain the probability that it would be the name of a male child born in summer. The questions of probability, in the treatises upon the subject, have usually been such as relate to balls drawn from urns, and games of cards, and so on, in which the question of the *independence* of events, as it is called — that is to say, the question of whether the probability of C, under the hypothesis B, is the same as its probability under the hypothesis A — has been very simple; but, in the application of probabilities to the ordinary questions of life, it is often an exceedingly nice question whether two events may be considered as independent with sufficient accuracy or not. In all calculations about cards it is assumed that the cards are thoroughly shuffled, which makes one deal quite independent of another. In point of fact the cards seldom are, in practice, shuffled sufficiently to make this true; thus, in a game of whist, in which the cards have fallen in sets of four of the same suit, and are so gathered up, they will lie more or less in sets of four of the same suit, and this will be true even after they are shuffled. At least some traces of this arrangement will remain, in consequence of which the number of "short suits," as they are called — that is to say, the number of hands in which the cards are very unequally divided in regard to suits — is smaller than the calculation

would make it to be; so that, when there is a misdeal, where the cards, being thrown about the table, get very thoroughly shuffled, it is a common saying that in the hands next dealt out there are generally short suits. A few years ago a friend of mine, who plays whist a great deal, was so good as to count the number of spades dealt to him in 165 hands, in which the cards had been, if anything, shuffled better than usual. According to calculation, there should have been 85 of these hands in which my friend held either three or four spades, but in point of fact there were 94, showing the influence of imperfect shuffling.

According to the view here taken, these are the only fundamental rules for the calculation of chances. An additional one, derived from a different conception of probability, is given in some treatises, which if it be sound might be made the basis of a theory of reasoning. Being, as I believe it is, absolutely absurd, the consideration of it serves to bring us to the true theory; and it is for the sake of this discussion, which must be postponed to the next number,* that I have brought the doctrine of chances to the reader's attention at this early stage of our studies of the logic of science.

§6. NOTES ON THE DOCTRINE OF CHANCES†

661. On reperusing this article after the lapse of a full generation, it strikes me as making two points that were worth making. The better made of the two had been still better made ten years before in my three articles in the [*Journal of Speculative Philosophy*] Vol. 2.‡ This point is that no man can be logical whose supreme desire is the well-being of himself or of any other existing person or collection of persons. The other good point is that probability never properly refers immediately to a single event, but exclusively to the happening of a given kind of event on any occasion of a given kind. So far all is well. But when I come to define probability, I repeatedly say that it is the quotient of the *number* of occurrences of the event divided by the *number* of occurrences of the occasion. Now this is manifestly wrong, for probability relates to the

* Ch. 7.

† 1910.

‡ See vol. 5, bk. II, chs. 1, 2, 3, particularly 5.355.

future; and how can I say how many times a given die will be thrown in the future? To be sure I might, immediately after my throw, put the die in strong nitric acid, and dissolve it, but this suggestion only puts the preposterous character of the definition in a still stronger light. For it is plain that, if probability be the ratio of the occurrences of the specific event to the occurrences of the generic occasion, it is the ratio that there *would be* in the long run, and has nothing to do with any supposed cessation of the occasions. This long run can be nothing but an endlessly long run; and even if it be correct to speak of an infinite "number," yet $\frac{\infty}{\infty}$ (infinity divided by infinity) has certainly, *in itself*, no definite value.

But we have not yet come to the end of the flaws in the definition, since no notice whatever has been taken of two conditions which require the strictest precautions in all experiments to determine the probability of a specific event on a generic occasion. Namely, in the first place we must limit our endeavors strictly to counting occurrences of the right genus of occasion and carefully resist all other motives for counting them, and strive to take them just as they would ordinarily occur. In the next place, it must be known that the occurrence of the specific event on one occasion will have no tendency to produce or to prevent the occurrence of the same event upon any other of the occurrences of the generic occasion. In the third place, after the probability has been ascertained, we must remember that this probability cannot be relied upon at any future time unless we have adequate grounds for believing that it has not too much changed in the interval.

662. I will now give over jeering at my former inaccuracies, committed when I had been a student of logic for only about a quarter of a century, and was naturally not so well-versed in it as now, and will proceed to define probability. I must premiss that we, all of us, use this word with a degree of laxity which corrupts and rots our reasoning to a degree that very few of us are at all awake to. When I say our "reasoning," I mean not formal reasonings only but our thoughts in general, so far as they are concerned with any of those approaches toward knowledge which we confound with probability. The result is that we not only fall into the falsest ways of thinking, but, what is often still worse, we give up sundry

problems as beyond our powers — problems of gravest concern, too — when, in fact, we should find they were not a bit so, if we only rightly discriminated between the different kinds of imperfection of certitude, and if we had only once acquainted ourselves with their different natures. I shall in these notes endeavor to mark the three ways of falling short of certainty by the three terms *probability*, *verisimilitude* or *likelihood*, and *plausibility*. Just at present I propose to deal only with Probability; but I will so far characterize *verisimilitude* and *plausibility* as to mark them off as being entirely different from Probability. Beginning with Plausibility,* I will first endeavor to give an example of an idea which shall be strikingly marked by its very low degree of this quality. Suppose a particularly symmetrical larch tree near the house of a great lover of such trees had been struck by lightning and badly broken, and that as he was looking sorrowfully out of the window at it, he should have happened to say, "I wonder why that particular tree should have been struck, when there are so many about the place that seem more exposed!" Suppose, then, his wife should reply, "Perhaps there may be an eagle's eyrie on some of the hills in the neighborhood, and perhaps the male bird in building it may have used some stick that had a nail in it; and one of the eaglets may have scratched itself against the nail; so that the mother may have reproached the male for using such a dangerous stick; and he, being vexed with her teasing, may have determined to carry the piece to a great distance; it may have been while he was doing this that the explosion of lightning took place, and the electricity may have been deflected by the iron in such a way as to strike this tree. Mind, I do not say that this is what did happen; but if you want to find out why that tree was struck, I think you had better search for an eyrie, and see whether any of the eaglets have been scratched." This is an example of as unplausible a theory as I can think of. We should commonly say it was highly improbable; and I suppose it would be so. But were it ever so probable in all its elements, it would still deserve no attention, because it is perfectly gratuitous to suppose that the lightning was deflected at all; and this supposition does not help to explain the phenomenon.

* Cf. 111, 269, 756f.

Eusapia Palladino had been proved to be a very clever prestigiateuse and cheat, and was visited by a Mr. Carrington,* whom I suppose to be so clever in finding out how tricks are done, that it is highly improbable that any given trick should long baffle him. In point of fact he has often caught the Palladino creature in acts of fraud. Some of her performances, however, he cannot explain; and thereupon he urges the theory that these are supernatural, or, as he prefers to phrase it, "supernormal." Well, I know how it is that when a man has been long intensely exercised and over-fatigued by an enigma, his common-sense will sometimes desert him; but it seems to me that the Palladino has simply been too clever for him, as no doubt she would be for me. The theory that there is anything "supernormal," or *super* anything but *superchérie* in the case, seems to me as needless as any theory I ever came across. That is to say, granted that it is not yet *proved* that women who deceive for gain receive aid from the spiritual world, I think it more plausible that there are tricks that can deceive Mr. Carrington than that the Palladino woman has received such aid. By Plausible, I mean that a theory that has not yet been subjected to any test, although more or less surprising phenomena have occurred which it would explain if it were true, is in itself of such a character as to recommend it for further examination or, if it be *highly* plausible, justify us in seriously inclining toward belief in it, as long as the phenomena be inexplicable otherwise.

663. I will now give an idea of what I mean by *likely* or *verisimilar*. It is to be understood that I am only endeavouring so far to explain the meanings I attach to "plausible" and to "likely," as this may be an assistance to the reader in understanding the meaning I attach to *probable*. I call that theory *likely* which is not yet proved but is supported by such evidence that if the rest of the conceivably possible evidence should turn out upon examination to be of a *similar* character, the theory would be conclusively proved. Strictly speaking, matters of fact never can be demonstrably proved, since it will always remain conceivable that there should be some mistake about it. For instance, I regard it as *sufficiently* proved that my name is Charles Peirce and that I was born in Cambridge,

* See Carrington's *Eusapia Palladino*, B. W. Dodge & Co., New York (1909).

Massachusetts, in a stone-colored wooden house in Mason Street. But even of the part of this of which I am most assured — of my name — there is a certain small probability that I am in an abnormal condition and have got it wrong. I am conscious myself of occasional lapses of memory about other things; and though I well remember — or think I do — living in that house at a tender age, I do not in the least remember being born there, impressive as such a first experience might be expected to be. Indeed, I cannot specify any date on which any certain person informed me I had been born there; and it certainly would have been easy to deceive me in the matter had there been any serious reason for doing so; and how can I be so sure as I surely am that no such reason did exist? It would be a theory without plausibility; that is all.

The history of science, particularly physical science, in contradistinction to natural science — or, as I usually, though inadequately, phrase the distinction, the history of nomological in contradistinction to classificatory sciences — this history ever since I first seriously set myself, at the age of thirteen, in 1852, to the study of logic,* shows only too grievously how great a boon would be any way [of] determining and expressing by numbers the degree of likelihood that a theory had attained — any general recognition, even among leading men of science, of the true degree of significance of a given fact, and of the proper method of determining it. I hope my writings may, at any rate, awaken a few to the enormous waste of effort it would save. But any numerical determination of likelihood is more than I can expect.

664. The only kind of reasoning which can render our conclusions certain — and even this kind can do so only under the proviso that no blunder has been committed in the process — attains this certainty by limiting the conclusion (as Kant virtually said, and others before him), to facts already expressed and accepted in the premisses. This is called necessary, or syllogistic reasoning. Syllogism, not confined to the kind that Aristotle and Theophrastus studied, is merely an artificial form in which it may be expressed, and it is not its best form, from any point of view. But the kind of reasoning which creates likelihoods by virtue of observations may render a likeli-

* Peirce read *Whately's Logic* at this time.

hood *practically* certain — as certain as that a stone let loose from the clutch will, under circumstances not obviously exceptional, fall to the ground — and this conclusion may be that under a certain general condition, easily verified, a certain actuality will be *probable*, that is to say, will come to pass once in so often in the long run. One such familiar conclusion, for example, is that a die thrown from a dice box will with a *probability* of one-third, that is, once in three times in the long run, turn up a number (either *tray* or *size*) that is divisible by three. But this can be affirmed with practical certainty only if by a "long run" be meant an endless series of trials, and (as just said) infinity divided by infinity gives of itself an entirely indefinite quotient. It is therefore necessary to define the phrase. I might give the definition with reference to the probability, p, where p is any vulgar fraction, and in reference to a generic condition, m, and a specific kind of event n. But I think the reader will follow me more readily, if in place of the letter, m (which in itself is but a certain letter, to which is attached a peculiar meaning, that of the fulfillment of some generic condition) I put instead the supposition that a die is thrown from a dice box; and this special supposition will be as readily understood by the reader to be replaceable by any other general condition along with a simultaneous replacement of the *event*, that a number divisible by three is turned up, and at the same time with the replacement of one third by whatever other vulgar fraction may be called for when some different example of a probability is before us. I am, then, to define the meanings of the statement that the *probability*, that if a die be thrown from a dice box it will turn up a number divisible by three, is one-third. The statement means that the die has a certain "would-be"; and to say that a die has a "would-be" is to say that it has a property, quite analogous to any *habit* that a man might have. Only the "would-be" of the die is presumably as much simpler and more definite than the man's habit as the die's homogeneous composition and cubical shape is simpler than the nature of the man's nervous system and soul; and just as it would be necessary, in order to define a man's habit, to describe how it would lead him to behave and upon what sort of occasion — albeit this statement would by no means imply that the habit *consists* in that

action — so to define the die's "would-be," it is necessary to say how it would lead the die to behave on an occasion that would bring out the full consequence of the "would-be"; and this statement will not of itself imply that the "would-be" of the die *consists* in such behavior.

665. Now in order that the full effect of the die's "would-be" may find expression, it is necessary that the die should undergo an endless series of throws from the dice box, the result of no throw having the slightest influence upon the result of any other throw, or, as we express it, the throws must be *independent* each of every other.

666. It will be no objection to our considering the consequences of the supposition that the die is thrown an endless succession of times, and that with a finite pause after each throw, that such an endless series of events is impossible, for the reason that the impossibility is merely a physical, and not a logical, impossibility, as was well illustrated in that famous sporting event in which Achilles succeeded in overtaking the champion tortoise, in spite of his giving the latter the start of a whole *stadion*. For it having been ascertained, by delicate measurements between a mathematical point between the shoulder-blades of Achilles (marked [by] a limit between a red, a green, and a violet sector of a stained disk) and a similar point on the carapace of the tortoise, that when Achilles arrived where the tortoise started, the latter was just 60 feet 8 inches and $\frac{1}{10}$ inch further on, which is just one tenth of a stadion, and that when Achilles reached that point the tortoise was still 6 feet and $8\frac{1}{100}$ inch in advance of him, and finally that, both advancing at a perfectly uniform rate, the tortoise had run just 67 feet 5 inches when he was overtaken by Achilles, it follows that the tortoise progressed at just one tenth the speed of Achilles, the latter running a distance in *stadia* of $1.111111111\dot{1}$, so that he had to traverse the sum of an infinite multitude of finite distances, each in a finite time, and yet covered the *stadion* and one ninth in a finite time. No contradiction, therefore, is involved in the idea of an endless series of finite times or spaces having but a finite sum, provided there is no *fixed* finite quality which every member of an endless part of that series must each and every one *exceed*.

The reader must pardon me for occupying any of his time

with such puerile stuff as that $0.1111 = \frac{1}{9}$; for astounding as it seems, it has more than once happened to me that men have come to me — every one of them not merely educated men, but highly accomplished — men who might well enough be famous over the civilized world, if fame were anything to the purpose, but men whose studies had been such that one would have expected to find each of them an adept in the accurate statement of arguments, and yet each has come and has undertaken to prove to me that the old catch of Achilles and the tortoise is a sound argument. If I tell you what after listening to them by the hour, I have always ended by saying — it may serve your turn on a similar occasion — I have said, "I suppose you do not mean to say that you really believe that a fast runner cannot, as a matter of fact, overtake a slow one. I therefore conclude that the argument which you have been unable to state, either syllogistically or in any other intelligible form, is intended to show that Zeno's reasoning about Achilles and the tortoise is sound according to some system of logic which admits that sound necessary reasoning may lead from true premisses to a false conclusion. But in my system of logic what I mean by bad necessary reasoning is precisely an argument which might lead from true premisses to a false conclusion — just that and nothing else. If you prefer to call such reasoning a sound necessary argument, I have no objection in the world to your doing so; and you will kindly allow me to employ my different nomenclature. For I am such a plain, uncultured soul that when I reason I aim at nothing else than just to find out the truth." To get back, then, to the die and its habit — its "would-be" — I really know no other way of defining a habit than by describing the kind of behavior in which the habit becomes actualized. So I am obliged to define the statement that there is a probability of one-third that the die when thrown will turn up either a three or a six by stating how the numbers will run when the die is thrown.

667. But my purpose in doing so is to explain what *probability*, as I use the word, consists in. Now it would be no explanation at all to say that it consists in something being *probable*. So I must avoid using that word or any synonym of it. If I were to use such an expression, you would very properly turn upon me and say, "I either know what it is to

be *probable*, in your sense of the term, or I do not. If I don't, how can I be expected to understand you until you have explained yourself; and if I do, what is the use of the explanation?" But the fact [is] that the probability of the die turning up a three or a six is not *sure* to produce any determination [of] the run of the numbers thrown in any *finite* series of throws. It is only when the series is endless that we can be *sure* that it will have a particular character. Even when there is an endless series of throws, there is no syllogistic certainty, no "mathematical" certainty (if you are more familiar with this latter phrase) — that the die will not turn up a six obstinately at every single throw. It might be that if in the course of the endless series, some friends should borrow the die to make a pair for a game of backgammon, there might be nothing unusual in the behavior of the lent die, and yet when it was returned and our experimental series was resumed where it had been interrupted, the die might return to turning up nothing but six every time. I say it *might*, in the sense that it would not violate the principle of contradiction if it did. It sanely *would not*, however, unless a miracle were performed; and moreover if such miracle *were* worked, I should say (since it is my use of the term "probability" that we have supposed to be in question) that during this experimental series of throws, the die took on an abnormal, a miraculous, habit. For I should think that the performance of a certain line of behavior, throughout an endless succession of occasions, without exception, very decidedly *constituted* a habit. There may be some doubt about this, for owing to our not being accustomed to reason in this way about successions of events which are endless *in the sequence* and yet are completed *in time*, it is hard for me quite to satisfy myself what I ought to say in such a case. But I have reflected seriously on it, and though I am not perfectly sure of my ground (and I am a cautious reasoner), yet I am more that what you would understand by "pretty confident," that supposing one to be in a condition to assert what *would surely be* the behavior, *in any single determinate respect*, of any subject throughout an endless series of occasions of a stated kind, he *ipso facto* knows a "would-be," or habit, of that subject. It is very true, mind you, that *no* collection whatever of single acts, though it were ever so many grades

greater than a simple endless series, can constitute a would-be, nor can the knowledge of single acts, whatever their multitude, tell us for *sure* of a would-be. But there are two remarks to be made; first, that in the case under consideration a person is supposed to be in a condition to assert what surely *would be* the behavior of the subject throughout the endless series of occasions — a knowledge which cannot have been derived from reasoning from its behavior on the single occasions; and second, that that which in our case renders it true, as stated, that the person supposed "*ipso facto* knows a would-be of that subject," is not the occurrence of the single acts, but the fact that the person supposed "was in condition to assert what *would surely be* the behavior of the subject throughout an endless series of occasions."[1]

668. I will now describe the behavior of the die during the endless series of throws, in respect to turning up numbers divisible by three. It would be perfectly possible to construct a machine that would automatically throw the die and pick it up, and continue doing so as long as it was supplied with energy. It would further be still easier to design the plan of an arrangement whereby a hand should after each throw move over an arc graduated so as to indicate the value of the quotient of the number of throws of three or six that had been known since the beginning of the experiment, divided by the total number of throws since the beginning. It is true that the mechanical difficulties would become quite insuperable before the die had been thrown many times; but fortunately a general description of the way the hand would move will answer our purpose much better than would the actual machine, were it ever so perfect.

After the first throw, the hand will go either to $0=\frac{0}{1}$ or $1=\frac{1}{1}$; and there it may stay for several throws. But when it once moves, it will move after every throw, without exception, since the denominator of the fraction at whose value it points will always increase by 1, and consequently the value

[1] Meantime it may be remarked that, though an endless series of acts is not a habit, nor a would-be, it does present the first of an endless series of steps toward the full nature of a would-be. Compare what I wrote nineteen[thirteen!] years ago, in an article on the logic of relatives [3.526ff].

of the fraction will be diminished if the numerator remains unchanged, as it will be increased in case the numerator is increased by 1, these two being the only possible cases. The behavior of the hand may be described as an excessively irregular oscillation, back and forth, from one side of $\frac{1}{3}$ to the other. . . .

CHAPTER 7

*THE PROBABILITY OF INDUCTION**

§1. RULES FOR THE ADDITION AND MULTIPLICATION OF PROBABILITIES[E]

669. We have found that every argument derives its force from the general truth of the class of inferences to which it belongs; and that probability is the proportion of arguments carrying truth with them among those of any *genus*. This is most conveniently expressed in the nomenclature of the mediæval logicians. They called the fact expressed by a premiss an *antecedent*, and that which follows from it its *consequent;* while the leading principle, that every (or almost every) such antecedent is followed by such a consequent, they termed the *consequence*. Using this language, we may say that probability belongs exclusively to *consequences*, and the probability of any consequence is the number of times in which antecedent and consequent both occur divided by the number of all the times in which the antecedent occurs. From this definition are deduced the following rules for the addition and multiplication of probabilities:

670. *Rule for the Addition of Probabilities.* — Given the separate probabilities of two consequences having the same antecedent and incompatible consequents. Then the sum of these two numbers is the probability of the consequence, that from the same antecedent one or other of those consequents follows.

671. *Rule for the Multiplication of Probabilities.* — Given the separate probabilities of the two consequences, "If A then B," and "If both A and B, then C." Then the product of the these two numbers is the probability of the consequence, "If A, then both B and C."

672. *Special Rule for the Multiplication of Independent*

* *Popular Science Monthly*, vol. 12, pp. 705–18 (1878), the fourth of a series of papers on "Illustrations of the Logic of Science." See 612n. Intended as Essay XI of the *Search for a Method* (1893).

Probabilities. — Given the separate probabilities of two consequences having the same antecedents, "If A, then B," and "If A, then C." Suppose that these consequences are such that the probability of the second is equal to the probability of the consequence, "If both A and B, then C." Then the product of the two given numbers is equal to the probability of the consequence, "If A, then both B and C."

To show the working of these rules we may examine the probabilities in regard to throwing dice. What is the probability of throwing a six with one die? The antecedent here is the event of throwing a die; the consequent, its turning up a six. As the die has six sides, all of which are turned up with equal frequency, the probability of turning up any one is $\frac{1}{6}$. Suppose two dice are thrown, what is the probability of throwing sixes? The probability of either coming up six is obviously the same when both are thrown as when one is thrown — namely, $\frac{1}{6}$. The probability that either will come up six when the other does is also the same as that of its coming up six whether the other does or not. The probabilities are, therefore, independent; and, by our rule, the probability that both events will happen together is the product of their several probabilities, or $\frac{1}{6}\times\frac{1}{6}$. What is the probability of throwing deuce-ace? The probability that the first die will turn up ace and the second deuce is the same as the probability that both will turn up sixes — namely, $\frac{1}{36}$; the probability that the *second* will turn up ace and the *first* deuce is likewise $\frac{1}{36}$; these two events — first, ace; second, deuce; and, second, ace; first, deuce — are incompatible. Hence the rule for addition holds, and the probability that either will come up ace and the other deuce is $\frac{1}{36}+\frac{1}{36}$, or $\frac{1}{18}$.

In this way all problems about dice, etc., may be solved. When the number of dice thrown is supposed very large, mathematics (which may be defined as the art of making groups to facilitate numeration) comes to our aid with certain devices to reduce the difficulties.

§2. MATERIALISTIC AND CONCEPTUALISTIC VIEWS OF PROBABILITY[E]

673. The conception of probability as a matter of *fact*, i.e., as the proportion of times in which an occurrence of one kind

is accompanied by an occurrence of another kind, is termed by Mr. Venn the materialistic view of the subject. But probability has often been regarded as being simply the degree of belief which ought to attach to a proposition, and this mode of explaining the idea is termed by Venn the conceptualistic view. Most writers have mixed the two conceptions together. They, first, define the probability of an event as the reason we have to believe that it has taken place, which is conceptualistic; but shortly after they state that it is the ratio of the number of cases favorable to the event to the total number of cases favorable or contrary, and all equally possible. Except that this introduces the thoroughly unclear idea of cases equally possible in place of cases equally frequent, this is a tolerable statement of the materialistic view. The pure conceptualistic theory has been best expounded by Mr. De Morgan in his *Formal Logic: or, the Calculus of Inference, Necessary and Probable.*

674. The great difference between the two analyses is, that the conceptualists refer probability to an event, while the materialists make it the ratio of frequency of events of a *species* to those of a *genus* over that *species*, thus *giving it two terms instead of one.* The opposition may be made to appear as follows:*

Suppose that we have two rules of inference, such that, of all the questions to the solution of which both can be applied, the first yields correct answers to $\frac{81}{100}$, and incorrect answers to the remaining $\frac{19}{100}$; while the second yields correct answers to $\frac{93}{100}$, and incorrect answers to the remaining $\frac{7}{100}$. Suppose, further, that the two rules are entirely independent as to their truth, so that the second answers correctly $\frac{93}{100}$ of the questions which the first answers correctly, and also $\frac{93}{100}$ of the questions which the first answers incorrectly, and answers incorrectly the remaining $\frac{7}{100}$ of the questions which the first answers correctly, and also the remaining $\frac{7}{100}$ of the questions which the first answers incorrectly. Then, of all the questions to the solution of which both rules can be applied —

both answer correctly $\frac{93}{100}$ of $\frac{81}{100}$, or $\frac{93\times81}{100\times100}$;

* Cf. 3.17.

the second answers correctly and the first incorrectly

$$\frac{93}{100} \text{ of } \frac{19}{100}, \text{ or } \frac{93\times19}{100\times100};$$

the second answers incorrectly and the first correctly

$$\frac{7}{100} \text{ of } \frac{81}{100}, \text{ or } \frac{7\ \times81}{100\times100};$$

and both answer incorrectly

$$\frac{7}{100} \text{ of } \frac{19}{100}, \text{ or } \frac{7\ \times19}{100\times100};$$

Suppose, now, that, in reference to any question, both give the same answer. Then (the questions being always such as are to be answered by *yes* or *no*), those in reference to which their answers agree are the same as those which both answer correctly together with those which both answer falsely, or $\frac{93\times81}{100\times100}+\frac{7\ \times19}{100\times100}$ of all. The proportion of those which both answer correctly out of those their answers to which agree is, therefore —

$$\frac{\frac{93\times81}{100\times100}}{\frac{93\times81}{100\times100}+\frac{7\times19}{100\times100}} \text{ or } \frac{93\times81}{(93\times81)+(7\times19)}.$$

675. This is, therefore, the probability that, if both modes of inference yield the same result, that result is correct. We may here conveniently make use of another mode of expression. *Probability* is the ratio of the favorable cases to all the cases. Instead of expressing our result in terms of this ratio, we may make use of another — the ratio of favorable to unfavorable cases. This last ratio may be called the *chance* of an event. Then the chance of a true answer by the first mode of inference is $\frac{81}{19}$ and by the second is $\frac{93}{7}$; and the chance of a correct answer from both, when they agree, is —

$$\frac{81\times93}{19\times7}, \text{ or } \frac{81}{19}\times\frac{93}{7},$$

or the product of the chances of each singly yielding a true answer.

It will be seen that a chance is a quantity which may have any magnitude, however great. An event in whose favor there is an even chance, or $\frac{1}{1}$, has a probability of $\frac{1}{2}$. An argument having an even chance can do nothing toward reënforcing others, since according to the rule its combination with another would only multiply the chance of the latter by 1.

676. Probability and chance undoubtedly belong primarily to consequences, and are relative to premisses; but we may, nevertheless, speak of the chance of an event absolutely, meaning by that the chance of the combination of all arguments in reference to it which exist for us in the given state of our knowledge. Taken in this sense it is incontestable that the chance of an event has an intimate connection with the degree of our belief in it. Belief is certainly something more than a mere feeling; yet there is a feeling of believing, and this feeling does and ought to vary with the chance of the thing believed, as deduced from all the arguments. Any quantity which varies with the chance might, therefore, it would seem, serve as a thermometer for the proper intensity of belief. Among all such quantities there is one which is peculiarly appropriate. When there is a very great chance, the feeling of belief ought to be very intense. Absolute certainty, or an infinite chance, can never be attained by mortals, and this may be represented appropriately by an infinite belief. As the chance diminishes the feeling of believing should diminish, until an even chance is reached, where it should completely vanish and not incline either toward or away from the proposition. When the chance becomes less, then a contrary belief should spring up and should increase in intensity as the chance diminishes, and as the chance almost vanishes (which it can never quite do) the contrary belief should tend toward an infinite intensity. Now, there is one quantity which, more simply than any other, fulfills these conditions; it is the *logarithm* of the chance. But there is another consideration which must, if admitted, fix us to this choice for our thermometer. It is that our belief ought to be proportional to the weight of evidence, in this sense, that two arguments which are entirely independent, neither weakening nor strengthening each other, ought, when they concur, to produce a belief equal to the sum of the intensities of belief which either would produce separately. Now, we have seen that the

chances of independent concurrent arguments are to be multiplied together to get the chance of their combination, and, therefore, the quantities which best express the intensities of belief should be such that they are to be *added* when the *chances* are multiplied in order to produce the quantity which corresponds to the combined chance. Now, the logarithm is the only quantity which fulfills this condition. There is a general law of sensibility, called Fechner's psychophysical law. It is that the intensity of any sensation is proportional to the logarithm of the external force which produces it. It is entirely in harmony with this law that the feeling of belief should be as the logarithm of the chance, this latter being the expression of the state of facts which produces the belief.

The rule for the combination of independent concurrent arguments takes a very simple form when expressed in terms of the intensity of belief, measured in the proposed way. It is this: Take the sum of all the feelings of belief which would be produced separately by all the arguments *pro*, subtract from that the similar sum for arguments *con*, and the remainder is the feeling of belief which we ought to have on the whole. This is a proceeding which men often resort to, under the name of *balancing reasons*.

These considerations constitute an argument in favor of the conceptualistic view. The kernel of it is that the conjoint probability of all the arguments in our possession, with reference to any fact, must be intimately connected with the just degree of our belief in that fact; and this point is supplemented by various others showing the consistency of the theory with itself and with the rest of our knowledge.

677. But probability, to have any value at all, must express a fact. It is, therefore, a thing to be inferred upon evidence. Let us, then, consider for a moment the formation of a belief of probability. Suppose we have a large bag of beans from which one has been secretly taken at random and hidden under a thimble. We are now to form a probable judgment of the color of that bean, by drawing others singly from the bag and looking at them, each one to be thrown back, and the whole well mixed up after each drawing. Suppose the first drawing is white and the next black. We conclude that there is not an immense preponderance of either color, and that there

is something like an even chance that the bean under the thimble is black. But this judgment may be altered by the next few drawings. When we have drawn ten times, if 4, 5, or 6, are white, we have more confidence that the chance is even. When we have drawn a thousand times, if about half have been white, we have great confidence in this result. We now feel pretty sure that, if we were to make a large number of bets upon the color of single beans drawn from the bag, we could approximately insure ourselves in the long run by betting each time upon the white, a confidence which would be entirely wanting if, instead of sampling the bag by 1,000 drawings, we had done so by only two. Now, as the whole utility of probability is to insure us in the long run, and as that assurance depends, not merely on the value of the chance, but also on the accuracy of the evaluation, it follows that we ought not to have the same feeling of belief in reference to all events of which the chance is even. In short, to express the proper state of our belief, not *one* number but *two* are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based.[1] It is true that when our knowledge is very precise, when we have made many drawings from the bag, or, as in most of the examples in the books, when the total contents of the bag are absolutely known, the number which expresses the uncertainty of the assumed probability and its liability to be changed by further experience may become insignificant, or utterly vanish. But, when our knowledge is very slight, this number may be even more important than the probability itself; and when we have no knowledge at all this completely overwhelms the other, so that there is no sense in saying that the chance of the totally unknown event is even (for what expresses absolutely no fact has absolutely no meaning), and what ought to be said is that the chance is entirely indefinite. We thus perceive that the conceptualistic view, though answering well enough in some cases, is quite inadequate.

678. Suppose that the first bean which we drew from our bag was black. That would constitute an argument, no matter how slender, that the bean under the thimble was also

[1] Strictly we should need an infinite series of numbers each depending on the probable error of the last.

black. If the second bean was also to turn out black, that would be a second independent argument reënforcing the first. If the whole of the first twenty beans drawn should prove black, our confidence that the hidden bean was black would justly attain considerable strength. But suppose the twenty-first bean was to be white and that we were to go on drawing until we found that we had drawn 1,010 black beans and 990 white ones. We should conclude that our first twenty beans being black was simply an extraordinary accident, and that in fact the proportion of white beans to black was sensibly equal, and that it was an even chance that the hidden bean was black. Yet according to the rule of *balancing reasons*, since all the drawings of black beans are so many independent arguments in favor of the one under the thimble being black, and all the white drawings so many against it, an excess of twenty black beans ought to produce the same degree of belief that the hidden bean was black, whatever the total number drawn.

679. In the conceptualistic view of probability, complete ignorance, where the judgment ought not to swerve either toward or away from the hypothesis, is represented by the probability $\frac{1}{2}$.[1]

But let us suppose that we are totally ignorant what colored hair the inhabitants of Saturn have. Let us, then, take a color-chart in which all possible colors are shown shading into one another by imperceptible degrees. In such a chart the relative areas occupied by different classes of colors are perfectly arbitrary. Let us inclose such an area with a closed line, and ask what is the chance on conceptualistic principles that the color of the hair of the inhabitants of Saturn falls within that area? The answer cannot be indeterminate because we must be in some state of belief; and, indeed, conceptualistic writers do not admit indeterminate probabilities. As there is no certainty in the matter, the answer lies between *zero* and *unity*. As no numerical value is afforded by the data, the number must be determined by the nature of the scale of probability itself, and not by calculation from the data. The answer can, therefore, only be one-half, since the judgment should neither favor nor oppose the hypothesis. What is true of this area is true of any

[1] "Perfect indecision, belief inclining neither way, an even chance." — De Morgan, p. 182.

other one; and it will equally be true of a third area which embraces the other two. But the probability for each of the smaller areas being one-half, that for the larger should be at least unity, which is absurd.

§3. ON THE CHANCE OF UNKNOWN EVENTS[E]

680. All our reasonings are of two kinds: 1. *Explicative, analytic,* or *deductive;* 2. *Amplifiative, synthetic,* or (loosely speaking) *inductive.* In explicative reasoning, certain facts are first laid down in the premisses. These facts are, in every case, an inexhaustible multitude, but they may often be summed up in one simple proposition by means of some regularity which runs through them all. Thus, take the proposition that Socrates was a man; this implies (to go no further) that during every fraction of a second of his whole life (or, if you please, during the greater part of them) he was a man. He did not at one instant appear as a tree and at another as a dog; he did not flow into water, or appear in two places at once; you could not put your finger through him as if he were an optical image, etc. Now, the facts being thus laid down, some order among some of them, not particularly made use of for the purpose of stating them, may perhaps be discovered; and this will enable us to throw part or all of them into a new statement, the possibility of which might have escaped attention. Such a statement will be the conclusion of an analytic inference. Of this sort are all mathematical demonstrations. But synthetic reasoning is of another kind. In this case the facts summed up in the conclusion are not among those stated in the premisses. They are different facts, as when one sees that the tide rises m times and concludes that it will rise the next time. These are the only inferences which increase our real knowledge, however useful the others may be.

681. In any problem in probabilities, we have given the relative frequency of certain events, and we perceive that in these facts the relative frequency of another event is given in a hidden way. This being stated makes the solution. This is, therefore, mere explicative reasoning, and is evidently entirely inadequate to the representation of synthetic reasoning, which goes out beyond the facts given in the premisses. There is,

therefore, a manifest impossibility in so tracing out any probability for a synthetic conclusion.

682. Most treatises on probability contain a very different doctrine. They state, for example, that if one of the ancient denizens of the shores of the Mediterranean, who had never heard of tides, had gone to the bay of Biscay, and had there seen the tide rise, say m times, he could know that there was a probability equal to

$$\frac{m+1}{m+2}$$

that it would rise the next time. In a well-known work by Quetelet,* much stress is laid on this, and it is made the foundation of a theory of inductive reasoning.

683. But this solution betrays its origin if we apply it to the case in which the man has never seen the tide rise at all; that is, if we put $m=0$. In this case, the probability that it will rise the next time comes out $\frac{1}{2}$, or, in other words, the solution involves the conceptualistic principle that there is an even chance of a totally unknown event. The manner in which it has been reached has been by considering a number of urns all containing the same number of balls, part white and part black. One urn contains all white balls, another one black and the rest white, a third two black and the rest white, and so on, one urn for each proportion, until an urn is reached containing only black balls. But the only possible reason for drawing any analogy between such an arrangement and that of Nature is the principle that alternatives of which we know nothing must be considered as equally probable. But this principle is absurd. There is an indefinite variety of ways of enumerating the different possibilities, which, on the application of this principle, would give different results. If there be any way of enumerating the possibilities so as to make them all equal, it is not that from which this solution is derived, but is the following: Suppose we had an immense granary filled with black and white balls well mixed up; and suppose each urn were filled by taking a fixed number of balls from this granary quite at random. The relative number of white balls in the granary might be anything, say one in three. Then in one-third of the

* *Théorie des Probabilités*, deuxième partie, §1.

urns the first ball would be white, and in two-thirds black. In one-third of those urns of which the first ball was white, and also in one-third of those in which the first ball was black, the second ball would be white. In this way, we should have a distribution like that shown in the following table, where w stands for a white ball and b for a black one. The reader can, if he chooses, verify the table for himself.

wwww.

wwwb.	wwbw.	wbww.	bwww.		
wwwb.	wwbw.	wbww.	bwww.		
wwbb.	wbwb.	bwwb.	wbbw.	bwbw.	bbww.
wwbb.	wbwb.	bwwb.	wbbw.	bwbw.	bbww.
wwbb.	wbwb.	bwwb.	wbbw.	bwbw.	bbww.
wwbb.	wbwb.	bwwb.	wbbw.	bwbw.	bbww.
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		
wbbb.	bwbb.	bbwb.	bbbw.		

bbbb. In the second group, where there is one b, there are
bbbb. two sets just alike; in the third there are 4, in the
bbbb. fourth 8, and in the fifth 16, doubling every time.
bbbb. This is because we have supposed twice as many
bbbb. black balls in the granary as white ones; had we sup-
bbbb. posed 10 times as many, instead of
bbbb.

bbbb. 1, 2, 4, 8, 16

bbbb.
bbbb. sets we should have had
bbbb.

bbbb. 1, 10, 100, 1000, 10000

bbbb.
bbbb. sets; on the other hand, had the numbers of black and
bbbb. white balls in the granary been even, there would have
bbbb. been but one set in each group. Now suppose two balls

were drawn from one of these urns and were found to be both white, what would be the probability of the next one being white? If the two drawn out were the first two put into the urns, and the next to be drawn out were the third put in, then the probability of this third being white would be the same whatever the colors of the first two, for it has been supposed that just the same proportion of urns has the third ball white among those which have the first two *white-white*, *white-black*, *black-white*, and *black-black*. Thus, in this case, the chance of the third ball being white would be the same whatever the first two were. But, by inspecting the table, the reader can see that in each group all orders of the balls occur with equal frequency, so that it makes no difference whether they are drawn out in the order they were put in or not. Hence the colors of the balls already drawn have no influence on the probability of any other being white or black.

684. Now, if there be any way of enumerating the possibilities of Nature so as to make them equally probable, it is clearly one which should make one arrangement or combination of the elements of Nature as probable as another, that is, a distribution like that we have supposed, and it, therefore, appears that the assumption that any such thing can be done, leads simply to the conclusion that reasoning from past to future experience is absolutely worthless.* In fact, the moment that you assume that the chances in favor of that of which we are totally ignorant are even, the problem about the tides does not differ, in any arithmetical particular, from the case in which a penny (known to be equally likely to come up heads or tails) should turn up heads m times successively. In short, it would be to assume that Nature is a pure chaos, or chance combination of independent elements, in which reasoning from one fact to another would be impossible; and since, as we shall hereafter see,† there is no judgment of pure observation without reasoning, it would be to suppose all human cognition illusory and no real knowledge possible. It would be to suppose that if we have found the order of Nature more or less regular in the past, this has been by a pure run of luck which we may expect is now at an end. Now, it may be we have no scintilla

* Cf. vol. 6, bk. II, ch. 1, §2.

† See 692.

of proof to the contrary, but reason is unnecessary in reference to that belief which is of all the most settled, which nobody doubts or can doubt, and which he who should deny would stultify himself in so doing.

The relative probability of this or that arrangement of Nature is something which we should have a right to talk about if universes were as plenty as blackberries, if we could put a quantity of them in a bag, shake them well up, draw out a sample, and examine them to see what proportion of them had one arrangement and what proportion another. But, even in that case, a higher universe would contain us, in regard to whose arrangements the conception of probability could have no applicability.

§4. ON THE PROBABILITY OF SYNTHETIC INFERENCES[E]

685. We have examined the problem proposed by the conceptualists, which, translated into clear language, is this: Given a synthetic conclusion; required to know out of all possible states of things how many will accord, to any assigned extent, with this conclusion; and we have found that it is only an absurd attempt to reduce synthetic to analytic reason, and that no definite solution is possible.

686. But there is another problem in connection with this subject. It is this: Given a certain state of things, required to know what proportion of all synthetic inferences relating to it will be true within a given degree of approximation. Now, there is no difficulty about this problem (except for its mathematical complication); it has been much studied, and the answer is perfectly well known. And is not this, after all, what we want to know much rather than the other? Why should we want to know the probability that the fact will accord with our conclusion? That implies that we are interested in all possible worlds, and not merely the one in which we find ourselves placed. Why is it not much more to the purpose to know the probability that our conclusion will accord with the fact? One of these questions is the first above stated and the other the second, and I ask the reader whether, if people, instead of using the word probability without any clear apprehension of their own meaning, had always spoken of relative frequency,

they could have failed to see that what they wanted was not to follow along the synthetic procedure with an analytic one, in order to find the probability of the conclusion; but, on the contrary, to begin with the fact at which the synthetic inference aims, and follow back to the facts it uses for premisses in order to see the probability of their being such as will yield the truth.

687. As we cannot have an urn with an infinite number of balls to represent the inexhaustibleness of Nature, let us suppose one with a finite number, each ball being thrown back into the urn after being drawn out, so that there is no exhaustion of them. Suppose one ball out of three is white and the rest black, and that four balls are drawn. Then the table in 683 represents the relative frequency of the different ways in which these balls might be drawn. It will be seen that if we should judge by these four balls of the proportion in the urn, 32 times out of 81 we should find it $\frac{1}{4}$, and 24 times out of 81 we should find it $\frac{1}{2}$, the truth being $\frac{1}{3}$. To extend this table to high numbers would be great labor, but the mathematicians have found some ingenious ways of reckoning what the numbers would be. It is found that, if the true proportion of white balls is p, and s balls are drawn, then the error of the proportion obtained by the induction will be —

half the time within	$0.477\sqrt{\frac{2p(1-p)}{s}}$
9 times out of 10 within	$1.163\sqrt{\frac{2p(1-p)}{s}}$
99 times out of 100 within	$1.821\sqrt{\frac{2p(1-p)}{s}}$
999 times out of 1,000 within	$2.328\sqrt{\frac{2p(1-p)}{s}}$
9,999 times out of 10,000 within	$2.751\sqrt{\frac{2p(1-p)}{s}}$
9,999,999,999 times out of 10,000,000,000 within	$4.77\sqrt{\frac{2p(1-p)}{s}}$

The use of this may be illustrated by an example. By the census of 1870, it appears that the proportion of males among

native white children under one year old was 0.5082, while among colored children of the same age the proportion was only 0.4977. The difference between these is 0.0105, or about one in 100. Can this be attributed to chance, or would the difference always exist among a great number of white and colored children under like circumstances? Here p may be taken at $\frac{1}{2}$; hence $2p(1-p)$ is also $\frac{1}{2}$. The number of white children counted was near 1,000,000; hence the fraction whose square-root is to be taken is about $\frac{1}{2000000}$. The root is about $\frac{1}{1400}$, and this multiplied by 0.477 gives about 0.0003 as the probable error in the ratio of males among the whites as obtained from the induction. The number of black children was about 150,000, which gives 0.0008 for the probable error. We see that the actual discrepancy is ten times the sum of these, and such a result would happen, according to our table, only once out of 10,000,000,000 censuses, in the long run.

688. It may be remarked that when the real value of the probability sought inductively is either very large or very small, the reasoning is more secure. Thus, suppose there were in reality one white ball in 100 in a certain urn, and we were to judge of the number by 100 drawings. The probability of drawing no white ball would be $\frac{366}{10000}$; that of drawing one white ball would be $\frac{370}{10000}$; that of drawing two would be $\frac{185}{10000}$; that of drawing three would be $\frac{61}{1000}$; that of drawing four would be $\frac{15}{1000}$; that of drawing five would be only $\frac{3}{1000}$, etc. Thus we should be tolerably certain of not being in error by more than one ball in 100.

689. It appears, then, that in one sense we can, and in another we cannot, determine the probability of synthetic inference. When I reason in this way:

Ninety-nine Cretans in a hundred are liars,
But Epimenides is a Cretan;
Therefore, Epimenides is a liar;

I know that reasoning similar to that would carry truth 99 times in 100. But when I reason in the opposite direction:

Minos, Sarpedon, Rhadamanthus, Deucalion, and Epimenides, are all the Cretans I can think of,
But these were all atrocious liars;
Therefore, pretty much all Cretans must have been liars;

I do not in the least know how often such reasoning would carry me right. On the other hand, what I do know is that some definite proportion of Cretans must have been liars, and that this proportion can be probably approximated to by an induction from five or six instances. Even in the worst case for the probability of such an inference, that in which about half the Cretans are liars, the ratio so obtained would probably not be in error by more than $\frac{1}{6}$. So much I know; but, then, in the present case the inference is that pretty much all Cretans are liars, and whether there may not be a special improbability in that I do not know.

§5. THE RATIONALE OF SYNTHETIC INFERENCE[E]

690. Late in the last century, Immanuel Kant asked the question, "How are synthetical judgments *a priori* possible?" By synthetical judgments he meant such as assert positive fact and are not mere affairs of arrangement; in short, judgments of the kind which synthetical reasoning produces, and which analytic reasoning cannot yield. By *a priori* judgments he meant such as that all outward objects are in space, every event has a cause, etc., propositions which according to him can never be inferred from experience. Not so much by his answer to this question as by the mere asking of it, the current philosophy of that time was shattered and destroyed, and a new epoch in its history was begun. But before asking *that* question he ought to have asked the more general one, "How are any synthetical judgments at all possible?" How is it that a man can observe one fact and straightway pronounce judgment concerning another different fact not involved in the first? Such reasoning, as we have seen, has, at least in the usual sense of the phrase, no definite probability; how, then, can it add to our knowledge? This is a strange paradox; the Abbé Gratry says it is a miracle, and that every true induction is an immediate inspiration from on high.[1] I respect this explanation far more than many a pedantic attempt to solve the question by some juggle with probabilities, with the forms of

[1] *Logique*. The same is true, according to him, of every performance of a differentiation, but not of integration. He does not tell us whether it is the supernatural assistance which makes the former process so much the easier.

syllogism, or what not. I respect it because it shows an appreciation of the depth of the problem, because it assigns an adequate cause, and because it is intimately connected — as the true account should be — with a general philosophy of the universe. At the same time, I do not accept this explanation, because an explanation should tell *how* a thing is done, and to assert a perpetual miracle seems to be an abandonment of all hope of doing that, without sufficient justification.

691. It will be interesting to see how the answer which Kant gave to his question about synthetical judgments *a priori* will appear if extended to the question of synthetical judgments in general. That answer is, that synthetical judgments *a priori* are possible because whatever is universally true is involved in the conditions of experience. Let us apply this to a general synthetical reasoning. I take from a bag a handful of beans; they are all purple, and I infer that all the beans in the bag are purple. How can I do that? Why, upon the principle that whatever is universally true of my experience (which is here the appearance of these different beans) is involved in the condition of experience. The condition of this special experience is that all these beans were taken from that bag. According to Kant's principle, then, whatever is found true of all the beans drawn from the bag must find its explanation in some peculiarity of the contents of the bag. This is a satisfactory statement of the principle of induction.

692. When we draw a deductive or analytic conclusion, our rule of inference is that facts of a certain general character are either invariably or in a certain proportion of cases accompanied by facts of another general character. Then our premiss being a fact of the former class, we infer with certainty or with the appropriate degree of probability the existence of a fact of the second class. But the rule for synthetic inference is of a different kind. When we sample a bag of beans we do not in the least assume that the fact of some beans being purple involves the necessity or even the probability of other beans being so. On the contrary, the conceptualistic method of treating probabilities, which really amounts simply to the deductive treatment of them, when rightly carried out leads to the result that a synthetic inference has just an even chance in its favor, or in other words is absolutely worthless. The color

of one bean is entirely independent of that of another. But synthetic inference is founded upon a classification of facts, not according to their characters, but according to the manner of obtaining them. Its rule is, that a number of facts obtained in a given way will in general more or less resemble other facts obtained in the same way; or, *experiences whose conditions are the same will have the same general characters.*

693. In the former case, we know that premisses precisely similar in form to those of the given ones will yield true conclusions, just once in a calculable number of times. In the latter case, we only know that premisses obtained under circumstances similar to the given ones (though perhaps themselves very different) will yield true conclusions, at least once in a calculable number of times. We may express this by saying that in the case of analytic inference we know the probability of our conclusion (if the premisses are true), but in the case of synthetic inferences we only know the degree of trustworthiness of our proceeding. As all knowledge comes from synthetic inference, we must equally infer that all human certainty consists merely in our knowing that the processes by which our knowledge has been derived are such as must generally have led to true conclusions.

Though a synthetic inference cannot by any means be reduced to deduction, yet that the rule of induction will hold good in the long run may be deduced from the principle that reality is only the object of the final opinion to which sufficient investigation would lead. That belief gradually tends to fix itself under the influence of inquiry is, indeed, one of the facts with which logic sets out.

CHAPTER 8

*A THEORY OF PROBABLE INFERENCE**

§1. PROBABLE DEDUCTION AND PROBABILITY IN GENERAL†

694. The following is an example of the simplest kind of probable inference:

About two per cent of persons wounded in the liver recover,
This man has been wounded in the liver;
Therefore, there are two chances out of a hundred that he will recover.

Compare this with the simplest of syllogisms, say the following:

Every man dies,
Enoch was a man;
Hence, Enoch must have died.

The latter argument consists in the application of a general rule to a particular case. The former applies to a particular case a rule not absolutely universal, but subject to a known proportion of exceptions. Both may alike be termed deductions, because they bring information about the uniform or usual course of things to bear upon the solution of special questions; and the probable argument may approximate indefinitely to demonstration as the ratio named in the first premiss approaches to unity or to zero.

695. Let us set forth the general formulæ of the two kinds of inference in the manner of formal logic.

* The Johns Hopkins *Studies in Logic*, edited by C. S. Peirce, Little Brown and Co., Boston (1883), pp. 126–181; intended as Essay XIV of the *Search for a Method* (1893).

† The headings of these sections were made by Peirce in his own copy of the Johns Hopkins *Studies*.

FORM I.

Singular Syllogism in Barbara.

Every M is a P,
S is an M;
Hence, S is a P.

FORM II.

Simple Probable Deduction.

The proportion ρ of the M's are P's;
S is an M;
It follows, with probability ρ, that S is a P.

It is to be observed that the ratio ρ need not be exactly specified. We may reason from the premiss that not more than two per cent of persons wounded in the liver recover, or from "not less than a certain proportion of the M's are P's," or from "no very large nor very small proportion," etc. In short, ρ is subject to every kind of indeterminacy; it simply excludes some ratios and admits the possibility of the rest.

696. The analogy between syllogism and what is here called probable deduction is certainly genuine and important; yet how wide the differences between the two modes of inference are, will appear from the following considerations:

(1) The logic of probability is related to ordinary syllogistic as the quantitative to the qualitative branch of the same science. Necessary syllogism recognizes only the inclusion or non-inclusion of one class under another; but probable inference takes account of the proportion of one class which is contained under a second. It is like the distinction between projective geometry, which asks whether points coincide or not, and metric geometry, which determines their distances.

(2) For the existence of ordinary syllogism, all that is requisite is that we should be able to say, in some sense, that one term is contained in another, or that one object stands to a second in one of those relations: "better than," "equivalent to," etc., which are termed *transitive* because if A is in any such relation to B, and B is in the same relation to C, then A is in that relation to C. The universe might be all so fluid and variable that nothing should preserve its individual identity,

and that no measurement should be conceivable; and still one portion might remain inclosed within a second, itself inclosed within a third, so that a syllogism would be possible. But probable inference could not be made in such a universe, because no signification would attach to the words "quantitative ratio." For that there must be counting; and consequently units must exist, preserving their identity and variously grouped together.

(3) A cardinal distinction between the two kinds of inference is, that in demonstrative reasoning the conclusion follows from the existence of the objective facts laid down in the premisses; while in probable reasoning these facts in themselves do not even render the conclusion probable, but account has to be taken of various subjective circumstances — of the manner in which the premisses have been obtained, of there being no countervailing considerations, etc.; in short, good faith and honesty are essential to good logic in probable reasoning.

When the partial rule that the proportion ρ of the M's are P's is applied to show with probability ρ that S is a P, it is requisite, not merely that S should *be* an M, but also that it should be an instance drawn *at random* from among the M's. Thus, there being four aces in a piquet pack of thirty-two cards, the chance is one-eighth that a given card not looked at is an ace; but this is only on the supposition that the card has been drawn at random from the whole pack. If, for instance, it had been drawn from the cards discarded by the players at piquet or euchre, the probability would be quite different. The instance must be drawn at random. Here is a maxim of conduct. The volition of the reasoner (using what machinery it may) has to choose S so that it shall be an M; but he ought to restrain himself from all further preference, and not allow his will to act in any way that might tend to settle what particular M is taken, but should leave that to the operation of chance. Willing and wishing, like other operations of the mind, are *general* and imperfectly determinate. I wish for a horse — for some particular kind of horse perhaps, but not usually for any individual one. I will to act in a way of which I have a general conception; but so long as my action conforms to that general description, how it is further determined I do not care. Now in choosing the instance S, the gen-

eral intention (including the whole plan of action) should be to select an M, but beyond that there should be no preference; and the act of choice should be such that if it were repeated many enough times with the same intention, the result would be that among the totality of selections the different sorts of M's would occur with the same relative frequencies as in experiences in which volition does not intermeddle at all. In cases in which it is found difficult thus to restrain the will by a direct effort, the apparatus of games of chance — a lottery-wheel, a roulette, cards, or dice — may be called to our aid. Usually, however, in making a simple probable deduction, we take that instance in which we happen at the time to be interested. In such a case, it is our interest that fulfills the function of an apparatus for random selection; and no better need be desired, so long as we have reason to deem the premiss "the proportion ρ of the M's are P's" to be equally true in regard to that part of the M's which are alone likely ever to excite our interest.

Nor is it a matter of indifference in what manner the other premiss has been obtained. A card being drawn at random from a piquet pack, the chance is one-eighth that it is an ace, if we have no other knowledge of it. But after we have looked at the card, we can no longer reason in that way. That the conclusion must be drawn in advance of any other knowledge on the subject is a rule that, however elementary, will be found in the sequel to have great importance.

(4) The conclusions of the two modes of inference likewise differ. One is necessary; the other only probable. Locke, in the *Essay Concerning Human Understanding*, hints at the correct analysis of the nature of probability. After remarking that the mathematician positively knows that the sum of the three angles of a triangle is equal to two right angles because he apprehends the geometrical proof, he then continues:* "But another man who never took the pains to observe the demonstration, hearing a mathematician, a man of credit, affirm the three angles of a triangle to be equal to two right ones, *assents* to it, that is, receives it for true. In which case, the foundation of his assent is the probability of the thing, the proof being such as, for the most part, carries truth with it; the man on

* Bk. IV, ch. 15, §1.

whose testimony he receives it not being wont to affirm anything contrary to or beside his knowledge, especially in matters of this kind." Those who know Locke are accustomed to look for more meaning in his words than appears at first glance. There is an allusion in this passage to the fact that a probable argument is always regarded as belonging to a *genus* of arguments. This is, in fact, true of any kind of argument. For the belief expressed by the conclusion is determined or caused by the belief expressed by the premisses. There is, therefore, some general rule according to which the one succeeds the other. But, further, the reasoner is conscious of there being such a rule, for otherwise he would not know he was reasoning, and could exercise no attention or control; and to such an involuntary operation the name "reasoning" is very properly not applied. In all cases, then, we are conscious that our inference belongs to a general class of logical forms, although we are not necessarily able to describe the general class. The difference between necessary and probable reasoning is that in the one case we conceive that such facts as are expressed by the premisses are never, in the whole range of possibility, true, without another fact, related to them as our conclusion is to our premisses, being true likewise; while in the other case we merely conceive that, in reasoning as we do, we are following a general maxim that will usually lead us to the truth.

697. So long as there are exceptions to the rule that all men wounded in the liver die, it does not necessarily follow that because a given man is wounded in the liver he cannot recover. Still, we know that if we were to reason in that way, we should be following a mode of inference which would only lead us wrong, in the long run, once in fifty times; and this is what we mean when we say that the probability is one out of fifty that the man will recover. To say, then, that a proposition has the probability ρ means that to infer it to be true would be to follow an argument such as would carry truth with it in the ratio of frequency ρ.

It is plainly useful that we should have a stronger feeling of confidence about a sort of inference which will oftener lead us to the truth than about an inference that will less often prove right — and such a sensation we do have. The celebrated law of Fechner is that as the force acting upon an organ of sense

increases in geometrical progression, the intensity of the sensation increases in arithmetical progression. In this case the odds (that is, the ratio of the chances in favor of a conclusion to the chances against it) take the place of the exciting cause, while the sensation itself is the feeling of confidence. When two arguments tend to the same conclusion, our confidence in the latter is equal to the sum of what the two arguments separately would produce; the *odds* are the product of the *odds* in favor of the two arguments separately. When the value of the *odds* reduces to unity, our confidence is null; when the *odds* are less than unity, we have more or less confidence in the negative of the conclusion.

§2. STATISTICAL DEDUCTION

698. The principle of probable deduction still applies when S, instead of being a single M, is a set of M's — n in number. The reasoning then takes the following form:

FORM III.

Complex Probable Deduction.

Among all sets of n M's, the proportion q consist each of m P's and of $n-m$ not-P's,

S', S'', S''', etc.; form a set of n objects drawn at random from among the M's;

Hence, the probability is q that among S, S', S'', etc. there are m P's and $n-m$ not-P's.

In saying that S, S', S'', etc., form a set drawn at random, we here mean that not only are the different individuals drawn at random, but also that they are so drawn that the qualities which may belong to one have no influence upon the selection of any other. In other words, the individual drawings are independent, and the set as a whole is taken at random from among all possible sets of n M's. In strictness, this supposes that the same individual may be drawn several times in the same set, although if the number of M's is large compared with n, it makes no appreciable difference whether this is the case or not.

699. The following formula expresses the proportion,

among all sets of n M's, of those which consist of m P's and $n-m$ not-P's. The letter r denotes the proportion of P's among the M's, and the sign of admiration is used to express the continued product of all integer numbers from 1 to the number after which it is placed. Thus, $4!=1\cdot2\cdot3\cdot4=24$, etc. The formula is:

$$q=n!\times\frac{r^m}{m!}\times\frac{(1-r)^{n-m}}{(n-m)!}$$

As an example, let us assume the proportion $r=\frac{2}{3}$ and the number of M's in a set $n=15$. Then the values of the probability q for different numbers, m, of P's, are fractions having for their common denominator 14,348,907, and for their numerators as follows:

m	Numerator of q.
0	1
1	30
2	420
3	3640
4	21840
5	96096
6	320320
7	823680

m	Numerator of q.
8	1667360
9	2562560
10	3075072
11	2795520
12	1863680
13	860160
14	122880
15	32768

A very little mathematics would suffice to show that, r and n being fixed, q always reaches its maximum value with that value of m that is next less than $(n+1)r$, [1] and that q is very small unless m has nearly this value.

700. Upon these facts is based another form of inference to which I give the name of statistical deduction. Its general formula is as follows:

[1] In case $(n+1)r$ is a whole number, q has equal values for $m=(n+1)r$ and for $m=(n+1)r-1$.

FORM IV

Statistical Deduction.

The proportion r of the M's are P's,

S', S'', S''', etc. are a *numerous* set, taken at random from among the M's;

Hence, *probably* and *approximately*, the proportion r of the S's are P's.

As an example, take this:

A little more than half of all human births are males;

Hence, probably a little over half of all the births in New York during any one year are males.

We have now no longer to deal with a mere probable inference, but with a *probable approximate* inference. This conception is a somewhat complicated one, meaning that the probability is greater according as the limits of approximation are wider, conformably to the mathematical expression for the values of q.

701. This conclusion has no meaning at all unless there be more than one instance; and it has hardly any meaning unless the instances are somewhat numerous. When this is the case, there is a more convenient way of obtaining (not exactly, but quite near enough for all practical purposes) either a single value of q or the sum of successive values from $m=m_1$ to $m=m_2$ inclusive. The rule is first to calculate two quantities which may conveniently be called t_1 and t_2 according to these formulæ:

$$t_1=\frac{m_1-(n+1)r}{\sqrt{2nr(1-r)}} \qquad t_2=\frac{1+m_2-(n+1)r}{\sqrt{2nr(1-r)}}$$

where $m_2>m_1$. Either or both the quantities t_1 and t_2 may be negative. Next with each of these quantities enter the table below, and take out $\frac{1}{2}\Theta t_1$ and $\frac{1}{2}\Theta t_2$ and give each the same sign as the t from which it is derived. Then

$$\Sigma\, q=\tfrac{1}{2}\,\Theta t_2-\tfrac{1}{2}\Theta t_1.$$

$$Table\ of\ \Theta t = \frac{2}{\sqrt{\ominus}} \int_0^t \ominus^{-t^2} dt.$$

t	Θt
0.0	0.000
0.1	0.112
0.2	0.22$\overline{3}$
0.3	0.32$\overline{9}$
0.4	0.428
0.5	0.520
0.6	0.60$\overline{4}$
0.7	0.67$\overline{8}$
0.8	0.742
0.9	0.79$\overline{7}$
1.0	0.84$\overline{3}$

t	Θt
1.0	0.84$\overline{3}$
1.1	0.880
1.2	0.910
1.3	0.934
1.4	0.952
1.5	0.966
1.6	0.976
1.7	0.98$\overline{4}$
1.8	0.989
1.9	0.99$\overline{3}$
2.0	0.995

t	Θt
2.0	0.99532
2.1	0.99702
2.2	0.9981$\overline{4}$
2.3	0.9988$\overline{6}$
2.4	0.99931
2.5	0.99959
2.6	0.99976
2.7	0.9998$\overline{7}$
2.8	0.99992
2.9	0.99996
3.0	0.9999$\overline{8}$

t	Θ
4	0.999999989
5	0.9999999999984
6	0.999999999999999982
7	0.99999999999999999999958

In rough calculations we may take Θt equal to t for t less than 0.7, and as equal to *unity* for any value above $t = 1.4$.

§3. INDUCTION*

702. The principle of statistical deduction is that these† two proportions — namely, that of the P's among the M's, and that of the P's among the S's — are probably and approximately equal. If, then, this principle justifies our inferring the value of the second proportion from the known value of the first, it equally justifies our inferring the value of the first

* There was no §3 in the original, and the present section formed part of §2.

† "these" is deleted in Peirce's own copy.

from that of the second, if the first is unknown but the second has been observed. We thus obtain the following form of inference:

FORM V

Induction.

S', S'', S''', etc. form a numerous set taken at random from among the M's,

S', S'', S''', etc. are found to be — the proportion ρ of them — P's;

Hence, *probably* and *approximately* the same proportion, ρ, of the M's are P's.

The following are examples. From a bag of coffee a handful is taken out, and found to have nine-tenths of the beans perfect; whence it is inferred that about nine-tenths of all the beans in the bag are probably perfect. The United States Census of 1870 shows that of native white children under one year old, there were 478,774 males to 463,320 females; while of colored children of the same age there were 75,985 males to 76,637 females. We infer that generally there is a larger proportion of female births among negroes than among whites.

703. When the ratio ρ is *unity* or *zero*, the inference is an ordinary induction; and I ask leave to extend the term "induction" to all such inference, whatever be the value of ρ. It is, in fact, inferring from a sample to the whole lot sampled. These two forms of inference, statistical deduction and induction, plainly depend upon the same principle of equality of ratios, so that their validity is the same. Yet the nature of the probability in the two cases is very different. In the statistical deduction, we know that among the whole body of M's the proportion of P's is ρ; we say, then, that the S's being random drawings of M's are probably P's in about the same proportion — and though this may happen not to be so, yet at any rate, on continuing the drawing sufficiently, our prediction of the ratio will be vindicated at last. On the other hand, in induction we say that the proportion ρ of the sample being P's, probably there is about the same proportion in the whole lot, or at least, if this happens not to be so, then on continuing the drawings the inference will be, not *vindicated* as

in the other case, but *modified* so as to become true. The deduction, then, is probable in this sense, that though its conclusion may in a particular case be falsified, yet similar conclusions (with the same ratio ρ) would generally prove approximately true; while the induction is probable in this sense, that though it may happen to give a false conclusion, yet in most cases in which the same precept of inference was followed, a different and approximately true inference (with the right value of ρ) would be drawn.

§4. HYPOTHETIC INFERENCE

704. Before going any further with the study of Form V, I wish to join to it another extremely analogous form.

We often speak of one thing being very much like another, and thus apply a vague quantity to resemblance. Even if qualities are not subject to exact numeration, we may conceive them to be approximately measurable. We may then measure resemblance by a scale of numbers from zero up to unity. To say that S has a 1-likeness to a P will mean that it has every character of a P, and consequently *is* a P. To say that it has a 0-likeness will imply total dissimilarity. We shall then be able to reason as follows:

FORM II (*bis*).

Simple probable deduction in depth.

Every M has the simple mark P,
The S's have an r-likeness to the M's;
Hence, the probability is r that every S is P.

It would be difficult, perhaps impossible, to adduce an example of such kind of inference, for the reason that *simple marks* are not known to us. We may, however, illustrate the complex probable deduction in depth (the general form of which it is not worth while to set down) as follows: I forget whether, in the ritualistic churches, a bell is tinkled at the elevation of the Host or not. Knowing, however, that the services resemble somewhat decidedly those of the Roman Mass, I think that it is not unlikely that the bell is used in the ritualistic, as in the Roman, churches.

705. We shall also have the following:

FORM IV (*bis*).

Statistical deduction in depth.

Every M has, for example, the numerous marks P', P'', P''', etc.,

S has an r-likeness to the M's;

Hence, probably and approximately, S has the proportion r of the marks P', P'', P''', etc.

For example, we know that the French and Italians are a good deal alike in their ideas, characters, temperaments, genius, customs, institutions, etc., while they also differ very markedly in all these respects. Suppose, then, that I know a boy who is going to make a short trip through France and Italy; I can safely predict that among the really numerous though relatively few respects in which he will be able to compare the two people, about the same degree of resemblance will be found.

Both these modes of inference are clearly deductive. When $r=1$, they reduce to Barbara.[1]

706. Corresponding to induction, we have the following mode of inference:

FORM V (*bis*).

Hypothesis.

M has, for example, the numerous marks P', P'', P''', etc.,

S has the proportion r of the marks P', P'', P''', etc.;

Hence, probably and approximately, S has an r-likeness to M.

[1] When $r=0$, the last form becomes

M has all the marks P,
S has no mark of M;
Hence, S has none of the marks P.

When the universe of marks is unlimited (see a note appended to this paper for an explanation of this expression [519]), the only way in which two terms can fail to have a common mark is by their together filling the universe of things; and consequently this form then becomes

M is P,
Every non-S is M;
Hence, every non-S is P.

This is one of De Morgan's syllogisms.

In putting $r=0$ in Form II (*bis*) it must be noted that, since P is simple in depth, to say that S is not P is to say that it has no mark of P.

Thus, we know, that the ancient Mound-builders of North America present, in all those respects in which we have been able to make the comparison, a limited degree of resemblance with the Pueblo Indians. The inference is, then, that in all respects there is about the same degree of resemblance between these races.

If I am permitted the extended sense which I have given to the word "induction," this argument is simply an induction respecting qualities instead of respecting things. In point of fact P', P'', P''', etc., constitute a random sample of the characters of M, and the ratio r of them being found to belong to S, the same ratio of all the characters of M are concluded to belong to S. This kind of argument, however, as it actually occurs, differs very much from induction, owing to the impossibility of simply counting qualities as individual things are counted. Characters have to be weighed rather than counted. Thus, antimony is bluish-gray: that is a character. Bismuth is a sort of rose-gray; it is decidedly different from antimony in color, and yet not so very different as gold, silver, copper, and tin are.

707. I call this induction of characters *hypothetic inference*,* or, briefly, *hypothesis*. This is perhaps not a very happy designation, yet it is difficult to find a better. The term "hypothesis" has many well established and distinct meanings. Among these is that of a proposition believed in because its consequences agree with experience. This is the sense in which Newton used the word when he said, *Hypotheses non fingo*. He meant that he was merely giving a general formula for the motions of the heavenly bodies, but was not undertaking to mount to the causes of the acceleration they exhibit. The inferences of Kepler, on the other hand, were hypotheses in this sense; for he traced out the miscellaneous consequences of the supposition that Mars moved in an ellipse, with the sun at the focus, and showed that both the longitudes and the latitudes resulting from this theory were such as agreed with observation. These two components of the motion were observed; the third, that of approach to or regression from the earth, was supposed. Now, if in Form V (*bis*) we put $r=1$, the inference is the drawing of a hypothesis in this sense. I

* Cf. 102.

take the liberty of extending the use of the word by permitting r to have any value from zero to unity. The term is certainly not all that could be desired; for the word hypothesis, as ordinarily used, carries with it a suggestion of uncertainty, and of something to be superseded, which does not belong at all to my use of it. But we must use existing language as best we may, balancing the reasons for and against any mode of expression, for none is perfect; at least the term is not so utterly misleading as "analogy" would be, and with proper explanation it will, I hope, be understood.

§5. GENERAL CHARACTERS OF DEDUCTION, INDUCTION, AND HYPOTHESIS

708. The following examples will illustrate the distinction between statistical deduction, induction, and hypothesis. If I wished to order a font of type expressly for the printing of this book, knowing, as I do, that in all English writing the letter *e* occurs oftener than any other letter, I should want more *e*'s in my font than other letters. For what is true of all other English writing is no doubt true of these papers. This is a statistical deduction. But then the words used in logical writings are rather peculiar, and a good deal of use is made of single letters. I might, then, count the number of occurrences of the different letters upon a dozen or so pages of the manuscript, and thence conclude the relative amounts of the different kinds of type required in the font. That would be inductive inference. If now I were to order the font, and if, after some days, I were to receive a box containing a large number of little paper parcels of very different sizes, I should naturally infer that this was the font of types I had ordered; and this would be hypothetic inference. Again, if a dispatch in cipher is captured, and it is found to be written with twenty-six characters, one of which occurs much more frequently than any of the others, we are at once led to suppose that each character represents a letter, and that the one occurring so frequently stands for *e*. This is also hypothetic inference.

709. We are thus led to divide all probable reasoning into deductive and ampliative, and further to divide ampliative reasoning into induction and hypothesis. In deductive reason-

ing, though the predicted ratio may be wrong in a limited number of drawings, yet it will be approximately verified in a larger number. In ampliative reasoning the ratio may be wrong, because the inference is based on but a limited number of instances; but on enlarging the sample the ratio will be changed till it becomes approximately correct. In induction, the instances drawn at random are numerable things; in hypothesis they are characters, which are not capable of strict enumeration, but have to be otherwise estimated.

710. This classification of probable inference is connected with a preference for the copula of inclusion over those used by Miss Ladd [Mrs. Christine Ladd-Franklin] and by Mr. Mitchell.[1] De Morgan established eight forms of simple propositions; and from a purely formal point of view no one of these has a right to be considered as more fundamental than any other. But formal logic must not be too purely formal; it must represent a fact of psychology, or else it is in danger of degenerating into a mathematical recreation. The categorical proposition, "every man is mortal," is but a modification of the hypothetical proposition, "if humanity, then mortality"; and since the very first conception from which logic springs is that one proposition follows from another, I hold that "if A, then B" should be taken as the typical form of judgment. Time flows; and, in time, from one state of belief (represented by the premisses of an argument) another (represented by its conclusion) is developed. Logic arises from this circumstance, without which we could not learn anything nor correct any opinion. To say that an inference is correct is to say that if the premisses are true the conclusion is also true; or that every possible state of things in which the premisses should be true would be included among the possible states of things in which the conclusion would be true. We are thus led to the copula of inclusion. But the main characteristic of the relation of inclusion is that it is transitive — that is, that what is included in something included in anything is itself included in that thing; or, that if A is B and B is C, then A is C. We thus get *Barbara* as the primitive type of inference. Now in *Barbara*

[1] I do not here speak of Mr. Jevons, because my objection to the copula of identity is of a somewhat different kind. [See *Studies in Logic*, pp. 17–69 and 72–106 for Miss Ladd's and Mr. Mitchell's papers.]

we have a *Rule*, a *Case* under the *Rule*, and the inference of the *Result* of that rule in that case. For example:

Rule. All men are mortal,
Case. Enoch was a man;
Result. ∴ Enoch was mortal.

711. The cognition of a rule is not necessarily conscious, but is of the nature of a habit, acquired or congenital. The cognition of a case is of the general nature of a sensation; that is to say, it is something which comes up into present consciousness. The cognition of a result is of the nature of a decision to act in a particular way on a given occasion.[1] In point of fact, a syllogism in *Barbara* virtually takes place when we irritate the foot of a decapitated frog. The connection between the afferent and efferent nerve, whatever it may be, constitutes a nervous habit, a rule of action, which is the physiological analogue of the major premiss. The disturbance of the ganglionic equilibrium, owing to the irritation, is the physiological form of that which, psychologically considered, is a sensation; and, logically considered, is the occurrence of a case. The explosion through the efferent nerve is the physiological form of that which psychologically is a volition, and logically the inference of a result. When we pass from the lowest to the highest forms of inervation, the physiological equivalents escape our observation; but, psychologically, we still have, first, habit — which in its highest form is understanding, and which corresponds to the major premiss of *Barbara;* we have, second, feeling, or present consciousness, corresponding to the minor premiss of *Barbara;* and we have, third, volition, corresponding to the conclusion of the same mode of syllogism. Although these analogies, like all very broad generalizations, may seem very fanciful at first sight, yet the more the reader reflects upon them the more profoundly true I am confident they will appear. They give a significance to the ancient system of formal logic which no other can at all share.

712. Deduction proceeds from Rule and Case to Result; it is the formula of Volition. Induction proceeds from Case and Result to Rule; it is the formula of the formation of a

[1] See my paper on "How to make our ideas clear." [Vol. 5, bk. II, ch. 5.]

habit or general conception — a process which, psychologically as well as logically, depends on the repetition of instances or sensations. Hypothesis proceeds from Rule and Result to Case; it is the formula of the acquirement of secondary sensation — a process by which a confused concatenation of predicates is brought into order under a synthetizing predicate.*

713. We usually conceive Nature to be perpetually making deductions in *Barbara*. This is our natural and anthropomorphic metaphysics. We conceive that there are Laws of Nature, which are her Rules or major premisses. We conceive that Cases arise under these laws; these cases consist in the predication, or occurrence, of *causes*, which are the middle terms of the syllogisms. And, finally, we conceive that the occurrence of these causes, by virtue of the laws of Nature, results in effects which are the conclusions of the syllogisms. Conceiving of nature in this way, we naturally conceive of science as having three tasks — (1) the discovery of Laws, which is accomplished by induction; (2) the discovery of Causes, which is accomplished by hypothetic inference; and (3) the predictio of Effects, which is accomplished by deduction. It appears to me to be highly useful to select a system of logic which shall preserve all these natural conceptions.

714. It may be added that, generally speaking, the conclusions of Hypothetic Inference cannot be arrived at inductively, because their truth is not susceptible of direct observation in single cases. Nor can the conclusions of Inductions, on account of their generality, be reached by hypothetic inference. For instance, any historical fact, as that Napoleon Bonaparte once lived, is a hypothesis; we believe the fact, because its effects — I mean current tradition, the histories, the monuments, etc. — are observed. But no mere generalization of observed facts could ever teach us that Napoleon lived. So we inductively infer that every particle of matter gravitates toward every other. Hypothesis might lead to this result for any given pair of particles, but it never could show that the law was universal.

* Cf. 643.

§6. INDUCTION AND HYPOTHESIS

INDIRECT STATISTICAL INFERENCES; GENERAL RULE FOR THEIR VALIDITY

715. We now come to the consideration of the Rules which have to be followed in order to make valid and strong Inductions and Hypotheses. These rules can all be reduced to a single one; namely, that the statistical deduction of which the Induction or Hypothesis is the inversion, must be valid and strong.

716. We have seen that Inductions and Hypotheses are inferences from the conclusion and one premiss of a statistical syllogism to the other premiss. In the case of hypothesis, this syllogism is called the *explanation*. Thus in one of the examples used above, we suppose the cryptograph to be an English cipher, because, as we say, this *explains* the observed phenomena that there are about two dozen characters, that one occurs more frequently than the rest, especially at the end of words, etc. The explanation is —

Simple English ciphers have certain peculiarities,
This is a simple English cipher;
Hence, this necessarily has these peculiarities.

717. This explanation is present to the mind of the reasoner, too; so much so, that we commonly say that the hypothesis is adopted *for the sake of* the explanation. Of induction we do not, in ordinary language, say that it explains phenomena; still, the statistical deduction, of which it is the inversion, plays, in a general way, the same part as the explanation in hypothesis. From a barrel of apples, that I am thinking of buying, I draw out three or four as a sample. If I find the sample somewhat decayed, I ask myself, in ordinary language, not "Why is this?" but "How is this?" And I answer that it probably comes from nearly all the apples in the barrel being in bad condition. The distinction between the "Why" of hypothesis and the "How" of induction is not very great; both ask for a statistical syllogism, of which the observed fact shall be the conclusion, the known conditions of the observation one premiss, and the inductive or hypothetic inference the other. This statistical syllogism may be conveniently termed the explanatory syllogism.

718. In order that an induction or hypothesis should have

any validity at all, it is requisite that the explanatory syllogism should be a valid statistical deduction. Its conclusion must not merely follow from the premisses, but follow from them upon the principle of probability. The inversion of *ordinary* syllogism does not give rise to an induction or hypothesis. The statistical syllogism of Form IV is invertible, because it proceeds upon the principle of an approximate *equality* between the ratio of P's in the whole class and the ratio in a well-drawn sample, and because equality is a convertible relation. But ordinary syllogism is based upon the property of the relation of containing and contained, and that is not a convertible relation. There is, however, a way in which ordinary syllogism may be inverted; namely, the conclusion and either of the premisses may be interchanged by negativing each of them. This is the way in which the indirect, or apagogical,[1] figures of syllogism are derived from the first, and in which the *modus tollens* is derived from the *modus ponens*. The following schemes show this:

First Figure.

Rule. All M is P,
Case. S is M;
Result. S is P.

Second Figure.		*Third Figure.*	
Rule.	All M is P,	*Denial of Result.*	S is not P,
Denial of Result.	S is not P;	*Case.*	S is M;
Denial of Case.	$\therefore S$ is not M.	*Denial of Rule.*	$\therefore$ Some M is not P.

Modus Ponens.

Rule. If A is true, C is true,
Case. In a certain case A is true;
Result. $\therefore$ In that case C is true.

Modus Tollens.	*Modus Innominatus.*
Rule. If A is true, C is true,	*Case.* In a certain case A is true,
Denial of Result. In a certain case C is not true;	*Denial of Result.* In that case, C is not true;
Denial of Case. $\therefore$ In that case A is not true.	*Denial of Rule.* $\therefore$ If A is true, C is not necessarily true.

[1] From *apagoge*, ἀπαγωγὴ εἰς τὸ ἀδύνατον, Aristotle's name for the *reductio ad absurdum*.

719. Now suppose we ask ourselves what would be the result of thus apagogically inverting a statistical deduction. Let us take, for example, Form IV:

The S's are a numerous random sample of the M's,

The proportion r of the M's are P's;

Hence, probably about the proportion r of the S's are P's.

720. The ratio r, as we have already noticed, is not necessarily perfectly definite; it may be only known to have a certain maximum or minimum; in fact, it may have any kind of indeterminacy. Of all possible values between 0 and 1, it admits of some and excludes others. The logical negative of the ratio r is, therefore, itself a ratio, which we may name ρ; it admits of every value which r excludes, and excludes every value of which r admits. Transposing, then, the major premiss and conclusion of our statistical deduction, and at the same time denying both, we obtain the following inverted form:

The S's are a numerous random sample of the M's,

The proportion ρ of the S's are P's;

Hence, probably about the proportion ρ of the M's are P's.[1]

721. But this coincides with the formula of Induction. Again, let us apagogically invert the statistical deduction of Form IV (*bis*). This form is —

Every M has, for example, the numerous marks P', P'', P''', etc.,

S has an r-likeness to the M's;

Hence, probably and approximately, S has the proportion r of the marks P', P'', P''', etc.

Transposing the minor premiss and conclusion, at the same time denying both, we get the inverted form —

Every M has, for example, the numerous marks P', P'', P''', etc.,

S has the proportion ρ of the marks P', P'', P''', etc.;

Hence, probably and approximately, S has a ρ-likeness to the class of M's.

[1] The conclusion of the statistical deduction is here regarded as being "the proportion r of the S's are P's," and the words "probably about" as indicating the modality with which this conclusion is drawn and held for true. It would be equally true to consider the "probably about" as forming part of the contents of the conclusion; only from that point of view the inference ceases to be probable, and becomes rigidly necessary, and its apagogical inversion is also a necessary inference presenting no particular interest.

722. This coincides with the formula of Hypothesis. Thus we see that Induction and Hypothesis are nothing but the apagogical inversions of statistical deductions. Accordingly, when r is taken as 1, so that ρ is "less than 1," or when r is taken as 0, so that ρ is "more than 0," the induction degenerates into a syllogism of the third figure and the hypothesis into a syllogism of the second figure. In these special cases, there is no very essential difference between the mode of reasoning in the direct and in the apagogical form. But, in general, while the probability of the two forms is precisely the same — in this sense, that for any fixed proportion of P's among the M's (or of marks of S's among the marks of the M's) the probability of any given error in the concluded value is precisely the same in the indirect as it is in the direct form — yet there is this striking difference, that a multiplication of instances will in the one case *confirm*, and in the other *modify*, the concluded value of the ratio.

723. We are thus led to another form for our rule of validity of ampliative inference; namely, instead of saying that the *explanatory* syllogism must be a good probable deduction, we may say that the syllogism of which the induction or hypothesis is the apagogical modification (in the traditional language of logic, the *reduction*) must be valid.

724. Probable inferences, though valid, may still differ in their strength. A probable deduction has a greater or less probable error in the concluded ratio. When r is a definite number the probable error is also definite; but as a general rule we can only assign maximum and minimum values of the probable error. The probable error is, in fact —

$$0.477\sqrt{\frac{2r(1-r)}{n}}$$

where n is the number of independent instances. The same formula gives the probable error of an induction or hypothesis; only that in these cases, r being wholly indeterminate, the minimum value is *zero*, and the maximum is obtained by putting $r=\frac{1}{2}$.

§7. FIRST SPECIAL RULE FOR SYNTHETIC INFERENCE.

SAMPLING MUST BE FAIR. ANALOGY

725. Although the rule given above really contains all the conditions to which Inductions and Hypotheses need to conform, yet inasmuch as there are many delicate questions in regard to the application of it, and particularly since it is of that nature that a violation of it, if not too gross, may not absolutely destroy the virtue of the reasoning, a somewhat detailed study of its requirements in regard to each of the premisses of the argument is still needed.

726. The first premiss of a scientific inference is that certain things (in the case of induction) or certain characters (in the case of hypothesis) constitute a fairly chosen *sample* of the class of things or the run of characters from which they have been drawn.

The rule requires that the sample should be drawn at random and independently from the whole lot sampled. That is to say, the sample must be taken according to a precept or method which, being applied over and over again indefinitely, would in the long run result in the drawing of any one set of instances as often as any other set of the same number.

727. The needfulness of this rule is obvious; the difficulty is to know how we are to carry it out. The usual method is mentally to run over the lot of objects or characters to be sampled, abstracting our attention from their peculiarities, and arresting ourselves at this one or that one from motives wholly unconnected with those peculiarities. But this abstention from a further determination of our choice often demands an effort of the will that is beyond our strength; and in that case a mechanical contrivance may be called to our aid. We may, for example, number all the objects of the lot, and then draw numbers by means of a roulette, or other such instrument. We may even go so far as to say that this method is the type of all random drawing; for when we abstract our attention from the peculiarities of objects, the psychologists tell us that what we do is to substitute for the images of sense certain mental signs, and when we proceed to a random and arbitrary choice among these abstract objects we are governed by fortui-

tous determinations of the nervous system, which in this case serves the purpose of a roulette.

The drawing of objects at random is an act in which honesty is called for; and it is often hard enough to be sure that we have dealt honestly with ourselves in the matter, and still more hard to be satisfied of the honesty of another. Accordingly, one method of sampling has come to be preferred in argumentation; namely, to take of the class to be sampled all the objects of which we have a sufficient knowledge. Sampling is, however, a real art, well deserving an extended study by itself: to enlarge upon it here would lead us aside from our main purpose.

728. Let us rather ask what will be the effect upon inductive inference of an imperfection in the strictly random character of the sampling. Suppose that, instead of using such a precept of selection that any one M would in the long run be chosen as often as any other, we used a precept which would give a preference to a certain half of the M's, so that they would be drawn twice as often as the rest. If we were to draw a numerous sample by such a precept, and if we were to find that the proportion ρ of the sample consisted of P's, the inference that we should be regularly entitled to make would be, that among all the M's, counting the preferred half for two each, the proportion ρ would be P's. But this regular inductive inference being granted, from it we could deduce by arithmetic the further conclusion that, counting the M's for one each, the proportion of P's among them must (ρ being over $\frac{2}{3}$) lie between $\frac{3}{4}\ \rho+\frac{1}{4}$ and $\frac{3}{2}\ \rho-\frac{1}{2}$. Hence, if more than two thirds of the instances drawn by the use of the false precept were found to be P's, we should be entitled to conclude that more than half of all the M's were P's. Thus, without allowing ourselves to be led away into a mathematical discussion, we can easily see that, in general, an imperfection of that kind in the random character of the sampling will only weaken the inductive conclusion, and render the concluded ratio less determinate, but will not necessarily destroy the force of the argument completely. In particular, when ρ approximates towards 1 or 0, the effect of the imperfect sampling will be but slight.

729. Nor must we lose sight of the constant tendency of

the inductive process to correct itself. This is of its essence. This is the marvel of it. The probability of its conclusion only consists in the fact that if the true value of the ratio sought has not been reached, an extension of the inductive process will lead to a closer approximation. Thus, even though doubts may be entertained whether one selection of instances is a random one, yet a different selection, made by a different method, will be likely to vary from the normal in a different way, and if the ratios derived from such different selections are nearly equal, they may be presumed to be near the truth. This consideration makes it extremely advantageous in all ampliative reasoning to fortify one method of investigation by another.[1] Still we must not allow ourselves to trust so much to this virtue of induction as to relax our efforts towards making our drawings of instances as random and independent as we can. For if we infer a ratio from a number of different inductions, the magnitude of its probable error will depend very much more on the worst than on the best inductions used.

730. We have, thus far, supposed that although the selection of instances is not exactly regular, yet the precept followed is such that every unit of the lot would eventually get drawn. But very often it is impracticable so to draw our instances, for the reason that a part of the lot to be sampled is absolutely inaccessible to our powers of observation. If we want to know whether it will be profitable to open a mine, we sample the ore; but in advance of our mining operations, we can obtain only what ore lies near the surface. Then, simple induction becomes worthless, and another method must be resorted to. Suppose we wish to make an induction regarding a series of events extending from the distant past to the dis-

[1] This I conceive to be all the truth there is in the doctrine of Bacon and Mill regarding different Methods of Experimental Inquiry. The main proposition of Bacon's and Mill's doctrine is, that in order to prove that all M's are P's, we should not only take random instances of the M's and examine them to see that they are P's, but we should also take instances of not-P's and examine them to see that they are not-M's. This is an excellent way of fortifying one induction by another, when it is applicable; but it is entirely inapplicable when r has any other value than 1 or 0. For, in general, there is no connection between the proportion of M's that are P's and the proportion of non-P's that are non-M's. A very small proportion of calves may be monstrosities, and yet a very large proportion of monstrosities may be calves.

tant future; only those events of the series which occur within the period of time over which available history extends can be taken as instances. Within this period we may find that the events of the class in question present some uniform character; yet how do we know but this uniformity was suddenly established a little while before the history commenced, or will suddenly break up a little while after it terminates? Now, whether the uniformity observed consists (1) in a mere resemblance between all the phenomena, or (2) in their consisting of a disorderly mixture of two kinds in a certain constant proportion, or (3) in the character of the events being a mathematical function of the time of occurrence — in any of these cases we can make use of an apagoge from the following probable deduction:

Within the period of time M, a certain event P occurs,

S is a period of time taken at random from M, and more than half as long;

Hence, probably the event P will occur within the time S.

Inverting this deduction, we have the following ampliative inference:

S is a period of time taken at random from M, and more than half as long,

The event P does not happen in the time S;

Hence, probably the event P does not happen in the period M.

The probability of the conclusion consists in this, that we here follow a precept of inference, which, if it is very often applied will more than half the time lead us right. Analogous reasoning would obviously apply to any portion of an unidimensional continuum, which might be similar to periods of time. This is a sort of logic which is often applied by physicists in what is called *extrapolation* of an empirical law. As compared with a typical induction, it is obviously an excessively weak kind of inference. Although indispensable in almost every branch of science, it can lead to no solid conclusions in regard to what is remote from the field of direct perception, unless it be bolstered up in certain ways to which we shall have occasion to refer further on.

731. Let us now consider another class of difficulties in regard to the rule that the samples must be drawn at random

and independently. In the first place, what if the lot to be sampled be infinite in number? In what sense could a random sample be taken from a lot like that? A random sample is one taken according to a method that would, in the long run, draw any one object as often as any other. In what sense can such drawing be made from an infinite class? The answer is not far to seek. Conceive a cardboard disk revolving in its own plane about its centre, and pretty accurately balanced, so that when put into rotation it shall be about[1] as likely to come to rest in any one position as in any other; and let a fixed pointer indicate a position on the disk: the number of points on the circumference is infinite, and on rotating the disk repeatedly the pointer enables us to make a selection from this infinite number. This means merely that although the points are innumerable, yet there is a certain order among them that enables us to run them through and pick from them as from a very numerous collection. In such a case, and in no other, can an infinite lot be sampled. But it would be equally true to say that a finite lot can be sampled only on condition that it can be regarded as equivalent to an infinite lot. For the random sampling of a finite class supposes the possibility of drawing out an object, throwing it back, and continuing this process indefinitely; so that what is really sampled is not the finite collection of things, but the unlimited number of possible drawings.

732. But though there is thus no insuperable difficulty in sampling an infinite lot, yet it must be remembered that the conclusion of inductive reasoning only consists in the approximate evaluation of a *ratio*, so that it never can authorize us to conclude that in an infinite lot sampled there exists no single exception to a rule. Although all the planets are found to gravitate toward one another, this affords not the slightest direct reason for denying that among the innumerable orbs of heaven there may be some which exert no such force. Although at no point of space where we have yet been have we found any possibility of motion in a fourth dimension, yet this does not tend to show (by simple induction, at least) that space has absolutely but three dimensions. Although all the bodies

[1] I say *about*, because the doctrine of probability only deals with approximate evaluations.

we have had the opportunity of examining appear to obey the law of inertia, this does not prove that atoms and atomicules are subject to the same law. Such conclusions must be reached, if at all, in some other way than by simple induction. This latter may show that it is unlikely that, in my lifetime or yours, things so extraordinary should be found, but [does] not warrant extending the prediction into the indefinite future. And experience shows it is not safe to predict that such and such a fact will *never* be met with.

733. If the different instances of the lot sampled are to be drawn independently, as the rule requires, then the fact that an instance has been drawn once must not prevent its being drawn again. It is true that if the objects remaining unchosen are very much more numerous than those selected, it makes practically no difference whether they have a chance of being drawn again or not, since that chance is in any case very small. Probability is wholly an affair of approximate, not at all of exact, measurement; so that when the class sampled is very large, there is no need of considering whether objects can be drawn more than once or not. But in what is known as "reasoning from analogy," the class sampled is small, and no instance is taken twice. For example: we know that of the major planets the Earth, Mars, Jupiter, and Saturn revolve on their axes, and we conclude that the remaining four, Mercury, Venus, Uranus, and Neptune, probably do the like. This is essentially different from an inference from what has been found in drawings made hitherto, to what will be found in indefinitely numerous drawings to be made hereafter. Our premisses here are that the Earth, Mars, Jupiter, and Saturn are a random sample of a natural class of major planets — a class which, though (so far as we know) it is very small, yet *may* be very extensive; comprising whatever there may be that revolves in a circular orbit around a great sun, is nearly spherical, shines with reflected light, is very large, etc. Now the examples of major planets that we can examine all rotate on their axes; whence we suppose that Mercury, Venus, Uranus, and Neptune, since they possess, so far as we know, all the properties common to the natural class to which the Earth, Mars, Jupiter, and Saturn belong, possess this property likewise. The points to be observed are, first, that any small class

of things may be regarded as a mere sample of an actual or possible large class having the same properties and subject to the same conditions; second, that while we do not know what all these properties and conditions are, we do know some of them, which some may be considered as a random sample of all; third, that a random selection without replacement from a small class may be regarded as a true random selection from that infinite class of which the finite class is a random selection. The formula of the analogical inference presents, therefore, three premisses, thus:

S', S'', S''', are a random sample of some undefined class X, of whose characters P', P'', P''', are samples,

Q is P', P'', P''';

S', S'', S''', are R's;

Hence, Q is an R.

We have evidently here an induction and an hypothesis followed by a deduction; thus:

Every X is, for example, P', P'', P''', etc.,	S', S'', S''', etc., are samples of the X's,
Q is found to be P', P'', P''', etc.;	S', S'', S''', etc., are found to be R's;
Hence, hypothetically, Q is an X.	Hence, inductively, every X is an R.

Hence, deductively, Q is an R.[1]

[1] That this is really a correct analysis of the reasoning can be shown by the theory of probabilities. For the expression

$$\frac{(p+q)!}{p!q!}\,\frac{(\pi+\rho)!}{\pi!\rho!}\cdot\frac{(p+\pi)!(q+\rho)!}{(p+\pi+q+\rho)!}$$

expresses at once the probability of two events; namely, it expresses first the probability that of $p+q$ objects drawn without replacement from a lot consisting of $p+\pi$ objects having the character R together with $q+\rho$ not having this character, the number of those drawn having this character will be p; and second, the same expression denotes the probability that if among $p+\pi+q+\rho$ objects drawn at random from an infinite class (containing no matter what proportion of R's to non-R's), it happens that $p+\pi$ have the character R, then among any $p+q$ of them, designated at random, p will have the same character. Thus we see that the chances in reference to drawing without replacement from a finite class are precisely the same as those in reference to a class which has been drawn at random from an infinite class.

734. An argument from analogy may be strengthened by the addition of instance after instance to the premisses, until it loses its ampliative character by the exhaustion of the class and becomes a mere deduction of that kind called *complete induction*, in which, however, some shadow of the inductive character remains, as this name implies.

§8. SECOND SPECIAL RULE FOR SYNTHETIC INFERENCE, THAT OF PREDESIGNATION

735. Take any human being, at random — say Queen Elizabeth. Now a little more than half of all the human beings who have ever existed have been males; but it does not follow that it is a little more likely than not that Queen Elizabeth was a male, since we know she was a woman. Nor, if we had selected Julius Cæsar, would it be only a little more likely than not that he was a male. It is true that if we were to go on drawing at random an indefinite number of instances of human beings, a slight excess over one-half would be males. But that which constitutes the probability of an inference is the proportion of true conclusions among all those which could be derived *from the same precept*. Now a precept of inference, being a rule which the mind is to follow, changes its character and becomes different when the case presented to the mind is essentially different. When, knowing that the proportion r of all M's are P's, I draw an instance, S, of an M, without any other knowledge of whether it is a P or not, and infer with probability, r, that it is P, the case presented to my mind is very different from what it is if I have such other knowledge. In short, I cannot make a valid probable inference without taking into account whatever knowledge I have (or, at least, whatever occurs to my mind) that bears upon the question.

736. The same principle may be applied to the statistical deduction of Form IV. If the major premiss, that the proportion r of the M's are P's be laid down first, before the instances of M's are drawn, we really draw our inference concerning those instances (that the proportion r of them will be P's) in advance of the drawing, and therefore before we know whether they are P's or not. But if we draw the instances of the M's first, and after the examination of them decide what we will select for the predicate of our major premiss, the

inference will generally be completely fallacious. In short, we have the rule that the major term P must be decided upon in advance of the examination of the sample; and in like manner in Form IV (*bis*) the minor term S must be decided upon in advance of the drawing.

737. The same rule follows us into the logic of induction and hypothesis. If in sampling any class, say the M's, we first decide what the character P is for which we propose to sample that class, and also how many instances we propose to draw, our inference is really made before these latter are drawn, that the proportion of P's in the whole class is probably about the same as among the instances that are to be drawn, and the only thing we have to do is to draw them and observe the ratio. But suppose we were to draw our inferences without the predesignation of the character P; then we might in every case find some recondite character in which those instances would all agree. That, by the exercise of sufficient ingenuity, we should be sure to be able to do this, even if not a single other object of the class M possessed that character, is a matter of demonstration. For in geometry a curve may be drawn through any given series of points, without passing through any one of another given series of points, and this irrespective of the number of dimensions. Now, all the qualities of objects may be conceived to result from variations of a number of continuous variables; hence any lot of objects possesses some character in common, not possessed by any other. It is true that if the universe of quality is limited, this is not altogether true; but it remains true that unless we have some special premiss from which to infer the contrary, it always *may* be possible to assign some common character of the instances S', S'', S''', etc., drawn at random from among the M's, which does not belong to the M's generally. So that if the character P were not predesignate, the deduction of which our induction is the apagogical inversion would not be valid; that is to say, we could not reason that if the M's did not generally possess the character P, it would not be likely that the S's should all possess this character.

738. I take from a biographical dictionary* the first five names of poets, with their ages at death. They are,

* Wheeler's *Biographical Dictionary.*

Aagard, died at 48.
Abeille, died at 76.
Abulola, died at 84.
Abunowas, died at 48.
Accords, died at 45.

These five ages have the following characters in common:

1. The difference of the two digits composing the number, divided by three, leaves a remainder of *one*.

2. The first digit raised to the power indicated by the second, and then divided by three, leaves a remainder of *one*.

3. The sum of the prime factors of each age, including *one* as a prime factor, is divisible by *three*.

Yet there is not the smallest reason to believe that the next poet's age would possess these characters.

Here we have a *conditio sine qua non* of valid induction which has been singularly overlooked by those who have treated of the logic of the subject, and is very frequently violated by those who draw inductions. So accomplished a reasoner as Dr. Lyon Playfair, for instance, has written a paper of which the following is an abstract. He first takes the specific gravities of the three allotropic forms of carbon, as follows:

Diamond, 3.48.
Graphite, 2.29.
Charcoal, 1.88.

He now seeks to find a uniformity connecting these three instances; and he discovers that the atomic weight of carbon, being 12,

$$\text{Sp. gr. diamond nearly} = 3.46 = \sqrt[2]{12}$$
$$\text{Sp. gr. graphite nearly} = 2.29 = \sqrt[3]{12}$$
$$\text{Sp. gr. charcoal nearly} = 1.86 = \sqrt[4]{12}$$

This, he thinks, renders it probable that the specific gravities of the allotropic forms of other elements would, if we knew them, be found to equal the different roots of their atomic weight. But so far, the character in which the instances agree not having been predesignated, the induction can serve only to suggest a question, and ought not to create any belief. To test the proposed law, he selects the instance of silicon, which

like carbon exists in a diamond and in a graphitoidal condition. He finds for the specific gravities —

Diamond silicon, 2.47
Graphite silicon, 2.33.[1]

Now, the atomic weight of silicon, that of carbon being 12, can only be taken as 28. But 2.47 does not approximate to any root of 28. It is, however, nearly the cube root of 14, ($\sqrt[3]{\frac{1}{2}\times 28} = 2.41$), while 2.33 is nearly the fourth root of 28 ($\sqrt[4]{28} = 2.30$). Dr. Playfair claims that silicon is an instance satisfying his formula. But in fact this instance requires the formula to be modified; and the modification not being predesignate, the instance cannot count. Boron also exists in a diamond and a graphitoidal form; and accordingly Dr. Playfair takes this as his next example. Its atomic weight is 10.9, and its specific gravity is 2.68; which is the square root of $\frac{2}{3}\times 10.9$. There seems to be here a further modification of the formula not predesignated, and therefore this instance can hardly be reckoned as confirmatory. The next instances which would occur to the mind of any chemist would be phosphorus and sulphur, which exist in familiarly known allotropic forms. Dr. Playfair admits that the specific gravities of phosphorus have no relations to its atomic weight at all analogous to those of carbon. The different forms of sulphur have nearly the same specific gravity, being approximately the fifth root of the atomic weight 32. Selenium also has two allotropic forms, whose specific gravities are 4.8 and 4.3; one of these follows the law, while the other does not. For tellurium the law fails altogether; but for bromine and iodine it holds. Thus the number of specific gravities for which the law was predesignate are 8; namely, 2 for phosphorus, 1 for sulphur, 2 for selenium, 1 for tellurium, 1 for bromine, and 1 for iodine. The law holds for 4 of these, and the proper inference is that about half the specific gravities of metalloids are roots of some simple ratio of their atomic weights.

[1] The author ought to have noted that this number is open to some doubt, since the specific gravity of this form of silicon appears to vary largely. If a different value had suited the theory better, he might have been able to find reasons for preferring that other value. But I do not mean to imply that Dr. Playfair has not dealt with perfect fairness with his facts, except as to the fallacy which I point out.

Having thus determined this ratio, we proceed to inquire whether an agreement half the time with the formula constitutes any special connection between the specific gravity and the atomic weight of a metalloid. As a test of this, let us arrange the elements in the order of their atomic weights, and compare the specific gravity of the first with the atomic weight of the last, that of the second with the atomic weight of the last but one, and so on. The atomic weights are —

Boron,	10.9	Tellurium,	128.1
Carbon,	12.0	Iodine,	126.9
Silicon,	28.0	Bromine,	80.0
Phosphorus,	31.0	Selenium,	79.1

Sulphur, 32.

There are three specific gravities given for carbon, and two each for silicon, phosphorus, and selenium. The question, therefore, is, whether of the fourteen specific gravities as many as seven are in Playfair's relation with the atomic weights, not of the same element, but of the one paired with it. Now, taking the original formula of Playfair we find

	Sp. gr. boron	$=2.68$	$\sqrt[5]{\text{Te}}$	$=2.64$
3d	Sp. gr. carbon	$=1.88$	$\sqrt[5]{\text{I}}$	$=1.84$
2d	Sp. gr. carbon	$=2.29$	$\sqrt[6]{\text{I}}$	$=2.24$
1st	Sp. gr. phosphorus	$=1.83$	$\sqrt[7]{\text{Se}}$	$=1.87$
2d	Sp. gr. phosphorus	$=2.10$	$\sqrt[6]{\text{Se}}$	$=2.07$

or five such relations without counting that of sulphur to itself. Next, with the modification introduced by Playfair, we have

1st	Sp. gr. silicon	$=2.47$	$\sqrt[4]{\frac{1}{2}\times\text{Br}}$	$=2.51$
2d	Sp. gr. silicon	$=2.33$	$\sqrt[6]{2\times\text{Br}}$	$=2.33$
	Sp. gr. iodine	$=4.95$	$\sqrt[3]{2\times\text{C}}$	$=4.90$
1st	Sp. gr. carbon	$=3.48$	$\sqrt[3]{\frac{1}{3}\times\text{I}}$	$=3.48$

It thus appears that there is no more frequent agreement with Playfair's proposed law than what is due to chance.[1]

[1] As the relations of the different powers of the specific gravity would be entirely different if any other substance than water were assumed as the standard, the law is antecedently in the highest degree improbable. This makes it likely that some fallacy was committed, but does not show what it was.

739. Another example of this fallacy was "Bode's law" of the relative distances of the planets, which was shattered by the first discovery of a true planet after its enunciation. In fact, this false kind of induction is extremely common in science and in medicine.[1] In the case of hypothesis, the correct rule has often been laid down; namely, that a hypothesis can only be received upon the ground of its having been *verified* by successful *prediction*. The term *predesignation* used in this paper appears to be more exact, inasmuch as it is not at all requisite that the ratio ρ should be given in advance of the examination of the samples. Still, since ρ is equal to 1 in all ordinary hypotheses, there can be no doubt that the rule of prediction, so far as it goes, coincides with that here laid down.

740. We have now to consider an important modification of the rule. Suppose that, before sampling a class of objects, we have predesignated not a single character but n characters, for which we propose to examine the samples. This is equivalent to making n different inductions from the same instances. The probable error in this case is that error whose probability for a simple induction is only $(\frac{1}{2})^n$, and the theory of probabilities shows that it increases but slowly with n; in fact, for $n = 1000$ it is only about five times as great as for $n = 1$, so that with only 25 times as many instances the inference would be as secure for the former value of n as with the latter; with 100 times as many instances an induction in which $n =$ 10,000,000,000 would be equally secure. Now the whole universe of characters will never contain such a number as the last; and the same may be said of the universe of objects in the case of hypothesis. So that, without any voluntary predesignation, the limitation of our imagination and experience amounts to a predesignation far within those limits; and we thus see that if the number of instances be very great indeed, the failure to predesignate is not an important fault. Of characters at all striking, or of objects at all familiar, the number will seldom reach 1,000; and of very striking characters or very familiar objects the number is still less. So that if a large number of samples of a class are found to have some

[1] The physicians seem to use the maxim that you cannot reason from *post hoc* to *propter hoc* to mean (rather obscurely) that cases must not be used to prove a proposition that has only been suggested by these cases themselves.

very striking character in common, or if a large number of characters of one object are found to be possessed by a very familiar object, we need not hesitate to infer, in the first case, that the same characters belong to the whole class, or, in the second case, that the two objects are practically identical; remembering only that the inference is less to be relied upon than it would be had a deliberate predesignation been made. This is no doubt the precise significance of the rule sometimes laid down, that a hypothesis ought to be *simple* — simple here being taken in the sense of familiar.

This modification of the rule shows that, even in the absence of voluntary predesignation, *some* slight weight is to be attached to an induction or hypothesis. And perhaps when the number of instances is not very small, it is enough to make it worth while to subject the inference to a regular test. But our natural tendency will be to attach too much importance to such suggestions, and we shall avoid waste of time in passing them by without notice until some stronger plausibility presents itself.

§9. UNIFORMITIES

741. In almost every case in which we make an induction or a hypothesis, we have some knowledge which renders our conclusion antecedently likely or unlikely. The effect of such knowledge is very obvious, and needs no remark. But what also very often happens is that we have some knowledge, which, though not of itself bearing upon the conclusion of the scientific argument, yet serves to render our inference more or less probable, or even to alter the terms of it. Suppose, for example, that we antecedently know that all the M's strongly resemble one another in regard to characters of a certain order. Then, if we find that a moderate number of M's taken at random have a certain character, P, of that order, we shall attach a greater weight to the induction than we should do if we had not that antecedent knowledge. Thus, if we find that a certain sample of gold has a certain chemical character — since we have very strong reason for thinking that all gold is alike in its chemical characters — we shall have no hesitation in extending the proposition from the one sample to gold in general. Or if we know that among a certain people — say the Icelanders — an extreme uniformity prevails in regard to all their

ideas, then, if we find that two or three individuals taken at random from among them have all any particular superstition, we shall be the more ready to infer that it belongs to the whole people from what we know of their uniformity. The influence of this sort of uniformity upon inductive conclusions was strongly insisted upon by Philodemus,* and some very exact conceptions in regard to it may be gathered from the writings of Mr. Galton. Again, suppose we know of a certain character, P, that in whatever classes of a certain description it is found at all, to those it usually belongs as a universal character; then any induction which goes toward showing that all the M's are P will be greatly strengthened. Thus it is enough to find that two or three individuals taken at random from a genus of animals have three toes on each foot, to prove that the same is true of the whole genus; for we know that this is a *generic* character. On the other hand, we shall be slow to infer that all the animals of a genus have the same color, because color varies in almost every genus. This kind of uniformity seemed to J. S. Mill to have so controlling an influence upon inductions, that he has taken it as the centre of his whole theory of the subject.

742. Analogous considerations modify our hypothetic inferences. The sight of two or three words will be sufficient to convince me that a certain manuscript was written by myself, because I know a certain look is peculiar to it. So an analytical chemist, who wishes to know whether a solution contains gold, will be completely satisfied if it gives a precipitate of the purple of cassius with chloride of tin; because this proves that either gold or some hitherto unknown substance is present. These are examples of characteristic tests. Again, we may know of a certain person, that whatever opinions he holds he carries out with uncompromising rigor to their utmost logical consequences; then, if we find his views bear some of the marks of any ultra school of thought, we shall readily conclude that he fully adheres to that school.

743. There are thus four different kinds of uniformity and non-uniformity which may influence our ampliative inferences:

(1) The members of a class may present a greater or less general resemblance as regards a certain line of characters.

* See Theodor Gomperz, *Herculanische Studien*, pt. I (1865). Cf. 761.

(2) A character may have a greater or less tendency to be present or absent throughout the whole of whatever classes of certain kinds.

(3) A certain set of characters may be more or less intimately connected, so as to be probably either present or absent together in certain kinds of objects.

(4) An object may have more or less tendency to possess the whole of certain sets of characters when it possesses any of them.

A consideration of this sort may be so strong as to amount to demonstration of the conclusion. In this case, the inference is mere deduction — that is, the application of a general rule already established. In other cases, the consideration of uniformities will not wholly destroy the inductive or hypothetic character of the inference, but will only strengthen or weaken it by the addition of a new argument of a deductive kind.

§10. CONSTITUTION OF THE UNIVERSE

744. We have thus seen how, in a general way, the processes of inductive and hypothetic inference are able to afford answers to our questions, though these may relate to matters beyond our immediate ken. In short, a theory of the logic of verification has been sketched out. This theory will have to meet the objections of two opposing schools of logic.

The first of these explains induction by what is called the doctrine of Inverse Probabilities, of which the following is an example: Suppose an ancient denizen of the Mediterranean coast, who had never heard of the tides, had wandered to the shore of the Atlantic Ocean, and there, on a certain number m of successive days had witnessed the rise of the sea. Then, says Quetelet, he would have been entitled to conclude that there was a probability equal to $\frac{m+1}{m+2}$ that the sea would rise on the next following day.[1] Putting $m=0$, it is seen that this view assumes that the probability of a totally unknown event is $\frac{1}{2}$; or that of all theories proposed for examination one half are true. In point of fact, we know that although theories are not proposed unless they present some decided plausibility, nothing like one half turn out to be true. But to apply correctly

[1] See Laplace, *Théorie Analitique des Probabilités*, [1812], livre ii, ch. vi.

the doctrine of inverse probabilities, it is necessary to know the antecedent probability of the event whose probability is in question. Now, in pure hypothesis or induction, we know nothing of the conclusion antecedently to the inference in hand. Mere ignorance, however, cannot advance us toward any knowledge; therefore it is impossible that the theory of inverse probabilities should rightly give a value for the probability of a pure inductive or hypothetic conclusion. For it cannot do this without assigning an antecedent probability to this conclusion; so that if this antecedent probability represents mere ignorance (which never aids us), it cannot do it at all.

745. The principle which is usually assumed by those who seek to reduce inductive reasoning to a problem in inverse probabilities is, that if nothing whatever is known about the frequency of occurrence of an event, then any one frequency is as probable as any other. But Boole has shown that there is no reason whatever to prefer this assumption, to saying that any one "constitution of the universe" is as probable as any other. Suppose, for instance, there were four possible occasions upon which an event might occur. Then there would be 16 "constitutions of the universe," or possible distributions of occurrences and non-occurrences. They are shown in the following table, where *Y* stands for an occurrence and *N* for a non-occurrence.

4 *occurrences.*	3 *occurrences.*	2 *occurrences.*	1 *occurrence.*	0 *occurrence.*
Y Y Y Y	*Y Y Y N*	*Y Y N N*	*Y N N N*	*N N N N*
	Y Y N Y	*Y N Y N*	*N Y N N*	
	Y N Y Y	*Y N N Y*	*N N Y N*	
	N Y Y Y	*N Y Y N*	*N N N Y*	
		N Y N Y		
		N N Y Y		

It will be seen that different frequencies result some from more and some from fewer different "constitutions of the universe," so that it is a very different thing to assume that all frequencies are equally probable from what it is to assume that all constitutions of the universe are equally probable.

746. Boole says that one assumption is as good as the other. But I will go further, and say that the assumption that

all constitutions of the universe are equally probable is far better than the assumption that all frequencies are equally probable. For the latter proposition, though it may be applied to any one unknown event, cannot be applied to all unknown events without inconsistency. Thus, suppose all frequencies of the event whose occurrence is represented by Y in the above table are equally probable. Then consider the event which consists in a Y following a Y or an N following an N. The possible ways in which *this* event may occur or not are shown in the following table:

3 *occurrences.*	2 *occurrences.*	1 *occurrence.*	0 *occurrence.*
$Y\ Y\ Y\ Y$	$Y\ Y\ Y\ N$	$Y\ Y\ N\ Y$	$Y\ N\ Y\ N$
$N\ N\ N\ N$	$N\ N\ N\ Y$	$N\ N\ Y\ N$	$N\ Y\ N\ Y$
	$Y\ Y\ N\ N$	$Y\ N\ N\ Y$	
	$N\ N\ Y\ Y$	$N\ Y\ Y\ N$	
	$N\ Y\ Y\ Y$	$Y\ N\ Y\ Y$	
	$Y\ N\ N\ N$	$N\ Y\ N\ N$	

It will be found that assuming the different frequencies of the first event to be equally probable, those of this new event are not so — the probability of three occurrences being half as large again as that of two, or one. On the other hand, if all constitutions of the universe are equally probable in the one case, they are so in the other; and this latter assumption, in regard to perfectly unknown events, never gives rise to any inconsistency.

Suppose, then, that we adopt the assumption that any one constitution of the universe is as probable as any other; how will the inductive inference then appear, considered as a problem in probabilities? The answer is extremely easy;[1] namely, the occurrences or non-occurrences of an event in the past in no way affect the probability of its occurrence in the future.

747. Boole frequently finds a problem in probabilities to be indeterminate. There are those to whom the idea of an unknown probability seems an absurdity. Probability, they say, measures the state of our knowledge, and ignorance is denoted by the probability $\frac{1}{2}$. But I apprehend that the expression "the probability of an event" is an incomplete one.

[1] See Boole, *Laws of Thought*, p. 370.

A probability is a fraction whose numerator is the frequency of a specific kind of event, while its denominator is the frequency of a genus embracing that species. Now the expression in question names the numerator of the fraction, but omits to name the denominator. There is a sense in which it is true that the probability of a perfectly unknown event is one half; namely, the assertion of its occurrence is the answer to a possible question answerable by "yes" or "no," and of all such questions just half the possible answers are true. But if attention be paid to the denominators of the fractions, it will be found that this value of $\frac{1}{2}$ is one of which no possible use can be made in the calculation of probabilities.

748. The theory here proposed does not assign any probability to the inductive or hypothetic conclusion, in the sense of undertaking to say how frequently *that conclusion* would be found true. It does not propose to look through all the possible universes, and say in what proportion of them a certain uniformity occurs; such a proceeding, were it possible, would be quite idle. The theory here presented only says how frequently, in this universe, the special form of induction or hypothesis would lead us right. The probability given by this theory is in every way different — in meaning, numerical value, and form — from that of those who would apply to ampliative inference the doctrine of inverse chances.

749. Other logicians hold that if inductive and hypothetic premisses lead to true oftener than to false conclusions, it is only because the universe happens to have a certain constitution. Mill and his followers maintain that there is a general tendency toward uniformity in the universe, as well as special uniformities such as those which we have considered. The Abbé Gratry believes that the tendency toward the truth in induction is due to a miraculous intervention of Almighty God, whereby we are led to make such inductions as happen to be true, and are prevented from making those which are false.* Others have supposed that there is a special adaptation of the mind to the universe, so that we are more apt to make true theories than we otherwise should be. Now, to say that a theory such as these is *necessary* to explaining the validity of induction and hypothesis is to say that these modes of infer-

* See *La Logique*, Paris (1855), vol. 2, pp. 196–97.

ence are not in themselves valid, but that their conclusions are rendered probable by being probable deductive inferences from a suppressed (and originally unknown) premiss. But I maintain that it has been shown that the modes of inference in question are necessarily valid, whatever the constitution of the universe, so long as it admits of the premisses being true. Yet I am willing to concede, in order to concede as much as possible, that when a man draws instances at random, all that he knows is that he *tries* to follow a certain precept; so that the sampling process might be rendered generally fallacious by the existence of a mysterious and malign connection between the mind and the universe, such that the possession by an object of an *unperceived* character might influence the will toward choosing it or rejecting it. Such a circumstance would, however, be as fatal to deductive as to ampliative inference. Suppose, for example, that I were to enter a great hall where people were playing *rouge et noir* at many tables; and suppose that I knew that the red and black were turned up with equal frequency. Then, if I were to make a large number of mental bets with myself, at this table and at that, I might, by statistical deduction, expect to win about half of them — precisely as I might expect, from the results of these samples, to infer by induction the probable ratio of frequency of the turnings of red and black in the long run, if I did not know it. But could some devil look at each card before it was turned, and then influence me mentally to bet upon it or to refrain therefrom, the observed ratio in the cases upon which I had bet might be quite different from the observed ratio in those cases upon which I had not bet. I grant, then, that even upon my theory some fact has to be supposed to make induction and hypothesis valid processes; namely, it is supposed that the supernal powers withhold their hands and let me alone, and that no mysterious uniformity or adaptation interferes with the action of chance. But then this negative fact supposed by my theory plays a totally different part from the facts supposed to be requisite by the logicians of whom I have been speaking. So far as facts like those they suppose can have any bearing, they serve as major premisses from which the fact inferred by induction or hypothesis might be deduced; while the negative fact supposed by me is merely the denial of

any major premiss from which the falsity of the inductive or hypothetic conclusion could in general be deduced. Nor is it necessary to deny altogether the existence of mysterious influences adverse to the validity of the inductive and hypothetic processes. So long as their influence were not too overwhelming, the wonderful self-correcting nature of the ampliative inference would enable us, even if they did exist, to detect and make allowance for them.

750. Although the universe need have no peculiar constitution to render ampliative inference valid, yet it is worth while to inquire whether or not it has such a constitution; for if it has, that circumstance must have its effect upon all our inferences. It cannot any longer be denied that the human intellect is peculiarly adapted to the comprehension of the laws and facts of nature, or at least of some of them; and the effect of this adaptation upon our reasoning will be briefly considered in the next section. Of any miraculous interference by the higher powers, we know absolutely nothing; and it seems in the present state of science altogether improbable. The effect of a knowledge of special uniformities upon ampliative inferences has already been touched upon. That there is a general tendency toward uniformity in nature is not merely an unfounded, it is an absolutely absurd, idea in any other sense than that man is adapted to his surroundings. For the universe of marks is only limited by the limitation of human interests and powers of observation. Except for that limitation, every lot of objects in the universe would have (as I have elsewhere shown)* some character in common and peculiar to it. Consequently, there is but one possible arrangement of characters among objects as they exist, and there is no room for a greater or less degree of uniformity in nature. If nature seems highly uniform to us, it is only because our powers are adapted to our desires.

§11. FURTHER PROBLEMS

751. The questions discussed in this essay relate to but a small part of the Logic of Scientific Investigation. Let us just glance at a few of the others.

752. Suppose a being from some remote part of the uni-

* See vol. 6, bk. II, ch. 1, §2.

verse, where the conditions of existence are inconceivably different from ours, to be presented with a United States Census Report — which is for us a mine of valuable inductions, so vast as almost to give that epithet a new signification. He begins, perhaps, by comparing the ratio of indebtedness to deaths by consumption in counties whose names begin with the different letters of the alphabet. It is safe to say that he would find the ratio everywhere the same, and thus his inquiry would lead to nothing. For an induction is wholly unimportant unless the proportions of P's among the M's and among the non-M's differ; and a hypothetic inference is unimportant unless it be found that S has either a greater or a less proportion of the characters of M than it has of other characters. The stranger to this planet might go on for some time asking inductive questions that the Census would faithfully answer, without learning anything except that certain conditions were independent of others. At length, it might occur to him to compare the January rainfall with the illiteracy. What he would find is given in the following table[1]:

REGION	January Rainfall	Illiteracy
	Inches	Per cent
Atlantic seacoast, Portland to Washington	0.92	11
Vermont, Northern and Western New York	0.78	7
Upper Mississippi River	0.52	3
Ohio River Valley	0.74	8
Lower Mississippi, Red River, and Kentucky	1.08	50
Mississippi Delta and Northern Gulf Coast	1.09	57
Southeastern Coast	0.68	40

[1] The different regions with the January rainfall are taken from Mr. Schott's work. [*Tables and Results of the Precipitation in Rain and Snow in the United States*, 1872.] The percentage of illiteracy is roughly estimated from the numbers given in the Report of the 1870 Census. [The maps originally published with this paper have not been considered worth reproducing.]

He would infer that in places that are drier in January there is, not always but generally, less illiteracy than in wetter places. A detailed comparison between Mr. Schott's map of the winter rainfall with the map of illiteracy in the general census, would confirm the result that these two conditions have a partial connection. This is a very good example of an induction in which the proportion of P's among the M's is different, but not very different, from the proportion among the non-M's. It is unsatisfactory; it provokes further inquiry; we desire to replace the M by some different class, so that the two proportions may be more widely separated. Now we, knowing as much as we do of the effects of winter rainfall upon agriculture, upon wealth, etc., and of the causes of illiteracy, should come to such an inquiry furnished with a large number of appropriate conceptions; so that we should be able to ask intelligent questions not unlikely to furnish the desired key to the problem. But the strange being we have imagined could only make his inquiries haphazard, and could hardly hope ever to find the induction of which he was in search.

753. Nature is a far vaster and less clearly arranged repertory of facts than a census report; and if men had not come to it with special aptitudes for guessing right, it may well be doubted whether in the ten or twenty thousand years that they may have existed their greatest mind would have attained the amount of knowledge which is actually possessed by the lowest idiot. But, in point of fact, not man merely, but all animals derive by inheritance (presumably by natural selection) two classes of ideas which adapt them to their environment. In the first place, they all have from birth some notions, however crude and concrete, of force, matter, space, and time; and, in the next place, they have some notion of what sort of objects their fellow-beings are, and of how they will act on given occasions. Our innate mechanical ideas were so nearly correct that they needed but slight correction. The fundamental principles of statics were made out by Archimedes. Centuries later Galileo began to understand the laws of dynamics, which in our times have been at length, perhaps, completely mastered. The other physical sciences are the results of inquiry based on guesses suggested by the ideas of mechanics. The moral sciences, so far as they can be called sciences, are equally

developed out of our instinctive ideas about human nature. Man has thus far not attained to any knowledge that is not in a wide sense either mechanical or anthropological in its nature, and it may be reasonably presumed that he never will.*

754. Side by side, then, with the well established proposition that all knowledge is based on experience, and that science is only advanced by the experimental verifications of theories, we have to place this other equally important truth, that all human knowledge, up to the highest flights of science, is but the development of our inborn animal instincts.

* Cf. 1.118.

CHAPTER 9

*THE VARIETIES AND VALIDITY OF INDUCTION**

§1. CRUDE, QUANTITATIVE, AND QUALITATIVE INDUCTION

755. Retroduction and Induction face opposite ways. The function of retroduction is not unlike those fortuitous variations in reproduction which played so important a *rôle* in Darwin's original theory. In point of fact, according to him every step in the long history of the development of the moner into the man was first taken in that arbitrary and lawless mode. Whatever truth or error there may be in that, it is quite indubitable, as it appears to me, that every step in the development of primitive notions into modern science was in the first instance mere guess-work, or at least mere conjecture. But the stimulus to guessing, the hint of the conjecture, was derived from experience. The order of the march of suggestion in retroduction is from experience to hypothesis. A great many people who may be admirably trained in divinity, or in the humanities, or in law and equity, but who are certainly not well trained in scientific reasoning, imagine that Induction should follow the same course. My Lord Chancellor Bacon was one of them. On the contrary, the only sound procedure for induction, whose business consists in testing a hypothesis already recommended by the retroductive procedure, is to receive its suggestions from the hypothesis first, to take up the predictions of experience which it conditionally makes, and then try the experiment and see whether it turns out as it was virtually predicted in the hypothesis that it would. Throughout an investigation it is well to bear prominently in mind just what it is that we are trying to accomplish in the particular stage of the work at which we have arrived. Now when we get to the inductive stage what we are about is finding out how much like the truth our hypothesis is, that is, what proportion of its anticipations will be verified.

* From ms. "*G.*" c. 1905.

756. It is well to distinguish three different varieties of induction. The first and weakest kind of inductive reasoning is that which goes on the presumption that future experience as to the matter in hand will not be utterly at variance with all past experience.[1] *Example:* "No instance of a genuine power of clairvoyance has ever been established: So I presume there is no such thing." I promise to call such reasoning *crude induction.** Bacon seems to refer to this when he speaks of "inductio quae procedit per enumerationem simplicem." But I hardly think he meant to say that that phrase exactly describes it. It certainly does not; since in most cases no enumeration is attempted; and the enumeration, even if given, would not be the reasoner's chief reliance, which is rather the *absence* of instances to the contrary.

757. Crude induction is the only kind of induction that is capable of inferring the truth of what, in logic, is termed a universal proposition. For what is called "complete induction" is not inductive reasoning, but is logistic deduction. We might further say, if we chose, that every crude induction concludes a universal proposition; but this would be merely the expression of a way of regarding matters. For any proposition concerning the general run of future experience may be regarded as universal, even if it be "A pair of dice will, every now and then, turn up doublets." The undipped heel of crude induction is that if its conclusion be understood as indefinite, it will be of little use, while if it be taken definitely, it is liable at any moment to be utterly shattered by a single experience; for a series of experiences, if the whole constitutes but a single one

[1] When I say that it goes on that presumption, I merely mean to describe the presumption of the reasoning as being that the particular uniformity as to a certain matter that has attached to past experience will be maintained in the future. I shall explain below how there is a certain justification in this, though a very slender one. I do not mean to say, as some logicians do, that the force of the induction is just the same as that of a syllogism whose major premiss should be, "Future experience will not violate the uniformity of past experience." For such a syllogism being a fallacy of the particularly atrocious kind called "logical fallacy," would have no justification whatsoever. For a sound syllogism must have a major premiss of definite meaning: otherwise it may be thrown into the form of a fallacy of undistributed middle. Now the induction in question, though weak, is by no means without justification.

* Cf. 111, 269, 662, 757n.

of the instances to which an inductive conclusion refers, is to be regarded as a single experience.[1]

758. From the weakest kind of induction let us pass at once to the strongest. This investigates the interrogative suggestion of retroduction, "What is the 'real probability' that an individual member of a certain experiential class, say the S's, will have a certain character, say that of being P?" This it does by first collecting, on scientific principles, a "fair sample" of the S's, taking due account, in doing so, of the intention of using its proportion of members that possess the predesignate character of being P. This sample will contain none of those S's on which the retroduction was founded. The induction then presumes that the value of the proportion, among the S's of the sample, of those that are P, probably approximates, within a certain limit of approximation, to the value of the real probability in question. I propose to term such reasoning *Quantitative Induction*. Now, if I were writing a treatise on logic, I should here be obliged, not only to teach the art of sampling, including all that Dr. Karl Pearson* and others

[1] Induction is such a way of inference that if one persists in it one must necessarily be led to the truth, at last. It is true that this condition is most imperfectly fulfilled in the *Pooh-pooh* argument. For here the unexpected, when it comes, comes with a bang. But then, on the other hand, until the fatal day arrives, this argument causes us to anticipate just what does happen and prevents us from anticipating a thousand things that do not happen. I engage a stateroom; I purchase a letter of credit for fifty thousand dollars, and I start off determined to have a good time. On the way down the bay, my wife says to me, "Aren't you afraid the house may be struck by lightning while we are gone?" *Pooh-pooh!* "But aren't you afraid there will be a war and Boston will be bombarded?" *Pooh-pooh!* "But aren't you afraid that when we are in the heart of Hungary or somewhere you will get the Asiatic plague, and I shall be left unable to speak the language?" *Pooh-pooh!* On the morning of the fourth day out there is a terrific explosion and I find myself floating about on the middle of the Atlantic with my letter of credit safe in my breast pocket. I say to myself, my *Pooh-pooh* argument broke down that time sure enough, but after all it made my mind easy about a number of possibilities that did not occur, and even about this one for three days. So I had better be content with my lot. This little parable is intended to illustrate how even the *Pooh-pooh* argument, the weakest of all sound induction, does satisfy the essential condition of saving me from surprises both positive and negative; that is from the happening of things not anticipated and the non-occurrence of imaginary disasters. — From a fragment c. 1902.

* Cf. *The Grammar of Science* (1892); *The Chances of Death* (1897).

have taught us about distributions of specific instances among general ones, and the consequent proper inferences in such cases, but I should have to state and expound the exact definitions of "real probability," "independent," "fair sample," "predesignate," etc. As it is, I will limit myself to a single needful explanation that, so far as I know, the reader could not find definitely stated in any of the books. It is that when we say that a certain ratio will have a certain value in "the long run," we refer to the *probability-limit* of an endless succession of fractional values; that is, to the only possible value from 0 to ∞, inclusive, about which the values of the endless succession will never cease to oscillate; so that, no matter what place in the succession you may choose, there will follow both values above the probability-limit and values below it; while if V be any *other* possible value from 0 to ∞, but *not* the probability-limit there will be some place in the succession beyond which all the values of the succession will agree, either in all being greater than V, or else in all being less.

759. The remaining kind of induction, which I shall call *Qualitative Induction*, is of more general utility than either of the others, while it is intermediate between them, alike in respect to security and to the scientific value of its conclusions. In both these respects it is well separated from each of the other kinds. It consists of those inductions which are neither founded upon experience in one mass, as Crude Induction is, nor upon a collection of numerable instances of equal evidential values, but upon a stream of experience in which the relative evidential values of different parts of it have to be estimated according to our sense of the impressions they make upon us.

Qualitative Induction consists in the investigator's first deducing from the retroductive hypothesis as great an evidential weight of genuine conditional predictions as he can conveniently undertake to make and to bring to the test, the condition under which he asserts them being that of the retroductive hypothesis having such degree and kind of truth as to assure their truth. In calling them "predictions," I do not mean that they need relate to future events but that they must antecede the investigator's knowledge of their truth, or at least that they must virtually antecede it. I will give an illus-

tration of such "virtual antecedence." Suppose that to avoid wasting a great deal of time upon a hypothesis which the first comparisons with the facts may show to be utterly worthless, an investigator of a certain conjecture draws up and resolves to follow a well-considered initial program for work upon the question, and that this consists mainly in working out and testing as many consequences of the hypothesis as he can work out by a certain mathematical method and can ascertain the truth or falsity of at a cost of not more than $100 for each. But suppose that among the half dozen predictions to which that method will carry him, there, quite unexpectedly, turns up one whose truth has long been known to him, though it is a surprise to him to find that it is deducible from the hypothesis under examination. What course does sound logic impose upon him under these circumstances? The answer is that he must reëxamine the process of retroduction that suggested the hypothesis; and if the fact that is now repredicted in any degree influenced that hypothesis, it has had its due effect, and must not be used again. But if not, will he then be free to use the prediction if he likes? Not at all: the validity of his Qualitative Induction will be found to depend upon his following a rational and decisive method; he has no more right, but rather less, to favor the inductive rejection of the retroductive suggestion, than to favor its inductive adoption; and he is bound, as a man who means to reason as honestly as the imperfections of his nature and training will permit, to admit the true prediction into his counsels. The predictions must *eventually* be so varied as to test every feature of the hypothesis; yet the interests of science command constant attention to economy, especially in the earlier inductive stages of research.

Having made his initial predictions the investigator proceeds to ascertain their truth or falsity; and then, having taken account of such subsidiary arguments as there may be, goes on to judge of the combined value of the evidence, and to decide whether the hypothesis should be regarded as proved, or as well on the way toward being proved, or as unworthy of further attention, or whether it ought to receive a definite modification in the light of the new experiments and be inductively reëxamined *ab ovo*, or whether finally, that while not true it probably presents some analogy to the truth, and that

the results of the induction may help to suggest a better hypothesis.

760. I will now state, with slight hints of argument, the conclusions which I have reached as to the warrant, or basis of validity, of the inferential processes in the three stages of inquiry. I have been actively studying this subject, for the sake of completely satisfying my own mind about it, for 50 or 51 years. To be sure, I have, some half dozen times during the half-century, let my mind lie fallow, as to this subject, during one or two dozens of months, hoping so to rid myself of any inveterate bad habits of thinking that I may insensibly have fallen into. I have six times published my views, in 1867,* 1868,† 1878,‡ 1882 [1883],§ 1892,¶ and 1902 [1901].|| The last of these publications, compared with my present brief abstract, shows that my last week of years has by no means been an idle one, and encourages me to hope that I may yet be able to detect errors and omissions in my views, even if others do not confer upon me the benefit of such amendments.

§2. MILL ON INDUCTION

761. In regard to the theory of the validity of Induction the great majority still follow the *System of Logic* set forth in 1843 by John Stuart Mill, who was certainly a clear thinker, and apparently a remarkably candid thinker, in spite of his long training in writing for one of the old "quarterlies," and his consequent unfortunate taste for and skill in controversy, which, combined with his having imbibed his father's sterilizing nominalism with his mother's milk, rendered him, for example, incapable of appreciating Whewell, whose acquaintance with the processes of thought of science was incomparably greater than his own. J. S. Mill's beautiful style, of truly French perfection, together with the bulk of the two volumes, prevent all but the keenest readers from perceiving that he unconsciously wavers between three (not to say four) incom-

* "On the Natural Classification of Arguments," ch. 2.
† "Some Consequences of Four Incapacities," vol. 5, bk. II, ch. 2.
‡ "Deduction, Induction, and Hypothesis," ch. 5.
§ "A Theory of Probable Inference," ch. 8.
¶ "The Doctrine of Necessity Examined," vol. 6, bk. I, ch. 2.
|| "Pearson's Grammar of Science," vol. 9.

patible theories of the validity of induction. The first (stated in Bk. III, Chap. 3, Sec. 1) is [that] the whole force of Induction is the same as that of a syllogism of which the major premiss is the same for all inductions, being a certain "Axiom of the uniformity of the course of nature" (so described in the table of "Contents"). This was substantially Whately's theory of 1826. The second theory (which seems to be usually uppermost in Mill's mind; especially in Bk. II, Chap. 3, Sec. 7 and in Bk. III, Chap. 4, Sec. 2), is that induction proceeds *as if* upon the principle that a predicate which throughout a more or less extensive experience has been uniformly found to be true of all the members of a given class that have been examined in this respect may, with little risk, be presumed to be true of every member of that class, without exception; and that while it is not necessary that the inductive reasoner should have this principle clearly in mind, the *logician*, whose business it partly is to explain why inductions turn out to be true, must recognize the fact that nature is sufficiently uniform to render that *quasi* principle true, and must recognize that [nothing] else renders induction a safe and justifiable procedure. This theory is little more than the old maxim that "we must judge of the future by the past," which Mill — into such unfairness can an inclination toward controversy betray even an eminently fair mind! — attacks as if it merely meant that future history will repeat past history, instead of what it has meant, that future experience must be presumed to resemble past experience under sufficiently similar conditions. The third theory (see Bk. III, Chap. 3, Sec. 3), is that nature as a whole is not absolutely uniform, variety being a far more prominent characteristic of it; and that such uniformity as there is, is "a mere tissue of partial regularities," each consisting in the fact that some classes of objects show a greater, and some a less, tendency to a resemblance of all their members in respect to certain lines of characters; and that whoever knows this "has solved the problem of Induction." This theory was original with Mill; and though it is not the sole, nor the main, support of induction, it certainly does bring a powerful additional support to many inductions. But it is curious that Mill should have chanced to say, whoever might be acquainted with this theory "knows more of the philosophy of logic than the wisest

of the ancients." For a quarter of a century later Gomperz* published so much as remained of the contents of a papyrus from Herculaneum, which was a defence of induction and a theory of its validity by the Epicurean Philodemus, under whose instruction Cicero studied; and the theory of Philodemus, like that of Mill, is that this kind of reasoning (the only valid reasoning in his opinion) derives its validity from the existence in nature of special uniformities. Only, the uniformities that attracted the attention of Philodemus, instead of characterizing certain classes, characterize certain characters, and consist in their having a special tendency to be present (or to be absent) throughout all the members of certain kinds of classes. In fact, still other types of uniformities may affect the strength of inductions.

A yet fourth theory of induction, that of Laplace, received, by implication, the assent of Mill; and since this theory is taught as correct in all the textbooks of the Doctrine of Chances, it behooves me, in adopting another, to state, with the utmost brevity wherein Laplace's theory is false and harmful. I shall also give my explanation of Mill's assenting to it.

762. If, upon any occasion, we were to devise a method of forming a numerous sample of any class, say the S's, which should be suitable for use in determining to a given degree of approximation what proportion of future experiences of S's would, in the long run, be found to have the character of being P, in case existing general conditions should undergo no alteration, then in case there were any definite reason to expect that, among S's coming to our attentive experience from any particular sub-class, say among the S's that belong to the sub-class of T's, a markedly different proportion would turn out to be P from the proportion among the S's that should not be T's, then that method of forming the sample, since we have supposed it to be "suitable" for showing the proportion of P's among all future experiences of S's, must needs insure that the proportion of S's that are T's should be nearly the same in the sample as it was destined to be among all the S's of our future experience; though, as I need not repeat again, this would be so only under the supposition of unchanged general circum-

* Theodor Gomperz, *Philodemi de ira liber* (1864); *Herkulanische Studien* (1865–6).

stances, and need not be more precisely true than would suffice to keep the errors of the concluded proportion of subsequently experienced S's that should be P within the intended limit of approximation. Moreover, should there be any serious reason to suspect that any identifiable S presenting itself for admission to the sample was so connected with any S already admitted as to have a special liability whether to being like or to being unlike that already admitted instance in respect to being or not being P (under the same limitation that is not to be repeated), then our "suitable" method would have to exclude that instance from the sample. And once again, should there happen to be any reason to suspect that an instance had attracted our attention owing to causes connected, whether directly or indirectly, with its being P, or to such causes as should be connected with its *not* being P, then our suitable method must exclude that instance. Furthermore, our suitable method must so operate that the sample shall contain a sufficient number of instances to give the intended degree of approximation. For instance, if it will suffice that the figure next following the decimal point in the decimal expression of the proportion among all the S's of such as are P should be exact, 9 instances may first be taken, and if these make the ratio less than 0.05 or greater than 0.95, they will suffice. If not, 14 more instances may be collected; and if the whole 23 make the ratio less than 0.15 or greater than 0.85, they will suffice. If not, then if 11 more instances being taken, the whole 34 make the ratio less than 0.25 or greater than 0.75, they will suffice. If not, add 7 more, and if the ratio appears as less than 0.35 or more than 0.65, the 41 will suffice. If not, take 4 more and if the ratio then appears as less than 0.45 or greater than 0.55, the 45 will suffice. If not, one more instance will in any case be enough. If the first two figures of the decimal fraction must be correct, a hundred times as many instances will be requisite.

763. Every person of common-sense must, upon reflexion, acknowledge, what is familiar to everybody habituated to inductive reasoning, that all the above precautions are requisite, except that the concluding rule need not have been so detailed, and that, if the instances were sufficiently multiplied, it would suffice that the other rules should not be too fre-

quently and grossly violated and that they should not prevailingly be violated in the same direction. Let all such diminutions from them be made, and it still remains true for sound reason, that *such an induction does not follow merely from the fact that* P *is true of such and such of the* S's *of a collection of* S's, *but that it is necessary to take account of the manner in which these* S's *were brought to the inquirer's attention.* This fixes a great gulf between Induction and Deduction. It is quite true that we may describe the general conditions of a valid quantitative induction, and may convince ourselves that if the sample be drawn strictly at random from among the S's and be made sufficiently numerous, then, the general conditions remaining unchanged, it necessarily follows that future experience, *under the same general conditions*, will on the average of an indefinite multitude of such inductions, bear out the Inductive conclusion. Still, this in no wise suffices to reduce the quantitative induction to any kind of induction [deduction?]. For even if I were to grant that the truth of the inductive conclusion would necessarily follow if the conditions of a fair sample were to be ideally fulfilled, which, for a reason that I will presently state, I find myself unable to do, still the person who really draws the inductive inference cannot possibly have any demonstrative evidence that those conditions are fulfilled even to the imperfect degree that is needful for an approximation to the true ratio. He knows, if you will, that [he] has made strenuous efforts to make his sample a fair one; but he cannot be quite sure that deep down in the caverns of his heart there may not lurk, unsuspected by him, a determination to force himself to believe in a certain value for the ratio, nor that this has not frustrated all his efforts to make the sample a fair one; and if he cannot be absolutely certain even of his own honesty, how can he so much as approach certainty as to the correctness of his concluded approximation to the ratio not having been destroyed by external circumstances? A theoretician — or rather, a papyrobite, a man whose vitality is that of sentences written down or imagined — may reply that that contingency is covered by proviso that general conditions remain sufficiently unchanged. But that is to overlook the principal end of inquiry, as regards human life. What is the chief end of man? *Answer:* To actualize ideas of the immortal, ceaselessly prolific kind.

To that end it is needful to get beliefs that the believer will take satisfaction in acting upon, not mere rules set down on paper, with lethal provisos attached to them. The inductive reasoner cannot possibly find any strictly demonstrative reasoning that could take the place of his induction, since every demonstrative is strictly limited to the field of that part of its copulate premiss that corresponds to the minor premiss of a syllogism; while to serve his purpose, that of forming a basis for conduct, it must transcend that limit in concluding future from past experience. Now every valid mathematical reasoning is demonstrative and is limited to an ideal state of things. The reasoning of the calculus of probabilities consists simply of demonstrations concerning "probabilities," which, in all useful applications of the calculus, are *real* probabilities, or ratios of frequency in the "long run" of experiences of designated species among experiences designated, or obviously designable, genera over those species; which real probabilities are ascertained by quantitative inductions from statistics laboriously collected and critically tabulated. But the phrase "the probability of an event," which is perpetually recurring in the treatises, and which is not free from objection, even when the real probability is meant (because it seems to refer to a singular experience considered by itself, and because it does not mention that *two* classes of experiences are essentially concerned), is used in various different senses, owing to the ambiguity of the word "probability"; and the writers of the mathematical treatises on the subject have not had sufficient power of logical analysis to found any useful theory upon it. . . .

764. Laplace maintains that it is possible to draw a necessary conclusion regarding the probability of a particular determination of an event based on not knowing anything at all about [it]; that is, based on nothing. When a man thinks himself to know nothing at all as to which of a number of alternatives is the truth, his mind can no more incline toward or against any one of them or any combination of them than a mathematical point can have an inclination toward any point of the compass. Suppose the question concerns the color of an object which we know has a high color, but are otherwise in a state of blank ignorance [about it]. Then, according to Laplace, if one were to draw two lines across a map of the spectrum, it

would be probable that the color did not match any part of the spectrum included between those lines; no matter how nearly they might include the whole spectrum. Laplace holds that for every man there is one law (and necessarily but one) of dissection of each continuum of alternatives so that all the parts shall seem to that man to be "*également possibles*" in a quantitative sense, antecedently to all information. But he presents not the slightest reason for thinking this to be so, and seems to admit that to different men different modes of dissection will seem to give alternatives that are *également possibles*. It is only by basing the theory of probability upon this doctrine, and thus rendering probability without interest except to a student of human eccentricities, that it is possible to assign any mathematical probability to an inductive conclusion. Much might be added in refutation of Laplace's position.

765. In the first edition of his *Logic*,* Mill presents arguments against Laplace's view; but in his third, without answering his former arguments, as far as I see, he abandons them, and thus assents to all that is necessary for calculating a necessary probability for the inductive conclusion, without any regard to the manner in which the instances have been collected.

766. I will now sketch one or more ways of refuting each of Mill's three professed theories of Induction.

To the first theory, that an Induction is equivalent to a syllogism whose major premiss is the axiom of the uniformity of nature, while its minor premiss states the observed facts about the instances, the conclusion being identical with that of the induction, each of the following objections is conclusive: *first*, that an induction, unlike a demonstration, does not rest solely upon the facts observed, but upon the manner in which those facts have been collected; *secondly*, that a syllogism infers its conclusion apodictically, while an induction does not; *thirdly*, that a syllogism enriches our knowledge of ideas, but not our information, which is what Kant meant in saying that it only explicates but does not amplify knowledge, while an induction does amplify our knowledge; *fourthly*, that the proposed syllogism would be fallacious, because its major premiss is vague, so that it could be fairly thrown into the form of a fallacy of

* Bk. III, ch. 18.

undistributed middle, since all we really know of the general uniformity of nature is that *some* pairs of phenomena (an apparently infinitesimal proportion of all pairs) are connected as logical antecedent and consequent; *fifthly*, because a sound syllogism must not conclude beyond the breadth, or logical extension of its minor premiss (when this is suitably stated), while to represent a true induction it must do so. There are other objections, fully as strong as these five; but it seems needless to mention them.

767. The second theory correctly describes the procedure of the mind in crude inductions, but in no others; and Mill's celebrated four methods (chiefly based on the *Novum Organum*), though they may be of some help to minds that need such aids, yet furnish nothing but crude inductions, after all. The principle of this theory also sufficiently explains how it is that we meet such frequent opportunities to draw crude inductions as we do. But the moment the attempt is made to apply this theory to *justifying*, or explaining the validity even of crude inductions (and it is still worse with other kinds of induction), it lays itself open to all the objections to the first method, including the five that were specified above. For this second theory, which is the point where Mill's vain attempt to make reasoning able to get along without generalization becomes the most futile, and verges closely upon overt absurdity, differs from the first merely in not allowing, as essential to induction, that it should have any of such force as it might derive from employing the uniformity of experience as a premiss. Now this point of difference cannot confer upon induction as explained by the second theory any validity that it would not have if it were explicable by the first theory.

768. The third theory presents two decided advantages. For it may remove entirely the vagueness of the general principle of uniformity; and in some cases makes the special uniformity predicate a probability, so as to render the refutation of the theory, on the ground that induction does not conclude apodictically, considerably more difficult in those cases. Moreover, this theory does correctly state a part of the argument for very many inductive conclusions. But this part of the argument is not inductive but deductive. For these special uniformities (such, for example, as that every chemical element

has the same combining weight, no matter from what mineral or from what part of the globe it has come), have only become known by induction, often only by elaborate investigations, and are not logical principles; so that they need to be stated as premisses when the argument is to be set forth in full. The special uniformities, when they become known, enable us to dispense with certain inductive inquiries that would otherwise be requisite. But they leave other inductions (such as that which led Mendeléeff to enunciate his periodic law), quite untouched, not explaining them in any sense.

769. The true guarantee of the validity of induction is that it is a method of reaching conclusions which, if it be persisted in long enough, will assuredly correct any error concerning future experience into which it may temporarily lead us. This it will do not by virtue of any deductive necessity (since it never uses all the facts of experience, even of the past), but because it is manifestly adequate, with the aid of retroduction and of deductions from retroductive suggestions, to discovering any *regularity* there may be among experiences, while *utter irregularity is not surpassed in regularity by any other relation of parts to whole*, and is thus readily discovered by induction to exist where it does exist, and the amount of departure therefrom to be mathematically determinable from observation where it is imperfect. The doctrine of chances, in all that part of it that is sound, is nothing but the science of the laws of irregularities. I do not deny that God's beneficence is in nothing more apparent than in how in the early days of science Man's attention was particularly drawn to phenomena easy to investigate and how Man has ever since been led on, as through a series of graduated exercises, to more and more difficult problems; but what I do say is that there is no possibility of a series of experiences so wanting in uniformity as to be beyond the reach of induction, provided there be sufficiently numerous instances of them, and provided the march of scientific intelligence be unchecked.

770. Quantitative induction approximates gradually, though in an irregular manner to the experiential truth for the long run. The antecedent probable error of it at any stage is calculable as well as the probable error of that probable error. Besides that, the probable error can be calculated from the

results, by a mixture of induction and theory. Any striking and important discrepancy between the antecedent and *a posteriori* probable errors may require investigation, since it suggests some error in the theoretical assumptions. But the fact which is here important is that Quantitative Induction always makes a gradual approach to the truth, though not a uniform approach.

771. Qualitative Induction is not so elastic. Usually either this kind of induction confirms the hypothesis or else the facts show that some alteration must be made in the hypothesis. But this modification may be a small detail.

772. Experiments* which I have conducted in great numbers and great elaboration have convinced me of the extremely important advantages of making use, in Qualitative Induction, of numbers in place of such adverbs of comparison of the intensity of feelings as, "slightly," "a little," "somewhat," "tolerably," "moderately," "considerably," "much," "greatly," "excessively," etc. It is not necessary to use the adverbs; but in some cases I have found it convenient to employ a few of them. What is necessary is to get certain feelings so fixed in one's mind that they can be exactly and severally reproduced in the imagination at any time, these feelings forming such a series of ten or so, beginning with the zero of intensity and running up to high intensities; and further being such that any one of them being contemplated by the investigator and compared with the next intenser in the series, the interval of intensity between them shall appear, to the contemplator's feeling, to be equal to the interval between any other one of the series and the member next intenser than it. It is certain that this can be done, since all sidereal astronomers since Ptolemy have practised this; and many psychologists beside me have done something similar for other feelings than that of luminosity. It has been demonstrated that a series of positive numbers, integer and fractional, expresses *in itself* nothing more than an order of succession. But this scale is made to express, besides, a feeling of a difference of feeling in one respect; and the experiments of many persons prove conclusively that people generally can form such a scale; and further that the scales of different persons are concordant to a pretty high degree. The next step

* See volumes 7 and 8.

has been executed by but few persons so far as I know; but the experiments of these few render it all but certain that all normal persons can do so with good accord. This consists in comparing a difference of feeling in one respect with a difference in a single other respect; such as luminosity and pressure-feelings, or the relative bitterness of two solutions of quassia and that of self-blame for two former actions. Such comparisons as these last are, to be sure, of no direct applicability so far as I am aware; but they are good exercises in that prescissive abstraction of intensity from its subject which is required for estimating the equality of two differences of intensity. Such estimations enable us to add and take the arithmetical mean of intensities referred to the same standard; and not only the practice of all photometricians, both astronomers and gas-examiners, but also very many thousands of experiments by me upon a wide variety of qualities of sensation, establishes, to my full satisfaction, the great utility of such applications of number in giving a control over qualitative inductions. I have not found multiplications of such numbers useful, for example, in establishing the laws of such comparisons as the relative photometric value of two lights of different colors, where I need not say that it is one thing to ask what intensity of a light A, of fixed hue and chroma but variable luminosity best matches a light B, that is altogether fixed, and quite another and independent question what photometric intensity of B, if this be made to vary, best matches an A of given fixed intensity. The meaning of the product of two differences of intensity which refer in general to different qualities is obvious enough: it is the number to be attached as a measure to a phenomenon which involves two feelings of the intensities indicated by the multiplicand and multiplier, these two feelings being [in] a certain fixed relation to one another in which they are as independent as possible. But there is no advantage in attaching any single measure to such a complex phenomenon, unless there are different ways of analyzing it, more or less similar to the different systems of coördinates in geometry. I mean that, for example, different horizontal areas are not only measured by the sum of the parallelogram into which [they] may be cut up, each parallelogram having its sides in the directions of ENE and N by W. Were that the sole method of measurement, nothing

would be gained by combining the linear measures in the two directions, but rather the reverse. But in fact we may measure the area by parallelograms in any other two dimensions; and the ratio between any two areas will be the same by any two such methods of measurement. Moreover, we may employ polar, in place of Cartesian, coördinates, and cut the area up into a circle and broken, concentric, and very thin rings. The area will be the sum of the areas, each of which will be $X \times Y$, where X is the difference of the two radii, while Y is the proportion of their sum, the proportion being that of the entire ring which forms a part of the area.

CHAPTER 10

NOTES ON AMPLIATIVE REASONING

§1. REASONING*

773. Reasoning is a process in which the reasoner is conscious that a judgment, the conclusion, is determined by other judgment or judgments, the premisses, according to a general habit of thought, which he may not be able precisely to formulate, but which he approves as conducive to true knowledge. By true knowledge he means, though he is not usually able to analyse his meaning, the ultimate knowledge in which he hopes that belief may ultimately rest, undisturbed by doubt, in regard to the particular subject to which his conclusion relates. Without this logical approval, the process, although it may be closely analogous to reasoning in other respects, lacks the essence of reasoning. Every reasoner, therefore, since he approves certain habits, and consequently methods, of reasoning, accepts a logical doctrine, called his *logica utens*. Reasoning does not begin until a judgment has been formed; for the antecedent cognitive operations are not subject to logical approval or disapproval, being subconscious, or not sufficiently near the surface of consciousness, and therefore uncontrollable. Reasoning, therefore, begins with premisses which are adopted as representing percepts, or generalizations of such percepts. All the reasoner's conclusions ought to refer solely to the percepts, or rather to propositions expressing facts of perception. But this is not to say that the general conceptions to which he attains have no value in themselves.

774. Reasoning is of three elementary kinds; but mixed reasonings are more common. These three kinds are *induction*, *deduction*, and *presumption* (for which the present writer proposes the name *abduction*).

775. *Induction* takes place when the reasoner already holds a theory more or less problematically (ranging from a pure interrogative apprehension to a strong leaning mixed with

* *Dictionary of Philosophy and Psychology*, vol. 2, pp. 426–28.

ever so little doubt); and having reflected that if that theory be true, then under certain conditions certain phenomena ought to appear (the stranger and less antecedently credible the better), proceeds to *experiment*, that is, to realize those conditions and watch for the predicted phenomena. Upon their appearance he accepts the theory with a modality which recognizes it provisionally as approximately true. The logical warrant for this is that this method persistently applied to the problem must in the long run produce a convergence (though irregular) to the truth; for the truth of a theory consists very largely in this, that every perceptual deduction from it is verified. It is of the essence of induction that the consequence of the theory should be drawn first in regard to the unknown, or virtually unknown, result of experiment; and that this should virtually be only ascertained afterward. For if we look over the phenomena to find agreements with the theory, it is a mere question of ingenuity and industry how many we shall find. Induction (at least, in its typical forms) contributes nothing to our knowledge except to tell us approximately how often, in the course of such experience as our experiments go towards constituting, a given sort of event occurs. It thus simply evaluates an objective probability. Its validity does not depend upon the uniformity of nature, or anything of that kind. The uniformity of nature may tend to give the probability evaluated an extremely great or small value; but even if nature were not uniform, induction would be sure to find it out, *so long as inductive reasoning could be performed at all.* Of course, a certain degree of special uniformity is requisite for that.

But all the above is at variance with the doctrines of almost all logicians; and, in particular, they commonly teach that the inductive conclusion approximates to the truth because of the uniformity of nature. They only contemplate as inductive reasoning cases in which, from finding that certain individuals of a class have certain characters, the reasoner concludes that every single individual of the class has the same character. According to the definition here given, that inference is not inductive, but is a mixture of deduction and presumption. Cf. Probable Inference [§4.] See also Scientific Method [vol. 7.]

776. *Presumption*, or, more precisely, *abduction* (which the

present writer believes to have been what Aristotle's twenty-fifth chapter of the second *Prior Analytics* imperfectly described under the name of ἀπαγωγή, until Apellicon substituted a single wrong word and thus disturbed the sense of the whole), furnishes the reasoner with the problematic theory which induction verifies. Upon finding himself confronted with a phenomenon unlike what he would have expected under the circumstances, he looks over its features and notices some remarkable character or relation among them, which he at once recognizes as being characteristic of some conception with which his mind is already stored, so that a theory is suggested which would *explain* (that is, render necessary) that which is surprising in the phenomena.

He therefore accepts that theory so far as to give it a high place in the list of theories of those phenomena which call for further examination. If this is all his conclusion amounts to, it may be asked: What need of reasoning was there? Is he not free to examine what theories he likes? The answer is that it is a question of economy. If he examines all the foolish theories he might imagine, he never will (short of a miracle) light upon the true one. Indeed, even with the most rational procedure, he never would do so, were there not an affinity between his ideas and nature's ways. However, if there be any attainable truth, as he hopes, it is plain that the only way in which it is to be attained is by trying the hypotheses which seem reasonable and which lead to such consequences as are observed.

777. Presumption is the only kind of reasoning which supplies new ideas, the only kind which is, in this sense, synthetic. Induction is justified as a method which must in the long run lead up to the truth, and that, by gradual modification of the actual conclusion. There is no such warrant for presumption. The hypothesis which it problematically concludes is frequently utterly wrong itself, and even the method need not ever lead to the truth; for it may be that the features of the phenomena which it aims to explain have no rational explanation at all. Its only justification is that its method is the only way in which there can be any hope of attaining a rational explanation. This doctrine agrees substantially with that of some logicians; but it is radically at variance with a common theory

and with a common practice. This prescribes that the reasoner should be guided by balancing probabilities, according to the doctrine of inverse probability. This depends upon knowing antecedent probabilities. If these antecedent probabilities were solid statistical facts, like those upon which the insurance business rests, the ordinary precepts and practice would be sound. But they are not and cannot, in the nature of things, be statistical facts. What is the antecedent probability that matter should be composed of atoms? Can we take statistics of a multitude of different universes? An objective probability is the ratio of frequency of a specific to a generic event *in the ordinary course of experience.* Of a fact *per se* it is absurd to speak of objective probability. All that is attainable are subjective probabilities, or likelihoods, which express nothing but the conformity of a new suggestion to our prepossessions; and these are the source of most of the errors into which man falls, and of all the worst of them. An instance of what the method of balancing likelihoods leads to is the "higher criticism" of ancient history, upon which the archaeologist's spade has inflicted so many wounds.

778. The third elementary way of reasoning is *deduction*, of which the warrant is that the facts presented in the premisses could not under any imaginable circumstances be true without involving the truth of the conclusion, which is therefore accepted with necessary modality. But though it be necessary in its modality, it does not by any means follow that the conclusion is certainly true. When we are reasoning about purely hypothetical states of things, as in mathematics, and can make it one of our hypotheses that what is true shall depend only on a certain kind of condition — so that, for example, what is true of equations written in black ink would certainly be equally true if they were written in red — we can be certain of our conclusions, *provided no blunders have been committed.* This is "demonstrative reasoning." Fallacies in pure mathematics have gone undetected for many centuries. It is to ideal states of things alone — or to real states of things as ideally conceived, always more or less departing from the reality — that deduction applies. The process is as follows, at least in many cases:

We form in the imagination some sort of diagrammatic,

that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination. If visual, it will either be geometrical, that is, such that familiar spatial relations stand for the relations asserted in the premisses, or it will be algebraical, where the relations are expressed by objects which are imagined to be subject to certain rules, whether conventional or experiential. This diagram, which has been constructed to represent intuitively or semi-intuitively the same relations which are abstractly expressed in the premisses, is then observed, and a hypothesis suggests itself that there is a certain relation between some of its parts — or perhaps this hypothesis had already been suggested. In order to test this, various experiments are made upon the diagram, which is changed in various ways. This is a proceeding extremely similar to induction, from which, however, it differs widely, in that it does not deal with a course of experience, but with whether or not a certain state of things can be imagined. Now, since it is part of the hypothesis that only a very limited kind of condition can affect the result, the necessary experimentation can be very quickly completed; and it is seen that the conclusion is compelled to be true by the conditions of the construction of the diagram. This is called "diagrammatic, or schematic, reasoning."

§2. VALIDITY*

779. The possession by an argumentation or inference of that sort of efficiency in leading to the truth, which it professes to have; it is also said to be "valid."

780. Every argument or inference professes to conform to a general method or type of reasoning, which method, it is held, has one kind of virtue or another in producing truth. In order to be valid the argument or inference must really pursue the method it professes to pursue, and furthermore, that method must have the kind of truth-producing virtue which it is supposed to have. For example, an induction may conform to the formula of induction; but it may be conceived, and often

* *Ibid.*, vol. 2, pp. 748–49, by Peirce and Mrs. C. Ladd-Franklin.

is conceived, that induction lends a probability to its conclusion. Now that is not the way in which induction leads to the truth. It lends no definite probability to its conclusion. It is nonsense to talk of the probability of a law, as if we could pick universes out of a grab-bag and find in what proportion of them the law held good. Therefore, such an induction is not valid; for it does not do what it professes to do, namely, to make its conclusion probable. But yet if it had only professed to do what induction does (namely, to commence a proceeding which must in the long run approximate to the truth), which is infinitely more to the purpose than what it professes, it would have been valid. Validity must not be confounded with *strength*. For an argument may be perfectly valid and yet excessively weak. I wish to know whether a given coin is so accurately made that it will turn up heads and tails in approximately equal proportions. I therefore pitch it five times and note the results, say three heads and two tails; and from this I conclude that the coin is approximately correct in its form. Now this is a valid induction; but it is contemptibly weak. All simple arguments about matters of fact are weak. The strength of an argument might be theoretically defined as the number of *independent* equal standard unit arguments upon the other side which would balance it. But since it is next to impossible to imagine independent arguments upon any question, or to compare them with accuracy, and since moreover the "other side" is a vague expression, this definition only serves to convey a rough idea of what is meant by the strength of an argument. It is doubtful whether the idea of strength can be made less vague. But we may say that an induction from more instances is, other things being equal, stronger than an induction from fewer instances. Of probable deductions the more probable conclusion is the stronger. In the case of hypotheses adopted presumptively on probation, one of the very elements of their strength lies in the absence of any other hypothesis; so that the above definition of strength cannot be applied, even in imagination, without imagining the strength of the presumption to be considerably reduced. Perhaps we might conceive the strength, or urgency, of a hypothesis as measured by the amount of wealth, in time, thought, money, etc., that we ought to have at our disposal before it

would be worth while to take up that hypothesis for examination. In that case it would be a quantity dependent upon many factors. Thus a strong instinctive inclination towards it must be allowed to be a favouring circumstance, and a disinclination an unfavourable one. Yet the fact that it would throw a great light upon many things, if it were established, would be in its favour; and the more surprising and unexpected it would be to find it true, the more light it would generally throw. The expense which the examination of it would involve must be one of the main factors of its urgency.

781. Returning to the matter of validity, an argument professing to be necessary is valid in case the premisses could not under any hypothesis, not involving contradiction, be true, without the conclusion being also true. If this is so in fact, while the argument fails to make it evident, it is a bad argument rhetorically, and yet is valid; for it absolutely leads to the truth if the premisses are true. It is thus possible for an argument to be valid and yet bad. Yet an argument ought not to be called bad because it does not elucidate steps with which readers may be assumed to be familiar. A probable deductive argument is valid, if the conclusions of precisely such arguments (from true premisses) would be true, in the long run, in a proportion of times equal to the probability which this argument assigns to its conclusion; for that is all that is pretended. Thus, an argument that out of a certain set of sixty throws of a pair of dice about to be thrown, about ten will probably be doublets, is rendered valid by the fact that if a great number of just such arguments were made, the immense majority of the conclusions would be true, and indeed ten would be indefinitely near the actual average number in the long run. The validity of induction is entirely different; for it is by no means certain that the conclusion actually drawn in any given case would turn out true in the majority of cases where precisely such a method was followed; but what is certain is that, in the majority of cases, the method would lead to *some* conclusion that was true, and that in the individual case in hand, if there is any error in the conclusion, that error will get corrected by simply persisting in the employment of the same method. The validity of an inductive argument consists, then, in the fact that it pursues a method which, if

duly persisted in, must, in the very nature of things, lead to a result indefinitely approximating to the truth in the long run. The validity of a presumptive adoption of a hypothesis for examination consists in this, that the hypothesis being such that its consequences are capable of being tested by experimentation, and being such that the observed facts would follow from it as necessary conclusions, that hypothesis is selected according to a method which must ultimately lead to the discovery of the truth, so far as the truth is capable of being discovered, with an indefinite approximation to accuracy.

§3. PROOF*

782. An argument which suffices to remove all real doubt from a mind that apprehends it.

It is either mathematical demonstration; a probable deduction of so high probability that no real doubt remains; or an inductive, i.e., experimental, proof. No presumption can amount to proof. Upon the nature of proof see Lange, *Logische Studien*, who maintains that deductive proof must be mathematical; that is, must depend upon observation of diagrammatic images or schemata. Mathematical proof is probably accomplished by appeal to experiment upon images or other signs, just as inductive proof appeals to outward experiment.

§4. PROBABLE INFERENCE†

783. Any inference which does not regard its own conclusion as being necessarily true (though the facts be as the premisses assert).

In such an inference the facts asserted in the premisses are regarded as constituting a sign of the fact stated in the conclusion in one or other of three senses, as follows: *i.e.*, that relation of the premissed facts to the concluded fact which is regarded as making the former a sign of the latter (1) may be such as could not exist until the conclusion was problematically recognized; this is inductive or experimental inference. Such a relation (2) may be altogether irrespective of whether the conclusion is recognized or not, yet such that it could not sub-

* *Ibid.*, vol. 2, p. 359.

† *Ibid.*, vol. 2, pp. 353–55.

sist if the concluded fact were not probable; this is probable deduction. Such a relation (3) may consist merely in the premissed facts having some character which may agree with, or be in some other relation to, a character which the concluded fact would possess if it existed; this is presumptive inference.

784. (1) The first case is that in which we begin by asking how often certain described conditions will, in the long run of experience, be followed by a result of a predesignate description; then proceeding to note the results as events of that kind present themselves in experience; and finally, when a considerable number of instances have been collected, inferring that the general character of the whole endless succession of similar events in the course of experience will be approximately of the character observed. For that endless series must have some character; and it would be absurd to say that experience has a character which is never manifested. But there is no other way in which the character of that series can manifest itself than while the endless series is still incomplete. Therefore, if the character manifested by the series up to a certain point is not that character which the entire series possesses, still, as the series goes on, it must eventually tend, however irregularly, towards becoming so; and all the rest of the reasoner's life will be a continuation of this inferential process. This inference does not depend upon any assumption that the series will be endless, or that the future will be like the past, or that nature is uniform, nor upon any material assumption whatever.

Logic imposes upon us two rules in performing this inference. The first is this: so far as in us lies, the conditions of the experience should remain the same. For we are reasoning exclusively from *experience*, that is, from the cognitions which the history of our lives *forces* upon us. So far as our will is allowed to interfere, it is not experience; so we must take pains that we do not, in taking the instances from which we are to reason, restrict the conditions or relax them from those to which the question referred. The second prescription of logic is that the conclusion be confined strictly to the question. If the instances examined are found to be remarkable in any other respect than that for which they were selected, we can draw no inference of the present kind from that. It would be merely an infinitely weaker inference of the third kind (below).

The present kind of inference derives its great force from the circumstance that the result is virtually predicted.

785. (2) The second kind of probable inference is, by the definition of it, necessary inference. But necessary inference may be applied to probability as its subject-matter; and it then becomes, under another aspect, probable inference. If of an endless series of possible experiences a definite proportion will present a certain character (which is the sort of fact called an objective probability), then it necessarily follows that, foreseen or not, approximately the same proportion of any finite portion of that series will present the same character, either as it is, or when it has been sufficiently extended. This is governed by precisely the same principle as the inductive inference, but applied in the reverse way. The same prescriptions of logic apply as before; but, owing to that being now inferred which was in the other case a premiss, and conversely, it is not here true that the relation of the facts laid down in the premisses to the fact stated in the conclusion, which makes the former significant of the latter, requires the recognition of the conclusion. This is probable deduction. It covers all the ordinary and legitimate applications of the mathematical doctrine of probability.

The legitimate results of the calculus of probability are of enormous importance, but others are unfortunately vitiated by confusing mere likelihood, or subjective probability, with the objective probability to which the theory ought to be restricted. An objective probability is the ratio in the long run of experience of the number of events which present the character of which the probability is predicated to the total number of events which fulfill certain conditions often not explicitly stated, which all the events considered fulfill. But the majority of mathematical treatises on probability follow Laplace in results to which a very unclear conception of probability led him. Laplace and other mathematicians, though they regard a probability as a ratio of two numbers, yet, instead of holding that it is the limiting ratio of occurrences of different kinds in the course of experience, hold that it is the ratio between numbers of "cases," or special suppositions, whose "possibilities" (a word not clearly distinguished, if at all, from "probabilities") are equal in the sense that we are

aware of no reason for inclining to one rather than to another. This is an error often appearing in the books under the head of "inverse probabilities."

786. (3) Probable inference of the third kind includes those cases in which the facts asserted in the premisses do not compel the truth of the fact concluded, and where the significant observations have not been suggested by the consideration of what the consequences of the conclusion would be, but have either suggested the conclusion or have been remarked during a search in the facts for features agreeable or conflicting with the conclusion. The whole argument then reduces itself to this, that the observed facts show that the truth is *similar* to the fact asserted in the conclusion. This may, of course, be reinforced by arguments of some other kind; but we should begin by considering the case in which it stands alone. As an example to fix ideas, suppose that I am reading a long anonymous poem. As I proceed, I meet with trait after trait which seems as if the poem were written by a woman. In what way do the premisses justify the acceptance of that conclusion, and in what sense? It does not necessarily, nor with any necessitated objective probability, follow from the premisses; nor must the method eventually lead to the truth. The only possible justifications which it might have would be that the acceptance of the conclusion or of the method might necessarily conduce, in the long run, to such attainment of truth as might be possible by any means, or else to the attainment of some other purpose. All these alternatives ought to be carefully examined by the logician in order that he may be assured that no mode of probable inference has been overlooked.

It appears that there is a mode of inference in which the conclusion is accepted as having some chance of being true, and as being at any rate put in such a form as to suggest experimentation by which the degree of its truth can be ascertained. The only method by which it can be proved that a method, without necessarily leading to the truth, has some tolerable chance of doing so, is evidently the empirical, or inductive, method. Hence, as induction is proved to be valid by necessary deduction, so this presumptive inference must be proved valid by induction from experience.

The presumptive conclusion is accepted only problemati-

cally, that is to say, as meriting an inductive examination. The principal rule of presumption is that its conclusion should be such that definite consequences can be plentifully deduced from it of a kind which can be checked by observation. Among the wealth of methods to which this kind of inference (perhaps by virtue of its experiential origin) gives birth, the best deserving of mention is that which always prefers the hypothesis which suggests an experiment whose different possible results appear to be, as nearly as possible, equally likely.

787. Among probable inferences of mixed character, there are many forms of great importance. The most interesting, perhaps, is the argument from Analogy, in which, from a few instances of objects agreeing in a few well-defined respects, inference is made that another object, known to agree with the others in all but one of those respects, agrees in that respect also.

§5. PREDESIGNATE*

788. (A word formed by Sir W. Hamilton by composition from Lat. *prae*, in front of, and *designatus*, marked out): (1) A term applied by Hamilton to verbal propositions whose quantity, as universal or particular, is expressed (*Lectures on Logic*, xiii).

789. (2) By C. S. Peirce applied to relations, characters, and objects which, in compliance with the principles of the theory of probability, are in probable reasonings specified in advance of, or, at least, quite independently of, any examination of the facts. See Probable Inference [785].

790. For example, the laws of England will, in the long run, cause the majority of English sovereigns to be males. In that sense it was unlikely that the successor of William IV would be a queen. But it would be absurd to say this after knowing that there was no heir to the crown so near as the Princess Victoria; and, in like manner, to say that it was not very unlikely that Queen Victoria's successor would be a queen was true enough as long as the character of her progeny was not known, or, if known was not taken account of, but false considering the number of her sons and grandsons. In such cases of deductive probable inference the necessity of the pre-

* *Ibid.*, vol. 2, pp. 324–25.

designation is too obvious to be overlooked. But in indirect statistical inferences, which are mere transformations of similar deductive consequences, and the validity of which, therefore, depends upon precisely the same conditions, the necessity of the predesignation is more often overlooked than remarked. Thus Macaulay, in his essay on the inductive philosophy, collects a number of instances of Irish whigs — which we may suppose constitute a random sample, as they ought, since they are to be used as the basis of an induction. By the exercise of ingenuity and patience, the writer succeeds in finding a character which they all possess, that of carrying middle names; whereupon he seems to think that an unobjectionable induction would be that all Irish whigs have middle names. But he has violated the rule, based on the theory of probabilities, that the character for which the samples are to be used as inductive instance must be specified independently of the result of that examination. Upon the same principle only those consequents of a hypothesis support the truth of the hypothesis which were predicted, or, at least, in no way influenced the character of the hypothesis. But this rule does not forbid the problematic acceptance of a hypothesis which has nothing to do with the theory of probability.

§6. PRESUMPTION*

791. In logic: a more or less reasonable hypothesis, supported, it may be, by circumstances amounting all but to proof, or, it may be, all but baseless.

Logical or philosophical presumption is non-deductive probable inference which involves a hypothesis. It might very advantageously replace hypothesis in the sense of something supposed to be true because of certain facts which it would account for. See Probable Inference [786].

* *Ibid.*, vol. 2, p. 337.

APPENDIX

MEMORANDA CONCERNING THE ARISTOTELIAN SYLLOGISM*

792. The Quantity of Propositions is the respect in which *Universal* and *Particular* Propositions differ. The Quality of Propositions is the respect in which *Affirmative* and *Negative* Propositions differ.

Names and Signs for Propositions.

Universal Affirmative:	**A**:	Any S is P.
Particular Affirmative:	**I**:	Some S is P.
Universal Negative:	**E**:	Any S is not P.
Particular Negative:	**O**:	Some S is not P.

Terms occupying the places of S and P in the above, are called the logical *Subject* and *Predicate.*

Relations of Propositions

793. In the following diagram, the different propositions are supposed to have the same logical Subject and Predicate. The lines connecting A with O, and E with I, are meant to indicate that these connected propositions contradict one another. The sign $\gtrdot$ has its broad end towards a proposition which implies another, and its point toward the proposition implied.

Rule, Case, and Result.

794. A syllogism in the first figure argues from a *Rule,* and the subsumption of a *Case,* to the *Result* of that rule in that case.

* Privately printed and "distributed at the Lowell Institute, Nov. 1866." Cf. Bk. III, ch. 2, Part II.

Rule: Any man is mortal,
Case: Napoleon III is a man;
Result: ∴Napoleon III is mortal.

The Rule must be universal; and the Case affirmative. And the subject of the Rule must be the predicate of the Case. The Result has the quality of the Rule and the quantity of the Case; and has for its subject the subject of the Case, and for its predicate the predicate of the Rule.

THE THREE FIGURES.

Figure 1.

Assertion of Rule, **A E**
Assertion of Case; **A I**
Assertion of Result. **E A O I**

Figure 2.

Assertion of Rule, **A E**
Denial of Result; **O I E A**
Denial of Case. **O E**

Figure 3.

Denial of Result, **I O A E**
Assertion of Case; **A I**
Denial of Rule. **O I**

The letters A, E, I, O, in the above diagram are so arranged that inferences can be made along the straight lines.

795. It is important to observe that the second and third figures are *apagogical*, that is, infer a thing to be false in order to avoid a false result which would follow from it. That which is thus reduced to an absurdity is a Case in the second figure, and a Rule in the third.

To *contrapose* two terms or propositions is to transpose them, and at the same time substitute for each its contradictory.

The second figure is derived from the first by the contraposition of the Case and Result, the third by the contraposition of the Rule and Result. The Rule and Case of the first figure cannot be contraposed, because they already occupy the same logical position, namely, that of a *premiss;* their contraposition in either of the other figures converts these figures into one another.

Let F, S, T denote syllogisms of the first, second, and third figures, respectively. And let s, t, f denote the processes of contraposition of the Case and Result, Rule and Result, and Rule and Case, respectively. Then

$$sF = S \qquad sS = F$$
$$tF = T \qquad tT = F$$
$$fS = T \qquad fT = S$$
$$s^2 = t^2 = f^2 = 1$$
$$f = st = ts \quad s = ft = tf \quad t = fs = sf$$

796. The following table exhibits all the moods of Aristotelian syllogism (varieties resulting from variations of the Quantity and Quality of the propositions). Enter at the top, the proposition asserting or denying the rule; enter at the side, the proposition asserting or denying the case; find in the body of the table the proposition asserting or denying the result. In the body of the table, propositions indicated by italics belong to the first figure, those by black letter to the second figure, and those by script to the third figure.

	I	A	E	O
E		𝔈	𝔄	
A	*I*	*A*	*E*	*O*
I	*A*	*I*	*O*	*E*
O		𝔒	𝔍	

Two moods of the third figure, namely, A A I and E A O, are omitted, for two reasons. The first is that they correspond

by contraposition to two moods in the first figure, A A I and E A O, never given by logicians, who, therefore, act inconsistently in admitting these. The second reason is, that, like those moods in the first figure, they are virtually enumerated already, if the change of a proposition from universal to particular be not an inference; but if it be, then, again like those moods of the first figure, the argument they embody may be analyzed into a syllogism and an inference from universal to particular.

797. The celebrated lines of William Shyreswood (?) are here given. The vowels of the first three syllables of each word indicate the three propositions of the syllogisms. He enumerates, along with the moods of the first figure, the Theophrastean moods (two of which we omit for the same reason that we do those two in the third figure):

Barbara: Celarent: Darii: Ferio: Baralipton:
Celantes: Dabitis: Fapesmo: Frisesomorum:
Cesare: Camestres: Festino: Baroco: Darapti:
Felapton: Disamis: Datisi: Bocardo: Ferison.

798. The diagram below shows the relations in which the second and third figures stand to the first. In order to understand the seven syllogistic formulas there set down, it is necessary to notice that propositions may be divided into four parts: first the *Any* or *Some*, second the Subject, third the *is* or *is not*, and fourth the Predicate. When a proposition admits of varieties in either of these parts, they are shown in the diagram by two words or letters, one above the other, as $\overset{\text{is}}{\text{is not}}$ in the rule of the first figure. Two independent variations may occur in one formula, and the variations of different parts are independent, but in the same part either the upper or lower line must always be read, in any one syllogism.

For example, the result in the first figure has four forms: any *or* some S is *or* is not P; but if *Some* has been read in the Case, *Some* must also be read in the Result. So, in the second figure, where a variation is possible in the quality of either premiss; but the same line of the third part of both propositions must be taken.

Figure 1.	*Figure* 2.	*Figure* 3.
Any **M** is / is not **P**	Any **M** is / is not **P**	Some / Any **S** is not / is **P**
Any / Some **S** is **M**	Some / Any **S** is not / is **P**	Any / Some **S** is **M**
Any / Some **S** is / is not **P**	Some / Any **S** is not **M**	Some **M** is not / is **P**
	Any not **P** / **P** is not **M**	Any Some **S** / **S** is not / is **P**
	Some / Any **S** is not **P** / **P** **M**	Some **M** is some **S** / **S**
	Some / Any **S** is not **M**	Some **M** is not / is **P**
	Any not **P** / **P** is not some **S** / **S**	Any **S** / some **S** is **M**
	Any **M** is not **P** / **P**	Some not **P** / **P** is **S** / some **S**
	Any **M** is not some **S** / **S**	Some not **P** / **P** is **M**

At the top of the diagram are given the formulæ of the first figure, and of the second and third, as derived from that of the first by contraposition of the propositions. Under the second and third figures, respectively, are given forms expressing the same arguments in the first figure. It is necessary to study carefully the manner in which this reduction to the first figure is effected.

799. It will be perceived that the arrangements of the terms in the three figures, as determined by the rules given in 794, are as follows: where the first letter of each pair indicates the subject of a proposition of the syllogism and the second its predicate:

	Figure 1.	*Figure* 2.	*Figure* 3.
First.	*B* *A*	*N* *M*	Σ Π
Second.	Γ *B*	Ξ *M*	Σ *P*
Third.	Γ *A*	Ξ *N*	*P* Π

800. It is plain that there are two ways of transposing the arrangements of the terms of the second and third figures without removing a term from the conclusion, so as to give the term the same arrangement as that of the first figure.

This is shown in the following table, where the columns headed *s* show the propositions whose terms are to be transposed, while those headed *m* show the propositions to be transposed.[1]

	Figure 2.		Figure 3.	
	s	*m*	*s*	*m*
Short Reduction	1st		2d	
Long Reduction	2d 3d	2d 1st	1st 3d	1st 2d

The effect of these transpositions is here shown.

SECOND FIGURE

	Short Reduction	*Long Reduction*
N M	*M N*	*M* Ξ
Ξ *M*	Ξ *M*	*N M*
Ξ *N*	Ξ *N*	*N* Ξ

THIRD FIGURE

	Short Reduction	*Long Reduction*
Σ Π	Σ Π	Σ *P*
Σ *P*	*P* Σ	Π Σ
P Π	*P* Π	Π *P*

It must next be shown how these transpositions may be made, in syllogisms themselves.

801. The short reduction of the second figure is shown in the second syllogism of that column of the large diagram headed Figure 2. The term not-P is introduced. This we define as that class to which some or any S belongs, when it is not P. Accordingly, for "some or any S is not P," we can substitute "some or any S is not-P," and this substitution is made

[1] "Ubicunque ponitur *s* significatur quod propositio . . . debet converti simpiciter . . . et ubicunque ponitur *m* debet fieri transpositio in præmissis." — *Petrus Hisp.*

in the reduction. But we cannot, on that account, substitute "any M is not-P" for "any M is not P." For "any M is not P," is substituted, in the reduction, "any P is not M;" and for "any M is P" is substituted "any not-P is not M." The only syllogisms by which these substitutions can be justified are these:

Any M is not P,	Any M is P,
Any P is P;	Any not-P is not P;
∴Any P is not M.	∴Any not-P is not M.

Both these are syllogisms in the second figure.

802. The short reduction of the third figure is shown in the second syllogism of the column headed Figure 3. The term some-S is introduced. The definition of this term is that it is that part of S which is or is not P when some S is or is not P. Hence, we can and do substitute "Any some-S is or is not P" for "Some S is or is not P," though we could not substitute "Any some-S is M" for "Some S is M." For "Some S is M" we substitute "Some M is S"; and for "Any S is M" we substitute "Some M is some-S"; and these substitutions are justified by inferences which can be expressed syllogistically only thus:

Any S is S,	Some S is some-S,
Some S is M;	Any S is M;
∴Some M is S.	∴Some M is some-S.

These are both syllogisms in the third figure.

803. The long reduction of the second syllogism is shown in the third syllogism of the column headed Figure 2. Here not-P is defined as that class to which any M belongs which is not P. Hence we can substitute "Any M is not-P" for "Any M is not P." Some-S is defined as in the short reduction of the third figure. Hence, for "Some S is or is not P," we can say "Any Some-S is or is not P." Then, we use the inferences which are expressed syllogistically, thus:

Any $^{\text{some S}}_{\text{S}}$ is not P,	Any $^{\text{some S}}_{\text{S}}$ is P,
Any P is P;	Any not P is not P;
∴Any P is not $^{\text{some S.}}_{\text{S.}}$	∴Any not P is not $^{\text{some S.}}_{\text{S.}}$

These are both syllogisms of the second figure. Substituting their conclusions for the second premiss of the second figure and transposing the premisses we obtain the premisses of the reduction. The conclusion of the reduction justifies that of the second figure, by inferences which are expressed syllogistically as follows:

Any M is not some-S,	Any M is not S,
Some S is some-S;	Any S is S;
∴ Some S is not M.	∴ Any S is not M.

Both these are syllogisms of the second figure.

804. The long reduction of the third figure is shown in the third syllogism of the column headed Figure 3. Some S is here defined as that part of S which is M when some S is M. Hence, for "Some S is M," we can substitute "Any Some-S is M." Not-P is defined as in the short reduction of the second figure. Hence, in place of "Some or any S is not P," we can put "Some or any S is not-P." In place of "Some S is P or not-P" we again substitute "Some P or not-P is S," and in place of "Any S is P or not-P" we substitute "Some P or not-P is some-S," in virtue of inferences which are expressed syllogistically thus:

Any S is S,	Some S is some-S,
Some S is $^{\text{not P;}}_{\text{P;}}$	Any S is $^{\text{not P;}}_{\text{P;}}$
∴ Some $^{\text{not P}}_{\text{P}}$ is S.	∴ Some $^{\text{not P}}_{\text{P}}$ is some S.

These are syllogisms of the third figure.

Then, the premisses being transposed, we have the premisses of the reduction. The conclusion of the reduction justifies that of the third figure by inferences which are expressed syllogistically thus:

Any not-P is P,	Any P is P,
Some not-P is M;	Some P is M;
∴ Some M is not-P.	∴ Some M is P.

These are syllogisms of the third figure.

805. The reduction called *reductio per impossibile* is nothing more than the repetition or inverse repetition of that con-

traposition by which the second and third figures have been obtained. It is not *ostensive* (that is, does not yield an argument with essentially the same premisses and conclusion as that of the argument thus to be reduced), but apagogical, that is, shows by the first figure that the contradiction of the conclusion of the second or third leads to the contradiction of one of the premisses. Contradiction arises from a difference in both quantity and quality. But it is to be observed that in the contraposition which gives the second figure, a change of the *quality* alone, and in that which gives the third figure, a change of the *quantity* alone of the contraposed propositions is sufficient. This shows that the two contrapositions are of essentially different kinds. The reductions *per impossibile* of the second and third figures respectively involve, therefore, these inferences:

Figure 2.

The Result follows from the Case;

∴ The negative of the Case follows from the negative of the Result.

Figure 3.

The Result follows from the Rule;

∴ The Rule changed in Quantity follows from the Result changed in Quantity.

These inferences may also be expressed thus:

Figure 2.

Whatever (S) is M is $\overset{\text{P;}}{\underset{\text{not P;}}{}}$

∴ Whatever (S) is $\overset{\text{not P}}{\text{P}}$ is not M.

Figure 3.

Any $\overset{\text{S}}{\underset{\text{some S}}{}}$ is whatever (P or not-P) M is;

∴ Some M is whatever (P or not P) $\overset{\text{some S}}{\text{S}}$ is.

And if we omit the limitations in parentheses, which do not alter the essential nature of the inferences, we have:

Figure 2.

Any M is $\overset{\text{P;}}{\underset{\text{not P;}}{}}$
$\therefore$ Any $\overset{\text{not P}}{\text{P}}$ is not M.

Figure 3.

Any $\overset{\text{S}}{\underset{\text{some S}}{}}$ is M;
$\therefore$ Some M is $\overset{\text{some S.}}{\text{S.}}$

We have seen above that the former of these can only be reduced to a syllogism in the second figure, and the latter only to one in the third figure.

806. The ostensive reductions of each figure are also apagogical reductions of the other. There are also the following:

Any not-M is $\overset{\text{not S,}}{\underset{\text{not some S,}}{}}$
Any $\overset{\text{not P}}{\text{P}}$ is not-M;
Any $\overset{\text{not P}}{\text{P}}$ is $\overset{\text{not S.}}{\underset{\text{not some S.}}{}}$

Any some-M is $\overset{\text{some S,}}{\text{S,}}$
Any $\overset{\text{some P}}{\underset{\text{some not P}}{}}$ is some-M;
Any $\overset{\text{some P}}{\underset{\text{some not P}}{}}$ is $\overset{\text{some S.}}{\text{S.}}$

But all these reductions involve the peculiar inferences we have found in those which have been examined, inasmuch as they are but complications of the latter.

807. Hence, it appears that no syllogism of the second or third figure can be reduced to the first, without taking for granted an inference which can only be expressed syllogistically in that figure from which it has been reduced. These inferences are not strictly syllogistic, because one of the propositions taken as a premiss in the syllogistic expression is a logical fact. But the fact that each can only be expressed in the second or third figure of syllogism, as the case may be, shows that those figures alone involve the respective principles of those inferences. Hence, it is proved that every figure involves the principle of the first figure, but the second and third figures contain other principles, besides.

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