These conditions are:

(i) $n_0 \in \Sigma$, $S(n_0) \in \Sigma$, $S(S(n_0)) \in \Sigma, \cdots$

where n_0 is a given element of Σ and S is the successor function.

(ii) $D \neq n_0$, $D \neq S(n_0)$, $D \neq S(S(n_0)) \cdots$,

(iii) $D \in \Sigma$.

Diagonal procedures are constructive and do not generate contradictions. Heterological procedures are nonconstructive and generate contradictions. A heterological procedure involves two nonconstructive steps each of which leads to a contradiction. Each of these contradiction-generating steps arises in the attempt to carry out an operation which is expressly excluded by the constructive conditions. The distinction between orders is not brought in *ad hoc* after the appearance of contradictions but is already present in the statement of constructive conditions. (*Received May 23, 1966.*)

PAUL WEINGARTNER. *Modal logics with two kinds of necessity and possibility.*

The system SS1 can be defined as the set of all sentences which are satisfied by the matrix Mat = $[T, F, N, A, L]$ where T(true) = $\{1, 2, 3\}$, F(false) = $\{4, 5, 6\}$. N, A and L are defined by the formulas:

$N(1) = 6$, $N(2) = 5$, $N(3) = 4$, $N(4) = 3$, $N(5) = 2$, $N(6) = 1$.

$A(1, 1) = A(1, 2) = A(1, 3) = A(1, 4) = A(1, 5) = A(1, 6) = A(2, 1) = A(3, 1) = A(4, 1) = A(5, 1) = A(6, 1) = A(2, 5) = A(3, 4) = A(4, 3) = A(5, 2) = 1.$

$A(2, 2) = A(2, 3) = A(2, 4) = A(2, 6) = A(3, 2) = A(4, 2) = A(6, 2) = 2.$

$A(3, 3) = A(3, 5) = A(3, 6) = A(5, 3) = A(6, 3) = 3.$

$A(4, 4) = A(4, 5) = A(4, 6) = A(5, 4) = A(6, 4) = 4.$

$A(5, 5) = A(5, 6) = A(6, 5) = 5.$ $A(6, 6) = 6.$

$L(1) = 1$, $L(2) = 3$, $L(3) = 6$, $L(4) = 6$, $L(5) = 6$, $L(6) = 6$.

INTERPRETATIONS. 1. *Modal.* N, A, L as negation, disjunction and necessity. p has the matrix 1 2 3 4 5 6. $Kpq = NANpNq$, $Cpq = ANpq$, $Epq = KCpqCqp$, $Mp = NLNp$. This gives a modal system with 14 modalities; the positive ones in implicational sequence: LLp, Lp, MLp, p, LMp, Mp, MMp. 3 kinds of implication—strong, strict and material—(corresponding to a threefold concept of consequence) are distinguished. Result: It is probable that all the statements of classical propositional calculus are at least strictly valid in SS1.

2. *Intuitionistic.* As in 1 except: for Np take $NLMp$ (=intuitionistic negation), for Cpq $CLMpLMq$, for Kpq $KLMpLMq$; Apq is not changed. Then SS1 turns into an intuitionistic system (the 11 axioms of Heyting are provable as strongly valid and the tertium non datur does not hold).

3. *Syllogistic.* As in 1 and with the "statistical interpretation" of modal operators by O. Becker. Then the same 15 syllogisms as in modern logic are strongly valid as modal statements in SS1.

4. *Tense-Logic.* As in 1 except: MLp as "for all t, it holds that p" and LMp as "for at least one t, it holds that p".

5. *Epistemic Logic.* As an extension of 1 (see my article in ***Notre Dame Journal of Formal Logic***, to appear). Consistency proof and decision procedure obtainable for all interpretations 1-5. (*Received June 27, 1966.*)

EDUARD WETTE. *Ein (relativ vollständiger) kanonischer Kalkül konstruktiver Arithmetik.*

Verf. gab einen Überblick über seinen ausführlichen Beitrag aufgrund einer Einladung der Ohio Academy of Science zu einem Symposium über ein Thema von Oppenheimer anläßlich des 60. Geburtstags von Gödel.

§1. Der 1960 auf dem Stanford-Kongreß bekanntgegebene Ansatz zur Aufzählung einer Kategorie von Definitionen durch Vorgängerinduktion, deren Rekursivität im Kalkül selbst mit Hilfe der Deduktion gewisser formaler Implikationen (1), (2) zu konstatieren ist, läßt sich inoffiziell in der Form (1), (2) $\Rightarrow$ (3) anschreiben. Die beim Einbau dieser Operation in ein formales System erwachsenden substitutionstechnischen Schwierigkeiten wurden unter Hinweis auf den kanonischen 96-Regel-Kalkül $\mathfrak{S}_1$ a. a. O. übergangen.

§2. *Übersetzung* der nach Takeuti formalisierten „niederen" Ordinalzahltheorie (ohne Ersetzungs- und Kardinalzahl-Axiomschema) in $\mathfrak{S}_1$. Dazu wird der 1957 beim Heyting-Colloquium vorgetragene und in Amsterdam veröffentlichte Niveauvergleich binärer Prädikate, für die Vorgängerinduktion deduzierbar ist, halbstabilisiert (ohne Definientes von Prädikaten anzutasten, die einen Ordinalzahlterm repräsentieren). Beschränkte und unbeschränkte transfinite Terme werden durch Prädikatformen und -schemata dargestellt, für die Induktionsformen bzw. relative Induktionen deduzierbar sind; halbformale Schlüsse scheiden aus. Der Sphärenkalkül $\mathfrak{S}_1$ macht insbesondere die Übersetzung des Axiomschemas der transfiniten Induktion ableitbar.

§3. *Umdeutung* der höheren Ordinalzahltheorie in $\mathfrak{S}_1$. Explizite Berechnung für Gödels P-Funktion laut Archiv-Notiz.[1] Transfinite Ausformung der quantorenfreien Kerne bei $\bigvee\!\bullet\!\bigwedge\!\bullet$-Umdeutung logischer Partikeln nach Gödel 1958.

Reduktion der Wf. des Bernays-Gödel-Systems auf die Konstitution eines (kanonischen) Kalküls, der „koerzible" Sphären und „obstruktive" Theorien innerhalb $\mathfrak{S}_1$ abkapselt.

§4. Prüfung des primären Wahrheitsgehalts von $\mathfrak{S}_1$ an der physischen Realität. Verf. plant ohne Übersetzung und Umdeutung eine (anläßlich des Kongresses in Warszawa-Kraków 1965 veranschaulichte) ad-hoc-Ansatz-freie induktive Konstruktion des „vollkommenen" geometrischen Kontinuums, die vermutlich die analytische Form der Grundvoraussetzungen der Physik erklären wird.

[1] Wie schon am 29.7.1966 (anläßlich der internationalen Tagung über 'Universelle Algebra' im Mathematischen Forschungsinstitut Oberwolfach/Lorenzenhof) vorgetragen, läßt sich $P(\gamma, 0)$ für transfinite γ noch auf eine einzige Formel zusammenziehen. (*Received April 29, 1966.*)

WOLFGANG YOURGRAU. *Gödel and physical theory.*

Implicit in every accepted physical theory is an undesirable completeness postulate: A physical theory should remain open and therefore incomplete. Such an open theory would be more realistic in the light of Gödel's results.

The semantic rules for a physical theory must allow for a theory of measurement and this involves arithmetic. According to Gödel, these rules will thus generate undecidable statements.

The correspondence between a formal system and its interpretation is relative and recursive. In the schema

$$I_0 \to I_1 \to \cdots \to I_n$$

I_0 is the set of statements in an uninterpreted formal system. I_k is uninterpreted with respect to I_{k+1} which is an interpretation of I_k. The relation is transitive, e.g. $I_0 \to I_3$. The converse, $I_{k+1} \to I_k$, amounts to abstraction. The relationship in the initial schema is irreflexive and strictly ordered. Thus, I_{k+1} is *semantically richer* and *logically stronger* than I_k.

The doctrine of partial isomorphism between the formal and the empirical realms should be replaced by the concept of partial homomorphism.

A number of pointers are submitted to stimulate further investigations:

1. The claim that the choice of an open or closed set of semantic rules is optional should be rejected in favor of adopting an open set. The open set is unable to describe its own semantics and hence allows for undecidable instances.

2. The set of rules should be incomplete and open at any arbitrary stage of development of a physical theory. This state permits ready adaptation of the theory to new data.

3. Pursuant to (1) and (2), there will always be statements about reality such that the theory is incapable of deciding whether they are or are not proper to the theory.

4. If the uninterpreted system F applies to the domain D, then F provides a set of inferential rules for the interpretation I, and I is intermediate between F and D. (*Received July 6, 1966.*)

The following papers were presented at the meeting, but no abstracts were submitted:

R. B. JENSEN. *Levels of the constructible hierarchy.*

I. LAKATOS. *On the significance of non-standard analysis for the historiography of mathematics.*