where $[x]^F_{x \in N}$ is a standard enumeration of all functionals Kleene primitive recursive in F. By iterating $\mathscr{J}$ over a simultaneously generated set of ordinal notations $O^{\mathscr{F}}$ we obtain a hierarchy of type $n + 1$ objects F_a, $a \in O^{\mathscr{F}}$.

THEOREM. *A functional of pure type $\leq n + 1$ is definable by a term of $T_0(\mathscr{F})$ if and only if it is primitive recursive in F_a for some $a \in O^{\mathscr{F}}$.*

For $\mathscr{F}$ of type 2 it follows from [WAINER, *A hierarchy for the* 1-*section of any type* 2 *object*, this JOURNAL, vol. 39 (1974), pp. 88–94] that the functions definable in $T_0(\mathscr{F})$ are precisely those recursive in $\mathscr{F}$. (This was first proved, more directly, by Feferman.) However, for $\mathscr{F}$ of type $n + 3$ it follows from [MOSCHOVAKIS, *Hyperanalytic predicates*, ***Transactions of the American Mathematical Society***, vol. 129 (1967)] that the functionals of type $\leq n + 2$ definable in $T_0(\mathscr{F})$ do not in general exhaust the $(n + 2)$-section of $\mathscr{F}$.

This immediately poses the problem whether infinite terms can be used to characterize full Kleene recursion in higher types. In connection with this Feferman has recently obtained a new characterization of higher-type recursion which although not formulated as a system of terms is nevertheless motivated by the idea of autonomous enumeration. We give a reformulation of his definition as a system of terms extending T_0.

PAUL WEINGARTNER, *Remarks on "intension" in the history of logic.*

(1) One of the most frequent traditional doctrines of extension and intension—held by Aristotle, the Middle Ages, Leibniz and De Morgan—can be stated as a simple thesis: To describe an entity x by the individuals of which it is predicated, by its elements or by its subsets is to describe it extensionally, while to describe x by its properties or by the supersets of which it is an element or a subclass is to describe it intensionally.

(2) The traditional concepts of extension and intension involved in the above thesis and in the inverse law can be defined in such a way that: (a) it can be introduced into the standard theory of classes; (b) the inverse ratio-principle is derivable as a theorem; (c) with the help of definitions of intensional inclusion, intersection and union a plausible theory of extension and intension which seems an adequate interpretation of traditional doctrines can be incorporated into the standard logic of classes.

In the history of logic this thesis is strongly connected with the so-called inverse ratio which is formulated by De Morgan thus: "The genus is said to be part of the species; but in another point of view the species is part of the genus. All animal is in man, notion in notion: all man is in animal, class in class" (*Syllabus of a Proposed System of Logic*).

(3) Suggested definitions: Class x is in class $y =_{\mathrm{df}} x \subseteq y$. Notion x is in notion $y =_{\mathrm{df}} x \sqsubseteq y$ (intensional inclusion); $x \sqsubseteq y =_{\mathrm{df}} (z)(x \subseteq z \rightarrow y \subseteq z)$, where $x \subseteq y$ is the standard class-inclusion. The inverse ratio then becomes the theorem $x \subseteq y \leftrightarrow y \sqsubseteq x$.

$$x \sqcap y = z \leftrightarrow (w)[(x \subseteq w \wedge y \subseteq w) \leftrightarrow z \subseteq w] \quad \text{(intensional intersection).}$$
$$x \sqcup y = z \leftrightarrow (w)[(x \subseteq w \vee y \subseteq w) \leftrightarrow z \subseteq w] \quad \text{(intensional union).}$$

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EDUARD WETTE, *A simplifying complication concerning my inconsistency-deduction within formalized arithmetic.*

I refer to my Varna report [5] on the fact that Heyting arithmetic deduces the consistency-critical proposition which arithmetizes 'I am unprovable'. Gödel's 1931 "undecidability" explanation was untrue, and Kolmogorov's 1932 interpretation of A → B as "solve the problem B, if a solution of the problem A is given" leads to a paradoxical solution of 'problem' within pure number theory. In order to save consistent calculations, you could abolish quantifiers and superimposed implications; the use of free variables and computable functions, of direct proofs and direct inductions will suffice, e.g. for explaining away quantum & relativity "principles" [4] through the optimal synthesis of infinitesimal distances and infinitesimal angles.

Remark concerning [4]. 2.8×10^{-13} cm is a diameter rather than a radius. The amplitude $\vartheta_0 = \frac{1}{2}$ corresponds to the half of maximal overfoldings in a pouch-wave. $\vartheta_0 = \frac{1}{2} = 1.4$ fm (1.3 fm) yields $\nu_0 = 2 = 0.2$ TTHz and $\hat{E} = 2 = \hbar\nu_0 = 22.4$ pJ (23.9 pJ).

§.1 ***Improvements of*** [2], [3], [5] ***versus*** [1], [2], [3]. Take $\bigvee \rightarrow 0 = 0$, $\mathrm{a} = \mathrm{b} \leftrightarrow {'\mathrm{a}} = {'\mathrm{b}}$, ${'\mathrm{a}} = 0 \vee 0 = {'\mathrm{a}} \rightarrow \bigwedge$, $+\mathrm{abc} \leftrightarrow (\mathrm{b} = 0 \wedge \mathrm{c} = \mathrm{a}) \vee \bigvee\mathrm{x} \bigvee\mathrm{y} \bigvee\mathrm{z}\, (\mathrm{a} = \mathrm{x} \wedge \mathrm{b} = {'\mathrm{y}} \wedge \mathrm{c} = {'\mathrm{z}} \wedge +\mathrm{xyz})$, $\cdot\mathrm{abc} \leftrightarrow (\mathrm{b} = 0 \wedge \mathrm{c} = 0) \vee \bigvee\mathrm{x} \bigvee\mathrm{y} \bigvee\mathrm{z}\, (\mathrm{a} = \mathrm{x} \wedge \mathrm{b} = {'\mathrm{y}} \wedge +\mathrm{zac} \wedge \cdot\mathrm{xyz})$ as arithmetical axioms. Change imperator, deductor, derivator from 4-, 4-, 9- into 2-, 3-, 4-place functors: $^{\Rightarrow}\mathrm{VU}$ instead of $^{\Rightarrow}\circ\, \mathrm{V}_2\mathrm{V}_1\mathrm{U}/^{\Rightarrow}\mathrm{x}\ \circ\circ\mathrm{U}$; an instruction for the last deductive step is computable in the number of each deduction; the new derivator receives the 5th, 6th, 9th, 4th place of the old derivator, all other places are computable after arithmetization.

$\mathfrak{R}_0$ consists of 75 + 70 (or 70 + 70) rules with $\leq 5 + 2$ premises on 5 or 4 pages; change of rule numbers in [2]: 12. 0 + 8, 5. 11 + 3, (6. 25 + 0). $\leq 8 + 6$ finitary variables per rule. First items concern $\mathfrak{Z}_0$ ($\ni \mathfrak{Z}$-deductions).

§2. **Last refinement.** Replace

$$\frac{^{\rightarrow}\mathrm{AB}\ \mathrm{U_A}(\mathrm{k}_1 \ldots)}{\mathrm{V_B}(\mathrm{k}_1 \ldots)} \text{ in [5, 1.1] by } \frac{^{\rightarrow}\mathrm{AB}}{^{\Rightarrow}\mathrm{U_A V_B}(\mathrm{k}_1 \ldots)};$$

extend $\mathrm{gl}^l(\mathrm{a})$ (: derivation *a* belongs to grading level *l*) [5, 3.1] to the new derivation step, even if its result $^{\Rightarrow}\mathrm{U_A V_B}(\mathrm{k}_1 \ldots)$ translates $^{\rightarrow}\mathrm{AB}$ on the production level $'l$ [5, 1.1]. The concept $\mathrm{ri}^l(\mathrm{a})$ in [5, 3.1] remains unchanged *formally*, but depends on the extended gl^{k}'s for $\mathrm{k} \leq l$.

§3. **Statistical intermezzo.** Each $\mathfrak{R}_0$-rule R has now $\leq 20 + 7$ occurrences of $\leq 8 + 3$ finitary variables; $\leq 1 + 4$ variables with $\leq 4 + 8$ occurrences altogether can be unfettered (: $\in \mathrm{sp_R}$, $\notin \mathrm{cc_R}$; $\mathrm{cc_R} \rightleftharpoons$ R-conclusion, $\mathrm{sp_R} \rightleftharpoons$ sum of R-premises). $\mathfrak{R}_0$ contains 8 + 89 such occurrences, plus 447 + 547 occurrences of fettered variables (: $\in \mathrm{sp_R}$, $\in \mathrm{cc_R}$): = 1091 communicators.

§4. **Complication.** The 23 transformations in the total elimination φ [5, 2., 2.1–4, 3.2] of $\ulcorner\mathfrak{Z}\urcorner$ base upon

$$\frac{\mathrm{U}\ ^{\Rightarrow}\mathrm{UV}}{\mathrm{V}}\text{-‘application’}$$

between $\ulcorner {^{\rightarrow}\mathrm{AB}} \urcorner$ and $\ulcorner \mathrm{V_B}(\mathrm{k}_1 \ldots) \urcorner$. The number of exponents' "floors" in explicitly used (primitive recursive) definientes, however, remains ≤ 13.

§5. **Simplification.** A more rapid $\mathfrak{Z}$-deduction of the arithmetized transformation lemma [5, 1.5, 3.3] is obtained.

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Lars Löfgren, *On Hempel's paradox of confirmation.*

Hempel's paradox concerns hypotheses of the form H: $\forall x(R(x) \Rightarrow B(x))$. If $R(x)$ stands for "x is a raven" and $B(x)$ for "x is black" we have the hypothesis that "all ravens are black." According to a criterion of Nicod, such an hypothesis is supported by every object c such that both $R(c)$ and $B(c)$, i.e., by every black raven in the actual example. H is logically equivalent with H_1: $\forall x(\neg B(x) \Rightarrow \neg R(x))$. Hence, by Nicod's criterion, any object d, such that both $\neg B(d)$