the global section operation for sheaves of structures over arbitrary spaces. (2) T is invariant under limits of structures i.e. under products and equalizers. (3) T is invariant under equalizers and T has models for finite, consistent presentations. (4) T has models for arbitrary consistent presentations. (5) T can be axiomatized by Horn sentences of the form $\forall\bar{x}(\beta(\bar{x}) \rightarrow \exists\bar{y}\alpha(\bar{x}, \bar{y}))$, where β, α are in $\bigwedge At(L)$, and for any such sentence in T there exists $\mu(x, y)$ in $\exists\bigwedge At(L)$ such that $T \vdash \forall\bar{x}(\beta(\bar{x}) \rightarrow \exists y(\alpha(\bar{x}, \bar{y}) \wedge \mu(\bar{x}, \bar{y})))$ and $T \vdash \forall\bar{x}\exists^{\leq 1}\bar{y}\mu(\bar{x}, \bar{y})$. (6) T has a definitorial extension T^* by predicates defined by formulas in $\exists\bigwedge At(L)$ such that T^* can be axiomatized by sentences of the form $\forall\bar{x}\,(\beta(\bar{x}) \rightarrow \exists^1\bar{y}\alpha(\bar{x}, \bar{y}))$ with β, α in $\bigwedge At(L^*)$ and the defining axioms for the new predicates.

Thus the set of sentences under limits resp. global sections is r.e. if L is countable. This and the equivalence of (1) and (5) solves a problem of Mansfield (this JOURNAL, vol. 42 (1977), pp. 241–250). In addition, the equivalence of (3) and (5) resp. (2) and (4) extends a result of Sabbagh resp. Fujiwara for universal theories. Moreover, there exists a theory T which is invariant under limits but which cannot be axiomatized by limit-invariant sentences (cf. Rabin's result for convex theories). Thus the side condition in (5) cannot be eliminated, since one has $T \neq T^*$ in this case.

The above theorm is based on the following results. (i) A characterization of the theories invariant under equalizers by a condition similar to (5). This improves a result of Bacsich. (ii) A characterization of the theories invariant under products and filtered colimits as those theories which satisfy (5) without the side condition. (iii) The observation that limits can be obtained as global sections of sheaves over partially ordered spaces.

The above results become simpler if one assumes invariance under homomorphic images. Further variants can be obtained using embeddings of structures instead of homomorphisms.

EDUARD W. WETTE. *The inconsistency of classical analysis.*

I exhibited a consistency-critical constructible set $\{{}^{\sim}\phi_m | m \in \omega_0\} \cup \{\{{}^{\sim}\phi_m \mid m \in \omega_0 \wedge p^c m = p^c m_0\} \mid m_0 \in \omega_0\}$ of solving sets with respect to $\bar{\bar{\mathfrak{S}}}_1$ [6, *0.2*] [5] [3] [4] [2]; its order $\dot{o}(< \omega_1)$ is a majorant of Λ, the limit of its element-orders. $\bar{\bar{\Lambda}} = \bar{\bar{\omega}}_0$ yields an $\bar{\bar{\mathfrak{S}}}_1$-induction up to ω_0 for $\ulcorner\bar{\bar{\mathfrak{S}}}_1$-consistency$\urcorner$, so that $\vdash_{\mathfrak{S}_1} 0 = 1$ [5, 4.] [3, 4.].

(1) [5, 1.]. $\bar{\bar{\mathfrak{S}}}_1$ was a subsystem of analysis translated into transfinite arithmetic: intuitionistic predicate calculus without equality; stability $\neg\neg\alpha \leq \beta \rightarrow \alpha \leq \beta$, transitivity $\alpha \leq \beta \wedge \beta \leq \gamma \rightarrow \alpha \leq \gamma$, [weak] connectivity $[\neg\neg]\,(\alpha \leq \beta \vee \beta \leq \alpha)$, infinity $\bigvee_{\zeta_0}\bigvee_{\zeta}(\zeta_0 < \zeta \wedge \bigwedge_{\xi(<\zeta)}\bigvee_{\eta(<\zeta)}\xi < \eta)$ $(:\alpha < \beta \leftrightharpoons \alpha \leq \beta \wedge \neg\beta \leq \alpha)$; transfinite induction $\bigwedge_{\xi(<\alpha)} A(\xi) \rightarrow A(\alpha) \Rightarrow A(\beta)$ [1, footnote 24, p. 186], transfinite recursion $(*)$ $\bigwedge_{\xi(<\alpha)}(\pi_1(\xi, \ldots) \leftrightarrow \pi_2(\xi, \ldots)) \rightarrow (A(\alpha, \ldots; \pi_1) \leftrightarrow A(\alpha, \ldots; \pi_2)) \Rightarrow \rightarrow_{\pi(\eta,\ldots)} A(\eta, \ldots; \pi)(\beta, \ldots) \leftrightarrow A(\beta, \ldots; \rightarrow_{\pi(\eta,\ldots)}A(\eta, \ldots, \pi))$; replacement $A(\beta_1, \alpha) \wedge A(\beta_2, \alpha) \rightarrow \beta_1 = \beta_2 \Rightarrow \bigwedge_{\xi_0}\bigvee_{\eta_0}\bigwedge_{\xi(<\xi_0)}\bigwedge_{\eta}(A(\eta, \xi) \rightarrow \eta < \eta_0)$.

$A \vee \neg A$ is *partially* eliminable: each premise of $(*)$ has to be deduced without $A \vee \neg A$ [1,*2.2*, pp. 159–164].

(2) [5, 1.]. $\bar{\bar{\mathfrak{S}}}_1$ can be reproduced in ZF-$\mathfrak{P}(\ldots \times \omega_1 \times \omega_1)$. Replacing constructible sets of ZF$^-$ (without power sets) by their ordinal labels transformed $\in$, $=$ into $(*)$-definable $\bar{\bar{\mathfrak{S}}}_1$-predicates $\dot{\in}$, $\doteq$. [Recursively defined] constructible classes can be $\bar{\bar{\mathfrak{S}}}_1$-represented through a predicate Q in $\dot{\in}$, $\doteq$, *if* the subset-property $(\dagger)$ $\bigwedge_{\xi}\bigvee_{\eta}\bigwedge_{\zeta}(\zeta \mathrel{\dot{\in}} \eta \leftrightarrow \zeta \mathrel{\dot{\in}} \xi \wedge Q(\zeta))$ is $\bar{\bar{\mathfrak{S}}}_1$-deduced.

(3) [5, 2.]. My transfinite extension of Gödel's Dialectica reinterpretation penetrated the recursor $\rightarrow_{\pi(\eta,\ldots)}$ with varying types $\tau(\beta, \ldots)$: [1, pp. 181–183, *Typen*, *Kerne*]. The solution of each reinterpreted $\bar{\bar{\mathfrak{S}}}_1$-deduction was produced from 6 functions by means of 3 + 2 functionals: ${}^1\phi^c x = 0$, ${}^2\phi^c x = x$, ${}^3\phi^c x = \omega_0$, ${}^4\phi^c\langle xy\rangle = x$, ${}^5\phi^c\langle xy\rangle = y$, ${}^6\phi^c\langle\dot{r}\dot{s}\rangle = \iota_{k(<2)}((\dot{r} \leq \dot{s} \wedge k = 0^+) \vee (\dot{s} < \dot{r} \wedge k = 0))$; ${}^{(1)}\phi_{\mathfrak{v}}{}^c\dot{r} = \Psi^{c\,(1)}\phi_{\mathfrak{v}} \upharpoonright \dot{r}$, ${}^{(2)}\phi_{\mathfrak{v}}{}^c\dot{r} = \mu_{\dot{w}} \wedge_{\dot{u}(\leq\dot{r})}\dot{w} > \varepsilon_{\dot{v}}(\Psi^c\langle\dot{v}\dot{u}\rangle = 0)$, ${}^{(3)}\phi_{\mathfrak{v}}{}^c x = \mu_{k(<\omega_0)}\,(Od^c\Psi^c\, k \leq Od\,{}^c\Psi^c k^+)$, ${}^{((1))}\phi_{\mathfrak{v}_1\mathfrak{v}_2}{}^c x = \langle\Psi_1{}^c x\Psi_2{}^c x\rangle$, ${}^{((2))}\phi_{\mathfrak{v}_1\mathfrak{v}_2}{}^c x = \Psi_2{}^c\Psi_1{}^c x$ [3, 3.] [2, p. 323, Hinweis]. $\bigwedge_y\bigwedge_z(y \in x \rightarrow y \subseteq x \wedge (z \in y \rightarrow z \subseteq y))$ expressed the property $\dot{x}$ ('x is an ordinal number'). Such constructible classes can be $\bar{\bar{\mathfrak{S}}}_1$-represented: $\vdash_{\mathfrak{S}_1}(\dagger)$.

(4) [5, 3.]. The closure ϕ of ${}^{1,\ldots,6}\,\phi$ under ${}^{(1,2,3)}\phi_{\mathfrak{v}}$, ${}^{((1,2))}\phi_{\mathfrak{v}_1\mathfrak{v}_2}$ was enumerated by ϕ_k with $\langle yxk\rangle \in \phi \leftrightarrow \phi_k{}^c x = y$, where $\nu(l^+) - \nu(l)$ functions ϕ_k, $\nu(l) \leq k < \nu(l^+)$, were built up from the $\nu(l)$ preceding ϕ_k, $0 \leq k < \nu(l)$; $\nu(0) = 0$, $\nu(l^+) = 6 + (3 + 2 \cdot \nu(l)) \cdot \nu(l)$. ϕ has a $\bar{\bar{\mathfrak{S}}}_1$-representation via subset labels or orders: Q_ϕ or $\varphi(\alpha) = \mu_\eta\bigwedge_\zeta(\zeta \mathrel{\dot{\in}} \eta \leftrightarrow \zeta \mathrel{\dot{\in}} \alpha \wedge Q_\phi(\zeta))$.

Restriction of arguments and values: $e = \bigcup_k e_k$, $e_k = \bigcup_l e_{kl}$, $e_{k0} = \{\bigcap\}$, $e_{k,l+1} = \bigcup_{m(<k)}\phi_m{}^{cc}e_k l$ $/ = e_{kl} \cup e_{kl} \times e_{kl} \cup \dot{e}_{k,l-1} \cup \bigcup_{m(<k\wedge C(m))} \bigcup \dot{u}_{(\leq\dot{e}_{k,l-1})} \{\phi_m \upharpoonright \dot{u}\} \leftarrow l$ even/odd; $\dot{e}_{kl} = \bigcup(e_{kl} \cap On)[\leftarrow 2|l]$, $C(m) \leftrightharpoons \bigvee_l 0 \leq m - \nu(l^+) < \nu(l^+) - \nu(l)$ singles out $\phi_m = {}^{(1)}\phi_{\phi_j}$, $\nu(l) \leq j = m - \nu(l^+) + \nu(l)$

$< \nu(l^+)$. $e, {}^{\sim}\phi = \phi \cap e \times e \times \omega_0$, $\dot{e} = \bigcup(e \cap On)$ are constructible *sets*; their orders ($< \omega_1$) have $\bar{\mathfrak{S}}_1$-descriptions on account of replacement (1).

Let δ be an $\dot{e}$-label; put $\gamma = \mu_{\xi(<\delta)} \neg \bigvee_{\eta(<\delta)} \eta = \psi_\delta(\xi)$, $\psi_\delta(\alpha) = \mu_{\eta(<\delta)} (\eta \mathrel{\dot{\in}} \delta \wedge \neg \bigvee_{\xi(<\alpha)} \eta \doteq \psi_\delta(\xi))$.

(5) [5, 3., 4.]. A set-theoretic representative $\tau_\in$ of the type form of ${}^{\sim}\phi_m = \phi_m \cap e \times e$ was the value $p^{\mathfrak{c}}m$ of a constructible set/function p; $\mathfrak{D}(p) = \omega_0$. 'Empty set' and 'emptiness type/function' are formally distinct: cf. [1, p. 182, line 23], put $\lambda_{\xi(<0)} \tau = \circ$, $\lambda_{\xi(<0)} f = \circ$, ${}^{\mathfrak{c}}c \circ = c$, ${}^{\mathfrak{c}}\circ a = \circ$, ${}_{\smile} \circ a = {}_{\smile} a \circ = a$ [5, note 5].

The solution ${}^{\smile\wedge}B(\beta, ..) \fallingdotseq D(\beta, ..; c^\tau, a^\upsilon)$ of each $\bar{\mathfrak{S}}_1$-deduced B (3) [1, pp. 182–183] was recapitulated in terms of an e-restricted pair $\langle {}^{\sim}\phi_{m_0} \{{}^{\sim}\phi_m \mid p^{\mathfrak{c}}m = \nu_\in\}\rangle$, where $p^{\mathfrak{c}}m_0 = \tau_\in$: $\vdash_{\bar{\mathfrak{S}}_1} \beta_1 < \gamma \wedge .. \wedge \beta_n < \gamma \rightarrow {}^{\smile\wedge}B(\beta_1, .., \beta_n)$. An $\in$-representative of the γ-bounded formula $\hat{a}\hat{\alpha}_1 .. \hat{\alpha}_n \, {}_\gamma D_B(\alpha_1, .., \alpha_n; {}_\gamma c, {}_\gamma a)$ was the value $d^{\mathfrak{c}\ulcorner} \vdash_{\bar{\mathfrak{S}}_1} B(\beta_1, .., \beta_n)^\urcorner$ of a constructible set/function d; $\mathfrak{D}(d) = \omega_0$. The orders ($< \omega_1$) of p, d have $\bar{\mathfrak{S}}_1$-descriptions $\leq \Lambda'$ ($> \Lambda$); $\vdash_{\bar{\mathfrak{S}}_1} \ulcorner \nvdash_{\bar{\mathfrak{S}}_1} \wedge \urcorner$ resulted from $d^{\mathfrak{c}}h = \{{}^{\sim}\phi_m \mid p^{\mathfrak{c}}m = \upsilon_\in(\ulcorner B \urcorner)\} \times (\dot{e})^{n(\ulcorner B \urcorner)}$ for several *formal* variables ranging over Gödel numbers $< \omega_0$ and ordinal labels $< \Lambda'$. Q.e.d.

(6) Bernays' oral comments: "Steine klopfen" / ".. buchstabieren .." (Oberwolfach, 1968 IV 03/1967 IV 07). For stronger inconsistencies [5, Introduction] and positive applications see e.g. [7] [10], [8] [9].

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MARIKO YASUGI. *A formalization of* Od(Ω).

We developed a generalized theory of ordinal diagrams, self-iterating schemes of ordinal diagrams Od(Ω) (announced in Wrocław, Poland, 1977). Very roughly, it is a theory of transfinite iteration along a wellordered set Ω of the theory of ordinal diagrams.

We have next formulated a system **S** of π_1^1-arithmetic with a kind of uniform inductive definitions (υ) in which a formal accessibility of Od(Ω) can be executed.

S is defined as follows. The basis is second-order arithmetic with the π_1^1-comprehension and the π_1^1-induction augmented by (υ): Let Ω be a primitive recursive (p.r.) set with a p.r. wellorder $<$. Let O and S be p.r. sets of pairs whose first entries are elements of Ω and which determine p.r. linear orders $<^r$ and $<_r$ (uniformly in r) respectively for the second entries of pairs belonging to O and S respectively. The requisites for S and O are as follows.

$\{a; (s, a) \in O\} \subset \{a; (r, a) \in O\}$ if $s < r$ for s and r elements of Ω;
$\{i; (r, i) \in S\} = \bigcup_{s<r} \{a; (s, a) \in O\}$;
$<^s = <^r \upharpoonright \{a; (s, a) \in O\}$ when $s < r$;
$<_r = \bigcup_{s<r} <^s$.