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Charles D. Brown, *Validity, soundness and implication.*

Validity transfers whatever 'lowest' degree of truth (a priori, analytic, empirical) the premisses of a sound argument have to the conclusion of the argument. Consider, now: If anything is either black or white then it is not red. For any conceivable thing, it is not the case that if it is red then it is yellow. Therefore, everything is red, and nothing is black or white or yellow. [The formulation and proof are straightforward.]

Because the argument is valid and the premisses are *at least* empirically true, the conclusion must be at least empirically true; but the conclusion is empirically false. A sound argument with a false conclusion is a paradox *par excellence.* Neither " $\sim (x)(Rx \supset Yx)$" nor "$(x)(Rx \supset \sim Yx)$" is an adequate (re)formulation of the second premiss, "$(x) \sim (Rx \supset Yx)$", which is the source of the logical problem since it yields the false "$(x)(Rx \cdot \sim Yx)$".

Implication may be defined as either

I. $p \supset q =_{\mathrm{df}} \sim p \vee q$, or

II. $p \supset q =_{\mathrm{df}} \sim(p \cdot \sim q)$.

The latter (which is equivalent to the former by DeM. and D.N.) yields

III. $(p \supset q) \equiv \sim(p \cdot \sim q)$,

and hence

V. $\sim(p \cdot \sim q) \supset (p \supset q)$, and

VI. $\sim(p \supset q) \supset (p \cdot \sim q)$.

Since VI has the false sentential interpretation, "$(x) \sim (Rx \supset Yx) \supset (x)(Rx \cdot \sim Yx)$", VI fails to satisfy standard conditions for logical correctness. Hence, logics employing VI (and thus V, III, II or I) cannot reflect valid human inference. Intuitionistic logic prohibits VI, V and III while permitting

VII. $\sim(p \supset q) \equiv \sim (\sim p \vee q)$

and hence

VIII. $\sim (p \supset q) \supset \sim(\sim p \vee q)$,

thus averting invalid inferences only by proscribing such 'intuitively clear' forms as "$\sim \sim p \supset p$" and "$\sim(\sim p \vee q) \supset (p \cdot \sim q)$".

Since neither classical nor intuitionistic logic reflects *both* valid *and* 'intuitively clear' inference, basic logic needs restructuring. Incidentally, abandoning VI, etc., eliminates the 'paradoxes' of material implication; but those 'paradoxes' are minor in comparison with the paradox of the sound argument with a false conclusion.

Eduard W. Wette, *The definition of an inconsistent primitive recursive function.*

§1. $\Omega_{4+}(l, e, k, a, d)$ [16] corresponds to $\varphi(l, a, d)$ [6, *3.2*] with $e = [a]_{1,1}$ (instead of $(a)_1$) [8, §§2, 4] and $k = 1 + k_a$ [6, *2.1*] : $[w]_j = \sum_{k=1..\lambda_j} a_{\lambda_0+..+\lambda_{j-1}+k} \cdot 19^{\lambda_j-k} = \ulcorner \mathrm{w}_j \urcorner$, $w = \ulcorner \mathrm{w} \urcorner = \ulcorner \mathrm{w}_0\mathrm{w}_1..\mathrm{w}_n \urcorner = \sum_{k=1..\lambda} a_k \cdot 19^{\lambda-k} = \sum_{j=0..n}[w]_j \cdot 19^{\lambda_{j+1}+\cdots+\lambda_n} = \langle \ulcorner \mathrm{w}_0 \urcorner, .., \ulcorner \mathrm{w}_n \urcorner \rangle$, $0 \leqq a_k < 19$, $\lambda = \|\mathrm{w}\| = \lambda_0 + .. + \lambda_n$ [11, §§2, 3]; $\lambda_0 + .. + \lambda_j = \mu_l\ (l \leq w + 1 \wedge \kappa(w, l) = n - j)$, $\kappa(w, 0) = 1$, $\kappa(w, l + 1) = \kappa(w, l) + n_{l+1} - 1$, n_k number of arguments of "a_k", $0 \leq n_k \leq 4$; $n_1 = n$, $\lambda_0 = 1$. I define Ω_{4+} by a 40-term sum [14] $\chi(e = 0 \wedge k = 0) \cdot \sum_{n=1..13} \omega_{13.n}\ (\Omega_*; l, a, d) + \chi(e > 0 \wedge k = 0) \cdot \sum_{n=0..25} \omega_{6.n}\ (\Omega_*; l, e, a, d) + \chi(e > 0 \wedge k > 0) \cdot \omega_{6.-}(\Omega_*; l, e, k, a, d)$, where $\omega_{13.n}$, $\omega_{6.n}$, $\omega_{6.-}$ are abbreviations for $\chi(cd_{13.n}(a, d)) \cdot \tau_{13.n}(\Omega_*; l, a, d)$, $\chi(cd_{6.n}(e, a, d)) \cdot \tau_{6.n}(\Omega_*; l, e, a, d)$, $\chi(cd_i(e, a, d)) \cdot \sum_{n=0,1} \chi(1 - (2 \dot{-} k) = n) \cdot \tau_{in}(\Omega_*; l, e, k, a, d)$.

§2. The conditions $cd_{13.n}$, $cd_{6.n/i}$ express $\mathrm{gl}_0(a) \vee \mathrm{gl}_0(d) \leq l \wedge dc^{\leq}_{13.n}(a) \wedge ps(a, d)$, $\mathrm{gl}_0(e, a) \vee \mathrm{gl}_0(e, d) \leq l \wedge dc^{\leq}_{6.n/i}(e) \wedge rs(e, a) \wedge ps(a, d)$; dc_i indicates $\ulcorner$induction step$\urcorner$, and $dc^{\leq}_{13.n/6.n}$ decipher the (first) $\mathfrak{R}_0$-rule $13.n/6.n$ [7]([6][4][8, §1]) applied in the last step of a $\ulcorner$derivation$\urcorner$ / $\ulcorner$deduction$\urcorner$; $rs(e, a) \leftrightarrows e = [a]_{1,1} \wedge dc_{13.10}([a]_1) \wedge dc_{13.12}(a)$, $ps(a, d) \leftrightarrows dc_{13.11,12}(\{d, [d]_1, [d]_{1,1}, .., a\})$, gl_0

minimal grading level (absolute / relating to e)—d exempts 6.10 from "+1" [12, §1], 'implication level' can stand for 'production level'. $\tau_{13.n/6.n/in}$ transform d into a *ri*-⌜derivation⌝ [12, §1] (: without 13.11—13, except for "inessential" 13.12-applications immediately after 13.10 for 6.16 with $\mathrm{w} \equiv \vee$) with the same ⌜result⌝ $[d]_3$. The deductively eliminable rule 6.0 substitutes k for x in $A(x) \to B(x)$.

§3. A retranslation τ_{dD} of ⌜U_A-derivation⌝ into ⌜$(\bigvee \to A)$-deduction⌝ eliminates those parameters d in 6.9-(exportation-) ⌜solutions⌝ [6, *2.3*] that anticipated an additional 13.12-application

$$\frac{U \Rightarrow V \quad ?U}{V}.$$

Concerning 6.10-(importation-) ⌜solutions⌝, m 13.12-applications

$$\frac{V_{1..10} \Rightarrow V_{1..11} \quad !V_{1..10}}{V_{1..11}}$$

$(0 \leq \|1\ ..\ 1\| < m)$ starting from B in $C \wedge A \to B$ refer back to $m + 1$ 13.12-applications starting from $A \to B$ in $C \to (A \to B)$: $\tau_{6.10}$ equals

$$\Omega_{3+}(l, [e]_2, b, \rho(a, d, el(s([e]_3) - 1, \varphi^{\cdot *}(s([e]_{2,3,2}), b, [f]_2)))),$$

where $b = \varphi^*([e]_2, [f]_1, q_8([a]_1))$, $f = \boldsymbol{el}(s([e]_{3,1}), \Omega_2(l, [a]_2))$, and $\Omega_{3+}(l, e, a, d) = \Omega_{4+}(l, e, 0, a, d)$, $\Omega_2(l, a) = \Omega_{3+}(l, 0, a, a)$; for s, el see [6, *2.2*], substituting $\check{s}$ for s^- in el yields $\boldsymbol{el}$; $\varphi^*(e, a^*, j) = \langle \ulcorner\vdash\urcorner, \ulcorner\vdash\urcorner, e, \ulcorner\circ\urcorner, p([e]_3, j), j\rangle, \boldsymbol{el}(s([e]_3) - 1, a^*), [p([e]_3, j)]_2, \ulcorner\circ\urcorner\rangle$, $p(b, j) = \chi(R_{3.}(b)) \cdot r_{5.}(r_{11.}(b), r'_{7.1.}(b), j)$ [6, *2.1*, *1.*], $\varphi^{\cdot *}(l, a, b^*) = \ulcorner\vdash\urcorner, \boldsymbol{el}(l, a), \boldsymbol{el}(l - 1, b^*), [\boldsymbol{el}(l, a)]_{3,2}, \ulcorner\circ\urcorner\rangle$; $\rho(a, d, b)$ is ⌜the result of replacing "a" in "d" by "b"⌝ with $[b]_3 = [a]_3$. (The q's specify/permute variable-free ⌜substitutes⌝ in ⌜tuples⌝ [6, *2.2*, *2.1*].) Noninterlaced nestings [13] occur in the argument-places a, d of Ω_*.

§4. $\Omega(a) = \Omega_2(\mathrm{gl}_0(a), a)$ is idempotent: $\Omega\Omega = \Omega$. The Ω-definition (via Ω_{4+}) has < 4000 symbols "over" some trivially definable auxiliary functions. If $a_\wedge$ ($\ll 1$ million-illion (i.e. n^n for $n = 10^6$, [13]) $< 1072198!$, [17]) denotes the "*unde*vicesimal" number of the $1 = 0$-derivation [6, *1.5*], then $\vdash \chi([a_\wedge]_3 = \ulcorner {=}'00 \urcorner) = 1$ / 'right', $\vdash \chi([\Omega(a_\wedge)]_3 = \ulcorner {=}'00 \urcorner) = 0$ / 'wrong', $\vdash [\Omega(a_\wedge)]_3 = [a_\wedge]_3$ evaluate 3 variable-free terms in 1-place primitive recursive functions; $\chi(b = c) = \overline{\mathrm{sg}}(|b - c|)$ computes different values for the "same" argument $b_1 = b_2$ (in different representations).

§5. Such computation trees are *theoretically* derivable within a 22-rule calculus $\mathfrak{R}$ [10]. 'Triangle excess' $\triangledown$ (instead of 'square excess' $\diamond$) economizes the composition of special 1-place functions [9][15] : 'triangle side' $\triangle(n) = [(\sqrt{8n + 1} - 1)/2]$ and ${}^\times\mathrm{f}_1\mathrm{f}_2(n) = \mathrm{f}_1(n) \cdot \mathrm{f}_2(n)$ contain $\|\triangle\| = 80 = 26 + 7 + 5 + 27 + 15$, $\|{}^\times\mathrm{f}_1\mathrm{f}_2\| = 43 + 11 + 10 + 49 + 26 + 3 \cdot (\|\mathrm{f}_1\| + \|\mathrm{f}_2\|)$ occurrences of $|\triangledown^{+c.}$; in $|\diamond^{+c.}$, $[\sqrt{n}]$ and $^\times$ require $144 + 20 + 22 + 141 + 82 = 409$, $200 + 28 + 30 + 205 + 110 + 4 \cdot (\|\mathrm{f}_1\| + \|\mathrm{f}_2\|)$ symbols. ($\mathfrak{K}_0$-derivation of "$k \cdot l$" via 12.3—8 [7] needs $\|{}^\times(k, l)\| = 8 + 7l + (k + l)(k + l + 15)/2 + 9l \cdot k(k + 1)/2 + k^2 \cdot l(l + 1/2)(l + 1)/3$ symbols; $\|{}^\times(j, j)\| \simeq j^5/3$.)

$$(k, l) = \binom{k + l + 1}{2} + k \ (= n)$$

and $\triangledown(n) = k$, $\triangle(n) = k + l$ guarantee an elegant / "uninterrupted" parameter-elimination.

§6. **§§1—5** settle [1, Problem 1-4, p.320][2][3]. Problem 5 l.c., a "positive" application, is halfway solved as well: [5][12, §§2, 2>].

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