

We introduce the notion of monadic second order definability (based on ideas of Büchi [3]) and use Theorem 1 to get the common characterization of G.t.f. for propositional calculi, finite automata and procedure-free programs (Theorems 3, 4 and 5). The similar problem for G.t.f. for predicate calculi is open.

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P. D. WELCH, Σ_3^1 wellfounded relations and the core model.

Martin proved the following results on Σ_3^1 sets of reals (identified with ${}^\omega\omega = N$) (in $ZF + DC + \forall a \in N (a^* \text{ exists})$; the cardinals in brackets refer to the result assuming full AC).

1) Every Σ_3^1 wellfounded relation $\subseteq N^2$ has length $< \aleph_{\omega+1}[\aleph_3]$ (and hence $\delta_3^1 = \sup(\text{lengths of } \mathcal{A}_3^1 \text{ prewellorderings of } N) \leq \aleph_{\omega+1}[\aleph_3]$).

2) Every Σ_3^1 set is the union of $\aleph_{\omega+1}$ sets in $B_{\aleph_{\omega+1}}$ (here B_κ = smallest Boolean algebra containing the closed sets and closed under unions of lengths $< \kappa$).

(AC) Every Σ_3^1 set is the union of $\aleph_2$ Borel sets.

If one assumes in addition $\neg 0^+$ (or indeed $\exists b \in N (\neg b^+)$) we obtain (without AC):

1.) The lengths above are $< \aleph_2; \delta_3^1 \leq \aleph_2$.

2.) Every Σ_3^1 set is the union of $\aleph_1$ sets each of which is the union of $\aleph_1$ Borel sets.

And so

(AC) Every Σ_3^1 set is the union of $\aleph_1$ Borel sets.

These figures depend completely on the computation of u_ω (the ω th uniform indiscernible), where $C = \langle u_i \mid i \in On \rangle = \bigcap_{a \in N} I^a$ (I^a = class of Silver indiscernibles for $L(a)$), since [TS] Martin showed that if $\forall a (a^* \text{ exists})$ then every Σ_3^1 set admits a $(u_\omega)^\omega$ -scale. The bounds in 1)–2') are all obtained by computing u_ω .

Assuming all reals have sharps and $\neg 0^+$, we may use Jensen's Σ_3^1 -absoluteness theorem for K , the core model (see [CM]). Using a Paris "patterns of indiscernibles" type result for mice we can compute that $u_\omega < \tau^*$, where τ^* = the height of the first admissible set that contains K_τ (where $\tau = \aleph_1^i$). Thus $\tau^* < (\tau^*)^K \leq \aleph_2$.

Indeed, defining $C^\tau = \langle u_i^i \mid i \in On \rangle = \bigcap_{a \in \mathcal{P}^{< \tau(t)}} I^a$ inside K , we obtain $\forall i \in On \ u_i = u_i^i$.

Thus (if all reals have sharps) complicated Σ_3^1 sets imply inner models with measures.

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EDUARD W. WETTE, 1977 *Wrocław abstract renewed: Control for the exhibition of inconsistent numbers*.

For the complete title cf. XLIV, p. 476, [12]. The abstract was written on April 30, 1977 (=C.F. Gauss* + 200a); it summarized my ICS/acm i.a. type-script (16 pp., 1976 IX 15). The renovation of §1 can now refer to my 1983 Salzburg contribution (7 ICLMPS, Section 1), entitled *The consistency-critical primitive recursive function and the inconsistent variable-free elementary term within formalized Peano arithmetic*. Cf. also Dümmlerbuch 4711, Bonn 1982 & 1983.

§1. Formal end of mathematics, logic: power-iteration discomputed.

The practice of primitive recursive definition with < 300 subfunctions or of binary computation with < 2300 variable-free polynomial terms annuls all the "semantic" claims of proof theory and, moreover, the truth of "proven" decision procedures. Gödel's comment on my talk (January 24, 1975): "Ende der Wissenschaft", no other words from Princeton to Washington D.C.! His main question "was machen wir jetzt mit den Permutationen? die sind für Chemie noch wichtig", when he telephoned me before, had been

answered from my geometro-static approach to the totality of 'motion' by "the" seamless 'warp & woof' on a *maximal* surface.

Counter-corollaries. The intrafinite Boolean operations $\cdot \times \cdot, \cdot + 1 \bmod 2, \Pi_{m < W} \delta_m$, where $\delta_m \in \{1, 0\}$, are inconsistent, if $W \geq \psi_3(5, 10^6)$; cf. Herbrand Symposium. The use of a language and of a free property in space & time confines us to $(\log a_A)^{1/4} \approx 56$ consistent steps for each dimension. ERGO: keep silence and save more degrees of freedom; re-educate your mind after a "refuting" walk.

Elementary paradoxes apply $< 0.1/10$ megabit of information within (i) the nonelementary function $\Omega(n) < \psi_3(n, 19^3) < \psi(4, 2n + 1)$, and its induction-property $rs(\Omega; a) \Leftrightarrow ri(\Omega(a)) \wedge [\Omega(a)]_3 = [a]_3 \wedge gl(\Omega(a)) \leq gl(a) - id(a) - 2 \cdot md(a)$, a polynomial predicate in Ω (4 depth-concepts restored minimal stilts!), (ii) the inconsistent number a_A (with $a_A^2 < {}^2 1000000$) whose main part encodes $\vdash_3 R_{1,3}(\xi) \rightarrow rs(\Omega; \xi)$. The < 2300 key-terms which compute a_A from $\ulcorner \text{predicate} \urcorner$ & $\ulcorner \text{proof} \urcorner$ -units, and the direct access to $\ulcorner \text{subwords} \urcorner$ guarantee a rapid control of the induction-free specialization $rs(\Omega; a_A)$ and of its consequence $49838 = [\Omega(a_A)]_3 \neq 49838$. A few minutes of data processing can verify such logic-free paradoxes; explicit computation of $\ulcorner \Omega \urcorner$ or a_A is a practicable problem, but the outputs (in few hours) cannot be controlled with the same certitude as a storage of 2300 key-addresses.

Note that W absorbs, e.g., a 'propositional' description for the number of the derivative solution to m^2 -computation with $\cong m^3/3$ symbols, etc., if $m \leq m_0 = {}^3 1000000$, as within my former inconsistency-computation; $W^2 \approx W$, whereas m_0^2 contains one more bit of information than m_0 ; $\Pi_{m < W} \delta_m$ eliminates v -computed m -selections $\Pi_{m: v(m) \leq m_0} \delta_{v(m)}$ (vs. XLII, p. 477, §1). *Logical reflection is untrue.*

"Neo-Sandrechnung" stated, 2186 years after Archimedes, that reckoning is consistent, IFF one observes (A) numbers are restricted to their numerical meaning, i.e. *no language, no meta-code*, & (B) all intermediate results have an explicit decimal value, i.e. *no implicit term without concretely accessible evaluation*.

W. ZADROŻNY, *Partial reflection.*

We consider the following problem: A device M is scanning some process A and produces information about A : $A(0), A(1), \dots$. However, some pieces of information in each $A(i)$ can be false. One wants to extract the "truth" about A . If T is a theory and A_i a sequence of structures then T is partially reflected if for some T^* such that $Cn(T^*)$ contains T , every sentence of T^* is true in infinitely many A_i 's. We prove that the theory of integer addition is partially reflected by Z_n 's (" $+$ " modulo n), and that this theory is the unique one which admits some random testing with respect to Z_n 's, and so can be viewed as being the "truth" given by Z_n 's. We show that the theory of any ultrapower of Z_n 's does not have these properties. It follows that sometimes we can extract correct information in a constructive way, even if we are given mutually contradictory facts.

PIOTR ZAKRZEWSKI, *On the classification of measure zero sets.*

Let S denote the family of all nonincreasing sequences of positive reals with converging series. We say that a sequence $(\varepsilon_n)_{n \in \omega} \in S$ covers a set $X \subset \mathbb{R}$ iff there exists a sequence $(Z_n)_{n \in \omega}$ of intervals closed from the left and open from the right with length $Z_n = \varepsilon_n$ and $X \subset \bigcup_{n \in \omega} Z_n$. $K(\varepsilon_n)$ denotes the family of all Lebesgue measure zero sets covered by a sequence $(\varepsilon_n)_{n \in \omega}$.

THEOREM 1. For every sequence $(\varepsilon_n)_{n \in \omega} \in S$ and every open interval $I \subset \mathbb{R}$ of length greater than $\sum_{n \in \omega} \varepsilon_n$ there exists a Lebesgue measure zero set $E \subset I$ not covered by $(\varepsilon_n)_{n \in \omega}$.

THEOREM 2. For $(a_n)_{n \in \omega}, (b_n)_{n \in \omega} \in S$, $K(a_n) \subset K(b_n)$ iff there exists a sequence $(P_n)_{n \in \omega}$ of pairwise disjoint, nonempty sets of natural numbers such that $a_n \leq \sum_{i \in P_n} b_i$ for every $n \in \omega$.

THEOREM 3. The ordering $\langle \{K(\varepsilon_n): (\varepsilon_n) \in S\}, \subset \rangle$ has the following properties:

- (i) There exists a family of power 2^ω consisting of pairwise incomparable elements.
- (ii) It is not a lattice.
- (iii) For any countable ordinal α there exists a well ordered chain of type α .
- (iv) For any uncountable ordinal α there does not exist any well ordered chain of type α .

IRVING H. ANELLIS, *A new proof of the strong version of the Löwenheim-Skolem theorem.*

Let Q be full quantification theory and let F_{Q^n} be the axioms of Q . We present each formula of Q in a Skolem normal form conjunction. If F_{Q^n} allows, for every finite k , a formula $F_{Q^n}^k$ which is k -satisfiable and is associated to the conjuncts of F_{Q^n} , then $F_{Q^n}^k$ is consistent. Now to a 1-satisfiable formula F^1 of Q , we conjoin an exhaustive list of formulas $F^2, \dots, F^{n-1}, F^n$ for Q consistent, and to each conjunct assign a