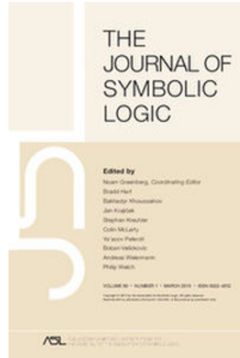

Meeting of the Association for Symbolic Logic, Marseilles, 1981 (the Herbrand Symposium)

J. Stern

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THE HERBRAND SYMPOSIUM

(MARSEILLES JULY 16 - JULY 24 1981)

Fifty years after the death of Herbrand, about two hundred people attended this colloquium which was also the European Summer Meeting of the Association for Symbolic Logic.

The symposium was sponsored by the CNRS (Centre National de la Recherche Scientifique), the division of logic, methodology, and the philosophy of science, the Association for Symbolic Logic. Financial support was also given by the Société Française de Logique and both universities in Marseilles.

The organization was beautifully handled by Professor A. Preller and the Marseilles Logic Group. Although the weather was not perfect, the participants could enjoy one of the nicest areas in France; part of them had the privilege of being among the first users of the CIRM (Centre International de Recherches Mathématiques) which had opened just before the conference.

The program committee was chaired by Professor R. Fraïssé. A large part of the program was devoted to invited lectures on the work of Herbrand and on the role of Herbrand's ideas in logic. The other invited lectures dealt with other topics of current research in mathematical logic.

The conference has tried to encompass fifty years of logic; because of the enthusiasm of the invited speakers and of the audience, this ambitious goal was achieved to a large extent.

J. STERN

IRVING H. ANELLIS, *Conjecture on Gödel incompleteness and countable models.*

Conjecture. For H^* countable finite but infinitary, if $\vdash_{H^*} X(Z^*(\langle \mathscr{F}_{G_k^*}, X_{k_1}, X_{H_{s_1}}, \mathscr{P}_{\mathscr{L}'} x_k(\mathscr{G}, \mathscr{K})\rangle))$, then $\vdash_{\mathscr{H}^* = \langle H^*, Z^{G^*}\rangle} \mathscr{G}$, for any countable k. The conjecture is based on the following fundamental theorems.

THEOREM 1. *For* $H = \langle X, \mathscr{H}\rangle$ *and* $X = \{\langle X_1, \ldots, X_n\rangle\}$, *we obtain* $H(X(\langle \mathscr{F}_{\mathscr{H}_1}, X_1\rangle), \ldots, (\langle \mathscr{F}_{\mathscr{H}_n}, X_n\rangle))$, *and* H *is a homomorphic model for* $\mathscr{H}$, *where* $\mathscr{F}_{\mathscr{X}_1}, \ldots, \mathscr{F}_{\mathscr{X}_n}$ *are functions of* $\mathscr{H}$.

THEOREM 2. $\mathrm{Hom}(H^*, G^*) = H^*(X(\langle \mathscr{F}_{G_1^*}, X_1, X_{H_{s_1}}\rangle, \ldots, \langle \mathscr{F}_{G_n^*}, X_n, X_{H_{s_n}}\rangle))$ *for* $\mathscr{F}_{G_1^*}, \ldots, \mathscr{F}_{G_n^*}$ *functions of* G^*, *and* $X_{H_{s_1}}, \ldots, X_{H_{s_n}}$ *subsets of compact Hausdorff space.*

THEOREM 3.

$$\mathrm{Hom}(H^*, \mathscr{L}') = H^*(X(\langle \mathscr{F}_{G_1^*}, X_1, X_{H_{s_1}}, \mathscr{P}_{\mathscr{Z}'} x_1 (\mathscr{G}, \alpha)\rangle, \ldots,$$
$$\langle \mathscr{F}_{G_n^*} X_n, X_{H_{s_n}}, \mathscr{P}_{\mathscr{Z}'}(\mathscr{G}, \omega)\rangle, \ \ldots, \langle \mathscr{F} G_\omega^*, X_\omega, X_{H_{s_\omega}}, \mathscr{P}_{\mathscr{Z}'}, x_\omega(\mathscr{G}, \sqsupset_{Z^{G^*}})\rangle)),$$

for $\mathscr{L}' = G\langle Z, \mathscr{G}\rangle$. *Thus* $\mathscr{L}'$ $\mathrm{Rep}(\mathrm{Hom}(G, \mathscr{G})Z))$.

THEOREM 4. $\vdash_{H^*}\mathscr{G}$ *if* $Z^{G^*} \Vdash \mathscr{G}$ *and* $\mathrm{Rep}(\mathscr{H}^*(Z^{G^*} \Vdash \mathscr{G}))$, *for* $\mathrm{Rep}(\mathscr{H}^*)$ *an algebraic representation of* H^*

CHRISTINE CHARRETTON AND MAURICE POUZET, *Comparison of Ehrenfeucht-Mostowski models of a theory with the strict order property.*

In the following, $C, C', C_1, \ldots$ will be chains, C^* will be the reverse of C and Q will be the chain of the rationals.

Let R be a binary relation on a set E, every element of E being in the range of some function mapping the strictly increasing n-tuples of C in E for some n; we say that (E, R) is a *C-invariant* binary relation if the elements of C are indiscernible for quantifier-free formulae.

THEOREM 1. *Any C-invariant partial ordering can be extended in a C-invariant linear ordering.*

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THEOREM 2. *If Q and C^* can be embedded into C, any C-invariant linear ordering can be embedded into $C^{<\omega}$ lexicographically ordered.*

Fact. To any chain C can be associated a chain C_1, containing Q and C_1^* as subchains, of the same power as C, and such that C is embeddable in C' iff $C_1^{<\omega}$ is embeddable in C_1'.

THEOREM 3. *If T is a theory with the strict order property, to every chain C of power $> |T|$ can be associated a model $F(C)$ of T, of the same power as C, such that $F(C)$ is elementary embeddable into $F(C')$ iff C is embeddable into C'.*

ZOÉ CHATZIDAKIS, *Elementary theories of boolean filtered powers.*

Let L be a language, L^* the two sorted language associated with L (V. Weispfenning, ***Journal of Algebra,*** vol. 36). L^* contains L, the usual language for boolean algebras, and an n-ary function symbol v_R for each n-ary relation R of L. If X is a boolean space and F is a closed subspace of X, define $B(X)$ to be the boolean algebra of clopen subsets of X, and $I(F)$ to be the ideal of clopen subsets of X which do not intersect F. If $A \subseteq B$ are two L-structures, we define $\mathrm{Th}^f(B, A)$ to be the set of sentences of the form $\overline{Qx} \in A\overline{Q'y} \in B\varphi(\bar{x}, \bar{y})$, where $\varphi(\bar{x}, \bar{y})$ is a quantifier free L-formula.

THEOREM. *Let $A \subseteq B$ be two L-structures, X a boolean space and F a closed subspace of X. Let M be the L-structure of locally constant functions from X to B which map F into A. Then the elementary theory T of $(M, B(X))$ in L^* is completely determined by* $\mathrm{Th}^f(B, A)$ *and* $\mathrm{Th}(B(X), I(F))$. *In addition* $\mathrm{Th}^f(B, A)$ *is interpretable in T, and so is* $\mathrm{Th}(B(X), I(F))$ *if $A \not\equiv B$.*

We indicate how to extend these results to the general case where we allow any finite number of closed subspaces of X and substructures of B.

S.D. COMER, *An application of algebraic logic to the study of Boolean combinations.*

The main success of algebraic logic has been to develop algebraic proofs of results known already in logic. The following result was discovered using ideas related to the theory of cylindric algebras.

THEOREM. *Given a theory Γ, a formula ψ, and a nonnegative integer k, ψ is equivalent (modulo Γ) to a Boolean combination of formulas, each depending on at most k variables, if and only if, in every model of Γ, the interpretation of ψ is a Boolean combination of definable (at most k-ary) relations.*

A similar result in which the Boolean combination is specified in advance also holds and, in fact, in this case the sets of variables admissible for each component can also be prescribed in advance. On the other hand, an example shows that the "if" implication fails when one tries to specify the sets of variables without prescribing the Boolean combination.

B.I. DAHN and H. WOLTER, *Towards an axiom system for ordered exponential fields.*

Let L be a language for fields with an additional unary function symbol E and let T_0 be an $\forall$-axiom system for fields of characteristic 0 augmented by the axiom $E(x + y) = E(x) \cdot E(y)$. The models of T_0 are said to be exponential fields. The most important examples are $(\boldsymbol{R}, e^x)$ and $(\boldsymbol{C}, e^z)$, the exponential fields of real and complex numbers, respectively. It is well known that the ring of integers is definable in $(\boldsymbol{C}, e^z)$ and hence the theory of $(\boldsymbol{C}, e^z)$ is rather complicated. In existentially complete models of T_0 one has a similar situation. $(\boldsymbol{C}, e^z)$ itself is not existentially complete.

In the following let T_1 be the $L(\leq)$-theory of ordered exponential fields augmented by the axiom $1 + x \leq E(x)$. This inequality plays an important role. Particularly, one can prove in T_1 the continuity of E and its differential equation (formulated in the ε-δ-language). This yields the uniqueness of e^x in $(\boldsymbol{R}, e^x)$. But in other T_1-models the exponential function is not yet determined uniquely. If $E_k(x) = \sum_{0\leq i\leq k} x^i/i!$ and $\varphi_k = \forall x\,(x \neq 0 \rightarrow E_k(x) < E(x))$, k odd, then we have among other properties for E_k: $(\boldsymbol{R}, e^x) \models \varphi_k$ and $T_1 \not\models \varphi_k$ if $k \geq 3$. Hence, T_1 is not strong enough to axiomatize the $\forall$-theory of $(\boldsymbol{R}, e^x)$. It seems to be natural to regard the axiom system $T_2 = T_1 \cup \{\varphi_k : k \text{ odd}\}$. If l is even then

$$T_2 \models \forall x((x < 0 \rightarrow E_l(x) > E(x)) \wedge (x > 0 \rightarrow E_l(x) < E(x))).$$

Since E is strongly monotone, E has an inverse function. Every model of T_2 is extendable to a real closed field which is a model of T_2 (this is not trivial!); particularly, the existentially complete models of T_2 are real closed fields, in which, of course, the intermediate value property is true for polynomials. We proved this property also for some other classes of terms.

The analogy between the model theory of ordered fields and ordered exponential fields is further investigated.

ETIENNE GRANDJEAN, *Complexité de la théorie des formules vraies dans presque toutes les structures finies.*

Soit S un ensemble fini de symboles de relations dont l'une est d'arité au moins 2. Une formule F du premier ordre, close et de type S, est dite "presque partout vraie" si $\lim_{m\to\infty} p_m(F) = 1$, où $p_m(F)$ est la proportion des modèles de F parmi les structures de type S et de domaine $\{1, 2, \ldots, m\}$. Soit Th(S) la théories des formules presque partout vraies.

Pour la première fois, semble-t-il, on étudie ici la complexité de la décision de Th(S). On donne une méthode de décision de Th(S), polynomiale en espace.

Par une méthode "d'arithmétisation" des machines de Turing (analogue à celle de L. Stockmeyer), on montre qu'une certaine classe de complexité polynomiale en espace, se "réduit" a l'ensemble Th(S) et on en déduit une borne inférieure de la complexité de Th(S), très proche de la borne supérieure.

On étudie aussi la complexité de deux variantes de la théorie Th(S), l'une de même complexité que Q.B.F. (l'ensemble des Formules Booléennes Quantifiées valides), l'autre indécidable.

Ces résultats complètent ceux de H. Gaifman et R. Fagin qui ont montré que Th(S) est une théorie axiomatisable et complète et en ont donné une méthode de décision par élimination des quantificateurs.

VALENTINA HARIZANOV, *Łoś's theorem for ultraproducts of models with monotone quantifiers.*

In this work we consider ultraproducts of models in logic $L(Q)$, the first order logic with a new quantifier symbol Q. By the reduced product of a family $\{(\mathscr{A}_i, q_i) \mid i \in I\}$ over a filter F (on I), in symbols $\prod_{i\in I}(\mathscr{A}_i, q_i)/F$, we understand the model $(\mathscr{A}, q)$ where

$$\mathscr{A} = \prod_{i\in I}\mathscr{A}_i/F \quad \text{and} \quad q = \{\prod_{i\in I}X_i/F \mid \text{for each } i \in I,\ X_i \in q_i\}.$$

A quantifier q on A is monotone (increasing) if it satisfies the condition: $X \subseteq Y \subseteq A, X \in q \Rightarrow Y \in q$. q is nontrivial if $q \neq \varnothing$ and $\varnothing \notin q$. The main theorem in the paper is:

THEOREM. *Let $\{(\mathscr{A}_i, q_i) \mid i \in I\}$ be a collection of models such that for each $i \in I$, q_i is a nontrivial monotone quantifier. If F is an ultrafilter on I and $(\mathscr{A}, q) = \prod_{i\in I}(\mathscr{A}_i, q_i)/F$, then*
(1) $(\mathscr{A}, q) \models \varphi[f_1/F, \ldots]$ iff $\{i \in I \mid (\mathscr{A}_i, q_i) \models \varphi[f_1(i), \ldots]\} \in F$;
(2) $(\mathscr{A}, q') \models \varphi[f_1/F, \ldots]$ iff $\{i \in I \mid (\mathscr{A}_i, q_i) \models \varphi[f_1(i), \ldots]\} \in F$,
where $q' = \{X \subseteq \prod_{i\in I}A_i/F \mid X \supseteq Y$ for some $Y \in q\}$.

COROLLARY. *The compactness theorem holds for models with monotone quantifiers.*

As a consequence we also obtain Łoś's theorems for the quantifier "there exist at least $\aleph_\alpha$" and for the equicardinal quantifier, which are due to G. Fuhrken. In the proof we use the following assertions.

LEMMA. *Let models $(\mathscr{A}, q_1)$ and $(\mathscr{A}, q_2)$ be such that $q_1 \subseteq q_2$ and every $Y \in q_2\backslash q_1$ is nondefinable over $(\mathscr{A}, q_1)$. Then $(\mathscr{A}, q_1) \prec (\mathscr{A}, q_2)$.*

LEMMA. *Let $\{(\mathscr{A}_i, q_i) \mid i \in I\}$ be a collection of models (for $i \in I$, $q_i\backslash\{\varnothing\} \neq \varnothing$ and $q_i^c\backslash\{\varnothing\} \neq \varnothing$). If (1) holds, then every definable set over $\prod_{i\in I}(\mathscr{A}_i, q_i)/F$ is of the form $\prod_{i\in I}X_i/F$ where for all $i \in I$, $X_i \in q_i$ or for all $i \in I$, $X_i \notin q_i$.*

DAVID MARKER, *Variations on Harrington's construction of models of arithmetic.*

Harrington constructed a nonstandard model M of PA with Diag(M) $\leq_T 0'$ and Th(M) non-arithmetic. We use variations on Harrington's construction to prove the following.

(1) There is a nonstandard $M \models$ PA with Diag(M) $\leq_T 0'$ and Th(M) $\equiv_T 0^{(\omega)}$.

(2) Suppose $\forall n\ d\ {}_T{\geq}\ 0^{(n)}$; then there is a nonstandard $M \models$ Th(N) with Diag(M) $\equiv_T d'$.

(1) answers a question of McAloon. (2) extends a result of Knight. We also examine several model-theoretic properties of Harrington's model.

ŽARKO MIJAJLOVIĆ, *Partial saturation of nonstandard models of arithmetic.*

An extended version of A. Robinson's Overspill Lemma is proposed, which might be considered also as a partial saturation of nonstandard models of Formal Arithmetic (P). This is

THEOREM. *For every positive integer k, every nonstandard model N of P realizes every recursive Σ_k type. (A set S of formulas is Σ_k if every formula in S is Σ_k.)*

From the above theorem various results on nonstandard models of P are easily obtained:

Robinson's Overspill Lemma; Note that $\{\varphi(x)\} \cup \{n < x : n \in \omega\}$ is a recursive Σ_k type for some k.

If X is an infinite r.e. subset of ω and X^* its nonstandard extension in a nonstandard model N of P, then $X^* - X$ is coinitial in $N - \omega$.

Let ω denote the standard model of P and let $N \models P$ be nonstandard. Then (1) and (2) are equivalent, where

(1) $\omega \prec_{\Sigma_1} N$.

(2) The intersection of all initial segments of N isomorphic to N is ω.

(J. Schlipf) Model of P which belongs to a non-ω-standard model $\mathcal{M}$ of ZF is recursively saturated. First it is proved, by use of the Truncation Lemma that every (standard) recursive set of formulas is a subset of some Σ_k set of formulas in $\mathcal{M}$ for some infinite k.

Most of the results can be extended to other structures, as to non-ω-standard models of KPU.

REFERENCES

[1] J. BARWISE, ***Admissible sets and structures,*** Springer, Berlin, 1975.

[2] H. Friedman, *Countable models of set theory*, ***Cambridge Summer School in Mathematical Logic,*** Lecture Notes in Mathematics, vol. 337, Springer, Berlin, 1973, pp. 539–573.

[3] J. S. SCHLIPF, *A guide to the identification of admissible sets above structures*, ***Annals of Mathematical Logic,*** vol. 12 (1977), pp. 151–192.

DIDIER MISERCQUE, *The nonhomogeneity of the E-tree. Answer to a problem raised by D. Jensen and A. Ehrenfeucht.*

We prove that the ordered system of all C^1EP's, under the order "admits embedding in" is not homogeneous. This answers a problem raised in [1].

Let L be the set of all $\forall_1$-sentences of Peano arithmetic (PA) modulo PA. By "E-tree" we mean the class of all prime filters of L under the partial ordering $\supset$. Obviously, there is an isomorphism between the E-tree and the C^1EP's. We prove the following results.

LEMMA 1. *The E-tree has an element F such that* (i) *F is not maximal*; (ii) *F is not minimal*; (iii) *F has no immediate predecessor*; (iv) *if B is any branch of the E-tree containing F, then F has an immediate successor in B.*

LEMMA 2. *If F is any maximal element of the E-tree, then F has no immediate predecessor.*

The nonhomogeneity of the E-tree (and of the ordered system of all C^1EP's) is an obvious consequence of these two lemmas.

REFERENCE

[1] D. JENSEN and A. EHRENFEUCHT, *Some problem in elementary arithmetics*, ***Fundamenta Mathematicae,*** vol. 92 (1976), pp. 223–245.

MIKE PREST, *Theories of modules: categories of substructures of models.*

I consider from a global point of view complete theories of (right) modules over a fixed ring R with 1. There is in some sense a largest such theory T_c, and since every complete theory is a "direct summand" of this theory, I concentrate on T_c.

Let $\mathcal{C}^*_{T_c}$ be the category whose objects are submodules of models of T_c and whose morphisms are the *pp*-type-preserving homomorphisms (if $\tilde{M}$ is a very saturated model of T_c then $\mathcal{C}^*_{T_c}$ "restricted to $\tilde{M}$" has as objects the small submodules of $\tilde{M}$ and as morphisms those R-homomorphisms which extend to endomorphisms of $\tilde{M}$).

Extending results of L. Gruson and C.U. Jensen I show that $\mathcal{C}^*_{T_c}$ naturally embeds in the category $\mathcal{AB}^{\mathcal{M}^{f.p.}_{R^{op}}}$ of additive functors from the category of finitely presented left R-modules to the category of abelian groups. The proof uses a variant of the tensor product construction which takes account of *pp*-type rather than just isomorphism type.

I then show how exploiting this equivalence allows a unified approach to results which have been proved independently, on the one hand by S. Garavaglia (using a model-theoretic approach), and on the other hand by L. Gruson and C.U. Jensen and by B. Zimmermann-Huisgen and W. Zimmermann (using an algebraic approach).

I go on to discuss connections between the model theory of modules and representation theory of algebras.

DAN SARACINO and CAROL WOOD, *Quantifier-eliminable nil-2 groups of exponent* 4.

THEOREM. *There exist continuum many countable quantifier eliminable groups nilpotent of class* 2 *and exponent* 4.

REMARK. All nil-2 exponent 4 groups are uniformly locally finite, hence all the groups in the theorem are $\aleph_0$-categorical. None of these groups, however, are stable: the formula $xyx^{-1}y^{-1} = 1$ has the independence property.

The proof of our theorem follows a general procedure due to Fraïsse. We obtain continuum many distinct classes of finite nil-2 exponent 4 groups, each closed under substructure and isomorphism and each having the amalgamation property. We then show that each class K gives rise to a distinct countable quantifier-eliminable group G_K whose finite subgroups are exactly those in K.

This result has appeared in ***Journal of Algebra,*** vol. 76 (1982), pp. 337–352.

JÜRGEN SAFFE, *A superstable theory with the dimensional order property has many models.*

All the notations of this paper follow closely [Sh 1], and a certain familiarity with stability theory is necessary for understanding. Let T denote a complete first order theory without finite models which is superstable. T has the dimensional order property (dop in short) iff there are $F^a_{\aleph_0}$-saturated models M, M_1, M_2, N and $p \in S^1(N)$ such that M is an elementary substructure of M_1 and M_2, $t(M_1, M \cup M_2)$ does not fork over M, N is $F^a_{\aleph_0}$-prime over $M_1 \cup M_2$ and p is orthogonal to M_1 and M_2. This is a reasonable notion, because a superstable theory without the dop satisfies some structure theorem (see [Sh 2]). Here we present an easy proof that a superstable theory with the dop has 2^λ nonisomorphic models for any $\lambda \geq |T| + \aleph_1$ by directly interpreting graph theory in such a theory, and by a suitable coding we get as many models as graphs. The basic lemma which simplifi s the proof of this result of [Sh 2] is the following:

LEMMA. *Suppose* T *(is superstable and) has the dop. Then there are* M, M_1, M_2, N *and* $p \in S^1(N)$ *as in the definition such that even the following hold*: $w(M_1, M) = w(M_2, M) = 1$ *and* p *is regular.*

The proof follows by choosing models which exemplify dop such that $w(M_1, M) + w(M_2, M)$ is minimal and looking at regular types which exemplify the definition of weight.

The next tool is to reduce the models of that lemma to some finite sets which have the important properties. This lemma gives an easy proof of the

THEOREM. *Suppose* T *has the dop. Then for all* $\lambda \geq |T| + \aleph_1$ *there are* 2^λ *nonisomorphic models of cardinality* λ.

REFERENCES

[Sh 1] S. SHELAH, ***Classification theory,*** North-Holland, Amsterdam, 1978.

[Sh 2] ———, $\aleph_\varepsilon$*-saturated models, the main gap* (to appear).

PETR ŠTĚPÁNEK, *Embeddings of Boolean algebras and automorphisms.*

Several important classes of Boolean algebras are defined by automorphism properties, e.g. the classes of rigid or homogeneous Boolean algebras. There are general embedding theorems for these classes of Boolean algebras.

THEOREM 1 (KRIPKE). *Every Boolean algebra can be completely embedded in a complete homogeneous Boolean algebra.*

THEOREM 2 (BALCAR, ŠTĚPÁNEK). *Every Boolean algebra can be completely embedded in a complete rigid Boolean algebra.*

If a BA B is completely embedded in C, we may ask whether every automorphism of B extends to an automorphism of C. Clearly, if C is rigid, then no nontrivial automorphism of B extends to an automorphism of C. On the other hand, it can be shown that in the case of Kripke's embedding, every automorphism of the smaller algebra can be extended to an automorphism of the larger one.

We say that a Boolean algebra B has no rigid or homogeneous factors if for every nonzero element u of B, the principal ideal $B \upharpoonright u$ of all $v \in B$, $v \leq u$, is neither a homogeneous nor a rigid Boolean algebra.

We have the following results.

THEOREM 3. *Every Boolean algebra B can be completely embedded in a complete Boolean algebra C with no rigid or homogeneous factors such that every automorphism of B extends to an automorphism of C.*

THEOREM 4. *Every Boolean algebra B can be completely embedded in a complete Boolean algebra C with no rigid or homogeneous factors such that no nontrivial automorphism of B extends to an automorphism of C.*

Thus we have two embedding theorems for the class of complete Boolean algebras with no rigid or homogeneous factors showing that this class of algebras shares some embedding properties with the classes of rigid and homogeneous algebras. Similar results hold if we restrict ourselves to the case where both B and C are κ-distributive for an arbitrary cardinal κ.

L.W. SZCZERBA, *Elementary interpretations.*

Let $\mathrm{St}_\sigma^{\mathrm{iso}}$ be a category of structures of signature σ (with morphisms being model-theoretical isomorphisms). Let $\mathrm{Sn}_\sigma^{\vdash}$ be a category of sentences of signature σ. The set of morphisms from φ to ψ is one element if $\varphi \vdash \psi$, and empty otherwise. Let Γ be a functor from a subcategory of $\mathrm{St}_\sigma^{\mathrm{iso}}$ into $\mathrm{St}_\tau^{\mathrm{iso}}$ and F a functor from $\mathrm{Sn}_\tau^{\vdash}$ into $\mathrm{Sn}_\sigma^{\vdash}$. We say that they are correlated if for any structure $\mathfrak{A}$ of signature σ and any sentence φ of signature τ we have $\Gamma\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models F\varphi$. In such a case we shall call Γ an interpretation and F a translation. Examples of interpretations are: definitional extension, reduct, restriction, quotient structure and Cartesian power. The interpretations which are finite compositions of interpretations mentioned above are called elementary interpretations. It is possible to characterise elementary interpretations in terms of categories. The condition turns out to be a generalisation of Beth's theorem.

C.M. WAGNER, *Martin's conjecture for theories or trees.*

Let T be a complete consistent theory in $L_{\omega\omega}$. Define $L_1(T)$ to be the smallest fragment of $L_{\omega_1\omega}$ containing $L_{\omega\omega} \cup \{\bigwedge_{\phi, p}\phi(\bar{x}) \mid p \in S_n(T),\ n < \omega\}$. For $\mathfrak{A}$ a model of T, let $T_1(\mathfrak{A})$ be the complete theory of $\mathfrak{A}$ in the language $L_1(T)$.

The Martin Conjecture. If T has fewer than $2^{\aleph_0}$ countable models, then $T_1(\mathfrak{A})$ is $\aleph_0$-categorical for all countable $\mathfrak{A} \models T$.

Note that Martin's Conjecture implies the Vaught Conjecture.

Throughout the following, a tree is a structure $\langle\mathfrak{A}, <\rangle$, with $\mathfrak{A}$ partially by $<$, such that for all $x \in |\mathfrak{A}|$, $\{y \in |\mathfrak{A}| \mid y \leq x\}$ is linearly ordered by $<$.

We prove the following theorem:

THEOREM 1. *Let T be a complete theory of trees with countably many unary relations. Then the Martin Conjecture is valid for T.*

It should be noted that work by J.R. Steel proves that the Vaught Conjecture is valid for theories of trees. That proof depends heavily on the use of admissible sets.

In previous work (Ph.D. thesis) we proved that Martin's Conjecture is valid for theories of a linear order with countably many unary relations (see M. Rubin for a proof of the Vaught Conjecture in this case). More precisely, we proved the following stronger result.

THEOREM 2. *Let T be a theory of a linear order with countably many unary relations such that T has* $< 2^{\aleph_0}$ *countable models. Let* $\mathfrak{A} \models T$. *Then* $\mathfrak{A}$ *is characterized up to isomorphism by a single statement in* $L_1(T)$.

The proof of Theorem 1 uses this result repeatedly by coding the theories of specific subtrees into a theory of a linear order with unary relations. This technique is coupled with a series of cutting and pastings to analyze the structure of the tree.

MARC BERGMAN, *Compiling algebraic abstract data types with Horn-clauses.*

During the last decade, the art of programming has changed in various ways. Among them, two aspects take a part growing and growing: the first is, following Hoare, the intensive use of abstract data types; the second is logic programming.

Our aim is to present the denotational and procedural semantics of Algebraic Abstract Data Types (AAT) in terms of rewriting systems and their programming in the PROLOG language.

An AAT is considered as an interpreter, the semantical actions of which are rewrite rules. The power of the methodology enables us to construct hierarchical types, including genericity and error treatment.

We start describing the formal semantics of an AAT in terms of a rewriting rules system. After recalling classical results we introduce the notion of T-reducibility, i.e. local reduction via a type T, clarifying the normalization for complex types.

The second part shows how the reduction property, using substitutions "equals by equals", may be programmed with Horn-clauses from a specification "à la Guttag", or, more precisely, how it may be automatically interpreted in PROLOG.

The third part describes how this methodology respects the independence of types and how it authorizes the implementation of hierarchical types in a way which takes its inspiration partly from the Martin-Löf theory of types (1973) and partly from Burstall and Goguen (1977). Finally we outline the error treatment.

Our work is an attempt to implement symbolic and algebraic manipulations as constructive mathematics, as in fact suggested by R. Loos (EUROSAM 74). This general viewpoint may be considered, for both user and designer, as a unique language.

These ideas are under implementation.

DANIEL BOQUIN, *Dilatateurs et le "dendroïde associé".*

On présentera l'équivalence entre une propriété de dilatateurs et une propriété du "dendroïde associé". Plus précisément:

DÉFINITION. $\overline{\mathrm{ON}}$ est la catégorie ainsi définie:

(*i*) Les objets sont les couples $(x, \sim)$ [abréviation, $\bar{x}$], où $\bar{x}$ est un ordinal et $\sim$ une classe d'équivalence dont les classes sont des intervals fermés.

(*ii*) Les morphismes $f \in I(\bar{x}, y)$ sont les applications strictement croissantes de x dans y telles que si $[a, b]$ est une classe de $\bar{x}$, alors $[f(a); f(b)]$ est une classe de $\bar{y}$.

Etant donné un dilatateur F, on peut associer à tout objet $\bar{x}$ be $\overline{\mathrm{ON}}$ un objet $F(\bar{x})$ de $\overline{\mathrm{ON}}$. Mais ceci peut-il définer un foncteur de $\overline{\mathrm{ON}}$ dans $\overline{\mathrm{ON}}$? On montre:

F est ainsi prolongeable si et seulement si $D = BCH(F)$ satisfait la propriété suivante:

$$\forall S \in D^*[s = (a_0 \ldots a_{2k+1}) \to \exists s_0 \in D\, \exists s_\infty \in D \forall t \in D$$
$$((S_0 \text{ prolonge } s \text{ et } s_\infty \text{ prolonge } S) \text{ et } (t \text{ prolonge } S \to S_0 \prec t \prec S_\infty))].$$

Pour bien situer cet énoncé on pourra en un premier temps nebrosser rapidement "le foncteur branchement" qui permet l'association d'un dendroïde à tout dilatateur.

ROBERT A. DI PAOLA, *The theory of partial α-recursive operators.*

Throughout the sequel, let α be an admissible ordinal.

Functionals are ordinal-valued maps defined on classes of functions. Operators are function-valued maps defined on classes of functions. The theory bifurcates into two subtheories: one about weak partial α-recursive operators and functionals, the other about partial α-recursive operators and functionals in a proper sense.

Let α^* be the Σ_1-projectum of α. A set A is weakly α-enumeration reducible to a set $B\,(A \leq_{w\alpha e} B)$ iff $(\exists \varepsilon < \alpha^*)(x)[x \in A \leftrightarrow (\exists \eta)[\langle x, \eta\rangle \in W_\varepsilon \text{ and } K_\eta \subseteq B]]$.

The mapping thus defined by any $\varepsilon < \alpha^*$ from 2^α into 2^α is said to be a *weak α-enumeration operator Φ_ε^W with index* ε. A set A is α-enumeration reducible to a set $B\,(A \leq_{\alpha e} B)$ iff

$$(\exists \varepsilon < \alpha^*)(\delta)[K_\delta \subseteq A \Leftrightarrow (\exists \eta)\,[\langle \delta, \eta\rangle\, \varepsilon W_\varepsilon \text{ and } K_\eta \subseteq B]].$$

For each $\varepsilon < \alpha^*$, we define a mapping Φ_ε from 2^α to 2^α:

$$\Phi_\varepsilon(A) = \bigcup\{K_\sigma \mid (\exists\eta)[\langle\delta, \eta\rangle \in W_\varepsilon \text{ and } K_\eta \subseteq A\}.$$

We define $V_\varepsilon = \{A/A$ is single-valued and $\Phi_\varepsilon(A)$ is single-valued$\}$.

Φ_ε is an *α-enumeration operator* if for all $A \in V_\varepsilon$, $\Phi_\varepsilon(A) \leq_{\alpha\varepsilon} A$. The (weak) partial α-recursive operators, (weak) α-recursive operators, and (weak) general α-recursive operators are defined by means of (weak) α-enumeration operators as they are in Rogers' book for the simpler ω-case; likewise for the corresponding classes of (weak) functionals.

In a lengthy paper from which the present summary is extracted, many topics in the theory of partial recursive operators have been examined, namely: I. Elementary Propositions and Theorems; II. Extension Theorems and Effective Operations; III. Computational Complexity; IV. Limit. Functionals. A general feature is the necessary difference treatment of partial α-recursive operators as opposed to partial α-recursive functionals.

Let C_A be the characteristic function of the set A. Under I, the composition of weak α-enumeration operators need not be a weak α-enumeration operator; also $A \leq_{W\alpha} B$ iff $C_A \leq_{W\alpha e} C_B$ and $A \leq_\alpha B$ iff $C_A \leq_{\alpha e} C_B$, and, for total functions f and g, $f \leq_{W\alpha} g$ iff $f \leq_{W\alpha e} g$. But we conjecture that there are α and total f, g such that $f \leq_{\alpha e} g$, but $f \not\leq_\alpha g$. Under II, there are serious anomalies in the theory of classes of α-resets (some classical proofs break down completely). This affects the formulation and status of the Myhill-Shepherdson and Kreisel-Lacombe-Shoenfield theorems at level α. Under III, we have lifted Constable's Operator Gap Theorem to all α. Under IV, a highlight is our generalization of Friedberg's theorem on the existence of a Banach-Mazur functional that agrees with no recursive functional on the class of recursive functions to all α such that $\lambda < \alpha^*$, where λ is the Σ_2-cofinality of α, and also to $\alpha = \omega_1^{CK}$.

JORGE HERRERA, *La méthode arborescente en démonstration automatique.*

Après que les origines de la Démonstration Automatipue furent établis, Newell, Shaw et Simon, Wang, Gilmore et Davis, et putnam pour la logique propositionnelle et Robinson pour la logique de premier ordre publièrent les points forts de la démonstration par machine. Tseitin avec la résolution avec extension et résolution regulière montra la puissance de la méthode à la Robinson et Quine.

On peut enmener une formule booleenne vers une formule en forme normale conjonctive avec au plus trois variables ou négation de variables par clause. En s'inspirant de la méthode des Arbres de Beth on peut simuler une exploration de l'arbre de Beth correspondant à la formule précédente en ordre préfixe.

Les règles de Davis et Putnam se combinent bien avec cette exploration donnant lieu à d'autres règles en accélerant les algorithmes classiques. On peut obtenir les avantages de la résolution regulière sans avoir une explosion en espace du a la résolution des clauses. Dans le cas de la logique de premier ordre on peut éviter—en utilisant toujours le théorème de Herbrand—la règle de "factoring". Pour le cas de formules de Horn la méthode est polynômiale.

ALBERT HOOGEWIJS, *The nondefinedness notion in a two-valued logic.*

In [1] and [2] we introduced a partial predicate calculus PPC, that is a three-valued logic, in order to obtain a formalization of the nondefinedness notion. In addition to the classical connectives we used the symbol Δ to express that a formula is defined.

For a model $\mathscr{M}$ for the language and a formula φ, $\mathscr{M} \models \varphi$ means that φ is valid in $\mathscr{M}$, that is, φ takes the truth-values 1 or 2 in $\mathscr{M}$. $\mathscr{M} \equiv \varphi$ means that φ is true in $\mathscr{M}$, that is, φ takes the truth-value 1. A formula φ is a *consequence* of a set Γ of formulas (*notation*: $\Gamma \models \varphi$) iff for all models $\mathscr{M}$ of Γ, $\mathscr{M} \models \varphi$.

DEFINITION. We call a formula φ *equivalent* with a formula ψ iff for all models $\mathscr{M}$ for the language, $\mathscr{M} \models \varphi$ iff $\mathscr{M} \models \psi$ (notation: φ equi ψ). A formula ψ is *counterpart* of a formula φ iff for all models $\mathscr{M}$, $\mathscr{M} \equiv \varphi$ iff $\mathscr{M} \equiv \psi$ (*notation*: φ counter ψ).

THEOREM. *For each formula φ there are formulas ψ_1 and ψ_2 such that φ equi ψ_1 and φ counter ψ_2, where for all models $\mathscr{M}$ for the language ψ_1 and ψ_2 take truth-values in* $\{0, 1\}$.

COROLLARY. *For each set of formulas $\Gamma \cup \{\varphi\}$ we can find a set Γ_1 of counterparts of the formulas of Γ and a formula φ_1 equivalent to φ such that $\Gamma \models \varphi$ iff $\Gamma_1 \models \varphi_1$.*

Now we are able to define a sublanguage Δ-PPC containing formulas which take truth-values in $\{0, 1\}$, such that each formula φ has an equivalent ψ_1 and a counterpart ψ_2 in Δ-PPC.

COROLLARY. *We can express all properties concerning satisfaction, validity and consequence by means of formulas of Δ-PPC.*

If we take the deduction calculus of [1] and [2], drop the rules which are trivial for formulas of Δ-PPC and add some rules concerning $\alpha \wedge \Delta\alpha$ and $\neg \alpha \wedge \Delta\alpha$, where α is atomic, we get a complete and sound deduction calculus (*notation*: $\Gamma \vdash \varphi$ for φ is deducible from Γ) in Δ-PPC with respect to the notion of consequence ($\Gamma \models \varphi$).

REMARK. We note that in Δ-PPC $\Gamma \models \varphi$ has the classical meaning, i.e. for all models of Γ, φ takes the truth-value 1.

REFERENCES

[1] A. HOOGEWIJS, *A calculus of partially defined predicates*, Seminar Notes, Gent, 1977.

[2] ———, *On a formalization of the non-definedness notion*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, Band 25 (1979), pp. 213–217.

SHIH-CHAO LIU, *The Heine-Borel theorem admits only nonconstructive proof in the formal set theory ZF.*

Let $R(x, y)$ be a recursive predicate of two variables. Kleene, in his paper of 1943 (***Transactions of the American Mathematical Society***, vol. 53, pp. 41–73), suggested that the statement $\forall x \exists y R(x, y)$ has a constructive proof if and only if a recursive function $f(x)$ can be found such that $R(x, f(x))$ holds for every natural number x. Thus for any predicate $S(x, y)$ (which may or may not be recursive) the statement $\forall x \exists y S(x, y)$ cannot have a constructive proof if for no recursive $f(x)$, $\forall x S(x, f(x))$ holds. In this sense Kleene gave an example $\forall x \exists y \forall z \bar{T}_3(f, f, x, y, z)$, which can be shown to be true but is not provable constructively. (For its explanation, see p. 70 of the above paper.)

In this paper we shall give some formulas of the form $\exists y f(y)$ which are provable in the Zermelo-Fraenkel axiomatic set theory ZF but are not constructively provable in ZF in a sense different from that of the above example. Specifically we shall first define a constant H in ZF such that the formula $\exists y(y \in \omega \;\&\; y = H)$ is provable in ZF, but for every constructive element $\ulcorner i \urcorner$ in ω, $\ulcorner i \urcorner = H$ is not provable in ZF. Then we say that $\exists y(y \in \omega \;\&\; y = H)$ admits only nonconstructive proof.

Here for any intuitive natural number i, $\ulcorner i \urcorner$ is defined recursively by $\ulcorner 0 \urcorner = 0$, $\ulcorner i + 1 \urcorner = S(\ulcorner i \urcorner)$ where $S(x)$ is the successor operation $x \cup \{x\}$.

Other than such artificial examples as this, we construct one which has mathematical meaning in analysis and topology whence we show that the classical Heine-Borel theorem admits only nonconstructive proof in ZF.

JAMES P. JONES and YU. V. MATIJASEVIČ, *Exponential diophantine representation of enumerable sets.*

Recent work of the authors will be presented. The following results have been proved.

THEOREM 1. *Every r.e. set can be represented in the form*

(1) $$\exists z, y[P(z, y) \leq Q(z, y)]$$

where $P(z, y)$ and $Q(z, y)$ are functions constructible from the variables z, y and natural numbers by addition, multiplication and the operation of raising 2 to a power 2^x.

The unknowns z and y range over natural numbers. z and y are unique (singlefold representation). (1) implies every r.e. set can be represented in the form $\exists z, y, x[P(z, y, x) = Q(z, y, x)]$ where P and Q are as above and x, y and z are unique. Alternatively, it is also possible to represent every r.e. set in the form

(3) $$\exists z \forall y[R(z, y) \leq S(z, y)]$$

and hence in the form $\exists z \forall y \exists x[R(z, y, x) = S(z, y, x)]$. Here the universal quantifier, $\forall y$ may be bounded. Also the interior existential quantifier $\exists x$ may be bounded. Kalmar elementary relations can be represented with one fewer quantifier, i.e. in the forms $\exists y[P(y) \leq Q(y)]$ and $\forall y[R(y) \leq S(y)]$ and hence in the forms $\exists y, \exists x[P(y, x) = Q(y, x)]$ and $\forall y \exists x[R(y, x) = S(y, x)]$.

KLAUS LEEB, *Diagonalization in the syntax of categories.*

For about four years now the illusion prevails among logicians that they had found a "strictly mathematical" statement beyond the reach of PA—one not involving any syntax. In this note I want to point out some advantages to the opposite attitude, namely of recognizing the syntax involved and making good use of it.

PH speak of solutions S to Ramsey's partition problem in the category Ord (of order embeddings between finite ordinals) having the property min $S \leq$ card S. To make my point right away, this is a diagonalization between the object syntax (card) and the morphism syntax (min) of Ord, both together better known as the Pascal theory of that category. Friedman, McAloon and Simpson struggle to get yet further by coloring more subsets (Galvin and Prikry) and then iterating. I would like to suggest that working in other categories and using their actually more complex syntax might bring the PH idea to full fruition. After all, the Pascal identity for Ord is just

$$\mathrm{Ord}\binom{1+X}{1+k} = \mathrm{Ord}\binom{X}{1+k} + \mathrm{Ord}\binom{X}{k}.$$

The three most successful ideas in Pascal theory have been Dualization, Labeling (i.e. looking at Comma categories, Higman-Kruskal-Nash = Williams), and Initial segments (Nash = Williams-Galvin-Prikry and Deuber-Voigt).

Dualization. Dualizing Ramsey's theorem led to the Hales-Jewett theorem and eventually to the proof of Ramsey theorems as diverse as for vector spaces and for Deuber solutions of Rado regular linear systems. Here is what it offers for our purpose.

(a) *Ramsey*: One disposes of Milliken's theorem for the category of order-preserving partial surjections and can try to impose rather strong PH-like conditions upon the solution morphism. A Milliken morphism looks like this:

$$\S\S\S\S 00\S 0\S\S 1\S 1\S 111\S\S 2\S 222\S 2 \cdots \S\S L\S LLL\S L\S\S\S$$

and hence the starting point of the first parameter can again control the size of the subobject, but internally one can iterate much more. The same applies to choice functions, a combination of Injections and partial Surjections.

(b) *Bqo*: In the Pascal identity for Trees,

$$\text{Trees } C\binom{xRtX}{kRtK} = \coprod_{i \in \mathrm{dom} X} \text{Trees } C\binom{Xi}{kRtK} + C\binom{x}{k} \times \text{OrdTrees } C\binom{X}{K},$$

replacing the category Ord by its *dual*, the Hales-Jewett category $[\varnothing]$, something Galvin probably told me a dozen years ago to do, one certainly will be able to speed up the growth of lengths of "skew antichains", studied by myself and Friedman, on the wqo of Trees.

Labeling: One can push further the self-reference of Pascal identities, by looking at (Trees labeled by)* C, and thus speed up the wqo-defined fast growing functions. Also one can enter the morphism syntax to make the wqo-guaranteed comparabilities harder to achieve.

Initial seqments: The initial-segment method of NW-G-P-Ellentuck resp. Deuber-Voigt provides the simplest proofs of the ω- resp. finite versions of Ramsey theorems for the various categories and thus should be given new careful attention.

HILBERT LEVITZ, *Polynomial functions with exponentiation.*

Let N be the set of natural numbers and $\mathscr{T}$ the smallest class of functions from N into N containing the constant function 1, the identity function x, and closed under addition, multiplication and exponentiation of functions. The *majorization* relation on $\mathscr{T}$ is defined by $f \prec g$ if and only if there exists $n_0 \in N$ such that $f(x) < g(x)$ for all $x \geq n_0$. Skolem and Tarski separately raised the question of whether $\prec$ is a well ordering of $\mathscr{T}$ and, if so, what its order type would be. Ehrenfeucht [1] showed it to be a well ordering and the speaker [2] established the bounds $\varepsilon_0 \leq \|\mathscr{T}\| \leq \mathscr{X}_0$ where $\mathscr{X}_0$ is the least solution of $\varepsilon_x = x$. In [3] the speaker used nonstandard methods to show that the order type of the initial segment determined by the function 2^{x^2} is exactly ω^{ω^3}. Even on this small initial segment the ordering is rather complicated and it is an open question whether or not it is recursive.

In an effort to improve on the results above, a detailed study of the structure of the initial segment determined by the functions 2^{x^3} has been made and will be reported on here. On the basis of these results some interesting conjectures about the full family $\mathcal{T}$ will be made.

REFERENCES

[1] A. EHRENFEUCHT, *Polynomial functions with exponentiation are well ordered*, ***Algebra Universalis***, vol. 3 (1973), pp. 261–262.

[2] H. LEVITZ, *An ordinal bound for the set of polynomial functions with exponentiation*, ***Algebra Universalis***, vol. 8 (1978), pp. 233–243.

[3] ———, *Calculation of an order type: An application of nonstandard methods*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 28 (1982), pp. 219–228.

ANDRZEJ JANKOWSKI, *Recursive closure spaces.*

A *pseudorecursive closure space* is an ordered pair $X = \langle X, \mathcal{F}\rangle$ such that:

(0) X is a recursive set,

(1) $\varnothing, X \in \mathcal{F} \subseteq \mathcal{P}(X)$,

(2) $\varnothing \neq \mathcal{R} \subseteq \mathcal{F} \Rightarrow \bigcap \mathcal{R} \in \mathcal{F}$.

EXAMPLE. Let $F = \langle \mathcal{L}, \mathcal{T} \cup \{\varnothing\}\rangle$ where:

$\mathcal{L}$ is the countable set of all propositional sentences for classical propositional calculi;

$\mathcal{T}$ is the set of all theories in language $\mathcal{L}$.

The *proper closure operator* for X is the function $C: \mathcal{P}(X) \to \mathcal{P}(X)$ such that:

$$C(Z) = \bigcap \{F \in \mathcal{F} \mid F \neq \varnothing \;\&\; Z \subseteq F\}.$$

An element $x \in X$ is a *tautology* (*antitautology*) in X, in symbols $x = 1_X$ ($x = 0_X$) provided that $x \in C(\varnothing)$ ($C(\{x\}) = X$). If there is no $x \in X$ such that $x = 1_X$ and $x = 0_X$, then X is *consistent*. We will say that $X = \langle X, \mathcal{F}\rangle$ *satisfies the compactness theorem* if the following condition is fulfilled: $y \in C(Y) \Rightarrow (\exists Y_0)(Y_0 \subseteq Y \;\&\; \bar{\bar{Y}}_0 < \omega \;\&\; y \in C(Y_0))$. *$X$ is a recursive closure space* provided the set $\{\langle y, Y\rangle \mid y \in C(Y) \;\&\; \bar{\bar{Y}} < \omega\}$ is recursive.

X_0 is *recursive embeddable* in X_1 if there is a recursive function $f: X_0 \xrightarrow{1\text{-}1} X_1$ such that:

(i) If $x \in X_0$ and $X \subseteq X_0$ then $x \in C_0(X) \equiv f(x) \in C_1(f(X))$.

(ii) If $x \in X_0$ then $x = 1_{X_0} \equiv x = 1_{X_1}$ and $x = 0_{X_0} \equiv x = 0_{X_1}$.

THEOREM. *A pseudorecursive closure space X is recursive embeddable in $\mathcal{F}$ iff X is a consistent recursive closure space which satisfies the compactness theorem.*

Let $\{z\} \cup Z \in X$. We will say that Z is the *conjunction (disjunction)* of Z in $X = \langle X, \mathcal{F}\rangle$, denoted by $z = \bigwedge_X Z$ ($z = \bigvee_X Z$) provided $C(\{z\}) = C(Z)$ ($C(\{z\}) = \bigcap_{z' \in Z} C(\{z'\})$); $f: X_1 \to X_2$ *preserves all conjunctions (disjunctions) of power less than η* if for every $z \in X_1, Z \subseteq X_1, \bar{\bar{Z}} < \eta$ we have $z = \bigwedge_{X_1} Z \Rightarrow f(z) = \bigwedge_{X_2} f(Z)$ ($z = \bigvee_{X_1} Z \Rightarrow f(z) = \bigvee_{X_2} f(Z)$);

F is prime for $X = \langle X, \mathcal{F}\rangle$ if $F \in \mathcal{F}$ and for every finite subset Z of X

$$(z = \bigvee_X Z \;\&\; z \in F) \Rightarrow Z \cap F \neq \varnothing.$$

$X = \langle X, \mathcal{F}\rangle$ is a *$\langle \omega, \omega\rangle$-regular closure space* provided there is a family $\mathcal{R} \subseteq \mathcal{F}$ such that:

(a) if $X \in \mathcal{R}$, then F is prime for X,

(b) X satisfies the compactness theorem,

(c) if $\langle X, \mathcal{F}'\rangle$ satisfies the compactness theorem and $\mathcal{R} \subseteq F$, then $\mathcal{F} \subseteq \mathcal{F}'$.

We will say that a closure space X_0 is *recursive embeddable with η-conjunctions* (*η-disjunctions*) in a closure space X_1 if there is a recursive function $f: X_0 \xrightarrow{1\text{-}1} X_1$ such that: (i), (ii) and f preserves all conjunctions (disjunctions) of power less than η. Observe that by the definition of a conjunction we have:

REMARK. If X is recursive embeddable in X_1, then for every η, X is recursive embeddable with η-conjunctions in X_1.

THEOREM 2. *A pseudorecursive closure space X is recursive embeddable with ω-disjunctions in F iff X is a consistent $\langle \omega, \omega\rangle$-regular recursive closure space.*

J. VAN DE WIELE, *Recursive dilaters and generalized recursions.*

DEFINITIONS. (i) The category ON:

objects: ordinals.

morphisms: strictly increasing functions.

(ii) The category ON $< \omega$ is the full subcategory of ON with finite ordinals as objects.

(iii) A dilator is a functor from ON to ON which preserves directs limits and pullbacks.

(iv) A recursive dilator D is a dilator which maps ON $< \omega$ into ON $< \omega$ and such that the restriction of D to ON $< \omega$ can be encoded by some recursive function (in the ordinary sense).

THEOREM 1. *If D is a recursive dilator then the function $x \mapsto D(x)$ from ordinals to ordinals is $(\infty, 0)$-recursive (in Hinman's terminology).*

THEOREM 2. *For any function F from sets to sets, which is uniformly-Σ_1-definable over all admissible sets, there exists some recursive dilator D such that for any ordinal x $\mathrm{rk}(F(x)) \leq D(\mathrm{rk}(x))$.*

COROLLARY 1. *Any function F from ordinals to ordinals which is uniformly-κ-recursive for all recursively regular ordinals κ is majorizable by some recursive dilator. Furthermore, F is $(\infty, 0)$-recursive.*

COROLLARY 2. *Any function from sets to sets, which is uniformly-Σ_1-definable over all admissible sets, is E-recursive (in Normann's terminology).*

To establish Theorem 2 we use J.Y. Girard's work in inductive logic which characterizes validity in all inductive models and for which we have a cut-elimination theorem.

REFERENCES

J. BARWISE, ***Admissible sets and structures,*** Springer-Verlag, Berlin, 1975.

J. Y. GIRARD, ***A survey of Π^1_2-logic*** (Hannover, 1978).

———, *Π^1_2-logic. I: Dilators,* ***Annals of Mathematical Logic,*** vol. 21 (1981), pp. 75–219.

———, . . . , Séminaire, Université de Paris-VII.

P.G. HINMAN, ***Recursion-theoretic hierarchies,*** Springer-Verlag, Berlin, 1978.

D. NORMANN, *Set recursion,* ***Generalized recursion theory. II (Proceedings of the Second Symposium, Oslo, 1977),*** North-Holland, Amsterdam, 1978, pp. 321–329.

J. VAN DE WIELE, Thèse de 3[ième] cycle (1981).

MARIE-CHRISTINE ZANDA-FERBUS and MARIE TORRIS, *Bornes fonctorielles pour l'élimination des coupures.*

On démontre le théorème d'élimination des coupures dans la logique $L_{\beta\omega}$ introduite par Girard, en donnant des bornes fonctorielles.

On choisit le calcul des séquents.

Pour tout ordinal x, on définit la notion de x-déduction d'un séquent. Si f est une fonction strictement croissante de y dans x et $\mathscr{D}$ une x-déduction d'un séquent $\Gamma \vdash \Delta$, on définit dans certains cas, une y-déduction, $f^{-1}(\mathscr{D})$, du séquent $f^{-1}(\Gamma \vdash \Delta)$.

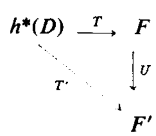
On définit la catégorie Dem dont les objets sont les triplets $(x, \Gamma \vdash \Delta, \mathscr{D})$ où $\mathscr{D}$ est une x-déduction de $\Gamma \vdash \Delta$ et les morphismes de $(y, \Gamma' \vdash \Delta', \mathscr{D}')$ dans $(x, \Gamma \vdash \Delta, \mathscr{D})$ les applications strictement croissantes de y dans x telles que $f^{-1}(\Gamma \vdash \Delta) = \Gamma' \vdash \Delta'$ et $f^{-1}(\mathscr{D}) = \mathscr{D}'$.

Une β-démonstration d'un séquent $\Gamma \vdash \Delta$ est un foncteur D de ON dans Dem tel que $D(x) = (x, \Gamma \vdash \Delta, D_x)$ et $D(f) = f$.

On définit la catégorie β-Dem dont les objets sont les β-démonstrations et les morphismes de D dans D' les transformations naturelles de D dans D'.

Si D est une β-démonstration, on définit un dilatateur $h^*(D)$, hauteur de D, et une majoration de D est un couple (F, T) où F est un dilatateur satisfaisant pour tout $x \in \mathrm{On}$ $h^*(D)(x) \leq F(x)$ et T une transformation naturelle de $h^*(D)$ dans F.

Si Φ est un foncteur de β-Dem dans β-Dem, une majoration de Φ est un couple (Ψ, θ) où Ψ est un foncteur de DIL dans DIL tel que pour toute β-démonstration D, pour toute majoration (F, T) de D, $(\Psi(F), \theta_T)$ est une majoration de $\Phi(D)$ et que si l'on a



on en déduit

$$\begin{array}{ccc} h^*(\Phi(D)) & \xrightarrow{\theta_\Gamma} & \Psi(F) \\ & \searrow^{\theta_{\Gamma'}} & \downarrow{\scriptstyle \Psi(U)} \\ & & \Psi(F') \end{array}$$

On construit un foncteur de normalisation N de β-Dem dans β-Dem tel que si D est une β-démonstration, $N(D)$ est une forme normale de D (i.e. pour tout $x \in$ On si $D(x) = (x, \Gamma \vdash \Delta, D_x)$ alors $N(D)(x) = (x, \Gamma \vdash \Delta, \varphi(D_x))$ où $\varphi(D_x)$ est une x-déduction de $\Gamma \vdash \Delta$ qui n'utilise pas la règle de coupure et pour tout morphisme f de ON, $N(D)(f) = f$).

On construit la majoration (Ψ, Θ) de N.

On procèdera en donnant des formulations ponctuelles, aussi bien dans la construction de N que dans celle de sa majoration: pour tout ordinal x, si $\mathscr{D}$ est une x-déduction, on définit un ordinal $h^*(\mathscr{D})$, hauteur de $\mathscr{D}$, et $\varphi(\mathscr{D})$, la normalisation de $\mathscr{D}$. Une majoration de $\mathscr{D}$ sera un couple (y, f) où y est un ordinal $\geq h^*(\mathscr{D})$ et f, une application strictement croissante de $h^*(\mathscr{D})$ dans y.

Une majoration de φ sera un couple (ψ, θ) où ψ est un foncteur de ON dans ON tel que pour toute x-déduction $\mathscr{D}$, pour toute majoration (y, f) de $\mathscr{D}$ $(\psi(y), \theta_f)$ est une majoration de $\varphi(\mathscr{D})$ et que si l'on a

$$\begin{array}{ccc} h^*(\mathscr{D}) & \xrightarrow{f} & y \\ & \searrow^{f'} & \downarrow{\scriptstyle g} \\ & & y' \end{array}$$

on en déduit

$$\begin{array}{ccc} h^*(\varphi(\mathscr{D})) & \xrightarrow{\theta_f} & \psi(y) \\ & \searrow^{\theta_{f'}} & \downarrow{\scriptstyle \psi(g)} \\ & & \psi(y) \end{array}$$

et de plus que cette majoration soit fonctorielle, relativement aux morphismes de β-Dem.

CARLOS A. DI PRISCO, *Several times huge and superhuge cardinals.*

A cardinal κ is λ-huge if there is an elementary embedding $j\colon V \to M$ such that κ is the critical point of j and M is $j(\kappa)$-closed. We call κ huge α times if there are cardinals $\lambda_0 < \lambda_1 < \cdots < \lambda_\xi < \cdots$ $(\xi < \alpha)$ such that κ is λ_ξ-huge for all $\xi < \alpha$.

PROPOSITION 1. *Let $\alpha \leq \kappa$. If κ is huge α times then for each $\xi < \alpha$ there is a normal ultrafilter on κ concentrating on cardinals which are huge ξ times.*

This shows that the first huge cardinal is λ-huge for a unique λ.

THEOREM 2. *Let μ be supercompact and $\alpha < \mu$. If κ is the first huge α times then $\kappa < \mu$ and the first α cardinals λ for which κ is λ-huge are also smaller than μ. If μ is the first supercompact and κ is the first cardinal μ times huge then $\kappa > \mu$.*

Call a cardinal superhuge if it is λ-huge for arbitrarily large λ's. If κ is superhuge then there is a normal ultrafilter on κ concentrating on supercompact cardinals.

J.M. HENLE, *Forcing on strong partition cardinals.*

We explore various generic extensions of models of ZF + DC containing a strong partition cardinal κ ($\kappa \to (\kappa)^n$ where $n \geq \omega$). Results include:

(1) A Magidor-like forcing which changes the cofinality of κ to δ, adds no bounded subsets of κ and collapses few (if any) cardinals; given $\kappa \to (\kappa)^\delta_\alpha$ for all $\alpha < \kappa$.

This improves theorems of Spector and Apter.

(2) A Radin-like forcing which adds a new member of $[\kappa]^\kappa$, preserving all cofinalities, with the same added properties as in (1); given $\kappa \to (\kappa)^\kappa_\alpha$ for all $\alpha < \kappa$.

(3) A new analysis of Spector forcing which shows that it collapses no cardinals and probably adds no well-ordered sets.

(4) A model in which $\kappa \to [\kappa]^{\aleph_1}$ but $\kappa \not\to [\kappa]^\omega$.

STANISLAW KRAJEWSKI, *Infinity of finite axiomatizations of ZF in the language of GB.*

The family $\mathscr{F}$ of all finitely axiomatizable theories in the language of GB which are conservative over ZF is quite complicated. It contains infinitely many members both above and below GB. There exist infinite pairwise inconsistent subfamilies of $\mathscr{F}$. $\mathscr{F}$ is densely ordered by inclusion. For every $A, B \in \mathscr{F}$ such that $A \subsetneqq B$ the family of all C with $A \subset C \subset B$ is an atomless Boolean algebra. This algebra includes an anti-isomorphic copy of the Lindenbaum algebra of the ZF-consequences of $A \wedge \neg B$. And $A \wedge \neg B \in \mathscr{F}$ for some A, B, while for others $A \wedge \neg B$ is not conservative. Many examples are given.

Identifying elements of $\mathscr{F}$ modulo mutual interpretability we still get an infinite structure: a lower semilattice w.r.t. interpretability. However many members of $\mathscr{F}$ get identified; in particular GB is mutually interpretable in a strong sense, namely preserving the set-universe up to isomorphism, with a finite axiomatization K of ZF made according to a general method introduced by Kleene.

The results hold not only for ZF but also for a much wider class of theories.

JEANLEAH MOHRHERR, *A question of Ershov.*

In the paper *A hierarchy of sets*, III, ***Algebra and Logic,*** vol. 9 (1970), pp. 20–31, Yu. L. Ershov suggests how the hierarchy of differences of recursively enumerable sets that is constructed along recursive ordinals can be relativized to an arbitrary oracle. We show that one of these suggestions fails if $A' \geq_T \mathcal{O}$. This work will appear titled, *A conjecture of Ershov for a relative hierarchy fails near $\mathcal{O}$*, in ***Algebra and Logic.***

ANDRZEJ PELC, *Measures on σ-algebras and Banach's problem.*

A σ-algebra $\mathscr{A}$ of subsets of the interval [0, 1] is called measurable if there exists a probability measure on $\mathscr{A}$ vanishing on atoms of $\mathscr{A}$. Banach asked if the union of two countably generated measurable σ-algebras can generate a nonmeasurable σ-algebra. This problem was solved positively by Grzegorek. The following answers a related question of Galvin and gives a strong solution of Banach's problem under the assumption of Martin's Axiom.

THEOREM 1. *Assume MA. Let $1 < \kappa \leq \omega$. There exist a countable family $\mathfrak{A} = \{A_n: n \in \omega\}$ of subsets of* [0, 1] *and a family $\{\mathscr{A}_\xi: \xi < 2^\omega\}$ of subsets of $\mathfrak{A}$ with the following properties*:

(i) *any union of $< \kappa$ sets $\mathscr{A}_\xi$ generates a measurable σ-algebra*;

(ii) *any union of $\geq \kappa$ sets $\mathscr{A}_\xi$ generates a nonmeasurable σ-algebra.*

The existence of such a family turns out to be strictly weaker than MA.

The second theorem answers an invariant version of Banach's problem and generalizes a result of Prikry and the author.

THEOREM 2. *Assume CH. Let G be a group of permutations of* [0, 1], $|G| \leq 2^\omega$. *There exist countably generated σ-algebras $\mathscr{A}_1$, $\mathscr{A}_2$ on* [0, 1] *and probability measures μ_1, μ_2 on $\mathscr{A}_1$, $\mathscr{A}_2$, respectively, such that*:

(i) *the μ_i vanish on atoms of $\mathscr{A}_i$*;

(ii) *measure completions $\bar{\mu}_i$ and σ-algebras $\bar{\mathscr{A}}_i$ of $\bar{\mu}_i$-measurable sets are G-invariant*;

(iii) *the σ-algebra generated by $\mathscr{A}_1 \cup \mathscr{A}_2$ is nonmeasurable.*

MICHAEL VON RIMSCHA, *Nonfounded constructible sets.*

We look at models of ZF^0 + Sext, where ZF^0 denotes the system ZF minus foundation and Sext stands for the axiom of strong extensionality ("sets with $\in$-isomorphic transitive closures are equal"). For such models one can define a "constructible" hierarchy as follows:

$$LS(\varnothing) = \varnothing;$$

$$LS(\alpha + 1) := \mathrm{Def}(LS(\alpha)) \cup \{r \mid \exists x \exists u \in LS(\alpha)\, (x \text{ trans} \wedge \in \textstyle\bigcap x^2 \text{ iso } u \wedge r \in TC(x))\};$$

$$LS(\lambda) = \bigcup_{\alpha<\lambda} LS(\alpha);$$

$$LS = \bigcup_{\alpha\in \mathrm{On}} LS(\alpha).$$

We call this hierarchy constructible because just as it is the case in the usual constructible hierarchy, at successor-stages on fills in those sets, which are definable from sets, which belong to lower stages. This is also true for the sets belonging to $\{r \mid \exists x \dots\}$—the axiom Sext guarantees uniqueness.

For an arbitrary ZF^0 + Sext-model $\mathscr{J} = \langle I, E\rangle$, the inner model $\mathscr{LS}(\mathscr{J}) = \langle LS, E\restriction_{LS}\rangle$ is a model of ZFC^0 + Sext. Furthermore one easily sees $\mathscr{LS}(\mathscr{LS}(\mathscr{J})) = \mathscr{LS}(\mathscr{J})$.

The structure—also of the well-founded part—of $\mathscr{LS}(\mathscr{J})$ depends widely on the structure of the nonfounded part of $\mathscr{J}$. But it is possible to define from 'outside' a constructible (in the above sense) hierarchy LA, s.t. the union over this hierarchy forms a model $\mathscr{LA}$ of ZF^0 + Sext + $U2$ ($U2$ is the strongest universality condition, consistent with ZF^0 + Sext). For $\mathscr{LA}$ one can show: $\mathscr{LS}(\mathscr{LA}) = \mathscr{LA}$. One has to see which properties of the constructible universe L carry over to $\mathscr{LA}$.

S. VUJOSEVIC, *On the notion of well ordering.*

Within the intuitionistic set theory several alternative definitions of a well ordering are possible. Following the ideas of D. Scott, the independence of the definitions is established by means of sheaf models for set theory. Different concepts of finiteness as well as properties of intuitionistic ordinals are discussed. It transpires that some of those concepts are too weak, and that one cannot justify the ε-recursion over the class of ordinals.

PIOTR BOROWIK, *On Gentzen's axiomatization of the reducts of many-valued logic.*

The presented axiomatization is a dual representation, so-called finitely generated, trees logic due to S.J. Surma (see [2], [3]). In the Gentzen approach additional functors needed in the trees logic are superfluous.

Let N denote the natural numbers, let $V = \{p_i : i \in N\}$ be a set of all sentential variables, and let F be a m-arguments sentential connective. The language on the alphabet $\{p_i : i \in N\} \cup \{F\}$ is an abstract algebra $\underset{\sim}{S} = \langle S, F\rangle$ type $\langle m\rangle$ freely generated by set V, when the set S is defined as follows:

$$\begin{aligned} S_0 &= V, \\ S_{k+1} &= S_k \cup \{x : \bigvee_{x_1, x_2, \dots, x_m \in S_k} x = Fx_1x_2 \dots x_m\}, \\ S &= \bigcup\{S_k : k \in N\}. \end{aligned}$$

Let $\underset{\sim}{E} = \langle E, f\rangle$ be a finite algebra, where $f: E^m \to E$. Without detriment to totality of problem we can assume that $E = \{1, 2, \dots, n\}$. Let $E^* \subset E$ be the set of the distinguished values i.e. $E^* = \{r, r+1, \dots, n\}$, $1 < r \le n$. The algebra $\underset{\sim}{S} = \langle S, F\rangle$ is similar to $\underset{\sim}{E}$. A propositional formula $x \in S$ is a tautology in the matrix $\mathscr{M} = \langle \underset{\sim}{E}, E^*\rangle$ iff $hx \in E^*$ for every homomorphism $h: \underset{\sim}{S} \to \underset{\sim}{E}$.

Let $S_1, S_2, \dots, S_n$ be subsets of S (in particular certain S_i can be empty for $1 \le i \le n$).

A sequent is an ordered n-tuple $\langle S_1, S_2, \dots, S_n\rangle$ of subsets of S. The sequents will be denoted by Σ (with indices if necessary)

The presented system consists of the following axioms and rules.

The sequent $\Sigma = \langle S_1, S_2, \dots, S_n\rangle$ is an axiom iff there exist $j, k \le n, j \ne k$, and a formula $x \in S$ such that $x \in S_j$ and $x \in S_k$.

Rules. Let $\Sigma = \langle S_1, S_2, \dots, S_n\rangle$ and let $k \in E$. For every $\alpha \in E^m$ so that $f(\alpha) = k$, let $\Sigma_\alpha = \langle S_1', S_2', \dots, S_n'\rangle$ be a sequent so that $S_i' = S_i \cup \{x_j : \alpha_j = i\}$ for every $i = 1, 2, \dots, n$, where α_j denotes the jth element of α.

Then the schema of the rule of appending of connective P to the set S_k of the sequence $\Sigma = \langle S_1, \dots, S_k, \dots, S_n\rangle$ will be the following form:

$$\frac{\{\Sigma_\alpha : \alpha \in E^m \wedge f(\alpha) = k\}}{\langle S_1, \dots, S_k \cup \{Fx_1x_2 \dots x_m\}, \dots, S_n\rangle}.$$

We shall call a system $D = \langle P, P', x_0, R\rangle$ a proof tree if:

(i) P is a finite set, the system $\langle P, R, x_0\rangle$ is the lower semilattice with the zero element x_0;

(ii) $P' = \{y : \{x : xRy\} = \emptyset\}$;

(iii) the cardinality of the set $\{x : xRy\}$ is at most n^m.

The sequent $\Sigma = \langle S_1, S_2, \dots, S_n\rangle$ is the end sequent if there exist a formula $x \in S$ and $j \le n$ so that for every i, $1 \le i \le n$, $i \ne j$, $S_i = \emptyset$, $S_j = \{x\}$. The end sequent Σ has a proof on the ground of the set of sequents Γ in the reduct of the logic over the $\underline{E}$ if and only if there exists a proof tree $D = \langle \Delta, \Gamma, \Sigma, R \rangle$ where $\Gamma \subset \Delta$ and the relation R is defined as $\Sigma R \Pi$ if and only if there exists a sequence of sequents $\Sigma_1, \Sigma_2, \ldots, \Sigma_k$ such that $\Sigma_1 = \Sigma$, $\Sigma_k = \Pi$ and for every i, $1 \leq i \leq k$, there exists a rule r_i such that Σ_{i+1} is the conclusion and Σ_i is one of its premises.

A propositional formula x is a theorem in the sequent calculus over $\underline{E}$ if and only if there exist sets of the axioms on the ground of which the following end sequents are proved:

$$\Sigma_1 = \langle S_{11}, S_{12}, \ldots, S_{1r-1}, S_{1r}, S_{1r+1}, \ldots, S_{1n} \rangle,$$
$$\Sigma_2 = \langle S_{21}, S_{22}, \ldots, S_{2r-1}, S_{2r}, S_{2r+1}, \ldots, S_{2n} \rangle,$$
$$\cdots\cdots\cdots$$
$$\Sigma_{r-1} = \langle S_{r-11}, S_{r-12}, \ldots, S_{r-1r-1}, S_{r-1r}, S_{r-1r+1}, \ldots, S_{r-1n} \rangle,$$

where

$$S_{ij} = \begin{cases} \{x\} & \text{if } i = j, \\ \varnothing & \text{if } i \neq j. \end{cases}$$

$\varnothing$ is the empty set.

THEOREM 1. *The formula $x \in S$ is a theorem of the Gentzen system over $\underline{E}$ iff x is a tautology over $\underline{E}$.*

REFERENCES

[1] R.M. SMULLYAN, ***First order logic***, Springer-Verlag, New York, 1968.

[2] S.J. SURMA, *A method of the construnction of finite Lukasiewiczian algebras and its application to a Gentzen-style characterization of finite logics*, **Reports on Mathematical Logic**, vol. 2 (1974), pp. 49–54.

[3] ———, *An algorithm for axiomatizing every finite logic*, **Reports on Mathematical Logic**, vol. 3 (1974), pp. 57–61.

M.R. DONNADIEU, *Semantics of conditional languages.*

A conditional language $\mathscr{L} = \bigcup_{n \geq 0} \mathscr{L}_n$ is defined in Rambaud [1]. A realization of $\mathscr{L}$ is first of all a partial algebra $\mathscr{A} = \langle A, (\bar{f}_n : D_n \subset A^p \to A)_{n \geq 0}, (\bar{R}_i \subset A^q)_{i \in I} \rangle$ having the additional property: D_n is the set of all $\bar{a} = \langle a_1, \ldots, a_p \rangle \in A^p$ satisfying the condition Φ_n. We achieve this by a simultaneously inductive definition of $\mathscr{A}_n$, $\mathscr{D}(t)$ (resp. $\mathscr{D}(\Phi)$) $t^n[\bar{a}]$ and $\mathscr{A}_n \models \Phi[\bar{a}]$.

Let $\mathscr{A}_{n-1}$ be a realization of $\bigcup_{p<n} \mathscr{L}_p$. Define:

1. $\mathscr{A}_n = \mathscr{A}_{n-1} \cup \{\bar{f}_n : D_n \to A\}$ where $D_n = \{\bar{a} \in A^p / \bar{a} \in \mathscr{D}(\Phi)$ and $\mathscr{A}_{n-1} \models \Phi[\bar{a}]\}$.

2. $\mathscr{D}(t)$ (resp. $\mathscr{D}(\Phi)$) is the set of all points where the term t (resp. the formula Φ) is realizable:

$$\mathscr{D}(f(t_1, \ldots, t_p)) = \{\bar{a} \in \bigcap_{1 \leq i \leq p} \mathscr{D}(t_i) / (t_i^n(\bar{a}))_i \in \mathscr{D}(\Phi_f) \text{ for } 1 \leq i \leq p \text{ and } \mathscr{A}_{n-1} \models \Phi_f[\bar{a}]\}.$$

3. The realization $t^n[\bar{a}]$ (resp. $\mathscr{A}_n \models \Phi\,[\bar{a}]$) for $\bar{a} \in \mathscr{D}(t)$ (resp. $\bar{a} \in \mathscr{D}(\Phi)$) by:

If $t = f(t_1, \ldots, t_n)$ then $f(t_1, \ldots, t_p)^n[a] = \bar{f}(t_1^n[\bar{a}], \ldots, t_p^n[\bar{a}])$.

If Φ is $R(t_1, \ldots, t_p)$ and $\bar{a} \in \mathscr{D}(\Phi)$ then $\mathscr{A}_n \models \Phi[\bar{a}]$ iff $(t_1^n[\bar{a}], \ldots, t_p^n[\bar{a}]) \in \bar{R}$.

If Φ is $\Phi_1 \vee \Phi_2$ and $\bar{a} \in \mathscr{D}(\Phi)$ then $\mathscr{A}_n \models \Phi[\bar{a}]$ iff $\mathscr{A}_n \models \Phi_1[\bar{a}]$ or $\mathscr{A}_n \models \Phi_2\,[\bar{a}]$.

If Φ is $\neg\,\Psi$ and $\bar{a} \in \mathscr{D}(\Phi)$ then $\mathscr{A}_n \models \Phi[\bar{a}]$ iff not $\mathscr{A}_n \models \Psi[\bar{a}]$.

If Φ is $\exists x \Psi$ and $\bar{a} \in \mathscr{D}(\exists x \Phi)$ then $\mathscr{A}_n \models \Phi[\bar{a}]$ iff there is b such that $(\bar{a}, b) \in \mathscr{D}(\Psi)$ and $\mathscr{A}_n \models \Psi[\bar{a}, b]$.

Then $\mathscr{A} = \bigcup \mathscr{A}_n$.

Finally $\mathscr{A} \models (\Phi, \mathscr{U})$ iff $\mathscr{A} \models \Phi[a]$ for all $\bar{a} \in \mathscr{D}(\mathscr{U})$.

THEOREM. *Let T be a conditional theory. Then $T \vdash \Phi, \mathscr{U}$ iff $T \models \Phi, \mathscr{U}$.*

PROOF. We take as universe A the subset of all terms of T which probably exist in T. There is a canonical partial algebra $\mathscr{A}$ on A which is also a realization of $\mathscr{L}(T)$ if T is a complete Henkin theory.

One aim of our work is to obtain a description of equational classes of conditional algebras [2].

BIBLIOGRAPHY

[1] C. RAMBAUD, *Syntax of conditional languages* (abstract), this JOURNAL, vol. 48 (1983), pp. 1226–1227.

[2] V.S. Poythress, *Partial morphisms on partial algebras*, ***Algebra Universalis***, vol. 3 (1973), pp. 182–202.

GEORGE K. GARGOV, *On path logics.*

Let $\langle K, R\rangle$ be a Kripke frame, i.e. $K \neq \emptyset$ and $R \subseteq K^2$. A finite or ω-sequence $\sigma = (x_0, \ldots, x_n, \ldots)$ in K is an R-sequence if $x_n R x_{n+1}$ for all n. The infinite and the maximal finite R-sequences are called *paths* in $\langle K, R\rangle$. Denote the set of all paths by $\mathscr{P}$. Then $\langle K, R, \mathscr{P}\rangle$ is a *path structure.*

In this paper we study multimodal propositional logics—*path logics*—with one unary modal operator $\Box$ connected with R in the usual way, and several other n-ary operators ($n \geq 1$) interpreted in path structures.

In particular we give an axiomatization of the minimal logic of $\Box$ and the bar operator ∇ introduced and investigated in another setting by Bowen and de Jongh. ∇ has the following semantics: $x \models \nabla A$ iff all paths σ starting from x contain a y such that $y \models A$. This connective is analogous to the *during* operator of Pratt's process logic. We consider also analogues of *throughout, preserve*, etc.

Finally, a descriptively universal language (cf. [1]) is introduced and the set of its formulas valid in all path structures is characterized.

REFERENCE

[1] H. Nishimura, *Descriptively complete process logic*, ***Acta Informatica***, vol. 14 (1980), pp. 359-369.

JEAN PORTE, *Fifty years of deduction theorems.*

The first printed statement (with proof) of the classical deduction theorem is in Herbrand's thesis (pp. 90-91 in van Heijenoort's edition)—even if Tarski and Jaśkowski had previously used the result.

The theorem, for the classical propositional calculus (PC), being written

(I) $$\text{if } x_1, \ldots, x_n, y \vdash z \text{ then } x_1, \ldots, x_n \vdash y \to z,$$

generalizations have been looked for in several directions:

(1) replacing PC by a weaker formal system;

(2) replacing deducibility ($\vdash$) by a stricter relation;

(3) replacing $y \to z$ by another function, $d(x, y)$;

(4) replacing (I) by a more complex statement linking premisses and conclusion.

The first extensions of the theorem were of type (4): Barcan (1946), Moh Shaw-Kwei (1950), Church (1951), Curry (1954)—see later the "indirect deduction theorems" of Surma (1967).

Reexamination of the works of Moh Shaw-Kwei and of Church led eventually to extensions of type (2) for the relevant implicational systems $R_\to$, $E_\to$, and $T_\to$: see Anderson and Belnap (1975).

Examination of problem (1) led Pogorzelski (1968) to describe the exact scope of the classical theorem among the subsystems of PC.

Extensions of type (3) were found by Porte (1961), for Łukasiewicz's three-valued logic ($d(x, y) = x \to (x \to y)$), and for modal systems S4 and S5, axiomatized with rules of material detachment and of necessitation ($d(x, y) = Lx \to y$, L being necessity). Those results were independently rediscovered by Pogorzelski (1964) and Żarnecka-Biały (1968), who extended them.

Pogorzelski (1964), Surma (1972) and Perzanowski (1973) have published surveys of results of various types in the fields of modal and other propositional systems.

Among problems not yet solved, conspicuous is the lack of negative results, particularly for extensions of type (3): How to prove that no satisfactory functions d exist when their nonexistence may be conjectured?

C. RAMBAUD, *Syntax of conditional languages.*

The mathematician working in foundations by category theory needs a conditional language on diagrams, especially if he wants to make interpretations of one category theory into another. A conditional language on diagrams has partial functional symbols which are defined only if a certain formula (the condition) and a certain graphical configuration (the diagram) are satisfied by the variables to which the symbol applies.

We present here a special case of conditional languages on diagrams, that is, conditional languages of classical logic. Related work can be found in Ebbinghaus [2] and Markwald [3].

A conditional language $\mathscr{L} = \bigcup_{n\geq 0} \mathscr{L}_n$ is given by its symbols $(R_i)_{i\in I}$ and $(F_n)_{n\geq 0}$. R_i is a predicate of arity P_i, belonging to $\mathscr{L}_0$, $F_n = (f_n, \Phi_n)$ where f_n is a functional symbol of arity P_n, and Φ_n (the condition of f_n) is a formula containing at most P_n free variables and belonging to $\bigcup_{0\leq k<n} \mathscr{L}_k$. $\mathscr{L}_n$ is made of all terms and formulas using f_n or symbols already in $\mathscr{L}_k$ for $k < n$. Φ_0 is the formula T (true). Notice that Φ_n may have occurrences of f_k for $k < n$, but there is no vicious circle: If $R(g(x))$ is the condition that f applies to x, then $f(x)$ cannot figure in the condition of g. Our semantics differs from [2] and [3]. In fact, we realize fewer formulas.

For each term t of $\mathscr{L}$ we define the existence condition C_t by induction, where

$$C_{f(t_1\ldots t_p)} = \bigwedge_{1\leq i\leq p} C_{t_i} \wedge \Phi_f(t_1 \ldots t_p) \wedge C_{\Phi_f}(t_1 \ldots t_p).$$

For each formula Φ of $\mathscr{L}$ we define the finite set $\mathscr{U}_\Phi$ of all terms occurring in Φ but having themselves no occurrence of a quantified variable.

Provability is a notion which cannot be defined for a formula Φ alone, but only for a couple $(\Phi, \mathscr{U})$ where $\mathscr{U}$ is a finite set of terms, including $\mathscr{U}_\Phi$. Symbolically, we write $\vdash \Phi; \mathscr{U}$.

The propositional axioms and rules are as usual except for the modus ponens: $\vdash \Phi; \mathscr{U}$ and $\vdash \Phi \to \Psi; \mathscr{V}$ implies $\vdash \Psi; \mathscr{U} \cup \mathscr{V}$. There is an extra rule: $\vdash \Phi; \mathscr{U} \cup \{t\}$ iff $\vdash C_t \to \Phi; \mathscr{U} \cup \mathscr{U}_{C_t}$ where $\mathscr{U}_\Phi \subset \mathscr{U}$. Axioms and rules of the predicate calculus are the usual equality axioms and $\vdash \Phi_x[t] \to \exists x\Phi; \mathscr{U}_{\Phi_x[t]}$ and $\vdash \Phi \to \Psi; \mathscr{U}$ implies $\vdash \exists x \Phi \to \Psi; \mathscr{U} \dot{-} x$ where x is not free in Ψ, quantifiable in $(\Phi, \mathscr{U})$, and where $\mathscr{U} \dot{-} x$ is obtained from $\mathscr{U}$ omitting all terms containing x.

BIBLIOGRAPHY

[1] Marie Renée Donnadieu; *Semantics of conditional languages* (abstract), this Journal, vol. 48 (1983), pp. 1225–1226.

[2] H.D. Ebbinghaus, *Über eine Prädikatenlogik mit partiell definierten Prädikaten und Funktionen*, ***Archiv für Mathematische Logik und Grundlagenforschung***, vol. 12 (1969), pp. 39–53.

[3] W. Markwald, *Prädikatenlogik mit partiell definierten Funktionen*, ***Archiv für Mathematische Logik und Grundlagenforschung***, vol. 14 (1971), pp. 16–23; vol. 16 (1974), pp. 15–22.

Z. Šikić, *Multiple forms of Gentzen's rules and some intermediate logics.*

Gentzen's sequential system is a formalization of classical or intuitionistic logic depending on whether we take its rules in multiple or singular form. Indeed, in the singular system extended by the initial sequents $\to A \vee \neg A$, it is possible to prove at once the permissibility of the multiple forms of all the inference rules.

An analysis of each rule separately shows that the multiple form of the negation or the implication introduction is sufficient for the formalization of classical logic. The multiple form of the universal quantifier introduction is not sufficient for the formalization of classical logic, but it is also too strong for the formalization of intuitionistic logic. The multiple forms of the other rules do not extend the intuitionistic system. The extension of the singular (intuitionistic) system by the multiple form of the universal quantifier introduction is therefore the formalization of an intermediate logic. We call this extension L_2.

We show that the system L_2 is related to Gödel's completeness theorem. Kleene's detailed analysis of the proof of the theorem shows that the only nonintuitionistic assertion used in the proof is of the form $\forall x A(x) \vee \exists x \neg A(x)$, for a certain predicate $A(x)$. Therefore, it is interesting to extend the singular (intuitionistic) system by the initial sequents of the form $\forall x A(x) \vee \exists x \neg A(x)$. We call this extension L_3.

Moreover, Kleene's analysis shows the predicate $A(x)$ to be decidable. Therefore, it is interesting to extend the singular (intuitionistic) system by the initial sequents of the form $\forall x(A(x) \vee \neg A(x)) \to \forall x A(x) \vee \exists x \neg A(x)$. We call this extension L_1.

We prove constructively that L_3 extends (possibly equals) L_2, which extends (possibly equals) L_1.

It is plain that L_3 properly extends L_1. The question remains: Is the system L_2 equal to L_1 or possibly to L_3?

ATWELL R. TURQUETTE, *Herbrand's Deduktionstheorem for M-valued logics.*

In Chapter 3 of his thesis, Herbrand proves what Hilbert and Bernays in their *Grundlagen der Mathematik* (Erster Band, p. 155) later called the *Deduktionstheorem* (herein denoted as DT). Using a minimal set of axiomatic conditions for DT, denoted as $\underline{\text{min}}$ DT, the present paper constructively defines a set Σ of M-valued implications which is such that an implication C is a member of Σ if and only if C satisfies $\underline{\text{min}}$ DT.

A subset Σ^* of Σ is such that for each C of Σ^*, $CMq = 1$. This latter equation may be interpreted as saying that the maximum value of an M-valued logic implies anything. It is shown that the C's of Σ^* can be used to construct elegant axiomatic systems of M-valued logic with S designated values ($1 \leq S < M$). Gödel's M-valued implication G is a member of Σ^*. Recall that $Gpq = q$ if $p < q$ and $Gpq = 1$ if $p \geq q$.

Dual formulas are introduced for effectively counting the number of different implications in both Σ^* and Σ. They reveal a rapid growth relative to M. For example, already for $M = 7$, there are 357 different implications in the set Σ^* and 4824 in the set Σ.

EDUARD W. WETTE, *After the elementary inconsistency-computation: a nontransformable solution to the main problem of applied mathematics.*

The number $\ulcorner(*)\urcorner$ of *one* formally general 'proof' (*) [5, 1.], [7, 1.] entails the (*)-counterexample $\xi = k_A$. (*), "k_A" use nonminimal stilt-patterns (vs. [3, note 6]) with succedents-balancing ⟨16, 0,.⟩'s in each antecedent. After [5, 5.] concerning $\Omega_{4+}(l, e, m, c, a)$ [4], [5, 3.], [7, 7.], [10], the term $\text{sg}(|[k_A]_3 - [\Omega(k_A)]_3|) = 1 \ \& = 0$ is implicitly ${}^7 2$-computable, since ${}^5 2 < k_A < {}^6 2 < \Omega(k_A) <$ ${}^7 2$ [7, 3., 6., 4., 2.]: ${}^n 2 = \psi_3\ (n, 2)$, *if* $\psi_{m+1}(0, a) = \text{sg}(m)$, $\psi_0(n, a) = a + n$, $\psi_{m+1}(n + 1, a) = \psi_m(\psi_{m+1}(n, a), a)$; Péter's $\psi(m + 1, n) = \psi_m(n + 3, 2) - 3$, $\psi(4, n) = {}^{n+3}2 - 3$. k_A can explicitly be evaluated [7, 2., 5.] from an implicit k_A-computation via $< 2^{11}$ variable-free subterms.

1. The pre-Socratic tradition ("everything is .") can be renewed in terms of 'curvature-distribution of the seamless netting on a closed *max*imal line/surface/. . ($\mathfrak{M}^n$)'. The problem *why* mathematical methods are applicable under the actual realities in nature, economics, etc.—Felix Klein 1908 vs. David Hilbert 1904—is solvable by a flattening and equalizing projection $\mathfrak{P}_{f_e}$ of the absolute and complete representation/diagram $\mathfrak{M}^4$ of primary 'motion' onto its own average [1], [5, [8]], [6], [9].

I use intrafinite recursive functions $(\varphi/\tau)(m_1, m_2, d)$ instead of "real" $f(u_1, u_2)$ with $u_1 : u_2 : 1 = m_1 : m_2 : d$, since harmonic triples $\langle \xi_0, \xi_1, \xi_2 \rangle\ (u_1, u_2)$ tear the net on a pouch-wave $\mathfrak{M}^2$ [1, Figure 2⊥] *in singularities.* φ, τ *lead to "fast" algorithms* [8], *whereas the inconsistent analysis* [2] *often yields* practically inefficient procedures.

2. A non-Bernoullian "elastic" line $\mathfrak{M}^1$ consists of a series of N fold-flings $\mathfrak{F}_q$ along a polygon $P_0 \ldots P_{N-1} P_0$ (without self-intersection), whose average forms a regular N-gon on a huge circle $\mathfrak{T}^1$ [4, [12, §2 p. 48]]. The computation of $\mathfrak{F}_q$ started from $\langle \xi, \eta \rangle (u) = {}_0^{s(u)}\!\int \langle \cos, \sin \rangle\, \alpha \cdot ds$, $ds/dv = \exp \varepsilon$, $\langle \alpha, \varepsilon \rangle (v) = {}^v\!\int \langle \sin, \cos \rangle\ \Theta(w) \cdot dw$ [6, 2., Figure 1], where now

$$\Theta(u) = \pi U(2B - AU) \wedge 0 \leq U = u/L \leq 1,$$

and $A = 2B - 1$, $\hat{\theta} = M\pi \geq \pi \rightarrow B = M + \sqrt{\ } M(M - 1) \geq 1$, so that $\Theta = \pi \leftrightarrow U = \tilde{U} \in \{1/A, 1\}$, $|\overline{P_{q-1}P_q}| = L \cdot \tilde{U}$. α, ε are linear combinations of Fresnel's integrals.

3. I calculate

$$\langle \alpha, \varepsilon \rangle (u) = L \cdot \sum_{0}{}_{\nu} \langle a_\nu, e_\nu \rangle \cdot U^\nu,$$

where

$$\langle a_\nu, e_\nu \rangle (M) = \nu^{-1} \sum\nolimits_{\mu = 0..[\nu/4] - \langle \delta^+, \delta^* \rangle(\nu)} (-)^{[\nu/2] - \mu - \langle 1 - \delta(\nu), 0 \rangle} (\nu - 1 - 4\mu - 2 \cdot \langle \delta(\nu), 1 - \delta(\nu) \rangle)!^{-1}$$
$$\times\ (2\mu + \langle \delta(\nu), 1 - \delta(\nu) \rangle)!^{-1}\ (2B\pi)^{\nu - 1 - 2\mu - \langle \delta(\nu), 1 - \delta(\nu) \rangle}\ (1 - 1/(2B))^{2\mu + \langle \delta(\nu), 1 - \delta(\nu) \rangle} \leftarrow \nu > 0,$$

$\delta(\nu) = \nu - 2 \cdot [\nu/2]$, $\delta^+(\nu) = 1 - \delta([\nu/2])$, $\delta^*(\nu) = (1 - \delta(\nu)) \cdot (1 - \delta^+(\nu))$, e.g. $a_1 = e_2 = 0$, and $\langle \xi, \eta \rangle (u) = L \cdot \sum_{1}{}_{\nu} \langle C_{\nu-1}, C_{\nu-1}^+ \rangle \cdot U^\nu/\nu$, where

$$\langle C_\nu, C_\nu^+\rangle\,(L, M) = \sum_0 {}_m L^m \cdot \sum_0 {}_{j_0,\ldots,k_0,\ldots;\lhd} \langle 1 - \delta(j), \delta(j)\rangle \cdot (-)^{[j/2]} \prod_0 {}_\mu a_\mu^{j_\mu}/j_\mu! \cdot \prod_0^\mu {}_\mu e^{k_\mu}/k_\mu!$$

with the indices-condition

$$\lhd \leftrightharpoons \sum_0 {}_\mu j_\mu + \sum_0 {}_\mu k_\mu = m \wedge \sum_1 {}_\mu \mu j_\mu + \sum_1 {}_\mu \mu k_\mu = \nu; \quad j = \sum_0 {}_\mu j_\mu.$$

4. The integration constants a_0, e_0 are determined by $\langle\alpha, \varepsilon\rangle$ (0) or from $\langle\alpha, \varepsilon\rangle$ $(L\tilde{U})$ of the immediately preceding "Faltenwurf" $\mathfrak{F}_{q-1}$, possibly via $\langle\alpha, \varepsilon\rangle(L\tilde{U})$ of $\mathfrak{F}_q$ (!), and, as to a_0, also from $\sphericalangle P_{q-2}\, P_{q-1} P_q$. The equalizing property

$$\sum_1 {}_\nu \langle C_{\nu-1}, C_{\nu-1}^+\rangle \cdot \tilde{U}_\nu/\nu = \langle \tilde{U}, 0\rangle$$

determines $\tilde{L}$, $\tilde{M}$; it yields $\langle\xi, \eta\rangle(\tilde{L}, \tilde{M}; \tilde{u}) = \langle\tilde{u}, 0)\rangle$, where $\tilde{u} = \tilde{L}\cdot\tilde{U}$. The "aesthetic" distribution of points $\langle\xi, \eta\rangle$ (m/d) on $\mathfrak{M}$. thus excludes any transformability. Virtual transformations enter in only after $\mathfrak{P}_{f_e}(\mathfrak{M}^1 \dashv \mathfrak{T}^1)$. Spectra of a_0, e_0, $\tilde{L}$, $\tilde{M}$ come from the closing property that $\mathfrak{F}_N$ precedes $\mathfrak{F}_1$.

5. The universal $\mathfrak{P}_{f_e}$-limit is a unique $++++$-hypertorus $\mathfrak{T}^4$ [4, [12]], [6, 3., Figure 2]. Descriptive geometry substantiates $\mathfrak{M}^1$ and 2-projections of $\mathfrak{M}^2$ [1] (or $\mathfrak{M}^3$, $\mathfrak{M}^4$), eliminates physical concepts, undermines probability-distribution and strategy statistics, rectifies measurement/observation [6], locates psychoanalytic concepts [9], disentangles language [11] and old-fashioned formulas.

REFERENCES

[2] EDUARD W. WETTE, XLV, pp. 412–413. Errata, p. 412: line −8, $\vdash\bar{\mathfrak{S}}_1$; line −3, ϕ_m "e_{kl}; line −2, for $\bigcup\dot{u}$ read $\bigcup_{\dot{u}}$. Errata, p. 413: line 6, for first $_\circ$ read ; line 9, for ν read υ.

[4] ———, XLIV, pp. 474–476. Errata, p. 475: line 16, insert $\langle$ behind first comma; line 17, for = read = $\langle$, and replace the 1st/3rd occurrence of ***el*** by *el*; correction, line 30, for 80, 26, 27, 43, 49 read 86, 29, 30, 48, 54, resp.

[5] ———, Second Leeds address, 1979, August 06. XLVI, pp. 443–445; delete second ri in (*).

[1][3][6] ———, XXXIX, p. 409 [1]; XLI, p. 273 [5] = p. 280 [2]; [5, [13]], 1979, August 27, section 8, pp. 201–205.

[7] ———, *Inconsistenza dell'induzione matematica* (translation of [5, [12]] by Alessandro Atti) ***Nominazione 1*** (La sfida di Peano), Milano 1980, pp. 153–158. Formal errata, 29-line paper.

[8] ———, *Critical remarks on the tradition of so-called "proofs" and on the future of mathematics*. Cf. also ***International Logic Review***, vol. 21 (1980), pp. 5–24.

[9] ———, *Morphe, prototype and aspect*, ***4th International Congress of Psychoanalysis (Le Semblant)***, Milano 1981, January 28-31.

[10] ———, *The refutation of number theory*. II. (Complete Ω-definition: >300 subfunctions, ~0.1 megabit of definitional information.)

[11] ———, *Metempsychosis mathematized*, ***5th International Congress of Pychoanalysis (Sex and Language)***, New York, 1981, April 30-May 02.

[12] ———, *Theometric consequences of the contradiction within 2-valued propositional logic*, ***6th International Wittgenstein Symposium (Language and Ontology)***, Kirchberg am Wechsel 1981, August 24-30.

GERHARD HEINZMANN, *La philosophie de la géométrie et de la logique de Ferdinand Gonseth* (1890–1975).

Une idée centrale dans l'oeuvre de Gonseth est d'essayer de considérer une connaissance géométrique comme "construction schématique". Dans cette construction les structures géométriques 'formelles' doivent être obtenues au moyen d'un "processus d'abstraction" à plusieurs niveaux à partir d'éléments concerts.

Ensuite la géométrie sert comme paradigme pour un modèle de connaissance générale—précisé pour la logique—conciliant "dialectiquement" empirisme et rationalisme rigide.

Tandis que dans la théorie réductive de la démonstration se fondant sur Hilbert, les suppositions plus ou moins 'évidentes' entrées dans une théorie mathématique se mesurent à la compréhension des procédés dans la métalangue acceptés comme 'constructifs', Gonseth détermine le surplus 'hypothétique' d'une théorie par le degré de schématisation des constructions dans le langage des objets. La notion de schéma utilisée rappelle le schème génétique de Piaget, mais sans que l'explication de iuris soit réduit à une explication de la genèse empirique.

Gonseth trouve la connexion entre la géométrie et la logique en définissant les objets de la logique comme des schémas développés dans différentes axiomatisations géométriques et considérés *en tant que* schémas. Par conséquent 'la' logique n'est pas une science empirique au sens usuel du mot, mais une "technique" avec schémas en tant que schémas, bref, une "physique de l'objet quelconque". L'expression gonsethien "l'objet quelconque *est* (*purement*)" et "l'objet quelconque *est* (*ordinairement*)" se laisse maintenant interpréter comme une diction métaphorique pour marquer un aspect 'type-token' au schéma en tant que schéma. D'autre part, la relation entre "être" et "vérite" correspond par la suite au rapport de la construction de schéma et d'une énoncé d'exactitude sur cette construction.

YVON GAUTHIER, *Le constructivisme de Herbrand.*

On sait que, sur le plan fondationnel, l'influence déterminante sur Herbrand à été la métamathématique de Hilbert qui rend compte de la prépondérance de la théorie des démonstrations dans l'oeuvre de Herbrand. Même quand il semble introduire des notions sémantiques ensemblistes comme champ infini [1, p. 135], Herbrand "constructivise" ces notions (par exemple, par le biais du concept de champ réduit [1, p. 138] ou partiel [1, p. 225]).

Par ailleurs, Herbrand pense tirer de sa méthode—conjonction et disjonction des formules—un théorème constructif général: les méthodes transcendantes (ou analytiques) ne peuvent permettre de démontrer en arithmétique de théorème qu'on ne puisse démontrer sans leur aide [1, p. 152]. On peut dire ici qu'avant les résultats de Selberg sur le théorème des nombres premiers et sur les séries de Dirichlet (1949), Herbrand a montré plus de clairvoyance qu'un Lautman, par exemple. En d'autres mots, les moyens constructifs assurent seuls la "constructibilité des objets mathématiques" [1, ibid.], ce qui va à l'encontre de la théorie hilbertienne des "ideale Elemente" que Herbrand veut défendre [1, p. 163]. Le finitisme hilbertien de Herbrand s'accommode bien cependant des aspects constructifs (c'est-à-dire non métaphysiques) de l'intuitionnisme brouwerien, comme il s'en explique dans une note [1, p. 225].

Il convient donc de montrer la cohérence de la position finitaire de Herbrand qui, malgré certaines erreurs techniques, s'est montré soucieux de défendre un point de vue constructiviste, sans doute plus près de Hilbert que de Brouwer, que son inclination philosophique lui a permis d'ériger non certes en forme de "programme", mais plutôt en termes d'un véritable projet mathématique.

REFERENCE

[1] JACQUES HERBRAND, ***Ecrits logiques,*** Presses Universitaires de France, Paris, 1968.

RADOVAN SEDMAK, *A note on poliautomata.*

It is well known that poliautomata are computationally universal. We use them to compute integrals of a certain class of functions on $\boldsymbol{Z}$ directly via a congruence relation defined that suggests a hierarchy of functions on $\boldsymbol{R}$ based on their integral complexity.

JACEK CICHOŃ, *On the compactness of some Boolean algebras.*

DEFINITIONS. For an infinite cardinal κ and $p \in [\kappa]^{<\omega}$ denote by $\hat{p}$ the set $\{q \in [\kappa]^{<\omega}: p \subseteq q\}$. The family $\{\hat{p}: p \in [\kappa]^{<\omega}\}$ is a basis for a filter F on $[\kappa]^{<\omega}$. Let $B_\kappa = P([\kappa]^{<\omega})/F$. We say that a Boolean algebra B is λ-compact if for each family $Z \subseteq B - \{0\}$, with $|Z| \leq \lambda$, if $\inf Z = 0$ then $\inf Z_0 = 0$ for some finite $Z_0 \subseteq Z$.

Results. (1) If $\mathrm{cf}(\kappa) = \kappa$ then B_κ is ω_1-compact.

(2) If MA holds and $\omega \leq \kappa < 2^\omega$ then B_κ is ω_1-compact.

(3) If $\kappa^\omega = \kappa$ then B_κ is not ω_1-compact.

(4) CON(ZFC) $\rightarrow$ CON(ZFC + $2^\omega = \omega_2$ + B_{ω_1} is not ω_1-compact).

Result (3) gives a negative solution of the problem 5c from a paper of B.M Benda, ***On saturated reduced products, Pacific Journal of Mathematics,*** vol. 39 (1971), pp. 557–571.

EWA GRACZYŃSKA, *Remarks on regular and symmetric identities.*

We shall consider varieties of algebras of type τ: $T \to N$ where N denotes the set of positive integers. Our nomenclature and notation are basically those of [3], [4]. Following J. Płonka an identity $p \equiv q$ is called regular if the set of variables occurring in the polynomial p is the same as that in q. An identity $p \equiv q$ is symmetric if $p = q$ or if none of p and q is a variable. If K is a variety then $E(K)$, $R(K)$, $S(K)$ denote the set of all (regular, symmetric) identities satisfied in K, respectively. For a given set Σ of identities of type τ, $E(\Sigma)$ and $S(\Sigma)$ denote the set of all (symmetric) consequences of Σ, respectively.

Given a variety K, with $E(K) \neq R(K)$ and an identity e from $E(K) - R(K)$. Our Theorem 1 describes a method of indicating a proof of an identity $p \equiv q$ of $E(K)$ starting from the set $R(K) \cup \{e\}$. In [1] we gave syntactic proofs of:

THEOREM 2. *For a given variety* K, $E(K) = E(S(K) \cup \{e\})$ *for any identity* e *from* $E(K) - S(K)$.

THEOREM 3. *Let* K_1, K_2 *be varieties of type* τ. *Then* $S(E(K_1) \cup E(K_2)) = E(S(K_1) \cup S(K_2))$ (*comp.* [2]).

Our procedure is effective.

REFERENCES

[1] E. GRACZYŃSKA, *Proofs of symmetric identities*, preprint.

[2] E. GRACZYŃSKA and F. PASTIJN, *Proofs of regular identities*, ***Houston Journal of Mathematics***, vol. 8 (1982), pp. 61–67.

[3] E. GRACZYŃSKA and A. WROŃSKI, *On normal Agassiz systems of algebras*, ***Colloquium Mathematicum***, vol. 40.1 (1978), pp. 1–8.

[4] G. GRÄTZER, ***Universal algebra***, Springer-Verlag, Berlin, 1979.

[5] J. PŁONKA, *On equational classes of algebras defined by regular equations*, ***Fundamenta Mathematicae***, vol. 64 (1969), pp. 241–247.

JAN BRZUCHOWSKI, *Point-arity—the new cardinality index for ideals.*

Let I be any σ-ideal with Borel basis on the real line **R**.

DEFINITION. $a(\mathrm{I}) = \min\{\kappa : (\exists X \subseteq \mathrm{I})(\forall \mathscr{A} \subseteq X) \bigcup \mathscr{A} \in \mathscr{B}(\mathrm{I})\ \&\ \bigcup X \notin I\ \&\ X \text{ is point } < \kappa\}$.

Recall (see e.g. BRZUCHOWSKI, CICHOŃ and WEGLORZ, *Some applications of strong Lusin sets*, ***Compositio Mathematicae***, vol. 43 (1981), pp. 217–224) that by $\alpha(\mathrm{I})$ we denote the least cardinality of a partition of some Borel set not in I into sets from I, by $\beta(\mathrm{I})$ we denote the least cardinality of some subset of **R** not in I, and by $\gamma(\mathrm{I})$ we denote the additivity of I. In BRZUCHOWSKI, CICHOŃ, GRZEGOREK and RYLL-NARDZEWSKI, *On the existence of nonmeasurable unions*, ***Bulletin de l'Académie Polonaise des Sciences Séries des Scienques Mathématiques***, vol. 27 (1979), pp. 447–448 we have proved that $\omega < a(\mathrm{I})$. Moreover the following hold:

THEOREM 1. $\beta(\mathrm{I}) < \alpha(\mathrm{I}) \Rightarrow a(\mathrm{I}) = \mathrm{On}$.

THEOREM 2. $\gamma(\mathrm{I}) = \alpha(\mathrm{I}) \Rightarrow a(\mathrm{I}) \leq \alpha(\mathrm{I})^+$.

THEOREM 3. $\pi b(\mathscr{B}(\mathrm{I})) \leq \alpha(\mathrm{I}) \Rightarrow \alpha(\mathrm{I}) \leq a(\mathrm{I})^-$.

ALEKSANDAR JOVANOVIĆ, *Real valued measures.*

The additivity of a measure μ is defined as

$$\mathrm{add}(\mu) = \min_{x \subseteq \mathrm{dom}(\mu)} |x| \ (\mu(\bigcup x) > 0\ \&\ \forall y \in x\ \mu(y) = 0).$$

The norm of a measure μ we define by $\|\mu\| = \min_{x \in \mathrm{dom}(\mu)} |x|\ (\mu(x) > 0)$. If the norm is equal to the cardinality of the measure index, the measure is uniform. The consequence of the well-known theorem of Ulam is that additivity of real valued measures is either $\leq 2^\omega$ or at least equal to the first measurable cardinal (when measure is atomic). However, there is no constraint on the measure norm. The above helps to define real valued large cardinals by simply replacing $\{0, 1\}$ by $[0, 1]$ measures in the definitions of large cardinals, other properties remaining the same. Consider for example real valued (strongly) compact cardinals. We can prove the consistency of the existence of a real valued compact cardinal $\leq 2^\omega$ relative the consistency of ZFC + "there is a compact cardinal". The following problem is open.

Con ZFC + "there is a RVC $\kappa \leq 2^{\omega}$" $\Rightarrow$ Con ZFC + "there is a compact cardinal"?

REFERENCE

R. Solovay, *Real-valued measurable cardinals*, ***Axiomatic set theory***, Proceedings of Symposia in Pure Mathematics, vol. 13, part I, American Mathematical Society, Providence, R.I., 1971, pp. 379–428.