Existential graphs and proofs of pragmaticism

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Abstract

We show how Peirce's architectonics folds on itself and finds local consequences that correspond to the major global hypotheses of the system. In particular, we study how the pragmaticist maxim (i.e., the pragmatic maxim fully modalized, support of Peirce's architectonics) can be technically represented in Peirce's existential graphs, well-suited to reveal an underlying continuity in logical operations, and can provide suggestive philosophical analogies. Further, using the existential graphs, we formalize — and prove one direction of — a "local proof of pragmaticism," trying thus to explain the prominent place that existential graphs can play in the architectonics of pragmaticism, as Peirce persistently advocated. Finally, we present a web of "continuous iterations" of some key Peircean concepts (maxim, classification, abduction) that supports a "lattice of partial proofs" of pragmaticism.

Keywords: *pragmaticism; existential graphs; proof; modalities; local; global*

1. Existential graphs reflections inside Peirce's architectonics

Back-and-forth osmotic processes are fruitful companions in Peirce's architectonics. In fact, constructing local reflections of global trends can be seen as a consequence of the permanent crossing of structural arches in Peirce's system (pragmaticist maxim, categories, universal semeiotics, indetermination-determination adjunction, triadic classification of sciences), a weaving that produces natural communicating hierarchies and levels in the edifice. One of the basic *abductions* that supports this proposal holds that Peirce's system constitutes a natural apparatus to *correlate* in a refined way global and local, total and partial, continuous and discrete. It is a hypothesis that can be supported by enough *inductions*. In this paper, we try to support it through local bounded

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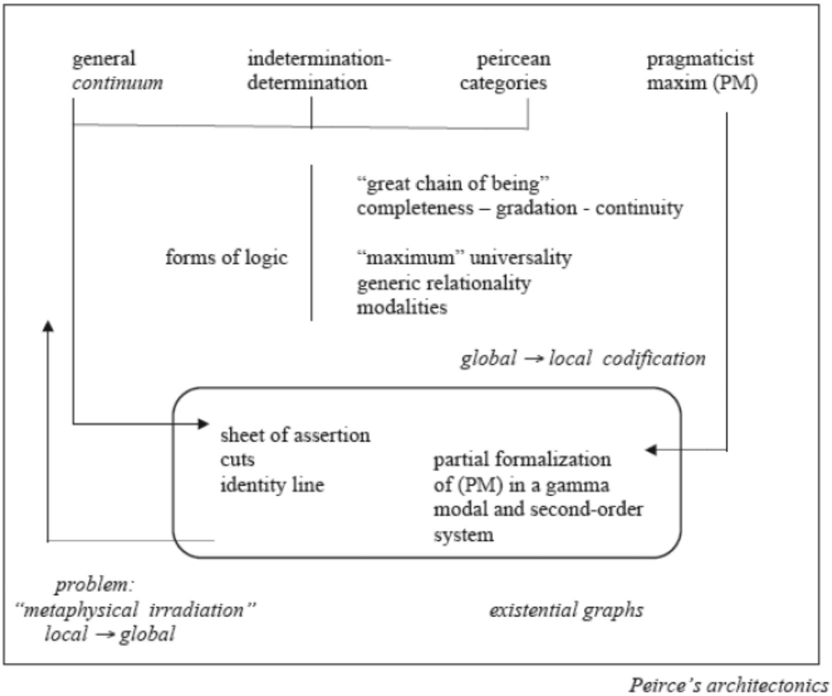


Figure 1. *Various level reflections of pragmaticist architectonics. The global continuum inside the local continuum of existential graphs. The modal form of the pragmatic maxim inside a system of gamma existential graphs.*

deductions. In figure 1, we synthesize a fold of Peirce's global architectonics on some of its local fragments.

Peirce's systems of existential graphs — his "*chef d'oeuvre*" (*NEM* 3: 2.885, 1908) — reflects *iconically* his entire philosophical edifice. The alpha sheet of assertion, a continuous sheet on which the graphs are marked, stands as an iconic reflection of *real* non-degenerate continuity (thirdness), while the beta line of identity, a continuous line that opens up the possibility of quantifying portions of reality, stands as an iconic reflection of *existence* degenerate continuity (secondness):

Since facts blend into one another, it can only be in a continuum that we can conceive this to be done. This continuum must clearly have more dimensions than a surface or even than a solid; and we will suppose it to be plastic, so that it can be deformed in all sorts of ways without the continuity and connection of parts being ever ruptured. (*CP* 4.512, 1903)

. . . the Phemic Sheet iconizes the Universe of Discourse, since it more immediately represents a field of Thought, or Mental Experience, which is itself directed to the Universe of Discourse, and considered as a sign, denotes that Universe. Moreover, it [is because it must be understood] as being directed to that Universe, that it is iconized by the Phemic Sheet. So, on the principle that logicians call "the *Nota notae*" that the sign of anything, X, is itself a sign of the very same X, the Phemic Sheet, in representing the field of attention, represents the general object of that attention, the Universe of Discourse. This being the case, the continuity of the Phemic Sheet in those places, where, nothing being scribed, no *particular* attention is paid, is the most appropriate Icon possible of the continuity of the Universe of Discourse — where it only receives *general* attention as that Universe — that is to say of the continuity in experiential appearance of the Universe, relatively to any objects represented as belonging to it. (*CP* 4.561, note 1, 1906)

Among Existential Graphs there are two that are remarkable for being truly continuous both in their Matter and in their corresponding Signification. There would be nothing remarkable in their being continuous in either, or in both respects; but that the continuity of the Matter should correspond to that of Significance is sufficiently remarkable to limit these Graphs to two; the Graph of Identity represented by the Line of Identity, and the Graph of Coexistence, represented by the Blank. (*NEM* 3: 4.324, c.1906)

These quotes show the importance Peirce assigned to self-reference processes inside his system. Adequate symbolic concretions of the self-reference principle "*nota notae*" are observed both in the empty sheet of assertion and in the line of identity, graphs that continuously match their forms and meanings. Looking closely to the line of identity, Peirce analyzes further its full richness as a general sign, where iconical, indexical, and symbolic tints blend together (*CP* 4.448, c.1903).

In fact, Peirce's line of identity can be considered fairly as the more powerful and "plastic" (in Peirce's *continuum* sense) of the symbolic conceptual tools that he introduced in the "topological" logic of existential graphs. Coherently with that plasticity, an adequate handling of a *thicker* identity line (existential quantifier in a second-order logic), will be the basis of our approach to a "local proof of pragmaticism." We follow Peirce's indication in *MS* 693, brought up by Don Roberts (1973: 75): "*The Gamma Part* supposes the reasoner to invent for himself such additional kinds of signs as he may find desirable." The thick identity line, representing second-order existential quantification, is such an "invention." Next, we remind briefly the basic properties of alpha, beta, and gamma existential graphs needed to proceed (for a full presentation, see Roberts 1963; Zeman 1964; Thibaud 1975; Burch 1991).

Through a pragmatic collection of systems, the existential graphs cover classical propositional calculus (system of alpha graphs and generic illative transformations), first-order classical logic over a purely relational language (system of beta graphs and transformations related to the identity line), modal intermediate calculi (systems of gamma graphs and transformations related to the broken cut), and fragments of second-order logic, classes and metalanguage handlings (specific “inventions” of new gamma graphs). Over Peirce’s *continuum* (generic space of pure possibilities), information is constructed and transferred through general action-reaction dual processes: *insertion-extraction*, *iteration-deiteration*, *dialectics yes-no*. The realm of Peirce’s *continuum* is represented by a blank sheet of assertion where, following precise control rules, some cuts are marked, through which information is introduced, transmitted, and eliminated. The diverse marks progressively

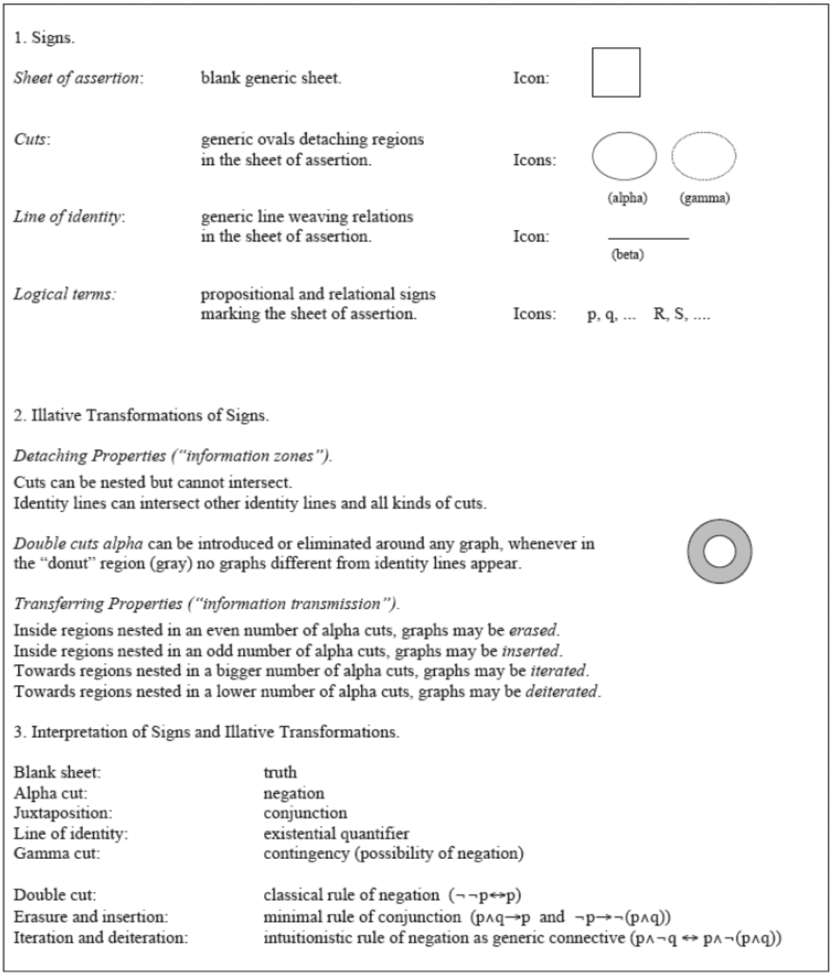
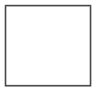
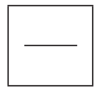
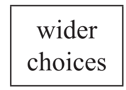


Figure 2. *Rudiments of existential graphs*

registered in the sheet of assertion allow logical information to *evolve* from indetermination to determination, thanks to a precise triadic machinery: (1) formal graphical languages, (2) illative transformations, (3) natural interpretations, all well-intertwined in a pragmatic perspective.

The existential graphs variety of formal languages and illative transformations can be turned into logical *calculi* if one assumes surprisingly elementary axioms:

– axioms:  (Alpha)  (Beta) 

(Gamma)

– *calculi*: Alpha ≡ Classical propositional calculus
Beta ≡ Purely relational first-order logic
Gamma$_{\text{I}}$ ≡ Intermediate modal logics
Gamma$_{\text{II}}$ $\supseteq$ Second-order logic.

The proofs of equivalences Alpha or Beta (we will use versals for alpha, beta or gamma considered as logical systems, with well-defined rules) are far from being obvious: the conjectures are due to Peirce, proofs to Roberts (1963) and Zeman (1964). The best treatment of Gamma modal systems is to be found in Zeman (1964: 140–177). Zeman shows that the Gamma calculus extending Alpha to the broken cut without restrictions in the iteration and deiteration rules corresponds to a Lukasiewicz modal calculus, while other Gamma extensions with *restrictions on iteration and deiteration* through broken cuts correspond to Lewis' systems S4 and S5. Peirce hoped that the existential graphs could help to provide a full "apology for pragmaticism" (*CP* 4.530, 1906). In fact, in all due justice, the very existential graphs looked at themselves — under the perspective that Roberts' and Zeman's completeness proofs have supplied — provide an outstanding apology for the deep pragmatic approach that Peirce undertook in logic (see figure 3).

Indeed, the *simultaneous* axiomatization of classical propositional calculus and purely relational first-order logic, with the *same five generic rules* (double

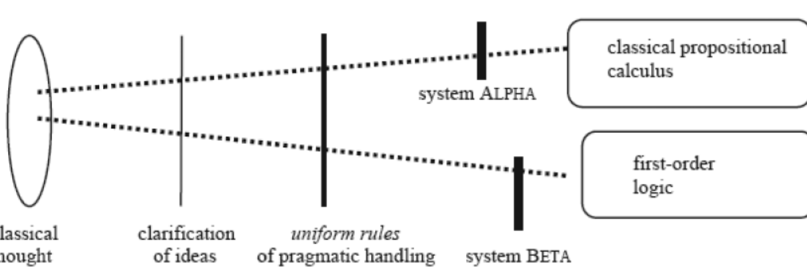


Figure 3. *Existential graphs as an "apology for pragmaticism"*

alpha cuts, insertion, erasure, iteration, and deiteration), renders explicit *technical common roots* for both *calculi* that have been ignored in all other available presentations of classical logic. The *same rules* detect, in the context of alpha language, the handling of classical negation and conjunction, and, in the context of beta language, the handling of the existential quantifier — something unimaginable for any logic student raised into Hilbert-type logic systems. Thus — in agreement with Peirce's pragmatic maxim and Peirce's "idealist" realism — the Alpha and Beta *calculi* show that there exists a *kernel*, a *"real general"* for classical thought, a kernel that, in some representational contexts, gives rise to the classical modes of *connection*, and that, in other contexts, gives rise to the classical modes of *quantification*. The common roots for classical connectives and quantifiers are revealed in *common* pragmatic action-reaction processes, global and general, which in *diverse* representational contexts generate derived rules, local and particular, proper to each context. We thus face a truly remarkable "revelation" in the history of logic, not yet fully understood nor valued in all its depth. It is, in a very precise way, the *only* known presentation of classical logical *calculi* that uses the same global and generic axiomatic rules to control the "traffic" of connectives and quantifiers.

In turn, the "apology for pragmaticism" obtained with the existential graphs shows the coherence of the synechist abduction, at least if it is restricted to the continuum underlying classical logic. In fact, the existential graphs show that the rules of classical connectives and quantifiers correspond continuously to each other over a generic bottom; their apparent differences are just contextual and can be seen as breaks on the underlying logical continuity. But even beyond the classical realm, we count on several mathematical supports to conjecture that the synechist hypothesis can span a wider range of validity, including — fair abduction — diverse progressive forms of the logical *continuum* (intuitionistic, categorical, peircean) up to — bold abduction — the cosmological *continuum*.

A pair of examples, where (going from local to global) we re-interpret some specific "marks" of the graphs, can be useful to show the possible interest of a "metaphysical irradiation" of the graphs. In first place, as shown in figure 4, the immediate comparison of axioms for the ALPHA, BETA, and GAMMA$_{II}$ (second order) *calculi*, shows symbolically that existence (first- and second-order lines of identity) can be seen, *simultaneously*, as a continuity break in the "real

Figure 4. *Axioms for existential graphs calculi*

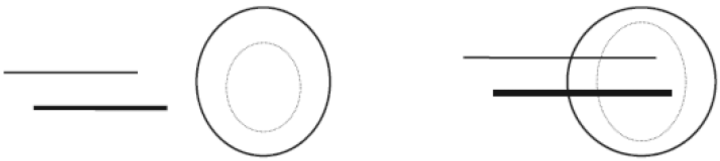


Figure 5. *Continuous iterations (and deiterations) of lines of identity. Existence (actuality, secondness) is continuously linked to real possibilities.*

general" (blank sheet of assertion), and as a continuity link in the "particular" realm (ends of the identity line).

The identity lines, continuous sub-reflections of the sheet of assertion, are self-reflexively marked on the general *continuum* and allow to construct the transition "from essence to existence" (Lautman 1977: 10–18). The elementary axioms of the basic systems of existential graphs thus support the idea — central in philosophy (from Pre-Socratics to Heidegger) — that a first self-reflection of "nothingness on nothing" (in Veronese's full intentional sense: a fluid primigenial *continuum*) can be the initial spark that puts in motion the evolution of the cosmos.

In second place, the continuous iterations of lines of identity (beta or gammaII) through cuts (alpha or gamma; see figure 5) show that existence is no more than a form to link continuously fragments of actuality inside the general realm of all possibilities. It would be fallacious, then, as Peirce severely advocated in his "disputes against nominalists," to think the existent, the actual, the given, *without previously assuming* a coherent continuous bottom of real *possibilia*, a bottom needed in order to guarantee the *relational emergence* of existence.

2. A local proof of pragmaticism

In 1903, in his Harvard lectures, Peirce thought he had imagined a "proof of pragmaticism" (1997; see also *EP* 2: 398–433, "Pragmatism," 1907). Of course, such a proof, in an absolute and global sense, could not be sustained and would go in opposite direction to the pragmatic maxim. Nevertheless, the impossibility of an absolute proof does not preclude that some fragmentary and local codings of the proof could, in principle, be realized. Peirce insisted that the existential graphs should help in that task, but it seems that he never fully completed the scattered indications left in his latter writings. The problem of the "proof of pragmaticism" has been one of the crucial open problems in Peircean scholarship (see, for example, Robin 1997: 145–146).

> I beg leave, Reader, as an Introduction to my defence of pragmatism, to bring before you a very simple system of diagrammatization of propositions which I term the System of Existential Graphs. For, by means of this, I shall be able almost immediately to deduce some important truths of logic, little understood hitherto, and closely connected with the truth of pragmaticism. (*CP* 4.534, 1906)

An immense majority of Peirce scholars considers "faulty" or mistaken the connections that Peirce sought between the existential graphs and proofs of pragmaticism (see, for example, Zeman 1964: 177). Our position, instead, seeks to retrieve and further advance the richness of those connections, following Esposito (1980: 228), who considers that the existential graphs "not only appear to establish the truth of the pragmatic maxim *philosophically* in the form of a deduction, but also *pragmatically* and *inductively* by affording an efficient logical system."

As Peirce writes, "It is one of the chief advantages of Existential Graphs, as a guide to Pragmaticism, that it holds up thought to our contemplation with the wrong side out, as it were" (*CP* 4.7, 1906). The fact that existential graphs help to contemplate "with the wrong side out" the proof of pragmaticism can be interpreted as an indication that the proof has to be strongly *modalized* (as here we try). The *reverse* of the sheet of assertion is not just the world of non-existence, but also the world of *possible* existence. In fact, the situation could further be enriched if Peirce's full geometry of the graphs would be implemented beyond the limitations of the actual planar interpretation: "Existential graphs . . . must be regarded only as *projection* upon a surface of a sign extended in three dimensions. *Three dimensions* are necessary and sufficient for the expression of all assertions" (*MS* 654, cited in Esposito 1980: 227).

We now present a translation of the "full modal form" of the pragmaticist maxim to the language of existential gamma graphs, indicating advances and limitations in our approach. In particular, a formalization of the maxim, *half-way provable in a modal second-order gamma system*, shows that the maxim can acquire new supports for its validity. Indeed, beyond the clear usefulness of the maxim as a global philosophical method (abductively stated, inductively checked), it is also of precious value to count on a reflection of the maxim as a valid local theorem (deductively inferred). Peirce's pragmaticist maxim, always considered by Peirce as a hypothesis, thus obtains a new confirmation by means of a logical apparatus. The three dimensions of reasoning (abduction-induction-deduction) become strongly welded together. If — in the future — the structural transfer from local to global fostered in part by pragmaticism becomes better understood, the local gamma proofs of pragmaticism could then acquire an unsuspected relevance to support the general architectonics of the system.

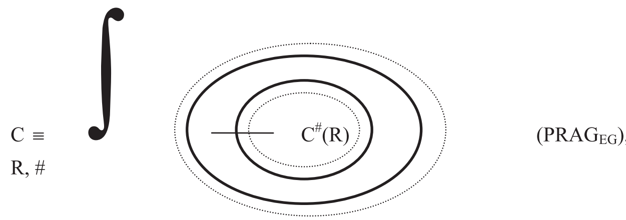


Figure 6. *Semiformal modal pragmaticist maxim*

In first instance, combining the notion of "integral" (relational gluing) and the formalism of gamma graphs, we can obtain an intermediate, *semi-formal*, statement of the pragmaticist maxim. The value of semi-formality (or "informal rigor") consists in allowing further refinements, depending on the way the "integral" is afterwards rendered symbolically in adequate gamma systems. An intermediate expression of the pragmaticist maxim is the universal closure of the following statement, obtained directly as a diagrammatic translation of the full modal form of the maxim to a "mixed" language with existential graphs (semi-formal "mixtures" involving symbols ≡ and ∫ will soon be deleted; see figure 6).

That is, for all C, "C is equivalent to the integral of all necessary relations between interpretants of C and elements of their contexts, running on all possible interpretative contexts." With the usual logical symbols this can also be written semi-formally:

$$\forall \mathrm{C}\left(C \equiv \int_{R,\#} \Diamond \exists x \, \Box \, \mathrm{C}^{\#}(\mathrm{R}, \mathrm{x}) \right).$$

The pragmaticist maxim, understood semi-formally as the (universal closure of) the intermediate statement ($\mathrm{PRAG}_{\mathrm{EG}}$), can then be implemented locally in diverse gamma fully formal systems, in which ($\mathrm{PRAG}_{\mathrm{EG}}$) may become a *theorem* of the system. As the implementation will be more *faithful*, and the gamma system will be more *universal*, the pragmaticist maxim will acquire greater deductive strength. We proceed now to an *elementary* implementation of the maxim in a *specific* gamma system, closely related to Peirce's general realism (scholastic reality of universals, where the possibly necessary becomes actual). The implementation is still far from being duly faithful (codifies all interpretants in just one universal sign), and the gamma system is still away from true universality (requires the axiom $\Diamond\Box\, p \leftrightarrow p$), but we think that an important step in a local proof of pragmaticism is here undertaken.

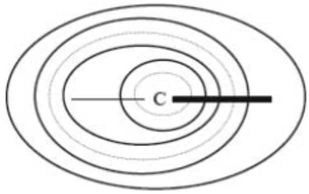


Figure 7. *Modal pragmatic understanding*

Consider the following:

$$(\text{PRAG}_{\text{EG}})\text{: } C \equiv \int_{R,\#} \Diamond \exists x \, \Box \, C^{\#}(\text{R, x}).$$

Identifying # with *identity* (use of the self-reference principle "*nota notae*": codification of all interpretants of a sign in the sign itself), and translating the integral ∫ as a *universal quantification* on all relations, we see that the right-hand side of (PRAG_{EG}) can be represented as in figure 7 (where the thicker line stands for a gamma second-order existential quantifier). The "second step" (construction of diagrams) in the mathematical proof of a theorem is thus fulfilled. Peirce considered that the construction of diagrams could be "the weakest point in the whole demonstration" (*MS* 1147; see Roberts 1978: 125). In our approach, after the diagram is constructed, experimentation, observation, and deduction follow, as advocated by Peirce.

Now, using the rules of erasure, deiteration, and double alpha cut elimination, it is shown that this diagram (which we can call the "pragmatic reading of C") illatively *implies* the diagram in figure 8. In the horizontal order of illative inference, the specific uses of the rules are: erasure of lines of identity (beta, gamma) in an even area (6 nested cuts alpha *and* gamma); deiteration of identity lines beta and gamma to regions with lower number of cuts (4 around beta line, 1 around gamma line); erasure of the beta identity line in an even area (4 cuts) and apparition of the gamma second-order axiom (thick line unenclosed); deiteration of the all gamma identity line; erasure of the gamma line in an even area (0 cuts) and twice double alpha cut elimination.

Thus, the diagram representing the "pragmatic reading of C" does in fact imply C, *in the case in which the double broken cut may be erased*, that is when the modality $\Diamond\Box$ can be eliminated.

This shows that *one* of the two implications in the equivalence that constitutes a local form of the pragmaticist maxim (the "positive" implication according to which the pragmatic knowledge of C guarantees the knowledge of C) can be proved in systems where $\Diamond\Box\, p \rightarrow p$; that is, in systems in which the possibly necessary implies the actual. On the other hand, the reverse implica-

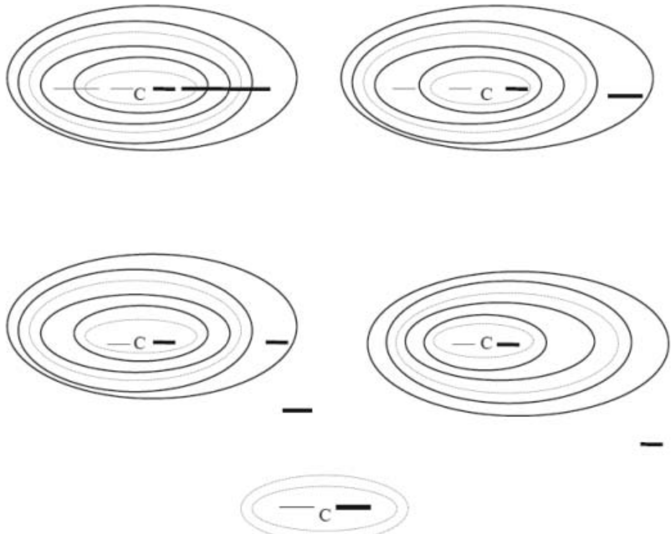


Figure 8. *Illative deductions from "the pragmatic reading of C"*

tion does not seem to be provable, not even in case we could count on *introducing* double broken cuts (corresponding to a full equivalence $\Diamond\Box$ p ↔ p). Observe that the problem lies in the first step of the deduction, which cannot be reversed: the *erasures* of the lines of identity in the 6 nested cuts area cannot be turned into *insertions* and gluings of extended new identity lines, for which we would need to be in an odd area. We can call this reverse implication the "negative" one: the denial of one of the conceivable characters of C implies not-C. Arguably, this "negative" implication can be considered the more interesting one from the perspective of a fallibilist architectonics such as Peirce's, showing that our advance in weaving *graphs-pragmaticism* is still a modest one. To obtain a fuller equivalence between C and its pragmatic reading, a finer implementation of the pragmaticist maxim would have to be achieved, but we hope our tentative implementation opens the way to such a *possibilia* realm.

Our reflection of the global pragmaticist maxim — half-way provable in a local setting of gamma graphs — can be considered as a further indication (*induction*) of the eventual correction of the general maxim. Peirce had proposed the maxim as a hypothesis (*abduction*) to be criticized, contrasted, and refined. An important trend of research would then consist in obtaining other interesting implementations of the maxim that could become theorematic (*deduction*) in other gamma systems. Our methodology follows closely the pragmaticist maxim itself: to capture the *actual* maxim, it has been locally represented in a given context and therein his *necessary* logical status has been studied. The vertical gluing of many theorematic implementations of the maxim would be closer to a generic "proof of pragmaticism."

3. Lattice of partial proofs of pragmaticism

A sound use of the pragmaticist maxim — applied reflexively to itself in a self-unfolding *continuum*, helping to understand better its eventual "proof" — shows that arguments in favor of pragmaticism can never be set in a definitive way, in an absolute space. Indeed, as the maxim itself advocates, any argument that hopes to attain a certain degree of *necessity* has to be set *locally* in a determined interpretation context. From this elementary observation, it follows that the "proof of pragmaticism" sought by Peirce *may* (in fact, *must*) be seen as a sophisticated *lattice of partial proofs*, where along diverse hierarchical levels converge local abductions, inductions, and deductions that *may* (*must*) correlate each other, but that can never be summarized in a unique "transcendental deduction." Peirce's architectonics shows, in fact, that knowledge is always constructed along different perspectives, floors, and levels — like Borges' tower of Babel, doubly-infinite, never comprised in a unique glance — without a "transcendental" or "absolute" vantage point from where a complete panorama could be stared at (observe that the non-existence of such a "point at infinity" is perfectly linked with the non-existence of privileged points in Peirce's *continuum*).

Inside Peirce's architectonics, it is thus natural to emphasize some *argumentative mixtures* (confluences abduction-induction-deduction) that build up the lattice of supports for pragmaticism. It may be said that all of Peirce's work

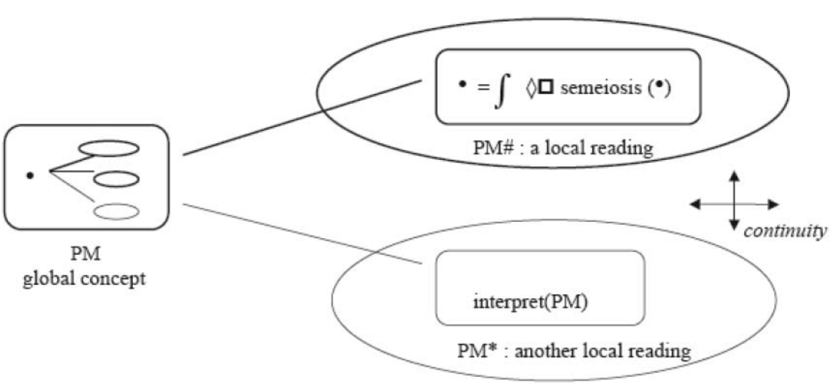


Figure 9. *The pragmaticist maxim (PM) applied to itself: PM(PM). Infinite ramification of Peirce's architectonics. Continuous lattice of local proofs of MP.*

— from his first timid logical comments to his final daring cosmological speculations — consists in the meticulous and perseverant construction of that lattice, always trying to enlarge consistently its range of validity, to extend its depth and to correlate its diverse "marks." Of course, we face *a lattice of marks sketched over a continuous bottom*, where, once again, what plays an extraordinary role is the *natural correspondence* between a general philosophical trend, the world that supports it, and the methods that seek to prove it. In the following, we will study just some of the marks supporting pragmaticism, which are closely related to the *continuum*: the existential graphs as "apology for pragmaticism," the central place of the pragmatic maxim in the classification of sciences, the self-referential and "fixed-point" arguments sustaining pragmaticism, and, finally, the "logic of abduction."

One of the finer marks in support of Peirce's pragmaticism is a natural "continuity interpretation" of some peculiar features of the existential graphs. On one side, the genesis of the graphs (Roberts 1973) shows clearly that they were constructed *continuously*, departing from diagrammatic experiments related to the logic of relatives, coming abductively to propose basic rules and ideas (entitative graphs), and afterwards making permanent corollarial illations, inductively contrasted and polished (entries in the *Logic Notebook*), up to constructing truly theorematic *systems* of existential graphs. It is interesting to notice that this process of discovery takes full advantage of the argumentative triad abduction-induction-deduction, and that it *only* uses that mixture. Since the result is the *simultaneous* re-construction of both classical propositional calculus and first-order logic, which can be considered as a neat basis for the main general qualitative and quantitative modes of thought, the construction of the existential graphs shows that Peirce's argumentative triad may include the *continuum of all possible types of arguments* representable in classical thought. In this way, the pragmaticist hypothesis stating that the triad abduction-induction-deduction *saturates* all inferential processes obtains an important backing: another "mark" in our lattice-type "proof of pragmaticism."

On the other side, the construction of the existential graphs should be understood as a full "apology" for pragmaticism and synechism, not only because of the unveiling of the "real general" for classical thought that we have already discussed, but also because of its ability to represent pragmatically — in its *language*, *rules*, and *axioms* — deep local reflections of the global continuous trends present in the architectonics. The language of existential graphs iconically reflects the cosmological *continuum* (thirdness), its continuity breaks (secondness), and its chance elements (firstness): the alpha sheet of assertion and the beta line of identity are plastic fusion operators (thirdness), the alpha cuts are segmenting marks that depart from the real general and give rise to actual existence (secondness), the gamma cuts are fissures that open the way to chance and possibility (firstness). The rules, or illative transformations, reflect in an outstanding pragmatic way the more elementary osmosis occurring in semeiosis: registering and forgetting information (rules of insertion and erasure), detaching and transgressing dual information zones (rules of introduction and erasure of double alpha cuts), transferring and recovering information (rules of iteration and deiteration). Finally, the axioms, as already mentioned, can be thought as a nutshell expression of Peirce's wider general synechism.

If, following Peirce, we understand the pragmatic maxim as a part of "methodeutics" ("studying methods to be followed in the search, exposition and application of truth," *EP* 2: 260, 1903), its place in the "perennial" classification of sciences lies naturally in the trichotomic subdivision 2.2.3.3, a prominent central place inside the classification that supports generality layers above it and profits from particularization layers below, as Richard Robin (1997: 145–146) has pointed out. Going deeper, and *extending continuously* Robin's fundamental remark, we may understand pragmaticism as a continuous irradiation of the maxim — more precisely, as its *continuous iteration and deiteration — from place 2.2.3.3 towards all other neighborhoods* of knowledge present in the classification (see figure 10).

The diagram in figure 10 suggests another useful argument to consolidate the *global web of local marks* that may support the "proof of pragmaticism."

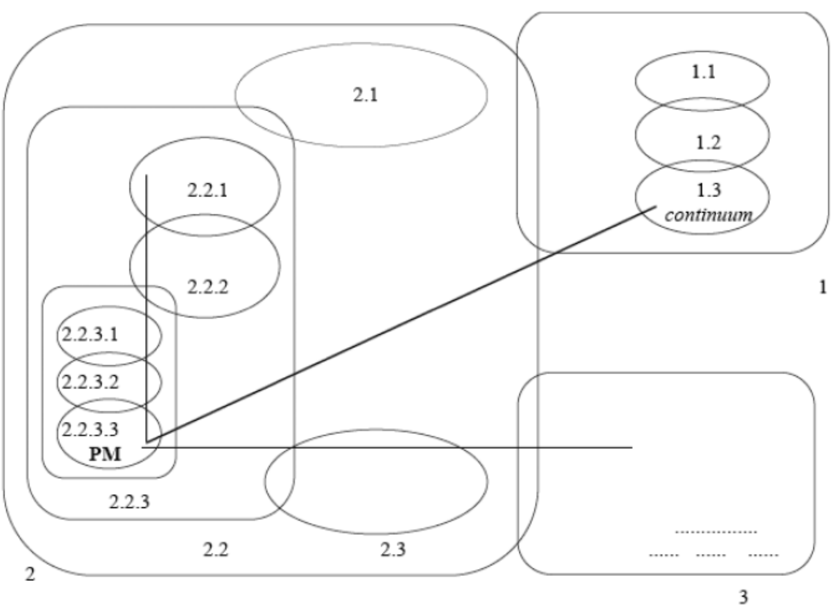


Figure 10. *Continuous iterations of the pragmatic maxim (PM) along a continuous unfolding of the triadic classification of sciences*

The diagram suggests to construct an adequate *translation* of the classification into existential graphs, a translation that should perhaps be *inverse* (or done in a sheet *verso*) to the one represented in figure 7 — where regions with *more* trichotomic ramifications in the classification tree should be surrounded by *less* cuts — in such a way that the pragmatic maxim could really be *iterated* towards all other neighborhoods in the classification. An even finer implementation would have to introduce also the *types* of gamma cuts that should be nested *iconically* around fragments of the classification: possibility (broken-alpha) cuts for trichotomies of type 1, actuality (alpha) cuts for trichotomies of type 2, necessity (alpha-broken-alpha) cuts for trichotomies of type 3. If this kind of translation could be done, we could pass from *discrete* models for the classification (trees with ramification 3) to *continuous* models (assertion neighborhoods, natural osmosis), producing thus a coherent sub-determination of Peirce's synechism. An effective continuous implementation of figure 7 could also help to understand, not only the central irradiation of the maxim in all fields of knowledge, but also the natural pre-eminence of some crossings between disciplines *in detriment* of others, constructing thus the prolegomena of a true "topographical" science that could determine "heights" and "access roads" in the continuous relief of knowledge. Such a continuous unfolding of Peirce's classification of the sciences seems here to be hinted at for the first time. In part, it corresponds to Pape's view that hypotheses should be considered as "singularities" in the space of continuous logical relations (Pape 1999: 248–269). We contend, in fact, that the discrete branching classification of the sciences may be seen as a sort of singularity, to be further embedded in the continuous space of gamma graphs. The embedding of the *discrete* triadic branching into *continuous* gamma graphs would also substantiate Hausman's forceful insight that *possibilia are loci of branching* (Hausman 1993: 185–189).

The central place of the pragmatic maxim in the classification of sciences allows one to perceive the maxim as a *balance environment* in a wide structure. In turn, pragmaticism can also be understood as a generic *fixed-point* technique, a reflexive and self-referential apparatus which, through each self-application, stratifies the field of interpretation. Peirce's fourth article in *The Monist* series was going to present

> a theory of Logical Analysis, or Definition [which] rests directly on Existential Graphs, and will be acknowledged, I am confident, to be the most *useful* piece of work I have ever done . . . Now Logical Analysis is, of course, Definition; and this same method applied to Logical Analysis itself — the definition of definition — produces the rule of pragmaticism. (*MS L* 77, 1909; Fisch 1986: 372)

Habits turn out to be fixed-points of the self-referential operator *definition of the definition*, since its definition resorts to the very same term which is being defined. Now, the fact that habits can be seen as fixed-points connects again in a very natural way the architectonics of pragmaticism with its underlying *continuum*. Indeed, it can be shown in modern mathematics that, underneath *any* fixed-point theorem lies a natural topology that renders *continuous* the fixed-point operator and that allows one to construct the fixed-point as a *limit* of discrete approximations. The local results of modern mathematics, abductively and continuously transferred to the global design of the architectonics, provide thus another "mark" that pulls taut the web of supports of pragmaticism. For future endeavors remains the task of modeling — inside the mathematical theory of categories — an integral translation of some the differential "marks" we have been recording: the "free" iconicity of existential graphs, the iterative "universality" of the pragmatic maxim, the "reflexivity" of habits.

The pragmaticist maxim, fully modalized, depends crucially on a range of *possible* interpretation contexts, where some hypothetical representations are subject to further deductive inferences and inductive contrasts. Peirce's *logic of abduction* — understood as a system to orderly adopt hypotheses with respect to *given* contexts — lies then at the very core of pragmaticism. From the natural correlation "pragmaticism :: logic of abduction" it follows that another "vague" proof of pragmaticism — another mark in its supporting web — should be looked for in an adequate continuous understanding of the "logic of abduction," that is, of the abductive inference, illation, and decidability processes. Abduction's "perfectly definite logical form" (1997: 245, 1903) arises in Peirce's early studies around "vague" variations of the Aristotelean syllogism (*W* 1: 505–514, 1866).

Understood as a *system* to provide reasonable hypotheses that could explain irregular states of things, Peirce's abduction develops between 1870 and 1910, accurately defining the system's tools in accordance with the general dictate of logic to evolve towards progressive determination. The "logic of abduction" refines Peirce's prior ideas on the "logic of discovery": its ability to undergo experimental testing, its capacity to explain surprising facts, its economy, its simplicity, its plausibility, its correlation with the evolved instinct of the species (*HP* 2: 753–754, 1901). Led by his breakthroughs in the logic of relatives, Peirce moves from describing analytically the *particular predicative* form of syllogistic abduction towards constructing synthetically abduction as a *general relational* system: contextual and contrasting handling of hypotheses, optimization, and decision "filters" to maximize the likelihood of adequate hypotheses, search of correlations between the complexity of hypotheses and their probability of correction.

The logic of abduction, as Peirce himself describes very precisely, tries to explain in a systematic way *regularity breaks* and homogeneity disorders, along given contexts, that go beyond simple casual (punctual) irregularities. In fact, explanation is only really needed when it goes *beyond particulars* and

Deductive systems	Abductive systems
$\dfrac{\Gamma \vdash \alpha \rightarrow \gamma}{\Gamma, \alpha \vdash \gamma}$	$\dfrac{\Gamma \vdash \alpha \rightarrow \gamma}{\Gamma, \gamma \vdash \Diamond\alpha}$
	$\Gamma, \gamma \vdash \mathrm{Prob}(\alpha)$

In general, there are important correlations between the conclusion's complexity *in context's eyes* (Γ-Compl(γ)) and the probability of the explanatory correction of the hypothesis (Prob(α)). The higher the complexity (Γ-Compl(γ)), the more plausible becomes the equivalence $\mathrm{Prob}(\alpha) \equiv \alpha$ along the context Γ, *reversing* thus the inference.

Figure 11. *Abduction as a system of logical approximation towards correctness and optimization of explanatory hypotheses*

when it fuses into the general (the *continuum*). Abduction reintegrates breach and context from a higher perspective, and fuses them in a common explanatory *continuum*. Thus, the deep task of the logic of abduction may be seen as locally *gluing* breaks in the *continuum*, by means of an arsenal of methods that select effectively the "closer" explanatory hypotheses for a given break and which try to "erase" discontinuities from a new regularizing perspective (see figure 12).

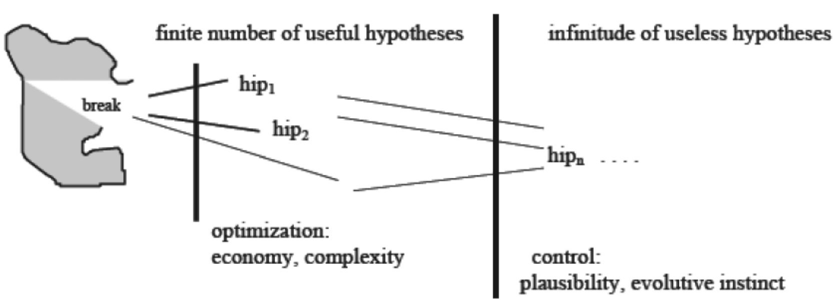


Figure 12. *Abduction as "gluing" breaks in the continuum. "Optimal" selection of explanatory hypotheses.*

Thus, the logic of abduction becomes in fact one of the basic supports of Peirce's pragmaticist architectonics and general synechism. Abduction serves as a regulatory system for the Real, for that plastic weaving (third) formed by facts (seconds) and hypotheses (firsts), where hypotheses are subject to complexity tests until they continuously fuse with facts. The logic of relatives —

which filters technically continuity and generality — serves also as a crucial "filter" in the logic of abduction: it is the natural apparatus that provides the *normal forms* of hypotheses, in order to study their adequate complexity.

Beyond Murray Murphey's famous judgment on the ineffective use of continuity to hold Peirce's architectonics ("Peirce was never able to find a way to utilize the continuum concept effectively. The magnificent synthesis which the theory of continuity seemed to promise somehow always eluded him, and the shining vision of the great system always remained a castle in the air" [Murphey 1961: 407].), we hope to have been able to show that Peirce's "castle" — very *real*, but not reducible to existence — is far from flying in the air.

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