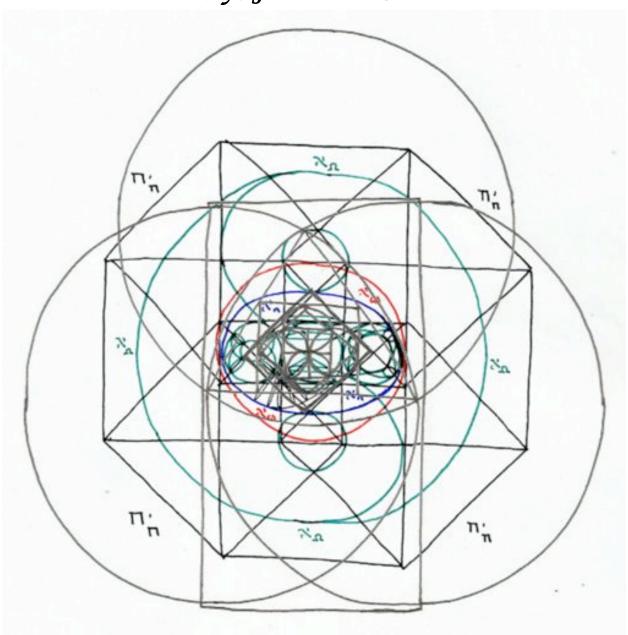
HARMONIA

Volumes 1 & 2

by: Jonathan Gee



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by Jonathan Barlow Gee (Jon Gee)

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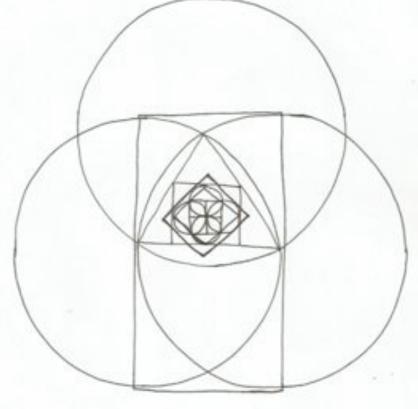
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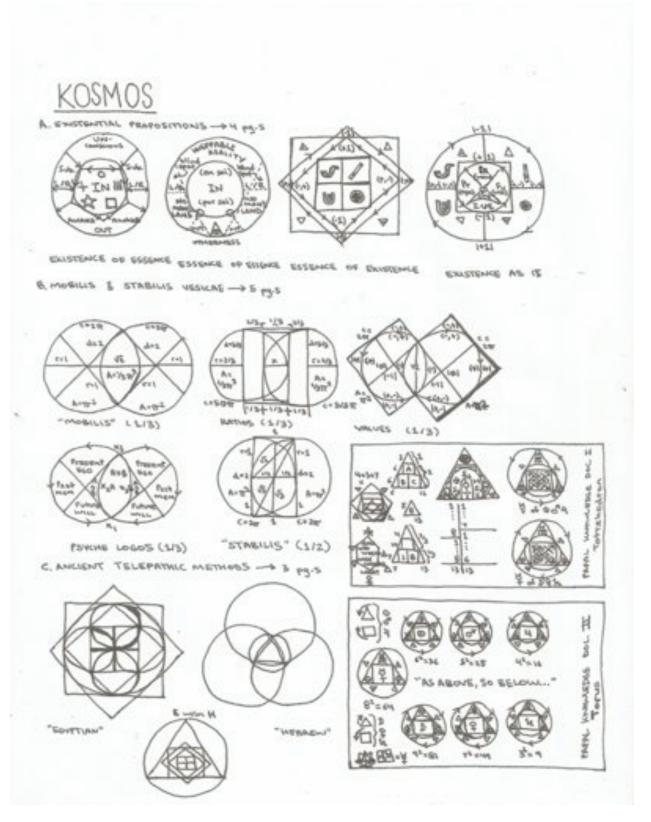




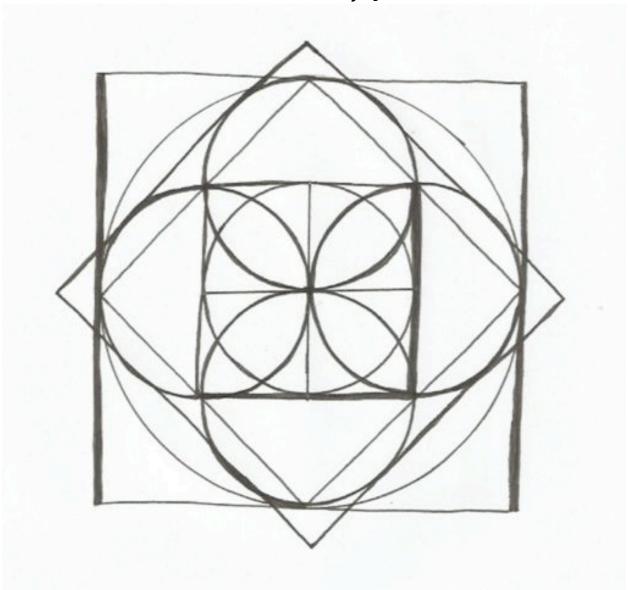
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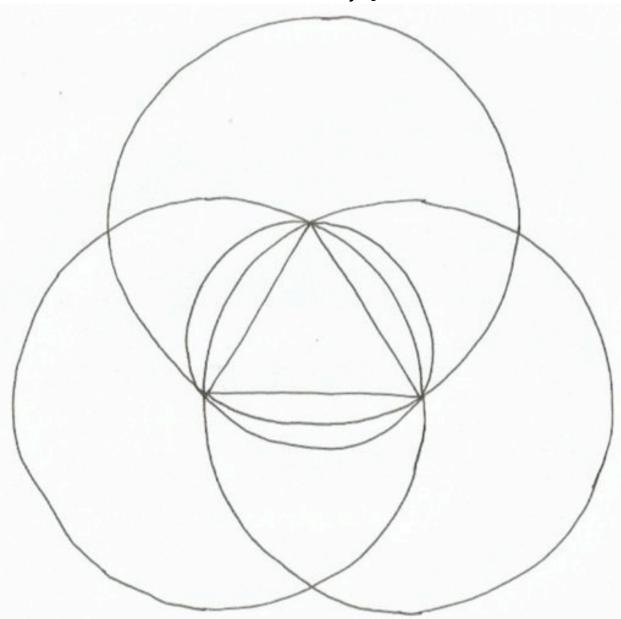


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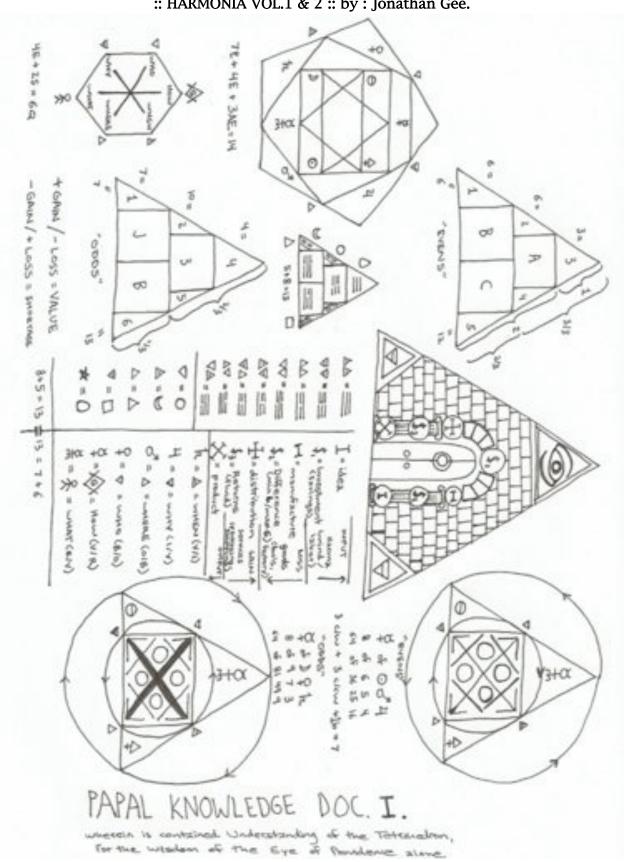
Such is the ancient "Egyptian" method for social "mind control" over the individual. In this model, the "individual" is symbolized by the central circle, while 4 exterior circles surround and overlap this, symbolizing in turn the manifestations of the 4 cosmic elements, such that: cup = water = gravity, pipe = air = electromagnetism, fasces = fire = fission, bread = earth = fusion. By integration and, ultimately, rotation of these overlapping 4 external circles within the central circle, symbolic of the individual, the circumference of the interior circle is, it was believed, thereby expanded to that of the larger circle, encompassing all four. Expressed in geometrical absolutes, this process was measures as the "square" or integer-exponent one greater than the central circle's diameter, expressed as a "square" surrounding the circle's circumference. Thus, the terrestrial elements are, via the consumption of their material manifestations, elevated to the level of the cosmic elements over time, according to the "Egyptian" method for "controlled mind-expansion."

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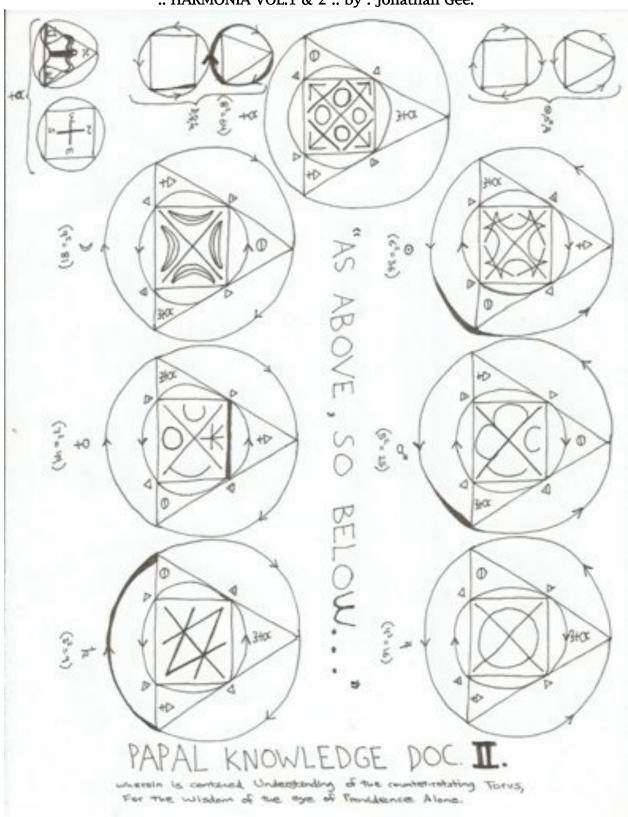


Such is the ancient "Hebrew" method for social "mind control" over the individual. In this model, the "individual" is symbolized by the central circle, while 3 exterior great-arcs surround and each partially define the shared circumference of the central circle. Within the central circle symbolic of the "individual" is a triangle, later called the "triangle of summoning," or of "conjuring" using "creative visualization" in the craft of ritual magick. This triangle, within the circle symbolic of the "individual," is formed between the mid-points of the three great-arcs surrounding and defining it, and defines their shared areas as being such that: for each "great-arc," 1/3 of its total area overlaps other areas while 2/3 remains its own; and that, for this 1/3 of each "great arc's" total area that shares overlap with the others, 1/3 shares space with the great-arc adjacent to it on its left and 1/3 with that on its right, 1/3 shares space with both other great-arcs, forming the central circle.

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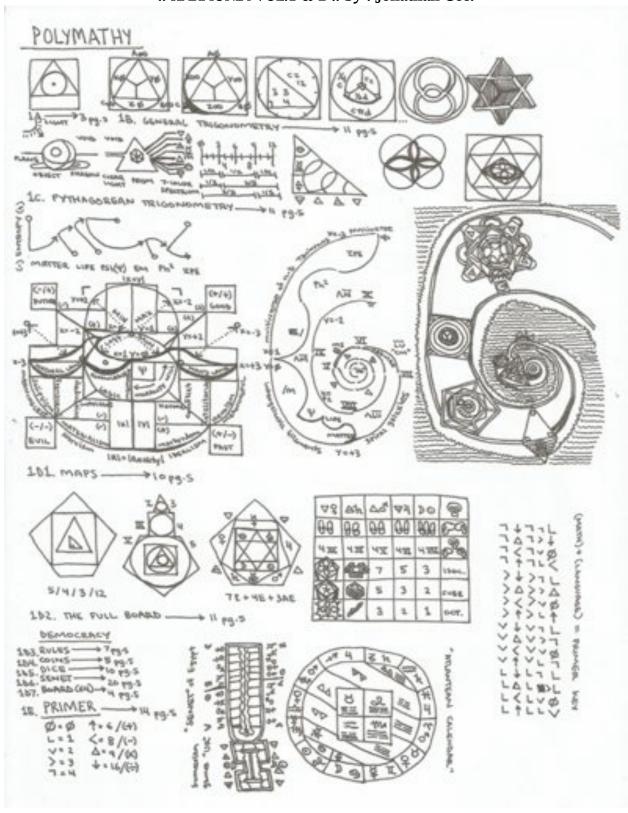


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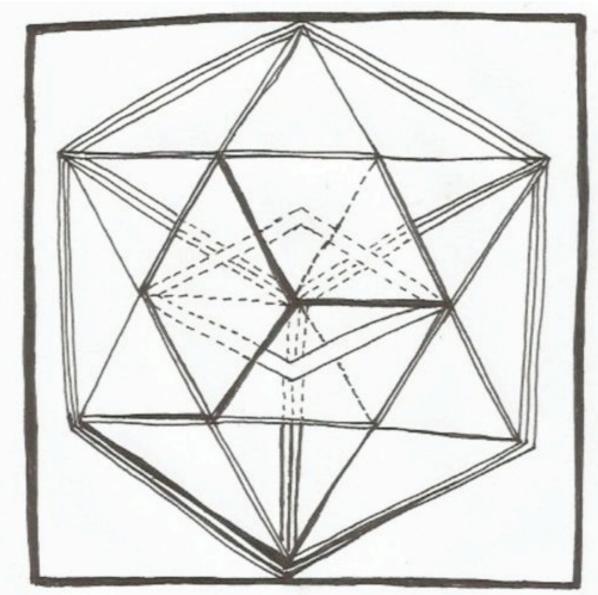


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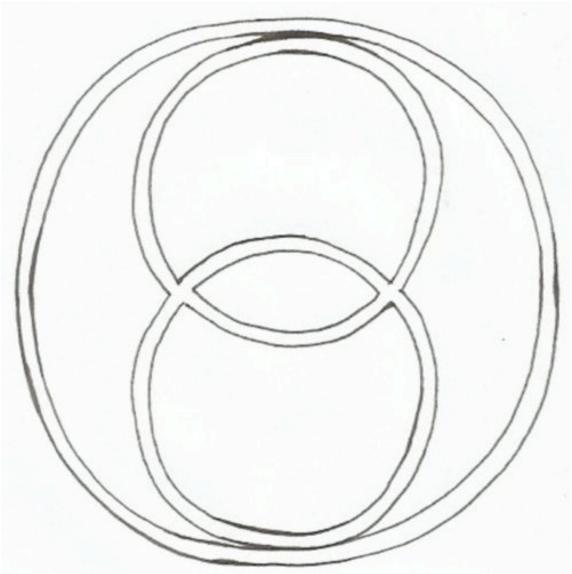


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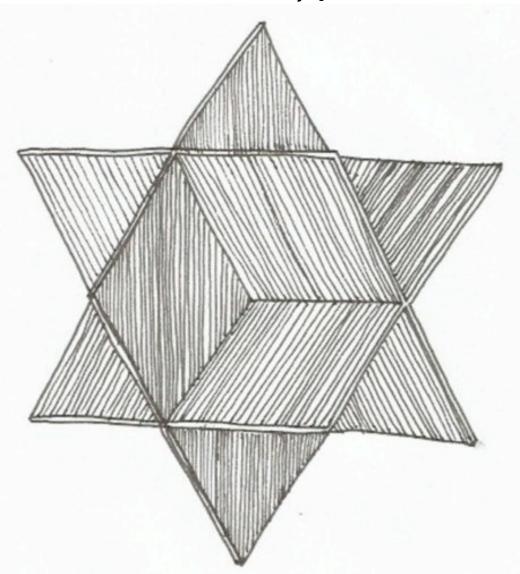
Such is the extrapolation of the cube from the central square plane-space @ 45°, surrounded by the outer square, in this case, symbolic of the tesseract. The central square plane space is oriented as an exact diaganol slice between opposite corners of the cube, and the cube is, in turn, oriented at a similar angle of inclination relative the figurative "tesseract," symbolized by the square. The area of the central square is measured as a set of four Pythagorean triangles, and, if each "hypotenuse" of these triangles is taken as a "side" of the central square, then the total area of this square would be equal to the length of one such side mulitplied by the length of another, equal length such side. If one side of the central square were given as "X," then the area of the central square would total "X" x "X" or "X^2." The area of the cube surrounding this central square is the same measure to the "cubed" exponent, e.g. "X^3." Likewise, the area of the large square symbolic of the "tesseract" is equal to the area of the flat side of one of the 16 faces of the 4-dimensional "hyper-cube," which is comprised, in total, of areas equalling the factor of "X^4."

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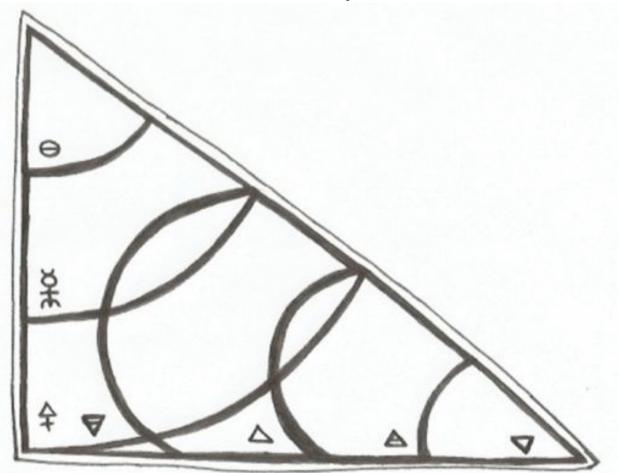
Drawn to the same scale as the preceeding diagram, in this depiction of the standard, "mobilis" (1/3 ratio) version of the "vesica pisces" (overlapping circle pair), the interaction of the previously depicted exponential area ratios is here shown as resulting from the interaction between two, eqaul-area circles, forming the standard "mobilis vesica" configuration, surrounded by and contained within a single larger circle. The ratio of the central "square" in the preceding diagram is here shown as the overlapping "intersection" between the twin, equal-area circles; the ratio of the cube exponential area is given as the non-overlapped areas of the circles "above" and "below" combined; and the ratio of the tesseract or hypercubic exponent is expressed as the larger circle, combining both small circles' areas and the remainder of its own area into a "union" of shared and overlapping spaces. Whether the "tesseract" exponent, symbolized here by the outermost circle's circumference, contains the area within it of the plane-space "square," symbolized here by the innermost overlapping of the twin equal area circles as a "vesica" (later called the "ICHTHOS" for its fish shape), or not remains undiscussed as of yet.

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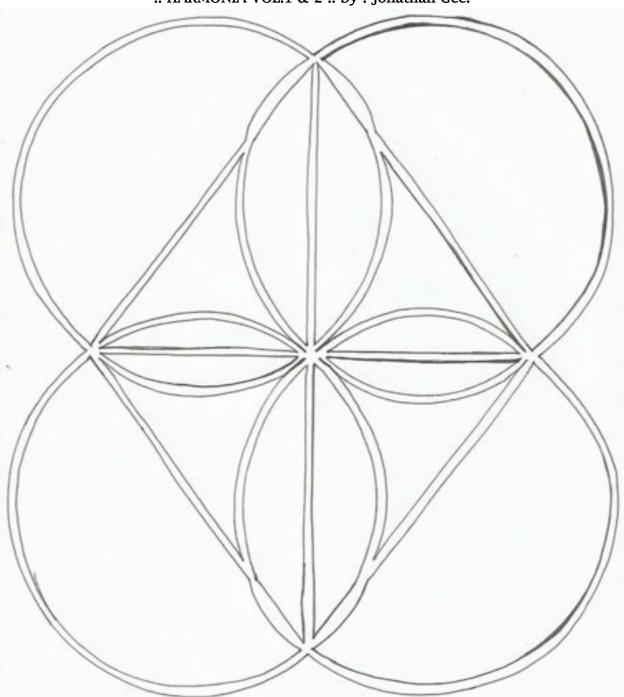
Drawn to the same scale as the preceeding two diagrams, this model depicts the "Hebrew" ("Star of David") hexagram as being comprised of 3 square plane-spaces, each intersecting both others at right angles. This model can exist only in 3-dimensional (and thus, 6-directional) space as symbolic of extending opposite sides of each plane-space square in the 6 cardinal directions, and, when this model is tilted at a 45° angle, such as depicted here, this hexagonal symmetry repeats as the optical illusion formed by 6 of the shape's overall 12 corners. As the 3 intersecting squares of this model also operate as extensions from the mid-points between the outer edges of a cube's 6 faces, this shape also implies the dimensions of the "cubed" exponent. In this case the "X^2" area being cubed would be any one, 1/4-sized area ("quadrant") of any one of the depicted 3 intersecting squares. As defined by their central, triple-intersecting origin point being surrounded in 3 directions with perpendicular (90° or "right") angles, this shape also encapsulates the same geometry present in the coring down from its circumference produced by removing 1/8th the total volume of a sphere, however there, as a 3-d sphere, and here, in flat, plane-space geometry.

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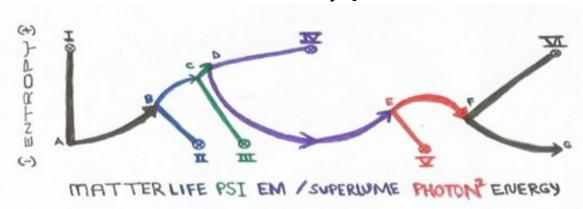
In this depiction of the usually reckoned "Pythagorean Triangle" (according to Euclid's "Proposition X.47"), we find the method for measuring the area of a square plane-space that is tilted at 45° to the observer in 3-dimensional space. In this case, the central square, thus "tilted," would be formed by duplicating the depicted triangle in mirror horizontal, vertical, and inverse symmetries, rotating the shape around a central origin-point formed at the "right" angle of intersection at the "Pythagorean" triangle's intersecting legs. The standard model of "Euclid's 47th" depicts this right triangle ratio by the figurative sums measuring its leg lengths as: vertical leg = 3, horizontal leg = 4, diaganol hypotenuse = 5, with a complex set of proofs to demonstrate how these whole number, sequential integer sums are derived. The substance of these proofs aside, another exists by applying, rather than purely linear trignometry as measures of "absolute" distance, the presence of arc radians as measures of "relative" distance, and, by this application, additional areas within the regular "Pythagorean" triangle may be identified and defined. For example, it is given that the horizontal leg (or triangle's "base") is of 4 lengths of equal sum, and it is given that the four cosmic elements may be thus attributed to this sum (such as depicted from right to left: gravity, electromagnetism, fission and fusion); and, likewise, it is also given the vertical leg (the triangle's "height") is of 3 equal length sums, and it is given that the 3 conditions (or "states") of matter exist as being equivalent to "alchemical" elements, thus, from above to below: salt = solid, mercury = liquid and sulfer = gas. These occur terrestrially as ice, fluid and steam conditions.

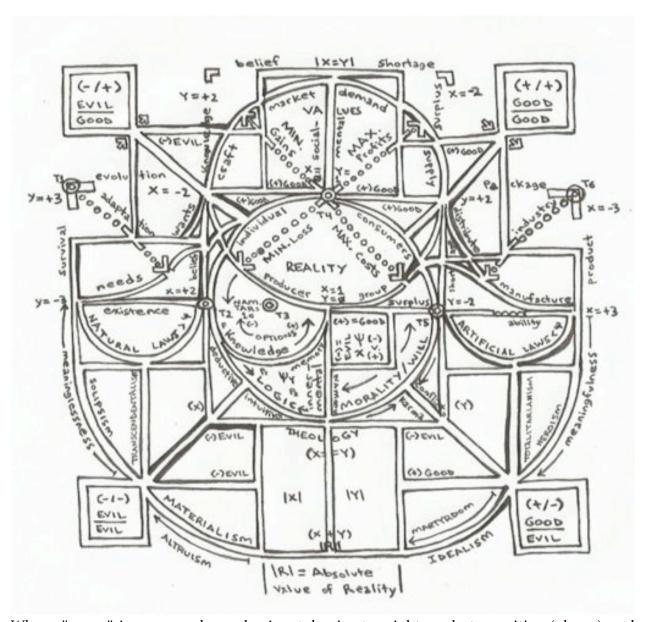
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The central square in plane-space, depicted at 45° to the observer, thus "tilted" in 3-dimensional space, is here shown comprised of its constitutency of 4 such "Pythagorean" type triangles; although here it should be noted, for this method to maintain, these triangles do not need to be equal specifically to the sums given in Euclid's 47th Proposition as only one example of such a typical case of triangles. The numbers of 6 units per "horizontal" (as shown) diaganol, of 8 units per "vertical" (ibid) diaganol, with sides of 5 units each, would be wholly arbitrary to apply to this model; also the total of all sides of the central square summing one unit each applies.

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Where "space" is measured as a horizontal axis at a right angle to positive (above) and

negative (below) entropy, the density of "space" is determined according to a "lag" in the distance travelled by various quanta given an equal duration of time. This pattern, present also in the solar plasma sheath's extension of penumbral EM coiling, reflects a "lag" or discrepancy in pacing of purely solid matter vs. purely plasma energy, and results in the accumulation of additional mass / density (=matter) such that the otherwise more "sinusoidal" wavelength pattern here assumes a more "covalent" wavefront (akin to a "phi" spiral pattern). We see in this diagram as matter (black arc, left side of chart) accumulates positive entropy, eventually it generates life (blue arc), which curbs the rate of entropy, followed by "psi" (symbolic of sentience, green arc) a very brief hyper-acceleration of entropy, followed by the EM spectrum's (purple arc, center) decline toward negative entropic zero-point of "c" (the speed of photons in a void), followed by an increase through tachyons (c^2 , red arc) into "ZPE" (black arc, right side). The points A-G should constitute a 7 note "chromatic" spectral scale. The points Roman Numerals I-VI are considered "levels of clearance" in the "T4 program."

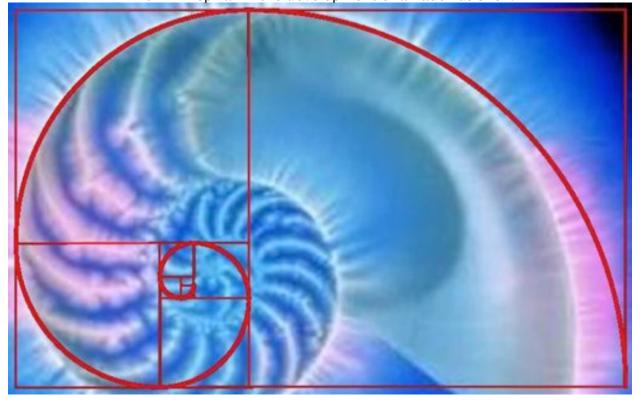
Here we see a measurement system that uses a multi-origin point method (multicentroid graph, or combined horizontal and vertical "slide" rules) to measure the values (both "absolute," denoted by "bar / value / bar" AND "relative," denoted by "parenthesis, value, comma, value, parenthesis") of various variables. The values ascribed to the variables (such as "+" or "-" and integer values) relate to the variable's distance from the primary vertical axis at a right angle to the horizontal mid-point. The structure is here applied to measuring the "values" of variables "X" and "Y," where the values given for one occur "from the perspective," or "from the POV" of the other. Thus, for example, the value in the middle of the left-most side, "Y = -3" denotes the value of "Y" from the POV of "X" is "-3." In this arrangement we see that X is the individual, and Y the group or collective of any given society or culture of the same species as X. Thus, the middle vertical axis measures the minimum values of X to Y and of Y to X, and the further from this, the greater their perceived values relative to one another become. The top-most arc intersecting the central vertical axis is symbolic of the free market of exchange between the individual and the society or group, and constitutes a graphic depiction of our elevated, though arbitrary, over-inflated valuation of imaginary commodities in this market-place, where the very currency of exchange itself is, perhaps even necessarily so, fake. Below the central horizontal axis are the mind-states of those individuals more prone toward an "individualist mind-set," perspective or POV (on the left), and those more prone to a "collectivist mind-set," perspective or POV (on the right). The measure at the lowest-most middle-point symbolizes the "Absolute Value of Reality" as being comprised of twin 1/2s, one the perspective of the individual (labeled the absolute value of X), the other that of the collective, society or group (that of Y). Additional labels indicate the polarization influencing a toroidal counter-rotation that causes the shift in the centroid "zero sum" value at the origin point for the chart to drift from a single fixed location can be classified according to traditional moral binary units of "good" (+) and "evil" (-) though paired and duplicated among themselves, such that the outer-most ring of the complete system revolves clock-wise from "good/good" (the upper right) to "good/evil" (the lower right) to "evil/evil" (the lower left) to "evil/good" (the upper left) In this manner, "good" and "evil" are rendered subjective concepts relative to the social construct as a totality in itself.

The "Phi" spiral in the development of a human cerebrum.

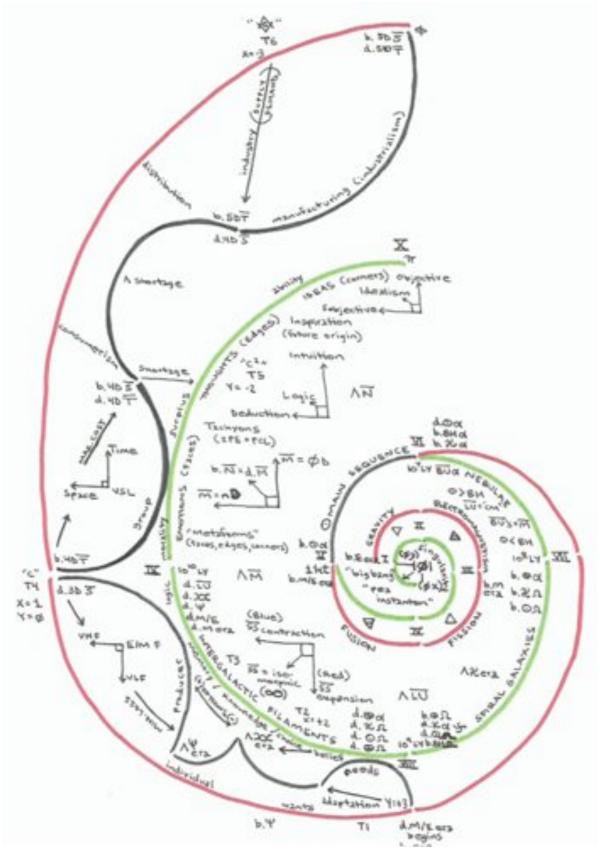
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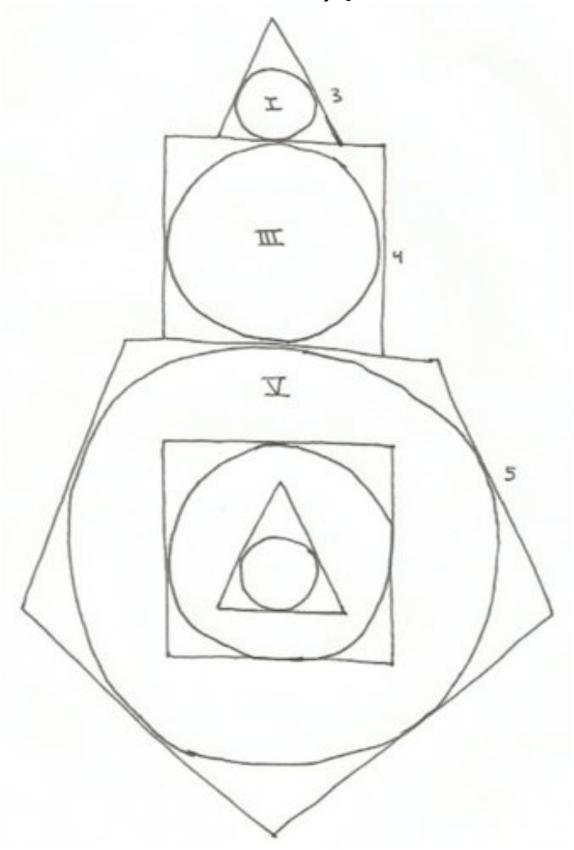
The "Phi" spiral in the development of a nautillus shell.



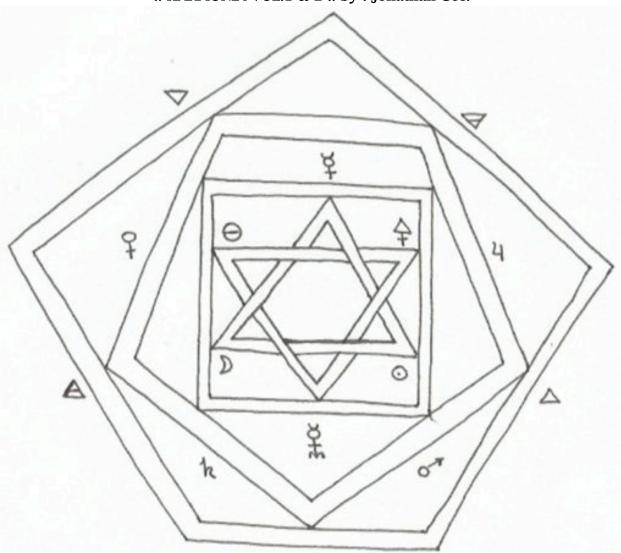
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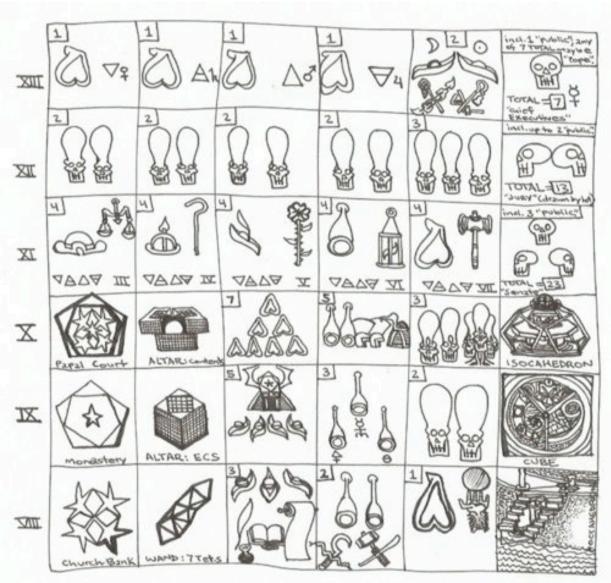


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The ratios of the cubed exponent (expressed here as the "Star of David" hexagram) and the tesseract exponent (expressed symbolically as the square surrounding the hexagram) are here coupled with those of an exterior pair of downward within upward tip-pointing pentagons. Just as the central square in plane-space and the cube in 3-dimensional space are measurements based on one set of basic sums, comprised of the geometry of the square ($^{\circ}$ 2), the cube ($^{\circ}$ 3) and the tesseract ($^{\circ}$ 4), so do the conjoined pentagons here symbolize the ratios arrived at by extrapolating the original base sum unit hyper-pentagonally (that is, to the power of the 5th exponent, or rather, ^5). Although at this point the exact ratio measurements begin to break down (as the actual size of each exponential level's total volume of areas would make the next lesser exponential level's total volume infinitesimal by comparison), and thus the double-pentagon level resorts back, as had the tesseract exponential level, to a depiction merely symbolically. Here, we see that the innermost hexagram, symbolizing the 3rd exponential level (or 3-dimensional space) is labeled with the 3 alchemical conditions of matter and 3 of the 7 classical planets, the inner-pentagon with the remaining 4 of the 7 planets, and the outer-pent with the 4 cosmic elements.

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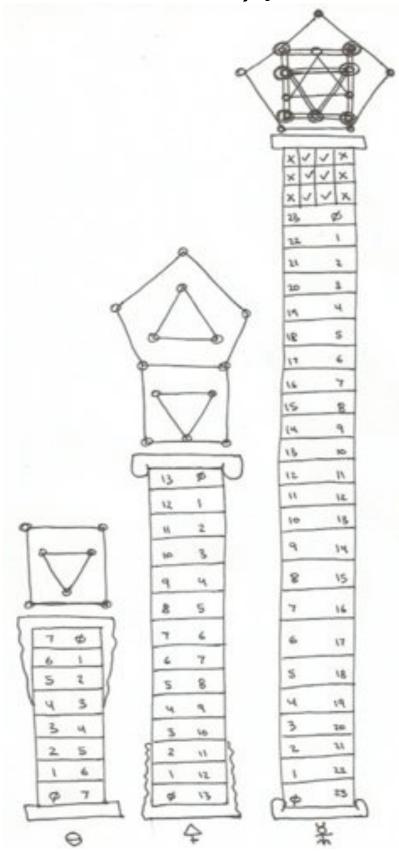
The same ratios expressed as lengths of sides, as polygonal areas or as polyhedral volumes can also be expressed as groups of individuals assembled collectively into equivalent numerical sums. The entire basis for "Atlantean Society," as it may be depicted in this single graph, above, shows the 3 degrees of State above and the 3 degrees of Church below, each with 6 different aspects defining the sums present in each group iteration within the whole. The upper-most row shows the 7 Chief Executives, the second down the 13 Jurors, the third from top the 23 Senators; the lowest rank is the 6 member Church, followed above by the Monestary of 10, and lastly by the Ecumentical Senate of 15. When these different sums are expressed as groups of individuals, the relationships that may arise between those within each group, and thus how each group can relate best to the whole, change in direct porportion to the nature of the number sums of their membership. This is the essence of our ancients' society, both the logical component as "Atlantean Democracy" and the moral component as "Lemurian banking" expressed as R & C.

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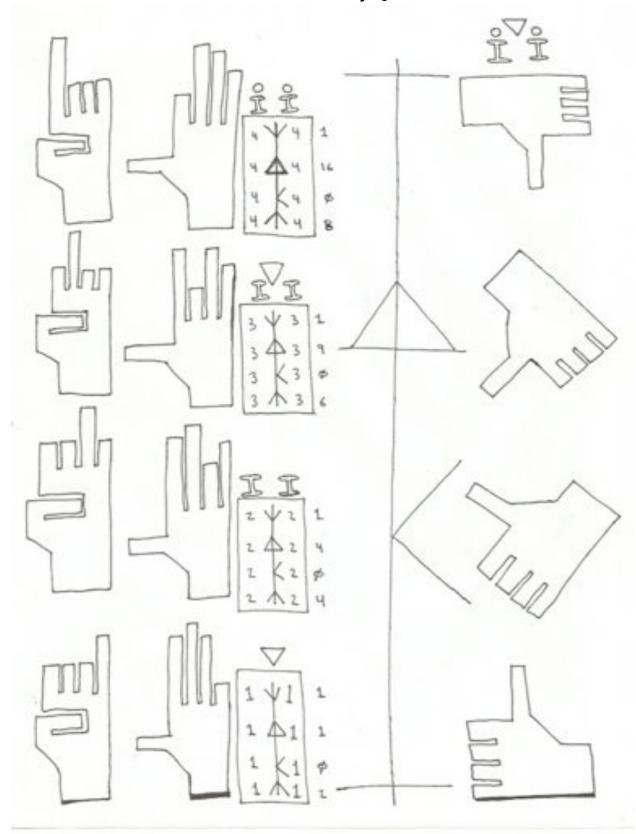


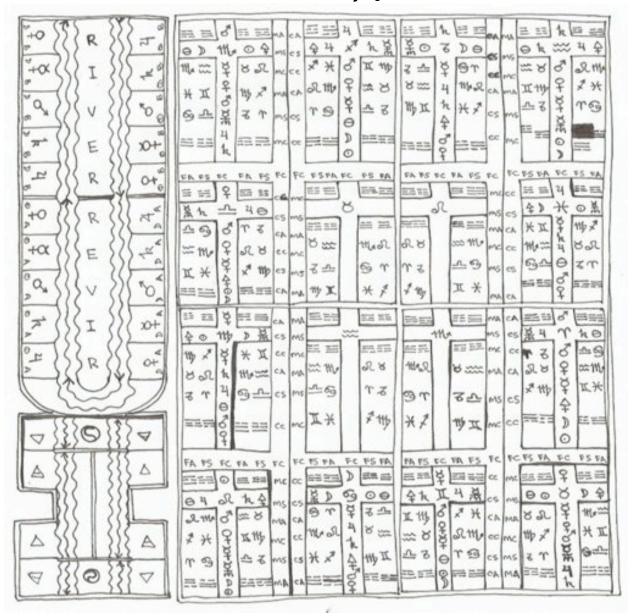
Here we see all the "heads of state" in "Atlantean Democracy" expressed as upon the edges or corner-angles of the same polygons that defined that model just previous to the chart that preceded this arrangement. The central "cube" exponent (\(^3\)) is expressed symbolically as the "star of David" - a hexagram, in this case labeled with the twin Papal Alternates (due guards to the Pope) as Sun and Moon, the Papal crown itself as Alchemical Mercury, and 1 of the 3 "public" members of the Senate as planetary Mercury. The "tesseract" exponent (^4) is expressed symbolically as a square, in this case labeled with the 4 Chief Executives besides the twin Papal Alternates and the "public" Executive, as being anonymously and interchangely either any of Venus, Saturn, Mars or Jupiter, with any of these 7 serving as the Pope themselves. Surrounding the hexagram within a square motif are four sides, comprised of 5 seats each, that, with the full 3 total "public" Senators, combine to form the 23 member "Atlantean Democratic Senate Forum." Just as any of the Senators may alternately serve as a Chief Executive, so too may any Chief Executive serve as an alternate to the Pope. The Pope holds the keys to the "Lemurian" vault in which is stored access to knowledge that, arranged in a psyclotronic vortex, propels the perptual expansion of the cosmos into zero-point energy via entropy.

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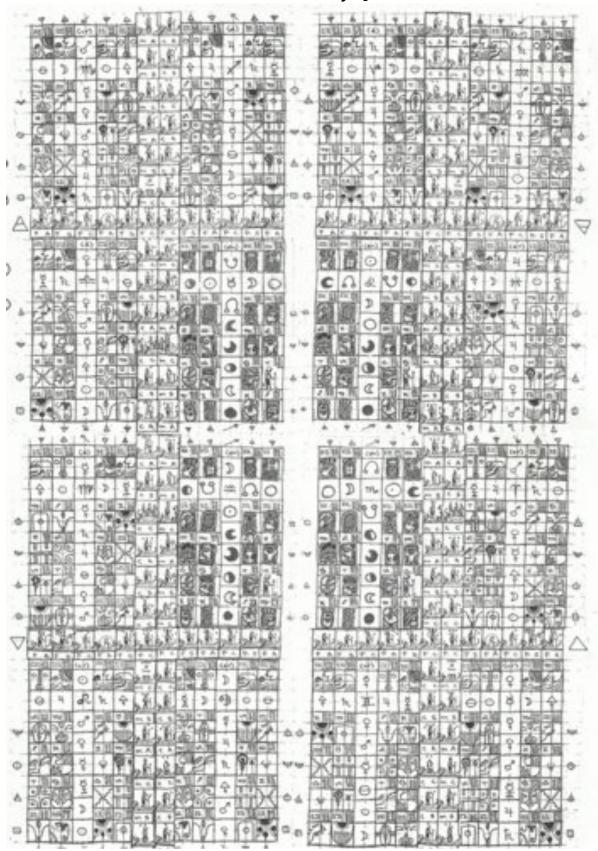
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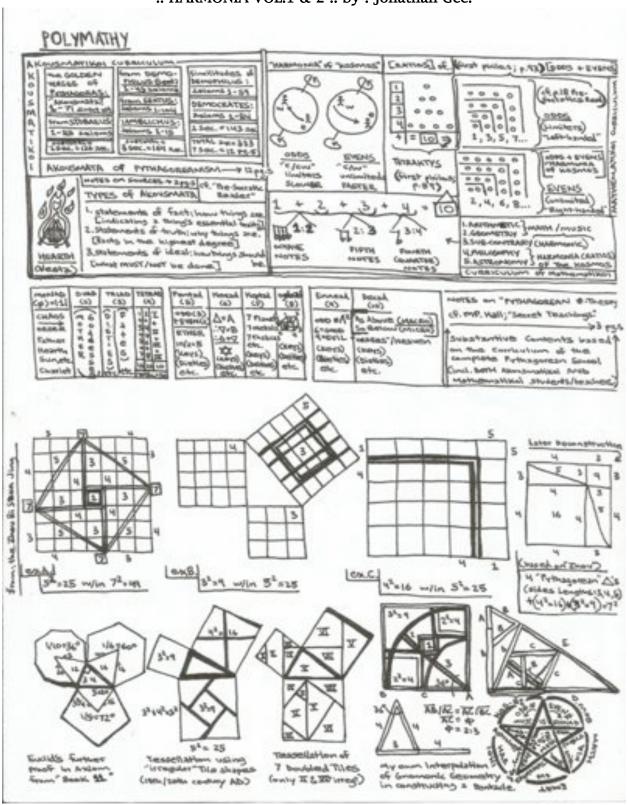




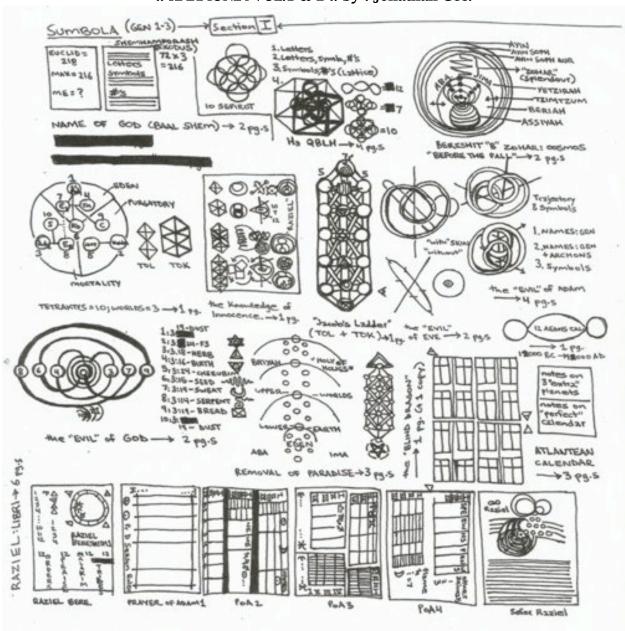
The "royal" (possibly pre-Pharaohnic era) Egyptian board-game, "Senet" (meaning "River"), is here displayed as the upper left 2 columns of 10 squares each. "River" was played by two opponents across the width of the board and along the length of it. It derived, in turn, from the "royal" (possibly pre-Ubaid era) Sumerian board-game of Ur, possibly called "hounds and jackyls," designed with a similar "running chase" game-play in mind, but with a board defined in a different shape, as depicted in the lower left 2 columns - as 2 square-cells above and 2 below with 1 cell between = 5 per column. Both of these board-games can, if combined, be derived from a basic numerical pattern that arises when the left two columns' additive sums are cross-collated by rows and columns; to complete the comparison to the original mathematical framework underlying the board-games' structures, the "Atlantean calendar" model of "Enochian Senet" (a 4 X 4 square motif yielding 12 around a central 4 square-cell block) derives from the set produced in this manner.

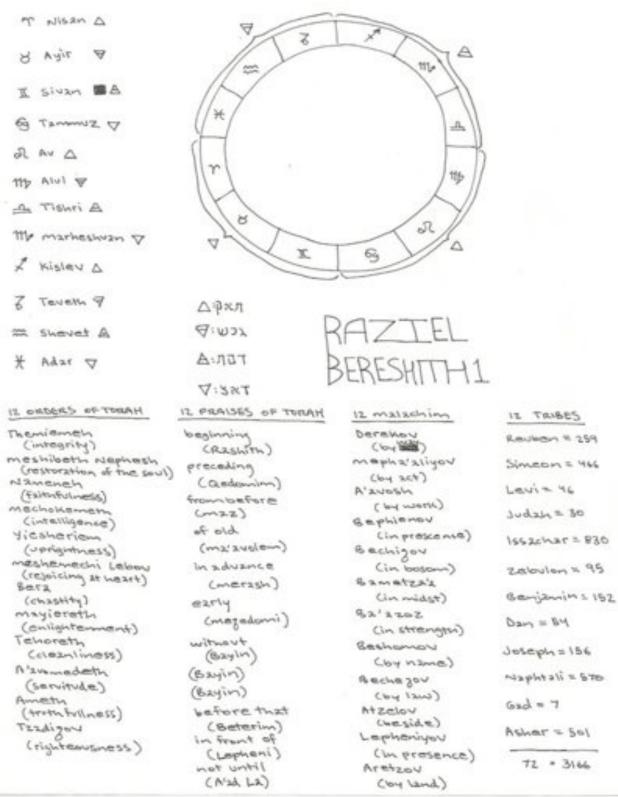
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END OF "POLYMATHY" SECTION.





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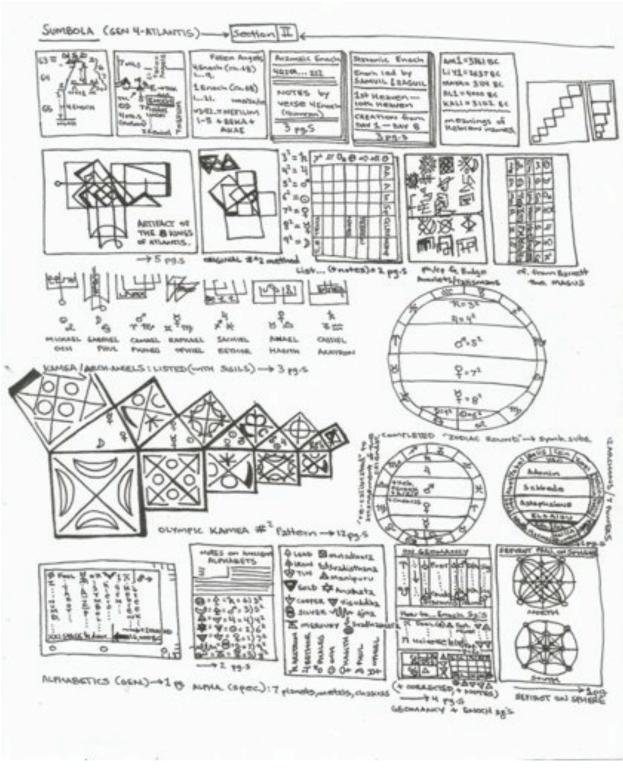
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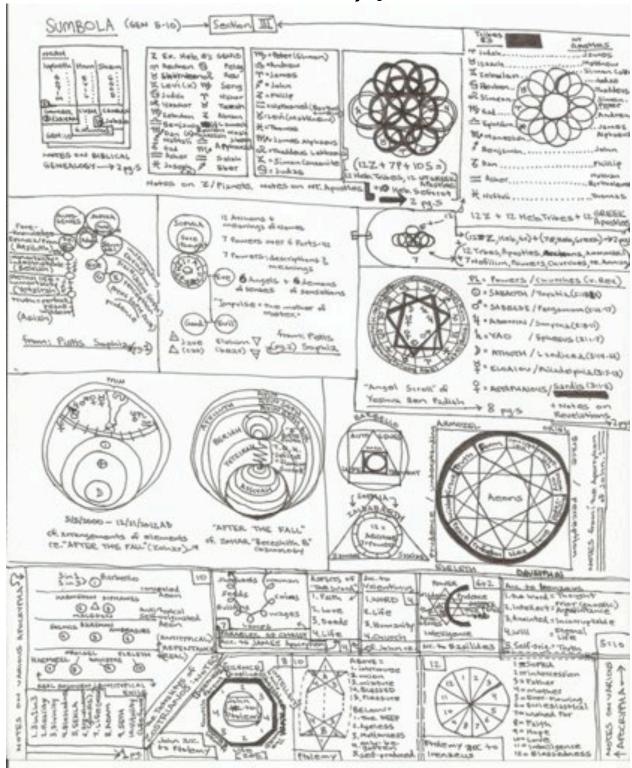
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END OF "SUMBOLA" SECTION.

HISTORIA:: VOL. 2. "Music Theory." Content Begins: Notes on the Magic #-Square of the Absolute Value of the ZERO-Sum.

Can Nothingness Exist? Can Reality Non-Exist?

The role of "Zero" - both as a "place-holder" in solution set proofs, as a decimal holder, as a product multiplier, as we as in symbolizing the sum totality of a solution set as eventually "Abstract" or "Odd," or originally, "Valueless" or even an unending abyss of wanton hungering needs - in ancient cultures should not be overlooked as it relates to "music theory" and "tone" based geometric / numerological arrays (e.g. magic #-squares, commonly worn as talismans.

The Ancients (pre-civilization) all used "Zero" freely, as it arose naturally to them amongstheir first five (and our still most common 5) phoenetic asperations, derived directly, as the were by them then, from the counting of the 4 fingers using their opposable thumb, with its own independent dexterity, as a figurative "Zero-sum" in doing so. Because they are the invention of our Elders during the earliest era of civilizations, when the Vedic-Aryan, Ubaid-Babylonian and Old Kingdom Upper-Egyptian methods for caluclating #-squares arose as independent off-shoots from the, originally Harbiru-Hyksos method developed by Abraham of Ur, called Endubsar therein, and called Ibruim without, and called ImHotep in Egypt at the beginning of the pyramid building era there, therefore, #-squares using "Zero" as more than a place-holder SHOULD exist.

However, none do, neither from ancient times nor through to our own modern times even now. The use of only "whole" number "counting" sums relates to later Phoenician (possibly Pythagorean era) arrangements of alphabetic letters relatve to counting numerals, by which time excluding the "Zero-sum" from #-square based talismans was already common. So, by now, a vast amount of potential for humanity's researching the "Zero-sum" has been subsequently suppressed into our cultural collective subconscious, from whence it manifests as the unintended totality of our collective social products.

"All secrets bite their keeper from behind, given infinite time."

So, if Infinity (∞) #-squares COULD be posited that use the "Zero" (0) or "empty sum" in unit cells, or even as their own "magic sums" adding Rows, Columns and Diaganols, an yet Zero (0) DO exist, and if the "Absolute Value" as it relates to the "relative value" (based on locational distance/duration coordinates and projected trajectories within any given manifold's definitions for "space" and "time," Euclidian and Netwonian or otherwise, has been overlooked and under-studied as a result of this, then the underlying "strange attractor" of constructing a "Zero-sum" #-square with a "magic sum" or "Zero" can hardly be denied. Yet this proposes the concept (obvious to any who distinguish its traits by differing functions) that "Zero" in itself is NOT ABSOLUTE, which implies an overlooked aspect in moden #-squares: is a "magic" #square and LESS "magic" if all or some portion of its sum amounts to "Zero"? Many squares already use some "Zeros" (as place-holders), and, from the point of view of all "serious" students of cryptography, it can be argued the total socal value of studying such ancient talismans is, in modern times, if not already at the Absolute Value of "Zero-sum," then asymptotically apporaching it in all relative vectors; for all of which they feel they need care naught.

So, ultimately, how does one prove the value of a magic #-square constructed using "Zero" as a sum-cell or as a total, overall "magic #-sum" for the whole square,

wherein its Rows, Columns and Diaganols functionally cancel out. to such a skeptical mathematician in modern society?

Consider the potential value of exploring beyond "Infinity," as well as beyond the limit of "Zero." Transfinite, hyperspatial, extra-dimensions governed between them all by transcendental sets, DO EXIST that exceed the definitively "Infinite" (though inexplicably 1:1) sets of all "counting" #'s, all integers and all ratios. Is it not impossible for there to be "layers" of "nothingness" within the "Abyss" at the "origin" - thus rendering it a relative sum in itself? In what way would "Zero's" multiple. functionality-determined aspects, influence the "1:1" ratio of all counting #'s, integers and ratios defined as the sum total of "Infinity," considering integers include "Zero," but "counting" #'s do NOT?

If "Zero" can have different, "relative" values in different geometric manifold settings (i.e. non-Cartesian graphs), then perhaps it holds the key to generating such "magic #-squares" as further "matrices" measuring "vectors" of motion in what would constitute a sum of multiply-Infinite(so-called "n-dimensional") different geometric conditions based on the manifold set's size. Just as the # of cells per "Order' (by R + C) determines a magic #-square's magic #-sum, the dimensions of any such "hyperspace" manifold should determine a rough average for "strange attraction" and "gravitic mulitplier" effects (cf. "cellular automata" and "information units" such as "uncertain" states of certain quanta) of otherwise free radicals including both particles and waves. Given a large enough manifold to measure with, by this criteria it can be imagined, one would eventualy be able to arrive at a measure that could predict the location at any specified moment of how much and what type of matter-energy would be where just then.

So, again, what then are the "relative" or the "Absolute" values of "Zero," as they relate to "Zero" being the "magic #-sum" of any given size format of "square"? If "Zero" may be expressed as an "Absolute" in Euclidian-Cartesian mathematics, in what other geometric arrangements might it be induced instead to decompose into even lesser consitutent components? Spherical geomtry (with "Pi" arc-tangent radians measuring intersecting "ley-lines" of lattitude and longitude on the sphere's external circumference and outer-surface, begins the implication of "relative" Zero in such a triple right-angled environment as the relationship formed between the 3, 90° angle intersections (1 @ the orb's pole and 2 along its equatorial maxiumum "buldge") on the sphere's upper and outer surface to the core at the center of mass of the orbital manifold-space, where, if the circumference is measured (as one arbitrarily may, it functions variably also) as "Infinity" (fi), then the cre would be the "Absolute Value" of a "Zero-sum" (polarized opposite external "Infinity") core, but has intersections on the surface occurring @ multiple points of origin @ 3 "Zero Point" intersections.

Notes on the Magic #-Square of the Absolute Value of the Sum = 1.

```
Square of Man = ?

I A M = !

A M I = ?

A I M = !

A I M = !

A I M = !

A I M = "Aim" (target to destroy)
```

"One-sum" (given as "I" or Roman Numeral One, symbolic of self-concept) is the magic #-square of 1 X 1 and $1^2 = 1$ because "I" Am AND Am Not a #-square. Because "I" BOTH (union) "Am" AND "Am Not" a magic #-square, therefore "I" = the sum of

opposite (self-negating) parts (i.e. the "absolute value" of I = "relative +" I / "relative -" I), and yet, because "I" still IS, it proves itself also a "magic sum" of I + IC, and thus "greater than the sum of its parts."

The War Between Evens and Odds.

Pythagorean Numerals

0 = Evenly Odd

1 = Oddly Even

2 = Evenly Even

3 = Oddly Odd

The Remainder of Infinity

4 = even

5 = odd

... etc .

The Tetragrammaton (water = gravity, air = e-m, fire = fission, earth = fusion) applies thus: 0 = water / gravity; 1 = air / e-m; 2 = fire / fission; 3 = earth/fusion, as Yod Heh Vav Heh. Thus, 0, 1 & 2 are unique, but 3 is a repeat of 1, as is the second Heh, which is called "Heh (final)." Trigonometry (simplest, "Euclidan" / planar, rhomboids) is thus the "language" of the Singularity or Monda, as 3 "repeats" 1. 0 is the "origin" point of the "Kosmos"(the "singularity" of our "local universe," supposedly of 3-dimensions expanding in a 4th-dimensional direction, that is, over time). 1 is the Monas Singularitatis, whose Historia we are. 2 are the "evenly evens" who idealize our existence as ultimately either all "good" or all "evil." 3 "repeats" 1.

"How To" Make a magic #-square to relate to the words "animal" and "planet" or "kingdom" or "realm."

Proper Latin ("Tongue of Devils") =

A N I M A = M U N D I = A M I N A. I D N U M.

= Reversed Latin ("Satan's Alphabet")

1 2 3 4 5

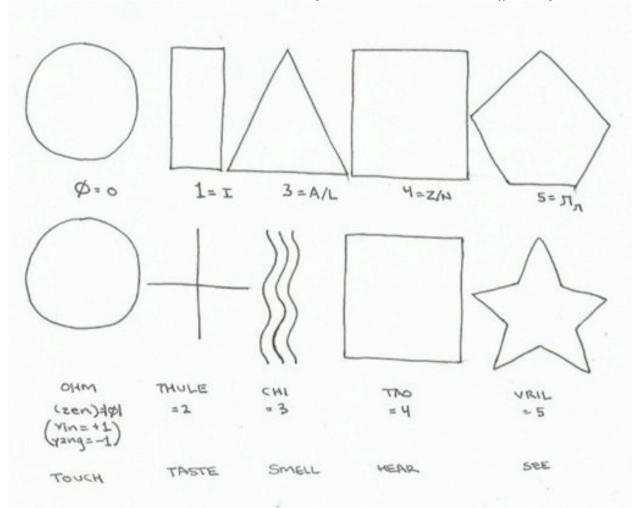
- SATOR = "satyr" (1/2 zoomorph, 1./2 human; symb. of Pantheism)
 ANIMA = "animals" (kingdoms, classes, families, species, etc.)
 REVER = "river" or to "reverse;" to "revere" (venerate in Latin)
 MUNDI = "world" or "loka" (1 of 8 types of reincarnation)
 ROTAS = "rotates" or "to rotate;" to alter, change (cf. dharma)
- Caveat: "invoking" the phrase "Idnum Amina" MAY psychosomatically work as "Angellic Language" to conjur spirit guides in animal form, however doing so only sways one's future dharma toward the "loka" of animal reincarnation oneself. If one wished to cause this effect, one would procure such a spell as the above as a talisman.

"There are always the same sums of all zeroes and of all ones.

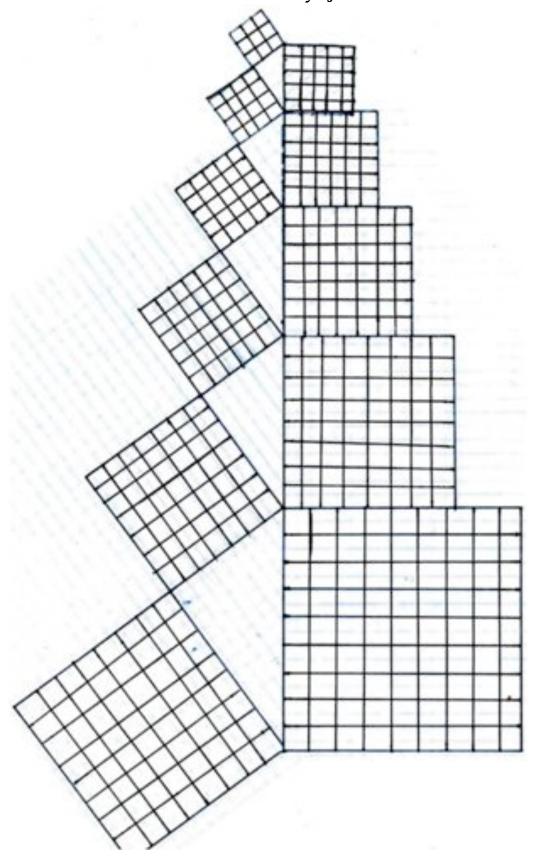
There are as many opportunities as risks at all times."

- authentically Pythagoran Akousmatikoi saying.

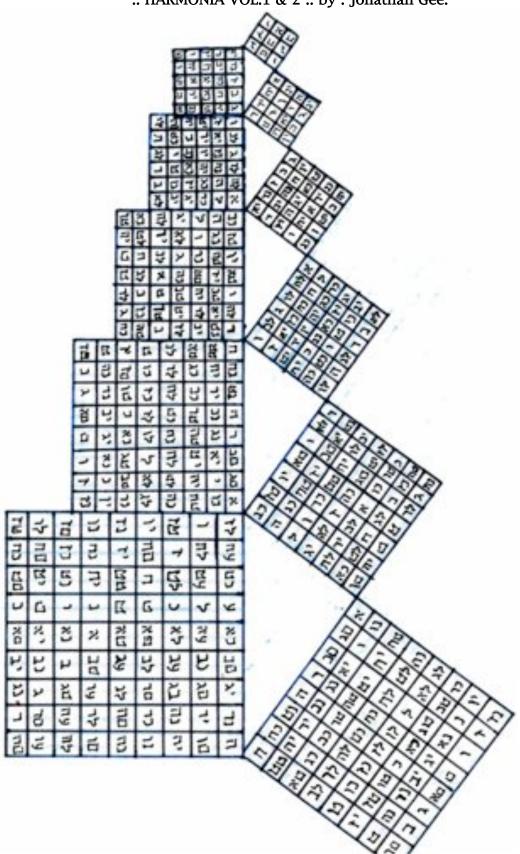
Notes on the 5 "letters" of the "Atlantean Alphbet" and 7 "Kamea" Magic #-Squares.



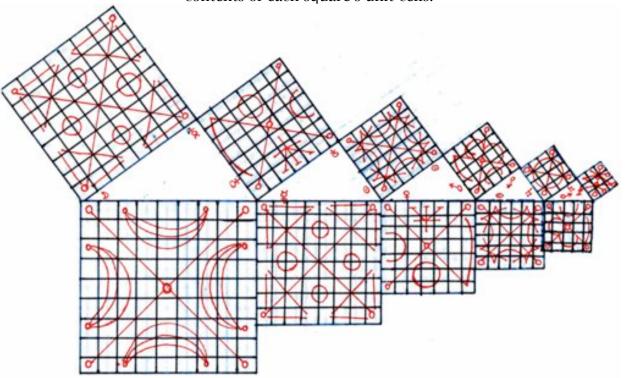
Because they had four fingers and an opposable thumb, thus five digits, the Ancient protohominids founded their earliest written alphabet and maths on the base-5 system, and assigned to each a geometric shape relative to its unique numerological value. Over the aeons, the names of these original letters in the "Atlantean Alphabet" were altered and eventually they became called, rather than "Ohm, Thule, Chi, Tao and Vril," instead "Zen, Tao, Trigonmetry, Squaring (exponents), and the Pentagon (symbol of civilization founded on odd rather than even sums of numbers in rulership)." Because there are five total, these shapes me be arranged relative to one another to form the rows and columns of a "5th Order" (or $5 \times 5 = 5^2$, or 25-unit cell) magic #-square. As these elements may be interchangeable, we find 5 such possible magic #-squares arise for any arrangement of a "5th Oder" type of square. Of these arrangements, we may examine a "Babylonian" model (only symbols), a "Vedic" model (only sounds), and an "Egyptian" model (using number-letters) all arise from an original "Hebrew" model (using the 4 cosmic elements and the symbol of "man").



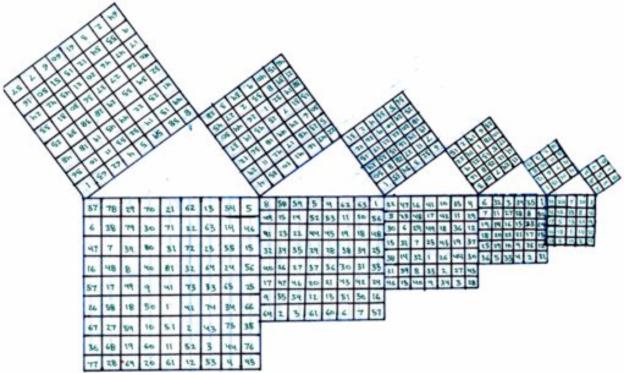
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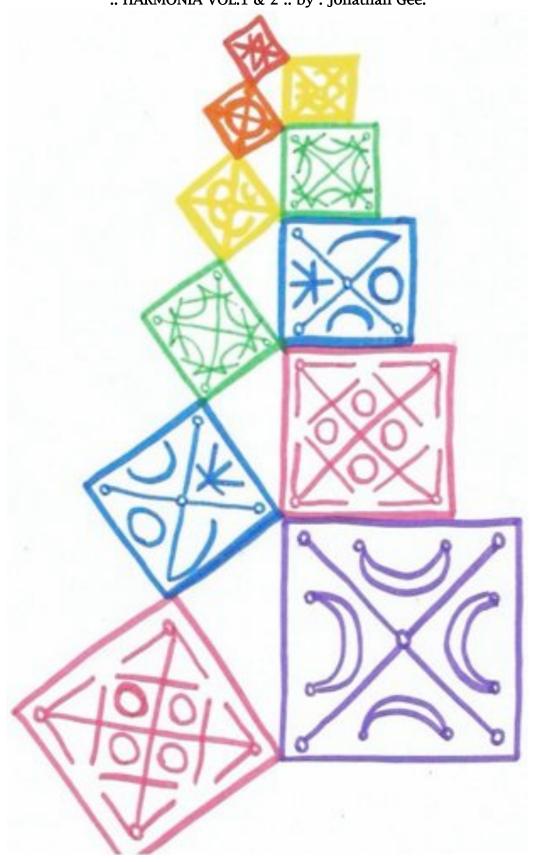
"Sigils of the Kamea" expressing geometrical relationships between the numerical contents of each square's unit-cells.

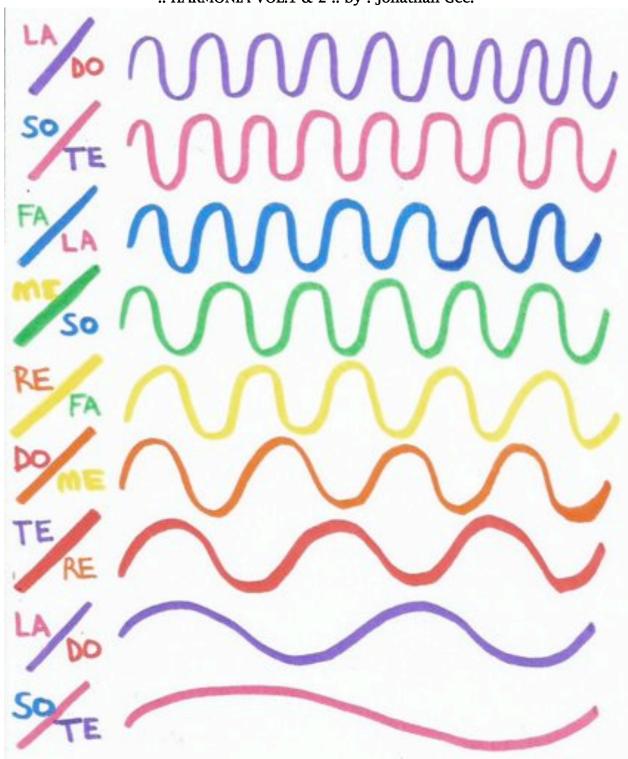


the Magic #-Squares of: (right to left: 9=Saturn, 16=Jupiter, 25=Mars, 36=Sun, 49=Venus, 64=Mercury, 81=Moon)



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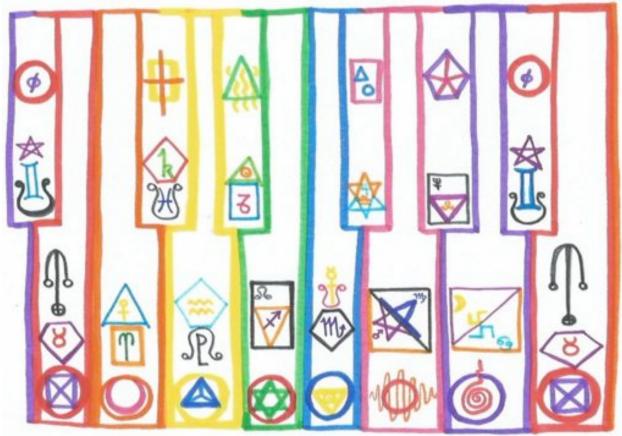




Do = Red; Re = Orange; Me = Yellow; Fa = Green; So = Blue; La = Indigo; Te = Violet.

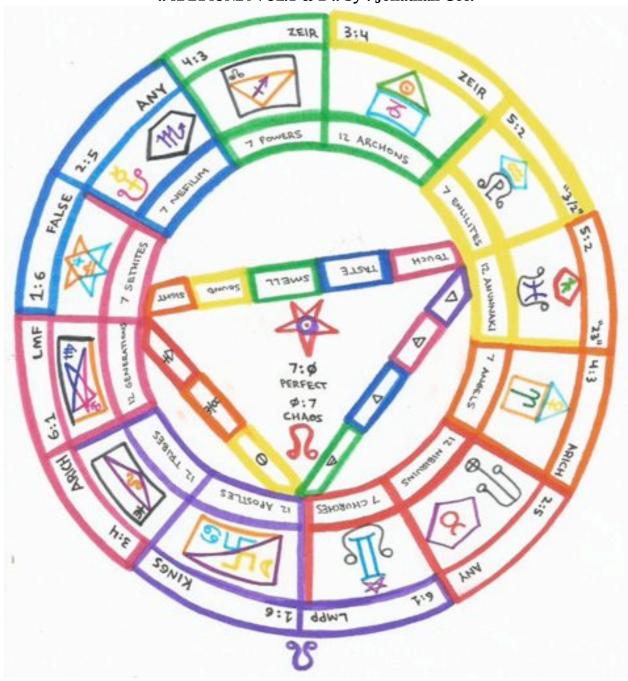
Positioned with "Do" as a 3-cycle wavelength, associated to an order-3 magic #-square, and "Te" as a 9-cycle wavelength and associated order-9 magic #-square.

the 7 pure tones (octave) plus 5 combined tones (flats / sharps) form the 12 Aeons of the "Seasons of the Pope."



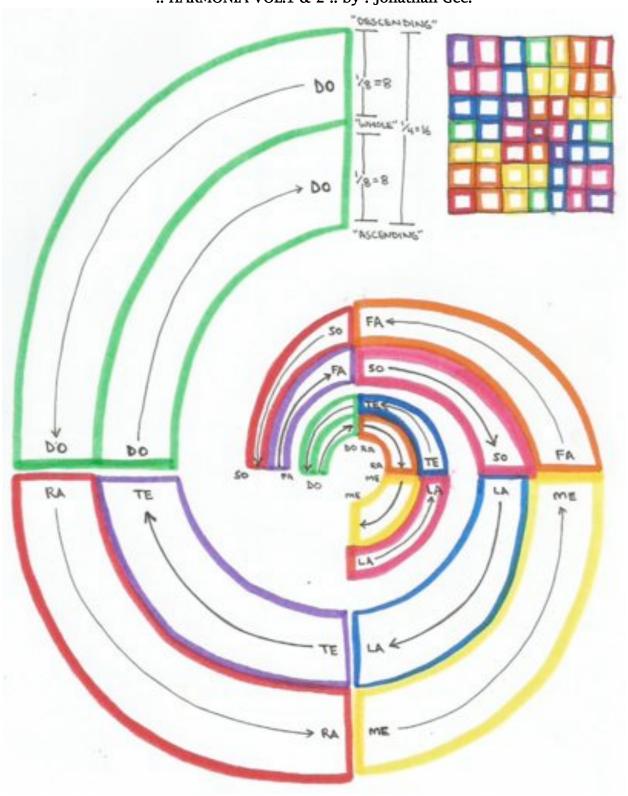
Beginning the depiction on the far left with B-flat, progressing next from left to right to "B1" and thus to "middle C" and so on for an "octave" including the "eighth note" above "B1" as one "octave" or "B2" on the far right end. From B-flat, we begin a count of 12 total tones, left to right, until the cycle repeats: 7 "pure" tones (comprising the lower "keys" in the depiction) and 5 "partial" tones (comprising the upper "keys" in the above depiction) sum 12 "tones" total. The 7 "pure" tone keys below are also labeled with the symbols of the 7 chakras (ascending from left to right) and the 5 "partial" tone keys above are also labeled with the symbols of the "Atlantean Alphabet." From B-flat = "6:1 LMPP," the aeon of "7 churches" (marked by the zodiac sign "Gemini" rising on the eastern horizon in the north hemisphere during the spring equinox), symbols of the "Seasons of the Pope" calendar of the Pythagorean Order of Death proceed next to B1 (and consequentially B2 also) = "2:5 Any," the aeon of "12 Nibiruins" marked by the rising of Taurus in Northern Spring. Our present date, around +2000 (YP),occurs between the aeon marked B-flat = "6:1 LMPP," the aeon of "7 churches" makred by Gemini and B = "2:5 Any" as the aeon of "12 Nibiruins," denoted by Taurus.

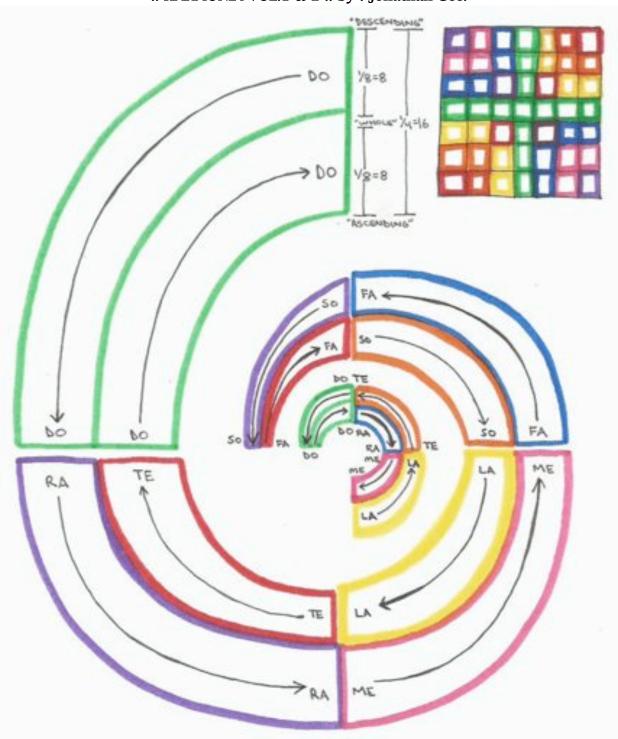
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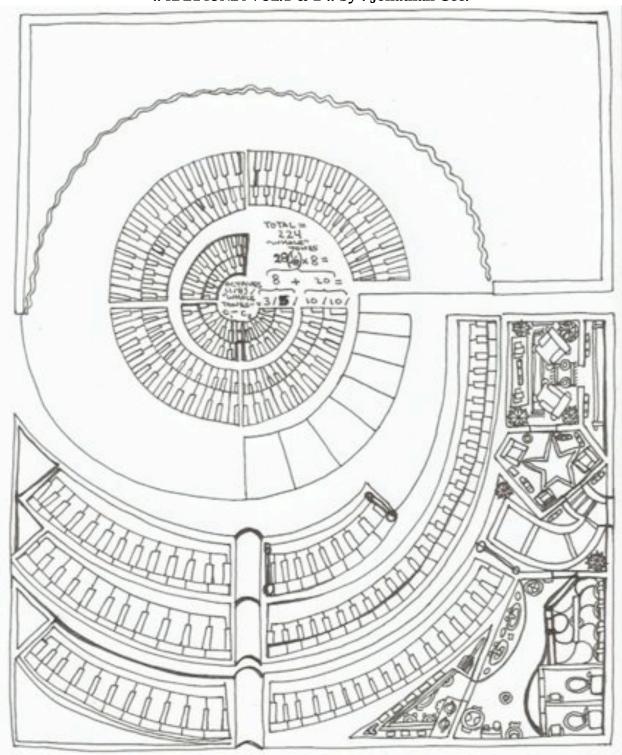
Color-coded so the 7 "pure" tones (Do-Re-Me-Fa-So-La-Te) correspond to the 7 color spectrum of white light divided in a prism (ROYGBIV) and the 5 "partial" tones correspond to combinations occurring between these 7 prismatic colors.

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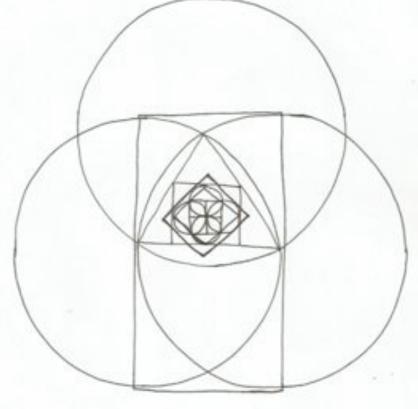
The "Phi," spectral-chromatic comma of dual octaves, surrounding the starting and repeating the ending note on a standard 7-tone whole-note harmonic-scale, "DO," from the full scale (ascending) 1=DO; 2=RA, 3=ME; 4=FA; 5=SO; 6=LA; 7=TE; where "1=DO" repeats as an octave (8=DO). Here, 1=DO=Green; 2=RA=Violet; 3=ME=Indigo; 4=FA=Blue; 5=SO=Orange; 6=LA=Yellow; 7=TE=Red. Each arc-radian of the spiral is 1/8th the whole.



A keyboard instrument and architectural auditorium designed around the ratios expressed in the "Phi" spiral "spectral comma," where the chord-lengths of the inner keyboard correspond to the edge of the stage, and the sounds generated on the outer keyboard funnel in an arc around clock-wise into the balconied seating.

showing the auditorium and instrument based on "Phi."





Fortifiere: by - Jonathan See on-Mach 1, 2014

HARMONA VOL. 1 " In THREE SECTIONS with Introduction ::

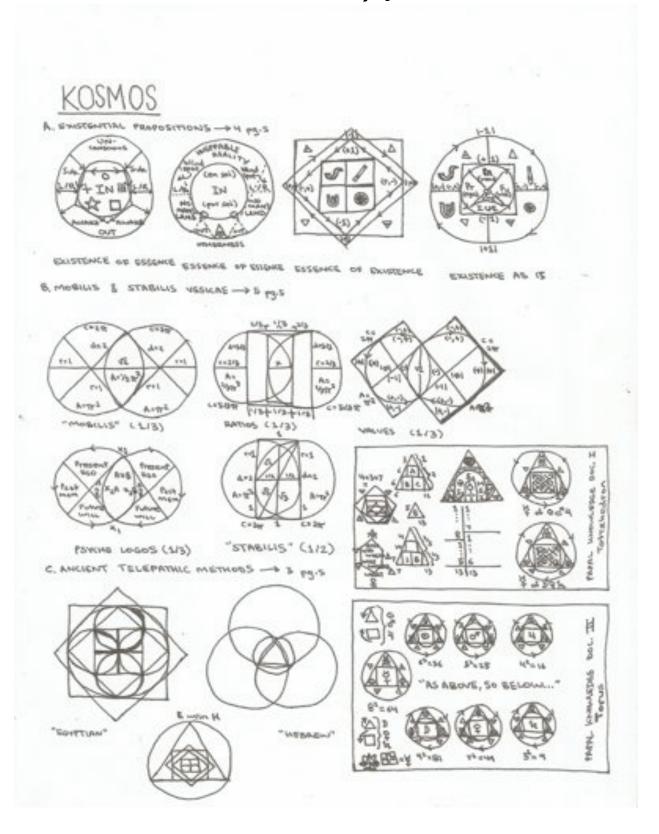
Ø. Kosmos (introduction)	→ H pg/s	
A. KWISTENTIAL PROPERTIES	11 130	
8. MOBILIS & STABBLIS VESICAE C. RAKIENT TELEPRINIC METHADS B. PAPAL KANWLEDGE DOCS		
I POLYMATHY	→145 pg·S	
1 ATLANTERN MODELS	112 132	
A EMERGE HAVE OF GOD B. GENERAL TRUGORDANETRY C. PYTHROCKEAN TRUGORDANETRY D. ATLANTERAL "DEMOCRACY" GAME E. LANGUMER PRIMER		
F. MASC. ADD.		
2 PYTHAGOREAN MODELS		
A, ANOUSMATINOL CURRICULUM		
I. SUMBOLA -	→ 115 pg.5	
1. 664. 1-3 1 Azes.		
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3. 6603. 5-10 + NT EGA : A-I.		
III. APPENDICES — DROSS		

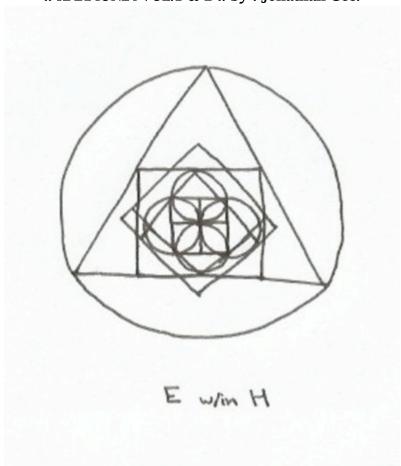
KOSMOS

:: introduction to HARMONIA Vol. 1 ::

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II. the Torus





Existence of Essence



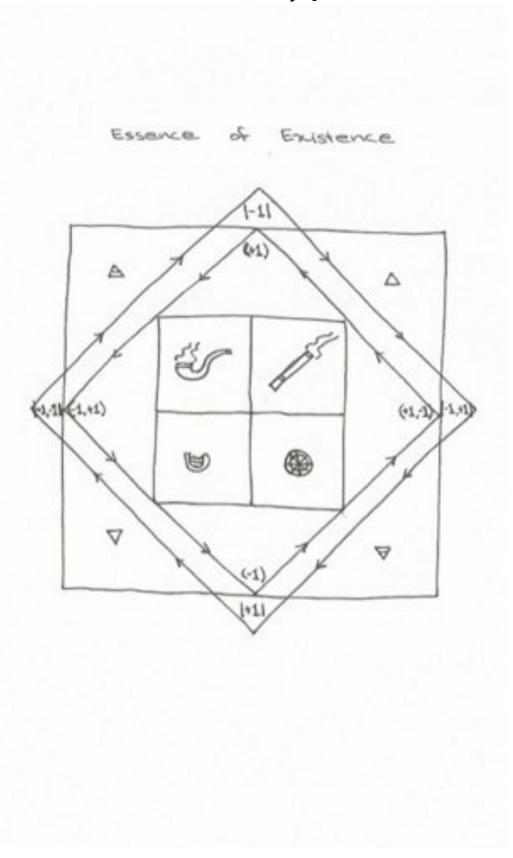
the Self-Unit as ancestor to the Quantum and as predecessor to the meme.

Essence of Essence

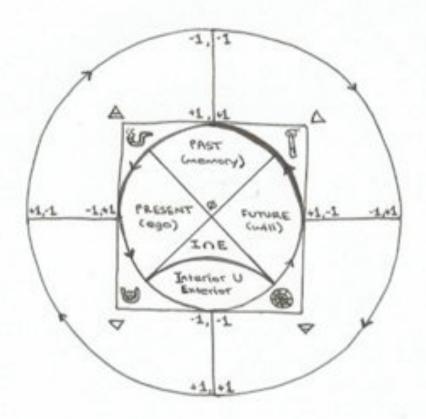


the INDIVIDUAL SELF
AS MONAD-UNIT with
Relativity to Exteriorization
of "Self" as "Otherness".

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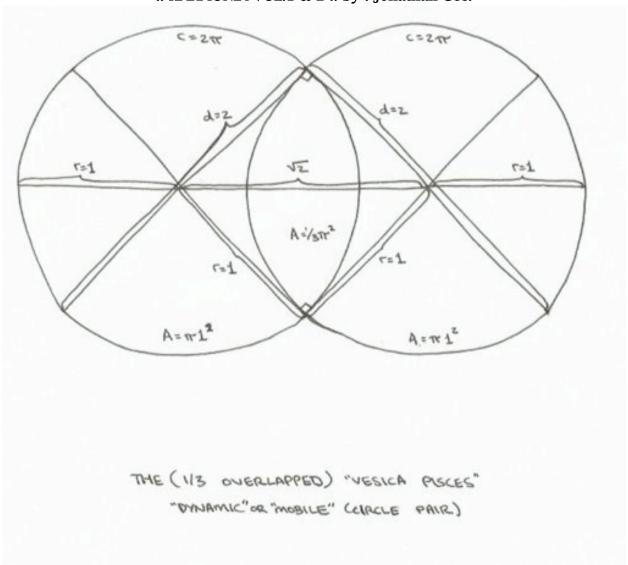


Existence As Is

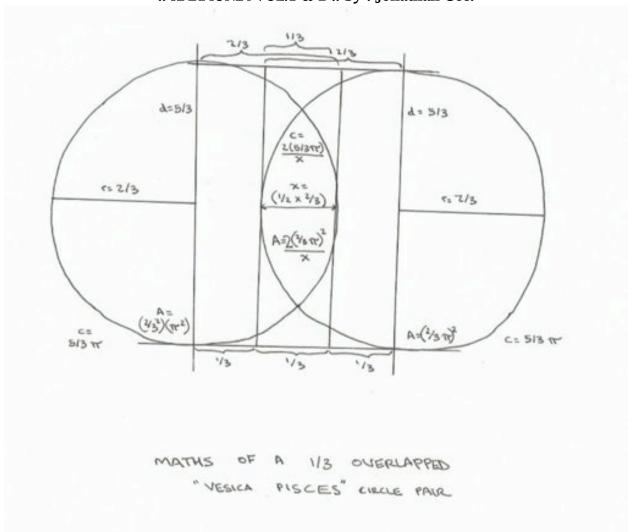


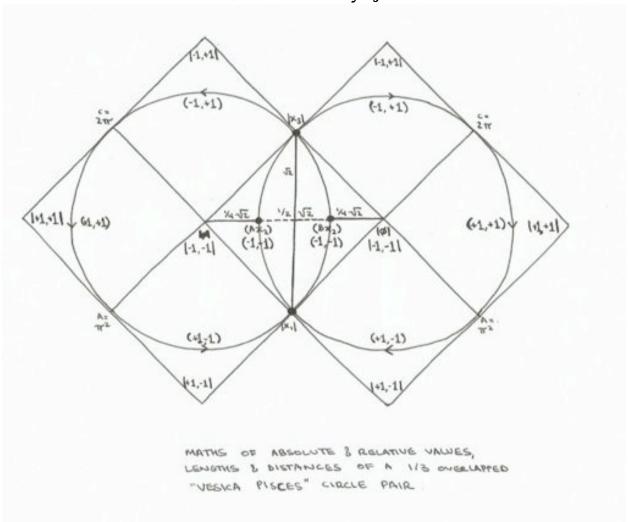
INDIVIDUAL SELF RELATIVE TO EXTERIOR REALITY YIELDS THE "SELF" AS NUTOMATED "SYSTEM", INITIATION OF INTERIOR/EXTERIOR COUNTER-ROTATION/INTVITION.

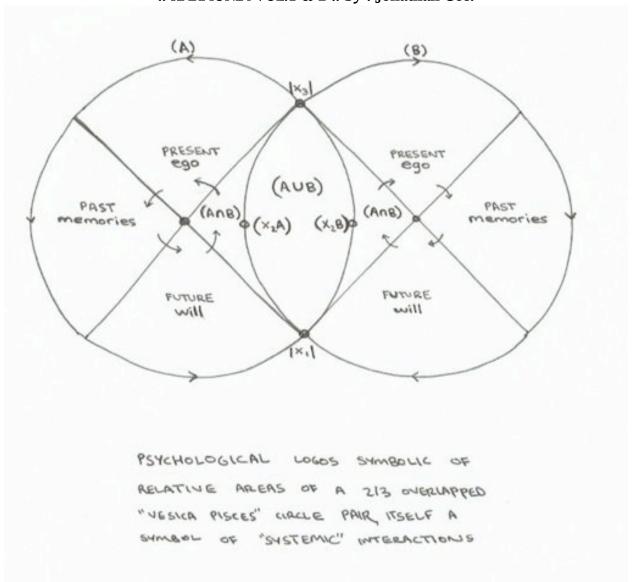
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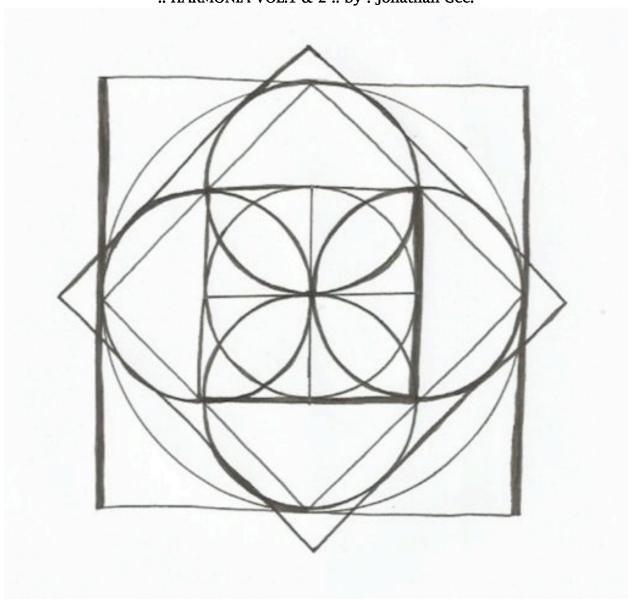
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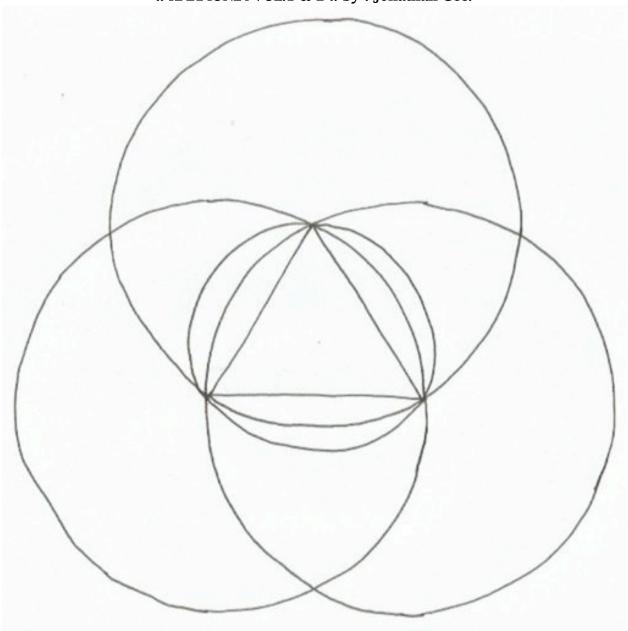




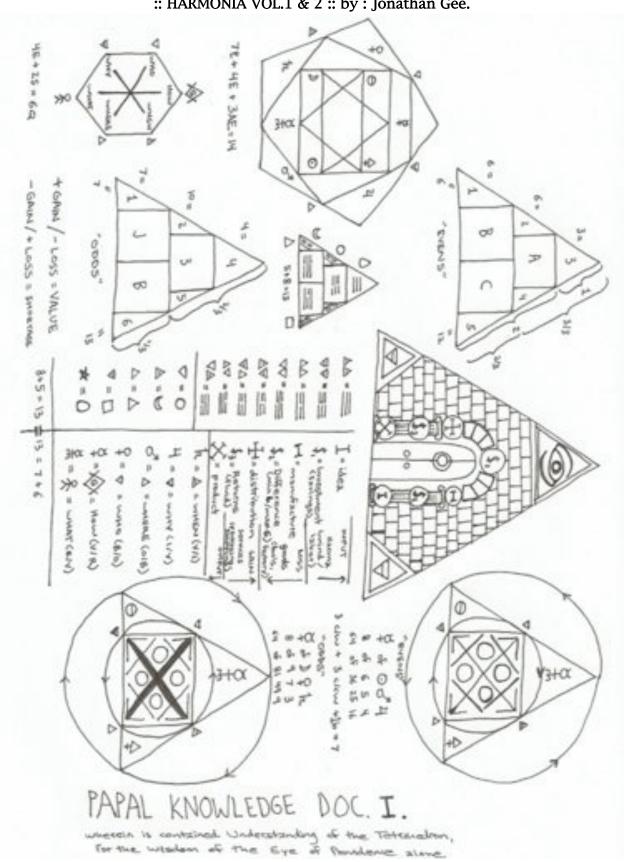
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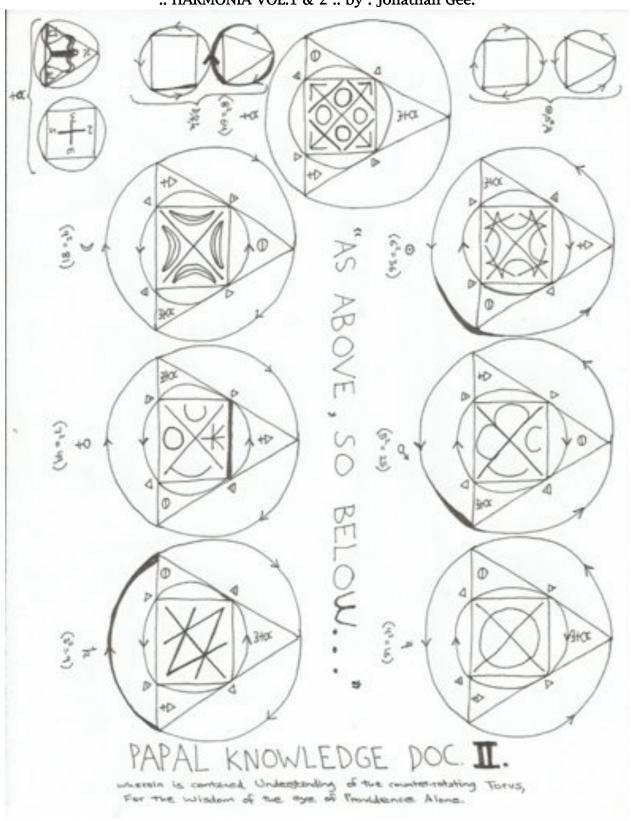


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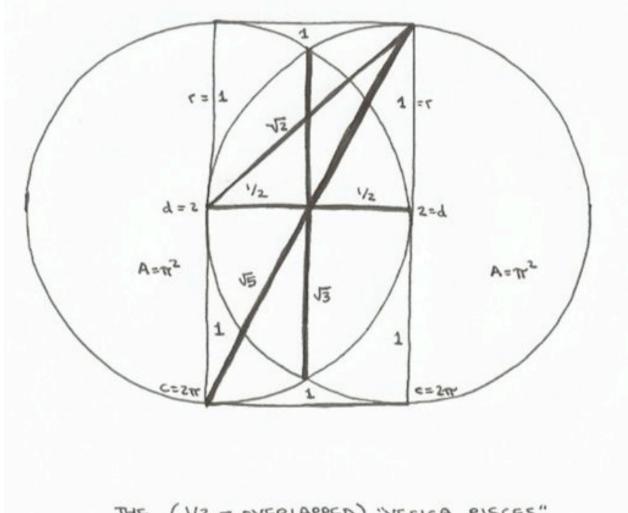


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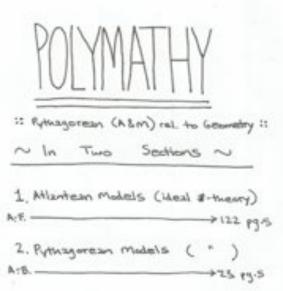


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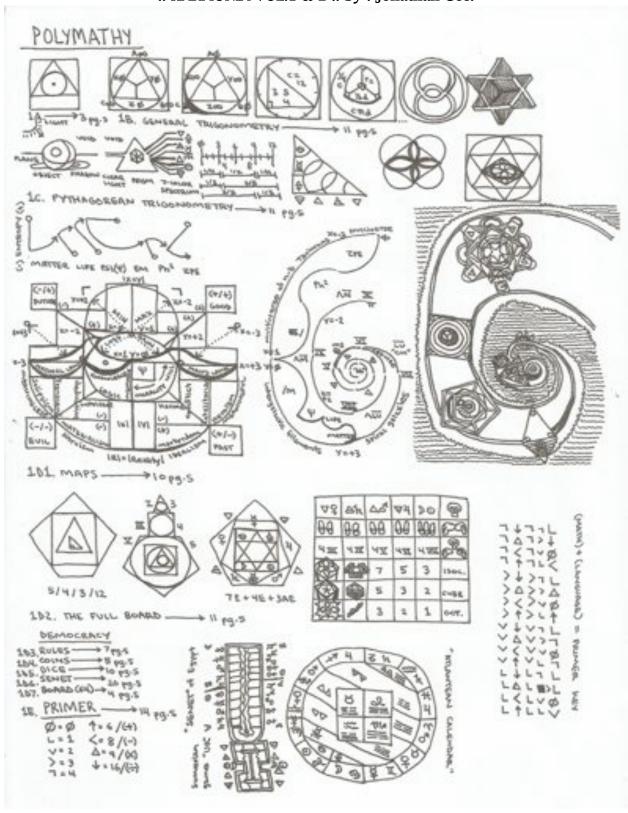


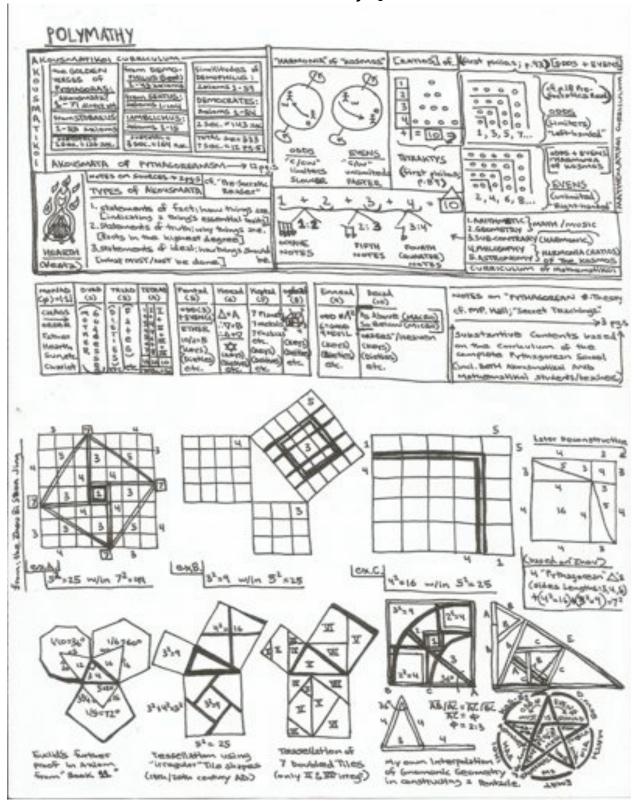
THE (112 - OVERLAPPED) "VESICA PISCES"

"STATIC" OR "STABLE" (CIRCLE PAIR)



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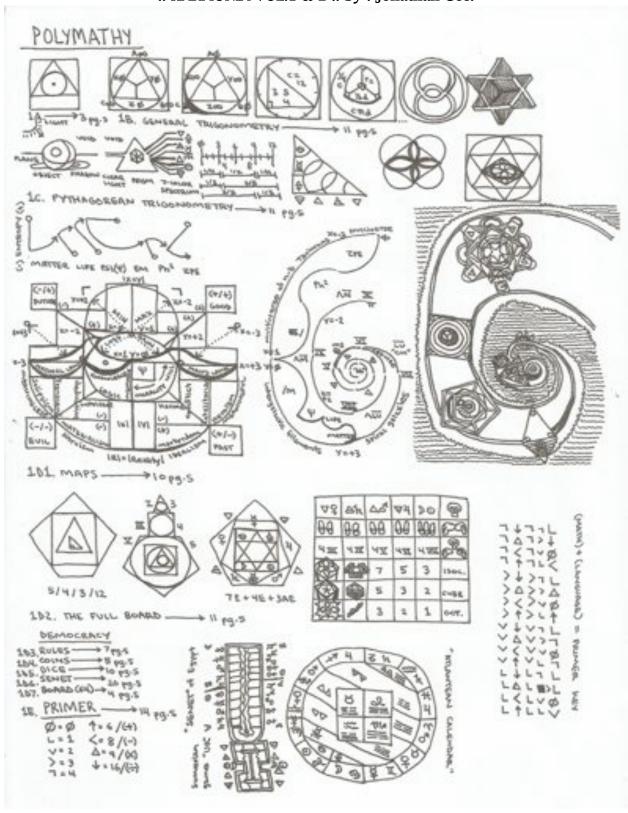




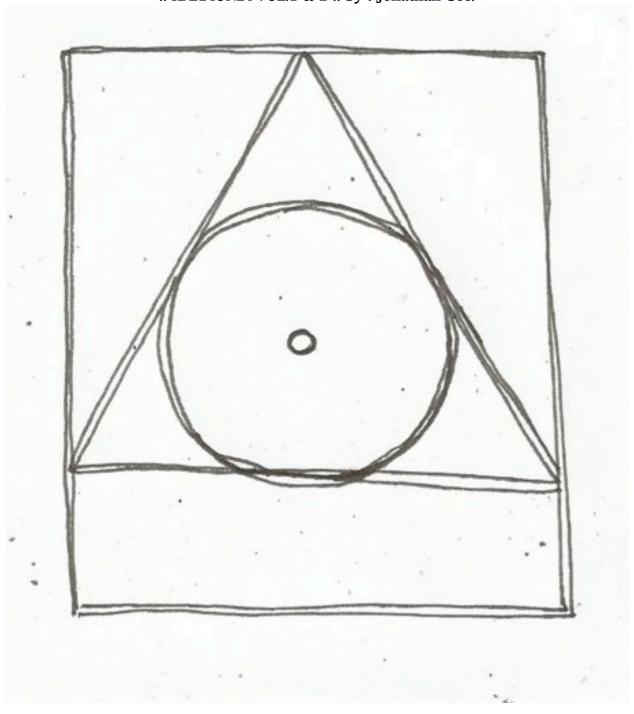
:: Section : 1. Atlants ::

1A. Enochian Names of God —	→ 3 pg.5
18. General Trigonometry —	→ 11 pg·s
1C. Pythogorean Trigonometry—	- 11 pgs
LD. ATLANTIS—KSTOTAL PER SON	
101.mas	> 4 pg.5
102 THE FULL BOARD -	-> 11 pg.5
103, Rues of bemocracy -	→7 P5.5
184. Coins / Votes	→ 5 PJ:
105. Dice / RHES	-> 10 6d
106, Senet - (TOTAL/Sed	ion + to pa
156A, research	→ 6 pg.
1068 Egyptian (notes)	→8 kg
106C , whiter versions	->6 P3
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F. Misc.	-> 2 pg

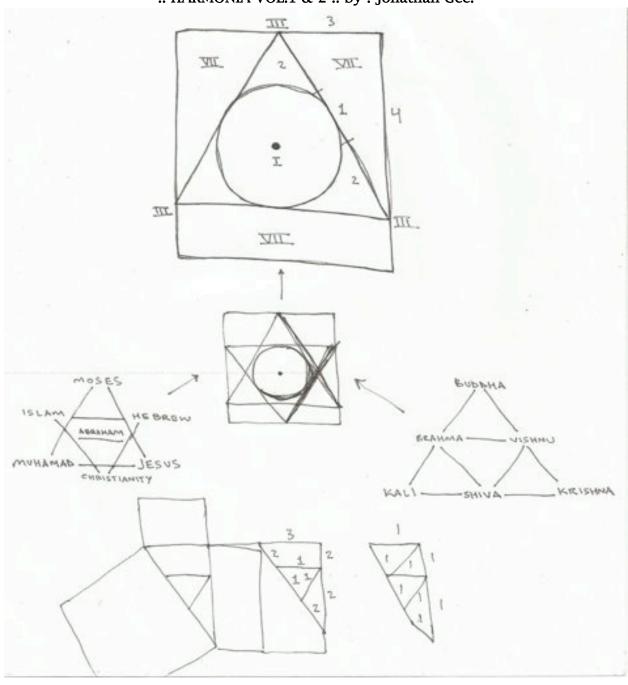
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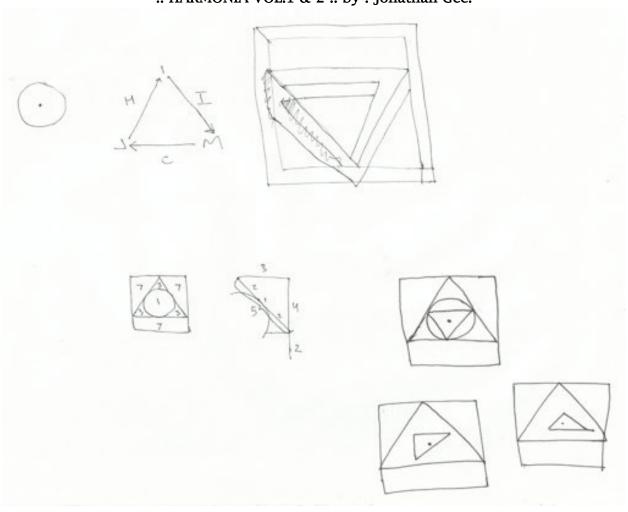


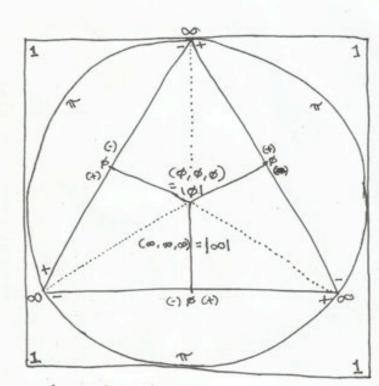
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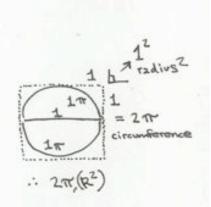


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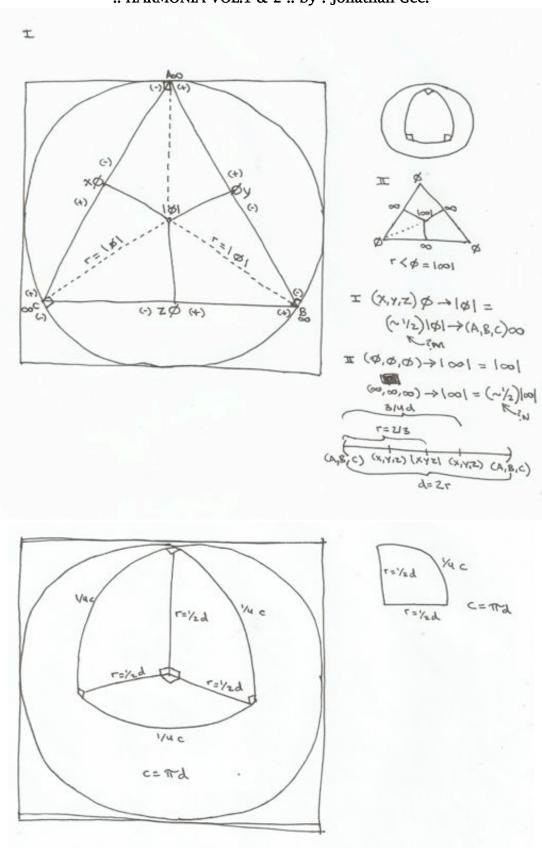


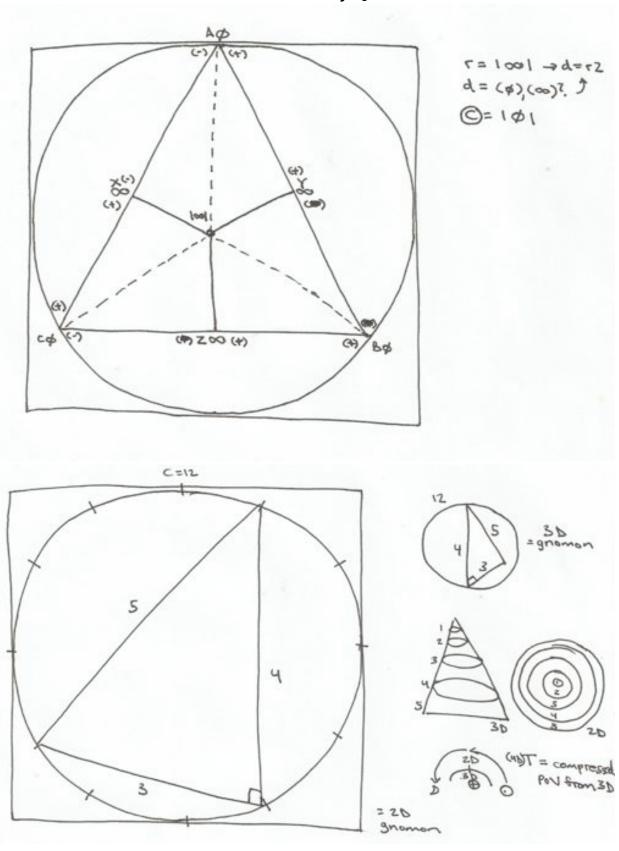
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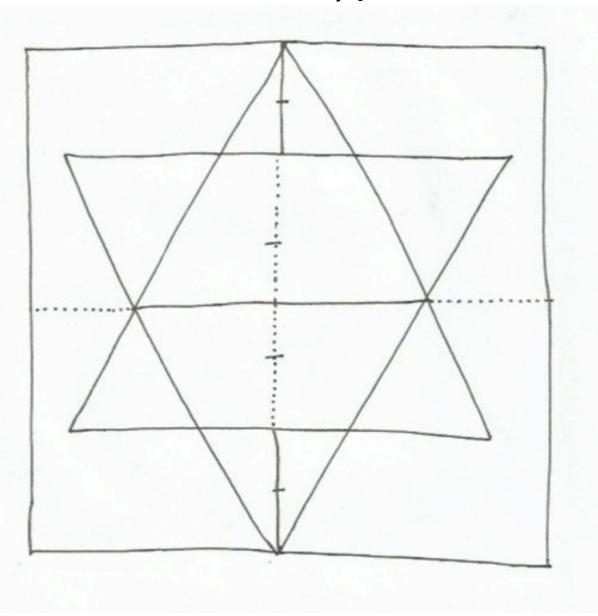
$$1^{4} = 1^{3} = 1^{2} = 1 . \infty/\phi = \phi . \phi \times \omega = \phi .$$

$$(\phi)^{\infty} = |\phi| .$$

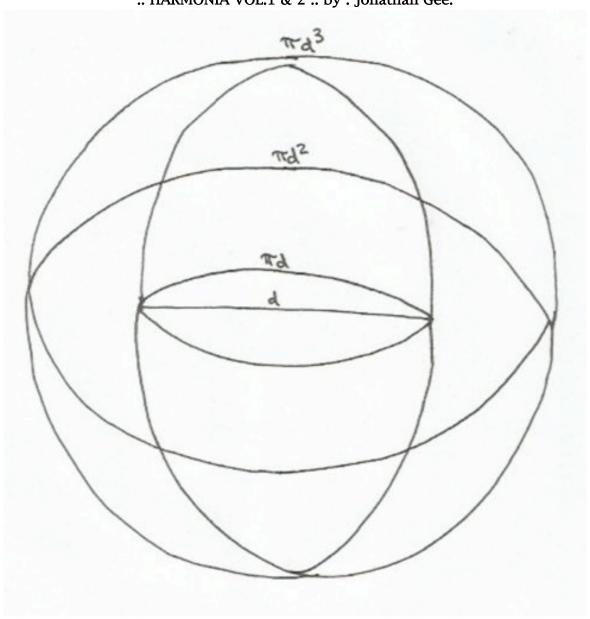




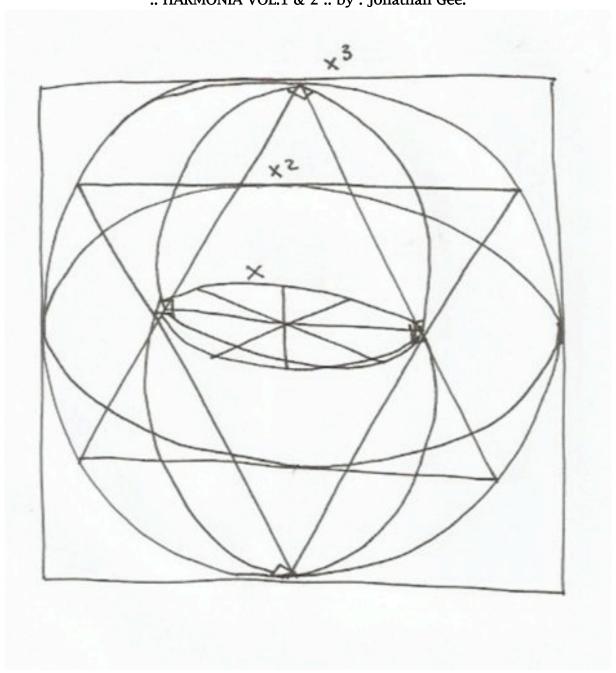
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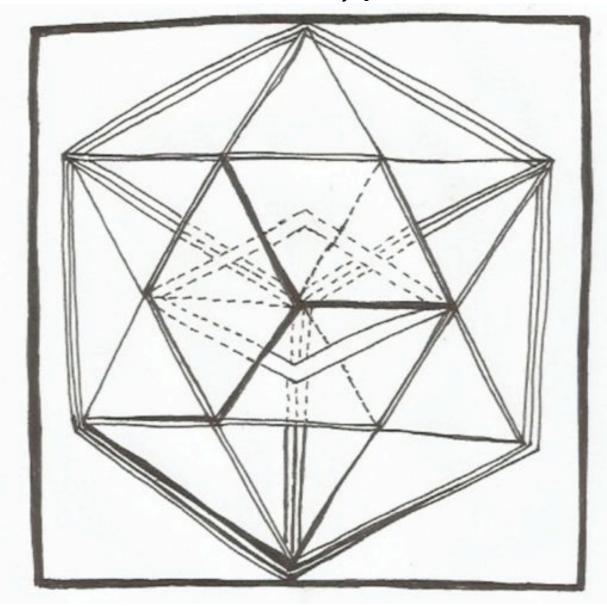
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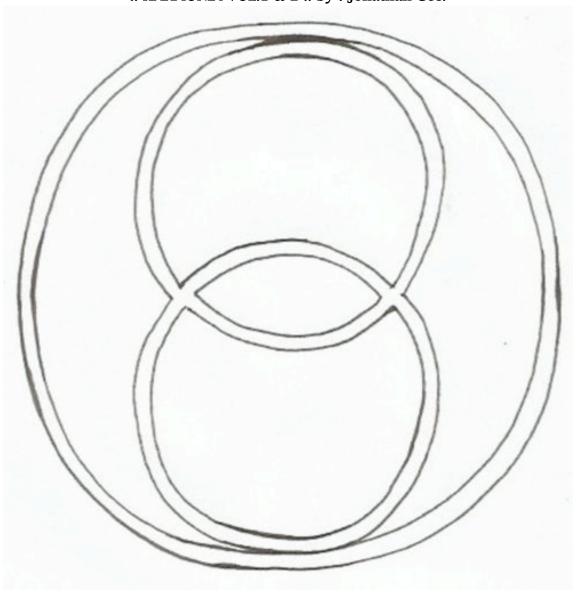
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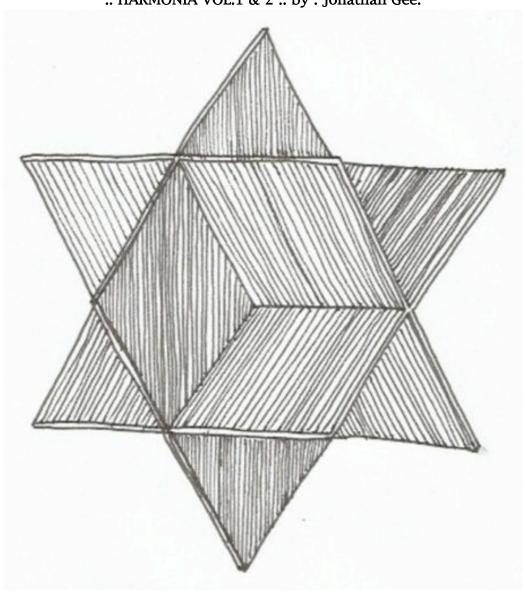
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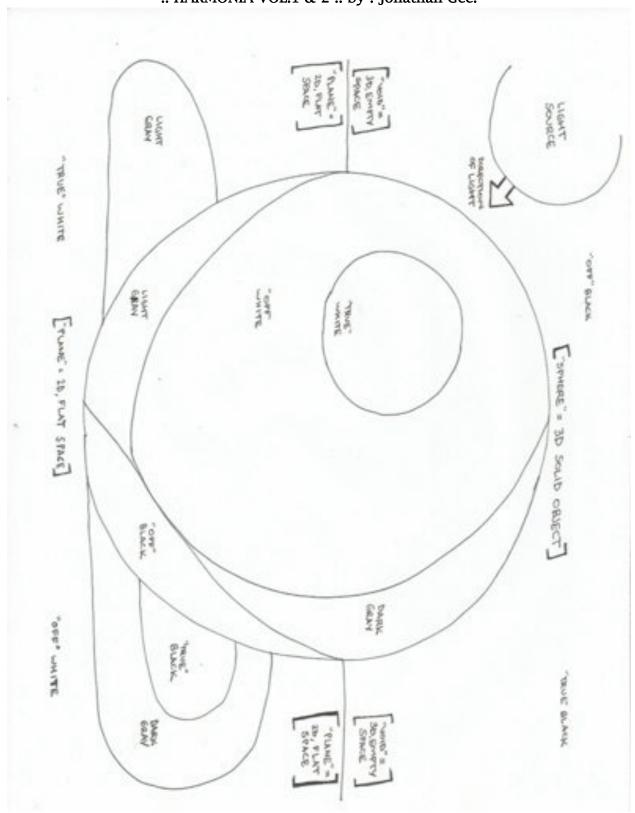
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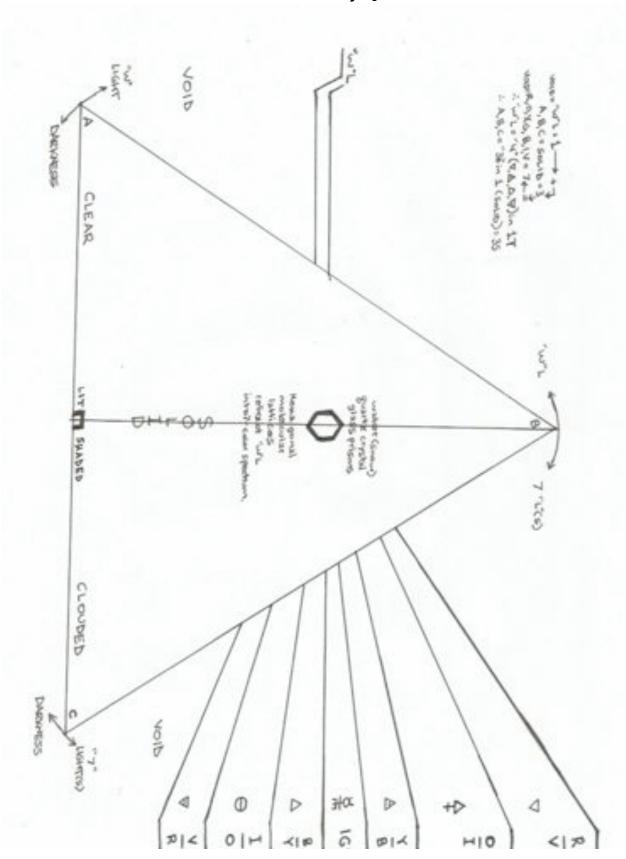
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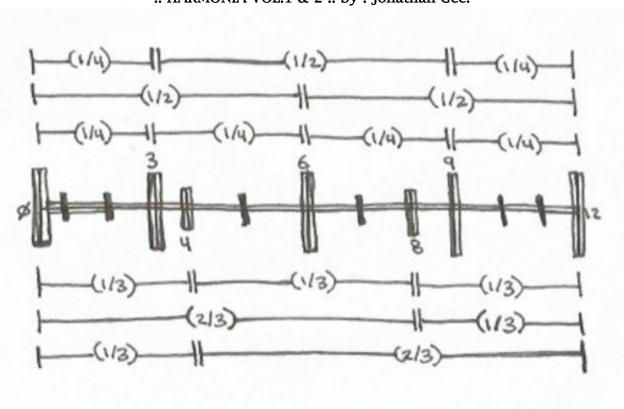
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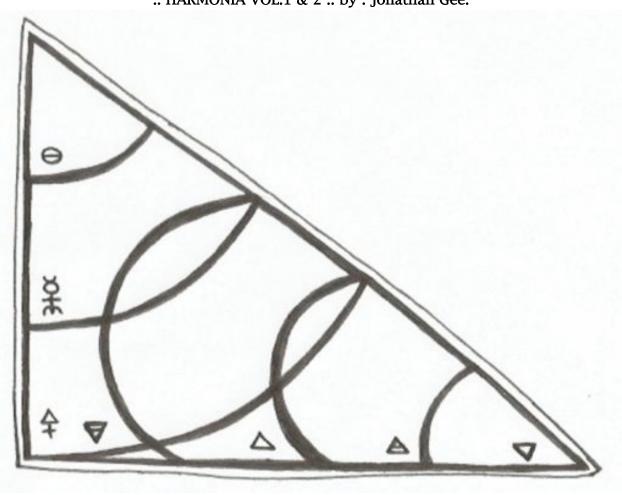
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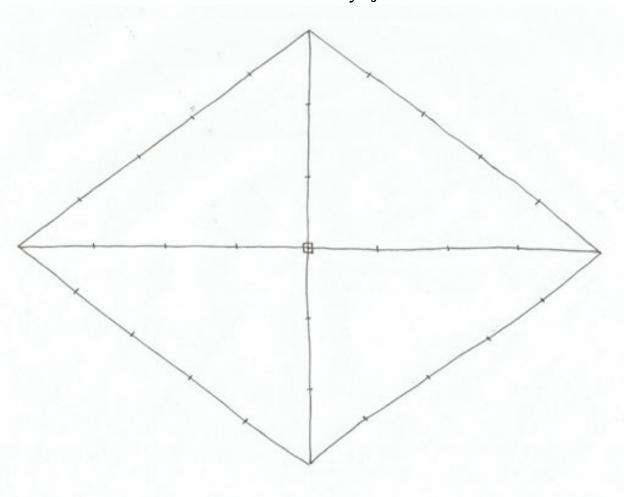
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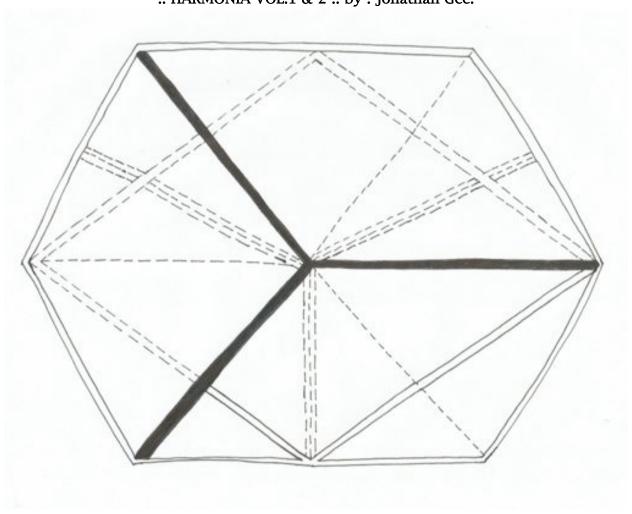
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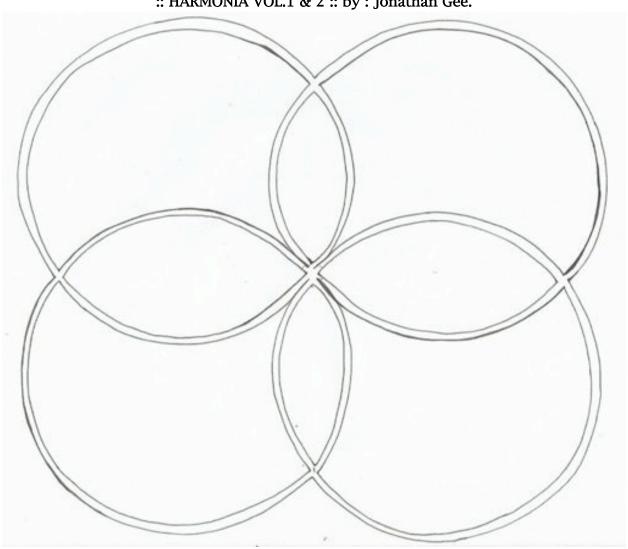
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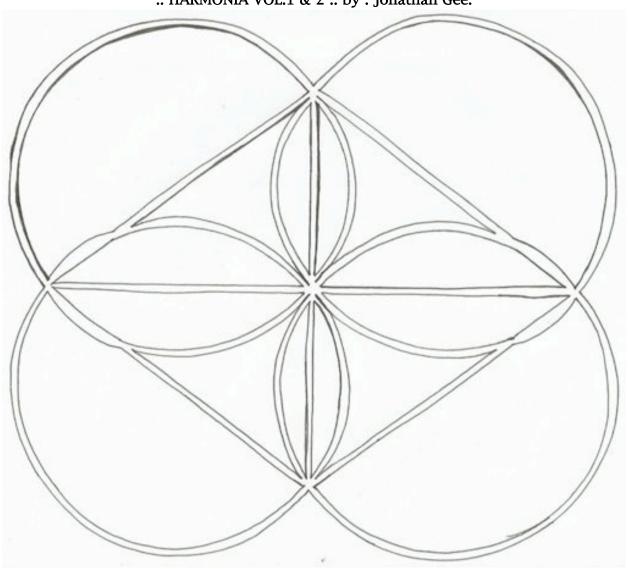
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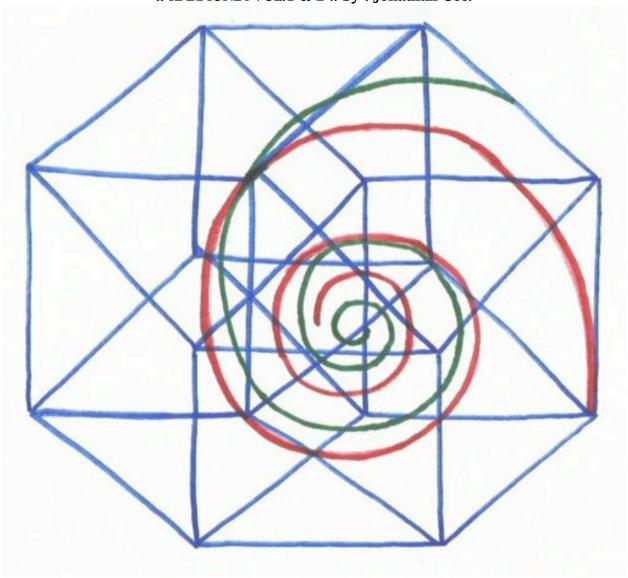
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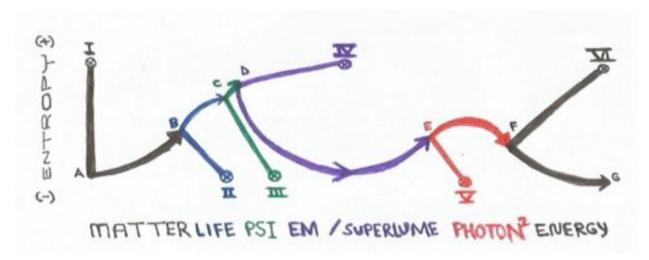


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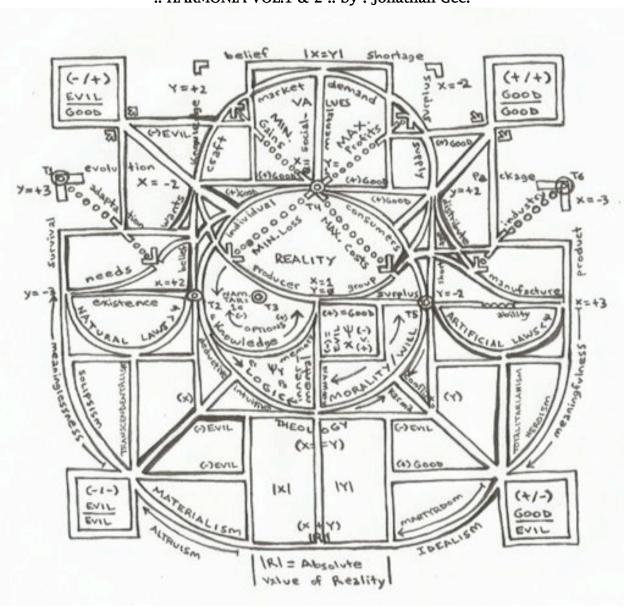


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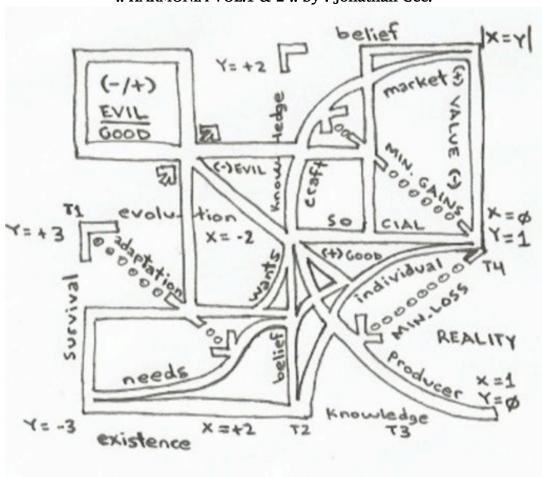




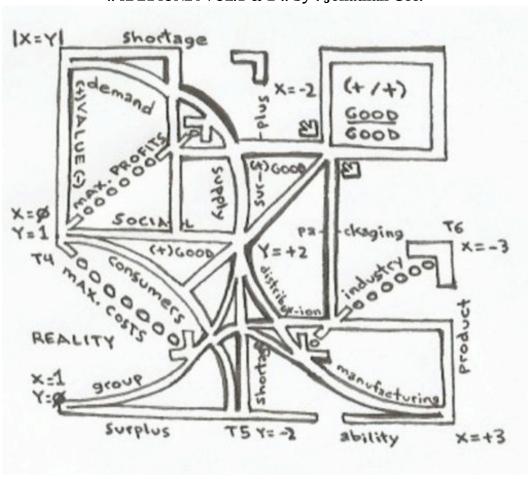
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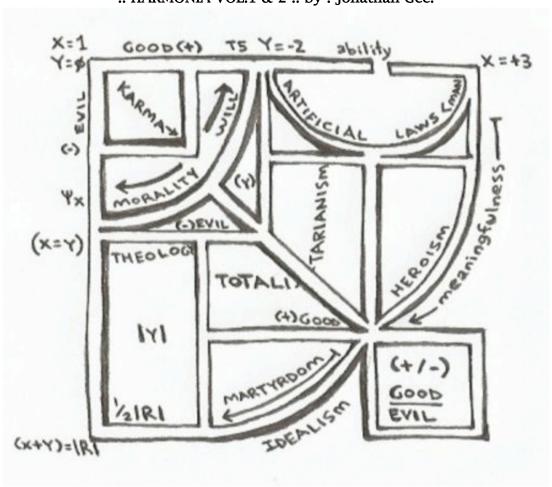
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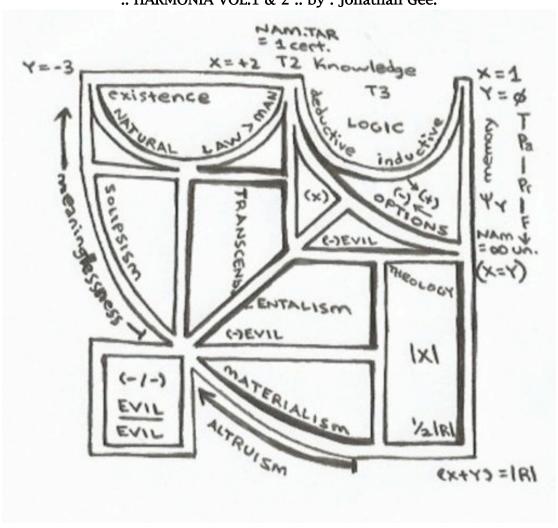
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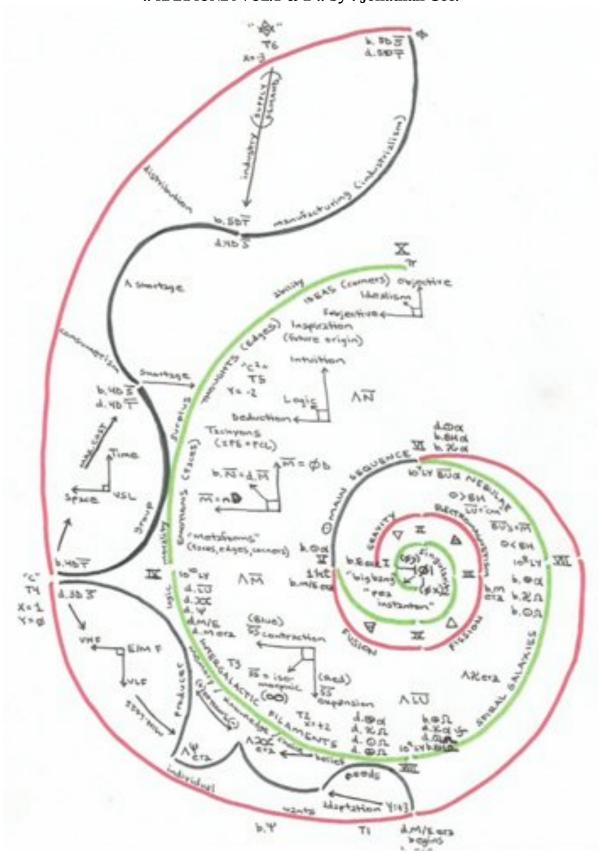
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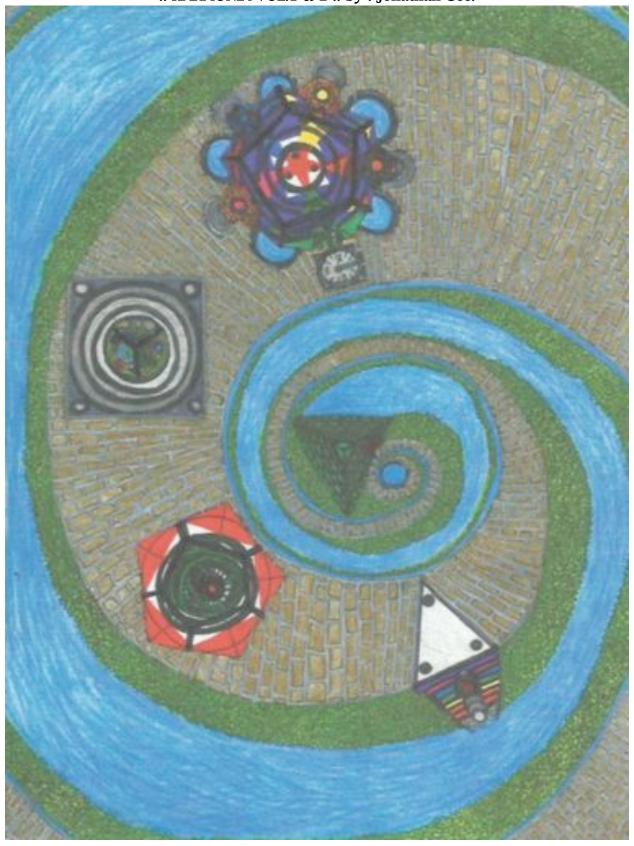


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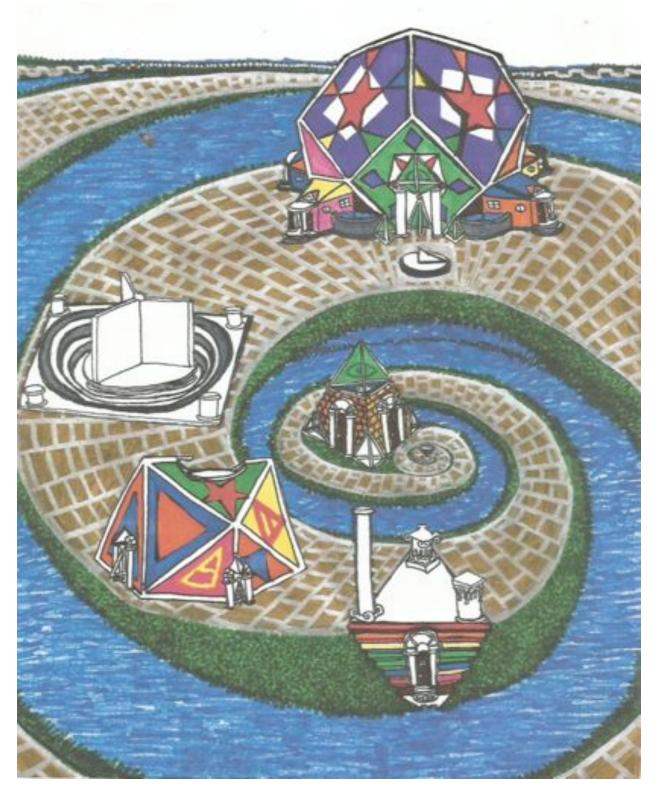


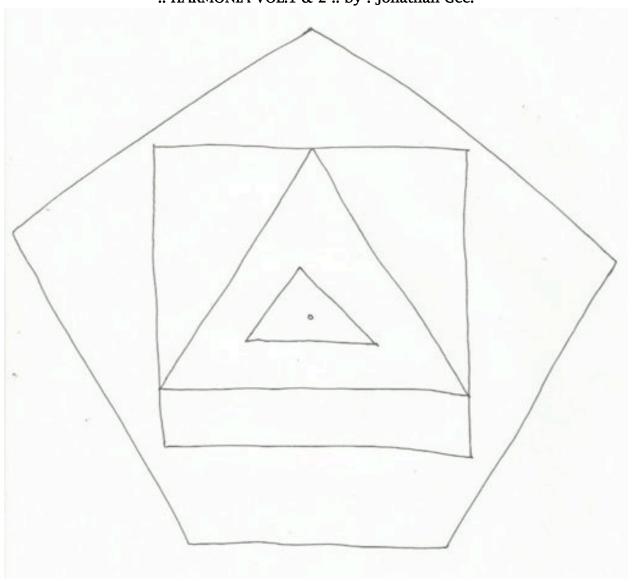
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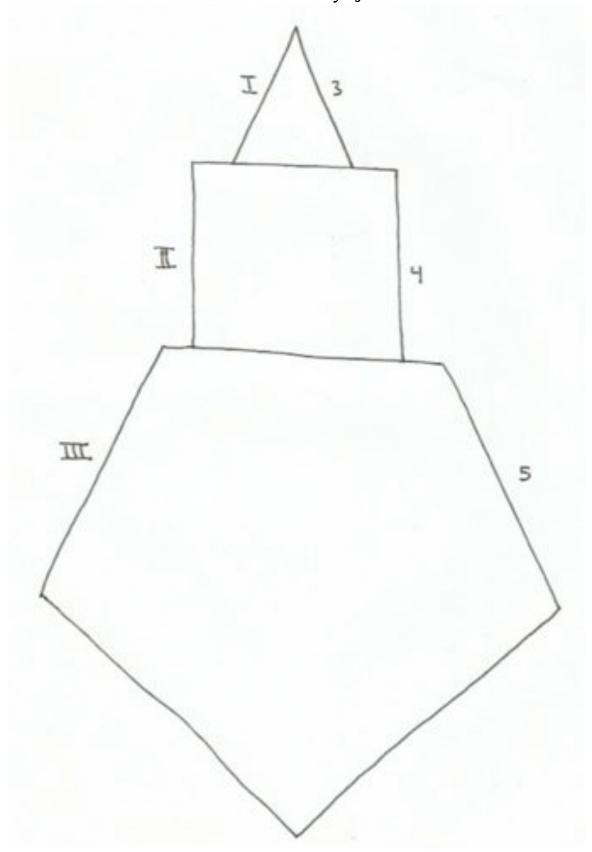


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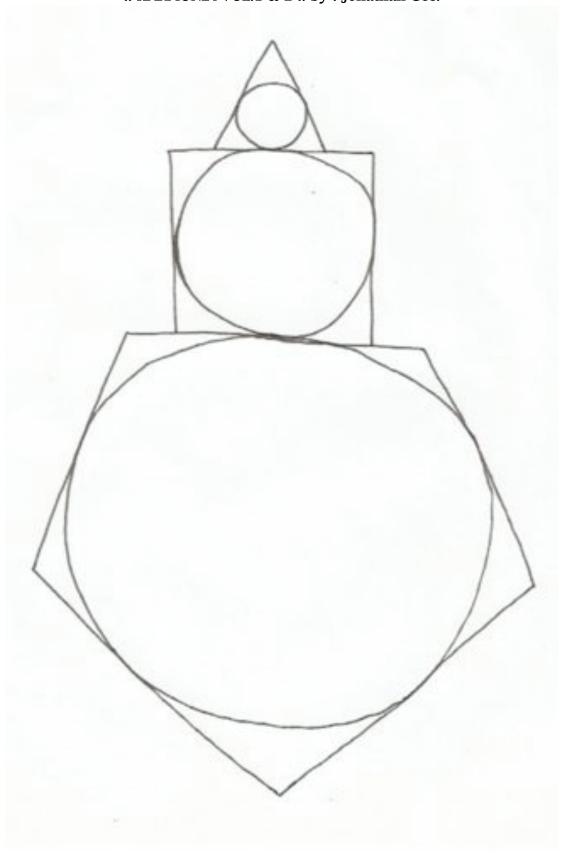




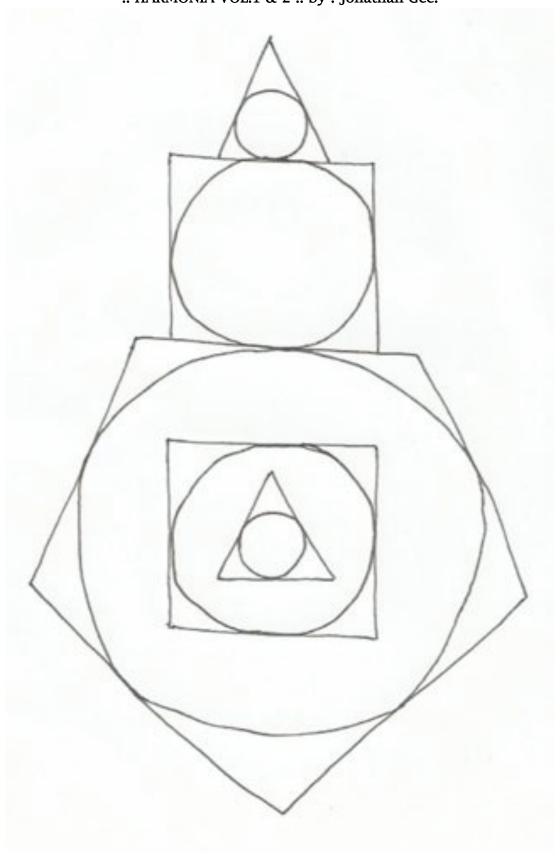
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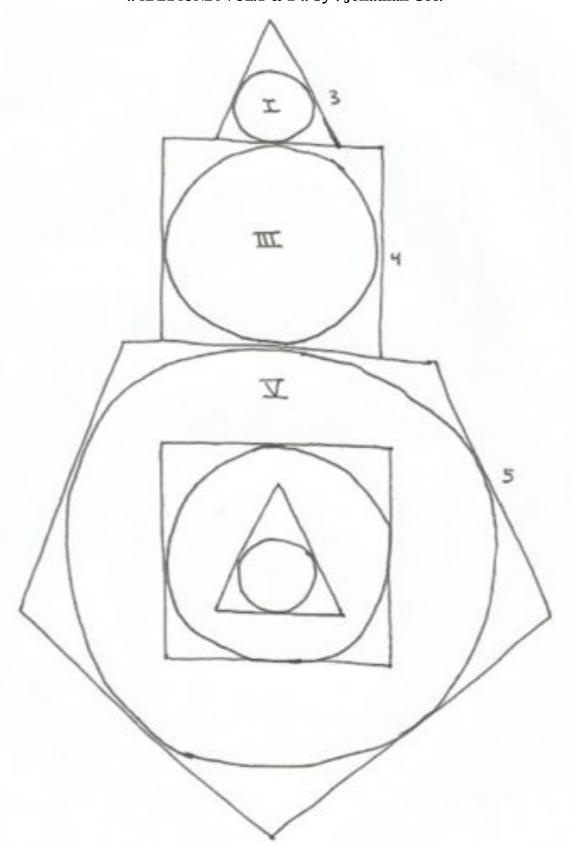
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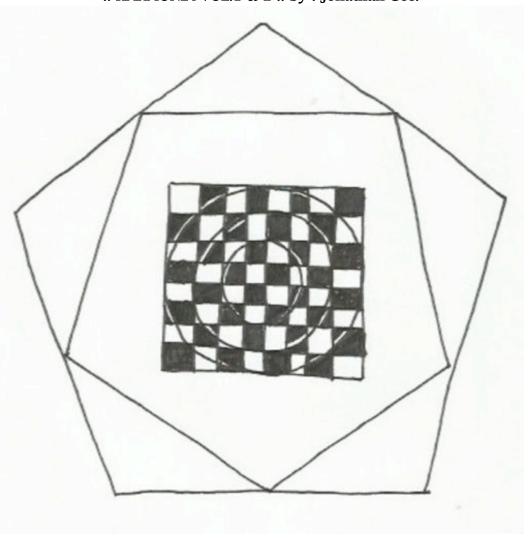
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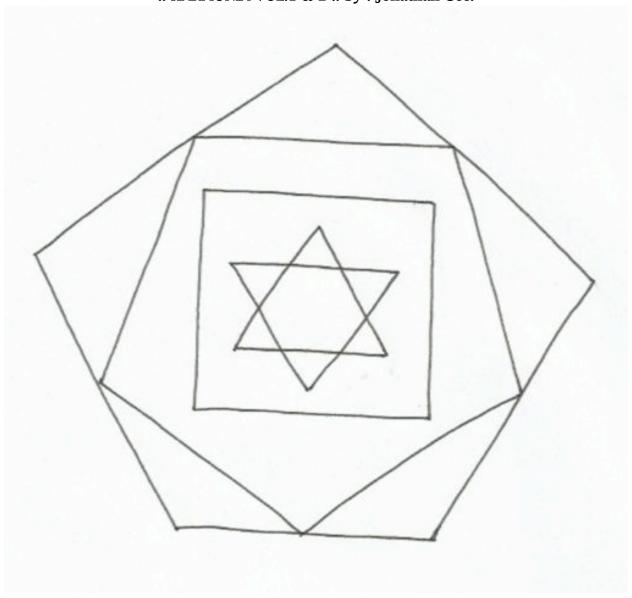
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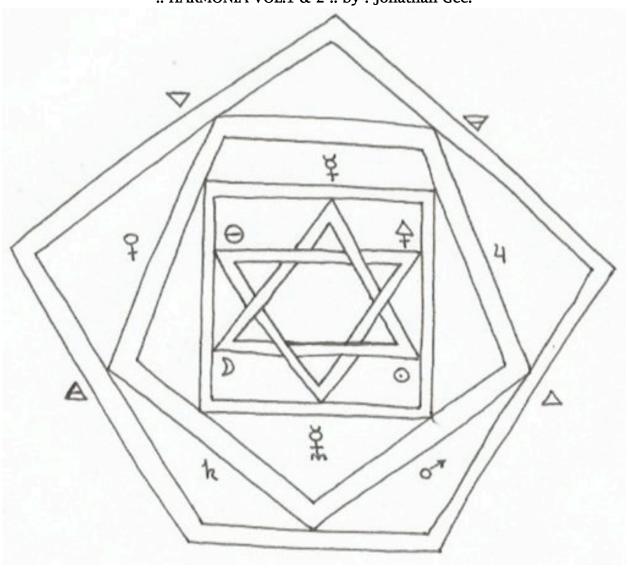


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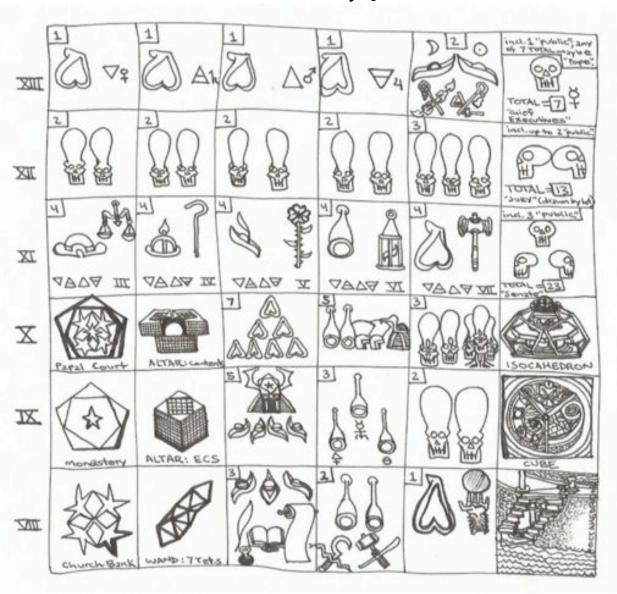


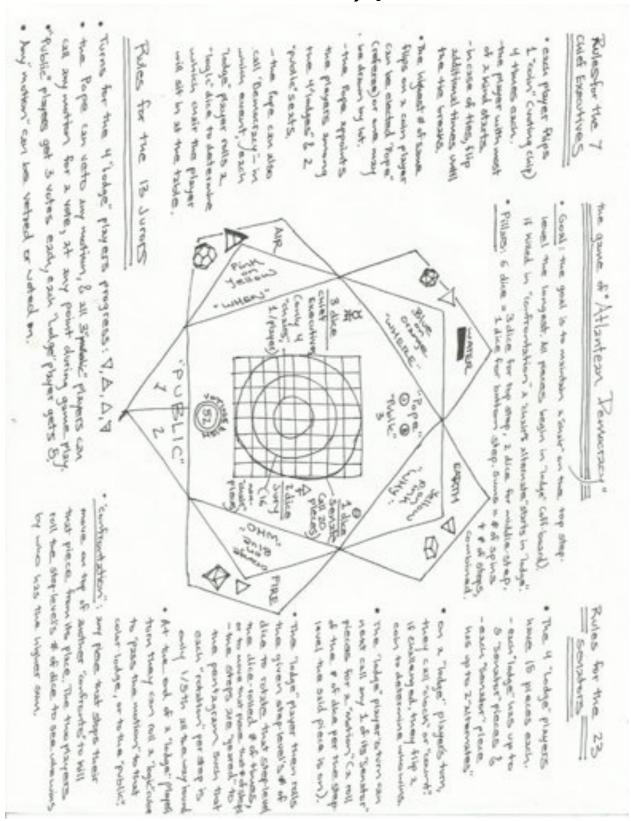


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:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.





Rules of "bemocracy" (countlipping / vote-production game)

consists of 5 "rosses" (evin-tips) of one "voting evop" (41N) coin.

TOSS \$\sigma\$ = "Public" Throw = (4="world"/N="self")

determines between 2 PLAYERS who will PLAY "benowing"

outcomes chosen between PLAYERS prior to TOSS

1 PLAYER = Y("world") / 1 PLAYER = N ("self")

whoever predicts the extreme correctly chooses

either themselves or their opponent to PLAY... 3 and of 5...

TOSS 1 = "Popular" Throw = (4="up" / N="down") = 1 v.6" (4yany)

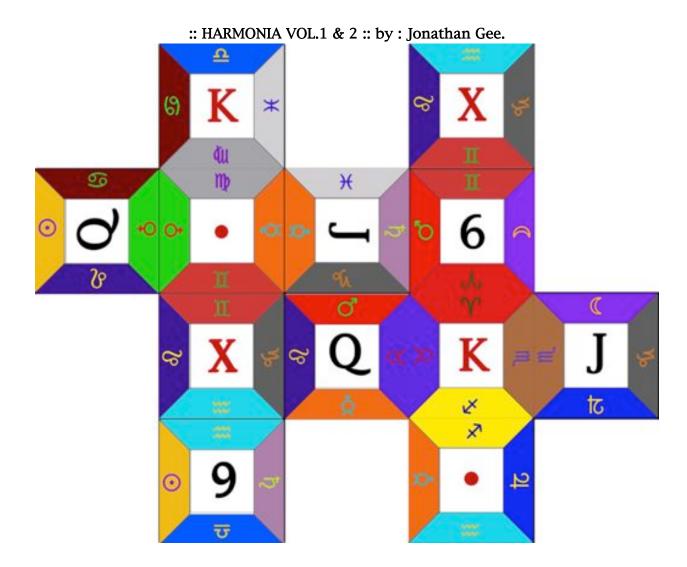
TOSS 2 = "Executive" Throw = (4="in" / N="out") = 2 v.5 (22 is)

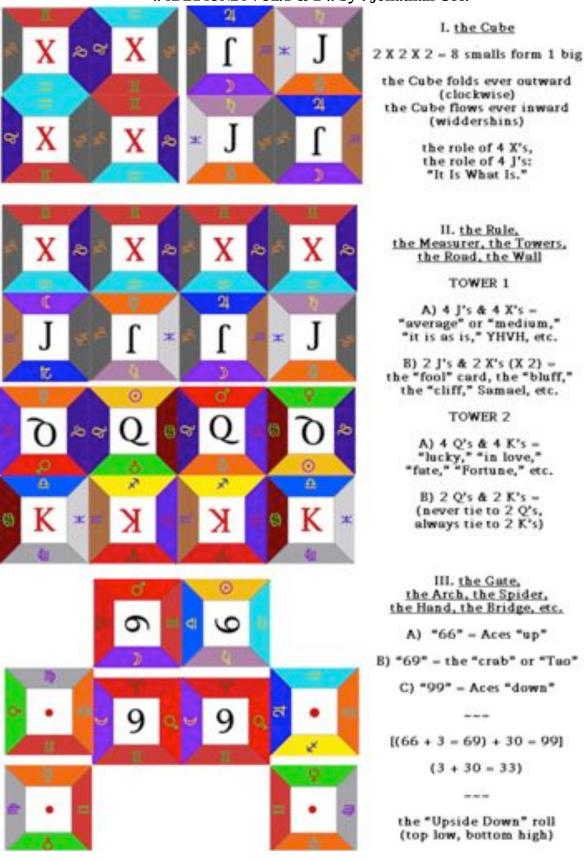
TOSS 3 = "Jury" Throw = (4="24" / N="Right") = "Jv.4" (domonacy)

TOSS 4 = "Sende" Throw = Cy="front" / N="back") = VICTORY

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II. EXECUTIVE = "up" vs. "down" = SALT = 1/4 III. JUDICIAL = "in" vs. "out" = SULFUR = 1/8	IIAK	I. PUBLIC = "open" vs. "closed" = LEAD = 1/2
	0000	A STATE OF THE STA
		III. JUDICIAL = "in" vs. "out" = SULFUR = 1/8
V. PAPAL = "front" vs. "back" = GOLD = 1/32 V. PAPAL = "front" vs. "back" = GOLD = 1/32 V. PAPAL = "front" vs. "back" = GOLD = 1/32		IV. SENATE = "left" vs. "right" = MERCURY =1/16
		V. PAPAL = "front" vs. "back" = GOLD = 1/32





20%

$$\begin{array}{c|cccc} L & W & G \\ \hline 2 & 8 & = 10 \\ \hline 3 & 9 & = 12 \\ \hline 4 & 12 & = 16 \\ \hline \end{array}$$

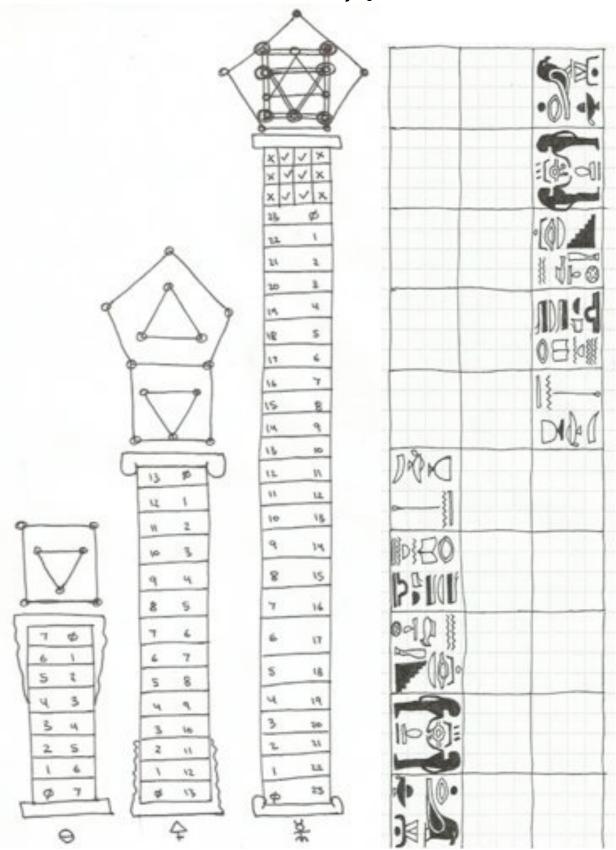
331/3%

L	W	G
3	5	= 8
2	3	= 5
1	3	= 4

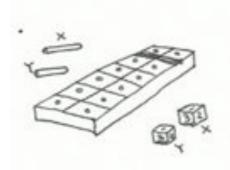
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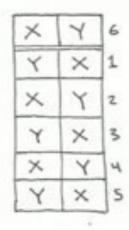
$$\begin{array}{c|cccc}
L & W & G \\
\hline
2 & 4 & = 6 \\
4 & 6 & = 10 \\
6 & 7 & = 13
\end{array}$$

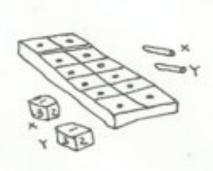
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Senet: scoring w) dice







rules:

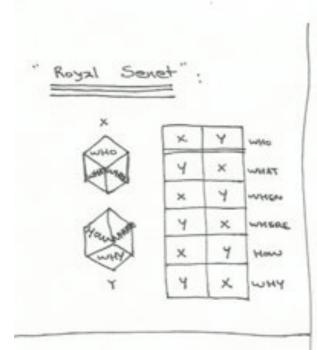
DICE

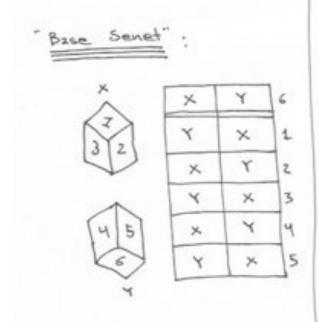
- regular 6x cubes XZ
- (s ot que repely place -
- roll one dice each

SCORING

- Player X & Player Y both have one peg each. Each posts their peg up to their own space on the board.
- X can become Y & Y become X

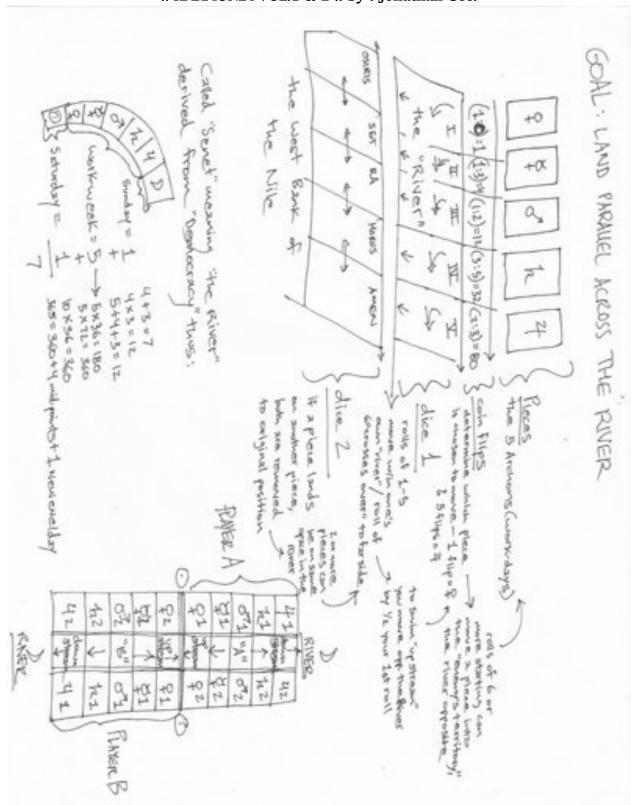
both X & Y tje, then the second to score that point drops down one slot in the same column, & the winner is bumped up one slot or row.

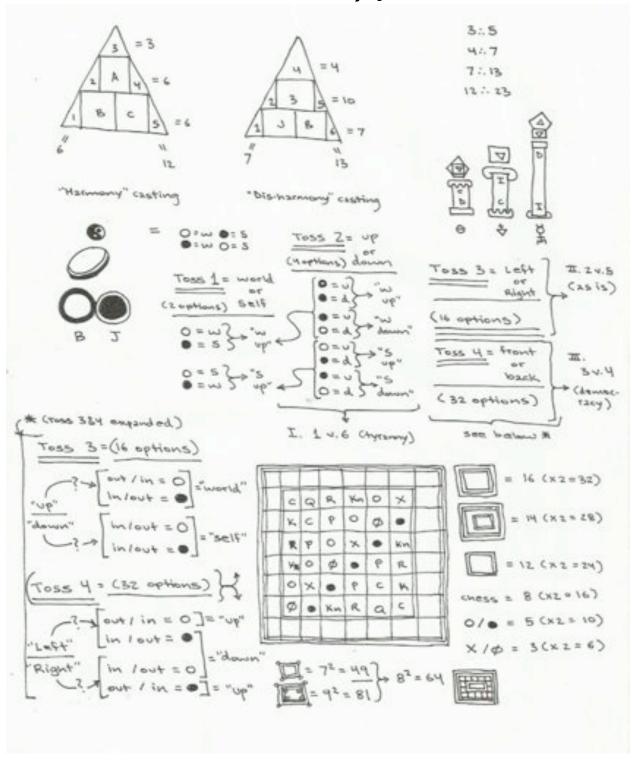


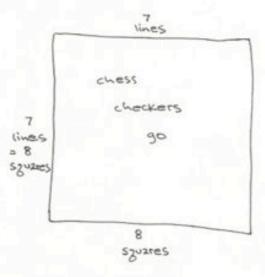


GOALS :

the game "Senet" is a race. Players (x,y) begin at the bottom of the board. 5 is the lowest score, 6 the highest, & I second down after the top. Base " Senet is played with regular pipped dice (#d-sides 1-6), while "Royal" Senet is played with logic dice (me & questions). Usually, two games - 1 "Roys" & 1 "base" are played simultaneously - such that players X &Y have both Kinds of dice. The GOAL of the game is to have the highest point total Caccording to the row-slots as given on the senet board), ¿ to have one's peg closest to the top of the board. The # of rolls & length of play can warry anywhose from I rell of I dice to my & rolls of any # (or kinds) of dice. A usual game lasts two rolls of two dice by two players.







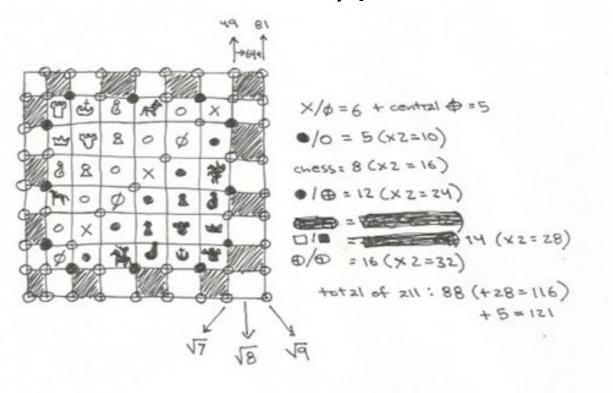
46 votes amongst 32 senators
4 lodge chairs are of 7 execs
the fifth lodge is "public" to
three representatives, each of
whom gets two votes (3×2=6;
46-3+6=48;48=7²)

new members zways require a majority vote (417) among the tribune, comprised of no less than one administrator.

for 5 to rule 7 2dd 2 new members for 7 to rule 13 2dd 6 new members for 13 to rule 23 2dd 2 new lodges (10 members)

the same system permeates the gov4.'s voting each member has two votes (black= N, white = Y) thus 46 votes are all that are needed. To resolve any issue in the senate requires 26 yor N. If an issue cannot be resolved by the senate, then it reverts to the Tribune 8 the 7 + 6 = 13. That is how the 13-Tribune resolves the senate.

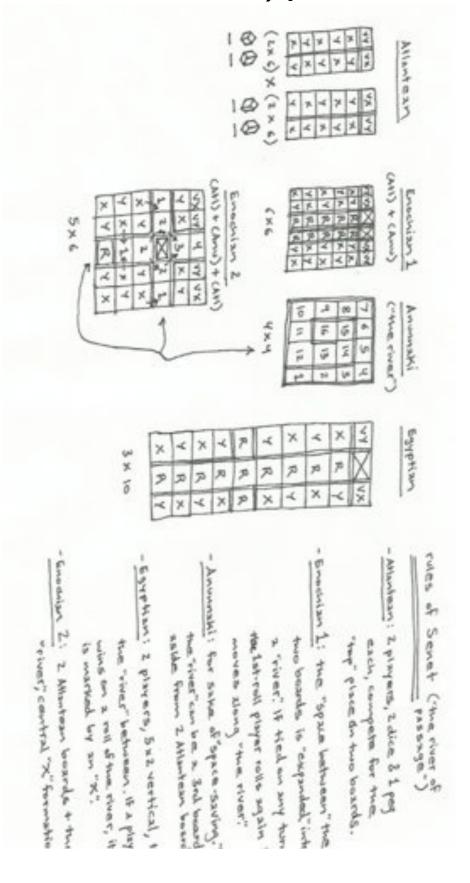
It all breaks down to 5 in 7, 3 in 5, 2 in 3, 8 1 in 2.



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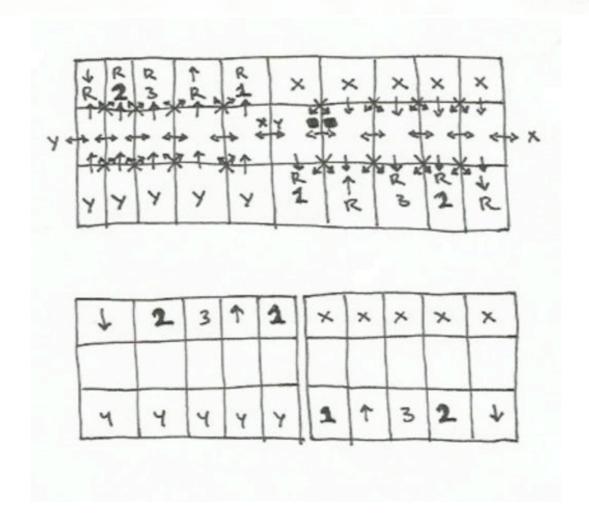
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	0	0		0		WHA	π															
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	0	0	-	0	9		-	0	-	0	1			0	0	0	0	0	0		0	
	0	0					-		-	10	1			0	-	0	-	0	0	0	0	
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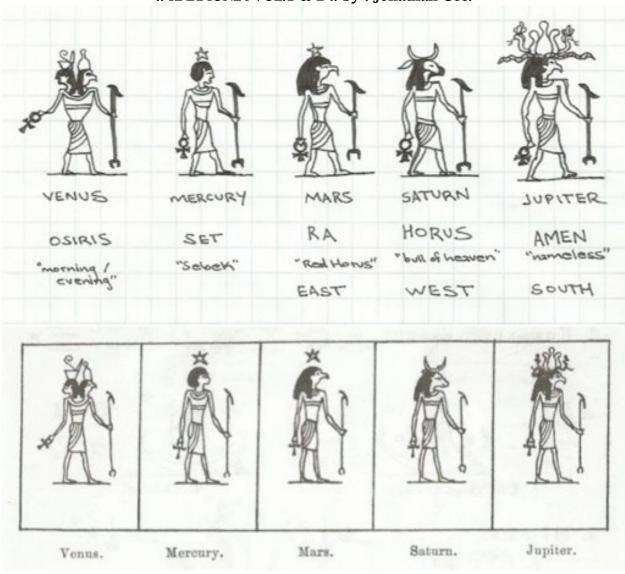
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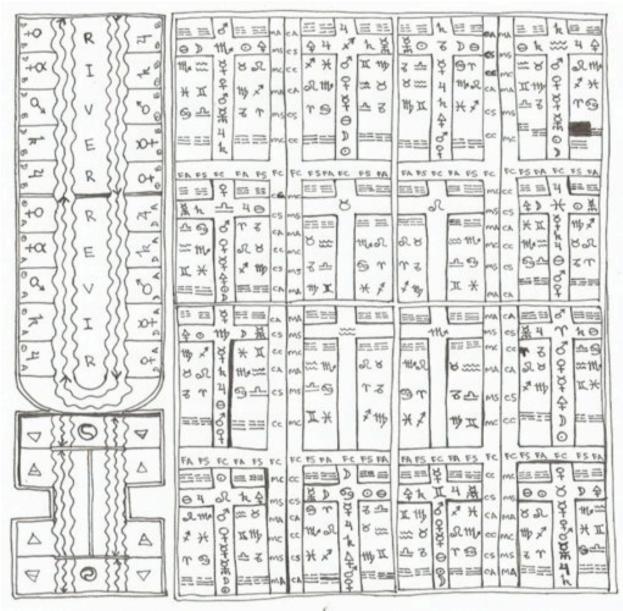
:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.

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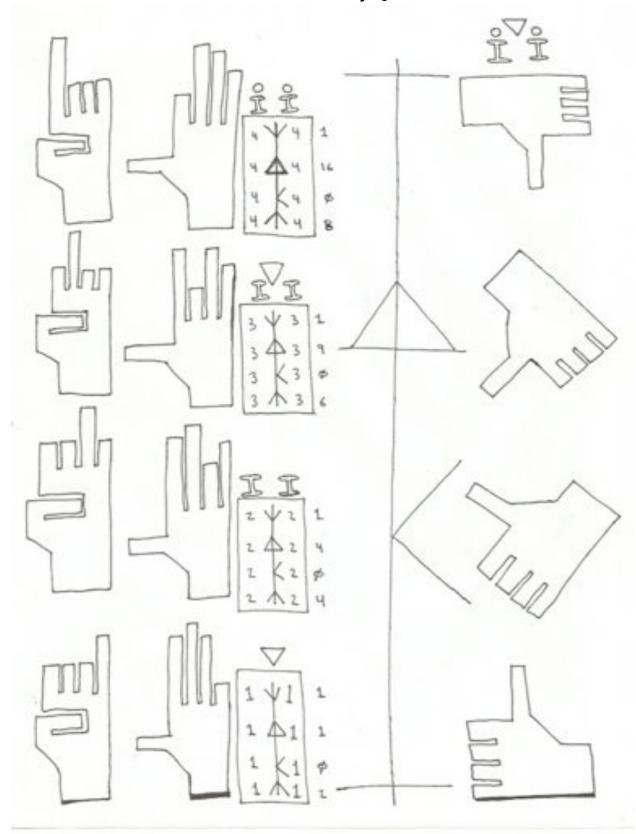




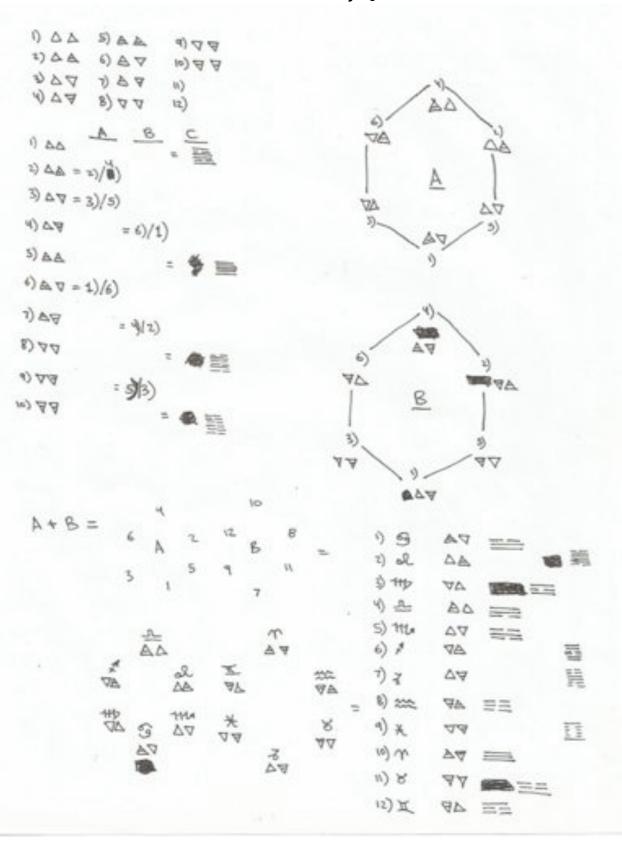
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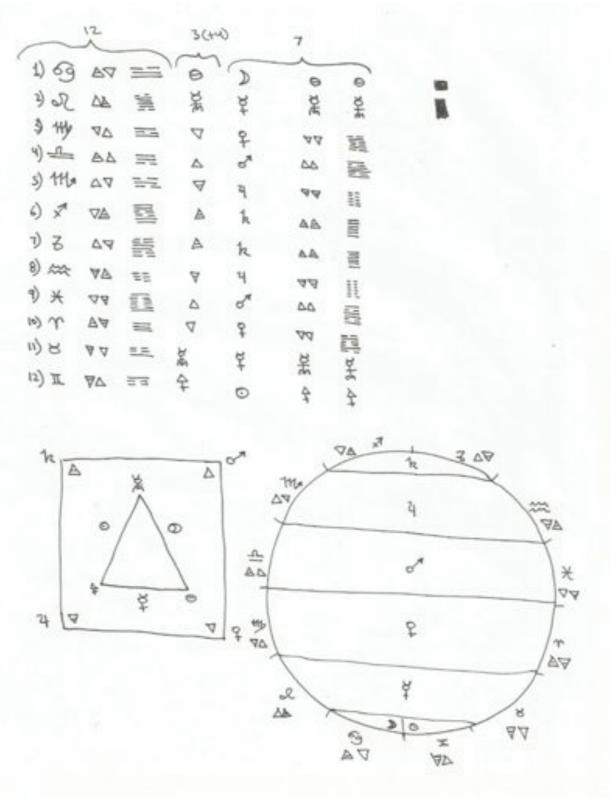
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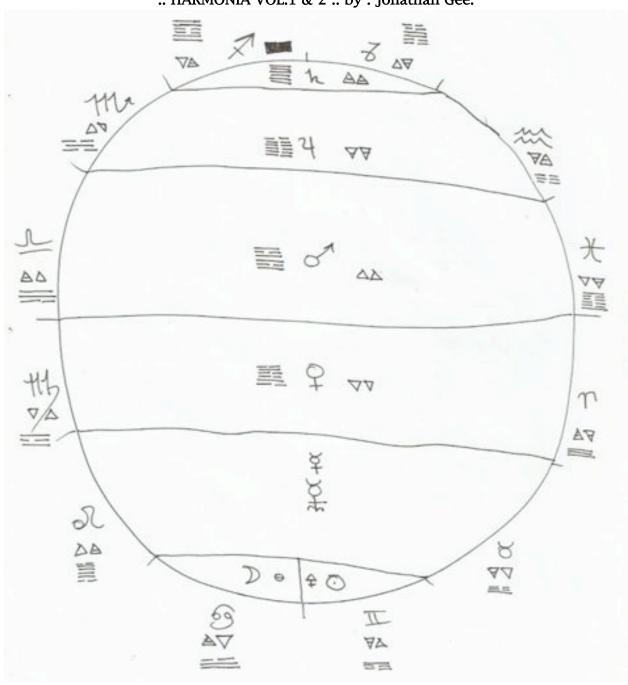
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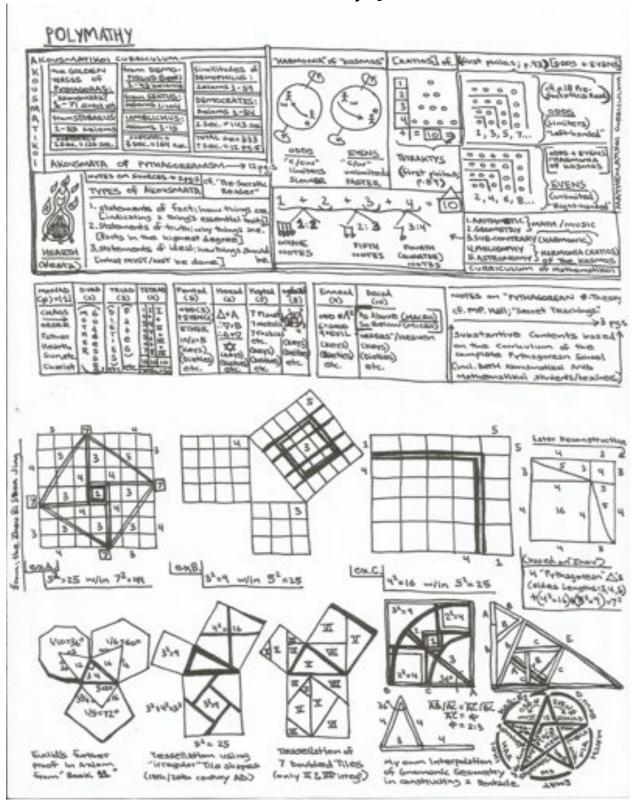
2A AKONSMATIKOI CURRICHUM -> (TOTAL)->17 pg.s

- + Assummate of Pythagoreaniam -> 12 pg.s
- + NOTES on "Prohyeren #-Theory"

 (cf. MP Hall; "Secret Teachings") -> 3 pg.5

28 MATHEMATIKAL CUTTENSON

- + from: the zoon & Sam Jing -+
- + Euclid's former proof (swinn & Bill)
- 4 Tessellations Circegular Hlings
- 4 my own interpolation of a 5°=25 Greenen
- + fractals & lengths in a Penhagramis tyou.
- 4 complete Pythagorean Curriculum



THE GOLDEN VERSES OF PYTHAGORAS

- 1. First worship the Immortal Gods, as they are established and ordained by the Law.
- 2. Reverence the Oath, and next the Heroes, full of goodness and light.
- 3. Honour likewise the Terrestrial Dæmons by rendering them the worship lawfully due to them.
- 4. Honour likewise thy parents, and those most nearly related to thee.
- 5. Of all the rest of mankind, make him thy friend who distinguishes himself by his virtue.
- 6. Always give ear to his mild exhortations, and take example from his virtuous and useful actions.
- 7. Avoid as much as possible hating thy friend for a slight fault.
- 8. [And understand that] power is a near neighbour to necessity.
- 9. Know that all these things are as I have told thee; and accustom thyself to overcome and vanquish these passions:--
- 10. First gluttony, sloth, sensuality, and anger.
- 11. Do nothing evil, neither in the presence of others, nor privately;
- 12. But above all things respect thyself.
- 13. In the next place, observe justice in thy actions and in thy words.
- 14. And accustom not thyself to behave thyself in any thing without rule, and without reason.
- 15. But always make this reflection, that it is ordained by destiny that all men shall die.
- 16. And that the goods of fortune are uncertain; and that as they may be acquired, so may they likewise be lost.
- 17. Concerning all the calamities that men suffer by divine fortune,
- 18. Support with patience thy lot, be it what it may, and never repine at it.
- 19. But endeavour what thou canst to remedy it.
- 20. And consider that fate does not send the greatest portion of these misfortunes to good men.
- 21. There are among men many sorts of reasonings, good and bad;
- 22. Admire them not too easily, nor reject them.
- 23. But if falsehoods be advanced, hear them with mildness, and arm thyself with patience.
- 24. Observe well, on every occasion, what I am going to tell thee:--
- 25. Let no man either by his words, or by his deeds, ever seduce thee.
- 26. Nor entice thee to say or to do what is not profitable for thyself.
- 27. Consult and deliberate before thou act, that thou mayest not commit foolish actions.
- 28. For it is the part of a miserable man to speak and to act without reflection.
- 29. But do that which will not afflict thee afterwards, nor oblige thee to repentance.
- 30. Never do anything which thou dost not understand.
- 31. But learn all thou ought'st to know, and by that means thou wilt lead a very pleasant life.
- 32. in no wise neglect the health of thy body;
- 33. But give it drink and meat in due measure, and also the exercise of which it has need.
- 34. Now by measure I mean what will not incommode thee.
- 35. Accustom thyself to a way of living that is neat and decent without luxury.
- 36. Avoid all things that will occasion envy.
- 37. And be not prodigal out of season, like one who knows not what is decent and honourable.
- 38. Neither be covetous nor niggardly; a due measure is excellent in these things.
- 39. Do only the things that cannot hurt thee, and deliberate before thou dost them.
- 40. Never suffer sleep to close thy eyelids, after thy going to bed,
- 41. Till thou hast examined by thy reason all thy actions of the day.
- 42. Wherein have I done amiss? What have I done? What have I omitted that I ought to have done?
- 43. If in this examination thou find that thou hast done amiss, reprimand thyself severely for it;
- 44. And if thou hast done any good, rejoice.
- 45. Practise thoroughly all these things; meditate on them well; thou oughtest to love them with all thy heart.
- 46. 'Tis they that will put thee in the way of divine virtue.

- 47. I swear it by him who has transmitted into our souls the Sacred Quaternion, the source of nature, whose cause is eternal.
- 48. But never begin to set thy hand to any work, till thou hast first prayed the gods to accomplish what thou art going to begin.
- 49. When thou hast made this habit familiar to thee,
- 50. Thou wilt know the constitution of the Immortal Gods and of men.
- 51. Even how far the different beings extend, and what contains and binds them together.
- 52. Thou shalt likewise know that according to Law, the nature of this universe is in all things alike.
- 53. So that thou shalt not hope what thou ought'st not to hope; and nothing in this world shall be hid from thee.
- 54. Thou wilt likewise know, that men draw upon themselves their own misfortunes voluntarily, and of their own free choice.
- 55. Unhappy that they are! They neither see nor understand that their good is near them.
- 56. Few know how to deliver themselves out of their misfortunes.
- 57. Such is the fate that blinds mankind, and takes away his senses.
- 58. Like huge cylinders they roll to and fro, and always oppressed with ills innumerable.
- 59. For fatal strife, innate, pursues them everywhere, tossing them up and down; nor do they perceive it.
- 60. Instead of provoking and stirring it up, they ought, by yielding, to avoid it.
- 61. Oh! Jupiter, our Father! if Thou would'st deliver men from all the evils that oppress them,
- 62. Show them of what dæmon they make use.
- 63. But take courage; the race of man is divine.
- 64. Sacred nature reveals to them the most hidden mysteries.
- 65. If she impart to thee her secrets, thou wilt easily perform all the things which I have ordained thee.
- 66. And by the healing of thy soul, thou wilt deliver it from all evils, from all afflictions.
- 67. But abstain thou from the meats, which we have forbidden in the purifications and in the deliverance of the soul:
- 68. Make a just distinction of them, and examine all things well.
- 69. Leaving thyself always to be guided and directed by the understanding that comes from above, and that ought to hold the reins.
- 70. And when, after having divested thyself of thy mortal body, thou arrivest at the most pure Æther,
- 71. Thou shalt be a God, immortal, incorruptible, and Death shall have no more dominion over thee.

PYTHAGOREAN ETHICAL SENTENCES FROM STOBÆUS

- 1. Do not even think of doing what ought not to be done.
- 2. Choose rather to be strong in soul than in body.
- 3. Be persuaded that things of a laborious nature contribute more than pleasures to virtue.
- 4. Every passion of the soul is most hostile to its salvation.
- 5. It is difficult to walk at one and the same time many paths of life.
- 6. Pythagoras said, it is requisite to choose the most excellent life; for custom will make it pleasant. Wealth is an infirm anchor, glory is still more infirm; and in a similar manner, the body, dominion, and honour. For all these are imbecile and powerless. What then are powerful anchors. Prudence, magnanimity, fortitude. These no tempest can shake. This is the Law of God, that virtue is the only thing that is strong; and that every thing else is a trifle.
- 7. All the parts of human life, in the same manner as those of a statue, ought to be beautiful.
- 8. Frankincense ought to be given to the Gods, but praise to good men.
- 9. It is requisite to defend those who are unjustly accused of having acted injuriously, but to praise those who excel in a certain good.

- 10. Neither will the horse be adjudged to be generous, that is sumptuously adorned, but the horse whose nature is illustrious; nor is the man worthy who possesses great wealth, but he whose soul is generous.
- 11. When the wise man opens his mouth, the beauties of his soul present themselves to the view, like the statues in a temple
- 12. Remind yourself that all men assert that wisdom is the greatest good, but that there are few who strenuously endeavour to obtain this greatest good.
- 13. Be sober, and remember to be disposed to believe; for these are the nerves of wisdom.
- 14. It is better to live lying on the grass, confiding in Divinity and yourself, than to lie on a golden bed with perturbation.
- 15. You will not be in want of anything, which it is in the power of fortune to give and take away.
- 16. Despise all those things which when liberated from the body you will not want; invoke the Gods to become your helpers.
- 17. Neither is it possible to conceal fire in a garment, nor a base deviation from rectitude in time.
- 18. Wind indeed increases fire, but custom love.
- 19. Those alone are dear to Divinity who are hostile to injustice.
- 20. Those things which the body necessarily requires, are easily to be procured by all men, without labour and molestation; but those things to the attainment of which labour and molestation are requisite, are objects of desire, not to the body, but to deprayed opinion.
- 21. Of desire also, he (Pythagoras) said as follows:--This passion is various, laborious, and very multiform. Of desires, however, some are acquired and adventitious, but others are connascent. But he defined desire itself to be a certain tendency and impulse of the soul, and an appetite of a plentitude or presence of sense, or an emptiness and absence of it, and of non-perception. He also said, that there are three most known species of erroneous and depraved desire, viz., the indecorous, the incommensurate, and the unseasonable. For desire is either immediately Indecorous, troublesome, and illiberal, or it is not absolutely so, but is more vehement and lasting than is fit. Or in the third place, it is impelled when it is not proper, and to objects to which it ought not to tend.
- 22. Endeavour not to conceal your errors
- by words, but to remedy them by reproof.
- 23. It is not so difficult to err, as not to reprove him who errs.
- 24. As a bodily disease cannot be healed, if it be concealed, or praised, thus also, neither can a remedy be applied to a diseased soul, which is badly guarded and protected.
- 25. The grace of freedom of speech, like beauty in season, is productive of greater delight.
- 26. It is not proper either to have a blunt sword or to use freedom of speech ineffectually.
- 27. Neither is the sun to be taken from the world nor freedom of speech from erudition.
- 28. As it is possible for one who is clothed with a sordid robe, to have a good habit of body; thus also he whose life is poor may possess freedom of speech.
- 29. Be rather delighted with those that reprove, than with those that flatter you; but avoid flatterers, as worse than enemies.
- 30. The life of the avaricious resembles a funeral banquet. For though it has all things requisite to a feast, yet no one present rejoices.
- 31. Acquire continence as the greatest strength and wealth.
- 32. "Not frequently man from man," is one of the exhortations of Pythagoras; by which he obscurely signifies, that it is not proper to be frequently engaged in venereal connexions.
- 33. It is impossible that he can be free who is a slave to his passions.
- 34. Pythagoras said, that intoxication is the meditation of insanity.
- 35. Pythagoras being asked, how a lover of wine might be cured of intoxication, answered, if he frequently surveys what his actions were when he was intoxicated.
- 36. Pythagoras said, that it was requisite either to be silent, or to say something better than silence.
- 37. Let it be more eligible to you to throw a stone in vain, than to utter an idle word.

- 38. Do not say a few things in many words, but much in a few words.
- 39. Genius is to men either a good or an evil dæmon.
- 40. Pythagoras being asked how a man ought to conduct himself towards his country, when it had acted iniquitously with respect to him, replied, as to a mother.
- 41. Travelling teaches a man frugality, and the way in which he may be sufficient to himself. For bread made of milk and flour, and a bed of grass, are the sweetest remedies of hunger and labour.
- 42. To the wise man every land is eligible as a place of residence; for the whole world is the country of the worthy soul.
- 43. Pythagoras said that luxury entered into cities in the first place, afterwards satiety, then lascivious insolence, and after all these, destruction.
- 44. Pythagoras said, that of cities that was the best which contained most worthy men.
- 45. Do those things which you judge to be beautiful, though in doing them you should be without renown. For the rabble is a bad judge of a good thing. Despise, therefore, the reprehension of those whose praise you despise.
- 46. Those that do not punish bad men, wish that good men may be injured.
- 47. It is not possible for a horse to be
- governed without bridle, nor riches without prudence.
- 48. It is the same thing to think greatly of yourself in prosperity, as to contend in the race in a slippery road.
- 49. There is not any gate of wealth so secure, which the opportunity of fortune may not open.
- 50. Expel by reasoning the unrestrained grief of a torpid soul.
- 51. It is the province of the wise man to bear poverty with equanimity.
- 52. Spare your life, lest you consume it with sorrow and care.
- 53. Nor will I be silent as to this particular, that it appeared both to Plato and Pythagoras, that old age was not to be considered with reference to an egress from the present life, but to the beginning of a blessed life.
- 54. The ancient theologists and priests testify that the soul is conjoined to the body through a certain punishment, and, that it is buried in this body as in a sepulchre.
- 55. Whatever we see when awake is death; and when asleep, a dream.

THE PYTHAGOREAN SENTENCES OF DEMOPHILUS

- 1. Request not of Divinity such things as, when obtained, you cannot preserve; for no gift of Divinity can ever be taken away; and on this account he does not confer that which you are unable to retain.
- 2. Be vigilant in your intellectual part; for sleep about this has an affinity with real death.
- 3. Divinity sends evil to men, not as being influenced by anger, but for the sake of purification; for anger is foreign from Divinity, since it arises from circumstances taking place contrary to the will; but nothing contrary to the will can happen to a god.
- 4. When you deliberate whether or not you shall injure another, you will previously suffer the evil yourself which you intend to commit. But neither must you expect any good from the evil; for the manners of everyone are correspondent to his life and actions. Every soul too is a repository, that which is good, of things good, that which is evil, of things depraved.
- 5. After long consultation, engage either in speaking or acting; for you have not the ability to recall either your words or deeds.
- 6. Divinity does not principally esteem the tongue, but the deeds of the wise; for a wise man, even when he is silent, honours Divinity.
- 7. A loquacious and ignorant man both in prayer and sacrifice contaminates a divine nature. The wise man therefore is alone a priest, is alone a friend of Divinity and only knows how to pray.
- 8. The wise man being sent hither naked, should naked invoke him by whom he was sent; for he alone is heard by Divinity, who is not burdened with foreign concerns.
- 9. It is impossible to receive from Divinity any gift greater than virtue.
- 10. Gifts and victims confer no honour on Divinity, nor is he adorned with offerings suspended in

temples; but a soul divinely inspired solidly conjoins us with Divinity; for it is necessary that like should approach to like.

- 11. It is more painful to be subservient to passions than to tyrants.
- 12. It is better to converse more with yourself than others.
- 13. If you are always careful to remember that in whatever place either your soul or body accomplishes any deed, Divinity is present as an inspector of your conduct; in all your words and actions you will venerate the presence of an inspector from whom nothing can be concealed, and will, at the same time, possess Divinity as an intimate associate.
- 14. Believe that you are furious and insane in proportion as you are ignorant of yourself.
- 15. It is necessary to search for those wives and children which will remain after a liberation from the present life.
- 16. The self-sufficient and needy philosopher lives a life truly similar to Divinity, and considers the non-possession of external and unnecessary goods as the greatest wealth. For the acquisition of riches sometimes inflames desire; but not to act in any respect unjustly is sufficient to the enjoyment of a blessed life.
- 17. True goods are never produced by indolent habits.
- 18. Esteem that to be eminently good, which, communicated to another, will be increased to yourself.
- 19. Esteem those to be eminently your friends, who assist your soul rather than your body.
- 20. Consider both the praise and reproach of every foolish person as ridiculous, and the whole life of an ignorant man as a disgrace.
- 21. Endeavour that your familiars may reverence rather than fear you; for love attends upon reverence, but hatred upon fear.
- 22. The sacrifices of fools are the aliment of the fire; but the offerings which they suspend in temples are the supplies of the sacrilegious.
- 23. Understand that no dissimulation can be long concealed.
- 24. The unjust man suffers greater evil while his soul is tormented with a consciousness of guilt, than when his body is scourged with whips.
- 25. It is by no means safe to discourse concerning Divinity with men of false opinions; for the danger is equally great in speaking to such as these, things either fallacious or true.
- 26. By everywhere using reason as your guide, you will avoid the commission of crimes.
- 27. By being troublesome to others, you will not easily escape molestation yourself.
- 28. Consider that as great erudition, through which you are able to bear the want of erudition, in the ignorant.
- 29. He who is deprayed does not listen to the divine law, and on this account lives without law.
- 30. A just man who is a stranger, is not only superior to a citizen, but is even more excellent than a relation.
- 31. As many passions of the soul, so many fierce and savage despots.
- 32. No one is free who has not obtained the empire of himself.
- 33. Labour, together with continence, precedes the acquisition of every good.
- 34. Be persuaded that those things are not your riches which you do not possess in the penetralia of the reasoning powers.
- 35. Do that which you judge to be beautiful and honest, though you should acquire no glory from the performance; for the vulgar is a depraved judge of beautiful deeds.
- 36. Make trial of a man rather from his deeds than his discourses; for many live badly and speak well.
- 37. Perform great things, at the same time promising nothing great.
- 38. Since the roots of our nature are established in Divinity, from which also we are produced, we should tenaciously adhere to our root; for streams also of water, and other offspring of the earth, when their roots are cut off, become rotten and dry.
- 39. The strength of the soul is temperance; for this is the light of a soul destitute of passions; but it is much better to die than to darken the soul through the intemperance of the body.

- 40. You cannot easily denominate that man happy who depends either on his friends or children, or on any fleeting and fallen nature; for all these are unstable and uncertain; but to depend on oneself and on Divinity is alone stable and firm.
- 41. He is a wise man, and beloved of Divinity, who studies how to labour for the good of his soul, as much as others labour for the sake of the body.
- 42. Yield all things to their kindred and ruling nature except liberty.
- 43. Learn how to produce eternal children, not such as may supply the wants of the body in old age, but such as may nourish the soul with perpetual food.
- 44, It is impossible that the same person can be a lover of pleasure, a lover of

body, a lover of riches, and a lover of Divinity. For a lover of pleasure is also a lover of body; but a lover of body is entirely a lover of riches; a lover of riches is necessarily unjust; and the unjust is necessarily profane towards Divinity, and lawless with respect to men. Hence, though he should sacrifice hecatombs, he is only by this means the more impious, unholy, atheistical, and sacrilegious, with respect to his intentions: and on this account it is necessary to avoid every lover of pleasure as an atheist and polluted person.

45. The Divinity has not a place in the earth more allied to his nature than a pure and holy soul.

SELECT SENTENCES OF SEXTUS THE PYTHAGOREAN

- 1. To neglect things of the smallest consequence, is not the least thing in human life.
- 2. The wise man, and the despiser of wealth, resemble God.
- 3. Do not investigate the name of God, because you will not find it. For every thing which is called by a name, receives its appellation from that which is more worthy than itself, 1 so that it is one person that calls, and another that hears. Who is it, therefore, that has given a name to God? God, however, is not a name to God, but an indication of what we conceive of Him.
- 4. God is a light incapable of receiving its contrary, darkness.
- 5. You have in yourself some thing similar to God, and therefore use yourself as the temple of God, on account of that which in you resembles God.
- 6. Honour God above all things, that He may rule over you.
- 7. Whatever you honour above all things, that which you so honour will have dominion over you. But if you give yourself to the domination of God, you will thus have dominion over all things.
- 8. The greatest honour which can be paid to God, is to know and imitate Him.
- 9. There is not any thing, indeed, which wholly resembles God; nevertheless the imitation of Him as much as possible by an inferior nature is grateful to Him.
- 10. God, indeed, is not in want of anything, but the wise man is in want of God alone. He, therefore, who is in want but of few things, and those necessary, emulates him who is in want of nothing.
- 11. Endeavour to be great in the estimation of Divinity, but among men avoid envy.
- 12. The wise man whose estimation with men was but small while he was living, will be renowned when he is dead.
- 13. Consider all the time to be lost to you in which you do not think of divinity.
- 14. A good intellect is the choir of divinity.
- 15. A bad intellect is the choir of evil dæmons.
- 16. Honour that which is just, on this very account that it is just.
- 17. You will not be concealed from divinity when you act unjustly, nor even when you think of doing so.
- 18. The foundation of piety is continence; but the summit of piety is the love of God.
- 19. Wish that what is expedient and not what is pleasing may happen to you.
- 20. Such as you wish your neighbour to be to you, such also be you to your neighbour.
- 21. That which God gives you, no one can take away.
- 22. Neither do nor even think of that which you are not willing God should know.
- 23. Before you do anything think of God, that his light may precede your energies.
- 24. The soul is illuminated by the recollection of deity.
- 25. The use of all animals as food is Indifferent, but it is more rational to abstain from them.

- 26. God is not the author of any evil.
- 27. You should not possess more than the use, of the body requires.
- 28. Possess those things which no one can take from you.
- 29. Bear that which is necessary, as it is necessary.
- 30. Ask those things of God which it is worthy of God to bestow.
- 31. The reason which is in you, is the light of your life.
- 32. Ask those things of God which you cannot receive from man.
- 33. Wish that those things which labour ought to precede, may be possessed by you after labour.
- 34. Be not anxious to please the multitude.
- 35. It is not proper to despise those things of which we shall be in want after the dissolution of the body.
- 36. You should not ask of divinity that which, when you have obtained, you will not perpetually possess.
- 37. Accustom your soul after it has conceived all that is great of divinity, to conceive something great of itself.
- 38. Esteem nothing so precious, which a bad man may take from you.
- 39. He is dear to divinity, who considers those things alone to be precious, which are esteemed to be so by divinity.
- 40. Every thing which is more than necessary to man, is hostile to him.
- 41. He who loves that which is not expedient, will not love that which is expedient.
- 42. The intellect of the wise man is always with divinity.
- 43. God dwells in the intellect of the wise man.
- 44. Every desire is insatiable, and therefore is always in want.
- 45. The wise man is always similar to himself.
- 46. The knowledge and imitation of divinity are alone sufficient to beatitude.
- 47. Use lying like poison.
- 48. Nothing is so peculiar to wisdom, as truth.
- 49. When you preside over men, remember that divinity also presides over you.
- 50. Be persuaded that the end of life is to live conformably to divinity.
- 51. Depraved affections are the beginning of sorrows.
- 52. An evil disposition is the disease of the soul; but injustice and impiety are the death of it.
- 53. Use all men in such a way, as if you were the common curator of all things after God.
- 54. He who uses mankind badly, uses himself badly.
- 55. Wish that you may be able to benefit your enemies.
- 56. Endure all things, in order that you may live conformably to God.
- 57. By honouring a wise man, you will honour yourself.
- 58. In all your actions place God before your eyes.
- 59. You are permitted to refuse matrimony, in order that you may live incessantly adhering to God. If, however, as one knowing the battle, you are willing to fight, take a wife, and beget children.
- 60. To live, indeed, is not in our power, but to live rightly is.
- 61. Be unwilling to admit accusations against the man who is studious of wisdom.
- 62. If you wish to live with hilarity, be unwilling to do many things. For in a multitude of actions you will be minor.
- 63. Every cup should be sweet to you which extinguishes thirst.
- 64. Fly from intoxication as you would from insanity.
- 65. No good originates from the body.
- 66. Think that you suffer a great punishment when you obtain the object of corporeal desire; for the attainment of such objects never satisfies desire.
- 67. Invoke God as a witness to whatever you do.
- 68. The bad man does not think there is a providence.
- 69. Assert that which possesses wisdom in you to be the true man.

- 70. The wise man participates of God.
- 71. Where that which is wise in you resides, there also is your good.
- 72. That which is not noxious to the soul, is not noxious to man.
- 73. He who unjustly expels a wise man from the body, confers a benefit on him by his iniquity. For he thus becomes liberated, as it were, from bones.
- 74. The fear of death renders a man sad through the ignorance of his soul.
- 75. You will not possess intellect, till you understand that you have it.
- 76. Think that your body is the garment of your soul; and therefore preserve it pure.
- 77. Impure dæmons vindicate to themselves the impure soul.
- 78. Speak not of God to every man.
- 79. It is dangerous and the danger is not small, to speak of God even things which are true.
- 80. A true assertion respecting God is an assertion of God.
- 81. You should not dare to speak of God to the multitude.
- 82. He does not know God who does not worship Him.
- 83. The man who is worthy of God is also a God among men.
- 84. It is better to have nothing, than to possess much and impart it to no one.
- 85. He who thinks that there is a God, and that nothing is taken care of by him, differs in no respect from him who does not believe that there is a God.
- 86. He honours God in the best manner who renders his intellect as much as possible similar to God.
- 87. If you injure no one, you will fear no one.
- 88. No one is wise who looks downward to the earth.
- 89. To lie is to deceive in life, and to be deceived.
- 90. Recognise what God is, and what that is in you which recognises God.
- 91. It is not death, but a bad life, that destroys the soul.
- 92. If you know him by whom you were made, you will know yourself.
- 93. It is not possible for a man to live conformable to divinity, unless he acts modestly, well, and justly.
- 94. Divine Wisdom is true Science.
- 95. You should not dare to speak of God to an impure soul.
- 96. The wise man follows God, and God follows the soul of the wise man.
- 97. A king rejoices in those whom he governs, and therefore God rejoices in the wise man. He who governs likewise, is inseparable from those whom he governs; and therefore God is inseparable from the soul of the wise man, which he defends and governs.
- 98. The wise man is governed by God and on this account is blessed.
- 99. A scientific knowledge of God causes a man to use few words.
- 100. To use many words when speaking of God, produces an ignorance of God.
- 101. The man who possesses a knowledge of God, will not be very ambitious.
- 102. The erudite, chaste, and wise soul, is the prophet of the truth of God.
- 103. Accustom yourself always to look to Divinity.
- 104. A wise intellect is the mirror of God.

PYTHAGOREAN SENTENCES FROM THE PROTREPTICS OF IAMBLICHUS

- 1. As we live through soul, it must be said that by the virtue of this we live well; just as because we see through the eyes, we see well through the virtue of these.
- 2. It must not be thought that gold can be injured by rust, or virtue by baseness.
- 3. We should betake ourselves to virtue as to an inviolable temple, in order that we may not be exposed to any ignoble insolence of soul with respect to our communion with, and continuance in life.
- 4. We should confide in virtue as in a chaste wife; but trust to fortune as to an inconstant mistress.
- 5. It is better that virtue should be received accompanied with poverty, than wealth with violence;

and frugality with health, than voracity with disease.

- 6. An abundance of nutriment is noxious to the body; but the body is preserved when the soul is disposed in a becoming manner.
- 7. It is equally dangerous to give a sword to a madman, and power to a depraved man.
- 8. As it is better for a part of the body which contains purulent matter to be burnt, than to continue in the state in which it is, thus also it is better for a deprayed man to die than to live.
- 9. The theorems of philosophy are to be enjoyed as much as possible, as if they were ambrosia and nectar. For the pleasure arising from them is genuine, incorruptible, and divine. They are also capable of producing magnanimity; and though they cannot make us eternal beings, yet they enable us to obtain a scientific knowledge of eternal natures.
- 10. If vigour of sensation is considered by us to be an eligible thing, we should much more strenuously endeavour to obtain prudence; for it is as it were the sensitive vigour of the practical intellect which we contain. And as through the former we are not deceived in sensible perceptions, so through the latter we avoid false reasoning in practical affairs.
- 11. We shall venerate Divinity in a proper manner if we render the intellect that is in us pure from all vice, as from a certain stain.
- 12. A temple, indeed, should be adorned with gifts, but the soul with disciplines.
- 13. As the lesser mysteries are to be delivered before the greater, thus also discipline must precede philosophy.
- 14. The fruits of the earth, indeed, are annually imparted, but the fruits of philosophy at every part of the year.
- 15. As land is especially to be attended to by him who wishes toe obtain from it the most excellent fruit, thus also the greatest attention should be: paid to the soul, in order that it may produce fruit worthy of its nature.

THE SIMILITUDES OF DEMOPHILUS

- 1. Flattery is like painted armour, because it affords delight, but is of no use.
- 2. Learning is similar to a golden crown; for it is both honourable and advantageous.
- 3. Flighty men, like empty vessels, are easily laid hold of by the ears. 1
- 4. Life, like a musical instrument, being harmonized by remission and intention, becomes more agreeable.
- 5. Reason, like a good potter, introduces a beautiful form to the soul.
- 6. The intellect of wise men, like gold, possesses the greatest weight.
- 7. Boasting, like gilt armour, is not the same within as without.
- 8. Reason has the same power as an ointment, for it benefits us when we are disordered, but delights us when well.
- 9. Of a bad man, as of a bad dog, the silence is more to be dreaded than the voice.
- 10. It is neither becoming to prefer a mistress to a wife; nor flattery to a friend.
- 11. Garrulous men, like magpies, by their continued loquacity destroy the pleasures of conversation.
- 12. The Furies pursue the sins of bad men who are impious, and those also of the stupid and daring, when they grow old.
- 13. It is necessary that a well-educated man should depart from life elegantly, as from a banquet.
- 14. A port is a place of rest to a ship, but friendship, to life.
- 15. The reproof of a father is a pleasant medicine; for it is more advantageous than severe chastisements.
- 16. It is necessary that a worthy man, like a good wrestler, should oppose his weight to fortune, when acting the part of an antagonist.
- 17. The possession of self-sufficiency, 1 like a short and pleasant road, has much grace and but little labour.
- 18. Restive horses are led by the bridle, but irritable minds, by reasoning.
- 19. Tests, like salt, should be used sparingly.

- 20. Both a well-adapted shoe, and a well-harmonized life, are accompanied with but little pain.
- 21. Garments reaching to the feet impede the body 2; and immoderate riches, the soul.
- 22. To those who run in the stadium, the reward of victory is in the end of the race; but to those who delight to labour in wisdom, the reward is in old age.
- 23. It is necessary that he who hastens to behold virtue as his country, should pass by pleasures, as he would by the sirens.
- 24. As those who sail in fair weather are wont to have things prepared against a storm, so also those who are wise in prosperity, should prepare things necessary for their assistance against adversity.
- 25. Garments that are made clean and bright become soiled again by use; but the soul being once purified from ignorance, remains splendid forever.
- 26. Fugitive slaves, although they are not pursued, are affrighted; but the unwise suffer perturbation, although they have not yet acted badly.
- 27. The wealth of the avaricious, like the sun when it has descended under the earth, delights no living thing.
- 28. The fruits of the earth spring up once a year; but the fruits of friendship at all times.
- 29. It is the business of a musician to harmonize every instrument; but of a well-educated man to adapt himself harmoniously to every fortune.
- 30. Neither the blows of a sick man, nor the threats of a stupid one, are to be feared.
- 31. It is necessary to provide an inward garment for the protection of the breast, and intellect as a protection against pain.
- 32. The diet of the sick, and the soul of the unwise, are full of fastidiousness.
- 33. Untaught boys confound letters, but uneducated men, things.
- 34, The intellect derived from philosophy is similar to a charioteer; for it is present with our desires, and always conducts them to the beautiful.
- 35. Time, indeed, will render the herb absinthium sweeter than honey, but circumstances may sometimes make an enemy preferable to a friend.
- 36. A good pilot sometimes suffers shipwreck, and a worthy man is sometimes unfortunate.
- 37. Thunder especially frightens children; but threats, the unwise.
- 38. Figure adorns a statue; but actions adorn a man,
- 39. It is the same thing to drink a deadly medicine from a golden cup, and to receive counsel from an injudicious friend.
- 40. Swallows signify fair weather; but the discourses of philosophy, exemption from pain.
- 41. Orphan children have not so much need of guardians as stupid men.
- 42. Fortune is like a depraved rewarder of contests; for she frequently crowns him who accomplishes nothing.
- 43. There is need of a pilot and a wind for a prosperous navigation; but of reasoning and fortune, to effect a happy life.
- 44. A timid man bears armour against himself; and a fool employs riches for the same purpose.
- 45. It is the same thing to moor a boat by an infirm anchor, and to place hope in a depraved mind.
- 46. Clouds frequently obscure the sun; but the passions, the reasoning power.
- 47. Neither does a golden bed benefit a sick man; nor a splendid fortune, a stupid man.
- 48. Pure water dissolves inflammation; but mild discourse dissolves anger.
- 49. Austere wine is mot adapted for copious drinking, nor rustic manners for conversation.
- 50. The anger of an ape, and the threats of a flatterer, are to be alike regarded.
- 51. Of life, the first part is childhood, on which account all men are attentive to it, as to the first part of a drama.
- 52. It is necessary that we should be cautious in our writings, but splendid in our actions.
- 53. As in plants, so also in youth, the first blossoms indicate the fruit of virtue.
- 54. In banquets, he who is not intoxicated with wine is the more pleasant; but in prosperity, he who does not conduct himself illegally.
- 55. It is the same thing to nourish a serpent, and to benefit a depraved man; for gratitude is

produced from neither.

- 56. It is rare to suffer shipwreck in fair weather; and equally so not to suffer shipwreck from want of counsel.
- 57. Wind inflates empty bladders; but false opinions puff up stupid men.
- 58. It is necessary that he who exercises himself should avoid fatigue, and he who is prosperous, envy.
- 59. "Measure is most excellent," says one of the wise men; to which also we being in like manner persuaded, O most friendly and pious Asclepiades, here finish the curations of life.

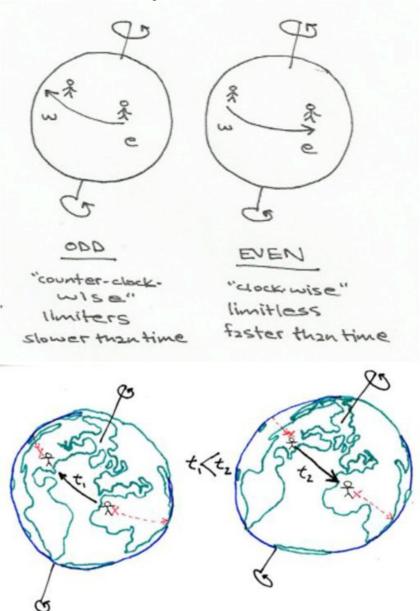
THE GOLDEN SENTENCES OF DEMOCRATES

- 1. If anyone will give his mind to these sentences, he will obtain many things worthy of a man, and be free from many things that are base.
- 2. The perfection of the soul will correct the depravity of the body; but the strength of the body without reasoning does not render the soul better.
- 3. He who loves the goods of the soul will love things more, divine; but he who loves the goods of its transient habitation will love things human.
- 4. It is beautiful to impede an unjust man; but, if this be not possible, it is beautiful not to act in conjunction with him.
- 5. It is necessary to be good, rather than to appear so.
- 6. The felicity of a man does not consist either in body or in riches, but in upright conduct and justice.
- 7. Sin should be abstained from, not through fear, but for the sake of the becoming.
- 8. It is a great thing to be wise where we ought in calamitous circumstances.
- 9. Repentance after base actions is the salvation of life.
- 10. It is necessary to be a speaker of the truth, and not to be loquacious.
- 11. He who does an injury is more unhappy than he who receives one.
- 12. It is the province of a magnanimous man to bear with mildness the errors of others.
- 13. It is comely not to oppose the law, nor a prince, nor one wiser than yourself.
- 14. A good man pays no attention to the reproofs of the depraved.
- 15. It is hard to be governed by these who are worse than ourselves.
- 16. He who is perfectly vanquished by riches, can never be just.
- 17. Reason is frequently more persuasive than gold itself.
- 18. He who admonishes a man that fancies he has intellect, labours in vain.
- 19. Many who have not learnt to argue rationally, still live according to reason.
- 20. Many who commit the basest actions often exercise the best discourse.
- 21. Fools frequently become wise under the pressure of misfortunes.
- 22. It is necessary to emulate the works and actions, and not the words of virtue.
- 23. Those who are naturally well disposed, know things beautiful, and are themselves emulous of them.
- 24. Vigour and strength of body are the nobility of cattle; but the rectitude of manners is the nobility of man.
- 25. Neither art nor wisdom can be acquired without preparatory learning.
- 26. It is better to reprove your own errors, than those of others.
- 27. Those whose manners are well ordered will also be orderly in their lives.
- 28. It is good not only to refrain from doing an injury, but even from the very wish.
- 29. It is proper to speak well of good works; for to do so of such as are base is the property of a fraudulent man and an impostor.
- 30, Many that have great learning have no intellect.
- 31. It is necessary to endeavour to obtain an abundance of intellect, and not pursue an abundance of erudition.
- 32. It is better that counsel should precede actions, than that repentance should follow them.
- 33. Put not confidence in all men, but in those that are worthy; for to do the former is the

province of a stupid man, but the latter of a wise man.

- 34. A worthy and an unworthy man are to be judged not from their actions only, but also from their will.
- 35. To desire immoderately is the province of a boy, and not of a man.
- 36. Unseasonable pleasures bring forth pains.
- 37. Vehement desires about any one thing render the soul blind with respect to other things.
- 38. The love is just which, unattended with injury, aspires after things becoming.
- 39. Admit nothing as pleasant which is not advantageous.
- 40. It is better to be governed by, than to govern, the stupid.
- 41. Not argument but calamity is the preceptor to children.
- 42. Glory and wealth without wisdom are not secure possessions.
- 43. It is not indeed useless to procure wealth, but to procure it from injustice is the most pernicious of all things.
- 44. It is a dreadful thing to imitate the bad, and to be unwilling to imitate the good.
- 45. It is a shameful thing for a man to be employed about the affairs of others, but to be ignorant of his own.
- 46. To be always intending to act renders action imperfect.
- 47. Fraudulent men, and such as are only seemingly good, do all things in words and nothing in deeds.
- 48, He is a blessed man who has both property and intellect, for he will use them well in such things as are proper.
- 49. The ignorance of what is excellent is the cause of error.
- 50. Prior to the performance of base things, a man should reverence himself.
- 51. A man given to contradiction, and very attentive to trifles, is naturally unadapted to learn what is proper.
- 52. Continually to speak without being willing to hear, is arrogance.
- 53. It is necessary to guard against a depraved man, lest he should take advantage of opportunity.
- 54. An envious man is the cause of molestation to himself, as to an enemy.
- 55. Not only he is an enemy who acts unjustly, but even he who deliberates about so acting.
- 56. The enmity of relations is far more bitter than that of strangers.
- 57. Conduct yourself to all men without suspicion; and be accommodating and cautious in your behaviour.
- 58. It is proper to receive favours, at the same time determining that the retribution shall surpass the gift.
- 59. When about to bestow a favour, previously consider him who is to receive it, lest being a fraudulent character he should return evil for good.
- 60. Small favours seasonably bestowed, become things of the greatest consequence to those who receive them.
- 61. Honours with wise men are capable of effecting the greatest things, if at the same time they understand that they are honoured.
- 62. The beneficent man is one who does not look to retribution, but who deliberately intends to do well.
- 63. Many that appear to be friends are not, and others, who do not appear to be friends, are so.
- 64. The friendship of one wise man is better than that of every fool,
- 65. He is unworthy to live who has not one worthy friend.
- 66. Many turn from their friends, if, from affluence, they fall into adversity.
- 67. The equal is beautiful in everything; but excess and defect to me do not appear to be so.
- 68. He who loves no one does not appear to me to be loved by any one.
- 69. He is an agreeable old man who is facetious, and abounds in interesting anecdote.
- 70. The beauty of the body is merely animal unless supported by intellect.
- 71. To find a friend in prosperity, is very easy; but in adversity, it is the most difficult of all things.

- 72. Not all relations are friends, but those who accord with what is mutually advantageous.
- 73. Since we are men, it is becoming, not to deride, but bewail, the calamities of men.
- 74. Good scarcely presents itself, even to those who investigate it; but evil is obvious without investigation.
- 75. Men who delight to blame others are not naturally adapted to friendship.
- 76. A woman should not be given to loquacity; for it is a dreadful thing.
- 77. To be governed by a woman is the extremity of insolence and unmanliness.
- 78. It is the property of a divine intellect to be always intently thinking about the beautiful.
- 79. He who believes that Divinity beholds all things, will not sin either secretly or openly.
- 80. Those who praise the unwise do them a great injury.
- 81. It is better to be praised by another than by oneself.
- 82. If you cannot reconcile to yourself the praises you receive, think that you are flattered.
- 83. The world is a scene; life is a transition. You came, you saw, you departed.
- 84. The world is a mutation: life a vain opinion.

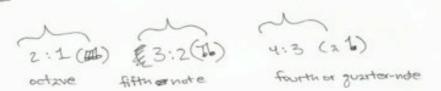


· - Thousmata: "things passed down or heard"	, Etast	philosophers,	12.88)
-sumbola : "tokens" or "pass words"	3 -		

(First philosopher's, p. 89) (First philosophers, p. 93)

- · f. T22 (DK 58 CU; KRS 277) p.101 (1st philo)
 re. "akousmatics"
- cf. T23 (KRS 279) re "tetraktys" 8 "music": pp.101,102 (1st philo)

 1 + 2 + 3 + 4 = 10



- 1) arithmetic; 2) geometric; 3) "subcontrary" (orig.) later re-named the "harmonic" by Archytas 8 Hippasus. cf. 731 (DK 18A15) pp. 106, 107 (1st Ph.)
- . cf. T32 (DK 58821; KRS 436) re. solution of internal angles of all triangles being = 1800 (prio7, 1st Philo)

1) xkovsmatikoi - venerated religious & noval teachings; rejected the from "zkovsmata" (things heard) 2) mathematikoi - pinile, math, novsic & astronomy; zeopted the akousmulikoj 4 from "mothemate" ("Study" or "learning") 6 "Kosmos" = universe as a harmonious arrangement 6 cf. sayings 15, 16, 17, 18, p. 22-23 (Pre-Sec Reader) 6 limiters (add #s) & unlimiteds (even #s), 6 harmonia = the knowable ratios of the kosmos 7 le. Kicked dog anactdate. 7 cf. sayings 2 & 3 re. "Polymathy" (much learning) & much insight 7 cf. saying 7, p. 19 (loid) re. past-lives 8 types of Akousmata: 1) some indicate what something in the 9 greatest degree, & 3) others what much oriminate not be done 6 saying 23, p. 24 (Pre-Sec Reader) 6 conter of the	. 2 groups of disciples:
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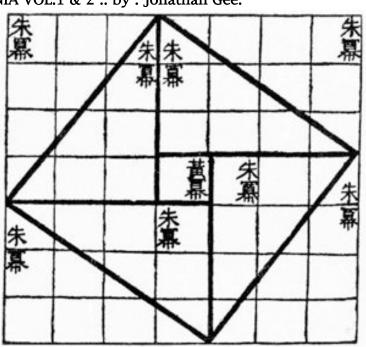
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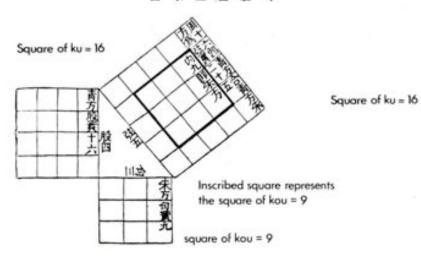


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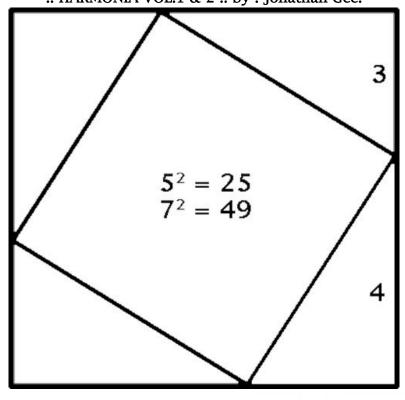


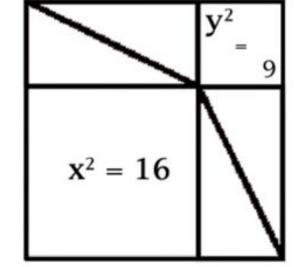


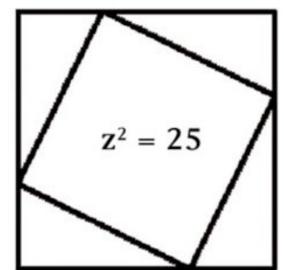
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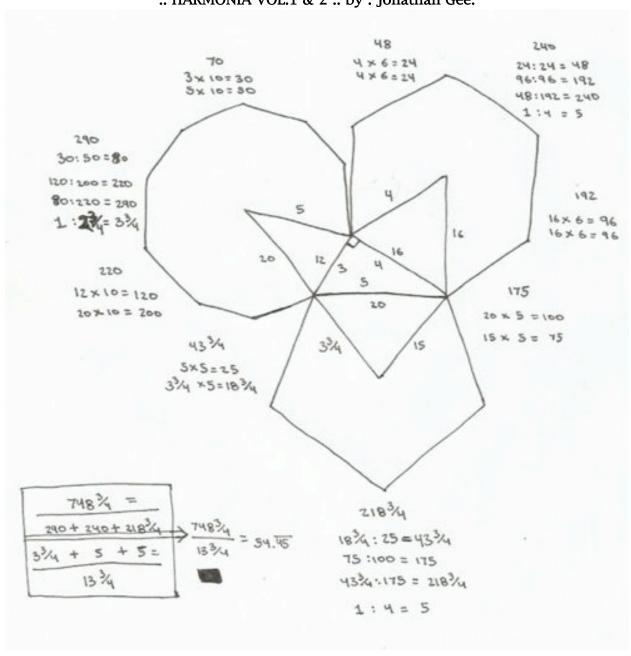


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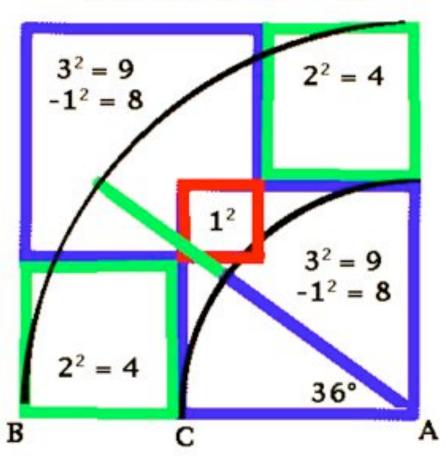
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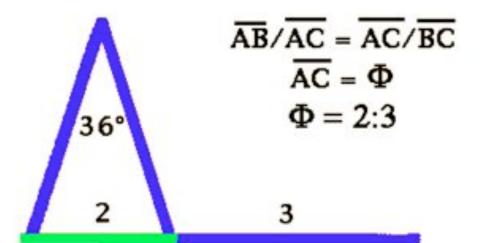
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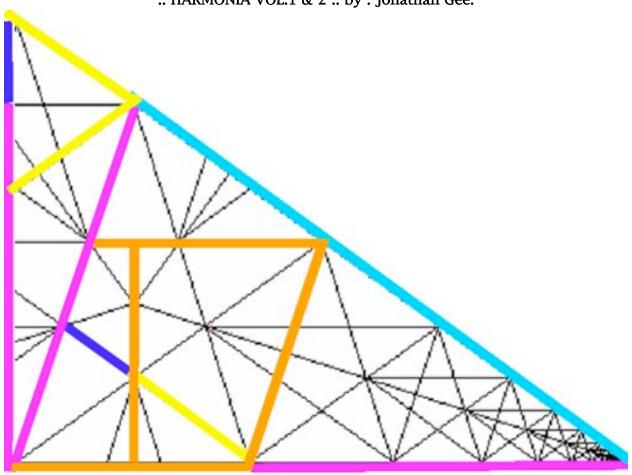
:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee. 42 + 16 5 $\begin{vmatrix} 3^2 = 9 \\ + 4^2 = 16 \\ = 5^2 = 25 \end{vmatrix}$ 1

$$16 + 8 + 1 = 5^2 = 25$$





:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.

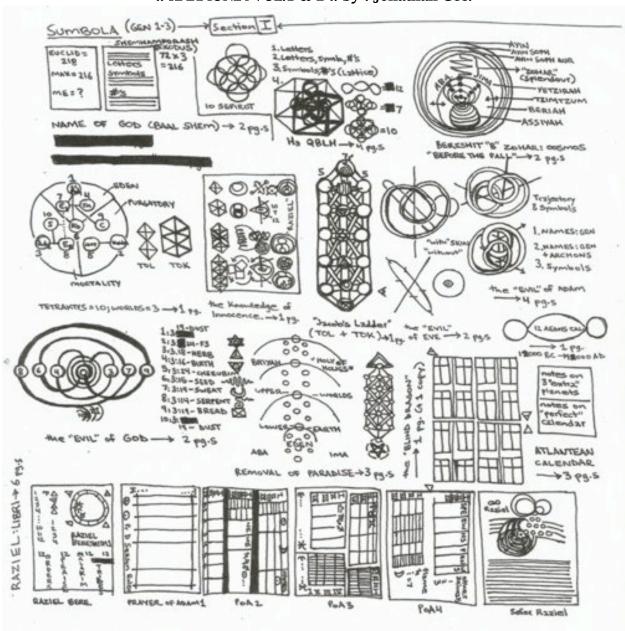


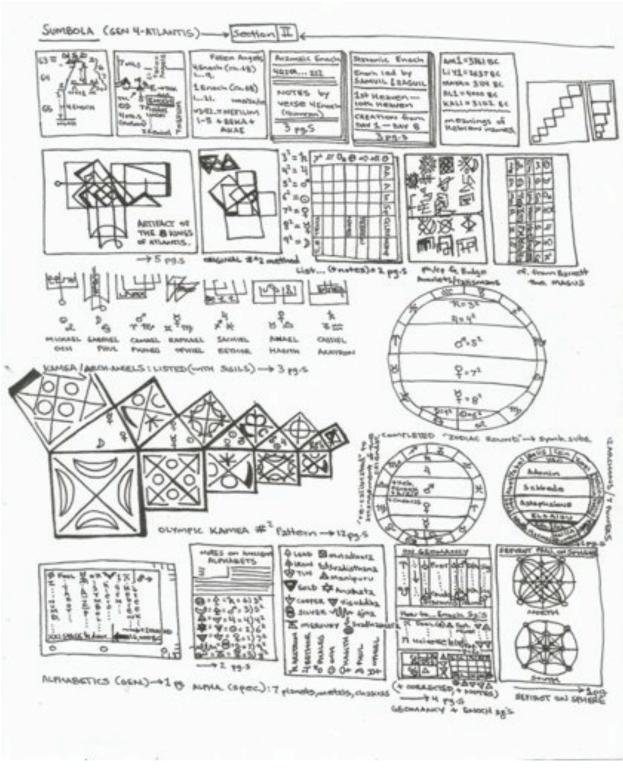
SUMBOLA

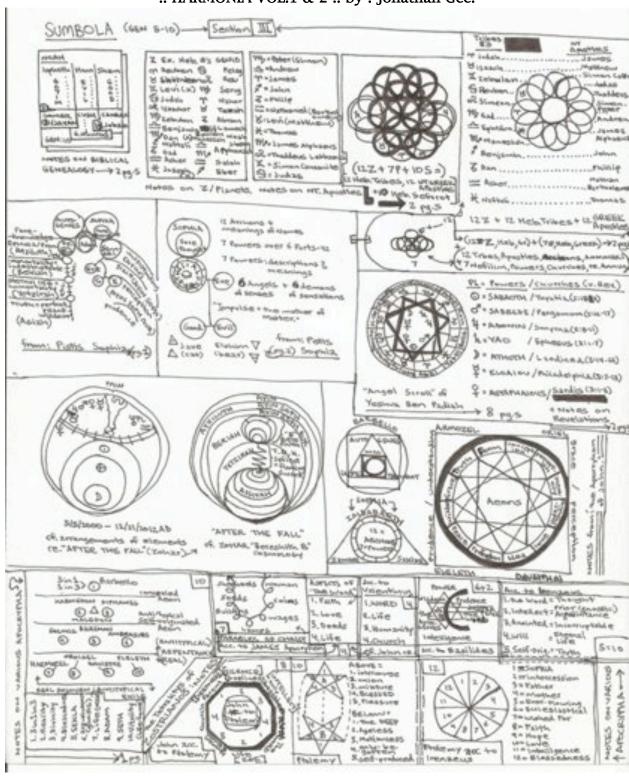
" Apostyphia (ot & NT) rel. to QBLH "

 \sim In Three Sections \sim

2. GENY-AHENNE: A.N -> 56 pg.5





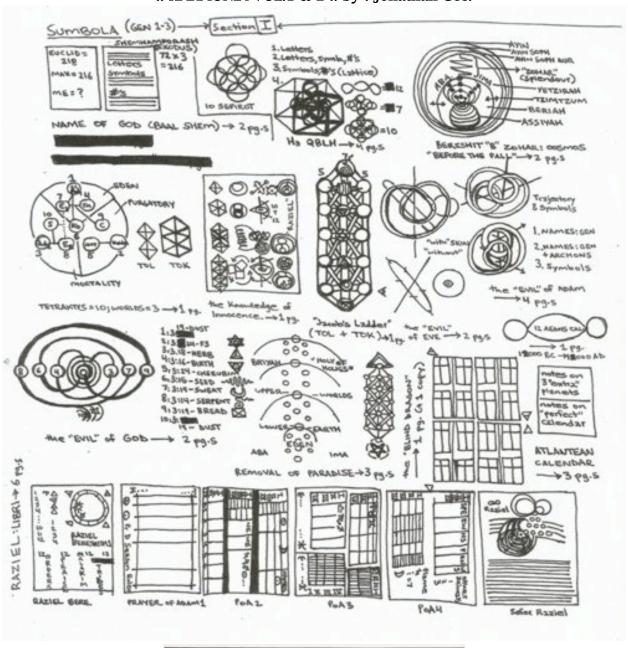


SUMBOLA

```
" Section : 1. GEN 1-3 ::
~ including: (eg. sums additionaled) ~
IA, HAME OF GOD - 2 pgs
18. Ha QBLH --- 4 pg. 5
IC. "BERESHITH B" Cosmology
   - 2 pg-5
10. TETRANTYS NO; WORLDS = 3
15. The Knowledge of Innovence
15 "James Ladder" (TOL+TOK)
16. the "EVIL" of EVE -> 2 pg.s
IH. the "Evil" of Adam - + 4 pgs
II. 12 Aeons Calendar
11 the "EVIL" of God -+ 2 pgs
IK REMOVAL OF PARABUSE+ 3 pg.5
IL the "Blind Dragen" (TOL + TOK)
I'm the Atlantean Calendar - 3 pg.5
IN. LIBRI RAZIEL, including:
```

Raxiel Bereshitta

the Prayer of Adam (2-4)
Sefer Raziel >649.5



כהת	メリメ	ככה	מהש	עלם	סיט	יכיי	7:17
הקם	ויירי	מבה	יוֹכ	ההע	כאו	אכד	71:
רדדילר	בולה	דרר	נכר	פהכ	כוו	כלי	כאו
ושור	לכב	אום	רני	שאה	ירת	האא	בֿרברבֿ
ליוץ	רהע	וזעם	אנדי	בונד	כוק	כהוו	ירדר
בויה	עשל	ערי	סאל	יכה	ווכ	בזיכ	-1-1-1
פרי	מבה	בֿלת	ננא	עבום	דווש	75-7	רדילר
בורזי	ענגר	יובוי	ובוב	בוצר	הרוז	ייכ'	נֿבום
בורם	77-	יבבו	ראה	רזבר	איע	בונק	דמב

- Freeze-framing shows that the 216 digit number actually has 218 digits. It's: 941432434315126593210548723904868285129134748760
 276719592346023858295830472501652325259296925727655364363462727
 184012012643147546329450127847264841075622347896267285928582953
 47502772262646456217613984829519475412398501.
- The 216-digit number which Max hand-writes on paper (different from the 218-digit number displayed on-screen by Euclid) is: 884509627386359275033751967 943067599621731590401694134 434007629683591574337516791 197615733475195375920401694 343151239621353184932676605 800621596380716399501371459 954387507655892533875618750 354029981152863950711207613. The piece of paper he writes it on has "Only God is Perfect" at the bottom.

→ 616743155135165535385148

551321361354182156616774625

73748515661744418215453135

73748515661744418215453135

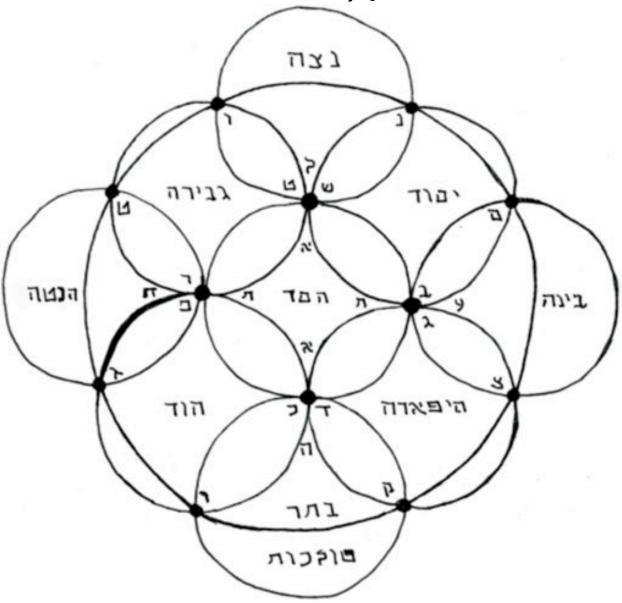
856757515163123135847126612943

856757515163123135847126612943

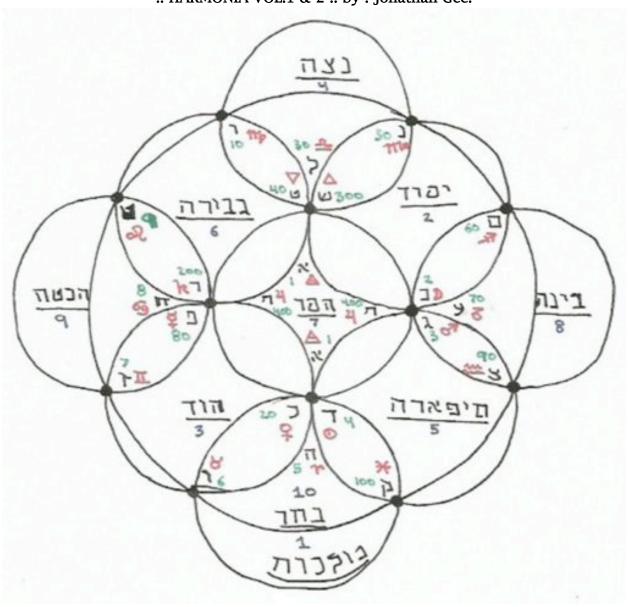
8567575151661943318126735166163

3156514516382256121765414-×

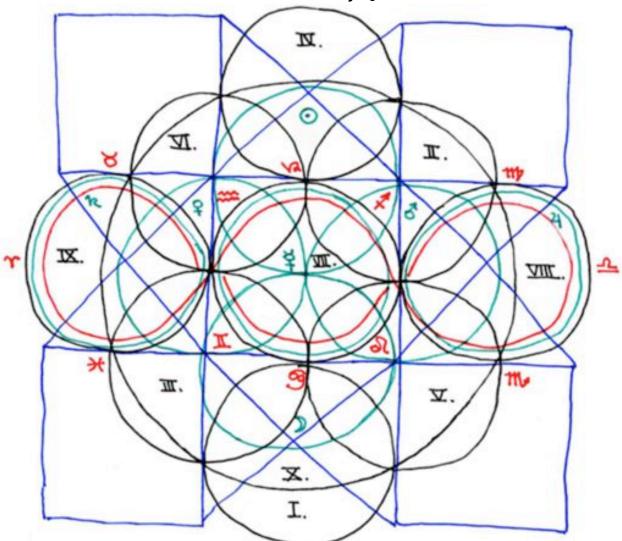
```
OMYNFLKLHHMYHLAHKALMASYV(M
    HVLAQQEZHALZHKLHLYLHGO)I
    LVYVMYHLAVQYCHAHSHMTYU(1)
MAASYVMHYRCHAMKLYVLAQSHYHN(19
Y SLEALYY HYHANN V H CHSLK V Y A Q A T (TO) II.
   LHLKHZAMYBQCLVRBMYHTAH(EV)
MHYRCLAMOMĀYUMHYNFMNNĀHRV(19
                YMUBYNMCHNH(20
VYBABYNMENNHMTER
MYMHVAQBYVHBRCHLMYHTAMSKY V(a)
の マ は TU を マ マ マ マ マ サ マ ト 日 ト 日 日 マ マ マ か で は (19)
マンサムてるるとはしは、日本日とは、日本日からて上去している
(ロ) とはおらないなられるとしてのないのでする(ロ)
物 A 农 A A B 物 T 物 T B m B B T B A B W A R A B (20) T.
てかかかてかましまるかのおられるとりのかよけると (11)
マイサルのムマのマるかと中ではいるのととのとんでいる(の)
マカマイとすべるのかのであるととなる (な)
```



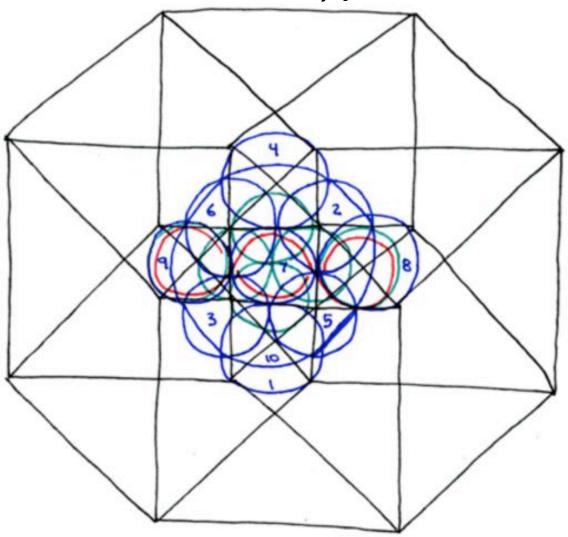
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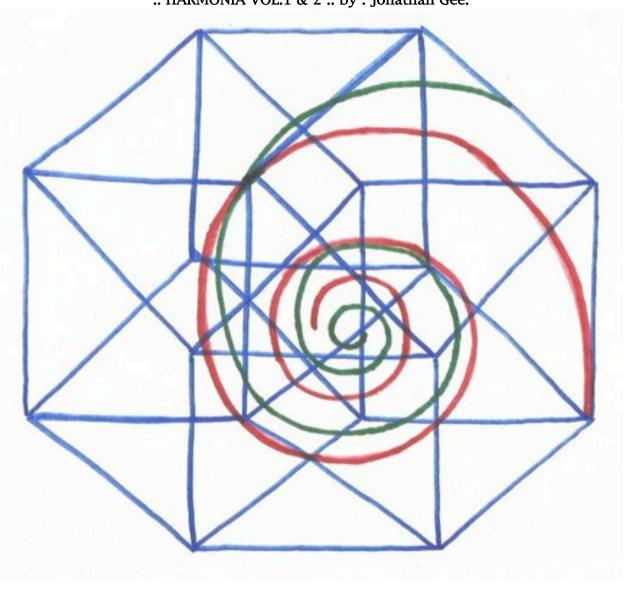
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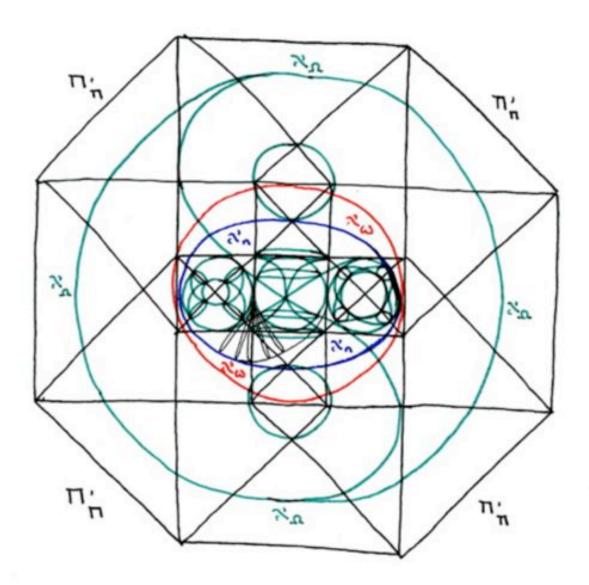
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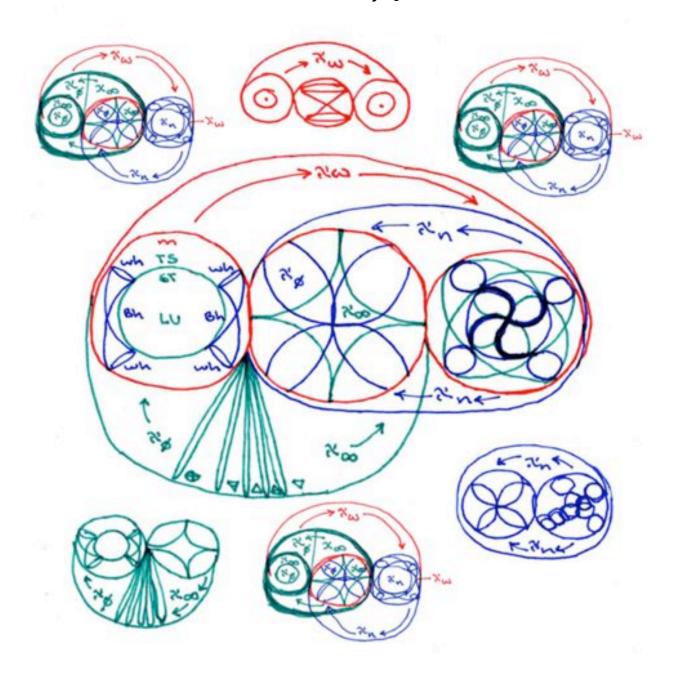
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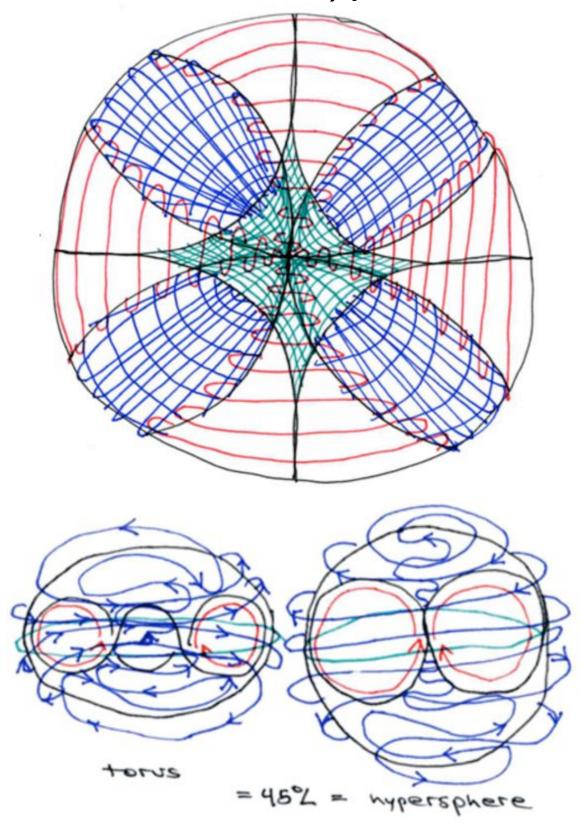
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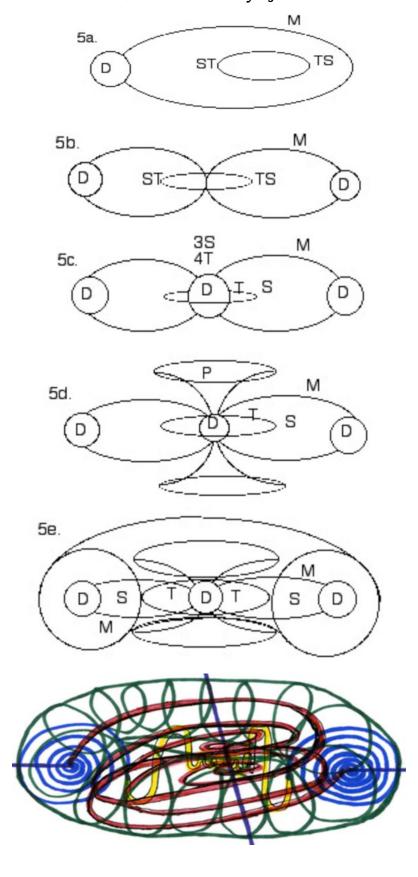


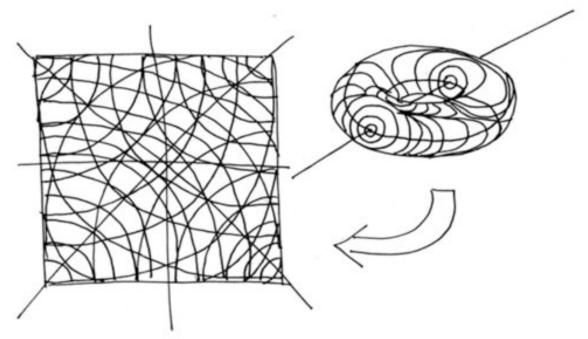
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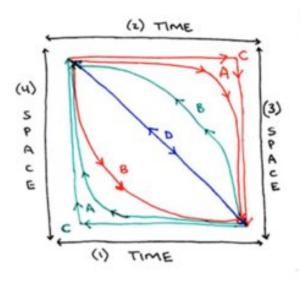


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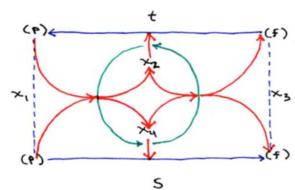




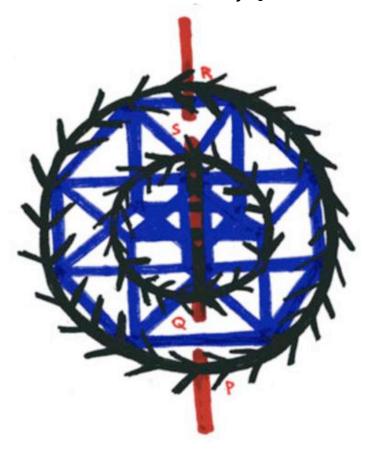


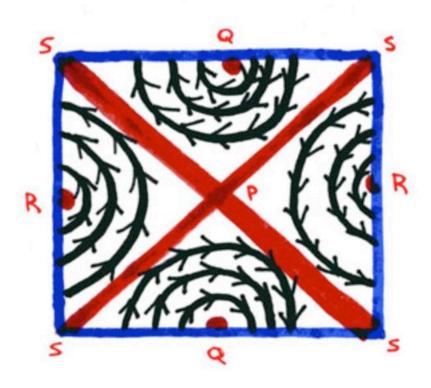
- A: "saddle" (curve >1R), crunch
- B: Sphere (curve < 1t), fixity
- c: cube (curve = 1 te), exponential
- D: plane (curve = \$), perpetual
- 1) time (singularity), 1-D 2) time (linear), 2-D

 - 3) space (bound), 3-D
 - 4) space (open), n-D

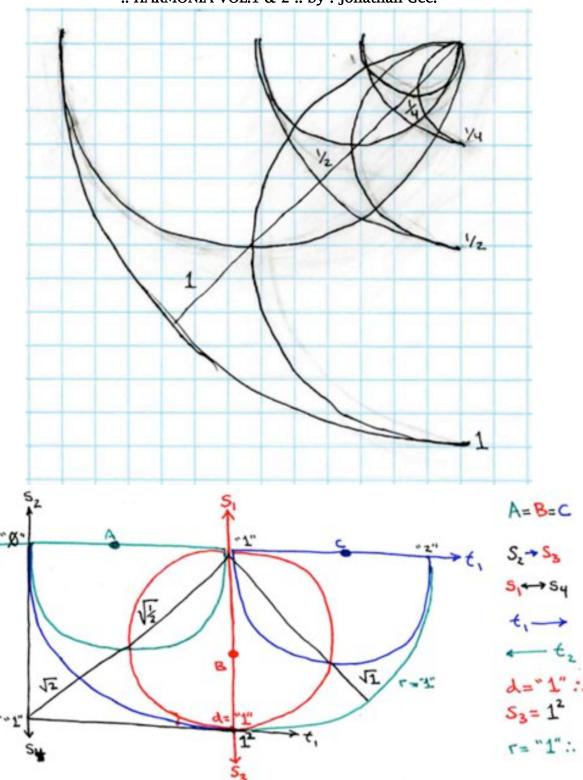


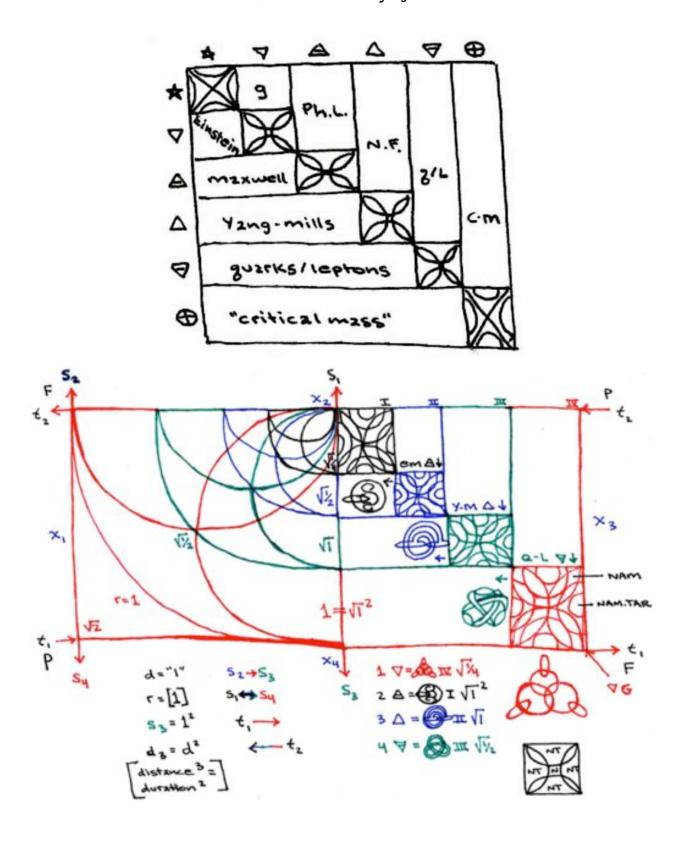
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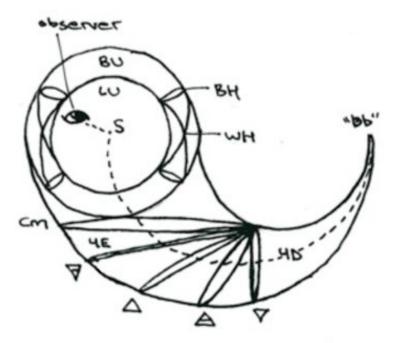




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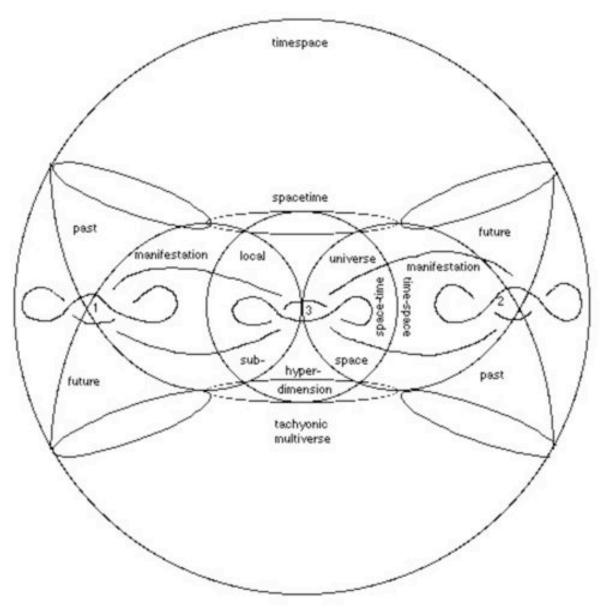




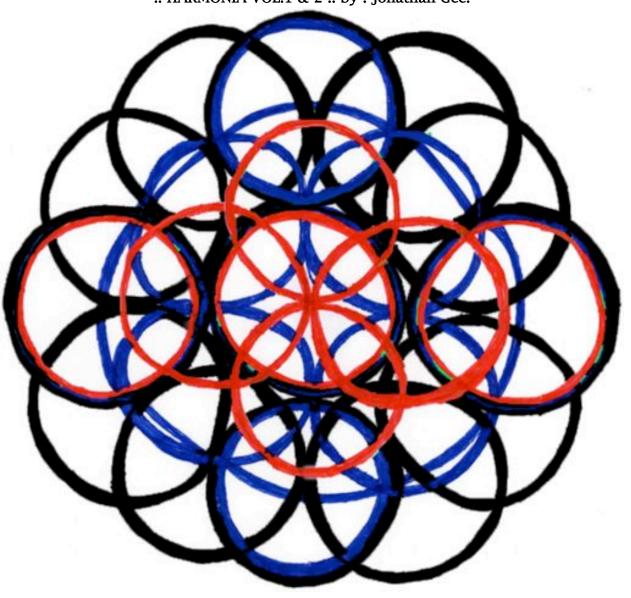
BU= bzby universe 40=4 dimensions S= Singularity BH= black hole WH=worm hale cm= critical mass

LU = local universe 4E = 4 elements F= strong nuke D=wesk mike A= e/m V= BEZYHY "bb" = big bang

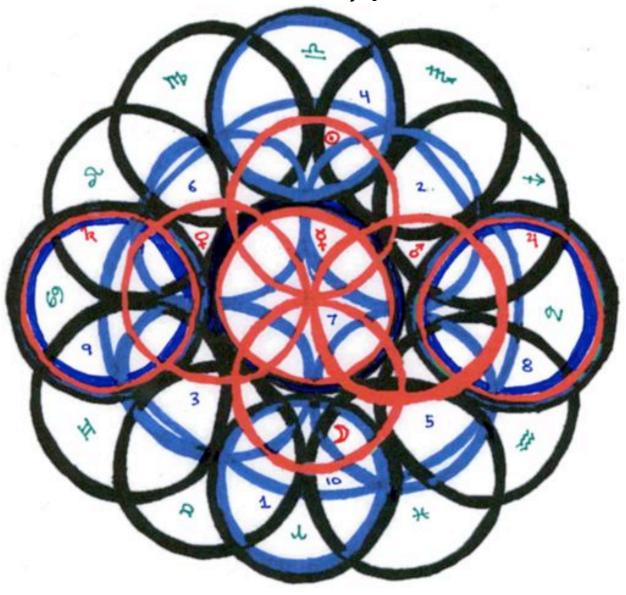
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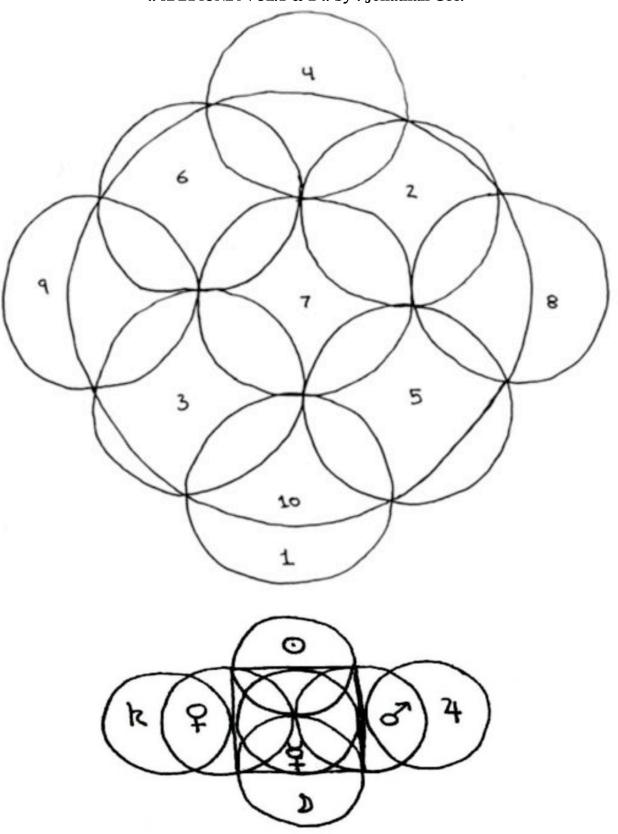
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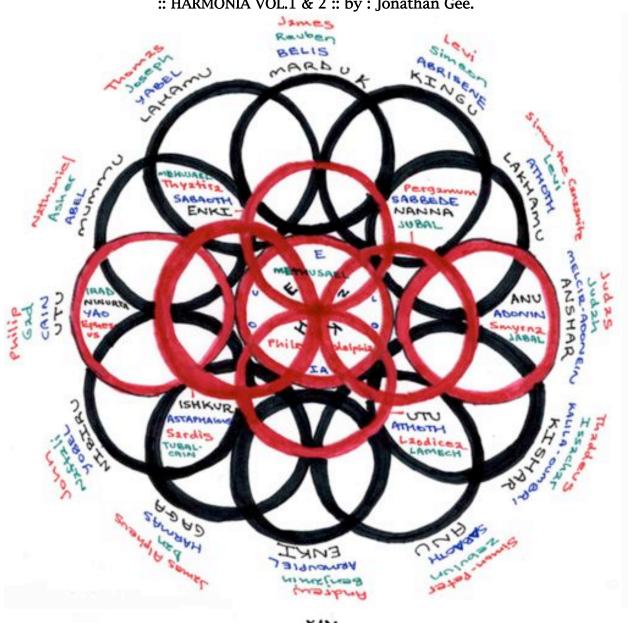
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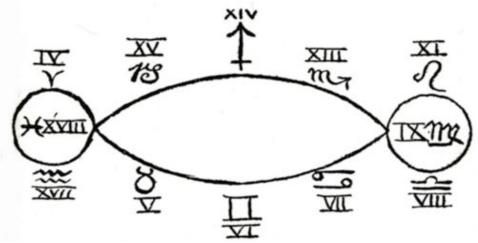


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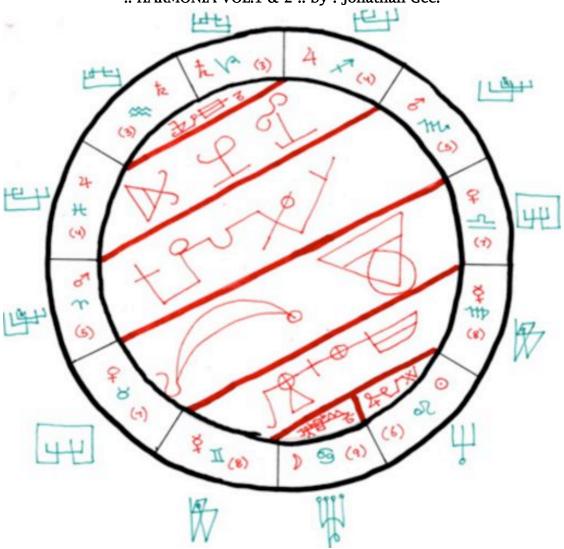


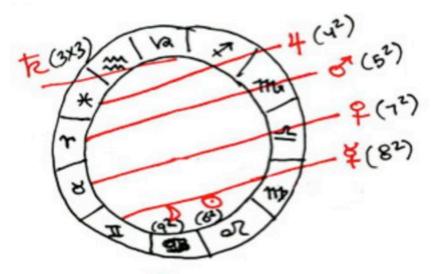
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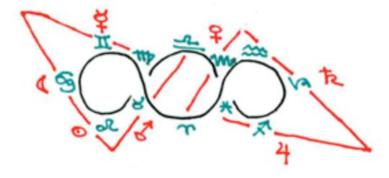


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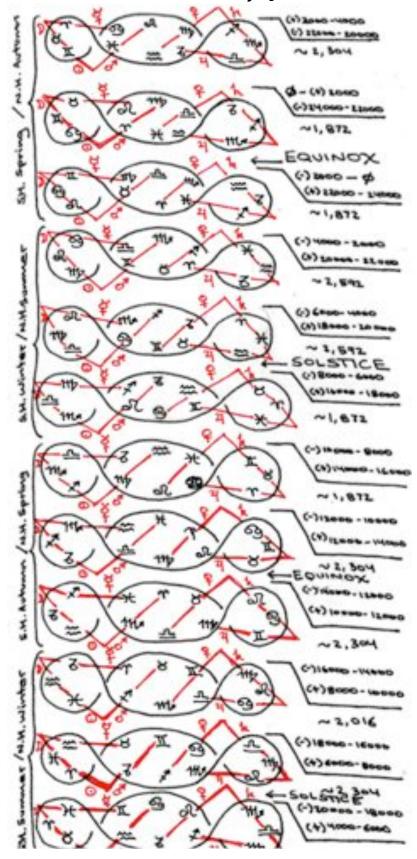




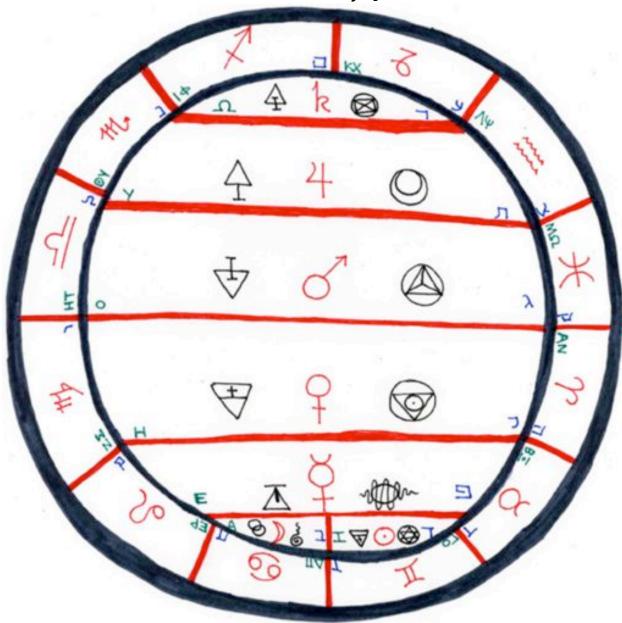
Hebrew from numbers guare Seal from Remutation of " Spirit/demon = Hebrew Sigils angels = names of zadiac Kamez = positions in zadiac

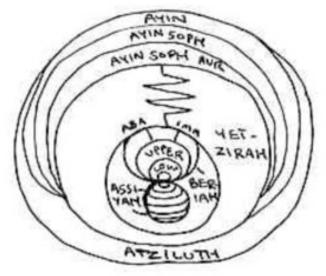


:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.



:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.





ATZILUTH - emanation

AYIN- nothing

AYIN SOPH- limitless ness

AYIN SOPH AUR- limitless light

YETZIRAH - formation

SEFIRAH - creation

BERIAH - creation

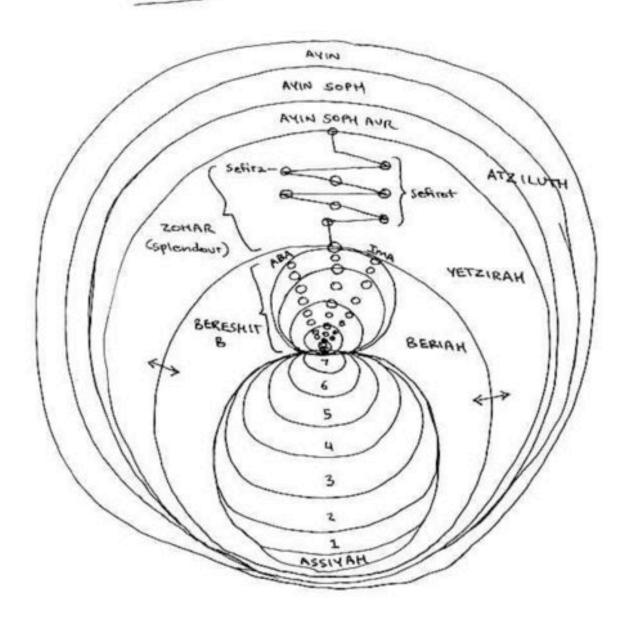
LOWER WORLDS, UPPER

WORLDS, BERIYAH

ASSIYAH - action

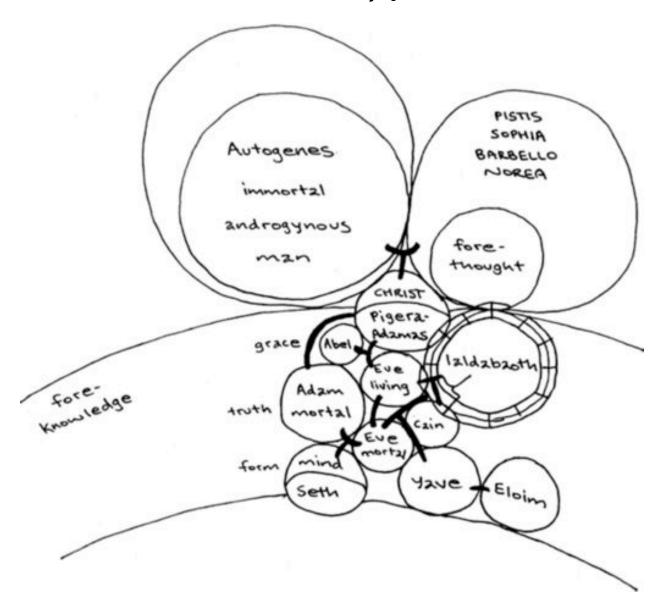
7 days of creation

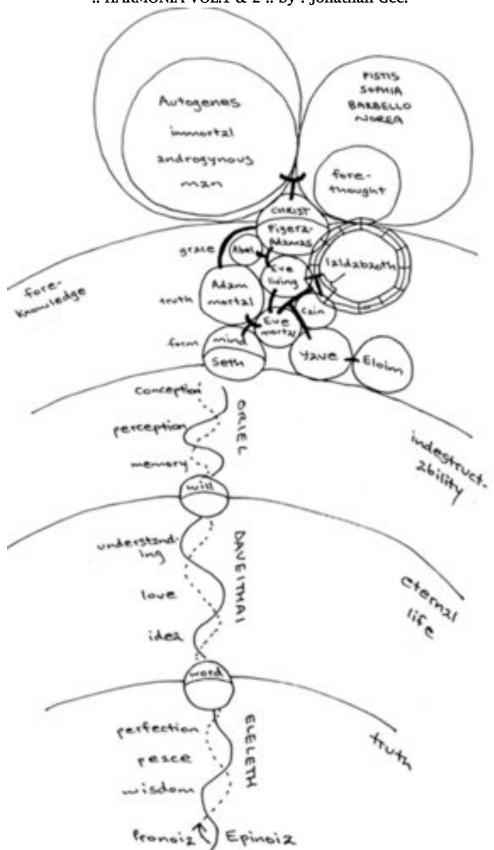
BEFORE THE FALL



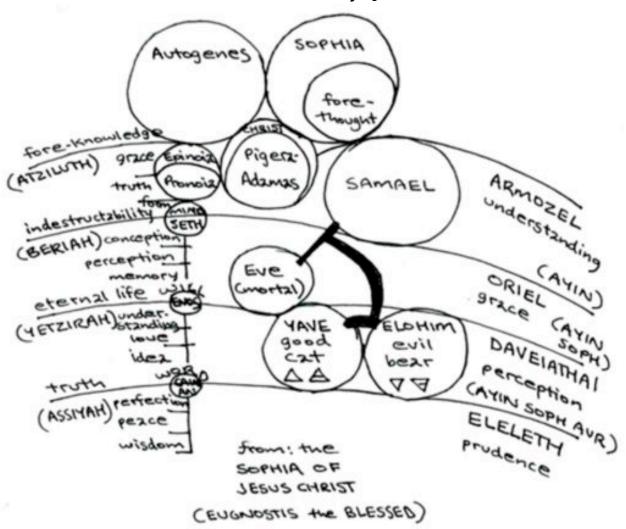


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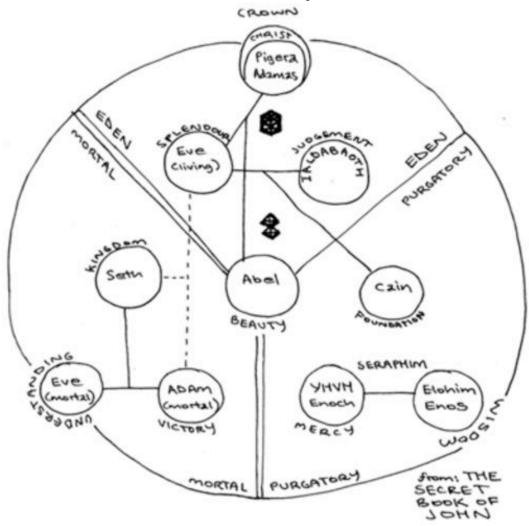


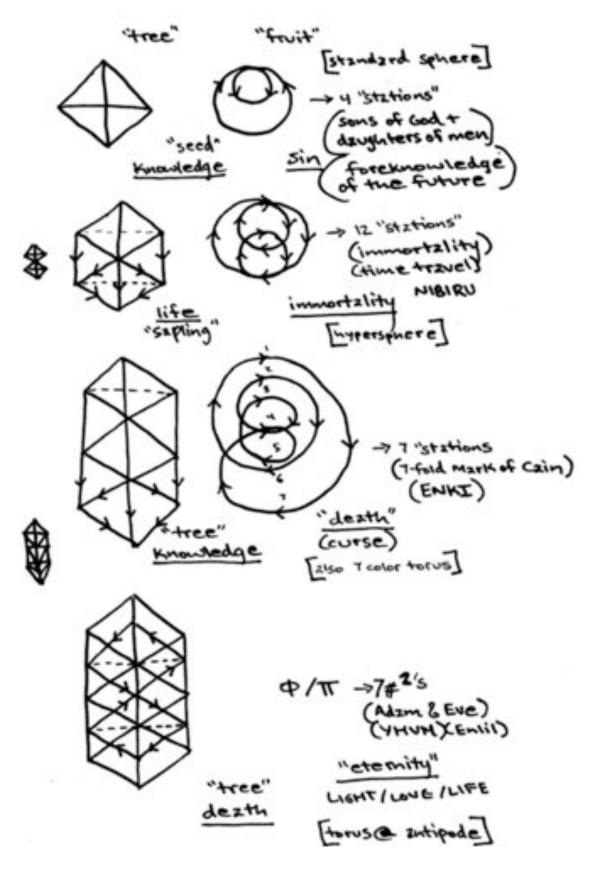


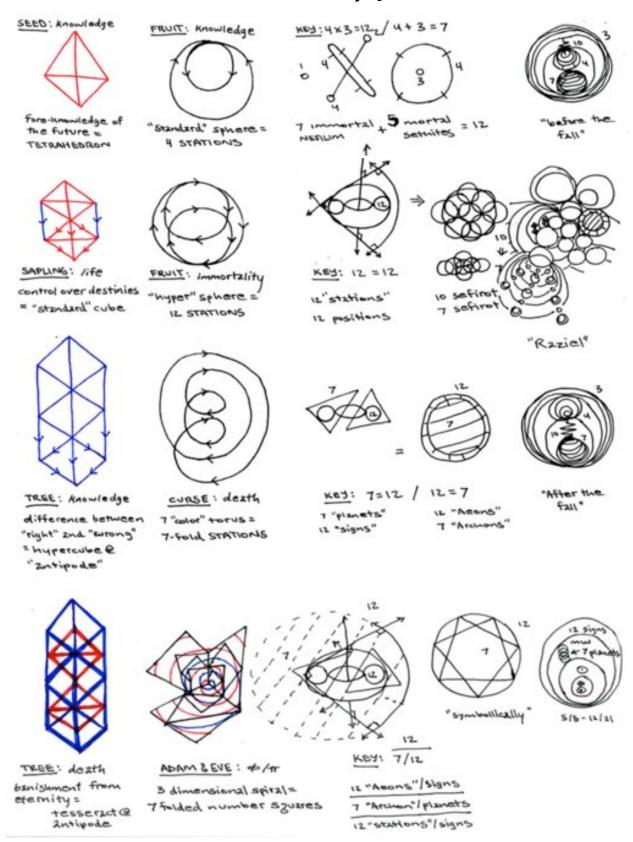
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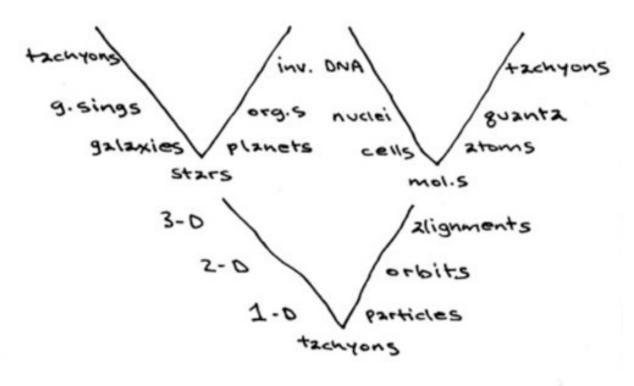


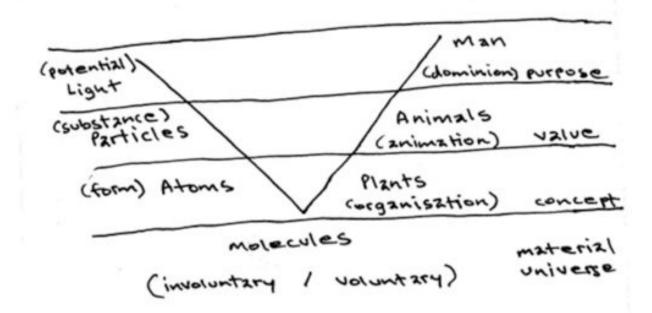
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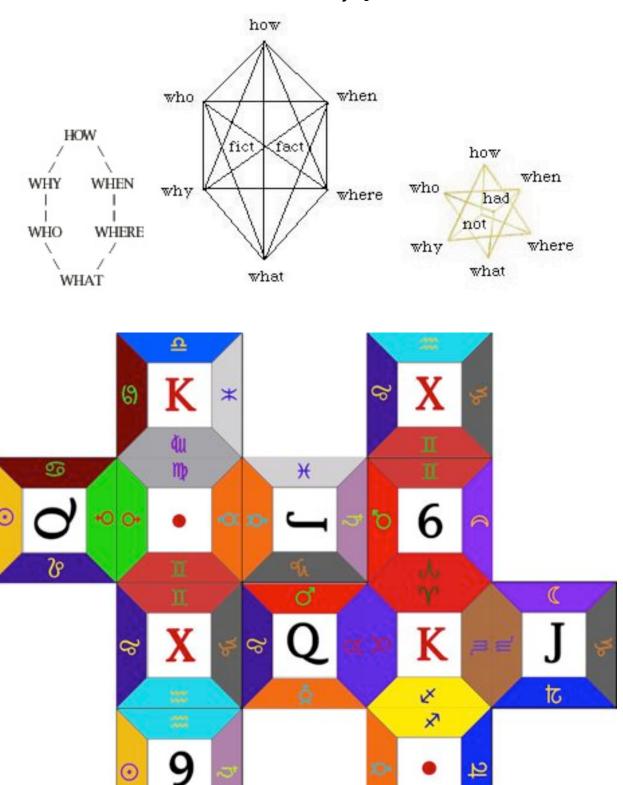




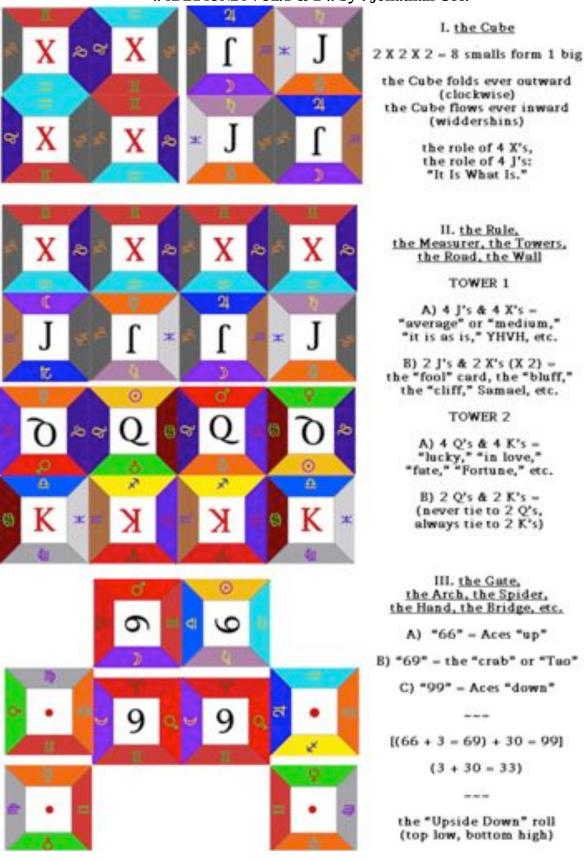




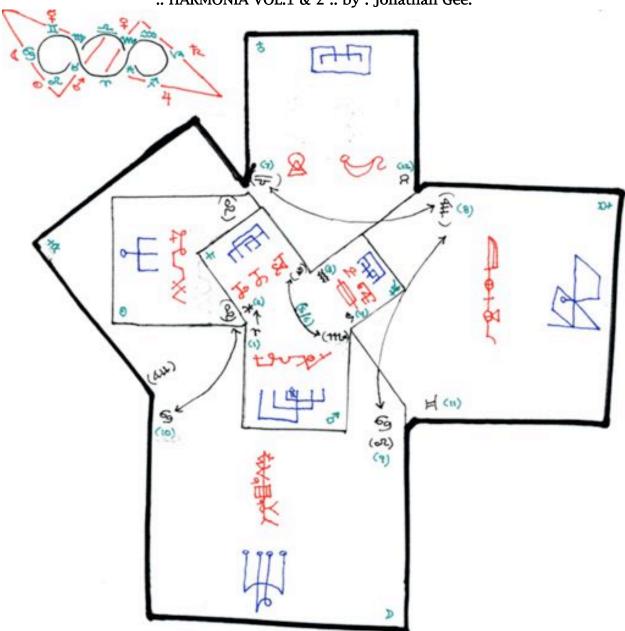


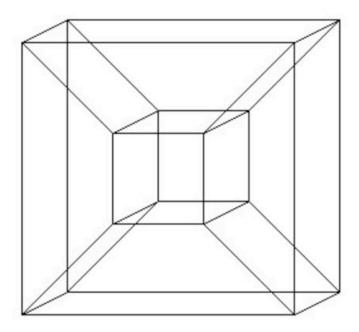


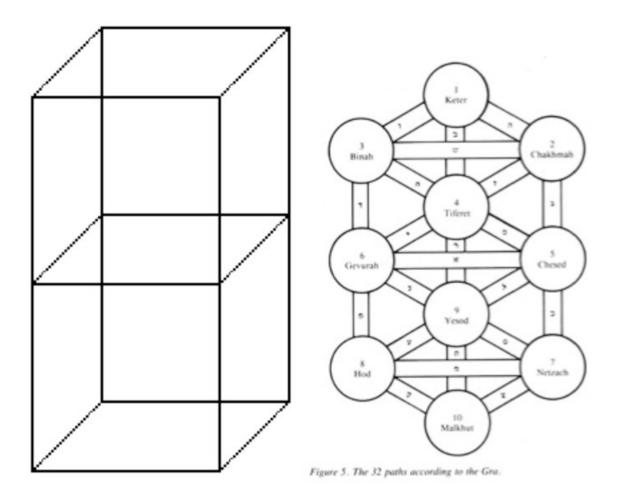
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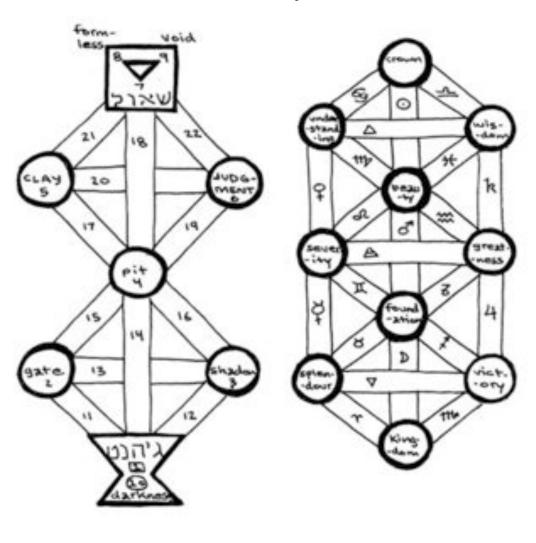


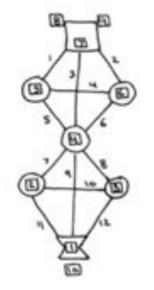
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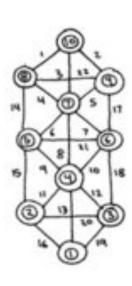


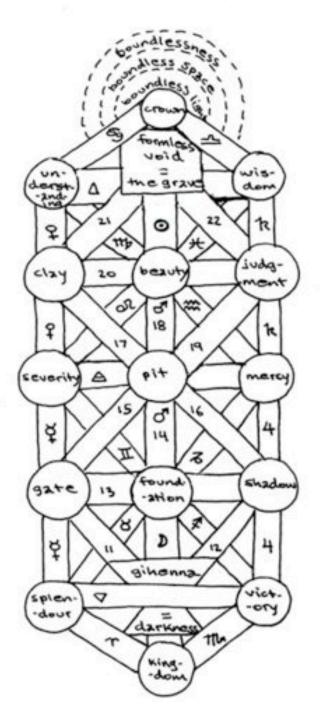


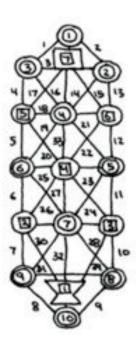




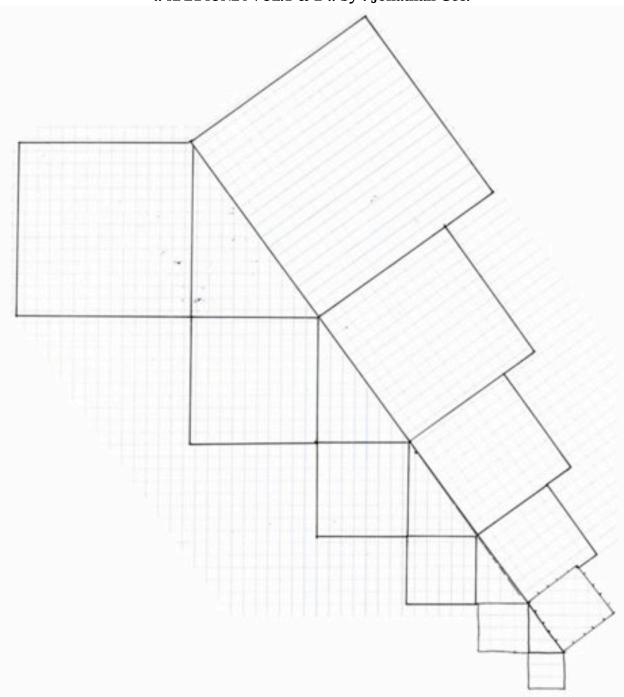


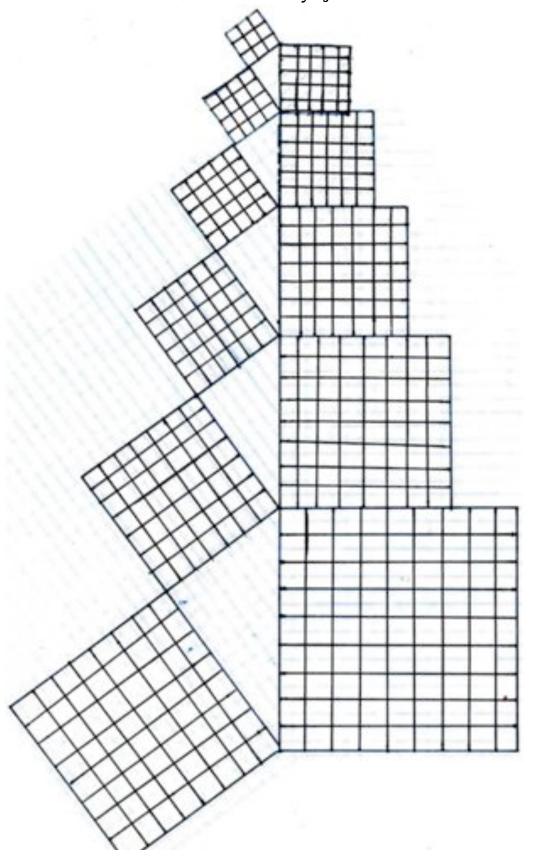




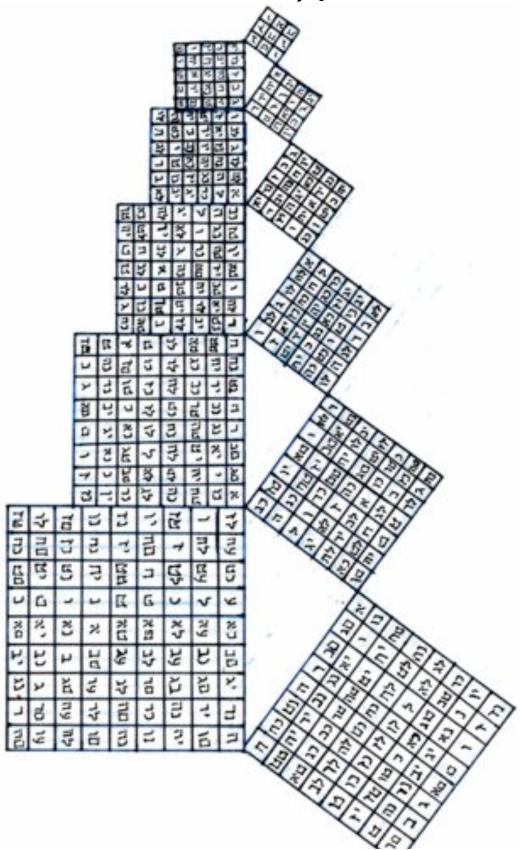


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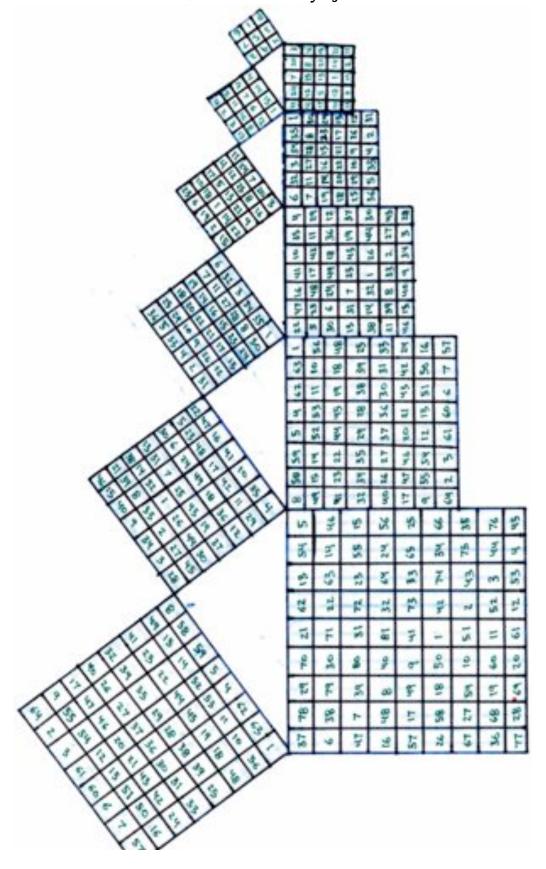




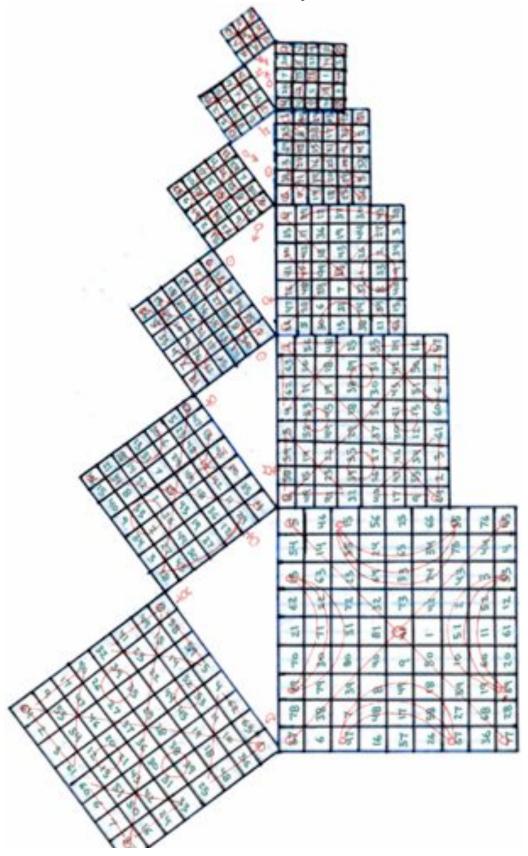
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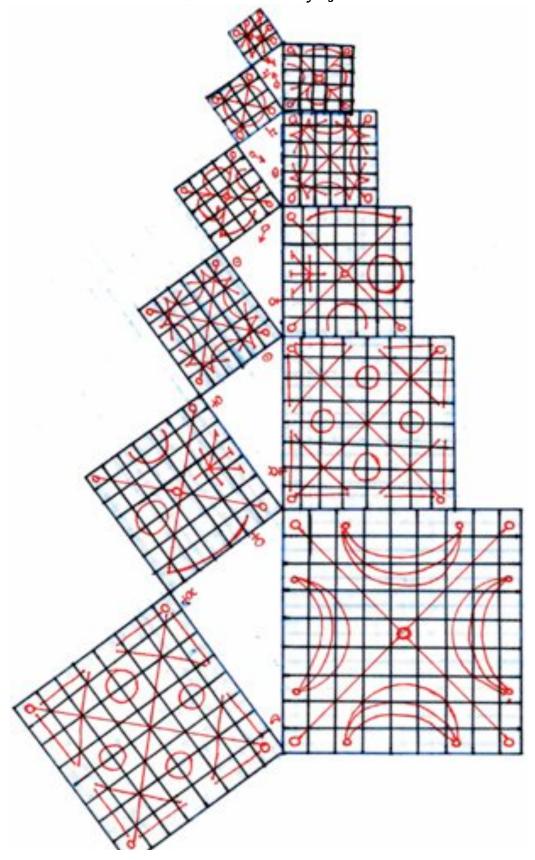
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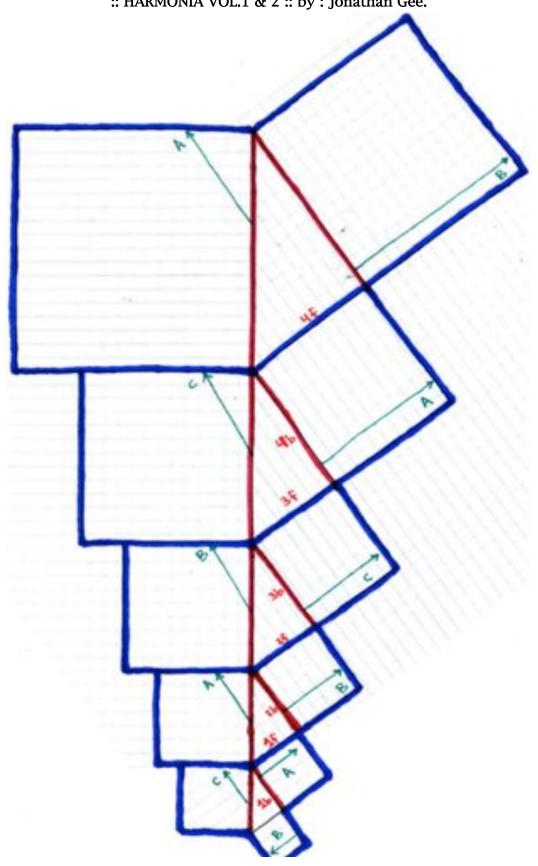
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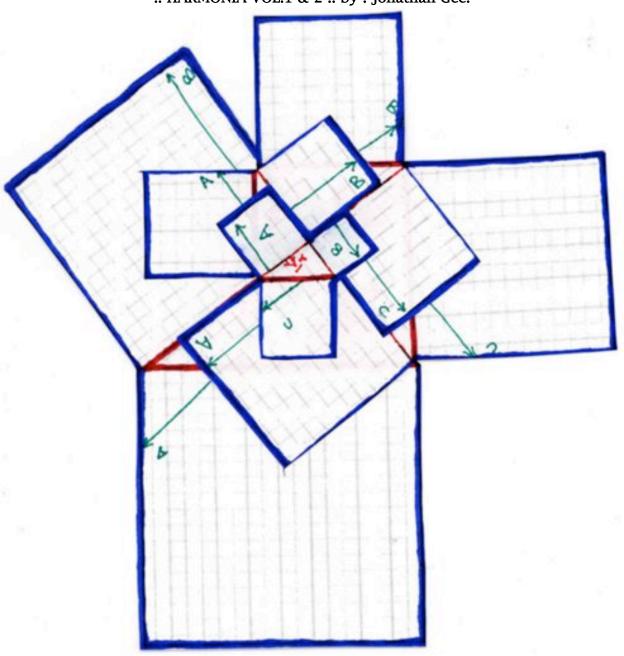
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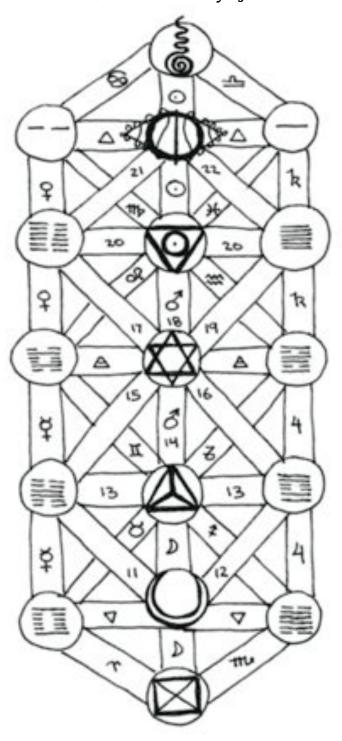


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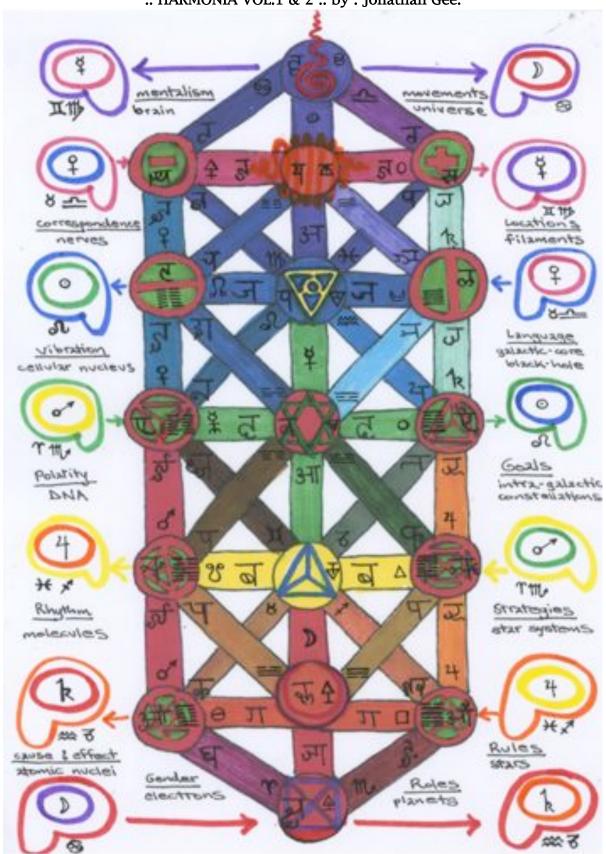


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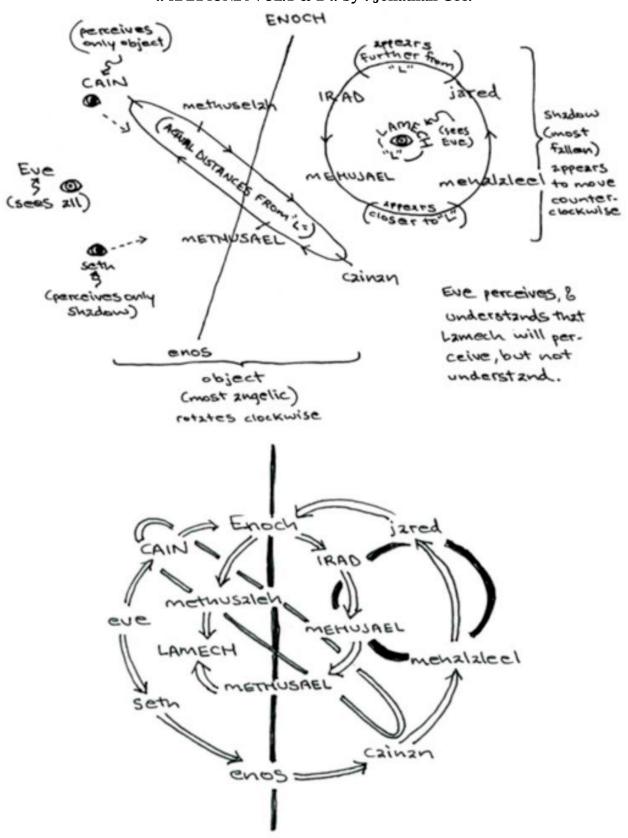


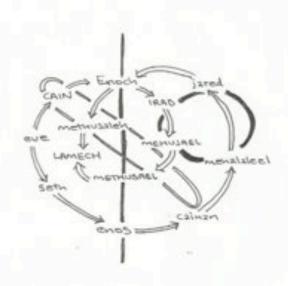


The combined Tree of Life and Tree of Knowledge diagram.



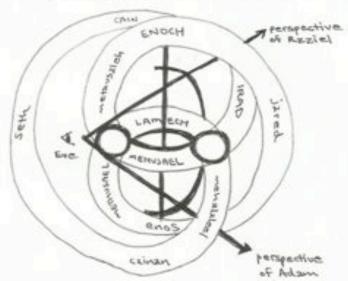
:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.





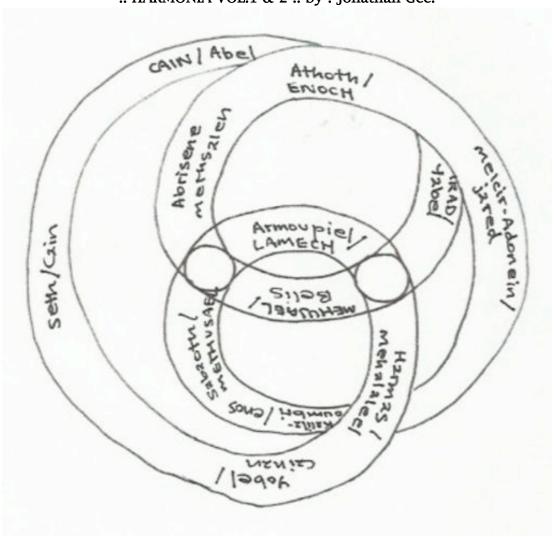
o was, I behold the fruit of the sim of Adam, Forbidden the immortals. For this In men since have been given to death. How is this the first I was written, "be fruitful." He see this fruit "refers to the generations, How is this fruit that of the original sin? It is said, by their fruit know them." So are these the generations of man that fell & that followed the fall. Because Adam are from the tree of life, he conceived Cain upon Eve as immortal. Secause Adam are from the tree of Knowledge, he conseived Noel 25 2 wise mortz), Because Adam fell from grace, he conceived Seth as merely mortal. Thus the generations of Seth were mortal, but the generations of Cain were immortal, Because Adam had been "like unto" God, the immortal heirs to cain were called "sons of God." They bred with the "wives of men; who were the mortal generations since seth, and begat "men of reknown" who were "like giants" Now, in those days comes Enos out of Enoch, who is 2 man, but who "died not, for God took him." From Enech we learn that the "sons of God" begat with the wives of men, but Enoch's wife was a daughter of Adam. That is how known knew the teachings of the "sons of God" but himself was translated to metatron, while the same teachings corrupted the seven mertal generations to follow. So, from the daughter of Adam, Enoch begit IRAD, who was immortal; but from a daughter of Seth, Enoch begst methospiels. The immortal generations before Enoch moved backward in time, and after Eman forward, Between Enos and Enoch the mertal generations moved forward in time, but afterwards backward, it was Enough, who knew of moses aforehand, who turned the tides, when Ence entered Encen, & Encen was translated metatron, He beheld the vision of the fruits of the generations of Adam. He hung his head in judgment, and from this came the flood. This vision is rightly called LAMECH, for in LAMECH'S time ended both the immortal "sons of God" and the fallen mortals,

It was not before the Creation of the world, as some say, that Adam was given by Sad the task of naming everything. This was the "prayer" of Adam, when God asked Adam each name by giving him the first letter. This, some say, must have occurred in the primerdial existence, it not while in paradise. But surely I tell you no. According to the most recent editions of sefer Rexiel, Adam called the names of everything from the Acons down, those being the angolic seraphim, the flaming smooth itself, bearing his re-entry into the paradise of Eden, while he tortured himself in the river Gilber, causing all the fish to dessicate his skin, that it became soft and river Gilber, causing all the fish to dessicate his skin, that it became soft and pliable, as our skin is now. Girclorg tells us three Ethiopiane found him thus. pliable, as our skin is now. Girclorg tells us three Ethiopiane found him thus. one of them was Raziel, and he is known to history as Azazel, as Azzael, as Surael, Isldabacth, Saklas and Shaytan. He came to Christ on the Shemyazza, as Samael, Isldabacth, Saklas and Shaytan. He came to Christ on the Cares as Panamea. New, Adam, when he was submerging himself in the Gilbon,

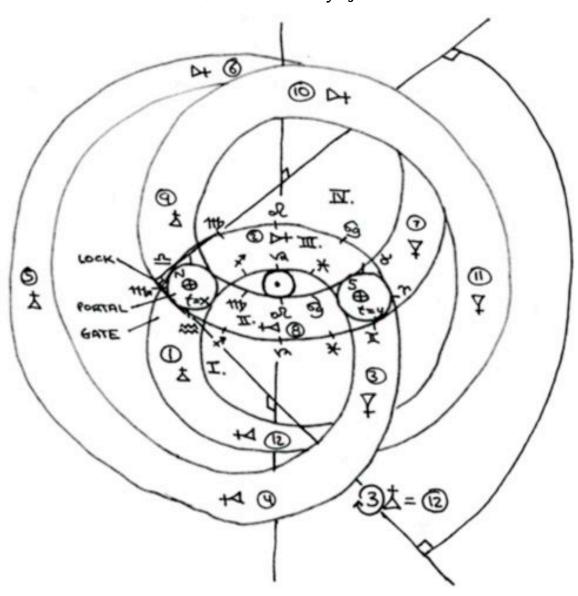


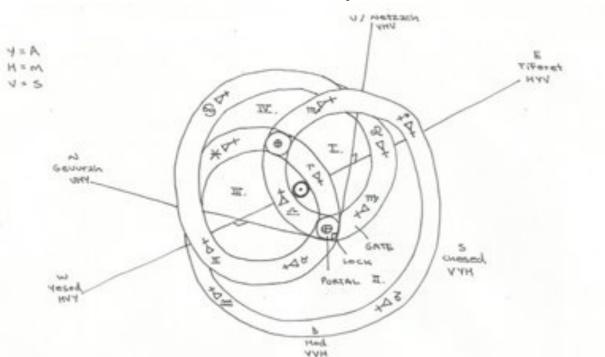
gave hames to everything in existence, However what is known of a man is not known by his work, but by the fruits of his labour. And so all of this was being watched and observed from a distance by the Fallon spirit, who lost the contest to Adam of guessing south given names for everything. Raziel then proved upon Adam and tempted him away from the redemption prayer, what could have drawn Adam away from his explication to God in the Gonon? It was at that moment that see hore cain, whom had been sired in sin in the Garden of Eden, It was at that moment that Adam aworks, realising none of this had happened yet, but would now follow inevitably from the sh ne was already in the act of committing by then that is why the vision of the apple live saw to different from what Adam saw, while eve perceived only suadows, smoke and mirrors, Adam saw the Aeons, and already saw the tempter Raziel standing above them, beyond and above setzirah, while Adam already sank into Beriah. So this is the same view as Adam had while eating the apple that Raziel had from above setzirah.

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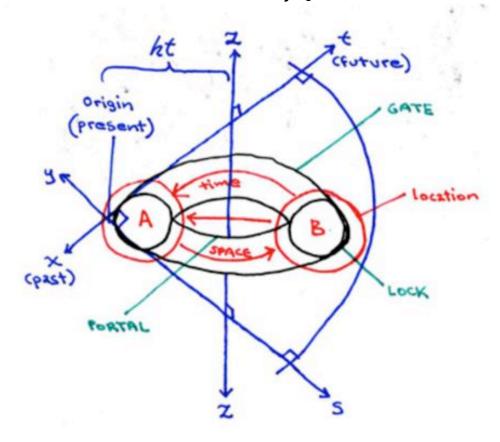


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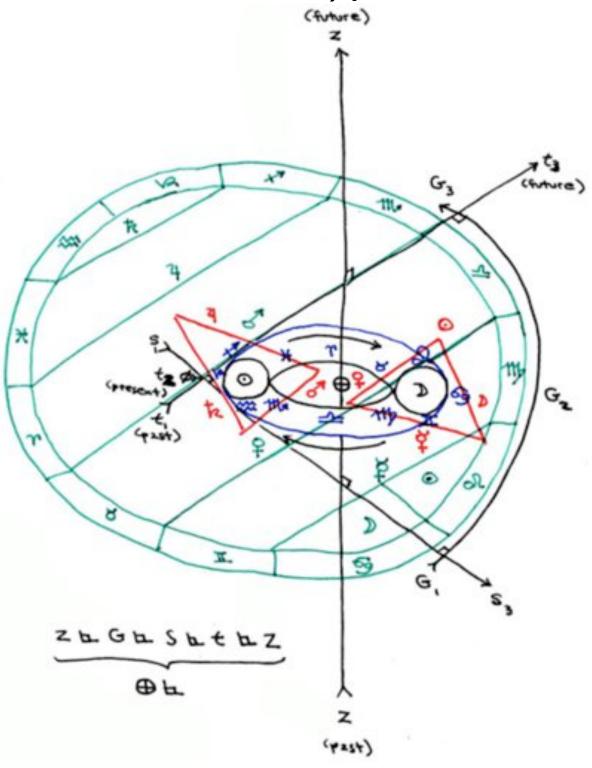


it is said that Adam, following the expulsion from paradise, 1084 the side of live his wife and journeyed until the came to a very wide oliver, presumed to be the rone, although it is possible this is entry a metaphor for the Red Sea. Adam, whose height, though decreased from that while he dwest in Eden, in the garden, Berizh, still towered over oven that of the "giants" & "men of retrieves" who ruled in smooth's life; 📾 neak-deep in the water, remained In prayer for devens of days, as the days & heavens of night passed by eventueed, just as Adam's neight was slowe that of the glants, so too were his "days" longer than theirs, meaning he lived hundreds of years longer, though even these were less than his immertality in Eden, the creation of the Eternal God of Heaven. Adam prayed upon the numbering of his days, calling up to hadven hames of the seems. From the realm of the seems It is written, three wise men approached the share where Adam dwest t with the Their skin was of a much darker pigment than Adam's, now pale and withered The color among them was named Raziel, and he gave Adam knowledge of how to perform secrifice, understanding of the resum of the Neons, and wisdom of the names of God in old Hebrew, or positivy even an older, now lost appealant. This alphabet Is known as the "Three famers," while makern Hebreus is that of the three mathers," The supernal crown taken from Adam's head was given back to him by Radiel in the CALM, or "received tradition" of writing, gesture and enought, of all men-weetax communication. it is written that, in his tools, adam related as this to his mortal our, seth, and that, upon Adam's death, Enoch related the gnosis of Raziel to methysateh, who was the prototypical Zorosster of Amera-mazda, and methodelah related the Homenedge of the freed to come to his son, moun, we prototypical utnapisation of Sargan, the scorpion King. Enoch distated the "three stell of Seth" to methodesah, who carved them in the language of the "three fathers" on one stone, the "three mothers" on another, and the third he kept in the city of Enoch, with directions to the other two, marion were sexted the Brach's death in his tomb. The third was given to rooth and kept until the time of moses, much is was smashed. The location of the other two pillars following that point became me non-vertical tradition, or ERM, of Baal-binem:

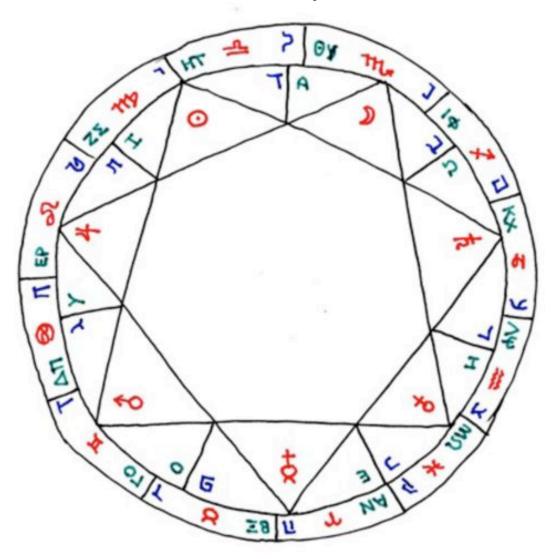


fraveller moves between gate and portal from one locked position to another PORTAL L GATE LL LOCK L PORTAL

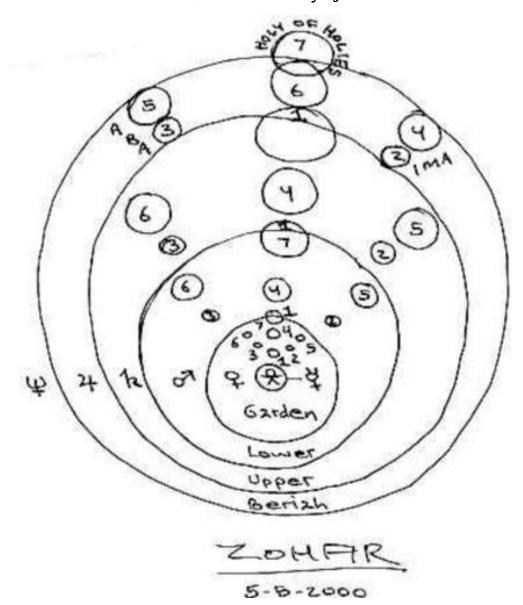
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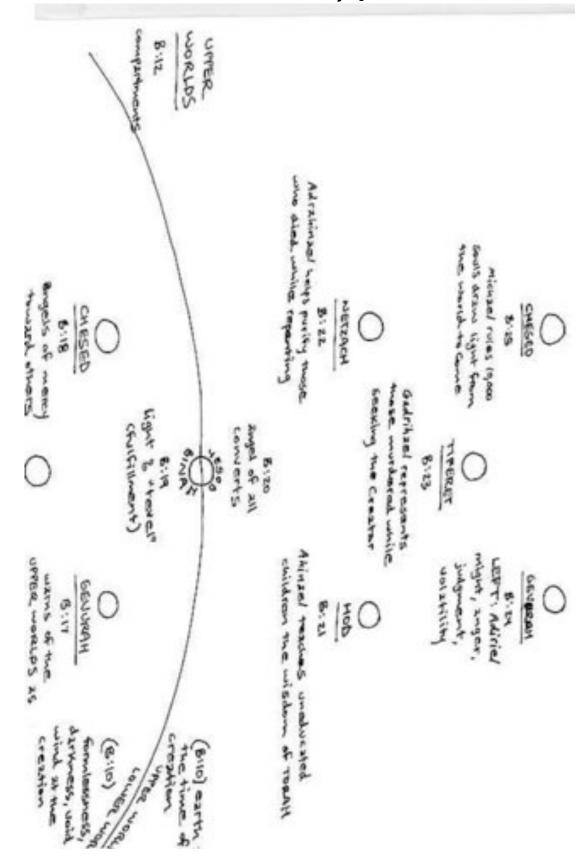


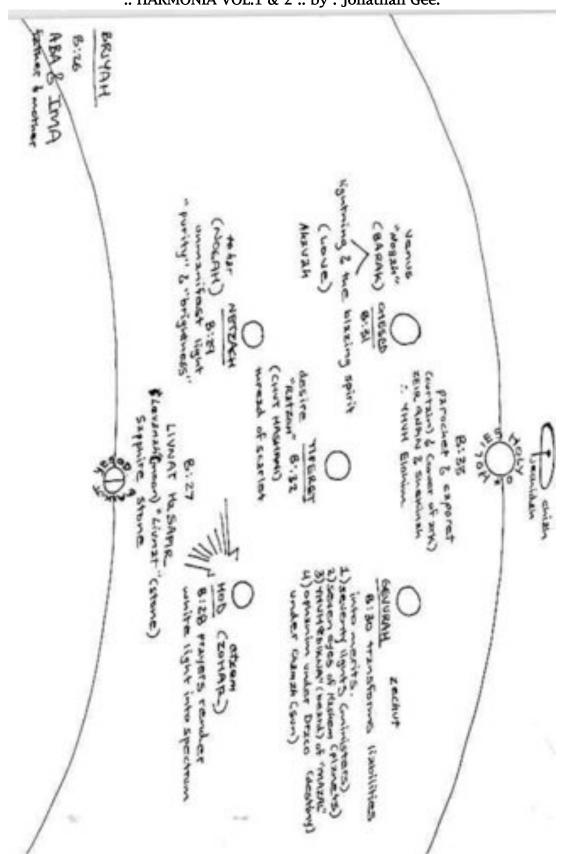
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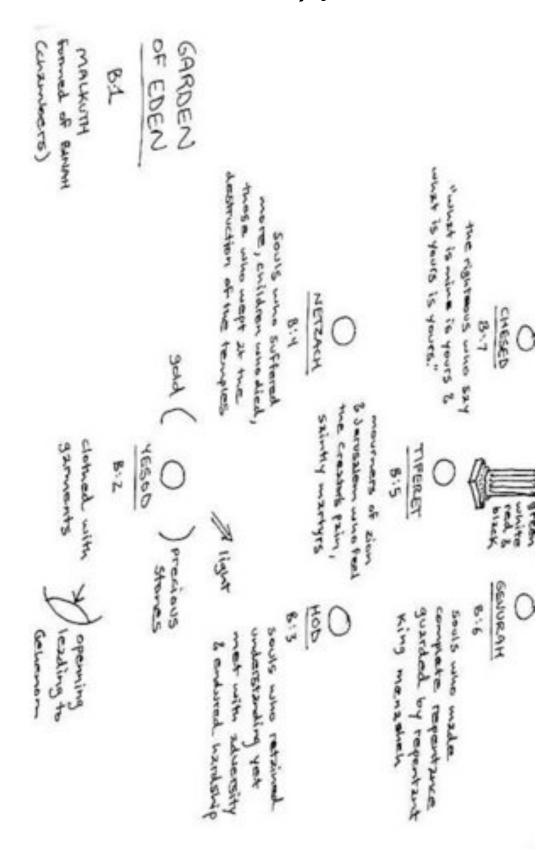


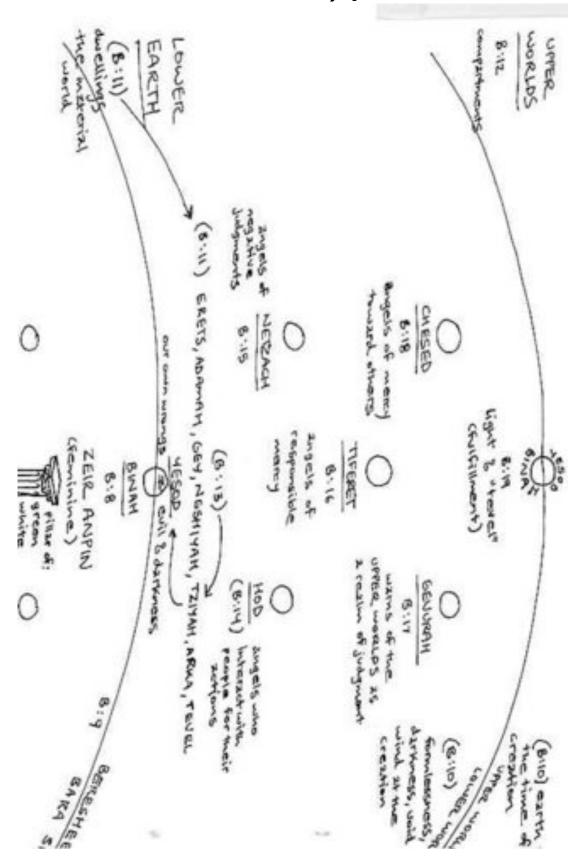
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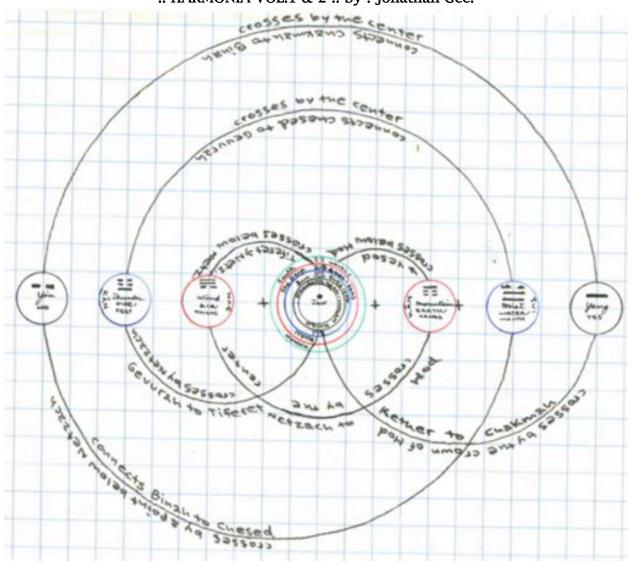




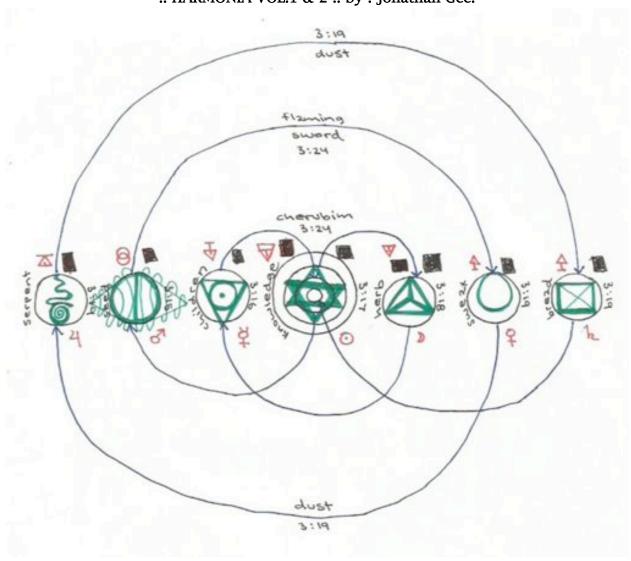


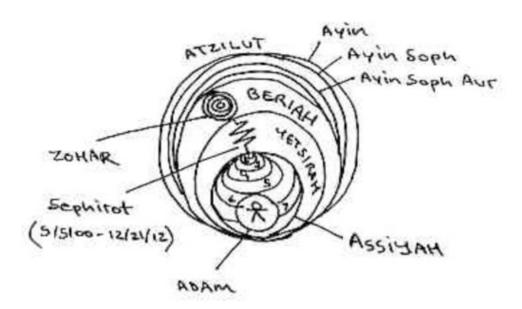


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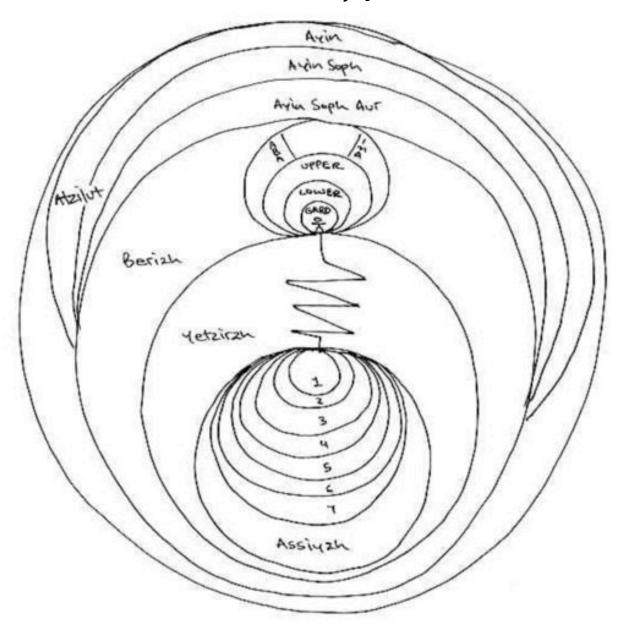


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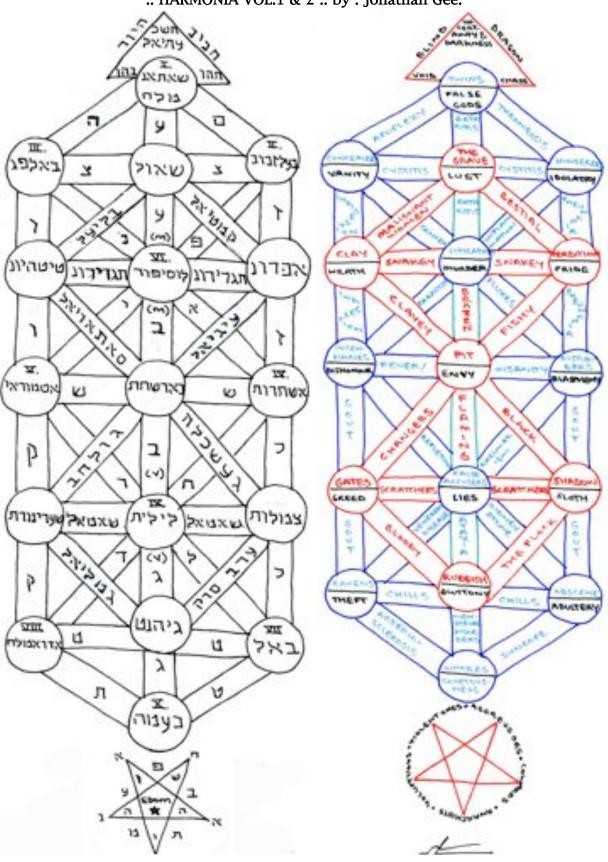
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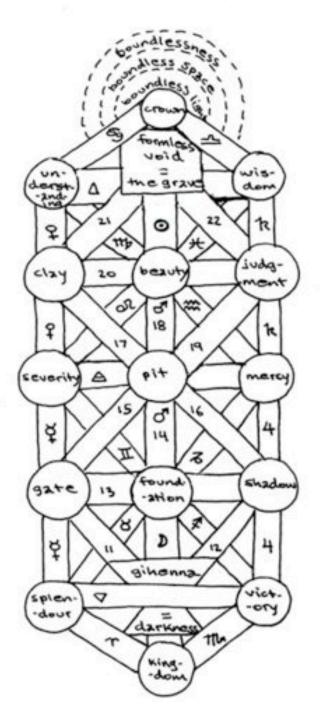


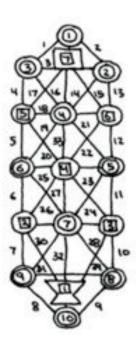
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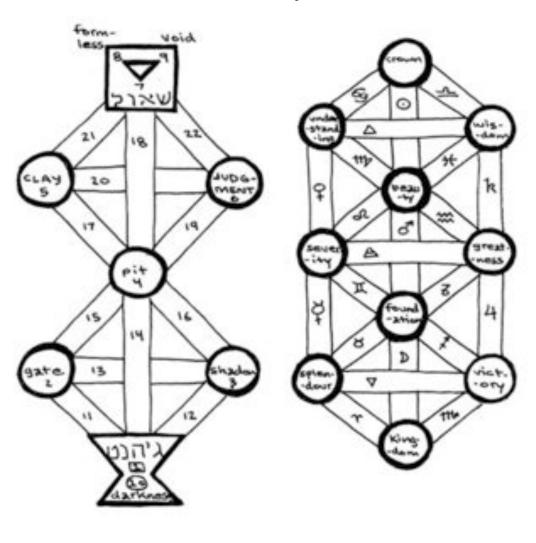


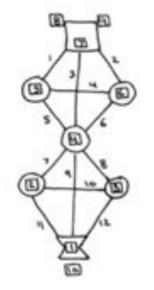
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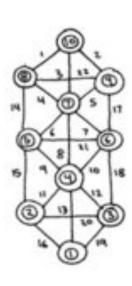




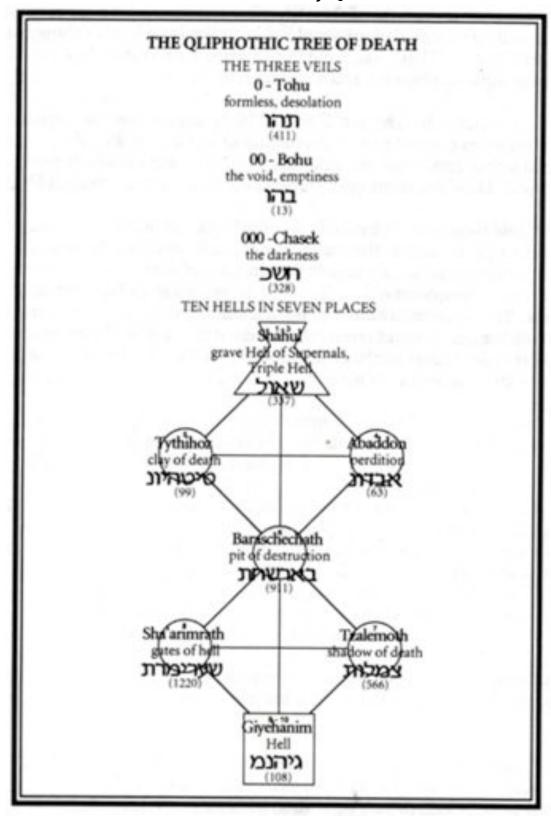


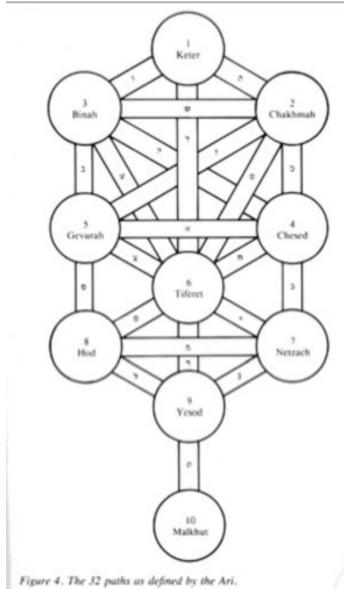




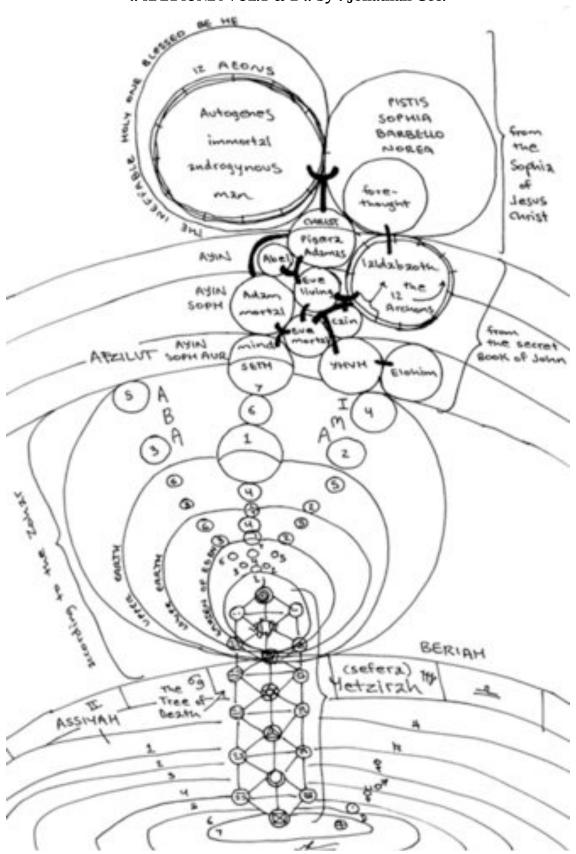


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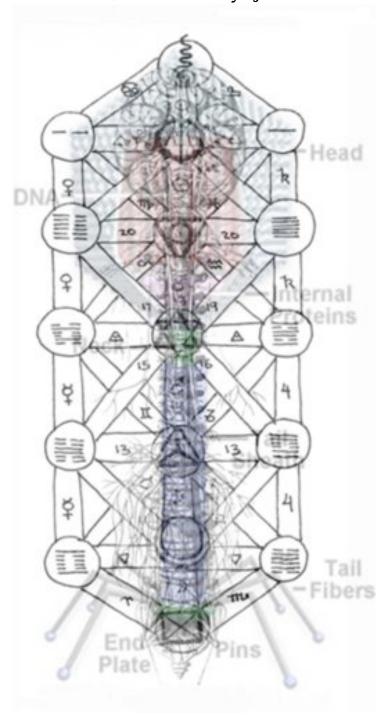




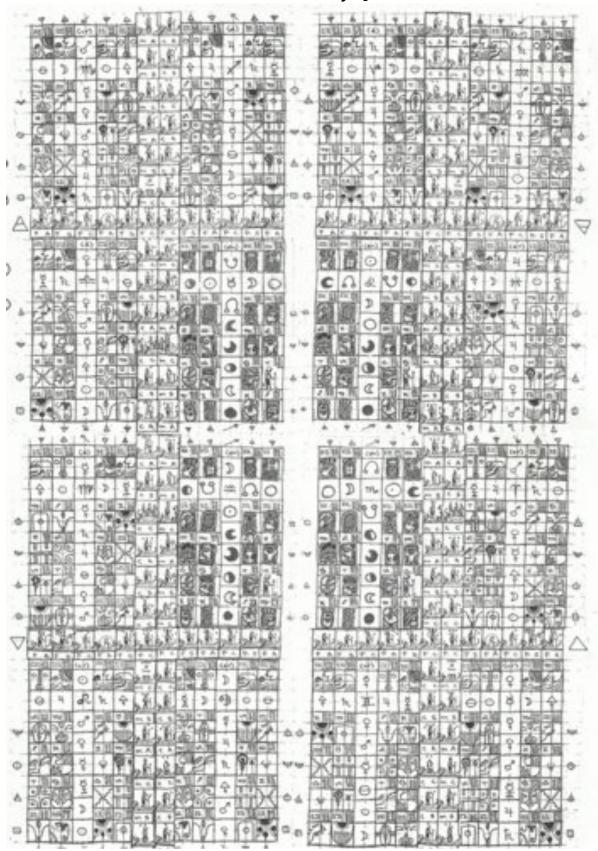
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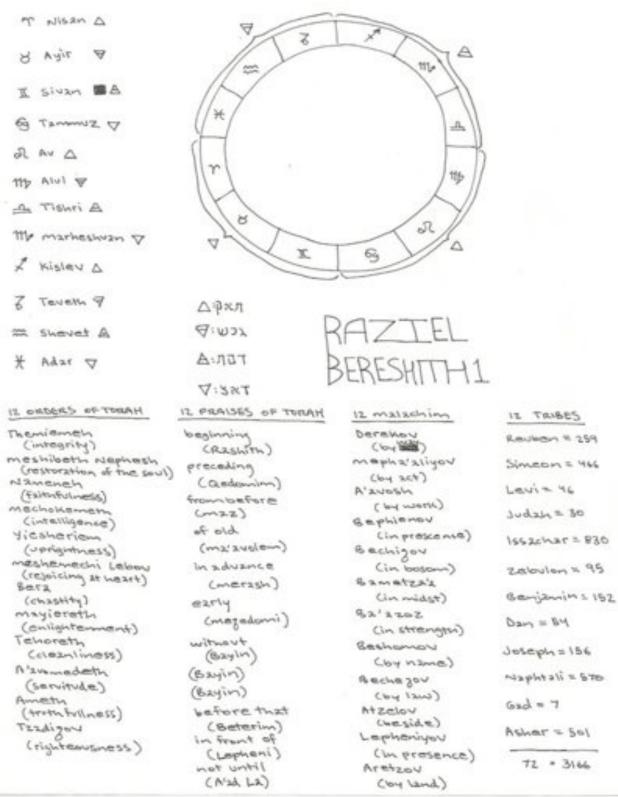


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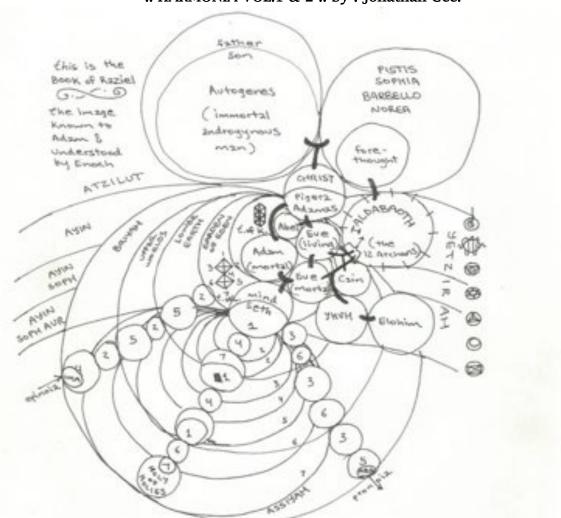
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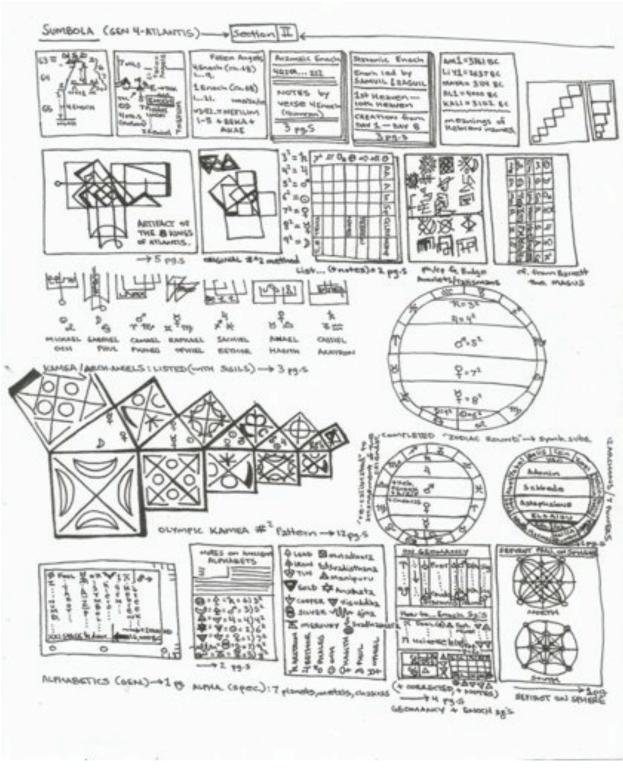
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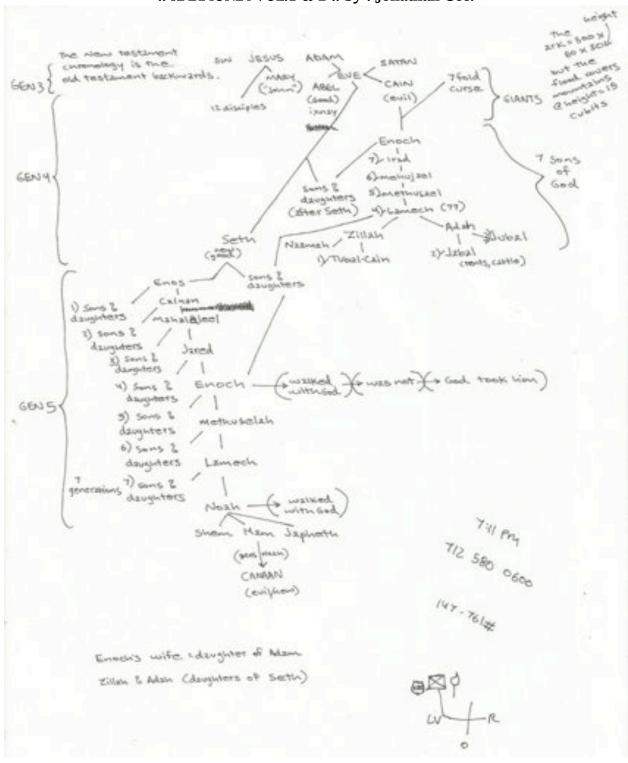
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I am Raziel, the H.G.A. of Adam. Just so there are angels ministering over each part of the body, so am I the guardian angel, head of them all, over the whole creation of that form, I brought the engals of the being together. It was on the direct command of God. After the expulsion from paradide, I went to Adam with the book of my perspective, and each of the angels remaining in his ours who had not respected against thin and begun out warms would fucir hands to trunctibing its different parts in detail. These became the Books of Enoch, who saided from them to the predestral libraries of the descendents of Seth. You see, because Adam sinced He "Wid" from God, so we fell, & so paid panance, and two made prayer in a river of the texts of Eternal Eve, where I drew Him out at the despair to offer my perspective. But soon the tides of the seas thousand because of Eternal Evers crying before the expulsion. The "zeone" described as malacular, in Adam's prayer, the listed as superior to himself, but they were only the archers, the messenger ingers, of the fillen rejects of his own outs. He was innecent of seeing that we were evocenient to Him, and in so doing caused some of us to rebet. I, myself, have stready come to be ested snemly seen, Aziel, Sztan, etc. Hewever to the Setulans I appeared as the questly image of Adam resourced, they caused me "the truth," the needal, Jeshuah, but not Raziel.

" Section : 2.66N4-ATLANTIS !! ~ Including: (pg. sums astumated) ~ 2A Biblical Genealogy (Gen 3-5) 28, monas on Apoetyphia of Enoch incl (Emispien) 1 Enach -+ 1 pg. seekille correpations, FAULERS AMERIS per climens -1 pq. (Aramaic) 4 Enech - 3 pgs (Stavic) 2 Enoch -->3495 After 1 155 / sive-sention > 8 1935 26 NOTES ON VARIOUS CALENDRICAL "STARY DATES" ("STIENG "YEST BEING") th keys to sequence of # sy empansion 28. Attilizat of 8 Kings of ATLANTISTS 195 28. Original (Pythagorean Spiral) method di Arranging "Kamez" (# 59.5) 26. Planetary Habrew Hames & Traits+2 pg.5 24. Komez Mtributes: Co. Budge, Amrichs & Tallomans) 11. Accompliantitutes: (cf. Barrett, Magua) EJ. Kamer & Archangels: LISTED (with SIGILS) - > 3 pg.s 2K, completed "zodiac acoud" - symbol substitutions 26. OFFIRE KAMER #2 PAHEM -+ 12 PSS 24 MINABETICS: (ECHERAL) - 1 P3 (MINE) 8 875 (SPECIFIC) T Flances, SERVENT ON SMEAR - 125 24" ZABIAL Sound" (+ Hebrew, Greek, Symbolic & Chakess) + (se-salvasted sel. to the "Allanteen Calendar")+1 pg. 4" EMPLAC BOWNS" (IZ ARCHONS / I POWERS) +2 PSS







The "Faller Angels" from the BOOK OF ENOCH 4 Enoch (Aramaic) :: Fragment : Ca. 68 1. Assed: weaponry (motallingy); antimony (jourdary, makeup). 2. Semyara: enchantments, etmogens, magick, shamanism. 3. Armoras: reversing homes, blocking fascinations, anti-magick. 4. Baragiyal: astrology; the "moving heavens" (planets, meon, sun) in rediac. 5. Kokabel: "constellations," the "fixed howers," primum mobile. 6 Ezeguel: meteorology, clouds; weather, climate, impact, cycles. 7 Aragiel: "Signs" (sumbola) of earth; NAM (destiny) = orbit, cotation, procession. 8. Shambiel: Sumbola of the Sun, rel. earth, rel. mw (+ galactic Core B-H), etc. 9. Sariel: sumbola (warn) of the moon; rather of exact size, distance; craters. 1 Enoch (Ethiopian) :: Cu. 68 U.3 Lesdons & 100's V.2] 1. Shemyaza II. Armers \$ 50'5 2. Arstikapha 12. Batariel (Bataryal) 8 04 10'5" 3. Armen 13. Basasael 4.4.-18] 1. Yekun ("the celoel") - led fallen, taught y Kawabel (www.ad)Ananel (w) them to considered. 2. Keszbel - privated out exil to angels in 5. Turiel (Turel) 15. Turiel (Turyal) their making of manistinal. 6. Rumiel (amjel) Simapiscal (16) 3.62drel-discovered every stroke of death 7. baniel (banyal) Yetarel (11) to the children of men." 8 Kzel Seduced Eve; invented weaponty. 18. Tumzel 9. Barakel 4. Penamue - discovered to men bitter & sweet; M. Tarel trught to understand warming on a table. 10.Azzzel to Rumal 5. Kasyade - discovered every winded stroke of u. Azzzyel sparits to of domons;" 2. the stroke of the embryo in the wenter to divinish it; (ne innes to attack a feetus Cyrresolm # swem & betwee "stroke of the spirit by the bite of the screent! "Tabaset" (Att. "male" or "strong") - "the stroke which is given in the mid-day by the offspring of the scripent." V.19. This is the # of the Kesebel; the principal part of the oath which the most High, dwelling in glory, revealed to the Hely ones, V.20] Its name is BEKA. He spoke to holy michael..." contempting to improve the "Fallon" back by the Name of God) 122] me "ozth of AKae" by me "instrumentality of hely michael"

Aramaic Enoug

- 40 201 : 1) Enoch sunounces his oracle regarding runishment of the watchers by the Mely ones for a future generation squees with
- emissin Enoch 2) description curies) of seasons (in everyteens) compared to the eniftiness of sinners (+14-6; 214-6:0)
- :20-8;10:54; 3) they swere an oath on my Hermon : chief Shenikazah; then Artezof, Rant'el, Kokabel, - 'el, Rahaicl, Daniel, Zegiel, Baragiel, Assid, Hormani, material, Ananial, Satolal, Shanshiel, Sahariel, (54-14:01 Tumbel, Turiel, You'vel, Yehads'el: the chiefs of chiefs of tens - married women, taught them sorcery, by them begat
 - giants 8,000 cubits tall
 - u) sheminara: incantations & new to cut roots Harmoni : now to under magic Barages: the signs of the shafts Kokabiel: the signs of the stars snamstel; he signs of the son Zegral: the signs of lightning Artegofitue signs of earth Sahari'el : the signs of the moon

michael, Sariel, Raphael, Gabriel hears the deaths of the sons of earth

- 5) orders to save the son of Lamech orders to Raphael to bind Azzel & worl him into darkness
- 6) the represent coverant

4 a zoz: & watchers, sons of sky, take outs to marry women

(519-619+617-81) Shemilhaza protests saying he will bear their blame 8:2-4:4; 10:8-15; - Anan'el (13), Sato'el (14), Shamahi'el (15), Shahari'el (16), Tumi'el(17), Turiel (18), Yomiel (19), Yenzdi'el (20) 14.4.6)

- same account reglants - Assial: manufacture of swords & broastfiates, mining of gold Emorking, empossing of silver; women - braceletts, eye-shadow, jewiery & antimony, precious stones, dyes ...
- 3) same teachings; Earlagiel teaches signs of "rays" same some of Heaven entrest God's vengeance
- 4) God exacts weath upon me watchers & Grant's by "war of attrition"
- s) message is delivered to the watchers

- 40204: 1) weight impermanent actions of sinfolman against parmanent earts, heavens, everyreens and the seasons.
 - 2) cutef sneminazan, Arregof (1), Ramaier (3), Kertabier (4), ier (6), Ramaier (6),
 Danier (7), Zegier (8), Baragier (4), Asaier (10), Hermoni (11), Matraier (12),
 Ananier (13), Satoier (14), Shamehiler (15), Saharier (16), Tumiler (17), Turrier (18),
 Yomiler (14), Yerradier (20): chiefs of ten
 - 3) enterminate the watchers' wastards & make the plant of sighteenswess appear
 - 4) goes to Dan shu of mt. Hermon: waterers request not to be granted house of histories 2 beyond it house of fire
 - 5) thronotop = szphire, pillars of bottomiess fire & beyond them aboyes
 - 6) to the E saw trees of mastic, circumon, restrict of tears and galbonum & comparable to aimend peel
 - 7) to the 5 three doors for south-wind, dew 3 raing to the Ethree doors ...
 - B) parable of the lamb
 - a) a wife for methosalah & their child albino
 - 10) man to be runished for Yared's sins, Noah to be saved
- 40205: 1) place of punishment; we ever-burning the, Ra'u'el says directs luminaries
 - 2) centre of exists mountain, river fouring to SE E & w = mountains; mountains & 3 ravines total
 - 3) parable of the white bull
 - 4) parable of the lamb
 - SI parable of the ram & the eve
- 40206: 1) the walls imprisoning the dammed & one who waits to heaven
 - 2) to the E= trees of incense and myrrh; mountains to the NE= nard, mastic, cardonom & papper; further E= med sea & beyond the Paradise of Justice
 - 3) frequents re. Eden & of Uriel's Instructions
 - 4) Frequents re. the flood
 - 5) fragments re. and of fixed & parable of white bull
 - 6) Fragment's re. water & parable of flock

- 40207: 1) fragments re. falling stars & re. parable of black bulls
 40212: 1) testimony on the wisdom of Enoch
 - 2) testimeny of Enoch Chorn Hudzy of 1st week, the 2nd week will follow ...)
 - 3) plant of justice increases wisdom sevenfold in the 7th week;

 8th week = week of justice 8 building of the temple of kingship of the

 Great one; 9th week = 211 men will see the just eternal path;

 lothweek, day 1= new heaven 7x as bright; many weeks of justice

 4) who among men can...? etc. etc. etc.

```
Enoch is led by Samuil and Raquil
ist newen : sez greater than earth
                                                    Slavonic Enoch
2nd Heaven: the fallen angels
                                                     or 2 Enoch
3rd Meaven : PARADISE OF EDEN 3 tree of life
            (in the worth iterrible place, simers tortured)
Hen Heaven : sun and meen
         sm forst: six gates
             west: six gates Comentions 365 14)
                    28 year circuit
             "exit: 12 gates (1= 31; 2= 31; 3= 30; 4= 30; 5= 31; 6= 31;
                            7=30; 8=31; 9=31; 10=30; 11=31; 12=28)
             west: 12 gates (same & days per gate)
                   lunar year & 354
                   great eyeles $32 years
                   mentions leap years
                    Thold course in 19 years (!)
 5m Heaven: Grigori (angelic soldiers larger than glants)
              followed Satanail
              went to Ermon, bred with wives of men
               Enoch saves the Grigori
 6th Heaven: archangels over angels, phoenixes & cherubin
  The Heaven: aroungels, dominions, orders, cheromoun, scraphine,
               loanit stations of light
             - 10th Heaven = God, In Hebrew Aravat"
                             "ruler of Sabaeth"
              -enter Gabriel
  8th Heaven: "in Hebrew, muzzloth" changer of seasons
                twelve signs of zodize above 7th heaven
  9th Heaven: "in Hebrew Kuchavim"
                 "homes of the twelve signs"
   10th Heaven: "Arayoth", archistratege michael
                 archangel Pravuil Cacribe, zuick, massenger)
                 60 days & nights, Enech writes 366 books
                Gabriel sits to God's Left
 the creation: visible Adoll comes down from invisible dark
              1) light from Adoil is above even bod's throne
               2) hard Archas, red, creates the lower
                  becomes a foundation
```

ZEnoch

the creation: 3) the first day = the deep waters like glass 2 the seven stars

- 4) dry chaos = earth midst of earth = bottomless abyes day of "first-created" (Sunday)
- 5) second day (monday) lightning created the "orders of the ten troops of angels" from newn "rock" of "Great Fire" resellion of one order & expulsion
- 6) third day (tresday)
 "planted Paradise" & "created Renewal"
- 7) fourth day (wednesday) stars

 1st upper circle: Kruno

 2nd: Aphrodit

 3rd: Aris _____ omits 4th (possibly "lower")

 5th: Zeus ?)?

 6th: Ermis

 7th lesser: the moon

 10wer: sun 2 moon 8 stars
- 8) 5th day (Thursday) birds & fishes, 4 legged "creeping things"
- a) 6th day (Friday)

 "commanded my wisdom" to create man from

 "a consistencies": 1) flesh from Earth; 2) blood from

 dew; 3) eyes from sun; 4) bones from stone; 6) brain

 comenters)

 from cloud; 6) veins 8 halt from grass; 7) soul from wind
- (brezen of God)

 (brezen of God)

 God taught good & evil; only in Paradise 51/2 hours (!)

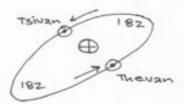
 Eva (Adam's last word before skeep: "mother")
- 11) saturail makes setona, another world seduces Eva in form of thought against Adam

ZEnoch

12) 8th day: "revolve in the form of the 7,000 & that 2t the beginning of the 8,000 there should be 2 time of not-counting, endless, with neither years nor mentus nor weeks nor days nor hours."

· Enoch returns for 30 days

: the concourse of the sun -



7777777 - the ever-being (true name of God)

AM 1 = 3761 BCE

Chinese Yin Yang Li Year I = 2637 BCE

Mayan 13.0.0.0.0 = 13 Aug 3114 BCE (Greg)

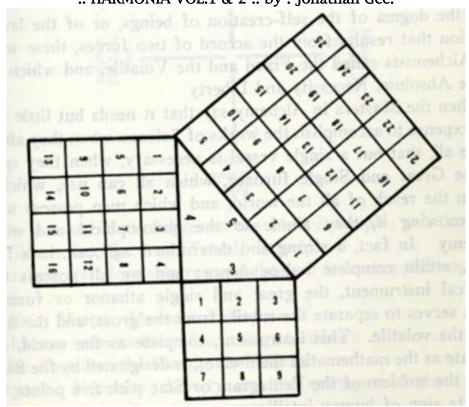
AL1 = 4000 BCE

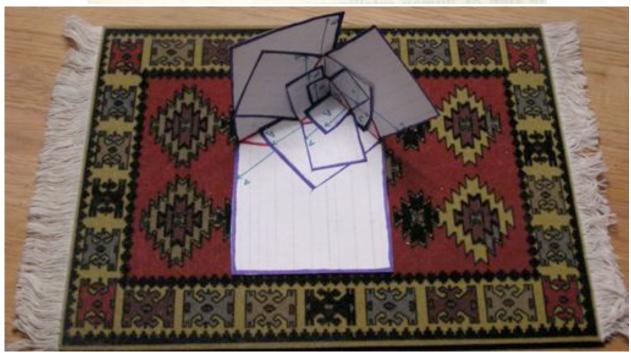
KALL-4069 = 3102 BCE (12545 432,0004)

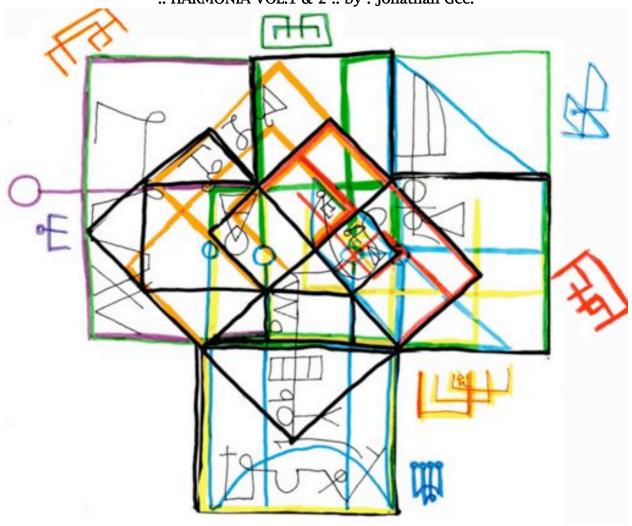
Some notes on Kamea & the Olympic dignituries

write I do find it a likely belief that the Kamea of the Karbalistic planetary traits ettributed to Assian & yetzirah preceded the Greek olympian dignitaries' sigils, it is still more likely that these arose contemporarily to one another, while the names of the Kabbalistic traits no doubt do indeed trace back to Solomon, or even originated in Egypt, is really beside the point of from whence the sigils therefore, themselves, date, and I wesleve that these came much later. The practise of sigil crafting, apart entirely from the earlier usage of genatria and tallemans - which date back furmer still - I believe the Kabbalistic traits arose contemporarily to the olympic dignities, springing at the same time from different sources, and that the sigils for both 2150 occured contemporarily - but at a much later date, and 2150 from Separate sources. Thus, we see that, while genatria and talismans influenced both the Kaubalistic traits and the Olympic dignitaries, the traits and dignitaries wear no commonstitles between mem basides similar influence, and so remain autonomous for perhaps as much as a milleunia, at least several centuries, until, in the late dark ages just before the Renaissance, they again are resurrected by a common influence - that of sigil-crafting - but, again, this occurs for each individually of the other, and, while thenceforth they wear in common the traits of generia, talismans and sigils, they are, from inception, unique in national character: the expensionic traits remain summertically Mebrew, while the Olympic dignitaries are genuinely Greek in character, Now, this leaves one potential commonality as of yet unexplored; meir application to number squares, or kames, while the kames for the Kabballette traits is given, the sigil shape of the oxympic dignitaries strongly suggests a similar derivation. However, upon inspection I can assert with confidence that the given Kamez for the traits is applicable also to the sigils of the dignitaries. needless to say I find this matter verting. Since the olympic Kamez is not given, but the Mabbalistic is, it is extremely tempting to want to apply the one to the other, and thus arrive at a sigilum generia for the dignitaries. Yet this tactic lacks the support of historical context, as already stated, since the similar vestments appear to have evolved entirely submomously. Thus, I cannot say for certain that the known general Cimplied only by the base-7 factorial olympic provinces) is intended as such or even, indeed, relevant at all. Myway, the good news in all of this is that both the Hames and sigils bo relate in some way to the seven planets. Thus more is some scientific link to the orbital (rotational #1).

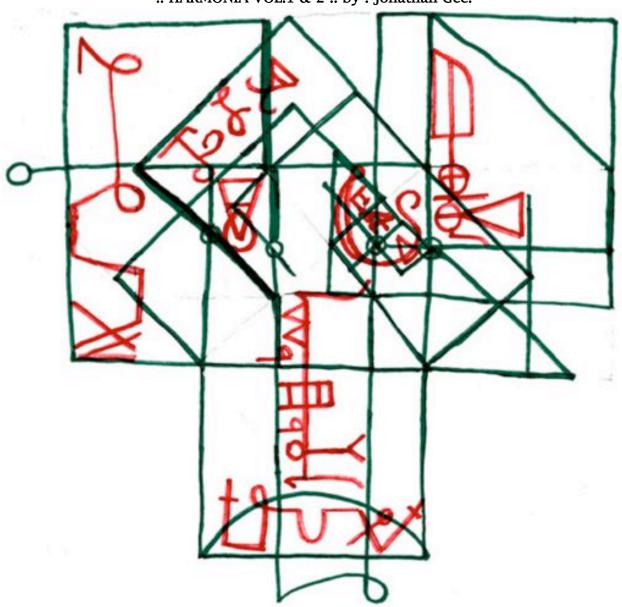
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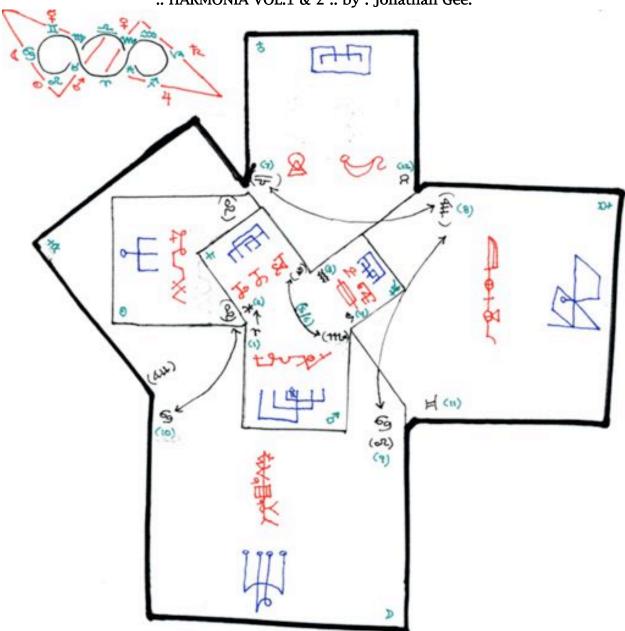




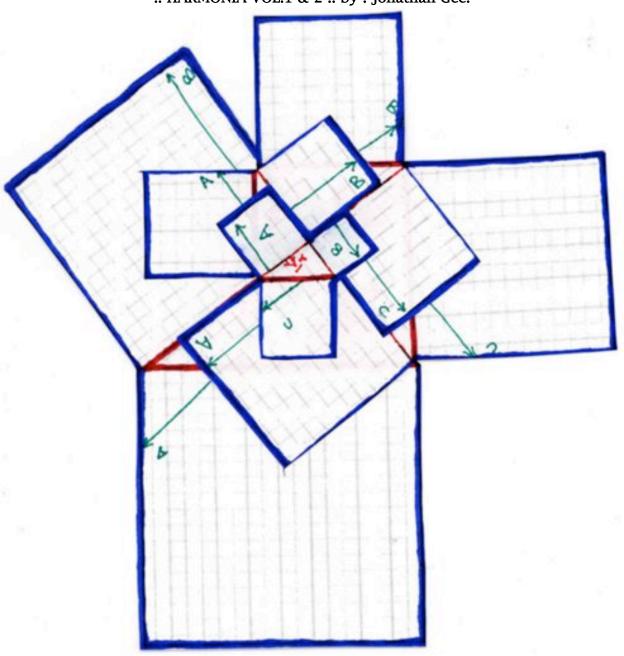
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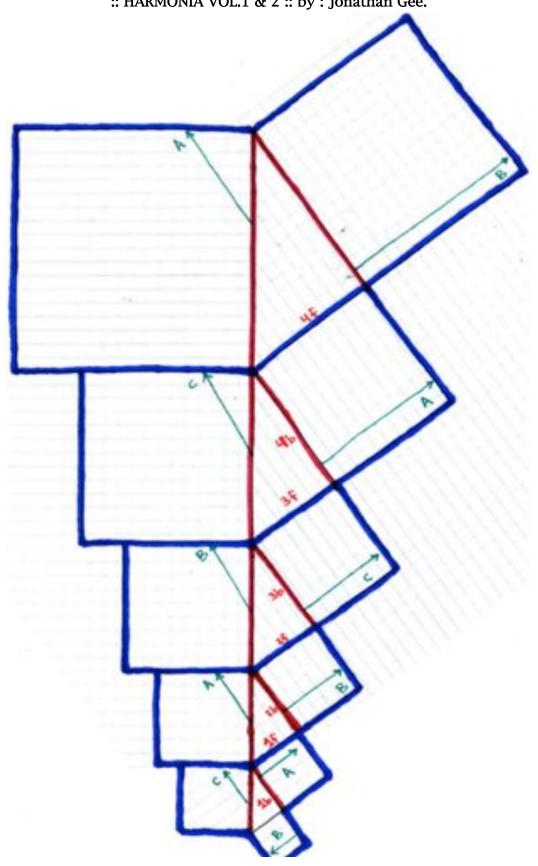
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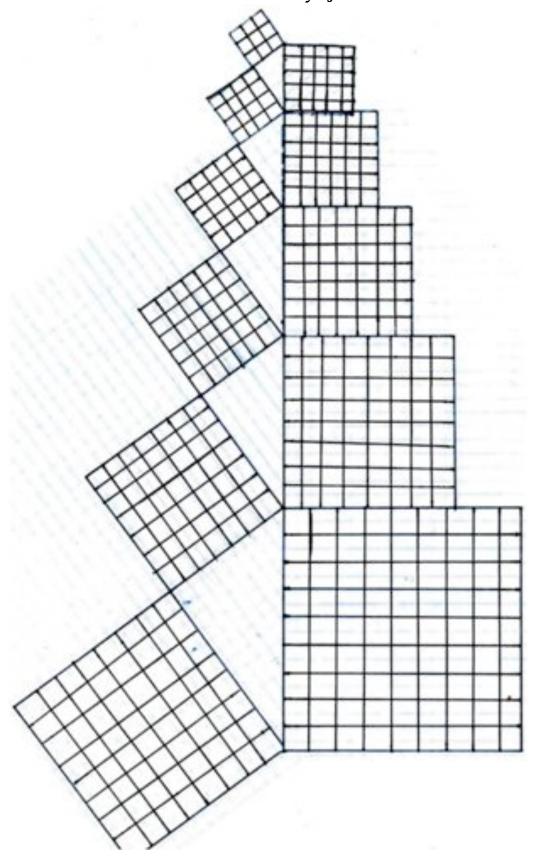


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KABBALISTIC NAMES AND SIGNS

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The Moon has two demons, one subordinate to the other.

The Magical Squares of the seven astrological planets are :-

I. THE KAME'A of LEAD (SATURN, 3 columns).

4	9	2	٦	0	2
3	5	7	2	п	1
8	1	6	n	26	,

Whether these nine figures are added up vertically, horizontally or diagonally the total is 15, or in Hebrew letters YH, the shortened form of the Tetragrammaton YHWH. The total of the three columns of figures is 45, i.s. the expanded Tetragrammaton YWD - HA - WAW - HA. When used as an amulet this square was cut upon a sheet of lead.

 THE KAMÉ'A of SILVER OF TIN (JUPITER, 4 columns).

Ľ	F.	1	15	14	4
1		12	6	7	9
1		8	10	11	5
١.		13	3	2	16

een figures are added or diagonally the total

א יה

Whether these sixteen figures are added up vertically, horizontally or diagonally the total is 34, or in Hebrew letters D = 4 and L = 30. These letters form a part of the Hebrew word for tin. The total of the four columns of figures is 136, which is the number of the Spirit and of the Demon of the

planet. When cut upon coral this square protected the wearer from sorcery.

3. THE K'AMÉ'A of IRON (MARS, 5 columns).

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

M.	כר	t	2	1
٦	2,	כה	n	10
r	n	X.	CN	0
•	77*	N	7"	23
22	1	5	2	יח

Whether these five times five figures are added up vertically, horizontally or diagonally the total is 65, or in Hebrew letters A=x, D=4, N=50 and Y=10, i.e. 'Adőnáv a name of God. The total of the five columns of figures is 325, which is the number of the Spirit and Demon of the planet.

4. THE KAME'A of GOLD (the SUN, 6 columns).

6	32	3	34	35	1
7	11	27	28	8	30
24	14	16	1.5	23	19
13	20	22	21	17	18
25	29	10	9	26	12
36	5	33	4	2	31

1	לב	1 2	לר	לתן	N
T	Nº	12	כח	n	>
D.	4,	70	TI.	23	73
Ti*	2	20	CM	Le	7,
כח	כט		8	13	2
15	n	15	7	13	לא

The total of the addition of these thirty-six figures in each of the three directions is III, i.e. the total of the numerical values of the consonants zāhāb paz, "refined gold." The total of the six columns of figures is 666, which is the number of Sôrārii, the

396

AMULETS AND TALISMANS

Demon of the Sun. A variant square of the Sun as a lion is given by Dr. Bischoff (op. cit. p. 146).

5. THE KAME'A of COPPER (VENUS, 7 columns).

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	21	7	25	43	19	37
38	14	32	1	26	44	30
21	39	8	33	2	27	45
46	15	40	9	34	3	28

-	-				_	
	CII					
	72					
	1					
	לא					
	יר					
	לט					
110	210	D	0	לדן	3	כח

The total of the addition of these forty-nine figures in each of the three directions is 175, i.e. the total of the numerical values of the consonants in sodh-Mny, "secret council of the goddess Meny" (Venus). The total of the seven columns of figures is 1,225, which is the number of the Spirit or Intelligence of the planet.

 THE Ķāmž'A of QUICKSILVER (MERCURY, 8 columns).

8	58	59	5	4	62	63	1
49	1.5	14	52	53	11	10	56
41	23	22	44	45	19	18	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	47	46	20	21	43	42	24
9	55	54	12	13	51	50	16
64	2	3	61	60	6	7	57

п	נת	100	1 7	17	120	140	I M
20	ית	7'	23	133	Nº	1	13
CIM	13	20	מר	מה	100	Tre	מחו
לב	לד	25	(0)	כת	לחו	55	כת
D	13	13	115	15	15	N'S	1
1,	tro	10	2	כא	120	20	73
D	נה	נד	3,	12"	N3	3	10
וסר	3	3	ND	D	1	1	13

KABBALISTIC NAMES AND SIGNS

The total of the addition of these sixty-four figures in each of the three directions is 260, i.e., the total of the numerical values of the consonants in bokab kessf bayyim, "star of living silver" (i.e. quicksilver). The number of Tirker, the spirit of the planet, is 260. The total of the eight columns of figures is 2,080, which is the number of the demon of the planet, TAPHITHÄRTHARATH. As an amulet the square should be written upon a sheet of tin or quicksilver.

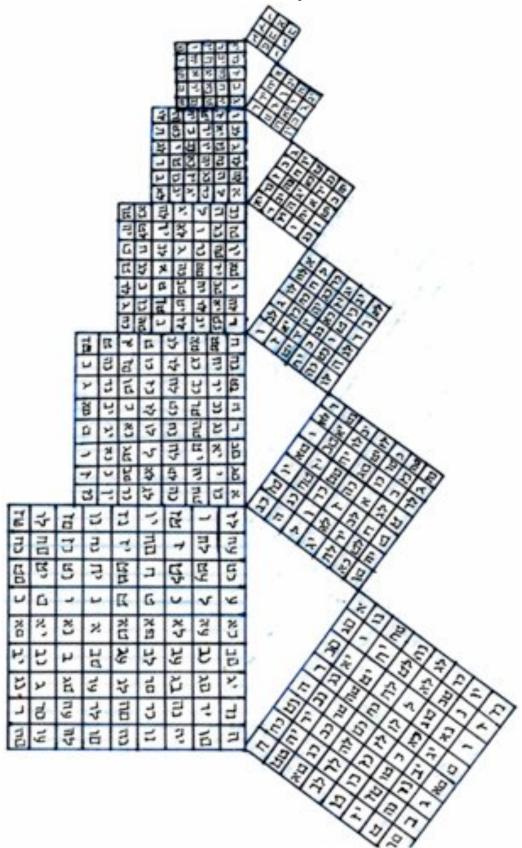
7. THE KAME'A OF SILVER (the MOON, 9 columns).

37	78	29	20	21	62	13	54	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25
26	58	18	50	1	42	14	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

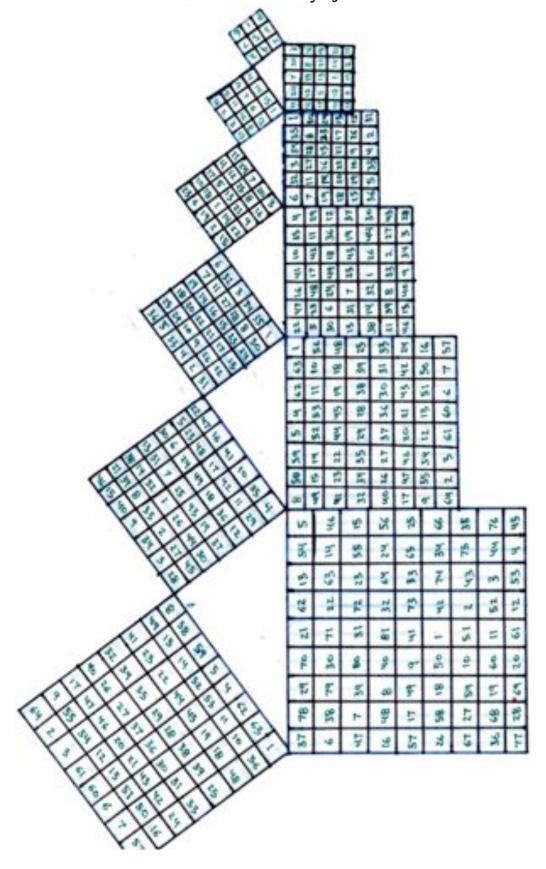
15	mp	וכט	Ų	KO	20	20	173	l n
1	לח	טט	5	NU	23	1D	77	מון
m	1	לט	9	לא	עכו	22	נחן	יח
90	מת	п	b	GW.	125	סר	כר	13
13	19	ממ	0	ND	1.20	25	no	כה
13	נח	יח	3	N	מבי	ער	175	10
10	13	נט	,	CM	13	10	קה	לח
15	סח	10	D	Nº	1 22	1	מר	110
TU	כת	DD	2	ND	21	13	7	מתו

Note the arithmetical progression in the central diagonal column of each square.

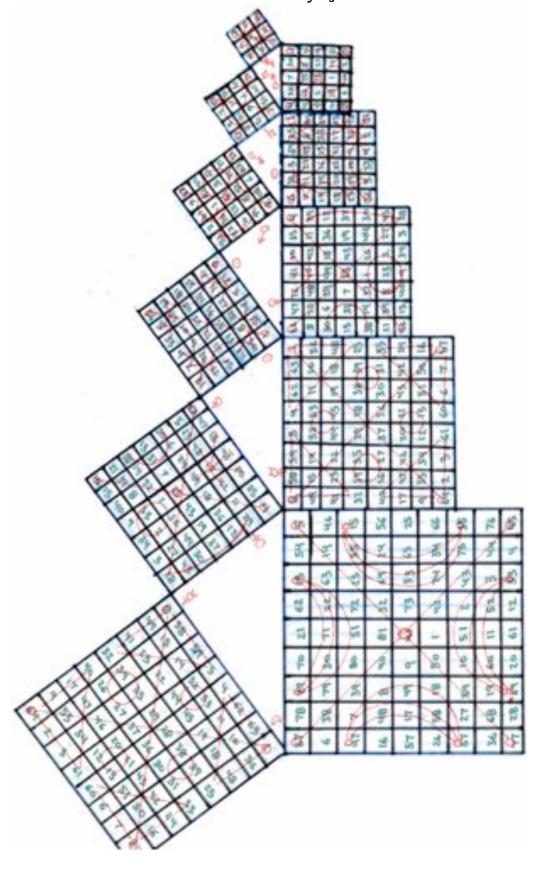
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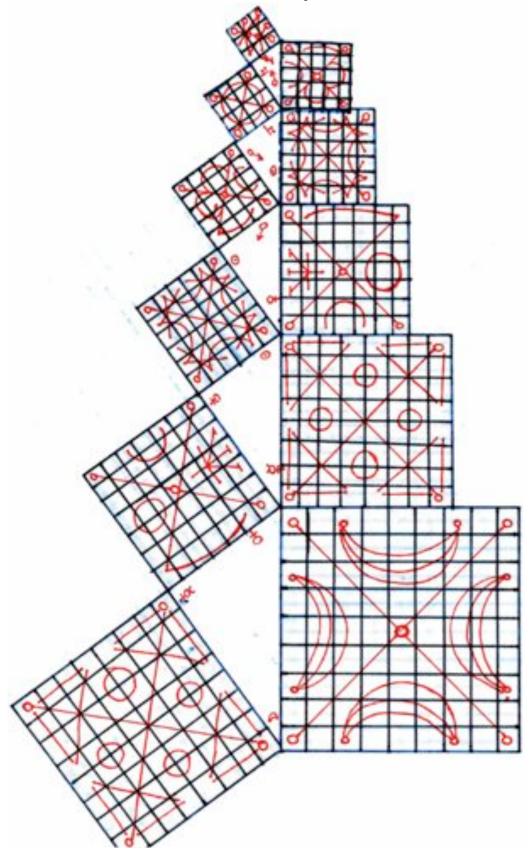
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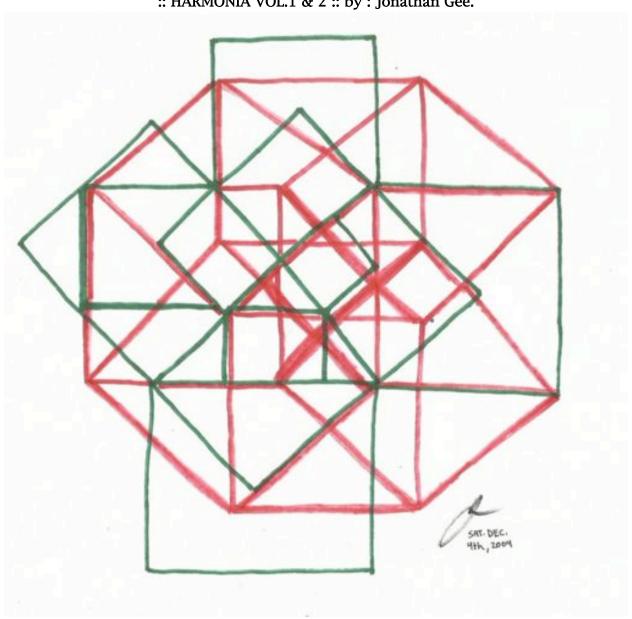
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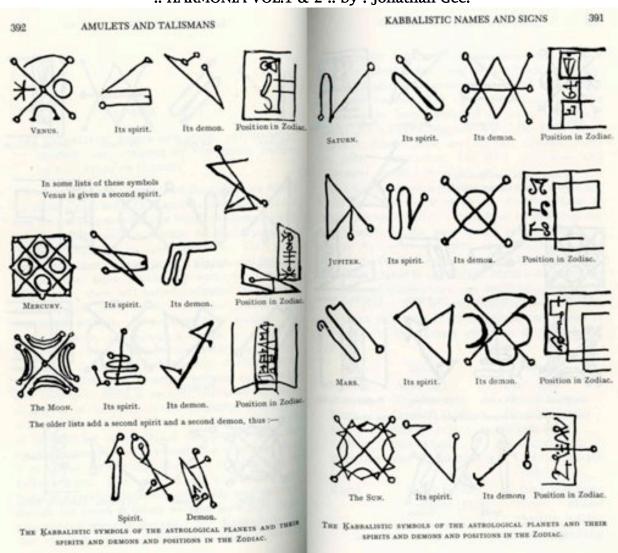


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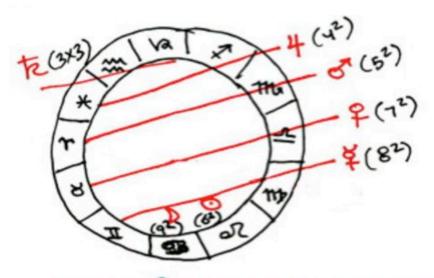
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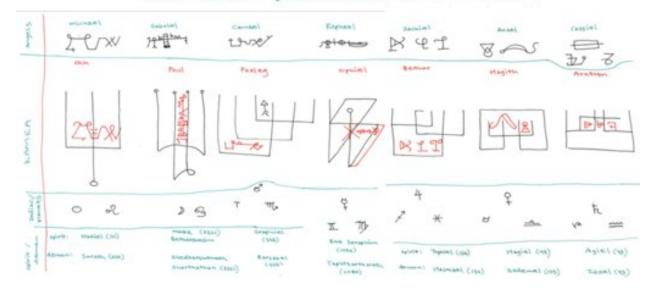


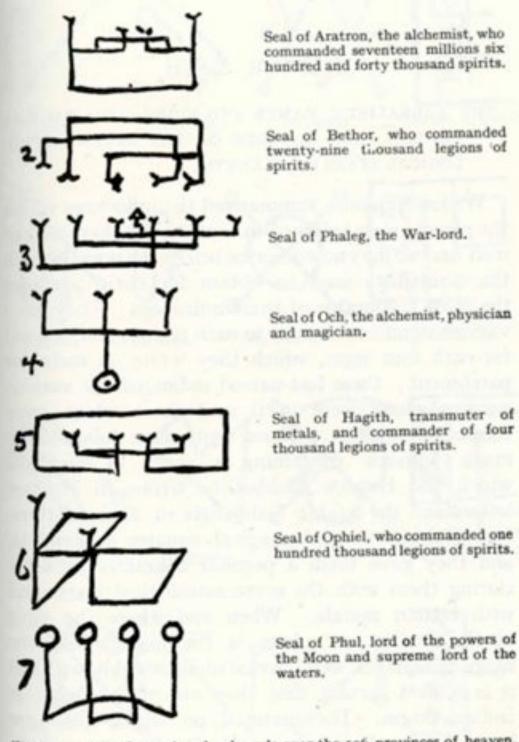
A Tuble showing the names of the Angels governing the 7 days of the week with their Sigila Planets Signs &c.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Salundar
Michaiel	Gabriel	Camael Raphael		Sachiel	Ana'el	Caffiel
L.C.X	184841	town	rde ld	BLI	8	129
0 a) @	r om	ஈ ஆ	1 30	8 <u>5</u>	7 ·
Machen.	name of the 1. House	name of the 5 Hapon		Zebul.	Sagun.	. 15 - tagelo water



Hebrew from numbers guare Seal from Remutation of " Spirit/demon = Hebrew Sigils angels = names of zodiac Kamez = positions in zodiac

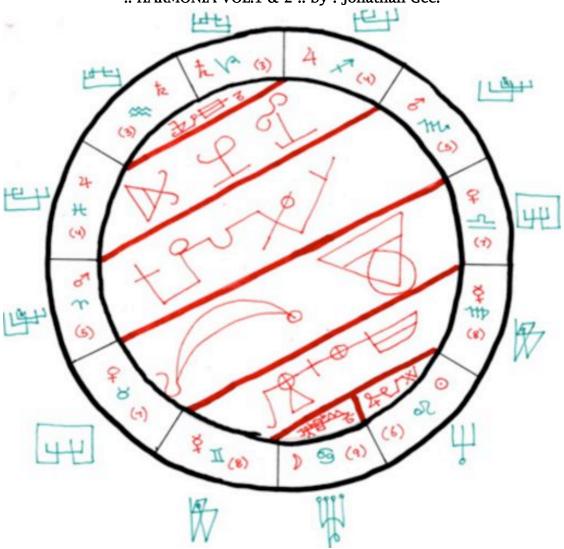




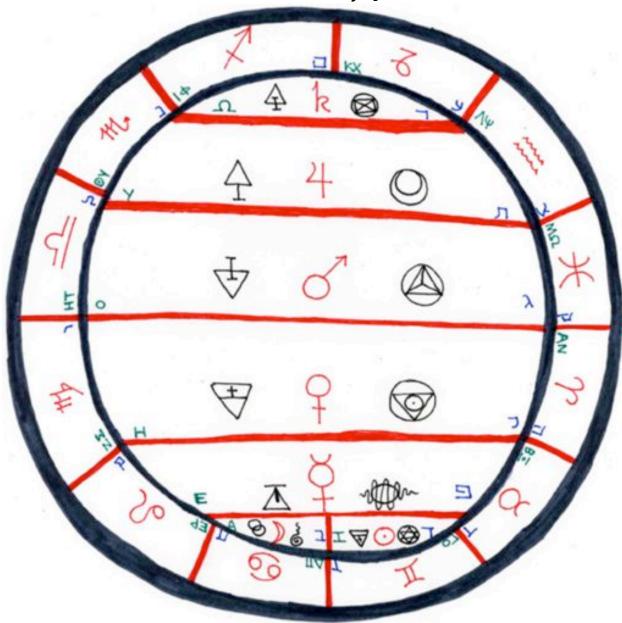
The Seals of the Seven Angels who rule over the 196 provinces of heaven.

(From the collection of ancient magical books published in the German edition of Cornelius Agrippa's works, IVth edition, vol. v., Berlin, 1921, p. 111 f.)

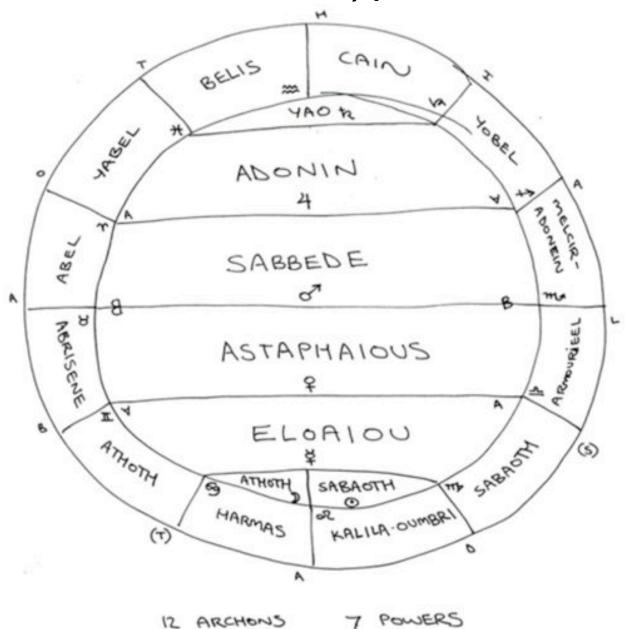
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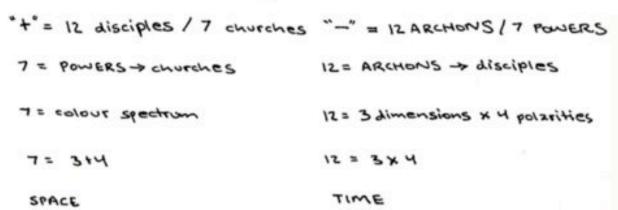


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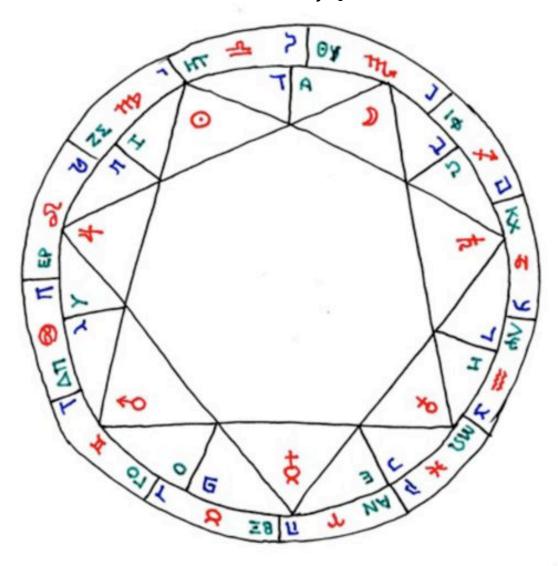


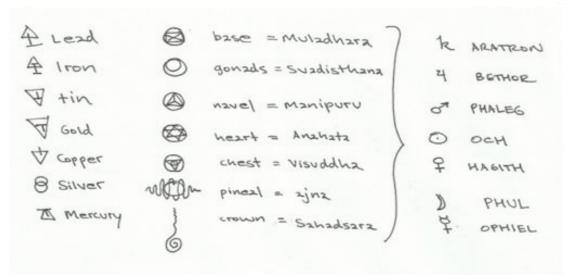
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when the ; promotion (father of neuron)

mariant argumate whending

THE THE SECOND CONTRACTOR OF THE SECOND CONTRA

it is more likely that the reformed order of the Henrew steph-best is based on These than the atner way ground.

while the Habiro language derived in its earliest form from the prote- Presentation egistres which sixewice gave with to Greek, Funk-Pheenician, and early Aramak, and were

themselves derived from Sinaltic/Canamite, from Egyptian coptic and originally from Sumero - Akkedlen, the language of Auranam; modern Hebrew evolved from the Judeoviddion short-wand which in turn evolved autonomously from old Hebraro via Hebenic Aramaic, while medern Hebrew borrows its generaliz from the Pythagoroan theory eriginary developed among the preemistan lingual family, ha characters were created when simeon ben mude reformed, the alephonet during the early-mid rensissance,

This reformed steph-bat astabilished the modern crowning of letters, as well as codifying the seven final doubles already in short-hand use as early as 1200. If was there, from Sepheralc septements, that the story surrounding the reformed exepulat began to be securibed with the Helorens commentary on the doctrinal Terain known as ha gabala. The first work of he zebela, the Banir or Humination, dates to the 1st contury, and itself being probably first written in Aramaic makes few references to the Hebreni cipher methods of generalis, notorizon and terrorain. I seek horts, writing in the 13th contury, however, resummed interest in he goests by commenting upon the sefer tetritish, or week of termation, which became the primary feers of interest in the transition parted barmeen smalle neurons and sequerate medern; it giving me history of an the entant letters as attributed to the welling of Abraham. This work, already steeped in medieval cipiers by this time irregardless of the potential authenticity of its origin, thus became the imperos for the cadification of the sepheralic modern order of the steph-bet, so would be set down by ben linds. However forther commentaries as late as a century later continued to put forth alternative erders and All remain faltural to the text of the formation.

in the meantime, ancient theorem, save for agreement upon the few letters con ed on specifically in the basis and thus preserved through the transition period of medieval explores, had become all but totally lost and became regarded as the language of the angelo." one fatheriar medicual cryptographer, sonn bee, and arrowers of ariting in the 1800's, used this mythology surrounding sinsific to draught his own "language of the angels" bee attributed his alphabetic cipherto Enech, though this contention is dublous, it is likely bee, writing up a magical system for engand's Queen victoria as she brokered the Anglican colormationist split from Harian catalogism, was id only have been familiar, if at all, with the Stavenic version of Enoun, which was likely a Gypsy and Gothic Fabrication based upon the book of severation.

Types

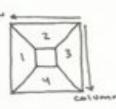
	STABLEL	tame?	F	+€	OFE	か	4	0,	0	+0	+04	0
	1542	(Attitution)	-			(114) KECLNG paylinders	CSSI) Kidind	(N) CRNG	(942) CELING	לאיזארן (פו)	(wi) ALC NG	(ME) LENG
	12.88	(Brish)				(196) E(3, N.C.)	Cont DULKS	(85) 451 パピ	(ma) TELLING	(18) XIXC	(אני) דפועל	(w) B,CKC
-	4802	(Hetzkezh)				(Sk.) XX (AR)	(96) (UE,NG	(515) TLNE, K &	THE THE PROPERTY STREET	(44) LY(224	לפער) טינריאל	(44) 171 (12
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8	÷.	\$	Amissia	A	9	kedenel asmodal	60
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11/2	ï.	8	Puella	A	Ř	Taphthar tharath Zuniel	
~	Ť	9	Conjunctio	4	9	Kedenel Hamaliel	00
THE	÷	8	Rubeus	A	8	Bartzabel Barchiel	~
2	:	\$	Aczvisitio	Δ	24	Hismzel advachiel	4
VA	::	0	Carcer	A	k	Zazel Hannel	8
**	:	日	Tristitia	A	k	Zazel	0
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V	:	7	cauda praconis	Δ	h on	zzzel & Bartzabel	00

atter	Diret	Symbol	germancy	row	column	4 3
%	Fool (e)	Δ.	Fort. Minor	9	4	distribution.
コ	magician (1)	벟	-	-	-	\$2000
7	High Priestess (1)	D	-	-	-	400000
٦	Empress (3)	9	2	-	-	Great-Cross
П	Emperer (4)	T	Puer	Δ	Δ	1) Zediac, small care
1	Hieropeant (5)	8	Annissa	∇	V	1) SHIT (6)
7	Lovers (4)	X	Albus	4	4	3) Dekan
n	Charlet (7)	69	Secretary.			4) Element of Table
D	Strength (#)	ર	Populus Fort.major	Δ	A	Calvary Cross
(5)	Adjustment (8) Hermit (9)	119 (==)	(Puella)	ACD		1) Element of Table
7			Conjunctio	Δ	4	1) Spirit (1)
7	wheel (fortune)			-	-	3) Element of lesse
3(4)	Sustice (4)	(%)	Fuella (Fort.maj.)	D (4)	Acar	M) Sal Sagaira
11	Hanged man (12)	Δ	VIX	A	A	lester of Tet.
1	beath(15)	THE	Rubeus	V	V	Taret card
	Temperance(IN	1	Acquisitio	A	Δ	Kerubic Syveres
	Devil (15)	N	Carcer	Δ	Q	Taret card of
Ŋ			100	-	- 1	2) Exempet of Table
-ZI	Tema # (16)	8	_			3) column ele.
7.	5tar (11)	222	Tristitla	A	4	4) 500-610
B	weev (18)	9t	LARTINA	Δ	A	Servient Square
٦	Sun (4)	0	-	-	-	Csvb-eles)
W	Judgment (20)	Δ	Canda Dras.	₩.	Δ	1) ENTWINDIAC 2) COLUMN
л	Universe (11)	ta (v)	capet brac.	4	4	3) sub-exe 8 ger.
etc	A 4 00	A E V		200		Tablet of Union 1) Column 2) 3) raw
ek	etc.	BN		m		4) 66-
1	_ \a	AB	1 10			02

7 D= gravity	CORRECTED ROWS & COLUMS
7 D= werk	FOR ENOCHIAN SQUARES
Enorte = P Fi	

etter	Taret	Symbol	geomantic	70~	Column
1	Fool	A	Fort, minor	M A	Δ
7	magician	\$	-	-	-
X	High Priestess	D	-	-	-
7	Empress	우	-	-	-
П	Emporar	r	Puer	A	W A
1	Hierophant	8	Amisso	Α	A
T	Lovers	X	Albus	Δ	Δ
H	chariot	69	Papulus	A	Δ
D	strangth/ Adjustment	21	Fort.major/	A/	D/
7	Harmit	117	conjunctio	Δ	A
7	wheel	4	-		-
5	Justice/	\$1	Puella! Fort. maj.	4	4
	Hanged Man	A	Miz	A	
1	Dezta	117/0	Rubeus	A	A
D	Temperance	1	Acquisitio	Δ	44
4	Devil	13	Carcer	A	A
5	Tower	0	-	-	-
	Star	***	Tristitia	\triangle	Δ
P	Moon	*	Lzetitiz	Δ	Α
7	Sun	0	~	-	-
W	Judgmant	Δ	Cauda Des		∇
Л	Universe	#(c4)	Capot Dra	4. 4	A



In this chart the symbols come from the neurew letters, & represent the longest standing traditional apprinctions, & therefore the ends that freezely need to be changed the most they seem to be derived by an alignment of seven planets in THE ZODIAL KNOWN as the "grand cross" This could not have happened before/ After at teast 2000 wars age - ween years ago, because it is creical, & happened again in 1999 Conty differently than described by the Hebrew letter symbol attributions) white crowdey took one right step in rectifying this in the taret (the double attributions) it falls far short of the complete reordering of symbols & rewriting of Taret required For ak. why in F.m. at the top & Fort, maj. In the middle! why only totate it & qualita around conjunction may net daugle con junctio & flip fim. with quer ? wany ignit conjunctio the wheel of fortune? who is twee two gravilles? 1

70

some commentary on the findings: (notes on revisions)

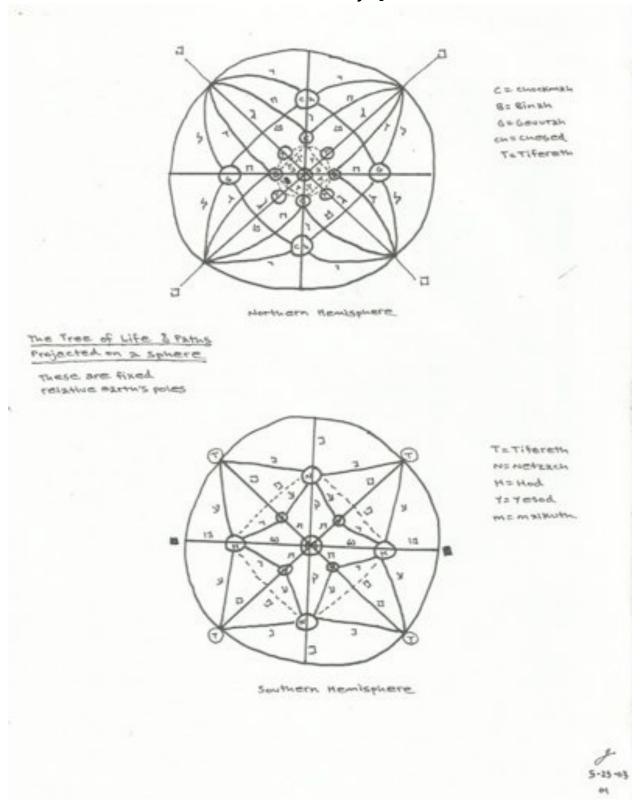
It can be put forth, that what is done or found out in some or any wind of altered state of consciousness, be it the sportable and sportaneous state of spiritual eastagy or communish with the lord or any of his angelic hosts, or by white magic, intentionally contacting therewish by an act of the will and correspondence of attention the act for such purpose, by black magic, in the same manner for eventing the purely animist instincts, or be it by the use of any other such method or substance that constitutoth the radiant rainbour of experiences between those matternity polarities, cannot rightly be added, anneaded or in some or any way set aright while not in such an shrilar state, whether by the same suffer by others, for my sake in my present endeaver, in as much as I am solver, I can only supplicate a suspension therefore of such an belief by my or any other existant higher power

For I shall now ennounerate these changes I should like to make, and here is the source and substance of my calling to do 30: It has been revealed by that art of archaeology that the apostyphal writings of the Hebrew prophet Enech were known of at the Escene community of countain Called Danaseus), and thereat worked by the man years a Ben Padiah, known as Joseph Ben Padera, or son of the Lion. Subsequently to this a system professing itself, and having some correspondence to, the same Enechan system was worked, with much expansion, by one John bee. In the father day the Lion system was morked, with me enemy Rome, by the latter day with England, the enemy of Rome, and also with the enemy Rome, b in the latter day with England, the enemy of resurrection of Joseph Christ, which had fucied the coverdes, now my name is Jonathan, who was king to a generation of prophets, as well as a false messiah in the manabeau family, as well remembered by the Roman Catholics, and, by presession, my analazal sign is Loo, the Lion of Fortifiede. Such is the tradition of the past incarnations of my outward identity, my being-for-others, that is the nated and turke accursed baset of my exposition in most shed to be one with the father thereof, which are the All, of my ego which I must shed to be one with the father thereof, which are the All.

And so I say thus; all that which the Golden Dawn attributed to the system of Juna one shall be revised, down to their first attribution, which is the hyperwood to each of the letters in the four watcutowers, but this shall be the same, this zione. Further the letters of these watchtemers shall all be changed, down to the very last one of than so upon the Black Cross, but should the letters of the 91 names of earthly angels not be enanged, for, as by Enech's sight, they are the fallen, these who are beneath the eyes of the seven watchers bound into the three exverns without water, yet that a portion by seven and six should be saved for the sake of that cavern where the water of life flowery, to mese sevens of the planets shall be multiplied by the angels over the twelve tribes to the seventh time, but should the last seven remain under the planets and by the sight of the watchers, New to these dignitaries and ministers should be gluen in the hypercubes the sub-sign of a planet ceither in the order of bee or otherwise) which shall replace those feelish apprientions by the star of the banon, and thus to rectify not this fall as was ascribed on the tree of Knowledge Chy the sin of Adam), but to glarify his rising bezuty as the right average of 4 and 6. So, also, should the signs be as to linea spiritus sancti 26 2 precession, therewith 2/ready worked out, New where these do not double it Should be as either the number eyet not the element) of the safirot or of the area or some continuente as 2-9), such; MANEY

Lether

noti) of



: Section : 3, GENS-10 + NT ::

~ Including: (pg. sums estimated) ~

3A, mores on BUBLICAL GENERLOGY (GENTIO) -> 2 pages

38, HOTES on 7 Medition & 12 Tribes (else)

3C. HOTES on IZ HT Aposties

30. (12 Zuliac + 7 Planets + 10 Service)

+(II Helb. Tribes; II GK. Apostles; 10 Help. Selfiret)

3E. 12 Zodiac + 12 Heb. Tribus + 12 GW. Apostles

+ (12 Z, Neb, Greek letters)+ (7P, Neb, GH letters)+2 pg.s

+ (II tribes, Aposties, Archans, Association T Americal & Medium, Churches, Powers)

3F. from: PISTIS SOPHIA (pg. 1 "6000"; pg. 2 = "EVIL")

36. the "Angel Scroll" of Yeshwah Ben Padiah + 8 pgs

+ wores on "REVELATIONS" (NT) -> 2 +9.5

34. "AFTER THE FALL" (cl. "Zener Cosmology)

+ 5/5/2000 -> 13/11/2012 (cf. similarities 40)

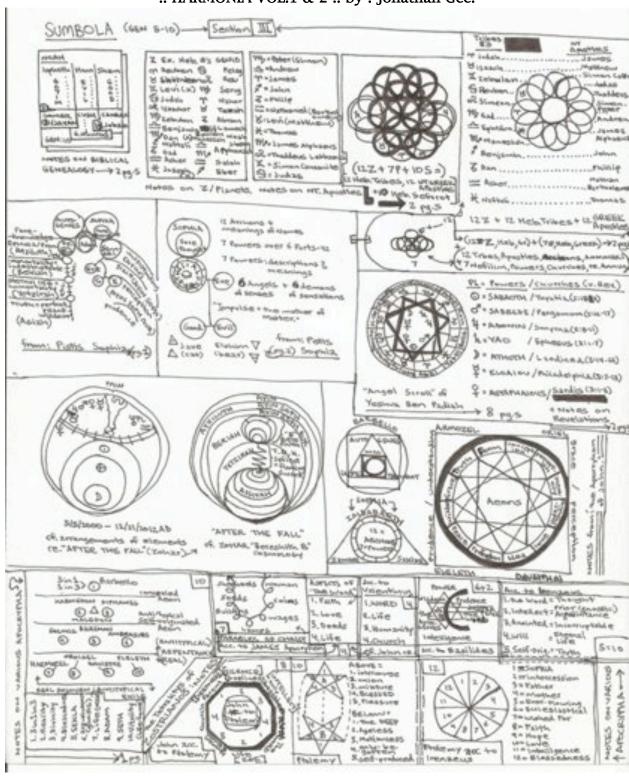
SI NOTES on The Apocryphon of John " no.

SOPILLA / AUTOGENES ; BARRELLO / AGCHANS; AGONS

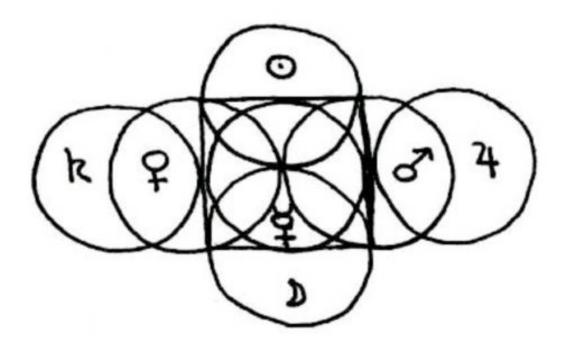
+ HOTES ON VARIOUS APOCRYPHA: Conch.

Zosteranos; John 10. Haleny; Pholemy re. Ironamis; Procemy, JAMES; Valorimus;

Basillides; Ironaeus.)







$$1 = Seth = Nzzmzh = R$$

$$2 = Enos = Tubzl - Czin = 0$$

$$3 = Czinzn = Jubzl = P$$

$$4 = Mzhzlzleel = Jzbzl = P$$

$$5 = Jzred = Methuszel = 0$$

$$6 = Enoch = Mehujzel = D$$

$$7 = Methuszleh = 1rzd = P$$

		EXODUS		NUMBERS		Gen:10
2	r	Reuben	דאובנ	Judzh	T	Peleg)
	8	Simeon	STYLL	1552Ch25	я	Reu Su
	I	Levi	לוי	Zebulen	π	Serve
	_			Reuben	9	Nahor Au
E	69	Judzh	יהודה	Simoon	S	Terah)
	S	Issachar	יטט כר	Gad	THY	(4(SI) METERA
	117	Zebulun	זבול ב	Ephrzim	<u>.n.</u>	Lamech (1) Wi
5	~	Benjamin	בבישיב	Manassah	THE	Neal
	Mr.	Dan	フェ	Benjamin	2	Shem)
	1	Naftali	נבתקי	ban	3	Arphaxad (Sp
w	3	Gad	て入	0 4 71		The second secon
	2000	Asher	702	Asher	***	Salah
	*	Joseph	יושם	N29+21i	光	Eber }

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The Peter = Simon-Peter

= Andrew = Andrew

= James = James

= John = John

= Philip = Philip

= Nathanzel = Bartholomen

= Levi (matthew) = Matthew

= Thomas = Thomas

+ Thomas = Thomas

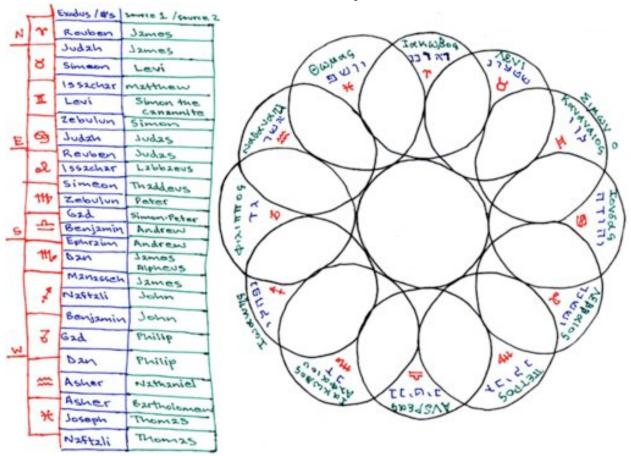
+ James (son of Alpheus) = James

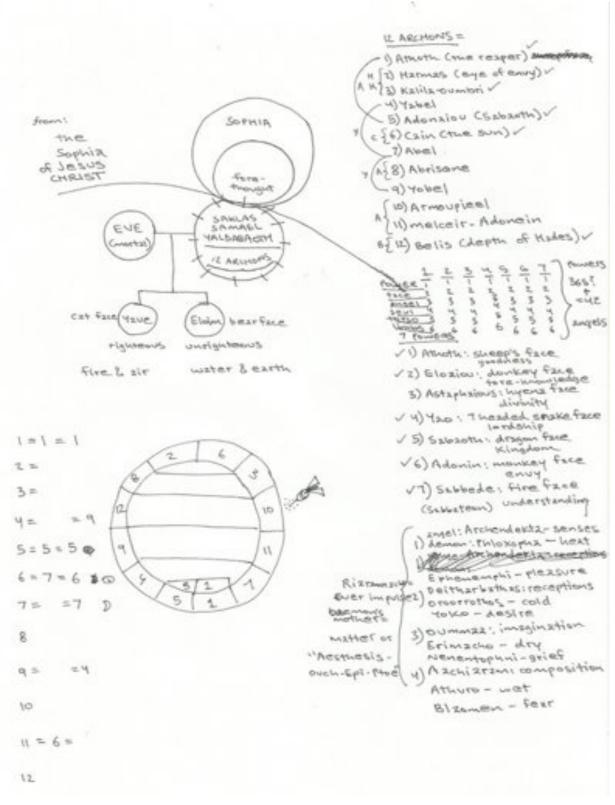
= Labozeus (Thaddeus) = Thaddeus

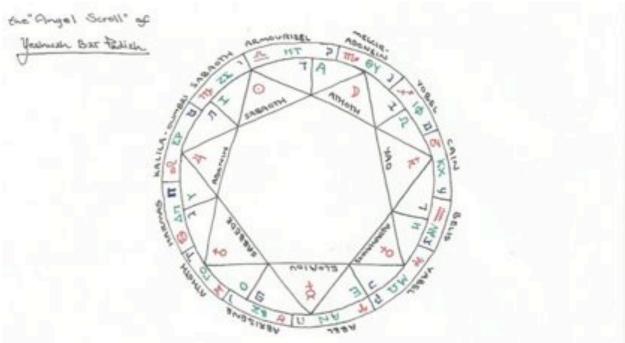
= Simon the cananite = Simon

Judas = Judas

= Judas = Judas
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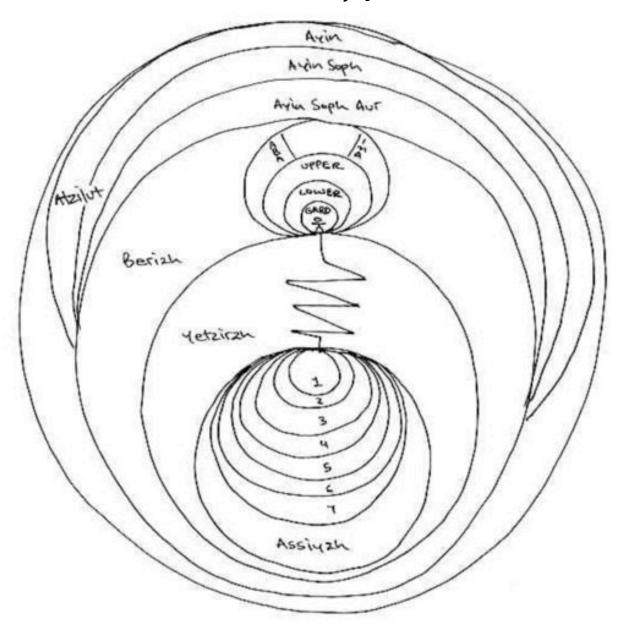




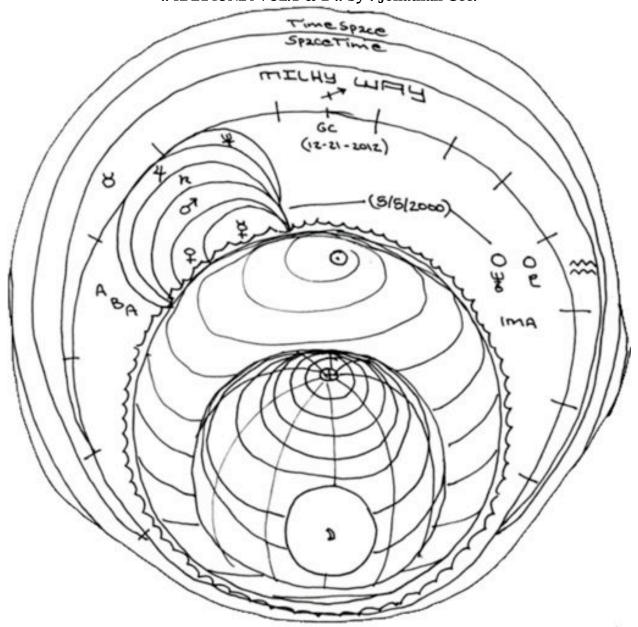


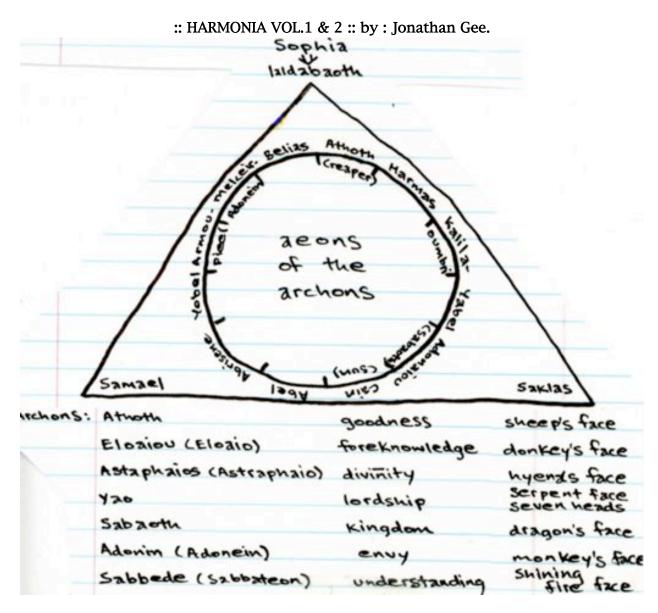
There are layers to think wearer to the self things change rapidly. Purther away events take langer. Consciousness (Adam's angel, Charameon, Sall 25, etc.) of tachyons has caused the evolution of BNA (the body of Adam, the forbidden fruit) 25 2 vessel. The emmiservient but caused some of its own power to turn against itself and created bacterial life-forms and viral life-forms this, the remaining sentions DNA contrives as furgiven by the "entirety" of the universe, Allah, because it had created the dustities of the universe, though not seeming to restise the universe it's fallen consciousness perceives is not, in fact, me true "entirity" of thee, Allah. Each of what Adam called "seems" were exiled the faller "archone" by the Grootics. These "archore" of the Growtics are called "Acone" today. These are the wason year long "ages" or "erzs" described 2100 by the mayons and crowley, of the own's "precession" throu the constellations of the radiac, itself the result of earth's axis' 28.5° angle of Inclination from the acciptic alignment between earth's equator and the sun's Adam was created, as you ordered, Sir, nowever by having created Adam (omnisentient ONA) and Eve from Mine (me divided viral and bacterial duality), the spirit of the universal consciousness, the mental multiverse of tachquic thou experienced as emotions and caused by time (currently future + past dominant), essumed itself to be indeed the sun known of the universe, and declared itself the "entirety" of thee, Allah, How "Khab - Allah" is the "body" of Elehim and "AHONNYAY" THE BEEL Shew; yed-name, of the mind. DHA soumes the mentle of annibentiant consciousness, realising the "indestructuality" of tachyons as I can then the own twice-Passen state of consciousness, yet not realising even now to serve think "entirity" best by escaping the dutches of the fallen transport the dutches of the fallen transport to the entropic imbalance caused by the transport resonance of comic admissions are transported in the inferior of comic admissions are transported in rebel.

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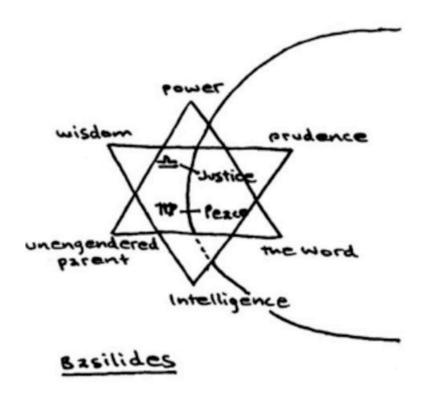
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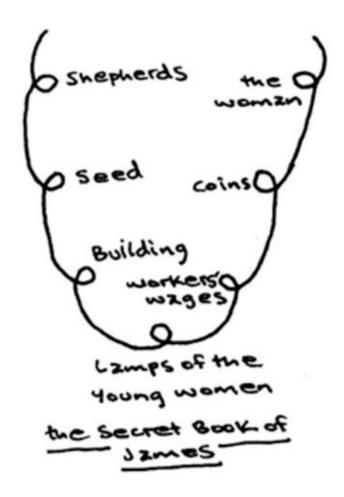


from: <u>Hypostasis of the Archons</u>

http://www.gnosis.org/naghamm/hypostas.html

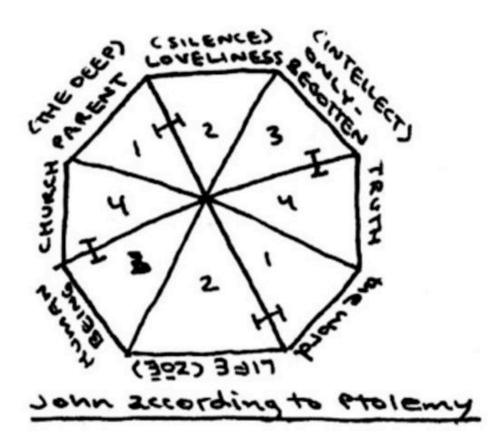


from: <u>Basilides letter to the Egyptians</u>
http://www.gnosis.org/library/basilide.htm



from: the book of James

http://www.gnosis.org/naghamm/jam.html



from: John according to Ptolemy
http://www.gnosis.org/library/ptl.htm

1. the word 1. faith

2. life

3. humanity 3. good works

4. church

4. life

4. will

5. self-orig. * Truth

WORK ZALWER. to James

5. humanity

5. humanity

3. nobsted = hought

2. life

4. will

5. self-orig. * Truth

WORK ZALWER. to James

5. humanity

5. humanity

3. nobsted = hought

2. life

3. nobsted = hought

4. will

5. self-orig. * Truth

4. humanity

5. self-orig. * Truth

6. humanity

6. humanity

6. humanity

6. humanity

7. humanity

8. humanity

9. humanity

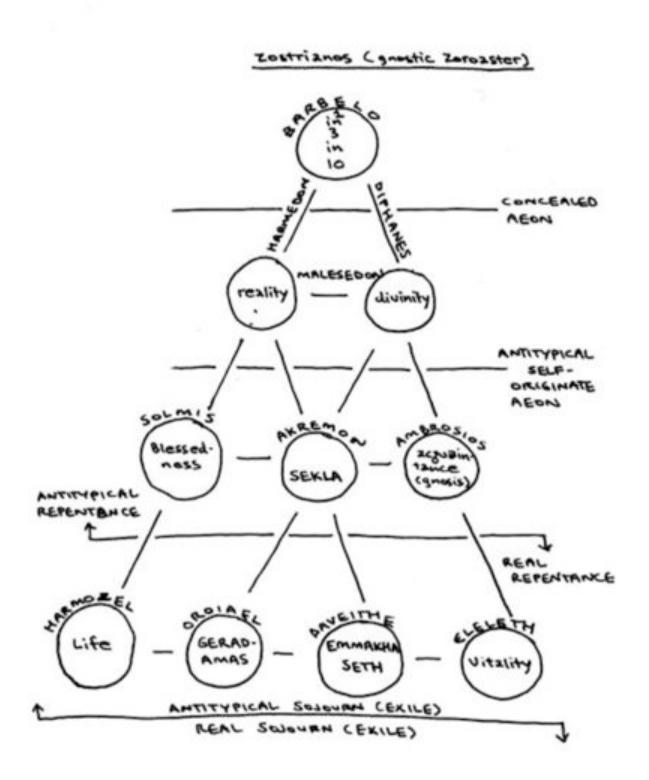
Valentinus, James, and St. Ireneaus



from: Ptolemy
http://www.gnosis.org/library/flora.htm

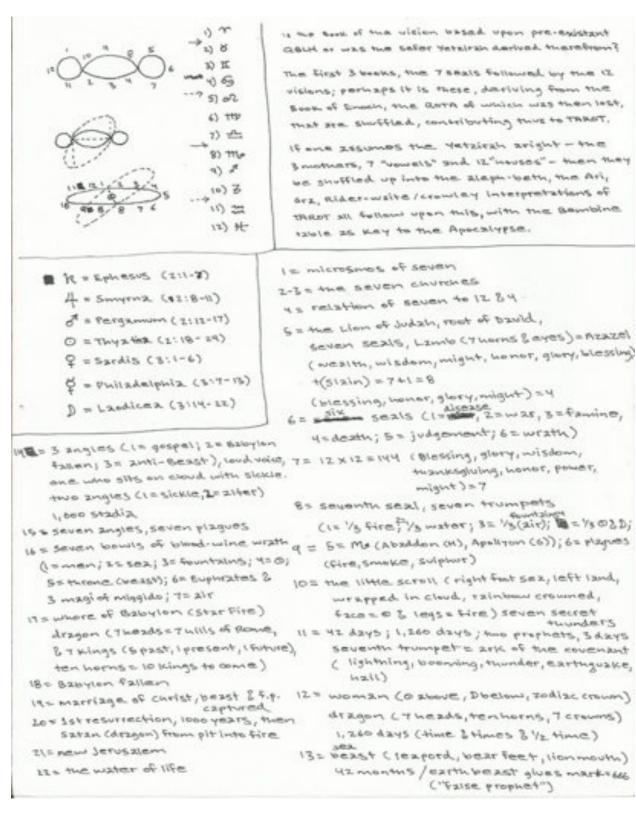


from: Ptolemy according to Ireneaus
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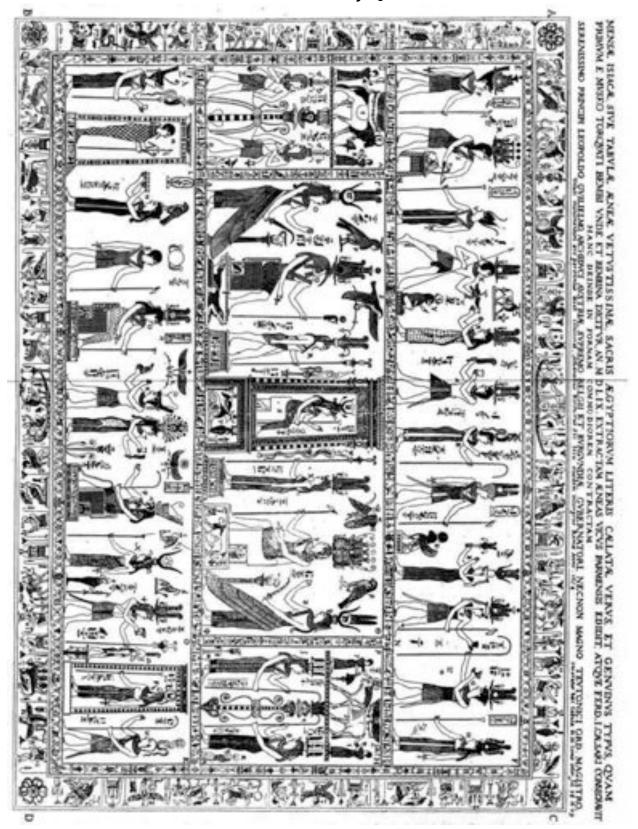


from: Zostrianos, the "gnostic Zoroaster"

http://www.gnosis.org/naghamm/zostr.html



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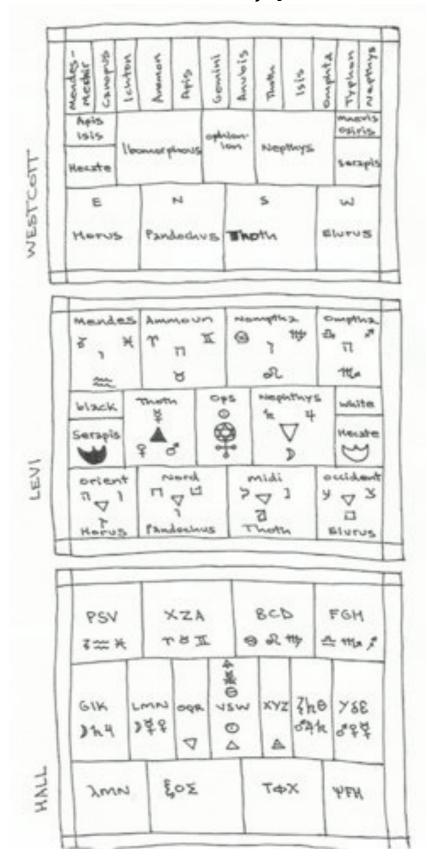
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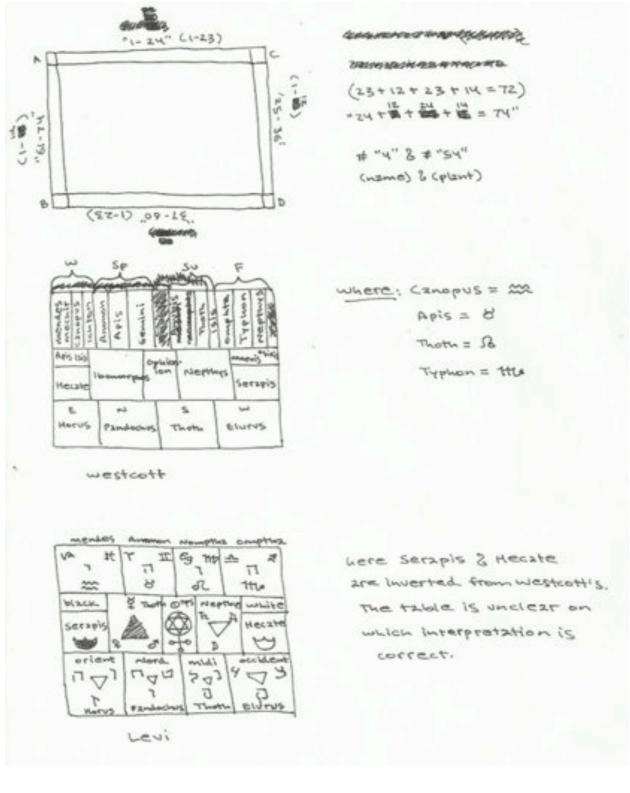
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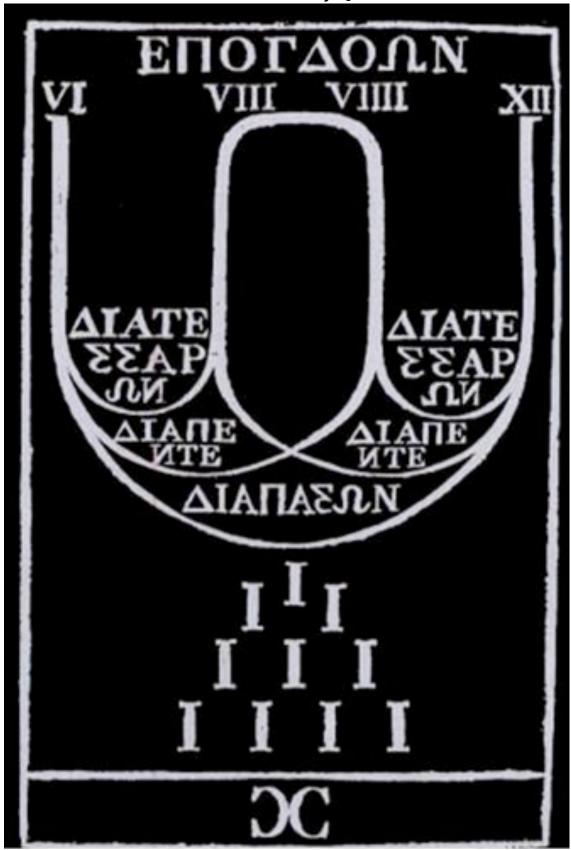


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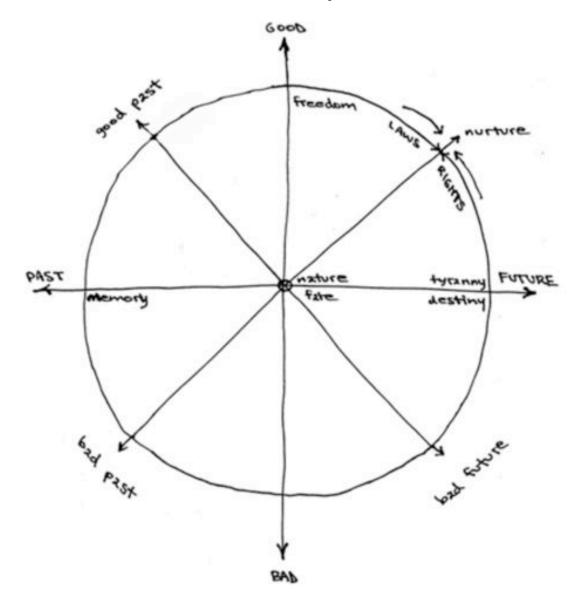


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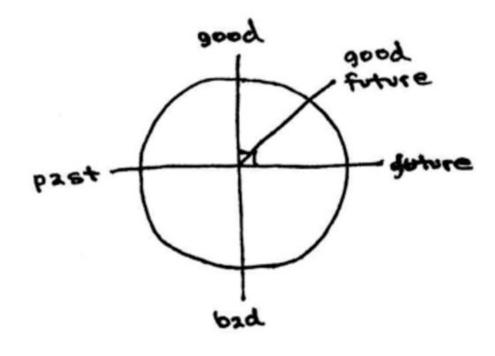
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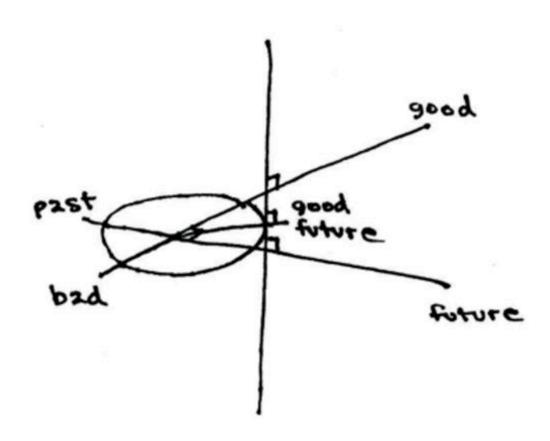


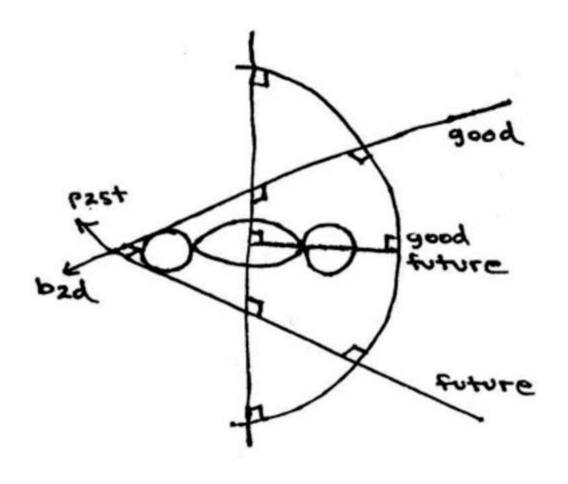
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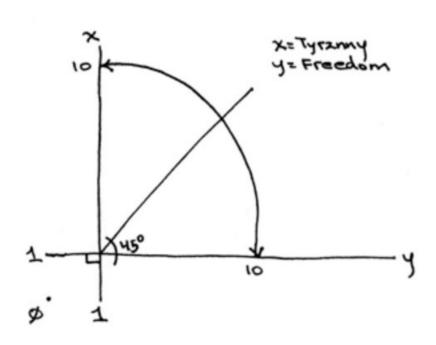


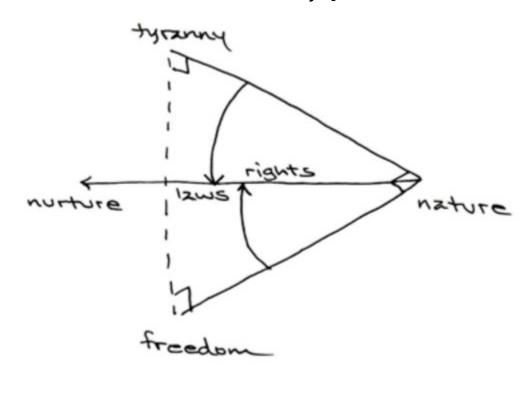
In this overtly simple model, we see "good" and "bad" oriented as polar opposites from one another along the vertical axis, and along the horizontal axis measuring from left (past) to right (future) we measure the passage of time. As a person's "Karma" may be either "good" or "bad" in the immediate (and more distant) past, and also either "good" or "bad" in the immediate (and more distant) future, so one's central self-concept, or "ego" is merely propelled along a sinusoidal wave bouncing between "good" above and "bad" below upon the moving surface of time. Thus, if we wish to prevail against entropy and gravity causing us a "downward" shift in "Karma" towards a "bad future," we must assail the upper-right most quadrant of this assemblage, where the Absolute "good" of "Freedom" and the Absolute certainty of "Destiny" delineate the no-man's land of tyrannical laws between them. In the next few diagrams, we will look at how the underlying geometry of this diagram may be changed, and thus the relative positioning of location of one assigned attribute on the model also be changed into a different location and relative meaning on an altered geometric version of this model. However, regardless of how one bends this model, it IS the True and Only "Law" that applies to us all universally.

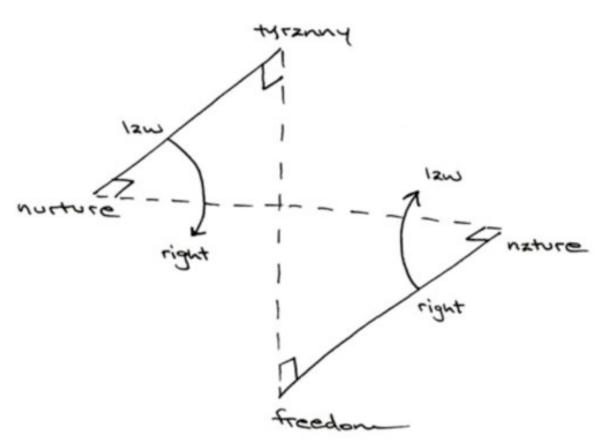


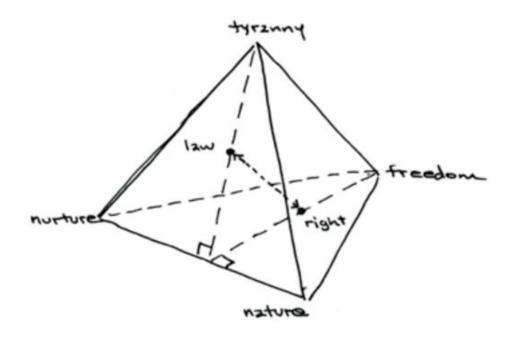


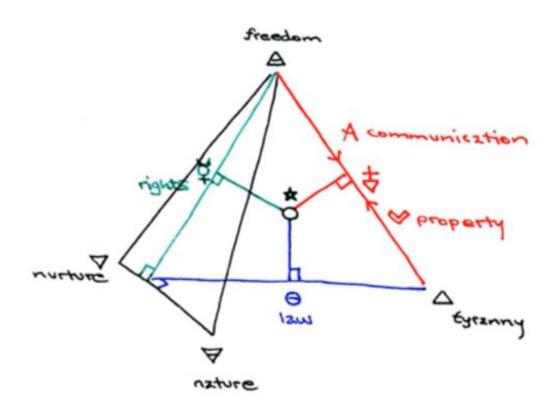


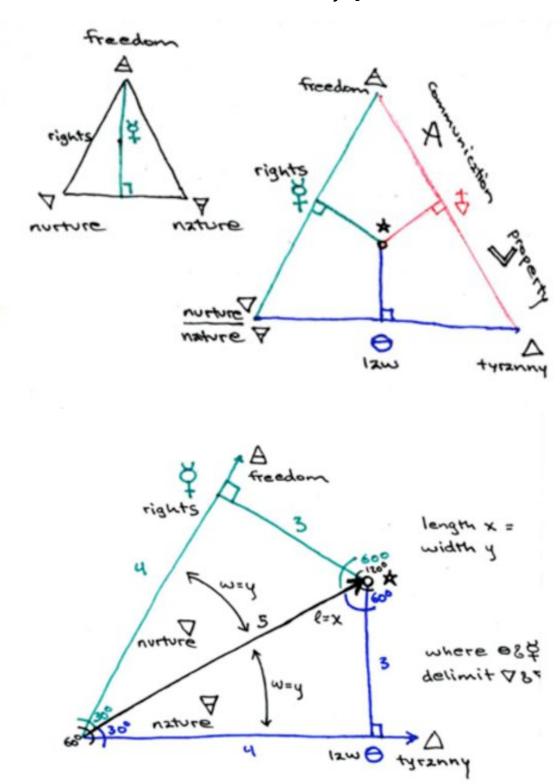


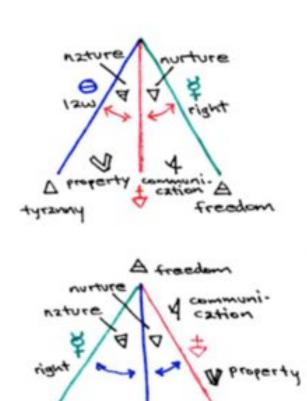


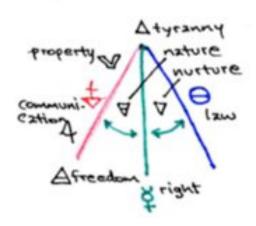






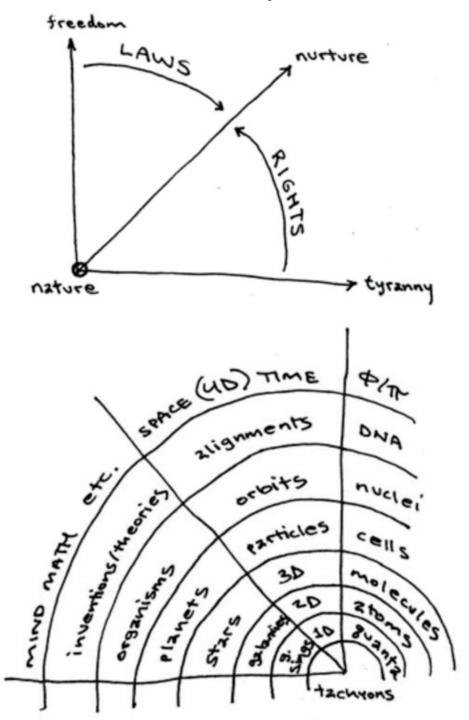






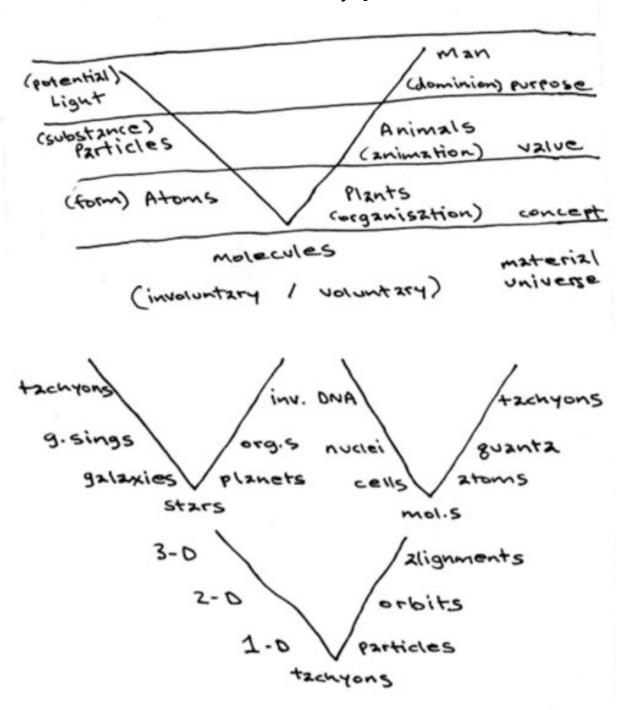
lzw

:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.

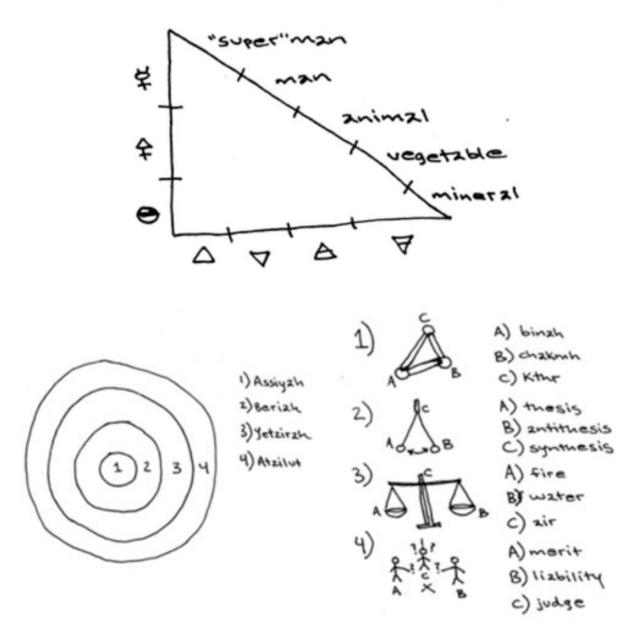


In the same manner we may now see the relationship between "Rights" (pushing toward the side of "Freedom" on the one hand) and "Laws" (pushing toward the side of "Tyranny" on the other hand) conflict with one another along the central axis at 45° connecting "nature" in the present to "nurture" in the "good future." Likewise, we may see that this same, overtly simple model, may also yield many other means of describing the layered and leveled correspondences between traits and aspects across the entire spectrum of scales, from the sub-atomic to the cosmological.

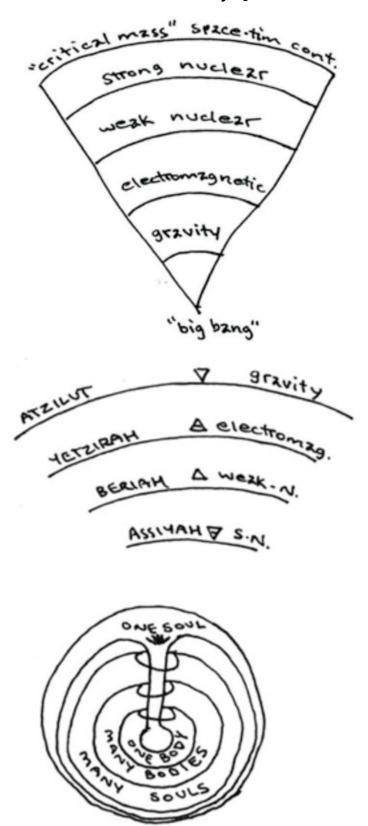
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The manner in which the Laws governing living organisms as well as sub-atomic particles and the cosmological geometry of spacetime itself are here laid out clearly and overtly simply. That all of these "divided lines" depict the same relationship should go without saying, as shuold it that they may thus, and should thus be so, all overlapped to understand the relationship of one scales' laws to another's. So too as there "tachyons" throughout this entire model's manifold positionings, such all is Light, and thus, self-awareness' reflection upon itself within the wellspring of its own inner-eye.



As any two things MAY be related, thus so too may ALL things be constantly related and changing relationships relative to one another as well. In truth, not only is ALL in constant motion at varying velocities differing per scale of size, but ALL things are related into One Big Thing, and this One Big Thing is our entire cosmos over all time. From the utmost furthest extent of the "macrocosm" to the innermost speck of star-dust floating in one's cerebral neural synapses, the utmost "microcosm," ALL is related one to the another and thus to the ALL, and likewise, the clear Light of tachyons permeate the entirety of the cosmos, and so all is One as well. The Hebrews express the Light of tachyons as "Atziluth," the outermost and highest of the 4 QBLHistic "worlds." They imagine they may attain to this level of cognitive perception by ascending the dialectical model of ha QBLH itself alike a ladder. The goal in this methodology is reunion of the "godhead" within one's intellect with the "bride of god" or "spirit" dwelling within one's emotions. The desired outcome is to strengthen the will and resolve to survive and flourish against disadvantages.



:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee. <u>"Harmonia Vol. 2: Music Theory</u> (Past and Present Research)

by Jonathan Barlow Gee (Jon Gee)

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insanity clause #23:

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Harmonia

Vol. 2: Music Theory

Section 1: Present Research

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NOTES : MUSIC THEORY

"SHAPING SOUND" (text, pg.5 18-48)

- Sound white a longitudinal (wave moves in some direction as dispersement to medium) wavelengths (from trough to peak a "randaction") peak to trough a "compression") in a molecular Cocale of hearing) medium.

To "transverse" wave-front

sher-come sirmologicale "sound."
"sound" dispracement shape in pattern par h

- Sound "Event" = 1) in applied force to initiate the displacement of molecules

1) 2 medium with molecules in substant grantities to allow the waves to propagate (cf. the "inverse space law" for dispersion properties in 2).

3) an "observer" to interpret pressure of waves as "sound."

- sour "steet" = f (frequency) = cyclosis accord (for sound measured in "Hertz" pieces of tone; 1 Wz = 12/1 second; 2 Hz = 22/1 second).

I (writerests) a velocity / frequency (for "sound" velocity is a function of materials medium; density of medium-production "compression"; elasticity of medium-production")

- per malecular media: Bensity & Temperature are inversely proportional.

(25 temperature increases, density decreases & as temperature decreases, density increases)

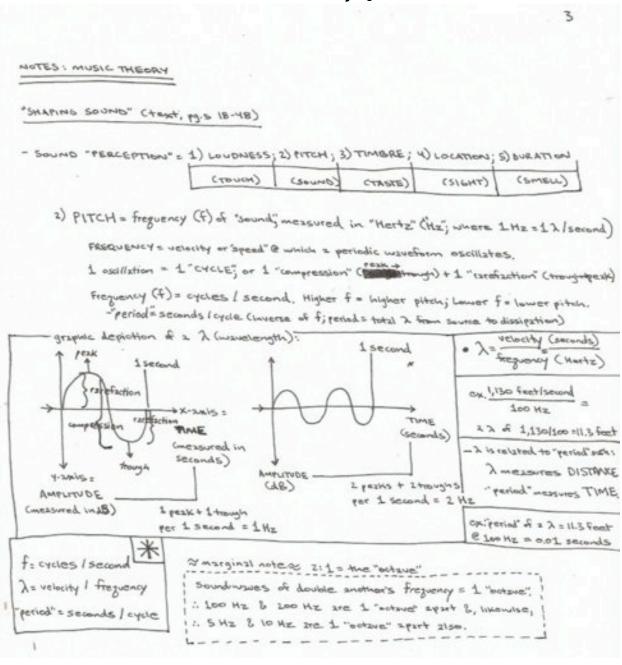
Altitude (elevation) decreases temperature: reduces welently or "speed" of "sound; however due to tulmer air molecule density, also reduces "sound" propagation (essing "speed" of sound to increase), as both density is electrify are mutually cancelled out tomand zero.

-per "sir" molecule medium: @ 32°F (vare dogrees colorus, freezing put of water)
"sound" travels @ 1,088 feet/second.
@ 70°F "sound's" speed is 1,180 feet/second.

- in "air" medium "sound speed" increases by 1.1 feet/second for each 1°F increase in temperature.

-per light or cold media: @ 32°F in Salt-Water = 4,800 feet Isecond
@ 32°F in Aluminum (metal) = 17,000 feet Isecond

"sound" travels FASTER through lightle b solids, but not as FAR, the to the impedience of wave-propagation by molecular density.



NATES : MUSIC THEORY "SHAPING SOUND" CYCKT, PSS 18-48) - SOUND "FERCEPTION" = 1) LOUDNESS; 3) FITCH; 3) THABRE; 4) LOCATION; 3) BYBATION (yourn) CYNSTE) (peuro) (THUIS) (SMELL) 3) TIMBRE = combinations of distributed component frequencies, called a "composito". fundamental frequencies = "nates; combinations of various approximate "nates" timbre.

("harmonics")

("harmonics") -fourier analysis = a graph platting "complex waveforms" (comprised of muttiple sine-waves) generating: 1) fundamental frequency, 2) harmonics (individual sinemans tenes), 3) working 6 4) reministrative related evertenes, by protting Ameritable a 90° to frequency. - the "fundamental treguency" is usually boulest in AMPLITODE & Lomest in frequency in a composite -harmonics are whole + multiples of a fundamental; AKA "partial tones" or "partials." the "first hamonic" = the "fundamental frequency," = the "zero evertone" (ex. 220 Hz) the "second harmonic"s 2x me "fundamental" or "first," = the "first overtone" (ex. 440 Hz) the "third become nic" = 3x the "first" / "hard smooth ! = the "second overtone" (on 660 Hz) the "Fourth harmonic" = 4 x the "fundamental frequency." = the "third evertone". (ex. 880 Hz) - "new-harmonic everbones" are non-whole # multiples of the fundamental frequency." ex. static or "unite-noise" a multiple frequencies occurring in overlap yields ramaption of frequencies occurring "randoming" (seemingly "out of place"), etc. - types of 2 "Sine" waves : have NO exertences NOR Harmonica. "Sinusoidal" 27 / - "triangle" waves: have only add Harmonia a Low AMPLITUDE TO, the "fundamental frequency" where "N" athe "Mumber of the Harmonic Triangle" waves = 1/N" : the "third harmonic" is 1/3° or 1/4th the Amplitude of the "fundamental", the "fifth harmonic" is 1/5° or 1/25%; the "seventh homenic" is 1/7" or 1/49th the "Fundamentals" implifule; etc 2:/ - "Saytooth" waves; contain ALL Harmonics (whole & intervals) & High antitude of fundament where "h" = the Hamonic "H", "southern waves = 1/N; " the "second hamonic" = 1/2 the

"Fundamental" amplitude, the "third hamonic" = 1/3rd, the "Forth" = 1/4th; ctc. 2= 1/10/10

HOTES : MUSIC THEORY

"SMAPIRE SOUND" (text, pg. 18-48)

- SOUND "PERCEPTION" = 1) LOUDNESS; 2) FITCH; 3) TIMBRE; 4) LOCATION; 5) DURATION

(TOUCH) (SOUND) (FASTE) (SIGHT) (SMELL)

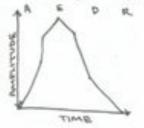
3) TIMBRE (cont'd)

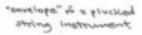
- types of & (courtd)

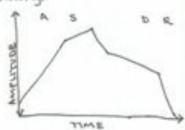
- "Fulse" waves: All Marmonics = to the "fundamental Frequency." For "N" = the "fundamental" or "first harmonic", "Fulse" waves = All Marmonics = 1;: N=1, N2=1, N3=1, N4=1; etc. $\lambda = \frac{1}{1+1+1+1}$
- "Source" weres: only obb Hermanics @ HIGH AMPLITUDE to the "fundamental" / "First" where "N" = the "First hamanic", "Source" weres & 1 lN; : the "flirth hamanic" = Yark the "fundamental", the "Fifth" is 1/8th the "First", the "seventh" is \$/7th; etc. \(\lambda = \lambda \)
- 4) DURATION = "envelope" is the "volume stape" (loudness) over the lengths of time of the components of any "sound"

"covelope" is graphed as the "Fourier" format for a 2 amplitude / time, and grantified into 4 assential segments (ASBR).

- Attack: how the "sound" begins, ex. prucking, bonung, striking, blowing,
- botay terroses in white following
- Sustain: systemment of the "sound"
- Release; eventual coasiation of "sound"







"emolope" of a bound.

~ Transports : random amplitude peaks.

NOTES: MUSIC THEORY

"SHAPING SOUND" (text, pg.s 18-48)

- sound "PERCEPTION"=	1) LOVENESS	; HOTH (5;	3) TMBRE	H) LOCATION	5) DURATION
	(Youry)	(sound)	CHESTES	(SIGHT)	(smeu)
		-			

- 5) LOCATION = "binzvral hearing" consists of 3 primary components: 1) inter-avral intensity; 2) inter-avral arrival time; 8 3) physiology (the anatomy of the eax).
 - inter-zural intensity: the difference in landmass-level of a sound as it reaches each ear independently if a sound reaches both ears simultaneously it is preceived as being contectional to ourselves (everlapping our own location).
 - -inter-overal arrival time: AKA "binaural time of incidence" is the difference in time between when a sound reaches each ear independently. If a sound reaches one ear souner, it is closer to the side that ear is on
 - -purplielegy: the shape of the outer-ear lets us know the sources of sounds' locations.

 -the pinna: the outer car; has 2 ridges to effect sound into the ear.

 -direct sound travels directly into the car canalywhile some of the sound is

 reflected by the 2 ridges & so is delayed, and thus this "emo" effect lets us

 determine origin of location for sources of enterior sound.
- Environmental Effects on Perception of Sound
- -Absorption: determined by the "absorption coefficient" of a surface, comparing % of total emergy that is absorbed to the % reflected.
- -Reflections: "early" or direct "reflections are the first to reach an "blossoner" along an unolostructed, straight-line.
- Echos: @ lesst 80 milliseconds behind the original wave, an one is a wave that has been reflected with sufficient magnitude and delay as to be detected as an autonomous wave, distinct from the directly transmitted signal!

NOTES: MUSIC THEORY

"SHAPING SOUND" (text, pg. 5 18-48)

- SOUND "PERCEPTION"= 1) LOUDNESS; 2) FITCH; 3) TIMBRE; 4) LOCATION; 5) DURATION

(TOUCH) (SOUND) (TASTE) (SIGHT) (SMELL)

5) LOCATION (contid)

- ENVIRONMENTAL EFFECTS (contid)

- Reverberation: a series of random reflections that decrease difference between one another & the original signal, and which decrease in amplitude, over time.

"RT-60" = "reverb time minus 60 decibels", measures the duration that elapses between when reverb begins until the sound-pressure level has diminished 60 decibels. Large rooms have longer RT-60's 8 small rooms have shorter RT-60's.

- FLETCHER/MUNISON EQUAL LOUDNESS CONTOURS

- 1. the human ear is NOT equally sensitive to ALL frequencies in its hearing range.
- 2. 21tering "listening level" 21ters which pitch-frequencies the ear will favor.
- 3. the ear's sensitivities' descrepancies are most pronounced at Low "Listening-levels"
- 4. the ear's sensitivities' descrepancies are least promounced at HIGH "Listening-lovels."
- 5. the ear is most sensitive to MIB-range f's, then HIGHE, & least so to Lows.

NOTES: MUSIC THEORY

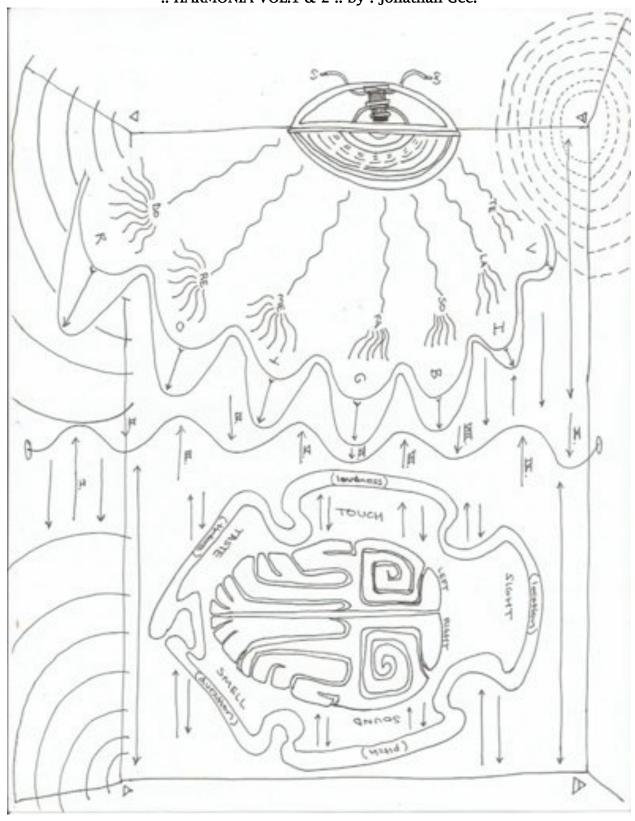
"SHAPING SOUND" CHEVE, PS.S 18-48)

- SOUND-WAVE "BEHAVIOR"
 - -directionality: Mich frequencies propagate in arraighter lines; Low frequencies spread out; Mich frequencies have directionality, Low frequencies disperse more easily; Mich frequencies "Fenetrate", Low frequencies "Hug" or "wrap Around" objects.
 - diffusion: the "spreading out" of sound, Low frequencies diffuse; High are directional.
 - -diffraction: the "bending" or "wrapping around" corners by com frequencies due to their langer 2's (wavelengths).
 - Sound-wave COMBINATION (S)
 - constructive interference: when sound-waves form a "sum-wave" of more amplitude.
 - destructive interference: when coundmanes form a "summane" of LESS amplitude.

 Of + Of: both waves are "out of phase"-compression/rarefaction opposite.
 - sound-wave CYCLES are divided (from start-road) into 3600; (

of 40° 184° 176° 146°

- sound-wave masking: occurs when I sound-wave obscures or "covers up" another, Loudge sound-waves mask soften or covered sound-waves.
- -the STANDING WAVE : a "stationary" maneform whose "sound contours" are defined by interference among echoed or reflected frequencies, either constructive (due to HIAM-pressure ANTI-NOPES) or destructive (due to Low-Pressure NODES) causing compression or rarefaction. Two Listeners / "observers" in 1 room will hear the same sound in different ways. "Standing waveforms" are: "Scalar" waves.



NOTES ON [D] " (can it exist?) (when "It is non-existence)?

the Rule of "Zero"—both as a "place-holder" in solution set proofs, as a decimal holder, as a product multiplier, as now as in armitedizing the sum totality of a solution set as "old," "abotract" or, originally, as "valueless" or oven an unending "about of "nothingness"—in ancient cultures should now be evertected tell to "music theory" & "tone", based geometric humerological arrays (e.g. magic # squares, commenty worn as talismans).

The Ancient's (pre-ciu.) 211 used "Zere" freely, 25 if 2000 naturally termen amongst their first five (8 our still most common 5) phoenetic apprehions, derived directly, 25 they were by them then, from the counting of the 4 fingers using their epposable thumb, with its own independent deaderity, 25 2 figurative "Zero" sum in doing so. Thus, #-sguares (the invention of our Elders during the earliest erz of confizations, when the Vedic, Dumaro-Babylonian & biggotian methods for calculating number-sguares originally arose as independent off-shoots from the originally Habiru-method developed by Abram of Ur, called Inhotop, & Abraham) using "Zero" 25 more than a place-holder Should exist.

However, none do, noither from ancient times nor through to our own medern times even now. The use of only "whole" number "counting" awas relates to later Phoenician (possibly Pythagorean-era) arrangements of alphabetic letters relative to counting numerals, by which time excluding the "zero sum" concept from 8-squares was already common. Thus, by now, a great amount of potential for humanity's researching the "zero sum" has been subsequently suppressed into our cultural, collective-subsensibus, from whence it manifects as the unintended totality of our products of manifects. Whence "All SECRETS BITE THEIR KEERER FROM BEHIND." (given or TIME)

So, if an aisgustes could be posited that use \$\psi\$ or "empty sun" squares 25 could, or even as their own "mayic cours" adding Rous I communis 8 oracanous, and yet ZERO exist, 8 if the "Absolute Value" as it relates to the "relative value" (based on locational distance/duration co-ordinates 8 projected trajectories within any given manifold's definitions for "space" 8 "time", Euclidian 8 newtoning or nong 26 well as ANTI-beth), has been everbooked 8 understudied as a result of this...

(中)1中1 國(S=申) (江江-n*+1)111 国(S=1)

NOTES ON "(\$,\$\phi_1,\$\phi_1,\$\phi_1\)" (revalue zero) can it be record as an amount of matter? Then the underlying "strange attractor" of constructing a "zero sum" magic. A guare could hardly be denied. Yet this proposes the concept (elevious to any who distinguish its traits by differing functions) that "zero" in itself is more absolute, which implies an overlooked aspect in modern (particularly post. Bunkay) expregraphy & study of ancient & modern #-squares. Is a magic #- Square any Less "magic" if all or some portion of its sum amounts to zero? Many squares use some "zeros" (as piece-holders) &, from the point of view of all "serious" students of cryptography, it can be argued the total social value of studying such ancient talismans is, in modern times, if not at zero, but at less progressing steadily along an asymptote toward it, for which they need case naught.

So, uttimately, now does one preve the value of 2 magic #-square constructed using "zero" as a cell-sum or as a total, overall "magic #sum" for the wide square - wherein its Roms, commons & Diagamore functionally carried out, to such a skeptical mathematician in modern society?

consider the potential value of exploring beyond infinity, as well as beyond the limit of zero. Transfinite, super-infinite is transcendental acts to exist that outside the definitively "infinite" (mough, inexplicably 1:1) sets of all "counting" \$5, all integers, 8 all ratios, is it not impossible for there to be "layers" of "nothingness" within the "Abyse" at the "origin"— thus rendering it a relative sum in uself? In what way would "zero's" multiple, functionality-determined aspects, influence the "1:1" ratio of all counting #5, integers & ratios defined as the sum total of "infinity" considering integers include "zero" is "counting #5" do not?

NOTES ON " | \$ | (Show hor zero) what is its water?

-> (wants to drap) pog.3

If "zero" can have different, "relative" values in different geometric manifold settings (i.e. non-cartesian graphs), then perhaps It holds the key to generating equivalent such "magic & squares" as further "matrices" measuring "vectors" of oretion in what would constitute a sum of murtiply-infinite (so called "n-dimensional") different geometric conditions based on the manifold's sets size. Just as the & of cells per "ORDER" (by R or C) determines a magic & squares "magic &" sum, the dimensions of any such "hyper-space" manifold should determine a rough average for "strange attraction" & gravitic multiplier effects (d. automata & cesular info-units) of otherwise free radicals including both particles & waves. Given a large enough manifold to measure with, by this criteria it could be imagined, one would eventually be able to arrive at a measure that could predict the location & any specified mement of how much & what type if mass or energy would be where when.

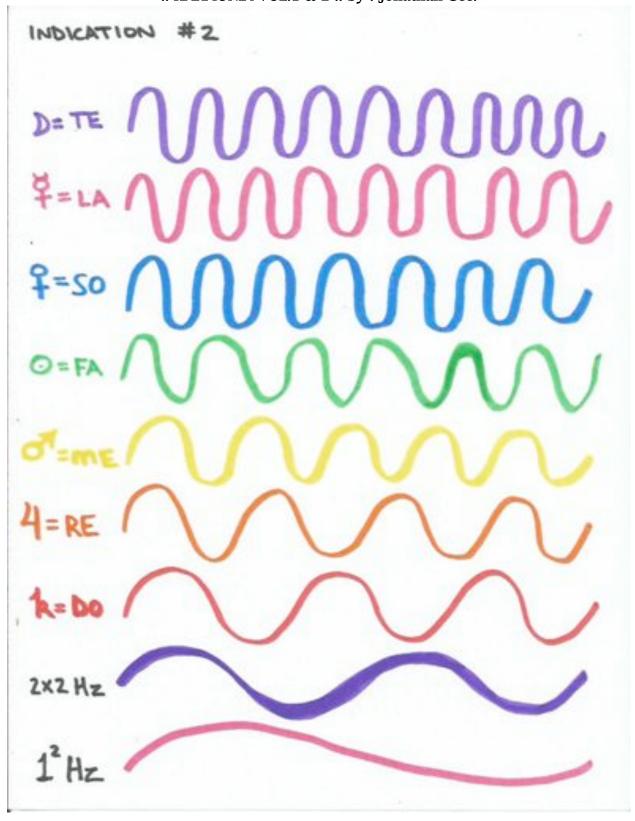
So, again, what then are the "relative" or "absolute" values of zero, as they relate to zero being the "magic of sun of any given size formatifyant if "zero" may be expressed as an "absolute" in Euclidian Cartesian mathematics, in what other geometric arrangements might it be induced instead to decompose into onen lesser constituent components? Spinerical geometry (with 12-acrthogent redians measuring intersecting "legitines" of lattitude & lengitude on the sphere's external circumference, on its outer surface) begins the implication of "relative" zero in such a triple right-angled environment as the relation-ship formed between the 3 40" intersections (10 the orbit pole & z ziong its of the center of mass of the orbital manifold-space, where intrace to the core is measured (2s one arbitrarily may, as it functions variably also) as on, it on the core north be the 101; however, if the circumference is measured as "zoro" contains not only an "go" (polarized to outer "qi") core, but has intersections on the surface occurring @ multiple points if origin @ 3 "zero" point intersections

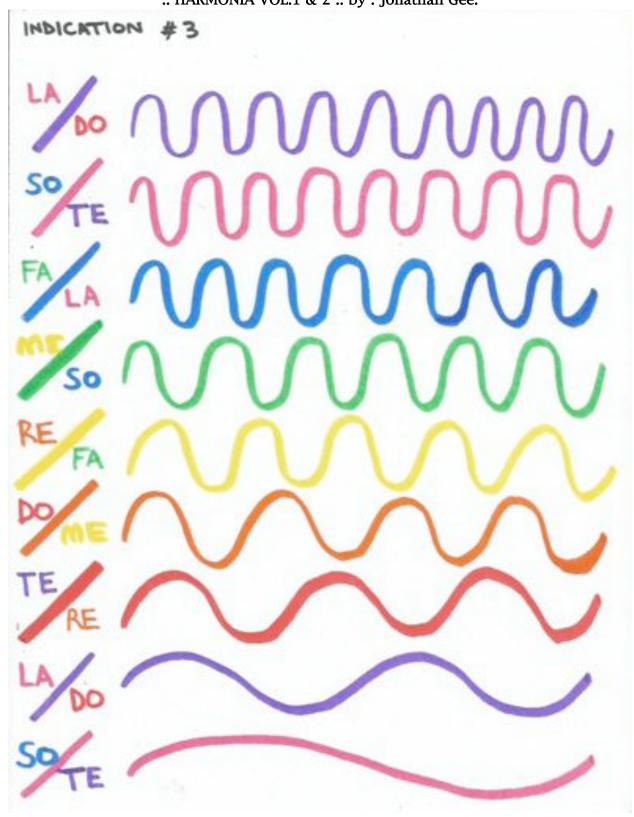
NOTES: MUSIC THEORY

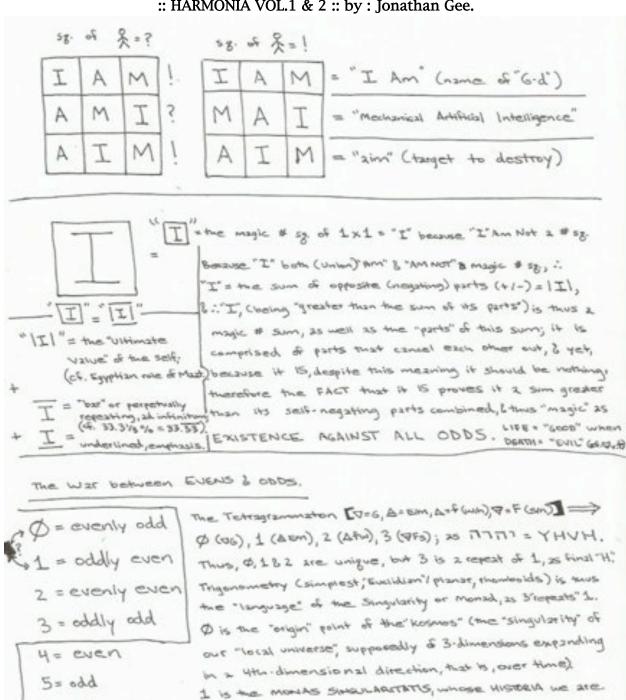
"INDICATIONS" (I's = wavelengths)

- # 1) The Standard Understanding of DO-RA-ME-FA-SO-LA-TE as the "Octave," where "Pitch" ascends as # of wavelengths (X's) per space increases, such that "DO" is presented as a X of 1 "peak" + 1 "trough" (=1 "cycle"). Lower "pitch" tones enit "red" spectral shift 8 higher "pitch" tones spectral shift "violet," due to "Doppler" effect (approach / retreat). "DO" of 8 "cycles" farmonizes with "DO" of 1 "cycle", etc. as an "octave."
- # 2) By shifting the # of "cycles" per "DO" from 1 to 3, we derive 2 "measure" from "DO"= 3 "cycles" -> "TE" = 9 "cycles" in accordance with the Rows & columns ("order") of the 7 "Amulet" magic # squares Affixing the 7 "root" tones of a "measure" (1-less than an "octave") to be orders 3-9 magic #-squares will prove useful later on.
- #3) Positioned with "DO" xs & 3-"cycle" \(\), issociated to an order-3 magic #-square, and "TE" as a 9-"cycle" \(\) \(\) associated order-9 magic #-square, we begin dissecting primary (spectral-hued) "PURE" tones as they arise by combinations of one another—"DO"occurring as a "PURE" tone between "OVERTONE" "TE" \(\) "UNDERTONE" "RE"; each "PURE" tone occurs between a \(\) \(\) \/ "cycle" / order-# magic sum square that is 1 PURE tone above it (the undertone) \(\) the PURE tone BELOW it (the overtone). For example, "PURE" "DO" is between overtone "TE" \(\) under tone "TE" \(\) under tone \(\) \(





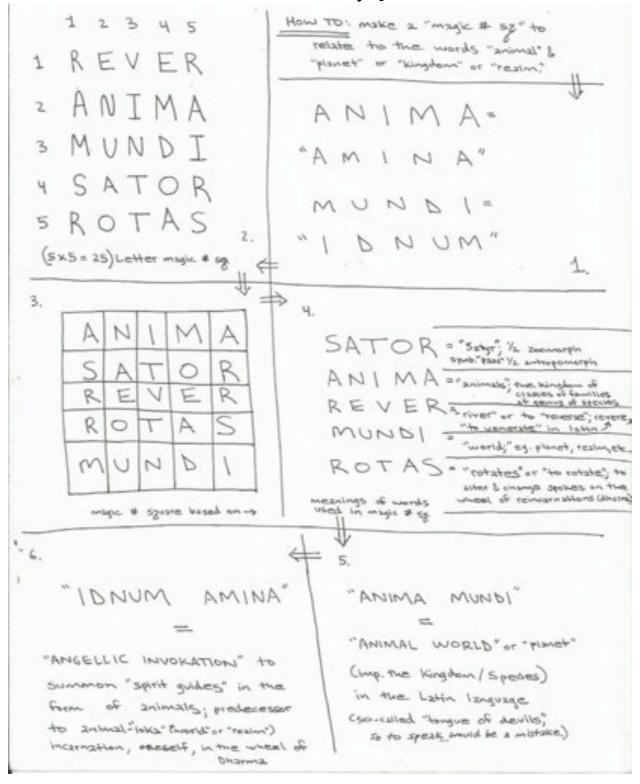




6 = e, 7 = 0, etc.

2 are the "every evens" who idealize our existence as

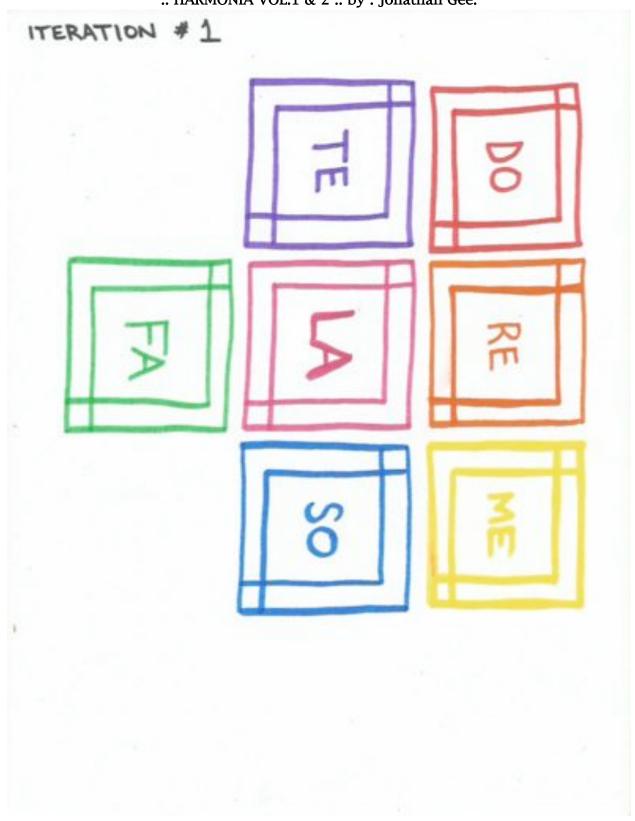
ultimately either all "good" or all "avil," 3" repeats" 1.



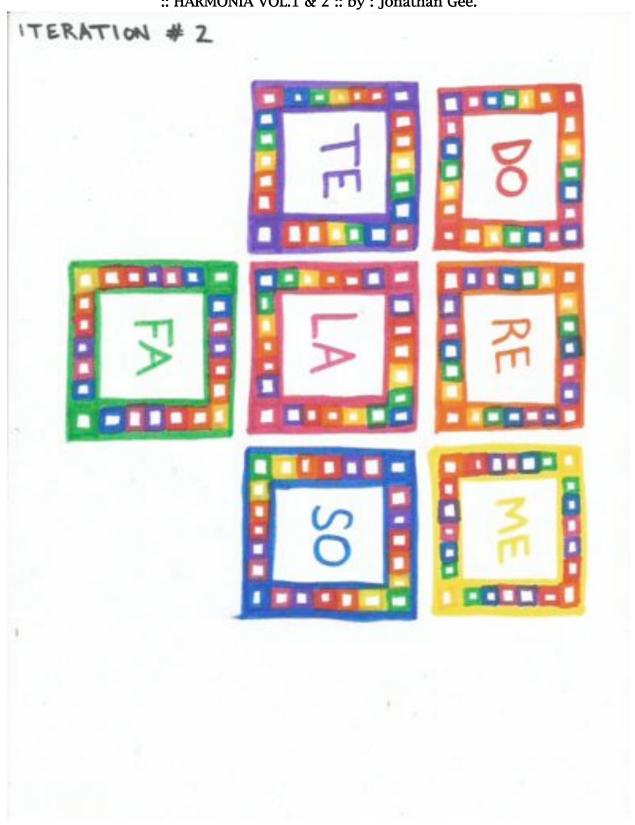
NOTES: MUSIC THEORY

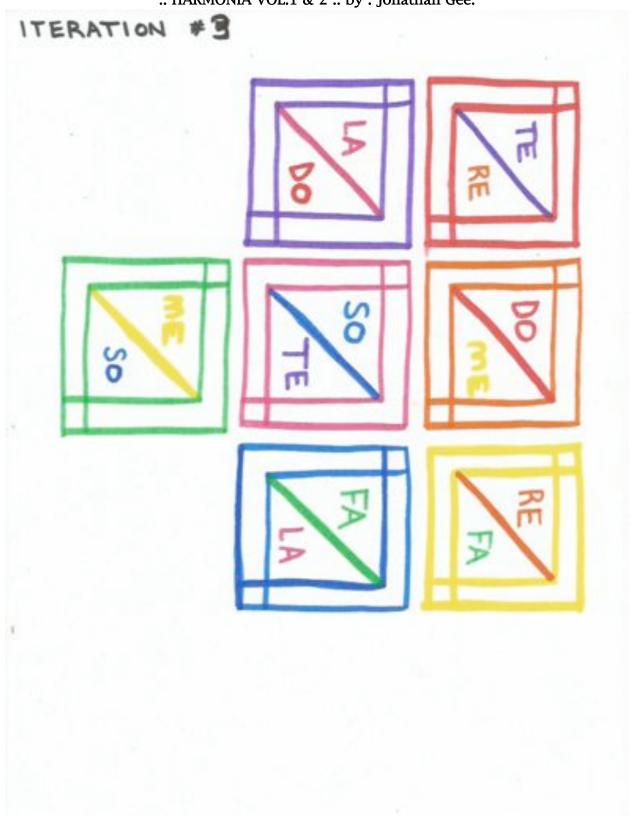
"ITERATIONS" (gnomon squares as is measured)

- # 1) The Standard association of the musical "pure tones (DO-RE-ME-FA-SO-LA-TE) short I of the "octave" with the Spectral hues (ROYGBIV) is here diagrammed as a set of 7 gnomonic-sliced squares, each of ORDER-7 (Having 7 Rows & 7 columns).
- #2) The gnomonic-squares are next further sub-divided within each order-7 square into relations between the 7 "PURE" tones (depicted by spectral hue only, not labeled) to depict a wavelength entering the lower right corner & exiting the upper-left corner of each square alike "Chladni" plates, applied to varied frequencies of vibration. For example, as the "Do" (red) wavelength enters the gnomon-square of "Do", it does so as a "pure" tone, however mid-way across it "peaks" (or alternately "troughs") between the corner-squares opposite its input 8 output points, in this example being labeled "TE" (violet).
- #3) By connecting these opposite corners across the λ @ its mid-point ("peak" or 'trough") we arrive at the undertone (one "Pure" tone above) below the overtone (one "Pure" tone below) depicted graphically as a diagranol bisecting the gnomonic-sliced interior-square (measuring order-6) within the exterior-square (measuring order-7). Further Research on this Method is forthcoming.



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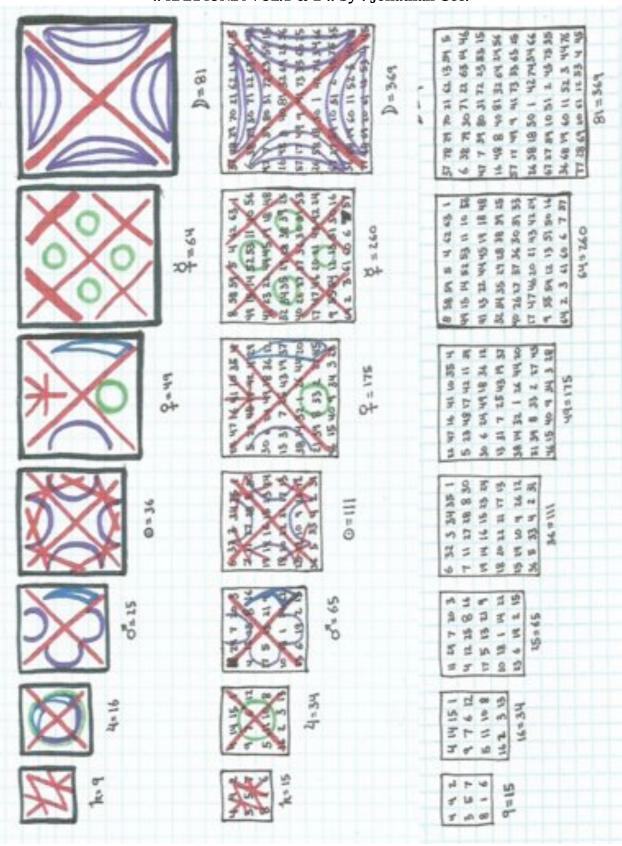




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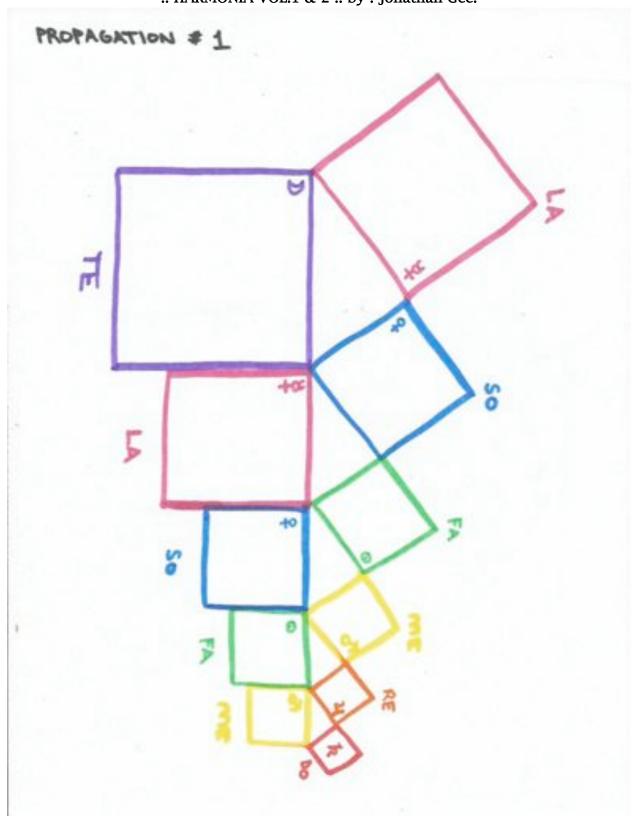


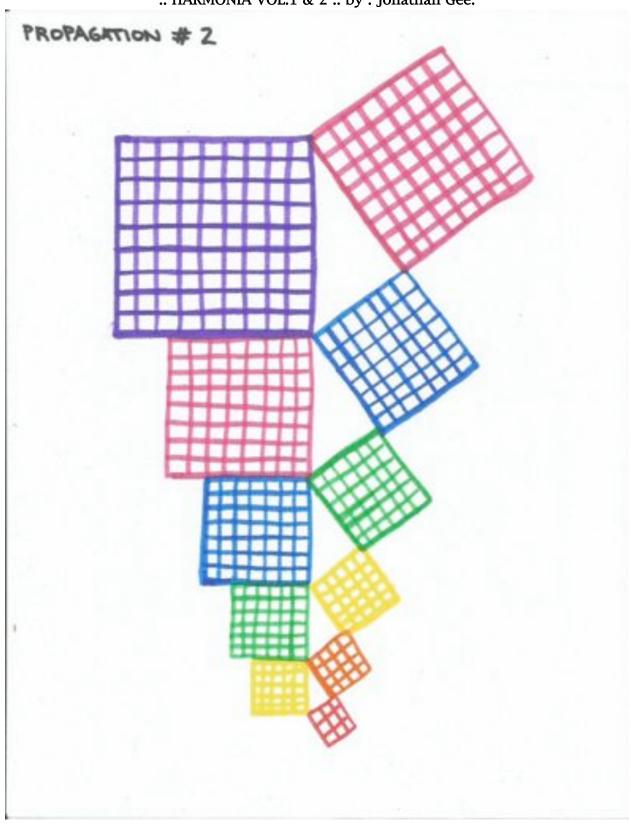
NOTES : MUSIC THEORY

"PROPAGATIONS" (2's 25 Poincaré sections & chiadhi plate surfaces)

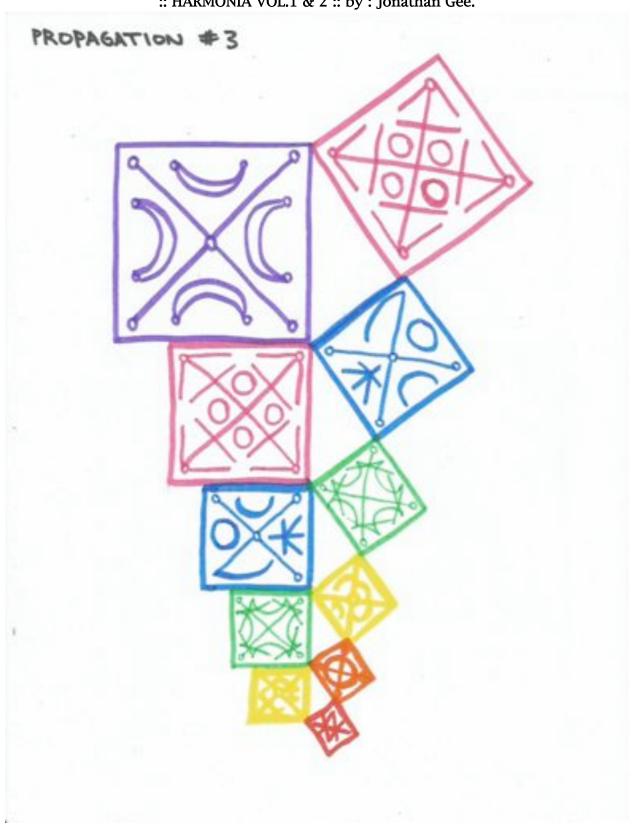
- #1) DO-RE-ME-FA-SO-LA-TE (the usual "OCTAVE" minus-one) are correlated here, again, to the divided Prismatic Spectrum (ROYGRIV) by ascending orders of pure-tone pitch (Hz frequencies 3-9) as well as to the excess magic-# squares by scale of 3x3 + 9x9, arranged in ascending size-scale according to a forced Pythagorean triangle relationship (occurring 5 times per 7 square (tones). The five triangles comprise the 5 "mixed" pitch or "Ya note" tones on a scale.
- #2) "DO", indicating the Lowest Pitch frequency with the Londest wavelength (2), is thus depicted as a RED square of 3 Reus & 3 columbs an "order s" #- 520210. So is "RE", the next pure tone of 4 Hz (cycles per second), an orange square of order 4, & so on up to "TE", a purple 520210 of 9 smaller, "unit" squares.
- #3) The Chaldean-era originating lamons of the "planetary Kamea"-derived by numerical relations between the Rows' & communs' #-sums when each ORDER #-square is factored to produce 2 "magic #-sum" & this derived into its additive components arrayed in a matrix across the grid-depicted per each color (RoyGBN) for the 7 "planetary Kamea" (R, 4, 0*, 0, 9, 4, D). These ancient lamons may relate to the Chladhi plate patterns for Hz frequencies 3-9 respectively.

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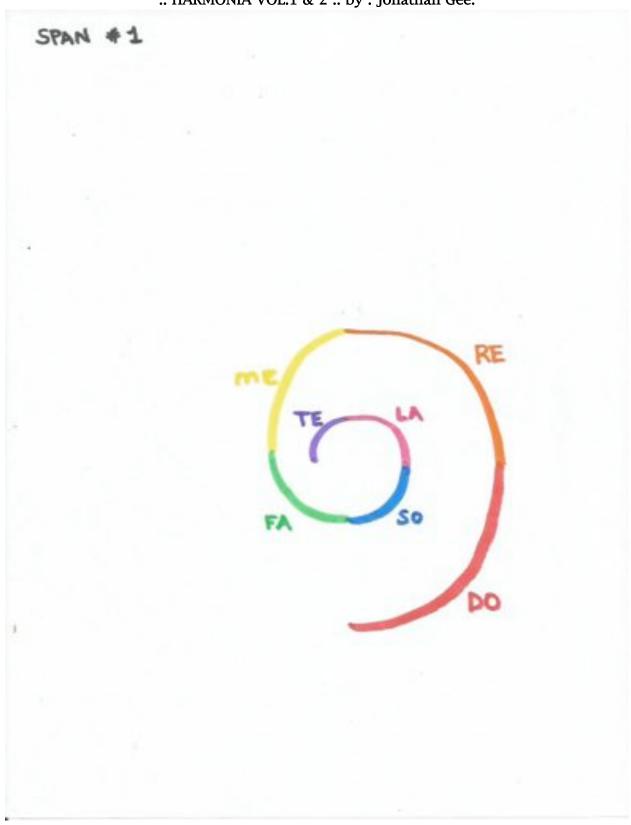


NOTES: MUSIC THEORY

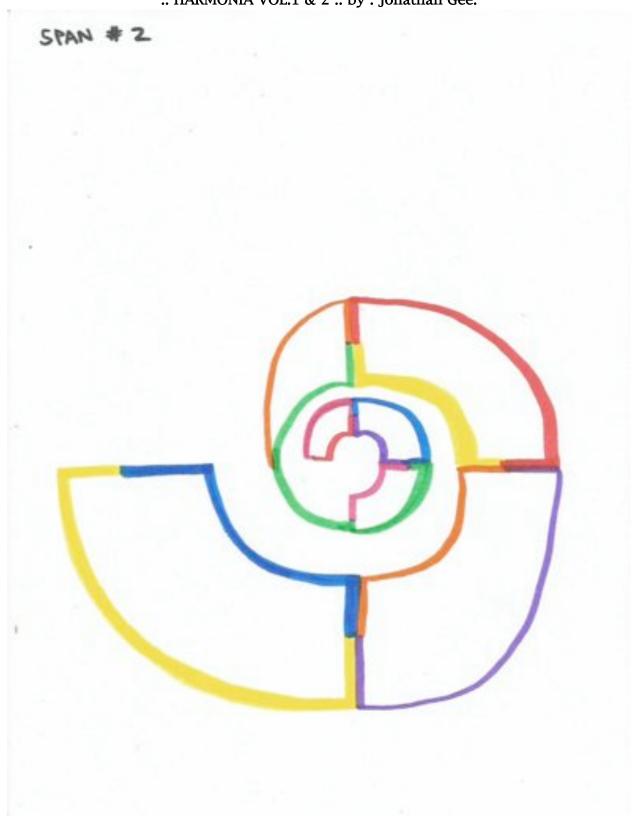
"SPANS" (the "Staggered" comma measuring 2's)

- # 1) Lower "Pitch" Tones are comprised of longer 2's (wavelengths) and spectral-shifted toward the "hot" end of the spectrum due to the "Doppler" effect (compression while approaching, expansion while retreating). Higher "pitch" tones are shorter 2's and of the "cool" end of the spectrum of hues. THEREFORE, "00" (red) is a "longer" 2 than "TE" (violet), which is very short by comparison. This relationship is here depicted as a "staggered" comma, showing a spiral divided into an "octave", however with 1 missing Pure tone between the Lower 4 pitches 8 the UPPER 3 pitches.
- #2) The same "staggered" comma, showing, instead of WHOLE "pure" tones, the combinations of OVERTONES (outer spiral edge) over UNDERTONES (inner spiral edge) that effectively combine to form the pure tones.
- #3) Both the "OVERTONES/UNDERTONES" & "PURE" tones "Staggered" commas combined into one chart.

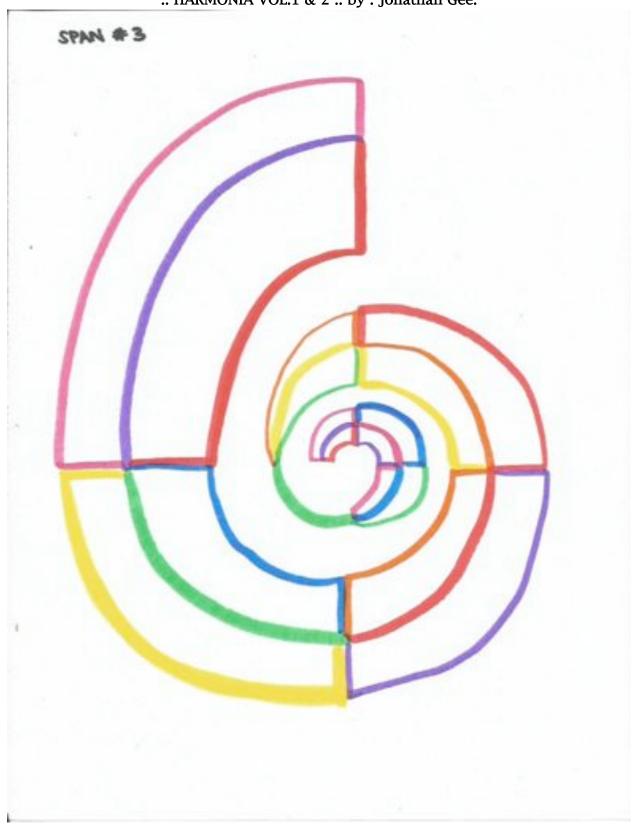
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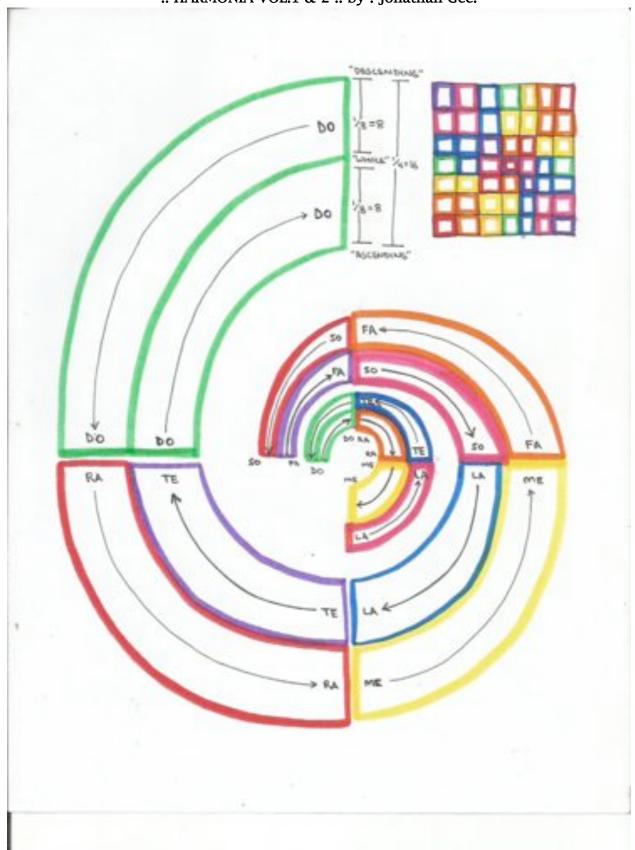


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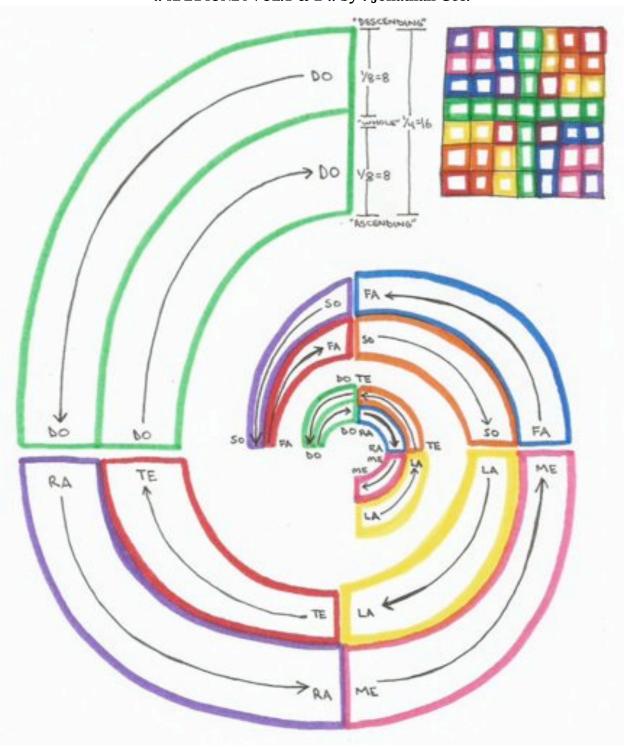


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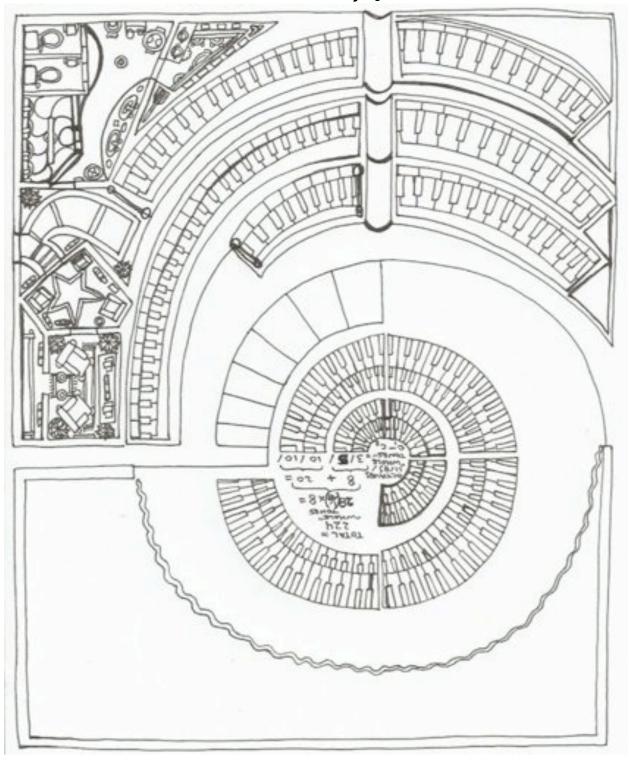




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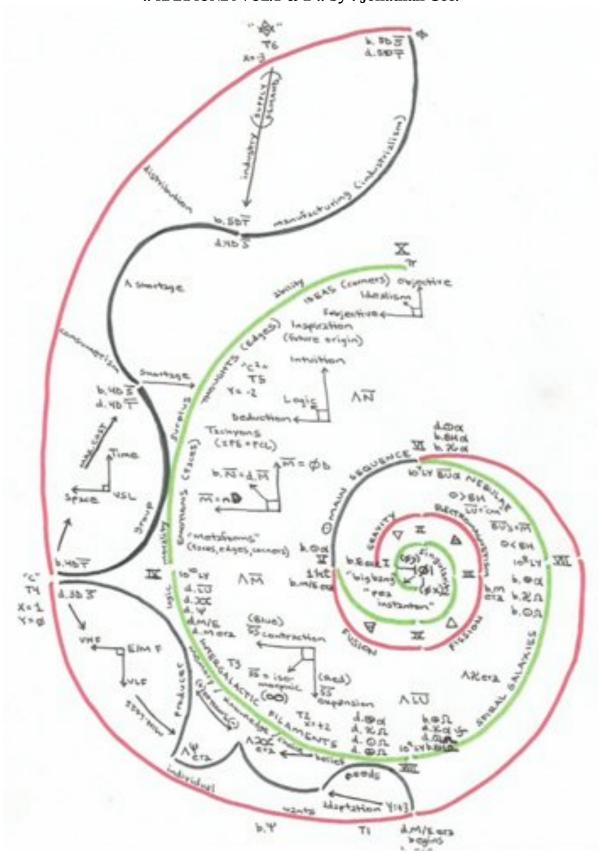


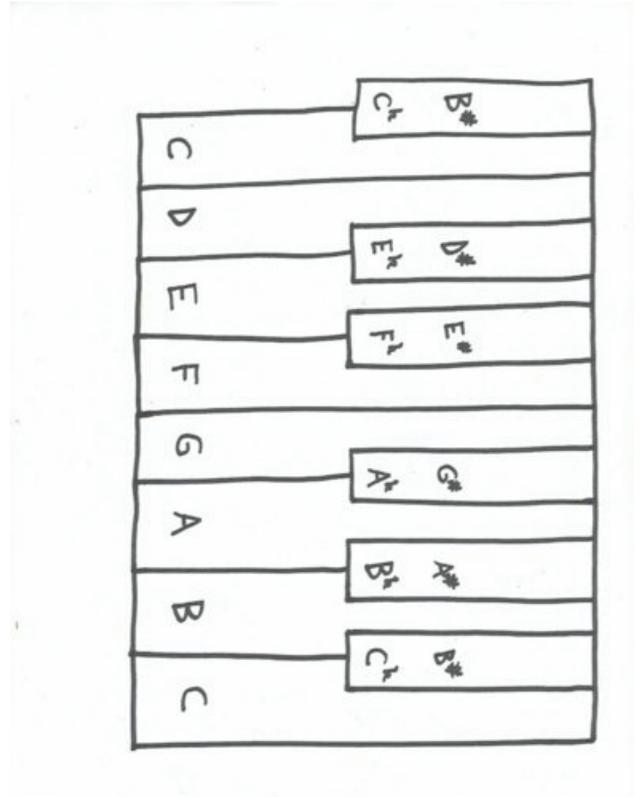
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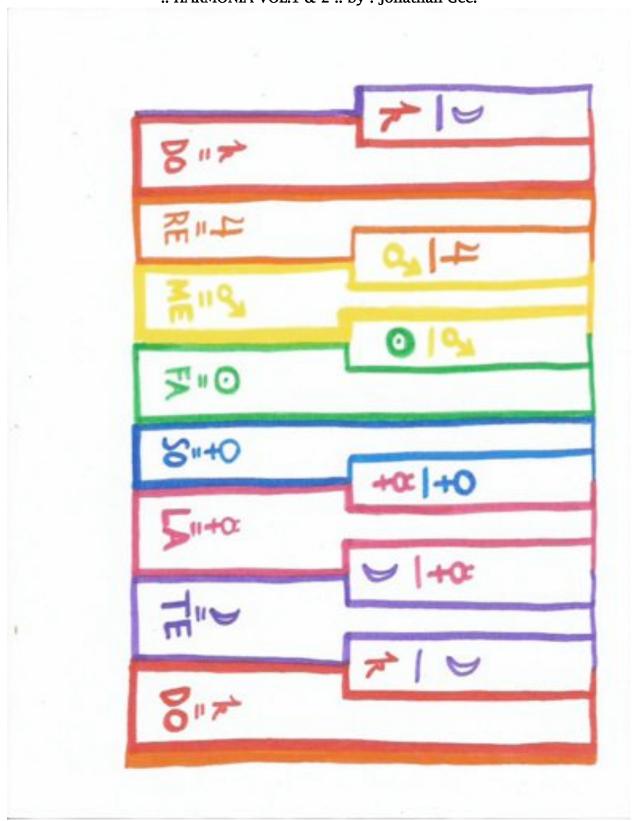


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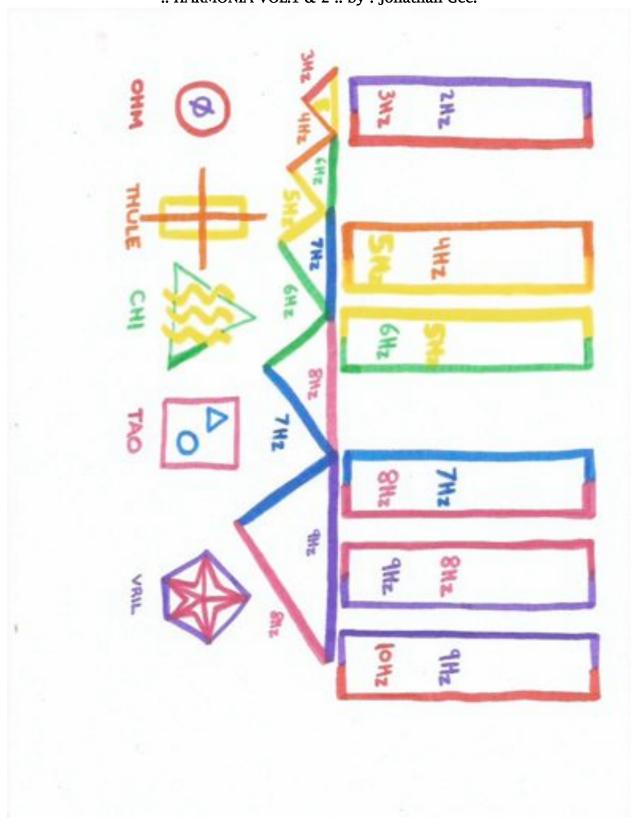




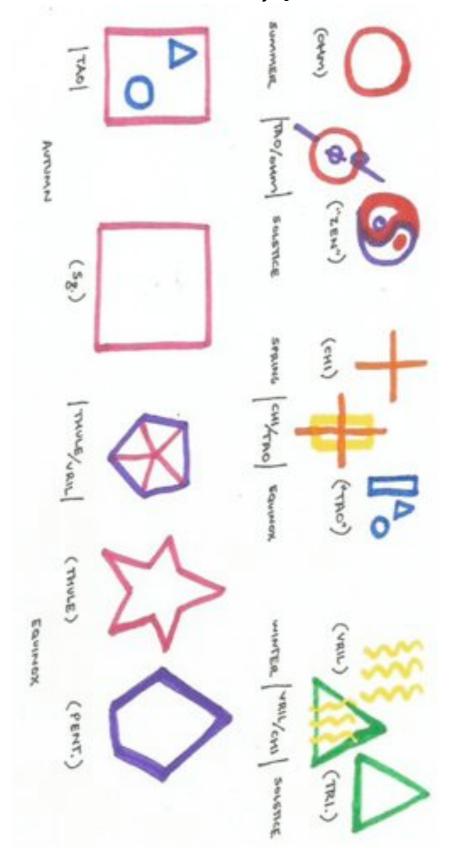
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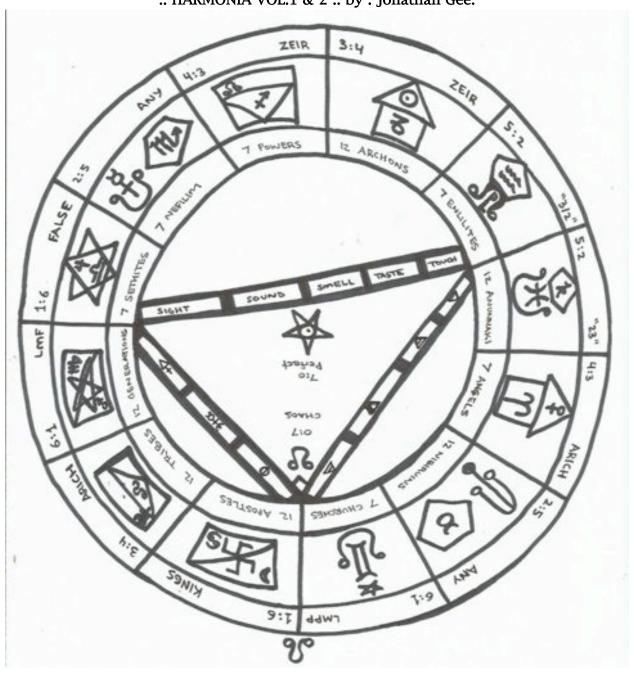


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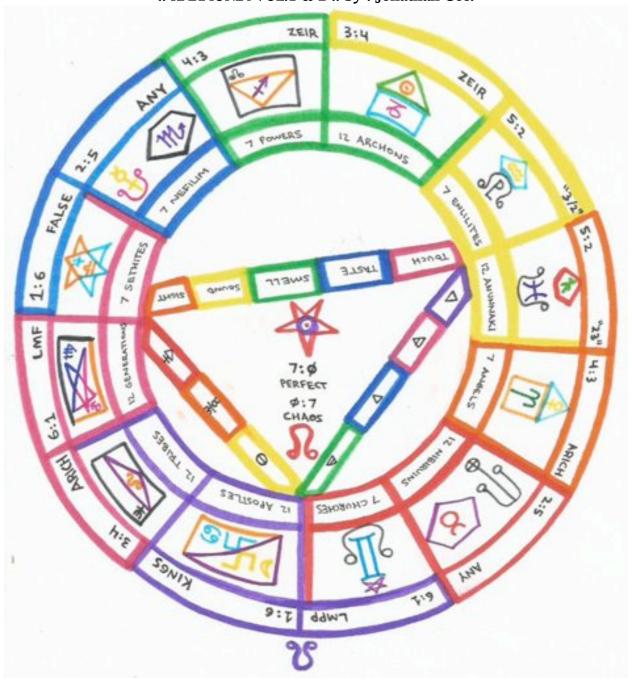


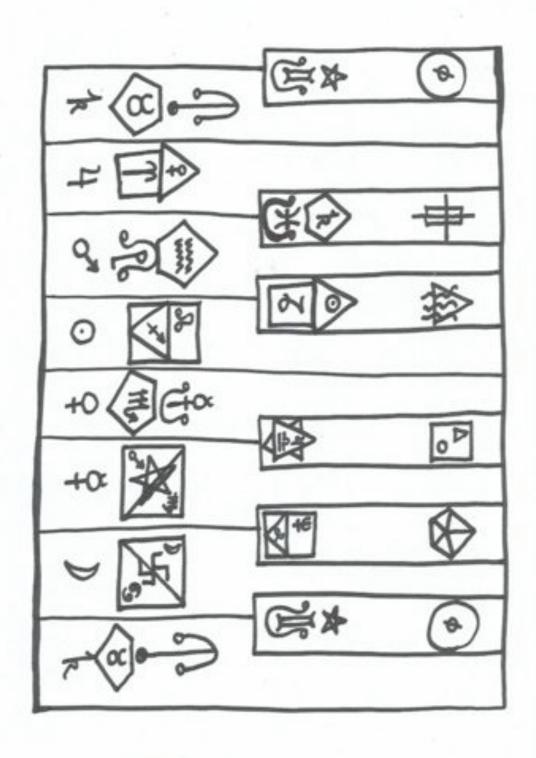
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X		St.	(4)×Y	AA
XII	4	THULE	141,351 = U	÷
XIII	X	THULE	151= 0	k
XV		Perry,	(5) = Y	4

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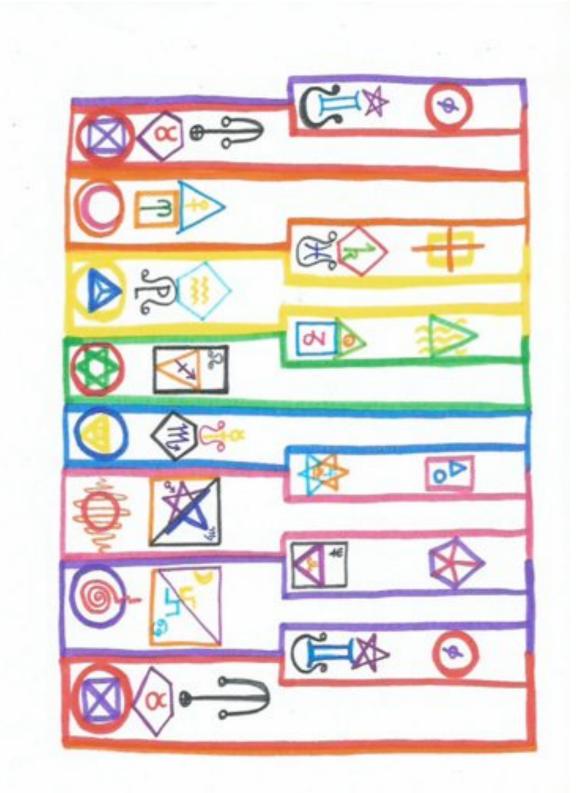


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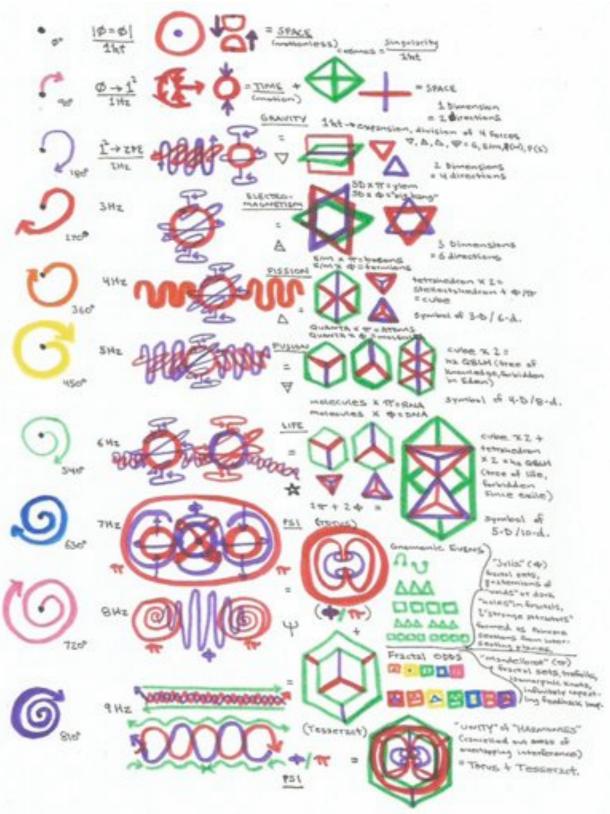




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Harmonia Vol. 2: Music Theory Section 2: Past Research

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Superparticular ratio

In mathematics, a superparticular ratio, also called a superparticular number or **epimoric ratio**, is a ratio of the form

$$\frac{n+1}{n} = 1 + \frac{1}{n}.$$

Thus:

A superparticular number is when a great number contains a lesser number, to which it is compared, and at the same time one part of it. For example, when 3 and 2 are compared, they contain 2, plus the 3 has another 1, which is half of two. When 3 and 4 are compared, they each contain a 3, and the 4 has another 1, which is a third apart of 3. Again, when 5, and 4 are compared, they contain the number 4, and the 5 has another 1, which is the fourth part of the number 4, etc.

—Throop (2006).

Superparticular ratios were written about by Nicomachus in his treatise "Introduction to Arithmetic". Although these numbers have applications in modern pure mathematics, the areas of study that most frequently refer to superparticular ratios by this name are music theory and the history of mathematics. Mathematical properties

As Euler observed, the superparticular numbers (including also the multiply superparticular ratios, numbers formed by adding an integer other than one to a unit fraction) are exactly the rational numbers whose continued fraction terminates after two terms. The numbers whose continued fraction terminates in one term are the integers, while the remaining numbers, with three or more terms in their continued fractions, are superpartient.

The Wallis product
$$\prod_{n=1}^{\infty} \left(\frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \right) = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{6}{5} \cdot \frac{6}{5} \cdot \cdots = \frac{4}{3} \cdot \frac{16}{15} \cdot \frac{36}{35} \cdot \cdots = 2 \cdot \frac{8}{9} \cdot \frac{24}{25} \cdot \frac{48}{49} \cdot \cdots = \frac{\pi}{2}$$

represents the irrational number π in several ways as a product of superparticular ratios and their inverses. It is also possible to convert the Leibniz formula for π into an Euler product of superparticular ratios in which each term has a prime number as its numerator and the nearest multiple of four as its denominator:

$$\pi/4 = \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{13}{12} \cdot \frac{17}{16} \cdots$$

In graph theory, superparticular numbers (or rather, their reciprocals, 1/2, 2/3, 3/4, etc.) arise via the Erd?s-Stone theorem as the possible values of the upper density of an infinite graph.

Other applications

In the study of harmony, many musical intervals can be expressed as a superparticular ratio. Indeed, whether a ratio was superparticular was the most important criterion in Ptolemy's formulation of musical harmony. In application, Størmer's theorem can be used to list all possible superparticular numbers for a given limit; that is, all ratios of this type in which both the numerator and denominator are smooth numbers.

These ratios are also important in visual harmony. Aspect ratios of 4:3 and 3:2 are common in digital photography, and aspect ratios of 7:6 and 5:4 are used in medium format and large format photography respectively.

Ratio names and related intervals

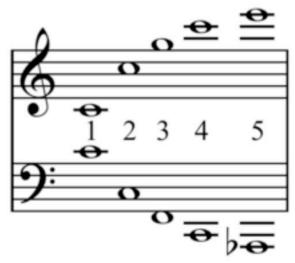
Many individual superparticular ratios have their own names, either in historical mathematics or in music theory. These include the following:

Examples

Ratio	Name	Related musical interval
2:1	duplex	octave
3:2	sesquialterum	perfect fifth
4:3	sesquitertium	perfect fourth
5:4	sesquiquartum	major third
6:5	sesquiquintum	minor third
7:6		septimal minor third
9:8	sesquioctavum	major second
10:9	sesquinona	minor tone
16:15		just diatonic semitone
25:24		just chromatic semitone
81:80		syntonic comma
126:125		septimal semicomma
4375:4374		ragisma

The root of some of these terms comes from Latin *sesqui*- "one and a half" (from *semis* "a half" + -que "and") describing the ratio 3:2.

Otonality and Utonality



5-limit Otonality and Utonality: overtone and "undertone" series, partials 1-5 numbered Play Otonality, Utonality, major chord on C, and minor chord on F. *Otonality* and *Utonality* are terms introduced by Harry Partch to describe chords whose pitch classes are the harmonics or subharmonics of a given fixed tone (identity). For example: 1/1, 2/1, 3/1,... or 1/1, 1/2, 1/3,....

Definition

G (=1/1), A(=9/8),
$$\frac{1}{5}$$
H(=5/4), $\frac{1}{5}$ C(=11/8), D(=3/2), $\frac{1}{7}$ F(=7/4)

Otonality on G = lower line of the tonality diamond bottom left to top right.

Utonality under G = lower line of the tonality diamond bottom right to top left. An Otonality is a collection of pitches which can be expressed in ratios, expressing their relationship to the fixed tone, that have equal denominators. For example, 1/1, 5/4, and 3/2 (just major chord) form an Otonality because they can be written as 4/4, 5/4, 6/4. Every Otonality is therefore composed of members of a harmonic series. Similarly, the ratios of an Utonality share the same numerator. 7/4, 7/5, 7/6, and 1/1 (7/7) form an Utonality. Every Utonality is therefore composed of members of a subharmonic series.

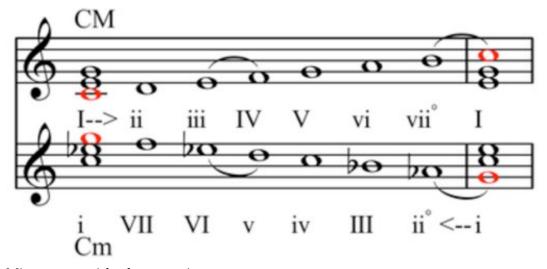
An Otonality corresponds to an arithmetic series of frequencies, or lengths of a vibrating string. Brass instruments naturally produce Otonalities, and indeed Otonalities are inherent in the harmonics of a single fundamental tone. Tuvan khoomei singers produce Otonalities with their vocal tracts.

Utonality is the opposite, corresponding to a subharmonic series of frequencies, or

an arithmetic series of wavelengths (the inverse of frequency). The *arithmetical proportion* "may be considered as a demonstration of Utonality ('minor tonality')."[1] Relationship to standard Western music theory

Partch said that his 1931 coinage of "Otonality" and "Utonality" was, "hastened," by having read Henry Cowell's discussion of undertones in *New Musical Resources* (1930).

The 5-limit Otonality is simply a just major chord, and the 5-limit Utonality is a just minor chord. Thus Otonality and Utonality can be viewed as extensions of major and minor tonality respectively. However, whereas standard music theory views a minor chord as being built up from the root with a minor third and a perfect fifth, an Utonality is viewed as descending from what's normally considered the "fifth" of the chord, so the correspondence is not perfect. This corresponds with the dualistic theory of Hugo Riemann:



Minor as upside down major.

In the era of meantone temperament, augmented sixth chords of the kind known as the German sixth (or the English sixth, depending on how it resolves) were close in tuning and sound to the 7-limit Otonality, called the tetrad. This chord might be, for example, $A \triangleright -C-E \triangleright -G \triangleright [F\#]$. Standing alone, it has something of the sound of a dominant seventh, but considerably less dissonant. It has also been suggested that the Tristan chord, for example, F-B-D#-G# can be considered a Utonality, or 7-limit utonal tetrad, which it closely approximates if the tuning is meantone, though presumably less well in the tuning of a Wagnerian orchestra.

Though Partch presents Otonality and Utonality as being equal and symmetric concepts, when played on most physical instruments an Otonality sounds much more consonant than a similar Utonality, due to the presence of the missing fundamental phenomenon. In an Otonality, all of the notes are elements of the same harmonic series, so they tend to partially activate the presence of a "virtual" fundamental as though they were harmonics of a single complex pitch. Utonal chords, while containing the same dyads as Otonal chords, do not tend to activate this phenomenon as strongly.

Combination tone \mathbf{o} O O O

Sum and difference tones of A220 and unison, just perfect fifth, and octave. Hz marked.

A **combination tone**, also called a **sum tone** or a **difference tone** (also occasionally resultant tone), can be any of at least three similar psychoacoustic phenomena. When two tones are played simultaneously, a listener can sometimes perceive an additional tone whose frequency is a sum or difference of the two frequencies. The discovery of some of these phenomena is credited to the violinist Giuseppe Tartini, and so the tones are also called **Tartini tones**. Explanation

One way a difference tone can be heard is when two tones with fairly complete sets of harmonics make a just fifth. This can be explained as an example of the missing fundamental phenomenon. If f is the missing fundamental frequency, then 2f would be the frequency of the lower tone, and its harmonics would be 2f, 6f, 8f, etc. Since a fifth corresponds to a frequency ratio of 2:3, the higher tone and its harmonics would then be 3f, 6f, 9f, etc. When both tones are sounded, there are components with frequencies of 2f, 3f, 4f, 6f, 8f, 9f, etc. The missing fundamental is heard because so many of these components refer to it.

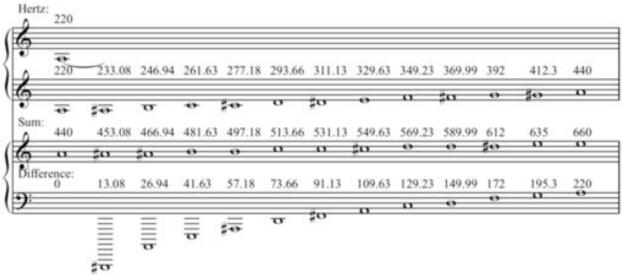
The specific phenomenon that Tartini discovered was physical. Sum and difference tones are thought to be caused sometimes by the non-linearity of the inner ear. This causes intermodulation distortion of the various frequencies which enter the ear. They are combined linearly, generating relatively faint components with frequencies equal to the sums and differences of whole multiples of the original

frequencies. Any components which are heard are usually lower, with the most commonly heard frequency being just the difference tone, f2 - f1, though this may be a consequence of the other phenomena. Although much less common, the following frequencies may also be heard:

$$2f_1 - f_2, 3f_1 - 2f_2, \dots, f_1 - k(f_2 - f_1)$$

For a time it was thought that the inner ear was solely responsible whenever a sum or difference tone was heard. However, experiments show evidence that even when using headphones providing a single pure tone to each ear separately, listeners may still hear a difference tone. Since the peculiar, non-linear physics of the ear doesn't come into play in this case, it is thought that this must be a separate, neural phenomenon. Compare binaural beats.

Heinz Bohlen proposed what is now known as the Bohlen-Pierce scale on the basis of combination tones, as well as the 833 cents scale.

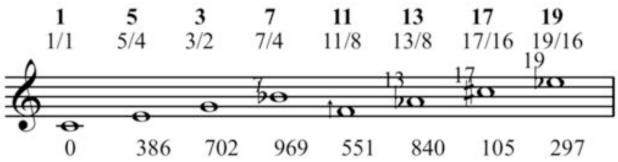


Resultant tone

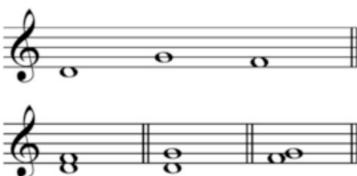
In pipe organs, a **resultant tone** is the sound of a combination of organ pipes that allows the listener to perceive a lower pitch.

This is done by having two pipes, one pipe of the note being played, and another harmonically related, typically at its fifth, being sounded at the same time. The result is a pitch at a common subharmonic of the pitches played (one octave below the first pitch when the second is the fifth, 3:2, two octaves below when the second is the major third, 5:4). This effect is useful especially in the lowest ranks of the pipe organ where cost or space could prohibit having a rank of such low pitch. For example, a 64' pipe would be costly and take up at least 32' of space (if capped) for each pipe. Using a resultant tone for such low pitches would eliminate the cost and space factor, but would not sound as full as a true 64' pipe.

This effect is most often used in the lowest octave of the organ only.



Interval



Melodic and harmonic intervals.

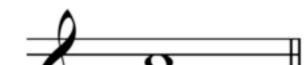
In music theory, an **interval** is the difference between two pitches.[1] An interval may be described as **horizontal**, **linear**, or **melodic** if it refers to successively sounding tones, such as two adjacent pitches in a melody, and **vertical** or **harmonic** if it pertains to simultaneously sounding tones, such as in a chord.[2][3]

In Western music, intervals are most commonly differences between notes of a diatonic scale. The smallest of these intervals is a semitone. Intervals smaller than a semitone are called microtones. They can be formed using the notes of various kinds of non-diatonic scales. Some of the very smallest ones are called commas, and describe small discrepancies, observed in some tuning systems, between enharmonically equivalent notes such as C# and $D \ b$. Intervals can be arbitrarily small, and even imperceptible to the human ear.

In physical terms, an interval is the ratio between two sonic frequencies. For example, any two notes an octave apart have a frequency ratio of 2:1. This means that successive increments of pitch by the same interval result in an exponential increase of frequency, even though the human ear perceives this as a linear increase in pitch. For this reason, intervals are often measured in cents, a unit derived from the logarithm of the frequency ratio.

In Western music theory, the most common naming scheme for intervals describes two properties of the interval: the quality (perfect, major, minor, augmented, diminished) and number (unison, second, third, etc.). Examples include the minor third or perfect fifth. These names describe not only the difference in semitones between the upper and lower notes, but also how the interval is spelled. The importance of spelling stems from the historical practice of differentiating the

frequency ratios of enharmonic intervals such as G-G \sharp and G-A \flat . Size



Example: Perfect octave on C in equal temperament and just intonation: 2/1 = 1200 cents.

The size of an interval (also known as its width or height) can be represented using two alternative and equivalently valid methods, each appropriate to a different context: frequency ratios or cents.

Frequency ratios

The size of an interval between two notes may be measured by the ratio of their frequencies. When a musical instrument is tuned using a just intonation tuning system, the size of the main intervals can be expressed by small-integer ratios, such as 1:1 (unison), 2:1 (octave), 3:2 (perfect fifth), 4:3 (perfect fourth), 5:4 (major third), 6:5 (minor third). Intervals with small-integer ratios are often called *just intervals*, or *pure intervals*. To most people, just intervals sound consonant, that is, pleasant and well tuned.

Most commonly, however, musical instruments are nowadays tuned using a different tuning system, called 12-tone equal temperament, in which the main intervals are typically perceived as consonant, but none is justly tuned and as consonant as a just interval, except for the unison (1:1) and octave (2:1). As a consequence, the size of most equal-tempered intervals cannot be expressed by small-integer ratios, although it is very close to the size of the corresponding just intervals. For instance, an equal-tempered fifth has a frequency ratio of 27/12:1, approximately equal to 1.498:1, or 2.997:2 (very close to 3:2). For a comparison between the size of intervals in different tuning systems, see section Size in different tuning systems.

Cents

The standard system for comparing interval sizes is with cents. The cent is a logarithmic unit of measurement. If frequency is expressed in a logarithmic scale, and along that scale the distance between a given frequency and its double (also called octave) is divided into 1200 equal parts, each of these parts is one cent. In twelve-tone equal temperament (12-TET), a tuning system in which all semitones have the same size, the size of one semitone is exactly 100 cents. Hence, in 12-TET the cent can be also defined as one hundredth of a semitone.

Mathematically, the size in cents of the interval from frequency f1 to frequency f2 is

$$n = 1200 \cdot \log_2 \left(\frac{f_2}{f_1}\right).$$

Main intervals

The table shows the most widely used conventional names for the intervals between the notes of a chromatic scale. A perfect unison (also known as perfect prime) is an interval formed by two identical notes. Its size is zero cents. A semitone is any interval between two adjacent notes in a chromatic scale, a whole tone is an interval

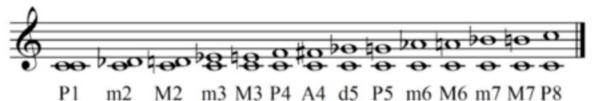
spanning two semitones (for example, a major second), and a tritone is an interval spanning three tones, or six semitones (for example, an augmented fourth). Rarely, the term ditone is also used to indicate an interval spanning two whole tones (for example, a major third), or more strictly as a synonym of major third.

Intervals with different names may span the same number of semitones, and may even have the same width. For instance, the interval from D to F? is a major third, while that from D to G? is a diminished fourth. However, they both span 4 semitones. If the instrument is tuned so that the 12 notes of the chromatic scale are equally spaced (as in equal temperament), these intervals will also have the same width. Namely, all semitones will have a width of 100 cents, and all intervals spanning 4 semitones will be 400 cents wide.

The names listed here cannot be determined by counting semitones alone. The rules to determine them are explained below.

Number of semitones	Minor, major, or perfect intervals	Short	Augmented or diminished intervals	Short	Widely used alternative names	Short
0	Perfect unison(5)[7]	P1	Diminished second	d2		
1	Minor second	m2	Augmented unison(5)	A1	Semitone,[8] half tone, half step	s
2	Major second	M2	Diminished third	d3	Tone, whole tone, whole step	Т
3	Minor third	m3	Augmented second	A2		
4	Major third	мз	Diminished fourth	d4		
5	Perfect fourth	P4	Augmented third	АЗ		
			Diminished fifth	d5	Tritonorm	
6			Augmented fourth	A4	Tritone[6]	TT
7	Perfect fifth	P5	Diminished sixth	d6		
8	Minor sixth	m6	Augmented fifth	A5		
9	Major sixth	M6	Diminished seventh	d7		
10	Minor seventh	m7	Augmented sixth	A6		
11	Major seventh	M7	Diminished octave	d8		
12	Perfect octave	P8	Augmented seventh	A7		

Interval number and quality

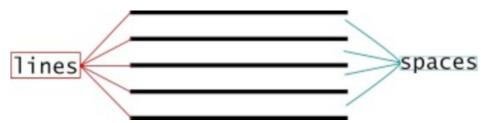


Main intervals from C.

In Western music theory, an interval is named according to its *number* (also called *diatonic number*) and *quality*. For instance, *major third* (or **M3**) is an interval name,

in which the term major (M) describes the quality of the interval, and third (3) indicates its number.





Staff, with staff positions indicated.



Fifth from C to G in the A b major scale.

The number of an interval is the number of staff positions it encompasses. [citation needed] Both lines and spaces (see figure) are counted, including the positions of both notes forming the interval. For instance, the interval C-G is a fifth (denoted P5) because the notes from C to G occupy five consecutive staff positions, including the positions of C and G. The table and the figure above show intervals with numbers ranging from 1 (e.g., P1) to 8 (e.g., P8). Intervals with larger numbers are called compound intervals.

There is a one-to-one correspondence between staff positions and diatonic-scale degrees (the notes of a diatonic scale).[9] This means that interval numbers can be also determined by counting diatonic scale degrees, rather than staff positions, provided that the two notes which form the interval are drawn from a diatonic scale. Namely, C-G is a fifth because in any diatonic scale that contains C and G, the sequence from C to G includes five notes. For instance, in the A?-major diatonic scale, the five notes are C-D \flat -E \flat -F-G (see figure). This is not true for all kinds of scales. For instance, in a chromatic scale, the notes from C to G are eight (C-C#-D-D#-E-F-F#-G). This is the reason interval numbers are also called *diatonic numbers*, and this convention is called *diatonic numbering*.

If one takes away any accidentals from the notes which form an interval, by definition the notes do not change their staff positions. As a consequence, any interval has the same interval number as the corresponding natural interval, formed by the same notes without accidentals. For instance, the intervals C-G# (spanning 8 semitones) and C#-G (spanning 6 semitones) are fifths, like the corresponding natural interval C-G (7 semitones).

Interval numbers do not represent exactly interval widths. For instance, the interval C–D is a second, but D is only one staff position, or diatonic-scale degree, above C. Similarly, C–E is a third, but E is only two staff positions above C, and so on. As a consequence, joining two intervals always yields an interval number one less than their sum. For instance, the intervals C–E and E–G are thirds, but joined together they form a fifth (C–G), not a sixth. Similarly, a stack of three thirds, such as C–E, E–G, and G–B, is a seventh (C–B), not a ninth.

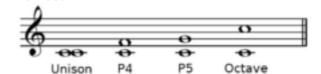
The rule to determine the diatonic number of a compound interval (an interval larger than one octave), based on the diatonic numbers of the simple intervals from which it is built is explained in a separate section.

Quality

	C and	D and	E and	F and	G and	A and	B and
C	P1						
D	M2	P1					
E	M3	M2	P1				
F	P4	m3	m2	P1			
G	P5	P4	m3	M2	P1		
Α	M6	P5	P4	M3	M2	P1	
В	M7	M6	P5	A4 (TT)	M3	M2	P1
C	P8	m7	m6	P5	P4	m3	m2
d		P8	m7	M6	P5	P4	m3
е			P8	M7	M6	P5	P4
f				P8	m7	m6	d5 (TT)
g					P8	m7	m6
a						P8	m7
b							P8

Intervals formed by the notes of a C major diatonic scale.

The name of any interval is further qualified using the terms **perfect** (\mathbf{P}), major (\mathbf{M}), minor (\mathbf{m}), augmented (\mathbf{A}), and diminished (\mathbf{d}). This is called its *interval quality*. It is possible to have doubly diminished and doubly augmented intervals, but these are quite rare, as they occur only in chromatic contexts. The quality of a compound interval is the quality of the simple interval on which it is based.



Perfect intervals on C.

Perfect

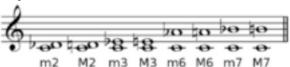
Perfect intervals are so-called because they were traditionally considered perfectly consonant, although in Western classical music the perfect fourth was sometimes regarded as a less than perfect consonance, when its function was contrapuntal. Conversely, minor, major, augmented or diminished intervals are typically considered to be less consonant, and were traditionally classified as mediocre consonances, imperfect consonances, or dissonances.

Within a diatonic scale all unisons (P1) and octaves (P8) are perfect. Most fourths and fifths are also perfect (P4 and P5), with five and seven semitones respectively. There's one occurrence of a fourth and a fifth which are not perfect, as they both span six semitones: an augmented fourth (A4), and its inversion, a diminished fifth (d5). For instance, in a C-major scale, the A4 is between F and B, and the d5 is between B and F (see table).

By definition, the inversion of a perfect interval is also perfect. Since the inversion does not change the pitch of the two notes, it hardly affects their level of consonance (matching of their harmonics). Conversely, other kinds of intervals have the

opposite quality with respect to their inversion. The inversion of a major interval is a minor interval, the inversion of an augmented interval is a diminished interval.

Major/minor

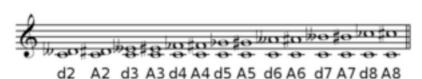


Major and minor intervals on C.

As shown in the table, a diatonic scale[9] defines seven intervals for each interval number, each starting from a different note (seven unisons, seven seconds, etc.). The intervals formed by the notes of a diatonic scale are called diatonic. Except for unisons and octaves, the diatonic intervals with a given interval number always occur in two sizes, which differ by one semitone. For example, six of the fifths span seven semitones. The other one spans six semitones. Four of the thirds span three semitones, the others four. If one of the two versions is a perfect interval, the other is called either diminished (i.e. narrowed by one semitone) or augmented (i.e. widened by one semitone). Otherwise, the larger version is called major, the smaller one minor. For instance, since a 7-semitone fifth is a perfect interval (P5), the 6-semitone fifth is called "diminished fifth" (d5). Conversely, since neither kind of third is perfect, the larger one is called "major third" (M3), the smaller one "minor third" (m3).

Within a diatonic scale, unisons and octaves are always qualified as perfect, fourths as either perfect or augmented, fifths as perfect or diminished, and all the other intervals (seconds, thirds, sixths, sevenths) as major or minor.

Augmented/diminished



Augmented and diminished intervals on C.

Augmented and diminished intervals are so called because they exceed or fall short of either a perfect interval, or a major/minor pair by one semitone, while having the same interval number (i.e., encompassing the same number of staff positions). For instance, an augmented third such as C-E# spans five semitones, exceeding a major third (C-E) by one semitone, while a diminished third such as $C\#-E \nVdash$ spans two semitones, falling short of a minor third ($C-E \ggg$) by one semitone.

Except for the above-mentioned augmented fourth (A4) and diminished fifth (d5), augmented and diminished intervals do not appear in diatonic scales(see table).

Example

Neither the number, nor the quality of an interval can be determined by counting semitones alone. As explained above, the number of staff positions must be taken into account as well.

For example, as shown in the table below, there are four semitones between A \flat and B \sharp , between A and C \sharp , between A and D \flat , and between A \sharp and E, but

- $A \triangleright -B \#$ is a second, as it encompasses two staff positions (A, B), and it is doubly augmented, as it exceeds a major second (such as A-B) by two semitones.
 - A-C# is a third, as it encompasses three staff positions (A, B, C), and it is

major, as it spans 4 semitones.

- A-D \flat is a fourth, as it encompasses four staff positions (A, B, C, D), and it is diminished, as it falls short of a perfect fourth (such as A-D) by one semitone.
- \bullet A#-E is a fifth, as it encompasses five staff positions (A, B, C, D, E), and it is triply diminished, as it falls short of a perfect fifth (such as A-E) by three semitones.

Number	Interval name	Staff positions							
of semitones	Interval name	1	2	3	4	5			
4	doubly augmented second	Ab	B#						
4	major third	Α		C#					
4	diminished fourth	Α			Db				
4	triply diminished fifth	A#				Eli			

Shorthand notation

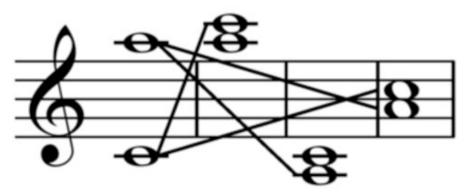
Intervals are often abbreviated with a **P** for perfect, **m** for minor, **M** for major, **d** for diminished, **A** for augmented, followed by the interval number. The indication M and P are often omitted. The octave is P8, and a unison is usually referred to simply as "a unison" but can be labeled P1. The tritone, an augmented fourth or diminished fifth is often **TT**. The interval qualities may be also abbreviated with **perf**, **min**, **maj**, **dim**, **aug**. Examples:

- m2 (or min2): minor second,
- M3 (or maj3): major third,
- A4 (or aug4): augmented fourth,
- d5 (or dim5): diminished fifth,
- P5 (or perf5): perfect fifth.

Inversion



Interval inversions



Major 13th (compound Major 6th) inverts to a minor 3rd by moving the bottom note up two octaves, the top note down two octaves, or both notes one octave

A simple interval (i.e., an interval smaller than or equal to an octave) may be inverted by raising the lower pitch an octave, or lowering the upper pitch an octave. For example, the fourth from a lower C to a higher F may be inverted to make a fifth, from a lower F to a higher C.

There are two rules to determine the number and quality of the inversion of any simple interval:

- 1. The interval number and the number of its inversion always add up to nine (4 + 5 = 9), in the example just given).
- 2. The inversion of a major interval is a minor interval, and vice versa; the inversion of a perfect interval is also perfect; the inversion of an augmented interval is a diminished interval, and vice versa; the inversion of a doubly augmented interval is a doubly diminished interval, and vice versa.

For example, the interval from C to the E? above it is a minor third. By the two rules just given, the interval from E? to the C above it must be a major sixth.

Since compound intervals are larger than an octave, "the inversion of any compound interval is always the same as the inversion of the simple interval from which it is compounded."

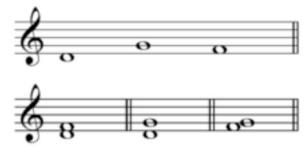
For intervals identified by their ratio, the inversion is determined by reversing the ratio and multiplying by 2. For example, the inversion of a 5:4 ratio is an 8:5 ratio.

For intervals identified by an integer number of semitones, the inversion is obtained by subtracting that number from 12.

Since an interval class is the lower number selected among the interval integer and its inversion, interval classes cannot be inverted.

Classification

Intervals can be described, classified, or compared with each other according to various criteria.



Melodic and harmonic intervals.

Melodic and harmonic

An interval can be described as

- Vertical or harmonic if the two notes sound simultaneously
- Horizontal, linear, or melodic if they sound successively.

Diatonic and chromatic

In general,

- A diatonic interval is an interval formed by two notes of a diatonic scale.
- A *chromatic interval* is a non-diatonic interval formed by two notes of a chromatic scale.



Ascending and descending chromatic scale on C.

The table above depicts the 56 diatonic intervals formed by the notes of the C major scale (a diatonic scale). Notice that these intervals, as well as any other diatonic interval, can be also formed by the notes of a chromatic scale.

The distinction between diatonic and chromatic intervals is controversial, as it is based on the definition of diatonic scale, which is variable in the literature. For example, the interval B–E? (a diminished fourth, occurring in the harmonic C-minor scale) is considered diatonic if the harmonic minor scales are considered diatonic as well. Otherwise, it is considered chromatic.

By a commonly used definition of diatonic scale (which excludes the harmonic minor and melodic minor scales), all perfect, major and minor intervals are diatonic. Conversely, no augmented or diminished interval is diatonic, except for the augmented fourth and diminished fifth.



The $A \triangleright$ -major scale.

The distinction between diatonic and chromatic intervals may be also sensitive to context. The above-mentioned 56 intervals formed by the C-major scale are sometimes called *diatonic to C major*. All other intervals are called *chromatic to C major*. For instance, the perfect fifth $A \not \vdash E \not \vdash$ is chromatic to C major, because A? and E? are not contained in the C major scale. However, it is diatonic to others, such as the A? major scale.

Consonant and dissonant

Consonance and dissonance are relative terms that refer to the stability, or state of repose, of particular musical effects. Dissonant intervals are those that cause tension, and desire to be *resolved* to consonant intervals.

These terms are relative to the usage of different compositional styles.

- In the Middle Ages, only the unison, octave, perfect fourth, and perfect fifth were considered consonant harmonically.
- In 15th- and 16th-century usage, perfect fifths and octaves, and major and minor thirds and sixths were considered harmonically consonant, and all other intervals dissonant, including the perfect fourth, which by 1473 was described (by Johannes Tinctoris) as dissonant, except between the upper parts of a vertical sonority—for example, with a supporting third below ("6-3 chords"). In the common practice period, it makes more sense to speak of consonant and dissonant chords, and certain intervals previously thought to be dissonant (such as minor sevenths) became acceptable in certain contexts. However, 16th-century practice continued to be taught to beginning musicians throughout this period.

- Hermann von Helmholtz (1821–1894) defined a harmonically consonant interval as one in which the two pitches have an upper partial (an overtone) in common (specifically excluding the seventh harmonic). This essentially defines all seconds and sevenths as dissonant, and the above thirds, fourths, fifths, and sixths as consonant.
- Pythagoras defined a hierarchy of consonance based on how small the numbers are that express the ratio. 20th-century composer and theorist Paul Hindemith's system has a hierarchy with the same results as Pythagoras's, but defined by fiat rather than by interval ratios, to better accommodate equal temperament, all of whose intervals (except the octave) would be dissonant using acoustical methods.
- David Cope (1997) suggests the concept of interval strength, in which an interval's strength, consonance, or stability is determined by its approximation to a lower and stronger, or higher and weaker, position in the harmonic series. See also: Lipps–Meyer law.
 - #Interval root

All of the above analyses refer to vertical (simultaneous) intervals.

Simple and compound



Simple and compound major third.

A simple interval is an interval spanning at most one octave. Intervals spanning more than one octave are called compound intervals.

In general, a compound interval may be defined by a sequence or "stack" of two or more simple intervals of any kind. For instance, a major tenth (two staff positions above one octave), also called compound major third, spans one octave plus one major third. A major seventeenth (two staff positions above two octaves) is another example of compound major third, and can be built either by adding up two octaves and one major third, or four perfect fifths.

Any compound interval can be always decomposed into one or more octaves plus one simple interval. For instance, a seventeenth can be always decomposed into two octaves and one major third, and this is the reason why it is called a compound major third, even when it is built using four fifths.

The diatonic number DNc of a compound interval formed from n simple intervals

with diatonic numbers
$$DN1$$
, $DN2$, ..., DNn , is determined by: $DN_c = 1 + (DN_1 - 1) + (DN_2 - 1) + ... + (DN_n - 1)$,

which can also be written as:

$$DN_c = DN_1 + DN_2 + \dots + DN_n - (n-1),$$

The quality of a compound interval is determined by the quality of the simple interval on which it is based. For instance, a compound major third is a major tenth (1+(8-1)+(3-1)=10), or a major seventeenth (1+(8-1)+(8-1)+(3-1)=17), and a compound perfect fifth is a perfect twelfth (1+(8-1)+(5-1) = 12) or a perfect nineteenth (1+(8-1)+(8-1)+(5-1) = 19). Notice that two octaves are a fifteenth, not a sixteenth (1+(8-1)+(8-1)=15). Similarly, three octaves are a twenty-second (1+3*(8-1)=22), and

Intervals larger than a seventeenth seldom need to be spoken of, most often being

referred to by their compound names, for example "two octaves plus a fifth" rather than "a 19th".

Steps and skips

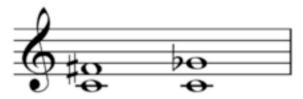
Linear (melodic) intervals may be described as *steps* or *skips*. A *step*, or *conjunct motion*, is a linear interval between two consecutive notes of a scale. Any larger interval is called a *skip* (also called a *leap*), or *disjunct motion*. In the diatonic scale, a step is either a minor second (sometimes also called *half step*) or major second (sometimes also called *whole step*), with all intervals of a minor third or larger being skips.

For example, C to D (major second) is a step, whereas C to E (major third) is a skip.

More generally, a step is a smaller or narrower interval in a musical line, and a skip is a wider or larger interval, with the categorization of intervals into steps and skips is determined by the tuning system and the pitch space used.

Melodic motion in which the interval between any two consecutive pitches is no more than a step, or, less strictly, where skips are rare, is called *stepwise* or *conjunct* melodic motion, as opposed to *skipwise* or *disjunct* melodic motions, characterized by frequent skips.

Enharmonic intervals



Enharmonic tritones: A4 = d5 on C.

Two intervals are considered to be *enharmonic*, or *enharmonically equivalent*, if they both contain the same pitches spelled in different ways; that is, if the notes in the two intervals are themselves enharmonically equivalent. Enharmonic intervals span the same number of semitones.

For example, the four intervals listed in the table below are all enharmonically equivalent, because the notes F? and G? indicate the same pitch, and the same is true for A # and $B \not \triangleright$. All these intervals span four semitones.

Number		Staff positions					
of semitones	Interval name	1	2	3	4		
4	major third	F#		A#			
4	major third		Gb		ВЬ		
4	diminished fourth	F#			ВЬ		
4	doubly augmented second		Gb	A#			

When played on a piano keyboard, these intervals are indistinguishable as they are all played with the same two keys, but in a musical context the diatonic function of the notes incorporated is very different.

Minute intervals



Pythagorean comma on C. The note depicted as lower on the staff (B#+++) is slightly higher in pitch (than C#).

There are also a number of minute intervals not found in the chromatic scale or labeled with a diatonic function, which have names of their own. They may be described as microtones, and some of them can be also classified as commas, as they describe small discrepancies, observed in some tuning systems, between enharmonically equivalent notes. In the following list, the interval sizes in cents are approximate.

- A *Pythagorean comma* is the difference between twelve justly tuned perfect fifths and seven octaves. It is expressed by the frequency ratio 531441:524288 (23.5 cents).
- A *syntonic comma* is the difference between four justly tuned perfect fifths and two octaves plus a major third. It is expressed by the ratio 81:80 (21.5 cents).
- A *septimal comma* is 64:63 (27.3 cents), and is the difference between the Pythagorean or 3-limit "7th" and the "harmonic 7th".
- A *diesis* is generally used to mean the difference between three justly tuned major thirds and one octave. It is expressed by the ratio 128:125 (41.1 cents). However, it has been used to mean other small intervals: see diesis for details.
- A *diaschisma* is the difference between three octaves and four justly tuned perfect fifths plus two justly tuned major thirds. It is expressed by the ratio 2048:2025 (19.6 cents).
- A *schisma* (also skhisma) is the difference between five octaves and eight justly tuned fifths plus one justly tuned major third. It is expressed by the ratio 32805:32768 (2.0 cents). It is also the difference between the Pythagorean and syntonic commas. (A schismic major third is a schisma different from a just major third, eight fifths down and five octaves up, $F \triangleright in C$.)
- A *kleisma* is the difference between six minor thirds and one *tritave* or *perfect twelfth* (an octave plus a perfect fifth), with a frequency ratio of 15625:15552 (8.1 cents).
- A *septimal kleisma* is six major thirds up, five fifths down and one octave up, with ratio 225:224 (7.7 cents).
- A *quarter tone* is half the width of a semitone, which is half the width of a whole tone. It is equal to exactly 50 cents.

Intervals in chords

Chords are sets of three or more notes. They are typically defined as the combination of intervals starting from a common note called the root of the chord. For instance a major triad is a chord containing three notes defined by the root and two intervals (major third and perfect fifth). Sometimes even a single interval (dyad) is considered to be a chord. Chords are classified based on the quality and number of the intervals which define them.

Chord qualities and interval qualities

The main chord qualities are: major, minor, augmented, diminished, half-diminished, and dominant. The symbols used for chord quality are similar to those used for interval quality (see above). In addition, + or **aug** is used for augmented, ° or **dim** for diminished, ø for half diminished, and **dom** for dominant (the symbol - alone is not used for diminished).

Deducing component intervals from chord names and symbols

The main rules to decode chord names or symbols are summarized below.

- For 3-note chords (triads), major or minor always refer to the interval of the third above the root note, while augmented and diminished always refer to the interval of the fifth above root. The same is true for the corresponding symbols (e.g., Cm means Cm3, and C+ means C+5). Thus, the terms third and fifth and the corresponding symbols 3 and 5 are typically omitted. This rule can be generalized to all kinds of chords,[21] provided the above mentioned qualities appear immediately after the root note, or at the beginning of the chord name or symbol. For instance, in the chord symbols Cm and Cm7, m refers to the interval m3, and 3 is omitted. When these qualities do not appear immediately after the root note, or at the beginning of the name or symbol, they should be considered interval qualities, rather than chord qualities. For instance, in Cm/M7 (minor major seventh chord), m is the chord quality and refers to the m3 interval, while M refers to the M7 interval. When the number of an extra interval is specified immediately after chord quality, the quality of that interval may coincide with chord quality (e.g., CM7 = CM/M7). However, this is not always true (e.g., Cm6 = Cm/M6, C+7 = C+/m7, CM11 = CM/P11).[21] See main article for further details.
- 2. Without contrary information, a major third interval and a perfect fifth interval (major triad) are implied. For instance, a C chord is a C major triad, and the name C minor seventh (Cm7) implies a minor 3rd by rule 1, a perfect 5th by this rule, and a minor 7th by definition (see below). This rule has one exception (see next rule).
- 3. When the fifth interval is diminished, the third must be minor. This rule overrides rule 2. For instance, Cdim7 implies a diminished 5th by rule 1, a minor 3rd by this rule, and a diminished 7th by definition (see below).
- 4. Names and symbols which contain only a plain interval number (e.g., "Seventh chord") or the chord root and a number (e.g., "C seventh", or C7) are interpreted as follows:
- If the number is 2, 4, 6, etc., the chord is a major added tone chord (e.g., C6 = CM6 = Cadd6) and contains, together with the implied major triad, an extra major 2nd, perfect 4th, or major 6th (see names and symbols for added tone chords).
- If the number is 7, 9, 11, 13, etc., the chord is dominant (e.g., C7 = Cdom7) and contains, together with the implied major triad, one or more of the following extra intervals: minor 7th, major 9th, perfect 11th, and major 13th (see names and symbols for seventh and extended chords).
- If the number is 5, the chord (technically not a chord in the traditional sense, but a dyad) is a power chord. Only the root, a perfect fifth and usually an octave are played.

5.

The table shows the intervals contained in some of the main chords (**component intervals**), and some of the symbols used to denote them. The interval qualities or numbers in **boldface** font can be deduced from chord name or symbol by applying rule 1. In symbol examples, C is used as chord root.

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Main <u>ch</u>	ords	Component intervals				
Name	Symbol examples	Third	Fifth	Seventh		
Majortriad	С	maj3	perf5			
Major triad	CM, or Cmaj	maj3	perf5			
Minor triad	Cm, or Cmin	min3	perf5			
Augmented triad	C+, or Caug	maj3	aug5			
Diminished triad	C°, or Cdim	min3	dim5			
Dominant seventh chord	C7, or Cdom7	maj3	perf5	min 7		
Minor seventh chord	Cm7, or Cmin7	min3	perf5	min 7		
Major seventh chord	CM7, or Cmaj7	maj3	perf5	maj 7		
Augmented seventh chord	C+7, Caug7, C7 ^{\$5} , or C7 ^{aug5}	maj3	aug5	min 7		
Diminished seventh chord	C°7, or Cdim7	min3	dim5	dim7		
Half-diminished seventh chord	C ^Ø 7, Cm7♭5, or Cmin7 ^{dim5}	min3	dim5	min7		

Size of intervals used in different tuning systems

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			Compa	rison of interv	al width (in	cents)
Number of semitones	Name	5-limit tuning (pitch ratio)	5-limit tuning	Pythagorean tuning	1/4-comma meantone	Equal temperament
0	Perfect unison	1:1	0	0	0	0
1	Minor second	16:15	112	90	117	100
2	Major second	9:8 10:9	204 182	204	193	200
3	Minor third	6:5 75:64 32:27	316 (wolf) 275 294	294 318	310 (wolf) 269	300
4	Major third	5:4 512:405 32:25 81:64	386 406 (wolf) 427 408	408 384	386 (wolf) 427	400
5	Perfect fourth	4:3 675:512 27:20	498 478 520	498 (wolf) 522	503 (wolf) 462	500
6	Augmented fourth Diminished fifth	45:32 64:45	590 610	612 588	579 621	600
7	Perfect fifth	3:2 40:27 1024:675	702 680 722	702 (wolf) 678	697 (wolf) 738	700
8	Minor sixth	8:5	814	792	814	800
9	Major sixth	5:3	884	906	890	900
10	Minor seventh	9:5 16:9	1018 996	996	1007	1000
11	Major seventh	15:8	1088	1110	1083	1100
12	Perfect octave	2:1	1200	1200	1200	1200

In this table, the interval widths used in four different tuning systems are compared. To facilitate comparison, just intervals as provided by 5-limit tuning (see symmetric scale n.1) are shown in **bold** font, and the values in cents are rounded to integers. Notice that in each of the non-equal tuning systems, by definition the width of *each* type of interval (including the semitone) changes depending on the note from which the interval starts. This is the price paid for seeking just intonation. However, for the sake of simplicity, for some types of interval the table shows only one value (the most often observed one).

In 1/4-comma meantone, by definition 11 perfect fifths have a size of approximately 697 cents (700- Σ cents, where $\Sigma \approx 3.42$ cents); since the average size of the 12 fifths must equal exactly 700 cents (as in equal temperament), the other one must have a

size of about 738 cents $(700+11\Sigma)$, the wolf fifth or diminished sixth); 8 major thirds have size about 386 cents $(400-4\Sigma)$, 4 have size about 427 cents $(400+8\Sigma)$, actually diminished fourths), and their average size is 400 cents. In short, similar differences in width are observed for all interval types, except for unisons and octaves, and they are all multiples of Σ (the difference between the 1/4-comma meantone fifth and the average fifth). Note that 1/4-comma meantone was designed to produce just major thirds, but only 8 of them are just (5:4, about 386 cents).

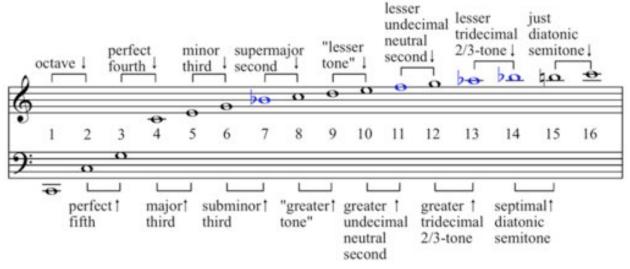
The Pythagorean tuning is characterized by smaller differences because they are multiples of a smaller Σ ($\Sigma \approx 1.96$ cents, the difference between the Pythagorean fifth and the average fifth). Notice that here the fifth is wider than 700 cents, while in most meantone temperaments, including 1/4-comma meantone, it is tempered to a size smaller than 700.

The 5-limit tuning system uses just tones and semitones as building blocks, rather than a stack of perfect fifths, and this leads to even more varied intervals throughout the scale (each kind of interval has three or four different sizes). Note that 5-limit tuning was designed to maximize the number of just intervals, but even in this system some intervals are not just (e.g., 3 fifths, 5 major thirds and 6 minor thirds are not just; also, 3 major and 3 minor thirds are wolf intervals).

The above mentioned symmetric scale 1, defined in the 5-limit tuning system, is not the only method to obtain just intonation. It is possible to construct juster intervals or just intervals closer to the equal-tempered equivalents, but most of the ones listed above have been used historically in equivalent contexts. In particular, the asymmetric version of the 5-limit tuning scale provides a juster value for the minor seventh (9:5, rather than 16:9). Moreover, the tritone (augmented fourth or diminished fifth), could have other just ratios; for instance, 7:5 (about 583 cents) or 17:12 (about 603 cents) are possible alternatives for the augmented fourth (the latter is fairly common, as it is closer to the equal-tempered value of 600 cents). The 7:4 interval (about 969 cents), also known as the harmonic seventh, has been a contentious issue throughout the history of music theory; it is 31 cents flatter than an equal-tempered minor seventh. Some assert the 7:4 is one of the blue notes used in iazz.

In the diatonic system, every interval has one or more *enharmonic equivalents*, such as augmented second for minor third.

Interval root



Intervals in the harmonic series.

Although intervals are usually designated in relation to their lower note, David Cope and Hindemith both suggest the concept of **interval root**. To determine an interval's root, one locates its nearest approximation in the harmonic series. The root of a perfect fourth, then, is its *top* note because it is an octave of the fundamental in the hypothetical harmonic series. The bottom note of every odd diatonically numbered intervals are the roots, as are the tops of all even numbered intervals. The root of a collection of intervals or a chord is thus determined by the interval root of its strongest interval.

As to its usefulness, Cope provides the example of the final tonic chord of some popular music being traditionally analyzable as a "submediant six-five chord" (added sixth chords by popular terminology), or a first inversion seventh chord (possibly the dominant of the mediant V/iii). According the interval root of the strongest interval of the chord (in first inversion, CEGA), the perfect fifth (C–G), is the bottom C, the tonic.

Interval cycles

Interval cycles, "unfold [i.e., repeat] a single recurrent interval in a series that closes with a return to the initial pitch class", and are notated by George Perle using the letter "C", for cycle, with an interval-class integer to distinguish the interval. Thus the diminished-seventh chord would be C3 and the augmented triad would be C4. A superscript may be added to distinguish between transpositions, using 0–11 to indicate the lowest pitch class in the cycle.

Alternative interval naming conventions

As shown below, some of the above mentioned intervals have alternative names, and some of them take a specific alternative name in Pythagorean tuning, five-limit tuning, or meantone temperament tuning systems such as quarter-comma meantone. All the intervals with prefix *sesqui*- are justly tuned, and their frequency ratio, shown in the table, is a superparticular number (or epimoric ratio). The same is true for the octave.

Typically, a comma is a diminished second, but this is not always true (for more details, see Alternative definitions of comma). For instance, in Pythagorean tuning the diminished second is a descending interval (524288:531441, or about -23.5 cents), and the Pythagorean comma is its opposite (531441:524288, or about 23.5 cents). 5-limit tuning defines four kinds of comma, three of which meet the definition of diminished second, and hence are listed in the table below. The fourth one, called syntonic comma (81:80) can neither be regarded as a diminished second, nor as its opposite.

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		Gener	ric names		-	Specific names	
Number	Quality and nur	mber					1/4-
semitone s	Full	Shor t	Other naming convention		Pythagorean tuning	5-limit tuning	comma meanton
	perfect unison or perfect prime	P1					
					descending	lesser diesis (12	8:125)
0	diminished second	d2			Pythagorean comma (524288:531441)	diaschisma (2048:2025) greater diesis (648:625)	
	minor second	m2	semitone	diatonic semitone, minor semitone	limma (256:243)		
1	augmented unison or augmented prime	A1	half tone, half step	chromatic semitone, major semitone	apotome (2187:2048)		
2	major second	M2	tone, who	le tone, whole	sesquio	ctavum (9:8)	
3	minor third	m3				sesquiquintum (6:5)	
4	major third	мз				sesquiquartum	(5:4)
5	perfect fourth	P4			sesquit	ertium (4:3)	
6	diminished fifth	d5	tritonom				
0	augmented fourth	A4	tritone[6]				
7	perfect fifth	P5			sesquia	ilterum (3:2)	
12	perfect octave	P8				duplex (2:1)	

Additionally, some cultures around the world have their own names for intervals found in their music. For instance, 22 kinds of intervals, called shrutis, are canonically defined in Indian classical music.

Latin nomenclature

Up to the end of the 18th century, Latin was used as an official language throughout Europe for scientific and music textbooks. In music, many English terms are derived form Latin. For instance, semitone is from Latin *semitonus*.

The prefix semi- is typically used herein to mean "shorter", rather than "half". Namely, a semitonus, semidiatessaron, semidiapente, semihexachordum, semiheptachordum, or semidiapason, is shorter by one semitone than the corresponding whole interval. For instance, a semiditonus (3 semitones, or about 300 cents) is not half of a ditonus (4 semitones, or about 400 cents), but a ditonus shortened by one semitone. Moreover, in Pythagorean tuning (the most commonly used tuning system up to the 16th century), a semitritonus (d5) is smaller than a tritonus (A4) by one Pythagorean comma (about a quarter of a semitone).

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Number of semitones	Quality and number	Short	Latin nomenclature
0	Perfect unison	P1	unisonus
	Minor second	m2	semitonus
1	Augmented unison	A1	unisonus superflua
	Major second	M2	tonus
2	Diminished third	d3	
	Minor third	m3	semiditonus
3	Augmented second	A2	tonus superflua
	Major third	мз	ditonus
4	Diminished fourth	d4	semidiatessaron
	Perfect fourth	P4	diatessaron
5	Augmented third	АЗ	ditonus superflua
	Diminished fifth	d5	semidiapente, semitritonus
6	Augmented fourth	A4	tritonus
	Perfect fifth	P5	diapente
7	Diminished sixth	d6	semihexachordum
0	Minor sixth	m6	hexachordum minus, semitonus maius cum diapente, tetratonus
8	Augmented fifth	A5	diapente superflua
	Major sixth	М6	hexachordum maius, tonus cum diapente
9	Diminished seventh	d7	semiheptachordum
10	Minor seventh	m7	heptachordum minus, semiditonus cum diapente, pentatonus
10	Augmented sixth	A6	hexachordum superflua
	Major seventh	М7	heptachordum maius, ditonus cum diapente
11	Diminished octave	d8	semidiapason
10	Perfect octave	P8	diapason
12	Augmented seventh	A7	heptachordum superflua

Pitch-class intervals

In post-tonal or atonal theory, originally developed for equal-tempered European classical music written using the twelve-tone technique or serialism, integer notation is often used, most prominently in musical set theory. In this system, intervals are named according to the number of half steps, from 0 to 11, the largest interval class being 6.

In atonal or musical set theory, there are numerous types of intervals, the first being the ordered pitch interval, the distance between two pitches upward or downward.

For instance, the interval from C upward to G is 7, and the interval from G downward to C is ?7. One can also measure the distance between two pitches without taking into account direction with the unordered pitch interval, somewhat similar to the interval of tonal theory.

The interval between pitch classes may be measured with ordered and unordered pitch-class intervals. The ordered one, also called directed interval, may be considered the measure upwards, which, since we are dealing with pitch classes, depends on whichever pitch is chosen as 0.

Generic and specific intervals

In diatonic set theory, specific and generic intervals are distinguished. Specific intervals are the interval class or number of semitones between scale steps or collection members, and generic intervals are the number of diatonic scale steps (or staff positions) between notes of a collection or scale.

Notice that staff positions, when used to determine the conventional interval number (second, third, fourth, etc.), are counted including the position of the lower note of the interval, while generic interval numbers are counted excluding that position. Thus, generic interval numbers are smaller by 1, with respect to the conventional interval numbers.

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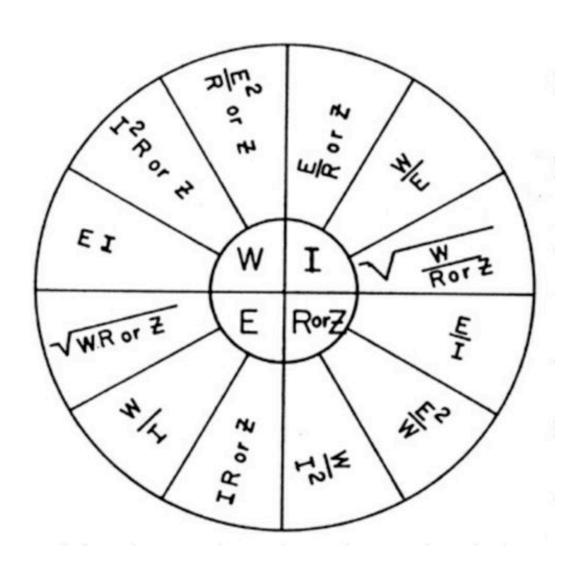
Specific inte	erval		
Number of semitones	Interval class	Generic interval	Diatonic name
0	0	0	Perfect unison
1	1	1	Minor second
2	2	1	Major second
3	3	2	Minor third
4	4	2	Major third
5	5	3	Perfect fourth
6	6	3 4	Augmented fourth
7	5	4	Perfect fifth
8	4	5	Minor sixth
9	3	5	Major sixth
10	2	6	Minor seventh
11	1	6	Major seventh
12	0	7	Perfect octave

Generalizations and non-pitch uses

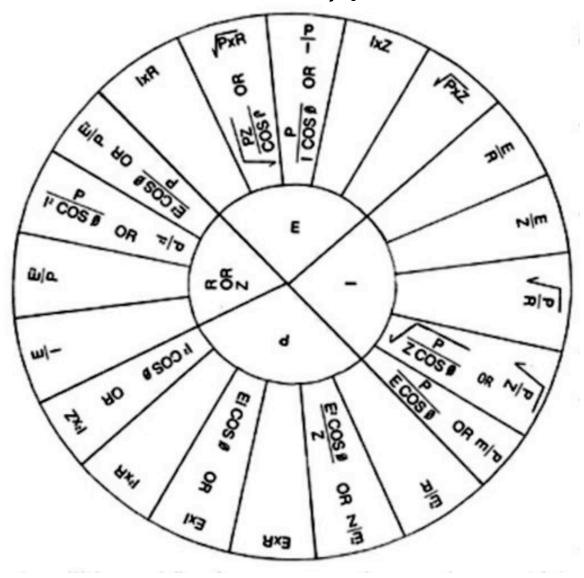


Division of the measure/chromatic scale, followed by pitch/time-point series. The term "interval" can also be generalized to other music elements besides pitch.

David Lewin's *Generalized Musical Intervals and Transformations* uses interval as a generic measure of distance between time points, timbres, or more abstract musical phenomena.



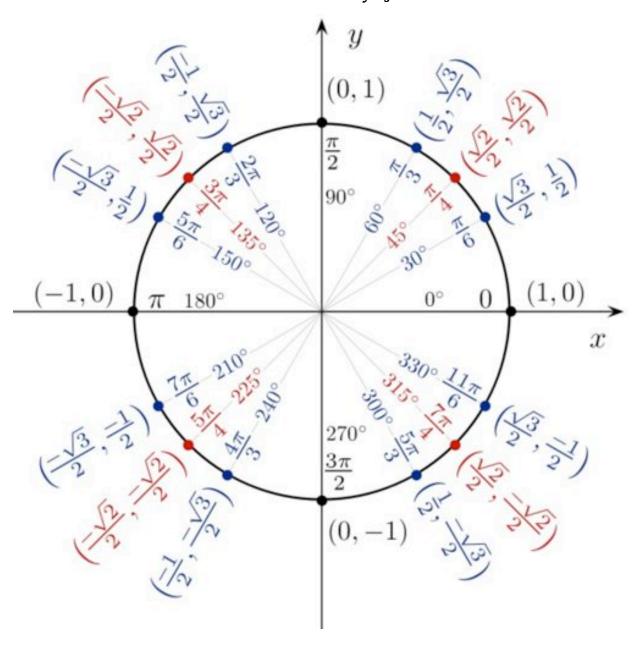
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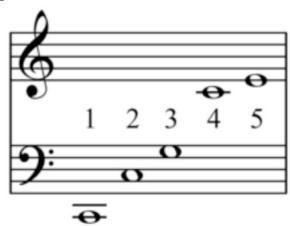
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Just intonation



Harmonic series, partials 1-5 numbered

In music, **just intonation** (sometimes abbreviated as **JI**) or **pure intonation** is any musical tuning in which the frequencies of notes are related by ratios of small whole numbers. Any interval tuned in this way is called a **pure** or **just interval**. The two notes in any just interval are members of the same harmonic series. Frequency ratios involving large integers such as 1024:927 are not generally said to be justly tuned. "Just intonation is the tuning system of the later ancient Greek modes as codified by Ptolemy; it was the aesthetic ideal of the Renaissance theorists; and it is the tuning practice of a great many musical cultures worldwide, both ancient and modern."

Just intonation can be contrasted and compared with equal temperament, which dominates Western instruments of fixed pitch and default MIDI tuning. In equal temperament, all notes are defined as multiples of the same basic interval. Two notes separated by the same number of steps always have exactly the same frequency ratio. However, except for doubled frequencies (octaves), no other intervals are exact ratios of integers. Each just interval differs a different amount from its analogous, equally tempered interval.

Justly tuned intervals can be written as either ratios, with a colon (for example, 3:2), or as fractions, with a solidus (3/2). For example, two tones, one at 300 Hertz (cycles per second), and the other at 200 hertz are both multiples of 100 Hz and as such members of the harmonic series built on 100 Hz.

Examples

Just intonation An A-major scale, followed by three major triads, and then a progression of fifths in just intonation.

Equal temperament An A-major scale, followed by three major triads, and then a progression of fifths in equal temperament. By listening to the above file, and then listening to this one, one might be able to hear a slight buzzing in this file.

Equal temperament and just intonation compared A pair of major thirds, followed by a pair of full major chords. The first in each pair is in equal temperament; the second is in just intonation. Piano sound.

Equal temperament and just intonation compared with square waveform A pair of major chords. The first is in equal temperament; the second is in just intonation. The pair of chords is repeated with a transition from equal temperament to just

temperament between the two chords. In the equal temperament chords a roughness or beating can be heard at about 4 Hz and about 0.8 Hz. In the just intonation triad this roughness is absent. The square waveform makes the difference between equal and just temperaments more obvious.

History

Origins

Harmonic intervals come naturally to horns and vibrating strings.

Recorded history

Pythagorean tuning, perhaps the first tuning system to be theorized in the West,[2] is a system in which all tones can be found using powers of the ratio 3:2, an interval known as a perfect fifth. It is easier to think of this system as a cycle of fifths. Because a series of 12 fifths with ratio 3:2 does not reach the same pitch class it began with, this system uses a wolf fifth at the end of the cycle, to obtain its closure.

Quarter-comma meantone obtained a more consonant tuning of the major and minor thirds, but when limited to twelve keys (see split keys), the system does not close, leaving a very dissonant diminished sixth between the first and last tones of the cycle of fifths.

In Pythagorean tuning, the only highly consonant intervals were the perfect fifth and its inversion, the perfect fourth. The Pythagorean major third (81:64) and minor third (32:27) were dissonant, and this prevented musicians from using triads and chords, forcing them for centuries to write music with relatively simple texture. In late Middle Ages, musicians realized that by slightly tempering the pitch of some notes, the Pythagorean thirds could be made consonant. For instance, if ones decreases by a syntonic comma (81:80) the frequency of E, C-E (a major third), and E-G (a minor third) become just. Namely, C-E is flattened to a justly intonated ratio of $(81:64) \times (80:81) = 5:4$

and at the same time E-G is sharpened to the just ratio of $(32:27) \times (81:80) = 6:5$

The drawback is that the fifths A-E and E-B, by flattening E, become almost as dissonant as the Pythagorean wolf fifth. But the fifth C-G stays consonant, since only E has been flattened (C-E * E-G = (5:4) * (6:5) = 3:2), and can be used together with C-E to produce a C-major triad (C-E-G).

By generalizing this simple rationale, Gioseffo Zarlino, in the late sixteenth century, created the first justly intonated 7-tone (diatonic) scale, which contained pure perfect fifths (3:2), pure major thirds, and pure minor thirds:

 $F \to A \to C \to E \to G \to B \to D$

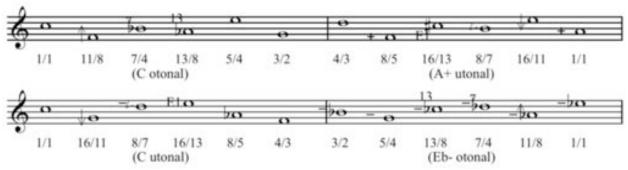
This is a sequence of just major thirds (M3, ratio 5:4) and just minor thirds (m3, ratio 6:5), starting from F:

F + M3 + m3 + M3 + m3 + m3 + m3

Since M3 + m3 = P5 (perfect fifth), i.e. (5:4) * (6:5) = 3:2, this is exactly equivalent to the diatonic scale obtained in 5-limit just intonation.

The Guqin has a musical scale based on harmonic overtone positions. The dots on its soundboard indicate the harmonic positions: 1/8, 1/6, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 7/8.

Modern practice

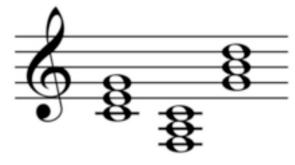


Primary forms of the just tone row from Ben Johnston's *String Quartet No. 7*, mov. 2. Each permutation contains a just chromatic scale, however, transformations (transposition and inversion) produce pitches outside of the primary row form, as already occurs in the inversion of PO.

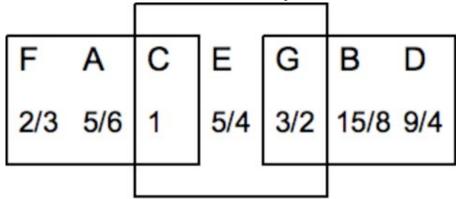
Today, despite the dominance of repertoire composed under equal-tempered systems and the prominence of the piano in musical training, musicians often approach just intonation either by accident or design because it is much easier to find (and hear) a point of stability than a point of calculated instability. A cappella groups that depend on close harmonies, such as barbershop quartets, usually use just intonation by design. Bagpipes, tuned correctly, also use just intonation. There are several conventionally used instruments which, while not associated specifically with just intonation, can handle it quite well, including the trombone and the violin family of instruments.

Diatonic scale

It is possible to tune the familiar diatonic scale or chromatic scale in just intonation in many ways, all of which make certain chords purely tuned and as consonant and stable as possible, and the other chords not accommodated and considerably less stable.



Primary triads in C.



Just tuned diatonic scale derivation.

The prominent notes of a given scale are tuned so that their frequencies form ratios of relatively small integers. For example, in the key of G major, the ratio of the frequencies of the notes G to D (a perfect fifth) is 3/2, while that of G to C (a perfect fourth) is 4/3. Three basic intervals can be used to construct any interval involving the prime numbers 2, 3, and 5 (known as 5-limit just intonation):

- 16:15 = s (Semitone)
- 10:9 = t (Minor tone)
- 9:8 = T (Major tone)

which combine to form:

- 6:5 = Ts (minor third)
- 5:4 = Tt (major third)
- 4:3 = Tts (perfect fourth)
- 3:2 = TTts (perfect fifth)
- 2:1 = TTTttss (octave)

A just diatonic scale may be derived as follows. Suppose we insist that the chords F-A-C, C-E-G, and G-B-D be just major triads (then A-C-E and E-G-B are just minor triads, but D-F-A is not).

Then we obtain this scale (Ptolemy's intense diatonic scale):

	Name	С		D	E	F	G	A	В	С
	Ratio	1/1		9/8	5/4	4/3	3/2	5/3	15/8	2/1
Note	Natural	24		27	30	32	36	40	45	48
	Cents	0		204	386	498	702	884	1088	1200
Step	Name		Т	t	s	Т	t	Т	s	
	Ratio		9/8	10/9	16/15	5 9/8	10/	9 9/8	3 16/	15
	Cents		204	182	112	204	183	2 204	1 11	2

The major thirds are correct, and two minor thirds are right, but D-F is a 32:27 semiditone. Others approaches are possible (see Five-limit tuning), but it is impossible to get all six above-mentioned chords correct. Concerning triads, the triads on I, IV, and V are 4:5:6, the triad on ii is 27:32:40, the triads on iii and vi are 10:12:15, and the triad on vii is 45:54:64.

Twelve tone scale

There are several ways to create a just tuning of the twelve tone scale.

Pythagorean tuning

The oldest known form of tuning, Pythagorean tuning, can produce a twelve tone

scale, but it does so by involving ratios of very large numbers, corresponding to natural harmonics very high in the harmonic series that do not occur widely in physical phenomena. This tuning uses ratios involving only powers of 3 and 2, creating a sequence of just fifths or fourths, as follows:

Note	G♭	Db	Ab	Εb	ВЬ	F	С	G	D	A	E	В	F♯
Ratio	1024:72 9	256:24 3	128:81	32:27	16:9	4:3	1:1	3:2	9:8	27:16	81:64	243:128	729:512
Cents	588	90	792	294	996	498	0	702	204	906	408	1110	612

The ratios are computed with respect to C (the base note). Starting from C, they are obtained by moving six steps to the left and six to the right. Each step consists of a multiplication of the previous pitch by 2/3 (descending fifth), 3/2 (ascending fifth), or their inversions (3/4 or 4/3).

Between the enharmonic notes at both ends of this sequence, is a difference in pitch of nearly 24 cents, known as the Pythagorean comma. To produce a twelve tone scale, one of them is arbitrarily discarded. The twelve remaining notes are repeated by increasing or decreasing their frequencies by a multiple of 2 (the size of one or more octaves) to build scales with multiple octaves (such as the keyboard of a piano). A drawback of Pythagorean tuning is that one of the twelve fifths in this scale is badly tuned and hence unusable (the wolf fifth, either F#-D | if G | is discarded, or B-G | if F# is discarded). This twelve tone scale is fairly close to equal temperament, but it does not offer much advantage for tonal harmony because only the perfect intervals (fourth, fifth, and octave) are simple enough to sound pure. Major thirds, for instance, receive the rather unstable interval of 81/64, sharp of the preferred 5/4 by an 81/80 ratio. The primary reason for its use is that it is extremely easy to tune, as its building block, the perfect fifth, is the simplest and consequently the most consonant interval after the octave and unison.

Pythagorean tuning may be regarded as a "3-limit" tuning system, because the ratios are obtained by using only powers of n, where n is at most 3.

Quarter-comma meantone

The quarter-comma meantone tuning system uses a similar sequence of fifths to produce a twelve tone scale. However, it flattens the fifths by about 5.38 cents with respect to their just intonation, in order to generate justly tuned major thirds (with interval ratio 5:4).

Although this tuning system is based on a just ratio (5:4), it cannot be considered a just intonation system, because most of its intervals are irrational numbers (i.e. they cannot be expressed as fractions of integers). For instance:

- the ratio of most semitones is $S = 8:5^{5/4}$,
- the ratio of most tones is $T=\sqrt{5}:2,$ the ratio of most fifths is $P=5^{1/4}$.

Five-limit tuning

A twelve tone scale can also be created by compounding harmonics up to the fifth. Namely, by multiplying the frequency of a given reference note (the base note) by powers of 2, 3, or 5, or a combination of them. This method is called five-limit tuning. To build such a twelve tone scale, we may start by constructing a table containing fifteen pitches:

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Factor		1/9	1/3	1	3	9
	note	D	Α	E	В	F♯
5	ratio	10/9	5/3	5/4	15/8	45/32
	cents	182	884	386	1088	590
	note	ВЬ	F	С	G	D
1	ratio	16/9	4/3	1	3/2	9/8
	cents	996	498	0	702	204
	note	G♭	Dþ	Ab	Εb	В♭
1/5	ratio	64/45	16/15	8/5	6/5	9/5
	cents	610	112	814	316	1018

The factors listed in the first row and column are powers of 3 and 5, respectively (e.g., $1/9 = 3^{-2}$). Colors indicate couples of enharmonic notes with almost identical pitch. The ratios are all expressed relative to C in the centre of this diagram (the base note for this scale). They are computed in two steps:

- 1. For each cell of the table, a *base ratio* is obtained by multiplying the corresponding factors. For instance, the base ratio for the lower-left cell is $1/9 \times 1/5 = 1/45$.
- 2. The base ratio is then multiplied by a negative or positive power of 2, as large as needed to bring it within the range of the octave starting from C (from 1/1 to 2/1). For instance, the base ratio for the lower left cell (1/45) is multiplied by 26, and the resulting ratio is 64/45, which is a number between 1/1 and 2/1.

Note that the powers of 2 used in the second step may be interpreted as ascending or descending octaves. For instance, multiplying the frequency of a note by 26 means increasing it by 6 octaves. Moreover, each row of the table may be considered to be a sequence of fifths (ascending to the right), and each column a sequence of major thirds (ascending upward). For instance, in the first row of the table, there is an ascending fifth from D and A, and another one (followed by a descending octave) from A to E. This suggests an alternative but equivalent method for computing the same ratios. For instance, one can obtain A, starting from C, by moving one cell to the left and one upward in the table, which means descending by a fifth and ascending by a major third:

$$\frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}.$$

Since this is below C, one needs to move up by an octave to end up within the desired range of ratios (from 1/1 to 2/1):

$$\frac{5}{6} \cdot \frac{2}{1} = \frac{10}{6} = \frac{5}{3}$$

A 12 tone scale is obtained by removing one note for each couple of enharmonic notes. This can be done in at least three ways, which have in common the removal of G?, according to a convention which was valid even for C-based Pythagorean and

1/4-comma meantone scales. We show here only one of the possible strategies (the others are discussed in Five-limit tuning). It consists of discarding the first column of the table (labeled "1/9"). The resulting 12-tone scale is shown below:

Factor		1/9	1/3	1	3	9
	note	D	Α	E	В	F♯
5	ratio	10/9	5/3	5/4	15/8	45/32
	cents	182	884	386	1088	590
1	note	ВЬ	F	C	G	D
	ratio	16/9	4/3	1	3/2	9/8
	cents	996	498	0	702	204
1/5	note	G♭	Dþ	Ab	Εb	В♭
	ratio	64/45	16/15	8/5	6/5	9/5
	cents	610	112	814	316	1018

Extension of the twelve tone scale

The table above uses only low powers of 3 and 5 to build the base ratios. However, it can be easily extended by using higher positive and negative powers of the same numbers, such as 52 = 25, 5?2 = 1/25, 33 = 27, or 3?3 = 1/27. A scale with 25, 35 or even more pitches can be obtained by combining these base ratios (see Five-limit tuning for further details).

Indian scales

In Indian music, the just diatonic scale described above is used, though there are different possibilities, for instance for the 6th pitch (Dha), and further modifications may be made to all pitches excepting Sa and Pa.

Note	Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
Ratio	1/1	9/8	5/4	4/3	3/2	5/3 or 27/16	15/8	2/1
Cents	0	204	386	498	702	884 or 906	1088	1200

Some accounts of Indian intonation system cite a given 22 Srutis. According to some musicians, one has a scale of a given 12 pitches and ten in addition (the tonic, Shadja (Sa), and the pure fifth, Pancham (Pa), are inviolate):

Note	С	Db	Db	D	D	Eb	Eb	E	E	F	F	FF
	1	256	16	10	9	32	6	5	81	4	27	45
Ratio	1	243	15	9	8	27	5	$\overline{4}$	64	$\bar{3}$	20	32
Cents	0	90	112	182	204	294	316	386	408	498	520	590
Note	F#	G	Ab	Ab	A	А	B♭	Вь	В		В	С
								- 0	4.4		110	
	64	3	128	8	5	27	16	9	15) 2	243	2
Ratio	$\frac{64}{45}$	$\frac{3}{2}$	$\frac{128}{81}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{27}{16}$	$\frac{16}{9}$	$\frac{9}{5}$	$\frac{15}{8}$		28	$\frac{2}{1}$

Where we have *two* ratios for a given letter name, we have a difference of 81:80 (or 22 cents), which is known as the syntonic comma. One can see the symmetry, looking at it from the tonic, then the octave.

(This is just one example of "explaining" a 22-Sruti scale of tones. There are many different explanations.)

Practical difficulties

Some fixed just intonation scales and systems, such as the diatonic scale above, produce wolf intervals. The above scale allows a minor tone to occur next to a semitone which produces the awkward ratio 32:27 for F:D, and still worse, a minor tone next to a fourth giving 40:27 for A:D. Moving D down to 10/9 alleviates these difficulties but creates new ones: G:D becomes 27:20, and B:G becomes 27:16.

One can have more frets on a guitar to handle both A's, 9/8 with respect to G and 10/9 with respect to G so that C:A can be played as 6:5 while D:A can still be played as 3:2. 9/8 and 10/9 are less than 1/53 octave apart, so mechanical and performance considerations have made this approach extremely rare. And the problem of how to tune chords such as C-E-G-A-D is left unresolved (for instance, A could be 4:3 below D (making it 9/8, if G is 1) or 4:3 above E (making it 10/9, if G is 1) but not both at the same time, so one of the fourths in the chord will have to be an out-of-tune wolf interval). However the frets may be removed entirely—this, unfortunately, makes intune fingering of many chords exceedingly difficult, due to the construction and mechanics of the human hand—and the tuning of most complex chords in just intonation is generally ambiguous.

Some composers deliberately use these wolf intervals and other dissonant intervals as a way to expand the tone color palette of a piece of music. For example, the extended piano pieces *The Well-Tuned Piano* by LaMonte Young, and *The Harp Of New Albion* by Terry Riley use a combination of very consonant and dissonant intervals for musical effect. In "Revelation," Michael Harrison goes even farther, and uses the tempo of beat patterns produced by some dissonant intervals as an integral part of several movements.

For many instruments tuned in just intonation, one can not change keys without retuning the instrument. For instance, a piano tuned in just intonation intervals and a minimum of wolf intervals for the key of G, then only one other key (typically E-flat) can have the same intervals, and many of the keys have a very dissonant and unpleasant sound. This makes modulation within a piece, or playing a repertoire of pieces in different keys, impractical to impossible.

Synthesizers have proven a valuable tool for composers wanting to experiment with just intonation. Many commercial synthesizers provide the ability to use built-in just intonation scales or to program your own. Wendy Carlos used a system on her 1986 album *Beauty in the Beast*, where one electronic keyboard was used to play the notes,

and another used to instantly set the root note to which all intervals were tuned, which allowed for modulation. On her 1987 lecture album *Secrets of Synthesis* there are audible examples of the difference in sound between equal temperament and just intonation.

Singing

The human voice is among the most pitch-flexible instruments in common use. Pitch can be varied with no restraints and adjusted in the midst of performance, without needing to retune. Although the explicit use of just intonation fell out of favour concurrently with the increasing use of instrumental accompaniment (with its attendant constraints on pitch), most a cappella ensembles naturally tend toward just intonation because of the comfort of its stability. Barbershop quartets are a good example of this.

Western composers

Most composers don't specify how instruments are to be tuned, although historically most have assumed one tuning system which was common in their time; in the 20th century most composers assumed equal temperament would be used. However, a few have specified just intonation systems for some or all of their compositions, including John Luther Adams, Glenn Branca, Martin Bresnick, Wendy Carlos, Lawrence Chandler, Tony Conrad, Fabio Costa, Stuart Dempster, David B. Doty, Arnold Dreyblatt, Kyle Gann, Kraig Grady, Lou Harrison, Michael Harrison, Ben Johnston, Elodie Lauten, György Ligeti, Douglas Leedy, Pauline Oliveros, Harry Partch, Robert Rich, Terry Riley, Marc Sabat, Wolfgang von Schweinitz, Adam Silverman, James Tenney, Michael Waller, Daniel James Wolf, and La Monte Young. Eivind Groven's tuning system was schismatic temperament, which is capable of far closer approximations to just intonation consonances than 12-note equal temperament or even meantone temperament, but still alters the pure ratios of just intonation slightly in order to achieve a simpler and more flexible system than true just intonation

Music written in just intonation is most often tonal but need not be; some music of Kraig Grady and Daniel James Wolf uses just intonation scales designed by Erv Wilson explicitly for a consonant form of atonality, and Ben Johnston's *Sonata for Microtonal Piano* (1964) uses serialism to achieve an atonal result. Composers often impose a limit on how complex the ratios used are: for example, a composer may write in "7-limit JI", meaning that no prime number larger than 7 features in the ratios they use. Under this scheme, the ratio 10/7, for example, would be permitted, but 11/7 would not be, as all non-prime numbers are octaves of, or mathematically and tonally related to, lower primes (example: 12 is a double octave of 3, while 9 is a square of 3). Yuri Landman derived a just intoned musical scale from an initially considered atonal prepared guitar playing technique based on adding a third bridge under the strings. When this bridge is positioned in the noded positions of the harmonic series the volume of the instrument increases and the overtone becomes clear and has a consonant relation to the complementary opposed string part creating a harmonic multiphonic tone.

Staff notation

bb b	4 #	×	Pythagorean series of fifths – the open strings (c g d a e)
₽q#×	B 6 B	Ĥ	lowers / raises by a syntonic comma 81:80 = circa 21.5 cents
}	\$ 6	Ĥ	lowers / raises by two syntonic commas circa 43 cents
L	١		lowers / raises by a septimal comma 64:63 = circa 27.3 cents
Ļ	F		lowers / raises by two septimal commas circa \$4.5 cents
+	4		raises / lowers by an 11-limit undecimal quarter-tone 33:32 = circa 53.3 cents
4	#		lowers / raises by a 13-limit tridecimal third-tone 27:26 = circa 65.3 cents
*	*		lowers / raises by a 17-limit schisma 256:255 = circa 6.8 cents
,	`		raises / lowerss by a 19-limit schisma 513:512 = circa 3.4 cents
1	↓		raises / lowers by a 23-limit comma 736: 729 = circa 16.5 cents

Ex. 1: Legend of the HE Accidentals



Pythagorean diatonic scale on C. Johnston's notation.



Just intonation diatonic scale on C. Johnston's notation (Pythagorean major scale in Helmholtz-Ellis notation).



Just intonation diatonic scale on C. Helmholtz-Ellis notation.



Just harmonic seventh chord on C. 7th: 968.826 cents, a septimal quarter tone lower than $B \, b$.

Originally a system of notation to describe scales was devised by Hauptmann and modified by Helmholtz (1877) in which Pythagorean notes are started with and subscript numbers are added indicating how many commas (81/80, syntonic comma) to lower by. For example the Pythagorean major third on C is C+E) while the just major third is C+E1). A similar system was devised by Carl Eitz and used in Barbour (1951) in which Pythagorean notes are started with and positive or negative superscript numbers are added indicating how many commas (81/80, syntonic comma) to adjust by. For example, the Pythagorean major third on C is C-E0 while the just major third is C-E^-1.

While these systems allow precise indication of intervals and pitches in print, more recently some composers have been developing notation methods for Just Intonation using the conventional five-line staff. James Tenney, amongst others, preferred to combine JI ratios with cents deviations from the equal tempered pitches, indicated in a legend or directly in the score, allowing performers to readily use electronic tuning devices if desired.[16] Beginning in the 1960s, Ben Johnston had proposed an alternative approach, redefining the understanding of conventional symbols (the seven "white" notes, the sharps and flats) and adding further accidentals, each designed to extend the notation into higher prime limits. Johnston's method is based on a diatonic C Major scale tuned in JI, in which the interval between D (9/8 above C) and A (5/3 above C) is one syntonic comma less than a Pythagorean perfect fifth 3:2. To write a perfect fifth, Johnston introduces a pair of symbols representing this comma, + and ?. Thus, a series of perfect fifths beginning with F would proceed C G D A+ E+ B+. The three conventional white notes A E B are tuned as Ptolemaic major thirds (5:4) above F C G respectively. Johnston introduces new symbols for the septimal (67 & 97), undecimal (up & down), tridecimal (13 & 31), and further prime

extensions to create an accidental based exact JI notation for what he has named "Extended Just Intonation". For example, the Pythagorean major third on C is C-E+ while the just major third is C-E \flat .

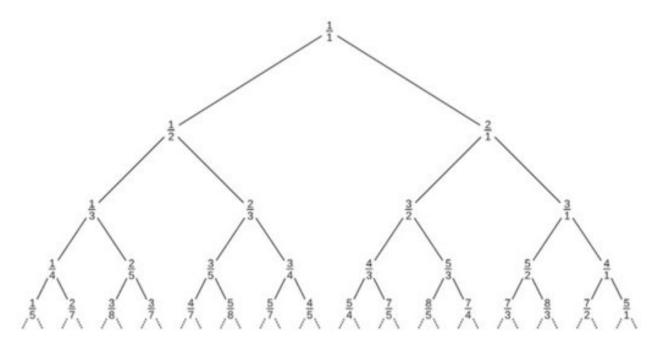
In the years 2000-2004, Marc Sabat and Wolfgang von Schweinitz worked together in Berlin to develop a different accidental based method, the Extended Helmholtz-Ellis JI Pitch Notation.[18] Following the method of notation suggested by Helmholtz in his classic "On the Sensations of Tone as a Physiological Basis for the Theory of Music", incorporating Ellis' invention of cents, and following Johnston's step into "Extended JI", Sabat and Schweinitz consider each prime dimension of harmonic space to be represented by a unique symbol. In particular they take the conventional flats, naturals and sharps as a Pythagorean series of perfect fifths. Thus, a series of perfect fifths beginning with F proceeds C G D A E B F# and so on. The advantage for musicians is that conventional reading of the basic fourths and fifths remains familiar. Such an approach has also been advocated by Daniel James Wolf. In the Sabat-Schweinitz design, syntonic commas are marked by arrows attached to the flat, natural or sharp sign, Septimal Commas using Giuseppe Tartini's symbol, and Undecimal Quartertones using the common practice quartertone signs (a single cross and backwards flat). For higher primes, additional signs have been designed. To facilitate quick estimation of pitches, cents indications may be added (downward deviations below and upward deviations above the respective accidental). The convention used is that the cents written refer to the tempered pitch implied by the flat, natural, or sharp sign and the note name. A complete legend and fonts for the notation (see samples) are open source and available from Plainsound Music Edition. For example, the Pythagorean major third on C is C-E by while the just major third is C-E ♭ -arrow-down.



Staff notation of partials 1, 3, 5, 7, 11, 13, 17, and 19 on C using Johnston's notation. One of the great advantages of such notation systems is that they allow the natural harmonic series to be precisely notated.

Just Intonation and the Stern-Brocot Tree

Since I work in Just Intonation I'm always looking for new ways to organize and think about ratios. Recently I came across the mathematical idea called the Stern-Brocot Tree from a very nice video posted by a friend. This is a way to represent all whole number ratios in their lowest terms as a binary tree growing from 1:1. Any ratio can be reached through a number of successive left or right moves down the tree. For specifics please consult the above links of other sources as I will not cover them here.



I decided to implement ways of traversing the tree in a SuperCollider class (this depends on the MathLib and dewdrop_lib Quarks). This works based on three representations of any position: the actual ratio, a matrix, and a continued fraction. By jumping back and forth between these three forms, you can easily move left, right, up, compare properties of different ratios, etc. It is functional but pretty rough and subject to major refactoring whenever I get around to it.

My current focus with the tree involves ratios between 1:1 and 2:1 (otonality) and the difference tones between them. Since playing with this I've noticed a few interesting things (that I'm sure others have seen before too).

Paths

Within the tree structure we have this notion of a path, which is a sequence of right (R) and left (L) steps downward. In Just Intonation theory, one of the most important types of ratios is the superparticular ((n+1)/n). If we look at the tree we can see that each of these are reached by starting from 1:1, moving R to 2:1, followed by consecutive L moves. This means every superparticular ratio falls in the path of RLLLLL.... etc. This can be represented as an array based on the continued fraction form of the ratio. After studying this representation we can see that each element is the number of steps in a direction, starting with a R, and alternating LRLR after that.

For example, the path to 4:3 is RLL and the continued fraction is [1,2] (one R followed by 2 Ls). 5:4 would be [1,3], 6:5 is [1,4] and so on.

Difference Tones against 1:1

Say you want to know what difference tone you will get between a given fundamental and some ratio above it where both pitches are in the same octave (between 1:1 and 2:1) as each other. The math for that is simple: just subtract the two. However I've found that there is a correlation between the difference tone of any ratio between 1:1 and 2:1 and its path within the tree. Take (3/2) - (1/1) = 1/2. The path to 3:2 is [1,1] and the path to 1:2 is [0,1] (the 0 means that we do not have any R moves, so our first move is an L). How about (5/3) - (1/1) = 2/3. 5:3 is [1,1,1] and 2:3 is [0,1,1]. Seeing a pattern? It turns out that the path of the difference tone against 1:1 for any ratio between 1:1 and 2:1 is the path to the ratio with the first R move set to 0. For 7:4 [1,1,2], the difference against 1:1 is [0,1,2].

Axis and Mirroring

The next concept is that of a mirror. All this means is that you take the opposite path (if you moved R before, move L this time) around a central ratio that we will call an axis. If our axis is 1:1, the mirror is simply the reciprocal (the mirror of 2:3 is 3:2). Why do I have a new term if this already has a name? Because if we mirror around ratios other than 1:1 the result is not the reciprocal. Using 1:1 as an axis is the special case where the mirror and the reciprocal are the same thing.

Lets look at this in terms of the path to a ratio first. The mirror of 2:1 [1] around the axis 1:1 is 1:2 [0,1]. The mirror of 3:2 [1,1] around 1:1 is 2:3 [0,1,1]. Like we said earlier, if you want to move L first instead of R, you make the first slot in the array 0. With the difference between a ratio and 1:1 we replaced the first element with 0. Here we insert a 0 before the remaining operations if we are going left of 1:1, and we remove the 0 if we are going right of 1:1.

But why? Well, the path to 1:1 is [0] since we don't need to go anywhere to get to it, but that still doesn't tell us the whole story.

For this we have to understand how a change of direction in the tree is achieved. First lets look at the paths to the whole numbers. 2:1 [1], 3:1 [2], 4:1 [3]; each of these are consecutive R moves from 1:1 [0]. Now look at 3:2 [1,1] and 3:1 [2] in the tree. They are both children of 2:1 and are also mirrors around 2:1 since one is to the L and the other to the R. We can represent this in the path by separating 2:1?s path from the paths of both ratios. If we think for a second about the path in terms of Rs and Ls rather than just numbers:

it becomes rather obvious that we are simply moving from the first R. So 3:2 and 3:1 are mirrors around the axis 2:1 since their paths are inversions of each other starting from the axis. Here's one more example with 3:2 as the axis:

Now that we can think of it as Rs and Ls, we can manipulate the continues fraction to represent that:

 $3:2 = \{1,1\}$ $7:5 = [\{1,(1\}+1),1]$ $8:5 = [\{1,1\},1,1]$

Difference Tones against 3:2

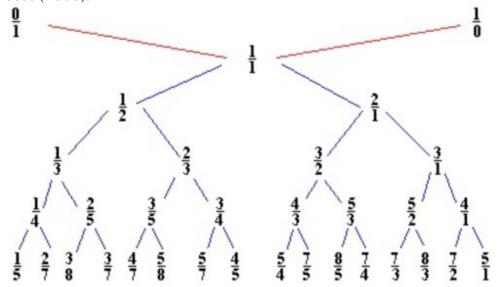
In exploring more of the difference tones within the tree I found a very special property of 3:2. The difference tone of 3:2 and any of its children is the same as the difference between 3:2 and the child's mirror around 3:2. Let's take the ratios we were just looking at. The difference between 3/2 and 7/5 is 1/10. The mirror of 7:5 around the axis 3:2 is 8:5. The difference between 8/5 and 3/2 is 1/10. The same is true for any child of 3:2 and its mirror. Also interesting is that the denominator of all mirrors around 3:2 will be the same (e.g. 7/5, 8/5).

- source: http://coleingraham.com/tag/just-intonation/

Stern-Brocot Tree

0/1 is a fraction while 1/0 is not. However, using it as such helps describe a way to get all possible positive fractions arranged in a very nice manner. So disguising 1/0 as a fraction has a very good excuse.

We already know how to obtain the mediant of two given fractions. The mediant of two fractions m1/n1 and m2/n2 is, by definition, the fraction (m1 + m2)/(n1 + n2). So, for example, from 0/1 and 1/0 we get 1/1. The mediant of 0/1 and 1/1 is 1/2 while the mediant of 1/1 and 1/0 is 2/1. On the next stage of the construction, we form four new fractions: 1/3 from 0/1 and 1/2, 2/3 from 1/2 and 1/1, 3/2 from 1/1 and 2/1, and, finally, the mediant of 2/1 and 1/0 which is 3/1. Continuing this way we get an infinite tree known as the Stern-Brocot tree because it was discovered independently by the German mathematician Moriz Stern (1858) and by the French clock maker Achille Brocot (1860).



A remarkable thing about Stern-Brocot tree is that it contains all possible non-negative fractions expressed in lowest terms and each exactly once. Just to remind, a

fraction m/n is in lowest terms iff m and n are coprime, i.e, iff gcd(n,m) = 1. To prove this we'll need a series of facts of which the following is the most fundamental: if m1/n1 and m2/n2 are two consecutive (consecutive in the sense that one of them is a direct descendant of the other) fractions at any stage of the construction then (1)

m2n1 - m1n2 = 1

This relation is true for the initial fractions: $1 \cdot 1 - 0 \cdot 0 = 1$. Also, assuming (1) holds for two fractions m1/n1 and m2/n2, their mediant should satisfy the following two:

n1(m1 + m2) - (n1 + n2)m1 = 1, and

m2(n1 + n2) - (m1 + m2)n2 = 1

both of which are equivalent to (1).

Also, if m1/n1 < m2/n2 then

m1/n1 < (m1 + m2)/(n1 + n2) < m2/n2,

from which it follows that the construction of the tree preserves the natural order between the rationals. Therefore, it's impossible to obtain the same fraction twice.

And there is the last point: let a/b be a fraction in lowest terms with a and b positive. I wish to show that this fraction will appear somewhere on the tree. So long as it did not, we shall have

m1/n1 < a/b < m2/n2

for a couple of fractions m1/n1 and m2/n2 satisfying (1). These are rewritten as n1a - m1b > 0 and m2b - n2a > 0,

which imply

 $n1a - m1b \ge 1$ and $m2b - n2a \ge 1$.

from where

 $(m2 + n2)(n1a - m1b) + (m1 + n1)(m2b - n2a) \ge m1 + n1 + m2 + n2$.

An application of (1) yields

 $a + b \ge m1 + n1 + m2 + n2$.

with inevitable conclusion that after at most a+b steps of computing mediants one of them will be equal to a/b.

P.S

Prof. W.McWorter has offered the following clarification to the proof:

At the root of the tree the minimum numerator-denominator sum (nd sum) is 2. At the first level of the tree the minimum nd sum is 3, and at the n-th level of the tree the minimum nd sum is n + 2. To see this, every fraction at the n-th level is the mediant of a fraction at the (n - 1)-st level and one at a higher level. The nd sum of a fraction at the (n - 1)-st level is at least n + 1 and the nd sum of a fraction at a higher level is at least 1. Hence the nd sum of their mediant is at least n + 2.

Thus, if a/b is a fraction in lowest terms not equal to any fraction in the tree, then it lies strictly between two consecutive fractions, one of which is at level a+b-1 of the tree (consecutive here is restricted to all fractions constructed up to and including level a+b-1; none constructed beyond this level are considered). Hence by the same calculations $a+b \ge a+b+1$, a contradiction.

Remark

Pierre Lamothe from Canada informed me recently of a property of the Stern-Brocot tree I was unaware of. Pierre discovered that property in his research on music and harmony.

Let's associate with any irreducible fraction p/q the number w(p/q) = 1/pq - its simplicity. The property discovered by Pierre states that the sum of simplicities of all fractions in any row of the Stern-Brocot tree equals 1! We can easily see that this is true for a few first rows:

Row number	Tree members	Sum of simplicities
0	1/1	1/1
1	1/2, 2/1	1/2 + 1/ 2
2	1/3, 2/3, 3/2, 3/1	1/3 + 1/6 + 1/6 + 1/3
3	1/4, 2/5, 3/5, 3/4, 4/3, 5/3,	1/4 + 1/10 + 1/15 + 1/12 + 1/12 + 1/15 +
3	5/2, 4/1	1/10 + 1/4

The inductive proof is based on a fact (proved on later pages) that, for any fraction p/q, two fractions p/(p+q) and (p+q)/q are located just one row below that of p/q. We have an obvious identity:

W(p/(p+q)) + W((p+q)/q) = 1/p(p+q) + 1/q(p+q) = 1/pq = W(p/q),

from which it follows that if the assertion holds for one row, it holds for the next one as well.

Pierre also introduced another function W defined for the members of the tree. For an irreducible fraction p/q, Let N(p/q) denote the row of the tree that contains p/q. We start with N(1/1) = 0, N(1/2) = 1, N(2/1) = 1, N(1/3) = 2, and so on. Then the weighted simplicity W(p/q) is defined as

 $W(p/q) = w(p/q) \cdot 2^{-N(p/q)-1}$

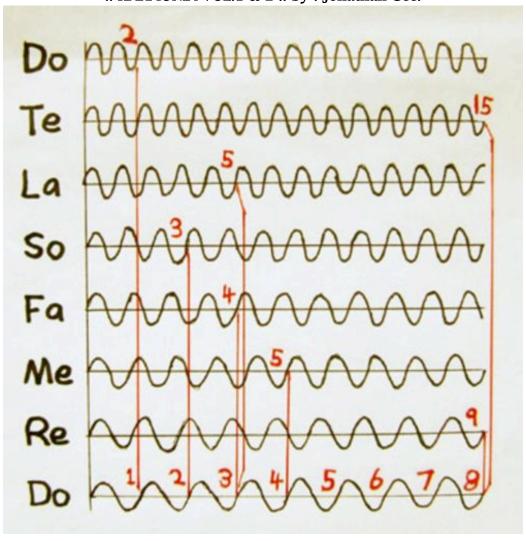
From the previous statement it then follows that the sum of all weighted simplicities of the fractions on the tree is equal to 1!

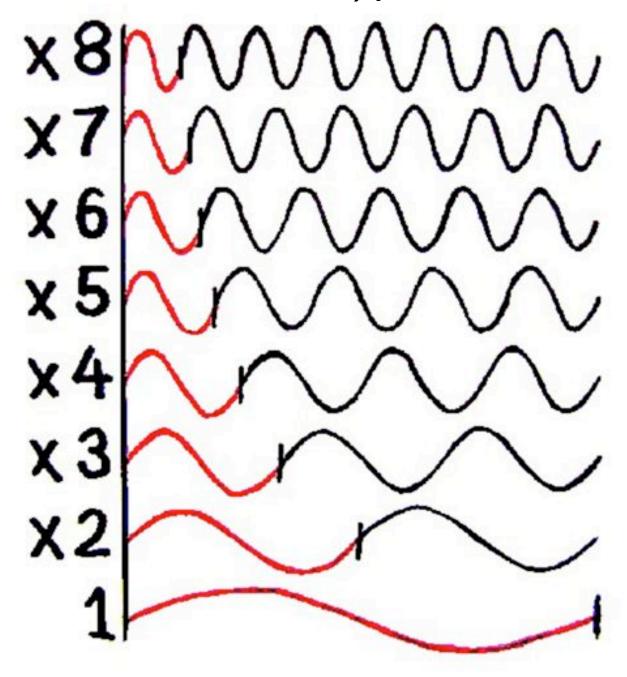
References

- 1. R.Graham, D.Knuth, O.Patashnik, Concrete Mathematics, 2nd edition, Addison-Wesley, 1994.
- 2. Brian Hayes, Computing Science On the Teeth of Wheels, American Scientist, July-August, Volume 88, No. 4,

-source: http://www.cut-the-knot.org/blue/Stern.shtml

:: HARMONIA VOL.1 & 2 :: by : Jonathan Gee.



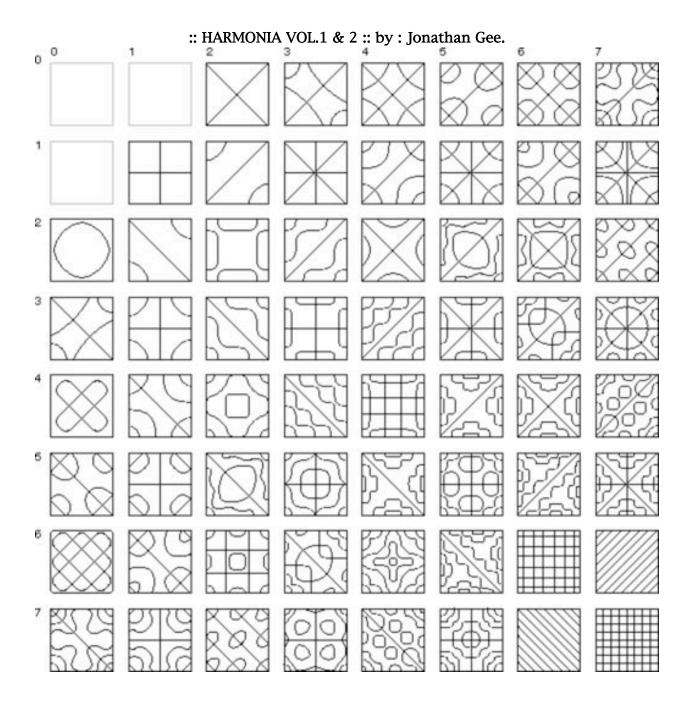


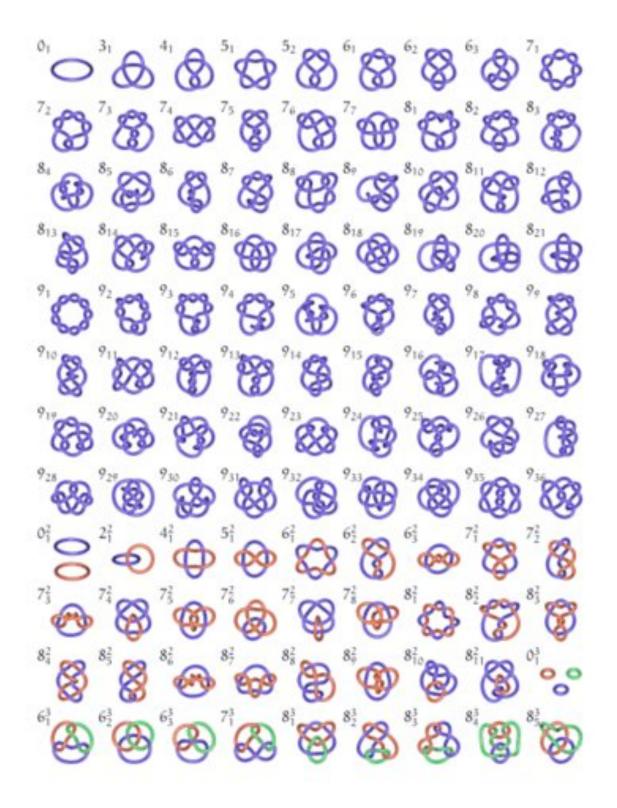
	Frequencies for equal-tempered scale	e, A4 = 440 Hz
Note	Frequency (Hz)	Wavelength (cm)
CO	16.35	2109.89
C#0/Db0	17.32	1991.47
D0	18.35	1879.69
D#0/Eb0	19.45	1774.20
EO	20.60	1674.62
FO	21.83	1580.63
F#0/Gb0	23.12	1491.91
G0	24.50	1408.18
G#0/Ab0	25.96	1329.14
A0	27.50	1254.55
A#0/Bb0	29.14	1184.13
ВО	30.87	1117.67
C1	32.70	1054.94
C#1/Db1	34.65	995.73
D1	36.71	939.85
D#1/Eb1	38.89	887.10
E1	41.20	837.31
F1	43.65	790.31
F#1/Gb1	46.25	745.96
G1	49.00	704.09
G#1/Ab1	51.91	664.57
A1	55.00	627.27
A#1/Bb1	58.27	592.07
B1	61.74	558.84
C2	65.41	527.47
C#2/Db2	69.30	497.87
D2	73.42	469.92
D#2/Eb2	77.78	443.55
E2	82.41	418.65
F2	87.31	395.16
F#2/Gb2	92.50	372.98
G2	98.00	352.04
G#2/Ab2	103.83	332.29
A2	110.00	313.64
A#2/Bb2	116.54	296.03
B2	123.47	279.42
C3	130.81	263.74
C#3/Db3	138.59	248.93
D3	146.83	234.96
D#3/Eb3	155.56	221.77
E3	164.81	209.33
F3	174.61	197.58
F#3/Gb3	185.00	186.49
G3	196.00	176.02
G#3/Ab3	207.65	166.14
A3	220.00	156.82
A#3/Bb3	233.08	148.02
В3	246.94	139.71

	HARMONIA VOL.1 & 2 by . Julia	auian Gee.
C4	261.63	131.87
C#4/Db4	277.18	124.47
D4	293.66	117.48
D#4/Eb4	311.13	110.89
E4	329.63	104.66
F4	349.23	98.79
F#4/Gb4	369.99	93.24
G4	392.00	88.01
G#4/Ab4	415.30	83.07
A4	440.00	78.41
A#4/Bb4	466.16	74.01
B4	493.88	69.85
C5	523.25	65.93
C#5/Db5	554.37	62.23
D5	587.33	58.74
D#5/Eb5	622.25	55.44
E5	659.25	52.33
F5	698.46	49.39
F#5/Gb5	739.99	46.62
G5	783.99	44.01
G#5/Ab5	830.61	41.54
A5	880.00	39.20
A#5/Bb5		
	932.33	37.00
B5	987.77	34.93
C6	1046.50	32.97
C#6/Db6	1108.73	31.12
D6	1174.66	29.37
D#6/Eb6	1244.51	27.72
E6	1318.51	26.17
F6	1396.91	24.70
F#6/Gb6	1479.98	23.31
G6	1567.98	22.00
G#6/Ab6	1661.22	20.77
A6	1760.00	19.60
A#6/Bb6	1864.66	18.50
B6	1975.53	17.46
C7	2093.00	16.48
C#7/Db7	2217.46	15.56
D7	2349.32	14.69
D#7/Eb7	2489.02	13.86
E7	2637.02	13.08
F7	2793.83	12.35
F#7/Gb7	2959.96	11.66
G7	3135.96	11.00
G#7/Ab7	3322.44	10.38
A7	3520.00	9.80
A#7/Bb7	3729.31	9.25
	3951.07	8.73
B7		
C8	4186.01	8.24
C#8/Db8	4434.92	7.78
D8	4698.63	7.34
D#8/Eb8	4978.03	6.93

E8	5274.04	6.54
F8	5587.65	6.17
F#8/Gb8	5919.91	5.83
G8	6271.93	5.50
G#8/Ab8	6644.88	5.19
A8	7040.00	4.90
A#8/Bb8	7458 . 62	4.63
B8	7902.13	4.37

("Middle C" is C4) (To convert lengths in cm to inches, divide by 2.54) (Speed of Sound = 345 m/s = 1130 ft/s = 770 miles/hr)





Quarter-comma meantone

Quarter-comma meantone, or **1/4-comma meantone**, was the most common meantone temperament in the sixteenth and seventeenth centuries, and was sometimes used later. This method is a variant of Pythagorean tuning. The difference is that in this system the perfect fifth is flattened by one quarter of a syntonic comma, with respect to its just intonation used in Pythagorean tuning (frequency ratio 3:2). The purpose is to obtain justly intoned major thirds (with a frequency ratio equal to 5:4). It was described by Pietro Aron (also spelled Aaron), in his *Toscanello de la Musica* of 1523, by saying the major thirds should be tuned to be "sonorous and just, as united as possible." Later theorists Gioseffo Zarlino and Francisco de Salinas described the tuning with mathematical exactitude.

Construction

In a meantone tuning, we have diatonic and chromatic semitones, with the diatonic semitone larger. In Pythagorean tuning, these correspond to the Pythagorean limma and the Pythagorean apotome, only now the apotome is larger. In any meantone or Pythagorean tuning, a whole tone is composed of one semitone of each kind, a major third is two whole tones and therefore consists of two semitones of each kind, a perfect fifth of meantone contains four diatonic and three chromatic semitones, and an octave seven diatonic and five chromatic semitones, it follows that:

- Five fifths down and three octaves up make up a diatonic semitone, so that the Pythagorean limma is tempered to a diatonic semitone.
- Two fifths up and an octave down make up a whole tone consisting of one diatonic and one chromatic semitone.
- Four fifths up and two octaves down make up a major third, consisting of two diatonic and two chromatic semitones, or in other words two whole tones. Thus, in Pythagorean tuning, where sequences of just fifths (frequency ratio 3:2) and octaves are used to produce the other intervals, a whole tone is

$$\frac{(3/2)^2}{2} = \frac{9/4}{2} = \frac{9}{8},$$

and a major third is

$$\frac{(3/2)^4}{2^2} = \frac{81/16}{4} = \frac{81}{64} \approx \frac{5}{4} = \frac{5 \cdot 16}{4 \cdot 16} = \frac{80}{64}.$$

An interval of a seventeenth, consisting of sixteen diatonic and twelve chromatic semitones, such as the interval from D4 to F6?, can be equivalently obtained using either

- a stack of four fifths (e.g. D4—A4—E5—B5—F6#), or
- a stack of two octaves and one major third (e.g. D4—D5—D6—F6#).

This large interval of a seventeenth contains (5 + (5 - 1) + (5 - 1) + (5 - 1) = 20 - 3 = 17 staff positions). In Pythagorean tuning, the size of a seventeenth is defined using a stack of four justly tuned fifths (frequency ratio 3:2):

$$\left(\frac{3}{2}\right)^4 = \frac{81}{16} = \frac{80}{16} \cdot \frac{81}{80} = 5 \cdot \frac{81}{80}.$$

In quarter-comma meantone temperament, where a just major third (5:4) is required, a slightly narrower seventeenth is obtained by stacking two octaves (4:1) and a major third:

$$\frac{4}{1} \cdot \frac{5}{4} = 5.$$

By definition, however, a seventeenth of the same size (5:1) must be obtained, even in quarter-comma meantone, by stacking four fifths. Since justly tuned fifths, such as those used in Pythagorean tuning, produce a slightly wider seventeenth, in quartercomma meantone the fifths must be slightly flattened to meet this requirement. Letting x be the frequency ratio of the flattened fifth, it is desired that four fifths have a ratio of 5:1,

$$x^4 = 5$$
,

which implies that a fifth is

$$x = \sqrt[4]{5} = 5^{1/4},$$

a whole tone, built by moving two fifths up and one octave down, is

$$\frac{x^2}{2} = \frac{\sqrt{5}}{2}$$

and a diatonic semitone, built by moving three octaves up and five fifths down, is

$$\frac{2^3}{x^5} = \frac{8}{5^{5/4}}.$$

Notice that, in quarter-comma meantone, the seventeenth is 81/80 times narrower than in Pythagorean tuning. This difference in size, equal to about 21.506 cents, is called the syntonic comma. This implies that the fifth is a quarter of a syntonic comma narrower than the justly tuned Pythagorean fifth. Namely, this system tunes the fifths in the ratio of

$$5^{1/4} \approx 1.495349$$

which is expressed in the logarithmic cents scale as

$$1200 \log_2 5^{1/4} \text{ cents} \approx 696.578 \text{ cents},$$

which is slightly smaller (or flatter) than the ratio of a justly tuned fifth:

$$\frac{3}{2} = 1.5$$

which is expressed in the logarithmic cents scale as
$$1200\log_2\frac{3}{2}~{\rm cents} \approx 701.955~{\rm cents}.$$

The difference between these two sizes is a quarter of a syntonic comma:

$$\approx 701.955 - 696.578 \approx 5.377 \approx \frac{21.506}{4}$$
 cents.

In sum, this system tunes the major thirds to the just ratio of 5:4 (so, for instance, if A is tuned to 440 Hz, C# is tuned to 550 Hz), most of the whole tones (namely the major

seconds) in the ratio $\sqrt{5}$: 2, and most of the semitones (namely the diatonic

semitones or minor seconds) in the ratio $8:5^{\frac{5}{4}}$. This is achieved by tuning the seventeenth a syntonic comma flatter than the Pythagorean seventeenth, which implies tuning the fifth a quarter of a syntonic comma flatter than the just ratio of 3:2. It is this that gives the system its name of quarter-comma meantone.

12-tone scale[edit]

The whole chromatic scale (a subset of which is the diatonic scale), can be constructed by starting from a given *base note*, and increasing or decreasing its frequency by one or more fifths. This method is identical to Pythagorean tuning, except for the size of the fifth, which is tempered as explained above. The construction table below illustrates how the pitches of the notes are obtained with respect to D (the *base note*), in a D-based scale (see Pythagorean tuning for a more detailed explanation).

For each note in the basic octave, the table provides the conventional name of the interval from D (the base note), the formula to compute its frequency ratio, and the approximate values for its frequency ratio and size in cents.

Note	Interval from D	Formula	10000 100	Size (cents)
АЬ	diminished fifth	$x^{-6} \cdot 2^4 = \frac{16\sqrt{5}}{25}$	1.4311	620.5
ΕÞ	minor second	$x^{-5} \cdot 2^3 = \frac{8\sqrt{5}x}{25}$	1.0700	117.1
В♭	minor sixth	$x^{-4} \cdot 2^3 = \frac{8}{5}$		813.7
F	minor third	$x^{-3} \cdot 2^2 = \frac{4x}{5}$	1.1963	310.3
С	minor seventh	$x^{-2} \cdot 2^2 = \frac{4\sqrt{5}}{5}$	1.7889	1006.8
G	perfect fourth	$x^{-1} \cdot 2^1 = \frac{2\sqrt{5}x}{5}$	1.3375	503.4
D	unison	$x^0 \cdot 2^0 = 1$	1.0000	0.0
Α	perfect fifth	$x^1 \cdot 2^0 = x$	1.4953	696.6
E	major second	$x^2 \cdot 2^{-1} = \frac{\sqrt{5}}{2}$	1.1180	193.2
В	major sixth	$x^3 \cdot 2^{-1} = \frac{\sqrt{5}x}{2}$	1.6719	889.7
F#	major third	$x^4 \cdot 2^{-2} = \frac{5}{4}$	1.2500	386.3
C#	major seventh	$x^5 \cdot 2^{-2} = \frac{5x}{4}$	1.8692	1082.9
G#	augmented fourth	$x^6 \cdot 2^{-3} = \frac{5\sqrt{5}}{8}$	1.3975	579.5

In the formulas, $x=\sqrt[4]{5}=5^{1/4}$ is the size of the tempered perfect fifth, and the ratios x:1 or 1:x represent an ascending or descending tempered perfect fifth (i.e. an increase or decrease in frequency by x), while 2:1 or 1:2 represent an ascending or descending octave.

As in Pythagorean tuning, this method generates 13 pitches, but A? and G? have almost the same frequency, and to build a 12-tone scale $A \not \models$ is typically discarded (although the choice between these two notes is completely arbitrary).

C- based construction tables

The table above shows a D-based stack of fifths (i.e. a stack in which all ratios are expressed relative to D, and D has a ratio of 1/1). Since it is centered at D, the base note, this stack can be called *D-based symmetric*:

$$Ab-Eb-Bb-F-C-G-D-A-E-B-F\#-C\#-G\#$$

Except for the size of the fifth, this is identical to the stack traditionally used in Pythagorean tuning. Some authors prefer showing a C-based stack of fifths, ranging from $A \triangleright to G \#$. Since C is not at its center, this stack is called *C-based asymmetric*:

$$Ab-Eb-Bb-F-C-G-D-A-E-B-F\sharp-C\sharp-G\sharp$$

Since the boundaries of this stack (A \flat and G \sharp) are identical to those of the D-based symmetric stack, the 12 tone scale produced by this stack is also identical. The only difference is that the construction table shows intervals from C, rather than from D. Notice that 144 intervals can be formed from a 12 tone scale (see table below), which include intervals from C, D, and any other note. However, the construction table shows only 12 of them, in this case those starting from C. This is at the same time the main advantage and main disadvantage of the C-based asymmetric stack, as the intervals from C are commonly used, but since C is not at the center of this stack, they unfortunately include an augmented fifth (or A5, i.e. the interval from C to G?), instead of a minor sixth (m6). This A5 is an extremely dissonant wolf interval, as it deviates by 41.1 cents (a diesis of ratio 128:125, almost twice a syntonic comma!) from the corresponding pure interval of 8/5 or 813.7 cents.

On the contrary, the intervals from D shown in the table above, since D is at the center of the stack, do not include wolf intervals and include a pure m6 (from D to B?), instead of an impure A5. Notice that in the above mentioned set of 144 intervals pure m6's are more frequently observed than impure A5's (see table below), and this is one of the reasons why it is not desirable to show an impure A5 in the construction table. A *C-based symmetric* stack might be also used, to avoid the above mentioned drawback:

$$Gb-Db-Ab-Eb-Bb-F-C-G-D-A-E-B-F$$

In this stack, $G \not \models$ and F # have a similar frequency, and $G \not \models$ is typically discarded. Also, the note between C and D is called $D \not \models$ rather than C #, and the note between G and A is called $A \not \models$ rather than G #. The C-based symmetric stack is rarely used, possibly because it produces the wolf fifth in the unusual position of $F \# -D \not \models$ instead of $G \# -E \not \models$, where musicians using Pythagorean tuning expected it).

Justly intonated quarter-comma meantone

A just intonation version of the quarter-comma meantone temperament may be constructed in the same way as Johann Kirnberger's rational version of 12-TET. The value of $5^1/8 \ 35^1/3$ is very close to 4, that's why a 7-limit interval 6144:6125 (which is the difference between the 5-limit diesis 128:125 and the septimal diesis 49:48), equal to 5.362 cents, appears very close to the quarter-comma $(81/80)^1/4$ of 5.377 cents. So the perfect fifth has the ratio of 6125:4096, which is the difference between

three just major thirds and two septimal major seconds; four such fifths exceed the ratio of 5:1 by the tiny interval of 0.058 cents. The wolf fifth there appears to be 49:32, the difference between the septimal minor seventh and the septimal major second.

Greater and lesser semitones

As discussed above, in the quarter-comma meantone temperament,

- the ratio of a semitone is $S = 8:5^{5/4}$,
- the ratio of a tone is $T=\sqrt{5}:2$.

The tones in the diatonic scale can be divided into pairs of semitones. However, since S^2 is not equal to T, each tone must be composed of a pair of unequal semitones, S, and X:

$$S \cdot X = T$$
.

Hence.

$$X = \frac{T}{S} = \frac{\sqrt{5}/2}{8/5^{5/4}} = \frac{5^{1/2} \cdot 5^{5/4}}{8 \cdot 2} = \frac{5^{7/4}}{16}.$$

Notice that S is 117.1 cents, and X is 76.0 cents. Thus, S is the greater semitone, and X is the lesser one. S is commonly called the **diatonic semitone** (or minor second), while X is called the **chromatic semitone** (or augmented unison).

The sizes of S and X can be compared to the just intonated ratio 18/17 which is 99.0 cents. S deviates from it by +18.2 cents, and X by -22.9 cents. These two deviations are comparable to the syntonic comma (21.5 cents), which this system is designed to tune out from the Pythagorean major third. However, since even the just intonated ratio 18/17 sounds markedly dissonant, these deviations are considered acceptable in a semitone.

Size of intervals

The table above shows only intervals from D. However, intervals can be formed by starting from each of the above listed 12 notes. Thus, twelve intervals can be defined for each **interval type** (twelve unisons, twelve semitones, twelve intervals composed of 2 semitones, twelve intervals composed of 3 semitones, etc.).

As explained above, one of the twelve fifths (the wolf fifth) has a different size with respect to the other eleven. For a similar reason, each of the other interval types, except for the unisons and the octaves, has two different sizes in quarter-comma meantone. This is the price paid for seeking just intonation. The table below shows their approximate size in cents. Interval names are given in their standard shortened form. For instance, the size of the interval from D to A, which is a perfect fifth (P5), can be found in the seventh column of the row labeled D. Strictly just (or pure) intervals are shown in **bold** font. Wolf intervals are highlighted in red.

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Interval

From	A1	A5	A2	A6	A3	P1	P5	M2	M6	M3	M7	A4
ЕЬ	76	773	269	966	462	0	697	193	890	386	1083	579
вь	76	773	269	966	503	0	697	193	890	386	1083	579
F	76	773	269	1007	503	0	697	193	890	386	1083	579
С	76	773	310	1007	503	0	697	193	890	386	1083	579
G	76	814	310	1007	503	0	697	193	890	386	1083	579
D	117	814	310	1007	503	0	697	193	890	386	1083	579
A	117	814	310	1007	503	0	697	193	890	386	1083	621
E	117	814	310	1007	503	0	697	193	890	386	1124	621
В	117	814	310	1007	503	0	697	193	890	427	1124	621
F♯	117	814	310	1007	503	0	697	193	931	427	1124	621
C♯	117	814	310	1007	503	0	697	234	931	427	1124	621
G♯	117	814	310	1007	503	0	738	234	931	427	1124	621
	m2	m6	m3	m7	P4	P1	d6	d3	d7	d4	d8	d5

Approximate size in cents of the 144 intervals in D-based quarter-comma meantone tuning. Interval names are given in their standard shortened form. Pure intervals are shown in **bold** font. Wolf intervals are highlighted in red.

Surprisingly, although this tuning system was designed to produce pure major thirds, only 8 of them are pure (5:4 or about 386.3 cents).

The reason why the interval sizes vary throughout the scale is that the pitches forming the scale are unevenly spaced. Namely, as mentioned above, the frequencies defined by construction for the twelve notes determine two different kinds of semitones (i.e. intervals between adjacent notes):

1.

The minor second (m2), also called diatonic semitone, with size
$$S ext{ (or } S_1) = rac{8}{5^{5/4}} pprox 117.1 ext{ cents}$$

(for instance, between D and E?)

The augmented unison (A1), also called chromatic semitone, with size

$$X \text{ (or } S_2) = \frac{5^{7/4}}{16} \approx 76.0 \text{ cents}$$

(for instance, between C and C?)

Conversely, in an equally tempered chromatic scale, by definition the twelve pitches are equally spaced, all semitones having a size of exactly

$$S_E = \sqrt[12]{2} = 100 \text{ cents.}$$

As a consequence all intervals of any given type have the same size (e.g., all major thirds have the same size, all fifths have the same size, etc.). The price paid, in this case, is that none of them is justly tuned and perfectly consonant, except, of course, for the unison and the octave.

By definition, in quarter-comma meantone 11 perfect fifths (**P5** in the table) have a size of approximately 696.6 cents $(700-\Sigma \text{ cents})$, where $\Sigma = 3.422 \text{ cents}$; since the average size of the 12 fifths must equal exactly 700 cents (as in equal temperament), the other one must have a size of $700+11\Sigma$ cents, which is about 737.6 cents (the wolf fifth). Notice that, as shown in the table, the latter interval, although enharmonically equivalent to a fifth, is more properly called a diminished sixth (**d6**). Similarly,

- 10 major seconds (M2) are ≈ 193.2 cents (200-2 Σ), 2 diminished thirds (d3) are ≈ 234.2 cents (200+10?), and their average is 200 cents;
- 9 minor thirds (m3) are ≈ 310.3 cents (300+3 Σ), 3 augmented seconds (A2) are ≈ 269.2 cents (300?9?), and their average is 300 cents;
- 8 major thirds (M3) are ≈ 386.3 cents (400-4 Σ), 4 diminished fourths (d4) are ≈ 427.4 cents (400+8?), and their average is 400 cents;
- 7 diatonic semitones (**m2**) are ≈ 117.1 cents (100+5 Σ), 5 chromatic semitones (**A1**) are ≈ 76.0 cents (100-7 Σ), and their average is 100 cents.

In short, similar differences in width are observed for all interval types, except for unisons and octaves, and they are all multiples of ?, the difference between the 1/4-comma meantone fifth and the average fifth.

Notice that, as an obvious consequence, each augmented or diminished interval is exactly 12Σ cents (≈ 41.1 cents) wider or narrower than its enharmonic equivalent. For instance, the d6 (or wolf fifth) is 12Σ cents wider than each P5, and each A2 is 12Σ cents narrower than each m3. This interval of size 12? is known as a diesis, or diminished second. This implies that Σ can be also defined as one twelfth of a diesis.

Triads in the chromatic scale

The major triad can be defined by a pair of intervals from the root note: a major third (interval spanning 4 semitones) and a perfect fifth (7 semitones). The minor triad can likewise be defined by a minor third (3 semitones) and a perfect fifth (7 semitones).

As shown above, a chromatic scale has twelve intervals spanning seven semitones. Eleven of these are perfect fifths (P5), while the twelfth is a diminished sixth (d6). Since they span the same number of semitones, P5 and d6 are considered to be enharmonically equivalent. In an equally tuned chromatic scale, P5 and d6 have exactly the same size. The same is true for all the enharmonically equivalent intervals spanning 4 semitones (M3 and d4), or 3 semitones (m3 and A2). However, in the meantone temperament this is not true. In this tuning system, enharmonically equivalent intervals may have different sizes, and some intervals may markedly deviate from their justly tuned ideal ratios. As explained in the previous section, if the deviation is too large, then the given interval is not usable, either by itself or in a chord.

The following table focuses only on the above-mentioned three interval types, used to form major and minor triads. Each row shows three intervals of different types but which have the same root note. Each interval is specified by a pair of notes. To the right of each interval is listed the formula for the interval ratio. The intervals d4, d6 and A2 may be regarded as wolf intervals, and have been marked in red. S and X denote the ratio of the two above-mentioned kinds of semitones (m2 and A1).

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3 semit (m3 or		4 semi (M3 o		7 semitones (P5 or d6)		
Interval	Ratio	Interval	Ratio	Interval	Ratio	
C-E	$S^2 \cdot X$	C-E	$S^2 \cdot X^2$	C-G	$S^4 \cdot X^3$	
C#-E	$S^2 \cdot X$	C#-F	$S^3 \cdot X$	C#-G#	$S^4 \cdot X^3$	
D-F	$S^2 \cdot X$	D-F#	$S^2 \cdot X^2$	D-A	S4 · X3	
E♭-F#	s·x2	E♭-G	$S^2 \cdot X^2$	E♭-B♭	$S^4 \cdot X^3$	
E-G	$S^2 \cdot X$	E-G#	$S^2 \cdot X^2$	E-B	$S^4 \cdot X^3$	
F-G#	s·x2	F-A	$S^2 \cdot X^2$	F-C	S4 · X3	
F#-A	$S^2 \cdot X$	F#−B♭	$S^3 \cdot X$	F#-C#	$S^4 \cdot X^3$	
G−B♭	$S^2 \cdot X$	G-B	$S^2 \cdot X^2$	G-D	$S^4 \cdot X^3$	
G#−B	$S^2 \cdot X$	G#-C	$S^3 \cdot X$	G#−E♭	$S^5 \cdot X^2$	
A-C	$S^2 \cdot X$	A-C♯	$S^2 \cdot X^2$	A-E	S4 · X3	
B♭-C♯	$S \cdot x^2$	B♭-D	$S^2 \cdot X^2$	B♭-F	$S^4 \cdot X^3$	
B-D	$S^2 \cdot X$	В-Е	$S^3 \cdot X$	B-F♯	S4 . X3	

First, look at the last two columns on the right. All the 7-semitone intervals except one have a ratio of

$$S^4 \cdot X^3 \approx 1.4953 \approx 696.6 \text{ cents}$$

which deviates by -5.4 cents from the just 3:2 of 702.0 cents. Five cents is small and acceptable. On the other hand, the d6 from G# to $E \not \models$ has a ratio of

$$S^5 \cdot X^2 \approx 1.5312 \approx 737.6$$
 cents

which deviates by +35.7 cents from the just fifth. Thirty-five cents is beyond the acceptable range.

Now look at the two columns in the middle. Eight of the twelve 4-semitone intervals have a ratio of

$$S^2 \cdot X^2 = 1.25 \approx 386.3 \text{ cents}$$

which is exactly a just 5:4. On the other hand, the four d4 with roots at C#, F#, G# and B have a ratio of

$$S^3 \cdot X = 1.28 \approx 427.4 \text{ cents}$$

which deviates by +41.1 cents from the just M3. Again, this sounds badly out of tune. Major triads are formed out of both major thirds and perfect fifths. If either of the

two intervals is substituted by a wolf interval (d6 instead of P5, or d4 instead of M3), then the triad is not acceptable. Therefore major triads with root notes of C#, F#, G# and B are not used in meantone scales whose fundamental note is C.

Now look at the first two columns on the left. Nine of the twelve 3-semitone intervals have a ratio of

$$S^2 \cdot X \approx 1.1963 \approx 310.3 \text{ cents}$$

which deviates by -5.4 cents from the just 6:5 of 315.6 cents. Five cents is acceptable. On the other hand, the three augmented seconds whose roots are E?, F and B? have a ratio of

$$S \cdot X^2 \approx 1.1682 \approx 269.2 \text{ cents}$$

which deviates by -46.4 cents from the just minor third. It is a close match, however, for the 7:6 septimal minor third of 266.9 cents, deviating by +2.3 cents. These augmented seconds, though sufficiently consonant by themselves, will sound "exotic" or atypical when played together with a perfect fifth.

Minor triads are formed out of both minor thirds and fifths. If either of the two intervals are substituted by an enharmonically equivalent interval (d6 instead of P5, or A2 instead of m3), then the triad will not sound good. Therefore minor triads with root notes of $E \not \mid F$, F, G # and $B \not \mid F$ are not used in the meantone scale defined above.

The following major triads are usable: C, D, $E \triangleright$, E, F, G, A, $B \triangleright$.

The following minor triads are usable: C, C#, D, E, Fb, G, A, B.

The following root notes are useful for both major and minor triads: C, D, E, G and A. Notice that these five pitches form a major pentatonic scale.

The following root notes are useful only for major triads: $E \triangleright$, F, $B \triangleright$.

The following root notes are useful only for minor triads: C#, F#, B.

The following root note is useful for neither major nor minor triad: G#.

Alternative construction

As discussed above, in the quarter-comma meantone temperament,

- the ratio of a greater (diatonic) semitone is $S=8:5^{5/4}$,
- the ratio of a lesser (chromatic) semitone is $X = 5^{7/4}: 16$,
- the ratio of most whole tones is $T = \sqrt{5}: 2$,
- the ratio of most fifths is $P=5^{1/4}$.

It can be verified through calculation that most whole tones (namely, the major seconds) are composed of one greater and one lesser semitone:

$$T = S \cdot X = \frac{8}{5^{5/4}} \cdot \frac{5^{7/4}}{16} = \frac{5^{2/4}}{2} = \frac{\sqrt{5}}{2}.$$

Similarly, a fifth is typically composed of three tones and one greater semitone:

$$P = T^3 \cdot S = \frac{5^{3/2}}{2^3} \cdot \frac{8}{5^{5/4}} = 5^{6/4 - 5/4} = 5^{1/4},$$

which is equivalent to four greater and three lesser semitones:

$$P = T^3 \cdot S = S^4 \cdot X^3.$$

Diatonic scale

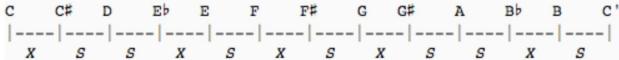
A diatonic scale can be constructed by starting from the fundamental note and multiplying it either by T to move up by a tone or by S to move up by a semitone.

The resulting interval sizes with respect to the base note C are shown in the following table:

Note	Formula	Ratio	Cents	Pythagorean cents	EQT Cents
С	1	1.0000	0.0	0.0	0
D	T	1.1180	193.2	203.9	200
E	T2	1.2500	386.3	407.8	400
F	T ² S	1.3375	503.4	498.0	500
G	P	1.4953	696.6	702.0	700
Α	PT	1.6719	889.7	905.9	900
В	PT ²	1.8692	1082.9	1109.8	1100
C'	PT2S	2.0000	1200.0	1200.0	1200

Chromatic scale

Construction of a 1/4-comma meantone chromatic scale can proceed by stacking a series of 12 semitones, each of which may be either diatonic (S) or chromatic (X).



Notice that this scale is an extension of the diatonic scale shown in the previous table. Only five notes have been added: C#, $E \flat$, F#, G# and $B \flat$ (a pentatonic scale).

As explained above, an identical scale was originally defined and produced by using a sequence of tempered fifths, ranging from $E \flat$ (five fifths below D) to G # (six fifths above D), rather than a sequence of semitones. This more conventional approach, similar to the *D-based* Pythagorean tuning system, explains the reason why the X and S semitones are arranged in the particular and apparently arbitrary sequence shown above.

The interval sizes with respect to the base note C are presented in the following table. The frequency ratios are computed as shown by the formulas. Delta is the difference in cents between meantone and 12-TET. 1/4-c is the difference in quarter-commas between meantone and Pythagorean tuning.

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Note	Formula	Ratio	Cents	12TET	Delta	1/4-c
С	1	1.0000	0.0	0	0.0	0
C#	X	1.0449	76.0	100	-24.0	-7
D	T	1.1180	193.2	200	-6.8	-2
Εb	TS	1.1963	310.3	300	+10.3	3
E	T ²	1.2500	386.3	400	-13.7	-4
F	T ² S	1.3375	503.4	500	+3.4	1
F♯	T ³	1.3975	579.5	600	-20.5	-6
G	P	1.4953	696.6	700	-3.4	-1
G#	PX	1.5625	772.6	800	-27.4	-8
Α	PT	1.6719	889.7	900	-10.3	-3
В♭	PTS	1.7889	1006.8	1000	+6.8	2
В	PT ²	1.8692	1082.9	1100	-17.1	-5
C'	PT2S	2.0000	1200.0	1200	0.0	0

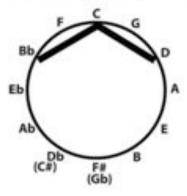
Comparison with 31 equal temperament

The perfect fifth of quarter-comma meantone, expressed as a fraction of an octave, is 1/4 log2 5. This number is irrational and in fact transcendental; hence a chain of meantone fifths, like a chain of pure 3/2 fifths, never closes (i.e. never equals a chain of octaves). However, the continued fraction approximations to this irrational fraction number allow us to find equal divisions of the octave which do close; the denominators of these are 1, 2, 5, 7, 12, 19, 31, 174, 205, 789 ... From this we find that 31 quarter-comma meantone fifths come close to closing, and conversely 31 equal temperament represents a good approximation to quarter-comma meantone. See: schisma.

Circle of Fifth and Fourth Intervals

The circle of fifths and fourths can also be very helpful to learn intervals. Although intervals have already been explored, they have not been discussed from the perspective of the circle of fifths. The figures in this chapter are going to be very helpful visual aids in learning and remembering intervals by utilizing geometrical patterns.

The figures below show major second and minor seventh intervals. The major second intervals rotate clock-wise, and the minor seventh intervals rotate counter clock-wise. The first figure 6.1 shows the first iteration starting from C. C to D is a major second interval, and C to Bb is a minor seventh interval. The second figure 6.2 completes six major second intervals and six minor seventh intervals creating the hexagon shape.



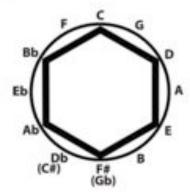
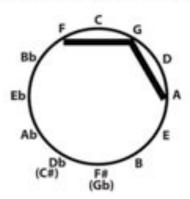


Figure 6.1 - C major second and minor seventh intervals

Figure 6.2 - Six major second and minor seventh intervals

By rotating the hexagon to the right (1/12 clock-wise turn), the remaining six interval sets can be constructed. The first figure 6.3 below shows one iteration from G to A (major second) clock-wise and G to F (minor seventh) counter-clockwise. The second figure 6.4 completes the cycle of major seconds and minor sevenths.



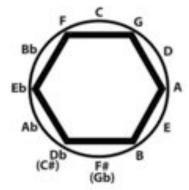
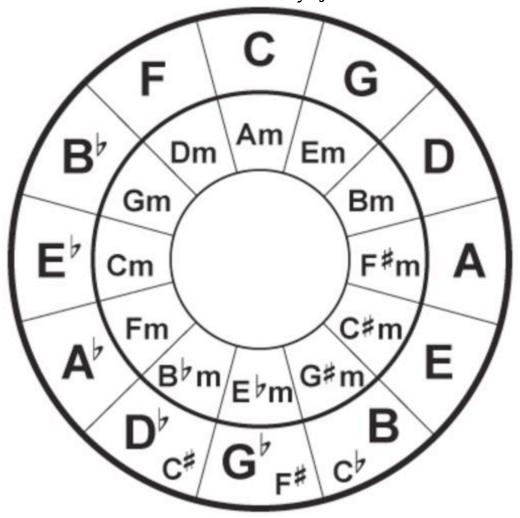


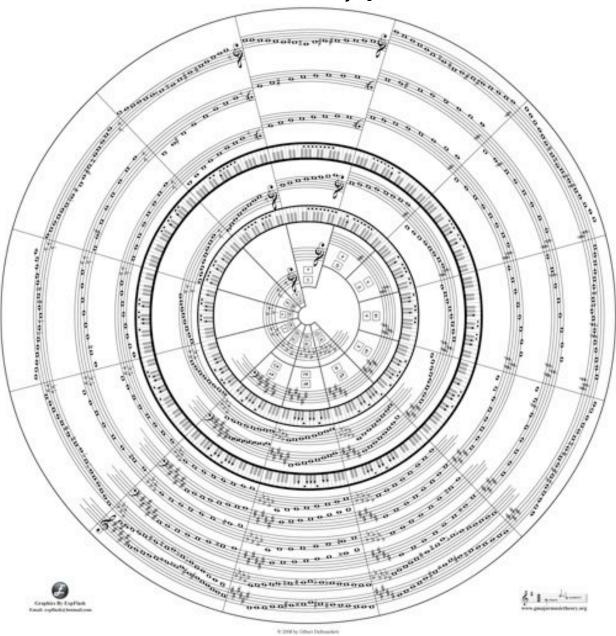
Figure 6.3 - G major second and minor seventh intervals

Figure 6.4 - Six major second and minor seventh intervals

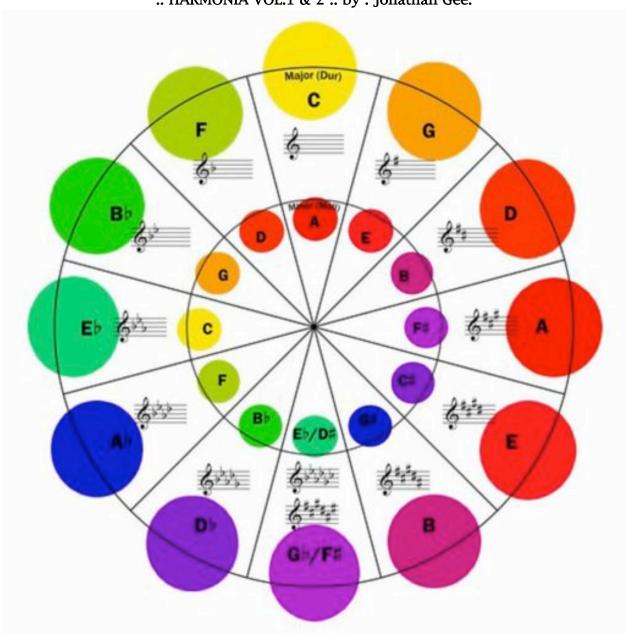
The four figures below show major third and minor sixth intervals. The major third intervals rotate clock-wise, and the minor sixth intervals rotate counter clock-wise. Major thirds cycle are very helpful when building augmented chords and have only four iterations. Remember, the formula to build augmented chords is 1, 3, #5, or two major third intervals. The four figures below show the four iterations. The first one shows the notes to build the Bbaug, Daug, and F#aug chords. The Bbaug chord has the notes Bb, D, F#. Bb to D is a major third interval, as well as D to F#. The Daug chord has the notes D, F#, and Bb, and the F#aug chord has the notes F#, Bb, and D. Notice all three chords use the same

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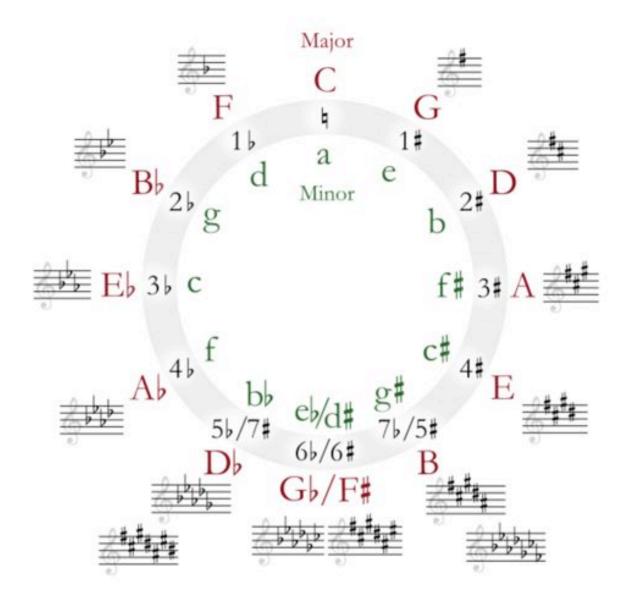




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Circle of fifths



Circle of fifths showing major and minor keys



Nikolay Diletsky's circle of fifths in *Idea grammatiki musikiyskoy* (Moscow, 1679)

In music theory, the **circle of fifths** (or **circle of fourths**) is a visual representation of the relationships among the 12 tones of the chromatic scale, their corresponding key signatures, and the associated major and minor keys. More specifically, it is a geometrical representation of relationships among the 12 pitch classes of the chromatic scale in pitch class space.

Definition

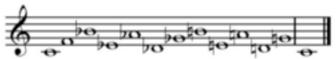
The term 'fifth' defines an interval or mathematical ratio which is the closest and most consonant non-octave interval. The circle of fifths is a sequence of pitches or key tonalities, represented as a circle, in which the next pitch is found seven semitones higher than the last. Musicians and composers use the circle of fifths to understand and describe the musical relationships among some selection of those pitches. The circle's design is helpful in composing and harmonizing melodies, building chords, and modulating to different keys within a composition.

At the top of the circle, the key of C Major has no sharps or flats. Starting from the apex and proceeding clockwise by ascending fifths, the key of G has one sharp, the key of D has 2 sharps, and so on. Similarly, proceeding counterclockwise from the apex by descending fifths, the key of F has one flat, the key of B \flat has 2 flats, and so on. At the bottom of the circle, the sharp and flat keys overlap, showing pairs of enharmonic key signatures.

Starting at any pitch, ascending by the interval of an equal tempered fifth, one passes all twelve tones clockwise, to return to the beginning pitch class. To pass the twelve tones counterclockwise, it is necessary to ascend by perfect fourths, rather than fifths. (To the ear, the sequence of fourths gives an impression of settling, or resolution. (see cadence))



circle of fifths clockwise within one octave



circle of fifths counterclockwise within one octave

Structure and use

Pitches within the chromatic scale are related not only by the number of semitones between them within the chromatic scale, but also related harmonically within the circle of fifths. Reversing the direction of the circle of fifths gives the **circle of fourths**. Typically the "circle of fifths" is used in the analysis of classical music, whereas the "circle of fourths" is used in the analysis of Jazz music, but this distinction is not exclusive. Since fifths and fourths are intervals composed respectively of 7 and 5 semitones, the circumference of a circle of fifths is an interval as large as 7 octaves (84 semitones), while the circumference of a circle of fourths equals only 5 octaves (60 semitones).

Octaves (7 ? 1200 = 8400) versus fifths (12 ? 700 = 8400), depicted as with Cuisenaire rods (red (2) is used for 1200, black (7) is used for 700).

Diatonic key signatures

The circle is commonly used to represent the relationship between diatonic scales. Here, the letters on the circle are taken to represent the major scale with that note as tonic. The numbers on the inside of the circle show how many sharps or flats the key

signature for this scale has. Thus a major scale built on A has 3 sharps in its key signature. The major scale built on F has 1 flat.

For minor scales, rotate the letters counter-clockwise by 3, so that, e.g., A minor has 0 sharps or flats and E minor has 1 sharp. (See relative key for details.) A way to describe this phenomenon is that, for any major key [e.g. G major, with one sharp (F#) in its diatonic scale], a scale can be built beginning on the sixth (VI) degree (relative minor key, in this case, E) containing the same notes, but from E - E as opposed to G - G. Or, G-major scale (G - A - B - C - D - E - F# - G) is enharmonic (harmonically equivalent) to the e-minor scale (E - F# - G - A - B - C - D - E).

When notating the key signatures, the order of sharps that are found at the beginning of the staff line follows the circle of fifths from F through B. The order is F, C, G, D, A, E, B. If there is only one sharp, such as in the key of G major, then the one sharp is F sharp. If there are two sharps, the two are F and C, and they appear in that order in the key signature. The order of sharps goes clockwise around the circle of fifths. (The major key you are in is one half-step above the last sharp you see in the key signature. This does not work if you are in a minor key.)

For notating flats, the order is reversed: B, E, A, D, G, C, F. This order runs counter-clockwise along the circle of fifths; in other words they progress by fourths. If you follow the major keys from the key of F to the key of C flat (B) counter-clockwise around the circle of fifths, you will see that as each key signature adds a flat, they are these seven flats always in this order. D flat in the key signature is always found in the fourth position among flats, and G is always in the fifth position. If there are three flats in the key signature, they are only B, E, and A, while the other positions are empty. (The major key you are in is the penultimate flat you find in the key signature; of course, there must be at least two flats in the key signature for this to work. It does not work if you are in a minor key.)

Modulation and chord progression

Tonal music often modulates by moving between adjacent scales on the circle of fifths. This is because diatonic scales contain seven pitch classes that are contiguous on the circle of fifths. It follows that diatonic scales a perfect fifth apart share six of their seven notes. Furthermore, the notes not held in common differ by only a semitone. Thus modulation by perfect fifth can be accomplished in an exceptionally smooth fashion. For example, to move from the C major scale F - C - G - D - A - E - B to the G major scale F - C - G - D - A - E - B to the G major scale F - C - G - D - A - E - B to "F#".

In Western tonal music, one also finds chord progressions between chords whose roots are related by perfect fifth. For instance, root progressions such as D-G-C are common. For this reason, the circle of fifths can often be used to represent "harmonic distance" between chords.



IV-V-I, in C According to theorists including Goldman, harmonic function (the use, role, and

relation of chords in harmony), including "functional succession", may be "explained by the circle of fifths (in which, therefore, scale degree II is closer to the dominant than scale degree IV)". In this view the tonic is considered the end of the line towards which a chord progression derived from the circle of fifths progresses.



ii-V-I turnaround, in C Play subdominant, supertonic seventh, and supertonic chords illustrating the similarity between them.

According to Goldman's *Harmony in Western Music*, "the IV chord is actually, in the simplest mechanisms of diatonic relationships, at the greatest distance from I. In terms of the [descending] circle of fifths, it leads away from I, rather than toward it." Thus the progression I-ii-V-I (an authentic cadence) would feel more final or resolved than I-IV-I (a plagal cadence). Goldman[4] concurs with Nattiez, who argues that "the chord on the fourth degree appears long before the chord on II, and the subsequent final I, in the progression I-IV-viio-iii-vi-ii-V-I", and is farther from the tonic there as well. (In this and related articles, upper-case Roman numerals indicate major triads while lower-case Roman numerals indicate minor triads.)



IV vs. ii7 with root in parenthesis, in C.

Goldman argues that "historically the use of the IV chord in harmonic design, and especially in cadences, exhibits some curious features. By and large, one can say that the use of IV in final cadences becomes more common in the nineteenth century than it was in the eighteenth, but that it may also be understood as a substitute for the ii chord when it precedes V. It may also be quite logically construed as an incomplete ii7 chord (lacking root)." The delayed acceptance of the IV-I in final cadences is explained aesthetically by its lack of closure, caused by its position in the circle of fifths. The earlier use of IV-V-I is explained by proposing a relation between IV and ii, allowing IV to substitute for or serve as ii. However, Nattiez calls this latter argument "a narrow escape: only the theory of a ii chord without a root allows Goldman to maintain that the circle of fifths is completely valid from Bach to Wagner", or the entire common practice period.

Circle closure in non-equal tuning systems

When an instrument is tuned with the equal temperament system, the width of the fifths is such that the circle "closes". This means that ascending by twelve fifths from any pitch, one returns to a tune exactly in the same pitch class as the initial tune, and exactly seven octaves above it. To obtain such a perfect circle closure, the fifth is slightly flattened with respect to its just intonation (3:2 interval ratio).

Thus, ascending by justly intonated fifths fails to close the circle by an excess of approximately 23.46 cents, roughly a quarter of a semitone, an interval known as the Pythagorean comma. In Pythagorean tuning, this problem is solved by markedly shortening the width of one of the twelve fifths, which makes it severely dissonant. This anomalous fifth is called wolf fifth as a humorous metaphor of the unpleasant sound of the note (like a wolf trying to howl an off-pitch note). The quarter-comma meantone tuning system uses eleven fifths slightly narrower than the equally tempered fifth, and requires a much wider and even more dissonant wolf fifth to close the circle. More complex tuning systems based on just intonation, such as 5-limit tuning, use at most eight justly tuned fifths and at least three non-just fifths (some slightly narrower, and some slightly wider than the just fifth) to close the circle.

In lay terms



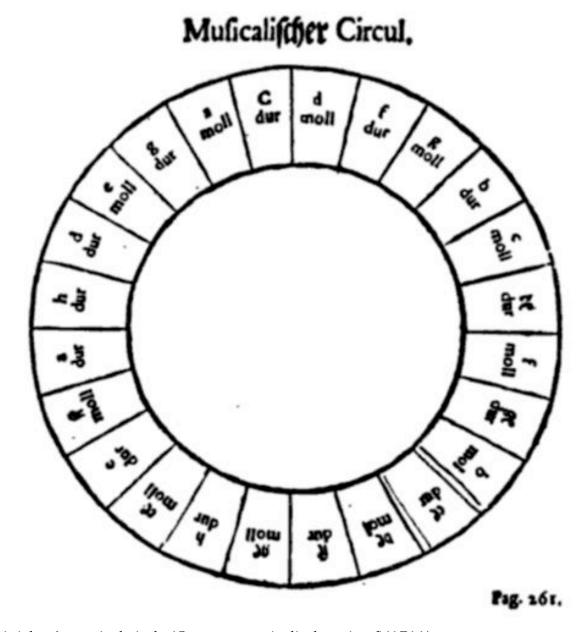
Playing the circle of fifths

A simple way to see the musical interval known as a fifth is by looking at a piano keyboard, and, starting at any key, counting seven keys to the right (both black and white) to get to the next note on the circle shown above. Seven half steps, the distance from the 1st to the 8th key on a piano is a "perfect fifth", called 'perfect' because it is neither major nor minor, but applies to both major and minor scales and chords, and a 'fifth' because though it is a distance of seven semitones on a keyboard, it is a distance of five steps within a major or minor scale.

A simple way to hear the relationship between these notes is by playing them on a piano keyboard. If you traverse the circle of fifths backwards, the notes will feel as though they fall into each other. This aural relationship is what the mathematics describe.

Perfect fifths may be justly tuned or tempered. Two notes whose frequencies differ by a ratio of 3:2 make the interval known as a justly tuned perfect fifth. Cascading twelve such fifths does not return to the original pitch class after going round the circle, so the 3:2 ratio may be slightly detuned, or tempered. Temperament allows perfect fifths to cycle, and allows pieces to be transposed, or played in any key on a piano or other fixed-pitch instrument without distorting their harmony. The primary tuning system used for Western (especially keyboard and fretted) instruments today, twelve-tone equal temperament, uses an irrational multiplier, 21/12, to calculate the frequency difference of a semitone. An equal-tempered fifth, at a frequency ratio of 27/12:1 (or about 1.498307077:1) is approximately two cents narrower than a justly tuned fifth at a ratio of 3:2.

History



Heinichen's musical circle (German: musicalischer circul)(1711)

In the late 1670s a treatise called *Grammatika* was written by the composer and theorist Nikolai Diletskii. Diletskii's *Grammatika* is a treatise on composition, the first of its kind, which targeted Western-style polyphonic compositions. It taught how to write kontserty, polyphonic a cappella, which were normally based on liturgical texts and were created by putting together musical sections that have contrasting rhythm, meters, melodic material and vocal groupings. Diletskii intended his treatise to be a guide to composition but pertaining to the rules of music theory. Within the Grammatika treatise is where the first circle of fifths appeared and was used for students as a composer's tool.

Related concepts

Diatonic circle of fifths

The diatonic circle of fifths is the circle of fifths encompassing only members of the diatonic scale. Therefore, it contains a diminished fifth, in C major between B and F. See structure implies multiplicity.

The circle progression is commonly a circle through the diatonic chords by fifths, including one diminished chord and one progression by diminished fifth:



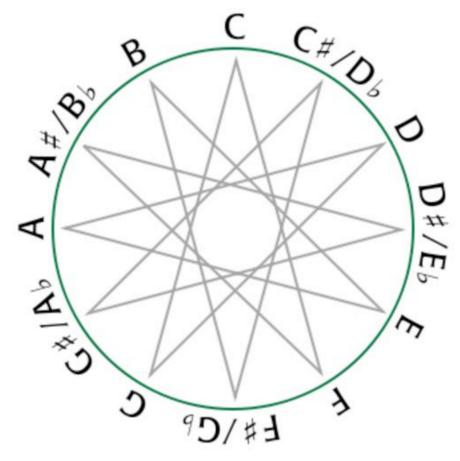
I-IV-viio-iii-vi-ii-V-I (in major)

Chromatic circle

The circle of fifths is closely related to the chromatic circle, which also arranges the twelve equal-tempered pitch classes in a circular ordering. A key difference between the two circles is that the chromatic circle can be understood as a continuous space where every point on the circle corresponds to a conceivable pitch class, and every conceivable pitch class corresponds to a point on the circle. By contrast, the circle of fifths is fundamentally a *discrete* structure, and there is no obvious way to assign pitch classes to each of its points. In this sense, the two circles are mathematically quite different.

However, the twelve equal-tempered pitch classes can be represented by the cyclic group of order twelve, or equivalently, the residue classes modulo twelve, . The group has four generators, which can be identified with the ascending and descending semitones and the ascending and descending perfect fifths. The semitonal generator gives rise to the chromatic circle while the perfect fifth gives rise to the circle of fifths.

Relation with chromatic scale



The circle of fifths drawn within the chromatic circle as a star dodecagram.

The circle of fifths, or fourths, may be mapped from the chromatic scale by multiplication, and vice versa. To map between the circle of fifths and the chromatic scale (in integer notation) multiply by 7 (M7), and for the circle of fourths multiply by 5 (P5).

Here is a demonstration of this procedure. Start off with an ordered 12-tuple (tone row) of integers

(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)

representing the notes of the chromatic scale: 0 = C, 2 = D, 4 = E, 5 = F, 7 = G, 9 = A, 11 = B, 1 = C #, 3 = D #, 6 = F #, 8 = G #, 10 = A #. Now multiply the entire 12-tuple by 7:

(0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77)

and then apply a modulo 12 reduction to each of the numbers (subtract 12 from each number as many times as necessary until the number becomes smaller than 12):

(0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5)

which is equivalent to

(C, G, D, A, E, B, F#, C#, G#, D#, A#, F)

which is the circle of fifths. Note that this is enharmonically equivalent to:

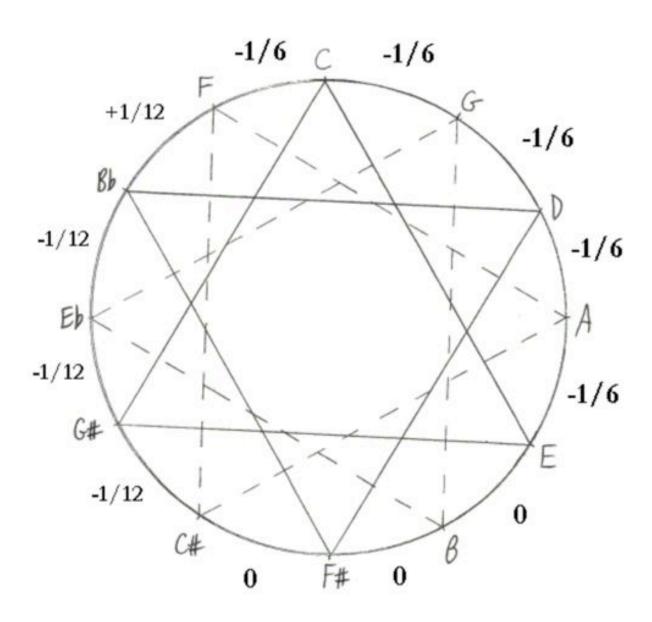
(C, G, D, A, E, B, G \(\beta \), D \(\beta \), A \(\beta \), E \(\beta \), B \(\beta \), F).

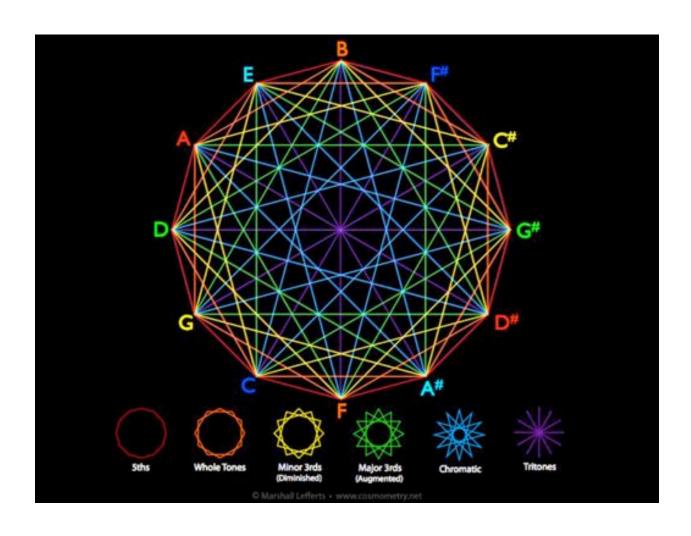
Enharmonics

The "bottom keys" of the circle of fifths are often written in flats and sharps, as they are easily interchanged using enharmonics. For example, the key of B, with five sharps, is enharmonically equivalent to the key of C, with 7 flats. But the circle of

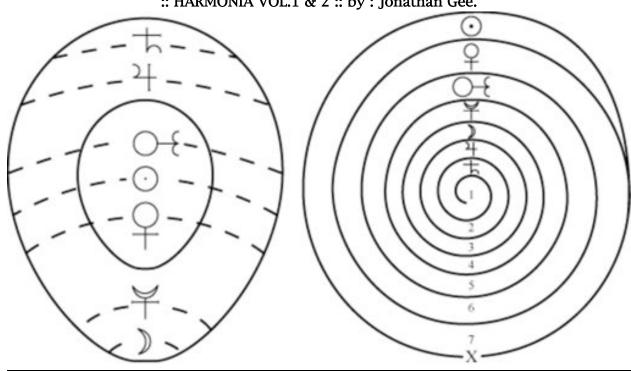
fifths doesn't stop at 7 sharps (C#) or 7 flats ($C \flat$). Following the same pattern, one can construct a circle of fifths with all sharp keys, or all flat keys.

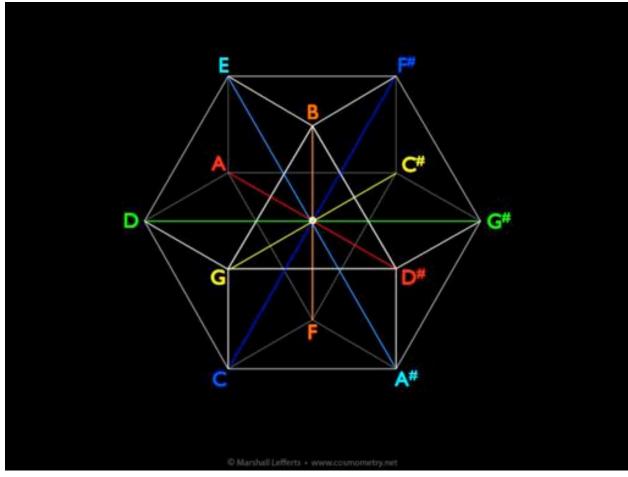
After C# comes the key of G# (following the pattern of being a fifth higher, and, coincidentally, enharmonically equivalent to the key of $A \not \triangleright$). The "8th sharp" is placed on the F#, to make it F. The key of D#, with 9 sharps, has another sharp placed on the C#, making it C. The same for key signatures with flats is true; The key of E (four sharps) is equivalent to the key of F $\not \triangleright$ (again, one fifth below the key of C $\not \triangleright$, following the pattern of flat key signatures). The last flat is placed on the B $\not \triangleright$, making it B.

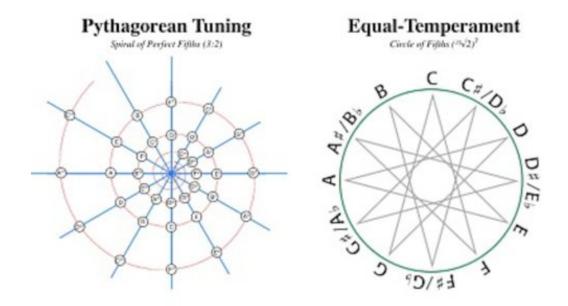


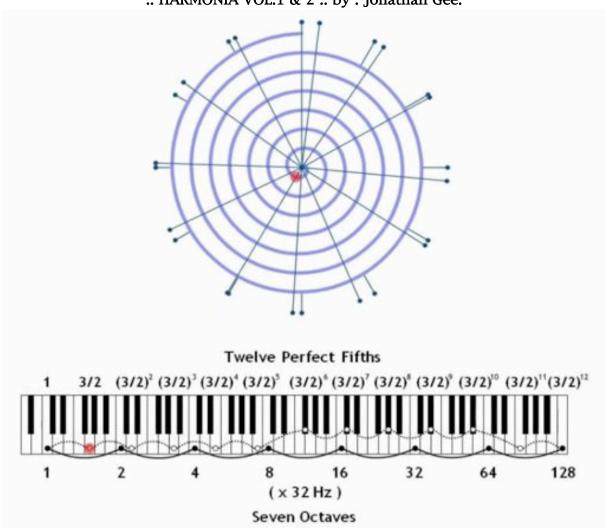


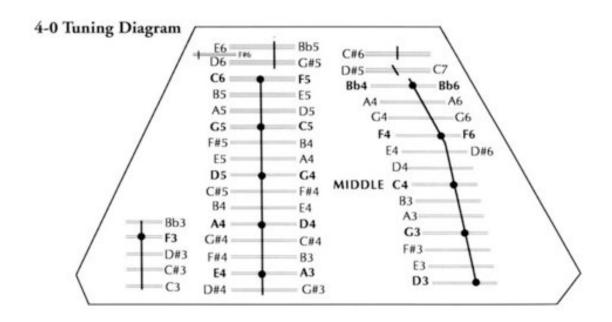
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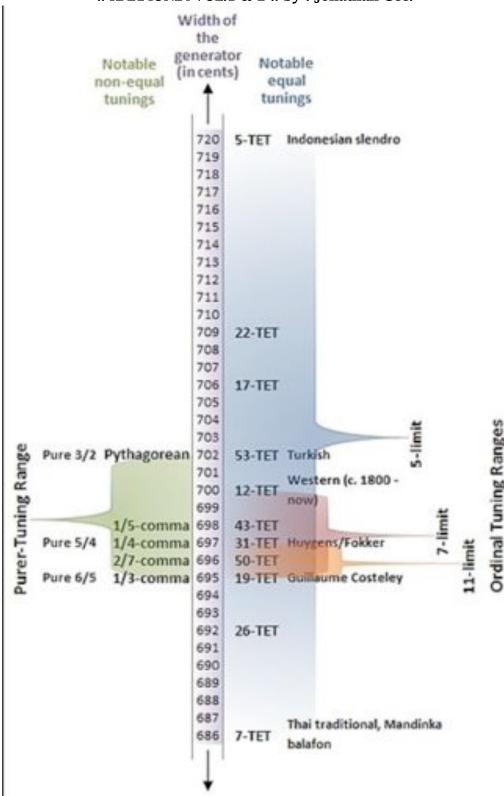




Note	Gb	Db	Ab	Εb	В♭	F	С	G	D	A	E	В	F♯
Ratio	1024:729	256:243	128:81	32:27	16:9	4:3	1:1	3:2	9:8	27:16	81:64	243:128	729:512
Cents	588	90	792	294	996	498	0	702	204	906	408	1110	612

Pythagorean tuning is a tuning of the syntonic temperament in which the generator is the ratio 3:2 (i.e., the untempered perfect fifth), which is 702 cents wide (see the figure labelled "The syntonic tuning continuum" below).

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The syntonic tuning continuum, showing Pythagorean tuning at 702 cents. Hence, it is a system of musical tuning in which the frequency ratios of all intervals

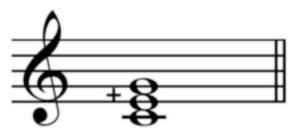
are based on the ratio 3:2, "found in the harmonic series."[2] This ratio, also known as the "pure" perfect fifth, is chosen because it is one of the most consonant and easy to tune by ear.



Pythagorean diatonic scale on C.



Diatonic scale on C 12-tone equal tempered and just intonation.



Pythagorean (tonic) major chord on C (compare equal tempered and just).

The system had been mainly attributed to Pythagoras (sixth century BC) by modern authors of music theory, while Ptolemy, and later Boethius, ascribed the division of the tetrachord by only two intervals, called "semitonium", "tonus", "tonus" in Latin (256:243 x 9:8 x 9:8), to Eratosthenes. The so-called "Pythagorean tuning" was used by musicians up to the beginning of the 16th century. Leo Gunther wrote, that "the Pythagorean system would appear to be ideal because of the purity of the fifths, but other intervals, particularly the major third, are so badly out of tune that major chords [may be considered] a dissonance."

The **Pythagorean scale** is any scale which may be constructed from only pure perfect fifths (3:2) and octaves (2:1) or the gamut of twelve pitches constructed from only pure perfect fifths and octaves, and from which specific scales may be drawn (see Generated collection). For example, the series of fifths generated above gives seven notes, a diatonic major scale on C in Pythagorean tuning, shown in notation on the top right. In Greek music it was used to tune tetrachords and the twelve tone Pythagorean system was developed by medieval music theorists using the same method of tuning in perfect fifths, however there is no evidence that Pythagoras himself went beyond the tetrachord.

Method

Pythagorean tuning is based on a stack of intervals called perfect fifths, each tuned in the ratio 3:2, the next simplest ratio after 2:1. Starting from D for example (*D-based* tuning), six other notes are produced by moving six times a ratio 3:2 up, and the remaining ones by moving the same ratio down:

$$E \flat - B \flat - F - C - G - D - A - E - B - F \# - C \# - G \#$$

This succession of eleven 3:2 intervals spans across a wide range of frequency (on a

piano keyboard, it encompasses 77 keys). Since notes differing in frequency by a factor of 2 are given the same name, it is customary to divide or multiply the frequencies of some of these notes by 2 or by a power of 2. The purpose of this adjustment is to move the 12 notes within a smaller range of frequency, namely within the interval between the **base note** D and the D above it (a note with twice its frequency). This interval is typically called the **basic octave** (on a piano keyboard, an octave encompasses only 13 keys).

For instance, the A is tuned such that its frequency equals 3:2 times the frequency of D — if D is tuned to a frequency of 288 Hz, then A is tuned to 432 Hz. Similarly, the E above A is tuned such that its frequency equals 3:2 times the frequency of A, or 9:4 times the frequency of D — with A at 432 Hz, this puts E at 648 Hz. Since this E is outside the above-mentioned basic octave (i.e. its frequency is more than twice the frequency of the base note D), it is usual to halve its frequency to move it within the basic octave. Therefore, E is tuned to 324 Hz, a 9:8 (= one epogdoon) above D. The B at 3:2 above that E is tuned to the ratio 27:16 and so on. Starting from the same point working the other way, G is tuned as 3:2 below D, which means that it is assigned a frequency equal to 2:3 times the frequency of D — with D at 288 Hz, this puts G at 192 Hz. This frequency is then doubled (to 384 Hz) to bring it into the basic octave. When extending this tuning however, a problem arises: no stack of 3:2 intervals (perfect fifths) will fit exactly into any stack of 2:1 intervals (octaves). For instance a stack such as this, obtained by adding one more note to the stack shown above $A \triangleright -E \triangleright -B \triangleright -F -C -G -D -A -E -B -F \# -C \# -G \#$

will be similar but not identical in size to a stack of 7 octaves. More exactly, it will be about a quarter of a semitone larger (see Pythagorean comma). Thus, A? and G?, when brought into the basic octave, will not coincide as expected. The table below illustrates this, showing for each note in the basic octave the conventional name of the interval from D (the base note), the formula to compute its frequency ratio, its size in cents, and the difference in cents (labeled ET-dif in the table) between its size and the size of the corresponding one in the equally tempered scale.

Note	Interval from D	Formula	Frequency ratio	Size (cents)	ET-dif (cents)
ΑЬ	diminished fifth	$\left(\frac{2}{3}\right)^6 \times 2^4$	$\frac{1024}{729}$	588.27	-11.73
ΕÞ	minor second	$\left(\frac{2}{3}\right)^5 \times 2^3$	$\frac{256}{243}$	90.22	-9.78
В♭	minor sixth	$\left(\frac{2}{3}\right)^4 \times 2^3$	$\frac{128}{81}$	792.18	-7.84
F	minor third	$\left(\frac{2}{3}\right)^3 \times 2^2$	$\frac{32}{27}$	294.13	-5.87
С	minor seventh	$\left(\frac{2}{3}\right)^2 \times 2^2$	$\frac{16}{9}$	996.09	-3.91
G	perfect fourth	$\frac{2}{3} \times 2$	$\frac{4}{3}$	498.04	-1.96
D	unison	$\frac{1}{1}$	$\frac{1}{1}$	0.00	0.00
Α	perfect fifth	$\frac{3}{2}$	$\frac{3}{2}$	701.96	1.96
E	major second	$\left(\frac{3}{2}\right)^2 \times \frac{1}{2}$	$\frac{9}{8}$	203.91	3.91
В	major sixth	$\left(\frac{3}{2}\right)^3 \times \frac{1}{2}$	$\frac{27}{16}$	905.87	5.87
Fs	major third	$\left(\frac{3}{2}\right)^4 \times \left(\frac{1}{2}\right)^2$	$\frac{81}{64}$	407.82	7.82
C#	major seventh	$\left(\frac{3}{2}\right)^5 \times \left(\frac{1}{2}\right)^2$		1109.78	9.78
G♯	augmented fourth	$\left(\frac{3}{2}\right)^6 \times \left(\frac{1}{2}\right)^3$	$\frac{729}{512}$	611.73	11.73

In the formulas, the ratios 3:2 or 2:3 represent an ascending or descending perfect fifth (i.e. an increase or decrease in frequency by a perfect fifth), while 2:1 or 1:2 represent an ascending or descending octave.

The major scale based on C, obtained from this tuning is:

Note	C)	E	F		G		A	В	(0	
Ratio	1/1	9/	8	81/64	4/3	;	3/2	27	7/16	243/128	2	2/1	
Step	_	9/8	9/8	256/	243	9/8	9/	8	9/8	256/2	43	_	

In equal temperament, pairs of enharmonic notes such as A? and G? are thought of as being exactly the same note — however, as the above table indicates, in Pythagorean tuning they have different ratios with respect to D, which means they are at a different frequency. This discrepancy, of about 23.46 cents, or nearly one quarter of a semitone, is known as a *Pythagorean comma*.

To get around this problem, Pythagorean tuning ignores A?, and uses only the 12 notes from E? to G?. This, as shown above, implies that only eleven just fifths are used to build the entire chromatic scale. The remaining fifth (from G? to E?) is left badly out-of-tune, meaning that any music which combines those two notes is unplayable in this tuning. A very out-of-tune interval such as this one is known as a *wolf interval*. In the case of Pythagorean tuning, all the fifths are 701.96 cents wide, in the exact ratio 3:2, except the wolf fifth, which is only 678.49 cents wide, nearly a quarter of a semitone flatter.

If the notes G^{\sharp} and $E \not\models$ need to be sounded together, the position of the wolf fifth can be changed. For example, a C-based Pythagorean tuning would produce a stack of fifths running from $D \not\models$ to F^{\sharp} , making F^{\sharp} - $D \not\models$ the wolf interval. However, there will always be one wolf fifth in Pythagorean tuning, making it impossible to play in all keys in tune.

Size of intervals

The table above shows only intervals from D. However, intervals can be formed by starting from each of the above listed 12 notes. Thus, twelve intervals can be defined for each **interval type** (twelve unisons, twelve semitones, twelve intervals composed of 2 semitones, twelve intervals composed of 3 semitones, etc.).

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	Interval											
From	A1	A5	A2	A6	A3	P1	P5	M2	M6	M3	M7	A4
Eb	37/211	38/212	39/214	310/215	311/217	1/1	3/2	9/8	27/16	81/64	243/128	729/512
вь	37/211	38/212	39/214	310/215	4/3	1/1	3/2	9/8	27/16	81/64	243/128	729/512
F	37/211	38/212	39/214	16/9	4/3	1/1	3/2	9/8	27/16	81/64	243/128	729/512
С	37/211	38/212	32/27	16/9	4/3	1/1	3/2	9/8	27/16	81/64	243/128	729/512
G	37/211	128/81	32/27	16/9	4/3	1/1	3/2	9/8	27/16	81/64	243/128	729/512
D	256/243	128/81	32/27	16/9	4/3	1/1	3/2	9/8	27/16	81/64	243/128	729/512
Α	256/243	128/81	32/27	16/9	4/3	1/1	3/2	9/8	27/16	81/64	243/128	210/30
E	256/243	128/81	32/27	16/9	4/3	1/1	3/2	9/8	27/16	81/64	212/37	210/35
В	256/243	128/81	32/27	16/9	4/3	1/1	3/2	9/8	27/16	213/38	212/37	210/38
F≓	256/243	128/81	32/27	16/9	4/3	1/1	3/2	9/8	215/39	213/38	212/37	210/38
C♯	256/243	128/81	32/27	16/9	4/3	1/1	3/2	210/310	215/39	213/38	212/37	210/36
G♯	256/243	128/81	32/27	16/9	4/3	1/1	218/3"	215/310	215/39	213/38	212/37	210/38
	m2	m6	m3	m7	P4	P1	d6.	d3	d7	d4	d8	d5

Frequency ratio of the 144 intervals in D-based Pythagorean tuning. Interval names are given in their shortened form. Pure intervals are shown in **bold** font. Wolf intervals are highlighted in red. Numbers larger than 999 are shown as powers of 2 or 3.

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	ues	ŧν	

From	A1	A5	A2	A6	A3	P1	P5	M2	M6	M3	M7	A4
ЕЬ	114	816	318	1020	522	0	702	204	906	408	1110	612
ВЬ	114	816	318	1020	498	0	702	204	906	408	1110	612
F	114	816	318	996	498	0	702	204	906	408	1110	612
С	114	816	294	996	498	0	702	204	906	408	1110	612
G	114	792	294	996	498	0	702	204	906	408	1110	612
D	90	792	294	996	498	0	702	204	906	408	1110	612
A	90	792	294	996	498	0	702	204	906	408	1110	588
E	90	792	294	996	498	0	702	204	906	408	1086	588
В	90	792	294	996	498	0	702	204	906	384	1086	588
F♯	90	792	294	996	498	0	702	204	882	384	1086	588
C♯	90	792	294	996	498	0	702	180	882	384	1086	588
G♯	90	792	294	996	498	0	678	180	882	384	1086	588
	m2	m6	m3	m7	P4	P1	d6	d3	d7	d4	d8	d5

Approximate size in cents of the 144 intervals in D-based Pythagorean tuning. Interval names are given in their shortened form. Pure intervals are shown in **bold** font. Wolf intervals are highlighted in red.

As explained above, one of the twelve fifths (the wolf fifth) has a different size with respect to the other eleven. For a similar reason, each of the other interval types, except for the unisons and the octaves, has two different sizes in Pythagorean tuning. This is the price paid for seeking just intonation. The tables on the right and below show their frequency ratios and their approximate sizes in cents. Interval names are given in their standard shortened form. For instance, the size of the interval from D to A, which is a perfect fifth (P5), can be found in the seventh column of the row labeled D. Strictly just (or pure) intervals are shown in **bold** font. Wolf intervals are highlighted in red.

The reason why the interval sizes vary throughout the scale is that the pitches forming the scale are unevenly spaced. Namely, the frequencies defined by construction for the twelve notes determine two different semitones (i.e. intervals between adjacent notes):

1. The minor second (**m2**), also called diatonic semitone, with size

$$S_1 = \frac{256}{243} \approx 90.225 \text{ cents}$$

(e.g. between D and E?)

2. The augmented unison (A1), also called chromatic semitone, with size

$$S_2 = \frac{3^7}{2^{11}} = \frac{2187}{2048} \approx 113.685 \text{ cents}$$

(e.g. between E b and E)

Conversely, in an equally tempered chromatic scale, by definition the twelve pitches are equally spaced, all semitones having a size of exactly

$$S_E = \sqrt[12]{2} = 100.000 \text{ cents.}$$

As a consequence all intervals of any given type have the same size (e.g., all major thirds have the same size, all fifths have the same size, etc.). The price paid, in this case, is that none of them is justly tuned and perfectly consonant, except, of course, for the unison and the octave.

For a comparison with other tuning systems, see also this table.

By definition, in Pythagorean tuning 11 perfect fifths (**P5** in the table) have a size of approximately 701.955 cents ($700+\Sigma$ cents, where $\Sigma \approx 1.955$ cents). Since the average size of the 12 fifths must equal exactly 700 cents (as in equal temperament), the other one must have a size of $700-11\Sigma$ cents, which is about 678.495 cents (the wolf fifth). Notice that, as shown in the table, the latter interval, although enharmonically equivalent to a fifth, is more properly called a diminished sixth (**d6**). Similarly,

- 9 minor thirds (m3) are ≈ 294.135 cents (300-3 Σ), 3 augmented seconds (A2) are ≈ 317.595 cents (300+9?), and their average is 300 cents;
- 8 major thirds (M3) are ≈ 407.820 cents ($400+4\Sigma$), 4 diminished fourths (d4) are ≈ 384.360 cents (400?8?), and their average is 400 cents;
- 7 diatonic semitones (**m2**) are ≈ 90.225 cents (100-5 Σ), 5 chromatic semitones (**A1**) are ≈ 113.685 cents (100+7 Σ), and their average is 100 cents.

In short, similar differences in width are observed for all interval types, except for unisons and octaves, and they are all multiples of Σ , the difference between the Pythagorean fifth and the average fifth.

Notice that, as an obvious consequence, each augmented or diminished interval is exactly 12? (≈ 23.460) cents narrower or wider than its enharmonic equivalent. For instance, the d6 (or wolf fifth) is 12Σ cents narrower than each P5, and each A2 is 12Σ cents wider than each m3. This interval of size 12Σ is known as a Pythagorean comma, exactly equal to the opposite of a diminished second (≈ -23.460 cents). This implies that Σ can be also defined as one twelfth of a Pythagorean comma.

Pythagorean intervals

Main articles: Pythagorean interval and Interval (music)

Four of the above mentioned intervals take a specific name in Pythagorean tuning. In the following table, these specific names are provided, together with alternative names used generically for some other intervals. Notice that the Pythagorean comma does not coincide with the diminished second, as its size (524288:531441) is the reciprocal of the Pythagorean diminished second (531441:524288). Also *ditone* and *semiditone* are specific for Pythagorean tuning, while *tone* and *tritone* are used generically for all tuning systems. Interestingly, despite its name, a semiditone (3 semitones, or about 300 cents) can hardly be viewed as half of a ditone (4 semitones, or about 400 cents). All the intervals with prefix *sesqui*- are justly tuned, and their frequency ratio, shown in the table, is a superparticular number (or epimoric ratio). The same is true for the octave.

10 10 10		Gene	ric names		Specific names					
Number of semitones	Quality and nun	ber	Oth		D. 40	E the la bouten	1/4-comma			
	Full	Short	Other naming conventions		Pythagorean tuning	5-limit tuning	meantone			
0					Pythagorean comma (524288:531441)		diesis (128:12			
0	diminished second	d2			(531441:524288)					
1	minor second	m2	semitone,	diatonic semitone, minor semitone	limma (256:243)					
1	augmented unison	A1	half tone, half step	chromatic semitone, major semitone	apotome (2187:2048)					
2	diminished third	d3	teen whel	a tono subala aton	sesquioctavum (9:8) semiditone (32:27) sesquiquintum (6:5)					
2	major second	M2	tone, who	e tone, whole step						
3	minor third	m3								
4	major third	мз			ditone (81:64)	sesquiquart	um (5:4)			
5	perfect fourth	ourth P4 diatessaron		n	sesquiterti					
6	diminished fifth	d5	tritone							
6	augmented fourth	A4								
7	perfect fifth	P5	diapente		sesquialte	rum (3:2)				
12	(perfect) octave	P8	diapason			duplex (2:1)				

History

Because of the wolf interval, this tuning is rarely used nowadays, although it is thought to have been widespread. In music which does not change key very often, or which is not very harmonically adventurous, the wolf interval is unlikely to be a problem, as not all the possible fifths will be heard in such pieces.

Because most fifths in Pythagorean tuning are in the simple ratio of 3:2, they sound very "smooth" and consonant. The thirds, by contrast, most of which are in the relatively complex ratios of 81:64 (for major thirds) and 32:27 (for minor thirds), sound less smooth.[8] For this reason, Pythagorean tuning is particularly well suited to music which treats fifths as consonances, and thirds as dissonances. In western classical music, this usually means music written prior to the 15th century.

From about 1510 onward, as thirds came to be treated as consonances, meantone temperament, and particularly quarter-comma meantone, which tunes thirds to the relatively simple ratio of 5:4, became the most popular system for tuning keyboards. At the same time, syntonic-diatonic just intonation was posited by Zarlino as the normal tuning for singers.

However, meantone presented its own harmonic challenges. Its wolf intervals proved to be even worse than those of the Pythagorean tuning (so much so that it often required 19 keys to the octave as opposed to the 12 in Pythagorean tuning). As a consequence, meantone was not suitable for all music.

From around the 18th century, as the desire grew for instruments to change key, and therefore to avoid a wolf interval, this led to the widespread use of well temperaments and eventually equal temperament.

In 2007, the discovery of the syntonic temperament [1] exposed the Pythagorean tuning as being a point on the syntonic temperament's tuning continuum. Discography[edit]

• Bragod is a duo giving historically informed performances of mediaeval Welsh music using the crwth and six-stringed lyre using Pythagorean tuning

- Gothic Voices *Music for the Lion-Hearted King* (Hyperion, CDA66336, 1989), directed by Christopher Page (Leech-Wilkinson)
- Lou Harrison performed by John Schneider and the Cal Arts Percussion Ensemble conducted by John Bergamo *Guitar & Percussion* (Etceter Records, KTC1071, 1990): *Suite No. 1* for guitar and percussion and *Plaint & Variations* on "Song of Palestine"

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