Bernays Project: Text No. 7

On Nelson's Position in the Philosophy of Mathematics (1928)

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(Über Nelsons Stellungnahme in der Philosophie der Mathematik, *Die Naturwissenschaften* **16** (1928), 142–145.)

Translation by: *Volker Peckhaus*

Comments:

Revised by Paolo Mancosu and Steve Awodey

||[142a] In connection with the preceding article by Otto Meyerhof, a few words on Nelson's significance for the philosophy of mathematics might be added.

Nelson was among those philosophers whose style of thinking resulted from a familiarity with the spirit of the exact sciences. Mathematics and physics represented the methodical ideal that he strove to achieve in elaborating his philosophical thoughts.

He considered the demand of systematic rigor to be ||[142b] completely satisfied in mathematical axiomatics, in particular in the form that Hilbert had given it in *Foundations of Geometry*. And he therefore endeavored to extend the reach of this method of axiomatics in the domain of philosophy.

In doing so Nelson avoided the unfruitful imitation of mathematics that was dominant in pre-Kantian metaphysics, which was based on the belief that knowledge could be conjured up from nothing by logical reasoning.

||[143a] As a follower of Kant, he held the doctrine of the *synthetic character* of mathematical knowledge; he stressed that the cognitive content of mathematics was captured in its axioms, which he considered the expression of knowledge deriving from *pure intuition*.

In various writings, in particular in the essay "Remarks on non-Euclidean geometry" (1906), he turned against the skeptical and the empiricist conceptions, which—in regard to the validity of the geometrical axioms—have found more and more adherents among scientists since the discovery of non-Euclidean geometry.

Here he shows how these views result from clinging to the old Aristotelian doctrine according to which all knowledge has its origin either in the senses as the source of experience or in the understanding as the source of logic.

If this disjunction, which in itself is not compelling, is dropped, one retains the possibility of recognizing extra-logical necessities, especially of an intuitive sort, which are expressed in synthetic propositions. In particular, concerning the parallel axiom, if this "dogmatic disjunction" is abandoned, it is by no means possible to infer from the logical possibility of a non-Euclidean geometry that the parallel axiom has no necessary validity, but only the that this axiom has a synthetic, i.e. non-logical, character.

These ideas were further elaborated by Nelson in a lecture "On the Foundations of Geometry," which he delivered in Paris in April 1914 (on the occasion of the foundation of the "Société internationale de philosophie mathématique."

Here Nelson supports his claim of the *intuitive but at the same time rational character of geometrical knowledge* by a series of arguments.

In particular, he points out that the difficulties presented by a conceptual description of (the continuity of) the continuum are a clear sign of the fact that this is a task posed to thought from without, i.e. through intuition.

He furthermore emphasizes that intuition cannot be charged with the typical geometrical errors such as, for instance, those which originate from overlooking the possibility of one-sided surfaces; rather, they result from hasty conceptual generalizations of intuitively grasped states of affairs.

In addition, he objects to the claim that non-Euclidean space can be grasped intuitively. In the familiar spatial presentations of non-Euclidean geometry, e.g. by the geometry of the interior of a sphere with a suitable definition of congruence, what is presented is |[143l] not a non-Euclidean space but only the satisfaction of the non-Euclidean laws by certain objects and relations of the Euclidean space.

If this argument is not accepted by many today, this is due to the fact that today's mathematicians and physicists have mostly lost sight the real meaning of the words "intuition" and "intuitive," so that one talks about intuitiveness in most cases only in a paled and blurred sense, according to which no distinction is drawn between real intuitive representation and mere intuitive analogy.

A weightier objection against Nelson's standpoint originates from the view that our spatial intuition is not perfectly sharp; therefore the geometrical laws are only approximately determined by intuition and are derived from the data of intuition only by a process of idealization.

Nelson argues against this claim as follows. It cannot be denied that the geometrical axioms represent an idealization with respect to the facts of observation. But this circumstance only speaks against the *empirical* character of geometrical laws. Their *intuitive* character is not thereby disputed (unless one relies on the dogmatic disjunction already mentioned).

On the contrary, an idealization presupposes an ideal. Only if such an ideal, in the sense of an epistemological norm, is given to us, does the abstraction that is to be carried out by the idealization have its definite distinctiveness, free of arbitrariness; and only then, as well, is the stability of the idealization vis-a-vis the extensions of our domain of experience guaranteed. Hence, it is the viewpoint of idealization that points to the fact of pure intuition, on the basis of which the process of idealization can simply be understood as the transition from sensory intuition to pure intuition.

From this doctrine of pure intuition as the norm for geometrical idealizations, Nelson draws the consequence that there is a fundamental difference between geometrical and physical idealization. In physical idealizations, the applicability to reality is always problematic, in the first place because the assumption of a limit for the idealizing limit process requires a justification through experience and therefore can only be shown as highly probable at best. By contrast, in geometrical idealizations the limiting entities are given to us in pure intuition, which guides the process of geometrical idealization; here the existence of a limit is for us certain, independently of experience. |[144l]

The independence from experience is not to be understood in the sense of pure immanence, so that one should, e.g., distinguish the apriori validity of geometry for intuition from the validity of "real" (physical) space. Rather, Nelson states explicitly—in this respect too, a true follower of Kant: "We know only *one* space. This is the space of geometry and in which physical bodies are."

Accordingly, the laws of geometry are binding for physics. They form the framework within which all natural science is bound, and only through which does the task of physical research receive its determination. This is because, as Nelson explains, if one makes geometry itself an object of experimental control, then one loses the possibility of drawing definite conclusions from physical observations. For, given a new observation, one can never know whether it expresses a previously unknown feature of space or some other physical fact. Nelson elucidates this by the following example. Let us assume that, when the Earth was thought to be a disk, one had established that the sum of the angles of earthly triangles was larger than two right angles; then one could equally have concluded from this result, according to the empirical conception of geometry, either a non-Euclidean property of space or the spherical shape of the earth.

What is said here in particular about geometrical laws similarly applies to all those laws which, according to the Kantian doctrine, are taken from pure intuition, i.e. also the laws of time and the geometric doctrine of motion (kinematics).

Because of his conviction about the binding apriori character of these laws for the physical explanation of nature, Nelson opposed the new physics, whose characteristic feature consists precisely in the increased freedom from the necessity of integrating all physical facts into the framework of the apriori fixed, spatio-temporal ordering, which resulted in the distinguished position of the geometric-kinematic laws vis-a-vis the physical laws.

However, this change in the methodological conception of physics forms only a part of the philosophical impact originating from the more recent development of the exact sciences. Another important influence comes from research on the *foundations of arithmetic*. Nelson was actively involved n the development of this research.

Nelson was |[144r] in close touch with the work resulting from *Cantorian set theory* through several members of the neo-Friesian school founded by him, especially Gerhard Hessenberg, who was one of the leaders in this development.

He dealt specifically with the *paradoxes of set theory*, the emergence of which he witnessed. These paradoxes had a special interest for Nelson because of their relation to certain dialectical modes of inference, which he often used for disproving antagonistic views — especially by showing an "introjected" contradiction, i.e. a contradiction which occurs in such cases where accepting the validity or insightfulness of a posited general claim already gives a counterexample to its validity.

The essay "Remarks on the paradoxes of Russell and Burali-Forti" (*Abhandl. d. Friesschen Schule*, II(3)), composed by Nelson together with Grelling, does not claim to solve the paradoxes; it served to state them more precisely and sharpen the given range of problems and reject unsatisfactory solutions. It is here that the very concise paradox related to the word "heterological" was presented for the first time.

Nelson was critical of attempts to found mathematics by pure logic. By contrast, he had a deep and active sympathy for the Hilbertian enterprise of a new foundation of mathematics. In this way of founding mathematics, Nelson welcomed the realization of the methodological principle of a *separation of critique and system*, i.e. the complete dissociation of the foundational procedure from the systematic deductive construction of mathematics, and the associated epistemological distinction between proper mathematical facts and "meta-mathematical" facts which have to be shown by the foundation. This agreement of the Hilbertian approach with the basic ideas of his own methodology, following Fries, was a source of great satisfaction for Nelson. Even shortly before the end of his life he expounded in a paper (56th convention of German philologists and schoolmen, Göttingen, September 1927) the methodological kinship of the Hilbertian foundation with the Friesian critique of reason.

There is, however, still another feature relating the Hilbertian foundation of mathematics to Nelson's philosophy: the "finitist attitude" demanded by Hilbert as methodological foundation must be characterized epistemologically as some sort of *pure intuition*, because, on the one hand, it is intuitive and, on the other hand, it goes beyond what can actually be experienced. |[145l]

The prerequisite of such a foundation of knowledge is, as such, still independent of the special nature of the Hilbertian conception; it holds for any finitist foundation of mathematics. A characteristic feature of the Hilbertian foundation, however, is that here the *finitist standpoint is related to the axiomatic foundation of the theoretical sciences.* The conditions of the finitist attitude present themselves thereby as the conditions *for the possibility of* *theoretical knowledge of nature*, quite in the sense of the Kantian formulation of the problem.

Once this connection is generally |[145r] recognized, it will be possible for the basic ideas of the Kantian critique of pure reason to be revived in a new form, detached from its particular historical conditions, from whose bounds theoretical science has freed itself.

Such a methodological clarification can help contribute to restoring what was correct in the rational tendencies that were always advocated by Nelson, but which are so one-sidedly disregarded today.