

Bernays Project: Text No. 18

**Some perspectives on the problem of the  
evident  
(1946)**

Paul Bernays

(DRAFT 1/21/01. *Synthese* 5 (1946), 321-326. German translation by the author in *Abhandlungen zur Philosophie der Mathematik* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1976).)

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Comments:

*This essay raises the question whether the French or the German version has greater authority. The former is clearly what Bernays originally wrote; most likely the German version was done in the course of his work on *Abhandlungen* many years later. It seems to me that the original character of the French version has greater weight than the fact that German was Bernays's native language; therefore the present translation is basically done from the French, but I have used the German from time to time when it seemed to clarify Bernays's meaning. The terms *vidence* and *Evidenz*, meaning the property of being evident, have always been problematic for translators into English. "Self-evidence" is sometimes used; that seems to me to have a much stronger and more specific meaning; I see no reason to suppose it captures Bernays's intention. I see no alternative to translating it simply as "evidence"*

in nearly all occurrences. But we might consider using Dallas Willard's expedient of capitalizing the word. (See "Knowledge", in Barry Smith and David Woodruff Smith (eds.), *The Cambridge Companion to Husserl* (Cambridge University Press, 1995).)

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The importance for philosophy of the question of the evident is hardly put in doubt. But it does not always happen that justice is done to the complexity of the problem. Evidence is often regarded as a quality that can simply be attributed or not to an axiom, a principle, or a mode of reasoning, and then the problem of evidence seems simply to be to decide where evidence is in fact to be found.

This simple aspect of the problem arises particularly under the influence of the idea of absolute evidence, which is substituted for the empirical concept of *de facto* evidence.

Refraining from introducing here an a priori postulate, we limit ourselves to *de facto* evidence - in other words we do not take as point of departure the idea of an absolute guarantee of truth, but we are content to observe that there are certain cases in our judgments and reasonings where we find a satisfying purchase or a point of departure given by a direct representation (which sometimes comes up spontaneously and sometimes requires some effort of the imagination). The object of evidence in this sense can be an existence or a relation. We know the distinction made in this respect by Leibniz, Hume, and others.

In taking account of the concrete character of evidence, we are led to recognize that evidence, originating from a mental situation, is relative to

the implicit suppositions that such a situation includes. Please note: the expression "relative" does not mean that there is here a sort of indifference of point of view. The mental situations at issue are those that knowledge passes through in its development, and it can very well happen that in reaching a position superior to a preceding one, we discover an implicit supposition that at the same time we find ourselves obliged to give up. In this way an evidence relating to a stage of the intellectual process can be lost at a more advanced stage.

In particular, this is the case for the evidence of outer sensation, such as it is found in the position of naive realism. At a more advanced stage we discover suppositions relative to that position that we must abandon because it turns out that:

1. Sensible qualities do not apply directly to reality;
2. The information that outer sensation gives us from objects does not have a character of immediacy.

It is known that the first theory to take account of this discovery is that of Democritus, revived by John Locke, a theory that introduces the distinction between real and apparent qualities. At this level a large part of the evidences of naive realism are still preserved. And one can say that it is the intention of the system of Kant to give a comprehensive philosophical interpretation to the situation as it thus presented itself.

As you know, there are theories that are opposed in a more radical way to naive realism: that of the phenomenalism of Mach and Avenarius and that of the philosophers of the school of Brentano, who deny completely the evidence

of outer sensation, only recognizing the evidence of inner intuition. It seems that this opposition to naive realism goes too far; in fact it is certainly not an adequate description of the facts simply to contest the evident experience of a reality surrounding us - "reality" taken here in a sense still unexplicated, and it must be observed that this primitive evidence is not shaken at all by the criticisms to which naive realism is subjected.

Looking at the matter more closely, it seems that the superiority of inner intuition over outer sensation does not in the first instance relate to the moment of existence: the existence of our Ego has originally scarcely more certitude and evidence than that of an external world. What constitutes the superiority of inner intuition is that the categories that it produces are immediately applicable to reality, which is not the case for those that derive from outer sensation. In fact it is evident that to feel, to see, to meditate, to doubt, to be glad, to be afraid, to feel pride or jealousy are possible states of a mental being; inner intuition, in supplying us with such categories, has a character less of sensation than of reason. (That is why it is almost impossible to separate the role of interpretation from that of perception in observations by inner intuition.)

Nevertheless you know that inner intuition, besides its qualities of evidence and rationality, also has its weaknesses:

1. There are also cases in experience of the mental where we are deceived by a direct impression.
2. Here as well, there is a kind of perspective, which distorts quantitative relations and hides important constituents of mental states.

3. Finally, an essential deficiency specific to inner intuition is that the attribution of mental states to the subject (that is to say to the Ego) is not made in an intuitive way.

Let us return to the main point: it was to determine the position resulting from the critique carried out on naive realism. Although one must give up almost all the realist evidence of outer sensation, something still remains: the indication of the existence of a reality that surrounds us, manifesting the variety of its content by the forms of contact that are revealed in our sensations.

We have here an important example of loss of evidence. But in the course of the intellectual process there are also evidences gained. At the outset naive realism is also an example, since the position of even this realism represents a stage in the acquisition of knowledge. But it isn't necessary to go so far back: in fact the evidences arising in mathematics are certainly almost all acquired evidences.

At issue here is evidence of relations, and the way in which they are formed is a special case of the general process of the origin of a dialectic, in the sense that Ferdinand Gonseth gives to that term.<sup>1</sup>

What distinguishes this case is that the dialectic is established in our mind in such a penetrating manner that it influences our intuitive imagination, that is to say that it influences the way in which we represent intuitively certain categories of objects. Thus the intentions of the dialectic find a sort of intuitive realization lying in spontaneous interpretations. In this way one also

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<sup>1</sup>(*Note added in the German version.*) In the meantime this use of the term in Gonseth's philosophy has been pushed into the background by another use.

understands that intuition can derive notions that surpass the possibilities of a complete effective control and whose conceptual analysis gives rise to infinite structures. In particular this is the case for geometric intuition, which engenders notions like that of symmetry, encompassing that of the middle, as well as the distinction between straight and curved lines. I believe that that way of looking at things is not in disagreement with the results to which Gerrit Mannoury was led by following out his distinction between choice and exclusion negation: one must concede that there are geometric notions that are not directly intuitive, such as that of straight lines that do not ever intersect which is precisely the usual definition of parallels. In general, it seems that geometric intuition has for its object only configurations of finite extension. (As is known, in the Euclid's Elements the axioms are formulated so as always to refer to finite figures; in the axiomatic system of Pasch, the rule to limit oneself to finite figures is observed intentionally.)

Concerning the theory of parallels, it must be noted nonetheless that the characteristic properties of Euclidean geometry can be expressed without introducing the negative notion of parallelism mentioned above. For example, the possibility, which children's games have made familiar to us, of juxtaposition of cubes (in such a way as to fill a portion of space without gaps) provides us with a formulation.

Thus our point of view permits us to recognize that the dialectic of Euclidean geometry has an intuitive evidence such that it is not found again in any other metric geometry. But one must also make the following remarks:

1. It should be understood that geometric evidence can no longer be considered - as was the case in Locke's philosophical position - as relating

immediately to physical reality (that is to say as expressing properties of physical space); rather, it is a case of phenomenological evidence, for whose genesis one can nonetheless assume external causes that lie in the structure of physical space.

2. It seems that there is a part of geometric evidence that has a more primitive and fundamental character: that is the evidence of topological relations. Let us observe in particular that in carrying out reasoning in elementary axiomatic geometry one uses in general more or less crude sketched figures; here what we represent to ourselves intuitively are only the topological properties of figures, while for the rest we proceed according to conceptual rules. It is clear that for this semi-intuitive manner of reasoning, Euclidean geometry has no privilege over the geometry of Lobachevsky.

3. It must be recognized that for the construction of mathematical theories in their present form one can do without geometrical evidence; in effect, it is eliminated from the mathematics of today as far as the foundations are concerned; the role that still devolves on it is on the one hand that of an interpretation of great value, and on the other hand, for topological evidence, that of giving directives for the conceptions of the general theory of spaces. But, so it seems, the tendencies of intuition can here only be satisfied approximately, in the sense of a compromise - and that whatever system of arithmetic one also uses.

We have envisaged geometric evidence as an example of acquired evidence. The same holds for the evidences guiding arithmetical methods; they are acquired at a certain stage of intellectual development.

It is true that there are, in the domain of purely formal relations, com-

pletely primitive statements, for example to the effect that, in applying the usual rules of elementary algebra, from the expression  $(a + b) * (a - b)$  one arrives at the different expression  $(a * a) - (b * b)$ . (That is not a tautology; in fact the indication of an operation to be carried out does not contain the indication of the result.)

These are the purest forms of evidence that we have at our disposal. But already the elementary theory of numbers goes beyond these primitive statements. There we encounter the general intuitive concept of natural number and the procedures of reasoning by complete induction and by recursive definitions that are connected with it. We have there already a full dialectic, which certainly did not exist from the beginning for the mind but which had to be tried out and dared at a certain stage.

Surely there is still a great distance between this dialectic of natural numbers and that by which we reason in the usual infinitesimal analysis. It must be conceded to Brouwer that this last dialectic does not have as fundamental an evidence as the former; moreover it must be admitted that it is not of a purely arithmetical character. Nevertheless we can state that it has succeeded quite well, that it constitutes a satisfying solution to the problems for which it was conceived, and that it too has engendered an evidence *sui generis*. What it lacks is only, with respect to the possible extensions of its methods, a leading idea appropriate for obtaining a delimitation that can be made without a conventional element.

The philosophy of intuitionism would suggest to us that we eliminate the usual dialectic in favor of a more strictly arithmetical procedure, as geometric evidence was eliminated. But in order that this idea should be accepted, it



would be necessary, according to the rules of knowledge, that the intuitionistic method should be shown to be superior in every respect to the usual method.

In any case, the possibility of eliminating an evidence in the foundation of a science is a remarkable fact. - Moreover, from our analysis of acquired evidences it follows that it is not an essential condition for the efficacy of a dialectic that it should be equipped with a specific mode of evidence.

One could conceive the idea of eliminating evidence completely from the foundations of the sciences, only keeping for it the role that it has in heuristics, analogy, and interpretation.

Nonetheless one notices soon enough that, for the foundations, one cannot do without primitive evidences concerning formal relations, because these are necessary to check the functioning of a dialectic and for noting contradictions. Moreover it is certain that for experimental sciences we need evidences from observation, that is to say some psychological evidences, but it must be pointed out that these evidences are not valid here in a direct way but are inserted in a complicated manner into the total process of empirical research.

As you know, one meets in the modern social sciences with the "behaviorist" tendency to eliminate the evidences of internal intuition as much as possible. It is pointed out that for the investigation of psychological facts indirect indication is often more reliable than the direct indication of internal intuition. Certainly one cannot reasonably dispute that, but this preference for external indications surely goes only up to a certain point, and in any case a psychology in terms of external objects and relations such as the extreme defenders of the behaviorist tendency contemplate has little chance of being

sufficient.

In mathematics there was the tendency of Hilbert, with his original conception of a theory of proof, to reduce all mathematical knowledge to primitive formal evidences. It was already a compromise to make use of the full finitist dialectic (incorporating the general concept of numeral), and it is [well] known that even that basis has been shown to be insufficient. Yet it is still possible that we might succeed in establishing a dialectic of constructive mathematics that would be equal to the requirements of proof theory.

But whatever the fate of these different attempts might be, in any case we are led to discuss the possibility of kinds of dialectic that do not have a character of evidence of their own. In order to work with such a dialectic a certain *understanding* is needed; we need to be in a position to attribute a sense to certain terms and to grasp relations resulting from the sense of these terms. (And the requirements of working with the dialectic are surely not the only ones!)

In this way we recognize the necessity of something like intelligence or reason, which will not be regarded as a container of [items of] a priori knowledge, but as a mental activity consisting in reacting to given situations by the formation of categories applied on a trial basis.

That reminds us of the tendency of Leonard Nelson, who (following the example of Kant and even more that of Fries) opposed making evidence the sole authority for knowledge. It was necessary to free this tendency from its subjection to traditional apriorism. That has just been attempted in the foregoing, in accordance with the ideas of the idoneism of Gonseth. It is also by this same idoneistic philosophy that we are led to recognize that it does

not suffice to have evidence, but reason in its totality is needed.