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On the role of language from an epistemological point of view (1961)

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Comments:

none

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[1] His book "The Logical Syntax of Language" occupies a predominant position in the philosophy of Rudolph Carnap. The conception [*Konzeption*] of the logic of science [*Wissenschaftslogik*] as the investigation of the language of science [*Wissenschaftssprache*] together with its concepts that is developed here form, so to speak, the initial framework [*Ausgangsrahmen*] for Carnap's further investigations. He has significantly revised the views that are expressed in the Logical Syntax in the course of these investigations. Also that framework for the considerations itself, together with its associated concept formations, have experienced heavy changes. The discussions with philosophers of related areas of research have contributed substantially to this.

[1]Contribution in honor of Professor Rudolph Carnap's seventieth birthday, received after Vol. XII no. 4 had been published *(editor's note)*.

These steps of revision of Carnap's philosophy stand for a successive dissociation from the exclusive and reductive tendencies of the initial program of the Vienna school. Already the Logical Syntax had brought significant corrections of its too simplifying theses. But here Carnap still defended the view that every epistemology, insofar as it claims to be scientific, has to be understood as being nothing else than the syntax of the language of science [*Wissenschaftssprache*], resp. the language itself. Since then he extended substantially the aim [*Aufgabestellung*] of the scientific philosophy by adding the semantics and pragmatics (following C. W. Morris) and, furthermore, by confronting [*gegenüberstellen*] the distinction between what is logical and what is descriptive with the other point of view of the distinction between theoretical and observational language. In the following the significance of the introduction of these extensions of the methodological framework for the shaping of Carnap's philosophy and also for its coming closer to more familiar philosophical views will be elucidated from several points of view; at the same time I want to point out certain questions that naturally suggest themselves in this context.

1

The composition [*Anlage*] of the Logical Syntax can be addressed as an extension of the approach of Hilbert's proof theory. For Hilbert the method of formalization ranges only over mathematics. However, in his lecture "Axiomatic Thought" Hilbert also said: "Everything at all that can be object of scientific thinking, falls under the axiomatic method, and thereby indirectly under mathematics, when it becomes mature enough to form theories." Carnap goes a step further in this direction in the Logical Syntax, by considering science as a whole as an axiomatic-deductive system which becomes a mathematical object through formalization: the syntax of the language of science is the metamathematics that is directed towards this object.

But the idealizing scheme of science that is used here is certainly not sufficient for epistemology. First, it represents only the finished result of science, not the entire process of scientific research [*wissenschaftlichen Geschehens*]. For the great mathematical theories an axiomatic-deductive presentation of the finished disciplines might display sufficiently well what is significant for them. But the circumstances are already fundamentally different in theoretical physics, since here the supreme principles of the theory in their mathematically precise formulation are not the starting point of the research, but the final result.

Moreover, for many areas of research it is an act of violence to to bring about the deductive structure [*für viele Gebiete der Forschung ist die Hervorhebung des Deduktiven gewaltsam*]. In these areas one does not even proceed deductively; rather, the logical reasoning is applied almost only for *heuristic* considerations, which motivate the formulations of hypotheses or of claims about matters of fact [*Tatsachen-Behauptungen*].

By the addition of *pragmatics* all of the above can be taken into account. It surely belongs to pragmatics to discuss the development of the sciences, not with regard to what is historical or biographical for sure, but in the sense of working out the methodologically significant trains of thought. Thus, here the heuristic considerations find their natural place [*Einordnung*].

Parenthetically I want to remind that the heuristic does not play a role only in the empirical sciences, but also in the purely mathematical research, which has been pointed out lately particularly emphatically by Georg Pólya. There exists a methodical analogy between the research in mathematics and in the natural sciences in the sense that also in mathematics there is a kind of empirical proceeding and a guessing of laws [*Gesetzlichkeiten*] based on a series of singular ascertainments [*Einzelfeststellungen*]. But such a formulation of a law is only of provisional character in mathematics, the more so as in number theory, where the individual case never can be singled out by irrelevant conditions (as those of place and time in physics), rather every single number has its own particular properties. But that it is possible even in number theory to gain convictions based on our practice with the numbers is shown by the example of the statement of the unique factorization of numbers into prime factors, which one tends to regard as completely self-evident (when one has not yet come across number theoretic proofs) from one's experiences by calculations [*Zahlenrechnen*]. Only at an advanced level the need for a proof for this statement is acknowledged, which is then satisfied accordingly.

2

It is useful for the consideration of the relation between syntax and semantics to bring to mind that, from the usual point of view, it is fundamental for a language as such that its words and sentences are oriented directly towards a sense [*unmittelbare Sinn-Bezogenheit haben*]. If the formation of forms [*Formbildungen*] of a language are objectified [*zum Gegenstand machen*] regardless of the meaning [*Bedeutung*] of the expressions, then this is a consciously effected, modifying abstraction.

In Carnap's Logical Syntax the exclusion of what is meaningful [*Sinnesmäßigen*] is compensated in part by stating [*statuieren*] "determinations of transformation" [*Umformungsbestimmungen*] as well as "determinations of form" [*Formbestimmungen*] as rules of the language. He not only counts those rules according to which a statement is transformed into a logically equivalent one as belonging to these determinations of transformation [*Umformungsbestimmungen*] of a formalized theory, but, more generally, all those that determine logical dependencies [*nach denen sich logische Abhängigkeiten bestimmen*], and moreover also the fixations [*Festsetzungen*] according to which particular statements have the role of logically universal propositions or *formalized axioms*.

Shortly afterwards, under the influence of Alfred Tarski's investigations and in connection with the extension of his methodical program, Carnap relegated the concept of logical entailment [*Folge*] from the syntax to the semantics.

The logical symbols obtain their meaning in the semantics through the "rules of truth", and the semantic concept of entailment [*Folgerungsbegriff*] is tied to these rules of truth. The formal deductions can be introduced from there by first noting [*vermerken*] the relations of consequence [*Folgerungsbeziehungen*] partly as propositions and partly as rules of inference [*Ableitungsregeln*], and then by axiomatizing the manifold of the obtained propositions and rules. In this way the concept of determinations of transformation [*Umformungsbestimmungen*] as primary rules of the language becomes basically dispensable, while the "rules of truth" should be seen as belonging to the characterization of the language.

The sharp contrast [*prägnante Gegenüberstellung*] between the semantic and the syntactic concept of entailment [*Folgerungsbegriff*] that is hereby obtained has great advantages for the presentation of mathematical logic — insofar this is not directed towards a constructive methodology from the outset — , and particularly Heinrich Scholz has emphasized [*zur Geltung gebracht*] this point of view.

It is often felt to be a shortcoming of the semantics that it is based on a non-constructive kind of concept formation [*Begriffsbildung*]. But being non-constructive is not specific for the semantics. One can practice [*betreiben*] a semantics in principle [*an sich*] also within an elementary framework of concept formation. On the other hand, it will be hardly possible to avoid transcending the elementary concepts [*Begrifflichkeit*], with or without a semantics, if one wants to fix a concept of "validity" [*Gültigkeit*], as Carnap intends to, such that for every purely logical proposition A (i.e., a proposition without extra-logical components) not only the alternative "A or not-A" is valid (in the sense of the principle of excluded middle), but in addition also that either the logical validity of A or of not-A holds.

The semantics is also criticized with regard to a different point, namely insofar it transgresses the domain of the logical considerations of extension [*? umfangslogische Betrachtungen*] and addresses questions regarding the sense and in particular regarding the relation between the extensional and the intensional. In particular Willard Quine claims that a scientifically inadmissible hypostasis [*Hypostasierung*] is performed by the introduction of contents of sense [*Sinngehalte*] (intensions) of expressions as objects [*Gegenständlichkeiten*], and that even by the reduction of questions about sense to those about sameness and difference of sense [*Sinngleichheit und Sinnverschiedenheit*] one still remains in the domain of what is difficult to make precise [*Gebiet des schwer Präzisierbaren*]. In this discussion Quine agrees with Carnap by tending to explain the sameness of the sense of two statements as their logical equivalence and accordingly to reduce the sameness of the sense of predicates and characterizations [*? Kennzeichnungen*] to logical equivalences. Thereby the concept of sameness of sense [*Sinngleichheit*] comes into a close relation to the analytic.

But such a definition [*Begriffsbestimmung*] of sameness of sense yields unwanted conclusions [*? Unzuträglichkeiten*], provided, as Carnap and many contemporary philosophers do, that the matters of fact of pure mathematics are regarded as logical laws. From this point of view any two valid statements of pure mathematics are logically equivalent and thus, if sameness of sense was the same as logical equivalence, any true [*zutreffend*] statements of pure mathematics, for example the statements that there exits infinitely many prime numbers and that the number π is irrational, would have the same sense—or, to take a simpler example: the statement $3 \times 7 = 21$ would have the same sense as the statement that 43 is a prime number.

But for this consideration we can even make ourselves independent from an opinion [*Stellungnahme*] with respect to the question of the purely logical character of arithmetic. Let us take an axiom system A and two totally different theorems [*Lehrsatz*], S and T, that are provable from these axioms. We would hardly be prepared to say that the claim "S follows logically from A" has the same sense as "T follows logically from A", even when both statements are true, thus both are logically valid and so both are logically equivalent.

Therefore, the sameness of sense by no means always coincides with the logical equivalence. On the other hand, in many cases, including mathematics, one surely would consider a logical transformation as not changing the sense. For example, one would consider the two statements "if a, b, c, n are numbers of the sequence of numbers beginning with 1 and $a^n + b^n = c^n$, then either $n = 1$ or $n = 2$" and "there do not exist numbers a, b, c, n of the sequence of numbers beginning with 1, such that $n > 2$ and $a^n + b^n = c^n$" to be formulations of the same mathematical claim (Fermat's great theorem).

With these examples we are confronted at first with the difficulty of delimiting what has to be considered to have the same sense. But at the same time we notice that this difficulty is based on the distinction of the kind of abstraction [*Abstraktionsweise*] which is peculiar to the different domains of inquiry [*Untersuchungsgebieten*]. We will profess two theoretical-physical [*theoretisch-physikalisch*] assertions to have the same sense, when one is obtained from the other by conversion [*Umrechnung*] of a mathematical expression it contains; but this is not permissible in general with mathematical assertions. We will say of a formulation of a mathematical proposition that its sense is not changed by an elementary logical transformation; but this will no longer hold when the elementary logical relations itself are considered. We have only considered the sameness of sense for statements; but the same can be stated for predicates and definitions [*Kennzeichnungen*]. Thereby the consideration of mathematical definitions [*Kennzeichnungen*] yields many examples in which the contrast [*Gegenüberstellung*] between extension and intension agrees with our usual scientific way of thinking. Let us take the representation of a positive real number by an expression of analysis, e.g., an infinite series [*? Reihe*] or a definite integral. The extension of this definition [*Kennzeichnungen*] is the real number itself, and the intension is a rule to determine this number, i.e., for it being contained [*Eingrenzung*] in arbitrary small intervals. As is well-known, one and the same real number can be determined by very different such rules; then we have the same extension with different intensions.

To mention also a predicate as a mathematical example for having the same extension with different intensions, the prime numbers among the numbers different from 1 can be characterized in two different ways: On the one hand, as those that have no proper divisor different than 1, on the other hand, as those that only divide [*? aufgehen*] a product, if they divide at least one factor. This results in two different intensions of a predicate with the same extension: The extension is the class of prime numbers, the intensions are the two definitions of the concept "prime number" that correspond to the characterizations. Analogous examples can also be found in the empirical sciences, e.g, if it is possible to characterize an animal species in different ways, so that different definitions result in the same concept of species [*Artbegriff*] and thus different intensions of the name of the species with the same extension.

On the one hand, our considerations show that there are ample classes of cases in which the concept of intension has a scientifically natural [*naturgemäße*] application. On the other hand, we have become aware of the difficulties with the concept of sameness of sense, which are related to the different attitudes in the different areas of research, whereby it does not suffice to contrast the logical with the extra-logical in order to meet [*Rechnung zu tragen*] the differences.

We can come closer to the matters of fact regarding this point if we bring to our mind the kind of abstraction which is relevant for the concept of intension.

Here one does not start with the separation of the expressions of the language as entities of form from their function to express something [*der Absonderung der Sprachausdrücke als Formgebilde von ihrer Ausdrucksfunktion*], but this is consciously [*geflissentlich*] retained. What is abstracted from are the particularities of means of expression [*Besonderheiten der Ausdrucksmittel*] that are irrelevant for this function and the variety [*Vielfältigkeit*] of formulations that are based on them which can be used to express the same content [*? den gleichen Ausdruckszweck*]. This manifold [*Mannigfaltigkeit*] of possibilities consists from a conventional point of view, on the one hand, in the multiplicity of languages, and on the other hand, in conceptual and factual equivalences that can hold between determinations [*? Bestimmungen*], properties, and relations. Such an equivalence warrants the substitution of an expression by another only if it is totally unproblematic in the framework of the exposition [*Darlegung*] or investigation in which the expression is used, i.e., if it belongs to the domain about which one does not have to discuss, but which is taken for granted. In fact, our efforts towards obtaining knowledge [*? Erkenntnisbemühungen*], at least at the stage in which reflection has developed [*im Stadium eines entwickelten Reflektierens*], are based on a certain supply (of which we are more or less conscious) of notions [*Vorstellungen*], points of view [*Ansichten*], and beliefs [*Überzeugungen*] to which we, either consciously [*mit Wissen*] or instinctively, hold on in our questions, considerations, and methods. Following Ferdinand Gonseth's concept "préalable", such notions [*Vorstellungen*], points of view [*Ansichten*], and beliefs [*Überzeugungen*] may be called "antecedent"

[*Vorgängig*].

The assumption of certain antecedent notions [*Vorstellungen*] and premises [*Voraussetzungen*] for any scientific discipline and also for our natural attitude in day-to-day life, is not subject to the same problematic [*Problematik*] as the assumption of *a priori* knowledge. It is not claimed that the antecedent notions are irrevocable [*unumstößlich*]. A science that is based on a premise [*Voraussetzung*] at first can lead us to abandon this premise in its further development, whereby we may be compelled to change the language of the science [*die Sprache der Wissenschaft*]. The scientific methodology [*Methodik*] also brings with it that we make ourselves aware of the antecedent premises and even make them the object of an investigation, resp., include them into the subject matter of an investigation.

These premises therby lose their antecedent character for the research area in question. In the course of the development of the theoretical sciences this leads to the fact that more and more of their premises are subjected to an investigation [*? Thematisierung*], so that the domain of what is antecedent becomes more and more narrow. The specially formulated [*statuiert*] starting-concepts [*Ausgangs-Begriffe*] and principles then take the place of the earlier, spontaneously formulated antecedents.

In contrast to the concept of *a priori*, the concept of antecedent is related either to a state of knowledge [*??? Erkenntniszustand*] or to a discipline; nothing is assumed that is antecedent in an absolute sense.

Accepting the concept of antecedent, one can formulate [*ansetzen*] the following definition of sameness of sense [*Sinngleichheit*]: two statements of a disciple have the same sense if the equivalence between them is antecedent to the discipline. The sameness of sense of predicates and characterizations [*? Kennzeichnungen*] would have to be explained correspondingly. In the definition the discipline can also be replaced by a situation of knowledge [*??? Erkenntnislage*] (state of knowledge [*??? Erkenntniszustand*]), in relation to which one could speak of an antecedent in a sufficiently determinate way.

It appears that the observed difficulties with the determination of the sameness of sense can be removed in this way. For sure, in this explanation one has to accept [*in Kauf nehmen*] that the sameness of sense of sentences depends on the discipline resp. the situation of knowledge [*??? Erkenntnislage*] in which it is considered. But, by closer inspection this turns out to be not so paradoxical.

3

Let us turn to the extension of the methodical framework of the logical syntax which Carnap obtains by contrasting the theoretical language with the observational language [*Beobachtungssprache*].

When considering the method of the natural sciences [*Naturwissenschaften*] we are used to contrast theory and experiment. But, in the initial form of logical empiricism, the moment [*Moment*] of the theoretical did not come into its own; only the discussions about the initial view [*Auffassung*], in which Karl Popper was involved in particular, have led to the preference of the point of view that the formulations of laws of nature figure as the proper sentences of the language of science in the revised standpoint of the logical syntax.

We understand that some resistance emerged against this when we make ourselves clear that with the acknowledgment of the role of statements of physical laws [*physikalischen Gesetzesaussagen*] as proper sentences the two-ness of "relations of ideas" and "matters of fact", that was proposed formerly by David Hume and which the Vienna school strived to maintain in a bit more precise form, turns out not to be exhaustive. On the one hand, the statements of laws [*Gesetzesaussagen*] of the natural science are not statements about "relations of ideas", i.e., not sentences of pure logic or of pure mathematics, on the other hand, they are neither statements [*Feststellungen*] of matters of fact since they have the form of general hypothetical sentences.

Using Carnap's diction this consequence says that the domain of the descriptive (the extra-logical) does not coincide with that of the factual, but rather that the domain of the factual is narrower.

The same facts [*Sachverhalt*] can be elucidated also from a different point of view. Carnap explains the concept of logical truth using "state description" in his book 'Meaning and necessity'. Thereby he follows Leibniz' idea of "possible worlds": what is necessary must hold in all possible worlds; and the "state descriptions" schematically represent the constitutions of the possible worlds [*möglichen Weltbeschaffenheiten*]. Thus, Carnap now defines: A sentence is logically true, if it holds for every "state description". In this consideration the concepts of necessity and possibility occur. But it is not agreed upon [*ausgemacht*] that one can speak of necessity and possibility only in the logical sense. Carnap himself mentions the investigation of those non-extensional operators which express physical and causal modalities [*Modalitäten*] under the open problems for semantics in the appendix to his 'Introduction to Semantics' (§ 38 d, p. 243). Physical and causal modalities concern what is possible within natural laws [*das naturgesetzlich Mögliche*] and what is necessary in nature [*das Naturnotwendige*]. Now if the laws of nature are stated [*statuiert*] as being valid in the framework of the language of science, and, furthermore, it is acknowledged that the laws of nature are not logically necessary, then a distinction between between what is necessary and what is actual [*Tatsächlichen*] follows which is different from that between what is logical and what is descriptive. Then we can consider "state descriptions" in a narrower sense by admitting only those that agree [*gemäß sind*] with the laws of nature and we thereby obtain a narrower manifold of possible worlds.

Thus, not only are the statements [*Feststellungen*] of what is factual contrasted with the statements of logical laws, but more generally with any statements of laws. We can now express this more general contradistinction [*Entgegensetzung*] using the concept of the theoretical, by contrasting the statements about what is actual with theoretical statements. Then the domain of what is theoretical contains what is logical as a proper subset.

What is specific of the theoretical surely does not only consist in the totality of statements which are acknowledged as being valid, but above all in a world of concepts [*Begriffswelt*] in the framework of which the theoretical statements take place [*erfolgen*]. Within the language of science the theoretical formation of concepts [*Begriffsbildung*] finds its place [*? findet ihren Niederschlag*] in what Carnap calls the "theoretical language".[2]

[2]By following Carnap in simply [*schlechtweg*] speaking here of "the theoretical language", the idea of an overall science [*Gesamtwissenschaft*] should not be implied. This is by no means the case also in Carnap's own explanations [*Ausführungen*] with regard to the theoretical language. He speaks of "methodological problems, that are connected to the build-up [*Aufbau*] of a theoretical system, like one for theoretical physics" (*Beobachtungssprache und theoretische Sprache*, Dialectica 47/48, p. 241–242).

Let us now take a closer look at the role that Carnap assigns to the theoretical language. In his view the theoretical language is not immediately [*unmittelbar*] interpreted, the theoretical terms rather obtain their significance only in connection with the "correspondence-postulates" [*Korrespondenz-Postulaten*], which establish the relations between the theoretical terms and the observational terms. However, these relations are not thought to be so extensive that they would define all the theoretical terms in the observational language [*Beobachtungssprache*]. Carnap rather joins the view of those who thought that the requirement that it should be possible to define every theoretical term experimentally and that it should be bound in its use by such a definition is too restrictive for theoretical research and also that it is not according to the actual proceeding in the theoretical sciences, as it has been done in the circle of neo-positivism in particular by Herbert Feigl and Carl Hempel.

A fundamental prerequisite for the freedom of the theoretical formation of ideas [*theoretische Gedankenbildung*] is hereby acknowledged. But it still remains the fact that the theory is not seen as a world of ideas [*Gedankenwelt*], but rather only as a language-apparatus [*Sprach-Apparatur*], so to speak. The reduction to the purely mathematical is added as another characteristic feature [*Moment*] to this only more technical aspect that Carnap attributes to the theoretical language. Whenever possible Carnap strives to reduce the theoretical entities to mathematical ones. This possiblity is shown in the domain of physics in a particular way by the presentation [*? Vorstellungsweise*] of field theory, whereby the physical events [*Geschehen*] consist of a succession of states in the space-time-continuum. The determination of states [*Zustands-*

bestimmung] is given by scalars, vectors, and tensors.

For example, the description of the physical state [*Zustand*] in the pure field theory of gravitation and electricity employs the symmetric tensor of the metric field, from which the measurement of length and time as well as the forces of inertia and gravitation are determined, and the antisymmetric electro-magnetic tensor which determines the electrical and magnetic forces. Material particles, either charged or uncharged, are understood here as particularly concentrated distributions of the magnitudes of the field [*Feldgrößen*] in a spacially narrow part of the world [*? einem räumlich engen Weltgebiet*]. The components of the tensors are functions of space-time points [*Stellen*], and when a coordinate system is introduced and units are chosen the magnitudes [*? Maßzahlen*] of the components become mathematical functions of the space-time coordinates;[3] let us call them "field-functions" [*Feldfunktionen*]. The physical laws of the field [*physikalische Feldgesetzlichkeit*] are formulated by the differential equations for these mathematical functions (in a way that is invariant with respect to the coordinate system), and the field-functions which represent the sequence of states of the system form a solution for this system of differential equations.

The connection [*Anknüpfung*] between the theory and the actuality of experience [*Erfahrungswirklichkeit*] is given through various kinds of relations:

1. those, on which the introduction of space-time coordinate systems and the possibilities of the values of the field-functions are based,

2. those which regard the effects of system states partly on our direct

[3]At the outset the components of the metric field are yet unnamed [*unbenannte*] numbers.

perceptions and partly on our experimental observations,

3. those which yield the instructions for the theoretical translation of a case that is observationally given [*beobachtungsmäßig gegeben*] (either only schematically or in a precise experimental determination) which should be investigated using the theory.

Carnap thinks that all these relations are axiomatizable by the correspondence-postulates in which the links [*Vernküpfungen*] between the field-functions and our observations are expressed. Such a system of correspondence-postulates can only be formulated if the manifold of the possible applications of field theory (of the differential equations of the field) to observations is axiomatizable.

Thus, with these reservations [*unter diesem Vorbehalt*] the possibility is given to wholly restrict the theoretical language of physics to mathematical concepts and to transfer everything that is specific to the physics partly into the observational language and partly into the correspondence-postulates. Then the theory of physics no longer makes statements about something that exists in the physical nature, it even does not state anything at all by itself, but it only yields a mathematical handle [*Handhabe*] for the predictions of observations on the basis of given observations. Strictly speaking, one should not talk here of a theoretical language at all.

But a kind of theoretical language can still be regained by introducing suitable physical names [*Benennungen*] for certain often recurring mathematical relations and expressions according to the meanings that they have in the contentually understood theory; then the procedure is analogous to interpreting geometrically the arithmetical relations and objects [*Gegenständlichkeiten*] of an (analytical) geometry that is constituted purely arithmetically.

What is perplexing in the described method of eliminating theoretical entities is the fact that it can be applied to any kind of formulation [*Ansetzen*] of natual objects [*Naturgegenständen*]: If the assumption of natural objects [*Naturgegenständlichkeiten*] is appropriate in the prevalent [*geläufig*] cases of daily life and, furthermore, if we extrapolate out prevalent methods of orientation in place and time to the notion of the four-dimensional space-time manifold, then it does not seem appropriate [*angängig*] to discontinue, so to speak, the formulation [*? Ansetzen*] of natural objects [*Naturgegenständlichkeit*] at a certain point and to replace the objects here by their mathematical descriptions.

However, Carnap can reply to this consideration that the difference of the methodical treatment does not relate to the differences of positions [*Stellen*] in the space-time manifold, but it refers to the different theoretical levels. What is meant by such a difference of level can be exemplified with the distinction between macro- and micro-physics. In general, a further theoretical level is present in the treatment of a domain of knowledge where the formation of concepts compels one to a greater transgression of what one is intuitively familiar with. Such a step of increased theoretization can be successful and proven satisfactory, and a practical safety can arise by the handling of the at first unfamiliar concept. But hereby the difference remains between what is methodically more or less elementar, i.e., between what is closer and farther away from the conrete and the observation.

It is obvious that quantum physics means an increased theoretization in the sense mentioned above compared with the previous "classical" physics. But the method for the elimination of entities described above cannot be applied directly also to quantum physics, since here the idea of a definite [*eindeutig*] sequence of states in the space-time manifold that is determined objectively and independently of experiments is lost. In a different respect quantum physics is well-disposed for the aims of the method of elimination, since here the idea of objecthood [*Gegenständlichkeit*] is already weaker and the mathematical considerations dominate the concept formations. Quantum physics also shows us how to implement the the different methodical treatments of diverse theoretical entities without offensive interruptions [*anstößige Zäsur*] by giving the role of the observational language, so to speak, to the theoretical language of the previous physics.

At the same time the idea is hereby suggested that it is reasonable to relate the distinction between observational language and theoretical languge to the level of the concept formations instead of understanding it to be absolute. If we think about the role [*? Bewandnis*] of the observational language in the scientific practice this idea is confirmed. When physicists talk among themselves about their experiments they surely do not speak only of objects of immediate perception. One talks maybe of a piece of wood, of an iron rod, of a rubberband or of a mercury-column [*? Quecksilbersäule*]. But the language in which physicists report their experiments goes much farther in this respect[4]. It is also noteworthy that the names of the concepts of physics

[4]Indeed, the thesis has been proposed that all experiments in physics come out to be statements about coincidences. But surely this claim has to be taken only *cum grano salis*: The statement of coincidence (or non-coincidence) is in each case only the last decisive step in the overall process of an experiment. Moreover this requires that the person doing the experiment [*Experimentator*] recognizes the equipment [*Apparatur*] as such and that he handles it correctly, also that this apparatus has been set up appropriately [*sachgemäß*].

(like "barometric pressure" [*Luftdruck*], "electrical current") have entered in great part the usual common language.

On the whole the facts [*Sachverhalt*] can be characterized by saying that an observational language of a science that is on a determinate level refers to an antecedent [*vorgängige*] world of ideas and concepts [*Vorstellungs- und Begriffswelt*]—"antecedent" in the sense that has been introduced in our section 2. The antecedent theoretical concepts obtain also their names [*Benennungen*] in the observational language at this level. We do not need to separate the observational language from the colloquial language [*Umgangssprache*] at all, I suppose. The observational language can rather probably be understood as a colloquial language that has been augmented by a greater set of terms [*größeren Reihe von Termini*].

The relativization of the observational language to a conceptual level [*begriffliches Niveau*] is fair also to that kind of opposition between what is empirical and what is theoretical that is intended by Ferdinand Gonseth's principle of duality. What is meant here is that there are no distinct empirical and theoretical domains, but that both moments come into play in every domain and in every stage of cognition [*Erkennen*]. The different points of view of the above considerations: the elimination of abstract entities, the distinction between theoretical levels [*Stufen des Theoretischen*], and the relativization of the observational language to a conceptual level all have their application in particular to mathematical proof theory. The latter assumes a distinction

Moreover the scientist [*Experimentierende*] should have sufficiently confirmed that no interferences occur etc. That all that which has to be understood and practiced in order to do this can be reduced to simple statements about coincidences is hardly the case. But, to be sure, this is not meant with that thesis.

between the "classic" method of mathematics that is applied in analysis, set theory, and the newer abstract mathematical disciplines and the more elementary methods that are characterized as "finite" [*finite*], "constructive" or "predicative", depending on the kind of demarcation [*Abgrenzung*]. In the proof-theoretic investigation of classical mathematics an elimination of the abstract entities is made possible by the method of formalizing propositions and proofs using the logical symbolism [*Symbolik*]. One tries to utilize this elimination to prove the formal consistency of classical theories from one of the more elementary standpoints. So far formal consistency proofs using constructive methods have been obtained only for such formal systems that can be interpreted at least predicatively. Recently it appears that a consistency proof for formalized impredicative analysis is possible from a wide version [*weiten Fassung*] of the constructive standpoint by a method developed by Clifford Spector.

The elementary "meta-language" in which such a consistency proof is carried out has the role of an observational language, as has been note by Carnap. Originally it was Hilbert's idea that this language should totally remain within the framework of concrete considerations, i.e., be an observational language in the absolute sense. But step-by-step one has been forced to include more and more theoretical terms [*Theroetisches*]. Already the "finite standpoint" uses strictly [*grundsätzlich*] more than Hilbert originally wanted to allow; but also this standpoint turned out not to be sufficient for the intended purpose, due to the results of Kurt Gödel. The result of this statement appears not to be so fatal for proof theory as it initially was seen if one accepts the idea of relating the observational language to a conceptual level [*Begriffsniveau*]. The acknowledgment of the methodical importance of the proof-theoretic investigations and in particular those about formal consistency is not tied to the view that the usual classical mathematics is dubious or to that standpoint of "formalism" after which classical mathematics is justified only as a pure technique for formulas [*Formeltechnik*]. Hilbert basically never thought in this way, despite of some remarks of his that point in this direction.—The task [*Aufgabenstellung*] of constructive consistency proofs is motivated by the high theoretical level [*Stufe des Theoretischen*] as it is present in classical mathematics.

In any case, an adherent [*Angehöriger*] of the constructive proof-theoretic direction of research can very well have the point of view which is favoured also by Carnap that the concept formations of classical mathematics have their justified application also when they are considered contentfully [*inhaltlich*]. But whether it is reasonable to accept all entities that are introduced by set theory as real [*eigentlich*] is open for discussion also from this standpoint. One would also not be inclined to award the positive position with respect to the theoretical concepts as a privilege to the mathematical concept formation: What is just for the mathematical classes and functions is equitable for the entities of the natural sciences insofar they are used [*angesetzt*] in a way that generates understanding.