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CHOICE AND DESCRIPTIONS IN ENRICHED INTENSIONAL LANGUAGES – I

Many intensional logicians have not abandoned, as unrealisable, the dream of something like Leibnitz's *characteristica universalis*, of an almost universal logical language, with simple components, and with a precise and acceptable semantics, within which the whole of (English) discourse can be expressed, and whose semantics provides theories of truth, consequence and meaning for the discourse expressed. Admittedly things did seem rather desperate when, after the initial successes of the pioneering days of Wittgenstein's *Tractatus Logico-Philosophicus* and Carnap's *The Logical Syntax of Language* had faded, it was shown that the theories of these texts fail at point after point. However, most of the objections which were taken as winning the day against these theories – for example the inability of the theories to cope at all adequately with intensional discourse, with context dependence, with non-significance and other discourse failures – are now seen to rest on the paucity of the logical, and especially semantical, equipment then available, not in inherent limits to logic and semantical analysis.[1] To make the dream a little more real then, both the logical syntax and the semantical framework will have to be much enriched.[2]

Realising the dream involves however a very ambitious program. We don't pretend to know whether it can be brought off, or how exactly. But we do believe that if the program is to stand a chance of succeeding its logical theory will have to include enrichments like those we go on to discuss. These enrichments in turn however lead to many new problems, several of them philosophical (and the regress in philosophical problems set up by technical solutions to earlier philosophical problems may well be vicious). We opt, in later sections of the paper, to concentrate on one important set of these problems, one which has a special bearing on the shape the grand semantics should take, the general admission of descriptions into intensional discourse, and of a choice descriptor in particular.

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I

We are still far from having even intensional formal systems which are rich enough to provide base structures for the analysis of more than a mere fragment of English discourse. On the so-called syntactical front there are two main classes of problems, that of determining an adequate morphological structure, and that of working out the associated axiomatic constraints. Ways of rectifying the worst morphological deficiencies of formal languages, as bases for natural language grammars, have been known since the Thirties, ways that again seem promising now that a transformational component offers hope of smoothing the remainder of the way to natural language grammars; but appropriate axiomatic treatment of fully intensional notions, as distinct from modal notions, is only in its infancy, and only recently have such important notions as entailment obtained partial formalisations. Even the question of the incorporation of descriptions and extensional identity into quantificational formalisations of these theories leads, as we shall see, to a series of so far largely undiscussed and unresolved issues.

There are two common ways of enriching the morphology of formal languages: either by enlarging the formation rules in the style of a phrase structure grammar which adds as required new grammatical categories and corresponding combination rules, or by switching to a functorial (i.e. categorial) grammar, where new grammatical categories and their controlling rules are derived functorially from a finite set of base grammatical classes. The methods can of course be combined, and in any case the second method amounts to but an important special case of the first since functorial grammars can always be reformulated as phrase structure grammars. The functorial method has great appeal to logicians; for it has, as Adjukiewicz [3] pointed out, considerable power combined with simplicity, only one or two rules being required to generate a wealth of forms, many of which require separate phrase structure rules; and it has the additional merit of running in tandem with functorial semantical methods.[3]

The rarely appreciated fact is that there is a ready-made formal and semantical theory which lends itself admirably to reinterpretation as a theory with a functorial grammar, which so reinterpreted takes up all that Adjukiewicz was striving for with his full theory of syntactic connexion [3], and which is ripe for intensional and also significance enlargements.

This theory is the simple theory of types, as formulated by Church [7] (in the form which excludes arithmetic) and furnished with a semantics by Henkin [6]. Let there be no mistake about what we are claiming and what we are not: simple type theory is inadequate as a theory of significance (which is what Russell thought type theory was), thoroughly unsatisfactory as a set theory and as a way of resolving the logical paradoxes (which is how the theory is usually regarded nowadays), but extremely promising as a basic logic for discourse. When the simple theory of types is so revamped linguistically, it provides a ready-made base for improvements much superior to quantification theory, which it of course includes. Nor is there anything particularly new about construing type theory as a logical theory built on a functorial grammar. Russell himself, after early vacillation, came down, in order to meet Black's objection to the theory, in favour of interpreting types syntactically (see [8], pp. 691–2), and the Hilbert school re-expressed the linguistic type rules as formation rules, thereby establishing the modern practice of treating type theory as providing a grammar. Unfortunately however the prevailing practice has also tended to recast type theory as a set theory and to interpret types ontologically rather than linguistically, thereby throwing away all the advantages and most of the initial plausibility of simple type theory. To avoid confusion of our construal of the theory with the prevailing one, we call our theory *structure theory*, and our labels not type symbols but structure labels or grammatical category symbols. As formulated by Church the theory has in effect only two basic grammatical categories, declarative sentences with structure label o, and subjects (proper names on a narrower – too narrow – construal) with structure label ι. There are however good reasons for not stopping at two, for including as basic categories at least general terms (including common nouns[4]), with structure label μ say, and perhaps also imperatival sentences and question sentences. We assume then that there are finitely many basic categories including at least those labelled by o, ι and μ. There is just one rule for generating derived grammatical categories, which we reflect in the following formation rule:

> If α and β are structure labels so is $(\alpha\beta)$,
> or in Adjukiewicz's notation α/β.

The derived category with label $(\alpha\beta)$ consists of parts of speech which take an expression with label β and yield one with label α; for example one-place predicates have label $(o\iota)$ since such predicates take a subject and deliver a sentence, and similarly two-place predicates have label $((o\iota)\ \iota)$. This functional mode of category deriviation is an outcome of the following recursive formation rule (which is but Adjukiewicz' central rule):

2. If E and F are wfps (well-formed phrases) with structure labels $(\alpha\beta)$ and β respectively then (EF) is a wfp with structure label α.

This rule is of course underpinned by a base rule

1. A variable or constant alone is a wfp with structure label as specified:

and this rule is in turn underpinned by a listing of primitive symbols and their respective structure labels. The primitive symbols consist of a (denumerable) set of variables or parameters for each structure label, parentheses, and a set of constants, which include in the extensional two-valued case the constants N (for negation) with label (oo), K (conjunction) with label $((oo)\ o)$, π (universalization, read 'everything') with label $(o(o\alpha))$ and ε (arbitrary selection [5]) with label $(\alpha(o\alpha))$, where for an expanded quantification theory α is ι (and where, for a more general theory, which however introduces expressions whose grammatical well-formation is decidedly questionable, α is any structure label). In Church's formulation the functional abstraction symbol λ is classified, along with parentheses, as an improper symbol, and the following formation rule is added for λ-expressions:

3. If E is a wfp with structure label α and w is a variable with label β then (λwE) is a wfp with label $(\alpha\beta)$.

However this rule can be reduced, at the cost of enlarging the syntax marginally, to a case of rule 2 by reclassifying λ as a constant with label $(((\alpha\beta)\ \alpha)\ \beta)$ which always applies to a variable.[6] Then, under the reduction, an occurrence of a variable w is bound if it is in a wfp of the form λw (the form of the *pure* bound variable); otherwise it is free. A wff (well-formed formula) is a wfp whose label is o, i.e. it is a, perhaps open, sentence.

Unlike Church but like Russell, Leblanc and Meyer [9], and written English, we generally refrain from writing structural labels on wff (although it is sometimes handy to so index variables). Whenever required a *labelled* wfp can (like a phrase marker) be derived by adding labels, as determined recursively by the formation rules, to the components, e.g. as subscripts.

It is now easy to explain how it is that structure theory encapsulates Adjukiewicz's theory so neatly. Adjukiewicz's theory of syntactic connexion can be seen as consisting of two parts, a theory of concatenation of structure labelled expressions which is mirrored faithfully in structure theory[7] because of rule 2, and a further tentative theory – designed to cope with expressions containing operators – of circumflex operators and constants such as π (with index $s/(s/n)$).[8] But the theory of circumflex functions sketched (which is based on Russell's use of the circumflex symbol in distinguishing functions from function values notationally) is none other than Church's theory of λ-conversion, with the rules of conversion clearly formulated (for the extensional case; [3], p. 229); and the further method of reducing quantifiers as operators to constants applied to circumflex expressions is exactly that Church adopts (in [7]). Though structure theory captures Adjukiewicz's theory it in fact goes much further, in as much as it is already fully axiomatised in the two-valued extensional case by Church's theory, for which Henkin has supplied a model-theoretic semantics.[9] The salient point however is that the morphology of structure theory or, what almost comes to the same, the grammar of Adjukiewicz's full theory, provides a very general framework for logico-grammatical investigations to which a range of axiomatic and semantic systems can be geared.

It is worth exhibiting a little of the power of structure theory for the formalisation of discourse, before turning to its weaknesses and to its axiomatic enrichment. The theory provides a functorial grammar adequate as a base for the formalisation of substantial fragments of English discourse (as the work of Montague [2], Lewis [5] and others indicates). For it not only includes quantifiers and some descriptors; by providing a general theory of syntactic connexion it includes a ready-made theory of modifiers, of adverbs and attributive adjectives, in all their iterations and profusion; and Henkin's work once again furnishes a semantics in the easy two-valued extensional case.[10] Enlarged by constants to reflect operators and connectives of English, the applied theory constitutes a theory (of varying adequacy) of all parts of speech.[11] For example tenses can be included as verb modifiers, conjoined and disjoined subjects and predicates can be included simply by letting constants K and A, of conjunction and alternation, have, as well as structure labels $((oo)\ o)$, labels $((\iota\iota)\ \iota)$ for subject concatenation and $(((o\iota)\ (o\iota))\ (o\iota))$ for predicates, and so on. Of course to obtain an axiomatic theory, postulates governing the connectives have also to be added, e.g. a lattice structure would be imposed in the case of compound subjects.

A crucial weakness of functorial grammars, from the point of view of natural language analysis, springs from the assignment of a single structure label to each expression. For instance a verb like 'rides' may be treated as mapping a single subject to a sentence (i.e. it will have label $(o\iota)$), or it can, in effect, map a pair of subjects to a sentence (i.e. it will have label $((o\iota)\ \iota)$), but it cannot do, what 'rides' does in English, both jobs, which depending on its sentence context. The same goes for adverbs and adjectives, and other derived parts of speech, where one and the same *unambiguous* English expression may, for example, map one or two or more place verbs onto the same and may also map other adverbs to adverbs and adjectives to adjectives (e.g. quietly, quickly, silently). This fundamental problem[12] can however be avoided (if not resolved) by allowing derived categories – not basic ones or effectiveness could be jeopardised – to have labels which may vary within specified limits. For example a predicate (general verb) can be characterised as an expression with structure label π_k (i.e. $(\ldots((o\iota)\ \iota)\ldots\iota)$ with k occurrences of ι) for some k; and the verb 'rides' would be assigned label π_k with k restricted to 1 or 2. Then in any given occurrence precisely which label 'rides' has will be determined by its frame of occurrence. In short the problem may be circumvented by making the *reduction* of an English sentence to logical form *frame* (or context) *sensitive.* Since the reduction is frame sensitive the language being reduced has really to be assigned at least a frame-sensitive functorial grammar. The structure labels of expressions of derived grammatical categories are determined in a sentence frame by the expressions upon which they operate; for example 'Tom rides Dobbin' is first recast in logical order as '((rides Tom) Dobbin)'; then since the whole is a sentence and 'Tom' and 'Dobbin' each have label ι, π_k is resolved to π_2[13]. The sentence is not ambiguous and unlike ambiguous sentences has only one reduction.

A serious weakness of structure theory, insofar as it simply reinterprets type theory, is its extensionality. The worst features of Church's theory can be avoided, as is well-known, by simply omitting the axioms of extensionality. Nonetheless it remains essential to intensionalise the resulting structural logic; for although intensional discourse can be formally represented once extensionality axioms are omitted, its main logical and semantical features cannot be revealed, and many logical relations are either destroyed or have to be specially postulated. As Ackermann explains ([14], pp. 16–18), without intensionalisation even simple entailments such as that John believes that Tom is tall and Tom is thin entails that John believes that Tom is thin, and simple reference statements such as that John's beliefs are about the same person, cannot be accommodated. A crucial corresponding semantical reason for intensionalisation is that otherwise the semantic evaluation of compound expressions cannot be obtained from the evaluation of their components. A merely extensional logic with an extensional semantics is inadequate to deal with the effect of connectives and modifiers in expressions such as 'necessarily $2+2=4$', 'walks rapidly', and 'possible president'. For example in the case of 'possible'

> one can find two common nouns ζ and η corresponding to the same set of individuals – or having as we should ordinarily say, the same extension (with respect to the actual world and the standard model) – but such that 'possible ζ' and 'possible η' have different extensions. (Montague [2], p. 220).

An immediate outcome of the compounding requirement is that modalisation alone is inadequate. For there are many connectives and modifiers, e.g. 'Tom believes (that)', 'it is interesting (that)', 'allegedly' for which substitution or interchange conditions are stronger than modal, i.e. than intersubstitutivity of strict equivalence of some brand – with the result that functionality once again breaks down if only intensionality of modal strength is required. By basing the theory not on a modal logic, but on a weak entailment logic, this problem can be largely avoided.[14]

The business of intensionalising structure theory is unproblematic, at least as far as the logical syntax is concerned. We add an implication F with structure label $((oo)\ o)$ and define $(A \rightarrow B)$ as $((FA)\ B)$ where A and B are wff; thus $(A \rightarrow B)$ is a wff. We replace the axioms, 1–4 of Henkin [6], for classical two-valued logic, by a set of axioms (without structure labels attached) for entailment (e.g. any set from [12]), the quantificational axioms, 5^{α}–6^{α}, by a set of axioms for quantification with entailment (as set out in [13], or in Part II of this paper), we abandon, of course, axioms 10, of extensionality, and we replace '$\supset$' in the ε-axiom, 11, and the rule of detachment, V, by '$\rightarrow$'. Otherwise the revamped Church system is kept; e.g. the rules for substitution and λ-conversion remain as formulated in [6], though it now follows of course that $(\lambda x A(x))(y) \leftrightarrow A(y)$, and weaker equivalences no longer guarantee their intersubstitutivity. In fact however, the postulates can be drastically chopped, since the whole quantificational structure and rule VI may be derived using the ε-axiom

(0) $A(x) \rightarrow A(\varepsilon A)$, where A has label $(o\alpha)$ and x label α, for any α.

But designing an adequate semantics for such an intensionalised structure theory is by no means so unproblematic.

The intensionalising of structure theory is only the first of a series of desired improvements. The next step of making the theory into a significance theory is easy when the starting theory is extensional structure theory; the axioms (of reinterpreted type theory) are simply replaced by those from systems QS_6 and QSE of [1], Chapter 7. But the matter of amalgamating significance and intensional enlargements is not so easy and remains to be done, as does the conversion of these logics into context logics (along the lines laid out in [1], Chapter 2).

As in the syntactical case, so also in the semantical case, though the framework for general semantics has been progressively and strikingly enlarged since the early pioneering days, the framework is still too austerely fashioned, in several important respects which severely limit its adequacy for the analysis of discourse and in providing a background setting for philosophical problems; namely, through

(i) the restriction to possible worlds,
(ii) the restriction to possible individuals,
(iii) the restriction to an underlying two-valued (and usually first-order) account,
(iv) the meagre role, if any, assigned to context.

Each of these restrictions can be removed, (i) by the inclusion of impossible and incomplete worlds and theories, and of set-ups generally, (ii) by the inclusion of inconsistent, incomplete and indeterminate items, whether individual or not (thus (ii) generalises on (i)), (iii) by enlarging the number of values, and (iv) by using context supplied functions in semantical evaluations. Moreover the admission of more of discourse forces the abandonment of these restrictions, e.g. the admission of belief and entailment functors forces the abandonment of (i), and the general admission of singular descriptions, which we go on to advocate, compels the abandonment of (ii), since some descriptions are not about possible individuals but are about items which are impossible. Naturally these desirable extensions of the semantical framework bring in their turn many new problems, some of them technical, such as the amalgamation of theories designed to surmount just one of the restrictions, and some of them philosophical.

There are two main reasons why these philosophical problems arise. Firstly, philosophers mistakenly committed to the ontological assumption that what they talk about must exist, are reluctant, to say the least, to talk about inconsistent or incomplete items, which of course do not exist; and hence are loath to use the enlarged semantical framework. Philosophy has generally – as part of the prejudice in favour of the actual – been opposed to indefiniteness, to the indeterminacy and ill-roundedness of objects: only inconsistency is considered worse. Secondly, the enlarged semantical framework is not only open to rival (metaphysical) construals, but is rich enough to incorporate views of rival philosophical positions. This emerges especially in that supposed paradigm of philosophical neutrality, the theory of truth. Each formal philosophical theory, each different theory of descriptions for example, will fashion its own theory of truth, and philosophical disputes will be reflected in different truth-values assigned and equivalences admitted, e.g. for sentences containing non-denoting descriptions. (Admittedly many – not those which disagree about what can be true or false, and not all many-valued theories – will lead, for some class of sentences, to versions of Tarski's convention T, but even these theories will disagree about what count as equivalent reformulations of the convention.) There will also be dispute about the formulation of the theory of truth, whether it should be model-theoretic, domainless or merely substitutional, about the relation of the metatheory to the object language, about the point of extensionality and the extent to which it is mandatory, and so forth. In short, the semantics, and the theory of truth it incorporates, cannot hope to be philosophically neutral, and cannot in general offer any resolution of philosophical issues but only, what is nonetheless very important, a sharpening and reformulation of them. One reason for interest in the wider semantical framework, connected naturally enough with its importance in supplying a semantics for more of discourse, is that it promises to supply a semantics for, and a sharpened formulation of, much of Meinong's theory of objects.

II

An important amalgamation problem, which generates many philosophical issues, is that of adequately intensionalising logics which are not existentially committed, that is to say of combining logics which can formalise discourse and assess arguments about what does not exist (namely most items) as well as what does exist, with logics which include fully intensional functions of at least the order of entailment. Singular descriptions play a central role in all this; for a very common way – not the only way because ordinary proper names will do as well – of talking and arguing about items that do not exist is by use of descriptions. Indeed a leading feature of, and part of the motivation for, the more adventurous of those theories (such as Meinong's theory of objects) and logics (such as that of [1]) which aim to go beyond the actual and to deal quite generally with discourse about what does not exist, has been the wholesale admission of singular descriptions of a wide range of sorts (e.g. definite, indefinite, generic, attributive) as (substitution) values of subject variables. The admission is warranted because singular descriptions are perfectly satisfactory, and generally uneliminable, subjects (the case for this is argued in [15] and also in [1]), and therefore ought to figure in the substitution and instantiation ranges of subject variables, both free and bound – and not just be admitted to the free variable positions as in free logic. The admission of such descriptions as logical subjects – which induces no logical disasters – at once enriches the underlying logic (e.g. structure logic), makes it plain why the logic has to be interpreted non-existentially, and shows how the logic differs from classical logics such as Russell's theory and standard description theories. For several principles of quantification logic, for example, automatically extend to descriptions as a consequence of the fact that descriptions are values of subject variables; e.g. the principles $(Ux)\, A \rightarrow A(\iota xB)$ (where '(Ux)' reads simply 'for every x' with 'every' ontologically neutral, and $(Ux)\, A =_{\mathrm{df}} \Pi(\lambda xA)$ in structure theory, and 'ι' represents a likewise neutral 'the') and $\iota xA = \iota xA$, both of which fail on Russell's theory because the item described may not exist, emerge immediately as correct principles. That these principles already hold in structure theory reflects the fact that this theory has effectively incorporated the assumption, which Church rightly accepted, that (at least ε- and ι-) descriptions are values of subject variables; this is a further reason for our interest in structure theory. More generally the admission of descriptions as values of variables implies that the principles of universal instantiation and its contraposition particularisation hold generally not only for all proper names but for arbitrary singular descriptions τxB (or equivalently $\tau(\lambda xB)$, or τA where A has label μ, for arbitrary descriptor τ), and in particular for logically interesting descriptions formed by using descriptors such as ι ('the'), ε('an arbitrarily selected' or, almost, 'any'), I (generic 'The', or capitalisation of one sort) § ('that'), κ ('a certain', as distinct from 'some or other') and $\wedge$ (Russell's class abstraction).

However, the issues at stake are not confined to the question of whether singular descriptions can be values of subject variables (where no distinction is drawn between the roles of free and bound variables), but turns more generally on whether descriptions and names can take on their full roles – roles some of them *do* have in discourse – in introducing and eliminating quantifiers within an intensional framework. That descriptions are full values of variables guarantees, as we saw, at least the straightforward cases of universal quantifier elimination and particular quantifier elimination, namely $(Ux)\,A \rightarrow A(\tau yB)$ and $A(\tau yB) \rightarrow (Px)\,A$ (possibly after an alphabetic change of bound variable in $A(\tau yB)$), where $(Px)\,A =_{\mathrm{df}} \sim(Ux)\sim A$. The remaining cases, familiar from natural deduction systems, $(Px)\,A \rightarrow A(\tau_a yB_b)$ and $A(\tau_c yC_d) \rightarrow (Ux)\,A$, will not hold generally, but only for particular specifications of descriptors τ_a and τ_c and of wff B_c and C_d. But will these latter principles hold generally for *any a*, *b*, *c*, *d*? If they do then a main part of the case we are going to argue will be made. However, it is known from semantical analyses of quantified modal logic that these principles are independent of quantified intensional logic, so the mere adoption of such a logic does not commit one to the principles. Thus the case for these principles has to be argued over and above the – difficult enough – case for quantified intensional logic.

A central argument will simply be that these principles are analytic; they hold in virtue of the intended sense of the descriptors in question, as reflected in the semantics which brings the principles out as logically necessary. For example, $(Px)\,A \to A(\varepsilon yA)$ holds in virtue of the intended sense of descriptor ε, as meaning 'an arbitrarily selected'. The relevant descriptions have already been admitted as values of variables, so the particular principles which distinguish distinct descriptors should also be formulable and should be incorporated in any theory which pretends to be adequate. In short, once descriptors are admitted as full values of variables, these further principles are forced upon us by the intended sense of the descriptors and the principles which characterise and distinguish them.

This naturally raises the more general question: which principles distinguish and characterise distinct descriptors? We know a little bit about this – for example, though

(0′) $\quad (Px)\,A \to A(\varepsilon xA)$, is correct,

$(Px)\,A \to A(\kappa xA)$ is doubtful, because κxA is a preselected element, not an arbitrarily selected one, and $(Px)\,A$ may not guarantee the relevant element at every world; and $(Px)\,A \to A(\iota xA)$ is not correct, but only $(P!x)\,A \to A(\iota xA)$[15] – but not nearly enough. What is lacking are axiomatic and semantic investigations of many of the more important descriptors, attention so far having been concentrated on the definite descriptor.

However, even our modest claims that (0) and its equivalent (0′) are correct are bound to be severely challenged, once their consequences are realised. For these principles, though apparently innocuous, though an integral part of intensional structure theory, and though vital to a full Meinongian logic, have as consequences principles which are commonly reckoned to be clear logical fallacies.

III

The principle of description

(1) $\quad (Py)\,[(Px)\,A(x) \to A(y)]$[16]

read: for some y, that for some x $A(x)$ entails that $A(y)$, though extensionally admissible, is rejected in standard intensional logics, notably in standard modal, intuitionistic and entailmental logics. But shouldn't it be a theorem?

For (1) to be true it is sufficient that

(2) $(Px)\, A(x) \rightarrow A(a(A))$

be true for some name or description $a(A)$ (with $a(A)$ of the form τxB in case a is a description). Then (1) follows by particularisation, just as (1) follows from (0′), and so from (0). But, to take the modal case first,

(3) $(Px)\, A(x) \supset A(a(A))$

is not just true – a point which can be admitted without enlarging the class of theorems of quantification logic – but necessarily true. For if $A(x)$ holds for some x then we can always and readily find, by logical means (e.g. by introducing a new proper name (for the sentence A), or by Hilbert's epsilon theory or by, what does the same, structure theory) a name or ε-description which we can correctly use to instantiate $(Px)\, A(x)$. It simply does not matter for the necessity of (3) how hazy or indeterminate the item described is, or whether it is intensionally specified: it is enough that it may be named or appropriately described. Such a "new name" move is used without question in ordinary argumentation and in mathematical reasoning and lies at the base of natural deduction formulations of quantification logic. Thus we argue: $A(x)$ holds for some x, say (call it) Alf, so, necessarily, A(Alf). Abandoning (3) and its necessitated version (2) would, then, deprive us of an important and seemingly correct method of reasoning which introduces free names or else descriptions as exemplary placeholders for particular quantifications; indeed abandoning (3) would strike at the heart of everyday mathematical argumentation.[17] Somewhat similar arguments may be invoked in the intuitionistic and entailment cases. Thus if $(Px)\, A(x)$ holds in a given (evidential) situation H then in that situation a description a can surely be assigned independently of H – we can call such an item a – such that $A(a)$ holds in H. In short, given the semantical analysis of intuitionist and entailmental logics, (2) should be true. Then (1) follows.

Not only, then, are description and new names perfectly proper (substitution) values of subject variables (as argued in [1]); but furthermore one cannot have an adequate intensional theory with these familiar devices *without* the principle of description. This is especially plain in the case of the ε-operator; for $\varepsilon x A(x)$, read 'an arbitrary x for which $A(x)$' can be logically characterised by the principle (0′) from which (1) follows.

Moreover this entailment principle is analytic on the intended sense of ε. Hence any theory which aims to avoid (1) will have to find grounds for rejecting descriptors such as ε, as somehow defective, and beyond the reach of what a logical theory of discourse should encompass.

Quine does object that selection operators of this sort 'add to the primitive notation, and what they add we do not even know how to translate into intuitive terms' [16], p. 224. But we do know how to cash the ε-operator intuitively, in terms of arbitrary selection and semantically in terms of functions (as e.g. in [6]) – not that the matter of an intuitive cashing should have bothered Quine overmuch since he pronounces in the same book (p. viii) 'here intuition *is* bankrupt'. Furthermore, the ε-operator need not enlarge the primitive apparatus since it can be used to eliminate quantifiers.[18]

If ε is not somehow excluded from discourse – and how can it be if we choose to introduce it, or to make use of closely related descriptors such as 'any'[19] – it is virtually impossible to avoid the strict form of (1). For the principle

(3′) $\quad (Px)\, A \supset A(\varepsilon x A)$

is logically true (on the standard semantics for ε provided in Asser [17], and also in [18], and much earlier in [6]). Hence by the widely accepted adequacy condition for logical necessity $\Box$ (proposed e.g. by Carnap, [19], 39-1) that '$\Box$ (...)' is true iff '...' is L-true,

(3″) $\quad (Px)\, A(x) \rightarrow\!\!\!\!\!\!\exists\; A(\varepsilon x A(x))$

is true, and indeed logically true. So it and its consequence (1) should be theorems. In any case (3″) follows from (3′) by the rule of necessitation, which is not in dispute as applied to non-modal *logical* truths.

Despite the pre-analytic appeal of (1), and our cogent defence of (1) and (2), there are a batch of countervailing considerations that have, so far, prevailed. Sadly the appeal of extensionalism has prevailed once again, though within the very confines of intensional logics.

Not surprisingly the first and most powerful objections to (1) are built on the models typically appealed to in showing that (1) is not a theorem of standard intensional logics; namely models which expand quantified expressions truth-functionally over finite domains. Call this objection the *truth-functional objection*, i.e. the objection from the truth-functional interpretation of quantifiers. Suppose, for instance, that a quantificational model has a two item domain $D_\iota = \{z_1, z_2\}$; then given a truth-functional expansion of quantifiers, with the particular quantifier P expanded disjunctively, (1) reduces to the form

(4) $(A \vee B \rightarrow A) \vee (A \vee B \rightarrow B)$

It is not difficult to show, by semantical or matrix means, that (4) is not – indeed it *ought not* to be – a theorem of intensional logics. But if, as we have contended, (1) is true, then all the argument so far reveals is that our quantified expressions do not expand nicely truth-functionally, as they do in the special case of extensional frames. Naturally if we try to build the truth-functional expansion into our understanding of quantifiers, by incorporating the expansion idea in the semantical rules for quantifiers, we shall fail (1). But one natural interpretation of quantifiers will then diverge from the envisaged, standard, interpretation of quantifiers.

The state of play is as follows: in intensional logics there are rival interpretations of quantifiers; the standard truth-functional interpretation where quantified expressions expand truth-functionally at least over finite domains but for which formulae such as (1) and (2) fail, and the enriched or intensional interpretation, yet to be spelled out in detail, for which the truth-functional expansion breaks down but for which (1) and (2) do hold. Only in the limiting case of classical logic, where all situations coincide with the actual situation, do these rival interpretations converge. The convergence shows at once that both interpretations are consistent. Moreover both of the rival interpretations are formally viable, as we shall see, and appropriate semantical rules can be given in each case for the quantifiers. Thus the choice between the interpretations, if a choice is made, is not just a formal or formal semantical matter. We have argued against the truth-functional interpretation that it blocks familiar and correct arguments, and therefore is unsuitable for a range of wanted applications, furthermore the truth-functional interpretation rules out the development of an adequate theory of ordinary names, terms and descriptions within the necessary framework of intensional logic, since it excludes such expressions as values of variables (see [15] and also part II).

Not wishing to be intolerant, however, we are prepared to admit truth-functional quantifiers, even though these quantifiers do not have the general applicability or the importance commonly ascribed to them. For while enriched quantifiers can be used to characterise truth-functional quantifiers, as restricted quantifiers, a converse strategy is bound to fail since enriched quantifiers have a larger domain of assignments than truth-functional quantifiers. This asymmetrical relation between the two sorts of quantifiers can be clarified by a preview of the model-theoretic semantics of subject terms. In the standard semantics each subject x is assigned an element of the non-null domain D_ι of items exactly as for extensional logic; but in the enriched semantics x is assigned an element of D_ι^K, where K is the non-null set of worlds. That is, x is assigned at each world a in K an element of D_ι, which may vary as a varies. Using some symbols, $V(x) \in D_\iota$ in the standard semantics; but $V(x) \in D_\iota^K$ in the enriched semantics, so that $V(x)(a) \in D_\iota$. More generally, the (enriched) model-theoretic semantics for intensional structure theory is obtained from Henkin's semantics for type theory (in [6]) by taking each basic domain to the power K; e.g. initial sentences are assigned elements of D_o^K, where D_o is the set of values $\{T, F\}$ in the usual 2-valued case (and the set $\{t, f, n\}$ in the 3-valued significance case); and initial common nouns are assigned elements of D_μ^K where $D_\mu = \{w : (Pf \in D_o^{D_\iota}) (w = \{z \in D_\iota : f(z) = T\})\}$. As in Henkin, the element assigned to an initial expression with structure label $(\alpha\beta)$ is a function in $D_\alpha^{D_\beta}$. Since $\mathrm{Card}(D_\iota^K) \geqslant \mathrm{Card}(D_\iota)$, and the inequality commonly holds, the domains associated with enriched quantifiers outrun those for truth-valued quantifiers. If however, that subset of subjects whose interpretation is world invariant can be delineated, then the values of truth-valued quantifiers can be defined in terms of those of enriched quantifiers. These invariants are simply the rigid subjects, an intensional extension of Kripke's rigid designators (of [21]), where for subject x, rigid (x) iff, semantically, $V(x)(a) = V(x)(b)$ for every a, $b \in K$. Whether or not ordinary proper names like 'Nixon' are rigid subjects (as Kripke claims, but we are inclined to doubt), there is no doubt that rigid subjects – Kripke names – can be concocted. It is obvious, given the semantics, that by introducing the predicate 'rigid' into the enriched theory (in cases where it cannot be defined) the truth-functional universal and particular quantifiers A and S can be defined thus (with '$\ni$' read 'such that'): $(Ax)\, B =_{\mathrm{df}} (Ux \ni \mathrm{rigid}(x))\, B$ and $(Sx)\, B =_{\mathrm{df}} (Px \ni \mathrm{rigid}(x))\, B$.

It is worth observing that if both an enriched and a truth-functional quantifier are taken as primitive (instead of defining the latter), then the formal combination of these different quantifiers is not simply the straightforward matter of adding usual axiomatisations of each to the one sentential base. Such a procedure would collapse the truth-functional quantifiers, and thereby lead to interpretational inconsistency, as follows:-

$(Ax)\,B \rightarrow B$,	by instantiation
$(Ux)\,[(Ax)\,B \rightarrow B]$,	by generalisation
$(Ax)\,B \rightarrow (Ux)\,B$,	by distribution, since x is not free in $(Ax)\,B$

Similarly $(Ux)\,B \rightarrow (Ax)\,B$; whence $(Ux)\,B \leftrightarrow (Ax)\,B$. Hence collapse ensues. Furthermore, (1) then holds for truth-valued quantifiers, upon substitution; but, by the intended interpretation, this is impossible. As either the semantics or the theory of restricted quantifiers reveals, $(Ux)\,B \rightarrow (Ax)\,B$ holds but its converse fails; and in the multiply quantified theory implications such as $(Ax)\,B \rightarrow B$ and $(Ux)\,[(Ax)\,B \rightarrow B]$ have to be qualified.

The fan of truth-functional quantification is usually not so tolerant: he will argue that enriched quantifiers license invalid arguments and, in any case, raise insoluble problems. These objections have been most clearly articulated against formulae deductively equivalent to (1), namely

(5) $\quad B \rightarrow (Px)\,A \rightarrow (Px)\,(B \rightarrow A)$, where x is not free in B,

and, in the modal case,

(6) $\quad \Box (Px)\,A \rightarrow (Px)\,\Box\, A.$

(1) follows from (5) taking B as $(Px)\,A$. Conversely (5) follows using (1) as follows: $B \rightarrow A \rightarrow .\; B \rightarrow (Px)\,A$, prefixing $A \rightarrow (Px)\,A$. Hence (Px) $(B \rightarrow A) \rightarrow (Px)\,(B \rightarrow (Px)\,A)$, by generalising and distributing P. Hence, since $(Px)\,C \rightarrow C$ where x is not free in C,

(7) $\quad (Px)\,(B \rightarrow A) \rightarrow .\; B \rightarrow (Px)\,A$, where x is not free in B.

Now given any system such as E or S3 where $A \rightarrow C \rightarrow .\; B \rightarrow A \rightarrow .\; B \rightarrow C$ (E Syll) holds, $(Px)\,((Px)\,A \rightarrow A) \rightarrow .\; (Px)\,[B \rightarrow (Px)\,A \rightarrow .\; B \rightarrow A]$, generalising and distributing P. Then where x is not free in B, (5) follows by detaching (1) and applying (7) to the result. Next given any system (e.g. E or S3) where $A \rightarrow B \rightarrow .\; \Box A \rightarrow \Box B$ holds, it follows from (1) that $(Px)\,[\Box (Px)\,A \rightarrow \Box A]$, whence (6) follows by (7). Finally in modal systems (5) follows using (6). Since $A \rightarrow .\; {\sim} B \vee A$, $(Px)\,A \rightarrow (Px)\,({\sim} B \vee A)$;

rather similarly $\sim B \rightarrow (Px)(\sim B \vee A)$. Hence $B \supset (Px) A \rightarrow (Px)(B \supset A)$. Thus by an S3 principle, $\Box(B \supset (Px) A) \rightarrow \Box(Px)(B \supset A)$. Hence by (6) $B \rightarrow (Px) A \rightarrow . (Px)(B \rightarrow A)$, where $C \rightarrow D \leftrightarrow \Box(C \supset D)$. Thus, as in classical logic, (5) follows.

Principle (6) has taken the brunt of the attack so far. There are well-known counterexamples, of which the following is typical, designed to unmask it as a modal fallacy: While it is logically necessary that some number n is the number that numbers the planets, it is false that, for some number n, n is necessarily the number that numbers the planets. But this is a dubious "counterexample", as the number that numbers the planets appears to provide an appropriate number. For as it is necessary that some number numbers the planets, it is logically necessary that the number that numbers the planets numbers the planets; and hence, by particularisation, for some number n it is logically necessary that n numbers the planets. Similarly, to take an example of Quine's (discussed in [22]), in games where it is necessary that some player wins, the winner, i.e. the player who wins, is bound to win; so for some player it is necessary that that player wins.

It is likely to be objected that if some number necessarily numbers the planets we ought to be able to say exactly which one it is, i.e. to make an identifying reference. But which one is it then? Zero, or one, or two, or three, or...? The question assumes however that the quantifiers are appropriately truth-functional, that it is quite in order to expand the quantified expression: for some natural number n $A(n)$, disjunctively to give $A(0) \vee A(1) \vee A(2) \vee \ldots$ (where $B(n)$ abbreviates: n numbers the planets, and $A(n)$ abbreviates $\Box B(n)$). But the expansion is not in order in the case at hand unless the disjunct A (the number that numbers the planets), $A(np)$ for short, is added; and $A(np)$ guarantees the enlarged disjunction because it is true. The fuller argument for $A(np)$ runs as follows: since $(Pn) B(n) \rightarrow B(np)$ by Church's choice axiom, $\Box(Pn) B(n) \rightarrow \Box B(np)$, whence $A(np)$ follows by detachment. Thus too the answer to the referentially-loaded question Which one is it? is: np. It is quite untrue that some pretty queer intensional objects are thereby introduced. There is nothing intensional about the description 'np'; nor does it differ from ordinary proper names in having different assignments in some different worlds. Moreover, $np = 9$; that is np is, as a matter of contingent fact, the number 9; 'np' and '9', though they have different meanings, are about the same item. But though np is accordingly an ordinary enough item, *in fact* the same as 9, one cannot argue correctly that as $\Box B(np)$, so $\Box B(9)$. For extensional identity does not warrant intersubstitutively in intensional frames such as $A(\ )$ (as [1] explains); for the reason that it is easy to envisage situations other than the actual one where, for example, $np = 10$.[20]

The reply to the planets objection illustrates what ought to be well-known, that interchange principles for modalities and quantifiers are very sensitive to the range of values of quantifiers: compare, for example the Barcan wff which is correct for wide quantifiers but breaks down for restricted quantifiers such as existentially-loaded quantifiers (as [23] explains).

The method of countering the planets objection also illustrates a general procedure that can, it seems, be applied to meet every alleged counter-example to (6); namely that of generating through the ε-axiom an ε-description which defeats the counter-example by supplying, to add to its allegedly complete listing of cases, a further case whose properties alter the truth-value of the key expression of the form $(Px)\,\Box A$. In the planets case a further number logically distinct from each of the natural numbers, performed just this task – at the cost, someone is bound to complain, of ω-completeness. But ω-completeness is not to be generally expected in intensional (or for that matter in non-existential) settings: its loss is no loss.

There remain, however, other variants on the principle of description which may look even less appealing than (6), notably the generalisation principles

(8) $(Py)\,[A(y) \rightarrow (Ux)\,A(x)]$ and its prenex form

(9) $(Py)\,(Uz)\,(B(z) \rightarrow B(y))$, with z not free in $B(y)$.

(8) is deductively equivalent to (1) by contraposition principles. Can we find a new name or description for which (8) holds good? Naturally ε-elimination of the particular quantifier in (8) provides such a description, but simpler still is an A-universality indicator δxA. Since δxA is a universality indicator, or a paradigm A,

(10) $A(\delta xA) \rightarrow (Ux)\,A$,

whence (8) follows. An alternative to such a paradigm A item is however already provided in ε-theory by, perhaps a little surprisingly, $\varepsilon x{\sim}A$; for if ever such an item, which does not satisfy A, has A then everything must. It is just this sort of device, the introduction of new names which are universality-indicating for given sentential frames, that is characteristic of Henkin-style completeness proofs both for extensional and intensional logics. Should such a valuable device be given up?[21] (9) is likewise readily vindicated given the general availability of singular descriptions as full subjects; for it is an immediate consequence, by quantification logic, of the ε-characterising axiom (0).

The formal theory (to be developed in the next part) will focus on the single descriptor ε, which is interpreted (as in [1]) non-existentially, since a number or unicorn does not have to exist in order to be selected (except in a naive sense of 'select'). The special features of ε as a descriptor are captured by the single scheme (0), i.e. $B(x) \to B(\varepsilon xB)$. Where ε is the sole descriptor it is possible to give a definitive answer to the question of the quantification principles corresponding to principles, in this case just (0), for the extraction of descriptive subjects. The addition of the principle of description to standard quantified intensional logics with E Syll is both necessary and sufficient; and otherwise for logics without E Syll (5) is. Perhaps less opaquely, ε-theory obtained by adding (0), with its means of eliminating quantifiers (through definitions like $(Px)A =_{df} A(\varepsilon xA)$) is a conservative extension of quantified intensional logics enriched by (5). But of course ε-theory outruns standard quantified intensional logics.

But even if some descriptions should be admitted as values of variables, it does not follow that ε-descriptions should be. And even if a case can be made out for modal versions of (1) and (3) it does not follow that ε-theory is thereby vindicated within an entailmental setting. For there are some specific objections to ε-theory based on relevance, to the effect that ε-principles are repugnant to relevant logics. These objections are not simply criticisms of the syntactical principle (0) which appears on the surface impeccable. After all if some term t satisfies A then this surely necessitates that an arbitrary item which satisfies A does indeed satisfy A. If something fs then surely an arbitrarily chosen f does? The objections depend rather on the intended interpretation of ε-term in terms of ε choice or a choice operation. Consider (0″) $f(a) \to f(\varepsilon xf(x))$, and consider first the case where some item a indeed fs. Then so the objection runs, $\varepsilon xf(x)$ is the chosen f-item. Let 'b' name that chosen item. Then $f(a) \to f(b)$ is, by (0″), true on logical grounds! There is something very funny about that. For a and b may have *nothing* to do with each other, and the *accidental* fact that b is the chosen f-item is quite insufficient for there being a relevant connection between $f(a)$ and $f(b)$. Even worse is the second case where no item is f. Then $\varepsilon x f(x)$ is just any item b – the number 0, say. But why should "Nixon is both treacherous and not" imply that 0 is both treacherous and not? There doesn't seem to be any non-circular argument.

The argument breaks down however at each critical point. On the one hand, if it *is* only accidental that $b = \varepsilon x f(x)$, i.e. if the identity, like that typically supplied by naming relations is merely contingent, then intersubstitution in an intensional sentence frame, such as a relevant implication provides, is illicit. (For contingent identities only hold at the actual world, whereas implication relations say something about a whole class of worlds other than the actual.) Hence it does *not* follow that $f(a) \to f(b)$ is true on logical grounds; and (as variation of the models show) it is not. If, on the other hand, $b \equiv \varepsilon x f(x)$, that is the identity is a logical one (which semantically persists through worlds), then though $f(a) \to f(b)$ is true on logical grounds, there is nothing funny or accidental about it. It is no more curious than that "7 is odd" entails "5+2 is odd", also of the same form, and no more irrelevant.

At this point the objection is likely to be given a more semantical twist, by appealing to the intended interpretation of ε, for example in terms of a choice function. In the extensional quantificational case the idea is that each interpretation of the logic supplies, as well as a non-null domain D_ι of items, a choice function c which for any non-null subset D' of D_ι selects an element of D' and otherwise, where D' is null, selects an element of D_ι. Then $V(\varepsilon x f(x)) = c\{V(x) : V(f(x)) = T\}$, i.e. for a given interpretation, $\varepsilon x f(x)$ is about what c selects from the elements of D_ι of which f is true. For intensional logics the evaluation rule has to be complicated to take account of situational variance, and there are rival ways of doing this. The common method in modal logic is the simplistic one of making subject evaluation world invariant; but this simple method is inadequate in the case of ordinary names and descriptions, for what these subjects are about may vary from world to world. However, it remains true that $V(\varepsilon x f(x))\,(a) = c\{V(x)\,(a) \in D_\iota : V(f(x))\,(a) = T\}$; and this may seem to concede enough for the objection to be remobilized. It doesn't.

Of course in a given modelling the chosen element may be b; and in the worse case the element selected may well be 0 (assuming then $0 \in D_\iota$). But the fact is that even if one does have in effect, in a given modelling, that Nixon is both treacherous and not ensures that 0 is both treacherous and not, this does not establish irrelevance, or, what is false, that the first statement implies the second. One might just as well argue that, because in certain models. e.g. those restricted to possible worlds, $p \& {\sim}p$ guarantees an arbitrary q, entailment logics are bound to be irrelevant, or that p & ${\sim}p$ implies q. These arguments are invalid; and their difficulties are defeated by considering a class of models which appropriately vary assignments, not just one or a select subset of modellings. Moreover it would be equally invalid to argue that, because each model may be defective, the set of all models may also be defective.[22]

An interesting side issue that the relevance objection does raise is that of criteria for relevance. What for example is the quantificational analogue of the sentential variable-sharing relevance? According to the criterion of *predicate* relevance, there should be no theorems of the form $A \rightarrow B$ where A and B are wff which fail to share a predicate parameter (with sentential parameters counted as zero-place predicate parameters). Thus forms such as $f(a) \rightarrow f(b)$ where $b \equiv \varepsilon y f(y)$ do not violate predicate relevance. If enriched quantified entailment logics, such as EQ enriched by (1), or by (0), are predicate relevant, as we conjecture, then the objection really falls flat. The onus of proof is on the objector to show how ε introduces irrelevance.

We have gradually shifted from considering the amalgation problem, properly enough, at a pre-analytical level to considering it semantically. It might be thought, however, that the issues, the fact of (1) and (6) in particular, can be quickly settled by appealing to the formal semantics for quantifiers. This is not so. Various quantificational semantics have been marketed, at least in the modal case, and one can buy semantics which invalidate (1) and (6), namely the standard thoroughly rigid semantics where for each subject x $V(x) \in D_\iota$, or one can buy alternative, more comprehensive and superior, semantics which do not, where $V(x) \in D_\iota^K$, or others. As is usual in the market place one pays one's money and takes one's choice – from what limited range of wares is available. Thus an appeal to semantics in itself settles nothing (apart from making it obvious that one can take both if one is rich enough); but it does set the scene for further, and maybe more sophisticated arguments, as to the merits of (0), (1), (6) and their kin. What these further arguments will reveal however, in our opinion, is that such theorems of intensionally enriched structure theory as (0) and (1) – which are part of the new metaphysics – are here to stay.

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NOTES

[1] This point is elaborated in [1], Chapter 4.

[2] Ideally these developments would be carried out in combination since, as Montague has emphasized (e.g. [2], p. 212), 'there will often be many ways of syntactically generating a given set of sentences, but only a few of them have semantic relevance'. Ideally – but there is a danger, in the current rudimentary state of semantical and context theory, that important notions, as well as discourse that cannot be properly digested by the theory, will be inadequately or roughly treated, if investigated at all.

[3] As Lewis [5] noticed but is already plain from Henkin [6].

[4] Lewis [5] does include these as a basic grammatical category and Adjukiewicz [3] was tempted to do so. As a penalty for not succumbing the Adjukiewicz theory will generate – when applied to English, not Polish – a stack of ungrammatical sentences, e.g. 'The man hit man', 'A rose hit man and rose smells'. What is worse, though it is not a punishment for succumbing, the Lewis theory fails to generate such sentences as 'Porky is yellow', 'Porky is a pig' (as distinct from 'A pig is Porky') and 'Porky is something' from the lexicon specified (on p. 172). But Lewis would no doubt try to argue that such sentences can be recovered transformationally.

[5] Church uses ι instead of ε, but we prefer to reserve ι for definite descriptions. For Church's ι, as it appears in Henkin's formulation, is not a definite description operator but none other than Hilbert's ε-operator, as Henkin' semantics at once reveals.

[6] We owe this observation to S. Read. However the reduction while simplifying the syntax adds bound variable wfps of the form λw, for w with label α, and complicates the semantics.

[7] Though the concatenation of structure theory is different because of the way n-place functors are represented.

[8] This theory is not the only one Adjukiewicz considers in order to deal with operator expressions, but his alternative is unsatisfactory. Furthermore he misses in his general argument on pp. 221–2 (which is accordingly flawed) an alternative to λ-theory which is now taken seriously, that of treating quantified phrases as general verbs.

[9] It goes further in other respects as well, e.g. it includes under the Church formulation quantification for every grammatical category – quantification that can however be satisfactorily interpreted quotationally in cases other than subject quantification.

[10] That Church's type theory provides a ready-made and attractive theory of modifiers was pointed out to us by M. K. Rennie, to whom we are generally much indebted. He convinced one of us that there was a worthwhile role for type theory after all (albeit a distinct role from the one he has elaborated in [10]). The many virtues of type theory as a theory of modifiers are brought out in his [10]. Structure labels such as π_k are taken from [10].

[11] For should there be any parts of speech that cannot be derived functionally they can as a last resort be treated as further basic categories. This is not to gloss over the very substantial class of unsolved problems where transformations of some effective sort will have to be applied in the reduction to logical form, e.g. a prominent case is that of predicative adjectives following inert verbs such as 'is', 'looks', 'grows' and 'becomes'.
[12] This is Rennie's description of the problem (in [10]). As he points out the problem appears to affect not only type theories but any theory, such as his theory of predicate modifiers based on quantification logic, which employs a set-theoretical semantics. The problem for quantification logic can however be escaped in semantic theories which are not set-theoretic; e.g. by buliding on a semantics like that of Church for quantificational logic (given in [11]) where predicates are simply assigned relations (not sets of ordered elements) which can reflect the varying adicity of the predicates they model.
[13] There will be corresponding semantic ado about obtaining the correct relations between 'rides' with label π_1 and 'rides' with label π_2. Moreover the method opens the way to other problems. Thus if 'rides' is the same in such sentences as 'Tom rides', 'Tom rides every day', 'Tom rides Dobbin' and 'Tom rides Dobbin every day', then such sentences as

Tom rides and in fact Tom always rides Dobbin

should be open to quantification to yield

For some f, Tom f and in fact Tom always f Dobbin,

something that type theory proper certainly excludes. But there is no reason why quotational (generalised substitutional) quantification, in this case for predicates of variable adicity and read 'for some predicate $qu(f)$', should not be invoked. Whether the type-theoretic restriction is mandatory or not depends on how the logical paradoxes are to be formally averted, and this depends on features of the axiomatisation, in particular on substitution principles. In contrast the set-theoretic paradoxes need cause little concern; for set theory can be axiomatised using the constant $\in$ with label $((o\iota)\,\iota)$ along standard lines.
[14] Problems as to exactly which substitution conditions obtain for ultra-modal functors, and how functors of more than entailment strength can be accommodated, are discussed briefly in [1], §7.2.
[15] In fact it seems that uniqueness need be only contextually supplied, but we omit such needed deepenings of the logic from the present discussion. Such deepenings are especially important for a proper discussion of definite descriptions which are about items that do not exist, since descriptions of non-existent items rarely, if ever, ensure uniqueness. And very often existential uniqueness of items that do exist is not required for the correct application of a definite description, contextually specified uniqueness is enough.
[16] An analogous principle for definite descriptions is: $(Py)\,[(P!x)\,A(x) \rightarrow A(y)]$. But again it is enough that uniqueness be contextually supplied.
[17] In contrast to abandoning Disjunctive Syllogism, a principle which appears to be rarely used outside areas such as axiomatic set theory.
[18] The Hilbert school was not alone in introducing operators like ε. Whitehead and Russell came close to introducing such an operator in their discussion of the principle of universal generalisation, where they say ([20], p. 132):

'We may express this primitive proposition by the words: "What is true in *any* case, however the case is selected, is true in *all* cases". We cannot symbolise this proposition because if we put $\phi y \supset (x)\, \phi x$ that means: "However y may be chosen ϕy implies $(x)\, \phi x$" which is in general false. What we mean is: "If ϕy is true however y may be chosen, then $(x)\, \phi x$ is true". But we have not supplied a symbol for the mere *hypothesis* of what is asserted in "$\vdash \phi y$", where y is a real variable, and it is not worth while to provide such a symbol, because it would be very rarely required. If for the moment we use the symbols $[\phi y]$ to express this hypothesis, then our primitive proposition is $\vdash [\phi y] \supset (x)\, \phi x$.'

$[\phi y]$ binds y so that values cannot be substituted for y. Thus $[\phi y]$ is best expanded in a way which makes the binding plain, e.g. as $\phi(\delta y \phi)$ where δ is a universality operator. In fact $\delta y \phi$ may be defined as $\varepsilon y \sim \phi$.

19 'Any' appears to differ from 'an arbitrarily selected' solely in its significance conditions. In particular its use supposes, as a significance condition, that it will be possible still to make the selection; hence the absurdity of such sentences as 'I met any man from Isfahan' as contrasted with 'I will meet any man from Isfahan'.

20 Indeed enriched quantification facilitates the semantical development of the theory of extensional identity (which delivers contingent identity in the actual situation) in combination with quantification (as [22], p. 196 ff. reveals, and then tries to block); and combined the two moves enable one to cut clearly through essentialist objections (as [23] indicates, and [21] can be used to show).

21 However, the appeal to principles used in Henkining, like the earlier argument in the case of intuitionist logic, is not completely conclusive. For it can be argued that with intensional logics all we should expect is situational-relativized Henkining – even if "genuine" proper names are not to be *interpreted* in a situationally-relative way! That is to say, for *each* situation a we can find an $x_0(a)$ such that if $V(A(x_0(a)))(a) = T$ then $V((Uy)\, A)(a) = T$; but, as $x_0(a)$ depends on a, we cannot generalise to validate (8), which requires some one $x_0 = x_0(a)$ which works uniformly for each a. The issue of whether new naming devices have to be situationally relative, or can be freed from such dependence, is crucial to the issue of evaluating enriched quantification. We think, however, that Kripke is right (in [21]) in holding that new rigid subjects can be introduced, stipulatively if need be; and these Kripke names suffice as situationally-independent new names.

22 This argument is invalid in much the way that sceptical arguments from *possibly each*... to *possible all*... are invalid.

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