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RICHARD ROUTLEY AND ROBERT K. MEYER

DIALECTICAL LOGIC, CLASSICAL LOGIC,
AND THE CONSISTENCY OF THE WORLD

Dialectical logic, especially Soviet logic, has customarily received sharp and summary treatment at the hands of Western critics. For example apparently devastating reviews of work in Soviet logic can be found in that bastion of classical logic, the *Journal of Symbolic Logic*; thus, e.g., Kline [15] :

...to the reviewer this seems a desperate, and hopeless, quibble ... resulting from the impossible attempt to uphold simultaneously the principles of formal logic and of dialectic.*

Even organs sympathetic to Soviet thought, such as *Studies in Soviet Thought*, frequently contain caustic remarks about dialectical logic (e.g., Bochenski asserts that "there is, obviously, no such thing as 'dialectical logic';" [1], p. 33), and presuppose that so-called 'dialectical logic' is not a viable logical enterprise (e.g. "These two books ... fail completely to touch the major question — the viability of dialectical logic as a logic" Blakeley [2], p. 166). To date classical logic — which is Western mainstream logic — has been strongly on the offensive in the ideological logical warfare between East and West, with many supporters in fact among the Soviets, and dialectical logic has been very much on the defensive. The object of this paper is to try to upset this ideological power structure by furnishing dialectical logic with the framework at least of a viable semantics, and at the same time to shatter the imperviousness of mainstream Western logic, and thereby to assist the cause of that newer, less orthodox and so far minor, logical theory — relevance logic.

It will emerge that the differences between the orthodox Western and Soviet positions cannot be satisfactorily represented as logic, or formal or classical logic, on the one side, and anti-logic or the rejection of formal logic on the other; each position can furnish viable, equally formal, but competing logical theories, and the differences between these positions will come down to *philosophical* differences about such highly debatable and empirically untestable matters as the consistency of the world.

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I. REDUCING THE PROBLEM OF FORMALISING DIALECTICAL LOGIC TO WORKABLE DIMENSIONS

A leading feature of dialectical logic as typically presented is ‘the dogmatic assertion that there are real contradictions’ ([3], p. 45), that at any given time contradictions may hold, both A and its negation $\neg A$ may be true. Thus (unless the negation in $\neg A$ is only apparent, and is eliminable through devices like those deployed by Russell in his theories of descriptions and classes) a dialectical logic is impossible within the framework of classical logic or of such leading alternatives to it as intuitionist or modal (classical strict) logics. For these logics all contain, as well as the detachment rule (modus ponens) for their implication connective ‘ $\rightarrow$ ’, a paradoxical principle (*ex falso quodlibet*)

$$(1) \quad A \ \& \ \neg A \rightarrow B$$

which spreads any contradiction everywhere – which converts simple inconsistency, of a sort dialectical logic can thrive upon, into absolute inconsistency, which would render the logic worthless and lead to the empirical falsification of applications. A framework for dialectical logic is, however, attainable in the shape of logics such as entailment logics, which rightly abandon (1).

It is not difficult to devise many-valued logics without spread laws such as (1) (cf. Goddard & Routley [4]), and various endeavours to formulate dialectical logics have taken the direction of many-valued logics (e.g., Asenjo¹ [8]). But, quite apart from the irrationality of many-valued sentential logics ([10]), many-valued logics are semantically inadequate to the task (as Rescher [16] pp. 143–8 has also observed). For a leading idea of Soviet logic is that such Aristotelian principles as that every assertion is either true or false and that no assertion is both are maintained. These consequences of a Tarski-style theory of truth – a Tarski-style theory of truth is also apparently acceptable – are ruled out by many-valued logics, where assertions take on other values besides truth and falsity, e.g. non-significance, non-truth-valuedness and so forth. A condition of adequacy, then, for a (basic) dialectical logic is that it has a two-valued non-classical semantical analysis, the values being just truth and falsity. In any case many-valued logics, with the appropriate connectives, all contain classical logic as their two-valued (truth-falsity) restriction, and hence give rise to precisely the same problems for real contradictions as two-valued classical logic.

In short, though dialectical logic is not the same as classical logic, it is

not adequately represented as a many-valued logic. The only real alternative is to view dialectical logic as a two-valued intensional logic (and anyway every many-valued logic can be re-presented as a two-valued intensional logic). Fortuitously, work on the semantics of implication has disclosed some promising candidates for static dialectical logics. And there are good independent reasons for thinking the semantical approach very promising, the main one being that the semantics reveals rather better than the associated syntax what is going on philosophically in orthodox dialectical logic. There is, perhaps unexpected, concurrence on the primary of semantics from the orthodox position.

The principal question of dialectical logic is the problem of truth. It investigates the forms of thought in regard to their content, and shows to what extent these forms mediate a true knowledge of the world ([11], p. 327 quoted in [6] p. 126)²

Certainly the question of truth, or propositions as forms of thought and of their content, are central issues in semantics. Such parts of semantics are not included in syntax, or formal logic, narrowly construed:

Formal logic is not the only science on thought – there is also dialectical logic whose main object is the problem of truth, not external relations but the laws of development of all material, natural and spiritual things ([11], p. 308; [2], p. 23).

The main thrust of dialectical logic, which includes formal logic, is then semantical.

Before beginning on the semantical investigation, it pays to cut the subject down to an initially workable size. An important and difficult part of dialectical logic, which can profitably be pruned off in initial semantical investigation, concerns the laws of development and more generally questions of changes over time. Taking the lead given by physics and later by economics let us distinguish static and dynamic components of logic. A *static* logic is the logic at a given time which does not relate the given time to other times: accordingly time can be eliminated from consideration giving the static time-less, or present tense, logics that are so familiar. Alternatively these logics can be (classically) construed as omnitemporal logics, for all times, since there are correspondences between a language at a given time and for all times. *Dynamic* logics enrich static logics by tensing (as in tense logics) or by what is more general given appropriate-ordering relations, the introduction of temporal variables and connectives, so that assertions can be assigned times, e.g. at which they are true. Although a static logic must sacrifice many of the

leading features of dialectic such as the theory of development, the most logically difficult features of Soviet logic arise already at the static stage.

Since it is known, in principle, how to tense or add temporal variables to logics to convert static logics into dynamic ones, it is a good conjecture that if a satisfactory static dialectical logic can be designed then it can be straightforwardly blown up into a dynamic one, the main business being a largely technical one (that can be left to logical technicians). In fact the fundamental problems with respect to dialectical logics arise at the sentential level: for already there the issue of real contradictions emerges. Furthermore once the problem is solved at the sentential level there are familiar ways of extending the solution to the quantificational level.

At the sentential level, however, an adaption of the semantics for weak entailment logics provides a suitable framework for static dialectical logic. Semantical analyses of entailment logics keep the classical evaluation rules for conjunction and disjunction — in a situationally or world relativised form, e.g.

$A \& B$ holds in situation (at world) a , iff A holds in a and B holds in a ; and, $A \vee B$ holds in a , iff either A holds in a or B holds in a —

but replace the classically inadequate rules for the evaluation of implication and negation by rules of the following form:

$A \rightarrow B$ holds in a iff, for every situation b and c such that $Rabc$, if A holds in b then B holds in c ; and

$\neg A$ holds in a iff A does not hold in a^+ .

Naturally three-place relation R and operation $^+$ on situations are subject to conditions — exactly which set of conditions depending on the entailment logic represented, e.g. commonly negation principles will result in the conditions that

$a^{++} = a$ and that if $Rabc$ then Rac^+b^+ .

What really happens semantically that is especially important dialectically, is that the semantics admits not only worlds which are incomplete, but worlds which are impossible in that contradictions hold in them (cf. [17], which explains the basic semantical ideas). The leading feature of static dialectical logics that we are concerned to capture can be restated semantically thus:

the actual situation or theory 0, *the base world, where truth is determined, is an impossible one.*

It is an immediate outcome that by no means any entailment logic will serve; systems such as Ackermann's system π^1 which, like Lemmon's systems of strict implication, contain a primitive rule of material detachment,

- (2) if A and $\neg A \vee B$ are theorems so is B ,

are ruled out. For this rule presupposes that the actual situation is a possible one. To grasp the syntactical picture suppose some contradictory pair, A_0 and $\neg A_0$ say, is provable; then A_0 and $\neg A_0 \vee B$ are theorems, whence by (2) so is B . Thus (2) reintroduces the effect of (1) in spreading contradictions about everywhere. There remain, however, features of the usual entailment systems without (2) which may appear to disqualify them as satisfactory bases for dialectical logic. Because the law of non-contradiction (or contradiction!)

- (3) $\neg(A \& \neg A)$

is usually a theorem, a contradiction such as $A_0 \& \neg A_0$ looks as if it will conflict with *logical* truths of the system. Hence, too, the system will contain 'logical contradictions', such as $(A_0 \& \neg A_0) \& \neg(A_0 \& \neg A_0)$, whereas the contradictions that hold are supposed, on some accounts at least, to be synthetic a priori and not analytical truths like the laws of logic³.

Actually, there is a choice: there are weaker dialectical logics without (3) and there are stronger dialectical logics with (3) as a thesis.⁴ The orthodox Soviet position (as presented, e.g., in [11]) appears to retain (3) as a thesis, thus appearing to classical logicians to generate intolerable tensions which destroy any prospect of a coherent logic. It is interesting to demonstrate that the matter is one of appearance only.

Though in the sections that follow only the stronger alternative will be investigated, the weaker position is also noteworthy since it provides an analysis of Meinong's notion of internal negation, in terms of which one can say truly both that the round square is round and also that the round square is not round, without inconsistency or conflict with logical laws. The fuller Meinongian two negation theory, with both external negation and internal negation, can be formalised by adding internal negation to a standard entailment logic (see [18]).

Logicians are characteristically reductionists, so not surprisingly much

resourcefulness has been wasted trying to reduce double negation theories to single negation theories. Classical logic and realism regularly endeavour to reduce all negations to a strict external negation ‘ $\sim$ ’ which satisfies the exclusion condition, i.e., for every world a

$\sim A$ holds in a iff A does not hold in a ;

while dialectical logic and idealism typically attempt to reduce negations just to an internal negation.

II. SIMPLE STATIC SENTENTIAL DIALECTICAL LOGICS DL AND DM

A sentential dialectical logic should have as connectives as well as the basic negation connective ‘ $\neg$ ’, required to express fundamental contradictions, at least a conjunction ‘ $\&$ ’, required to unite contradictions and required to formulate, what the orthodox position accepts, Aristotle’s principle of non-contradiction $\neg(A \& \neg A)$, and also an implication or entailment relation ‘ $\rightarrow$ ’, representing the logical consequence relation that the orthodox position assumes throughout standard works such as [11]. Alternatively, a compatibility or consistency relation, symbolised ‘ $\subset$ ’, also regularly assumed in orthodox works, can be taken as primitive and implication defined in good Lewis style thus:

$$A \rightarrow B =_{\text{Df}} \neg(A \subset \neg B).$$

Where, as in what follows, ‘ $\rightarrow$ ’ is primitive, $A \subset B =_{\text{Df}} \neg(A \rightarrow \neg B)$.

In order to be able to represent contradictions sententially, the sentential language of DL is assumed to have, as well as a connective set $\{\neg, \&, \rightarrow\}$ and a denumerable stock of sentential parameters (variables): $p, q, r, p', q', r', p'', \dots$, a denumerable stock of sentential constants: $p_0, q_0, \dots$ (i.e. where p is a sentential variable, p_0 is a sentential constant). Thus it can be ensured that DL contains real contradictions by taking $p_0 \& \neg p_0$ as an axiom.⁵ $p_0 \& \neg p_0$ will be some representative contradiction, e.g. the conclusion of a Kantian antinomy or logical paradox with both p_0 and $\neg p_0$ proved by pure reason from admittedly true premisses, or the conclusion of one of Zeno’s paradoxes with $\neg p_0$ (say ‘Achilles does not overtake the Tortoise’) proved by pure reason and p_0 established by observation.

Though standard dialectical logics are committed to at least the sentential framework so far introduced, there is considerable flexibility in the axio-

matic constraints they impose on the connectives adopted. It is more than a little tempting to argue that there is no unique system of dialectical logic even at the sentential level, any more than there is a unique system of relevant implication. There are, however, some minimal requirements on a dialectical logic which can be extracted from the informal arguments standard texts employ. In particular conjunction behaves in a firmly classical way, e.g. implication, $A \& B \rightarrow A$, holds without any indicated restrictions, and composition, if $A \rightarrow B$ and $A \rightarrow C$ then $A \rightarrow B \& C$, is unrestrictedly assumed. To set the scene for discussion a rather minimal, though not uncontroversial and not uniquely determined, dialectical logic DL is selected. DL can be enriched to yield stronger systems along the same lines that weaker relevant logics can be enriched (to provide a hierarchy of systems); but there is, here at least, no philosophical advantage to be won from such greater strength, only losses to be incurred, as may become apparent.

DL has the following axiom schemes and rules:

- | | |
|---|--|
| D1. $A \rightarrow A$ | D6. $A \& (B \vee C) \rightarrow (A \& B) \vee (A \& C)$, |
| D2. $(A \rightarrow B) \& (B \rightarrow C) \rightarrow A \rightarrow C$ | where $A \vee B =_{df} \neg(\neg A \& \neg B)$. |
| D3. $A \& B \rightarrow A$ | D7. $\neg \neg A \rightarrow A$ |
| D4. $A \& B \rightarrow B$ | D8. $A \rightarrow \neg B \rightarrow B \rightarrow \neg A$ |
| D5. $(A \rightarrow B) \& (A \rightarrow C) \rightarrow A \rightarrow (B \& C)$ | D9. $A \rightarrow B \rightarrow \neg(A \& \neg B)$
(equivalently: $(A \rightarrow \neg A) \rightarrow \neg A$). |
| AD1. $p_0 \& \neg p_0$ | |
| RD1. $A, A \rightarrow B \rightarrow B$, i.e. where A and $A \rightarrow B$ are theorems so is B . | |
| RD2. $A, B \rightarrow A \& B$ | |
| RD3. $A \rightarrow B, C \rightarrow D \rightarrow B \rightarrow C \rightarrow A \rightarrow D$ | |

A great deal of light is cast on DL by its semantical analysis, which takes the following lines:

A DL *model structure* (DL m.s.) is a frame $\mathfrak{M} = \langle O, K, <, + \rangle$ where K is a set of worlds (situations, theories, scenarios, or whatever), O is a subset of K (providing the regular worlds, where all the theorems of DL hold), 0 (the actual or real world, the Absolute⁶) is an element of O , $<$ is a three-place relation on K (such that for any worlds a, b, c , $<_a b c$, rewritten $b <_a c$, orders b and c from the perspective of a) and $+$ is a 1-place operation on K . Two ordering relations should be distinguished: $b <_a c =_{df} <_a b c$; and $b \subseteq c =_{df} (Px)(Ox \& b <_x c)$, i.e., b is included in c iff with respect to some regular

world b precedes c $\mathfrak{M}$ is subject to the following conditions:

1. Basic conditions.
 - p1. $a \subseteq a$
 - p2. If $a \subseteq d$ and $b <_d c$ then $b <_a c$
 - p3. $a = a^{**}$
 - p4. If $b <_a c$ then $c^+ <_a b^+$
2. Extra conditions.
 - p5. (for D2). If $b <_a c$ then, for some world x , $b <_a x$ and $x <_a c$; i.e. D2, which is an optional extra, semantically imposes a density requirement on $<_a$.
 - p6. (for D9). $a^+ <_a a$.
 - p7. For $x \in O$, $x \neq x^+$.

The smallest frame for DL is the following two-element model structure: $K = \{0, 0^+\}$, $O = \{0\}$, $a^+ = a$ for $a \in K$, $a <_c b$ for $a, b, c \in K$ except for $0 <_0 0^+$. It is a straightforward matter to verify that p1-p7 hold, though $0 <_0 0^+$ does not hold. The frame leads to 4-valued matrices for DL, and thence to a finitary consistency proof.

A *valuation* v in $\mathfrak{M}$ is a function which assigns a value from $2 = \{0, 1\}$ to each parameter and constant P at each world of K subject to the requirements

- (i) If $a \subseteq b$ and $v(P, a) = 1$ then $v(P, b) = 1$, and
- (ii) $v(p_0, x) = 1 \neq v(p_0, x^+)$, for $x \in O$.⁷

According to (i) if a is subworld of b and P holds in a then it holds in b , while according to (ii) p_0 holds in the actual world 0 but not in its image 0^+ and likewise for other worlds in O .

An *interpretation* I extending v from initial formulae to all formulae of DL is defined as follows for every a in K :

- (li) $I(P, a) = v(P, a)$
- (lii) $I(A \& B, a) = 1$ iff $I(A, a) = 1 = I(B, a)$,
i.e. $A \& B$ holds at a iff, A holds at a and B holds at a .
- (lii) $I(A \rightarrow B, c) = 1$ iff, for every a and b of K , if $a <_c b$ and $I(A, a) = 1$ then $I(B, b) = 1$.
- (liv) $I(\neg A, a) = 1$ iff $I(A, a^+) \neq 1$.

A model structure $\mathfrak{M}$ together with a valuation v in $\mathfrak{M}$ provides a model for DL, i.e. a *DL model* is a structure $\langle 0, O, K, <, +, v \rangle$ for short, where $\mathfrak{M} = \langle 0, O, K, <, + \rangle$ is a DL m.s.

A formula A holds at world a in model $\mathfrak{S}$ iff $I(A, a) = 1$; A is true in $\mathfrak{S}$ iff $I(A, 0) = 1$ and false in $\mathfrak{S}$ iff $I(A, 0) \neq 1$, and then $\mathfrak{S}$ provides a *counter-model* to A . Formula A is *DL-valid* iff it is true in all DL models, and otherwise *DL-invalid*.

The main result connecting the logic and its semantics (effectively proved in [18]) is naturally: *a formula A is a theorem of DL iff it is DL-valid*. It follows from the soundness of DL that it is absolutely consistent – in case anyone had doubts about the consistency of a logic containing real contradictions. However DL is not simply consistent (with respect to its negation $\neg$): not only are both p_0 and $\neg p_0$ theorems, but there are infinitely many logically distinct contradictions which are theorems. Define $p_n = (p_{n-1} \ \& \ \neg p_{n-1})$, for n a positive integer; then $p_m \ \& \ \neg p_m$ is a theorem for each non-negative integer m , in virtue of the principle of non-contradiction $\neg(A \ \& \ \neg A)$, an immediate consequence of D1 and RD2. Thus a single contradiction generates infinitely many,⁸ but by no means everything, i.e., contradiction does not spread into absolute inconsistency. The system DL also shows that the inclusion of (even infinitely many) contradictions does not result in total system disorganisation.

The semantics furnishes a theory of truth for dialectical logic. Of course, the theory of truth does not conform to the strict canons of a Tarski-type truth definition inasmuch as it appeals to worlds beyond the actual in providing the recursive definition: but the strict canons are too strict to admit any intensional logic, and the dialectical deviation is no worse than that for other intensional logics.

Choice of DL as a basis for discussion is likely to encounter the criticism that while the positive $\{\&, \vee, \rightarrow\}$ logic of DL is dialectically pretty unexceptionable, the negative logic is excessively classical and does not take into proper consideration the dialectical law of negation of negation or the peculiar character of dialectical negation $\neg^9$. It is certainly true that a logic such as DL which contains both double negation principles, D8 and its converse $A \rightarrow \neg\neg A$, will be unacceptable to dialecticians who reject one of these principles; and then the only course is to furnish a basic logic which eschews the damaging principles. Accordingly, in the quest for a fairly generally acceptable dialectical logic, a retreat is made to a weaker dialectical logic DM included in DL. For most of the philosophical points we are concerned to argue can be made in terms of DM.

In DM disjunction, ‘ $\vee$ ’, is no longer definable and must be taken as primi-

tive; so appropriate disjunction axioms have to be included. The basic DM system, BM, has as postulates the rules of DL, the axiom schemes D1, D3, D4, D5, D6, and these further postulates:

- D10. $A \rightarrow A \vee B$
 D11. $B \rightarrow A \vee B$
 D12. $(A \rightarrow C) \& (B \rightarrow C) \rightarrow (A \vee B) \rightarrow C$
 D13. $\neg A \& \neg B \rightarrow \neg(A \vee B)$
 D14. $\neg(A \& B) \rightarrow \neg A \vee \neg B$
 RD4. $A \rightarrow B \rightarrow \neg B \rightarrow \neg A$.

DM results upon adding D2, D7, D9, and AD1. In sum, DM differs from DL in replacing D8 by D10–D14 and RD4.

DM modellings differ from DL modellings simply by weakening basic condition p3 to

$$p3^1. a^{**} \subseteq a$$

DM is sound and complete, i.e. a formula is a theorem of DM iff it is DM-valid. ABM model structure is naturally weaker; it need satisfy only p1, p2 and p4¹:

$$\text{If } b \subseteq c \text{ then } c^* \subseteq b^*.$$

The modelling conditions corresponding to double negation and contraposition principles of DL are then as follows:

$$\begin{array}{ll} \text{D7. } \neg\neg A \rightarrow A & p3^1. a^{**} \subseteq a \\ \text{D7}^1. A \rightarrow \neg\neg A & a \subseteq a^{**} \\ \text{D8}^1. A \rightarrow B \rightarrow \neg B \rightarrow \neg A & p4. \text{ If } b <_a c \text{ then } c^* <_a b^*. \end{array}$$

The thesis D7 of DM captures part of the logical force of the dialectical law of negation of negation; for it amounts to the assertion included in the law that the content of the negation of the negation of A includes that of A . Since, however, the content of $\neg\neg A$ is claimed to be logically richer than that of A , the converse double negation principle D7¹ presumably fails, and therefore postulate D8 of DL too fails. DM is, then, a most promising initial system on which to build static sentential dialectical logic.

Whereas intuitionism rejects the negation principle $\neg\neg A \rightarrow A$, dialectical logic DM retains D7 but rejects, what intuitionism keeps, $A \rightarrow \neg\neg A$. These formal differences derive from deeper semantical differences, in that whereas intuitionists regard the real world 0 as incomplete (as concerns non-finitary

matters) dialecticians take 0 to be inconsistent (beyond local stable regions).

III. THE EDUCTION OF SOME CHARACTERISTIC FEATURES OF STATIC DIALECTICAL LOGIC

Before developing some of the leading and most controversial theses of dialectical logic, it is worthwhile introducing the notion of a full model, of a canonical model, and of absolute (semantical) discourse. A *canonical* model $\mathfrak{M}$ (e.g. for DL) is a model such that for every formula A , $I(A, a) = 1$ iff $A \in a$. A *full* (canonical) model is a (canonical) model where 0 is the class of all and only true statements expressible and there is a distinct world in the model for each combination of true and false statements of 0 that the logic discriminates. In a formal language, such as that of DL, with a denumerable set of sentential constants, the class of truths expressible is as large as that of natural languages. (We dodge, however, the controversial question of whether such denumerable languages can *express* all truths.) It is often valuable, too, to be able to talk semantically of a formal language or logic in an absolute (i.e., model-independent) way, such absolute discourse being explainable as discourse relative to an assumed full canonical model. In these terms, which will be taken advantage of, *the world* is the base of 0 of the full canonical model; that is, it is the class of true statements.¹⁰ Plainly every valid statement is true, absolutely.

There are contradictory statements which are simultaneously true, indeed valid. For both p_0 and $\neg p_0$ are valid: Yet p_0 and $\neg p_0$ are contradictory, and demonstrably inconsistent.

For as $A \rightarrow A$ is a theorem, so is $\neg(A \subset \neg A)$, i.e. A and $\neg A$ are provably inconsistent. p_0 & $\neg p_0$ is accordingly a real contradiction, and real contradictions hold in the actual case.

Mainstream Western philosophy from the pre-Socratics on has denied that this can be so: indeed Parmenides and Plato assumed 'that the absence of contradiction is a key test of reality' (Rescher [5], p. 88). The classical Western defence was made by Aristotle in his *Metaphysics* and *On Interpretation* where he argued that 'Of contradictory statements it is necessary for one or the other to be true or false'. But Aristotle's main argument depends upon adopting the classical negation principle, that $\neg A$ is true iff A is false, i.e., canonically,

$$I(\neg A, 0) = 1 \text{ iff } I(A, 0) = 0, \text{ for every } A.$$

But this implies¹¹ (in fact is equivalent to) the assumption that $0^+ = 0$, and

hence assumes the point at issue. For the classical negation principle is precisely what dialectical logic repudiates. Most of the other standard classical arguments are variants on this argument, obtained by appealing to some classical intuitive modelling of negation, for example the exclusion picture, according to which A and $\neg A$ exclude each other, or the erasure or annihilation picture according to which putting up $\neg A$ erases or wipes out A , and so forth.¹² But what these intuitive modellings (some of which can be given a dialectical twist) all come down to when they are made explicit is the classical negation principle for world 0.

Rescher's basic strict, and allegedly incontrovertible, version of 'the traditional Law of Contradiction' is

the 'exclusion principle' that a proposition (say, p) and its negation ($\neg p$) cannot both be true together: at most one can be true, so that the truth of one excludes or precludes that of the other.

Certainly in this (very weak) form no known system of logic violates the principle. And for good reason. For if both ' p ' and ' $\neg p$ ' were true, then one could scarcely maintain $\neg$ as a viable mode of negation [5] p. 102.

Evidently dialectical logic violates the exclusion principle, though dialectical logic is by no means unknown. Is it the case that ' $\neg$ ' is not a viable mode of negation? The connective ' $\neg$ ' does conform to a general condition that can be laid down for characterising negation determinables (as given in [12] and [13]). It is a little swift then to rule ' $\neg$ ' out as not a negation connective, and it is certainly logically viable. The argument from meaning, according to which negation by its very meaning must conform to the exclusion principle, may get a grip at this point. It has several variants, a common one being that a one-place connective ' N ' is not a genuine negation unless it conform to the exclusion rule, for every world a , that $I(NA, a) = 1$ iff $I(A, a) = 0$. This, however, is to try quite illegitimately to read a possibility requirement on worlds, a completeness and consistency assumption, into the very meaning of negation. The requirement is far too strong and no natural language negation can meet it. For in situations where connectives more highly intensional than modal ones are evaluated, natural language negation determinables cannot meet the requirement (see e.g. [13] and also [17] and [12]). In any case the argument from meaning is likely to turn into a terminological quibble with the word 'negation' being ceded to vested interests. Call ' $\neg$ ' a *negatiom* then, in which case dialectical logic has no negation, i.e. classical negation, but does have as a surrogate a *negatiom*.

DL refutes outright Rescher's suggestion ([15], p. 103) that a system in which a proposition and its negation can both be true — 'is simply not a system of logic, whatever else its merits might be' — unless '¬' is once again disqualified as a negation. DL and its extensions also do permanent damage to Rescher's thesis that

no system of logic that tolerated the concurrent truth of a proposition and its negation could answer to the purposes for which such systems are instituted within the framework of rational inquiry. ([5], p. 103).

As will emerge, DL not only has a role in the formulation of Soviet metaphysics but also offers a base for several simply inconsistent but hitherto dismissed theories, such as ideal set theory, the classical theory of infinitesimals, and one version of Zeno's paradoxes, and perhaps for physical theories such as the Everett-Wheeler interpretation of quantum mechanics.

Aristotle's principle of non-contradiction is correct, both in syntactical and semantical formulations. In open sentential form Aristotle's principle $\neg(A \ \& \ \neg A)$ is a theorem, hence valid, hence true. In its semantical formulation (one, but not the only, historical formulation) it asserts that no statement is both true and false, that is in the full canonical model, it is never the case that $I(A, 0) = 1$ and $I(A, 0) = 0$: but this is guaranteed by the bivalent features of the semantics.

It is now a straightforward matter to inflate DL so as to validate in the framework some of the most controversial dogmas (from an internal viewpoint) of Marxism-Leninism. For the semantics furnished extends to provide a truth-valued semantics for a zero-order formulation of DL, i.e., a formulation where the language is enlarged to include subject-predicate and relational forms so long as bound variables do not occur. Let the zero-order formulation have as an axiom (which may replace AD1), $f_0x_0 \ \& \ \neg f_0x_0$, i.e., item x_0 has property f_0 and also it is not the case that x_0 has property f_0 . This thesis expresses a contradiction in things. Hence, in the framework of extended DL

the principle of contradiction ... is correct but Aristotle, who first expressed it, made a mistake when he concluded there are no contradictions in things. ([7], p. 19).¹³

It is worth seeing what goes wrong in the usual Western criticism that "it is hardly possible to give a rational foundation to such an assertion" (Lobkowicz [13], p. 45; also Rescher [5], p. 91). Lobkowicz having remarked that the orthodox position validated above, has to take refuge in a "curious 'conceptualism' ", contends that

we find such a 'conceptualism' in [11] ... where it is said ... that Aristotle had correctly formulated the principle of contradiction of formal logic, but that he had erroneously inferred that the real could not be contradictory. In other words, the conjunction of two actually contradictory propositions is always false and, in spite of this, it does not follow that one and the same thing cannot have two mutually contradictory properties ([3], p. 45).

The mistake lies in the claim that follows the words 'in other words', which smudges over a non-identity. For it does *not* follow from the fact that $\neg B$ is true that B is false, in particular it does not follow from the fact that $\neg(A \ \& \ \neg A)$ is true that $A \ \& \ \neg A$ is always false. Lobkowicz has mistakenly assumed that dialectical negation must be classical, that $\neg A$ excludes A , that $\neg A$'s holding at 0 rules out A 's holding, that $I(\neg A, 0) = 1$ implies $I(A, 0) = 0$, that $0 = 0^+$. However $I(\neg A, 0) = 1$ only implies $I(A, 0^+) = 0$; and while $0^+ \subseteq 0$ by p7, the converse inclusion does not hold generally, and so $I(A, 0^+) = 0$ does not generally guarantee $I(A, 0) = 0$. The classical fallacy is exposed in detail in [17] in showing that the principle of Disjunctive Syllogism, $A \ \& \ (\neg A \vee B) \rightarrow B$, is invalid. The orthodox Marxist view of negation is not dissimilar:

the dialectical negation is not a simple negation which rejects the entire previous development but, rather, the base for a development which retains all that was positive in the previous state ([7], § 9.23, p. 22. Cf. also § 9.11).

Rescher has added his weight to the traditional thesis that the consistency of 0 is a matter of rational intelligibility (though he immediately tries to give this a conceptualist turn). He argues thus:

Suppose ... that reality were inconsistent: ... something both is and is not so.... Clearly, only one rationally viable upshot is now possible. We have no choice but to regard *the very hypothesis we are making* as self-destructive, as simply annihilating itself. That is the demands of intelligibility constrain us to the position that it is not *nature* that is self-contradictory (and ultimately unintelligible) but the very assumption that we are being asked to make about it. ([5], p. 91).

In short the hypothesis that something is both so and also not so, that both p_0 and $\neg p_0$ to fit it into the discussion framework, is (i) self-contradictory, and hence (ii) self-annihilating and (iii) unintelligible. But self-contradictoriness does not imply unintelligibility; contradictions are significant even if, or because false (as [4] argues). Nor does self-contradictoriness imply self-annihilation unless a view like the exclusion view of negation is also assumed whereby $\neg A$ wipes out A . The main point, however, is that the hypothesis is not self-contradictory, but embeddable in a consistent framework. One does not need to go as far afield as dialectical logic to show this; less esoteric

systems of relevant logic will suffice. The semantical evaluation of "Routley considered the hypothesis that p_0 and $\neg p_0$ " takes us to an inconsistent situation distinct from 0 but does not impugn the consistency of 0. Compare too the assertion: 'Meyer's hypothesis was that $E = NR$ ': Meyer could consistently hold his hypothesis even though it is logically impossible that $E = NR$. Even if it is conceded to the classical position that contradictions cannot hold true this does not imply that the making of the hypothesis that contradictions hold is thereby ruled out. On the contrary any adequate semantical analysis of intensional notions has to admit this (thus possible world analyses are inadequate).

Dialectical logic yields an account of logical consequence which, unlike the classical account, is paradox-free. A leading feature of the account (which is similar to that elaborated in [22]) is that arbitrary statements are not logical consequences of contradictions and that logically necessary statements do not follow from every statement whatsoever. The main principles of a dialectical theory of logical consequence – essentially specifying logical consequence in terms of inclusion of logical content – may be worked out from the metatheoretic connection:

B is a logical consequence of $A_1, \dots, A_n$ iff $A_1 \& \dots \& A_n \rightarrow B$ is a theorem of dialectical logic, where $A_1, \dots, A_n$ and B are zero-degree formulae, i.e. they contain no occurrences of ' $\rightarrow$ '.

Thus, in particular, an arbitrary formula B is not a (dialectical) logical consequence of the contradictory pair p_0 and $\neg p_0$, a point which destroys such conventional criticisms of contradictions in nature as that advanced by Acton:

Hegel's views on contradictions are difficult to understand and have been interpreted in various ways. If intended to argue that contradictory propositions could both be true, that 'both p and not- p ,' then he was wrong and so was Engels in following him. For it can be proved that from any pair of contradictory propositions any conclusion we like can be deduced and hence that if contradictories are true, *anything* can be true. ([21], p. 392)

For this undesirable result can only be proved through importing classical assumptions into the accounts of logical consequence and its converse, deducibility.

Classical sentential logic is a special case of dialectical sentential logic DL.

The official view nowadays (though it does not go uncontested even in Soviet philosophy) is that both formal and dialectical logic alike have their rights, but that formal logic

represents, in a sense a special case of dialectical logic, much as Euclidean geometry represents a special case of non-Euclidean geometry. ([6], p. 125).

Wetter, who here understates the extent of controversy over the relation of dialectical to formal (classical) logic, shares the prevailing Western commentators' view that there is no hope for dialectical logic as a logic:

...the attempt was made to supplement formal logic by a dialectical logic, and to 'complete' the former by means of the latter. Approximating as it did to a squaring of the circle, this undertaking ended by provoking the most violent controversies among the Soviet philosophers... ([6], p. 125).

Classical logic holds for sublogics of dialectical logic obtained by deleting the isolated contradictions. In the case of DL an exact relation is easy to formulate. Let CL be that sublogic of DL obtained by omitting AD1 (or alternatively by deleting constant ' p_0 ' and all postulated conditions on p_0). Then *CL is a sublogic of DL which contains classical logic* (expressed in terms of connective set $\{\neg, \&\}$); in this sense dialectical logic contains classical logic as a special case. What has to be demonstrated then is that some formulation of classical sentential logic is derivable in CL. One way of doing this, semantically, is by showing that *normal* models suffice to model CL where a normal model $\mathbf{D}$ is one where $0^* = 0$, i.e., that A is a theorem of CL iff A is valid in all normal models. Just this is demonstrated, however, in [18] in the course of showing that material detachment (the rule (γ))

$$(2) \quad A, \neg A \vee B \rightarrow B$$

is a derived rule of CL. (The argument breaks down for DM, which does not include CL.) It is also easy to show using normal models that some set of axioms for classical sentential logic, Rosser's axioms for example, are valid. Since these axioms together with (2) provide a postulate set for classical logic the result is established. Since every classical theorem is thereby a theorem of CL, and CL is a sublogic of DL, *DL has as theorems all theorems of classical sentential logic*. But since DL has further theorems beyond those of classical logic, DL genuinely extends classical logic, and in a sense does penetrate deeper. (In another and weaker sense, however, dialectical logic DL says nothing that cannot be represented classically in relation theory; for the semantics given provides a translation into classical relation theory and thereby a classical view of dialectic. But though the semantics can be represented classically it does not have to be: it can be carried out in the framework of

various nonclassical logics. Thus an appeal back to classical logic, or to classical negation, is by no means inevitable and can be avoided in the semantics.)

The simple consistency of sublogic CL of DL shows that DL is simply consistent except in the neighbourhood of p_0 . A major objective in the investigation of such extensions of DL as ideal set theory (obtained by adding an unqualified abstraction scheme to quantified DL) would be to show that such contradictions as the Russell paradox and its consequences could be decently isolated.¹⁴

Though rule (γ) is a derived rule of CL it is not a derived rule of DL. For if it were then as $p_0 \ \& \ \neg p_0$ and $\neg(p_0 \ \& \ \neg p_0) \vee B$ are theorem schemes of DL, B would be a theorem scheme of DL, for any formula B , contradicting the absolute consistency of DL. The inadmissibility of rule (γ) also explains what may puzzle some, how one can add $p_0 \ \& \ \neg p_0$ to a strongly complete logic such as classical sentential logic and not obtain absolute inconsistency: the answer is that proofs of strong completeness depend upon extending (γ) to apply to the enlarged logic considered, in this case, classical logic

$$+ \{p_0 \ \& \ \neg p\}.$$

The failure of (γ) is one of the most distinctive features of dialectical logic as here explicated. It is what makes the addition of contradictions, to the logic possible. By leaving the question of the admissibility of (γ) open, to be determined in each application, relevance logics provide a reconciliation of a sort, a synthesis so to say, of dialectical logic, which rejects (γ) in order to account for real contradictions, and classical logic, which accepts (γ) and so has to assume the consistency of every theory to which it applies.

The thesis now to be advanced is that the relevance logic position is the rational one, that both dialectical and classical positions are overly dogmatic. This turns the dispute into a three-way one. How is it to be settled, if it can?

IV. WHAT IS AT STAKE IS THE QUESTION OF THE CONSISTENCY OF THE WORLD, 0

For if the world is consistent, if $0 = 0^*$ (i.e. canonically for every A , $A \in 0$ iff $\neg A \notin 0$), then the classical position is correct; but if the world is inconsistent, then the dialectical position is correct. Whichever is the case, however, the relevance position does not go wrong. This provides a major reason why the relevance position is more rational than the other positions, should it turn

out that the matter of the consistency of the world cannot be definitely settled. But *the question of the consistency of the world cannot be conclusively settled in a non-question-begging way* — so at least it will be argued — hence the keeping-options-open strategy of relevance logic is the rational one from a decision-making viewpoint.

What is at stake is *not* the *absolute* consistency of the world, or the logic or theory whose truth it reflects. All the positions can agree that the world is absolutely consistent — which is as well for them, since the absolute consistency of the world can be empirically verified. The statement, q_0 , “Routley is in Belnap’s rooms at the University of Pittsburgh on May 2, 1974”, for example, can be empirically falsified. Hence q_0 is false and q_0 does not belong to 0. Therefore, 0 is absolutely consistent.

It might be thought that given the absolute consistency of the world, the (simple) consistency of the world is also proved, and the classical position thereby established. However the ‘proof’ of simple consistency must depend on the rule

$$(4) \quad A, \neg A \rightarrow B$$

which transforms simple inconsistency into absolute. But this rule is tantamount to rule (γ), even in weak sublogics of CL, and is semantically equivalent to the assumption that $0 = 0^+$ (given absolute consistency or non-degeneracy of the world). Hence the question is begged by rule (4), since precisely what is at issue is rule (γ) and the consistency of 0.

The question of the consistency of the world is not, it seems, empirically decidable. It is a metaphysical thesis, but a perfectly significant one. Since it is a universal claim — to the effect that for every statement A exactly one of A and $\neg A$ belongs to 0, i.e. holds true — it would seem that the dialectician is in a stronger position to establish his claim than the classicist since universal claims can be falsified in principle by a single counterexample. To establish his thesis, however, the classicist has to establish a claim that extends over all true theories, whether scientific, mathematical, evaluative or whatever. Should a contradiction lurk deep in some still unpenetrated mathematical or other theory then the classicist is undone. Only a priori ‘evidence’ that such a contradiction or inconsistency cannot occur would seem to exclude the possibility: thus the classical position if true is not empirically so in any simple way, no endless search is contemplated, nor would its results be relied upon. The dialectician, on the other hand, has the chance of falsifying the

consistency hypothesis by a solitary counterexample. But, as any dialectician soon finds out, a simple falsification of the consistency hypothesis is not so easily achieved, as each alleged contradiction can be avoided, by theoretical shifts if need be (even without appeal to conventionalist stratagems). In this way it quickly becomes apparent that the consistency hypothesis is a highly theoretical claim.

Characteristically dialecticians appeal to paradoxes to establish the inconsistency thesis. Hegel, it seems, was convinced both by the Kantian antinomies and by Zeno's paradoxes of motion; Soviet thinkers are more impressed by Zeno's paradoxes and invest heavily in the contradictions they observe in motion (which generalise to development). None of these cases are, however, decisive, and even if the classicists do not have really convincing solutions, say to some of Zeno's paradoxes, they do have alternative classical resolutions which so far get by. More convincing examples may perhaps be drawn from the logical and semantical paradoxes, but dialecticians do not seem to have made much appeal to such examples. Nor would they be conclusive: the classicist will say of any such theory which contains a contradiction that it is not true, that it is (if perchance absolutely consistent) a theory distinct from the actual one. Thus counterexamples lead to a stalemate: the consistency hypothesis is not straightforwardly falsifiable, or indeed falsifiable at all.

A useful example of the theory-saving methods of the classicist in the face of apparent inconsistency in nature is provided by Rescher [5], who, confronted by the incompatible actual states assumed by the Everett-Wheeler theory of wave packet reduction in quantum physics, casts around for a *difference of respect* between the incompatible states in order to save the law of non-contradiction, and 'locates', i.e. postulates, such a suitable distinguishing respect in a further time dimension.¹⁵ Rescher's difference-of-respect procedure (a method that goes back to the Socratic dialogues) thus offers further confirmation for the popular claim that a logical or mathematical theory can always be saved — at varying costs — by making sufficient changes or complications in scientific theories, since logical principles rarely confront empirical data in isolation and generally only do so rather indirectly in combination with other theoretical assumptions. But though a non-empirical principle, such as the consistency hypothesis, never directly encounters the hard empirical data and can always be saved in one way or another, with greater or less cost, by changes elsewhere, the costs may be too high, and it may be better to give up the principle. A convincing microphysical theory based on dialectic-

tical logic might provide such a reason. For success with theories based on classical logic has always been at the finite macro-level (as Brouwer would have said): there is, in principle at any rate, no reason why classical logic should not go the way of Euclidean geometry – satisfactory locally in regions of consistency, but globally defective. Admittedly thus far there is little persuasive evidence that a conceptual revolution is called for: however, it should now be obvious that such a change would be perfectly possible.

If the consistency hypothesis is not straightforwardly falsifiable, it is not readily verifiable either. By virtue of a by-product of the logical paradoxes, the situation appears to look still blacker for the classical position. Consistency is a logical matter, and if it can be established it will presumably be by logical means. Suppose, with a view to proving consistency, that the theory of $\mathcal{O}$ is formalised, and suppose that the truths of $\mathcal{O}$ include those expressible in a sufficiently rich language (expressed, e.g., by way of constants); then according to Gödel the consistency of $\mathcal{O}$ is not incontrovertibly provable except by means at least as demanding as the resources of $\mathcal{O}$ (nor can they exceed the resources of $\mathcal{O}$ without appealing to falsehoods?). The same problem arises each time a reformalisation is attempted aimed at incorporating syntactically outstanding truths and thus approximating syntactically to $\mathcal{O}$. In short, the classicist can never conclusively establish his position; for, any formal proof of his hypothesis will use means at least as powerful and *accordingly as questionable* as his claim. Although we can be sure that the world is locally consistent for most regions we work in, like locally Euclidean (more exactly there are consistent subtheories that we commonly use),¹⁶ globally we can have no such confidence, thanks to the fine injection of scepticism Gödel's results provide.

Therefore the question of the consistency of the world cannot be conclusively resolved; which was one of the theses to be argued. This in turn suggests the uncharitable proposition that belief in the consistency of the world is a mere act of faith, a part, so to speak, of the classical religion. While the belief may be an act of faith for many people, it doesn't have to be; for, the consistency hypothesis may be part of a correct scientific theory, and sufficiently comprehensive scientific theories are in much the same kind of epistemological plight as the consistency hypothesis. There can be good evidence for such theories even when they cannot be conclusively verified. The uncharitable proposition then rests on a false dichotomy between conclusive resolution or else mere act of faith.

One remaining puzzle — which we here leave unresolved — concerns the character of the consistency principle: is it a contingent statement, or a purely logical one, or (most likely, as synthetic non-empirical) something else again? It does not appear to be empirical in the way that the deeper assumptions of physics such as invariance principles are. The alternative, on the simplistic classification that positivism so obligingly imposes, seems even less satisfactory, namely that the hypothesis is logical, and so if true analytic¹⁷ and if false logically false. The alternative does, however, have the virtue of grouping the consistency hypothesis with a range of other principles which raise similar epistemological problems, in particular the axiom of choice and axioms of infinity. The consensus that seems to have been reached regarding these principles is that we reason as far as possible without them, note carefully where they are used, try to establish whether or not they are essential where they are used, mark out those cases where they are essential, and so on. And the consensus view reflects the rational course of action, namely to minimize questionable logical assumptions. The same method would incline one to eschew the consistency hypothesis as far as practicable, and to work, where consistency cannot be established, with a relevant logic; for this is the rational course of action.

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NOTES

*Compare also Philipov's view as reported by Church [14]: it is the author's prediction that Soviet logic will not develop fruitfully or successfully, and for these three principal reasons are alleged:

Lack of freedom for creative thought, extreme Russian nationalism, and incompatibility of logic and dialectic.

¹ Asenjo's 'calculus of antinomies' is nothing other than Kleene's 3-valued logic K_1 discussed in [4], p. 335 ff., and Asenjo's argument for his matrices (which can be defined in terms of the pair $\{\sim, U\}$) simply repeats, in muddier form, Aqvist's argument for the matrices of the larger system obtained by adding connective 'T' to the connective set (the larger logic coinciding with Łukasiewicz's 3-valued logic). Asenjo's reading of his third value 'P' ('paradoxical') as 'both true and false' is unsatisfactory since the negation of an assertion which has value P likewise has value P and accordingly should be neither true nor false: the reading of 'P' as non-truth-valued would avoid this problem. But the change in reading cuts any tenuous links of the logic with a dialectical one. Moreover the requirement on dialectical logic that for some formula A both A and $\sim A$ are true is excluded by the matrix for ' $\sim$ ' of K_1 .

² Wetter goes on in [11] to assume the older mistaken view that the inclusion of semantics in logic 'transforms logic into the theory of knowledge' ([6], p. 126). Little hangs on this point, however, since dialectic contains both the theory of knowledge and logic ([10], p. 304), and it is sometimes even asserted that logic and theory of knowledge are identical (a rather congenial thesis for those who think that epistemology is but applied logic).

³ It could be argued that the German idealist position which took the conclusions of Kant's antinomies as establishing contradictions in nature only, and not in thought, was committed to just such a position. But this point does not really force exponents of such a position into adopting a weak dialectical logic without (3); for they could still contend, with but slight modification of this position, that while the primary contradictions, such as those generated by Kant's and Zeno's antinomies, are synthetic a priori (or contingent non-empirical truths), these generate derived logical contradictions.

⁴ N. da Costa (e.g. in [9] and [23]) has opted for the weaker alternative: indeed he goes so far as to require that 'the principle of contradiction, $\neg(A \ \& \ \neg A)$, must not be a valid schema', thus ruling out orthodox dialectical logics. This is one of the unsatisfactory restrictions on da Costa's fine vision of a general theory of inconsistent formal systems. Other deficiencies of the approach so far concern

(i) the very limited set of inconsistent systems, the C systems, studied as part of the program. In fact all of the C systems are negation extensions of Hilbert's positive logic. Since these systems are shot through with paradoxical implication principles, they provide a quite inadequate, because excessively strong, basis for many important philosophical applications. Moreover the strength of the positive system excludes an adequate negation theory, such as that of DL; thus da Costa's inconsistent systems exclude a symmetrical treatment of negation, with full principles of double negation and contraposition.

(ii) the limited semantical investigations undertaken. The C systems themselves are easily supplied with semantics, the semantics being elaborations of Kripke's semantics for positive logic. The semantics have the advantage of bringing out the relations of da Costa's systems and intuitionistic and Fitch logics, but at the same time they help reveal the intuitive weaknesses of the systems as candidates for formulating Meinongian and dialectical logics.

For other pioneering work on the theory of inconsistent formal systems, see [19] and [20]. As to the philosophical importance of inconsistent theories, see [12], [17] and [18].

⁵ There are well-known alternatives to the use of constants for this purpose, e.g. use of sentential quantifiers; but appeal to such quantifiers not only complicates the language but raises a number of fresh semantical and philosophical problems, which are separable from issues as to dialectical logic, e.g., problems as to the meaning of such quantifiers and as to semantical representations and the completeness of relevant logics with such quantifiers.

⁶ The Absolute is perhaps better defined as the (calculus of individuals) fusion of the statements of 0. To get such calculus of individuals' notions as *fusion* off the ground in this area a part-relation needs, however, to be defined; but this can be done as follows:

A is a part of B iff $A \rightarrow B$ is a theorem. This interconnexion helps explain the adoption of D2 needed, at least in rule form, to guarantee transitivity of the part relation.

⁷ In general the roles of constants will be restricted not just over 0 but for classes of worlds. For example, to guarantee $\Box(p_0 \ \& \ \neg p_0)$, i.e., that the contradiction AD1 is necessarily true, (ii) would have to hold not just for x in 0 but for all worlds related to

x by the modal alternativeness relation. Semantically a constant differs from a variable simply in being constrained in some situations.

⁸ Though $p_n \& \neg p_n \rightarrow p_{n-1} \& \neg p_{n-1}$ is a theorem, the converse does not hold. The contradictions thus form an infinite chain.

⁹ The same criticisms apply to Thomason's formalisation [24] which offers a semantics like that of [17] for a first-degree part of DL. We owe these points to Newton da Costa. To take account of his further suggestion that some Marxist philosophers regard $\neg^{n+1}A$ as richer than $\neg^n A$ (for $n \geq 0$) would require the addition of a richness relation to the logic.

¹⁰ It would perhaps be less misleading to call 0 the (true) Theory, since, despite Wittgenstein, there is a strong proclivity to confuse the World with the class, or fusion, of things. The World 0, which is everything that is the case, and not the totality of things, does, however, have associated with it a domain $d(0)$ which is the totality of things.

0, at least, is invariant over full canonical models. Whether, however, a unique full canonical model can be determined is not so easily determined, and will depend on the language modelled. It is tempting to suppose that suitable uniqueness is assured for sufficiently rich languages whose components are fully analysable.

¹¹ The argument is as follows: Since $I(\neg A, 0) \neq 1$ iff $I(A, 0) = 1$ for every A , $I(A, 0^*) = 1$ iff $I(A, 0) = 1$ for every A . Hence for every A , $A \in 0^*$ iff $A \in 0$; whence as worlds represent sets of statements, $0^* = 0$.

¹² A more detailed discussion of negation, the negation metaphors and intuitive modelings, will be found in [12].

¹³ The formulation of "the principle of contradiction", however, which is omitted from the quotation cited, appears defective. According to 8.11 the principle of contradiction "forbids the simultaneous acceptance of propositions which contradict each other". But this formulation is not only historically inaccurate: it is impossible to reconcile with the logical theory developed, or, it would seem, with any consistent logical theory.

For p_0 and $\neg p_0$ contradict each other, and, since $\vdash p_0 \rightarrow \neg \neg p_0$, p_0 is inconsistent with $\neg p_0$; but $\vdash p_0 \& \neg p_0$, so the conjunction is true (canonically) and both p_0 and $\neg p_0$ can be accepted.

¹⁴ p_0 could then be replaced by such statements as the class of classes not members of themselves is a member of itself. Such a proof would go some way towards vindicating Wittgenstein's later position on the logical paradoxes, and would offer an alternative way of justifying correct mathematical practice which, *prima facie*, is to work with a naive, inconsistent, set theory though generally *not* with a logical basis with anything approaching the excessive power of classical logic.

Da Costa has made a detailed study of various inconsistent but apparently non-trivial set theories within the framework of certain special inconsistent logics (see [23]). In these systems, however, the simplicity of naive set theory is sacrificed in favour of (what is far less important) strength in the basic positive logics used, which would admit Curry-type paradoxes were an unqualified set abstraction axiom adopted. Because of Curry-type paradoxes (e.g. in $DM + A \& (A \rightarrow B) \rightarrow B +$ Ideal Set Theory), the absolute consistency problem is not always escaped even in dialectical or relevance logics.

¹⁵ To what extent the Everett-Wheeler theory can be satisfactorily constructed on the basis of a dialectical logic is an interesting question that takes us beyond the scope of this paper. The same goes for questions as to whether such discredited theories as the classical theory of infinitesimals can be 'satisfactorily' reformulated in the framework of dialectical logic, and the connected question of the extent to which the admission of inconsis-

tencies at the infinitesimal level would help to resolve Zeno's paradoxes. For each of these theories, each of which leads to at least isolated inconsistencies at micro-levels, would demand a substantial amount of technical development before a worthwhile discussion could get off the ground.

¹⁶ The official Soviet view is not dissimilar; e.g., classical logic does apply in the case of simple, relatively stable objects and relations; see the discussion of Kédrov's position in [15], p. 84.

¹⁷ Forcing the principle to be analytic takes us back through the meaning-of-negation circuit we have already been quickly around.

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