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Necessary limits to knowledge: unknowable truths

Richard Routley

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Abstract The paper seeks a perfectly general argument regarding the non-contingent limits to any (human or non-human) knowledge. After expressing disappointment with the history of philosophy on this score, an argument is grounded in Fitch's proof, which demonstrates the unknowability of some truths. The necessity of this unknowability is then defended by arguing for the necessity of Fitch's premise—viz., there this is in fact some ignorance.

Keywords Knowability · Fitch's paradox · Verificationism

Whereas it is impossible to know “everything” we do know the method by which anything whatsoever can be known. Such is the [rather, *a*] claim, implicit or explicit, which underlies the origins of modern (and much of contemporary) philosophy.¹

¹ Rosen (1974, p. xiv). Rosen's discussion of the modern claim is however disappointing.

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In what follows the modern claim is refuted.² It is shown that, given that it is impossible to know “everything”, some truths are unknowable, so there is no method by which anything whatsoever can be known.

1 The theme of a method versus the theme of limits and of limitations to knowledge

“From the perspective of modernity, classical ... thought ... [was] preoccupied with the notion of limits”,³ limits to the size of cities and states, limits on wealth and poverty, limits to avoid both excess and insufficiency, and limits to knowledge: thus, for example, Aristotle’s view that a universal (science of) science is impossible. The classical preoccupation was “replaced by a modern preoccupation with freedom as a progressive liberation of man from all traditional and natural limits”, and a modern view of unrestricted progress, of unlimited opportunities for humans, and of unimpeded domination of nature.⁴ Impressive advances in science and technology encouraged the (erroneous) idea that limits could be removed, an idea reinforced by theoretical presumptions as to the solvability of every problem, and the availability of a method—the “scientific method”—by which everything could be known.

Recently these modern assumptions have been challenged, and subjected to serious criticism. Several limitations have become very conspicuous, especially a range of ecological constraints upon “progress”, but also theoretical limitations upon technological advance and upon problem resolution.⁵ A further limitation of theoretical importance is that upon knowledge and upon scientific method.

The theme of a method, sponsored by the founding fathers of “modern” philosophy of science such as Descartes and Bacon, and very widely propagated, did not go entirely unquestioned, even before contemporary logical and ecological developments. For the idea of limits to *human* knowledge has remained an element both in Western philosophical traditions, emanating especially from Kant, and in Eastern traditions, especially those of Buddhist orientation. The limits intended are not merely contingent (else they would not tell against the method theme). It is true, and not without philosophical interest, that there are non-denumerably many things which humans do not *in fact* know, many of which they might know. Even Anglo-American empiricism acknowledges that (human) knowledge is partial.⁶ What is intended in such limitation

² While the paper does not pretend to originality, it is presented in the hope that it includes some news. My thanks to W. Hart who rescued the main argument from archival oblivion, and to S. Sommerville who drew my attention to it and some problems with it, and for comments to R. K. Meyer, P. Roeper, C. Mortensen and P. Hanson.

³ Rodman (1980, pp. 52 and 61). Rodman also gives a worthwhile expansion of the points summarised in the text.

⁴ See, for example, the assumptions of the Dominant Western Worldview as presented in Catton and Dunlap (1980, Table 1).

⁵ As to the latter, see, e.g. Routley (1980b).

⁶ Thus, e.g. Russell (1948), which ends: it is “a doctrine by which empiricist philosophy has been inspired: that all human knowledge is uncertain, inexact, and partial. To this doctrine we have not found any limitation whatever”. Despite the title of his work Russell has comparatively little to say about the scope of human knowledge, and almost nothing as to its limits. It is not hard to guess however roughly what sort of rough

claims, and more interesting philosophically, are limits to what humans—and sometimes other knowers—*can* know. Alleged limitations are not difficult to come by: “People cannot know what the true cause of pine blight is, nor can they know the ultimate consequences of their ‘remedy’ ”, says Fukuoka (1978, p. 39f). There is much that is right in what Fukuoka has to say, about humans’ vast lack of knowledge, and partly consequent thereupon, modern unwarranted technological optimism and overconfidence as regards what can be successfully accomplished, especially with ecosystems. Even so, his examples do not establish all he intended or other than contingent limitations. For we can know—and for some types of pines and blights we *do* know—the causes, perhaps even the “true” causes, of pine blight (namely, that it is such and such a fungus, e.g. *Dothistroma pini* in the case of *Pinus radiata*, operating in such and such a fashion); and we can also know many of the consequences of “remedies”—though not all the consequences given that they disperse into an endless future—and in *some* cases (where ‘ultimate’ is sufficiently well-defined) the “ultimate” consequences, e.g. where the pines are wiped out and the ecosystem destroyed.

Nothing so simple, straightforward, and apparently easily countered, emerges in the esoteric limitations of knowledge outlined in Western philosophers in the Kantian tradition. According to Kant what must remain unknown are matters that go beyond the (legitimate) bounds of understanding, for instance, such theoretic objects as noumena and their properties. Thus there are, presumably, true statements concerning, for instances, noumena, which are not just humanly unknown, but humanly unknowable. For human understanding “cannot know these noumena through any of the categories, and ... it must therefore think them only under the title of an unknown something. [Thus] it ... sets limits to itself...”⁷ and limits to human knowledge, since knowledge is restricted to the understanding (Kant 1781, pp. 156, 171). There are good reasons, it is widely (and correctly) considered, for rejecting this whole approach: the arguments for it, where they can be unravelled, lack cogency; such an approach is not necessary, or desirable, and can be avoided.⁸ As such a sympathetic commentator on Kant as Strawson remarks of “the grand strategy”, which is supposed to demonstrate, among many other positive things, “the confinement of knowledge to the empirical”, “All that is actually achieved is negative: ... the exhibition of certain cosmological questions as at least highly problematic in character. The rest is unpalatable claim and fallacious argument.”⁹ If the Kantian approach is to carry weight in a way that is widely taken seriously, it will have, at the very least, to be reworked in a more precise and rigorous way than has yet been achieved: otherwise the approach will only impose “bounds” on human knowledge that can be got around or defeated. It is doubtful

Footnote 6 continued

empiricist bound he would *wish* to impose. Such empiricist bounds are no real bounds: so it is argued in Routley (1980a, Chap. 9).

⁷ Kant (1781, p. 156). The quote is slightly rearranged. See also p. 152.

⁸ Substantiating these claims would involve a considerable detour, but can certainly be done. That Kant’s arguments lack cogency is established by examination of the arguments, as has been done by many commentaries on the relevant arguments (e.g. recently, Strawson’s cited below). As to grounds for the larger claims, see Routley (1980a, Chaps. 8 and 9).

⁹ Strawson (1966, p. 156). Kant’s “grand strategy” is outlined on p. 155.

that such a reworking (which would have to be considerable) can be accomplished (cf. footnote 8).

Other metaphysical theories than those of Kantian cast also try to impose rather sharp limits upon what can be known: certain atomistic systems (such as that of Wittgenstein's *Tractatus*,¹⁰ considered below) have this feature as well as representational systems. As affording evidence of limitations of knowledge such metaphysical theories are not a solid base from which to operate: far too many dubious, or downright false, assumptions are involved in the theories; and, furthermore, they are opposed on the issue by rival metaphysical theories, such as Cartesianism and mainstream positivisms—especially those theories which have envisaged a universal science ultimately encompassing in principle all truth, apparent truths resisting this framework of knowledge being dispensed with as illusion or nonsense or rubbish.

Similar points apply to limits on knowledge discerned under certain linguistic theories. For instance, limits on human competence, in acquiring language, have been insisted upon (e.g. by Chomsky); indeed such limits form an essential backdrop to the idea of a universal grammar promoted. Again the basis lacks solidity, more than limited acceptability, etc.

There is also another serious deficiency in most philosophical (and linguistic) work hitherto on limits of knowledge, namely its lack of generality through its restriction to *human* knowledge. In an investigation of the limits of knowledge, this parochiality should be avoided. Knowledge is not just confined to humans;¹¹ and what is of interest and philosophical concern is not just human knowledge but all knowledge, including the knowledge of terrestrially remote intelligent creatures and of such giant-brained terrestrial creatures as whales. (This is not to say that there is not a place for, and much point in, such works as *Cetacean Knowledge: Its Scope and Limits*, which make heavy use of limiting features of cetacean neurophysiology: but they would not exactly be philosophical works). An investigation of the limits of knowledge, as distinct from human knowledge should not depend at all on *human* faculties or facilities, in the way that Kant's investigations and those of his successors did and those who also mistakenly psychologised logic did. Avoiding Kantian narrowness take us from philosophical psychology to logic.

¹⁰ Cited and briefly considered below. The theme of a method, evident in the *Tractatus*, persisted into Wittgenstein's later philosophy, though the method changed from a logical to a linguistic one. Thus Lee (1979, p. 218) has the following to report (slightly reorganised): "Wittgenstein thought that there was a method [of investigation] in philosophy, and presumably, that he had found it ('today philosophy has found its method'). Namely: 'In philosophy all that is not gas is grammar'."

¹¹ See, e.g. Routley (1981). A recent example of the parochiality is provided by Bradley and Swartz (1979, p. 140). Apart, however, from the restriction to human knowledge, Bradley and Swartz argue for limitations on knowledge similar to those defended below. Moreover, they explain humanly possible knowledge in terms of *capacities*, that is, using the same modality as that invoked below—given which the restriction to humans can drop out for most philosophical purposes. But if the restriction *is* duly removed, the Bradley–Swartz example of "a contingent proposition which is in principle unknowable" (Bradley and Swartz 1979, p. 167) fails, since there are finite nonhuman creatures which can know the proposition. As a result the argument they offer against positivism is without force, since such logical positivists as Carnap did not limit experience to human experience.

2 The main argument to limitations

Sounder bases for limitations on knowledge can be derived using logical theory and its ramifications, such as recursion and automata theories. A very elementary logical argument, which, in contrast to more powerful theories, makes but few assumptions, none of them in real doubt, is applied in what follows to show that, as a matter of logic, some truths are unknowable.

The main argument, an “unjustly neglected logical gem”,¹² is formulated first as a *direct* argument in *sentential* logic. The underlying logic assumed is the very weak modal system S0.5 (formulated by Lemmon, in an attempt, among other things, to axiomatise the functor ‘It is tautologous that’); in fact the even weaker but less well known system S0.5° will suffice.¹³ The modal connectives of the underlying logic are *not* interpreted for the most part, in (quite) the usual way, i.e. with $\Diamond$ interpreted ‘It is (logically) possible that’ and $\Box$ as ‘It is (logically) necessary that’. For the object will be to formalise arguments involving such terms as ‘verifiable’, ‘provable’, ‘confirmable’, etc., and this will require a breakdown of *-able* terms into a modal functor plus a base functor, for example, of *confirmable* into a capacity functor + *confirmed*, on the ground that ‘confirmable means, ‘able to be confirmed’, ‘capable of being confirmed’. This break down procedure is fraught with difficulty, as Mill’s notorious decomposition of ‘desirable’ shows, and Kielkopf’s less renowned decomposition of ‘permissible’ into ‘possibly permitted’ equally reveals.¹⁴ ‘Confirmable’ does *not* mean ‘possibly confirmed’, or ‘provable’, ‘possibly proved’, any more than (!) ‘desirable’ means ‘possibly desired’ or ‘capable of being desired’, but rather ‘worth desiring’ (cf. OED). The functorial decomposition adopted is artificial, but, it is to be hoped, adequate: ‘it is V-able that’ is rendered ‘It can be that it is V(-ed) that’, ‘It is able to be the case that it is V(-ed) that’ and the like. Thus $\Diamond$ and $\Box$ are interpreted respectively as, for instance, ‘it can be that’ and ‘It cannot but be that’, as *capacity* modalities. It is evident enough that the axioms and rules of S0.5 hold under this interpretation (subject to usual restrictions to consistent situations).

¹² According to Hart (1979). The argument has a history of passing interest (cf. Hart 1979, p. 164). It was published, in essentials, by Fitch (1963), where the result is attributed to an anonymous referee of an earlier 1945 paper that Fitch did not publish. Nothing is made of the result in Fitch; it is not even commented upon. In Hart (1979) the result is applied but against verificationism, in an evident aside from the main business of the paper. Although the argument is revived (again, in an indirect form) in Hart (1979), it is again a place where it may well be neglected. So also may the present paper. Hart would like to know the identity of Anon, the anonymous referee. Meyer conjectures, what seems to be unlikely, that Anon=Gödel.

¹³ On system S0.5, see Lemmon (1957, 1959). The interpretation of S0.5 is debated by Cresswell (1966, 1970), Rennie (1971), and Routley (1968a,b) System S0.5 adds to classical sentential logic *S* the one-place connective $\Box$ and has a postulates any set of (schematic) postulates for *S* together with the following postulated:

$$\Box A \supset A$$

$$\Box(A \supset B) \supset \Box A \supset \Box B$$

$$\Box C, \text{ where } C \text{ is a theorem (or tautology) of } S.$$

System S0.5° results by deleting the schema $\Box A \supset A$.

¹⁴ On Kielkopf’s decomposition, see Routley (1983).

The underlying logic is extended by a one-place connective K , i.e. where A is a wff so is KA , and the postulates apply to wff containing K . Call the resulting system KS0.5. The main intended interpretation of K is as a knowledge functor, either

- (i) as the neutral functor ‘It is known that’ or as a quantified epistemic functor such as ‘Someone knows that’ or ‘Someone at some time knows that’ or ‘Every creature knows that’; or
- (ii) as a creature-relativised functor, i.e. as representing K_a , read ‘ a knows that’, for some given creature a ; or as a creature- and temporally-relativised functor, such as $K_{a,t}$ ¹⁵ read ‘at time t a knows that...’.

However other interpretations will prove important (e.g. “It is provable (proved) that”) or interesting (e.g. “ a rationally believes that”). Indeed the functional interpretations permitted are restricted only by the further conditions K is now assumed to satisfy, namely:

A0: $q \supset \Diamond Kq$, i.e. (under main interpretation (ii)) whatever is true is possibly known.

A1: $Kq \prec q$, i.e. necessarily, whatever is known is true; or more weakly

A1*: $AKq \prec \sim K \sim Kq$, i.e. for whatever is known, necessarily, it is epistemically possible that it is known.

Here A1* is a weaker alternative to A1, and strict implication is defined, as usual: $A \prec B = Df \Box(A \supset B)$.

A2: $K(q \& r) \prec Kq \& Kr$, i.e. necessarily, each conjunct of what is known is known.

The adequacy of these assumptions—all commonly taken to be true, or at least highly plausible—will shortly be a matter of much concern. For what is shown in the first argument is that the assumptions, added to KS0.5, lead to the evidently false conclusion

C: $p \supset Kp$, i. e. whatever is true is known.

The *first argument* runs thus (for every p and every q):

- | | |
|---|---|
| 1. $\sim \Diamond(q \& \sim q)$ | Necessitated Non-Contradiction,
from S0.5. |
| 2. $\sim \Diamond(Kp \& \sim Kp)$ | Substitution in 1. |
| 3. $K \sim Kq \prec \sim Kq$ | By A1*, or from A1. |
| 4. $Kq \& K \sim Kq \prec \cdot Kq \& \sim Kq$ | From 3, by Composition. |
| 5. $\sim \Diamond(Kq \& \sim Kq) \supset \sim \Diamond(Kq \& K \sim Kq)$ | From 4, using $A \prec B \supset \cdot \Box A \supset \Box B$ |
| 6. $\sim \Diamond(Kp \& K \sim Kp)$ | by 2 and 5. |
| 7. $K(p \& \sim Kp) \prec Kp \& K \sim Kp$ | Substitution in A2. |
| 8. $\sim \Diamond(Kp \& K \sim Kp) \supset \sim \Diamond K(p \& \sim Kp)$ | From 7, using $A \prec B \supset \cdot \sim \Diamond B \supset \sim \Diamond A$. |
| 9. $\sim \Diamond K(p \& \sim Kp)$ | By 6 and 8. |
| 10. $\sim(p \& \sim Kp)$, i. e. $p \supset Kp$ | From 9 by A0. |

The argument can be made fully strict in form, by making A0 strict. Then the underlying logic is S4 and the assumptions are $\Box A0$, and A1 and A2. The conclusion $p \prec Kp$ results. An S4 principle is needed to ensure that $\Box \sim \Diamond(q \& \sim q)$ in step 1: otherwise only S2 principles are required. The characteristic confusion of implication

¹⁵ It is inessential that knowledge be formally represented through a functor, rather than as say, a predicate in the style of R. Montague.

with modality, embodied in strict implication, which here forces the use of S4 in place of S2, is avoided in relevant reformulations of the argument. The argument may be rendered *relevantly* valid—and so *truly* valid (in relevant settings, which lack the rule of material detachment)—by replacing $\prec$ and also $\supset$ throughout (except for $\supset$ in 10) by $\rightarrow$. The argument may then be conducted in a (weak) relevant logic with systemic connective $\Box$.¹⁶ Naturally material detachment using 10 is illegitimate in inconsistent situations: but for much of what follows it will be assumed—as usual, and with good reason—that we are not working in inconsistent situations.

More intriguing than such variations on the argument, is the matter of what is wrong with it. Where does the dubious step in the argument lie, and where the dubious assumptions? Let us (anticipating things) work back up the argument line by line. The conclusion, line 10, is said to be ‘preposterous’: is line 9 preposterous also? The answer certainly seems to be: No, for it is *true* that it is impossible that it is known that p is true but not known. The truth can be demonstrated, though in ways that are (of course) not assumptionless. $\sim\Diamond K(p \ \& \ \sim Kp)$ is a theorem of quite basic epistemic logics, including relevant ones. Most simply it is equivalent, since K distributes over $\&$, to $Kp \prec \sim K\sim Kp$. But this is $A1^*$, and is a consequent of $Kq \prec q$, i.e. $A1$, by substitution upon $q \prec \sim K\sim q$. Alternatively, the truth of $\sim\Box K(p \ \& \ \sim Kp)$ can be shown, without syntactical detour, by semantical methods.

The mistaken assumption is then $A0$, which leads from 9 to 10, from what is true to what is false. Somewhat independently, remaining assumptions $A1$ – $A2$ used in the argument to C are extremely hard to fault. Also $A0$ is readily countermodelled, and so shown to be false, in epistemic logics.

The main conclusion that some truths are unknowable has been virtually reached, but the argument to it remains less straightforward than it would be were advantage taken of sentential (or assertoric) quantifiers. Let neutral sentential quantifiers, interpreted in truth-valued fashion (or substitutionally), and subject to the usual postulational constraints, be adjoined to $KS0.5$. Consider now the extension $K+$ of $KS0.5$ so quantified, obtained by adding both $A2$ and either $A1$ or $A1^*$, and

$A3$: $(Pq)(q \ \& \ \sim Kq)$, i.e. for some q , q is true but not known.

Then the *second argument* may be given theorem form.

Theorem (Anon) $(Pq)(q \ \& \ \sim\Diamond Kq)$, i.e. *there are unknowable truths, is a theorem of $K+$* .

Proof As in the first argument

- | | |
|--|--|
| 9. $\sim\Diamond K(p \ \& \ \sim Kp)$, | whence generalising |
| 10'. $(r)\sim\Diamond K(r \ \& \ \sim Kr)$. | |
| 11. $(Pr)(r \ \& \ \sim Kr)$, | i.e. $A3$ |
| 12. $(Pr)[\sim\Diamond K(r \ \& \ \sim Kr) \ \& \ (r \ \& \ \sim Kr)]$ | from 10' and 11 by |
| | $(r)A \ \& \ (Pr)B \supset (Pr)(A \ \& \ B)$. ¹⁷ |

¹⁶ On relevant logics with systemic connectives, see Routley et al. (1982, Chaps. 7 and 10). Note too that the argument in no way depends upon the adoption of a particular conception of truth, e.g. upon a classical (logical) conception of truth.

¹⁷ For $C \ \& \ (Pr)B \supset (Pr)(C \ \& \ B)$ where r is not free in C , and $(r)A \supset A$. Again the argument may be rendered relevantly valid by replacing $\supset$ by $\rightarrow$.

13. $(Pq)(q \& \sim \Diamond Kq)$, from 12 by particularisation.

Alternatively, the argument can be expositively clarified through (and infact use of sentential quantifiers replaced by) use of sentential constants. Thus A3 either yields or is displaced by

A3^c: $r_0 \& \sim Kr_0$ (for suitable constant r_0).

A3^c replaces 11 in the proof. From 9,

14. $\sim \Diamond K(r_0 \& \sim Kr_0)$, whence
 15. $(r_0 \& \sim Kr_0) \& \sim \Diamond K(r_0 \& \sim Kr_0)$.
 16. $q_0 \& \sim \Diamond Kq_0$, from 15 where q_0 is $r_0 \& \sim Kr_0$.¹⁸

Observe that the proofs also furnish information as to the character of such unknowable truths, namely that *where r is an unknown truth, $r \& \sim Kr$ is an unknowable truth*. This recipe (only one among many) for generating unknowable truths does not exhibit or reveal a way of exhibiting an unknowable truth. But it is not so evident, as one may be at first inclined to think, that unknowable truths cannot be exhibited. Certainly an unknowable truth r cannot be proved or established constructively, that is (in this case) by exhibiting such. Constructively $(Pr)(r \& \sim \Diamond Kr)$ is established by exhibiting an instance, r_1 say, for which it is *proved* that $r_1 \& \sim \Diamond Kr_1$. But if r_1 is proved, then its truth is established on excellent grounds, so r_1 can be known; that is, $\Diamond Kr_1$, contradicting $\sim \Diamond Kr_1$. Much the same holds for unknown truths. Exhibiting is, however, substantially weaker than establishing by or while exhibiting, and a statement may be exhibited without being established. In particular, truths which are not in fact known may be presented. This may be done regularly, and perhaps reliably, by a truth diviner.

Consider a fairly reliable truth diviner, Sheva. (She differs from fortune tellers, and so on, chiefly in her reliability. Sheva may be compared with a water diviner who divines unproven water.) Then some of the assertions divined by Sheva will be true though not known. Let r be one such: then $r \& \sim Kr$ is an unknowable truth, which can be exhibited. It could be exhibited in fact, through a representative inscription, in Sheva's gallery of the unknown. The gallery is, of course, like most galleries, one that changes over time: inscriptions will come down, before what they state is known, and others will go up. The same applies, unfortunately (?), to the "unknowable" truths so far advanced, since these are only *relatively* unknowable, unknowable relative to what is unknown. (Subsequently evidence for absolutely unknowable truths will be assembled.) Observe that Sheva's gallery puts us beyond the reach not merely of constructivism (which it will, with due supplementation, refute) but also of wider verifiability and provability methods which though they permit nonconstructive methods (such as *reductio ad absurdum*) are still far too restrictive. These observations indicate subsequent directions of the dialectic.

Now $K+$ is true. For its axioms are true (under the intended interpretations) and its rules preserve truth.¹⁹ Therefore, by an inductive argument, all theorems of $K+$ are true. Hence the

¹⁸ A different interesting but not unproblematic example of q_0 , suggested by M. J. Cresswell is this: Everyone is always asleep, in a situation where everyone (every potential knower) is always asleep.

¹⁹ With one important qualification, that material detachment ($A, A \supset B \Rightarrow B$) be restricted to consistent situations—a qualification which can be removed by making things duly relevant.

Corollary *It is true that there are unknowable truths.*

3 Limitations versus freedom theses

Given the evidence showing that A0 is false, that some truths are unknowable, why is A0 tempting? What are the difficulties and costs of its abandonment? A0 is tempting, for *one* reason, because of certain freedom (or no limitation) theses, such as Meinong's thesis that there is no limitation on what we can assume, we can assume anything we like.²⁰ But if we can assume or think what we like, then similarly we can believe anything we like, can't we? (The answer could be, No: belief and thought are not so similar. But it is not.) And similarly again, reason about anything we like: there are no bounds on reason. At least these themes are appealing to anyone who has glimpsed the *considerable* virtues of paraconsistent theory. And for present purposes, linking freedom of belief for example with A0, only a rather restricted form of the freedom of belief thesis is needed (a form even fascists can admit, after a fashion), that we are free to believe whatever we like that is true (since they can manipulate "truth"). Now let r be any alleged counterexample to A0 which is true, that is r is true but alleged unknowable. Then r can, by the freedom principle, be believed, and surely good grounds can be found for believing it since it is true. So r can (applying some suitable variant upon Russell's analysis of knowledge) come to be known, contradicting its alleged unknowability. Well, the argument is not tight, since we can't, without further ado, be *so* sure that good grounds can be found; and r 's alleged unknowability casts some doubt on the availability of good grounds.

The question of good grounds can however be bypassed, since the theorem applies as well to true belief as to knowledge. Define the functor K_w (for piss-weak knowledge) thus: $K_w A = \text{Df } BA \& A$, where B represents belief. Then K_w satisfies A1–A3 (rewritten for K_w). A1 is immediate by simplification; A2 depends upon a similar property for belief (argued for elsewhere: see Routley 1980a, p. 8.12; and for A3 let q be an arbitrary assertion that is true but not believed, i.e. $q \& \sim Bq$. whence $\sim K_w q \& q$. Hence, by the corollary, there are truths which are not piss-weak knowable. Doesn't this do damage to the idea that anything is believable, and so to the freedom of assumption thesis? It would, were the following principle,

R1: $\Diamond BA \& A \prec \Diamond (BA \& A)$,

(or its material form) correct. For then where q is not piss-weak knowable but true, q is not believable. In fact R1 (or its material form, if we're classical) is exactly what

²⁰ On and in defence of these freedom theses, see Routley (1980a), especially 6.4. Contrary to Wittgenstein (1961),

*3.02 There are no limits in what can be thought. What is impossible is thinkable also.

*3.031 We can say what an illogical world would look like.

The principles of logic are not restricted to, and do not bound, the possible. Wittgenstein's has continued a long history of conflation of three different contrasts: possible/impossible logical/illogical, and thinkable/unthinkable. The results assembled in this paper also augment the case against Wittgenstein's 6.5, already countered by unsolvable problems. For where p is an unknowable truth, p ? can be asked but not definitively answered.

is required to transfer such unknowability to unbelievability. However R1 is false, as may be shown independently of freedom principles, by a three-worlds countermodel. The reason for the failure of R1 is easily seen in outline: BA is rather independent of the truth of A (that is what allows freedom), and so can be represented (roughly) by B , but the principle $\Diamond B \& A < \Diamond(B \& A)$ is, and should be, rejected (for take A as $\sim B$).

Furthermore, the theorem does not hold for belief and assumption, since not just A1 but its weakening A1* fails²¹ (in the case of assumption A2 is also in doubt). Thus requisite freedom principles—for assumption, belief, thought, reason, and so on—emerge, so it appears, unscathed, thus disposing of my main reason for being tempted by the likes of A0.

4 Discerning necessary limitations, in two ways

A0 is tempting for *another* reason, an historically more important reason, because of the unlimited vista of knowledge it offers, because the modern claim of a method requires it, because of the (illusory) prospect of a universal science. Given, it will generally be conceded, there are serious limitations, perhaps of a contingent character, on what we do know, there are no limits on what we might or could know. But what the corollary shows is there are similar limitations, again *perhaps* contingent character, on what we might know. In this respect the argument does not so far establish a necessary limitation on knowledge, but only a *conditionally* necessary one, that in certain correct systems (strengthening KS0.5 without A3), $(Pr)(r \& \sim Kr) < (Pq)(q \& \sim \Diamond Kq)$. If $\Box(Pr)(r \& \sim Kr)$, then $\Box(Pq)(q \& \sim \Diamond Kq)$, but if $\nabla(Pr)(r \& \sim Kr)$, i.e. it is contingent that something is not known, then the truth of $(Pq)(q \& \sim \Diamond Kq)$ may also be contingent, so far as information supplied goes.

For *necessary* limits to knowledge, for absolute limits, further argument is required. Can such argument be supplied? It can, I'm *inclined* to say, *given* that the necessity modality is a capacity one.²² The argument is in essence the following collection of platitudes: The universe is vast; knowledge-acquiring creatures inevitably constitute only a small and bounded part of it; so inevitably the knowledge they can pool is partial, is always incomplete. Before enlarging on the argument, it is important however to dispose of counter considerations based on the oft-claimed possibility of omniscience,²³

²¹ It may be argued that A1* holds, i.e. $Bq < \sim B \sim Bq$, whatever is believed it is doxastically possible that it is believed. For the narrower technical notion of belief that satisfies this principle, the full freedom of belief principle fails, but that is not the ordinary conception. The general reason for the failure of iterated belief principles like A1* (with B) are presented in Routley (1980a, 8.12). More specifically, counterexamples to $B \sim Bq < \sim Bq$ may be designed.

²² For alethic (i.e. logical) modalities the issue is different. On the face of it, we can simply select a possible world where everything true is known (the world will indeed be consistent and deductively closed, and if not maximal can be represented by a set of maximal extensions; but it will in general be neither finite nor decidable). And such a world is suitably modally accessible, since logical modalities have an S5 structure. So it is, it certainly seems, *logically* possible that for every p , p is known.

²³ It is not as obvious as it is commonly taken to be philosophically that omniscience is possible. Much as there are “paradoxes” of omnipotence (e.g. a creature unleashes a power it cannot contain or control, ties a knot it cannot untie), so there appear to be “paradoxes” of omniscience; for instance, a wise being discovers that there are truths that it doesn't, and can't, know. A truly wise creature will, like Socrates, realise that it knows that there are things it does not know; so a truly wise creature cannot be omniscient.

namely that for some x , e.g. God, $\Diamond(r)(r \supset K_x r)$, contradicting $\Box(Pr)(r \& \sim K_x r)$, and so, if ‘known’ is spelt out to include x , $\Box(Pr)(r \& \sim Kr)$. A critical question is how ‘It is known that’ is elaborated (or, put differently, what are the appropriate truth conditions). As ordinarily deployed, ‘ K ’ certainly seems to exclude God and other omniscient objects (e.g. we don’t say that it’s known how to perfect a long distance electric car because God knows how to), but to be nonetheless rather fuzzy at the boundaries, e.g. there is often a margin of localism in its application. When, after all, is something known? When *some people hereabouts* know that it is so? Consider, for instance, whether we say it is known that p when the fox knows that p (e.g. that there is a rabbit in the burrow) but no human does, when an Ituri pigmy knows that p but no urbanite does. There is no need here to attempt to resolve such indeterminacy in *which* knowers serve to make something known. For an initial (uncontroversial) *outer* bound, which will serve for the argument, is had by restricting the class of creatures that can render something known to some (past or present) time existent creatures, roughly that is to creatures that have at some time lived (in this part of this world) or do now (so) live. Then, over any finite stretch of time—which is all that is relevant, given the outer bound, to settling whether something is known—only a finite set of finite creatures are involved. In short, only finitely many knowers (likewise finitely many observers, verifiers, etc.) can be enlisted to try and exhaust knowledge. This, it seems certain, they cannot achieve. For there are going to be thousands upon thousands of things about the world that no such creatures are in fact going to know. Why not? The argument turns on the range of things there are to know, and the limited capacities and discriminating powers of finite collections of finite creatures.²⁴ As to the first, there are thousands and thousands of things in almost any surroundings which (though they have a truth value) are not known.²⁵ Consider, to illustrate, the wealth of *how many* questions, which attract no known answer. For example: How many molecules are there in the wooden beam? How many blades of grass in that lawn? Worms under the lawn? Leeches in the adjacent forest? Etc. The answers, my friends, are not blowing in the epistemic wind. Despite the best efforts of many humans to put humans everywhere, there aren’t enough finite knowers (and, in a good sense, couldn’t be enough)

Footnote 23 continued

A neat argument, due to C. B. Daniels, that no creature (including God) is omniscient, goes as follows: the argument deploys self-reference but is perfectly acceptable (at least paraconsistently). Let x be any allegedly omniscient creature, i.e. anyone who knows every statement that is true, and consider the statement written in the box below.

The statement written in this box is not known to be true by anyone.

Then (1) the statement concerned, t_0 say, is true; and (2) x does not know t_0 . Hence x is not omniscient. As to (1), suppose t_0 is not true. Then the statement in the box is known to be true by someone; so t_0 is true. Therefore in any case (whether t_0 is true or not), t_0 is true. As to (2), since t_0 is true, no one, including x , knows t_0 to be true.

²⁴ Here it may seem that, despite earlier pronouncements, the way is weaving back towards Kant. It is not. Nothing depends on humans, on their features, faculties, categories, etc.

²⁵ It’s not said, of most of those things, that they couldn’t be known. In *each* case they mostly could be, but not in *every* case. There is a crucial difference between: “In each case it is possibly known that p ” and “In all cases it is possibly known that p ”, a difference between distributive and cumulative quantifiers that can be represented, in simple cases such as the present, as the difference between $(c)\Diamond Kp(c)$ and $\Diamond K(c)p(c)$.

to monitor every terrestrial instant and give an answer to all the questions that arise. This is not to consider extraterrestrial issues, or times before life began, details of which are largely beyond the compass of what will ever be known. The other feature relied upon in this sketch of an argument is the finite character of finite creatures, their limited capacity to acquire information, to discern and digest details of their surroundings and of unoccupied surroundings, etc. Is this merely a contingent feature of living creatures, or can it be made analytic on, e.g. being human, being a dolphin, being a creature. Seemingly it can, and given it can, not only will there always be truths that are unknown, but further there are, as Fukuoka observed, *bound* to be some truths that are unknown.

The argument sketched can be reinforced by limitations differently arrived at, from variations upon semantic paradoxes. Suppose, to introduce matters, *a* is a creature so contrived that the only truths of mathematics it can know are those that are proved systemically. Then, classically, by the limitative theorems, there are, as a matter of logic, truths *a* does not know. So $\Box(Pr)(r \ \& \ \sim K_a r)$. Similar necessary, and not merely contingent, limitations apply to other sorts of creatures and items, certainly to those items whose joint abilities or epistemic means are effectively restricted, in the way that automata are and humans and other living creatures seem to be. Objects of these sorts at least will be subject to epistemic limitative results.

The Finsler–Gödel argument for undecidable propositions depends upon locating, characteristically by diagonalisation, a statement which “says” of itself that it is unprovable (provability replacing truth or analogues in semantic paradox arguments). Analogous epistemic limitative results turn upon statements which say of themselves, in one way or another, that they are unknowable (or *alternatively* not known), for example, most simply upon determination of a *p* such that

$$F1: p \leftrightarrow \sim \Diamond K p.$$

Such statements can easily be found, if self-reference is not unjustly outlawed. Consider, for instance, the statement

F2: This very statement (F2) is not knowable.²⁶

Then since F2 is the statement that F2 is not knowable, F2 satisfies F1. Now any *p* conforming to F1 is true but unknowable. The argument requires, apart from elementary logic, but one further correct assumption, namely, for every *p*,

²⁶ The alternative,

F2*: This very statement (F2*) is not known, forces paraconsistency (and the use of $\rightarrow$ in place of $\supset$ and $\prec$) in much the way that the semantic paradox,

F: This very statement (F) is not true,

does (as P. F. Hanson in effect pointed out). For, as in the text,

- i. It is proved that $F2^* \& \sim K F2^*$, and so
- ii. $F2^* \& \sim K F2^*$. Hence, by the main argument,
- iii. $\sim \Diamond K (F2^* \& \sim K F2^*)$.

But since what is proved is known and so possibly known (at least on usual assumptions), by i,

- iv. $\Diamond K (F2^* \& \sim K F2^*)$,
- contradicting iii. Similarly, in the case of F, it is provable that
- v. $F \& \sim F$.

What the hardened paraconsistentist says is that F and $\Diamond K (F2^* \& \sim K F2^*)$, though inconsistent, are nonetheless coherent, that this is how things are: some (but not too many) inconsistencies hold true.

F3: $\sim p \rightarrow \sim \Diamond K p$. i.e. false statements are not knowable.²⁷

The alternative parallel argument to unknown truths simply appeals to A1($\rightarrow$) again. Let p_ϵ be an arbitrary p satisfying F1. Then²⁸

1. p_ϵ is true. If p_ϵ is true then it is true. If $\sim p_\epsilon$ is true then, by F1, $\sim \sim \Diamond K p_\epsilon$, so $\Diamond K p_\epsilon$, whence, by F3 and contraposition, p_ϵ is true. Hence in any case, p_ϵ is true (by $(p \rightarrow r) \& (\sim p \rightarrow r) \therefore r$).
2. p_ϵ is not knowable (by 1 and the other half of F1).

The argument and the alternative argument supply truths that are unknowable and unknown as a matter of logic, and so *both* (the alternative using the main argument) furnish truths that *cannot* be known.

Unlike the main argument, this subsidiary argument is under suspicion because of its affinity to the semantic paradoxes. Can the argument be “cleaned up” in the way Gödel sanitised the argument to undecidable propositions? Under certain conditions it looks as if it can, namely where the means of acquiring knowledge can be effectively specified and represented. Consider, for instance, extended empiricism (of a Lockean or Kantian cast) where all knowledge is obtained either (i) by initial sense perception, yielding a finite set of axioms of knowledge or (ii) by effective processing from (reflection upon) what derives from (i), i.e. by effective rules for the further generation of knowledge. Obviously, (i) and (ii) mirror the axioms and rules of logics which (when suitably classically or intuitionistically based) succumb to Gödel’s theorems. Since knowledge includes arithmetic, generated (somehow) through (ii), Gödel’s detailed arguments can, in principle at any rate, be mimicked to yield unknowable truths. Similarly any philosophical position which, like empiricism, duly confines epistemic means to effective methods, is subject to serious epistemic limitations: the knowledge the position admits cannot exhaust the truth, but is necessarily incomplete. Perhaps, in truths that cannot be known lies “the mystery at the heart of things”.

5 Applying the argument against verificationism and against other philosophical positions

A0 has also been found widely tempting because it emerges straightforwardly on verificationist, and idealist, positions—positions that are not nearly as divorced as is often supposed. In fact, Hart has argued that A0 is “a weak consequence of verificationism and a weak thesis of idealism” and so, antilogising, used the fact of unknowable truths to refute verificationism—if it needed further refutation.²⁹ His argument is the following sorites:

²⁷ Again the capacity interpretation of the modality $\Diamond$ is important. For under an alethic interpretation of $\Diamond$, it is possible that contingent falsehoods are true and so, in certain cases logically possible that they are known.

²⁸ The argument resembles the preliminary account of Gödel’s theorem in Kleene (1962, p. 205).

²⁹ Hart (1979, p. 156). That verificationism *seems* to require further refutation is evident from the arrival of such “new verificationists” as Dummett and Putnam.

H1: What is meaningful is verifiable, since according to verificationism, what is not verifiable is nonsignificant.

H2: What is true is meaningful.

H3: What is verifiable can be known.

Therefore, A0, what is true is knowable.

But A0 is false, H2 and H3 are true, and hence (classically) H1 is false. Unfortunately for Hart's "knockdown" argument, H1 is not true (nor is the derived assumption, what is meaningful can be known). $E = NR$ is meaningful but logically false and so not verifiable, and, excluding noncontingent assertions, it is at least doubtful that what is false is verifiable. H1 should be corrected to

H1': What is meaningful is verifiable or falsifiable,

since it follows from the verification principle, according to Russell (1956, p. 275), "that what cannot be verified or falsified is meaningless". But this amendment destroys the argument given, since the analogue of H3, that what is falsifiable can be known, is false.

The argument can be reconstructed readily enough, however, and in a telling general form, using

H4[G]: What is true is verifiable [ψ able].

Hence, by H3 or H3G, A0 follows. Moreover, H4 follows from emendations of H2 and H1; namely what is true is meaningful and true, and what is meaningful and true is verifiable.

In fact H4 can be refuted directly without appeal to H3. For an assertion is verifiable iff it can be verified (for an appropriate 'can'), and the main theorem can simply be applied directly with K interpreted as 'It is verified that'. It suffices to show that K satisfies requisite conditions when so interpreted (it is then tantamount, near enough, to von Wright's functor V). If does (as von Wright, 1951, explains). What is verified is true, and if a conjunction is verified each conjunct is. Certainly also, some statements are true but not verified. Hence, *there are unverifiable truths*, contradicting H4.³⁰

A similar argument, showing that there are un ψ able truths, works for any functor ψ satisfying the requisite conditions A1 or A1*, A2 and A3 (for ψ). There are several such functors, some bizarre, some important; as well as some notable exceptions, both important and bizarre. Examples include:

- (i) *Constructible, mathematically constructible*, etc.: As a corollary to the theorem, intuitionism is false. For at any time there are truths, of intuitionistic mathematics, that are not yet mathematically (intuitionistically) constructed. This refutation of mathematical intuitionism, which only adds a short proof to the substantial brief against theses of the position, is however an outsider's refutation. Insiders will reject A3, which deploys a nonintuitionistic quantifier, that,

³⁰ This tells equally against "new verificationism" as against older verificationism. For such "new" people as Dummett and Putnam apparently want to say that an (empirical) statement already has a truth-value, and so may be true, even if its (well-defined and executable) confirmation procedure has not been carried out; i.e. there are true statements which are not (constructively) verified. And V satisfies the requisite conditions even on constructive readings.

insiders will (have to) claim, does not make intuitionistic sense. On a constructive interpretation of quantifier P , A3 is false (as we have seen). However A3 makes perfectly good (substitutional) sense understood nonconstructively.³¹

- (ii) *Provable*: There are truths that are not provable. A much more sophisticated version of this sort of result (not without substantial, and rejectable, classical assumptions) is given by limitative theorems. The result makes its modest contribution to undermining the (already much savaged) assumption that mathematical truth can be equated with provability.
- (iii) *Confirmable*: There are unconfirmable truths, i.e. there are true statements that cannot be confirmed. Showing that the requisite conditions are met involved a new turn; for A1 fails, Cp does not guarantee p (with C read ‘It is confirmed that’). However $A1^*$, $C \sim Cp \prec \sim Cp$ (that it is confirmed that it is not confirmed that p strictly implies that it is not confirmed that p), *appears* to hold. A2 and A3 are not in doubt for C .
- (iv) *Probable*: There are truths that are not probable: that is hardly news, and it is more readily illustrated by alternative methods. What is captivating about the example is that we are obliged to invent a new verb, ‘to prob’ say and to investigate some of the properties of the functor ‘It is probed that’ (cf. the derivation of ‘probable’ from ‘probare’: but ‘proved’ has now gone its separate way).
- (v) *Ineffable*: The argument, *if* it succeeded, would show that there are ineffable truths, and so could underwrite a sort of mysticism. Consider ‘ineffable’ in the (OED) dictionary sense as ‘unutterable’ or ‘too great for words’, as quite distinct from ‘unknowable’. what has to be looked at are properties of the functor ‘It is effed ($\approx$ uttered) that’. Then while A3 holds both A2 and A1* appear false. In short, the argument fails with ineffability as it did with believability.
- (vi) *Sustainable*: There are truths that cannot be sustained. Again the main doubt focusses on A1*.
- (vii) *Ascertainable, idealistically ascertainable, empirically ascertainable* etc., etc., for other restrictions on methods of ascertainability: There are for example, truths that are not idealistically ascertainable; hence idealism is inadequate.³²

The latter result is hardly news, but it is obtained in a very elementary fashion. It is an outcome of idealism, as usually understood, that every thing consists, so to say, of ideas. This severely limits idealistic truth-definitions: whether an assertion is true will depend in every case on determinisation through ideas. So truth is circumscribed by

³¹ The question arises as to whether an insider’s proof of non-constructible truths can be found using intuitionistically admissible apparatus, e.g. $\sim(Ur) \sim$ in place of (Pr) . If intuitionism is to stand the answer is, presumably, No. But a serious difficulty for insiders is that the principles required in the second argument, so reconstructed, appear to be intuitionistically valid. In particular, the key distributional principle, $C \& \sim(Ur) \sim A \rightarrow \sim(Ur) \sim(C \& A)$, with r not free in C , is intuitionistically valid.

³² According to Stace (1924, p. 49), presenting both his own and Hegel’s idealist theses, there is nothing that is unknowable, “*there is nothing in the universe which the human mind cannot know*, neither the infinite, nor the Absolute, nor the thing-in-itself”. On the face of it, these theses have been refuted. But in showing that “*any theory of unknowable truth will give rise to the same contradictions*”, Stace (1924, p. 45) appeals to a high redefinition of ‘unknowable’, to *absolute* unknowability. In this high sense, an “unknowable truth” would not be unknowable, since in being true it would be known in some respect and so not “absolutely” unknown!

idealistic ascertainability. But as application of the main theorem reveals, truth is not so circumscribed. Similar points hold as regards empirical ascertainability, and generally, as regards *ascertainability by limited means*. Truth always exceeds what is ascertained by limited means, including epistemic means. There is no (epistemic) method.³³

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³³ *Added in proof*: (1) The argument of the first paragraph can be made lighter by drawing upon results from cybernetics: specifically, using the law of requisite variety, and the much greater variety of the world than the creatures in it have the capacity to handle. (2) The neat argument of footnote 23 is anticipated (in main outline) in earlier work of B. Mates.