On The Algebraic Representation of Causation

Giving Mathematical Form to Philosophical Decisions

Part I

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Part I

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Roy Lisker

I. Introduction

It is always the case when a causal framework is introduced into the descriptions of the natural sciences that a philosophical analysis,
(sometimes a decision on one side or the other of a traditional philosophical problem) , is being translated into a mathematical representation. Ideally this will be quantitative, although this is not always necessary. For example, the teleological arguments coming from the 2-fold Darwinian mechanisms of adaptation and natural selection - whenever it is argued that certain structures are

present in a living organism *because* they enhance (or once did enhance) survival - are not strictly quantitative. In general however, these suggest a formal dialectic which can readily be given mathematical form. Examples are the morphological transformation schemes of d'Arcy Thompson, the epigenetic landscapes of C.H. Waddington, and the modeling methods of Catastrophe Theory,

The philosophical analysis clarifies the causal concept , the philosophical decision selects among various hypotheses and classical ideological positions, while the mathematical representation translates the conceptual foundations into symbolic, functional or schematic forms . These may now take on a life of their own, with no further appeal to their philosophical origins.

One of the real strengths of mathematics is that it is free of any attachment to its motivating concerns. The Theory of Probability originated in the desire to predict the outcomes of card games and other types of gambling; yet today's probabilists are not obliged to take an interest in gambling , or even know how to play cards.

In the same way, the most rudimentary causal schemes are based on a primitive "before and after" relationship. These lead in a natural fashion to the study of totally and partially ordered sets: lattices, ordinals and order types. These can be studied quite apart from any application to causal structures, or may even suggest other possibilities, (such as those present in J.L. Borges famous story : "The Garden of the Forking Paths") , for causation that may eventually be applicable to natural phenomena.

The paradigm [1] for all such procedures for the translation of causal notions into mathematics is the Quantum Theory. Only a partial list can be given of all of the mathematical structures which have emerged from the fundamental insight that energy is emitted in discrete particles or quanta:

(1) Heisenberg's Matrices
(2) Schrödinger's Wave Equation
(3) Dirac's Bra-Ket Formalism
(4) von Neumann's Operator Algebras
(5) Feynman's Amplitudes; Diagrams; Integral
(6) Jauch-Piron Propositional Lattices
(7) Bohm's Hidden Variables
(8) Irving Segal's Jordan Algebras
(9) Mackey's Measure Spaces
(10) Reichenbach's 3-valued logic
(11) Popper's Scatter Ensembles
(12) Gudder's Non-Standard Arithmetics

In this paper we will spend some time examining the first 4 of these, with occasional references to the others. Yet even in those sciences which are not readily quantifiable: Psychology, Economics, Sociology, Anthropology, Journalism, the so-called "human sciences" , one can trace the development of this process

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[1] Thomas Kuhn has forever altered the meaning of the word "paradigm". In our usage however we adhere to the older, more traditional uses of the word as defined in the following dictionary references:

(1) paradigm...pattern, example...serving as a pattern or example
-The Barnhart Dictionary of Etymology H.W. Wilson 1988

(2) paradigm... An example; a pattern followed; a typical example; an epitome... paradigm case : a case or instance to be regarded as representative or typical.
-The New Shorter Oxford Dictionary, Vol.2 , Clarendon Press 1993)

It is evident for example in Geology, which possesses a conflict as inherent to its character as a Kantian antinomy , between Uniformitarianism and Catastrophism. It is present in the stimulating controversies of Biology, those surrounding the mechanisms of Evolution, those challenging the authenticity of the fossil record (Paleontology) , those which debate the relative roles of Nature vs. Nurture . Cosmology has its exotic mix of great explosions , inflationary scenarios , a bushelful of solution spaces for Einstein's field equations , and homogeneity and isotropy principles.

In the writing of History one uncovers every species of methodological dilemma: observer/ observed , determinism vs. free will , teleology vs. chaos , gradualist/catastrophist synchronic vs. diachronic perspectives , objectivity/ subjectivity, community versus the individual versus the idea as historical determinants . Indeed, considered as a science, history is so forbiddingly difficult that one imagines that only the most disciplined and gifted scholars would dare to enter the field; which is why one is always making the sad discovery that most historians are little more than mediocre jingoists, vaunting monarchy, war, "great man" theories , "imperial glory" delusions , tabloid psychologism and so on. [2] **Yet good or bad, a historian must be first and foremost practice competent philosophizing on the mechanisms of historical causation. All the same , unless he is a Marxist or at least some kind of Hegelian, the translation of his framework of historical causation into logical , algebraic or functional schemes, cannot easily be envisaged .**

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[2] **We hasten to exclude from this demonization such names as Herodotus, Thucydides , Ibn Khaldun , Fernand Braudel, Simon Schama , H. R.Trevor-Roper , Burchart, Arthur Symons, and many others.**

For most of the human sciences mathematical representations of causality cannot advance beyond a surface level, even though the schemes that do emerge may be very complex: Keynesian economics, game theory, glottochronology, the lattice structure of the hominid tree, the kinship structures of organized societies around the world

By making a close examination of the traditional interpretations of causality in different scientific fields one quickly discovers that ***many possibilities for mathematical modeling of causal schemes, consonant with standard philosophical preferences , are often overlooked*** **. An even more controversial finding** ***is that the ideal of causality which is incorporated into the conceptual framework of a certain science may not correspond to the assumptions about causality actually employed in the daily practice of that science*** **.**

A fundamental idea underlies everything in this essay : a certain kind of philosophical perspective on causation leads to a certain kind of mathematics. Different philosophical positions lead to different mathematics. In every philosophical system for causation that is rich enough to serve as the foundation for some natural science one generally finds an abundance of equivalent mathematical representations: quantum theory being , once again, the paradigm . In specific practical applications these may reduce to algorithms for predicting the future from the present. Thus , in the elucidation of causal structure, mathematics plays an intermediary role between philosophy and observation. Most significantly it rarely, if in fact it ever does, arise from the uninterpreted data of observation.

II. Causal Function Algebras

The construction of a Causal Algebra of functions , algorithms and/or operators is the key to any mathematical representation of causal principles. This is a set or space [3] of entities which, through their permitted modes of combination relative to the dimension of time, model the causal structure.

Causation necessarily takes place in time. However the topological character of the time dimension, (infinite in one or both directions, open or closed, circular, branching, etc.) need not be specified in advance but may be derivable indirectly from the nature of the entities in the causal algebra.

Other independent magnitudes entering into calculations of the forms and functions of the causal algebra will be the vectorial quantities that define the atemporal representation spaces for the phenomena under consideration. Their ranges may be anything: real or complex numbers, other functions, indeterminates, etc., even ideas such as "wave", or "particle". An example of the latter is found in the following metaphorical algebraic construction:

Let O stands for " 1-slit apparatus"
T " " "2-slit apparatus
E " " " Electron "
P " " " Particle "
W " " " Wave"

Then we have the 'equations' :

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[3] In our terminology:

Set : Any entity constructible through and consistent with the Zermelo-Fraenkel Axioms of Set Theory.

Space : A collection of indeterminate entities that becomes a set through the specification of n of them, where n is finite or countable. Thus, Euclidean 2- space becomes a set when any three of its points (say $\boxed{p_1 = (0,0), p_2 = (0,1)\ and\ p_3 = (1,0)}$ **. The purpose of the 3rd point is to establish the direction of positive orientation.**

O (E) = P

T(E) = W

The "space" for this Causal Algebra is *The Space of Apparatuses* for determining the particulate or wave nature of elementary particles. Its "domain" could be the collection of all electrons in the universe, whereas its "range" consists of the two terms "particle" and "wave". Thus we see that a Causal Algebra need not be expressed in terms of operators or functions ultimately acting on numbers. In this example, the temporal dimension enters into the various ways in which One- and Two-Slit Apparatuses , (and presumably others) can be combined in parallel and series, to produce outputs of "particle" or "wave" or combinations of these with differing probabilities, on ensembles of particles.

Causal Algebras have both an Extrinsic and an Intrinsic Structure.

(a) The Extrinsic Structure:

Any Causal Algebra , C will, in general, contain functions , f,g,..., function spaces F, G,....operators H , and Operator Spaces H . For example, in the full elaboration of the Quantum Theory, one has :

- Schrödinger Wave Functions , { ψ_α }

- Complete families of Orthogonal Functions , { Ω_α }

- Observable Operators : Energy, Momentum, etc.

- Families of Unitary Linear Operators : scalar, vector, tensor, differential forms, integral transformations, von Neumann Algebras, Jordan Algebras, etc.

All of these things will be covered by the generic term "agent", since it is their common attribute that they all act on

something beyond themselves, a space, a geometry, real or complex numbers, etc.

The extrinsic structure of a Causal Algebra , $\mathbf{C}$, is the set of rules by which agents may be put together in combination to produce other agents. Thus, let $\mathbf{U}$ be the class of hypermaximal self-adjoint operators acting on the closed Hilbert Space $\mathbf{H}$. Then if K and L are operators in $\mathbf{U}$, then K+L will also be in $\mathbf{U}$, but KL will only be in $\mathbf{U}$ if they commute. Likewise, if $\mathbf{O}$ is the class of invertible matrices, and A, B, ε $\mathbf{O}$, then AB and BA will be in $\mathbf{O}$, but A+B may not be .

The extrinsic structure of a causal algebra therefore consists of the complete set of combination rules under which its set $\mathbf{S}$ of agents is closed. which are closed in $\mathbf{S}$. This definition naturally extends the standard notion of a function algebra , closed under multiplication, addition and subtraction, multiplication by scalars and functional composition.

If $\mathbf{C}$ is to be a appropriate model for some conception of causation, ideally every composition rule will reflect some feature of that conception.

Examples:

[A] : Any space E of functions modeling "entropy" must respect the 2ND Law of Thermodynamics : The absolute quantity of entropy in an isolated system always increases. E then must be a subclass of the space $\mathbf{M}$ of all monotonically increasing functions of time, which is closed under addition, multiplication by a positive number, composition and time translation. Neither E nor $\mathbf{M}$ are not closed under subtraction:

$$\boxed{\neg(\forall f, g \in E \Rightarrow f - g \in E)}$$

This statement does in fact reflect an important property of thermal systems: heat does not move from a cold to a warmer body.

[B] : Let T be the class of all 'motions' of a particle p, along a single spatial dimension, x, as a function of time, t. The Light Principle of Special Relativity states that energy cannot be transported across space at a speed exceeding c, the speed of light. The subclass T of T which models the Light Principle for material objects consists of functions x = f(t) such that $\boxed{|f'(x)| < c}$ **for every value of t. This class can be enlarged to include functions for which**

$\boxed{f'_+(t) \neq f'_-(t)}$ **at certain 'jump points ' of the derivative. To be precise we define T as the class of functions such that Max|f '(x)|=m < c , and** $\boxed{\bar{T}}$ **as the boundary of T : this includes those functions which equal c at some points, or may converge to ± c as t goes to + or - infinity.**

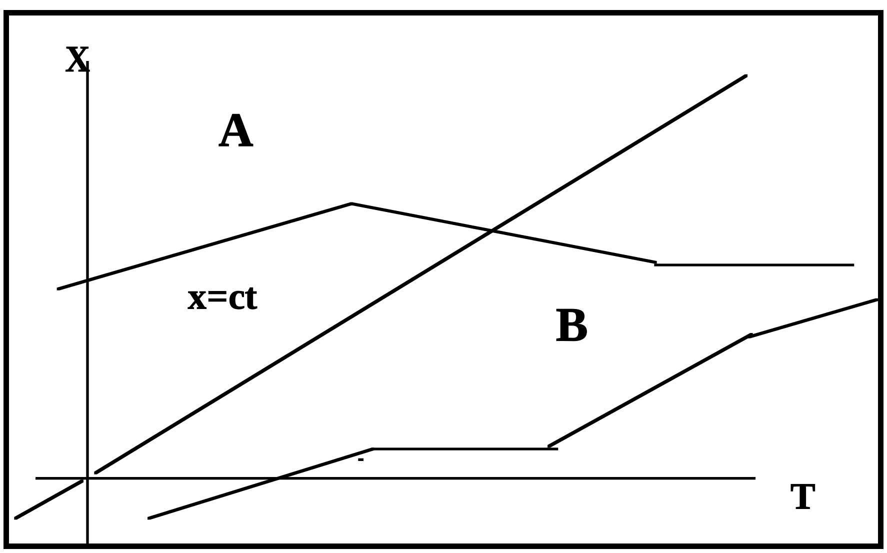

Figure 1

(Functional arcs in half-plane A never cross the light cone)

Note that, although jumps of the *derivative* are allowed, spontaneous jumps *along the trajectory of f* are forbidden; also that non-right or left-differentiable functions -at any point! - are also forbidden.

What is the extrinsic structure of this class of functions? If f,g ε T , and x = h(A, B ; t)) = Af(t) + Bg(t) , then

x' = Af'(t) + Bg'(t)

| Af'(t) + Bg'(t) | ≤ |A| |f'(t)| + |B||g'(t)| ≤ c (|A| + |B|) .

Therefore if A| + |B| ≤ 1 , then h (A,B ; t) will always be in T .

This is an example of an extrinsic law, but it is not 'natural' to Relativity.

In general h ε T if $Max\left|Af'(t)+Bg'(t)\right| = m < c$ **Here we are ignoring the subtleties of jumps in the derivative, but this extension is easily made. Differentiating, we find that the critical points of this expression are at:**

$$\begin{aligned}
&Af''(t)+Bg''(t)=0,\ or\\
&A/B = -\frac{g''(t)}{f''(t)} = a(t)\ ,that\ is\\
&Ba(t)=A
\end{aligned}$$

There is now some intrigue to this problem, because we must solve the equation for "a" , for every combination of values A and B, and single out those for which the expression for the first derivative is bounded away from c. However, since

$$\begin{aligned}
&Max\left|Af'(t)+Bg'(t)\right| = m < c \Rightarrow\\
&Max\left|kAf'(t)+kBg'(t)\right| = km < kc
\end{aligned}$$

we know that for any pair of values (A,B) which satisfy this condition, the entire segment of values (kA, kB) |k|≤ 1 also satisfy that condition. So far we have been considering *mathematical composition laws* for the spaces T and $\bar{T}$ **.**

However, ***relativistic motions do not compose under these relations but under the hyperbolic tangent law*** **:**

$$(+) \quad h' = \frac{f' + g'}{1 - f'g'/c^2}$$

Under the normal interpretation of relativistic composition of velocities, the previous relations have no physical meaning. What other composition laws are there in this space which have a meaningful interpretation in Special Relativity? Consider the motion $x = m\cos t$ **, where m < c. The derivative of this is** $x' = -m\sin t$ **, which has a maximum of m . This can be interpreted as a relativistic "clock" that 'ticks' at each reversal of direction along the x-axis, at the times t = 0 , ±2nπ . If we modify the first equation to** $x = mA\cos(t/A)$ **, then the system will still change direction at the maximum value m in each direction, but its period will be reduced to P = 2π/A . Since these are permissible relativistic motions, this fact has the following interpretation:** ***There is nothing in the theory of Special Relativity which prohibits the construction of clocks of any period*** **.**

One is thus led to make a distinction between two kinds of composition laws for the motions of T ($\bar{T}$ **)**

I . Mathematical Composition Laws, those suggested by the symmetries inherent to the function space. An example is:

$$(m) \quad |A| + |B| = 1, and\ f, g\varepsilon T \Rightarrow$$
$$h = Af + Bg\varepsilon T$$

II. Relativistic Composition Laws such as

$$(+) \quad f, g\varepsilon T \Rightarrow h\varepsilon T, where$$
$$h' = \frac{f' + g'}{1 - f'g'/c^2}$$ **, and**

$$(p) \quad m < c, A \neq 0 \Rightarrow x = mA\cos(t/A)\varepsilon T$$

Hence, T *, as a Causal Algebra for Relativity* , contains (+) and (p) in its extrinsic structure, whereas (m) is not in the extrinsic structure.

There is a related Causal Algebra in which (m) does fit , which we may label $\hat{T} \subset T$. $\hat{T}$ bears the roughly the same relationship to T that a Lie Algebra bears to a Lie group. Let κ be a constant with the property that the relativistic effects on objects moving at speeds κc or less are *undetectable* . We then define $\hat{T}$ as the subset of T consisting of motions f , such that m = Max|f'| ≤ κc/2 . In this subspace the margin of error between relativistic addition (+) and ordinary addition (m) is undetectable. Observe that, although addition is a valid extrinsic composition law for $\hat{T}$, it is not closed under addition.

(b) The Intrinsic Structure

What really distinguishes one scientific discipline from all others, apart from its subject matter, (which may overlap or sometimes even be identical) is its catalogue of representation spaces. Let us look at some standard examples : Statistical Mechanics studies the behavior of ensembles of systems in Phase Space ; Special Relativity is enacted in Minkowski Space-Time; General Relativity in Riemannian 4-manifolds with inertial index -2; Quantum Mechanics lives in Position Space, Momentum Space , Hilbert Spaces, and spaces of operator algebras, or Banach Spaces ; String Theories employ very elaborate and abstract representation spaces; Lagrangian Mechanics operates over isolated systems in Configuration Space , while Hamiltonian Mechanics operates on single systems in Phase Space, or on bundles formed from the same system translated along a continuous spectrum of initial conditions .

The representation spaces for Biology are likewise interpenetrated with graphs and lattices: hereditary trees, Aristotelian and Linnaen classification schemes, graph structures for chemical reactions , genetic codes ....

This characterization is no less true of the Human Sciences than of the Natural. Scientific theories do address themselves directly to observations or raw data, but to the pictures we build upon these observations and data . Such pictures are sketched , modeled, elaborated, then displayed in a "plenum " whose nature is purely conceptual , which in its ultimate refinement is mathematical. It is from these images that theories arise which, when applied to them, generate predictions which are *tested against* further observations of natural phenomena.

Consider a science such as psycho-analysis, which many, (myself included), prefer to classify as a pseudo-science. Sigmund Freud's many representation schemes are crude to the point of embarrassment . All of his models are rife with internal inconsistencies , while being in contradiction with one another.

Yet any one of them , whether it be the libido theory, the instinct theory , the mechanism of repression, the Oedipus Complex, can be reduced without much difficulty to a collection of schematic diagrams like engineer's blueprints, similar to those that depict the workings of a steam engine, a hydraulic pump or a storage battery, some sort of mechanical system.

In the repression theory for example, the Unconscious is a "place" , into which "painful thoughts" are "pushed", or "buried", and kept there by a kind of "censor" that looks remarkably like a steam engine's safety valve. In the schematology of the Oedipus Complex, illicit sexual desire is "balanced" by the fear of patriarchal punishment. In this picture all the emotions,

love, jealousy, fear, hatred, guilt, anxiety are ingeniously coupled in a dynamic though stable equilibrium like the ropes, cams, gears and pulleys of an elaborate waterwheel.

No science ever makes predictions other than through appeal to the abstract model in its appropriate representation space. Predictions are made *on* the model , *then* tested through interaction with external reality.

The Intrinsic Structure of a Causal Algebra therefore consists of the set of predictive mechanisms , constructed from agents in permissible combinations, acting in the representation space to produce , from present data , hypothetical models in other locations of space-time which can then be tested against experience or experiment.

Using quantum theory as an example, its agents are Operators acting over Hilbert Spaces and functions acting over Configuration Spaces (position or momentum) .

The *Extrinsic Causal Structure of Quantum Theory* holds the rules for composing operators and families of operators . For Spin Operators these would be the properties of the tensor product.

The *Intrinsic Causal Structure of Quantum Theory* consists of the rules and algorithm whereby one calculates eigenvalues and probabilities. Thus $\boxed{< E >= \int \phi^* |E| \phi dx^3}$ **would be in the Intrinsic Structure.**

In the case of Special Relativity, the extrinsic structure contains the Lorentz group; the intrinsic structure applies the functions of this group to the calculation of length contractions, time dilations, and mass increases in Minkowski Space.

Even historiography can be shown to exhibit representation spaces , agents, an extrinsic structure and an intrinsic structure.

Since interpretation is an inherent part of this science, these will differ from one historian to the next. No Hegelian would dispute this. However all historians relate their epics within a framework of assumptions about the 'motors' or 'forces' that have shaped the world. They also avail themselves of various theories about the way human nature works; these can on occasion be quite involuted. Still, there never was a historian for whom it was possible to describe the course of events in past or present society without speculation on human motives.

Our focus in this essay is not on the subject matter of the sciences per se, but on their underlying structure of causation. All of science depends ultimately on prediction. Even mathematics is restricted by the possibility that errors may turn up at some point in demonstrations or initial assumptions. Back-reconstructive sciences such as geology depend on the uncovering of new evidences to decide between conflicting models of the past. Journalism, in the best sense the historiography of the present, may be either invalidated or confirmed by new evidence .

It is therefore the case that, for every science, one can always derive, from their formal description, the structure of one or more causal algebras , $\mathbf{C} = (\mathbf{A}, \mathbf{E}, \mathbf{I}, \mathbf{R})$, where

$\mathbf{A}$ is the class of agents (functions, operators, etc.)

$\mathbf{E}$ is the extrinsic structure

$\mathbf{I}$ is the intrinsic structure

$\mathbf{R}$ is the class of representation spaces

One does not find the representations of the causal concepts in the class of agents or their representation spaces, which are incorporated in the extrinsic structure (rules by which the agents

combine among themselves), and the intrinsic structure (actions of the agents on the representation spaces generating predictions) .

What is meant by extrinsic versus intrinsic causation can best be seen through specific examples , to which we now turn:

Example I

"Every effect is also a cause."

The statement "Every effect is also a cause" has numerous interpretations. Corresponding to each is an appropriate causal algebra. The Kantian perspective [4] notwithstanding , the concept contained in this assertion is neither self-evident nor a *synthetic apriori* . In those sciences for which teleological descriptions are the standard , it may be either invalid or irrelevant. With the general acknowledgment of Darwin's 2-stage evolutionary mechanism the community of biologists inherited a predilection for teleological "explanations" for the presence of even the most anomalous animal body parts. The result has been a bizarre union of a random mechanism (adaptation) , with a determinist tautology (selection) , Not all of them are 'adaptationists', the extreme right wing, one might say, of this position, but many of them arrogate to themselves a peculiar talent for diagnosis, a special ability for discovering *what things were "for "* , essentially a form of Platonism through the back door. Of course if one asks them directly, none of them would ever confess to a belief that objects in the natural world hold the imprint of heavenly intentions.

[4] By the "Kantian perspective" , or "Kantian paradigm", or more generally "Leibniz/Kantian paradigm" we shall mean the interpretation of causation as expressed in Analogies 1 , 2 and 3, (Analogy 2 in particular) , in the *Critique of Pure Reason* .

In the worse cases, tautology blends with Panglossianism in an intimate weave: *Why do creatures have eyes?* To see with of course . *Why do most mammals have tails?* To enhance balance, to flick flies, as sexual ornaments, to extend the chassis of horizontal organization, to wrap around trees when climbing, to express joy or anxiety. Whatever reasons particular biologists may advance for special cases , the Kuhnian paradigm of biology asserts , (with appropriate cautions) *that tails increase the chances for survival ,otherwise animals wouldn't have them* ! Therefore human beings don't have them, because if we did, they would hinder our survival, etc.

The philosophical challenges and dilemmas of these forms of circular reasoning have been exhaustively discussed by many others. One cannot deny the validity of the Darwinian mechanism in small-scale adaptation and change, and most of the arguments center around large-scale historical phenomena.

We will continue our discussion of the causal postulate " *Every effect is also a cause*" , under 4 headings , in conjunction with interpretive mathematical models in current use:

(i) Multi-Agents

(ii) Causal Chains

(iii) Feedback Loops

(iv) Self-generating motions: Vector Fields

Multi-Agents

These are entities which, depending upon their use or context, can serve as operators, functions, vectors or scalars. The distinction between 'cause' and 'effect' thereby becomes blurred, and may even disappear altogether :

Once again it is quantum theory that gives us the richest selection of examples: the "pure state" vector ψ , which is both a *function* in its own right, the *argument* of the set of observable operators , and a *tool* for calculating eigenvalues , can also be interpreted as an *operator* in its own right,
namely the corresponding projection operator $\boxed{P_{|\psi\rangle}}$ The action of this operator on its set of eigen-vectors yields their probabilities:

$$\boxed{|c_i|^2 =< \psi_i | P_{|\psi\rangle} \psi_i >}$$

A rather different example of a multi-agent is the speed of light , c, in Special Relativity . It is first and foremost a scalar, a universal constant of the natural world, which can and has been measured to many places. This makes it an entity in the intrinsic structure.

Yet in the statements of the theory of special relativity, it actually functions as an *indeterminate* , essentially a *letter* expressing the ratio of the universal unit of time with that of length. In this form it has no specific value but rather serves a function, that of expressing the combination rules for the agents of special relativity, in particular the identities of the Lorentz group. . In this capacity it belongs to the extrinsic structure.

In the same way, the number "1" carries the dual function of being the measure of a single unit of some magnitude, together with that of the idea of "identity" in the abstract theory of groups.

Causal Chains

Loosely speaking, a Causal Chain looks like this: An initial cause C_1 , acts on a medium M_1 to produce an effect E_1 . (Example: A potter acts on clay to produce a vase) . By acting on

yet another medium M_2 , E_1 becomes C_2 : the change of context has turned an effect into a cause.
(The vase is taken to market and is the cause of the potter's receiving a good sum of money.) . This in turn produces an effect E_2 (The money received from the sale) . In yet another context M_3 , E_2 becomes a cause C_3 , producing an effect E_3 . (The potter uses the money to dine in restaurants and grows fat) .....
Schematically:

$$\boxed{\begin{array}{llll} \textit{potter} & & & \\ \downarrow \to \textit{vase} & & & \\ \textit{clay} & \downarrow \to \textit{money} & & \\ & \textit{market} & \downarrow \to \textit{obesity...etc.} & \\ & & \textit{food} & \end{array}}$$

The basic unit in this process may be written:

$$\boxed{C_k = \left| \begin{array}{c} E_{k-1} \\ \downarrow \\ M_{k-1} \end{array} \right| \to E_k}$$

The "effect" does not turn into a "cause" until there is a substrate or domain on which it acts in a natural fashion to produce another effect.
Analytically one pictures a class of functions F = { f_α } , acting on a set of domains Δ = { D_α } to produce objects in a set of ranges { R_α } , which are either scalars or functions, depending on the domains to which they are applied.

Let M^k be the space of all mxn rectangular matrices with real or complex entries, such that m , n ≤ k . If A , B e M^k , define an "action" of A onto B by $\boxed{A \uparrow B \equiv BA^T}$, when this matrix multiplication is defined. Symbolically, $\boxed{[m \times n] \uparrow [m \times j] = [j \times n]}$. Therefore , actions are possible only between matrices of equal horizontal length, producing another matrix which serves both as a “scalar” and as another “function”. The “causal chains” described above can therefore be modeled somewhat as follows by sequences of matrices in $\mathbf{M}^{\mathbf{k}}$:

$$\boxed{\begin{array}{l} \underset{C_1}{[m_0 \times m_1]} \uparrow \underset{M_1}{[m_0 \times m_2]} \uparrow \underset{M_2}{[m_1 \times m_3]} \uparrow \underset{M_3}{[m_2 \times m_4]} \ldots \\ E_1 = (C_1 \uparrow M_1) = C_2 \\ E_2 = (C_2 \uparrow M_2) = C_3 \\ \ldots\ldots\ldots\ldots \\ E_k = (C_k \uparrow M_k) = C_{k+1} \end{array}}$$

Every matrix except the first is a “medium”. The accumulated products are the “effects” which, acting on the next medium, becomes a “cause” , etc...

Feedback

In the example of the potter and the vase one can imagine that, after the potter has sold the vase, he uses the money to buy more clay to make another vase. Such a “causal coupling” is usually combined with the increase in absolute value of some magnitude, producing the phenomena of positive and negative feedback.

In a typical scenario in the theory of dynamical systems, there is a fixed function, for example $\boxed{\Phi(z) = \lambda z(1-z)}$. Any of its arguments ω , can be considered to be the “cause” of the value $\omega' = \phi(\omega)$. In this interpretation the complex number ω “acts on” the medium, the fixed function ϕ .

Vector Fields

Let $\boxed{v|_x = \sum_{i=1}^{n} \xi_i \, \partial / \partial x_i}$ be a vector field defined over an n-dimensional manifold M. The field generates a flow $\boxed{\Psi(\varepsilon, x)}$ from any starting point x on the surface. The fundamental properties of this flow are:

$$\boxed{\begin{array}{l} (1)\ \Psi(\varepsilon, \Psi(\delta, x)) = \Psi(\varepsilon + \delta, x) \\ (2) \Psi(0, x) = x \\ (3) \dfrac{d\Psi(\varepsilon, x)}{d\varepsilon|_{\varepsilon = 0}} = v|_x \end{array}}$$

Identifying the parameter ε with time, the above list of properties states that in some fashion, *the flow acts on itself* . If one allows that the increase in elapsed time from any given starting point is an irreversible process , (It is a misnomer to say that “time moves”, any more than “space moves”. However systems “move through space”, and that motion is never instantaneous, but always a ‘flow’ through time.) , it follows that ε must increase, in elapsed time to a quantity ε + δ , which can be interpreted as the action of the flow $\boxed{\Psi(\delta, x)}$ *on* the flow $\boxed{\Psi(\varepsilon, x)}$.

Other interpretations of the statement “Every effect is also a cause” can be advanced, but these suffice for the purposes of illustration.

Example II

Additivity

"Causal additivity" has three aspects, as depicted in the following instances:

(1) Two ounces of water combine with three ounces of water to give five ounces of water. (The algebra of magnitudes)

(2) A 5-pound brick together with another 5-pound brick gives two 5-pound bricks, not one 10-pound brick. (The algebra of sets)

(3) A 220 Hertz pitch P_1 of amplitude A , played simultaneously with a 330 Hertz pitch P_2 of amplitude B, produces a sound in which both pitches maintain their independence, the amplitude of which is a highly non-linear function of A , B , P_1 and P_2 . (Superposition principles)

As we will be making frequent reference to this last example we will examine it in more detail:

The amplitude of a sound wave is not as simple a notion as one has been misled to imagine in generic education. At one extreme, any extremal critical point on the wave shape is an amplitude. The habit of designating the maximum of the absolute value of the sound pressure as "the amplitude" relies on the assumption that the sound wave is periodic, the period being so minuscule that variations in the amplitude can be ignored.

Consider two pure tones:

$$\phi_1 = A\sin\omega_1 t$$
$$\phi_2 = B\sin\omega_2 t$$

These are 'coupled' to produce a wave shape:

$$\psi_1 = A\sin\omega_1 t + B\sin\omega_2 t$$

The amplitudes are at the critical points. Setting the derivative to 0, we find that these are at times

$$\frac{\cos\omega_2 t}{\cos\omega_1 t} = -A\omega_1 / B\omega_2$$

(1) The two frequencies are commensurable. Then the wave shape is periodic. Making the additional assumption that the period is so short enough that the amplitudes at the various critical points can't be distinguished. We let μ stand for a time at which the absolute value of ψ attains its global maximum, C . The calculation is straightforward and one obtains:

$$C^2 = \frac{AB\omega_2 \sin\omega_1\mu + \sqrt{B^2\omega_2^2 - A^2\omega_1^2\cos^2\omega_1\mu}}{B\omega_2}$$

(2) If on the other hand, the period is very long, then the amplitude will appear to 'fluctuate' between a set of eigenvalues. The phenomenon is so far from being additive that the "sum" of the amplitudes A and B has generated not one, but an entire set of amplitudes

(3) If ω_1 and ω_2 are not commensurable, then the wave shape is not periodic but almost periodic. Then any number of possibilities may present themselves. The expression for C gives an absolute bound for the greatest possible amplitude, but the actual effect of the sound on a listener may differ greatly as a function of the two ratios A/B and ω_1 / ω_2 .

We now return to an examination of the 3 categories of additive phenomena:

(i) Magnitude Algebras . Quantitative Addition :

Let us start with 3 glasses of water. We have no way of measuring the volumes of water, but we know that glass G_3 is so

large that it can hold all the water from G_1 and G_2 combined. Let these amounts be M_1 , M_2 and M_3 respectively. The extrinsic structure of the Causal Algebra C , for magnitudes has no numbers, but it does have order relations, and operations + and -

(a) It is possible to set M_1 , M_2 and M_3 in increasing order, with
$M_1 \leq M_2 < M_3$

(b) $M_3 = M_1 + M_2$

(c) If water in G_3 is now poured back into G_1 until it is full, the remainder can be poured back into G_2 until it is full. This is what is meant by an additive magnitude, and we can write
$M_1 = M_3 - M_2$, and
$M_2 = M_3 - M_1$.

An essential feature of the notion of a 'magnitude' , is that the different parts of the water, from the viewpoint of the magnitude algebra, are indistinguishable.

If we take the glass G_3 and, prior to pouring it back into G_1 and G_2 , we shake it vigorously, this will have no effect on the result. We therefore define Magnitude as a kind of space of indistinguishable entities, on which it is possible to place an additive measure.

II. Concatenation Algebras. Set Theory :

Sets, as per Zermelo -Fraenkel, allow for no confusion or overlap in their elements. Two bricks remain two bricks, no matter what their weight is. 2 green bricks, 3 green bricks and an orange brick cannot be recombined (without pulverization or recasting, which are not set theoretic operations !) into 3 green bricks, 2 orange bricks and a red brick.

If an order-type be associated with a set, then one has the more familiar idea of a concatenation: a sequence of bricks laid out in a row, or a set of beads producing a pattern, tiling, etc.

The extrinsic structure of a concatenation algebra will contain the operations of union, intersection, complement, and so forth. More generally, one can speak of Boolean lattices as extrinsic structures.

The intrinsic structure may contain selection rules for choosing subsets (the elements of the power set) , or patterns and order relations between the various elements of the set.

It is these subtle differences between Point Set Topology and Measure Theory , between additive algebras of type II and those of type I, which produce the paradoxes and pathologies of Lesbesgue Measure and Lesbesgue Integration: Baire Sets, non-Measurable Sets, the Tarski sphere construction, non-standard arithmetics and even the Continuum Hypothesis .

III. Superposition Principles:

Superposition combines " measure and set inclusion " , with
" magnitude and Cartesian product" : superpositions are sums which can be both decomposed back into their prime components while working as additive magnitudes. To do this a background of irreducible components needs to be given in advance. For example, the number "2" , *as magnitude* , is a prime in the ring Z of integers, though composite in the ring Z(i) . As a *measure* the value "2" is unaffected by its decomposition over Euclidean rings : 2 = 1 + 1 in all of them.

The situation is similar with respect to the wave functions of Quantum Theory, and caution is advised with speaking of "the

superposition principle" without specifying the context in which it is operative. Recalling our previous example,

$$\psi \;= A\sin\omega_1 t + B\sin\omega_2 t$$

To effect a unique decomposition of this shape into component functions,

$\phi_1 = A\sin\omega_1 t, \phi_2 = B\sin\omega_2 t$ **, one has to specify the orthonormal basis relative to which there decomposition is being made. If, for example, ω_1 and ω_2 are incommensurable , ω_1/ω_2 = irrational , one can express (at least within a finite time frame) the component functions as**

$$\sin\omega_1 t = \sum_{-\infty}^{\infty} r_n e^{(in)t}$$

$$\sin\omega_2 t = \sum_{-\infty}^{\infty} s_n e^{(in)t}, \quad and$$

$$\psi = \sum_{-\infty}^{\infty} (Ar_n + Bs_n) e^{(in)t}$$

Expressed against this set of basis vectors, ψ can be written as the sum of two orthogonal functions in an endless number of ways. If one knows that the eigen-frequencies of y are ω_1 and ω_2 , one can apply the theory of almost-periodic functions to derive ϕ_1 and ϕ_2 .

The extrinsic structure of a*Superposition Causal Algebra* (SCA) precedes looks something like this:

(1) One has a pre-established collection of irreducible components , such as primes, basis vectors, building blocks , generating a vector space over a ring , field , algebra, etc.

(2) The agents of the SCA are finite or infinite "sums" of these components, weighted by their coefficients. The word *sum* is used in

two senses :

(i) As the Cartesian product, (vectorial sum) of the distinctive components. As in the way sets are augmented by the operation of *union* , these retain their identity and may be retrieved by a process which, very generally, may be called "co-addition", or "co-multiplication"

(ii) As an algebraic sum obtained by "evaluating" the components in the ring of coefficients at some point in space-time and thereby forming an infinite series over that ring, with appropriate criteria for convergence, etc.

In the situation presented by the quantum theory the wave function ψ is both a vector and a magnitude. "Decomposing" the vector into unique components *requires the presence of an operator which is relevant to the quantity* , possessing a complete set of eigenfunctions that form the basis of the closed linear manifold which is the natural setting, (the background of natural phenomena) for ψ .

What do we mean by a "relevant operator"? If the operator K represents electron spin , and our particle isn't an electron, or doesn't have the set of internal freedoms known as spin, then the "action" of K on ψ is without meaning.

However, associated with ψ is a magnitude , or "norm", given by

$< \psi, \psi > = \int_V \psi^* \psi dx^3$ **. This quantity, like the water shaken in the glass is without distinguishable parts: It cannot be "decomposed" in the absence of a relevant operator.**

Many examples from daily life come readily to mind. The evaluation process corresponds to an "unfaithful" transformation, reducing a collection of distinguished components to an undifferentiated magnitude, rather analogous to the way in

which a Black Hole transforms all the different kinds of matter and energy entering it to a uniform "substance" (if that is the appropriate word). Example : In calculating income tax, one translates all of one's assets into monetary equivalents. "Money" then operates like an undifferentiated "magnitude" under the extrinsic rules of a magnitude algebra. It is because of this useful property that the detestable institution of money will no doubt always be with us.

EXAMPLE 3: Strict Determinism

The first two examples have dealt with the extrinsic structures of certain standard causal algebras. The following assertion is the victor in the popularity contest for intrinsic causal postulates:

" If S is a state variable describing the behavior of an isolated system K , then all past and future values of S may be computed from the values of the magnitudes in S and all their time derivatives , at any instant of time. "

The statement says nothing about the combination rules for its agents of a causal algebra . What it does mandate is a set of procedures for extracting predictions from the range of magnitudes in its representation spaces at an arbitrary time t. It is for this reason that the normal assumption in most of physics is that the intrinsic structure of such causal algebras contain nothing but analytic functions, representable as a Taylor Series of some non-vanishing radius of convergence. Simply : the entire history of an analytic function may be computed from information about its derivatives at any given instant.

It may not always be the case that every function in such algebras is composable with every other function. Rules that

restrict permissible combinations create what's sometimes a "groupoid" , though it is high time mathematical terminology revealed more imagination.

However such structures are natural to physics. An electromagnetic field E "acts on" a positively or negatively charged particle yet has no effect on a neutral particle such as a neutron N . As particles themselves may be treated as fields, the action of field E on field N may be deemed impermissible. It is somehow the case that , with respect to all forces except gravitation, individual bits of matter have the freedom to decide which forces they care to be subjected to!

If P signifies the "particle", σ the "parity of charge" (= -1, 0, 1) , and $\boxed{\vec{v}}$ its position vector then, symbolically $\boxed{E(P_{S;\vec{v}}) = \sigma\vec{u} + \vec{v}}$, $\boxed{\vec{u}}$ **being the displacement .**

EXAMPLE 4:

Invariance Under Time Translation

Let U^A be a causal algebra obedient to a postulate of time translational invariance. If F is any agent of U^A , then

$$\boxed{\forall F \forall t_0 (F(t) \in U^A \rightarrow f(t \pm t_0) \in U)}$$

This is the condition of temporal independence. It is not applicable in any space-time region of our universe in which the Hubble expansion, or conscious awareness play a significant role in events. Its normal domain of application is to small, isolated parts of the cosmos to which we give the suggestive name of "laboratories". Thermodynamic dissipation is assumed to move in the same arrow of time; according to Stephen Hawking, this process sets the direction for the former two. Entropy increase does retain a "time invariance" not present in the others , because

one is free to begin a dissipative thermal process at any arbitrary point of space-time .

The condition of temporal invariance is unusual in that it is both an extrinsic rule of combination for the agents of U^A , yet can also be employed in the intrinsic extraction of predictive information from particular situations. Thus, the integral: $\boxed{\sigma(t) = \int_{t_0}^{\infty} w(s-t) f(s) ds}$ has a dual interpretation as either

(i) A calculation made along the path of f for all time beyond the present, or:

(ii) An integration across a transverse cross-section of the family of functions $\{ f_t (s) \} = \{ f(t+s) \}$ at the instant t , weighted by the function w. In the first case, σ is a convolution of two vectors "w" and "f " . In the second case, w is a probability density.

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In the elucidation of both extrinsic and intrinsic structures one is guided by certain "criteria of reality" akin to those of Einstein, Podolsky and Rosen, themselves a simple application of Ockham's Razor to the quantum domain.

(E_a) : Every composition rule of the extrinsic structure should correspond to invariant features of the observed universe.

(E_b) : Every invariant feature of the observed universe should find a representation in the agents and combination rules of the extrinsic structure

(I_a) : Every algorithm of the intrinsic structure should correspond to some empirically derived law of nature, (Hooke's Law, the Navier-Stokes equation, Avogadro's Law, etc.)

(I b) : The mathematical form of every empirical law of the observed universe of the science under consideration should be present in the intrinsic structure of the causal algebra.

In the ideal case every entity that goes into the establishment of a causal algebra : agents, extrinsic and intrinsic structures, representation spaces and data of observation will correspond to some feature of what one might call the "description": the world picture prior to interpretation shared in the understanding of all persons working in the particular field. It would be assumed, for example, that persons working in Zoology would agree that there exist animals, human, primates, birds, snakes and so forth. The evolutionary scenarios and hereditary trees that hypothetically unify this heterogeneous collection of random facts would belong to the causal algebra: ideally every tree would correspond to some feature of the living kingdom; to every known feature of that kingdom there would correspond an entity in the causal algebra.

In the best of situations the representation is a faithful isomorphism. In real scientific practice one is generally satisfied with much less: in advancing a particular science one doesn't wait for its principles to be identified in advance. Investigators are usually quite satisfied to have a serviceable homomorphism: everything in the model refers to something in the description, though the reverse may not be true.

Even this is not always possible, and in many instances one works with a "best approximation". Recall the cliché associated with quantum electrodynamics: "The best theory we have." This is quite acceptable provided one is given in advance some purposes to which the theory will be applied.

The 3 Temporal Modes

Every interpretation of the notion of causation must consider its application with respective to the 3 temporal categories of present, future and past . These correspond to the scientific procedures of:

(I) Description

(II) Prediction

(III) Back-reconstruction

Of course any particular science will specialize in one or more of these modes , and we can speak of the "Predictive Sciences" (Physics, Chemistry) , "Descriptive Sciences" (Journalism or Photography which can be considered 'sciences of the present') , and "Back Reconstructive Sciences" such as History and Geology. However no science is really comprehensive without orienting itself relative to all 3 temporal categories.

Our concerns in this essay begin at the place after most of these things have been established as pre-requisites. The correct formulation of causal assumptions often serves as merely the starting point of irresolvable controversies central to a particular science. Consider the situation in geology: largely a back-reconstructive science, historically its vision has been obstructed by an antinomy, that of Uniformitarian versus Catastrophist interpretations of the origins of features of the earth's topography. There are so many situations for which an inevitable logic appears to lead to either catastrophist and uniformitarian scenarios demonstrates that the basic concepts of causation are never self evident, and that several non-compatible causal descriptions , (along with their corresponding algebraic structures) can be advanced.

“Uniformitarianism” and “catastrophism” are irreconcilable causal mechanisms. A Uniformitarian maintains that every feature of the earth’s past can be understood by appeal to processes at work on the earth as it is today. A Catastrophist will stress the existence of present features which oblige one to envisage past processes of which there no traces, or very few, in today’s world. This kind of trade-off is intrinsic to any back-reconstructive science. For, where can we turn to for our understanding of the past, if not the present? Yet, if back-reconstruction suggests formative processes which are not in existence now, how does one deal with the requirement of all science that explanations be testable against experience? It comes as no surprise to see the same debate resurfacing with respect to the cosmic inflation scenarios of Alan Guth and others, that postulate the past existence of a “Higgs scalar field” in the first micro-second of the Big Bang, which has since disappeared “ leaving not a jot behind”.

Consider the distinctive character of the kinds of differential equations one might develop to model uniformitarian versus catastrophist approaches. The Uniformitarian might begin by making an inventory of all terrestrial processes and forces, convection, tectonic plate movements, volcanic activity, erosion, and so forth, that start juggling permutations and couplings of these to see how well they can reproduce the world around us. The Catastrophist might begin with a catalogue of all conceivable processes, governed say by every imaginable combination of differential forms expressed as Hamiltonians, to see if they produce better models. To take a well-known example, there is the famous capsizing

catastrophe of Hugh Auchincloss:

“ In a series of broadsides circulated to congressmen, government leaders, scientists, and journalists over half a century, as well as in a book published at his own expense, Brown argued that the Earth capsizes at intervals of about 8000 years, each time wiping out whatever civilization has managed to emerge. The next one, he says, is overdue, and the Eskimos may be among the few survivors, because the polar areas will be the least subjected to catastrophic water action.”

(Walter Sullivan, “ Continents in Motion”, pg. 22, see Bibliography)

One must therefore permit a considerable amount of leeway in the formation of the Extrinsic Structure of a causal algebra . The Intrinsic Structure is easier to deal with: any set of practical or operational formulae that produce sufficiently close a correlation to the data are admissable.

Modal Predicates

For the purposes of this essay, the *possible* is defined as the collection of permissible universes induced by the action of the agents of the causal algebra on the representation spaces. The *actual* is a special subspace of this, in which all unbound variables of the agents are replaced by their domain specifications in the raw database of observations.

In a departure from the customary employment of the term, the *impossible* is taken to mean the set of *all extensions of* CA *which can be expressed in its language , but lie outside the*

world description, or actual **. Anything else will be labeled** ***inconceivable*** **.**

For example: The Relativity Principle states that no material particle can attain , in any reference frame, the speed of light, c . One may easily construct "extensions" of relativity in which c is given a value larger than 299,729 km./sec. "Relativistic possibility" is that theory which replaces the specific value of light in our universe with an indeterminate constant, c, whereas "relativistic actuality" replaces c by the experimentally obtained value.

One might then identify the "inconceivable" (vis-a-vis relativity) as a universe in which there is no radiation. " impossibility " and
" inconceivability" are therefore relative labels .

This picture may not satisfy the rigorous standards of sophisticated philosophers of science; nor am I totally satisfied with it myself. We are content if we can provide a fresh insight into the procedural assumptions which underlie scientific practice, assumptions all too often taken for granted and which merit further scrutiny.

Modal Calculi

We describe 3 symbolic calculi for representing the notions of possibility, necessity, actuality, knowability, etc. These are very tentative, and no axiomatic development from first principles is presented at this point. These formalisms will be employed in various places in this essay, as needed:

I. Possibility versus Necessity

(a) $\boxed{(q \xrightarrow{p} p)}$ **" It is possible that q implies p" . The following formation is taken to be a tautology:**

$$\boxed{(p \to q) \to (q \xrightarrow{p} p)}$$

" p implies q " implies that " "q implies p" is possible " .

(b) We also want $p \Rightarrow q$ " p *necessarily* implies q" . This states that if p exists, then q must exist. It does not mean that p need exist. In other words, this notation is a short hand for $\exists p \Rightarrow \exists q$. The way in which these are used is best illustrated by examples.

(1) Apple trees cause apples. However, a particular apple tree may be infertile and not produce any apples. *However* : if I hold an apple in my hand, I know *for a certainty* that there must have been a tree that produced it . So, using the above notation, one has:

$$\boxed{\begin{array}{c}(apple \Rightarrow tree)\\ but\\ (tree \xrightarrow{p} apple)\end{array}}$$

Clearly there are many situations in which the effect necessarily implies the cause but not the reverse. Generalizing, one recognizes 4 distinct situations:

(1) Apples *necessarily* imply apple trees, the converse is *possible* only:

$$\boxed{\begin{array}{l}(Effect \Rightarrow Cause) \quad but\\ \rightarrow (Cause \xrightarrow{p} Effect)\end{array}}$$

(2) A living body *necessarily* implies a future corpse . A dead carcass *necessarily* implies the former existence of a living body. [5] $\boxed{(p \Leftrightarrow q)}$

(3) A field strewn with corpses implies that there *may* have been a massacre, but a massacre implies there must be dead bodies.

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[5] **Until such time as we are able to combine soups of DNA and other molecules to exactly reconstruct the corpse of an animal !**

$$\boxed{\begin{array}{l}(Cause \Rightarrow Effect) \quad but \\ \rightarrow (Effect \xrightarrow{p} Cause)\end{array}}$$

(4) Education may produce wisdom; wisdom may have come from education.

$$\begin{array}{l}(Cause \xrightarrow{p} Effect) \quad and \\ \rightarrow (Effect \xrightarrow{p} Cause)\end{array}$$

(2) is roughly equivalent to "necessary and sufficient" in mathematics. The essential paradox associated with the notion of past time, is that only the past is certain, cannot be known only reconstructed. Lets give labels to these things:

(1) Back - reconstruction

(2) Formal equivalence

(3) Determinism

(4) Correlation.

II. The Knowable, the Unknowable and the Known

The situations peculiar to "The New Physics" (the expression of A. d'Abro) require us to spend much of our time explaining or justifying the notion of the "possible but unknowable". This idea is foreign to classical science.

(a) Consider Proposition P_1 : *X is unknowable and X --> Y* .

Question: Is this proposition true or false?

Example: Let X be " *The evidence burned in the fireplace proves Q's guilt* ". Let Y be " *There are 24 hours in a day.* " Aristotelian material implication of Aristotelian logic states that the truth of Y is sufficient to determine the truth of P_1 . However, what if Y is false? Examples

(i) "Since the evidence that was burned in the fireplace proves the guilt of Q, Q is not guilty . "

(ii) "Since the evidence that was burned in the fireplace proves the guilt of Q, Q is guilty."
By material implication , and assuming that Q *is* guilty, which one of these statements is true? Both? Neither? One or the other?

It impresses me that since X is unknowable, we must deem both (i) and (ii) *formally unknowable* .

Proposition P_2 : " Q's guilt proves that the evidence burned in the fireplace proves the guilt of Q. " This is obviously false, as is

" Q's innocence proves that the evidence burned in the fireplace proves the guilt of Q. "

The calculus of unknowables therefore seems to indicate that any true or false statement can imply an unknowable statement, but that no unknowable statement can imply either a true or a false statement. If both X and Y are unknowable, we must value the whole statement "unknowable" even those X and Y may be identical! Examples:

A. " 'The first name of the unknown soldier is Jack' implies that his first name has four letters."

B. " 'The evidence burned in the fireplace proved the guilt of Q implies that the evidence in the fireplace proved the guilt of Q."

Let T = true , F = false , U = unknowable.

P ---> Q

P \ Q	**T**	**F**	**U**
T	**T**	**F**	**T**
F	**T**	**T**	**T**
U	**F**	**F**	**U**

Example 2 : This is based on the fact that, according to Special Relativity, if a light beam is sent from earth at time t=0 , is bounced off a distant star about which we know nothing, and returns to earth at time t = T ,then it is intrinsically impossible to know the time at which the beam hit the surface of the star.

"The time on a clock on earth, at which a light beam hit a distant star, exists."

This is *very* controversial, because the epistemology of relativity really seems to imply that this "time" does not exist , since there is no way to know it.

Example 3 :

"The individual spins of a pair of entangled electrons exist."

Another extremely controversial statement.

Example 4 :

" The only evidence for Q's guilt was burnt in the fireplace. However either Q's guilt exists or Q's innocence exist, but not both "

One can construct a consistent logic in which 4 is false.

Lets introduce a few more predicates. The first is the conventional "existence" predicate : $\exists A$, "A exists."

The second predicate we will designate by " ∑ :

" ∑T " means *There exists a way of measuring T* .

What are the truth values of these in combination? I send a light beam out to a distant star about which I know nothing. I send this out at 12 noon; it bounces off the star and returns to me at 6 PM. By relativity the time when it bounced off the star is intrinsically unknowable. Consider the following combinations:

(1) $\exists A$"There exists a time at which the light beam bounced off the star."

(2) $\neg\Sigma\exists T$ " We cannot measure this existent time." Since there is no way to determine (1) , the question arises, does (2) have any meaning ? (3) $\neg\Sigma T$ "T can't be measured."

The problem is that before relativity, Kantian and other systems of causation maintained that all events, measurable or not, gave rise to a trail of indirect effects that, sooner or later would become known. Just before his death, Q might write off a confession and send it to the newspapers. An eye-witness to the crime may surface 20 years later. etc. Even if these things did not happen, the possibility that they might happen was always open. But relativistic epistemology opened the door to events *that were intrinsically unknowable because of the structure of physical theory* . This is a totally new phenomenon. The other example I gave, of entangled electrons, is not so unusual, for one can argue that the "individual identity" of the electrons is just a fiction. However, when causation itself is propagated at a finite speed, one generates paradoxes that have never before been seen.

(4) $P{:}(\exists T \wedge \neg\Sigma T) \to \# T = U$ **, where #T stands for the 'truth value of T' (true, false, unknowable) . Such entities will be deemed 'intrinsically unknowable' .**

We introduce an example from psychology: For a great many traits , it is a fact that by knowing of them we automatically change them . Therefore we can never know our present state but only a past one: if I'm killing myself because I drink contaminated water , then I can be called a "self-destructive human being". But the moment I learn that this activity is destroying me, I stop it. Learning that I am a self-destructive human being means that I am no longer such.

Schematically "P exists and P is unknown. When P becomes known, it ceases to exist." In the calculus we've been developing, one has:

$$\exists P \to (\# P = "U")$$
$$(\# P \neq "U") \to \neg\exists P$$

III. The Possible and the Conceivable

In this context, "possibility" relates only to entities which may be freely combined in thought experiments, or systems in isolation, that are not prohibited by the restrictions on causation. We notate the modal quantifier "P is possible" as >P< . This means : ***P is an element of the conceptual universe (or collection of universes) constructible from the agents and rules of the causal algebra*** C^A **.**

Example: Consider the Uniformitarian Postulate for Geology : ***"Every feature of the Earth's past can be explained by processes at work in the Earth as it is today. "***

Owing to the astronomical increase in astronomical knowledge in the modern world, the scope of geology is extended

to include any sort of rock in the universe, notably the planets, moons and asteroids in the solar system, etc.

Relative to this principle, what is “possible” and what isn’t?

(1) The complete melting away of the ice caps *is* possible, since we can explain that by the melting, even the disappearance, of glaciers in our own day. This despite the fact that history has never recorded the disappearance of the ice caps.

(2) Brown’s hypothesis of spontaneous capsizings, whereby the poles are flipped every 8,000 years is “not possible”. However, if it can be given a mechanism other than invisible blue devils at the earth’s core, or a fifth fundamental force of nature, etc., it may be deemed “conceivable”

Suppose we use the symbol: $\cup C^{A}$ to signify the collection of conceptual universes generated by $\mathbf{C^{A}}$. Then $>P<$ means that $P \in \cup C^{A}$.

“Not Possibility ” will be given a slightly different definition from the conventional one , and will be notated $\overline{> P <}$. This means that P is expressible in the language of $\mathbf{C^{A}}$, but not an element in any of the conceptual universes it generates. The collection of *linguistic* universes generated by CA will be notated $\mathrm{I}\,C^{A}$.

Therefore: $\overline{> P <} \Rightarrow P \in \mathrm{I}\,C^{A} \wedge P \notin \cup C^{A}$.

Example: “Unicorns” are conceivable in terms of the language of Zoology, but are not possible . They have no relevance to the language or concepts of Geology; we have another term for that: we say that “unicorns” are not *conceivable* in the language of Geology , for which we use the symbol (P)*. The “impossible” will be taken to be anything that is either not possible or inconceivable

Simply stated $(P)^{*} \Rightarrow P \notin \mathrm{I}\,C^{A} (\supset \cup C^{A})$.

The "actuality" quantifier : as defined in this essay actuality does *not* refer directly to the actual world. To say that "P" is actual shall mean: *P is a calculation from established domains of inputs into the arguments, functions and agents of* C^A . The "world of actuality" , $\otimes C^A$ is yet another collection of universes derivable from the causal algebra . It contains only descriptions, not interpretations or laws. These descriptions can then be compared with the world of observation and experience for the purposes of falsification and prediction. Symbolically "P is actual" will be notated #P# (The "number sign" indicating "quantification") .

This is the complete modal calculus for the description of the algebraic structure of causation. Everything is inclusion, exclusion and negation; the calculus is a simple form of a Boolean Algebra. To summarize:

$$
\begin{aligned}
&\otimes C^A \subset \cup C^A \subset \mid C^A \\
&\# P \# \Rightarrow P \in \otimes C^A \\
&> P < \Rightarrow P \in \cup C^A \\
&\overline{> P <} \Rightarrow P \in \mid C^A , P \notin \cup C^A \\
&(P)^* \Rightarrow P \notin \mid C^A
\end{aligned}
$$

All this is fairly self-evident. One need merely point out that "not possible" and "impossible" are different categories. The "impossible" includes the "not possible" as well as the "inconceivable".

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Appendix to Chapter II

Algebraic Causation in Quantum Theory : Four Formalisms

I. Heisenberg Formalism

(a) Principles of Causation:

1. Position and Momentum are non-commuting

2. Time and Energy are non-commuting

3. State vectors are time independent

4. Observables are matrix elements of infinite matrices and vibrate harmonically

5. Nothing exists at the quantum level beyond the probabilities computable from the action of the observables on the state vectors

(b) Agents

1. Hermitian Matrices

2. Unitary Matrices

3. Classical Hamiltonians

4. Vectors

(c) The Extrinsic Structure

(i) Quantization

$$\boxed{\begin{array}{l} q \leftrightarrow x\,\bullet \\ p \leftrightarrow i\hbar\,\partial()/\partial x \\ E \leftrightarrow i\hbar\,\partial()/\partial t \end{array}}$$

(ii) Superposition

(iii) The algebra of inner products and matrices in Hilbert Space

(d) Representation Spaces

(i) Configuration Space

(ii) Momentum Space

(iii) Hilbert Space

(e) The Intrinsic Structure

(i) $\boxed{i\hbar\frac{dA_{jk}}{dt} = \varpi A_{jk}}$

(ii) $\boxed{i\hbar\frac{dA}{dt} = [H, A]}$

(iii) $\boxed{i\hbar\frac{dU(\Psi)}{dt} = HU(\Psi)}$

II. Schrödinger Formalism

The key differences of the Heisenberg and Schrödinger formalisms are

(a) Principles of Causation

*** Simultaneous presence of all eigenvalues; wave equation "smeared" over space**

*** Observables are time independent**

*** State vectors are vibrating harmonic oscillators**

(b) Extrinsic structure

*** Observables are linear *differential forms***

(c) Representation Spaces

*** The evolution of the state vector is analogous to that of the path of light ray in Hamiltonian phase space**

(d) Intrinsic Structure

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi}$$

3. Dirac Formalism

The Dirac Formalism does not differ conceptually from the others, but introduces an intelligent notation in the extrinsic structure which allows one to work with it more effective

(c) Extrinsic Structure

$$\boxed{\begin{array}{l} Bras - \langle n| \\ Kets - |m\rangle \\ Scalars: \langle n|m\rangle \\ Operators: |m\rangle\langle n| \\ Tensors: |a\rangle|b\rangle \end{array}}$$

4. von Neumann Formalism

(a) Principles of Causation

(1) Existence of Sharp Values of Eigenvalues even when unobserved

(2) Collapse of the Wave Packet on Observation

(b) Agents

Integral Linear Transformations **as Observables**

Projection Operators **as states**

(c) Extrinsic Structure

(1) von Neumann Algebras

(d) Intrinsic Structure

Stieljes Integrals

The Trace Formula

$\boxed{Prob.() = Tr.(P_{|\psi\rangle} \bullet P_W)}$ **(check this in Redhead)**

The Statistical Operator

$\boxed{W = \sum c_i P_{|\Delta\psi_i\rangle}}$ **for mixtures**

❋❋❋❋❋❋❋❋❋❋❋❋
❋❋❋❋❋❋❋❋❋❋❋❋

II. The Lagrange/Hamilton Paradigm

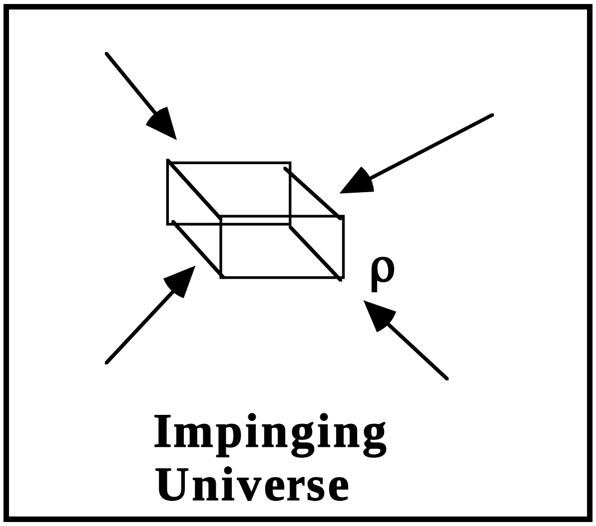
(a) Unpacking the Instant: The Null Set as Infinite Well of Possibility.

The *instant* is an abstraction which, as with most useful notions of science, corresponds to nothing in the phenomenal world. That the nexus of change is always instantaneous is a basic assumption of the physical sciences. In fact, an *instant* in the trajectory of a monotonic state variable S(t) is best understood as the length of time between acquisition of a particular value v, and its infinitesimal passage to the value v + dv .

In and of itself an instant is null and void. Nothing 'happens' within the confines of the instant: indeed, to speak of events happening 'inside' the instant is deemed a violation of language . Yet classical physics ascribes an enormous content to the instant. There , and in hidden variables formulations and semi-classical limits of quantum theory, the entire history, the so called world line of a system K in isolation is considered to be present in the configuration of K at any arbitrarily selected instant. We call this *The Lagrange/Hamilton Paradigm* .

Gottfried Leibniz and Immanuel Kant envisaged an extreme form of the Lagrange/Hamilton paradigm, one in which the entire cosmos, from inception to extinction, is entirely present at every point of space-time, in every instant of every location. The mirroring of the Macrocosm in the Microcosm, the arbitrarily great in the vanishingly small, is universally present.

Such a conception is theoretically testable: take an arbitrarily small region ρ around some location, arbitrarily chosen, and explore how much of science can be deduced from the information available in ρ .



Without arguing either the pros or the cons of this conception we would merely remark that it corresponds to what science in fact labors to accomplish. Relative to the known or knowable universe, the earth is a negligible speck. Relative to the vast reaches of time, (back to the Big Bang or the foreseeable future), the entire history of the human race is but an instant. Yet, it is from the information available here that we endeavour to determine all else that is happening everywhere.

The major difficulty with the Leibniz/Kant paradigm is that it leaves no room for thought experiments. Here indeed we are touching upon an irreducible contradiction within classical physics. On the one hand: we like to imagine that Universe so acts on Earth that, from this earthbound prison we can divine the universal laws.

On the other hand: it is clear that science is unworkable, if not inconceivable, without allowing for the possibilities of the ***thought experiment*** **and the** ***system in isolation*** **. Both of these require that, by an act of mental visualization** ***one can escape all biasing local conditions*** **, and place our laboratories in hypothetically ideal regions of space purified of all alien**

influences: gravity-free flat regions of space-time; perfect black boxes impervious to external fields; ideal 'Schrödinger cats' which have no way of letting the world know if they are alive or dead; containment vessels in which all the products of any reaction may be swept up to demonstrate the conservation of matter and energy.....

Another implication of the Leibniz/Kant paradigm is that every particle feels every force. There are no neutral particles, no neutrons. Both the strong and the weak nuclear forces are really coupled to infinity, although the coupling be so weak that we'll never be able to detect it. Note that this is not so far off from Richard Feynman's stance in Quantum Electrodynamics: the manifested paths of all moving entities, including photons, are actually cancellations of the amplitudes of all possible paths. ***Thereby all neutrality is in effect cancellation*** **; anything that can happen does, though almost all of it with vanishing probability.**

We have inadvertently put our finger on the essential distinction between the Lagrange/Hamilton paradigm and the Leibniz/Kant paradigm. Lagrangian mechanics, as enriched to its modern form by William Rowan Hamilton, rests securely on the notion of the system in isolation. The substance of a Lagrangian Configuration Space of 3N dimensions, (where each particle is represented by 3 coordinate axes corresponding to the x,y,z coordinates in physical space) is such that , excepting only these N points, the rest of the universe is excluded in advance. The Lagrangian differential form $L = U - V$ embodies the full teleology, the initial conditions, the motor of time evolution, and the ultimate fate of any dynamical system under its command.

The situation does not change materially when Configuration Space is enlarged to Hamilton's Phase Space by the

introduction of new variables, the generalized momenta, which are generated directly out of the Lagrangian through the substitutions

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

The Hamiltonian world-line moves, in a manner analogous to a sound wave through an incompressible fluid, from time $-\infty$ to time $+\infty$, as if Phase Space were itself the sensorium of experience, a universe complete and closed on itself that is never intended to hook up with processes in the real world.

Lagrange/Hamilton systems are clocks; their very conception in an agreement with the notions of a clockwork universe so dear to the rationalists of the 18th century. Unlike terrestrial clocks, they cannot be influenced by tides, sunspots, cosmic rays, mechanical failures, entropy, expansions, inflations, and so on. Of course it is true that the variability of the potential V, itself derivable from a gradient that is distributed over space, does represent , albeit in a very simplified or reduced fashion, the influence of the rest of the universe on the system of N particles.

However, in the full Leibniz/Kant paradigm, it is not only the universe which affects the system, but the system with also influences the rest of the universe, therefore also changes the potential. In addition, the universe is constantly interacting on itself, which means that the value of V at every point must be a function of the value of V at all other points.

Furthermore: given that Hamiltonian systems are, in general, not only C^∞ but analytic - otherwise stated, that the fibration of Phase Space
by the bundle of world-lines determined by all possible initial conditions at a moment t = 0 , is holomorphic - it follows that

knowledge of all time derivatives at that initial instant is sufficient to determine all of its trajectories through all of time. Once again, a Hamiltonian system figures as a closed universe, Phase Space as a mental construct.

We will be looking at various alternative models of the Lagrange/Hamilton paradigm in the following pages. Gottfried Wilhelm von Leibniz himself. ("Leibniz" G. MacDonald Ross, Oxford UP 1984 pgs. 88-100) constructed an algebraic model for Leibniz/Kant causation . The monads, present at every point of space, each containing others in a descending chain, are essentially observers, spectators for the entire cosmos. In line with his view that Ultimate Reality consists of the sum total of all existent mental viewpoints, the cosmos itself is nothing but this infinite congregation of witnesses. It is their acts of observation that bring the cosmos into being.

> "In short, there exist only monads, and monads are nothing other than actualised sets of perceptions defined by a particular point of view."
>
> (Macdonald Ross, pg. 95)

> "...It was one of the main theses of his philosophy that objective truth is the summation of the different viewpoints of all individuals ."
>
> (Ibid , pg. 75)

> " We normally understand the world as consisting of objects of perception separate from and common to different perceivers. Leibniz held that such objects were only mental constructs. "

(Ibid, pg. 90)

The mathematical tools for constructing models or realizations of the programme of Leibniz's Monadology have only been developed quite recently, in the Fractal Geometries of Benoit Mandlebrot and his school.

Kantian epistemology, an uniquely brilliant fusion of psychology with metaphysics, has been under attack from its inception. Indeed, it is to its credit that so much of it *is* falsifiable and has in fact been falsified. Euclidean Geometry was dethroned from its exalted station as a *synthetic apriori* by Gauss, Bolyai and Lobatchevsky in the 1870 's , (although all non-Euclidean and Riemannian geometries are locally Euclidean .) Causal interconnectedness in the literal sense of Leibniz' monadology was superceded by Special Relativity: once again, Bell's Theorems and the Aspect Experiment have reunited the cosmos via a 'correlation' which , people such as Eberhardt and Stapp assure us , can't convey information, yet which nevertheless remains mysterious.

It should also be noted that Mach's Principle, which plays a large role in General Relativity, reaffirms the principle of a totally interdependent, interconnected universe. The idea is as old as religion, viz. "the fall of a sparrow..." Neo-Kantians like Ernst Cassirer and Idealists like Alfred North Whitehead have proposed various ways of reconciling the Kantian ideal with the new physics.

In theory, neither Quantum Theory nor classical Thermodynamics violate Leibniz/ Kant causality. One cannot deny that physicists in these fields, when setting up experiments in their laboratories, would no doubt consider it a hindrance to have to take into account the influence of distant stars.

Throughout this essay we will be taking a critical stance towards metaphysical opinions advanced by scientists concerning the postulates of causal governance in their disciplines which lie too far afield of real applications in their daily work .

Summarizing: whatever conclusions one may be inclined to reach about the "Large Scale Structure" of the universe, the Leibniz/Kant paradigm has little application in either the physical or the conceptual laboratory. It fails to deliver with respect to two crucial requirements:

(1) Thought experiments are indispensable in the sciences.

(2) Most of the phenomenological world is and will remain inaccessible to us.

(b) Difficulties with the Leibniz/ Kant Paradigm

(i) *Cancellation Points:* If one maintains that the history of the cosmos may be computed from all the information potentially available at a single point in space-time one is led to deny the impossibility of all symmetries leading to the cancellation of forces at that or any other point.

The theoretical possibility of systems in isolation leads one to the opposite conclusion : *Any compact, isolated system of material entities must have a force barycenter, a cancellation point for all forces operative in the system's internal dynamics* . We are indebted to the mathematician René Thom for pointing this out to us.

To simplify the arguments, let S be a self-contained system of N particles. Define P as a generalized Phase Space of 9N coordinates specifying positions, velocities and accelerations in the 3 directions of space x, y, z. Normally one might say that positions and momenta ought to be sufficient. Let me counter this by

making that observation that even in the simplest and earliest of dynamical theories, that of Newtonian gravitation, *the accelerations are given first* . Momenta calculated from them through integration of Newton's second law and the boundary conditions. Let the particles have positions, velocities and accelerations :

$$X_1, \dot{X}_1, \ddot{X}_1, Y_1, \dot{Y}_1, \ddot{Y}_1, Z_1, \dot{Z}_1, \ddot{Z}_1,$$

$$X_2, \dot{X}_2, \ddot{X}_2, \ldots\ldots\ldots X_N, \dot{X}_N, \ddot{X}_N$$

, with masses $M_1 \ldots M_N$, charges $e_1 ,\ldots\ldots e_N$, etc. Since the system is bounded within a compact region of space at any given instant, one can compute the collective moment from any point in the 9N-dimensional space, given as the inner product of the coordinates with certain functions of the various masses, charges ,etc.

$$M = \frac{\sum(f_j^x x_j + f_j^y y_j + f_j^z z_j) + \sum g_j^x \dot{x}_j + \ldots + \sum h_j^x \ddot{x}_j + ..}{\sum(f_j^x + ..) + \sum g_j^x + .. + \sum h_j^x + ..)}$$

It is an elementary result from Affine Geometry, that there exists a unique barycenter $B = \{x_j^0, y_j^0, z_j^0, \ldots\}$

at which M = 0 .

This demonstrates that there is an instantaneous cancellation point. If we translation our coordinate system to the point B by (1) spatial translation (2) change of reference frame and (3) introduction of Coriolis-like "fictive" force fields, then we can place the observer at a point at which, within the closed universe of the isolated system, all forces and influences are canceled. At that point the observer can detect no motion through space; his "lookout" is permanently set at generalized phase coordinates

$$B = (\underbrace{0,0,0,\ldots,0,0,0}_{9N})$$

All of this follows simply from the assumption that position, momentum and force are vector quantities, that is to say, linear differential operators on a finite Hilbert Space. At such cancellation points, neither Leibniz/Kant causation nor Lagrange/Hamilton causation can hold, because one cannot extract any information out of them. In particular, all individual massive particles , considered as systems in isolation, must have their own space/momentum/force barycenter where all the resultant sums of static, kinematic and dynamical magnitudes are annihilated. One might try to retain a form of total causal interconnectedness for most of the universe by treating such cancellation points as singularities, that is to say as exceptions. However, since each individual particle has its force barycenter, *the point set on which Kantian causation may be presumed to hold must be in those domains of space-time in which matter is absent!*

For the Leibniz/Kant paradigm to maintain consistency one is led to postulate an unbounded universe: a bounded universe must have force barycenters. In our modern cosmological models these are ready to hand: the Black Holes which, following the Hawking-Penrose interpretation, are timeless stable configurations at the boundary of space-time. Since Hawking radiation derives from Quantum Field Theory, which is statistical, it does not enter into this discussion, which is concerned only with the two basic paradigms for determinism.

Following the standard cosmological model a universal cancellation point is clearly present at the moment of the Big Bang.

Although an unmodified strict Leibniz/Kant paradigm implies an unbounded universe , one might still be able, in theory, to construct systems asymptotically in isolation. One has to send them as far beyond the Hubble horizon as one needs to render negligible the forces acting upon them from the local universe. This construction is frequently employed in the theory of Black Holes, where one often speaks of objects or radiation arriving "from infinity".

On the other hand , since the Lagrange/Hamilton formalism for isolated systems does imply the existence of cancellation points, one might as well allow for the existence of ***indefinitely large independent regions of space-time*** **in which all universe forces are, (like the amplitudes of Quantum Electrodynamics) , canceled. These would be ideal as regions into which to "move" or "drop" thought experiments and systems in isolation.**

The concept of the world-line, the backbone of both classical and quantum physics, requires a philosophically sound definition of the system in isolation. Although there is no reason why the real universe should "make space" for us to put our thought experiments into it, yet there is something incurably wrong-headed about positing one system of causal connections for the real world, and yet quite another one for thinking about it.

One might call it, " the fundamental dilemma of algebraic causation theory."

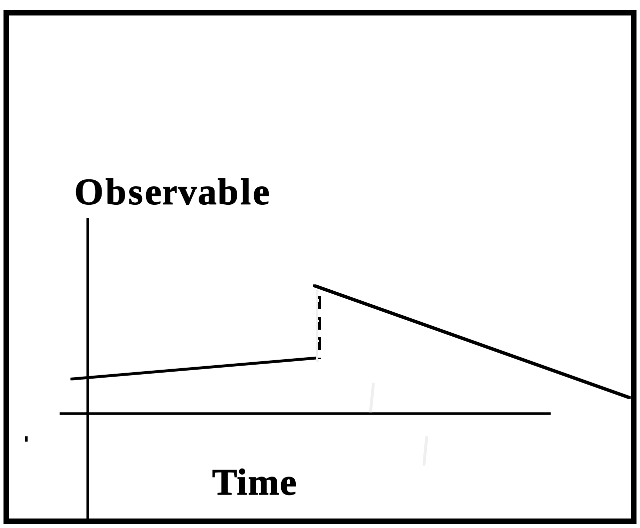
(ii) Singularities

Singularities in the mathematical representations of dynamical processes (whether in physics or the other sciences), are classifiable under many species and genera, of which the following is but a sampling:

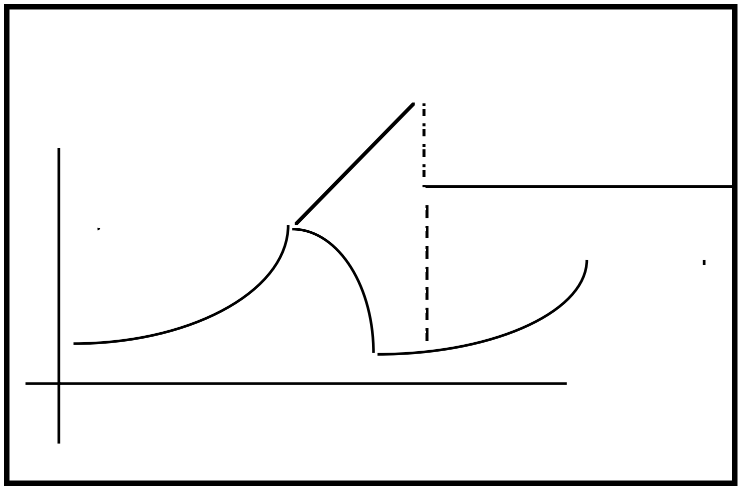
(a) One or more of the state variables go to infinity. These singularities may be “removable” by changing the representation space. For example, the “line at infinity” bounding the projective plane can be eliminated by a map that transfers it homeomorphically onto the equator of the projective, or Kleinian, sphere.

(b) One or more of the state variables vanish . If a vector field vanishes at a point there is no way of determining the direction of a flow from that point, and the dynamics automatically stops.

(c) Jump discontinuities :



(d) A simple function such as y = f(t) , becomes multi-valued at some point t* . This can manifest itself as a sets of discontinuous jump, or as a continuous branching into several paths, or some combination of the two.



(e) A real-valued magnitude becomes imaginary or complex, and thus no longer corresponds to a quantity in the real world. In relativity, speeds larger than light are ruled out because they produce imaginary values for $\boxed{\beta = \sqrt{1-v^2/c^2}}$

(f) The above case has several interesting generalizations. One way of looking at the transition x ---> z = x+iy , from the real to the complex numbers, is to think of the imaginary part of z , or y, as a new dimension. Thus, one may consider any transition from k-vectors to n-vectors , n ≠ k , as an singularity. This may be treated as a continuous transition by identifying the origins of the respective vector spaces V_k **and** V_n **.**

(g) By transferring this idea of a singularity to configuration space, one can model the spontaneous breakup of a particle into smaller entities by transitions from 3 coordinates (x,y,z), to 3N coordinates.

(h) The concept of an *essential* singularity will be the focus of our attention at various places in this essay: *A singularity of a state variable S at a point t* is called essential if it attains, in the limit of all sequences of neighboring points to it , every possible value of its range.* The value at the point at infinity of the function w= e^z is an essential singularity.

(i) Sometimes the Schrödinger wave equation is treated as a *distributed essential singularity* which is "smeared" over all of space with a certain probability. "Particles" are configured at every location simultaneously. Quantum Electrodynamics carries this interpretation one step further: all possible paths of a sub-atomic transition are considered as existing, even complex and time reversed ones , with probability amplitudes which must be summed to obtain the total picture. Indeed, situations in which all state variables are essential singularities at all points must uniquely characterize the quantum theory.

A similar construction applies to historiography: all interpretations of past events can be considered correct if one assigns to each of them a "plausibility factor" which functions like a probability.

Let us now spend some time - not nearly enough unfortunately - examining the treatment of singularities in the customary causal schemes of Physics and Biology:

Physics

(i) Black Holes

The vexing issue of the existence of irreducible singularities at the very core of the causal description of nature, has been with physical theory since Kepler, Galileo and Newton refashioned physics into its present state. The conceptual device of concentrating the mass of a finite collection of particles at its centroid allows one to define a gravitational potential U(r) over the entire universe. U goes to infinity at this centroid. Theoretically when one point- particle becomes trapped in the gravitational field of another their relative speed at collision will be infinite. One can only give thanks to the creator(s) of our universe(s) for having supplied its real material particles with

enough extension, density and hardness to prevent such catastrophes !

It is as if the universe had made a decision to divide by zero at certain privileged locations . The dilemma is not readily renormalizable. Indeed, it emerged as the core issue in Arthur Eddington's attack on Subramanyan Chandrasekhar's discovery, in the 1920's, of the theoretical inevitability of Neutron Stars and Black Holes. When a star's gravitational field overcomes all counter-active forces, it must automatically implode into this singular state of matter. One might argue that the existence of such entities proves that the unthinkable can occur, and the universe yet survive! For the Black Hole is the material realization of the infinite value of the gravitational potential function at the barycenter of an isolated system. The entire object is a singularity. According to most observational astronomers, it does exist: there may even be one at the center of our own galaxy.

Not everyone agrees: (***Quote Phillip Morrison: "I'll believe in one when I 'see' it ."*** **). The observation that , in the visible world at least, only finite amounts of any magnitude can exist appears to be universal . By an argument first propounded by Anaximander, an infinite amount of any substance implies an infinite potential for Becoming in that substance. Since the theory of the origin of the universe in a Great Explosion is well established, what we call "time" began at some particular moment in the past. Therefore the forces producing Black Holes have only had a finite time in which to develop and operate, and could not have produced infinity of any substance, including that of a Black Hole.**

(ii) The Big Bang

It was previously observed that the essential singularities of Schrödinger's interpretation of quantum theory are disbursed throughout the universe in obedience to a probability distribution . The Big Bang is the unique paradigm for a *non-distributed* essential singularity . Gazing back to the first infinitesimal splinter of time from the initial explosion, the "Planck instant" (between 10^{-33} to 10^{-42} sec.) , one beholds a singular region of space-time containing all the matter and radiation of the universe at infinite temperature . Assuming that the amount of matter/radiation in our is finite*, then it is inherent in the Maxwell-Boltzmann statistical model heat and temperature, that the phrase "infinite temperature " is equivalent to the presence of all potential configurations of matter and radiation in that single instant.* In other words , an essential singularity.

Although essential singularities embody the most complete breakdown of traditional causation imaginable, it is possible, (we shall show this in the final section of the essay), by constructing causal algebras based on them, to model the possible forms of determinism that may emanate over time from this massively acausal event. This original Ansatz provides, we believe, a solution to the fundamental paradox inherent in the Big Bang theory, namely that from the Chaos of the global essential singularity involving everything in the universe, there can emerge the coherency and determinism connecting all phenomena at the level of normal observation, i.e. non-quantum. Such *essential singularity causal algebras* may turn out to be useful as models for the various inflationary scenarios that have gained currency in recent years.

It is highly doubtful that Leibniz's scheme of causation in the Monadology [6], or that of Kant as set forth in the Third Analogy of Experience in the Critique of Pure Reason, [7] (both of which posit the simultaneous influence of all things in all things), can be consistently applied in any universe, closed or open, without the requirement of singularities, void domains, force or matter vacua, or essential singularities.

If all the matter in the universe were contained in a compact region, the arguments already set forth would be sufficient to establish this point. If one assumes otherwise, that matter is distributed throughout an infinite cosmos, one quickly runs up against an Ölbers Paradox : if every particle is obliged to 'feel' a perturbing influence coming from every other particle, then it must be under the perturbing influence of a potentially unlimited number of forces. No object could withstand the accumulated pressure of such forces, which would either dissipate all matter to an infinite entropy, or crush everything into a Black Hole, or push all things to the speed of light.

So, if the total force acting at every point in space be finite, there must be some mediating factor that dampens the contributions of other particles, and it is obvious that this mediating force must be an exponential expansion field. If the force in question is the electromagnetic field, we are dealing with

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[6] "Thus, in saying that at any given time the state of each monad expresses the states of all the others, Leibniz is just asserting that, given a complete knowledge of the state of any particular monad at any particular time, a sufficiently discerning mind could read off the state of any other monad at that time. Further, each state of a monad similarly reflects all past and future states of that monad. Consequently, if one knew completely the condition of any single monad at any time, and if one had adequate logical powers, one could determine the states of that and all other monads at all times. This is the sense in which 'everything is connected with everything.' " Mates: Philosophy of Leibniz, pgs 38 , Oxford U.P. 1986

[7] " All substances, so far as they coexist, stand in thorough-going community, that is, in mutual interaction" Immanuel Kant, Critique of Pure Reason, pg. 233 , Norman Kemp translation

the standard resolution of the Ölbers paradox: back reconstruction to the origins of the expansion field leads to a Big Bang singularity.

The investigation of all the possibilities for the construction of causal algebras for the modeling of Leibniz/ Kant causation would undoubtedly unearth some interesting mathematics. Yet a strict adherence to their requirements raises so many conceptual difficulties, and is so contrary to the way we actually think about our world , that there will be no further discussion of them in this essay.

(iii) Particles as Systems in Isolation

Our attention has been drawn to this significant observation : it is precisely at the cancellation points of isolated systems that one finds the infinite singularities of the gravitational potential : the mass-weighted barycenters. Otherwise stated the two singular violations of Kantian causation:

(i) Cancellation points (Inertia)

(ii) Infinite values of the potential (Gravity)

occur at the same points in space-time. This is not unconnected to the equivalence of gravitational and inertial mass, which is the cornerstone of General Relativity. Paradoxically, whereas (i) , being the nullification of a vector sum of momenta, is quite consistent with mathematical continuity, (ii) , as an infinite quantity, is alien to our conception of a finite universe.

Therefore every particle, treated a world unto itself , contains all violations of Kantian causality. Conceptually there is little difference between a 'particle' and a 'system in isolation' . No difficulties are anticipated if we sometimes use the two words interchangeably.

Particles violate strict Kantian causation: it is not possible, from accumulating data on the inner dynamics of a single particle, to intellectually project the history of the entire universe in either direction of time. If we enlarge the notion of a particle to encompass this tiny patch of earth in the brief interval human science, our quest for a Theory of Everything is unalterably vain.

BIOLOGY

The definition and identification of causal singularities and cancellation points in biological systems is a subject of considerable interest. What is it that distinguishes the animate from the inanimate? If this were an essay on ethics or metaphysics, the author would define a living organism as any physical object which it is immoral to injure. Since our focus is, properly speaking, on the mathematics of causation we can't really use this fundamental line of demarcation between the animate and the inanimate, and will retreat to the far more restricted territory of the cell theory. This combines *the hypothesis of Schleiden (1838) and Schwann (1845) ,* that the cell is the fundamental unit of living structure, with *Virchow's axiom (1858) ,* stating that all living things are formed from pre-existing living cells. Up to the present day, these are the cornerstones of biology. The viruses may be exempted from this broad agenda, or one might argue that their relationship between the animate and the inanimate is not yet well understood. In this section we will only be speaking of organisms, that is, creatures formed from one or more cells.

By appropriately extending the conception of Death, we can introduce an additional axiom stating that all organisms die. For the purposes of the present analysis, "Death" shall signify the cessation of the individuality of the organism. There are two ways in which this occurs:

(i) Death in the ordinary sense

(ii) The splitting of the organism into two or more units, either by mitosis, or the more complex process of meiosis. Likewise, when an earthworm is cut in two and regenerates two

living individuals, we will consider that the original earthworm has died, and two new ones born.

Since the eventuality of death is an intrinsic characteristic of all organisms, one reasons that the total state S of an organism cannot be identical at any two moments $t_1 < t_2$, of its living existence. The condition $S(t_1) = S (t_2)$ is equivalent to periodicity. This (as long as the nutritive substrate and chemical composition of the environment are maintained and replenished) would imply a potentially eternal existence . Note that this statement is true, even when the individual cells in the organism die off and are replaced by formally identical ones. Eternal identity may be a plausible hypothesis for the elementary particles, but it has no place in the definition of living organisms.

The Life-Expectancy Function

Because of the absence of cycles, the propagation of biological systems within their appropriate phase spaces must occur along non self-intersecting world lines. Theoretically it should be possible, from the information available at any single arbitrary moment in time t, to define a monotonically decreasing *maximum life expectancy function* , $L(t)$, at that instant, and a *minimum lower bound, $B(t)$* on the date of birth, calculated backwards from that instant . Organisms have built-in clocks with an irreversible count-down in the direction of the arrow of time.

Biological and physical systems exhibit major differences on other particulars as well:

(i) The concept of the system in isolation is meaningless for biological systems. Organisms cannot be defined apart from the particular *substrates* off which they feed, or the range of external conditions of their evolutionary niche: All biological systems are characterized by *dependence*

(ii) Each species of plant and animal possesses a characteristic life-span, defined as an upper bound beyond which all members of the species must perish. Despite the propaganda surrounding Methuselah, and despite the astonishing advances of modern medical technology, no human being to our knowledge has ever living 200 years.

This upper bound is independent of all external conditions. If as a thought experiment we idealize an environment E, in which the potential for longevity is maximal, all other environments must detract from that potential. Thus if, at age 60, I know I cannot live another 90 years, then it may turn out that something happening to me at age 70 (a disease, etc.) may reduce this upper bound to 20 years. But nothing can raise it to 80 years!

When external circumstances bring about an abrupt truncation of life span, one can model the world line of the organism as a jump discontinuity to the ground state. Even without the intervention of fatal accidents, the 'vitality trajectory' of an organism points monotonically downwards. Although it is customary, particularly in societies of advanced technology, to believe in the existence of optimal environments for maximizing the life span, our medical knowledge is very primitive in this area. It is known that the inhabitants of the Caucasus mountains tend to be long lived, which may be correlated with vigorous mountain climbing from an early age. This knowledge has made no impact on the social customs of scientifically advanced societies, where far too many persons spend much of their time sitting in automobiles and behind desks, and rarely bother even to walk.

Yet it is undeniable that public medicine in the industrialized world has made great strides in the eliminate of famine, disease, ignorance and war, and that people in them have a expectation age for longevity at least twice that of the Third World. Although we all seek to live as long as possible, we also realize that, even in the Caucasus, there is an upper limit to life. It is a safe assumption that for every person on earth there is a function L (t), defined as the longest possible time that person can live upon reaching the age $A= t - t_0$, where t_0 is the time of birth. Because of its substrate dependence, *this function is not Lagrangian* . one cannot, from a knowledge of its value and that of all its derivatives at any time, chart its future.

Whether comprehensive information about the organism's state function S at time t is sufficient to describe the value of S *at all previous times* back to birth, is an open question. To a qualified doctor, the signs of prolonged malnutrition can enable him to make an essentially accurate picture of a hunger victim's condition as it was before the onset of famine. How far into the past this can be carried is conjectural. One does find psychiatrists who claim to be able to discern from a woman's gait that she was molested as a child; obviously they are arrant frauds.

It can be seen from this short analysis that there is more than a little subtlety and complexity in the Darwinian catch-phrase : "The Survival of the Fittest" . Apart from vague generalizations and certain large-scale projections, little seems to be known about how vitality is affected by circumstances. There is a genre of automatic thinking among many biologists, because of which they assume all to readily that organisms behave in such a way as to maximize their survival. All to often, this constitutes circular reasoning : it may be convenient for the science, (much like the "hog theory" of economists which states that all human needs are insatiable always), but falls short of explaining what we see in nature.

Chain smokers have known for at least 30 years that every drag they take on a cigarette is bringing them closer to the grave; yet a great many have concluded that even the contemplation of the ordeal of quitting is too painful. In addition, some of them even continue to *enjoy* smoking, bringing in an added complication. Mixing pain, pleasure, survival and reproduction properly in the Darwinian equation takes considerable skill: the biologist's model for pain as an enhanced defense mechanism is simple-minded indeed.

(iii) Upon death the organism undergoes dissolution. As when the keystone of an arch is dislodged , a lynch-pin pulled, a ridge-pole broken, some key ingredient to the functioning of the holism of the organism is put out of commission. This causes a rapid shutting down of all sub-systems, followed by the eventual disintegration of the various components of the functioning organism into simple chemical compounds and elementary particles: dust unto dust. [8]

In contrast, the dynamical systems of physics do not experience dissolution. Nor do quantum or thermodynamic processes behave in quite the same way. Atomic reactions bring about the transmutation of particles into other particles. Although matter and energy may transform freely in some isolated physical system, the total amount of matter plus energy remains the same. Likewise, the theory of heat, energy and entropy is a statistical one. Fluctuations are permitted and they are known to occur. *Yet the universal dying of all living organisms is not considered to be a statistical law like the flow of heat from warmer to colder bodies, but an absolute law like the law of gravitation* .

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[8] **One sandstorm in the Sahara says to the other: "Human to human" !**

Causal Algebras for Biological Systems

The preceding discussion has shown us that the Causal Algebras employed in the modeling of biological systems should contain families of constants, variables, functions and operators governed by the following restrictions :

(1) *Substrate dependency*. The metaphysic of the world-line has no relevance to biological systems. The nearest equivalent to this entity is the *interaction surface* expressing the co-dependency of organism to substrate. Example : the Logistic Difference Equation of population biology $y = \lambda x(1-x)$, and the corresponding Feigenbaum chart of period doubling.

(2) *Irreversible temporal direction* : Periodic, or even stationary, equilibrium states are prohibited in the functional descriptions of organic trajectories.

(3) *Temporal upper and lower bounds* on the back-reconstruction of 'birth' and the predicted time of 'death'. These bounds must be computable from the local configuration surrounding any moment of the organism's life span .

(4) The abrupt disintegration of the organism at death should be modeled as the resultant consequence of damage, injury or destruction to some unifying, hierarchic principle within the organism-substrate interaction. This takes the form of a jump discontinuity in entropy, with a corresponding drop of the energy of the system down to the ground potential of inert matter. We see that this collection of recipes for biological causation is far removed from the simplistic world-lines of physical systems, as described by a Hamiltonian flow in phase space.

There are special difficulties associated with condition (1): substrate dependency. Relative to pre-determined and fixed substrates (air, water, food, etc.) the living organisms tied to them (For example, animals in the vicinity of a watering hole, etc.) can, in theory, be assigned a " Lagrangian" . The local configuration of the organism at any moment generates a function beginning from some ground state and terminates at some precise instant. In between these cut-offs the trajectory is roughly deterministic. *Such a scenario cannot be modeled by families of analytic functions, which lack these properties.*

We will not, at this moment, further elaborate on the construction of Causal Algebras for biological systems, though we intend to do so in subsequent editions. The subject is truly enormous, perhaps as vast as life itself which is limitless: epigenetic landscapes, Catastrophe Theory, Chaos Theory, the Michaelas equation, evolutionary dynamics, even wide-ranging philosophical systems such as that of Teilhard de Chardin, must all enter into such a project. Key issues are substrate/organism interactions, ecological dynamics , the respective roles of teleology, freedom and accident, the identification of the "fundamental particles", or "systems in isolation" appropriate to the life-sciences (genes, cells, organisms, etc.) . Even the moral issues specific to biology (and to no other "hard science") may be amenable to algebraic descriptions.

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END OF PART I

On The Algebraic Representation of Causation

Giving Mathematical Form to Philosophical Decisions

Part II

Table of Contents

I. Lagrange/Hamilton Paradigms

(a) Mathematical Representations of the Concept of the World-Line

(i) Elaboration of the Dynamical Trajectory

More suitable to the workaday world of scientists than the Leibniz-Kant paradigm, the Lagrange- Hamilton paradigm has been the *modus vivendi* for the physical sciences since the time of Galileo. It has four parts:

(i) It is permissible to conceptualize arbitrarily large yet bounded regions R of our universe, over arbitrarily large yet finite intervals of time

$T = [-\tau, \tau]$. The combination of the spatial and temporal interval produces a space-time region $\boxed{V = R \otimes T}$. The duration of T, 2τ, is centered on a distinguished point $t = 0$, the origin, or "now" moment of time. R may be expanded or diminished to accommodate the world-lines of any system placed within it, as a function of τ. In the modern terminology of General Relativity, the 0-point of time may be interpreted as a "Cauchy surface", a region in which the notion of a "present moment" is definable.

The presence of all forces , particles or fields originating outside V is either negligible or proscribed. What this means in practice is that these entities can be neglected as 3rd order or higher perturbations. One must go to the 3rd order , dx^3 , because Newton's Laws are first and second order in the time derivatives.

More generally one can speak of arbitrarily shaped "force vacuums" on which external force fields exert at most a k^{th} order perturbation $k \geq 3$.

(ii) It is permissible to assert the presence of physical *systems in isolation* S, idealized from real observation, together with their world lines, operative in these force vacua V . Generally speaking, Lagrangian dynamics is a local theory and does not deal with infinitely large spaces or processes of infinitely long duration . One can of course admit them as idealizations in the description of strange attractors and related phenomena.

(iii) Let S be a system in isolation. *In the Lagrangian formulation* it is permissible possible to represent S, at some moment t=0 in time, as a single point in an affine *Configuration Space* of finite dimension 3N , N being the number of point masses . *In the Hamiltonian formulation* one may speak more generally of representations in a *Phase Space* of 6N-k dimensions , whose coordinates cover the positions and generalized momenta of the particles, less the number k of constraints. S's temporal trajectory through either the Lagrangian configuration space or Hamilton's phase space is called the system's world-line , L_S .

The term "affine" is employed here because no apriori metric properties are assumed to hold *apriori* between points of these spaces. Their elucidation often gives rise to measurement problems characteristic of specific disciplines. It is customary to place metric,

differential and simplectic structures on phase spaces, particularly in Statistical Mechanics, but these are not essential to the notion of the world line and the system in isolation.

(iv) In both directions from the "present " t = 0 of a hypothetical observer , L_S is a single-valued function of its coordinates at that moment, (know as "initial conditions") , and its behavior in any infinitesimal neighborhood around that moment . This is true whether one is dealing with configuration space, phase space, or any of the prolongation spaces suitable to the equations of motion.

The principles governing the local determinism of Lagrangian dynamics may therefore be summarized in the 3 key expressions *system in isolation* , *world-line* , and *initial conditions* .

It is (iv) which supplies the crucial Lagrange-Hamilton principle of causation that governs the construction of Causal Algebras of agents
(operators and functions) over phase spaces. Notice that (iv) is not inherent in (i) , (ii) and (iii) . These rather express *independence from*
***principles of global determinism* , i.e. such things as Mach's Principle, the Hubble Expansion Field, the Cosmological Constant, the Anthropic Principle, Cosmic Inflation and so forth. Indeed, as we have seen from the work of Hawking, Ellis, Penrose, Geroch and others who have sought to describe the large-scale structure of the universe on the basis of Einstein's Field Equations, Lagrangian scenarios are not sufficient to this task. A considerable amount of extrinsic topological structure must also be imposed , entailing many difficulties in the designation of singularities.**

Condition (iv) may be called the *principle of local determinism* . As was shown by Hume , it is not capable of demonstration or proof. Despite this, ever since Laplace, many scientists have accorded it the status of an article of dogmatic faith that approaches religion.

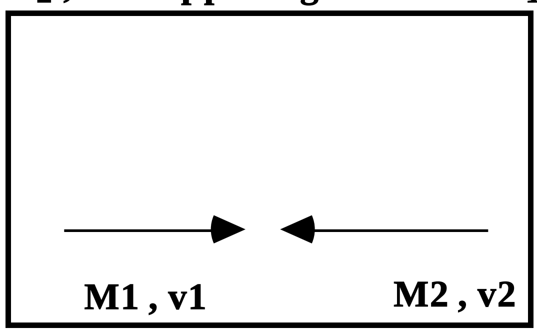
The distinction between ‘direct’ and ‘indirect’ , or ‘manifest’ and ‘derived’ information about the configuration of L_S in the neighborhood of an instant is crucial to Lagrangian causation. The *direct information* consists of the actual values of the 6N-k coordinates at the origin: the initial conditions. In and of themselves, these are insufficient: a photograph taken of the system at this moment will tell us nothing about its potential evolution or antecedents . The shape of L_S before and after the present is *indirectly* constructed by drawing a tiny hypersphere centered on the location of S in the *configuration space* V at the temporal origin. The distinction between “phase space” and “configuration space” mirrors the distinction between “direct” and “indirect” information.

Indeed, the complete specification of momenta and positions represented by a single point in the phase space is required only for the laying down of initial conditions of the trajectories in the configuration space . In this loose sense, phase space acts like a tangent plane , or more accurately, a symplectic space, in which are housed the Lie algebra of vector fields that determine the flow of world-lines in the configuration space. One then applies some computational process over or within that hypersphere that records local invariants obtained through its shrinkage back to the origin. This is what is meant by indirect information. Traditionally this has been assumed to be the taking of derivatives. As we shall see, there exist other possibilities as well.

(ii) Collisions

Although collisions are characterized by discontinuous jumps in momentum in the encounters of particles, they do not violate the principle of local determinism. Let us carefully examined the situation :

Consider the moment of impact of particles, P_1 and P_2 , with masses M_1 , M_2 , and opposing velocities v_1 , v_2 :



This may be identified in Phase Space by means of a single coordinate

$$(x_1, x_2, p_1, p_2)$$
$$p_1 = M_1 v_1 \; , \; p_2 = M_2 v_2$$

Before the collision, the equations of motion are simply

$$x_1 = A + v_1 t$$
$$x_2 = B + v_2 t$$

where A and B are arbitrary starting points. The world line is determined by a pair of polynomials in two variables, invariant first integrals of in the momenta :

$$Momentum = K = p_1 + p_2 = M_1 v_1 + M_2 v_2$$

$$Energy = E = \frac{p_1{}^2}{2M_1} + \frac{p_2{}^2}{2M_2} = \frac{M_1 v_1{}^2 + M_2 v_2{}^2}{2}$$

We can use these equations to determine the new momenta and velocities after collision, and from this compute the values of the x-coordinates. There is a transfer of momentum in the amount

$$\mu = \pm \frac{2(M_2 p_1 + M_1 p_2)}{M_1 + M_2}$$

to each of the particles, the new total momenta being therefore

$$p_1^* = p_1 - \mu$$
$$p_2^* = p_2 + \mu$$

with corresponding velocities

$$v_1^* = v_1 - \frac{2M_2(v_1 + v_2)}{M_1 + M_2}$$
$$v_2^* = v_2 + \frac{2M_1(v_1 + v_2)}{M_1 + M_2}$$

A collision is therefore a singularity of a special kind. **It is still true that the time evolution may be calculated from information in an infinitesimal hypersphere drawn around the instantaneous location.** ***However the forward and backward shapes of the world-line must be derived separately*** **. Thus although the invariant integrals of the motion are analytic everywhere, the specific equations of motion are only forward (backward) analytic from the moment of collision, at which time there is a jump caused by the transfer of a discrete quantity of momentum between the colliding particles .**

Leibniz expressed dissatisfaction with this traditional treatment of collisions. He pointed out that since perfect "hardness" does not exist in nature, collisions are never totally abrupt but have to involve a deformation in the shape of each of the particles. The trajectories of their elastic rebound can therefore be hypothesized as being analytic:

This is what is meant by saying that a Lagrange-Hamilton continuum is a symplectic manifold: it combines the affine structure of configuration space with its tangent bundle. Via the Poisson bracket, the collection of invariant integrals of the

Hamilton , or equivalently, the set of functions that commute with the Hamiltonian, form a Lie algebra which, through the principle of local determinism, completely generates the dynamic flow of world-lines through V.

The attribution of physical reality to the concept of the infinitesimal neighborhood is a central feature of those notions of causation which are traditional to science since the 17th century . The *instant* , *place* , *infinitesimal displacement* and *infinitesimal neighborhood* are the magnitudes fundamental to this world view. The proper topology for the representation spaces of Lagrange-Hamilton causation is over the Cartesian products of points and neighborhoods, as exemplified by the "jet spaces" of Singularity Theory and Differential Topology.

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(b) Algebraic Models for the Lagrange-Hamilton Paradigm

Most of the differential equations of classical physics are second order in their derivatives and partial derivatives: Newton's field equations, Maxwell's equations, the wave equation, heat equation and so on. Classical solutions for them are preferentially those which are not only C^N for some integer N , but analytic everywhere save at perhaps a collection of isolated singularities that can be calculated from the equations themselves.

The motivation for this preference is straightforward : the unfolding of their entire history in both directions in time may be computed from the information available in an infinitesimal neighborhood about any point. Analytic functions or families of analytic functions exemplify the basic property of the Lagrange-Hamilton paradigm. In line with the previous discussion

concerning collisions, singularities of a certain kind may be admitted. Though they require special treatment, they do not violate the principle of local determinism.

The general meromorphic function is determined by a Taylor-Maclaurin series up to its radius of convergence. By analytic extension its domain may often be extended over the sheets of their appropriate Riemann surfaces. Such extensions may sometimes (as in Feynman's Q.E.D.), have a physical interpretation.

This remarkable consonance between the conditions for Lagrange-Hamilton causation and the properties of analytic functions has produced a consensus in the physics community over the last 300 years, to the effect that for any situation in which local determinism is applicable, the universe will conveniently throw in some algebraically structured collection of analytic or meromorphic functions for its governance .

To take a simple example: under the assumption of analyticity, if a function ϕ and all its derivatives at points t_0 and t_1 are identical , then ϕ must be periodic with period $\omega = |t_0 - t_1|$. Yet there are alternative possibilities: consider the standard Helmholtz equation $\boxed{dx^2/d^2t = -A^2x^2}$, A real .

If admissible solutions are restricted to analytic functions, they will be taken from the family of trigonometric equations:

$$\boxed{x(t) = R\sin(At+\alpha) + S\cos(At+\beta)}$$

Nothing inherent in the equation itself obliges us to make such a restriction. There may arise situations for which solutions need only be at most C^k for a fixed integer k . Examine the following list of differential equations:

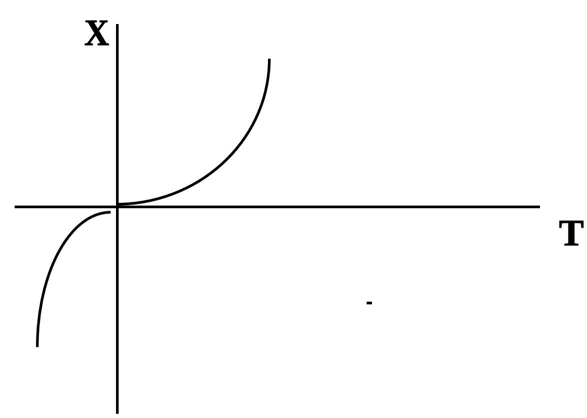
$$\boxed{\begin{array}{l}(1)\ x'(t) = x(t)\\(2)\ tdx - 2xdt = 0\\(3)\ x'^2 = \tfrac{3}{2}xx''\\(4)\ x'' = -A^2x\end{array}}$$

The form of (1) compels its solutions to be analytic: iteration of the derivative produces the sequence $\boxed{x^{(n)} = x^{n-1} = \ldots\ldots = x' = x}$ **with solutions of the form $x = Ae^t$.**

However (2) possesses both analytic and non-analytic solutions. The analytic solutions are $\boxed{x = {}^{g}\!/\!_{2}\, t^2}$ **, g an arbitrary constant historically associated with gravitation .**

A class of functions which are also solutions of this equation is :

$$\boxed{x = ({}^{g}\!/\!_{2})t|t|}$$



x=(1/2)gt|t|

These functions are continuous and first differentiable; the *second* derivative becomes singular at t=0 . One might argue that strict causality breaks down at this point. *Yet it is not causality per se, but the Lagrangian paradigm that fails* : the trajectory in the neighborhood of any time to the left of the origin does not contain enough information to predict the shape of the trajectory to the right of origin. One could imagine a particle in free fall

against the earth's gravity which at a certain moment is deflected by an electromagnetic field. One can speak of a breakdown of causation, yet one can also invoke the intervention of a new C^1 determinate force. In any case, without the restriction to analytic solutions the mere form of equation (2) is not sufficient in itself to describe a Lagrange-Hamilton causal trajectory .

As for equation (3) , it also admits an alternative, non-analytic solution :

$$\boxed{x = k|t|^3}$$

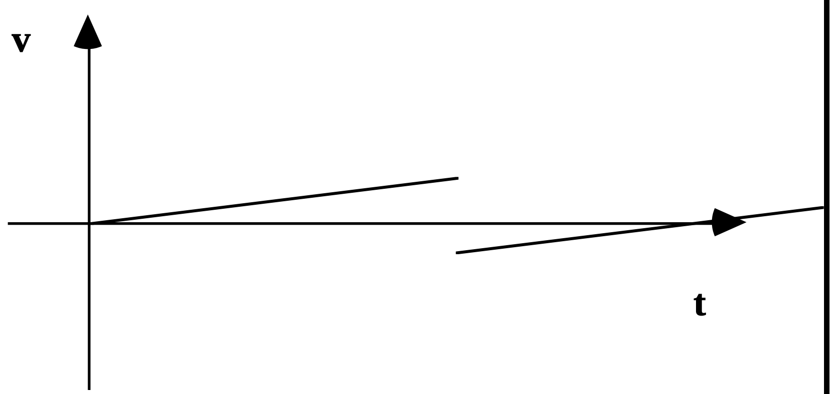
This has derivatives up to the second order at 0 but not beyond.

Equation (4) is the traditional Helmholtz equation. By introducing a slight modification of the definition of an n^{th} order derivative, a modification reflecting the difference between the bi-directional time of theoretical physics, and the mono-directional time of real measurements, we will construct non-analytic solutions of (4) :

Definition: ***Suppose that a function x(t) has a well-defined n^{th} order derivative everywhere in the neighborhood of (0) , but not necessarily at 0 itself. If*** $\boxed{\lim_{t\to 0^+}\frac{d^n x}{dt^n} = \lim_{t\to 0^-}\frac{d^n x}{dt^n}}$ ***, given that both limits exist, then we will call this common value the n^{th} derivative of the function at this point.***

The advantage of this definition is that it is possible to define n^{th} derivatives at points for which not all previous derivatives are well-defined. Such a convention does in fact reflect the usual way of measuring acceleration. Acceleration is not measured "after the fact" : change is always in the forward direction of time. Thus , the convention of taking the t --> 0+ limit of the rate of change of the velocity as the "left" acceleration at

that point, seems the most reasonable. If this is the same as the "right" acceleration, then we are justified in calling it simply, ' the acceleration' , despite the possible existence of a "jump" in the velocity :



The Velocity Jump Leaves Acceleration Invariant

Given this convention, one can admit the following class of non-analytic solutions for (4) :

$$\boxed{x = r\sin(A|t| + \alpha) + s\cos(A|t| + \beta)}$$

These might, for example, describe harmonic wave fronts which "spontaneously" reverse direction at t=0 in such a way that the energy flux is unaltered.

(i) Non-Differentiable Models for Lagrange-Hamilton Causation

Nothing inheres in the structure of Lagrange-Hamilton causation that requires modeling by families of analytic or meromorphic functions:

ANY algebra A of functions of time such that , for every element $f(t)$ ε A, there exists a means of extracting sufficient information from the infinitesimal neighborhoods of arbitrary points in the domain of f to reconstruct and back-reconstruct f's complete trajectory , can serve as a model for Lagrangian causation.

Although the combination rules preserving analyticity are of great generality, making them the natural choice for almost also situations in the real world, they do possess inherent limitations because of which they may be unsuitable in certain instances. Let us review the rules of extrinsic structure for some familiar collections of analytic functions in the complex plane:

(A) Extrinsic combination rules for the set C^R of all analytic functions of radius of convergence ≤R , R > 0

If f, g ε C^R , then:

(i) af +bg ε C^R , a,b complex constants

(ii) fg ε C^R

(iii) f(g) ε C^R , when $|g| \leq R$

(iv) If f has an infinite radius of convergence , then f(g) ε C^R

(v) Let S ≠ R, both >0 . Via the homogenous transformation

z --> (S/R)z all classes of analytic functions with finite radii of convergence are equivalent: $\boxed{f(z) \in C^R \leftrightarrow f(Rz/S) \in C^S}$

(vi) S > R --> $\boxed{C^S \supset C^R}$

(vii) f' and the anti-derivative definable from the Taylor's series are both ε C^R .

(viii) One can also develop a set of extrinsic rules based on functions defined on the coefficients of the corresponding Taylor's series. For example, if $\boxed{f = \sum a_n z^n}$ **is ε C^R, then the function g given by** $\boxed{g = \sum a_n^2 z^n}$ **is also in this class. In general any function of the coefficients which, after a certain point, diminishes their absolute value, will produce a new element of C^R .**

(B) Additional extrinsic combination rules for the collection D of analytic functions of infinite radius of convergence.

(i) $\boxed{f,g \in D \rightarrow f(g) \in D}$

(ii) **f ε D , g ε C^R --> f (g) ε D**

Limitations on classes of analytic functions become apparent when we examine their situation relative to spaces

(a) F^R of all complex functions defined in a circle |z| < R, and

(b) F of all functions defined for all complex z . The function algebras C^R and D do not constitute *ideals* , (under any of the composing extrinsic relations) as subspaces of the above. This means that if $\boxed{\phi \in C^R, \text{ and } \psi \in F^R}$, then $\boxed{a\phi + b\psi \text{ , } \phi\psi \text{ , } \phi(\psi) \text{ , or } \psi(\phi)}$, will not in general be analytic with radius of convergence R .

In the next section, we will examine some causal function algebras with the following properties:

(1) They model Lagrangian causality.

(2) They do not, in general, have the range of extrinsic structures found in closed algebras of analytic functions.

(3) Some are right ideals under functional composition in the space F_M of *bounded real functions on the real line* . In other words If K is such an algebra, f ε K , and g ε F_M , then g(f) ε K .

(c) Densely Periodic Function Algebras

By a densely periodic function on the real line we mean a periodic function that has no smallest period. That such functions exist is apparent from this simple example :

$$\boxed{\xi = \begin{cases} 1 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}}$$

Another example is supplied by the classic non-measurable Vitali set V from Lesbesgue measure theory. If A is an arbitrary set of reals, let

{ x + A } designate the set obtained by adding the number x to every element of A . Q is , as is customary, designates the set of all rationals on the real line . If r_i and r_j are distinct rational numbers then the Vitali set has the following properties :

$$\boxed{\begin{array}{l} (a)\{r_i + V\} \cap \{r_j + V)\} = \varnothing \\ (b) Q + V = \mathbb{R} \\ (c) Q \cap V = \varnothing \end{array}}$$

Each rational therefore translates V into a unique non-overlapping set on $\boxed{\mathbb{R}}$; the complete collection of translates cover the real line. Let P(x) be any real function defined on the domain V, and extend it to the rest of $\boldsymbol{R}$ by the construction $\boxed{\forall x \in V, r \in Q : P(x+r) = P(x)}$. Every rational is therefore a period of P, and there is no smallest period.

Theorem I: **Let L be the collection of all periodic functions on R which have no smallest period. Then L satisfies the conditions for an algebraic model for Lagrange-Hamilton causation.**

PROOF: Let ρ be an element of L, x any point in its domain. By definition the values of ρ in arbitrarily small neighborhoods N around x go through its entire range. Thus, its entire trajectory over all of R can be constructed from the translates of N by any of its periods.

It follows that any natural phenomenon which can be modeled by a family of functions of L is governed by local determinism causality.

In passing we may remark that "causation" is not inherent in the structure of the world line alone, but also depends on the relationship of the function modeling that world-line to the class of all the agents of its causal algebra. The causal structure of a system S may be interpreted as a form of local determinism when derived from a causal algebra A, yet subject to some other forms of causation if the functions that produce it are elements of another causal algebra, B :

Consider the differential equation:

$$\boxed{dx/dt = \frac{kx}{t^3}}$$

with solutions

$$\boxed{x = Ae^{-\frac{1}{2kt^2}}}$$

This is not analytic at t = 0, although it is C^∞ . All of its derivatives vanish at t= 0 , therefore no causal information may be obtained from them. However if, rather than working in the algebra of analytic functions, we work in the algebra of functions defined by a Laurent Series , we can represent the above class of solutions as

$$\boxed{\begin{aligned}&x = f(t) = 0\,,(t=0)\\&= A\left(1-\frac{1}{2kt^2}+\frac{1}{2!(2kt^2)^2}-\frac{1}{3!(2kt^2)^3}+..+\frac{(-1)^n}{n!(2kt^2)^n}+\ldots\right),(t\neq 0)\end{aligned}}$$

Densely periodic functions may appear to be somewhat " far-fetched" in terms of modeling natural phenomena, yet they figure in the abstract theory of time measurement and clock construction:

Let D be the integral domain of all real numbers of the form:

$$\boxed{\begin{array}{l} d = l + m\alpha + n\beta \\ (l, m, n = 0, \pm 1, \pm 2, ..,) \end{array}},$$

where α is any non-rational algebraic number, and β any transcendental number. Let

$$\boxed{\Gamma(t) = \{\begin{array}{ll} 1 & t \in D \\ 0 & otherwise \end{array}}$$

Γ is a member of the causal algebra of functions defined by such integral domains, D. Let's say that our theory of causation implies that all of our modeling functions will belong to D . Since D is everywhere dense in R, knowledge of the behavior of Γ in any infinitesimal interval gives enough information to compute α and β , and thereby the whole trajectory of Γ .

One can interpret such functions Γ as models for the pulsation of a system of 3 coupled clocks with incommensurable periods. When Γ = 1, one can imagine a bell rings. Two such rings set up an interval which, at it will be translated periodically, can define a unit of temporal measurement. When Γ=0 there is silence, and no unit can be established. The function Γ provides us with a model for quantized time.

THEOREM II : If P(x) is a *continuous* periodic function on R, then it either has a smallest period or is a constant:

Proof : If P(x) has no smallest period, then those values x for which

P(x) = P(0) will be dense on the real line. Since P(x) is continuous, it must be a constant.

Corollary : If periodicity is the essential feature of local determinism for a system, then its causal function algebras will contain only constants or elements of L .

Let us now examine some interesting sub-algebras of L :

1. Functions with no smallest period, all of whose periods are commensurable.

2. Functions which have periods of length $1/2^k$, k = 1,2,..... These take on an identical value in their range at all points of the domain expressible as a finite binary decimal.

Example : **Let:**

(a) f (x) = 1 , when x is a finite binary decimal

(b) If x is rational, and x = p/q in lowest terms, then let

f(x) equal k , where q = $k2^n$, and n is the highest exponent of 2 in q.

(c) If x irrational, then f(x) = 0

In certain situations , such non-analytic Lagrange-Hamilton models may be a better reflection of the discrete semigroup structure of actual experimental time, than the "everywhere continuous time" which is usually the precondition for algebras of analytic functions.

3. Functions with two, more, or even an infinite number of mutually incommensurable periods. These have already been discussed in connection with the sets, D.

4. Generalizations of the Vitali construction: let $\mathbf{B} = \{ \beta\mu \}$, be a *Hamel Basis* for the real numbers. The construction of a Hamel basis, the details of which need not concern us but which depends upon the Axiom of Choice, turns the real line into an infinite dimensional vector space over the field of the rationals, Q . Any real number ζ , can be decomposed in a unique fashion as the dot product of a finite number of elements $\boxed{b_{n_1}, b_{n_2}, \ldots b_{n_k}}$ from the basis . with rational numbers $\boxed{r_1, r_2, \ldots, r_k}$ from Q , such that $\boxed{x = r_1 b_{n_1} + r_2 b_{n_2} + \ldots . r_k b_{n_k}}$

Starting with a basis B , we separate it arbitrarily into a pair of arbitrary disjoint subsets B_1 , B_2 . Let C_1 be the collection of reals built on Q and B_1 , while C_2 is the collection of reals built from Q and B_2 . Then $\boxed{\begin{array}{l} C_1 \cap C_2 = \varnothing \\ \{C_1 + C_2\} = \mathbb{R} \end{array}}$

Define a function $\sigma(x)$ arbitrarily on C_1 . If μ is any element of C_2 , x any real number, then we define $\sigma(x+\mu) = \sigma(x)$. This is an element of L .

It is unlikely that Hamel bases have much application to physics, but they fill out the mathematical picture. We should not forget that applications for non-standard arithmetic have been found in Quantum Theory, and categories in Quantum Field Theory. At a certain point, the application of exotic mathematical disciplines to physics becomes more than a little fanciful, but what may appear exotic today can become the stuff of high school education a few millenia down the future.

(i) Composition Laws for Densely Periodic Function Algebras

(a) . In general, ϕ and ψ ε L does not imply that $\theta = a\phi + b\psi$ will be so. However, if ϕ and ψ have the *same set* of periods, or if the set of periods of one of them is a subset of the periods of the other , then θ will be a member of L .

(b) If ϕ ε L, then functions of the form $\lambda = a\phi(kt + r) + l$, a, k, r , l arbitrary constants, will belong to L with a different period set and a phase. L allows for translations in both time and space .

(c) *L is a left ideal in the space of all bounded functions , B* . This property is common to all periodic functions. It is, naturally,

a one-sided ideal only : if ϕ is in L , and ψ is any bounded function whatsoever, then
$\theta = \psi(\phi)$ will be a member of L , although $\kappa = \phi(\psi)$ may not be so .This property expresses an important feature of this form of Lagrange-Hamilton determinism. Observe that the space of all periodic functions does not, in and of itself, constitute a causal algebra for modeling Lagrange-Hamilton causation: one can, for example, envisage functions that are completely random in some closed interval [a,b] , which randomness is then propagated periodically over all time. Conversely, although the analytic function algebras are Lagrange-Hamilton , they are not ideals in B

The next theorem expresses another important feature of L .

Definition : **By a *systematic disturbance* is meant any perturbation of a world-line which systematically modifies positions, times and velocities but leaves the Hamiltonian, H, itself invariant. The notion of what constitutes a Hamiltonian is of course generalized to any sort of action principle from which the laws of motion may be obtained. Algebraically a systematic disturbance Δ may be thought of as a set of functions of one variable,**

$$\boxed{\begin{matrix}\Delta_0,\Delta_1,\Delta_2,....,\Delta_n,\\ \Omega_0,\Omega_1,\Omega_2,....\Omega_n\end{matrix}}$$

such that

$$\boxed{\Delta_0 H(q_i,p_i) = H(\Delta_1 q_1,....,\Delta_n q_k;\Omega_1 p_1,...\Omega_n p_n)}$$

Theorem iii : **Let W be a universe whose causal laws are modeled by functions from L. Then arbitrary systematic disturbances can be admitted into W without violating its causal structure.**

As examples of systematic disturbances one may cite some universal transformation that instantaneously doubles all

velocities, or one that shifts all light spectra to the red. By acting on the left, the invertible functions of $\mathbf{B}$ are systematic disturbances: periodic phenomena continue to be periodic and there is no change in the causal structure.

Since the subspace of analytic functions is not an ideal in $\mathbf{B}$, either right or left, systematic disturbances in an analytic model may violate causality. Let $\mathbf{W}$ be a universe operative under a cyclic time, whose expansion-contraction cycle imposes a fixed period ρ on all systems of $\mathbf{W}$, and let Ψ be a harmonic oscillator, or clock, constructed to oscillate at a period ζ which is incommensurable with ρ .(Note that clocks in $\mathbf{W}$ with commensurable periods do not imply Lagrange-Hamilton determinism , but that incommensurable ones do! This is because they must pulse with two periods the pulsation points of which form an integral domain dense in the time continuum .) No left-acting systematic disturbance on $\mathbf{W}$ will alter its period from ρ , nor will it alter the nowhere continuous dual periodic structure of Ψ .

II. Substrate-Dependent Lagrange-Hamilton Causation

One can also consider non-standard Lagrange-Hamilton causal algebras $\mathbf{A}$ relative to a fixed energy substrate. These may find applications in biology, notably in the description of biological clocks.

The agents of such algebras will be dependent on a substrate domain D which, by convention, can be interpreted as some combination of energy sources. In addition to the basic forces of nature, D would carry information on nutrients, chemicals, air and water, soil, sunlight, and other renewable or non-renewable resources. In the simplest models such as one finds

in the study of local ecologies, the functions of the substrate algebra will share the domains of the same "survival parameters".

For the application of non-standard models for local determinism over a substrate, one can look at a more general class of functions Ξ including L , ***with the property that every value of their range is attained in every neighborhood of their domain*** **: densely periodic functions, everywhere continuous nowhere-differentiable functions, space or volume filling functions, fractals, etc. Because of the imaginative work of Benoit Mandelbrot and his followers, such objects are no longer considered pathological.**

Fix a function φ in Ξ . We define Λ_φ as the class of all functions $\theta \in \Xi$, such that for any two moments s and t ,

$$\boxed{\varphi(s) = \varphi(t) \leftrightarrow \theta(s) = \theta(t)} \,.$$

Under the condition that the complete trajectory of φ is given, known in advance, Λ_φ is a substrate -dependent causal algebra. One need compute the values of θ only in any tiny sub-interval of the domain, then use its homology with φ to extrapolate over its entire graph.

For more specific applications, we can weaken the conditions on Ξ as follows: we admit elements α ***for which an interval N exists*** **in which α attains to every value in its range in every subinterval . Fix one such function, α to be called the** ***standard referent function*** **. Construct, before, the function algebra $\Lambda_{N,\alpha}$. This is a Lagrangian causal substrate algebra, the substrate now being the region N. The elements of $\Lambda_{N,\alpha}$ will be determined in N, but not necessarily beyond it .**

Λ_φ and $\Lambda_{N,\alpha}$ have more extrinsic structure than Ξ : if f and g are in Λ_φ , then h = af + bg , a , b constants, will also be in Λ_φ . Λ_φ is also an ideal in B .

Notice, however, that it is not the case that

$$\boxed{q(t) \in \Lambda_{\varphi} \to q(kt+h) \in \Lambda_{\varphi}}$$

for constants k and h ! Substrate algebras are not invariant under time translations or affine space transformations.

The whole notion of substrate algebras, or causal models whose predictive algorithms depend on knowledge of a previously predicated process or family of functions, can be richly elaborated within the context of Lagrangian and other general causation schemes.

For example, let γ be some fixed referent function, uniformly continuous and non- constant in any interval (a,b) , and $\mathbf{A}$ the class of all analytic functions. Then $\mathbf{A}\gamma$, the class of all functions of the form

f = g (γ (t)) , where g is analytic, is a substrate algebra. At any given time t, the value of γ is known. By construction it is locally invertible, either from the left or the right. This allows us to *locally* calculate all derivatives of g . Since the configuration of γ is known one can construct the complete trajectory of f from local information .

Substrate algebras, as already stated, find their natural domain of application in biology. In the above example γ could be temperature, $\mathbf{A}\gamma$ a class of "metabolism functions" dependent on temperature.

(a) General Observations on Lagrangian Causal Algebras

A reasonable requirement for any member of a Lagrange-Hamilton causal algebra is that all the procedures which allows the construction of a world line from the neighborhood of a point be computable. Then it is possible to replace infinitesimal neighborhoods by suitably chosen countably dense subsets,

thereby eliminating the need for exotic or 'pathological' constructions involving Hamel bases, etc.

Let $t_1 < t_2$ be distinct moments in time on the world line of a system S modeled by some Lagrange-Hamilton causal algebra. Let
$\boxed{H(q_1,\dots q_n; p_1,\dots p_n)}$ **designate some generalized form of Hamiltonian which, via some version of the Principle of Least Action, generates the equations of motion X:**

$$\boxed{\begin{array}{l} q_j = \phi_j(t) \\ p_j = \psi_j(t)\,;\, j = 1,2,\dots n \end{array}}$$

H generates a world-line in phase space. All of the above functions belong to the causal algebra. To derive H(t_2) from H (t_1) one needs to know the values of

(i) $\boxed{\phi_j(t)\,;\, \psi_j(t)\,;\, j = 1,2,\dots n}$

(ii) A countable, indexed set of parameters { a_n (t_1) } computable from the defining processes of the causal algebra

(iii) Some invertible connection between t_1 and t_2 which, basically a formal scheme like a power series in time, or a collection of periodic cycles, which we write as C(t_1 , t_2) . The general structure of a Lagrange-Hamilton causal algebra may then be described in the most general terms by the metaphorical equation:

$$\boxed{H(t_2) = \Phi[H(t_1);\{a_j\};C(t_1,t_2)]}$$

Here the procedure Φ is the same for all agents of the causal algebra.

III. Essential Singularities and Point-Source Algebras

A *Lagrangian Point-Source Causal Algebra* over a system S is defined as follows:

(i) Lagrange-Hamilton causation is no longer assumed to hold in S for *all* moments on the time continuum. Instead one admits *only a discrete set of points Ω* , which may be countable or finite *, or simply a unique starting point t=0 ,* at which the Lagrange-Hamilton property holds. From the infinitesimal neighborhoods of the moments belonging to the set Ω it is possible to compute that portion of the world line of S going from it to the right (present to future) , up till the next element of Ω . The distinguished moments belonging to Ω will be called "seeds" .

On The Algebraic Representation of Causation

Part III

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I. Causality and Coding

Codes

Let A_L be some Lagrangian, or substrate dependent Lagrangian Causal Algebra. γ(t) is any function in A_L , with domain $0 \leq t < \infty$, and uniformly bounded range. Let $t_1 < t_2$ be a pair of distinguished moments in the forward direction of time. Since γ(t) complies with Lagrange-Hamilton causality we can express the process whereby it constructs its trajectory from local information. information as:

$$\boxed{\Xi(\gamma(t_1),\chi(t_1,t_2)) = \gamma(t_2)}$$

Ξ belongs to the extrinsic structure of A_L , and represents the algorithms involving in computing a countable set Π = {α_j } of structural parameters (analogous to Taylor's coefficients or Fourier coefficients) from the configuration of γ(t) in an infinitesimal neighborhood of t_1 χ is a "connection operator" , (infinite series , Σ , for example) that calculates the value of γ(t_2) from t_1 , t_2 , and the set Π .

Let $\boxed{S = s_0, s_1, s_3, \ldots . = \{s_n\}, 0 \leq n < \infty}$ **be a sequence of half-open intervals of the positive real line :**

$$\boxed{\begin{array}{l} s_j = [a_j, b_j) \; with \\ 0 \leq a_j < b_j \end{array}}$$

There is no other restriction on these intervals, which may overlap or even be identical.

A *back-reconstructible code* @ will be defined as a map from the set of non-negative integers Z^+ onto itself. such that

(i) @ is a computable function Z^+ ---> Z^+

(ii) @ is back-reconstructible from any positive integer k . Given the values of @ (j) , for j ≥ k , there is a recursive formula for calculating @(k-1) . Thus, each range value of @ at integers k, can be derived from both :

(i) recursions on previous values 0 ≤n ≤ k-1 , and

(ii) recursions on all values k+1 ≤ n ≤∞ .

Here is an example of such a code:

C = 0,1,0,1,2,0,1,2,3,,0,1,2,3,4 ,......

From any segment of this sequence, one can figgure out that one is dealing with an arithmetic progression, which can then be back-reconstructed. These are examples of codes which cannot be back-reconstructed:

(a) C = 1 ,1,1,1,1,1,1,2 3,4,2,3,4,2,3,4,2,3,4,2,3,4,..... the sequence 2,3,4 going on forever. Since the infinite segment starting at entry 8 is identical to the one starting at entry 11, there exists no unique back-reconstruction from the infinite fragment 2,3,4,2,3,4,........

(b) C = 2,0,1,2,3,0,1,2,3,4,......

Although one recognizes this as an arithmetic progression, there is no way from an arbitrary sub-segment, to back-reconstruct the place at which the progression breaks off.

The application of @ to the indices of the set of half-open intervals S will build the arc of a new function λ (τ) , from sections of the arc of γ (t) . This suggests that the function γ must be

of distinguishable character **: Given a < b , there do not exist a pair of real numbers c < d , with l = b-a = d- c , and**

$$\boxed{\gamma(a+t) = \gamma(c+t)\,, \forall t\{0 \le t \le l\}}$$

Thus, no segment of the arc of γ is congruent to any other segment by horizontal translation. Next, subdivide the domain of γ into segments determined by the elements of S, and concatenate these arcs in a sequence determined by the application of the code @ . This process generates the arc of the derived function λ . Since @ is a causal code, knowledge of the shape of λ after any time T, combined with knowledge of the behavior of the initial

function γ , allows one to reconstruct the entire arc of λ from T back to the origin t = 0 .

Functions of the form ξ (t) = λ (1/t) provide us with an algebra for modeling point-source causality . They map ∞ into the origin, and intervals of time from T to ∞ into intervals from 1/T to 0 . As determined by the causal code , an infinitesimal neighborhood of the origin of ξ will contain infinitely many sections of the arc
of γ (1/t) .

II. Patterned Functions

The class of patterned functions is formed from the set of all functions over [0 , ∞) by means of a causal code designated the "pattern index function". They are of interest because they suggest numerous applications in the algebraic theory of causation. They can model point-source causality from essential singularities, branching causality and the braided causalities to be discussed in the final part of this essay.

DEFINITION: P (t) will be said to be patterned if, for any real number M>0, there is an L>0 such that P(t) = P(t+L) $0 \leq t \leq M$. In other words, any section of the arc of P is congruent by horizontal translation to a section of arc at some other location. Since the arc including these copies and the intervening interval is also a piece of the arc of P, it follows that any section of P's arc has an infinite number of congruent equivalents all along its range.

Periodic and constant functions are obviously patterned. Patterned functions are the simplest generalization of the notion of periodicity.

(a) The pattern index function

Let n be any positive integer, and factor it as $\boxed{n=(2k+1)2^{m}}$. m is the *exponent of 2 in n* . ψ(n)= m is the pattern index function . Its first few value are:

n	=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\varphi(n)$	=	0	1	0	2	0	1	0	3	0	1	0	2	0	1	0	4	0	1

Extending the index function to non-positive values becomes complicated: bi-directional patterned functions are much more complicated than uni-directional ones . For examples, it is possible to construct a patterned function for which the complete functional arc less than 0 is completely arbitrary. One then slices off progressively larger pieces of the negative portion of the arc from the origin, then concatenates them on the right side of the origin by means of the pattern index function.

The way in which the index function organizes the range of a uni-directional patterned function over the reals works as follows:

Let A_0 be some arbitrarily shaped arc over an initial domain $s_0 = [a_0 ,b_0)$:

To this we adjoin another arc A_1 over domain s1 $=[a_1 , b_1)$ then tack A_0 onto it :

A third arc , A_2 is added, and the arc $A_0A_1A_0$ repeated:

The method of construction is clear:

$$\boxed{P = A_0A_1A_0A_2A_0A_1A_0A_3A_0A_1A_0A_2A_0A_1A_0A_4.....}$$

The indices reproduce the range of the patterned index function. Equivalently, the code determined by ψ (j) has been applied to the set formed from pieces of the associated function A(t) .

(b)Terminology

The *section point set* Θ = {aj } is the collection of places at which the domain of the associated function is cut.

FIGURE

The places where new material is introduced into the arc of P , produces a set M={Mj} called the *pattern set* .

FIGURE

The pattern set is not unique: different versions of the pattern set, relating to different ways of defining the section point set, may still yield the same patterned function.

FIGURE

Patterned function with two pattern sets , { M_i } and { M_i'}

FIGURE

Corresponding associated functions and section points

If P(t) is not too pathological one can define a unique *minimal pattern set* M , which allows one to back-reconstruct a unique associated function A (t) .

Given a pattern point M_j , the *pattern length* Lj is defined as the largest interval , (starting from 0 and less than Mj) on which :

$$\boxed{P(t) = P(t + M_j),\ 0 \le t \le L_j}$$

Lastly there is the "D-series", D = {Dj}. These are the places at which arcs of the associated function are spliced onto those of the patterned function:

Assuming that M is the *minimal* pattern set, each of these entities can be calculated from the others. The formulae are presented below, without proof, since the methods for deriving them are straightforward:

(c) Formulae for patterned functions

$$(1.)\, M_k = L_k - L_{k-1}$$

$$(2.)\, a_k = M_k + \sum_{j=0}^{k-1} (j+1-k) M_j$$

$$(3.)\, M_k = a_k + \sum_{j=0}^{k-2} 2^{k-j-2} a_j$$

$$(4.)\, a_k = L_k - \sum_{j=0}^{k-1} L_j$$

$$(5.)\, L_k = a_k + \sum_{j=0}^{k-1} 2^{k-j-1} a_j$$

$$(6.)\, D_k = \sum_{j=1}^{k-1} (a_{\varphi(j)+1} - a_{\varphi(j)})$$

(i) The Algebraic Structure of Patterning

The translational congruence of sub-arcs of patterned functions gives rise to an algebraic structure which is a generalization of the group of translations characteristic of periodic forms.

Let $\boxed{N = n_1, n_2, n_3, \ldots\ldots, n_k, \ldots}$ **be a patterned sequence of positive integers, that is to say, a sequence with the property that any finite segment is exactly reproduced at infinitely many places.**

Assume that N has more than one distinct entry. If S_1 is a finite segment of N starting from n_0, then we define the operator T as the "gaped concatenation" of S_1 with its next distinct copy

along the length of N. That is to say $\boxed{T = S_1 \underset{(j)}{\wedge} S_1^{'}}$ **. The letter j indicates the size of the gap between the two copies of** S_1 **, while "^" is the symbol for concatenation.**

The operator "E" fills the gap with the next section from the associated function, S_2 **. Thus** $\boxed{ET(S_1) = S_1 \wedge S_2 \wedge S_1^{'}}$ **By convention E(E) = E. E is idempotent. If there is no gap in the concatenation, then E doesn't change the form of the section of the patterned function. It is the operator "E" which distinguishes a patterned function from a periodic function, which is built up through repeated applications of T without gaps.**

Applying the concatenation operators T and E in succession to (A,S), the associated function with section point set , one has

$$\boxed{\begin{array}{l} T(A,S) = S_1 \underset{j}{\wedge} S_1^{'} \\ ET(A,S) = S_1 S_2 S_1^{'} \\ TET(A,S) = S_1 S_2 S_1^{'} \underset{j}{\wedge} (S_1 S_2 S_1^{'})' \\ ETET(A,S) = S_1 S_2 S_1^{'} S_3 (S_1 S_2 S_1^{'})', \\ etc.... \end{array}}$$

The algebra of operators generating a patterned function from the associated function with section point set, consists of combinations of T and E: T applied to any segment of the domain of P , gap-concatenates it with the next identical segment. E takes any gapped form and fills the missing space with the relevant sections of the associated function.

(ii) Composition Rules for Patterned Functions

The combination rules for patterned functions determine the extrinsic structure, the "modes of possibility" , of their

corresponding causal algebra. Let D be the collection of bounded patterned functions with minimal associated functions on [0 , ∞) , and

let P ε D :

(1) If P is patterned, then aP(bx+c) is patterned

(2) Periodic and constant functions are sub-algebras of D

(3) If P and Q share the same set of pattern points (this need not be the *minimal set* in either case), then F(f,g) is patterned, where F is any function of two variables with domain in the Cartesian product of the ranges of f and g .

(4) D form a *right ideal* in the space F of all functions defined over $\boxed{R_0^+}$. If f ε D , g ε F , then g(f) ε D . This is true for all functions produced by the coding method: coded functional transformations are unaffected by any systematic alteration on their range.

(5) The algebra of patterned functions is structured by Boolean operations on the section point sets : If f and g are patterned functions derived from the same associated function ,A , but with different

section point sets, S_f and S_g , then one may construct patterned functions,

notated $\boxed{h = f \cup g, l = f \cap g}$, built up from A via the section point sets $\boxed{S_h = S_f \cup S_g, S_l = S_f \cap S_g}$.

For a more detailed picture of the dependency of the patterned functions on their associated functions and section point sets we display the appropriate inversion formulae.

(iii) Inversion Formulae

The analytic expression for the pattern index function φ(x), the exponent of 2 in x, is: $\boxed{\varphi(n) = \sum_{j=1}^{\infty}([n/2^j]-[n-1/2^j])}$**. For negative values, m = -n, this becomes**

$$\boxed{\begin{aligned}\varphi(m) &= \sum_{j=1}^{\infty}([1-m)/2j]-[-m/2j]) \\ &= \varphi(1-m) = \varphi(n+1)\end{aligned}}$$

The formula breaks down for n=0 as one expects for a multiplicative function. It is obvious from the above that

$$\boxed{\varphi(2^{\varphi(n)}) = \varphi(n)}$$

Since the j^{th} summand in these infinite series has period 2^{j}, one can compute a Fourier series for the function $\boxed{\varphi(x) \equiv \varphi([x])}$**:**

$$\boxed{\varphi \sim 2\pi + 2/\pi \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{\sin(k\pi x/2^m)\cos(k\pi(2x-1)/2^m)}{k}\right)}$$

Define functions $\boxed{[x]_D, \{x\}_D,}$ **as follows: If $D_j \le x < D_{j+1}$, then**

$$\boxed{\begin{aligned}&[x]_D = D_j, and \\ &\{x\}_D = j\end{aligned}}$$

It can be shown that the relationship between the patterned function, P, and its associated function A is given by

$$\boxed{\begin{aligned}&P(x) = A(x-[x]_D + a_{\varphi(\{x\}_D+1\}} \\ &A(x) = P(x + L_{\{x\}_S} - [x]_S)\end{aligned}}$$

(d) Braided Patterned Functions

These inversion formulae are not sufficient, without further information, to reconstruct the missing arc of a patterned function from any given point back to the origin. The “braiding” of the coding algorithm must also be considered :

We assume that we have complete information about the shape of a mono-directional patterned function in the interval [T, ∞)

Under what circumstances can the missing arc, L, be reconstructed, knowing only that P is a patterned function? There are in fact two classes of patterned functions:

(1) Those which, given the forward arc from T, permit the reconstruction of a unique arc from 0 to T, consonant with the patterning requirement, and

(2) Those which allow 2, 3, or even an infinite number of possible back-reconstructions.

(i) Functions which back reconstruct a unique arc, *from any point T*, will be called "unbraided" .

(ii) If the arc reconstructed from the point T_1 is unique , but there are several reconstructions possible from T_2 , then we will say that P is unbraided at T_1 and braided at T_2 .

A function which is braided everywhere will be referred to , simply, as a "braided patterned function" .

The alternative back-reconstructions P_1 , P_2 , P_3 ... for a patterned function at point T will be called the "braids" of the function at that point. On the basis of the previous discussion one sees that *unbraided functions can model point source causality*

We present two ways of constructing braided patterned functions:

[I .] Let A, B, C, D,.... be a set of distinct, finite functional arcs with domains on the x-axis. We will construct functions P_1 and P_2 from concatenations of these arcs. Apart from the requirement that the length of the domain of A be equal to that of B, the domains of the remaining arcs in the set can be arbitrary .
P_1 starts with A, P_2 with B

P_1 : A

P_2 :B

Adjoin C to both functions as “filler” , then adjoin A to both sequences:

P_1 : A C A

P_2 :B CA

Insert D as filler, and adjoin B to both functions:

P_1 : A C A DB

P_2 :B CAD B

Insert E as filler, then repeat ACA in both sequences:

P_1 : A C A DE ACA

P_2 :B CAD E ACA

Insert F as filler, then repeat BCADB in both functions:

P_1 : A C A DBE ACA FBCADB

P_2 :B CADB E ACAF BCADB

Insert G as filler and repeat ACADBEACA in both sequences. The lower bracketting indicates the patterning :

$$\boxed{\begin{array}{l} P_1 : A\underbrace{C}ADBE \,||\, A\underbrace{C}A \,||\, FBCADEG\underbrace{ACADBEACA} \,||\, HBCADBE \\[2em] P_2 : \underbrace{BCADB}EACAF\underbrace{BCADB}GACADBEACAH\underbrace{BCADBEACAF} \end{array}}$$

The method of construction is clear. Having reached a pattern point, of either P_1 or P_2 , one inserts a new length taken from the set of sections (the operator E) . Then one applies the patterning process, (the operator T) , to the other function.

The associated functions for P_1 and P_2 are quite different:

A_1 : A • C • DBE • FBCADBG • HBCADBEACAFBCADBI •....

A_2 : B •CAD •EACAF •GACADBEACAH •....

[II.] : Let κ and ρ and be two functional arcs of distinguished character over the x-axis. Form the set of all possible combinations of κ and ρ , enumerated by some counting method. For example:

$$\boxed{\begin{aligned} &C_0 = \kappa\ , \\ &C_1 = \rho\ , \\ &C_2 = \kappa\kappa\ , \\ &C_3 = \kappa\rho\ , \\ &C_4 = \rho\kappa\ , \\ &C_5 = \rho\rho\ , \\ &C_6 = \kappa\kappa\kappa\ , \\ &C_7 = \kappa\kappa\rho\ , \\ &C_8 = \kappa\rho\kappa\ , \\ &C_9 = \rho\kappa\kappa\ , \\ &C_{10} = \rho\kappa\rho\ , \\ &C_{11} = \rho\rho\kappa\ , etc. \end{aligned}}$$

Concatenate the C's into an unbroken sequence $C_0C_1C_2$ .... *No matter how the enumeration is done* , the resulting functional arc will be that of a patterned function that is braided everywhere. Also, the number of braids increases to infinity as the initial point T moves to infinity. Indeed, between 0 and T any combination of κ 's and ρ 's adding up to the length T , can be combined and the resultant function will be patterned.

Braided patterned functions will turn out to be very convenient for modeling some of the forms of branching causation described in the next section.

THEOREM 1: **Let P be a non-periodic *analytic* function over the positive reals. Then P cannot be a patterned function**

Proof: **Analytic functions model local determinism. Therefore any repetition of local conditions must produce an identical future. P must therefore be either periodic or non-analytic.**

Clearly, it is the very property which makes analytic functions suitable for the modeling of Lagrangian causality, that makes it impossible to cut and splice them to produce other analytic functions.

Theorem 2 : **Let Ω (z) be some non-periodic, analytic function over the reals. If Ω , in combination with an arbitrary section-point set Θ , is the associated function for some patterned function, P, then P cannot be braided anywhere.**

Proof : **Since A(z) is not periodic by hypothesis , all the points of a pattern set , M, must be non-analytic points on the domain of P**

The set of points at which P fails to be locally analytic is a subset of the D-series $\{D_j\}$, namely $\{ D_{2n} \}$. To derive the pattern point set

$M = \{M_j\}$ one can employ a simple congruence method: Since A(z) is neither periodic nor constant, *identical arcs on P must have come from the same section of A* .

Via this procedure one can reconstruct the section point set, and from this, combined with the analyticity of A, one can completely back- reconstruct P from any point , T. Therefore P has a unique back-reconstruction everywhere and is braided nowhere.

If A is periodic, then braided configurations can be produced because of the ambiguity of the locations of the section points .

III.Branching Causation

(a) Overview

One discovers that there exists a marked disparity between the assertions made by scientists about their axiomatic assumptions , and the methods employed in their actual research. For example, scientists working in different fields usually assume they're talking about the same thing when they speak of causality. However this is belied by the manner in which they interpret their findings and discoveries. We illustrate this observation through a brief survey over several major scientific disciplines:

(i)Mathematics

Mathematics is inherently acausal. A tautological relationship between primitive elements and pre-established axioms is not normally treated as causal. Causality implies both:

(i) Temporal dependence

(ii) Temporal asymmetry

Note that whereas (i) is basic to any discussion of causality, (ii) has been the subject to extensive debate ever since the origin of Thermodynamics. The facile resemblance of

" '*p implies q*' does not imply '*q implies p*' "

$$\boxed{\neg[(p \rightarrow q) \rightarrow (q \rightarrow p)]}$$

with

" '*h causes k*' does not imply that '*k causes h*' "

$$\boxed{\neg[(h \Rightarrow k) \rightarrow (k \Rightarrow h)]}$$

although both are asymmetries, does not introduce a causal structure into mathematics. The material implication " p implies q"

gives one no information concerning either the temporality or atemporality of p or q. Thus, the statement:

S: " If it rains today I had eggs for breakfast yesterday"

, is true regardless of the reversal of normal temporal orientation, provided only that it does rain today and that I did have eggs for breakfast yesterday. Indeed, for Aristotelian implication it is sufficient that eggs were on the menu of yesterday's breakfast!

That these observations are not trite can be seen from their relevance to the theory of questions. There are two kinds of questions: Logical Questions, which are addressed to "the universe" and thus have atemporal answers (such as "What are the first four places of the decimal expansion of the square root of 2?") ; and Contingent Questions, which do not have answers until the respondent answers them.

Example: I arrive at the opened door of a house, and shout " Is anybody home?" This has various answers. Note that "Yes" and "No" give the same information, as does "I'm coming", or "I'm not receiving visitors today." However *silence* has two interpretations : either no-one is at home, or *the respondent has used his freedom of choice* to remain silent . The interrogator cannot tell which of the two alternatives he is being presented with.

From the viewpoint of the interrogator this kind of question exhibits a temporal dependence, akin to causation in the natural sciences. That the question is asked *before* the answer is given is fundamental. The situation is analogous to that of the measurement problem in Quantum Theory: that the measurement is made *before* the data is recorded establishes a fundamental asymmetry, which is not present either in mathematics nor in classical theoretical physics.

Mathematics is acausal by its very nature. The suitability of mathematical frameworks for the modeling of phenomena is due to its acausality. If mathematical deduction were space-time dependent it could hardly be of much use to science.

(ii)Reconstructive Science
History, Geology , Paleontology, Archaeology

Sciences with a predominantly historical emphasis employ back-branching models and other forms of reconstructive causation, Their conceptions are generally much broader than the indirect proof of mathematics. It sometimes happens that an historic reconstruction leads to a unique scenario for some past event, the so-called "smoking gun". In practice, researchers in reconstructive science are permitted considerable leeway, when not laxity, in the multiplication of alternatives. There are many important questions for which definitive answers are never found. Either the evidence is too difficult to ferret out with current limitations on technology, or it has been permanently destroyed, by death, fire, erosion, etc.

Who were the first hominids? Were the dinosaurs birds or reptiles? What brought about the collapse of Mayan civilization? What are the origins of the Basque language? Did Mileva Maric give Einstein his ideas on relativity? Did Sally Hemmings tell Thomas Jefferson what to write in the Declaration of Independence? In reconstructive science one can expect to encounter a climate of contention, that can and frequently does turn nasty, centered around equally defensible models for some inaccessible past event.

Although many of these controversies do get resolved through new discoveries, the antinomies inherent to all reconstruction of the past will always remain . The most basic of all is, of course , the antinomian couple: *"The past does/ does not exist"* . In terms of its practical consequences the debate over *Uniformitarianism versus Catastrophism* , of which Geology is the paradigm, is the most important.

***Uniformitarianism* is an article of faith without which there can be no reconstructive science. It has two forms: the first asserts that it is possible to reconstruct the past by assuming that the laws of nature are time independent, and were therefore the same in the past as they are today. A more specific form, current in Geology since the work of James Hutton in the 18th century, and William Whewell and Charles Lyell in the early 19th, is that the processes at work on the earth in the distant past are still present today. Even more restrictive is the view that none of the geological formations of the past were radically different from those in today's world. Today there is too much evidence in the geology textbooks to give this credence: apart from the now well-established Alvarez Asteroid Hypothesis, it is believed that the Mediterranean Sea periodically becomes an immense dehydrated trench, with a drop between it and the Atlantic Ocean thousands of metres deep. No such formations exist on today's earth .**

***Catastrophism* comes in many variants. At its extreme fringe one finds the Creationists, who insist that we must all believe that the universe was formed by divine ukases exactly as stated in the Old Testament. The more scientific form of Creationism derives from George Cuvier in the 19th century: differences in fossil formations at various stratigraphic levels are due to autonomous acts of creation preceded by total cataclysms.**

The catastrophism that one finds in the Inflationary scenarios of Alan Guth and others, simply claims that there are features of the early universe which require the assumption that certain past events (the near instantaneous cooling of the universe), and the causes for these events (the "Higgs field") have permanently disappeared from view.

Explanation by catastrophe has always been popular in Geology: Earth collides with Venus (Velikowski) ; Ice Ages are produced through spontaneous flip-flops of the Earth's poles (Brown) ; life arrives on this planet through its colonization by charioted aliens ; (Von Daniken); the sudden submergence of entire continents (Plato).

More credible catastrophist hypotheses are usually related to the Great Extinctions, collisions with meteors and asteroids, and gigantic volcanic eruptions. The problem with most catastrophist scenarios is that they tend to elaborate considerably over a paltry database. Many uniformitarian fiats suffer from similar defects : one thinks of all those "isotropy", "homogeneity" , "uniformity" or "steady-state" principles which claim some sort of scientific foundation, but which are really the expression of the need to believe that the universe is not so unstable as to make science inconceivable.

Cosmology is much more accommodating to catastrophes than Geology. Geologists are hampered to some extent by not being able to appeal to the mathematics of 4-dimensional differential geometry, by which one can conceptualize pictures that no-one can ever hope to see, even in the mind's eye. Lacking a mathematics which enables one to conclude anything without committing oneself, geologists are obliged to be more cautious.

***A dynamic tension between Uniformitarian and Catastrophist philosophies lies at the vital core of every reconstructive science*. To understand why this is so requires that we conduct an examination of the distinguishing features of the two temporal categories of the Unknown : Past and Future.**

Everyone recognize that the methods appropriate to reconstruction are not the same as those applicable to prediction: the verification of a reconstructive hypothesis depends on predictive procedures , not conversely.

All theories are tested or falsified by the arrival of some observable, predicted, ideally reproducible, event. If the event is *sui generis* , one of a kind , then the circumstances that make it possible must be so fine tuned that alternative explanations have vanishing probabilities . This is almost always the case in sciences such as archaeology and history, which often depend on unique documents or structures. Sometimes homogeneity principles substitute for reproducibility: millions of fragments of Greek vases have given us a very good idea of how such objects were manufactured.

In the presence of evidence that contradict them, such theories will be modified or abandoned. Faced with equally fortified alternative explanations it is comforting to believe that new discoveries must eventually dispose of all but at most one of them. [9] Since in practice the hypotheses surrounding a finding are not mutually exclusive, the final consensus will usually be some combination of all of them.

In any case the verification of any reconstructive hypothesis must of necessity be very indirect. It consists of two phases:

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[9] **Although Villon's "Where are the snows of yesterday?" still awaits an answer.**

` (1) A model of some region U of space, in some interval T of past time , may produce consequences for the present that one can look for. If the Native Americans did in fact cross the Bering Straits from East Asia many thousands of years ago, we should find genetic similarities between , say, Tibetans and Navahos; which we do . We expect that additional data from the Hubble Telescope will help us to decide between a number of models of the early universe.

(2) One also looks for *preserved objects* to fill in the pieces of a reconstructive puzzle. The hypothesis that Richard the Second was responsible for the murder of two children who obstructed his way to the English throne , (a view contested in Josephine Tey's detective novel, "Daughters of Time") , was much strengthened by the discovery of skeletons in the debris of a staircase in the Tower of London that had, with a high probability, belonged to them .

Two principles are present in any attempt to put together a credible account of past events:

[A.] Things (distinguishable entities) don't change unless there is a reason for them to change.

[B.] Things don't stay the same unless there is something to prevent them from changing.

We need [A] to give meaning to the expression, "preserved object from the past". Without it there could never be any grounds for saying that anything , from a Leakey skeleton to a Dead Sea scroll, was a link to the past.

[B] is invoked for justification of the various methods of dating objects that were presumably created at some time in the past , potassium-argon , carbon-14 , dendrochronology , glottochronology, Cepheid variable stars (also used for gauging distances because of the universal value of the speed of light), or by the indirect association with other things in the same environment which can be dated by these means. If Permanence were not embedded in Change , there would be no way to compare the scientific value of Lucy's skeleton to that of Piltdown Man, or that of a Dead Sea scroll to a Xerox of a Dead Sea scroll.

Example: In the New York Times of February 14th, 1985, it was reported that a bundle of old letters was discovered under the staircase of a house that once belonging to Abraham Lincoln's son. Most of them are copies of letters one has reason to believe were written by him. In assessing the worth of this discovery many assumptions are made:

(a) Stability of language : American English has not changed so drastically that significant words or phrases have become totally misleading.

(b) That certain universal criteria of truthfulness will enable competent historians to judge when the author was telling the truth or when he was lying.

(c) That the entire cache is not just somebody's hoax. It is known that hoaxes don't have to be very clever to succeed: van Maegheren's faked Vermeers , dozens of works falsely attributed to van Gogh or Haydn, Piltdown Man , the Shroud of Turin, etc.

(d) The existence and reliability of independent means of dating paper and ink, the handwriting, mouse droppings, and so on.

(f) Comparative dating by allusions to contemporary events; checking for anachronisms, (a method that quickly disposes of most art forgeries) .

That these letters do indeed constitute a solid link with the past can therefore be asserted only in a context saturated with theoretical considerations. Was Lincoln's son as truthful as the legendary Honest Abe? Since the writer of a letter may have as many reasons for lying as for telling the truth, the long term effect of this discovery may be disinformative, temporarily (or permanently!) *increasing* our ignorance of the 19th century.

To summarize: confirmation of a reconstructive hypothesis requires not one but several acts of faith. Among them is the belief that there exists evidence which has mysteriously escaped the ravages of time, that will eventually turn up to cast the deciding vote between equally likely hypotheses. If such evidence does surface, it will itself have meaning only within a framework of many theoretical assumptions.

For centuries it was felt that certain passages in the published versions of Bach's chamber music (notably the orchestral suites) were wrong. Professional musicians were afraid to tamper with them. Autograph manuscripts were recently discovered that confirm what the more perceptive and independent-minded had always maintained. A pretentious mystique surrounds the works of any great artist, and acts of true bravery are required of anyone who asserts that what we possess may not always be the just expression of the artist's intentions. The public for art directs its critical judgment to the task of penetrating into the deeper meaning of scripture , whether it be a Brandenburg Concerto, a play of Shakespeare's. or an alleged Rembrandt . Thus, both predictive and reconstructive models depend on future evidence .It is because of the complementary/ contradictory principles [A] and [B] that there will always be a Uniformitarian/Catastrophist controversy at the heart of all

reconstructive science. Note that a major discovery of new evidence from the past generally produces far-reaching disruptions *within* any reconstructive science. Catastrophism may or may not characterize the content of a reconstructive science, but it always characterizes its own history! It is incautious to set too much store by models which are too uniform. New evidence has a way of demolishing established theories with terrifying regularity.

This is also true in physics and the other hard sciences; yet there is no comparison between the ways in which Einstein's theory of gravitation supercedes Newton's theory of gravitation, and the way in which, for example, the discovery of Viking artifacts in New England completely demolishes the theory that Christopher Columbus was the first European to visit the New World. Since there is a high degree of catastrophism in the daily conduct of the reconstructive sciences, one should always allow for a degree of catastrophism in the theories themselves.

At the same time, if everything has got to be explained by miraculous interventions or random occurrences, the field quickly degenerates into a mass of *ad hoc* hypotheses, ceases indeed to be a science. If the processes at work in the past are as inaccessible to us as the events they are marshaled to explain, there is the risk of setting up an indefinite regress of explanation. Hypotheses that do not, in one way or another, have their roots in the present, lose all scientific worth.

***Why does the sun shine?* God created it. *How did he create it?* He said, "Let there be light!" *Why did he decide to say that?* Because he was "well pleased" with Himself for doing so. Each 'explanation' in the chain gets pushed further back in its degree of inaccessibility. We encounter no gods walking around today**

who can create suns by saying "Let there be light!" We therefore have no way of knowing if these activities gives them pleasure. Inaccessibility is being "explained" by further inaccessibility.

In other words, principle [A]*"Nothing changes without a reason."* opens the door for potential 'explanations by catastrophe' whenever continuity with the present is not rigorously established. Likewise principle [B] stresses the necessity for uniform processes that guarantee the invariance of objects, structures and natural forces from one period to the next.

"Creationism" may well be bogus science, not even a science at all, yet whatever value it does have lies not so much in its fanciful Biblical reconstructions, as in its critique of the fatuously "smooth" evolutionary chains advanced by the paleontologists. Working in another reconstructive science, archaeologists are only now beginning to look at the way global weather patterns have brought about seemingly disparate historical events all over the world. One contemporary theory relates the rise of Islam, the appearance of bubonic plague, the destruction of Aztec temples, crop failures around the world and the Avar invasion of Europe to the weather conditions created by the gigantic explosion of Krakatoa in the 7th century. Such a theory combines uniformitarian with catastrophist elements.

The Uniformitarian/Catastrophist antinomy is inherent to reconstructive science. Dogmatism on either side of the divide is inadvisable.

(iii)Descriptive Science Journalism, Meteorology, Structuralism

The excessively reductive, yet convenient, representation of Time as a 1-dimensional Euclidean manifold imposes a natural subdivision at each instant into Past, Present and Future. There are, in addition, two forms of the present: the abstract or relative present used in physics, and the absolute or real present of conscious awareness.

The subtleties of the *abstract present* , neither fully present nor fully past, are to be seen in the multitude of tenses, such as those expressing an action begun in the past and on-going ("I have been watching the cows") , or begun and completed in the past , (The perfective: "The roast is burnt", as opposed to "The roast was burned"), or simply past with no indication as to its continuance ("He sat on the chair" - he may still be sitting there for all we know), or on-going but interrupted ("I was seeing the light when you walked into the room") , along with combinations of these.

Written French also uses an "absolute literary past" , a kind of timeless time appropriate to fiction : *Il observa les fleurs sans regret* . "He was observing the flowers without regret "; All tenses, including "He observes the flowers...", and "He observed the flowers" are contained in this expression by being placed in the realm of the imagination.

The *real* present is the present of individuated consciousness : I am alive now, and I am aware of it . The "now" in which I am writing these sentences is assumed, perhaps falsely , to be the same as that of all other persons sitting in or walking around in the cafe-bookstore where I happen to be working . It is the same 'now' which one used to assume could be objectified as " *9:37 Junes 17,2000* " throughout the entire cosmos, yet which, since the appearance of the Theory of Relativity has become an

open question. Is my subjective present identical with yours? Is the sum total of all subjective presents identical with an objective present? And so forth. This essay evades all such issues in by restricting its focus to the abstract time of science. This overly simplified but useful fiction is reflected in the way in which one may classify all sciences as Reconstructive, Descriptive and Predictive.

Descriptive sciences likewise divide into pure and applied . One finds pure and applied mathematics, theoretical and practical economics, descriptive and Chomskyian linguistics [10] , the theory of disease and the description of disease , etc. Among the applied descriptive sciences on can include Journalism, Anthropology, Classificatory Biology, Meteorology, etc.

Even as Mathematics is an entirely descriptive pure science, Journalism is an entirely descriptive applied science. Mathematics is atemporal, Journalism dedicated to the momentary description of transient events. What links these two fields is that neither of them makes predictions or incorporates causality.Of course there is lots of bad journalism, infected with vindictive moralism, which assigns blame with free-wheeling impunity. The best journalists record the facts as they occur, in such a way as to permit the reader to draw his own conclusions. Although the conjectures raised by mathematicians do resemble the predictions of physics, refutation by contradiction is not quite the same as refutation by evidence from a failed experiment.

Every science partakes in various degrees in all 3 temporal modes. The focus of a descriptive science is on the accurate description of present or presented phenomena, pure diagnosis,

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[10] **Which I happen to believe is fraudulent; yet its goal is to turn Linguistics into a theoretical science, with “language” subject to mathematical laws**

setting aside all historical speculation or future consequence . Speaking informally, one might suggest that Reconstruction makes use of the mathematics of modal logic, Prediction the mathematics of analysis and probability, and Description the relational structures of Algebra: lattices, groups, partitions, etc.

All of the disciplines that flourish under the vague label of "Structuralism" are descriptive. Simply described, structuralism is a crash program much in vogue in the 60's and 70's, that aspires to raising the immense accumulation of uninterpreted data in the warehouses of the human sciences, (Anthropology, Linguistics, Psychology, Sociology, Political Science) to the level of pure or theoretical science. The search for universal structures in human institutions and artifacts may be seen also as a search for the appropriate causal algebras to be employed in back-reconstruction and prediction.

IV. Branching Causality

Branching Causality arises when :

(1) A decision in favor of one out of several proposed models of the past cannot be made on the basis of available knowledge.

(2)Probabilities can be assigned to these models

(3) It can be rigorously demonstrated, or is highly likely that in the time at one's disposal, it will not be possible to set up a procedure for selecting a unique past from the options available. This can mean several things:

(a) The observer disturbs or destroys the observed in the course of the experiment, as in

(i) Archaeological excavations which destroy their sites

(ii) Biologists who kill their specimens

(iii) Particle physicists generating quantum uncertainties.

(b) Time has destroyed too much evidence to allow for a unique reconstruction:

(i) Erosion

(ii) Death of witnesses

(iii) Information destroying processes in the formation of Black Holes

(c) The requisite knowledge is intrinsically inaccessible, hence unknowable

(i) Cosmic censorship inside a Black Hole

(ii) Schrõdinger's Cat

(iii) Life after death, or before birth

Back-branching causality appears explicitly in Quantum Theory in many contexts. The classical picture of von Neumann of the "collapse of the wave packet" assigns definite probabilities to each value in a spectrum of eigenvalues. All of them are assumed to "exist" before any observation is made.

The reductive simplicity of quantum physics cannot serve as a model for human history. Strange, is it not, that although a historian must be committed , even more than a physicist, to the belief in the past's uniqueness, it is he, more than any other scientist, who must keep alive all the alternative interpretations of past events?

Persons who care to do so are free to speculate whether branching causality exists only in the representation spaces, or reflects some intrinsic structure in real time. Can the present really be the product of a collection of independent pasts? In sciences such as history, archaeology or paleontology one is frequently obliged to represent the state of one's knowledge as a collection of

autonomous pasts. The theoretical belief in a unique past must often be abandoned in practice.

Let's imagine that Chaucer and Froissart somehow got together in France during the Hundred Years War of the 14th century. Chronicler and fiction writer conspired to play a joke on mankind by concocting an account of a ferocious battle set near a little village named Agincourt.

Though this makes the battle of Agincourt pure fiction, belief in its reality has had an immense influence on subsequent history, from Shakespeare to Winston Churchill to Desmond Seward to Barbara Tuchman. Isn't that enough? Must historians maintain that a real battle must leave consequences visible today? Yet how otherwise does one distinguish a battle invented by Froissart from the ones he actually witnessed ? Unlike the situation in physics and chemistry , one can't experiment with history.

Since the "past" is always a back-reconstruction, back-branching causality can never be eliminated from the representation spaces of the reconstructive sciences.

It is a commonplace of detective fiction, and all too often of crime in the real world, that the criminal will murder his victim for the sole purpose of rendering a unique reconstruction of the crime impossible. A strict Lagrange-Hamilton description of cause and effect via local determinism may exist in that case only in the mind of the perpetrator. For all others, judge, jury, lawyers, journalists and the public, a scenario of branching causality leading to and from the event may be the best one available.

Note that there is nothing within even the strictest formulation of classical causality that guarantees that enough information must be left lying around to allow us to make a

unique reconstruction of the past. The local determinism of the Lagrange-Hamilton paradigm implies that different pasts must result in different presents. It does not give us any help in reconstructing that past.

Indeed, there is little in our experience to cause us to believe that the past has to be unique. If two entirely different pasts can be shown to produce virtually identical presents, why do we need an axiom to tell us that only one of these must be correct? For the most part causality is only concerned with future connections: a unique future must come out of a unique present. One does not even have to assume that one is living in a unique present: all that a workable axiomatic framework for physics has to assert is that *given* a unique present, then a unique future is inevitable. Mathematical frameworks for back-branching causality can be developed without consideration of the philosophical issues concerning either the uniqueness or multiplicity of the past.

One of the things which Thermodynamics seems to be saying is that the past, no more than the present, is not static: it, too, can be lost. Since the ideas of entropy and of a measure of information content are closely related, it is possible to interpret the dissipation of mechanical energy into non-reconvertible heat as the destruction of knowledge, that is, loss of the past. A consequence of the existence of inaccessible energy is that the calculation of the coefficients of local determinism, (whether by differentiation or other means) can only be approximated. Thus, both past and future thermal trajectories are of necessity branched. This may in fact be the real meaning of the concept of "dissipation" .

Similar observations can be made relative to the Heisenberg Uncertainty Principle. Any real or conceptual physical

experiment in which the observer *is placed within the system in isolation* , leads unavoidably to perturbations in all measuring processes. As in quantum theory, these find expression in a multitude of pasts converging causally on the present.

The determination of stellar distances requires a form of branching description that is intrinsic to the character of Astronomy as a science. In the absence of compensating factors, it is formally impossible to distinguish a bright star which is very far away from a dim star that is close at hand. Until more external evidence becomes available, all possible distance determinations must be kept in the model, and there is no a priori reason to believe that such external evidence will show up.

(a) Mathematical Models

Fix some moment of time T > 0 away from the temporal origin. Let t stand for the quantifiable time variable, P the class of all real-valued patterned functions defined on the interval I = [T , ∞) . Let Λ be the sub- algebra of unbraided patterned functions, that is to say, all patterned functions f defined on I which allow one to make a unique back-reconstruction from T to the origin:

$$\boxed{f,g \in \Lambda \to [(f(t)=g(t), t \geq T) \to (f(t)=g(t), 0 \leq t \leq T)]}$$

Choose f ε Λ , and assume that f is not pathological. Then it comes from a minimal associated function A , and minimal section point set, S. Since f is unbraided, the domain of A can be uniquely extended back from T to 0 . Given the function A , one can combine it with arbitrary section point sets Σ , to derive a collection of patterned functions Ω all derivable from the same associated function, A .

Some of the patterned functions derivable this way may be braided. However, there exists a class of patterned functions Λ^*, which are generated from associated functions such that none of the patterned functions derivable from them will be braided. We will say that the functions in Λ^* are *absolutely reconstructible*, whereas those in Λ will be called, simply, *reconstructible*.

Let $f \varepsilon \Lambda^*$ and examine the behavior of $g = f(1/t)$ around the origin in the interval $(0, 1/T]$, it is possible to reconstruct all of g from knowledge of its shape in this interval. From that we can reconstruct all of f. And from f we can derive a unique associated function A, provided the following list of conditions are satisfied:

(i) g is non-constant on any interval

(ii) $f = g(1/t)$ is uniformly continuous

(iii) The associated function is also continuous

Among all possible candidates, we naturally select the with the minimal pattern set.

(b) Examples of Models for Branching Causality

Construction A:

(1) "Time" begins at an essential singularity - a "Big Bang".

(2) This sets up a point-source algebra; The entire future can be derived from calculations made in the neighborhood of that origin.

(3) Until time $T > 0$, one is dealing with a phenomenon combining a pair of criss-crossing branches, or braids.

(4) At T the two branches merge into a unique causal flow, unto eternity

(5) In the region [T , ∞) causation obeys Lagrange-Hamilton local determinism : any neighborhood around any instant contains complete forward predictive information.

Start with any analytic, non-periodic function defined over the positive temporal line $y = \gamma\,(t)$, real and bounded away from 0 . Let $\theta\,(t) = \gamma\,(1/t)$. Then the arc of γ from some fixed point a, to infinity, will be in 1-1 correspondence with that of θ from 1/T to 0 .

Let U and V be distinct section point sets

$$\boxed{\begin{gathered} U = \{0, a, s_1, s_2, s_3, ..., s_k, ...\} \\ V = \{0, a, r_1, r_2, r_3, ..., r_k, ...\} \end{gathered}}$$

Let α be the associated function for γ . Using U and V as section point sets, we form two patterned functions:

$$\boxed{P_1(\alpha, U)\,;\, P_2(\alpha, U)}$$

Then the functions

$$\boxed{\begin{gathered} Q_1(t) = P_1(1/t) \\ Q_1(t) = P_1(1/t) \end{gathered}}$$

will satisfy the above recipe for simple branching causality:

(i) Since both P_1 and P_2 are unbraided, they model strict causality outwards from the essential singularity t =0 , of Q_1 and Q_2 .

(ii) Between t =0 and t =T , Q_1 and Q_2 form independent paths, alternately recombining and separating.

(iii) For t>0, they merge into a unique flow

(iv) In the region [T , ∞) , $Q_1 = Q_2 = \alpha\,(1/t)$ is analytic: if f has no singularities from 0 to T , then $\alpha(1/t)$ will have no singularities from 1/T to ∞ , any t > 0 .

Illustration :

For associated function we choose $\alpha(x) = \sin(x)$; $T = 2\pi$

The two section point sets will be given by the roots of the equations

$$\boxed{\begin{aligned}U(t) &= t_1 \sin(\sqrt{2}t) + \sin(\sqrt{3}t)\\ V(x) &= t_2 \sin(\sqrt{5}t) + \sin(\sqrt{7}t)\end{aligned}}$$

t_1 and t_2 are calculated from the conditions:

$$\boxed{\begin{aligned}&U(0) = V(0) = U(2\pi) = V(2\pi) = 0;\\ &t_1 = -\frac{\sin(2\sqrt{3}\pi)}{\sin(2\sqrt{2}\pi)}, t_2 = -\frac{\sin(2\sqrt{7}\pi)}{\sin(2\sqrt{5}\pi)}\end{aligned}}$$

The "inverted argument" functions

$$\boxed{Q_1 = P_1(1/t;\alpha;U), Q_2 = P_2(1/t;\alpha;V)}$$

are continuous everywhere and analytic everywhere save at a countably infinite set of discrete points converging to the origin .

Construction B :

Branching Causality Converging to a *Future* Essential Singularity

(1) From time t = 0 , there is a unique causal trajectory up to some fixed moment T.

(2) Before the zero-point, there were a pair of braids L_1 , L_2 each of them converging with local determinism fashion to the same forward arc.

(3) After t = 0 the system does not obey local determinism. However, as one approaches T it is possible to reconstruct the path with ever greater accuracy.

(4) Complete knowledge of the shape of the world-line may be obtained by calculations made in any neighborhood of the future terminal point T . One can thereby reconstruct not only the

segment from 0 to T, but the two braids from zero to minus infinity.

(5) At the moment t =T the system collapses into an essential singularity.

This recipe can be modeled by braided pairs of patterned functions.

Let P_1 , P_2 be a pair of braids derived from an analytic non-periodic function Z(t) in the manner described in the example on page 14 . Keeping with the same notation, the front section of P_1 is designated A, that of P_2 is designated B. We require that the domains of A and B have the same length, L. Define

$$\boxed{\begin{aligned} Q_1 &= P_1\left(-\frac{1}{(t-1/L)}\right) \\ Q_2 &= P_2\left(-\frac{1}{(t-1/L)}\right) \end{aligned}}$$

These functions in combination reproduce all the features of the recipe:

DIAGRAM

(1) The branches merge at t=0 into the "inverted argument form" of the congruent portions of the pair of braids P_1 and P_2 .

(2) In the inverted argument form, the two braids translate back into independent branches from 0 to $-\infty$.

(3) There is an essential future singularity at t = 1/L

(4) As one approaches this singularity, and more information becomes available to us, one can back reconstruct over greater reaches of the past with greater accuracy

(5) If one could somehow compute from the future point to any other point close to it in time, the complete trajectory of the system for all past time would become known .

This picture has much in common with the way real science advances . Initially one knows nothing about the pas. Knowledge about what happened historically accumulates gradually as one advances into the future.

Likewise, predictions concerning the future only take one forward a short distance in time. One would have to go all the way to the end of time before really understanding everything that is happening in one's present environment . As one goes further into the past, the number of branching or alternative scenarios increases: the braids become frayed in proportion to the distance of past events from us.

We see that this picture is, in certain respects, more faithful to our state of knowledge at any given moment, than the fiction of a smooth, uniquely determined scheme of things that we all would like to imagine really exists over the universe. .

The employment of braided and unbraided patterned functions allow one to model an enormous variety of branching models: point-source causation, strict Lagrangian or substrate causation, back-branching causation, etc. By investigating larger classes of automorphic functions, and employing the full diversity of coding schemes that are totally or partially back-reconstructible from key moments, one should be able to generate useful models for every imaginable form of universal causal structure.

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Each seed injects a new causal process into the flow. This may cancel or overshadow all previous inputs, or become integrated with them. The process is analogous to the way in which a new instrumental sound introduced into the texture of a symphony may blend with the flow, or soar above, dominate it, or perform a solo cadenza while all others remain silent. When the new input only makes a contribution to the whole without supplanting them, predictions of the future course of the system demand knowledge, not only of the algorithmic process for extracting causal information from a specific seed, t_n , but information about the application of such procedures to all previous seeds t_{n-1} , t_{n-2} ,......

This conception has a close correspondence with our daily experience. Indeed this is the way science works in practice. For example, no competent historian would claim to explain the political geography of the world in terms of the outcome of a single war. In his quest to understand and the national boundaries of today's world , he would study all wars and treaties

as far back as necessary or reasonable. He would recognize that the Roman Empire, the Holy Roman Empire, the Hundred Years War, Thirty Years War, Napoleon, World War I and II, etc., have all made their contributions to today's political landscape. Eventually he would come to the understanding that the fictive homogeneity which one attributes to the nations of, say, Western Europe, (those whose boundaries have not changed significantly since the decade after WWII) , is no more than a convenient illusion . Active separatist movements exist today in Northern Ireland , Spain, France (the Bretons), Belgium and elsewhere. As for Eastern Europe, a country such as Yugoslavia never was anything but an idea constrained by a label. As of this date its disintegration is very much with us. All national boundaries of all countries are the residues and reflections of previous wars, invasions, explorations and political negotiations . Point-source causation is the only mode of determinism suitable for historiography.

(a) Autonomous and Determinable Seeds

Point source causation always presupposes a set of privileged moments of emanation or seeds, at which some kind of Lagrangian computational process may be applied for predictive purposes. Seed moments are of two kinds

A. Autonomous seeds, which are not computable or predictable from previous collections of seeds.

B. Determinable seeds, which are determinable from computations on one or more seeds in their past.

In most situations of interest , seeds manifest themselves in the form of *explosions* : essential singularities of

one or more of the measurable observables. As a general rule, although there are algorithms for deriving the world line of a system S from their neighborhoods , knowledge of the configuration of S at those moments is unattainable.

The standard model of the Big Bang is the paradigm for the explosive form of point source causation . One may also site the Hawking model for the decay of a Black Hole from the moment when it becomes isolated from the rest of universal causality. The Hawking-Penrose Cosmic Censorship Hypothesis states that no acausality can radiate from the interior of a Black Hole. However, 2nd quantization Hawking radiation is in effect a kind of ***radiation of autonomous causation*** **from its surface.**

> **" The singularities produced by gravitational collapse occur only in places, like Black Holes, where they are decently hidden from outside view by an event horizon....it protects observers who remain outside the Black Hole from the consequences of the breakdown of predictability that occurs at the singularity...."**

Pg. 88 ***A Brief History of Time*** **; Stephen W. Hawking, Bantam Books, 1988**

The conditions for point source causal algebras are :

(i) The possibilities for constructing the world-line from local information, are restricted to a discrete set, fine, countable or unique, of seeds. In general one requires information about the configuration of the system in all seeds proceeding the one under consideration. In teleological models, which are not strictly Lagrangian, (as all such neighborhoods are intervals of infinite

length) information obtainable from neighborhoods around eternity, or the end of time, allow one to back-reconstruct a world-line.

(ii) Events separated by a seed moment are not causally connected. For events *between* two seed moments there may be some small-scale connection to refer back to the left-hand seed to relate them causally.

(iii) From any instant τ outside Ω there is no Lagrangian procedure for constructing the world line beyond the immediate neighborhood of that instant.

Examples from daily life are legion. Consider the correspondences in the vocabularies of two cognate languages, such as Spanish and French. Beyond a certain point it is not possible to establish their direct connection save by appeal to their parent source, which is Latin.

Micro-organisms like amoebae which reproduce by splitting, materially represent the sorts of natural branching tree diagrams associated with point-source causation.

If , as is believed, our universe originated in a great explosion, the reconstruction of its history from the present back to the Big Bang isn't possible without some direct evidence from the neighborhood of that event. That this is possible to obtain is due to the peculiar relationship of time to distance in a relativistic universe. By looking ever further out in space, say with the Hubble telescope, one also looks back in time, However, there is absolutely no way one can predict what one will see at any distance beyond that which has already been penetrated; and what one does see must materially alter ones picture of the entire evolution of the cosmos up to the present moment.

Point-source algebras are most appropriate for the modeling of explosions. A terrorist plants a bomb in some office building, which explodes with terrifying force. Someone situated at some distance from the explosion, who is hit by debris at time t_1 cannot say , from any amount of analysis of this debris, anything about whatever may be coming his way at a later time t_2 . However, from the viewpoint of the terrorists, knowledge of the composition and capabilities of the bomb , the character of the structure being destroyed, and the way in which the bomb was placed within the structure, has given them quite a lot of information about the way in which the debris will be scattered over time and space.

Another obvious area for the application of point-source algebras is Catastrophe Theory. The localized differential topology on one of the sheets of the 3-D surface of a standard bifurcation catastrophe gives no information about the structure of the flow of world-lines on any of the other sheets. But study of an infinitesimal neighborhood around any of the bifurcation singularities will provide complete causal information about the unfolding of events on all the sheets emanating from it.

(b) Computation Schemes for Essential Singularities

Let $\Pi(t)$ be bounded and periodic with period $\omega = 1$ for convenience . Π is assumed continuous . Also , $\Pi(t) \neq 0$ for all t. Under these conditions , which are easily generalized, we will construct a causal algebra consisting of agents of the form $\Lambda(t) = \Pi(1/t)$.

Three methods are presented for assembling a table of numbers, computed in the neighborhood of the origin, by which

one may construct the arc of $\Lambda(t)$ for all positive t. All of them derive from the following observation:

Let $\{t_j\}$ be an indexed, countably dense subset of points in [0,1]. Since Π has period 1, we have for all pairs of positive integers N, j :

$$\boxed{\Lambda(1/N+t_j)=\Pi(N+t_j)=\Pi(t_j)=\Lambda(1/t_j)}$$

Therefore, by making N sufficiently large so that $1/(N+t_j)$ is in the interval $(0,\varepsilon)$, we can, from the values Λ in that interval, calculate the value of $\Pi(t_j)$.

Next let N = j, and let Δ be the collection of values ,{ $1/(N+t_N)$ }. Given ε, only a finite number of the elements of Δ will lie outside the interval $(0,\varepsilon)$. Therefore, evaluating Λ at the points of Δ creates a countably dense set of ordinates for $\Pi(x)$ in (0, 1) . Since Π is assumed to be continuous, we thereby obtain sufficient information to compute its entire arc.

METHOD 1: Enumerate all the rationals in the interval (0,1) . For example one might use the Cantor J-function. If r = a/b, then

$$\boxed{J(r)=\frac{(a+b)^2+3a+b}{2}}$$

METHOD II: The inverse binary method. Let B be the collection of all binary decimals of finite length in (0,1) . If b ε B , then we may write it as

$$\boxed{\begin{array}{l} b=0.E_1^bE_2^bE_3^b.....E_k^b, with \\ E_j^b=\{{}^1_0, j\neq k, and \\ E_k^b=1 \end{array}}$$

Under this representation of b, define the “reversed binary integer”, b*, given by $\boxed{b^*=E_k^bE_{k-1}^bE_{k-2}^b.....E_1^b}$, interpreted as a binary integer. The collection of integers {b*} is Z^+. This sets up

a natural 1-1 correspondence between the integers and a countably dense set in (0,1) .

Define $\tau_b = b + b^*$, and $\sigma_b = 1/\tau_b$. Referring back to the functions Π and Λ , we have $\Lambda(\sigma_b) = \Pi(\tau_b) = \Pi(b)$. The values of b* can be used to index the set $\{\sigma_b\}$. As this index augments, the finite binary decimals sweep back and forth across the unit interval in a net of increasing refinement:

EXAMPLE:

$\Pi(x) = Sin(x); \Lambda(x) = Sin(1/x)$

b	b^*	τ_b	σ_b	$\Lambda(\sigma_b) = \Pi(\tau_b)$
1	$1/2$	$3/2$	$2/3$	0.99
2	$1/4$	$9/4$	$4/9$	0.77
3	$3/4$	$15/4$	$4/15$	–0.57
4	$1/8$	$33/8$	$8/33$	–0.83
5	$5/8$	$45/8$	$8/45$	–0.61
6	$3/8$	$51/8$	$8/51$	0.09

The systematic list of coefficients on the right fulfills, for point-source causality, the same role as the coefficients of a Taylor series for Lagrangian causality.

METHOD III: **Let ζ be some irrational number. Then the numbers**

$\boxed{k_n = n\zeta - [n\zeta]}$ **will be uniformly dense in (0,1) . Let** $\boxed{b_n = 1/n\zeta}$

, and look at the values of $\Lambda(b_n)$. Since

$\boxed{\Lambda(b_n) = \Pi(nz) = \Pi(n\zeta - [n\zeta])}$ **, a large enough quantity of these will produce a close approximation to Π(ζ) in (0.1).**

(i) Fourier Summability

If Π(x) is Fourier-summable, one may calculate Λ(x) from its behavior at the origin. The method employed closely resembles

that of calculating the differential coefficients of a Taylor's series. Let ω = 2 π , and assume that Π(t) may be represented by a Fourier series of the form :

$$\boxed{\Pi(t) = {a_0}/{2} + \sum_{k=1}^{\infty}\{a_k \cos(kt) + b_k \sin(kt)\}}$$

Then the corresponding series for Λ (x) is given by

$$\boxed{\Lambda(t) = {a_0}/{2} + \sum_{k=1}^{\infty}\{a_k \cos(k/t) + b_k \sin(k/t)\}}$$

By constructing an algorithm for calculating these coefficients around the origin, we can plot the rest of the arc of Λ (t) throughout all positive time. Through Fourier analysis we know that for each k,

$$\boxed{a_k = {1}/{\pi}\int_0^{2\pi} \Pi(s)\cos(ks)ds}\ .$$

Since Π(x) is periodic this integral can also be written in the form:

$$\boxed{a_k = \lim_{L\to\infty} {1}/{\pi L}\int_{2\pi L}^{4\pi L} \Pi(s)\cos(ks)ds}$$

By a change of variables, this becomes:

$$\boxed{a_k = \lim_{L\to\infty} {2}/{L}\int_{L}^{2L} \Pi(2\pi s)\cos(2\pi ks)ds}$$

Let z = 1/s , ds = -(dz)/z^2 . Since Λ(z) = Π(1/z) , one obtains:

$$\boxed{a_k = \lim_{L\to\infty} {2}/{L}\int_{1/2L}^{1/L} \frac{\Lambda(z/2\pi)\cos(2\pi k/z)dz}{z^2}}$$

Letting ε = 1/L ,

$$\boxed{J_k(\varepsilon) = \frac{1}{2\int_{\varepsilon/2}^{\varepsilon}\frac{\Lambda(z/2\pi)\cos(2\pi k/z)dz}{z^2}}}$$

Finally : $\boxed{a_k = \lim_{\varepsilon\to 0} \frac{\varepsilon}{J_k(\varepsilon)}}$ **. The coefficients { a_k } and {b_k } may therefore be derived from any arbitrary neighborhood of an**

essential singularity in a manner thoroughly analogous to that used for the computation of Taylor or Fourier coefficients.

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