Time, Euclidean Geometry and Relativity

Space-Time Topology and Constructible Clocks

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1. Introduction

The paradoxes of the theories of Special and General Relativity derive from the conflict between our intuitive notions of time, and its operational role as a parameter in the equations of physics. Relativity's representation of space-time as a 4-dimensional pseudo-Euclidean space (Minkowski space) employs a narrowly specialized, quantifiable simplification of the time dimension. In many instances this departs significantly from our common understanding of it. That our senses are immersed in the same temporal flow as all other physical, natural or cosmic events, is a further complication.

However, relativistic time, in addition to being counter-intuitive, does not always provide a satisfactory explanation for the attributes that we do find in the time of everyday experience . Theories that lead to non-intuitive conclusions must also incorporation explanations for their intuitive counterparts. Einstein himself called this the *criterion of reality* in the celebrated paper co-authored by himself, Rosen and Podolsky .[1]

The indefinite metric of Special Relativity for flat space-time is given by:

$$\boxed{-ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2}$$

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[1] **Can a quantum-mechanical description of physical reality be considered complete? A. Einstein, P. Podolsky , N. Rosen Physical Review, vol 47; May 15,1935**

Our strong sense of the many ways in which time measurement differs from space measurement is reduced to a pair of minus signs . Even for purely scientific purposes, this characterization appears poverty-stricken. The local arrow of time has been replaced by the 'imaginary' status of the square root of minus one. The global arrow of time has been eliminated altogether. Simultaneity has been abolished, although we still need it for the purposes of measurements at the quantum scale. Time in Quantum Theory is designated a parameter although length is treated as an operator. It is thereby very different from space.

Lacking simultaneity what is one to make of quantum non-locality? The contrast between the use of the translational invariance of rulers in the measurement of length and the kinematics of clocks in the measurement of duration? The important role of time in causality versus the essentially acausal features of space?

In particular, the assumption that the time variable possesses linearity , that is to say, all the affine and metric properties of Euclidean spatial length, an assumption implicit in the above equation for proper time , over-simplifies our experientially derived concept of time.

One might even , through a form of reverse implication, argue that the assumptions of linearity and homogeneity in time *imply* that space and time are linked together in some sort of geometry. Space-time would not then be so much a discovery of modern physics, as the revelation of a central classical assumption.

This article examines certain deficiencies of the Einstein-Lorentz-

Poincaré-Minkowski model for space-time. In order that the time dimension serve as an equal partner with the 3 spatial dimensions in a 4-dimensional global continuum, one needs to make explicit important assumptions which have not been articulated in this model . These assumptions are far from being self-evident, and the standard representation of space-time cannot be accepted without reservations. Even the habit of placing the time differential , dt, the "nexus of change" on an equal footing with the spatial differentials dx , dy , dz , must be viewed critically to the extent that it prejudices our understanding of nature.

The author does not thereby maintain that Minkowski space ought to be rejected as a mathematical representation. His only concern is that its essential inadequacies be addressed. Specifically:

(1) Linearity in time demands the underpinning of a system of postulates setting forth the relationship of time to the state descriptions of mechanical systems, as well as a system of axioms governing the measurement of temporal duration,(time reckoning) . Four such axiom systems will be considered in this paper: Homogeneous Time, Cyclic Time , Euclidean Time, and Relativistic Cyclic Time.

(2) Those features of Minkowskian space-time which have no correlatives in observation must somehow be explained, modified or eliminated.

Quantum Time
A long footnote

The time observable in Quantum Theory is highly ambiguous, even unreliable. Time is not so easily dismissed by calling it like mass, a parameter. Quantities of matter can be physically sub-divided ,

combined or compressed. Mass measurements can be made through scattering experiments, based on the law of conservation of momentum, or by weighing on scales, based on the law of gravity. (The experimental confirmation of the identity of gravitational and inertial mass is of course the cornerstone of General Relativity.) It is not so easy to imagine how two instants can be 'slammed together' , or how one might weigh the 'heaviness' of a year relative to that of a second. Evidently the only place in the universe where mass and time come together under the same umbrella is in the formalism of Quantum Theory!

The performance of *simultaneous* measurements of complementary or conjugate variables, the "sacred ritual " of quantum theory, presupposes the notion of simultaneity. Simultaneity in the form of a direct collision of particles or isolated systems may be unproblematical, but difficulties arise when one wishes to compare events by means of identical readings on referent clocks. Clocks, being dynamical systems, function in compliance to a Hamiltonian of the form H = U + V . In this equation U (kinetic energy, a function of momenta), and V (potential energy, a function of positions) , are conjugate variables. Because of the Uncertainty Principle it is not possible, through simultaneous measurement, to know the position and momentum of, for example, the centroid of an isolated system, with an uncertainty less than

$$\Delta C = \Delta q \Delta p > h/4\pi \, . \qquad (I.2)$$

It follows that referent clocks cannot be well defined in the formalism of quantum theory. Indeed, cleaning up the definition of time in quantum theory is such a difficult undertaking that, even if I

thought I had a way of doing so, there would not be enough room in this article to present it. Such issues must therefore be deferred to another article at some later date.

Discontinuity And Time Measurement

Clocks are defined on the basis of a perception of sameness of state. Yet there must also be a deviation from some initial state before returning to it. If the initial state never changes, one has permanence, not change. Therefore a particle moving on an inertial path at a uniform velocity cannot be employed *directly* as a clock:

(i) Its' state is variation free.

(ii) In the frame of the particle its' trajectory obviously won't work as a clock.

(iii) The conclusion that the particle *is* moving with a uniform velocity must itself be derived from the readings of autonomous rulers and clocks.

(iv) If the rest observer and moving particle are taken together as a single system K, its' state description changes through time and, (in a non-cycling universe) , is never the same. If $x = vt$ is the location of the particle, then K's state description is $S= (x,v)$. Using S as a clock means that we acknowledge a linear mapping between time and length. However, if v is not uniform, waxing and waning for example with a high frequency, this can't be ascertained from examining the values of S.

Hence a second system (v^*, x^*) is needed to monitor the first - and so on in an infinite regress. Yet, by using a system consisting of a pulsing clock, the criterion of *sameness of state after deviation* suffices

for the

(discontinuous) observation of the passage of time.

Basically, time measurement is a discrete, discontinuous process, although the time dimension itself may be a continuum. Indeed, without the postulate of relativity asserting the constancy of the speed of light in all frames , there are no grounds for calling time a continuum.

2 . Homogeneous Time

" A static representation of time is necessary for its use as a variable in the sciences"

- Café conversation with René Thom, rue de la Montagne St. Geneviève, Paris , July, 1986

The basic distinction between spatial measurement and time reckoning is the following:

(1) In the world of our (admittedly limited) experience, *lengths* admits of isometric translations, rotations, and reflections, in all 3 dimensions; whereas *time-intervals* can only be displaced , in thought or theory, by treating them as lengths. In reality there is no way to recover the past by moving it up to the present.

Compare for example the procedure for finding the mid-point of a finite line segment L, with that of locating the exact midpoint of a time interval of duration T, (both in appropriate units) . T can be presented either in the form of a sound signal (or light flash. etc.) which goes off at the endpoints of the swings of a pendulum, or as a continuous musical message, (motion picture, etc.) that abides for exactly this length of time, then repeats itself.

L, likewise, can be represented as two dots on a plane, or as the solid straight line between them. If our universe were one-dimensional, the process of designating the exact mid-point of L would be quite difficult, yet still easier than that of locating the mid point of T: One finds a ruler, R, about which nothing is known save that it is less than L and larger or equal to 1/2 L (If R is less than 1/2L, one lays off copies of R until their concatenation falls just short of L , then re-designates this entity as "R") .

If R is exactly equal to 1/2L we are finished. Otherwise, we designate the remainder interval L-R as R' , and begin laying off R' against L. If R' exactly covers L we count the number of copies: if this is even we are finished. If odd, then the problem of finding the mid-point of L has been replaced by that of finding the midpoint of R'.

If R and L happen to be incommensurable, then each reapplication of the measurement process leads to a new remainder R', R'', R''' ,...$R^{(k)}$ dividing L in increasingly many segments of amounts $N_1 < N_2 < N_3 ...., < N^k$, where Nk ---> ∞ as k---> ∞ .

Then the lengths set of lengths $L_k = R^{(k)} [N_k/2]$ (greatest integer function) will converge to the point L/2

This method breaks down completely when applied to T . As before, we find a clock with period Q, where $T/2 \leq Q < T$. Rewind the two clocks and set them pulsing together. If Q pulses exactly twice in the interval (0, T) we are finished. If Q does not, then the residue interval is Q' = T-Q .

How does one construct a clock that pulses with the exact period Q' = T-Q ? **The only way of doing so is to 'couple' a clock pulsing at period T with one pulsing at period -Q , and if we admit this kind of**

time reversal we haven't got what we call time anymore. In other words, there is no problem with constructing a clock , out of clocks with periods Q and T, that pulses at period T+Q, but it is impossible to construct one by coupling that pulses at period T-Q . This is true even if Q and T happen to be commensurable.

Unlike the situation in 1-dimensional space, one cannot just 'push back' the Q' interval to the origin and begin to lay it off in equal periods.

The only method available to us for finding the midpoint of a time interval, without availing ourselves of some 1-1 functional relationship between time and space , is to build clocks of successively smaller periods,

$Q_1 > Q_2 > Q_3 > ......Q_n > ....$, and count the number of recyclings of each within the interval T. However we have no way of insuring that

$$\boxed{\lim_{N\to\infty} Q_N = 0} \quad !$$

Conclusion: **It is impossible to locate the midpoint of a temporal duration without first translating time reckoning into spatial measurement via a physical theory governing the behavior of mechanical systems. This is exactly what we do when we consult the hands on a watch dial.**

Moving next to a 2-dimensional space ('Flatland') , the determination of the exact midpoint of a line segment L , can be done without making appeal to some potentially infinite algorithmic process: the familiar construction using parallel lines dropped from the end-points of equal lengths cut off on another line intersecting, at some arbitrary angle, the initial endpoint of L, will suffice.

In 3 dimensions one can use a ruler (translation group) and compass (rotation group) to construct the perpendicular bisector to L, as well as its intersection with L. [2]

It is always possible, either physically, theoretically, or in a thought experiment, to compare the lengths of two linear entities E_1 and E_2 in the same rest frame. Either one moves E_1 up to E_2 and correlates their end-points; or one takes measurements on E_1 with a ruler on the first, then carries the ruler to E_2 for the correlation . By definition the translation of a material object under the action of the Euclidean group does not cause any distortion of its metric properties.

Congruence translations of this sort are not possible in the temporal dimension: it is not possible to move "yesterday" up to "today" , and compare their endpoints. All that we ever have is the *now* . One might argue that this is because we are inhabitants of the universe we are trying to observe. Yet this begs the question, since there is no guarantee than any observer 'outside' our world would see things any differently. The empirical evidence is consistent in showing that direct, (as opposed to theoretical) congruence translations are not possible in time.

Yet all peoples acknowledge quantitative features in time. That the idea of "equal intervals of time" transcends cultural differences can be discerned through the great regularity and sophistication of all the world's musics, from the most technologically advanced to the most primitive society . It can only be because of its overwhelming success in the prediction of future events that the idea of equal time intervals is

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[2]**Is it not now apparent to everyone that the gods purposed to put us into a 3-dimensional space so that we could bisect finite line segments without difficulty?**

taken seriously. My watch tells me that it is 4:00. In two hours I will be going home. At 6:30 I will turn on the television and watch the newscast that has been scheduled for that time. The people who publish the TV guide , the producers of the news program, the makers of my watch, and myself are all in agreement that 6 and a half regular hours pass between noon and the time of this broadcast. Our consciousness of a universal time dimension has evolved historically through many levels of refinements of our observations , of the rise and decline of the sun, the seasons, the regularity of the heart beat and other bodily rhythms , observations on freely falling objects, the swinging of pendulums, ultimately the vibrations of atomic clocks.

This indirect evidence of the quantitative character of a universal time dispersed through any fixed reference frame, is so strong that no sensible person would dismiss it. However , we would like to stress that the evidence is indirect only. The belief in the existence of a quantitative, universal time, like that of the square root of two, depends upon the law of the excluded middle: that is to say, it is acquired through indirect reasoning. Consequently time reckoning is dependent on certain assumptions inherent in the way we think about it . These assumptions can be expressed by axioms and postulates , in which time figures as an implicit variable. Its' measurement is likewise indirect, dependent on spatial geometry and the action principles of local physics.

Clocks

Time is measured with clocks. A *clock* will be defined to be any harmonic oscillator or isolated periodic system that somehow distinguishes the completion of its periodic cycle, of length P, by the

emission of a pulse at times P, 2P, 3P.....

Not all forms of space-time permit clocks. Their existence depends upon the *principle of sameness of state* , defined and developed through the postulates in the newxt section.

It is assumed that clocks can be rewound and initiated at any well-defined instant of time. That is to say: given any instant t , (distinguished by some event E) , it is always possible to reset any clock to time the initiation of its cycle with that event. In particular, a clock with period U can be reset to *initiate* its cycle simultaneously with the initiation of the cycle of a clock of period V. *However, clocks cannot be set to coincide at the common termination of their periods* . This is a major distinction between clocks and rulers.

Clocks with periods P/N , N positive integer. will also be considered to have period P. A clock's irreducible period is its' smallest sub-period.

Clocks can also be coupled. If C_1 has period P, and C_2 has period Q, then a clock C_3 of period U=P+Q is formally constructible via a sequence of rewindings whereby C_1 is timed to recommence at each pulsing of C_2 , and C_2 is timed to recommence at each pulsing of C_1 .

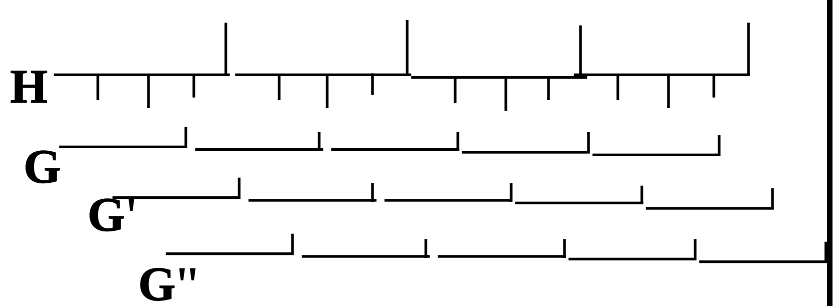
Clocks with identical irreducible periods will be deemed *indistinguishable* in the following sense: If C_1 initiates a P-cycle at t = 0 , and C_2 initiates a P-cycle at time t = T , then this situation is treated as indistinguishable from that obtained by initiating C_2 at t = 0 and C_1 at t = T. that this is not merely an academic quibble can be seen from the following interesting:

THEOREM I : Given clocks H, G , with periods P and 3/4 P respectively, it is possible to construct a clock that eventually pulses

with period 1/2P.

PROOF: **If H and G are initiated at time 0 , then G will pulse at the half-period moment of H's cycles number 2 , 5 , 8 ,...... The assumptions enunciated above enable us to construct a copy, G' of G , which initiates at time P , and therefor pulsates at the half-period moments of H's cycles number 3, 6 , 9 ,......**

Another copy of G, G'', can now be built and initiated at time 2P. Using the indistinguishability principle, the system I = {H,G,G',G''} can be converted into a functioning clock by permuting the identities of G,G' and G'' at each period instant 2P, 3P, 4P.... ! Notice that our system cannot locate the half-period moment of the first cycle, nor can it fit the strict definition of a clock until the third cycle.



Composite Clock System
K= [H; G; G'; G'']

This is a special case of a more general theorem which does not need to be proven :

THEOREM II: **If H has period P, and G has period AP/B , A/B a rational fraction in lowest terms, then it is possible to build a clock that will *eventually* cycle in periods P/B .**

We will be talking about the 'topology' of the time dimension.

The cosmology establishment, Hawking, Penrose, Thorne, Wheeler and so on, have their own sense of the meaning of this expression, but we define it by the possibilities and limitations imposed by physical law on the constructibility of clocks. Hopefully this will be clarified by the numerous examples that we will be presenting. In some abstract or global sense, time may be very different from the ways in which it can be measured, yet for scientific purposes, the two are co-extensive.

The four postulates stated below are requirements for any space-time. The 4 axioms that follow are specific to homogeneous time. By homogeneous time we mean any topology of the time dimension which is homeomorphic to an unbounded 1-dimensional linear continuum, which can, in other words, be parametrized by the real number system.

Definition A:

The term isosystem refers to a dynamical system I which, in the interval of time under consideration, is , in conception or reality, causally isolated from the rest of the universe. We want I to be isolated in a *compact* region of space because we want to be able , by means of its centroid or something similar, to localize it. In the real world, approximate isolation serves to define an isosystem.

Definition B:

A *Space-Time, W* , shall be called a *Void* , if the physical laws governing the behavior of systems in W do not rule out the possibility of moving the initial conditions of any isosystem to any point in space, or any instant in time. For example, a universe dependent upon its origins in a unique Great Explosion, is not a perfect void, though certain regions may be deemed void to all effects and purposes.

Loosely speaking , a Void universe is a world of possibility, the

place where we drop our thought experiments. Worlds of actuality are in general not voids, though in our theories they may be dependent on the existence of a background void structure.

Definition C:

A *state description S* , or simply, a state ,of an isosystem I, shall mean a set of numbers or mathematical relations, by which it is possible to reconstruct I , localized at coordinates V= (x, y, z,t) at any other point V' = (x', y', z', t') in a void space-time W. S contains relative, not absolute coordinates of masses, energies, and other basic observables. Though S changes with time, it does not contain time explicitly but only in the form of time derivatives of relative spatial locations. The state description allows us to move a system around in time and space, and also to tell us when two systems in different places are identical.

3.Temporal Postulates

Postulate I :

Time is subject to a local total ordering: around any instant t , one can define a Hausdorff neighborhood, N , in which time naturally decomposes into the categories of 'past', present' and 'future'.

This investigation requires that spatial measurement by rulers, and temporal measurement by clocks, be distinguished precisely by the lack of congruence motions from past to future. The neighborhood N will be called the "local context". Within it one can employ the usual notation of a total ordering $x < t < z$, to designate past , present and future.

Note the requirement that the instant t be given. In Special Relativity, persons in different reference frames may not be able to agree that t and t' are the same instant.

We cannot assert the existence of a global temporal ordering. Cyclic time can be oriented, therefore be given a local temporal ordering, but one cannot impose a global temporal ordering on a cyclic time dimension, even as one cannot (globally) characterize points on a circle as "before" and "after".

Postulate II (Simultaneity):

All time intervals can be compared in the limited sense that one can state that their beginning and endpoints either do, or do not coincide.

In the context of Special Relativity one must adjoin the modifying phrase " relative to a given reference frame".

Postulate III:

The existence of a length λ , of temporal duration in a given universe W, is equivalent to the possibility (in reality, theory or conception), of constructing a clock with λ as its period.

For example, in a universe W with a minimum time quantum ψ , one cannot build a clock which pulses at intervals $\psi/2$. Notice that this postulate does not say anything about the existence of specific 'moments', or 'instants', which may be correlated with specific 'events. What is says is that, if we have two events E_1 and E_2 , at distinct moments in time t_1 and t_2 , and a clock to measure the time between t_1 and t_2 can't be built , then one cannot assert anything about the length of time between them, beyond the bare fact that $t_1 \neq t_2$.

Postulate IV:

All past and future states of an isosystem I, (though not necessarily their times), can be derived from the complete state description at any given instant.

This can be taken as an alternative definition for isolation. If future states can't be derived from a present state alone, then something else influences the future state. Hence the system is not truly isolated.

4. Axioms of Homogeneous Time

" Physics is Geometry plus an Action Principle"

-Tullio Regge, off-hand remark made at the Einstein Centennial Symposium , IAS, Princeton, March, 1979

Axiom I: (Uniqueness)

The phase-space trajectory J of any isosystem I is always single valued for each instant of isolation.

Commentary : The assumption of the existence of initial conditions doesn't always mean that we know what should go into their makeup. Clearly the values of the constants of nature c , h, μ , e, should be part of that specification; also the relative spatial locations of the point masses ; and all of the local time derivatives

$$\boxed{x, dx/dt, d^2x/dt^2, \ldots; y, dy/dt, d^2y/dt^2, \ldots} \ldots.$$

It can be seen that "time" enters indirectly into the initial state, through infinite sequences of constants which are interpreted as

time derivatives. When the equations of motion are expressed in the form of a vector field or Hamiltonian flow, then time is in fact indirectly adjoined to the list of independent variables as the parameter of the linear subgroup determined by the initial conditions.

The first derivative in particular, dx/dt, expressing the homogeneity of time with space, is a magnitude whose dimension is given by the ratio of the unit of length with the unit of time. Following Zeno, these cannot, without further qualifications, be compared. The rates of change of geometrical magnitudes such as lengths, volumes, tensors, etc. , depends on the topology of the time dimension, which is itself derived from the dynamical structure of the given universe. For those conserved fluxes derivable from the symmetries of the Hamiltonian , (mass, energy, momentum, and others), the first time derivative vanishes and the special characteristics of the time dimension do not affect them. Therefore, in a universe acting in cyclic time, or quantized time, or in homogeneous time, one does not have to modify the properties of conserved quantities.

The assertion of the existence of a unique state S, of a system I, at some given moment in time, t, is more than a tautology : it can be represented by a table of numbers, obtained through observation and experiment . Since the impossibility of making a direct comparison of temporal intervals, also implies that it is equally impossible to state with certainty that the state of a system at time t_1 is identical to that at t_2 , the interpretation of states must also take into account the constructibility of clocks. To use an analogy: my possession of ten dollars 25 years ago, does not have the same consequences as my possession of that same amount today: an unchanging income implies a

continual worsening of one's economic situation, not a status quo! Likewise, one needs some kind of axiom to guarantee that the reappearance of an identical table of state values at a future time has the same meaning that it does now. This is the intent of Axiom I .

Ultimately, there can be no completely satisfying solution to the problem of time translation. One has to assume that it is possible to make a faithful representation of the essential features of a physical system by a set of numbers known as the state description, and that, through this representation , one can say that two systems I_1 and I_2 , at two different moments in time t_1 and t_2 , with the same state descriptions, will behave in essentially the same way.

In pragmatic terms , all of the numbers in the state description will be derived through interactions between the observables of the system with measuring instruments.

Postscript on 'sameness of state' **: Doing science is much like going to the theater: in witnessing a play one sees only brief episodes in the lives of the characters on stage. From these one has to deduce a hidden context and all its interconnections. Likewise, the entire history of human observations of a physical entity gives us only a finite, and indeed rather small, list of reliable characteristics. However the list of attributes of most objects is so large as to be beyond human comprehension. Therefore the criteria for 'sameness of state' cannot be fixed for all time, although they may become progressively sharper through history. Scientific time is actually an almost periodic variable . The 'perfect clock ' can only be established after an infinite number of observations , requiring an infinite amount of time to make them - by which time it will no longer**

be of any use to anyone.

AXIOM II (Connectedness)

If t_1 and t_2 are distinct instants, then there is at least one local context N including both t_1 and t_2. In this context $t_1 < t_2$, or $t_1 > t_2$ but not both.

This relationship of relative past and future need not be the same for every local context containing t_1 and t_2. For example, time may be both circular and oriented, in which case there is a context in which t_1 comes before t_2 , and another in which t_1 comes after t_2 .

Corollary:

Let I be an isosystem in state S at time t_1 If I returns to state S at any time t_2 in the future of t_1 , then there is an instant
$t_3 > t_2$ which, provided I remains in isolation through the entire interval $[t_1,t_3]$, has the following properties:

(i) $S(t_3)=S(t_2) =S(t_1)$

(ii) There is a continuous strictly monotonic map ϕ from the interval $[t_1,t_2]$ onto $[t_2 ,t_3]$ such that , for $x \varepsilon [t_1,t_2]$, $S(x) = S(\phi (x))$

This corollary follows from combining Axiom II with Postulate IV.

Axiom II represents a weakened form of periodicity which allows us to define the concept of equal time intervals. By iteration along the future (past) we generate a sequence of (possibly identical) instants $\{ t_n \}$, and intervals $\{ [t_n , t_{n+1}]\}$ which we are able to "equalize" through

this corollary and the other axioms. Let us emphasize that these are neither definitions nor tautologies. Axiom II makes the non-trivial assertion that *states* and *temporal durations* are related in this fashion.

Axiom II has this further consequence: given two isosystems I_1 and I_2 , with initial states S_1 and S_2 at time $\tau = 0$. As systems in isolation, they will also be isolated from each other. If at some future time, σ , while remaining in isolation, they both return to their initial states, then they will continue to oscillate together in time intervals of length σ for as long as their isolation persists . In other words, the combined state of a pair of isolated systems will be the Cartesian product of each individual state and a *universal time variable* . If they are not isolated one cannot, of course, make such an assertion: imagine two clocks being thrown towards each other across a room so that they smash together in mid-air at time τ . The heavier clock may survive the collision while the lighter is, perhaps, destroyed by it. One cannot then say that they will pulse together for all time with period τ !

Nowadays no-one considers the existence of a universal time dimension self-evident. Relativity made the assumption untenable for global space-time and limited its applicability to a fixed reference frame. We take account of this in:

Axiom III

(A)(Comparability):

Let A and B be isolated clocks . Suppose that there exists,
(within a local context) , a duration l T = [t_1, t_2], such that:

(i) A's period recycles k times in the interval T

(ii) B's period recycles m times in the interval T

Then K = [A,B], the combined non-interacting system, is an isolated clock with period

$$\boxed{P = {km}/{g.c.d(k,m)} = l.c.m.(k,m)}$$

Temporal homogeneity requires such an axiom. Without it clocks cannot be compared. If every isolated system operated under its 'own' time, one would not be able to exclude situations in which two clocks oscillate together until a certain moment after which they begin to diverge. This bizarre situation does not occur when rulers are used to measure length. Moving a ruler R along a line from point x to y, one is perfectly free to say R is the same ruler at y that it was at x, or that a new ruler R' at y has been compared to the one at x and found to be equal.

Only with Axiom III can one speak of a congruence structure on the temporal dimension that works like the familiar congruence structures of Euclidean geometry in 3-space. It is through Axiom III that we can maintain the fiction of *moving through time* , although no such phenomenon exists in reality. Axiom III also excludes the possibility that different parts of the universe might "slow down" at different rates.

Axiom III has other consequences: Let W be some universe in which Axiom III does not hold. 3 clocks, C_1 , C_2 , and C_3 in a fixed reference frame are wound up at and released at the same initial time

t = 0 . C_2 pulses at the endpoints of a time interval, T . C_1 pulses at the endpoints of T, and *once within T* , at a point which, by Axiom I , we can designate as T/2 . C_3 also pulses at the endpoints of T , as well as once within the interval T; *yet this inner pulse is not simultaneous with the inner pulse of C_1 !*

This could happen in the absence of a universal time , so that each clock operates according to its own time dimension. Space- time might then be modeled by means of a 2-dimensional continuum, with a temporal coordinates (τ ,σ) at every instant, each individual system defining a one-parameter subgroup in the temporal plane. We do not have to look far afield for such a model: the relative motion of two frames in Special Relativity requires two variables: "time" (t) , and "proper time" (s) .

In a world of independent temporal trajectories, one cannot always construct a clock C_4 , by which the 3 clocks, C_1 , C_2 , and C_3 , can be compared. i.e., which functions like a transportable ruler. This fact is used in all proofs of the non-existence of simultaneity in Special Relativity, including Einstein's . Axiom III guarantees that the anti-simultaneity thought experiments of Special Relativity do not also apply to *fixed* reference frames.

(B) (Continuity):

If clocks A and B do not have a common period , then we assert that they are comparable without always being able to provide an algorithmic process for making the comparison.

If, however, future time is unbounded, then a count of the number of pulses of A in the intervals

produced by the pulses of B will, *in the infinite limit, (at eternity)* , converge to a unique limit μ , which may be considered their (incommensurable) ratio .

We will say that any space-time W , incorporating Axioms I ,II and III A *abides in Homogeneous Time* . Axiom IIIB depends on the existence of an infinite extent of future time, which belongs properly to what we shall describe as *Euclidean Time* . Without Axiom IIIB, there appears to be no simple way of comparing incommensurable durations , which may cause theoretical difficulties but does not have any effect on practical, or real world, time measurements.

We make a further requirement, that within W, the presence of homogeneous time should be *detectable* : Each of the axioms must translate into some identifiable feature in W:

Axiom I: No double-valued temporal systems. Every system in isolation must be in one and only one state at every moment of time.

The superposition of states in quantum theory is not compatible with our definition of homogeneous time.

Axiom II: For all pairs of isosystems, (in their interval of isolation) , identical initial conditions imply identical future states.

Axiom III : All clocks can be compared.

Homogeneous time allows one to construct a linear metric over the time dimension, such that systems returning to identical states in periods of length P continue to pulsate in intervals of P.

Given clocks C_1 and C_2 with periods P_1 and P_2 : if P_1/P_2 is rational, it is possible to use Theorem II to construct a clock C_3 which shares the irreducible periods of both C_1 and C_2.

If $P1 /P2 = \mu$, irrational, Axiom IIIB states only that C_1 and C_2 can be compared, but doesn't really tell us how the construction is made. The difficulties that arise in this situation demonstrate once again how very different spatial measurement is from temporal measurement. The ratio of incommensurable lengths in space is readily computed by means of the Euclidean Algorithm. This generates a convergent continued fraction expressing the ratio of the lengths to any degree of accuracy. Lengths can be moved back and forth on the real line, segments can be cut away, superimposed and counted: there is no need to travel "out to infinity" to compute this ratio. The comparison of commensurable durations requires Axiom IIIA , that of incommensurable durations requires Axiom IVB.

Time Topology And Clock Constructibility

Clocks are periodically pulsating isosystems. Clocks with infinite periods cannot be admitted, as these have an initial but no terminal pulse. Such a clock is really an 'event' that can be moved around in space-time. Since constructibility only means compatibility with the laws of physics, conceptual systems or thought experiments are also considered constructible.

Examples:

(i) Let W designate a void space-time. If all isosystems in W are periodic with the same period, P, we will say that *time itself is circular* .

(ii) If all isosystems must be periodic, but with different periods, we can show that all periods must be of the form $r\psi$, where ψ is some

arbitrarily chosen duration from the period set:

Let C_1 have period U, C_2 period V. Then we may construct, in a permissible thought experiment, the isosystem $C_3 = [\ C_1 , C_2\ ]$.
Since every isosystem of W must be periodic, C_3 's period Z must be an integral sum of periods of C_1 , and an integral sum of periods of C_2 :

$$Z = kU = mV$$

$$U/V = m/k = r, \text{ rational.}$$

ψ can then be chosen to be either U or V, or any rational linear combination of them. If an isosystem of period rψ is constructible for every rational r, one can describe the time dimension as *linear, nowhere connected, and countably dense* .

(iii) Suppose that a *minimal time quantum* q is operative in W. Then all clocks pulsate with period Nq , N = 1 ,2 , 3, ... Let Σ be a standard clock, of period q , which has been arbitrarily "set" to begin pulsating at some time t = 0 ; and K an observer whose observations depend on Σ . Since all observations are based on measurements, and all measurements derive from coincidences, intersections, simultaneities, or collisions , etc.[3] , *one can assume that W, as seen from K's perspective, moves in discrete jumps of duration q* .

An observer K' , might choose some intermediate time $kq < t' < (k+1)q$, as zero-point, with corresponding clock Σ '. However , Σ and Σ' do not describe the same universe. One might even argue that K and K' cannot communicate their findings to one another, since their sense organs also move in discrete jumps of duration q.

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[3] **If one accepts that "two things cannot occupy the same space at the same time, one might define 'measurement' as a failed attempt at doing so! We did not have to wait from John von Neumann's "collapse of the wave packet" to tell us that all measurements introduce singularities; the difference being that classical measurements are removable singularities, whereas quantum measurements are irreducible singularities**

The simplest model for such a quantized time is one in which all clocks start pulsating, in cycles of N time quanta , N = 1,2,3,... at the same moment, which can be taken as the starting point of the universe. Our own universe, with its starting point at the moment of the Great Explosion, combined with the phenomenon known as 'quantum leaps' is a prime candidate for some kind of discrete time quantum.

So far "time" has been characterized as circular, countably dense, or discrete, entirely on the basis of constructible periodic isosystems.

(iv) Let us say that our time dimension has a starting point at α , and a "hole" somewhere along the line. A given instant of time τ will be deemed non-existent if no clock can be constructed which, starting from α , has a period of length τ .

It is easy to see that none of the instants τ / N can exist either, since a clock with such a period also has τ as a period. One thus has a special structure with a discrete infinite set of moments between t and the birth of the universe, at which causality breaks down. If one admits the principles of coupling and indistinguishability, one concludes that clocks that either initiate or terminate that these key moments cannot be built. Thus the entire countably dense set of instants { p/q τ } is excluded from the time continuum: causal breakdown will be dense within causality.

(v). Next, suppose that time does not have a starting point, yet it is impossible to build a clock with period τ . Arguments similar to those employed in (iii) and (iv) now show once again that the set of causal breakdowns is dense in the time dimension.

(vi) *Quantum Uncertainty Time :* What are the logical consequences of the Uncertainty Principle when applied to clocks?

Normally "states" include masses, positions, momenta and energies, but we can imagine a simplified state defined by a clock's energy levels only. Consider these 3 possibilities:

(A) A precise time T records a vague state S of the clock I

(B) A vague time interval records a precise state.

(C) A vague time interval records a vague state.

For each of these we ask the question : What happens to the temporal topology?

(A): At distinct instants 0 and t, states S_0 and S_t are measured. These states are approximately equal, yet there is an irreducible error due to the uncertainty principle. This error must amplify with each recycling at S_{2t} , S_{3t} ,....until S no longer functions as a clock. For the purposes of measurement in a homogeneous time, all quantum clocks eventually become worthless.

(B) $S_0 = S_t$ exactly, but there is an error in t due to the uncertainty principle. Then C is a clock, but its period is variable. *Time ceases to be homogeneous.* A system with exactly equal states will have variable periods. Time becomes a fuzzy line.

(C) combines (A) with (B) : Clocks are worthless *and* time is fuzzy !

A similar situation analysis can be made of the consequences of General Relativity. Let clock I be transported on an accelerating vehicle. Within its' own frame, I appears to be speeding up with the passage of time. (This follows if the speed of light, c, is to remain 1 in relativistic units.) Identical states cycle in increasingly shorter periods; identical time periods do not correspond to the re-appearance of identical states. Unlike the quantum scenario these discrepancies are deterministic. *The*

observer on board the vehicle must conclude that I is not an isosystem . **The Principle of Equivalence states this in another way: a gravitational field is interposed to rob the system of its isolation. One senses the enormity of the problem of constructing a proper clock for Quantum Gravity: (a) Dysfunctional clocks**

(b) Non-homogeneous time

(c) No sameness of state

(d) Non-isolated systems !!

These examples are consistent with the thesis that time is an implicit variable within the laws of nature , whose properties are given by the limitations on constructible mechanical systems.

5. Cyclic Time

Let W be a space-time within which every real or possible isosystem must be periodic with the same period P. W itself is therefore a clock. If W is a void then any configuration in W can initiate at any time within the complete temporal cycle:any possible system can be dropped into W , provided it oscillates with period P. We will see that it is not possible to give W the structure of a void in most of the space-times which are of interest to us.

W is best studied by embedding it in another space-time Ω , subject to a continuous, homogeneous linear temporal flow that is potentially infinite in the forward direction.

A resonating caesium atom might serve as a model for an isosystem in W. Any watch or clock K will serve just as well, provided we abolish all the laws of Thermodynamics, and pay a janitor to comes around every P-days to rewind it . Or omit the janitor, keep the watch, and take comfort in the thought that we won't be needing the watch

for all time, only long enough to illustrate our arguments.

W's space should be large enough to place uncountably many ($\aleph_1$) copies of K without interference, each of which begins its round at a different instant in the P-cycle. All this is easily accomplished; it's a matter of approaching the NSF when it's in a good mood.

Having conceptualized W, embed it in Ω . Ω contains a person Y who wanders around picking up watches, examining them and writing down what he sees. As he does so he consults his own wrist-watch, which has the remarkable property of recording a potentially infinite stretch of time from, say, the moment that he begins his observations.

For the most part Y is only looking at a single watch at a time. Sometimes he will be running several of them concurrently , which he initiates at different moments. Any *finite* set of isosystems with period P, initiated at different times, will *eventually* produce a possible configuration in W.

Corresponding to observer Y in Ω , there is an observer, X, in W . There are many ways of thinking about X. He can live a short lifespan, Λ , relative to W , (say Λ = 80 years, P = 16 billion years). After lying dead for billions of years, he might , after going through a brief gestation, re-emerge in W at the initiation of its new cycle. Λ could also be an integral fraction of P, $\Lambda = P/N$. X could then be recycled N times in a single period, provided that with each recycling he is teleported to a new region of W whose initial conditions are identical to those of his previous life-time.

Or he can be allowed to live out the full extent of W's period, as long as he, and all his mental contents, return to their initial state at the beginning of each cycle. This implies two options: X can "die" shortly

before the recycling of W, in which case his mental contents abruptly, (and discontinuously) disappear.

Or X may be able to function as an analytic, or C^{∞} system, in which case he need not be born nor pass away. Y observes a continual arising and passing away of knowledge. All that X knows is forgotten, all that is forgotten must be relearned in an unending cyclic process. This enables X to study the behavior of a watch that initiates at time t, and also another watch initiating at time s, where the ratio of $r = |t-s|$ to P is irrational. X merely forgets everything he knows about the first watch at a certain moment, while still retaining his knowledge of the working of the second watch. Thus, X never stops learning, even as he never stops forgetting. However, he cannot always connect what he observes at one instant with what he observes at another.

The purpose of this epistemological analysis is merely to show that, in order to properly describe the global structure of a universe W subject to cyclic time, one has to imagine oneself from the vantage of Y, situated in

Ω outside of W and looking in.

Constructing The Ideal W- Observer, X

X does not need consciousness, only those attributes of a human observer that enable it to construct a scientific theory on the basis of its observations.

X's 'mind' consists of 3 parts:

(A) A data collector E : E makes all measurements with clocks and rulers, then records its findings in:

(B) Memory , M. M stores all the data acquired in some fixed

interval of time $T < P$. The memory contents of X at time t are given by $\mu(t) = M(t-T, t)$. μ alters through time. Clearly if $t < s < t+T$, the contents of $\mu(t)$ and of $\mu(s)$ will overlap in the region $[s, t+T]$. Thus M systematically "forgets" all of its data between t and s by the time it gets to $s+T$. In addition to the data, M also contains:

(C) A theory $\Theta(t)$. This theory consists of all necessary consequences drawn from the data in the memory at time t , through the application of postulates I - IV. One might also call this theory " the paradigm" .

Even as systems and memory recycle with period P, so do paradigms. We see examples of this in our own sciences, which claim to abide in the homogeneous linear time of scientific history - so called 'scientific progress' . Paradigms often contain the seeds of their own demise, as well as the promise of their re-occurrence at some future date. Typical of such paradigms are: 'wave' versus 'particle' models for radiation ; ' psychic' versus 'biological' models for mental illness; the absence or presence of a universal ether; 'catastrophist' versus uniformitarian models in geology; 'nature' versus 'nurture' models in the formation of human character; 'vitalist' versus 'mechanist' models for the behavior of living organisms; and others.

The origin of these cyclic shifts in scientific paradigms lies in the fact that each paradigm alone is unable to give a satisfactory description of Nature. The reigning paradigm gives rise to theories and experiments which tend to undermine it, until the preponderance of the evidence leads to its' abandonment . Because the memories of scientists,

(particularly in the modern world of information overload and

intense pressure to stay on the cutting edge) , are as limited as everyone else's, the new or dominant paradigm quickly eradicates centuries of discovery, which can either sit comfortably in libraries, rotting away, or even be entirely destroyed.

Then, (and it happens frequently) , someone digs through old books and journals and finds an idea which, neglected for centuries, admirably resolves some current scientific controversy. This ancient idea is hailed as a new discovery, accompanied perhaps with some condescending praise of the brilliance of earlier thinkers, who were able, quite by accident, to hit upon our advanced notions !

Examples of ideas which are constantly being "lost" and "rediscovered" include : the unconscious mind; the atomic structure of matter; the quantum of action (Aristotle's *minima*); monads (from Anaxagorus' seeds to Leibniz' monads to Schrödinger's probability distributions) ; chaos (Heraclitus to Mandlebrot); and so on. This quasi-predictable recycling of scientific ideas is modeled in simplified form by the tri-partite structure of the observer X, with his sense organs capable only of recording an instantaneous *now* , finite memory M , and progressive recycling of paradigms Θ.

Nothing captures this phenomenon of determinism in the recycling of dominant paradigms better than the rise and fall of systems of government as described in Plato's Republic. The descent from Platonic Republic, to Monarchy, to Democracy, to Oligarchy, to Tyranny, thence back to Republic , is driven by a simple mechanism that ensures its' continuance in a perpetual chain: the 'son' of the republican,

(monarchist, etc.) , aware of the liabilities in his 'father's' system of

government, leaves home to set up his own state based on the reform of his father's errors. This can serve as a good model for the way in which the mind of an X-observer in cyclic W-space systematically alters his theories through time so that they recycle in period P.

X versus Y's Perspectives

Of the many differences in perspective between X and Y, the following are fundamental:

(1) *Periodicity* . If X is aware of the periodicity of his own cosmos, it is because he has developed a theory, based on his limited observational base, which concludes that this is so. We see this in contemporary cosmologies that incorporate a theory of the Big Crunch. Y does not need a theory: his conviction of the periodicity of W is a matter of simple observation. All he needs do is examine readings on his watch at the endpoints of W's cycle, then check earlier recordings of the states of W's systems to verify that they are identical.

Let us suppose that X has been lucky enough to derive equations which correctly describe his space-time, W. A logical paradox now results from X's realization that all systems in W, *including himself* , are periodic with the same period. He may then realize that he has reached this conclusion infinitely often in the past. He might then decide that it is a waste of time to go on collecting knowledge, since he is bound to lose it all anyway, (another decision he's made infinitely often in the past); or he may hope that some mistake will appear in his equations, or that some miracle will occur this time that will nullify the periodicity of W.

The important point is that, whatever kind of delightful fictional character we want to make of X , all of his ideas about the eschatology

of W must be inferred on the basis of theory.

For Y the situation is much simpler. He simply observes that X recycles his complete mental contents in intervals of P or less.

(2) Continuity : **Ω abides in a homogeneous time that is both linear and continuous. When the isosystems of W , (such as a standard watch, K) , are embedded in Ω , the observer Y, who lives in Ω , will observe that all of the states of K proceed in a continuous, uninterrupted flow. To propose an analogy , imagine two persons , X and Y, observing the yearly return of the 4 seasons. Y comments on the differences between this year's spring and the last one because he retains memories from the previous year. But X suffers from a form of Alzheimer's Disease, and always thinks of each spring as a new event which he is unable to relate to anything in his past. Generally speaking, if X observes an event at time T_1 , he cannot compare this to the identical event at time $T_2 = T_1 + P$.**

This has many consequences: All of the topological features in

examples (ii) and (iv) above apply to W: from X's viewpoint, time is linear , nowhere connected and countably dense. Causal breakdown is dense in the time dimension.

Furthermore, suppose X has a starting point, or 'date of birth' in W. Then X's memory must begin from that moment. A possible model is $\mu(t) = M(0,t)$, for $t<T<P$, and $\mu(t) = M(t-T,t)$, for $T<t<P$. At the end of W's cycle, M must suddenly empty out its contents and start from scratch. If we let T approach P, then the final paradigm $\Theta(P) = \Theta(0)$ is simultaneously X's discovery of the complete structure of W, and the

complete eradication of all of his knowledge! If we want X = (E,M,Q) to be continuous without causal breakdowns, then one has to somehow incorporate a gradual fading away of memory, with m(t) --> ϕ, the null set, as t --> P. Each paradigm in succession incorporates the latest discoveries on an increasingly eroded database.[4]

Inertial Paths, Frames, Time Scales

W is a void. All conceivable systems which do not contradict its laws can be placed in it anywhere. Such uniformity implies spatial translational invariance, and temporal translational invariance in the forward direction. In his laboratory where he experiments with W, Y can initiate any W-system at any time. Assuming only the conservation of mass, these space-time symmetries imply, by Noether's theorems, the conservation of momentum and the conservation of energy.

We intend to demonstrate that:

(1) The inertial paths in W must be finite orbits.

(2) The "spatial" geometry of W is an elliptic Riemannian manifold

(3) The "space-time" geometry of W can be modeled on a (flat) torus.

(4) An oriented cyclic space- time void is incompatible with particle physics, and should be modeled by a wave mechanics. Inertial paths become the resonant circuits we normally characterize by quantum numbers.

What is the relevance of the concept of an inertial path? The classical definition of inertial paths, frames and time-scales consists of two parts:

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[4]Like so much of today's science !

(1) In an inertial frame, every freely moving particle describes a straight line. These lines are inertial paths.

(2) An inertial time scale is one such that any freely moving particle tracks equal distances in equal times.

Systems move along inertial paths with uniform velocities. Where there is uniform velocity, one can set up a 1-1 correspondence between space units and time units. If time is cyclic, all spatial trajectories take the form of closed loops. For the moment we assume that they can be of any length, and may be self-intersecting.

The allowable path lengths of particles moving with initial velocity v, are therefore given by

$$\boxed{L_N = vP / N, (N = \pm 1, \pm 2, \ldots\ldots)}$$

It is apparent also that N ought to be a function of v, *as it is in the nature of an inertial path that its configuration be uniquely determined by its velocity relative to an observer at rest* . Hence

$$\boxed{\begin{array}{l} N = F(v) \\ L_N = (v / F(v))P \end{array}}$$

Applying elementary considerations to inertial motions , it is possible to show that W must have the geodesic geometry of an Elliptic Riemannian space and a toroidal geometry in space-time:

(1) ***It is important that inertial motions form a group, G : W ,as void, must look the same to someone on an inertial path, as it does to someone at rest. In particular, the class of inertial paths is not changed , only permuted, by motion along any one of them.***

(2) ***Any point on the path of an inertial motion can be taken as the origin in time and space for that motion.***

This rules out self-intersecting loops: every *space-time* location on

an inertial path should be topologically indistinguishable from every other location. However a path is allowed to recircle itself any finite number of times.

(i) Let the dimension of W be 1+1 : one spatial and one temporal dimension. The above considerations lead to the conclusion that W's space is a ring of fixed length, L. We can give W a 'flat' metric by using , as our model, the geodesic circle on a flat cylindrical surface obtained through cutting the surface by a plane orthogonal to its axis.

W's inertial group is therefore:

$$\boxed{\begin{array}{l} v = Nc, N = 0, \pm 1, \ldots. \\ c = L/P \end{array}},$$

where L and P are the fixed length and fixed period respectively. All such inertial motions "resonate harmonically", performing an integral number of revolutions around the space-time torus in the universal time period, P. In a non-relativistic periodic universe only this discrete set of velocities is permitted.[5]

Interactions Of Inertial Systems In W

Further considerations on particle scattering within W confirm that a wave, rather than particle, mechanics, is the appropriate framework for a cyclic universe. Take an observer, O, at rest at some point on W, watching a particle p_1 of some standard mass , M, circulating W at some speed which is an integral multiple of c , v = kc . (k can be larger than 1. There need not be particles circulating at velocity v = c.)

By moving reference frame p_1 , the void character of W tells us

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[5] **We discuss a relativistic model based on length distortion in the final section of this article.**

that there is another particle p_2 which, relative to p_1 , moves at velocity kc, therefore relative to O with velocity 2kc. In this way we derive the existence of particles p_1 , p_2 ,...p_n , all of mass M, moving with velocities kc , 2kc,........nkc . Thus, any velocity v* = nkc is produceable once one allows v = kc.

We now subdivide the ring L into equal lengths L/n, and, at each division point, place a particle at rest. p_0 , p_1 ,......, p_{n-1}. The particle p_0 is now set in motion with velocity kc. It hits p_1 , loses all its momentum and comes to a halt. p_1 picks up at velocity kc and moves along to hit p_2 , etc...... The process continues around the ring until, after a length of time equal to the period P/k , every particle p_j is in the place formerly occupied by p_{j+1} ; (j is naturally (mod n)) Thus, in time P/k , the configuration along the ring has been shifted forward by an amount L/n. *Although clocks with identical periods are indistinguishable, particles of identical mass are not* . The total amount of time needed for the return of the configuration to its initial state is therefore nP/k . *If n is chosen so that n= rk, where r is some integer > 1 , this system will not be periodic with period P.*

The only solution is to require that k = n, that is to say, no particle in our space can move with a speed less than nc. However, we are free to make n as large as we please, merely by adding more particles, which implies that the "minimum" speed must be infinite!

Conclusion: **Wave mechanics, which permits the superposition principle, is the only compatible framework for cyclic time.**

Authentic Versus Deficient Clocks

Let W be a void cyclic space-time with period P. An *authentic*

clock is defined to be a periodic isosystem that is potentially isolated throughout the entire period of W. This concept of *potential isolation* is an important one. Let us suppose that a clock C has been constructed to pulse with period $\sqrt{2}P/2000$. C cannot pulse throughout the entire length of W's period because it's period is incommensurable with that of P. C is isolated but cannot remain so. We say that C is not potentially isolated, thus not authentic, but *deficient* .

Let us suppose, however, that C pulses for 1000 cycles, then smashes into a wall , Γ . C buzzes along, out of control, for the remainder of W's cycle before starting all over again. It would seem that we are able to construct a clock to measure incommensurable periods , at least for part of W's history. The problem with this is that, in the context of W, C is not an isosystem. *The full system, from the very beginning of C's history, consisted of C plus the wall, Γ , into which it was destined to crash.* Otherwise C could not have been constructed. *Clocks in potential isolation , of periods incommensurable to P are unconstructible.* Authentic clocks must be potentially capable of cycling indefinitely in the universe in which they are placed.

Now, what happens if we construct the system "clock plus wall", I = C+Γ , within W, and use this to measure intervals of time of incommensurable duration for some limited period of time U < P ? Since such systems require the eventual disposal of C as a clock , their operation is dependent on a violation of Postulate IV, expressing an important feature of causality: *All past and future states of an isosystem I, as well as their temporal ordering, (which may not be isometric), can be derived from the complete state description of I at any given instant.*

The regular return of C to its initial state over finitely many cycles

does not imply that it will continue to pulsate in regular intervals forever (modulo W) .

Isolated clocks which are not potentially isolated must experience a singular breakdown in causality at some point in their history.

All clocks which are not *authentic* will be called *deficient* . There are two kinds of deficient clocks: The first are those which, as described above, are isolated but not potentially isolated. The second are potentially isolated as well as isolated, but which happen to interact with other systems at some point in their career. The first are deficient by construction and may be called *intrinsically deficient* . The second are deficient through the particular W- world-line on which they happen to be moving, and may be called *extrinsically deficient* .

In the context of a Big Bang- Big Crunch model, all systems in isolation, possible or actual, eventually interact. The moment of total destruction functions as a convenient 'causal breakdown' that wraps everything up nicely: our universe can proceed as if it operates under an unimpeachable linear or homogeneous time, even as we rest secure in our knowledge that, sooner or later, a global catastrophe will wipe the slate clean and wrench the universe back into a cyclic mode!

This is overly simplistic. Only intrinsically deficient clocks need a catastrophe to put them back on the right track. Furthermore, deficient clocks don't all have to break down at the same time, provided that the total time of "operation plus catastrophe plus rewind" for each deficient system is some integral fraction of the total period .

Properties of Authentic Clocks in an Oriented Cyclic Void Space-Time, W

Authentic clocks are subject to major structural restrictions:

(i) Authentic clocks C_1 , C_2 such that C_1 's period does not divide C_2 's , can't be coupled.

(ii) Individual authentic clocks demarcate sets of indistinguishable instants. Their inertial time-scale is therefore not homogeneous.

(iii) Without the aid of an intrinsically deficient clock to serve as a counter , authentic clocks cannot determine an Arrow of Time. They cannot distinguish between past, present and future.

(iv) Without the aid of an intrinsically deficient clock, any system of time reckoning based on authentic clocks will violation Axiom I for homogeneous time: states may be multi-valued functions of time.

(iv) Authentic clocks cannot be continuously deformed into one another.

(i) *Coupling* . All authentic clocks have periods which are subperiods of the universal period P. Yet they need not be subperiods of each other. Imagine a clock C_1 with period P/4 , C_2 with period P/5. A coupled system is one in which the period of C_2 is concatenated with that of C_1 to produce a total period of P/4 + P/5 = 9P/20, This is equivalent to constructing an authentic clock of period P/20 , "counting" off 9 periods, and constructing another clock from the repeated cycles of that counter. However, it is not possible to turn such a counter into an authentic clock, since its period is 20 P, contrary to the restrictions on space-time.

(ii) *Non-homogeneity* : Authentic clocks in W must have periods P, P/2, P/3,...... Assuming that all such clocks are present, we can assemble a system C_G for measuring time, combining clocks C_N , for

each permissible sub-period. We've invented our clocks so that clock C_N , at each recycling (at instants kP/N , k $\leq$ N) , will emit a whistle of loudness A/N^2 . "A" is just some standard amplitude . This whistle being periodic, it cannot differ from one cycle to the next.

Consider now the 'sound' emitted at time P/2 . It consists of the accumulation of whistles of all the clocks of periods P/2j , j = 1,2, ... This has amplitude

$$A/2^2 (1 + 1/2^2 + 1/3^2 + \ldots) = (A/4)\sum_{k=1}^{\infty} 1/k^2 = A\pi^2/24$$

In general, at time P/N, the total sound intensity will be $A\pi^2/6N^2$, which is different for each N. C_G can therefore *distinguish between* all the times 0=P, P/2 , P/3 ,...... However, there can be no way, for C_G to distinguish the instants:

$$q_j P/N, \; j = 1,2,\ldots..$$

,where the q's range over the set of integers Δ(N) less than and relatively prime to N.

(iii) *Orientation* . Given events E_1 and E_2 : for them to be recorded by the same authentic clock, their temporal separation must be a rational fraction of P. The clock C_N that does this cannot distinguish between the interval T that goes from E_1 to E_2 , and the interval P-T that goes from E_2 to E_1 .

(iv) *Counters* . Since C_G records all the pulses of Δ (N) as a single

instant, any system in W whose behavior is being timed by C_G can

have
multi-valued states, violating Axiom I for homogeneous time. If we employ an intrinsically deficient clock system to serve as a counter, the history of its state description must incorporate one or several moments of causal breakdown, thereby violating Axiom II for homogeneous time.
(Identical states produce identical futures)

(v) *Deformability* : We have built a clock with period P/2 , using springs, transistors, whatever is available in our space W. Now we want to construct a slightly heavier replica which will cycle with period (P/2)(1- ε), ε arbitrarily small . This isn't possible in general, since the next permissible period is P/3 . *Thus, the restrictions on time place equal restrictions on the deformability of all mechanical systems.* Slight perturbations of parameters such as mass, speed, energy, etc., are ruled out. *Indeed, the mere existence of a cyclic constraint on the time dimension automatically quantizes all observables* .

Summary of the Properties of Non-Relativistic Cyclic Time

I. The inertial paths of all moving entities, (particles, wavefronts, fields), are topological circles. If W is a 1+1 space-time, then there is an absolute unit for time (the period, P), an absolute length L, and a *minimum* velocity c = L/P. (If P is very large compared to L, this quantization is not noticed, and speeds may appear to vary continuously.)

II. If W is a void, then all of its dynamical systems must obey the laws of a wave mechanics, and not a particle mechanics.

If the inertial motions of W don't form a group , one can construct

particularized universes allowing for a very restrictive particle mechanics

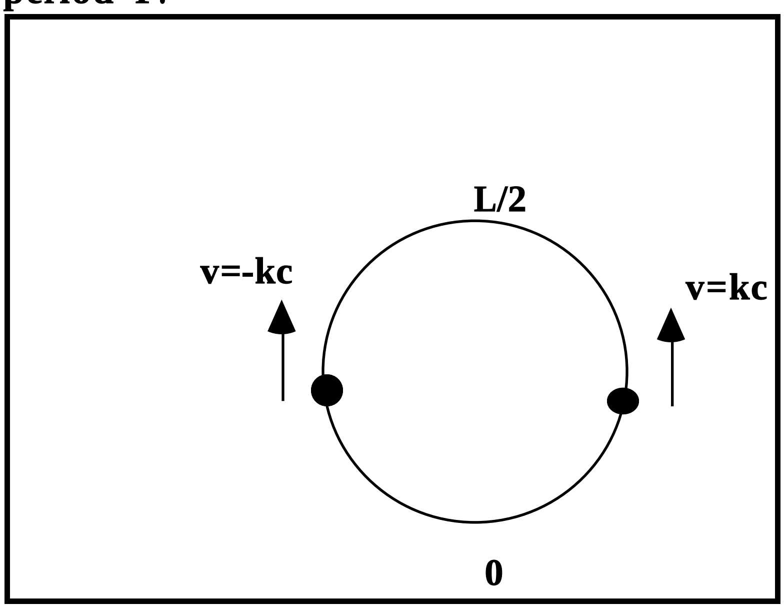
Examples:

(a) W is a static universe

(b) W contains a single particle, of mass M, orbiting with fixed velocity kc , where k is some integer ≥ 1 .

(c) By placing a set of n particles, moving around the ring of W all in the same direction, and at fixed velocity kc , k ≥ 1.

(d) Number 2n equally-spaced particles in succession. If all the even-numbered particles move clockwise, and all the odd numbered move counter-clockwise. then with 2n particles on the ring, moving at velocities ±nc , the entire configuration of motions and collisions will recycle exactly in the period time, P . For example, the following system recycles with period P:



III. If W's laws forbid the construction of intrinsically deficient clocks then ,relative to any clock system C_G :

(a) Only rational instants rP , r = q/n <1 , can be observed

(b) One must allow for the possibility of multi-valued states for sets of equivalent instants of the form (q_j P)/ n, qj relatively prime to n.

(c) There is no well-defined arrow of time

(d) All observables are quantized

IV: If some inauthentic, extrinsically deficient clocks are constructible, *then causal breakdown must be built into the space-time of W* . This may take the form of a Big Bang, Big Crunch, radioactive decay, Hawking radiation, etc. Note that the existence of moments of causal breakdown within W's cycle is incompatible with a void structure.

V. An internal observer X is defined to be a system X= [E ;M ;Θ] . *Internal observers never have enough information to develop a global picture of W* .

The theories Θ developed by Γ are derived as necessary conclusions from the information provided by the databank M, which, by a *principle of conservation of information* , is always discarding and adding new data. VI. *Although X is able to identify a clock by invoking the postulates dealing with sameness of state, it cannot discriminate between authentic and inauthentic clocks* . X's theories do not allow it to predict causal breakdowns. Y, the observer in the linear homogeneous universe Ω , is under no such constraint. His theories can always be tested against future observations.

VII. Paradigms. If X constructs theories about W, Θ_1 and Θ_2 at different times $t_1 < t_2$, where $t_1 + t_2 \geq P$, each is capable of refuting the other: in t_2 's theory , t_1 comes before t_2 , in t_1 's theory t_2 comes before t_1. This sets up a dynamic of paradigm recycling similar to

Plato's description of the transformations of governments from republic through to tyranny and back again.

This short list of characteristics of non-relativistic cyclic time shows that cyclic time cannot be homogeneous .

In fact, cyclic time violates all the axioms for homogeneous time:

Axiom I (Uniqueness) : For space-times without causal breakdown, multi-valued states must be allowed. For space-times with causal breakdowns, the state S is singular at these breaks: multi-valued, infinite, or non-existent.

Axiom II (Reproducibility) : Without a well defined arrow of time, the instant t_3 , (in the Corollary to Axiom II) may well be identical with t_1 . Repetition of state and recurrence of time are thereby confounded.

Axiom III (Comparability) :

Clocks cannot, in general be compared. If clock C begins its cycle at time t_1 , and an identical clock C*'s initiation moment is moved to time t_2 , where (t_2 -t_1) / P is irrational, then no clock can be built which pulses at the time interval between a pulse of C and any pulse of C*. Such irrational time intervals may be undetectable by X, but they will be constructible by Y.

One cannot build authentic counters even for rational intervals . Using inauthentic clocks one can build counters, but they must crash at the inevitable causal breakdowns. The most one can say is that some clocks can be compared over limited periods.

6. Axioms Of Euclidean Time

The enunciation, in the century before Socrates, of a set of fundamental space-time paradoxes has traditionally been attributed to

Zeno [6] , a prominent figure in the school of Parmenides at Elea. It is not unreasonable to conjecture that Zeno's paradoxes will always be with us. Seemingly trite, almost childish in their initial presentation, they become more opaque , not more transparent, with each re-examination. Straddling the borderlines of logic, geometric intuition and common sense, they have , and will continue to play an active role at the birth of major areas of scientific investigation: atomism; Archimedes' method of exhaustion; Newton's infinitesimals; Euler's theory of infinite series; issues of continuity and convergence; Cantor's transfinite arithmetic; Russell's Theory of Classes; Dedekind's theory of cuts; quantum physics; contemporary chaos theory .....

Two of Zeno's paradoxes are of particular interest to us: the *Achilles* paradox , and the *Dichotomy* paradox:

> **"... Achilles is racing against a tortoise that has been given a head start, and it is argued that Achilles, no matter how swiftly he may run, can never overtake the tortoise, no matter how slow it may be. By the time that Achilles will have reached the initial position of the tortoise, the latter will have advanced some short distance; and by the time Achilles will have covered this distance, the tortoise will have advanced somewhat farther; and so the process continues indefinitely, with the result that the swift**

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[6]**Florian Cajori: "History of Zeno's Arguments On Motion" American Mathematics Monthly 22 (1915)**

Achilles will never overtake the slow tortoise..."

-A History of Mathematics , Carl B. Boyer, pg. 75 . John Wiley,1991

It has become customary to 'resolve' this paradox by chiding Zeno for not knowing enough about Cartesian Analytic Geometry. The solution, such commentators tell us, is 'obvious' to a modern audience: let the Abscissa represent Time, the Ordinate Space, and on the graph defined by them draw lines showing, respectively, the trajectories of the tortoise and of Achilles. Their intersection point gives both the time at which they will meet and the distance they will have run.

Lancelot Hogben waxes lyrical over this explanation in "Mathematics for the Million." (pgs. 11,12; W.W. Norton , 1971)

" The Greeks were not accustomed to speed limits and passenger-luggage allowances. They found any problem involving division very much more difficult than a problem involving multiplication [...] For all those and other reasons which we shall meet again and again, the Greek mathematician was unable to see something that we see without taking the trouble to worry about whether we see it or not. "

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To our way of thinking, it is not this graph which represents the solution to the Achilles paradox; rather it is the graph itself which expresses what is paradoxical in the Achilles paradox! :

(i) What grounds are there for assuming that time is a 1-dimensional smooth manifold, topologically homeomorphic to length?

(ii) This being granted, what further grounds are there for assuming the existence of a natural 1-to-1 linear isometry mapping from time to length?

Achilles and the tortoise might be residents of a cyclic universe W. Let us say that, from some mutual starting point , the tortoise sets out first with velocity v_1 . Achilles waits for a time interval of length σ before running after it at velocity $v_2 >> v_1$. Let P be the recycling period of W, and let:

$$v_2\sigma/(v_2 - v_1) > P, or$$

$$\sigma > (1 - {}^{v_1}\!/_{v_2})P$$

In this case the entire universe is obliged to rewind and start all over again long before Achilles overtakes Mr. tortoise! (Needless to say, neither Achilles nor the tortoise are authentic isosystems.)

Similar objections may be raised concerning the standard "infinite series" solution that is most often proposed for the Dichotomy paradox.

In *Mathematics and Western Culture* , (*pgs.. 405-404, Oxford University Press, 1953*) , Morris Kline even proposes to solve the Dichotomy with Russell's class constructions and Cantor's transfinite arithmetic!

> **"...before a moving object can travel a given distance, it must first travel half this distance; but before it can cover this, it must travel the first quarter of the distance; and before this, the first eighth, and so on through an infinite number of subdivisions. The runner wishing to get started must make an infinite number of contacts in a finite time; but it is impossible to exhaust an infinite collection, hence the beginning of motion is impossible."**
>
> ***-Carl Boyer, op. cit., pages 74-75***

We have a different take on the Dichotomy:

If the runner advances 1 foot in the first second, 1/2 foot in the next 1/2 second, and so forth, what justifies the unstated assumption that the semi-group operation which concatenates these lengths to produce a total length of one foot, has anything to do with the semi- group that concatenates temporal durations? How can one speak of measuring, let alone adding , intervals of time if it turns out to be impossible to build clocks to measure them?

Indeed, in line with our investigation of cyclic time, how can we even justify the assumption that a velocity of 2 feet in 2 seconds is the same as 1 foot in 1 second, or 1/2 foot in 1/2 second, etc. ?

Differing time intervals s and r, respectively, can only be defined

when clocks with periods s and r are constructible. This is already not self-evident. How then can it be taken for granted that it is possible to put together an assemblage of clocks that will perform the infinite sum:

$$\sum = 1\text{sec.} + \tfrac{1}{2}\text{sec.} + \tfrac{1}{4}\text{sec.} + \ldots \ ?$$

The kind of time one can represent on a Cartesian graph along with length or other magnitudes, depends upon a system of axioms peculiar to itself. We call this *Euclidean Time* .

Such a system of axioms for Euclidean time is presented below. Their independence, consistency and completeness is discussed to some extent but we freely admit that more work needs to be done towards establishing these requirements.

Notational convention:

Let K_1 , K_2 be isosystems, at least in some arbitrary time interval, [U , V] . Then:

K3 = [K1 ; K2] will designate the isosystem formed from them in the interval [U,V] .

K_4 = K1 + K2 will designate the combined system in that interval of time [R,S] including [U,V] ($R \leq U$, $S \geq V$) , in which they do not interact with the rest of the universe, (but may interact with each other) .

Euclidean Time is assumed to be homogeneous. We therefore incorporate all of the axioms of homogeneity as Axioms I, II and III.

Axioms Of Continuity

AXIOM IV :

Given K_1 , K_2 isosystems, their initial conditions represented by complete state descriptions $S_1(0)$, $S_2(0)$. Let them be mutually

isolated at time t = 0. Then the state S_3 of the composite system $K_3 = K_1 + K_2$ has the following form:

(i) In the time interval [U,V] surrounding 0 , (when K3 = [K1 ; K2]) , the state description S_3 (t) is given by S_1(t) , S_2(t) , the *relative* spatial locations of K_1 and K_2, and the time t ε [U,V].

In the remainder of the time interval [R,S] , S_3(t) is *a single-valued function* of S_1(t) , S_2(t) , the *relative* spatial locations of K_1 and K_2, and the time.

AXIOM V:

All observables , q, p, m, E, etc., are right continuous and right differentiable. That is , if O is an observable defined in an interval (t, s), and x is a point in this interval, then $\lim_{z \to x^-} O(z) = O(x)$ .Thus, nature is 'not too discontinuous', and all velocities, computed as time moving, from the right, back to the instant, exist. This is the procedure followed by J.v. Neumann in *Mathematical Foundations of Quantum Theory* , in his definition of the *resolution of the identity* . (*pgs. 113,119, Princeton University Press, 1935*)

Axioms of Orientation

Axiom VI: (Time's Arrow) :

There exists at least one standard isosystem J , which is never periodic. Such a system will be called a *universal counter* . J is therefore isolated through all time. In its state description is included a monotonically increasing state variable , S , which functions as a calendar for the history of the universe. Traditionally this is taken to be the direction of cause and effect, of living consciousness, of the increase in entropy, of the Hubble expansion of the universe, etc.

Axiom VII (Principle of Relativity) :

The ratio of the spatial metric unit to the temporal metric unit is an invariant of nature. It is referred to as the 'standard velocity', and can be measured by the motion of a standard isosystem along an inertial path, a standard ruler and a standard clock.

In a relativistic universe, the standard isosystem is the light quantum. It is Axiom VII which permits us to assert that a speed of 2 inches in 2 minutes is indeed the same as 1 inch in 1 minute, and so on.

Commentary:

It might appear that Axiom III, (Comparability) and Axiom VI , (the existence of J), are equivalent, or that at least the existence of a counter guarantees that all clocks can be compared: VI implies III. However this is not so. One can imagine a world in which every instant has some unique or distinguishing characteristic, yet in which one cannot, even in principle, construct a clock pulsating at some regular period, Λ; or one in which sameness of state is unachievable ; or one within which periodic systems in potential isolation are ruled out : all constructible isosystems must interact with something else at some point in time.

Conversely, Axiom III cannot be used to build a counter J. Space-times in which all systems are periodic, with no largest period, have already been discussed.

Axiom VII (Relativity) is the only satisfactory way of dealing with the Achilles paradox. The presence of a standard velocity shifts the burden of the problem of the continuity of time onto the continuity of space , which can then be handled via the methodology

of Dedekind cuts . One need no longer speak of comparing clocks "at zero" (construction of infinitesimal clocks) , or "at infinity"(counting of relative period pulses) . Axiom VII combined with Axiom III allows for the comparison of clocks by back-reconstruction to an initiating moment, that is to say, 'at zero' .

Axiom VII in combination with Axiom V makes possible the meaningful definition of a uniform velocity. Otherwise stated, the dimensional unit D/T (= distance unit /time unit) , corresponds to some physical reality. Fixing a reference frame, let ρ be a particle moving at a uniform velocity c . The axioms of Euclidean time enable us to determine the value of c by inspecting clocks set up along ρ 's path. We are also permitted, in theory at least, to build up a universal collection C_G of clocks, each one of which has a distinct period corresponding to some instant within the unit time interval. The combined system K = [*J* ; C_G] extended over all time, can now be employed in the performance of all arithmetical operations. K functions in time like a ruler on the Euclidean line.

Example:

Given two time intervals U_1 and U_2 of different lengths and different initiating points, their "sum" may be constructed in the following manner: Using the standard inertial system I = (c, ρ) , one translates these time intervals into lengths $L_1 = cU_1$ and $L_2 = cU_2$. The length $L_3 = L_1 + L_2$ can then be constructed using a markable ruler, or ordinary ruler and compass. From the system C_G we now select a clock of period $U_3 = L_3 /c$, which is the time it takes the standard particle ρ to move the length L_3 .

It must be emphasized that Axiom VII cannot be derived from

the other axioms. Even as the measurement of physical length is based on the assumption of the invariance of rigid bodies under the action of the Euclidean group E , so is the measurement of time absolutely dependent on the assumption that all of the periods of an isolated periodic system are equal, which is a form of invariance under a discrete group Γ . ***The groups E and Γ are so different that one must invoke an independent axiom to guarantee that the ratio of the units involved in these two forms of measurement is a natural invariant.***

The invariance of the speed of light leads to anthropic speculations to the effect that Nature wanted to make sure that Achilles would overtake the tortoise in all reference frames! We seem to need Relativity to solve problems raised by the paradoxes of Zeno. Once again it appears to us that space-time is not so much an advance over classical physics, as the consequence of a central previously unstated assumption.

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7. Relativistic Cyclic Time

Are there relativistic models for cyclic time in which a continuum of velocities is allowed? Note, once again, that we are speaking of inertial velocities: extrinsically deficient velocities are always possible by making allowances for causal breakdowns.

Our models should preserve these essential features of inertial systems:

I. An inertial path of a system is a uniform curve in space-time. There is no inherent way of distinguishing one location from any

other as the origin of motion.

II. Inertial motions should form a group. This means that the conceivable world (void universe) must look the same for observers on all inertial paths.

The Aristotelian Model

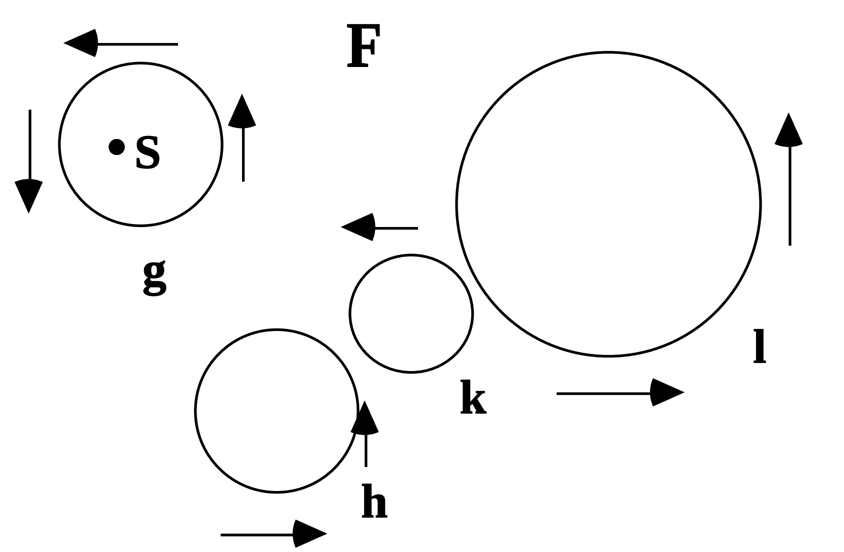
Our first model for cyclic relativity has been inspired by the rotation of the planets in the solar system against the frame of the fixed stars : Aristotle's cosmology .We hypothesize a 2 -dimensional absolute oriented rest frame, F , relative to which all inertial systems (particles) appear to rotate counter-clockwise all with the same period, P . ***The velocity of a particle on an inertial path is proportional to the circumference of its orbit .***

Relative to F, a typical particle g, at location (x,y) and with velocity vector v, will rotate counter-clockwise in a circle of radius R, where $v = 2\pi R/P$ **. Any circle in the F-plane can be the inertial path of a particle.**

Obviously the world must look very different to someone moving with the particle itself. Relative to g, another particle h will not appear to move along a simple uniform curve with a uniform motion. Indeed the complexity of its circuit requires a computational scheme similar to that of Ptolemaic epicycles.

However, the observer on g can reason that the apparent motion of h and other particles is not the real motion. **The real motion is obtaining by imagining the absolute frame to be at rest and re-calculating all other orbits from this vantage. In g's rest frame it is the absolute frame which appears to rotate with period P . He therefore finds, through calculation or observation, that location, perpendicular to his plane of**

observation,which rotates about him in a perfect circle. This location, S, may be designated g's "sun". The distance from g to S is his "solar unit". In these units, g's orbit has length 2π. Calculated from S at rest, his velocity is $v_g = 2\pi/P = c$, ***the speed of the absolute frame relative to all inertial systems in W*** **. By converting from the Ptolemaic to the Copernican viewpoint, all inertial systems appear to move on uniform circular paths, at velocities proportional to their radii. Thus the universe cycles with period P.**



Aristotelian Model for Cyclic Time

Aristotelian inertial paths satisfy many of the traditional requirements for inertial systems:

(1) ***The state of an inertial path is described by the velocity vector, calculated in the absolute frame*** **.**

This is a single-valued but unfortunately not constant function of time. There may be a way of modifying the metric so that all circles will

be flat in the manifold

(2) *All inertial paths are uniform shapes in absolute space-time.*

This means that any point along the path can be taken as the origin of the motion and the frame of its observer

(3) *The universe, (scaled modulus the radius of the observers' distance to the 'sun', or 'solar unit'.) looks the same from the viewpoint of an observer at any point of any particle's trajectory.*

Like radiation in Special Relativity, the absolute frame stands outside of mechanics. On any inertial orbit, the observer will describe the universe with the same physical laws as an observer on any other orbit.

"Collisions" wreak havoc with this model. Once again it is suggested that a wave mechanics, with superposition of states, is most appropriate.

The Conic Model

Special Relativity includes both length and time distortion. However, in a cyclic universe *in which velocity is the only variable entering into the state description of an inertial motion* , there can be no time distortion. In particular there isn't any Twin's Paradox:

Let Y be moving away from X along a loop of finite length L . In a cyclic universe all inertial paths, (indeed all world lines) must be orbits. This means that no acceleration need be applied to Y to bring it back to X. The state of all of Y's functioning clocks and rulers must be identical to what they were when he first moved away from X at some initial moment that , by mutual consent, was set to t = 0 .

Since inertial motions form a group, the statements

"*Y is moving away from X at velocity v* " ,

and

"X is moving away from Y at velocity -v "

are equivalent. Therefore, any speeding up or slowing down of Y's clocks relative to X is accompanied by a equal speeding up or slowing down of X's clocks relative to Y. At their next meeting they will record no relative divergence .

Length distortion is still a possibility . Let us fix a standard reference velocity, c . A standard particle, α , orbiting a one-dimensional loop at this velocity, will appear, to an observer O at rest, to return to the origin in time P. The distance traveled by q will then be L = cP . If another particle , β, moves relative to O at velocity v $\neq$ c , then for β to orbit W in time P there must be an apparent length distortion given by

$$\boxed{L' = vP = (v/c)L}$$

The path-length is therefore proportional to velocity **. We recognize in this relation the Hubble formula for the expansion of the universe. It is easy to show that it governs a group:**

Consider 3 systems: U at rest, V traveling away from U with velocity v_1 , and W traveling away from V with velocity v_2 . In time t (no time distortion, therefore universal) , V has moved a distance $d_1 = s_1 t$ from U , while W has moved a distance
$d_2 = s_2 t$ from V. Since $d_1+d_2 = s_1 t + s_2 t = (s_1 + s_2) t$, the composition laws
$s_3 = (s_1 + s_2)$, and $d_3 = d_1+d_2$, will be arithmetically consistent. That is to say, either distance and speed add by normal arithmetic, or neither of them do.

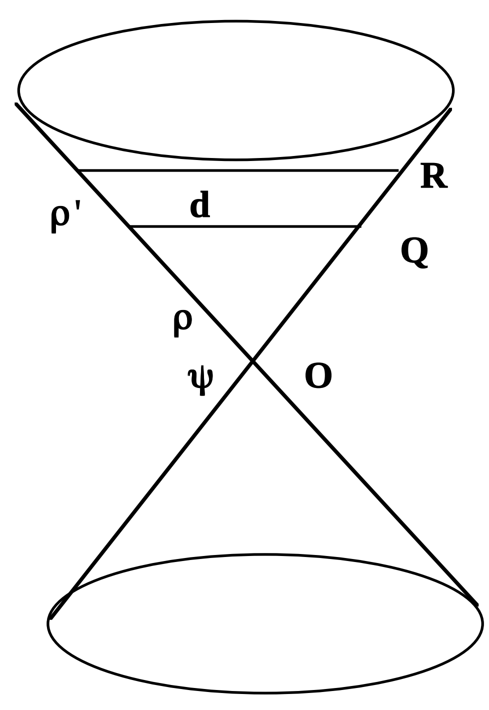
Note that distance is not required to add in the ordinary fashion,

since in fact d_1 and d_2 are on different orbits! If, for example, we wanted velocity to add via the relativity composition formula ,

$$s_1(+)s_2 = (s_1 + s_2) \big/ (1 + s_1 s_2)$$

, then distance would follow the same formula .

The space of the observer O can be pictured as a conic surface, on which particles move along inertial paths that are constrained to circular paths formed by planes intersect the cone at right angles to its axis. The observer is located at the vertex. None of the orbits intersect him but he can see them all. The upper half- cone contains the orbits of all systems moving with positive velocity. On the lower cone they move in the opposite direction.



Relativistic Space Time seen by Observer at O
Q and R are inertial paths of orbiting particles

By boosting the momentum of a ball moving around a specific orbit Q , one raises its trajectory to a higher orbit R in direct proportion

to the momentum increase. *Thus the recycling period remains constant.* The principle resembles that of a gyroscope or pendulum. Consider how the world looks from the point of view of an observer Δ on Q's orbit. Δ sees himself as stationary. He measures the lengths of orbits above and below him as a function of their relative distance from him. If Q's distance from the vertex is ρ , and the distance of R from the vertex is ρ', then he will measure the distance of R from himself as d= ρ' - ρ . If the base angle of the cone is arbitrarily taken as ψ , then Q will give the orbit of R relative to himself a trajectory of ψd.

This creates a consistent space-time geometry. The line elements are not geodesics on the cone because they are circular arcs, but they are geodesics in the polar coordinates (ρ, θ). It works particularly well when we consider the relative motion of orbits beneath Q . What happens is that Q now operates as a new vertex, with all lower orbits moving in the opposite direction. The rest observer, O , measures the length of the trajectory of Q as ψρ . The sum ψd+ψρ = ψρ' expresses the simple addition law governing *both* velocity and distance.

How "large" is W ? The question cannot be easily answered, since *velocity creates distance* . The 'rest frame' consists of a single point! Topologically, loops of differing circumferences can't be contained in one another. Therefore each individual particle moving at a different speed creates its own orbit relative to the fixed observer. Since motion creates space, the customary concept of a rest frame with objects all at various distances and all at rest relative to one another, not longer makes sense.

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