Topological Paradoxes of Time Measurement

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The ideas and constructions presented here are taken from the treatise *Time, Euclidean Geometry and Relativity* written in 1967 and revised several times, the latest being in the year 2000. It has formed the basis for presentations at the Institut Poincaré (1968), Trinity College in Dublin(1970), and Wesleyan University (2000).

* Distance in Space is measured with Rulers

* Duration in Time is measured with Clocks

* Clocks are dynamical systems, that is to say, Machines. Their functioning therefore depends on the particular mechanical laws which govern the universe in which they are employed.

* Since different universes are governed by differing principles of dynamics, restrictions arise on the kinds of machines, therefore clocks, that can be constructed in these universes.

* Therefore the study of time measurement leads to an examination of the collection of constructible clocks, which is a subset within the collection of constructible machines.

* Of particular interest to us are the constructible clocks in a relativistic universe, a non-relativistic universe, a quantum universe, a cyclic universe and so forth.

In the original paper we describe the axiomatic structure of time, and the associated issues of clock construction, in a linear, homogeneous, relativistic, cyclic , and linear-cyclic, (which bears some resemblance to a quantum) , universe.

Definition: A linear universe U is one in which there is some way translating the temporal dimension into a spatial dimension. That is to say, the structure of U allows the measurement of duration by clocks to be replaced by a measurement of distance by rulers. An obvious example of a linear universe is the universe of Special Relativity, or any universe in which the Postulate of Relativity (constancy of the speed of light) applies.

Let W be a non-linear universe, one in which no isomorphism can be established between time measurement and length measurement. We make the following assumptions:

1. Given a length L , it is possible to fashion a ruler with the exact length L.

2. Given a ruler of length R, it is always possible to fashion a ruler of some length R'< R (This does not assume that an R' can be specified as some given proportion to R, only that one can always make a ruler shorter than any given ruler.)

3. Given a clock measuring duration T, it is always possible to fashion a clock measuring some shorter duration T'< T. Once again there is no assumption about the ratio of T to T'

4. No time reversal. Time is always measured in the forward direction, which is asummed to be known.

5. It is assumed that *Spatial* sub-spaces of W, those subspaces defined at any given instant of time, are Euclid-Hilbert. (Finite or Infinite)

6. No such restriction applies to space. Rulers can be freely transported in all directions, freely rotated, etc.

Theorem:

Given the above set of assumptions for a a non-linear universe W it is not possible, from the existence of a clock Co measuring a duration of length T, to construct, save by trial and error a clock measuring a duration of length $d = (1/2) T$. More generally, it is not possible to construct a clock measuring an interval of time aT, where a is any constant $0< a < 1$. By the measurement of a duration by a clock, one means that the clock ticks only at the beginning and the end of the duration T.

However, in either a 1-dimensional, 2-dimensional or 3-dimensional Euclidean space, it is possible , using a ruler, to determine,from a given length L, the midpoint $l= (1/2) L$.

Corollary:

If I have a finite collection of clocks ticking off durations $t_1 < t_2 < t_3 .... < t_n$, then I cannot, save by accident or trial and error, construct a clock which ticks in any predetermined interval less than the minimum t_1 .

Clocks versus Rulers

(A) *RULERS*

Definition : A *Ruler* is a mechanical system that functions as instrument for measuring lengths. It's basic property is that the distance between its endpoints remains invariant under rotations, translations and reflections.

Fix a moment in time. Let L be a pre-assigned distance in W between end-points p_1 and p_2 . The axioms of a Euclid-Hilbert universe allow one to construct the entire line segment S connecting p1 and p2 . We look at methods for constructing the midpoint of S in

1.One dimension

2. Two dimensions

3. Three dimensions

THE ONE DIMENSIONAL CONSTRUCTION:

To determine the midpoint of a segment S = [p_1, p2] of length L, by the motions of rulers in one spatial dimension:

Our assumptions allow us to make a ruler of length R < L. Lay off integral lengths of R along S, starting from p1. If R goes into L an integral number of times n , then L = nR . If n is even our work is finished.

If n > 1 is odd, or if R does not divide L exactly, then make a ruler of length R*< R , and compare the following lengths:

(1) R_a = R* ;

(2)R_b = R-R*. It is important to note that the property of free translation in space has made it possible to construct the length R_b .

Place the leftmost endpoints of rulers R and R* next to each other. If y is the terminal point of R*, z the terminal point of R, then

R_b = Length of (y,z) .

Either R_a or R_b must be less than (1/2)R in length.

Choose the shorter of these two lengths, and label it R_1. Next lay off R_1 against L . If R_1 goes into L exactly, then we can compute a new number n1 such that $L = n_1R_1$. If n_1 is even we are finished. If r_1 does not exactly divide L , or if n_1 is odd, then make a shorter ruler R_1* The minimum of the two lengths R_1** and $R_1 - R_1$* will be our next ruler R_2.

This process, known as the Euclidean algorithm, can be continued indefinitely One thereby builds up a sequence of remainder lengths, R_1, R_2, R_3..... If R and L are incommensurable, this sequence is infinite. It must converge to zero however since each remainder is less than or equal to 1/2 of the previous one.
Each R_k goes into L a certain number of times, say nk =$[L/R_k]$. Let h_k = $[(1/2)n_k]$ and locate the point on the segment S at the distance $d_k= h_kR_k$. Then the sequence of lengths {dk} must converge to the point (1/2)L .

Since there are no temporal restrictions on the measuring process, one can get around the Zeno Paradox by positing that each operation takes half the length of time of the previous. It is sufficient for our purposes to observe that the succession of rulers converges to zero.

THE TWO DIMENSIONAL CONSTRUCTION:

Here One can find the midpoint of any segment by using the familiar construction from Euclidean Geometry involving parallel lines. All that is needed is a way of constructing parallel lines. This can be done with marked rulers, which are certainly permitted from our initial assumptions. One can restate this as follows: since rulers are postulated to be able to move about freely they can be employed effectively on the plane as compasses . The issues surrounding the use of compasses or marked rulers have nothing to do with the mechanical laws governing the space of the plane in which the construction takes place.

THE THREE DIMENSIONAL CONSTRUCTION.

The compass as a mechanical system is allowed by the assumptions governing the universe W.

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(B) *CLOCKS:*

It is clear that the mechanical process of finding the mid-point of a temporal duration T, in the absence of the postulate of relativity,(or some other unambiguous way of mechanically setting up an isomorphism between temporal duration and spatial length.) involves a host of new difficulties

* How do clocks measure "equal intervals of time"? There is only one way of doing this. Given a dynamical system M localized in a compact region of space, one computes its collection $\{v_j\}$of relevant state variables, (invariants or symmetries) at some instant which one arbitrarily sets at t=0. Then one has to wait until the values of

$\{v_j\}$ reproduce themselves exactly at some later time T.

Lots of assumptions about Causation, and the relationship of the values of a state variable S to the determination of the behavior of a system M are involved here, but we will not go into them. I refer the interested reader to my paper On the Algebraic Structure of Causation The following Axiom is crucial:

* Any isolated and closed system M with an identical complete set of state variables V at two distinct moments in time t_1 and t_2 , will pulse forever from and to this state in durations of equal length

$T = [t_1, t_2]$. One may in fact take this as the definition of what is meant by an equal interval of time. Note that it does not depend on the measurement of time, nor on the assigning of a numerical metric to the instants t_1 and t_2 . This axiom is itself dependant on numerous conditions discussed in the papers cited above. It is also assumed of course that W is deterministic, not quantum.

* Therefore: in a deterministic non-quantum, non-relativistic, non-linear universe W , all clocks are periodic non-reversible dynamical systems .

* In a relativistic or linear universe, the Postulate of Relativity allows one to escape this periodicity requirement precisely because it asserts that every light quantum is a system with a constant dynamical state variable, c.

We will now show that under our set of assumptions for a non-linear W that, given a clock C which pulses in periods of duration T, there can be no procedure (other than lucky accident) for constructing a clock C* of period $d= (1/2)T$.

By assumption, given C with period T a clock C_1 with period $T_1 < T$ can always be constructed. We wind up both clocks and set them going simultaneously at time $t = 0$. Obviously T_1 and T tick together in the first period of C , we can compute n such that $T = nT_1$. If n is even we are finished. Note that C_1 was a lucky accident.

If n is odd, we select a new clock, call it C_1 and start again. If T_1 does not exactly divide T there is an integer m >1 such that $0 < (m-1)T_1$. The interval J = Duration [y,z] , between the terminal pulse of mC and the terminal pulse of C_1 could, in the spatial situation, be used in the production of a Euclidean algorithm process leading to convergence to a midpoint, perhaps in the infinite limit. However because of the irreversibility of time there is no way, given only the existence of C and C_1 , to construct a clock pulsing in the period J.

Any such construction must involve some way of "pushing" the initial point of the second cycle of C back to the terminal point of C_1 , a mechanical action that is easily achieved with a ruler. (This statement, which is clear yet informal here, is given a more rigorous treatment in the original paper.)

Note that, even if it were possible to construct a clock with period J, and J were incommensurable with T, the convergence of remainders cannot be guaranteed . This is because the procedure of building clocks to select between $T_a = J$, and $T_b = C_1 - J$, cannot be made without bringing in time reversal.

It is perhaps a supercilious play on words to say that the carrying out of the full Euclidean algorithm process could "never" be accomplished because it would require an infinite amount ot time! In fact it isn't even possible to set it up.

In the world of daily life in which both quantum and relativistic effects are disregarded there is no way to subdivide the periods of a clock without treating time as a spatial dimension by making the assumption of constant velocity in some mechanical system that effectively replaces time duration with spatial length. Since length is reversible while duration is not, one is in some sense 'cheating' by doing so. But this is what is in fact done.

Bringing back relativity there are no absolute velocities, and one is obliged to rely on the speed of the light quantum as the only reliable way to get around the limitations of clocks as periodic systems, and subdivide arbitrary intervals of time.

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Reference:

Time, Euclidean Geometry and Relativity. On-Line Philosophy of Science Archive, University of Pittsburgh : <http:phil-sci/archive/00001291/Euclid.pdf>