Somit ist

$$\tau_{(0)} = \tfrac{1}{12} R I_1^2 S_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Auf dieselbe Art erhält man aus (29), (30) und (31)

$$\underset{\mu,\nu}{T_{(1)}} = \underset{\mu}{L_{(3)}} \underset{\nu}{M_{(2)}} + I_1 \underset{\mu}{L_{(0)}} \underset{\nu}{M_{(1)}} - \underset{\mu}{L_{(2)}} \underset{\nu}{M_{(3)}} \quad . \quad . \quad . \quad . \quad (37)$$

$$\underset{\mu,\nu}{T_{(2)}} = \underset{\mu}{L_{(3)}} \underset{\nu}{M_{(1)}} - \tfrac{1}{2} R \underset{\mu}{L_{(0)}} \underset{\nu}{M_{(2)}} - \underset{\mu}{L_{(1)}} \underset{\nu}{M_{(3)}} \quad . \quad . \quad . \quad . \quad (38)$$

$$\underset{\mu,\nu}{T_{(3)}} = \underset{\mu}{L_{(2)}} \underset{\nu}{M_{(1)}} + \underset{\mu}{L_{(0)}} \underset{\nu}{M_{(3)}} - \underset{\mu}{L_{(1)}} \underset{\nu}{M_{(2)}} \quad . \quad . \quad . \quad . \quad . \quad (39)$$

und hieraus die Differentialinvarianten zweiter Ordnung

$$t_{(1)} = P_{3,2} + I_1 P_{0,1} - P_{2,3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

$$t_{(2)} = P_{3,1} - \tfrac{1}{2} R P_{0,2} - P_{1,3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

$$t_{(3)} = P_{2,1} + P_{0,3} - P_{1,2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

und

$$\tau_{(1)} = I_1 \quad (-\tfrac{1}{2} I_1 S_1 - I_2 S_2 - I_3 S_3) = \tfrac{1}{6} I_1^2 S_1 \quad . \quad . \quad . \quad (43)$$

$$\tau_{(2)} = \tfrac{1}{2} R \; (-I_1 S_1 - I_2 S_2 - I_3 S_3) \;\; = -\tfrac{1}{6} R I_1 S_1 \quad . \quad . \quad (44)$$

$$\tau_{(3)} = \qquad -I_1 S_1 - I_2 S_2 - I_3 S_3 \quad = -\tfrac{1}{3} I_1 S_1 \quad . \quad . \quad (45)$$

Aus (36), (43), (44) und (45) folgt

$$\tau_{(0)} : \tau_{(1)} : \tau_{(2)} : \tau_{(3)} = R I_1 : 2 I_1 : (-2R) : (-4).$$

Ueberschiebt man (27) mit $M_{(1)}$ bzw. $M_{(2)}$, so entstehen die Relationen

$$R I_2 S_3 = 2 I_3 S_2 \text{ und } 3 I_1^2 S_3 = 4 I_3 S_1 \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Weitere Relationen zwischen R, I_1, I_2, I_3 und $P_{i,k}$ folgen schliesslich aus den projectiven Invarianten von $L_{(0)}$, $L_{(1)}$, $L_{(2)}$, $L_{(3)}$, $M_{(1)}$, $M_{(2)}$ und $M_{(3)}$.

Aus (18) und (18*a*) kann noch gefolgert werden, dass $t_{(0)}$, $t_{(1)}$, $t_{(2)}$ und $t_{(3)}$ folgenden Gleichungen genügen:

$$\begin{array}{lllll} & -I_1 t_{(1)} & +I_2 t_{(2)} & -I_3 t_{(3)} & = 0 \\ +I_1 t_{(0)} & & -I_3 t_{(2)} & -\tfrac{1}{2} R I_2 t_{(3)} & = 0 \\ -I_2 t_{(0)} & +I_3 t_{(1)} & & -\tfrac{1}{2} I_1^2 t_{(3)} & = 0 \\ +I_3 t_{(0)} & +\tfrac{1}{2} R I_2 t_{(1)} & +\tfrac{1}{2} I_1^2 t_{(2)} & & = 0 \end{array}$$

Mathematics. — *On relativistic thermodynamics.* By D. VAN DANTZIG.

(Communicated by Prof. J. A. SCHOUTEN).

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1. Let $d\mathfrak{B}_i$ [1] [2] be a 3-dimensional element in space-time, provided with an exterior orientation [3], $N^{dV} = \mathfrak{N}^h \, d\mathfrak{B}_h$ the algebraic number [4] of molecules of a chemically homogeneous fluid, the worldlines of which intersect $d\mathfrak{B}_i$, $P_i^{dV} = \mathfrak{P}^{h}_{\cdot i} \, d\mathfrak{B}_h$ the total momentum and energy of these molecules (i.e. the sum of their momenta and energies and of their interaction-momenta and -energies); $\mathfrak{N}^h$ is the particle-current, $\mathfrak{P}^{h}_{\cdot i}$ the tensor density of stress, momentum and energy. If dx^h is an element along a worldline of the macroscopical motion, $\mathfrak{N}^h$ is proportional with dx^h.

The amount of heat flowing through $d\mathfrak{B}_i$, as measured by an observer using x^4 as time-coordinate, is defined if $d\mathfrak{B}_i$ does not intersect the macroscopical worldlines, i.e. if

$$N^{dV} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and then is equal to [5]

$$Q^{dV} = -\frac{P_i^{dV} \, dx^i}{dx^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Evidently it is not invariant, though $Q^{dV} dx^4$ and of course also the *proper* heat-flow $Q_0^{dV} = c \, Q^{dV} \dfrac{dx^4}{ds} = -c \, i^j \, P_j^{dV}$ are, where $i^h = \dfrac{dx^h}{ds}$ is a unit vector, directed along the macroscopical worldlines. The invariant condition $Q^{dV} = 0$ is equivalent with

$$P_i^{dV} \, dx^i = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

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[1] $h, i, j, k, l = 1, 2, 3, 4$.

[2] All densities occurring here are W-densities. Cf. J. A. SCHOUTEN, Proc. Kon. Ned. Ak., Amsterdam, **41**, 568—575 (1938); J. A. SCHOUTEN and D. VAN DANTZIG, On ordinary quantities and W-quantities. Classification and geometrical applications. To appear in Compositio Mathematica.

[3] I.e. an orientation is given for all arcs, having one point in common with $d\mathfrak{B}_i$; the 3-dimensional element itself is not oriented. Cf. O. VEBLEN and J. H. C. WHITEHEAD, The foundations of differentialgeometry, Cambr. Tracts. in Math., N⁰. 29 (1932), p. 56.

[4] I.e. the difference of the numbers of molecules passing through $d\mathfrak{B}_i$ in positive and in negative sense. If in particular $d\mathfrak{B}_i$ is purely spatial (with respect to some observer) and oriented according to increasing time, N^{dV} is the number of molecules *contained in* this element.

[5] For a detailed discussion of these and other relations cf. D. VAN DANTZIG, On the phenomenological thermodynamics of moving matter, to appear in Physica.

It is easy to prove [5a]:

Theorem I. *Condition* (3) *is satisfied for each element* $\mathrm{d}\mathfrak{V}_i$ *for which* (1) *holds, if and only if* $\mathfrak{P}^{h}_{.\,i}$ *has the special form*

$$\mathfrak{P}^{h}_{.\,i} = \mathfrak{N}^{h}\,\pi_i + \mathfrak{p}\,A^{h}_{i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In this case, where no transport of heat occurs except by convection [6], the fluid is called *perfectly perfect.*

In (4) $\mathfrak{p}$ is the pressure and π_i is the vector of average momentum and enthalpy pro molecule [7].

2. The energy-tensor of a relativistically perfect fluid is well-known, viz., using our notations,

$$\mathfrak{P}^{h}_{.\,i} = -(\varrho + \mathfrak{p})\,i^h\,i_i + \mathfrak{p}\,A^{h}_{i},. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

ϱ being the proper energy-density. The signature of g_{ij} is supposed to be $---+$. Evidently (5) is a particular case of (4) with

$$\varrho + \mathfrak{p} = -\mathfrak{N}^{i}\,\pi_i\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence:

Theorem II. *A relativistically perfect fluid is always perfectly perfect.*

Corollary. *A fluid is relativistically perfect if not only the shearing stresses, but also the conduction of heat vanishes.* In other words: if in a fluid as seen by a certain observer the viscosity is negligible but the conduction of heat is not, then it is not perfectly (hence not relativistically) perfect; an other observer may see the flow of heat (i.e. of energy) as accompanied by a flow of momentum, viz. by a shearing stress. The energy tensor of such an imperfectly perfect fluid (e.g. liquid helium) will be, instead of (5):

$$\mathfrak{P}^{h}_{.\,i} = -\{(\varrho + \mathfrak{p})\,i^h + \mathfrak{Q}^h\}\,i_i + \mathfrak{p}\,A^{h}_{i} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

with $\mathfrak{Q}^{[h}\,i^{j]} \neq 0$. As measured by a co-moving observer the MINKOWSKian force upon a surface-element $\mathrm{d}\mathfrak{D}_{ij}$ at rest with respect to the fluid is

$$k_i = \mathrm{d}\mathfrak{D}_{hj}\,i^j\,\mathfrak{P}^{h}_{.\,i} = \mathfrak{p}\,\mathrm{d}\mathfrak{D}_{ij}\,i^j - \mathfrak{Q}^h\,\mathrm{d}\mathfrak{D}_{hj}\,i^j\,i_i\,. \quad . \quad . \quad . \quad . \quad (8)$$

Its *spatial* part reduces to $\mathfrak{p}\,\mathrm{d}\mathfrak{D}_{ij}\,i^j = \mathfrak{p}\,n_i\,\mathrm{d}\mathfrak{D}$, if n^h is a unit vector orthogonal to the surface-element and to i^h and $\mathrm{d}O = \sqrt{-\mathfrak{g}}\,\mathrm{d}\mathfrak{D}$, $\mathrm{d}\mathfrak{D} = \sqrt{\tfrac{1}{2}\,\mathrm{d}\mathfrak{D}_{ij}\,\mathrm{d}\mathfrak{D}_{kl}\,g^{ik}g^{jl}}$, its magnitude, so that $\mathrm{d}\mathfrak{D}_{ij} = 2\,n_{[i}\,i_{j]}\,\mathrm{d}\mathfrak{D}$. Hence the fluid is perfect. The second term in (8) corresponds with the flow of heat which pro unit of proper-time is equal to $Q_0\,\mathrm{d}O = c\,\mathrm{d}\mathfrak{D}_{hj}\,i^j\,\mathfrak{Q}^h = c\,n_h\,\mathfrak{Q}^h\,\mathrm{d}\mathfrak{D}$.

[5a] Cf. l.c. [5] § 4.

[6] Radiation is supposed to be negligible.

[7] The *enthalpy* or *heatcontent* of a volume V of gas is $E + \mathfrak{p}\,V$, E being the *total* energy. Like the energy it always occurs in 4-vectors with reversed sign.

For an observer in motion with respect to the fluid this terms leads to a shearing stress equal to $-\frac{1}{c}\,Q_0\,\mathrm{d}O\,i_i$. Hence it is directed along the worldlines, equal to $Q_0\,\mathrm{d}O\,\frac{u}{c^2\sqrt{1-u^2/c^2}}$, if u is the relative velocity. and directed so as to retard the hotter matter. The value is too small to be measurable.

The total *entropy* $S^{\mathrm{d}V} = \mathfrak{S}^h\,\mathrm{d}\mathfrak{V}_h$ of the matter passing through $\mathrm{d}\mathfrak{V}_i$ has been shown by PLANCK to be an invariant. For simplicity we measure it as a pure number, i.e. we denote the ordinary entropy by $k\,S^{\mathrm{d}V}$ instead of $S^{\mathrm{d}V}$, $k = 1{,}372 \times 10^{-6}$ erg/grade being BOLTZMANN's number. Hence, as for a quasi-statical process $\delta S = \frac{\delta Q}{k\,T}$, and as (taking $x^4 = t$) $\delta Q\,dt$ is an invariant according to (2),

$$\boxed{d\theta = k\,T\,dt = \frac{k}{c}\,T_0\,ds} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

is an invariant differential, the integral of which, taken along the macroscopical worldlines, will be called the *thermasy*. It is the time-integral of the absolute temperature (measured in energy-units) and has the dimension of an action. It is useful to introduce the vector

$$\vartheta^h = \frac{1}{kT}\,\frac{dx^h}{dt} = \frac{c}{kT_0}\,i^h, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which we call the *temperature-vector*; its time-component is the *reciprocal* absolute temperature (measured in energy-units); its spatial part is the product of this reciprocal temperature with the velocity [7a].

3. We consider a variation of the state of a perfectly perfect fluid consisting of a variation:

1⁰. $\delta\mathfrak{N}^h$ of the density and velocity, 2⁰. $\delta\mathfrak{P}^{h}_{.\,i}$ of the stress, momentum and energy, 3⁰. $\delta\,\mathrm{d}\mathfrak{V}_i$ of the position and magnitude of the 3-element under consideration without change of place and time of the point to which it is applied ($\delta x^h = 0$). Such a variation is considered as a *virtual* process, i.e. as a comparison of two states of perfectly-perfectness, and is therefore analogous to a quasi-statical process. It can be proved then [7b]:

Theorem III. *The variation* $\delta S^{\mathrm{d}V}$ *of the amount of entropy of a perfectly perfect fluid flowing through an element* $\mathrm{d}\mathfrak{V}_i$, *under variation of* $\mathfrak{N}^h$, $\mathfrak{P}^{h}_{.\,i}$, $\mathrm{d}\mathfrak{V}_i$ *with* $\delta x^h = 0$ *is given by*

$$\delta S^{\mathrm{d}V} = -\vartheta^i\,\delta P^{\mathrm{d}V}_{i} - \lambda\,\delta N^{\mathrm{d}V} + \mathfrak{p}\,\delta\,\mathrm{d}\mathfrak{V}_i, \quad . \quad . \quad . \quad . \quad (11)$$

[7a] Cf. l.c. [5] § 6.

[7b] Cf. l c. [5] § 7.

where $-\lambda k T = \mu$ is the average value pro molecule of the so-called chemical potential of the fluid.

(11) leads to

$$\delta \mathfrak{S}^h = -\vartheta^i \, \delta \mathfrak{P}^h_{.\,i} - \lambda \, \delta \mathfrak{N}^h . \tag{12}$$

and, introducing

$$\mathfrak{Z}^h = \mathfrak{S}^h + \mathfrak{P}^h_{.\,i} \, \vartheta^i + \lambda \, \mathfrak{N}^h . \tag{13}$$

to

$$\mathfrak{Z}^h = \mathfrak{p} \, \vartheta^h . \tag{14}$$

Hence by (12)

$$\delta \mathfrak{Z}^h = \mathfrak{P}^h_{.\,i} \, \delta \vartheta^i + \mathfrak{N}^h \, \delta \lambda .$$

Hence λ and ϑ^h can be taken as independent variables, characterising the state of the fluid, and we have:

$$\mathfrak{P}^h_{.\,i} = \frac{\partial \mathfrak{Z}^h}{\partial \vartheta^i} \qquad \mathfrak{N}^h = \frac{\partial \mathfrak{Z}^h}{\partial \lambda} \tag{15}$$

or, inserting (14) and comparing with (4):

$$\mathfrak{N}^h = \mathfrak{p}' \, \vartheta^i \qquad \left(\mathfrak{p}' = \frac{\partial \mathfrak{p}}{\partial \lambda} \right) \tag{16}$$

$$\mathfrak{P}^h_{.\,i} = \vartheta^h \frac{\partial \mathfrak{p}}{\partial \vartheta^i} + \mathfrak{p} \, A^h_i ; \qquad \pi_i = \frac{1}{\mathfrak{p}'} \frac{\partial \mathfrak{p}}{\partial \vartheta^i} . \tag{17}$$

Moreover (13) and (14) lead to

$$\mathfrak{S}^h = \eta \, \mathfrak{N}^h , \tag{18}$$

where η is the average entropy pro molecule, equal to

$$\eta = -\lambda - \pi_i \, \vartheta^i . \tag{19}$$

It is sometimes useful to take π_i and η instead of ϑ^h and λ as independent variables. Then we obtain $\mathfrak{p}' = -\frac{\partial \mathfrak{p}}{\partial \eta}$ and

$$\mathfrak{N}^h = -\frac{\partial \mathfrak{p}}{\partial \pi_h} , \tag{20}$$

$$\mathfrak{P}^h_{.\,i} = -\frac{\partial \mathfrak{p}}{\partial \pi_h} \pi_i + \mathfrak{p} \, A^h_i ; \qquad \vartheta^h = \frac{\partial \mathfrak{p}}{\partial \pi_h} \Big/ \frac{\partial \mathfrak{p}}{\partial \eta} . \tag{21}$$

Often $\mathfrak{p}$ will be independent of the velocity i^h, i.e. a function of λ and ϑ_0 or of η and π_0 alone, where $\vartheta_0 = \frac{1}{c} \sqrt{\vartheta^i \vartheta_i}$ is the reciprocal *proper* temperature and $\varepsilon = -\pi_0 = -c \sqrt{\pi_i \pi^i}$ is the *proper* enthalpy pro molecule. Then we obtain, the suffix always denoting the proper values [8])

$$\mathfrak{N}_0 = \frac{\partial \mathfrak{p}}{\partial \lambda} \vartheta_0 = \left(\frac{\partial \mathfrak{p}}{\partial \varepsilon} \right)_\eta , \tag{22}$$

$$\pi_i = -\varepsilon \, i_i , \qquad \varepsilon = -\frac{\partial \mathfrak{p}}{\partial \vartheta_0} \Big/ \frac{\partial \mathfrak{p}}{\partial \lambda} , \qquad \vartheta_0 = -\left(\frac{\partial \mathfrak{p}}{\partial \varepsilon} \right)_\eta \Big/ \left(\frac{\partial \mathfrak{p}}{\partial \eta} \right)_\varepsilon . \tag{23}$$

$$\eta + \lambda = \varepsilon \, \vartheta_0 . \tag{24}$$

4. The *compressibility-coefficient* (under constant temperature and velocity) of a fluid is $\varkappa = m \frac{\delta \mathfrak{N}^4}{\delta \mathfrak{p}}$, m being the proper mass of one molecule and the variation being performed under constant ϑ^h. Evidently it is not an invariant (though of course the *proper* compressibility-coefficient is). It is however related to the invariant

$$\gamma = \frac{\mathfrak{p}''}{\mathfrak{p}'} = \frac{\partial}{\partial \lambda} \log \frac{\partial \mathfrak{p}}{\partial \lambda} . \tag{25}$$

by

$$\varkappa = \frac{m \gamma}{k T} . \tag{26}$$

Also the voluminar *dilatation-coefficient* (under constant pressure and velocity) $\alpha_\mathfrak{p} = \frac{\delta \, d \mathfrak{B}_4}{d \mathfrak{B} \, . \, \delta T} = -\frac{\delta \mathfrak{N}^4}{\mathfrak{N}^4 \, \delta T}$ is not an invariant, but, putting

$$\beta_\mathfrak{p} = \alpha_\mathfrak{p} \, T \tag{27}$$

$\beta_\mathfrak{p} = \frac{1}{\mathfrak{p}'} \frac{\delta \mathfrak{N}^4}{\delta \vartheta^4}$ is, as $\delta \mathfrak{N}^h$ is proportional with $\delta \vartheta^h$.

We obtain:

$$\beta_\mathfrak{p} = -\frac{\partial \eta}{\partial \lambda} = 1 + \vartheta^i \frac{\partial \log \mathfrak{p}'}{\partial \vartheta^i} - \frac{\mathfrak{p}''}{(\mathfrak{p}')^2} \vartheta^i \frac{\partial \mathfrak{p}}{\partial \vartheta^i} . \tag{28}$$

The *specific heat under constant pressure* (and velocity) pro molecule and under measurement of temperature in energy-units is $c_\mathfrak{p} = \frac{1}{R} C_\mathfrak{p}$, where R is the gas constant and $C_\mathfrak{p}$ the ordinary specific heat (pro mol); $c_\mathfrak{p}$ is a dimensionless number. We obtain $c_\mathfrak{p} = \frac{1}{R} \frac{\delta Q}{\delta T} = T \frac{\delta \eta}{\delta T}$ under the conditions $\delta \mathfrak{p} = 0$, $\vartheta^{[h} \, \delta \vartheta^{i]} = 0$ which leads to

$$c_\mathfrak{p} = -\left(\vartheta^i \vartheta^j \frac{\partial^2 \lambda}{\partial \vartheta^i \, \partial \vartheta^j} \right)_\mathfrak{p} . \tag{29}$$

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[8] The lower index to the right of round brackets envelopping a partial differential coefficient denotes as usually the quantity kept constant.

If (as will usually be the case) $\mathfrak{p}$ does not depend upon the velocity i^h, the operation $\vartheta^i \frac{\partial}{\partial \vartheta^i}$ in (28), (29) is equivalent with $\vartheta_0 \frac{\partial}{\partial \vartheta_0} = - T_0 \frac{\partial}{\partial T_0}$, so that then $c_{\mathfrak{p}} = - \left(T_0^2 \frac{\partial^2 \lambda}{\partial T_0^2} + 2\, T_0 \frac{\partial \lambda}{\partial T_0} \right)_{\mathfrak{p}}$. The analogous *specific heat under constant volume* $c_V = \frac{1}{R} C_V$ is obtained by implying the condition $\delta \mathfrak{N}^h = 0$ instead of $\delta \mathfrak{p} = 0$. A small calculation [8a] shows that it is related to $c_{\mathfrak{p}}$ by the invariant relation:

$$c_{\mathfrak{p}} - c_V = \beta_{\mathfrak{p}}{}^2 / \gamma \, . \qquad (30)$$

5. An *ideal gas* (inasfar as it is perfectly perfect, i.e. moves adiabatically) is defined here as a perfectly perfect fluid, satisfying BOYLE—GAY—LUSSAC's law $\mathfrak{p} V = R\, T$, which is equivalent with

$$\mathfrak{N}^h = \mathfrak{p}\, \vartheta^h \, , \qquad (31)$$

i.e. with

$$\frac{\partial \mathfrak{p}}{\partial \lambda} = \mathfrak{p} \, . \qquad (32)$$

From (31) follows the physical meaning of the coordinate-temperature $T = \frac{1}{k\, \vartheta^4} = \frac{\mathfrak{p}\, \mathrm{d} \mathfrak{B}_4}{k\, N^{\mathrm{d}V}}$ as measured by an observer in relative motion with respect to the fluid. Except for a constant factor $k\, N^{\mathrm{d}V}$ *the coordinate-temperature is the product of the pressure*[9]) *and the* apparent *volume* (as measured by the observer) *of an ideal gasthermometer in relative rest with respect to and in thermodynamic equilibrium with the fluid.* Because of (32) $\mathfrak{p}$ is an exponential function of λ and has the form $\mathfrak{p} = e^{\lambda}\, \mathfrak{q}\, (\vartheta^h)$. By (22), (28) for an ideal gas

$$\gamma = 1, \; \beta_{\mathfrak{p}} = 1 \, . \qquad (33)$$

and by (27)

$$c_{\mathfrak{p}} - c_V = 1 \, . \qquad (34)$$

The invariant relations (33) correspond with the usual uninvariant ones:

$$\varkappa = m / k\, T, \quad \alpha_{\mathfrak{p}} = 1 / T,$$

whereas (34) corresponds with the ordinary relation $C_{\mathfrak{p}} - C_V = R$.

An *incompressible fluid* is defined by

$$\gamma = 0 \, . \qquad (35)$$

Hence $\mathfrak{p}$ is a linear function of λ. By (29), (30) it implies that either c_V becomes infinite or that $\beta_{\mathfrak{p}} = 0$. In the last case (which usually is assumed) and under assumption that $\mathfrak{p}$ is independent of the velocity, $\mathfrak{N}_0$ is independent of λ as well as of ϑ^h. In that case η becomes a function of ϑ_0 alone and all specific heats become equal to $c_{\mathfrak{p}}$.

[8a]) Cf. l.c. [5]) § 9.

[9]) The pressure, being a *scalar* density, is invariant under LORENTZ-transformations.

6. Finally we remark that the larger part of the paper is altogether independent of the relativity principle and therefore remains unaltered in classical theory.

With the exception of 1^0 all quantities bearing an index 4 and those defined by means of such quantities (viz. dt, T, V, $Q^{\mathrm{d}V}$, δQ, μ, $\varkappa$, $\alpha_{\mathfrak{p}}$) which are related to a particular frame of reference and 2^0 all quantities bearing an index 0, as well as g^{ij}, $d\,s$, i^h, n^h, ϱ, ε, k_i, $\mathrm{d}\mathfrak{O}$, $\mathrm{d}\,O$, which are defined by means of the fundamental tensor, all quantities introduced in the paper are independent of the choice of any particular system of coordinates as well as of any metric or connexion. Questions referring to this independence will be discussed more completely in another paper [5]).