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ONTOLOGY AND THE LOGISTIC ANALYSIS OF LANGUAGE

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ONTOLOGY AND THE LOGISTIC ANALYSIS OF LANGUAGE

An Enquiry into the Contemporary Views on Universals

Revised edition

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ONTOLOGIE UND LOGISTISCHE ANALYSE DER SPRACHE

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To my Parents

PREFACE

It is the aim of the present study to introduce the reader to the ways of thinking of those contemporary philosophers who apply the tools of symbolic logic to classical philosophical problems. Unlike the "continental" reader for whom this work was originally written, the English-speaking reader will be more familiar with most of the philosophers discussed in this book, and he will in general not be tempted to dismiss them indiscriminately as "positivists" and "nominalists". But the English version of this study may help to redress the balance in another respect. In view of the present emphasis on ordinary language and the widespread tendency to leave the mathematical logicians alone with their technicalities, it seems not without merit to revive the interest in formal ontology and the construction of formal systems.

A closer look at the historical account which will be given here, may convince the reader that there are several points in the historical development whose consequences have not yet been fully assessed: I mention, e.g., the shift from the traditional three-level semantics of sense and denotation to the contemporary two-level semantics of representation; the relation of extensional structure and intensional content in the extensional systems of Wittgenstein and Carnap; the confusing changes in labelling the different kinds of analytic and apriori true sentences; etc. Among the philosophically interesting tools of symbolic logic Leśniewski's calculus of names deserves special attention. Despite the pioneering efforts of Professor C. Lejewski, philosophers still have not caught on to it so far.

I wish to repeat here my gratitude to my teacher, Professor J. M. Bocheński, to whose inspiring guidance this book owes its origin, and to the late Professor E. W. Beth, whose great learning and equally great kindness I remember vividly. I am indebted also to Professors N. Goodman, H. Hermes, A. Heyting, R. Ingarden, N. Luyten, and C. Regamey, whose teaching and advice I had the privilege to enjoy.

The revised edition has profited from remarks by Professor Beth, Dr. F. von Kutschera and Dr. G. Knauss, who not only took the trouble

to read the German edition, but were kind enough to put their criticism on paper. To Professor B. Sobociński go my thanks for reading the chapter on Leśniewski and suggesting several improvements. I also wish to express my gratitude to Mrs. Clare Mays for undertaking the difficult task of translating the book and for calling my attention to several passages which needed clarification. Last but not least, special thanks to that expert typist and helpful secretary, who is also my dear wife.

GUIDO KÜNG

Notre Dame, Indiana
April 1966

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0. INTRODUCTION

0.1 The linguistic and logical interests of contemporary philosophy

The widespread tendency to approach philosophical problems from a linguistic and logical point of view, the focussing of attention on the use of words and signs and on the rules governing linguistic systems – these features will be immediately apparent to anyone who takes an interest in contemporary philosophy. Influential philosophical works have such titles as *Logische Untersuchungen*, *Tractatus Logico-Philosophicus*, *Der logische Aufbau der Welt*, *Das literarische Kunstwerk*, *Ethics and Language*, *Logic and Language*, *From a Logical Point of View*, to mention only a few characteristic works among many.[1]

In contrast to post-cartesian philosophy, contemporary philosophy comes thereby again closer to the aristotelian and scholastic tradition.[2] For there, too, words had been the starting-point for philosophical argument, and special attention had been given to formal logic.

Much that we find in Aristotle can be interpreted as language analysis, and there is an intimate connection between his ontology and the forms of language. For instance, his categories reflect an attempt to differentiate various meanings of the copula '*ἔστι*'.[3] Scholastic philosophy, too, shows a constant preoccupation with expressions and terms. From the *nominales* of early scholasticism to the "terminists" of late scholasticism there were schools which claimed explicitly to continue the aristotelian emphasis on language and logic. But the non-nominalists, too, were well versed in logic, semantics (the theory of suppositions) and "speculative grammar"[4], since these subjects figured prominently in the education of

[1] HUSSERL 1900/01; WITTGENSTEIN 1921; CARNAP 1928; INGARDEN 1931 (with the subtitle: *Untersuchungen aus dem Grenzgebiet der Ontologie, Logik und Literaturwissenschaft*); STEVENSON 1945; FLEW 1951/53; QUINE 1953.

[2] This affinity has been noticed, e.g., by G.E.M. ANSCOMBE and P. T. GEACH (1961).

[3] ARISTOTLE *An. Pr. A* 37, 49a 6 ff.; *Top. A* 9, 103b 20 ff.

[4] See below, p. 121 and p. 7.

every student. The thomistic doctrine of analogy, for instance, was developed under the title *De nominum analogia* (On the analogy of names) and not as a metaphysical theory divorced from its linguistic context.[5]

Because of their predilection for detailed and formally exact distinctions, scholastics had been rebuked for triviality by humanists and post-cartesian philosophers, just as today modern logicians and analytic philosophers are sometimes accused as proponents of a soulless formalism and as word-mongers.[6]

The contemporary renewal of interest in language and logic has not been confined to a single school of thinking. This appears especially in the fact that since the turn of the century a general theory of signs has been developed independently in different quarters. It has been initiated by thinkers as divergent in their background as C. S. Peirce, E. Husserl, F. de Saussure; logicians and professional linguists, analytic philosophers and phenomenologists have been concerned with it.[7]

The fundamental role which language and logic play in philosophy has gained new recognition at a time when the special sciences of these fields have undergone a profound change. After a long period of stagnation, logic has witnessed a rebirth in the form of mathematical or symbolic logic, also called logistic[8], and the 20th century has brought an entirely new approach to linguistics.

While the new, so-called structural linguistics has not as yet had a great influence on philosophers concerned with language[9], the impact of symbolic logic has been important. Many philosophers consider the artificial languages of symbolic logic to be more perfect than the natural languages and try to formulate their theories in such artificially regulated logistic languages. On the other hand there are philosophers who

[5] Cf. BOCHEŃSKI 1948; MCINERNY 1961.

[6] Cf. BOCHEŃSKI 1954.

[7] Since these parallel developments and their interconnections are not common knowledge, their outline is given in a special appendix to this chapter (see below, p. 18–20).

[8] The term 'logistic', proposed in 1904 by Itelson, Lalande and Couturat to the International Congress of Philosophy as a name for mathematical logic, is not frequently used in English and must not be confused with the term 'logistics', the name of a branch of military science having to do with moving, supplying, and quartering troops. We will use 'logistic' mainly as an adjective which is useful in combinations like 'logistic language', 'logistic philosopher', etc., meaning "language of mathematical logic", "philosopher using the tools of mathematical logic", etc.

[9] An exception is ZIFF 1960.

are in reaction against this trend and who do not wish to go beyond the bounds of "naturally" evolved languages. Normally they do not deny the scientific value of the logistic calculi; but they relegate them to mathematics and specialized fields of science and deny their philosophical significance. To this group belong the phenomenologists and, in English-speaking countries, the followers of the later Wittgenstein, the so-called ordinary language philosophers. The latter especially are sometimes very radical and believe that all philosophical problems arise merely from a misuse of ordinary language, and that their solution lies in a careful examination of correct linguistic usage; their procedure is often *ad hoc* and they do not want to undertake the construction of a systematic body of knowledge.[10]

We think that the rejection of the use of logistic languages in philosophy is unjustified. It has its roots in a false estimate of the relationship between natural and artificial languages; in the idea that they are basically diverse and hence incomparable.

0.2 Natural and logistic languages

It is true that an artificial language can never be a colloquial one. This is so if only for the simple reason, that a language used in everyday life is of necessity subject to constant diachronic changes (i.e. changes of the system occurring with the lapse of time): it never remains static. Changes in the nature of its sounds and in meanings lead to inner conflicts; their resolution can lead to irregularities elsewhere in the system, and these in their turn may by way of compensation upset the linguistic equilibrium yet elsewhere; and so forth.[11] Colloquial language cannot be bound by academic rulings but develops in unpredictable and often quite fortuitous ways.

In principle, however, the difference between a colloquial language as it exists synchronically at any particular stage of its development, and a logistic language is merely one of the precision and logicality of their grammatical rules. As A. Church once put it: ". . . speaking in principle, and leaving all questions of practicality aside, the logician must declare

[10] Examples of their method will be found in BLACK 1949; FLEW 1952/53; FLEW 1956, and elsewhere.

[11] Cf., e.g., DE SAUSSURE 1916, part 3.

it a mere historical accident that you and I learned from birth to speak English rather than a language with less irregular, and logically simpler, syntactical rules, similar to those of one of the familiar logistic systems in use today. . . . The difference of a formalized language from a natural language lies not in any matter of principle, but in the degree of completeness that has been attained in the laying down of explicit syntactical and semantical rules and the extent to which vaguenesses and uncertainties have been removed from them."[12]

The vagueness and multiplicity of meaning in ordinary language, however, must not be evaluated merely negatively. Although a scientific language should be a logically regulated, unambiguous one, this does not mean that, for example, poetry should similarly be written in an unambiguous language. Thus Ingarden refers to the artistic value of the "opalising effect" of the multiplicity of meaning in literary works of art.[13] And in any case, if we want to remove the ambiguities of the words of natural languages, we can usually achieve this by placing them in an appropriate context. Everything that can be said at all, can be said in a natural language. Even the illogical variety of structure of a natural language, with which we are familiar from youth, can reveal itself as a certain plasticity that allows the varied aspects of phenomena to be more readily expressed in a first description. On the other hand, every description which is intended to become part of the whole of our well-established knowledge should be expressible in terms of a clearly and consistently structured system, and for this purpose it is best to use a precisely regulated logistic language with formal inter-relationships that are easily grasped and rationally motivated. Just as in everyday life we find that verbal directions, useful as they are to the traveller, do not exclude the desirability of a systematic map, so in the field of philosophy, analysis in ordinary language and constructions in a logically ideal, artificial language, are mutually complementary.[14] Thus natural, colloquial languages and artificial logistic languages each have their proper spheres of application.

It is often thought that an essential difference between natural and logistic languages is the fact that the sentences of the latter are written

[12] CHURCH 1951a, p. 106.
[13] INGARDEN 1931, p. 150.
[14] GOODMAN 1956, p. 85.

not in words, but in a symbolism similar to that used in mathematics. The use of well chosen ideographic signs is indeed of great practical importance, since it eases considerably the comprehension of involved logical relationships. It is only when written in this way that complex sentences are readily grasped by our limited intellectual powers. Mechanical "calculation", i.e., the transformation of an expression in accordance with logical rules, becomes much simpler. However, the fact that "calculations" can be performed at all, does not depend on the use of an ideographic instead of a phonetic notation, but on the simplicity and precision of the logical rules of the language. In any case, every symbolic formula could be transcribed immediately into a phonetic script. Those who wish to maintain that there is an essential difference between natural and artificial languages cannot therefore base their argument on the fact that the latter use an ideographic symbolism.

Fortunately it seems that modern linguistic study has developed in a way that promises to bring to the fore the affinity between the natural languages and the languages of symbolic logic, without blurring their real differences in the process.[15] Since F. de Saussure (1857–1913) introduced the synchronic method into linguistics[16], the so-called structuralism of the Prague and Copenhagen schools has shown awareness of the need to describe the structure of the system of inter-relationships which characterizes a living language at a particular point in time.[17] And important advances have been made by the American School which, especially under the influence of L. Bloomfield (1887–1949), had become engaged in the purely formal description of linguistic structure.[18] A method is being developed which allows the grammatical structures of natural languages to be traced without imposing any pre-conceived schemata. The method is neutral in the sense that the input of linguistic material (the data, i.e., the statements of the "informant" whose language is being investigated) alone determines the nature of the resulting classifications. The grammatical categories of the various languages so to speak "ex-

[15] On the question of the different tasks of the linguist and the logician cf. among others, CHOMSKY 1955.
[16] Cf. DE SAUSSURE 1916.
[17] For an original, far-reaching and philosophically interesting program of this task cf. HJELMSLEV 1943.
[18] Cf. HARRIS 1951; HARRIS 1957; CHOMSKY 1957.

hibit" themselves, unimpaired by philosophical speculations of the grammarian.

This permits the structure of natural and logistic languages to be compared more accurately than has been possible so far, so that points of agreement and difference can be precisely stated. It is, for example, interesting to compare the way in which all the sentences of the English language can be derived from a number of so-called kernel sentences by means of so-called transformations[19], with the way in which the sentences of logistic systems are formed from atomic sentences in accordance with rules of formation. Deduction rules, too, appear to be comparable to a special kind of transformations.

Only recently attempts have been initiated in linguistics to develop a semantical theory which by means of a dictionary and projection rules assigns readings (meanings) to all well-formed expressions produced according to the formation and transformation rules of a given grammar. This development is similar to the one which occurred when logistic syntax became enlarged into semantics, and it gives rise to still further comparisons between natural and logistic languages.[20]

0.3 The concern of the present study

The present work is concerned with questions arising in philosophy through the use of logistic languages. The technique of constructing entire systems in a logically exact language appears to introduce hitherto unknown possibilities into philosophy, and to cast a new light on old problems.

Hegel, for example, freely "deduced" the structure of the entire universe by the device of developing one concept out of another "dialectically". Things are different today. It is no longer a mere "façon de parler" to speak of the structure of systems. Just as a physicist states precisely the mathematical calculi used in the formulation of his theory, so the contemporary logistic philosopher clearly indicates the linguistic tools he is using and gives an account of his methods of construction.[21] The question thus arises: in what relation does the syntactic structure of the linguistic description stand to the ontological structure of the described world?

[19] HARRIS 1957; CHOMSKY 1957.
[20] KATZ-FODOR, KATZ 1964.
[21] An instructive example of this way of procedure is GOODMAN 1951.

0.31 *Speculative grammar*

So far as *natural languages* are concerned, philosophers have in the past frequently attempted to relate the structure of language to the structure of reality – grammatical categories to ontological categories. Contemporary linguists[22], however, accord little value to these attempts at a philosophical foundation of grammar, as put forward especially in the *grammaticae speculativae* compiled by the Scholastics from the 12th century onwards. These speculative grammarians, whose treatises *De modis significandi* earned them the name of *modistae*, believed that there was essentially only one grammar, which re-appeared in individual languages with merely accidental modifications, and that the philosopher could ascertain the details of this grammar *a priori* by considering the ontological nature of things.

Thus Roger Bacon writes: "Grammar is in its substance one and the same in all languages, although it is subject to accidental variations" (*grammatica una et eadem est secundum substantiam in omnibus linguis, licet accidentaliter varietur*). And: "Therefore it is not the grammarian but the philosopher who by diligent consideration of the true nature of things, will discover the (true form of) grammar" (*Non ergo grammaticus, sed philosophus, proprias naturas rerum diligenter considerans ... grammaticam invenit*).[23]

As we know today, after a world survey of natural languages, there exist essentially different grammatical systems. To mention only *one* fundamental difference: apart from languages in which sentences are constructed on a bipolar subject-predicate schema, such as the European languages with which we are familiar, there are others, so-called "ergative" languages (for example Basque, classical Tibetan, and Eskimo) which are uni-polar and where a nounlike verb is qualified by the rest of the sentence. For instance, whereas we would say in English: "Buddha has taught the doctrine", or "The doctrine has been taught by Buddha", there is only one construction in classical Tibetan, a more literal rendering of which is: "There has been teaching (nounlike verb) with respect to the doctrine (absolutive) by Buddha (ergative)." And where we would say in English: "The scholars like science", or "Science is liked by the schol-

[22] Cf. ROBINS 1951.
[23] Latin texts quoted from ROBINS 1951, p. 77.

ars", a literal rendering of the classical Tibetan construction is: "There is liking (nounlike verb) with respect to the scholars (absolutive) for science (dative)." As can be seen, grammatical forms such as "active", "passive", "nominative", "accusative", which are basic for Indo-European grammar, do not occur in the grammar of the ergative languages.[24]

Similarly, it is futile to attempt to ascribe an invariable semantical significance to all grammatical categories of ordinary language. The correlation, for example, of the masculine, feminine and neuter genders of Indo-European nouns with the masculine, feminine and neuter sex of objects, results in so many inconsistencies and accidental features that its semantical value is severely restricted. For what semantical reasons should *nauta* (Latin for 'the sailor') have a feminine declination and *das Weib* (German for 'the woman') be neuter? Why does one say *pluvium* (neuter), *la pluie* (feminine) and *der Regen* (masculine)? There are accidental features, too, with regard to singular and plural. What is the semantical significance, for example, of the fact that in English the word 'wheat' is a singular, whereas oats are always talked of in the plural? And there are even so-called "isosemantic" grammatical categories, for which not even an approximate semantical distinction has been found: for example, strong and weak verbs in German.[25]

It is possible that in the early days of mankind grammatical features of natural languages reflected accurately features of reality as experienced by the speakers, but we have no knowledge of a primitive language of this kind. All natural languages known to us are products of a long development in the course of which any clear-cut semantical significance that grammatical categories may have had, has been distorted by the occurrence of exceptions and inconsistencies. Contemporary linguists therefore aim first to determine grammatical categories by purely formal criteria and only then to attribute to them an *approximate* semantical content.[26]

0.32 *Logistic languages and ontology*

Because they are artificially made to conform to a logical ideal and be-

[24] Cf. REGAMEY 1952; REGAMEY 1954. There are languages with ergative as well as non-ergative sentence constructions, for example Hindi and Chinese.
[25] Cf. WHORF 1945 on "isosemantic" and on other properties of grammatical categories.
[26] Cf. ROBINS 1951; CHOMSKY 1957.

cause of the precision of their rules, logistic languages present a completely different case than do the natural languages. We still find a wide range of possible syntactical systems or "grammars" – the basic difference, for example, between Leśniewski's system and a system in the style of Frege and Russell will be discussed in Chapter 8 – but these consciously contrived logistic languages are all alike in the fact that they are determined explicitly by the definite logical and ontological views of their inventors.

The translation from a natural into a logistic language, i.e., the logistic analysis of language, involves an ontological commitment for every sentence translated. The categories of logistic signs are intended to stand in a systematical correlation to the categories in which we see the world, so that it is meaningful to investigate this correlation between the precise syntactic structures of a logistic system and the ontological structure of the world that serves it as a model.[27]

It would seem, therefore, that modern symbolic logic was of immediate interest to aristotelian and scholastic philosophy. But at first both mathematical logicians and traditional philosophers had believed the logistic approach to be incompatible with aristotelianism, since it supposedly contradicted aristotelian logic.[28] Only slowly, thanks to the pioneering work of J. Łukasiewicz[29] and the endeavours of men like H. Scholz, J. M. Bocheński, P. Boehner, R. Feys, I. Thomas, and others, have the misunderstandings been removed, so that mathematical logic has largely become recognized even in traditional circles as the contemporary form of formal logic.

However, this does not mean that the relevance of logistic systems for traditional ontology and metaphysics[30] has likewise obtained wide rec-

[27] This important difference between natural and logistic languages is foreshadowed already in the linguistic discussions of Greek philosophers on the two topics "by nature or by convention?" and "analogy or anomaly?". There the Stoics combined, with respect to ordinary language, the positions of "naturalism" and "anomalism", holding that only correct, i.e., corrected language, but *not* the anomalous ordinary usage, reflected the nature of things. Cf., *inter alia*, ROBINS 1951, p. 16.

[28] A witty refutation of arguments brought forward in support of this belief can be found, *inter alia*, in the "aristotelian dialogue" of BANKS 1950, where an aristotelian with a knowledge of mathematical logic argues with a Mr. Palaeo and Mr. Neo.

[29] Cf. ŁUKASIEWICZ 1951.

[30] We prefer to use the terms 'ontology' and 'metaphysics' in the sense, in which metaphysics is the science making existential assertions about the different kinds of

ognition. This is not the case, despite the fact that already in the 'thirties J. Salamucha (a young Polish thomist, 1903–1944) and H. Scholz (1884–1956) had pointed out the intimate connection between logistic systems and metaphysics in the aristotelian sense (i.e. ontology).[31] Scholz had come to the conclusion that a sentence like the following theorem from predicate logic:

[1] $(x) \sim (Px. \sim Px)$

(i.e. 'For all individuals x: it is not the case that x both has the property P and has not the property P') is really an ontological statement, since it makes a general assertion about all possible individuals. It is a formulation of the ontological principle of non-contradiction. So-called predicate logic is therefore not really logic but ontology in the best sense of the word. And statements about *secundae intentiones* in the true scholastic logical tradition, will be found in so-called metalogic. Corresponding to the ontological theorem [1], for example, we have the logical, i.e., metalogical, theorem:

[2] 'All sentences of the form '$\sim (Px \sim Px)$' are true'.

J. M. Bocheński has stressed how the construction of logistic systems has brought traditional philosophical problems once more into focus, giving them a new precise form.[32] Thus the systematical ambiguity of signs in Russell's theory of types has been recognized as a contemporary form of analogy. And the question of universals, to give a further example, has undergone a kind of renaissance. Particularly among logistic philosophers living in the United States the discussion has been renewed with vigour: Carnap, Church, Goodman, Quine, and others, have taken part in it.[33]

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entities which ontology merely describes without deciding whether they actually exist. Cf. in this connection INGARDEN 1947/48, vol. 1, § 5.

[31] MICHALSKI 1937; SCHOLZ 1938; SCHOLZ 1940; SCHOLZ 1941; SCHOLZ 1944; cf. also HASENJAEGER 1962, ch. 2 'Logik als Ontologie'; and see below, pp. 103–104, on Łukasiewicz's and Leśniewski's distinction between logical and ontological (or metaphysical) principles. For biographical data of Salamucha cf. SOBOCIŃSKI 1958.

[32] Cf. BOCHEŃSKI 1959.

[33] Cf. for example QUINE 1953; BOCHEŃSKI-CHURCH-GOODMAN 1956; CARNAP 1950, and others; further references will be given in subsequent chapters. Some of the main points of the discussion have been dealt with in German by STEGMÜLER 1956/57, who

But how many traditional philosophers are aware that what the mathematician studies under the title of 'set theory' with the aid of the most modern techniques, is in fact the formal aspect of the old-established relation of participation, and that it is identical with the logical class-member relation? Although, as we shall see, most mathematical logicians use a platonistic language[34], there is still a rooted prejudice that a logistic treatment of basic philosophical problems necessarily involves nominalism. Those of us who find this suprising will want to know the reason for this deep-seated mistrust; and will wonder how the members of the Vienna Circle – for it is their writings that are primarily responsible for this view – managed to fix the equation "logistic analysis of language = positivism" so firmly in the minds of traditional philosophers.

The situation is similar on the side of the mathematical logicians. On the one hand, they are now ready to admit that aristotelian syllogistic is an integral part of formal logic, and that research into stoic and medieval logic is of historical interest. But as far as philosophy proper, the body of traditional ontological knowledge, is concerned, interest is often slight and only rarely is there a feeling of community of purpose. This is so despite the fact that logistic philosophy has revived traditional terms such as 'ontology', 'platonism', 'nominalism', 'universal', 'particular', 'abstract', 'concrete', and so forth, and has given them new technical currency. In consequence logistic philosophers have only a superficial knowledge of traditional ontology, particularly with regard to the internal structure of things. They have no clear conception of the difference between inhering properties on the one hand and abstract, ideal properties on the other, and have never heard of the distinction between *accidens praedicamentale* and *accidens praedicabile*.[35]

Enquiries made in the course of conversation about this lack of interest often lead to a blank wall of prejudice. Although apart from extensional logistic calculi, there are also intensional[36] ones dealing expres-

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gives a lucid exposition and relates the modern version of the problem of universals to its history from the Greeks to the present day.

[34] Cf. below, p. 131 f. Note that the adjective 'platonistic' refers to platonism in the special sense of "doctrine which is the opposite of nominalism", while the adjective 'platonic' refers historically to the philosophical doctrine(s) taught by Plato.

[35] Cf. p. 170.

[36] The words 'inten*s*ion' and 'inten*s*ional' must not be confused with the words 'inten*t*ion' and 'inten*t*ional'. 'Intension' is complementary to 'extension', cf. LEIBNIZ

sedly with relations between meanings, and with modal logic, logistic philosophers for the most part dissociate themselves from traditional analyses of "essences", and regard phenomenological descriptions of objective concepts as a psychologistic aberration of the kind that should no longer occur at the present day.

How is this estrangement between logistic thinking and traditional philosophy to be explained, since both sides discuss similar questions? An initial reason might be found in the different languages used. Anyone who wishes to concern himself closely with traditional philosophy should be able to read Greek and Latin texts; and those who wish to participate in logistic discussions must understand the artificial symbolic languages of modern logic. It is nonetheless incomprehensible why so disproportionately few philosophers of the one tradition ever learn the language of the other.

The reason for this misunderstanding and misplaced mutual criticism appears to lie in a peculiar difference in ways of thinking. As will be shown in this book, each side thinks within a different semantical framework. Whereas traditional philosophers distinguish three things: the sign, the objective meaning, and the designatum, most modern logicians[37] make only a two-fold distinction of sign and the reality represented.[38] Those who think in three-level semantics put abstract entities on a special semantical level, viz. as objective meanings, and in consequence they are inclined to overlook that in a two-level semantical system the level of represented reality can include abstract as well as concrete entities: that the use of a two-level semantical system does not necessarily involve a nominalistic standpoint. Conversely, the users of a two-level semantical system tend to confuse the objective meanings of a three-part system with subjective concepts, and in consequence accuse all holders of such a view of psychologism. However, the distinction between three-level and two-

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1875–1890, vol. 5, p. 469. The intension of a term consists of the qualities which go to make up the concept corresponding to this term, while the extension consists of the things which fall under this term.

[37] A. Church, who uses a three-level semantical system modelled on Frege's (cf. CHURCH 1946) forms an important exception. But Church is also an exception in the sense that he is extremely well acquainted with the history of logical thinking.

[38] Strictly speaking this holds only for names, not for sentences. As we shall see three levels have always to be distinguished for entire sentences.

level semantics does not correspond to that between psychologism and anti-psychologism, nor to that between a view accepting abstract entities and a nominalistic view. This is borne out by the fact that both traditional and logistic philosophers attack psychologism and defend similar positions concerning the problem of universals.

0.4 Plan of the book

An initial aim of this study will therefore be to give an exhaustive account of the transition from three-level semantics to the *two-level semantics of the relation of representation*, and thus to throw light on the historical background of contemporary ontological discussion. Thereby we hope to make the new logistic way of formulating philosophical problems understandable to those unacquainted with it.

It will be shown that an interpretation of semantical relationships on the basis of a two-level system is bound up with the desire for a language that makes as few assumptions as possible, i.e., whose representational function can be explained with a minimum presupposition of represented entities. This present-day *reductionism* tends above all to restrict the number of abstract entities posited. As we shall see, "poorer" nominalistic and "richer" platonistic languages are distinguished.

Secondly we shall therefore turn to the *question of abstract entities*. This will allow us to show how a traditional problem takes on a new look under a logistic approach, the question of abstract entities being nothing less than the *problem of universals*, a problem that has been the subject of lively discussions in the golden age of Greek philosophy and again in the medieval period of scholastic philosophy.

This question of abstract entities also plays an important part in distinguishing the formal ontological point of view, as expressed in logistic systems, from the epistemological and psychologistic attitude of post-cartesian philosophy. As already mentioned, the question of semantical levels, for example, is easily confused with that of universals.

The logistic formulation of the problem of universals takes its lead from the predicate signs of logistic languages, i.e., from those signs that correspond to ordinary language expressions such as 'is red', 'is intelligent', 'walks', and so forth. But as will be seen, predicate signs in our sense include not only predicates in the narrower sense, such as 'is red', 'walks',

but also relational expressions – known as many-place predicates – such as 'is father of', and even substantival designations of properties and relations, i.e., designations that can stand in the subject place of sentences, like 'redness', 'fatherhood', etc. In the language of *Principia Mathematica* they are all represented by Greek or capital Roman letters.[39]

The question, then, concerns the semantical relation of these signs to the reality designated by them: what do the signs represent? Substantival predicate expressions like 'the concrete property red', 'the class of all red things', 'the abstract property red', etc., indicate the kinds of entities for which, in the opinion of philosophers, predicate signs can stand. Different answers to the above question are possible, but present-day philosophers have not made an exhaustive comparison of them; they have for the most part been content to assume implicitly one or other of the possible views without in any special way describing it or justifying it. A detailed study of the *relation of representation of predicate signs* appears to us, therefore, to have an independent value.

The state of affairs outlined above determines the plan of the present study. It consists of two main parts. The first part is intended as an introduction to the ways of thinking underlying logistic analysis of language, and as a description in particular of the transition to the two-level semantics of representation. The second part will deal specifically with the relation of representation of predicate signs, i.e., with the problem of universals in its logistic formulation.

The first part is divided into four chapters, which lead up to a description of the relation of representation.

The first chapter begins with a description of the break with post-cartesian philosophy as it occurred in England, where the so-called analytic trend originated. Russell played a decisive part: with his analysis of relational facts he set up a parallel between the ontological structure of reality and the logical structure of sentences, regarding the sentences as pictures of reality.

The second chapter is intended to show how the problems of post-cartesian philosophy have been given a different sense by the logistic ana-

[39] Cf. below, p. 132, footnote 17. There are also logistic languages with syntactical categories that differ essentially from the predicate calculus of *Principia Mathematica*. An example is Leśniewski's so-called "ontology", the general *names* of which translate as predicate signs into the language of the *Principia*; cf. below, p. 112.

lysis of language. Instead of putting forward epistemological arguments about the existence of the external world, Russell formulates language systems – so-called constructional systems – the relation of representation of whose basic terms refers only to immediately experienced entities. He refers to Ockham's razor to justify his preference for a system with a minimum of presuppositions about reality. The distinction that we can here see taking shape between languages which make a greater or smaller number of ontological presuppositions, is basic from the point of view of contemporary discussions of universals.

The third chapter is more restricted in scope and deals in detail with semantical relations. We shall see how the disappearance of concepts as an intermediary stage between sign and designatum makes the relation of representation stand out more clearly. Two stages will be distinguished. First, it will be shown how anti-psychologism does away with subjective ideas: Frege's three-level semantics will be discussed at this point. Secondly it will be shown how in his theory of descriptions Russell drops even the objective meanings of words, and how Tarski's semantical system too – the first to be strictly formulated and free of contradictions – is based on the two levels of a correspondence theory of truth.

Lastly, Chapter 4 deals with the systematic description of the relation of representation undertaken by Wittgenstein in the *Tractatus*.

The second part of the book deals, as already mentioned, with the relation of representation of predicate signs.

We shall begin in Chapters 5–9, by surveying the views on this question of philosophers who have paid special attention to the relation of representation, viz. Bertrand Russell, Ludwig Wittgenstein, Rudolf Carnap, Stanisław Leśniewski, W. V. Quine, and N. Goodman. There will be two main lines of enquiry: *How* is the relation of representation structured – are the predicate signs genuine names or are they "syncategorematic" expressions. *What* is represented by predicate signs – are the entities represented concrete or abstract, extensional or intensional? It will be shown that Russell and Carnap regard predicate signs as genuine names of intensional entities. According to Quine and Goodman, on the other hand, predicate signs are to be regarded as syncategorematic; and there is Quine's well-known criterion for determining which signs of a particular language are to be regarded as genuine names. Wittgenstein appears to discount predicate signs altogether in the ideal case. Finally,

Leśniewski is important because his theories appear to have reference only to concrete individuals. Mereology concerns itself not with the platonistic class-member relation but with the part-whole relation between concrete individuals; and the theory which he calls "Ontology" describes the relations between empty, unshared and general names to the concrete individuals subsumed by them.

In Chapter 10 we shall present in systematic form all the possible answers to the question: "How and what do predicate signs represent?" We shall also attempt to explain the so-called "syncategorematic" functioning of predicate signs. Four views will be discussed: the view of predicate signs as genuine names, the equality and the similarity interpretation, and the extreme reistic conception.

The conclusion will consist of an assessment, in the light of the present study, of the connection between ontology, logistic analysis and traditional philosophy.

Appendix I / Brief historical survey of logistic philosophy

Readers who are less familiar with the logistic philosophers treated in this book, will find a short sketch of the historical context of value. (A chronological table will be found at the end of the book.)

Disregarding the great precursor, Leibniz, we can say that mathematical logic was founded around the middle of the 19th century by G. Boole (1815–1864) and A. de Morgan (1806–1878). It was, however, G. Frege (1848–1925) who most significantly influenced its development. He made three contributions to the new branch of learning: firstly, accuracy – by formulating his proofs with utmost precision; secondly, philosophical depth – by basing his formalism on platonistic intuitions; thirdly, a task – by drawing up the programme of logicism, according to which mathematics is to be formulated in purely logical terms and its laws deduced from purely logical axioms.

However, the symbolism used by Frege in his writings was not an easy one to understand. At the time he was therefore less well known than the Italian G. Peano (1858–1932), whose logical symbolism proved more congenial to mathematicians. It was through meeting Peano that one of the most influential contemporary philosophers, Lord Russell (*1872) became interested in the new logic. But Russell is also one of the few who rec-

ognized the vital significance of Frege's work during the latter's lifetime. He set himself the task of carrying on Frege's logicist programme, and wrote the monumental *Principia Mathematica* in collaboration with A. N. Whitehead (1861–1947) with this aim in view.[40] As we shall show in detail, Russell appealed to the methods of mathematical logic in all his philosophical thinking and thus gave to analysis in the tradition of Locke, Berkeley, and Hume an entirely new look.

L. Wittgenstein (1889–1951), a pupil of Russell's, is of particular importance from the point of view of the present work. He studied systematically the relation of the new logic to philosophy, and in the process made important observations about the relations between signs and represented reality. His book, the *Tractatus Logico-Philosophicus*, exercised a widespread influence, becoming in effect the bible of the so-called Vienna Circle, the neo-positivist movement that developed during the 1920's and especially the 1930's. This group's active propaganda and crusading zeal gave the impression that they had taken over the new logic for their exclusive use, with the result that even at the present day many non-logicians are of the mistaken belief that mathematical logic is identical with positivism. Most mathematical logicians, however, are not positivist in their views, and R. Carnap (*1891) is the only leading mathematical logician to have been a member of the Vienna Circle. Also, particularly since his settling in the United States, his views have in many respects developed beyond the narrow framework of the original positivist dogmas.[41]

The so-called Polish school represents a line of logical thought that is largely independent of Russell. Stanisław Leśniewski (1886–1939), in particular, worked out a comprehensive system based entirely on his own original views. In contrast to Russell, he formulated precisely what meaning he gave to his signs, thus becoming a founder of modern semantics. Unfortunately he himself published little, but his pupil A. Tarski (*1902), who has given the first non-contradictory formulation of the concept of truth, contributed decisively to the development of metalogical research.

These various lines of development can all be traced today in the United

[40] For a less technical introduction to the *Principia Mathematica* cf. RUSSELL 1919.
[41] Cf. below, p. 86.

States. Thus A. Church (*1905) effectively continues Frege's platonism; Carnap develops his own standpoint; while the line of development starting from Russell leads on to the work of W. V. Quine (*1908) and N. Goodman (*1906), who, like Leśniewski, object to abstract entities.

Appendix II / The different traditions of contemporary semiotics

The fact that since the turn of the century a general theory of signs has been developed in various parts of the world, reflects the increasing interest in language and logic. The various aspects of this development have as yet hardly been taken account of, and a brief attempt to outline them therefore seems desirable.

In the United States, the mathematical logician C. S. Peirce (1839–1914), founded a general theory of signs ("semiotics"), of which the investigation of linguistic signs formed only a part. Furthermore, he concerned himself not merely with the bare relation of sign to thing signified, but also with the pragmatic context within which the signs are used. He classified signs under ten different headings. The most interesting is perhaps his generic distinction of icon, index and symbol: an icon resembles the thing signified; an index is a sign that stands in a causal relationship with it; and a symbol is a sign that is assigned to it according to a rule. His views did not at first become widely known, as on the whole he published little. On his theory of signs he published only the two articles 'Sign' and 'Index' in Baldwin's *Dictionary of Philosophy and Psychology* in 1902.[42]

It was through the work of Charles Morris (*1901) that from 1938 onwards interest in this general theory of signs became more widespread. Morris's distinction between syntactics, semantics, and pragmatics, is generally accepted nowadays. Syntactics deals with the relations between signs; semantics with the relations between signs and things signified (it can thus be said to include syntactics); and pragmatics concerns itself with signs, with things signified, and with sign-users. Another important contribution was made in 1945 by Charles L. Stevenson, who defined pragmatical meaning as a dispositional property of signs, and made out a clear case for the distincion between descriptive and emotive meaning.

[42] Cf. PEIRCE 1931–1935, pp. 2304 f.

At about the same time, but independently of American developments, several beginnings were made in Europe. In 1892 Frege published his well-known paper on the distinction between *Sinn* (meaning) and *Bedeutung* (denotation). Since it and the further developments of logistic semantics will be treated below, in Chapter 3, we will not elaborate further.

Another interest in semantical relations developed among the disciples of F. Brentano (1839–1917). Already in 1894 K. Twardowski had investigated the semantical and pragmatical functions of names, but it was E. Husserl who made them the subject of a particularly penetrating phenomenological analysis in the second volume of his *Logische Untersuchungen* of 1901. Understandably, neither Twardowski nor Husserl use Morris's terminology. Twardowski distinguishes between *Benennen* (naming), *Bedeuten* ("meaning", here in the psychologistic sense of evoking a representational experience in the listener) and *Ausdrücken* (expressing). And Husserl uses the terms *gegenständliche Beziehung* (object relation), *Ausdruck* which he later changed to *Bedeutung* (meaning) and *Kundgabe* (making known: the expressive function).

A third European line of thought can be traced back to the lectures on general linguistics given by Ferdinand de Saussure at the University of Geneva in 1906, 1908 and 1910, which were published posthumously in 1916. De Saussure considered speech as a coded message transmitted from a sender to a receiver and regarded linguistics as a part of a general theory of signs (*sémiologie*). Long before Wittgenstein he compared language to a game of chess, stressing that the value of a language sign is defined by its relation to all other language signs belonging to the same system. The theory of signs of De Saussure has especially been developed by the Danish linguist L. Hjelmslev. His "glossematics" endeavours to formulate algebraically the two parallel structures of "expression" and "content" (meaning) of language signs.[43]

In K. Bühler (*1879), a pupil of O. Külpe, the *Denkpsychologe*, the traditions of Husserl and De Saussure met. His distinction between the three functions of human language which he introduced in 1918, became particularly well known through his book of 1934 on the theory of language. These functions are *Darstellung* (representation), *Ausdruck*

[43] Publications from 1935 onwards, but the major programmatic publication is HJELMSLEV 1943.

(expression)[44], and *Appell* (appeal). A language sign is a "symbol" as far as it represents an object or a state of affairs, it is a "symptom" as far as it expresses the state of the "sender", and it is a "signal" as far as it appeals to the "receiver".

R. Ingarden (*1893), a pupil of Husserl's, finally distinguished five different functions of linguistic signs, viz. two representational functions: purely "conceptual" abstract representation, and "imaginative" intuitive representation; the expressive function (Ingarden emphasizes that it is the way in which the words are uttered rather than their meaning that counts here); and two appellative functions: the "communicative function", which makes the listener cognitively aware, and the "influencing function" which, through the meaning of the words, affects him emotionally, or brings about a specific action, etc.[45]

Finally still another tradition of semiotics has to be mentioned, namely the reformers' movements of "Significs" and "General Semantics" which are especially concerned about the sociological consequences of the misuse of language signs. Significs goes back to 1896, when Lady Welby (1837–1912) published an article in *Mind* and offered a prize for an essay. She also corresponded with C. S. Peirce. The best-known book of this tradition is C. K. Ogden and I. A. Richards's *The Meaning of Meaning*, published in 1923. The movement became very active in Holland, where G. Mannoury (1867–1956), professor of mathematics in Amsterdam, developed his own original semiotic theory[46], and where intuitionist mathematicians and analytic philosophers became members of the group.

[44] Cf. BÜHLER 1934, p. 28; in BÜHLER 1918 he had used the husserlian term *Kundgabe* as a name for the expressive function.
[45] Cf. INGARDEN 1955, p. 145 f. and INGARDEN 1931 (1960), p. 407.
[46] Cf. MANNOURY 1948, VUZSJE 1953.

PART ONE

THE LOGISTIC ANALYSIS OF LANGUAGE AND THE RELATION OF REPRESENTATION

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1. A PHILOSOPHICAL REVOLUTION

This chapter will begin with a brief sketch of the historical origin of the analytic trend in contemporary philosophy. We shall then indicate the novel and peculiar features of Russell's method, as revealed in his analysis of relational facts and of relational propositions, and show how he was led to mathematical logic.

1.1 The birth of contemporary analytic philosophy

It was Leibniz who first thought of basing philosophy on a logistic analysis of language. He speculated about a *characteristica universalis*, an ideal symbolic language with precise rules of transformation and of definition (*calculus ratiocinator* and *ars combinatoria*). This was to be used for the formulation of an exact system of the principles of all human knowledge (a *mathesis universalis*), by means of which all philosophical problems could be solved.[1] This project, however, was a mere dream: the essential means – logistic languages – had not yet been developed, and in any case, as can to-day be precisely demonstrated, a mechanical decision procedure for all problems is logically impossible.[2]

Frege was the first important thinker who answered truly philosophical questions (such as, for example, of the nature of number, of existence, and so forth) by means of a logistic system. He was, however, primarily interested in the foundations of mathematics, and his ideas were too novel and too difficult to have an immediate influence on philosophical thinking. Only in the form given to it by Russell did the logistic analysis of language find a place in general philosophical discussion.

This came about as follows. At the turn of the century a philosophical revolution occurred in England.[3] G. E. Moore discredited the idealism which at the time dominated the thinking of English-speaking philoso-

[1] Cf. Leibniz 1875–1890, vol. 1, p. 57; vol. 7, p. 185, p. 200.
[2] Cf. below, p. 182 f.
[3] Cf. Ryle 1956; Urmson 1956.

phers. His critical ideas began to take shape towards the end of 1898[4], and in 1903 his paper 'A Refutation of Idealism' appeared in *Mind*. It had a devastating effect on British idealism. Moore adopted a realist position, but his mode of procedure was as important as the standpoint he took up. It revealed a new way of philosophizing. Moore advanced his argument in small steps, and painstakingly, word for word, examined each thesis put forward. He also took care always to state explicitly the logical structure of his argument. This analytical approach with its concentration on detail was in complete disagreement with the monism of the idealist Bradley. In Bradley's view any distinction of details involved falsification since reality, in contrast to appearances, was a mystic, unanalysable unity.[5] In opposition to monistic idealism Moore adopted a standpoint of pluralistic realism and insisted on the correctness of the analytic approach, thus becoming the originator of a new philosophical trend, the so-called analytic school.

Bertrand Russell at once gave his support to this new way of thinking: as he says, he "followed closely in his [Moore's] footsteps".[6] Russell, too, strongly opposed Bradley's denigration of analysis: "One is often told that the process of analysis is falsification, that when you analyse any given concrete whole you falsify it and that the results of analysis are not true. I do not think that is a right view."[7] His approach was one of ontological pluralism, according to which reality is in fact composed of different entities, so that analytic differentiation of detail does not involve falsification. He intended to save himself at all costs from the disastrous consequence of monistic idealism: the admission that two contradictory sentences are equivalent in the sense of being two equally imperfect approximations to the one integral, inexpressible truth.[8] He found the inexpressible too easy a solution to be satisfied with: "It seems to me that philosophical investigation, as far as I have experience of it, starts from that curious and unsatisfactory state of mind in which one feels complete certainty without being able to say what one is certain of.

[4] Russell 1959, p. 54. Cf. the platonistic pluralism of Moore 1899.
[5] Bradley 1883, p. 95: "It is a very common and most ruinous superstition to suppose that analysis is no alteration." Cf. Bradley 1893.
[6] Russell 1959, p. 54.
[7] Russell 1918/19, p. 495.
[8] Russell 1910, pp. 155–156.

The process that results from prolonged attention is just like that of watching an object approaching through a thick fog: at first it is only a vague darkness, but as it approaches articulations appear and one discovers that it is a man or a woman, or a horse or a cow or what not. It seems to me that those who object to analysis would wish us to be content with the initial dark blur. Belief in the above process is my strongest and most unshakable prejudice as regards the methods of philosophical investigation."[9]

Moore is not counted among modern logicians; but the attempt to justify a standpoint of pluralistic realism led his colleague Russell to mathematical logic. This proved to be of decisive significance, for with his wide range of interests and with his lively personality Russell was admirably suited to become the leader of a new philosophical movement, and to stimulate large numbers of philosophers to take an interest in modern logic. His thinking has thus exerted an extraordinary influence, and has determined the course of contemporary philosophy.

It will therefore be of value to investigate more closely how it came about that Russell's attention was drawn to modern logic. This leads us to the topic that is basic to the whole of Russell's philosophy: the analysis of relational facts.

1.2 Russell's analysis of relational facts

Three things will appear here: first, Russell relies on his analysis of relational facts to justify his analytic procedure: he sees in ontological pluralism the ground for the admissibility of analysis and seeks to make this ontological standpoint plausible by referring to relational facts; second, the fact that he regards relations as irreductible basic elements causes him to turn his attention to mathematical logic; and third, this early example of logical analysis already shows how Russell co-ordinates the logical structure of sentences with the ontological structure of things.

The discussion of idealism turned on the question of the so-called "internal" and "external" relations. The literature on the topic is too extensive to allow us to discuss the various opinions held. Moreover, philosophers

[9] Russell 1959, p. 133.

vary in their definitions of the terms in question. We shall therefore restrict ourselves to an exposition of Russell's analysis.

Russell became aware of the importance of the theory of relations in 1899, when he lectured on the philosophy of Leibniz in the Easter Term.[10] He discovered that Leibniz's peculiar conception of relations was the key to the understanding of Leibniz's philosophical system, and he believed he could see the origin of this conception in traditional logic, which is restricted to sentences of the subject-predicate form.

This linguistic form where the predicate is said to be contained in the subject was, he argued, satisfactory only for sentences concerning attributes that are entirely "internal" to a subject. It can express a relation only if this is completely reducible to attributes internal to the subject; if, in fact, it is an "internal" relation. A genuine relation, such as the of a to b when a is greater than b, is an "external" relation which cannot be reduced to attributes. One might ascribe an attribute r_1 to a and an attribute r_2 to b; but if 'ar_1' and 'br_2' are to express the fact that a is greater than b, it would be necessary to state that the attributes r_1 and r_2 have a particular relationship to each other; that is, a certain relation between r_1 and r_2 would have to be stated, which brings one back to the beginning of the problem. The subject-predicate form, therefore, cannot express external relations, and leads to a view which regards substances as unrelated monads. (Russell found this monadic view not only in Leibniz but also in Lotze.[11])

The monadic conception is unsatisfactory, since it is forced to assume an artificially pre-established harmony in order to explain the actual interplay between substances. One might therefore prefer to adopt a form of monism and regard all relations as the internal relations of a single substance.[12] But even then, as Russell shows, it is not possible to do justice to genuine relations. This is particularly clear in the case of an asymmetrical relation, as when a is greater than b. On a monistic view a and b would be contained within the one subject, in symbols: '(ab)'; and the relation would somehow have to be ascribed to this whole as an attribute: '$(ab)r$'. But since '(ab)' and '(ba)' refer to the same whole, '$(ab)r$' and '$(ba)r$' would have to have the same value, i.e., the asymmetry

[10] RUSSELL 1900 (1937), p. XIII. For the following cf. RUSSELL 1903, pp. 221–224.
[11] Cf. RUSSELL 1900, p. 14.
[12] BRADLEY 1893, pp. 29–30.

of the relation could not be expressed. The difficulties of allowing for real relations remain as before.[13]

Russell did not accept the easy solution of saying that relations exist only in the mind and not in reality: "If I believe *A* to be *B*'s father, when this is not the case, my belief is erroneous; and if I believe *A* to be west of *B*, when westerliness in fact exists only in my mind, I am again mistaken."[14] He therefore rejected the ontological conceptions of monadism and of monism, as well as the limitation of logic to the subject-predicate form of traditional logic.

Where was he to find an acceptable linguistic form for relation sentences? He found it in the logic that had been developed by mathematicians since the middle of the 19th century. As Russell himself reports, his interest in it was aroused in 1900 at the International Congress of Philosophy in Paris, where he became acquainted with the school of G. Peano and was fascinated by the precision of its method: Russell was a mathematician himself but had been disillusioned by the inaccuracy of mathematical arguments.[15] Peano's logic was not restricted to considering the extension of concepts on the subject-predicate model, but was based on monadic and polyadic functions, such as are expressed by '*x* is a number', '*x* is the successor of *y*', and so on. After all, most mathematical sentences deal with relations such as "greater than", "less than", "successor of", and so forth. This was precisely what Russell was looking for. It is therefore not surprising that he was enthusiastic and at once began to elaborate further the theory of polyadic functions, i.e., the logic of relations. (The results of this work are incorporated in *The Principles of Mathematics* and in *Principia Mathematica*.)

Let us review this first example of russellian analysis. The essential feature of his method is that he analyses the relational structure of a fact and shows that his structuring is basic, i.e., irreplaceable by any other

[13] It is of course possible to say that every sentence ascribes a predicate to a subject in the sense of JØRGENSEN 1959, viz. if we admit that a subject can have true parts. Jørgensen distinguishes between purely attributive sentences, where the subject is designated by a single symbol, and relational sentences, where the various parts of the subject are designated by different expressions (p. 130). Cf. also LEWIS 1944, p. 244, and LEWIS 1946, pp. 60–61.

[14] RUSSELL 1903, p. 447.

[15] RUSSELL 1944, p. 12; RUSSELL 1959, p. 65 ff.

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combination of elements: real relations cannot be reduced to attributes.[16]

Moore, incidentally, had argued in a similar way to demonstrate the indefinability, i.e., the irreductible simplicity, of the concept "good": "good" cannot be reduced to a combination of other qualities; nor can "good" be identified with pleasure (or any other quality), for then the view that "good" is pleasure would be a mere tautology; "good" must therefore be one unique simple object of thought.[17]

It needs to be noted that Russell regards an expression such as, for example, '*ar*' as an exact image of a state of affairs: to each element of the expression there corresponds a particular component of the state of affairs. A similar idea can be found in Leibniz[18], and Russell was acquainted with the relevant texts. Leibniz speculated about an "alphabet" that would represent each of the basic concepts of human knowledge by a single letter, so that these could be manipulated like numbers. For Leibniz, however, symbols refer primarily to concepts or ideas and only mediately to things. Leibniz clearly thinks in terms of three-level semantics. Russell, on the other hand, analyses actually existing states of affairs and thus co-ordinates symbols with real entities. In his opposition to idealism, he is especially concerned with the *truth* of sentences, with their correspondence to reality.

A further important point must be made. Russell found that language can represent categorial differences. For him, the linguistic duality of subject-sign and predicate-sign reflects an ontological structuring into substance and attribute. Here we see that a significant change has taken place: whereas in traditional syllogistics a term could stand both in the position of subject and of predicate, modern logic usually stipulates precisely whether an expression shall be either an individual name, or a predicate expression, or a relational expression, depending on whether an individual, a property, or a relation is to correspond to it.[19] This corre-

[16] Once the introduction of classes is allowed, relations can be defined. A diadic relation, for example, can then be defined as a class of ordered pairs, and an ordered pair $\langle a, b\rangle$ can be defined as a class of two classes: $\langle a, b\rangle = df((ab)(a))$, as stated by WIENER 1914 and KURATOWSKI 1921. But even in such a system some undefined signs for relational functions remain, e.g., '$\in$', the sign for the membership function.

[17] MOORE 1903a, pp. 5—17.

[18] Cf. above, p. 23.

[19] A russellian predicate expression can, however, stand in the subject position of a higher-level sentence, for a property may itself have properties of a higher level.

spondence between logical and ontological structure is indicative of the close connexion between logic and ontology, and shows how the logical analysis of language and the ontological analysis of reality are related to each other. It is therefore not surprising that Russell stresses the importance for philosophy of language analysis: "The influence of language on philosophy has, I believe, been profound and almost unrecognized."[20] "That all sound philosophy should begin with an analysis of propositions, is a truth too evident, perhaps, to demand a proof."[21]

[20] RUSSELL 1924, p. 367.

[21] RUSSELL 1900, p. 8. Admittedly Bradley had already based his philosophy on considerations of subject and predicate, but his language analysis was hampered by the fact that he possessed only the very inadequate techniques of so-called "classical logic" ("Classical" logic must not be confused with traditional logic in its entirety; cf. BOCHEŃSKI 1956, § 36).

2. FROM THE THEORY OF KNOWLEDGE TO THE LOGICAL ANALYSIS OF LANGUAGE

The aim of this chapter is to show how logistic analysis has altered the approach to philosophical problems. We will see how Russell turned the epistemological problem of the external world into a question of comparing the different languages of constructional systems. But first we must explain the logicist definition of number, which is the prototype of a further kind of russellian analysis, that of so-called reduction. A knowledge of this is essential for an understanding of the question that has replaced the problems of the theory of knowledge, viz. the question of so-called "unnecessary entities".

2.1 The logicist definition of number

Initially Russell had regarded numbers as primitive platonistic entities: "Arithmetic must be discovered in just the same sense in which Columbus discovered the West Indies, and we no more create numbers than he created the Indians."[1] Peano provided him with an axiomatic system of arithmetic with five axioms and only three non-logical primitive terms, 'number', 'unity', and 'successor':

[1] '1 is a number.'
[2] 'Every successor of a number is a number.'
[3] 'No two numbers have the same successor.'
[4] '1 is not the successor of a number.'
[5] 'If 1 is contained in a class, and granted that if a number is contained in this class then its successor is also contained in it, then this class contains all numbers.'[2]

Russell, however, was not satisfied, because Peano's axioms do not completely characterize the meaning of what we ordinarily understand by 'number', 'unity', and 'successor', but hold for any progression what-

[1] Russell 1903, p. 451.
[2] Cf. Peano 1891.

ever.[3] For example, they remain true if to the term 'number' in the axioms we give the meaning "even number", to the term 'unity' the meaning "two", and to the term 'successor' the meaning "result of the operation +2".

Russell therefore sought a better solution, and like Frege he found that numbers could be defined explicitly in terms of classes.[4] According to this view a number is a certain class (e.g., the number 2 is the class of all pairs), and numbers are predicated of classes (e.g., number 2 is predicated of each pair; i.e., each pair is an element of number 2; i.e., each pair is an element of the class of all pairs).

The following definition can be set up: the (cardinal) number of a class α is the class of all classes that are equivalent to α (i.e., that have exactly as many members as α):

$$C'\alpha = df \quad \hat{\beta}(\beta E \alpha).$$

The suspicion that the definition is circular is not justified, since the equivalence of classes can be defined in purely logical terms without using the term 'number': α is equivalent to β if and only if a one-one correspondence can be set up between the members of α and those of β[5]:

$$\alpha E \beta = df \quad (\exists R)(\alpha = D'R \cdot \beta = \breve{C}'R \cdot R \in 1 \rightarrow 1).$$

A relation is a one-one correspondence if and only if it is true that if x and x' have the said relation to y, then x is identical with x'; and that if x has the said relation to y and to y', then y is identical with y':

$$\begin{aligned} 1 \rightarrow 1 = df \quad \hat{R}\{&(x)(x')(y)(xRy \cdot x'Ry \supset x = x') \cdot \\ &\cdot (x)(y)(y')(xRy \cdot xRy' \supset y = y')\}. \end{aligned}$$

Thus the term 'number' can be defined by means of terms of logic alone.

Particular numbers also can be defined by purely logical means. For instance, number 2, i.e., the class of all pairs, can be defined as the class of all those classes for which it is true that (a) they contain an x and a y, where x and y are not identical, and (b) every z contained in them is iden-

[3] RUSSELL 1903 (1937), pp. V-VI.

[4] FREGE 1884, § 62 ff.: RUSSELL 1903, §§109–111; cf. RUSSELL's remarks on his relation to Frege in RUSSELL 1903, appendix A.

[5] This idea can already be found in HUME 1739, Book 1, part 3, section 1, p. 71, quoted by FREGE 1884, p. 73.

tical with either x or y:

$$2 = df\, \hat{\alpha}\{(\exists x)(\exists y)\,(x \in \alpha \cdot y \in \alpha \cdot x \neq y \cdot \\ \cdot (z)(z \in \alpha \supset z = x \vee z = y))\}.$$

This definition of the term 'number' by means of terms of logic alone can be interpreted as supporting the thesis of *logicism*, i.e., the thesis that mathematics is reducible to logic. This definition tells us that instead of talking about numbers we can talk about classes of equivalent classes, therefore it is said that it *reduces* numbers to equivalent classes, or that it *constructs* numbers on the basis of equivalent classes. Viewed in this light the definition is a further kind of logistic analysis, which explains an entity (here: a number) as a complex structure of simpler entities (here: equivalent classes).

As far as logicism is concerned, victory is not yet complete: it must still be shown that all mathematical laws are deducible from purely logical axioms. Russell and Whitehead in their *Principia Mathematica* set out to carry this deduction through. But they encountered grave difficulties, and for two of their axioms, that of infinity and that of choice, it remains questionable whether they are logical in character.[6]

Our main interest here, however, is not logicism as such, but the general method of performing reductions, of which the logicist definition of number is only one example. Whitehead used the same method of reduction in his description of the physical world and defined spatio-temporal points as classes of mutually overlapping physical events.[7]

The essential feature of a reduction is that an explicit definition is given, stating how a name for a particular entity (e.g., the name 'number' or the name 'spatio-temporal point') is to be replaced by the expression for a class (e.g., the expression 'class of equivalent classes' or the expression 'class of mutually overlapping events') formed from other entities (e.g., from equivalent classes or from mutually overlapping events). In this way the number of primitive terms is reduced, the name for the particular entity in question need no longer be listed "beside" the other names. Instead, however, a class name belonging to a level "above" the level of primitive terms is introduced.[8]

[6] For a first, less technical introduction to the *Principia Mathematica*, cf. RUSSELL 1919.
[7] WHITEHEAD 1919; according to RUSSELL 1959, p. 103, Whitehead was the first to apply the method outside mathematics.
[8] For further information concerning other equivalence relations see below, pp. 164–165 and pp. 170–174.

2.2 Logical constructions in place of epistemological inferences of existence

Throughout the history of modern philosophy from Descartes and Locke onward, the problems of the theory of knowledge had been regarded as central, and the critical question concerning the dependence of the object of knowledge on the knowing subject, had been put before all other questions. To-day, however, we live in a different philosophical climate, where the epistemological approach with its psychological involvements has lost much of its appeal and has given place to differently orientated interests. The development of logistic methods has played a decisive part in bringing about this change of attitude, as will be shown below.

As is well known, the persistent question of the theory of knowledge was how to make the transition from the data of immediate experience to knowledge of a trans-subjective world. RUSSELL 1914 cast a new light on the issue. Inspired by the logicist definition of number and Whitehead's constructions in physics, he remarked that instead of taking the risk of *inferring* the existence of entities not directly experienced, it would be preferable to *construct* entities from the data of immediate experience.[9] "The supreme maxim in scientific philosophizing is this: whenever possible, logical constructions are to be substituted for inferred entities."[10] He suggested the construction of a system, the non-logical terms of which would refer only to what he considered to be immediately given, namely to sense-data and the subject which is aware of these sense-data.[11] The things of the external world with which, according to Russell, we have no immediate acquaintance, were not to be named by primitive terms of this system. But nothing seemed to be lost, since, although it was impossible to talk in this system about physical things, Russell suggested we could talk instead about certain classes of sense-data. Using the method of reduction, physical things were to be replaced by classes of those sense-data which we normally describe as caused by the physical things in question.

Basically the project is similar to that put forward by the British empir-

[9] RUSSELL 1914; cf. RUSSELL 1959, p. 103.
[10] RUSSELL 1918, p. 155.
[11] On the question of sense-data cf. MARC-WOGAU 1945. To be precise we must add that for Russell universals, too, belong to the immediately given; see chapter 5 below.

icists of the 18th century, according to whom every idea must originate directly in sense impressions or must be composed of ideas directly rooted in sense impressions. For them, too, physical things were just certain bundles of impressions. The decisive novelty, however, of Russell's approach is that with the aid of the methods of mathematical logic he attempts to state precisely the manner in which the ideas are composed.

Of course, in order to get a system rich enough to accommodate all our knowledge about the world, a single layer of constructions will not do, but constructions will have to be built upon constructions. By repeated applications of the method entire hierarchies of definitions, entire hierarchies of classes, classes of classes, classes of classes of classes, etc., have to be set up. The actual working out of such a *constructional system* is a highly complicated and difficult task.[12]

Russell later modified his system and abandoned the solipsistic view that a description of the world could be based exclusively on the philosopher's sense-data. He now chose events (possible as well as actual objects of sensation) as the referents of his primitive terms. He continued, however, to replace physical things by logical constructions, and in 1918 he even ceased to regard the self (the subject) as an unanalyzable entity, and replaced it by a certain class of events, namely by the class of those events which are the sensations occurring in the philosopher's body. Such a system, where the basic elements are neutral with respect to the subject-object dichotomy and where the subject is only constructed on a higher level, corresponds to the epistemological position called neutral monism, according to which the self is not real, but a mere appearance. Russell had encountered this epistemological position before in William James.[13]

However, from the point of view of future developments the arguments belonging to the traditional theory of knowledge which underlie Russell's systems, are not of primary importance. What counts is, on the contrary,

[12] Cf. the attempts made by CARNAP 1928 and GOODMAN 1951. Russell himself never gave more than a first sketch of a constructional system. The term 'çonstructional system' was adopted by GOODMAN 1951, p. XIII. Carnap used the term *Konstitutionssystem*, which is related to German epistemological terminology. He preferred the expression 'to constitute' because he thought it was neutral in regard to the distinction between making up and finding out (cf. CARNAP 1928, § 5).

[13] Cf. RUSSELL 1959, p. 105 ff., p. 134 ff.; JAMES 1904. For more details concerning the evolution of Russell's epistemological views cf. FRITZ 1952.

how Russell's logical technique brings about a new approach to the old questions.

In connexion with his constructs Russell refers to "Ockham's razor": "Entities are not be multiplied without necessity."[14] He interprets this maxim as meaning that in constructional systems one should aim at a minimal language, i.e., one into which all sentences can be translated but which has as few primitive terms as possible and thus makes a minimum of assumptions about reality. He is satisfied if, through the method of reductions, terms representing certain entities become unnecessary, and therefore reference need not be made to these entities. The important point is that Russell needs neither to assert nor to deny the existence of disputed entities: he can leave the question open.[15]

This reductionist attitude is in complete contrast to that of the post-cartesian epistemological theorists. For them, the decision regarding the existence of an external world, of a cognizing self, etc., was of the utmost significance and could not be left open. Their entire philosophical endeavour was directed towards finding a definite answer to these questions. The new approach does not oppose two metaphysical views between which a decision must be taken, but compares the *languages* in which these views are formulated. Thus with Russell we see coming up a new insight: in their representational function different systems make more or fewer assumptions about the nature of reality depending on whether the primitive terms of their language represent more or fewer entities of different kinds. Not only people can be classified according to their philosophical views, but languages, too, can be arranged according to the different metaphysical presuppositions implicit in them. And languages have the advantage of being more easily surveyable than people's thoughts. Furthermore, since they are the tools of intellectual activity, they can be classified according to their usefulness. One can ask

[14] RUSSELL 1914, p. 112, p. 153; RUSSELL 1959, p. 13. The wording of Ockham's razor as *Entia non sunt multiplicanda praeter necessitatem* can, e.g., be found in LEIBNIZ 1875–1890, vol. 4, p. 158. But according to PH. BOEHNER 1957, p. XXI, it has not been found in Ockham's own writings. Instead Ockham uses the formulation *Pluralitas non est ponenda sine necessitate* ("Plurality is not to be posited without necessity") and less frequently *Frustra fit per plura quod potest fieri per pauciora* ("What can be explained by the assumption of fewer things is vainly explained by the assumption of more things").
[15] Cf. also GOODMAN 1951, p. 103 ff.; GOODMAN 1956, p. 78 ff.

whether a given language is adequate as a scientific language. This indicates a possible pragmatic criterion for metaphysical assertions: the truth of a metaphysical standpoint that denies assumptions essential to any scientific language is highly improbable.[16]

The entities whose assumption Russell wants to render "unnecessary" are *real* entities (e.g., physical things) and he replaces them by *abstract* constructs. As we shall see later, reductionism can also function in the opposite way, eliminating the assumption of *abstract* entities and making reference instead to certain *real* and *concrete* entities. The question whether all assumptions of abstract entities can thus be avoided is the contemporary formulation of the problem of universals.[17]

2.3 Philosophy as logical analysis of language

In view of what has been said above, it will not surprise the reader that in subsequent systematic methodological reflection the nature of philosophy was viewed differently than in the post-cartesian period, and that the privileged position of the theory of knowledge was explicitly rejected.

Russell's disciple and collaborator Wittgenstein stated clearly that the aim of philosophy was the logical clarification of language.[18] Unfortunately, although in the *Tractatus* Wittgenstein deals not only with the analysis of sentences but parallel to it with the analysis of facts, i.e., with ontological questions, he speaks only of *logical* clarification and has no special term for the *ontological* aspect of analysis.[19] As a result the valuable notion that ontology finds expression in the logic of language was obscured. Thus in the Vienna Circle Carnap stipulated that ontological statements were misleadingly formulated, that the only correct mode of speech for philosophical questions was the so-called formal mode in which one speaks exclusively of relations between words or symbols.[20]

Wittgenstein defined the theory of knowledge as the philosophy of

[16] See below, pp. 140 ff.
[17] See below, pp. 148 ff.
[18] WITTGENSTEIN 1921, 4.112, 4, 4.001; 4.0031.
[19] WITTGENSTEIN 1921, 1–2.063 are purely ontological propositions. The reason why Wittgenstein did not distinguish between logical and ontological analysis appears to be that he believed the logical and the ontological form to be identical (cf. below, p. 54).
[20] CARNAP 1934, §§ 78, 79.

psychology, stressing that psychology was a branch of the natural sciences no closer to philosophy than any of the others, and thus deprived epistemology of its pre-eminent status.[21] Furthermore, he said that most of what had hitherto been written about philosophical matters was neither true nor false, but nonsensical.[22]

The Vienna Circle took over these views of Wittgenstein and disseminated them actively. Carnap divided the heterogeneous content of post-cartesian philosophy into logical (= philosophical) propositions, psychological propositions, and emotionally motivated, meaningless pseudo-propositions.[23] Among the pseudo-problems he counted especially the epistemological controversy between idealists and realists.[24]

[21] WITTGENSTEIN 1921, 4.1121.

[22] WITTGENSTEIN 1921, 4.003. The distinction true-false-meaningless was already clearly used by FREGE 1892, p. 200: "The sentence 'Julius Caesar exists' is neither true nor false, but meaningless." Nowadays a distinction is usually made between syntactically meaningless and empirically meaningless sentences. A sentence is syntactically meaningless if its arrangement of signs fails to qualify as a sentence according to the rules of formation of syntax; it is empirically meaningless if it precludes empirical verification as, for example, the sentence 'Inside the atom there is a nucleus, which, however, has no perceptible effects'. Wittgenstein did not make this distinction. He was interested only in an ideal language and was convinced that in such a language an empirically meaningless sentence could not be syntactically correct.

[23] Cf. the schema in CARNAP 1935.

[24] CARNAP 1928a.

3. FROM THE PSYCHOLOGICAL CONCEPT TO THE GRAPHICAL SIGN

This chapter narrows the field of enquiry to that of semantics, i.e., it will be concerned with signs and their representational functions. Once again a change with respect to epistemologically oriented post-cartesian philosophy must be noted: an important achievement of contemporary philosophy has been its elimination of psychologism, of the confusion of subjective and objective concepts. Indeed, many philosophers reacted so strongly against psychologism that they not only relegated subjective concepts from the sphere of logic to that of psychology, but also abandoned objective concepts (Frege's sense) and restricted themselves to the two-level semantics of linguistic signs and represented reality. Thus, as we shall see, Russell's theory of descriptions put forward the view that it is unnecessary to assume, in addition to the reference, a special level for the sense of the words. (His theory, incidentally, will also serve as example for a third kind of logistic analysis.) And Leśniewski's disciple, Tarski, whose system is the first strictly formulated and non-contradictory semantical system, also proposes a two-leveled correspondence theory of truth, where not the concepts, but the graphical signs are correlated with reality.

3.1 The elimination of psychologism and Frege's semantics

Ever since Descartes and Locke modern philosophy had been severely handicapped by its tendency to psychologism. Subjective sense perception and subjective concepts, both part of the individual cognizing consciousness, were confused with objective meaning, with the object of the act of cognition. One consequence, among others, was the attempt to explain the laws of logic in terms of psychology. Although there were always opponents of psychologism[1], it was left to Husserl's phenomenology and Moore's analysis to eliminate psychologism effectively. Espec-

[1] HUSSERL 1900/01 (vol. 1, p. 227) quotes especially B. Bolzano and H. Lotze.

ially Husserl's *Logische Untersuchungen*, published in 1900 and 1901, is a classic in this respect.

A further important factor in the movement against psychologism was the development of mathematical logic. Frege, who exercised a strong anti-psychologistic influence on Husserl, was the first contemporary logician to deal with semantical questions without confusing them with psychological issues. We shall therefore briefly survey his views as they appear in the essay entitled 'Über Sinn und Bedeutung' (On sense and reference).[2] Not all logicians shared Frege's views, but they all had to come to terms with them, so that even from a purely historical view his writings are of considerable importance. It is, for example, not insignificant from the point of view of later development, that in the above essay Frege investigated the semantics of only two categories of expressions: proper names, i.e., names of objects, and sentences, but not of signs for concepts and relations, i.e., predicate signs.[3]

Frege makes a four-fold distinction with regard to signs: the *sign* (*Zeichen*) itself, the *reference* (*Bedeutung*), the *sense* (*Sinn*), and the *idea* (*Vorstellung*). A sign is, for Frege, an event or an object having a specific form.[4] The reference of a sign which is a proper name, is the object designated by it. Its idea (*Vorstellung*), Frege holds, is wholly subjective. In between the reference and the idea lies the sense, which is not subjective like the idea but equally is not the object itself. Frege effectively compares reference, sense and idea with three different things that can be distinguished when one observes the moon through a telescope: the moon; the real image in the telescope, giving a definite view of the moon, which is objective insofar as it can be seen by several observers; and the retinal image of each individual observer. Frege concerns himself with ideas, for reasons of anti-psychologism: ". . . to ensure that the idea

[2] FREGE 1892. We shall quote M. Black's translation throughout. Cf. also the interesting monographs by THIEL 1965 and BIRJUKOV 1961. A warning concerning the meaning of the German word *Bedeutung* is in order. In Frege the word *Bedeutung* means "reference", "nominatum", "denotation". But in non-fregean texts the usage is different; there the word means usually "meaning" and is synonymous with the word *Sinn*. What Frege wants to say when he uses the word *Bedeutung* is usually expressed with the phrase *das Bezeichnete*.

[3] Frege treated the semantics of predicate signs in another paper of the same year: FREGE 1892a. But is it written in polemical form, answering objections, and not as a straight systematic explanation. It is therefore not of the same clarity as FREGE 1892.

[4] FREGE 1892, p. 26.

aroused in the hearer by a word shall not be confused with its sense or reference".[5] From the point of view of logic, only sense and reference are of importance. They correspond approximately to the traditional distinction between intension and extension, connotation and denotation.[6]

It was mainly the case of identity sentences that decided Frege to posit a special level of sense apart from reference. Is '$a = b$' equivalent to '$a = a$'; are both equations mere tautologies? Or does '$a = b$' say more, by giving the additional information that two different *names*, 'a' and 'b', denote the same object? Both interpretations are inadequate. For example, the equation 'morning-star = evening-star' is no mere tautology and equally what it reports is not a matter of terminology alone, but a scientific discovery made after a large number of astronomical observations. Frege allows for this fact by introducing the distinction between sense and reference. Thus he holds that 'a' and 'b' have the same reference but different senses. Sense and reference belong, in his view, to quite different spheres[7], and he distinguishes terminologically between 'to express' (*ausdrücken*) and 'to designate' (*bezeichnen*): "By means of a sign we express its sense and designate its reference".[8]

Frege also applies the distinction between sense and reference to sentences, the sense and reference of a sentence being made up of the senses and references respectively, of the signs contained in it.[9] He calls the sense of a sentence "thought" (*Gedanke*); the reference of a sentence is its truth-value.[10]

A second reason for Frege's distinction between the two levels of sense and reference is that he wished to uphold the general validity of the so-called principle of extensionality.[11] According to this principle the reference of a compound expression remains the same if a sign occurring

[5] FREGE 1892, pp. 30–31.
[6] Cf., *inter alia*, MILL 1843.
[7] FREGE 1892, p. 35: "By combining subject and predicate, one reaches only a thought, never passes from sense to reference, never from a thought to its truth value. One moves at the same level but never advances from one level to the next. A truth value cannot be a part of a thought, any more than, say, the Sun can, for it is not a sense but an object."
[8] FREGE 1892, p. 31.
[9] FREGE 1892, p. 35. Frege expressly uses the terminology of whole and part, though with appropriate modification.
[10] FREGE 1892, p. 32, p. 34.
[11] Cf. BIRJUKOV 1961, p. 544.

in it is replaced by another with the same reference. For example, 'the tutor of Alexander the Great and disciple of Plato' has the same reference as 'the Stagirite and disciple of Plato', since both 'the tutor of Alexander the Great' and 'the Stagirite' refer to Aristotle. There are, however, contexts where the substitution of signs with normally identical reference results in a different reference for the expression as a whole. For example, the sentence: 'George IV once asked whether Walter Scott was the author of *Waverley*' is true, whereas the sentence 'Georege IV once asked whether Walter Scott was Walter Scott' is false. The two sentences have different references although Walter Scott was, in fact, the author of *Waverley*, so that 'Walter Scott' and 'the author of *Waverley*' would normally have the same reference.[12] Thus in respect of the context 'George IV once asked whether Walter Scott was . . .', either the principle of extensionality does not hold, or the names substituted do not have their normal reference in it. As Quine puts it, it is an "opaque" context.[13] Frege resolves the difficulty of "opaque" contexts by adopting the second of the two above solutions: in such a context the signs do not have their normal reference but refer to what is normally their sense. Thus by positing sense, Frege is able to maintain the general validity of the principle of extensionality without restriction.

In order to obviate the case where graphically alike signs have different references (as happens, for example, in natural language when a sign 'the author of *Waverley*' occurs in an extensional context and a sign 'the author of *Waverley*' occurs in an opaque context), A. Church later developed a system where a new series of expressions is introduced in order to refer to the senses of prior expressions. But what we have said above about identity sentences applies also to these new expressions. Therefore they, too, must have a sense different from their reference, which sense would have to be designated by a third series of expressions, and so on, ad infinitum. Church's system is thus somewhat complicated and little used in practice.[14]

Whereas a realm of reference is easily accepted, this is different for the realm of sense: senses are more controversial entities, and have presented Frege's successors with considerable difficulties. Frege's sense seems to

[12] The example is Russell's.
[13] Cf. QUINE 1939.
[14] CHURCH 1946; CHURCH 1951; by way of introduction cf. CHURCH 1951a.

combine two features. On the one hand Frege says that a sense can be understood: a sense can be grasped by anybody who is familiar with the language to which it belongs. On the other hand, a sense appears to be something that is directly expressed in the sign: the sense, Frege says, contains the mode of presentation of an object. He gives the example of the point of intersection of two straight lines *a* and *b*, coinciding with the point of intersection of *b* with a third straight line *c*: here the expression 'the point of intersection of *a* and *b*' and 'the point of intersection of *b* and *c*' have different senses but the same reference, and the different mode of presentation of the reference can be clearly read off from the expressions themselves.[15] We shall return to these questions later.[16]

3.2 Russell's theory of descriptions

The next logician, after Frege, to concern himself with semantical questions was Russell. To an even greater extent than Frege, Russell makes the written sign the starting-point of his investigations. We have already shown in Chapter 1 how he sets up a parallel between relational signs and relations. He is not at home in the domain of senses, of objective meanings, and characteristically finds little use for phenomenological analysis. Although he is acquainted, for example, with Brentano's analysis of perception, with its distinction between act, content and object, Russell considers it unnecessary to distinguish between the objective content and the object of a perception.[17] His aims are of a quite different nature, viz. he wants to set up a constructional system on a minimal basis. It is therefore not surprising that he had no misgivings about leaving the senses of words out of consideration, when he found a theory that resolved the difficulties which at first had seemed to force him to accept them.

Russell is convinced of one thing: if a sentence is to be meaningful for us, then something must exist about which it says something, to which the terms contained in it refer, and with which we can become immediately acquainted. From what we have said about the analysis of relational sentences, the reader will already be familiar with Russell's notion that every single term represents something and thus has existential import,

[15] FREGE 1892, pp. 26–27.
[16] Cf. below, pp. 55 ff.
[17] RUSSELL 1959, p. 134.

i.e., indicates an existing entity; and the view that all knowledge must be based on direct experience, will bring to his mind Russell's attempt to reduce the objects of the external world to sense-data.

However, negative existence statements such as, for example

[1] 'The present king of France does not exist'

presented Russell with a difficulty, for here we speak of something that does not exist. Russell at first decided that the *non-existing* must nevertheless somehow *be*, since otherwise it would be impossible to say anything about it - for example, deny its existence. He therefore posited a sphere of *being* in addition to that of *existing*.[18]

This, however, was not a satisfactory solution, for descriptions of "non-existing beings" conflicted with the laws of logic. Meinong, for example, had gone so far as to posit a round square, an entity which was both round and square, i.e., not round.[19] And not only contradictory expressions presented difficulties.[20] Take, for example, the sentence:

[2] 'The present king of France is bald'.

As at the present time there is no reigning king of France, the question arises: is [2] true or false? If we assume that [2] is false, which seems the most obvious answer, then according to the law of the excluded middle, the assertion

[3] 'The present king of France is not bald'

is true. But the assertion that the present king of France has a head of hair seems to be as false as [2]: [2] and [3] seem to violate the law of the excluded middle. To assume that [2] and [3] are both true would be even worse, since this would violate the law of contradiction.

If the laws of logic are not to be infringed, the meaningfulness of [2] and [3] in their present form must be questioned. Russell reaches the

[18] RUSSELL 1903, pp. 449–50.

[19] Cf. RUSSELL 1905a, i.e., Russell's discussion of Meinong. In connexion with Russell's difficulty, note that although entities with contradictory properties are impossible, entities with contradictory characterizing marks seem possible. Cf. below, p. 104 f.

[20] For the following cf. RUSSELL 1905 and WHITEHEAD-RUSSELL vol. 1, pp. 66–71. The essay of 1905 is not clearly written, and we shall therefore present the argument in our own way. In 1905 Russell used the term 'denoting phrase', but we shall replace it by 'description', which is the term that later became established.

conclusion that 'the present king of France' and in general every description of the form 'the so-and-so' is in itself meaningless. He can afford to take this view, as he has found a way of nonetheless allowing for sentences containing descriptions: every such sentence can be replaced by an equivalent conjunction of sentences in which no description occurs and which does not violate the laws of logic. For example, sentence [2] can be replaced by:

[4] 'There is an x such that x is at present king of France, and every y that is at present king of France is identical with x, and x is bald.'

or in symbols:

$$(\exists x)\{Kx \cdot (y)(Ky \supset y = x) \cdot Bx\}$$

In this way Russell solves the problem of the functioning of descriptions, which play so important a part in enabling us to speak about things with which we are not directly acquainted.[21] Names of things that do not exist in reality no longer occur. The replacing of [2] by [4] is a so-called "*definition in use*" of a description: accordingly a description is an *incomplete symbol* that designates nothing taken by itself, but whose use is defined within the context of an entire sentence.[22]

A definition in use may be regarded as a third kind of logistic analysis. Here we are not concerned with the demonstration of the existence of an irreducible basic element, as in the analysis of relational facts; nor with the explicit reduction or construction of an entity, as discussed in Chapter 2; but with the explaining away of a misleading turn of phrase.

The theory of descriptions also explains negative existence statements. Thus [1] is replaced by:

[5] 'There is no x for which it is true that: x is at present king of France, and every y that is at present king of France is identical with x.'

[21] RUSSELL 1912, p. 92: "The chief importance of knowledge by description is that it enables us to pass beyond the limits of our private experience. In spite of the fact that we can only know truths which are wholly composed of terms which we have experienced in acquaintance, we can yet have knowledge by description of things which we have never experienced."

[22] WHITEHEAD-RUSSELL vol. 1, p. 66.

or in symbols:

$$\sim (\exists x)\{Kx \cdot (y)(Ky \supset y = x)\}$$

No name for a "non-existing being" occurs in [5], but only the sign 'x'. This 'x' is a variable, i.e., a sign to which a so-called value-range of entities is assigned. [5] says no more than that for each of the entities belonging to the value range of 'x' it is not the case that it has the unique property of being at present king of France. And since no such phantom entity as the present king of France occurs in the value-range of 'x', [5] makes reference only to actually existing individuals.

Russell emphasizes the importance of variables: "I take the notion of the variable as fundamental."[23] We shall see below[24] how Quine develops this idea of Russell's.

Russell's theory of descriptions thus rendered untenable the specious argument that a special domain of senses must be posited in order to fill the gap caused by the absence of referents. A missing referent cannot be replaced by a sense, for – as Russell rightly says – a sentence does not speak *about* the *sense* of the words contained in it, but *about* their *referents*. For example, if I say: "The dog has bitten me", I do not wish to make a remark about the sense of the word 'dog'; and if I say that there are no centaurs, I do not intend to deny the existence of the sense of the word 'centaur'.[25]

If we wish to speak *about* the senses of certain words, then these senses must be *referred* to by new expressions.[26] As we have pointed out above, a self-consistent theory of sense and reference is quite complicated. It is therefore understandable that Russell failed to see things clearly and spoke

[23] RUSSELL 1905, p. 479.

[24] Cf. below, p. 130.

[25] These examples are not Russell's. It is, of course, possible to interpret the statement that there are no centaurs so as to mean that the real world contains no "realization" of the ideal sense of the word 'centaur'. This is the platonistic interpretation of the sentence 'There is no x which is a centaur'. But even on this interpretation we do not really speak *about* the ideal sense – for the name of the sense is not the subject of the sentence – but *about* the real entities in the value-range of 'x', in which the ideal senses are "realized".

[26] In ordinary language words occurring in certain types of subordinate clauses refer to what is usually their sense. But in a well-regulated logistic language signs of the same graphic form should always have the same reference. Cf. above, p. 41.

of Frege's distinction between sense and reference as "an inextricable tangle".[27]

However, how can Frege's difficulty with regard to identity sentences be resolved without assuming a domain of senses?[28] For in this case, as, e.g., Wittgenstein has pointed out, a sentence like '$a = b$' would have to be regarded as a mere rule for the use of signs, and as telling us nothing about the designatum of 'a' and 'b'.[29] Indeed, in Russell's language one and the same object should not have two or more proper names. But there is a way to take care of the different senses of names such as 'morning star' and 'evening star', namely by means of descriptions like 'the x which has the property M (the property of being a star and of disappearing from the sky in the morning later than any other star)' and 'the x which has the property E (the property of being a star and of appearing in the sky in the evening before any other star)', or in symbols: '$(\imath x)(Mx)$' and '$(\imath x)(Ex)$'. Thus what in ordinary language appears as the sense of a proper name is in Russell's language expressed by means of predicate expressions. In place of '$a = b$' we get

'the x with the property A = the x with the property B'

or in symbols:

$$(\imath x)(Ax) = (\imath x)(Bx).$$

If we eliminate the descriptions, which in Russell's language are only incomplete symbols defined in use[30], then we arrive at the sentence:

> 'There is an individual x and there is an individual y such that x has the property A and no other individual has the property A, and y has the property B and no other individual has the property B, and x is identical with y'

or in symbols:

$$(\exists x)(\exists y)\{Ax \cdot (z)(Az \supset z = x) \cdot By \cdot (z)(Bz \supset z = y) \cdot x = y\}.$$

Here the sign of identity stands only between variables.

[27] RUSSELL 1905, p. 87.
[28] Cf. above, p. 40.
[29] WITTGENSTEIN 1921, 4.241, 4.242.
[30] Cf. above, p. 44.

Russell's theory of descriptions excluded senses or meanings as an intermediary domain between words and designata, and this was of far-reaching importance for the development of contemporary philosophy. Attention became concentrated on the theory of reference, at the expense of the theory of meaning, and the investigation of relations of sense was at first neglected.[31] But apart from the negative there was also a positive side, for the immediate juxtaposition of sign and designated reality drew attention to the co-ordinations between them, and made possible the systematic investigation of the relation of representation which Wittgenstein was the first to undertake.

Russell does not deny all meanings; it is only the meanings of words that he refuses to admit. He explicitly admits the senses of sentences, describing propositions as something which is common to different sentences and can be expressed in different languages.[32] But he finds it difficult to decide what kind of entities propositions are. They cannot be identical with the facts of the real world, since "For each fact there are two propositions, one true and one false."[33] Russell initially concluded that propositions must belong to a world other than the real one.[34] Wittgenstein, too, as we shall see, distinguished between states of affairs, that need not be realized, and facts. Since sentences are combinations of words, it will not be surprising that their semantical relations differ from those of their constituent elements.[35]

The *Vienna Circle*, however, vehemently proclaimed its denial of all "metaphysical" entities, and firmly upheld Neurath's physicalism.[36] Every effort was made to refer solely to material objects, and to correlate signs only with physical entities. Some members of the Circle explicitly iden-

[31] Thus Quine emphasizes the distinction between a "theory of reference" and an unreliable "theory of meaning". Cf. QUINE 1953, p. 130f., and also below, p. 141.
[32] RUSSELL 1940, p. 12.
[33] RUSSELL 1918/1919, p. 7. This, of course, applies equally to the negative facts assumed by RUSSELL, *ibid.*, p. 23, and by WITTGENSTEIN 1921, 2.06.
[34] RUSSELL 1918/19, p. 23. Cf. Church in BOCHEŃSKI-CHURCH-GOODMAN, pp. 3–11, where propositions are interpreted as eternal, ideal entities. It is, however, also possible to think of them as abstract but existentially heteronomous entities that come into existence and go out of it (cf. the "unities of meaning", *Bedeutungseinheiten*, in INGARDEN 1931, pp. 98–110).
[35] Cf. below, pp. 56 ff.
[36] NEURATH 1931.

tified the meaning of a sentence with its method of verification.[37] This standpoint has been rightly criticized, since verifiability, being a possibility, is not itself a physical entity.[38] At a later date Russell, too, gave a physicalistic-psychologistic definition of propositions as classes of events occurring in the brain.[39] Carnap was a strict physicalist during his Vienna period[40], but later returned to a standpoint close to Wittgenstein's.[41] It is interesting to note that at the beginning of his career, i.e., in his dissertation, Carnap speaks of a phenomenological intuition of essences (*Wesenserschauung*), and thus of synthetic a priori judgments, which are based on connexions between objective meanings.[42] Later, however, he showed himself unable to distinguish between subjective and objective concepts, and accused phenomenology of psychologism.[43]

3.3 Tarski's definition of the concept of truth

Neither Frege nor Russell formalized their semantical views in a strict, non-contradictory system. In connexion with the central semantical concept of truth the well-known antinomy of the liar had been discovered already by the Greeks.[44] Ancient as well as medieval philosophers pro-

[37] Thus cf. WAISMANN 1931, p. 229; but CARNAP 1931a is not explicit on this point. On the history of the verification principle cf. in particular HEMPEL 1950; CARNAP 1936/37; CARNAP 1956.

[38] Cf. INGARDEN 1934. On the difficulties of a nominalistic interpretation of possibility cf. GOODMAN 1947.

[39] RUSSELL 1940, ch. 13A.

[40] Cf. CARNAP 1931.

[41] CARNAP 1942, p. 235, explicitly accepts propositions and gives a list of quotations from other writers; but cf. in particular CARNAP 1947, p. 30 f.

[42] CARNAP 1921, p. 23.

[43] "In our view propositions are either empirical or analytic: there is no third kind. The supposed third kind of proposition appears to us to be the result of a confusion between a psychological and a logical approach. It is our belief that phenomenology has not yet really overcome the psychologism against which it has so decidedly set itself. The development of phenomenology from its founder, Husserl, to its contemporary representatives has shown clearly enough the danger of allowing an initially scientific approach to lead to entirely subjective, if not meaningless, views" (CARNAP 1934, p. 143).

[44] Epimenides, the Cretan, is supposed to have said: "I am lying." Did he lie or tell the truth? If he lied, then what he said cannot be true, i.e., he spoke the truth; if, on the other hand, he spoke truly, then what he said must be true, i.e., he lied. Cf. in this connexion RÜSTOW 1910; RIVETTI-BARBÒ 1961; VON KUTSCHERA 1964.

pounded a number of attempted solutions[45], but no one of them appears to have been generally accepted; and during the logically unsophisticated period of post-cartesian philosophy the problem ceased to be of interest.

When Russell discovered the antinomy of the class of all classes which are not members of themselves[46], and thus brought the foundations of mathematics into question, all kinds of antinomies began once more to be collected, emphasis being laid at first on their common features. F. P. Ramsey, however, distinguished between two kinds of antinomies, which later became known as "logical antinomies" and "semantical antinomies". Whereas semantical expressions such as 'define', 'name', 'assert' need not occur in the formulation of logical antinomies (such as e.g. Russell's antinomy), these expressions are essential in the formulation of semantical antinomies (such as the antinomy of the liar). Ramsey made this distinction because he believed he could disregard semantical antinomies from the point of view of logic (he did not yet think in terms of an independent discipline of semantics, distinct from psychology and epistemology).[47]

It was Leśniewski who first formulated a special system of semantics. Its technical elaboration owed much to the results of the metamathematics founded by D. Hilbert and developed above all by K. Gödel. Just as *meta*mathematics stands *above* mathematics, so a semantical system makes statements *about* the system of the object-language and its function of representation, and is thus *metalogical*. Unfortunately Leśniewski himself published very little, but his disciple A. Tarski put forward a semantical system free of antinomies. By consistently distinguishing between language and metalanguage, he became the first to formulate a non-contradictory definition of the concept of truth[48], a definition in which truth is conceived along aristotelian lines as correspondence. Tarski, however, does not select the relation of correspondence between thing and *concept* (*adaequatio rei et intellectus*), as did the scholastics, but instead sets up a parallel between sentences about things and sentences

[45] Bocheński 1956, pp. 151–153, pp. 277–292.
[46] Cf. below, pp. 72 ff.
[47] Ramsey 1946, p. 20 f. In Ramsey's view logical antinomies could be resolved by the ordinary theory of types, Russell's complex theory of types being unnecessary.
[48] Tarski 1933; cf. also Tarski 1936; Tarski 1944; Stegmüller 1957.

about sentences. His standard example is the following: ''It is snowing' is a true sentence if and only if it is snowing.' As is clearly apparent, Tarski uses a two-level semantical system containing no designations of meanings but only designations of signs and the signs that designate reality.

The precision of his terminology is indicated by the fact that Tarski is aware of the distinction between a concrete series of signs (a *token*) and a whole class of such series which are of equal shape (a *type*). He states explicitly that he finds it more convenient to stipulate that the word 'sentence' and the expressions in inverted commas, are to indicate such classes of series of signs.[49]

The example quoted above gives a formulation of truth conditions for one concrete instance only. The difficulty, however, is to find a non-contradictory *general* definition, comprising *all* individual cases. Tarski succeeded in doing this *via* the concept of satisfying. It would be a digression to give a detailed account of the relevant arguments, and we shall limit ourselves to some brief indications. The following is a simple example of what is meant by "satisfying": 'For every a – a satisfies the sentential function 'x is white' if and only if a is white'; thus, e.g., snow satisfies the sentential function 'x is white'.[50] This shows that Tarski's semantics not only is two-levelled like Russell's, but also that variables, and the domain of objects that can be values of these variables, likewise play an important part in it.

Another concept central to modern metalogic is that of a *model* or a realization. If all non-logical constants in a sentence have been replaced by variables, thus making it into a sentential function, then any sequence of objects that satisfies the said function is called a model or a realization of the sentence in question.[51] In the sentence-model dichotomy we once again have the characteristic two-levels with which modern semantics is almost exclusively concerned.[52]

[49] TARSKI 1956, p. 156. CARNAP 1934, § 4, does the same. The distinction between token (*sinsign*) and type (*legisign*) had already been made in 1903 by PEIRCE 1931–1935, 2.243, 4.537. Cf. also below, p. 64 f.

[50] TARSKI 1956, p. 190.

[51] TARSKI 1956, p. 416–417, and elsewhere.

[52] Cf. below, pp. 136 ff., where the ontological classification of the various models is discussed.

4. THE RELATION OF REPRESENTATION

As a result of the exclusion of subjective psychological concepts as well as of the objective meanings of words, the direct relationship between signs and what they stand for came clearly to the fore. In the *Tractatus* Wittgenstein has investigated this relation in detail, and has drawn attention to its essential features.[1] In speaking of it, he uses the German verb *abbilden*, the literal translation of which is 'to picture', 'to depict'. He calls a sentence a picture (*Bild*), because he wants to compare it with a broad spectrum of other examples of pictures, ranging from a *tableau vivant* (*lebendes Bild*: a silent and motionless group of persons, etc., arranged to represent a scene) to a mathematical projection (*Abbildung*).[2] For our own terminology we prefer the more neutral term '*representation*'. We want to give it a broader meaning so that it covers not only Wittgenstein's picturing-relation, but also the corresponding semantical relations in Russell, Carnap, etc.

Our aim in this chapter will be to point to two main features of the relation of representation as examined by Wittgenstein: the sharing of structure and form and the representing of the content. Since the *Tractatus* is a very condensed and difficult text, we do not claim to give a final interpretation and refer the reader for further studies to the many detailed commentaries available.[3]

[1] WITTGENSTEIN 1921. We consider here only Wittgenstein's *Tractatus*. In a later period Wittgenstein abandoned logistic philosophy and held that the descriptive function of language had no privileged position, but was only one beside a great many other "games" that could be played with ordinary language.

[2] The terminology seems to stem from Frege: "It would be desirable to have a special term for signs having only sense. If we name them, say pictures (*Bilder*), the words of an actor on the stage would be pictures; indeed the actor himself would be a picture" (FREGE 1892, p. 33). Cf. how for Wittgenstein sentences have only sense (see below, p. 56) and how he compares a sentence with a *tableau vivant* (WITTGENSTEIN 1921, 4.0311). Other fregean expressions in the *Tractatus* are *Sinn* ('sense'), *Bedeutung* ('reference'), *Gedanke* ('thought').

[3] BLACK 1964; FAVRHOLDT 1964; GRIFFIN 1964; PITCHER 1964; STENIUS 1960; ANSCOMBE 1959; MASLOW 1961 (written in 1933 by a student of M. Schlick).

4.1 The sharing of structure and form

The direct relation established between signs and that for which they stand, drew attention to their common features. "A sentence (*Satz*) is a picture of reality"[4], Wittgenstein claims, and: "The configuration of objects in a situation corresponds to the configuration of simple signs in the propositional sign."[5] Here we see Wittgenstein's close relation with his teacher, Russell.[6] For it is relational sentences, of which we have already had occasion to speak, that Wittgenstein has in mind: "It is obvious that a sentence of the form *aRb* strikes us as a picture."[7] "That '*a*' stands to '*b*' in a certain relation says *that aRb*."[8] And he holds that the correspondence between the structure of a sentence and that of a state of affairs is more readily apparent in a formula written in logistic language. In ordinary language matters are much more complicated; for there the outward form of the dress – as Wittgenstein puts it – has been fashioned for quite different purposes than to reveal the form of the body.[9]

Wittgenstein develops his views on how language functions within the framework of an entirely general theory of representation; thus his reference to the series: gramophone record – musical idea (*Gedanke*) – written notes – sound-waves, the members of which all stand in a pictorial relation to each other.[10] "The pictorial (*abbildende*) relationship consists of the correlations of the picture's elements with things."[11]

The following diagram and formula may spell out more fully what Wittgenstein has in mind (neither the diagram nor the explanatory remarks are his):

[4] WITTGENSTEIN 1921, 4.021. We quote the translation of D. F. Pears and B. F. McGuinness, except for the fact that we render Wittgenstein's *Satz* by 'sentence' and not by 'proposition'.

[5] WITTGENSTEIN 1921, 3.21.

[6] But cf. below, p. 68 and p. 81, for an important divergency.

[7] WITTGENSTEIN 1921, 4.012.

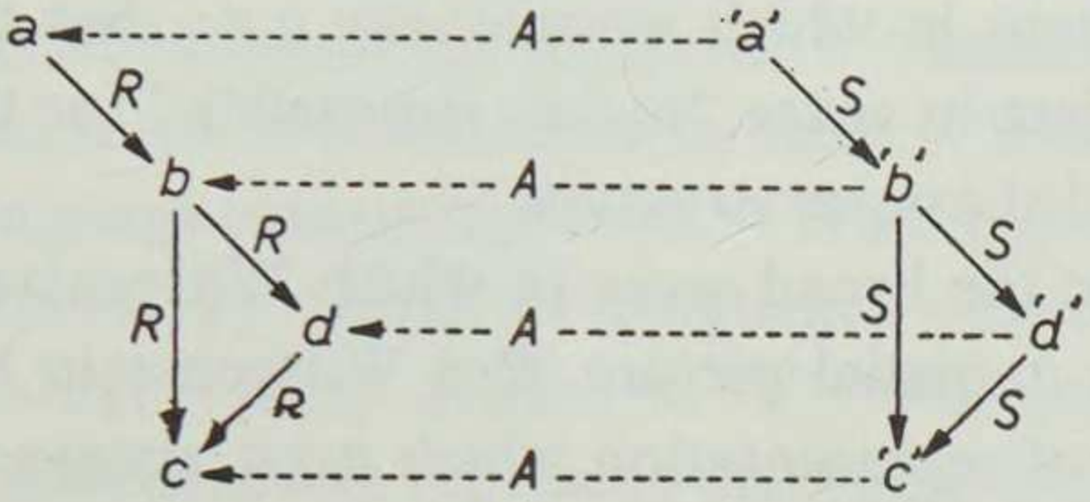
[8] WITTGENSTEIN 1921, 3.1432; cf. also the negative version in 5.5151.

[9] WITTGENSTEIN 1921, 4.002.

[10] WITTGENSTEIN 1921, 4.014.

[11] WITTGENSTEIN 1921, 2.1514.

Chart of facts



R = relation holding between the objects occurring in these facts.

Chart of picture of these facts

A = pictorial relation.

Linguistic picture of these facts

a	*R*	*b*
b	*R*	*c*
b	*R*	*d*
d	*R*	*c*

S = relation holding between the elements occurring in a picture of these facts, e.g., the syntactical relation of positional order holding between names occurring in the same atomic sentence formula.

In russellian terminology A is a one-one relation whose converse domain is the field of R and of which it holds that the relative product of A, R and the converse of A is equal to S (i.e., that the representation preserves the order of that which is represented):

$$A \in 1 \to 1 \cdot \mathcal{D}'A = C'R \cdot A|R|\breve{A} = S$$

Two relations between which a representational relation of this kind holds, are called isomorphic. The structure of a given relation may be defined as the class of all relations that are isomorphic to it.[12]

However, Wittgenstein does not use class terminology, but says simply that *structure* is the way in which the objects in a state of affairs or the elements of a picture are connected together.[13]

But Wittgenstein mentions not only the structure or configuration, he talks also about the form of the representation (*die Form der Abbildung*) which a picture and the depicted reality must have in common.[14] For instance, a spatial picture and the spatial reality depicted by it have space as a form of representation in common, and a coloured picture shares

[12] Cf. CARNAP 1929, § 22; BOCHEŃSKI-MENNE § 21.4.
[13] WITTGENSTEIN 1921, 2.032, 2.034 and 2.15.
[14] WITTGENSTEIN 1921, 2.2; cf. also 2.161, 4.032, 4.04.

with coloured reality the characteristic of belonging into a framework of colours.[15] Wittgenstein calls the form, the possibility of the structure.[16] Probably this is meant in the sense in which we may say e.g., that the general property of being an object in space "makes it possible" for the object to have this or that spatial shape or structure.

However, not every picture (in the broad sense in which Wittgenstein uses the word) is a coloured or a spatial picture. But Wittgenstein believes that there is a basic form of representation which must always be present: the *logical form*.[17] It is most clearly shown by the sentences of logistic languages. Take as an example the sentential formula '*aBb*' describing the fact that Peter is beating Paul.[18] The formula is clearly a relational sentence with two proper names, i.e., it has the general logical form *xRy*. The same form can be abstracted from the fact of Peter beating Paul, which is an example of a relation between two individuals, and therefore has the form *xRy*. By analogy with spatial form, logical form circumscribes a whole "logical space" of possibilities. Each substitution for the variables of '*xRy*' constitutes one "point" of this "logical space", i.e., one possible sentence formula, or one possible state of affairs.[19] Each set of compatible possible states of affairs (*Sachverhalte*) constitutes one possible world.[20]

It is important to notice how for Wittgenstein the so-called logical form is not merely the syntactical form of sentences, but at the same time the ontological form of the world of facts. He calls it explicitly "the form of reality".[21] Among the positivists of the Vienna Circle this ontological side of Wittgenstein's doctrine was lost and ontology reduced to mere syntax.

Wittgenstein insists strongly that the logical form can only be displayed (*aufgewiesen*), mirrored or reflected (*gespiegelt*), shown (*gezeigt*), and

[15] Cf. WITTGENSTEIN 1921, 2.171.
[16] WITTGENSTEIN 1921, 2.033, 2.15, 2.151.
[17] WITTGENSTEIN 1921, 2.18.
[18] The example is not Wittgenstein's and is inappropriate, as it is too "crude"; we should really take an example from atomic physics, but this would complicate matters unduly. Cf. WITTGENSTEIN 1929, p. 165. On the interpretation of predicate signs, cf. below, p. 82 f.
[19] WITTGENSTEIN 1921, 3.315, 3.4; WITTGENSTEIN 1929, p. 162, p. 169.
[20] Cf. WITTGENSTEIN 1921, 1.13, 2.04–2.06.
[21] WITTGENSTEIN 1921, 2.18.

that it cannot be depicted (*abgebildet*), represented (*dargestellt*), said (*gesagt*): "A picture cannot, however, depict its pictorial form (*Form der Abbildung*): it displays it."[22] "Sentences (*Sätze*) cannot represent logical form: it is mirrored in them. – What finds its reflection in language, language cannot represent. – What expresses *itself* in language, *we* cannot express by means of language. – Sentences *show* the logical form of reality. – They display it."[23] "What *can* be shown, *cannot* be said."[24] "Sentences can represent the whole of reality, but they cannot represent what they must have in common with reality in order to be able to represent it – logical form. – In order to be able to represent logical form, we should have to be able to station ourselves with sentences somewhere outside logic, that is to say outside the world."[25]

Wittgenstein concludes that nothing can be said *about* logical form. He was so strongly convinced of this point that he was ready to regard his own philosophical assertions, which in effect are propositions about logical form, as pseudo-propositions.[26] Nowadays it is generally agreed, as Russell had already remarked in his introduction to the *Tractatus*, that although we cannot step *outside* any logical form – Wittgenstein is right on this point – we can, nevertheless, make statements *about* the formal system of any language within the system of a metalanguage. Carnap, for example, has shown this in detail in a book written in express opposition to Wittgenstein.[27]

4.2 The question of the content

Wittgenstein uses the verb 'to show' also in talking about the *sense*, i.e., about what is expressed by the *structure*, the configuration of the elements of a sentence[28]: "A sentence (*Satz*) *shows* (*zeigt*) its sense".[29] However, in another passage he says that a picture *represents* (*stellt dar*)

[22] Wittgenstein 1921, 2.172.
[23] Wittgenstein 1921, 4.121.
[24] Wittgenstein 1921, 4.1212.
[25] Wittgenstein 1921, 4.12.
[26] Wittgenstein 1921, 6.54.
[27] Carnap 1934.
[28] Cf. Wittgenstein 1921, 3.1431.
[29] Wittgenstein 1921, 4.022; italics in Wittgenstein.

its sense.[30] This twofold terminology is puzzling since, as we have seen, for Wittgenstein showing and representing seem to exclude one another. But there may be a clue to its understanding in the following quotation, where Wittgenstein distinguishes between two aspects of the sense, its form and its content: "A sentence does not actually contain its sense, but does contain the possibility of expressing it. . . . A sentence contains the form, but not the content, of its sense."[31]

In order to find out what Wittgenstein could mean by the form and the content of the sense, we must first see how his conception of sense (*Sinn*) and reference (*Bedeutung*) has developed away from Frege's.

Whereas Frege talks of all categories of expressions, of proper names as well as of sentences, as having both sense and reference, Wittgenstein does not. He says on the one hand that names have reference (*Bedeutung*), and on the other hand that sentences have sense.[32] We have given a detailed account in connexion with Russell's theory of descriptions, of how objective meanings (i.e., senses) of names came to be disregarded.[33] Wittgenstein rejects the notion of the reference of sentences as well. He holds that truth-values, which on Frege's view are what sentences refer to, are not objects. And facts cannot be regarded as the referents of sentences either, since he says that states of affairs (*Sachlagen*) can be only described and not named.[34] Thus, although Wittgenstein, like Frege, calls the sense of a sentence a thought [35], his standpoint differs from Frege's in a number of ways.

According to Wittgenstein the reference of names depends on the sense of the sentences in which they occur: "Only in the nexus of a sentence does a name have reference."[36] The reason for this opinion seems to be the following: "Objects can only be *named*. Signs are their representatives. I can only speak *about* them: I cannot *put them into words* (*sie aussprechen*

[30] Cf. WITTGENSTEIN 1921, 2.221; no italics in Wittgenstein.
[31] WITTGENSTEIN 1921, 3.13.
[32] WITTGENSTEIN 1921, 3.144, 3.3, 6.124.
[33] Cf. above, p. 42 f.
[34] Cf. WITTGENSTEIN 1921, 4.431, 3.144.
[35] "A thought is a meaningful sentence" (*Der Gedanke ist der sinnvolle Satz*) (WITTGENSTEIN 1921, 4).
[36] WITTGENSTEIN 1921, 3.3. We translate *Bedeutung* by 'reference'.

kann ich nicht)."[37] What is present in a sentence is only the configuration of its elements which *shows* the form of the described fact. The objects of the fact, on the contrary, cannot be displayed in the sentence. The only thing names and the objects they stand for have in common, is that the names occupy places in the configurations of the elements of sentences which correspond to places occupied by their objects in the configurations of elements of facts. This is the only feature which connects names and objects. As far as a sentence is a picture, it depicts only the structural and formal properties of the objects; it depicts them, so to speak, in a "colourless" way.

However, Wittgenstein not only maintains that the objects are *represented* in a colourless way; he says more, namely that the objects *are* colourless.[38] Distinctions between objects, apart from their categorial "logical forms" and their "external properties" (i.e., the "properties" of their happening to be part of the structure of certain facts), are reduced to mere discreteness: "If two objects have the same logical form, the only distinction between them, apart from their external properties, is that they are different."[39] As a consequence of what we may call Wittgenstein's structuralism he thinks that the world could be described completely by using quantified variables only[40], since formal and structural properties are shown equally well by variables.[41]

The doctrine of the "colourlessness" of objects has other important consequences. For it says that objects have no special intensional essence that can be intuitively understood. But if this is so, then there are no essential, necessary connexions between individual facts, and individual elementary sentences are then not dependent upon each other: "Each item can be the case or not the case while everything else remains the same."[42] "There is no compulsion making one thing happen because

[37] WITTGENSTEIN 1921, 3.221. Italics in Wittgenstein.
[38] WITTGENSTEIN 1921, 2.0232. This view is similar to that prevalent in modern physics, where "secondary qualities" like colours are reduced to relational structures.
[39] WITTGENSTEIN 1921, 2.0233. On the logical form of objects, cf. p. 84 below.
[40] Cf. WITTGENSTEIN 1921, 5.526.
[41] Concerning the possibilities of a structuralistic point of view see below, pp. 87 ff.
[42] WITTGENSTEIN 1921, 1.21. Wittgenstein later restricted the independence of elementary sentences to some extent: two sentences such as 'Brown is now sitting in this chair' and 'Jones is now sitting in this chair' might exclude each other in consequence of their logical form. For further details cf. WITTGENSTEIN 1929, p. 168 f.

another has happened. The only necessity that exists is *logical* necessity."[43] "Superstition is the belief in the causal nexus."[44] By logical necessity Wittgenstein means the necessity stemming from the laws of combination that hold for logical truth functions; these form the only kind of connexion between elementary sentences.[45]

Even if it is assumed that the objects corresponding to names are colourless, nevertheless, in relating the names to these objects we get a formula which does more than merely display a logical form: it also represents a content (cf. the difference between '*xRy*' and '*aBb*'). However, this "colourless" content is quite "thin" and must be distinguished from the rich intensional content which, e.g., phenomenologists claim to discover in intuitive understanding (*Wesensschau*).[46]

We can now answer the question why Wittgenstein speaks of the sense both as being shown and as being represented. The reason appears to be that Wittgenstein's concept of sense contrasts with different other concepts. In one way the sense of *sentences* is opposed to the reference of *names*. A sentence for Wittgenstein is a combination of names[47]; what is specific about the sense of a sentence, in contrast to the reference of the names, lies precisely in what is expressed by the juxtaposition of the names, i.e., in the displayed form. We can thus understand why Wittgenstein talks of the sense as being shown.

On the other hand, the reference of the names is, in Wittgenstein's view, also part of the sense of a sentence. A sentence is intended as saying something about reality, and the co-ordinations existing between names and objects are, as Wittgenstein puts it, feelers with which the picture touches reality.[48] Only in the case of logical sentences, which Wittgenstein contrasts with meaningful sentences, are these co-ordinations unimportant.[49] Logical sentences are, in his view, empty tautologies that say

[43] WITTGENSTEIN 1921, 6.37.
[44] WITTGENSTEIN 1921, 5.1361. We prefer Ogden's translation here.
[45] "A sentence is a truth-function of elementary sentences (*Elementarsätze*)" (WITTGENSTEIN 1921, 5).
[46] Cf. below, pp. 89 ff.
[47] Cf. WITTGENSTEIN 1921, 4.22.
[48] WITTGENSTEIN 1921, 2.1515.
[49] WITTGENSTEIN 1921, 6.1263.

nothing about the actually existing world.[50] From this point of view, i.e., in contrast to logical sentences, it is precisely the content given by the reference of the names that characterizes a meaningful sentence; and this content cannot be shown, but only represented.

There is an important disproportion between the sense of a sentence and the reference of a name. A name is a name only if something exists that is named by it.[51] The case of a sentence, i.e., of the statement of a connexion between elements, is different: a picture of a connexion is possible as soon as the elements with their names are given; the connexion itself need not have actual existence in reality. In a sentence a situation is, in Wittgenstein's words, constructed by way of experiment as in a *tableau vivant*.[52] Thus it is possible to have false sentences that are meaningful but do not stand for any actual fact, whereas in the case of elementary names there must always be a referent.[53]

This gives us a third contrast for Wittgenstein's concept of sense: that between the sense of a sentence and an actual fact. Wittgenstein understands the sense of a sentence, i.e., a possible state of affairs, in close analogy to the facts of the actual world: as facts fill physical space, so possible states of affairs fill logical space.[54] Wittgenstein often uses the verb 'to depict' (*abbilden*) for the relation of sentences to facts and to actual reality, and the verb 'to represent' (*darstellen*) for the relation of sentences to possible states of affairs[55], but the usage is not without exception.[56]

[50] WITTGENSTEIN 1921, 6.1, 6.11. But Wittgenstein does say that something *shows* itself in them: "The fact that the propositions of logic are tautologies *shows* the formal – logical – properties of language and the world" (6.12).

[51] This, of course, applies only if by a name we understand an elementary expression, and not a description; hence the importance of Russell's analysis of descriptions reviewed above in ch. 3, p. 42 f.

[52] WITTGENSTEIN 1921, 4.031, 4.0311.

[53] Cf. also CARNAP 1947, p. 30 f.

[54] WITTGENSTEIN 1921, 2.202.

[55] WITTGENSTEIN 1921, 2.201, and many other places.

[56] WITTGENSTEIN 1921, 4.12.

PART TWO

THE RELATION OF REPRESENTATION OF PREDICATE SIGNS AND CONTEMPORARY VIEWS ON UNIVERSALS

With the beginning of this part we reach the heart of our subject matter, and shall now describe in detail the relation of representation of predicate signs[1], so as to give the reader an insight into contemporary views on universals.

The first four chapters have been preparatory in the sense that they have introduced the reader to the relation of representation that exists between the symbols of a text and the reality described by it. They have served to emphasize the importance of linguistic analysis, and to show that the structure of logistic sentences reflects ontological features of reality. This leads to a new approach to the problem of universals. We hope that the reader will find it refreshing to disengage his attention from psychological issues, to leave aside for once the complex problems of the theory of knowledge as developed by post-cartesian philosophers and instead to pay increased attention to graphic signs and formal connexions.

Between the graphic signs contained in books and the reality which they describe, there are correlations that can be described without noting in detail the separate acts of cognition that have given rise to them[2]; just as there are relations between a map and the range of mountains represented by it, which can be described without entering into the personal experiences of the cartographer or of the viewer of the map. And when an archaeologist tries to decipher a newly discovered text, he relies on the graphic signs and not on the mental experiences of the author, who died long ago. He studies the signs of the text and the syntactical rules according to which the text is structured; his task is, guided by certain clues, to find a correlation between the structure of the text and the structure of certain states of affairs, which the text might describe.

[1] Designations of properties and relations; cf. above, p. 13 f.

[2] For a detailed phenomenological account of the psychological processes involved in the cognition of a literary work, cf. INGARDEN 1937.

So far we have avoided as far as possible the question of the relation of representation of predicate signs. It is true, we have seen [3] how Russell thinks of the sign 'r_1' in the expression 'ar_1' as representing a property, and how he wants special relation signs to stand for relations. We have not, however, examined the precise nature of these representations. This is not easy, since various interpretations are possible, depending on the attitude which is adopted concerning universals. Unfortunately, philosophers have not put forward their views on the relation of representation of predicate signs by discussing systematically the different possibilities, and often they have simply made implicit assumptions.

The reason for this lack of clear conceptions seems to be that most modern philosophers have only very inadequate views about the ontological entities (things, properties and relations) and their connexions, which have to be represented. Strongly influenced by Hume and Locke, who had described our idea of a thing in psychologistic fashion as a collection of ideas of properties, most of them are acquainted only with the superficial ontological conception that things are "bundles of qualities".

However, what are we to understand by "bundles of qualities"? As we shall see, Russell attempts to give a precise sense to the notion, by constructing it as a class. Apart from the class-member relation, there is also the formally different schema of the whole-part relation: with its help general qualities can be conceived as certain concrete wholes and individual things as parts of these wholes. In both cases – in that of the class-member interpretation as well as in that of the whole-part interpretation – it is assumed that equal predicate signs always stand as names for identically the same entity. However, as will be shown, it appears to be equally possible to correlate equal signs with numerically different entities according to their occurrence in concrete sentential contexts.

In order to be able to distinguish accurately these various interpretations of the relation of representation, it is essential to have a terminology that allows a clear distinction to be made between the *many* (concrete) *sign-inscriptions* ("tokens", e.g., the many tokens '*P*') that occupy different spatio-temporal positions, and the *one* (abstract) *sign-shape* ("type", e.g., the one type '*P*'). We shall therefore use the expressions 'sign' and 'predicate sign' as synonymous with 'sign token' and 'predicate sign

[3] Cf. above, p. 26.

token', respectively; and we shall explicitly use the designation 'type' when we wish to refer to abstract sign-*shapes*.[4]

Thus, on a closer view the relation of representation of predicate signs gives rise to a whole series of problems. Special importance attaches to two questions, viz. the "how" and "what" of representation. By "how" we mean the way in which representation functions, the kind of formal structure it has, whether equal signs always stand for one and the same entity. By "what" we mean the entities represented; we shall ask whether they are concrete or abstract, extensional or intensional. In the following chapters our task will be to gather material: in other words, to make clear the views of individual philosophers on the relation of representation of predicate signs. In the final chapter we shall be able to give a systematic answer to the question: "How and what do predicate signs represent?"

[4] So far as the use of inverted commas is concerned, we make no explicit distinction between token and type, but use single inverted commas for both kinds of material supposition. For example, we shall write both 'the sign (token) '*P*'', and 'the type '*P*'', since the addition of expressions such as 'sign (token)' and 'type' to the single inverted commas removes any possible misunderstanding. (Double inverted commas are used to characterize expressions that are quotations or are used in an unusual sense.) Similarly, no explicit distinction between token and type will be made with regard to terms like 'expression', 'sentence', 'term', 'designation', 'inverted commas', etc., as the correct interpretation will without difficulty be inferred from the context. Cf. PENTTILÄ-SAARNIO for a method of symbolically characterizing type and token, as well as material and formal supposition.

5. BERTRAND RUSSELL

The first logistic philosopher to think in terms of the semantical relation of representation was Bertrand Russell, and we shall therefore begin our survey by finding out how he interprets predicate signs. The preceding chapters have shown how he persistently analyses sentences and states of affairs into their elements. He introduced the designation 'logical atomism' to characterize the view underlying this kind of analysis. As we shall see, he considers the assumption of universals as logical atoms to be unavoidable. He regards predicate signs like individual names, as genuine names of determinate entities, so that on his view two *equal* predicate signs always designate identically the *same* entity.

Three further points must be made in connexion with Russell's interpretation of predicate signs: (a) It is not, as one would expect, the universals but rather the individual substances that give him trouble; (b) Since the theory of classes has led into difficulties, he tries to dispense with classes; (c) He regards substantival predicate expressions as improper.

5.1 Universals as logical atoms

Among Russell's most basic convictions is the (aristotelian) conception of truth as correspondence with the facts, where to the structure of true sentences there corresponds a specific structuring of the facts.[1] Immediately after the discrediting of idealistic monism he became an extreme platonist, assuming every single word of a language to stand for a special entity, which it named. Thus, for example, the fact that we have number words indicated to him that numbers could all be found "sitting in a row somewhere".[2] He even thought that "... the word 'the', for example, denotes some curious kind of object which the virtuous logician may hope to meet in the Platonic heaven".[3] He regarded

[1] RUSSELL 1959, p. 157.
[2] RUSSELL 1959, p. 62.
[3] RUSSELL 1959, p. 160. – Compare this with the platonism of MOORE 1899, p. 181: "All that exists is thus composed of concepts necessarily related to one another in

all words as genuine names, the only semantical relation being that of denoting.[4]

Soon, however, Russell's domain of denoted entities became smaller. By replacing natural language by an exact, formalized language he was able, as we have seen, to make a large number of words, and hence entities, unnecessary by explicit definitions (as in the logistic definition of number) or by definition in use (as in the theory of descriptions). But he retained his original viewpoint with regard to all undefined signs. Like Leibniz he was convinced that every state of affairs could ultimately be analyzed and described by a sentence consisting only of basic signs, each one of which denotes one determinate simple entity.[5] He called the view that accepts final, unanalyzable entities "logical atomism".[6]

As Russell himself emphasizes, his systematic development of this view was strongly influenced by his pupil, Wittgenstein.[7] But the essential features of logical atomism can already be clearly traced in Russell's analysis of relational sentences, which dates from the early days of his revolt against idealistic monism.[8] Already at that time Russell thought of individual words as corresponding to unanalyzable elements that are combined together into states of affairs. Russell took over this basic standpoint from Leibniz, defended it with his theory of descriptions, and then passed it on to Wittgenstein. Wittgenstein concerned himself consistently and in detail with the elaboration of this view, and this could

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specific manners, and likewise to the concept of existence." (Notice that Moore is talking here about facts and not about propositions.)

[4] Cf. for example RUSSELL 1959, p. 158, where he says that words *denote* things, properties and relations.

[5] RUSSELL 1959, p. 165.

[6] RUSSELL 1918/19. Russell's more recent opinion is that the question whether there are simple entities, is unnecessary: ". . . although many things can be known to be complex, nothing can be *known* to be simple, and, moreover, statements in which complexes are named can be completely accurate, in spite of the fact that the complexes are not recognized as complex. . . . It follows that the whole question whether there are simples to be reached by analysis is unnecessary" (RUSSELL 1959, pp. 165–66). For example, two colours, C and C', occurring at different places, are known to be different if and only if C is distinguishable from C', or if there is a colour C'' that is distinguishable from C but not from C', or from C' but not from C. "But since I can never know that I have surveyed *all* colours, I can never be sure that C and C' are identical" (RUSSELL 1940, p. 107).

[7] RUSSELL 1914, pp. 8–9; RUSSELL 1918/19, p. 18.

[8] Cf. above, p. 25 f.

not but influence Russell in return. Wittgenstein's logical atomism, however, differs essentially from Russell's precisely as regards properties and relations.[9]

Russell distinguishes two kinds of logical atoms: *particulars* and *universals*.[10] In the language of the *Principia Mathematica* the former are represented by small Roman letters (the designations of individuals), while Greek letters (the designations for properties) and capital Roman letters (the designations for relations) stand for the latter. The predicate signs thus represent universals.[11] Since, in Russell's view, it is impossible to give a complete description of reality without including predicate signs, he believes that universals are indispensible.

As we have seen in Ch. 1, Russell attaches special importance to relational signs. On the other hand, he believes that designations of properties are not altogether necessary. For example, the expression 'is red' could be replaced by the phrase 'is similar in colour to the standard thing *a*', where *a* is a red individual.[12] However, the relation-word 'similar' is indispensible and something must therefore correspond to it in the real world.

In Russell's view identically the same entity must correspond to this word in all instances, i.e., the entity must be a platonistic universal. His argument is that otherwise an infinite regress would result.[13] Suppose, for example, that it is true of the individuals *a*, *b* and *c*, that *a* and *b* are similar to one another, that *b* and *c* are similar to one another, and that *a* and *c* are similar to one another. If the word 'similar' does not stand for identically the same entity in all three instances, but for three numerically different entities, then there must be another reason why the same word 'similar' is used in all three instances. It must be because these three numerically different relational entities designated by the word 'similar' are somehow similar to one another. In describing how these relational entities are similar to one another we again use the word 'similar' several times. (Or perhaps in place of 'similar' we might use 'similar$_2$', since we are talking now of a similarity of a higher order.) And

[9] Cf. below, pp. 81 ff.

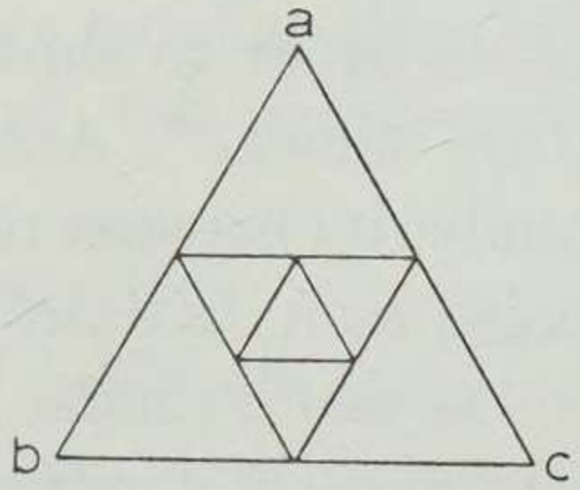
[10] WHITEHEAD-RUSSELL vol. 1, p. XIX.

[11] This was also the opinion of G. E. Moore, cf. MOORE-STOUT-HICKS, p. 95 f.

[12] Cf. RUSSELL 1918/19, p. 18.

[13] RUSSELL 1940, p. 346; RUSSELL 1959, p. 172.

again in all these new instances either the word 'similar' (i.e., 'similar$_2$') stands for identically the same entity, namely for a platonistic universal, or else it stands for numerically different entities, i.e., for entities which in turn have to be similar (similar$_3$) to one another; and so on *ad infinitum*. Our illustration may help to visualize the situation.



Russell does not think that an infinite regress of this kind can justifiably be accepted, and therefore regards the predicate signs 'similar' as denoting an abstract entity, a platonistic universal. This bears out our earlier assertion that in a two-levelled semantics the domain of represented reality can include abstract entities such as occur in the domain of sense in a three-levelled semantics.[14]

The rejection of the infinite regress of similarities, used as an argument for the existence of platonistic universals, can already be found in Husserl's *Logische Untersuchungen*.[15] Husserl even quotes John Stuart Mill, who had argued along similar lines[16]: "What, then, is the common something which gives a meaning to the general name? Mr Spencer can only say, it is the similarity of the feelings: and I rejoin, the attribute is precisely that similarity. . . . Every general name, whether abstract or concrete, denotes or connotes one or more of those resemblances. It will not, probably, be denied, that if a hundred sensations are undistinguishably alike, their resemblance ought to be spoken of as one resemblance, and not a hundred resemblances which merely *resemble* one another. The things compared are many, but the something common to all of them must be conceived as one, just as the name is conceived as one, though corresponding to numerically different sensations of sound each time it is pronounced." But in spite of this argument, Mill does not want to accept

[14] Cf. above, p. 12.
[15] HUSSERL 1900–1901 (1913), vol. 2, part 1, pp. 196–197.
[16] HUSSERL 1900–1901 (1913), vol. 2, part 1, pp. 116–119; MILL 1843 (1961), p. 118, footnote. Cf. also below, p. 166, footnote 11.

universals as existing entities and warns that for him an attribute is merely "a particular mode of naming our sensations".[17] Husserl rightly criticizes this as an unconvincing evasion.

Russell's rejection of the infinite regress of similarities also resembles an argument by Aristotle, the so-called "*τρίτος ἄνθρωπος*" ("the third man").[18] But while Russell tries to reduce *ad absurdum* the notion of the similarity of similarities, in order to show the need for assuming an abstract platonistic entity "similar", Aristotle tries to reduce *ad absurdum* the notion of the similarity between the platonic idea of "man" and the concrete men subsumed by it, in order to reject platonic ideas!

Finally it is worth mentioning that a similar infinite regress is possible with respect to yet another ontologically important relation, namely the relation of a property (or a relation) to an individual.[19] It makes no difference whether it is the relation of inherence of a concrete property or whether it is the relation of participation of an abstract, platonistic property. Thus, for example, if we have a property P of an individual a, then there seems to exist a relation R (of inherence or participation) of P to a; but R itself seems also to have a relation R' (of inherence or participation) to a; if this is so, then R' should also have a further relation R'' to a; and so on *ad infinitum*.

$$\underbrace{P \qquad\qquad a}_{R}\quad \underbrace{\qquad}_{R'}\quad \underbrace{\qquad}_{R''}\quad \underbrace{\qquad}_{\vdots}$$

As in the regress of similarities rejected by Russell, the members of this regressive series are relations of higher and higher levels, i.e., relations in an analogical sense.[20]

We shall see later that as far as symbolization is concerned, the infinite regress can be "stopped", both here and in Russell's case of similarities, by picturing a relation instead of designating it.[21]

[17] MILL 1843 (1961), p. 117, footnote.
[18] Cf. below, p. 104.
[19] This regress is described and rejected in BRADLEY 1893, p. 27. See also below, p. 168.
[20] They belong to different types; cf. below, p. 77.
[21] Cf. below, p. 168.

5.2 Qualia as individuals

Russell regards things as "bundles of qualities"[22] in the empiricist tradition. Thus, in representing qualities by predicate signs such as 'φ', 'ψ' etc., he leaves names of individuals with the sole function of standing for "a mere unknowable substratum"[23], that is, an x. Russell finds a hidden entity like this more puzzling than properties; and, like Hume[24], he regards this "substance", this unknowable support of properties, as unnecessary.

Russell's picturesque description of substance typifies his – and incidentally most empiricists' – odd conception of it. It is, he says, something like "an invisible peg from which properties would hang like hams from the beams of a farmhouse".[25] This description is mistaken, since a substance is *not* hidden *behind* its accidents, i.e., the properties, but shows itself *in* them: accidents are *forms* of the substance. The circumstance that Russell is unfamiliar with the model of the matter-form relationship and therefore ignores individualization by inherence, explains why he so light-heartedly identifies properties with platonistic entities.

From a thomistic standpoint, Russell's conception of the ontological structure of objects is inadequate for the further reason that apart from the *accidental forms* of a substance, there is also the *substantial form* of a thing. If both kinds of form are represented by predicate signs such as 'P', 'Q' etc., two cases can be distinguished: either the predicate signs, for example 'P', stand for an accidental form, in which case the 'x' in 'Px' corresponds to the substance that shows itself in the form P; or the predicate signs stand for a substantial form, and then 'x' corresponds to the *materia prima*.

After some hesitation, Russell proposes an unusual way of disposing of the "unknowable" substance: not things but phenomenal qualia such as experienced redness, hardness, pleasantness are to be regarded as individuals of his constructional system.[26] Significantly, indications of position in the visual field (e.g., 'centrality', 'rightness', etc.) are also treated as names of phenomenal qualia, i.e., as designations of individuals of the

[22] Russell 1959, p. 161, e.g.
[23] Russell 1959, p. 161.
[24] Hume 1739, book 1, part IV, section III.
[25] Russell 1959, p. 161.
[26] Russell 1959, pp. 161–172.

system. Thus, for example, 'This is red', where 'this' points to the central position, can be replaced by 'Redness is compresent with centrality'. Substances, i.e., things, are no longer represented by basic signs of the system but by class expressions, where the classes correspond to "bundles" of co-occurring qualia.

In this system it is easy to become confused over questions concerning universals. The phenomenal *qualia* considered by Russell are what *ordinarily* would be called universals, e.g., identically the same *quale* red can occur in my experience of this ball and that book. But *in the system* these *qualia* appear as individuals and not as universals. For instance, the fact that this ball and that book look red is expressed in the system by saying that the same *quale* red (the same individual) is a member of two different classes, and not by saying that two different individuals, the ball and the book, fall under the same *quale* red (the same universal). There is no individual in the system which could fall under the *quale* red, i.e., in the system the *quale* red is no universal.

But the system does, of course, contain other universals. The *qualia* themselves are subsumed under genera (e.g., the *quale* red is a colour *quale*), and are inter-related (e.g., the *quale* red and the *quale* orange are similar). Such genera (e.g., colour) and relations (e.g., similar) are universals even in this system, since individuals of the system (the *qualia*) fall under them. Furthermore, as we shall see in more detail[27], classes are platonistic universals. Therefore things (like, e.g., this ball or that book), which *ordinarily* are considered concrete individuals, but which in the system appear as certain classes of *qualia*, are *in the system universals*.

5.3 Antinomies in the theory of classes

Russell at first favoured classes. As we have seen in Ch. 2, he was greatly impressed by the method of constructing entities as classes, instead of inferring them. But the faith he had placed in classes was later so badly shaken, that he tried to dispense with them altogether and to refer only to properties.

In June 1901 Russell[28] made a surprising discovery in the course of his logicist studies: he found that a paradox could be derived from what ap-

[27] Cf. below, pp. 136 ff.
[28] RUSSELL 1903, p. 13.

peared to be valid premises in the logic of classes. This was the antinomy of the class of all classes which are not members of themselves: "Let *w* be the class of all those classes which are not members of themselves. Then whatever class *x* may be, '*x* is a *w*' is equivalent to '*x* is not an *x*'. Hence giving to '*x*' the value of *w*, '*w* is a *w*' is equivalent to '*w* is not a *w*'." [29]

The astonishing thing was the simplicity of the argument and the fact that the concepts involved had seemed to be intuitively clear. The notion of a class being a member (or not a member) of itself did not seem dubious. Was is not similar to the notion of a catalogue of books listing (or not listing) its own title along with other books? If one could talk about the class of all pairs (Frege's number two[30]), could one then not also talk about the class of all classes with more than two members; and was it not obvious that the class of all pairs was not itself a pair (i.e., was not a member of itself), whereas the class of all classes with more than two members certainly had more than two members (i.e., was a member of itself)?

The discovery of this antinomy, therefore, was a serious blow to the belief in the reliability of intuition, and this in a field where traditionally the most convincing examples of intuitively evident necessary truths are taken from. Further, in making an unrestricted assumption of classes untenable, it challenged platonism, the belief in abstract platonistic entities. Indeed, the effects of the discovery of this antinomy on contemporary philosophy have been extremely far-reaching.

As an immediate consequence one had to be more cautious about assuming classes. Russell at first contemplated the possibility of two types of restrictive criteria[31]: (a) a class must not be too large; (b) the propositional function corresponding to a class must have a certain simplicity (the

[29] WHITEHEAD-RUSSELL, vol. 1, p. 60. We have replaced Russell's double quotation-marks by single ones and have added quotation-marks to the '*x*' in the phrase 'giving to '*x*''. The antinomy of the class of all classes which are not members of themselves was not the first and is not the only antinomy discovered in modern logic and in studies on the foundations of mathematics. But because of its simplicity it was the one which received the most general attention. For a detailed description and bibliography of the various antinomies, with which we cannot deal further here, cf. BETH 1959, pp. 481–518. See also VON KUTSCHERA 1964.

[30] Cf. above, p. 31.

[31] RUSSELL 1906.

intensional, so-called zig-zag theory). The first criterion was adopted by Zermelo among others, the second by Quine. Russell himself, however, eventually adopted a solution that was typical of him, viz. the "no-class theory", which attempts to make all designations of classes unnecessary.[32]

Russell's solution is a further example of his way of dealing with questions concerning problematic entities: instead of answering these questions, he shows how to avoid talking about the entities in question. His definition in use for class expressions is based on the fact that between classes and properties there is a one-many correlation. Every property determines a class whose members consist of those and only those entities that have the property in question; but the same class may be determined by different properties. Two properties P and Q that determine the same class are called equivalent: it then holds for every entity that it has the property P if and only if it also has the property Q. Every sentence about a class can therefore be replaced by a sentence about a property, so long as it is borne in mind that nothing is asserted about the property that could not equally be asserted about any other equivalent property whatsoever. Russell therefore gave the following definition in use of classes[33]:

$$F\{\hat{z}(Qz)\} = df(\exists P)\big\{(x)(P!x \equiv Qz) \cdot F\{P!\hat{z}\}\big\}.$$

A rendering of this formula closer to natural language would be: "A sentence of type F about the class of z's determined by a property Q, is replaced by a sentence stating that there is a property P, which is equivalent to Q and about which a sentence of type F is asserted."

The sign '!' in '$P!x$' indicates that P must be a predicative property.[34] In their endeavour to avoid the antinomies, Poincaré and Russell had formulated in 1905–1906 the so-called "vicious circle principle" according

[32] Cf. GÖDEL 1944, p. 131 f. A further original answer to the antinomies has been worked out by LEŚNIEWSKI, see below, p. 109 f. For a detailed discussion of the various systems of class theory and a comprehensive bibliography cf. FRAENKEL-BAR-HILLEL 1958. WANG-MCNAUGHTON gives a particularly concise and clear survey with bibliography of the various systems. QUINE 1963 pays special attention to the ontological commitment of different systems.

[33] WHITEHEAD-RUSSELL *20.01. According to CARNAP 1947, p. 146 f., it is preferable to use a general quantifier in the definition rather than an existential one, so that we get the definiens: $(P)\{(x)(P!x = Qx) \supset F\{P!\hat{z}\}\}$.

[34] This and the following six paragraphs sum up more technical matters and are not essential to the understanding of the remainder of the book.

to which only predicative properties may be defined. A predicative property is a property whose definition is predicative, i.e., does not refer *via* a quantifier to a totality, to which the property to be defined itself already belongs.

The following is an example of an impredicative definition[35]:

$$Hx = df\ (Q)\{Q7 \cdot (y)(z)((Qy \cdot z = y+1) \supset Qz) \supset Qx\}$$

i.e. 'Hx' states by definition: "For every property Q, x has the property Q, if it holds that if 7 has the property Q then every successor of 7 has the property Q" (or in short: "x possesses all hereditary properties of 7").

The definition is impredicative because it talks about all properties, i.e., also about H:

$$H7 \cdot (y)(z)((Hy \cdot z = y+1) \supset Hz) \supset Hx$$

is an instance of the universal formula which is the definiens.

For a precise formulation of the prohibition of impredicative definitions, a certain number indicating their *order* has to be assigned to all predicative expressions in such a way that in a well-formed predicative definition the predicate sign in the definiendum will always be of a higher order than any predicate sign occurring in the definiens. The expressions then form a constructive hierarchy, the so-called *ramified theory of types*.

The existence of an equivalent predicative property P for every property Q cannot be proved by constructive means, but must be assumed axiomatically. This is the debated axiom of reducibility[36]:

$$(Q)(\exists P)(x)(P!x \equiv Qx)$$

According to Russell, if the existence of classes were assumed, then the axiom of reducibility could be deduced, since the membership of x in a class α ($x \in \alpha$) could be interpreted as a predicative property of x (as $P!x$). On the other hand, the existence of classes cannot be inferred from

[35] Cf. CARNAP 1934, § 44, where it is also proved that 'H5' is false and hence meaningful; see also VON KUTSCHERA 1964, pp. 101–107.

[36] Cf. WHITEHEAD-RUSSELL *12.1. CHWISTEK 1921 was the first to find the simultaneous assumption of the axiom and of the distinction of the predicative ramifications into orders superfluous and proposed, in CHWISTEK 1924–1925, instead of the ramified theory of types a simple theory of types (cf. below, p. 77). A similar view was stated by RAMSEY 1926.

the axiom of reducibility. Russell affirmed, therefore, that the assumption of the axiom of reducibility was a smaller assumption than the assumption that there are classes.[37]

In the second edition of *Principia Mathematica* Russell tried to omit the axiom of reducibility. This seemed possible since, following a suggestion of Wittgenstein's, he used only truth functors in the definition of higher order predicates. Truth functors are extensional, and with them equivalent properties can be regarded as identical, so that there is no practical difference between properties and classes.[38]

With his "no-class" theory Russell manages to avoid a reference to classes. He does not wish to deny the existence of classes, but merely to show that their assumption is unnecessary.[39] Russell's abolition of classes, which (as we shall see in more detail later[40]) are abstract platonistic entities, had been welcomed by nominalists.[41]

But does the fact that Russell refers only to properties really further the nominalistic cause? As we have shown above, on Russell's view the entities represented by predicate signs are identically the same for different individuals[42], so that his properties, too, must be abstract entities. But for a nominalist, abstract platonistic properties seem even less desirable than classes. For as we have already mentioned, there are more properties than classes; the assumption of abstract properties leaves us with a larger number of abstract entities. And there seems to be no justification for regarding abstract properties and classes as two disjunct kinds of entities. Abstract properties, it seems, are also classes but have, in addition, an intensional content.[43]

That abstract properties are inseparable from classes is shown by the fact that antinomies analogous to those of classes can arise in the case of abstract properties. In order to avoid them, Russell had to devise a special theory in which he distinguishes different "logical" types.

[37] WHITEHEAD-RUSSELL, vol. 1, p. 58, p. 166.
[38] Cf. WHITEHEAD-RUSSELL, vol. 1, p. XIV, p. XXXIX.
[39] WHITEHEAD-RUSSELL, vol. 1, p. 72.
[40] See below, pp. 136 ff.
[41] HAHN 1929; cf., on the other hand, QUINE 1941.
[42] This is also shown clearly by the fact that Russell quantifies predicate signs, as, e.g., in his definition in use of classes. Cf. Quine's criterion, discussed below, pp. 127 ff.
[43] For further details cf. below, p. 138 f.

5.4 The hierarchy of types

Although Russell is willing to regard properties as simple abstract entities that can form a domain of quantification[44], he is careful to stress the difference between universals and individuals. The antinomies had arisen precisely because classes and individuals had been put indiscriminately on the same level. Russell therefore distinguishes between different types that must not be confused.

He starts from the consideration of expressions of so-called propositional functions, such as, e.g., "x is beating y". Frege had called both the expressions and the functions expressed "unsaturated" (*ungesättigt*).[45] Russell says that the expressions are incomplete symbols that signify or represent nothing by themselves, since they contain ambiguities – indicated in the above example by the variables 'x' and 'y'. He says that it is only when the ambiguity of a functional sign has been removed – e.g., in the above case by substituting the designations 'Peter' and 'Paul' for 'x' and 'y' – that the resulting sentence becomes meaningful as a whole. Function and argument must fit together in such a way that the ambiguity is completely removed.[46] For example, 'is beating is beating' is not the expression of a proposition, since beating belongs to a type that requires individuals as arguments and not functions, which again contain ambiguities.

The various types (in traditional terminology they would be called categories), form a hierarchy: there are individuals, one-place functions of individuals (that correspond to properties), two-place functions of individuals (that correspond to two-term relations), ... functions of functions of individuals, ... and so on.

This interpretation in terms of functions shows that properties and relations are not completely autonomous entities but occur only in connexion with individuals. "... if we try to descry some entity denoted by these relation-words as capable of some shadowy kind of subsistence outside the complex in which it is embodied, it is not all at clear that we can succeed."[47] Russell holds that predicate signs may not ultimately

[44] Cf. p. 131 below.
[45] FREGE 1892a, p. 205; FREGE 1904, p. 663 f.
[46] WHITEHEAD-RUSSELL, vol. 1, pp. 47–48; RUSSELL 1924, p. 376; RUSSELL 1959, pp. 68–69.
[47] RUSSELL 1959, p. 173.

stand in subject-place in a sentence; this place may only be occupied by the names of individuals, and indeed this peculiarity defines the type of individuals.[48] He regards the designation of a function by a noun as improper: "There is not an entity 'Plato's love for Socrates' but only the fact that Plato loves Socrates."[49] All sentences in which the designation of a property or a relation appears to be the subject, are meaningful only if they can be re-formulated. "... relation-words ought only to be employed as actually relating and ... sentences in which such words appear as subjects are only significant when they can be translated into sentences in which the relation-words perform their proper function of denoting a relation between terms."[50]

Russell so emphasizes the dependent nature of the entities corresponding to predicate signs that sometimes he comes close to denying the existence of universals as separate "logical" atoms: "... the matrix $\varphi!x$ has the peculiarity that, when a value is assigned to x, this value is a constituent of the result, but when a value is assigned to φ, this value is absorbed in the resulting proposition, and completely disappears." But the quotation continues by affirming the existence of a similarity: "We may define a function $\varphi!\hat{x}$ as that kind of similarity between propositions which exists when one results from the other by the substitution of one individual for another."[51]

Russell is not very clear in what he says about functions, and sometimes confuses signs and designated entities. "Propositional functions", as their name suggests, appear to be similar in kind to propositions, which are abstract entities. Russell says that when the ambiguity of propositional functions is removed, they become propositions. Sometimes, however, he states explicitly that functions are signs: "Thus the proper symbol for "yellow" (assuming for the sake of illustration that this is an attribute) is not the single word "yellow", but the propositional function

[48] WHITEHEAD-RUSSELL, vol. 1, p. XIX.
[49] RUSSELL 1924, p. 375.
[50] RUSSELL 1959, p. 173. "Some propositions containing the word 'similarity' can be replaced by equivalent propositions containing the word 'similar', while others cannot. These latter need not be admitted" (RUSSELL 1940, p. 347)('propositions' is here mistaken: Russell should have written 'sentences').
[51] WHITEHEAD-RUSSELL, vol. 1, p. XXX. Compare Russell's formulation with the interpretation according to which like predicate signs may stand for many similar concrete properties. See below, p. 175 f.

"x is yellow", where the structure of the symbol shows the position which the word "yellow" must have if it is to be significant."[52] The following restatement (!) of the doctrine is typical of the existing confusion: "A propositional function is an expression containing a variable and becoming a proposition as soon as a value is assigned to the variable. For example, 'x is a man' is a propositional function. If, in place of x, we put Socrates or Plato or anybody else, we get a proposition. ... A propositional function ... can form part of a sentence. ... The phrase 'propositional function' sounds perhaps unnecessarily formidable. For many purposes one can substitute the word 'property'."[53]

Russell has admitted explicitly that in his theory of types he was defining categories of entities, not syntactical categories of signs: "My definition was wrong, because I distinguished different types of *entities*, not of *symbols*."[54] He calls this a mistake, because the theory of types must also apply to "type" itself and his definition of the entity "type" violated the theory of types: if different types are entities of different type, then different types of "type" must be distinguished, which Russell failed to do. If the theory of types is constructed as dealing with signs, then there is no such difficulty, i.e., the word 'type' does not violate the theory of types. This is so because all different types are then syntactical properties of expressions, i.e., all types belong to the same type.

A consistently formulated doctrine of syntactical categories was first worked out by Leśniewski and the Polish logistic school.[55]

[52] RUSSELL 1924, p. 376. The double quotation-marks are Russell's.
[53] RUSSELL 1959, pp. 68–70.
[54] RUSSELL 1944, p. 691.
[55] See footnote 33, pp. 111–112 below. Cf. how GÖDEL 1944, p. 126, deplores that the *Principia* are "so greatly lacking in formal precision".

6. LUDWIG WITTGENSTEIN

As mentioned in the preceding chapter, Russell has been influenced in his logical atomism by Wittgenstein. There seems, however, to be an important difference between them: whereas Russell includes relations among logical atoms and thus assumes platonistic entities, Wittgenstein apparently refers only to particulars. But neither Wittgenstein nor Russell drew attention to this difference. Our first aim in this chapter will therefore be to establish our interpretation in detail. It will give rise to further questions, viz.: "Why does Wittgenstein nevertheless write '*aRb*' and not simply '*ab*'?", and "If there are only names of individuals, how can there be general statements?" [1]

6.1 The ideal language without predicate signs

For Wittgenstein, as for Russell, the relation of representation is of primary importance. But whereas Russell initially had correlated *every* undefined sign of his language with a represented entity, Wittgenstein suggested that only elementary sentences had a representative function. On the latter's view the connectives 'and', 'or', etc., do not stand for anything in reality, since if we know all elementary sentences, then molecular sentences formed with the aid of truth functors supply us with no new information about reality. They merely develop further the subject-matter already contained in the elementary sentences. The philosopher's task is to analyze these latter.[2] Russell accepted this view gladly. By thus making the study of molecular sentences unnecessary, Wittgenstein considerably simplified the programme of logical atomism.

So far as atomic or elementary sentences are concerned, Wittgenstein holds that the configuration of simple signs in a sentence corresponds to

[1] The reader is reminded that our concern is solely with Wittgenstein's *Tractatus*. His later writings are not relevant to our subject matter, viz. the problem of universals in logistic philosophy.

[2] WITTGENSTEIN 1929, pp. 162–63, and elsewhere.

the configuration of elements in a state of affairs. Implicit in this is another distinctive feature of Wittgenstein's conception, but in contrast to the above-mentioned view regarding logical connectives, Russell did not take it over; he did not even notice it. (There is no mention of it in Russell's introduction to the *Tractatus*.) The difference is the following: whereas on Russell's view there are both individuals and universals (especially relations) among the elements (or "logical" atoms) of states of affairs, Wittgenstein considers only the configuration of *objects*. "A state of affairs (a state of things) is a combination of objects (things)." "In a state of affairs objects fit into one another like the links of a chain."[3] And, correspondingly, for Wittgenstein elementary sentences consist exclusively of names of individuals: "An elementary sentence consists of names. It is a nexus, a concatenation of names."[4]

It must not be thought that Wittgenstein regards predicate signs, too, as names of objects; he uses consistently only individual constants and individual variables to designate objects.[5] Concerning the material properties of things he says that they are represented only by the sentences, that they are formed only by the configuration of objects.[6]

It is characteristic of Wittgenstein's view that, when discussing the formula '*aRb*', he makes no special reference to the predicate sign '*R*'. He merely says: "*That* '*a*' stands to '*b*' in a certain relation says *that aRb*."[7] He does not wish to denote the relation, but only to picture it by

[3] WITTGENSTEIN 1921, 2.01, 2.03; cf. 3.21.
[4] WITTGENSTEIN 1921, 4.22.
[5] Cf. in particular WITTGENSTEIN 1921, 4.24.
[6] WITTGENSTEIN 1921, 2.0231.
[7] WITTGENSTEIN 1921, 3.1432. According to our interpretation Wittgenstein's ideal language contains no designations of relations. But this view is not universally accepted, as is shown by the controversy to which this passage from the *Tractatus* has given rise. DAITZ 1953, taking a russellian line, posits *four* elements in the expression: '*a*', '*b*', '*R*', and the relation of ordered sequence of the signs; and analyzes the represented state of affairs into *three* entities: the relation and its two terms. STENIUS 1960, p. 130 ff., gives the same number of elements (but he distinguishes two "keys of interpretation", the second of which does not regard '*R*' as a name). EVANS 1955 and EVANS 1959 goes even further and claims that Wittgenstein posits *four* elements not only in the expression but also in the state of affairs, viz. the relation, its two terms and the structure connecting these three entities. However, COPI 1958, p. 159, interprets this passage from the *Tractatus* in our sense, and says quite clearly: "There are no relation words in the "logical notation" recommended by Wittgenstein." ANSCOMBE 1959, pp. 98–102; FAVRHOLDT 1964, p. 76, p. 201; GRIFFIN 1964, p. 52 f; SIMPSON 1964, p. 85; hold a similar view. BLACK 1964, p. 106, rejects Copi's interpretation.

the way in which the names are ordered: "Situations can be described but not *given names*." The model that he has in mind is that of a diagram of arrows: "Names are like points; sentences like arrows – they have sense."[8] The arrow is clearly a simile of the ordered sequence of the names in the sentence, since Wittgenstein also says that the spatial position of the names in relation to each other expresses the sense of a sentence.[9]

Why, then, do *formulae* containing predicate signs (e.g., '*aRb*', '$f(x)$', '$g(x, y)$') nevertheless occur in Wittgenstein's *Tractatus*?[10] The most obvious reason that comes to mind is that such formulae are in general practical use. Wittgenstein's view – as we interpret it – that sentences are combinations of individual names only, does not hold for the sentences that are used in actual fact. Wittgenstein is aware that the language he outlines is an unrealized ideal. Referring to its elementary sentences, i.e., to its simplest sentences which assert the existence of a state of affairs[11], and of which all other sentences are compounded, he says: "Elementary sentences consist of names. Since, however, we are unable to give the number of names with different reference, we are also unable to give the composition of elementary sentences."[12] Here he explicitly refers to an ideal language that cannot be specified, and states that its sentences consist of names.

However, Wittgenstein did not state *explicitly* that the non-occurrence of predicate signs in his ideal language distinguishes it from languages actually used in science and in everyday life. And he never gave a systematic answer to the question how the predicate signs of other languages can be explained in terms of his ideal language.

6.2 The interpretation of predicate signs of non-ideal languages

A first answer to the question of how Wittgenstein might have explained the predicate signs of non-ideal languages, can be found in his statement that ordinary language disguises logical structure. In this connexion he compares representation in ordinary language to a geometrical projection

[8] WITTGENSTEIN 1921, 3.144.
[9] WITTGENSTEIN 1921, 3.1431. Cf. also our discussion of sense above, pp. 55–59.
[10] WITTGENSTEIN 1921, 3.1432, 4.24, and elsewhere.
[11] WITTGENSTEIN 1921, 4.21.
[12] WITTGENSTEIN 1921, 5.55. We translate *Bedeutung* by 'reference'.

that is normalized in such a way, that rectangles and ellipses are translated into squares and circles. The subject-predicate form and relational forms are the norms of our particular language, and into these "we project in ever so many different ways ever so many different logical forms". Actual analysis reveals logical forms that bear little resemblance to the forms of ordinary language. Wittgenstein gives the following examples of sentences of ordinary language, where the most heterogeneous forms are forced into the same schema: 'This paper is boring', 'The weather is fine', 'I am lazy'.[13]

However, this explanation will not do, since the subject-predicate schema and predicate signs occur not only in ordinary language, but also in logistic languages. Wittgenstein's above-mentioned remark, that the totality of names making up an elementary sentence cannot be given, indicates a different answer: the structure of reality is too complex to be precisely analyzed in all its details. The unattainable ideal of an adequate description would consist in a set of formulas where every individual atom would be given a proper name, preferably a number.[14] (The word 'atom' may here be understood both in the sense of a logical atom and in that of an ultimate particle in physics.[15]) But in the sentences that we actually use, whole complexes are left unanalyzed and the structures of the states of affairs are indicated in crude outline only.[16] The predicate expressions of actual languages may thus be used to mark the distinction between different structures of the same over-all outline.

In this sense the physicist's language comes closer to Wittgenstein's ideal than ordinary language, since it analyzes further many types of structure ordinarily indicated by adjectives, and reduces them to configurations of atoms. Wittgenstein considers the word 'red', for example, to be an unanalyzed expression[17], and refers to the "logical structure of colour".[18]

[13] WITTGENSTEIN 1929, p. 163 f.

[14] Cf. WITTGENSTEIN 1929, p. 165 f., where Wittgenstein states that the adequate rendering of the logical form of states of affairs requires the use of a language of co-ordinates.

[15] Cf. how Wittgenstein attaches great importance to natural science: "The totality of true sentences (*Sätze*) is the whole of natural science" (WITTGENSTEIN 1921, 4.11).

[16] Cf. WITTGENSTEIN 1921, 3.24.

[17] WITTGENSTEIN 1929, p. 166.

[18] WITTGENSTEIN 1921, 6.3751.

This helps also to understand what Wittgenstein might have in mind, when he implies that two objects can have different logical forms.[19] The ultimate logical atoms, which must be points with no further internal structure, must all have the same logical form; the only distinction possible is that they are numerically different and that they have different external properties, i.e., that they enter *de facto* into different configurations. The objects, however, which are spoken of in actual languages, are entire unanalyzed complexes, which can very well differ in their internal structure and which therefore can have different logical forms.

The usefulness of an over-all knowledge, even for science, seems to be hinted at when Wittgenstein refers to probability as a kind of generalization, where facts are not known in their entirety but *something* is known about their form.[20]

6.3 Some consequences of Wittgenstein's conception

The non-occurrence of predicate signs has several consequences. A positive result, from the point of view of the problem of universals, is that real *relations* are pictured in the sentences by syntactical *relations*, viz., by the juxtaposition of names. In this way the problem of naming the relations as if they were things does not arise and there is no need to hypostatize them.

For Russell the designation of relations by relational words was unavoidable, although he was aware of the problem and found it regrettable. "A great deal of the confusion about relations comes from the fact that relations are indicated, not by relations, but by words, which are as substantial as other words."[21]

Wittgenstein's approach, however, gives rise to some doubts. How can different relations be distinguished without the use of predicate signs? I. M. Copi points to the example of mathematics, where different spatial relations between two signs 'x' and 'y' – such as in 'xy' and 'x^y' – are used to show different mathematical relations, such as that of product and that of power. "Since", he argues, "a relation can be represented

[19] WITTGENSTEIN 1921, 2.0233. Cf. above, p. 54.
[20] WITTGENSTEIN 1921, 5.156.
[21] RUSSELL 1927, p. 275; cf. also p. 264.

by a relation other than itself, indefinitely many relations among objects can be represented by the indefinitely many spatial relations among the names for these objects without any ambiguity in the symbolization."[22] But in view of Wittgenstein's structuralism, according to which facts are merely configurations of "colourless" things, it seems probable that he had only one kind of relation in mind: some structural relation in "logical space". In this case every configuration can be represented by an *n*-tuple of individual names.

A further problem is the following: how can general sentences be formulated in a language consisting only of individual names? A language that pictures precisely every individual state of affairs, would not be very useful. We need general laws that allow a systematical survey of the many individual facts.

In our discussion of logical form[23], we have already mentioned a way in which Wittgenstein generalizes the individual sentences of his language: he replaces constants by variables, so that the correlation between names and objects is no longer determinate but indeterminate. In this way *one* formula, *one* schema, can be used to refer to different but formally alike states of affairs.[24] Thus, for example, Wittgenstein says that mechanics is concerned "never with *specific* material points but always only with *any* points *whatsoever* (*irgendwelchen*)".[25]

[22] COPI 1958, p. 158.
[23] Cf. Ch. 4 above, p. 54.
[24] Cf. WITTGENSTEIN 1921, 3.315.
[25] WITTGENSTEIN 1921, 6.3432.

7. RUDOLF CARNAP

It has always been Carnap's view that predicate signs represent something; and without hesitation he correlates equal signs with one and the same entity. This, of course, is not a new approach, but falls in line with Russell's view. However, something else can be learnt from Carnap: namely that the reality represented by predicate signs must have an intensional content. Although Carnap prefers to restrict himself to extensions, he is nevertheless forced to take intensions into account. The way in which his views have developed over the years, provides us with an excellent example of how precise formulation and clear thinking can bring about the abandonment of restrictively narrow schemata and lead to real progress.[1]

The question of intensions arises in two cases: in connexion with "well founded" relations and in the case of synonymity. Furthermore, although Carnap refers to the representation of intensional entities, he does not believe that the assertion of the existence of such entities is an assertion about the nature of reality; this is part of his peculiar pragmatic conventionalism and of his dislike of metaphysics, which we shall discuss in the final section of this chapter.

7.1 "Well-founded" relations

The most important work of Carnap's early period – *Der logische Aufbau der Welt* (The Logical Structure of the World)[2] – represents an important experiment: here for the first time a "logical" map of the world, i.e., a completely general constructional system[3], was not only postulated but actually worked out, at least approximately. Inspired by the plans

[1] A similar development can be discerned, e.g., in Carnap's treatment of the question of verification: from the narrow requirement of verifiability, *via* the admission of so-called "reduction sentences" (CARNAP 1936/37) to his present standpoint (CARNAP 1956), his views have progressively become subtler and more adequate.
[2] CARNAP 1928; cf. GOODMAN 1951, for an excellent critical commentary and review.
[3] Cf. above, p. 34.

sketched by Russell, Carnap tried to restrict himself to a single undefined predicate sign, this being a sign for the relation of recognition-of-similarity (*Ähnlichkeitserinnerung*). The relation of recognition-of-similarity holds between two experiences (*Erlebnisse*) of the knowing subject, i.e., between two individuals of the system, if and only if the first of them is remembered as in part similar to the second.

As can be seen, the system is solipsistic. But this solipsism is freely chosen. Carnap calls it merely a "methodological solipsism" and says that another basis, e.g., with electrons or world-points as individuals, would have been equally possible.[4]

However, the epistemological reasons that motivated Carnap's choice do not concern us here. All that needs to be noted is that the system of the *Aufbau* is based on a domain of individuals ordered by a single primitive relation.

Carnap assumes that this primitive relation is given through a list of its instances, i.e., a list of all pairs of experiences between which a relation of recognition-of-similarity holds. This list reminds one of Wittgenstein's ideal language, since only names of individuals need occur in it. And, like Wittgenstein, Carnap thinks in terms of a geometric interpretation of this list in an arrow diagram, a map of the network of relations.[5] Every individual (i.e., every experience) would be represented on this map by a point, and those points that correspond to individuals linked by the primitive relation (that of recognition-of-similarity) would be connected by an arrow. In order to make his meaning clearer, Carnap compares this map to the familiar case of the map of a railway network: here, too, every station (every individual) is represented by a point, and every rail connection between two stations (every relational state of affairs) is indicated by a straight line connecting the two points. If we disregard the written text, then the individuals on such a map are characterized solely by their relative position within the network as a whole, and the primitive relation is pictured only by the structural form of the network.

[4] We would then have had a "methodological materialism". Cf. CARNAP 1928, §§ 59–64.

[5] CARNAP 1928, § 14. It is interesting to note that exactly the same idea of a network of relations and connected points underlies e.g. L. Hjelmslev's "glossematics". He is likewise of the opinion that reality can be scientifically described only in terms of a structure of purely formal relations; and he explicitly states his agreement with the views put forward in this connexion by Carnap in the *Aufbau* (cf. HJELMSLEV, 1947, p. 75).

Carnap then asked himself the following question[6]: is the map of the primitive relation of a constructional system capable of only one interpretation? For example, is recognition-of-similarity the only relation having the structural form pictured by the map; does a comparison between the map and reality necessarily lead to the conclusion that the arrows on the map represent the relation of recognition-of-similarity? Are the individuals characterized unambiguously by their topological position within the structural form of the relation of recognition-of-similarity?

To answer these questions in the affirmative would be to say that a basic relation like recognition-of-similarity can be unambiguously characterized by its *structural form* as this is pictured by the map.[7] The answer that Carnap is forced to give to this question has not received the careful consideration which, in our opinion, it deserves. Carnap himself did not lay particular emphasis on it, as it does not fit into his general line of thought; and he never returned to the question in his later work.

As a primitive relation such as that of recognition-of-similarity orders the domain of a very large number of individuals, forming a highly complex network, it would seem improbable that there could be a second relation with exactly the same structural form, and it also seems likely that every individual is characterized by a unique relative position within the total configuration of individuals.

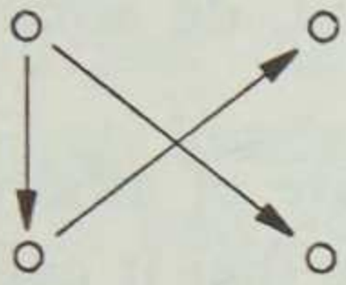
However, a moment's thought shows that for every relation we can stipulate a whole series of equally structured, isomorphic relations. All

[6] CARNAP 1928, § 154. That this question was at the time a central one for Carnap can be gathered also from the article CARNAP 1927, in which he put together his views on material and formal concepts.

[7] Of course, in order to be able to read the structural form off the map, we must (a) understand the syntax of the map, i.e., know, for example, how to segment: know which parts of the inscription count as *one* sign; and (b) have been informed which ontological categories correspond to which syntactical categories. "... no characterization of a system is purely formal" in the sense that it presupposes no information about its intended interpretation. "For if we are given just an arrow-diagram, without any interpretative information whatever, then we do not even know that the arrows represent relationships or that the letters represent elements. We can tell nothing at all about the system in question or even that there *is* a system in question; the diagram might be a hex sign or a complex character serving as the proper name of a single element" (Goodman in BOCHEŃSKI-CHURCH-GOODMAN, p. 22). Cf. also below, p. 159, and footnote 38, p. 137.

that is required is a one-one transformation of the totality of individuals in themselves; the new relation will then be that relation whose list is the transformed list of the original relation. The following simple example should make this clear:

[1]



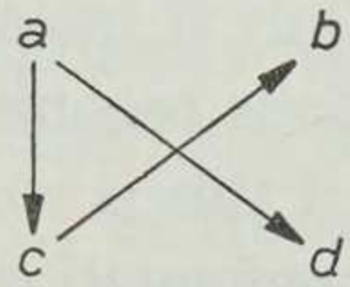
Map

[2]

a c
a d
c b

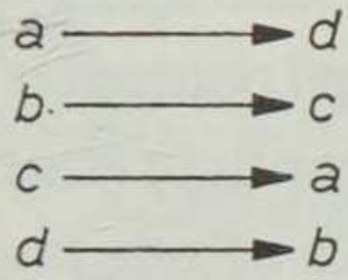
List of relation *R*

[3]



Interpreted map of the relation *R*

[4]



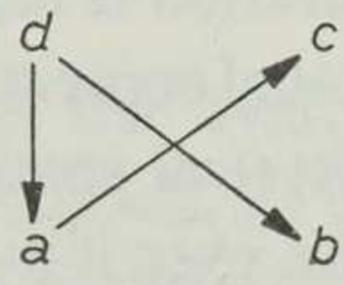
Transformation

[5]

d a
d b
a c

List of new relation *R'*

[6]



Interpreted map of new relation *R'*

As can be seen, *R* and *R'* have the same structural form; maps [3] and [6] both have the structural form [1].

The new relation obtained by transformation is well-defined through its list, and classes can be formed on its basis in the same way as on the basis of the original relation. However, such a relation obtained by transformation usually has the peculiarity that it is not an "*experience-able*" (*erlebbare*) relation, that there is no specific experience (sensation, feeling or understanding) of it. Normally the only feature which all the pairs of individuals on its list and no other pairs of individuals have in common, is the feature of being listed on this list.

Carnap tries to rule out "abnormal" interpretations of the map in terms of such undesirable relations, by requiring that the relation represented must be experienceable. He proposes the introduction of a special primitive sign ('fund') which, if predicated of a relation, indicates that the relation is well-founded, i.e., experienceable.

This means, in our view, that in order to ensure the uniqueness of the representation by the map, Carnap refers to the intensional content of the relation represented, stipulating that it must have a high degree of

simplicity and intuitiveness. He does this, even if there is no further explicit reference to the intensional aspect of relations within the extensional system of the *Aufbau*.

This brings us to an important point: the mere fact of the existence of the individuals means that all their configurations are in some sense realized. Only if intensional meaning is taken into account, do some configurations become more outstanding than others.

But if this is so, how can Wittgenstein regard a state of affairs as a mere configuration of objects and nevertheless assume that it can exist or not exist in reality? It seems that the distinction between existing and non-existing states of affairs can only be maintained if the relation of configuration is regarded not merely as extensional but also as having an intensional content. On Wittgenstein's view this intensional content is probably physical spatiality. A particular physical-spatial configuration need not exist, even if the individual objects exist; the existence of *one* physical-spatial configuration even precludes the simultaneous existence of a different physical-spatial configuration of the same objects. Wittgenstein's failure to point out the intensional nature of physical spatiality can be explained by the fact that in his system physical spatiality is the only intensional content: it is therefore not distinguished from other intensions and hence easily overlooked.

This way of showing that even somebody who uses only extensional sentences must nonetheless presuppose the existence of intensions, is especially valuable, since it is precisely the attempt to think out systematically the extensionalistic view (according to which there are no intensional meanings) that leads here to the conclusion that this view is untenable.

7.2 Synonymity

Carnap's thinking about the correlations existing between an extensional system and reality in the *Aufbau*, drew attention to the existence of intensional meanings. It is in a different connexion that Carnap came to consider the intensional aspect of represented reality more closely, viz. in connexion with synonymity and *a priori* statements.

7.21 *Kinds of a priori statements*

The dissertation which Carnap wrote in Jena under Bruno Bauch, and which shows the influence of Frege, was an examination of the kantian doctrine of space as an *a priori* form of intuition (*Anschauung*). Carnap adopted in it Kant's three-fold division of true judgements into analytic *a priori*, synthetic *a priori* and synthetic *a posteriori*.[8] Moreover, he did not interpret the intuition of synthetic *a priori* judgements narrowly in a purely kantian sense, but explicitly referred to Husserl and his "intuition of universal essences (*Wesenserschauung*)": "... as Husserl has shown, we are concerned here not with facts in the sense of facts of empirical reality, but with the essence ("*eidos*") of specific data, the particular nature of which essence can be grasped already on one single occasion."[9]

However, after his dissertation Carnap dismissed the synthetic *a priori*.[10] Adopting Wittgenstein's easily remembered criterion, he divided all true sentences into two categories: (a) those which are true for every possible state (*Sachlage*), i.e., for all possible worlds, and which therefore say nothing about actual reality; and (b) those which are true only for a limited number of possible states, so that it has to be verified empirically whether actual reality is one of these states. Sentences of the first kind are true for logical reasons alone, while those of the second kind are factually true.[11]

The dichotomy is an old one which can already be found in Leibniz (*vérités de raison*, resp. *identiques*, and *vérités de fait*) or Hume (truth concerning "relations of ideas" and truth concerning "matters of fact").[12] But whereas Leibniz was thinking in terms of syllogistic, Wittgenstein has the logic of truth functions in mind. For him a tautology is a sentence put together from atomic sentences with the aid of truth functors in such a way that it is true, whatever the truth-value of the atomic sentences. For example, a sentence of the form '$\sim(p \cdot \sim p)$', i.e., 'not both p and not p' is true, irrespective of whether it is true that p or it is false that p.[13]

[8] CARNAP 1921, p. 63.
[9] CARNAP 1921, pp. 22–23.
[10] Cf. footnote 43, p. 48 above.
[11] Cf. WITTGENSTEIN 1921, 4.462, 6.1.
[12] Cf., e.g., LEIBNIZ 1875–1890, vol. 5, p. 343; HUME 1902, p. 25. See also PAP 1958.
[13] WITTGENSTEIN 1921, 4.46.

In kantian terminology the tautological sentences express analytic *a priori* judgments; the factually true sentences, synthetic *a posteriori* judgments. An intermediate type of synthetic *a priori* judgments being excluded, the terminology became simplified and true sentences became simply referred to as either analytic or synthetic.

However, upon closer examination the complexity of the situation was again discovered. For example, the sentence 'No bachelor is married' certainly is not a factual sentence whose truth is contingent on the fact that so far no married bachelor has been found. But it is also not analytic in the same sense as the tautology 'no non-triangular figure is triangular', since 'bachelor' has not explicitly been introduced into the system as an abbreviation for 'unmarried man'. It is therefore customary nowadays to distinguish two kinds of analytic sentences: (a) analytic sentences which are simply logically true, and (b) analytic sentences which are not explicit tautologies, but which must nevertheless be regarded as necessarily true on account of their meaning, of synonymity, etc.[14]

Among the examples given for the second kind of analytic sentences, we find not only what we might call "implicit tautologies" like 'No bachelor is married' but also a sentence like 'Man is a rational animal', which is traditionally known as the "real definition" of the essence of man; or the sentence 'Everything green is extended', the standard example of a synthetic *a priori* statement (which, confusingly enough, is now discussed among the analytic statements!).[15]

The following diagram may help to clarify the situation:

EXAMPLES:	Married men are married. No man is both married and not married	A bachelor is an unmarried man. Man is a rational animal	Everything green is extended. Man has the ability to laugh.	Socrates is sitting.
KANTIAN TERMINOLOGY:	analytic *a priori*		synthetic *a priori*	synthetic *a posteriori*
LOGISTIC TERMINOLOGY:	analytic (*a priori* would have been better)			synthetic
	logically true in the narrow sense.			

[14] WHITE 1950; QUINE 1953, p. 22; CARNAP 1947 (1956), p. 222 and elsewhere. See also STEGMÜLLER 1954, PAP 1958.

[15] CARNAP 1947, 1; QUINE 1953, p. 32; cf. also below, p. 156.

Several empiricists, who feel uneasy about this come-back of traditional problems, deny that the whole analytic-synthetic distinction has any objective value and see in it merely a pragmatically motivated classification with no sharp dividing lines.[16] Carnap, on the other hand, strongly defends the value of this distinction, the details of which he has gradually worked out in the course of his work on the logistic formulation of scientific theories.

7.22 *Synonymity in logical syntax and semantics*

Carnap had at first restricted himself to *syntax*, i.e., to the description of the relations holding between the words of a language, the symbols of a theory, where no reference is made to what the words or symbols represent.[17] Not that Carnap would at that time have denied the semantical relation between signs and represented reality. Thus, for example, he said that predicate signs were so to speak proper names for properties.[18] However, he considered the investigation of the co-ordinations between words and things to be the task of the empirical natural sciences – of psychology and sociology.[19] He thought that whatever was logical about these co-ordinations could be translated into syntactical sentences. Thus Carnap said[20] that the semantical sentence

'The word 'luna' in the Latin language *designates* the moon'

can be replaced by the syntactical sentence

[16] Thus Goodman, Quine, White, and others. Cf. QUINE 1960, p. 67, for historical details. WHITE 1950, p. 339, denounced explicitly the "dualism shared by both scholastics and empiricists" and advocated "dropping the myth of sharp distinction between essential and accidental predication (to use the language of the *older* Aristotelians) as well as its contemporary formulation – the sharp distinction between analytic and synthetic".

[17] It seems that it was Wittgenstein who took over the designation 'syntax' from the field of grammar of ordinary language, into that of logic, and he said that logical syntax must never concern itself with the reference of signs. Cf. WITTGENSTEIN 1921, 3.325, 3.33.

[18] CARNAP 1934, § 3; and cf. CARNAP 1947, p. 18, where he explicitly says that equal predicate signs stand for identically the same entity; cf. also our footnote 25, below on p. 95.

[19] CARNAP 1934, § 1.

[20] CARNAP, 1934, § 75, example 4.

> 'There is an equipollent expressional translation of the Latin into the English language in which the word 'moon' is the correlate of the word 'luna''.

Questions concerning the logical aspect of the semantical relation between language and represented reality, could therefore be incorporated into syntax by making them into questions concerning the possibility of translation between two languages.

Instead of referring to possible translations from one language into another, we can also make statements about synonymity within one and the same language. According to Carnap[21] the sentence

> 'The word 'daystar' *designates* the sun'

can be replaced by the sentence

> 'The word 'daystar' is synonymous with 'sun''.

And this is where the distinction between *a priori* and *a posteriori* re-appears. The example of 'morning star' and 'evening star' has been discussed ever since Frege first introduced it, and the above example of 'daystar' and 'sun' bears some resemblance to it. But whereas the identity of daystar and sun seems to be determined *a priori*, that of morning star and evening star represents an astronomical discovery and therewith an *a posteriori* truth. Carnap allows for this difference by distinguishing between P-synonyms, the reasons for which are physical, i.e., factual; and L-synonyms, which are based on "logical" reasons. 'Daystar' and 'sun' are L-synonymous; 'morning star' and 'evening star', P-synonymous.[22]

What, however, is meant by "logical" reasons in this context? In the subsequent semantical period of Carnap's logical investigations, the dependency of L-synonyms on relations between intensional meanings came clearly to the fore.

It was Tarski's work – in particular his famous paper on the definition of the concept of truth – that persuaded Carnap to turn his attention to *semantics*, for it showed him that semantical relations could be treated in as precise a manner as syntactical relations.[23] From 1938 onwards

[21] CARNAP 1934, § 75, example 2.
[22] CARNAP 1934, § 75, example 7.
[23] TARSKI 1933; cf. CARNAP 1942, p. VI. See also above, p. 48 f.

Carnap adopted Ch. Morris's distinction between syntactics (or syntax) which concerns itself with the relations between signs; semantics, which deals with the relations between signs and the reality represented (and thus includes syntax as part of its subject-matter); and pragmatics, which includes the language users within its field of inquiry. Carnap's logic thus ceases to be restricted to syntax; it becomes, in fact, semantics.[24]

It contains now special rules of designation and of truth.[25] These allow more detailed proofs of synonymities – or equivalences, as he now calls them – to be given. Carnap gives the following example[26]:

[1] '*H*' and '$F \cdot B$' are equivalent, but not L-equivalent
[2] '*H*' and '*RA*' are L-equivalent

The rules of designation of the signs '*H*', '*RA*', '*F*' and '*B*' are explicitly stated[27]:

'*Hx*' is a symbolic translation of '*x* is human (a human being)'.
'*RAx*' – '*x* is a rational animal',
'*Fx*' – '*x* is (naturally) featherless',
'*Bx*' – '*x* is a biped'.

However, the justification of the difference between [1] and [2] depends essentially on the following additional statement: "The English words here used are supposed to be understood in such a way that 'human

[24] CARNAP 1938; CARNAP 1942, p. 8, p. 250.
[25] Carnap likes to use the word 'designation' in a wide sense, so that it covers all forms of relations of representation. In CARNAP 1950, p. 33, footnote, he wrote: "The term 'designation' is used in the present article in a neutral way; it may be understood as referring to the name-relation or to the intension-relation or to the extension-relation or to any similar relation used in other semantical methods." In CARNAP 1947 he distinguishes his own semantical method from what he calls the name-relation method. So far as predicate signs are concerned, the difference is the following: Frege and Russell had introduced special signs for classes, apart from those for properties; a particular sign is either the name for a class or the name for a property, but not for both at the same time. (Although Russell – as we have seen in Ch. 5, p. 74 – finally makes the assumption of classes unnecessary.) Carnap, on the other hand, intends to use only one kind of sign, a particular sign having at the same time a definite extension and a definite intension. That is, he defines relations of extensional and of intensional equivalence of signs by referring to what the signs designate. Thus it is not only extensions but intensions that are designated.
[26] CARNAP 1947, propositions 3–8 and 3–11.
[27] CARNAP 1947, p. 4.

being' and 'rational animal' mean the same."[28] In respect to [1], on the other hand, Carnap makes the following assumption, which he characterizes as a biological fact and thus as *a posteriori*: "All human beings are featherless bipeds and vice versa."[29] Thus the L-equivalence is clearly based on an extra-systematic explanation of the relations holding between the intensions represented by the words in question.

Carnap eventually included explanations like the above explicitly within his semantical systems as so-called meaning postulates.[30] These determine the sense of the descriptive constants of the system. Thus, for example, it can be stated *a priori* that for all x, x is a human being, if and only if x is a rational animal:

$$(x)(Hx \equiv RAx);$$

and similarly that for all x, if x is green, then x is extended:

$$(x)(Gx \supset Ex),$$

etc. Also formal properties, like the transitivity and irreflexivity of "being warmer than", that are necessarily connected with the intensions of certain relations, can be explicitly expressed within the system by meaning postulates:

$$(x)(y)(z)(Wxy \cdot Wyz \supset Wxz),$$

i.e.: 'For all x, y and z: if x is warmer than y, and y is warmer than z, then x is warmer than z';

$$(x)(\sim Wxx),$$

i.e.: 'For all x: x is not warmer than x.' Recursive definitions, such as can, e.g., be found in arithmetic, and Carnap's so-called reduction sentences[31], which it would otherwise not be easy to classify, can also be regarded as meaning postulates.

The introduction of meaning postulates allows "analytic" sentences of the second kind to be clearly defined: they are those sentences that can only be inferred with the help of meaning postulates.

[28] Carnap 1947, p. 4.
[29] Carnap 1947, assumption 3–6.
[30] Carnap 1952.
[31] Cf. Carnap 1936/37.

7.3 Conventionalism and positivism

Although Carnap holds that "the theory of intension of a given language L enables us to *understand* the sentences of L"[32], he denies that meaning postulates are based on a phenomenological analysis of the intensional content of the given, but stresses that they represent conventions regarding the use of words. And although he says that predicate signs represent properties and classes, and thus talks about abstract entities, he nevertheless rejects as meaningless the metaphysical assertion of the existence of such entities. Where does this curious refusal to go all the way to the acceptance of intensions come from?

Carnap's attitude concerning meaning postulates is in part explained by the fact that he is primarily concerned with the transition from ordinary language to an explicit system. Explication – the choice of a precise definition within the system for a word from ordinary language – is largely a matter of subjective opinion. Should the word 'raven', for example, be defined in such a way as to be applicable only to black birds; should the meaning postulate be:

$$(x)(Rx \supset Bx),$$

i.e.: 'For all x: if x is a raven, then x is black'? Or should the possibility that 'raven' might designate, e.g., a blue bird not be excluded?[33] Carnap's definition of intension stresses the conventional aspect by its reference to the voluntary disposition of a language-user X: "The intension of a predicate 'Q' for a speaker X is the general condition which an object y must fulfil in order for X to be willing to ascribe the predicate 'Q' to y."[34]

From Carnap's point of view, one explication is preferable to another if it proves itself the more useful (e.g., more useful in the development of biology). Scientifically-minded as he is, he regards our knowledge of intensions, i.e., of universal essences, as a matter of the progressive improvement of "the explicit conventions" (the meaning postulates) of the language of empirical science. For example, he mentions how in chemistry

[32] CARNAP 1947, p. 234. Italics in Carnap.
[33] CARNAP 1947, p. 225.
[34] CARNAP 1947, p. 242. For an explication of meaning as a dispositional property cf. in particular STEVENSON 1945, pp. 37–80.

"slowly ... certain properties were selected as essential" for the various chemical elements.[35]

Like a great many contemporary philosophers, Carnap is sceptical with respect to the possibility of an immediate intuition of essential connexions. He has learned that a first flash of "intuition" (cf., e.g., the "insights" of "naive" set theory) does not represent the end, but only the beginning of the philosopher's work. It provides an inspiration, but it is not until this inspiration has been precisely formulated in the terminology of a system and incorporated in it without contradiction, that it can be said to be genuine.

Even a phenomenologically inclined philosopher will concede this last remark. The requirement of precise formulation is made by husserlian phenomenologists and logistic philosophers alike; they differ only in their choice of the language in which to carry through their formulations. Phenomenologists, who train their minds to seize the manifold aspects of phenomena, prefer the adaptable, flexible language of every-day speech; on the other hand, logistic philosophers are interested primarily in achieving a strict system of classifications and freedom from contradiction, and therefore use an artificially regulated, logistic language.[36]

In order to explain why Carnap does not want to be a metaphysician who accepts abstract entities, we have to go into more historical details. His position can be traced back to two sources: the methodology of the natural sciences and the reaction of certain philosophical circles against the way metaphysical problems were treated in post-cartesian philosophy.

We are all familiar with the idea, now that newtonian physics has been replaced by Einstein's, that in the natural sciences a number of different theories can explain the same phenomena, their probability being determined by purely pragmatical criteria of simplicity. Carnap had made a special study of euclidean and non-euclidean geometries in his dissertation, and had concluded that the question which geometry to use in physics could be decided only in terms of the principle of scientific economy. (He quotes Ernst Mach in this connexion.[37]) In his early work he made a detailed study of the nature of measurement as dis-

[35] CARNAP 1947, pp. 241–242.
[36] Cf. also below, p. 180.
[37] CARNAP 1921, p. 36.

cussed by philosophers of science like Henri Poincaré and Hugo Dingler. These philosophers stress that arbitrary stipulations (*wahlfreie Festsetzungen*) must be made, which are neither confirmed nor falsified by experience.

The example that Carnap gives of such a stipulation concerns the decision which physical paths to regard as straight. It is customary to stipulate that light travels in a straight line, but it is possible to stipulate differently, and a different set of spatial relationships will result in each case. Carnap even mentions a theory according to which the earth's surface is described as a plane, so that from the point of view of a "normal" description all spatial relationships undergo a queer transformation where only the topological situation remains the same, and where light rays follow a strange curved path.[38]

Carnap thus regards every statement about reality as a fusion of two components: The material, i.e., the actual data of experience (*Tatbestand der Erfahrung*) and a freely chosen form (*wahlbestimmte Form*).[39]

He had been so strongly influenced by *conventionalism* with its stress on the necessity of arbitrary stipulations in physics, that he relied on this doctrine in order to prove the inadequacy of pure empiricism.[40]

Non-euclidean geometries and the theory of relativity were heterodox theories appearing in particular fields of knowledge. But soon non-classical theories were also formulated in the discipline which is basic for all rational knowledge, namely in logic. In 1917 Łukasiewicz constructed a system of many-valued logic, and in 1930 Heyting formalized intuitionist logic, where the law of the excluded middle is not generally valid.[41] It was especially with a view to distinguishing his own position from that of intuitionism, that Carnap extended the conventionalism of physics to the sphere of logic. Just as in physics a standard is stipulated to decide which physical lines are to be regarded as straight, so in logic the criteria, i.e., the axioms and rules that determine which sentences are deducible in a system, can, on Carnap's view, be arbitrarily laid down. In 1934 he announced the so-called *principle of tolerance*, which he later said would

[38] Carnap 1921, p. 46.
[39] Carnap 1921, p. 39.
[40] Carnap 1923, p. 90.
[41] Cf. Łukasiewicz 1918, where he mentions that he discovered three-valued logic in 1917; Heyting 1930.

be more suitably named *principle of conventionalism*[42]: "It is not our business to set up prohibitions, but to arrive at conventions." "In logic, there are no morals."[43]

The principle of tolerance involves, as a consequence, the rejection of metaphysical assertions; for it not only asserts that the choice of any one system is a matter of pragmatical convention, but also that the sentences of the system deducible from these conventions, the so-called "logical" or "analytic" sentences, say nothing about the actual data of experience.

This brings us to the second, the positivist source of Carnap's position. As we have shown in Ch. 2, Russell changed metaphysical questions into questions about language, and Wittgenstein analyzed post-cartesian epistemological metaphysics into logical analysis of language, psychology and meaningless pseudo-propositions.

It was under the influence of this anti-metaphysical, neo-positivist trend that Carnap initially translated metaphysical assertions such as 'There are numbers', 'There are properties', etc., into syntactical sentences like 'There are number expressions', 'There are adjectives', etc., which speak about language systems.[44]

Later Carnap became more liberal and allowed sentences like 'There are numbers', 'There are properties', etc., as sentences *within* his systems. He thought he could do this by distinguishing between what he called internal and external questions.[45] According to this view a question such as whether there are properties, can be formulated in exactly two ways. First, it can be put as an internal question *within* the framework of a specific system. If the system contains a predicate 'is a property' and variables of higher type, then the sentence 'There are properties' can be

[42] CARNAP 1942, p. 247.

[43] CARNAP 1934, §17. Carnap refers to K. Menger, who, in the dispute over the foundations of mathematics, had effectively put forward the principle of tolerance and had apparently been the first to do so. Menger justifies his "implicationist standpoint" as follows: "Whether mathematician *A* declares the axiom of choice to be "admissible", "believes" in it and applies it; whether mathematician *B* rejects it as "unconstructive" or "because he cannot attach any meaning to it": these facts are of interest from the point of view of the biographies of mathematicians *A* and *B*, perhaps of history, but certainly not of mathematics and logic. These latter are concerned only with what follows from the axiom of choice" (MENGER 1930, pp. 324–325).

[44] Cf. CARNAP 1934, § 78.

[45] CARNAP 1950, pp. 30–31.

formulated in the system, and will be trivially deducible from the logical axioms of the system (unless the system is an empty one); i.e., it will be logically true and assert nothing about reality. Secondly, the question can be put as an external question prior to the choice of a specific system. The question is then, which system is better suited as a framework for the recording of experience: one in which the sentence 'There are properties' can occur, or one in which it cannot occur. This is a question of utility, and not of truth or falsity. A metaphysical question distinct from the internal and external questions, is on this view meaningless, and the traditional problem of universals is considered as a pseudo-problem.[46]

However, the above account of sentences like 'There are properties' is unsatisfactory. As we shall see in Chapter 9, a detailed study of the relation of representation of such sentences shows that under regular interpretation they make assumptions concerning the nature of an ontological model which is supposed to be real.

It seems that Carnap finally accepts this conclusion of Quine's: since 1956 he tries to comply with the demands of nominalism (in the sense of Quine and Goodman).[47] Reviving the old distinction between observation language and theoretical language[48], he asks that the observation language be nominalistic and finite, i.e., make as few ontological assumptions as possible, while the theoretical language can be platonistic. This satisfies nominalism, as he stipulates that only the observation language be given a regular interpretation. The theoretical language is interpreted only indirectly; it is connected with reality only through the rules of co-ordination linking it with the observation language and may be regarded largely as an abacus, as a mere calculus the rules of which are determined by pragmatical criteria. Carnap thus includes platonistic sentences such as 'There are properties' in the theoretical language, without being committed to the acceptance of abstract entities.

[46] CARNAP 1950, p. 33.
[47] CARNAP 1956; CARNAP 1958.
[48] CARNAP 1923.

8. STANISŁAW LEŚNIEWSKI

Russell, and initially also Carnap, unhesitatingly accepted references to abstract entities. Wittgenstein did not touch the question at all, since his ideal language says nothing *about* properties and relations, these being merely *pictured*. Leśniewski, on the other hand, regarded abstract entities as problematical from the very beginning.

Indeed, in his pre-logistic period he formulated a strict proof to the effect that there cannot be a "general object", since such an object would be contradictory. In place of the theory of classes he developed a system of the part-whole-relation, *mereology*, where reference is made only to concrete objects. Similarly, Leśniewski did not accept the predicate calculus of modern logic, constructing instead a system according to his own intuitions, the so-called "*ontology*". Although his "ontology" is comparable with the simple theory of types, it seems that it cannot be considered as committed to the explicit assumption of abstract entities. The names of the system are merely said to stand for several individuals, for one individual, or for none, and the functors are not considered as naming anything. Leśniewski has a highly original conception of the quantifiers, which differs considerably from Frege's and Russell's.

Leśniewski aimed at a system free from any non-logical presuppositions. In his view a logical system should be entirely neutral, since it should provide the framework for the formulation of all possible kinds of theories. T. Kotarbiński, however, based his explicitly metaphysical theory of so-called *reism*, according to which only things exist and nothing else, on Leśniewski's system. The question therefore arises whether and to what extent Leśniewski's system can be called "nominalistic".[1]

[1] Because of the unusual character of Leśniewski's system and the limited number of written sources, the author is especially indebted to the following disciples of Leśniewski: B. Sobociński, C. Lejewski, H. Hiż, as well as to Lejewski's disciple E. C. Luschei, and wishes to thank them for their willingness to anwer his questions and to provide him with valuable information. See also footnote 28 on p. 109 below.

8.1 The contradictory nature of so-called "general objects"

Like Russell and later Quine, Leśniewski concerned himself in his early writings with the problems connected with existence statements. His initial conclusions, however, were odd and not very useful. Thus in his first paper he interprets Mill's terminology of denotation and connotation in such a way that all negative existence statements turn out false.[2] Later he went still further, and considered all positive existence statements to be likewise false.[3]

Leśniewski explicitly dissociated himself from these early works, when he began to construct logistic systems, and admitted the "bankruptcy of their "philosophico-grammatical approach".[4] He retained only one proof from his pre-logistic period: that of the impossibility of assuming a so-called "general object" (*przedmiot ogólny*).[5] A "general object" is defined as an object that possesses those and only those properties that are common to all the individual objects corresponding to it. Suppose that *A* is a property common to some but not to all the individual objects in question; then by definition the corresponding "general object" cannot possess the property *A*. But neither can it possess the property of not possessing *A*, i.e., it must possess *A*, which is contradictory to the above. It follows that there can be no "general object".

This proof assumes the principle that "For each property it holds that every object either possesses or does not possess it", which excludes any indeterminateness in the object. Leśniewski calls this principle the *metaphysical* or *ontological* principle of the excluded middle, and distinguishes it from the *logical* principle of the excluded middle, which he formulated: "At least one of two contradictory propositions must be true." This distinction between the ontological and the logical, is indicative of the precision with which Leśniewski expresses himself – very much in contrast to Russell.[6]

[2] LEŚNIEWSKI 1911; he obtained his Ph.D. for it in 1912 under K. Twardowski at Lwów University. Cf. MILL 1843, vol. 1, ch. 2, § 5.
[3] Cf. LEŚNIEWSKI 1913. We shall return to this point below, p. 121.
[4] LEŚNIEWSKI 1927–1931, vol. 30, p. 183. They are, however, still worth reading for the precise and effective reasoning which they contain.
[5] LEŚNIEWSKI 1913, pp. 7–8. Cf. LUSCHEI 1962, pp. 308–310.
[6] LEŚNIEWSKI 1913 considered the metaphysical principle to be true, but he believed

Let us add at once some words of criticism. Leśniewski's argument against "general objects" holds if these are defined as above, i.e., if they are regarded to be exactly like concrete objects. However, an abstract object (a class, a universal idea, that is to be exemplified in many individuals) must be something quite different. It cannot be defined as possessing the properties of the concrete individuals subsumed under it, for it must possess properties that cannot be attributed to any of the individuals in question: it is abstract, timeless, exemplified in such-and-such a number of individuals, etc. The properties that are common to the individuals subsumed under it, belong to the abstract object not as *properties*, but as *characterizing marks* (Latin: *notae*, German: *Merkmale*) which indicate what kind of individuals fall under it.

It is interesting in this connexion to consider the classical "third man" objection that has been raised against Plato's doctrine of ideas.[7] If the idea of man is introduced in order to explain the similarity between individual men, and if this idea is taken as being itself similar to individual men, then in strict logic a second idea of man (a higher-order idea), i.e., a "third man", would have to be introduced to mediate between any individual man and the first-order idea of man; and so on *in infinitum*. – It is therefore important to stress that normally an idea does *not* have properties similar to those of the individuals subsumed under it. Stegmüller, for example, rightly says: "The idea of redness is not itself red, the class of red things is not itself a red thing."[8] It is mistaken to think of an idea as of a picture, as if it were another concrete object, which shares properties with what it pictures.

The distinction between properties and characterizing marks of abstract objects is an old one. In modern logic it is made, e.g., by Frege, who refers to the properties (*Eigenschaften*) and the characterizing marks (*Merkmale*) of objective concepts (*Begriffe*).[9] R. Ingarden has given a detailed phenomenological description of this special dual structure of

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that the logical principle was false, since he held that all positive as well as all negative existence statements were false. Cf. how ŁUKASIEWICZ 1910, too, had distinguished between the ontological and the logical principle of contradiction.

[7] PLATO, *Parmenides* 132A–133A; ARISTOTLE, *Met.* A9 990 b 18, M4 1079 a 13.

[8] STEGMÜLLER 1956, p. 204.

[9] FREGE 1884, p. 64.

ideas within the framework of a philosophical ontology.[10] He shows how the universality of an abstract object is explained by the fact that its characterizing marks include variables as well as constants. Thus, for example, it is part of the idea of man that man has size, hair-colour, and so on; on the other hand, in the universal idea the precise determination of the size, the hair-colour and so on, is left open. The properties of an idea, however, are in no way indeterminate: if an idea is to be regarded as an object then it must have a closed structure. Thus the *tertium non datur* holds for the properties of abstract objects (though not for their characterizing marks) without producing any paradox. Leśniewski's argument therefore fails if abstract objects are conceived in this way.

The metaphysical question nevertheless arises whether there are in fact objects with such a dual ontological structure; whether the existence of such entities can be assumed.

8.2 Mereology

Because of the inaccuracy of Russell's formulations, mathematical logic at first seemed unintelligible to Leśniewski. Already in 1911, J. Łukasiewicz, who was then "Privatdozent" at Lwów University, had drawn his attention to mathematical logic. But even Łukasiewicz could not give an exact meaning to Russell's explanations concerning the antinomies, and Leśniewski therefore directed his attention instead to the study of J. S. Mill and Husserl.[11]

In 1914, however, he became interested in the foundations of mathematics, and tried to find his own solution to the antinomies of the theory of classes.[12] The result was a new and special theory. No antinomies occur in it, as it does not deal with genuine classes, but only with concrete entities: instead of references to abstract classes it contains references to concrete collective totalities, to "wholes".

As Leśniewski sees in these concrete totalities one possible explication of what the much-used word 'class' may indicate, he applies the designation 'class' to them despite their difference from the entities of the traditional

[10] INGARDEN 1948, ch. 11; the essentials can already be found in INGARDEN 1925.
[11] LEŚNIEWSKI 1927–1931, vol. 30, p. 165 f.
[12] LEŚNIEWSKI 1914; the first axiomatisation of mereology is given in LEŚNIEWSKI 1916.

calculus of classes.[13] In order to avoid confusion we shall in the following add the index '*L*' to the words 'class' and 'member', when used with reference to concrete totalities in Leśniewski's sense.

Leśniewski's theory is distinguished from the ordinary calculus of classes by a number of special features. For example, whereas a sphere, the class of its halves and the class of its quarters, would normally be regarded as three different entities, on Leśniewski's view a sphere is identical with the class_L of its halves and the class_L of its quarters. The sentence that a half is a member_L of the class_L of quarters is true.[14] Thus a member_L of the class_L of objects of type *A* need not necessarily be an *A*. In contrast to the classical class-member relation, Leśniewski's class_L-member_L relation is transitive: if *X* is a member_L of the class_L of objects of type *A*, and if the class_L of objects of type *A* is a member_L of the class_L of objects of type *B*, then *X* is a member_L of the class_L of objects of type *B*. The expressions 'the class_L of objects of type *A*' and 'the class_L of the class_L of objects of type *A*' have the same designation, viz. the collective totality of all *A*'s.[15] A unit-class_L is identical with its member_L, and there is no mereological null-class_L.[16] Leśniewski's theory is akin to a generalized Boolean algebra without null element.[17]

Leśniewski was aware of the non-traditional nature of his "theory of classes_L". He referred to it as the doctrine of part-whole relations, and gave it the special title of '*mereology*' after the Greek word for 'part': '*μέρος*'.[18] From a philosophical point of view, an essential feature of his theory is that it extends the notion of an object. A collective totality – a concrete "heap", as Quine was to say later[19] – counts as one concrete object in the same way as any component part that may be "cut out" of it. The components of a "heap" need not "hang together"; for example, all the cats in the world at the present moment together form *one* such heap, and can be designated as *one* object in this sense. Furthermore mereology can also be applied to non-material objects. If there are

[13] For a further explication cf. below, pp. 108 ff.
[14] Leśniewski 1914, p. 66.
[15] Leśniewski 1914, p. 69.
[16] Leśniewski 1927–1931, vol. 30, p. 187, p. 186.
[17] Cf. Tarski 1935a.
[18] Leśniewski 1927–1931, vol. 30, p. 165.
[19] Quine 1953, p. 114.

angels, then we can speak, e.g., of the mereological whole of all angels.

In 1926 Tarski drew Leśniewski's attention to the similarity existing between his mereology and Whitehead's theory of events.[20] Whitehead, too, refers to part-relations; one event can be part of another; two events can overlap. In the United States, mereology is known as the "calculus of individuals" – a designation that is etymologically somewhat paradoxical, since the objects of mereology are anything but indivisible *individua*. It was developed there independently of Leśniewski by N. Goodman and H. S. Leonard around 1930.[21] J. H. Woodger, the English biologist, constructed his own, similar theory and applied it to biology.[22]

It is not surprising that interest in mereology should have developed independently in different parts of the world. This is readily explained by the nominalistic trend in contemporary philosophy: mereology, the theory of concrete totalities, can in many cases be applied in place of the theory of classes, which makes explicit reference to abstract objects.[23] Unfortunately Leśniewski's work, the first in the field of mereology and the most detailed and precise, remained largely unknown outside Poland until 1937, when Tarski brought it to the attention of a wider public.[24] Leśniewski's theory is being developed further by B. Sobociński and C. Lejewski; and we look forward to the publication, in due course, of a detailed monograph on mereology.[25]

[20] LEŚNIEWSKI 1927–1931, vol. 31, p. 286. Cf. WHITEHEAD 1919. Already in 1902, in a letter to Russell (dated 28th July, quoted in BARTLETT 1961, pp. 43–44) Frege emphasized the distinction between a whole and a class and described it in clear terms. Frege, however, did not construct a calculus of part-whole relations.

[21] However, the calculus of individuals was not published until 1940 in LEONARD-GOODMAN. For evidence of the earlier origin of the work, cf. GOODMAN 1951, footnote p. 42. It is possible that Whitehead's ideas partly inspired the work: he was teaching at Harvard at the time, with Leonard working under him.

[22] Cf. WOODGER 1937.

[23] On these possibilities and their implications, cf. in particular GOODMAN-QUINE and GOODMAN 1951. We shall give a detailed account of the American "nominalists" in the next chapter.

[24] In WOODGER 1937, where Tarski gives a simplified version of mereology in an appendix. Tarski had already based his paper TARSKI 1929 on a special formulation of mereology.

[25] A book by Sobociński on mereology has for some time been announced in the series "Studies in Logic" (Amsterdam). Cf. the papers SOBOCIŃSKI 1954/55; LEJEWSKI 1954/55; LEJEWSKI 1955/56; LEJEWSKI 1963a, and the dissertation CLAY 1961.

8.3 Ontology$_L$

8.31 *The distributive conception of totalities*

On Leśniewski's view membership in a concrete totality can be expressed not only in terms of mereology, by saying that something is a part of a collective whole: a whole can also be conceived distributively.[26] In order to gain a clearer idea of the meaning of the collective-distributive distinction, let us consider the following sentences[27]:

[1] 'Socrates is a component part of the concrete totality formed by all mortals',

[2] 'Socrates is a component part of the concrete totality formed by all Greek tribes';

or using the terminology of 'class' and 'member':

[1′] 'Socrates is a member of the class of mortals',

[2′] 'Socrates is a member of the class of Greek tribes'.

Evidently, [2′] is equivalent to the true sentence [2] only if 'member' and 'class' are understood mereologically. Strictly, we should have to write:

[2′] 'Socrates is a member$_L$ of the class$_L$ of Greek tribes'.

On the other hand, [1′] is true even according to the ordinary theory of classes.

On Leśniewski's view there is a further important distinction between [1′] and [2′]: in [1′] the words 'class' and 'member' are eliminable; the sentence can be re-formulated as:

[1″] 'Socrates belongs to the denotation of 'mortal'',

[1‴] 'Socrates is a mortal being',

or simply:

[1⁗] 'Socrates is mortal'.

[26] Cf. in particular Sobociński 1954/55, p. 2; also Sobociński 1949/50, p. 239 f.; Kotarbiński 1929, p. 13.

[27] We have formulated [1], [1‴], [2], and [2′] ourselves; [1′], [1″] and [1⁗] can be found in Sobociński 1954/55, p. 2.

That is, the class-member relation in [1′] can be rendered by the logical copula 'is'; no special non-logical functor like 'part of' need be introduced, as in mereology. Here, then, we have a second explication of the concept of class. This is the so-called "distributive conception". The "class", the concrete totality, is regarded as made up of certain concrete component parts: as "distributed" in a specific way.

The laws holding for distributive wholes are different from those holding for the collective wholes of mereology. Leśniewski thus had to develop another new theory, to which he gave the name '*ontology*', since it is the theory of the copula 'is'. In order to avoid confusion with the philosophical discipline of ontology, we shall refer in the following to Leśniewski's theory by means of the name 'ontology$_L$'. There is, in fact, a connexion between ontology$_L$ and ontology, in that Leśniewski believed that his ontology$_L$ realized the aristotelian project of a "first philosophy", i.e., of a completely general theory of objects.[28]

Leśniewski believed that the antinomies which Russell had discovered,

[28] KOTARBIŃSKI 1919, p. 254. LEŚNIEWSKI developed his ontology$_L$ after the mereology; systematically, however, ontology$_L$ is prior to mereology. Whilst working on the *mereology* (1914–1917), it became clear to Leśniewski that he needed a formalized logic which corresponded to his intuitions. He therefore developed a theory of names, the *ontology$_L$* (1919–1921), and finally a sentential calculus with quantifiers and variable functors, the *protothetics* (1923). Systematically the protothetics comes first. It is presupposed by ontology$_L$; and mereology presupposes the two theories of protothetics and ontology$_L$. In contrast to protothetics and ontology$_L$, mereology is not a part of logic. It contains, as we have seen above, a special non-eliminable functor which is non-logical, and with the aid of which the proper name of an individual object, viz. the name 'universe', can be defined (SOBOCIŃSKI 1954–1955, p. 2, p. 5; LEJEWSKI 1957, p. 255).

Leśniewski developed a special symbolism for his system, which he axiomatised. As a result of his own efforts and the research of his pupils, the number of axioms has been reduced to three: one for each theory (cf. SOBOCIŃSKI 1949/50, p. 257; SOBOCIŃSKI 1960/61; and LEJEWSKI 1963a).

Leśniewski's system is more precise and simpler than that of *Principia*, the second edition of which appeared in 1925; but as Leśniewski published so very little, his work has had no direct effect outside Poland. It was not until 1929 and 1930 that two articles of his appeared in German; and of LEŚNIEWSKI 1938 and LEŚNIEWSKI 1938a only a few copies of special reprints were distributed. However, through his teaching at the University of Warsaw, where he was professor of mathematics from 1919 until 1939, Leśniewski has exerted a decisive influence on the logicians of the Polish School.

To-day the only book on Leśniewski is LUSCHEI 1962. Further material can be found in the publications of Kotarbiński and of Leśniewski's disciples Lejewski, Słupecki, Sobociński and Tarski. See also Clay, Grzegorczyk (but cf. the criticism of LUSCHEI 1962, pp. 154–166), Jordan, Kearns, Prior, Sinisi.

could be explained as arising from a confusion of the collective and distributive conceptions of a "class".[29] The sentence:

[3] 'Every member of the class of objects of type *A*, is an *A*',

which is characteristic of the theories in which the antinomies had occurred, is tautologically true on a distributive interpretation of the word 'class':

[3′] 'Whatever is *A*, is *A*'.

On the other hand, on a collective-mereological interpretation it results in the false sentence:

[3″] 'Every member$_L$ of the class$_L$ of objects of type *A*, is an *A*.'

The two interpretations must not be confused.

Like mereology, ontology$_L$ differs from ordinary class calculi. Its sentences are based on the schema '*A est B*'[30], which can be read: '*A* is *B*', '*A* is a *B*', '*A* is one of the *B*'s', '*A* is a member of the distributive class of *B*'s'. The distributive class of *B*'s is for Leśniewski not an abstract entity, but like the mereological whole of the *B*'s, it consists of the *B*'s.

Of course, it may happen that for a given '*B*' there is exactly one object which is a *B*. For example, if '*B*' is an abbreviation for 'moon' (this word being taken in its most ordinary sense as applying to the natural satellite of the earth), then the *B*'s are identical with this one *B*, i.e., the distributive class is identical with its only member. In this case '*B est B*', i.e., '*B* is one of the *B*'*s*' ('The moon is one of the moons') is true. The functor '*est*', the *copula* of ontology$_L$, unlike the membership relation of a class calculus based on the theory of types, is thus not irreflexive.

If '*A*' and '*B*' are two terms which apply each to exactly one, and both to one and the same object, then '*A est B*' is true. If furthermore '*B est C*'

[29] Sobociński 1949/50, p. 239 f.

[30] This is the notation used in Kotarbiński 1929. His exposition of ontology$_L$ has the virtue of being readily intelligible, and has the explicit approval of Leśniewski (Leśniewski 1927–1931, vol. 34, p. 160). Leśniewski's notation is '$\varepsilon\{Ab\}$'. This is more complicated, and the use of *epsilon* is liable to cause confusion since it is customarily used to designate the membership-relation of ordinary class theory. On Leśniewski's distinction between capital and small letters cf. below, p. 113, footnote 41. In order to facilitate understanding, we will use neither Leśniewski's nor Kotarbiński's special notation for quantifiers and sentential connectives, but render these in the usual peano-russellian symbolism.

is true, then '*A est C*' is true. Thus unlike the membership relation of a class calculus based on the theory of types, the functor '*est*' is not intransitive. (In fact, it is transitive: in *all* cases in which formulas of the form '*X est Y*' and '*Y est Z*' are true, the formula of the form '*X est Z*' is true, too.)

If the term '*B*' happens to be an empty term which applies to no object, i.e., if there are no *B*'s, then we may say that "the class of the *B*'s is an empty class". There is then nothing in reality corresponding to '*B*', but there still is the term '*B*'. Therefore, while there simply is no mereological null-class$_L$, "there are" in some sense "null classes" in ontology, namely in the sense that there are empty terms. If '*B*' in '*A est B*' is an empty term, then *A* cannot be one of the *B*'s and '*A est B*' is certainly false. Also if '*A*' is an empty term, then it is false to say that *A* is one of the *B*'s, no matter what the *B*'s are. Thus one might not only say that in ontology$_L$ "null classes do not have members", but also that "null classes are not members of any class".

8.32 *Shared, unshared and fictitious names*

However, to speak in the above way in terms of classes is more misleading than illuminating. The arguments '*A*' and '*B*' in '*A est B*' are not considered as proper names of classes, but rather as referring to concrete individual objects. They are usually called "names"; however, the word 'name' is here not used in the narrow sense of "genuine name proper to exactly one entity", but in the broader sense of "term". Ontology$_L$ is a calculus of names (*rachunek nazw*)[31], where one name may refer to several objects, to exactly one, or to no object: i.e., to use J. H. Woodger's terminology, it deals with shared, unshared and fictitious names.[32]

From the point of view of logic, this division of names is not to be regarded as basic, since it depends on the factually existing state of the world. Leśniewski therefore puts all names into the same semantical category.[33] His ontology$_L$ resembles in this respect scholastic logic, where

[31] KOTARBIŃSKI 1929, p. 227.

[32] Cf. WOODGER 1952, p. 196; LEJEWSKI 1957, p. 240; LEJEWSKI 1958, p. 154.

[33] In contrast to Russell's, Leśniewski's metalogical formulations are extremely precise (cf. above, p. 78 f. and p. 103 f.); in fact, he developed an explicit metalogical system. He was inspired by Husserl's doctrine of the *Bedeutungskategorien* in *Logische*

it is also the case that all terms, singular as well as general, belong to the same category.[34] Leśniewski does not distinguish between names which in a *meaningful* sentence can stand *before* '*est*', and those that can stand *after* '*est*'; every name can stand in each of these places.

This peculiarity distinguishes the ontological$_L$ sentences of the form '*X est Y*' from the sentences of modern predicate calculi, where equal signs can never stand both in the subject and in the predicate place of a meaningful sentence, but where the syntactical categories of individual names and of predicate designations are always kept strictly apart.[35]

It is probably not a coincidence that ontology$_L$ has been developed by a Pole; for Polish, like Latin but unlike English (or German, French, Italian, Spanish) has no indefinite article, so that the same grammatical form applies to the predications of common nouns and to identity sentences. For example, '*Sokrates jest człowiekiem*' ('*Socrates est homo*') is constructed in the same way as '*Sokates jest Sokratesem*' ('*Socrates est Socrates*'), whereas in English the forms of 'Socrates is *a* man' and 'Socrates is Socrates' are different.[36]

A further peculiarity of the russellian systems has to be mentioned: the individual names of a meaningful sentence are normally not allowed to be empty, only predicates may be "fictitious", i.e., may apply to no

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Untersuchungen and referred to the categories as *semantical*, not as syntactical categories of signs. For although his theories are presented as strictly formal calculi, so that sign-categories can be determined purely in terms of syntactics, he nevertheless attached great importance to the intuitive interpretation, to the semantical dimensions of his system, which he wanted to be more than a mere formalism (cf. Leśniewski 1929, p. 6, p. 78; see also Luschei 1962, § 4, for a detailed discussion of this aspect). Later, however, after Leśniewski, the designation 'syntactical category' became prevalent. Cf. Tarski 1935, p. 335 f.; Adjukiewicz 1935; Bocheński 1949. See also below, pp. 123 ff.

[34] For the relation between syllogistics and ontology$_L$ cf. Lejewski 1963.

[35] Although the distinction between terms under which falls exactly one individual and terms under which fall several individuals depends on the factual state of the world, it is still possible to make a distinction between individual names and predicate signs, which is not accidental. For every predicate *can* (logical possibility) apply to several individuals (and a non-atomic predicate *can* refer to no individual) even if, like the predicate 'is a natural satellite of the earth', it refers *de facto* to exactly one individual; while on the other hand, every individual name can only refer to a single individual. Cf. Frege 1884, p. 63; see also Lewis 1944, p. 239; Lewis 1946, p. 45, who makes the intensionality of his distinction between singular and general terms explicit.

[36] To be precise, in ontology$_L$ the English sentence 'Socrates is Socrates' can be rendered in two different forms: either as the predication 'Socrates *est* Socrates' or as the identity sentence 'Socrates=Socrates', where '=' is defined: $(X)(Y)\ (X=Y \equiv X\ est\ Y \cdot Y\ est\ X)$.

individual. The arbitrariness of this exclusion of empty individual names has been compared with the arbitrariness of the exclusion of empty terms in Aristotle's syllogistics, and some authors have proposed systems of so-called "free logic", i.e., of predicate logic in the russellian style but admitting empty individual constants.[37]

Whereas on Leśniewski's view the *meaningfulness* of a sentence does not depend on how many objects are subsumed under the names contained in the sentence, this factor is vitally important in determining the *truth or falsity* of sentences. For instance, as we have seen above[38], if either '*A*' or '*B*' (or both) is an empty name, then '*A est B*' is false. Leśniewski also requires that '*A*' must not be a shared name. He interprets the sentence '*A est B*' explicitly as saying: "Every *A* is *B* and at most one object is *A*".[39] In a *true* sentence, therefore, the name standing *before* '*est*' must always be an unshared name or a description of a single actually existing object.[40] Thus, while from the rules of formation for meaningful sentences we cannot learn which names are in fact proper to a single individual, we can find this out by investigating which names occur in the subject place of *true* ontological$_L$ sentences of the form '*X est Y*'.[41]

The one axiom of ontology$_L$ guarantees these truth conditions by stipulating:

$$(X)(Y)\{X\ est\ Y \equiv (Z)(Z\ est\ X \supset Z\ est\ Y) \cdot (\exists Z)(Z\ est\ X) \cdot (Z)(W)(Z\ est\ X \cdot W\ est\ X \supset Z\ est\ W)\}$$

[37] Cf. LEONARD 1956 (cf. CHURCH 1963); HINTIKKA 1959; HINTIKKA 1966; LEBLANC-HAILPERIN and the papers by K. LAMBERT. The term 'free logic' has been coined by LAMBERT 1958–1964, vol. 13, p. 52. Although these systems allow individual names to be empty, they nevertheless retain the russellian type of quantification (see below, p. 117 f.).

[38] Cf. above, p. 111.

[39] LEŚNIEWSKI 1927–1931, vol. 34, p. 164. Leśniewski does not say "Every *A* is *B*, and exactly one object is *A*", since for him "Every *A* is *B*" already implies that at least one object is *A*.

[40] Already LEŚNIEWSKI 1913, p. 13, had assumed that every sentence whose subject designates nothing (as, e.g., in the case of 'centaur', 'square circle', etc.) is a false sentence.

[41] SOBOCIŃSKI 1949/50, p. 98f. Leśniewski distinguishes unshared names from shared names in his notation by using capital and small letters. But the fact that Kotarbiński omits this notational distinction in his exposition of ontology$_L$ shows that it is not essential to the system, and that all names belong to one and the same syntactical category. However, already the introductory axiom of a name constant may specify that it is an unshared name.

i.e.: 'For all X and for all Y: Z is Y if and only if (a) it is true for all Z that if Z is X, then Z is also Y; and (b) it is true for at least one Z that Z is X; and (c) it is true for all Z and for all W that if Z is X, and W is X, then Z is W'.[42]

Since the characteristic of being an object is not *shown* by a special syntactical category (i.e., by a category to which all proper names would belong) the ontological name 'object' occurs in Leśniewski's theory like any other name. Symbolically it is written 'V', being the most general name. Ontology$_L$ therefore seems eminently suited to be the language of the philosophical discipline of ontology. The name 'object' can be defined in purely logical – or rather: ontological$_L$ – terms:

$$(X)\,(X\ est\ \mathsf{V} \equiv X\ est\ X),$$

i.e.: 'For all X: X is an object if and only if X is X'; or alternatively:

$$(X)\,\{X\ est\ \mathsf{V} \equiv (\exists Y)\,(X\ est\ Y)\},$$

i.e.: 'For all X: X is an object if and only if it is true for some Y that X is Y'.[43] And the predication of the name 'object' is not trivial, for in ontology$_L$ a sentence of the form of the definiens '$X\ est\ X$' is not always true, i.e.,

$$(X)\,(X\ est\ X)$$

is false.[44] So there is no temptation to believe, with Wittgenstein and Carnap, that "object" is a mere "pseudo-concept".

It is also possible to define a name which is necessarily empty, viz., the name 'contradictory object' (symbolically: 'Λ'):

$$(X)\,(X\ est\ \Lambda \equiv X\ est\ X \cdot \sim X\ est\ X),$$

i.e.: 'For all X: X is a contradictory object if and only if it is true that both X is X and X is not X'.[45] As the definiens is clearly contradictory, the

[42] Leśniewski 1927–1931, vol. 34, p. 158; Kotarbiński 1929, p. 227.
[43] Sobociński 1949–1950, p. 248.
[44] See below, that '$\Lambda\ est\ \Lambda$' is logically false.
[45] Sobociński 1949–1950, p. 248. Cf. Prior 1953. The condition '$X\ est\ X$' in the definiens is necessary in order that the definition be well-formed. The formula '$(X)(X\ est\ \Lambda \equiv \sim X\ est\ X)$' would be a logically false sentence, since for any empty name or for any shared name in the place of 'X', the form '$X\ est\ \Lambda$' becomes a false sentence and '$\sim (X\ est\ X)$' becomes a true sentence.

following holds:

$$(X)\,(\sim X\ est\ \Lambda).$$

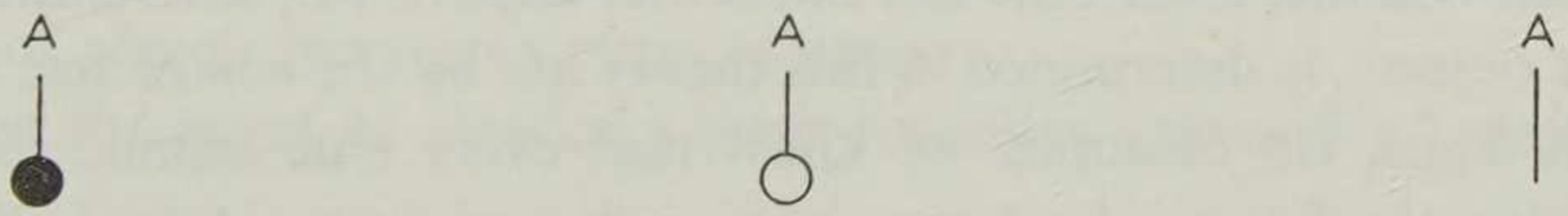
Even 'Λ *est* Λ' is an *a priori* false sentence, since 'Λ' is not the proper name of any individual object.

8.33 *Functors and existential import*

In our above exposition we have presented ontology$_L$ as the theory of the functor '*est*', since the theory did in fact develop out of an analysis and explication of the copula. It is, however, also possible to base the theory on different functors.[46] For, from a completely general point of view, ontology$_L$ is nothing else but a calculus of names; a theory of the different sentential forms, viz., of the different possible relations, which can hold in a sentence between the names in view of their respective extensions.

Leśniewski's theory thus continues the tradition of the scholastic logic of supposition[47], with the important difference that it does not analyze the suppositions of the names of a previously existing language (e.g., of scholastic Latin), but instead – and this is one of the characteristics of contemporary logic – orders and classifies the relations within the consistently structured system of an "artificially" constructed language.

Taking into account the three cases of a name being a shared, an unshared or a fictitious name, 16 different situations can occur with respect to the extensions of two names '*A*' and '*B*'. Lejewski has put together a table of diagrams.[48] The three types of names are symbolized as follows:



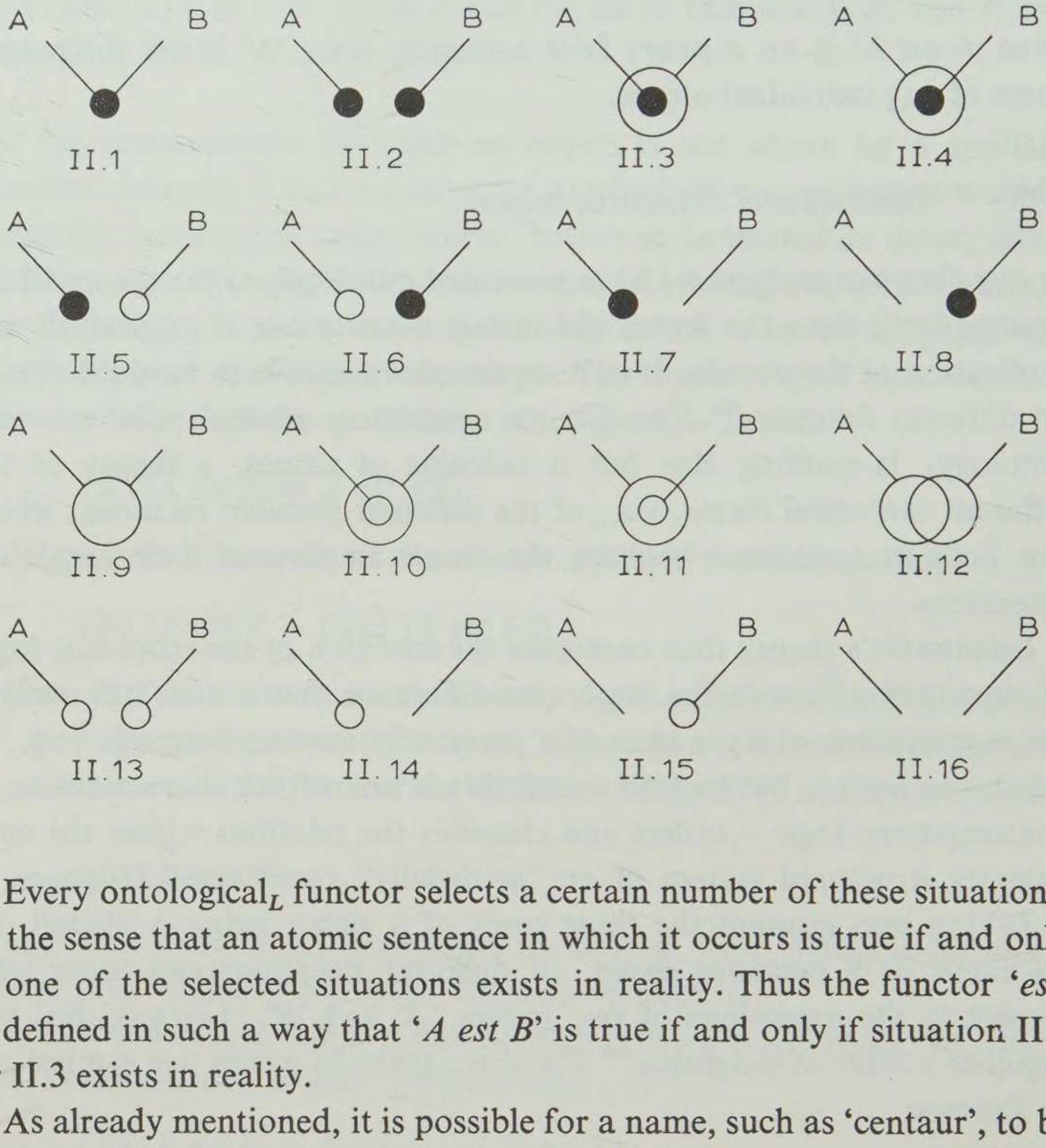
I.1. Unshared name

I.2. Shared name

I.3. Fictitious name

[46] Cf., e.g., the various axioms in LEJEWSKI 1958, p. 164f.
[47] See below, p. 121. Leśniewski, however, does not discuss his medieval antecedents.
[48] LEJEWSKI 1958, p. 155.

Then the 16 possible combinations are the following:



Every ontological$_L$ functor selects a certain number of these situations, in the sense that an atomic sentence in which it occurs is true if and only if one of the selected situations exists in reality. Thus the functor '*est*' is defined in such a way that '*A est B*' is true if and only if situation II.1 or II.3 exists in reality.

As already mentioned, it is possible for a name, such as 'centaur', to be fictitious. Whether a sentence has existential import, i.e., asserts the existence of objects, is determined in this theory *not* by the *names* but by the *functor*. Thus, for example, we know that every true atomic sentence containing the functor '*est*' presupposes the existence of an individual object, i.e., that this functor has existential import. The following one-place functors can be rendered in ordinary language by 'exists' [49]:

[49] Cf. LEJEWSKI 1958, p. 158. In the usual russellian type of system there are no such functors and there are no formulas at all which would correspond to singular existence statements. See below, p. 128.

'*ex* (*A*)' is true if and only if situation I.1 or situation I.2 occurs; it can be read: "There is at least one *A*."[50]

'*sol* (*A*)' is true if and only if situation I.1 or situation I.3 occurs; it can be read: "There is at most one *A*."

'*ob* (*A*)' is true if and only if situation I.1 occurs; it can be read: "There is exactly one *A*."[51]

It is of course also possible to define functors without existential import: for example, a one-place functor that forms a true sentence if and only if situation I.3 occurs, or a two-place functor that forms a true sentence if and only if situation II.16 occurs.

8.34 Quantifiers without existential import

The fact that in ontology$_L$ names have no existential import, also affects the interpretation of quantifiers. The particular quantifier, which ordinarily is referred to as the "existential quantifier", asserts here nothing about the existence of objects. Thus, for example, from the factually true sentence

[1] $\sim ex\,(Pegasus)$

or from the logically true sentence

[2] $\sim ex\,(\Lambda)$

we can infer:

[3] $(\exists X)\,(\sim ex\,(X))$.

[3] is not to be read: "There is an object that does not exist", but simply: "Something does not exist", or more explicitly: "It is true for some *X* that *X* does not exist", where the variable '*X*' does not extend over a range of *objects* but over a range of *names*.

From the point of view of a frege-russellian system[52], Leśniewski's

[50] '*ex*(*A*)' is true if and only if '*A*' is a non-fictitious name. But '*ex*(*A*)' is not a sentence *about* names. Compare with what is said (below, p. 118) about Leśniewski's interpretation of the existential quantifier.

[51] Compare this functor with the name 'V'. The functor 'ob' is sometimes written '1'. Further functors '2' ("There are exactly 2 . . ."), '3', etc., can also be defined in ontology$_L$.

[52] Cf., e.g., FREGE 1879, p. 22, where he explicitly translates the existential quantification of a negative sentence as: "There are some things that do not have the property *X*".

quantified sentences hover curiously between object-language and meta-language. For example, one is tempted to translate [3] meta-linguistically as: "It is true of some name that if it is substituted for '*X*' in the sentential form '*X* does not exist', then a true sentence is obtained." But such a meta-linguistic formulation can be expressed explicitly in Leśniewski's system, and does not coincide with the object-language sentence [3].

The possibility of interpreting the particular quantifier in this way has been noted by others[53], but so far as I am aware only Leśniewski has developed it into a strict and consistent system. And no one, it seems, has drawn attention to the curious combination of *meta-linguistic* quantification over a range of (meaningful) signs, with *object-language* sentences speaking about things.

Leśniewski himself apparently never considered any other interpretation of the quantifier. Already in 1914, for example, he used the formulation "For a certain meaning of the expression '*a*' ... *K* is *a*" (*Przy pewnym znaczeniu wyrazu "a" ... K jest a*) and considered the substitution of 'square circle' for '*a*'.[54] He never made any reference to the distinctive features of his interpretation of the quantifiers; on the contrary, in his 'Outline of a new system of the foundations of mathematics' he quoted, without disagreeing, a passage from Tarski where the latter refers to the Mitchell-Peirce interpretation of the quantifiers, which is identical with the Frege-Russell interpretation.[55]

Lejewski calls the russellian interpretation of the quantifiers a restricted one, since it covers only unshared names, and he shows how the russellian formula '$(\exists x)(Px)$' can be translated into the ontological$_L$ formula '$(\exists x)(ex(X) \cdot X\ est\ P)$'. However, he does not discuss the question how the ontological$_L$ formula '$(\exists X)(X\ est\ P)$' is to be rendered in russellian language.[56]

Leśniewski's special interpretation applies not only to the particular but also to the universal quantifier. 'For some *X*' and 'not for all *X* not' are

[53] Cf., for example, MATES 1950, p. 223.
[54] LEŚNIEWSKI 1914, p. 64, p. 67, p. 71.
[55] LEŚNIEWSKI 1929, p. 12. Kotarbiński in his popular introduction stated that '$(\exists X)(X\ est\ A)$' means "One can find such a name for '*X*', that its denotation (*desygnat*) falls under *A*", but he does not point out the unusual nature of this interpretation (cf. KOTARBIŃSKI 1929 (1961), p. 229).
[56] LEJEWSKI 1954.

equivalent also in Leśniewski's theory. As a matter of fact, he even included only the universal quantifier in the "official" version of his formal system, because the particular quantifier is only one of a whole series of restricting quantifiers (others are, for example: 'for every X which ...', 'for at least nX', 'for at most nX', and so on) and he found it impossible to elaborate a precise rule which would govern the introduction of all of them.[57] An example which is characteristic for Leśniewski's conception of the universal quantifier, is the formula '$(X)(X\ est\ X)$', which is a *false* ontological$_L$ sentence. It is to be read: "For every X: X is X", the variable 'X' again extending not over a range of objects but over a range of names. A falsifying instance is 'Pegasus *est* Pegasus' or the logically false sentence 'Λ *est* Λ'.

In contrast to the ordinary standard systems of logic[58] in the style of *Principia Mathematica*, Leśniewski's ontology$_L$ holds for all possible worlds of objects. It is a strictly logical system, making no assumptions about the existence of objects, and is trivially valid also for an empty universe. In the system of *Principia* the existence of an object is logically demonstrable via the generally valid theorem

$$(x)(Px) \supset (\exists x)(Px),$$

i.e.: 'If for every x, x has the property P, then there exists some x that has the property P'. With the aid of this theorem it is possible to deduce from a logically true formula like

$$(x)(x = x)$$

the assertion

$$(\exists x)(x = x),$$

i.e.: 'There exists some object x that is identical to itself'.

In ontology$_L$, on the other hand, it is impossible to deduce *a priori* the existence of an object from the corresponding theorems

[1] $(X)(X\ est\ F) \supset (\exists X)(X\ est\ F)$

i.e.: 'If for every X: X is F, then for some X: X is F', and

[2] $(X)(f(X)) \supset (\exists X)(f(X))$

[57] Cf. LUSCHEI 1962, p. 117; SOBOCIŃSKI 1960/61, p. 68, footnote 3.
[58] Cf. CHURCH 1958, p. 1013.

i.e.: 'If for every X the function f holds, then for some X the function f holds'. In [1] the antecedent '$(X)(X\ est\ F)$' is logically false, therefore '$(\exists X)(X\ est\ F)$' can never be detached. In [2], if a sentence of the form '$(X)(f(X))$' is logically true[59], then it must be the case that the functor has no existential import; but if the functor has no existential import, then '$(\exists X)(f(X))$' similarly has none, since existential import is not a matter of the quantifier '$(\exists X)$' but of the functor.[60]

The sentential calculus of "protothetics", which is presupposed by the name calculus of ontology$_L$, forms a further part of Leśniewski's logic.[61] And here too the quantifiers are interpreted in the same way: sentential variables refer to a value range of sentences, although the quantified sentences in which they occur are sentences of the object-language, and not of the meta-language. It is interesting to note that although Russell appears to have been the first to introduce the quantification of sentential variables[62], subsequently he did not develop the theory further. Perhaps this was due to the fact that here only an interpretation like Leśniewski's is satisfactory.[63]

It is interesting to note that both for Russell and for Leśniewski the problem of existence statements, particularly of negative existence statements, provided the starting-point for philosophical speculation. Their ways of solving this problem were, however, very different.

Russell did not like the fact that in a sentence such as 'Pegasus does not exist', a fictitious name stands in the subject place. As we have seen, he was convinced that every name represents an object in reality, and thus has existential import. He found a way out of this dilemma by means

[59] If '$X = Y$' is defined as '$X\ est\ Y.\ Y\ est\ X$', then '$(X)(X = X)$' is logically false. But there are other functions of X, for which '$(X)(f(X))$' is logically true, e.g., '$(X\ est\ X) \vee \sim (X\ est\ X)$'.

[60] For a limited theory of quantification where a russellian interpretation of quantifiers is combined with validity for the empty domain, cf. CHURCH 1951, p. 18; HAILPERIN 1953; QUINE 1954; HINTIKKA 1959. See also the papers by K. LAMBERT.

[61] Cf. footnote 28, p. 109.

[62] RUSSELL 1906a; a quantified sentential calculus was taken up again by ŁUKASIEWICZ 1929; cf. also ŁUKASIEWICZ-TARSKI; but Leśniewski's protothetics is the first complete system, which also contains variable sentential functors. It is, up to the present moment, the only worked-out system of this kind.

[63] Cf., below, p. 133.

of the theory of descriptions, which allows a "name" like 'Pegasus' to be regarded as a bundle of predicate expressions – 'horse', 'winged' etc. – all of which stand for universals actually occurring somewhere.[64] Because of the existential import of names, the particular quantifier, too, has existential import, and a special predicate 'exists' becomes unnecessary.

On the other hand, Leśniewski, as we have already mentioned, started from Mill's terminology of denotation and connotation. The difficulty, from his point of view, was that 'existing being' appeared to be a name that connotes nothing.[65] For, assuming further that a sentence is true if and only if the object denoted by its subject-term possesses all the attributes connoted by its predicate-term, he concluded that all existence statements had the same truth value, namely he thought that they were all false.[66] Because of this odd conclusion Leśniewski subsequently left connotation out of account, and developed a logical system, ontology$_L$, based solely on the relation of denotation.[67] He continued, however, to count 'existing being' or 'object' as an admissible expression of his system and thus kept particular quantification and assertion of existence separate.

8.4 Leśniewski's nominalism

The scholastic terms corresponding to (though not synonymous with) 'denotation' and 'connotation' are '*suppositio personalis*' (or '*supposito formalis*') and '*suppositio simplex*'. Although the earlier medieval scholastics bestowed much attention on *suppositio simplex*, later scholastics neglected it. The reasons for this were ontological. Whereas a platonist like Petrus Hispanus regarded a general term in *suppositio simplex* as representing a universal essence, nominalists like Ockham and Albert of Saxony would at most consider it as standing for a mental concept; and Buridan, in all consistency, left *suppositio simplex* entirely out of account.[68]

[64] Cf. p. 44 f., p. 68.
[65] LEŚNIEWSKI 1913, p. 22.
[66] LEŚNIEWSKI 1913, p. 12.
[67] KOTARBIŃSKI 1958, p. 4, draws attention to this change of approach.
[68] Cf. BOEHNER 1952; MOODY 1953, especially p. 33 f., where Moody mentions the difference between the platonistic "inherence theory" and the nominalistic "identity theory" of the copula. (The "inherence theory" is rather a "participation theory", cf. below, p. 163).

As already mentioned, Leśniewski had proved the definition of a general object to be contradictory, and it may well be that nominalistic considerations played some part in his decision to abandon connotation and to restrict himself to the relation of denotation. However, the "bankruptcy" of his pre-logistic period made him even more cautious, and unwilling to enter any "unscientific" metaphysical discussions.

Tadeusz Kotarbiński, on the other hand, not only adopted Leśniewski's logical system, but also wrote a considerable number of essays putting forward the metaphysical standpoint of reism.[69] He summarizes reism in the following three theses: "1. Every object is a thing. 2. No object is a property (*cecha*), a relation, an event or any of the other so-called objects alleged to belong to some ontological category other than that of things. 3. The terms 'property', 'relation', 'event' as well as all other alleged names of alleged objects supposed to belong to an ontological category other than that of things, are pseudo-names."[70]

This is a standpoint of categorial monism, the only category assumed being that of things. Kotarbiński does not reject, however, the subject-predicate sentences of ordinary language: "We assume, of course, that there is snow; that there is white snow; that show is white. It is only the alleged "whiteness of snow" that we refuse to accept."[71] He is thus primarily concerned to avoid naming abstract entities, and for this he relies on Leśniewski's ontology$_L$. He also refers to his viewpoint as '*concretism*', for he assumes no abstract, but only concrete entities. Further designations are 'pansomatism', since for Kotarbiński all things are material bodies, and 'radical realism', because he denies the existence of such things as mental images.[72]

However, the question arises whether it is proper to appeal to Leśniewski's logic in order to support a reistic standpoint. As we have seen, it is true that ontology$_L$ is based on the semantical relation between names and concrete individual objects, and that the particular quantifier only

[69] Kotarbiński first used the designation 'reism' in KOTARBIŃSKI 1929, p. 67. He mentions Leśniewski, Leibniz, and Franz Brentano as precursors of reism (cf. KOTARBIŃSKI 1958a, p. 39, p. 110 f.).
[70] KOTARBIŃSKI 1958a, p. 104–105. For Kotarbiński the designation 'object' is the most general name for "a being (*byt*)", "a something (*coś*)".
[71] KOTARBIŃSKI 1929, p. 65.
[72] KOTARBIŃSKI 1958.

affirms the existence of names, not of objects. An evaluation such as Quine's [73], according to which ontology$_L$ is a class calculus asserting explicitly the existence of abstract entities, overlooks the peculiar nature of a system based on *suppositio personalis* and fails to appreciate the special meaning of its particular quantifier, which is not an existential quantifier in the usual sense.

But although in ontology$_L$ the only primitive semantical categories are the category of names and the category of functors such as the copula '*est*', the theory allows the successive introduction of constants and quantified variables of every possible semantical category, and this makes it comparable with the simple theory of types.

For instance, the functor '*ex*' mentioned above[74], can be introduced by the definition

$$(X)\,(ex(X) \equiv (\exists Y)(Y\ est\ X))$$

i.e.: 'For every X: an X exists if and only if for some Y: Y is X'.

An empirical expression of the same semantical category as '*ex*' would be 'human' as introduced by the following definition on the basis of the *name* 'Human':

$$(X)\,(human\ (X) \equiv X\ est\ Human)$$

and this formula might be read: 'For every X : X has the property "*human*" if and only if X is human'. Note, however, that '*human* (X)' seems not to have any new existential import, it is merely viewed as another linguistic way of expressing that X is human. Furthermore, if 'A' is a shared name, then '*human* (A)' is not considered as meaningless, but merely as false.

Once constants of a specific semantical category are introduced into the system, then quantified variables of this semantical category may also be used, since Leśniewski's quantifiers assert only the existence of

[73] QUINE 1952. (Incidentally Quine once stayed in Warsaw and met Leśniewski personally.) Even A. N. Prior, who has been instrumental in promoting the study of Leśniewski's system (cf. PRIOR 1955), writes in one place that ontology$_L$ "is just a broadly russellian theory of classes deprived of any variables of Russell's lowest logical type" and reports approvingly the interpretation of Jerzy Łoś, according to which '*A est B*' expresses simply "the *inclusion* of a unit class in another class" (PRIOR 1965, p. 150, p. 151). However, two pages later Prior has to denounce the term 'class name' as "unfortunate" and speaks instead of "common names".

[74] Cf. pp. 116–117.

expressions. For example since '*human* (*Socrates*)' is true, the following is also true:

$$(\exists f)\,(f(Socrates))$$

which might be read: 'For some f: Socrates has the property f'.

However, the semantical category of names does not represent the ontological category of being an individual object, since it includes also common nouns; similarly the semantical category of the variable 'f' does not represent the ontological category of being a property of individual objects, because the same semantical category includes also functors which together with common nouns as arguments form true sentences. But as we defined a name 'object', so we can try to define a higher level functor 'Property', e.g.[75]:

$$(f)\{Property(f) \equiv (X)(f(X) \supset X\ est\ X)\}$$

This might be read: 'For every f : f is a property if and only if for every X it holds that if $f(X)$, then X is X'. The definition specifies that '*Property* (f)' is a true formula if and only if the argument of 'f' in a true atomic formula is an unshared name.

Instead of applying higher-level functors to lower-level functors we can also explore, in ontology$_L$, the possibilities of predicating functors of the same level of one another by means of a higher-level copula. An example of such a copula is '*est**', defined in the following way[76]:

$$(f)(g)\{f\ est^*\ g \equiv (\exists X)(\exists Y)(X\ est\ Y \cdot f(Y) \cdot g(Y) \cdot \cdot (X)(Y)(f(X) \cdot f(Y) \supset (Z)(Z\ est\ X \equiv Z\ est\ Y))\}$$

i.e.: 'For every f and for every g: $f\ est^*\ g$ if and only if (1) for some X and for some Y, X is Y and $f(Y)$ and $g(Y)$, and (2) for any X and for any Y, if $f(X)$ and $f(Y)$, then any Z which is X is also Y and vice versa'. The definition specifies (in the second part of the conjunction of the definiens) that '$f\ est^*\ g$' is a true formula only if the arguments with which the functor 'f' can form true atomic formulas must all have the same extension. That 'f' must represent exactly one extension corresponds to the requirement that if '$A\ est\ B$' is true, then 'A' must name exactly one object. It can

[75] Cf. SŁUPECKI 1955, p. 56.
[76] Cf. LEJEWSKI 1957, p. 249; see also SŁUPECKI 1955, for further analogues of '*est*'.

be proved that '*est**' is an exact analogue of '*est*', i.e. that if a formula with '*est*' and name variables is logically true, then a corresponding formula with '*est**' and functor variables is true.

Lejewski suggests that, e.g., the English sentence 'Man is a species' has the logical form '*f est* g*'. He believes that in this sentence 'man' and 'species' are only pseudo-names which name nothing and which in a strict logical formulation should be replaced by functors, so that we get: 'Forming-the-class-of-men est* Forming-a-species'. Lejewski does not give definitions of the functors in question, but the first can easily be defined in terms of the name 'Human':

$$(X)\{\textit{Forming-the-class-of-men}\,(X) \equiv (Y)(Y\ \textit{est}\ X \equiv Y\ \textit{est}\ \textit{Human})\}$$

i.e.: 'For every *X*: The *X*'s form the class of men if and only if any *Y* that is an *X* is also human and vice versa'. The definition specifies that in a true atomic formula the argument of the functor must have the same extension as 'Human'.

Although Leśniewski's system allows the successive introduction of constants and quantified variables of ever higher level, this seems not to increase the explicit ontological commitment of its formulas. It seems only to add new ways of speaking. The higher-level symbols are not considered as *naming* anything. The semantical categories of Leśniewski are not said to represent ontological categories but are considered merely as different grammatical "parts of speech".

Higher-order quantification, too, seems not to introduce a new ontological commitment, since the quantifiers range only over symbols. And there seems to be no danger of platonism here, because the symbols can be regarded as concrete tokens: in keeping with the "constructive nominalism"[77] of Leśniewski's system, the rules or "terminological explanations" refer only to sign tokens and stipulate that the deduction or introduction of new sentences can only be based on previous sentences which are effectively available, i.e., which have actually been written down. Although the system and therewith the number of signs may constantly increase, the number of signs at any one moment is always finite.

But if Leśniewski's system makes no explicit reference to abstract

[77] Cf. Luschei 1962, p. 125 f.

entities, this does not mean that reism (i.e., the assumption that there are only concrete things) can give a satisfactory justification of its working. The question, for example, *why* two things are subsumed under the same shared name is left unanswered by reism. As will be shown below, the predication of a general predicate can be justified only by assuming an additional ontological category, distinct from the category of things: one of abstract entities or at least one of concrete properties.[78]

Furthermore, the definitions in Leśniewski's system are more than merely convenient abbreviations which in principle could be dispensed with. Many of them allow us to replace one statement by an equivalent statement which contains symbols belonging to different semantical categories than the symbols in the first. Such definitions cannot be eliminated from the system, because they are "*creative*": through the addition of such a definition to the system sentences can be proved which would not have been provable without it, though these sentences contain neither the defined symbol nor a symbol defined with its aid.[79] To regard such indispensible higher level reasonings merely as calculations with graphical tokens seems not satisfactory. (Leśniewski, who always insisted on the intuitive meaningfulness of every formula of his system, would be the first to admit this.) To do full justice to the formulas of such a proof, it would seem that one must consider them not only as marks, but as marks which express a special *sense*; that like the Stoics one must go beyond reism and accept λεκτά, propositions.

However, it remains true that whereas a russellian type of system commits its user explicitly to the assumption of abstract entities into his universe of discourse, Leśniewski's system makes no such explicit commitment. One might think of clarifying the ontological commitment of Leśniewski's system by translating its sentences into russellian language. But this encounters the difficulty that Leśniewski's distinction between quantification and existential assertion cannot be rendered in russellian symbolism.[80]

[78] Cf. below, p. 178.

[79] Cf. LEŚNIEWSKI 1931; SŁUPECKI 1953, p. 51; SŁUPECKI 1955, pp. 64–65; SOBOCIŃSKI 1954/55, footnote 13; LUSCHEI 1962 p. 132 f. (Luschei prefers the term 'fruitful' to 'creative'). Concerning creative definitions see also MYHILL 1953.

[80] Cf. above, p. 118.

9. W. V. QUINE AND N. GOODMAN

For Quine and Goodman the distinction between names that name something and so-called syncategorematic signs, is of special importance. In order to answer the question "How and what do predicate signs represent?" in their sense, we must first know whether predicate signs are genuine names or merely syncategorematic words. As we shall see, Quine has found a criterion that indicates precisely which words the user of a frege-russellian language regards as genuine names: they are those words in respect of which quantification is permitted; i.e., those that name an object in the value-range of the variables.

We shall find that there are languages whose predicate signs purport to name something, and others with syncategorematic predicate signs. The former are known as platonistic, the latter as nominalistic languages. Mathematicians have long been in the habit of distinguishing between more or less "rich", more or less "platonistic" ontological models. These models may be regarded as value-ranges for the variables of more or less "rich" languages. To nominalistic languages correspond the universes described by Leśniewski's mereology.

As a nominalistic language makes fewer explicit ontological assumptions than a platonistic one, the question as to its adequacy as a language of science will arise. This constitutes a pragmatical criterion for the metaphysical standpoints underlying nominalistic and platonistic languages. Quine and Goodman would prefer a nominalistic language, where predicate expressions are regarded as syncategorematic. However, as we shall see, their account of the syncategorematic functioning of predicate signs is not very satisfactory.

9.1 Quine's criterion

9.11 *To be is to be the value of a variable*

The starting point for Quine is the same as that from which Russell de-

veloped his theory of descriptions.[1] His aim, however, is different from Russell's. The latter was concerned, as we have seen above, to eliminate those definite descriptions and apparent proper names to which no individual corresponds in reality. For this purpose he had developed his definition-in-use. And whereas previously he had regarded all words as genuine names, he now came to realize that there are what he called "incomplete symbols", i.e., signs that name nothing by themselves.

Quine, on the other hand, tends from the start to assume syncategorematic words, that do not name anything. One of the examples that he gives of such a word, is 'up'. The question he asks himself is not how such words can be eliminated, but in what respect their use differs from that of non-syncategorematic words, i.e., how one can tell which words a language-user employs as genuine names.

For example, someone who uses the word 'Bucephalus' as a genuine name, being convinced that a horse bearing this name occurs at some point in world history, will accept the following sentence as true: 'There is such a thing as Bucephalus'. And, e.g., the sentence 'There is such a thing as *the* property horse' will be denied by a nominalist, who does not accept abstract entities.

Quine warns against the danger of confusing a sentence of the form 'There is such a thing as so-and-so' or simply 'There is so-and-so' with a sentence of the form 'There is a so-and-so'. Thus 'There is such a thing as horse' (i.e.: 'There is such a thing as the property horse') is not equivalent to 'There is a horse', logistically: '$(\exists x)(x$ is a horse)'.[2] Quine calls a sentence of the first kind, a singular existence statement; a sentence of the second kind, a general existence statement. Frege and Russell had rejected singular existence statements as meaningless. Since they regard every constant expression as normally naming an existing entity, a sentence like 'a exists', where 'a' is a constant, is from their point of view superfluous.[3]

[1] This section is based on QUINE 1939, where the criterion was first formulated. Quine subsequently dealt with it in a number of papers; cf. in particular QUINE 1943 and the collected volume QUINE 1953.

[2] Note that the quantifiers are here used in the frege-russellian, and not in Leśniewski's sense.

[3] FREGE 1892a, p. 220; RUSSELL 1918/19, p. 46. In "free-logic" (cf. above, p. 113, footnote 37), which admits also empty individual names, a singular existence statement concerning an individual x is formulated '$\mathsf{E}!x$'. There the formula '$\mathsf{E}!x \equiv (\exists y)(x = y)$'

Quine, however, discovered that the assumption of a singular existence statement involves a consequence which one would not normally be prepared to accept with respect to all constants of a sentence. For example, assuming that there is such a thing as Bucephalus, a sentence containing the word 'Bucephalus', e.g., 'Bucephalus is a fiery horse', entails a general existence statement, in this case '$(\exists x)$ (x is a fiery horse)'.[4] If, on the other hand, the singular existence statement is false – as it would be, e.g., in the case of 'Pegasus', since there is no such thing as Pegasus – then the existential generalization may lead from a true sentence to a false sentence; e.g., from the true sentence 'Nothing is identical with Pegasus' to the false sentence '$(\exists x)$ (nothing is identical with x)'.[5]

A singular existence statement in respect of a specific entity thus amounts precisely to the admission of existential generalizations of sentences concerning this entity. If it is assumed that there is such a thing as appendicitis, then 'Appendicitis is dreaded' entails the sentence '$(\exists x)$ (x is dreaded)'. If, on the other hand, the word 'appendicitis' is regarded as syncategorematic and not as a genuine name, then this consequence cannot be accepted.

In a standard system the admissibility of existential generalization also entails the admissibility of so-called specification. That is, if the transformation of a sentence of the form '... a ...' into one of the form '$(\exists x)(\ldots x \ldots)$' is admissible, then the transformation of '$(x)(\ldots x \ldots)$' (i.e.: 'For all x: ... x ...') into '... a ...' is similarly admissible. As the converse holds also[6], it can be stated generally that singular existence statements in respect of an entity named by a constant 'a' correspond precisely to the admission that this entity may be regarded as a value of a variable 'x'.

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holds; it can be read: "to exist as an individual is to be the value of an individual variable", which is a special case of "Quine's criterion" (see below, p. 130). Cf. also above p. 116–117 for the functors which predicate existence in Leśniewski's logic.

[4] The example is ours. It seems best to introduce Quine's idea first by way of a case that is unconnected with the question of universals.

[5] This, of course, holds only if the quantifier '$(\exists x)$' is interpreted in the frege-russellian sense. If ('$\exists x$)' is read as a particular quantifier in Leśniewski's sense, then the last-mentioned sentence is not false but true. Cf. above, p. 117.

[6] QUINE 1939 does not mention the converse. In Leśniewski's view, the deduction of an existence statement from a general sentence runs counter to the nature of a logical theory (cf. above, p. 119).

Quine thus reaches the following important conclusion: "Here, then, are five ways of saying the same thing: 'There is such a thing as appendicitis'; 'The word 'appendicitis' designates'; 'The word 'appendicitis' is a name'; 'The word 'appendicitis' is a substituent for a variable'; 'The disease appendicitis is a value of a variable'. The universe of entities is the range of values of variables. *To be is to be the value of a variable*." [7]

The final sentence of the passage quoted above is the well-known formulation of what we have called Quine's criterion. It asserts that it is precisely those entities that are values of variables, whose existence is explicitly assumed in a language.

9.12 *Different kinds of variables*

If Quine in his criterion refers to *quantified variables*, and not to constants, it is not only because in his view constants can be apparent names, i.e., syncategorematic signs that do not name anything, but for other reasons as well. One of them is that the number of entities assumed within the range of the quantified variables of a specific theory, may be larger than the number of entities that can be designated by name constants with the aid of the linguistic potential of the theory. Thus the real numbers of classical mathematics form an infinity larger than the totality of names constructible in any language.[8] Furthermore it is possible that in some language there are no genuine names at all. As Quine has shown[9], the use of proper names is entirely avoidable, if these are replaced by descriptions which in turn are reducible by Russell's method. For example, instead of referring to Pegasus, we can speak of the only x that "pegasizes". ('To pegasize' means roughly the same as to be-a-winged-horse-caught-by Bellerophon').

Unquantified variables do not involve any special ontological assumptions. An expression in which such variables occur is never a sentence, but at most a sentential *form*. A sentential form of this kind is a schema that allows us to see at a glance the structure of all sentences obtainable by

[7] QUINE 1939, p. 708. The italics are ours.
[8] QUINE 1951, p. 67.
[9] QUINE 1953, p. 8.

putting constant expressions into the spaces occupied by the unquantified variables.[10]

In a language formulated as a predicate calculus quantification applies either only to variables of individuals or also to *predicate variables*: in the first case only individuals occur in the value-range of the variables, whereas in the second case explicit reference is made to the existence of entities under which different individuals are subsumed, and which thus are universal, abstract entities.[11] A nominalist claiming to use meaningfully a language of this latter kind would involve himself in an explicit contradiction.[12] Languages of this second kind are therefore called "platonistic languages", in distinction to languages of the first kind, which are called "nominalistic languages".

Calling a language "platonistic" does not imply that its users must accept all of Plato's theory of ideas. The word 'platonistic' is used here in a wider sense.[13] Thus, e.g., aristotelianism is in this sense a platonistic conception, since it makes reference not only to concrete individuals but also to abstract entities, which are designated by names. The question of how these abstract or ideal objects are perceived, is irrevelant here. And different views are also possible within what here is termed the platonistic conception, as regards their existential dependence or independence.[14] Thus (a) ideal objects may subsist in themselves as Plato

[10] Cf. QUINE 1953, p. 108. Quine has found a suggestive name for such variables: he calls unquantified predicate variables 'dummy predicates'.

[11] For a more detailed discussion, cf. below, p. 162.

[12] Apart from systems making type distinctions between different categories of variables (e.g., between variables of individuals and of predicates), there are systems where all variables belong to the same syntactical category. (Because of their formal simplicity, Quine himself has made a detailed study of the construction of such systems which are in the tradition of Zermelo. The greater formal simplicity of these systems, however, involves a corresponding loss of semantical clarity.) Despite the different "grammatical" formulation of such systems, the question concerning the acceptance of abstract entities continues to be relevant, but changes from a "category question" into a so-called "sub-class question": abstract entities, if presupposed, simply form a part of the single value-range comprising all entities.

In order to avoid contradictions hierarchical differences among the entities must, nevertheless, be taken into account. Formulae must be "stratified", i.e., be regulated by special postulates which exclude expressions such as, e.g., '$x \varepsilon x$'. Cf. QUINE 1951, p. 68 f.; QUINE 1953, p. 90 f., and elsewhere.

[13] Cf. above, p. 11, footnote 34.

[514] Cf. in this connexion the doctrine of existential moments of INGARDEN 1947/48, vol. 1.

thought (like real things, but in an even stronger sense); (b) the existence of some ideal entities may depend on that of other ideal entities; (c) the ideal entities may depend on the existence of the entities subsumed under them; (d) the ideal entities may be dependent on the existence of an actual conscious being; etc.[15]

Once predicate signs are quantified, they can also be put in the subject-place in higher-order sentences; for if predicate signs are regarded as genuine names of abstract objects then statements can be made about these objects, i.e., second-order properties can be predicated of them.[16] This corresponds to the use of abstract noun forms in natural language. For example, if the sentence 'The rose is red' means that there is something, namely the property Red, which belongs to the rose, then we can also make statements about this property Red, e.g. 'Red is a colour species', 'Red includes coloured', 'Red is universal', 'Red is a property', etc.[17]

As we have shown, Russell tended to regard all words as genuine names and he quantified predicate variables, thus using a platonistic language. On the other hand, Quine and Goodman prefer to use, if possible, a nominalistic language, where predicate signs are used only "syncategorematically".[18] Further below we shall discuss in detail the possibility of such a restriction to a nominalistic language.[19]

In set theory *special* terms and (quantified) *variables for classes* are used, beside the usual predicate expressions. Russell, as we have seen[20],

[15] For further details concerning the "nominalistic" – "platonistic" distinction, cf. below, pp. 136 ff.

[16] Quine 1963 investigates the consequences of these different possibilities in terms of set theory.

[17] Lewis 1946, pp. 42–43 calls ordinary-language expressions such as 'red', which can occur both in predicate and in subject place, "attributives"; and says that they are concrete terms when used in predicate place, abstract terms when used in subject place. However, the concrete – abstract distinction is best referred to whether or not we can quantify: for example, even if 'red' is used only in predicate-places, it must under certain conditions be regarded as representing an abstract entity. This holds for a russellian interpretation of ordinary language. If the word 'is' is interpreted in the sense of Leśniewski's ontological$_L$ functor, then 'red' in the assertion 'The rose is red' is always regarded as standing for concrete entities, viz. for red things.

[18] Goodman – Quine p. 105. Traditionally only logical constants and quantifiers, but not predicate signs, are called "syncategorematic". We shall therefore inquire what might be meant by the "syncategorematic" functioning of predicate signs. Cf. below, pp. 155 ff. and pp. 163 ff.

[19] Cf. below, p. 148 ff.

[20] Cf. above, p. 74.

introduced this class terminology by a definition-in-use in terms of quantified predicate variables. But often mathematical logicians take the class terminology as primitive. Names of particular classes can then be defined in terms of unquantified predicate signs[21]:

$$\hat{x}(Px) = df\ (\iota\alpha)(x)(x \in \alpha \equiv Px)$$

i.e.: 'The class of objects satisfying the sentential function Px is that entity α for which it holds that for all x, x is a member of α if and only if Px'. The introduction of '$\hat{x}(Px)$' is called an "abstraction" of a class, because it defines the name of an abstract entity on the basis of the functional expression 'Px', the 'P' of which makes no explicit platonistic assumptions since it is used only "syncategorematically". However, the *definiens* of '$\hat{x}(Px)$' already contains class variables and the symbol '$\in$' for class membership. The existence of the classes whose names are introduced in this way must be presupposed. It is postulated by special so-called comprehension axioms (also called *axiomes de séparation*), which have the general form:

$$(\exists\alpha)(x)(x \in \alpha \equiv Px).$$

Certain restrictions are, however, necessary: because of the possibility of antinomies not every 'P' is acceptable.[22]

Sometimes also quantified *sentential variables* are interpreted as extending over a value range of abstract entities. These abstract entities are said to be propositions belonging to the domain of the Stoic $\lambda\varepsilon\kappa\tau\grave{\alpha}$.[23] However, this interpretation is not in line with the russellian interpretation of the other quantified variables, since propositions are not that *about* which one normally speaks, as one speaks *about* things and properties.[24] We can thus understand why Russell, although he was the first to explore the possibility of quantifying sentential variables, subsequently dropped the idea.[25]

In any case, so far as *extensional* sentential calculi are concerned, nominalists need not worry about sentential quantification. Tarski has

[21] Cf., e.g., QUINE 1953, p. 87.
[22] Cf., e.g., WANG-MCNAUGHTON.
[23] Cf. above, p. 47, footnote 34.
[24] Cf. above, p. 45.
[25] Cf. above, p. 120, footnote 62.

shown how such quantifiers can be reduced quite simply: every sentence of the form '$(p)(\ldots p \ldots)$' or '$(\exists p)(\ldots p \ldots)$' can be interpreted as an abbreviation of a sentence of the form '$\ldots S \ldots \ \ldots \sim S \ldots$', or '$\ldots S \ldots \vee \ldots \sim S \ldots$', where '$S$' is an abbreviation of any sentence whatsoever.[26]

9.13 *On the precise formulation of Quine's criterion*

Quine's criterion has been accepted by "platonists" as well as by "nominalists"; thus by Alonzo Church, one of the foremost platonists; and by Nelson Goodman, a declared nominalist, who rejects abstract entities as unintelligible, and who, in collaboration with H. S. Leonard, has developed a "calculus of individuals" related to Leśniewski's mereology.[27]

In a paper 'Ontological commitment', Church quotes in detail from contemporary philosophical writing on ontological questions, and shows how assumptions inadvertently made because of the language, the phrases used, openly contradict the ontological theses put forward. He reaches the following conclusion: "No discussion of an ontological question, in particular of the issue between nominalism and realism, can be regarded as intelligible unless it obeys a definite criterion of ontological commitment."[28]

With respect to the criterion, Church specifies the following[29]:

(a) A *language* carries the ontological commitment of any of the sentences that are "analytic" in the language in question (i.e., whose truth follows logically from the semantics of the language; that are deducible from the axioms of the language with the aid of its semantical rules).

(b) Ontological commitment is carried in the first place by affirmative sentences with an existential quantifier (i.e., by general existence statements); for there are systems, not counted among ordinary standard systems of logic, where negative existence sentences or sentences with general quantifiers carry no ontological commitment.[30]

(c) A sentence of the form '$(\exists x)(Px)$' assumes such entities x for which it holds that Px. For example, '$(\exists x)(x > 10^{1000})$' assumes not only

[26] Cf. QUINE 1953, p. 118.
[27] Cf. above, p. 107.
[28] CHURCH 1958, p. 1012.
[29] CHURCH 1958, pp. 1013–14. The numbering is not in Church.
[30] Cf. above, p. 120, footnote 60.

positive integers but asserts more, viz. the existence of integers greater than 10^{1000}.

(d) "Ontological commitment" is an intensional notion, for it makes reference to a class concept (an intension) rather than merely to a class. For example, to assume unicorns is not the same thing as to assume purple cows, even though the two classes are empty and therewith extensionally identical.

This intensionality is also revealed if one tries to find a precise and adequate formulation of the criterion, which itself makes no undesirable ontological assumptions. Investigations have shown that such a formulation will probably have to be made in intensional language.[31] For consider the following extensional version of the criterion[32]:

[1] "'$(\exists x)(Px)$' presupposes objects of kind K if and only if (a) there are objects x that are of kind K and for which 'P' is true, (b) there are no objects x that are not K and for which 'P' is true.'

Here the user of the criterion can only ascribe to a theory the assumption of objects of whose existence he is himself convinced. Thus, for example, he could not say that '$(\exists x)$ (x is a unicorn') presupposes the existence of unicorns, for he would then be committed to asserting, in accordance with the first half of condition (a), that there are objects that are unicorns. It also follows from this version of the criterion that all false existence statements, such as '$(\exists x)$ (x is phlogiston)', '$(\exists x)$(x is Pegasus)', etc., make no ontological assumptions at all (and are therefore equivalent in this respect) since the second half of condition (a), asserting that there are objects for which it is true that they are phlogiston, Pegasus etc., is never true in the case of such sentences.

The following, second extensional version of the criterion is more satisfactory[33]:

[2] "'$(\exists x)(Px)$' presupposes objects of kind K if and only if for every object x: 'P' is true for x if and only if x is of kind K.'

[31] CARTWRIGHT 1954; ANDERSON 1957; SCHEFFLER-CHOMSKY 1959.
[32] Cf. CARTWRIGHT 1954, p. 323 (IV); Cf. QUINE 1953, p. 131.
[33] Cf. CARTWRIGHT 1954, p. 323 (V). In the definiendum we have replaced implication by equivalence, thus avoiding the inference that any false existence statement makes all kinds of assumptions, e.g., that rabbits exist, etc.

It no longer follows from [2] that one can ascribe to a theory only the assumption of objects of whose existence one is oneself convinced, and that false existence statements carry no ontological commitment. But it still follows that false existence statements all carry the same commitment: the sentence '$(\exists x)$ (x is a unicorn)' for example, assumes not only unicorns, but also phlogiston, etc.

A satisfactory criterion must be *intensional*, like, e.g., the following[34]:

[3] "$(\exists x)(Px)$' of theory T presupposes objects of kind K if and only if it follows *from the semantical rules of* T that for all objects x: if 'P' is true for x, then x is of kind K.'

9.2 Ontologically different universes of discourse

As we have seen, nominalistic and platonistic languages can be distinguished according to the types of entities assumed in the value-ranges of their variables. The distinction is one of universes of discourse, of different worlds presupposed as underlying the sentences. More specifically, these worlds are models of varying degrees of complexity.

9.21 *Individuals and classes*

Let us suppose, for example, a universe of three (or n) atomic individuals a, b, and c. From a nominalistic point of view it would be regarded as made up of seven (i.e., $2^n - 1$) different atomic and molecular (i.e., mereologically composed) individuals: a, b, c, ab, bc, ac, abc.[35] A platonist would likewise accept these concrete mereological "heap formations", but it is characteristic of his standpoint that he is further able to unify or "compress" entities into higher-order entities, viz. into abstract classes (sets).[36]

[34] Cf. CARTWRIGHT 1954, p. 354 (VI). [3] is intensional, since replacement after 'it follows from the semantical rules of T that' of an expression (e.g., of 'K') by an extensionally equivalent expression, may change the truth-value of the sentence (cf. below, p. 140).

[35] Concerning the characteristics of a nominalistic universe cf. especially Goodman in BOCHEŃSKI-CHURCH-GOODMAN 1956.

[36] Logicians usually prefer the term 'class', mathematicians the term 'set' (i.e., Cantor's term *Menge*), but the two terms are ordinarily used synonymously. Note, however, that in set theory the term 'class' is sometimes used in a special sense: for a plurality of

On the basis, for example, of the three (or n), atomic individuals a platonist can form seven (i.e., $2^n - 1$) different classes of individuals: (a), (b), (c), (a, b), (a, c), (b, c), (a, b, c), and will in most cases also assume an additional null class ( ). This, however, is only a beginning. On the level of classes of classes of atomic individuals, there will be (with the inclusion of null-classes) $2^{2^3} = 256$ (i.e., 2^{2^n}) further platonistic entities, such as $((a))$, $((a), (b))$, $((a), (a, b))$, and so on. At the subsequent level of classes of classes of classes of atomic individuals, the number of platonistic entities in our example become astronomical. And this is only the modest start of an unending process of class-formation. In view of the infinity thus opened up by the three atomic individuals, the nominalists' attitude of scepticism towards such explosive "procreation" becomes rather more understandable.[37]

What a nominalist finds unintelligible about platonism is the way it distinguishes between entities that have the same "content", i.e., that refer back to exactly the same individuals.[38] For example, in place of the class of individuals (a, b), the classes of classes of individuals $((a), (b))$, $((a), (b), (a, b))$, $((b), (a, b))$, $((a, b))$, etc., and of all other higher-order classes that can be formed on the basis of the two individuals a and b, the nominalist's world contains only the one mereological whole ab, consisting of the two individuals in question.

A class belongs to an ontological category different from that of the individuals subsumed under it. This is clearly shown, e.g., by the fact that

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objects which is said to have elements, but which itself is not allowed to be an element of a still higher entity, i.e., which cannot be "compressed" so that it becomes a regular set. The notion of a class in this special sense has been introduced into some set theories because in view of the antinomies not every definition of a plurality of objects is acceptable as the definition of a set. The expression 'compressible' was coined by H. Hermes (cf. BETH 1955, p. 147).

[37] It appears that models are also possible in which no atomic individuals occur, either because every individual can be subdivided mereologically, or because they contain only classes that have further classes as members. Cf. in this connexion GOODMAN 1951, p. 86, p. 177.

[38] Goodman applies the term 'generating relation' to both the mereological part-relation $\ll$ and the "ancestral relation" $\in^*$ of the platonistic element-relation, since they both lead from simple entities to complex ones; cf. GOODMAN 1958. In BOCHEŃSKI-GOODMAN-CHURCH, p. 21, he states incorrectly that in order to determine whether a given system is nominalistic or platonistic, we must know *either* which are its atoms *or* which is its generating relation. In fact, it is necessary to know both the one and the other.

a class of material objects is not subject to physical laws, as are its elements. For instance, it is meaningless to say that it expands when heated. With respect to abstract objects, a dual ontological structure must be assumed, distinguishing the properties of the abstract object from its characterizing marks, which refer to the properties of its elements.[39]

Since nominalists tend to be of a cautious disposition and have a predilection for easily surveyable systems, they also often adopt a *finitist* position, assuming only a finite number of atomic individuals (and therewith of individuals in general).

9.22 *Classes and intensions*

A universe like the one described above with its proliferation of classes, is not yet the ontologically richest kind of universe. Although a platonist who accepts such a universe is not bound by the nominalistic restriction that entities generated on the basis of the same atomic individuals cannot be different, he does accept the *extensionalistic* thesis that classes having exactly the same members cannot be different. *Intensionalistic* platonism, on the other hand, distinguishes between abstract entities which have exactly the same members: e.g., between the abstract property (the objective concept, the intension) of being a rational animal and the abstract property of being a featherless biped. Ontological universes and ontological points of view can thus be graded as in the table on p. 139.

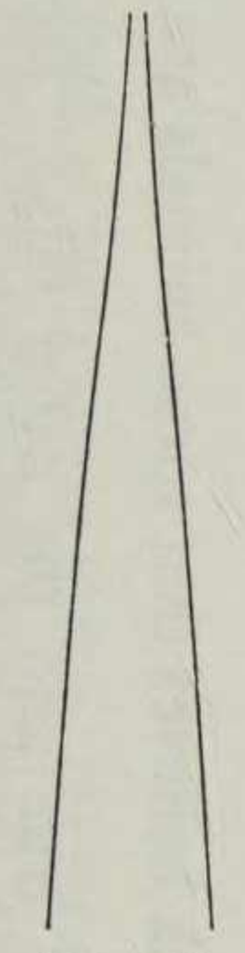
A good example of a convinced intensionalistic platonist is A. Church: "For just as an opaque body may be seen, so a concept may be understood or grasped. And the parallel between the two cases is indeed rather close. In both cases the observation is not direct but through intermediaries – light, lens of eye or optical instrument and retina in the case of the visible body, linguistic expressions in the case of the concept ..."[40]

It seems that the criteria for the identity of intensions, that is for the synonymity of expressions, can be interpreted ever more narrowly until finally no synonyms remain. Consider, for example, the following two sentences:

[1] 'Everyone who believes that all unmarried men are happy, believes that all unmarried men are happy.'

[39] Cf. above, p. 104 f.
[40] CHURCH 1951a, p. 104.

ONTOLOGICAL POINTS OF VIEW AND KINDS OF LANGUAGES:



			Examples which indicate the discriminatory force:				
extensionalism	nominalism	finitism	mereological whole of all featherless bipeds	=	mereological whole of all human beings	=	mereological whole of all human cells
	extensionalistic platonism		class of all featherless bipeds	=	class of all human beings	≠	class of all human cells
intensionalism (intensionalistic platonism)			abstract property of being a featherless biped	≠	abstract property of being a human being	≠	abstract property of being a human cell

[2] 'Everyone who believes that all unmarried men are happy, believes that all bachelors are happy.'

It seems possible that [1] is true and [2] false; which means that the expressions 'bachelor' and 'unmarried man' are not absolutely synonymous.[41] An extreme intensionalistic platonist might therefore distinguish two abstract properties, one of being a bachelor and another of being an unmarried man.

9.3 A new way of judging ontological points of view

Corresponding to the different ontological standpoints there are thus precisely differentiated languages, the value-ranges of whose variables are in each case restricted to the model of a specific ontological universe. This makes possible a new way of judging ontological points of view, a kind of pragmatic test. If an ontological point of view is to claim our serious attention, then the language corresponding to it must be demonstrably sufficient as a language of science. We shall examine below first the position in this respect of extensionalism and secondly that of nominalism.

9.31 *Intensionalism and extensionalism*

From a formal point of view, the intensionality of a language is reflected in the following distinction: whereas in an extensional language, quantifiable expressions that have the same extension, i.e., subsume the same individuals (such as, e.g., 'human being' and 'featherless biped'), can be substituted for each other without restriction and without altering the truth value of sentences, in an intensional language such substitution can lead from true sentences to false sentences. For example, of the sentences

'Jack believes that some animals are featherless bipeds'

and

'Jack believes that some animals are human beings',

one may be true and the other false. 'Jack believes that' is a non-

[41] Cf., e.g., MATES 1950, p. 215. See also below, p. 142 f. for Goodman's view on synonymity.

extensional context. Therefore in the sentence

'$(\exists P)$(Jack believes that some animals are P)'

the variable 'P' must have a value-range of intensions, not merely of classes.

Extensional languages meet the requirements of the above-mentioned test of adequacy for the formulation of valid scientific data, to a significant degree. They suffice for the construction of mathematical theories. The important systems of the foundations of mathematics, such as those of Frege, Whitehead-Russell and Leśniewski, use an extensional language. Many sentences which contain non-extensional contexts can be replaced by extensional sentences. But Carnap's *thesis of extensionality*, formulated in 1934, which states that for any non-extensional system in science there is an extensional system into which the former can be translated, has not yet been established. The problem of whether it holds or not is still unsolved.[42]

If the thesis of extensionality should turn out true and if therefore we could get along by using merely extensional languages, this would be a point in favour of extensionalism as a metaphysical point of view, but it would not decide the metaphysical question. On the contrary, as we have seen earlier[43], even for extensional languages questions of intensionality arise in connexion with the well-foundedness of relations and the synonymity of expressions.

Also in the semantical meta-theory of his language the extensionalist can refer only to extensional, but not to intensional entities, as designated by the words of his language, and this seems not satisfactory for the justification of semantical statements concerning the synonymity and the non-tautological analyticity of expressions.

It is therefore significant that Quine emphatically distinguishes two types of semantical theory: *the theory of meaning*, which is concerned with synonymity, analyticity, entailment, etc., and the *theory of reference*, which

[42] Cf. CARNAP 1934, § 67; CARNAP 1942, p. 249; CARNAP 1947, p. 141; cf. also RUSSELL 1940, ch. 19. A statement which resists translation into extentional language is the criterion of ontological commitment itself. It would seem that it can be satisfactorily formulated only in an intensional language (cf. p. 135 f. above).

[43] Cf. above, Ch. 7, pp. 86–96.

deals only with truth, denotation, truth-functional consequences, etc.[44] The theory of reference, the basic concepts of which have been formalized by Tarski[45], is in Quine's view a sound theory; but the theory of meaning he finds strongly suspicious, since it seems only to make sense if there is an intuitive understanding of intensions.

Goodman who, as a convinced nominalist, is particularly sceptical of the over-rich ontology of intensionalism, has published two important contributions, in which he attempts to give an extensionalistic criterion for the synonymity of terms and for the so-called projectability (related to Carnap's well-foundedness) of predicates respectively.[46]

The criterion for the *synonymity* of terms is based on the following principle: "... while it is clear that difference in meaning of two terms '*P*' and '*Q*' is not always accompanied by difference in extension, I think it is always accompanied by difference in the extension of certain terms other than '*P*' and '*Q*'."[47] Thus, for example, it is not the case that all pictures of unicorns are also pictures of centaurs, i.e., the extensions of 'unicorn-picture' and 'centaur-picture' are different, although the extension of 'unicorn' is equal to that of 'centaur', viz. empty. Goodman gives the following definition: two terms are synonymous if and only if neither their own extensions nor the extensions of compounds obtained by making identical additions to them, are different; i.e., if they have the same primary *and secondary* extensions.

However, it follows from this definition that different terms are never exactly synonymous. Thus the expressions 'bachelor' and 'unmarried man', ordinarily regarded as synonymous, have different secondary extensions. For example, the inscription token 'a bachelor who is *not* an unmarried man' (it is to be noted that this inscription token is a concrete physical object) belongs to the extension of 'bachelor-description' but not to that of 'unmarried-man-description'. Goodman thus replaces the notion of absolute synonymity by one of relative degrees of similarity-in-meaning.

[44] Cf. QUINE 1953, p. 130 f. On the relation between extensional and intensional semantics cf. among others CHURCH 1951a.
[45] Cf. TARSKI 1933, and by way of introduction TARSKI 1944.
[46] Cf. GOODMAN 1949, and GOODMAN 1955.
[47] GOODMAN 1949, p. 70.

A useful degree of synonymity can be defined by the following modification of the above-mentioned criterion: two terms are synonymous if and only if they have the same primary and the same *important* secondary extensions. "Unimportant" in this context are the extensions of those compounds for which it holds that corresponding compounds of any two terms always have differing extensions.[48]

Whereas Carnap arrived at the problem of the *well-foundedness of predicates* in trying to answer the question how the map of a relational network was to be co-ordinated with reality, Goodman encountered it while studying inductive method, namely in connexion with the central question of projectability.[49] For example, on the basis of previous experiences and in accordance with scientific practice, the following hypothesis can be set up, i.e., projected into the future:

[1] 'All emeralds are green.'

What, however, is the position of the hypothesis:

[2] 'All emeralds are grue'.

where 'grue' is defined as follows:

'grue' = *df* 'green until 31st December 1970 and afterwards blue'?

All experiences made up to day confirm [2] in exactly the same way as [1]. But no scientist would regard [2] as a projectable hypothesis.

Evidently the reason for this is, that in contrast to the meaning of 'green' the meaning of 'grue' is a fortuitous mixture. By what criterion, however, is one to decide whether a given word has a sufficiently coherent meaning to be projectable? The answer that this is a question of simple intuition of the intensional meaning of the word concerned, does not satisfy Goodman, who wants a more "tangible" criterion.

The definiens of 'grue' contains a concrete time specification ('31st December 1970'), and one might be inclined to regard this as a negative

[48] Cf. GOODMAN 1952a.

[49] For the following cf. GOODMAN 1955, in particular p. 74, p. 79 f. We wish to emphasize that we have selected only *one* argument from among the many to be found in this rewarding book.

criterion for the purpose of rejecting 'grue' as a general term in science. The same objection could, however, be raised against the normal and projectable predicate 'green'. For just as 'grue' can be defined with the aid of 'green', 'blue' and a date specification, so 'green' can be defined in terms of 'grue', 'bleen' and a date specification. ('Bleen' means "blue until 31st December 1970 and afterwards green".) The definition reads:

'green' = *df* 'grue until 31st December 1970 and afterwards bleen'.

Goodman finally bases his criterion on the fact that "well-behaved" and well-founded predicates like 'green' have become more and more "entrenched"[50] in the course of previous scientific practice, i.e., have become established in scientific vocabulary. He indicates a method for determining the degree of "entrenchment" of a predicate. Its projectability could then be read off from the degree of entrenchment.

Goodman is aware that not everyone will be satisfied with merely noticing a *de facto* entrenchment. One might wish to know *why* one predicate has become entrenched and not another.[51] However, Goodman does not believe this to be a fruitful line of enquiry. In his view the task of the philosopher consists in replacing vague intuitions, like those concerning the synonymity of terms or the projectability of predicates, by explicitly formulated criteria which regulate the application of the corresponding terminology (e.g., of the words 'synonymous' or 'projectable') in all important cases, and which thus give to the words a precise (if merely extensional) meaning.

9.32 *Platonism and nominalism*

9.321 Degrees of platonism

The designation 'platonism', which in logistic philosophy has almost entirely replaced that of 'platonistic realism', gained currency especially through its use by mathematicians in the context of set theory. Thus P. Bernays[52] distinguishes the following different degrees of platonism:

[50] GOODMAN 1955, p. 95.
[51] GOODMAN 1955, p. 98.
[52] BERNAYS 1935.

absolute, moderate (*restreint*) and constructive platonism. *Absolute platonism* assumes not only the existence of classes (sets), classes of classes, etc., but also believes that all these entities form a fixed totality where the law of the excluded middle holds. It underlied the first naive, "intuitive" version of set theory, which the discovery of the antinomies has shown to be contradictory.[53]

Moderate platonism has an ontological model, or universe, where classes are ordered in a hierarchy of ever higher types of classes. From the point of view of traditional philosophy the distinction between moderate and absolute platonism represents a rediscovery of the doctrine of the analogy of 'being': the term 'class' does not univocally designate entities which are all the same kind of beings, but entities that belong to essentially different levels. The ambiguity between the different levels, however, is not a random one, but is systematic. The relations between the entities of one level and the relations between the entities of another level, reveal common features; despite their essential differences, the various levels are linked by specific relations. The term 'class' is analogous.[54] This discovery represents an important addition to the doctrine of analogy, for modern theories of classes or sets, are formulated in a highly developed technical language, and thus provide examples of analogies of a much greater precision than have hitherto been available.

There are at present many different forms of moderate platonism under discussion, special emphasis being laid on the distinction between constructive and non-constructive forms. For example, in a *non-constructive* version of moderate platonism, a so-called quasi-combinatorial process can be repeatedly applied[55]: with each totality of entities, the next-higher totality of all combinatorially possible classes of these entities is also given (cf. the above-mentioned example, where the individuals a, b, c give rise to the classes (a), (b), (c), (a, b), (b, c), (a, c), (a, b, c)). The process of class-formation is called "quasi-combinatorial", because it starts with a totality consisting of infinitely many entities (e.g., the totality of all non-negative integers), so that the combinations cannot be effectively listed, since at once a non-denumerably infinite number of classes

[53] Cf. above, p. 72 f.
[54] On the equivalence between the "systematical ambiguity" mentioned in *Principia mathematica* and analogy, cf. BOCHEŃSKI 1948.
[55] BERNAYS 1935, cf. also MCNAUGHTON 1957, p. 71 f.

of these entities is obtained. *Constructive* versions, on the other hand, admit only classes that can be effectively constructed, e.g., by means of so-called predicative definitions[56], and contain no totalities with a more than denumerably infinite number of entities.

Among mathematicians, it is especially the *intuitionist* school that maintains a standpoint of constructive platonism. The school was founded in 1907 by L. E. J. Brouwer (*1881).[57] Brouwer's intuitionism has frequently been compared from an epistemological point of view with the conceptualist approach of kantianism.[58] Quine continues this tradition, distinguishing the conceptualism of intuitionism from the platonistic realism of non-constructive mathematics.[59] However, in the broad sense in which we use the term here, the intuitionists are platonists who accept the existence of some abstract entities. They even accept the existence of entities which have never been actually constructed in the mind. For example, it is unlikely that anybody has ever had the time actually to construct the number $10^{10^{10}}$. The intuitionists accept the existence of any abstract entity for which a method can be specified by which "in principle" it could effectively be constructed.

Whereas in a platonistic language, the entities in the value ranges of the variables belong to the different levels of one of the platonistic models mentioned above, the entities presupposed by a *nominalistic* language are all from the "ground floor" of such hierarchies: here quantification only extends univocally over a range of individuals (including individuals mereologically "composed" out of individuals); there are no classes of individuals.

9.322 Systems and experience

Even when a specific ontological structure (a nominalistic model or one of the platonistic models) has been selected, the question still remains, with which entities of experienced reality the entities of this structure are to be identified. There are at first no restrictions with regard to what is to count as an individual in a constructional system.[60] The natural tendency

[56] Cf. above, p. 75.
[57] Cf. HEYTING 1934; BROUWER 1952; HEYTING 1955; HEYTING 1956, for competent introductions to the work of this interesting school.
[58] Cf. BETH 1955, p. 150, and elsewhere.
[59] QUINE 1953, p. 14, p. 127 f.
[60] Concerning constructional systems cf. above, p. 34.

is to treat particular entities, such as physical objects, as individuals. However, one may also try to treat as individuals some of the entities that are ordinarily designated as abstract, and that are normally assigned to higher-order levels of platonistic systems. Thus we have seen that Russell treated *qualia* as individuals, and things as platonistic classes of qualia.[61] In fact, it seems that whatever has been regarded as a class, may also be regarded as an individual; with the provision, of course, that in any one system, the same entity of experienced reality may not *at the same time* appear *both* as a class and as an individual.[62]

At some time in the future it may perhaps be possible to demonstrate in a technical way that certain ontological classifications of the entities of experienced reality are more natural, i.e., simpler, than others. But in view of our lack of experience concerning the construction of such systems, and concerning the use of the predicate 'simpler'[63], we can as yet reach no sound conclusions on this point.

As Goodman emphasizes, the modern technical term 'nominalistic' is thus applicable to systems that each assume very different entities as basic individuals.[64] He mentions the following types of constructional systems, all of which can also be nominalistic:

(a) particularistic-physicalistic systems, where physical objects are assumed as basic individuals;

(b) particularistic-phenomenalistic systems, that assume phenomenal *concreta*, e.g., elementary experiences (*Elementarerlebnisse*), as basic individuals;

(c) realistic-physicalistic systems, where the basic individuals are properties of things;

(d) realistic-phenomenalistic systems, that assume phenomenal *qualia* as basic individuals.[65]

Here an important complication must be mentioned: an individual

[61] Cf. above, p. 71 f.

[62] Thus Goodman in BOCHEŃSKI-CHURCH-GOODMAN, pp. 17–18: "Whatever can be constructed as a class can indeed be constructed as an individual."

[63] For a measure of simplicity cf. GOODMAN 1951, 2nd. ed. 1966, pp. 66–123.

[64] The basic individuals of a constructional system are those individuals that are basic units of the system. And the basic units of a system are those entities to which is ascribed one of the undefined predicates of the system. Cf. GOODMAN 1951, p. 86.

[65] GOODMAN 1951, p. 101 f.; p. 143 f.

of a system may *in* the system be called "abstract", and an abstract entity of a system, such as a class, may *in* the system be called "concrete". This happens if, for instance, in a realistic-physicalistic system we define physical objects as certain classes of properties, i.e., as certain classes of individuals of the system. Because then we may introduce the predicates 'concrete' and 'abstract' in such a way into the system, that in agreement with the ordinary way of talking, 'abstract' is predicated of the properties and 'concrete' is predicated of those constructs which we labelled 'physical objects'.[66]

Goodman calls the task of defining entities which are ordinarily called "concrete" (resp. of entities which are ordinarily called "abstract") on the basis of entities which ordinarily are called "abstract" ("concrete"), "the problem of concretion" ("the problem of abstraction").[67] There are thus, so to speak, two problems of universals: (a) the question put from "outside" the system, asking whether the system is formulated in a platonistic language, i.e., whether it is committed to an ontological model containing not only individuals, but also classes of individuals; (b) the question whether within the system a predicate 'abstract' or 'universal' has been defined to apply to certain entities.

9.323 Nominalistic reformulations

In view of the fact that most logistic systems used so far in practice, have been formulated in a platonistic language, Quine and Goodman investigated in detail the possibilities of converting a platonistic system into a system with a narrower nominalistic basis.[68] Every platonistic language contains a nominalistic language as sub-language, but in addition admits sentences with quantified predicate signs. The problem,

[66] GOODMAN 1951, pp. 199–202. As a matter of fact predicates like 'concrete', 'abstract', 'particular', 'universal' are given a more precise meaning within the system than they ever had in ordinary philosophical discourse.

[67] Cf. GOODMAN 1951, pp. 110–113, pp. 160–172.

[68] QUINE 1947; GOODMAN-QUINE; GOODMAN 1951, in particular p. 33 f.; QUINE 1953, ch. 6. R. M. Martin's so-called "nominalistic" systems (cf. MARTIN 1943; MARTIN 1949; MARTIN 1949*a*; MARTIN 1950; MARTIN-WOODGER) are not genuine, i.e., are not nominalistic systems in the sense of Goodman and Quine (cf. MYHILL 1950); their ancestral quantifications (cf. MARTIN 1943, p. 4), though original, are inadmissible from a nominalistic point of view; also Martin assumes the existence of infinitely many individuals.

therefore, is to replace these specifically platonistic sentences by equivalent formulations in nominalistic terminology.

Quine has shown how the sentences of a theory of classes based on a *finite* number n of individuals, can be regarded as abbreviations of nominalistic sentences about these n individuals.[69] An odd complication arises from the fact that the number of symbols needed to write down these nominalistic sentences turns out to be greater than n. Therefore, if we assume (as most nominalists would) that sentences exist only if there is an actual inscription of them, then Quine's proposal works only if the n individuals do not exhaust the number of individuals in the actual universe.

Platonistic systems, however, are usually based on infinitely many individuals, and then no general method of translation into nominalistic language is available. Nevertheless, a nominalistic rendering of platonistic sentences is often possible in concrete instances, as Goodman has found. The method is to replace platonistic sentences about classes by sentences about mereological wholes, and to develop the so-called calculus of individuals in the place of the theory of classes.[70] From the wealth of material available, we shall discuss two examples which show the possibilities and limitations of the procedure particularly clearly.

Our first example concerns the definition of the ancestor relation.[71] This definition allows the reduction of one important predicate of individuals to another predicate of individuals, which is of considerable importance from the point of view of economy. It would, however, be quite in order to have both predicates as undefined primitives in a nominalistic system, since as long as predicate signs are syncategorematic, they do not offend against nominalistic standards. At a first glance it might therefore seem all the more surprising that the nominalistic formulation of a definatory relation between two such predicates is particularly difficult.

Let us find out, then, how the relational predicate 'ancestor' can be defined with the aid of the relational predicate 'parent'. Frege gives the following formulation in terms of the theory of classes:

[69] QUINE 1947, p. 81 f.
[70] Cf. above, pp. 105–107.
[71] After GOODMAN-QUINE, pp. 108–9; GOODMAN 1951, p. 40.

$$a \text{ is ancestor of } b = df$$
$$a \neq b \cdot (x)\{b \in x \cdot (y)(z)(z \in x \cdot y \text{ is parent of } z \supset y \in x) \supset a \in x\},$$

i.e., '*a* is an ancestor of *b* if and only if *a* is different from *b*, and if it is true for all classes *x* that if *b* is a member of *x* and all parents of members of *z* are members of *x*, then *a* is a member of *x*'.

A nominalistic version is the following one:

$$a \text{ is ancestor of } b = df$$
$$a \neq b.\ (\exists u)\ (a \text{ is parent of } u).\ (\exists w)\ (w \text{ is parent of } b).\ (x)\ \{b \text{ is part of } x\ .\ (y)(z)\ (z \text{ is part of } x \cdot y \text{ is parent of } z \supset y \text{ is part of } x) \supset a \text{ is part of } x\},$$

i.e., '*a* is an ancestor of *b* if and only if *a* is different from *b*, and if *a* is a parent, and if *b* has a parent, and if it is true for all mereological individuals *x* that if *b* is a part of *x* and all parents of parts of *x* are parts of *x*, then *a* is a part of *x*'.

Here the definiens has been expanded by the addition of the condition

$$(\exists u)(a \text{ is parent of } u) \cdot (\exists w)(w \text{ is parent of } b)$$

This ensures that *a* and *b* are discrete human beings, and not proper mereological parts of one human being or mereological "cuts" from "heaps" of several human beings. The relational predicate 'ancestor' thus applies to precisely those individuals which, on the class-theoretical view, are members of the classes referred to.

A serious drawback of the above nominalistic definition is that, unlike the class-theoretical definition of the ancestral relation, it cannot be transferred generally from the human ancestral relation to any predecessor relation whatsoever. (For any two-term relational predicate '*R*', there is always a predecessor predicate which correlates every individual with those individuals that are connected with it through a chain of *R*-relations.) The above nominalistic definition works only in the case of a relation *R* where every individual has at least one part unshared by any "predecessor". For example, if *n* is a successor of an individual *m*, and *n* is a proper part of *m*, and *m* has a predecessor and *n* has a successor, then *n* would be falsely counted among the predecessors of *m*. Up to the present, it has not been possible to find a generally valid nominalistic schema of definitions of predecessor predicates.

Our second example is a numerical sentence and concerns the nominalistic formulation of such apparently simple sentences as 'There are more dogs than cats'.[72] In view of the extreme platonism of the logicist definition of numbers as classes of classes, it is not surprising that such nominalistic formulations involve special difficulties.

As in the case of our first example, there is no problem if one does not mind accepting an excessively large number of undefined predicates. The sentence 'There are more '*A*'s than *B*'s could be trivially formulated in nominalistic language as follows:

$$(x)\{(y)(Ay \vee By \supset y \textit{ is part of } x) \supset \textit{Has-more-A's-than-B's-as-parts } x\},$$

i.e.: 'For all x: if it is true for all y that if y is A or y is B, then y is a part of x, then x has more *A's* than *B*'s as parts'. However, for every pair of constants (such as '*A*' and '*B*') an additional predicate (such as 'Has-more-*A*'s-than-*B*'s-as-parts') would have to be introduced, and this would be very uneconomical indeed.

A strategically preferable procedure would therefore be to use a predicate that can be applied in a large number of instances, such as the relational predicate 'is greater than' ('>'). With its help, the predicates 'is as great as' and 'is an *A*-*B*-bit' can be defined:

$$a \textit{ is as great as } b = df \sim a > b \cdot \sim b > a,$$

i.e.: '*a* is as great as *b*, if and only if *a* is not greater than *b* and *b* is not greater than *a*'.

$$a \textit{ is an A-B-bit} = df\ (\exists y)\{a \textit{ is as great as } y \cdot (Ay \vee By) \cdot (z)(Az \vee Bz \supset \sim z > y)\};$$

thus, for example: '*a* is a dog-cat-bit if and only if *a* is as great as the smallest animal that is a dog or a cat.' 'There are more dogs than cats' can now be rendered as follows:

$$(x)\{(s)(s \textit{ is a dog} \supset (\exists t)(t \textit{ is part of } s \cdot t \textit{ is a dog-cat-bit} \cdot t \textit{ is part of } x)) \supset (\exists y)((u)(u \textit{ is a cat} \supset (\exists v)(v \textit{ is part of } u \cdot v \textit{ is a dog-cat-bit} \cdot v \textit{ is part of } y)) \cdot x > y)\}.$$

[72] GOODMAN-QUINE, pp. 109–111; GOODMAN 1951, pp. 36–39.

i.e.: 'For every mereological whole that contains a dog-cat-bit of every dog, there is a smaller mereological whole that contains a dog-cat-bit of every cat'.

In the same way we can render sentences like 'There are more districts than cantons' and 'There are more telephone areas than cantons'. (A telephone area can incorporate bits of different Swiss cantons.) The method indicated above works so long as neither the *A*'s nor the *B*'s overlap; so long as we can be sure that it cannot happen that in the contrasted mereological wholes a single *A*-*B*-bit stands for two different *A*'s or two different *B*'s. By a slight alteration, the method can be so far generalized as to be applicable in all cases where every *A* has a part unshared by any other *A*, and every *B* has a part unshared by any other *B*.[73] But no generally valid method of translating 'There are more *A*'s than *B*'s' into a nominalistic language, applicable to all *A*'s and *B*'s whatsoever, has (so far) been found.

These examples show that, if a serious effort is made, in many cases a nominalistic rendering of platonistic sentences about concrete individuals can be found, and that with some practice a nominalistic reduction of some undefined basic signs can also be achieved. On the other hand, nominalistic methods cannot be generalized with the same ease as their platonistic counterparts. Further advances will no doubt result from future research, but it does seem that there are certain insurmountable difficulties inherent in the nominalistic approach.

The above examples involve sentences in which mathematical expressions are applied to concrete objects. (The predecessor-relation is also basic to mathematics, e.g., with respect to the number series.) Mathematical sentences proper, on the other hand, are ordinarily interpreted as referring to numbers, i.e., to entities usually regarded as abstract objects – at least if physical things are regarded as individuals[74] – and as such they are unacceptable on a nominalistic view. Until some other interpretation is found, nominalists have no choice, therefore, but to regard

[73] This alteration (a) replaces 'is an *A*-*B*-bit' by a predicate applicable to bits as great as the smallest of the above-mentioned unshared parts, and (b) formulates the condition that a single such bit in the contrasted mereological wholes may never stand for two different *A*'s or two different *B*'s.
[74] Cf. p. 146 f.

mathematical formulae as objects in a game of symbols and to use them only like an abacus, like an uninterpreted calculus.

The idea of axiomatizing mathematics and of investigating the resultant system of formulae without reference to its interpretation, can be found already in the mathematical school of formalism founded by Hilbert, where the aim was to find a proof of non-contradiction, i.e., a proof that not any formula whatsoever, is deducible in the system.[75] Formalism is therefore often referred to as "nominalistic"[76], but this designation is incorrect. For the signs of the formalized system are the object of a higher-level intuitive theory, viz. metamathematics. And this latter must be platonistic, at least in the constructive sense, if it is to contain a proof of the non-contradiction of formalized mathematics.

If a nominalist wishes to describe the syntax of a calculus, then he must use a nominalistic language. If, for example, he regards physical objects as individuals, then only concrete inscriptions (or tokens or marks), and not general expressions or types, may occur in the value ranges of the variables ("marksism"). Reference to the form of the inscriptions may be made only with the aid of syncategorematic predicates. Goodman and Quine have dealt with this question in detail, and have shown how a nominalistic syntax can be elaborated.[77]

Again, the curious complication mentioned above is encountered: account must be taken of the total number of individuals making up the physical world. A nominalistic syntax can deal only with actually available inscriptions, and these cannot, of course, have more elements than there are physical individuals in the universe. And a nominalistic sentence can only be regarded as an abbreviation of another sentence, or as a theorem deduced from another sentence, if this other sentence is actually available somewhere in the universe.[78] A platonistic syntax, on the other hand, encounters no such difficulties, since it can refer not only to the types exemplified in actually available inscriptions, but has access to an infinite world of classes containing abstract entities corresponding to all possible combinations of basic signs of the language.

[75] Cf. HILBERT 1904, and HILBERT-BERNAYS.
[76] Thus, among others, QUINE 1953, p. 14.
[77] GOODMAN-QUINE, p. 112 f.
[78] QUINE 1947, pp. 83–84; GOODMAN-QUINE, p. 106.

In conclusion, the following may be said about the usefulness of platonistic and nominalistic languages, respectively, which will amount to a pragmatic evaluation of the metaphysical views underlying these languages:

(a) A language corresponding to absolute platonism is contradictory.

(b) A nominalistic language can say surprisingly much. This is so because in principle any "syncategorematic" predicate expression whatsoever is admissible. And with some ingenuity it is in many cases possible to reduce the number of undefined basic signs considerably; even if, in comparison with the possibilities of platonistic languages, there appear here certain specific limitations. However, the decisive drawback of nominalism seems to be that it can interpret mathematics only as an abacus.

(c) A language of moderate platonism appears to be the most advantageous, but it is not clear precisely in what way absolute platonism is to be restricted.

Quine, who tends to be pragmatic and "tolerant" in his philosophical thinking, does not insist on a strict nominalism under these circumstances, but favours a compromise with platonism, though he would like his platonism to be as moderate as possible.[79] Goodman, on the other hand, is more conscientious: "The scientist may use platonistic class constructions, complex numbers, divination by the inspection of entrails, or any claptrappery that he thinks may help him get the results he wants. But what he produces then becomes raw material for the philosopher, whose task is to make sense of all this: to clarify, simplify, explain, interpret in understandable terms. The practical scientist does the business but the philosopher keeps the books. Nominalism is a restraint that a philosopher imposes upon himself, just because he feels he cannot otherwise make real sense of what is put before him."[80]

Goodman emphasizes the successes achieved with respect to nominalistic constructions. Thus, for example, there is even a beginning of an economical treatment of sentences with shape predicates.[81]

[79] Cf. QUINE 1953, pp. 173–74; QUINE 1960, p. 243, p. 269.
[80] BOCHEŃSKI-CHURCH-GOODMAN, p. 28.
[81] Cf. GOODMAN 1951, p. 203 f.

9.4 The syncategorematic functioning of predicate signs

On the view of Quine and Goodman predicate signs can be regarded as syncategorematic. But what exactly does this mean, and what is the origin of this view?

Traditionally only logical constants and quantifiers, and not predicate signs, are called "syncategorematic". Thus the earliest reference to the term '*syncategoremata*' has been found in Priscianus, the 6th-century grammarian, who says, referring to the teaching of the "*dialectici*" (the Stoics?): "Speech consists of two parts, the *nomen* and the verb, for these alone, through the mere fact of being joined together, form a complete utterance; the remaining parts they call *syncategoremata*, i.e., "consignifiers"."[82] But already Aristotle had not only distinguished between nouns (*ὀνόματα*) and predicates (*ῥήματα*), as had Plato, but had also made allowance for words that have no independent meanings (*σύνδεσμοι*), i.e., literally: "conjunctions".[83]

9.41 *Quine's views*

Quine's thinking on semantics, like that of most contemporary logicians, starts from Frege's distinction between sense (meaning) and reference (naming). But Quine applies Frege's schema to predicate signs in his own way.[84] Corresponding to sense and reference of genuine names of individuals, Quine distinguishes the meaning and *extension* of predicate signs, where the extension of a predicate sign consists of those individuals to which the predicate sign in question can be ascribed truly. Thus, for Quine, the function of a syncategorematic predicate sign which corresponds to the naming function of an individual name, is that of being true of individuals.[85] This is similar to Leśniewski's view concerning general

[82] "*partes orationis sunt duae, nomen et verbum, quia haec solae et iam per se coinunctae plenam faciunt orationem; alias autem partes syncategoremata, hoc est consignificantia, appelant*" (PRISCIANUS, II, 4, 15).

[83] Cf., e.g., ROBINS 1951, p. 19. On the scholastic usage of the term 'syncategorematic' cf., e.g., BOEHNER 1952, p. 19 f.; MOODY 1953, p. 16 f.

[84] In his most well-known essay on sense and reference (FREGE 1892), Frege does not deal with predicate signs. Elsewhere, however, he says that concepts (*Begriffe*) are the reference (*Bedeutung*) of predicate signs; cf. FREGE 1892a, p. 198.

[85] QUINE 1953, p. 21.

names, which we have seen above.[86] But as in the case of Leśniewski's system, so the question arises here: why is a certain predicate sign true for some individuals and not for others? Let us therefore investigate whether Quine's views on the *meaning* of predicate signs will provide an answer to this question.

Like most modern logicians, Quine finds the notion of objective meaning, i.e., that of Frege's sense, a difficult one. He notices the connexion between the notion of meaning and the traditional notion of essence, and remarks wittily: "Meaning is what essence becomes, when it is divorced from the object of reference and wedded to the word."[87]

Quine gives the following reason for this divorce. According to Aristotle, men, for example, are essentially rational, whereas being two-legged is only accidental for them. This, Quine thinks, can only be interpreted as meaning, if anything, that being rational is involved in the meaning of the word 'man', whereas being two-legged is not. However, in Quine's view a man could equally well be designated by the word 'biped', in which case being two-legged is involved in the meaning of the word, whereas being rational is not. Therefore Quine thinks that speaking of things instead of words, an individual designated both by 'man' and by 'biped' would have to be regarded both as being essentially rational but only accidentally two-legged, and as being essentially two-legged but only accidentally rational, i.e., one would get an explicit contradiction.[88]

From a traditional point of view, however, the two words are not of equal weight in explaining the individuals in question: The word 'man' with the meaning "rational animal" expresses a more profound understanding of them than the word 'biped'. The former is said to indicate the essence of the individuals, while the latter only points to one of their external features. Of course, the external feature of bipedity essentially includes two-leggedness (since it is even identical with two-leggedness), but this does not mean that the individuals have to be essentially two-legged.

[86] p. 111 f.

[87] QUINE 1953, p. 22.

[88] This argument (cf. also WHITE 1950, p. 330 and above, footnote 16, p. 93) appears to have been first put forward by C. I. Lewis (LEWIS 1944, p. 238; LEWIS 1946, p. 41). However, by the 'signification' of a term, Lewis explicitly understands the properties present *in things* that justify the application of the term to the things (cf. LEWIS 1946, p. 39, p. 133 footnote), while for Quine the meaning of a word is nothing tangible.

Divorce, for Quine, is not a solution. Separated from the objects of reference, the meanings cannot find a new home with the words[89], but become for Quine "obscure intermediary entities"[90] that are neither things nor signs. Quine gives up all belief in their existence; there are for him no meanings in the sense of abstract entities that can be named, but only linguistic expressions of which it can be said that they are meaningful and, in certain cases, synonymous with one another. The words 'meaningful' and 'synonymous' themselves are on this view like other predicate signs, syncategorematic expressions that do not name anything.[91]

Quine attributes meaningfulness first and foremost to entire sentences. He bases his standpoint on that of Frege, who, he says, had put an end to the old-established term-by-term empiricism.[92] Frege had analyzed propositions into functions and arguments – not into separate, independent terms – and had stressed the unsaturated nature of functions.

Russell, who, as we have seen[93], adopted Frege's analysis, used in connexion with functions the designation 'incomplete symbols'. He applied the same designation to descriptions[94], since in his view these, too, were meaningful only in the context of an entire sentence and not by themselves. However, functions and descriptions are two very different cases which should not be subsumed under the same designation: a functional sign is an integral part of a sentence, whereas a description can be eliminated. But Russell's ontological views were not greatly influenced by his views on incomplete symbols. He considered analysis into functions and arguments important insofar as it brings out type differences; but on the whole he preferred, as we have seen, to think in terms of logical atomism and to analyze states of affairs into individuals, properties and relations. The second kind of incomplete symbols, viz. descriptions, he discounted as soon as he found that they could be eliminated.

Quine's reaction, on the other hand, is different. From his point of view, Russell's theory of descriptions demonstrates the important fact that there

[89] Cf. above, p. 57, on how the intensional content cannot be contained in the signs.
[90] QUINE 1953, p. 22.
[91] QUINE 1953, p. 11.
[92] QUINE 1953, p. 42, p. 39.
[93] Cf. above, p. 77.
[94] Cf. above, p. 44.

actually are "incomplete symbols", i.e., syncategorematic expressions; and that it is not necessary for a word to have an independent sense in order to be used in a meaningful sentence. He therefore believed that he could justifiably give up the notion of the separate meaningfulness of individual words.[95]

But the meaningfulness of entire sentences poses new problems. As we have seen[96] it is customary to distinguish between synthetic (*a posteriori*) sentences, that have an empirical meaning and are true or false in the light of factual experience, and analytic (*a priori*) sentences, which can be regarded as true independently of empirical verification. But Quine is unable to find a criterion for distinguishing between synthetically meaningful and analytically meaningful sentences. This is not astonishing since, as we have seen, the truth of some analytic sentences is dependent on meaning relations between individual words, the separate meaningfulness of which Quine has given up.

The solution which Quine adopts in the face of this situation, is typical of his general way of thinking: even an entire sentence, he claims, is meaningless if taken in isolation; only the totality of sentences asserted by science can be said to be meaningful. It is only the system as a whole that can be confronted with reality and tested for its truth. It is "a man-made fabric, which impinges on experience only along the edges".[97] Some sentences have a central place in the system, i.e., they are not easily displaced by new findings, while others are more peripheral, i.e., are less resistant if confronted with conflicting experience. The former correspond approximately to analytic, the latter to synthetic sentences, but no fixed boundary separates the two groups: it is possible that experiences will force us one day to revise even our most central assumptions. For Quine the two assertions, viz. that our knowledge is reducible to separate, individually meaningful sentences, and that analytic and synthetic sentences can be precisely distinguished, are unfounded dogmas.[98]

However, it seems that by transferring meaningfulness from predicate signs to entire sentences and finally in a pragmatic way to the whole

[95] QUINE 1953, p. 6, p. 39.
[96] Cf. above, p. 91 f.
[97] QUINE 1953, p. 42.
[98] Cf. QUINE 1953, p. 41 f.; QUINE 1951.

system, Quine does not explain, but avoid the issue. Quine explains neither for one word nor for the whole system in what way it is meaningful, i.e., *how* it applies to reality.[99]

9.42 *Goodman's views*

Goodman, too, refers to predicate signs as syncategorematic without giving a *positive* explanation of their semantical function. On his view, as on Quine's, 'syncategorematic' means the same as 'not referring' and is therefore only a negative description.

Goodman is convinced that a comparison between reality on the one hand and a descriptive system on the other, is impossible: and he therefore does not make any sustained investigation of semantical relations. "There is", he says, "no such thing as the structure of the world for anything to conform or fail to conform to".[100] We only know about the world insofar as we have described it. And to describe something means, for Goodman, to express it in a conventionally schematized, abbreviated way; to construct a map. There is no such thing as a total rendering, free of all abbreviation, wherein *the* structure of the world would be revealed, for this would no longer be a rendering, but reality itself. "There is no such thing as a completely unabridged map; for abridgment is intrinsic to map making."[101] And in order to read the map we must possess the clue to the correct interpretation, for without it every picture can be made to yield whatever information we please: "With suitable principles of correlation, Constable's landscape painting could provide an enormous amount of information about a pink elephant."[102]

Indeed, not only the description of our experience but our experience itself is more than a simple registration of reality: "Without some schema there is no perception" and "that we know what we see is not truer than that we see what we know. ... "There is no innocent eye"."[103]

However, Goodman's conventionalism is not exactly like Carnap's.[104]

[99] Note that Goodman, too, accords only metaphorical value to Quine's saying that a system "impinges on experience only along the edges". Cf. GOODMAN 1952, p. 164.
[100] GOODMAN 1960, p. 56.
[101] GOODMAN 1956, p. 84.
[102] GOODMAN 1960a, p. 599.
[103] GOODMAN 1960a, p. 596.
[104] Cf. above, pp. 97–101.

Whereas Carnap tends to regard the choice of logical apparatus, the decision between a nominalistic and a platonistic language, as a convention to be judged by pragmatic standards, Goodman categorically rejects all platonistic systems as unintelligible. On Goodman's view, it is not the choice of the logical framework which is conventional, but rather the decision in which way to put reality into the framework; e.g., the decision which particular elements of experienced reality to take as the basic individuals of the constructional system.[105]

On the basis of his conviction that there is no such thing as *the* structure of reality, Goodman rejects the correspondence theory of truth[106], where the structure of sentences is contrasted with that of reality, and the two are compared for points of correlation. However, it seems to me that even on the assumption that several descriptions of reality are equally workable, it would still make sense to investigate the nature of the semantical relation of representation holding in each case. Even if it is impossible to confront the expressions of a system with "naked" reality, it is still possible to correlate these expressions to *reality as described in terms of this very system*. To see whether reality as described in the language of a given system explains in a satisfactory way why certain sentences of this language can be said to be true and others false, may even be considered as a further test of the workability of that system.

[105] Cf. above, pp. 146–148.

[106] Cf. GOODMAN 1952, p. 165; GOODMAN 1960, p. 53. He accepts, however, Tarski's definition of truth, which is based on the semantical relation of satisfaction or non-satisfaction of a sentential function by given objects. But he finds that the expression 'satisfaction', like other semantical expressions, is in need of further explanation. GOODMAN 1952, p. 166, makes a brief reference to a tentative definition, according to which a sentence is true if it signals a certain (future) event.

10. THE INTERPRETATIONS OF PREDICATE SIGNS

We have now come to the end of the survey in the course of which we have touched on a number of contemporary philosophers, from Russell to Quine and Goodman, and have discussed the nature of nominalistic and platonistic languages. Our aim has been to bring together various views on the relation of representation, so as to gain a new perspective to the problem of universals.

Because we have consistently approached our problem from the point of view of the relation of representation, it has been possible to state some questions in very precise terms, and this will help to clarify matters. As has been shown in the first part of this book, Russell and the early Wittgenstein provide the basic concepts of our investigation. But the different possibilities of the relation of representation of predicate signs have never previously been compared in a systematic way, despite – or perhaps precisely because of – the fact that they involve basic ontological attitudes. It still remains for us to approach the question systematically and to order the different points of view. This will allow also to fill in some gaps in what has been said so far and thus to obtain a more comprehensive picture.

With this aim in view, our method will be the following: we shall take an example – a concrete sentence formula – and with its aid we will show how the different interpretations work. The example we choose is the sentence formula '$Pa \cdot Pb$' (e.g., '*a* is red and *b* is red' or 'Albert is intelligent and Bruno is intelligent'). It is a conjunction containing the names of two different individuals *a* and *b*, so as to show clearly how two equal predicate signs (note that these are *tokens*[1]) function with respect to two *different* individuals.[2] For each interpretation we shall give a detailed reading of the sentence, and answer the question as to how and what the

[1] Cf. above, p. 64.

[2] Our considerations will also apply, *mutatis mutandis*, to relational signs containing two or more terms. In this case, a sentence formula like '*Rab. Rcd*' would have to be taken as example.

predicate signs represent, i.e., whether their relation of representation is unambiguous or ambiguous, and whether they stand for concrete or abstract entities. Attention will be paid to the representational function of the two significant relations between sign tokens: positional order and equality (in shape):

$$\overbrace{\underbrace{Pa}\cdot Pb}^{\text{equality}}$$

positional order

The equality relation, in particular, has so far not been sufficiently considered.

So far as we have been able to determine, there are two main interpretations of the relation of representation of predicate signs. First, there is the standpoint adopted by Russell and Carnap, according to which predicate signs are genuine names of specific entities, so that equal predicate signs name identically the same entity. Secondly, there is the interpretation which Quine and Goodman tend to adopt, but which is still rather vague. All we know is that on this interpretation predicate signs are said to be "syncategorematic". We shall deal first with the former interpretation.

10.1 Predicate signs as genuine names

On this interpretation '$Pa \cdot Pb$' means "The entity P belongs to a, and the same entity P belongs to b". 'P' thus names one single entity.

As pointed out by Quine, the method of quantification of Frege and Russell indicates clearly which signs are used as genuine names. For example, the sentence '$(\exists Q)(Qa \cdot Qb)$' is read as: "There exists a Q and this Q belongs to a and this Q belongs to b": And '$(Q)(Qa \cdot Qb)$' is interpreted as "For every Q it holds that this Q belongs to a and this Q belongs to b". Explicit reference is made here to *one* entity that belongs to *several* individuals.

But what kind of an entity is this? It is sometimes possible to regard it as a concrete mereological individual, of which the individuals a and b are parts, but this interpretation in terms of wholes and parts holds only in special cases.[3] And it would not explain why two syntactical categ-

[3] Cf. above, p. 149 f.

ories, predicate signs and individual names are distinguished, since a mereological whole and its parts belong to the same ontological category.

We must look for the One and the Many on different levels. In fact, the formal relationship here between the one entity and the many individuals is none other than the mathematical relation holding between a class and its members. Predicate signs, if they are genuine names, name classes, i.e., abstract platonistic entities.

But there is still a further question: are extensionally equivalent classes to be regarded as identical, or may they be further distinguished according to their intensional meanings? In the latter case we are concerned not simply with classes of individuals but with intensionally differentiated classes, i.e., with intensions or abstract properties. As we have seen, so-called intensional languages explicitly assume the existence of intensions, and there are even considerations concerning extensional languages which indicate that the existence of intensions must be presupposed.[4]

If predicate signs are regarded as genuine names, what do the syntactical relations between the signs represent? The juxtaposition of a predicate sign and an individual name pictures a class-member relation or, in other words, a relation of participation.[5] On the other hand, the relation of equality between signs, both in the case of names of individuals and in that of predicate signs, does not correspond to a relation in what is represented. It merely indicates that two sign-tokens represent identically the same entity. In the case of the predicate signs there is thus here a one-one correlation between the (abstract) sign-types and the abstract entities represented by them.

10.2 Syncategorematic predicate signs

The obscure notion of the "syncategorematic" functioning of predicate signs may become clearer, if we call to mind what has been pointed out by Wittgenstein, viz. that relations between signs can *picture* relations between the entities represented by the signs. Thus on Wittgenstein's

[4] Cf. above, p. 140 f.

[5] The fact that the juxtaposition represents something in reality has also been stressed by G. Bergmann. The "nexus of exemplification" between the "individual" named by the individual name and the universal "character" named by the predicate sign is for him an additional "constituent" of the state of affairs. Cf. BERGMANN 1964, p. 47 f.

view the positional order of names pictures the structure of states of affairs.[6] But this in no way exhausts the possibilities of "picturing", for positional order is not the only significant relation between signs: the equality or inequality of signs can also be taken into account. However, Wittgenstein, who, in our opinion, assumes ideally only one category of terms, viz. names of individuals[7], does not think of the relation of equality between signs as picturing something. For him the equality of two '*a*'s, for example, merely indicates that the same individual *a* is represented in both cases.[8]

But if two categories of signs are distinguished, e.g., individual names and predicate signs, then the possibilities of representation are increased, and also become more complicated. In this case, the equality of signs of the one category will still indicate numerical identity, but the equality of the signs of the other category may then have the function of picturing something. Thus two equal signs of this latter category, for example two '*P*'s, may represent two different entities between which a certain relation holds, the equality of the signs picturing the relation in question.

This means that the relation of representation of such a sign is ambiguous; it only becomes unambiguously determined by an adjacent sign of the first category. For example, in such an interpretation the relation of representation of a sign '*P*' taken in isolation is ambiguous. But the relation of representation of each of the two '*P*'s in the sentence '$Pa \cdot Pb$' is unambiguously determined by the individual name ('*a*', resp. '*b*') adjacent to it.

It would seem that this is what is meant by the "syncategorematic" functioning of predicate signs. For a predicate sign is thus indeed "meaningless" if taken in isolation, i.e., does not name a determined entity. Precisely what a single predicate sign stands for, is determined only in the context of a sentence.

The syncategorematic functioning of predicate signs may be interpreted in two ways, depending on whether the relation pictured by the equality of the signs is an equality or merely a similarity.

Whether a given relation is an equality or a similarity depends on its formal properties. An equality is an equivalence, i.e., a symmetrical and

[6] Cf. above, p. 52.
[7] Cf. ch. 6, p. 80 f.
[8] Cf. WITTGENSTEIN 1921, 5.53.

transitive relation. A similarity (or resemblance) on the other hand, is always symmetrical, but need not be transitive: *a* may resemble *b*, and *b* resemble *c*, without *a* resembling *c*.

A relation is symmetrical if it fulfills the following condition: if *x* has the said relation to *y*, then *y* has this relation to *x*:

$$(x)(y)(Rxy \supset Ryx).$$

The condition of transitivity of a relation is usually formulated as follows: if *x* has the said relation to *y*, and *y* has this relation to *z*, then *x* has this relation to *z*:

$$(x)(y)(z)(Rxy \cdot Ryz \supset Rxz)$$

However, for philosophical reasons we prefer a restricted form of transitivity. As formulated above, symmetry and transitivity entail a further formal property, that of reflexivity. That is to say, if a symmetrical relation fulfills the above condition of transitivity, then it can be inferred that it must also fulfill the condition that for all *x*, *x* has this relation to itself:

$$(x)(Rxx).$$

A reflexive relation, however, is not a real relation, but rather a relation of reason based on the conceptual duplication of an entity that in reality is a single entity.

If we want to talk about real relations, then we have to exclude the entailment of reflexivity. This we can achieve by adding to the condition of transitivity a restriction, specifying that *x* be not identical with *z*:

$$(x)(y)(z)(Rxy \cdot Ryz \cdot x \neq z \supset Rxz)$$

Carnap has introduced the term 'aliotransitivity' to distinguish this restricted transitivity from the ordinary one.[9] We thus define an equivalence as a symmetrical and aliotransitive relation. And with regard to similarity, too, we do not make the usual assumption of reflexivity, but specify only that it be symmetrical.

10.21 *The equality interpretation*

On this interpretation '$Pa \cdot Pb$' says "An entity P^a belongs to *a*, and

[9] CARNAP 1929, 16b.

another entity P^b, which is equal to P^a, belongs to b". Equal, but numerically distinct, entities are said to belong to the two individuals. For instance, 'Albert is intelligent and Bruno is intelligent' means here that Albert and Bruno both have intelligence, but the intelligence of Albert is considered to be a different entity from the intelligence of Bruno; the predicate 'intelligent' is not assumed to stand for a single abstract entity.

This interpretation is compatible with the nominalistic rejection of platonistic entities. But at the same time it provides what the contemporary nominalists like N. Goodman fail to provide, namely an ontological justification for distinguishing between true and false predication. The entities it assumes can be thought of as certain entities "*in*" the individual things, through which the things are as they are (and not otherwise). We call these entities "*concrete properties*"[10] and their being equal justifies the use of equal predicate signs. For instance, the presence in reality of the concrete intelligence of Albert and of the concrete intelligence of Bruno is the reason why both men can truly be said to be intelligent. Remark how two equal concrete properties may be independent from one another: there are happenings which affect the intelligence of Albert but not the intelligence of Bruno, and vice versa.

If equal concrete properties are so useful in explaining predication, why have they not been assumed in nominalistic semantics?[11] There are many reasons; let us mention some of them.

(a) One reason seems to lie in the fact that we are accustomed to say that two things are equal, because they are "the same" with regard to some properties. Thus we are inclined to believe that there always mus.

[10] G. F. Stout speaks of "particular characters" and reserves the adjective 'concrete' for what is "a subject to which characters belong and which cannot itself be a character of anything else" (MOORE-STOUT-HICKS, p. 114). We prefer 'concrete' because 'particular' can easily be misunderstood to mean "special".

[11] Cf., however, the following passage from MILL 1843 (1961), p. 117, footnote: "Mr. Herbert Spencer (*Principles of Psychology*, pp. 125–7) . . . charges me with falling into the error pointed out by Archbishop Whately and myself, of confounding exact likeness with literal identity. . . . Mr. Spencer is of the opinion that because Socrates and Alcibiades are not the same man, the attribute which constitutes them men should not be called the same attribute; that because the humanity of one man and that of another express themselves to our senses not by the same individual sensations, but by sensations exactly alike, humanity ought to be regarded as a different attribute in every different man." For Mill's own point of view cf. our quotation on p. 69 above.

be a further reason why two entities which are numerically different, are equal. In application to properties this would mean that equal properties are either identical, or there are properties of properties in regard to which the properties given are "the same". But from the point of view of our above interpretation, this is not required. The equality of concrete properties may here be considered as a basic relation which is in no need of further justification.

Perhaps this view can be made clearer by comparing equality with similarity. It seems that we have less difficulty in accepting the similarity of two properties (e.g., the matching of the colours of two objects) as a basic fact which needs no further justification. But as has been mentioned above, equality is like similarity except for the fact that an equality is also aliotransitive.

(b) Another reason for overlooking the possibility of the existence of concrete properties, may lie in the fact that the usual logistic languages contain no proper names for them. As we have seen in the discussion of the first interpretation, if the ordinary signs for properties such as '*P*', '*Q*', etc., are regarded as genuine names, then they must be regarded as naming abstract properties. In order to name equal, but numerically distinct, concrete properties we had to form *ad hoc* the new names 'P^a' and 'P^b'.

The fact that we do not ordinarily use genuine names for concrete properties, is also the reason why we do not quantify over a special range of concrete properties; why the concrete properties are not singled out in our universe of discourse. Note, however, how Church in his refined formulation of Quine's criterion specifies that a sentence like '$(\exists x)(Px)$' does not merely assert the existence of an individual x, but of an individual of which 'Px' is true.[12]

(c) Furthermore the assumption of concrete properties may be rejected out of fear of an infinite regress. If we introduce concrete properties of individuals and if we speak of their equality, then at once the further question of the status of this equality relation comes up. Have we to do

[12] Cf. above, p. 134. Bocheński, too, emphasizes the need to assume properties *in* the things; cf. BOCHEŃSKI-CHURCH-GOODMAN, p. 49, where he says: "For a property, to be is not to be a value of a variable but *to qualify the value of a variable*." My thoughts concerning concrete properties have been inspired by this paper. However, Bocheński does not consider equality, but only similarity and different types of identity. AARON 1952 similarly distinguishes only identity and resemblance (cf. p. 218).

here with one abstract entity (namely with one abstract relation), or shall we say that there are several concrete entities (several concrete relations) inherent in the different concrete properties? In the first case we would fall back into a platonism which accepts genuine names of abstract entities. In the second case the concrete relations, which are "the same" inasmuch as they are all equality relations, must also be related to one another – probably by an equality relation of higher order; and we can continue asking about the status of this equality of higher order; and so on *in infinitum*. (Remember the analogous regress with respect to similarity, as described by Russell.[13])

The nominalists, especially, who regard with suspicion the "teaming" infinity of abstract entities, will be very reluctant to end up in this way with another infinity of concrete entities. However, it seems not at all impossible that our concrete world should include an infinity of concrete entities of different levels, an infinity which our limited intellect will never be able to exhaust explicitly. So far as symbolization is concerned, the infinite regress can be "stopped", because the equality of the concrete properties is not *represented* by some further symbol but is "*pictured*" through the equality of the tokens of the predicate expressions.

It is of interest to recall here that there is a problem of an infinite regress also in regard to the other relation which is pictured through significant relations of the signs. I mean the relation between properties and individuals (inherence of concrete properties or participation of abstract properties) which is pictured by the positional order between predicate signs and individual names. A. Church, for example, accepts this infinite regress, despite the impossibility to represent it exhaustively by genuine names.[14]

Explicit reference to equal but numerically distinct entities in things can be found in the aristotelian-thomistic philosophy of matter and form.

[13] Cf. above, p. 68 f.

[14] Cf. CHURCH 1956, p. 35. See also SIMPSON 1964, p. 75. But G. Bergmann, e.g., rejects the infinite regress. He counts "exemplification" as an additional "constituent" of the states of affairs, but he calls it a "nexus" or a "fundamental tie". Whereas a relation (a "thing" that can be "named" by a relation-sign) needs additional ties to tie it to the individuals, a nexus for Bergmann is an entity of a special sort "which ties others into complexes . . . without need of a further entity to tie it to what it ties". BERGMANN 1964, p. 197.

For instance St. Thomas writes: "In the case of an equality it is not numerically, but only specifically the same quality that is present in each of two equal individuals" (*in similitudine non est eadem qualitas numero, sed specie, in utroque simili*).[15] The passage continues by saying that this holds not only for equality in quality (*similitudo*), but also for equality in quantity (*aequalitas*)[16], and that since the subjects of the relation of equality are numerically distinct, the relations themselves have to be numerically distinct (i.e., they are concrete relational *accidentia*).[17]

It is therefore possible to correlate different occurrences of equal predicate signs with these equal but numerically distinct qualities and relations. The aristotelian forms are characterized by many distinctions – neglected, for the most part, by modern philosophers, who simply treat all "properties" alike. Thus there is the substantial form of a thing, and

[15] *Summa theologica, suppl. q.* 44 *a.* 2 *ad* 3. In scholastic philosophy this doctrine of equal, but numerically different, properties goes back to William of Champeaux who elaborated it, when the argument of Peter Abelard had shaken the then traditional theory of platonistic realism, which he had defended before. Notice that William's term for the concept of equality is '*indifferentia*': *Vides has duas voces,* '*unum*' *scilicet et* '*idem*', *duobus accipi modis secundum indifferentiam et secundum identitatem, eiusdem prorsus essentiae; secundum indifferentiam, ut Petrum et Paulum idem dicimus esse in hoc quod sunt homines; quantum enim ad humanitatem pertinet, sicut iste est rationalis et ille, et sicut iste est mortalis et ille. Sed si veritatem confiteri volumus, non est eadem utriusque humanitas, sed similis, quam sint duo homines*, i.e., "You see that these two expressions, 'one' and 'the same' that is, can be understood in two ways, according to the equality (*indifferentia*) and according to the identity of entirely the same essence; according to the equality, as when we say that Peter and Paul are the same in that they are men; because as far as humanity is concerned, the one is as rational as the other, and the one is as mortal as the other. But if we want to tell the (exact) truth, (then we must say that) the two have not the same humanity, but (each has) an equal (one), since they are two men" (LEFÈVRE 1898, p. 14, p. 25).

[16] The scholastic distinction between *similitudo* and *aequalitas* does not correspond to the contemporary distinction between similarity and equality, but is based on the distinction between quality and quantity, as the following passage shows: *Sicut dicit Philosophus unum in substantia facit idem, unum in quantitate aequale, unum in qualitate facit simile*, i.e., "As the philosopher [i.e., ARISTOTLE, 5 *Metaph. c.* 15 1021 a 11–12] says, being one in substance is being identically the same; being one in quantity is being equal in quantity; being one in quality is being equal in quality" (THOMAS AQUINAS, I *Sent. d.* 19 *q.* 1 *a.* 1).

[17] *. . . et iterum subiecta similitudinis sunt duo numero; et similiter est de aequalitate: ideo et aequalitas et similitudo omnibus modis est alia numero in utroque similium et aequalium*, i.e., ". . . and also the subjects of equality are two in number; the same holds for equality in quantity; therefore the equality in quality as well as the equality in quantity is in every way numerically distinct in each of the two individuals which are equal in quality, resp. equal in quantity" (*Summa theologica, suppl. q.* 44 *a.* 2 *ad* 3).

its accidental forms; and the latter are further subdivided intensionally into accidental forms necessarily related to the substantial form (the so-called *accidentia propria*), and accidental forms not necessarily related to the substantial form (the so-called *accidentia communia*, which are not only *accidentia praedicamentalia* but also *accidentia praedicabilia*).[18]

From this point of view, the juxtaposition of a predicate sign and an individual name pictures the inherence relation between a concrete form and its subject. It is revealing, in this connexion, to compare the traditional doctrine of a formless *materia prima* with the way in which, in mathematical logic, an individual may be represented by a mere variable '*x*', all its determining characteristics being represented by predicate signs.[19]

10.22 *Equalities as a foundation for abstraction*

The assumption of equal concrete properties and relations clarifies not only nominalistic semantics: it is also valuable for a platonist, because he can show that these concrete entities provide an adequate foundation for the introduction of corresponding abstract entities. On the basis of the equality of the concrete entities he can define certain classes in such a way that all entities which are equal to one another are always members of identically the same such class. In this sense Husserl is right when he asserts: "It is a fact that wherever there is equality, there is also identity in the strict and true sense."[20] There are well-known theorems of the theory of equivalence relations, which can be applied to the equalities of concrete entities, and which prove in a strictly formal way that the classes in question form an exhaustive and non-overlapping classification.[21]

In order to show this, we first define the classes which will be called "equivalence classes":

$$e(x) = df\ \hat{y}(Exy \vee y = x)$$

i.e., 'The equivalence class of the individual x is the class of all the ele-

[18] Cf. Gredt 1901/2, no. 142, 172.
[19] Cf. above, p. 71 and 130.
[20] Husserl 1900–1901 (1913), vol. 2, p. 112.
[21] Cf. Scholz-Schweitzer; this monograph contains also a detailed account of the historical development of this method of classification. See also Carnap 1929, § 20; Carnap 1958, p. 136; Beth 1955, § 10; and elsewhere.

ments y which are equivalent with x or which are identical with x'.[22] The alternative '$\vee\, y = x$' has been added because according to our definition an equivalence is not necessarily reflexive[23], and we want nevertheless that for every x the equivalence class of x contains x as a member:

[1] $\quad (x)(x \in e(x))$.

The following theorem also holds:

[2] $\quad (x)(y)(z)(x \in e(z) \cdot y \in e(z) \cdot x \neq y \supset Exy)$

i.e., 'Two distinct elements of the same equivalence class are always equivalent'. The proof runs as follows: According to the definition $x \in e(z)$ means that Exz or $x = z$. Let us first consider the case that Exz. From $y \in e(z)$ we learn that Eyz or $y = z$. If $y = z$, then together with Exz we obtain Exy, the desired result. If Eyz, then because of the symmetry of equivalence relations also Ezy, and together with Exz and $x \neq y$ we get, because of the aliotransitivity of E, the result Exy. Let us now consider the case that $x = z$. Then because of $x \neq y$, not $y = z$. Therefore Eyz. Therefore Ezy, and because of $x = z$, we again obtain the result Exy.

Further theorems which can be proved in similar ways are:

[3] $\quad (x)(y)(z)(x \in e(z) \cdot Exy \supset y \in e(z))$

i.e., 'Every element that is equivalent to a member of an equivalence class is also a member of this same equivalence class'.

[4] $\quad (x)(y)(z)(z \in e(x) \cdot z \in e(y) \supset e(x) = e(y))$

i.e., 'If two equivalence classes have an element in common, then they are identical''. In other words: 'Equivalence classes do not overlap'.

[5] $\quad (x)(y)(Exy \supset e(x) = e(y))$

i.e., 'Two equivalent elements represent one and the same class'.

These theorems of the theory of equivalence relations show that every equivalence relation gives rise to an exhaustive and non-overlapping classification, such that every element can be regarded as representative of the equivalence class to which it belongs.

[22] Cf. above, p. 133, for the definition of expressions of the form '$\hat{x}(Px)$'.
[23] Cf. above, p. 165.

The conjunction of [2] and [3] is often taken as defining the predicate 'is an equivalence class' ('Equ'):

[6] $Equ\,(\alpha) = df\ (x)(y)(x \in \alpha \cdot y \in \alpha \cdot x \neq y \supset Exy) \cdot$
$\cdot\,(x)(y)(x \in \alpha \cdot Exy \supset y \in \alpha)$

We prefer a slightly different (but equivalent) definition, which has a more interesting analogue with respect to similarity[24]:

[6′] $Equ(\alpha) = df\ (x)(y)(x \in \alpha \cdot y \in \alpha \cdot x \neq y \supset Exy) \cdot$
$\cdot\,(y)\{(x)(x \in \alpha \cdot x \neq y \supset Exy) \supset y \in \alpha\}.$

i.e., 'α is an equivalence class if and only if every two members of α are equivalent and every element that is equivalent with every member of α (with which it is not identical) is a member of α', i.e., 'α is an equivalence class if and only if it is a maximal class where every member is equivalent with every other member'.

An example of an application of this method of classification is the logistic definition of number, of which we have given a detailed account in an earlier chapter.[25] There a higher-order domain of classes of numerically equivalent classes, i.e., of numbers, is introduced above the second-level domain of classes of individuals.

For the application to our present case, the classification of equal concrete entities, a further clarification concerning our basis is necessary. For it seems advisable to take as our equivalence relation not the relation of equality between concrete properties, but the corresponding relation of equality between things, e.g., to take instead of the equality of concrete colour properties the equality-in-respect-to-colour of concrete things.[26] In this way we can avoid talking about equivalence classes which have

[24] Cf. below, p. 176–177. Note that according to [6] and [6′] the null-class, which is not an $e(x)$, is also an equivalence class.

[25] Cf. above, pp. 30–32.

[26] It is worth mentioning that an equality-in-respect-to-P which is an equivalence has to be distinguished from a *part equality*. The latter is not an equivalence but only a similarity, since it is not transitive. For example equality-in-respect-to-colour of uniformly coloured spheres is an equivalence, but the part-equality-in-colour of cubes with sides of different colours is not: if cube a is part-equal to cube b (if, e.g., a and b each have a red side), and if b is part-equal to cube c (e.g., if b and c each have a blue side), then it does not follow that a is necessarily part-equal to c (that a and c each have a side of the same colour).

concrete properties as members[27] and talk simply about classes of *things*.

The operation of constructing (or discovering) the equivalence classes of things with equal concrete properties corresponds to what is traditionally called abstraction. For instance, an equivalence class of things with equal concrete colour properties can be considered a colour, i.e., an abstract property. Another often quoted example is the classification of straight lines on the basis of their being parallel. Here the equivalence classes are the different directions.

On this view the concrete equality of the concrete properties is that aspect in concrete reality on which abstraction is based (the "*fundamentum in re*" of abstraction!) and the theorems concerning the classification on the basis of equivalence relations describe the formal ontological side of the aristotelian theory of abstraction.[28]

[27] One is here reminded of the "*collectio*"-doctrine ascribed to Joscelin of Soissons (12th century), which identifies the universals with collections of concrete forms: *speciem igitur dico esse non illam essentiam hominis solum quae est in Socrate, vel quae est in aliquo alio individuorum, sed totam collectionem ex singulis aliis huius naturae conjunctam*, i.e., "I say that the species is not only that essence of man which is in Socrates, or which is in some other of the individuals, but the whole collection made up (also) of each single further (essence) of this kind" (*De generibus et speciebus*, COUSIN 1836, p. 524). This doctrine is easily confused with a doctrine where the species man is identified with the mereological whole of all men. The author of our text contributes himself to this confusion, since in order to explain how a *collectio* can be one and many at the same time, he gives as an example a nation (*populus*) i.e., a collection of many persons. Cf. also the 20th-century doctrine of G. F. Stout: ''To have a nose' is a general term, standing not for a single character, but for a class or kind of characters. When I say that I have a nose I assert that some particular character or other which is a member of this class belongs to me. So when I say that you have a nose I assert that you have a character which is an example of the same sort or class. Whatever is a shareholder in the general class of characters 'having a nose', by possessing an example of it thereby belongs to the class 'things which have noses'." But notice how Stout is unable to explain how the class of characters originates: "A thing belongs to a certain class only because a character of a certain kind is predicable of it. But we cannot, without moving in a vicious circle, go on to say that characters themselves can belong to classes or kinds only because other kinds of characters are predicable of them. What I maintain, therefore, is that qualities and relations belong to classes or kinds just because they are qualities and relations" (MOORE-STOUT-HICKS, p. 116).

[28] A difficulty arises from the fact that on the aristotelian-thomistic view a universal concept can be abstracted from one single individual: [Aristoteles] *dicit quod universale est quod natum est pluribus inesse, non autem quod pluribus inest; quia quaedam universalia sunt quae non continent sub se nisi unum singulare, sicut sol et luna*, i.e., "Aristotle says that the universal is that which is capable of being in many things, but not that which is in many things; for there are some universals that subsume only one single thing, such as sun and moon" (THOMAS AQUINAS, *In Metaph. Arist. lib.* 7,

Peano actually gave the name 'definition by abstraction' to the method of defining equivalence classes, and Russell refers to the basic law underlying these classifications as "the principle of abstraction". Similarly Carnap once called the equivalence classes "classes of abstraction" (*Abstraktionsklassen*).[29] However these logicians were not familar with the notion of concrete properties in the things, and therefore did not draw attention to the fact that relations of equality between things are rooted in concrete properties of these things.

The method of definition enables us for the first time to give an exact formulation of the relationships between concrete entities and the entities abstracted from them, so that now strict proofs of their formal properties can be given. This is an important achievement. But, of course, not all questions of the problem of universals are questions of formal ontology. The description of the formal relationship is not yet an explanation of the psychological process of abstraction; nor does it give a metaphysical justification for the existence of abstract entities.

Since a platonist can justify the introduction of abstract entities on the basis of the concrete entities which are presupposed by the equality interpretation of predicate signs, it is easy for him to move from the equality interpretation to the first-mentioned interpretation of predicate signs

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lect. 13, 1574; Cf. also HUSSERL 1900/01 (1913), vol. 2, p. 114). However, the view that abstraction is always based on a real concrete relation of equality can be maintained, if we remember the thomistic doctrine of real possibility: then even an individual which happens to be equal to no other individual, can have a real relation of equality, namely in the sense of a real possibility(as a so-called "transcendental relation").

[29] PEANO 1894, § 38; RUSSELL 1903 (1937), p. 166, p. 220; RUSSELL 1914, p. 42; CARNAP 1929, § 20b. Cf. also FREGE 1884, p. 73 f. Remember the "class abstraction" (explained above on p. 133) and note that not every class abstraction is a "definition by abstraction" in the sense of Peano. – Also, as Goodman has shown, not every equivalence class is a "class of abstraction" in the sense that an abstract property in the usual sense corresponds to it. Take, e.g., the following simple example: in a universe of three discs *a*, *b* and *c*, each of which has differently coloured sides, *a* being blue and red, *b* red and green, *c* green and blue, "part-equal-in-colour" happens to be an equivalence, since it is here not only symmetrical but in fact also transitive. And all three things belong to a single equivalence class. Nevertheless no one colour is common to all three discs. (Cf. GOODMAN 1951 (1966), p. 169.) Only if a relation is *necessarily* an equivalence, e.g. if it is an equality, can we be certain that we get "well-founded" (cf. above, pp. 86–90) "classes of abstraction". This shows again how in constructing "normal" extensional systems we must be guided by presystematic intensional knowledge.

as genuine names of abstract entities. This explains further why the interpretation of predicate signs in terms of equality is hardly ever found in practice.

The situation is different in the case of similarity, to which we shall now turn.

10.23 *The similarity interpretation*

If we formulate it in analogy to the equality interpretation, then the similarity interpretation of the sentence '$Pa \cdot Pb$' reads as follows: "An entity P^a belongs to a, and a similar entity P^b belongs to b." As on the equality interpretation, predicate signs stand for concrete properties, or forms, in the things. But now the two properties are not assumed to be equal; they are only regarded as similar.

Of course, this may also apply to the signs: they, too, might not be regarded as equal in shape, but only as similar. For example the two 'P's in the above sentence might be regarded as similar. One must then say that similar signs represent similar concrete forms; that the similarity between the signs pictures a similarity between concrete forms.

The objections against the similarity interpretation are like those made against the equality interpretation. Philosophers who reject the similarity interpretation usually view every similarity categorically as a part-identity: for them two things are similar because they have a property in common; e.g., two coloured things are similar because they have the same colour, or at least because they have colouredness in common.

But while the equality interpretation of predicate signs is seldom found in practice, there are many philosophers who hold a modified form of the similarity interpretation. The situation with respect to similarity is different mainly for two reasons, one linguistic and the other formal.

We shall deal first with the linguistic reason and its consequences. Ordinarily we say that two things are equal only after we have acquainted ourselves with the properties because of which they are equal; the phrase 'they are equal' is in fact used as an abbreviation, as an elliptic way of saying 'they are equal with respect to the properties under consideration'. If we say that a and b are the same without specifying particular properties with respect to which a and b are the same, then we are taken to mean

that *a* and *b* are the same in all respects, i.e., that *a* and *b* are identical. The same is not true for similarity. We can say that *a* resembles *b* without giving any further information concerning this similarity. We may ourselves not know anything more specific about it.

This linguistic fact makes it easier to accept the ontological point of view, according to which similarity can be ontologically basic i.e., according to which the fact that two entities are similar does not necessarily presuppose that they must have some property in common (that they are part-identical).

Furthermore, the fact that two different things can be said to be similar without further specification, is especially welcome to philosophers who do not want to accept properties because they reject abstract entities, and are unfamiliar with the notion of concrete properties. But if one wishes to refer only to things and their similarities, then '$Pa \cdot Pb$' must be interpreted as an abbreviation for '$Sfa \cdot Sfb$', which can be read "*a* resembles the standard object *f* and *b* resembles the same standard object *f*". Of course, the '*S*'s are predicate signs which in turn must be interpreted, either as genuine names of an abstract entity, of an abstract relation "similar"[30], or as "syncategorematic" signs representing concrete entities, namely equal concrete relations or similar concrete relations.

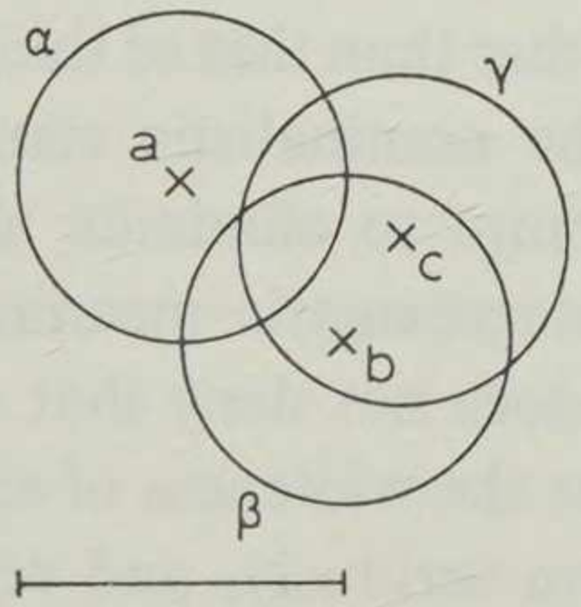
In an earlier chapter we have seen that proposals have been made to build a constructional system with only one special similarity-predicate as primitive.[31] If '*S*' is the only primitive sign, then it need not be written down, since in this case the similarity can simply be pictured by the positional relation of the individual names. This brings us to the interpretation which we attributed to Wittgenstein.[32]

The case of the similarity interpretation differs from the case of the equality interpretation also for a formal reason. As has been mentioned, a similarity is not necessarily transitive resp. aliotransitive.[33] For example if there are three coloured objects *a*, *b*, *c*, and *a* matches *b*,

[30] This was Russell's choice; cf. above, p. 68 f., for his argument rejecting the infinite regress.
[31] The constructional systems of Russell and Carnap; cf. above, p. 87.
[32] Cf. above, p. 85.
[33] Cf. above, p. 165.

and b matches c, then it is not necessarily the case that a matches c.

This is important from the point of view of classification. The method of definition by abstraction cannot simply be extended to non-transitive relations. For if instead of the equivalence classes (the maximal classes where every element is equivalent with every other element) we form what has been called "similarity circles" (the maximal classes where every element resembles every other element), then we usually find that the similarity circles overlap. It may even happen that there are as many different similarity circles as there are elements in the field of the similarity relation. This can be seen clearly in the adjacent diagram, where the similarity relation is a neighbourhood relation. The points in the plane constitute the field of the relation. Two points are said to be related if they are not too far from each other; a standard distance indicates the maximal reach of the similarity, i.e., points further apart are not regarded as related. α, β, γ are examples of similarity circles having as centers the points a, b and c respectively. It will be easily understood that here every point of the plane is the center of another similarity circle.



Standard of reach of similarity

This is not to say that a similarity can in no way lead to a classification of things. The possibilities of classification on the basis of special similarity relations have been explored by Carnap and Goodman in their constructional systems. But Goodman has encountered two serious difficulties, the so-called "companionship-difficulty" and the "difficulty of imperfect community", which every such attempt must face.[34]

The fact that similarity relations, unlike equalities, lead to no simple and neat classification procedures, suits the basic convictions of those phi-

[34] CARNAP 1928; GOODMAN 1951, esp. p. 123, p. 125.

losophers who are usually labelled 'nominalists'. For these philosophers tend to regard the borderlines in all classifications as arbitrary and emphasize the individuality of things, maintaining that there are no features that are exactly alike; that every object has, so to speak, its own highly individual nature. If, however, the designation 'nominalism' is taken in the precise technical sense as the view that rejects the existence of abstract entities, then both the equality interpretation and the similarity interpretation are compatible with it. On the other hand similarity classes as well as equivalence classes are unacceptable on this view, since all classes are abstract entities.

10.3 The strictly reistic interpretation

Since the reality of similarity relations is often overlooked, a philosopher who assumes only things and their being similar, can easily reach a purely reistic point of view, according to which only individual things exist.

Kotarbiński, who coined the designation 'reism', explicitly rejects all ontological categories other than that of things.[35] In practice, however, his view coincides with the nominalistic standpoint of Goodman and Quine, who make no attempt to eliminate all predicate signs, but explicitly accept their syncategorematic meaningfulness. Remember how Kotarbiński says that he does not deny that snow is white, but merely that there is such a thing as the whiteness of snow.[36]

If we were to take reism seriously, and reject the existence both of properties and of relations, then we should no longer be able to find any grounds in represented reality for subsuming two individual things under the same designation; the sharing of names would become a matter of pure convention.

In actual fact, the subsumption, for example, of certain plants under the general designation 'vegetable' seems to be a case of pure convention. It appears that no one can explain which kinds of plants are vegetables, except by enumerating them.[37] But such a case is the exception and not the rule.

[35] Cf. above, p. 122.
[36] Cf. above, p. 122.
[37] According to J. M. Bocheński the example is due to L. Petrażycki.

A further consequence of strict reism would be that the reading of sign tokens, too, would become a matter of pure convention. Every single sign written on a page would require the making of a new arbitrary convention to determine whether it was to be named an '*a*' or a '*b*', or a '*P*', or a '*Q*', etc.

Obviously, the extreme reistic interpretation is quite untenable; we have mentioned it merely for the sake of the completeness of our account.

11. CONCLUSION

In the preceding chapters we have tried to familiarize the reader with the relation of representation of logistic languages, and have systematically worked through the various possible correlations of linguistic and ontological entities with respect to predicate signs. To conclude, we shall give a brief account of the importance of logistic analysis for philosophy. We shall stress in particular the value of exact formulation, the fundamental importance of logistic analysis from the point of view of formal ontology, and the fact that the use of a logistic language in no way indicates a narrowly formalistic attitude on the part of its user: upon reflection it leads, on the contrary, to the acknowledgment of an intuitive content beyond what is merely formal.

11.1 The value of exact formulation

Every rational philosophy is put forward in terms of a language, for thought becomes clear only when linguistically formulated. Unformulated inspiration is not the goal of rational activity but only the starting point. It is like a spark that starts the process of thought, forcing it to find expression in an objective formulation, which may then be judged as to its merit by universally recognized criteria.

Contemporary logistic philosophers are particularly sceptical of the reliability of intuitions. The antinomies have taught them a lesson: what seemed to be an intuition concerning the nature of classes, proved to be contradictory upon formulation.

Phenomenologists, too, are aware of the need for precise formulation, and strive to express their intuitions in accurate descriptions. Husserl "sought lucid testable intuitions, that he could grasp and fix with rational means. Intuitions were indispensible to him, but he did not yet regard them as scientific knowledge. They had first to be understood and ex-

pressed in clear-cut rational structures."[1] The opinions of logistic philosophers and phenomenologists differ only in regard to the question of what kind of formulation to choose: whereas phenomenological descriptions are formulated in ordinary languages, logistic philosophers construct their systems in artificial languages.[2]

One advantage of an artificial, logistic language is that through its rules of formation and definitions, its expressions stand in a strict relationship to each other. Thus the meaning of what has been grasped intuitively is rendered precise in a special way, since the meaning of an expression is characterized by the linguistic contexts in which the expression can meaningfully occur. Furthermore, because of the explicitly formulated rules of inference, hidden presuppositions are revealed and false assumptions are readily detected, since they entail undesirable consequences or even contradictions.

Of great philosophical significance is the way in which the syntactical categories of logistic languages stand in a systematic relationship of representation to the structure attributed to reality: this feature of logistic systems has been discussed in detail in the present work. As has been shown, the construction of logistic systems poses in a very explicit way the basic problems of representation. The correlations between the symbolic expressions of the system (the descriptive "map") and the reality described, can be investigated in precise detail. With the development of a two-level semantics, it has become possible to translate problems regarding described reality into questions concerning a symbolic system, i.e., into questions of the logistic analysis of language.[3] "In this way, the linguistic version of a philosophical problem may be a useful heuristic device for attacking the problem, just as in deciding certain questions of geography it is useful to translate them into questions regarding the positions of marks on a map. But just as it would be ludicrous to suppose that maps

[1] "[Husserl] wollte nachprüfbare, geklärte Intuitionen haben und sie mit rationalen Mitteln fassen und festlegen. Die Intuitionen waren für ihn unentbehrlich, aber sie bildeten für ihn noch keine Wissenschaft. Sie mussten erst verstanden und in präzis bestimmten Vernunftgebilden erfasst werden" (INGARDEN 1959, p. 461).

[2] Cf. also above, p. 3 f., p. 97 f.

[3] Cf. Carnap's translations from the material into the formal mode of speech (CARNAP 1934, § 75 f.). However, he oddly regards linguistic categories as primary and terms ontological categories "quasi-syntactical".

constitute the entire subject matter of geography, so also it would be a great mistake to suppose that philosophy is or ought to be nothing more than the study of language."[4]

11.2 The intensional background of formal structures and the incompleteness of all formalizations

Among non-specialists the prejudice is often found that logistic philosophy is necessarily nominalistic and opposed, by its very nature, to all genuine philosophical thought. This prejudice is unfounded.

So far as the charge of nominalism is concerned, we have seen that it is true that some logistic philosophers attempt to maintain a standpoint which is nominalistic in a precisely defined sense. Most logistic systems, however, are platonistic; and the fact must not be overlooked that a number of prominent logicians adopt intensional systems, that are not only non-nominalistic but even non-extensional.

The fact that there are intensional systems shows most clearly that logistic analysis can do justice to intuitively recognizable essential connexions. However, nominalistic or extensional languages are quite acceptable from an intensionalistic point of view; they are in no sense "wrong", but merely more economical. What is more, as we have seen in our discussion of Carnap[5], careful reflexion upon the presuppositions of an extensional system leads to the recognition that the well-foundedness of the extensional relations which such a system represents and the truth of its non-tautologous analytic sentences, must depend on intuitively recognizable essential connexions. The formal structures represented by an extensional system must have an intensional background.

Furthermore, because of the development of metalogical investigations the following has – unexpectedly – been proved in a rigorous way: a theory describing a structure which is as complex as, e.g., the structure of the arithmetics of natural numbers, is in principle never completely formalizable, i.e., it can never be completely formulated as an axiomatic system. Gödel's proof states that for every attempted formalization of such a system, it is possible to formulate a sentence which is intuitively true but

[4] MATES 1950, p. 226.
[5] Cf. above, Ch. 7, pp. 86–96.

which is undecidable, i.e., can neither be proved nor disproved in the proposed formalization.[6] It is, of course, possible to extend the calculus in such a way, that the sentence in question is added to the axioms. But in a calculus thus enlarged yet another intuitively true sentence could be formulated, which would be undecidable in the enlarged calculus; and so on *in infinitum*. No single calculus, therefore, can exhaust the entirety of mathematical truths. An endlessly "growing" series of calculi seems to be required for this purpose.[7]

Since to every calculus there corresponds a machine and vice versa, the theorem of Gödel seems to prove nothing less than that the human mind is more powerful than any possible computer; that the human mind can always know the truth of at least one mathematical sentence, for which a given computer can furnish neither a derivation nor a disproval. This refutes any mechanistic philosophy which believes that the human mind can be explained entirely in terms of a mechanical model.[8]

It would be perfectly possible to extend a calculus in such a way that the negation of Gödel's undecidable sentence would be added to the axioms. The resulting new calculus would still be consistent, but it could no longer be interpreted as dealing with what we normally call the natural numbers. Instead it would describe a strangely "pathological" non-standard model. The fact that mathematicians regard *one* specific arithmetical model as the true one and all others as non-standard, although formalized descriptions of them are equally possible, shows that mathematicians, too, are guided by basic intuitions. Thus, even for a non-empirical theory like mathematics, a standpoint of extreme formalism, which regards a theory as a mere uninterpreted abacus, seems to be in-

[6] GÖDEL 1931; cf. NAGEL-NEWMAN (but cf. PUTNAM 1960); ROSSER 1939; MYHILL 1952; LADRIÈRE 1957; STEGMÜLLER 1959, and others.

[7] Cf., e.g., WANG 1954.

[8] Cf. LUCAS 1961; NAGEL-NEWMAN, p. 10; and many others. However, PUTNAM 1960 denies that Gödel's theorem refutes mechanism. For a mechanist's attempt to get around this difficulty see SMART 1961 or SMART 1963, p. 116–120. He admits that a computer cannot give a rigorous proof of Gödel's undecidable sentence, but he believes that human minds are not better off; in order to see the truth of Gödel's undecidable sentence they have to rely on their intuitive knowledge of meta-linguistic rules, and Smart thinks that this, too, is not rigorous knowledge but merely an empirically justified conviction.

sufficient. There too, the calculi are measured against the background of an intuitive model.[9]

But a consistent logistic philosophy will not only make evident the existence of intensional connexions: by describing their formal structure, it actively contributes to a clearer and more systematic understanding of these connexions. If ordinary language appears to be more suited for the first formulation of subtle intuitions, the second stage, that of systematizing, requires logistic analysis, the translation from ordinary language into the rationally constructed categories of a logistic language.[10]

11.3 Logistic analysis and formal ontology

The formal relationships of logistic languages picture the ontological structuring of reality. As mentioned earlier, it was H. Scholz who pointed out that, e.g., the logistic predicate-calculus embodies a theory of formal ontology.[11] It is an axiomatic system that characterizes all true propositions concerning the formal relations between things and properties, just as Hilbert's axiomatic system of geometry characterizes all true propositions about the formal relations between points and straight lines. Admittedly, these axiomatic systems in no way answer all the philosophically relevant questions concerning ontology or geometry; but both systems are of fundamental significance for anyone who is seriously interested in the philosophical investigation of either of these fields.

Because of the focussing of attention on questions of linguistic formulation, the psychologistic theory of knowledge has lost much of its appeal in favour of formal ontology. For example, so far as the problem of universals is concerned, this change in attitude is reflected in the abandonment of the epistemological distinction of "nominalism-conceptualism-platonic realism" and its replacement by the formal ontological dichotomy of "nominalism-platonism", with platonism being further subdivided into structurally different forms.[12] Philosophers are no longer

[9] McNaughton speaks of "conceptual schemes". Cf. McNaughton 1954; McNaughton 1957. In connexion with non-standard models, cf. among others Beth 1955, p. 106; Myhill 1953a. Cf. also the discussion in Copi 1949; Turquette 1950 and Copi 1950 on "Gödel and the synthetic *a priori*".

[10] Cf. above, p. 4.

[11] Cf. above, p. 10.

[12] Cf. above, p. 144 f.

primarily concerned to know "where" abstract entities exist: whether in the mind of man, in a platonic heaven, or in the mind of God; but they want to know what formal relations abstract entities have to each other and to the individuals subsumed by them, and whether it is useful to assume the existence of entities characterized by such formal relationships.

When the first naive intuitive attempt to determine these relationships foundered with the discovery of the antinomies, different schools tried to find the weak spot in the theory and to close it with all the aids of modern logistic technique. Only one of these schools, intuitionism, started from a critical analysis of the process of mathematical thinking; as we have seen, it has been referred to as conceptualistic, since it claims to accept only those mathematical entities that can be effectively constructed in the mind.[13] But even this epistemologically minded school finally formulated its logical and mathematical propositions in a strictly formalized system.[14] In this way even those who do not appreciate the epistemological scruples of the intuitionists, can gain a precise understanding of the intuitionist way of restricting extreme platonism. And they will be able to evaluate the formalized system of intuitionism in comparison with their own and other formalized systems.

The logistic analysis of language, i.e., the translation into logistic language, has at least the following three advantages, from the point of view of formal ontology:

(a) It gives information about the formal ontological standpoint adopted by a philosopher. As has been explained, logistic languages carry with them an explicit ontological commitment. The ontological model presupposed by a logistic language can be precisely determined by means of Quine's criterion.

(b) The connexion between logistic languages and ontological standpoints makes possible a new kind of pragmatical evaluation of the latter. Thus it appears that the most advantageous combination of economy in ontological assumptions with possibility of serving as the language of science, is probably realized in some language of a moderate platonism. However, important as the pragmatical criterion is, we do not wish to

[13] See above, p. 146.

[14] HEYTING 1930 and HEYTING 1930a. However, Heyting takes care to add the proviso that one can never be sure whether a given formal system takes account of all possibilities of mathematical thinking (cf., for example, HEYTING 1956, p. 9).

claim that it is of primary importance. There are also logistic philosophers who place still greater value on the criterion of intelligibility: "[The philosopher] is driven not by practical needs but by an impractical desire to understand ... a system works for him only to the extent that it clarifies."[15]

(c) Logistic analysis sharpens and increases our ontological knowledge. This will perhaps be seen most clearly in what we have said about the connexion between concrete and abstract entities.[16] By means of the logico-mathematical theory of definition by abstraction, we were able to show exactly how the equality relation between concrete individuals constitutes the *fundamentum in re* of the universality of abstract entities.

We should like to draw the attention of philosophers to the wealth of new technical aids that have become available in the fields of logic and mathematics, and which allow relationships of formal ontology to be formulated with increasing precision. Even if it is true that ontology involves the understanding of intensions and not only of purely formal connexions, this does not alter the fact that formal structures are so to speak the backbone of ontology, without which all further ontological insights would lack a firm basis.

Thomas Aquinas, for example, had availed himself of the mathematical techniques of his time in formulating his concept of analogy.[17] The formal sciences of our own day have developed to such an extent, that they have all but merged with the most abstract philosophical disciplines. Mathematics has long included much more than merely devices for the counting of apples and the surveying of land. Structures of the greatest possible generality have been investigated and new fields, such as algebra, set theory, topology, etc., have been opened up. Husserl explicitly recognized this and said that modern mathematics was nothing else than a kind of formal ontology.[18] But many traditionally minded phi-

[15] Goodman in BOCHEŃSKI-CHURCH-GOODMAN, p. 29.
[16] Cf. above, pp. 170–174.
[17] Cf. BOCHEŃSKI 1948.
[18] HUSSERL 1929, p. 68. Cf. also HUSSERL 1900/01, vol. 1, p. 252 (2nd ed.): "Only those who are not acquainted with mathematics as a modern science, especially with formal mathematics, and who take their measure only from Euclid and Adam Riese, can remain attached to the general prejudice that the essence of the mathematical lies in number and quantity."

losophers still repeat Aristotle's doctrine of the three degrees of abstraction and consider mathematics merely as the science of quantity.

An up-dating of traditional philosophy by the expedient of reformulating it in logistic language would also benefit contemporary logistic philosophy. For there the meager ontological inheritance from post-cartesian days is clearly inadequate.

I hope that what has been said in this book about language, logic and mathematics, will have convinced the reader that the treatment of these subjects is not of interest only to the specialist, the technical linguist, logician and mathematician, but enters into the discussion of central philosophical questions.[19]

[19] That even existentialists, little inclined as they are towards formal logic and mathematics, cannot overlook the new developments is shown by KNAUSS 1954, the dissertation of a student of K. Jaspers, in the bibliography of which we find such names as Ackermann, Cantor, Carnap, Fraenkel, Frege, Gödel, Hilbert and Bernays, Ramsey, Russell, Skolem, Weyl.

BIBLIOGRAPHY

AARON, J. R.: 1952, *The Theory of Universals*. Oxford.

AJDUKIEWICZ, K.: 1935, 'Die syntaktische Konnexität', *Studia Philosophica* **1**, 1–28.

ANDERSON, A. R.: 1957, Review of CARTWRIGHT 1954, in *The Journal of Symbolic Logic* **22**, 393–394.

ANSCOMBE, G. E. M.: 1959, *An Introduction to Wittgenstein's Tractatus*. London.

ANSCOMBE, G. E. M., and GEACH, P. T.: 1961, *Three Philosophers: Aristotle, Aquinas, Frege*. Oxford.

BANKS, P.: 1950, 'On the philosophical interpretation of logic: an Aristotelian dialogue', *Dominican Studies* **3**, 139–153. (Also in BOCHEŃSKI 1959.)

BARTLETT, J. M.: 1961, *Funktion und Gegenstand. Eine Untersuchung in der Logik von Gottlob Frege*. Diss. München.

BENACERRAF, P. and PUTNAM, H., ed.: 1964, *Philosophy of Mathematics, Selected Readings*. Englewood Cliffs, N. J.

BERGMANN, G.: 1964, *Logic and Reality*, Madison, Wisc.

BERNAYS, P.: 1935,' Sur le platonisme dans les mathématiques', *L'Enseignement mathématique* **34**, 52–69. English translation in BENACERRAF-PUTNAM, pp. 274–286.

BETH, E. W.: 1955, *Les fondements logiques des mathématiques*. Paris-Louvain. 2nd éd.

BETH, E. W.: 1959, *The Foundations of Mathematics*. Amsterdam.

BIRJUKOV, B. V.: 1961, 'Teorija smysla Gotloba Frege', in *Primenenie logiki v nauke i technike* (ed. by the Philosophical Institute of the Academy of Sciences, Moscow), pp. 502–555. (English translation in BIRJUKOV 1964.)

BIRJUKOV, B. V.: 1964, *Two Soviet Studies on Frege* (ed. I. Angelelli). Dordrecht.

BLACK, M.: 1949, *Language and Philosophy*. Ithaca, N. Y.

BLACK, M.: 1964, *A Companion to Wittgenstein's 'Tractatus'*. Ithaca, N. Y.

BOCHEŃSKI, J. M.: 1948, 'On analogy', *The Thomist* **11**, 474–490. (Also in BOCHEŃSKI 1959).

BOCHEŃSKI, J. M.: 1949, 'On syntactical categories', *The New Scholasticism* **23**, 257–280. (Also in BOCHEŃSKI 1959.)

BOCHEŃSKI, J. M.: 1951, *Europäische Philosophie der Gegenwart*. Bern. 2nd ed. (English translation: Berkeley-Los Angeles 1956.)

BOCHEŃSKI, J. M.: 1954, 'Spitzfindigkeit', in *Festgabe an die Schweizer Katholiken*. Fribourg, pp. 334–352.

BOCHEŃSKI, J. M.: 1956, *Formale Logik*. Freiburg-München. (English translation: Notre Dame, Ind., 1961.)

BOCHEŃSKI, J. M.: 1959, *Logisch-philosophische Studien* (ed. A. Menne). Freiburg. (English edition: Dordrecht 1962.)

BOCHEŃSKI, J. M., CHURCH, A., and GOODMAN, N.: 1956, *The Problem of Universals. A Symposium*. Notre Dame, Ind. (Bocheński's paper also in BOCHEŃSKI 1959 and Goodman's paper also in BENACERRAF-PUTNAM.)

BOCHEŃSKI, J. M., and MENNE, A.: 1954, *Grundriß der Logistik*. Paderborn.

BOEHNER, PH.: 1952, *Medieval Logic* (*1250–1400*). Chicago-Manchester.

BIBLIOGRAPHY

BOEHNER, Ph.: 1957, *Ockham. Philosophical Writings*. Edinburgh.

BRADLEY, F. H.: 1883, *The Principles of Logic*. London. (2nd ed.: 1922.)

BRADLEY, F. H.: 1893, *Appearance and Reality*. London.

BROUWER, L. E. J.: 1907, *Over de grondslagen der wiskunde*. Amsterdam-Leipzig.

BROUWER, L. E. J.: 1952, 'Historical background, principles and methods of intuitionism', *South African Journal of Science* **49**, 139–146.

BÜHLER, K.: 1918, 'Kritische Musterung der neueren Theorien des Satzes', *Indogermanisches Jahrbuch* **6**.

BÜHLER, K.: 1934, *Sprachtheorie. Die Darstellungsfunktion der Sprache*. Jena.

CARNAP, R.: 1921, *Der Raum. Ein Beitrag zur Wissenschaftslehre*. Diss. Jena. (Also Supplementary vol. 56 of *Kantstudien*, Berlin 1922.)

CARNAP, R.: 1923, 'Über die Aufgabe der Physik und die Anwendung des Grundsatzes der Einfachstheit', *Kantstudien* **28**, 90–107.

CARNAP, R.: 1927, 'Eigentliche und uneigentliche Begriffe', *Symposion* **1**, 355–374.

CARNAP, R.: 1928, *Der logische Aufbau der Welt*. Berlin. (2nd ed. Hamburg 1961; English translation: Berkeley-Los Angeles 1967).

CARNAP, R.: 1928a, *Scheinprobleme in der Philosophie. Das Fremdpsychische und der Realismusstreit*. Berlin. (Also in 2nd ed. and in English translation of CARNAP 1928.)

CARNAP, R.: 1929, *Abriß der Logistik. Mit besonderer Berücksichtigung der Relationentheorie und ihrer Anwendungen*. Wien.

CARNAP, R.: 1931, 'Überwindung der Metaphysik durch logische Analyse der Sprache', *Erkenntnis* **2**, 219–241.

CARNAP, R.: 1931a, 'Die physikalische Sprache als Universalsprache der Wissenschaft', *Erkenntnis* **2**, 432–465.

CARNAP, R.: 1933, 'Über Protokollsätze', *Erkenntnis* **3**, 215–228.

CARNAP, R.: 1934, *Logische Syntax der Sprache*. Wien. (English translation: London 1937.)

CARNAP, R.: 1934a, 'Die Methode der logischen Analyse', in *Actes du 8ème Congrès International de Philosophie à Prague 1934*. Prague 1936, pp. 142–145.

CARNAP, R.: 1935, *Philosophy and logical syntax*. London.

CARNAP, R.: 1936/1937, 'Testability and meaning', *Philosophy of Science* **3**, 419–471; **4**, 2–40. (Reprinted New Haven 1954.)

CARNAP, R.: 1938, 'Logical foundations of the unity of science', in *International Encyclopaedia of Unified Science* **1**, no. 1. Chicago, pp. 42–62.

CARNAP, R.: 1942, *Introduction to Semantics*. Cambridge, Mass.

CARNAP, R.: 1947, *Meaning and Necessity*. Chicago. (2nd ed. 1956.)

CARNAP, R.: 1950, 'Empiricism, semantics and ontology', *Revue Internationale de Philosophie* **4**, 20–40. (Reprinted in the 2nd ed. of CARNAP 1947.)

CARNAP, R.: 1952, 'Meaning postulates', *Philosophical Studies* **3**, 65–73. (Reprinted in the 2nd ed. of CARNAP 1947.)

CARNAP, R.: 1955, 'Meaning and synonymy in natural languages', *Philosophical Studies* **7**, 33–47. (Reprinted in the 2nd ed. of CARNAP 1947.)

CARNAP, R.: 1956, 'The methodological character of theoretical concepts', in *Minnesota Studies in the Philosophy of Science*, **1**: *The Foundations of Science and the Concepts of Psychology and Psycho-analysis* (ed. by H. Feigl and M. Scriven). Minneapolis, pp. 38–76.

CARNAP, R.: 1958, 'Beobachtungssprache und theoretische Sprache', *Dialectica* **12**, 236–248.

CARNAP, R.: 1958a, *Introduction to Symbolic Logic and its Applications*. New York.
CARTWRIGHT, R. L.: 1954, 'Ontology and the theory of meaning', *Philosophy of Science* **21**, 316–325.
CHOMSKY, N.: 1955, 'Logical syntax and semantics: their linguistic relevance', *Language* **31**, 36–45.
CHOMSKY, N.: 1957, *Syntactic Structures*. The Hague.
CHURCH, A.: 1946, 'A formulation of the logic of sense and denotation', *The Journal of Symbolic Logic* **11**, 31.
CHURCH, A.: 1951, 'A formulation of the logic of sense and denotation', in *Structure, Method and Meaning. Essays in Honor of Henry M. Scheffer* (ed. by P. Henle, H. M. Kallen and S. K. Langer). New York, pp. 3–24.
CHURCH, A.: 1951a, 'The need for abstract entities in semantic analysis', *Proceedings of the American Academy of Arts and Sciences* **80**, 100–112.
CHURCH, A.: 1946, *Introduction to Mathematical Logic*, part I. Princeton.
CHURCH, A.: 1958, 'Ontological commitment', *The Journal of Philosophy* **55**, 1008–1014.
CHURCH, A.: 1963, Review of LEONARD 1956, in *The Journal of Symbolic Logic* **28**, 259 ff.
CHWISTEK, L.: 1921, 'Antynomje logiki formalnej, *Przegląd Filozoficzne* **24**, 164–171.
CHWISTEK, L.: 1924/25, 'The theory of constructive types', *Rocznik Polskiego Towarzystwa Matematycznego* [Annales de la Société Polonaise de Mathématique] **2**, 9—48; **3**, 92–141.
CLAY, R. E.: 1961, *Contributions to Mereology*. Diss. Notre Dame.
COPI, I. M.: 1949, 'Modern logic and the synthetic a priori', *The Journal of Philosophy* **46**, 243–245.
COPI, I. M.: 1950, 'Gödel and the synthetic a priori: a rejoinder', *The Journal of Philosophy* **47**, 633–636.
COPI, I. M.: 1958, 'Objects, properties and relations in the "*Tractatus*", *Mind* **67**, 145–165.
COUSIN, V., ed.: 1836, *Ouvrages inédits d'Abélard*. Paris.
DAITZ, E.: 1953, 'The picture theory of meaning', *Mind* **62**, 184–201. (Reprinted in FLEW 1956.)
EVANS, E.: 1955, '*Tractatus* 3.1432', *Mind* **64**, 259–260.
EVANS, E.: 1959, 'About "aRb"', *Mind* **68**, 534–538.
FAVRHOLDT, D.: 1964, *An Interpretation and Critique of Wittgensteins Tractatus*. Copenhagen.
FEIGL, H. and SELLARS, W., ed.: 1949, *Readings in Philosophical Analysis*. New York.
FLEW, A. G. N., ed.: 1951/53, *Logic and Language*, 2 vols. Oxford. (Reprinted in 1 vol. Garden City, N. Y., 1965.)
FLEW, A. G. N., ed.: 1956, *Essays in Conceptual Analysis*. London.
FODOR, J. A. and KATZ, J. J., ed.: 1964, *The Structure of Language. Readings in the Philosophy of Language*. Englewood Cliffs, N. J.
FRAENKEL, A. A. and BAR-HILLEL, Y.: 1958, *Foundations of Set Theory*. Amsterdam.
FREGE, G.: 1879, *Begriffsschrift. Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*. Halle. (2nd ed.: Darmstadt 1964.)
FREGE, G.: 1884: *Die Grundlagen der Arithmetik, eine logisch-mathematische Untersuchung über den Begriff der Zahl*. Breslau. (Reprinted Darmstadt 1961. German with English translation: Oxford-New York 1950.)
FREGE, G.: 1891, *Funktion und Begriff*. Jena. (Reprinted in FREGE 1962. English translation in FREGE 1952.)

BIBLIOGRAPHY

FREGE, G.: 1892, 'Über Sinn und Bedeutung', *Zeitschrift für Philosophie und philosophische Kritik* **100**, 25–50. (Reprinted in FREGE 1962. English translation in *Philosophical Review* **57** (1948) 209–230, in FEIGL-SELLARS, and in FREGE 1952.)

FREGE, G.: 1892a, 'Über Begriff und Gegenstand', *Vierteljahrsschrift für wissenschaftliche Philosophie* **16**, 192–205. (Reprinted in FREGE 1962. English translation in *Mind*, **60** (1951) 168–180, and in FREGE 1952.)

FREGE, G.: 1904, 'Was ist eine Funktion?', in *Festschrift L. Boltzmann gewidmet zum 60. Geburtstag*, Leipzig, pp. 656–666. (Reprinted in FREGE 1962. English translation in FREGE 1952.)

FREGE, G.: 1952, *Translations from the Philosophical Writings of Gottlob Frege*, by P. GEACH and M. BLACK. Oxford-New York.

FREGE, G.: 1962, *Funktion, Begriff, Bedeutung. Fünf logische Studien* (ed. G. Patzig). Göttingen.

FRITZ, Ch. A.: 1952, *Bertrand Russell's Construction of the External World.* London.

GÖDEL, K.: 1931, 'Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme', *Monatshefte für Mathematik und Physik* **38**, 173–198.

GÖDEL, K.: 1944, 'Russell's mathematical logic', in SCHILPP 1944, pp. 123–153. (Reprinted in BENACERRAF-PUTNAM.)

GOODMAN, N.: 1947, 'The problem of counterfactual conditionals', *The Journal of Philosophy* **44**, 113–128. (Reprinted in GOODMAN 1955, and LINSKY 1952.)

GOODMAN, N.: 1949, 'On likeness of meaning', *Analysis* **10**, 1–7. (Reprinted in LINSKY 1952 and MACDONALD 1954.)

GOODMAN, N.: 1951, *The Structure of Appearance.* Cambridge, Mass. (2nd ed.: Indianapolis-New York-Kansas City 1966.)

GOODMAN, N.: 1952, 'Sense and certainty', *Philosophical Review* **61**, 160–167.

GOODMAN, N.: 1952a, 'On some differences about meaning', *Analysis* **13**, 90–96. (Reprinted in MACDONALD 1954.)

GOODMAN, N.: 1955, *Fact, Fiction and Forecast.* Cambridge, Mass. (2nd ed.: Indianapolis 1965.)

GOODMAN, N.: 1956, 'The revision of philosophy', in *American Philosophers at Work* (ed. S. HOOK). New York, pp. 75–92.

GOODMAN, N.: 1958, 'On relations that generate', *Philosophical Studies* **9**, 65–66. (Reprinted in BENACERRAF-PUTNAM.)

GOODMAN, N.: 1960, 'The way the world is', *The Review of Metaphysics* **14**, 48–56.

GOODMAN, N.: 1960a, Review of H. GOMBRICH, *Art and illusion* (New York 1960), in *The Journal of Philosophy* **57**, 595–599.

GOODMAN, N., and QUINE, W. V.: 1947, 'Steps toward a constructive nominalism', *The Journal of Symbolic Logic* **12**, 105–122.

GREDT, J.: 1901/02, *Elementa philosophiae aristotelico-thomisticae*, 2 vols. Freiburg i. Br. (11. ed.: Freiburg-Barcelona 1956.)

GRIFFIN, J.: 1964, *Wittgenstein's Logical Atomism.* Oxford.

GRZEGORCZYK, A.: 1955, 'The systems of Leśniewski in relation to contemporary logical research', *Studia Logica* **3**, 77–97.

HAHN, H.: 1929, *Überflüssige Wesenheiten.* Wien.

HAILPERIN, Th.: 1953, 'Quantification theory and empty individual-domains', *The Journal of Symbolic Logic* **18**, 197–200.

HARRIS, Z. S.: 1951, *Methods in Structural Linguistics.* Chicago.

HARRIS, Z. S.: 1957, 'Co-occurrence and transformation in linguistic structure', *Language* **33**, 283–340.

HASENJAEGER, G.: 1962, *Einführung in die Grundbegriffe und Probleme der modernen Logik*. Münster.

HEMPEL, C. G.; 1950, 'Problems and changes in the empiricist criterion of meaning', *Revue internationale de philosophie* **11**, 41–63. (Reprinted in LINSKY 1952.)

HEYTING, A.: 1930, 'Die formalen Regeln der intuitionistischen Logik', *Sitzungsberichte der Preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse*, 42–57.

HEYTING, A.: 1930a, 'Die formalen Regeln der intuitionistischen Mathematik', *Sitzungsberichte der Preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse*, 57–71, 158–169.

HEYTING, A.: 1934, *Mathematische Grundlagenforschung. Intuitionismus. Beweistheorie*. Berlin.

HEYTING, A.: 1955, *Les fondements des mathématiques. Intuitionnisme. Théorie de la démonstration*. Paris-Louvain. (French translation of HEYTING 1934, with additions).

HEYTING, A.: 1956, *Intuitionism. An Introduction*. Amsterdam.

HILBERT, D.: 1904, 'Über die Grundlagen der Logik und der Arithmetik', in *Verhandlungen des Dritten Internationalen Mathematiker-Kongresses in Heidelberg vom 8. bis 13. August 1904*. Leipzig 1905, pp. 174–185.

HILBERT, D., and BERNAYS, P.: 1934–1939, *Grundlagen der Mathematik*, 2 vols. Berlin.

HINTIKKA, J.: 1959, 'Existential presuppositions and existential commitments', *The Journal of Philosophy* **56**, 125–137.

HINTIKKA, J.: 1966, 'Studies in the logic of existence and necessity, I: Existence', *The Monist* **50**, 55–76.

HJELMSLEV, L.: 1943, *Omkring sprogteoriens grundlaegelse*. Kobenhavn. (English translation: *Prolegomena to a Theory of Language*, Supplement to *International Journal of American Linguistics*, vol. 19, no. 1, Baltimore 1953.) (Revised ed.: Madison, Wisc., 1961.)

HJELMSLEV, L.: 1947, 'Structural analysis of language', *Studia linguistica* **1**, 69–78.

HUME, D.: 1888, *A Treatise of Human Nature* (ed. L. A. Selby-Bigge). Oxford. (Reprinted 1958.)

HUME, D.: 1902, *Enquiries concerning the Human Understanding and Concerning the Principles of Morals* (ed. L. A. Selby-Bigge). 2nd ed. Oxford. (Reprinted 1951.)

HUSSERL, E.: 1900/01, *Logische Untersuchungen*. 2 vols. Halle. (2nd ed.: 1913–1921.)

HUSSERL, E.: 1929, *Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft*, Halle. (Also in *Jahrbuch für Philosophie und phänomenologische Forschung* **10**.)

INGARDEN, R.: 1921, 'Über die Gefahr einer *Petitio Principii* in der Erkenntnistheorie', *Jahrbuch für Philosophie und phänomenologische Forschung* **4**, 545–568.

INGARDEN, R.: 1925, 'Essentiale Fragen. Ein Beitrag zum Problem des Wesens', *Jahrbuch für Philosophie und phänomenologische Forschung* **7**, 125–304.

INGARDEN, R.: 1931, *Das literarische Kunstwerk. Eine Untersuchung aus dem Grenzgebiet der Ontologie, Logik und Literaturwissenschaft*. Halle. (2nd ed.: Tübingen 1960.)

INGARDEN, R.: 1934, 'Der logistische Versuch einer Neugestaltung der Philosophie', in *Actes du 8ème Congrès International de Philosophie à Prague 1934*. Prague 1936, pp. 203–208.

INGARDEN, R.: 1937, *O poznawaniu dzieła literackiego*. Lwów. (Reprinted in R. INGARDEN, *Studia z estetyki*, vol. 1, Warszawa 1957, pp. 1–251.)

BIBLIOGRAPHY

INGARDEN, R.: 1947/48, *Spór o istnienie świata*, 2 vols. Kraków. (2nd ed.: Warszawa 1960. German ed.: *Der Streit um die Existenz der Welt*, 2 vols., Tübingen 1964/65. English translation of parts of vol. 1 in INGARDEN 1960.)

INGARDEN, R.: 1955, 'O tłumaczeniach', in *O sztuce tłumaczenia* (ed. M. Rusinek). Wrocław 1955, pp. 127–190.

INGARDEN, R.: 1959, 'Edmund Husserl zum 100. Geburtstag', *Zeitschrift für philosophische Forschung* **13**, 459–463.

INGARDEN, R.: 1960, *Time and Modes of Being* (ed. H. R. Michejda). Springfield, Ill.

JAMES, W.: 1904, 'Does "consciousness" exist?', *The Journal of Philosophy* **1**, 477–491.

JORDAN, Z.: 1945, *The Development of Mathematical Logic and of Logical Positivism in Poland between the Two Wars*. London.

JØRGENSEN, J.: 1959, 'Towards a theory of inference', *Theoria* **25**, 123–147.

KATZ, J. J.: 1964, 'Analyticity and contradiction in natural language', in FODOR-KATZ, pp. 519–543.

KATZ, J. J., and FODOR, J. A.: 1963, 'The structure of a semantic theory', *Language* **39**, 170–210. (Reprinted in FODOR-KATZ, pp. 479–518.)

KEARNS, J. T.: 'The contribution of Leśniewski', *Notre Dame Journal of Formal Logic* (forthcoming).

KNAUSS, G.: 1954, *Gegenstand und Umgreifendes*. Basel.

KNEALE, W. and M.: 1962, *The Development of Logic*. Oxford.

KOTARBIŃSKI, T.: 1929, *Elementy teorii poznania, logiki formalnej i metodoligii nauk*. Lwów. (2nd ed.: Wrocław-Warszawa-Kraków 1961.)

KOTARBIŃSKI, T.: 1948/49, 'Sur l'attitude réiste (ou concretiste)', *Synthese* **7**, 262–273.

KOTARBIŃSKI, T.: 1957, *Wykłady z dziejów logiki*. Łodź.

KOTARBIŃSKI, T.: 1958, 'Fazy rózwojowe konkretyzmu', *Studia Filozoficzne* **4** (7), 3–13. (Reprinted in 2nd ed. of KOTARBIŃSKI 1929.)

KOTARBIŃSKI, T.: 1958a, *Wybor pism*, 2 vols. Warszawa.

KURATOWSKI, K.: 1921, 'Sur la notion de l'ordre dans la théorie des ensembles', *Fundamenta mathematicae* **2**, 161–171.

KUTSCHERA, F. von: 1964, *Die Antinomien der Logik. Semantische Untersuchungen*. Freiburg-München.

LADRIÈRE, J.; 1957, *Les limitations internes des formalismes. Etude sur la signification du théorème de Gödel et des théorèmes apparentés dans la théorie des fondements des mathématiques*. Louvain-Paris.

LAMBERT, K.: 1958–1964, 'Notes on E!', *Philosophical Studies* **9**, 60–63; **12**, 1–15; **13**, 51–59; **15**, 85–88.

LAMBERT, K.: 1963, Existential import revisited', *Notre Dame Journal of Formal Logic* **4**, 288–292.

LAMBERT, K.: 1965, 'On logic and existence', *Notre Dame Journal of Formal Logic* **6**, 135–141.

LEBLANC, H., and HAILPERIN, T.: 1959, 'Nondesignating singular terms', *Philosophical Review* **58**, 239–243.

LEFÈVRE, G.: 1898, *Les variations de Guillaume de Champeaux et la question des universaux. Etude suivie de documents originaux*. Lille.

LEIBNIZ, G. W.: 1875–1890, *Die philosophischen Schriften von G. W. Leibniz* (ed. C. J. Gerhardt). Berlin.

LEJEWSKI, Cz.: 1954, 'Logic and existence', *British Journal for the Philosophy of Science* **5**, 104–119.

LEJEWSKI, CZ.: 1954/55, 'A contribution to Leśniewski's mereology', *Rocznik Polskiego Towarzystwa Naukowego na Obczyznie* (London) **5**, 43–50.
LEJEWSKI, CZ.: 1955/56, ' A new axiom of mereology', *Rocznik Polskiego Towarzystwa Naukowego na Obczyźnie* (London) **6**, 65–70.
LEJEWSKI, CZ.: 1956, Review of SŁUPECKI 1955, in *The Journal of Symbolic Logic* **21**, 188–191.
LEJEWSKI, CZ.: 1957, 'Proper names', in *Proceedings of the Aristotelian Society*. Suppl. vol. 31, pp. 229–256.
LEJEWSKI, CZ.: 1958, 'On Leśniewski's ontology', *Ratio* **1**, 150–176.
LEJEWSKI, CZ.: 1960, 'A re-examination of the Russellian theory of descriptions', *Philosophy* **35**, 14–29.
LEJEWSKI, CZ.: 1963, 'Aristotle's syllogistic and its extensions', *Synthese* **15**, 125–154.
LEJEWSKI, CZ.: 1963a, 'A note on a problem concerning the axiomatic foundations of mereology', *Notre Dame Journal of Formal Logic* **4**, 135–139.
LEONARD, H. S.: 1956, 'The logic of existence', *Philosophical Studies* **7**, 49–64.
LEONARD, H. S., and GOODMAN, N.: 1940, 'The calculus of individuals and its use', *The Journal of Symbolic Logic* **5**, 45–55.
LEŚNIEWSKI, ST.: 1911, 'Przyczynek do analizy zdań egzystencjalnych', *Przegląd Filozoficzny* **14**, 329–345.
LEŚNIEWSKI, ST.: 1912, 'Próba dowodu ontologicznej zasady sprzeczonści', *Przegląd Filozoficzny* **15**, 202–226.
LEŚNIEWSKI, ST.: 1913, 'Krytyka logicznej zasady wyłączonego środka', *Przegląd Filozoficzny* **16**, 5–28.
LEŚNIEWSKI, ST.: 1914, 'Czy klasa klas, nie podporządkowanych sobie, jest podporządkowana sobie?', *Przegląd Filozoficzny* **17**, 63–75.
LEŚNIEWSKI, ST.: 1916, 'Podstawy ogólnej teoryi mnogości I', *Prace Polskiego Kola Naukowego w Moskwie, s. mat.-przyrod.* Moskwa.
LEŚNIEWSKI, St.: 1927–1931, 'O podstawach matematyki', *Przegląd Filozoficzny* **30**, 164–206 (1927); **31**, 261–291 (1928); **32**, 60–101 (1929); **33**, 77–105 (1930); **34**, 142–170 (1931).
LEŚNIEWSKI, ST.: 1929, 'Grundzüge eines neuen Systems der Grundlagen der Mathematik', *Fundamenta mathematicae* **14**, 1–81.
LEŚNIEWSKI, ST.: 1930, 'Über die Grundlagen der Ontologie', *Comptes rendus des séances de la Société des Sciences et des Lettres de Varsovie*, Cl. 3, **23**, 111–132.
LEŚNIEWSKI, ST.: 1931, 'Über Definitionen in der sogenannten Theorie der Deduktion', *Comptes rendus des séances de la Société des Sciences et des Lettres de Varsovie*, Cl. 3, **24**, 289–309.
LEŚNIEWSKI, ST.: 1938, 'Einleitende Bemerkungen zur Fortsetzung meiner Mitteilung u. d.T. "Grundzüge eines neuen Systems der Grundlagen der Mathematik"'. Offprint of the never published *Collectanea Logica* (Warszawa). (Copies in Harvard and Münster, Westfalen.)
LEŚNIEWSKI, ST.: 1938a, 'Grundzüge eines neuen Systems der Grundlagen der Mathematik § 12.' Offprint of the never published *Collectanea Logica* (Warszawa). (Copies in Harvard and Münster, Westfalen.)
LEWIS, C. I.: 1944, 'The modes of meaning', *Philosophy and Phenomenological Research* **4**, 236–249. (Reprinted in LINSKY 1952.)
LEWIS, C. I.: 1946, *An Analysis of Knowledge and Valuation*. La Salle, Ill.
LINSKY, L., ed.: 1952, *Semantics and the Philosophy of Language*. Urbana, Ill.
LUCAS, J.: 1961, 'Minds, machines and Gödel', *Philosophy* **36**, 112–126. (Slightly

revised in *The modeling of mind. Computers and intelligence*, ed. K. M. Sayre and F. J. Crosson, Notre Dame 1963.)

ŁUKASIEWICZ, J.: 1910, *O zasadzie sprzeczności u Arystotelesa. Studyum Krytyczne*. Kraków.

ŁUKASIEWICZ, J.: 1918, 'Treść wykładu pożegnałnego wygloszonego w auli Universitytu Warszawskiego 7 marca 1918', *Pro Arte et Studio* (Warszawa) **11**. (Reprinted in *Wiadomości*, London, 501.)

ŁUKASIEWICZ, J.: 1929, *Elementy logiki matematycznej*. Lithogr. Warszawa. (2nd ed.: Warszawa 1958.)

ŁUKASIEWICZ, J.: 1951, *Aristotle's Syllogistics from the Standpoint of Modern Formal Logic*. Oxford.

ŁUKASIEWICZ, J. and TARSKI, A.: 1930, 'Untersuchungen über den Aussagenkalkül', *Comptes rendus des séances de la Société des Sciences et des Lettres de Varsovie*, Cl. 3, **23**, 1–21. (English in TARSKI 1956.)

LUSCHEI, E. C.: 1962, *The Logical Systems of Leśniewski*. Amsterdam.

MCCALL, ST., ed.: *Polish Logic*. Oxford (forthcoming).

MACDONALD, M., ed.: 1954, *Philosophy and Analysis*. Oxford.

MCINERNY, R. M.: 1961, *The Logic of Analogy. An Interpretation of St. Thomas*. The Hague.

MCNAUGHTON, R.: 1954, 'Axiomatic systems, conceptual schemes, and the consistency of mathematical theories', *Philosophy of Science* **21**, 44–53.

MCNAUGHTON, R.: 1957, 'Conceptual schemes in set theory', *The Philosophical Review* **66**, 66–80.

MANNOURY, G.: 1948, *Handboek der analytische signifika*, 2 vols. Bussum.

MARC-WOGAU, K.: 1945, 'Die Theorie der Sinnesdaten: Probleme der neueren Erkenntnistheorie in England', *Uppsala Univ. Årsskrift*.

MARTIN, R. M.: 1943, 'A homogeneous system for formal logic', *The Journal of Symbolic Logic* **8**, 1–23.

MARTIN, R. M.: 1949, 'A note on nominalism and recursive functions', *The Journal of Symbolic Logic* **14**, 27–31.

MARTIN, R. M.: 1949a, 'A note on nominalistic syntax', *The Journal of Symbolic Logic* **14**, 226–227.

MARTIN, R. M.: 1950, 'On virtual classes and real numbers', *The Journal of Symbolic Logic* **15**, 131–134.

MARTIN, R. M. and WOODGER, J. H.: 1951, 'Toward an inscriptional semantics', *The Journal of Symbolic Logic* **16**, 191–203.

MASLOW, A.: 1961, *A Study of Wittgensteins Tractatus*. Berkeley, Calif.

MATES, B.: 1950, 'Synonymity', *University of California Publications in Philosophy* **25**, 201–226. (Reprinted in LINSKY 1952.)

MENGER, K.: 1930, 'Der Intuitionismus', *Blätter für deutsche Philosophie* **4**, 311–325.

MICHALSKI, K., ed.: 1937, *Myśl katolicka wobec logiki współczesnej*. Studia Gnesiensia 15, Poznan.

MILL, J. ST.: 1843, *A System of logic*. London. (8th ed. reprinted 1961.)

MOODY, E. A.: 1953, *Truth and Consequence in Medieval Logic*. Amsterdam.

MOORE, G. E.: 1899, 'The nature of judgement', *Mind* **8**, 176–193.

MOORE, G. E.: 1903, 'The refutation of idealism', *Mind* **12**, 433–453.

MOORE, G. E.: 1903a, *Principia ethica*. Cambridge.

MOORE, G. E., G. F. STOUT and G. D. HICKS: 1923, 'Symposium: Are the characteristics of particular things universal or particular?' *Proceedings of the Aristotelian*

Society. Suppl. vol. **3**, pp. 95–128.
MORRIS, CH.: 1938, *Foundutions of the Theory of Signs*. Encyclopedia of Unified Science 1, no. 2, Chicago.
MYHILL, J. R.: 1950, Review of MARTIN 1949 and MARTIN 1949a, in *The Journal of Symbolic Logic* **15**, 153.
MYHILL, J. R.: 1952, 'Some philosophical implications of mathematical logic', *The Review of Metaphysics* **6**, 165–198.
MYHILL, J. R.: 1953, 'Arithmetic with creative definitions by induction', *The Journal of Symbolic Logic* **18**, 115–118.
MYHILL, J. R.: 1953a, 'On the ontological significance of the Löwenheim-Skolem theorem', in *Academic Freedom, Logic, and Religion*. Philadelphia, pp. 57–70.
NAGEL, E. and NEWMAN, J. R.: 1959, *Gödel's Proof*. London.
NEURATH, O.: 1932, 'Physikalismus', *Scientia* **50**, 297–303.
OGDEN, C. K. and RICHARDS, I. A.: 1923, *The Meaning of Meaning*. London.
PAP, A.: 1958, *Semantics and Necessary Truth, An Inquiry into the Foundations of Analytic Philosophy*. New Haven.
PEANO, G.: 1891, 'Sul concetto di numero', *Rivista di Matematica* **1**, 87–102, 256–267.
PEANO, G.: 1894, *Notations de logique mathématique*. Turin.
PEIRCE, C. S.: 1931–1935, *Collected Papers of Charles Sanders Peirce* (ed. Ch. Hartshorne and P. Weiss). 6 vols. Cambridge, Mass.
PENTTILÄ, A., and SAARNIO, U.: 1934, 'Einige grundlegende Tatsachen der Wortheorie nebst Bemerkungen über die sogenannten unvollständigen Symbole', *Erkenntnis* **4**, 28–45, 139–159, 164.
PITCHER, G.: 1964, *The Philosophy of Wittgenstein*. Englewood Cliffs, N. J.
POINCARÉ, H.: 1905/06, 'Les mathématiques et la logique', *Revue de Métaphysique et de Morale* **13** (1905), 815–835; **14** (1906), 17–34, 294–317.
PRIOR, A. N.: 1953, Review of SOBOCIŃSKI 1949/50, in *The Journal of Symbolic Logic* **18**, 331–333.
PRIOR, A. N.: 1955, *Formal Logic*. Oxford. (2nd ed.: 1962.)
PRIOR, A. N.: 1963, 'Existence in Leśniewski and in Russell', in *Formal systems and recursive functions. Proceedings of the 8th Logic Colloquium, Oxford, July 1963* (ed. J. N. Crossley and M. A. E. Dummett). Amsterdam 1965, pp. 149–155.
PRISCIANUS DE CAESAREA: 1855–1859, *Institutio de arte grammatica* (ed. M. Hertz), in H. KEIL, *Grammatici latini* vols. 2 and 3. Leipzig.
PUTNAM, H.: 1960, Review of NAGEL-NEWMAN, in *Philosophy of Science* **27**, 205–207.
QUINE, W. V.: 1939, 'Designation and existence', *The Journal of Philosophy* **36**, 701–709.
QUINE, W. V.: 1941, 'Whitehead and the rise of modern logic', in *The Philosophy of A. N. Whitehead* (ed. P. A. Schilpp). New York.
QUINE, W. V.: 1943, 'Notes on existence and necessity', *The Journal of Philosophy* **40**, 113–127. (Reprinted in LINSKY 1952.)
QUINE, W. V.: 1947, 'On universals', *The Journal of Symbolic Logic* **12**, 74–84.
QUINE, W. V.: 1951, 'On Carnap's views on ontology', *Philosophical Studies* **2**, 65–72.
QUINE, W. V.: 1951a, 'Two dogmas of empiricism', *The Philosophical Review* **60**, 20–43. (Reprinted in QUINE 1953.)
QUINE, W. V.: 1952, Review of K. AJDUKIEWICZ, 'On the notion of existence', *Studia philosophica*, vol. 4 (1951), pp. 7–22, in *The Journal of Symbolic Logic* **17**, 141–142.
QUINE, W. V.: 1953, *From a Logical Point of View*. Cambridge, Mass.
QUINE, W. V.: 1954, 'Quantification and the empty domain', *The Journal of Symbolic Logic* **19**, 177–179.

QUINE, W. V.: 1960, *Word and Object*. Cambridge, Mass.-New York.
QUINE, W. V.: 1963, *Set Theory and its Logic*. Cambridge, Mass.
RAMSEY, F. P.: 1926, 'The foundations of mathematics', *Proceedings of the London Mathematical Society* **25**, 338–384. (Reprinted in F. P. RAMSEY, *The Foundations of Mathematics and Other Logical Essays*. London 1931.)
REGAMEY, C.: 1952, 'Text of contribution', in *Proceedings of the 7th International Congress of Linguists, London Sept. 1952*. London 1956, pp. 129–130.
REGAMEY, C.: 1954, 'A propos de la "construction ergative" en indo-aryen moderne', in *Sprachgeschichte und Wortbedeutung, Festschrift für A. Debrunner*. Bern, pp. 363–381.
RIVETTI-BARBÒ, F.: 1961, *L'antinomia del mentitore nel pensiero contemporaneo da Peirce a Tarski*. Milano.
ROBINS, R. H.: 1951, *Ancient and Mediaeval Grammatical Theory in Europe. With particular reference to modern linguistic doctrine*. London.
ROSSER, B.: 1939, 'An informal exposition of proofs of Gödel's theorem and Church's theorem', *The Journal of Symbolic Logic* **4**, 53–60, IV.
RUSSELL, B.: 1900, *A Critical Exposition of the Philosophy of Leibniz*. Cambridge. (2nd ed.: London 1937.)
RUSSELL, B.: 1903, *Principles of Mathematics*. Cambridge. (2nd ed., with introduction: London 1937.)
RUSSELL, B.: 1905, 'On denoting', *Mind* **14**, 479–493. (Reprinted in RUSSELL 1956.)
RUSSELL, B.: 1905a, Review of A. MEINONG, *Untersuchungen zur Gegenstandstheorie und Psychologie* (Leipzig 1904), in *Mind* **14**, 530–538.
RUSSELL, B.: 1906, 'On some difficulties in the theory of transfinite numbers and order types', *Proceedings of the London Mathematical Society* **4**, 29–53.
RUSSELL, B.: 1906a, 'The theory of implication', *American Journal of Mathematics* **28**, 159–202.
RUSSELL, B.: 1910, *Philosophical Essays*. London.
RUSSELL, B.: 1912, *The Problems of Philosophy*. London.
RUSSELL, B.: 1912a, On the relation of universals and particulars', *Proceedings of the Aristotelian Society* **12**, 1–24. (Reprinted in RUSSELL 1956.)
RUSSELL, B.: 1914, *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*. London. (2nd ed.: 1926.)
RUSSELL, B.: 1918, *Mysticism and Logic and Other Essays*. New York. (2nd ed.: London-New York 1921.)
RUSSELL, B.: 1918/19, 'The philosophy of logical atomism', *The Monist* **28**, 495–527 (1918); **29**, 32–63, 190–222, 345–380 (1919). (Reprinted: Minneapolis 1949 and in RUSSELL, 1956).
RUSSELL, B: 1919, *Introduction to Mathematical Philosophy*. London.
RUSSELL, B.: 1921, *The Analysis of Mind*. London.
RUSSELL, B.: 1924, 'Logical atomism', in *Contemporary British Philosophy* (ed. J. F. Muirhead), vol. 2, pp. 356–383. (Reprinted in RUSSELL 1956.)
RUSSELL, B.: 1927, *An Outline of Philosophy*. London.
RUSSELL, B.: 1940, *An Inquiry into Meaning and Truth*. New York-London.
RUSSELL, B.: 1944, 'My mental development' and: 'Reply to criticism', in SCHILPP 1944.
RUSSELL, B.: 1956, *Logic and Knowledge, Essays 1901–1950* (ed. R. Ch. Marsh). New York.
RUSSELL, B.: 1959, *My Philosophical Development*. London-New York.

RÜSTOW, A.: 1910, *Der Lügner*. Leipzig.
RYLE, G., ed.: 1956, *The Revolution in Philosophy*. London.
SAUSSURE, F. de: 1916, *Cours de linguistique générale* (ed. Ch. Bally, A. Sechehaye and A. Riedlinger). Lausanne-Paris. (2nd ed.: Paris 1922.)
SCHEFFLER, I., and CHOMSKY, N.: 1959, 'What is said to be', *Proceedings of the Aristotelian Society* **59**, 71–82.
SCHILPP, P. A., ed.: 1944, *The Philosophy of Bertrand Russell*. Evanston, Ill.
SCHILPP, P. A.: 1963, *The Philosophy of Rudolf Carnap*. La Salle, Ill. – London.
SCHOLZ, H.: 1938, 'Die mathematische Logik und die Metaphysik', *Philosophisches Jahrbuch der Görresgesellschaft* **51**, 257–291.
SCHOLZ, H.: 1940, *Was ist Philosophie? Der erste und der letzte Schritt auf dem Wege zu ihrer Selbstbestimmung. Frege-Studien*. Schriften zur Grundlagenforschung und zur Geschichte der Logik no. 1. Berlin-Wien. (Also: *Archiv für Rechts- und Sozialphilosophie* vol. 33 (1939/40), pp. 1–55. Reprinted in SCHOLZ 1961.)
SCHOLZ, H.: 1941, *Metaphysik als strenge Wissenschaft*. Köln. (Reprinted: Darmstadt 1965.)
SCHOLZ, H.: 1944, 'Logik, Grammatik, Metaphysik', *Archiv für Rechts- und Sozialphilosophie* **36**, 393–433. (Reprinted in *Archiv für Philosophie* (Stuttgart), vol. 1 (1947), pp. 39–80, and in SCHOLZ 1961.)
SCHOLZ, H.: 1961, *Mathesis universalis. Abhandlungen zur Philosophie als strenger Wissenschaft* (ed. H. Hermes, F. Kambartel and J. Ritter). Basel.
SCHOLZ, H., and SCHWEITZER, H.: 1935, *Die sogenannten Definitionen durch Abstraktion. Eine Theorie der Definitionen durch Bildung von Gleichheitsverwandtschaften*. Forschungen zur Logistik und zur Grundlegung der exakten Wissenschaften, fasc. 3. Leipzig.
SIMPSON, T. M.: 1964, *Formas lógicas, realidad y significado*. Buenos Aires.
SINISI, V. F.: 1962, 'Nominalism and common names', *Philosophical Review* **71**, 233.
SINISI, V. F.: 1964, 'Kotarbiński's theory of genuine names', *Theoria* **30**, 80–95.
SŁUPECKI, J.: 1953, 'St. Leśniewski's protothetics', *Studia Logica* **1**, 44–111, 299.
SŁUPECKI, J.: 1955, 'St. Leśniewski's calculus of names', *Studia Logica* **3**, 7–76.
SMART, J. J. C.: 1961, 'Gödel's theorem, Church's theorem and mechanism, *Synthese* **13**, 105–110.
SMART, J. J. C.: 1963, *Philosophy and Scientific Realism*. London.
SOBOCIŃSKI, B.: 1934, 'O kolejnych uproszczeniach aksjomatyki "ontologji" prof. St. Leśniewskiego', in *Fragmenty filozoficzne. Księga pamiątkowa ku 15-lecia pracy nauczycielskiej w Uniw. Warszawskim prof. T. Kotarbińskiego*. Warszawa, pp. 143–160.
SOBOCIŃSKI, B.: 1949, *An Investigation of Protothetics*. Cahiers de l'Institut d'Etudes Polonaises en Belgique 5. Bruxelles.
SOBOCIŃSKI, B.: 1949/50, 'L'analyse de l'antinomie russellienne par Leśniewski', *Methodos* **1**, 94–107, 220–228, 308–316 (1949); **2**, 237–257 (1950).
SOBOCIŃSKI, B.: 1953/54, 'Z badan nad aksjomatyką protothetyki St. Leśniewskiego', *Rocznik Polskiego Towarzystwa Naukowego na Obczyźnie* (London) **4**, 18–20.
SOBOCIŃSKI, B.: 1954/55, 'Studies in Leśniewski's mereology', *Rocznik Polskiego Towarzystwa Naukowego na Obczyźnie* **5**, 34–43.
SOBOCIŃSKI, B.: 1955/56, 'On well constructed axiom systems', *Rocznik Polskiego Towarzystwa Naukowego na Obczyźnie* **6**, 54–65.
SOBOCIŃSKI, B.: 1956, 'In memoriam Jan Łukasiewicz', *Philosophical Studies* (Ireland) **6**, 3–49.

BIBLIOGRAPHY

SOBOCIŃSKI, B.: 1956/57, 'Jan Łukasiewicz 1878–1956', *Rocznik Polskiego Towarzystwa Naukowego na Obczyźnie* **7**, 3–21.

SOBOCIŃSKI, B.: 1957, 'La génesis de la escuela polaca de lógica', *Revista Oriente* (Madrid) **7**, 83–95.

SOBOCIŃSKI, B.: 1958, 'Jan Salamucha (1903–1944): a biographical note', *The New Scholasticism* **32**, 327–333.

SOBOCIŃSKI, B.: 1960/61, 'On the single axioms of protothetics', *Notre Dame Journal of Formal Logic* **1** (1960) 52–73; **2** (1961) 111–126, 129–148.

STEGMÜLLER, W.: 1954, 'Der Begriff des synthetischen Urteils a priori und die moderne Logik', *Zeitschrift für philosophische Forschung* **8**, 535–563.

STEGMÜLLER, W.: 1956/57, 'Das Universalienproblem einst und jetzt', *Archiv für Philosophie* **6**, 192–225 (1956); **7**, 45–81 (1957).

STEGMÜLLER, W.: 1957, *Das Wahrheitsproblem und die Idee der Semantik. Eine Einführung in die Theorien von A. Tarski und R. Carnap*. Wien.

STEGMÜLLER, W.: 1959, *Unvollständigkeit und Unentscheidbarkeit. Die metamathematischen Resultate von Gödel, Church, Kleene, Rosser und ihre erkenntnistheoretische Bedeutung*. Wien.

STENIUS, E.: 1960, *Wittgenstein's 'Tractatus'. A critical exposition of the main lines of thought*. Ithaca, N.Y.

STEVENSON, CH. L.: 1945, *Ethics and Language*. New Haven-London-Oxford.

TARSKI, A.: 1929, 'Les fondements de la géometrie des corps', in *Księga pamiątkowa Pierwszego Polskiego Zjazdu Matematycznego*. Suppl. to *Annales de la Société de Mathématique*, Krakow. (English in TARSKI 1956.)

TARSKI, A.: 1933, *Pojęcie prawdy w jezykach nauk dedukcyjnych*. Prace Towarzystwa Naukowego Warszawskiego, wydział 3. (In German under the title: 'Der Wahrheitsbegriff in den formalisierten Sprachen', *Studia philosophica* vol. 1, 1936, pp. 261–405; off prints dated 1935. German text reprinted in RIVETTI-BARBÒ 1961. English translation in TARSKI 1956.)

TARSKI, A.: 1935, 'Zur Grundlegung der Booleschen Algebra I', *Fundamenta Mathematicae* **24**, 177–198. (English in TARSKI 1956.)

TARSKI, A.: 1936, 'Grundlegung der wissenschaftlichen Semantik. Über den Begriff der logischen Folgerung', *Actes du Congrès International de Philosophie Scientifique* **3**, 1–8; **7**, 1–11, Paris. (English in TARSKI 1956.)

TARSKI, A.: 1944, 'The semantic conception of truth', *Philosophy and Phenomenological Research* **4**, 341–375. (Reprinted in LINSKY 1952.)

TARSKI, A.: 1956, *Logic, Semantics, Metamathematics. Papers from 1923 to 1938*. Oxford.

THIEL, Ch.: 1965, *Sinn und Bedeutung in der Logik Gottlob Freges*, Monographien zur philosophischen Forschung vol. 43. Meisenheim am Glan.

THOMAS AQUINAS: 1888–1906, *Opera omnia iussu impensaque Leonis XIII P. M. edita* (vol. 4–12: *Summa theologica*). Rome.

THOMAS AQUINAS: 1929–1947, *Scriptum super Libros Sententiarum Magistri Petri Lombardi Episcopis Parisiensis* (ed. P. Mandonnet and M. F. Moos). Paris.

THOMAS AQUINAS: 1935, *In Metaphysicam Aristotelis commentaria* (ed. M.-R. Cathala). Turin. 3rd. ed.

TURQUETTE, A. R.: 1950, 'Gödel and the synthetic a priori', *Journal of Philosophy* **47**, 125–129.

TWARDOWSKI, K.: 1894, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*. Wien.

URMSON, J. O.: 1956, *Philosophical Analysis. Its Development between the two World Wars*. London-Oxford.

VUYSJE, D.: 1953, 'Significs, its tendency, methodology and applications', *Proceedings of the American Academy of Arts and Sciences* **80**, 223–270.

WAISMANN, F.: 1932, 'Logische Analyse des Wahrscheinlichkeitsbegriffes', *Erkenntnis* **1**, 228–248.

WANG, H.: 1954, 'The formalization of mathematics', *The Journal of Symbolic Logic* **19**, 241–266.

WANG, H. and MCNAUGHTON, R.: 1953, *Les systèmes de la théorie des ensembles.* Paris-Louvain.

WHITE, M. G.: 1950, 'The analytic and the synthetic: an untenable dualism', in *John Dewey: Philosopher of science and freedom* (ed. S. Hook), New York. (Reprinted in LINSKY 1952.)

WHITEHEAD, A. N.: 1919, *An Enquiry concerning the Principles of Natural Knowledge.* Cambridge.

WHITEHEAD, A. N. and RUSSELL, B.: 1910–1913, *Principia mathematica.* Cambridge. (2nd ed. 1925–1927). 3 vols.

WHORF, B. L.: 1945, 'Grammatical categories', *Language* **21**, 1–11.

WIENER, N.: 1914, 'A simplification of the logic of relations', *Proceedings of the Cambridge Philosophical Society* **17**, 387–390.

WITTGENSTEIN, L.: 1921, 'Logisch-philosophische Abhandlung', *Annalen der Naturphilosophie* **14**, 185–262. (German text with English translation by C. K. Ogden and introduction by B. Russell under the title: *Tractatus logico-philosophicus*, London 1922. With new English translation by D. F. Pears and B. F. McGuinness, London 1961.)

WITTGENSTEIN, L.: 1929, 'Some remarks on logical form', *Proceedings of the Aristotelian Society.* Suppl. vol. 9, pp. 162–171.

WOODGER, J. H.: 1937, *Axiomatic Method in Biology.* Cambridge-New York.

WOODGER, J. H.: 1952, 'Science without properties', *British Journal for the Philosophy of Science* **2**, 193–216.

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