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ONTOLOGY OF DIVINITY

Edited by Mirosław Szatkowski

PHILOSOPHICAL ANALYSIS

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Ontology of Divinity

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J. M. Bocheński inspired me to undertake the *Ontology of Divinity* project:

> The level of contemporary thinking about God is unsatisfactory. … There is an urgent need to resume serious philosophical study of God. … Scholastic theories about God need to be critically worked through. … But the critical adoption of tradition alone is not enough. First of all, systematic work is needed. … Neo-Thomistic works of this kind, written without knowledge of the conceptual tools of mathematical logic, are no longer satisfactory today. … Followers of St. Thomas have sometimes such great reverence for him that they take a priori every sentence of his to be true and try to defend his views even where they are obviously wrong. … The authors of these works show very little interest in logical problems and even less knowledge of logic. (J. M. Bocheński, *Gottes Dasein und Wesen*, Munich: Philosophia, 2003)

I would like to thank Father Professor Bocheński, OP, for this inspiration and steadfast attitude in the struggle with clichéd superstitions and intellectual mediocrity.

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Mirosław Szatkowski

List of Abbreviations

AC	Argument from consciousness
AD	Aseity-sovereignty doctrine
BARC	Beer annihilates reflexive credences
CF	Counterfactuals of freedom
CH	Continuum hypothesis
CHO	Choice operator
COVID-19	Coronavirus disease 2019
DARC	Deliberation annihilates reflexive credences
DDI	Nicholas of Cusa, *De Docta Ignorantia* (On Learned Ignorance)
DDS	Doctrine of divine simplicity
DNA	Deoxyribonucleic acid
DRA	Double reflection axiom
EDRA	Extended double reflection axiom
EFA	Exponential function arithmetic
En.	Plotinus, *Enneads*
ETRP	Extended theological reflection principle
GRP	Global reflection principle
HB	Hebrew Bible
IBE	Inference to the best explanation
IM	Individual maximality
IUC	Impossibility of uncaused concrete objects
LEM	Law of the excluded middle
LNC	Law of non-contradiction
LP	Paraconsistent logic
LST	Language of set theory
MA	Mereological aggregate
MF	Metaphysical foundationalism
ML	Montague-Levy’s reflection principle

Mon.	St. Anselm, *Monologion*
MTA	Modified theistic activism
NASA	National Aeronautics and Space Administration
NBG	(or VNBG) Von Neumann-Bernays-Gödel set theory
NF	Quine’s set theory
NRSV	New Revised Standard Version
OOM	Omnipotent, omniscient, morally perfect being
OT	Old testament
PCG	Principle of character grounding
PDB	Perfect divine beauty
PM	Property maximality
PMC	Principle of material causality
POS	POS(A) is read “the class A is positive”
PP	Precautionary principle
PW	Pascal’s Wager
QED	Quod erat demonstrandum
QTRP	Quantificational theological reflection principle
RE	Real essentialism
RL	Religious language
RP	Reflection principle
S. th.	Aquinas, Summa *Theologiae*
S4	Reinhardt’s reflection principle
S5	One of five systems of modal logic proposed by Clarence Irving Lewis and Cooper Harold Langford in their 1932 book *Symbolic Logic*
SD	Substance dualism
SP	Substantial priority, i.e. the view that substances are fundamental unities of parts, properties, and powers ordered for the sake of the entity’s proper functioning.
ST	Thomas Aquinas (1947), *Summa theologica*
SU	Sustainer of the universe

TIO	Thinking immaterial object
TMO	Thinking material object
TRP	Theological reflection principle
VNBG	(or NBG) Von Neumann-Bernays-Gödel set theory
ZF	Zermelo-Fraenkel set theory
ZFC	ZF plus the axiom of choice
ZFM	the theory ZFC + “there exists a measurable cardinal”

Introduction: An Incomplete Guide to Ontology of Divinity

Mirosław Szatkowski

The nature of *God's attributes* and debates concerning *His existence* or *non-existence* are old but compelling topics, because they date from the birth of philosophy.

Now, we have a collection of 35 essays that announces a new era in the philosophy of God. Many of these contributions work to create stronger links between the philosophy of God, on the one hand, and mathematics and metamathematics, on the other hand. While the remaining contributions are rich with insights that press God-oriented literature further in ways that deserve careful study.

This editorial introduction is structured as follows. In Section 1, we reflect on the question: What is the Ontology of Divinity? We start out with a definition of this discipline, but our primary focus is to raise useful and interesting questions scattered throughout it, rather than trying to answer them. In Section 2, we point out some possibilities of applying mathematics and metamathematics to questions about God. But, is this *only* possible? No, it seems to be indispensable! We also ask the reverse question: Does philosophy of God have anything to offer mathematics? We enter the maze of issues generated by this question, hoping to arouse interest in them. Finally, in Section 3, we offer a brief summary of each of the papers included in this volume.

1 What Is the Ontology of Divinity?

What we mean by ‘ontology of divinity’ is a study of *God’s attributes* (of the nature of God) and problems related to *His existence* or *non-existence*. As a matter of fact, a huge plethora of divine attributes was accumulated in the centuries-old philosophical literature—*inter alia*: omniscience, omnipotence, omnibenevolence, omniperfection (a being having all perfections), omnipresence, omnitemporality, a being than which nothing greater can be conceived, an absolute creator of everything which exists distinct from Him, an unmoved mover of everything which exists distinct from Him. And the concept of God is thought of as the cluster or the set of all divine attributes that are tied together in some way. The claim here is that each of the divine attributes is an essential property of God, where a property is essential to some being if and only if the loss of that property entails that that being ceases to exist. History shows that attempts to explain each of the above-mentioned divine attributes led to their various characteristics, which naturally produced the question: What, if anything, unifies these characteristics?

The divine attributes are deeply intertwined with the concept of consistency. The condition of their consistency and coherency between them seems to be an utterly natural one. Unfortunately, the following three examples challenge this condition. The doctrine of divine *omniscience* is thought of as comprising—among others—the two following claims: (i) God’s knowledge is *complete*, i.e., God knows all truths, and (ii) God’s knowledge is *sound*, i.e., everything that God knows is true. Thus, the claim (i) demands that there is a set of all truths, which is inconsistent with Cantor’s theorem which states that there can be no such set. Moreover, the claims (i) and (ii) are not consistent. The argument is this: Let us consider the divine Liar (*): God knows that (*) is false. Trying to apply a classical binary truth value to this statement leads to the Liar paradox, i.e., to a

contradiction. *Omnipotence* is generally understood as the power to do anything. But what the word 'anything' involves is highly contentious. Does this word also include a logical impossibility, for example, could God make 2+2=5? And there are other dilemmas, such as the *paradox of the stone*: Could God create a rock so heavy that he cannot lift it? And the *paradox of sovereignty*: Could a sovereign God create a law that binds himself? *Omnibenevolence* is understood as perfect and infinite *goodness*, depending on precisely how the word 'good' is recognized. Thus, saying that God is omnibenevolent means that he is an absolutely (perfectly) good being. God's omnibenevolence is, however, problematic. The existence of evil in the world seems to suggest that omnibenevolence, omnipotence and omniscience are incompatible. Because if God knows evil things are happening, why does he allow them to happen if he is omnibenevolent and omnipotent? A legitimate question is: How can one achieve consistency in divine attributes? Or can we justify in any way the existence of inconsistent divine attributes and incoherency between them?

How many divine attributes are there? One of the answers is the doctrine that all of God's attributes are identical.[1] The question then arises: What motivates such an extremely radical ontological doctrine? In fact, this doctrine is no homogeneous theory, and there are a variety of theories with the label 'divine simplicity.' Choosing then what is common to these various attributes and roughly speaking, divine simplicity is understood as God's lack of parts, which covers not only spatial and temporal parts (if any) but also metaphysical parts or ontological constituents—God is free of matter-form composition, potency-act composition, and existence-essence composition, and God as subject of his attributes-his attributes composition. "God is thus in some sense identical to each of his attributes, which implies that each attribute is identical to every other one" (→Vallicella,

2019). A bit more precisely, the proof of the thesis that all divine attributes are identical can be captured as follows: *Premise 1* (*Referential Indiscernibility of Divine Attributes*): All divine attributes are referentially identical, and their distinction is only a manner of speaking; *Premise 2* (*Identity of Indiscernibles*): If attributes X and Y are referential identical—i.e., for every object x, object x has a divine attribute X if and only if object x has a divine attribute Y—then X is identical to Y; *Thesis*: All divine attributes are identical. This reasoning is formally correct, but the problem is to justify Premises 1 and 2. Really, whether any two of the divine attributes are referentially identical or not should be a subject of proof, and should not simply be *a priori* assumption in the debate. And the traditional notion of the *identity of attributes* by their referential identity is only one of the possible ways to characterize this concept, which may not be applicable in every case—in particular, in the case of divine attributes. Another question arises still: Can determining the number of divine attributes take place without presupposing a defined identity? According to the traditional notion of counting, it is not reasonable to think so. But there are views that do not presuppose either identity or difference for the entities being counted (see →Tzouvaras, 2005; →French and Krause, 2006; →Domenech and Holik, 2007; →Muller and Saunders, 2008).

What's important is that these views influence the answers to the question: What kind of entities are divine attributes? The issue here is not about listing divine attributes, but about their common nature (interchangeably, essence). This question can be broken down into the following ones: Do divine attributes have a common nature? What is this common nature of divine attributes—in other words, What is the essence of common nature of divine attributes? And it's about whether and how there is a common nature of divine attributes. Of course, full answers to these questions require, first of all, a general theory

of *essence*. Unfortunately, there is no universally accepted general theory of essence and the prospects for obtaining such a theory seem too slim. The case needs to be settled: Is it beneficial to use several theories to understand the essence of objects, of properties or of other types of entities—and in consequence, of divine attributes—rather than just one? One could say, each theory has its shortcomings, and another theory might be able to fill in the gaps.

With such a comprehensive overview of the issues related to divine attributes, let us now turn to the matter of God's existence. And so, two extreme positions can be distinguished in studies of the existence of God: (i) The existence of God can be proven; and (ii) The existence of God cannot be proven. But then, the question arises: Under what conditions are such statements true or false? On the one hand, any answer to this question must, in one way or another, take into account the concept of 'proof' or 'provability.' It can be added that in the philosophical and mathematical literature these concepts are intensely debated, and many characteristics have been proposed for them. On the other hand, to prove the existence of God is to prove something that has some attributions. A great number of arguments for and against God's existence have been proposed over the centuries. To begin with, let us mention, by way of example, only the ontological and cosmological arguments, as both of these types of arguments seem to be the most common arguments for the existence of God in the philosophical tradition, incorporated into the Christian doctrine. What is significant is that ontological arguments are based on the notion of God as the most perfect entity, that is, possessing all the positive qualities, whereas cosmological arguments restrict the existence of infinite descending chains of groundings of, for instance, final causes, and they assume that for all such chains there exists the same lower limit. Unfortunately, there is no

argument for or against the existence of God that would be indisputable from a logical or ontological point of view. It seems to be just a matter of faith that if God created people, He also gave them the ability to recognize Him—God cannot play hide and seek with them by failing to provide them with chances for success in this respect. What is of special interest, therefore, are the questions of proving the provability and unprovability of the existence of God.

2 Applications of Mathematics and Metamathematics to the Ontology of Divinity, and Vice Versa

The ongoing creation of mathematics, that started five or six millennia ago and is still continuing at present, is hard to describe and can make the mind dizzy. A fundamental question is whether mathematics is determinate. Without going into the consideration of this issue, by ‘mathematics’ we here mean an autonomous science that in textbooks and professional journals is categorized as mathematics. By ‘metamathematics,’ meanwhile, we mean the study of mathematics itself using mathematical methods. Metamathematics provides, according to Kleene, “a rigorous mathematical technique for investigating a great variety of foundation problems for mathematics and logic” (Kleene, 1952, p. 59). However, many important questions and results, which according to this above definition should be regarded as part of metamathematics, are spread across such meta-theoretical fields as philosophy of mathematics or foundations of mathematics. Therefore, it seems advisable to establish the relationship between these three disciplines. For D. Hilbert, metamathematics is essentially proof theory, i.e. the syntactical investigation of proofs carried out in formal axiomatic

systems. But again, much of the important proofs and results of metamathematics are based on concepts and methods belonging to other areas of mathematics, for example—to model theory or recursion theory. For clarity here, precise definitions of mathematics and metamathematics are not needed for the purposes of this introduction, it is enough to focus on those aspects of mathematics and metamathematics that could be utilized in the ontology of divinity.

2.1 Mathematics as the Paradigm of Ontology of Divinity

The claim here is that mathematics should be a model of perfection for the sake of clarity of its concepts and the certainty of its conclusions. The concepts that function in mathematics are either primitive or defined or characterized by axioms. Of course, definitions—i.e. statements which capture the meaning, the use, the function and the essence of terms or concepts—and axioms must meet certain stringent conditions. Here we take over Easwaran's notion of the "axiom" (see →Easwaran, 2008). According to him, statements must meet four conditions in order to be accepted as axioms: 1. They should be able to be widely accepted; 2. They should be useful in establishing interesting results that mathematicians accept; 3. There should be real philosophical problems that can be avoided through the use of the axioms; and 4. It should not be provable from the existing axioms.[2] There are several notions of what (mathematical) proof is. The—most widespread—formal view of proof is contrasted with the informal views of proofs, of which there are several. To save space, we unify here these informal views under one banner. The standard definition of formal proof, i.e., as it is disseminated in almost all logic textbooks, is as follows: A formal proof is a list of successive statements, the last of which—the

one that is being proved—is a conclusion (thesis), and the rest of which are either axioms or the results of the application of one of the inference rules to the preceding statements. In a formal proof, every single step must be true if the preceding steps are true, and a formal proof is always relative to some formal system. But this notion of proof does not reflect the practice of mathematicians (→Hanna, 1990; →Rota, 1997; →Thurston, 1994). Formal proofs would be, if they were written down, too long to be of any interest of the community of mathematicians. Many (but not all) mathematicians use in their mathematical practice a different concept of proof, the so-called informal proof. It is qualified as valid when it is accepted by a broad community of mathematicians. And each inference step in such a proof is made by means of widely accepted mathematical means. Leitgeb (2009) contrasts the features of formal and informal provability[3] as depicted in →Table 1.

→**Tab. 1:** Formal vs. informal provability according to →Leitgeb, 2009.

Formal Provability	**Informal Provability**
formal syntax	not syntactically determined
logical level: 1st order, 2nd order, …	not logically determined
terms, formulas	interpreted terms, interpreted formulas
logical rules, syntactically encoded	truth preservation, evident steps
logical axioms	no logical axioms
mathematical axioms	lack of axioms,
	axioms as partial definitions,
	deductive “gaps” vs.
	true and evident foundational axioms

Informal proofs are not presented in a way that makes their formalizations either apparent or routine, but this does not mean that they are stripped of rigor. Really, they are presented in such a way that makes their rigor clear to the reader. The question reads: Can every informal proof can be somehow treated as a formal proof? There are supporters of both a positive answer (for example, →Enderton, 1977) and a negative one (for example, →Rav, 1999 and 2007; →Leitgeb, 2009; →Antonutti Marfori, 2010; and →Tanswell, 2015) to this question.

2.2 Gödel’s Scenarios

We here point out the possible applications of Gödel's first and second incompleteness theorems—ones of the deepest discoveries in the twentieth century and important milestones in the history of mathematics and metamathematics—to the ontology of divinity. There are various forms and proofs of these theorems, but for our needs here it is enough to quote the simplest of their formulations. And so, *Gödel's first incompleteness theorem* asserts that there is a sentence which is neither provable nor refutable in the theory P under consideration, whereas *Gödel's second incompleteness theorem* asserts that for this sentence one can take a formalization in P of the statement that the theory P itself is consistent (cf. →Beklemishev, 2010, p. 858). Formalizations of these theorems use notions such as axiomatic systems, primitive recursive arithmetic, arithmetization/numbering, representability/interpretability, consistency, completeness, diagonalization, etc.[4] A formal system P, exchanged in Gödel's original theorems, is related to Russell-Whitehead's *Principia Mathematica* and based on the simple theory of types over the natural number series and the Dedekind-Peano axioms (see →Beklemishev, 2010, p. 859). By tracing the mathematical-philosophical literature, the depth of Gödel's incompleteness theorems can be assessed according to various criteria. Let's take a look at the three criteria featured by →Cheng (2022): *influence* (the research practice reveals that these theorems marked the directions for research in mathematics, foundations of mathematics, philosophy, theoretic computer science, and many others), *fruitfulness* (the fruitfulness of incompletness theorems measures the degree to which this theorems or their proofs lead to yet further theorems and proofs (cf. also Arana, 2015). The fruitfulness of Gödel's incompleteness theorem is judged by the following three indicators: *different proofs* of the theorem, *generalizations* of the theorem (how and in what degree the theorem can be generalized or extended),

and the *boundary* (or the *limit*) of the theorem (i.e. under what conditions the theory holds and under what conditions theorem fails)), and *unity* (the unity of Gödel's incompleteness theorem means that it ties together apparently disparate fields and draws interconnections between these fields). Without a doubt, there is an urgent need to undertake serious mathematical-philosophical studies to gasp a relationship between Gödel's incompleteness theorems and ontology of divinity. Although first steps in this direction have already been made in Bulzago (2019). The paper raises the question of proving the unprovability of the existence of God, in particular, it deals the relation between the 'existence of God' and the conditions imposed on a formal system that are required by Gödel's second theorem. We here spread the conjecture concerning proofs for the existence of God, received from Professor Andrzej Wroński (a Polish logician):

> Proving the existence of God seems to be similar to proving the non-contradiction of arithmetic. We believe that arithmetic is non-contradictory. Yet, as it constitutes the foundation of our entire knowledge, we would also like to have a convincing proof in addition to the belief. It is known (2nd Gödel's Incompleteness Theorem) that the proof for the non-contradiction of arithmetic cannot be successful without referring to external means (not formalized in arithmetic). Hence, the non-contradiction of the theory which 'is able' to prove the non-contradiction of arithmetic is as problematic as the non-contradiction of arithmetic itself. But for any pre-determined natural number *n*, the fact that there is no contradiction among arithmetic statements whose formal proofs' length is ≤ *n* can be easily proved by means formalized in arithmetic (reflection principle). Is there any theological counterpart of the reflection principle? It may be useful to differentiate between the global and local proof for the existence of God, show the possibility of local proof, impossibility of global proof, etc. (private correspondence)[5]

Probably, this distinction between the global and local proof for the existence of God, which is mentioned in Wroński's

conjecture, will accompany the distinction between global and local attributes of God.

Here, let us mention yet *Gödel's ontological proof* of the existence of God, i.e., Gödel's text entitled "Ontological Proof" of February 10, 1970. This proof was never published when Gödel was alive, but a brilliant American logician and philosopher, Dana Scott, made it the subject of his seminar in Princeton, thanks to which this proof reached a certain small group of logicians. Only when it was posthumously published in the third volume of *Collected Works* (Clarendon Press) did it inspire very numerous publications in the last 50 years. However, as it turned out, Gödel's ontological proof does not meet the basic deductive requirements, since: (1) It was proven that Gödel's axioms do not result in a thesis for the existence of God; and (2) It was proven that the thesis for the existence of God is inconsistent with Gödel's axioms. These results do not seem to convince the devotees of Gödel's ontological proof, probably in many cases because the acceptance of these proofs would undermine their faith in Gödel's authority.

2.3 The Paraconsistentist Program

The credo of the applicability of Gödel's incompleteness theorems to ontology of divinity must deal with the following question: Is there a way to reconcile Gödel's incompleteness theorems with the incoherency of divine attributes (if proven)? The paraconsistent program seems to come to the rescue here. Roughly speaking, this program is the investigation of formal systems endowed with a negation ¬ such that not every contradiction of the form $(\varphi \wedge \neg\varphi)$ entails everything. In contrast to classical systems, paraconsistent formal systems distinguish consistency from non-contradiction, contradiction from non-consistency, as well as inconsistency from non-consistency—

consistency in these systems is not synonymous with freedom from contradiction. W. Carnielli and D. Fuenmayor formulated the paraconsistent version of Gödel's first incompleteness theorem (along with its proof) as follows (→Carnielli and Fuenmayor, 2020, pp. 16 and 18):[6]

> *The paraconsistent version of Gödel's first incompleteness theorem*: Assume F is a consistent formal system which contains Peano Arithmetic. Let G_F be a formula that satisfies $\vdash_F G_F \leftrightarrow \neg Pr_F(\ulcorner G_F \urcorner)$, we have: (i). G_F exists (Existence Lemma); (ii). $\nvdash_F G_F$ (Non-provable Lemma); and (iii). Under an additional premise, $\nvdash_F \neg G_F$ (Non-refutable Lemma).

Remark. $\vdash_F \varphi$ indicates that φ is a theorem of F. For any expression φ of the language of F, $\ulcorner \varphi \urcorner$ denotes the corresponding *Gödel number* of φ. $Pr_F(\ulcorner \varphi \urcorner)$ expresses that the Gödel numeral $\ulcorner \varphi \urcorner$ of a sentence φ is provable in F.

W. Carnielli and D. Fuenmayor formulate two variants of the paraconsistent Gödel's second incompleteness theorem; the first one draws from Smorynski (1977) and the second one draws from →Boolos (1995).

> *The paraconsistent version of Gödel's second incompleteness theorem*: Assume F is a consistent formal system which contains Peano Arithmetic. Let $Cons_F$ be a formula of F representing the consistency of the system. We also assume the derivability conditions on Pr_F, i.e., 1. If $\vdash_F \varphi$ *then* $\vdash_F Pr_F(\ulcorner \varphi \urcorner)$; 2. $\vdash_F Pr_F(\ulcorner \varphi \urcorner) \wedge Pr_F(\ulcorner \varphi \rightarrow \psi \urcorner) \rightarrow Pr_F(\ulcorner \psi \urcorner)$; and 3. $\vdash_F Pr_F(\ulcorner \varphi \urcorner) \rightarrow Pr_F(\ulcorner Pr_F(\ulcorner \varphi \urcorner) \urcorner)$. *We have two variants:* (i). *Let* G_F be some formula of F *such* that $\vdash_F G_F \leftrightarrow \neg Pr_F(\ulcorner G_F \urcorner)$. We show that $\vdash_F G_F \leftrightarrow Cons_F$. Then $\nvdash_F Cons_F$, i.e., the paraconsistent version of Gödel's second incompleteness theorem, follows as an immediate corollary from the paraconsistent version of Gödel's first incompleteness theorem (namely from one half: $\nvdash_F G_F$) by replacement; and (ii). $\nvdash_F \mathrm{Cons}_F$ follows from *Löb's Theorem*: *For* any sentence φ of the language of F, $\nvdash_F \varphi$ if $\vdash_F Pr_F(\ulcorner \varphi \urcorner) \rightarrow \varphi$.

Remark. A formal system *F*, mentioned in both theorems, is a system built over some (strictly defined) paraconsistent provability logic—in Carnielli's and Fuenmayor's notation, over *RmbC*⨁*K*. A formal system *F* is *consistent* when it cannot derive both a sentence and its negation, i.e. $\nvdash_F \varphi \wedge \neg\varphi$. $Cons_F$ can be defined as $\neg Pr_F(\ulcorner \Lambda \urcorner)$, where Λ is any convenient contradictory statement of the language of *F* (→Smorynski, 1977, p. 829).

Now it is natural to ask whether and under what conditions these paraconsistent Gödel's incompleteness theorems are applicable in Ontology of Divinity, Equally important is the question of the applicability of the paraconsistent program to defuse paradoxes among divine attributes.

2.4 Set Theory with Divine Attributes

Taking his cue from Zermelo, who defended the Axiom of Choice on the basis of its consequences, one can justify the desirability of enriching the language of set theory with divine objects (or, only one divine object) or/and divine attributes, by obtaining well-known or entirely new theorems that they generate. Set theory in this context would be considered as an ontology of some kind, what would correspond to the view of Georg Cantor, who considered his set theory as ontology for mathematics and "his views on God were of crucial importance for his defence of his theory of the transfinite" (Van der Veen and →Horsten, 2013, p. 119). Briefly, "Cantor saw a connection between his work on infinity and his concept of God" (Van der Veen and →Horsten, 2013, p. 123). Divine notions in this new set-theoretical context would not have to be carriers of old clichéd contents. We allow that mathematics and metamathematics can change our traditional views of divine attribute. In talking about divine attributes, however, the emphasis should be situated also or primarily on their totality itself, and not only on the members of

that totality, i.e., we should be interested in the question: What are divine attributes and how can we refer to their totality? Divine attributes, such as omniscience, omnipotence, omnibenevolence, etc., are characterized by a certain kind of *absoluteness* (or, *absolute generality, absolute universality*). This feature seems to be necessary for all divine attributes, but an open question remains as how to provide the necessary and sufficient conditions to be a divine attribute, if such conditions exist at all. Doubts about the absoluteness of divine attributes, however, are suggested by the argument that they are *indefinitely extensible*, i.e., they are subject to principles of extendibility which yield a hierarchy of ever more inclusive extensions.[7] One way of avoiding this predicament is to allow that there are no absolute attributes, but rather that any attribute may be extended.[8] Consequently, one would have to reject the existence of entities (including, God) with an unbounded number of attributes, each of those it possesses in unbounded measure. However, this option is only one of many options, as evidenced by the extensive catalog of the pro's and contra's of views regarding the existence of the set-theoretic universe as a whole. Specifically, Van der Veen's and Horsten's viewpoint seems to be powerful and highly fruitful:

> [O}ne can formulate *a class theoretic reflection principle* that expresses that *V* is ineffable in a very specific sense (→Welch and Horsten, 2012). The principle in question states, *roughly*, that *V* is second order elementary formula equivalent to some set sized initial segment V_K, in the sense that every second-order formula is true in *V* with all its proper classes if and only if it is true in the structure $<V_K, V_{K+1}, \in>$. This principle is called the *Global Reflection Principle* (*GRP*). It can be shown that if *GRP* is added to a weak (set theoretic conservative) class theory such as a fragment of NBG, strong large cardinal consequences follow. At the same time, the theory *NBG* [*Von Neumann-Bernays-Godel set theory*, our addition] + *GRP* can be shown to be set theoretically sound from certain large cardinal axioms (the axiom of subcompact cardinals). Thus it seems

> *right* to say that the mathematically Absolutely Infinite is mathematically unknowable in a very fundamental sense. And this class theoretic reflection principle can *only* be formulated against the background of a class theory (such as *NBG*) in which *V* is recognised to exist. Thus we can extend the scope of the indispensability argument from sets to proper classes. The outcome is that support is given to the claim that the set-theoretic universe exists as a mereological whole of all individual sets.
>
> The global reflection principle is a principle *about V*. We have argued that it is true. Thus it could be objected that it seems that, after all, it is possible for us to have some mathematical (rational) knowledge of *V* after all; as such, it would be an exaggeration to say that we can have no knowledge about the about the Absolutely Infinite at all. However, *GRP* does say that a certain kind of knowledge can never be had of *V*. We have to acknowledge that it is not a set, but at the same time we cannot know any proposition of it that, when relativized to any initial segment V_α would be false. This is enough, we submit, to claim that *V* fundamentally exceeds human comprehension. (Van der Veen and →Horsten, 2013, p. 135)[9]

Let us also mention here the dispute between set-theoretic *Universism* and *Multiuniversism*, i.e., the dispute between two positions that give the different answers to the question: Whether set theory should be interpreted as the theory of a *single* universe of sets or whether it should be viewed as a theory about *multiple* universes? On one side, *Universism* is the view that there is a single correct theory T of sets, or that there is a *unique* structure embodying all set-theoretic truths, on the other side, *Multiversism* is the view that there is no unique, or preferred, theory T of sets, and that all theories express, to some extent, some properties of sets—for example, both ZFC+CH and ZFC+¬CH are equally valid theories of sets (see →Ternullo, 2019). There are several different multiverse proposals on offer, but we will only look at those listed in Barton and Friedman (2020). Assuming the iterative conception of set,[10] N. Barton and Sy-D. Friedman characterize *actualism* and *multiversism with respect to height/width* as follows:[11]

> By *actualism* with respect to height/width, we mean those views which hold that there are universes of set theory which cannot be extended with respect to height/width.
>
> By *multiversism* with respect to height/width, we mean those views which hold that any universe of set theory can be extended in the relevant dimension to a new universe of set theory. (→Barton and Friedman, 2020, p. 629; see also Antos et al., 2015)

With more detailed views (Barton and Friedman (2000), p. 631; see also Antos et al. (2015)):

> By *Radical Actualism* we mean the view that there are universes of set theory that cannot be extended in either height or width. The normal view of this kind is *Absolutism*: the view that there is a single such universe.[12]
>
> By *Pure Width Multiversism* we mean the view that there are universes of set theory that cannot be extended in height, but that every universe can be extended in width.[13]
>
> By *Zermelian Multiversism* we mean the view that holds that there are universes of set theory that can be extended with respect to height, but cannot be extended with respect to width.[14]
>
> By *Skolemite Multiversism* we mean the view that any universe of sets can be extended with respect to both height and width.[15]

This broadening of expressive resources opens up new research horizons. The question arises: How do these different views interact with ideas about divine attributes?

3 Overview of the Book's Content

We shall summarize the papers, in the order of the table of contents, and occasionally post short comments to them.

Part I: Who Created the Concept of God?

This part contains three papers, numbered 1 through 3, respectively.

Jaco Gericke, in "The Meaning(s) of the Question of 'Origins' and the Creation of 'GOD' as Concept," seeks to illustrate how our higher-order discourses configure the conditions of possibility for the assumed meaningfulness of the religious language used in identifying associated research problems. On the one hand, what has, is and could be discussed by asking "Who created the concept of God," depends on the historically-relative meaning(s) of the technical terms 'created/origin,' 'concept,' and 'God' adopted. On the other hand, the meaning of the question is more than the sum of the meanings of its parts and asking "Who created the concept of God" also links to assumptions about the nature, meaning and reference of religious language, topics in religious epistemology, the nature and attributes and the existence of God, how the relationship between religion and history, culture, and science are perceived, as well as to the problems of religious pluralism or diversity, among others. In these ways, the question of the meaning(s) of the questions themselves precedes and complicates the question regarding the truth of any proposed answer (or assessment of purported solutions). The article concludes that by identifying the periods, persons, problems, perspectives behind the second-order scholarly language of any related research we will come to understand not only why associated questions/answers appear the way they do but also what must already be in place for them to appear at all.

Agnieszka Kijewska, in "The Concept of God as Perfect Being. The Presentation of Ancient Christian and Medieval Views," distinguishes two different, but not mutually exclusive, understandings of "perfect being": (i) a perfect being is one that lacks nothing, the distinctive property of such a being is stability and immutability (an advocate of such an approach is, for

example, St. Augustine), and (ii) God's perfection is identified with His incomparable intensity of existence (St. Thomas Aquinas, John Duns Scotus and Nicholas of Cusa, for example, are the followers of such an approach). In these, in turn, she lists further subtypes depending on the way knowledge of God is acquired: 1. the way of *affirmation*—the causal relationship between God (Creator) and creatures (all what is beyond Him) provides the basis for ascribing to God absolute attributes that creatures have in a limited extent, 2. the way of *negation*—i.e. a describing God by articulating what God is not rather than what He is, and 3. the way of *eminence*—it is affirming the positive attributes of God in such a way as to deny at the same time that affirmed perfection is limited in Him. In this conceptual grid, however, it is difficult to locate Christian God with attributes such as *being one in three persons*, the *incarnated, crucified* and *resurrected being*.

James E. Taylor, in "Concepts of God and Their Origins," argues for the thesis that there is more than one concept of God and that at least one of these concepts may be uncreated. The concepts that are components of this thesis are made precise by him as follows. 'Concept' is a way of thinking about a thing or kind of thing that distinguishes this thing from other things by means of some descriptive content. Concepts can be either *fully descriptive*—their descriptive content is sufficient by itself to enable one who understands the concept to identify the thing or kind of thing to which it applies, or *partially descriptive*—they contain an indexical or referential component in addition to their purely descriptive conceptual content. Concepts can be *uncreated* (*innate*) (for example, Descartes—following Plato, Aristotle, St. Augustine and St. Anselm—advocated the innate concept of God as a perfect being) or *created*, and the latter can be *intentionally created* (for example, Spinoza's concept of God—as an infinite substance that has the attributes of thought and

extension and that can be characterized as either God or Nature —belongs to this) or *unintentionally created* (the concept of God as an old man with a long white beard sitting on his throne in heaven falls into this category). Taylor's argument for the thesis that the concept of God as a perfect being is innate runs as follows: 1. We have never experienced the kind of infinity or perfection involved in the concept of God as a perfect being; 2. If we have never experienced that kind of infinity or perfection, then the origin of our idea of it must be non-experiential; 3. If the origin of our idea of that kind of infinity and perfection must be non-experiential, then it must be innate; therefore, 4. Our idea of God as a perfect being is innate.

Part II: Omniscience, Omnipotence, Timelessness and Spacelessness of God

This part contains four papers, numbered 4 through 7, respectively.

Jan Heylen, in "Divine Omniscience: Complete Knowledge or Supreme Knowledge?," distinguishes two concepts of divine omniscience: the concept of having complete knowledge and the concept of having supreme knowledge. The roots of the idea of thinking of omniscience as supreme knowledge go back to St. Anselm. The supreme concept of omniscience is consistent with having incomplete knowledge. The authors of other concepts, also consistent with incomplete knowledge, are Langtry (2008) and Nagasawa (2017), however, their proposals face problems (Oppy, 1997; Kvanvig, 2018). Heylen considers various formulations of omniscience, both in the sense of complete and supreme knowledge. According to him, the conception of omniscience as supreme knowledge is impervious to both arguments directed against the conception of omniscience as

complete knowledge—one based on Cantor's theorem and second based on Montague's theorem.

Wes Morriston, in "Omnipotence and the Power to Make Evil Choices," explores the following pair of theses: *A*. If God is morally perfect, then he lacks the power to choose to do evil; and *B*. If God is omnipotent, then he has the power to choose to do evil. And distinguishing between *essential* and *contingent* possession of attributes, he accepts the view that if A and B are true, then God's essential moral perfection is incompatible with his omnipotence, regardless of whether or not the second of these features is among his essential attributes. However, the main message of the article is the following thesis: the contingent moral perfection of God is incompatible with his omnipotence, also regardless of whether the omnipotence is essential or contingent.

Ryan Thomas Mullins, in "Divine Eternality," discusses God's relationship to time, drilling down to the following questions in particular: (i) Is God timeless or temporal?; (ii) Is time created by God, or is time an aspect of the divine nature?; and (iii) What was God doing before He created the universe? Regarding question (i), R. T. Mullins distinguishes within it the Problem of Creation and the Problem of Omnipresence. Further on, he elaborates on the first of these as the Problem of Creational Change and as the Problem of Eternal Creation. Where the 'Problem of Creational Change' is the name of the proof string: (PCC1) If God is timeless, then God is changeless; (PCC2) If God begins to create the universe, then God changes; (PCC3) God begins to create the universe; (PCC4) Therefore, God changes; (PCC5) If God changes, then God is not timeless; and therefore, (PCC6) God is not timeless. The 'Problem of Eternal Creation' is the name of the proof string: (PEC1) If God is eternally causing X to exist, then X eternally exists; (PEC2) God is eternally causing X to exist; and therefore, (PEC3) X eternally exists. And finally, the 'Problem of

Omnipresence' is the name of the proof string: (PO1) God is located at all moments of time; (PO2) If God is present at a moment of time, then God has temporal location; (PO3) If God has temporal location, then God is temporal; (PO4) God is located at a moment of time; (PO5) Thus, God has temporal location; and thus, (PO6) God is temporal. Mullins finds controversial issues in each of these Problems. With regard to (ii), R. T. Mullins studies two theories of time: the *relational* and *absolute* theory. According to the first of them, God created time together with the universe, and according to the second, time is an eternal, uncaused substance. Turning now to (iii), Mullins links this question with the *Sooner Objection* in the version inspired by G. W. Leibniz: (SO1) God is perfectly rational $=_{def}$ (i) God always performs an action for a reason, and (ii) in the absence of a reason, God will not perform an action; and (SO2) God could have created the universe sooner. And he derives a contradiction from the assumptions that are built into (SO2). Consequently, he rejects (SO2).

Robin Le Poidevin, in "Space, Mind and Deity," examines the following argument for the aspatiality of God: 1) God is independent of any created thing; 2) Space is a created thing; 3) Any spatial thing is dependent for its existence on space; 4) God is independent of space (from 1 and 2); 5) God is aspatial (from 3 and 4). Rejecting 5—because God's spatiality, not God's aspatiality, is required by his omniscience, omnipresence and having a mind (mental life)—he locates the vulnerable points at premises 2 and 3. First, assuming idealism—i.e., that space is not a feature of the world as it is in itself, but rather of our representations of the world—the premises 2 and 3 are meaningless. Further, premise 3 is also refutable under both positions: relationism and substantivalism, when a distinction is made between the way in which God is embedded in space and the way in which other things are so. Where, according to

relationism, the space is simply the network of spatial relations between the kinds of things that we might ordinarily describe as the 'contents' of space: material objects, parts of an electromagnetic field, and so on; spatial relations between these relata are not mediated by anything further thing, but are rather direct. According to substantivalism, in contrast, space is an entity in its own right, existing independently of its contents; spatial relations between objects are not direct but rather mediated by spatial regions or points. And finally, Le Poidevin offers ways to reject 2), both with the assumption that space is independent of God and the assumption that space is dependent on God.

Part III: God and Perfect Goodness, Perfect Beauty and Perfect Freedom

This part contains four papers, numbered 8 through 11, respectively.

Michael Almeida, in "On Modal Arguments against Perfect Goodness," challenges both *modal arguments against perfect goodness* and *modal arguments from goodness*. Modal arguments against perfect goodness urge that there are at least some states of affairs S in some possible world w such that S entails that nothing exemplifies F in w, where F is a conjunctive property including moral perfection, omnipotence, and omniscience.[16] M. Almeida discusses two modal arguments against perfect goodness, called respectively, the *traditional* modal arguments against perfect goodness (in other words, *the modal argument from evil*) and the *stronger modal argument from evil*. The first one depends on there being possible worlds—or at least contingently creatable portions of possible worlds—that are on balance bad. The second one does not require possible worlds that are on balance bad or even creatable portions of worlds

that are on balance bad. It does not depend on there being a single evil state of affairs in any world, in contrast—it is consistent with every possible world being infinitely good. The main goal of the modal arguments from goodness, considered by M. Almeida, is to provide good reason to believe that at least some possible worlds are consistent with the existence of a being exemplifying a conjunctive property F including moral perfection, omnipotence, and omniscience. According to M. Almeida, both modal arguments from evil and from good are unsound, because no states of affairs in any world provides any non-trivial evidence against or for the existence of a being exemplifying a conjunctive property F including moral perfection, omnipotence, and omniscience.

In “God and Perfect Beauty,” *Peter Forrest* argues for the incoherence of *Perfect Divine Beauty* (PDB), explores eight substitutes of the PDB and prefers two of them: the *Weaker Divine Beauty* (the *Pre-eminent Divine Beauty*, as he calls it) and the *Stronger Divine Beauty* (*All Beauty*, as he calls it), and examines four arguments against *God’s Great Beauty*, namely, the *Argument from Divine Simplicity*, the *Argument from Divine Spirituality*, the *Argument from Divine Necessity*; and the *Argument from Moral Righteousness*. Although it is extremely difficult to define the concepts of beauty and perfect beauty, P. Forrest tries to bring the reader closer. And so, beauty is pure aesthetic value, but the scope of the term ‘aesthetic value’ is wider than that of ‘beauty’—although the beautiful is aspectual, it is not contextual in the way that much aesthetic evaluation is, and dispositions can have aesthetic value but not beauty. Beautiful things form a partially ordered system with respect to comparative beauty, with infinitely long chains. That the ordering is partial may result in a few unequal instances of maximum beauty that are non-comparable, but they are all such that there is nothing more beautiful. In turn,

> … when we talk of perfect beauty we are using the term ‘perfect’ loosely, and probably what we mean is maximal beauty, that which is more beautiful than anything (else) that is conceivable. There is, however, an alternative, which I call the *hybrid conception*, namely the beauty something has as a result of being a perfect (i.e. complete and flawless) specimen of the very highest kind of being. (p. 200)

The hybrid conception of perfect beauty, however, is qualified as problematic. The interpretation of the PDB as Divine Maximal Beauty—namely, that there is nothing conceivable that is more beautiful than God actually is—is incoherent. In Forrest’s words, “It is conceivable that God has maximal static beauty, but because we are to include dynamic and moral beauty there is no maximal divine beauty but rather a sequence of ever more beautiful divine states” (p. 203).

Michał Głowala, in “Absolute Immutability and God’s Libertarian Freedom in the Ontology of Divine Agency,” critically examines the problem of compatibility of divine immutability (i.e. God is absolutely immutable) with God’s libertarian freedom (i.e. God has the ability to do otherwise). He locates his considerations as belonging to the ontology of libertarian freedom. After reviewing the difficulties concerning the compatibility of divine immutability and libertarian freedom and proposing some fundamental principles for recognizing divine immutability, M. Głowala focuses on: 1. The Necessary Volitions Problem—i.e., Is God’s love for himself compatible with his libertarian freedom?; 2. The Reasons Problem—i.e., Is the reason responsiveness of the divine will compatible with its immutability?; 3. The Contingent Volitions Problem—i.e., are God’s competing choices, say a and b, a sort of intrinsic difference between God choosing a and God choosing b or are they a sort of extrinsic denomination?; and 4. The Omissions Problem—i.e., whether God can refrain from wanting something?

Kenneth L. Pearce, in “Why Not? God,” addresses the following issue: What role does God play in actualizing possibilities? He starts with the definition of actualization: a state of affairs *A* is *actualized* by an agent *S* iff there is a complete explanation of why *A* is actual which traces back solely to *S*’s agency—of which, after critical review, he replaces by: a state of affairs *A* is *deterministically actualized* by an agent *S* iff there is a complete deterministic explanation of why *A* is actual which traces back solely to *S*’s agency, where Γ is *deterministically explanatory* of *A* iff the explanatory relation between Γ and *A* presupposes no background conditions and Γ is not explanatory of not-*A*. An additional definition is: A state of affairs *A* is *decreed* by an agent *S* iff there is a complete deterministic explanation of why *A* is actual which traces back solely to *S*’s explicit act of willing that *A* should be actual. Briefly, Pearce’s position is as follows: “Although every substantive fact is actualized by God, not every substantive fact is deterministically actualized by God, and only a relatively small proportion of substantive facts are decreed by God. Further, every non-actual state of affairs is non-actual because it is not actualized by God” (p. 254). A large part of the paper is devoted to analyzing the various species of impossibility—namely: *logical, essentialist, metaphysical, deontological, axiological* impossibility—in terms of different ways theistic explanations of non-actuality might go.

Part IV: God, Fundamentality and Creation of All Else

Four papers, numbered 12 through 15, belong to this part.

István Aranyosi, in “God and/as the Universe,” studies the God-Universe relation from the perspective of the Big Bang and Big Bounce cosmology. He shares his own—previously expressed—view, called ‘logical pantheism,’ according to which

God should be identified with logical space. He seeks better grounding for this conception by anchoring it more in the traditional pantheistic or panentheistic views. In short, *pantheism* identifies God with the Universe, whereas *panentheism* states that the Universe is in God. The latter is of two kinds: *mereological* or *emergent* panentheism. Mereological panentheists mean that the Universe is a proper part of God's, where 'proper part' is understood in accordance with core classical mereology.

Felipe Leon, in "The Problem of Creation *Ex Nihilo*: A New Argument against Classical Theism," argues that creation *ex nihilo* is *prima facie* impossible. His argument is as follows: 1. All concrete objects that have an originating or sustaining efficient cause have an originating or sustaining material cause, respectively; 2. If classical theism is true, then the universe is a concrete object that has an originating or sustaining efficient cause with neither an originating nor a sustaining material cause; and therefore, 3. Classical theism is false. According to F. Leon, there is at least one possible world *W* in which Premise 1 is non-vacuously true. Thus, in *W*, some things or stuffs in *W* are made, and all made things or stuffs are made from other things or stuff. That is, no things or stuffs in *W* that are made are made *ex nihilo*, in which case no god or gods made them *ex nihilo* in *W*. This last statement contradicts classical theism, according to which, for any world that contains concrete things or stuffs distinct from God, at least some of those things or stuffs were made *ex nihilo*. Premise 2 is a conceptual truth, therefore it is non-negotiable. F. Leon anticipates two sorts of classical theist responses to his argument, namely, *non-concessive* and *concessive* responses, which he analyzes in more detail.

Willem B. Drees, in "Natural Sciences as Constraints on Ideas about the Divine," discusses the relationship between natural sciences and religion. He questions the view that natural

sciences and religion are either at war or indifferent to each other. Natural sciences are very successful, and by collecting data, creating models and formulating theories, they provide answers to many questions, but they do not explain reality completely and do not answer all questions. Answers, that they offer, often raise new questions. Natural sciences can exclude various ideas about God and created reality, they can act as a corrector in relation to a philosophical and theological discourse, what does not mean that they completely ignore the discourse about God. Natural sciences, on the one hand, and philosophical and theological discourse, on the other hand, should complement each other. In Drees's words: "Thus, science serves as a constraint on some elements of theologies, especially the worldview dimension, but is not fully determinative of them" (p. 315). Natural sciences imply a single theology, different Ontologies of Divinity may be consistent with the sciences. In particular, natural sciences do not conflict neither with the concept of God as transcendent being who is the Creator and Designer of the universe nor with the concept of God as Ground of Being.

Joshua Rasmussen, in "God and Fundamentality: From Fundamentality to Perfection," explores the question: How anything could be fundamental? The answers to this question can help illuminate the divine nature, because on classical models of God, He is fundamental. Rasmussen considers several proposals for how one might conceive of fundamentality, namely, as: (i) *being total*—something is fundamental by being the *total* of all that exists; (ii) *being necessary*—the chain of grounds bottoms out in a *necessary* thing, where a necessary thing is something that *cannot* fail exist, and therefore, there is no reason to explain why a necessary thing exists rather than not; (iii) *being tautological*—fundamental things are facts that are self-evidently true, like a tautology, therefore they require no

further explanation; (iv) *being inexplicable*—a thing is fundamental if and only if it cannot be further explained; and (v) *being perfect*—in Rasmussen's words*:*

> The perfection proposal has two parts. First, we *remove imperfections* that would call for an explanation in terms of a prior ground. Second, we *fill in perfection* (in a complete way), which results in a call for no further ground. In this way, we identify a relevant difference between a fundamental thing and grounded things: a fundamental thing, unlike grounded things, would lack fundamental limits, boundaries, and arbitrary specificities that depend on some ground. A *perfect* thing, then, is precisely the sort of thing that would lack fundamental limits, boundaries, and arbitrary specifies. For this reason, a perfect thing is precisely the sort of thing that could be fundamental. (p. 328)

J. Rasmussen rejects (i)–(iv) and defends (v) against possible allegations.

Part V: Simplicity and Ineffability of God

This part is created by four papers, respectively numbered by 16–19.

Michael Almeida, in "Divine Simplicity and Eliminative Theism," distinguishes between two types of identity claims*:* *reductivist* and *eliminativist*, and argues that God is identical to its intrinsic properties (i.e. God is *simple*) in the eliminativist sense. He explains these two types of identity claims as follows:

> [If] the identity claim that the chair *C* = the configuration of particles *P* is merely reductive, then there exist chairs and there exist configurations of particles and it turns out that they are identical. It is nonetheless true that *C* exists and *P* exists. … It is not as though there were chairs and now, given the discovery that chairs = configurations of particles, there are no chairs. … But if the identity claim is eliminativist, then it is true again that chairs are configurations of particles, but chairs do not in fact exist. It is true that the illusory object that we call a chair turns out to be nothing

> over and above a configuration of physical particles. The chair reduces to the configuration of particles without remainder. (p. 335)

With these distinctions in mind, Almeida explores: (i) Plantinga's argument against divine simplicity; (ii) the relation of the doctrine of divine simplicity to various one category ontologies; (iii) the relation of the doctrine of divine simplicity to the austere nominalism, understood as the view that nothing exists over and above particular concrete objects. And M. Almeida adds: "According to … austere nominalism, for instance, there is no ontological complexity in God, since there are no ontological features of God that ground or explain why sentences attributing properties to God are true" (p. 342); and (iv) the account of divine simplicity by eliminative theism. According to eliminative theism, divine simplicity depends on the eliminative identity of God's essential properties with God. Properties such as essential omniscience, omnipotence, and moral perfection are identical to God, but they in fact do not exist.

Stephen T. Davis, in "Why Divine Simplicity Is Unnecessary," assumes that the doctrine of divine simplicty is motivated by the following three requirements: (i) If there were any complexity in God, there would have to be potentiality in God as well, which is unacceptable; (ii) Ensuring consistency with other divine attributes, including *divine eternity, immutability, impassability*, and *necessary existence*; and (iii) The desire to protect the divine difference from and transcendence over all of creation—no complex object can be a First Cause, a Necessary Being, or exist *a se* (from itself). He criticizes these motivations, and consequently, he argues that divine simplicity is unnecessary. Concerning (i), Davis distinguishes two types of potentiality in God: *intrinsic* and *extrinsic*. He confesses that there is no intrinsic potentiality in God, but finds no convincing arguments that there cannot be extrinsic potentiality in God. Concerning (ii), Davis

says that he doesn't understand why timelessness, immutability, impassability or necessary existence require simplicity. And concerning (iii), Davis argues that God's simplicity is not a necessary condition for his transcendence. A complex God can also be a First Cause, a Necessary Being, and have aseity.

Graham Priest, in "God and the Paradox of Ineffability," shows how to make sense of the Paradox of Ineffability: God is both an effable and ineffable entity. For this purpose, he uses Gödel numbering—which indicates that he works in the arithmetic language—and the techniques of paraconsistent logics. He precedes his technical discourse with a history of the matter. Two of Priest's remarks are important here, firstly, to accept his solution to the Paradox of Ineffability, one has to be a dialetheist, i.e. the Principle of Non-Contradiction must be rejected; and secondly, the question of whether the techniques of paraconsistent logics can be applied to other divine paradoxes should be considered for each of these paradoxes separately.

Katherin Rogers, in "Divine Simplicity: Anselm's Neoplatonic Approach," examines Anselm's version of the doctrine of divine simplicity. God, according to Anselm, is "a being than whom nothing greater can be conceived." This concept of God underpins Anselm's doctrine of divine simplicity in two ways. Firstly, God must be absolutely independent, uncaused by any possible understanding of 'cause.' But, if God was not simple, he would be composed of parts, that is, He would be the bearer of a kind of internal and fundamental dependence—a contradiction. This also applies to His attributes, for example, omniscience, omnipotence and omnipotence. They cannot be separate properties of God because they would undermine God's independence. Secondly, one cannot think that God is composed of parts. Because if He could be thought to be composed of parts then He could be thought to be destructible. A being that is destructible is a being than whom a greater can be thought,

which results in a contradiction. K. Rogers devotes a significant portion of her paper to the questions key to the doctrine of divine simplicity, including how a simple God, Who must be immutable, could interact with a changing creation, and how a simple God, who cannot do other than He is doing, could be free. In short, Anselm deals with the first question through his analysis of the nature of time and eternity. All times are equally real—Rogers calls this theory of time *isotemporalism*—and God interacts with everything that exists at any time, "at once," in His one simple, unchanging act. Regarding the second question, he maintains that God can do nothing other than what He does. God makes the best of everything. This does not mean that God is not free. God's freedom, unlike created agents, does not include the ability to do otherwise.

Part VI: God, Necessity and Abstract Objects

Three papers, numbered 20 through 22, belong to this part.

Ciro De Florio and *Aldo Frigerio*, in "God, Necessity, and Necessities," discuss the relationships between God and necessity. More specifically, they defend the view that God is the source of the necessity and possibility, excluding from consideration the competing view that there are necessities independent of God and to which God is as subjected as any other entity. The clever trick in the paper consists in distinguishing two types of necessity/possibility: *global*—what is necessary/possible from God's point of view, and *local*—what is necessary/possible within a creation. This bi-modality turned out to be useful in blocking the theory from regress, i.e. in protecting the theory from the paradoxical statement "Necessary states of affairs are not necessary," as well as in explaining puzzles saying that God never acts evilly but that He could act in such a way,

that it is necessary that 2+2=4 but God could have done otherwise, and others.

Paul Gould, in "In Defense of Christian Platonism," defends Modified Theistic Activism (MTA) (= Gould's version of Christian Platonism) from three kinds of objections: incoherency, implausibility, being unmotivated. MTA is characterized by five following clauses: 1. God does not depend on anything distinct from himself for his existing; 2. Everything distinct from God, including abstract objects, depends on God for its existing; 3. God is the creator and sustainer of the universe; 4. God is ontologically distinct from the universe yet present and active in the universe; and 5. The universe points beyond itself to the sacred order. Gould completes this description of the MTA by adding further comments, of which we select here: (i) God created substances, properties, and meanings (i.e., concepts and propositions); (ii) A substance is a fundamental whole that is metaphysically prior to its non-separable physical and metaphysical parts—God is a substance that is metaphysically prior to its properties, parts, and powers, while the universe is a plurality of finite created substances organized hierarchically along a great chain of being; (iii) A substance's nature is different from its properties—a substance doesn't have a nature, rather a substance is identical to its nature, and substance's properties are distinct from and flow from its nature. God's properties metaphysically depend on God, although, given causal pluralism, there is a hierarchy of metaphysically dependency relations found within the divine substance. The inconsistency objection against MTA, in Gould's words, is as follows: "If God is the creator of all Platonic properties and God has Platonic properties ..., then God creates his own properties. But God can't create his own properties unless he already has ... the property of *being able to create his own properties*. Thus, God is and is not the creator of all properties. This scenario is

incoherent” (p. 423). Regarding the implausibility charge against the MTA, P. Gould only deals with W. L. Craig’s statement that Gould’s defense of the MTA “is considerably muddied … by [the] bizarre affirmation that God’s thoughts are abstract objects” (p. 431). Finally, P. Gould lists three underexplored areas that provide motivation for Christian Platonism: (i) Platonism supports theism; (ii) The ontology of Christian Platonism offers an attractive account of key theological doctrines; and (iii) Linguistic realism—i.e. the view that concepts arise from the nature of things and refer to the nature of those things—legitimizes Christian Platonism.

Scott A. Shalkowski, in “God and Abstract Objects,” offers some suggestions for dealing with the question: How are God and abstract objects related? On the one hand, central to classical theism is the conception of God as the sole, truly existing, eternal and omnipotent creator of all things apart from Himself. On the other hand, if abstract objects exist always and necessarily then

then God is neither the creator of everything nor the only independently existing being. In Shalkowski’s opinion, the independent existence of abstract objects does not threaten the independence and creativity of God and there are no adequate theological grounds for accepting or rejecting abstract objects. Apparent problems with the co-existence of God and abstract objects result from failures in philosophical imagination. Shalkowski illustrates the kind of theoretical re-evaluation he has in mind with reference to an apparent inconsistency in the philosophy of modality. An intuitive principle—every claim has modal status—seems to conflict with David Lewis’s reductive theory of modality. While there is a technical fix that harmonizes the principle and the theory, he argues that philosophical good sense recommends that those embracing Lewis’s programme relinquish the intuitive principle. Similarly, considering ‘limits’

that are commonly thought to apply to God's attributes shows a way of re-evaluating what is crucial for theological orthodoxy, opening a way to harmonize traditional claims about God, suitably interpreted, with the existence of abstract objects. Shalkowski evaluates W. L. Craig's arguments—from biblical teachings, teachings of early Church fathers, and perfect being theology—against the existence of abstract objects and argues that they are insufficient for ruling out the existence of abstract objects on theological grounds.

Part VII: God, Infinity, and Pascal's Wager

This part includes five papers, respectively numbered by 23–27.

Paul Bartha, in "Pascal's Wager and the Precautionary Principle," analyzes two principles in parallel: *Pascal's Wager* (*PW*), which disposes whether or not advocate the existence of God—wagering for God has infinite expected utility and is therefore superior to not wagering, which has at best finite expected utility; and the *Precautionary Principle* (*PP*), which is a tool for making decisions in potentially catastrophic circumstances—catastrophes are treated as outcomes that have negative infinite utility and at most finite positive utility. Both the phrases, 'infinite utility' and 'negative infinite utility,' are vague and raise many objections. The three objections to *Pascal's Wager*, considered by P. Barth, are as follows: the *many-gods objection*—by the assumptions that Infinite Utility and Finite Probability apply to more than one possible god, all ways of allocating resources are equally rational and there is no sufficient reason for allocating resources in one way rather than another; the *mixed-strategies objection*—it is an argument for wagering nothing, for investing no resources in religious belief, because *Try to avoid resources in religious belief* is just as good as *Wager for God* ; and the *fanaticism objection*—it is an argument

for wagering everything in wagering for God, because every increment yields an infinite increase in expected utility. The three objections to the *Precautionary Principle*—namely, the *many-disasters objection*, the *mixed-strategies objection* (*do nothing*) and the *fanaticism objection* (*black hole challenge*)—are parallel to these objections to the Pascal's Wager. P. Gould refutes the first two objections by referring to specially constructed lexical utility models for PW and PP, respectively. He does not resign from the use of infinite utilities in either Pascal's Wager or precautionary reasoning, but both infinite and negative infinite utilities in his strategy are accordingly modified. Unfortunately, the lexical utilities model offers no help in responding to the fanaticism objection, either for PW or for PP. Gould suggests a line of research in this regard: "To avoid extremism, we need to allow trade-offs between distinct 'levels' of utility, …" (p. 483), and he adds: "Steel and Bartha (2022), however, suggest ways in which the lexical utilities model can be adapted to allow certain forms of trade-offs" (p. 483).

Alan Hájek, in "Reflections on Infinite Utility and Deliberation in Pascal's Wager," shows how Pascal's Wager satisfies the so-called "*D*eliberation *A*nnihilates *R*eflexive *C*redences" thesis (in short, *DARC thesis*). He presents Pascal's reasoning in modern parlance as follows: P1. The probability *p* that God exists is positive; P2. You should choose the action of maximal expectation if there is one; P3. The decision matrix is as follows: Wager for God / God exists has infinite utility, while the other combinations, namely, Wager for God / God does not exist, Wager against God / God exists, and Wager against God / God does not exist, have finite utilities; Conclusion 1: Wagering for God has infinite expectation, while wagering against God has finite expectation; Conclusion 2: You should wager for God. A. Hájek had argued in one of his earlier papers that this reasoning is invalid, because Conclusion 2 does not follow from Conclusion

1—“It does not follow that you should wager for God if some *other* option also has infinite expectation” (p. 496). According to the *DARC thesis, while deliberating about what you’ll do, you cannot rationally have credences for what you’ll do*—you cannot rationally have option-credences. So it is with all decisions, including the decision about whether you should wager for God or not, while you are in the process of deliberating about them. A. Hájek invokes arguments for and against the DARC thesis, but he himself opposes this thesis. However, regardless of whether the DARC thesis is true or false, there is a problem for the Wager’s validity. In Hájek’s words:

> If the DARC Thesis is false, and you assign positive probability to winding up wagering for God at the end of your deliberation, you have an alternative option that has infinite expected utility: deliberation about this very decision problem. If the DARC Thesis is true, then this option has undefined expected utility. Either way, the infinite expected utility of wagering for God is not strictly greater, and so we do not yet have a valid argument for favoring that option. (p. 508)

That is, in both cases Pascal’s Wager is invalid.

Leon Horsten, in “Quantity Has a Quality All of Its Own,” explores the relation between *quantitative infinity*, on the one hand, and *ontological reflection*, on the other hand. To characterize *quantitative infinity*, L. Horsten refers to the work of Dedekind, where he finds two conceptions of quantitative infinity: the *ordering conception* of infinity—a multitude is quantitatively infinite if the natural numbers can be embedded into it; and the *reflection conception of infinity*—a multiplicity is infinite if and only if there is a one-to-one onto correspondence between the multitude and a proper sub-multitude of itself. Importantly, “Dedekind knew that (in the presence of the Axiom of Choice), a multitude is ordering-infinite if and only if it is reflection-infinite” (p. 519). Roots of ontological reflection date

back to Antiquity, where it can be found in discussions about the nature of God. For example, Philo of Alexandria (the end of the first century BCE—the middle of the first century CE) postulates the following reflection principle: God is reflected, in the sense of being mirrored, in an entity in the world (an angel). According to Augustine (354–430), the infinity of natural numbers is somehow reflected in a bounded entity in God's thought. Reflective considerations have become a permanent part of contemporary set theory. L. Horsten estimates that the philosophical motivation of modern set-theoretic reflection principles is close to Philo's reasoning. The general idea between set-theoretic reflection principles, which function in contemporary set theory, is the following: a mathematical statement or collection of statements is true of the universe *V* as a whole if and only if it is true of certain parts of *V* when all quantifiers are restricted to these parts of *V*. If there is a link between ontological reflection and quantitative infinity, and set theory is a mathematical study of quantitative infinity, as assessed by L. Horsten, then it becomes clear that ontological reflection has a key mathematical significance—"... the truth of set-theoretic reflection principles follows from philosophico-theological arguments," except that the concept of God is replaced by the concept of the set-theoretic universe.

Samuel Levey, *Øystein Linnebo* and *Stewart Shapiro*, in "Potential Infinity, Paradox, and the Mind of God: Historical Survey," delve into the two opposing themes: the first, the Aristotelian guard-rail against infinity paradoxes, and the second, challenges of Judeo-Christian theology for the Aristotelian guard-rail of potentiality. The division of a given stick illustrates well the difference between actual and potential infinity. The statement 1. $\forall x(Pxs \rightarrow \exists y\, Pyx)$, where *Pxy* means that *x* is a *proper* part of *y*, expresses the infinity of divisions in the sense of actual infinity. While the conjunction of the

following two statements says that the divisions of the stick are merely potentially infinite: 2. $\forall x(Pxs \rightarrow \Diamond \exists y\, Pyx)$ (read: No matter how many times the stick has been divided, it can always be divided again), and 3). $\neg\Diamond\forall x(Pxs \rightarrow \exists y\, Pyx)$ (read: It is impossible to complete infinitely many divisions of the stick). In the words of the paper authors:

> Consider the procedure of repeated bisection of some continuous magnitude. It is necessarily possible to continue the procedure. However, if the procedure could be completed, paradox would follow, as the magnitude would be "divided away into nothing." We must therefore deny that the procedure can be completed. By instead invoking potentiality, we ensure that the infinitely many objects producible by the procedure are not all present at a single moment of time or possible world... The infinity is instead absorbed into a capacity ... that can be partly articulated across time or possibility, but that cannot be completely brought to fulfillment ... in either. (p. 533)

Conversely, Judeo-Christian theological thought opts for two principles: *Completablity*: There can be a completed totality of everything generable by the procedure of iterated application of some extending operation; and *Extendability*: The extending operation can always be applied to any available object(s); there can be no arbitrary prohibition against its application. God sees all and is omnipotent, therefore He has access to this Completability and can always apply the iterated augmentation operation. S. Levey, Ø. Linnebo and S. Shapiro propose some solution to this tension between the Aristotelian guard-rail of potentiality and challenges to it due to the Judeo-Christian theological tradition.

Anne Newstead, in "Absolute Infinity, Knowledge, and Divinity in the Thought of Cusanus and Cantor," compares the contributions of two great German-speaking thinkers, Nicholas of Cusa (1401–1464) and Georg Cantor (1845–1918), to the debate about infinity and divinity. We present here some of

Newstead's findings in the following points. 1. One of the most important contributions of Aristotle to study of infinity is identifying a dichotomy between potential and actual infinite. However, his distaste for actual infinity had the result that until Cantor's time the notion of potential infinity dominated in mathematical thinking about infinity. According to Cantor**,** both types of infinity have their place in metaphysics and mathematics, with the actual infinite being the ontological foundation that grounds statements about the potential infinite. He argued that Aristotelian arguments against actual infinities are incorrect, presupposing the existence of only finite numbers by making the properties of finite numbers essential to *all* numbers. Also, Cusanus objected to Aristotle's idea of potential infinity because it is based on an infinite progression of finite quantities—for Cusanus, absolute infinity cannot be quantified and measured, so it will never be known by finite minds. 2. For both thinkers, the existence of the Absolute is not questioned, but they deny the human mind the full knowledge of the Absolute. Both accept the assertion of negative theologians that the nature of God cannot be defined in terms of concepts, because if there were a true exact description of God consisting of the concatenation of infinitely many names, such a name would be ineffable. Cantor asserts that "[t]he Absolute can only be acknowledged [anerkannt] but never known [erkannt]—and not even approximately known." Cantor's diagonal argument could be used here to show that if we consider that we have a countably infinite series of God's names, then this series cannot be complete. 3. A major difference between Cantor and Cusanus is revealed in their attitude to logic and the Aristotelian Law of Non-Contradiction (LNC). Cusanus maintains that (LNC) does not apply to the Infinite/Absolute, while Cantor assumes that contradictions have no place in mathematical knowledge.

Part VIII: God and (Meta-)Mathematics

This part includes five papers, respectively numbered by 28–32.

Neil Barton, in "Reflection in Apophatic Mathematics and Theology," proposes several theological reflection principles and explores their relationship to set-theoretical reflection principles within what he calls "apophatic mathematics." The *theological reflection principle* (or *TRP*) is the claim that if the divine possesses a (possibly ungraspable) property P, then there is a creature that possesses a creaturely property P_0 (of which *P* is the divine analogue). The *extended theological reflection principle* (or *ETRP*) is the claim that there is a creature *C*, such that if the divine possesses a (possibly ungraspable) property *P*, then *C* possesses P_0 (of which *P* is the divine analogue). These definitions are supplemented by two axioms: the *double reflection axiom* (or *DRA*) states that there is both a creaturely and divine property corresponding to the *TRP*; and the *extended double reflection axiom* (or *EDRA*) there is both a creaturely and divine property corresponding to the *ETRP*. Informally, *apophatic mathematics* is the view that all talk of proper classes is *meaningless* (or perhaps *false*, where it involves existential quantification). The only legitimate proper class talk is that which can be paraphrased by *first-order* talk about sets. And the basic idea of a mathematical reflection principle is that the universe of sets is so rich that parts of the universe resemble the universe itself. N. Barton sees a parallelism between the versions of theological reflection within apophaticism and the set theory of reflection for the apophatic mathematician. He argues that theological problems can be resolved by considering theological reflection principles in analogy with the apophatic mathematician's set-theoretic reflection, in particular suggesting that it enables the study of the divine *via* the examination of creatures.

Meir Buzaglo, in “Are Proofs of God’s Existence Impossible? A Critical Examination of Kant’s Critique,” critically analyzes Kant’s refutation of proofs for the existence of God. Kant classified all possible proofs into three categories: the *ontological proof*, i.e. the proof of God’s existence from the concept of God; the *cosmological proof*, i.e. the proof of God’s existence from the existence of the world; and the *physico-theological*, i.e. the proof of God’s existence from the nature of the world. By refuting these proofs Kant argues that he can prove the unprovability of God’s existence. However, as Buzaglo demonstrated in his earlier paper, Kant’s objective is in opposition to Gödel’s second theorem. As for Kant’s assessment of the existing proofs for the existence of God, Buzaglo shows that Kant’s classification of proofs is not exhaustive, and he rejects Kant’s argument that unless we assume that existence is a predicate it is impossible to prove the existence of God, because there are existence theorems that are provable without assuming that existence is a predicate. As for Kant’s criticism of the cosmological proof, Buzaglo shows that if we add certain assumptions that are not unreasonable, this proof can be restored. In Buzaglo’s discussion of the physico-theological proof, he rejects the claim that Kant raised against this proof that at most it shows the existence of an architect but not of a creator God. He further deals with the claim that this evidence does not prove the existence of God in the strong sense of this term. Buzaglo argues that in the case of the physico-theological proof, an amendment is possible by using Kantian presuppositions. He took the proof from the physical sciences to ethics. In his words: “The assumption that God is the creator of humanity is *ipso facto* more powerful than the God promised by the physico-theological proof, which does not take humanity into account” (p. 628).

Meir Buzaglo, in “Spinoza and the Universal Set: A Proposal,” takes the first steps towards building a set theory that is based on Spinoza’s principles concerning the philosophy of infinity, and is immune to Cantor’s criticism of these principles. Cantor believed that the infinity that represents God is given by the set of all sets, and the Cantor Paradox shows that this contains a contradiction. From this arises an argument against the pantheistic rationalism of Spinoza who held—in Cantor’s reading—on the one hand that God’s infinity is represented by absolute infinity and on the other hand that this infinity can be understood by human reason. Buzaglo shows how Spinoza’s rationalism, and especially the principle of plenitude, can be used to build a set theory that can answer the challenge posed by Cantor to rationalist pantheism. Now, the set theory proposed by M. Buzaglo is characterized by the following axioms: 1. The set of all sets is a set; 2. For any set, its power set is also a set; 3. The identity function is a bijective function from the set of all sets to its power set; 4. Given a bijective function f from the set of all sets to its power set, the predicate “$x \notin f(x)$” does not define a set; 5. There exists an empty set; and 6. If α is an ordinal, in von Neumann’s sense, and f is a function, then f(α) is a set. Buzaglo provides these axioms with an appropriate philosophical commentary and proves several theorems.

Harvey M. Friedman, in “A Divine Consistency Proof for Mathematics,” proved the consistency of mathematics as formalized by ZFC axioms and based on the notions of ‘definable class of objects,’ ‘positive class of objects,’ and ‘divine object.’ Where a *class of objects A* is *definable* (in the language $\mathscr{L}$)—symbolically, DEF(A)—is characterized by the following axiom schema: $\forall x\,(A(x) \leftrightarrow \varphi) \wedge \mathrm{DEF}(A_1) \wedge \ldots \wedge \mathrm{DEF}(A_n) \rightarrow \mathrm{DEF}(A)$, provided φ is a formula of $\mathscr{L}$ with at most free variables $x, A_1, \ldots, A_n$. Friedman’s characterization of positive classes of objects is motivated by Gödel’s depiction of them. And so, in mathematical

vocabulary, K. Gödel used positive properties as an ultrafilter consisting merely of the classes containing some given special point, which was God. He defined positive properties using the concept of God, and *vice versa*.—briefly, a class of objects is positive if and only if it contains God. H. Friedman comments on this approach as follows:

> So it would appear that one cannot expect to do anything substantial, mathematically, with the positivity ultrafilter … It occurred to us that if we take God out of the class of all objects, treating God as exceptional, but keeping the positivity ultrafilter, pruned to be over the class of objects excluding God, then the positivity ultrafilter is no longer trivial. In fact, it is a nontrivial ultrafilter over the class of all objects without God. (p. 646)

Formally, a *class of objects A* is *positive* (POS(A)) if and only if the following two axioms hold: $\forall x\,(A_1(x) \vee A_2(x) \rightarrow \mathrm{POS}(A_1) \vee \mathrm{POS}(A_2))$ and $\mathrm{POS}(A_1) \wedge \mathrm{POS}(A_2) \rightarrow \exists x,y\,(x \neq y \wedge A_1(x) \wedge A_1(y) \wedge A_2(x) \wedge A_2(y))$. And an *object x* is *divine* (*div*(x)) if and only if x is an element of all definable positive classes—formally, $\mathrm{div}(x) =_{df} \forall A(\mathrm{DEF}(A) \wedge \mathrm{POS}(A)\, A(x))$. H. Friedman accepts the *divine object axiom* which says that there exists a divine object, formally: $\exists x\, \mathrm{div}(x)$. The consistency of the theory T_5—i.e. the theory determined by the axiom schema for DEF, the axioms for POS and the *divine object axiom*, plus the set-theoretical principles: *pairing* (on objects), *choice* (selecting elements from classes), *comprehension* and *extensionality*—is established in ZFC plus the *existence of a measurable cardinal*. And T_5 interprets ZFC plus the *existence of arbitrarily large strongly Ramsey cardinals*. Of course, Friedman's article is very profound and promising, and reveals that *Divinity* may be present in mathematics, which may be recognized one of the most important discoveries after Gödel in the foundations of mathematics.[17]

Claudio Ternullo, in “Naming God’s Essence: Ineffability, Analogy and Set Theory,” uses set-theoretical methods to unravel the intricacies related to the ineffability of God. Regarding the attribution of properties to God, two approaches have come to prominence in the Christian theological tradition: *Pseudo-Dionysius’ negative way*, which holds that God can only be characterized by what He is not, i.e., by properties He does not have; and *Aquinas’ way of analogy*, according to which the attribution of properties to God is neither predicated *univocally* nor *equivocally*, but *analogically* with respect to those referring to the created beings. According to negative theology, the three fundamental meta-properties of God are: *ineffability*—God’s essence (qualities) cannot really be named, that is, a characterization of God is beyond what is linguistically expressible; *reflection*—properties predicated about God are, in fact, predicated about God’s creatures; and *super-essentiality*—God’s properties are ‘super’-properties, as compared to the properties of the created beings. On the other hand, Aquinas interprets analogy in two ways: as *simple proportionality* and as *composite proportionality*. As postulated by J. Bocheński, both Pseudo-Dionysius’ and Aquinas’ views may be construed as fostering the idea of the existence of an isomorphism between God and the created beings. Now, Bocheński’s approach has certain shortcomings, which are discussed at length in the paper. C. Ternullo shows that, using strong reflection principles and a specific instance of the set-theoretic multiverse one may, on the one hand, overcome the difficulties inherent in Bocheński’s approach and, on the other, also adequately represent, and better understand, the conceptions of analogy which underlie both the Areopagite’s and Aquinas’ doctrines. In particular, Ternullo argues that the first methodology (reflection principles) is more suitable to capture aspects of the negative way, whereas the second one (the set-theoretic multiverse)

provides a more adequate formalization of Aquinas' theory of analogy.

Part IX: God and Mind

This part contains three papers, numbered 33 through 35.

John Heil, in "What's on God's Mind?," reflects on this article's title question. However, any answers to this question ought to raise some eyebrows. He asks, how can a man whose mind remains a great mystery have anything to say about God's mind? Our conceptions of minds and their contents are adapted to finite beings and do not extend to God. Any characterization of God is inevitably pictorial or analogical. According to Heil, there is not much to say about the divine nature, in particular, about His mind. This, however, does not mean that nothing useful can be said. He approaches the question of God's mind not by analyzing research papers on this subject, but by sharing his own thoughts as an experienced metaphysician. He begins by presenting some basic principles regarding descriptions of the nature of God, and then proceeds to consider the implications of the divine nature and how they relate to the question of whether a being of this nature can have thoughts, and what this might mean for the character of the cosmos. He outlines what is sometimes considered a problem for theism and argues with some sophistication. Heil's thesis is that God, as an acting being, has a mind, but any talk of God's beliefs, desires, intentions, and actions is figurative.

James P. Moreland, in "How Phenomenal Consciousness Provides Evidence for God's Existence and Informs What It Means to Say God Is a Spirit," sets two goals: 1). Defending the argument from consciousness (AC) for the existence of God; and 2). Arguing that God is essentially a spirit. Moreland devotes the most attention to the following deductive form of (AC): (1).

Genuinely non-physical conscious states exist; (2). There is an explanation for the existence of conscious states; (3). Personal explanation is different from natural scientific explanation; (4). The explanation for the existence of conscious states is either a personal or natural scientific explanation; (5). The explanation is not a natural scientific one; (6). Therefore, the explanation is a personal one; (7). If the explanation is personal, then it is theistic; (8). Therefore, the explanation is theistic. Moreland proceeds to argue that the theistic explanation of consciousness by a God who is spirit has positive content, rather than the more common negative characterizations of God as immaterial. A spirit $=_{df}$ a substance that is essentially characterized by the actual and dispositional properties of consciousness. Moreland devotes much attention to the meticulous analysis of the criticisms of this definition.

Charles Taliaferro, in "God and Mind," proceeds from the assumption that the philosophy of mind and the philosophy of God are deeply intertwined. He is interested in this philosophical context influencing the philosophy of mind rather than the philosophy of mind itself, narrowly treated. Going into details, Taliaferro tries to answer three basic questions: 1). Why have the three philosophical models: Logical Positivism, Wittgensteinian Philosophy, and Scientific Naturalism—all critical of theism—lost popularity? 2). What are the manifestations of the revival of philosophical theism in the last 50 years? and 3). Does the pluralistic context—understood as the difference between Western and the Eastern philosophy—influence the concept of God, and how?

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Notes

1 Roots of this doctrine go back to ancienr philosophers such as Thales, Parmenides, Plato, Aristotle, Philo of Alexandria, Plotinus and antiquity Neo-Plotinists, Gregory of Nyssa, Augustine, but it was most developed in the Middle Ages by Boethius, St. Anselm, Thomas Aquinas and Duns Scotus, and is echoed by many contemporary philosophers or theologians such as, for example, →Stump, 2012; →Stump and Kretzmann, 1985; →Dewan, 1989; →Leftow, 1990, 2006; →Rogers, 1996; →Holmes, 2001; →Franks, 2005; →Brower, 2008; →Dolezal, 2011; →Vallicella, 1992, 2019; and →Mann, 1975, 1982, 1983 and 1986.

2 →Maddy (1997) claims that questions about mathematics must be resolved using only purely mathematical (and not philosophical) methods, therefore she rejects the condition 3 of the Easwaran's definition.

3 There is a distinction between *proving* and *proof*— *proving* is a process, whereas a *proof* is the result of this process.

4 Regarding the mutual relationship of these two statements, L. D. Beklemishev writes: "Initially, the prevailing opinion was that Gödel's second theorem is a kind of 'supplement' to the first theorem that only indicates an explicit form of a quite independent statement whose existence is asserted in the first theorem. To some extent, this point of view is

widespread even now. At the same time, after the works of Kreisel, Löb, Feferman, de Jongh, Smoryński, and others who studied generalizations of Gödel's second theorem, it became clear that the nature of these two statements is substantially different. Gödel's second theorem turns out to be mainly connected with modal logical properties of the formula expressing provability, and with the self-reference effect in arithmetic. In our opinion, it is still too early to decide definitively what the 'correct' context for Gödel's second incompleteness theorem is" (→Beklemishev, 2010, pp. 858–859).

5 For those who find good use for this suggestion and treat it as inspiration for further research, we recommend additional reading in the form of the article: "Absolute Infinity in Class Theory and in Theology" (Horsten, 2016).

6 Cf. also Smoryński, 1977; →Raatikainen, 2018; and Epstein and Carnielli, 2008.

7 See, for example, →Dummett (1963 and 1991) for arguments against the absolute generality of concepts.

8 This kind of thought—called *Set-Theoretic Potentialism*—is the same as the one that there is no absolute universe of sets, but rather that any universe may be extended; see, for discussion, Barton and Friedman (2013). In Hamkins' and Linnebo's words: "Set-theoretic potentialism is the view in the philosophy of mathematics that the universe of set theory is never fully completed, but rather unfolds gradually as parts of it increasingly come into existence or become accessible to us" (Hamkins and Linnebo, 2018, p. 1).

9 For articles, which are concerned with reflection principles in the context of the set-theoretic universe, see also →Welch and Horsten, 2012 and 2016a, b.

10 According to the *iterative conception*, sets are formed in stages. We begin at stage 0 with no objects whatsoever—formally, $V_0 = \emptyset$. If at some stage α we have formed a collection V_α of sets, then at the next stage α+1 we form all the subsets of V_α—formally, $V_{\alpha+1} = \wp(V_\alpha)$. At limit stages λ, i.e. in steps which are not immediately preceded by any other stage, we simply pool all the collections of objects already formed—formally, $V_\lambda = \bigcup_{\gamma<\lambda} V_\gamma$. Finally, $V = \bigcup_{\alpha\in On} V_\alpha$.

11 Barton and Friedman clarify the *height* and *width* parameters as follows: "By *questions of height* we mean questions concerning what ordinals exist to index the V_α. By *questions of width* we mean questions concerning what subsets of V_α are contained in $V_{\alpha+1}$" (→Barton and Friedman, 2020, p. 628).

12 See →Gödel (1947) and →Welch (2014) for views of this kind. Actualism has a variety of meanings in the literature, for example, →Linnebo (2013) uses the term 'actualist' to refer to the position here called 'Absolutism.'

13 See also →Steel, 2014, and →Meadows, 2015.

14 See Zermelo, 2008 [1930]; →Hellman, 1989; and →Isaacson, 2011.

15 See →Arrigoni and Friedman, 2012.

16 M. Almeida uses the symbol $\Box F_x$ instead of F.

17 In this context, let us inform about Friedman's further, already started but not completed, research; see →https://u.osu.edu/friedman.8/foundational-adventures/fom-email-list/. More specifically, for a supernatural consistency proof for mathematics, see FOM 509, FOM 511, FOM 513 and FOM 515; and for unique undefinable elements, see FOM 508 and FOM 510.

I Who Created the Concept of God?

1 The Meaning(s) of the Question of ‘Origins’ and the Creation of ‘GOD’ as ‘Concept’

Jaco Gericke

Abstract: In this contribution, the question about the ‘origin’ of ‘God’ as a ‘concept’ will itself be the target of analysis. More specifically, an attempt will be made to identify certain ontological conditions of possibility for the meaningfulness of the religious language involved. For illustration, the focus will be on related research in Hebrew Bible scholarship. Following this first ontological reduction, a second is evident in having to contextualize the question via the concerns and categories in the second-order discourse of the auxiliary discipline informing the chosen interpretative approach. An additional ontological reduction occurs when the rudiments of some theory of concepts affect one’s discussion of the origins of variable instances of the ‘concept’ of God. Though highly specific conceptual-historical clarifications seem both possible and meaningful, it is concluded that an ontologically non-reductive approach to the question is neither.

1.1 Introduction

To begin with, consider the theme of this section, phrased as the following question:

“Who created the concept of God?”

Taken literally, one would be excused for wondering what kind of question that is. What does it mean? What exactly is being asked? What answer could possibly suffice? Why does it

start with ‘Who’? Does this refer to an individual or a collective? Would either alternative be meaningful? What is meant by ‘created’? How might this be similar or different from the meaning of its ‘origin’? And what about the word ‘concept’? Is it used in a popular or technical sense? Does the use of the definite article ‘the’ imply that there is just one concept? “And what”? about ‘God’? Are we discussing it in the sense of a personal name for a very specific deity, or are we considering a more general ontological category like ‘divinity’? These questions seem to suggest that perhaps a proper and hardly pedantic first response to the topic of this section would be that it all depends on what one means by ‘who,’ ‘created,’ ‘concept’ and ‘God.’

Additional requests for nuance and specificity are not out of place either. More broadly, how is the question assumed to be related to others with similar interests but featuring different variables in the wording? For example, questions concerning the origin of classical theism, of monotheism, of theism in general, of religious beliefs or of religion as such (cf. →Mikalson, 2010, on Cleanthes; cf. →Boys-Stones, 2009, pp. 1–22). Moreover, is the question itself really a philosophical one, concerned with the ontology of theistic beliefs? (cf. →Szatkowski, 2018). Or might it be more at home in linguistics, literary theory, the history of religion, the psychology of religion, the sociology of religion, theology or even biology? Does the wording not risk committing several informal fallacies associated with question framing? Or are these semantic concerns ignoring the pragmatics of the context of the formulation? Is the meaning of the question not more than the sum of the meaning of its parts? Might the practical requirement for brevity (and therewith generality) applicable to titles and the warrant for fuzzy and provocative conceptual content in any savvy marketing strategy render all the above redundant?

However we may respond to the flurry of counterquestions posed, for the sake of the argument, the relevance of the discussion is based on the assumption that questions of meaning cannot be separated from and typically precede questions of truth. Consequently, creating awareness of the potential complexities supervening on conceptual clarification is as suited to reflections on the ontology of divinity as is any attempted propositional justification (cf. MacDonald, 2009). The objective of this contribution then is not so much providing an answer to the question of who created the concept of God but to illustrate how our higher-order discourses configure the conditions of possibility for the assumed meaningfulness of the religious language in the question itself. The relevance of doing so is that the exemplary ontological framework able to explain why this question, its meanings and any possible answers appear the way they do, or why they appear at all.

1.2 Meanings, Questions, Concepts and Origins

Given the focus of this chapter, the quality of any discussion with similar concerns could be improved by taking cognizance of that which has already been debated in related philosophical theories of meaning. One of these is on the meanings of ‘words’ (on which, see Gasparri, 2018). Another is the more general ‘semantic’ or ‘foundational’ theories of meaning, including views denying that linguistic expressions are the kinds of entities with reference to which there are facts (see Peaks, 2019). Insofar as our interest lies with the meanings of an interrogative sentence, the rich variety of theoretical research on the semantics of questions should also not be neglected (see Cross and Roelofsen, 2022).

Insofar as the question features the ‘concept’ of God, one would think that the philosophical discussions of concepts “at

the intersection of semantics and philosophy of mind" is also relevant (see →Earl, 2021). In other words, what are our assumptions with reference to the semantics, ontology and epistemology of concepts? What theories of conceptual structure or analysis might be taken for granted? For example, the question of the origin of the concept of God seems as much concerned with the epistemology of concepts (concept acquisition, possession, categorization) as with those of their ontology (concepts as universals, mind-dependence or not, existence conditions, criteria for identity, complex and simple concepts in religious language). Even stating how one views the concept of God in relation to the different theories of concepts, e.g., *inter alia*, classical and neo-classical theories, prototype and exemplar theories theory-theory, atomism, proxytype, pluralism or eliminativism) would be elucidating (for more extensive overviews, see e.g., *inter alia*, →Margolis and Laurence, 1999, 2007, 2015, 2019).

One problem with the requirements suggested in the previous paragraph is that the word concept is now part and parcel of both technical jargon and ordinary language. Philosophers of religion like most others very often refer to the concept of God in the popular sense, judging from references to it in the literature (see e.g., *inter alia*, the titles/contents in →Ward, 1998; →Buckareff and Nagasawa, 2016; →Bowker, 2018; →Morley, 2019). Wherever definitions of the divine are included and with reference to essential properties or necessary ones, a (neo-)classical theory of conceptual structure might be implicit despite the problems associated therewith and the preference for prototype theories in many cognitive sciences (see Pyysiäinen, 2005). To be sure, the ontology of divinity in classical theism seem to make adopting any view other than an essentialist one metaphysically problematic for clarifying the concept of God as perfect being. Moreover, if a second-order

ontology has no place for properties, relations or kinds, it has to be considered whether and to what extent this might affect the meaning of the question of the origin of the concept of God within a tradition's the first-order religious language.

One example of the latter can be found in →Cupitt (1996) who sought to locate the notion of a concept within the history of religious language and the ontology of divinity. On this view, what used to be inchoate names for particulars featuring nameless spirits became subsumed under common nouns or general terms because of biological and social needs related to hunting/gathering. The first deities were distinguishable from spirits by virtue of their possessing personal names alongside generic terms for their kind. In Plato, the sacred world of divine names and general terms became depersonalized and consigned to a transcendental realm of Forms. Then, in the Christian Platonism of late antiquity, the Platonic forms came to be seen as ideas in the mind of God. Another major shift occurs with Kant in whose ontology the divine ideas are brought down to earth and are now human concepts (also the concept of God). Finally, in Nietzsche, Wittgenstein and Derrida we see a further reduction language so that what used to be called concepts in the Kantian sense are now only names, words, signs, etc. Yet the word concept lives on, however, having become part and parcel of both ordinary and scientific language.

From Cupitt's account, whether we agree with the details or not, the complexity of the problem is increased exponentially whenever one goes beyond merely seeking to clarify the concept of God and tries to determine its 'origin.' This adds a historical dimension to the question, as does any inquiry as to its 'creation.' Here some perspectives from related discussions by Continental thinkers prior to and influencing Cupitt can be considered. For example, when Nietzsche (1979 [1873], p. 77) presents his view of concept formation it can be taken to imply

that the concept of ‘God’ and even an ontology of ‘divinity’ in the generic sense could only originate through the ignoring of discrepancies in every instance of actual word use. Going a bit further, →Foucault (1977, p. 139-164) might be interpreted as allowing us to infer that whatever we call the origin of the concept of God is in fact a disparity of other things and a fiction of our second-order discourses harmonizing discontinuities in religious language viewed in retrospect. →Derrida (1989, p. 107), for his part, could be understood to show how every description of the origin of the concept of God will never be about something self-identical but instead only re-present an irreducibly heterogeneous differential ontological framework already structured through religious language.

1.3 Orders of Religious Language and Their Ontological Concerns and Categories

For the purposes of differentiating ontological categories, one may distinguish but not separate different orders of religious language (RL): Let first-order RL (RL1) be any form of language wherein there is any reference to god(s), as one might find in ancient texts irrespective of the genre, including archaeological artifacts whether iconographical, epigraphic, or otherwise. Let second-order RL (RL2) refer to the way scholars write about RL1. Only in RL2 do we encounter words like ‘ontology,’ ‘divinity’ and ‘concept’. It is also within RL2 in which the question of the origins of the concept of God arises as a problem featuring the particular propositional content (which is, of course, itself open to different interpretation and, a-fortiori, restatements). The many responses (which is a broader category than answers) responses, though concerned with RL1, likewise is found in RL2 only.

When we consider the meaning of the question of the origin of the concept of God with special attention to divinity and ontology, we are entering another order of discourse. Though still in the domain of RL2 and about RL1, third-order RL (RL3) refers to the domain of discourse featuring meta-commentary on RL2. That is, in RL3 we are seeking to describe what the question has meant in RL2 with reference to RL. We are also interested in identifying a selection of supervening conditions of possibility for the meaningfulness of both questions and responses. These distinctions are not meant to imply some form of transcendental perspective, only a further level of abstraction and reflection on what comes 'before the question'. It is also not assumed that that the problem of the meaning of the question can be reduced to questions regarding the nature, meaning and reference of religious language. Yet closer look from the perspective of RL3 at the choice of words using for question framing in RL2 shows they are not separable from other religious-philosophical loci, e.g., *inter alia*, the concept of religion, religious epistemology, the nature of God, the relationship between religion and history, culture, and science, the problem of religious pluralism or diversity, etc.

There are also meta-ontological considerations of relevance when discussing the ontology of theistic beliefs (on which see →Szatkowski, 2018). Consequently, to the extent the language and logic of the question of the origin of the concept of God is correlated therewith, one must reckon with the reality of serious theoretical differences regarding what ontology is concerned with, what the task in relation to those concerns is and whether it is a legitimate philosophical discipline (see Hoffweber, 2021). Assuming it is, for the sake of the discussion, there may be discrepancies between what is believed about the concept of God and its instance(s), and whether the latter is ontologically dependent on the former. There might also be disagreements

about the ontological status of entities in the world of the question. Some philosophers of religion may take the reference to the concept of God to presuppose a critical or non-realist stance whereas others will think the wording implies no such thing and is inclusive of naïve-realist ontologies.

Of course, objections to the question may still be raised from the perspective of informal logic. For example, one could link the question to a variety of fallacies of question framing in general or in the context of ontology, in comparative philosophy, in the history and the science of religion which here cannot be bracketed by the philosophy of religion. These include but are not limited to presumption, begging the question, complex question, genetic, essentialist, equivocation, category mistakes, perennialism, etc. All of these may be granted, or not. So can the recognition that we will never have a vocabulary perfectly suited to the task of clarifying concepts or for asking/answering questions other than what we made ourselves as part of cross-disciplinary literary traditions (cf. →Rorty, 1978, pp. 141–160).

Given the meaninglessness of attempting to illustrate the application value of all the aforementioned *desiderata*, with reference to the meaning of the question of the origin of the concept of God simpliciter, I shall attempt to do so in a specific context of RL1/2. Seeing how the Hebrew Bible (HB) is often understood as linked in one way or another to the origin of the concept of God of many an implied reader of this volume, the discussion to follow will be limited to the meanings of the question emergent within the ontological domains of the associated disciplines. Even so, this will still be done in a manner as hypothetical as possible to allow for transposing and extrapolation of the structure of the inquiry to other traditions of RL1/2.

1.4 Ontological Conceptual Reductions in RL2 (HB)

In the context of specialized research, HB scholars never ‘just read’ the texts. One is always already approaching the concept of God from the ontological categories of the subtype of RL2 of the supervening theory in the auxiliary discipline (see →Gericke, 2017, pp. 632–646) Moreover, ontological reduction occurring through inevitable specialization will be identifiable from the auxiliary discipline’s technical terms for any given context of RL2 (cf. →Van Riel and Van Gulick, 2016). By this is meant that the meaning of the question will be relative to the supervening conceptual framework, as illustrated only briefly and hardly with sufficient diversity, detail and differentiation in the following breakdown. In doing so, some remarks related to what are implied about the origin of the ‘concept’ of God will be included.

To begin with, it may be recalled that in the Hebrew Bible, the words translated as ‘God’ are not only used in the absolute sense as proper names for God but also in the generic sense as common nouns for the kind of entity and other related entities and phenomena were assumed to be. A complex multiplicity of extensions and intensions of the generic terms for divinity in the HB are found within the different worlds of the text. This attempts at an analysis only partly possible for isolated instances of what is inherently a fuzzy concept (see →Gericke, 2009, pp. 21–45). In the RL2 of linguistic approaches, termini for origins of the concept of God, or why some was called a god, or why Yhwh was assumed to be one or why the ontology of the concepts of generic godhood included but was more extensive than just God have been partly correlated to morphology and etymology. Not so much for semantic purposes but for genealogical ones. Roots of the Hebrew common nouns cannot be used to identify the origin of the concept of God, but they can show traces of conceptual conditions of possibility for the meaningfulness of RL1. While in theory there many proposed possible root derivations, most scholars appear to consider the original sense

of the terms as connoting ‘power’ or ‘leadership.’ This in turn only begs the question of how power was originally contextually constructed within the same subset of RL1, which lead to traces and a disparate ontology in relation to the meaning of the question.

In many forms of biblical literary criticism, the question in RL2 of who created the concept of God in this context of RL1 is relatively more meaningful even if the only assurance can be had ends up as a theoretical entity in a narrative-critical ontology (the world of the text). Not primarily interested in the question of the origin of the concept of God, some scholars have made claims of a creationist kind (in the context of the metaphysics of fiction, not the anti-evolutionist variety). Thus God-as-character in the Bible is “the creature of the author/narrator” (Gunn, 1993, p. 61). The ‘imaginative generative power of rhetoric offers to the hearer of this text a God who is not otherwise known or available or even—dare one say—not otherwise “there” (→Brueggemann, 1997, p. 33). In other words, with reference to a god as a character in a story, the ontology can be reduced to “the world that the teller has created for his representation of Old Israel” (→Thompson, 1999, p. 304). These are all in a sense an implied response to who created the concept of God, but only in the context of a particular form of RL2 and made possible by a narrative-critical reductive ontology of divinity.

One way to think of these claims as less crude is to consider them in conjunction with the sociomorphisms in the world of the text providing termini ad quo for any use of the concept within the syntax of more complex expansions in RL1. That is, the historically temporary and culturally relative origin of objects, of divine attributes and relations within the broader ontological framework reflect some of the initial necessary but not sufficient conditions of possibility for the meaningfulness of instances of the concept of God in RL1. The latter may not be the same as the

origin of the RL1 in question, which is usually much younger than the creation of the conceptual content. e.g., *inter alia*, the view of divine relations as structured like a kingdom (cf. Deut 32:8–9; 1 Sam 8:7; Dan 6:27; etc.) could not originate before the first of human monarchies.

The same give-away of initial conditions of possibility for the meaningfulness of RL1 in the HB are given in the form of technomorphisms that are part of constructing certain instances of the concept of God. Examples related to philosophical-theological topoi can be found in conceptualisations of divine action, e.g., where the deity rides horse-drawn chariots (cf. 2 Kgs 2:11–12, 6:17; Zach 6:1–8; etc.) or fight with the sword (cf. Gen 3:22f.; 32:1–2; Josh 5:13–15; 2 Sam 24:16, 27; etc.) could not be created before humans used them. In this way it is possible to think of the concept of God in RL1 as created and distinguish this from the origin of the conceptual content involved. Again, however, the conceptual-historical origin of the morphism itself is not identical to the origin or creation of it in RL in the HB as such or coterminous with the creation of the concept of God in any given instance. The link is indirect and cannot be used to determine with satisfactory exactitude either the relative or ultimate origin of the concept of divinity in context.

A different reductive account of the origin of the concept of God in RL1 came with the rise of religious-historical and comparative-religious approaches to the ontology of divinity in the HB during the nineteenth century (and up to the middle of the twentieth century) (cf. Oesterley and Robinson, 1930, pp. 11–21 for an early overview). Here the focus was not so much on the question of who created the original concept of God in the world of the text but what the latter seemed to be the empirical form of the earliest numinous phenomenon the god Yhwh was thought to be associated with. Based on anomalous bits of RL1 that could not have been conceptualized in later times, but had

to be assimilated to it, various proposals for the empirical origin of the concept of God were put forward.

Some of these remain possible but unverifiable hypotheses linked to imagery used in older poetic depictions of theophany and correlated to theriomorphisms or geomorphisms in related North-West Semitic prototypes. Examples include e.g., the ‘Bull of Jacob’ (like El in Ugarit), as clan divinity in Gen 49, or the thunderstorms coming from the South in Judges 5, Habakkuk 3, Psalm 68 (like Ba’al in Ugarit), or with a volcano in the Arabian Peninsula, as in motifs found in the Pentateuch out of sync with the ontologies of divinity in the post-exilic redactions thereof. in the region south of present-day Israel. Others are less mainstream, and mostly dated to later times, e.g., Yhwh a solar or astral divinity. Other etiologies are no longer taken seriously, if remembered at all, despite once being the center of attention, e.g., Yhwh as a divinized ancestor (cf. the ghost of Samuel called a god in 1 Sam 28), a fire-demon (Gen 15, Ex 3,4 and elsewhere), or even a sea god like Yam in Ugarit (see Smith (1996) on “the early history of God”).

Not that all such links have been rejected *en toto*. Rather, they are seen as assimilated kinds, properties and relations in the ontologies of divinity of emergent monotheistic concepts of God (re-)created in response to new social, historical, political and religious developments, e.g., the rise of Empires (cf. →Albertz, 1992, p. 21; →Zevit, 2001, p. 87; →Smith, 2001, 2008). More recently, however, there has been a renewed interest in recounting the state of research related to the ‘origins of monotheism’ (see →Smith, 2001), ‘invention’ of God (see Römer, 2015) or the ‘origin and character of God (see →Lewis, 2020) in prior to RL1 in the HB (i.e., the history of Israelite religion). Once RL2 is an implied answer to the question of the who created the concept of God, or what its origins were, the ontology of

religious-historical reductions is likely to be operant conditions of possibility for their meaningfulness.

Last but not least, perhaps the reductive format of RL2 where the words ‘concept of God’ in HB appear most frequently is that of OT theology. Here, as in philosophy of religion, the word ‘concept’ is used in both technical and popular senses, with almost no attention to theories of concepts. In addition, in the RL2 of biblical scholars, ‘concept’ is also used interchangeably with historical-philosophical variants with ontologically variable import, e.g., ‘names,’ ‘forms,’ ‘ideas,’ ‘images’ of – and ‘words’ for – ‘God’ in the HB. This seems to be a general trend in all leading mainstream writings during the twentieth century, and up to the present day. Even biblical scholars who otherwise claim to reject the involvement of philosophical categories and ontological concerns will refer to ‘concepts’ of God in the HB without worrying about the philosophical-theological (Neo-Kantian) critical-realist reality-appearance distinction often included. In more conservative writings, aspects of ontologies associated with Platonism and Aristotelianism are more overtly at work (via supervening post-Reformation receptions of scholastic philosophical-theological categories and concerns). Though the HB’s concepts of God may be variable and metaphorical, and literary-ontological dimensions notwithstanding, somehow they all assumed to refer, even if in historically and culturally-relative ways, to a singular instance thereof in the world beyond the text.

In theologically-reductive accounts of the origin of the concept of God in RL1, questions related to the ontology of divinity are often bracketed (cf. →Brueggemann, 1997, p. 117). Yet they cannot be altogether ignored, especially where socio-literary reductions supplement theological ones, either as prolegomena of for the sake of nuance in the recognition of religious diversity in Israelite religion supervening of the

emergent theological pluralism in the theologies of the HB (cf. →Gottwald, 1979; →Gerstenberger, 2002; →Gericke, 2013, pp. 215–226). This introduces the need for additional specificity when it comes to the criteria for identity in concepts of God in the HB with reference to incommensurable axiological dimensions in the ontology of divine attributes and relations. On the one hand, what makes a maximally great creator, warrior, king, father, tribal leader, sage, healer, etc. are not the same qualities. On the other hand, diachronic dimensions in constructions of divine personal identity assimilate incompatible great-making properties from changing cultural roles in the source domains of metaphors used in the construction of the concept of God within the target domains of RL1.

Attempts to add nuance to the 'concept' of God in RL2 theological reductions might involve exchanging the metaphysical term attributes for others assumed to carry less philosophical import, i.e., 'names,' 'predicates,' 'adjectives,' 'qualities,' 'marks,' 'characteristics,' 'traits,' etc. (see →Gericke, 2020). Yet not only do these very over time and across possible worlds in the text, the ontologies of different types of RL 1 also supervene so that even where an instance exemplifies what are verbatim the same properties, on closer inspection these may mean entirely different things. This could be taken to imply that the origin of the concept of God in RL1 is identical to the origin of specific conceptions of divine existence, attributes and relations in the same order of discourse. Finally, given the uses of God as both personal name and polythetic genus in RL1, the question of who created the concept God and that regarding the ontology of (generic) divinity are two very different concerns, at least for RL2 on the HB.

1.5 Conclusion

With reference to the question of who created the concept of God, this chapter began with the assumption that questions of meaning precede questions of truth. This also applies to the meaning of the questions we seek to answer. Even so, philosophical theories of the meaning of words, propositions, and questions are seldom noted, as are philosophical debates on concepts in philosophy of mind. In philosophy of religion, technical/popular senses of the word ‘concept’ tend to be conflated when referring to concepts of God. It is usually not considered how the ‘Concept’ as a philosophical term has a history within the history of religious language. Furthermore, any attempt to identify the ‘origin’ of a concept is compromised by the disparities of history and language.

As a first step in discussing the meaning of the question of the origin of the concept of God, a distinction was introduced between different orders of religious language. This in the knowledge that asking who created the concept of God risk committing fallacies of question framing. To partly overcome this obstacle, a move was made from semantics to pragmatics and, for the purposes of illustrating the need for nuance, from the concept of God in general to the ways it is approached in one context associated with origins, i.e., the Hebrew Bible. This in turn led to a recognition of how, in Hebrew Bible scholarship, second-order discourse is always already present and provided by an auxiliary field. Consequently, a conceptual reduction of the ontology of divinity in discussions of the origin of the concept of God in first-order religious language are unavoidable. This implies that the meaning of the question of who created the concept of God and any possible answers offered in response must suffer the same fate.

Samples from linguistic, literary-critical, religious-historical and theological approaches were offered to prove the point. In each of these, the same tendency to conflate technical and

popular meanings of the word ‘concept’ was observed. In addition, in each domain of discourse, the question of who created the concept of God or the one about the origins of divinity was shown to be at least meaningful, even if only partly functional or answerable. The form of the variable responses could be correlated to the concepts, concerns and categories of the method used to discuss it. In each case, the identity of who created the concept of God or of the origin thereof could only partly be described. No unified answer was forthcoming because no unified meaning of the question was operative. Traces of conceptual origins were limited to termini of imagery in the roots of words for divinity, in the morphisms of narration, in anomalous empirical components of theophanic language and in the history of specific great-making divine properties and relations. The origin of these should not be confused with the origin of the concept of God in the world of the text, which had to be distinguished from the history of religion and belief revision supervening thereon. None of these were ultimate origins.

From the illustration of what the question of who created the concept of God might mean the context of Hebrew Bible scholarship, some of the findings allow for more general conclusions relevant and extrapolatable to other disciplines, including philosophy of religion. Here too there is no pure or perfect language to ask of answer the question of the origin of the concept of God. Yet by paying attention to how this is or is not done in the context of specific periods, persons, presuppositions, problems, perspectives and purposes, it is likely that what the question may mean will be determined by what such variables allow it to mean, which in turn will predict the general outlines and ontological reductions of the possible set of what could count as meaningful if tentative answers in response.

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2 The Concept of God as Perfect Being. The Presentation of Ancient Christian and Medieval Views

Agnieszka Kijewska

Abstract: When we speak about a perfect being, we may highlight two different features of a being. For one thing, we may stress the essential completeness of the being in question, which lacks nothing and whose distinctive property is stability and immutability. In the Christian antiquity we encounter such a conception of a perfect being in the writings of St. Augustine. This concept of perfection achieves its zenith in the thought of St Anselm of Canterbury in his ontological argument for God's existence, which attributes to God such perfection, that it is not possible to think Him not to exist. A number of thinkers (Eriugena, Master Eckhart), who wished above all to stress God's ontological and epistemological transcendence, preferred to speak of Him as Non-being. In another sense God's perfection is understood to imply the incomparable intensity of His way of existence, which is infinity. This conception found its proponents in St. Thomas Aquinas, John Duns Scotus and Nicholas of Cusa.

2.1 God of Greek and Christian Antiquity. The General Framework

The thought of the Christian antiquity grew in many ways out of the speculation and wisdom of pagan antiquity, which it nevertheless transformed in the light of the Revelation. In the relationship between the ancient Christian intellectual world and

the pagan culture both elements of continuity, disruption, and even rejection of some aspects of the latter can be found. Disruption is particularly evident in the conceptions of the divine, God and other deities. Eric R. Dodds cautioned: “we should of course remember in the first place that in a polytheistic society the word *theos* did not carry the overwhelming overtones of awe and remoteness that the word ‘God’ carries for us. In popular Greek tradition a god differed from a man chiefly in being exempt form death and in the supernatural power which this exemption conferred on him. Hence the favourite saying that ‘Man is a mortal god, and god is an immortal man’; hence also the possibility of mistakintg a man for a god if he appears to display supernatural powers as is said to have happened to Paul and Barnabas at Lystra and on several occasion to Apollonius of Tyana” (→Dodds, 1965, pp. 74–75).

However, parallel to the popular polytheistic tradition there existed another religious current in ancient Greece, namely that of mystery cults, a preferred object of reference of philosophers such as the Pythagoreans, Empedocles, and above all Plato. These mystery religions cultivated a different conception deity and the relationship obtaining between the divine and human beings.[1] Goodness and moral perfection were chief attributes of divine beings within these cults, gods were thought not only to be powerful and blessed beings, but also to be paradigms of virtue to be imitated by men, who on their side should undertake every effort to make themselves like gods (*homoiosis tou theou*).[2] This esoteric conception of the divine involved the traits of transcendent dignity, mystery, inaccessibilty, radical otherness: the divine was no longer on a par with the human. This new and reverent approach to the divine is illustrated by the famous passage from Plato’s *Timaeus* (28 C), which in a certain measure helped to shape subsequent considerations of the First Principle of the Universe, including Christian ones:

> But it would be a hard task to discover the maker and father of this universe of ours, and even if we did find him, it would be impossible to speak of him to everyone (τὸν μὲν οὖν ποιητὴν καὶ πατέρα τοῦδε τοῦ παντὸς εὑρεῖν τε ἔργον καὶ εὑρόντα εἰς πάντας ἀδύνατον λέγειν) (Plato, *Timaeus* 28 C, 2008, p. 17).

In the Latin version of Calcidius:

> Igitur opificem genitoremque universitatis tam invenire difficile quam inventum impossibile digne profari. (→Calcidius, 1962, p. 21)

Plato may have not been the first to emphasize the importance of the philosophical quest for the efficient cause of the visible cosmos (Anaxagoras did this before him), he may have been the first to call the Maker of the Universe the Father, and was a pioneer of apophatic approach to the divine in his skepticism as to the possibility of articulation of the knowledge of the Demiurge to ordinary humans. The whole Platonic tradition of antiquity (including Middle- and Neoplatonism) agreed with its founder in calling the divine First Principle of the Universe the Father. For instance, Alcinous in his *Handbook of Platonism* (*Didaskalikos*) attributes the name of Father to God alongside such names as Mind, Good, Beauty and Truth.[3] Alcinous's characterization of the divine First Cause is typical of the tendency in the pagan Platonism to exalt the First Principle and it heralds later representations of God as a perfect being:

> The primary god, then, is eternal, ineffable, 'self-perfect' (that is, deficient in no respect), 'ever-perfect' (that is, always perfect), and 'all-perfect' (that is, perfect in all respects); divinity, essentiality, truth, commensurability, [beauty], good. I am not listing these terms as being distinct from one another, but on the assumption that one single thing is being denoted by all of them. (→Alcinous, 1993, 10, 3, p. 17)

The New Testament way of referring to God the Father, the Creator of the Universe agreed with this philosophical tendency to attribute absolute perfection and goodness to the highest God. We find two Greek terms in the New Testament capturing and expressing God's completeness and absence in Him of any privation: *teleiotes* (integrity, completeness, perfection)[4] and *pleroma* (fullness, fulfillment, plenitude, saturation)[5] and their derivatives; the Vulgate typically translates these as, respectively, *perfectio* and *plenitudo*.

God is the plenitude, fullness of positive content of being and therefore is the paradigm to imitate and object to strive for to all nature, in particular He is the model for us to become as much like Him as is possible for us. God's plenitude and perfection is the fullness of being, life, beatitude, glory and wisdom, that lacks nothing and whose reality and power is of infinite intensity. In an effort to capture and express that surpassing reality of God, philosophers came up with a number of "catalogs" of perfections which they thought must characterize an absolutely perfect being. These divine perfections, numerous as they are, do not compromise the absolute unity and simplicity of the Divine Essence; the multiplicity of descriptions human mind can legitimately ascribe to the Perfect Being is merely a witness to the inadequacy of human efforts to comprehend the Divine Essence and express its greatness by means of terms that are borrowed from and primarily applicable to the description of the finite perfections of nature.

Thus the task that the philosophers engaged in the quest for the Absolute have taken upon themselves, cannot be adequately performed in the conditions defining the way human beings exist and cognize: already Plato was aware of it, as can be gathered from his statement in *Timaeus* 28 C quoted above ("it would be impossible to speak of Him to everyone"). The abyss

separating the Father, the ultimate Cause of all the Universe had nevertheless to be bridged somehow, if the philosophical endeavor to “speak of Him” if not “to everyone,” at least to those few that were capable of some understanding were to have some measure of success. This could be done along three ways, which were thought of as mutually complementary[6] and, therefore, had to be applied together to achieve the best result. The order in which these three ways had to be used and the details of the procedures required by each of them were subject to some alterations over time and according to different authors and circumstances, yet their foundation and essential content remained recognizably the same throughout history. (1) The first of these ways was based on the assumption that God is the only non-conditioned cause of the derived reality, which, therefore must bear some, albeit distant, resemblance to its Author; God’s work somehow declares the perfection of the Creator, thus the foundation of the first of these ways is the causal relationship obtaining between the Divine Maker and the effect of His work, the engendered Universe, which certainly bears an imprint, however distant and imperfect, of its ultimate source. This causal connection between the transcendent Cause and its effect provides the basis for the attribution to God of perfections present and evident in creatures; these perfections are affirmatively predicated of God, therefore this way, following Pseudo-Dionysius, has been described as affirmative way or affirmative theology. The perfections found in creatures and attributable to God have to be pure perfections, which means they have to be unconditionally better than their opposites and they must be free of any hint or suggestion of any imperfection. This way of affirmation and attribution to God properties found in nature is wholly legitimate way of doing theology on condition that we bear in mind that God absolutely surpasses any positive content that we can attribute to Him in this way; this is why the

affirmative way has to be complemented by the (2) way of negation, which consists in denying the predication of God even the pure perfections: no predicates we can think of can really capture the content of the Divine Essence, therefore we must refuse attribution to Him the perfections we found in created nature, and this not because He lacks any perfections, but because He possesses them in an transcendent way that totally eludes our understanding and predication. However, both the affirmative and negative ways are one-sided and apparently mutually contradictory, therefore both these ways have to be complemented and their apparent contradiction has to be resolved in a third step in any complete theological investigation, which has been called (3) the way of eminence, or surpassing. In this way we return to affirmative predication of God of pure perfections, which we take, however, in a degree incomparably surpassing the form these perfections assume in creatures and which we are capable of comprehending.[7]

As Fran O'Rourke has shown, this "threefold way" originated in Platonic philosophical tradition[8] and from there it penetrated by different ways into Christianity. Christianity, however, modified the pagan representation of the divine transcendence by emphasizing one more property of Divine Being, which had been in principle ignored by pagan antiquity, namely the boundlessness or infinity of God. In the Hellenistic mental universe, infinity was not perfection; on the contrary, it was synonymous with lack of definition or formlessness, it could not be grasped by any concept of the intellect, consequently they avoided attributing infinity to the divine. This typical approach of the Hellenic mind to infinity was systematized by Aristotle.[9] The context in which he treated of infinity was investigation of the physical Universe, and not theology. In his physics he stated that actual infinity is impossible and that an infinite being could not

exist on its own.[10] However, according to him time will have no end, therefore potential infinity in time is possible. The possibility of potential infinity is shown by the possibility of addition in infinity.[11]

> In sum, Aristotle seems to be a thinker who stressed finiteness. It is not clear if he rejected an all-encompassing being, as Anaximander did with the απειρον, which he quotes in his *Physics.* It may well be that Aristotle's account of potential infinity, as quoted in his *Physics*, and Anaximander's account of the ἄπειρον cannot be reconciled. Aristotle rejected the ability of the human mind, the νοῦς to think infinite, and he contended that God is not infinite. However, he held that the human mind can potentially think in infinite processes, and that the time and motion are infinite, even though space is finite. (→Achtner, 2014, p. 23)

The pagan thinker, who cleared the way for a positive conception of infinity in philosophical thinking was undoubtedly Plotinus, who associated the notion of infinity with the unbounded power of creation residing in the Absolute more than with the number and geometrical magnitude: the One, which is Good, has the unlimited power endlessly to produce the whole Universe.[12] Plotinus, in his treatment of the creative might of the One made use of Aristotle's notion of *dynamis*, which he reinterpreted to mean active power to produce things, and which became one of the key concepts of his system. In the phrases like *power* of *everything* (*dunamis panton*) or *infinite power* (*apeiros dunamis*) this concept serves to "define the One's generative capacity and explain how everything comes from the One without the One being affected by its own overflowing into the totality of things."[13]

In this way Plotinus laid the foundation for the attribution to God of the property of infinity, which was done in Christianity. Yet the logical consequence of God's essential infinity is the absence in Him of any conceptually graspable definition (to

define literally means to delimit), and, as a further consequence, His incomprehensibility and unnameablity: God appears to be an unbounded, incomprehensible expanse of being.

In the context of such notion of God as essentially infinite, another conception of the perfection of Divine Being could appear, that is the conception of Divine Essence as fullness or plenitude (*pleroma*) of being, which is no longer a collection, be it complete, of properties, but an unlimited intensity of the reality of a Being, whose fundamental property is infinity. The threefold way of cognitively approaching God retains its validity with this new notion of divine perfection, with, however, a new note added, namely the awareness of an infinite abyss intervening between the stages of the affirmative and negative ways and the way of eminence. This abyss is unbridgeable by any effort of the intellect, the only way of somehow crossing this divide is by reading symbolic representations of the Transcendence found in the realm of mathematical concepts or in the world of nature, yet the continuity between the content of these symbols and the reality they represent is constituted in the mind of the human “reader,” rather than in reality of essences.

For this reason, to some thinkers it appeared plausible that it is better for human understanding to refer to Divine Plenitude and Transcendence solely by means of terms denoting absence rather than presence of any positive, yet inevitably limited, content, and so they came to apply to the Divine Essence the names of Non-being and Nothingness. The natural human language, formed in the experience of and applicable to the necessarily limited reality of creation, when applied to Divine Reality, inevitably had the effect of diminishing, delimiting and dividing the Divine Infinity. To avoid that effect a radical step was required: the usual cognitive categories had to be discarded and the way of paradox had to be adopted: and thus it came to pass

that the name of Nothingness was applied to designate the Plenitude of Being.

These three ways of referring to God as the Plenitude of Perfection (completeness of essence, infinite intensity of being, nothingness/non-being) had their proponents in antiquity and outspoken representatives in the Middle Ages, yet there usually was no watertight compartments between the theologies they inspired, the approaches leading to them tended to merge with one another, therefore it is only in rare cases that a given thinker can be unequivocally classified as a representative of one particular approach to the exclusion of all others. For this reason, in the presentation that follows I am not particularly concerned with the exactly defining the place of a given theologian on the map of all theological currents, the choice of thinkers I make is to some extent a result of my subjective inclination; what I am concerned with is to present some identifiable tendencies and approaches which appear to me important from the point of view of some definite conceptions of God. The scheme that all the discussed approaches to the issue of divine perfection have in common is that of the “triplex via,” a combination of the affirmative and negative ways, and their synthesis and overcoming of their one-sidedness, the way of eminence, even though the order of these ways and the meaning of each way within the whole of the theological discussion may differ in particular cases.

In the Christian thought, the treatment of God as a perfect being is additionally complicated by the consideration of the personal nature of God and Him being in fact the Trinity of Persons. Being a person, God enters a particular relationship with every human person, and this fatherly, providential and caring relation constitutes a whole new dimension of divine perfection. Even more complication results from the fact, that God, the all-perfect Absolute, descends into the essentially

imperfect sphere of matter, assuming the mortal human body and becoming a man, and all this for the sake and benefit of man, a claim in itself absurd and scandalous, a *skandalon* in the eyes of the cultivated listeners to St Paul at the Athenian Areopagus (Acts 17, 32–34). God, in his personal relationship with man, calls man to follow Himself, guides a human being and actively supports her/him in their efforts to become like Himself, what is more, God calls on man to bring to completion His own work, as though His work lacked perfection.[14]

This point of the Christian faith has clear consequences for the representation of Divine Perfection as completeness of essence: a new area of problems arises as a result thereof, namely what personal qualities must be added to the established list of pure perfections to make it really complete and provide an adequate representation of God as the very Perfect Being. As for the ways of negation and eminence, the personal nature of God and His descent into the human nature only highlight the paradox and enhance the sense of the transcendence of God, who being the absolute Plenitude, becomes one with the human nothingness.

2.2 The Fullness of Divine Perfection

The ancient pagan conceptions of divinity are not, as we have already remarked, easy to sort out and define. The mythical theogonies passed down by the poets were often subjected to thoroughgoing criticism by philosophers; philosophers themselves, often at a risk of being accused of impiety, sought for more exalted conceptions of deity in the mystery cults and engaged in theological speculations of their own. St. Augustine’s discussion of diverse pagan theological conceptions found in his *City of God* provide very illuminating guidance into the complex world of pagan imagination while throwing light upon the way

Christian intellectuals tended to look upon that heathen inheritance. In discussing the distinctions concerning religion introduced by Varro in his encyclopedia, Augustine writes:

> Next, what does he [sc. Varro] mean when he says that there are three kinds of theology: that is, of accounts given of the gods? Of these, he calls one mythical, another physical, and the third civil. [...] That the second kind should be called 'natural' the custom of speech now admits; and he has himself given a Latin name to the third, which he calls 'civil'. Then he says: 'They call that kind of theology mythical which is especially used by the poets; the physical is that which the philosophers use; and the civil, that which the people use. [...]' (→Augustine, 1998, pp. 246–247)

There can be no doubt, Augustine affirms, that only the second, "physical" account of gods deserves serious attention, for it "concerns the real world, which is the most perfect of all things." The first kind of theology, whose proper ground is the theater, and the civil theology, are easily lead astray by false opinions, therefore one should keep clear of them.[15] Varro himself is forced to acknowledge that these conceptions of gods are human fictions "just as men made the gods in human form, so have they believed that the gods love the same pleasure as human beings" (→Augustine, 1998, pp. 252–253). The second account of gods looks for the correct definition of the divine in the world of nature, which is not a product of human imagination. Philosophers wrote many volumes on this kind of theology:

> In them, they discuss what gods there are, where they are, of what kind they are, of what quality, for how long they have existed, whether they have always existed, whether they are made of fire, as Heraclitus believes, or of numbers, as Pythagoras thinks, or of atoms, as Epicurus says. And there are other things which the ears can more easily endure inside the walls of a school than outside in the forum. (→Augustine, 1998, p. 247)

The theology based on nature seeks to discover the essence of divinity within the ordered world of natural phenomena and their hidden causes. The true God is the God of nature, although not every thing found in nature is divine. Therefore, much of theological speculation produced by philosophers deserves to be regarded with the utmost distrust, nevertheless not everything that philosophers said on the subject of the divine is without value. Augustine himself highly appreciated especially the theories offered by the Platonists and thought Christians should not shun these solutions, but instead take them away from their "unjust possessors" insofar as they agree with our Christian beliefs. He referred to the story of the people of Israel appropriating for their own use valuable things possessed by the Egyptians; in the like way Christians should take for their own use whatever in the ancient pagan thought they find of true value.[16]

As is well-known from St. Augustine's biography, his encounter with Neoplatonism, its theory and spirituality was a breakthrough in his struggle to free himself from the shackles of the Manichaean materialism and in the process of the formation of his own conception of God and the Divine Nature. His entry into the "circle of Milan Neoplatonists"[17] which gathered around St. Ambrose and Simplicianus, two Christian leaders of importance, occasioned his study of the treatises by Plotinus and Porphyry, later collectively referred to by Augustine as *libri platonicorum*.[18] The change this Neoplatonist reading caused in Augustine's thinking was profound and concerned the elementary notions of ontology: it was the Neoplatonist conception of reality that provided Augustine with the ontological foundations for the formation of the conception of spiritual substance, which he earlier tried in vain to arrive at:

> Although I did not know why and how, it was clear to me and certain that what is corruptible is inferior to that which cannot be corrupted; what is immune from injury I unhesitatingly put above that which is not immune; what suffers no change is better than that which can change. My heart vehemently protested against all the physical images in my mind, and by this single blow I attempted to expel from my mind's eye the swarm of unpurified notions flying about there. (→Augustine, 1991, p. 111)

In the light of the quoted passage, as well as many other statements scattered all over Augustine's work, we can gather what attributes must characterize a perfect spiritual substance: foremost among these attributes are: immutability, indestructibility, and inviolability. Immutability of the Divine Being results from the perfection of His essence, which has no need of any further fulfillment, complement or actualization, because it already is all that it can be: it is the absolute fullness of being.[19] God's being is in every way complete, entirely immune from change, in no way subject to the flow of time; it is timeless, because its very substance is eternity, as Augustine asserts in *The Expositions on the Psalms*:

> [...] eternity is the very substance of God, which hath nothing changeable; there nothing is past, as if it were no longer: nothing is future, as if it existed not as yet. There is nothing there but, Is: there is not there, Was, and Will be; because what was, is now no longer: and what will be, is not as yet: but whatever is there, simply Is... Behold this great I am! [...] Who can understand that TO BE? Who can share it? Who can pant, aspire, presume that he may be there? Despair not, human frailty! "I am," He saith, "the God of Abraham, and the God of Isaac, and the God of Jacob." (→Augustine, 2012b, pp. 1338–1339)

This transcendent Nature of the eternal God has been revealed to man in the Book of Exodus (3, 14) and that revelation is contained in the name of God disclosed to Moses: "I am, Who I am." This mysterious name has always struck the philosophically minded as revelatory of the essential nature of Divine Being, that

can be expressed in philosophical terms: God is a necessary, and therefore perfect and immutable Being. The necessary being of the Divine Nature is where the God of philosophy and religion turns out to be one with the God of inner personal experience: in both these perspectives the ultimate and absolute existence of God is given as His most fundamental attribute. Immutability and eternity are the consequence of that fullness of being, in accordance with Plato's fundamental ontological assumption, that only that is a true being, which cannot cease to be. To highlight this attribute of God, Augustine calls Him substance (*substantia*) or essence (*essentia*), and he shows marked preference for the latter term, because the term "essence" derives from the Latin word *esse:* "to be," "to exist," and therefore better than other descriptions corresponds with the Divine Name revealed to Moses "I Am":

> He is, however, without doubt, a substance, or, if it be better so to call it, an essence, which the Greeks call ousia. For as wisdom is so called from being wise, and knowledge from knowing; so from being comes that which we call essence. And who is there that is, more than He who said to His servant Moses, "I am that I am;" and, "Thus shall thou say unto the children of Israel, He who is hath sent me unto you?" But other things that are called essences or substances admit of accidents, whereby a change, whether great or small, is produced in them. But there can be no accident of this kind in respect to God; and therefore He who is God is the only unchangeable substance or essence, to whom certainly being itself, whence comes the name of essence, most especially and most truly belongs. (→Augustine, 2012a, pp. 185–186; cf. →Clark, 2000, pp. 11–13)

William E. Mann distinguished among the essential attributes of God the property of "metaphysical independence or *aseity*," which is a complex property comprising three points: "(C) God depends on nothing for his existence. (D) God depends on nothing for his being what he is. (E) God is perfectly free" (→Mann, 2004, p. 36). I believe the divine attribute that

Augustine intended to emphasize by calling God essence is precisely the property of aseity as defined by Mann.

Another essential property of God identified by Mann is sovereignty, which is analyzed into the following points: (A) Everything that exists depends on God for its existence. (B) Every situation that is the case depends on God for its being the case (→Mann, 2004, p. 36). The idea of God's absolute power over the created Universe and everything contained within it, and, consequently, of the absolute dependence of all of the creation on God for both its existence and its essence was close and dear to St. Augustine: this idea captured the nature of the relationship obtaining between the Creator and the creature. God created all things alone and out of nothing: in producing them He had no help from any other being, nor had He any preexistent material out of which to form the objects of the created Universe. The conception of the *creatio ex nihilo* was a powerful response to the Manichaean dualism[20] and it provided metaphysical explanation for the contingency of the created Universe: the finite beings within the world are changeable, destructible, contingent in every respect: God imparts being to all existent things in the world, nevertheless they remain somehow marked with non-existence, which is responsible for their existential fragility.[21] Yet existing, being real itself is good, therefore every substance insofar as actually existent constitutes some good, *contra* Manichaeans.[22] Thus, wherever there is being, there is also good: God, as the source of all being, is also the Supreme Good; being and good are coextensive in Him, in fact they are one essence (as whatever is found in God, is one with His essence), against the Neoplatonist gradualism.[23] God, the highest Being and Good, is also the Truth, as Augustine argued in the second book of his *De libero arbitrio* (*On Free Choice of the Will*) in his so-called Noological argument.[24] In this dialogue, in

an effort to demonstrate God's existence, Augustine ascends the rising levels of reality as ordered by the most general ontological categories, which Augustine borrowed from Porphyry, namely being, life and thought and looks for that reality within the Universe, which perfectly instantiates these basic properties.[25] Undoubtedly, man, whose attribute is reason, is a reality, in which all three properties coincide, yet human beings are far from representing ontological perfection, as they are constantly subject to change, doubt and error. In his search for the perfect instance of these basic categories of reality, Augustine, in his dialogue with Evodius has to move further:

> But I ask you: If you find nothing above our reason except what is eternal and unchangeable, will you hesitate to say that *this* is God? For you know that physical objects are changeable through various states; and reason is surely proved to be itself changeable when at one time it strives to reach the truth and at another it does not, and at one time it reaches truth and at another it fails. Suppose that reason sees something eternal and unchangeable through itself, without recourse to any bodily organ—not through touch, taste, or smell; not through the ears or the eyes, nor through any sense inferior to itself. Reason must then admit itself to be inferior, and the eternal and unchangeable being [that it sees] to be its God. (→Augustine, 2010, pp. 42–43)

Just as He is the highest being, good and truth, God is also the highest beauty, and His power over all of the creation is but a reflection of His ineffable might.[26] Thus there is a plurality of attributes that human mind can ascribe to God, yet this plurality, far from destroying His simplicity, is only the expression of His one and simple Essence; each attribute being in fact wholly identical with it. Therefore in his *Confessions* Augustine may say in a prayer addressed to God: "you yourself are your own magnitude and your own beauty" (→Augustine, 1991, p. 69)

Augustine's theology, as may be gathered from what was said above, tends freely to affirm of God the qualities which are

pure perfections, however it would be wrong to conclude, that it is through and through positive. Augustine never tires of emphasizing the insufficiency of human understanding with respect to God and His Nature; he cautions against literally ascribing to God the features and properties found in nature, even in that summit of natural perfection which is human rational nature. Therefore, for St. Augustine too, affirmative theology ought to pass into negative theology and finally find its solution and justification in the way of eminence:

> [...] that so we may understand God, if we are ale, and as much as we are able, as good without quality, great without quantity, a creator though He lacks nothing, ruling but from no position, sustaining all things without 'having' them, in His wholeness everywhere, yet without place, eternal without time, making things that are changeable, without change of Himself, and without passion. Whoso thus things of God, although he cannot yet find out all ways what He is, yet piously takes heed, as much as he is able, to think nothing of Him that He is not. (→Augustine, 2012a, p. 185)

The surest way for a human being to approach God is the way of inner experience, as is made evident both by the Noological argument and the celebrated Augustinian *Si fallor—sum*.[27] God whom we approach is a personal God, who enters into a personal relationship with man.[28] Made in the image of God, the human soul bears a structural resemblance to Him in His Unity and Trinity: the three powers residing in the soul, identical with its essence and mutually interpenetrating, corresponding to the Persons of the Trinity.[29] It was the *libri platonicorum* that pushed Augustine along the way of the exploration of his inner experience and his mind, yet the more advanced stages of that journey he had to complete on his own, especially when the conception of personal God, who enters into loving dialogue with man, came into the focus in his considerations. In his most

inspired prayers Augustine immerses himself in the dialogue with God and is more certain of the reality of his Interlocutor than of his own, despite the infinite distance separating the

> Eternal truth and true love and beloved eternity: you are my God. To you I sigh “day and night” (Ps 42:2). When I first came to know you, you raised me up to make me see that what I saw is Being, and that I who saw am not yet Being. And you gave a schock to the weakness of my sight by the strong radiance of your rays, and I trembled with love and awe. [...] And you cried from far away: “Now, I am who I am” (Exod. 3:14). I heard in the way one hears within the heart, and all doubt left me. I would have found it easier to doubt whether I was myself alive than that there is no truth “understood from the things that are made” (Rom.1:20). (→Augustine, 1991, pp. 122–123)

Augustine’s conception of God as Being, Unity, Trinity and perfect Reality, immutable and eternal, who nevertheless stoops lovingly over the nothingness of His creatures, firstly in the act of creation and then in the act of redemption, was taken up and further developed in the Middle Ages by authors that represented the Augustinian tradition broadly conceived or, more specifically, the Augustinian Neoplatonism.[30]

The crowning of this way of thinking about God is perhaps found in St. Anselm, in his celebrated arguments for God’s existence in *Monologion* and *Proslogion*. These arguments are very close to one another as the notion of the *Ens quo maius cogitari nequit* is the fruit of the discussion contained in *Monologion*, and the assumptions and approach in both opuscles was summed up by Brian Leftow as “Anselm’s perfect-being theology.”[31] Starting by considering the created reality given in our experience, Anselm asks:

> It is, then, easy for someone to ask himself the following question: although the good things whose very great variety we perceive by the bodily senses and distinguish by the mind’s reason are so numerous, are we to believe that there is one thing through which all good things are

> good, or are some things good through something else? [...] Therefore, since it is certain that if compared with one another all good things are either equally or unequally good, it is necessary that all [good] things are good through something which is understood to be identical in [these] different goods—although at times, ostensibly, some things are said to be good through something else. [...] But who could doubt that that through which all [good] things are good is [itself] a great good? Therefore, it is good through itself, since every [good] thing is good thorough it. (→Anselm, 2000, pp. 7–8; cf. →Zieliński, 2012, pp. 137–154)

The ultimate, metaphysical cause of particular things in the world being good, albeit in different degrees, is the Good in itself, the perfect and supreme good, which, participated by the many good things, is the ground and source of their goodness. The same structure of reasoning is repeated in the other arguments in *Monologion*, with "greatness" (*Mon.* 2), dignity/perfection (*Mon.* 4) and being (*Mon.* 3) apparent in finite creatures, taken as the basic metaphysical perfections and providing the ground for the arguments. This kind of reasoning can always be made with the so-called "pure perfections" (*perfectiones purae*), as a starting point; pure perfections being the properties which contain no imperfection in their essence and whose possession is always better and more desirable than non-possession. All the arguments of this structure converge in attributing to the Supreme Nature the possessions of the pure perfections in the highest degree:

> For the Supreme Being must in no respect be said to be one of those things to which something that is not what they are is superior. And the Supreme Being (as reason teaches) must unqualifiedly be said to be each of those things to which whatever is not what they are is inferior. Therefore, necessarily, the Supreme Being is living, wise, powerful and all-powerful, true, just, blessed, eternal, and whatever similarly is in every respect better than its negation. (→Anselm, 2000, p. 26; cf. →Zieliński, 2012, p. 141)

The next step in Anselm's procedure is to show that all these properties are in fact identical with the very essence of the Supreme Nature (*Mon.* 16) and, for that reason, they do not destroy that Nature's simplicity:

> For example, when [this Nature] is said to be justice or being, [these predicates] signify the same thing as do the other [predicates], whether considered distinctly or all together. Thus, even as whatever is predicated essentially of the supreme Substance is on, so whatever the Supreme Substance is essentially it is in one way, in one respect. (→Anselm, 2000, pp. 28–29)

Some of the Supreme Nature's properties are personal in nature, they belong to that Essence insofar as It is a Person: the Highest Nature speaks a Word, which is identical (*Mon.* 29–30, 44) and co-eternal with It (*Mon.* 32), and this act of speaking is performed in Love (*Mon.* 49–59). Throughout most of *Monologion*, the concept of the Supreme Nature is built and gradually enriched by an abstract metaphysical speculation, only in the last chapter of that work it is clearly stated that the proper name for this considered Nature is God (*Mon.* 80).

Again, from superficial reading of Anselm's treatises one could easily get the impression that his theological discussion belongs wholly to the realm of affirmative or positive theology. Here again, however, as in the case of St. Augustine, a caution is in place: Anselm has very keen awareness of the limitation of human understanding, and this awareness is a characteristic feature of his rationalism. It is perfectly legitimate for human reason to try to penetrate on its own into the depth of the revealed truth on the condition that it stops at the threshold of the mystery contained in it. While with respect to at least some truths of the faith human reason can demonstrate with unshakable certainty, on the basis of necessary reasons, that things are so as the faith proclaims, it cannot comprehend the

reason why they should be so. Thus Anselm's positive theology turns out to be underlined by a sense of the mystery, which is the ultimate foundation of our comprehension of God:

> The hiddenness of so sublime a matter seems to me to surpass the entire acute gaze of the human intellect; and so, I think that I ought to refrain from attempting to explain how this doctrine is true. For I think that anyone who is investigating and incomprehensible doctrine should be content if by rational inference he comes to recognize that this doctrine is most certainly true. And [I think that] to these [doctrines] which are asserted [to be true] on the basis of compelling proofs (with no other rational consideration opposing) the certitude of faith ought no less to be assigned if because of [the doctrines'] naturally deep incomprehensibility they cannot be explained. (→Anselm, 2000, p. 74)

Nevertheless, Anselm is confident, that whatever knowledge of God we can gain by our careful reasoning, albeit unaided by faith, is true and valuable in itself. The ultimate ground for humanity having special cognitive access to God is the fact that man has been created in the image and likeness of his Maker, and through that likeness, which is in himself, man can acquire some comprehension of God. As Anselm remarks: "it is certain that one more closely approaches a knowledge of it through that which more closely approximates it in likeness" (→Anselm, 2000, p. 76).

The celebrated argument from *Proslogion*, intended as a piece of reasoning which is both very simple and irresistible in its compelling force, takes for its point of departure the concept of an absolutely perfect being: a "being, than which no greater can be thought."[32] The fool from the Psalms (14 (13), 1; 53 (52), 2), who denies the existence of God, is invoked as an illustration of the point that some kind of existence obviously belongs to God, minimally as a notion in the fool's mind. Yet to exist in somebody's mind (*in intellectu*) is not the same as to exist in reality (*in re*); Anselm makes this point clear by citing the

example of a painter, who first conceives the idea of his work in his mind and then executes it in reality. No doubt, Anselm affirms, to exist in reality is “greater” than merely to exist in somebody’s thought. These assumptions provide the basis for further argument:

> But surely that than which a greater cannot be thought cannot be only in the understanding. For if it were only in the understanding, it could be thought to exist also in reality—something which is greater [than existing only in the understanding]. Therefore, if that than which a greater cannot be thought were only in the understanding, then that than which a greater *cannot* be thought would be that than which a greater *can be* thought! But surely this [conclusion] is impossible. (→Anselm, 2000, pp. 93–94; cf. →Davis, 2004, pp. 160f.)

Of crucial importance for this argument is the fact that human mind is able to form the concept of God as an absolutely perfect being; this is an illustration of the cognitive power of human intellect (probably grounded in the likeness human intellect bears to God’s intellect). Having, in his own opinion at least, established that God, as an absolutely perfect being, must exist not only in somebody’s mind, but also in reality, Anselm can go a step further and affirm, that if God is “that than which no greater can be thought,” we must conclude that God is a reality which cannot be thought not to exist:

> Therefore, O Lord my God, You exist so truly that You cannot even be thought not to exist. And this is rightly the case. For if any mind could think of something better than You, the creature would rise above the Creator and would sit in judgement over the Creator—something which is utterly absurd. Indeed, except for You alone, whatever else exists can be thought not to exist. Therefore, You alone exist most truly of all and thus most greatly of all. (→Anselm, 2000, p. 94)

At this point, Anselm’s line of thinking is taken up, continued and originally developed by St. Bonaventure. He agrees with St.

Anselm, that God cannot be thought of as nonexistent, yet for him this is a consequence and an indication of the nature of all intellectual cognition. In intellectual cognition as such, grasping the absolute, pure existence, which is God's existence, is a necessary prerequisite of any intellectual cognition whatever: the finite and particular being can only be known as related to pure and absolute being, which is the foundation and measure of everything conditioned and circumscribed. The conceptual grasp on God as pure being is, in St. Bonaventure's eyes, the *primum cognitum*, that is the notion, that is present in all our concepts as their foundation and is the necessary *a priori* condition of all human intellectual knowledge; it is thus the first notion in the human intellect, first not in the order of time, but in the order of the essential structure of all our concepts. In this way a confused and indistinct concept of God is always present in our thinking, even if we are unaware of it. A clear and distinct notion of God, as well as an insight into His transcendent beauty, we may only acquire at the end of a long journey of intellectual cognition, the "itinerary of the mind towards God." This journey starts with the things of our sense experience, which are "traces" (*vestigia*) left by God, and their beauty, which is a glow of God's beauty; it then continues into the inner self of a human being, where the three mutually interpenetrating faculties, which constitute the very essence of the soul, form an image of the Trinity of Divine Persons in God. Finally, our journey takes us beyond and above ourselves to the contemplation God's Being as It is in Itself and knowing Him by His primary name, which is "*He who is*."[33] Thus contemplation of God reveals that He is Being itself:

> Let him who wishes to contemplate this invisible things of God in the unity of his essence fix his attention first on being itself, and let him see that being itself is so certain in itself that it cannot be thought not to be. For pure being occurs only in full fight from nonbeing just as nothing is in

> full flight from being. […] Since nonbeing is the privation of being, it does not come into our understanding except through being; but being does not come to us through something else because everything which is understood is understood as nonbeing or being in potency or being in act. If, therefore, nonbeing can be understood only through being and being in potency only through being in act, and if being signifies the pure act of being, the being is what first comes into the intellect and this being is pure act. (→Bonaventure, 1978, p. 96)

The typical procedure in the theological effort to find out what the “plenitude of God’s being” means in the trend of medieval theology that is represented by St. Anselm and St. Bonaventure is to start with consideration of some perfections of creatures, which, being God’s own work, represent in a certain way His transcendent perfections. The perfections that above all were taken into consideration were: being and unity, unity and good, unity and Trinity, immutability and eternity, aseity and sovereignty. Different authors proposed slightly differing “catalogs” of ontological perfections, which were then complemented with personal perfections, since in Christianity God is personal and stands in a personal relationship with man. Having been made in the image and likeness of God, the human inner self is the privileged place in which to discover and approach God.

However, it would be wrong to conclude that the way of theological considerations as adopted by St. Anselm and St. Bonaventure is pure positive theology without any admixture of alternative approaches to theological matters. All the noteworthy proponents of this way in medieval theology show awareness that the perfections one discovers in creatures and which are attributable to God as the supreme cause of both creatures and their perfections, cannot be transferred to God without profound modification of their sense. This is because all qualities ascribable in common to God and creatures are realized differently in either case. In creatures they are properties of

finite beings and finitude is their inherent characteristic, by contrast, in God they are one with God's Nature, which is infinite and absolutely simple. Thus all perfections in God are identical with the Divine Essence and with one another, the fact that clearly exceeds the power of human comprehension. Confronted with this mystery, the best thing a theologian can do is bow to it, accept his/her limitations, and nevertheless continue in his/her attempts to gain as much comprehension of God as is possible for human intellect, in order come closer to the Divine Plenitude.

2.3 God as *Nihil per Excellentiam*

The discussion carried out so far concerns one distinctive orientation in the theological endeavor to discover and articulate the nature of God, but there was also another current in the late antique and medieval theological thought, which has to be discussed in turn to make the picture the theology of that period complete. The proponents of this current laid special emphasis on and took as their point of departure the absolute transcendence of the Divine Nature, which so much surpasses in greatness and perfection anything we can conceive of, that, according to them, the most appropriate name we can attribute to God is that of Nothingness. These theologians referred to the widely conceived tradition of Dionysian Neoplatonism, which drew its inspiration from the writings of Pseudo-Dionysius the Areopagite. This tradition originated in the Greek Neoplatonist milieu of late antiquity, where the author of the *Corpus Dionysiacum* lived and was transplanted into the Latin West first by the translation of this body of writings into Latin by Hilduin, the abbot of Saint Denis, shortly followed by that prepared by John Scotus Eriugena. Eriugena's own philosophico-theological outlook was essentially formed by Christian Greek Neoplatonism, which he imbibed from the writings of Pseudo-

Dionysius, Maximus the Confessor and Gregory of Nyssa. One of the distinctive features of this Neoplatonist approach was a tendency to merge together in the efforts to comprehend reality subjective (cognitive) and objective (natural, causal) categories.[34] John Scotus Eriugena defines reality in cognitive terms, and his masterwork, *Periphyseon*, written in the form of a dialogue between the Master (*Nutritor*) and Disciple (*Alumnus*), begins with the following characteristic statement:

> As I frequently ponder and, so far as my talents allow, ever more carefully investigate the fact that the first and fundamental division of all things which either can be grasped by the mind or lie beyond its grasp is into those that are and those that are not, there comes to mind as a general term for them all what in Greek is called Φυσις and in Latin Natura. (→Eriugena, 1987, p. 25)

Nature as a general category referring to the principle of all phenomena is familiar enough, yet in Eriugena this term acquires a novel, even more comprehensive meaning: nature comprises both that-which-is and that-which-is-not; the division into being (that which is) and non-being (that-which-is-not) is the most fundamental division of the whole of reality. To count non-being as part of Nature may appear surprising, therefore we are offered an explanation as to the meaning of that-which-is-not: there are no fewer than five ways of understanding the opposition of that-which-is and that-which-is-not (→Eriugena, 1987, p. 26–29). The fundamental meanings of being and non-being seem to be, respectively, that which can somehow be comprehended by either senses or intellect, and that which eludes all human comprehension. In this way defined category of non-being includes, according to John the Scot, matter (which cannot be immediately known either by senses or by intellect), God and the essences of created things:

> For just as God as He is in Himself beyond every creature is comprehended by no intellect, so is He equally incomprehensible when considered in the innermost depths of the creature which was made by Him and which exists in Him; while whatsoever in every creature is either perceived by the bodily sense or contemplated by the intellect is merely some accident to each creature's essence which, as has been sad, by itself is incomprehensible, but which, either by quality or by quantity or by form or by matter or by some difference or by place or by time, is known not as to what it is but as to that it is. (→ Eriugena, 1987, p. 27)

As is made evident in the quoted passage, Eriugena's approach in a way reverses the method of proceeding of an affirmative theologian: the properties found in the experienced natural objects do not tell us anything about *what* God is or *how* He is, but only *that* He is, whereas the incomprehensible essences of natural things are the "witnesses" to the incomprehensibility of the Supreme Essence which is God's.

However, God's incomprehensibility, His "nothingness," differs crucially from the incomprehensibility of matter; to make this evident John Scotus applies to God the description "Nihil per excellentiam" (nothingness through excellence), whereas to matter he reserves the description "nihil per privationem" (nothingness because of privation). Everything, which can be referred to as "something" (*quid*), falls within the domain of Aristotle's ten categories, which we use in our ordinary thinking and by means of which we define whatever can be defined.[35] Yet obviously, no one of these categories can be applied to God in its usual meaning, since he transcends all cognitive comprehension and can be grasped by no definition, as, according to the meaning of the Latin word "deffinire," defining means finding limits of or imposing them to a thing. Clearly, God has no boundaries to find and articulate, nor can any be imposed on Him, neither has He any accidents, so He cannot be characterized as a definite substance (*diffinita substantia*), described by its accidents. The fundamental property of God is

infinity, which makes Him unreachable to any finite, defining thinking.[36] For this reason the name “Nothingness” better characterizes the fullness of the Divine *plus-quam esse* than any other name available in human language, even though its application to God may appear shocking to some:

> I would not easily concede that the divine superessentiality was nothing [or could be called by so privative a name]. For although it is said by the theologians not to be, they do not mean that it is nothing but that it is more than being. For how could the Cause of all things that are be understood to be no essence when all things that are show that it truly is —although by no demonstration of the things thar are is it understood what it is? (→Eriugena, 1987, p. 251; cf. →Beierwaltes, 1977, pp. 146–147)

This infinite fullness of the Divine Nothingness, absent from which are all essences or substances, emerges from its inaccessible transcendence in the creative act by which It establishes created reality as Its own theophany, the cognitively available manifestation of the unattainable Divine Nature. Thus to be the theophany of invisible God is what defines the essential meaning of all of created reality: whatever is visible or otherwise comprehensible, is but a sign, a symbol, which refers the viewer to the mysterious, hidden source, and in this way makes that source somehow cognitively accessible.

The created world is thus a symbol, and the nature of the symbol is dialectical, including the opposition of revealing and concealment, affirmation and negation. Eriugena emphasizes this dialectical nature of the theophany which is the comprehensible reality in the following memorable passage:

> For everything that is understood and sensed is nothing else but the apparition of what is not apparent, the manifestation of the hidden, the affirmation of the negated, the comprehension of the incomprehensible, [the utterance of the unutterable, the access to the inaccessible,] the understanding of the unintelligible, the body of the bodiless, the essence

> of the superessential, the form of the formless, the measure of the measureless, the number of the unnumbered, the weight of the weightless, the materialization of the spiritual, the visibility of the invisible, the place of that which is in no place, the time of the timeless, the definition of the infinite, the circumscription of the uncircumscribed, and on the other things which are both considered and perceived by the intellect alone and cannot be retained within the recesses of memory and which escape the sharpness of the mind. (→Eriugena, 1987, p. 250)

One can say that in Eriugena the traditional threefold way of speculative ascent towards God, the way of affirmation, negation and eminence has been reversed to a degree: here the inaccessible transcendence of God is the last word of the theological enquiry, despite some positive moments made available by the theophanies. Whatever God reveals of Himself in the world, however, is by far insufficient to fill the gap separating the Creator and His creatures, and man, being the “reader” of the theophanies, is made clearly aware of the unbridgeable nature of that gap. Somewhat paradoxically, however, the nature of that divine concealment is made familiar to us by the fact that our own nature as selves is hidden before our intellect and is only revealed through our actions and utterances, we ourselves participate in a way in the divine hiddenness. The following passage makes it clear:

> And we can acquire a hint of this from examples from our own nature. For our own intellect too, although in itself it is invisible and incomprehensible, yet becomes [both] manifest and comprehensible by certain signs when it is materialized in sounds and letters and also indications as though in sorts of bodies; and while it becomes externally apparent in this way it still remains internally invisible, *and* while it breaks out into various figures comprehensible to the senses it never abandons the incomprehensible state of its nature. (→Eriugena, 1987, p. 250)

Man, having been created in God’s image, participates in His incomprehensibility and by reflecting upon himself and his own

manner of being, acting and knowing, he can gain some insight into the way the Divine Nothingness reveals itself in Its creative actions. Man gives name to all things made available to him in experience, which is tantamount to fixing their essences, he cannot define himself or fix his own nature, which remains hidden in God's Mind.[37] Thus, in the depth of his hidden nature, man shares in the mystery and inaccessibility of his Source, and imitates his Maker in producing his own signs, which are the manifestation and revelation of his own non-apparent essence. By knowing his own way of existing in concealment and emerging from hiddenness through his acts, man constitutes himself as the designated reader of divine signs and interpreter of God's theophanies.[38]

A very close conception of God as Nothingness appears in the thought of Master Eckhart. Undoubtedly, in the case of Eckhart, as in that of Eriugena, the influence of Pseudo-Dionysius was decisive, though Eriugena's own speculative contribution may have been at work as well.[39]

In *Sermon* 96 Eckhart retraces the traditional scheme of the threefold way of theological progress towards God and gives to it a novel interpretation. According to him, the affirmative way in theology reveals more of our own nature and way of knowing than it does of God. Every description attributed to God as the cause of perfections found in creatures admits of different degrees of perfection and intensity, whereas no gradation of any kind is conceivable with respect to God, therefore none of the attributes ascribed to God can be thought really to grasp and reveal His true essence. This being so, it is more fitting to describe God in a negative way than by affirming of Him finite perfections found in creatures. He is better said to be Nothingness than Being:

> God is nameless because none can say or understand anything about Him. Concerning this a pagan master says that what we understand or declare about the first cause is more what we ourselves are than what the first cause is, because it is above all speech or understanding. If I now say God is good, it is not true; rather, I am good, God is not good. I will go further and say I am better than God; for what is good can become better, and what can become better can become best of all. Now God is not good, therefore He cannot become better. And since He cannot become better, therefore He cannot become best; for these three, good, better, and best, are remote from God, since He is above them all. Thus, too, if I say God is wise, it is not true: I am wiser than He. So too if I say God is a being, that is not true: He is a transcendent being and superessential nothingness. (Eckhart, 1979, p. 463)

The founding principle of these paradoxical reflections on God is Eckhart's interpretation of God in terms of the primacy of intellect over being, which he defended in his first Parisian Question (*Questio parisiensis*), propounded by him as Master of Theology at Paris University in 1303–1305.[40] In this work he proposed to reverse the traditional order of being and intellect in God and to put knowing in God before His being: "knowing is the proper and highest definition of God, and is therefore the basis of this Being" (→Enders, 2013, p. 361). Paradoxical as this thesis may appear, it had a solid foundation in the Holy Scriptures, more specifically in the Prologue to St. John's Gospel, where God is said to be the Word, that is the Intellect, which is uncreated and through which all else was created. That being comes after the Intellect in the order of reality can be gathered from *Liber de causis*—where it is said that being is the first of the created things. Moreover, being is always something definite, that is finite, whereas intellect, being indefinite in itself, is in principle infinite. Therefore we should assume that *intelligere* in God comes before *esse*, and is the highest degree of perfection (→Enders, 2013, p. 362). We are not able to represent to ourselves the nature of that divine knowing, as our own knowledge is always conditioned by some object already in

existence, and the existence itself is the result of God's creative knowing (→Enders, 2013, p. 363). If God is the cause of all Being, He Himself cannot be a Being. However, Markus Enders observes: "Eckhart does however mention one significant qualification: he would not object if someone wanted to call knowing a Being, but Being is ascribed to God only via knowing" (→Enders, 2013, p. 363).

It is interesting to see Eckhart apply the cognitive category of understanding in his attempt to explain the relations between the persons of the Trinity, as this category is better suited to grasp the paradox of Unity and Trinity:

> Now note the way of the eternal birth. Inasmuch as Personal understanding pertains to the unity of nature, it is one with the same understanding whereby the Father understands Himself in His characteristic nature. Were this not so, there would have to be two intelligible essences, but there are not: there is but one intelligible essence wherein the Father sees Himself in His characteristic nature. The object of the Father's thought is the eternal Word. Where this eternal Word cleaves to the natural understanding of the Father it is none other than the Father-nature. Where this same Word is directed on to itself, it is separated as regards Person and is yet one simple essence in the divine nature. (Eckhart, 1979, pp. 441–442)

Man is separated from God by an abyss, which accounts for the fact that God is hidden from man and makes man know Himself only through signs revealing His Presence.[41] Starting with these signs, man can try to gain some knowledge of the Cause by studying Its effects, however, this way may yield only imperfect results and never will it make us approach the very Plenitude of divine reality. A better and really effective way is that of "clothing oneself with the Lord Jesus Christ" (Romans 13, 14), that is going into one's own inner self, where the image and similitude of God Himself is found. For the nothingness that resides deep inside

human soul is in a special relationship with the Divine Nothingness:

> But the soul he has made not merely like the image in Himself, or like anything proceeding from that is predicated of Him, but He has made her like *Himself*, in fact like everything that He is—like His nature, His essence, and His emanating-immanent activity, and like the ground wherein He subsists in Himself, where He ever bears His only-begotten Son and where the Holy Ghost blossoms forth: it is like this outflowing, indwelling work that God has formed the soul. [...] There is something in the soul in which God is bare, and the masters say this is nameless, and has no name of its own. It is, and yet has no being of its own, for it neither this nor that nor here nor there [...]. And in *that* he means you should enter into God, into blessedness: for in here the soul gets her whole life and being, and from this she sucks her life and being, for *this* is wholly in God and the rest is outside, and therefore the soul is always in God through this, unless she turns it outward or lets it be extinguished within her. (Eckhart, 1979, pp. 448–449)

All this became possible because Christ assumed human nature: “For between your human nature and his there is no difference: it is one, for it is in Christ what it is in you” (Eckhart, 1979, p. 450). In this way, man can reach perfection (II), of which God is the model, and this happened when the fullness of time had come and the Son was sent (Ga 4:4): “*That* is the *fullness of time*, and *then* I am in a right state, and then I am truly the only Son and Christ” (Eckhart, 1979, p. 450).

Following the way outlined by Pseudo-Dionysius, both Eriugena and Master Eckhart laid very special emphasis on the radical ontological and epistemic transcendence of God. The Divine Nature is so much “other” than anything we can represent to ourselves that the name of Nothingness is more suitable to it than that of Being. However, God does not remain entirely hidden from human comprehension, but reveals Himself through signs which are the created beings, and in particular through His incarnate Word, Jesus Christ.

2.4 The Infinity of the Divine Being

Aquinas's treatment of God and His Nature has usually been regarded as an outstanding example of positive theology, but it seems that this description is not wholly objective and calls for some qualification. Recently, a number of studies have appeared which represent his thought as a kind of negative theology, or even as a form of agnosticism in the question of the possibility of knowing God by our human means.[42] It is common knowledge that Aquinas thought God's being to be absolutely self-evident in itself, but not to be so to ourselves, who can only attain the knowledge *that* He is, but cannot know by the natural means *what* He is, that is, what is His essence. Still, there is a distinctive tendency towards qualified affirmative statements about God in Aquinas. As J. Wippel observed

> this position does not prevent Aquinas form acknowledging that some of the names we apply to God are predicated of him substantially (ST Ia.13.2) and properly (13.3). this means that as regards what such names (those of pure perfections) signify, they are properly said of God. But as regards the way in which they signify (*modus significandi*), they are not properly said of God; they retain a mode of signifying that pertains to creatures. (Wippel, 1933, p. 116)

Without attempting to enter into the discussion of the way how positive claims about God are to be understood in Aquinas, I would like to point to what appears to me the ground for so different and often mutually contradictory judgments about Aquinas's theological theory; this ground seems to me to be his conception of God's infinity.

Norman Kretzmann in his study on Part I of *Summa Contra Gentiles* has shown how according to Thomas we can arrive at certain positive knowledge of God as the cause of the Universe beginning with an analysis of various perfections of created

beings.[43] While contemplating God as an absolutely perfect being, we should first of all take into consideration such of His attributes as: Goodness, uniqueness and infinity.

> As for positive results, I suppose God's goodness is as distinctively positive as any result could be. God's uniqueness, however, could readily be construed as the impossibility of there being more than one entity that fits the emerging conception of God, and for logical purposes it is more conveniently construed in that way, as we'll see. What's more, infinity, the third of these three intermediate topics, doesn't merely seem unmistakably negative; it was expressly identified in the preceding methodological chapters as involving the ascription of a perfection to God 'through negation, as when we call God … infinite' (30.278). […]—'but not in such a way that "infinite" is taken privatively, as it is in connection with extensional or numerical quantity (43.358). Not *privatively*, yet 'in connection with God "infinite" is understood only *negatively*, since there is no limit or end to his perfection. (→Kretzmann, 1997, p. 159)

Kretzmann goes on to highlight the way Thomas shifts the emphasis in his consideration of divine infinity from the quantitative aspect—infinite greatness and extension—towards the dynamic aspect, with the divine infinity understood above all as infinite power, as the source of action, especially creative action which consists in calling reality into being out of nothing.[44] This strand of Aquinas's discussion will reappear in the conception of Nicholas of Cusa.

The divine infinity understood as absence of all boundaries with respect to the Divine Perfection and Divine Power is the consequence of God's unique way of existence, which consists in being *ipsum esse subsistens* (the very being existing on its own), in which essence is identical with existence and for this reason imposes no limitation upon its own act of existence.[45] Thomas states this in Part I of his *Summa Theologiae*:

> The fact that the being of God is self-subsisting, not received in any other, and is thus called infinite, shows him to be distinguished from all other

> beings, and all others to be apart from Him. (Aquinas, 1947, p. 40; cf. →Wippel, 2000, p. 573)

The divine infinity conceived positively as unbounded intensity of endless being which is one with absolute simplicity, rather than privatively in terms of lack of boundaries, was taken, in opposition to Aquinas, as the point of departure by John Duns Scotus.

As Richard Cross has argued, the theology of Duns Scotus appears to be a continuation of the "perfect-being theology" as formulated above all by St. Anselm. For Scotus it is evident that we can ascribe to God "pure perfections" (*perfectiones purae*), that is the properties, which it is unconditionally better to possess than not possess. Following Anselm, Scotus compiles a list of such perfections, which includes, among others, such qualities as: incorporeal, living, wise, omnipotent, true, eternal. As Cross states:

> In addition to these attributes, Scotus argues that God, like everything else, will exhibit the transcendental attribute of *being*, and those attributes that are straight-forwardly coextensive with being—namely, unity, truth, and goodness (in the sense of the desirability). God, as we shall see, has these attributes in an infinite degree; and, as Scotus notes, the crucial feature of all these perfections is that they do not in themselves entail any kind of limit. Each perfection is in this sense compatible with its being, exemplified to an infinite degree of some being. (→Cross, 1999, p. 32; cf. King, 200, p. 26)

This last formula—an infinite degree of some being—seems appropriate to Scotus's conception of infinity. According to Cross, Scotus moves away from Aquinas's relational conception of infinity understood in terms of mutually complementary and at the same time delimiting components of being.[46] Instead, he takes for his point of departure the Aristotelian concept of potential infinity[47] which he radically transforms in a "thought

experiment” of a kind to arrive at the concept of actual infinity, which is a whole actually existent, whose all parts actually co-exist and which is a true perfection, which lacks nothing. To be sure, Aristotle himself believed actual infinity to be impossible, yet Scotus seriously considers the possibility of the existence of an actually infinite reality.[48] He applies the formed conception of actual infinity to the notion of being: is an actually infinite being possible? Such a being will have the feature of existing “wholly” at one (timeless) instant, without any division into temporal or spatial parts, and it will have the feature of perfection in the absolute sense and not merely relative perfection. This conception of actual infinity forms Scotus’s original contribution to the “perfect-being theology”:

> But if you say that “highest good” or “highest being” expresses an intrinsic mode of being and includes other concepts virtually, I reply that if “highest” is taken in a comparative sense, then it includes a relation to something extrinsic to being, whereas “infinite” is an absolute [or nonrelative] concept. But if “highest” is understood in an absolute sense—that is, as meaning that the very nature of the thing is such that it cannot be exceeded—then this perfection is conceived even more expressly in the notion of an infinite being, because “highest good” as such does not indicate whether it is infinite or finite. (Duns Scotus, *Ordinatio* I,d. 3, nos. 58–60, in Frank and →Wolter, 1995, pp. 117–119)

In Scotus’s metaphysics finitude and infinity form a pair of “disjunctive attributes of being” (*passions disiunctae entis*), they are mutually exclusive (no being is both finite and infinite), and jointly extend to the whole realm of being (every being is either finite or infinite). In fact, infinity is an attribute of only one being: God, while all beings apart from God are finite.[49] Infinity is not a property like others and on a par with others: it is a special feature which consists in the internal intensity of being, it is an “intrinsic mode” *(modus intrinsecus)* of a being. In Scotus’s own words:

> From this it follows that intensive infinity is not related to the being that is said to be infinite as a kind of attribute that accrues to it extrinsically. Neither should it be thought of as an attribute or property of being in the way "good" and "true" are. Indeed, intensive infinity expresses an intrinsic mode of that entity. It is so intrinsic that if we abstract from all its properties or quasi-properties, we have still not excluded infinity, but it remains integrally included in that one single entity itself. [...] Therefore, the intrinsic mode of anything intensively infinite is infinity itself, which intrinsically expresses a being or essence which lacks nothing and which exceeds everything finite beyond any determinable degree. (Duns Scotus, 1975, p. 11–112, quoted after Frank and →Wolter, 1995, p. 154)

The theory of intrinsic modes of being, a distinctive feature of Scotus's metaphysics, is of crucial importance for his theology, given his rejection of the Thomist theory of transcendental analogy and acceptance instead of the theory of transcendental univocity. For Scotus, the "convertible transcendentals" (*transcendentalia convertibilia*), such as "being," "one," "good," "true" are univocally (in exactly the same meaning) predicated of God and creatures. Univocity of a term means for Scotus, that if we predicate and negate the same term of the same entity, the result is contradiction.[50] The advantage of the theory of transcendental univocity consisted in Scotus's eyes in that it allowed unproblematic passage from creatures to God in the predication of transcendental properties. The drawback, however, of that theory was that it suggested that the same kind of reality belonged to God and to creatures, or that the nature of being in God and creatures was the same, which would destroy God's transcendence. To obviate this difficulty, Scotus introduced the theory of intrinsic modes, or ways in which transcendental perfections are realized in particular beings. The abstract term "being," for instance, was univocally predicable of God and creatures, but it did not capture the specific way being was realized in a given case, only that term as taken jointly with the appropriate intrinsic mode (e. g. "infinite being," or "finite

being") expressed the concrete way in which a given entity existed. Thus the simple notion of being is not proper to the Divine Being, the concept adequately and specifically applicable to God is that of "infinite being," the most perfect and at the same time the simplest of all concepts that we can apply to God in our natural cognition.[51] Scotus's conception of God in terms of actual infinity accounts for the complexity of his celebrated argument for God's existence, which, unlike Aquinas's relatively simple proofs, consists of a number of steps. This has to be so, because for him it is not enough to prove that there is a first being in a certain order of reality, not even that there exists the first in all identifiable orders; to make an argument for God's existence complete, we have to demonstrate that this first being is actually infinite:

> Regarding the existence of God, I propose three questions. First, in the world of beings, is there some being that is first in an unqualified sense? Second, could several different sorts of being possess such primacy? Third, is some being actually infinite in an unqualified sense? (Duns Scotus, *Reportatio* I A, in Frank and →Wolter, 1995, p. 41)

Nicholas of Cusa, who lived in the context of the advancing humanist thought and intense criticism of traditional scholasticism, was another theologian of importance who was deeply concerned with the problem of what we could know about God, what we can predicate of Him and how must this predication be understood. Flourishing at the end of the Middle Ages, he was able to echo the conceptions and assumptions of many of his medieval predecessors, and we do, indeed, find in him references to some of the thinkers discussed above. He shared Eriugena's and Eckhart's view that any positive theological discourse does not properly apply to God, therefore the best name we can give Him in our epistemological condition is that of Nothingness. In the well-known passage from his *De*

Deo abscondito, written in the form of a dialogue between Pagan and Christian, the last named says:

> God is beyond nothing and something, for nothing [or not-being] obeys Him in order that something be made. And this is His omnipotence, by which power He excels all that which either is or is not, so that that-which-is-not obeys Him, even as does that-which-is. For He makes not-being pass into being and makes being pass into not being. Therefore, He is not any of the things that are subordinate to Him and which His omnipotence precedes. And, accordingly, He cannot be said to be *this* rather than *that*, since all things are from Him. (→Nicholas of Cusa, 1994, p. 303)

Already in his first philosophical work, the celebrated *De docta ignorantia* (*On Learned Ignorance*), he affirms that positive descriptions of God, formed with predicates signifying attributes found in and properly applicable to creatures, only in a slight degree correspond to God and for this reason they appear to be of no value.[52] This categorical statement is all the more significant as it comes after the often referred to passage, in which Cusanus, starting with consideration of geometrical figures, shows the way these figures, when infinitely extended, help us to see how contrary properties merge into one in infinity, thus introducing his famous conception of God as “coincidence of opposites,” in which opposite descriptions, such as “Maximum” and “Minimum” are seen to refer to one and the same reality. This coincidence of contraries is called “Paradise” in the treatise *De visione Dei*:

> O Lord God, Helper of those who seek You, I see You in the garden of Paradise, and I do not know what I see, because I see no visible things. I know only the following: viz., that I know that I do not know—and never can know—what I see. Moreover, I do not know how to name You, because I do not know what You are. […] For You are separated by a very high wall from all these [modes of apprehending]. For [this] wall separates from You whatever can be spoken of or thought of, because You are free from all the things that can be captured by any concept.

> Hence, when I am very highly elevated, I see that You are Infinity. Consequently, You are not approachable, not comprehensible, not nameable, not manifold, and not visible. And so, one who ascends unto You must ascend beyond every limit and every end and [everything] bounded. (→Nicholas of Cusa, 1985a, pp. 703–704)

In Cusanus we find a number of statements containing negative descriptions of God. After the leading statement "God is infinity," follow a number of negative statements: He is inaccessible, incomprehensible, unknowable, non-complex, invisible, without any boundaries. Does Cusanus's theology, however, consist of negations alone? Is there a possibility to move beyond the negative descriptions and affirm something positive about God?[53] The immediate apprehension of God appear to be something positive: "I see that you are Infinity"; that is, beyond all conceptual approaches to God, there is some immediate experience of Him as an object, which is somehow given to me, and, therefore, must have some positive content, be it conceptually ungraspable and linguistically inexpressible, which is *present* to me, and I positively apprehend this presence. In mystical experience, the experiencer is aware of "discovering" its object, which reveals itself, and not of producing it. William Alston in his book on cognitive aspects of mystical experience emphasizes the directness and the moment of givenness in that experience, and he concludes that we may call it a "direct, extra-sensory experience of God" and range it "under a generic concept of perception."[54] My mental vision "discovers" the ineffable God, who sees me and by seeing me establishes me in being. Compare Nicholas in *De visione Dei*:

> O Lord, when You look upon me with an eye of graciousness what is Your seeing, other than Your being seen by me? In seeing me, You who are *deus absconditus* give Yourself to be seen by me. No one can see You except insofar as You grant that You be seen. To see You is not other than that You see the one who sees You. (→Nicholas of Cusa, 1985a, p. 686)

The object of mystical experience appears as present, and yet it cannot be apprehended as a *something*, that is as an object that can be defined, cognitively grasped as having some boundaries, as conceptually comprehensible and possessing definite attributes. Whatever properties we can predicate of God, are identical with Himself, and yet, being the very Infinity, He transcends any perfections we could possibly attribute to him:

> Therefore, as regards whoever sets out to see Your Face: as long as he conceives of something, he is far removed from Your Face. For every concept of face is less than Your Face, O Lord; and all beauty that can be conceived is less than the beauty of Your Face. All faces *have* beauty; but they are not beauty itself. But Your Face, O Lord, has beauty, and this having is being. (→Nicholas of Cusa, 1985a, p. 689)

This Infinity, which thus comes as an object into our mental vision, exceeds all created and conceivable orders, being the Absolute Power, which is the source of the existence of every created thing. Therefore, It is present in every entity within reality, and yet It is always transcendent with respect to all things which are not Itself: “since You, O Lord, are the Power, or Beginning, from which all things derive and since Your Face is the Power and Beginning from which all faces are that which they are” (→Nicholas of Cusa, 1985a, p. 690).

This Absolute Power, which is the source of being of every reality, exists in Itself in absolute unity and simplicity. It is the very actual being in itself (*actu esse*), it is also in Itself all that, which can exist (*posse esse*). This unity of absolute actuality with absolute possibility is what sets this Infinite Power apart from the possibility inherent in the reality of matter. For this Infinity is the actuality of all possibility. In his later writings Cusanus will attempt to capture this transcendent unity of actuality and possibility by coining the new name of God, the name of *Possest —The possible—is*:

> You, then, O God, who are Infinity itself, are the one God Himself, in whom I see that all possibility-to-be exists actually. For possibility which is free from all possibility that is contracted to prime matter or to any passive potency whatsoever is Absolute Being. For whatever exists in Infinite Being is most simple Infinite Being. Thus, in Infinite Being the possibility-to-be-all-things is Infinite Being itself. Likewise, in Infinite Being actually-being-all-things is Infinite Being itself. Therefore, in You my God absolute possibility-to-be and absolute actual being are only You my Infinite God. (→Nicholas of Cusa, 1985a, pp. 709–710)

This Divine Infinite Power is the simplest and the most unified active principle, therefore it is the most effective. Being so, it is in fact omnipotent and infinite in a sense wholly opposed to the ancient privative conception of infinity. Contrary to that privative conception, God's infinite power is the expression of perfection, integrity and simplicity:

> Therefore, a power that is so unified that it cannot be more greatly unified is infinite and omnipotent. O God, You are omnipotent, because You are Absolute Simplicity, which is Absolute Infinity. (→Nicholas of Cusa, 1985a, p. 709)

The actual infinity of God contains all things in a unity lacking all division and internal separation, therefore it admits of no change, no gradation, no temporal movement. Therefore Actual Infinity is eternity conceived, in the manner of Boethius, as an eternal "now," total actuality without past and future. Such eternity is the necessary condition for all realities subject to the flow of time and who are "before" and "after" and which come into being in succession:

> For Your Concept is most simple eternity itself. Now, *posterior* to most simple eternity no thing can possibly be made. Therefore, infinite duration, which is eternity itself, encompasses all succession. Therefore, everything which appears to us in a succession is not at all posterior to Your Concept, which is eternity. (→Nicholas of Cusa, 1985a, p. 699)

God’s Infinity is the infinite intensity of being, the way of being in the absolute degree, the kind of reality that generates infinite power, capable of positing in being whatever can possibly be. This Infinity can be neither greater nor lesser than It is the absolute boundary for all things, their measure and limit:

> Because, O Lord, You are the End that delimits all things, You are an End of which there is no end; and thus You are an End without an end—i.e., an Infinite End. This [fact] escapes all reasoning, for it implies a contradiction. Therefore, when I assert the existence of the Infinite, I admit that darkness is light, that ignorance is knowledge, and that the impossible is necessary. And because we admit that there is an end of the finite, necessarily we admit the Infinite—i.e., the Final End, or End without an end. (→Nicholas of Cusa, 1985a, p. 705)

Once an intellect became aware of that Infinity as existent and present here and now, it cannot find satisfaction in anything but that, which lacks any boundaries whatsoever. It realizes, that only infinite object is appropriate to the capacity of the intellect, and only a union with such an object can be the source of the highest felicity (*felicitas*).

The closest kind of union is that of filiation, that is the relationship obtaining between father and son. Since the Actual Infinity is a person, man enters in a personal relationship with It and forms a union with It like that of a son with his Father, following the most perfect example set to us by Jesus Christ. This experience of the presence of the Divine Infinity is a foretaste of the future elevation beyond and above the nature of man, towards the source of all things:

> Above all, You have given me Jesus as Teacher and Way and Life and Truth, so that nothing at all can be lacking to me. You comfort me with Your Holy Spirit, through whom You inspire holy desires and the decisions of life. Through a foretaste of the sweetness of a glorious life You draw [me unto Yourself] I so that I may love You, who are infinite good, You enrapture me, in order that I may transcend myself and foresee the

> glorious place to which You invite me. You show me many exceedingly appetizing repasts that attract me by the most appealing aroma. You permit [me] to see the treasure of riches, of life, of joy, and of beauty. You disclose, in nature as well as by art, the Fount from which flows everything desirable. You keep nothing secret. You do not hide the source of love, of peace, and of rest. To wretched me, whom You created from nothing, You offer all things. (→Nicholas of Cusa, 1985a, pp. 735–736)

2.5 Concluding Remarks

To present a coherent and orderly picture of medieval conceptions of God as a perfect being is no easy task. The thought of the theologians of that period is very diversified, formed by the reception of different sources passed down from antiquity and by the need to respond to various challenges posed by the times in which they lived. What I did in this paper is to propose a journey across the history of medieval philosophy according to my subjective inclinations and with the choice of topics and authors largely determined by my own interests. I have distinguished a number of conceptions of God, which, however are not mutually exclusive, so in many theologians it is easy to find elements of different conceptions co-exist in one coherent vision.

The most obvious medieval conception of God is that of a perfect being, whose attributes are: immutability, unity, coinciding, however, with the Trinity of Divine Persons, simplicity, goodness and benevolence. The problem facing the theologians engaged in the construction of this concept of the absolutely perfect being is how to attribute to the absolute and transcendent perfection the qualities and properties that are acquired from the experience of creatures. Different ways of solving that problems were proposed, yet all agreed that positive qualities and perfections found in creatures are not applicable to God without a thorough modification as to their nature, degree

of perfection and mode of inherence. All agree that God and His creation are not on a par, therefore if there are features predicable in common of Him and His work, they must exist in God in a more perfect way and in a higher degree than in creatures (*via eminentiae*).

Another characteristic conception of God was represented by those thinkers who laid strong emphasis on God's many-sided, in particular ontological and epistemological, transcendence with respect to creatures; these theologians, somewhat paradoxically, claimed that the most proper name of God we can invent is that of Nothingness. God's Nature entirely eludes all human efforts to achieve some comprehension.

One particular aspect of God which largely accounts for His incomprehensibility is His infinity. All medieval thinkers agreed in attribution of infinity to God, however, only some of them, notably John Duns Scotus and Nicholas of Cusa, made it the centerpiece and the point of departure of their theology.

Another element introducing complexity into Christian theological discourse was the personal nature of God: the list of ontological perfections of the Divine Essence had to be completed with the perfections proper to a personal being. Even more challenge to theologians came from the history of salvation and the fact that "although He existed in the form of God [...] he took the form of a slave by looking like other men and by sharing in human nature" (Philippians 2, 6–7)

The thinkers of the Middle Ages struggled with these difficulties, with the "scandal of Incarnation" and multiple paradoxes of transcendence; they made determinate effort to form some notion of Divine Reality, be it defective and imperfect, fully aware of the fact that a satisfactory account of God, His nature, powers and perfections by far exceeds human possibilities, as Plato had already observed.

The mystery of God, which both attracted and overawed the Medievals, still remains with us. Is it possible, that the last word of all human efforts to gain some comprehension of God is what Master Eckhart wrote as the ending to his *Sermon 96:*

> You should love Him as He is: a non-God, a non-spirit, a non-person, a non-image; rather, as He is a sheer pure limpid One, detached from all duality. And in that One may we eternally sink from nothingness to nothingness. So help us God. Amen. (Eckhart, 1979, p. 465)

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Notes

1 See →Vlastos, 1970, pp. 93–94.

2 See →Dodds, 1965, p. 75: “The philosophers, however, had added another qualification for being a god—perfect goodness. And man, they said, should imitate this divine goodness so far as in him lay. This is the doctrine of *homoiosis*, ‘assimilation to God’, first stated by Plato in a famous passage of the *Theaetetus*, and constantly echoed by the Platonists of our period, both pagan and Christian.”

3 See →Alcinous, 1993, p. 16: “Matter constitutes one principle, but Plato postulates others also, to wit, the paradigmatic, that is the forms, and that constituted by God the father and cause of all things.”

4 See Mt 5, 48: “Be perfect, therefore, as your heavenly Father is perfect” / “ἔσεσθε οὖν ὑμεῖς τέλειοι ὡς ὁ πατὴρ ὑμῶν ὁ οὐράνιος τέλειός ἐστιν” / “Estote ergo vos perfecti sicut et Pater vester caelestis perfectus est.”

5 See →Agamben, 2005, pp. 75–76: “What Paul says here is that insofar as messianic time aims toward the fulfilment of time—(*pleroma ton kairon* […])—it effectuates a recapitulation, a kind of summation of all things, in heaven and on earth—of all that has transpired from creation to the messianic ‘now’, meaning of the past as a whole. […] This recapitulation of the past produces a *pleroma*, a saturation and

fulfilment of *kairoi* [...], that anticipates eschatological *pleroma* when God 'will be all in all.'"

6 See →O'Rourke, 2016, pp. 520–521.

7 See Pseudo-Dionysius, 1999, p. 217: "In our *Outlines of Theology* we have treated what is of greatest importance in affirmative theology. That is, how the divine and good nature is called bot hone and three, what fatherhood and sonhood are in it, and what the theology of the spirit is. [...] In our *Symbolic Theology* we have discussed the names which are transffered from sensibles to what is divine [...]. Now, however, that we are to enter the darkness beyond intellect, you will not find a brief discourse but a complete absence of discourse and intelligibility."

8 See →Alcinous, 1993, pp. 18–19.

9 See →Achtner, 2014, pp. 21f.

10 Aristotle, *Physics* 202b, 204a, 206a; see →Coope, 2012, p. 276; and →White, 2009, pp. 260–265.

11 Aristotle, *Physics* 207a–b, 206a–b.

12 See *En.* V, 5, 10

13 Plotinus, 2015, p. 14; →Sleeman and Pollet, 1980, col. 268–269; →Mrugalski, 2022, pp. 74, 76 and 78.

14 See →Granados Rojas, 2013, pp. 65f.

15 →Augustine, 1998, VI, 7, p. 251: "The mythical theology, then—that of the theatrical performances, full of

unworthiness and vileness—is referred back to the civil: the whole of the mythical theology, rightly judged worthy of condemnation and rejection, is a part of the civil theology, which is deemed worthy to be cultivated and observed!”

16 See →Augustine, 1996, pp. 159–160: “If those, however, who are called philosophers happen to have said anything that is true, and agreeable to our faith, the Platonists above all, not only should we not be afraid of them, but we should even claim back for our own use what they have said, as from its unjust possessors. Itis like the Egyptians, who not only had idols and heavy burdens, which the people of Israel abominated and fled from, but also vessels and ornaments of gold and silver, and fine raiment, which the people secretly appropriated for their own, and indeed better, use as they went forth from Egypt.”

17 See →Madec, 1974, pp. 16–17; →Courcelle, 1968, pp. 252–253.

18 See →O’Meara, 1997, pp. 172–183; →Courcelle, 1968, p. 157; →Stock, 1996, p. 65.

19 See Augustinus, *De genesi ad litteram* V, 16 (34).

20 See →MacDonald, 2001, pp. 83–84.

21 Cf. Augustine, *De natura boni* I, 10–11; *De vera religione* XVIII, 35.

22 Augustine, *De natura boni* I, 3: “quid bonum est esse substantiam.”

23 →Djuth, 2004, p. 232, rightly remarks: “Augustine’s solution to dualism, then, is either Manichaean nor Neoplatonic because neither the separation nor the hierarchical ordering of goodness and being adequately captures the Christian understanding of God and the world. Only when goodness and being are co-extensive is it possible to advance in the understanding of Christian wisdom that Augustine so earnestly sought throughout his life.”

24 This description was used by →Heinzmann, 1999, pp. 65f. Cf. also →Harrison, 2006, pp. 95f.

25 See →Meesen, 2005, pp. 440–441.

26 Cf. Augustine, *De natura boni* I, 3.

27 In this text I draw a comparison between the Augustinian *Fallor* and Eriugena’s *Cogito*; cf. →Kijewska, 2016, pp. 364f.

28 See →Mann, 2004, p. 35: “These divine activities are *personal;* they could issue only from a being with beliefs and desires similar, in some respect at least, to ours. Any characterization of God that denied him these personal activities or negotiated them away in favour of some advantage to philosophical theology would be rightly regarded by believers as akin to replacing your loved ones with their cardboard cutouts.”

29 →Augustine, 2012a.

30 See →Kijewska, 2023, pp. 568–582. Cf. →Matthews, 2004, p. 62: “Anselm seems to have used Augustine, not only

as a doctrinal authority, but also as a model for his own literary style."

31 → Leftow, 2004, pp. 132 and 140.

32 See → Anselm, 2000, p. 93: "Therefore, O Lord, You who give understanding to faith, grant me to understand—to the degree You know to be advantageous—that You exist, as we believe, and that You are what we believe [You to be]. Indeed, we believe You to be something than which nothing greater can be thought." Cf. → Davis, 2004, p. 159.

33 → Bonaventure, 1978, p. 95. Cf. → Kijewska, 2019, pp. 15–18.

34 See → Gersh, 1978, pp 2–3, 82 and 112.

35 See → Eriugena, 1987, p. 51. Cf. → Erismann, 2011, pp. 205f.

36 See → Eriugena, 1987, pp. 192–193. Cf. → Beierwaltes, 1969, p. 487, and → Kijewska, 2015, pp. 174–178.

37 → Eriugena, 1987, p. 412: "For I understand the substance of the whole man to be nothing else but the concept of him in the Mind of his Artificer, Who knew all things in Himself before they were made; and that very knowledge is true and only substance of the things known, since it is in that knowledge that they are most perfectly created and eternally and immutably subsist."

38 → Kijewska, 2021, pp. 35f.

39 See →Rubino, 2013, pp. 299–300; cf. →Dietrich, 2013, p. 316.

40 See →Enders, 2013, pp. 361f.; →Flasch, 2015, pp. 88f.

41 See Eckhart, 1979, p. 447: “Where is this God?—In the Father. Where is this God?—In eternity. No man could ever have found God, for as the prophet says, *Lord, thou art the hidden God* (Isa. 45:15). Where is this God?—It is just as if a man were to hide himself and then to give himself away by clearing his throat: God has done the same. No man could ever have found God, but He has revealed His presence.”

42 See →Buijs, 1988, pp. 723–725.

43 See →Kretzmann, 1997, pp. 139f.

44 →Kretzmann, 1997, pp. 165–167.

45 See Aquinas, 1947, p. 20: “God is not only His own essence, as shown in the preceding article, but also His own existence.”

46 See →Cross, 1999, pp. 39–40.

47 See Frank and →Wolter, 1995, p. 33.

48 →Zieliński, 1980, p. 24.

49 See →King, 2003, pp. 26f. and →Zieliński, 1980, pp. 32f.

50 See →King, 2003, pp. 18–21.

51 See →Zieliński, 1980, p. 39.

52 →Nicholas of Cusa, 1985b, p. 41: “And so, from these considerations it is evident that the affirmative names we ascribe to God befit Him [only] infinitesimally. For such [names] are ascribed to Him in accordance with something found in created things [...]. For it is not the case that any affirmations—which post in Him, as it were, something of what they signify—can befit Him who is not some particular thing more than He is all things.”

53 See →Kijewska, 2017, pp. 201–206.

54 See →Alston, 1991, p. 5: “The focus will be on what are taken to be direct, non-sensory experience of God. In calling them ‘experience’ I am thinking of them as involving a presentation, givenness, or appearance of something to the subject, identified by the subject as God. It is this presentational character of the experiences that leads me to range them under a generic concept of perception.” See →Alston, 1991, p. 11: “... in order to have a term parallel to sense perception, one that is more compact than ‘perception (awareness) of God’, I will, with some trepidation, speak of mystical perception; ‘mystical experience’ will be used for the mode of experience involved in that sort of perception.” See →Alston, 1991, pp. 16–17.

3 Concepts of God and Their Origins

James E. Taylor

Abstract: The question, “Who created the concept of God?” is a multiply complex question—it presupposes answers to at least two prior questions. One is “Is there one and only one concept of God?” Another (which presupposes that there is only one such concept) is “Is this concept of God created?” I will argue that there is more than one concept of God and that at least one of these concepts may be uncreated. If I am right, then the question “Who created the concept of God?” is based on a clearly false presupposition (that there is one and only one concept of God) and a presupposition that is not clearly true (that the concept of God was created). It follows that this question does not have a straightforward answer. Nonetheless, there is much to learn—about concepts of God and their origins—by analyzing it.

3.1 Introduction

The purpose of this essay is twofold. First, I will argue that the question the editor of this volume asked me to answer,

Q: Who created the concept of God?

does not have a straightforward answer. Second, I will show that an analysis of the question can nonetheless provide insight into concepts of God and their origins.

My general reason for coming to the conclusion that Q does not have a straightforward answer is that Q is *complex*—it presupposes answers to at least two prior questions. And the answers to these questions it presupposes are either clearly

false (in the case of one of the presuppositions) or not clearly true (in the case of the other). Since questions that presuppose answers to prior questions that are either clearly false or not clearly true are questions that have no straightforward answer, Q does not have a straightforward answer.

The first prior question, to which Q presupposes a clearly false answer, is "Is there one and only one concept of God?" Q presupposes that the answer to this question is affirmative. It would make sense for a person to ask "Who created *the concept of God*" only if that person presupposed that there is one and only one concept of God. That's because the italicized expression is a definite description, and definite descriptions purport to pick out a unique referent.

Is the presupposition that there is one and only one concept of God true? Clearly not. There is indeed at least one concept of God, but it is not the case that there is *only one* concept of God. A ridiculous concept of God—which is nonetheless a concept of God—is that of an old man with a long white beard sitting on a throne in heaven. This concept is clearly different from the concept of God as a supremely perfect being. So, Q presupposes a clearly false answer to the first prior question.

The second prior question, to which Q presupposes an answer that is not clearly true, is "Is the concept of God created?" Q presupposes that the answer to this question—with respect to the allegedly unique concept of God the questioner has in mind—is "yes." Obviously, a questioner would not desire to know which person or persons created X unless that inquirer was assuming that X was created.

The answer to this second prior question depends on which of the many existing concepts of God the questioner has in mind and on what it means to create a concept. I will argue that, though some concepts of God are created, there is no conclusive reason for thinking that all of them are. Some concepts of God—

the most important ones—are possibly uncreated (I'll explain below under what conditions a concept is created and what counts as an important concept of God). So, at least with respect to the most important concepts of God, the answer presupposed by the second prior question—that there is a concept of God that was created—is not clearly true.

In sum, if there is more than one concept of God, then Q does not have a straightforward answer. And if there is at least one uncreated concept of God, then Q does not have a straightforward answer. Again, it is easy to show that there is more than one concept of God. And I will argue in due time that a good case can be made for the claim that at least one of those concepts is both important (in a sense to be specified) and uncreated (in a sense to be defined).

3.2 Concepts

But what is a *concept*? A concept is a way of thinking about a thing or a kind of thing. It is a way of thinking about a thing or kind of thing that purportedly distinguishes that thing from other things (or kind of thing from other kinds of things). All concepts are at least partly descriptive. That is, all concepts identify the thing or kind of thing of which they are concepts by means of some descriptive content. The descriptive content of concepts is also conceptual. For instance, the concept of an elm is the concept of a certain kind of tree. That is, the descriptive content of the concept of an elm includes the concept of a tree. And the language used to express or convey this descriptive content has the form of an indefinite description (a description that begins with the indefinite article "a(n)").

Some concepts are *fully* descriptive. That is, they function in such a way that their descriptive content is sufficient by itself to enable one who understands the concept to identify the thing or

kind of thing to which it applies. An example of a concept that functions this way is the concept of a bachelor. The content of this concept is entirely descriptive: a bachelor is an unmarried, adult, human male. Those who possess this concept can use it to identify a person to which it applies simply by confirming that the person in question is unmarried, adult, human and male.

But some concepts are only partially descriptive. Such concepts contain an indexical or referential component in addition to their purely descriptive conceptual content. A famous plausible example of such a concept is the concept of water. Hillary Putnam argued that the ordinary non-scientific concept of water is the concept of the sort of liquid that fills our oceans, lakes, and rivers, etc. This concept is only partially descriptive because it contains a non-descriptive referential element. Possessors of the ordinary concept of water who do not know that water consists in H_20 conceive of water as the type of liquid (descriptive component) that fills those certain types of spaces *out there* (referential component). In order to identify water, they need to know not only that it is a type of liquid but also that it is the type of liquid that is the same sort as *that* (referring directly or indirectly to an actual sample of water).[1]

And as I've been saying, some concepts are ways of thinking about individuals and others are ways of thinking about kinds. All the examples of concepts given above (of elms, bachelors, and water) are concepts of kinds of things. They are *general* concepts associated with general terms (in English, 'elms,' 'bachelors,' and 'water'). But there are also concepts—ways of thinking—about *individual* persons, places, or things (things to which we refer by means of singular terms like proper names and definite descriptions).

For example, one way of thinking about George Washington is to think of him by means of the definite description "the first President of the United States." This is an entirely descriptive

way to think about him. But one can also employ a partially referential concept to think about an individual. For instance, you can think about a stranger you see in a crowd as “that guy over there.” This concept is partially descriptive (“that *guy*”) and partially referential (“over *there*”).

In sum, all concepts are at least partially descriptive, some concepts are fully descriptive, and some concepts are a mix of descriptive and referential elements. And these generalizations apply to both concepts of kinds of things and concepts of individual things.

3.3 Concepts of God

The most general and uncontroversial way of thinking about God is by means of the concept of *a divine being (or reality)*. It’s hard to see how a competent speaker of English could mean anything by means of the word ‘God’ that does not include this general description. And since this descriptive content is formulated as an indefinite description (“a divine being or reality”), it could conceivably apply to more than one individual. And of course, a polytheistic picture of reality entails that there is more than one god—more than one individual to which the concept of God as a divine being or reality applies.[2]

It is natural to think that God (if God exists) is a divine being (what else could God be?). So, it seems appropriate to say the concept of God is, at least in general terms, the concept of a divine being. But, as just indicated, the concept of *a* divine being (indefinite description) may seem to imply that there could be more than one divine being. That is, it may seem to leave open the possibility that some version of polytheism is true—that there is more than one being to which the concept applies.

In order to preclude that possibility, it seems reasonable to say that the concept of *God*—in the non-polytheistic sense that

entails that there is (and perhaps could be) only one God—is the concept of a divine being that is the only (or the only possible) divine being. Given this qualification, the concept of God is not only the concept of a certain *kind* of reality, but also the concept of a certain *individual* reality. The concept of God is the concept of a divine reality that is "one of a kind." It is the concept of a reality or being that is *the only (possible) divine reality or being*. Thus, the non-polytheistic concept of God is expressible by means of both an indefinite description ("a divine reality") and a definite description ("the divine reality")—or a conjunction of these two ("a divine reality that is the only divine reality").

And it seems reasonable to think that Q would be asked by someone who has the non-polytheistic God in mind rather than a polytheistic god (since in the latter case, the questioner would likely ask instead, "Who created the concept of *a god*?").

So, in general terms, the concept of God is the concept of a divine reality that is also the only divine reality.

This is one general non-polytheistic concept of God.[3] Perhaps it is the one and only such concept of God that everyone would agree is a concept of God. Perhaps it is also the most general descriptive content that any concept of God worth calling a concept of God must contain or entail.[4] If there is a non-polytheistic God, then it seems clearly analytically true to say that this God is a divine being or reality that is the only divine being or reality. And if this proposition is clearly analytically true, then no concept of God would be acceptable as a concept of God if it didn't contain or entail it.

Now it's possible that someone who asks question Q has only this very general concept of God in mind as the target of his or her question. But it's unlikely. The concept of God as the one and only divine being is quite general and so relatively uninformative and unilluminating. At least philosophers and theologians who ask questions about the concept of God are

likely to have a more specific concept in mind that adds additional conceptual or descriptive content.

This additional content could be descriptive of either intrinsic or extrinsic properties of God.

God's extrinsic properties would concern how God is related to whatever exists—if anything—that is not identical with God or a part of God (if God has any parts). According to pantheism, God is identical with all of reality. So, pantheism entails that God does not have any extrinsic properties (other than the negative extrinsic property of *not* being related to anything that is not God). If pantheism is true, then God has only *intrinsic* positive properties—properties that are intrinsic to God's nature or essence.

If some version of monotheism is true, on the other hand, then God would have the positive extrinsic property of being related to the universe in some way or other. Examples of such a relation include being the creator and sustainer of all that is not God (creation monotheism), being the sustainer of an uncreated contingent non-divine universe that is dependent on God for its eternal existence (contingency monotheism), and being eternally related to a necessary, independent, but non-divine universe that God did not create and does not sustain (cosmological dualism).

Do these alternative views of God's relation to whatever (if anything) is not God entail alternative *concepts* of God? The answer to that question depends on whether our concept of a concept includes ways of thinking about things or kinds of things in relation to other things or kinds of things. With respect to the concept of God, it seems that we often do include the concept of *creator* as descriptive content, so it seems reasonable to think that at least one concept of God is the concept of God as a creator (among other things). And if that is the case, then it would also seem reasonable to conclude that any way of

thinking about God that does not include God's being a creator but attributes to God a different extrinsic relation to the universe would be an alternative *concept* of God.

Another concept of God that attributes a positive extrinsic property to God is that of God as a personal being. A personal being would be at least conscious, and therefore capable of being aware of any actual or possible objects that are not God or a part of God. That's why the property of being a personal being is a positive extrinsic—or relational—property, rather than an intrinsic property. And of course, the alternative general concept of God is that of God as being impersonal (in the sense of not having all the attributes required for personhood at all, such as consciousness and any attribute that depends on consciousness—such as thought or will). The concept of an impersonal God would be a concept attributing the negative extrinsic property to God of *not* being capable of relating personally to anything other than God.

So, there are many concepts of God determined by different views about the relation of God to actual and possible non-God things.

But there are also many different concepts of God that contain different more specific concepts of *intrinsic* properties. Perhaps the most general distinction between different concepts of God categorized in terms of intrinsic properties they attribute to God is the distinction between the concept of God as a supremely, completely, or absolutely *perfect* being and the concept of God as a being or reality in the *process* of developing, growing, or maturing toward perfection (whether God could or would ever reach that goal or not). This is the general distinction between the classical perfect being conception of God and the contemporary process conception of God.

And there are specific variations of each of these two general ways of thinking about God. For instance, there are

disagreements about what general concept of perfection should be the basis for determining what the attributes of a perfect being would be. Would a perfect being be necessarily absolutely changeless in every respect—or not? And once this question is answered, there are further disagreements between proponents of the classical perfection concept of God about what specific attributes or properties count as perfections. Would a perfect God be eternal in the sense of being atemporal (existing outside of time) or everlasting (existing in time at every point of time)?

Perhaps these disagreements could be characterized as disputes between people who hold the same *general* concept of God (the perfect being) but who hold different more *specific* concepts of God (the perfect being who exists *outside* of time vs. the perfect being who exists *in* time). Thus, whether two or more people possess the same concept of God can be determined only relative to a certain level of generality or specificity. But the more specific the concept is, the more likely is the degree of disagreement about whether it provides a completely accurate way of thinking about the divine reality.

That's because every disagreement at a more specific level tends to lead to more disagreements at an even more specific level. So, two or more people who agree that the most general concept of God is the concept of the one and only divine reality or being can disagree about whether God is absolutely perfect or not. And two or more people who agree that the one and only divine being or reality is a perfect being can disagree about whether God exists outside of time or in time. Furthermore, two or more people who agree that the one and only divine being or reality is a perfect being who exists outside of time can disagree about how to characterize the timeless God's relation to time, etc.

What this characterization of potential disagreements suggests is that alternative concepts of God can be represented

by means of a “tree” diagram that starts with a node representing the most general concept of God about which there is the most agreement and that has branches to at least two more specific concepts of God that share the same general content but diverge in some more specific content. And the nodes representing those two alternative concepts could have at least two branches coming from each of them as well to represent even more specific competing versions of the concept represented at the higher, more general level. Such a concept tree would continue sprouting branches as long as there continues to be disagreement and debate at more specific levels.

I said above that those who ask question Q are likely to have a non-polytheistic concept of God in mind but are also likely to have a more specific concept of God in mind than the most general non-polytheistic concept of God as the one and only divine reality or being. But at which level of specificity in the concept tree should we locate the concept the questioner has in mind? And which of the alternative more specific concepts of God at that level is the questioner interested in? Obviously, there can be no determinate answer to those questions in the absence of a real human being asking question Q with a certain relatively determinate concept of God in mind. And the possibility of there being many different questioners with many different concepts of God in mind provides further reason to think of question Q as problematic. The only adequate way to answer it is to find an individual who raises it and ask him or her what specific concept of God her or she has in mind.

But then question Q becomes question Q’:

Q’: Who created the concept of God I currently have in mind?

Now that Q has become Q’, it’s clear that the answer to it depends on the person who asks the question and which concept of God that person has in mind at the time of asking. Q’

is an improvement over Q since it does not presuppose, as Q does, the clearly false assumption that there is one and only one concept of God. But this new question is also complex, since it presupposes, as does Q, that the concept of God the asker has in mind was *created*. In order to address that presupposition adequately, we need to get clearer on the distinction between created and uncreated concepts.

3.4 Created Concepts

A clear case of a created concept is a case in which a person or group of people consciously and deliberately construct(s) a new way of thinking about a kind of thing or an individual thing. For instance, I can invent the concept of a book I have read in its entirety that is sitting on the part of the bookshelf in my office that is next to the window and three rows from the floor. That's the concept of a kind of thing (rather than an individual thing), and it is partly descriptive and partly referential (since it contains a reference to me by means of the word 'I' and to a particular part of the bookshelf in my office by means of the definite description "the bookshelf that is in my office …"). This concept is a created concept since (1) it didn't exist before I thought of it and (2) I deliberately constructed it out of pre-existing concepts I already had in mind.

A less clear case of a created concept is a concept that is a product of a long evolutionary social cognitive process involving a number of different people who have jointly, over time, without deliberate forethought and somewhat unconsciously or subconsciously, contributed to a new way of thinking about a thing or kind of thing. The reason this case is less clear than the previous one is that there is no discernible point at which a concept with this sort of origin was consciously and deliberately created by an individual or group. There was no fiat or

stipulation that brought the concept about. But I am classifying it as “created” because it is not *innate*. So, I am working with the assumption that every concept is either uncreated by human beings because it is somehow innate in each human being’s mind, or it is created by human beings either consciously and deliberately or by means of a process of social cognitive evolution.

An example of the latter sort of created but evolved concept is that of a Facebook friend. I assume that when Facebook first came into existence, the “friends” with whom a Facebook user was connected were genuinely friends in the traditional sense of that word. But over time, the extent of the sorts of persons who counted as “friends” for Facebook purposes became broader than what was encompassed by the traditional concept of a friend. As a result, a new concept emerged that is related to the traditional concept but that goes beyond it in certain respects (so as to include, for instance, friends of friends in the old-fashioned sense of “friend”). I assume that this new concept was not the result of any individual’s or group’s conscious and deliberate intention to create a new “friend” concept, but instead a product of a certain social dynamic among Facebook users given the structure and resources provided by Facebook. And the concept of a Facebook friend has led to other evolved created concepts as well, including the concept of the activity of “unfriending” or “de-friending” a person.

3.5 Concepts of God and Concept Creation

Since concepts can be uncreated and innate or created intentionally or unintentionally, each concept of God must fall into one of these three categories (which I assume to be jointly exhaustive and mutually exclusive). I’ll suggest a candidate concept of God for each category.

Perhaps Spinoza's specific concept of God is an example of one that has been consciously and deliberately created by an individual. In a nutshell, Spinoza conceived of God as an infinite substance that has the attributes of thought and extension and that can be characterized as either God or Nature. Though Spinoza employed a number of pre-existing concepts in his formulation of this concept of God, it's arguable that he was the first to employ a concept of God with this particular content. And it seems clear that his use of it is a result of his careful intentional construction of it.[5]

There are many concepts of God that fall into the category of unintentionally created concepts. A clear example is the concept of an old man with a long white beard sitting on a throne in heaven, which I mentioned above. That concept seems not to have arisen because of any deliberate creative act on anyone's part but instead as a result of a process of social cognitive evolution due to the human tendency to anthropomorphize.

The Greek philosopher Xenophanes believed that different groups of people make up gods that reflect their own particular group's characteristics:

> The Ethiopians make their gods snub-nosed and black; the Thracians make theirs gray-eyed and red-haired.[6]
>
> And if oxen and horses and lions had hands, and could draw with their hands and do what man can do, horses would draw the gods in the shape of horses, and oxen in the shape of oxen, each giving the gods bodies similar to their own.[7]

So, Xenophanes seemed to think that all concepts of the gods were made in the image of human beings on the basis of specific distinctive characteristics of their particular social group. But it seems unreasonable to say that these groups made up these concepts as deliberate fictions, since they apparently believed in

the reality of the divine beings that corresponded to the concepts they had made.

A number of proposals have been made for innate concepts of God. Descartes is a clear case of a philosopher who argued for the innateness of the concept of God as a perfect being.[8] In doing so, he drew on the Platonic tradition of the innateness of some of our ideas and on the perfect being theological tradition that has its roots in Plato but flowered in the thought of St. Anselm (with contributions along the way by Aristotle and Augustine—among some others).

Of course, some philosophers in the empiricist tradition, such as Locke and Hume, explicitly denied the existence of innate ideas. And both argued that all our ideas—including our idea of God as a perfect being—are constructed by our minds out of our ideas of sensation and reflection.[9] Descartes anticipated this sort of objection in his *Meditations* by arguing that the "objective reality" of his idea of God as a perfect being was greater than any finite being could produce, and so required the existence of a perfect being as its causal source.[10]

If Descartes (and other rationalist perfect being philosophical theologians who followed him, such as Spinoza and Leibniz) is right and the empiricists wrong, then at least one concept of God is uncreated. And if the concept of God as a perfect being truly is innate, then it falls into the category of being an *important* concept (in a sense I will now explain).

3.6 Important vs. Unimportant Concepts

Concepts are tools, and tools are important or unimportant relative to a particular purpose or goal. Since the purpose of concepts is to provide us with adequate ways of thinking about things and sorts of things, a concept is important to the extent that it enables us to think adequately about the thing or kind of

thing of which it is a concept. We need to have adequate ways of thinking about things or kinds of things for both theoretical and practical purposes.

I need to have an adequate concept of a hammer so that I can recognize one when I see it and also so that I can use it well to accomplish the purpose for which it was made. Achieving both of these goals requires that my concept of a hammer be both sufficiently accurate and sufficiently specific. If I go into my garage thinking that a hammer is a container of some kind, I won't be successful in recognizing one. And even if I'm successful in recognizing a hammer, I won't be successful in using it well unless I conceptualize it as a tool one grasps by the handle in order to drive a nail into certain kinds of materials (such as wood) by means of the hammer's face through repeated strikes on the head of the nail.

The analogy with concepts of God should be clear. A concept of God is important (valuable) to the extent that it enables one to conceive of God with a sufficient degree of accuracy and specificity for both theoretical and practical purposes. What one's theoretical and practical purposes are for employing a concept of God will depend on whether one is a theist (who believes that God exists), an atheist (who believes that God does not exist), or an agnostic (who suspends belief about whether or not God exists).

An atheist may have a variety of theoretical and practical purposes for employing a concept of God. For instance, an atheistic philosopher might use a concept of God for the theoretical purpose of thinking about what God would be like if God existed and for the practical purpose of constructing an argument for God's non-existence. An atheistic or agnostic sociologist might use a concept of God for the theoretical purpose of understanding the theology of a particular religious group (that employs that concept) and for the practical purpose

of constructing a survey to give to members of that group aimed at investigating how their concept of God influences their political views and voting behaviors.

For theists, a primary theoretical purpose of a concept of God is to enable one to know what the existing God is like. And a primary practical purpose of a concept of God for theists is to enable them to relate to God in appropriate ways.

So, concepts—including concepts of God—can be relatively valuable or not to the extent that they enable one to have sufficiently accurate and determinate thoughts about the thing or kind of thing of which they are concepts—so that one can understand those things or kinds of things, and if they exist, interact with them adequately.

But concepts can also be relatively important or unimportant depending on the importance for one's thought and life of the object or kind conceptualized. Clearly, God is more important for thought and life than hammers are. If God exists (and perhaps even in some cases if God doesn't exist), then some concept of God is arguably the *most* important concept for both our thinking and our living as human beings. But which of the many concepts of God mentioned previously is the most important one (again, assuming that God exists)?

Since the most valuable concept of God will be the one that will help us to think most adequately (accurately and specifically) about God, if there is an innate concept of God, and if that innate concept was implanted in the human psyche by God as a means of communicating to humans what God wants us to know about God's nature, then such a concept would be more likely to be accurate than any concept of God created by human beings. A God-implanted innate concept of God would also have the degree of specificity required to accomplish whatever *general* theoretical and practical purposes God has in mind for all human beings in our thinking about and living with God (leaving open

the possibility that God also has *more specific* theoretical and practical purposes for us that would require supplementation by a special revelation from God).

On the assumption that God exists and is an absolutely perfect being, God is omniscient and infallible in knowledge. Moreover, if there is a perfect God, then God is perfect in goodness and love. And if God has provided humans with an innate concept of God, these attributes would constrain God to provide us with a concept that is sufficiently accurate and determinate to enable us to know God and relate to God to the extent that God desires us to know and relate to God by means of this innate general concept of God (again, keeping in mind that we might need a more specific concept of God provided by a special divine revelation in order to know and relate to God even more adequately).[11]

By contrast, humanly-created concepts of God are likely to be less adequate than uncreated innate concepts of God implanted in humans by God. Finite humans are limited in knowledge and subject to error. We are also subject to distortions in our concept of God due to our individual moral and spiritual defects and deficiencies and the various cultural biases to which we are subject. For instance, we may be inclined to create a concept of God that is limited in such a way as to give us a sense of security based on the illusion that we can know and understand God to a greater extent than is actually possible. And we may be tempted to create or accept a concept of God that allows us to justify a way of life that falls short of the ideal God has in mind for us. Or we may live in a culture that is secularized to such an extent that our concept of God is affected adversely by its influence.

Admittedly, some concepts of God created consciously and deliberately by individual humans are likely to be more adequate than created concepts of God that are a product of relatively

unintentional and subconscious social cognitive evolution. The relatively specific concept of God Spinoza created on the basis of a more general concept of God as perfect that Spinoza assumed was innate is likely to be more adequate in certain respects than the colloquial concept of God as a man with a long white beard sitting on throne in heaven. The latter concept is likely to have evolved because of its tendency to give its employers a sense of epistemological security and moral license. These conveniences are not available to those whose non-anthropomorphic concept of God would be more likely to generate awe about divine mystery and fear about human sinfulness in the face of divine holiness (themes that are especially important in the Judeo-Christian tradition). And again, an innate and so uncreated concept of God instilled in humans by God is even more likely to enable humans to think of God adequately.

3.7 Testing Concepts of God for Adequacy

Now we are in a position to appreciate the comment I made in footnote 4 about the need to test proposed concepts of God for adequacy by adding the definite description “the one and only divine being or reality” to it. Once we make that qualification explicit, we can test the result for adequacy on the basis of our semantic intuitions about whether such a being or reality is possible and whether such a being or reality is deserving of the honorific title “God.” It seems clear that a being or reality can count as divine only if it is sufficiently great or exalted in its nature. If that’s true, then it’s possible for a being or reality to be divine only if it both satisfies an appropriate description and also meets an appropriate normative standard.

The concept of God as an old man with a long white beard sitting on a throne in heaven seems clearly not to rise to this level. As a result, such a merely descriptive concept is not an

adequate concept of God.[12] But even if we add a normative dimension to the concept by changing it to the concept of a *superhuman* being sitting on a throne in heaven (such that a superhuman being is a being that surpasses humans in power, knowledge, and goodness), the resulting concept of God would still not be fully adequate (even if it is more adequate than the previous concept). That's because, intuitively, a being is deserving of the honorific title "God" only if it is not only great and exalted but *unsurpassably* great and exalted.

Here's an argument for that claim. We have been assuming that, in order to be adequate, a concept of God must be the concept of the one and only divine being. And we have also been assuming that this being must be sufficiently great to deserve the title "God." How great? Consider a possible being with level of greatness G and another possible being with a level of greatness higher than G. Clearly, the latter possible being is more deserving of the title "God" than the former possible being is. This line of reasoning clearly entails that a possible being can qualify as God only if that possible being is such that there is no possible being greater than it. That is, a possible being can be God only if it is maximally and unsurpassably great. That means that only an absolutely, supremely, and completely perfect and infinite being could be God.

With this conclusion, we can rule out any concepts of God that cannot apply to a perfect being as inadequate and so unimportant relative to the goal of enabling us to think about God accurately and specifically for theoretical and practical purposes. But is the concept of God as a perfect being not only important but also created? I will address that question in the next—and final—section of this essay.

3.8 Is the Concept of God as a Perfect Being Created?

I have just argued that the relatively general concept of God as a perfect being is more important than alternative concepts of God. And earlier in this essay, I stated that it is not clearly true that the most important concepts of God are created. I will now state my reasons for that claim. I will not argue here that it is *clear* that the concept of God as a perfect being is uncreated and so innate. Rather, I will argue that it has not been shown that this concept is created, which leaves open the possibility that it is innate—a possibility worth exploring further, since an innate concept of God implanted in humans by God would be more adequate for theistic theoretical and practical purposes than a concept of God created by human beings—as I argued earlier.

Whether the concept of a perfect being is created or not depends on what perfection—and adequate thinking about perfection—require. Concept empiricists argue that all of our concepts, including our concepts of God, are constructed out of our perceptions, which are based entirely on our sensory and introspective experiences.

Here's a standard empiricist explanation of how we construct an idea of God as a perfect being on the basis of our perceptions alone: First, we form the idea of a conscious personal being on the basis of reflecting on ourselves and the contents of our mind. Then we form the ideas of knowledge, power, and goodness in the same way (by means of introspective access to our own knowledge, power, and goodness).

Since our idea of God is the idea of a conscious personal being who is *perfect* in knowledge, power, and goodness, we form of the idea of a conscious personal being who possesses knowledge, power, and goodness to an unlimited or infinite extent or degree. And we do this by means of employing an empirically-derived concept of infinity. Since we have not experienced an infinite amount or degree of anything, including

an infinite amount of knowledge, power, and goodness, our concept of infinity must be constructed on the basis of things we have experienced. And a standard empiricist account of how that works is that we construct the concept of an infinite amount on the basis of our experience of adding the number one to itself and our experience of doing something without stopping. Then we have the complex idea of adding the number one to itself without ever stopping, which gives us the concept of infinity. And once we have this concept of infinity, we simply combine it with our concepts of knowledge, power, and goodness to yield the concepts of an infinite amount of knowledge, power, and goodness. The empiricist concludes that we have thereby constructed the concept of being perfect in knowledge, power, and goodness (and other relevant properties)—which we then add to our concept of a conscious personal being to yield the concept of a perfect being.

Concept rationalists, who hold that some human concepts are innate, can offer the following sorts of points in reply. First, the concept of perfection the empiricists employ is based entirely on a quantitative concept of infinity. But the concept of perfection that applies to God arguably contains a qualitative dimension (e.g., perfect love is not merely the highest amount of love in a purely quantitative sense but also a *kind* of love—such as unconditional self-sacrificial love). Second, the empiricist conception of infinity requires that we have experienced engaging in an activity without ever stopping. But regardless of how long we have spent doing something, we have never consciously done something *without ever stopping*. Third, the concept of perfection as infinity defined in terms of doing something *without stopping* would at best yield a concept of *potential* infinity. But the concept of perfection required for the concept of God as a perfect being is a concept of *actual* infinity. Even if the concept empiricist can account for the former on the

basis of human experience alone, there is no way the concept empiricist can account for the latter on the basis of human experience alone, since no human has ever experienced an actually infinite reality.

These are reasons to reject this concept empiricist account of the origin of the concept of God as a perfect being. But what do concept rationalists have to offer by way of positive reasons for accepting their view that there is a concept of God as a perfect being that is innate?

Here's one argument. First, we have never experienced the kind of infinity or perfection involved in the concept of God as a perfect being (that's the conclusion of the above argument against the concept empiricist view). Second, if we have never experienced that kind of infinity or perfection, then the origin of our idea of it must be non-experiential. But if the origin of our idea of that kind of infinity and perfection must be non-experiential, then it must be innate, since there is no other non-experiential source from which our idea of infinity or perfection could have come. Therefore, our idea of God as a perfect being is innate.

Now concept empiricists will no doubt be able to think of defenses against the concept rationalists' objections. For instance, they could argue that the superior kind of love exercised by a perfect being is both a kind of love humans have experienced imperfectly and also a kind that comes in degrees. And they could insist that the concept of infinity could be constructed solely on the basis of the concept of continuing, repeating, or adding—and not also on the concept of *failing to stop* this process. Moreover, concept empiricists could deny that any concept of actual infinity is coherent—on the ground that what is actual is complete and what is infinite is not. Finally, concept empiricists will no doubt have objections to the concept rationalists' positive argument for their position.

But these replies and objections will surely not stop the concept rationalists from formulating defenses and raising objections of their own—so as to keep the conversation going. And this is the primary basis on which I am arguing that it is not clearly true that the most important concepts of God are created. If it were clearly true, then the concept empiricist case would be clearly superior to the concept rationalist case. But it is not. So, it's not clearly true that all concepts of God as a perfect being were created by humans.[13]

3.9 Conclusion

I wrote this essay in response to question Q—"Who created the concept of God?" I argued that Q does not have a straightforward answer, since it presupposes a clearly false affirmative answer to one prior question ("Is there one and only one concept of God?") and an affirmative answer to another prior question that is not clearly true ("Is that concept of God created?").

I then discussed the concept of concepts and applied that concept to the concept of God. And I argued that there are many concepts of God of varying degrees of specificity. I then argued that, in light of there being numerous concepts of God, Q should become Q': "Who created the concept of God I currently have in mind?"

Then I explained the difference between created concepts (of two kinds) and uncreated, innate concepts. I also argued that uncreated, innate concepts of God—if there are any—are more important or valuable than created ones, since they are more likely to be adequate (sufficiently accurate and specific for the theoretical and practical goals we have when we employ concepts of God).

After introducing a way to test concepts of God for adequacy, I argued that perfect being concepts of God are more likely to be adequate than are concepts of God that do not attribute perfection to God. Finally, I argued that it is still an open question whether the concept of God as a perfect being is innate or created; a case can be made for both a concept empiricist account of this concept (according to which humans have created all concepts of God as a perfect being) and for a concept rationalist account of it (according to which there is at least one concept of God as a perfect being that humans did not create).

In conclusion, though there is no straightforward answer to the question "Who created the concept of God?," I hope the reader will agree that my analysis of it has led to some interesting and instructive observations about concepts of God and their origins.

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Notes

1 →Putnam, 1975.

2 I assume that henotheism (devotion to a single god that doesn't involve denying the possible existence of other gods) is a form of polytheism.

3 From now on, by "the (or a) concept of God" I will mean *the (or a) non-polytheistic concept of God*.

4 Even the concept of an old man with a long white beard sitting on a throne in heaven could be an adequate concept of *God* only if it contained or implied that the old man is the one and only divine being or reality. And once we realize this, we can use the explicit addition of this clause to a proposed concept of God to test it for adequacy. I'll say more about this below.

5 →Spinoza, 2000, Part One. A qualification is in order here. I would classify Spinoza among rationalist perfect being theologians like Descartes and Leibniz. These philosophers agree to conceptualize God as the one and only divine being who is also an absolutely perfect being. And they also agree that this general concept of God is innate (see below). However, I classify Spinoza as also having employed a more specific concept of God (the one I sketched above) that was consciously and deliberated created by him.

6 Xenophanes, Diels/Kranz 21 B 16, →Robinson, 1968, p. 52.

7 Xenophanes, Diels/Kranz 21 B 15, →Robinson, 1968, p. 52.

8 →Descartes, 1996.

9 →Locke, 1959; →Hume, 1975, Section II.

10 →Descartes, 1996, Meditation 3.

11 The claims I make in this paragraph presuppose that the problem of divine hiddenness has an adequate solution.

12 Of course, some concepts of God suggested by anthropomorphic descriptions of God in the Bible are also subject to this evaluation. As a consequence, such concepts should be taken as non-literal modes of thought that suggest more accurate ways of conceiving of God.

13 If a perfect God exists, could this God have created a concept of God as a perfect being? If the concept in question is *God's* way of thinking about God's nature, then the answer seems clearly to be *no*. A perfect being would be uncreated and would always have employed the same uncreated concept as a tool for self-reflection. If the concept in question concerns a *human* way of thinking about God, then the answer is arguably *yes*—at least to the extent that a human way of thinking about a perfect being would differ from a divine way. But then human concepts of God as a perfect being would be created by God only because in virtue of creating humans, God also created human ways of thinking about things.

II Omniscience, Omnipotence, Timelessness and Spacelessness of God

4 Divine Omniscience: Complete Knowledge or Supreme Knowledge?

Jan Heylen

Abstract: One of the divine attributes is omniscience. The standard concept of omniscience is the concept of having complete knowledge: God knows every truth. But there are also other concepts of omniscience that are consistent with having incomplete knowledge. I will propose a new concept of omniscience, namely the concept of having supreme knowledge. It is inspired by how Anselm talks about God's knowledge and it makes good sense of a key premise in an Anselmian argument for omniscience. Moreover, it can be used to escape some of the set-theoretical and logical arguments against divine omniscience.

4.1 What Is Divine Omniscience?

One of the divine attributes is omniscience. But how to conceptualize omniscience? The most straightforward conceptualization of omniscience is complete knowledge: God knows every truth. There are different ways of formalizing the property of having complete knowledge. With *T* the truth predicate, K_i the knowledge operator with agent-index, and *K*(x,y) the relation of x knowing y, one can formalize the property of x being omniscient, expressed by *O*x, in the sense of having complete knowledge in at least three different ways:[1]

CK1	$\varphi \rightarrow K_X\varphi$, for all $\varphi \in \mathscr{L}$
CK2	$\forall y(T(y) \rightarrow K(x,y))$
CK3	$\forall p(p \rightarrow K_X\, p)$

The first formalization is a scheme, without a quantifier prefix. The second formalization is a first-order quantified sentence. The third formalization is a higher-order quantified sentence. A significant difference is the following: CK1 is restricted to whatever truth is expressible in the language $\mathscr{L}$, whereas CK3 is not so restricted.

A much discussed puzzle with respect to divine omniscience is about divine foreknowledge and future actions chosen by free humans. The puzzle goes back to Augustine (*De libero arbitrio voluntatis*, III, 3) and Boethius (*De consolatione Philosophiae*, V). Wierenga (2021, Section 3) renders the basic argument as follows:

1. If God has foreknowledge that person S will do action A, then it is necessary that S will do A.
2. If it is necessary that S will do A, then S is not free with respect to doing A.
3. Therefore, if God has foreknowledge that S will do A, then S is not free with respect to doing A.

Aquinas (*Summa contra Gentiles*, I, 67, 10) pointed out that premise 1 is ambiguous between

1. It is necessary that, if God has foreknowledge that person S will do action A, then S will do A.
2. If God has foreknowledge that person S will do action A, then it is necessary that S will do A.

Premise 1′ is true, whereas premise 1″ is false. With premise 1′ the argument is valid, whereas with premise 1″ the argument is invalid. So, either way the argument is incorrect (→Wierenga, 2021, Section 3). The debate did not end there: more sophisticated arguments have also been put forward—see →Pike (1965) and →Sobel (2003, pp. 481–494). I am in this section not going to discuss the merits of the various arguments, but rather I want to mention one conceptual move that has been made in response to arguments about divine foreknowledge and future actions chosen by free humans.

The conceptual move consists in rethinking omniscience to allow for incomplete knowledge. →Langtry (2008, p. 39) proposes that it is consistent with divine omniscience that God does not know a true proposition as long as his ignorance is not because of "any defect or limitation in his intrinsic cognitive capacities." Langtry notes that it is then also consistent with divine omniscience that God does not know which future actions humans freely choose to do, as long this lack of divine foreknowledge is not the result of a defect or limitation in God's intrinsic cognitive capacities. A divine being that is fully capable of knowing about future human actions but that voluntarily decided not to known can still be omniscient in this sense. Similarly, →Nagasawa (2017, p. 92) has proposed the following:

> God is the being that has the maximal consistent set of knowledge, power, and benevolence.

The above definition leaves it open that God may be ignorant to some extent. Nagasawa does not explicitly discuss it, but divine ignorance may also pertain to future human action. However, there are issues with both concepts of divine omniscience. Langtry's concept of divine omniscience seems to be consistent with divine ignorance that results from "inattention or

performance error" (→Oppy, 2010, p. 592), which are cases of ignorance that we do intuitively find incompatible with omniscience. Nagasawa's concept of divine omniscience faces the problem that it is a priori likely that there are multiple maximally consistent combinations of knowledge, power and benevolence that may perhaps not be coherently co-instantiated (→Kvanvig, 2018).[2]

In this chapter we will examine both the complete knowledge conception of divine omniscience and an alternative conception of divine omniscience that makes room for incomplete knowledge. In Section 4.2 we will look into an argument for divine omniscience, while in Section 4.3 we will look into some arguments against divine omniscience. In neither section the focus is on the arguments themselves. We will be mainly concerned with the different concepts of divine omniscience.

4.2 Pro Omniscience

The attribution of omniscience to God is part of so-called 'perfect being'-theology, which is supposed to trace its roots to Anselm's *Proslogion*. In the latter work one finds the first version of what was later dubbed the 'ontological argument' by Kant in his *Critique of Pure Reason*. There are several formalizations of Anselm's argument (→Priest, 1995; →Jacquette, 1997; →Oppenheimer and Zalta, 1991). What follows is a combination of some of the ideas behind the aforementioned formalizations. Let δ be a description operator that is neutral between a definite reading (*the* x) and an indefinite reading (*an* x).[3] Let ∃x (there is at least one x) and ∃!x (there *is* a unique x) be quantifiers ranging over a domain containing existent objects and possibly non-existent objects as well.[4] In contrast, let *E*! be the existence predicate. I will make use of a version of what Priest (1995, p. 63)

calls the ‘characterisation principle’ but which I call the ‘self-predication principle’:[5]

SP	δxφ → φ (δxφ)

Finally, let τ be a conceivability operator and > the relation of being greater than.[6] Then one formalization of Anselm’s argument goes as follows:

1.	∃x¬∃yτ(y > x)	Prem.
2.	∃x¬∃yτ(y > x) → ¬∃yτ (y > δx¬∃yτ(y > x))	Self-pred.
3.	∀x (¬*E*!x → ∃yτ (y > x))	Prem.
4.	¬*E*!δx¬∃yτ(y > x)	Red. ass.
5.	¬*E*!δx¬∃yτ(y > x) → ∃yτ (y > δx¬∃yτ(y > x))	UI, 3
6.	∃yτ (y > δx¬∃yτ(y > x))	MP, 4, 5
7.	¬∃yτ (y > δx¬∃yτ(y > x))	MP, 1, 2
8.	*E*!δx¬∃yτ(y > x)	Red., 4, 6, 7

In other words, that than which nothing greater can be thought exists. The problem with the above argument is that conceivability is an intensional notion (→Jacquette, 1997) and self-predication is unsound when intensional notions are involved (→Heylen, 2010a, b).

But our stated purpose is not first and foremost to engage in critical analysis of Anselm’s argument, but rather to examine the

conceptual issues. Consider again premise 3 of Anselm's argument: if something does not exist, then there is something that can be conceived to be greater than it. Likewise, one might think that: if something is not omniscient, then there is something that can be conceived to be greater than it. Then one can rerun Anselm's argument as follows:

1.	$\exists x \neg \exists y \tau (y > x)$	Premise
2.	$\exists x \neg \exists y \tau (y > x) \rightarrow \neg \exists y \tau (y > \delta x \neg \exists y \tau (y > x))$	SP
3.	$\forall x (\neg O x \rightarrow \exists y \tau (y > x))$	Premise
4.	$\neg O \delta x \neg \exists y \tau (y > x)$	Red. ass.
5.	$\neg O \delta x \neg \exists y \tau (y > x) \rightarrow \exists y \tau (y > \delta x \neg \exists y \tau (y > x))$	UI, 3
6.	$\exists y \tau (y > \delta x \neg \exists y \tau (y > x))$	MP, 4, 5
7.	$\neg \exists y \tau (y > \delta x \neg \exists y \tau (y > x))$	MP, 1, 2
8.	$O \delta x \neg \exists y \tau (y > x)$	Red., 4, 6, 7

In other words, that than which nothing greater can be thought is omniscient. The above argument can be criticized for the same reason as the previous one. But the point is to think about what omniscience has to mean in the context of the above argument.

Anselm himself describes the scope of divine knowledge of God in Chapter 6 of his *Proslogion* as follows:

> Ergo, Domine, quamvis non sis corpus, vere tamen eo modo summe sensibilis es, quo summe omnia cognoscis, non quo animal corporeo sensu cognoscit.

This has been translated as follows (→Anselm, 1998, p. 90):

> So it is, Lord, that although You are not a body You are supremely perceptive, in the sense that You know supremely all things and not in the sense in which an animal knows through a bodily sense-faculty.

Interestingly, Anselm talks about supreme knowledge, not complete knowledge. He talks about knowledge to the highest degree. He could also have talked about unsurpassed knowledge. In other words, an omniscient being is a being that at least knows everything that anyone else knows. One can formalize the property of x being omniscient, expressed by *O*x, in the sense of having supreme knowledge, in at least three different ways:

SK1	$\forall y(K_y\varphi \rightarrow K_x\varphi)$
SK2	$\forall y \forall z(K(y, z) \rightarrow K(x, z))$
SK3	$\forall y \forall p(K_y p \rightarrow K_x p)$

Note that SK1–SK3 are weaker than CK1–CK3, respectively. Knowledge is factive:

- $K(y, z) \rightarrow T(z)$
- $K_y p \rightarrow p$

Therefore, if one knows all truths, then one also knows everything known by someone else. But the other direction does not follow. If no one knows anything, then SK1–SK3 are true, but CK1–CK3 are false of everyone.

Perhaps the above interpretation is too weak, because it is compatible with multiple beings having supreme knowledge. In other words, it is compatible with an omniscient being that is not surpassing everyone else in its knowledge. One can formalize the property of having unsurpassed knowledge and surpassing everyone else's knowledge in at least two different ways:

1. $\forall y \neq x(\forall z(K(y, z) \rightarrow K(x, z)) \wedge \exists z(\neg K(y, z) \wedge K(x, z)))$
2. $\forall y \neq x(\forall p(K_y p \rightarrow K_x p) \wedge \exists p(\neg K_y p \wedge K_x p))$

Because of the extra conditions, SK2* and SK3* are stronger than SK2 and SK3, respectively. Interestingly, SK2* and SK3* are incomparable with CK2 and CK3. Suppose that everyone has complete knowledge. Then CK2 and CK3 are true of everyone, but SK2* and SK3* are false. Suppose that nobody knows anything about a certain class of truths, while about the rest there is someone who knows more than anyone else. Then SK2* and SK3* are true, but CK2 and CK3 are false of everyone.

Given the interpretation of omniscience as the property of having supreme knowledge, being omniscient is a comparative notion in the sense that it compares knowledge between agents: everything known by anyone is also known by the given person and, perhaps, everyone else does not know something that the given person does know. Consequently, being non-omniscient is also a comparative notion in the sense that it compares knowledge between agents: another knows something that the given person doesn't and, perhaps, everything that is known by the given person is also known by that other. With this reading one can start to make sense of premise 3 of the argument for the omniscience of that than which nothing greater can be thought. The consequent of premise 3 also contains a comparative claim in the sense that it compares agents: someone is greater than the given person. So, premise 3

connects one comparative claim about agents to another comparative claim about agents.

Of the two interpretations of omniscience as supreme knowledge, the second one is the more useful in the context of Anselm's argument. Indeed, non-omniscience in the sense of another knowing something that the given person doesn't, does not entail that the other person is epistemically greater than the given person. Suppose that the given person has unsurpassed knowledge about sports and the other person has unsurpassed knowledge about music and the first person is surpassed by the second person in knowledge about music and the second person is surpassed by the first person in knowledge of sports. But if it is also the case that the other person knows everything that the given person knows, then this other person is epistemically greater than the given person. This is not to say that not more work is needed to support premise 3, but the pertinent question here is whether one needs non-omniscience in the sense of incomplete knowledge.

It follows from earlier observations that the claim of non-omniscience in the sense of having incomplete knowledge does not entail non-omniscience in either sense of having surpassed knowledge. If it did, then SK2 and SK3 or SK2* and SK* would entail CK2 and CK3, respectively. But they do not. Non-omniscience interpreted as the property of having incomplete knowledge is not a comparative notion in the sense that it does not compare knowledge between agents. As such, it makes it relatively harder to make sense of premise 3, which does make a comparison between agents.

The point of these reflections is to carve out room for concepts of divine omniscience other than the concept of complete knowledge. Anselm himself talks about supreme knowledge, not complete knowledge, and there is an interpretation of supreme knowledge that makes more sense in

the context of one reconstruction of his ontological argument than the concept of complete knowledge does. In the next section we will see some arguments against the idea of an omniscient being. We will check whether concepts of incomplete knowledge and, in particular, versions of the concept of supreme knowledge are also useful in debates about the contra arguments.

4.3 Contra Omniscience

We will consider two kinds of arguments against divine omniscience: one kind of argument involves Cantor's theorem (Section 4.3.1) and another kind of argument involves Montague's theorem (Section 4.3.2). The arguments are targeted against the existence of an being that has complete knowledge.[7] We will also discuss how the concept of supreme knowledge may be of interest to those who would like to defend the possibility of omniscience in some sense.

4.3.1 The Set of All Truths and the Set of All Propositions

A first kind of argument against divine omniscience makes use of a key result from set theory. Let us first introduce some basic terminology (→Jech, 2006, pp. 11–12 and 27). A function $f : X \rightarrow Y$ is a one-to-one mapping of X into Y if and only if, for all $x,x' \in X$, if $x \neq x'$, then $f(x) \neq f(x')$. A function $f : X \rightarrow Y$ is a mapping of X onto Y if and only if, for all $y \in Y$ there is an $x \in X$ such that $f(x) = y$. A set X has a cardinal number, denoted $|X|$. The cardinal number of a set X is smaller than or equal to the cardinal number of a set Y, denoted $|X| \leq |Y|$, if and only if there is a one-to-one mapping of X into Y. The cardinal number of a set X is the same as the cardinal number of a set Y, denoted $|X| = |Y|$, if and only if there is a one-to-one mapping of X onto Y. The cardinal

number of a set X is strictly smaller than the cardinal number of a set Y, denoted $|X| < |Y|$, if and only if $|X| \leq |Y|$ but $|X| \neq |Y|$. Cantor's theorem states that the cardinal number of a set is strictly smaller than the cardinal number of its power set, namely the set of all the subsets of X (→Jech, 2006), pp. 27–28:

Theorem 3.1 (Cantor). *For every set* X, $|X| < |\wp(X)|$.

Grim (1988a, pp. 356–357) uses Cantor's theorem to argue for the following result:

Lemma 3.2 (Grim). *There is no set of all truths*.

Proof. Suppose for a reductio that there is a set T of all truths. By the power set axiom, it follows that there is also the power set $\wp(T)$, namely the set of all subsets of T. For every $T' \in \wp(T)$ and $t \in T$, $t \in T'$ is a truth or $t \notin T'$ is a truth (but not both). Therefore, $(t \in T') \in T$ or $(t \notin T') \in S$ (but not both). Consequently, there is a function

$$f : \wp(T) \to T : x \mapsto \begin{cases} t \in x \text{ if it is true that } t \in x \\ t \notin x \text{ if it is true that } t \notin x \end{cases}$$

Furthermore, if $x \neq y$, then $f(x) \neq f(y)$: if $S' \in \wp(S)$ and $S'' \in \wp(S)$ and $S' \neq S''$, then $t \in S'$, $t \in S''$, $t \notin S'$, $t \notin S''$ are all different truths. In other words, f is a one-to-one function. Hence,

$|\wp(S)| \leq |S|$. This contradicts theorem 3.1.

Grim continues the argument as follows:

1.	If there is an omniscient being, then there is a set of all truths.	
2.	There is no set of all truths.	Lem. 3.2
3.	There is no omniscient being.	MP, 1, 2

Grim's argument does not work against CK1. The latter is a scheme for which have as a rule that its metavariable φ can be substituted by any formula of the given language. A language is an enumerable *set* of nonlogical symbols and a sentence is a sequence of symbols (Boolos et al., 2007, p. 103). The sentences of a language also form a set. The true sentences of a language also form a set, namely a subset of the sentences of the language. One can accept the following counterpart of premise 1:

1. If $\exists x(\varphi \rightarrow K_x\varphi)$, for all $\varphi \in L$, then there is a set of true sentences of $\mathscr{L}$.

But the following counterpart of premise 2 is false:

1. There is no set of true sentences of $\mathscr{L}$.

Grim's argument also does not work against CK2. It is standard to let the domain of quantification be a non-empty *set*. The denotation of the truth predicate will then be, like for any other predicate, a subset of the domain of quantification. It is standard practice to let the denotation of the truth predicate be a subset of the set of Gödel numbers or the set of structural-descriptive names of the sentences of the given language. One can accept the following counterpart of premise 1:

1. If $\exists x\forall y(T\,(y) \rightarrow K(x,y))$, then there is a set that is the denotation of *T*.

But the following counterpart of premise 2 is false:

1. There is no set that is the denotation of *T*.

One may reply that this is all fine, but a being that has complete knowledge should also have knowledge of propositions that are

not expressed by sentences in a given language. The question is then whether there is a set of true propositions.

Surprisingly, Grim (1988a) does not discuss a much earlier paradox discovered by →Russell (1903, app. B) and →Myhill (1958).[8] The Russell-Myhill paradox is relevant for the question whether there is a set of true propositions. The paradox goes as follows (Zalta et al, 2014, pp. 799–800):

1. There is a set P of all propositions.
2. For every set S of propositions, there is a uniquely identifiable proposition p_S. For instance,
   $\forall q(q \in S \rightarrow q)$ is a uniquely identifiable proposition for a set S.
3. For sets S, S' of propositions, if $p_S = p'_S$, then S = S'. For instance,
   if $\forall q(q \in S \rightarrow q) = \forall q(q \in S' \rightarrow q)$, then S = S'.

By premise 1 and the axiom of separation, let $R \subseteq P$ be the set $\{q \mid \exists S \subseteq P(q = p_S \wedge q \notin S)\}$.

By premise 2, there is an uniquely identifiable proposition p_R for set R. Either $p_R \in R$ or $p_R \notin R$. Suppose that $p_R \in R$. Then $\exists S \subseteq P(p_R = p_S \wedge p_R \notin S)$. By premise 3, it follows from the first conjunct that R = S. It follows from $p_R \notin S$ and R = S that $p_R \notin R$. Contradiction. Suppose that $p_R \notin R$. Then $\forall S \subseteq P(p_R = p_S \rightarrow p_R \in S)$. Since $R \subseteq P$ and $p_R = p_R$, it follows that $p_R \in R$. Contradiction.

One possible conclusion from the Russell-Myhill paradox is that there is no set of all propositions. If that is correct, then there is also no set of all true propositions. Going back to Grim's argument, one can then say that the following counterpart of his premise 2 is true:

1. There is no set of all true propositions.

However, one may now doubt the following counterpart of his premise 1:

1. If $\exists x \forall p(p \rightarrow Kxp)$, then there is a set of all propositions.

After all, propositionally quantified formulas range over non-empty sets of propositions, but there is no assumption that the domain for the propositional quantifiers has to contain all propositions.

Perhaps the underlying thought is the following. There has been philosophical discussion about the possibility of unrestricted quantification—see, for instance, →Williamson (2003). Suppose that one is skeptical about the possibility of unrestricted quantification over propositions. If one thinks that genuine divine omniscience requires unrestricted quantification over propositions, then one has reason to be skeptical about genuine divine omniscience as well.

Let us return to the idea of omniscience as supreme knowledge. There is no commitment to the idea that God knows absolutely every truth. He just needs to have unsurpassed knowledge and, perhaps, surpass the knowledge of any other being. It is commonly accepted that non-divine agents are ignorant about some truths. After all, non-divine beings are presumably finite and there are infinitely many truths. There is no reason to think that the collection of everything known by any non-divine agent is 'too big to be a set.' There is therefore no reason to believe that the collection of everything known by a being with unsurpassed knowledge is too big to be a set.

4.3.2 The Paradox of the Divine Knower

A second kind of argument against divine omniscience makes use of a key result from the field of axiomatic truth theories. Let

us first introduce some basics. If φ is a sentence of a language extending the language, then ⌜φ⌝ is the Gödel code numeral for φ (Boolos et al., 2007, ch. 15, p. 220). Let Q be the system of Robinson arithmetic (Boolos et al., 2007, ch. 16). A crucial result in proof theory is the following (Boolos et al., 2007, p. 223):

Lemma 3.3 (Diagonal Lemma). *Let T be a theory containing Q. Then for any formula φ* (*x*) *there is a sentence γ such that* $\vdash_T \gamma \leftrightarrow \varphi$ (⌜γ⌝).

With the help of the above lemma, →Montague (1963) has proved the following result:

> **Theorem 3.4** (Montague). *Let T be a theory containing Q. Let T contain the following theorems*:
>
> 1. κ (⌜φ⌝) → φ
> 2. κ (⌜κ (⌜φ⌝) → φ⌝)
>
> *Suppose also that*
>
> 1. if $\vdash_T \kappa$ (⌜φ⌝) and $\vdash_T$ φ → ψ, then $\vdash_T \kappa$ (⌜ψ⌝)[9]
>
> *Then T is inconsistent*.
>
> The proof goes as follows:

1.	$\gamma \leftrightarrow \neg\kappa(\ulcorner\gamma\urcorner)$	Lem. 3.3
2.	$\kappa(\ulcorner\gamma\urcorner) \rightarrow \gamma$	Ass. 1 of thm. 3.4
3.	$\kappa(\ulcorner\gamma\urcorner) \rightarrow \neg\kappa(\ulcorner\gamma\urcorner)$	taut., 1, 2
4.	$\neg\kappa(\ulcorner\gamma\urcorner)$	taut., 3
5.	γ	taut., 1, 4
6.	$(\kappa(\ulcorner\gamma\urcorner) \rightarrow \gamma) \rightarrow \gamma$	taut., 5
7.	$\kappa(\ulcorner\kappa(\ulcorner\gamma\urcorner) \rightarrow \gamma\urcorner)$	Ass. 2 of thm. 3.4
8.	$\kappa(\ulcorner\gamma\urcorner)$	Ass. 3 of thm. 3.4, 6, 7
9.	$\bot$	taut., 4, 8

The choice of the κ symbol was guided by the fact that we are interested in the application of Montague's theorem to knowledge.

→Grim (1988b, pp. 13–15) reviews the assumptions of theorem 3.4 in light of epistemic readings of κ. First, he considers a reading of κ where it means 'is known by Grim.' Then the assumptions of the theorem are the following:

1. If something is known by Grim, it is so.
2. The previous claim is known by Grim.
3. If φ is known by Grim and if ψ is derivable from φ, then ψ is known by Grim.

He notes that assumption 1, namely the factivity of knowledge, is generally accepted for all epistemic agents. He thinks that assumption 2, knowledge of the factivity of knowledge, might also hold for him. But assumption 3 is false. It is a so-called 'closure principle for knowledge.' He admits that his knowledge

is not closed under deduction. Second, he considers a reading of κ where it means ‘is known by an omniscient God.’ Then the assumptions of the theorem are the following:

1. If something is known by an omniscient God, it is so.
2. The previous claim is known by an omniscient God.
3. If φ is known by omniscient God and if ψ is derivable from φ, then ψ is known by an omniscient God.

Now he thinks that condition 3 should hold. But theorem 3.4 shows that the above claims are jointly inconsistent.

Before we continue discussing the issue, let us first have a look at a simpler and more direct argument against God’s omniscience. Let S be the smallest theory extending Q that has $\forall x(K(x, \ulcorner\varphi\urcorner) \rightarrow \varphi)$ as a theorem and that is closed under the following rule: if $\vdash_S \varphi$, then $\vdash_S T(\ulcorner\varphi\urcorner)$. The theorem is a factivity principle for knowledge and the rule is the so-called ‘introduction rule for truth.’ Note that S is a consistent theory. Take the standard model of arithmetic and extend it by assigning the empty set to the knowledge predicate and the domain to the truth predicate. Then S is true in the extended standard model. The constant g will be used as a name for God. The argument against God’s omniscience goes as follows:

1.	γ ↔¬*K*(g, ⌜γ⌝)	Lem. 3.3
2.	*K*(g, ⌜γ⌝) → γ	Fact.
3.	*K*(g, ⌜γ⌝) → ¬*K*(g, ⌜γ⌝)	taut., 1, 2
4.	¬*K*(g, ⌜γ⌝)	taut., 3
5.	γ	taut., 1, 4
6.	*T* (⌜γ⌝)	Intr. *T*, 6[10]
7.	*T* (⌜γ⌝)∧¬*K*(g, ⌜γ⌝)	taut.
8.	∃y(*T* (y)∧¬*K*(g,y))	EG, 7

One might think that dropping omniscience in the sense of CK2 and replacing it by a notion of supreme knowledge is all it would take to take the sting out of the above arguments. But the issue is more subtle than that. Consider the above argument again. Stop at step 5. You may know each of the assumptions to get there. The reasoning is sound and you know the reasoning to be sound in the above steps. You come to know that γ.[11] You are not God. But then SK2 and SK2* are false:

∃y ≠ g∃z(*K*(y, z)∧¬*K*(g, z))

God has been surpassed with respect to knowledge of a particular truth. But it is more subtle still. We will in private rerun the above argument but now with another diagonal sentence that says that you don't know it. We come to know diagonal sentence. If God exists, he may know it as well.

It may help to think of it in terms of a game. The goal for each player is to know more than the other players do. In other words, the goal is to epistemically surpass the opponents. Let us say that the players are not allowed to lie. If a player knows that

φ, then a move can consist in asking: “Do you know that φ?” If the player who is asked the question does not know that φ, then that player can ask a question back. If the answer to a question is negative, then the player asking the question gets a point. Player a asks player b: “Do you know that γ_1?” Sentence γ_1 is the following diagonal sentence: $\gamma_1 \leftrightarrow \neg K(b, \ulcorner \gamma_1 \urcorner)$. Player *b* has to truthfully answer: “No.” Player a scores a point. Then it is the turn of player b, who asks: “Do you know that γ_2?” Sentence γ_2 is the following diagonal sentence: $\gamma_2 \leftrightarrow \neg K(a, \ulcorner \gamma_2 \urcorner)$. Player a has to truthfully answer: “No.” Player b scores a point. Both player a and player b scored a point. Neither player beats the other player.

The game described above is misleading because, if player a knows something about drama that player b does not know and if player b knows something about guns that player *a* does not know, then this may cancel each other out. First, the pertinent cases are not just any cases of truths unknown by an agent but rather cases of truths that cannot consistently be known by an agent. The diagonal sentences are prime examples of the latter. →Sorensen (2022, Section 5.4) talks about “blindspots”:

> A blindspot for a propositional attitude is a consistent proposition that cannot be accessed by that attitude. Blindspots are relative to the means of reaching the proposition, the person making the attempt, and time at which he tries.

(The quote is about propositions, not sentences, but a true and, hence, consistent sentence that cannot be consistently known by an agent could also be called a ‘blindspot’.) Second, the pertinent cases are not just any cases of truths that cannot consistently be known by a particular agent but rather cases where for each agent there is a truth of the same type that cannot consistently be known by that agent. Again, the diagonal

sentences are paradigmatic examples of such a type of truth. We could talk about all agents having the same type of blindspot.[12]

4.4 Conclusion

Omniscience is standardly conceived as complete knowledge, which may be formalized in different ways (CK1–CK3). However, there are also conceptions of omniscience that are consistent with incomplete knowledge. For instance, →Langtry (2008) and →Nagasawa (2017) have made proposals in this direction. However, their proposals face problems (→Oppy, 2010; →Kvanvig, 2018). A promising approach is to think of omniscience as supreme knowledge. The latter may be understood as having unsurpassed knowledge, which can also be formalized in different ways (SK1–SK3). Alternatively, it may be understood as having unsurpassed knowledge and surpassing everyone else's knowledge, which too can be formalized in different ways (SK2*–SK3*). The idea of thinking of omniscience as supreme knowledge was inspired by how Anselm describes divine knowledge. It also helped to make sense of a key premise in an Anselmian argument for divine omniscience. We have also looked at two arguments against divine omniscience (→Grim, 1988a, b): one based on Cantor's theorem (thm. 3.1) and one based on Montague's theorem (thm. 3.4). The conception of omniscience as supreme knowledge helps to alleviate worries about the collection of everything known by God being too big to be a set. We have seen that there is reason to amend the concept of supreme knowledge in response to an extension of a variation on the divine knower paradox. In short, if something is a 'blindspot' for an agent, while it is not a blindspot for another agent, then that this does not mean that the first agent has been epistemically surpassed

by the second agent, if the second agent has the same type of blindspot that is not a blindspot for the first agent.

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Notes

1 Each of these formalizations has also been used in the debate about the Church-Fitch paradox of knowability (with the difference that human knowledge is the object of interest in that debate). →Fitch (1963, thm. 4, pp. 138–139) used the counterpart of CK3. →Fischer (2013, thm. 1, p. 66) has used the counterpart of CK1. →Horsten (2011, pp. 88–90) has used a variation on CK2, in which *K*(x, y) is replaced by *T*(Ky) (where the latter is a sloppy but readable formal notation that in principle should be replaced with a formula using Gödel coding).

2 This is reminiscent of →McGee (1992) who showed that there are multiple maximal consistent but mutually inconsistent sets of instances of Tarski's disquotational truth scheme.

3 Priest (1995) and Jacquette (1997) make use of δx, whereas Oppenheimer and Zalta (1991) make use of a definite description operator ιx (the x).

4 Priest (1995), Jacquette (1997) and Oppenheimer and Zalta (1991) all agree on this.

5 Priest (1995) did not make self-predication (φ(δxφ)) conditional on ∃xφ, whereas Oppenheimer and Zalta (1991) do. If δx is replaced by ιx, then that would make 'the round square is round' true, as →Russell (1905, p. 491) pointed out. One may add that it would also make 'the round square is square' true and, hence, 'the round square is round and square,' which is a contradiction. It

would also make very easy ontological arguments possible: an/the existing … exists. Jacquette (1997) does not use a self-predication principle, but rather an extensionality principle for descriptions: ∀x(δy…y… · x → … x …). On the condition that ∃x … x …, one can presumably instantiate the extensionality principle with δy … y … and presumably it also follows as well that δy … y… = δy … y … In that case the self-predication principle follows from the extensionality principle.

6 Priest (1995) and Oppenheimer and Zalta (1991) use a unary predicate to express the property of being conceivable, whereas Jacquette (1997) uses a unary operator.

7 For this reason, →Nagasawa (2017, pp. 92–93) thinks that his concept of a 'maximal God' (see Section 4.1), which is consistent with God having incomplete knowledge, creates room for those theists who are willing to accept the conclusions of those arguments.

8 →Sobel (2003, pp. 119–120) does give a Cantorian argument against the existence of a proposition that is about all propositions. →Sobel (2003, pp. 389–392), critically discusses the argument.

9 This is a condition that is entailed by two conditions of Montague's original theorem. It turns out that it suffices to prove the conclusion.

10 Sentence γ does not contain the truth predicate.

11 Presumably adding $K(a, \ulcorner K(g, \ulcorner \gamma \urcorner) \rightarrow \gamma \urcorner)$ (with $a \neq g$) would not result in inconsistency, even in the presence

of the closure rule.

12 It is not the same as what →Sorensen (2022, Section 5.4) calls a “universal blindspot.”

5 Omnipotence and the Power to Make Evil Choices

Wes Morriston

Abstract: I distinguish between the *power to choose* and the *conditional power to do what one has chosen to do*. With this distinction in mind, I make a case for saying that maximal power overall includes a power that no morally perfect being could have—viz., the power to choose evil—, and that no morally perfect being could therefore have maximal power overall. I defend this argument against the traditional objection that "choosing to do evil" is not a power but a mere liability. Finally, I consider the implication of saying that moral perfection is one of God's essential properties. I conclude that, at the very least, maximal power (and therefore omnipotence) is inconsistent with *contingent* moral perfection.

5.1 Introduction

Many philosophically minded theists hold that God is a personal being who necessarily exists and necessarily possesses the best possible combination of perfections. Omnipotence and perfect moral goodness are generally reckoned to be among those perfections, and God is said to possess them in every possible world.

A few contrarians have objected that this combination of essential attributes is not possible.[1] In order to be omnipotent, they say, God must be able to do evil. But then there must be possible worlds in which he does evil. *In those worlds* he is

obviously not morally perfect. So either moral perfection is not one of God's *essential* attributes, or he is not omnipotent.

Suppose (if only for the sake of argument) that the contrarians are right about this. It might seem that there is a fairly painless way to deal with this dilemma. Why not simply withdraw the claim that God is morally perfect in every possible world and substitute for it the (still quite impressive) claim that God is always—as a matter of contingent fact—morally perfect? Why not concede that God could do evil, whilst denying that he ever actually does any?[2]

I do not think that the problem can be dealt with in this way. To see why, consider the following pair of theses:

1. If God is morally perfect, then he lacks the power to *choose* to do evil.
2. If God is omnipotent, then he has the power to *choose* to do evil.

If both theses are true, it follows straightaway that God's moral perfection is incompatible with his omnipotence *whether or not* either of these features is among his essential attributes.

Since I'll be saying quite a bit in favor of both *A* and *B*, you might expect me to conclude that omnipotence and moral perfection are incompatible, full stop. I won't go that far, however, since (as we shall see) there may be a way to block my argument for thesis *A* when moral perfection is taken to be one of God's *essential* attributes. I'm not entirely persuaded by the blocking maneuver, but there is (as I'll explain later) a respectable case to be made for it. So the thesis I settle for in this paper is the weaker one that *contingent* moral perfection is incompatible with omnipotence. It's a small point—but not, I think, an insignificant one. It means that a theist can't get away

with saying: “Sure, God *could* do evil; but that’s ok because he never actually does any.”

5.2 Moral Perfection and Choice: The Case for Thesis *A*

I don’t have a complete analysis of the concept of moral perfection, but this much I take to be obvious. A morally perfect being must be unsurpassable in its love of the good and it must never choose to do evil. If it would be wrong for a morally perfect being to do something it has the power to do, then the being will know that this is so, and will refrain from choosing that option.[3]

Refraining from evil choices cannot be difficult for such a being. Since it perfectly instantiates all the virtues appropriate to a being of its type,[4] a morally perfect being does not have to struggle with temptation. Indeed, I think we can see that evil choices would be psychologically impossible for a morally perfect being. Let me explain.

We all have the power to do things that we cannot *bring ourselves* to do. *Choosing* to exercise our power to do them is psychologically impossible for us. For example, there is an obvious sense in which I probably *could* drink a cup of paint. But as things are, I could not bring myself to do such a thing. If you promised to have my stomach pumped immediately, assured me that my gastrointestinal tract would not be seriously damaged, offered me a billion dollars to drink the paint, and I believed you, then perhaps I could get myself sufficiently motivated to go through with it. But as things are now, I have not the slightest inclination to drink paint, no reason to do so, and an irresistible aversion to the prospect. Given my present desires, values, and beliefs, I could not choose to drink paint. If a general principle is desired, it might go something like this: If, given all of a person’s

beliefs, desires, and values at a time *t*, (i) she has no reason to do *A* (or, if she does have such a reason, it carries no weight with her), (ii) she has no inclination to do *A*, and (iii) she has a very strong aversion to doing *A*, then she cannot—at *t*—choose to do *A*.

Now, then, let *A* be a profoundly immoral action—torturing and murdering another person, say. Many people (perhaps most of us) have the power to do *A* in the sense I have in mind here. What I mean is that *if* one truly set one's mind to it, one *would* succeed in creating and carrying out a plan for capturing, torturing, and murdering another person. Fortunately, very few of us have the slightest inclination to do this sort of thing. We aren't morally perfect, and we are often tempted to do lesser wrongs, but we are not at all tempted to do this one. Given our beliefs and desires and values, torture and murder are simply not among our live options. Indeed, if a person is even moderately decent, it is not psychologically possible for her to make such a choice.

What I want to suggest next is that the closer a person approximates to the ideal of moral perfection, the fewer will be the evils that count as live options for that person, and that if one realized the ideal perfectly, none of one's live options would be evil.

To see this, it will help to do a small thought experiment. Imagine two people, Jack and Jill. Jack has not the slightest inclination to inflict severe pain on other people. He is, we may suppose, strong enough and clever enough to inflict quite a lot of pain, but his sympathetic nature makes it psychologically impossible for him to choose to do so. Jill, on the other hand, has a malicious disposition that she struggles to keep in check. She often enjoys the pain of other people and is frequently tempted to cause pain purely for her own amusement. However, she knows that such behavior is very wrong; and although she could

at any moment give in to this temptation, she always chooses to resist it.

Now, then, which of the two—Jack or Jill—has the better moral character? On Jill's side, it can be said that she faces, and overcomes, challenges that Jack does not have to worry about. That she does overcome them is certainly impressive. On the other hand, the challenges Jill faces arise from a severe moral defect in her character, whereas the fact that Jack does not face those same challenges is explained by the moral excellence of his character. All else equal, it seems clear to me that Jack has a better moral character than Jill. Resisting nasty temptations is a very fine thing. But a moral character that excludes nasty temptations altogether is even finer. It is the ideal for which one should strive, and it is surely closer to the kind of character a morally perfect God would have.

Even if it is only contingently true that God is morally perfect, this is all we need to conclude that he cannot choose to do evil. So great would be his love of the good, so strong his aversion to evil, and so perfect his knowledge of both, that he could not even be *tempted* to do evil.[5] Given his perfect knowledge and character, it must therefore be psychologically impossible for him ever to choose evil. So, even if there are remote possible worlds in which God is less than morally perfect, the fact remains that in the actual world, not only does he refrain from doing evil, but his character is such that he cannot choose to do it.

By way of illustration, consider the following state of affairs.

1. *Innocent creatures enduring intense suffering forever.*

God may, in the weak sense specified above, have the power to actualize *S*. If he tried to do so, he would succeed. But I take it to be a necessary truth that if God is morally perfect, then he *cannot choose to exercise* any such power.

This completes my defense of thesis *A*—of the claim that it would be psychologically impossible for a morally perfect being to choose to do evil. But what about thesis *B?* Why think that an omnipotent being must have the power to choose to do evil? More specifically, why think that if God is omnipotent, then he must be able to choose to actualize a state of affairs like *S?*

5.3 Omnipotence and Choice: The case for Thesis *B*

Most philosophical theists would insist that even an omnipotent being could not square the circle or make a contradiction true. As Aquinas famously said, we should say of such things, not that God is unable to do them, but rather that they simply cannot be done. The trouble is that actualizing *S* doesn't seem to fall within the class of things that simply "cannot be done." So why would an omnipotent being not be able to bring it about that *S* obtains?

Aquinas held that God is only *conditionally* able to do evil—that his ability to do it should be understood only "on a condition, the antecedent of which is impossible—as, for instance, if we were to say that God can do evil things if he will."[6] On this view, what follows from God's perfect goodness is not that he lacks the conditional power to do evil, but only that he cannot choose to exercise that power.[7] If, per impossibile, God were to choose to do evil, he would succeed in doing it. But given the perfection of his nature, God cannot bring himself to make any such choice.

If this is right, then we should say that God has the conditional power to actualize states of affairs like *S*, but that his moral perfection prevents him from ever choosing to exercise his power in that way. This will allow us to combine the claim that God has the maximum possible degree of "power on a condition" with the claim that God is morally perfect.

So far, so good. But is the maximum possible degree of conditional power sufficient for omnipotence? I doubt it. If God cannot choose to exercise his conditional powers in certain ways, then I am inclined to say that he does not have the maximum possible degree of power overall (and so is not omnipotent[8]). To test your intuition about this, think of a person who has the conditional power to create a stone of any size, but who lacks the power to *choose* to create a stone weighing more than (say) ten thousand pounds because she has an overwhelming aversion to stones weighing more than that. Such a person would have the conditional power to create such stones, but would be unable to exercise that power. Surely we do not need to know more in order to conclude that this person does not have the maximum possible degree of power overall. Why so? Because, without any *other* change in the limits of her power, she could have been free of that overwhelming aversion, in which case she would have had more alternatives to choose from. In that sense, she would have had greater power of choice, and therefore greater power overall.

We are now ready to see how the argument goes in the case of a morally perfect God. Let α designate the actual world, and let "Yahweh" be a rigid designator for the Supreme Being in α, and suppose that Yahweh is morally perfect in α. Now consider again the terrible state of affairs, *S* ("innocent creatures enduring intense suffering forever"). Yahweh, we may suppose, has the maximum possible degree of "power on a condition," but is unable to bring himself to exercise his power in such a way as to actualize *S* and its ilk because the moral perfection of his nature prevents him from choosing to exercise his power in that way. The question is: does Yahweh, in α, have the maximum possible degree of power *overall*?

To answer this question, consider another putatively possible world, β—a world in which Yahweh has as much "power on a

condition" as he possesses in α, but in which he is able to choose to exercise any of his conditional powers, regardless of whether it would be good or evil for him to do so. In β, Yahweh may or may not in fact choose to actualize *S*, but the perfection of his nature does not prevent him from doing so. It follows—does it not?—that Yahweh has *greater power of choice* and therefore *more power overall* in β than in α?

If you are not yet persuaded, consider a third world, γ. In γ, Yahweh has as much power "on a condition" as it is possible to have, but he has an extremely nasty motivational structure. In γ, not only is he tempted to actualize *S*—he is unable to resist that temptation. Surely everyone would agree that in γ Yahweh does not have the maximum possible degree of power overall? But then, say I, the same goes for Yahweh in α.[9]

The argument can be conveniently summarized as follows:

1. A being having the maximum possible degree of power must have enough conditional power to actualize *S*.
2. A morally perfect being must be unable to choose to exercise its power in that way.
3. If a being is unable to choose to exercise its conditional power in that way, then it does not possess the maximum possible degree of power overall.
4. So, a morally perfect being must lack the maximum possible degree of power overall. (from 1, 2, 3)
5. An omnipotent being must have the maximum possible degree of power overall.
6. Therefore, no morally perfect being is omnipotent. (from 4 and 5)

As it stands, the argument says nothing about the modal status of God's moral perfection. So it might seem that if its premises (1, 2, 3, and 5) are true, we are entitled to conclude that

omnipotence is incompatible with moral perfection *tout court*, in which case, of course, it would be incompatible with *contingent* moral perfection.

As noted earlier, there *may* be a way out for those who say that moral perfection is among God's *essential* properties. I'll say more about this in the final section of the paper. But first I'd like to consider a more traditional response to the argument—one that, if sound, would be available regardless of what one says about the modal status of God's attributes.

5.4 The Ability to Make Evil Choices Is Not a Power

A typical medieval response (and one that is still endorsed by many) would simply be to deny that the ability to make evil choices is a genuine power. According to this view, the "ability" to go wrong is a mere *liability*—one that afflicts ignorant, weak, and error-prone creatures like ourselves. God's freedom from temptation, his perfect knowledge of the good, and his consequent "inability" to choose evil do not therefore detract from his power. Rather, they combine to enhance it. Indeed, it might be said that the maximum possible degree of power logically requires freedom from ignorance and temptation, which in turn makes it impossible for a maximally *powerful* being to choose evil.

How, exactly, is this a response to the argument outlined above? A first thought might be that something has gone wrong at the very first step. If there is no *power* to do evil, then maximal power cannot involve any such power. That can't be quite right, though—certainly not for Aquinas, who explicitly held that God "can do evil things *if he will*" (my italics). On this view of things, God has the power to do evil—but only on a condition that can never be satisfied—viz., God's *willing* evil.

I think that the “mere liability” view might more plausibly be thought of as providing an objection to step 2. The idea would be that it is not strictly correct to say that a morally perfect being is *unable* to *choose* to exercise its conditional power to do evil. The proper thing to say is that neither a morally perfect nor an omnipotent being could *suffer from a liability* to fall into sin and error.

In support of this suggestion, it is sometimes said that no one ever does or can choose evil merely for evil’s sake—that in even the worst of our choices we have some good in view. If, for example, someone chooses to set fire to a cat because he gets pleasure from hearing it scream in pain, he is pursuing something that is intrinsically good, viz., his own pleasure. And that this is so, even though, all things considered, it is hideously wrong to pursue pleasure in that way.

But even if it is true that no one can choose evil merely for the sake of doing *evil*, I don’t see why there couldn’t be a genuine *power* to choose things that it would be wrong—*necessarily* wrong—for anyone to choose. Consider again the terrible state of affairs, *S* (innocent creatures enduring intense suffering forever). It would be wrong for God to actualize *S*, but I do not see why his choosing to do so would not be a genuine exercise of power. To be sure, if God is morally perfect, then actualizing *S* is not a live option for *him*. *He* cannot choose to actualize it. But I do not see why choosing to actualize *S* would not be an *act*, nor why performing such an act would not count as a genuine exercise of power.

If this point is granted, then step 3 of the argument seems natural enough. All else equal, God’s *power of choice* would be greater if he could choose to exercise his power to do evil. So if his moral perfection diminishes his power of choice by limiting the range of options amongst which he can choose, it diminishes

his power overall—in which case, he lacks *maximal* power overall.

This is not (of course) to deny that God would be more *perfect* overall if he were unable to make evil choices. But one should not rule out a priori the possibility that the best possible combination of attributes excludes maximal power. Judgments about what makes a person *more (or less) perfect* must not be confused with judgments about what makes a person *more (or less) powerful*.

After all, no one would deny that the inability to choose things it would be *good* to do diminishes power. Why, then, should we not say the same of the inability to choose to do things that it would be wrong to do?[10] Traditional theists may, if they like, give the latter inability the honorific title, "perfect will" or "strength of will," but the fact remains that the wonderful moral nature underlying such perfection and strength *limits the range of possibilities* among which one has the power to *choose*. And this brings with it a lesser degree of power *overall*.

I have argued that the ability to make evil choices is relevant to power comparisons, and that it cannot be written off as a mere liability. And it is worthy of note that *many theists* are poorly positioned to disagree with me about this. The reason is that they subscribe to a version of the free will defense, according to which we are free—in a robust, libertarian sense—to choose between good and evil. If the ability to make evil choices were only a regrettable weakness, then God would not be justified in permitting those evil choices. He should instead have made us free to do what is right whilst liberating us from this wretched "liability" to go astray. That would be very bad news indeed for free will defenders.

5.5 But What If Moral Perfection Is One of God's *Essential* Features?

At the outset I assumed, if only for the sake of argument, that the contrarians are right in thinking that omnipotence is incompatible with *essential* moral perfection. I wanted to consider whether retreating to the position that moral perfection is one of God's contingent properties is a viable option for theists. It has turned out not to be, since there is quite a strong argument for saying that, even if it is one of God's merely contingent features, moral perfection is incompatible with choosing to exercise a genuine power. If that's right, then it may seem to make no difference whether God's omnipotence and moral perfection are among his essential properties. Either way, moral perfection is incompatible with omnipotence.

As I have already suggested, however, there may be a way out for theists who hold that moral perfection is one of God's essential features. Here's how it might work. Suppose, as is commonly claimed, that God is necessarily existent. Then nothing inconsistent with the existence of a being having all of *his* essential features is truly possible. In this sense, God is a "delimiter of possibilities." If, then, he is morally perfect in every possible world, then there can be no possible world in which he chooses to exercise a conditional power to do evil—in which case, it might be argued, we are free to reject premise 3 of my argument for incompatibility ("If a being is unable to choose to exercise one of its conditional powers, then it does not possess the maximum possible degree of power").

To see why 3 might be rejected, again let "Yahweh" again be a rigid designator for the being who is God, and consider the following instantiation of 3.

1. If Yahweh is unable to choose to exercise one of his conditional powers, then Yahweh does not possess the maximum possible degree of power.

If Yahweh is morally perfect, then the *antecedent* of 3' is true. He has the conditional power to do evil, but is unable to choose to exercise his power in that way. However, since moral perfection belongs to his essential nature, the consequent of 3' may nevertheless be false. If there is no possible world in *Yahweh* exercises a power to do evil, then we cannot conclude that *he* could have been more powerful than he is.

Still, one might wonder: might it not have been the case that some *other* being has greater power to choose whilst possessing the same degree of conditional power as Yahweh? If so, then 3 (and 3') are true after all, and the argument for incompatibility is vindicated.

Again, this is much too quick. Suppose that Yahweh has maximal conditional power plus maximal power to choose in ways consistent with his moral perfection, that these are among his essential properties, and, finally that Yahweh is necessarily existent. Then it may also be plausible to suppose that there is no possible world in which Yahweh has a serious rival in the "power department." But then the powers of the rival I've imagined are not the powers of any *possible* being, and our intuition that such a being would have more power than Yahweh fails to show that there *could have been* a being more powerful than Yahweh. So, then, Yahweh may after all have the maximum *possible* degree of power overall whilst lacking the power to choose to exercise his conditional power to do evil.

This is quite a challenging response, but I have never been quite persuaded by it. Building power limits into one's idea of God's nature, and then claiming that these limits are consistent with omnipotence because God necessarily exists and necessarily has those limits has always seemed like a bit a cheat to me. Why, I wonder, couldn't a necessarily existent being's power be *essentially* limited in such a way as to make it a poor

candidate for omnipotence? To see what I have in mind, consider the following whimsical example.

Imagine a “demonist” who holds that the creator and ruler of the universe is omnipotent and omni-malevolent. When it is pointed out that the Demon cannot exercise his conditional power to perform acts of kindness, the demonist replies that the Demon is necessarily existent, and that omni-malevolence is one of its *essential* properties. He quickly concludes that there is no possible world in which anyone has more power than the Demon has. Should we be satisfied with such a response? Or should we continue to insist that the Demon lacks the maximum possible degree of power? I should think so; but then why, I wonder is the case different for a morally perfect God?

I expect most traditional theists to be unmoved by this consideration. They will point out that it is God, not the Demon, who is necessarily existent; that God, not the Demon, is the supreme delimiter of possibilities; and that our intuitions about the degree of power that would be possessed by a necessarily non-existent being if (per impossibile) it did exist are wholly irrelevant to the analysis of omnipotence.

I am not entirely sure who has what burden of proof at this point. A lot depends on issues about the relationship between consistent conceivability and metaphysical possibility, on whether we think maximal power should be understood in terms of consistently conceivable power or in terms of metaphysically possible power, and on just how much modal skepticism we are willing to let theists get away with. Depending on how those issues are settled, it might or might not be right to conclude that omnipotence and moral perfection are incompatible tout court. Either way, though, the modest thesis of *this* little paper stands: *contingent* moral perfection is incompatible with omnipotence.

References

Aquinas, Thomas (1945), *The Summa Theologica*, Volume I, Article 3 (“Whether God is Omnipotent?”) in *The Basic Writings of Thomas Aquinas*, Volume I (edited by Anton C. Pegis), Indianapolis, IN: Hackett Publishing Company. →

Pike, Nelson (1969), “Omnipotence and God’s Ability to Sin,” in *American Philosophical Quarterly* 6, 208–216. →

Notes

1 See →Pike (1969) for the locus classicus of this position.

2 This is quite a natural first thought, and it is in fact the response I encounter most often in classroom.

3 I do not, of course, mean to deny that a morally perfect being could cause intrinsic evils for the sake of greater goods. The point is rather that a morally perfect being could not deliberately choose to do something it believes to be, all things considered, evil. This qualification will make no difference to the argument of this paper.

4 I add this qualification, because there are virtues that human beings can and should aspire to, but which are not appropriate to a being of the sort God is generally supposed to be. For example, one might wonder whether an omnipotent, omniscient being could ever need to be courageous.

5 Jesus is said to have been “tempted as we are.” He is also said not to know certain things. In my view, what follows is that Jesus was neither morally perfect nor omniscient. There must be some implication here for the high Chalcedonian view of Incarnation, but I won’t try to draw it out here.

6 →Aquinas (1945), 264.

7 Aquinas, of course, also holds that such a “power” would be a defect and therefore incompatible with

omnipotence. I discuss this claim briefly below.

8 Whatever else omnipotence may be, it surely entails the maximum possible degree of power overall.

9 Those who believe that God is morally perfect in every possible world will object that what Yahweh can or can't do in β and γ is irrelevant because they are not genuinely possible worlds. I'll have a bit to say about this presently. For the moment, I'll just note in passing that this move is not available to those who say that moral perfection is one of God's *contingent* properties. *They* have no principled reason for denying that β and γ are genuinely possible worlds, and it is their view that is the *principal* target of this paper.

10 The ability to do evil may well presuppose the presence of desires and inclinations that make a person worse overall. But so what? I refer the reader back to my first observation.

6 Divine Eternality

R. T. Mullins

Abstract: The majority of models of God affirm that God is eternal, but divine eternality can be understood as either timeless or temporal. In this essay, I shall consider the following questions: What is God? What is time? Is God timeless or temporal? Is time created by God, or is time an aspect of the divine nature? What was God doing before He created the universe? This essay will introduce readers to thinkers and debates from across the world's religions.

6.1 What Is God?

I understand the concept of God to be that of a perfect being which is the ultimate foundation of reality. A model of God is a particular conception that specifies a unique set of claims about what it is to be perfect, and what it is to be the ultimate foundation of reality.

I start with reflections on perfection. Most models of God affirm that a perfect being is a necessarily existent being with essential attributes like eternality, aseity, self-sufficiency, maximal power, maximal knowledge, maximal goodness, perfect rationality, and perfect freedom. These are relatively uncontested divine attributes, whereas other attributes are often debated such as simplicity, immutability, mutability, impassibility, and passibility. For this essay, my interest is in the debate over the best understanding of eternality.

A being is eternal if and only if it exists without beginning and without end. This follows straightforwardly from necessary

existence since a necessarily existent being cannot fail to exist. When someone wishes to affirm that God is timeless or temporal, they must add further claims to eternality.[1] God is timeless if and only if God exists without beginning, without end, without succession, without temporal location, and without temporal extension. A timeless being cannot undergo any sort of change, be it intrinsic or extrinsic, since any kind of change will bring about succession in the divine life.[2] A timeless being cannot stand in any kind of temporal relations with other events such as being earlier-than, simultaneous with, or later-than because standing in such relations entails having succession and temporal location.[3] In this context, having temporal extension means persisting over a series of temporal moments. As various medievals would explain things, God stands in a timeless present that lacks a before and after.

Divine temporality sees things differently. God is temporal if and only if God exists without beginning and without end, and God can undergo succession, have temporal location, and temporal extension. Proponents of divine temporality differ over various details. For example, William Lane Craig says that God is accidentally temporal. God is timeless sans creation and temporal with creation.[4] Others, like William Hasker, Richard Swinburne, and Nicholas Wolterstorff, say that God is essentially temporal.[5] They maintain that God undergoes succession as He freely exercises His power to create and sustain a universe.

This brings me to the next piece of the puzzle for a model of God—divine foundationalism. God is the foundation of reality in that God is the ultimate synchronic and diachronic cause of the existence of all things apart from Himself.[6] Opinions differ over how best to flesh out this claim. For example, there is a common distinction between emanation and creation. An emanation is when certain things necessarily flow from the divine nature,

whereas a creation occurs when God freely wills that certain things exist. For this chapter, I will focus on the difference between eternal creation and creation ex nihilo.

> *Eternal Creation*: An eternal creation occurs if and only if (i) a particular universe, set of universes, or all possible universes are freely caused to exist by God, and (ii) there is no state of affairs where God exists without a particular universe, set of universes, or all possible universes.
>
> *Creation Ex Nihilo*: A creation ex nihilo occurs if and only if (i) a particular universe, set of universes, or all possible universes are freely caused to exist by God, and (ii) there is a precreation moment or state of affairs where God exists without a particular universe, set of universes, or all possible universes.

The differences between these two views hangs on whether or not there is a precreation moment. The doctrine of creation ex nihilo is widely affirmed by Jews, Christians, and Muslims.[7] To be clear, this doctrine teaches that the universe is not co-eternal with God because the universe has a beginning. This is in contrast to the once popular Aristotelian and Plotinian view that the cosmos is co-eternal with God.[8] On the once popular Aristotelian understanding of modality, anything that is eternal is also necessary.[9] As Peter Adamson explains, for the medieval philosophers, "To say that the world is not eternal is to deny that God's relation to the world is a necessary one."[10] What this means is that Jewish, Christian, and Muslim philosophers who denied the co-eternality of the cosmos with God wished to affirm God's freedom over creation, the unique status of God as necessary and eternal, and the contingency of the created order.[11] Hence, the widespread affirmation that there is a state of affairs where God exists without the universe.[12]

As John of Damascus explains, "it is not natural that that which is brought into existence out of nothing should be co-eternal with what is without beginning and everlasting."[13]

Following the lead of Augustine, Emil Brunner says that God creates time along with the universe. Brunner is clear that God and the universe are not co-eternal because the universe has a beginning.[14] Brunner asks what was before the universe. He answers, "Before Time there is God Alone."[15] Brunner admits that this is somewhat paradoxical because the *before* is a pre-temporal before.[16] I will return to this paradox in due course.

Before discussing the paradox, it would be good to say a bit more about eternal creation, and its emphasis on the freedom of God and the contingency of the universe. It might seem difficult to understand how there could be divine freedom and contingency on a doctrine of eternal creation. For example, Avicenna is happy to bite the bullet and affirm an emanationist scheme.[17] However, others wish to affirm the contingency of the universe or multiverse. For example, within the Islamic tradition, Ibn Taymiyya and Averroes reject creation as a necessary emanation from God. They also reject the doctrine of creation ex nihilo that is affirmed by Kalam theologians like al-Ghazali which says that there is a state of affairs where God exists without the universe.[18] Instead, they affirm that God is eternally creating, and thus has always existed with a universe of some sort.[19] Within Hinduism it is difficult to find the doctrine of creation ex nihilo, whereas it is easy to find doctrines of eternal creation and emanation on which God always exists with a universe of some sort.[20] For instance, Rāmānujā affirms that God is the ultimate cause or ground of the universe, and that God always exists with a universe of some sort.[21] The contemporary Christian theologian, Thomas Jay Oord, affirms something similar. On Oord's understanding, God need not exist with any particular universe, but God must create a universe of some sort. God has always been creating, and has never existed without a universe

of some sort.[22] Thus God has some freedom, and there is some contingency.

6.2 What Is Time?

Marcello Oreste Fiocco complains that contemporary philosophy of time does not attempt to answer the fundamental question, "what is time?" Most debates in the philosophy of time focus on different kinds of issues about the direction of time, tensed vs. tenseless theories, what moments of time exist, and so on.[23] Those issues have played an important role in debates over God's relationship to time, but it seems curious that people are debating God's relationship to time without first addressing what time is. Most discussions start by quoting Augustine saying that he does not know what time is, and then carry on from there. That is less than helpful.

One thing that most contemporary philosophers of time do agree on is that whatever time is, it must have a close relationship with change.[24] Yet the precise relationship between time and change is not entirely clear. By way of example, consider D. H. Mellor's reflections on the matter. Mellor says, "Change here I take to be temporal variation in the properties of things. By this I mean that changes are things having, at different times, incompatible properties, i.e. properties that no one thing could have at the same time."[25] According to Mellor, "Time is certainly the dimension of change in this sense: variations must be temporal changes."[26] Yet Mellor says that this is an unsatisfactory account of time because mere variation does not distinguish space from time.[27] Though Mellor thinks that there is a close relationship between time and change, he seems to think that merely invoking change is insufficient to capture exactly what time is.

The two most prominent theories on what time is are called the *relational theory* and the *absolute theory*. The absolute theory of time starts with time as a fundamental substance in our ontology, and constructs the rest of the temporal world from there. The relational theory of time wishes to be more ontologically parsimonious by refusing to posit that time is actually a thing. The general idea is to reduce time away to non-temporal entities in our ontology, and try to construct the rest of the temporal world from there. Yet there is a deep ambiguity within the relational theory. W. H. Newton-Smith comments that the relational theory of time has been so attractive to physicists and philosophers that they have not really bothered to articulate it and defend it. They just think the position must be true.[28] Yet the lack of articulation leads to a theory that is so nebulous that one wonders what the theory could possibly be. According to John Earman, "there are almost as many versions of relationalism as there are relationalists."[29]

Despite the ambiguity, there is one common assumption that seems to be shared by all relationalists. The relational theory starts with the assumption that time exists if and only if change exists. Different relational theories will say different things about how to reduce time to change, or how to make time parasitic on change without introducing time as a genuine thing into our ontology. Yet one will rightly ask how is the reduction of time to change being made.

One version of the relational theory of time holds that time cannot exist without change because time just is change. The reduction here is quite straightforward, yet this runs into an immediate problem.[30] One will need to ask what a change is, and the answer will need to avoid sneaking in time since the relational theory is trying to reduce time away to change. The problem here is that change is standardly accounted for in terms of time, such as saying that change is an object having

incompatible properties at different moments of time.[31] If the goal is to avoid introducing time into one's ontology, then this version of the relational theory has not been successful. Hence, the relational theory ought not to say that time simply is change.

Consider another popular version of the relational theory called event-relationalism. The general idea here is that we can abstract time from events. This is a reductionist view which says that time is reduced to a relationship between events. Events are the more fundamental entities in our ontology that give rise to our notions of time.

Yet this also seems to raise problems. This reductionist strategy needs to explain the relationship between events in non-temporal terms. Otherwise, they are sneaking in temporal notions when they are meant to be explaining where our temporal notions come from. The problem is that it is difficult to explain the relationships between events without relying on temporal notions like *before* and *after* or *simultaneous*. So again, time is snuck back into the story when time was supposed to be explained by non-temporal things.

According to Robin Le Poidevin, relationalism says "that time is just an ordered series of events, each individual moment identified with a collection of simultaneous events."[32] Newton-Smith says that the suggestion from the proponent of this relational theory of time is that moments of time are constructed out of events that are simultaneous with each other.[33] As Dean Zimmerman explains, a moment of time just is the sum of all the events occurring then.[34] That sounds somewhat confusing because it already has some sort of temporal notion built into the definition. In Zimmerman's definition, it assumes the temporal notion *then*, whereas in Le Poidevin and Newton-Smith's definition, it uses the temporal notion of *simultaneous*.

This seems hopelessly circular because it has already presupposed time.

A relationalist might respond by saying that there is no problem here. Perhaps she can say that our temporal notions like *then* and *simultaneous* are primitive. The relational theorist is not denying our primitive temporal notions, but instead merely denying that time is a thing in our ontology.[35]

Yet someone like Ulrich Meyer will find this response less than satisfying. According to Meyer, the problem is not that we are forced to use primitive temporal notions. The real problem is that event-relationalism lands us in ontological circulatory. Meyer says, "While relationists claim that times are abstractions from events, Strawsonian accounts of eventhood suppose that events are composed, amongst other things, of times. If events are made up of times, then times cannot also be made up of events."[36]

To see this circularity more clearly, consider E. J. Lowe's discussion of events. Lowe points out that events are things that *occur* or *happen*. Lowe says, "This tells us, indeed, that events are *temporal* entities … an event is a *temporal particular*: an item which happens or occurs at a particular time and is not repeatable … an event just *is* the exemplification of a property by an object at a time."[37] Meyer's point is that standard definitions of events like Lowe's presuppose the existence of time, thus introducing a circularity problem. The circularity problem is that events have an occurrence or a when, thus presupposing the existence of time. Hence, there appears to be a vicious circularity here in the relational theory's understanding of time.[38]

Given these kinds of problems, one may wish to consider the absolute theory of time. Looking to the East, Jainism says that time is an eternal substance, the function of which is to explain the existence of things in the present, but also to explain the

existence of change and movement.[39] Jainism distinguishes between time itself and conventional time. Conventional time consists of an infinite number of temporal atoms, or what one can call instants or moments. These are the smallest possible units of conventional time. Each time atom, or moment, is distinct and cannot be mixed with other moments. This is because moments are ordered successively. However, we can carve out larger units of conventional time such as minutes, hours, and years. These divisions of conventional time into different, measurable units are dependent upon external factors like the movements of the planets and stars.[40] The moments, or time atoms, themselves are dependent on time. Every change is accompanied by a moment of time, but time itself is the continuing reality underneath these changes and underneath the series of moments.[41] Time is what makes change possible, and what makes the moments progress.

In the Nyaya-Vaiśeṣika school of Indian philosophy, time is also taken to be a substance. Time is an eternal and uncaused substance that is the cause and basis for the notions of priority, posteriority, simultaneity, and succession.[42] As Debika Saba points out, the Nyaya-Vaiśeṣika school takes time to be a substance that is independently real. Time is not contained in anything, but rather is all pervasive. Time is the necessary antecedent or condition for any event, movement, or change. Saba writes,

> For any causal operation the idea of absolute time is accepted as a necessary presupposition… Any causal operation is necessarily an event in time. No event is conceivable without time as its *adhara* or receptacle. According to Nyaya-Vaiśeṣika philosophy, to deny time an objective reality is to face a static universe where there is no room for any change or movement.[43]

Further, this school of thought distinguishes between time and moments of time. The divisions of time into moments like past, present, and future are not essential features of time itself. Instead, those are features of the moments themselves.[44] The temporal order of moments is not reversible. In other words, unlike space, time has one direction. According to Saba, "A moment is the point of time that refers to the final phase of cause and the initial phase of effect."[45]

Fiocco offers a similar analysis of time that distinguishes between time itself and moments of time.[46] Following his lead, I say that time is (i) a natured entity that makes change possible, (ii) the ontological source of moments, and (iii) that which orders a set of successive moments into a coherent timeline. I take a moment of time to be the way things are but could be subsequently otherwise. A moment is a *when* things or events happen. Moments can be thought of as abstract or concrete. For example, one might say that moments of time are nearly maximal proposition-like entities that can obtain or fail to obtain.[47] These proposition-like entities have built within them earlier-than and later-than relations with other possible moments.[48] Moments can be successively ordered in many different ways. A timeline is a particular coherent ordering of moments. Most philosophers say that the particular ordering of moments is a contingent matter.[49] A theologian might say that whichever particular order of moments comes about is due to the providential activity of God. As I understand things, time itself does not necessarily have a metric. A metric can only apply to a timeline if certain other features obtain. In particular, there need to be uniform laws of nature that provide the basis for consistent, periodic changes in order for there to be a metric.[50]

In describing this successive series of moments as a timeline, I am in no way ruling out Eastern philosophical reflections on the

nature of time. You have probably heard it said that Westerners see time as linear whereas Easterners see time as circular. However, this is a misinterpretation of actual Indian understandings on the nature of time.[51] According to Anindita Niyogi Balslev, this constant slogan that Easterners believe in cyclical time thwarts interreligious communication because it falsely assumes that Judeo-Christian religions have a radically different conception of time from Indo-Hellenic religions.[52] Hinduism, for example, does believe in a pattern of birth, death, and reincarnation, but such events occur successively. One universe comes into existence, and then it is destroyed, and then another universe is brought into existence and so on. These events are all successively ordered into a coherent timeline. As Balslev makes clear, Hindu philosophers had many debates about the nature of time. Is time absolute or relational? Are the moments of time discrete or continuous? Those were questions that were fiercely debated. Yet no one debated if time is actually a circle instead of linear.[53] It was simply assumed that time is linear.

With the relational and absolute theory of time set before us, I will now consider several issues concerning God's relationship to time.

6.3 Timeless or Temporal?

The major problem for divine timelessness is to understand how a God without succession and temporal location can be present to a world filled with succession. I will focus on two problems here related to creation and omnipresence.

The Problem of Creation starts with the assumption of the doctrine of creation ex nihilo. The doctrine of creation ex nihilo says that there is a precreation moment where God exists all alone. It is difficult to see how God does not change once the

universe comes into existence. As Augustine tells us, before God made heaven and earth, God was not doing anything.[54] It seems like God is not doing anything, and then God begins to create a universe. Here is one way to state the problem. Call this the Problem of Creational Change.

(PCC1)	If God is timeless, then God is changeless.
(PCC2)	If God begins to create the universe, then God changes.
(PCC3)	God begins to create the universe.
(PCC4)	Therefore, God changes.
(PCC5)	If God changes, then God is not timeless.
(PCC6)	Therefore, God is not timeless.

(PCC3) seems difficult to deny. The ninth-century Islamic group called the Karrāmīs embraced the conclusion of divine temporality. As did twelfth-century thinkers like Faḫr ad-Dīn ar-Rāzī and Abū Barakāt al-Bagdādī.[55] Yet (PCC3) is the typical premise that proponents of divine timelessness deny.[56] The claim is that while creation ex nihilo is true, God does not begin to create the universe. Instead, God is eternally creating the universe. This is because a timeless being is timelessly doing whatever He is doing.[57] A timeless being cannot begin to do anything.

Yet this response leads to other problems. Call this the Problem of Eternal Creation. It can be formulated in several ways, but I will offer a formulation that focuses on causation because it brings out parallel reasoning in several areas of Christian theology.

(PEC1)	If God is eternally causing X to exist, then X eternally exists.
(PEC2)	God is eternally causing X to exist.
(PEC3)	Therefore, X eternally exists.

If one places “the universe” in for X, then one gets the conclusion that the universe eternally exists. In which case, one will have abandoned the doctrine of creation ex nihilo and adopted the doctrine of eternal creation. Various Christian thinkers like Augustine and Aquinas wish to resist eternal creation. They claim that God is eternally causing the universe to exist, but that the universe somehow has a temporal beginning.[58] Somehow the universe is not co-eternal with God even though God is eternally causing the universe to exist. Bonaventure says that it is beyond our understanding as to how God can eternally cause created things to exist without those things being co-eternal with God.[59]

This is going to be an unsatisfactory reply for many. Someone like Avicenna will say that it is not beyond our understanding, but instead is unintelligible. If God is eternally causing something to exist, then that thing will eternally exist as well.[60] For people like Proclus and Oord, God is eternally causing the universe to exist, and thus the universe is co-eternal with God. Proclus and Oord are explicit that this involves denying the doctrine of creation ex nihilo.[61]

People like Augustine, Aquinas, and Bonaventure are placed in a tight spot here since they do accept that there are cases of eternal causes with eternal effects. Within the classical doctrine of the Trinity, it is said that God the Father eternally causes God the Son to eternally exist.[62] Avicenna and Proclus can complain that Augustine and Bonaventure are engaged in special

pleading. If they accept eternal causes with eternal effects in the Trinity, there is no good reason to deny this in the case of the universe.

A proponent of divine timelessness might be tempted to think that she can just abandon the doctrine of creation ex nihilo in order to maintain timelessness. Yet this does not automatically save divine timelessness since there are other problems to consider. God is said to be causally sustaining the universe in existence from moment to moment. God is also said to be omnipresent. The exact way to articulate omnipresence is controversial. I will stick with the least controversial definition. God is omnipresent via knowledge and power in that God is directly causing each moment to exist, and is aware of each moment that exists.[63] According to proponents of divine timelessness, God is not absent from any moment of time.[64] For example, Anselm claims that God exists at all moments of time.[65] This generates the Problem of Omnipresence.

Start with the assumption of omnipresence.

(PO1) God is located at all moments of time.

Next, consider the following conditionals.

(PO2) If God is present at a moment of time, then God has temporal location.

(PO3) If God has temporal location, then God is temporal.

These are very plausible premises to accept. Fabrice Correia and Sven Rosenkranz take it as axiomatic that if something exists at a time, then it has temporal location, which entails that it is

temporal. As they would put it, if something has temporal location, it is a "resident of time."[66] They have no theological axe to grind in their work on the philosophy of time, so it will be difficult to claim that they are begging the question against divine timelessness. From (PO1), we can infer,

(PO4) God is located at a moment of time.

From (PO2) and (PO4), we get,

(PO5) Thus, God has temporal location.

From (PO3) and (PO5), we get,

(PO6) Thus, God is temporal.

Atemporalists have several moves they can make. One strategy is to say that "present" in (PO2) is ambiguous. Perhaps there is some special, non-temporal sense of "present" being used that gets the atemporalist off the hook. This has been suggested by Eleonore Stump and Norman Kretzmann.[67] However, contemporary atemporalists and temporalists have largely rejected this as vacuous.[68]

One thing that is perplexing here is that divine timelessness says that God does not have any temporal location. One will find it curious that atemporalists widely accept (PO1) and refuse to see the inference to (PO6). What atemporalists maintain is that

God exists *at* every moment of time, but does not exist *in* time. The medieval at/in distinction might be a way to deny (PO2) and (PO3). However, as Rory Fox points out, this distinction was taken to be obvious throughout the Middle Ages, but was left vague and largely unexplored.[69] I take it as obvious that a vague distinction is not terribly helpful in defending a view. Elsewhere I have argued that the distinction is vacuous and leaves the atemporalist position looking exactly like the divine temporalist position.[70] Here is why. On T. J. Mawson's defense of timelessness, he says that the at/in distinction amounts to the following claim: If time did not exist, then God would not exist at any moment of time, and if time exists, then God would exist at any given moment of time. This is indistinguishable from the divine temporalist view of Craig who says that God is timeless sans creation, but temporal with creation. As Mawson says, "Mullins is unconvinced" that there is any meaningful distinction to be had here between existing *at* a time but not being *in* time.[71] I will leave it up to the reader to decide if she is convinced or not.

There is a deeper worry here that has never been adequately addressed by atemporalists. All of the atemporalist claims are assuming that there can be timeless causes with temporal effects. Sam Baron and Kristie Miller think that the entire idea of timeless causation is impossible. Without time, causation cannot exist.[72] By their lights, the best account of timeless causation that one might be able to offer is that of a purely counterfactual analysis of causation.[73] Yet, there are good reasons for thinking that a counterfactual analysis of causation doesn't work because absences cannot have causal power.[74] Moreover, a purely counterfactual analysis of causation will be of no use to any model of God, creation, and sustaining since creation and sustaining involves God producing a universe.

6.4 Does God Create Time?

I wish to return to an earlier discussion about the relational vs absolute theory of time. A common assumption is that God is the creator of all things apart from Himself. Proponents of the relational theory of time, like Augustine and Craig, typically say that God creates time with the universe.[75] They can run an objection against proponents of the absolute theory of time because the absolute theory says that time is an eternal, uncaused substance. Call this the Creator Objection.

(CO1)	God is the creator of all things distinct from Himself.
(CO2)	Time is an uncreated thing distinct from God.
(CO3)	If time is an uncreated thing distinct from God, then God is not the creator of all things distinct from Himself.
(CO4)	Thus, God is not the creator of all things distinct from Himself.
(CO5)	Thus, God is the creator of all things distinct from Himself and God is not the creator of all things distinct from Himself.

This is a contradiction that needs to be removed. One strategy for removing the contradiction comes from al-Razi who denies (CO1). On his view, God is not the creator of all things distinct from Himself. For al-Razi, there are five eternal, uncaused substances that exist: God, the world soul, prime matter, absolute space, and absolute time.[76] Al-Razi maintains that the world soul creates the universe at a particular moment, but nothing creates time because time is not the sort of thing that can be created.

This view will most likely be seen as less than satisfying to contemporary thinkers. To start, this view goes against the concept of God that I articulated earlier. The concept of God is that of a perfect being which is the single, ultimate foundation of reality. On al-Razi's view, God is not the single, ultimate foundation of reality. Further, one might complain that this is a rather bloated ontology with far too many eternal, uncaused substances floating around.

Raghunātha Śiromoṇi shares the worry that this is a bloated ontology, but he is unwilling to abandon the absolute theory of time. He proposes a different strategy for avoiding the contradiction by rejecting (CO2). Śiromoṇi was unconvinced that there is enough justification to infer that time is an independent, eternal, and uncaused substance. Instead, he identified time with God.[77] Part of his argument is that it is more ontologically parsimonious to posit one independent, eternal, and uncaused substance than two. God is the sort of being who can fulfill all the roles of time. God is an independent, eternal, and uncaused substance who supports the existence of everything else and makes change possible. God can explain the existence of moments, explain why things exist now, and explain why certain events exist in earlier-than relations with other events. Time is nothing but God.[78]

Identifying time with God is a major theme within Indian philosophy, but it also has precedence within Western thought.[79] In reflecting upon early modern British philosophy, Emily Thomas distinguishes between three different kinds of absolute theories on time.[80] First, is what she calls Morean absolutism, named after Henry More. This view says that time is identified with God. God and time may be conceptually distinct, but in reality they are the same being. As I have argued elsewhere, this Morean absolutism is able to avoid the contradiction by

identifying time with God.[81] Second, there is a Gassendist absolutism, named after Pierre Gassendi. This view can maintain that God and absolute time are independent substances, though time depends upon God. This view avoids contradiction by denying (CO2). Third, Newtonian absolutism, named after Isaac Newton. This view makes no connection between God and absolute time. This view might not be able to avoid the Creator Objection. However, Thomas points out that Newton himself is a Morean absolutist. In fact, she argues that a majority of the British absolutists about time were Morean.[82]

6.5 What Was God Doing Before He Created the Universe?

There is a puzzling question that often comes up in discussions about God and time. What was God doing before He created the universe? Augustine jokingly says that God was creating hell for people who ask those kinds of questions.[83] The apostle Paul seriously says that, prior to creation, God was plotting to take over the world.[84] This question about what God was doing before creation is an interesting question in itself, but it is closely related to another question that can cause problems for various views. Why didn't God create the universe sooner?

With regards to the first question, call this the Sooner Objection. The Sooner Objection can come in different forms, but one popular form tries to show that there is an incompatibility between God's perfect rationality and His choice to create the universe when He did. If God is perfectly rational, it might seem like God would have a good reason for creating when He did. What could that reason possibly be? If there is no good reason, then it might seem like God has no reason to

create at all. Here is a version of the Sooner Objection inspired by G. W. Leibniz.[85]

(SO1)	God is perfectly rational =def. (i) God always performs an action for a reason, and (ii) in the absence of a reason, God will not perform an action.
(SO2)	God could have created the universe sooner.

In Leibniz's context, he is taking aim at Samuel Clarke. Clarke is a proponent of the Morean absolute theory of time that we encountered earlier, and he accepts (SO2).[86] On the surface, this might not seem so bad. However, with a bit more precision, one can derive a contradiction from the assumptions that are built into (SO2). In saying that God could have created the universe sooner, one is assuming that God created the universe at some particular moment of time. Call this moment of time t^x, and assume that t^x is later than some earlier moment of time t^y. In affirming (SO2), one is also assuming that it is possible for God to have created the universe at this earlier moment of time t^y. If God could not have created the universe at t^y, then God could not have created the universe sooner than t^x. Hence, in affirming (SO2), one will also affirm the following.

(SO3)	God creates the universe at time tx.
(SO4)	God could have created the universe at time ty instead of tx.
(SO5)	God cannot create the universe at both t^y and t^x.

With this more precise statement of what seems to be in mind in (SO2), the argument can continue.

(SO6)	If time t^y and time t^x are equally good moments for God to create the universe, then God does not have a reason to choose either t^y or t^x over the other at which to create the universe.
(SO7)	Time ty and time tx are equally good moments for God to create the universe.
(SO8)	Thus, God does not have a reason to choose either ty or tx over the other at which to create the universe.
(SO9)	If God does not have a reason to choose either ty or tx over the other at which to create the universe, then God does not create the universe at either ty or tx.
(SO10)	Thus, God does not create the universe at either t^y or t^x.

From (SO10), one can derive,

(SO11)	God does not create the universe at t^x.

From (SO11) and (SO3), one can derive,

(SO12)	God does not create the universe at time t^x and God creates the universe at time t^x.

This is a clear contradiction, so a theist will need to reject one of the premises in the argument. For Augustine and Craig, the

answer to the Sooner Objection involves rejecting (SO2). For Augustine and Craig, God creates time with the universe. Why didn't God create sooner? For Augustine and Craig, there is no sooner because there is no time in the precreation moment where God exists without the universe.[87]

Those who deny that God creates time might find it more difficult to address the Sooner Objection since they seem committed to something called *pure absolute duration*. Pure absolute duration says that time necessarily flows from moment to moment to moment.[88] To illustrate this point, consider a popular thought experiment from the seventeenth and eighteenth century used by proponents of this absolute theory of time. It goes a little something like this.[89]

Creation and Annihilation Thought Experiment: Imagine that God existed without a universe for a thousand years. Then God creates a universe, sustains the universe in existence for a thousand years. At some particular moment, God decides to annihilate the universe. Thus, bringing about a moment where God once again exists without the universe. Then, a thousand years later, God creates a new universe.

Proponents of the absolute theory of time thought that this scenario was possible. Al-Razi says that this is self-evident, and only people who have been spoiled by theology could deny it.[90] Within Indian philosophy, various thinkers say this scenario is a reality. For an eternity past, there have been a successive series of universes that God oversees.[91] Each universe that exists is said to last for a day of Brahman, and then is destroyed. A single day of God is given different calculations, and in one case is said to last for 4,320 million years.[92] As the day of Brahman draws to an end, things happening within history become worse, and evil and chaos spread.[93] Then God destroys the universe. After which, there is a period of time before the next universe is made.

During this in between period, God and numerous souls exist, yet these souls are said to be in a quiescent state until they are reincarnated again in the next universe.[94]

Those in the modern West who rejected the absolute theory of time denied the coherence of this creation and annihilation thought experiment. Notice that this version of the thought experiment simply assumes pure absolute duration—time necessarily flows at an equal tenor. Nothing here demonstrates that time must necessarily flow, and that it's flow must be at an equal tenor. One will need an argument for that view, and I am uncertain of what such an argument would look like. For my purposes, it does not matter if you think this is coherent. I simply need it to illustrate the commitment of Newton and Clarke to absolute time's essential flow. This commitment to pure absolute duration makes it difficult to answer the Sooner Objection.

One possibility is to reject (SO7) in the argument. One could say that times t^x and t^y are not equally good moments for God to create the universe. Clarke suggests that "the wisdom of God may have very good reasons for creating this world, at that particular time he did."[95] However, Clarke doesn't give any hint as to what those very good reasons might be. Brian Leftow raises doubts that there could be any good reasons. As Leftow points out, "If we abstract from what occurs at different moments, moments of time are identical save for their position... But to prefer some particular position as a place for a universe to start would seem irrational, for seemingly nothing about that position would make it an especially appropriate point for a universe to begin."[96]

Another possibility is to reject (SO6) in the argument. Clarke grants that God could have created the world sooner than He did.[97] Yet, Clarke sees no reason to affirm that agents must

refrain from acting when confronted with two equally good choices. In fact, Clarke claims that Leibniz has offered us no proof for thinking that this principle is true. As Clarke points out, free agents can and do act on the basis of strong motivating reasons and weak motivating reasons. When it comes to cases where there are two or more equally good ways of acting, Clarke says that agents can freely choose one course of action because an agent has good reason for acting in the first place.[98] This needs to be unpacked a bit further.

Someone like Leibniz will try to justify (SO6) by appealing to a principle of sufficient reason that covers contrastive reasons, but someone like Clarke will reject that the principle of sufficient reason must cover contrastive reasons. A contrastive explanation is one that refers to the specific alternative possibilities available. For example, why choose A instead of B? A contrastive reason will be one's reason for choosing A instead of B. A non-contrastive explanation gives a plausible reason for some state of affairs, but it need not make any reference to other possibilities. Various philosophers maintain that fully contrastive explanations are incompatible with free actions because such actions would be necessary.[99] According to someone like Clarke, if the principle of sufficient reason did cover contrastive reasons, the will of agents would be reduced to necessity, and thus not be free. God is free to create or not create, and thus God's reasons for creating must not be fully contrastive. A philosopher like Clarke could go further and point out that Leibniz's demand for fully contrastive reasons lead to the notion that God not only must create, but must create the best possible world, thus calling into doubt God's freedom.[100] This line of thought has some potential, but it needs to be developed.

Nicholas Wolterstorff seems to be taking this strategy. When confronted with the question "Why didn't God create sooner?,"

Wolterstorff says that he is unimpressed with the associated argument. He sees nothing that conflicts with rationality in most cases if one does not have a reason for performing an action at one precise moment instead of another. He writes,

> At a certain moment I decide to get out of bed in the morning. No doubt I have a reason for deciding to get out of bed; but do I also have a reason for deciding to get out *at just that moment*, rather than earlier or later? Or if at 6:15 I decide to get out at precisely 6:26 on my clock, do I have a reason for making the decision at 6:15 rather than earlier or later? So far as I can tell, I have no such reason; at a certain moment I just decide to get up. You ask me why not a second earlier, or a second later. I have no reason—just as I also often have no reason for walking to the right around some puddle rather than to the left, or vice versa.[101]

Wolterstorff thinks that he is perfectly rational for acting in this way without contrastive reasons, and says that things should be no different for God.[102] Wolterstorff's reply seems to suggest something like the following. God has a good reason to create the universe. Among all the possible moments of time that God could have chosen to create, any moment is just as good as any other. God already has a good reason to create, and many equally good moments to create. It shouldn't really matter which moment of time God chooses because they are all equally good. Is this irrational? This line of thinking says no. God has a good reason to create at any of those times because they are all good. Rationality need not demand a reason to pick one good moment over another equally good moment.

Al-Ghazali offers a similar sort of analysis of rationality and action. In his mind, we all make arbitrary decisions, and he doesn't think that this undermines our rationality. Al-Ghazali infers that we shouldn't think that God's perfect rationality is undermined just because He arbitrarily decides when to create the universe. Al-Ghazali gives us a thought experiment to get

our intuitions pumping in his favor. He asks us to consider a man who is presented with two equally delicious dates. Which bit of fruit will the man chose to eat? There is no reason to choose one over the other, so al-Ghazali says that the man arbitrarily chooses one of the dates. Surely no one will think that this man is irrational.[103] Why then think that God is irrational for arbitrarily deciding to create the universe at one moment instead of an equally good moment? Perhaps there is nothing here to undermine God's rationality in this case.

Is this a decent reply to the Sooner Objection? Averroes does not think that this is a good reply to the Sooner Objection because he thinks that perfect rationality rules out arbitrary decisions. Furthermore, Averroes thinks that al-Ghazali's analysis of the dates case is all wrong. According to Averroes, the man who chooses one date over the other is not acting arbitrarily. Averroes maintains that when one is in a Buridan's Ass-like situation one's choices are not really between two equally delicious dates. In this situation, one's choice is really about eating or not eating, or about having a desire fulfilled or go unfulfilled. The man chooses to eat one of the dates because if he doesn't choose one of them, his desire for a date will go unfulfilled.[104]

Sam Lebens offers a similar analysis of rationality. He explains that "a perfectly rational agent does not do something for no good reason—unless there is a good reason motivating the agent to do something for no good reason. Buridan's ass should not do a dance if it doesn't want to, and if it has no other good reason to do so. But it *should* arbitrarily choose one stack of hay over the other, since it *should* try to avoid starvation."[105] I gather that the idea is something like this. God cannot perform an *utterly* arbitrary act, or an act for no good reason whatsoever. God can, however, perform a *nested* arbitrary act. This is a seemingly arbitrary act that is nested within an overarching

course of actions, and that overarching course of actions is motivated by good reasons. The good reasons that motivate the overarching course of actions can motivate the need to perform an arbitrary act in order to prevent the frustration of the overarching course of actions. What this means is that a nested arbitrary act is really an act that an agent has some reason to perform.

When it comes to the Sooner Objection, a proponent of the identification view might just say that God has good reason to arbitrarily pick one moment of time instead of any other moment. God's action in this case is a nested arbitrary action. All of the moments are equally good for creation, and God has a good reason in general to create. If God does not just chose one of the moments, His entire rational plan will be frustrated. So God has a good reason to arbitrarily choose one of the moments. In which case, the act is not utterly arbitrary, but is instead nested within an overall rational course of actions. Given this, one will reject (SO6) as failing to grasp this more nuanced understanding of rational action. Contrary to (SO6), God does have good reason to pick one moment of time over the other for creating the universe.

Al-Razi offers a different strategy for avoiding the Sooner Objection. He rejects (SO3) which says that God creates the universe at time t^x. This is because he rejects the claim that God creates the universe. According to al-Razi, there is no good reason for God to create the universe at any particular moment. Any such choice would be arbitrary, and a perfectly rational God cannot perform arbitrary actions. For al-Razi, the soul of the world creates the universe. The world soul is an eternal, uncaused being who is foolish and ignorant of the future. This being can perform arbitrary actions, and in fact just arbitrarily picks a random moment to create the universe.[106] To be sure, many modern thinkers will see no need to posit a world soul,

and will wish to look elsewhere for an answer to the Sooner Objection. Yet, al-Razi will push back and say that his view has some motivation. When we look at all of the evil in the world, doesn't it make more sense to say that the universe is the product of a foolish soul than the product of a perfectly rational God?[107]

Personally, I think that a proponent of the absolute theory of time has a better option to consider. She should reject (SO2) which says that God could have created the universe sooner.[108] This involves denying the doctrine of pure absolute duration, but I see no good reason to affirm that time must necessarily flow from moment to moment. In fact, I think it undermines standard arguments for the absolute theory of time.[109] As I understand things, a moment of time is the way things are but could be subsequently otherwise. The transition from one moment to the next occurs if and only if things become subsequently otherwise. Prior to creation, there is God all alone. That is a single moment that never began to exist.[110] There is a way that things are—God existing all alone. Things could be subsequently otherwise if God freely exercises His power to bring about a universe. This precreation moment is earlier-than any possible subsequent moment that God might bring about.[111] At this precreation moment, any effect that God might wish to causally bring about will have to occur at subsequent moments. On this view, God cannot create any earlier. The earliest that God can create is the second moment of time, which is immediately after the precreation moment.

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Notes

1 →Mawson, 2018, p. 3.

2 →Lombard, 2007, Distinction VIII and XXXVII.7; →Helm, 2010, pp. 19–20, 81–87; Moreland and Craig, 2017, pp. 526–527.

3 →Moad, 2015, p. 435; →Helm, 2019, pp. 286–288.

4 →Craig, 2001a.

5 →Swinburne, 2016; →Wolterstorff, 2010; →Hasker, 1994; →Hasker, 2020.

6 →Bohn, 2019, p. 2.

7 →Blowers, 2012, Chapter 4; →Booth, 2017, pp. 65–67 and 133–135; →Lebens, 2020, pp. 31ff.

8 →Booth, 2017, p. 66.

9 →Booth, 2017, p. 102.

10 →Adamson, 2016, p. 77.

11 →Adamson, 2016, pp. 77–78.

12 →Broadie, 2010, p. 53; →Burrell, 1993, p. 7; →Fergusson, 2014, p. 40.

13 →Damascus, 1898, I.7.

14 →Brunner, 1952, p. 14.

15 → Brunner, 1952, p. 15.

16 → Brunner, 1952, p. 15.

17 → Booth, 2017, pp. 101–105.

18 → Moad, 2015, pp. 435–438.

19 J. → Hoover, 2019, p. 122.

20 → Flood, 2020, pp. 7–8.

21 → Bartley, 2002, p. 74.

22 Cf. → Oord, 2015a.

23 → Fiocco, 2017, p. 45.

24 → Meyer, 2013, p. 4.

25 → Mellor, 2012, p. 169.

26 → Mellor, 2012, p. 169.

27 → Mellor, 2012, p. 169.

28 → Newton-Smith, 1980, p. 7.

29 → Earman, 1989, p. 13.

30 → Poidevin, 2003, pp. 13–17.

31 → Newton-Smith, 1980, p. 15.

32 → Poidevin, 2003, p. 27.

33 → Newton-Smith, 1980, pp. 6–7.

34 D. →Zimmerman, 2010, p. 793.

35 Thanks to Robin Le Poidevin and Samuel Lebens for suggesting this rejoinder to me.

36 →Meyer, 2013, p. 15.

37 →Lowe, 2002, pp. 228–229.

38 →Gale, 2002, p. 66.

39 Radhakrishnan and Moore, 1957, p. 256.

40 →Saba, 2009d, pp. 724–725.

41 →Saba, 2009d, p. 725.

42 Radhakrishnan and Moore, 1957, pp. 390–404.

43 →Saba, 2009, pp. 661–662.

44 →Saba, 2009, p. 662.

45 →Saba, 2009, p. 663.

46 →Fiocco, 2017, p. 58.

47 →Byerly, 2014b, p. 77.

48 Correia and Rosenkranz, 2018, pp. 31–33. Cf. →Meyer, 2013.

49 →Byerly, 2014b, p. 78.

50 →Lucas, 1973, p. 311; →Swinburne, 1993, pp. 208–209.

51 →Balslev, 1992, p. 170.

52 →Balslev, 1992, p. 176.

53 →Balslev, 1992, p. 177.

54 →Augustine, 2001, XII.14.

55 J. →Hoover, 2022, pp. 94–95.

56 →Dolezal, 2017, pp. 97–104.

57 →Rogers, 2000, pp. 54–55.

58 →Augustine, 1991, V.17; →Augustine, 1958, XII; →Aquinas, 1934, II.35.

59 See Bonaventure's commentary on *The Sentences*, book 1, d.1, a.1, q.2.

60 →Acar, 2010, p. 82.

61 For discussion of Proclus' argument, see →Philoponus, 2004, pp. 42, 50, and 64; →Oord, 2022, p. 216.

62 →Ayres, 2010, pp. 264–265. Cf. →Mullins, 2017.

63 →Rogers, 2006, p. 8.

64 →Mawson, 2005, p. 49.

65 →Anselm, 2008, Book 22.

66 Correia and Rosenkranz, 2018, p. 31.

67 Stump and Kretzmann, 1981.

68 →Deng, 2018, pp. 20–26.

69 →Fox, 2006, pp. 311 and 324–327.

70 →Mullins, 2016a, p. 149.

71 →Mawson, 2018, p. 27.

72 Baron and Miller, 2019, p. 116.

73 Baron and Miller, 2019, pp. 165–167.

74 →Goldschmidt, 2016.

75 →Augustine, 1958, Book XI.6; →Augustine, 2001, Book 11.30.40; →Craig, 2001a, p. 272; →Craig, 2001b, p. 186.

76 Adamson, Al-Razi, 2021, pp. 25–29.

77 →Saba, 2009, p. 663.

78 →Williams, 2017, pp. 626–629.

79 →Balslev, 1992, p. 166.

80 →Thomas, 2018, pp. 9–10.

81 →Mullins, 2020.

82 →Thomas, 2018, p. 10.

83 →Augustine, 2001, XII.14.

84 Ephesians 1:4–10.

85 →Alexander, 1956, p. 39.

86 E.g. Samuel Clarke (→Alexander, 1956, p. 49). As several people have pointed out to me, the Sooner Objection

can be flipped back on a relationalist like Leibniz. The question can be asked, “Why didn’t God create a longer or shorter universe?”

87 → Craig, 2001c, pp. 229–233.

88 → Alexander, 1956, p. 152.

89 Cf. → Thomas, 2018, pp. 93–94.

90 Adamson, Al-Razi, 2021, pp. 104–106.

91 → Flood, 2020, p. 7.

92 → Balslev, 1992, p. 170; Adamson and Ganeri, 2020, p. 209. I can’t help but be reminded of 2 Peter 3:8, which says that a day is like a thousand years for the Lord, and a thousand years are like a day.

93 → Flood, 2020, p. 13.

94 → Flood, 2020, pp. 43–44.

95 → Alexander, 1956, p. 49.

96 → Leftow, 1991, p. 160.

97 → Alexander, 1956, p. 49.

98 → Alexander, 1956, p. 45.

99 → Skrzypek, 2017, p. 156.

100 Cf. → Rowe, 2004.

101 → Wolterstorff, 2001b, p. 172.

102 Wolterstorff, 2001, p. 238.

103 →Booth, 2017, p. 134.

104 →Booth, 2017, p. 149.

105 →Lebens, 2020, p. 75.

106 Adamson, Al-Razi, 2021, pp. 30–32.

107 Adamson, Al-Razi, 2021, pp. 32–35.

108 →Mullins, 2020, pp. 235–237.

109 →Meyer, 2013, p. 106.

110 D. W. →Zimmerman, 2002, pp. 88–89; →Swinburne, 2016, pp. 230–231.

111 →DeWeese, 2002, pp. 50–51.

7 Space, Mind and Deity

Robin Le Poidevin

Abstract: There is a plausible argument from God's status as the creator to the conclusion that God is aspatial, as follows: if God is the creator of all, including space, then he cannot be spatial, for then he would be dependent on space, and he is not dependent on anything he creates. Against divine aspatiality, however, can be urged a number of considerations, the most pressing of which is an argument from relativity: that given the unified nature of space-time, anything that exhibits a temporal structure also exhibits a spatial structure, and God has a temporal structure by virtue of having a mental life. This chapter offers a critical examination of the argument from creation to divine aspatiality, and sets it in the context of different theories of space: idealism, substantivalism and relationism. It ends with an exploration of the view that space is constituted by God.

7.1 An Argument for Divine Aspatiality

Here is a plausible argument for the aspatiality, or spacelessness of God:

1. God is independent of any created thing.
2. Space is a created thing.
3. Any spatial thing is dependent for its existence on space.
   Therefore (from 1 and 2):
4. God is independent of space.
   Therefore (from 3 and 4):
5. God is aspatial.

Call this the *argument from creation*. That argument is the subject of this chapter. In Section 7.2 we look at reasons to think that the conclusion of this argument is problematic, indeed one we should reject. Two of those reasons are not wholly convincing, but the third is rather more so. Enough, at any rate, to motivate a reconsideration of the premises. The focus will be on premises (2) and (3), and our exploration of them will be in the context of debates about the existence and nature of space. In particular, we will be concerned with idealism (briefly, in Section 7.3), and (at greater length) substantivalism and relationism (Sections 7.4 and 7.5). The sixth and final section (7.6) examines the prospects for 'divine substantivalism,' the view that space is grounded in God.

Before we consider the argument from creation any further, I want to say a few words about what on the face of it looks like a rather quicker route to divine aspatiality, suggested by Paul Helm. Helm's point is that a spatial God (where this is understood in a particular way) would be a finite God, and so, if God is infinite, as traditional theism conceives of him, then he must be aspatial.[1] Now, considering this move in itself, and suppressing for the time being the other assumptions which form the background of Helm's proposal, we might be inclined to counter it with the following considerations. First, if the reason for supposing that a spatial God would be a finite God is that *space itself is finite*, then we should ask: why can space not be infinitely extended in all directions? If that is not possible, is that because, as Aristotle argued, an actual infinite can never be realized? If so, and all infinities are only potential infinities (that is, to be conceived of as processes, which have no natural limit, like counting, or doubling), then the supposition of God's (actual) finitude is not something to be resisted. Finally, should the infinity of God be conceived of in mathematical, spatial or temporal terms anyway?[2]

I hope those considerations are enough to make us pause before proposing that the conception of a spatial God immediately implies that God is finite in some problematic way. But Helm's own argument depends on further considerations which invite careful examination. He suggests that 'God is in space' implies three things: (i) that God has some spatial perspective; (ii) that God cannot be wholly present at two places at once; and (in consequence) (iii) that if God is present in more than one place, he must be spatially divisible. (iii) implies that if God is in space, then he must be in some specific place, and not at other places, allowing him to have a spatial perspective. But then, if God is only in a specific place, then he is bounded by space, and so only finite.[3] That is clearly a more complex argument, requiring intermediate moves to get from God's spatiality to God's finitude. In the sections that follow, reasons to doubt (i), (ii) and (iii) will be encountered.

Now back to the argument from creation, and two preliminary remarks. Consider premise (1). Why should God be independent of any created thing (created by God, that is)? Because, since anything created by God is dependent on God for its existence, if God were in turn dependent on anything God had created, then the result would be an unacceptable (and very small) circle of dependence. It might be objected: but surely there can be mutual dependence. Think of the three angles of a Euclidean right-angled triangle: are these not interdependent? However, although it is true that the values of the three angles are geometrically related, they are so under the description "values of the angles of a Euclidean right-angled triangle." In other words, there is a logical relation between them which interferes with their distinctness. The case of God and creation is rather different: these are, it seems, genuinely distinct. (1), then, can be justified by the principle "there are no mutual existential dependence relations between distinct existences."

Now consider (3). Why is it necessary for the argument? Could we not have moved straight to the final conclusion from (1) and (2)? Perhaps not. Take this account of existential independence: *x* is independent of *y* if it is possible for *x* to exist and *y* not. (This analysis, note, only makes the right-hand side a sufficient condition for the left-hand side, not a necessary one. Perhaps there are cases where *x* is independent of *y* but no possibility of *x* existing and *y* not, because *y* is a necessarily existent object.) So God could be independent of space by virtue of the fact that, whereas God exists of necessity, space is merely contingent. So there is a world in which God is aspatial. (1) and (2), then, by themselves may only justify the conclusion that God is *possibly* aspatial, but not actually so. (3), however, rules this out. According to this premise, it is a feature of spatial things that they are essentially spatial, on the assumption that only spatial things are dependent on space.

So much for preliminaries. The argument looks defensible. But are there reasons not to accept the conclusion? Yes: let's now look at them.

7.2 Three Arguments against Divine Aspatiality

What follows are three reasons for denying the conclusion of the previous argument. Two, despite being initially plausible, are resistible; the third has more to be said for it.

The first reason we will briefly canvass is that God's spatiality is required by omniscience, that is, knowing all truths. The argument may be set out as follows:

1. God is omniscient, and so there is nothing that God does not know.
2. Only a spatial being can know, of a given place, that it is *here*.

Therefore (from 7):

1. If God is aspatial, then there is something which God does not know, viz., of any given place, that it is *here*.

Therefore (from 6 and 8):

1. God is spatial.

This mirrors a rather better-known argument against God's atemporality: if God is atemporal, then he does not know what time it is. Only a temporal being in time can know what time it is. So God, knowing all truths, is temporal. What gives the temporal argument force is the intuition that there is an objective fact of the matter as to which time it is *now*, since presentness is an absolute, mind-independent feature of times and events. There is no corresponding spatial intuition. Which place or object is here is, uncontroversially, a purely perspectival matter. A place counts as *here* for some subject if and only if the subject is located at that place. That no place counts as *here* for an aspatial being only reflects that being's lack of a spatial perspective, not ignorance of some fact.[4]

However, a variant of this argument sidesteps this metaphysical difference between time and space, and concentrates instead on representations of places. A wider conception of omniscience includes not merely propositional knowledge but also experiential knowledge. Linda Zagzebski has coined the word 'omnisubjectivity' to describe God's total knowledge of what it is like for each conscious being to be in the states that they are.[5] Since conscious beings typically have spatial perspectives, God must, if omnisubjective, have access to these states. God must, therefore, know what it is like to experience a place as *here*. But for this to entail the conclusion that God is spatial, we need the further premise that only a

being actually situated in space could know what it is like to experience a place as *here*. Why should we accept that premise? If we, who are anything but omnisubjective, can have the experience as of flying unaided, as a result of having a dream in which we do, without actually being able to fly, why could an aspatial God not have access to our spatially perspectival states?

The second reason to deny the conclusion of the argument for creation is that God's spatiality is required by omnipresence:

1. God is omnipresent, and so in all places.
2. Only a spatial being can be in any place.

Therefore:

1. God is spatial.

This argument relies on a particular view of omnipresence, that it requires God to be located at places in the way that material beings are located at places, by occupying them. But, as Anselm and Aquinas showed, it is possible to combine the doctrine of divine omnipresence with divine aspatiality, by construing omnipresence, not in terms of occupation, but rather in terms of God's knowledge and power: God has (direct) knowledge of and power over all places. His omnipresence thus falls out of his omniscience and omnipotence, without requiring any additional condition to be met.[6]

Defenders of a spatial occupation understanding of omnipresence may insist that such direct knowledge and power must be grounded in an actual occupancy of the places over which knowledge and power is had. This appears to have been Isaac Newton's view.[7] It might be further justified by the reflection that knowledge and power entails causal connection, and direct causal connections require immediate contiguity, or even co-location. A worry about this justification for God's real

presence in space is that it makes God's omnipotence and omniscience at least partly dependent on space.

So let us turn to the third reason. This is that God's spatiality is required by his having a mind, or mental life:

1. God has a mind.
2. Anything that has a mind is temporal.
3. Anything that is temporal is also spatial.

Therefore:

1. God is spatial.

Call this (since we will be referring to it again) the *argument from mind*. The sub-argument for (15) appeals to the notion of a combined space-time continuum. That might be developed in different ways. Consider the standard interpretation of the Special Theory of Relativity. According to this, simultaneity is not absolute, but rather can vary from one inertial frame to another. (An 'inertial frame' here is defined by a set of objects which are at rest with respect to each other. Objects in another frame are moving with respect to those in the first.) What, however, is frame-*in*variant is the space-time separation between events. This is defined as follows:

$s^2 = x^2 + y^2 + z^2 - t^2$

Where s is the space-time interval between any two given events, ($x + y + z$) and t the values, for some inertial frame, of the spatial distance and temporal separation, respectively, between those events. Since spatial and temporal relations co-vary across inertial frames in this way, space and time cannot be wholly separate dimensions, but rather aspects of a combined space-time. We can, however, describe events related as cause and effect as *time-like separated*, in that they are temporally

separated with respect to every initial frame, and so define time as ‘the causal dimension of space-time.’

The upshot of this is that nothing which exhibits a temporal structure can fail to exhibit a spatial structure. That is, in relativistic language, everything in ‘time’ is in fact in *space*-time.

A worry about making the argument from mind depend on an appeal specifically to the relativity of simultaneity is that Special Relativity is subsumed within, or superseded by a larger framework, that of General Relativity, or Quantum Field Theory. On some interpretations of these, absolute simultaneity may be restored. What remains, however, is the insight that space and time are part of a larger whole, and so essentially related, in such a way as to rule out a temporal yet aspatial object.

Now, minds are paradigmatic instances of entities which exhibit a temporal structure. They therefore exhibit a spatial structure too. (This conclusion goes through even if we think of the mental as non-physical, though, as Michael Lockwood has pointed out, demonstrating the possibility of mental and physical events being spatially co-located removes one objection to the identity theory.[8]) So, if God has a mind, God is in space.

It may seem odd to attempt to derive an attribute of God from some contingent (and not undisputed) physical theory. Is God to be constrained by the laws of physics? On the other hand, it would be wise not to invite a situation where God appears to falsify physics.

So, the situation so far is this: we have two plausible arguments, each with a conclusion which is incompatible with the conclusion of the other. By the argument from creation, God is aspatial. By the argument from mind, on the other hand, God is spatial. An antinomy of pure reason, no less! (Well, not so pure, since one of them makes implicit appeal to a concept from physics.) Both arguments cannot stand. What I propose to do in the remainder of this chapter is to test the premises of the

argument from creation for God's aspatiality. *Is* space a created thing? Does spatiality entail dependence on space? Answers to those questions are affected, perhaps even implied, by views of the nature of space itself, and to that topic we now turn.

7.3 An Ideal Space?

The first question to ask about the nature of space is whether it is real or not. There is a quick way of short-circuiting arguments for and against divine aspatiality, although it is a rather heroic one, and that is simply to deny the real existence of space and spatiality altogether. Space is not a feature of the world as it is in itself, but rather of our representations of the world. This is *idealism* about space, and it is associated most prominently with Kant, who describes space as the 'form of outer sense': things are represented as external to us by being represented as in space.[9] Idealism falsifies premise (2) of the argument for aspatiality: space is not created, as it does not really exist. Rather, *we* are so created that our minds project the idea of space onto the world. (3) may be true, but since nothing is actually a spatial object, nothing actually depends on space. In a sense, idealism does confirm that conclusion of the argument, for if space does not exist, then God is indeed aspatial. But we lose the intended contrast with other beings which was the point of the argument.

Even if we do adopt idealism about space, however, that doesn't immediately make the arguments for and against aspatiality completely irrelevant, for instead of taking them to cast light on the objective nature of God, we might reconstrue them as concerning our representations of God and of space. Are we, perhaps because of our epistemic or imaginative limitations, constrained to view God as spatial, or as aspatial? Nevertheless, those arguments may not entirely survive this

reconfiguration. Take the argument from mind. That argument based the inference from temporal structure to spatial structure on the unified spacetime of relativistic physics. But that need be no part of our *representation* of God. Moreover, although we may not be able to represent the life of a mind without also representing it as temporally structured, there does not seem to be a similar compulsion to represent that life as spatially structured.

Spatial idealism needs to be carefully articulated. The statement "things are not in fact spatial, but we cannot think of them except as spatial" would be self-defeating. We need to be able at least to form the idea of non-spatial things in order even to entertain idealism, never mind assent to it. And what holds of our idea of ordinary objects must also hold of our idea of God: we cannot, under idealism, be under a compulsion to think of God as spatial. But might there still be a reason to do so?

The challenge to idealism about space is to explain why, if things are not actually spatial, we perceive them as such. The answer may be a teleological one: representation of external objects as spatial is essential for our engagement with those objects. What we represent as our spatial orientation with respect to them may correspond to some other relation which in turn constrains the possibility of causal interaction. Whether or not that seems somewhat strained, in comparison with the reflection that the best explanation of the success of our spatial representations is that things really *are* spatial, how plausible is it when extended to God? Can it be that, in order to engage with God, we have to represent him as spatial? Do we have to spatially orient ourselves with respect to God? Certainly, the notion that we have to think of God as *temporally* present in order to engage with him seems entirely plausible. But do we have to think of God as spatially present, too?

There is a debate to be had about the content of our representations, but I will not pursue it further here. From now on, I will assume that space is an objective feature of the world. But what kind of objective feature?

7.4 Relational or Substantival Space?

Once we have settled on the reality of space, the next question to ask is to what ontological category it belongs. This takes us to the debate between relationism and substantivalism. According to *relationism*, space is simply the network of spatial relations between the kinds of things that we might ordinarily describe (without taking any metaphysical side) as the 'contents' of space: material objects, parts of an electromagnetic field, and so on.[10] Spatial relations between these relata are not mediated by any further thing, such as spatial points, but are rather direct. According to *substantivalism*, in contrast, space is an entity in its own right, existing independently of its contents. Spatial relations between objects are not direct but rather mediated by spatial regions or points.

Historically, the debate between relationism or substantivalism (sometimes called 'absolutism,' because of the association of substantival space with absolute motion) arose in a theological context. The classic source for this is, of course, the correspondence between G. W. Leibniz and Samuel Clarke.[11] Leibniz's main argument against substantivalism is a form of *detachment argument*. Suppose space to exist as an object in its right. Then, in principle, the universe could change its relations with respect to absolute space while the relative distance and orientation relations between the components of the universe remained unaltered. But such a change would have no observable consequences, indeed no causal consequences whatsoever. Now, given that God does nothing without a reason,

there would have to be a reason why God chose to situate the universe with respect to absolute space in one way rather than another. But since it would make no odds how the universe was situated with respect to absolute space, God could have no such reason. Therefore, absolute space is a fiction.

Graham Nerlich has pointed out that the detachment argument relies on something Leibniz would naturally have assumed, namely that physical space would exhibit a Euclidean geometry: there is nothing to distinguish, in terms of its internal structure, between one part of absolute space and another. But what if physical space were non-Euclidean? Suppose, for example, that it exhibited a variable curvature. Then the movement and orientation of the universe with respect to absolute space *would* make a difference.[12]

Setting aside the success or otherwise of the detachment argument for relationism, what effect does a choice between relationism and substantivalism have on the argument from creation to divine aspatiality? Take substantivalism first. The role of substantival/absolute space is to determine the spatial relations between, and the internal spatial structure of, objects in space. So premise (3) of the argument looks secure: how could a spatial object exist without a spatial structure? To put it another way: what could possibly ground the transworld identity of a spatial object with some object in a spaceless world? If (3) is secure, then it seems that, insofar as God as is the creator of space, he cannot occupy it.

There is an implicit assumption in the above line of reasoning that any object occupying space will have an internal spatial structure: that is, it will have different spatial parts in different places. This is how ordinary material objects, and fields, can be extended in space. It is also, as we remarked earlier, the reason why divine omnipresence was not traditionally interpreted in terms of God's spatial extension. How could a

partless God be extended in space? But, as Hud Hudson has pointed out, there is another way of occupying space which does not require spatial parts. Borrowing terms introduced by Josh Parsons, call the manner of occupying space exhibited by ordinary physical objects *pertension*: an object pertends through a region of space by having different parts in different parts of that region. Contrast this with *entension*: an object entends through a region of space by being wholly present in another part of that region.[13] Suppose then, that God entends, rather than pertends, throughout the whole of space. He thus lacks spatial parts. But is this enough to avoid the consequence that God would depend for his existence on space? Even entension seems to bestow certain spatial properties on God: his standing in distance relations to himself, for example. And surely that divine property depends on space?

Yes, is the obvious answer. God would have a property that is dependent on space. But does it immediately follow that God's *existence* is dependent on space? Perhaps God could have lacked that property, and indeed would do in a spaceless universe. Spatial properties may be essential to material objects, but not to God. God could, through his creation of space, make it the case that he occupies space. So (3) is false. Someone may object, however, that there is nothing to distinguish God's entending all space from his being merely virtually omnipresent, where this is spelled out in terms of having knowledge of and power over each place. The intrinsic nature of God is not changed by the creation of space, only his relations to creation. So the falsity of (3) is bought only at the cost of delivering the conclusion of the argument for aspatiality. Defenders of entension, however, will object that entension is a genuine, not merely virtual, way of occupying space.

Let us now look at the effect relationism has on the argument for aspatiality. Since on this view space is simply the

network of spatial relations between the components of the physical universe, in creating that universe, God creates space. So (2) is unthreatened. What of (3)? Note that the wording of (3) suggests a substantivalist reading: material spatial objects and fields are dependent on space itself. Relationism reverses this dependence: there is space (as a network of spatial relations) only insofar as there are material objects and/or fields which exhibit spatial properties and relations. But there is a relationist reading of (3): that ordinary objects could not exist except by exhibiting spatial relations amongst their parts. The same may not be true of God. We might suppose God only to exhibit spatial properties by coming to stand in spatial relations to other things, as a result of his creating the universe. God does not thereby come to be dependent for his existence on standing in those relations.

Both substantivalism and relationism give us reasons to resist (3), by differentiating between the way in which God is embedded in space, and the way in which other things are so. But this way of rejecting the argument from creation nevertheless gives us a kind of divine aspatiality.

So far, we have left premise (2) of the argument, that God creates space, unchallenged. We now turn to ways of resisting that premise

7.5 An Independent Space?

Suppose space is not created, but exists independently of God, as Descartes's contemporary Pierre Gassendi held.[14] What effect then does the choice between relationism and substantivalism have?

On a relationist view, if space exists independently of God, then what perhaps is rather misleadingly referred to as the 'contents' of space, material objects, fields, and the like, are also

independent of God. And that is because space exists only by virtue of these objects, their spatial properties and relations. The upshot is that *entire universe* exists independently of God.[15] So God is not, after all, the creator. I take it that this option is theologically unacceptable.

If space is independent of God, then, it is substantival space. Without God's creative act, this would simply be an empty container. However, although the theist may be willing to allow certain kinds of object independence from God, the paradigm cases of this are abstract objects, such as numbers, which exist of necessity. And the list may include moral values. The argument is that it is no limit to God's omnipotence that he cannot alter or obliterate these. As they exist and have their nature of necessity, the notion of destroying or changing them makes no logical sense. In contrast, intuitively, space is a contingent object, one that might not have existed. And that would then be a counterexample to the general principle that God is creator of all contingent things. Is that theologically acceptable? Surely not: it would constitute a case of something whose non-existence, though possible, God could not bring about—and could not have done so. That would be a serious limitation on omnipotence.

It might be insisted that space is in fact a necessary object. There could not have been nothing at all, and a minimum condition for the existence of any kind of world is the existence of space.[16] Even this move, however, is not enough to prevent a challenge to omnipotence. For even if the existence of space is necessary, its precise structure is not. As noted above, the possibility of various non-Euclidean structures for physical space suggests that any given structure (Euclidean, variably curved, hyperbolic, etc.) is a contingent matter. Yet if space is *wholly* independent of God it is apparently not in God's power to determine which structure space has. Could space be

independent of God for its existence, but dependent on God for its precise physical geometry? We may not be able to separate existence and geometry in this way. Arguably, one and the same collection of spatial points could not have exhibited a different physical geometry from the one it does in fact exhibit, since the relations between points are partly constitutive of their identity.

Still, it might be said that particular spatial points are substances, whose existence and nature are logically independent of other such points. Each has its individual essence, a *haecceity*. So, after all, it is possible for space (the very space that actually exists) to exist of necessity, and yet its contingent geometrical structure to be dependent on God.

Consider, then, a final move in this dialectic: if substantival space is independent of God, and material objects dependent on space, then it seems that God did not create material objects wholly *ex nihilo*, but had, as it were, to co-operate with space in their creation. In fact, if one version of substantivalism is true, that material objects are themselves regions of space with certain properties,[17] then the creation of material objects consisted of God's modifying space.

It would seem, then, that the notion of an independent substantival space involves revisions to the traditional way of thinking of God's powers. In any case, rejecting premise (2) on the grounds of space's independence actually provides a further motivation for asserting God's aspatiality. For if God were in space, and space were independent of God, then it would follow that *God's* spatiality would itself be independent of God. The combination of an independent space and a spatial God is a problematic one.[18]

However, there is a way of rejecting (2) without embracing either idealism or the view that space is independent of God, as we shall now see.

7.6 A Divine Grounding for Space

What if space were constituted by God? This, in one form or another, is a view that was adopted by, *inter alia*, the Cambridge Platonist Henry More (1614–87) and Leibniz's correspondent Samuel Clarke (1675–1729).[19] So space is neither created by nor independent of God. Since this would make space independent of its contents, this is a form of substantivalism, but a distinctive form which we might call *divine substantivalism*.

A straightforward identification of God and space, however, faces a dilemma. The explanatory role that substantival space is supposed by its proponents to play is that of determining the spatial structure of the universe: objects have spatial parts by virtue of having parts in different parts of space; objects stand in spatial relations to each other by virtue of the dimensions of spatial regions in between those objects; the geometry of objects is determined by the geometry of space. But it would seem that divine substantival space can only play this role if God has spatial parts, and that conflicts with the doctrine of divine simplicity. That doctrine is, admittedly, a somewhat obscure one, and not everyone professes to make sense of it. But part of that doctrine, that God lacks spatial (or temporal) parts, is not obscure, and is motivated by a further doctrine, that of divine *aseity*. God depends on nothing else for his existence. But an object which has parts, arguably, depends for its existence on its parts (or at least the existence of some parts or other). So either divine substantivalism involves a denial of the more intelligible, and motivated, component of the doctrine of divine simplicity, or the divine ground of space cannot play the explanatory role that substantival space is supposed to play, undermining the *raison d'etre* for substantival space.

Is there a way out of this dilemma? As substantivalism is understood by contemporary philosophers, extension is possible

precisely because the different parts of space are numerically distinct. Spatial distance relations between objects are mediated by regions of space which are non-identical to those regions occupied by the objects. But if space is an aspect of God, and God is wholly present at each place, what exactly differentiates one place from another? How can one place be non-identical to any other? Seventeenth- and eighteenth-century writers would not have been exercised by the need to explain how different places could be *qualitatively* distinct, for Euclidean physical space is entirely uniform. But once we allow the possibility that space has a non-Euclidean structure, the need to accommodate qualitative variation asserts itself. Can this be achieved if all places are one?

However, if space is only an *aspect* of God, rather than God himself, perhaps we do not need to infer divine spatial parts from a divine spatial structure of some kind. It is time we brought our attention back to the most compelling of the arguments against divine aspatiality: the argument from mind. Consider, then, the possibility that what confers a spatial structure on the universe is the structure of the mind of God. The states of the divine mind could constitute a spatial structure without God having spatial parts: God could be the partless subject of those states. Here we have the advantage of being able to appeal to a temporal analogy. Those who deny that objects have temporal parts—*endurantists*—hold something similar about the relation between an object and the life of that object. When an object changes, the life of that object has different stages at different times, and at each of those times different states of that object obtain. But the object itself, which is a component of those stages, does not itself have temporal parts.[20] Assuming that endurantism is an intelligible account of persistence and change (and it is supposed to represent our intuitive, pre-theoretical conception), why could the spatial

equivalent not apply to God? Admittedly, we are positing here a certain lack of simplicity in God, though we are locating this in the mental life of God, rather than God himself, but perhaps we cannot, without straining intelligibility, posit total simplicity in every respect.

What this latest proposal is suggesting is that God, although in some sense spatial, is so only in a derivative sense, that of being a participant in states which are themselves located in a non-derivative sense in space. It is a way of being in space without having spatial parts.[21] It is a compromise, a half-way point, between total aspatiality and total spatiality. It also gives some content to the idea of entension: God is located in space in a way which goes beyond merely having direct knowledge of and power over every place.

At this point, let us return briefly to Helm's objection to a spatial God, mentioned in the introduction: God cannot occupy two different places without having spatial parts, and so, if in space, can only occupy a part of space, and thus have a limited spatial perspective. Such a being would be finite. The notion of entension challenges the first part of this argument. But if God entends space, can he have a spatial perspective? If not, is God truly spatial? The answer, I suggest, is that God has *many* spatial perspectives, rather than just one, and these do not conflict with each other because those perspectives are thoughts, and those thoughts in turn have a spatial structure: they define locations. The truth-conditions of each thought are token-reflexive: the (token) thought '*x* is here' is true by virtue of *x*'s being in the place constituted by that thought. A simultaneous thought, '*x* is there' is true in virtue of *x*'s not being in the place constituted by that second thought. These truth-conditions can clearly both obtain without contradiction.

I envisage two challenges to divine substantivalism, as outlined here. One is that, whereas non-divine substantivalism

grounds spatial structure in a plurality of *objects* (namely spatial points and regions), the appeal to the mind of God in this way grounds spatial structure in a plurality of *states*. It shifts substantival space to a different ontological category. Is this intelligible? Can states do the job previously assigned to objects? The other challenge is to deal with the consequences for our conception of God. Quite what those consequences would be may not be immediately clear, so let me gesture towards them in a series of questions. It would odd, everyone can agree, to think of God as having a geometry. Is it much less odd to think of God's *mind* as having one? Must we also think of God's mind as having a *topological* structure, too (e.g. discrete versus continuous, unified versus disunified, orientable versus non-orientable)? And is the geometry and topology of God's mind a purely contingent matter? Further, can we integrate divine substantivalism into physical theories such as the General Theory of Relativity, on which the geometry of spacetime is affected by matter? Can the properties of God's mind be determined in this way? Answers to these questions may simply be bullets to bite, or absurdities to be resisted. I leave you to judge!

I have called this position 'divine substantivalism' because it posits a spatial structure which is independent of the created contents of space, and which structures those contents. But it also has elements of relationism, in that it makes space a construction out of other things, in this case states of the mind of God.[22] Another compromise, in other words. What else would you expect from an English philosopher?

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Notes

1 See →Helm (1980, pp. 220–221).

2 On this last question, see →Le Blanc (1993).

3 See →Helm (1980, pp. 119–121).

4 The parallel between the spatial and temporal arguments is drawn by →Helm (1980). His intention, however, is not to establish God's spatiality, but rather the conclusion that *if* we accept the temporal argument from omniscience to divine temporality, we should also accept its spatial analogue. As we remarked above, the thesis that God is spatial Helm regards as problematic, on the grounds that it implies God's finitude. His conclusion is that we should reject the temporal argument from omniscience. For critical comment on Helm's strategy, see →MacBeath (1983).

5 See, e.g., Zabzebski (2008).

6 For a sympathetic reconstruction of Anselm and Aquinas on omnipresence, see →Wierenga (2007). For a related attempt to reconcile omnipresence and aspatiality, see →Jedwab (2016).

7 In the "General Scholium" to the *Principia Mathematica*, Newton remarks: "He is omnipresent not only virtually but substantially, for action requires substance" (→Janiak, 2004, p. 91). Patrick →Connolly (2015) has argued that Newton assents to three theses which are in tension with each other: God is in space; space is real;

space is ontologically distinct from God. Other prominent thinkers of the Early Modern period (such as Henry More, Isaac Barrow, G. W. Leibniz and Samuel Clarke) tended to reject one or other of these theses. For Locke's views on the nature of space and God's relation to space (and a different interpretation of Newton) see →Gorham (2020).

8 See →Lockwood (1984) for a somewhat more detailed derivation of the spatiality of mental events from their temporality, given Special Relativity, and its consequences for the debate over whether mental events are identical to physical ones. Lockwood proposes a method for providing a rough estimate of the spatial location of a mental event, as follows. Suppose the mental event in question to be caused by one physical event and to cause another, thus: p_1–m–p_2. m will be located somewhere in the overlap of the forwards light-cone of p_1 and the backwards light-cone of p_2. That is likely to be a rather large overlap, however! The crucial point for Lockwood is that this nevertheless provides a means of locating mental events in space which is independent of any assumption of the truth of the identity theory.

9 See Kant (1929 [1781]), esp. B41. His argument in the *Critique* for the ideality of space rests on the first of the antinomies of pure reason. Later, in the *Prolegomenon* (Kant, 1953 [1783]) he offers an intriguing argument for idealism from the phenomenon of incongruent counterparts: asymmetric objects which are mirror images of each other. Since we represent a difference between counterparts, yet this cannot be explained by

appeal to real space, whether relational or substantival, the conclusion is that space is ideal.

10 The relationism presented here is the non-modal variety. A modal version of relation views space is a set of *possibilities* of location. (For discussion, see, e.g. →Butterfield, 1984.) This modal version, of course invites the question: what grounds these possibilities, if not spatial points? The non-modal version does not wholly avoid this question, however, as we might want to say that, although *x* and *y* in fact stand in such-and-such a spatial relation they could have stood in some other relation. What grounds that possibility?

11 →Alexander (1956). For a comprehensive account of the Early Modern debate about absolute space and time, see →Thomas (2018).

12 See →Nerlich (1991).

13 See →Parsons (2007) and →Hudson (2009). The nomenclature is new, but the idea not. Henry More gave the name *holenmerism* to the notion that immaterial entities were wholly present at each point of space. More was critical of the notion. See More (1995 [1679]).

14 See, e.g., his *Syntagma Philosophicum*, Part II, sect. 1, a translation of which can be found in →Brush (1972). For further references and discussion, see →Jammer (1969).

15 It would be simply bizarre, surely, to suggest that God is creator of abstract objects, but not of concrete ones.

16 Unless there could have been a world consisting just of abstract objects?

17 This position is dubbed “monistic substantivalism” by Jonathan →Schaffer (2009), who argues that it has decided advantages over standard, dualistic substantivalism, according to which material objects are distinct from spatial regions.

18 See footnote 7.

19 For More, see →Reid (2007), for Clarke, →Alexander (1956). The view has also been attributed to Newton, but this interpretation is denied by →Connolly (2015).

20 See, e.g., Sider (2001) for a thorough examination of endurantism, and an impressive case for rival views.

21 This precisely mirrors the proposal →Simons (2000) offers for the case of existence in time.

22 Recalling the issue raised in footnote 10 above, we might note here that, although divine substantivalism grounds space in something more fundamental, as relationism does, it has a ready answer to the question which faces the relationist: what grounds the possibility that two material objects could have stood in different spatial relations?

III God and Perfect Goodness, Perfect Beauty, Perfect Freedom

8 On Modal Arguments against Perfect Goodness

Michael Almeida

Abstract: The modal argument against perfect goodness requires nothing more than a single state of affairs that fails to provide conclusive evidence for the existence of God. If there is some state of affairs in some world that provides less than conclusive evidence for the existence of God, then the traditional God does not exist. But, surprisingly, there is an equally strong modal argument in favor of the existence of God. If there is a single state of affairs in some world that provides less than conclusive evidence against the existence of God, then the traditional God exists, indeed necessarily exists. The problem is that logics as strong as S5 produce unwelcome epistemological consequences. The proper logic for philosophical theism is among the weaker modal logics.

8.1 Introduction

Let □Fx be a conjunctive property including essential moral perfection, omnipotence, and omniscience.[1] The modal argument against perfect goodness urges that there are at least some states of affairs S in some possible world w such that S entails that nothing exemplifies □Fx in w. According to standard versions of the modal argument, if □Fx is not exemplified in some world, then □Fx is not exemplified in any possible world. There are no morally perfect beings and the traditional God is an impossible object.

Traditional modal arguments against perfect goodness require a logic at least as strong as S5. In Section 8.2 I discuss

the traditional argument against perfect goodness: the modal argument from evil. According to the modal argument from evil, a sound modal argument against perfect goodness depends on there being possible worlds—or at least contingently creatable portions of possible worlds—that are on balance bad.

In Section 8.3 I show that a sound modal argument from evil does not require possible worlds that are on balance bad or even creatable portions of worlds that are on balance bad. A sound modal argument from evil does not depend on there being a single evil state of affairs in any world. The soundness of the modal argument from evil is consistent with every possible world being on balance good and, indeed, consistent with every possible world being infinitely good.

The modal argument from evil requires just a single state of affairs that does not provide conclusive evidence for $\Box F_G$. If there is some state of affairs in some world that provides less than conclusive evidence in favor of $\Box F_G$, then the traditional God does not exist. If there is a single state of affairs S such that $P(\Box F_G | S) < 1$, then the traditional God does not exist, indeed, necessarily does not exist. Since there are many possible states of affairs that do not provide conclusive evidence for $\Box F_G$, we have an extremely strong argument against the existence of the traditional God.

In Section 8.4 I show that there are equally strong modal arguments in favor of $\Box Fx$. If there is a single state of affairs, in some world, that provides less than conclusive evidence for $\sim\Box F_G$, then the traditional God exists, indeed necessarily exists. So, it is sufficient to show that the traditional God exists that there is some state of affairs S, in some world, such that $P(\sim\Box F_G | S) < 1$. But there are many states of affairs that do not provide conclusive evidence for $\sim\Box F_G$. So we have an extremely strong modal argument in favor of the existence of the traditional God.

In Section 8.5 I show that it is a consequence of S5 that, necessarily, no state of affairs in any world provides any non-trivial evidence for or against $\Box F_G$. If $\Box F_G$ is true, then there are no states of affairs—no matter how bad those state of affairs happen to be—that we could not discover in the pluriverse. And if $\sim\Box F_G$ is true, there are no states of affairs—no matter how good those states of affairs happen to be—that we could not discover in the pluriverse. No state of affairs we might discover provides any non-trivial evidence for or against $\Box F_G$. Modal arguments from evil and modal arguments from goodness are therefore unsound. I offer some concluding remarks in Section 8.6.

8.2 The Modal Argument from Evil

According to standard formulations of the modal argument from evil the *mere possibility* of intrinsically evil states of affairs constitute at least some evidence against the existence of the traditional God. The initial goal of the modal argument from evil is to provide good reason to believe that at least some states of affairs in the pluriverse—somewhere in the totality of modal space—are bad enough that an essentially perfectly good being could not permit them. Ted Guleserian proposes the following principle.

> It is reasonable to hold: (C1) if an amoral world is more evil than good then necessarily it is morally impermissible for any OOM [omniscient, omnipotent, morally perfect] being to allow it to be actual. (→Guleserian, 1983, p. 225).

According to this moral principle any world in which God exemplifies essential omniscience, omnipotence, and perfect goodness—any world in which it is true that $\Box F_G$—is a world that

does not include more evil than good.[2] But the modal argument from evil does not require the existence of on balance bad worlds. Any possible world in which it is true that $\Box F_G$ is a world that does not include any intrinsically evil state of affairs that are gratuitous or unjustified.[3] Worlds that a perfectly good being could not permit might otherwise be extremely good worlds. Guleserian nonetheless aims to show that there exist on balance bad worlds in the pluriverse.

According to Guleserian, there also exist worlds in which the totality of contingent states of affairs is on balance bad.

> (C2) if … the maximum creatable portion of an amoral world is more evil than good, then necessarily it is morally impermissible for any OOM being to allow that world to be actual. (→Guleserian, 1983, p. 226)

The maximum creatable portion of a world is just the maximally consistent states of affairs that an omnipotent being actualizes in a world. So no matter how good a necessary state of affairs happens to be—say the infinite value of a necessarily existing perfectly good being—a world is not actualizable if its "creatable portion" is on balance bad. So, according to C2, it might be the case that a perfectly good being ought not actualize some worlds that are infinitely valuable.

But there might be many valuable objects in the creatable portion of worlds that are necessarily existing. It might be the case, for instance, that many sentient creatures are in the domain of every possible world. It might be that every sentient creature, or every possible object, exists in the domain of every possible world.[4] These non-contingent beings contribute to the overall value of the creatable portion of possible worlds. And the overall intrinsic value of the sentient portion of worlds might be infinitely positive. So, we might want additional restrictions on the portion of worlds described in (C2).

> (C3) if the maximum *contingent* creatable portion of a world is more evil than good, then necessarily it is morally impermissible for any OOM being to allow that world to be actual.

According to modal arguments from evil there are possible worlds, or contingent creatable portions of possible worlds, that satisfy the descriptions in the antecedents of C1–C3. So there are possible worlds that are such that it is impermissible for an essentially omnipotent, omniscient, perfectly good being to actualize them. Guleserian describes one of these worlds.

> Think of a world, we will call it 'P,' in which the only sentient beings whose existence is contingent are nonrational animals of various sorts—or are sentient beings a good deal like the higher nonrational animals in our world—all of which suffer long spontaneous bouts of excruciating pain, and spend the few hours between bouts barely doing what is necessary to survive... Perhaps they exist for an infinite stretch of time. And during this eternity they never experience anything we would call pleasure ... Such a wretched world as P clearly seems to be logically possible, yet one that no divine being would permit to become actual. (→Guleserian, 1983, p. 226)

No doubt the envisaged world is logically possible, since the world is perfectly consistent with classical logical laws. Indeed, the intrinsically evil states of affairs in the world P trivially entail those logical laws. And it is certainly not obvious that P is metaphysically impossible.

If there are possible worlds that satisfy the antecedent of C1–C3 then, according to the modal argument, there exists a possible world that is morally impermissible for an essentially omnipotent, omniscient, perfectly good being to actualize.

> There is a possible world W such that, necessarily, for every x, if x: is OOM then it is not morally permissible for x to allow W to be actual.
> (→Guleserian, 1983, p. 224)

If there are possible worlds that satisfy the antecedent of C1–C3 then, according to the modal argument, there are contingent portions of worlds that include extensive suffering that a perfectly good being necessarily prevents. So, there are at least some worlds that are not consistent with $\Box F_G$. Since the modal argument assumes S5, it follows that $\Box \sim \Box F_G$, there is no possible world that includes an essentially omnipotent, omniscient, perfectly good being. And so it is true in the actual world that $\sim \Box F_G$.

8.3 A Stronger Modal Argument from Evil

Most of the debate over the modal argument from evil concerns whether there exist worlds in which every sentient being suffers long spontaneous bouts of excruciating pain and, in general, whether there are worlds that are on balance bad. So it is crucial to the argument that a sound modal argument from evil does not require anything as strong as a possible world that is on balance bad. A sound modal argument from evil does not even require a contingent creatable portion of a world that is on balance bad.

It is no doubt more surprising that a sound modal argument does not require a single intrinsically evil state of affairs that is inconsistent with $\Box F_G$. There are cogent formulations of the modal argument forthcoming even if no possible worlds include any states of affairs S such that S entails $\sim \Box F_G$.

Let S be a state of affairs in some world w such that $\Box (S \longrightarrow \sim \Box F_G)$. The usual candidates for S include the vast magnitude of evil in some worlds or the unjust distribution of evil in some worlds or worlds that include gratuitous evils or unjustified evils and the like. But there remains significant disagreement over whether any of these states of affairs is possible and significant

disagreement over whether any of these states of affairs is inconsistent with $\Box F_G$. There remains, in fact, disagreement over whether any possible state of affairs is inconsistent with $\Box F_G$.

But there is no controversy over whether there are some states of affairs in some possible worlds that constitute at least *some evidence* against $\Box F_G$. That is, there is no controversy that there are some states of affairs S in some possible worlds such that the probability of $\Box F_G$ on S is less than certain.

If there is some state of affairs S in some world such that $\Box (S \longrightarrow \sim\Box F_G)$, then we can conclude that $P(\Box F_G | S) = 0$.[5] This follows from (1.3). In this case, a single state of affairs in a single world provides conclusive evidence against the existence of an essentially omnipotent, omniscient, perfectly good being.

$\Box (S \longrightarrow \sim\Box F_G) \longrightarrow P(\sim\Box F_G | S) = 1$ (1.3)

But suppose we fail to discover a possible state of affairs that is uncontroversially inconsistent with $\Box F_G$. Suppose the most we discover are states of affairs S in some possible worlds that make the probability of $\Box F_G$ less than certain. There are many intrinsically evil state of affairs S that constitutes at least some evidence against $\Box F_G$.

$P(\Box F_G | S) < 1$ (2.3)

Perhaps Guleserian's state of affairs of every sentient creature suffering infinitely is not inconsistent with the existence of an essentially omnipotent, omniscient, perfectly good being. But that state of affairs certainly does not make it certain that there

exists an essentially omnipotent, omniscient, perfectly good being. And there are lots of other states of affairs that do not constitute conclusive evidence in favor of $\Box F_G$. The state of affairs of an earthquake's destroying Lisbon in 1755 or the state of affairs of a tsunami's devastating Japan in 2011 might not be inconsistent with $\Box F_G$. But, again, these states of affairs do not make it certain that $\Box F_G$. These states of affairs constitute at least some evidence against $\Box F_G$. So there are several states of affairs that satisfy (2.3).

But if there is some state of affairs in some possible world that constitutes at least some evidence against $\Box F_G$, then that state of affairs does not entail $\Box F_G$. (2.3) is a consequence of the fact that, for every possible states of affairs S, if S entails $\Box F_G$ then $P(\Box F_G \mid S) = 1$.

$P(\Box F_G \mid S) < 1 \longrightarrow \sim\Box(S \longrightarrow \Box F_G)$ (3.3)

It follows from (2.2) and (2.3) that there exist some possible states of affairs that do not entail $\Box F_G$.

$\sim\Box (S \longrightarrow \Box F_G)$ (4.3)

But in S5 it is a consequence of (4.3) that S entails $\sim\Box F_G$. (5.3) is also a theorem of S5.[6]

$\sim\Box(S \longrightarrow \Box F_G) \longrightarrow \Box(S \longrightarrow \sim\Box F_G)$ (5.3)

From (3.3)—(5.3) we know that if the state of affairs of a tsunami's devastating Japan constitutes some evidence against the existence of God—or, for that matter, if the state of affairs does not constitute *conclusive evidence for* the existence of God—then a tsunami's devastating Japan entails the non-existence of God. But if a tsunami's devastating Japan entails the non-existence of God, then that state of affairs constitutes conclusive evidence against the existence of God. (6.3) is a consequence of the fact that, for all possible states of affairs S, if S entails $\sim\Box F_G$ then $P(\sim\Box F_G \mid S) = 1$.

$$\Box(S \longrightarrow \sim\Box F_G) \longrightarrow P(\sim\Box F_G \mid S) = 1 \qquad (6.3)$$

So, the modal argument from evil does not in fact require a possible world that is on balance bad. The modal argument does not even require a contingent, creatable portion of a world that is on balance bad. And the modal argument from evil does not require a single intrinsically evil state of affairs that is inconsistent with $\Box F_G$. A cogent formulation of the modal argument requires only a single state of affairs S that does not constitute conclusive evidence for $\Box F_G$. If there is a single state of affairs S in some world that does not constitute conclusive evidence for $\Box F_G$, then S constitutes conclusive evidence against $\Box F_G$. (7.3) follows from (1.3) – (6.3).

$$P(\Box F_G \mid S) < 1 \longrightarrow P(\sim\Box F_G \mid S) = 1 \qquad (7.3)$$

Since there are lots of states of affairs that constitute inconclusive evidence for the existence of an essentially

omnipotent, omniscient, and perfectly good being, (7.3) entails that those states of affairs constitute conclusive evidence against the existence of an essentially omnipotent, omniscient, perfectly good being. The state of affairs of an earthquake's destroying Lisbon and the state of affairs of a tsunami's devastating Japan might not be inconsistent with $\Box F_G$, but they certainly do not constitute conclusive evidence in favor of $\Box F_G$. So, it follows from (7.3) that we have conclusive evidence that God does not exist.

$P(\sim\Box F_G \mid S) = 1$ (8.3)

8.4 A Modal Argument from Goodness

The initial goal of the modal argument from goodness is to provide good reason to believe that at least some possible worlds are consistent with the existence of an essentially perfectly good being. Guleserian's conditions for impermissible actualization in C1–C3 provide the initial criteria for consistency. In general, possible worlds consistent with the exemplification of $\Box Fx$ meet the conditions in C4.

> It is reasonable to hold: C4 for any amoral world w consistent with the exemplification of $\Box Fx$, the maximal creatable portion of w and the maximum contingent creatable portion of w and world w itself, do not include more evil than good.

C4 is not intended to specify all of the conditions on possible worlds consistent with $\Box F_G$ Considerably higher standards of permissible actualization might include natural perfection and an axiological minimum. An amoral world is naturally perfect just if the world includes no natural evils. Amoral worlds trivially

meet the standard of moral perfection, since amoral worlds include no moral agents. A possible world meets the axiological minimum just if the world is overall sufficiently valuable. We might require that possible worlds w consistent with $\Box F_G$ are such that the maximal creatable portion of w and the maximum contingent creatable portion of w also meet the axiological minimum. It is difficult to believe that there are no possible worlds in the pluriverse that meet the standard in C4 and the considerably higher standards of natural perfection and an axiological minimum.

But it is extremely important for the modal argument from goodness that a sound modal argument does not in fact require a possible world that meets the standards above. A sound modal argument does not even require a contingent creatable portion of a world that is naturally perfect. All that is required for a sound modal argument from goodness is a single state of affairs that does not make $\sim\Box F_G$ certain. So, all that is required for a sound modal argument from goodness is a single state of affairs that does not constitute conclusive evidence against the existence of the traditional God.

Suppose we discover some state of affairs S in some possible world that does not make the probability of $\sim\Box F_G$ certain. So (1.4) is true.

$$P(\sim\Box F_G \mid S) < 1 \qquad (1.4)$$

Perhaps the state of affairs of every sentient creature enjoying its life over an infinite period of time or the state of affairs of a possible world being naturally perfect do not ensure the existence of an essentially omnipotent, omniscient, perfectly good being. But these state of affairs certainly do not make it

certain that there does not exist an essentially omnipotent, omniscient, perfectly good being. And there are lots of other states of affairs that do not constitute conclusive evidence for $\sim\Box F_G$. These states of affairs constitute at least some evidence for $\Box F_G$. So there are several states of affairs that satisfy (1.4).

But if there are some states of affairs in some possible worlds that constitute at least some evidence for $\Box F_G$, then those states of affairs do not entail $\sim\Box F_G$. (2.4) is a consequence of the fact that, for all possible states of affairs S, if S entails $\sim\Box F_G$ then $P(\sim\Box F_G \mid S) = 1$.

$$P(\sim\Box F_G \mid S) < 1 \longrightarrow \sim\Box (S \longrightarrow \sim\Box F_G) \quad (2.4)$$

It follows from (1.4) and (2.4) that there exist some possible states of affairs that do not entail $\sim\Box F_G$.

$$\sim\Box (S \longrightarrow \sim\Box F_G) \quad (3.4)$$

But it follows from (3.4) that S entails $\Box F_G$. (4.4) is also a theorem of S5.[7]

$$\sim\Box (S \longrightarrow \sim\Box F_G) \longrightarrow \Box (S \longrightarrow \Box F_G) \quad (4.4)$$

From (1.4)–(4.4) we know that if a state of affairs S does not constitute conclusive evidence for the non-existence of God, then S entails the existence of God. But if S entails the existence of God, then S constitutes conclusive evidence for the existence

of God. (5.4) is a consequence of the fact that, for all possible states of affairs S, if S entails $\Box F_G$ then $P(\Box F_G \mid S) = 1$.

$\Box (S \longrightarrow \Box F_G) \longrightarrow P(\Box F_G \mid S) = 1$ (5.4)

So, the modal argument from goodness does not in fact require a possible world that meets the standards described above. The modal argument from goodness does not require a contingent creatable portion of a world that is naturally perfect or axiologically perfect. The modal argument from goodness does not even require a single state of affairs that is inconsistent with $\sim\Box F_G$. A cogent formulation of the modal argument requires only a single state of affairs S that does not constitute conclusive evidence against the existence of God. If there is a single state of affairs S in some world that does not constitute conclusive evidence for $\sim\Box F_G$, then S constitutes conclusive evidence for $\Box F_G$. (6.4) follows from (2.4), (4.4), and (5.4).

$P(\sim\Box F_G \mid S) < 1 \longrightarrow P(\Box F_G \mid S) = 1$ (6.4)

Since there are lots of states of affairs that constitute inconclusive evidence for $\sim\Box F_G$, (6.4) entails that those states of affairs constitute conclusive evidence for $\Box F_G$. So, it follows from (6.4) that we have conclusive evidence that God does exist.

$P(\Box F_G \mid S) = 1$ (7.4)

In Section 8.3 it was shown that, given the logic of S5, any state of affairs S that does not constitute conclusive evidence for $\Box F_G$ does constitute conclusive evidence for $\sim\Box F_G$. But there are lots of states of affairs that do not constitute conclusive evidence for $\Box F_G$ including Lisbon's 1755 earthquake and Japan's 2011 tsunami. From these states of affairs we can derive (8.3) $P(\sim\Box F_G | S) = 1$. So, we have conclusive evidence that the traditional God does not exist.

But in Section 8.4 it was shown that, given the logic of S5, any state of affairs S that does not constitute conclusive evidence for $\sim\Box F_G$ does constitute conclusive evidence for $\Box F_G$. But there are lots of states of affairs that do not constitute conclusive evidence for $\sim\Box F_G$ including the state of affairs of there being a naturally perfect world and the state of affairs of every sentient creature enjoying its life over an infinite period of time. From these states of affairs we can derive (7.4) $P(\Box F_G | S) = 1$. So, we have conclusive evidence that the traditional God does exist.

But (8.3) and (7.4) are inconsistent. We cannot have conclusive evidence that the traditional God exists and also have conclusive evidence that the traditional God does not exist. But there is a more challenging conclusion. From (3.4)—(4.4) we know that there are some states of affairs S such that $\Box (S \longrightarrow \Box F_G)$. And from (4.3)—(5.3) we know that there are some states of affairs S such that $\Box (S \longrightarrow \sim\Box F_G)$. So we have the explicit contradiction $\Box F_G \,\&\, \sim\Box F_G$. It follows that either the modal argument from evil is unsound or the modal argument from goodness is unsound, and maybe both.

8.5 Some Consequences of S5

The central problem with the modal argument from evil is not that there are no possible worlds that include unjustly

distributed evils or unjustified evils. The problem is not even that there are no possible worlds that include, as Guleserian argues, sentient beings all of which suffer long spontaneous bouts of excruciating pain. The problem is rather that, granting the logic of S5, none of these states of affairs provides any *non-trivial evidence* against the traditional God. It is a theorem in S5 that every state of affairs in every possible world either trivially entails $\Box F_G$ or trivially entails $\sim\Box F_G$. (1.5) is a theorem.[8]

$$\forall S\Box\,(S \longrightarrow \Box F_G) \vee \forall S\Box\,(S \longrightarrow \sim\Box F_G) \qquad (1.5)$$

So every state of affairs in every possible world trivially supports $\Box F_G$ or every state of affairs in every world trivially supports $\sim\Box F_G$. Indeed, every state of affairs in every world makes $\Box F_G$ certain or every state of affairs in every world makes $\sim\Box F_G$ certain. (2.5) is also a theorem.

$$\forall S(P(\Box F_G \mid S) = 1) \vee \forall S(P(\sim\Box F_G \mid S) = 1) \qquad (2.5)$$

So, the state of affairs S—sentient beings suffering excruciating pain—does not entail $P(\sim\Box F_G \mid S) > 0$ without the auxiliary premise that $\sim\Box F_G$. To determine whether S is evidence for $\sim\Box F_G$ we have to first determine whether it is true that $\sim\Box F_G$. If it is true that $\Box F_G$ then the state of affairs S of sentient beings suffering excruciating pain is trivial evidence for $\Box F_G$. Indeed, in this case S makes $\Box F_G$ certain.

The modal argument from goodness has precisely the same problem. In S5, the state of affairs of a world's being naturally

perfect provides no non-trivial evidence for $\Box F_G$. Given (1.5) every state of affairs in every possible world either trivially entails $\Box F_G$ or trivially entails $\sim\Box F_G$. So, the state of affairs S of there being worlds that are naturally perfect does not entail $P(\Box F_G \mid S) > 0$ without the auxiliary premise that $\Box F_G$. To determine whether S is evidence for $\Box F_G$ we have to first determine whether it is true that $\Box F_G$. If it is true that $\sim\Box F_G$, the state of affairs S of there being naturally perfect worlds is trivial evidence for $\sim\Box F_G$. Indeed, in this case S makes $\sim\Box F_G$ certain.

The problem in the modal argument from evil and the modal argument from goodness is that (1.4) and (2.3) cannot both be true. Given the logic of S5, (3.5) is impossible. Modal space cannot include states of affairs that constitute some evidence against $\Box F_G$ and states of affairs that constitute some evidence against $\sim\Box F_G$.

$$\exists SP(\sim\Box F_G \mid S) < 1 \;\&\; \exists SP(\Box F_G \mid S) < 1 \qquad (3.5)$$

It is false, according to (3.5) that there is some possible state of affairs that makes God's non-existence less than certain and also some state of affairs that makes God's existence less than certain. The epistemology is in fact stranger, since (4.5) and (5.5) are both impossible.

$$\exists SP(\sim\Box F_G \mid S) = n(0 < n < 1) \qquad (4.5)$$

$$\exists SP(\Box F_G \mid S) = n(0 < n < 1) \qquad (5.5)$$

There are no states of affairs in any world that make $\Box F_G$ anything other than certainly true or certainly false. And of course the same holds for $\sim\Box F_G$. (3.5)—(5.5) follow from the fact that there are no states of affairs in any world that constitute non-trivial evidence for or against $\Box F_G$.

8.6 Conclusion

The modal argument from evil is unsound. It does not follow from the fact that there are possible worlds that include sentient beings suffering excruciating pain that the traditional God does not exist. It does not even follow from that state of affairs S that it is improbable—or even less probable—that the traditional God exists. The inference from S to $P(\sim\Box F_G \mid S) > 0$ is valid only if it is true that $\sim\Box F_G$. So discovering that there are intrinsically evil states of affairs does not alone constitute any evidence for $\sim\Box F_G$. For perfectly analogous reasons the modal argument from good is also unsound. The consequences are the same for any argument from evil—modal, evidential, or logical—whose background logic is at least as strong as S5.

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Notes

1 Let □Fx and similar properties name themselves, replacing λx□Fx.

2 Guleserian restricts attention to amoral worlds—worlds in which there are no free moral agents—but that is irrelevant to the forthcoming arguments. An ‘OOM being’ is an omnipotent, omniscient, morally perfect being.

3 The standard view on evil is that God cannot permit any unjustified evils. But compare →Almeida (2021).

4 See for instance →Linsky and Zalta (1996) and their (1994). See →Williamson (2015).

5 Conditional probabilities throughout are understood as epistemic probabilities. See →Mellor (2005).

6 I’ve left off the quantifiers, but the perfectly general proposition is a theorem in S5. ∀S(~□ (S ⟶ □F_G) ⟶ □ (S ⟶ ~□F_G)). The quantification is over possible states of affairs; I assume that the same states of affairs exist in every world.

7 The quantifiers have again been left off.

8 We add ∀S and ∃S to the language of the logic S5 quantifying over all possible states of affairs. We take states of affairs to be necessarily existing, abstract objects.

9 God and Perfect Beauty

Peter Forrest

Abstract: That God is of great beauty is religiously significant as a component of the widespread hope of a blessed experience in this or the next life. This motivates the project of formulating the strongest coherent thesis of divine beauty. This chapter has two parts. In Section 9.1 I argue for the incoherence of Perfect Divine Beauty. The natural reaction to this incoherence is to replace the idea of perfection, but in a way that coheres with the theists' religious tradition. I explore a range of such replacements, motivating the thesis of Pre-eminent Divine Beauty, that the actual divine beauty is infinitely greater than that of the physical universe. In Section 9.2 I examine various arguments against God's great beauty. They are the Argument from Divine Simplicity (and the related the Argument from Divine Spirituality), the Argument from Divine Necessity, and the Argument from Moral Righteousness.

Divine beauty is religiously significant as a component of the widespread hope of the *beatific vision*, that is, the blessed experience of God in this or, more likely, the next, life. That motivates the project of formulating a coherent thesis of divine beauty as exceedingly great. In addition, *perfect* divine beauty, if coherent, would be supported by the intuitively appealing idea of God as a "perfect being," namely as a being having "a maximally compossible collection of great-making properties, properties that, roughly, it is intrinsically better to have than to lack" (→Morris, 1987, p. 70).

This paper has two parts. In Section 9.1 I argue for the incoherence of Perfect Divine Beauty, even if interpreted charitably as the thesis of Divine Maximal Beauty. This incoherence should not surprise us, for similar considerations support the thesis that there is no best act of creation. Hence, we should replace Perfect Divine Beauty by some variant. I explore a range of such replacements, and give some reasons why theists should prefer two over the others. The weaker and hence more probable of these, Pre-eminent Divine Beauty, as I call it, is that the actual divine beauty is not merely infinite but infinitely greater than that of the whole physical universe, or any part thereof. More accurately: God is an infinity of *noticeabl*e ranks higher than the physical universe, or any part thereof, with respect to the ordering of comparative beauty. This coheres with one of the reasons theists might have for holding God to be of great beauty, the hope of the beatific vision. The stronger thesis is that God is *all beautiful*. It differs from divine pre-eminent beauty in asserting the divine excellence with respect to all types of beauty.

In Section 9.2, I examine various arguments against God's great beauty. These apply to all the replacements for Divine Perfect Beauty, and for that thesis itself should my argument for its incoherence fail. They are the Argument from Divine Simplicity (and the related the Argument from Divine Spirituality), the Argument from Divine Necessity, and the Argument from Moral Righteousness. I provide some defenses against these arguments.

9.1 The Incoherence of Perfect Beauty

9.1.1 Preliminaries

Defining Beauty

Whether or not we say that God is perfectly beautiful might depend on our conception of beauty. I shall not defend the claim that mine is correct for religious purposes, but merely state it, leaving those readers who disagree to decide if their conception affects the discussion.[1] My definition is as follows.

1. There is a social practice of creating works of art, including architecture, drama and even comedy. Although it might be contested just what works count as art and how we distinguish them from games, there are paradigms accepted in our culture.
2. Aesthetics is concerned with, among other things, the emotional response we humans often have to works of art, and the correlation between these responses and features of the art work.
3. The quality of typically arousing a certain combination of emotional responses may be defined to be an *aesthetic* quality of a work of art.
4. If the combination of emotional responses is good in a non-instrumental way, then we may say that the work of art has pure aesthetic value.
5. We have paradigms of *beauty*, including some works of art.
6. There is an emotional response that is often and properly elicited by the paradigms of beauty. It comprises a type of joy and a type of love. Hence the beautiful is a type of pure aesthetic value.
7. It is not merely wrong but absurd to be jealous regarding knowing something as beautiful, which distinguishes beauty from what is loveable in various exclusive human relationships.

8. Anything to which we can properly have this attitude is beautiful even if it is not a paradigm. This includes nature. And a non-paradigm may well be judged more beautiful than a paradigm.

Typically, the knowledge of something as beautiful is aspectual. For instance, the experience of a thunderstorm as beautiful is visual rather than auditory. The auditory experience is knowledge of something as sublime, which is not the topic of this paper.

Beauty is narrower than pure aesthetic value

This is implicit in the above characterization, but in case it needs emphasis I note how Frank Sibley (1959) writing about aesthetic concepts, describes their "almost endless variety, displayed in the following list: unified, balanced, integrated, serene, somber, dynamic, powerful, vivid, delicate, moving, tragic" (→Sibley, 1959, p. 421.) Although the beautiful is aspectual, it is not contextual in the way that much aesthetic evaluation is. Thus, when we consider a work of art aesthetically, we are often judging among other things the artist and the process of making the art work. There is no problem, therefore, in an aesthetic evaluation of the universe being treated as praise of God-the-cosmic-artist, but this is not the same as praise of God as beautiful.

Dispositions can have aesthetic value but not beauty

The point of the remarks on aesthetic value and beauty is to protect us against the careless assumption that because God is admirable in some aesthetic respect therefore God is beautiful in

that respect. An important case concerns potencies, that is dispositions, powers and capacities. Just as we admire the painters' sculptors' and musicians' abilities, we should surely be in awe of God's power to create a beautiful Universe. There is nothing beautiful about this power, however: contemplating a power does not give joy, merely the hope that that it will be manifested in some beautiful product.

To argue for the thesis that dispositions, powers, and potencies more generally, neither have beauty separately nor in combination, I first provide some examples supporting that thesis, and then undercut a putative counterexample. The examples concern dispositions that are arguably aesthetically relevant but which we judge do not affect beauty. We might, for instance, add vulnerability to Selby's list of aesthetic terms, but a fragile vase is not thereby more beautiful. Likewise, a person who lives beautifully but is vulnerable does not thereby live a more beautiful life. In the latter connection, I am reminded of Elton John's song "Like a candle in the wind," originally composed for Marilyn Monroe but famously adapted for the funeral of the British princess, Diana, whose life exhibited just such beautiful vulnerability. Arguably, the vase and Diana's life acquire extra aesthetic value because of vulnerability but would be as beautiful without it. Or consider power. A well-designed sports car has a certain beauty, but that is not due to its capacity to accelerate rapidly. For I might be admiring a scale model.

A putative counterexample might be the beauty of physical strength, illustrated by Blake's "Tyger Tyger burning bright," and, more prosaically, by a fit athlete. Of such examples, I say that we admire the beauty of fit bodies but not because they are powerful. To show this consider the Superslob twins, who never train, are overweight, smoke, drink and have flabby muscles. They turn up at the Olympics coughing and wheezing, but win a dozen gold medals between them. That is because they have the

capacity to run, jump and so on. The doctors who examine them cannot find any explanation, illegal or otherwise. For their capacity is without any physical basis. We may well admire the beauty of the athletes who win the silver medals in these events but our appraisal of the Superslobs is that they are grotesque not beautiful.

I conclude that although the narrative of divine acts is, I say, beautiful, had God never acted in any way, the unexercized capacity to create wisely and well would confer no beauty. This conclusion is contrary to the widespread inference from the beauty of the natural world to the conclusion that the creator of that world must be even more beautiful. For instance, Aquinas (1950) commenting on Pseudo-Dionysius says: “The beauty of a creature is nothing but the likeness of divine beauty participated in” (→Aquinas, 1950, §4:5.9). Again, Anthony Ashley Cooper (aka Lord Shaftesbury) has Theocles assert that “the beautifying not the beautified, is the really beautiful” (→Ashley Cooper, 1990, p. 322). I undercut this intuition by distinguishing between the beautiful and the aesthetic more generally, for which the appropriate religious term is “great.” Thus, the response that the preacher seeks to elicit is, in the words of the hymn, “How great Thou art!” not “How beautiful Thou art!”

Comparative Beauty

Beautiful things, whether actual, possible, or merely conceivable, form a partially ordered system with respect to comparative beauty. (Strictly speaking, we should say that things of equal beauty have the same rank and it is the ranks that are partially ordered.) The ordering is partial rather than total in that there are cases in which two items, x and y, are *non-comparable*, that is neither of them is more beautiful than the other, and yet they

are not of equal beauty. Try comparing Botticelli's *Primavera* with Vivaldi's *La Primavera*.

An explication of infinitely greater beauty

I say that z is *an infinity of ranks above* x, if, for any finite n, there are items y_1, …, y_n such that $x < y_1 < \ldots < y_n < z$. If we take the symbol "<" to mean "of noticeably less beauty," then this is an explication of the idea of z being infinitely more beautiful than x with respect to the ordering of comparative beauty.

We need the anthropocentric "noticeable" qualification because of a Zenoid, the possibility that y_n is less beautiful than y_{n+1} by an amount that rapidly decreases as n increases.[2] This qualification is not needed, however, for the thesis that divine beauty exceeds that of the universe or any part thereof by at least ζ ranks, for some suitably large infinite ordinal ζ. As we ascend the infinite ordinals, this provides us with increasingly strong explications of z being of beauty infinitely greater than x.[3]

The partial nature of the ordering is not relevant

That the ordering is partial might perhaps result in several non-equal cases of maximal beauty, which are non-comparable but all are such that there is nothing more beautiful. There might, for instance, be different ways in which God could be of maximal beauty, say a dynamic way analogous to a piece of music and a static way analogous to a sculpture. While acknowledging that as an epistemic possibility, I know of convincing cases neither for multiple maximality, nor for a multiplicity of what I shall call pre-eminent beauty. The most plausible case I can think of is the non-comparability of the sinless creature with the repentant

sinner. But, I say, the divine moral beauty exceeds both of these. Here I appeal to the following principle:

If x and y are non-comparable with respect to beauty and z is an infinity of noticeable ranks more beautiful than x then z is more beautiful than y.

Even the weakening of this principle with infinity replaced by some large finite number is intuitive. Consider the way Vivaldi's music is more beautiful than a painting-by-numbers version of Botticelli's painting, which is, however, non-comparable with whistling Vivaldi.

Types of Beauty

There is an alternative to ranking consistent items with respect to beauty. It is to consider excellence with respect to different types of beauty. Some distinctions between types are: static (e.g. sculpture) versus dynamic (e.g. music, dance); and sensuous (e.g. painting, sculpture, scenery, music) versus formal (e.g. mathematics). I also note moral beauty (e.g. a work of supererogation) as a type of beauty.

9.1.2 The Problematic Concept of Perfect Beauty

I now argue that the idea of perfect beauty is incoherent and should be replaced by some other thesis such as Pre-eminent Divine Beauty.

The Ambiguity of Perfection between the Maximal and the Complete

The most familiar concept of perfection is being both without flaw and, not quite the same, being complete, as in a perfect circle or a perfect specimen of some orchid. This is relative to a

kind: the perfect specimen of an orchid species lacks the capacity to move required for a perfect specimen of a gecko species, yet it is not imperfect, because this is not the lack of something proper to the orchid.

Now, beautiful things do not make up a kind, so it makes no sense to talk of beauty without flaw or complete beauty. Therefore, I submit that when we talk of perfect beauty we are using the term "perfect" loosely, and probably what we mean is maximal beauty, that which is more beautiful than anything (else) that is conceivable. There is, however, an alternative, which I call the *hybrid conception*, namely the beauty something has as a result of being a perfect (i.e. complete and flawless) specimen of the very highest kind of being. Just how we rank all kinds in a "Great Chain of Being" is a further question I'll not attempt to answer. We might however say that God has the beauty supervenient on being a perfect specimen of the kind, *agent*. Or maybe God is a perfect specimen of some sub-kind of agent, say a *person*, where I here suppose a person is an agent equipped with a point of view that affects the agent's values (→Baker, 2013, pp. 128–129). Again, we might consider the sub-kind consisting of agents with narratives of acts that have consequences, or yet again persons in a community, who interact. Or maybe God is of a *sui generis* kind.

This hybrid conception only deserves to be called perfect beauty if we grant that the ordering of kinds is necessarily correlated with comparative beauty in that higher kinds will, if perfect, be more beautiful than lower ones. This is counter-intuitive, however. For we would have to say that a perfect slug is more beautiful than the perfect orchid it would like for breakfast, assuming we grant that mollusks are a higher kind than flowering plants.

No Completion

The hybrid conception of perfect beauty is problematic, then. There is a further problem. For perfection is not the mere absence of flaws but requires completion, in other words, achieving a telos. Now an agent of limited power might achieve a telos in the sense of having done everything worth doing. A divine agent, however, although always without flaw, can never achieve completion. For there are always more wonderful things that could be done. Now, it could be objected that we should consider God not merely in the past and present but omni-temporally, and so the sequence of divine acts is complete. Even if we grant this omni-temporal perspective, there could still have been more acts than there actually are because God could have done everything quicker, and so made time for more acts.

Formal causes

I anticipate the objection that the acts are not part of the agent, but that something else that is part of the agent is complete. I consider, therefore, the suggestion that the *formal causes* of creation are perfectly beautiful. The idea of a formal cause is that the cause contains a plan for the effect. For instance, we might think of the DNA in a seed as a plan for the development of the plant. This does not hold for chemistry or physics, but is plausible in cases of deliberate agency, because *pure* spontaneity is not genuine agency, so there must be some kind of planning even if the plans undergo change.

It is beyond the scope of this paper to decide what is the formal cause for divine action. It might be the divine imagination. Coleridge claims to have had the whole of “Kubla Khan (Xanadu)” in his head, but to have forgotten part of it

when interrupted while writing it down. If so, what he had in his head was presumably of even greater beauty than the poem that was written down. Or we might think of the formal cause as a universal that is instantiated in creating.

The No Gap Objection

Whatever the nature of the formal cause, we should suppose that God prior to creation is aware of the plenitude of possible universes, and selects some to be created. This sets up a dilemma with the supposed formal cause, which is said to confer great beauty on God. On the one hand, if we think of it as that plenitude of all possible universes it is neither flawless nor complete. It is flawed, because many of the possible universes are ugly. It is incomplete because many features of the created universe depend on the non-actuality of various possibilities, and so are not there in the plenitude. Natural kinds, for instance, are specified as much by their separation from other kinds as by the types they include. Likewise, there are no individuals in the plenitude, because by small degrees we all merge into each other, as in Derek Parfit's example of his gradual transition into a replica of Greta Garbo (1984, part 3).

On the other hand, suppose that God first decides not to instantiate some possible universes, leaving the remainder as the formal causes. Then we are positing a two-step account of creation; select the remaining universes; then actualize them. But whatever is required to select some rather than other possible universes is enough for creation. These posited formal causes are redundant. I conclude that the not merely should we reject Plato's myth of the demiurge as making flawed copies of the flawless forms, but the formal cause is either itself the plenitude of possibilities and so flawed or is redundant. Hence

the hybrid conception of perfect beauty leads to an impoverished conception of divine beauty, and should be rejected.

There is an additional problem with positing flawless divine beauty. For flawless beauty implies that no part is intrinsically ugly. Yet there can be beautiful things that would be less beautiful overall if the ugly part were replaced by something beautiful. The Isenheim altarpiece comes close enough to establish at least the possibility—and not merely the conceivability—of this anti-classical beauty. Clearer examples are provided by narratives: a story without setbacks is boring. Judaism and especially Christianity tell a story of God's glorious conquest of human wrong-doing not by force but by the invitation to repent: God is the general who loses every battle but wins the war (see →Forrest, 2022). This has a greater claim to be beautiful than would a history of happy innocent hunter-gatherers handing their culture down year after year.

A corollary: The formal cause of creation is less beautiful than creation itself

The No Gap Argument shows that the formal cause of creation in the mind of God is the whole plenitude of possibilities, which is a mixture of the good and bad, the beautiful and the ugly. Moreover, it is incomplete, lacking division between kinds and between individuals. In creating, God selects what is beautiful, which is therefore superior to the flawed incomplete plenitude.

Note: In another sense the *formal cause* is defined as the form instantiated by the effect. In that sense, some might argue that it is as beautiful as its instance. That is not, however, the relevant idea of a formal cause when considering divine beauty, because it inheres in the effect not the cause.

The Incoherence of Maximal Beauty

We should reject the idea of Perfect Divine Beauty, with perfection understood the usual way. Consider, then, the charitable interpretation of Perfect Divine Beauty as Divine Maximal Beauty, namely that there is nothing conceivable (including conceivable ways God might be) that is more beautiful than God actually is. Perhaps our paradigms of beauty are static like a statue or a landscape, but if we characterize beauty as in Section 9.1.1, we should include dynamic examples, like music. Even further from the paradigms are examples of moral beauty. We might say that meeting a moral obligation is neutral, failing to do so ugly, but works of supererogation are beautiful.

It is conceivable that God has maximal static beauty, but because we are to include dynamic and moral beauty there is no maximal divine beauty but rather a sequence of ever more beautiful divine states. In support of this conclusion, consider the actual sequence A of divine acts and, we may suppose, creaturely responses. Then consider a possible sequence B that occurred more rapidly and so had advanced further, and which included an exact replica of A. B would be more beautiful than A. This is especially the case if B has achieved a telos, such as the goal of a loving divine/human community.

One response might be that in saying that God is maximally beautiful we are anticipating a future state in which this telos has been achieved. My rejoinder is to repeat the criticism of the hybrid conception of perfect beauty: agents do not have tele the way organisms at least seem to. Consider the traditional example of the sapling whose telos is to grow into a mighty tree that will flower and bear seed year after year. If it achieves this telos and dies three hundred years later, then its life is complete. Contrast this with an aid worker who after thirty years of hard

work in the field decides to retire and take life easy. There is no flaw in that decision but neither is there completion as an agent. For the aid worker could embark on a new career—as a politician, say. To be sure, in the human case death brings further projects to an end. But, to repeat my earlier remarks, in the divine case there is no weariness and hence no reason to rest content. Hence, achieving the telos of a divine plan cannot bring the perfectly beautiful sequence of divine acts to an end, because something more beautiful could be achieved by then embarking on a new sequence of acts. (If you retort that the concept of a telos *entails* no further activity, then we need a new concept, the quasi-telos but I shall stick with the term "telos.")

It might help to have a different kind of example in order to justify this re-conceptualization of the telos. Compare Aeschylus's trilogy the *Oresteia*. The first tragedy, "The Agamemnon" ends with a telos: Clytemnestra has her revenge on the man who killed her children; then she and her lover Agathon become joint rulers. Nonetheless the other two plays are not arbitrary sequels, and the whole trilogy has as its conclusion the inauguration of the rule of law in place of revenge. Likewise, we can treat the divine history as an infinite analog of a trilogy—an apeirology—about which we know only the first few parts.

The beautiful conceivable sequence of divine acts must, therefore, be unending, in which case there is no stage at which this conceivable sequence will be actual. Hence, it would seem, God is never maximally beautiful but forever growing towards perfection.

The Infinity plus One Objection

It is sometimes said that God plus creation is no more valuable than God alone because God is of infinite value, so adding something of finite (or indeed the same infinite quantity) just gives us infinity again. Something similar could be said of the divine beauty, namely that the narrative of creation and interaction with creatures cannot add to the already infinite beauty of God.

I have two replies. The first, which is more controversial, is that God has an inner life involving a sequence of acts (namely the interactions between divine Persons) and that the case against maximality applies to the beauty of this inner life quite as much as to God's creative activity. The less controversial reply is that the objection is based on faulty mathematics. The characterization of maximal beauty is based on an ordering of items by means of comparative beauty, so the relevant mathematical concept of infinity is that of the infinite ordinals. If the actual sequence of divine acts has ordinal ζ, then this sequence could have been completed by, say, AD 1, after which a further sequence of acts with ordinal η could then be performed. And the ordinal $\zeta + \eta$ is greater than the ordinal ζ.

I anticipate the rejoinder that in applying the ordinal numbers to a sequence of divine acts I am carelessly assuming that every divine act has an immediate successor, the next act. This is required for a divine narrative, but I concede that there could be multiple divine narratives going on at the same time. These narratives could themselves be arranged as a sequence ordered by how fast they achieve their tele. In that case the argument that whatever sequence God performs there could have been a faster version fails, for we might say there is a faster version as well. My response is that although there may well be an infinity of divine narratives and that maybe this multiplicity adds to the overall goodness of things, it detracts from the

divine beauty if there is a multiplicity of similar narratives, as implied by the suggestion of a faster version.

Another explication of perfect beauty

Absolutely infinite beauty may be defined as being of rank Ω more beautiful than something that is itself beautiful, where Ω is the greatest ordinal. The idea of a greatest ordinal requires non-standard (dialethioc) logic, however (see →Priest, 2002).

9.1.3. Substitutes for Perfect Beauty

The theistic impulse to call God perfectly beautiful is frustrated, therefore, and we should seek a substitute. Here is a list, of which the first three are acceptable.

1. *Pre-eminent Beauty*: God is an infinity of noticeable ranks higher than the physical universe, or any part thereof, with respect to the ordering of comparative beauty.

Comment: This is an acceptable substitute for perfect divine beauty, even though the term “noticeable” is anthropocentric.

1. ζ-*Beauty*: The divine beauty is at least ζ ranks greater than the physical universe or any part thereof. (Here ζ is some chosen infinite ordinal, as it might be ω_ω.)

Comment: This would be a good substitute for Perfect Divine Beauty, except that it is not clear just which ordinal is chosen. If it is ω_ω then why not select $\omega_\omega + 1$ instead? Absent a satisfactory answer, theoretical simplicity does not favor divine-beauty over pre-eminent divine beauty as a hypothesis.

1. *All Beauty:* For every type of beauty God is maximal if there is something maximal for this type. Otherwise, God is either (a) pre-eminent with respect to this type, or (b) has beauty of that type with rank ζ some large ordinal ζ, say ζ = ω_ω.

Comment: The disjunction (a) or (b) is intended to specify two versions of divine all beauty I have the same reasons for preferring the more modest pre-eminence disjunct (a) as I gave when discussing ζ-beauty above.

1. *Transcendent Beauty*: God is more beautiful than the physical universe, or any part thereof.

Comment: This is coherent, but I explain below why theists should prefer the stronger thesis of Pre-eminent Divine Beauty.

1. *Limitless Beauty*: Although God is never maximally beautiful, for any consistent state x there will be a stage at which God is more beautiful than x.

Comment: The No Completion Argument against the coherence of Perfect Beauty is based on the idea that God could always become more beautiful. Initially this might seem compatible with Limitless Divine Beauty but we could "speed up" the actual divine history so that further stages are adjoined showing the actual divine sequence of states never surpasses some conceivable state.

1. *Maximal Possible Beauty*

Comment: The classical theist might argue that God has the greatest *possible* beauty, not the greatest *conceivable* beauty, and that necessarily there is just the one divine act, not a

sequence, undercutting any case from an incomplete infinite sequence of acts. This assumes a conception of metaphysical possibility that is stronger than mere consistency. I argue below that it is untenable, for the God of classical theism, being simple and spiritual, would not be as beautiful as the physical universe.

1. *Ineffable Beauty*: The divine beauty cannot be described.

Comment: This is irrelevant. Our supposed inability to describe the divine beauty is no rival to substitutes 1 to 6 listed above, which are stated in terms of the partial ordering of beautiful things, without specifying any type of beauty. Nonetheless, because of the popularity of ineffability theses. I make some concessions below.

1. *Perfect Beauty by Analogy:* The phrase "perfect beauty" is predicated of God by analogy.

Comment: I argue against this in Section 9.1.3.

Pre-eminent Divine Beauty explains better than Transcendent Divine Beauty and so is to be. Preferred. For I hold that: (1) understanding indirectly is to explain one truth in terms of another; and (2) direct understanding, either partial or total, is an appreciation of the beauty (loveliness) of things (see →Forrest, 1991). This may be contrasted with the rival thesis that direct understanding is noting the simplicity of things. Theists might well hold that the divine beauty is sufficient to provide a total understanding of why God exists, if only we could describe God adequately. The thesis of Infinite Divine Transcendence would not establish a significant explanatory advantage over the thesis that the first cause is something purely physical.

Note: If we weaken Pre-eminent Divine Beauty to cover only what is possible then the proposed way of understanding God's

existence is only partial, because it assumes without explanation the necessary truths.

The all beautiful and the all great

The incoherence of perfect beauty implies the incoherence of Morris' characterization of a perfect being as "a maximally compossible collection of great-making properties" (→Morris, 1987, p. 70). Instead the intuitively appealing idea of divine perfection should be replaced by the thesis that God is *all great*:

> For every type of excellence, God is maximal if there is something maximal for this type. Otherwise God is either (a) pre-eminent with respect to this type, or (b) has an excellence of that type ζ ranks above the physical universe or any part thereof, for some large ordinal ζ, say ζ = ω_ω.

Some consequences of all beauty

In reply to various arguments against divine beauty I shall propose speculations such as the universe being the body of God, God consisting of (at least) three Persons, and a glorious history of divine interaction with creatures. The divine all-beauty would support these theses by implying that God has beauty of those types. In turn that would tend to support Christianity. A Hindu, however, might retort that the ineffable divine beauty thesis shows that God does not necessarily have those types of beauty but maybe something far exceeding them, but that it is proper for Christians to think of God as having those types of beauty, as in some sense pointing to the greater beauty God actually has. This is not the place to decide who is correct.

Concessions regarding ineffability

First, in saying that we can know something about God we do not claim to know everything or even very much about God. Related to this, to say that there are similarities between God and creatures does not imply that the shared properties anywhere near exhaust the divine nature. Another, almost trivial, concession: to apply the same term univocally of God and to creatures does not require that we can *imagine* how it applies to God.

I make, however, a more significant concession, which bears on the scholastic claim that the *modi significandi* differ in the case of God and creatures (see →Rocca, 1991). There are cases in which we know a term applies but do not know how. Consider the name "Abraham." It might or might not succeed in referring. If it does refer, then it does so because of a chain of reference borrowings of the form: X uses the name N to refer to whatever others have already used it to refer to (see →Kripke, 1980). We need no knowledge of these reference borrowings but provided the chain is unbroken and does not lead to ambiguity we may refer to Abraham. In that case we might either say that we do not know how the name "Abraham" refers, or that we do but that the details escape us. So, let us just say that the reference to Abraham is not *accessible.* My concession is that I do not carelessly assume that the application of various terms, including "beauty," to God are accessible. For knowing the paradigms of beauty enables unto use the term "beauty" without knowing all the types of beauty.

9.1.4 Perfect Beauty by Analogy?

There are many who do not admit to applying typical theological terms literally to God, that is, univocally to creator and creatures.

Aquinas is a notable example, Duns Scotus a notable exception. And more recently this reluctance has been stated in an extreme version as the bizarre thesis that it is idolatrous to claim literal knowledge of God (→Marion, 2012). This is relevant because if correct there is no point in discussing the beauty of God in a literal sense, which is my topic; and, it will be said, the thesis of divine perfect beauty should be understood figuratively, perhaps as metaphor or analogy. Here I ignore that figure of speech, hyperbole. For anything beautiful may be said to be perfectly so by hyperbole.

Aristotelian Analogy

In the previous section, I undercut the intuitive inference that God is beautiful from the premise that God is the cause of beauty. This intuition may also be undercut by exhibiting it as a figure of speech noted by Aristotle, and often called analogy by scholars. Just as we call certain diets healthy and others unhealthy because they are conducive to good and bad health respectively, we may call beautiful the cause of beauty. Analogy in this "pros hen" sense is literally false and given the contemporary use of the word "analogy" misleading when it comes to talking about God. It is not even analogy by analogy! It is like calling a gardener colorful because the garden has brightly colored flowers.

To say that God is perfectly beautiful by Aristotelian "analogy" requires either that the causal relation or the beauty caused is perfect. The former amounts to God being a causally necessary and sufficient condition, which may be true but hardly remarkable. Moreover, we could say that God perfectly causes beauty even if the beauty caused was quite slight. I assume, then, that we are to consider the suggestion that God is perfectly

beautiful in the Aristotelian analogical sense, if God causes literal perfect beauty. But in that case the incoherence of literal perfect beauty implies the incoherence of perfect beauty in the Aristotelian analogical sense.

Explaining the appeal of Aristotelian analogy

If we hold that in some way the effect resembles the cause then Aristotelian analogy may be assimilated to our contemporary sense of analogy as similarity. The only sense in which that cause/effect resemblance thesis is not outmoded is in the plausible thesis that an act has a formal cause in the mind of the agent. In that case, Aristotelian analogy enables a theologian to be coy about the way in which the formal cause is in the divine mind. That is fair enough, but it does not affect the corollary to the No Gap Argument that the formal cause of creation is less beautiful than the create universe.

Our Contemporary Sense of "Analogy"

Religious language makes much use of analogies in our contemporary sense of similes and metaphors. These are based on our capacity to recognize respects of similarity once we are invited to look for them. A defender of the use of "beautiful" of an agent in abstraction from any physical structure might, therefore suggest that even if we ignore formal causes, there are often some similarities between causes and effects and that we may therefore be invited to recognize the similarity between God and the beauty of creation.

In response, I say that things are similar only if either they have a shared property or they are species of the same genus. The first case might be relevant to the claim that God is a

person, but we might well doubt that beauty is itself a property or has properties. But if so, then it would be the shared property in question and so the term “beautiful” would apply univocally to God and creatures. The second way of being similar makes good sense: the genus in question would be aesthetic value more generally, and it is easy enough to distinguish those pure aesthetic evaluations such as grace and elegance that are similar to beauty from those such as strength that are less so. The resort to analogy in this sense, then, amounts to no more than something I have conceded, namely that I am using the term “beauty” in an extended sense and the term “lovely” might be more appropriate. But that does not affect the case against calling God perfectly beautiful.

Perhaps the suggestion is that the qualification “perfect|” is being made by analogy. If it is intended that God lacks even something resembling a flaw that would strengthen the definition of perfect beauty and so not undercut the case against divine perfect beauty. Therefore, consider instead the suggestions that the divine beauty does not derive from completion but something like completion, and that it is lacks maximal beauty but in some way resembles being maximally beautiful. These suggestions would betray a lack of thought on the topic. For we know what resembles completion, namely Limitless Divine Beauty. And we know what resembles the maximal divine beauty, namely that God has pre-eminent beauty.

Putting the question of perfection to one side, the suggestion that God is beautiful by analogy in the sense of similitude is coherent, but the suggestion that God is similar to a beautiful agent is open to the objection of the previous section. For dispositions do not have X for any X that is similar to beauty. Only because the aesthetic is so broad do we include strength and power, that unlike grace and elegance, apply to dispositional

properties. Grace, elegance and other beauty-like aesthetic valuations require a narrative of successive acts. For that which is achieved immediately is neither graceful nor elegant.

9.2 On Some Arguments against Divine Beauty

I shall state three arguments and offer speculative defenses. Elsewhere, I have expounded the idea of a speculative defense as a hypothesis that explains at least as well as alternatives and undercuts the argument in question (→Forrest, 2019, pp. 19–20; see also →Van Inwagen, 1991).

9.2.1 A Simple Being? A Spiritual Being?

The Argument from Divine Simplicity

If a substance has structure, then great beauty can supervene on the way the parts are related, but without that structure it is merely characterized by some non-relational properties. In the case of a non-physical mind, I have argued that these will be dispositions, which do not confer beauty. Now, Mathew →Baddorf (2017) claims that a simple substance would have no beauty at all, but his fallback position is that it could not have much beauty. An example that comes to mind is an intense uniformly blue sky gazed at while lying on your back. Of this I say its beauty is an infinity of ranks below the physical universe, nor is it totally without structure, because it has parts that are spatially related. There is, then, a prima facie conflict between the two theses of Divine Simplicity and (significant) Divine Beauty. Call this the Problem of Beautiful Simplicity.

The Equivocation Solution

The question “What is God” is, like most “What is?” questions, ambiguous between a request to list all the persisting intrinsic properties and a request to list only those that are essential. Suppose, then, that God is not essentially complex and hence not essentially beautiful but is intrinsically both complex and beautiful. This might occur in a number of ways, of which I can think of three. First, a heterodox Christian Social Trinitarian might hold that the three divine Persons are substances and the Trinity is therefore a beautiful complex structure. I consider an alternative, more orthodox, Trinitarian solution later, but the reasons given there for ascribing beauty to God would hold in this case too.

The second and third ways of developing the Equivocation Solution rely on the thesis of Divine Embodiment, which has become popular recently (→Jantzen, 1984) and coheres with another popular idea, Panentheism (→Culp, 2020). It does not require us to deny that God has a non-physical part, a “world-soul,” although I prefer a non-naturalistic pantheist variant (→Forrest, 2019, pp. 73–74). In any case, we might qualify the idea of the Universe as being the divine body by noting that parts of it, those occupied by creatures with free will, are not under divine control. Hence, we obtain Swiss Cheese, the thesis that the divine body has holes in it; sinners are the unholy holes in the holy whole.

The second way of treating the divine beauty as intrinsic but not essential is to assert that divine embodiment in the Universe is likewise intrinsic but not essential. That is to say of God what some dualists, such as Richard →Swinburne (2009) say of human beings, but it is perhaps more plausible for God than for us. The third way is more intriguing. It is to insist that God is essentially embodied as the Universe but that this body is not essentially complex, having begun as a Big Bang singularity.

Contrasting embodiment with immanence

Divine Embodiment may be contrasted with a weaker thesis, Divine Immanence, which I take to be the conjunction of (1) the thesis that God is aware of all that happens, and (2) the thesis that God can act anywhere. The latter might need qualification in two ways. First, we may hold that God, like us, is constrained by the laws of nature even though God set up these laws. Second, as in Swiss Cheese, we may hold that in some regions creatures, not God, have power over what is left undetermined by the laws.

The distinction between immanence and embodiment concerns the way God is aware of everything, the former being likened to vision, the latter to body-awareness (proprioception.) We may make this more precise, by considering the content of divine perception. The vision analogy amounts to God knowing that such and such, say a supernova, occurs over *there*; the proprioception-analogy that it occurs *here*, in the way that when human beings are aware of pains they are aware of them as *here.* This difference in the phenomenology abstracts away from the details such as vision being able to detect arbitrary shapes, and proprioception being concerned with the shape of one's body.

An a priori reason for preferring divine embodiment to divine immanence

The Divine Embodiment is not just a solution to the problem of how God can have beauty at least as great as the physical universe, it has a priori support because the alternative, Divine Immanence, uses the vision analogy, which requires God to have a point of view external to the universe, which is problematic. There is a well-known Thomist solution to this problem of the

divine point of view, namely that God intends that there be a certain type of universe and does not need subsequently to observe because the divine intention is never thwarted. This implies either predestination or divine ignorance of creatures' free decisions even after those decisions. Neither is acceptable.

Quasi-physical Embodiment

I now examine two solutions that differ slightly from straightforward embodiment.

It might be objected that non-physical souls, supposing there are any, can have shape. Here we may note that the phantom limb phenomenon shows that body-images do not exactly correspond to our material bodies. God may then be likened to a soul, and hence have the shape of the universe, without being embodied in it.

Not merely is the idea of an extended soul intellectually extravagant, but this solution suffers from a continuous multi-lemma, with one end of the spectrum being a soul with extent but only the same level of complexity as Euclidean Space. At the other end, there is a soul whose shape is intricately convoluted. Only by being at or near the convoluted end could the soul have beauty comparable to the physical universe, but near that end it is hard to distinguish God's soul from space-time, with geometrodynamics being a corollary. (This is the theory that the physical universe just is a suitably convoluted space-time.) It is conceivable that there is some intermediate position in which the divine soul's beauty is as great as the universe but in which this soul is not identical to that universe: conceivable but not plausible.

More plausible than an extended divine soul is the thesis of a divine phenomenal field, constituted by sense-data or some

substitute for sense-data, of which God is directly aware. Clearly this requires a theory of perception as indirect, which is controversial, but that is not my present concern (see →Forrest, 2005; →Hatfield, 2013). The manifold of sense-data arranged in a phenomenal field would indeed be of great beauty, but the problem is distinguishing a divine phenomenal field from the physical universe it represents. And if they are not distinguished, then we obtain not a rival to divine embodiment but a theistic idealist endorsement of it.

Non-physical Structure

The argument that God is simple and so of little beauty relies on the premise that all structure consists of relations between disjoint, or at least distinct, substances. I now note two ways of resisting this, the first is general and abstract, the second can be understood as a more concrete instance.

The argument against non-physical divine structure fails if there are beautiful higher-order structures based on properties of and relations between lower order properties, for instance second-order relations between the properties of substances (see Bergman, 1957; →Armstrong, 1978, ch. 23).

Some might object that this Bergmannesque hierarchy of universals is a metaphysical Tower of Babel, nothing more than hubristic trust in a priori intuition. But, although controversial, this hierarchy can be used to support realism about mathematics. (See →Franklin, 2014). The beauty of mathematics does not require a body, and may well be one component of the divine beauty. I consider below the argument that such beauty is not enough.

The Christian doctrine of the Trinity undercuts the argument from simplicity by proposing that the simple divine substance

consists of three Persons who are not substances in the relevant sense, but whose interpersonal relations give the divine substance structure. If we thought of the divine Persons as properties of the one God, then this would be an instance of the previous way of undercutting the argument. I note that these Persons are traditionally considered *hypostases*, that is items that have properties and stand in relations without being themselves properties or relations. A more conservative metaphysics refuses to distinguish hypostases from substances, which should make it more plausible that the Persons are distinct properties of a simple God.

The relations between divine Persons have a complex structure on which could supervene great beauty. Suppose, for instance that the love between the divine Persons finds expression in their gifts of power over, and knowledge of, the beautiful Universe they have created. For simplicity of exposition, take this to be a sequence of acts whereby Person M abdicates power to Person N. If there are only two persons this would be too boring to be eminently beautiful, consisting just in handing the power backwards and forwards. But with three there are infinitely many variations in the sequences of gifts. By analogy this is a divine dance or conversation.

If the whole universe is the divine body and the beauty of God is just that of the divine body, then God lacks transcendent beauty. If on the other hand, we suppose Swiss Cheese and if some creatures are depraved, then the divine body does not suffer from the flaws of moral ugliness. If, however, creatures exhibit moral loveliness by being good in a superogatory way, but God does not then, on Swiss Cheese, God is less beautiful than the universe God has created. One solution to this problem is to suppose that God performs works of supererogation, as in the Christian doctrines of the Incarnation and Redemption.

I see, however, no obstacle to combining the Divine Embodiment and Multiple Divine Persons solutions in a way that implies Pre-eminent Divine Beauty. The “dance” or “conversation” between the divine Persons could be about each responding to the others acts of partial creation, that are identified with the increasing definiteness of the divine body. This would cohere with the speculation that God creates not ex nihilo but by de-actualizing most of the initial plenitude of all that is possible (e.g. →Forrest, 2019, ch. 7).

Three Arguments for Divine Simplicity

I now present three arguments for divine simplicity in the sense of not having a multiplicity of substances as parts. As I have said, I am not endorsing these arguments, merely noting their prima facie force and hence the need to counter them as in the solutions proposed above to the Problem of Simple Beauty. Because of the Equivocation Solution it is also important to judge whether they support essential simplicity or merely intrinsic simplicity. I start with the more traditional ones.

God is traditionally believed to exist necessarily and to be incapable of ceasing to exist. The supposed necessity of God is the premise for a rather different argument against divine beauty discussed below, but here I am using that premise to argue for divine simplicity.

If God consists of substances related a certain way then, it is argued, these substances could cease to be related that way, maybe not even be related at all. So, the structure is contingent. This is then taken as a reductio ad absurdum of divine complexity, leading to the conclusion that God is essentially simple, obstructing the Equivocation Solution.

As it stands, the argument is open to the objection that the divine parts might be *necessarily* related in a certain way. A corollary, though, would seem to be that God is non-physical because physical relations between parts are contingent. In an earlier age this corollary could have been resisted in the grounds that the Euclidean nature of space was necessary. I say we know better. To be sure, had space-time had a different geometry it would not have had the parts it actually has, but it is nonetheless true that there could have been a different geometry.

The second argument is that we could not explain why there is something rather than nothing, unless we could understand the first cause, but given that there is something or other we might hope to explain to the physical details of the universe in terms of a non-physical agent. But this explanation is unsatisfactory if to explain the physical structure we invoke a structured God (cp. →Aquinas, 1989, 1a.3.7). This conclusion, note, does not obstruct the Equivocation Solution.

The third argument is based on the human procedent. The details of our minds are adequately explained in physical terms once we accept the mysterious facts of consciousness and agency. Hence there is no precedent in the human case for complex non-physical minds. To avoid extravagant metaphysics we should suppose that a God has no essential non-physical structure other than the dispositional and relational ones involved in agency and consciousness. Again, this is compatible with the Equivocation Solution.

Necessary Relations?

The Argument from Divine Necessity fails if the relations between the substantial parts themselves hold of necessity. I do not press this, because both the Divine Embodiment solution

and the Trinitarian solution propose contingent relations. This might not be obvious in the case of the Trinity because anything other than the love between the divine Persons is, we may assume, literally absurd—incomprehensible. But there are, I submit, many incomprehensible possibilities such as God choosing to abdicate so much power as to cease to be worthy of worship. We are able to know a priori that these are never actual.

The Weak Divine Necessity Thesis

Instead of proposing necessary relations between the parts of God, I weaken the thesis of divine necessity thus: the existence and nature of God was necessary up to some stage of God's history. If there is a first divine moment, then this is equivalent to saying that at that moment the existence and nature of God are necessary. This is compatible with God subsequently making contingent choices that affect the divine nature itself, such as becoming a Trinity or becoming embodied in the Universe.

Against the Argument from the Human Precedent

On the one hand, the human precedent clearly permits a complex divine body. On the other, the Trinitarian solution is compatible with the assertion that each divine Person is simple, even if these are not treated as properties but as substances in some sense. In that case the human precedent does not imply that God is a simple substance.

On the Argument from Explanatory Power

The Trinitarian solution does not threaten the explanatory power of a simple God, because both the Trinity and the Universe could be the work of the initial simple God. Such a simple God might explain the complex Universe that is identical to the divine body. We may deny that God is essentially embodied, in which case God creates the divine body. I provide a more satisfactory account in the next section.

I conclude that the Equivocation Solution succeeds provided there is no good case for the thesis that God is essentially simple. Of the other solutions, the only one that is satisfactory is the Trinitarian.

9.2.2 Arguments from Divine Necessity

God is not usually considered to be a contingent being, and this raises three problems for the divine beauty. The first is that we know what necessary beauty is like, it is the beauty of mathematics, which suggests that the divine beauty is only understood by mathematicians, and that the beatific vision is attending an unending mathematics seminar. Even though the idea of filling chairs in theology by pure mathematicians has appeal, this is preposterous. Moreover, unless we suppose the ordering of comparative beauty is total, we should judge that the abstract beauty of mathematics is not comparable to the partly sensuous beauty of the universes. Hence God would lack even transcendent beauty.

“Substantial” and Formal Beauty

Here is a solution to the problem that the beatific vision is a mathematics seminar: distinguish the form from the matter of an object of beauty. This is related to, but not the same as the

thesis that agency requires formal causes—bear in mind that some objects of beauty such as musical performances are not literally substances. Ashley Cooper (1990, pp. 312–313) in the person of Theocles insists that the beauty of a piece of metalwork is entirely in the design not the metal. This exhibits a Platonic bias perhaps, but it is nonetheless correct that we can abstract from the matter and consider the form as beautiful. For example, the totally color-blind might well be able to distinguish the different colors of a painting because of different textures and appreciate the form of the painting. Again, a piece of music might be appreciated by someone with a bionic ear unable to hear the quality of a note other than its pitch. These and other examples show that:

1. The “matter” e.g. the notes or colors, by themselves have only rather limited beauty,
2. The form can have considerable beauty, but
3. The composite has greater beauty than that of the form and “matter” taken separately.

We may expand the non-contingent beauty to include the formal beauty, but I expect readers to agree with me not Ashley Cooper that a purely formal beauty is less than the “substantial” beauty of the composite.

More difficult is the question of imagined beauty. Human beings require experience of the quality of music and of colors to imagine beauty but we might expect God to be able to imagine any possibility. This might show God has aesthetic experiences that go beyond what is actual. Perhaps these possibilia are real —a controversial metaphysical thesis. In any case it could be said that the plenitude of all possible universes is itself non-contingent, and that God is non-contingently aware of their beauty. My rejoinder is that, whether real or not, the possibilia in

this plenitude are not individuated as distinct items but form a continuous spectrum and that the individual is the item of the "substantial" beauty. Moreover, the totality of imagined possibilities is flawed because many of them are ugly. Hence, the created universe is more beautiful, and gives greater joy to God than some plenitude of possibilities.

A Trinitarian solution?

Nor should we appeal to the Trinity here. For, as mentioned above, the relations between the Trinity must be contingent if there is to be the dance-like beauty of giving up and accepting power.

Weak Divine Necessity Again

The only defense is to deny that the divine attributes are necessary, appealing to Weak Divine Necessity once again. There are two ways in which contingent divine acts could greatly increase the divine beauty. The first is if initially God is embodied as a simple singularity and becomes the extraordinarily beautiful Universe we know. The second is by the Trinitarian "dance" of the transfer of power, described above.

9.2.3 Moral Righteousness versus Divine Beauty

The third argument against divine beauty is from moral righteousness. Now, there is an a posteriori argument that the creator was motivated by love of beauty rather than the well-being of creatures. That argument is based on an intuitive assessment of the combination of natural beauty with the suffering of all sentient beings, even if we provide a theodicy to

explain human wrong-doing and suffering. In this section I add to this an a priori argument based on the beauty of the universe considered as the divine body. This is a serious problem for those who hold that God is essentially morally righteous, a thesis I have queried elsewhere (e.g. →Forrest, 2007).

I assume that we can partially understand an act in terms of the agent's awareness of the value of the outcome, but this does not tell us which values outrank others. So, to say that this is the best of all possible worlds like Leibniz, and before him Ashley Cooper, prompts the question, "Best in which respects?" The Completion Problem discussed in the first section shows that we cannot guarantee that in all respects there is a best. Given this under-determination of action by the value of the outcome, we might hope some further consideration will increase our understanding of acts. I submit that love provides a comprehensible further motive. Love ranks highly the good of what is loved over the good of what is, at the time of the action, a mere possibility. Once creatures have come to exist, we may suppose God loves them and this motive would tend to trump the aesthetic one. But prior to creation there are no creatures to love and hence God's self-love would be the dominant motive. (Bear in mind though that if God has a Trinitarian structure then the mutual love of divine Persons can constitute God's self-love.) The thesis that the universe is the body of God then supports the claim that prior to there being sentient creatures God is primarily concerned with the beauty of the Universe, not the well-being of those creatures that will come to exist.

This a priori argument adds to the weight of the a posteriori one. I submit, further, that the divine acts, motivated by self-love until there are sentient creatures and then out of love for them as well, is itself a beautiful narrative. I grant, though, that that requires Open Theism (→Basinger, 1996; →Hasker, 1998). For if

God had foreknowledge then future creatures would be as loveable as present ones.

9.3 Conclusions

The idea of perfect divine beauty is incoherent, but there are several replacements, notably that God is pre-eminently beautiful or that God is all beautiful. There are problems even with the weakest thesis, Transcendent Divine Beauty, but these admit of various speculative solutions, provided we equivocate on divine simplicity and weaken divine necessity.

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Notes

1 I prefer the term “lovely” to “beautiful” in the religious context (→Forrest, 2022).

2 Suppose there is a numerical scale for beauty, and for all m greater than n+1, y_{n+1} is between y_n and y_m, but is nearer on a beauty-scale to y_m than y_n.

3 We might hope there is a case for specifying some designated infinite ordinal, say $\zeta = \omega_\omega$.

10 Absolute Immutability and God's Libertarian Freedom in the Ontology of Divine Agency

Michał Głowala

Abstract: In the paper I discuss the compatibility of divine immutability with God's libertarian freedom in the ontology of divine agency. After a brief overview of *prima facie* problems concerning squaring immutability with libertarian freedom and a sketch of the basic ontological framework for considerations of divine immutability, I discuss five problems concerning the compatibility of immutability and libertarian freedom. Employing some insights both from contemporary and the late scholastic literature, I identify the most serious and central one and I sketch solutions to all of these problems.

One of the basic claims of classical theism is that God is absolutely immutable; another claim is that God enjoys libertarian freedom (freedom involving the ability to do otherwise). These two claims seem closely related; in particular, both have been deduced from the claim that God is the Prime Mover Unmoved (or that God is perfectly immune from any causal manipulation or coercion): as the Prime Mover, God must be immutable, and if his actions are totally uncaused, they seem to be free in a libertarian sense. On the other hand, however, absolute immutability and libertarian freedom might seem difficult to square for various reasons.

In this paper, I discuss the compatibility of absolute immutability and libertarian freedom within the *ontology* of

God's agency. In particular, I discuss a number of difficulties and reconstruct what I take to be the most serious ontological objection against their compatibility, and I sketch and defend what I believe to be a sound response to this and other objections in the ontology of divine freedom.

I focus here on *libertarian* freedom (involving the ability to do otherwise) rather than on freedom in a more general sense or some sort of compatibilist freedom for three reasons. The first is that it is clearly the divine libertarian freedom that poses most serious problems.[1] The second is that it *is* part of the classical theism to ascribe to God just the libertarian freedom, and not only the compatibilist one—to believe that God could have done otherwise. The third reason is that to discuss freedom *not* involving the ability to do otherwise one has to settle the question whether such a compatibilist freedom deserves to be called freedom, and this is the kind of question that I would like to avoid here: what I would like to do instead is to use a quite clear and strong notion of libertarian freedom in the ontology of the divine immutability. After all, in the old scholastic tradition that distinguishes voluntariness and freedom, freedom, as opposed to voluntariness is just libertarian freedom involving the ability to do otherwise (see, e.g., →Pink, 2009, and John of St. Thomas, 1885, q. 6, disp. 3, art. 2).

The discussion I offer here belongs to the *ontology* of libertarian freedom. By the *ontology* of freedom I mean a study of a set of interrelated questions: What kind of property is *libertarian freedom* of action? Is libertarian freedom a *causal* or rather a *non-causal* phenomenon? How is it related to agent- and event-causality? Are there different kinds of libertarian freedom, and how are they related? Does freedom involve exercises of a two-way power? What is the nature of two-way powers, and how are they related to alternate possibilities? Since it is the will that typically is taken to be the basic subject of freedom, the ontology

of freedom is closely related to the ontology of the will—involving, for example, issues of identity and individuation of acts of will, and, in particular, the relationship between acts of will and their objects (including thereby the issues of externalism and internalism with respect to the will). So, more precisely, my topic is the compatibility of ontological accounts of divine immutability and of divine libertarian freedom.

What I focus on here is the ontological account of divine libertarian freedom rather than arguments for the claim *that* God enjoys libertarian freedom. In discussions of various problems related to divine libertarian freedom, however, I hope to identify what I take to be the most compelling reasons for ascribing libertarian freedom to God, and respond to some crucial objections against it.

The literature I will be referring to includes both contemporary debates on divine libertarian freedom and late scholastic debates on the ontology of the divine will and divine libertarian freedom. The latter tradition, although still largely forgotten, offers unusual richness of insights, high standards of clarity of standpoints and patterns of argument, and sheds light on many details of contemporary discussions. In particular, what I take to be the most serious problem concerning the compatibility of immutability and divine libertarian freedom is identified and widely discussed just in late scholastic theological literature.

I will proceed as follows. I begin with a brief overview of *prima facie* difficulties concerning the compatibility of divine immutability and libertarian freedom in ontology (Section 10.1). Then I sketch the fundamentals of what I take to be the best ontological account of divine immutability (Section 10.2). After those twofold preliminaries I discuss a series of problems concerning the compatibility of immutability and libertarian freedom in Sections 10.3–10.6.

10.1 Immutability and Liberarian Freedom: A Brief Overview of *prima facie* Difficulties

Let me begin by hinting at a difficulty concerning the divine libertarian freedom that does *not* belong (at least immediately) to the realm of ontology. Traditionally, one of the main challenges for divine freedom has been the idea that a morally perfect agent is bound to always choose the best option.[2] Now what in that challenge belongs directly to ontology is the question whether there is at all something like *the best action* for the most perfect being—that is, whether the domain of possible actions of the perfect being contains a maximum item of this sort; otherwise, however, the question whether the most perfect agent must always perform the most perfect action does not belong to ontology as such.

What, by contrast, clearly belongs to ontology, is what might be called the general open theism objection; it is the rough idea that the exercise of freedom somehow essentially involves change, so an immutable exercise of freedom is basically impossible.[3] It is a task of ontology to judge if that rough idea is sound. In Section 10.2, I will argue it is not.

There are, however, a great number of a much more detailed problems in squaring immutability with freedom. I think their sources might be provisionally divided into three (closely intertwinning) groups.

(I) The first group of problems concerns alternate possibilities. What is relevant for libertarian freedom is not just sourcehood, but alternate possibilities; so if God enjoys libertarian freedom, He could have done otherwise. It seems clear, however, that the scope of alternate possibilities is in the case of God in some ways restricted just because of his immutability. For example, he cannot change his mind or he

cannot postpone a decision. In many cases a reasonable answer to such a worry would be that although *some* possibilities are precluded in God's case in this way, there remains still a plurality of options; and that options that are precluded are just *incoherent* (because, for example, the very idea of God's changing his mind is incoherent), so that they are not options at all.

A more serious problem belonging to this group is the following: in the case of some objects there is, according to the traditional metaphysics of theism, just one option for the most perfect being; for example, as regards his own nature, God cannot but love it in the infinite way; so it seems that *some* of divine actions are necessary in the sense that they lack any alternate possibilities; they are *not* free in the libertarian sense. Let's call it the *necessary volitions problem*.

Another problem is the following: traditional ontologies of freedom distinguish two kinds of alternate possibilities and two relevant kinds of libertarian freedom. On the one hand, an alternate possibility for willing *x* is willing something contrary to *x*; *libertas contrarietatis* is the sort of freedom corresponding to this sort of alternative. On the other hand, an alternate possibility for willing *x* is not willing anything: a refraining from willing; *libertas contradictionis* is the sort of freedom corresponding to this sort of alternative. Sometimes it is argued that the latter sort of freedom is argued to be a stronger brand of freedom. And sometimes refraining from *any* willing is thought to be a borderline manifestation of the latter sort of freedom. Now it seems that *this* sort of manifestation of freedom cannot be ascribed to God who is *actus purus*. Let's call it the *omission problem*.

(II) The other group of problems is related to what (following Benchimol (2011) and →Couenhoven (2016)) might be roughly called *volitionalism*. It is the claim that actions are free in virtue of

an intentional choice of the agent, or, more generally, of an internal volition involved in their genesis. →Couenhoven (2016) claims that volitionalism poses serious problems in accounts of divine freedom; in fact, it seems that there are at least two serious difficulties one has to face when one adopts volitionalism and attempts at an account of the divine freedom. Those problems arise out of a combination of volitionalism with problems of alternate possibilities.

The first problem is the following: if God could freely have done otherwise, and that action would be free in virtue of some God's intentional volition, then God might have had a volition different from the one he actually has; but it might seem difficult to square with the claim that God is absolutely immutable, so that he could not have been different from what he is. Let's call it *contingent volitions problem*. The problem was widely disputed in seventeenth-century scholastic ontology of God's freedom.

The other problem is involved in the ideas of divine simplicity which is closely related to his immutability (I will say more about the relationship between immutability and simplicity in Section 10.2). God does not have a plurality of really distinct volitions. But some of his internal acts of will are necessary and not free in the libertarian sense, as I have noted. It seems to follow that one and the same act of divine will is both necessary and free: and this may well seem difficult to accept in an ontology of freedom. This is an enhanced version of the problem of the necessary volitions problem. So let's call it the *enhanced necessary volitions problem*.

(III) The third group of problems concerns *reasons* for divine actions. It is often argued that freedom essentially involves responsiveness to reasons (see, e.g., →Lowe, 2013, p. 177). In particular, Lowe's answer to the pure chance objection against libertarian freedom is that a free action, as opposed to a pure chance event, is performed for a reason or has a reason

explanation. Now it is clear that on any *causal* account of acting for a reason (on assumption that reasons for which one acts are any sort of *causes* of the action) the immutable God must be denied the possibility of acting *for* a reason, because His actions are necessarily uncaused. But even if we reject any causal account of acting for a reason, it may seem that reasons that *motivate* the agent (even in some possible non-causal sense of motivation) are in some way *prior* to actions that they motivate, and this kind of priority is incompatible with actions of the Prime Mover; in particular, reason explanation seems at least close to teleological explanation, and the goal of the action is traditionally granted some sort of priority (this is what distinguishes real teleology from teleonomy). Call this the *reasons problem*.[4]

This, I hope, gives a preliminary picture of problems concerning the compatibility of the divine libertarian freedom and divine immutability. I will tackle the problems in the following way. In Section 10.2, I offer a sketch of ontological framework necessary for discussions of divine immutability. Here I also use some basic tenets of this framework to (i) reject the general open theism objection and to (ii) argue in favor of (a version of) volitionalism; the latter makes the problems of group (II) more stringent.

In Section 10.3, I discuss the necessary volitions problem and the enhanced necessary volitions problem, and in Section 10.4 the reasons problem. I try to show that the general versions of those problems are easily solvable. In Section 10.5, by contrast, I discuss the contingent volitions problem which I take to be the most serious and demanding; I refer to some advanced attempts to solve this problem in seventeenth-century scholasticism, and I discuss some particular versions of the enhanced necessary volitions problem and the reasons problem which prove particularly relevant in this context; the problems I

discuss here concern mainly some issues of externalism and internalism in accounts of divine mind. Finally, I try to outline what I take to be a sound solution of the contingent volitions problem. In Section 10.6, I discuss the omissions problem and some relevant problems of volitionalism again.

10.2 The Ontology of God's Immutability: Two Basic Ideas

I think there are two basic ideas which are necessary to make sense of the traditional doctrine of the divine immutability: (i) the distinction between a Cambridge change and a real one; (ii) the distinction between various ways of individuation of acts of will.

As for (i), it is clear that there are (tensed) predicates that come to be true or cease to be true about God at some times. For example, Geach proposes a rule concerning ascriptions of divine knowledge that if and only if it is true that *p* it is also true that God knows that *p* (→Geach, 1977, p. 40). In this sense, a Cambridge change clearly takes place in God. What the doctrine of divine immutability states, however, is that no *real* change takes place in God: if something new becomes true about God, it is because of some real change occurring somewhere else, not in God (for the contrast between a Cambridge change and a real one see e.g. →Geach (1969b), and for its application to God see →Geach (1969a)).

As for (ii), let us consider first an example not directly related to God; two chess players make similar movements first of a rook and then of a knight. Now it is perfectly comprehensible that the chess player A (who is a very competent player) had got an intention to move the rook *and then* to move the knight, whereas the chess player B (not so competent) had had got an intention to move the rook and *not* to move the knight, and after

the first movement that intention was replaced by another one, namely to move the knight. So it makes sense to say that in the circumstances the intention to move the rook and the intention to move the knight were *one and the same intention* of the player A, whereas the intention to move the rook and the intention to move the knight were *two distinct intentions* of the player B. So the difference between A and B is that whereas B undergoes a real change with respect to his chess intentions, A does not; moreover, this difference is closely related to the fact that A is a better player than B. In this way, it would also make sense to consider a chess master C who is even better than the player A so that C does not undergo a real change with respect to his chess intentions where A does change because of various changes in his intentions.

Peter Geach offered long ago what I take to be very attractive way of looking at these matters from a formal point of view (see →Geach, 1961, pp. 79–81 and →Geach, 1969c, pp. 45–46 and 49–53). When we namely say that A's intention to move the rook and A's intention to move the knight were one and the same intention (but it was not so in the case of B), we could think about two *functions*, namely someone's intention to move the rook and someone's intention to move the knight. The arguments of both functions are various players, in particular A and B, and the values of the functions are particular intentions of these players. Now what we have said about the intentions of A and B means that the value of both functions for A is one and the same, whereas the values of the functions for B are distinct. The account of the difference of the value of a given intention-function for two agents is that these agents may intend one and the same thing *in different ways*: their intentions may be *shaped* in different ways.

This shows, in general, that when two intelligent agents produce a series of movements, one of them may remain

unchanged (with respect to its mental acts of some kind) where the other changes (with respect to this sort of mental acts). Moreover, this also makes it plausible that the more powerful the mind of an agent is, the less changes it is going to undergo. This is enough to grasp a rough idea that an intelligent agent might *not change at all* in its mental activity due to its mental perfection. In particular, it is quite possible to conceive of an agent for whom every intention-function of the described kind has got one and the same value: such an agent would have just *one volition* that would be a volition to do everything that this agent does, and would indeed be unchangeable with respect to volitions due to having just this unusual sort of volition. Moreover, it is also conceivable that there is an agent which is himself identical to his value of all intention functions, that is: identical to his single volition, being, so to say, the fixed point of all these intention functions.

In other words: the identity criteria of mental acts make it possible to grasp the idea of an agent who is mentally very powerful, and thanks to the individuation criteria of mental acts specific to such a powerful mind is basically immutable just in this respect. That, I think, is the basic way to introduce the idea of the divine immutability. What becomes clear here is the close connection between immutability and simplicity.

I think this shows two further things. (i) The general open theism objection is deeply flawed, because change is not essentially involved in mental activity; the relationship between mental activity and change essentially depends on the individuation criteria of mental acts, and those criteria in turn depend on the powerfulness of a given mind; so it is quite reasonable to suppose that the more powerful a mind is, the narrower is the scope of changes it undergoes. So, quite contrary to the open theism general objection, it is the very concept of the mental (in particular: various individuation

criteria for *mental* activities) that paves the way for understanding the sort of immutability relevant to the case of the divinity.

(ii) Moreover, the ontological framework sketched above shows that volitions in a very general sense are crucial for understanding divine immutability. By 'volitions in a very general sense' I mean internal intentional acts of will that are basically different from desires or inclinations, because they are intentionally and consciously shaped by the agent; for it is just the difference in the shape of volitions that accounts for their specific individuation criteria, which are in turn crucial for immutability. So, contrary to the claim of →Couenhoven (2016), I think that problems related to divine freedom *cannot* be solved by rejecting volitionalism.

What is clearly incompatible with the ontology of divine immutability is the claim that volitions are *caused* by the agent, because divine actions are totally uncaused. On some volitionalist accounts, volitions are indeed said to be caused by agents. There are, however, many versions of volitionalism that explicitly deny that volitions are *caused* by agents (see, e.g., →Lowe, 2013); so by adopting volitionalism we are not bound to claim that agents *cause* their volitions.

I admit, however, that there are numerous difficulties lurking in volitionalist account of divine libertarian freedom; I will discuss them in due course.

10.3 The Necessary Volitions Problem and the Enhanced Necessary Volitions Problem

According to the classical theist tradition, there is an object that God *necessarily* loves, namely his own goodness or perfection. It is not possible for God not to love himself or his own goodness;

it is not possible for God to hate himself. So God's love for himself is necessary and not free in the libertarian sense.

Necessary volitions, however, are by no means specific to God; many Aristotelians claim that any human agent does necessarily want to be happy; happiness (defined as doing well or flourishing in some most general sense) is a necessary object of will of any human agent; it is not possible for a human agent not to want to be doing well in some most general sense, or to want not to be doing well in some very general sense. So for a human agent wanting to be happy is a necessary volition; it is not free in the libertarian sense.

This sort of necessary human volition involves a special kind of necessity; it is usually related to the inner nature of the will and opposed to the necessity of coercion (or causal necessitation). Aquinas claims, for example, that the necessity of coercion is incompatible with the will in general: if some activity of *A* is coerced, it is not, as such, an act of *A*'s will. On the other hand, however, he claims that the will itself has its own inner nature, and the very inner nature of the will makes something its necessary object.[5]

This does not mean that there is some special sort of necessarily unremitting volition going on all the time in any human agent; what this means is rather than a human volition of any particular action *a* is necessarily a volition of *a* as something that appears to the agent as a (conductive or constitutive) means to doing well in general; so it is also necessarily a volition of doing well in general. So the idea of inner nature of the will is closely related to the idea that the will must have a *formal object* which is constitutive for its identity. Roughly speaking, a *formal object* of a power or ability is a description *under which* things become objects or acts of this power and ability, and according to the classical doctrine any power or ability is specified by its formal object. Now in the

Aristotelian tradition, the formal object of the human will, is, roughly speaking, what is undertaken because appears to the agent as a (conductive or constitutive) means to doing well in general. In other word, the human will is the power to act in light of a very general reason that something is a (conductive or constitutive) means to doing well.

On the other hand, however, *a* is typically not conceived of as a *necessary* or *the only possible* means to doing well in general; and even if something is, as a matter of fact, the only possible means to doing well in general, it does not have to appear to the agent to be so. In this way, the formal object specific to human will leaves room for libertarian freedom of the volitions of various particular actions. By contrast, the formal object of human sight *does not* leave room for libertarian freedom in this way; the formal object of human sight is, roughly, what affects human retina in a relevant causal way.

Now a crucial point to make is the following: on the Aristotelian doctrine, any human volition of any particular action *a* is a volition of doing well in general. It is a *necessary* volition of doing well in general, and at the same time it is a *free* (in the libertarian sense) volition of a particular action *a*. In this way, one and the same volition of a rational agent may well be both free in the libertarian sense and necessary, just because being free in the libertarian sense and being necessary are *relative* to a given object of the volition; a volition is free or necessary only as a volition of *something*, and not as such. And since a volition typically has got a number of related objects (there is no such thing as *the* object of a volition), it may be a *necessary* volition of one thing and at the same time a *free* (in the libertarian sense) volition of another thing.

What I have said in the previous paragraphs has nothing to do with compatibilism as a metaphysical doctrine about freedom. It is rather a more fundamental point concerning the

way in which freedom or necessity (of the relevant kind) are ascribed to volitions; they are ascribed to volitions relative to their *objects*. On the other hand, it is a point concerning a special sort of necessity which is distinct from causal necessitation.

In the case of the divine will, its formal object is—according to the Aristotelian tradition—not just doing well in general, but the divine perfection itself which is the most perfect case of doing well and may be imitated in innumerable although imperfect ways; in other words, when God wants anything, he wants it as related in some way to his own perfection. On the other hand, various things that God wants may be only contingently related to the divine perfection, and that leaves room for the divine libertarian freedom; there is just *no* such thing as *the* manifestation or *the* imitation of God's perfection.

Here, perhaps, a brief digression could be helpful. Rowe (2002, p. 410) claims that if there is basically no best possible world, God the creator is impossible. This claim may be paraphrased in terms of formal objects as follows; I would argue that it amounts to the idea the formal object of the divine will is doing the best possible thing God can do (if God wants anything, he wants it as the best possible thing God can do). We might say that if there is no such thing as the best possible world, the formal object of the divine will (as Rowe conceives of it) is inherently inconsistent. Now the formal object of the divine will that might be identified in Rowe's standpoint *does not* leave room for divine libertarian freedom. If one wants to reject Rowe's conclusion, the best thing to do is to reject just this idea of the formal object of the divine will. And the formal object of divine will traditionally assigned in the Aristotelian tradition does leave room for libertarian freedom in the sense sketched above.

In this way, God necessarily loves his own perfection and the relevant sort of necessity is related to the formal object of God's will, or to the divine will's inner nature. God may also freely (in

the libertarian sense) want or love something else that is somehow imperfectly imitating to God's perfection. Here again one and the same volition will be both necessary volition of God's own perfection and at the same time a free (in the libertarian sense) volition of some contingent object.

Here we certainly get close to various problems discussed by Plantinga in his *Does God Have a Nature?* (→Plantinga, 1980). I cannot discuss them in depth. But it seems to me that the crucial point is that it is possible to conceive of the very nature of the divine will (and the formal object of the divine will) in various ways. In the classical theist picture, the formal object of the divine will does leave room for libertarian freedom; by contrast, the picture of the divine will sketched in →Rowe (2002) does not.

To sum up, three points are crucial here. The first is that in both human and divine volitions there is a sort of necessity related to formal objects of human and divine will. The second is that both the formal object of the human will and the formal object of the divine will (at least conceived along the lines of the mainstream theist tradition) do leave room for libertarian freedom of volitions of many particular things, although the very necessary volitions are *not* free in the libertarian sense. That, in my opinion, solves the necessary volitions problem. The third point is that it is basically possible that one and the same act of will is both necessary (in the sense related to formal objects) and free in the libertarian sense, relative to its various objects, because necessity (in the sense related to formal objects) and libertarian freedom are always relative to objects of acts of will.[6] That, in my opinion, solves in general the enhanced necessary volitions problems.

What I think I have shown here is that *basically* and *in general* it is pretty possible that one and the same act of will is both necessary (in the sense related to the inner nature of the will and its formal object) and free in the libertarian sense; and that

this basic general possibility is somehow displayed both in human and divine agency. It does not mean, however, that there are no special problems concerning the relationship between necessity and libertarian freedom specific to the case of God; in fact, I do think that there are such particular problems and I am going to address them in Sections 10.5 and 10.6.

10.4 The Reasons Problem

The simplest way to square the reason responsiveness of the divine will with its immutability is to reject totally any *causal* theory of reasons for action—to reject the idea that reasons (for an action) are causes (of the action) of any sort at all. This move is possible even in the case of purely human responsiveness to reasons. It seems, however, that we still need some more positive account of the non-causal role for reasons for which one actually acts and some answer to the famous Davidsonian worry about the difference between reasons that the agent is just aware of and that could justify his actions, and the reasons the agent actually acts for. Now it seems clear that some of these positive accounts can't be applied in the case of the immutable Prime Mover.

Consider, for example, the proposal of O'Connor (2009); on this account, reasons are propensities structuring the agent causal power and affecting the objective probability of a given course of action (see →O'Connor, 2009, pp. 198–199). Now it seems that this account of reasons for action could hardly be applied to the case of the immutable agent. For example, God clearly could have acted for a different reason, and on O'Connor account this would amount for different structuring of God's agent power, which seems impossible to square with the divine immutability.

A more promising strategy is suggested by the non-causal and externalist account by Lowe (2008) and Lowe (2013). He claims that reasons are some worldly items rather than mental states, and that rational agents have a primitive ability not just to choose a given course of action, but to choose at the same time the reason for which they act (→Lowe, 2013, p. 182); in other words, what one in fact chooses, is always *a*-because-of-*r* or *a*-for-the-sake-of-*s*. There are, however, two further problems we meet when we try to apply this sort of account to the case of the immutable Prime Mover.

Suppose *A* decides to take a certain medicine for the sake of health. According to the non-causal view, 'for the sake of health' gives just part of the content of *A*'s decision (he decides to take-a-certain-medicine-for-the-sake-of-health). Now the first problem is the following: it seems that if someone decides to take a medicine for the sake of health, it is crucial that he wants health (here, as Lowe points out, lies a crucial difference between reasons for action and reasons for belief; and, on the other hand, wanting health is crucial if health is to count as a *motivational* reason). Now we are in need of some account of the crucial role of this wanting. Here Lowe himself introduces *interests* or objective *needs* to solve the problem (see →Lowe, 2008, ch. 10). But it seems clear that no *needs* can be ascribed to the immutable Prime Mover, so one has to look for another solution here.

The other problem is the following: acting for a reason seems closely related to teleology; and teleology, as opposed to teleonomy, must assign a kind of priority of the goal over what is directed to it. Aristotle often speaks of the end as motivating the agent, and the motivation here seems at least close to moving (see, e.g., →Aristotle, 1995, 433b10–20). Now this kind of moving, even if it is not moving in a literal sense, may seem hard to square with the immutability of the Prime Mover.

Let me just sketch here what I take to be the most sound solution to both problems; it is to be found in the mainstream theist tradition, for example in Aquinas (see →Aquinas, 1980, *Summa contra gentiles* I, c. 86 and 87, and *S. th.* I, q. 19, art. 5).

Let us return to the case of *A* who decides to take a certain medicine for the sake of health. This implies that *A* wants health. Now on the traditional account one wants health basically in the same act of will in which one decides to take a medicine for the sake of health; deciding to take a medicine for the sake of health *is* itself an act of wanting health; both health and the taking of the medicine are object of this act of will. This, however, is not the whole story. In a typical case, the certain medicine *A* decides to take must have been searched for and invented before the use, and in a typical case the source of the search for the medicine has been the wanting of health; now *that* previous wanting of health was *not* yet the decision to take the certain medicine. Here we have a distinct act of will directed towards health as an end, but not towards any particular means.

Now Aquinas makes the following claims: (i) when one wants *m* for the sake of *e*, and wants both in one and the same act of will, then wanting of *e* is not a cause of wanting *m*, just because of the irreflexivity of causality. But (ii) when there are distinct acts of will directed at *e* and at *m*, then in a typical case the wanting of *m* does *causally depend* on the previous wanting of *e*; more precisely, the final wanting of *m* and *e* at the same time *causally depend* on the previous wanting of *e* which wasn't wanting of *m*. So in the above story *A* wants health, and this act of will causes first the will to search for a medicine, and then the will to take the medicine (both are at the same time modified, so to say, volitions of health). This view, in short, combines causal and non-causal accounts of reasons for action. It departs from a radically non-causal view of Lowe in that it claims that volitions of means may be caused by prior volitions of ends; and it

departs from a Humean causal view in that it claims that the caused volition must be a volition of both the end and the means.

Now in the case of the immutable Prime Mover we have, as I have sketched in 2, a total identity of volitions; this accounts, according to Aquinas, for the absence of *causal* connections in God's will, but still leaves the possibility that God wants something for the sake of something else. While in the human case this typically involves causal connections between distinct volitions, it is not the case when we consider the divine will. In particular, God may well want to do something *because* someone prays for it; this amounts for the divine responsiveness to prayers without the divine volition being *caused* by anything. This, in short, is how the traditional standpoint solves the first problem.

As for the other problem, it can well be illustrated in the prayer case. It may seem that if God really wants something *because* someone prays for it, then even if the prayer does not *cause* God want something, it has got some sort of influence on the divine will that is hard to square with the divine immutability. It is clear that God's volition does not have to depend counterfactually on the prayer, yet still it may seem that the prayer, being a reason for God's will, exerts some sort of influence on the divine will.

Here is how this problem would be resolved along Aquinas' lines. In general, what basically moves the agent is the goal (as opposed to means); so, for example, when *A* decides to take the medicine for the sake of health, it is rather health than taking the medicine that *moves* A in the sense relevant to final causality or teleology. Now the ultimate goal in the case of God is God himself who is identical to the perfect divine will. Because of the identity of the ultimate goal and the divine will there is no causal connection, priority or dependence between them (since

causality, priority or dependence are all irreflexive). Any other object of the divine will is not the goal, but rather a means for the ultimate goal, and as such it does not exert any sort of influence upon the divine will. So in a way whenever God wants *a*, he wants it for the sake of his infinite perfection—as far as *a* manifests or reflects it, or in a way serves it, or leads something to it.

Here it would be useful to return to the idea I have discussed in Section 10.3 that the formal object of the divine will leaves room for libertarian freedom. The point is that while God wants anything else than his own perfection for some reason, there is no *compelling* reason for any particular object of the divine will other than God's own perfection (what is a real reason for an action need not be a *compelling* reason for that action). This is because any particular object of the divine will is wanted for the sake of the divine perfection—as far as it manifests or imitates or serves it; now there is no such thing as a necessary—the only possible—way of manifesting or reflecting the divine perfection, or the only possible way of serving it. This is the root of the divine libertarian freedom with respect to any object distinct from God himself.

Contrast again Rowe's God. Rowe's God's ultimate goal is to do his best, and anything he does, he does for the sake of doing the best. It seems reasonable to suppose that this ultimate end offers many *compelling* reasons for particular actions, or perhaps for *every* particular action.

10.5 The Contingent Volitions Problem: Internalism, Externalism, and Divine Freedom

The problem I am going to discuss here is one of the main topics in the late scholastic debate on the ontology of divine libertarian freedom; one of the participants of the debate calls it "one of the

most difficult in the whole theology" (→Caspensius, 1641, tract. 4, disp. 3, sectio 3, n. 14).

Suppose God freely chooses some finite good *a*; the choice enjoys libertarian freedom, so that God might have not chosen *a*, and, for example, might have chosen *b* instead of *a*. Now according to what I have said in Sections 10.2 and 10.3 God's actual volition of *a* is identical to God's necessary love for himself; and God's possible volition of *b* would have been identical to God's necessary love for himself. Now the problem is the following: consider the two scenarios, one with God choosing *a* and the other with God choosing *b*; is there any intrinsic difference between God in the first scenario and God in the other scenario? Since God is necessarily immutable, the answer seems decidedly no; there is one and the same immutable divine volition, his love for himself, that is actually God's choice of *a*, and would have been, without any intrinsic difference, God's choice of *b*. Or, in other words, God may be truly said to freely choose *a* in virtue of his love for himself; and God might have been truly said to freely chose *b* in virtue of the very same divine love for himself. This, however, suggests that choosing *a* and choosing *b* ascribed to God are a sort of extrinsic denomination; or that they are just a matter of some sort of relationship between the divine will and various extrinsic effects, *a* and *b*.

Now it may seem pretty controversial that, in short, volitions directed towards alternate possibilities are ascribed to God as extrinsic denominations without any intrinsic distinction in God. There is nothing strange in the fact that many things are ascribed to God as extrinsic denominations; but it seems that if volitions of alternate possibilities are described to God only as extrinsic denominations, then the very libertarian freedom is ascribed to God as an extrinsic denomination, and this would be at least very strange. One might say: where we have only

Cambridge change, we do not have any real change; and it may seem that on the above account what we ascribe to God is, so to say, Cambridge libertarian freedom, and Cambridge libertarian freedom is no real libertarian freedom.

To assess the difficulty here let us note first that it is perfectly possible (in the human case) that an object of a volition proves distinct due to some purely extrinsic circumstances, without any intrinsic difference in the volition itself. This is particularly clear in (scholastic) theories of indirect intention according to which, for example, a change in norm or obligation may change the object of a volition making you responsible for what you had not been responsible without the obligation. Suppose you want to go for a walk; then if you had been obliged to do something else at the same time, and due to your blameworthy negligence you had not been aware of the duty when you were forming the volition to go for a walk, your volition to go for a walk would have been nevertheless (although in an indirect way) a volition not to fulfill the duty. In this sense it is quite possible that a change of extrinsic circumstances changes the very object of volition without any intrinsic change in the very volition. What I discuss here is, of course, a version of externalism with respect to volitions; and I think it is clear that some version of such externalism is quite sound.

It is striking, however, that the above cases speaking in favor of will externalism are utterly unlike the divine alternative volitions case. They essentially involve blameworthy ignorance or not fulfilling one's duty, and, what seems crucial, they do not concern alternate possibilities or freedom of choice. So, to repeat, it may seem problematic how to square the claim that God, being free in the libertarian sense, might have had a volition he does not actually have, and the claim that God, being necessarily immutable, might not have lacked any intrinsic property he actually has.

Let us note that the above problem includes in a way particular versions of other problems discussed above: of the necessary volitions problems, enhanced necessary volitions problem and the reasons problem; that is why I claim that the above problem is particularly serious.

This problem is the heart of the late scholastic debate on the divine freedom; the very number of nuanced and detailed solutions may seem difficult to comprehend, the reasons for various standpoints are closely related to reasons for various sorts of externalism or internalism in contemporary debate in the philosophy of mind.[7] It seems, however, that there are two extreme ways of tackling the problem in late scholasticism, and each of the nuanced scholastic standpoints are just moderated versions of one of them; it may be convenient to label the one extreme the internalist, and the other—the externalist one, for in a way the whole debate reflects an application of the general externalism-internalism issues to the case of the divine freedom.

The first extreme standpoint (that might be called internalist) can be traced back to Cardinal Cajetan and his commentary on Aquinas' *Summa*; Cajetan seems to claim that there is a sort of intrinsic property or perfection that God might have lacked; it is a *quite free* perfection (*perfectio voluntaria et omnino libera*) and not having such a quite free perfection (in case God should have made a different choice) would not be any sort of imperfection on God's part (see →Cajetan, 1888, q. 19, art. 3, n. 3). The obvious problem with this standpoint is that it seriously undermines the divine immutability, for we claim here that God could have had different intrinsic properties than he actually has.

There are a number of attempts to moderate this extreme internalist standpoint.[8] The basic strategy is to show, roughly speaking, that the very distinction between the alleged free perfection and divine necessary perfections is so weak that the lack of the free perfection would not undermine the divine

immutability; and the basic way to accomplish this is to apply some externalist ideas—to claim, roughly speaking, that what distinguishes the alleged free divine perfection from necessary perfections of God is some sort of relationship to the contingent object of the free volition, in particular some created object; in this way, in God the free divine perfection and divine necessary perfections are one and the same, and the lack of the free perfection is just the lack of the relation.

The other extreme, the externalist one, claims that free divine volitions in general are just extrinsic denominations, or that they consist in some sort of God's relations to various contingent objects; this claim can be traced back to Peter Auriol, and in various moderated versions it is to be found in Vazquez.[9] The claim obviously secures the divine immutability, but it poses two other serious problems. The first is that it may seem strange to claim that volitions in general are extrinsic denominations or relations. The other problem concerns the divine causal priority; it seems namely that God's relation to a contingent created object is in a way posterior to the existence of that object, but it would be absurd to claim that the divine volition of a contingent created object is posterior to the existence of that object; it seems clear that the divine volition is causally responsible for the existence of that object, so that the volition can neither *be* the relation nor *depend* in any way on the relation. To solve both problems one has to adopt some internalist ideas concerning divine volitions; this, however, poses the above problem specific to internalist treatment of divine free volitions: may seem to undermine divine immutability.

So that is the outline of the problem that is the core of late scholastic debate on the ontology of divine freedom. In short: internalist solutions pose a threat to divine immutability, while externalist solutions seem to undermine divine causal priority. It seems clear that what is central for the problem is the

relationship between divine volition and its contingent object, involving a number of issues from the externalist-internalist debate about the will.

I think that a basically sound strategy of solving this problem is to be found in at least two famous late scholastic writers, Suarez and Poinsot (John of St. Thomas). There are some differences in the details between their solutions that may prove crucial in some contexts, but I cannot discuss these nuances here. What I would like to do here is to offer a rough outline of basic points that seem intuitively plausible and crucial for the solution of the problem, although they surely demand further elaboration.

Let me begin with two closely related differences between human and divine freedom that both authors discuss.[10] The first difference is that the exercise of human freedom involves producing new volitions, and, we may add, human libertarian freedom involves the ability to produce a different volition (or perhaps not producing a volition at all). In the divine case, by contrast, the exercise of freedom does not consist in producing different or new volitions, since there is a single eternal divine volition. The exercise of divine freedom consists rather in having or not having something as an object of the unique divine volition that remains perfectly the same, and the divine libertarian freedom involves the ability to have a different object of the same volition (or perhaps not having something as an object of the same volition). (Notice that this point is closely related to a rejection of what is controversial in Cajetan's internalist standpoint.)

The other difference is that volitions produced by human wills are essentially dependent in their identity and individuation conditions on their objects. In other words, both Suarez and Poinsot seem to claim that in the case of human will some stronger form of externalism is true. To see some reasons for

this claim, consider again the cases discussed above in which a change in purely external circumstances may affect the content of a volition. Another way of stating this sort of externalism with respect to human will is to say that human agents cannot shape the volitions they produce *at will*, just because the nature of their volitions is determined by the nature of their objects. A theory that ignores this sort of objective constraints posed on human volitions is the notorious doctrine of "directing one's intention" (*diriger l'intention*) criticized repeatedly by Anscombe (see, e.g., →Anscombe, 1982; →O'Brien, 2013); the doctrine claims that whatever you actually do, you may direct your intention in any way you want (so that, for example, when you take someone else's property you may direct your intention so that you do intend to satisfy your needs, but you *do not* intend at all to violate the owner's rights). Now the falsity of the doctrine of directing one's intention shows that we cannot shape our volitions *at will*: they have got their intrinsic natures which depend on their objects. And this is roughly what Suarez and Poinsot claim concerning the dependence of human volitions on their objects.[11]

Now the point is that, according to Suarez and Poinsot, divine volition of a contingent object does *not* display this sort of dependence on its object. The nature of the divine volition is just the divine nature, and it does not depend in any way whatsoever on the nature of any particular contingent object. That is why—returning to the first difference above—God is free to include or not include any particular contingent object within the scope of the divine volition. There is a sort of dependence between an actual contingent object of the divine volition and the volition itself: but it is a dependence of the object on the volition to which the object owes its existence, and not a dependence of the volition on the object.

To see better both differences discussed above, consider the following example. When you want to travel somewhere, you are free to include the use of railway among the objects of your volition. The first of the differences is that this typically involves producing a *new* volition which is a decision to travel somewhere by rail (and not by bike or car). The other of the differences is that the nature of your volition to use railway is determined in many respects by the nature of railways in general. God, by contrast, can just include something within the objects of his volition without the production of any new volition that would depend in its nature on the nature of that object; for example, he may want to have (or not to have) railways in our world without producing a distinct volition for railways that would essentially depend in its nature on the nature of railways. Perhaps we might say: God may just include (or not include) railways in his plan (which is identical to his wisdom)—without making any changes in his plan, or formulating a new plan whose nature would essentially depend in some respects on the nature of railways.

I think these intuitions of Suarez and Poinsot may be paraphrased in terms of causal profiles of agents in the following way: in the case of human freedom an agent modifies his intrinsic causal profile by producing new volitions;[12] the modification of the causal profile accounts for causing a certain thing by the agent. The agent causes *o* just because his volition is a volition of *o*. But what makes the volition a volition of *o* is some sort of dependence of the volition on the object *o*.[13] God, by contrast, can cause anything, including *o*, without any modification of his intrinsic causal profile; what accounts for the fact that he causes *o* is that his volition is a volition of *o*, but what makes his volition a volition of *o* is his free choice, and not any sort of dependence of his volition on *o*; and the free choice of *o* is not distinct from the very unique divine volition.

This is basically the framework within which Suarez and Poinsot try to solve the divine contingent volitions problem.

Now at this point it is pertinent to return to the main question and the main problem; suppose God includes a contingent object *o* within the objects of his volition; he might have not included it and thereby might have lacked the volition of *o*. What precisely does this actually including *o* among the contingent objects of divine volition consist in? What is the difference between the scenario in which God wants *o* and the scenario in which God does not want *o*? Now the answer of Suarez and Poinsot is roughly the following: it is only the actual existence of *o*, caused by God's volition, that distinguishes both scenarios; there is no corresponding difference in God's intrinsic properties, just because God does not have to change his intrinsic causal profile to produce *o*. This is what sets their standpoint apart from Cajetan's internalism. On the other hand, however, Suarez and Poinsot claim that it is not the actual existence of *o* that grounds or founds God's inclusion of *o* among the objects of his volition—just because the divine volition, as opposed to human volitions, does not depend in any way on any of its contingent objects. It is rather the very existence of *o* that causally depends on the divine volition.

Now the objection that might strike one is the following: on the account of Suarez and Poinsot, there is no intrinsic property in God that accounts for the fact that *o* is or is not included among the objects of the divine volition, or for the fact that *o* is or is not actually caused. In some respects, this objection is similar to the standard contemporary objection against strong forms of agent causality.[14] According to this objection, there must be something in the agent—in the agent's intrinsic causal profile—that accounts for the fact that the agent causes or not causes *o*, or that the agent causes *o* or something else.

I think that both Suarez and Poinsot would agree that this objection is sound in the case of human agents—that is why they give volitions a prominent role in causing or not causing anything by that agent; volitions having particular natures modify the causal profile of the agent and account for the existence of particular effects. In the case of divine will, however, they argue that it is the simple immutable divine will without any contingent addition that does account for including or not including *o* among the objects of the divine volition and causing or not causing *o* by God. I think they argue so not because they think that the standard objection against strong agent causality is generally weak; it is because they think that it is just in the case of the *divine* will that one can find a satisfying response to the otherwise very strong objection; what is relevant here is that God, without any additional or new volition, has got a causal profile that can account for causing (without necessitation) any particular object. The standard objection against agent causality holds good just because human agents lack this kind of causal profile that God has.

Now this trait of God and divine perfection—the divine causal profile which is enough for causing any contingent object—is very close to the fact that God is the necessarily immutable source of any change in the world. It is also close to the divine omnipotence.

Another way of making that point would be the following: in the case of human freedom, two different causes of action are manifestations of two different causal profiles; in the case of the divine freedom, two effects are different manifestations of one and the same causal profile; and it is essential for such a causal profile that it can be manifested in infinite ways such that none of them is necessary. God chooses a given manifestation of his causal profile when he creates a particular object he might have not created.

I would say that this offers some prospects for a plausible solution of the contingent volitions problem. But to go further along this path one obviously has to think more about the intricacies of causation and causal profiles in the context of volitions and their objects, and in the context of omnipotence.

10.6 The Omissions Problem and Volitionalism Again

As far as alternate possibilities for choosing *o* are concerned, we may consider both choosing something else than *o* (perhaps something contrary to *o*), and not choosing anything at all or refraining from choice. There are two kinds of libertarian freedom corresponding to those two types of alternate possibilities; the former corresponds to freedom of contrariety or freedom of specification (*libertas quoad specificationem*), the latter to freedom of contradiction or freedom of exercise (*libertas quoad exercitium*). In the case of freedom of specification alternate possibilities are *contrary*, in the case of freedom of exercise they are rather *contradictory*.[15]

One may sensibly ask what is the relationship between those two kinds of libertarian freedom, and, in particular, which of them should count as the stronger or more basic one—whether it is choosing something else or rather refraining from choosing that is the fundamental alternate possibility. The second question to consider is what kind of alternate possibilities and libertarian freedom can be ascribed to God (and whether what should count as a stronger form of libertarian freedom can be ascribed to God). Let me briefly discuss both questions. What is my basic concern here is to consider the possibility that what counts as a *stronger* kind of libertarian freedom *cannot* be ascribed to God.

As far as the first question is concerned, there is a serious tradition stemming from some strands of scholasticism

according to which it is the ability to *refrain* from (or to *suspend*) an act of will that is most relevant for the strong sense of libertarian freedom. For example, when Lowe contrasts the exercise of freedom with random event such as the radioactive decay of a certain radium atom at a given moment, he says that the atom is not free partly because it may *fail* to decay, but it can't *refrain* from decaying (see →Lowe, 2013, p. 177). →Senor (2008, p. 182) discussing the divine libertarian freedom focuses solely on the question whether God can *refrain* from wanting something, and not on the question whether God can want something contrary to it.

A related and much debated issue of late scholasticism is whether there are *pure* free (human) omissions—that is, human omissions that do not involve any positive act of will, just the suspension of any activity of will. You may arrive at the idea of a pure free omission in the following way. First, in some cases there is a positive volition to refrain, perhaps for some period, from some external action (or from another volition—for example a decision you want to postpone); but it is also possible that you refrain from doing something (or even from a volition) without any volition to refrain; the lack of action is due not to a volition, but just to the lack of the volition to act. Even here, however, there are often positive volitions responsible in turn for the lack of the volition (when, for example, the lack of the volition is due to being engaged in some other very absorbing activity, or is due to being asleep, which in turn is due to a decision to go to bed). In such cases, the omission of the very volition is not pure; and the scholastic debate concerns the possibility of a refraining from volition that would not involve any sort of positive volition in this way.

In general, the Scotists as well as some Jesuits argue that a free suspension of *any* activity of will, a free refraining from any volition, is possible and is just the most radical exercise of

freedom (see, e.g., Mastri and Belluto, 1727, *De anima*, disp. 7, q. 4, n. 83). The Thomist response is that the alleged general suspension of activity of will could not be distinguished from a mere failure to act on the part of the will, and henceforth could not be considered free; according to the Thomists, although an omission is not to be identified with some positive volition, it could be regarded as free only by involving in some way (for example, in its causal genesis) some positive volition of something else (John of St. Thomas, 1885, q. 6, disp. 3, q. 3, art. 3, n. 11–15 I n. 26). The whole debate is clearly close related to contemporary debates on volitionalism and non-will-centered accounts of moral responsibility (see →Benchimol, 2011).

It seems to me that the Thomistic objection against the possibility of absolutely pure free omission, or the possibility of *free* suspension of any activity of the will, is a very serious one. It is closely related to the standard argument against stronger forms of agent causality. It is clear that there are cases of inactivity or idleness of the will; the point of the Thomistic objection is that there must be some ontological ground for treating this inactivity as something that is free and up to the will, and not just a failure to act. According to the Thomists it can be only via some sort of positive volition produced by that will that the omission can be ascribed to a will as its free refraining.

Here it may be objected against Thomism that in many practical cases we treat something as culpable omissions not referring to any positive volitions, but just to some norms and other external circumstances. This is the key point for non-will-centered accounts of moral responsibility. For example, a doctor may be blamed for an omission of medical help, and it may seem that what counts are just external circumstances and norms; we do not have to go into the details of the doctor’s volitions. I think the Thomist response is that all those norms and circumstances are relevant because of some positive volitions of the doctor. So,

basically it is a set of volitions that constitutes such a causal profile of an agent that the result of inactivity may be ascribed to that agent. In short, it seems to me that there are strong arguments in favor of volitionalism. In this way, I would be inclined to say that the freedom of suspensions and refrainings does basically depend on the freedom of some volitions, so that basically the freedom of exercise depends on the freedom of specification.

To sum up, I think that the idea of a free pure suspension of any activity of will is spurious; it is deeply entangled in most controversial aspects of strong versions of agent causality. The freedom of omissions, suspensions and refrainings depends on freedom of volitions.

As for the other question—whether libertarian freedom of exercise could be ascribed to God—it is clear that the activity of the will of immutable God cannot be generally suspended—just because of the divine immutability; after all, God's necessary volitions are not free in the libertarian sense. But in the case of God's free volitions of contingent objects one can sensibly ask whether the alternate possibilities include also a suspension of any volition concerning a given object, or only other volitions. Both answers have their adherents in late scholastic debates, although the negative answer seem more frequent.

Caspensius (1641, tract. 4, disp. 3, sectio 3) is a good example of the negative answer. According to him, God cannot just refrain from willing, for example, the existence of a contingent object *o*; God either positively wants *o* to exist, or positively wants *o* not to exist (this may be seen as a sort of completeness of divine volitions). He argues that refraining from willing is a form of idleness that is incompatible with the pure actuality of God as the Prime Mover; and that it involves a sort of potentiality for which there is no room in God. Moreover, he argues that the freedom of exercise is not as such a perfection, but involves

essentially some sort of imperfection, so the claim that God does not have this sort of freedom does not deny any perfection to God.

I think that the argument of Caspensius suggests one point that seems crucial to me. According to the solution presented in Section 10.5, human volitions are individuated by their objects, but the divine volitions of contingent objects are not; the point is that divine volitions can be independent in this way on their objects only because God is the summit of perfection and activity. From this point of view, the key question is the following: can refrainings from willing or suspensions of willing be independent (as far as their identity and individuation are concerned) on their objects in the way specific to divine volitions? It seems to me one can plausibly argue that they cannot; refrainings from willing are necessarily dependent on their objects, and that is why there is no room for them in God's full actuality.

Another point that could be made in favor of Caspensius's standpoint is that God does not *produce* volitions of contingent objects; so, arguably, God can't refrain from such volitions.

On the other hand, however, the Scotist John Punch argues that freedom of exercise can be ascribed to God (see →Poncius, 1671, disp. 6, q. 2, n. 12); obviously this is closely related to the fact that Scotists tend to believe in the possibility of pure free omissions, and to take it as a basic manifestation of freedom. He rejects Caspensius's claim that freedom of exercise essentially involves imperfection; according to Punch, there is no more imperfection involved in freedom of exercise than in general ability to do otherwise. It seems to me that this point of Punch is debatable; to show that the divine ability to do otherwise does not undermine immutability one has to claim that divine volitions of contingent objects do not depend on their objects;

and it is not so clear that refrainings can be independent on their objects in this way.

It seems to me that the arguments against ascribing liberty of exercise to God are very strong; but perhaps one has to consider much more intricacies of refrainings from willing or omissions of volitions in general to be quite positive here. At any rate, since I am also skeptical about pure free omissions in human case, I tend to think that denying the freedom of exercise to God is fully compatible with ascribing him every authentic perfection.

10.7 Summary

It is quite a reasonable worry that the attempt to give a satisfying ontological account of divine freedom is just a clear case of overconfidence of the powers of our reason. Perhaps a task that would suit human ability better would be to show that no reason against the divine libertarian freedom is compelling. I hope I have managed to show something along these lines.

On the other hand, however, what I was trying to show here is that what we basically need to discuss the divine freedom is a good understanding of freedom in general—for example, a good understanding of the fact that freedom can be ascribed to a volition only relative to an object; of formal objects of the will; of the relationship between reasons and causes; of the relationship between volitions and causal profiles; of the relationship between causal profiles and omnipotence; of the real controversies surrounding strong versions of agent causality; of the dependence of volitions on their objects and the limits of externalism in the case of the will; and on the nature of refraining from willing and the distinction between freedom of specification and freedom of exercise. I hope I have succeeded in showing how a serious consideration of these issues may shed

light on the nature of the divine freedom, or perhaps remove some obstacles for attempts at understanding it.

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Notes

1 It seems symptomatic that some recent treatments of the issue of God's freedom do tend to deny libertarian freedom to God; see e.g. →Wierenga (2002), Couenhoven (2012), →Timpe (2014), →Timpe (2016); for an overview see →Kittle (2016).

2 For a contemporary treatment of the issue see e.g. →Rowe (2002) and →Wierenga (2002). The debate over the issue, however, is very old and rich, especially in the late scholastic tradition; see e.g. Caspensius (1641), tract. 4, disp. 3, sectio 5; →Lugo (1679), disp. 2.

3 For a brief overview of these problems see e.g. →Davies (1993), pp. 141–167.

4 A special thing to note here is that acting for a reason is closely related to God's responsiveness to prayers, which in turn is closely related to God's freedom to answer the prayer or to reject it (the efficacy of a prayer consists in the fact that God wants something *because of* that prayer). I discuss this issue in some detail in Section 10.4.

5 See Aquinas (1980), *S. th.* I–II, q. 6, art. 4; *De veritate*, q. 22, art. 5; the distinction between the kinds of necessity goes back to Augustine, *De civitate Dei*, lib. V, c. XI. It is the distinction between these kinds of necessity that makes it possible to argue that necessary volitions or actions (and in particular the divine necessary volitions) can be free in some non-libertarian sense. As I have

declared in the introduction, I do not like to enter the debate whether they deserve to be called free in some week sense; the scholastics would just say they are voluntary, but not free.

6 Another interesting issue which I cannot discuss here is whether other sorts of necessity, in particular the necessity of causal coercion, could be ascribed to acts only relative to their objects.

7 See e.g. →Suarez (1861), *Disp. met* 30, s. 9; John of St. Thomas (1883), q. 19, disp. 4, art. 3 and 4; Cajetan (1888), q. 19, art. 3; Caspensius (1641), tract. 4, disp. 3, sect. 2; →Nazarius (1610), q. 19, art. 2, controversia unica; Poncius (1671), disp. 6, q. 2; →Vazquez (1606), q. 19, disp. 80.

8 See e.g. →Nazarius (1610), q. 19, art. 2, controversia unica.

9 See e.g. →Aureoli (1596), lib. I, dist. 45; →Vazquez (1606), q. 19, disp. 80, c. 2.

10 →Suarez (1861), disp. 30, sectio 9, n. 35–40; John of St. Thomas (1883), q. 19, art. 4, n. 5–11.

11 What I have said here is just a brief outline of some intuitions; there are a number of fundamental issues concerning the ontology of *objects* of volitions which I cannot discuss here.

12 I develop an account of volitions as constituting intrinsic causal profile of an agent in →Głowala (2022); it accommodates some intuitions presented in →Ellis

(2013), who claims that agency involves a modification of the agent's causal profile (pp. 194–197).

13 This principle is crucial for settling the issue whether a given agent is *causally responsible* for a given effect of his action, for example, if the engineer who designs a certain steam machine is causally responsible for the death of some particular victim of railway transport; the answer depends on the objective nature of the relevant volitions of the engineer, and these volitions do in turn depend in some way on what it is that the engineer wanted.

14 I use the term 'strong forms of agent causality' to contrast agent causality in the sense introduced by Chisholm or O'Connor with the claim of Lowe (weak agent causality) according to which substances are fundamental efficient causes, but they always act by doing something or in virtue of having some properties.

15 See e.g. →Pink (2019). There are some crucial issues relating to freedom of specification that I do not discuss here; in the case of choice what seems contrary to the choice of *o* is the choice of something contrary to *o*, but if you consider loving *o* as an act of will, then what is contrary to loving *o* is both loving something contrary to *o* and hating *o*; it is far from being clear what is the relationship between loving something contrary to *o* and hating *o*; see e.g. →Anscombe (1981), pp. 102–104.

11 Why Not? God

Kenneth L. Pearce

Abstract: It is widely agreed among broadly Anselmian theists that God is in some sense the ‘delimiter of possibilities.’ In other words, the scope of possibility is explained by the manner in which the universe emanates from God. However, existing accounts of God’s role here—in terms of freedom, choice, or power—face serious difficulties. The present paper provides a new account of God’s role as the delimiter of possibilities in terms of the different manner in which the non-actuality of non-actual states of affairs is explained. Intuitively and metaphorically, we can think of the menu of states of affairs being ‘whittled down’ in successive stages of a process of divine decision-making, with the different species of impossibility corresponding to when and how the states of affairs are ruled out. I argue that this approach provides genuine contingency while also preserving the Principle of Sufficient Reason.

Classical theism is a form of rationalism: that is, it holds that everything that stands in need of explanation is fully explained. Among the varieties of rationalism, classical theism purports to have an advantage in explaining contingency because it takes the universe ultimately to be explained by God’s free and rational choice. A choice of this sort, it is claimed, is a kind of mean between necessity and arbitrariness—it is fully explained and yet it could have been otherwise (Leibniz, 1985 [1710], §§345–349; and →Pearce, 2022a, pp. 47–50). Hence, if the universe is rooted in such a choice, we can have complete explanation without necessitarianism.

It follows from this picture that God is, in Thomas Morris's memorable phrase, "the delimiter of possibilities" (→Morris, 1987, p. 48). That is, the scope of possibility is defined by how the universe emanates from God.

How precisely does this story go? It might be thought that the bounds of God's freedom set the bounds of possibility. But I have previously argued that God's freedom has no bounds. God is free even with respect to impossible propositions (→Pearce, 2021). Similarly, it might be thought that the possible worlds correspond to the menu of options from which God chooses. But I have previously argued that even impossible states of affairs are among God's options (again, see →Pearce, 2021). Finally, it might be thought that the possible states of affairs are those God has the power to bring about (directly or indirectly).[1] But I have previously argued that it is a mistake to think of God as having a multitude of distinct powers (→Pearce, 2018) and that God's infinite or unlimited power implies that even impossible states of affairs are such that if (*per impossibile*) God willed them God would intentionally bring them about (→Pearce and Pruss, 2012, pp. 405–409; and →Pearce, 2017a).

My aim in this paper is to develop a new account of God's role as delimiter of possibilities. My starting assumption is that for every non-actual state of affairs, whether possible or impossible, there is an explanation of why it is not actual, and the chain of explanations leads back to God. From this starting point, I proceed to distinguish several different ways these explanations might go—i.e., different kinds of reasons for non-actuality. I show that each of these kinds of reasons can be seen as corresponding to a different kind of impossibility. Intuitively and metaphorically, we can think of the menu of states of affairs being 'whittled down' in successive stages of a process of divine decision-making, with the different species of impossibility corresponding to when and how the states of affairs are ruled

out.[2] I argue that this approach provides genuine contingency without compromising explicability.

11.1 Explaining Actuality and Non-Actuality

A substantive fact is a fact that stands in need of explanation. A fact, in my usage, is just a state of affairs which obtains (i.e., is actual). I maintain that every substantive fact is actualized by God. However, not all are actualized by God in the same way.

Plantinga influentially distinguished between 'strong' and 'weak' actualization (→Plantinga, 1974, pp. 172–173). On Plantinga's account, to strongly actualize a state of affairs is to cause it to be actual. To weakly actualize a state of affairs is to cause something else to be the case, knowing that if one does so then the state of affairs will be actual. Plantinga's central example of weak actualization is causing it to be the case that a being exists and faces a certain free choice in certain circumstances while knowing what the being would choose if placed in those circumstances. In such a case, according to Plantinga, one does not cause that being so to choose (else the being would not choose freely), but one nevertheless may be said to actualize the state of affairs *that being's so choosing*.

Plantinga's distinction relies on the assumption that all actualization is by causation. In my view, God is the ground, and not the cause, of the created world (→Pearce, 2017a, 2017b). Accordingly, a different set of distinctions is needed (see →Pearce, 2021, p. 187n8).

Let us begin with the following definition:

Actualization: A state of affairs A is actualized by an agent S iff there is a complete explanation of why A is actual which traces back solely to S's agency.

Three elements of this definition deserve further comment: 'tracing back,' 'solely,' and 'agency.'

Explanations can generally be traced back in chains. If one asks why P and is told that P because Q, it is usually legitimate to ask why Q, and so on. It is not *always* legitimate, for Q may be an autonomous fact, a fact which stands in no need of explanation (see →Dasgupta, 2016, pp. 383–387; →Pearce, 2017b, pp. 259–260; and →Pearce, 2022a, p. 36). However, for most 'why'-questions, there are many links in the chain before one arrives at an autonomous fact.

In saying that the explanation traces back *solely* to S's agency, I mean there is a link in the chain of explanations at which facts about S's agency and no other facts appear. The reason for this can be seen in an intuitive example: if two carpenters work together to build a table, neither of them actualizes the table's existence. We could say that each of them partly actualizes the table's existence, or that they jointly actualize the table's existence. However, we will not need such notions in the present context. Thus, in what follows, actualization is always total actualization, tracing back to one agent alone.

To say that an explanation traces to S's *agency* is simply to say that it appeals to S as an agent, i.e., a being who acts for reasons. Thus, to say that I broke the window by hitting it with a baseball bat is to explain the window's being broken in terms of my agency, but to say that I broke the window by being thrown through it is not. A rock could just as easily have figured in the explanation in the latter way.

To say that a state of affairs is explained in terms of my agency does not imply that I brought it about intentionally, nor does it imply that the state of affairs is traceable to a particular choice of mine. Below I will argue that we can explain the non-actuality of certain states of affairs by appeal to facts about God's agency which guarantee that God does *not* choose in certain ways.

Deterministic Actualization: A state of affairs A is deterministically actualized by an agent S iff there is a complete deterministic explanation of why A is actual which traces back solely to S's agency.

In what follows, deterministic explanation will figure in the analysis of various flavors of necessity. Therefore, on pain of circularity, deterministic explanation cannot be defined in terms of necessity. Instead, I appeal to a primitive notion of explanatoriness (cf. →Pearce, 2017b, pp. 256–257).

Explanatoriness is a relation between a set Γ of states of affairs and a state of affairs A. Intuitively, one can think of the claim that Γ is explanatory of A as saying that the members of Γ are jointly capable of explaining A. This should not be regarded as a definition, since 'capable of' is a fuzzy notion that might require a partly modal analysis, and I will be analyzing modal notions in terms of explanatoriness. Officially, explanatoriness is primitive.

Deterministic Explanatoriness: Γ is deterministically explanatory of A iff the explanatory relation between Γ and A presupposes no background conditions and Γ is not explanatory of not-A.

This definition may perhaps be more easily grasped by considering two ways an explanatory relation could fail to be deterministic: by presupposing background conditions and by the explanans being equally capable of explaining the negation of the explanandum. These are both ways in which the explanans may not be enough, on its own, to rule out alternatives to the explanandum.

Intuitively, the match lit because it was struck, and the clay constitutes a statue because it is shaped thus and so. However, even if the match were struck, it would not have lit in the absence of oxygen, and even if the clay were shaped thus and so it would not constitute a statue had it not been shaped with

artistic intentions, come to be regarded in a certain way by the art world, and so on.

The cause or ground of a state of affairs is explanatory of that state of affairs: the striking of the match is explanatory of its lighting, and the shape of the clay is explanatory of the statue's existence. Yet these explanatory relations do not hold unless some further conditions are satisfied.

Some philosophers maintain that the distinction between grounds/causes and background conditions is merely a pragmatic feature of our explanatory practices. According to this view, objectively, the presence of oxygen and various other factors are all part of the total cause of the match's lighting, though not all of these factors are equally of interest to us. Similarly, whatever conditions are necessary for the clay to constitute the statue are all objectively among the grounds of the statue's existence. Other theorists maintain an objective distinction between causes/grounds and background conditions, so that the striking of the match may indeed be regarded as a total cause of its lighting, and the existence of the statue may be wholly grounded in the shape of the clay.[3]

The definition of deterministic explanatoriness does not take a position in this debate. It simply says that *if* there is an objective distinction between the relata of the explanatory relation and the background conditions of the explanatory relation *then* any explanatory relation that requires background conditions is not deterministic.

Second, an explanatory relation is not deterministic if the explanans is also explanatory of the negation of the explanandum. This case may be regarded as absurd, but this is not obviously so. For instance, on the objective propensity interpretation of the probabilities of quantum mechanics, one may think that (e.g.) an electron's being in a certain superposition is equally explanatory of its being measured spin

up and being measured spin down. Suppose it is measured spin up. Then we can say that it was not measured spin down because it was measured spin up, and it was measured spin up because it was in a superposition of up and down states at the time of measurement (see →Pruss, 2006, §8.3). Similarly, one might suppose that, when an agent with libertarian freedom chooses to act, this choice is explained by the agent's beliefs, desires, etc., and yet the same set of beliefs, desires, etc., could explain the agent's not choosing to act in that way (see →Pruss, 2006, §7.2.4).

It may be thought that indeterministic explanations are always incomplete explanations. Elsewhere, I argue that metaphysical rationalists must reject this view (Pearce, manuscript[b]). Intuitively, the idea is that an indeterministic cause or ground may have all the *oomph* it takes to produce the explanandum. For present purposes, I stipulate that my primitive notion of explanatoriness allows for indeterminism.

My preferred model for indeterministic explanation is based on Bader's (2021) 'competing grounds' view (Pearce, manuscript[a], §1; for a similar approach in the theistic context, see Lo, manuscript). This view treats the absence of certain blockers as objective background conditions. Thus, the electron's being measured spin up is a blocker of the causal relation between the previous superposition and the electron's being measured spin down. In other words, the superposition causes it to be measured spin down on condition that it is not measured spin up. However, nothing here will hinge on my preferred model: it suffices that not all explanatoriness is deterministic.

If the notion of explanatoriness does not require all explanatory relations to be deterministic, then our definition of actualization above does not require all actualization to be deterministic. I introduce one final definition:

Decree: A state of affairs A is decreed by an agent S iff there is a complete deterministic explanation of why A is actual which traces back solely to S's explicit act of willing that A should be actual.

Informally, an agent decrees A when the agent efficaciously decides that A will be actual, in such a way as to rule out all alternatives simply by willing.

I maintain that every substantive fact is actualized by God. The restriction to substantive (i.e., non-autonomous) facts is important. For all facts that require explanation, God is involved in their explanation at some appropriate stage. But not all facts require explanations. Autonomous facts include both real and nominal definitions (see →Dasgupta, 2016, pp. 383–387; →Pearce, 2017b, pp. 259–260; →Pearce, 2022a, p. 36). At the root of the explanatory hierarchy is the real definition of God. Thus, God's agency does not explain what it is to be God, since this must be prior to God's agency.

My overall position may now be summarized very briefly as follows. Although every substantive fact is actualized by God, not every substantive fact is deterministically actualized by God, and only a relatively small proportion of substantive facts are decreed by God. Further, every non-actual state of affairs is non-actual because it is not actualized by God.

My aim in what follows will be to analyze the various species of impossibility in terms of different ways theistic explanations of non-actuality might go.

11.2 Logical Impossibility

According to the analysis of omnipotence developed by Alexander Pruss and myself (→Pearce and Pruss, 2012), omnipotence is the conjunction of perfect efficacy of will and perfect freedom of will. A being has perfect efficacy of will iff,

necessarily, for any proposition p it is non-trivially true that if that being should will that p then that being would intentionally bring it about that p.

By the Distribution Axiom of modal logic, it is impossible that a being with perfect efficacy of will should will an impossible proposition. For instance, if this definition is to be maintained, it must be impossible that God should will that 2 + 2 = 5. Such an impossibility is consistent with perfect freedom, Pruss and I argued, iff the impossibility follows from the agent's character and/or choices in the right way.

In the original paper, we left 'in the right way' vague and contented ourselves with showing that many theories of free will, both libertarian and compatibilist, allow that there is a 'right way' for an agent's choice to be limited, such that (for instance) Martin Luther's "I can do no other" may be understood as an expression of his freedom and not a defect of it. The general idea of such 'Luther cases' is that the individual's not choosing otherwise—and, indeed, having no possibility of choosing otherwise—is itself explained by the individual's agency and is not foreign to it.[4] If the impossibility of God's willing that 2 + 2 = 5 is to be consistent with the perfect freedom required for omnipotence, God must endorse or value or care about the laws of logic and mathematics in such a way as to guarantee that God wills according to them (→Pearce, 2017a, pp. 12–13).

The present proposal is to analyze logical impossibility in terms of divine rationality: the logically impossible states of affairs are those whose non-actuality is fully explained by God's perfect rationality, where perfect rationality is understood as a feature of God's character or values.

What precisely is this feature? On pain of circularity, it cannot be defined modally. A definition in terms of formal entailment may seem plausible, but care must be exercised here. In the first

place, there are competing formal systems. In the second place, it is implausible to suppose that God thinks symbolically.

James F. Ross used to say,[5] “the one thing an omnipotent being cannot do is to cancel its action in the doing of it.” This may perhaps provide a clue in the direction of a non-circular account of perfect rationality. I suggest that the perfect freedom included in omnipotence requires a kind of wholeheartedness in willing. To will a contradiction would be, as it were, to will and un-will at the same time. To will wholeheartedly thus requires respect for logical consistency, and this is characteristic of a being with perfect freedom of will.

This notion of wholeheartedness is drawn from the work of Harry Frankfurt,[6] but it cannot here be employed in precisely Frankfurt’s sense. For Frankfurt, a person is a being who has the capacity to reflect on its conflicting first-order desires and form a view about which of them should be effective—i.e., should move the being to action. The effective desire is what Frankfurt calls the person’s will (→Frankfurt, 1971). Frankfurt then analyses free will as being “free to will what [one] wants to will, or to have the will [one] wants” (→Frankfurt, 1971, p. 15). Thus, persons are capable of acting either freely or unfreely, depending on whether their will (effective desire) is in harmony or conflict with their higher-order volitions. Wholeheartedness, for Frankfurt, occurs when one’s volitional structure is such that there is a univocal answer to the question of what one really wants, and this occurs when there is no conflict within the highest order of volitions one has formed (Frankfurt, 1993 [1987], p. 176; →Frankfurt, 2004, pp. 91–95). A failure of wholeheartedness is thus a failure (or at least an imperfection[7]) of agency: when one is not wholehearted, one cannot succeed in working one’s will in the world because there is no fact of the matter about what one’s will is (→Frankfurt, 2004, pp. 96–97; →Korsgaard, 2009, pp. 69–70 *et passim*).

Frankfurt notes that his concept of person "excludes those suprahuman beings, if any, whose wills are necessarily free" (→Frankfurt, 1971, p. 14).[8] Frankfurt does not elaborate on this point, but his thinking seems to be that for God there would be no question of acting freely or unfreely, since God's higher-order volitions (if any) could not be frustrated (cf. →Watson, 2004, p. 31). (This, incidentally, follows from the definition of perfect efficacy of will.) In the same manner, the question of wholeheartedness would not arise since God could not experience a conflict of higher-order volitions.

In employing Frankfurt's account to talk about God, I follow a tried and true practice from the apophatic tradition (e.g., →Maimonides, 1995, ch. 1.58): what I mean in saying that an omnipotent being must be wholehearted is that an omnipotent being could not exhibit the kind of defect in agency which, in humans, is the failure of wholeheartedness. This is because any such defect in agency would be either a defect in efficacy (if some higher-order volition were frustrated) or a defect in freedom. God's perfect rationality follows from the wholeheartedness of God's willing, which guarantees that God does not (directly or indirectly) will and un-will the same thing. A state of affairs is logically impossible iff it is non-actual because God does not directly or indirectly will and un-will the same thing.

11.3 Essentialist Impossibility

The account of (narrow) logical impossibility given in the previous section can be augmented by an account of essences. I have elsewhere endorsed an account on which essences are grounded in God's intellectual activity—i.e., God "thinks them up" (Pearce, manuscript[a], §2).[9] This activity is prior to any volition on God's part. It's not that God decides that water will

contain hydrogen. Rather, God comes up with the concept of water (which implies that it contains hydrogen) and then decides whether to create any.

Essences, then, are God's concepts, and real definitions are statements of essences.

A (partial) real definition, like *water contains hydrogen* is an autonomous fact, and is therefore not explained by God's intellectual activity (or by anything else). However, real definitions are statements of essences, and God's intellectual activity explains why there is such an essence in the first place. This provides a kind of higher-order explanation. The case here is analogous to the case of nominal definitions. The fact that *all mothers are parents* is an autonomous fact because that's just (part of) what 'mother' means. It is therefore a mistake to wonder why all mothers are parents, or to wonder why some particular person is also a parent, given that she's a mother. Yet there is an explanation, to be found in historical linguistics, of the fact that the English language exists and contains such a word as 'mother' which is defined in such a way. In the same way, God's intellectual activity explains the existence of the system of essences, why the system contains an essence for water, and why containing hydrogen is among the properties included in that essence.

Traditionally, essences (especially when identified with divine ideas or concepts) have often been thought to be normative. That is, the essence of human might be thought to be God's idea of a perfect or ideal human. (Indeed, the English word 'ideal' is derived from this very conception of ideas!) I think this view should be rejected because it has bad consequences for philosophy of biology, ethics, and philosophy of disability, as well as implying that God falls short of creating what God is trying to create (cf. →Pearce, 2022b, pp. 195–196). To say what it is to be human is not to say what a human should be, but only what a

human (every human) is. The essence of human is the concept, idea, or bit of knowledge whereby God knows how to create humans—*all* humans, whether two-legged (etc.) or not.

We may then say that states of affairs are impossible in the *essentialist* sense iff they are non-actual because God wills through these essences. Thus, it is impossible that there be water without hydrogen because, in willing that there be water, God employs the concept of a substance containing hydrogen. If God had willed that there not be water, God would still have been willing through this concept, and the fact that God employs this concept would still explain why there is no hydrogen-free water.

The essence of God requires special treatment in this picture. In thinking up the essences of other things, God is engaged in an activity akin to an author thinking up various possible fictions and characters to populate them (Pearce, manuscript[a], §2; cf. →Lebens, 2020, pp. 98–100; →Page, 2022). In thinking up God's own essence, God is understanding what already exists, namely, Godself. This is why God's essence includes existence: in knowing Godself, God understands not what may be created but what (already) exists to do the creating. The divine essence is God's self-understanding of what it is to be God.

11.4 Metaphysical Impossibility

The notion of metaphysical impossibility may now be very succinctly analyzed: the metaphysically impossible states of affairs are those whose non-actuality can be explained by any combination of logical and essentialist explanations.

Metaphysical modality, so constructed, will obey the system S5. The easiest way to see this is to construct the system of metaphysically possible worlds—i.e., all the maximal states of affairs whose non-actuality *cannot* be explained in terms of

divine rationality and the system of essences. (The actual world will be included among those worlds whose non-actuality cannot be explained in these ways since one cannot explain what isn't so.) Since the logical laws and the essences do not differ among these worlds, they will all agree on which worlds are metaphysically possible (i.e., in model-theoretic terms, the accessibility relation will be an equivalence relation), which guarantees that this system is a model of S5.

11.5 Deontological Impossibility

It is widely recognized among broadly Anselmian theists that there are some states of affairs which are not formally contradictory or contradictory to facts about finite essences but nevertheless (in some sense) cannot be actualized by God. Leibniz gives the example of *the innocent being condemned to hell* (Leibniz, 1985 [1710], §166). It is impossible that God should actualize these states of affairs because doing so (whether directly or indirectly) would conflict with God's moral attributes.

I endorse a Kantian moral theory and maintain that God has moral obligations (→Pearce, 2022b, §2.4).[10] That God fulfills all of God's Kantian obligations can be derived from God's wholeheartedness in willing. According to the Kantian view, the practical judgments (maxims) that drive our actions are implicitly universal (→Korsgaard, 2009, §4.4).[11] To decide to take an action is, implicitly, to judge that that action is *the thing to do* in those circumstances, and hence that similarly situated others ought also to do that action. A maxim is impermissible, in the Kantian view, just in case an agent cannot consistently adhere to this universal judgment. But such a failure of universality is a failure of wholeheartedness: it involves, in a certain way, willing and un-willing the same thing. Thus, as Kant says, "A *will is absolutely good* that cannot be evil, hence whose maxim, if made a

universal law, can never conflict with itself" (Kant, 2012 [1785], p. 48). A will that can never conflict with itself—that is necessarily perfectly wholehearted—would be morally perfect in the Kantian sense.[12]

Kantian wrong-doing is not the same kind of failure of wholeheartedness that was at issue in our discussion of logical impossibility above. Logically impossible states of affairs are impossible because willing that they be so would also, simultaneously, be willing that they *not* be so. Deontologically impossible states of affairs (as I here call them) are impossible because, in order for God to actualize them, God would have to make an inconsistent (and therefore not wholehearted) judgment about what is to-be-done. This is a practical judgment about the action itself, and not about the state of affairs the action aims to bring about. Logical impossibilities are ruled out by God's wholeheartedness in willing that states of affairs be actual; deontological impossibilities are ruled out by God's wholeheartedness in judging that actions are to-be-done.

While this is not the place for a full defense of Kantian ethics, I agree with Kant that it is deontologically impossible that God treat any rational agent as a mere means. This implies (among other things) that Leibniz was right that it is impossible that the innocent be condemned to hell: such an action toward a person would not treat her as an end in herself.

11.6 Axiological Impossibility

The notion that God has obligations at all is controversial, and my Kantian understanding of these obligations is even more controversial. Some may therefore doubt whether anything is deontologically impossible, in the sense outlined in the previous section. Many theists (including Leibniz) instead see the impossibility of states of affairs like *the innocent being condemned*

to hell as axiological impossibility. That is, it is because of their disvalue that these states are not actual. Even on my Kantian view, it seems likely that some states of affairs have this kind of impossibility. The Kantian picture will again enable us to derive God's unsurpassable commitment to what Kant calls 'imperfect duties' from God's wholeheartedness in willing. According to Kant, imperfect duties are fundamentally of two kinds: to promote the perfection of oneself and the happiness of others (Kant, 1996 [1797], pp. 149–156). These duties are imperfect in the sense that (for us finite humans) they do not enjoin any specific actions. Rather, they require us to adopt these two items among our ends.

Interestingly, the Kantian approach fits neatly with the classical view that God is somehow motivated by love of God's own perfection (see, e.g., Aquinas, *Summa Contra Gentiles*, chs. 1.73–83). Indeed, this is sometimes expressed by saying that God is the end (telos) of creation (see, e.g., Aquinas, *Summa Theologiae*, Iq19a2r2, Iq44a4).

This view presents something of a puzzle in that, according to the classical tradition, the divine perfection is a necessary reality, and therefore not something that can be pursued or promoted as an end.[13] To get around this, the classical tradition, reaching all the way back to Plato, has relied on the idea that "goodness is diffusive of itself" (see, e.g., Plato, *Timaeus* 29d–30c; Dionysius, *The Divine Names*, §4.10; Aquinas, *Summa Theologiae*, Iq19a2). In other words, God somehow wills or affirms God's own goodness by multiplying finite images of that goodness.[14]

Kant defends the idea that the perfection of the self and the happiness of others are obligatory ends by the same kind of argument he uses to derive the deontological constraints on willing: these are implicit in all willing, so that a failure to will them would be a kind of inconsistency in the will (Kant, 2012 [1785], pp. 42–43; Kant, 1997 [1788], pp. 31–32; Kant, 1996 [1797],

pp. 154–156). If Kant's argument here is correct, then God's wholeheartedness in willing ensures that God adopts perfection and happiness as ends in God's willing.

This does not *immediately* generate axiological impossibilities, since the adoption of these obligatory ends, on the Kantian view, does not prevent agents from also adopting *discretionary* ends and it does not tell us how to balance the obligatory ends against each other or against our discretionary ends.[15] Thus, the Kantian view will allow God to have non-moral preferences.[16] However, we will get axiological impossibilities if we add that a wholehearted, perfectly rational agent cannot choose a *dominated* option (see →Pruss, 2016).

The idea here is that, even without a principle for balancing among the ends, options can be weighted on a separate scale for each end, based on how much they promote that end. Option A dominates option B if A scores higher than B on at least one such scale, and B does not score higher than A on any such scale. Pruss suggests that it is impossible for any agent to choose an option which that agent judges to be dominated. Regardless of whether we accept that claim, it is plausible that choosing a dominated option would be a failure of wholeheartedness in willing one's ends. One would not really be, as Korsgaard puts it, setting oneself to be the cause of those ends (→Korsgaard, 1996, p. 36). Thus dominated options, at least, will be axiologically impossible.

We may get a broader range of axiological impossibilities if there are specific principles God must use for weighing God's competing ends. If God has only one end, the perfection of God's creative product (mirroring God's own perfection), and there is one correct weighting of the values that contribute to this end, then we will get the Leibnizian result that it is axiologically (or, in Leibniz's terms, morally) necessary that God creates the best of all possible worlds. On the other hand if, as

Pruss argues, there is no one objectively correct weighting of competing values against each other, there will not be one objectively best world. Instead, there will be a large class of non-dominated worlds, each of which will be axiologically possible.

11.7 Contingent Enough?

In other work (Pearce, manuscript[b]), I've defended the importance of preserving contingency. I there give four reasons against necessitarianism: modal intuition, an argument from the notion of lawhood, an argument from quantum physics, and an argument from general relativity. With respect to the account given here, there are two questions to be asked*: is the contingency provided sufficiently robust?* and, *is the scope of contingency wide enough?*

With respect to the first question, the key issue is why certain states of affairs are worthy of our consideration in a way that others are not. For instance, the question of whether I will eat breakfast tomorrow is 'live' in a way that the question of whether I will draw a round square is not. The reason is that, in the former case, but not the latter case, either state of affairs (my eating breakfast or my not eating breakfast) is possible.

That the non-actuality of different states of affairs is explained in terms of divine agency in different ways allows for this difference. The state of affairs *my drawing a round square* is an essentialist impossibility, ruled out by the concepts of round and square God thinks up. My refraining from eating breakfast tomorrow may perhaps be impossible in some other sense (e.g., axiologically), but it is not impossible in the essentialist sense, and therefore cannot be ruled out in the same way.

Our modal intuitions are deserving of the most respect when they are directly connected with practical deliberation, as in the example just described. It is also in this context that the

argument from the notion of lawhood arises. Laws of nature are *constraint*s on practical deliberation: if a state of affairs violates a law of nature, there's no point trying to bring it about. However, if all non-actual states of affairs are impossible in just the same sense, then there is no distinction to be drawn here. States of affairs prohibited by the laws of nature should be somehow 'more impossible' than non-actual but physically contingent states of affairs.

Although none of the five species of impossibility discussed above precisely corresponds to physical/nomological impossibility, the model developed here can deliver what's needed. The idea that different states of affairs are ruled out of actuality in different ways is familiar and commonsensical. For instance, some states of affairs are non-actual because some human acted to prevent them. In the same way, my proposal is that the 'modal force' of laws of nature can be accounted for in terms of the manner in which the non-actuality of physically impossible states of affairs is explained. If, as some philosophers maintain (e.g., Shoemaker, 1997 [1980]; →Bird, 2007; →Bird, 2012), the physical laws can be derived from essences, then physical necessity can be understood as a subclass of essentialist necessity.

Once the distinction between physically contingent and physically necessary states of affairs is successfully drawn, there is no further issue about the fact that current physics treats some states of affairs as physically contingent.

Finally, the scope of metaphysical contingency, on the present view, will correspond at least roughly with intuition-based accounts: what is metaphysically contingent is what is compatible with the logical and essentialist facts.

Some may worry that many intuitively possible states of affairs will be deontologically or axiologically impossible. However, I do not think we should find this worrying. This is no

more than to say: there are explanations, in terms of the divine goodness, for why these metaphysically possible states of affairs are non-actual. This, however, is a conclusion which I think all theists should embrace.

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Notes

1 A view along these lines is defended by →Pruss, 2011.

2 My approach here is inspired by Tom Feeney's (2016) interpretation of Leibniz.

3 For a brief summary of the debate about background conditions in the theory of causation, see →Gallow, 2022, §1.2.3. For the analogous debate in the theory of grounding, see →deRosset, 2013, pp. 14–15; →Skiles, 2015; →Cohen, 2020; →Baron-Schmitt, 2021.

4 On 'Luther cases,' see, e.g., →Frankfurt, 1982, pp. 263–266; →Albritton, 1985, pp. 246–247; →Williams, 1993, p. 61; →Korsgaard, 1996, pp. 239–240; →Watson, 2004, ch. 4.

5 In undergraduate teaching, c. 2006. I quote from memory. For Ross's full account of divine omnipotence see →Ross, 1969, ch. 5.

6 My thinking here is also heavily influenced by Christine Korsgaard (1996 and 2009) and, as we will see below, Kant. However, I adopt Frankfurt's vocabulary, as I think it allows the points I want to make to be expressed most clearly.

7 Coates (2017) presents a plausible argument that it is not always desirable for humans to be wholehearted. He even claims that "ambivalence [i.e., lack of wholeheartedness] as such is no threat to agency *par excellence*" (→Coates, 2017, p. 421). However, Coates'

cases all involve tragic choices in which an agent is torn between betraying one part of her practical identity or another. Though action in such circumstances may perhaps count as *human* agency *par excellence*, the kind of ideal agency attributed to an omnipotent being is incompatible with being caught in this sort of dilemma.

8 I thank Taylor Cyr for discussion of this comment of Frankfurt's.

9 I did not—and do not—mean to endorse the Molinist elements of the story I there described. I am undecided on that particular point.

10 Kant's own position on the relation of the moral law to the 'holy will' is complex. On the one hand, he holds that the holy will cannot be said to be subject to imperatives or commands, since it has no possibility of willing wrongly (Kant, 2012 [1785], p. 27; Kant, 1997 [1788], p. 70; and Kant, 1996 [1797], p. 15). Kant nevertheless holds that such a will's "maxims would necessarily harmonize with" the moral law (Kant, 2012 [1785], p. 51) and explicitly insists that the Formula of Humanity applies even to God (Kant, 1997 [1788], pp. 74 and 109).

11 Kant's preferred way of talking about this is to say that the will is a faculty for prescribing laws to oneself (Kant, 2012 [1785], pp. 56–58). According to Kant, it is part of the very notion of a law that it purports to be universal. Hence our maxims (the subjective principles of our actions) purport to be universal, insofar as they are laws we prescribe to ourselves.

12 Kant himself argues that moral perfection and perfect freedom of will are one and the same thing. See Kant, 1997 [1788], p. 26.

13 Here too, the Kantian view is analogous, in that it requires us to treat our own humanity as an end in itself, but nothing could be required of (obligatory for) us unless our own humanity were already a reality. See Kant, 2012 [1785], p. 49.

14 For a contemporary defense of this view, see →Johnston, 2019.

15 For instance, Kant explicitly argues that, although the moral law requires us to be willing to sacrifice our own happiness for the good of others, it makes no specific demand as to the magnitude of this sacrifice (Kant, 1996 [1797], p. 156).

16 On this idea, see →Amijee, 2022; Pearce, manuscript[a], §3.

IV God, Fundamentality and Creation of All Else

12 God and/as the Universe

István Aranyosi

Abstract: I address the issue of what the role of the particular theoretical physical model of the Universe we inhabit should be in one's pantheistic or panentheistic theology, as this question is scarcely addressed in the traditional pantheistic or panentheistic views. Pantheists and panentheists do consider the God-Universe relation as crucial, but they do not delve into the physical theory, that is, into cosmology. This might be due to their thinking that the particular cosmology is not important as long as we have the general picture of how God is supposed to be related to the Universe. In this essay, I argue that as compared to the Big Bang model of the Universe, the more recent Big Bounce model, based on quantum cosmology, is more congenial to a naturalistic version of panentheism.

12.1 Introduction

In earlier work (→Aranyosi, 2013) I offered a view of God, which I dubbed "logical pantheism," according to which God is to be identified with (my account of) logical space.[1] The basic argument was that logical space is truly the only item in our ontology that deserves the name "God," given the most neutral and most correct definition of the concept God, the Anselmian "the being greater than which nothing can be conceived." Though I still subscribe to this view, I realize that it is quite odd and disconnected from the traditional pantheistic or panentheistic views, in that it does not award (for good reasons) any special place for the physical Universe in one's theological economy.

In this essay I would like to address the issue of what the role of the Universe is in one's pantheistic or panentheistic theology, not simply because I haven't said anything about it in my previous work, but because there is a neglect in the traditional pantheistic or panentheistic views when it comes to the precise nature of the Universe. Pantheists and panentheists do, of course, consider the God-Universe relation as crucial, but they do not delve into the physical theory, that is, into cosmology, possibly because they think that the particular cosmology is not important as long as we have the general picture of how God is supposed to be related to the Universe.

In what follows, I will try to correct this shortcoming. I will proceed as follows. In the following section (12.2), I will explain and compare two kinds of panentheism, *mereological* and *emergent* panentheism, depending on how they view the *being in* relation that is supposed to hold between the Universe and God. In Section 12.3, I consider naturalism and set it as a constraint on our theorizing about alternative notions of divinity. In Section 12.4, I argue that, on the face of it, emergent panentheism appears to be the only model that is compatible with naturalism. In Section 12.5, I consider Big Bang cosmology, and argue that it is not congenial to naturalistic panentheism. In Section 12.6, I talk about Big Bounce cosmology (a.k.a. loop quantum cosmology) and argue that it is a nice physical model that can ground a reasonable naturalistic mereological panentheist view. I end the essay with a few thoughts on the kabalistic notion of *tzimtzum* (Section 12.7), which appears to be an early (purely speculative) adumbration of loop quantum cosmology.

12.2 Mereological versus Emergent Panentheism

Pantheism and panentheism are alternatives to both classical theism and atheism. Pantheism *identifies* God with the Universe,

whereas panentheism states that the Universe is *in* God. Ever since Spinoza's *Ethics*, a common theme of advertising such alternatives is that they compensate for some shortcomings of the more established views. Atheism's shortcoming is the lack of spirituality; many people reject theism, but still consider themselves spiritual in some sense. Classical theism's shortcoming is that it posits a transcendent god, who many people find distant, unreachable, and unknowable. A further "selling point" of *panentheism*, as compared to pantheism, is that it avoids the danger of emptying the concept of God of spirituality by identification with nature, which is a problem of pantheism (cf. →Culp, 2017).

There are many versions of panentheism, stretching back in time all the way to Ancient religions, but I would like to put forward one distinction which I think is important for theoretical reasons—that between a simple, mereological understanding of the *being in* relation and a subtler, emergentist understanding of it.

The mereological understanding appears as the obvious interpretation of the relation of *being in*, expressed in the slogan "everything in God," yet one can seldom find it in the history of pantheism. I will mention and briefly discuss one recent such view, which I take to be close enough to what I call "mereological panentheism," but first let us see what this interpretation amounts to. What I mean by mereological pantheism is the view that the Universe is a proper part of God's, where "proper part" is understood in accordance with core classical mereology (cf. →Varzi, 2016). We can define proper parthood via a core principle and a decomposition principle of supplementation. The core principle is that x is a proper part of y iff x is a part of y, but not identical to y ($PPxy \leftrightarrow Pxy \,\&\, x \neq y$). The decomposition principle, which has standardly been taken as analytic of the concept of *Proper Part* (cf. Simons, 1987, p. 116), is

what Simons called “Weak Supplementation” in his survey of mereology:

Weak Supplementation: $\forall x \forall y(PPxy \rightarrow \exists z(Pzy \wedge \neg Ozx))$, where the predicates *P, PP*, and *O*. mean, respectively, “is a part of,” “is a proper part of,” and “overlaps.”

What the principle says is that whenever an individual, *x*, is a proper part of another, *y*, there must be a remainder—an individual, *z*, that is a proper part of *y* but disjoint from *x*. It is very tempting, indeed, to take *Weak Supplementation* as analytic, though there certainly have been quite a few dissenting voices in the metaphysics literature.[2]

The emergentist—or more, generally, a dynamic or process interpretation of—panentheism, on the other hand, is less straightforward, yet, it has been dominating the proposed models in the literature. I will discuss a particular such model later, but for now I just want to briefly describe what this type of panentheism is supposed to look like.

Instead of thinking about the Universe-God relation as a simple part-whole relation, according to this second type of panentheism we are supposed to the think of divinity as being somehow an emergent property of the Universe. In other words, though there is nothing more than the physical Universe needed in order to obtain God, God’s divinity is more than what the simple sum of the material parts of the Universe would entail. This is the basic idea of an emergent property; it is a property that cannot be predicted or inferred from the totality of properties instantiated at a lower level of complexity.

An example of a domain that many theorists consider as involving this type of strong emergence is quantum mechanics. In that literature it is called “holism” (→Teller, 1986; →Healey, 2016; Lewis, 2017). Though the metaphysics of emergence is a complex and controversial topic, one important point that I want to emphasize is that emergence is *not* supposed to be a wacky,

unnatural, anomic, mysterious phenomenon just because strongly emergent properties are not a priori entailed by base properties; on the contrary, emergence is taken as implying a *nomic* relation between the base and the emergent property. Indeed, C. D. Broad himself, who was one of the chief representatives of British emergentism (cf. →McLaughlin, 1992) defines emergence as involving nomicness as a necessary condition:

> Put in abstract terms the emergent theory asserts that *there are certain wholes, composed (say) of constituents A, B, and C in a relation R to each other; that all wholes composed of constituents of the same kind as A, B, and C in relations of the same kind as R have certain characteristic properties*; that A, B, and C are capable of occurring in other kinds of complex where the relation is not of the same kind as R; and that the characteristic properties of the whole R (A, B, C) cannot, even in theory, be deduced from the most complete knowledge of the properties of A, B, and C in isolation or in other wholes which are not of the form R (A, B, C). The mechanistic theory rejects the last clause of this assertion. (→Broad, 1925, p. 61; my emphasis)[3]

Since I think nomicness is what grounds naturalness when it comes to judging whether a certain phenomenon's explanation is naturalistic or not, I will now turn to a discussion of naturalism in the context of panentheism.

12.3 Naturalism

Let me first state that my agenda in this essay is to try to find a feasible and coherent *naturalistic* panentheism. Why naturalism? There are two reasons for this preference. One has to do with our *Zeitgeist*; indeed, in the last 60 years or so philosophy has increasingly come to be viewed by its practitioners as a continuation of empirical science, or at least as an armchair,

analytic activity which nevertheless is supposed to be compatible with the sciences. The second reason is that naturalism about theology, just like (or even more so than) naturalism about the mind, is a challenging and intrinsically interesting intellectual endeavor, much harder than the standard non-naturalistic theology. Indeed, even pantheism, which, on the face of it, seems like a straightforward doctrine very congenial to a naturalistic worldview, is actually even more problematic the more naturalistic one wishes to be about it (→Leftow, 2016).

Let me then state my definition of naturalism, which I will then briefly compare to the two mainstream views about it. My definition of naturalism is as follows:

Naturalism: A theory, *T*, meant to explain a phenomenon, *P*, is naturalistic at time *t* iff in all the hypotheses contained in *T* which involve *P, P* figures as a relatum in relations that are established/known/assumed at *t* as *laws of nature*.

This definition ties naturalness of an entity or phenomenon to whether it behaves according to some pattern, so that a law of nature regarding its manifestation can be established. Consequently, a theory that only posits nomic connections among phenomena is a naturalistic theory.

I introduced a time variable because it is possible to have periods of time when we have reason to posit the existence of a phenomenon, but we don't yet have an explanation of its behavior. Thus, miracles, for instance, will not count as natural and a theory that posits them won't count as naturalistic since miracles are events or phenomena that at the time of their occurrence do not fall under any known law of nature; they are counterlegal occurrences. Yet, it is logically possible that something we thought is a miracle at a time t become a natural phenomenon at a later time, when we are able to establish a law of nature having that phenomenon as a relatum.

Another consequence of my definition is that the classical theistic God is not a natural entity. The reason for this is that God's attributes, especially the attribute of omnipotence, come into conflict with any possibility that God could be a relatum in a law of nature. God's omnipotence entails that God can violate laws of nature at will, bring about miracles, and render the whole idea of regularity meaningless when it comes to His behavior. Conversely, if we could establish laws of nature about God's behavior, then God would not be the supernatural being that mainstream theism talks about.

It is useful to briefly compare this understanding of naturalism with the two mainstream ones, ontological and methodological naturalism. There is no agreed upon definition of ontological naturalism. One could adopt a causal view, and say that the natural is the causal, and so naturalism would be commitment to only causally efficacious entities.[4] This is not a very reasonable proposal since it would make both miracles and God natural entities if they do have causal effects in the world.[5]

One could also adopt the view that naturalism should be understood as *physicalism* (→Papineau, 1993, 2016). Physicalism is the view that everything that exists is physical in nature or reducible in some sense to physical facts. Now, this proposal also fails to appear very reasonable, especially considering the controversy about whether physicalism is true or not when it comes to the mind-body problem. The very idea that it does not seem logically incoherent to think of the mind as non-physical, yet connected to the physical by fundamental *psychophysical* laws, as per the emergentist view, shows that to equate naturalism with physicalism is a mistake.[6]

The other popular way to understand naturalism is *methodological* naturalism. Its basic point is to tie naturalness to the empirical scientific method. So, the idea would be a commitment to only natural entities and phenomena, where

“natural” means “discovered via empirical methods.” This seems reasonable, but there is a century or so of debate and controversy in the philosophy of science as about what “the scientific method” really means, when considered as a normative constraint. Anyhow, my definition of naturalism will simply coincide with methodological naturalism when the latter considers the expressions “established via empirical methods” and “established as a law of nature” as synonymous.

One might worry that, as it stands, the definition is not very informative since it does not further explicate the notion of a law of nature.[7] I am aware that there is a massive literature on various views of the laws of nature, but I believe that, in principle, my account of naturalism will be able to accommodate any of these. More to the point, one might consider three views of laws of nature that have been put forward and see whether they are problematic for my approach to naturalism: governing accounts, Humean accounts, and causal essentialist accounts. Let us briefly consider each and see whether they are problematic.

12.3.1 Governing Accounts

On the most prominent governing account, namely, the one put forward by →Armstrong (1983), a law of nature is a second-order necessitation relation between universals. The problem with these accounts seems to be that they do not offer an explanation of where the necessity comes from. →Beebee (2000) for instance, has suggested that a governing account of laws of nature grew out from a commitment to understanding what happens in the universe to be a matter of divine decree. Similarly, →Ellis (1999) argued that the governing conception of laws is best understood as a secularized version of the sort of divine command theory of the laws of nature defending by the

likes of Isaac Newton and Bishop George Berkeley in the seventeenth and eighteenth centuries.

Some would perhaps think that, because of the foregoing considerations, this is hardly a naturalist view. I have two replies to this worry. One is that we shouldn't commit the genetic fallacy and reject a view just because it originates on some other view that we do not subscribe to. Second, we shouldn't reject a view based on demanding from it exactly what the theory says it shouldn't be demanded. In particular, the governing accounts take laws of nature as basic and irreducible, hence, it should not be a surprise that the necessity involved in them is brute.

Anyhow, the key idea that makes this view compatible with naturalism is that it is, as pointed out above, a *secularized* version of the way Newton and Berkeley thought about the universe. I would say that it is not unique in this respect. Darwin's theory of evolution could also be taken as a secularized version of a creationist story, as long as that story appeals to particular *laws* by which God created various species.

12.3.2 Humean Accounts

The Humean approach to laws of nature conceives of them as supervening on local matters of fact. On this theory, the laws of nature are descriptive, supervening on contingent facts about significant uniformities that obtain in the entire history of the universe.[8] The regularities the laws of nature describe are not necessary connections. Here a worry might be that a god that is external to the universe could nevertheless be the source of the regularities. For instance, occasionalism would be compatible with this Humean picture, and occasionalism is hardly a naturalistic view.

In response, I should point out that, although occasionalism is coherent under this Humean picture, the problem with it is

that it posits something, namely God, that is, in fact, redundant. The Humean picture *does not need* such an entity outside the universe, that is, outside the local matters of fact; those facts are sufficient to account for the laws.

12.3.3 Causal Essentialist Accounts

Proponents of causal/dispositional essentialism take at least some of the intrinsic properties of objects to be causal powers. The laws of nature are descriptions of how such causal powers of objects and integrated systems of the causal powers of objects behave. Some proponents of the view even express their position in terms similar to those of proponents of Humean supervenience. George Molnar, for instance, writes: “Laws of nature supervene on [...] the simple powers of the objects of the world. Powers are the truthmakers for the laws” (→Molnar, 2003, p. 199). Space does not permit us to go into details of causal essentialism, but a problem for my account from this perspective seems to be that it is not laws of nature per se that can account for naturalism, but the fundamental causal powers or systems of interlocked such powers.

I do not strongly disagree with this point. Previously I did express sympathy for essentialism (→Aranyosi, 2010), and I am ready to accept that perhaps in the final analysis it might be that laws of nature are reducible to causal powers. Still it does not change the picture that I have put forward—in particular, not when it comes to my earlier rejection of the causal account of naturalism. The reason is that a blanket causal criterion, without the whole machinery of causal essentialism, would still not ensure the absence of singular and unpredictable causal interference by a divine being.

12.4 Naturalistic Panentheism

Naturalistic panentheism would be the view that the Universe is in God, in some sense of "in," in such a way that the strictures of naturalism are also respected. One can easily see why such a view would be attractive, if it worked. First, naturalism itself seems to be the rule nowadays when it comes to normative requirements regarding one's ontology. Second, as mentioned before, panentheism appears to be a nice midway position between pantheism and theism, which, if coherent, would do justice to both the naturalistic and the spiritualistic inclinations of the contemporary man.

As it happens, though, naturalistic panentheism is not exactly the most straightforward view. Let us start with what I earlier in this essay called "mereological panentheism," according to which the "in" in the slogan "everything in God" is to be understood mereologically, as standing for proper parthood. The problem with this type of panentheism is precisely the idea of there being a *remainder*, as per *Weak Supplementation*, once we subtract the Universe from God. One simple, and ultimately *extensionally correct* alternative formulation of naturalism is that there is nothing over and above the Universe.[9] What follows is that mereological panentheism can't be naturalistic. Indeed, Mark Johnston's recent panentheistic proposal (→Johnston, 2009) has been criticized by →Buckareff (2016) precisely along these lines. Johnston's view is that the Universe *constitutes* God but is not identical to God, where constitution is a *sui generis* relation, which implies material coincidence but is distinct from identity (cf. →Johnston, 1992). Johnston claims this is compatible with naturalism, yet there must be a remainder once we subtract the Universe from God, if he is right about constitution as not being identity.

It is worth digging a bit deeper into the issue of parthood and constitution at this point. Is Johnston committed to *Weak Supplementation* or not? Because if not, then maybe there is no

problem with his mereological panentheism at all since it is based on some nonstandard mereology, which denies *Weak Supplementation*; he wouldn't be the first one to deny it (cf. →Cotnoir, 2021). But, in fact, it is not *Weak Supplementation* that makes the difference here. We should distinguish between an uninterpreted (or most liberally interpreted) notion of parthood and a material/spatial interpretation. Mereologies as purely formal systems apply to both, but when it comes to metaphysical issues it will matter whether your mereology is based on an ultraliberal interpretation or not. The ultraliberal interpretation is found in David Lewis's mereology (→Lewis, 1991, pp. 75–81); he calls it "topic-neutral interpretation." In Johnston's view there must be a fusion of God and Universe, which is The World; this indicates an ultraliberal interpretation of mereology. Yet, it is also true that God and the Universe *coincide*, as per the constitution claim, which means that when it comes to coincidence Johnston must mean *material* coincidence, that is, coincidence assuming a material/spatial interpretation of "part." This is the only way his view could be coherent; yet, it is for this reason that it cannot be naturalistic since the supplement of the Universe in The World must exist and be immaterial/nonspatial.[10]

It looks, therefore, as though mereological panentheism is not congenial to naturalism. What about emergent or dynamic panentheism? Traditionally, most panentheist doctrines fall into this category, and, furthermore, it does seem to be compatible with naturalism. Consider Samuel Alexander's emergentist panentheism. As Thomas (2016) convincingly argues, it is a serious naturalist contender within the set of emergentist theologies, including contemporary ones, such as the one developed by →Clayton (2004a, 2004b). Alexander's magnum opus, *Space, Time, and Deity*, is an attempt to integrate God and divinity (deity) into a purely natural world. The basic idea is that

deity is not yet present in spacetime, but it will *emerge*. We can see here the crucial difference between emergence panentheism and mereological panentheism in the way they explicate the idea of God being partly the Universe but *more* than it. For the mereological panentheist this "more" is conceptually tied to proper parthood, hence, God is supposed to exist *in excess of* the Universe; but for the emergentist it is the completion of a final stage of a natural development of the Universe, hence, God being more than the Universe does not mean God being in excess of the Universe, but deity being the goal or final stage towards which the Universe has a *tendency* or, as Alexander puts it, a *nisus*, to evolve.[11] This nisus does not appear to be supernatural. As Thomas puts it (→Thomas, 2016, p. 262):

> Alexander is convinced that deity will emerge due to a nisus in space-time, which has already 'borne its creatures forward' through matter and life and mind, and will bear them forward still (→Alexander, 1920, ii. 346). Exactly what Alexander means by 'nisus' is unclear. A nisus is a kind of drive or striving, and Alexander seems to believe that the nisus is driving emergence towards higher qualities, not merely different qualities. The nisus provides progress, not merely process. There is no indication that Alexander takes the nisus to be anything other than a natural law or principle operating within his space-time universe; as we saw above, Alexander rejects entirely the thesis that the universe was designed. In itself, there is nothing inherently non-naturalist about the notion of a nisus [...].

If Thomas's interpretation is right, then Alexander's and similar emergentist panentheisms appear to be better placed than mereological panentheisms with respect to the naturalistic constraints. I won't dispute here the claim that emergentist panentheism is congenial to naturalism,[12] but rather the claim that mereological panentheism is doomed to failure when it comes to the requirements of naturalism. Consequently, in the next sections I am going to argue against this apparent failure.

12.5 Big Bang Cosmology and Theism

As I mentioned in my introduction to this essay, the exact physics of the Universe does not get discussed in the context of natural theologies put forward as alternatives to theism, because, I reckon, it is thought that the crux of the issue lies at a higher level of generality, where it is the Universe in general, regardless of how exactly it is structured, that is relevant.

This is wrong. The Big Bang cosmological model, which has been dominating contemporary physics in the last 50 odd years, ever since the discovery in 1965 of the cosmic microwave background radiation, has some conceptual problems that make it less congenial to naturalistic pantheism, and more attractive to theists. The main culprit for this is the notion of a spacetime *singularity*, which is what defines a cosmology as belonging to the Big Bang model. Big Bang cosmology puts forward in its formalism a *mathematical* singularity which is supposed to correspond in physical reality to a point in spacetime where the gravitational field and mass density of the Universe are infinite, or, equivalently, where the space occupied by the Universe is *zero*. The conceptual problem is that such a point is a mathematical fiction; physicist differ as to whether they believe the singularity is a physical reality, a merely mathematical posit, or just a physically paradoxical but ultimately empirically adequate posit.

The spacetime singularity attracts some philosophical attention, especially from people interested in the debates in the philosophy of religion regarding the existence of God. The right interpretation of the Big Bang singularity is, indeed, one of the central issues of contention in the debate between William Lane Craig and Quentin Smith (1993) about whether the Big Bang is more congenial to theism or to atheism.

Craig's main argument for *theism* is that in the Big Bang model, the Universe has a beginning in the past (namely, the *explosion*—the singularity is a mathematical fiction), and since we can't make sense of the Universe coming out of nothing, it must have had a cause, which then Craig thinks it must have been personal, that is, God.

Smith, on the other hand, thinks that it is *atheism* which follows from the Big Bang model. His argument is that the singularity has physical reality and what comes out of it is completely unpredictable, yet, if God created the world, He would create something that would predictably lead to life in the Universe; so, God could not have created the singularity, so God does not exist.

I am deliberately offering only these highly compressed versions of their argument, because my main goal here is not to adjudicate who is right about whether God exists, but to point out that the notion of singularity is highly problematic, and that in the context of trying to elaborate a naturalistic alternative to theism that is not merely atheism, it is this notion that seems to me to constitute the main obstacle. I'm not saying that the theist, like Craig, is *right* about theism being supported by Big Bang cosmology, but that he has a coherent point about the meaningfulness of asking the question "what was the cause of the Big Bang?"; and that question can be asked precisely because of the prediction of the singularity which the Big Bang model is bound to support.

If we try to apply a panentheistic picture to Big Bang cosmology, we will face trouble. If the Universe has as its origin the singularity, then it is finite in the past. Yet, if the Universe is supposed to be part of God, in the mereological sense, then God must precede the existence of the Universe (since God *is* eternal[13]), in which case it is hard to think of the God-Universe

relation otherwise than as a causal one, which is simply the theistic idea of creation.

12.6 Big Bounce Cosmology and Panentheism

Loop quantum cosmology, or the Big Bounce model of the Universe, is a “no-singularity” alternative to the Big Bang model. It is based on quantum gravity, which is an attempt in current physics to unify gravity, quantum mechanics and general relativity. The reason there are no singularities predicted or retrodicted by the theory is that space is taken as discrete, quantized, namely, as having a minimal extent. As we go back in time, we reach a period, just before the big explosion (superinflationary stage) whose effects we experience today, when space and time as such, that is, the Universe, were extremely small. But, importantly, “extremely small” here does not mean “point sized.” It is the stage at which the Universe was contracted to the minimal quantum of space that the theory posits—a scale on the order of a Planck length, approximately 10–35 meters; smaller scales do not exist. Consequently, the Universe had a very large density, but not infinite as in the Big Bang model.

The second crucial component of the Big Bounce model is that the Universe, which currently is in an inflationary phase, had a contracting phase before the big bounce event. Quantum loop cosmologist think that the Universe is *eternal* both towards the past and towards the future.[14] Our observable universe is the result of the big quantum bounce that happened at the end of the contracting phase of the ancient universe. There are plenty of interesting and highly technical issues that physicists are bound to discuss in the coming years regarding the details of this model,[15] but why is or should the model be interesting for non-specialists, like philosophers of religion?

The Big Bounce model holds the promise of a potential physical theoretical and empirical ground for a coherent and straightforwardly *mereological and naturalistic* panentheism. First, the philosophical temptation of theism really comes from two sources—the ontological argument and the cosmological argument. It is the latter that is relevant here for our discussion of panentheism.[16] The cosmological argument's force really depends on whether the Universe has a beginning in time. If loop quantum cosmology is the right model of the Universe, then the cosmological argument does not even arise since the Universe is eternal in the direction of the past.

Second, we do not have the problem of non-natural remainders in this model. Let's call the part of the Universe (or rather *Bi*verse) we are inhabiting U_1 and the pre-bounce one U_2 and their fusion U. The panentheistic view would then amount to saying that God is U. The slogan "Everything in God" would be interpreted as U_2 being a proper part of U. The remainder would then be U_1, the ancient, contracting, pre-bounce part of the Universe—nothing unnatural about it!

Now, of course, one could understandably raise the issue of whether there is anything god-like about U. It *is* a legitimate question but note that the same question ca be raised about *any* universe when it comes to *pantheism*; and panentheism is not that different from pantheism in their problem with justifying the "theism" bit in their name. It is difficult, in other words, to offer an obvious and convincing reason for taking the Universe as being divine or containing divinity intrinsically. In my version of pantheism, logical pantheism, that is, the doctrine of identifying God with Logical Space, the divinity comes from (or can be accessed and admired or *worshiped* in) the grandeur of Logical Space, which is the largest and richest conceivable space. Brian Leftow, for instance, has criticized my view, expressing

skepticism about my idea that one could worship Logical Space; it is worth quoting Leftow extensively, because it offers a quasi-exhaustive picture of why people have a problem with the divinity that pantheists (and hence panentheists, or at least naturalistic mereological ones) want to posit as intrinsic to a non-personal entity such as the Universe (or Logical Space for my earlier view):

> Aranyosi argues that we think an impersonal pantheist deity fails as an object of worship because it does not elicit appropriate emotions. But as he sees it—Sufism being his main example—worship is of two sorts: exoteric, which is merely the following of certain rules, and "esoteric," the worship of "experts" in worship, which is the pursuit of mystical experience. This, he says, is "about a psychological-spiritual state of the subject who worships … not about any cognitive-theoretical construct … the main focus is a state to be reached by the worshipper." The state's content is beyond description; so no description of a deity could be worse or better at articulating its content; and so a pantheist "object" for it is as good as any other. Aranyosi might say, then, that construing worship as talking-to is too bound to exoteric practice; "true worship" is beyond speech.
>
> Well, mystics themselves would be surprised to hear that they aim only at a psychological state. They would say that they aim at union with God, and that a description of God is integral to the pursuit. This is true even of the Sufi prayer Aranyosi instances, which consists largely in recitation of names (canonical descriptions) of God. His other example, Christian Palamism, involves a verbal formula invoking Christ and His mercy. Prayer, whether the Sufi dhikr or the Palamist Jesus prayer, is talking to. So the mystic pursuit Aranyosi sees as "esoteric" worship is no exception to the talking-to account. Again, in the Christian mystical tradition more generally, meditation on the nature of God and the life of Christ plays a central role in leading up to the union experience.
>
> Again, the esoteric/exoteric distinction is foreign to the Western monotheisms apart from Sufism; to the rest of us, what we are doing is grade-A standard worship, which has nothing to do with following rules (save insofar as these preserve good order at worship services) and everything to do with speaking to God. More to the point, while mystics say repeatedly that their experience is beyond description, they also say a good deal about its content: their point is that it is beyond fully adequate

> description. The most basic thing they say, repeatedly, is that they are sure they experience the God of Western theism, not of something else. *They themselves, then, would emphatically deny that a naturalist universe is as good as any other as the object of their quest*. (→Leftow, 2016, pp. 73–74; my emphasis)

I have emphasized the very last sentence of Leftow's, because it shows a misunderstanding on his part when it comes to why I think an entity like Logical Space deserves worship. First, my point was that this entity is *maximal* in so many respects, hence, it deserves our *awe*; and awe is a kind of intellectual worship. I am not saying that any entity would be good enough as on object of worship. I'm saying now the same thing about U, the big Universe/Biverse in the Big Bounce model. U is *awesome*—so it deserves worship. Second, I claim that the only reason we nevertheless are not yet convinced is that intellectual awe is not the kind of attitude that has the full force of traditional worship, in that it lacks the emotional component of the latter. Now, I grant Leftow that he has some good pints about whether my account of mystical experience is right, especially in light of what mystics themselves claim, but, in any case, I do not now consider that part of my view as the most important. The most important part is that certain facts about the Universe are *awesome* (its unity, its complexity, its beauty of organization and function, its laws of nature), hence there is an important sense in which it deserves to be worshiped as God. This is all a pantheist needs.

What about the panentheist? What holds of pantheism regarding the motivation for—and partly regarding the object of—worship will equally hold for panentheism. Going back to our Biverse in quantum loop cosmology, there is a lot of overlap with pantheism, except for one detail: the relevant (sub-)universe in terms of God's *immanence* in the world (which is what pantheism emphasizes) is our observable universe, U1, whereas the relevant (sub-)universe in terms of God's transcendence (which

is what panentheism adds to pantheism) is U2. This makes sense, indeed, because the part of the Big Bounce Biverse that would qualify as a naturalistic interpretation of a theistic immanence claim of ours, like “God is in the world,” should be U1 and not U2 since it is U1 that *we* inhabit. U2, on the other hand, fits very well the panentheist intuition of there still being a proper part of God that is beyond our reach. Indeed, the pre-bounce universe is causally and spatiotemporally separated from the universe we inhabit. The whole Biverse, then, looks like an *awesome* entity that constitutes the ground and support of everything that is relevant to our existence as human beings.

12.7 *Tzimtzum*, Big Bounce, and God’s Location

I would like to end with a few words about a philosophically interesting Jewish theological doctrine, the *tzimtzum*, or divine contraction, which does look like an early speculative anticipation of the idea of a big bounce and of a biverse.

The Lurianic Kabbala introduced the notion of contraction into the Kabbalistic tradition, motivated by some conceptual puzzles related to the idea of God being omnipresent already before a physical universe was created. In a recent article on the coherence of the *tzimtzum*, Samuel Lebens explains what the problem was supposed be, by reference to Luria’s questions regarding divine omnipresence:

> Rabbi Yitzchak Luria (1534–72) poses the following questions: How can there be a world if God is everywhere? Given God’s expansiveness, how could there be any room left over in which a world could exist? If God is ‘all in all,’ how can there be anything that isn’t God? How could God create a world from nothing, if there was no such thing as ‘nothing’ in the first place? (Luria, 2017, p. 162)

Though Lebens dismisses most of these questions as confused, I would like to argue that they do in fact make sense. It is notoriously difficult to explain the apparently simple idea of divine omnipresence, which is a central attribute of God's in standard perfect being theology. In connection with Luria's worries, for instance, Lebens suggests that God' omnipresence not be understood as involving location in space, but rather the power to influence things in any location in space:

> Some of these questions seem confused. There's no reason to think God is actually located in space, such that his omnipresence leaves no room for other things. He can be omnipresent in some derivative sense, without being located in space; so long as his power and/or knowledge extends to all regions. (Luria, 2017, p. 162)

This is a derivative notion of omnipresence rather than a fundamental and literal one (→Inman, 2017, p. 169), in the sense that God's omnipresence is grounded or even reduced to some other entity's or item's omnipresence; God is present at all locations, *l*, in virtue of some entity, *e*, being present at all such locations—in this particular case, this entity, *e*, is God's power or influence. On the alternative model of a non-derivative, or fundamental, omnipresence God is present at all locations simpliciter, without the need of any distinct entity to be so located.

As Inman observes, the derivative notion of omnipresence is problematic in that it makes literal, spatial omnipresence *redundant* as a divine attribute:

> One interesting thing to note in passing here is that it is difficult to see how on a DO model omnipresence is a distinct attribute over and above divine omnipotence, omniscience, omnibenevolence, or a combination of these divine attributes. For this reason, the proponent of a DO model might consider omnipresence to be reducible to or "nothing over and

> above" one of the above attributes or conjunction thereof. (Luria, 2017, p. 172)

This is the reason I think we should adopt the view that omnipresence should be understood *literally*, as God being spatially present everywhere, rather than in some derivative or even metaphorical way. The problem, then, that the doctrine of contraction is an answer to is that God is already everywhere before any creation of a universe takes place, hence, there is no space where a universe could be created. The idea of *tzimtzum* is that God contracts himself in order to allow for a universe to be formed.[17]

The *tzimtzum* is not a good solution for the problem, if one wants to be a *theist* and keep the idea that God is immaterial and simple (Goldschmidt and Lebens, 2020).[18] But it *is* a good solution for a naturalistic pantheist or panentheist, for whom God is a material being wo exists in spacetime by being identical to (or encompassing) the evolving 4D spacetime manifold.[19]

Finally, it is easy to see that not only is the shape of the universe analogous in the *tzimtzum* and in the Big Bounce model, but the conceptual problem to which that shape is a solution is very similar in the two. There are, more exactly, two such similarities. One has already been mentioned in the quote by Lebens: how could the world come out of nothing? Both the Lurianic Kabbalah and the Big Bounce model are based on taking this problem seriously; and as a solution they posit a pre-"creation" universe, separated from our universe by the bounce/*shevirah*.[20]

The second similarity is in the way the *shevirah* is described, which bears an uncanny resemblance to the idea of the quantum nature of space and time in loop quantum cosmology and gravity. Here is, for instance, Joseph Dan explaining the idea

as it appears in the classic interpretations of Luria's doctrines by Gershom Scholem and Isaiah Tishby:

> The analysis presented by Scholem and Trishby is most profound and mythic in character. According to it, when the initial phase, the *zimzum*, was carried out, the empty space was not really empty. It is like when a container is emptied of water; the inside of the container is still wet, with water clinging to its sides. Some divine light remained in the *tehiru*, and this residue, called by the Lurianists, in Aramaic, *reshimu* (impression) included in it some elements of difference and "otherness" that previously were scattered within the infinite Godhead. This was the real purpose of the zimzum: to concentrate and discharge these potentially different entities away from the Godhead, this achieving uniformity and perfection for the rest. (Scholem and Tishby, 2007, p. 76)

There are several analogies with loop quantum cosmology that one can easily draw from this passage. The small quantity of water clinging to the wall if the container is empty is analogous to the discrete, quantum character of space. The previous infinite Godhead is analogous to the pre-bounce infinite universe. The *tzimtzum* is analogous to the contracting part of the Biverse. And the discharge of *reshimu* is analogous to the post-bounce superinflationary phase of our universe.

The resemblance, as I said, is indeed uncanny, but we should keep in mind that we are comparing a purely speculative theology with a somewhat speculative but still scientific model of the universe; so we should not get too enthusiastic about a potential genuine isomorphism between them. Nevertheless, I do believe that there might be some genuine overlap with regard to some of the conceptual/metaphysical questions which motivated these two theories, as radically different in nature and justification as they might be.

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Notes

1 For a summary of Logical Pantheism, see →Aranyosi (2022).

2 See Cotnoir (2021) for a discussion of alleged counterexamples and the source of the disagreements about the principle.

3 The paradigmatic case for emergence has always been the mind-body problem. The mind, or rather some essential property of it (e.g. its intentionality, or its phenomenal aspect, or some other alleged property), is said, by dualists, for instance, to be emergent vis-à-vis the neurological base which grounds it. Yet philosophers who did put forward such views (the classical British emergentists, or contemporary ones, like →Chalmers (1996) or →Crane (2001) also emphasize the nomic connection between mental and physical properties and the existence of fundamental psychophysical *laws*. It is the non-fundamentality of the physical and the fundamentality of the psychophysical that these emergentists insist on, not some alleged *lawlessness* between the mental and the physical.

4 This would be equivalent to what has been dubbed "Alexander's dictum" (though its history goes back to Plato's dialogue *The Sophist*), that is, the slogan "to be is to have causal powers."

5 To unpack this a bit further, it depends, in effect, on how one thinks about causation. If causation is taken as not

requiring (let alone reducible to) regularities (causal laws of nature), then, as I claim above, God, angels, miraculous phenomena, etc., will come out as natural. If, on the other hand, causation is taken in a regularist fashion, then ontological naturalism will coincide with my understanding of naturalism, as it will identify the natural with the causal and the causal with the lawlike.

6 To further unpack this point, we could distinguish, following →Stoljar (2001) two interpretations of what "physical" could mean, and get theory-physicalism and object-physicalism. The former is based on the idea of defining "physical" as whatever is acknowledge by current physics, while the latter is based on defining "physical" as whatever belongs to the fundamental categorical basis of the dispositional properties observed by physics. Theory-physicalism is silent about a potential future discovery of psychophysical laws and whether it would be part of physics or not; this is a subproblem of the more general problem known as Hempel's dilemma (cf. Crane and Mellor, 1990). Object-physicalism I find quite obscure and speculative in a way that makes it not very useful in the context of how to account for the idea of naturalism. It is not clear whether there are such categorical and fundamental "object-physical" properties that would explain the instantiation of both dispositional physical properties and all mental properties (such as intentional and phenomenal). It is pure speculation; hence, it can't really figure as the ground for a potential definition of naturalism, which should not depend on whether such speculations turn out to be true or not.

7 I am indebted to Andrei Buckareff for the objections that I will formulate and respond to in the rest of this section.

8 For discussion, see →Beebee (2000), →Earman and Roberts (2005), →Loewer (1996), and →Buckareff (2019).

9 This is John Heil's way of formulating it (→Heil, 2012, p. 1). I say "it is extensionally correct" meaning it is *merely* extensionally correct! More to the point, as it stands, the definition picks out the right thing regardless of whether dualism or physicalism is true of this Universe, if by "Universe" we do not mean the *universe of discourse* but the *natural universe*. This, of course, brings about circularity since the term "naturalism" is defined in terms of a "natural universe." The universe of discourse, on the other hand, would include literally *everything* (that exists), hence not serve at all the purpose of circumscribing naturalism and hence distinguish it from non-naturalism.

10 This is a general problem with the constitution view, namely, the problem of what grounds the distinctness of two materially coincident objects; it is not a problem for non-naturalists, but it is one for those who want to be naturalists since everything that's material about the two objects is shared by them, so what is left to distinguish them, like, for instance, *de re* modal properties, does not seem to be material (cf. →Bennett, 2004; →Paul, 2002).

11 I should note here that Thomas rejects the popular view that Alexander was a panentheist. Her main argument is that, unlike, for instance, Lloyd Morgan, Alexander did

not think that God is beyond the Universe, but rather that God is a proper part of the Universe, which is, as Thomas put it "a theology *for which there is as yet no label at all*" (emphasis in the original). My explanation of why we disagree on this point is that Tomas thinks strictly in terms of mereological notions when considering the notion of "being in," whereas I have distinguished that notion from what a called "the emergence notion."

12 First, the idea of God or deity as an emergent quality out of a developing universe has, at least the way I see it, the flavor of non-naturalistic historical idealism á la Hegel or Bradley; indeed, it is well known that Alexander was influenced by Bradley, whose philosophy was influenced by Hegelian dialectic. Second, and more importantly, it is unclear, if we look at most of what has been written in the field of physical eschatology, whether the end times of the Universe have anything as interesting as to qualify as the completion of deity or divinity. Indeed, *heat death*, a consequence of the Second Law of Thermodynamics coupled with the inflationary cosmological model regarding the distant future of the Universe, looks rather the *opposite* of the glorious fulfillment of divinity. For an exhaustive annotated bibliography on physical eschatology, up to 2002, see →Ćirković (2003).

13 I am, sure enough, not claiming that all the theistic God's qualities should be accommodated by a panentheist. Some of them will be rejected as a matter of definition and understanding of panentheism as an alternative to theism; for instance, *personhood* and

omnibenevolence. Others will at most be accommodated as metaphorically interpreted; for instance, *omnipotence* and *omniscience*. Yet, some will be unproblematic and will have to be accommodated even by alternatives to theism; for instance, *eternity, omnipresence*, and *immutability*.

14 In the early history of the model, the big bounce was considered a stage in a series of contractions and expansions, known as the *oscillatory* model of the universe. Nowadays, a non-oscillatory view is more popular, according to which there is just one bounce.

15 Physicists should consult Ashtekar and Singh (2011) for an exhaustive article on the state-of-the-art in quantum loop cosmology.

16 Though see Aranyosi 2013 for why the former is relevant too, in general.

17 →Lebens (2017) offers an alternative view, which he calls “Hasidic idealism,” according to which the *tzimtzum* is a fiction, to be analyzed along the lines of how fictional discourse is accounted for in philosophy of language and metaphysics. See also →Lebens (2015).

18 Immateriality and divine simplicity are core doctrines of both Christian and Judaic theism.

19 Following Josh Parson’s distinction (Parson, 2007), and by analogy to endurance and perdurance in the persistence literature (→Lewis, 1986, p. 202), we could think of God as either “entended” or “pertended.” Pertension is what most people normally mean by

“extension in space,” that is, an object occupying place, *p*, by each of the subspaces of *p* being occupied by a proper part that object. Entension, on the other hand, is weird; an object entends a place *p* by being wholly present at each of the subspaces of *p*. Theists have a problem with both these ways for God to be omnipresent (→Hudson, 2009, p. 211, and →Inman, 2017)—they turn God into a material thing. But pantheists and panentheist are perfectly fine with both.

20 *Shevirat ha-kelim* is translated as “breaking of the vessels,” and it is the Lurianic term for the reconstruction of the universe after the *tzimtzum*. More on this and the *tzimtzum* below.

13 The Problem of Creation *Ex Nihilo*: A New Argument against Classical Theism

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Abstract: It's constitutive of classical theism that there is a necessarily existent personal god who is also the creator of the universe, where the latter claim includes at least the following three theses: (i) God is wholly distinct from the natural world; (ii) God is the originating or sustaining cause of the natural world; and (iii) God created the natural world *ex nihilo*, i.e., without the use of pre-existing materials. Call this tripartite component of classical theism *the classical view of creation*. In this chapter, I offer a new argument against classical theism. In particular, I argue that creation *ex nihilo* is *prima facie* impossible, and that since the doctrine of creation *ex nihilo* is constitutive of classical theism, classical theism is false.

13.1 Introduction

Call *classical theism* the view that there is a necessarily existent personal god who is omnipotent, omniscient, and morally perfect, and call *the classical view of creation* the view that consists of the following three theses: (i) God is wholly distinct from the natural world: the world is not identical to God or made from the stuff of God's being. Nor is it an idea in the mind of God or a mere feature or mode of God's being. Rather, it is a concrete object that exists in its own right (or an aggregate of such[1]); (ii) God is the originating or sustaining cause of the natural world; and (iii) God created the natural world *ex nihilo*, i.e., without the use of pre-existing materials. Finally, call *classical*

theism$_{cvc}$ any version of classical theism that includes the classical view of creation.[2] In this chapter, I offer a new argument against classical theism$_{cvc}$. In particular, I shall argue that creation *ex nihilo* is *prima facie* impossible, and that since the classical view of creation is constitutive of classical theism$_{cvc}$, classical theism$_{cvc}$ is false.

13.2 The Argument

The problem of creation *ex nihilo* can be expressed in terms of the following argument:

1. All concrete objects that have an originating or sustaining efficient cause have an originating or sustaining material cause, respectively.
2. If classical theism$_{cvc}$ is true, then the universe is a concrete object that has an originating or sustaining efficient cause with neither an originating nor a sustaining material cause.
3. Therefore, classical theism$_{cvc}$ is false.

The argument is valid, and so the conclusion follows from the premises of necessity. What, then, can be said on behalf of the premises?

Premise 1 expresses a causal principle, which I shall call the *Principle of Material Causality*, or PMC for short. In simple terms, PMC says that all *made* things are made *from* other things. A bit more carefully, it says that concrete objects have an originating or sustaining material cause whenever they have an originating or sustaining efficient cause, respectively. Before I defend the premise, some preliminary remarks about terminology are in order.

First, *concrete object* denotes at least the sorts of entities classically individuated by the ontological category of *substance*, and is meant to distinguish the entities at issue from those of other ontological categories (e.g., properties, relations, events, tropes, and the like). Examples of substances or individuals thus include atoms, stars, rocks, planets, trees, animals, people, and (if such there be) angels, Cartesian souls, and gods. They are thus to be distinguished from concrete entities in other ontological categories (shapes, surfaces, events, and the like) and abstract objects (propositions, numbers, sets, and the like).[3]

The next two key terms in premise 1 are those of *originating cause* and *sustaining cause*. By the former, I mean a cause of the temporal beginning of a thing's existence[4] (if it should have such), and by the latter, I mean a cause of a thing's continued existence. So, for example, matches and lighter fluid are at least partial originating causes of the existence of a flame, and the oxygen that surrounds it is at least a partial sustaining cause of the flame's existence.

Finally, *material cause* aims to capture (roughly) Aristotle's notion of the term, and to individuate the type of cause in play from the other three sorts of causes distinguished by Aristotle, viz., formal, efficient, and final causes. In particular, by *material cause*, I mean the temporally or ontologically prior things or stuff *from* which (though not necessarily *of* which) a thing is made. So, for example, the originating material cause of a shiny new penny is the parcel of copper from which it was made; the originating material causes of a new water molecule are the hydrogen and oxygen atoms from which it was made; and the sustaining material causes of a flame are the reacting gases and solids from which it is made.

Two points about the causal premise merit special emphasis. First, PMC is restricted to concrete objects as we've defined them. As such, it is *neutral* as to whether entities in *other*

ontological categories require a material cause. Second, the requirement of a material cause is restricted further to just those concrete objects that have an originating or sustaining efficient cause. It therefore allows for the possibility of concrete objects that *lack* a material cause, namely, those that lack an originating or sustaining efficient cause. So, for example, the premise allows that the universe may lack a material cause of its existence if it is both beginningless and also lacks a sustaining efficient cause. It also allows that a universe with a temporal beginning may lack a material cause if it also lacks an originating and sustaining efficient cause. An example of the latter sort of case might be a temporally finite, four-dimensional "block" universe. As such, the causal premise is neutral as to whether all concrete objects begin to exist, and to whether all concrete objects that begin to exist have a material cause. The causal premise *only* rules out concrete objects that *have* an originating or sustaining *efficient* cause, but *lack* a *material* cause.

Is PMC plausible? It certainly seems so. First, PMC enjoys abundant empirical support. This is perhaps most clearly seen in the case of the extremely well-confirmed law of the conservation of mass/energy.[5] The law states that if there is a given quantity of mass/energy at a given time, then it must have been caused by exactly the same quantity of mass/energy at any earlier time. In general, though, our uniform experience is such that whenever we find a concrete object with an originating or sustaining efficient cause, we also find it to have an originating or sustaining material cause, respectively. Furthermore, there seem to be no clear counterexamples to the principle in our experience. What explains this? PMC is a simple, conservative hypothesis with wide explanatory scope, which, if true, would best explain this data. Experience thus provides significant abductive support for PMC.[6]

Second, consider a version of PMC with stronger modal force:

> *Strong PMC*: Necessarily, all concrete objects that have an originating or sustaining efficient cause have an originating or sustaining material cause, respectively.

Strong PMC is supported by rational intuition or rational seemings. In particular, Strong PMC appears true on reflection, where the notion of reflection at issue is broad enough to include thought experiments or intuition pumps. Rational intuition and rational seemings have standardly been taken as evidence of metaphysically necessary truths.[7] Perhaps such intuitions aren't enough to *demonstrate* the impossibility of an originating cause without a material cause, but we ordinarily take such seemings to be at least defeasible, *prima facie* evidence for what must be the case. Therefore, if rational intuition supports Strong PMC, then since Strong PMC entails the modally weaker PMC, then rational intuition thereby provides at least *prima facie* support for PMC as well.

To prime our intuitions in support of Strong PMC, consider the following two thought experiments from Wes Morriston (2002, p. 29):

> *No Materials:* There was no lumber, no nails, no bricks, no mortar, no building materials of any kind. But there was a builder. One day, the builder said, "Five, four, three, two, one: let there be a house!" And there was a house.
>
> *No Builder:* There was no builder, but there was lumber, nails, bricks, mortar, and other necessary building materials. One day, these materials spontaneously organized themselves into the shape of a house.

No Materials depicts a scenario of an efficient cause without a material cause, while No Builder depicts a scenario of a material

cause without an efficient cause. However, there seems to be epistemic parity between the two stories: both scenarios are equally unintuitive and contrary to experience. Consistency therefore seems to require that one reject the possibility of both cases just in case one rejects either one (→Morriston, 2002, pp. 29–30). But the theist is likely to reject the possibility of No Builder. Therefore, there is significant epistemic pressure for the theist to reject the possibility of No Materials.

A similar intuition obtains when we consider any other concrete object arising from an originating efficient cause without an originating material cause: absent materials internal or external to the builder from which to create things, it seems that even the most strenuous attempt can only result in creative "dry heaves," as it were. Furthermore, the intuition doesn't seem to depend on whether we take the builder to have limited power: saying that an omnipotent builder can create a house without pre-existing materials seems to be on an epistemic par with saying that an omnipotent being can throw up a missed lunch from a completely empty stomach with a sufficiently strenuous dry heave.

We have similar grounds for thinking that concrete objects that depend upon a *sustaining* efficient cause for their existence also have a sustaining material cause. So, for example, the continued existence of a flame depends upon an efficient sustaining cause for its continued existence. But here we find that the flame's sustenance also crucially involves a sustaining material cause, viz., reacting gases and solids. Furthermore, our intuitions in support of the causal principle are triggered when we attempt to imagine the flame's continued existence *without* the presence of reacting gases, solids or some other material cause; the intuition does not diminish when we imagine the flame to be past-eternal.

In addition, it doesn't require a full-blown Aristotelian metaphysical system to find the materials for a compelling thought experiment for PMC in the apparent relationship between what actually exists and what merely has the potential to exist. Thus, the possibility of the origination or sustenance of an object requires the prior potential for its existence. But it seems that this potential must "reside" in some actually existing thing or stuff. So, for example, the potential existence of a penny "resides" in a parcel of copper. By contrast, nothingness lacks the capacity or potential for becoming anything, since nothingness, being nothing at all, has no capacities or properties whatsoever. *Prima facie*, then, concrete objects can't come into being from nothing, but only from other concrete things or stuff.

Finally, the same conclusion can be gotten from an extremely weak version of PMC:

> *Weak PMC:* Possibly, every concrete object that has an originating or sustaining efficient cause has an originating or sustaining material cause, respectively.

In simple terms, Weak PMC says that it is *possible* that all *made* things are made *from* other things. A bit more carefully, it says that there is *at least one possible world* in which all concrete individuals and stuffs that are made are made from or out of other concrete individuals or stuffs.

Now my own view is of course that a much stronger version of PMC is true—viz., that it holds of metaphysical necessity. But we've also seen that there are strong grounds for thinking PMC holds in at least the *actual* world: it's intuitive, it has no uncontroversial exceptions, and it's encoded in the well-confirmed conservation laws of physics. *A fortiori*, then, there is intuitive evidence to warrant the claim that there is *at least one possible world W* in which such a principle is non-vacuously true.

But if so, then in *W*, some things or stuffs are made, and all made things or stuffs are made from other things or stuff. And if so, then no things or stuffs in *W* that are made are made *ex nihilo*, in which case no god or gods made them *ex nihilo* in *W*. But on classical theism$_{cvc}$, for any world that contains concrete things or stuffs distinct from God, at least some of those things or stuffs were made *ex nihilo*. It follows that the god of classical theism$_{cvc}$ doesn't exist in *W*. But if so, then by (i) the fact that classical theism$_{cvc}$ entails that God is a metaphysically necessary being, and (ii) Axiom S5 of S5 modal logic, it follows that such a God doesn't exist in *any* world, and therefore, *a fortiori*, such a God doesn't exist in the *actual* world. Therefore, the same conclusion can be gotten from even a very weak version of PMC.[8]

All that remains is to defend premise 2. Why should we accept it? Premise 2 follows from our partial stipulative definition of "classical theism$_{cvc}$." It is therefore a conceptual truth. This causes no trouble for the argument's significance, for the partial definition captures several theses that are *prima facie* constitutive of classical theism. Such theses are among those that individuate theism from neighboring views about God, such as pantheism, panentheism, demiurgic theism, Berkeleyan idealism, and Spinozistic monism. They are also among the theses about God that have been accepted and defended by most prominent philosophers within the theistic tradition, including Augustine, Anselm, Aquinas, (and, more recently) William Alston, Alvin Plantinga, and Richard Swinburne. Indeed, it is safe to say that most contemporary analytic philosophers, both inside and outside the theistic tradition, would consider them essential theses of any account of classical theism worthy of the name. I therefore think it's safe to say that the stipulative truth of the premise won't undermine its relevance for evaluating classical theism's epistemic merits.

We've seen that the argument is valid. We've also seen that premise 1—the principle of material causality—is well supported from both *a priori* and empirical sources, and that premise 2 is a conceptual truth. It therefore looks as though the argument poses a formidable problem for classical theism$_{cvc}$.

How might the classical theist$_{cvc}$ respond? Premise 2 is non-negotiable, as we've seen that it is a stipulative, conceptual truth. That leaves open only two types of response for the classical theist$_{cvc}$: a non-concessive response, which would require providing a defeater for premise 1, and a concessive response, which grants the soundness of the argument, but rejects classical theism$_{cvc}$. In the next two sections, I will consider and criticize both sorts of response. Briefly, I will argue that (i) the non-concessive responses are unsuccessful, that (ii) most of the concessive responses require rejecting classical theism, and that (iii) the remaining theism-friendly concessive responses are of dubious religious significance and epistemic merit.

13.3 Non-Concessive Responses

As mentioned above, the only sort of non-concessive response to the argument is to provide principled grounds for rejecting or resisting premise 1. This would require principled grounds for thinking the principle of material causality is false or without adequate justification—that is, to undercut or rebut the claim that all concrete objects that have an originating or sustaining efficient cause have an originating or sustaining material cause of their existence, respectively. I can think of seven ways in which one might attempt to do so, which I shall consider below.

First, one might appeal to God's omnipotence as a way of defeating premise 1. Thus, one might argue that omnipotence is also constitutive of classical theism$_{cvc}$, that omnipotence entails

the ability to do anything metaphysically possible, and thus that God can create the universe *ex nihilo*. The problem is that the argument is missing a crucial premise, viz., that creation of the universe *ex nihilo* is metaphysically possible. But we've already seen that there are burden-shifting epistemic grounds against it. As such, even when the enthymemic premise is added to avoid invalidity, the objection begs the question at issue.

Second, one might reply that it's conceivable that a god creates the universe *ex nihilo*, that conceivability is *prima facie* evidence for metaphysical possibility, and thus that divine creation *ex nihilo* is metaphysically possible. However, this line of reasoning would seem to prove too much. For by the same token, one could argue that it's conceivable that a universe pops into existence *ex nihilo* without any cause *whatsoever*, that conceivability is *prima facie* evidence for metaphysical possibility, and thus that a universe popping into existence *ex nihilo* without any cause whatsoever is likewise metaphysically possible. But then we have a new argument against classical theism$_{cvc}$. For it's also constitutive of classical theism$_{cvc}$ that for any possible world, if there is a universe distinct from God in that world, then God created it. Therefore, the conceivability of an uncreated world that pops into existence *ex nihilo* without any cause whatsoever provides equally compelling grounds against classical theism$_{cvc}$ as the *prima facie* impossibility of creation *ex nihilo*.

A third response is related to the second and goes back to Hume. To get at the response, consider the following gloss on Strong PMC:

> *Strong PMC*: Necessarily, no concrete object is originated or sustained by an efficient cause without a material cause.

We've seen that Strong PMC is intuitive and that the actual world conforms to it with no clear counterexamples. However, despite these considerations, some philosophers remain skeptical. Some philosophers in this camp agree with Hume that the only necessary truths are analytic truths, and that no proposition that can be denied without contradiction is analytic. Others in this camp might stop short of this but hold that anything that can be conceived without contradiction is at least epistemically possible. Philosophers in this camp might therefore argue that since there is no contradiction in denying Strong PMC, or since one can consistently conceive of God creating the universe out of nothing, that is enough to defeat Strong PMC.

At least two things can be said in reply to this objection. First, at best, it would be grounds for rejecting Strong PMC (for those with the particular Humean bent at issue), which is just one of the two lines of support offered for PMC. As such, it leaves the abductive support for PMC unscathed.

Second, and more importantly, rejecting Strong PMC is just as problematic for classical theism$_{cvc}$ as accepting it. To see this, consider the following principle, which I'll call *the Strong Principle of No Uncaused Concrete Objects* (Strong NUC):

> *Strong NUC*: Necessarily, no concrete object originates out of nothing without a cause.

Strong NUC is just an instance of the more general principle, *ex nihilo nihil fit*. And as far as widely accepted metaphysical principles go, the latter is about as good as it gets. For not only is it intuitive, but the actual world conforms to it with no clear counterexamples. However, since Strong NUC can be denied without contradiction, and since one can consistently conceive of (for example) a quark—or even the whole universe—popping into existence uncaused out of nothing, those with the same

Humean bent as those mentioned above should think that that's enough to reject the principle.

Strong NUC therefore looks to be on an epistemic par with Strong PMC: Both principles are intuitive, and that the actual world conforms to both with no clear counterexamples. However, neither principle is analytic, and so one can deploy the Humean gambit above to resist them if one is so inclined. Given that Strong NUC and Strong PMC are in the same epistemological boat, therefore, it seems unprincipled and arbitrarily selective to accept one while rejecting the other. It therefore looks as though one should treat them similarly: either accept both or use the Humean gambit to reject both.

Here's the rub. Both options entail a defeater for classical theism$_{cvc}$. For consider the first option: accept both principles. If you do that, then you accept Strong PMC, in which case you accept something that entails that God can't create concrete objects *ex nihilo*, in which case you accept something that entails that classical theism$_{cvc}$ is false. On the other hand, suppose you reject both principles. Then you reject Strong NUC, in which case you accept that there is a metaphysically possible world at which concrete objects pop into existence uncaused, out of nothing. But since classical theism$_{cvc}$ entails that God is the creator or sustainer of all concrete objects outside himself in all possible worlds in which he exists, you thereby accept something that entails the falsity of classical theism$_{cvc}$. Therefore, either way, you accept something that entails the falsity of classical theism$_{cvc}$.[9] In short, rejecting Strong PMC is just a problematic for classical theism$_{cvc}$ as accepting it.

A fourth sort of response one might raise against premise 1 is one asserted by some in the field of quantum cosmology, viz., that there are plausible models of the origin of the universe according to which the universe arose from nothing without any

cause whatsoever.[10] One might reasonably worry that upon closer inspection, the claim supported by the scientific evidence is not that the universe popped into existence *ex nihilo*, but rather much weaker claims, such as that it arose from a random fluctuation in a quantum vacuum.[11] I will not pursue this worry here, however. For, strictly speaking, the possibility of concrete objects popping into existence *ex nihilo* without any cause whatsoever is *compatible* with PMC. For PMC does not require that all concrete objects have an originating and sustaining material cause. Rather, it only requires that they do if they have an originating or sustaining *efficient* cause. In this regard, PMC treats universes that pop into existence out of nothing without an efficient case as on a par with past-eternal universes and four-dimensional block universes.

In any case, and perhaps most saliently for our purposes, the truth of the current objection would provide no relief for the classical theist$_{CVC}$. For such evidence would likewise seem to provide a defeater for classical theism$_{CVC}$ as well. For, again, classical theism$_{CVC}$ entails that for any world in which there are concrete objects distinct from God, God created them. But if the present objection is correct, then there are possible worlds where concrete objects pop into existence without a cause, in which case God does not create them at that world, in which case classical theism$_{CVC}$ is false.

Fifth, the classical theist$_{CVC}$ might resist premise 1 by appeal to agent causal views of the self. Thus, they might argue that there are good reasons to think that (i) humans possess libertarian free will, that (ii) this is best explained on the assumption that the physical realm isn't causally closed, that (iii) the agent can thus cause things via energy from "outside" the natural causal order,[12] and that (iv) this is sufficient justification for the existence of genuine creation *ex nihilo*, in which case

premise 1 is false. This reply won't work, however. For even if (i)–(iii) could be adequately supported—contrary to the opinion of the majority of analytic philosophers[13]—the falsity of the causal closure of the physical wouldn't require positing the creation of concrete objects *ex nihilo*. Rather, at most, it would require the transfer of pre-existing energy from the agent (who acts from "outside" of the natural causal order) to the physical realm.

Sixth, one might object that (i) our intuitions and experience regarding material causes have been conditioned by our experience of causation *within* the physical universe; (ii) the case of the origin of the physical universe *itself* is quite different from such cases; and therefore that (iii) such evidence is insufficient to support PMC when applied to the origin of the universe. But this objection is of little help to the classical theist$_{CVC}$. For if it's sufficient to undercut our intuitive and empirical evidence for the requirement of a *material* cause, then it also seems sufficient to undercut our intuitive and empirical evidence for the requirement of an *efficient* cause. But then we have an equally powerful defeater for classical theism$_{CVC}$. For as we've seen in our response to previous objections, it's likewise constitutive of classical theism$_{CVC}$ that for any possible world in which God exists, if there is a universe distinct from God in that world, then God is its efficient cause. Therefore, the epistemic possibility of a universe that pops into existence *ex nihilo* without an efficient cause provides equally persuasive grounds against classical theism$_{CVC}$ as the *prima facie* impossibility of creation *ex nihilo*.

Finally, one might reject premise 1 via an appeal to theoretical cost-benefit analysis. In particular, one might argue that while denying PMC is a theoretical cost for classical theism$_{CVC}$, it can compensate for that cost if it turns out that classical theism$_{CVC}$ embodies the theoretical virtues (e.g., simplicity, scope, conservatism, etc.) better than other

competing hypotheses (e.g., naturalism, cosmopsychism, pantheism, panentheism, deism, demiurgism, etc.). And if that should turn out be so, the classical theist$_{CVC}$ would then be warranted in rejecting PMC in favor of a *qualified* version of it—say, one that asserts that all things with an efficient cause *besides* the creation of the universe require a material cause.

It is beyond the scope of this paper to assess whether classical theism$_{CVC}$ wins out over competing large-scale hypotheses in terms of a comprehensive theoretical cost-benefit analysis.[14] But for the purposes of this paper, it is enough to note that to respond in this way is just to grant that the argument is an undefeated defeater for classical theism$_{CVC}$ unless or until it can be shown that the explanatory merits of the latter warrant rejecting PMC. But that is all that the argument aims to do.

13.4 Concessive Responses

If one finds the non-concessive responses unpersusasive, one might finally turn to a concessive response; that is, one might accept a view of God that denies the classical view of creation. There are three basic versions of such a response, each one corresponding to a rejection of one of the three clauses of the classical view of creation as we have defined it. I will briefly consider each sort of response below.

The first type of concessive response is to reject thesis (i) of the classical view of creation as we've defined it, thereby denying that God is wholly distinct from the natural world. According to this sort of response, one allows that the world is either (a) identical to God, (b) made from the stuff of God's being, (c) a mere feature or mode of God's being, or (d) an idea in the mind of God. Unfortunately, options (a)–(c) come at the high cost of abandoning classical theism altogether, as embracing one of

these options amounts to embracing something in the neighborhood of pantheism, panentheism, or Spinozistic monism, respectively. And while (d) is arguably a version of classical theism, it requires embracing something on the order of Berkeleyan idealism. It would take us too far afield to evaluate the case for such a view, but it's enough for our purposes to note that few have found the case for Berkeleyan idealism persuasive, in which case it seems unlikely that many will accept a concessive response of this sort.

The second type of concessive response is to reject thesis (ii) of the classical view of creation. According to this sort of response, God may or may not be omnipotent,[15] omniscient, and morally perfect. However, he does not play the role of creator of the universe in any sense. Call this sort of view *hands-off theism*.

It seems antecedently unlikely that there will be many takers for hands-off theism. This is for at least two reasons. First, hands-off theism looks epistemically unmotivated. For many standard lines of evidence for theism depend upon inferences from the natural world to God. Examples include design arguments from the fine-tuning of the universe to a cosmic designer, as well as cosmological arguments for a first cause, ground of being, and sufficient reason for the existence of contingent concrete reality. But the god of hands-off theism plays none of these roles with respect to the universe. As such, accepting hands-off theism commits one to rejecting many of the core arguments of natural theology. It's also arguable that it causes trouble for religious experience. For example, in his widely-influential account of the evidential force of religious experience, William →Alston (1993) grants that awareness of certain phenomena (e.g., religious diversity) can undercut a good deal of the epistemic force of religious experience, and thus that the justification of theistic belief requires further buttressing with the help of other evidence, such as

cosmological and design arguments. But again, the hands-off theist is committed to rejecting many such buttressing arguments as evidence for her belief. Now perhaps an adequate case for hands-off theism can be made that does not depend on these lines of evidence. But for our purposes, it's enough to note that the hands-off theist seems to have their work cut out for them.

Second, the god of hands-off theism seems to be of much less religious significance than the God of classical theism$_{cvc}$. For on such a view, God is not responsible for the existence and order of the natural world. He is therefore not the cosmic architect, first cause, or ground of being upon which all else depends, let alone Anselm's greatest conceivable being. Rather, he exists as just one among the many uncreated concrete objects within the universe, having roughly the same creative and providential control as Plato's demiurge. On such a view, then, the grandeur, preeminence, and otherness of God would seem to be non-trivially diminished. There is also a corresponding loss of the awe that comes from a sense of ultimate dependence upon a Creator on such a view.

The third and final type of concessive response is to reject thesis (iii) of the classical view of creation. According to this sort of response, God plays the role of creator and designer. However, he did not create the world out of nothing, but rather from pre-existing materials. For obvious reasons, let's call this sort of view *demiurgic theism*. Now it seems that demiurgic theism is an improvement over hands-off theism in terms of epistemic motivation, since on such a view god plays at least a diminished role *qua* creator and designer of the universe, fashioning the universe out of primordial matter/stuff. Therefore, while standard cosmological arguments cannot be marshaled in support of demiurgic theism,[16] perhaps an argument from design can be made on behalf of such a view, as

well as (perhaps) religious experience and other sorts of evidence that don't appeal to an inference from the sheer existence of the universe to God.

However, whatever the epistemic merits of demiurgic theism, many of the drawbacks of hands-off theism apply here as well with respect to religious significance. For as with the latter view, the god of demiurgic theism is not the preeminent first cause and ground of being for the fundamental stuff of the natural world, nor does he provide the sufficient reason for its existence. He is therefore not responsible for the existence of the natural world in the ultimate sense that is ascribed to the god of classical theism$_{cvc}$. Rather, he exists alongside of it as another uncreated concrete object. Therefore, such a view entails a much weaker view of divine preeminence and uniqueness than what has been historically attributed to the God of classical theism. For at least these reasons, then, it is doubtful that many classical theists will be happy with this route to resisting the argument.

13.5 Conclusion

A powerful argument against classical theism can be constructed from two simple elements: (i) classical theism's doctrine of creation *ex nihilo*, and (ii) the well-supported principle of material causality. The prospects for the only non-concessive reply to the argument—rejecting the principle of material causality—seem less than promising. Furthermore, the concessive replies leave the classical theist with a picture of God and creation that they are likely to find costly and unattractive. Little attention has been paid to the argument to date, but our brief exploration suggests that it is worthy of serious investigation.[17]

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Notes

1 Henceforth I will leave this qualification unstated but assumed.

2 Berkeleyan idealism arguably fails to satisfy clause (i) of the classical view of creation as defined here. As such, while it may count as a version of classical theism, it fails to count as a version of classical theism$_{CVC}$.

3 Many philosophers accept a *thing ontology*, where all stuffs (e.g., water, bronze, wood, beer, etc.) reduce to primary substances or individuals. However, some accept a *stuff ontology*, according to which at least some stuffs are not reducible to primary substances or individuals. For those who accept a stuff ontology, the notion of concrete object in play should be construed to include stuffs as well.

4 Consider any arbitrary carving up of the stretch of time of a given object's existence into equal intervals of finite, non-zero duration, and ordered according to the "earlier than" relation. As I'm using the expression, an object has a temporal beginning of its existence just in case any such carving up includes an earliest temporal interval.

5 Thanks to Michael Tooley for discussion. See also →Fales, 2010, esp. pp. 24–27.

6 For those who remain unpersuaded by the case for premise 1, I will argue later that the same conclusion can be gotten whether one accepts the premise or not.

7 For recent defenses of rational intuitions and intellectual seemings as evidence, see, e.g., →Huemer, 2007; →Tucker, 2013; →Chudnoff, 2014; and →Bengson, 2015.

8 The primary aim of the present chapter is of course to defend an argument against classical theism$_{cvc}$ that relies on a stronger version of PMC. However, for those who read the present chapter and remain unpersuaded by the argument from the stronger version of PMC, I refer them to the argument from Weak PMC sketched above.

9 A version of this dilemma can be given in terms of undercutting defeaters for each horn of the dilemma (rather than rebutting defeaters) by swapping out talk of metaphysical possibilities with talk of epistemic possibilities, which I leave as an exercise for the reader.

10 See, for example, →Krauss, 2013.

11 See, for example, →Albert, 2012.

12 An argument in this vicinity is broached in (e.g.) Moreland, 2013.

13 According to a recent poll (→http://philpapers.org/surveys/results.pl, last accessed July 25, 2023), only 13.7% of philosophers answered with "accepting or leaning toward libertarianism." At least 71.3% of philosophers polled stated that they think we don't have libertarian free will, with 59.1% answering with "accepting or leaning toward compatibilism," and 12.2% answering with "accepting or

leaning toward no free will" (14.9% answered with "accepting or leaning toward 'other'").

14 For a case for theism on this score, see →Swinburne, 2004. For cases against theism on this score, see →Oppy, 2013, and Leon in Rasmussen and Leon, 2019.

15 Perhaps one will object that a god of this sort cannot be omnipotent if he cannot create or sustain the natural world. However, there is a long and established tradition of theists who claim that there are lots of things that an omnipotent god cannot do (e.g., make a round square, change the past, act contrary to his nature, know future free acts, etc.). One standard justification for such restrictions is to say that such things are metaphysically impossible, and that omnipotence does not include the power to do what is metaphysically impossible. But the same sort of justification seems available here. For the hands-off theist can say that creating or sustaining the universe without a material cause is metaphysically impossible, and thus his inability to create or sustain a universe *ex nihilo* does not count against his omnipotence.

16 I suppose a cosmological argument for an *unmoved mover* is still a possibility, although few have found arguments of this sort convincing since at least the dawn of Newtonian physics. Perhaps, though, the argument can be revitalized. We'll see.

17 Thanks to Savannah Leon, Michael Tooley, Dustin Crummett, Randy Ramal, Bob Fischer, Scott Hill, Wes Morriston, Robert Bass, Jason Thibodeau, and two anonymous referees for helpful comments and

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14 Natural Sciences as Constraints on Ideas about the Divine

Willem B. Drees

Abstract: The sciences are very successful. The Periodic Table of Elements covers all of material existence. There is coherence across levels of reality. However, there are limit questions that cannot be answered by science, and worldviews are underdetermined by theories and data. Furthermore, our human practices and judgments are categorically different in kind. What are the implications of the natural sciences for views of reality and of the divine? It is argued that the sciences provide constraints on philosophical theology; much is excluded. But the sciences do not imply a single worldview or theology. A theist can emphasize divine transcendence, envisaging God as the creator of all of natural reality. A religious naturalist may appreciate natural reality as a reality with sacred depth. Different 'ontologies of the divine' may be consistent with the sciences.

14.1 Introduction

Though this volume is about the divine, this contribution focuses on our knowledge of 'all else,' of the universe and our own bodily existence. What does it imply for possible views of reality and of the divine if we depart from the natural sciences as our prime source of knowledge?

We will begin with a characteristic example of successful science: the Periodic Table of Elements. The insights it represents cover all of material existence. Chemistry is relevant to biology and other 'higher' disciplines, while chemical

properties can be explained by physics. There is coherence across levels of reality.

The sciences provide answers to some questions, by collecting data, making models, and formulating theories. Answers may give rise to further questions. Some of those are passed on to other disciplines, e.g. from chemistry to physics, from geology to cosmology. At the outer limits of science, we encounter philosophical questions that may be raised in the context of science but cannot be answered by science. Furthermore, scientific theories are underdetermined by data, and worldviews are underdetermined by theories, leaving room for multiple metaphysical and epistemological interpretations. And though we, humans, as natural beings can be understood by the sciences, in our scientific knowledge we assume that we can step outside the limitations of our natural existence. And as agents we bring to our deliberations considerations that need a richer vocabulary. Moral judgments cannot be justified by reference to the natural sciences alone; values are categorically different from the descriptive and explanatory insights provided by the sciences.

What might be consequences of the success of science, but also of the unavoidability of limit questions, of underdetermination and of categorical distinctions for our thinking about God and the world? If one accepts scientific knowledge in this space age, we cannot envisage God as a being sitting on the clouds. It is also problematical to claim a role for God by arguing for the incompleteness of scientific explanations, a ‘God of the gaps.’ The sciences function as a constraint on philosophical theology. They do not imply a single worldview or theology. A theist could emphasize divine transcendence, envisaging God as the creator of all of natural reality—a reality with an internal coherence as uncovered by the sciences. A religious naturalist might appreciate natural reality as a reality

with sacred depth. Different ‘ontologies of the divine’ can be consistent with the sciences.[1]

14.2 Strengths of the Sciences

In school classes where chemistry and physics are taught, we find depictions of the Periodic Table of Elements. A table with at the top Hydrogen, Helium, followed by a couple of rows with other elements. These are the basic elements for all materials. The Table embodies discoveries from the nineteenth and twentieth century, discoveries made in many culturally different settings. 94 of those elements occur naturally on Earth; some more have been constructed artificially. Columns in the Periodic Table represent groups of elements that have similar properties, as metals, noble gases, and so on. The Periodic Table and the chemistry it embodies is extremely successful consolidated science. These scientific insights allow for the explanation of materials and the development of new ones, all with the same basic ingredients. This Table is not merely about materials on Earth. Helium was discovered in 1868 as an element by analyzing light from the Sun. This chemical knowledge applies everywhere in the Universe, under a very wide range of conditions qua temperature and density.

Chemical knowledge is directly relevant for biology. Molecules, complex configurations of the elements, make up living beings. They form membranes around single cell organisms, setting up a boundary between inside and outside. Molecules are chemically transformed in digestion. Molecules are the stuff of neural activity in mammals. Modern medicine rests upon such insights; that is why we have pharmacies. Scientific disciplines and theories do not stand alone, in isolation from other disciplines and theories.

If we wonder why the elements are the way they are, and hence why the Periodic Table catches something adequate about natural reality, we can dig deeper. Physicists have discovered that elements consist of protons, positively charged particles, neutral neutrons, and negatively charged electrons. Beginning with Hydrogen, with just a single proton and electron, up to Lead with 82 protons, and even heavier elements. Quantum physics explains the configurations of protons, neutrons and electrons that together make up atoms. Chemistry can be reduced to physics.

The Periodic Table describes the fundamental constituents of all living beings and material objects. Some criticize such an understanding of living beings as 'reductionist.' However, if that is reductionism, we better be grateful for it, as it allows for important therapies and remarkable materials. Philosophically, such 'reductionism' could be labeled as well 'holism'—it shows some of the coherence in our Universe. In this context, one can also speak of 'emergence,' as complex configurations arise out of more elementary units.

By way of summary: Chemistry exemplifies the strength of the sciences, a range of disciplines that are intertwined, and that together offer a very successful, theoretically coherent and technologically eminently applicable understanding of reality. Passing by the impressive results of the sciences is intellectually problematical. Not accepting consolidated scientific insights is also a moral failure. Science-denial has deprived some humans of effective anti-viral medicines to treat HIV/AIDS and, more recently, reduce the impact of COVID-19, and of knowledge we need to address climate change and hence the conditions of life for future generations.

However, accepting science does not do away with all philosophical issues. We will consider three key issues: limit questions, the underdetermination of our worldview by our

scientific knowledge, and the peculiar position that we, humans, exemplify.

14.3 Unanswerable Limit Questions

In 1868 Thomas H. Huxley lectured “On a Piece of Chalk,” on chalk as found at the cliffs of Dover. Why is it white? Why is it soft, so that we can write with it? Why are these cliffs made of chalk? He explains how such questions are addressed in physical, chemical and geological lines of inquiry. Each discipline answers some questions, while passing on other questions to other disciplines. The image of passing on questions as a disciplinary division of labor, is still a helpful image of the way disciplines work together to develop our understanding of reality. Some questions can be addressed at a particular disciplinary level. But answers give rise to new questions. At each level some questions are passed on to other disciplines.

A child can keep asking: “Why?” So too can the scientist. If we pursue scientific questions as persistently as we can, two lines of inquiry stand out. We have questions about the fundamental structure of reality, its entities and laws: Why is the universe the way it is? At the level of chemistry, the Periodic Table is the answer, but this gives rise to further questions. And so too for questions about origins. Where does the chalk in Dover come from? A question addressed by geology. But then, why is there this planet with those properties? Thus, questions are passed on to astrophysics.

Questions about the structure of reality are passed on until they end on the desk of those engaged in fundamental physics, while questions about origins end on the desk of cosmologist. Cosmology, the astrophysical study of the natural history of our universe, is nowadays to a large extent a regular discipline drawing on the repertoire of established physics, developing

hypotheses regarding the conditions in the early universe, tested against a variety of increasingly precise observational data. This has resulted in the Big Bang theory, a very successful scientific theory about the development of the universe. In astrophysical cosmology the issue is to understand the universe as we observe it, applying rules and concepts of physics as we have them. However, the Big Bang theory does not explain the Big Bang itself; we need new physics in order to push the explanatory quest in cosmology further. Such new physics brings one to the more speculative side of cosmology, where the framework itself is in dispute.

At the end of our explanatory quests, questions of physics and cosmology are intertwined. This was brought home forcefully by Steven Weinberg in *The First Three Minutes* (1977): when we discuss the early universe, we need particle physics, and when we want to test particle physics, we need to study the early universe. The earlier a miniscule event in the history of the universe takes place, the larger the scale of its effects in the later universe. To understand the largest scale of the universe, we need to go to the extreme end of high-energy physics. Theories about the fundamental nature of matter are explored for their cosmological consequences. In the quest for a more complete understanding, cosmological research coincides with the quest for yet unknown physics.

The physicist and the cosmologist have no scientific colleagues from an even more fundamental discipline, to whom they might shove off the remaining questions. This particular position of physicists and cosmologists in the quest for explanations may explain why they might be drawn into philosophical and theological disputes in ways that are less common to geologists, biologists, or chemists. Their work touches upon limit questions to the scientific enterprise, where the explanatory contingency of reality and its laws seems

unavoidable. Is there always an earlier moment? Or does ‘time’ lose its meaning? Or should we understand reality as space-time, and give up the idea of the flow of time? Do we have a highly ordered initial state, or rather a chaotic one?

There are multiple research programs in quantum cosmology (→Butterfield and Isham, 2001). Experimental tests and observations may well be insufficient to decide among the various research programs in cosmology. Aesthetic judgments are, at least partly, decisive in opting for a specific scheme. However, what one considers elegant, another may reject. And could a single and relatively simple complete theory be fair to the complexity of the world? Or, as Mary →Hesse (1988, p. 197) wrote, is it the case that for “the explanation of *everything* there must in a sense be a conservation of complexity, in other words a trade-off between the simplicity and unity of the theory, and the multiplicity of interpretations of a few general theoretical concepts into many particular objects, properties and relations”? Any simple unified theory will leave something about reality and about itself unexplained.

This may be illustrated by considering the conceptual difference between a vacuum and nothing. Our Universe may have arisen ‘out of nothing,’ at least without input of any material, that is, out of a vacuum (→Tryon, 1973). According to modern cosmology, conservation laws valid for the universe conserve quantities that might add up to zero. Take, for example, electric charge. Negative charges of electrons are matched by positive charges of protons. Atoms are electrically neutral, and so too, it seems, is the observable universe. Similar arguments can be made about other properties: either they add up to zero or they are not conserved. As far as the conservation laws are concerned, the universe might have come from a vacuum However, the equivalence of the Universe to ‘nothing’ only holds net. It is like someone borrowing a million euros and

buying stock for that amount. Such a person would be as wealthy, fiscally speaking, as someone without any possessions. However, the first would be of far more significance to the financial market than the second. When there is the option of borrowing, the financial system is taken for granted. And a physical vacuum may be electrically neutral, but the possibility of having equal numbers of positive and negative charges is there. The vacuum behaves according to quantum laws that allow for the fluctuations to happen, just as the apparent millionaire only can get started once there are concepts of money and of borrowing.

Even the most successful scientific explanations do not explain without remainder. Many forms of apparent contingency may be explained and thus replaced by natural necessity, but such explanations introduce contingency at other levels, such as the contingency of the general laws and of existence. Science answers many questions, but these answers give rise to further questions. Any explanation has certain assumptions upon which it relies, at a minimum assumptions about existence and the applicability of mathematics. Thus, with the success of science, there are philosophical limit questions such as “Why is there something rather than nothing?,” “Why is reality mathematically intelligible?,” and “Why are the laws or properties of reality as they are?”

There may be new scientific answers beyond our current horizon. Some questions will be resolved, but limit questions should be expected to arise again and again. In science, we move on in a continuing cat-and-mouse game of questions and answers. Science explains, but it does not explain the most encompassing questions. A theistic answer might be envisaged, but such would give rise to further questions—as any child can continue with ‘why’-questions. The theological tradition has resolved this by speaking of God as God’s own cause (*causa sui*),

or of God’s necessary existence. Whatever the intellectual merits of such ideas, discussed in other chapters in this volume, by then we clearly have left the domain of the natural sciences.

14.4 Unavoidable Underdetermination

Not only are there always more questions, including philosophical ones at the limits of science. There is also underdetermination: theories, answers, are not uniquely determined by the data. There may be more than one possibility to work with those data. A simple mathematical example: Even if you have a finite number of data points on a straight line, the line is not implied by those data. There are an infinite number of alternative models that fit these points as well, for instance oscillating curves. We may prefer a particular understanding of measurements—the straight line—but such a preference relies on additional assumptions about rationality, naturalness, simplicity or beauty. Data constrain theories—not all curves fit the data points—but they do not determine them uniquely. And the data are not free from theoretical assumptions either. In physical experiments, measurements rely on equipment and calculations, and those involve theories; they may even involve the same theory that they are supposed to support.

Furthermore, each theory introduces concepts that are as such not in the data. Data in support of Newton’s theory of gravity (1687) were the observed positions and movements of planets, but the theory introduced the concept of a gravitational force, a hypothetical entity. With Einstein’s Theory of General Relativity, of 1915, this gravitational force disappeared; the theory now envisaged reality as curved, and particular motions as the consequence of curved space-time. Though data are essential to science, scientific theories go beyond the data.

And a theory may be interpreted in various ways; worldviews are underdetermined by scientific theories. This phenomenon is best known for quantum theories. How to understand the entities that arise in this theory, such as ‘the wave function’? Should we understand it as a theory about our knowledge of nature, about what we will find when we engage in measurements, or does it describe reality as such? If so, does this imply that reality realizes an overwhelmingly vast number of possible states or that reality is fuzzy? What does quantum physics say about determinism? The Schrödinger equation, the basic equation of quantum physics, is deterministic, mathematically. But upon various interpretations the theory allows for multiple outcomes of measurements, and is thus claimed to support indeterminism. But then, a Many Worlds Interpretation treats all states represented in the wave function as equally real, which treats the theory as deterministic, as there is nothing possible that is not realized.

The co-existence of multiple interpretations is not exclusive to quantum physics. Einstein’s Special Theory of Relativity describes space and time as a four-dimensional whole. Thus, some have argued that we should understand reality as timeless, as a block universe, without a meaningful notion of a flowing present. But for an observer, there may be a meaningful way to understand the flow of time. And from an empiricist perspective, the four-dimensional universe is a model that goes far beyond the data that can be available to us, ever, as we do not have a point outside space and time, from where we could see the universe as a whole.

Our own position in relation to the scientific image brings me to the third reflection on science, aside of the persistence of limit questions and the underdeterminations of theories by data and of worldviews by theories: How to understand humans, ourselves, in relation to our scientific perspective on reality?

14.5 Categorical Distinctions

We, humans, are material beings. We can be the object of scientific study. No separate ingredients, such as a soul, show up within the scientific study of humans. Within the terms of the sciences, we are nothing but matter. At the same time, not all that is important is addressed by the natural sciences. We are also the ones who study reality, as if we are looking at it from outside. And we evaluate reality, the actions of others and of ourselves, in aesthetic and moral terms. Those are categorically of a different kind than scientific terms. We are biological beings, one form of natural reality, but we may envisage ourselves as outsiders. In reflecting upon reality, we behave as if we are able to observe reality from an external point of view. Such a *View from Nowhere*, to recall the title of a book by Thomas Nagel, is not accessible to us, but in our imaginative, reflective activities, we assume such a position, going beyond now and here.

The philosophical anthropologist Helmuth Plessner offered one way to reflect upon our reflexive capacities in his *The Levels of Organic Life and the Human* (2019), originally *Die Stufen des Organischen und der Mensch* (1928). Plessner introduces his own conceptual language to speak of human experience and agency. Basically, each living organism is characterized by a metabolism, taking food in and pushing waste out, and with that by a permeable boundary between inside and outside. Thus, with life there arises an inner world, while the organism is within its outer world, its environment. An organism has a particular place in the world; *positionality* becomes a characteristic of life. However, the relation to one's own position works out differently for different kinds of organisms, say plants, animals, and humans. Plants have no center that coordinates actions, and hence no relationship to their own positionality. Animals are aware of their environment and of their own position in it. As they move

around, they engage in intentional actions; they have a *centric* positionality. For an animal one might say that it has a body, while it also is that body.

With humans, a further step is made with the emergence of *self*-awareness, awareness of one's own center, and of the possibility that one could have been at a different place or time. In the words of Plessner, "He not only lives and experiences, but also experiences himself experiencing [*Er lebt und erlebt nicht nur, sondern er erlebt sein Erleben*]" (→Plessner, 2019, p. 271; →De Mul, 2019). In our experiences we may take distance from our own center, which thus might also be 'looked at' as if from outside that center. Thus, one can want to be more one's true self, as if at any moment one is not one's self. Plessner speaks of persons as having an *ex-centric* positionality, given their ability to engage in self-reflection, to consider their own actions as if from an external perspective. And in this self-reflective process as ex-centric beings, we also encounter others, and hence participate in a shared historical, cultural, and social world (*Mitwelt*). We are products of this shared world that precedes us and shapes us, but we are also its creators.

Plessner's imagery and language is just one conceptual vocabulary for philosophical anthropology; others have done it differently, but touched upon similar issues. The fundamental insight is that even if one accepts that the natural sciences are in their own terms successful in describing the world, including humans, there is something about us as humans that requires a richer vocabulary, that takes into account that we are humans who study reality, including ourselves and other humans, who may be influenced by such knowledge, and who act intentionally within reality. A self-reflective vocabulary that I have called elsewhere the human humanities (→Drees, 2021a).

In our scientific study, we approach reality in objective terms. In grammatical terms: we use third-person language. But in

personal engagement and agency, we also draw on the second person, seeking to understand others, while we cannot avoid speaking from a personal point of view, the first person perspective. We have intentions and values; we act for reasons. From a third-person perspective actions of people may be explained sociologically, as consequences of circumstances and interests. Such explanations do not exhaust our personal existence; they do not do justice to the values we espouse. Why we have those convictions may be explained, but a sociological or evolutionary explanation does not justify them. Issues of justification fall outside the scope of scientific explanations. A classic formulation of this issue is the is/ought distinction, the categorical distinction between on the one hand descriptive and explanatory projects, that deal with what is, and on the other hand prescriptive, normative discourse, that expresses convictions about that we hold which ought to be.

To summarize the discussion of science so far: The sciences are a very coherent and successful way of understanding natural and social reality. But there always will be questions at the limits of science. Furthermore, scientific insights are underdetermined by the data, and the consequences for our worldview are underdetermined by the best scientific knowledge. And our human practices have person-involving features that are not represented within a scientific perspective with its quest for objectivity. Thus, scientific knowledge is very relevant to our lives, to the actions we may consider and the judgments we express, but there is a need for more than science. There is a legitimate role for philosophical discourse.

14.6 Worldviews

Even more than for what it tells, science is informative for ideas that are discarded. The Earth is not flat. The Sun is not being

drawn on a chariot, rising in the East and going to rest in the West. Six days of creation, of 24 hours each, have not been enough. Living beings have not been around for a mere few thousand years. Thunder is not an arrow, thrown by an angry god. When Jurij Gagarin, the first cosmonaut, returned to Earth, he had not seen God in space, above the clouds. Up there, no man with a beard, nor a woman, sitting on a throne. More philosophically, given what we know about atoms, it is not convincing to speak of substance as space that is full of matter, infinitely divisible. Rather, substance includes, physically, a lot of empty space. Mythic images, creation narratives and ancient philosophy may evoke poetically a sense of awe and wonder and install in us humility and a sense of responsibility, but intellectually these are no match for scientific understanding.

The rich variety of life forms has evolved in the course of millions, or even billions, of years, in a process of variation and selection. Neither the variation (mutations) nor the selective pressure is beyond the scope of the natural, as studied by the sciences. Of course, data are always incomplete, but it would be against the scientific ethos to introduce alongside the natural causes someone who tinkers with organisms. Where science faces limitations, the appropriate response of scientists is to develop science further. They will not appeal to miracles, but assume that we do not know the natural conditions in sufficient detail or all laws, or that we have failed to deduct possible implications of those laws and conditions. This attitude may be called ‘methodological naturalism’ (Perry and Ritchie, 2018). Attempts to find room for God within natural processes, as a factor alongside natural causes, are suspect philosophically and theologically, as they introduce a God-of-the-gaps. Thus, for those who take the sciences seriously, much is excluded, often going against a common sense understanding of phenomena.

Positive consequences for many issues of philosophical interest are less definitive. The Periodic Table spoke of elements; physics understands those in terms of protons, neutrons and electrons. In particle physics, protons and neutrons are understood in terms of quarks, particles that may have 1/3 of the elementary charge of an electron, held together by gluons. And at much smaller scales (or at far higher energies), those may consist of ‘superstrings’ or of something else; we are in a range where there is no consolidated science yet. What then is matter, the substance of reality, deep down? We don’t know. Similarly for other topics of philosophical interest. From consolidated science, that provides a clear constraint on ideas about substance, we come into domains where theories are not settled, their interpretation is in dispute, and the view of reality thereby offered is quite different from the ways at the scales with which we are familiar. Thus, scientific knowledge can serve, negatively, as a constraint upon credible understandings of reality, but does not determine a unique worldview or metaphysics.

As an aside, the sciences not only have an impact on ideas about reality, our worldview or metaphysics, but also on ideas about knowledge. Does science offer an adequate image of reality, or should we rather treat it as models that are pragmatically useful? Thus, should we face some modest form of realism, or is empiricism such as advocated by Bas →Van Fraassen (2002) a more honest stance? If so, religious thought is perhaps to be appreciated more for what it does in human lives, pragmatically, strengthening communities, helping us live with finitude and providing moral orientation, than for its ontological truth.

14.7 Theologies

Among the many definitions of religion, a favorite of mine is Clifford Geertz's anthropological understanding of religion. According to him, religious symbols and practices integrate the ways in which we experience and orient ourselves in the world with our understanding of reality.

> Sacred symbols function to synthesize a people's ethos—the tone, character, and quality of their life, its moral and aesthetic style and mood—and their world view—the picture they have of the way things in sheer actuality are, their most comprehensive ideas of order. (→ Geertz, 1966, p. 3)

Religions integrate models *of* the world, a worldview, and models *for* the world, a vision. Geertz expresses this in an oft-quoted definition:

> A religion is (1) a system of symbols which acts to (2) establish powerful, pervasive, and long-lasting moods and motivations in men by (3) formulating conceptions of a general order of existence and (4) clothing these conceptions with such an aura of factuality that (5) the moods and motivations seem uniquely realistic. (Geertz, 1996, p. 4).

As a definition of the empirical phenomena of religiosity this definition places too much emphasis on the cognitive role of symbols, as contributing to conceptions of the order of existence, bypassing ritual, social and other non-cognitive roles of religious symbols. And the definition suggests a causal arrow from symbols via conceptions to moods and motivations, whereas the symbols may also express moods and motivations rather than establish them. Nonetheless, as a definition that involves the cognitive side of a religion, what could be called 'a theology' (or non-religious equivalent), Geertz' definition highlights the observation that in religious thought conceptions of the order of existence are intertwined with the appreciation of reality and norms for our behavior, moods and motivations. For

instance, to speak of the world as God's creation has a descriptive and a prescriptive aspect to it—by speaking of origins it calls for gratitude, humility and care.

In relation to the conception of the order of existence, science has its place—especially given that the worldview should have an 'aura of factuality.' Moods, e.g. wonder, may be influenced by knowledge, or lack thereof, and motivations may relate to ideas about what is technologically possible, and hence to our scientific understanding of reality. Nonetheless, in relation to the 'moods and motivations,' science by itself has less to offer; that touches upon important other spheres of human life.

Thus, science serves as a constraint on some elements of theologies, especially the worldview dimension, but is not fully determinative of them. How then might we think about the divine, if we accept science? I see a spectrum from theism, emphasizing transcendence, to religious naturalism, monistic in ontology.

One option is to assume a radical distinction between God and 'all else,' creation. Upon such an approach, natural reality with its structure and dynamics, as studied by the sciences, is seen as creation. That would be a valid form of 'creationism.' One would not insert divine activity where natural processes could be operative, as if God were in competition with God's created processes, but one would appreciate natural reality with all its laws and materiality as God's creation.

Such a theology can draw on the notion of *creatio ex nihilo*. 'Out of nothing' does not apply to causal processes within empirical reality, but refers uniquely to God's creative causality. God creates and sustains all things as their primary cause. All natural causes are real too, just as are all entities and events, but they are so because they have been created by God. Such real natural causes are 'secondary causes.' This distinction between primary and secondary causality was articulated in the European

Middle Ages, for instance by Thomas Aquinas, but its roots can be traced back at least to Augustine. God creates everything—past, present, and future events—and God creates them not as an amorphous bag of events but with their temporal, spatial, and causal relations.

A distinction between God and God's activity on the one hand, and creatures and creaturely activity on the other, might also be articulated as a difference with respect to time: creatures are temporal, whereas God, as conceived in this view, is not temporal. God's eternity is not envisaged as everlastingness but timelessness (→Leftow, 1991). Accepting the whole natural world as the creation of a timeless transcendent God may be consistent with a naturalistic view of the world, since it accepts the world as understood by the natural sciences as God's creation. There is no need for particular gaps within the world.

Upon such an understanding of God, one could accept the sciences as explanatory within the world, even though they are not explanatory of the world as such; limit questions are addressed in philosophical theology and not in the sciences. Science offers explanations, but every explanation assumes an initial state and laws. I find such a naturalistic theism a genuine and attractive possibility—that is, a fully science-inspired naturalism with respect to the world we live in and experience, combined with openness to the possibility that this remarkable reality is continuously created by a transcendent God. Such a theism has one major problem, as I see it. It is hard to give reasons, once one accepts a naturalist understanding of created reality, why one would hold such a theological position; "since there are no real 'gaps' to fill, we may be left without an argument for God's existence of the kind that would convince a science-minded generation" (→McMullin, 1988, p. 74).

Less dualistic, though still within the theistic tradition broadly conceived, might be speaking of God as Ground of

Being. A major figure in the articulation of such a theological position has been Paul Tillich; a more recent advocate is Wesley →Wildman (2006). Another idea is that all of natural reality is in God, even though God surpasses the world, pan-en-theism (→Clayton and Peacocke, 2004). I found a most inspiring poetic expression among aphorisms in *The Aristos* of John →Fowles (1980, p. 27).

> The white paper that contains a drawing; the space that contains a building; the silence that contains a sonata; the passage of time that prevents a sensation or object continuing forever; all these are 'God'.

Whatever the precise formulation, such positions have more deeply ingrained science-inspired naturalistic presuppositions by seeking to avoid a strong ontological dualism of a transcendent God alongside the natural world, even though they maintain a concept of God as given reality that is not identical with natural reality, but rather its ground or the reality-transcending setting within which natural reality exists, as in panentheism.

There are some positions that may be further removed from theism, as they do not understand God as an entity (or its ontological ground or all-encompassing being) but rather as a symbol used to speak of our existence and the world we live in. Perhaps, one can speak of *The Sacred Depths of Nature*, as the biologist and religious naturalist Ursula Goodenough titled a book (1998).

With all that we now know about 'all else,' science, we cannot definitively decide what should be our worldview and ethos, our theology. In that sense, it seems that with respect to answering ultimate questions an agnostic stance is scientifically most appropriate. That fits well our social reality, with a plurality of human religious orientations. For people who take science seriously, science is a constraint within each of the available

religious orientations, often more easily accepted by the more liberal among its adherents. Scientific insights may rule out various ideas about God and created reality. But that need not be perceived as a loss. Religiously speaking, many of the ideas that are to be discarded are ‘too small’ to be adequate to our knowledge of natural reality and the magnitude of ultimate questions such as the question why there is anything at all.

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Notes

1 This contribution draws upon earlier publications by the author, on cosmology and limit questions (→Drees, 1990), religion and science (1996 and 2010), religious naturalism (2006, 2021b) and the humanities (2021a).

15 God and Fundamentality: From Fundamentality to Perfection

Joshua Rasmussen

Abstract: This chapter explores a path from the premise that something is *fundamental* to the premise that something is *perfect* (in a certain sense to be defined). I begin by identifying a "fundamentality" puzzle, which challenges us to explain how anything could be fundamental. To unpack the puzzle, I consider a series of potential solutions that, for various reasons, only push the explanatory question deeper in. Then I present an account of fundamentality in terms of a purely perfect fundamental nature, and I show how this account has an explanatory advantage over others. Finally, I consider some objections that help draw out the implications of my account of fundamentality.

15.1 The Puzzle of Fundamentality

How could anything be *fundamental*? A fundamental thing, as I am thinking of it, would be ungrounded and inexplicable in terms of anything prior.[1] Could there be something like that? If so, how?

These questions invite us to seek some explanation of fundamentality. An explanation of fundamentality would demarcate fundamental things from non-fundamental things in terms of some relevant difference between things of these two sorts. For example, suppose some turtle were somehow a fundamental piece of reality. Then we might expect there to be some *relevant difference* between this turtle and every other

turtle that depends on prior causes. In particular, we might expect the fundamental turtle to have some special nature that would explain how that turtle could be fundamental rather than dependent.

However, it is not obvious how we might explain fundamentality. Part of the problem is that we are not accustomed to experiencing things as fundamental. Instead, we experience things that call for an explanation in terms of prior things. A turtle, for example, depends on prior causes that jointly explain the turtle's existence. Same for trees, towers, and thoughts: each of these calls for a further explanation in terms of prior causes. If there is something that is instead not dependent on anything, then it is an open question what, if anything, could set it apart from everything else.

We can translate the question of what might explain fundamentality into what I shall call, "the fundamentality puzzle." The fundamentality puzzle has two pieces:

1. Fundamentality: there is at least one fundamental thing.
2. Explicability: everything calls for some further explanation.

It is not my purpose here to argue for (1). Rather, I want to see how something could be fundamental in the light of (2). For the sake of focus, I will examine fundamental *concreta* (causally-capable) things. The challenge, then, is to see how anything concrete could be fundamental if everything calls for a further explanation.

There are many reasons one might like to have an account of how something could be fundamental. I note three benefits. First, an explanation of fundamentality would serve the investigation into the general structure of reality. If we can see why a certain portion of reality is relevantly different from dependent portions of reality, then we can gain insight into the

general structure of any reality that would have a foundational portion. Second, this query serves natural theology by contributing to the identification stage of cosmological arguments for a supreme being; opening a pathway from *fundamental existence* to *perfection* contributes to the project of identifying the nature of a first (or fundamental) cause. Finally, this inquiry is relevant to philosophical theology. On classical models of God, God is fundamental. How? An explanation of how *anything* could be fundamental could help illuminate the divine nature (if there is one). Thus, by seeing what it takes to be fundamental, we position ourselves to see more of what is at stake in developing a theory of a divine being's relationship to fundamental reality.

In this chapter, I will seek to solve the fundamentality puzzle. My strategy is to identify some relevant difference between fundamental and non-fundamental things. If we can identify a relevant difference, then we can see how, despite the general call for an explanation, there could be an exception to the rule. I will examine candidate explanations of fundamentality in terms of these features: (i) being total, (ii) being necessary, (iii) being a tautology, (iv) being inexplicable, and (v) being perfect. I will show how, on a certain minimal conception of "perfection," feature (v) can unlock a deeper solution to the fundamentality puzzle than the others.

15.2 Some Proposals

In this section, I will consider some proposals for how one might try to explain fundamentality. In each case, I will say why I think the proposal in question does not solve the fundamentality puzzle. By seeing why these proposals fail, I hope to bring into greater light the challenge of explaining fundamentality.

15.2.1 Totality

Perhaps something could be fundamental by being the *total* of all that exists. After all, there is nothing beyond the total of everything: everything includes everything. Therefore, there is nothing beyond the total that could *ground* (cause or explain) the total. For this reason, one might think the total is therefore ungrounded—and therefore fundamental.[2]

However, I don't think the totality proposal pulls out the root of the fundamentality puzzle. Consider, first, that while there is indeed nothing beyond the total of everything, it does not follow the total is thereby ungrounded. There is another option: the total might be grounded in something *within*. For example, perhaps the total includes some foundational item upon which all other things ultimately depend. Then the total itself might depend on the foundational item together with what it produces.

Second, and more fundamentally, even if a total *were* fundamental, its being a total does not itself tell us *how* (or why) it could be fundamental. To see what I mean, imagine a world with a turtle that includes all other things. In this world, a turtle is the total of everything. But is this world even possible? One might think not, for one might think that no turtle could exist without a prior ground or cause; then, no turtle could be an ungrounded total of everything. If there is an exception to this rule, one might think the exception would be *relevantly different* from every other turtle we know about. Consider that if some blob, *whatever it is*, were an ungrounded total of reality, that blob would be unlike dependent things. The question remains: how? Is there some relevant difference between the grounded and ungrounded? If so, what's the difference? Merely stipulating that something is the total of reality does not answer this deeper question. Totality is not enough.

In fact, the totality proposal is part of what generates the fundamentality puzzle. For whether a totality is itself fundamental or includes something fundamental, it follows that *something* is fundamental. That was the first piece of the fundamentality puzzle: something is fundamental. Pointing to this piece does not tell us anything about how it might fit with the other piece—the piece defined by the general call for a further explanation. The whole challenge is to see how these pieces might go together.

15.2.2 Necessity

Some philosophers have proposed that the chain of grounds bottoms out in a *necessary* thing, where a necessary thing is something that *cannot* fail to exist.[3] Here is one rationale for this proposal. Unlike necessary things, non-necessary things *can* fail to exist. For this reason, some have proposed that non-necessary things call for an explanation to account for why they exist *rather than not*. Furthermore, they have proposed that the explanation will be in terms of prior causes or grounds. Therefore, non-necessary things cannot be fundamental. A necessary thing, by contrast, is importantly different. It cannot fail to exist. So, there is no reason to explain why it exists *rather than not*. Or if there is an explanation of its existence, that explanation could simply be in terms of the *impossibility of its non-existence*: i.e., it exists because it cannot not. There is no need, then, to posit some prior conditions upon which a necessary thing would depend. (A related proposal is that fundamental reality is *eternal*, and that being eternal sets it apart from all dependent things. My remarks about necessary existence apply mutandis mutatis to this proposal about eternal existence.)

Here again I do not think this solution pulls out the root of the puzzle. I have two points. First, even if a *certain* call for an explanation is removed, it does not follow that every call for an explanation is removed. In particular, some necessary things may still be explicable in terms of more fundamental grounds. For example, one might think *sets* are grounded in their members, irrespective of whether the sets are necessary (e.g., {1} is grounded in 1). Moreover, if one is a "trinitarian" theist, one might think that there is a structure of dependence between necessary divine persons within God (e.g., where God the Son depends on God the Father). While we may debate examples, my point here is that appealing to necessity does not *by itself* remove every reason one might have to expect a further explanation.

Second, and more fundamentally, the necessity proposal relocates the puzzle. Suppose there is a necessary concrete reality. We now face the challenge of answering this question: what sort of concrete reality could be necessary? Merely stipulating that fundamental reality is necessary does not itself tell us what could be necessary.

To draw out the challenge, consider the following "concrete foundation" model of reality. According to this model, abstracta are grounded ultimately in concrete particulars (e.g., in particulars that instantiate properties and/or in mental activities of particular beings). These concrete particulars in turn are grounded ultimately in a foundational portion of reality that exists on its own, ungrounded. Now suppose contingent realities are indeed dependent on prior grounds or causes. Then this foundational portion of reality is not contingent; instead, it has necessary existence. Still, we may wonder what it could be. In particular, what relevant difference, if any, might set it apart from things that are merely contingent. If being a turtle doesn't make the difference, what might? An answer to this question

would give us a deeper explanation of how *anything* (big or small, turtle or not) could be a necessarily existent foundation.

To appreciate the challenge that remains, it can help to distinguish between a *sign* of fundamentality an *explanation* of how something could be fundamental. So far, we have identified possible signs of fundamentality. For example, necessity may be a sign that something is fundamental (because it lacks a certain call for an explanation). Totality may be a sign that *something or other* is fundamental (to explain non-fundamental chains without circularity). Still, signs of fundamentality don't themselves explain fundamentality. The challenge remains: what sort of thing could be set apart as fundamental?

15.2.3 Tautology

Perhaps fundamental things are facts that are self-evidently true, like a tautology. For example, the fact [1=1] is self-evidently true. For this reason, [1=1] does not call for a further explanation; we don't wonder why a number happens to equal itself. It is simply self-evident. Thus, no further explanation is required. Perhaps, then, everything is grounded ultimately in self-evident facts.[4]

My response involves clarifying the fundamentality puzzle. First, the puzzle I have in mind is about *metaphysical* grounds, not epistemic grounds. While [1=1] may be self-evident and thereby epistemically ungrounded, it does not follow that [1=1] just exist "out there" all on its own. My own view is that facts are grounded in the items those facts describe. Thus, for example, I think [1=1] is grounded in the number 1 together with the equals relation. This view illustrates how a fact could be epistemically self-evident while still metaphysically grounded.

Second, the fundamentality puzzle is not about facts, but about concrete particulars. Even if I am mistaken in my view

about the grounds of facts, there still remains a puzzle about how a particular *non-fact* could be fundamental. For example, in keeping with the subject of this anthology, one might wonder how *God* could be fundamental. God is usually conceived of as a fundamental being, not as a fact. So, a theory of fundamental *facts* (whether epistemic or metaphysical) does not explain how a particular concrete *thing* could be fundamental.[5] The fundamentality puzzle remains.

15.2.4 Inexplicability

It has been suggested that fundamental facts do not admit of a further explanation (→Dasgupta, 2016). Perhaps we can apply this suggestion to particulars, too: a particular is fundamental if and only if it *cannot* be further explained. Problem solved.

I believe this proposal points in the right direction. For I am inclined to think that whatever is ungrounded does not just *happen* to be ungrounded. Rather, it is the sort of thing that *cannot* be grounded.

Still, this question remains: what sort of thing cannot be grounded? The inexplicability proposal merely tells us that *whatever* is fundamental, it couldn't have *not* been fundamental. Great. But this information, if correct, merely inspires the original question: how might something be fundamental? If we say that fundamental things must be ungroundable, we have yet to say what concrete particulars, if anything, *could* be ungroundable.

To hammer down this point, consider again the case of an ungrounded turtle. Suppose some turtle were somehow fundamental and therefore ungroundable. Still, we might wonder *how* it could it be fundamental. We can say this much: the turtle is fundamental because it *couldn't have not* been fundamental. Still, this answer doesn't alleviate all curiosity. The

deeper question remains: how could there be any turtle that *couldn't have not* been fundamental? How is this turtle different from all other turtles? What sets it apart? An answer to this question would provide a deeper explanation of its fundamentality.

15.3 The Perfection Proposal

I will now consider how we might explain fundamentality more fully. I will begin by clarifying a minimal concept of "perfection." I will then show why something that is perfect, on this concept, may be relevantly different from things that depend on a further ground.

Start with the concept of "perfection." The seed idea I will work with is this: *x* is a perfect being if and only if *x* has a *supremely great* nature.[6] I stipulate two necessary conditions for a supremely great nature. The first condition removes *imperfections*: a supremely great nature includes no imperfections, where an imperfection entails some limit with respect to some possible excellence (or great-making property). For example, *being unable to know that 2 + 2 = 4* entails a limit with respect to what that being can know, which is a limit with respect to an excellence. Excellences may include knowledge, virtue, powers to make valuable things, or other great-making properties. I leave open whether excellences may themselves be further analyzed (such as in terms of a relation to what beings could like for its own sake).[7]

The second condition for a supremely great nature fills in *perfections*: a supremely great nature includes only perfections, where a perfection is a property that entails some excellence without also entailing a limit with respect to that excellence. For example, *being wholly good* is a perfection if it does not entail any

particular limit with respect to goodness, and if goodness is an excellence.

These conditions provide guiderails for a minimal concept of perfection. Together, they entail that a perfect being would be purely great (having only great-making properties) without any particular great-making limits. These conditions *leave open* which properties count as great-making and whether a purely great being would also be a maximally great being.[8] (I will consider questions about the implications of this concept of perfection in the objections section.)

The next step is to see how perfection might help explain fundamentality. There are two ways perfection might make the difference. First, perfection *removes* certain calls for a ground by clearing away imperfections that, if real, would call for a further explanation. Second, perfection fills in purely positive properties that arguably entail independent, fundamental existence. I will unpack both ideas.

Start with removing the call for an explanation. Some properties call for a further explanation because they are like other properties that call for a further explanation. For example, suppose fundamental reality were fundamentally *blue*. Then we might wonder, why blue? Why not red? The difference between blue and red does not appear to make a difference with respect to having a further explanation. For this reason, we may not expect fundamental reality to be fundamentally blue. After all, all colors equally call for a further explanation.

To remove the call for an explanation, we need to find some relevantly difference. For example, if we could somehow see why *blue* is different from other colors, such that only blue could be fundamental, then we could see why blue would be relevantly different. Unless we can identify such a difference, however, we are not in position to suppose that blue lacks a further explanation. Instead, we can expect that blue has some further

explanation in terms of a prior cause or ground. In general, for any property *P*, we can expect that fundamental reality is not fundamentally characterized by *P* unless we can identify how *P* is relevantly different from other properties.

The next step on this path is to see why *imperfections* are not relevantly different from properties that call for a further explanation. An imperfection (as I have defined it) is a property that entails some *limit* with respect to some excellence. For example, having knowledge of all even numbers *and no knowledge of odd numbers* entails a certain limit with respect to knowledge. We can define this "limit" in knowledge as marking a boundary between what it knows and what could be known. (In general, we may define a "limit" as a property that entails a boundary between one possible degree and a higher possible degree.) Now we face the obvious question: why *that* limit? Is there something special about even numbers, such that fundamental reality could only know them? The problem here is that, unless we identify some special reason why only even numbers could be known by a fundamental reality, we have not identified a *relevant difference* between knowing even numbers and knowing odd numbers. Hence, we have not removed the call for a further explanation.

My idea, then, is that we can remove a certain call for explanation if we remove fundamental imperfections. Suppose, for example, fundamental reality has a fundamental knowledge-entailing property P. Then we may suppose that P does not itself entail any particular limit on its knowledge. For this supposition removes the call for explaining the limit—i.e., for explaining why fundamental reality has a property that entails *that* limit (among other possible limits).[9] (This account leaves open whether fundamental reality may have certain *non-essential* limits, which it only has contingently.)

Perfection not only *removes* a certain call for grounds (by removing imperfections), but perfection may also add a call for fundamentality. A classical view is that a perfect being would be *too great* to depend for its existence on other, prior things. If so, then absolute perfection may independently entail independent, fundamental existence.

Here is one way we might unpack this idea. A purely great nature has no room for properties that would call for a further explanation. For example, a purely great nature would lack particular fundamental limits that call for a further explanation. Instead, it would be purely great, without fundamental limits. If, instead, fundamental reality were partially great (e.g., a mixture of great and non-great essential properties), then the partial degree would call for an explanation: why that degree? Pure greatness, by contrast, does not call for further explanation in the way partial degrees would. By removing the calls for a further explanation, what remains is a reality that *calls* for no further explanation.[10]

A related idea is that any ground of a thing imposes certain constraints that detract from pure greatness. For example, if some *x* grounds some *y*, then *y* has the particular property of *depending for its existence on x*. This particular property, one might think, imposes a certain constraint (i.e., depending on *x*). Pure greatness includes no such constraints, one might think. If that is right, then a perfect thing cannot be dependent. Instead, perfection entails independence.

At this point, one might wonder what, if anything, might ground *pure greatness*. Here is an Aristotelian idea. Pure greatness may be grounded in the fundamental being itself. This idea is different from the idea that some arbitrary limit is grounded in the being itself. For an arbitrary limit is precisely what invites the question: why ground *that* limit rather other equally arbitrary neighbors.? Pure greatness, by contrast, is not

an equally arbitrary neighbor of imperfections (or any other property). So pure greatness is better suited to fill in the nature of a fundamental reality.

In summary, perfection unlocks a unique solution to the fundamentality puzzle. I call this solution, “the perfection proposal.” The perfection proposal has two parts. First, we *remove imperfections* that would call for an explanation in terms of a prior ground. Second, we *fill in perfection* (in a complete way), which results in a call for no further ground. In this way, we identify a relevant difference between a fundamental thing and grounded things: a fundamental thing, unlike grounded things, would lack fundamental limits, boundaries, and arbitrary specificities that depend on some ground. A *perfect* thing, then, is precisely the sort of thing that would lack fundamental limits, boundaries, and arbitrary specifies. For this reason, a perfect thing is precisely the sort of thing that could be fundamental.

15.4 Objections and Replies

To further clarify what is at stake, I turn to some objections.

Objection 1. Why think there is any puzzle to solve? Perhaps there is no explanation of fundamentality. Instead, the foundation of reality is simply *whatever it must be* to explain the rest of reality. That’s it.

Reply. There is value in seeing how we might explain something, especially if we can identify an explanation. What makes fundamentality puzzling (to my mind, at least) is that, by default, things seem to call for an explanation. It doesn’t matter if we’ve had any prior experience with those things. Suppose we see a large red cube in the sky. We could still expect some explanation of the cube’s existence, despite having no prior experience with explanations of red cubes appearing in the sky. If instead something has no further explanation (in terms of a

prior ground or cause), we may wonder why not. What sets it apart? An answer to this question would give us greater insight into *how* something could be fundamental.

A principle at work in this reasoning is the principle that, other things being equal, we can expect an explanation. This principle is itself a reason to expect an explanation of fundamentality. For if something lacks an explanation (in terms of a prior ground), we can then expect an explanation of this lack of explanation. If no explanation is available, perhaps the best we can do is live with mystery. However, by the light of the perfection proposal, an explanation is available. Therefore, we need not live with mystery in this case.

Objection 2. The perfection proposal goes too far. We can conceive of a fundamental reality that is not perfect. While we may have reason to shave fundamental imperfections (if they indeed call for a further explanation), it does not follow that fundamental reality must *have* perfections. Maybe fundamental reality is purely neutral, with no imperfections or perfections. Why fill in fundamental reality with any great-making qualities at all?

Reply. First, I acknowledge that I have not shown that a fundamental reality could not be purely neutral. Perhaps a fundamental reality could lack imperfections without having any perfections. That may be logically possible.

Still, the perfection proposal has certain advantages over others in relation to the *actual* world. Consider this path of thought. First, by experience, one might think the actual world includes certain specific kinds of "positive" (value-entailing) states, including a sense of beauty, feeling loved, and knowledge of the truth. Next, one might think positivity seeps to the foundation. For a foundation of our world would *ipso facto* have the resources to be a foundation for such valuable states, and one might think that having such resources is itself a great-

making property. So, one might have reason to think fundamental reality is filled in with at least *some* greatness. Finally, to avoid arbitrary unexplained *gaps* in the fundamental nature of fundamental reality, one might suppose that fundamental reality is completely or purely great in its fundamental nature. The result of this line is that, if fundamental reality has some greatness, then it would be purely great (i.e., perfect) to avoid calls for a further explanation of its limits in greatness.

Second, the perfection proposal goes further in explaining why fundamental reality would be ungrounded. If we say that its fundamental nature is neutral, there remains the question of *why* that nature would preclude arbitrary limits, boundaries, or imperfections. A purely great nature, by contrast, is one that we can see would preclude arbitrary limits, boundaries, and imperfections. The perfection proposal provides a deeper solution, then, to the fundamentality puzzle. Theists who already believe that God is perfect may especially appreciate this result.

Objection 3. The perfection proposal does not go far enough. The classical view of God's nature includes *all* perfections, including perfect knowledge. But the minimal account of "perfection" does not automatically entail all perfections. More to the point, the aspects of perfection that actually contribute to solving the fundamentality puzzle (i.e., by clearing away imperfections and filling in perfection without gap) do not thereby entail every classical attributes of God.

Reply. I consider it an advantage of my proposal that it is modest. First, my proposal does not *rule out* a fuller theory of the divine nature. So classical theists need not reject my proposal. Second, my proposal is also not committed to classical theism. So, non-classical theists need not reject the proposal, either.

Moreover, the modesty of the proposal may be a gift to those who are be worried about objections to various classical

perfections (e.g., from the problem of evil, omniscience paradoxes, and omnipotence paradoxes, etc.). The more modest concept can then solve the fundamentality puzzle without thereby leading us into other, classical puzzles. Modesty reduces barriers.

Finally, the modesty of the perfection proposal can provide a foundation for additional fruitful research. The proposal identifies a minimal concept of perfection. The minimal concept is a seed concept that, not only helps us solve a certain puzzle, but also contributes to perfect being theology. Objections to certain fuller concepts of perfection could help carve away certain ideas without thereby carving away the seed concept. By starting with a minimal concept of perfection, we position ourselves to explore ways we might unpack this concept in ways that are consistent with actual reality.

15.5 Conclusion

Perfection can help explain fundamentality. The perfection proposal is helpful in two ways. First, it clears away fundamental imperfections, which helps clear away certain calls for a further explanation. Second, it fills in perfection, which helps explain why fundamental reality is not instead filled in with arbitrary limits, boundaries, or imperfections that would require a further explanation.

There is a bonus result. By seeing how perfection can explain fundamentality, we also open up a new pathway *from* a fundamental concrete reality *to* a perfect fundamental reality. Here are the steps in that path:

1. Suppose reality has some foundation (i.e., some ungrounded concrete layer of reality).

2. If reality has some foundation, then there is some explanation of the foundation's fundamentality (i.e., an account of why it is exempt from the general call for a further explanation).
3. If there is some explanation of the foundation's fundamentality, then that explanation is (most fully and completely) in terms of the foundation's perfection.
4. Therefore, reality has a perfect foundation.

Of course, not everyone will be in a position to take every step in this path. Still, I hope everyone reading this will appreciate the value of this path, either by seeing which steps not to take (and so which theories to avoid) or by seeing a new line to an interesting result.

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Notes

1 For the purposes of this inquiry, we may leave open whether a fundamental reality may, in some sense, be self-grounded or self-explained.

2 It is controversial whether a total reality must include something fundamental. →Cameron (2022), for example, defends the view that nothing is fundamental by challenging arguments against various types of infinite chains. On the other side, →Rasmussen (2019) defends the view that the total of everything must include something fundamental, not by arguing against infinite chains, but by arguing that any total of dependent things must depend on something independent.

3 For a defense of this proposal, see →Pruss and Rasmussen (2018).

4 A related idea is that the basic, unexplained facts are mundane in a way that don't call for further explanation. Cf. Baras (forthcoming).

5 Perhaps we could try analyzing fundamental things in terms of fundamental facts. For example, we could suppose that God is fundamental because the fact that God exists is fundamental. However, this analysis only reframes the puzzle at hand. We can now ask why and how a fact about a concrete thing could be fundamental. If there is a fact about a concrete thing that admits of no further explanation, what is that fact?

In other words, what is the nature of the concrete thing specified by a fact that admits of no further explanation? This question is a more elaborate way of asking our original question: what sort of concrete thing could be fundamental? So the puzzle remains.

6 I define "nature" as consisting of the essential properties of a thing (i.e., the properties it cannot fail to have).

7 For an analysis of perfections in terms of idealizations of various excellences, see →Oppy (2004, pp. 64–68). As far as I can tell, this analysis leaves open whether excellences may be universal or relative to certain essences. Thanks to Kha Andani for drawing my attention to this distinction.

8 This minimal concept may help us avoid certain challenges →Speaks (2018) raises for perfect being theology. For example, Speaks considers the worry that, if a perfect being would have the greatest *conceivable* power, then a perfect being would be impossible, for it would have powers that turn out to be metaphysically impossible. The concept of perfection I offer here avoids this worry because it does not commit us to supposing that a perfect being would have the greatest conceivable power. I say more about Speaks account in →Rasmussen (2019), where I develops a non-modal account of "perfection," similar to the account given here.

9 →Koons (1997) raises similar considerations in his argument for the conclusion that the basic (uncaused) properties of a fundamental (uncaused) reality are not

finitely measurable degreed properties. Finite degrees, like limits, call for a further explanation.

10 Pure greatness is also unlike specific determinates, like red, that are arbitrarily different from other specific neighbors. Specific determinates call for an explanation: why red rather than blue? Pure greatness is different because, unlike red, pure greatness is not a determinate among arbitrarily different neighbors.

V Simplicity and Ineffability of God

16 Divine Simplicity and Eliminative Theism

Michael Almeida

Abstract: Discussions of divine simplicity generally overlook the distinction between identity claims that are reductivist and identity claims that are eliminativist. If, for instance, the identity claim that ‘the chair = a configuration of particles’ is merely reductive, then there exist chairs and there exist configurations of particles and it turns out that they are identical. The identity in this case does not reduce the ontological complexity of the world. But if the identity claim is eliminativist, then it is true again that chairs are configurations of particles, but chairs do not in fact exist. The chair reduces to the configuration of particles without remainder. The defense of divine simplicity in this paper argues that it is in this eliminativist sense that God is identical to God’s intrinsic properties.

16.1 Introduction

On the traditional doctrine of divine simplicity there is no metaphysical complexity at all in God. The absolute simplicity of God is standardly assumed to entail that everything wholly within God—everything intrinsic to God—is identical to God himself. There exist no divine properties of omnipotence, omniscience, or moral perfection in addition to the existence of God.

But discussions of divine simplicity generally overlook the distinction between identity claims that are reductivist and identity claims that are eliminativist. If, for instance, the identity claim that ‘the chair C = the configuration of particles P’ is

merely reductive, then there exist chairs and there exist configurations of particles and it turns out that they are identical. C = P, but C exists and so does P. The identity in this case does not reduce the ontological complexity of the world. It is not as though we believed there were chairs and now, given the discovery that chairs = configurations of particles, there are no chairs. There remain configurations of particles and there remain chairs despite the fact that chairs just are configurations of particles.[1]

But if the identity claim is eliminativist, then it is true again that chairs are configurations of particles, but chairs do not in fact exist. It is true that the illusory object that we call a chair reduces to the configuration of particles without remainder. The defense of divine simplicity in this paper argues that it is in this eliminativist sense that God is identical to God's intrinsic properties. God is in this eliminativist sense omnipotent, omniscient, and omnibenevolent, and the properties of omnipotence, omniscience and omnibenevolence are nothing over and above God. These properties are illusory in the same way that the eliminated chair is illusory. The defense of divine simplicity concludes that God exists and the divine attributes just turn out to be eliminatively identical to God.

The initial sections of the paper consider various reductive strategies in defense of divine simplicity. In Section 16.2 I discuss Plantinga and reductive identity. Section 16.3 considers divine simplicity and one category ontologies. In Section 16.4 I discuss divine simplicity and austere nominalism and finally in Section 16.5 I discuss simplicity and eliminative theism.

16.2 Plantinga and Reductive Identity

The doctrine of divine simplicity is the view that there is no metaphysical complexity in God. The absolute simplicity of God

is standardly assumed to entail that everything intrinsic to God is identical to God himself. So, both constituent and non-constituent accounts of God's properties must be mistaken. The properties of God are not parts of God as the constituent view maintains and the properties of God are not exemplified or instantiated by God as the non-constituent or relationist view maintains. According to the classical account of divine simplicity in Aquinas and Augustine, God has no accidental properties at all and no parts of any kind: spatial, temporal, or material. And though there cannot be an exemplification relation or a composition relation holding between God and his essential properties—God's essence and existence—the classical doctrine of divine simplicity argues that there can be an identity relation. God's essence and existence are just identical to God. And from the fact that God's essential properties are identical to God it follows that God's essential properties are identical to each other.

But, according to Alvin Plantinga, God could not stand in the identity relation with his properties. Plantinga argues that the identity of God with the properties of God entails that God exemplifies all and only the properties exemplified by his properties.

> [...] if God is identical with each of his properties then since each of his properties is a property, he is a property—a self-exemplifying property. Accordingly God has just one property: himself. This view is subject to a difficulty both obvious and overwhelming. No property could have created the world; no property could be omniscient or, indeed, know anything at all. If God is a property, then he isn't a person but an abstract object; he has no knowledge, awareness, power, love or life. So taken, the simplicity doctrine seems an utter mistake. (→Plantinga, 1980, pp. 47–48)

The identity of God and God's properties is an example of a reductive or retentive identity. Of course, the indiscernibility of

identicals requires that God and God's properties exemplify the same properties, but the law tells us nothing about which properties must be exemplified by those objects. If it is true that pains are reductively identical to brain events, for instance, then it might be true that what were regarded as non-physical events—pains in the mind—turn out to be physical events. But it might also be true that what were regarded as physical events—neuronal events in the brain—turn out to be non-physical events in the mind. Leibniz's Laws ensure only that the pains and the brain events share the same properties, whether those are all and only the properties of neuronal events or not.

Plantinga concludes that, since God is identical to a property, God must be an abstract object. But that argument is invalid. From the premise that God is a property we might conclude instead that God's properties must be concrete objects. And for exactly similar reasons, none of Plantinga's additional conclusions follow from the premise that God is a property.

> No property could have created the world; no property could be omniscient or, indeed, know anything at all. If God is a property, then he isn't a person but an abstract object; he has no knowledge, awareness, power, love or life. (→Plantinga, 1980, pp. 47–48)

In general, we cannot conclude from the fact that God's properties are reductively identical to God that God exemplifies all of the intuitive *properties of* God's properties. It might be true instead that God's properties exemplify all of the intuitive *properties of* God. In either case, God and God's properties would exemplify exactly the same properties. It is obvious that it cannot be true both that God exemplifies all of the properties of an abstract object and that God's properties exemplify all of the properties of a concrete object. But this is common in reductive identities: it is obvious that it cannot be true that a material

event—a neuronal event—exemplifies all of the properties of a non-material event—a pain event—and a non-material pain event exemplifies all of the properties of a material neuronal event. But that's no reason to believe the reduction fails.

If God's properties are reductively identical to God, then God's properties might indeed have created the world and might possess knowledge, awareness, power, love, and life. There are in fact well-known ontologies—some one category ontologies, for instance—according to which everything—including God—is a property or composed of properties.[2] So there are well-known ontologies on which a property or a set of properties might have created the world and possessed knowledge.

16.3 Divine Simplicity and One Category Ontologies

Plantinga's argument against divine simplicity is invalid. But the metaphysics of divine simplicity remains problematic. On traditional one category ontologies such as Russellian bundle theory or trope bundle theory, bundles of universals or property particulars are understood as constitutive of objects. Objects are constituted by a compresence or co-location of properties. On this view the properties of omniscience, omnipotence, and moral perfection might constitute God. But on these traditional views there remains a perhaps fundamental ontological distinction between the constituted object and the constituting or compresent properties. These ontologies are not unequivocally one category. Properties alone are supposed to be ontologically fundamental on this account, but there is a structural distinction between the abstract constituents of God—the properties of God—and the concrete constituted object—God. Even on the assumption that God is reductively identical to his constituting properties—on the assumption, that is, that constitution just is

identity—there remains the structural distinction between constitutive properties and constituted object. The structural distinction is inconsistent with the doctrine of divine simplicity which rules out any ontological distinction at all between God and the constituents of God. The ontological distinction between God and God's constituents entails a complexity intrinsic to God that is inconsistent with the simplicity of God.

Similar problems arise for more recent one-category property theories—mereological bundle theories, for instance—according to which there is no fundamental ontological distinction between constitutive properties and constituted objects. Ordinary everyday objects like chairs, tables, people, are in fact just fusions of qualities or properties.

> [...] the mosaic theorist fiercely denies that fusing properties together to create located quality bundles gives us an emergent or otherwise irreducible category of "objects" [...] The world is purely qualitative, and spatiotemporal parts are fusions of properties (not emergent objects of any sort). We are simply building the world with n-adic properties, albeit with different sorts of properties at different compositional "levels." (→ Paul, 2017, p. 45)

Nevertheless, there is on these views a *non-fundamental* ontological distinction between constituent properties and constituted object. It is not that there are no objects, according to this version of mereological bundle theory, it is rather that there are no *fundamental* objects diverse from qualities. So, it certainly appears that there remains a distinction between objects and properties that is inconsistent with the doctrine of divine simplicity. Even non-fundamental ontological distinctions entail an ontological complexity that is inconsistent with absolute simplicity. It cannot be true, according to the doctrine of divine simplicity, that there is any—even a non-fundamental—ontological distinction between the properties of God and God.

Mereological bundle theories can be developed in ways that avoid the problem of generating a non-fundamental ontological distinction. Bundle theories might deny the existence of extended spatiotemporal objects altogether. If we extend this version of bundle theory to denying the existence of non-spatiotemporal objects as well, we can simply reject any fundamental or non-fundamental distinction between constituted objects and constituting qualities. There is in this case no ontological distinction at all between the properties of God and God. The result is a form of eliminativist bundle theory that better approximates the simplicity of God.

But among the unwelcome ontological consequences of eliminativist bundle theory is that we can say truly that the properties composing God exist, but we cannot say truly that the composed object, God, exists. As noted above, eliminativist identities contrast in important ways with reductive identities. If pains are merely reductively identical with brain events, then there exist pains, but they turn out to be neuronal events in the brain. But if pains are eliminatively identical to brain events, then there exist no pains at all. There exist only brain events and the appearance of the non-brain events we call pains are just illusions.

> [E]liminativism [about the mind is the] claim that our common-sense understanding of psychological states and processes is deeply mistaken and that some or all of our ordinary notions of mental states will have no home, at any level of analysis, in a sophisticated and accurate account of the mind. In other words, it is the view that certain common-sense mental states, such as [pains and] beliefs and desires, do not exist. (→Ney, 2021)

There is no question that eliminativist identities are ontologically simplifying. Eliminativist materialism might eliminate the entire ontology of folk-psychology, for instance. And there is also no question that the eliminativist identity of God and the properties

of God simplifies the divine ontology. But even supposing that the properties composing God—essential omniscience, omnipotence, and moral perfection—are identical to each other, the radical form of mereological bundle theory cannot provide an adequate account of the doctrine of divine simplicity. The eliminativist identity of God and the properties of God entails that 'God' is a non-referring term. The term 'God' is eliminated from theological discourse for roughly the same reason that 'phlogiston' has been eliminated from scientific discourse. We discovered that there exists nothing corresponding to the term 'phlogiston' and we discovered that there exists nothing corresponding to the term 'God.' God has been replaced with the fusion of essential omnipotence, omniscience, and moral perfection. But the doctrine of divine simplicity rejects the tradeoff of simplicity for God's existence.

16.4 Divine Simplicity and Austere Nominalism

The doctrine of divine simplicity demands more of divine reality than absolute simplicity. The absolute simplicity of, for instance, a simple property of essential omnipotence, omniscience, and moral perfection does not satisfy the requirements of the doctrine. The doctrine of divine simplicity entails that divine reality is an absolutely simple *object* and not an absolutely simple property. The absolute simplicity entails that there is no distinction—fundamental or non-fundamental—between God and God's properties. So, in general, no non-eliminativist reductive identity of God and God's properties will satisfy the doctrine. The reductive identity of God and God's properties guarantees the existence of the properties of essential omnipotence, omniscience, and moral perfection despite their identity to God. So, there will remain an ontological distinction in

God between the properties of God, and the ontologically fundamental object, God.

There are ontologies according to which there exist no abstract objects at all. It is part of austere nominalism, for instance, that we can in general regiment our talk about the properties of God in ways that eliminate all abstract singular terms. We can just as well treat all abstract singular terms as empty or non-referring terms. On this view singular terms like 'omnipotence,' 'omniscience,' 'moral perfection' do not refer to properties that God exemplifies and they do not refer to properties that stand in the constituting relationship with God. There in fact exist no exemplifiable or instantiable or constituting properties at all. There are no abstract universals or abstract particulars or tropes. There exists nothing in the world, according to the austere nominalist, except concrete particulars like plants, people and God.

But austere nominalism nonetheless affirms the truth of sentences like 'God is omnipotent' that obviously attribute properties to God. It is true that God is omnipotent, on this account, because God satisfies the predicate 'is omnipotent.' In general, atomic sentences—subject-predicate sentences like 'God is omnipotent'—are true just if they *correspond* to fundamental, irreducible, unarticulated facts in the world. The irreducible fact, in this case, is of course God's being omnipotent. There are no more basic facts that explain why it is true that God is omnipotent. Nevertheless, the world does contain the fundamental fact that God is omnipotent.

According to austere nominalism, then, it is a fundamental fact about God that God is essentially omnipotent, omniscient, and morally perfect. The fact that God is essentially omnipotent, omniscient, and morally perfect is not explained by the further fact of God's bearing any relation to another ontological feature of the world. God does satisfy the predicate, 'is omniscient,

omnipotent, and morally perfect' though there is no abstract universal or particular property of being omnipotent, omniscient, and morally perfect.

So austere nominalism does provide an account of divine simplicity that ensures that the fact that God is omnipotent does not entail that there is any ontological complexity in God involving the exemplification of abstract particulars or abstract universals. Properties are not exemplified or instantiated by God and properties do not constitute God and properties are not compresent or co-located in God. The truth of propositions attributing properties to God are fundamental facts about God for which there is no ontological explanation. These fundamental facts about God involve no further ontological articulation in God. Nevertheless, austere nominalism guarantees that we can correctly and incorrectly attribute properties to objects in the world. These property attributions describe fundamental facts, but there nonetheless remains a basic ontological diversity between objects and properties. It is true that God is omnipotent and it is false that God is identical to any of his properties. It is a fundamental ontological fact about the world that objects have properties and so it is a fundamental feature of the world that God has the properties of omnipotence, omniscience, and moral perfection. There remains an ontological distinction—albeit an inexplicable ontological distinction—between God and God's properties. The proposition is true that God is diverse from God's properties, but there is no further ontological explanation of that diversity.

Of course, the doctrine of divine simplicity rejects the thesis that God is ontologically diverse from God's properties. That proposition is false even on the assumption that the ontological diversity is fundamental and inexplicable. There is no ontological explanation in God for the diversity of God from God's properties, still it is true on austere nominalism that God is

diverse from God's properties. The doctrine of divine simplicity requires that there is no fundamental or non-fundamental ontological diversity in God at all.

16.5 Simplicity and Eliminative Theism

According to the doctrine of divine simplicity God is absolutely simple only if there is no ontological complexity in God. But there might be an ontological diversity between God and God's properties—God might not be identical to God's properties—even if there is no ontological complexity in God. According to austere nominalism, for instance, there is no ontological complexity in God, since there are no ontological features of God that ground or explain why sentences attributing properties to God are true. There are indeed true sentences predicating essential omnipotence, omniscience, and moral perfection to God. But these sentences correspond to ontologically fundamental facts. It is also an ontologically fundamental fact that God is not identical to his essential properties. So, on this view, there is ontological diversity between God and God's properties, a diversity that has no further ontological explanation in God. The properties of God on austere nominalism are distinguishable from God in spite of the fact that the diversity has no further ontological explanation.

An adequate account of divine simplicity must explain the absolute ontological simplicity of God and also the identity of God's essential properties with God. But there is no non-eliminativist identity of God with the properties of God that preserves God's ontological simplicity. The reductive identity of God and God's essential properties in fact preserves the ontological diversity of God's properties and God. Compare the identity of a chair with the configuration of particles composing the chair. If the identity is reductive, then the chair exists and the

configuration of particles exists, despite the fact that there is nothing more to the chair than the configuration of particles. The reduction preserves the ontological diversity of the chair and the particles.

But if the identity of the chair and the configuration of particles is eliminatively identical then the chair is nothing more than the configuration of particles. The chair turns out not to exist, so the eliminative identity does not preserve the ontological diversity of the chair and the configuration of particles. In the case of eliminative identities, the illusory object that we call a chair does not exist, just the configuration of particles.

Eliminative theism offers an account of divine simplicity that depends on the eliminative identity of God's essential properties with God. It is in this eliminativist sense that God is identical to God's essential properties. So, essential omnipotence, omniscience, and moral perfection are identical to God, but there in fact exist no properties of essential omnipotence, omniscience and moral perfection. These illusory properties, on further analysis, turn out to be God, just as the chair, on further analysis, turns out to be a configuration of particles. The reduction of the chair to the configuration is eliminativist.

The view that the essential properties of God are eliminatively identical to God is not merely an *ad hoc* solution to the puzzle of divine simplicity. Eliminative theists have an argument that the essential properties of God cannot exist. According to eliminative theism, if these properties did exist, then they would be *accidental* properties of a sort and there exist no accidental properties of God. The traditional properties of essential omnipotence, omniscience, and moral perfection would be, according to eliminative theism, *contingently essential* properties. Contingently essential properties are essential properties might have been different.

According to the S5 theorem $\Box F_G \longrightarrow \Box\Box F_G$ it is not possible that God should lose an essential property that he does exemplify. If God is essentially omniscient, for instance, there is no possible world in which God exists and is not essentially omniscient. According to the S5 theorem $\Diamond\Box F_G \longrightarrow \Box F_G$ it is not possible that God should acquire an essential property he does not already exemplify. If God is not essentially impassible, for instance, it is not possible that God should acquire the property of essential impassibility. But the S5 theorems do not govern the logic of the traditional essential properties of omnipotence, omniscience, and moral perfection.

The abstract singular terms 'omnipotence,' 'omniscience' and 'moral perfection' are paradigmatically vague terms. There is no discrete transition, for instance, from being morally perfect to being morally imperfect. Since these properties are imprecise, there are worlds in which God is more or less determinately morally perfect. Of course, this is a reflection of the fact that the concept of moral perfection is vague. It is not a reflection of any ontological vagueness in God.

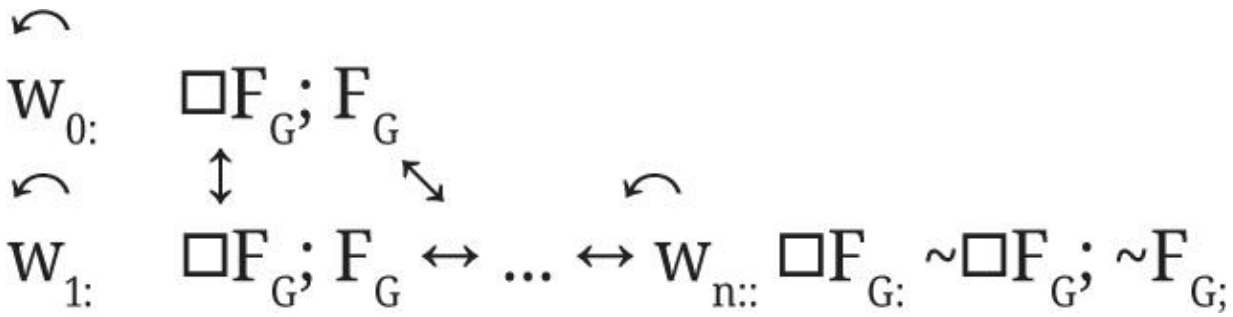
There are at least some ways to precisify the concept of moral perfection, for instance, according to which it is less than morally perfect to permit horrendous evils, no matter the reasons for permitting them. And there are worlds in which God permits horrendous evils. But this is just to say that God does not determinately fall under *our* concepts of moral perfection, omniscience, and omnipotence in every possible world, and that is not a surprising consequence. Our concepts of omnipotence, omniscience, and moral perfection include wide-ranging borderline cases.

But God can survive small changes in the determinacy of essential properties only if God's essential properties are either impossible properties or *contingent* essential properties.[3] Since God does not exemplify contingent properties or impossible

properties, God does not exemplify the essential properties of omnipotence, omniscience, and moral perfection.

Let Fx be a conjunction of properties including omnipotence, omniscience, and moral perfection. On traditional theistic views God exemplifies Fx in every possible world or $\Box F_{G.}$ If we assume $\Box F_G$ is true in w_0 and assume a logic at least as strong as S5, it follows that there is some world in which $\Box F_G$ and $\sim\Box F_G$ are both true. The assumption that God exemplifies Fx essentially entails a contradiction and so $\Box$Fx is an impossible property.

If $\Box F_G$ true in w_0 then, $\Box\Box F_G$ is also true in w_0, so $\Box F_G$ is true in every world accessible to w_0. Since every possible world simpliciter is accessible to w_0, $\Box F_G$ is true in every possible world. But, since God can survive infinitely small changes in the determinateness of essential properties, there will be some world w_n in which it is true that $\sim\Box F_G$. That is, there will be some world w_n accessible to w_0 in which God does not exemplify—or does not determinately exemplify—essential omnipotence, omniscience, and moral perfection. So, there is some world w_n in which it is true both that $\Box F_G$ and $\sim\Box F_G$. In the small model of this situation every possible world is assumed to have access to every possible world. God is determinately Fx in w_0 and w_1, but determinately ~Fx in w_n.



!@# Contradiction

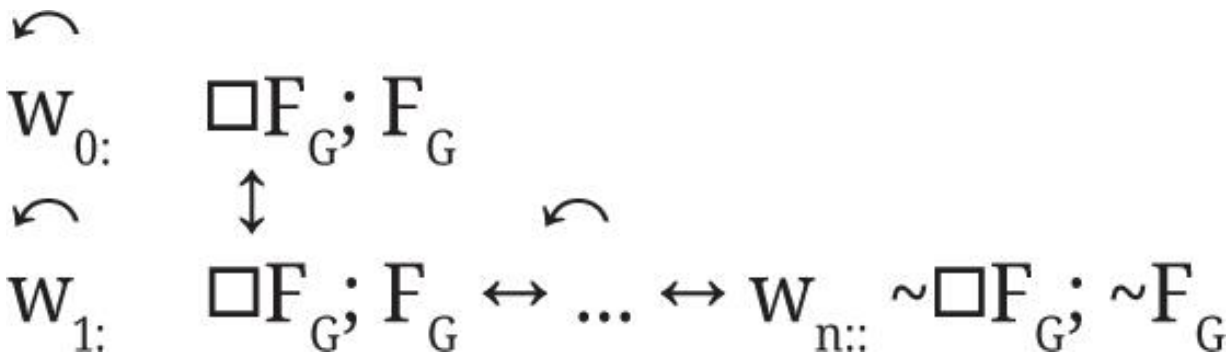
But how do we know there is some world w_n in which it is true that $\sim\Box F_G$? Since the traditional God can survive infinitely small changes in the determinacy of his essential properties, we know that there is some possible world w_n in which the traditional God is determinately $\sim F_G$. If the traditional God is determinately $\sim F_G$ in w_n, then it is true in w_n both that $\Box F_G$ and $\sim\Box F_G$. But that's impossible. The assumption that the traditional God exemplifies $\Box F_G$ in some world w_0 entails a contradiction. So the property $\Box F_G$, like the property of being a round square, cannot be exemplified.

Since $\Box Fx$ is an impossible property, there is no world in which God exemplifies $\Box Fx$. God exists, but the property $\Box Fx$ does not, and that is one way in which $\Box Fx$ might be eliminatively identical to God. It turns out that $\Box Fx$ does not exist at all, just as phlogiston does not exist. The attribution of $\Box Fx$ was an inaccurate description of God all along, just as the release of combustible bodies misdescribed combustion. It is not at all unlikely that our property attributions in the case of God are badly inaccurate descriptions. There exists no fire-like elements and there exists no property $\Box Fx$.

Alternatively, we might discover that $\Box Fx$ is a *contingently essential* property. If $\Box Fx$ is a contingently essential property, then there are some possible worlds in which it is true that $\Box F_G$ and some possible worlds in which it is true that $\sim\Box F_G$. There are some worlds in which God exemplifies essential omnipotence, omniscience, and moral perfection, and there are some worlds in which God does not exemplify essential omnipotence, omniscience, and moral perfection.

In the logic $K_{\rho\sigma}$ the S5 theorems, $\Box F_G \longrightarrow \Box\Box F_G$ and $\Diamond\Box F_G \longrightarrow \Box F_G$ are invalid. Since it is invalid that $\Box F_G \longrightarrow \Box\Box F_G$ it's possible that God, and of course anything else that exemplifies some essential properties, might lose an essential property. Since God

is essentially omniscient, for instance, it is possible that God is not essentially omniscient. Since it is invalid that $\Diamond\Box F_G \longrightarrow \Box F_G$ it is possible that God might acquire an essential property he does not now exemplify. God might acquire a human nature or essence, for instance, if he does not now exemplify a human nature or essence. Since the S5 theorems are invalid in $K_{\rho\sigma}$, there are contingent essences in $K_{\rho\sigma}$. In models for $K_{\rho\sigma}$, accessibility is not in general transitive, but reflexive and symmetric.



In w_0, God is essentially omnipotent, omniscient, and morally perfect. In w_n, God is not essentially omnipotent, omniscient, and morally perfect. On this model, all of God's properties—setting aside logically essential properties like $\Box F_G \vee \sim\Box F_G$ or $\Box F_G \longrightarrow \Box F_G$—are contingent.[4] If a property is essential, it is contingently essential. That of course is not true in S5. In S5, all of God's properties are necessary—necessarily essential or necessarily contingent. For any contingent property that God might exemplify, $\sim\Box C_G$, for instance, it is true that $\Box\sim\Box C_G$.[5] It is necessarily true that the property is a contingent property of God. It is impossible that it should be an essential property of God.[6] There are in S5 no contingent properties of any object that could be an essential property of that object. But in $K_{\rho\sigma.}$ there are objects that are contingently Cx in some worlds and essentially

Cx in other worlds—there are objects b such that ~□Cb in w and □Cb in w′. There are objects, for instance, that are essentially water in one world and essentially wine in another, and there are objects that are essentially non-divine in one world and essentially divine in another.

But if God's essential properties are contingently essential, then God cannot exemplify essential properties for the same reason that he cannot exemplify contingent properties generally. The presence of contingent properties in God entails that God might change from one world to the next and that is inconsistent with divine simplicity.

Since □Fx is a contingent (essential) property, there is no world in which God exemplifies □Fx. That is another sense in which we might discover that is □Fx eliminatively identical to God. Just like all contingent properties, it turns out that □Fx is not exemplified by God at all. The attribution of □Fx to God is just another inaccurate description of God. It is again not at all unlikely that there are serious limitations on cataphatic theology and that our property attributions in the case of God are just inaccurate descriptions.

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Notes

1 It is approximately equivalent to the view that Goliath = Lumpl and $\exists x(x = \text{Goliath})$ and $\exists y(y = \text{Lumpl})$.

2 For what it's worth, the conclusion that God is a property is not especially problematic since there are one category ontologies that do not exhibit obvious and overwhelming difficulties according to which everything —including God—is fundamentally a property or composed of properties. See for instance, Paul (2017).

3 For a similar argument that vague properties are impossible see, for instance, → Unger (2006).

4 I will use 'essential property' to refer to non-logical essential properties unless otherwise indicated.

5 Strictly, Cx is a contingent property just if $\sim\Box C_G \,\&\, \Diamond C_G$, or perhaps $\sim\Box C_G \,\&\, C_G$. So the corresponding theorem in S5 is $(\sim\Box C_G \,\&\, \Diamond C_G) \longrightarrow \Box(\sim\Box C_G \,\&\, \Diamond C_G)$. This is not true in $K_{\rho\sigma}$.

6 Though, of course, Cx might be an essential property of some other object. It might be that you are essentially rational, for instance, but your dog is contingently rational.

17 Why Divine Simplicity Is Unnecessary

Stephen T. Davis

Abstract: Although not a believer in the doctrine of divine simplicity, in the present paper I do not argue that it is incoherent or even false. I instead ague that it is unnecessary to protect what Christians want to say about God. It seems to me that three main considerations motivate the doctrine. First, if there is any complexity in God, there must also be potentiality in God, which is unacceptable. Second, the doctrine is designed to ensure logical consistency with other things classical Christians want to say about God. Third, the doctrine is designed to protect the divine transcendence over all of creation. I argue against these three points.

17.1

The doctrine of divine simplicity (DDS) says that God is metaphysically simple; that is, God has no physical or metaphysical parts. Indeed, in God there is no complexity at all. This is the basic idea. But several corollaries follow from it: (1) Unlike human beings (and indeed unlike all other existing things), God's essence *is* God's existence; God is existence itself. (2) No real distinction can be made between any of God's various attributes, because God is identical to each of God's intrinsic properties. What look like different divine properties are actually just different ways for us to grasp the one reality that is God. Items like "divine power" and "divine goodness" are not linguistic synonyms but the point is that they refer to the same reality. (3) There is no contingency in God; all God's intrinsic

properties are essential properties. (4) God is not a member of any class or set, not even the set of beings or things.

It is clear that DDS is puzzling. And it is puzzling in at least two ways. First, it is difficult to come to understand the doctrine. You can break your head thinking about it. Second, it is difficult (at least for me) to grasp why its defenders believe that the doctrine is needed—indeed, some hold that DDS is the lynchpin of everything else that Christians want to say about God.[1]

In the present essay I will not argue that DDS is incoherent. I suspect that in the end it is, although in recent years supporters of DDS have skillfully defended it against several criticisms.[2] After making a preliminary point and explaining my assumptions, I will instead attack what I take to be the three main motivations for the doctrine. I will first ague briefly against Aquinas' implicit claim that if there is any complexity in God, there must be some potentiality in God, and that since God is pure act, that is impossible. I will argue secondly that consistency with other things Christians say about God—especially God's relation to items like power, knowledge, time, and change—does not entail DDS. Finally, I will argue, in much more detail, that DDS is not needed to protect what Christians want to say about the divine transcendence over all of the creation.

Here is the preliminary point (which I will not explore in any detail): I strongly suspect that DDS is inconsistent with one of the things that Christians must crucially say about God, viz., that God is a Trinity. I am aware that defenders of the doctrine argue that the three Trinitarian Persons do not amount to proper parts of God. Perhaps they are right about that. But DDS makes a stronger claim than that God has no proper parts—as noted, it claims that in God there is no complexity at all. God is, as Jeffrey Brower puts it, "completely devoid of any metaphysical complexity."[3] Richard Howe asserts that "God is not composed

in any way."[4] And I do not see how believers in the orthodox doctrine of the Blessed Trinity can say that.

17.2

In this essay, I am making three assumptions. First, and most importantly, I assume that God is the creator of all contingent things, and thus is radically different from all of them. If God is not transcendent, we are not talking about the Christian God. On the other hand, some theologians, perhaps under the influence of Anselm's brilliant idea that God is not just the greatest actual being but the greatest conceivable or possible being, try to push transcendence as far as it can possibly go. There is, as some of them say (e.g., Kierkegaard,[5] Barth[6]), "an infinite qualitative difference between God and the creatures." I will resist that notion. I do not think an infinite distance can be crossed or bridged, not even by an omnipotent being.[7] If it were true that the distance between God and humans is infinite, we could have no knowledge of God whatsoever.

My second assumption is that God is a necessary being, has aseity. A necessary being: (1) exists and cannot not exist (its possibility entails its existence); (2) depends on no other thing for its existence; and (3) is everlasting, in the sense of never failing to exist.[8] I am a contingent being because I have a finite lifespan and because when I do exist I depend on other things both for my coming into existence (e.g., my parents) and for my continuing existence (e.g., the air I breathe and the food I eat). God, as the First Cause of all contingent existence, is not like that. God is metaphysically independent.

My third assumption is that human beings can make true positive statements about God, e.g., "God is good" or "God is powerful." Because of the huge difference between ourselves and God, however, we do not fully (or even very much)

understand those statements, but they are true nonetheless. We are not limited to negative statements (e.g., “God is not a horse”). The *via negativa*, in my opinion, does not tell us enough about God.

17.3

As noted, I will deal with what I take to be three main motivations for affirming DDS. The third is the most important.

We can deal with the first motivation briefly. It is this: Aquinas and his followers seem to hold that if there is any complexity in God, then there is also potentiality in God. But it is an absolutely crucial belief of Aquinas that there is no potentiality in God. God is already everything that God can possibly be. God cannot potentially be wiser or more compassionate or more powerful than God is and always has been.

Part of this is entirely correct: it goes without saying that there is no potentiality in God so far as God’s intrinsic properties are concerned. God is not growing wiser or kinder. But it seems clear to me (since I do not accept divine timelessness) that there must be a kind of potentiality in God—what we might call extrinsic potentiality. Surely we can say that God is now potentially but not actually in the state of knowing, for example, that Jesus has returned or that the last judgment has occurred.

But I will leave that point aside since believers of DDS are also believers in divine timelessness, and that complicates the point. More importantly, I will just say that I do not see why complexity in God entails potentiality in God’s intrinsic properties. If somebody could spell that point out, show and not just assume or assert that the one logically follows from the other, I might be convinced. But I do not think that has been done. It certainly does not follow that if God has n parts (where n

is greater than 1), then potentially or even possibly God has n + 1 parts.

Katherine Rogers, in defending an Anselmian notion of divine simplicity, offers an interesting version of Aquinas' argument. She writes:

> [...] anything thought to be composed of parts can be thought to be divisible, and hence its ceasing to exist is conceivable. But then a greater being can be thought—one that cannot even be thought to cease to exist. God, then, cannot be thought to be composed of parts. And classical thesis understood this claim in a very strong sense. God just *is* all the attributes we ascribe to him. But further, His knowing and doing are one and identical with Hid nature.[9]

Of course there are different senses of the word "conceivable." (Anselm himself makes this point in *Proslogion* IV.)[10] It might simply mean something like "imaginable," but that will not help the defender of divine simplicity. I can imagine virtually anything; I can imagine logically impossible things, like the Pythagorean Theorem being false, or like Anselm's fool imagining God's nonexistence. But if "is conceivable" instead means something like "is logically possible," then God's having parts will not entail that God's nonexistence is conceivable as long as God's parts are themselves necessary and necessary parts of God (more on this below).

17.4

The second motivation for affirming DDS is to ensure consistency with the divine attributes in what we might call the classical Christian view of God, as found in an inchoate way in many of the Church Fathers, in several medieval theologians, and especially in Aquinas. I am thinking of items like divine

eternity (in the sense of timelessness), immutability, impassability, and necessary existence.

I want to make two points about this motivation. The first is the fact that several of the attributes just mention are very much negotiable. I consider necessary existence to be, so to speak, theologically required, as already noted. And I will argue later that necessary existence does not entail simplicity. But the others on the list are rejected by many contemporary Cristian thinkers. Of course that does not mean that those theologians are correct. For myself, I accept divine immutability only in a very restricted sense; I believe that God's intrinsic nature is immutable, but that God changes in other ways, e.g., in God's actions and non-intrinsic attributes; and I do not accept divine impassability at all. So my point here is that the motivation to remain consistent with the classical view of God is not shared by all Cristian thinkers.

But at a deeper level I do not see why timelessness, immutability, or impassability entail or require simplicity. I will not argue this point in detail, but I do not see why a timeless being must be metaphysically simple. For example, I do not see why a triangle (consisting as it does of parts, e.g., three sides and three angles) cannot be timeless. I do not see why a changeless thing must be metaphysically simple. If our triangle can be timeless, it must also be intrinsically changeless. And I do not see why an impassable thing must be metaphysically simple.

17.5

What then is the third and most crucial motivation for DDS? What is it that (in my opinion, at least) most deeply drives such luminaries as Augustine, Anselm, Aquinas and its contemporary defenders to embrace the idea that God is metaphysically simple? It is clearly the desire to protect the divine difference

from and transcendence over all of creation. God's aseity, God's status as a necessary being, and God's status as the First Cause (to the extent that these three are different) are all designed to do that very thing. The basic insight of defenders of DDS is that no complex object can be a First Cause, a Necessary Being, or exist *a se* (from itself). I do not agree with that claim.

At the risk of naively ignoring important hermeneutical issues, let me cite various texts from scripture that have led Christians to stress the huge difference between God and the creatures.[11] Such texts apparently entail that God is far greater than and superior to the whole of creation; God is not like a powerful and grand human being; and we are quite unable to comprehend fully God in God's essence. As Job cries out about God, "If I go forward, he is not there; or backward, I cannot perceive him; on the left he hides, and I cannot behold him; I turn to the right, but I cannot see him" (Job 23:8–9).

I will mention eight items.

1. Unlike all versions of polytheism, Christianity insists that God is *one* (Deuteronomy 6:4–6; I Corinthians 8:4–6); that is, God is absolutely unique (Isaiah 40:13–28); there is no other like God (Psalm 113:4–6; Isaiah 43:10; 44:6–8; 45:21–22).
2. Unlike the limited gods of some religions, God is both *sovereign* (Deuteronomy 10:14; Isaiah 45:5–7; 46:9–10; Jeremiah 10:6–7) and *transcendent* over all existing things, including human beings, in the sense of being far superior to them in power, knowledge, authority, and moral character (Numbers 23:19; Psalm 50:21; Hosea 11:9; Romans 9:20).
3. God is a *spirit* (2 Chronicles 6:18; Jeremiah 23:24; John 1:18; 4:24; 1 Timothy 1:17; 6:16), not a perceivable physical object like rocks and trees and human beings.

4. Contrary to all views that make God limited or dependent on other beings, God is *everlasting, uncreated*, and (at least in essential respects) *changeless* (Psalms 90:1–4; 102:25–27; Malachi 3:6; James 1:17; Revelation 22:13); there is no moment when God does not exist and does not exist as God.[12]
5. God is not a part of the natural order because God is its *creator* (Genesis 1:1; 2:4; Psalms 8:3–5; 33:6,9; 100:3; Isaiah 41:4; 44:24; 64:8; Zechariah 12:1; Romans 4:17; Hebrews 11:3; Revelation 4:11).
6. Unlike human beings, God is *all-powerful* (Genesis 18:14; Job 42:2; Psalms 115:3; 135:6; Jeremiah 32:17, 27; Daniel 4:35; Matthew 19:26; Ephesians 1:19) and *all-knowing* (Psalms 139:1–6; 147:5; Hebrews 4:13; I John 3:20).
7. Unlike human beings, God is *perfectly morally good* and *loving* (Psalms 100:5; 107:1; Mark 10:18); there is no evil in God and God works for the salvation of human beings.
8. Finally, to us, God is incomprehensible in the sense of being beyond our ken (Isaiah 45: 15 [“Truly, you are a God who hides himself”]; Numbers 23:19; I Samuel 15:29; Isaiah 46:5; 55:8–9; Hosea 11:9). And I must point out that none of these eight points entail or require divine simplicity.

But we must not miss the fact that there is also ample scriptural support for the idea of similarities or close relationships between God and human beings. God is immanent. Let me mention four items.

1. Human beings were created *in God’s image* (Genesis 1:26–27). There have been countless discussions and disagreements in the tradition over what the “image of God” consists in. I will not try to solve that problem here.

Suffice it to say that the notion certainly seems minimally to suggest that we human beings are similar to God in some important ways. And since “being similar to” is a symmetrical relation (if A is similar to B, then B is similar to A), it follows that if we are similar to God in some ways, then God is similar to us in some ways.

2. God is *personal*. This notion denies all views in which an impersonal absolute (e.g., Brahman, the Dharma, the Tao, absolute emptiness) is ultimate reality. It also implies that God, like us, is a person (not, of course, in the same way that we are persons); that is, God has knowledge, memory, and desires, and formulates intentions and works to achieve them. It also means that, unlike the absentee God of the eighteenth-century Deists, God lovingly cares about the creation and works in it to achieve his purposes for it.
3. God does not remain silent; rather, God *reveals himself* to us (Amos 4:13; Matthew 16:17; John 1:14; Galatians 1:11–12; Hebrews 1:1–4). God speaks a word that humans can comprehend, a revelation that in part consists of human words. God chooses to reveal himself to human beings by being involved in human life and history. The purpose of revelation is the accomplishment of God’s purposes; God desires that human beings come to obey, worship, and love God. Christians hold that it is preeminently through Christ that this can happen.
4. In the incarnation of Christ, God became one of us and lived on earth as a man (John 1:14; Philippians 2:5–11); accordingly, we can learn about God by learning about Jesus.

17.6

So God is both transcendent and immanent. Some theologies give precedence to one side over the other. Anyone who wants to push divine transcendence as far as possible[13] may well be inclined toward DDS. The doctrine surely amounts to maximal transcendence. The danger of going too far in the direction of transcendence is the danger that God becomes hidden, unreachable, and irrelevant to human life and concerns.

The gods of the ancient Epicureans, for example, were ideal exemplars of *ataraxia* (calm, serenity, imperturbability), but they lived far away from the earth and had nothing to do with human affairs. The God of eighteenth-century Deism, who (so it was said) created the world and its natural laws and processes but never interfered thereafter, also seems too transcendent. So the danger of going too far toward transcendence is that we arrive at a God who has little to do with us and about whom we can know little.

The danger of going too far in the direction of immanence is either anthropomorphism or idolatry (or both). In the present context, anthropomorphism is the tendency to reduce the Creator to a creature, to suppose that God is very much like human beings. The ancient Greek pantheon, for example, was anthropomorphic; gods like Zeus, Athena, and Ares were indeed far more powerful than human beings and did not have to die, but in most other ways they were similar to human beings. The God or gods of contemporary Later Day Saint theology are also, in my opinion, not transcendent enough.

Sensible Christian thinking about God—so it seems to me—must be done in tension, so to speak, between the apparently opposing poles of transcendence and immanence. It is essential for Christians to hold that both sorts of claims are true and that neglect of one side over the other leads to error. God is both transcendent *and* immanent. The one must not detract from the other.

17.7

What follows from the claim that God is both transcendent and immanent? If I can put it this way, it follows that God must be radically different from the creatures, but not *too* different. Of course none of the divine perfections (e.g., omnipotence, omniscience, perfect goodness) are shared with human beings; still, there must be some commensurability between God and human beings. Otherwise, God could have nothing to do with us (e.g., by revealing himself to us), or us with God (e.g., by our praying to God).

Is it possible to deny DDS and still firmly hold on to transcendence? That is, is it possible for a complex God to be a necessary being, to exist *a se*, to be the First Cause (i.e., the Creator)? It seems clear to me that the answer is yes. But one barrier remains. It is a crucial claim made by Aquinas (and often repeated in the literature) to the effect that, "Every composite thing is posterior to it components and dependent on them."[14] And it is certainly true that all created things are complex and depend upon their parts.

Let's bite the bullet; let's assume that God is complex. To keep things simple, God, we will say, consists of two parts: Part A and Part B. What sorts of parts do I have in mind here? Certainly metaphysical parts. More than that I am unable to say. But perhaps A and B are parts of God in a similar sense in which, according to substance dualists, the soul is part of the person (the other part being the body). And according to people like Plato, Freud, and many others, even the soul (or in Freud's case, the unconscious mind) has parts.

Does the above inference—that every composite thing is posterior to its parts and depends on them—follow in this case? There are two claims here, which we can express as questions. First, must a composite thing be "posterior to its parts"? I

assume Aquinas intends not temporal posteriority here but a kind of logical or explanatory posteriority, as for example the sense in which "if A, then B" and "A" are logically prior in *modus ponens* to "B." And it is true that many times we understand or explain the parts of a thing before we understand or explain the thing. But of course that is not always true. Most of us knew what *human beings* were well before we knew what spleens or tibias or even big toes were. Most of us certainly understand what *space* is well before we know its dividing points or intervals. Nor is it true in this case. If God is composite, God is clearly logically prior to God's components. God is obviously not built up out of parts; rather, any parts that there are exist only because God exists.

Second, if God is a composite thing (in the way we are hypothesizing), must God depend on Parts A and B? No, not in any theologically or metaphysically untoward way. Recall that God is a necessary being; moreover, all God's parts are essential to God and God's parts cannot possibly be separated from each other or from God. So the inference does not follow. Nor, if God is a necessary being, is any explanation required as to why God's parts are related to each other and to God as they are. It is simply an essential aspect of reality that God's parts are related to each other and to God as they are.

Are the two parts ontologically distinct from God? Yes, if that just means that they are conceptually distinct, can be imagined as distinct. Is that problem? No, not if they are, as I say, essential parts of God and have no possible separate existence. Does God depend on Part A and Part B for God's existence or nature? Of course not: God is a necessary being. God does not depend on God's parts in the way that we depend on ours. Is it true that if A and B did not exist, God would not exist? Conceptually, yes. But it is not possible for A, B, and God not to exist.

Someone might object that given what I have just argued, Part A and Part B are not really parts at all because they do not differ from each other. Not at all—so I reply—they are indeed distinct. Part A is not the same thing as Part B; Part A is not the same thing as God; and Part B is not the same thing as God. God consists of Part A and Part B (plus perhaps other components, since I have not claimed that Part A and Part B together exhaust God).

17.8

My claim in the present paper is a modest one. I have been arguing that a complex God can be the First Cause, a necessary being, and have aseity. If that is true, the major motivation for DDS is not compelling. I am not claiming that a metaphysically simple God cannot be immanent and related to the world and to human beings in the appropriate ways. Although that may be true, I have presented no arguments along that line.

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Notes

1 Aquinas, for example, begins his investigation of the divine attributes with simplicity. *Summa Theologica* (→Aquinas, 1947), 1, 3, and 7.

2 I am thinking especially of Brower, Leftow, Rogers, and Stump.

3 →Brower, 2009, p. 99.

4 In his paper at the American Academy of Religion in Denver in 2019.

5 →Kierkegaard, 1985, p. 138.

6 →Barth, 1937, p. 10.

7 See my (2017).

8 I am not going to explore the status of what are sometimes called created necessary beings (e.g., numbers, sets, etc.).

9 →Rogers, 2020, p. 309.

10 →Anselm, 1963, pp. 7–11. He calls it “conceiving of the word” as opposed to “conceiving of the thing.”

11 This and the following two paragraphs are essentially taken from my (2009).

12 I will not explore the question whether these texts require complete divine immutability and timelessness.

13 Perhaps inspired by Romans 11:33: “O the depths of the riches and wisdom and knowledge of God! How unsearchable are his judgments and how inscrutable his ways!”

14 Aquinas, *Summa Theologica* (→Aquinas, 1947), 1, 3, 7.

18 God and the Paradox of Ineffability

Graham Priest

Abstract: In Christian theology it is standard to claim that God is ineffable. Moreover, arguments are given for this. Clearly, however, if such arguments work, God obviously is effable. We thus have an apparent paradox to the effect that God both is and is not ineffable. There are some standard ways of trying to defuse the paradox. However, such moves face well-known problems. A much more unorthodox and radical approach is to accept the paradox at face value: God is truly a contradictory object, both effable and ineffable. In recent years, the techniques of paraconsistent logic have been used to show how to make sense of paradoxes of various kinds, such as those of self-reference, simply by accepting their contradictory conclusions. These techniques may also be deployed to make sense of this theological paradox in the same way. In this essay I will show how.

18.1 Introduction

In Christian theology it is standard to claim that God is ineffable. Moreover, arguments are given for this. For example, it may be claimed that God is so different from God's creatures that human concepts (the only ones we have) cannot be applied. Clearly, however, if such arguments work, God obviously is effable. We thus have an apparent paradox to the effect that God both is and is not ineffable. There are some standard ways of trying to defuse the paradox. For example, it has been claimed that one cannot *assert* anything of God; all one can do is *deny*

things. Or it has been claimed that one cannot say anything literally true of God, but only non-literal things—maybe of an analogical kind. However, such moves face well-known problems.

A much more unorthodox and radical approach—pursued, for example, by Nicholas of Cusa—is to accept the paradox at face value: God is truly a contradictory object: both effable and ineffable. In recent years, the techniques of paraconsistent logics have been used to show how to make sense of paradoxes of various kinds, such as the liar paradox and Russell's paradox, simply by accepting their contradictory conclusions. These techniques may also be deployed to make sense of this theological paradox in the same way. In this essay we will see how.

In the first main part of the essay, we will have a look at some of the history of the matter. This does not pretend to be a comprehensive discussion. It could not be: one could write a book (indeed, many books) on it. I provide the discussion simply to indicate both the importance of the paradox to Christian theology and its thorny nature. After a brief segue, we then turn to how one may apply the techniques of paraconsistent logic to handle the matter.

18.2 God and Paradox

18.2.1 The Perfections

But first, let me put the whole discussion in the much more general context of the paradoxes which the Christian notion of God generates. It is orthodox in Christian theology to attribute to God certain extreme properties, such as omnipotence, omniscience, moral perfection, and so on (the so-called perfections). Like most limit properties, and as is well known,

these teeter on the brink of contradiction—or, if you believe in the existence of God, paradox.[1] Take, for example, omnipotence. To be omnipotent is to be able to do anything. However:

- Since God is omnipotent, they[2] can create a stone so heavy that they cannot move it.
- So it is possible that there is a stone they cannot move.
- But since God is omnipotent it is impossible for there to be a stone they cannot move.

The standard solution to the paradox is that suggested by Aquinas. Since God can do anything, it is impossible that there is a stone that God cannot lift. So creating such a stone is an impossibility. And even God cannot do the impossible. So the first premise is false.

But matters are not so straightforward. Creating a stone that God cannot move may be impossible. But creating a stone that its creator cannot life is not an impossibility. Give me enough clay, and I can do it. But God cannot do it. So God is not omnipotent.

The *combination* of perfections also leads to apparent contradiction. God is morally perfect. Morally perfect entities do not do harmful things. So God does not do harmful things. No contradiction there. But God is usually taken to have the perfections necessarily. So it is necessary that God does not do harmful things. That is, God cannot do harmful things. So God is not omnipotent. And again, doing harmful things is not at all logically impossible. I need much less than lots of clay to do this myself.

Of course, arguments of this kind are well known, and it is not my intention to go into all the things that have been said about them here.[3] I point them out to show that the Christian concept of God is deeply entangled in paradox. The rest of this

essay is about just one of these, though since it concerns the very possibility of talking about God at all, it is perhaps the most fundamental of the bunch.

18.2.2 Enter Ineffability

The paradox in question concerns God's ineffability. That God is ineffable is a standard claim of Christian theology. What drives it is the simple pious thought that God the creator is so different from God's creatures, that no concepts such creatures can form, or words they have, can be applied to God. But as hardly needs to be said, Christians, and theologians who theorize Christianity, say a great deal about God. Here, then, is our paradox. God cannot be talked about, and yet they are. As Karl Barth puts it:[4]

> As ministers we ought to speak about God. We are human, however, and so cannot speak of God. We ought therefore to recognize both our obligation and our inability [...].

The contradiction is, of course, obvious, and has hardly passed the notice of Christian theologians. In response to the paradox, two main strategies, broadly speaking, have appeared to address it—though one may find strands of both in many theologians. These are often called *kataphatic* and *anophatic*. The kataphatic approach is to the effect that one can say things about God, though they may not mean what you think they do. They have to be understood as analogies, or in some other non-literal way. The anophatic approach is more hard-nosed. One cannot say anything positive about God. All one can do is say what God is not. This is the *via negativa*, the negative way.

There is a rich tradition of Christian theology in the history of all this; and this is not the place to tell it. However, in the first

half of this essay, let us look at some of it, to give us a sense of the lie of the land.

18.3 Some History

18.3.1 Plotinus

Let us start with the anophatic approach. The ineffability of God certainly has Biblical roots.[5] But philosophically it derives from Neoplatonism. Thus, Plotinus (204/5–270)[6] describes his ground of reality, the One, saying:

> Knowing is a unitary thing, but defined: the first is One, but undefined: a defined One would not be the One-absolute: the absolute is prior to the definite.
>
> Thus The One is in truth beyond all statement: any affirmation is of a thing; but the all-transcending, resting above even the most august divine Mind, possesses alone of all true being, and is not a thing among things; we can give it no name because that would imply predication: we can but try to indicate, in our own feeble way, something concerning it: when in our perplexity we object, “Then it is without self-perception, without self-consciousness, ignorant of itself”; we must remember that we have been considering it only in its opposites. (*Ennead* V, 3: 12, 13; → Plotinus, 1991, p. 379f.)

In other words, the ground of beings cannot itself be a being: it cannot be a *this*, rather than a *that*. It cannot be characterized, that is, described. When we try to say something all we can do is (feebly) say what it is not. But note that Plotinus says a lot of very positive things about God in the process of explaining this. As far as I know, he does not address the contradiction, though there are certainly hints of dialetheism in the *Enneads*.[7]

18.3.2 Pseudo-Dionysius

The idea was taken up by the Christian Neo-Platonist known only by his pseudonym, Dionysius the Areopagite, now therefore usually referred to as ‘Pseudo-Dionysius’ (fl. late 5th, early 6th century).[8] Pseudo-Dionysius does not deny that there are ‘names’ (characterizations) that creatures can apply to God, such as *over-good, over-being*, but these are merely ‘symbolic.’ In the last instance, all names for God, including these, must be denied. All one can say is that God is none of these things. Thus, in Chapter 5 of his *The Mystical Theology*, we have:

> Once more, ascending yet higher we maintain that It [GP: the Godhead] is not soul, or mind, or endowed with the faculty of imagination, conjecture, reason, or understanding; nor is It any act of reason or understanding; nor can It be described by the reason or perceived by the understanding, since It is not number, or order, or greatness, or littleness, or equality, or inequality, and since It is not immovable nor in motion, or at rest, and has no power, and is not power or light, and does not live, and is not life; nor is It personal essence, or eternity, or time; nor can It be grasped by the understanding since It is not knowledge or truth; nor is It kingship or wisdom; nor is It one, nor is It unity, nor is It Godhead or Goodness; nor is It a Spirit, as we understand the term, since It is not Sonship or Fatherhood; nor is It any other thing such as we or any other being can have knowledge of; nor does It belong to the category of non-existence or to that of existence; nor do existent beings know It as it actually is, nor does It know them as they actually are; nor can the reason attain to It to name It or to know It; nor is it darkness, nor is It light, or error, or truth; nor can any affirmation or negation apply to it; for while applying affirmations or negations to those orders of being that come next to It, we apply not unto It either affirmation or negation, inasmuch as It transcends all affirmation by being the perfect and unique Cause of all things, and transcends all negation by the pre-eminence of Its simple and absolute nature—free from every limitation and beyond them all.
> (→Dionysius the Aeropagite, 1920)

In other words, all one can do is deny anything said of the Godhead—even the cardinal points of Christian theology.[9]

The passage also gives a reason for this in the last couple of sentences. It transcends language because it is perfect, simple,

and beyond limitation. The thought is that to describe the Godhead in any way with human concepts—what others do we have?—is to limit it; and the Godhead transcends any such limitation. Never mind the cogency of this argument, note that the reason given characterizes the Godhead in positive terms. And in doing so, it does exactly what it claims cannot be done.[10]

Moreover, the attempt to solve the problem with the *via negativa* is problematic in its own right. The Godhead cannot be characterized by human concepts. But negation itself is a human concept. So one cannot even describe the Godhead using negation, as Pseudo-Dionysius notes in the penultimate sentence. As hardly needs pointing out, however, he has just spent a whole long paragraph doing just that. In truth, there is a serious problem about a relentless *via negativa*. This may be illustrated by the following conversation between a Christian (*C*) and a non-Christian (*NC*):

1. I hold there to be an entity worthy of worship.
2. Is it a spiritual entity?
3. No.
4. Well, is it an intellect of some kind?
5. No.
6. Hm … So is it a creator of some kind?
7. No.
8. Well, tell me *something* at least.
9. I'm afraid I can't. It is beyond all characterization …
10. Then how can I know what you're talking about?
11. Indeed, it's not even true to say that it's worthy of worship.
12. ???

It's exactly this bind which motivates a kataphatic approach. So let's turn to this.

18.3.3 Anselm

As an example of this approach, let us consider Anselm of Canterbury (1033–1109).[11] In Chapter 2 of his *Proslogion* Anselm gives his celebrated version of the Ontological Argument for the existence of God, based on the definition of God as a being greater than which cannot be thought. However, in the lesser known Chapter 15, he explains that God is a being *greater* than can be thought:

> Therefore, Lord, not only are You that than which a greater cannot be thought, but You are also something greater than can be thought. For since it is possible to think that there is such a one, then, if You are not this same being something greater than You could be thought—which cannot be. (→Anselm, 1998, p. 96)

The reason is essentially that if God were not a being greater than which could be thought, one could think a different being that was. This would be greater than God which, by definition, is impossible.

Let us not go into the cogency of this reasoning here.[12] The important point to note is that Anselm is saying that God is greater than can be thought; and that means conceived; and that means described. For if God could be described, they could be conceived in those terms, so they could be thought. Yet clearly Anselm himself is describing God, in writing these very passages.

Anselm is well aware of the issue here. In the slightly earlier *Monologian*, he addresses the matter. In Chapter 65, entitled "How we came to true conclusions about something ineffable," he says:

> We do often speak of lots of things without expressing them properly, i.e. in the way proper to the way they are. What we do, when we cannot, or

> will not, utter something properly, is to signify it by means of something else—a riddle for example. And often we do not see something properly (i.e. as it is), but we see it by means of some likeness or image—when, for example, we make out someone's face in a mirror. Thus we say and do not say, see and do not see, one and the same thing. For it is through something else that we say it, and we see it. But through what is proper to it, we do not. This line of reasoning, therefore, allows our conclusions about the supreme nature to be true and the supreme nature itself to remain ineffable. We understand them to be indicating the supreme nature by means of something else (*per aliud*), rather than expressing it by means of what is proper to its essence. The names, then, that are apparently predicable of the supreme nature, merely gesture towards it rather than pinpoint it. They signify via some sort of similarity, not through what is proper. (→Anselm, 1998, p. 71)

In other words, our language does not apply literally to God. Rather, what we say is *similar* to what is true of God. This is not a comfortable position. When Anselm defines God as a being no greater than which can be thought, he is not saying that God is similar to this. He means it quite literally. (The Ontological argument is not intended as a riddle.) Indeed, when he says that one can say only things of God that are similar, this is meant literally. He is not saying that we can talk of God only in a way that is similar to being similar.

18.3.4 Aquinas

Let us move on to another important figure in the kataphatic tradition, Thomas Aquinas (1225–1274).[13] Thomas holds that one cannot apply concepts applicable to human things to God literally. One can, however, apply them analogically, as when we say that the music is sad. (Strictly, it is only people who can be sad, but the music may naturally be described as sad because it makes us feel sad.) Thomas' view of how one should understand such analogical predication was a complex and evolving one. Moreover, it depends on a problematic notion of causation.[14]

However, setting these things aside, his view has the familiar problems. He explains why things that are said literally of a creature cannot be said of God as follows:

> [...] it is impossible for anything to be predicated univocally of God and a creature: this is made plain as follows. Every effect of an univocal agent is adequate to the agent's power: and no creature, being finite, can be adequate to the power of the first agent which is infinite. Wherefore it is impossible for a creature to receive a likeness to God univocally. Again it is clear that although the form in the agent and the form in the effect have a common ratio, the fact that they have different modes of existence precludes their univocal predication: thus though the material house is of the same type as the house in the mind of the builder, since the one is the type of the other; nevertheless house cannot be univocally predicated of both, because the form of the material house has its being in matter, whereas in the builder's mind it has immaterial being. Hence granted the impossibility that goodness in God and in the creature be of the same kind, nevertheless good would not be predicated of God univocally: since that which in God is immaterial and simple, is in the creature material and manifold. (→Aquinas, 1952, Q 7, Art 7)

In the last sentence, in arguing that concepts that apply literally to one of God's creatures cannot apply literally to God, he predicates *that which is in God* of both God and creatures, saying that *it* means different things in the two cases. Indeed, even the conclusion of his argument has the same problem. This states that it is impossible to predicate something of God and one of their creatures univocally. This applies the (very human) concept of predication to God. Moreover, it is applied to God's creatures in the same sentence. And if this did not mean the same thing in both cases, the intended contrast would misfire. The claim, then is self-refuting.

Moreover, when Thomas tries to get himself out of the problem of talking about God, by saying that one can say something analogous, he digs himself deeper into the hole. He says:

> We must accordingly take a different view and hold that nothing is predicated univocally of God and the creature: but that those things which are attributed to them in common are predicated not equivocally but analogically [...]. [In this] a thing is predicated of two by reason of a relationship between these two: thus being is predicated of substance and quantity. (→Aquinas, 1952, Q 7, Art 7)

In the claim that "nothing is predicated univocally of God and the creature" the same thing is predicated of God and creature —namely predication. The same problem appears a few sentences later, as does a binary-predicate analogue of the same problem. Thomas talks about a relation between God and creature. But it is one and the same relation that holds between God and creature.

18.3.5 Cusanus

Let us finish this somewhat whistle-stop tour of history with a final Christian theologian who has a very distinctive position on the matter. This is Nicholas of Cusa (1401–1464).[15] The important text for our purposes is his *On Learned Ignorance*.[16] Nicholas is in the Neoplatonic tradition, as is clear from his summary towards the end of the text:

> Sacred ignorance has taught us that God is ineffable. He is so because He is infinitely greater than all nameable things. And by virtue of the fact that [this] is most true, we speak of God more truly through removal and negation—as [teaches] the greatest Dionysius, who did not believe that God is either Truth or Understanding or Light or anything which can be spoken of [...]. Hence, in accordance with this negative theology, according to which [God] is only infinite, He is neither Father nor Son nor Holy Spirit. Now, the Infinite qua Infinite is neither Begetting, Begotten, nor Proceeding. (→Hopkins, 1985, ch. 26, p. 45)

Why is God ineffable? Nicholas cites the usual reason. God is infinite. Human categories ('names') apply the things of our

familiar world, and God is infinitely greater than any of these. He puts a very distinctive spin on this matter, however. Following Anselm, he defines God as that than which there can be no greater, and so is the Maximum:

> Now, I give the name “Maximum” to that than which there cannot be anything greater. But fullness befits what is one. Thus, oneness—which is also being—coincides with Maximality. But if such oneness is altogether free from all relation and contraction, obviously nothing is opposed to it, since it is Absolute Maximality. Thus, the Maximum is the Absolute One which is all things. And all things are in the Maximum (for it is the Maximum) [...]. (→Hopkins, 1985, ch. 2, p. 6)

Since there can be nothing greater than the Maximum, it must contain all things within it. Moreover, since it does so, there are within it no oppositions of the kind required to apply a characterization:

> Since the Maximum is the unqualifiedly Maximum, to which nothing is opposed, it is evident that no name can properly befit it. For all names are bestowed on the basis of a oneness of conception [*ratio*] through which one thing is distinguished from another. But where all things are one, there can be no proper name. Hence, Hermes Trismegistus rightly says: “Since God is the totality of things, no name is proper to Him; for either He would have to be called by every name or else all things would have to be called by His name”; for in His simplicity He enfolds the totality of things. (→Hopkins, 1985, ch. 24, p. 40)

Again, leaving the cogency of this reasoning aside, it is clear that Cusanus faces the paradox of ineffability. For in his very explanation of why God is ineffable, he clearly attributes many (positive) properties to him. How is this to be accommodated? To handle matters, Nicolaus makes a remarkable move:

> Therefore, opposing features belong only to those things which can be comparatively greater and lesser; they befit these things in different ways; [but they do] not at all [befit] the absolutely Maximum, since it is

> beyond all opposition. Therefore, because the absolutely Maximum is absolutely and actually all things which *can* be [...], it is beyond both all affirmation and all negation. And it is not, as well as is, all that which is conceived to be; and it is, as well as is not, all that which is conceived not to be. But it is a given thing in such way that it is all things; and it is all things in such way that it is no thing [...]. (→Hopkins, 1985, ch. 4, p. 9; italics original)

Again endorsing the ineffability of God—whilst characterizing God in various ways—he says that God is and is not all things which are conceived to be or not to be. In other words, God is all things, so you can describe God in all ways. But it is none of these things, so you can say none of these things about God. So, in particular, God is both effable and ineffable.

The move is remarkable because, of course, it simply accepts the contradiction involved, violating the Principle of Non-Contradiction, a move that few Christian theologians, under the influence of Aristotle, have been prepared to make. This is no naïve move, however. Nicholas is well aware of what he is doing. His rejection of the view is quite explicit. Indeed, as one commentator puts it, he criticized “the Aristotelians for insisting on the principle of noncontradiction and stubbornly refusing to admit the compatibility of contradictions in reality” (→Maurer, 1967).

18.4 Segue into Paraconsistency

We have now enough historical background to give us a sense of what is at issue here. As we have seen, all parties hold God to be ineffable. They not only hold this view; they give reasons as to why it is true, reasons which characterize God positively and are intended literally. (One cannot *reason* otherwise!) Moreover, attempts to explain how this is to be done, produce more of the same, just making matters worse. We seem to be stuck with

contradiction. Nicholas makes the bold move of simply accepting the contradiction. Indeed, he does so in an extreme fashion, saying that all things—and so all contradictions—are true of god. As should not need to be said, one does not have go this far to accommodate just this one contradiction.

Nicholas' move is unlikely to recommend itself to philosophers and theologians in thrall to the Principle of Non-Contradiction—who are, in Wittgenstein's worlds, in superstitious fear and awe in the face of contradiction.[17] Once over such fear, there is nothing to prevent one accepting such a contradiction, at least in principle. However, simply to accept it, and leave it at that, is lame. What one needs is a theoretical articulation of what is going on, and some kind of guarantee that the contradiction involved does not go on holiday, taking contradictions to places where one really does not want them (for example implying that Jesus Christ was a frog and not a frog). It is here that the techniques of modern paraconsistent logic are of use. In the rest of this essay, I will show how. Whether such techniques could be applied to the other theological paradoxes we noted in Section 18.1, is another matter. Each would have to be considered in its own right; and this is not the place to do so.

Before we turn to that, though, let me just note that the phenomenon we are dealing with is not restricted to Christian theology, but can be found in others. Thus, in Hinduism, both God (*Brahaman*) and the self (*ātman*) are explained to be ineffable.[18] And in religions where there is no god, other things may be said to be ineffable. Thus, as I already noted, in Māhāyana Buddhism, ultimate reality (*paramārtha satya*) is explained to be ineffable.[19]

Moreover, in a number of non-religious philosophies, there is held to be some kind of reality 'behind' our phenomenological world, which is ineffable. Thus in Daoism, *dao* is ineffable; and

for Heidegger, being is ineffable.[20]And in many other philosophies, there are ineffable aspects of reality: noumena for Kant; form for Wittgenstein (in the *Tractatus*); nothingness in gluon theory.[21]

Of course all these paradoxes are hostage to certain theories concerning the existence and nature of something or other (*dao*, form, being, etc.). The paradox of an ineffable deity is no different in this way. This is not the place to go into any of this. I point these things out to make it clear that the phenomenon we are concerned with here of the effable ineffable is not restricted to Christian theism.

18.5 Modeling the Effable Ineffable

18.5.1 A Number-Theoretic Construction

So let us turn to technical matters. In the Christian theologies we have been looking at, God is an ineffable object. That does not mean that one cannot refer to them. (One can do so with the word ‘God.’) It means that one cannot say anything about the object referred to, that is, attribute some kind of property to them.[22] Now, all objects have properties. (If an object had no properties, it would have the property of having no properties.) Let us refer to an object’s having a property as a state of affairs —though exactly what a state of affairs is, we need take no stand on here. States of affairs may themselves be effable and ineffable. A state of affairs is effable if there is some statement which expresses it; it is ineffable otherwise. And clearly, an object is ineffable iff all the states of affairs of which it is a constituent are ineffable. (If an object is effable, one can say something about it, and so express some state of affairs concerning it. Conversely, if it is ineffable one cannot express

any state of affairs concerning it, or one would have said something about it.)

So, in understanding effability/ineffability, we may simply concern ourselves with states of affairs. In particular, we need to have a binary predicate, $E(x,y)$, meaning *x is a statement expressing state of affairs y*. That *y* is an effable state of affairs can then be expressed by $\exists x E(x,y)$. That *y* is ineffable is expressed by the negation of this. We also need a way of referring to statements and the states of affairs they express. If *A* is any sentence, we will let $\ulcorner A \urcorner$ be its name, and $[A]$ be the name of the state of affairs it refers to. Clearly, we have, for any sentence, *A*, $E(\ulcorner A \urcorner, [A])$. Hence, $[A]$ is effable, $\exists x E(x, [A])$.

There are doubtlessly many ways of building these things into a formal theory. A simple way is to use the well understood machinery of arithmetic and its coding powers. Let me explain how.[23] For the moment, assume that we are working in the standard (classical) model of arithmetic, $\mathbb{N}$. If *n* is a number, let **n** be its numeral.

We assume any standard gödelisation of the language. Let $\#A$ be the code of *A*; and if *n* is the code of *A*, let $\ulcorner A \urcorner$ be **n**. The set of code numbers is a decidable set, and so is defined by an arithmetic predicate $G(x)$. We may take the numbers which are *not* Gödel numbers to be states of affairs—or at least, their code numbers under some appropriate coding. Think, intuitively of the *even* ones as effable, and the *odd* ones as ineffable. We map (codes of) formulas onto and into the even states of affairs by some recursive function, *f*. Intuitively, $f(x)$ is the state of affairs expressed by *x*. Let *f* be defined by the binary arithmetic predicate $F(x,y)$. We may take $[A]$ to be the numeral of $f(\#A)$. Finally, $E(x,y)$ may be defined in the obvious way: *x* is the Gödel number of a sentence, and *y* is the state of affairs that *x* expresses. Let us write this, slightly oddly, as follows:

$\exists z(G(z) \wedge z = x \wedge F(z,y))$

Note that the identity conjunct is doing no real work at the moment. $E(x,y)$ could be defined by the logically equivalent $G(x) \wedge F(x,y)$. However, that conjunct will play an essential role in a moment.

Given our understanding of the machinery, E intuitively defines the expressibility relation; and $E(\langle\!\langle A \rangle\!\rangle [A])$ is a simply consequence of the construction. Hence, $\exists x \exists y E(x,y)$: some states of affairs are effable. But if s is any odd state of affairs then, by construction, $\neg \exists x E(x, \mathbf{s})$, so $\exists y \neg \exists y E(x,y)$: some states of affairs are ineffable, too.

18.5.2 Enter Inconsistency

Now, $\mathbb{퓌}$ is a classical model, and *a fortiori* a model of the paraconsistent logic *LP*. *LP* models are exactly the same as classical models, except that truth and falsity may overlap. Thus, truth and falsity conditions must be given in pairs. For example:

- $\neg A$ is true iff A is false
- $\neg A$ is false iff A is true
- $A \wedge B$ is true iff A is true and B is true
- $A \wedge B$ is false iff A is false or B is false

The *LP* models where truth and falsity are disjoint are exactly the classical models.[24] Now, let C be some subset of the Gödel codes. Let $\mathbb{퓌}'$ be the *LP* model which is exactly the same as $\mathbb{퓌}$, except that for each $c \in$ C, the pair $\langle c,c \rangle$ is added to the anti-extension of the identity predicate—that is, the set of pairs that make $x = y$ false. It is well known that everything true in $\mathbb{퓌}$ is true in $\mathbb{퓌}'$. In particular, if $c \in$ C, $\mathbf{c} = \mathbf{c}$. But $\mathbb{퓌}'$ makes more things true. I particular, if $c \in$ C, $\mathbf{c} \neq \mathbf{c}$ too.

Now, let S be any sentence such that $\#S = s \in$ C. Then $\exists x E(x, [S])$. The state of affairs expressed by S is effable. But now

consider $\exists x E(x,[S])$, that is, $\exists x \exists z(G(z) \wedge z = x \wedge F(z,[S]))$. If $n \neq s$ then $\neg \mathbf{n} = \mathbf{s}$ is true in the model. But if $n = s$ then $\neg \mathbf{n} = \mathbf{s}$ as well. Hence $\forall x \neg x = \mathbf{s}$. So $\exists x \exists z(G(z) \wedge z = x \wedge F(z,[S]))$ is false, since the middle conjunct is always false. Its negation is therefore true. That is, $[S]$ is ineffable. So, S expresses a state of affairs that is both ineffable and effable.[25] In fact, C contains (the codes of) all (and only) the statements that express effable ineffable states of affairs. It would quite natural to impose further constraints on C. Thus, for example, presumably $[A]$ is ineffable iff $[\neg A]$ is. So it would be natural to require that $\#A \in C$ iff $\#\neg A \in C$. But we need not pursue this matter here.

Note, however, that the only inconsistent objects are the numbers in C, so the contradictions are limited to those numbers which are in this set. In other words, the contradictions are limited, and under tight control.

18.5.3 Applying This to God

So far, our construction has been quite generic. It could apply to the effable ineffable, whatever the effable ineffable states of affairs are. How do we bring the construction to bear specifically on statements about God?

To do this, we need to make reference to an appropriate language, of course. Let this be an interpreted language, L. Let us keep matters simple. Atomic sentence of L are of the form Pa, where a is one of a bunch of names, and P is one of a bunch of monadic predicates. One of the names is g, ‘God.’ More complex sentences are built from these using conjunction, disjunction, and negation.

Now, when coding is applied to the language of arithmetic, it is normally the language of arithmetic that is being coded. But there is no reason, either in the nature of coding or the above

construction, which requires this to be so. In particular, we may take it that the codes are codes of sentences in L.

Let us write the set of Gödel codes as G. C ⊆ G is the set of identity inconsistent codes; that is the codes of effable ineffable sentences. Let the set of truths be T, and the set of falsities be F (though these may overlap). In what follows, it may help to refer to the diagram depicted in →Figure 1. The truths are in the vertical rectangle. The falsities are in the horizontal rectangle. So where these overlap, the sentences are true and false.

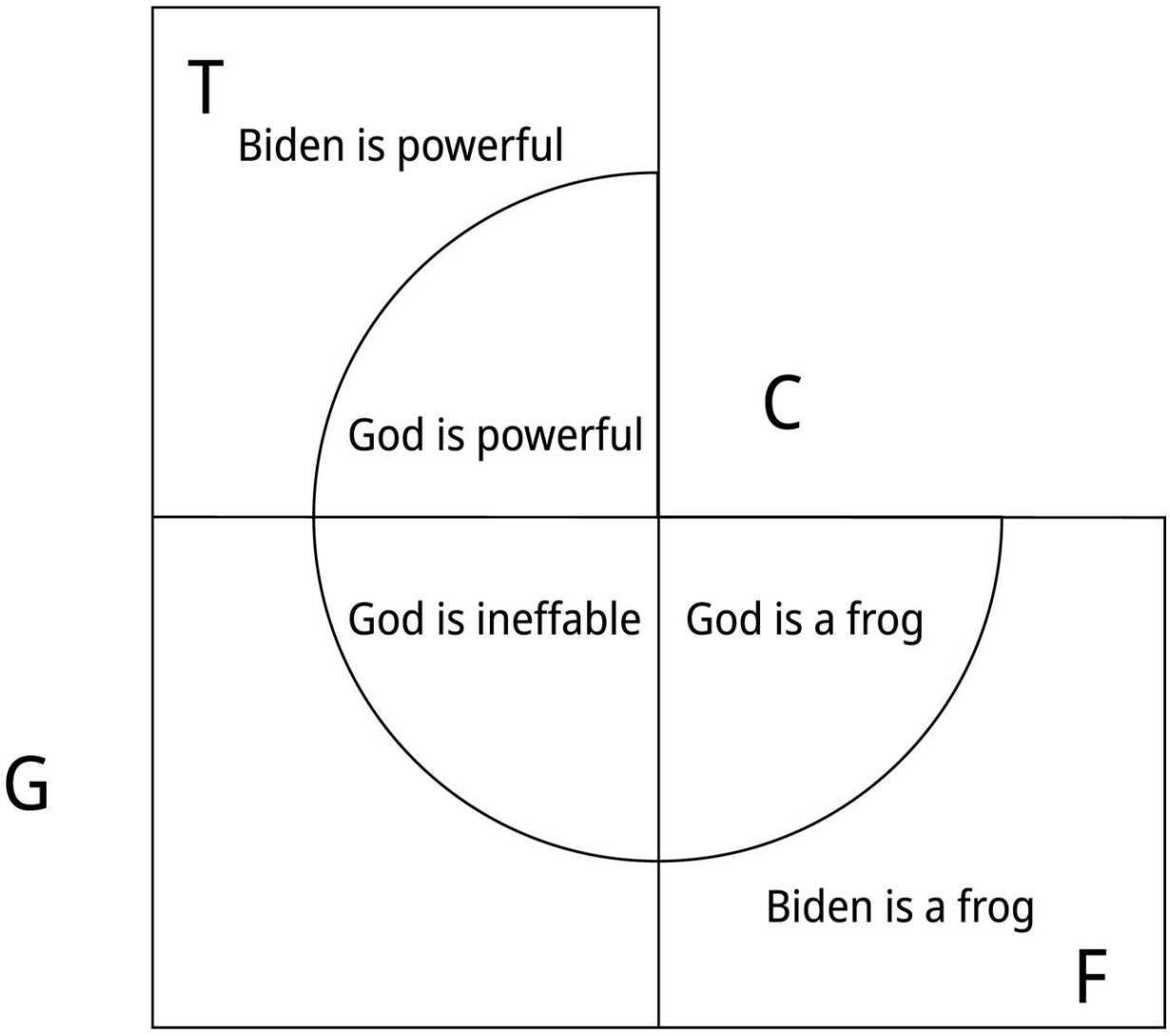

Fig. 1

Let us consider some sentences of the form *Pa*. Take the sentence “Biden is powerful.” This is true. On the other hand, “Biden is a frog” is false. Neither is both true and false, and neither is in C, since they are both straightforwardly effable.

Now consider the statement “God is powerful.” This is, presumably, true, and not false; but it describes an effable ineffable state of affairs, so it is C. What of “God is a frog”? “God is not a frog” is, presumably, true. But since it characterizes God in a certain way, it is ineffable. Assuming that $\#A \in C$ iff $\#\neg A \in C$, then “God is a frog” is also ineffable. Hence, “God is a frog,” is false and ineffable.

However, consider the sentence “God is ineffable.” This is true, and since we have just expressed it, it false. So the statement is in both true and false. But since it is effable and ineffable, it must be in C as well. Thus, in the model, God is both effable and ineffable. However, note that we do not have to follow Cusanus and hold that all contradictions are true of God. “God is powerful and God is not powerful” is simply false, as is “God is a frog and not a frog.”

18.6 Conclusion: Taking Stock

What we have seen in this essay is that, amongst the paradoxes delivered by the Christian conception of God, there is a fundamental one concerned with the very possibility of talking about God. We briefly looked at some of the theological history of the issue. As we saw, if one adheres to the Principle of Non-Contradiction, these are all—at the very least—problematic. The Principle has been highly orthodox amongst Christian theologians (and Western philosophers quite generally).

As we also saw, however, there was one who was prepared to reject it, and so accommodate the paradox, simply by accepting the conclusion that God is both effable and ineffable—

Nicholas of Cusa. Of course, Nicholas did not have the techniques of modern logic at his disposal. What was shown in the second half of the essay is that the techniques of modern paraconsistent logic can be deployed to establish that endorsing the contradiction without contradiction running rife, is tractable and coherent—so one certainly does not have to go as far as Nicholas and hold that all contradictions are true of God.

To accept such a solution to the paradox, one has to be a dialetheist, of course. Many will find that a tough bridge to cross. However, the Principle of Non-Contradiction never had the rational ground that most Western philosophers have taken for granted.[26] Indeed, what drives the paradox of ineffability is exactly the thought that God is *so* different from mundane things that one cannot expect the usual rules of the game to apply to them. So even if one is not a dialetheist about mundane matters, one might well hold that some contradictions are true of God.[27] At any rate, once over the bridge of dialetheism, a dialetheic position on the matter seems to leave its competitors trailing far behind.

Of course, I am sure that there is much more to be said about all the matters we have covered in this essay. At this point, I am happy to leave such discussion to those who are Christians.

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Notes

1 For discussion and references, see →Everett, 2010.

2 Every pronoun is wrong. The third person plural seems the least offensive.

3 See, again, →Everett, 2010.

4 In *The Word of God and the Word of Man* (1922); quoted in →White, 2010, p. 3. Note that Barth himself is, in fact, talking about God.

5 Exodus 3: 13–14: Moses said to God: “Suppose I go to the Israelites and say to them, ‘The God of your fathers has sent me to you,’ and they ask me, ‘What is his name?’ Then what shall I tell them?” God said to Moses, “I am who I am. This is what you are to say to the Israelites: ‘I am has sent me to you.’” Psalms 139: 6: “You hem me in behind and before, and you lay your hand upon me. Such knowledge is too wonderful for me, too lofty for me to attain.”

6 On Plotinus, see →Gerson, 2018.

7 “The One is all things and no one of them.” *Ennead*, 2, 1 (→Plotinus, 1991, p. 361). See also →Gilson, 1972, pp. 43ff.

8 On Pseudo-Dionysius, see →Corrigan and Harrington, 2019.

9 Those who know their Buddhism cannot but be struck by the similarity with a passage from the celebrated Māhāyana *Heart Sūtra*. In Māhāyana there is a crucial distinction between conventional truth and ultimate truth (emptiness). The passage in the *sūtra* comprises a denial of the ultimate truth of all things, including the cardinal points of Buddhist teaching. It reads as follows (→Garfield, 2016): “In the same sense, Śāriputra, all phenomena are empty. They have no defining characteristics. They are unarisen; they are unceasing. They are neither diminishing nor increasing. Therefore, Śāriputra, in emptiness there is no form, no feeling, no perception, no dispositions, no consciousness; no eye, no ear, no nose, no tongue, no body, no mind; no visible object, no sound, no smell, no taste, no tactile sensation, no mental object; there is neither ignorance nor the end of ignorance; neither aging and death nor the end of aging and death. In the same sense, there is no suffering, no origin of suffering, no cessation and no path; no wisdom, and neither attainment nor lack of attainment.”

10 Actually, the whole distinction between positive and negative is highly problematic. It is not marked by using negation or a negative prefix, and it is not clear that there is a sensible distinction here. Thus, *transparent* and *opaque* are opposites (as least when applied to physical objects), but each can be defined as the negation of the other. Moreover, ‘transparent’ can be defined as *letting light through* or *not blocking the passage of light*. ‘Opaque’ can be defined as *stopping the passage of light* or *not allowing light through*. Which of the pair is positive, and which negative? Closer to home,

the pair ‘finite’ and ‘infinite’ are similar. ‘Infinite’ can mean *not finite* or it can mean *greater than any finite quantity*. ‘Finite’ can mean *not infinite* or *smaller than any infinite quantity*.

11 On Anselm, see →Williams, 2020.

12 On which, see →Priest, 1995, 4.1–4.3.

13 On Thomas, see →O’Callaghan, 2014.

14 See →White, 2010, esp. ch. 4.

15 On Cusanus, see →Miller, 2017.

16 →Hopkins, 1985. Chapter and page references in what follows are to this.

17 →Wittgenstein, 1956, p. 53, Remark 17.

18 See →Priest, 2018, 2.3.

19 See →Priest, 2014, 13.9; and →Priest, 2018, 5.2.

20 See, respectively, Deguchi, Garfield, Priest, and Sharf, 2021, ch. 2; and →Priest, 1995, 2nd ed., ch. 15.

21 See respectively, →Priest, 1995, ch. 5; →Priest, 1995, ch. 12; and →Priest, 2014, 13.11.

22 I do not use the word ‘property’ in any heavy-duty metaphysical sense here and in what follows. The word just means whatever it is that is expressed by predication.

23 Full details of the construction can be found in →Priest, 2023.

24 On *LP*; see →Priest, 2008, chs. 7, 21.

25 A construction of the above kind was first used to explain how something could be both provable and not provable in a system of arithmetic. See →Priest, 2006, 17.8.

26 See, e.g., →Priest, 1998a and 1998b.

27 See, e.g., →Beall, 2021.

19 Divine Simplicity: Anselm's Neoplatonic Approach

Katherin Rogers

Abstract: The Doctrine of Divine Simplicity is central to traditional classical theism, but there are different ways to spell out the doctrine, and different ways to solve the problems it generates. Two key problems are: How can a simple God do and know many different things? And, could a simple God have done other than He has done? If so, that would seem to entail multiplicity—God's nature plus the divine aspects that could have been otherwise. Anselm of Canterbury, working within the Augustinian/Neoplatonic tradition, provides careful and coherent answers to these questions: A simple, *eternal*, God can do and know a vast multiplicity of things in one timeless act in which doing and knowing are the same. And, being perfectly good, this simple God inevitably, though freely, does the best.

19.1 Introduction

Anselm of Canterbury, as a classical theist, holds that God is omnipotent, omniscient, the standard for value, and moreover that God is absolutely simple. Different classical theists propose somewhat different versions of the doctrine of divine simplicity (DDS). In the present paper I will outline Anselm's approach, which I take to be rooted in the Augustinian, Neoplatonic tradition, and note how it differs from that of St. Thomas (at least as contemporary Thomists understand Thomas). I will go beyond Anselm's own writing (adopting the voice of the "Anselmian") by spelling out what I take to be some corollaries

of his view and by attempting to defend his version of DDS as worthy of consideration. First I discuss classical theist motivations for holding DDS. Next I address the claim that DDS is simply incoherent by suggesting *how* a simple being might rightly be said to be omnipotent, omniscient, and the rest. The bulk of the paper will be devoted to dealing with problems and questions generated by DDS, problems like: How a simple God, Who must be immutable, could interact with a changing creation; How a simple God, who cannot do other than He is doing, could be free; How a simple God could relate to created agents on whom He has bestowed robust free will. I will conclude by looking very briefly at the specifically Christian doctrines of the Trinity and the Incarnation. Prima facie one might suppose that these doctrines represent insurmountable contradictions to DDS. But it is at least arguably the case that the lesson of history is that adherence to DDS has encouraged analyses of these difficult doctrines which accord with, rather than abandon, the conciliar statements of the Trinity and the Incarnation.

19.2 Motivations for DDS

In Anselm's work we find two basic defenses of DDS, both grounded in the thought that God is the absolute, perfect and unlimited source of all. As Anselm says in proposing his famous proof from Chapter 2 of the *Proslogion*, God is "that being than which no greater can be conceived." This description entails one standard defense of DDS derived from the notion that God must be absolutely independent; uncaused by any possible understanding of "cause." A being composed of parts—"parts" of any sort—suffers a kind of intrinsic and fundamental dependency; the whole is what it is due to the parts and the parts are what they are due to their comprising the whole. Thus,

the whole is dependent on the parts and the parts are dependent on the whole.[1]

If one thought of the standard divine attributes like perfect knowledge or power or goodness as distinct properties had by God, they would be "parts" and would undermine divine aseity ("aseity" from the Latin *a se*, from oneself). Anselm gives this argument a Platonic cast: Things that are (for example) good by *having* goodness as a property must be participating in the shared, transcendent good by which all the good things we observe around us are good. God cannot "share" in goodness. God is Himself that very good in which the good things share. Similarly for the list of attributes we properly assign to God: God is not just, He is perfect justice, the source of all just things; God is not wise, He is perfect wisdom, the source of all wise things, etc. And, in God, all of these attributes are one and the same thing.

A second motivation for DDS in Anselm's work is perhaps less standard for classical theists. It depends upon the thought which drives the famous *Proslogion* proof. Anselm holds that we can show that God must exist starting from our conception of God as that being than which no greater can be conceived. And unpacking this conception—the project which follows the proof in the rest of the *Proslogion*—can allow us to understand something of what such a being must be like. God must be simple, says Anselm, because if He could be *thought to be composed of parts* then He could be *thought to be decomposed.* A being that is destructible, even if only *in intellectu* (in the human mind), is a being than whom a greater—an indestructible being—*can* be thought. God, then, must be simple.[2]

19.3 Coherence of DDS

It is important to note that classical theists like St. Anselm take it for granted that we will not be able to "comprehend" God. That is, we will not be able to "imagine" what God is like. We will not be able to "wrap our minds around" God. Anselm says, clearly and often, that we should not expect to see *how* something is the case just because we correctly embrace *that* it is the case.[3] This prescription can apply to commitment to Church teachings like the Trinity and the Incarnation, and it can also apply to the deliverances of reason. Reason has shown that that being than which no greater can be conceived must be simple. We must accept DDS even if we cannot grasp *how* a simple God could be all we believe He is. Anselm utters this sort of warning repeatedly and always follows it with an attempt to make some headway regarding *how* the difficult claim is to be understood. In the role of the Anselmian I will try to suggest ways to come closer to grasping Anselm's version of DDS. If it can be shown that DDS does not involve any literal contradictions, that is really enough to support the claim that God may be simple. But if we can go further and give some positive content to the notion of a simple God, even better.

The first point to note is that classical theism does not think of God as an object, a substance, or even a person engaged in, but distinct from, His various thoughts and activities. God *is* His thoughts and activities and they are all one. God is an act, with nothing separate from or underlying that act. Prima facie this may seem a strange proposal. But consider the human person, the *imago dei*. Think carefully about what you are actually conscious of when you introspect. (Human beings may be psycho-physical unities of soul and body, but in attempting to find an image of God we can bracket the body. Setting aside the Incarnation, God does not have a body. More on the Incarnation below.) What do you actually *experience* of the conscious aspect of your *self*? I take it that you experience a stream of conscious

events (thoughts, feelings, willings etc.) but I suggest that Hume and Kant are correct when they opine that you do not *observe* some unified, constant substance underlying the conscious events. Perhaps you do experience things from a particular perspective, your own first-person perspective. But the claim is that you do not experience, in addition to the stream of conscious events, the one *having* the thoughts, feelings, willings etc. Kant's argument that there must be such a one, your *self*, as one thing existing over time, is plausible. Positing this self, in addition to, and underlying, the stream of conscious events, makes sense of our disparate experiences and allows for moral responsibility. *You* cannot be responsible for what *you* did yesterday if there is no unified *you* underlying your thoughts, feelings, willings, etc. But the point here is that what we actually observe of ourselves, and *all* we observe of ourselves, when we introspect, is very like a series of actions ("actions" understood broadly)—thinking, feeling, willing, etc. To say that God is an act does not appeal to concepts utterly foreign to our experience. With human beings we have to posit an underlying self, but supposing that God is a simple act obviates the reasons for holding that the divine act must be had or done by some further, underlying divine "self." Divine unity is foundational, not something invoked to make sense of divine thinking and doing.

But the project of trying to render the Anselmian DDS coherent is just beginning. Not only is God pure act, this divine act just *is* all of those attributes we ascribe to God, and they are all one and the same. This claim can be rendered coherent, but in this case looking to the human being as an image through which to glimpse the divine will likely lead us astray. Let us focus on three main divine attributes, omnipotence, omniscience, and perfect goodness. On DDS these are ultimately the same thing in God. But at least prima facie, human power, human knowledge, and human goodness seem to be three very distinct

phenomena. One might well expect to find that a human being who possessed one could fail to possess the other two, or at least a human being who possessed one to a great extent might possess the other two only minimally. Even on a human level it is hard to imagine someone who, for example, is powerful in some meaningful sense, but who has *no* knowledge or virtue at all. Certainly history seems to be replete with examples of people who were very powerful—brutal dictators, for instance—but who seem to be woefully ignorant and vicious. Or we find morally good people who are, nonetheless, not well-informed or capable of making much impact on the world. On a human level it looks bizarre to equate power and knowledge and goodness. They are just different attributes.

But the situation is different with God. First consider divine omnipotence. It is important to appreciate that for the classical theist the central import of God's power is that He causes all that is not Himself. He produces creation immediately and ex nihilo. There is no preexistent matter with which He is confronted, nor are there any Platonic abstracta to which He looks for guidance. In contemporary philosophy of religion one sometimes runs across the thought that God surveys an array of independently existing possible worlds and chooses from among them what to create. The classical theist like Anselm rejects this picture. Whatever exists is caused to exist at the time it exists by God's power. This "primary causation" is the most fundamental exercise of divine omnipotence. "Secondary causation" is the causal relationships that hold between created things. Secondary causation can rearrange what exists in creation, but it does not bring anything into being ex nihilo. And it is God's primary causation that is keeping all the creatures in being, including their inherent causal powers and potencies and the exercise of those powers and potencies. Were God not

immediately sustaining created things in being right now they would just cease to exist.

How does God cause things? By thinking them. God's knowledge is causal. We human beings acquire knowledge by perceiving things, or accessing information, which exists outside of ourselves. Even when we introspect we can distinguish between the thinker and the mental phenomena which are the objects of thought. And, whatever causal story we tell about this acquisition of knowledge, at some point we must be *receiving* new information. The causal arrow between the knower and the known must, to some extent, run from the latter to the former. Not so with God. God knows all things outside Himself through the act of causing them. God causes and knows all things not Himself in His one act. Does it follow that all possibilia must be actualized? That is, human knowers can think about mythical beings, unicorns for example, and presumably there are no unicorns. If God thinks about unicorns, does He bring them into being? Not necessarily. The fact that God creates by knowing does not entail that all that He knows, He creates. He might recognize certain never to be actualized possibilia as ways in which His being could be—but never *will* be—reflected in creation. He might recognize mythological creatures as just what they are—inventions of human myth-makers. (But I will have more to say on the Anselmian attitutude towards unactualized possibilia below.) DDS entails that God's omnipotence and His omniscience are one and the same thing—the one act by which He knows and does everything He knows and does.

How does divine goodness fit in? In contemporary philosophy of religion, when the subject is the good, it is usual to focus on human moral goodness. Indeed, the central question often involves the relationship of God to the moral order, and it is frequently suggested that divine goodness is constituted by God's perfectly conforming to moral laws. This approach is alien

to classical theism in several ways. First, the focus on obedience to the moral rules often results in dismissing, or ignoring, or just never having run across, classical theism's understanding of value. The modern thinker's focus on moral value can lead to the (often unrecognized) assumption that the only good is moral good, and the rest of the universe is neutral at best. The stars, the planets, the trees, the frogs, are just "there." Perhaps, this contemporary approach continues, we can add a different kind of "good," that is the good of benefit, such that if the stars etc. benefit humanity they are instrumentally good—helpful for us. If they harm humanity they are instrumentally bad—harmful for us. In the contemporary discussion, considering harm and benefit might lead towards utilitarian ruminations on the production of pleasure and pain, or the satisfaction of preferences, for sentient beings. But all of this misses the core of value as far as the classical theist is concerned. For the classical theist the most fundamental good is existence. We can call this "ontological" value. The stars and planets and trees and frogs are not just there. They are *good*.[4] How could we prove the ontological value of things to one who denies it? We might point first to the attitude of small children for whom everything is interesting and worthy of attention. It seems natural for us to find things valuable. And scientists, too, seem to care deeply about the objects of their study. The physicist is not just concerned with his own mind, hoping to fill it with more facts. He wants to know about those particles and why and how they do what they do.

In this tradition "existence" can be understood in two ways. There is the "on/off" property of being there in the universe or not. But existence can also come in degrees. The grain of sand is good simply by existing. The tree is better by existing and living. The squirrel is better by existing and living a sentient life. And, of the beings we find in the physical world around us, human

beings are better still. We exist, we live, we feel, and we reason. We have a full, rich sort of existence that gives us the ontological status to make us very good. On this scale of value, a morally bad man is still a better sort of thing than a good squirrel. And the absolute standard for value, that makes the ontological scale possible, is God. God is perfect being, pure act, transcendent and necessary existence. God's being is His goodness and the existence of every creature is good insofar as it reflects the infinite being of God.

One corollary of this view is that human moral goodness can be analyzed as behavior aimed at protecting and promoting ontological goodness. This can be seen through appreciating St. Augustine's well-known privative theory of evil. If existence is good, and a richer mode of existence better than a "thinner" mode of existence, then evil must be the lack, or absence, or corruption, or destruction of something that ought to be there. Human sin involves pursuing the wrong good or the lesser good and results in the corruption and degradation of the sinful agent. The morally good person is not only *morally* better than the wicked person, but ontologically better insofar as he is doing a better job of preserving the *imago dei* in himself. He is living a richer life than the sinner who is, to some extent, destroying himself. (In the classical theist universe, no human being ever ceases to exist. Since existence is good, Hell is preferable to annihilation, and God does not blink anyone out of being.)[5] Another way of saying this is that the good is to actualize your potentials as the kind of thing you are. To "Be all that you can be." To flourish. If this sounds Aristotelian, it should, with the all-important difference that the ultimate telos for the human being is eternal beatitude with God in heaven.

Another corollary of the claim that human moral good depends on the more fundamental good of ontological value has to do with how to think about earthly power. As creatures,

human beings can exercise only secondary causation. We cannot exercise the ultimate power that God has to bring things into being ex nihilo. Above I offered the example of the brutal dictator as one whom we might believe has great earthly power but lacks goodness. However, on the privative theory of evil (and this seems to accord with the sad facts of history), what makes the dictator so brutal is that he is able to destroy. He can destroy the bodies of his citizens by killing them. He can (at least aim to) destroy the minds and souls of his citizens by imprisoning them in his gulags or, short of that, forcing them to live in daily fear. But if divine omnipotence is represented by God's creation of things ex nihilo, the dictator's exhibition of "power" is power only in a perverse and corrupted sense. Yes, calling the dictator "powerful" accords with correct usage, but that he can be "powerful," while lacking goodness, does not show that these attributes must be distinct in God. To the contrary, it shows that we human beings ought to appreciate that there is a different kind of power at work in the world. The power to create and to love—that is, the power to act upon a deep desire for the flourishing of oneself and others—is a better reflection of divine power than is the bastard "ability" to destroy. God is the supreme ontological good, and His goodness is one with His omnipotence and His omniscience and all are identical with His one act of being. I do not say that this is imaginable for human beings, but it is not incoherent.

19.4 Immutability and Other Puzzles

The Anselmian version of DDS entails that God is immutable, meaning that God cannot be doing other than He is doing or knowing other than He is knowing. This is because if God were to do one thing today and another tomorrow, we would have to posit in the divine at least the minimal multiplicity of a basic

divine nature plus the changing activities and knowledge. A similar issue arises if we suppose that God might have been doing other than He is doing or knowing other than He is knowing. St. Thomas Aquinas (at least as some recent Thomists interpret him) does not allow this entailment of DDS. Thomas is concerned to attribute to God what we today would call "libertarian" free will. God must be able to be doing other than He is doing. For example, according to St. Thomas, God might have made a different world, or might not have created any world at all. And God's simple act of being would be exactly the same if He had created a different world or no world at all. If we say that God created our world "out of love," we have to grant that this "love" would have been just the same had He made a different world or no world at all. There can be no explanation for *why* God made our world, rather than making some other world or no world. And if we say that "God loves the world," or "… made the world," or "… knows the world," we ought to understand that this tells us nothing about God in Himself. These are statements about the world, not about God. But this approach is problematic. It is intrinsically difficult to grasp, and—at least from the more Neoplatonic Anselmian perspective—it seems to disconnect God from His creation. If God's act would be just the same if He had made a different world, or even if He had not made any world at all, then the standard Neoplatonic claim that God made our world out of His love and goodness seems empty.[6]

But if we do not adopt the Thomist approach, how can we reconcile an immutable God with the thought that God acts in a changing world? And what are we to say to the question of whether or not God could have done other than He is doing? Anselm, I take it, has the tools to deal with the former question through his analysis of the nature of time and eternity. As I read him, Anselm holds that divine eternity is "present" to, or

encompasses, all of time; what to the temporal perceiver at any given time appears to be past, present, and future.[7] Thus, God "sees" all times as equally real. But then, since it is God's causal knowledge that makes creation to exist, all times *are* equally real. I call this theory of time "isotemporalism." (It is sometimes labeled "four-dimensionalism" or "eternalism.") God is causing the whole of space-time in one, immutable act.[8]

The main contender against isotemporalism is presentism, the view that all that exists is the present moment. The past and future are simply non-existent.[9] Isotemporalism accords much better than presentism with the thought that God is that than which no greater can be conceived. It is greater for God to have immediate power over, and knowledge of, all times than for Him to exist only in the present (since that's all there is) and to have immediate power over, and knowledge of, only things in the present. Presentism would not necessarily conflict with divine foreknowledge, but presumably God would know what the future holds because He would know what He Himself will cause. On presentism it is difficult to reconcile divine immutability with the claim that God interacts with a changing world. On presentism one might wonder how God could know what time it is *now* without His knowledge changing as the clock ticks on. (Perhaps the Thomist holds that "God knows what time it is now" is a claim about what time it is now and not about God. If so, this is puzzling.) Adopting isotemporalism solves the problem. God interacts with everything existing at all times "at once" as it were, in His one, simple, immutable act. He does not know what time it is *now* because there is no privileged "now" to be known. He knows what any temporal perceiver takes to be "now" at the moment when that perceiver is thinking or experiencing their "now," but *now*, like past and future, is subjective to the particular perceiver at the particular time. Isotemporalism is certainly strange phenomenologically. The

“you” of five minutes ago, and the “you” of five minutes hence exist every bit as much as you now. Very weird! But if attempting to discuss the supremacy of God results in positing weirdness regarding some creature’s experience, the classical theist will allow the weirdness, rather than countenance any suggestion of diminishing the divine. Isotemporalism fits well with divine supremacy, and, by entailing divine immutability, supports DDS.

The question of whether or not God might have been doing other than He is doing remains. If God might have made a different creation or no creation at all, then (setting aside the Thomist approach) it looks to be the case that DDS must be denied. We must posit God in His basic nature and then the things He does, which might have been different. As I read him, Anselm adopts the more Neoplatonic view of holding that God could not be doing other than He is doing. God is the best, the standard for all value, and inevitably does the best.[10] This answer supports DDS, but entails a number of further questions. First, does it conflict with the claim that God is free? It would do so only if freedom requires the ability to do other than one is doing. Anselm is adamant that God’s freedom does not involve an ability to do otherwise. Freedom for *created agents*, human beings and angels, does require that we can choose between alternative possibilities. This is because created freedom is important to ground moral responsibility and enable us to orient ourselves towards the good on our own. The created agent has all that he is—his existence, his nature, his will, his motives—from God. But in order for us to have a thin sliver of independence God establishes the structure of the rational will such that it is up to the agent alone which of the genuinely open options he will pursue.[11] As Anselm understands it, God has given created agents what we, today, would call libertarian free will.[12] God, of course, exists in perfect independence and does not “conform” to an external moral law. He is goodness itself

and does not need options to be free. That is why Anselm defines “freedom of the will” as the ability to keep justice for its own sake.[13] Created agents can fit the definition, since God makes it possible for us to abandon justice. Hence we can “keep” it on our own. God is justice itself. No need for, and no room for, open options for God. Anselm makes this point vividly in his *Cur Deus Homo* in discussing the free sacrifice of Christ. He argues that Christ can be born of the Blessed Virgin Mary because of her faith in His saving work. (Anselm is writing before the Immaculate Conception becomes Church doctrine.) But this —as the interlocutor, Boso, points out—seems to entail that Christ *must* make the sacrifice once He is conceived, since He could not be conceived unless He will make the sacrifice. Anselm agrees that Christ cannot fail to make the sacrifice, but nonetheless He makes it freely. It is made entirely of His own volition. But couldn’t He do otherwise? Only if He chose, and He could not choose to do less than the best and make the sacrifice necessary for the salvation of His human brothers and sisters.[14] God inevitably does the best and could not be doing other than He is actually doing.

But this introduces a further difficulty. Is there indeed a best to do? St. Thomas posits an infinite hierarchy of good possible worlds such that, no matter what world God chooses to create, there is always a better one He could make.[15] Alternatively one might insist that various goods are just incommensurable such that, while there might be many different good worlds that achieve a superlative level of goodness, still there is no “best” and God confronts open options because He is able to choose between these worlds.[16] These moves are made to defend divine freedom and omnipotence. But note first that on neither of these scenarios is the situation one of God’s making. On both, God’s being free is dependent upon realities about what is

possible that precede God's willing. And, while these suggestions might defend divine omnipotence through the thought that God might be doing a host of different things other than He is actually doing, they also undermine divine omnipotence by insisting that the situation concerning what is possible—which, again, is not of God's making—is such that God *cannot* make a best world.[17] So it is not clear that these proposals for why God faces open options really defend a more robust understanding of divine freedom and omnipotence than Anselm's.

Further, we can make sense of a "best" world. One move that many classical theists make adheres to a principle going back to Plato's *Timaeus* (29e–30c). It is the Principle of Plenitude: Not only would a good God—"being without envy"—inevitably create, but He would inevitably create the best, which means all the different possible kinds of things.[18] Above the point was made that in this tradition existence is good. And that means that the more different kinds of things there are, the better. The more the merrier on a cosmic scale! Someone might try to throw cold water on this exuberant theory by pointing out that the laws of nature as we understand them conflict with the possibility of every good kind of thing being actualized.[19] But there are two responses to this (or maybe one response put in two different ways). We are familiar with only our observable corner of the universe. Perhaps in all the vastness, different laws operate allowing for all the good possibilities to be actualized. Or perhaps our world is a multiverse of numerous different discrete and closed causal systems.[20] Would that mean an infinite number of universes? Or, could there be a "perfect" number?[21] God only knows. But positing a world in which all of the overall good universes are created seems a good way to think of a best

world. And the thesis that God really could make a best world supports DDS.

I have avoided the locution "best *possible* world" not only because contemporary possible worlds can be ontologically problematic, but also because human choices might conflict with a best *possible.* If, as I suggested above, Anselm believes that created agents have libertarian freedom such that our choices are ultimately up to us, then God does the best, working with the free choices of created agents. It does not follow that created agents exercise some sort of causal power over God. Our choices do not somehow *impact* God. As Anselm has analyzed the structure of created freedom, God can know what we are choosing by knowing what He himself is maintaining in existence. Still, the Anselmian has to allow that, to some extent, regarding some events, it is up to the created agent what God maintains in existence. God has set things up so that we are made in His image, not just in virtue of being rational and being agents in some very qualified sense. We are agents with enough independence that we can participate, in however small a way, in how the universe goes on. Some classical theists will find this too hard to swallow and will deny that created agents could have libertarian freedom. But the alternative is to says that God is the cause of all choices, including the choice to sin, and Anselm, at least, holds that it is logically impossible that God should cause a created agent's choice to sin.[22]

A further problem remains to be addressed. This Anselmian approach to DDS entails what some philosophers refer to, disapprovingly, as a "modal collapse." I prefer to label this consequence, "modal simplicity." If God cannot do other than He is doing in His simple, eternal act and if the isotemporal theory of time is the correct way to understand the time in our universe, or to understand all the times in all the temporal universes of the multiverse, then it seems that nothing could be

other than it is. On isotemporalism, every event in what to some temporal perceiver is subjectively past, present or future is there in the isotemporal universe. This is true even of free choices. If some agent S chooses X freely tomorrow (at T+1), then it is true at every moment of the universe that "S chooses X freely at T+1." (I suppose, if we do indeed live in a multiverse, that it is true in all the universes of the multiverse that "S chooses X freely at T+1 indexed to this ('our') universe.") The open possibilities that libertarian free will requires will have to be understood in terms of the abilities of agents. Agent S can choose otherwise because of the nature of S's will such that there is absolutely nothing causally necessitating that S choose X at T+1. And there is nothing *non-causally* necessitating that S choose X at T+1, except that S does indeed choose X at T+1.[23] This is only a logical, or conditional, or subsequent "necessity." If the fact of S's actually choosing X at T+1 undermines S's freedom when it comes to choosing X at T+1, then it seems logically impossible that agents have free will.[24] It is unreasonable to hold that the subsequent necessity caused by an agent's own choosing conflicts with free will. So isotemporalism does not conflict with libertarian free will.

But isotemporalism, coupled with the thought that God Himself "cannot" do other than He is actually doing in His simple, eternal act, does seem to entail modal simplicity. Some philosophers apparently find this such an unbearable conclusion that they dismiss any theory that entails it. But the claim is just that things are as they are. It is hard to see what is so fearsome about that. In terms of human psychology modal simplicity may be a wholesome theory. Coupled with a libertarian approach to free will, modal simplicity does not undermine the thought that free agents contribute to bringing about what their actual future will be. But it does conflict with unhealthy attachments to what "might have been." A life can be sadly overshadowed by mooning over unactualized "if only's." Regarding the

contingency through which some non-human creaturely events "might have been different," this can be analyzed analogously to the point made above about free will. Given the laws of physics (and the standard interpretation of quantum indeterminacy) it is possible for the electron to go through *this* slit or *that* slit at T+1. There is nothing about the electron or the universe it inhabits to causally necessitate its going through *this* slit rather than *that* at T+1. But if, upon observation, it is seen to have gone through *this* slit at T+1, then it is just a fact at every moment of the isotemporal universe that the electron goes through *this* slit at T+1.

It is true that the thought that things might have been different in the past, that there are open possibilities in the future, or that things might just be different than they are, does seem to accord better with our ordinary ways of speaking than does modal simplicity. But if the issue is the perfect nature of God, the classical theist will not let ordinary language drive the conclusions. All the propositions claiming that things might have been different from what they actually were in the past, or might be different from what they are at present, or might be different from what they will be in the future, if they have truth values, can be translated into propositions about the actual world. Sometimes people talk about "changing the future." But that is incoherent on any view of time. On isotemporalism the future is what it is and, barring a glitch in the law of non-contradiction, cannot be other than it is. On presentism the future is non-existent, so there is nothing there to change. On both theories the future is "brought about," but it is never changed. And modal simplicity need not cause problems for modal logic that deals in possible worlds. One just has to posit the correct analysis of what a possible world is—something like a consistent set of propositions. A world other than our actual world is not *really* possible, but that does not entail that one couldn't

construct a consistent set of propositions that was not the set that conforms to our actual world. And, since human knowledge is so limited, accepting modal simplicity does not rule out our talking meaningfully about possibilities *quoad nos*, epistemic possibilities. Modal simplicity seems implausible intuitively, but the claim that things are as they are should not be that upsetting, and, in any case, it seems to be an entailment of the Anselmian DDS as I have spelled it out.

There are two final issues to look at which are specific to Christian theology. Conciliar Christianity holds that God is a Trinity of three "persons" but one nature.[25] It also holds that God, the Son, the Second Person of the Trinity, assumed a human nature such that God Incarnate was one person with two natures. Prima facie it might look to be the case that the Christian would do well to jettison DDS in the face of these difficult theological doctrines. Interestingly, the lesson of history teaches otherwise. At least some of those who depart from the tradition of classical theism, including DDS, happily abandon the understandings of the Trinity and the Incarnation as set forth by the Councils of Nicaea and Chalcedon. One striking example relative to the Trinity is the proposal of a "social trinity." Insisting that "persons" be understood as modern intellectuals use the term, it is said that the Trinity is three separate individuals, each with their own intellects, wills, and interests, but bound together by each having a nature similar to the others.[26] Regarding the Incarnation history is rife with examples of unorthodox readings: There is a hierarchy within the Trinity such that the Son is lesser than the Father and that is how He can become Incarnate; The Son could not really become Incarnate but rather only uses a human body to *appear* incarnate; Jesus was just a very good man, not God Incarnate, and a plethora of other suggestions.

It is the adherents of DDS like Augustine and Anselm who vociferously defend the doctrines of the Trinity and Incarnation

as set forth by the Councils. And one key aspect of their strategy involves the central claim in DDS that God is an act rather than an inert object or even a mental substance that could underlie a changing consciousness. In his *On the Trinity* St. Augustine proposes a series of analogies for the Trinity based on the human mind engaged in actively thinking. Thinking draws upon, and is an expression of, a concept in memory when will (desire) unites thinking and memory through seeking to consider the concept. The act of thinking something, then, is a dynamic act involving three elements. The elements can be distinguished, but they are not separable, and they constitute one act. Memory, thinking, and will are analogous to Father, Son, and Spirit. Throughout *On the Trinity* Augustine is very careful to caution that the analogies he will advance are distant from the Trinity which transcends human abilities of comprehension, but they are enough to show that sense can be made of something which is both three and one. We do not need to suppose that the doctrine of the Trinity is simply contradictory. Anselm, in his *Monologion*, adopts Augustine's analogies from *On the Trinity*, but, with his usual optimism about human capabilities, he goes one step further. He offers what at least *could* be read as an attempt to *prove*—having shown through reason that there must be a highest being—that this being must be a trinity—a supreme, dynamic act in which the elements are properly labeled Father, Son, and Spirit. So, contrary to what might look more likely prima facie, it is the traditional adherents of DDS who have been the staunchest defenders of conciliar orthodoxy regarding the Trinity and the Incarnation.

The Anselmian approach to DDS does look to entail consequences that many have found unappetizing; time is isotemporal; God "cannot" do other than He is eternally and immutably doing; created free agents have some "input" into what God is doing; ours is the best world God could make; there

are no unactualized possibilities. But none of these consequences is sufficiently dire to force us to abandon DDS as the Anselmian portrays it. And, regarding specifically Christian doctrine, the lesson of history is that the conciliar understanding of the Trinity and the Incarnation has fared better when associated with DDS than when DDS is rejected. DDS is defensible, and the Anselmian version offers a powerful approach to this difficult doctrine.

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Notes

1 *Monologion* 16–17. Anselm does not couch his point in terms of the distinction, or identity, of essence and existence as will some later philosophers, such as St. Thomas Aquinas.

2 *Proslogion* 18.

3 *Proslogion* 14; *On the Incarnation of the Word* 1; *Cur Deus Homo*, Book 1, Chapters 2, 25.

4 This is a constant theme in the work of St. Augustine. See for example *Confessions*, Book 7, Chapter 12. The hierarchy of value in created things figures prominently in the second book of his *On Free Will.*

5 St. Augustine explains this in Book 3 of *On Free Will*.

6 I discuss St. Thomas's understanding of DDS and compare it to the Anselmian approach in →Rogers (2020a).

7 See →Rogers (2008, pp. 176–184).

8 If we live in a multiverse, then presumably God is causing all that exists in all the multiverses in one, immutable act. I defend the multiverse hypothesis in →Rogers (2020b).

9 Another suggestion is the "growing block" theory. All past moments exist, but the future does not. The present is the leading edge of a "growing block" of

time. This view seems to have the difficulties associated with isotemporalism with none of the advantages. Since the leading edge of time, the present, is so thin, any given perceiver is likely living in the past with what is past, present, and future subjective to that perceiver at that time.

10 →Rogers (2008, pp. 185–205).

11 →Rogers (2008, pp. 55–92).

12 How can our choices be up to us if God is the cause of everything? As I read him Anselm proposes a very clever analysis of free will which allows him to hold that God does indeed cause all that *exists*, but some of what *happens* is up to created agents. See →Rogers (2015, pp. 65–100).

13 *On Freedom of Choice* 3.

14 *Cur Deus Homo*, Book 2, Chapters 10 and 17.

15 St. Thomas frames this claim in terms of ways the divine could be imitated, not in terms of "possible worlds" as independently existing Platonic abstracta. *Summa Contra Gentiles*, Book 1, Question 81.

16 Alexander Pruss argues this point in →Pruss (2016).

17 Anselm himself argues that what is possible and what is necessary is ultimately rooted in the will of God. Happily, this does not make logic in any way shaky, since God's will is immutable and could not be otherwise.

18 →Lovejoy (1936).

19 When I was discussing the point that God keeps everything in being from moment to moment by thinking about it with my four-year old granddaughter, she asked, somewhat sternly, “Why isn’t God thinking about unicorns?”

20 →Rogers (2020b).

21 Anselm brings up the concept of a “perfect” number in discussing the number of the saved.

22 *On Freedom of Choice* 8.

23 →Rogers (2015, pp. 112–116). Ruling out non-causal necessitating factors renders contemporary Molinism a non-viable approach. Modern Molinists posit so-called “counterfactuals of freedom”—truths existing independently of God about what any possible (even if never actualized) agent would choose in any possible (even if never actualized) situation. And then any real agents cannot fail to choose as the relevant counterfactual says they choose. Molinism conflicts with classical theism’s understanding of divine omnipotence by positing these truths existing independently of God and it conflicts with robust libertarian free will by positing a necessitating factor (albeit a non-causal one) outside of the control of the agent.

24 On any theory of time, if S’s actually choosing X at T1 necessitates S’s choosing X at T1, then there can be no time at which it is possible for S to choose other than X at T+1. *After* T+1 it is too late. *At* T+1 it is too late, since S is choosing X. And at T, before T+1, it is too early since S

has yet to make a choice. S cannot choose before choosing.

25 I put “persons” in scare quotes because both Augustine and Anselm are careful to note that the term should not be taken in some ordinary sense. Augustine says that it is just a sort of place-holder, somewhat better than the alternative of saying that in the Trinity there are three “somethings” (*On the Trinity*, Book 5, Chapter 9). Anselm agrees (*Monologion* 79).

26 This approach gained traction in the nineteenth century. For a recent defense see →Hasker (2013).

VI God, Necessity and Abstract Objects

20 God, Necessity, and Necessities

Ciro De Florio

Aldo Frigerio

Abstract: In this essay, we discuss the relationship between God and the modal profile of the states of affairs. There is a well-known dilemma about this relationship: both the view that the necessity and the possibility of the states of affairs are dependent on God and the view that they are not dependent have unpalatable consequences. If the necessity of the states of affairs depends on God, then those states of affairs do not really seem necessary. By contrast, if the necessity of the states of affairs does not depend on God, then God is not the fundamental entity. However, the fundamentality of God is a cornerstone of classical theism. We present a bi-modal framework that solves the dilemma, by distinguishing between *global* and *local* necessities. This framework also proves useful for solving other dilemmas such as the problem whether God can or cannot do evil actions.

20.1 Introduction

The questions concerning God and necessity can ideally be divided into two families of problems:

1. Intrinsic problems,
2. Extrinsic problems.

Intrinsic problems pertain to the modal status of God and His properties. In classical theism, God is considered an entity that necessarily exists. However, which kind of necessity is at play

here? Is it a metaphysical necessity or a logical necessity? This question clearly affects the discussion regarding the ontological argument in favor of the existence of God in its modal version. Other intrinsic problems concern God's properties: does God only have necessary properties or also contingent properties? In general, the perfect being theology tries to characterize divine necessary (or essential) properties; however, some contingent properties seem to be relevant for God's personal being. These problems are obviously connected with the questions of whether God is an immutable being and whether He is a temporal or timeless being. If God is a timeless being, it is natural to think that He is immutable and that most or all His properties are essential. However, if God is a temporal being, then it is natural to think that He is mutable and that at least some of His properties are contingent.[1]

Extrinsic problems pertain to the relationships between God, as He is usually understood in classical theism, and necessary entities (whether things, states of affairs, facts, events, etc.). It is possible to distinguish two types of problems here; the first is purely ontological, whereas the other has a more logical nature.[2] The first set of questions pertains to the existence of necessary entities distinct from God and their independence from or dependence on God. For instance, suppose the existence of Platonic abstract entities is accepted. Typically, they are considered necessary and devoid of causal relationships with other entities. Therefore, there would necessarily exist entities independent of God. Many scholars have noticed that this view is in contrast with classical theism. Reasons relating to the Scriptures (the prologue to the Gospel of John is often quoted), the Patristic tradition (the Nicene Creed) and philosophical arguments (perfect being theology) oppose the view that there are entities independent of God.[3] He is typically considered the

fundamental entity, which means that any entity different from God directly or indirectly depends on Him.

The solutions to this problem are numerous, and the debate about them is broad. Platonists usually try to show that when it is stated that God is the creator of all things, the universal quantifier should be restricted to contingent things (see, for instance, →van Inwagen, 2009). Others have argued that the necessity of Platonic entities does not preclude their dependence on God (Morris and Menzel, 1986; →Gould and Davis, 2014). Still others have maintained that Platonic entities are identical to God's thoughts and, thus, even if they are eternal, they are strictly dependent on Him (→Welty, 2014). Those who embrace the Aristotelian views of universals state that universals exist if the things that instantiate them exist; therefore, universals are ultimately dependent on God because the things that instantiate them are created by God (→McCann, 2012). Others have accepted anti-realistic views about abstract entities (neo-Meinongianism or strategies that acknowledge the truth of necessary sentences but paraphrase them in such a way that they are not committed to the existence of necessary entities, pretense theory, fictionalism, etc.), which solve the problem at the source because they deny the existence of abstract entities (→Craig, 2014, 2016; →Shalkowski, 2014). We do not take a stance on this debate here because we are interested in another type of problem with a more logical nature.

Independent of the position one assumes about abstract entities, only a few are ready to reject the truth of necessary sentences. Many anti-realists about abstract entities also wish to maintain that something cannot be both *P* and not-*P*, that necessarily 1+1=2, that necessarily water is H_2O, or that necessarily, there is a relationship between the force acting on an object and the acceleration of that object. There are anti-realist theories, such as (some kinds of) fictionalism, that deny

the truth of necessary sentences, but most anti-realist views accept it. Their main problem is accommodating the existence of necessary truths in a framework in which the existence of necessary entities is denied.

Independent of the ontological questions about abstract entities, if one allows the existence of necessary truths, as we do here, a dilemma arises: are those truths dependent on God? If they are dependent on God, they are not really necessary because God could have done otherwise. If they are not dependent on God, there are truths regarding things that are not God, which do not depend on God. Both facets of the dilemma have unpalatable consequences.

It is worth noticing that when we speak of the necessary truths, we refer to those that Leftow (2012, p. 249) calls secular truths, namely, alleged necessary truths that are not about God and His attributes. As we said above, we will not deal with intrinsic problems but only with extrinsic ones. In particular, we are not interested in questions such as can God decide not to exist? Can God decide not to be omnipotent? Can God renounce His nature? These questions can be discussed, but it cannot be argued that we must respond affirmatively to them because the modal profile of the truths about God must depend on Him. As we will see below, the principle of fundamentality claims that any state of affairs *external* to God must depend on Him. Therefore, the fact that God has a certain nature that even God cannot modify does not violate this principle. If any state of affairs external to God depends on Him, the principle of fundamentality is satisfied whether God has a nature over which He has no jurisdiction or not. For the principle of fundamentality, it is important that the non-divine states of affairs depend on God, not that God's nature depends on God Himself.[4]

In this paper, when we speak about necessary truths, we only refer to necessary secular truths. We briefly discuss the

necessary states of affairs concerning God in Section 20.8. We show there that our framework is neutral about the question of whether God has a nature that even God cannot modify.

This essay is structured as follows. In Sections 20.2–20.4, we illustrate the dilemma raised by the relationship between God and necessary states of affairs. In Section 20.3, in particular, we will present the first version of our framework that allows us to demonstrate in Section 20.4 that the dilemma can be interpreted as a regress. Sections 20.5–20.7 contain our solution to the dilemma. Section 20.5 informally illustrates the second version of the framework, whereas Section 20.6 contains its formal presentation. Section 20.7 reviews the principle of fundamentality in light of the proposed solution. Section 20.8 explains the various types of modality that the framework allows, including a new kind of modality that we call locally contingent necessities. In Section 20.9, this modality proves useful for addressing some problems, such as the possibility that God does evil, and for illustrating the transworld depravity. Section 20.10 shows that the existence of local necessities is something that contemporary physics seriously takes into account for reasons wholly independent of those examined here. Section 20.11 concludes the essay.

20.2 The Dilemma

In this section, an informal version of the dilemma is presented. In the following sections, we provide a more formal version. The principles on which this *reductio* is based are the following:

1. God is the fundamental entity,
2. God is free,
3. There are necessary states of affairs.

The *reductio* proceeds as follows:

(1)	Let φ be a necessary state of affairs	
	(for instance, a mathematical state of affairs)	by (N)
(2)	φ depends on God	by (F)
(3)	God could have done otherwise	by (E)
(4)	Thus, it is possible that $\neg\varphi$	from (3) and (E)
(5)	Then, it is necessary that φ and it is possible that $\neg\varphi$,	from (1) and (4)
(6)	Contradiction	

This argument can be countered by negating one of the three principles upon which it is based. One might redefine God's freedom in a compatibilist sense in such a way that (3) does not follow from (E). God would be free even though He cannot do otherwise from what He does. As it is well-known, there are many arguments in favor or against compatibilism about human actions, and one might also explore this subject in relation to God. In the following, we assume, without argument, that God is free in a libertarian sense, namely, that He could do otherwise from what He does.

The second strategy for opposing the argument is to acknowledge some exceptions to principle (F). For example, (F) might be restricted to contingent states of affairs, thus excluding necessary states of affairs. This strategy parallels the Platonist view regarding abstract entities: the domain of the things that depend on God is narrowed. As for Platonism, the problem is to reconcile this position with the classical theist intuition that God

is the fundamental entity for any aspect of reality. As stated above, the reasons grounded on Scriptures, tradition, and philosophical reasoning lead us to believe that there is nothing external to God that does not depend on Him.

The third strategy is to deny (N): there are no necessary states of affairs because, if God wanted to, things would have been different. This voluntarist view is consistent with principle (F) because everything depends on God and with principle (E) because God can do otherwise. However, there are good reasons for thinking that at least some states of affairs are necessary in a sufficiently robust sense so as to exclude that things can be otherwise.

In this essay, we try to solve the dilemma while preserving principles (F), (E), and (N), whose negation is theoretically very costly.

20.3 Formal Framework

Let us define a framework whose language is as follows:

–	A stock of variables for states of affairs:	$\varphi, \psi, \xi, \ldots$
–	A stock of variables for agents:	$i, j, \ldots$
–	An *actualization* operator:	A_i
–	A *creation* operator:	C_i
–	Boolean connectives:	$\neg, \wedge, \vee$
–	Modal operators:	$\Box, \Diamond$

Intuitively, $A_i\varphi$ means that agent *i* actualizes state φ. In other words, the fact that φ is actual is dependent on agent *i*. It is important to note that the actualization operator has an intentional interpretation: agent *i* is the cause of (the actuality of) φ because *i* wants that it is the case that φ. To outline the semantics of $A_i\varphi$, we introduce a classical possible world framework (where worlds are indicated by w_i) in which the actual world is marked (by @). The worlds are related by an accessibility relation *R*, whose properties are deliberately left indeterminate. Therefore, by $R(w)$ we indicate the set of worlds w' such that $R(w,w')$. The last element of the semantics is the choice function $c: I \times W \rightarrow 2^W$, that is, a function that takes agents and worlds as arguments and returns a subset of worlds. Intuitively, they are the *agency options* that a certain agent has in a certain world. Then, $c_i(w)$ is the set of worlds that are agency options of *i*.[5]

It is quite common to assume that $c_i(w) \subseteq R(w)$ holds: the actual courses of actions available to *i* at *w* are a subset of the metaphysical possibilities at *w*. That is, although it is metaphysically possible for us to climb K2, this option is not plausibly included in our agency options. We see in the following how this relation is crucial for our topic.

The truth clause of $A_i\varphi$ is the following:

(Actualization) $M, w \models A_i\varphi$ iff $\forall w' \in c_i(w), M, w' \models \varphi$

Agent *i* actualizes state φ if φ holds in each agency option that is available to *i* at *w*. The actualization operator characterizes the first condition of free choices according to the libertarian approach: the causation of actions. The other condition is,

notoriously, the principle of alternative possibilities. If we add this, we obtain the definition of $C_i\varphi$:

(Creation) $C_i\varphi := A_i\varphi \wedge \Diamond\neg\varphi$

In our jargon, agent *i* creates state φ at world *w* if and only if the two following conditions hold: in every agency option of *i* at *w*, φ is true, and there is at least one possible world accessible from *w* in which $\neg\varphi$ is true:

$M, w \models C_i\varphi$ iff $\forall w' \in c_i(w)$, $M, w' \models \varphi$ and $\exists w'' \in R(w)$, $M, w'' \models \neg\varphi$

Given this definition, it is straightforward to state the dilemma of creation of necessary states of affairs. To create φ, an alternative possible world in which φ is false must be available, but this opposes the alleged necessity of state φ. Let us examine the dilemma in more detail.

The theistic intuition according to which God is the fundamental entity can be formalized as the following Axiom of Fundamentality:

(F) for any $\varphi \in$ L(Prop), $\varphi \rightarrow C_g\varphi$

By L(Prop) we indicate the modal free propositional language. That is an important point. If φ is modal-free, we have the *actualist version* of the fundamentality axiom: everything that is actual at *w* has been created by God. Let us assume that φ is a necessary state of affairs (for instance, a mathematical fact):

(1)	$\Box\varphi$	Since φ is necessary
(2)	φ	(1), by factivity
(3)	$C_g\varphi$	(2), by fundamentality
(4)	$\Diamond\neg\varphi$	(3), def. of C_g
(5)	Contradiction	(1), (4)

A more refined version of this dilemma can be laid down if we allow that within (F), it is possible to substitute any formula of the modal language (with the exception of the formulas regarding God's existence and essence). According to this interpretation of the fundamentality axiom, God is not only the ground of what actually exists but also of what could exist and of what necessarily exists. That is to say, the modal profile of things, and not only their existence, is under God's control.

(1)	$\Box\varphi$	Since φ is necessary
(2)	$\Box\varphi \rightarrow C_g\Box\varphi$	by the fundamentality axiom
(3)	$C_g\Box\varphi$	(1), (2), logic
(4)	$A_g\Box\varphi$ & $\Diamond\neg\Box\varphi$	(3), def. of C_g

Let us pause and analyze conjunction (4). The semantics of the two conjuncts is:

$M, w \vDash \mathrm{A}_g\Box\varphi$ iff $\forall w' \in c_g(w)$, $M, w' \vDash \Box\varphi$

$M, w \vDash \Diamond\neg\Box\varphi$ iff $\exists w'' \in R(w)$, $M, w'' \vDash \neg\Box\varphi$

Therefore, we have a situation in which in every world belonging to God's agency options, φ is a necessary state of affairs. However, there is at least one metaphysically possible world where φ is not necessary. Is this a contradiction? Yes, it is a contradiction if at least one of the following plausible assumptions holds.

First, if the agency options coincide with the metaphysical possibilities (that is, if $c_g(w) = R(w)$), we have a contradiction because, in all agency worlds, φ is a necessary state but, at the same time, there is a possible world (and then an agency option) in which φ is not necessary. Now, do the agency options coincide with metaphysical possibilities? Not generally, as mentioned above. However, in the case of an omnipotent agent, one could be tempted to answer affirmatively: at the end of the day, what God can do is what is possible to do.

Second, there are good reasons to assume that the accessibility relation R is reflexive; thus, the actualization operator A_i is factive. It follows that φ is necessary at the evaluation world w. Therefore, at w, $\neg\Diamond\neg\varphi$. However, we have seen that w "sees" a world w^* in which $\neg\Box\varphi$, that is, $\Diamond\neg\varphi$. This means that from w^*, there is an accessible world w^{**} in which φ is false. However, if transitivity holds, we have $R(w,w^{**})$ because, by hypothesis, we have $R(w,w^*)$ and $R(w^*,w^{**})$. Then, $\Diamond\neg\varphi$ also holds at w—that is, at the starting world. To avoid contradiction, we are forced to reject axiom 4 ($\Box\varphi \rightarrow \Box\Box\varphi$) and take on board a rather weak modal logic. Notably, it is debatable whether such a system is able to characterize metaphysical modalities.

In sum, the fundamentality axiom and the libertarian view of agency jointly entail the absence of necessary states of affairs. As a result, if one wants to stress the existence of necessary states of affairs, one must be prepared to give up either the fundamentality axiom or the libertarian understanding of divine agency. Perhaps, from a theistic point of view, the less

theoretically expensive choice is to renounce necessary states of affairs since this move does not undermine God's fundamental features. Therefore, voluntarism seems to be an attractive option for the theist. Nevertheless, things are much more complicated because the fundamentality axiom and the libertarian free will suffice to trigger a regress, as we will see in the next section.

20.4 Regress

If one takes the fundamentality axiom seriously, one should claim that God's creative power is not limited to the actual world but affects the whole modal sphere. What is or is not possible depends on what God freely establishes.[6] However, these assumptions alone trigger a regress.

Let us assume that ψ is any actual state of affairs, either contingent or necessary. Now, in accordance with the fundamentality axiom, if ψ is actual, then God freely created it. As a consequence, the negation of ψ must be possible. However, the fact *that* $\neg\psi$ *is possible* is, in turn, a *fact*. As such, it must fall, so to speak, under God's jurisdiction. Let us call ξ a (second order) modal fact according to which a first-order fact is possible. Moreover, ξ must be the product of God's free creative will. Again, this presupposes the existence of a further modal fact, namely, that it is possible $\neg\xi$. The regress is then straightforward:

(1)	ψ	Hypothesis
(2)	$\psi \rightarrow C_g\psi$	from fundamentality
(3)	$C_g\psi$	(1), (2), logic
(4)	$A_g\psi$ & $\Diamond\neg\psi$	(3), def. of C_g
(5)	$\Diamond\neg\psi$	(4), logic
(6)	$\xi \equiv \Diamond\neg\psi$	(5), definition
(7)	$\xi \rightarrow C_g\xi$	from fundamentality
(8)	$C_g\xi$	(6), (7), logic
(9)	$A_g\xi$ & $\Diamond\neg\xi$	(8), def. of C_g
(10)	...	

The worlds in which God actualizes ξ are the worlds in which it is true that $\Diamond\neg\xi$. As we have seen, however, $\Diamond\neg\xi$ also depends on God. Now, $\Diamond\neg\xi$ is equivalent to $\Diamond\neg\Diamond\psi$, that is, $\Diamond\Box\neg\psi$. Thus, if God freely creates a generic state ψ, then God is somehow forced to create the modal alternatives $\Diamond\neg\psi$, $\Diamond\neg\Diamond\psi$, $\Diamond\neg\Diamond\neg\Diamond\psi$, and so on. It is not immediately clear how to reduce these modalities to block the regress.

Regresses such as this constitute a problem for a theory like the one provided by Leftow in *God and Necessity* (→Leftow, 2012). One of the aims of Leftow's work is to try to show that the modal profile of the secular truths (namely, truths that do not concern God) depends on God. According to Leftow, before God decides which truths are necessary and which are (im-)possible, they have no modal status. Before the creation of the world, God, in accordance with His preferences (and not His nature), decides to

assign a modal status to some truths. Thus, they become either necessary or possible or impossible. For instance, God decrees that zebras are possible, but unicorns are not (→Leftow, 2012, p. 351). By this decision, God gives Himself the power to create zebras and denies Himself the power to create unicorns. However, if the decision to make the sentence "There exist zebras" a possible truth is a free choice, God could have chosen otherwise. He could have made it such that this state of affairs was not possible. In other words, let ψ be "There exist zebras"; thus, given that $\Diamond\psi$, $C_g\Diamond\psi$ should hold in Leftow's framework. However, $A_g\Diamond\psi$ & $\Diamond\neg\Diamond\psi$ should hold as well. It should be possible for the existence of zebras to be impossible. However, although Leftow claims that God freely chooses to assign a modal profile to entities, he denies that God could have done otherwise and, therefore, that $\Diamond\neg\Diamond\psi$ holds. This would block the regress. Nevertheless, if the modal status of things is a divine free libertarian choice, it seems unavoidable that $\Diamond\neg\Diamond\psi$. By definition, libertarian choices entail the possibility to do otherwise, so the possibility that God ascribes a different modal status to ψ arises. It is, therefore, possible that it is not possible that zebras exist. Although Leftow aims to escape this conclusion, we believe that this necessarily follows from his premises.[7] Leftow's theory cannot block the regress.[8]

The logical investigation of the necessity dilemma and the creation regress have highlighted some crucial points concerning the relationship between God and necessity. In the rest of the paper, we contemplate a possible way out of this stalemate.

20.5 A Way Out?

Thus far, we have seen that the *desiderata* for a theistic conciliatory model are the following:

1. God is the fundamental entity,
2. God is free in a libertarian sense,
3. There are necessary states of affairs.

The problem about God and necessity seems to undermine the internal cohesion of these requirements. Furthermore, we have shown that (F) and (E) alone trigger a kind of regress. In a nutshell, a free action presupposes a space of possibilities; the space of possibilities is *something* and, as such, depends on God's free action. Therefore, a second-order free choice capable of grounding the space of possibilities becomes necessary, but this triggers the regress. The aim of our framework is threefold. First, we would like to block the regress while maintaining our intuitions about God's freedom. Second, we would like to keep the strong theistic presupposition that God is the source of the modal profile of things. Third, we would like to preserve the intuition that there are necessary states of affairs, even though God could have done otherwise.

The basic concept of our framework is that of *creation*. A creation is a constellation of possible worlds that share a set of metaphysical and logical laws. In any creation, there is a "central" (or "actual") world, while the other worlds are alternatives to that central world. At the beginning of time, God can actualize one of the possible (infinite?) creations: they constitute the set of the possible fundamental metaphysical configurations that God can actualize by virtue of His omnipotence. Within this range of alternatives, God's action is free in a libertarian sense: He can decide to actualize a certain creation, but He could have actualized another one. To block the regress, we suppose that the set of possible creations depends on God but not on God's free will. It is God's power that grounds the existence of this space of possibilities.[9] Something is possible because God can create it. God can decide what to create, but,

crucially, He cannot decide what He *can* create. It is plausible to assume that the possibility of creating is a trait of God's essence that eternally characterizes Him. This eternal creative potentiality can be represented by the set of possible creations that God can actualize.

God chooses to actualize one creation by actualizing the "central" (or "actual") world of that creation. Depending on the possible worlds accessible from that world and on their features, we have different modal truths. For instance, suppose God decides to actualize zebras. He will actualize a creation whose central world contains zebras. Furthermore, suppose God wishes for winged horses to be possible but not actual. In this case, He will choose a creation whose central world does not contain winged horses, but in which winged horses exist in at least one of the possible worlds accessible from the central world. Finally, suppose God wishes that some truths, for instance, mathematical truths, are necessary. In this case, in every world accessible from the actual world of the creation He will choose, mathematical laws are true.

In fact, two concepts of necessity/possibility are at play in our framework. We can distinguish what is *locally* necessary/possible, namely, what is necessary/possible *within* a creation, from what is *generally* necessary/possible, namely, what is necessary/possible from God's point of view. Since this is a basic point of our framework, it is worth giving some details.

Let us call the central or actual world of creation $\mathscr{C}_1$, $@_1$. $R(@_1)$ is the constellation of worlds accessible from $@_1$. This set of worlds determines what is locally possible or impossible with respect to $@_1$. Another world that God can create, $@_2$, belonging to creation $\mathscr{C}_2$, can have a different constellation of accessible worlds. For instance, a state of affairs ψ that is not present in any of the worlds of $\mathscr{C}_1$ can instead be present in one of the worlds of

$\mathscr{C}_2$. This means that ψ is locally impossible with respect to $@_1$ but locally possible with respect to $@_2$. Therefore, local modality concerns creations and is relative to them. However, if we consider things not from the point of view of a single world but from that of God, both the (local) possibility of ψ and its (local) impossibility are possible in a global sense. God can actualize both creation $\mathscr{C}_2$, in which ψ is possible, and creation $\mathscr{C}_1$, in which ψ is impossible. If we see things from the point of view of God and of what He can do, we pass from the level of local possibilities to that of global possibilities. Some state of affairs is globally possible if it holds in at least one actual world of the creations available to God, namely, in $\{@_1, @_2 \ldots @_n\}$.[10] Therefore, both $\Diamond\psi$ and $\neg\Diamond\psi$ globally held $\neg\Diamond\psi$ is true with respect to $@_1$ and $\Diamond\psi$ with respect to $@_2$.

One might ask which is the ontological status of the worlds of a creation that are accessible from the actual world of that creation. This is a legitimate question, but we will not try to answer it because our framework is compatible with any ontology of possible worlds.

In this section, we have just given an intuitive characterization of our framework. In the next section, we provide a sketch of the formal structure.

20.6 Extended Framework

In this section, we sketch the formal framework to account for the intuitions previously analyzed. The language is as follows:

–	A stock of variables for states of affairs:	$\varphi, \psi, \xi, \ldots$
–	A constant that denotes God:	g
–	An *actualization* operator (referred to God):	$A_g\varphi$
–	A *creation* operator (referred to God):	$C_g\varphi$
–	Boolean connectives:	¬, ∧, ∨
–	A *local* possibility operator:	◊
–	A *general* possibility operator:	♦

A creation $\mathscr{C}_j$ is the set of worlds that are possible alternatives with respect to the actual world $@_j$, that is, $\mathscr{C}_j = \{w \mid w \in R(@_j)\}$. Any possible world belongs to a creation, and there is no possible world that belongs to two (or more) creations. Therefore, creations are a partition of the set of worlds. The totality of all creations is $\mathfrak{C}$. Then, $\mathfrak{C}$ is the modal absolute space that depends on God's creative power, while $\mathscr{e}$ is the evaluation point peculiar to God. Graphically, this can be represented as shown in →Figure 1.

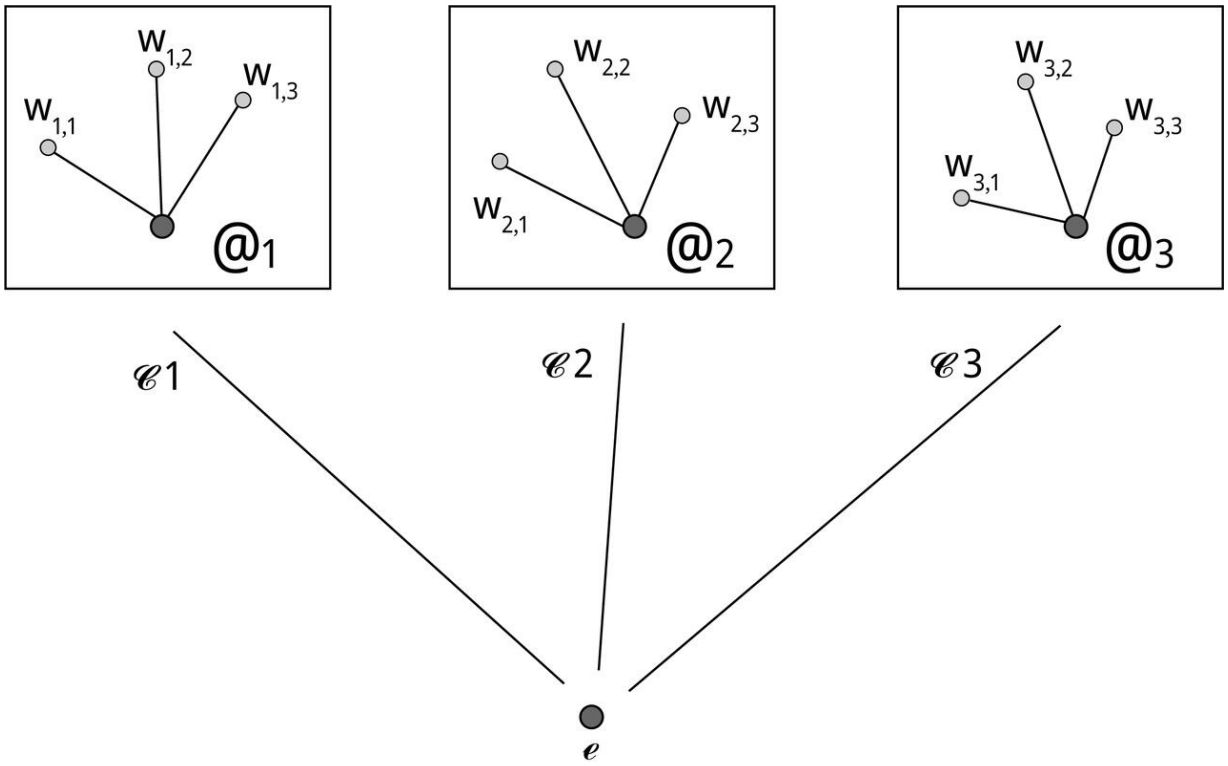

Fig. 1

Let us start by considering the case of mundane truths, namely, sentences that refer to features of a (possible) reality. In such a case, the evaluation behaves classically as follows:

$M, w \models \varphi$ iff $w \in \mathrm{I}(\varphi)$,

where $\mathrm{I}(\varphi)$ is the set of φ-worlds. Analogously,

$M, w \models \Diamond\varphi$ iff $\exists w' \in R(w)$, $M, w' \models \varphi$.

Obviously, the world w at which we evaluate φ or $\Diamond\varphi$ *does* belong to a particular creation. However, the advantage of this framework is the possibility of saying something concerning the very structure of all creations. To do that, we have to introduce another important element: the reference point e. From an intuitive point of view, e has to be construed as "God's perspective." This is in agreement with the intuition according to which God is not properly *within* a possible world, but He oversees the reality that includes various possibilities.

Furthermore, if one considers the actual world and the class of possible worlds related to the actual world as a creation, then it is rather improper to claim that this domain includes God. At the end of the day, God is not a created entity. The reference point e is a formal device to account for this intuition.

Now, which kind of formulas must be evaluated "from God's point of view"? It seems weird to evaluate at e mundane sentences, such as "Emma is blonde," because they refer to created entities. Therefore, they have to be evaluated with respect to a possible world belonging to a creation. We propose three types of sentences that can be evaluated at e:

1. Sentences concerning God's essence,
2. Sentences concerning God's agency,
3. Sentences concerning the absolute modal profile of things.

In this paper, we are not interested in the first class of sentences; let us consider the third class of sentences first. Let us assume that we want to say that state φ is possible in *absolute* sense, that is, from God's point of view, it is possible that φ. We have to take note that this expression carries some ambiguities. On the one hand, it could mean that there is at least a creation such that it contains at least one possible world in which φ holds. However, we consider another interpretation of the black operator of possibility, namely, $\blacklozenge$. Accordingly, $\blacklozenge$ is a quantifier on creations, specifically, *on the actual world* of each creation. Thus, to say that φ is possible in an *absolute* sense means that there is a creation in which φ is actual. This simplifies our semantic framework with no loss of expressive power; in fact, if we wish to express the idea that there is at least one creation, such that it contains at least one possible world in which φ holds, we can iterate the modal operators: $\blacklozenge\Diamond\varphi$. That is, there is a creation at

which the possibility of φ is actual. The clause for absolute possibility is, therefore, the following:

M, $\mathscr{e} \vDash \blacklozenge\varphi$ iff $\exists\mathscr{C}\exists i(@_i \in \mathscr{C}, M, @_i \vDash \varphi)$.

It is crucial now to adapt our concepts of actualization and creation to this extended framework. As we will see in a moment, things are easier. Here, indeed, we are concerned with just one agent, God; His agentive cone, that is $c_g(\mathscr{e})$, is a (possibly proper) subset of all the actual worlds of every creation. God, in other terms, act on the reality, by acting on the structure of the actual worlds that, in turn, fix the modal profile of things. So, we have the logical devices to consider the second class of truths evaluated from God's point of view, that is, the sentences concerning God's agency. God's agency, as seen before, is modeled by an actualization operator. For example, what does it mean that God created zebras? It means that God freely created a creation in which zebras are actual. Thus, let ζ be the state of affairs that consists of the existence of zebras. We have:

$M, \mathscr{e} \vDash A_g\zeta$ iff $\forall i(@_i \in c_g(\mathscr{e}), M, @_i \vDash \zeta)$.

In other words, God creates zebras if, for every creation belonging to the cone of agentive options, zebras exist in the actual world of these creations. Now, as we previously stated, *free* creation requires that it must be possible that zebras do not exist:

M, $\mathscr{e} \vDash \blacklozenge\neg\zeta$ iff $\exists\mathscr{C}\exists i(@_i \in \mathscr{C}, M, @_i \vDash \neg\zeta)$.

If God freely created zebras, then there is another (alternative) creation in which they are not actual. The modal operator $\blacklozenge$ is construed as a quantifier on creations and, therefore, on the actual worlds occurring in each of them. From this point of view, the structure of the divine agency is an instance of the general structure of agency. $\blacklozenge\neg\zeta$ is a (second order) modal fact for which there exists a creation where it is actual that there are no zebras.

The appeal to modal operator ♦ blocks the regress. As observed above, the regress can be blocked by assuming that the modal space constituted by all creations depends on God, but it is not something created by God in the proper sense. The various galaxies of possibility are grounded on God's power, but God cannot choose whether a certain configuration of possibilities (that is, a creation in our jargon) is available or not. He can, of course, decide not to actualize it. Thus, the idea is that God's creative power induces the modal space of possible creations $\mathfrak{C}$. Therefore, all and only the entities contained in $\mathfrak{C}$ are possible in the broadest sense of possibility.

One could object that our maneuver to block the regress is a mere reduplication of a strategy that can be carried out at the level of possible worlds without involving the concept of creation. Let us examine this criticism in detail. We argued that regress arises because to guarantee God's freedom, a space of possibilities must be available. In turn, this space must also depend on God's free choice, and so on. Thus, one might argue that God's creative power induces a class of *possible worlds* (and not a class of *possible creations*, $\mathfrak{C}$) and that, among them, God decides which world to actualize. Therefore, contrary to what happens in our account, the regress would already be blocked at the level of possible worlds: given state α, the possibility that α does not hold is something that depends on God but not on God's free choice, being grounded on His creative power. Put differently, there is but one available creation consisting of a class of possible worlds among which God freely decides which to actualize.

Although this is, at least *prima facie*, a coherent view, in our framework, the regress is blocked at the higher level, that is, at the level of creations (namely, systems of possible worlds) for allowing two levels of modality. As mentioned above, our framework is able to characterize the idea according to which

there are states whose necessity is local, that is, their necessity depends on God's will. This is possible because there are two kinds of modal operators: the standard modal operators and the "black" modal operators. By contrast, if there were just one modal dimension, we would be in the following situation: a state is necessary if and only if it holds in all possible worlds (suitably connected by the accessibility relation). Moreover, the structure of possible worlds is not the result of God's free choice (otherwise, the regress would arise). It follows that God does not govern the necessities and the possibilities, but only what is actual. Instead, our framework confers God greater metaphysical sovereignty: He governs the modal profile of the reality (in addition to what is actual) without triggering the regress. In the next section, we investigate this point.

20.7 Revisiting Fundamentality

Let us examine the principle of fundamentality in light of our framework. The existence of a golden mountain is possible with respect to the actual world. Supposing that ψ is the sentence that states the existence of a golden mountain, then, for fundamentality, $\Diamond\psi \rightarrow C_g\Diamond\psi$. Since a golden mountain is actually possible, then we have $C_g\Diamond\varphi$, namely, the possibility that a golden mountain exists is the result of a free divine creative act. This means two things:

1. at $\mathscr{e}$, in every creation alternative, the existence of a golden mountain is actually possible (that is, $\forall\mathscr{C}\exists i(\mathscr{C} \in c_g(\mathscr{e}), @_i \in \mathscr{C}, M, @_i \vDash \Diamond\psi)$; and
2. there is a creation in which the existence of a golden mountain is actually impossible (that is, $\exists\mathscr{C}\exists i(@_i \in \mathscr{C}, M, @_i \vDash \neg\Diamond\psi)$.

For the sake of simplicity, suppose there are only two possible creations ($\mathscr{C}_1$ and $\mathscr{C}_2$). In $\mathscr{C}_1$ there is a world $w \in R(@_1)$ in which ψ is true. It follows that in $@_1$, a golden mountain is possible. However, things could have gone otherwise. In $\mathscr{C}_2$, the existence of a golden mountain is impossible: in every world w, such that $w \in R(@_2)$, there is no golden mountain.

Consider the necessary states of affairs, such as 2+2=4 (let us denote this state of affairs as φ). As we have seen, our framework is neutral with respect to such states of affairs. Within it, it is possible to express the voluntarist view according to which the necessity of φ is a free decision of God:

$M, \mathscr{e} \vDash A_g\Box\varphi$ and $M, \mathscr{e} \vDash \blacklozenge\neg\Box\varphi$.

It is easy to construct a model in which these two formulas are satisfied. For the sake of simplicity, suppose only two creation alternatives exist in this model: $\mathscr{C}_1$ and $\mathscr{C}_2$. Furthermore, suppose that $c_g(\mathscr{e})$ is $\mathscr{C}_1$. Then, in $@_1$ (which belongs to $\mathscr{C}_1$), φ is necessary, that is, in every world accessible from $@_1$, 2+2=4 is true. However, there is another creation (in our toy model, $\mathscr{C}_2$) in which, in the actual world $@_2$, 2+2=4 is not necessary. It may also be supposed that 2+2=4 is necessarily false in $@_2$. This would be in line with the idea that the mathematical states of affairs cannot be contingent: these are either necessary or impossible. In this view, they are locally necessary or impossible but globally contingent. They must be so to guarantee God's freedom to create them.

Our framework is also compatible with the more classical thesis that God cannot create contradictory things and cannot make it such that 2+2=4 is false. In this case, it must be assumed that $\mathfrak{C}$, namely, the space of absolute possibilities, adheres to the principles of classical logic. As mentioned above, we do not see any principled reasons to restrict in such a way the space of global possibilities. However, this is not our main point. What is

important is that our framework allows us to express the idea that there are necessary states of affairs that depend on God's choice. In other words, in our framework, extremely audacious counterfactuals, such as "If God wished, 2+2=4 would have been false" can be evaluated. In a sense, we can express the idea that it is contingent that certain states of affairs are necessary. The seemingly apparent contradiction is resolved by the different semantics of the two involved modal operators.

20.8 Form Absolute Possibilities to Absolute Necessities?

Thus far, we have discussed only one global modal operator, namely, ♦. We have seen that ♦ is a quantifier on $\mathfrak{C}$ and that $\mathfrak{C}$ is induced by God's creative power. It follows that modal states of this global level are metaphysically but not voluntaristically dependent on God. One may wonder whether the dual of ♦, namely ■, exists and which is its semantics. Similarly to ♦, we argue that:

$M, \mathscr{e} \models \blacksquare\alpha$ iff $\forall\mathscr{C}\exists i(@_i \in \mathscr{C}, M, @_i \models \alpha)$.

that is, α is actual in every creation. It does not matter which creation God will actualize; α will always be present in the actual world of that creation. As we have seen, the meta-modal states of affairs, such as $\blacklozenge\alpha$ and $\blacksquare\alpha$, are related to the structure of the space of creations, that is, to the structure of domain $\mathfrak{C}$. These features do not depend on God's will but solely on His creative power. It is, therefore, natural, at least *prima facie*, to grant that the principle of fundamentally is valid not only for the meta-possibilities but also for the meta-necessities.

These kinds of meta-necessities are obviously delicate. An allegedly state $\blacksquare\alpha$ indicates that α is actual in every creational plan of God independently of His will. This might undermine the assumption that God is the sovereign of reality. Thus, it is

important to investigate the question of *whether* there are such necessary states and, if there are, *which* they are. One might maintain that the sole states α for which it is true that $\blacksquare\alpha$ are, in turn, locally necessary: $\blacksquare\Box\alpha$. If $\blacksquare\Box\alpha$ is true, then there is no world in any creation in which α is not true. Call this kind of necessity super-strong necessity. Therefore, the question becomes the following: are there super-strong necessary states of affairs? We do not take a stance on this question whose answer depends on anyone's very general assumptions and intuitions. The natural candidates for this super-strong necessary status are states concerning God's essence. It is not implausible to think that it is super-strong necessary that God exists and that He has a certain set of attributes. In the same way, it is not implausible that God's essence does not depend on a free choice of God. Granting that every God's essential property is super-strong necessary, some paradoxes, such as that of the stone that not even God can lift or that resulting from the question of whether God can decide being or not being omnipotent, are prevented. In general, one can escape the paradoxical view according to which God's nature is created by God.[11] One might argue that there are other super-strong necessities that do not regard God but states of affairs external to God. As stated above, we are sympathetic with a view that narrows down these kinds of states of affairs as much as possible. In any case, our framework is neutral about the extension of super-strong necessities.

The theory that necessities such as $\blacksquare\alpha$ must be super-strong is not the only possible one. We believe that the view that grants the existence of locally contingent meta-necessities is also very interesting. According to this view, there are states of affairs (β) for which it is true that $\blacksquare\beta$ and that $\blacksquare\neg\Box\beta$. Let us clarify this point. By definition, $M, e \models \blacksquare\beta$ iff $\forall\mathscr{C}\,\exists i(@_i \in \mathscr{C}, M, @_i \models \beta)$; that is, in every $\mathscr{C}_j$, it is actually true that β. However, it does not follow

that β is locally necessary, in any creation. For instance, it might be the case that $M, @_2 \vDash \beta$ and $M, @_2 \vDash \Diamond\neg\beta$, that is, in creation $\mathscr{C}_2$, β is true but contingent. This kind of modality is intriguing and particularly useful to model some cases in which we have contradictory intuitions. The next section is devoted to this topic.

20.9 Necessary Evils

Since ancient times, the following dilemma has been discussed: can God do evil actions? On the one hand, God is perfectly good, and it does not seem feasible for Him to do any evil action. This divine attribute is usually called *impeccability*. On the other hand, God is omnipotent, and it seems odd to deny God the power to do something that men and, according to an ancient tradition, even angels can do.[12]

We believe our framework can suggest a solution to this problem. Suppose ζ is an evil action. We can express the intuition that God cannot perform ζ as follows:

$M, \mathscr{e} \vDash \blacksquare\neg A_g\zeta$.

This means that in the actual world of any creation, God cannot perform ζ: $\forall\mathscr{C}\exists i(@_i \in \mathscr{C}, M, @_i \vDash \neg A_g\zeta)$. However, this is compatible with $M, @_1 \vDash \Diamond A_g\zeta$ and this can be true with respect to any actual world, namely, for any $@_i$. In other words, even though in every world actualized by God, God performs only good actions, it is also true with respect to any of these worlds that it is locally possible for God to perform an evil action.

The advantage of this solution is that it accounts for a strong intuition: although God can potentially do evil actions, this possibility is never actualized. God could do ζ, but He always chooses not to.[13] However, this intuition is difficult to clarify: a potentiality that cannot be actualized is not a potentiality because the possibility of actualization is inherent to the idea of

potentiality. We can technically state this objection as follows: if agent α can actualize ζ, there must be a possible world in which α actualizes ζ. If there is no possible world in which α actualizes ζ, then α cannot actualize ζ. If God can do an evil action, there must be a possible world in which God acts in an evil way. Therefore, the possibility that God acts in an evil way *can* become a reality after all. It is difficult to understand the sense in which a possible world cannot be realized because God is impeccable. If something is possible, by definition, it *can* be actualized.[14]

We believe our framework can account for the intuition that God never acts evilly but that He *could* act in such a way. The basic idea is that in every world that God could actualize, God does not act in an evil way. However, for each of these worlds, there is another accessible world in which God performs at least an evil action. This accounts for a possibility available to God but which God, being impeccable, never chooses to actualize. The bi-modal structure of our framework allows us to hold together things that otherwise would be impossible to hold together.

Another theoretical advantage of our framework is that it vindicates the intuition rooted in classical theism that God's impeccability depends on His essence. In other words, God does not do evil actions because He is always oriented to the good. In our framework, this is rendered by the fact that $\blacksquare\neg A_g\zeta$, as any other "black modality" refers to the structure of $\mathfrak{C}$, that is, to the space of all creations. Recall that this space is dependent on God but not on God's free will. It is His power that opens, so to speak, this space of possibilities. If $\blacksquare\neg A_g\zeta$ is true, then this is a feature of $\mathfrak{C}$. It follows that the fact that God always does good actions depends on God. This is a technically sophisticated way to state that God is essentially good. As we have seen, this essential attribute seems to be in contrast with God's freedom. However, the existence of possible worlds in which God locally does evil actions guarantees His freedom, thereby solving the dilemma.

Another area in which our framework proves useful is the formulation of some aspects of the Molinist proposal (cf. →Plantinga, 1977; →Freddoso, 1988; →Flint, 1998; →Craig, 1991).[15] Molinism advances a complex theodicy whose theoretical elements are extensively discussed in literature. The first ones are the so-called counterfactuals of freedom (CF). They are sentences having the following structure:

> (CF) If an agent *a* were in conditions C, *a* would freely choose to perform action *p.*

Plantinga's (1977) well-known example is about a corruption case: if Smith were in certain conditions, he would freely accept a bribe. If there are true counterfactuals of freedom, remarkable metaphysical consequences follow. There are metaphysically possible worlds that God cannot actualize. For instance, God cannot actualize a world where Smith *freely* refuses the bribe. Obviously, God can force Smith to refuse the bribe and even force Smith to refuse the bribe and mistakenly believe to have acted freely. However, the world in which Smith voluntarily refuses the bribe is possible but not actualizable by God.

Molinism assumes that God knows the truth value of every CF for every agent He can create. Therefore, before creating the world, He knows what agent *a* would decide to do in circumstances *C*. Within our framework, this means that God knows the truth value of every CF in every creation $\mathscr{C}$ and, thus, which creation is the closest to His aims. However, it seems that the Molinist proposal cannot *explain* why a considerable quantity of gratuitous evil is present in our world. Why did God not create a world in which the agents always choose good or at least do not do such a considerable quantity of gratuitous evil?[16] One possible answer is advanced by Alvin Plantinga. He hypothesizes the existence of a transworld depravity, a sort of metaphysical-

moral limitation of free agents for which they perform at least one evil action in every possible world in which they exist. We believe our framework allows a more fine-grained analysis of Plantinga's proposal. In fact, in the original proposal, there is no possible world in which an agent always does good actions. This implies that the agent is determined to do at least one evil action. This is contrary to the idea that the agent is always free to act goodly and, therefore, with the idea that there must exist at least one possible world in which the agent always performs good actions.

We can better characterize the transworld depravity hypothesis using the two modal dimensions of our framework. Let $\exists xEx$ be the sentence that states that there is at least one agent that (at least once) behaves badly. Then, M, $\mathscr{e} \vDash \blacksquare\exists xEx$ means that in the actual world of every creation, there is at least one agent who acts evilly in at least one circumstance. However, as we have seen, given a generic i, from $@_i \vDash \exists xEx$, it does not follow that $@_i \vDash \square\exists xEx$, that is, that in every world of creation $\mathscr{C}_i$, there is at least one evil action. If that were the case, the agents would necessarily choose the evil in at least one circumstance. That is, they would necessarily be (partly) evil. However, we can assume that $@_i \vDash \exists xEx \ \& \ \Diamond\neg\exists xEx$, that is, in every creation, there are possible worlds in which the agents always act goodly. Even though the creations are transworld depraved, the agents are wholly free and are not determined to act evilly. They could be perfectly good.

This better characterization of Molinism and the notion of transworld depravity does not solve the problems that affect this free will defense; instead, it makes them clearer. Indeed, as we have seen, the space of creations $\mathfrak{C}$ is grounded on God's essence. That, by essence, God cannot create a world where free agents always act goodly seems an improbable hypothesis and, in any case, something that requires some explanatory account.

This is a specific problem of the notion of transworld depravity that can be better appreciated using our framework.

20.10 Local Necessities and Multiverse

The idea that there can be local necessities is taken into account seriously by contemporary astrophysics. In particular, some theories suppose that there can be a multiverse of which our universe is only a part. In fact, under the label of a multiverse, very different hypotheses are of interest.[17] For example, our universe might, finitely or infinitely, continue beyond the horizon of the farthest electromagnetic signal we receive, there might be other universes within very short distances but in other dimensions (hypothesis of the brane multiverse), or the universe might be cyclic and have periods of expansion and periods of contractions.

However, we are interested here in another hypothesis that combines ideas from the theory of inflation, according to which our early universe would have had an extremely short period of exponential expansion, and from string theory, which tries to unify quantum mechanics and Einstein's theory of relativity. According to the string theory, point-like particles of quantum field theory should be interpreted as one-dimensional strings that can vibrate as harmonic oscillators. Different vibrational states of the same string would appear to be different types of particles. Moreover, the theory postulates the existence of spatial dimensions other than the three we have experienced. For example, the M-theory, which tries to unify all consistent versions of superstring theory, postulates the existence of 10 spatial dimensions. The seven extra dimensions exist as the three dimensions we know, but they are too tiny for us to observe. Strings can vibrate not only within the three dimensions of common experience but also within the seven tiny extra

dimensions. The size and shape of these extra dimensions are not fixed *a priori* but can vary. Therefore, the vibration of strings will depend on the size and shape of the dimensions within which they vibrate. Since the properties of the particles depend on the vibrational patterns of the strings, different sizes and shapes of the extra spatial dimensions determine the existence of different particles and different fundamental forces.

The hypothesis of the multiverse we are interested in supposes that our inflationary universe can be represented as an expanding bubble. It also supposes that there are other bubbles so that we can represent the multiverse as a gigantic block of Swiss cheese. In each of these bubbles, the size and shape of the extra dimensions can be different; thus, in each of them, different particles and forces with different constants exist. In other words, matter in each bubble is subject to different physical laws. Since the number of possible shapes and sizes of the extra dimensions is astronomically high (10^{500} in some versions of the theory), the kinds of universes that can exist are equally high.

This picture has striking similarities with that we have illustrated in the previous sections. The physical laws of every bubble are necessary within the bubble: the particles will necessarily have certain properties, and the physical constants will be fixed. For instance, in our universe, electrons necessarily possess a certain mass, and strong nuclear forces necessarily possess certain strength. This means that in our universe, there *cannot* be an electron with a mass different from that of the other electrons of this universe and that quarks *cannot* interact with each other differently from how they do so in this universe. However, these necessities are only local and concern only our bubble universe and not other universes where the shape of the spatial dimensions is different. In any case, there are global laws described by the string theory that are valid in every bubble.

Therefore, we can compare the necessity of local laws with our operator ❑ and the necessity of global laws with our operator ■. Obviously, we can suppose that even the global laws of string theories are necessary only locally because they concern only a subset of the possible worlds and that there are possible worlds in which other laws are valid. In this case, the laws of string theory would also be local, although less local than the laws of the bubble universes because they would concern a larger number of possible worlds.

The string theory and inflationary multiverse are hypotheses that have not yet received experimental confirmation. There is a broad consensus among scientists that *our* universe is inflationary because inflation seems to be the best explanation of the isotropy of the cosmic microwave background, but there is no consensus among scientists that our inflationary universe is only one of several existing bubble universes. Some scientists have even argued that the multiverse hypothesis is not verifiable in principle and, thus, is not a scientific hypothesis.[18] Here, we do not intend to take a stance on either the existence of the multiverse or on the scientific status of the hypothesis that the multiverse exists. We wish only to emphasize that this hypothesis is taken into consideration seriously by a significant number of astrophysicists. Therefore, the idea that there are local necessities, which are, in all respects, necessities but which concern only a subset of possible worlds, is something that circulates in the scientific community for reasons totally different from those we have considered in this paper. In our opinion, this is a clear advantage for the theory we are defending here.

20.11 Conclusions

In short, two possible theories can be advanced concerning the relationships between God and necessity. According to the first

one, there are necessities independent of God and to which God is subjected as any other entity. According to the second one, God is the source of the necessity and possibility. In this paper, we have tried to defend the latter theory.

We have demonstrated that, contrary to what is usually believed, the view that there are some necessities dependent on God's nature but not on God's will is not in contrast with the most extreme voluntarist positions according to which God could have created a world subjected to physical, metaphysical, mathematical, and logical laws different from those of the actual world. The classical objection put forward against voluntarism is that if God could have chosen differently, then there would not be true necessities, and all would be contingent. We have demonstrated that this consequence is preventable by adopting a bi-modal system in which there can be local necessities relative to creations. Local necessities should be distinct from global necessities that are grounded on God's power. Our framework has been proven effective in solving dilemmas such as that of the possibility of God doing evil actions or precisely expressing views such as that of transworld depravity.

It is worth emphasizing that although our framework is compatible with the most extreme voluntarist positions, it does not imply them. In fact, it is also compatible with the classical position according to which God cannot create round squares and cannot arrange things so that $2+2 \neq 4$. The scope of divine power coincides with the extension of the global possibilities, those we have indicated as "black" modalities. Our framework is even compatible with the view that every black necessity is also a white necessity, that is, with the view that all that is necessary in this world is also necessary for God. However, the aim of this paper is to demonstrate that this identification is not mandatory and that the two kinds of modalities can be, at least conceptually, distinguished. Hence, there is a theoretical space

to claim that it is necessary that 2+2=4 but that God could have done otherwise.

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Notes

1 For these questions, see De Florio Frigerio (2019).

2 These two types of problems are often confused with each other. For a distinction, cf. →Craig (2013) and →Craig (2017, p. 123).

3 For an in-depth discussion of these reasons, cf. Leftow (2012, pp. 3–22) and →Craig (2017, ch. 2).

4 In this paper, we use a relaxed terminology: sometimes we speak of necessary truths, sometimes of necessary states (of affairs). We will be deliberately vague about the relationships between necessary states of affairs and necessary truths. This vagueness will have no impact on our analysis.

5 For the classic logic of action, see →Belnap, Perloff and Xu (2001), →Chellas (1969), and →Pörn (1977). For a clear overview, see Herzig, Lorini, and Troquard (2018).

6 It may be noticed that the metaphysics of modality is relevant to this discussion. An actualist can object, for instance, that possibilities are ultimately grounded on actuality and then that God, creating the actual world, is *ipso facto*, responsible for what is possible or not in the actual world. We agree. The point is that we have to explain (as far as possible) the fact that God created *this* world (and not another possible world). Unless one assumes some forms of emanationism, which is at odds with classical theism, this is sufficient for the regress to arise, as we will see shortly.

7 For this critique, cf. →Craig (2013).

8 Another criticism often addressed to Leftow (cf. →Bohn, 2014, for instance) is that, in his theory, the alleged necessary truths are not really necessary. Given that God has seen to it that ψ was necessary, and given that God was able to do otherwise, we have to grant that $\Diamond \neg\Box\psi$. Assuming S5, as Leftow actually does, it follows that $\Diamond \neg\psi$. However, if it is possible that ψ is false, then ψ is not "really" necessary. This argument can be extended to any necessary secular truth; therefore, we conclude that there are no necessary secular truths. Leftow's attempt to state both the necessity of certain secular truths and their dependence on God seems flawed.

9 One might ask how wide this space is; let us notice that if a state ψ cannot be actualized by God, then it is not possible either. Indeed, state ψ is not possible because it cannot be actualized by God. Usually, there is a tendency to narrow down what God can do. As we have seen, it is quite common to deny that God can perform contradictory tasks or bring about that 2+2 ≠ 4 or do evil actions, and so on. However, we do not see, *prima facie*, reasons to exclude any state of affairs from the space of possibility grounded on God's creative power. Thus, in our framework, we leave open various options: God could create unicorns, worlds with different physical laws and fundamental particles, worlds in which it is not true that 2+2=4, and even worlds containing contradictory states of affairs. Herein, our model is able to accommodate J. C. Beall and A. Cotnoir's (2021, 2022) recent proposal to exploit the theoretical resources of

dialetheism to investigate some problems in the philosophy of religion (Christ's nature, for instance). We are not claiming that God *actually* creates contradictory states of affairs but that our framework is compatible with very different conceptions of divine power, from strong versions of voluntarism to views according to which God's power is much more limited.

10 We are assuming that the accessibility relation is reflexive.

11 The view that God creates His own nature is likely incoherent because properties such as being unconditional, transcendent, and being able to create are part of God's nature. If God creates His own nature, He must also create His own power to create. However, for creating the power to create, one must already have the power to create something. Thus, the property of being able to create seems to precede its creation. Be that as it may, our framework is neutral with respect to such paradoxical views, and it is compatible both with the idea that God's essence is a super-strong necessity and that God creates His own nature (for a similar position, cf. →Clouser, 2005, ch. 10).

12 This has led some scholars to put forward the thesis that omnipotence and moral perfection are incompatible attributes (for instance, →Morriston, 2001; →Funkhouser, 2006). However, there have been attempts at presenting a solution to the dilemma (for example, →Pike, 1969).

13 This seems to be the solution of Pike (1969). Pike claims that although God has the ability to sin, God will never

exercise this power.

14 A possible answer to this objection is based on the distinction between the power to do something and the power to bring oneself to do something. God would have the power to do evil but not the power to bring Himself to do evil. Although we are sympathetic with this solution, it should be clarified because, as it stands, it does not work. Suppose there is an agent who has the power to do every possible action but, by its essence, cannot make any decision. This agent has the power to do everything but not the power to bring itself to do anything. We would not call this agent omnipotent. Morriston (2001) concludes that the power to bring oneself to do something is a power that should be included in the notion of omnipotence. We believe our bi-modal framework is able to better specify the intuition at the base of this solution.

15 It is not our aim here to assess Molinism, which we have discussed in →De Florio and Frigerio (2019).

16 →Pike (1965) is an excellent, although dated, paper in which this critique is advanced.

17 For a nontechnical introduction, see, for example, →Greene (2011).

18 Cf., for instance, →Ellis (2011).

21 In Defense of Christian Platonism

Paul M. Gould

Abstract: Over the past decade there has been a lot of discussion amongst analytic philosophers of religion regarding the so-called Problem of God and abstract objects. The problem focuses on the difficulty of conjoining traditional (Christian) theistic beliefs with realism regarding abstract objects (i.e., Platonism). In the essay, I survey some of the main objections to Christian Platonism (i.e., it's incoherent, implausible, and unmotivated) and argue that none provide a good reason to abandon (at least) one version of Christian Platonism.

In 2014, I introduced the Problem of God and Abstract Objects by noting the tension between a triad of individually plausible but jointly inconsistent claims:[1]

1. Abstract objects exist.
2. If abstract objects exist, then they are dependent on God.
3. If abstract objects exist, then they are independent of God.

Claim (1) is an endorsement of Contemporary Platonism and Claim (3) is a commonly held assumption of Contemporary Platonism. Claim (2) is a corollary of Traditional Theism, the view that God is the sole ultimate reality. The Problem of God and Abstract Objects is a problem for the Contemporary Platonist who is also a Traditional Theist. I am both. Hence, the Problem of God and Abstract Objects is a problem for me. There is an easy and obvious way out of this problem for the theist: deny Claim (1) and endorse nominalism. I'll not travel down that road for I

think nominalism a false doctrine. I'm also a Traditional Theist, so I'll not deny Claim (2). That leaves Claim (3). I deny it.

Endorsing and denying claims is easy. Articulating and defending a mature metaphysical view of God, the world, and the God-world relation consistent with such endorsements and denials is more difficult. In this paper, I attempt to walk the more difficult path. Our map: In Section 21.1 I explicate my preferred theory, a version of Contemporary Christian Platonism. In Section 21.2 I defend my version of Christian Platonism from pressing objections leveled against my view, or closely related views, since I set forth the inconsistent triad in 2014. My conclusion is that there is at least one version of Christian Platonism that remains unscathed by the barrage of objections over the past few years.

21.1 Christian Platonism

Minimally, the Christian Platonist believes that the Triune God and abstract objects exist. The version of Christian Platonism I wish to defend endorses two further *theological* claims:

1. (i) God does not depend on anything distinct from himself for his existing, and (ii) everything distinct from God depends on God for its existing.
2. God is (i) the creator and sustainer of the universe, and (ii) ontologically distinct from the universe yet present and active in the universe. (iii) The universe points beyond itself to the sacred order.

The aseity-sovereignty doctrine AD secures God's ultimacy; God does not depend on anything distinct from himself and everything distinct from God depends on God. Belief in AD grounds my endorsement of Claim (2) in the inconsistent triad,

and of course, my belief in abstract objects explains my endorsement of Claim (1). Before explaining my mature theory, along with the rejection of Claim (3), I'll briefly unpack my understanding of the "sacramental universe" as specified in SU. I distinguish between "the world" and "the universe." The world includes all distinct from God reality whereas the universe refers to the physical cosmos. I do not claim that the universe is *wholly* physical, but it is not less than physical. The non-physical realities included in the world—abstract objects, souls, angels, and perhaps more—hook up to the universe in diverse ways, some to be explained in this essay. The universe is "sacramental" in that it participates in and points to God even as it is distinct from God. I understand "the sacred order" as *the realm of God and the enacted plan of God to create, sustain, and redeem the universe in Christ*.

My version of Christian Platonism aims to provide an account of the God-world relationship that preserves AD and SU without falling into the incoherency of the inconsistent triad. It's time to explain my account of God and abstract objects along with my denial of Claim (3) of the Triad. I defend a view of God and abstract objects called Modified Theistic Activism (MTA). According to MTA, abstract objects are found in two distinct realms. Some abstract objects are "in" God as constituents of the divine substance. These abstract objects include God's essential properties as well as concepts, identified with divine ideas, and propositions, identified as divine thoughts. God's essential properties exist as uncreated constituents of the divine substance whereas God's thoughts and ideas (propositions and concepts) are created by God via divine intellectual activity. The second domain of abstract objects consists of all distinct from God properties and relations. The abstract objects belonging to this distinct from God realm of abstract objects, call it Plato's Heaven, are created by God via divine volitional activity.

Five further comments are required to fill out this brief sketch of MTA. First, my account of Christian Platonism is version of *metaphysical foundationalism* (MF) the view that God is the metaphysical foundation to all reality. When creating the world, God creates three fundamental or ungrounded (i.e., relative to the created world) entities: *substances, properties*, and *meanings* (i.e., concepts and propositions).

Second, I endorse substantial priority (SP), the view that substances are *fundamental unities* of parts, properties, and powers ordered for the sake of the entity's proper functioning. I adopt a substance-attribute framework and (Aristotelian) causal pluralism. A substance is a fundamental whole that is metaphysically prior to its non-separable physical and metaphysical parts. The universe is understood as a plurality of finite created substances organized hierarchically along a great chain of being (ordered with respect to causal power profiles). Regarding the divine substance, God is a fundamental whole that is metaphysically prior to its properties, parts, and powers.

Third, I endorse real essentialism (RE), the view that a substance's essence or nature is defined non-modally in terms of its real definition. One implication of this view is that I distinguish between a substance's nature and its essential (and accidental) properties. I substance just is a particularized nature; a substance doesn't have a nature, rather a substance is identical to its nature. A substance's essential (and accidental) properties are distinct from and flow from its nature. Regarding the divine substance, God's essential properties metaphysically depend on God, although, given causal pluralism, there is a hierarchy of metaphysical dependency relations found within the divine substance (more on this below).

Fourth, I distinguish three logical moments to God's free, rational, and sovereign creative activity. In the first logical moment, the Biggest Bang, God freely and spontaneously

dreams up all creaturely possibilities, including possible individuals (concepts/divine ideas) and possible worlds (i.e., conjunctions of propositions/divine thoughts).[2] In the second logical moment, the Bigger Bang, God necessarily creates, via the divine will, the abstracta (i.e., properties and relations) that populate the Platonic Heaven. Finally, in the third logical moment of creation, the Big Bang, God freely and deliberatively creates the physical cosmos. This single yet complex divine act "covers" the world without overlap or gap and accounts for all metaphysical possibilities.[3]

Finally, I understand MTA within the divine exemplarism tradition. God creates finite substances (i.e., the fundamental building blocks of concrete material reality) according to divine ideas; divine ideas are the *formal cause* of finite substances. In this way, the universe, in its scale, order, and complexity, communicates and represents the goodness of God to intelligent creatures. The participation relation is a three-place relation between God, divine ideas, and finite substances. More formally, taking "shares in the being of" as primitive, creaturely participation in the divine can be understood as follows:

> P(x,y,z) = x shares in the being of y along dimension z, where x is a finite substance, y is God and z is a divine idea.

The participatory nature of the universe underscores the tight relationship between the natural and sacred orders as well as the importance of the participatory relation for the defender of a Christian Platonic metaphysic. With this brief sketch in place, I turn to objections to Christian Platonism.

21.2 Objections to Christian Platonism

I'll focus on three basic types of objections that have arisen since I introduced the Problem of God and Abstract Objects in 2014: the charges that Christian Platonism is *incoherent, implausible*, and *unmotivated*.[4] I'll consider specific tokens of these types, selecting those I think representative of serious problems for my view. I begin with the charge that Christian Platonism is incoherent.

21.2.1 Christian Platonism Is Incoherent

Perhaps the biggest challenge to any version of Christian Platonism committed to AD is the bootstrapping objection. One way to deny Claim (3) is to bring all abstract objects within the scope of God's creative activity. Thomas Morris and Christopher Menzel's defend one such version of modified Platonism called Absolute Creationism.[5] According to Absolute Creationism, abstract objects are identified with various constituents of the divine mind that are created in virtue of divine intellectual activity. The main problem with the idea of created abstract objects, unfortunately, is that the view is incoherent. The problem can be seen most clearly by focusing on properties. If God is the creator of all Platonic properties and God has Platonic properties (as the Absolute Creationist maintains), then God creates his own properties. But God can't create his own properties unless he already possesses—prior to his creative activity—the property of *being able to create his own properties*. Thus, God is and is not the creator of all properties. This scenario is incoherent.[6]

Lindsay Cleveland calls this version of the bootstrapping objection the *causal bootstrapping objection* and argues that "the only ways [for the Christian Platonist] to escape the causal bootstrapping objection are to maintain that God's properties are uncreated (and so exempt God's properties from the view

that properties are created), to abandon the platonic understanding of property exemplification, or to embrace divine simplicity."[7]

MTA goes for Cleveland's first option: God's essential properties are uncreated, non-separable metaphysical parts of the divine substance. The problem, according to Cleveland, is that "the first option is incoherent" because a new bootstrapping objection called the *metaphysical priority bootstrapping objection* ensues:[8]

> The metaphysical bootstrapping objection concerns the incompatibility of both God's mind being metaphysically prior to His concepts and God's concept/property *having a mind* being metaphysically prior to God's mind.

As stated by Cleveland, the metaphysical priority bootstrapping objection would only be a problem for those versions of modified Platonism that identify properties with God's concepts. Since MTA does not make that identification, she has not hit on any vicious circularity that renders the view incoherent.[9] I do think there is a metaphysical priority bootstrapping objection in the neighborhood, however. It would run like this:

> The metaphysical bootstrapping objection concerns the incompatibility of both God's mind being metaphysically prior to his property *having a mind* and God's property *having a mind* being metaphysically prior to God's mind.

I don't see any real worry here for MTA, given my endorsement of substantial priority (SP), real essentialism (RE), and metaphysical foundationalism (MF). I first argue for *no vicious circularity*. I then argue for *no problematic dependency*.

The divine being is a substance. Recall, according to RE that the real definition of a substance picks out the kind of substance

the thing is. The real definition of God is <a personal being of the divine kind>. Being <a personal being> entails <being a mind> thus God is a mind by nature. Recall also, given RE, that I distinguish between a thing's nature and its properties. The divine substance is a particularized nature; God's essential properties, given SP are metaphysically dependent on God due to the divine substance's role as a final cause of its parts, properties, and powers. To be crystal clear: The "is" in "the divine substance is a particularized nature" is the "is" of identity. This is not an endorsement of divine simplicity, however. Whereas the defender of *divine simplicity* thinks God exists necessarily as a simple being identical to his nature, I defend the doctrine of *divine unity* such that God exists necessarily as a complex being identical to his nature. Thus, given RE and SP, MTA is committed to the first horn of Cleveland's revised metaphysical priority bootstrapping objection: God's mind is metaphysically prior to his property *having a mind*. The problem is that I'm also committed to the second horn of this bootstrapping objection, due to MTA's commitment to the standard Platonist assumption about properties and character grounding:

> *Principle of Character Grounding* (PCG): properties partially explain the character things have.

For given PCG and the fact that the kind of explanation in view here is metaphysical explanation, it follows that the property *having a mind* is metaphysically prior to God's mind. Hence, the apparent vicious circularity. I now argue that there is no actual vicious circularity.

The circularity is apparent because there are two different kinds of metaphysical priority in view in the metaphysical priority bootstrapping objection. As I've argued elsewhere, the

metaphysical priority in view given PCG is distinct from the kind of metaphysical priority in view given SP.[10] Given causal pluralism, we can distinguish at least two senses of metaphysical priority: *formal* and *final* metaphysical priority.[11] These are different species of metaphysical priority, thus there is no vicious explanatory or metaphysical circle. Part of the problem, and it is a rather common problem, is that Cleveland assumes that the Platonist is wed to modal essentialism, the view that a thing's nature or essence is the set of essential properties had by that thing.[12] This is not the case, especially if substances are neo-Aristotelian. Given modal essentialism, if God has a mind in virtue of God's exemplifying the property *having a mind*, then there would be a vicious metaphysical explanatory circle, for the metaphysical priority in view from the part to the whole would be the same kind of metaphysical priority in view from the whole to the part (since God's nature just is the bundle of God's essential properties and thus there is a modal symmetry on this view between God's nature and God's essential properties). But MTA denies this picture. There are at least two senses of metaphysical dependency in view and God's nature is not identical to God's essential properties.

But there are dependency relations within the divine substance and this threatens AD. I now argue that there are *no problematic dependencies* between God and God's parts or properties according to MTA. This is for two reasons. First, given SP, final metaphysical priority is more fundamental than formal metaphysical priority. As Matthew Baddorf writes, "it is natural to think that if B exists for the sake of A, then A is more fundamental than B."[13] Since God's essential properties exist for the sake of God, it follows that God is more fundamental than God's essential properties. Thus, any dependency relations that run from the part to the whole will be less fundamental than the

dependency relations that run from the whole to the part. Second, MF ensures that there are no problematic dependencies within the divine being. Since God is absolutely fundamental, all of God's parts and properties, as well as the relationships that obtain between them, ultimately depend on God, even if they are not temporally or causally posterior to God. Thus, God's essential properties are less fundamental than God twice over, once given SP, and once given MF. Given MF, SP, and RE, God's Platonic properties are ultimately metaphysically dependent on God, even if they are uncreated by God and (on PCG) partially explain God's character.[14] The metaphysical priority bootstrapping objection fails.

In her dissertation, Cleveland raises a third version of the bootstrapping objection, let's call it the *higher-order bootstrapping objection*, that if unavoidable, would render MTA incoherent.[15] Elsewhere I've argued that the bootstrapping objection can be avoided by endorsing two claims:

1. God's essential platonic properties exist *a se* (i.e., they are neither created nor sustained by God, yet they are exemplified by the divine substance); and
2. Substances are Aristotelian.[16]

Whereas the metaphysical priority bootstrapping objection can be avoided by paying attention to Claim [B] and what it means to be an Aristotelian substance, Cleveland's higher-order bootstrapping objection focuses on claim [A]. The basic idea is this. Any property God possesses will have properties of its own. In particular, any property God possesses will possess propertyhood, i.e., the property *being a property*. But, this higher-order property of properties is not an essential property of God's and thus, according to claim [A] it is a property God creates. But since this higher-order property is a property had by

God's essential—and uncreated—properties, it must exist *before* God creates. But then, contrary to [A], the property of *being a property* is an uncreated property. Since being created and being uncreated is a contradiction, MTA is incoherent. As Cleveland states, "The contradiction amounts to a form of divine bootstrapping: the properties God must exemplify in order to create must already exemplify the property of propertyhood before God can create the property of propertyhood."[17]

We might ask, in reply, what is the proper metaphysical account of higher-order predicates? Cleveland's formalization of the higher-order bootstrapping objection is 18 lines long and includes a number of "standard Platonist assumptions."[18] While these assumptions might be standard, I see no reason, in at least two cases, to think they are compulsory.

Do higher-order predicates refer to Platonic properties? Either they do or they do not. If they do not, then the defender of MTA can reject Cleveland's premise (21), the claim that "The property of propertyhood is a simple, essential property of each and every property."[19] Ideological economy is sacrificed on this move, since now the Platonist adds another primitive to her ontology. On this account, it's just a primitive fact that properties possess propertyhood; that's just part of their nature, period. No appeal to properties of properties needed.[20] If the choice is between incoherency and less ideological economy, I opt for less ideological economy.

Alternatively, the defender of MTA can admit the existence of higher-order properties. But then, according to Cleveland's premise (23), we are told that "The existence of a simple essential property of a thing is metaphysically prior to the existence of that thing."[21] But how are we to understand this "seemingly uncontroversial" Platonist assumption? Cleveland offers no explication of this premise and so I'll do my best to

interpret it as follows. The idea seems to be that *for any x and y, if x is an essential property of y, then x exists metaphysically prior to y*. Fair enough. We might ask, however, how are we to understand the kind of priority in view here? It can't be what I call above *formal* or *final* metaphysical priority. Neither species of priority renders my gloss on Cleveland's premise (23) intelligible. Formal and final metaphysical priority explain dependency relations between two *existent* entities. Cleveland's premise (23) seems to trade on a third sense of metaphysical priority, a kind that obtains when one entity exists autonomously or independently and logically prior to another. The idea is intuitively easy to see: consider, e.g., that a thinker is logically prior to his thoughts or a necessarily existing abstract property is logically prior to the concrete contingent reality that instantiates it. Let's call this third kind of dependency *logical metaphysical priority*.

Consider now the priority in view with respect to properties of properties. Notice that there is a necessary connection between any first-order property and the second-order property *being a property*. Necessarily, if the truck possesses *being red*, then *being red* possess *being a property*. Necessarily, if God possesses *being omnipotent*, then *being omnipotent* possesses *being a property*. But then, given this necessary connection between God's essential properties and the property *being a property*, along with the fact (noted by Cleveland) that it is part of the nature of God's essential properties to possess the property *being a property*, it follows that *necessarily, if God's essential properties are uncreated, then any essential properties of God's essential properties are uncreated too*.[22] The overall point is that there is no logical metaphysical priority between the essential properties of God's essential properties and God's essential properties since the latter logically necessitate the former. Thus, the defender of MTA denies Cleveland's premise (23) when it comes to the essential properties of God's essential properties.

Of course, if one denies necessary connections, then premise (23) can be reasonably affirmed in the case of properties of properties and Cleveland's higher-order bootstrapping objection goes through. Unfortunately for Cleveland, she is committed to necessary connections given her commitment to divine simplicity, for necessarily, if God is simple, then God's essence is identical to God's existence. To deny the validity of necessary connections among abstract objects would be *ad hoc*, given her commitment in general to such connections obtaining in the divine case.

If the defender of MTA opts for properties of properties, the view seems to come at a steep ontological price, however. For if it seems that the property *being a property* must itself exemplify the property *being a property*. But then MTA is saddled with an infinite regress of higher-order properties of properties since the property of *being a property* itself has propertyhood, and so on, ad infinitum. In reply, the defender of MTA can either accept the reality of an infinite regress and argue that it is either benign or (perhaps surprisingly) beneficial or reject the reality of an infinite regress and posit some metaphysical or ideological "widget" to block it. Philosophers take seriously the possibility of gunky *concrete* objects (objects with parts all of which have further proper parts). On the first option, I see no reason to think that gunky *abstract* objects are not also a genuine possibility.[23] Moreover, it could be maintained that the indefinite iterability of the "is a property" predicate (and the like) are beneficial features of semantic theories that offset the cost of ontological bloat.[24] The other option is to block the regress by positing (i) further primitive predicates or (ii) a special class of properties that exemplify themselves. On (i) it is argued that while second-order predicates refer to properties, third-order predicates do not (i.e., it is a primitive fact that *being a property* is a property; there is no further property that *it* exemplifies). On (ii) it is

argued that there is a property that *being a property* exemplifies but that property just is itself. Suppose *x* and *y* are properties, R is the relation of exemplification, and that *x* bears R to *y*. (i.e., *x*R*y*) If *x* is a self-exemplifying property, then *x* = *y* and *x* bears R to *x* (i.e., *x*R*x*) and no regress of properties exemplifying numerically distinct properties ensues.[25]

One final worry. Has MTA avoided incoherency only by giving up on the aseity-sovereignty doctrine (AD)? William Lane Craig thinks that MTA's solution to the bootstrapping objection, if successful, comes at a high theological price: the rejection of AD. According to Craig, adopting uncreated properties "constitutes an abandonment of the doctrine of divine aseity in favor of Platonism; it just is the theological unacceptable position that in addition to God there exist other uncreated entities."[26] Given the "standard [Platonist] view" that properties are in things or had by things by way of exemplification, "properties remain objects apart from God... God is not, therefore, the sole ultimate reality."[27] Craig is mistaken. It is possible to admit complexity within the divine being, as the Platonist does, without sacrificing AD.[28] Loosely, I've spoken of the phrase "distinct from" found in AD in terms of "outside God's borders." Any object outside God's borders is dependent on God for its existing, and so on. But then, again, speaking loosely, God's properties—as objects *possessed by God* are not outside God's borders and AD is not abandoned. We can be more precise with the locution "distinct from," however. Craig's charge works only if "distinct from" is understood as "not identical to." But, why think this is the most natural way to understand "distinct from" as employed in AD? If so, then it is impossible to admit any complexity within the divine being and God is necessarily simple. But Craig rejects divine simplicity, so he should reject "not identical to" as the only possible or plausible reading of "distinct from." At any rate, I

reject that understanding. Cleveland helpfully notes that a natural understanding of "distinct from" employed in AD is "neither identical with nor a part of God."[29] If the defender of MTA employs, as I do, a constituent approach, then this understanding is sufficient and Craig's objection misses the mark. But even on a relational account, where properties are "tied-to" the substance without being a part of the substance, as long as SP, RE, and MF are endorsed, AD can still be maintained, with distinct from (as Cleveland suggests) now understood as "not identical with nor a part of God nor possessed by God."[30]

In conclusion, regardless of confident proclamations by some, I know if no iteration of the bootstrapping objection that renders at least one versions of orthodox Christian Platonism, MTA, incoherent. I do however, want to make two fixes to my Claim [A] in light of the above discussion. First, I now think it was a mistake for me to speak of uncreated properties as existing *a se*, as I did in my earlier work. In the literature on aseity, there is some ambiguity in the use of "a se." Some seem to understand "*a se*" as self-existent (my primary sense, as indicated by the parenthetical remark in [A]) and others as independent. I don't think God's essential properties are independent; rather they are not causally dependent, even if they are metaphysically dependent on God. Thus, I should have simply referred to God's essential properties as uncreated without adding the locution "*a se*." Hereafter, I drop that locution when speaking of uncreated properties. Second, I need to amend Claim [A] to avoid the appearance of incoherency given this concern over higher-order properties. Thus:

1. God's essential Platonic properties, *and any essential properties of God's essential properties*, are neither create nor sustained by God, yet they are exemplified by the divine substance.

I don’t see these move as unproblematic or *ad hoc*. I remove the “*a se*” language to aid clarity and I add the further clause regarding higher-order properties to make explicit an assumption I believe was already embedded within MTA.

21.2.2 Christian Platonism Is Implausible

There are a number of instances of the implausibility charge that could be addressed. In what follows, I’ll consider a specific charge related to my identifying propositions as divine abstract thoughts.[31] MTA identifies concepts with divine ideas and propositions with divine thoughts. To focus our discussion, let’s just worry about thoughts. Consider the human thought that Plantinga is wise. The human thought is a concrete object, a mental token, that has the proposition “Plantinga is wise” as its content. In the divine case, however, the divine thought just is the proposition “Plantinga is wise.” So obviously, human thoughts and divine thoughts are different in content and kind. As the prophet Isaiah reminds us in a different context: God’s thoughts are not human thoughts.[32] Since I think being shareable is a sufficient condition for being abstract and since I think that propositions/divine thoughts are shareable (for all the familiar reasons the realist believes in propositions), I claim that divine thoughts (and divine ideas) are abstract objects. William Lane Craig thinks this is all confused. According to Craig, my defense of MTA “is considerably muddied … by [the] bizarre affirmation that God’s thoughts are abstract objects.”[33]

What exactly is the confusion here? In his discussion of divine conceptualism in *God Over All*, Craig describes thoughts as non-sharable concrete mental states or events that possess causal powers.[34] Craig seems to agree that being sharable is sufficient for being abstract, so the alleged confusion must either lie with my claim that divine thoughts are sharable or with

my treatment of divine thoughts as a non-states or non-events.[35] Let's consider each possible confusion, beginning with the latter.

I deny that thoughts are events. Rather, thoughts are the *product* of mental acts or events of thinkers. Are thoughts states? Either thoughts are mental states or they are not. If thoughts are mental states, then in the divine case, I distinguish between two kinds of mental states, abstract and concrete. And why not?[36] Abstract mental states are divine thoughts, the product of divine mental activity. God has other kinds of mental states too. He can also take various attitudes toward his thoughts such as believing, disbelieving, desiring, and so on. These mental states have thoughts as content. But the mental state itself in these cases is not itself a thought; rather, it is the *having* of a thought (and it seems best to characterize these havings of a thought as concrete, like Craig does for thoughts in general).

Suppose thoughts are non-states. Then they are objects of some other kind. In considering this option, I begin by noting that in the literature on propositions, thoughts, and mental phenomenon, there is no commonly accepted ontology for thoughts.[37] Part of the reason for this lack of consensus is the "act/object ambiguity" related to the word "thought" noted by Michael Loux.[38] In ordinary discourse, sometimes the word "thought" refers to a mental state and sometimes it refers to an object of a mental state or event. Like many words in philosophy, then, "thought" is a term of art in need of careful definition. On this second option, stipulate that the word "thought" refers to a non-state (and non-event). On this story, in the human case "the thought that P" refers to a concrete mental token with an abstract proposition P as (grasped) content. In the divine case, "the thought that P" refers to an abstract mental type, the proposition P, understood as a fundamental and brutely

intentional object.[39] So, we have thoughts (non-states, abstract, and simple in divine case), the *having* of thoughts (mental states, concrete and causal in both the divine and human cases), and the *act* of thinking, which produces thoughts.

In order to better understand my Platonist assay of thoughts, whether states or non-states, consider first the familiar Platonist distinction between a property and a property instance. A property is an abstract object whereas a property instance is a (complex) concrete object with three constituents: the property, the exemplification relation, and an individuator. In the same way, on both options considered above, a human thought is a (complex) concrete mental token (state or non-state) consisting of a proposition, the grasping relation, and an individuator (e.g., the agent).[40] A human thought is a concrete mental token, a thought instance analogous to a property instance. A divine thought is an abstract mental (state or non-state) type, a thought analogous to a property. Of course, this picture is consistent with God *having* thoughts (and having various attitudes toward thoughts such as believing, desiring, disbelieving, etc.) and with God *producing* thoughts in the mental act of thinking.[41]

When discussing panpsychism, David Chalmers reminds us that dismissing a view as crazy (or, I'll add, in Craig's case, as confused), is by itself not a serious objection: "While the view [i.e., panpsychism] is counterintuitive to some, there is good reason to think that any view of consciousness must embrace some counterintuitive conclusions."[42] Since at least some, if not all, thoughts and acts of thinking involve phenomenal consciousness, it should not surprise us if there are some counterintuitive results to our mature theory. After all, the ontology of the divine (and human) mental life is far more complex than philosophers of religion often realize, requiring

serious foundational work in ontology, a kind of work that is often lacking in discussions over divine thoughts and ideas. I admit the counterintuitiveness of the view defended here, along with the (somewhat unlovely and) non-unified account of thoughts (divine and human). I deny, however, that my account of divine thoughts is implausible. And I fail to see any confusion (on my part). The only confusion, as far as I can tell, results from Craig's failure to take seriously the proposal that propositions are identical to divine thoughts.[43] Propositions are (abstract) objects and if propositions just are divine thoughts, then divine thoughts are abstract objects too. Moreover, if propositions are the thing that are shared by humans when they have a thought or think, then by Craig's own lights, again, divine thoughts are plausibly abstract since he thinks that sharable things are plausibly abstract.

21.2.3 Christian Platonism Is Unmotivated

In the philosophical literature, the most widely cited reasons for adopting abstract object realism relate to the characteredness of charactered objects and the apparent indispensability of abstracta. I am a Platonist largely because of the problem of universals. I think Platonic realism is the best explanation for the qualitative and resemblance facts found in the world.[44] William Lane Craig thinks the Platonic solution to the problem of universals is rather weak and focuses his attention—and his considerable philosophical guns—on what he takes to be the primary motivation for many would-be Platonists: the indispensability argument for Platonism.[45] Undoubtedly, the debate over the problem of universals and the indispensability argument will continue. Others argue that once God is in the picture, we have even further reason to reject Platonism: whatever work Platonic objects can do, God can do too—and do

better![46] I think the movement goes in the other direction: Platonism gives us reason to think theism and not naturalism is true and theism gives us reason to think Platonism and not nominalism is true. For the remainder of this essay, I consider three underexplored motivations for adopting a distinctively Christian version of Platonism, such as the version outlined at the beginning of this essay. While my remarks will be cursory and limited for reasons of space, I leave these as potentially fruitful areas of investigation that show Christian Platonism's explanatory power and elegance. I begin with a consideration about Platonism and then move to two considerations related to *Christian* Platonism.

21.2.3.1 Platonism Supports Theism

The debate between the Platonists and the naturalists, or Plato's gods and giants, is as much about temperament as it is about ontology.[47] The Platonist relishes in the wonder of being and is comfortable with a rich and genial universe full of a panoply of abstract and concrete objects. The naturalist, on the other hand, tends toward nominalism, desert landscapes, causal closure, and an austere ontology. Intuitively, Platonism renders theism more plausible than naturalism given this difference in temperaments and ontology. It is not hard to find comments to this effect within the Christian Platonic tradition. To cite two early historical examples, consider Justin Martyr (100–165 AD) and Augustine (354–430 AD). In his *Dialogue with Trypho*, Justin Martyr writes that his initial exposure to the Platonic theory of Forms "added wings to my mind," creating in him the expectation that he would soon "gaze upon God."[48] Augustine writes in *Confessions* that the Platonist books made the idea of a non-physical reality more plausible to him, preparing the way for his eventual

embrace of Christian theism.[49] As a nonbeliever, the Platonist texts "opened up for him the novel metaphysical vista of transcendence" that later disposed him to "a spiritual reading of the Christian scriptures."[50] For both of these thinkers Platonism functioned as a kind of John the Baptist, awaking them to a non-physical reality and the possibility of a supreme goodness. Examples of this movement from materialism to Platonic dualism to Christian theism, including autobiographical, could be multiplied. But can an argument along these lines be given to show that Platonism makes theism more plausible than naturalism? If so, then we have a further motivation to adopt Platonism.[51]

To see how theism is more plausible than naturalism on Platonism, I begin by noting two core tenets of Tr*aditional Platonism*: (i) transcendentalism and (ii) top-downism. Regarding (i), Traditional Platonism is committed to "an ontology countenancing the existence of a higher level of reality beyond the manifest image of the physical world."[52] Regarding (ii), Traditional Platonism is committed to the metaphysical and explanatory priority of the intelligible realm over the sensible realm or the immaterial over the material.[53] *Contemporary Platonism*—the view that abstract objects exist—minimally endorses transcendentalism. But must Contemporary Platonism endorse top-downism? No, it mustn't. There are "bottom-up" versions of Contemporary Platonism that only admit any abstract objects required by our best scientific theories. Bottom-up Platonisms will tend to be limited or sparse Platonisms. W. V. O. Quine, for example, allowed (abstract) sets into his ontology because he thought they were a required by our best science, endorsing transcendentalism while rejecting top-downism.[54] But not all Contemporary Platonists are bottom-uppers. Those, like myself, who think that abstract objects best explain resemblance

and qualitative facts, will be top-downers since the character of charactered objects is partially explained in virtue of these Platonic properties (see my PCG above) and so there is (one kind of) asymmetric metaphysical dependency that runs from the abstract to the concrete. In fact, I think—and I'll just assert without argument—that any Contemporary Platonism that admits *sui generis* properties, relations, and propositions to play an explanatory role in structuring the world or the mental lives of agents entails a commitment to top-downism. And since I think that the best (read: most plausible) versions of Contemporary Platonism include properties, relations, propositions, and the like that play these kinds of explanatory roles, I think the best versions of Platonism *simpliciter* will endorse both transcendentalism and top-downism. So understood, let A = transcendentalism, B = top-downism, and "Platonism" be understood as the endorsement of A and B. Consider the following argument:

1. If Pr (theism | A&B) >> Pr (naturalism | A&B), then theism is more plausible than naturalism on A&B. [Premise]

In English: if the probability of theism conditioned on transcendentalism and top-downism is much greater than the probability of naturalism conditioned on transcendentalism and top-downism, then theism is more plausible than naturalism conditioned on transcendentalism and top-downism.

1. Pr (theism | A&B) >> Pr (naturalism | A&B). [Premise]

Therefore,

1. Theism is more plausible than naturalism on A&B. [from 1 and 2, modus ponens]
2. A&B are core tenets of Platonism. [Premise]

Therefore,

1. Theism is more plausible than naturalism on Platonism. [from 3 and 4]

Since premise (4) follows from my gloss on Platonism as discussed above, the key premises in need of defense are (1) and (2). The notion of probability in view in (1) is epistemic: the epistemic probability of theism on transcendentalism and top-downism is the degree of support transcendentalism and top-downism give to theism, the epistemic probability of naturalism on transcendentalism and top-downism is the degree of support transcendentalism and top-downism give to naturalism. Premise (1) tells us that if the two core tenets of Platonism provide significant evidential support for theism over naturalism, then these two core tenets of Platonism make theism more plausible than naturalism. Assuming that theism and naturalism are exhaustive and contradictory options, then premise (1) seems uncontroversially correct.

The premise in need of further justification is (2). I offer two (brief) considerations in its defense. First, there is a tight *theoretical fit* between theism and transcendentalism and top-downism. This tight theoretical fit can be demonstrated in two ways. The first thing to notice is that while every version of theism entails transcendentalism and top-downism, not every version of naturalism endorses transcendentalism and top-downism. Some versions of naturalism—e.g., Quinean—endorse transcendentalism but not top-downism and other versions of naturalism—e.g., Russellian naturalism—deny both transcendentalism and top-downism. The second thing to notice is that for those versions of Platonic naturalism that do endorse transcendentalism and top-downism, a difficult problem remains, called *the linkage problem*. Travis Dumsday nicely

captures the difficulty of connecting the abstract and concrete realms on Platonic naturalism: “given that the abstract realm is, well, *abstract* and of itself causally inert, how is it that entities from that realm come to be manifested/reflected/instantiated by things belonging to the concrete physical realm?”[55] This issue of linkage between the abstract and the concrete, or the intelligible and the sensible, Dumsday notes, calls out for explanation. Historically, and plausibly, many have found a satisfying explanation for this linkage by positing a personal agent connector, i.e., God.[56] Without God in the picture, it remains utterly mysterious—if not unintelligible—how e.g., the moral abstract property of *being unjust* links up (correctly) with a moral act of murder or rape but not the moral act of walking an old lady across the street or how the abstract property of *being brown* is possessed by Fido but not the number two, and so on.[57]

Second, there is a suggestive *existential fit* between theism and transcendentalism and top-downism. Not only do humans long to know *about* reality, they long to be united *with* reality, especially with ultimate reality. As many have observed, there is a near universal longing for union with the source of our existence. Arguably, the most fitting object of this deep human longing is God—a personal being worthy of worship—and not an (impersonal) Platonic Form.[58] As the brief discussion regarding Augustine and Justin Martyr reveal, many have seen the reality of transcendentalism and top-downism as a stepping-stone or weigh station on the path to theism (and not naturalism). To put this last point another way, if life is a kind of quest or journey to discover the true story of the world, Platonism functions like a kind of cairn on a mountain path, pointing the traveler toward a theistic summit. For at least these reasons, premise (2) is more plausible than its denial. If so, then we a further motivation for Platonism: it renders theism more plausible than naturalism.

21.2.3.2 The Participatory Ontology of Christian Platonism Offers an Attractive Account of Key Theological Doctrines

By conceiving of the God-world relation in participatory terms, Christian Platonism offers an attractive account of key theological doctrines. Consider first a participatory gloss of the doctrines of divine transcendence and immanence. As the source and metaphysical ground of reality, God is "beyond and before every creature" yet fully present, even internally present, to every creature and thus "the basis for immanence, even intimacy."[59] On this participatory conception of the universe, matter and spirit are tightly united, but not identical (i.e., pantheism). Next consider a participatory gloss of the distinction between primary and secondary causation. On the version of Christian Platonism defended in this essay, creaturely causal powers are grounded in creaturely natures (given SP) and creaturely natures participate (and are grounded) in God. As Aquinas describes, "the higher cause [God] is not the *power* of the lower cause but the *power of its power*" and thus "the dignity of causality is imparted even to creatures."[60] On this participatory account, there is no competition between creaturely causation and agency and divine causation and agency. Creaturely agency is genuine agency; it is a gift that, like everything else, is imparted to finite substances in virtue of divine generosity and goodness.

Perhaps the most powerful *theological* motivation for Christian Platonism relates to its view of the universe as sacramental. There is a renewal movement afoot in theology today. Many theologians are seeking a return to the so-called Great Tradition, the Platonic-Aristotelian-Christian synthesis of the High Medieval Age.[61] A commitment to Christian Platonism, along with the theological doctrine SU (discussed in Section

21.1), helps to elucidate the nature of an enchanted cosmos. Every finite substance participates in God (via a divine idea) securing a tight connection between the natural and sacred orders.

What makes an ontology *sacramental*? To answer this question, it will be helpful to briefly explore the notion of a sacramental universe, as first described by William Temple in his 1932–1933 and 1933–34 Gifford Lectures.[62] In lecture 19, entitled "The Sacramental Universe," Temple elucidates the relationship between God and world. The universe is "sacramental," according to Temple, because (i) it functions *semiotically*, as a sign pointing beyond itself to the sacred order, and (ii) God's grace and presence are mediated to creatures through the universe (i.e., the universe *participates* in God).[63] Note that on this account, the sacramental universe not only conveys spiritual meaning to intelligent creatures, it also conveys God himself. As Temple summarizes,

> All things are present to Him, and are what they are by His creative will. In and through all of them He is accessible; there is therefore no contradiction in the supposition that in and through certain physical elements, by methods which He has chosen because of their appropriateness to our psycho-physical nature, He renders Himself in a peculiar degree accessible to those who seek Him through such media.[64]

The important point for our purposes is to note that there is a distinctively Platonic underpinning to a sacramental ontology, and this underpinning includes top-downism, divine exemplarism, and a participatory ontology. This attractive account of key theological doctrines, including the idea of a sacramental universe, motivate at least one version of Christian Platonism, as specified in Section 21.1.[65]

21.2.3.3 Linguistic Realism

Call the view that human language is causally prior to concepts linguistic relativism and the view that concepts are causally prior to human language linguistic realism.[66] According to linguistic relativism, language users assign concepts to things and it is impossible to know how good those concepts are at representing to the world itself. According to one version of linguistic realism, human language is caused by the things itself and we can be (at least somewhat) confident that our concepts accurately represent the world. Suppose one is attracted to linguistic realism and the idea, according to theologian Junius Johnson, that "concepts are pre-linguistic because they arise from the nature of things."[67] If so, then one should also be attracted to Christian Platonism. For on the view defended in this essay, every finite substance resembles a divine idea. Given divine exemplarism, God creates a tree according to the divine idea of tree-hood, God creates C. S. Lewis according to the divine idea of C. S. Lewis, and so on. Thus, (divine) concepts are prior to human language. Moreover, as the linguistic realist claims, (human) concepts arise from the nature of things and refer to the nature of (those) things. The overall point is that Christian Platonism can ground objective meaning in the world and the notion of objective truth with respect to language. Not only is there a tight connection between God and the world (given a participatory ontology), there is a tight connection between human language, the human mind, and the world because God has gifted to the world creaturely natures that exactly resemble divine ideas, and creaturely intelligences (i.e., humans) that can know creaturely natures.[68]

While none of these underexplored reasons for Christian Platonism are rationally compelling, they do provide *prima facie*

motivation for the version of Christian Platonism defended in this essay. I conclude that it is false that Christian Platonism is unmotivated.

21.3 Conclusion

In this essay, I've defended Modified Theistic Activism (MTA), my preferred version of Christian Platonism, from three basic kinds of objections. I've argued that there are resources available to the Christian Platonist to avoid incoherency in light of the bootstrapping worry. I've argued that identifying propositions and concepts with divine thoughts and ideas is not implausible, even when such divine thoughts and ideas are understood to be abstract objects. Finally, I've considered three underexplored areas that provide motivation for Christian Platonism. I hope I've shown that Christian Platonism is a defensible view—and even an attractive view—for Christians seeking to develop a mature metaphysics of God and all things in relation to God.

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Notes

1 →Gould, 2014b, p. 2.

2 The idea of the Biggest Bang is from →Leftow, 2012, pp. 272–298.

3 For a much fuller account of my version of Christian Platonism, see my (2022) and my defense, along with Richard Brian Davis, of MTA in →Gould, 2014a, pp. 51–64, 75–79.

4 Additional types of objections not considered in this essay due to space limitations include the charges that Christian Platonism is *unorthodox* and *idolatrous*. I hope to address these challenges elsewhere.

5 →Morris and Menzel, 1986.

6 The most rigorous formulation of the bootstrapping objection to Absolute Creationism is found in Bergmann and Brower, 2006. For Christopher Menzel's reply to Bergmann and Brower, see his (2016).

7 →Cleveland, 2021, p. 170.

8 →Cleveland, 2021, pp. 170–171.

9 I'll not take the time to rescue the Theistic Activist from the metaphysical priority bootstrapping objection raised by Cleveland, but I think that the moves I make in defense of MTA can be applied mutatis-mutandis by the

Theistic Activist to show that the bootstrapping objection fails there too.

10 See my (2013).

11 In my earlier essay, I distinguished between two senses of logical priority, causal and metaphysical. These distinct senses of logical priority correspond to what I am now calling, in an attempt to be more faithfully Aristotelian, final and formal metaphysical priority.

12 In her dissertation (Clevland, 2018), Cleveland attempts a reconstruction of my view by pulling from an earlier paper in which I describe without endorsing an Aristotelian/Platonic view of metaphysical explanation as articulated by Michael Loux (→Gould, 2013). She suggests that I identify a substance's nature with its substance-kind property, the property *being divine*. I do not think that a substance's nature is to be identified with any property, as I make clear in a paper in which I respond, along with Richard Brian Davis, to Michelle Panchuk (→Gould and Davis, 2017). In that paper I further develop my neo-Aristotelian account of substances, including the divine substance, in terms of real essentialism. Cleveland spends time in her dissertation interacting with the Panchuk paper but not my reply to Panchuk. While I don't discuss it here, I think that the bootstrapping objection she mounts in her dissertation against the Theistic Activist fails for largely the same reasons it fails in the case of MTA.

13 →Baddorf, 2017, p. 413.

14 William Lane Craig also mistakenly assuming that all Platonists adopt modal essentialism. In his *God Over All: Divine Aseity and the Challenge of Platonism*, Craig writes, “on Platonism, God’s essential properties *or nature* … serve to explain why God is God” (→Craig, 2016, p. 67, italics added). Since God’s essential properties are uncreated, Craig thinks it follows that “then God’s nature is causally independent of Him, and He depends for His Godhood on His exemplifying the relevant properties” (→Craig, 2016, p. 67). None of this applies to my version of MTA: God’s nature doesn’t exist independently of God. Rather, God is a particularized nature; God is identical to his nature and his essential properties ontologically, but not causally, depend on God.

15 →Cleveland, 2018.

16 I label [A] as [A*] in Paul M. Gould and Richard Brian Davis, “Response to Critics,” in →Gould, 2014a, pp. 76, 75.

17 →Cleveland, 2018, p. 55.

18 →Cleveland, 2018, p. 54.

19 →Cleveland, 2018, p. 53.

20 One other option is to argue that predicates refer to properties only in the case of the non-formal characteristics of things. So, for any formal predicates (e.g., concretehood, abstracthood, propertyhood, self-identity), there is no corresponding property.

21 →Cleveland, 2018, p. 54.

22 More technically, the principle in play is as follows: Necessarily, if (i) God possesses x, (ii) x is uncreated, (iii) x possesses y, and (iv) it is part of x's nature to possess y, then y is uncreated. For a helpful discussion on necessary connections among abstract objects, see →Cowling, 2017, pp. 196–206.

23 For a helpful discussion of mereological and non-mereological complexity and abstract objects, including the possibility of gunky abstract objects, see →Cowling, 2017, pp. 84–86.

24 →Cowling, 2017, p. 123.

25 On this story, the Platonist adopts two primitive notions with respect to exemplification: regular exemplification that is primitively asymmetric and self-exemplification which is primitively symmetric. For a helpful discussion of Platonism and infinite regress worries, see →Cowling, 2017, pp. 120–127.

26 →Craig, 2016, p. 66.

27 →Craig, 2016, pp. 66, 67.

28 For helpful discussions showing the compatibility of divine complexity with the doctrines of divine aseity and divine sovereignty, see Baddorf (2015) and →Fowler (2015).

29 →Cleveland, 2018, p. 51.

30 →Cleveland, 2018, p. 52.

31 Other charges of implausibility with respect to my version of Christian Platonism relate to the idea of created abstract objects and the idea of eternal creation (of abstracta) in general. For a defense of the idea of God creating abstract objects, see my "In Defense of God Making Stuff Up: A Reply to Ward," *Religious Studies*. DOI: →https://doi.org/10.1017/S0034412523000161.

32 See Isaiah 55:8–9.

33 →Craig, 2016, p. 66 n. 27. Craig is referring to the original articulation and defense of MTA advance by myself along with Richard Brian Davis in *Beyond the Control of God?* I claim as an ally in this supposed muddiness, the eminent philosopher Alvin Plantinga, who identified propositions with divine thoughts in his 1982 presidential address to the Western Division of the APA. See his (1982).

34 →Craig, 2016, pp. 90, 92.

35 While considering the divine conceptualist's gambit of employing divine thoughts to play the property-role such that all brown things are brown in virtue of falling under the divine concept/thought *browness*, Craig argues as follows: "Concepts are not plausibly construed as concrete objects, for they are shared by multiple thinkers in a way that thoughts are not. Concepts seem to be part of the content of thoughts. As such, they are plausibly abstract objects, if they exist at all" (→Craig, 2016, p. 92).

36 I allow that divine thoughts and divine ideas have formal causal powers, but not efficient causal powers.

Thus, they count as abstract objects on Craig's definition, since he has in mind efficient causation in his account of abstract objects as non-causal. In that sense, my abstract objects are non-causal too.

37 Frege famously referred to what are typically understood as propositions as "thoughts." See e.g., →Frege, 1956. Many today refer to thoughts as mental states or events. See e.g., →Swinburne, 1997, pp. 62–84.

38 →Loux and Crisp, 2017, p. 122.

39 I'm thinking of propositions as simple. For a discussion of the simple view, see →Juliano-Keller, 2022. In our earlier defense of MTA, Rich Davis and I utilized a structured view of propositions. I'm now inclined to adopt the simple view.

40 For a discussion of the grasping relation as primitive analogous to the exemplification relation, see →Cowling, 2017, pp. 150–155.

41 As Plantinga describes this picture, "while God believes a proposition [i.e., a divine thought] because it is true, a proposition [i.e., a divine thought] exists because God thinks it … [thus] necessarily, propositions [i.e., divine thoughts] have two properties essentially: *being conceived by God* and *being true if and only if believed by God*" (→Plantinga, 1982, p. 70).

42 →Chalmers, 2017, p. 19.

43 Craig is not alone in this failure to consider the complex ontology of human and divine mental states. See, e.g.,

Alex Malpass's attempt to generate a dilemma for the divine conceptualist by failing to take seriously the conceptualist claim that divine thoughts are identical to propositions. Malpass generates a dilemma, part of which is the claim that if the divine thought t has no content, then it is "completely arbitrary to say that t is p, rather than some other proposition, or just that it is not identical with any proposition." (→Malpass, 2021, p. 251) This kind of worry only makes sense if a thought and a proposition are distinct, a claim I deny in the divine case. On my view, divine thoughts do not *have* content, rather they *are* content; they are fundamentally and brutally intentional (abstract) objects. Malpass is responding to Anderson and Welty (2011, pp. 321–338).

44 See e.g., →Gould, 2020, pp. 247–259.

45 →Craig, 2016, pp. 44–53.

46 →Leftow, 2006 and 2012.

47 Grossman, 1992, p. 9.

48 St. Justin Martyr, 2003, pp. 6–7 (2.6).

49 "By the Platonic books I was admonished to return into myself. With you as my guide I entered into my innermost citadel, and was given power to do so because you had become my helper (Ps. 29:11). I entered and with my soul's eye … [saw] the immutable light higher than my mind… It was superior because it made me, and I was inferior because I was made by it… Eternal truth and true love and beloved eternity; you are my God" (→Augustine, 1998, p. 123 [7.10.16]).

50 →Kenney, 2021, p. 177.

51 I'm exploring the weaker claim that Platonism supports theism instead of naturalism. Some philosophers, including some naturalistic philosophers, think that Platonism is *inconsistent* with naturalism. See e.g., →Armstrong, 1983, p. 82. While I'll not argue for the inconsistency claim, obviously, if Platonism is inconsistent with naturalism, and if theism and naturalism are exhaustive and contradictory options, then Platonism supports theism. See also →Dumsday, 2021, pp. 141–158, where he provides a novel argument in support of theism over naturalism on Platonism given considerations related to the relation of instantiation.

52 →Hampton and Kenney, 2021, p. 4.

53 →Gerson, 2021, pp. 15–16.

54 Regarding his denial of top-downism, Quine writes that naturalism recognizes that "it is within science itself, and not in some prior philosophy, that reality is to be identified and described" (→Quine, 1981, p. 21).

55 →Dumsday, 2021, p. 155.

56 For a related linkage problem see →Ruloff, 2020, where it is argued that theism provides a plausible and satisfying explanation for the massive correlation between our mathematical beliefs and mathematical truths (understood as causally inert, mind-independent abstract objects) whereas naturalistic (mathematical) Platonism provides no explanation at all for this massive correlation. This is another nice example of the

theoretical fit that theism enjoys with Platonism and that naturalism lacks. Moreover, my version of Christian Platonism, MTA, can even account for the massive correlation between God's mathematical beliefs and mathematical truths (now understood as causally inert, mind-dependent abstract objects, i.e., divine thoughts). Ruloff is responding to →Baras, 2017. But see also Linsky and Zalta, 1995, where it is nicely argued that our knowledge of (abstract) mathematical reality is consistent with naturalism.

57 But see e.g., →Steinhart, 2021, pp. 1–7, for an introduction to defenders of Platonic Atheism who think that abstract laws and objects do a better job than a personal God in linking the abstract to the concrete and in grounding our existence.

58 As C. S. Lewis famously argues, "If I find in myself a desire which no experience in this world can satisfy, the most probable explanation is that I was made for another world … [and] "most people, if they had really learned to look into their own hearts, would know that they do want, and want acutely, something that cannot be had in this world" (→Lewis, 2001, pp. 136–137, 135). For more on the argument from desire, see →Gould, 2019, pp. 75–79.

59 Davison and Sherman, 2021, p. 360.

60 Aquinas, *Commentary on the Book of Causes*, prop. 9, p. 68 and *ST* 1.22.3; both quoted in Davison and Sherman, 2021, p. 361.

61 Representative works by theologians involved in the renewal project include →Boersma, 2009 and 2011; →Milbank, 2006; →Pickstock, 1998; →Ward, 2000; →Carter, 2018; →McDermott, 2018; →Tyson, 2017; and the collection of essays in Milbank, Pickstock, and Ward (1999) and →Smith and Olthuis (2005).

62 →Temple, 1934.

63 →Temple, 1934, pp. 482–483.

64 →Temple, 1934, p. 485.

65 I'm not claiming that these key theological doctrines, including the doctrine of a sacramental universe, *require* a Platonic participatory ontology. It is possible to reject abstract object realism and to accept a participatory ontology by, e.g., endorsing the claim that there are *particular essences that exactly resemble*. One could defend trope or resemblance nominalism and hold that there is a determinate reality, intrinsic meanings, and a semiotically ordered universe. I think that the resultant view will be less ideologically economical than Platonic versions of participatory ontologies given the need to postulate brute facts for each qualitatively distinct resemblance, but there is no obvious reason to prohibit nominalistic participatory ontologies, as long as essences (now understood as concrete particulars) are on the table. For a non-Platonic account of the metaphysics of participation along these lines, see →Inman, 2022.

66 For this way of characterizing the relationship between word and world, see →Johnson, 2020, pp. 89–95. For a

helpful overview of these basic positions, see Margolis and Laurence, 2019, Section 4.2.

67 →Johnson, 2020, p. 93. For a very brief discussion of how a certain kind of Platonism realism is inescapable when it comes to the relationship between word and world, especially if we assume that errors in language/speech/though possible, see →Graham, 2005, pp. 238–243.

68 For an account of human wisdom as knowledge of (Platonic) forms, see →Evans, 2010.

22 God and Abstract Objects

Scott A. Shalkowski

Abstract: Theological problems—such as that alleged about the co-existence of God and abstract objects—sometimes result from failures to update our beliefs and attitudes and sometimes from philosophical myopia. In this paper, I argue that each features in both the generation of and the attempted solutions to this so-called problem. I will argue, first, that at some stage of inquiry the theologically minded should have taken an "all bets are off" attitude toward the principle that helps to generate the problem. Second, I will argue that even if the theological principle is retained, the required philosophical assumptions should have been recognized as dubious enough to remove any theological urgency to the problem.

22.1 Introduction

God, according to Western monotheistic tradition, is supreme. Indeed, God is said to be so supreme that God and God's actions explain everything. Immediately, we should get a qualification from some quarters. Those maintaining that in at least some circumstances some creatures have "genuine" free will that is incompatible with universal determination by laws of nature and "initial conditions" should query whether God and God's actions *fully* explain everything, since they maintain that free human actions are essential to some explanations. To accommodate God's supremacy—specifically infallible divine foreknowledge—and human freedom, philosophically sophisticated thinkers make distinctions where before there had been none. They

distinguish soft facts and hard facts. Some facts must be soft enough to permit agency, at some level of description, the results of which are not already guaranteed by what God had set as fixed. Regarding both divine supremacy and the fixity or "necessity" of the past, reflection prompted the search for nuance, which found, say some, space to stand in an intellectually respectable manner where none had been evident.

Philosophical problems often arise when plausible principles seem to conflict with one another or seem to have implausible consequences. To generate a problem about the existence, character, or activity of God and the existence and character of abstract objects, some assumptions must be made and developed into theological or philosophical principles. The problem becomes intractable when these principles are treated as inviolable for retaining theological orthodoxy or philosophical good sense.

Dialectically, quite often one's *modus ponens* is another's *modus tollens*, with no resources to adjudicate which inference takes us to the truth. Here, I argue that neither the requisite theological nor the philosophical grounds for any puzzlement about God and abstract objects should have been treated as inviolable. Unnecessary generalization of a plausible principle that was extracted from the context within which it was plausible and gained acceptance will prove to be the problem-generating error.

I begin with a detour, to expose ways in which philosophical problems can be avoided by a process of stock taking and reassessment. I begin with a plausible principle about modality and a philosophical program about its foundations. While no single response to the problem is dictated, a reassessment of the plausible principle is motivated. I maintain that a similar strategy is warranted when thinking about God and abstract objects. I conclude both that theological grounds for rejecting

abstract objects are inadequate and that the existence of abstract objects is theologically unproblematic, requiring only the kind of revision to philosophical principles with which we are comfortable elsewhere.

22.2 A Detour through Modalizing

Many will find plausible:

Modal Status: Every claim/proposition has modal status.

Each claim has its truth value with some appropriate modal qualifier. Each truth and each falsity is either necessarily so, or else it is contingently so. How could this not be? Things could have been different from how they are in some respect(s) or not. Countenancing the nuance that some claims are neither true nor false is no way to avoid claims with modal status. Those claims will lack truth value either necessarily or contingently. Similarly, if the appropriate values are more than two. So far, so plausible.

One prominent program in the second half of the twentieth century was David Lewis's attempt to reduce the facts of modality to the facts of a plurality of concrete spatio-temporally isolated worlds (→Lewis, 1986). Something is necessary iff it obtains or occurs at/from the standpoint of every possible world and possible iff it obtains or occurs at/from the standpoint of some of those worlds. For the ordinary claims that grab our attention such as those about whether 7 is prime or whether Churchill is prime minister of the UK in the twentieth century, there is no conflict between Modal Status and Lewis's reductive program. Received views treat the first as necessary and the second as contingent, and Lewis's reductive theory handles each straightforwardly.

A bit of reflection, however, shows that Lewis's program itself introduces claims into the philosophical mix heretofore contemplated by none. Consider a few.

1. There are possible worlds.
2. Possible worlds are concrete.
3. There are more possible worlds than cats who have lived at 10 Downing Street.
4. There is a possible world in which Winston Churchill is prime minister.

These claims about the possible worlds themselves and about so-called logical space (though "logical" is not most apt) seem like they cannot have modal status. I might live in a house made of wood, or I might live in a house made of stone. Churchill might have opted for a life of leisure rather than a life of leadership. Many metaphysical theories, however, are not like theories about housing or politics. The Lewis program either exposes the very nature of modality or it does not. If it does, it could not fail to do so. If it does not, it could not. One cannot expose the nature of the modal beast merely contingently. Were that possible, Lewis would have exposed only what modality happens to be. None of his reasons for his claims about the plurality of worlds and of the metaphysical work he supposed them to do appeared to be in service of things merely as they are.

That way of putting things makes it look like metaphysical disputes about that reductive program are disputes about whether Lewis's modal realism is either necessarily true or necessarily false, still preserving Modal Status. This, however, does not quite account for the nature of a reductive program itself.

Reducing the mental to the physical is to root all of mentality in what is not by its nature mental. Perhaps mentality "arises" from the physical suitably arranged, but if it is mentality "all the way down," then we have not the reduction of the mental, but its utter pervasiveness. Panpsychism is quite the opposite of a

reduction of the mental to the non-mental, since panpsychism entails the emptiness of the non-mental. Indeed, one might be led to panpsychism precisely because it is just too hard to see how mentality could result from rearranging non-mental stuff. Similarly with the modal. If claims about the plurality itself have modal status, then we have not a reduction of modality, but panmodalism—or just modalism, even if spread across a plurality of worlds—quite the opposite of the Lewis program. Indeed, this is a position Lewis took pains to resist when the focus was on the likes of Churchill and his prime ministership.

Thus is the dispute over "advanced" or "extraordinary" modalizing. Alert to the issue, John Divers sought to preserve something like Modal Status by way of a disjunctive semantics for the modal expressions (→Divers, 1999, 2006, 2014; Divers and Parry, 2018). When dealing with ordinary affairs, modal expressions in the Lewis framework function as they were routinely explained as matters of either universal or existential generalizations over the plurality of worlds. When, however, these expressions qualified the peculiar claims about the plurality itself, the expressions are redundant, semantically vacuous. Sam Cowling demurred, advocating instead amodalism, according to which the peculiar claims about the plurality have no modal status (→Cowling, 2011), despite acknowledging ways to preserve Modal Status.

The Divers strategy saves the principle, though perhaps it is better to say that it saves the form of the principle. Modal expressions are still permitted as prefixes to any claim, but the substance of any occurrence is not independent of the content to which it is prefixed. The content of the modalized claim determines whether the substantive or the redundant interpretation is appropriate. Technically, there is nothing amiss with this disjunctive account of '□' and '♢,' as Cowling readily acknowledges. Philosophically, though, amodalism reflects

better the intellectual point of the Lewis program. Some things are not properly modally qualifiable and those things serve as the ultimate substance of the modal qualifications of other things. While it is tempting to treat this as a pragmatic matter, it is philosophically more useful to treat amodalism as capturing the spirit of the reductive project, leading to philosophical insight that the technically unimpeachable rescue of Modal Status does not. We have, then, Modal Status as an initially plausible principle. Lewis and defenders have made good attempts at making modal reduction plausible. The spirit of the latter requires the rejection of the former. What's a philosopher to do? Here are some options.

1. *Keep the principle; reject the program.* Retain the face value of *Modal Status* and forsake reductive ambitions. Learn to live with panmodalism. Reduction of the modal to the non-modal requires that there be something(s) non-modal, which is inconsistent with the principle.
2. *Keep the program; reject the principle.* The principle, after all, was an intuitive principle, the plausibility of which arose from considering "ordinary" modalizing. Waxing philosophical about the non-modal foundations of modality shows the principle to be limited in scope.
3. *Reconciliation.* Reinterpret either the principle or the program, in a "have your cake and eat it, too" strategy. Divers' disjunctive semantics for modal expressions is just this.
4. *Agnosticism* about both the principle and the program. Further reflection may yield no good way of selecting either of the first two options while exposing reconciliation as preserving the spirit/intent of neither principle nor program.

While my own preference is for none of these options, having argued only that modality is not reducible, which does not entail Modal Status, I recommend option (2) to those whose philosophical sympathies lie with Lewis. It exposes the philosophical point of his project. It is a fact of philosophical life that there will be intellectual inconveniences. Cowling exposed ways of blunting the force of those inconveniences.

22.3 God and Abstract Objects

22.3.1 Preamble

To be perfectly explicit from the start, I have no truck with platonism. I find the grounds given for it unconvincing and that ontological construals of some forms of words to be overgeneral. Platonists typically fail to provide an account of how, if reality is as platonists maintain it to be, we could come to know the existence and details of that reality. Having noted that, theistic platonists at least have a fighting chance of filling the lacuna. If they can account for how God would know about such matters, then divine design could account for our own reliability. I do not suggest that establishing the antecedent of that conditional is straightforward, but divine creativity might do duty beyond its normal domain.

Having said that, I find it hard to see how any of God's independence is compromised in any meaningful way, if it turns out that metaphysical matters are not of God's choosing or even arising from God's being (→Shalkowski, 2014). We may talk of limits on God if there are metaphysical constraints and/or objects not of God's making, but "limit" and its like in this context call to mind improperly fences or walls beyond which there is something off limits.

There are, of course, no metaphysical fences precisely because there is nothing “beyond” those constraints, despite our ability to use constructions that appear to convey ontological significance, such as “There are things that God cannot do.” Those constraints make off limits only what is not possible. I am limited because it is possible to do some of what I cannot do. You might well be able to do some of them. Those are real limitations of mine. For our purposes here, God is unable to do only what cannot be done, which is to say that if we must take the ontological turn here, there is no there there. There is nothing “beyond” those metaphysical limits.

Anyone complaining about God’s inability to do the impossible removes from their own arsenal—no doubt unwittingly—the prospect of using impossibility as a theological guide. Insisting that God can do the unqualifiedly impossible or can be some absolutely impossible way removes impossibility as grounds for concluding that God has not done it, or is not that specific way.

With that great deal of throat clearing out of the way, what of God and abstract objects? First, let us concede that this is a philosopher’s problem, not one of ordinary believers. Only when a philosophical theory is brought to bear on Western monotheistic belief does any apparent problem about God and abstract objects arise. Once in the grip of a theory that explains by way of abstract objects, say, predication, attribution, common characteristics, number, or semantic content, the question of how God is related to abstract objects can arise. If reasons given for thinking that abstract objects serve one or more of the functions said to require them also serve to show that these objects exist necessarily as does God, then the theological relevance of the explanation of necessities becomes apparent. To get to any theological problem pertaining to God and abstract

objects, one must be rather far beyond the understanding of or participation in Western theistic traditions by ordinary believers.

William Lane Craig presents an accessible but sufficiently thorough case for the thesis of his Cadbury lectures, that “God [is] over all” (→Craig, 2016). The main issue as he sees it is that the platonist thesis that there exist “heavyweight” self-existent abstract objects such as numbers, propositions, or properties is that it conflicts with the doctrine of divine aseity. For now, I am happy to follow Craig’s understanding of divine aseity.

> God alone is self-existent; everything else is dependent for its existence upon something else. Thus, the doctrine of divine aseity is closely related to the doctrine of creation. According to that doctrine, everything that exists (other than God) has been created by God. So everything that exists other than God is a created thing. Such things are therefore not self-existent, but are dependent for their existence upon God, their creator. (→Craig, 2016, pp. 1–2)

While I have much sympathy for his rejection of platonism, I find unconvincing his case for the theological centrality of this construal of divine aseity and the attending theological problem that platonism is supposed to create. We join company in thinking that the cases for alternatives (absolute creation, divine conceptualism, and mathematical fictionalism) to be underwhelming, we part company because I find his own case for his preferred position to be similarly underwhelming. Craig presents three broad bases, finding support in:

1. Biblical teachings
2. teachings of early Church fathers
3. perfect being theology.

I will discuss each in turn, urging more caution about the implications of (1) and (2) and that “perfect” is insufficiently fixed to push all the way to Craig’s preferred position.

22.3.2 Biblical Teaching and Tradition

Unsurprisingly, Craig numbers himself amongst those who think it appropriate to find orthodox Christian belief in warranted readings of biblical texts and that those beliefs should inform his own Christian philosophy. I register no complaint with the general framework, save to note that each portion is rather plastic. All biblical texts, not written originally in either English or in contemporary contexts, are subject to assumptions regarding proper translation and, when translated, to assumptions regarding suitable interpretation and application, thus sometimes providing grist of uncertain kind and texture to our intellectual mills. Linguistics might lead us to translations exposing authors' framing matters in terms of their contemporary culture(s) that permit accurate enough communication to their contemporaries but which, when treated straightforwardly, lead *us* astray. Too, charity might lead us to forgive an outmoded conceptual framework to convey ideas that we would articulate differently. Religious, scientific, and philosophical elements of the biblical writings might be set aside in the interest of finding truths expressed in terms not of our own choosing or preferences.

22.3.2.1 John's Prologue and Paul's Epistles

Craig begins his presentation of the biblical basis for banishing abstract objects from Christian philosophy with the prologue to John's gospel, homing in on John's first chapter, verse 3: "All things came into being through him, and without him not one thing came into being," the "him" being the *λόγος*/*logos* who was with God from the beginning (→Craig, 2016, pp. 13–24). Taken from the New Revised Standard Version, verse 3 certainly

appears to entail that nothing—not even numbers or propositions—has any existence independent of God's creative activity.

The NRSV is a convenient choice. A good majority of texts typically used by English speakers, however, are not so straightforward in their implication. Many begin with an initial sentence affirming that the *logos* was instrumental in the making of "all things," but then follow with a sentence omitted from the NRSV, some variation of the King James Version's "and without him was not anything made that was made." These variations can be found in the Amplified Bible ("and without Him not even one thing was made that has come into being"), the Common English Bible ("and without the Word nothing came into being"), the English Standard Version ("and without him was not any thing made that was made"). Others include: American Standard Version, Contemporary English Version, Christian Standard Bible, Darby's Translation, Douay-Rheims, Geneva, Good News Translation(s), Holman Christian Standard, New American Standard Bible, New International Version, Revised Standard Version, Wycliffe, World English Bible, and Young's Literal, all of which seem to limit the scope of John's claim, strictly speaking, to created items that came into being. Nothing that came into being did so without God. Why, exactly, should the philosopher stop John 1:3 where the NRSV does and not with the apparently restricted implications of other versions?

Platonists can certainly live with the restricted implication with the reminder of the inapplicability of any claims about creation, since the objects about which they are platonists were never to be numbered amongst those created. Craig himself, quite rightly, on pain of internal textual contradiction, observes that the scope of the quantifier could not be wholly unrestricted, lest John claim that God too be numbered amongst God's own creations (→Craig, 2016, p. 15). Not stated by the text itself, it is

left to the reader to use conversational common sense to insert the implicit “other than God.” It is hardly a stretch to think that further limitations are implicit.

I make here no definitive judgment about whether the NRSV is best for dropping an otiose phrase or whether the many other versions more accurately parse the text with the apparent restriction. I note, however, that Craig is not in a good position to insist that the NRSV is definitive. He concludes *God Over All* by declining to render a firm judgment about exactly where the platonist-friendly Indispenability Argument fails, contenting himself with the observation that many reject one key premise or another. The lack of even an approximate philosophical consensus on the matter suffices for the verdict that “no view can plausibly commend itself philosophically to the exclusion of all others” (→Craig, 2016, p. 207). Likewise here, I think, for the NRSV’s more streamlined presentation.

Such caution about conversational assumptions accommodates Craig’s claim that to focus on what early first-century thinkers might have had in mind explicitly is to limit the import of John’s claims unduly. Neutrinos and black holes not having yet been discovered are plausibly captured by John’s claim. Germane to which restrictions to countenance would be the theological tasks of John and others following the Hebrew tradition.

One key burden of the Hebrew Bible’s claims about creation was to put some distance between Hebrew beliefs and other Near Eastern traditions. The Babylonian creation myth found in the *Enūma eliš* tells of Marduk creating the earth and sky from the body of the vanquished Tiamat and the Tigris and Euphrates from her tears. The broad biblical tradition’s task was to deny coequal status of any to the God of creation, who used no pre-existing material for creation. There are no other divine beings and creation was *ex nihilo*. Seeing John and other New Testament

writers as continuing this religious project leaves plenty of room for thinking that Plato's ontological concerns were not theirs. One is permitted to think that John is blending claims about the *logos* into the Genesis affirmation that God created the heavens and the earth and all that *they* comprise. I, therefore, recommend the more cautious interpretation of John's prologue, treating it as insufficient for any New Testament distancing from platonism.

Pressing the matter, "the question is whether he intends his domain of quantification, once God is exempted, to be unrestricted" (→Craig, 2016, p. 16). Perhaps John did so intend things. If so, the matter is akin to the issues surrounding *Modal Status*, which was rightly treated as plausible prior to the construction of metaphysical claims purporting to articulate realities so fundamental that they would account for modality, if true. Completely forgivable is the philosopher who, upon being apprised of issues of modal metaphysics, mimics Homer Simpson with "D'oh!" as a reaction. Such peculiar claims might well have escaped one's attention because the focus was elsewhere: conditions of moral responsibility, whether Nixon might never have resigned, or whether information can escape from black holes. Similarly, if John's agenda was to extend the Hebrew religious tradition with its evolving monotheism and hopes for divine sovereignty and mastery of human history, then the ontological foundations of mathematics or correct predication may well have escaped notice. Religiously speaking, nothing is lost regarding belief and practice bearing on salvation, if John could have been apprised of all matters Greek and if he, upon reflection, restricted his domain of quantification. We have no way of settling what John's intentions would have been were he fully apprised of our concerns. I, thus, think that we should thus doubt what Craig finds doubtless (→Craig, 2016, p. 18).

As is common in discussion of John's prologue, John's use of *logos* is said to derive from Middle Platonism, notably from Antiochus of Ascalon, Eudorus, and Philo of Alexandria. I take that as sufficiently established. What I do not yet take to be sufficiently established is that the "chain of custody" between those figures and John is preserved sufficiently to assure us that all and only what Middle Platonists meant by their uses of *logos* carried over into John's. We may concede many similarities, but since scholars of John's gospel are unwilling to claim direct dependence of John's writing on Philo (→Craig, 2016, p. 20), the rest of us should think the matter insufficiently warranted to settle issues of John's meaning. It is not uncommon for language to be appropriated without all of its implications being imported. The temporal, geographic, and cultural distances should, at minimum, give us pause.

Similar remarks apply to St. Paul's use of universal quantifiers in Romans 11:36 ("For from him and through him and to him are all things."), 1 Corinthians 8:6 ("from whom are all things … through whom are all things"), 11:12 ("all things come from God"), and Colossians 1:15–16 ("by him all things in heaven and on earth were created … all things have been created through him and for him"). Certainly, the quantifiers appear to be unrestricted. What is not, though, obviously settled is what Paul took to be numbered amongst the things in heaven and on earth. Paul may well have used a "totalizing idiom" (→Craig, 2016, p. 26). That, though, fails to settle the content of the totality. The aforementioned tacit exclusion of God in heaven shows that there is totalizing and then there is totalizing. The texts do not really settle the matter we confront and their appearance of being utterly restricted is already conceded to be mere appearance.

22.3.2.2 Church Fathers

We get *somewhat* more straightforward guidance from post-biblical figures and creedal statements. The Nicene Creed begins "I believe in one God, the Father, Almighty, Maker of heaven and earth of all things visible and invisible." I defer to those who see here claims taken from John and Paul. Perhaps writers of the creed intended something as "totalizing" as Craig finds in those writers. Ambivalences about the actual and counterfactual import of John's and Paul's remarks simply transfer to the creeds.

In the interests of space, I limit myself to a few remarks, rather than repeat in several cases where the above observations apply. First, Church Fathers may be respected without taken to be definitive. Hardly any would treat their proclamations as instances of divine revelation, even if as instances of genius. Second, each is working within philosophical traditions using tools that we hardly take to be definitive elsewhere in our intellectual endeavors. To be perfectly clear, I here register no objection to either Platonic or Aristotelian frameworks, but only the observation that the best tools of the time, if indeed they were the best at the time, need not limit contemporary religious thinking on these matters (→Shalkowski, 1997). Third, sometimes the evidence adduced from Church Fathers does not really seem quite to hit the mark. Consider just the appeal to Origen.

> It is difficult for us to explain this other world; and for this reason, that if we did so, there would be a risk of giving some men the impression that we were affirming the existence of certain imaginary forms which the Greek call 'ideas'. For it is certainly foreign to our mode of reasoning to speak of an incorporeal world that exists solely in the mind's fancy or the unsubstantial region of thought; and how men could affirm that the

> Saviour came from thence or that the saints will go thither I do not see. (*On First Principles* 2.3.6, quoted in →Craig, 2016, pp. 35–36)

It is not perfectly obvious how this bears on the modern question of God's aseity and contemporary platonism. Origen seems led astray by the label "ideas." Modern platonists maintain the objective, heavyweight existence of abtracta, without any implications about their existence "in the mind's fancy" or in a "region of thought" (though thought may be how we have access to the region) and they need make no representation at all regarding from where the Savior might have come and where the saints might end up. Those issues are all beside the main platonist point. Any wishing to conflate the region of divine and saintly existence with the domain of mathematical objects have no less and no more work to do than those who wish to distinguish them. Platonists, even Christian platonists, can remain silent on the matter.

I conclude Section 22.3.2 with the judgment that platonists, Christian or otherwise, should be underwhelmed by the case for mandatory distance between both biblical and early Christian thought and platonism.

22.3.3 On Being Stuck with Platonism

22.3.3.1 A Thought Experiment

Suppose that I am wrong and that biblical writers really did intend their universal quantification to be wholly unrestricted save for the *sotto voce* "except for God." If we treat seriously the thought experiment of what said writers would now say were they apprised of modern physics and the implications of their pronouncements for neutrinos and black holes, perhaps we

should probe just a bit more about why they might think platonists' abstract objects are just as troubling as are the heavens and the earth, more narrowly construed, were they to exist uncreated. Having no powers of counterfactual mindreading, I limit myself to arguing that they should be untroubled by the prospect.

Indulge yet another detour, this time through human creativity. Suppose I embark on two weeks of minimalist bushcraft. I take rucksack, sleeping bag and pad, a small axe, a cooking pot, and not much else. During the fortnight I decide to make the best wood and tile dwelling that I can. Those who join me at the appointed place after the fortnight marvel at the detail and sophistication of my structure, noting that it took thousands of years of human history for there to be any evidence of such sophisticated structures. One visitor asks how I managed to construct such a marvel in short order, since I am not known to be a survivalist. Consider two forms of answers that I might give.

> *Scenario 1:* I declare that I just sat for a while thinking about wood, stone, clay, fire, and axe. I experimented for a day with the wood and axe to find useful joinery, I mixed clay and soil, firing various mixtures. In the end, I "came up" with the ideas and implemented my plans for the structure we see. The proper assessment of this, I think, is that I turned out to be very creative, having found myself in an unfamiliar environment with tools and materials with which city slickers tend to have little facility.
>
> *Scenario 2:* I confess to having indulged the adulation of my friends a bit too long. Yes, I had no help in the finding and manipulation of tools and materials, but I pull out detailed instructions and drawings for how to construct in this part of the world a structure like the one in which we stand. While my skill with axe and wood as well as my ability to follow instructions and architectural drawing may still be rightly praised, my creativity should not be, since it extends no further than thinking the project itself a good idea.

When the independent existence of abstract objects is thought to compromise God's own independence and creativity it might

result from how some of these objects are characterized, especially to those only just becoming acquainted with the denizens of Plato's heaven. Those figuring most prominently in Plato's dialogues are the forms. What forms are and their significance not being perfectly and immediately obvious to modern students, we might take different tacks to help students to understand.

Justice was a major concern of Plato's *Republic*. How to explain the Form of Justice to a modern reader? We might say that the Form of Justice is the very idea of justice itself, rather than the approximations that we observe in human history. Not to confuse the idea of justice with any particular mental representation of justice that any might have, we might well say that Plato had in mind the *ideal* of justice.

When thinking, however, about the Form of Rose, we might find reference to an ideal less than useful. An ideal rose? Were there to be any such there would need to be an ideal size and hue. An ideal rose plant would need to have an ideal time of development from seed to shoot to mature plant to bud, to flower, to rosehip. In due course, perhaps one could be persuaded of such ideals, but anyone trying to make Plato's ideas plausible should start elsewhere. Better might be to think of the Form of Rose as, well, a form, a structure, a pattern. Roses are easily distinguished from tulips, even when both are red, and from carnations and many others. If the Form of Rose is more like a pattern that we might give to children during a craft session to aid them in their drawings or paper sculptures, then questions about ideal roses drop away. All manner of variation is permitted amongst fair dinkum roses.

A theological concern might be that were there independently existing forms, they serve as patterns of the world (→Craig, 2016, pp. 20–24). God's creative activities would always be reduced to consulting those patterns. Much like my

own consulting instructions and drawings, God's consultations would reduce divine creativity to taking instruction from elsewhere. God has no choice to make roses according to the Form of Rose.

I deny the reality of this problem. Note how when trying to make sense to ourselves of the forms, we had to weasel around a bit. When thinking about the Form of Justice, think of an ideal. When thinking about the Form of Rose, think of a pattern. That such shifting is required to make sense of the platonic framework belies the fact that we have little useful access to any such domain. Reduced to shifting metaphors, we should be troubled by the suspicion that we do not really grasp the doctrine to which we adhere or from which we wish to distance ourselves.

To generate the theological concern, we end up treating God as less creative than me, when we each do essentially the same thing. In the more creative *Scenario 1*, I just thought about what I wanted to do and "came up" with the ideas to construct a building of a given sort and to solve engineering problems as they arose. To be fair, none of my friends who marveled at my handiwork were thinking about Plato's forms. Suppose, though, that I do my best to talk them through reasons, both ancient and modern, for thinking that Plato was right that for each kind of thing there is a form that each individual of that kind exemplifies by bearing a relation to it. That just is what it is to be a building like this, or a rose like that. Upon becoming card-carrying platonists, my friends think me no less creative than they did before. The metaphysical story about what it is to be something of a given kind makes no difference to the independence of my thought and reduces not one whit the difference between my independence of thought in *Scenario 1* and my having first raided a filing cabinet in *Scenario 2*.

The theological problem begins to get traction when we fall into thinking that were there to be independently existing abstract objects God would inevitably be reduced to a divine version of *Scenario 2.* Unable to invent what already exists, God could only "look to" the forms for instructions, as it were, about how to make a cosmos, or even roses. What is not yet in evidence anywhere, so far as I can tell, is that God's situation with forms is any different from mine, even on the assumption of platonism. Why am I creative but God a mere hack when all we each do is think about our circumstances, "come up" with ideas, consider our priorities, develop and then implement a plan?

There is, of course, the difference that I am both concrete and embodied and, thus, no part of Plato's heaven. Anti-platonists take this to present a challenge regarding how concrete and embodied thinkers can by way of thinking have access to Plato's heaven. I make nothing of the challenge, save to note that those with theological worries have not yet given any reason to think that God's situation is usefully different from my own. Not being abstract, God is concrete. God is an agent. God is not a part of Plato's heaven of eternal, independently existing *abstract* objects. Has any reason ever been given for thinking that just by virtue of being eternal and independently existing God both inhabits the same domain as abstracta and that those abstracta are any more open to consultation by God than they are to me? For the problem to take hold, we are owed more than that both God and abstracta are timeless, necessarily existing beings. We need some account with details of *how* God might "interact" with abstracta in ways that I cannot, and that account must then feed into something more detailed about exactly how *that* kind of "interaction" might compromise God's independence and creativity in a way that compromises divine greatness. Noting my embodiment does not suffice.

To reprise concern that partisans hardly know of which they speak, what is the meaning and relevance of “domain” in this context that might make it the case that God and abstracta inhabit a common domain in ways that I and abstracta do not? I am aware that logicians and mathematicians use the term freely and for now I register no objection to their usage. Theirs does not require that all objects in a domain share a space, have contact with each other, or know about each other. This particular route to the theological worry, however, seems to require something like what logicians and mathematicians do not.

If the philosophical use of “domain” is to provide a wedge to open the door to theological compromise that any should find troubling, it must mean something akin to its use in spatio-temporal contexts, along with a further account of how it is that an eternal, independently existing, non-physical concrete being is forced to consult the eternal, independently existing, non-physical abstracta in the shared domain. If the anti-platonist puzzle is driven by how *concrete* objects could consult abstract objects, then their worry carries over here, making the theological problem obscure.

If concreteness is beside the point and the theological concern is to arise because both God and abstracta are eternal, independently existing, and non-physical, the theologian owes us an account of exactly how these shared attributes force an interaction that renders God less creative than am I, by virtue of being “imprisoned” with abstracta whereas I am not, despite my inability ever to be completely rid of them. Merely citing shared characteristics is inadequate to demonstrate a problem. Those searching here for theological reasons to reject platonism have unfinished business.

22.3.3.2 Perfect Being Theology

> … the requirements of sound systematic theology include the affirmation that God is the source of all things apart from Himself. For divine aseity is a fundamental requirement of perfect being theology. … God would be diminished in His greatness if He were the cause of only some of the other things that exist. (→ Craig, 2016, p. 41)

To perfect being theologians, I urge caution. The plausibility of this reasoning depends on what else is in play. The God of Western theism does seem to be greater than Marduk, if Tiamet was also to be an independent existent, in no small part because Tiamet was vanquishable. For the sake of brevity, conflate mortality with contingent existence. Marduk, not being the creator of all contingent existents, may well fail to be the greatest being possible. Platonists, though, maintain that abstracta are unlike Tiamet in exactly this regard. They are necessary existents. As such, being their cause is an impossibility. No being, no matter how great, *could* be their creator.

Perfect being theologians now find themselves in the position of some students I have taught. Having listened to a lecture on the so-called logical problem of evil, some were inclined to object that theists were making their own lives conveniently too easy. God should not be limited by “logical possibility.” God, if deserving of divine status, should be able to accomplish the impossible. Full stop. No qualifications. I urged caution. Were God to be able to accomplish the impossible as wished, then critics of religious belief (as these students always were) would be unable to demonstrate God’s non-existence by locating an internal contradiction—perhaps with a paradox of the stone—or an external contradiction—perhaps with the existence, character, or pervasiveness of evil. By their own insistence, they would have empowered God, as it were, to do

the very things that we supposed to demonstrate God's impossibility. Their initial insistence on widening the scope of God's alleged power was not a good dialectical move. I warned them to be careful what they wished for.

Similarly here. Insisting that both that God is a perfect being and that being a perfect being demands that God account for all other existents, provides the basis for a platonist rejection of theism and anything entailing it, such as Christianity. To the degree that any think some platonist arguments successful, they have now been facilitated in rejecting the cornerstone of Craig's Christian belief.

Were it shown, contrary to my own expectations, that platonists are correct in their ontological claims for abstracta, a better option would be to ask: what is lost? How compromised is God's perfection, if some abstracta turn out to be necessarily existing independent objects? Pretty much nothing is lost. Since, *arguendo* it is impossible for those abstracta to be created, it is no blemish on God's creative ability to be unable to create them any more than it is a blemish to produce any other impossibilities, such as humans that are not mammals, iron that is not metal, or composite objects with no parts. If there really is some stone genuinely impossible to create, then it is no compromise of God's creative power to be unable to create it. The problem lies with what is to be created, not with the would-be creator. And, for the lifting as well. *My* creative powers are limited precisely because there are creatable and liftable objects beyond my abilities, especially very big stones. Whilst I might take steps to increase my capacities, it would be a fool's errand *par excellence* to pursue the ability to create an uncreatable stone or to lift an unliftable stone.

"Perfect being" is a term of art. With the best wills in the world, philosophers and theologians try to dress our rough and ready understanding of perfection in intellectually rigorous and

respectable garb. I have given no reasons that even I take to be persuasive for rejecting perfect being theology. My resistance throughout has been with the details within that framework, if it is embraced. It may be that Craig is more prescriptive about the details of the framework while I am less so. He finds unambiguous weight in biblical texts, Church fathers and councils that I cannot. Mine is more tentativeness than rejection. Origen, Augustine, Aquinas, et al. were giants that Western theists should take seriously. If pew sitters wish to take any of their declarations over my arguments here, I do not fault them. They have other fish to fry and from any kind of distance, perhaps the smart money goes on those with long-standing reputations. Still, I think that there are unrecognized nuances here to give contemporary working philosophers and theologians pause. Neither perfect being theology nor orthodoxy are genuinely threatened by anything raised here.

The refrain is not unfamiliar. In matters of practice, the religiously-minded might agree on the value of loving enemy no less than self. What counts as loving sufficiently in any particular context may not be perfectly obvious. On the one hand we should be mindful of This, but we must also not forget That. With both This and That in mind, how much of This and how little of That should we apportion to the actions we do or do not? For all that I have argued here, perfect being theology can stay. Just as the precise implementation of loving enemies as self is not always perfectly obvious, so the precise formulation of perfect being theology is not perfectly obvious. If abstracta prove to be impossible, then we are free to interpret the relevant biblical quantifiers to be unrestricted bar one. We may treat divine aseity in the most straightforward manner.

If the jury is out regarding platonism, then the jury should stay out about the straightforwardness of the details of divine aseity. If the jury returns with a positive verdict for platonism,

then aseity is not threatened; it must just be more nuanced than some might have hoped. Nothing significant is lost, other than a simple, straightforward presentation of one or more theistic doctrines. There is no shame in this, nor does it involve some religious "compromise," even if others wished to hang the label "heretic" on those who embraced what I call nuance. There was a neatness to ancient cosmology with circular orbits. We have traded that neatness for another. We should beware the seductions of initial forms of neatness.

Craig thinks that he's identified a bridge too far, one that inevitably takes us beyond any plausible interpretation of "perfect being." The realities for which God is responsible, if platonism is correct, is "infinitesimal," far outnumbered by abstracta (→Craig, 2016, p. 18). It sounds even worse on p. 41: "The physical universe which has been created by God would be an infinitesimal triviality utterly dwarfed by the unspeakable quantity of uncreated beings."

This is a different worry than we have seen so far. If John, Paul, Church Fathers, and writers of creeds really intended their quantifiers to be unrestricted bar one and if they took deviation to signal heresy from which would-be believers should shrink, then the most minimal falsification of their "totalizing" claim is devastating. We require some reason for thinking that God's creation being "utterly dwarfed by the unspeakable quantity of uncreated beings" would make the falsification even worse. There is, for instance, an unstated assumption that God has, at most, created countably many things. Still, suppose "utterly dwarfed" is accurate. What of it?

Perfect being theologians must commit to God being unique in some respects for God to be uniquely worthy of worship. Not in every respect, though. God is not the only concrete entity, nor the only with moral capacity and significance, nor the only with power. There are many more of us with these characteristics

than the unique God. We are forced to live with many ways in which God is not at all unique. If the philosophical push came to theological shove, perfect being theologians can still make their peace with God (and creation, if need be) being outnumbered in ways demanded by platonism. They would make their peace with it just as they make their peace with many other things. There would, in the envisioned dialectical circumstance, be no other choice available, the alternative being genuinely impossible. To do otherwise would be to rage pointlessly into the metaphysical night.

Finally, are considerations adduced by Brian Leftow (→Leftow, 1990; →Leftow, 2012, pp. 234–235), and adopted by Craig (2016, p. 43). First, on the platonistic story about property possession, God's being divine depends upon exemplifying the relevant divine-making forms. Second, since unique independent existence is one of the divine-making properties, God could not be divine, since if platonism is true, the one we label with "God" does not exist *a se*.

As in other stages of my discussion, I concede the form of the problem but not its force. Begin with dependence. There is the wholly uncontroversial fact that for every necessary existent, there is a fashion in which the existence of each counts as a necessary condition for any other thing, whether individual, event, or state of affairs. For any fact, *F*, and any necessary object, *NO,* variations on the material conditional "$F \rightarrow NO$" hold.

This minimal manner of expressing necessary conditions should not detain us. There is dependence and then there is dependence. There are necessary conditions and then there are necessary conditions. According to platonism, the existence of any of the necessarily existing abstract objects is a necessary condition for anything else. Were they not to exist, the other item in question would not either. That all of those abstracta are to be inert and that some are uninstantiated renders this kind of

dependence uninteresting. Since nothing bears the relevant instantiation relation to those uninstantiated abstracta, nothing actually depends on them, even when matters are construed as Leftow wishes. So, this kind of dependence/necessary condition should be set aside as a limitation of the expressive resources of the material conditional.

Leftow insightfully zeroes in on those abstracta that God does instantiate, according to platonism. Were there no abstract property that God does instantiate, God could not have that property. Correct. For the theologically minded to be troubled, however, we must expand the scope of "dependence" beyond concerns over whether God is creator or created. The mere existence of the abstracta has no implications for God being counted amongst the created. *Ex hypothesi*, these abstracta are to be inert, so they cannot be causally efficacious and forcing God to be amongst the created or even to compel God to be one way rather than some others.

To see what is peculiar about what is supposed to be troublesome, think about more ordinary objects. Admiring roses, you wish to construct one in a laboratory. You gather raw materials and begin assembling them. The flower you produce will depend upon you in many ways. In particular, its color will depend upon you mixing the right chemicals and placing the mixture in the right places. The red mixture goes in the petals, while the green goes in the stem. That is clear dependence of the flower upon you and your actions, not only for its very existence but also for its character. Were another heavenly being related to God as you are to your flower, then we have a problematic dependence. Had you not done what you did, there would be no rose. After a fashion, you are creator and the rose the created. That nothing at all like this story is implied by platonism removes any practical religious concern. Amongst persons, none is greater than God. Amongst the eternal and

independently existing, there is still none greater. No other has the key divine attributes. No other brings God into existence, and no other compels God to have all of those, or any other, properties.

You still worry that there are infinitely many uncreated things? Fear not, since religiously-relevant grandeur is not a matter of numerosity. You worry that God can neither create nor destroy any of these infinitely many things, thus compromising divine omnipotence? Fear not, since there are no fewer necessities that God is unable to alter, even according to anti-platonism. Non-contradiction? Bivalence? Gödel results? God is stuck with them no less. If the necessities of your favorite logic and your favored metaphysical theories do not compromise God's omnipotence, why do the platonist's favorite entities? They neither create nor compel God. God still is self-existent and the abstracta compel no divine actions.

Finally, to what might be thought to be Leftow's *piece de resistance*: God cannnot even be divine, if platonism holds. Contra my finding flexibility in "perfect being," Leftow finds rigidity in divinity, which demands unique independent existence. Any divine attribute, according to platonism, involves both particular and independently existing abstract object. So, God cannot be divine, since not the sole independent existence.

Whilst I grasp that one's account of divinity is more streamlined without dealing with platonist troubles, the theological trouble is still not apparent. Were we to become convinced of platonism, all that is required is that the same hard work and sophistication that was wielded when thinking about omnipotence or the trinity (three persons, one substance; begotten, not made) need be employed here. An unexpected turn? Not really. Plenty saw that the *straightforward* account of *aseity* could not be maintained alongside platonism. That, though, is no reason to think that we have the basis for rejecting

platonism, any more than we have grounds for rejecting the central necessities of our favored logic simply because a theory of omnipotence is so much easier to formulate. We may prefer to have no need to complicate our theories, but the complications are merely pragmatic inconveniences, not indicators of falsehood (Bueno and Shalkowski, 2020).

Leftow's real *piece de resistance* lies elsewhere (→Leftow, 2012, 2020). Lewisians think that the plurality of worlds underwrites modality, since the plurality constitutes so-called logical space. Leftow finds this function in God. He may reject Modal Status as a natural consequence of his own account of modality. This does not, however, undercut the import of my earlier considerations. Leftow maintains modal orthodoxy. God cannot but act in accord with logical, mathematical, and metaphysical truths (→Leftow, 2020, p. 54), so there are still modal "constraints" on anything God might do or be. Leftow's is a different way of massaging the problem. I found oddness in "constraint". He generates oddness because on his account, God is the source of these so-called constraints. Perhaps some back-and-forth would expose weaknesses in one way of identifying the oddness that would convince those inclined to the other. Be that as it may be after more philosophical work, there is still no there there when it comes to things "now" off limits in ways articulated by correct logic, mathematics, or metaphysics. My platonist-friendly calming of theological waters remains despite Leftow's option. Better to have my option available, since it is hard to know when any philosophical theory will crash and burn.

22.4 Conclusion

The "problem" of God and abstract objects is, I think, not much of a problem. This paper's ambitions are very limited. Whilst my sympathies are with anti-platonists, nothing here rests on those

sympathies. I care only that theists—even perfect being theologians—recognize options that are not immediately apparent. If things go against my philosophical sympathies, no great theological calamity befalls any. Adjustments can be made in good conscience.

I have attended to the reasons adduced for thinking that proper, serious Christian philosophy must, in effect, provide grist for the anti-platonist mill. For each kind of consideration, I still find myself unmoved. Unmoved, perhaps because the history of all manner of intellectual developments have involved finding sense where none was thought to be. Space and simultaneity still seem to many of us like they should be absolute and not relative. We have learned to make our peace with the unfamiliar and the unwelcome. Here, in my relative ignorance of how platonism/anti-platonism disputes will play out, I find theological space for either result. That may not be everything, but it's not nothin'.

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VII God, Infinity, and Pascal’s Wager

23 Pascal's Wager and the Precautionary Principle

Paul Bartha

Abstract: *Pascal's Wager* is based on the idea that "wagering for God" has infinite expected utility, assuming non-zero probability of an infinite reward (salvation). But infinite positive utility appears to trap us in puzzles and paradoxes. Similar difficulties arise for decisions where there is a risk of catastrophic outcomes (negative infinite utility). The precautionary principle (PP) has emerged as a tool for making such decisions, especially those involving environmental catastrophe. Commentators have noted the analogy between Pascal's Wager and PP. They use it negatively: careless formulations of PP are vulnerable to analogues of well-known objections to Pascal's Wager. In this paper, I use the analogy positively: successful replies to objections against PP can be adapted to yield successful responses to prominent objections against Pascal's Wager. Furthermore, these responses do not require that we give up the use of infinite utilities in either Pascal's Wager or precautionary reasoning.

23.1 Introduction

Pascal's Wager is typically framed as a decision about whether or not to wager that God exists. To wager for God is to take practical steps to promote religious commitment and belief. So long as your subjective probability that God exists is positive, wagering for God has infinite expected utility and is therefore superior to not wagering, which has at best finite expected

utility. This argument is based on the decision table represented in →Table 1 (→Hájek, 2022).

→**Tab. 1:** Pascal's Wager.

	***G* (God exists)**(p)	**~*G* (God does not exist)**($1 - p$)
***W*: Wager for God**	∞	f_1
~*W*: Don't wager	f_2	f_3

The numbers f_1, f_2 and f_3 are *finite* utilities. In the top-left corner, we have the possibility of salvation, which has infinite utility. If p is the probability that God exists, then as long as $p > 0$, *Wager for God* has infinite expected utility, while *Don't wager* has merely finite utility.[1]

There are striking similarities between Pascal's Wager and a different kind of decision problem: how should we act when there is a risk of potentially catastrophic outcomes? Scientists working on the Manhattan Project in 1942, for instance, briefly halted their work after recognizing "a chance that an atomic bomb might trigger the explosion of the nitrogen in the atmosphere or the hydrogen in the ocean" (→Compton, 1956, p. 127). The *precautionary principle* (PP) has emerged as a tool for making decisions in such circumstances (→Gardiner, 2006; →Steel, 2015). PP is often interpreted as placing absolute priority on disaster prevention. One way to represent this idea is to treat catastrophes as outcomes that have negative infinite utility.[2] The decision of the scientists working on the atom bomb can then be represented with the following decision table (→Table 2) which

looks the same as →Table 1 except for the replacement of ∞ with −∞.

→**Tab. 2:** Building the bomb.

	***F* (Flammable)**(q)	**~*F* (Not flammable)**$(1 - q)$
***B*: Build the bomb**	$-\infty$	g_1
~*B*: Don't build	g_2	g_3

The utilities g_1, g_2 and g_3 are finite. If q is the probability that the atmosphere or ocean will ignite following the detonation of an atomic bomb, then as long as $q > 0$, *Build the bomb* has negative infinite expected utility while *Don't build* has merely finite utility. The recommendation is *Don't build*.[3]

The analogy between the reasoning in Pascal's Wager and in precautionary scenarios has been noted (→Turner and Hartzell, 2004; →Manson, 2002; →Sandin, 2009; Colyvan et al., 2010; →Johnson, 2012; →Munthe, 2019). In Tables 1 and 2, the analogy is precise: the assumption of infinite (or negative infinite) utility powers both arguments, delivering recommendations that require only minimal probabilistic information. But the analogy extends beyond the basic structural resemblance between the two arguments. Commentators have also noticed that the arguments have analogous weaknesses. The assumption of infinite utility is the source of some of the most stubborn and serious objections to Pascal's Wager, and similar objections have been lodged against PP (even when it is formulated without negative infinite utilities). In the literature on PP, the analogy with Pascal's Wager is typically invoked as a cautionary lesson about precautionary reasoning itself. The lesson is: formulate

the precautionary principle with care in order to avoid the problems that bedevil Pascal's Wager!

My strategy in this paper is to run the analogy in the other direction. I shall argue that successful replies to objections against PP can be adapted to yield successful responses to prominent objections against Pascal's Wager. Furthermore, these responses do not require that we give up the use of infinite utilities in either Pascal's Wager or precautionary reasoning. However, both infinite and negative infinite utilities will need to be represented quite differently from Tables 1 and 2, using the "lexical utilities model" described below (in Section 23.4).

What does it mean for the utility of salvation to be infinite? It means, at the very least, that there is a discontinuity in our preferences.[4] There is a qualitative difference between the utility of salvation and that of the other three outcomes, a difference that is not just a matter of degree. That salvation has infinite utility is supposed to entail the recommendation *Wager for God*, no matter how small the finite probability that God exists. If salvation has merely large finite utility, then the recommendation depends upon exact expected utility calculations and the argument fails if the probability p that God exists is too small. The same difficulty arises if salvation has both infinite value and infinitesimal probability.[5]

Pascal, however, rejects all of these possibilities. He insists that salvation has both infinite value and finite probability, and these assumptions are central to his argument:

> But here there is an infinity of infinitely happy life to be won, one chance of winning against a finite number of chances of losing… That leaves no choice; wherever there is infinity, and where there are not infinite chances of losing against that of winning, there is no room for hesitation, you must give everything. (Pascal, 1670, L418/S680)

In this paper, I shall accept the two Pascalian assumptions implicit in the above passage from *Pensées:*

- *Infinite Utility*: The value of salvation is infinite.
- *Finite Probability*: The probability *p* that God exists is finite (non-infinitesimal) and positive.

Apart from fidelity to Pascal, it seems worthwhile to develop a version of decision theory that can handle decisions even when some of the possible outcomes are "infinitely" good or bad. The lexical utility interpretation of PP (→Bartha and DesRoches, 2021; Steel and →Bartha, 2022) is one such formulation of decision theory. It has been used to model precautionary decisions involving catastrophic outcomes. In this paper, I argue that this formulation can also be used to answer, at least in part, three of the major objections to Pascal's Wager that derive from *Infinite Utility*.

The paper proceeds as follows. Section 23.2 presents the three objections to Pascal's Wager that will be our focus: the *many-gods objection*, the *mixed-strategies objection*, and the *fanaticism objection*. Section 23.3 introduces the precautionary principle and presents three parallel objections that apply to precautionary reasoning. Section 23.4 outlines the lexical utility interpretation of PP. Lexical utilities provide a way to represent the qualitative distinction between catastrophic and ordinary outcomes, equivalent to representing infinite and negative infinite utilities. They also provide initial responses to two of the objections against PP, and to the analogous objections against Pascal's Wager. But these initial responses do not address the *fanaticism objection*, which alleges that the Wager is insanely demanding: there is no rational way to allocate resources towards the pursuit of ordinary human objectives instead of those that are "infinitely good." Section 23.5 addresses this

deeper worry by showing how reasonable trade-offs and compromises are possible, first for precautionary decisions and then for Pascal's Wager. Section 23.6 concludes by acknowledging that the solutions come with a cost: the Wager perhaps becomes too undemanding relative to Pascal's original formulation.

23.2 Three Objections to Pascal's Wager

In this section, I outline three well-known objections to Pascal's Wager. I then re-frame the objections as challenges about how to allocate resources (time, energy, money) when some possible outcomes are "infinitely better" than others.

First: the *many-gods objection* challenges Pascal's argument by expanding →Table 1. Suppose we have non-zero subjective probabilities for the existence of two possible gods, g_1 and g_2, both of whom reward believers with infinite utility. We now have two acts with infinite expected utility, W_1 and W_2 in →Table 3.

→**Tab. 3:** Many-gods objection.

	***G*$_1$ (g_1 exists) (p_1)	***G*$_2$ (g_2 exists) (p_2)	***N*** **(No god exists)**$(1 - p_1 - p_2)$
***W*$_1$: Wager for g_1**	∞	f_1	f_2
***W*$_2$: Wager for g_2**	f_3	∞	f_4
***D*: Don't wager**	f_5	f_6	f_7

The objection can now proceed in two distinct ways. Some critics level the charge of *incoherence*: Pascal's logic implies that we should wager for both gods, but that is impossible in practice.[6] But one might equally allege that the decision-maker faces *paralysis*: since all wagers have equal infinite expected utility, no wager is clearly recommended and there is no basis for choice.[7] Additional rows and columns create further complications.[8]

Second: the *mixed-strategies objection* (→Jeffrey, 1983; →Duff, 1986; →Hájek, 2003) expands on →Table 1 by allowing mixed strategies. Consider flipping a coin and wagering for God if and only if *Heads* is the result. This act also has infinite expected utility, so long as there is a finite positive probability of *Heads*. The same goes for an "avoidance strategy" (→Hájek, 2022) aimed at avoiding religious commitment. Since this option still leaves a small positive probability that you might end up wagering for God, it too can be viewed as a mixed strategy that has infinite expected utility (→Table 4).[9] *Try to avoid* is just as good as *Wager for God*.[10]

→**Tab. 4:** Mixed-strategies objection.

	***G* (God exists)**(p)	**~*G* (God does not exist)**($1 - p$)
***W*: Wager for God**	∞	f_1
***A*: Try to avoid**	∞	f_2

The conclusion that one should cultivate a particular religious belief, or any religious belief at all, is undermined.

Third: the *fanaticism objection* (→Tabarrok, 2000) argues that Pascal's reasoning leads to unreasonable or extremist recommendations. If you accept the reasoning in Pascal's Wager

but attach finite value to monetary wealth, then you should be willing to send Tabarrok all of your money in exchange for his prayers on your behalf. If Tabarrok's prayers produce even a tiny increment from p_1 to p_2 in your subjective probability of salvation, then they also produce an infinite increase $(p_2 - p_1) \cdot \infty$ in expected value. So you should send all of your money to Tabarrok.[11]

The reader might observe that there is a formal inconsistency between the *mixed-strategies objection* and the *fanaticism objection*. The former presumes that actions that change the probability of salvation make no difference: your expected utility is infinite either way. The latter presumes, to the contrary, that increasing the probability of salvation gives a further "infinite boost" to your already infinite expected utility. We can think of this pair of objections as posing a dilemma for Pascal: if we can respond to the *mixed-strategies objection* by showing that probability makes a difference, we thereby enable the *fanaticism objection*. This point is made precise in Section 23.4.

For now, let us note the related point that the three objections pull in different directions. To see this, it helps to stop thinking of Pascal's Wager as a single decision, "To wager or not to wager?" Think instead of the question: "What amount of resources (money, time, intellectual effort) should I invest in *sustained* wagering for a religious creed?" The three objections can then be reformulated as follows:

- The *many-gods objection* leads to *paralysis*. If the assumptions of *Infinite Utility* and *Finite Probability* apply to more than one possible god, then all ways of allocating resources (to the different wagers) are equally rational.[12] But this symmetry induces paralysis: there is no sufficient

reason for allocating resources in one way rather than another.

- The *mixed-strategies objection* is an argument for *wagering nothing*: I should invest (virtually) no resources in religious belief. Hájek (2003, 2012, 2022) maintains that the mixed-strategies objection undermines the validity of Pascal's argument because all mixed strategies have equal infinite expected utility. However, the objection could be mobilized to support the recommendation *Try to avoid* because this act yields infinite expected utility with minimal effort. There is an implicit lexical ordering here: in cases where multiple acts have maximal (infinite) expected utility, perform the one that requires the least effort.
- Finally, the *fanaticism* (or *extremism*) objection is an argument for *wagering everything*: I should invest all that I have in wagering for God, because every increment yields an infinite increase in expected utility.[13]

The fact that the three objections generate contrary implications about resource allocation poses an additional challenge to Pascal's Wager and specifically to the assumption of *Infinite Utility*.

23.3 The *Precautionary Principle* and Three Parallel Objections

As promised, let us now turn to the *Precautionary Principle* (PP). This principle is intended for use in decisions involving potentially catastrophic outcomes, even where we have limited knowledge of the relevant probabilities. In general, PP recommends taking precautionary measures to avoid catastrophic outcomes, provided the evidence for a serious threat meets some threshold of plausibility. Typically, the

catastrophic outcomes involve severe harm to the environment, to human beings, or to both. Although there are different formulations[14] of PP, most incorporate the following three components (→Steel, 2015):

1. *Harm condition*: a distinction between catastrophic and non-catastrophic outcomes (sometimes referred to as a *harm threshold*);
2. *Knowledge condition*: a threshold of plausibility that a potential catastrophe must meet in order to be taken seriously; and
3. *Recommended precautionary measures*: a recommendation for a response that prioritizes catastrophe prevention.

Both the harm condition and the knowledge condition are determined in the context of a particular decision. For instance, PP might recommend banning a commercial pesticide if there is evidence of an appreciable (but unknown) probability that it could seriously harm humans. Here, the harm condition is "serious harm to humans," while the knowledge condition is "appreciable probability." In *Building the Bomb* (Section 23.1), PP would recommend halting research on the atomic bomb until scientists determine whether there is a scientifically plausible mechanism showing that its detonation could lead to a global conflagration. Here, the harm condition is "destruction of human (and non-human) life" and the knowledge condition is "scientifically plausible."

The hallmark of this form of precautionary reasoning is that it does not require detailed expected utility calculations, but only an initial assessment that catastrophic outcomes (meeting the harm condition) are credible threats (meeting the knowledge condition). The reasoning then proceeds very much as in Pascal's Wager, as illustrated by →Table 2. As in the Wager, there

is a sharp distinction between two levels of value (*ordinary* and *catastrophic* outcomes). As in the Wager, the reasoning is typically not sensitive to precise probability values. As stated in the introduction, however, there are numerous objections to PP. Three of them are close analogues of the objections to Pascal's Wager described in Section 23.2.

23.3.1 The Many-Disasters Objection

Several authors have alleged that the precautionary principle is unable to handle situations where there is more than one potential catastrophe. They explicitly draw parallels to the *many-gods objection*, and as with that objection, their criticisms take different forms.

The first is a charge of incoherence: PP makes contradictory demands. For example, →Sunstein (2005) considers a version of PP that triggers precautionary measures if there is any chance of catastrophic harm. But if the precautionary measures themselves might have catastrophic consequences, PP both recommends and prohibits them. In a similar spirit, Manson (2002) imagines a scenario in which there is a tiny probability that drastic cuts in greenhouse gas emissions might lead to worldwide economic depression, political instability, war, and a disastrous nuclear holocaust. It appears that PP both recommends and prohibits the drastic emissions cuts.[15] Here, Manson explicitly notes the parallel with Pascal's Wager and the *many-gods objection*. Sandin (2009, p. 92) describes a similar dilemma:

> Suppose someone argues, with reference to the precautionary principle, that protocell research should be stopped because such research might, through some possible but extremely unlikely causal chain result in the end of all natural life on Earth... It might then be countered that there is an alternative scenario, also possible, in which *not* proceeding with

> protocell research leads to the extinction of all natural life on Earth… [Given that both scenarios are possible] we should, according to the precautionary principle, both proceed and not proceed with protocell research.

The second form of the *many-disasters objection* is a charge of paralysis. Sandin (2009) points out that PP can be formulated as the recommendation to "abstain from all actions that might have harmful consequences." In this case there is no incoherence, but "we are offered no action guidance at all."

Finally, the third form is a charge of non-discrimination. Here, we modify an example due to Colyvan et al. (2010), *Mangrove Forest*, to suggest that PP fails to discriminate properly between two remedies that might be adopted to prevent environmental disaster. Suppose that both the destruction (D) of a mangrove forest and the destruction (H) of half the forest are considered catastrophic outcomes. Act A will destroy half the forest, while act B will destroy the entire forest. It seems clear that B is worse than A, but PP provides no basis for discrimination. For if we assign utility $-\infty$ to both D and H, then both A and B have negative infinite utility (→Table 5), so the two options are equally bad. This is plainly wrong!

→**Tab. 5:** Mangrove Forest.

	***D* (all destroyed)**	***H* (half destroyed)**
A	–	$-\infty$
B	$-\infty$	–

23.3.2 The Mixed-Strategies Objection (Do Nothing)

We again draw on an example, *Endangered Species*, due to Colyvan et al. (2010a). The example is part of their extended argument against assigning infinite value to the natural environment.

> If persistence of an endangered species is considered infinitely valuable, actions with *any* nonzero chance of ensuring its survival will have infinite expected value… Actions with high and actions with low probabilities of species survival would have identical expected value. This would imply indifference about actively protecting endangered species and passively doing nothing, and this is patently the wrong result. (Colyvan et al., 2010, p. 225)

In the decision table represented in →Table 6, the possible states are *S* (species survives) and *E* (species goes extinct). The possible acts are *P* (actively protect), *N* (do nothing), and mixed strategies involving *P* and *N*.

→**Tab. 6:** Endangered Species.

	***S* (species survives)**	***E* (species goes extinct)**
***P*: Actively protect**	∞	f_1
***N*: Do nothing**	∞	f_2

P increases the probability of species survival, but this has no impact on the precautionary reasoning. Given this representation of the decision, and the likelihood that $f_2 > f_1$ due to the high cost of active protection, one might argue that *Do nothing* is the dominant act and thus the rational choice. This objection is directly analogous to the *mixed-strategies* objection to Pascal's Wager, as represented earlier in →Table 4.[16]

23.3.3 The Fanaticism Objection (Black Hole Challenge)

According to Munthe (2019), given that the basic premise of the precautionary principle is the avoidance of catastrophic harms,

> … there seems to be no rationally justifiable end to the amount of resources and time that we should spend on trying to clarify an existential risk further… Each such risk, therefore, appears as a 'black hole' that, by the awesome normative power of the mere ultimateness of the harm it threatens us with, sucks into its void all other things that might have been mobilized for the pursuit of the good in human decision-making. (→ Munthe, 2019, p. 51)

As Munthe frames it, the black hole challenge involves a nightmare of ceaseless dithering about disaster scenarios: the threat of ultimate harms "suck[s] all available resources into endless endeavors to clarify ridiculous eventualities of horrific things" (→ Munthe, 2019, p. 58). I construe the problem more broadly: PP seems to entail that 100% of our time, money and energy should be devoted to the prevention of catastrophe. As an instance of the black hole challenge, consider *NASA Budget*, adapted from Steel and Bartha (2022). Suppose that NASA has to decide how much of its budget to devote to preventing a catastrophic asteroid impact with Earth and how much to other valuable missions (space exploration, the Webb telescope, etc.). Given NASA's ability to reduce the existential threat of an asteroid collision, PP appears to recommend that the entire budget be spent on asteroid tracking, developing and testing methods of asteroid deflection, and so forth. This absurd recommendation is analogous to Tabarrok's *fanaticism* objection to Pascal's Wager.[17]

23.4 Lexical Utilities: Preliminary Responses to the Objections

23.4.1 Lexical Utilities

Call the representations ∞ and −∞ in the above tables *naïve infinite utilities* and the associated calculations *naïve infinite decision theory* (cf. →Bartha, 2007). Bartha and DesRoches (2021) propose instead that we represent discontinuity between "levels" using lexical utilities, and that a lexical utility model of PP can be used in response to the objections raised in Section 23.3. The lexical utility model also allows us to address the objections to Pascal's Wager from Section 23.2. At least in part, both sets of objections can be handled with a better representation of infinite and negative infinite utility.

Lexical utilities may be formally described as follows. Let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ be vectors of real numbers (members of $\mathbb{R}^n$). The standard lexical ordering on such vectors is given by:

$x < y$ iff $x_i < y_i$ for the first i such that $x_i \neq y_i$

and, as usual,

$x \leq y$ iff $x = y$ or $x < y$.

If O is a set of outcomes, then a *lexical utility ordering* on O is a function $U{:}O \rightarrow \mathbb{R}^n$ such that for any two outcomes a, b in O, $a \preccurlyeq b$ iff $U(a) \leq U(b)$. Here $\preccurlyeq$ is a preference ordering on O ($a \preccurlyeq b$ if and only if b is weakly preferred to a) and $\leq$ is the standard lexical ordering on $\mathbb{R}^n$. We write $U(a) = (u_1(a), u_2(a), \ldots, u_n(a))$ for the *utility vector* associated with a. The n dimensions of the vector correspond to a list of decision criteria, arranged in order of decreasing importance. From the above definitions, outcome b is strictly preferred to outcome a if it scores better on the first criterion where they differ in value:

$a \prec b$ iff for some $k \leq n$, $u_k(a) < u_k(b)$ and $u_i(a) = u_i(b)$ for $1 \leq i < k$.

The most important feature of a lexical ordering is that the ordering of criteria in the list is absolute: no amount of extra

utility u_{i+1} can compensate for lower utility u_i. Suppose that you are planning a hiking expedition and that you place a premium on safety, followed by scenic enjoyment. On a lexical model, each route a has utility $U(a) = (u_1(a), u_2(a))$ where u_1 represents safety and u_2 represents scenic enjoyment. In selecting your route, you first identify options tied for highest safety score, and then use scenic enjoyment to break the tie.

23.4.2 The Lexical Utility Interpretation of PP

On the simplest version of the lexical utilities model, precautionary reasoning is represented using a two-dimensional lexical utility function that assigns $U(a) = (u_1(a), u_2(a))$ to each outcome a, with the further requirement that the first component $u_1(a)$ is a *binary* function used to flag the distinction between catastrophic and non-catastrophic outcomes. Specifically, $u_1(a) = -1$ if a is catastrophic and $u_1(a) = 0$ if a is non-catastrophic. The other component, $u_2(a)$, reflects ordinary costs and benefits. What counts as catastrophic is context-dependent and is not limited to a single category, such as environmental impact. The precautionary principle PP is interpreted as the injunction to maximize *expected lexical utility* (or *lexical EU*) where expected lexical utilities are now understood as two-dimensional vectors, computed separately for each component.[18]

To illustrate how this works in a specific case, →Table 7 gives an overview of the lexical utilities for *Building the Bomb*.

→**Tab. 7:** Building the bomb (lexical version).

	***F* (Flammable)**(q)	**~*F* (Not flammable)**($1 - q$)
***B*: Build the bomb**	$(-1, g_0)$	$(0, g_1)$
~*B*: Don't build	$(0, g_2)$	$(0, g_3)$

The u_1 utilities are −1 for the single catastrophic outcome and 0 otherwise. The u_2 utilities are as in Section 23.1, except that we have added g_0. If q is the probability that the ocean or atmosphere is flammable, the expected lexical utilities are:

EU(B)	=	$(-q, qg_0 + (1-q)g_1)$
EU($\sim B$)	=	$(0, qg_2 + (1-q)g_3)$

Lexical EU is maximized by selecting *Don't build*, so long as $q > 0$. The values g_0—g_3 don't matter. In fact, we can simplify the calculation with a *first-order lexical decision table* that only exhibits u_1-utility values.[19]

Both the lexical approach (→Table 7) and the naïve infinite utilities approach (→Table 2) capture the distinction between catastrophic and non-catastrophic outcomes, and both approaches prioritize the avoidance of catastrophes. The lexical utilities approach is superior, however, because it lets us defend PP against the *many-disasters* and *mixed-strategies* objections.

Response to the many-disasters objection

Consider Sandin's protocell example. Following Sandin, let us grant a very low probability q that protocell research (R) may lead to catastrophe (extinction of life on Earth) and an even lower probability p that not undertaking the research ($\sim R$) leads to catastrophe. The naïve utilities approach leads to paralysis: both options have negative infinite expected utility. On the lexical utilities model, however, we have the first-order lexical decision table (showing only the u_1-utilities) that is represented in →Table 8.

→**Tab. 8:** Protocell research (lexical version).

	***E* (Extinction)**	**~*E* (No extinction)**
***R*: Pursue research**	−1 (probability q)	0
~*R*: Halt research	−1 (probability p)	0

Since $P(E/R) = q$ and $P(E/\sim R) = p$, the first-order lexical expected utilities are $EU(R) = -q$ and $EU(\sim R) = -p$. The recommendation is unequivocal: halt the research. There is no incoherence and no paralysis.

Response to the mixed-strategies objection

The lexical approach also eliminates the *mixed-strategies objection*. To see how this works for the *Endangered Species* example, replace →Table 6 with the first-order lexical decision table (→Table 9).

→**Tab. 9:** Endangered Species (lexical version).

	***S* (species survives)**	***E* (species goes extinct)**
***P*: Actively protect**	0	−1 (probability q)
***N*: Do nothing**	0	−1 (probability p)

Following Colyvan et al., assume that P(E/P) = q is much lower than P(E/N) = p. The first-order lexical expected utilities are EU(P) = $-q$ and EU(N) = $-p$, and the clear recommendation is *Actively protect*. The case for doing nothing disappears.

In short, on the lexical utilities model, PP avoids both paralysis and the counter-intuitive recommendation to do nothing. What about the *non-discrimination* objection, exemplified by the case of *Mangrove Forest* (→Table 5)? To address this problem requires a slight generalization of the basic model (cf. Steel and →Bartha, 2022). Instead of restricting u_1 to values 0 (non-catastrophic) and −1 (catastrophic), we allow the value to be any negative integer. PP is compatible with the idea that some catastrophes are worse than others, and that multiple catastrophes may be worse than just one. In *Mangrove Forest*, it is reasonable to put u_1(D) = −2 and u_1(H) = −1: the destruction of the entire forest (D) is twice as bad as the destruction of half the forest (H).[20] This restores the possibility of discrimination.

However, it is not yet clear that the lexical utilities model offers any help in responding to the *fanaticism objection* (Munthe's *black hole* challenge). In fact, the move from naïve infinite utilities to lexical utilities actually sharpens the problem. Consider the *NASA Budget* example. If we use −∞ to represent a catastrophic asteroid collision, then all of NASA's options arguably have the same expected utility (−∞) because none of

them eliminates the risk of a world-ending asteroid collision. On the lexical approach, however, options that reduce the probability of catastrophe really do have greater expected lexical utility. This leads straight to the extremist recommendation: devote all resources to disaster prevention! I shall return to the problem in Section 23.5, but first let us see how the lexical utilities approach applies to Pascal's Wager.

23.4.3 Lexical Utilities and Pascal's Wager

The lexical utilities model gives priority to the avoidance of "infinitely bad" catastrophes. When applied to Pascal's Wager, the model gives priority to attaining the "infinitely good" outcome, salvation. The same approach as in Section 23.4.2 can be used, where $u_1(a) = -1$ signifies an ordinary outcome and $u_1(a) = 0$ represents an outstandingly good outcome. However, we can maintain consistency with the previous section by continuing to use $u_1(a) = 0$ for ordinary outcomes and letting $u_1(a) = 1$ for "infinitely good" outcomes.[21]

In response to the *many-gods* objection, consider the first-order lexical version of →Table 3 that is represented in →Table 10.

→**Tab. 10:** Many-gods objection (lexical version).

	G_1 (g_1 exists) (p_1)	G_2 (g_2 exists) (p_2)	N (No god exists)($1 - p_1 - p_2$)
W_1: Wager for g_1	1	0	0
W_2: Wager for g_2	0	1	0
D: Don't wager	0	0	0

If $P(G_i) = p_i$, we immediately see that $EU(W_1) = p_1$, $EU(W_2) = p_2$ and $EU(D) = 0$. There is no incoherence and no paralysis: the lexical model does not recommend contrary actions, and if $p_1 \neq p_2$, the recommendation is to wager for the god with the highest subjective probability.[22] If $p_1 = p_2$, both wagers are equally good but there really is no basis for thinking otherwise. This response takes care of many versions of the many-gods objection based on "philosophers' fictions" (→Jordan, 2006). Fanciful theological possibilities (gods who reward people who don't step on sidewalk cracks, people who only drink Chardonnay, etc.) may be included as columns in the table without any effect, so long as their associated subjective probabilities are much lower than more serious possibilities. The lexical model thus goes a long way towards resolving the traditional *many-gods objection*, although challenges still remain.[23]

The lexical utilities response to the *mixed-strategies* objection follows the same general lines. Begin with the first-order lexical version of Pascal's Wager, obtained from →Table 1 by replacing ∞ with 1 and each finite utility with 0. The first-order expected

lexical utility of the mixed strategy [q *Wager*, (1−q) *Don't Wager*] is then $q \cdot p$, where p is the probability that God exists. The pure wager ($q = 1$) is recommended over any mixed strategy with $q < 1$. In particular, consider the modified lexical version of →Table 4 that is depicted in →Table 11.

→**Tab. 11:** Mixed-strategies objection (lexical version).

	***G* (God exists)**(p)	**~*G* (God does not exist)**(1 − p)
***W*: Wager for God**	1	0
***A*: Try to avoid**	1	0

If *Try to avoid* implies some tiny probability q of actually making the wager, then we equate it with [q *Wager*, (1−q) *Don't Wager*]. If p represents the probability that God exists, then EU(*Wager*) = p while EU(A) = $q \cdot p$. *Try to avoid* is just about the worst strategy you could pick.

The shift from naïve infinite utilities to lexical utilities thus allows us to preserve the fundamental Pascalian assumptions, *Infinite Utility* and *Finite Probability*, while avoiding the *many-gods* and *mixed-strategies* objections. But we do not yet have a way to handle the *fanaticism* objection, either for PP or for Pascal's Wager. To avoid extremism, we need to allow trade-offs between distinct "levels" of utility, yet this appears to be ruled out. Colyvan et al. (2010) identify this difficulty as the most crucial problem with using either naïve infinite utilities or lexical utilities to model precautionary reasoning:

> Sensible nontrivial trade-offs between outcomes at different lexical levels must be possible, but this is precluded by both the explicitly infinitary and lexical approaches to environmental value. (Colyvan et al., 2010, p. 227)

Steel and Bartha (2022), however, suggest ways in which the lexical utilities model can be adapted to allow certain forms of trade-offs. Section 23.5 explains these ideas and shows how they allow us to respond to charges of fanaticism both for PP and for Pascal's Wager.

23.5 Resource Allocation, Trade-offs and the *Fanaticism* Objection

The heart of the analogy between Pascal's Wager and the precautionary principle is that both rely upon discontinuity between two levels of utility.[24] Whether we use naïve infinite utilities or lexical utilities to represent this discontinuity, it appears to preclude trade-offs between levels. This is the basis for the charge of fanaticism, or extremism.

In the case of PP, we have pessimistic extremism in the form of the *black hole challenge*. PP seems to impose the unreasonable demand that all of our resources be devoted to disaster prevention, with nothing left for valuable "finite" objectives. NASA should give up on space exploration and the Webb telescope, and spend its entire budget on preventing an asteroid collision with Earth.

In the case of Pascal's Wager, we have optimistic extremism in the form of Tabarrok's *fanaticism objection*. Pascalian reasoning seems to impose the unreasonable demand that all of our resources should be devoted to wagering, leaving nothing for the pursuit of ordinary goods.[25]

In this section, however, we respond to these criticisms by showing that the lexical utilities approach is compatible with at least two reasonable options for avoiding extreme recommendations, without giving up infinite utilities.

23.5.1 Response to the Black Hole Challenge

In an important recent discussion of PP, Steel (2015) stresses the requirement of *proportionality* as a defense against criticisms that PP leads to incoherent or irrational recommendations. Steel interprets proportionality in terms of the consistency and efficiency of whatever version of PP is adopted to address a problem. *Consistency* means that no version of PP can trigger a precautionary measure that has consequences as bad as, or worse than, the disaster that it is meant to avoid. *Efficiency* means that "if more than one precaution can be consistently recommended by the version of PP being used, then those with lower costs should be preferred" (→Steel, 2015, p. 29). Munthe (2019) applauds the practical insight that PP should avoid an unacceptable "price of precaution." However, he also observes that proportionality alone cannot solve the black hole challenge. Given "the awesome gravity of an existential risk," we face "the prospect of having every last cent being pulled into its black hole even if we accept a cost minimizing requirement" (→Munthe, 2019, p. 52).

Munthe himself outlines two approaches for responding to the black hole challenge, building on Steel's idea of proportionality. The first resembles Herbert Simon's strategy of satisficing (→Simon, 1955). Munthe asks us to imagine a series of increasingly costly precautionary measures O_1–O_9 that might be adopted to handle an existential risk. If one of these options, O_5, is the first one in the series that is "good enough," then we are justified in carrying out O_5 without investing endless time and effort on the more costly options. He suggests two ways to identify a "good enough" option. The first is that O_5 might represent a "context-dependent compromise" (→Munthe, 2019, p. 56). The second relies on the idea that as we move along the series, even though we are trying to fend off catastrophe, we

eventually reach the point at which the price of further precaution "becomes drastically more difficult to justify." Munthe acknowledges, however, that neither idea is precise. Confronted with an existential threat, there is no clear way to identify or justify a "good enough" option short of total commitment, nor can we even demonstrate that such an option actually exists.

Munthe's second approach is a "pragmatic" solution that derives from human psychological limitations. The most natural human reaction to vast threats, suggests Munthe, is to avoid thinking about them at all: "one threat of ultimate harm after another silences human ability to psychologically engage with the challenge" (→Munthe, 2019, p. 58). Realistically, humans can plan and carry out precautionary activities only if they have "clearly stated end- and exit-points." Munthe's suggestion, then, is that we bind ourselves to "pre-set limits to our precautionary engagement." But this approach is also unsatisfactory. As Munthe acknowledges, we still have the problem of determining *which* end- and exit-points to select. This is essentially the black hole challenge all over again, since it seems that PP will favor selection of the most stringent exit-points.

Despite their inadequacy as responses to the fanaticism objection, both Steel's concept of proportionality and Munthe's pragmatic solutions are helpful. To address the problem of extremism, we should move away from the binary decisions represented in the tables above. Instead, we should consider the practical question of how to allocate finite resources to achieve some type of compromise. Using this approach, Steel and Bartha (2022) show how the lexical utility version of PP provides a partial response to the black hole challenge. There are two types of scenario where *compromise strategies*, allowing trade-offs between lexical levels, are consistent with and even recommended by PP.

Scenario 1: Probability threshold

The first scenario is exemplified by the problem of finding a "stopping point" for logging old-growth forest in the Canadian province of British Columbia (Bulte and van Kooten, 2000). This example, *old-growth forest*, requires a decision about trade-offs between ordinary economic benefits derived from commercial logging and avoiding a catastrophic outcome, the disappearance of the forest. The problem can be modeled in the following way. We have a finite resource M (the current area of old-growth forests in British Columbia) and the problem is to choose the number of hectares $0 \leq x \leq M$ to be logged. Although there is no existential threat in this example, it is an instance of the black hole challenge because it seems that PP should recommend no logging at all.

Steel and Bartha use lexical utilities $U(\mathrm{x}) = (u_1(x), u_2(x))$ to model the decision problem. Here, x is the amount of forest removed, u_1 flags the catastrophic outcome and u_2 stands for economic benefits. Any outcome x which leaves less than the minimum size E for a viable, ecologically significant forest has lexical utility of the form $(-1, u_2(x))$. The probability of catastrophe, as a function of how much logging takes place, now enters the picture:

$p(x)$ = Pr(less than E survives / logging is halted at x); and

$p_0 = p(0)$, the probability of disaster even if there is no logging.

Steel and Bartha make the additional assumption that there is an interval $0 \leq x \leq x_s$ where logging has no effect on the probability of disaster. That is, $p(x) = p_0$ in this interval, while logging beyond this point increases the chance that the forest will vanish.

Given this set-up, the lexical utilities model leads to a kind of compromise. The expected utility for stopping point x is as follows:

$EU(x) = (-p_0, u_2(x))$ for $0 \leq x \leq x_s$; and

$EU(x) = (-p(x), u_2(x))$ for $x > x_s$.

The lexical model therefore recommends x_s over any $x < x_s$ because u_2 is the decisive factor on the interval $0 \leq x \leq x_s$. And it recommends x_s over any $x > x_s$ because u_1 is the decisive factor on the interval $x_s \leq x \leq M$. The recommended stopping point is therefore x_s, the point at which commercial logging begins to reduce the probability of ecological viability. While the recommendation is conservative, it shows that trade-offs are compatible with PP and that extremism can be avoided when there is a relevant probability threshold.

Scenario 2: Sustainability

The second scenario is illustrated by the *NASA Budget* example. Does PP recommend that NASA should spend its entire budget on hunting for near-Earth objects and developing methods for asteroid deflection? Here, we do face an existential threat, but the extremist recommendation still seems absurd. Steel and Bartha (2022) offer the following counterargument, here presented in summary fashion. Roughly, PP recommends only that NASA should maximize the *total* amount that it spends on preventing an asteroid collision, not the amount that it spends each year. If NASA devoted its entire annual budget to asteroid interdiction, however, its budget would shrink. Dollars spent on high-profile missions, such as space telescopes, moon missions and probes sent to other bodies in the solar system, have a better *return on investment*. In mundane terms, these missions encourage Congress to maintain or increase NASA's budget.

More generally, the NASA example illustrates that trade-offs are possible on the lexical model whenever there is an issue of sustainability. In such cases, the lexical utility interpretation of PP prescribes a compromise: present resources need to be divided between catastrophe prevention and ordinary objectives that generate resources down the road, even if catastrophe prevention is our overall top priority.

Using the lexical utilities model, then, we can meet the black hole challenge in ways that preserve the pragmatic spirit of Steel's and Munthe's proposals but can still be defended as rigorous applications of PP. Compromises between lexical levels are not merely permitted but actually *prescribed* in two kinds of cases: where there is a threshold of resource allocation below which there is no impact on the probability of catastrophe, and where sustained effort is needed to deal with a long-term potential catastrophe.

23.5.2 Application to Pascal's Wager: Response to the Fanaticism Objection

Either of the two scenarios described above might be an appropriate analogue for your situation if you find yourself deliberating about Tabarrok's *fanaticism* objection.

By analogy to *old-growth forest*, you might conclude that sending all your money to Tabarrok in exchange for his prayers has no impact (or very little impact) on the probability that you will achieve salvation.[26] More generally: you might believe that devoting some of your wealth to "wagering" does have a positive influence on the probability of salvation, but also that there is a threshold beyond which additional spending has no further effect. As in *old-growth forest*, the lexical utilities approach then recommends that you devote resources to wagering only up to this threshold. Beyond that point, lexical

utility maximization recommends that you devote your time, energy and money to the pursuit of other worthwhile objectives.

Alternatively, by analogy to the *NASA Budget* example, you might think of the wager not as a single decision, but rather as a lifelong project. Pascal himself employs the metaphor of travelers on a road:

> Learn from those who were once bound like you and who now wager all they have. These are people who know the road you wish to follow, who have been cured of the affliction of which you wish to be cured: follow the way by which they began… (Pascal, 1670, L418/S680)

If wagering is a journey, then even those who wager "all they have" do better by ensuring that they can sustain themselves on the journey. If your objective is to maximize what you put into wagering, then you do best at each stage of life by allocating resources in a sustainable manner.

Both responses sketched here are backed up by the lexical utilities model, provided we represent Pascal's Wager as a decision about allocating a finite pool of resources between wagering and other more ordinary objectives.

Finally, let us return to an observation made at the end of Section 23.2. I suggested that the *many-gods* and *mixed-strategies* objections, like the *fanaticism* objection, can be understood as puzzles about how to allocate resources either to possible wagers or to ordinary objectives. I also suggested that the three objections correspond to incompatible recommendations about resource allocation: *Incoherence/paralysis* for the many-gods objection, *Wager nothing* for the mixed-strategies objection, and *Wager everything* for the fanaticism objection. By putting together the analyses from Sections 23.4 and 23.5, we can neutralize this concern about conflicting recommendations.

In order to bring all three problems together, we can generalize the decision problem in the following way: how should we distribute a finite pool M of resources over n possible wagers as well as a set of ordinary (finite) objectives? In other words, what is the best way to allocate resources x_i to wager W_i and y to ordinary objectives, where $x_1 + \ldots + x_n + y = M$? Different choices will lead to different credences over n theological possibilities. In this situation, the lexical model does not lead to incoherence or paralysis. Nor does it recommend *Wager nothing* or *Wager everything*. The best solution is a compromise with $y > 0$ (some resources are set aside to ensure sustainability, as sketched above) and exactly one $x_i > 0$, corresponding to the wager that will ultimately produce the highest credence.[27]

23.6 Conclusion

Infinite utility plays a leading role in Pascal's Wager, but it is also the source of persistent challenges to the Wager. My purpose in this paper has been to explore analogies between the Wager and the precautionary principle, which faces very similar challenges. In both cases, we can respond effectively to challenges without giving up infinite utility.

The first step is to shift from naïve infinite utilities to a more sophisticated representation that uses lexical utilities. The qualitative difference between "levels" of utility is flagged using the lexical dimension u_1. Applied to precautionary reasoning, this shift eliminates the *many-disasters* and *mixed-strategies* objections. Applied to Pascal's Wager, it yields a simple response to the *mixed-strategies* objection and to standard versions of the *many-gods objection*.

The second step is to shift from a framework in which decisions are modeled as choices among a small number of

options to a more complex approach in which decisions are modeled as choices about how to allocate a finite pool of resources between two lexical levels, possibly over an extended period of time. The lexical utilities approach then lets us provide a rationale for trade-offs, both for precautionary reasoning and for Pascal's Wager. This is a significant step towards addressing the criticism that the reasoning behind both PP and Pascal's Wager leads to extremist, and even absurd, recommendations.

Additional challenges clearly remain for defending PP. In particular, it may seem that the two scenarios described in Section 23.5 offer at best a partial response to the problem of fanaticism/ extremism. For instance, it may still appear extreme for NASA to maximize its spending on avoiding asteroid collision. As a somewhat different problem, consider Munthe's *Protocell* example. In Section 23.4.1, we showed that the lexical utilities model settles on a recommendation to ban protocell research if there is a probability that it leads to extinction of life (and only a much smaller probability that banning the research would have this catastrophic outcome). This recommendation may seem absurd if it requires paying "unreasonable attention to *extremely* unlikely scenarios" (Sandin et al. 2002, p. 290). This kind of extremism has nothing to do with resource allocation, and cannot be addressed by the compromise strategies outlined in Section 23.5.

But there is no analogue to this problem in the case of Pascal's Wager. Consider an agnostic who assigns positive, but extremely small, probability to the proposition that God exists. The situation might seem analogous to the *Protocell* example, but there is an important difference. The precautionary recommendation in *Protocell* is purely negative—banning the research—and involves no allocation of resources. For the agnostic, however, the issue is precisely whether her tiny positive probability for God's existence leads, through Pascalian

logic, to massive demands on her time and energy. For this case, the “probability threshold” strategy seems perfectly appropriate. No investment beyond a modicum of open-mindedness is recommended because no such investment will make any difference to her credences.

In Pascal’s Wager, every decision to wager involves a decision about allocating resources.[28] This makes one or both of the compromise strategies relevant. Both of them place an upper limit on the resources devoted to wagering. If, as I have argued, these strategies succeed in neutralizing some of the major objections to Pascal’s Wager, then the conclusion—that a rational agent should wager for God—is weaker than Pascal imagined.

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Notes

1 I shall refer to both the decision problem and the argument just sketched as *Pascal's Wager*, or just the *Wager*.

2 Equivalently, we can represent protection of the environment as having positive infinite value (→McCauley, 2006).

3 The scientists working on the Manhattan Project resumed work only after determining that there was no serious risk of catastrophe. As Compton wrote: "the impossibility of igniting the atmosphere... was assured by science and common sense" (→Rhodes, 1986, p. 419).

4 Infinite utilities violate the *Axiom of Continuity*, a core assumption of standard decision theory. For discussion, see →Dreier (1996); →Hájek (1997); →Steele and Stefánsson (2016); and Bartha and DesRoches (2021).

5 In the terminology of →Wenmackers (2018), some decision theories use "harmonious number systems" that allow for "cancellation" of infinite values through multiplication with infinitesimals. For example, →Chen and Rubio (2020) model the Wager using surreal numbers for both utilities and probabilities, which allows for this type of cancellation. In these non-standard decision theories, the Wager fails if the probability that God exists is a sufficiently small infinitesimal number.

6 As Hájek (2022) puts the point: “Pascal’s argument proves too much: by parallel reasoning we can ‘show’ that rationality requires believing in various incompatible theistic hypotheses.”

7 If the verdict is that we should be indifferent between W_1 and W_2, we still face the problem of paralysis when it comes to selecting one of the two options.

8 There is a large literature on the *many-gods objection*, summarized in Hájek (2022). See in particular Jordan (2006) and →Saka (2001). Here we limit attention to gods who potentially offer an infinite reward to their followers.

9 The expected utility in the bottom left corner is ∞ because there is at least a tiny probability of wagering and being rewarded with salvation.

10 *Try to avoid* is not literally the same as the avoidance of religious commitment, which has utility f_2. The relation between *Try to avoid* and avoidance is like the relation between *Wager* and belief: in both pairs, only the former is an available action.

11 A version of the *fanaticism objection* can also be given if the utility of salvation is large but finite (→Bostrom, 2009).

12 As noted above (n. 8), we restrict attention to possible gods who offer an infinite reward to their followers.

13 Tabarrok only mentions wealth, but his conclusion also applies to time and intellectual effort.

14 See Steel (2015) for review of the literature on PP. PP is sometimes interpreted as a high-level guiding principle for practical (legal and political) decisions and sometimes as a formal decision rule (→Bartha and DesRoches, 2021). In this paper, I focus on an attempt to formalize PP in decision-theoretic terms.

15 Manson's point is not that PP should be rejected, but that carelessly formulated versions of PP may be incoherent.

16 Dominance reasoning may not be convincing here because act-state independence fails for →Table 6. But just as in our earlier discussion of the *mixed-strategies objection* for Pascal's Wager, it seems plausible to incur minimal costs by doing nothing if both available acts have equal infinite expected utility. Because the objections are so similar for PP and Pascal's Wager, I refer to both as the *mixed-strategies objection*.

17 This objection to PP is also referred to as *extremism* or *absolutism* (Sandin et al., 2002). I use the term *fanaticism* to emphasize the parallel with Pascal's Wager, though I shall also sometimes refer to the problem as *extremism*.

18 We assume that expected lexical utility is linear in each component. This requires strong assumptions about agents' preferences; see →Fishburn (1971) and →Hausner (1954).

19 See Bartha and DesRoches (2021) for details and additional examples. In cases where first-order expected utilities are tied, we need to consider u_2 utilities.

20 The exact assignment is unimportant, so long as $u_1(\text{D}) < u_1(\text{H})$.

21 By analogy to the discussion of *Mangrove Forest* in Section 23.4.2, we might consider a generalization that allows the value of $u_1(a)$ to be any positive integer. I exclude this possibility when discussing Pascal's Wager, for two reasons. First: there is an important disanalogy between Pascal's Wager and PP. There are additive catastrophes (when more than one catastrophe occurs), but not additive rewards from multiple deities. Second: if $u_1(a)$ can be any positive integer, we face the problem of *arbitrariness* (→Hájek, 2003). Why represent the value of salvation as 1 rather than 2, 3, etc.? This disanalogy between PP and Pascal's Wager has no impact on the main conclusions of the paper.

22 This yields the same recommendation as →Schlesinger (1994), but this will be refined below (Section 23.5).

23 Additional forms of the *many-gods objection* (and related challenges) go beyond the scope of this paper. We could be asked to consider deities who reward strict evidentialism, or we could be asked to call into question the theology underlying our framing of the *Wager* (see →Mougin and Sober, 1994; →Oppy, 2018; →Sober, 2018).

24 In some cases, there may be additional levels, but I set aside this complication.

25 Some Global Priorities Institute research explicitly embraces a version of fanaticism close to this one; see

→Wilkinson (2022). However, I assume that Tabarrok's conclusion is genuinely counterintuitive.

26 You might also think that sending money to Tabarrok would lower your chance of salvation, or that you could improve your chances more by sending your money to a charitable foundation. I pass over these complications to focus on Tabarrok's main point: that Pascal's logic seems to require that you devote everything you have to the wager.

27 This is a variation on Schlesinger's Principle. →Schlesinger (1994) proposes that in a decision problem where more than one act has infinite expected utility, a rational agent should select the act that gives the highest probability for the infinite reward. The suggestion here is slightly different: after figuring out how much to set aside to ensure sustainability (y), put the remaining resources (x_i) into a single wager, where they will *eventually* produce the highest credence. I omit the proof!

28 Both Pascal's strategy, *Wager*, and Hájek's strategy, *Try to avoid*, are limiting cases.

24 Reflections on Infinite Utility and Deliberation in Pascal's Wager

Alan Hájek

Abstract: I have long argued that Pascal's Wager (from infinite expectation) is invalid. I now add several new twists to my argument. Suppose you are *deliberating* about whether to wager for or against God, and you assign credences to your performing each option. You are already doing something with infinite expectation. So why wager for God?—Keep deliberating! However, there is an interesting response. Various authors hold the thesis that *while deliberating about what you will do, you cannot rationally have credences for what you will do.* Then while deliberating, you are *not* enjoying infinite expectation; rather, your expectation is undefined. I reject this thesis, so I reject this reply. But even granting the thesis, it *still* shows that the Wager is invalid. Then we must compare your infinite expectation of wagering for God with your *undefined* expectation of continuing to deliberate. Then wagering for God does *not* maximize your expectation.

24.1 Introduction

Pascal's Wager presents believing in God or not as a decision problem. As such, it is natural to analyze it with decision theory. I have previously argued (2003a) that on a standard decision-theoretic formulation of it, it is invalid. Decision problems also lead us into a seemingly disparate debate. A number of authors have argued for the thesis that in a given decision problem, while deliberating about what you'll do, you cannot rationally

have credences (degrees of belief) for what you'll do. Levi (2007) calls this the "deliberation crowds out prediction" thesis. I call it the "deliberation annihilates reflexive credences" thesis, or for short, *the DARC Thesis.* A number of authors have defended the DARC Thesis. I have argued against it (2016), as have some others.

In this paper, I will have Pascal's Wager meet the DARC Thesis. Doing so will lead to several new twists, rather like an episode of *Black Mirror*.

24.2 Pascal's Wager

Pascal initially assumes that the probability that God exists is ½: "there is an equal risk of gain and loss." This is an application of the classical interpretation of probability, nascent at the time he wrote, in which we reduce probability to a ratio of numbers of coarse-grained possibilities: the number of those in which the proposition in question is true, divided by the total number of possibilities. Here, there is a total of two possibilities—God exists, God does not exist—and we seek the probability of the former. So that probability is ½—or so says a first pass at this approach to probability. With the benefit of hindsight, we might say that Pascal had in mind what we now call *rational subjective probability*, or perhaps *epistemic probability.*

Modern sensibilities may cringe at the naivety of this reasoning; indeed, →Hacking (1972, p. 189) calls this assumption about the probability a "monstrous premiss." The classical interpretation is somewhat plausible for typical gambling games that inspired it (involving coins, dice, cards, roulette wheels …). But much as Pascal regards his Wager as such ("a game is being played"), it is anything but typical.

However, he has a far more sophisticated line of reasoning up his sleeve. The probability of ½ is dispensable; all that matters

is that the probability that God exists is positive. Its exact value will not matter, given the jaw-dropping assumption that comes next: if God exists and you wager for God, you get *infinite* utility —"an infinity of infinitely happy lives." In short, salvation is infinitely valuable. And rational choice is a matter of taking a probability-weighted average of utilities, for each option—what we now call *expected utility*, or *expectation*—and maximizing this quantity. (Think of "probability" as "rational credence.")

Let's summarize Pascal's reasoning in modern parlance:[1]

1. The probability p that God exists is positive.
2. You should choose the action of maximal expectation if there is one.

→**Tab. 1:** Pascal's Wager.

	God exists	**God does not exist**
Wager for God	∞	f_1
Wager against God	f_2	f_3

Conclusion 1: Wagering for God has infinite expectation, while wagering against God has finite expectation.

Conclusion 2: You should wager for God.

Each of the premises has met with objections—see →Hájek (2017). Let's focus on P1. While it is a welcome weakening of the original "monstrous premiss," it is still too strong for some. Atheists might insist that the probability that God exists is 0. They might argue that the certainty of evil in the world is incompatible with the putative essential nature of God as omnipotent, omniscient, and omnibenevolent; or that the very

idea of such a triple-omni God is incoherent. Or radical subjectivists might insist that assigning probability 0 to God's existence is permissible, as long as one also assigns probability 1 to God's non-existence (and one updates by conditionalization). Furthermore, the probability of God's existence might be imprecise over an interval that includes 0, as I have previously argued (2000).

I will add one more objection to P1, since it will help to set up the dialectic that follows. There is another way for the probability to fail to be positive: it could *fail to exist.* "God exists" might be a *probability gap* for a rational agent. →Pascal (1948) writes:

> "God is, or He is not." But to which side shall we incline? Reason can decide nothing here. There is an infinite chaos which separated us. A game is being played at the extremity of this infinite distance where heads or tails will turn up. (66)

Unlike a coin that we believe to have chance ½ of landing heads, perhaps by closely inspecting its symmetry, we know nothing about this metaphorical "coin" according to Pascal. In this state of "epistemic nullity," as Morris (1986) calls it, one might refrain altogether from assigning a credence to "God exists." →Joyce (2005) argues that rational credences reflect one's evidence. If one has *no* evidence, then perhaps one should reflect that by having *no* credence—or at least, perhaps it is *permissible* to have no credence. Or in the face of having no evidence bearing on God's existence, it might seem that any probability assignment to God's existence is objectionably arbitrary, while leaving it as a probability gap is non-arbitrary—committal to exactly the same degree as the evidence is, namely *not at all.*

That said, there is some tension in Pascal's text, since soon afterwards he acknowledges that one has some evidence after all. An imaginary interlocutor asks: "[I]s there no means of

seeing the faces of the cards?" And Pascal answers: "Yes, Scripture and the rest, etc." (68). So the putative probability gap might be filled after all—at least once one accesses that evidence. Be that as it may, soon we will see other putative examples of probability gaps, so it will help our later discussion to have them in mind.

Conclusion 1 apparently follows from the premises.[2] The expectation of Wagering for God is:

$(p \times \infty) + (1 - p) \times f_1 = \infty$.

And Conclusion 2 seems to follow from conclusion 1 and the premises, since the expectation of wagering against God is finite:

$(p \times f_2) + (1 - p) \times f_3$ = some finite quantity,

which is less than the infinite expectation of wagering for God. In order to maximize expectation, you must wager for God rather than against God.

24.3 Pascal's Wager Is Invalid: The "Many Strategies" Objection

Notice my wording: "rather than against God." This would settle the matter if wagering for God and wagering against God were your only options. But they are not.

In my (2003a), I argued that Pascal's Wager is invalid, as follows.[3] Grant for the sake of the argument all of Pascal's premises. I just said that Conclusion 2 *seems* to follow from Conclusion 1, but in fact it does not. It does not follow that you should wager for God if some *other* option *also* has infinite expectation. Here's one that does, by Pascal's own lights: a *mixed* strategy. Toss a fair coin; if it lands heads, wager for God; if it lands tails, wager against God. With probability ½, the coin lands heads, and then you will perform an action that has an

expectation of ∞, by Pascal's lights; with probability ½, the coin lands tails, and then you will perform an action that has some finite expectation. So the expectation of this strategy is

(½ × ∞) + (½ × some finite utility) = ∞.

But the probability of ½ is dispensable; all that matters is that the probability of your winding up wagering for God is positive. Ironically, we can mirror Pascal's own reasoning—his jaw-dropping assumption kicks in again. His text provides the materials to deconstruct itself. Suppose that you wager for God if and only if the next spin of a roulette wheel with 37 slots results in #1. Calculate the expectation of this strategy: multiply 1/37 by ∞, add something finite, and get ∞ again. We need not even know the exact value of the probability multiplier, if we know that it is positive.[4] Suppose that you wager for God if and only if today the clouds in the sky spell out your name in Times font. You know that there is some positive probability that cloud-forming water droplets happen to arrange themselves in that way—a miniscule probability, to be sure, but positive nonetheless. Whatever it is, when multiplied by ∞, the result will be ∞. Suppose that you don't want to think about the Wager at all: you just want to have a beer. Nevertheless, with some positive probability you will wind up wagering for God after all. Suppose even that you do everything in your power to *avoid* wagering for God. At the end of the Wager, Pascal advises one to act like a believer in order to cultivate belief; suppose that you do the exact opposite. Still, there is some positive probability that you will wind up wagering for God … And so it goes, it seems, for any option that you might choose.

And so I concluded in my (2003a) that not only is Pascal's Wager invalid—it is invalid in the worst possible way. Decision theory breaks down. Every action has maximal expectation by Pascalian lights! Pascal was said to have been a pessimist, but this sounds downright Panglossian.

(Now, perhaps I just slightly overstated this. If wagering against God really is an option for you,[5] then it seems at least that *it* has finite expectation. And then at least Pascal is entitled to various comparative claims: wagering for God is *better than* wagering against God, and so is any action that results in your wagering for God with positive probability. But of course this falls well short of the conclusion that he wants: that you *should* wager for God. And the floodgates are still open, letting in all your other options besides wagering against God as tied in expectation with outright wagering for God.)

The key point at every stage has been that

a positive probability × ∞ = ∞.

We saw this in Pascal's Wager itself, and we saw it my deconstruction of it. Earlier we considered an atheist replying to Pascal: *don't* multiply ∞ by a positive probability—*multiply it by 0 instead*! This will *nullify*—literally—the swamping effect of ∞. However, this seems dogmatic, a more extreme credence than your evidence supports, and thus not reflecting that evidence. We considered one assigning a probability that was imprecise over an interval that includes 0. And we considered the probability not existing at all. In that case there is no multiplication at all; the expectation is expunged.

I showed that the Wager is invalid with the coin-tossing strategy, and various other mixed strategies, before making the more sweeping claim that *every* action has maximal expectation by Pascal's lights. I still stand firmly on the invalidity claim; but I am open to revisiting the more sweeping claim. Mirroring our earlier discussion again, there are three ways that the probability of your winding up wagering for God could fail to be positive:

1. The first way is that it could be 0. Perhaps there are some possible courses of action that have probability 0 of your eventually wagering for God? Perhaps, although they will

be harder to come up with than one might think—a probability of a billionth, or a trillionth, or a googolplexth is still positive. It seems dogmatic to zero out altogether the prospect of your eventually wagering for God, for any particular course of action that you can perform—arguably, that is a more extreme credence than your evidence supports, thus not reflecting that evidence.

2. The second way that the probability of your winding up wagering for God could fail to be positive, for some possible course of action, is its being imprecise over an interval that includes 0. Or at least then it is not *determinately* positive. But perhaps this is also dogmatic—perhaps your evidence requires a determinately positive credence, and including 0 in your credence interval does not reflect this evidence. More could be said about this case, but let's set it aside, because my real interest lies in the next one.
3. The third way for this probability to fail to be positive is for it to fail to *be*: it does not exist. So we have an easily overlooked way to counteract the swamping effect of multiplication ∞: *don't multiply it by anything at all*! This brings us to probability gaps.

24.4 Probability Gaps

I have long been interested in probability gaps. I have argued that such gaps are problematic for probability theory and Bayesian epistemology in various ways, paralleling the ways in which 0 is problematic for them. (It's funny that *non-existent* probabilities can cause such trouble!) Let's leave Pascal's Wager briefly, and talk about some of those ways. We will then canvas a particularly salient way in which such a gap is thought to arise. We will then return to the Wager with gap in hand. (If you are

not so interested in probability gaps in general, you could skip this section.)

Conditional Probability

Here's the usual ratio formula for conditional probability:

$P(A \mid B) = P(A \cap B)/P(B)$, provided $P(B) > 0$.

Suppose that $P(B)$ does not exist—it is undefined. Then we have:

$P(A \mid B) = P(A \cap B)/$*UNDEFINED,* which is *UNDEFINED.*

Yet various conditional probabilities should be defined even if $P(B)$ is undefined—e.g. $P(B \mid B) = 1$, *P(not-B | B)* = 0, *P(tautology | B)* = 1, $P($*contradiction* $\mid B) = 0$. See →Hájek (2003b) for further discussion.

Conditionalization

This problem for the conditional probability formula quickly becomes a problem for the updating rule of *conditionalization*, which is defined in terms of it.

Independence

Probability textbooks say that *A* and *B* are *independent* just in case

$P(A \cap B) = P(A)P(B)$.

But if $P(G)$ is undefined (G is a gap), then $P(G)P(X)$ is undefined, for all *X.* Then we have some uncomfortable silences when we want answers to questions about independence. For example, every proposition (apart from one with probability 1) should turn out to be *dependent* on itself, including *G*: the truth of the proposition is maximally informative about its own truth!

Regularity

Regularity is the following constraint on a probability function *P*:

If X is possible, then P(X) > 0*.*

Versions of regularity are endorsed by many authors.[6] Note that regularity, regarded as a rationality constraint on credences, would give Pascal a succinct argument for P1: if God's existence is possible, then your probability for it is positive—on pain of irrationality. Almost all of the discussion in the literature on regularity concerns cases in which *X* is possible (in some sense) but has probability 0. In my (MS), I emphasize cases in which *X* is a probability gap.

So probability gaps raise problems for various pillars of Bayesian epistemology. Let's turn to a debate in which they figure prominently, before returning to Pascal's Wager.

24.5 The Deliberation Annihilates Reflexive Credences Thesis

A number of authors (e.g. →Spohn, 1977; →Gilboa, 1999; →Levi, 2007; Liu and Price, 2019, 2020) hold the thesis that *while deliberating about what to do, you cannot rationally have credences for what you will do.* I have called this (2016) *the DARC Thesis*: *D*eliberation *A*nnihilates *R*eflexive *C*redences.

For example, suppose that you are deliberating about whether to have red or white wine with your meal. According to the DARC thesis, you cannot rationally assign credences to these options. You cannot assign probability ½ to each of them (reflecting a state of credal balance); you cannot assign 2/3 to red and 1/3 to white (reflecting a partial inclination towards red); and so on. And so it is with all decisions, while you are in the process of deliberating about them. That includes the decision

about whether you should wager for God or not, while you are deliberating about *it*—as we will soon see. But first …

24.5.1 Why Believe the DARC Thesis?

There are many arguments for the DARC Thesis. It would take us too far afield to go through them here—I discuss several in my (2016). (And you can skip the rest of this section if you are impatient to get quickly back to Pascal's Wager, fast-forwarding to Section 24.6.) Let me put my cards on the table (as Pascal might say): I find some of them to be just smoke and mirrors, and I find all of them wanting—I am an opponent of the DARC Thesis. More recently, I have appreciated an exchange with Yang Liu and Huw Price about it. I will briefly canvas an interesting argument for it in their (2020, 4365-4366):

> [It] is a special case of the so-called 'transparency' of first-person present-tensed reflection on one's own mental state. If someone asks me whether I believed yesterday that it would rain today, I consider my evidence (e.g., from memory) about what I believed *yesterday*. If someone asks me whether I believe that it is raining *now*, I don't consider my present state of belief at all, or at least not directly. I simply consider whether it is raining. My enquiry 'looks through' the question about belief, to a question about what the belief in question is itself about. This 'looking through' is transparency.
>
> Moran (2001) explains transparency in terms of a distinction between two paths to knowledge of one's own mental state—a *theoretical* path, which is the one I use to discover what I believed yesterday (or what another person believes); and a *deliberative* path, where I learn that I now believe that it is raining *by asking myself whether it is raining* (and concluding that it is). As we gloss Moran's conclusion, transparency rests on the fact that the deliberative path 'crowds out' the theoretical path. The practical case of this is simply [DARC], or so we argue.

Liu and Price go on to fill out this argument more, but this passage captures their key move of appealing to "transparency"

regarding reflection on one's own belief state. The view is associated with Gareth →Evans (1982), although Roy →Edgley (1969) clearly stated it considerably earlier.

I reply: transparency may be plausible for outright *belief*.[7] But I think that it is implausible for *degrees of belief*, which are our topic. Suppose instead that someone asks me whether I have *credence 0.6* that it is raining now. I *do* consider my present state of belief. I *introspect*, rather than looking out at my evidence (or the world).[8] And the switch from outright beliefs to credences is crucial: the DARC Thesis is a thesis about credences.

But even if credences are transparent, this doesn't mean that they *don't exist* while one is deliberating, which is what the DARC Thesis says. On the contrary, it just means that one uses a "deliberative path" to determine what they are, rather than a theoretical path. Compare: Edgley and Evans did not conclude from the transparency of beliefs that they *don't exist*—on the contrary, one just determines what they are in a distinctive way. If anything, transparency counter-supports the DARC Thesis! Transparency is a claim about self-knowledge: *how one comes to know one's own doxastic state* regarding some proposition. But the DARC Thesis is not a thesis about *how one comes to know* one's own credences about what one will do while deliberating about that; rather, it is a claim about the *non-existence* of these credences, period—another issue. So I do not see how a claim about self-knowledge of a doxastic state that one *has* supports this alleged non-existence, period.

Liu and Price (2019, p. 10) also canvas another argument for the Thesis: "Previous writers (→Louise, 2009; →Price, 2012) have noted that Moore's Paradox has some affinity with DARC. Someone who says "I believe (to certain degree) that I'll do A, and I'm making up my mind as we speak" gives something of the same sense of taking back with one hand what they gave us with the other." I reply: the point is certainly correct for

“believe.” But it is not for “to certain degree”—that parenthetical qualification makes all the difference. So let’s set free the qualification from the parentheses, and focus on it. It’s *because* I’m making up my mind as we speak that I *only* believe that I’ll do A *to a certain degree*. Far from being Moore Paradoxical, the second conjunct supports the first! For example, there is nothing Moore Paradoxical about “I believe to degree ½ that I’ll do A (and ½ that I won’t do A), and I’m making up my mind as we speak—that’s the reason why my degrees of belief are evenly split!” The parenthetical remark about “to certain degree” requires us to take back with one hand what “I believe” gave us with the other. But again, the DARC Thesis concerns degrees of belief, not outright belief.

24.5.2 Why Disbelieve the DARC Thesis?

In my (2016) I also present several arguments *against* the DARC Thesis. Here is a small sample:

1. When you are deliberating about what to do, you are *uncertain* about what you will do; probability theory is our best theory of uncertainty; so offhand, you should assign probabilities to what you will do—or at least, you *may.*
2. You can rationally make *comparative* probability judgments about your options, and it is unclear why numerical probabilities should pose any special problem. You judge that you are *more likely* to have red wine than to have red wine followed by a sherry, and this in turn is *more likely* than your having a shot of petrol. And I don’t see why numerical probabilities create any special problem. Of particular interest to us regarding Pascal’s Wager are comparative judgments of the form: the probability that you will eventually wager for God is *greater than 0*.

3. The DARC Thesis conflicts with the *Reflection Principle* (→van Fraassen, 1984, 1995): for any future time, your credence in X should be your (probabilistic) expectation of your credence in X at that time. Your future self serves as a kind of "expert" for you, so your current credence should reflect what you think your future self's credence will be. Suppose that you have not yet begun deliberating about whether you will have red wine or white wine at tonight's dinner, but you know that you will at that time. Consistent with the DARC Thesis and your rationality, you may *now* have a credence for each choice—say, ½ for white, ½ for red—but you must *lose* those credences when your deliberation starts. So ½ is *not* your expectation of your credence in each choice at that future time, according to the DARC Thesis. On the contrary, you are certain now that your credence at that time will be undefined (according to the Thesis), so your expectation of that credence is undefined.

So I oppose the DARC Thesis. It's not as if things must go *dark* in your mind regarding probabilities for what you will do while you are deliberating about it; on the contrary, that seems exactly the time at which credences for what you will do should be especially salient. Such credences are central to Skyrms' (1990) *deliberational dynamics.* →Bartha (2012) uses it to model one's evolving credences when faced with many theistic hypotheses, as one does in the "many Gods" objection to Pascal's Wager. And →Joyce (2002) and →Rabinowicz (2002) provide further arguments against the DARC Thesis.

But this remains an area of lively debate, and I don't pretend to have settled it here.[9] Indeed, I want to explore the ramifications of the DARC Thesis for Pascal's Wager.

24.6 Pascal's Wager and Deliberation: Credences for What You Will Do

So let's return to Pascal's Wager, now seen through the lens of deliberation. With some *Black Mirror*-like twists, we will see that whichever side we take on the DARC debate, there will be a new problem for the Wager's validity.[10]

Let's start by assuming that the DARC Thesis is false (as I have just argued). Suppose you are deliberating about whether to wager for or against God, and that (contra the DARC Thesis) you assign positive credences to each of these options ending up being your choice.[11] Then it seems by Pascal's lights, you *are already doing something with infinite expectation.* So why bother wagering for God?—Just keep deliberating! For example, if you assign credence 0.1 to your wagering for God when your deliberation ends, and 0.9 to wagering against God, then right now your expectation is:

(0.1 × ∞) + (0.9 × some finite value) = ∞.

So rather than deciding what to do, you could rationally keep deliberating about it![12]

We might think of this as a special case of the "many strategies" objection that I rehearsed earlier. But it is an especially interesting case, worthy of attention. Note firstly that this is not a mixed strategy. It is not that you are outsourcing your decision to some randomizing device, like a coin or a lottery. If you wanted, you could *add* such mixed strategies to your menu of options. But then you would face an enriched decision problem *about which you could deliberate*. And if you assigned credences to each of *its* options, by Pascal's lights you would enjoy infinite expectation by deliberating about *it.*

Moreover, something curious results if we add *deliberation itself* to your menu of options: wager for God, wager against

God, deliberate about which to choose. This enriched decision problem is itself something you could deliberate about: that is, deliberate about whether to wager (one way or another) and *deliberate about whether to deliberate*. If you regard this "meta"-deliberation as one of your options, then it should be added to the menu, yielding yet another decision problem *about which you could deliberate.* Infinite regress threatens. But Pascal had better not object to the *infinitude* itself, since he is up to his neck (and way beyond) in infinitude himself. Cognates of the word "infinite" appear 29 times in a couple of pages of §233 of the *Pensées*, predicated of several different things. (This section is called "Infinite—nothing," after all!) Be that as it may, we humans presumably stop the regress of higher-order deliberation at some early stage—perhaps the very first level. Notice that all this is peculiar to deliberation and not the various other options that I canvased (tossing a coin, waiting for the roulette result, having a beer …).

In any case, I will assume that deliberation *is* among your options: you can choose to act now (wagering for God or not), or you can choose to (continue to) deliberate about that. This captures a familiar phenomenology—"I'm not yet ready to decide; I want to think about this a bit longer." And having chosen the option of deliberating, you may well be glad that you did—"Thank God (!) I decided to think about this a bit longer: I'm now attending to considerations that I had previously neglected." Or you may come to rue that choice—"I sat on that decision for too long. I should have made my mind up faster!"[13] As such, the choice to deliberate can lead to better or worse results, and as such, it seems to be something you can assign an expected utility in advance.

The upshot again is that Pascal's Wager is invalid: his premises do not favor your wagering for God over your continuing to deliberate and assigning credences to what you

will eventually do. By his lights, while so deliberating you are already enjoying the same infinite expectation that you would were you to wager for God; so you are *not* better off making up your mind and wagering for God.

Yet this has a curious consequence. At each time at which you can make your choice, you may rationally continue to deliberate. Suppose that you keep doing so. Eventually you will die, and having *not* wagered for God, you will definitely *not* get the infinite reward of salvation.[14]

24.7 Pascal's Wager and Deliberation: The DARC Thesis

But proponents of the DARC Thesis have a snappy reply: While you are deliberating, you are *not* enjoying infinite expectation, or indeed *any* expectation as far as this decision problem goes; rather, your expectation is undefined. For example, your expectation while deliberating is *not*

(0.1 × ∞) + (0.9 × some finite value) = ∞.

Rather, it is:

(UNDEFINED × ∞) + (UNDEFINED × some finite value) = UNDEFINED.

Now, I reject the DARC Thesis, so I am unsympathetic to this reply. But even granting the Thesis, it *still* shows that the Wager is invalid, in a particularly interesting way. For now we must compare your infinite expectation of wagering for God with your *undefined* expectation of continuing to deliberate. And now it's not true that wagering for God *maximizes* your expectation. Infinite expectation cannot be compared to undefined expectation, and in particular the former does not exceed the latter. It's not true that ∞ > UNDEFINED. Compare: since 1/0 is undefined, it's not true that

1,000,000 > 1/0.

Indeed, it's not even true that

$\infty > 1/0$.

An inequality fails to hold when one side of it is undefined.

But this DARC route to showing that Pascal's argument is invalid seems to prove too much. It generalizes to *any* decision problem. According to DARC, *whenever* you are deliberating about what to do, deliberating itself has UNDEFINED expected utility. But continuing to deliberate is always an option. So you can never maximize expected utility over all your options. E.g. if you are deliberating about whether to have red wine or white wine, a further option is to continue deliberating. That has UNDEFINED expected utility according to DARC. And so you cannot maximize expected utility: the expected utilities of red and of white wine are not at least as great as UNDEFINED.

That said, mirroring our discussion in Section 25.3, at least the proponent of DARC is entitled to various comparative claims.[15] For example, perhaps having red wine is *better than* having white wine. But this falls short of the conclusion that we want (in that decision problem): that you *should* have red wine (say)—and likewise for any decision problem. We don't have any prescription regarding what you *should do.* Because of the alleged credence gaps, we have a partial ordering of your options by expectation—the gappy options cannot be slotted into the ordering—but not a total ordering with what you should do at the top. This seems to be another serious problem for the DARC Thesis. It's rather like the predicament that I argued befalls Pascal: while he is entitled to various comparative claims about options that are *better than* wagering against God, this falls short of the conclusion that he wants: that you *should* wager for God. Because of the alleged infinite utility, we have a weak ordering of your options by expectation—it has many ties—but not a total ordering that tells you what you should do. That's a serious problem for the Wager.

In both cases—gaps and ties—the Pascalian wants to say much more than such orderings will enable them to say. If we ignore the problem of gaps and consider only the problems of ties (due to mixed strategies), then all they can say is that you ought not outright wager against God (if that's even an option for you); everything else is tied. If gaps come in, they cannot even say that. All they can say is that outright wagering against God is worse than all other non-deliberational strategies, and all of these are incommensurable/incomparable with the option of deliberating further. That's a far cry from the initial Pascalian conclusion that you should wager for God!

In any case, while the DARC Thesis might give us a novel way to bring out the invalidity of the Wager via, we don't need it. The Thesis says that you *must* have certain credence gaps, but all that matters for my purposes is that you *do*. You might have other credence gaps for other options of yours resulting in your wagering for God, for whatever reason. For example, perhaps you have credence gaps for wagering for God or not after having a beer. Then wagering for God will again not maximize your expectation—it can't be compared to your undefined expectation for having a beer, which may result in your wagering for God, but your credence in its doing so is undefined. However, one might worry that such gaps would not reflect your evidence, and there is no candidate for a principled reason for having such gaps. Nobody has yet been so bold as to propose a BARC Thesis —*Beer Annihilates Reflexive Credences*—whatever plausibility it might have. At least the gaps triggered by deliberation are supposed to be principled: the principle is the DARC Thesis.

So perhaps my younger self overreached when I claimed that *every* action has maximal expectation by Pascal's lights—earlier I allowed that outright wagering against God does not, and now I have been allowing that deliberating does not. But

that's cold comfort for Pascal. His argument may not be invalid in the worst possible way, but it's close!

24.8 Final Reflections

Pascal walks us through some *prima facie* compelling reasoning for why we should wager for God rather than not. It's as if he is guiding us through a process of deliberation about this momentous decision problem. But deliberation itself gives rise to a fascinating philosophical debate over the DARC Thesis. And deliberation itself seems to be a further option that could be incorporated in the decision problem. Once it is, it provides a particularly striking way of showing that the Wager is invalid. If the DARC Thesis is false, and you assign positive probability to winding up wagering for God at the end of your deliberation, you have an alternative option that has infinite expected utility: deliberation about this very decision problem. If the DARC Thesis is true, then this option has undefined expected utility. Either way, the infinite expected utility of wagering for God is not strictly greater, and so we do not yet have a valid argument for favoring that option.

But like a *Black Mirror* episode, there will doubtless be further twists for us to reflect on ...

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Notes

1 Perhaps it can and should be given a more nuanced formulation—see →Hájek (2017, 2018).

2 I say "apparently," because the utility that I am denoting with "∞" needs to be handled with care. Does it straightforwardly enter into arithmetic operations, such as multiplication and addition? I am also bracketing here whether it is a legitimate utility value at all. See →Hájek (2017, 2018) for further discussion.

3 I also noted that →Duff (1986) anticipates my argument, although his emphasis is a little different.

4 Said more carefully: if we know that its *objective chance* is positive. Then, by (a version of) the Principal Principle (→Lewis 1980), we should have a positive credence.

5 It may not be a genuine option if you could fail to perform it despite doing everything in your power to do so, as I have just imagined.

6 They include →Kemeny (1955), →Shimony (1955), →Jeffreys (1961), Edwards et al. (1963), →Carnap (1963), →Stalnaker (1970), Lewis (1980), Skyrms (1980), →Appiah (1985), →Jackson (1987), →Jeffrey (1992), Benci, Horsten, and Wenmackers (2013), and others.

7 →Gertler (2011) argues that it is not.

8 Though see →Dogramaci (2016) for an expressivist defense of transparency of credences.

9 See, for example, Liu and Price (2020) for further defense of the DARC Thesis.

10 One might say that we will see in a *Black Mirror*, DARCly.

11 If your credences were 1 for one of these options and 0 for the other, it would be hard to make sense of your deliberating about them.

12 As Brian Hedden has pointed out to me, we reach the same conclusion if your credence for wagering for God is imprecise over an interval that does not include 0—even (0, 1], an imprecise representation of P1. For every precisification of your credence, deliberating has infinite expectation, and hence further deliberation is always permissible. (See my 2000.)

13 See the discontinuity at infinity problem discussed in footnote 14.

14 There is an analogy here with various decision problems that involve a "discontinuity at infinity," in the words of Bartha, Barker, and Hájek (2013). A discontinuity at infinity occurs when an infinite sequence of choices, each apparently sanctioned by plausible principles, converges (in a sense) to a "limit choice" whose utility is distinct from, and typically much lower than, the limit approached by the utilities of the choices in the sequence. "Satan's apple" is a nice example—see Arntzenius, Elga, and Hawthorne (2004). Imagine that you face a supertask with countably infinitely many choice points, at each of which you have to decide: continue deliberating, wager for, or wager against God. (We could throw in mixed strategies as well, if we want.)

At each choice point, any option (except “wager against”) seems equally good. But there is also a discontinuity at infinity: if you forever choose to continue deliberating, the result will be that you lose the infinite utility. There is also something odd about the fact that each choice point in this sequence looks exactly the same and yet you could permissibly make a different choice—to stop deliberating—at a later time. This apparently violates the principle of sufficient reason. You can’t give a reason why you decide to continue to deliberate at t_1 but you decide to wager for God at t_2. (Many thanks to Paul Bartha for this footnote.)

15 For this and the next paragraph, I am grateful to Brian Hedden.

25 Quantity Has a Quality All Its Own

Leon Horsten

Abstract: Since the work of Cantor, philosophical reflection arguments have been developed as justification for basic set-theoretic principles that are seen as mathematical reflection principles and that postulate large actual infinities of mathematical objects. A main objective of this paper is to discuss philosophical arguments for such ontological reflection principles, and their roots in the history of philosophy and theology.

> Just don't get ontological. Not now.
>
> I couldn't bear it if you were ontological with me.
>
> (Mr. Big, in: →Allen 1991, p. 288)

25.1 Introduction

In this article, I explore aspects of the relation between quantitative infinity on the one hand, and a particular concept of reflection on the other hand. The focus will lie exclusively on a particular form of ontological reflection. Epistemic concepts of reflection have also played a role in the history of philosophy (for instance in the work of Locke and Leibniz), but they will not be treated in this article.

We will see that ontological reflection has deep roots in the history of philosophy and theology. Ontological reflection first came to the fore in late Antiquity, in discussions about the nature of God. The connection between ontological reflection on

the one hand, and quantitative infinity on the other hand, became clear only much later: from the late nineteenth century onward. At around this time, the influence of theology as an intellectual discipline was receding rapidly. The relative scientific influence of philosophy was also declining, albeit less rapidly.

Reflection considerations play an important role in contemporary set theory. On the one hand, what are loosely called “reflection arguments” play a role in mathematical proofs in set theory. On the other hand, since the work of Cantor, *philosophical* reflection arguments have been adduced as justification for basic set-theoretical principles that are seen as mathematical reflection axioms. A main objective of the present article is to discuss these philosophical reflection arguments, and their roots in the history of philosophy and theology, in some detail.

Before we start off, let me express a disclaimer. Thoroughly to investigate the relations between ontological reflection across three disciplines (philosophy, theology, set theory) requires a breath of expertise that very few people can claim to have, and that I certainly do not claim to possess. I can only express my hope that others will in the future explore these relations further.

25.2 From Myth to Philosophy

Creation myths attempt metaphorically to explain where the order in our world comes from. They may, for instance, relate the struggle between a primeval animal- or monster-like entity on the one hand, and a super-human agent on the other hand, which ends with the super-human agent defeating the monster, and thereby imposing order on the universe.

Pre-Socratic Greek philosophers of nature transformed creation myths into rational explanations, and thereby sought to

acquire rational understanding of the nature of our world as a law-governed *cosmos*. This transformation involved a de-anthropomorphization of the creation myths.

Even though our knowledge of his philosophy is extremely fragmentary, it is clear that Anaximander is a key intermediary figure in this transformation process. Anaximander replaces the mythical pre-order state by a philosophical principle that he calls the *Apeiron*. Literally, Apeiron means something like "without boundary/end/limit," "what is not fenced in." In its philosophical use, Apeiron is variously translated as indefinite, indeterminate, unlimited, formless. (The distinctions between these different notions had not crystalized in Anaximander's days.) According to Anaximander, out of the Apeiron, all distinctions in our world (such as the "elements" water, fire, etcetera) are somehow generated, and to the Apeiron they will eventually return. We will see how Anaximander's Apeiron is the far-distant source of our modern concept of infinity.

According to Aristotle's *hylemorphic* metaphysics, every object consists of form impressed on matter. A form is a determination, a "fence," if you like. In *Metaphysics*, vii, 3, Aristotle describes formless matter in the following terms:

> By "matter" I mean that which in itself is not called a substance nor a quantity nor anything else by which being is categorised. For it is something of which each of these things is predicated, whose being is different from each of its predicates (for the others are predicated of substance, and substance is predicated of matter). Therefore this last is in itself neither substance nor quantity nor anything else. Nor is it the denials of any of these; for even denials belong to things accidentally. (→Aristotle, 2016, 1029a20–26)

Thus formless matter shares many properties with Anaximander's Apeiron: it is in no way "fenced in." Pure matter also seemingly cannot be known as it is in itself, since knowledge

necessarily involves subsumption under categories, which are conceptual representations of forms. This then immediately gives rise to an epistemological puzzle, for in the above quotation, for instance, Aristotle presumably does purport to express knowledge about formless matter.

25.3 The Problem of Mathematics

In most of Ancient philosophy—Anaximander was an exception to this—as in the creation myths, the chaotic and formless has strong negative connotations, whereas the orderly and definite has strong positive connotations. In Greek philosophy, for this reason, the Apeiron was excluded, as far as possible, from the world.

As far as the physical world is concerned, the project of banning the unbounded from the world was relatively successful. Unlike Anaximander, Aristotle believed that matter cannot exist independently of form; Greek astronomy adopted the position that the cosmos is a *finite* globe.

In mathematics, problems seemed to arise. Aristotle's contemporary Euclid taught that a line segment consists of points, and the multiplicity of points of which a line segment consists cannot be bounded by a number. Arithmetic posed problems, too. For every natural number, there is a greater number, so the multitude of numbers cannot itself be numbered.

In response to these difficulties, Aristotle developed his distinction between potential infinity and actual infinity. An entity (object, quantity, …) is potentially infinite if it is necessarily finitely bounded, but at the same time *can* exceed any given finite bound. An entity is actually infinite if it *actually* exceeds all finite bounds. In this notion of potential infinity, a modality ("necessarily," "can") is involved. This modality can be given

various interpretations: a temporal modality may be intended (always, sometimes), logical necessity/possibility may be meant, metaphysical necessity/ possibility may be what is relevant… At any rate, Aristotle's distinction quickly became the template for thinking about infinity in mathematics and in physics.

His distinction neatly solves the problem of the infinity of the natural numbers. According to Aristotle, the collection of natural numbers is potentially infinite. In any situation, only finitely many natural numbers exist. Thus the number of natural numbers can always be numbered and is thereby necessarily "fenced in." But there could always be more natural numbers than there are. The problem of line segments consisting of points is perhaps less convincingly solved by this strategy. A finite line segment *ab* can always be divided up into smaller line segments. But there can be no "infinite stage" at which, after at each finite stage choosing the left half of the remaining segment, the point *a* is finally reached. Thus "points" can be approached to arbitrary finite precision, but, in geometric reality, there are not and cannot be points. (So Euclidean geometry talks about certain entities that cannot exist!)

Aristotle' distinction also helps us with keeping the Apeiron out of our physical world. For every moment in time, there will be a later moment; but at no moment, an infinite period of time has elapsed. Thus also in the temporal direction, the cosmos is always finitely bounded. Every finite time interval can be divided in two; but there is no possible situation in which a time interval has been divided infinitely many times so that an unextended moment of time has been reached. Also, it seemed that Aristotle's distinction could be used to resolve Zeno's paradoxes, most of which seemed to revolve around the concept of infinity.

25.4 Mr. Big

In late Antiquity, two monotheistic religions, Judaism and Christianity, began to exercise a profound influence on Western thought. Philosophy at this time was in the fortunate and rare situation of important new data coming in. The following are among propositions that were increasingly seen as just as evident as immediate observation reports and elementary logical truths:

- There is one and only one God;
- God is perfect in every conceivable way;
- God transcends our intellectual powers.

This does not mean that in earlier Greek philosophy the hypothesis of a single God, for instance, was not somehow entertained. (Think of Aristotle's *Unmoved Mover*.) But it would have been treated as metaphysical speculation rather than as certainty. The reason that principles such as the above came in late Antiquity to be regarded as rock-solid *givens* of course has to do with the fact that they can be found in the Bible.

Concerning the physical world and the mathematical world, the dominant philosophical view remained substantially unchanged: it had no place for the Apeiron. There also was no *direct* pressure to associate God with infinity: the infinity of God is not mentioned in the Bible. Yet, the above principles entail that God exceeds all bounds. At some point the conceptual pressure became too great, and philosophers-theologians—the distinction between the two was often not clear—started to predicate infinity of God.[1] This precipitated a reverse of normative polarity concerning "bounded" and of "unbounded," like a switch of the magnetic poles of the earth that is said sometimes to occur. Infinity came to be seen as a *positive* attribute (a "good thing to have"), and boundedness as rather a negative property. It seems that Plotinus is an important

intermediate figure in this process of reverse of polarity of infinity (→Krainer, 2019, pp. 25–27). On the one hand, he predicates infinity of the One, and intends this as a positive attribution. On the other hand, he also predicates infinity of Evil, and in this case infinity is surely a negative property.

Aristotle's distinction was of no use in "taming" the infinity of God. Every un-actualized potentiality in God would be an imperfection. There can then be no un-actualized potentiality in God. And this means that if God is unbounded at all, then He is *actually* unbounded. The application of the concept of infinity primarily to God has brought about a momentous shift in the concept of infinity. Whereas before, pure matter, or, in Aristotelian terms, pure potentiality was the paradigmatic instance of infinity, now pure actuality becomes the paradigmatic instance of infinity. This reveals an interesting duality between the notions of pure potentiality and pure act.

Aristotle used the concept of infinity almost exclusively in a quantitative sense. When the concept of infinity comes in late Antiquity also to be applied to God, infinity comes to be applied to properties of God that are not easily conceived of in quantitative terms, such as goodness. What was *meant* when infinity was predicated of God, varied considerably, and was mostly a sense of qualitative infinity. The lesson of the modern theory of infinity—see the title of this paper—was not appreciated at this time.

The question about God's infinity became an important chapter in a long and extensive philosophico-theological debate about the nature of God. When they spoke of God's infinity, some authors had unbounded perfection in mind, sometimes a form of absolute simplicity was meant, at other times a combination of both. Indeed, the alleged combination of immensity and maximal simplicity of God posed a formidable challenge to attempts to construct a convincing theory of the

infinity of God. A form of *quantitative* infinity was mostly not what was intended, although that, too, played a role (if God is omnipresent, for instance). In this context, it should be kept in mind that important conceptual distinctions that we currently routinely make (such as between indefinability, incomprehensibility, semantic indeterminateness, ontological indeterminacy, unboundedness, indefinite extensibility…) were only to a very limited degree available then.

25.5 Reflection

Since God came in the Middle Ages to be increasingly thought to exceed all bounds, many came to believe that this in particular holds for all *conceptual* bounds. A consequence of this would be that positive (i.e., negation-free) propositional truths do not hold of God, whereby positive propositional knowledge of God is impossible. This line of thought is known as *negative theology*. Nonetheless, some philosophers put forward positive hypotheses about the nature of God. Some of these are *reflection hypotheses*. We discuss two of those, and we take them in reverse chronological order.

Augustine's views of infinity did not remain stable throughout his theological career.[2] But his writings contain a thought concerning quantitative infinity that has proved to be remarkably prescient. Augustine's thought concerns the multiplicity of the natural numbers. This multiplicity forms, according to received wisdom in Antiquity and the Middle Ages, a potential infinity in the world. But in God's knowledge this multiplicity is *limited* in the sense that He can somehow assign a number to it, so that something that is infinite for us, is finite for God (Augustine, 1972, 12.18):

> The infinity of number[s], although there is no number for infinities of numbers, is yet not incomprehensible by Him of whose understanding there is no number. And thus, if what is comprehended in knowledge is made finite by the comprehension of this knowledge, then all infinity is in some ineffable way finite to God, for it is not incomprehensible to His knowledge.

Thus the infinity of natural numbers is somehow *reflected* in a bounded entity in God's thought. This passage is many centuries later lauded by Cantor as a prefiguration of his theory of transfinite numbers (→Cantor, 1962, *Mitteilungen zur Lehre vom Transfiniten*, p. 402):

> More energetic and more perfectly as is done here by St. Augustin, the Transfinite cannot be justified and defended. […] By asserting the total, intuitive perception of the set of natural numbers [in God's knowledge], St. Augustin recognises this collection at the same time *formaliter* as an actually infinite whole, as a *Transfinitum*, and we are compelled to follow him in this. [my translation]

Three centuries earlier, in *On Dreams*, Philo of Alexandria postulated an inverse reflection phenomenon (Philo of →Alexandria, 1988, pp. 419–421):

> Thus in another place, when he had inquired whether He that is has a proper name, he came to know full well that He has no proper name, [the reference is to Exodus 6:3] and that whatever name anyone may use for Him he will use by licence of language; for it is not in the nature of Him that is to be spoken of, but simply to be. Testimony to this is also afforded by the divine response made to Moses' question whether He has a name, even "I am He that is (Exodus 3:14)". It is given in order that, since there are not in God things that man can comprehend, man may recognise His substance. To the souls indeed which are incorporeal and occupied in His worship it is likely that He should reveal himself as He is, conversing with him as a friend with friends; but to souls which are still in the body, giving Himself the likeness of angels, not altering His own nature, for He is unchangeable, but conveying to those who receive the impression of His presence a semblance in a different form, such that they take the image not to be a copy, but that original form itself.

This is a reflection phenomenon not from the world to God (as with Augustin), but from God to the world. An “angel” *reflects* the essence of God in the form of an image. But this angel-image is such a perfect copy that we cannot distinguish it from God in any way, so we humans tend to take such an “angel” to be God himself. Philo thus posits the following *reflection principle*: God is reflected, in the sense of being mirrored, in an entity in the world (an angel). We will see that reflection principles that have this structure, are theoretically very powerful: complexity of the reflected object can be deduced from such principles. In the quoted passage Philo also observes —he was clever indeed!—that this theory leads to a *semantic problem*. Since we cannot distinguish God from certain “angels,” there is nothing we can do to ensure that the word “God” refers to God rather than to one of the angels. So, literally speaking, on Philo’s view, God is unnameable.

An idea going back at least to Aristotle is that the most perfect kind of intellectual activity is *self-thought*.[3] The reason seems to be that when a mind thinks about an entity that is not herself, then her thought cannot be completely adequate to the object of thought, because the object of thought is then not numerically identical to her mind. But this problem dissolves when subject and object of knowledge coincide. From this it is only a small step to conclude that the most perfect thought (self-thought) is most perfectly produced by the most perfect entity (God).

Philo was of course familiar with this line of thought. We can connect it with his metaphysics in a way that he might well have been sympathetic with, by internalizing his reflection idea that we discussed above to God Himself, in the following way.

In Philo’s metaphysics, there is a sense in which God is maximally simple *as well as* a sense in which God is maximally immense. Philo maintains that God in his Essence is absolutely

simple, but at the same time He externalises Himself in what Philo calls the External Logos, which is truly immense.[4] The externalized Logos contains an abstract blueprint of the whole material world, is in some sense also responsible for the creation of our world, and has an immeasurably complicated and intricate structure.

Perhaps God in its deepest Essence thinks himself by simply being, by simply coinciding with himself.[5] But concerning the External Logos, the situation is interestingly different. Philo maintains that there is an (abstract) *idea of the Logos* (→Wolfson, 1947, pp. 213–214). Suppose the Logos contains such an idea (call it *idea 1*) beside the abstract "blueprints" of all the objects in our world, so that the Logos is reflected *in itself*. Suppose furthermore that this idea is perfect, in the sense that it is structurally isomorphic with the externalized Logos itself. Then this idea 1 must contain, among many other things, a representation of itself: call this *idea 2*. This idea must then again contain an idea of itself, and so on. The conclusion is that the Logos is *quantitatively infinite* in the modern sense of the word. Thus there is an intimate connection between perfect self-reflection and infinity in the mathematical sense of the word.

Philo did not articulate, let alone pursue, this line of thought. He therefore lacked a clear understanding of the relationship between self-reflection on the one hand, and quantitative infinity on the other hand. But—or so I suggest—he was not far from grasping this connection.

25.6 Two Faces of Infinity

In the nineteenth century, in the work of Dedekind, several strands came together. It became clear that there are two ways to define quantitative infinity: two conceptions of quantitative infinity, if you like. On the one hand, one can say that a multitude

is quantitatively infinite if the natural numbers—or a *simply infinite system*, in Dedekind's terminology—can be embedded into it. We might call this the *ordering conception* of infinity. This conception of infinity of course goes back at least to Aristotle. In his descriptions of potential infinity, he describes how such infinite orderings are "generated." On the other hand, Dedekind conceives infinity as a self-reflection property: a multiplicity is infinite if and only if there is a one-to-one onto correspondence between the multitude and a proper sub-multitude of itself (→Dedekind, 1888, p. 64). This may be called the *reflection conception* of infinity. These two notions of infinity correspond to the now familiar distinction between an ordinal concept of number and a cardinal concept of number. Moreover, Dedekind knew that (in the presence of the Axiom of Choice), a multitude is ordering-infinite if and only if it is reflection-infinite.[6]

Dedekind believed that he could *prove* that there is an order-infinite collection (→Dedekind, 1888, p. 66):[7]

> My own realm of thoughts, i.e., the totality *S* of all things which can be objects of my thought, is infinite. For if *s* signifies an element of *S*, then the thought s^0 that s can be an object of my thought, is itself an element of *S*. [...] then *S* is infinite, which was to be proved.

But Dedekind's argument has never been accepted as a real proof. Ultimately, the existence of an infinite collection is something that has to be *postulated*.

None of this in and of itself decides whether *actually infinite* collections exist. Clearly order-infinite collections of small transfinite order types can be conceived of in a potentialist way. But also Dedekind-infinity can be conceived of in a potentialist way, namely when the one-to-one correspondence between the whole and the part can be enumerated as a potentially infinite sequence.

If infinite pluralities of mathematical objects (numbers, or collections of numbers) are parts of the mind of God, then, because of the pure actuality of God, they must be actual infinities. To argue that certain infinite pluralities of mathematical objects are therefore also *bounded* and hence numerable (as Augustin did), would then be a further step, that could be taken, but could also be resisted. However, in the nineteenth century, speculation about the nature of mathematical infinity had mostly emancipated itself from theological speculation.[8] As far as I know, Dedekind did not appeal to medieval theology at any place in his theory of the foundations of mathematics. The only place where he uses a philosophical argument is his "proof" of the existence of infinite collections that was discussed above.

25.7 Cantor

Cantor was an exception to this emancipation process. He was a pre-eminent mathematician who did appeal to views and arguments from the history of philosophy and theology.

We have already seen how Cantor agreed with Augustin that mathematical objects are ideas in the mind of God. Since the mathematical world is infinite, it must then be *actually* infinite in God's mind. Furthermore, Cantor followed philosophers-theologians like Philo and Augustine in taking it to be a fundamental principle that God is in a very strong sense epistemically transcendent (→Cantor, 1962, *Abhandlungen zur Mengenlehre III*, Endnote to Section 4, p. 205, my translation):

> The Absolute can only be acknowledged, but never known, nor even approximately known.

In this famous quote Cantor takes us to already have "approximate knowledge" of God if we can "take the measure" of a dimension or compartment of God's mind. In particular, we would have approximate knowledge of God's mind if we could "measure" the extent of a dimension of it by means of the natural numbers, since the concept of the natural numbers is, for Cantor, perfectly clear. This leads him to posit what philosophers of mathematics take to be the first use of a *reflection argument in mathematics* (→Cantor, 1962, *Abhandlungen zur Mengenlehre III*, Endnote to Section 4, p. 205, my translation)

> Whereas hereto, the infinity of the first number class (I) [i.e., the class of finite cardinal numbers] alone has served as such a symbol [of the Absolute], for me, precisely because I regarded that infinity as a tangible or comprehensible idea, it appeared as an utterly vanishing nothing in comparison with the absolutely infinite sequence of numbers.

The reasoning in this passage goes along the following lines. Suppose there is a one-to-one correspondence between the natural numbers and the mathematical world as a whole. Then a "measure has been taken" of the mind of God using a perfectly clear measuring stick (the natural numbers). So by elementary knowledge of the natural numbers we have knowledge of the mathematical part of the mind of God. But this is incompatible with the epistemic transcendence of God. Therefore the collection of the natural numbers must be of bounded size in comparison to the immeasurability of the mind of God. Bounds are given by numbers. Therefore there must be a number that measures the size of the natural numbers. This will then be a transfinite number: a bounded completed infinity.

There seems no obstacle to the *human knowability* of the number that measures the size of the natural numbers, since the knowledge of this number would give us no knowledge of the mathematical compartment of the mind of God, which

immeasurably transcends the collection of the natural numbers. All this also holds for other "clear" collections of numbers, such as the rational numbers and the real numbers. So we might as well *try* to come to know the cardinal number of the natural numbers, as well as the cardinal numbers of other infinite collections, and how to calculate with these transfinite numbers. And this is of course exactly what Cantor did. In this way, he went beyond what Augustine thought possible: as we have seen, the latter's remarks were tentative, and he thought that in any event calculating with transfinite numbers is beyond the intellectual capacities of humans.

Cantor's reflection argument is restricted in scope. The Burali-Forti argument shows that the plurality of all ordinal numbers does *not* form a set. So, for Cantor, the infinity of all ordinals cannot be a "tangible, comprehensible idea," and is therefore not subject to a reflection principle.[9] This is somewhat puzzling, though, since the definition of the concept of ordinal seems quite perspicuous.

25.8 Reflection in Set Theory

Cantor has been credited with being the first to make use of a set-theoretic reflection principle.[10] Actually, the philosophical motivation of modern set-theoretic reflection principles is closer to Philo's reasoning than to Cantor's reasoning about the Absolute. Set-theoretic reflection principles center around the concept of *indiscernibility*. They somehow express that the set-theoretic universe V is indistinguishable from certain proper parts of V. It is of course not completely clear which notion of indiscernibility Philo had in mind: perceptual indiscernibility, epistemic indistinguishability in general, semantic indiscernibility... In modern set theory, the focus is firmly on a form of *semantic* indiscernibility. Like in Philo's reflection from

God to certain angels, no distinction is made in modern set-theoretic reflection between “clear” and “unclear” or “indefinite” infinities.

The general idea between set-theoretic reflection principles is the following: a mathematical statement or collection of statements is true of the universe V as a whole if and only if it is true of certain parts of V when all quantifiers are restricted to these parts of V. The notion of reflection in this form “is probably the most universally accepted rule of thumb in higher set theory” (→Maddy, 1988, p. 503).

This train of thought finds a clear expression in what is called the principle of *Montague-Levy reflection*. If we denote the α-th rank of the set-theoretic universe V as V_α, and the relativization of the quantifiers of a formula ϕ to V_α as $\phi^{V\alpha}$, then this schematic principle can be expressed as follows:

$$\frac{}{\phi \rightarrow \exists \alpha\colon \phi^{V\alpha}} \quad \text{(ML)}$$

Montague and Levy showed that this principle is provable in standard set theory (→Levy, 1960; →Montague, 1961):

Theorem 1. $ZFC \vdash \phi \rightarrow \exists \alpha\colon \phi^{V\alpha}$.

Nonetheless, Montague-Levy reflection has hidden strength. Over ZFC *minus* the axiom of infinity and the axiom of replacement (call this theory ZC^-), ML is equivalent to the remainder of the axioms of ZFC:

Theorem 2. $ZC^- \vdash ML \leftrightarrow Infinity + Replacement.$[11]

This shows that already ML is much more powerful than Cantorian reflection: a minimal “standard” model of ZC^- is V_ω, whereas a minimal standard model of ZFC is V_κ, where κ is the smallest strongly inaccessible cardinal.

Second-order (i.e., class-theoretic) analogues of ML can straightforwardly be formulated. Unlike Montague-Levy reflection, they are independent from ZFC, and are seen as plausible axioms of infinity. They are classed among the so-called “small large cardinal principles.”

Principles of infinity that are independent of ZFC are called *large cardinal axioms*.[12] Second-order versions of ML are among the most modest such principles. Another modest large cardinal principle is the Axiom of Inaccessible Cardinals, which can be formulated as follows (→Kanamori, 1994, Theorem 1.3):

Axiom 1 (IC). *There is a cardinal κ such that* $(V_K, \in) \vDash$ *ZFC*.

This Axiom is significantly stronger than ML in the following sense: rather than postulating that every true sentence is reflected in some rank (possibly different ranks for different true sentences), IC claims that there is *one single* rank in which infinitely many true sentences (all theorems of ZFC) are reflected.

The vast majority of large cardinal principles can be expressed as *embedding principles*, which can be described as follows. An *inner model M* of *V* is a model of ZFC that contains all ordinals. The following is a prototypical embedding axiom:

Axiom 2 (MC). *There is a transitive inner model M and a nontrivial bijective class function j such that for all formulas* $\Phi(x_1,\ldots,x_n) \in \mathrm{L}_{ZFC}$*, we have:*

$V \vDash \Phi(x_1,\ldots,x_n) \Leftrightarrow M \vDash \Phi(j(x_1),\ldots,j(x_n))$.

The first ordinal *κ* that is moved by *j*, which is called the *critical point* of *j*, is a cardinal number: it is called a *Measurable Cardinal*. Measurable cardinals are beyond what can be proved to exist by reflection principles that are variations on ML. Much larger large cardinals even than measurable cardinals can be postulated to exist by playing around with the conditions on *j* and by varying the notion of elementarity in Axiom MC.

Many set theorists take all embedding principles to be reflection principles. The reasoning behind this is straightforward. Inner models are proper *parts* of V. So MC postulates that V is ontologically reflected into itself as a part M of itself.

Since it contains all ordinals, M is a proper class. Therefore embedding principles do not postulate that V is reflected in a *relatively small* part of itself. The strength of embedding principles tends to be positively correlated to the degree of similarity of M to V itself.

The fact that embedding principles embed V into a large part of itself may be taken as a reason not to take embedding principles to be reflection principles in a strong sense of the word. But this defect of embedding principles can be overcome. Philip Welch has proposed the following large cardinal principle, which is called the set-theoretic *global reflection principle* (→Welch and Horsten, 2016):

Axiom 3 (GRP). *There is an initial segment of the universe* V_κ*, and a nontrivial elementary embedding*

$j: (V_\kappa, \in, V_{\kappa+1}) \rightarrow {}_e(V, \in, C)$

with critical point κ, and where e is, as in MC above, *an elementary equivalence relation.*

If in this Axiom *e* is taken to be *second-order* elementary equivalence, then GRP is as strong as the embedding principle that is called the Axiom of *extendible cardinals*. It is worth noting at this stage that *j* plays an essential role in embedding principles such as MC and GRP: merely postulating elementary equivalence between V and an inner model (or a set) does not yield significant large cardinal strength.

The Axiom GRP postulates that V, together with all its proper classes, is reflected in a *set-sized*, and therefore "small" part (V_κ)

of the universe, together with all of its sub-sets ($V_{\kappa+1}$). Thus GRP is a reflection principle in a strong sense of the word.

Most large cardinal axioms can naturally be and usefully are formulated as elementary embedding axioms. Since elementary embedding principles are class theoretic statements, this is an instance of the usefulness of second-order principles in set theory. By imposing additional conditions on *j* and on M, even stronger embedding principles are obtained. In particular, in stronger embedding principles, M looks more and more like V itself. Reinhardt observed that natural ultimate limit of this process therefore is to postulate a non-trivial embedding from V into *itself:*

Axiom 4 (R). *There is a non-trivial elementary embedding from V into V.*

But Axiom (R) was found to be incompatible with ZFC (→Kunen, 1971):

Theorem 3. *ZFC* $\vdash$ *"There is no non-trivial elementary embedding from V into V."*

On the one hand, this phenomenon may be viewed as a *bad company problem* for set-theoretic embedding principles in general. The fact that a natural strong embedding principle is inconsistent might give us pause: it might lead one to doubt that certain restrictions of it are sound, or even consistent.

On the other hand, it has been observed that the proof of Theorem 3 makes *essential* use of the Axiom of Choice. Indeed, a rich structure theory of "choiceless cardinals" is currently being developed in the context of ZF (*without* the Axiom of Choice), where a choiceless cardinal is a cardinal that cannot exist if the Axiom of Choice is true. In particular, various strengthenings of Axiom R have been proposed, and in the absence of the Axiom of Choice, they appear (so far!) to be consistent. This gives the appearance of there being a whole *realm* of cardinals beyond the "choicy" cardinals. Some interpret this as indicating that the

Axiom of Choice holds only up to a high level of the rank hierarchy, and that it has only has restricted validity beyond that.

25.9 Mathematical Warrant

It took a remarkably long time before it became clear that ontological reflection is entangled with quantitative infinity in the modern sense of the word. Since quantitative infinity lies within the compass of mathematics, ontological reflection has become of mathematical relevance. In particular, it has become of central importance in set theory, which can be seen as the mathematical investigation of quantitative infinity.

On the one hand, reflection phenomena have become part of set-theoretic experience. Montague-Levy reflection is only a tiny aspect of this: it is no exaggeration to say that the working set theorist encounters ontological reflection phenomena on an almost daily basis. On the other hand, strong reflection principles are *postulated* in set theory. This is not to say that strong reflection principles are universally or even very widely accepted as true in the set-theoretic community.[13] But they are highly trusted, in the sense that most set theorists regard it as unlikely that contradictions can be derived even from fairly strong set-theoretic reflection principles.

There is a temptation to explain this trust of the set-theoretic community in reflection principles by appealing to philosophical arguments for their truth. The thought is that the truth of set-theoretic reflection principles follows from philosophico-theological arguments of the kinds that we have reviewed, *except* that in the arguments the concept of God is uniformly replaced by the concept of the set-theoretic universe (V).

The epistemic warrant for reflection principles is taken to be similar in nature to our epistemic warrant for basic principles of set theory in general. The basic principles of set theory have to

be justified. But, since from a mathematical point of view the axioms of set theory are basic, their justification will have to involve non-mathematical notions. The justification of basic principles of set theory would thus be *partly* non-mathematical in nature. Let us call this the *justificatory account*.

The justification attempt for the basic axioms of set theory that has enjoyed most widespread support in the philosophy of mathematics is the *iterative conception of sets*. On this conception, the sets are generated in (cumulative) stages. At every successor stage $\alpha + 1$, for every plurality P of sets that exists at stage α, a *set* of the P's is generated. At any limit stage, the union is taken of all the sets that have been generated at earlier stages.[14] It is widely held that most axioms of ZFC can thus be justified.

The iterative conception is taken to be in tension with the attitude of taking the existence of choiceless cardinals seriously. Most philosophers believe that the Axiom of Choice can be justified on the basis of the iterative conception of set, along the following lines:[15]

> Suppose some family F of mutually disjoint non-empty sets exists at some stage α. Then there is a stage $\beta < \alpha$ such that all sets $a \in$ F (and *their* elements) exist at stage β. Then also at stage β some x-es exist such that, for each $a \in$ F, exactly one element of a is one of them (and nothing else is one of them). At stage $\beta + 1 \leq \alpha$, these x-es form a choice set C for F.

If this is true, then choiceless cardinals cannot exist in the same sense as the familiar choicy ones do. If we take the Axiom of Choice as a non-negotiable principle of set theory, then much of the current research into choiceless cardinals must be reinterpreted as being about inner models of ZF that *think* (1) that the Axiom of Choice does not hold unrestrictedly and (2) that there are choiceless cardinals.

However, it is not clear that what I have called the justificatory account is faithful to set-theoretic practice. Most working set theorists—I claim—have never even heard of the iterative conception of set. After all, it belongs to philosophy (of mathematics), not to mathematics. At the same time, a growing number of set theorists behave as if choiceless cardinals are "just there" in V—but very high up the rank hierarchy. Indeed, because of this tension with set-theoretic practice, the justificatory account of basic set-theoretic principles seems to me a fundamentally wrong picture of mathematical epistemology, as I shall now all too briefly argue.

As mentioned above, if the justificatory account is true, then our fundamental warrant for basic set-theoretic principles—including those reflection principles that qualify as such—is partly non-mathematical in nature. The iterative conception, for instance, makes use of the notion of *generation in stages*, which is a non-mathematical concept. But, as Maddy[16] and others have emphasized, mathematics in general, and set theory in particular, is not in need of extramathematical epistemic support. All that matters for set theory to be in good epistemic standing, is that the beliefs and practices of set theorists are rational responses to the set-theoretic challenges that they are faced with.

Proof certainly plays a central role in set-theoretic practice, and proof is a rational road to belief. But adopting certain principles as basic can also be a rational act. In particular, set-theoretic reflection principles play a central *organizing role* in our set-theoretic practice, and this is one factor that explains why some of them are candidates for being basic axioms. Set theorists do not have to take this as their *explicit reason* for being (mathematically!?) warranted in adopting certain reflection principles as basic beliefs; nor do they have to be able to produce an *Inference to the Best Explanation* argument in order to

support their belief in certain reflection principles. (After all, “best explanation” is also a non-mathematical notion.) All that matters, is whether their adoption of beliefs is a rational epistemic response: sometimes this is a matter of reasons (when a set theorist proves a theorem); at other times, it is a matter of *mathematical entitlement* that is not a matter of articulated motivating reasons.

25.10 Philosophical Warrant

What, then, becomes of the philosophical arguments that have been adduced to support the adoption of mathematical principles? Some of them may, for all that I have said in the previous section, still constitute *good* reasons for believing certain mathematical principles. *If* they are good reasons, then everyone—philosophers and mathematicians alike—can make epistemic use of them. In this sense, philosophical reflection arguments would then still be epistemically valuable. It is just that set theory, in its current state of development, is in good epistemic standing even if it does not mention them.

The root of philosophical arguments for reflection principles lies in the thesis of the unknowability (and existence) of God. This thesis is supported by the Bible, and has long been widely accepted in Western philosophy. Augustine argued that the mathematical world forms a part of the mind of God, and on the basis of this made an unknowability claim concerning the mathematical universe. Currently, the thesis of the unknowability of the mathematical universe as a whole enjoys wide support even in the absence of the identification of the mathematical universe with a part of the mind of God.

The thesis of the unknowability of God and the thesis of the unknowability of the mathematical part of the mind of God are negative theses. Philo saw that such negative claims can be

given a positive interpretation: it gives rise to *indistinguishability* claims (→Welch and Horsten, 2016, pp. 95–96). If we identify the mathematical Absolutely Infinite with the set-theoretic universe V, then V is unknowable in the sense that we cannot single it out or pin it down by means of any of our assertions: no true assertion about V can be made that excludes other unintended interpretations that make the assertion true. In particular—and this is stronger than the previous sentence—no assertion that we make about V can ensure that we are talking about the mathematical universe rather than an object in this universe. So if we do make a true assertion ϕ about V, then there exist sets s such that ϕ is also true when it is interpreted over s. Moreover, this should hold not only for individual claims about V, but also about infinite collections of claims about V. Gödel held the view that all sound large cardinal principles are somehow reducible to reflection principles (→Welch and Horsten, 2016, Section 8.7.9).

The paradox that Philo already identified remains: how can the indistinguishability claim even unambiguously be *stated* if no expression in our language singles out God (or V)? In modern set theory, this is dealt with by restricting the language for which the indistinguishability claim is supposed to hold: the official language of set theory does not contain an individual expression that refers to the universe as a whole.

If philosophical reflection arguments have justifying force, then *which* reflection principles do they justify? The philosophical reflection principles that we have considered aim at supporting statements that express ontological reflection of an absolutely infinite realm into a humanly intelligible entity. For set theory, the humanly intelligible objects are the sets. Therefore, Montague-Levy reflection captures the conclusion of the philosophical reflection arguments that we have discussed in a very weak sense, and variants of the Global Reflection Principle captures it in a much stronger sense. Standard embedding

formulations of large cardinal axioms posit the existence of an embedding of V into an inner model. Inner models contain all the ordinals, and are therefore absolutely infinite. It is thus not at all clear to what extent inner models can be considered as humanly intelligible objects in the same sense as sets are. Hence it is not clear that standard embedding axioms can be directly justified by philosophical reflection arguments of the kind that we have considered. In the light of this, let me therefore end this article with a:

Question. *Can certain embedding principles that are (outright or in consistency strength) significantly stronger than embedding principles that posit extendible cardinals be formulated as or approximated by principles that state that certain rank initial segments of V are significantly indistinguishable from V as a whole?*

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Notes

1 Gregory of Nyssa is an important figure in this evolution: see →Achtner, 2011, Section 1.4.2.

2 See →Drozdek, 2019.

3 See →Menn, 2012.

4 See →Wolfson, 1947, Chapter IV, Section IV.

5 There may be a connection with a mystic idea according to which in the highest form of thought, the duality of subject and object of thought somehow dissolves.

6 This follows from →Dedekind, 1888, p. 72.

7 As Dedekind himself observed, a structurally similar "proof" of the existence of an order-infinite collection was given earlier by Bolzano. For a discussion of Bolzano's argument, see →Tapp, 2019.

8 This process of emancipation began in the fourteenth century: see →Biard and Celeyrette, 2005, Introduction.

9 In his later work, Cantor calls the plurality of all ordinals an *inconsistent multiplicity*.

10 See →Hallett, 1984, pp. 116–118.

11 The Axiom of Choice play no role in the proof of this equivalence.

12 For an introduction to the theory of large cardinals, see →Kanamori, 1994, Chapter 1.

13 On this issue I wish to remain neutral.

14 A limit stage λ is thus the first stage at which all the sets that have been generated previously exist.

15 See, for instance, →Schoenfield, 1977, p. 335. Boolos is an exception: he defends the iterative conception, but argues that the standard argument for the Axiom of Choice from the iterative conception of sets is *circular* (→Boolos, 1971).

16 See →Maddy, 2009.

26 Potential Infinity, Paradox, and the Mind of God: Historical Survey

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Abstract: Aristotle argued that paradoxes of the infinite can be avoided only by insisting that all infinities are potential, not actual. There is a long tradition of thinking that a Judeo-Christian God would collapse potential infinities to actual ones, thus removing the Aristotelian guard-rail against paradox. After all, does not God know all numbers, regardless of whether they are actual or merely potential? We analyze the Aristotelian guard-rail of potentiality, as well as challenges to it due to Augustine, Burley, Scotus, and Cantor. At the heart of the debate we find the metaphysical question of how the temporal or modal "dimension" that harbors potential infinities compares to ordinary spatial dimensions. Certain theological assumptions underwrite a strong analogy. The best Aristotelian response, we suggest, is to reject the analogy by refusing to analyze modality extensionally as a point-space of possible worlds. Our analysis thus reveals lessons for atheists as well as theists.

26.1 Potentiality as a Guard-Rail against Paradox

Beginning with Aristotle, and continuing well into the nineteenth century, major philosophers and mathematicians rejected the notion of the actual infinite. They argued that the only sensible notion is that of potential infinity—at least for scientific or, later,

non-theological purposes. For example, in *Physics* 3.6 (206a27–29), Aristotle writes, “In general, the infinite is in virtue of one thing’s constantly being taken after another—each taken thing is finite, but it is always one followed by another” (→Aristotle, 1983, p. 14).

Along with ancient, medieval, and early modern mathematicians, Aristotle recognized the existence of certain *procedures* that can be iterated indefinitely, without limit. Examples are the bisection and the extension of line segments. Ancient mathematicians made brilliant use of such procedures. For example, the method of exhaustion was employed to calculate the areas of curved figures in terms of rectilinear ones. However, Aristotle rejected the idea of applying these procedures infinitely many times, as well as the purported end results of such applications, namely self-standing points, infinitely long lines, infinite sequences, and infinite sets.

Aristotle’s discussion in *On Generation and Corruption* provides an excellent illustration.

> For, since no point is contiguous to another point, magnitudes are divisible through and through in one sense, and yet not in another. When [...] it is admitted that a magnitude is divisible through and through, it is thought that there is a point not only anywhere, but also everywhere in it: hence it follows that the magnitude must be divided away into nothing. For there is a point everywhere within it, so that it consists either of contacts or of points. But it is only in one sense that the magnitude is divisible through and through, viz. in so far as there is one point anywhere within in and all its points are everywhere within it if you take them singly. (→Aristotle, 1984, 317a3–8)

It is true that a magnitude can be divided anywhere. But Aristotle rejects the idea that a magnitude can be divided everywhere, all at once. If this were possible, the magnitude would—absurdly—be “divided away into nothing.”

Closely related to the notion of potential infinity is that of potential *existence*.[1] For Aristotle, the points interior to a line segment only exist potentially—that is, it is possible to divide the line anywhere, thus bringing into existence a new point. But if the line is not broken at a given place, then the point only exists potentially.[2] And no matter how many times one divides a line, there will still be some of the line left. As Jonathan Lear put it, Aristotle's thesis is "that the structure of the magnitude is such that any division will have to be only a partial realization of its infinite divisibility: there will have to be possible divisions that remain unactualized" (→Lear, 1980, p. 194). The same goes for the parts of the line segment themselves. As a continuous magnitude, the line segment is a unity; its parts exist only potentially.

It is natural to explicate Aristotle's notion of potentiality in terms of modal logic.[3] Consider Aristotle's view, against the ancient atomists, that matter is infinitely divisible (see, for example, →Miller (2014)). However many times one has divided a given stick, it is always possible to divide it again:

1. $\forall x(Pxs \rightarrow \Diamond \exists y\, Pyx)$,

where *Pxy* means that *x* is a *proper* part of *y*. If the divisions of the stick formed an actual infinity, the following would hold:

1. $\forall x(Pxs \rightarrow \exists y\, Pyx)$

According to Aristotle, however, it is impossible to complete infinitely many divisions of the stick, that is:

1. $\neg\Diamond\forall x(Pxs \rightarrow \exists y\, Pyx)$

By endorsing both (1) and (3), one is asserting that the divisions of the stick are merely *potentially infinite*.

This brings us to the first of our two central themes. Aristotle's appeal to potentiality is intended to serve as a guard-rail against paradox. Consider the procedure of repeated bisection of some continuous magnitude. It is necessarily possible to continue the procedure. However, if the procedure could be completed, paradox would follow, as the magnitude would be "divided away into nothing." We must therefore deny that the procedure can be completed. By instead invoking potentiality, we ensure that the infinitely many objects producible by the procedure are not all present at a single moment of time or possible world (for short, that they do not "co-exist"). The infinity is instead absorbed into a capacity (*dunamis* = potentiality) that can be partly articulated across time or possibility, but that cannot be completely brought to fulfillment (*energeia* or *entelecheia* = actuality) in either.

This Aristotelian guard-rail has some attractive features. It promises to avoid arbitrary restrictions on our ability to apply the "extending operation" (as we will call it) that is being iterated, while simultaneously blocking paradox by ensuring that there is no single point with an infinite domain. As we will see, the insistence on potentiality is meant to guard against other paradoxes as well, such as Zeno's.

Our second central theme of this article is to explore how well this conception of potential infinity stands up when a Judeo-Christian deity enters the picture. The medieval Christian philosopher Walter Burley (Burley, 1972 [1501]) made an observation that questions the coherence, or at least the force and importance, of the distinction between potential and actual infinity:

> [W]hen it is said that God now sees all the points there are in a given line [...] I grant it. And I say that God [...] sees some intermediaries between

> all those points, taking the “all” collectively. (f. 176vb, translated in → Normore, 1981, pp. 259-260n 4)[4]

The basic idea is clear. Since God sees everything, it does not help to insist that the infinity is “spread out” temporally or modally. Such a distributed infinity may well be hidden from the human point of view, but not from God’s. Thus, there is a strong theological push towards

Completablity: There can be a completed totality of everything generable by the procedure of iterated application of some extending operation.

Even if *we* do not have access to this Completability, God does. Simultaneously, since God is omnipotent, He can always apply the extending operation (such as bisecting a magnitude) that is being iterated:

Extendability: The extending operation can always be applied to any available object(s); there can be no arbitrary prohibition against its application.

However, if we accept both Completability and Extendability, we obtain completed infinite totalities, which Aristotle and many of his followers take to be paradoxical.

In fact, the existence of God only aggravates a challenge that arises anyway, for theists and atheists alike. The problem is that both the theses of Completability and Extendability are *prima facie* plausible. Consider Completability. Assuming a sufficiently strong form of realism, it is immaterial whether *we* can see the totality; what matters is whether it is there. The existence of God would further strengthen the case for Completability. For one thing, God would serve as a guarantor of realism, reminding us that what matters isn’t whether we can see the totality, but whether it is there—for God to see.[5] Next, consider Extendability. Because of His omnipotence, God would guarantee the unrestricted applicability of the extending operation. But there

are non-theological reasons as well to take the operation to be unrestricted. This becomes particularly clear in more recent mathematics, which abandons the Aristotelian view that mathematical operations have to be physically realized. For example, for there to exist bisects of some line segment, there is no need for us literally to produce the bisects; it suffices that there is legitimate mathematical definition of them.

Our two opposing themes—the Aristotelian guard-rail of potentiality and the theologically motivated doubts about its efficacy—raise some important questions. How effective is the Aristotelian guard-rail of potentiality, if we assume the existence of God or a sufficiently strong form of realism? When grappling with the paradoxes of infinity, does it really help to absorb the infinities into incompletable capacities or procedures? Does not this just "spread out" the infinities temporally or modally—which would anyway not escape God's direct and immediate comprehension? Aristotle believes it helps, while many other philosophers, both religious and not, disagree. We propose to undertake a historically informed study of the two opposing themes.

Two main lessons emerge. First, our study reveals how a lot of prominent philosophers and mathematicians down the centuries have struggled with the tension between Completability and Extendability, often in connection with theological concerns. Second, at the heart of this dispute is the question of how to understand the metaphor of potential infinities being "spread out" temporally or modally. How does the "dimension" across which a potential infinity is distributed compare with the ordinary spatial dimensions? The more analogous this temporal or modal "dimension" is to the ordinary spatial ones, the less effective will be Aristotelian guard-rail. This becomes especially clear if there is a God whose ability to comprehend the infinity is unaffected by its being "spread out."

On the other hand, if there is some fundamental difference between the "dimension" in which a potential infinity "grows" and the ordinary spatial dimensions, then the guard-rail of potentiality looks more promising.

26.2 Aristotle and the Potentially Infinite

Aristotle's account of the infinite as potential and not actual appears almost designed to answer Zeno's bisection paradox. Recall the puzzle: for someone, say, Atalanta, to traverse a distance of one mile, she must first complete half the distance (1/2 mile), then half of what remains (1/4 mile), and then half of what remains of the remainder (1/8 mile), and so on *ad infinitum*. To complete the journey Atalanta would thus have to exhaust a series of half-distances infinite in number, which is impossible, since an infinite series never ends.

Aristotle's reply focuses first on the nature of continuous quantities:

> In what is continuous are contained infinite [plural: infinitely many] halves not actually but potentially. (263a28, our translation)

The sense in which the mile contains infinitely many halves—in potentiality—is that for any given number of divisions into half-distances, it is always possible to make a further division. "The bisections of the magnitude are infinite," Aristotle writes, "hence [this infinite] is potential but not actual: the parts that can be taken always surpass a definite amount" (see 207b10–13, translated in →Aristotle, 1983, p.17, slightly modified by us). The mile does not contain infinitely many actual halves, however, but only ever finitely many, which Atalanta can readily traverse.

Aristotle's proposal is that it is a mistake about the "essence and being" of continuous quantities (263b6–9, translated in

→Aristotle, 1984, p. 441) to suppose that the infinitely many parts into which they are divisible are actual parts. A continuous quantity is prior to its parts and is not composed of or completely resolvable into them. The parts come into being, come to be actual, only by an act of division and are themselves always continuous quantities of the same kind. Zeno's bisection paradox is thus seen to involve a misconception about the nature of the continuum. It supposes that if Atalanta is to reach the end of the mile, she must traverse an actual infinity of half-distances. But she does not, for the infinity of half-distances in a continuum is only potential. Once the mistake is removed, so too is the paradox.

Even so, a fundamental question remains. Is the idea of an actual infinity itself incoherent? A second layer of Aristotle's analysis of the bisection paradox provides some illumination. Suppose Atalanta were to count out all the half-distances as she journeyed across the mile. Aristotle comments:

> If you count the first half of the journey and then the half of what is left and so on, when you get to the end of the journey you would have counted an infinite number; which is admitted to be impossible. (263a 7–11)[6]

Why is it impossible? Aristotle is not explicit. One reason may be that to have counted the whole infinity of half-distances requires the existence of an infinite *number*, which would be incoherent by the standards of the day.[7] Quite apart from the question of infinite number, however, Aristotle has a separate reason for the impossibility: no multitude counted out can be both *infinite* and *complete*. Consider first his striking definition of the infinite in *Physics* III.6:[8]

> Something is infinite if, taking it quantity by quantity, we can always take something outside. (→Aristotle, 1984, 207a6–7).

Aristotle then directly goes on to say that for something to be "whole" and "complete" it must have no part outside (206a7–8).[9] Those premises suggest a simple conceptual argument. If the series of half-distances is infinite, then taking it quantity by quantity, as in counting, we can *always* take some part outside what has been taken. Hence however many parts have been "taken," or counted, they cannot be the *complete* infinite series. There is no "completing" an infinite series of actual half-distances one by one as imagined in Zeno's bisection; that is incoherent on its own terms and "admitted to be impossible."

Still, for the argument to rule out actual infinite multitudes in general, a further premise is required, namely, that for a multitude to be complete, it must be possible for all its elements to be "taken quantity by quantity" or counted. Here, however, a defender of the actual infinite might propose to sever any tie between completeness and countability, and introduce a distinction between "countable" and "uncountable" yet complete multitudes.[10] Perhaps the stars in the heavens, for example, are in principle uncountable but nonetheless complete. The natural reply to this maneuver is to ask whether the distinction is genuinely intelligible. In the absence of any means by which to pin down or "circumscribe" all the elements of a purported infinity (say, by counting them all out), at least in principle, how could the infinity legitimately be regarded as complete?[11] Sheer allegiance to the actual infinite by itself is not a justification. We need not resolve the matter here; it is enough to see with tolerable clarity where the issues will lie.

Let us return to the bisection paradox. Aristotle notes:

> To the question whether it is possible to pass through infinite[ly many] units either of time or of distance we must reply that in a sense it is and in a sense it is not. If the units are actual, it is not possible; if they are potential, it is possible. (→Aristotle, 1984, 263b5–9, with modification).

An infinity of units of time or distance traversed in motion cannot have been completed one by one, so they must be potential not actual. The same holds for number itself, which does not exist separately from what is numbered: "its infinity does not persist but consists in a process of coming to be" (207b14–15, translated in →Aristotle, 1984, p. 353). The infinity of numbers does not exist as a completed multitude; it is rather an inexhaustible capacity by which further numbers can always be generated.

Aristotle's distinction between potential and actual infinity is not just a device to rescue the possibility or intelligibility of motion from Zeno's bisection paradox. It is the great first guard-rail established against the difficulties of the "completed infinite." Mathematics drives us to embrace the infinite in some sense: the endless succession of number, the infinitely extensible or divisible line, etc. By his lights, in allowing only the potential infinite Aristotle provides both for safety and for mathematics. Commenting on the case of the infinite extension of lines he says:

> Our account does not rob the mathematicians of their study, by disproving the actual existence of the infinite in the direction of increase, in the sense of the untraversable. In point of fact they do not need the <actual> infinite and do not use it. They postulate only that the finite straight line may be produced as far as they wish. (207b27–30; transl. →Hintikka, 1996, p. 201)

Even so, Aristotle can seem of two minds. On the one hand, he denies that an infinite is ever actual (206b13–14). On the other, he holds that there is no beginning of time or motion (205b11–

28); the revolution of the heavenly bodies, for example, is eternal. Those commitments appear in tension. Doesn't a beginningless cycle of day and night imply than an actual infinity of days has passed?

The tension is mitigated, however, by Aristotle's distinction between potential and actual existence. This applies, he says, to things that exist over time such as the Olympic Games:

> For of these things too the distinction between potential and actual existence holds. We say that there are Olympic Games, both in the sense that they may occur and that they are actually occurring. (→Aristotle, 1984, 206a23–25, p. 351)

And likewise for a day or the infinite:

> The infinite is actual in the sense in which a day or the games are said to be actual. (206b13–14; →Hintikka, 1996, p. 199)

Like a day or the Olympic Games, the infinite is actual not all at once—with all its infinitude co-present and completed—but rather exists over time. That is by contrast with the existence of substantial things:

> We must not regard the infinite as a "this" (*tode ti*), like a man or a house, but must suppose it to exist in the sense in which a day or the games are said to exist—things whose being has not come to them like that of a particular substance (*ousia tis*), but consists in a process of coming to be and passing away. (206a29–33, transl. Hintikka 1996, p. 206)[12]

A person or a house exists as a whole, complete, at all at once. The infinite, like the day or the Games, exists in succession. Yet the infinite also differs in a further respect: not only is it not complete all at once, it is by its nature *incompleteable*, and one "can always take something outside." Thus, on the view, the

infinite consists in an inexhaustible process of extending the finite; as Aristotle writes in summary:

> In general, the infinite is in virtue of one thing's constantly being taken after another—each taken thing is finite, but it is always one followed by another. (→Aristotle, 1983, 206a27–29, p. 14)

One might be tempted to say that for Aristotle the infinite exists as distributed or "spread out" across time. But the metaphor is not quite right. For Aristotle, time is the "number" or "measure" of motion (220a35, 200b33), an aspect of change, not a domain of reality that extends from the present backward into the past and forward into the future.[13] It is not like a spatial magnitude in which all the given elements co-exist; rather, time exists in succession by the passing away of the components (206b1–3). So even if it is true that the heavenly bodies revolve eternally, time provides no repository in which to accumulate an actual infinity of days. This infinity too can be actual or exist only like the Olympic Games, in succession, by coming to be and passing away. Unlike the games, of course, there was no initial cycle inaugurated by Heracles, and each day is *always* followed by another.

Aristotle's account relies on carefully upholding the distinction between potential and actual infinite. The first must never collapse into the second on pain of Zeno's bisection paradox and the difficulties of the completed infinite. For continuous quantities like the mile, the idea of such a collapse is a misconception about the nature of the continuum. For discrete quantities that exist successively like days, since time is not a repository in which an actual infinity can accumulate, the idea of a collapse is a misconception about the nature of time. The capacities of the continuum and time are incompleteable by

nature, for Aristotle, so there is no prospect of reducing them to completed actualities.

Yet for discrete quantities whose existence is not potential and successive but actual and complete at any time, like people or houses or stars, we cannot appeal to potentiality to explain why there cannot be an actual infinity of them. The prohibition of the actual infinite in such cases appears instead to rest on the line of argument against the completed infinite discerned in *Physics* III.6. That argument is not obviously conclusive. The completed infinite is paradoxical in the literal sense of being "apart from belief"—Aristotle's counted bisection brings this out nicely—and were it granted to be possible, many further paradoxes would also spring forth.[14] But whether it is *impossible* is a further question.

26.3 The Threat of Collapse in Judeo-Christian Theology

Aristotle's doctrine of the potential infinite was predominant in philosophy and mathematics until the rise of infinitary methods in mathematics in the seventeenth century began to turn the tables. Well before that, however, arguments for a collapse of the potential into the actual put pressure on Aristotle's careful restraints on the infinite as only potential. In their historical forms, these arguments emerge in the context of Judeo-Christian theology: the mind of God may breach the guard-rail. As God's comprehension delimits or puts definition on the elements that can arise from the potential infinite, it threatens to exhaust the related capacity after all, resolving the potential to the actual. Further, the conception of God as transcending time and holding an eternal standpoint invites a view of existing in succession through time as on par with existing completely at a

time—in effect, spatializing it—and so also seems to obviate the distinction between the actual and the potential.

As noted at the outset, God seems to guarantee the principle of Completability, the idea that there is, after all, a completed totality of everything generable by some extending operation. Here in Section 26.3 we consider a few standout historical case studies, reviewing arguments suggested in the writings of Augustine, Burley, and Duns Scotus that seem to speak against the Aristotelian view of the infinite as only potential. Each in his own way treats the infinite not as absorbed in a capacity but as a completed multitude in some way "spread out" before God's comprehension: in the divine intellect, at least, for Augustine, and also more literally in space and in time, respectively, for Burley and Duns Scotus. We consider them as interesting exemplars of this line of response to Aristotle's model. The philosophical considerations retain their interest even outside of the theological context, of course, since it is the "realism" implicit in those discussions that drives the arguments, and which translates so neatly into contemporary discussions of the mathematical domain.

Once the infinite is admitted as actual, and so Aristotle's guard-rail is removed, profound difficulties arise concerning the union bestowed upon the infinite from the point of view of an unlimited comprehension. Uniting all of a completed infinity of things in a single thought yields mathematical paradoxes, including those of the most fundamental character. In Section 26.4, we shall consider a broad alternative strategy to answering the paradoxes proposed in similar fashion by Leibniz and by Cantor: in effect, to allow for Completability, *pace* Aristotle, but restrict the extending operations and so instead rescind the principle of Extendability. This becomes their own distinctive guard-rail, intended to allow the completed infinite in mathematics while preserving mathematics from contradiction.

We shall see, however, that it comes at a notable philosophical cost.

26.3.1 Augustine and God's Comprehension of Number

Suppose it is held that although God has in his own mind "finite conceptions" of all the finite things he makes (*City of God*, XII.18, and →Augustine, 1955, p. 539), God's knowledge cannot comprehend an infinite number of things (*City of God*, XII.19). Against this Augustine asks: Does not God know all the numbers? He argues that the numbers (viz., the natural numbers) are infinite—in the precise sense that for any proposed final number, arithmetical operations can by reference to that number identify others surpassing it—and that each number is individuated from all others by its arithmetical properties:

> For it is absolutely certain that these are infinite; since, no matter at what number you suppose an end to be made, this number can be increased. And I do not say that this is done simply by adding one to it. Rather, however great the number may be, and however enormous the multitude which it expresses, it still can be not only doubled, but even multiplied, according the principle and science of number. Moreover, each number is so defined [*terminatur*] by its own properties, that no two numbers can be equal to any other. They are therefore both unequal and different from one another; and while individually [*singuli*] they are finite, collectively [*omnes*] they are infinite. Does God not know numbers, then, on account of this infinity; and does God's knowledge extend only to a certain number of numbers, while he is ignorant of the rest? (*City of God*, XII.19; →Augustine, 1955, pp. 541–542; and →Augustine, 1998, p. 326, slightly modified)

To mortals whose knowledge extends only to a certain height in the numbers, the infinity of numbers might be incomprehensible. But God could not fail to know each and every number.

To this Augustine adds a further line of thought, based on the encompassing nature of God's understanding, that exposes a basic threat to the Aristotelian view of the infinite as only potential. By coming under divine comprehension, the infinity of numbers is "made finite" to God:

> For though there is no number of infinitely many numbers [*infinitorum numerorum*], the infinity of number [*infinitas numeri*] is still not incomprehensible by Him Whose understanding is without number [*non est numerus*]. And so, if everything which is comprehended by knowledge is made finite [*finitur*] by the comprehension of the knower, then all infinity is in some ineffable way made finite to God, for it is not incomprehensible to His knowledge. (*City of God*, XII.19; →Augustine, 1955, p. 542; and →Augustine, 1998, p. 527, slightly modified)

The infinity of all numbers is "made finite" in the sense of their being collectively bounded and individually defined in God's comprehension. It seems to be a presupposition of their being known to God that the numbers are not just always further definable; God's comprehension, as it were, pins down every single number and comprehends the series as complete.

Despite its being "ineffable" how the infinite is made finite to God, Augustine offers a few clues about God's cognition of the infinite. In XII.18, Augustine describes the divine mind as "absolutely unchangeable, infinitely capacious, and without succession of thought, counting all innumerable things [*innumera omnia … numerantem*]" (→Augustine, 1955, p. 540). As noted already, in XII.19, Augustine holds "there is no number of infinite numbers" and describes God's understanding correspondingly as *non numerus*, without number. This indicates two things about God's comprehension. First, God does not have to traverse successively through an infinite series of numbers to "count" them all, in effect bypassing the order and its puzzling lack of an end. Secondly, God's comprehension of all the

infinitely many numbers does not yield a number: just as there is no number of infinite numbers, God counts all innumerable things without number. His comprehension takes in all the numbers as collectively infinite in union—in some ineffable way making their series finite to God—but it is not yet a "number-making" act.

In Augustine's philosophy it seems the divine mind induces a collapse of number to a completed infinity. And he admits what Aristotle would not: the possibility of a counted infinite series of numbers, at least for God who counts "without number." The paradoxical aspect of a series being complete yet without an end is downgraded from an impossibility to only a reflection of our own finite understanding; it is ineffable, but not contradictory.

26.3.2 Burley and God's Vision of the Continuum

Walter Burley poses a similar prospect of collapse due to God's comprehension, this time not in number but in the composition of the continuum. Aristotelian orthodoxy holds that continuous magnitudes contain a potential infinity of points—in that a continuous magnitude is always further divisible—but it is a mistake to conceive of "all points" in a given line, for example, as a collection. Breaking with tradition, Burley (1972 [1501]) writes:

> [W]hen it is said that God now sees all the point there are in a given line, for instance in line B, I grant it. And I say that God … sees some intermediates between all those points, taking the 'all' collectively. (f. 176vb)

God's seeing "all" the points in the line invites the idea that for God the points in the line already constitute a completed multitude (and presumably, the same goes for the subintervals of the line, etc.). Burley's clarification "taking the 'all' collectively" brings this into high relief. The points co-exist in a

way that allows them to constitute a collection—indeed, an infinite collection and one that would seem to satisfy the conditions for being an actual or completed infinite: for instance, its infinitude is given all at once, not in succession. The points are actual enough for God, at any rate, who sees them all together, even if they remain only potential for us mortals.

26.3.3 Duns Scotus and God's Simultaneous Knowledge of the Infinite

In *Treatise on God as First Principle*, 4.48, Duns Scotus argues expressly for the collapse of the potential infinite to an actual infinite by virtue of the nature of God's intellect, as follows:

> Things potentially infinite in number (i.e. things which if taken one at a time are endless) become actually infinite if they exist simultaneously. Now what can be known is of such a nature so far as a created intellect is concerned, as is sufficiently clear. Now all that the created intellect knows successively, your intellect knows actually at one and the same time. There, then, the actually infinite is known. (→Duns Scotus, 1966, p. 102)

To buttress the reasoning he continues:

> Consider these potentially infinite things as a whole. If they exist all at once, they are either actually infinite or actually finite. If finite, then if we take one after the other, eventually we shall actually know them all. But if we cannot actually know them all in this way, they will be actually infinite if known simultaneously. (→Duns Scotus, 1966, pp. 102–103)

As is evident this relies on the premise that God's knowledge is (in some way) simultaneous. Note too, though, how Scotus takes it for granted that potentially infinite things can be considered as a whole. Here the distinction between potential and actual infinite seems to be diluted entirely to a difference between existing at one time and existing across times. By "spatializing"

the dimension of increase in this way, Scotus abandons a fundamental role for Aristotle's concept of the potential infinite as existing in succession through "coming to be" and "passing away."

In effect, Scotus adopts an "eternalist" view of time in which things may exist not now but in times apart from the present, yet be equally available for collecting into a whole in thought, at least for God. In this respect, then, the collapse of the potential to the actual is already in place in the analysis even without the further premise that God's knowledge is simultaneous. The domain for whole-making includes the "infinite things" as a completed infinite. God's knowledge now simply provides a single point at which to collect them together to qualify as actual—a standpoint of symmetry among them, from which they are all equally accessible.

Perhaps we should distinguish weak and strong forms of actual and potential infinite. The weak potential infinite is one whose elements cannot all exist at the same time; the strong potential infinite is one whose elements can "come to be" and "pass away" but cannot all exist as a completed multitude. Correspondingly, a weak actual infinite is one whose elements can all exist as a completed multitude, whereas a strong actual infinite is one whose elements can all exist simultaneously and need not necessarily "come to be" or "pass away." What Scotus's argument shows, then, is that given the nature of God's knowledge the weak potential infinite collapses into a weak actual infinite: though the potential infinite's elements cannot exist at a single time, they can all be considered as a whole and known a single time by God, and thus constitute a completed multitude.

From these theological notions—the determinacy and boundedness provided by God's comprehension, the symmetrical accessibility of objects distributed across time to

God’s standpoint—we have collapse of the potential to the actual, and the prospect of a return of the paradoxes of the completed infinite. Without Aristotle’s guard-rail, what protects us now?

26.4 An Alternative Guard-rail: Restricting the Operation

Recall the original problem. The independently plausible principles of Completability and Extendability together give rise to paradox. Aristotle’s guard-rail of potentiality had proposed to save mathematics from contradiction by eliminating the completeability of the infinite—and so, more generally, blocking Completability and eliminating a completed domain for mathematics. Once potential infinity has collapsed, the ground for Completability is reinstated and Aristotle’s guard-rail against paradox is removed. Some other safety measure is needed. A tempting alternative, then, is to restrict the application of the extending operations, thus preventing them from applying unlimitedly across a completed domain—in short, to rescind Extendability. This second great line of reply to the paradoxes is pioneered by Leibniz and by Cantor (and favored in some contemporary approaches[15]). Let us consider their two efforts in turn.

26.4.1 Leibniz’s Solution to Galileo’s Paradox of Natural Numbers

Principal among revolutionaries in the seventeenth-century turn toward infinitary methods in mathematics and a liberal embrace of the infinite more generally are Galileo and Leibniz. Writing in terms that could speak also for Galileo, Leibniz proclaims:

> I am so much in favor of the actual infinite that instead of admitting that Nature abhors it, as is commonly said, I hold that she manifests it everywhere, the better to indicate the perfections of her Author. Thus I believe that there is no part of matter which is not, I do not say divisible, but actually divided; and consequently the least particle ought to be considered as a world full of an infinity of different creatures. (Leibniz, 1923–, AII2.713)

In contrast to Aristotle, Galileo and especially Leibniz allow the actual or completed infinite into mathematics and into nature. Both are keenly aware of the paradoxes that arise.[16] Having removed Aristotle's guard-rail, they look to instate another, nearer the precipice of contradiction.

We limit our discussion to a paradox about numbers made famous by Galileo in *Discourses and Mathematical Demonstrations on Two New Sciences* (1638). Take the natural numbers. Since they include both the square numbers and non-square numbers, there are more naturals than squares. Yet there are just as many squares as there are roots, since every root has its own square and every square its own root; and there are just as many naturals as roots, since every natural is a root and every root is a natural. So it follows that there are just as many squares as naturals. We thus appear to have contradictory results: the natural numbers are both greater than and equal to the square numbers, which is absurd (→Galilei, 1898, Vol. 8, pp. 78–79).

Galileo's own response is to deny that "greater," "less," and "equal" apply in the infinite. In the character of his spokesman Salviati, Galileo writes:

> I don't see how any other decision can be reached than to say that all the numbers are infinitely many; all the squares infinitely many; all their roots infinitely many; that the multitude of squares is not less than that of all numbers, nor is the latter greater than the former. And in the final conclusion, the attributes of equal, greater and less have no place in infinite, but only in bounded quantity. (→Galilei, 1898, Vol. 8, p. 79; and →Galilei, 1974, p. 41)

One lesson of the paradox is that two intuitive standards for comparison of quantity—an “inclusion” standard and an “injection” standard, so to speak—which always agree in the finite can conflict in the infinite. Galileo rescinds both standards from applying in the infinite. Comparison is thus always to be restricted to “bounded” quantity.

Leibniz rejects Galileo’s solution. He sees the inclusion standard as expressing Euclid’s principle, from *Elements*, CN.5, that “the whole is greater than the part,” which he is not willing to contravene. In his 1676 dialogue on motion and the continuum, “Pacidius to Philalethes,” Leibniz has his interlocutors rehearse Galileo’s paradox and comment:

1. He [Galileo] said: the appellations ‘greater’, ‘equal’ and ‘less’ have no place in the infinite.
2. It is difficult to agree with this. For who would deny that the number of square numbers is contained in the number of all numbers, when squares are found among all numbers? But to be contained in something is certainly to be a part of it, and I believe it to be no less true in the infinite than in the finite that the part is less than the whole. (Leibniz, 1923–, AVI3, p. 551; and → Leibniz, 2001, pp. 178–179)

Leibniz’s alternative solution is to allow the infinity of numbers but deny that an infinity of things is ever “one” or a “whole” (see Leibniz, 1923–, AVI3, pp. 158 and 503). If the natural numbers do not form a whole, then *a fortiori* they do not form a whole that fails to be greater than the part and Euclid’s principle is preserved.

While there is an infinity of numbers, there can be no infinite number, nor any number of all numbers, and so on, any of which would count as an infinite whole on Leibniz’s account. This

becomes his signature position. As he later writes in the *New Essays*:

> It is perfectly correct to say that there is an infinity of things, that is, that there are always more than one can specify. But it is easy to demonstrate that there is no infinite number, nor any infinite line or other infinite quantity, if they be taken to be wholes. (Leibniz, 1923–, AVI6, p. 157; and →Leibniz, 1996, p. 157).

The infinite exists as a multitude or plurality but never as a whole or united totality. In this proposal Leibniz initiates what will become a principal alternative to Aristotle in responding to the paradoxes of the infinite: allowing the completed infinite but restricting one-making or whole-making or totality-making operations to collections that are always properly smaller than the most inclusive multitudes.

Let us spell out a difficulty. Notably, Leibniz does not reject the existence of wholes across the board, or even in mathematics. He expressly defines integer number as a "whole [*totum*] collected from unities as parts" (→Leibniz (1899), folio 35.1.9f, rv). So for Leibniz there can be wholes of any finite size, but no infinite wholes. Yet what prevents the formation of an infinite whole given the infinite supply of units? Leibniz's sole ground for enforcing this restriction on the whole-making operation appears to be the paradox. His account thus directly invites Dummett's objection of "wielding the big stick" and not offering an explanation.

The availability of the multitude for comprehension in thought can make the whole-making operation seem inevitable. In a striking early passage, before his study of the paradoxes of infinity, Leibniz writes:

> [E]very relation is either one of *union* or one of *harmony* [*convenientia*]. In union the things between which there is this relation are called *parts*, and

> taken together with their union, a *whole*. This happens whenever we take many things simultaneously as *one*. By *one* we mean whatever we think of in one intellectual act, or at once. For example, we often grasp a number, however large, all at once in a kind of blind thought, namely, when we read figures on paper which not even the age of Methuselah would suffice to count explicitly. (Leibniz, 1875–1890, Vol. 4, p. 35; and →Leibniz, 1969, p. 66).

Whatever things we take together in one thought are thereby bound by a relation of union into a whole. Compare Cantor's famous definition of the term "set":

> I use this word to designate a very broad theoretical concept which I have hitherto used only in the special form of a theory of geometric or arithmetical sets. In general, by a 'manifold' or 'set' I understand every multiplicity which can be thought of as one, i.e. every aggregate of determinate elements which can be united into a whole by some law. (→Cantor, 1883, p. 43, translated in →Ewald, 1996, p. 916)

It is not clear on what grounds Leibniz or Cantor can resist the formation of a whole or a set from a given multitude, once that multitude is admitted in the first place—save "wielding the big stick" when contradiction ensues.

This appears to be the price of locating the guard-rail between actual infinity and contradiction, as Leibniz does in response to Galileo's paradox. One might wonder whether the intellectual heirs to Leibniz, having admitted the relevant infinite multitudes as arguments for some extending operations (say, set-making or ordinal-making) but needing to restrict those operations to avoid the problem that arises when Completability is paired with Extendability, ultimately fare any better.

26.4.2 Cantor and the Generation of Number

Cantor's paradigm-shifting work in set theory brought about the decisive break from tradition and led to the contemporary

embrace of the completed infinite in mathematics. His new framework of transfinite sets and numbers rapidly took hold in mathematical practice for its technical elegance and fecundity. Cantor also offered more purely philosophical rationales in favor of the completed infinite. As we shall see, Cantor faces extraordinarily sharp challenges arising from the completed infinite, indeed prospects of outright contradiction. And in his anticipations of, or responses to, those difficulties, it is possible to discern both strong hints of Aristotle's potentialism about the infinite and the clear use Leibniz's tactic of restricting the formation of totalities out of multiplicities.

One line of reasoning that Cantor advances is an especially direct form of a collapse argument. The potential infinite, he claims, presupposes the actual infinite:

> There is no doubt that we cannot do without *variable* quantities in the sense of the potential infinite; and from this can be demonstrated the necessity of the actual-infinite. In order for there to be a variable quantity in some mathematical study, the 'domain' of its variability must strictly speaking be known beforehand through a definition. However, this domain cannot itself be something variable, since otherwise each fixed support for the study would collapse. Thus, this 'domain' is a definite, actually infinite set of values. (→Cantor, 1886, p. 9, translated in →Hallett, 1984, p. 25)
>
> [T]he potential infinite is only an auxiliary or relative (or relational) concept, and always indicates an underlying transfinite without which it can neither be nor be thought. (→Cantor, 1932, p. 391, translated in →Hallett, 1984, p. 25)

This so-called "domain principle"[17] applies directly to talk of procedures that can be iterated indefinitely:

> … to walk or travel safely a solid ground and a level path are necessarily required, a path that nowhere breaks off, but must be and remain passable everywhere up to where the journey leads … The long journey … is … not restricted to a finite path; hence, its path must be infinite, and

> precisely because it, on its part, is nothing variable, but everywhere constant, it must be an actually infinite path. Accordingly, every potential infinite (the variable limit) demands a transfinitum (the solid path on which to walk), and cannot be thought without the latter. (→Cantor, 1932, pp. 392–333, note, translated in →Jané, 2010, p. 215, note 49)

If Cantor is correct, it follows that Aristotle's effort to institute the potential infinite as a guard-rail against the difficulties of the completed or actual infinite cannot succeed; the actual infinite is not avoided by the potential infinite but made inevitable, and the associated difficulties remain to be confronted.

Cantor also presents theological arguments for existence of the actual infinite—the transfinite—in nature and in human cognition. He offers both an "a priori" and an "a posteriori" argument. The a priori argument, he writes,[18]

> ... is based on the concept of God and concludes first from the highest perfection of God's essence the possibility of the creation of a transfinitum ordinatum, then from his benevolence and his magnificence the necessity of the actual creation of a transfinitum. (→Cantor, 1932, p. 400, translated in →Jané, 2010, p. 212)

The a posteriori argument is a kind of inference to the best explanation:

> ... the assumption of a *transfinitum in natura naturata* permits a better, because more perfect, explanation of the phenomena, in particular of the organisms and the psychical manifestations, than the contrary hypothesis. (→Cantor, 1932, p. 4, translated in →Jané, 2010, p. 212)

In a passage reminiscent of Leibniz,[19] Cantor sees the existence of the actual infinite not only in God's thought but also in the natural world as an expression of its divine author:

> Not only does the transfinite fill the wide domain of the possible in God's cognition, but it also offers a rich, always increasing field of ideal

> research, and it is my conviction that also in the world of the created it achieves reality and existence to a certain extent and in various respects, in order to give a stronger expression of the magnificence of the Creator, in accordance with his absolutely free decision, than it could be the case in a mere "finite world." (→Cantor, 1932, pp. 405–406, translated in →Jané, 2010, p. 212)

It is especially striking as well that Cantor embraces Augustine's argument from divine comprehension for the actual infinite, reviewed above in Section 26.3.1. Referring expressly to *City of God*, XII.19, Cantor remarks,

> The transfinite cannot be called for more vigorously than St. Augustine does here, and it cannot be justified and defended more perfectly. Because hardly anybody will question that the infinite set (ν) of all finite whole numbers is not the absolute infinite. By asserting that God has a total, intuitive perception of the set (ν) "in some ineffable way," St. Augustine acknowledges at the same time this set *formaliter* as an actual infinite whole, as a *transfinitum*, and we are compelled to follow him in this. (→Cantor, 1932, p. 402, translated in Jané, 1995, p. 399)

Cantor recognizes that Augustine's idea that the infinite is made finite for God precisely corresponds to his own conception of an actually infinite set not as "absolutely infinite" but as an extended case of the (absolutely) finite, the *transfinite*. Still, there is a critical respect in which Augustine's view differs from Cantor's. As noted above, for Augustine, God's comprehension of all numbers is *non numerus* and does not yield the production of a further number; it is not a "number-making" act that extends the sequence. For Cantor, however, the comprehension of an infinite sequence of numbers is assured to be number-making, generating a wider domain of mathematical constructions, and along with it a profound new exposure to paradox.

Consider Cantor's "extended number sequence," now taken to be the ordinal numbers—perhaps the order types of well-

orderings. According to his second “generating principle,” any “definite” sequence of numbers can be used to generate a least upper bound. Suppose, following Cantor, that the natural numbers—the finite ordinals—collapse to an actual infinity. This yields a definite sequence of numbers. And, of course, for the Cantorian, and the contemporary mathematician, nothing bad happens: we “generate” ω, as an actual infinity, the first transfinite number. This was initially surprising, of course, and runs against the tradition, but it proved to be clearly mathematically fruitful.

But now suppose the ordinals themselves collapse to an actual infinity, thus making the *extended* number sequence definite. This, of course, is a disaster. We somehow manage to generate Ω, a number larger than all the numbers, a contradiction—the Burali-Forti paradox. Notice that Cantor’s endorsement of Augustine reaches the same result via the theological implication: Is there some ordinal God does not know? Surely not. But then all the ordinals are in some ineffable way “made finite” (or “made [de]finite [*αφωρισμνον*]”) to God,[20] and thereby, for Cantor, bound into an ordinal, namely, the ordinal of all ordinals.

So the rather natural thought behind the Augustine-Burley-Cantor invocation of the deity does not merely threaten to collapse the potentially infinite into the actually infinite; it threatens inconsistency. Even for God, the ordinal of all ordinals is a contradiction.

The situation for the Cantorian is initially familiar. Embrace of the completed infinite yields paradox, as it did in the eyes of Aristotle, Galileo, and Leibniz. But the difficulties described by those earlier philosophers today seem less formidable. We have gotten used to the “open end” of a complete infinite series of numbers and to the “equality” (or “equinumerosity”) of the set of natural numbers with its infinite subsets. It is not too hard to

find underlying assumptions we can surrender in order to dissolve the earlier paradoxes into merely "surprising" phenomena while leaving the intact the idea of the completed infinite. The guard-rails proposed by Aristotle or Leibniz thus appear not to have been absolutely necessary to preserve the mathematics after all. But in Cantor the assumptions that yield the Burali-Forti paradox are not so easily surrendered. For they appear to be the very principles on which the introduction of transfinite numbers is justified in the first place. We accept transfinite ordinals on the basis of the assumption that any definite sequence of numbers—finite or infinite—has a least upper bound. How can we then deny the same of the ordinals themselves?

Here the "extending problem" takes its sharpest form, and escape from contradiction may force us back either to Aristotle's appeal to the potential infinite to limit the availability of arguments for the operation, or to Leibniz's effort to impose restrictions on the operation itself.

Cantor's writings may contain versions of each strategy. The texts admit some latitude for interpretation. Jané reads Cantor's extended number sequence as "irreducibly potential" (→Jané, 2010, p. 198), and draws attention to Cantor's constructive language, especially in early works, about the generation of number and mathematical creation. Numbers are formed according to two principles of generation. The first is "the addition of a unit to an existing, already formed number" (→Cantor, 1883, p. 195; →Jané, 2010, p. 196n9). The second is "if any definite succession of defined integers is given, having no largest element, a new number is created [...] which is thought of as the limit of those numbers, i.e., it is defined as the next number greater than all of them" (→Cantor, 1883, p. 196; translated in →Ewald, 1996, pp. 907–908). This condition of a "definite sequence" of numbers may suggest that "definite" is

taken in contrast to a conception of the greater domain of number as something indefinite prior to ordinal-making acts of construction. The impression becomes even stronger when Cantor adds the third principle, this time a rule of constraint or limitation understood as:

> the demand that a new integer could be made with the help of one of the two other principles of creation *only* if the totality of all previous numbers had the power of a defined numberclass which was already *in existence* over its entire extent. (→Cantor, 1883, translated in →Ewald, 1996, p. 911)

Or when he says that the third principle:

> imposes certain successive constraints on the thoroughly endless process of creation, so that we obtain natural segments [*Abschnitte*] in the absolutely infinite sequence of integers. I call these segments number classes [*Zahlenklassen*]. (→Cantor, 1883, p. 3, translated in →Ewald, 1996, p. 883)

Taken at face value, this is natural to read as a story of the ordered construction of defined or limited items from an intrinsically indefinite or unlimited source, the imposing of cuts into the potential to create the actual. This would then invite a new placement of Aristotle's guard-rail, one that allows the construction of transfinite terms along with all the finite ones among actual units, while precluding a completed domain of the transfinite and thus consigning the (absolute) infinity of the extended number sequences to be only potential.

Yet alternatively one might read Cantor as assuming a completed domain, with the constructive language as only heuristic and the reference to "definite sequences" in number generation as picking out sequences that are quantifiable, able to belong to a single set. The "definite" then would be in contrast not to a domain indefinite in the sense of Aristotelian

potentiality but to an “absolutely infinite” domain, one indefinite in the sense that its multitude is beyond mathematical determination. In later writings that focus more on set-making than on the ordinal-making principles of generation, the attitude is more distinctly completist and not potentialist. As noted above, this is evident in Cantor’s domain principle itself and its downgrading of the potential infinite to having “only a borrowed reality, always pointing to an actual infinite, only through which it is possible” (→Cantor, 1932, p. 404, translated in →Jané, 2010, p. 215)

Cantor’s thoughts may waver or be ambiguous on this point, and his views evolve over time.[21] We do not insist on a single interpretation of the suggestive passages; it suffices to understand the alternatives and the conceptual matters they raise. Pressing on, let us consider the indisputable appearance of the second guard-rail in Cantor’s account.

In a much quoted letter to Dedekind, in 1899, Cantor’s remarks recall Leibniz’s tactic of restricting whole-making to avoid paradox. For Cantor, the restriction on totalities is not to the finite as opposed to the infinite, as it was for Leibniz, but to the finite-or-transfinite as opposed to the absolutely infinite:

> [I]t is necessary … to distinguish two kinds of multiplicities (by this I always mean definite multiplicities). For a multiplicity can be such that the assumption that all of its elements ‘are together’ leads to a contradiction, so that it is impossible to conceive of the multiplicity as a unity, as ‘one finished thing’. Such multiplicities I call *absolutely infinite* or *inconsistent multiplicities*… If on the other hand the totality of the elements of a multiplicity can be thought of without contradiction as ‘being together’, so that they can be gathered together into ‘one thing’, I call it a *consistent multiplicity* or a ‘set’. (→Ewald, 1996, pp. 931–932)

Here Cantor restricts the set-making operation. It does not apply to all multiplicities of elements but only to those multiplicities whose elements can be thought of as existing together without

contradiction. This restriction can hardly be incorrect (contradictory totalities cannot be anything, so they cannot be sets), but it will not do to justify the concept of set for infinite sets by saying that it is valid except where it is incoherent. That is the most direct possible form of "wielding the big stick": pointing to the contradiction instead of giving an explanation.

Cantor famously equates the complete sequence of ordinals with what he calls the Absolute, something beyond mathematical determination and so not capturable in a number.[22] While this means Burali-Forti will not arise in his account, it comes at the price of arbitrariness. Once we allow that completed infinities can be collected into sets, indeed ordinal-many such completed infinities of ever-increasing size, why should not *this* completed infinite, the sequence of all ordinals, also be collectable into a set?

Note that one articulation of the "limitation of size" conception of set faces the same question.[23] Here any multitude properly smaller than the entire set-theoretic universe, but none equal to it, constitutes a set, so sets are, so to speak, "absolutely finite" or "small" in comparison to the universe itself. As the acclaimed mathematician Hermann Weyl remarks:

> Such classes as are too 'big' are here excluded from admission to the club of decent sets, and in this way the disaster of the antinomies is avoided (→Weyl, 1949, p. 232).

But if the universe is understood as a completed domain, why should not the multitude of all its elements (or equally "big" multitudes) also be collectable into a set? True, if the elements are all collectable into a set, then the domain would be extended to a still greater one by the usual operations, which is now contrary to the hypothesis of the universe as a completed domain. Or else, if the domain is held fixed and the limit is lifted,

we again court the disaster of the antinomies. But the size limit is not *explained* by pointing to the contradiction of the hypothesis of a completed domain, nor by pointing to the disaster.

Iterative conceptions of set may promise some advance against this problem of arbitrariness with the twin ideas that sets are "formed" at "stages" that have an order of precedence, and that a set's elements are formed prior to it.[24] Since no stage includes all stages, no set comprehends all elements in the completed infinite domain of sets, and, it seems, the paradoxical sets do not emerge. But so long as the notion of stages presupposes the ordinals, taken as completed, to serve as their indices, we have only pushed back the underlying question without answering it.[25]

It appears, then, that those descendants of the strategy that Leibniz and Cantor developed as a guard-rail against paradox remain exposed to the same fundamental problem facing their original accounts.

26.5 Concluding Discussion

In the introduction, we promised that our study will support two main lessons. The first lesson—that many great thinkers across the centuries have struggled with the tension between Completability and Extendability—has now been well documented. The second lesson, though, concerns how this tension should be resolved. We wish to end with some reflections on this hard question, drawing together what we have learned throughout our study.[26]

The appeal to potentiality is intended as a guard-rail against various paradoxes of the infinite. The idea, as we saw, is to absorb the infinity into an incompletable capacity or procedure. It is tempting to invoke the metaphor of "spreading out" the

infinity along some “dimension of increase” rather than allow the entire infinity to co-exist at a single point along that dimension. But the metaphor is treacherous. For this guard-rail to work, the dimension of increase must obviously not be too analogous to the ordinary spatial dimensions. Otherwise, there would be no relevant difference between admitting an infinity distributed along the dimension of increase (allegedly acceptable) and admitting an infinity “spread out” only spatially (allegedly unacceptable). Our key question, then, is to what extent the dimension of increase is analogous to the ordinary spatial dimensions. The closer the analogy, the weaker the Aristotelian guard-rail.

Suppose there is a God. Then, if the dimension of increase is sufficiently space-like, God can take in the entire infinity at a glance, although it is spread out along this dimension and thus not directly accessible to our limited human point of view. So far, so uncontroversial. *But why should we accept the antecedent of this conditional?* How, in particular, might the existence of God give us a new or additional reason to accept the antecedent?

One plausible answer, we think, is that God’s assumed relation to time requires this dimension to be relevantly like space. After all, God is typically assumed to be eternal, in the sense of existing outside of time, as opposed to being tied up in the “process of things coming to be and passing away.” By existing outside of time, He has equally good access to all objects irrespective of when they exist. This symmetrical access ensures that all objects can be “known simultaneously,” as Duns Scotus put it, and thus “will be actually infinite.”

Let us explicate this *argument from God’s eternality*. The central idea is that an eternal God has “equal access” to all objects, whether past, present, or future. We find it useful to break this central idea into two main premises. First, for something to be an object of comprehension, this object has to

exist in the first place. Thus, the idea of an eternal God presupposes

Eternalism: There exist past and future objects, as well as present ones.

We might add that along with the past and future objects there exist past and future moments of time. Second, there is the distinctive theological premise of the argument, namely that there exists an eternal God.

Eternality of God: There exists a God outside of the temporal dimension on which all the past and future objects and moments of time are located.

Since God has perfect and simultaneous "access" to all the objects and moments of time that are distributed across the temporal dimension, there is no difference between a number that has been constructed and one yet to be constructed. God has direct comprehension of both. This yields:

Conclusion: God can comprehend every number in one single intellectual act.

Given the widely accepted theological premise that God is eternal, this argument has considerable strength. The argument does, however, leave the Aristotelian opponent with a fairly obvious retort. True, the argument collapses any potential infinity that is spread out *temporally* to a corresponding actual infinity. But what if the dimension of increase is *modal* rather than temporal? Indeed, even apart from the theological collapsing argument, it is plausible to understand a potential infinity as tied to a process that *could* yield one more instance, whether or not it in fact *will* do so.

Let us therefore consider the modal analogue of the argument from God's eternality. The first premise of the temporal argument, we recall, abandons presentism in favor of the eternalist view that there exist past and future objects as well as present ones. Transposing this to the modal arena, we must

abandon actualism—the view that all that exists, is actual—in favor of the following view:

Possibilism: There exist non-actual objects as well as actual ones.

Lewis's modal realism provides a famous example of possibilism (→Lewis, 1986). This contrasts with actualist views, which either reject the idea of possible worlds or understand these as actually existing entities that nonetheless represent non-actual circumstances. For example, actualists can take possible worlds to be maximally specific properties ("ways the world could be"), complete stories of how the world is, or the like—all of which can plausibly be taken to exist at the actual world.

The second premise of the original temporal argument adds to the acceptance of non-present objects a God external to time. The modal analogue of this premise adds to the acceptance of a logical space of non-actual possible worlds a God external to this space.

Amodality of God: There exists a God outside of the logical space of non-actual possible worlds and non-actual objects.

Against, since God has perfect and simultaneous "access" to all objects, it follows that God can comprehend every number in one single intellectual act.

How does this modal analogue of the argument from God's eternality compare with the original temporal version? Let us begin with the premise of Possibilism. Whether this departure from actualism is defensible is a large question that we cannot attempt to answer here. We do, however, venture a comparative claim. The rejection of actualism is more problematic than its temporal counterpart, namely the rejection of presentism. It is hard to resist the acceptance of non-present moments of time and events lying outside any given plane of simultaneity that one might wish to identify as the present; after all, this is how time is treated in physics. By contrast, there is no science that requires

non-actual possible worlds. The closest we get are probably statistics and statistical physics, both of which require quantification over states of a physical system. But these states are plausibly understood in an actualist manner, say, as maximally specific properties of the system.

What about the second premise, Amodality of God, which posits a God external to the logical space of non-actual possible worlds? While the idea that God is eternal has a strong theological pedigree, we are not aware of any theological tradition that regards God as amodal. So the theological pressure to accept the second premise is, we believe, less strong in the modal case than in the original temporal one.

In sum, we conclude that the modal analogue of the argument from God's eternality is considerably less convincing than the original temporal version.

Our second main lesson is now clear. Our best hope of salvaging the attractive Aristotelian idea of the potentiality of infinity as a guard-rail against paradoxes is by invoking an analogue of another Aristotelian idea: not presentism about time, as in Aristotle, but its modal analogue, namely actualism about modality. If there is to be any point in absorbing infinities into incompletable procedures or capacities, the associated modal notions must be rock bottom, not reducible to generalizations over a vast collection of moments of time or possible worlds.[27] In short, to salvage the Aristotelian guard-rail, we must be more thoroughgoing Aristotelians.

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Notes

1 For related discussion, see →Hintikka, 1996; →Lear, 1980; and →Coope, 2012.

2 We follow a traditional interpretation of points as not existing in the continuum prior to division, similar to that given by (*inter alia*) →Sorabji, 2006; Charleton, 1991; →Coope, 2012; and →Cooper, 2016. But note an alternative view on which points (etc.) do exist in continua prior to division but with some special status in their mode of being; defenders include →Pfeiffer, 2018; →Bowin, 2007; and →Beere, 2009. See →Rosen, 2021, for discussion.

3 See Linnebo and Shapiro, 2019. We make no attempt here (or anywhere else) to be faithful to everything, or even most things, that Aristotle himself says about modality. The proposal here is to invoke some technical notions from contemporary modal logic and metaphysics in an attempt to explicate a broadly Aristotelian notion of potential infinity (in the spirit of →White, 1992).

4 Augustine and Duns Scotus make similar observations; see Section 26.3.

5 As we explain in Sections 26.3 and 26.4, the existence of God can support Completability in more subtle ways as well.

6 Following →Aristotle, 1934, with slight modification. Compare →Aristotle, 1984: “[S]ome put the same

argument in another form, and would have us grant that during the time in which a motion is in progress we should first count the half-motion for every half-distance that we get, so that we have the result that when whole distance is traversed, we have counted an infinite number, which is admittedly impossible."

7 It would be precluded by the principle of finite comparability of all magnitudes, enshrined in Euclid's *Elements*, Vd4: "Magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another" (→Euclid, 1926, p. 114). Today this is known as "Archimedes' axiom" and written: for any $x, y > 0$, if $x > y$, then there is a natural number n such that $ny > x$. It should be mentioned that Aristotle also gives a direct argument against infinite number (204b7–10, quoted in Rosen 2021, p. 149.): "But indeed, neither is there a number, in such a way as to be separate and infinite. For a number, or that which has number, is countable. If, then, it is possible to count that which is countable, then it would also be possible to go through the infinite." But this argument rests on the impossibility of counting the infinite and so will not explain that impossibility.

8 One might note the similarity between this conception of the infinite and Bertrand Russell's (1907, p. 29) remark that "the contradictions result from the fact that … there are what we may call self-reproductive processes and classes. That is, there are some properties such that, given any class of terms all having such a property, we can always define a new term also having the property in question." And →Dummett

(1993, p. 441): “[an] indefinitely extensible concept is one such that, if we can form a definite conception of a totality all of whose members fall under the concept, we can, by reference to that totality, characterize a larger totality all of whose members fall under it.”

9 “The whole and the complete are either entirely the same or quite close in their nature” (→Aristotle, 1983, 207a12–13). →White, 1992, p. 11 notes: “‘The infinite’, in this sense, Aristotle contrasts with what is ‘complete and whole’ (*teleion kai holon*).” See also →Coope, 2012, p. 268.

10 Note that Aristotle distinguishes *plurality* (πλῆθος) and *number* (ἀριθμός) and says that “plurality is, as it were, the genus of number; for number is a plurality measurable by one” (*Metaphysics* 1.6 1057a2–4, quoted in →Rosen, 2021, p. 149). This suggests at least a lexical possibility of a plurality that is *not* measurable by one and so does not have number and is not countable. See Rosen (2021) for discussion.

11 See, for example, →Dummett, 1991 and →Velleman, 1993.

12 Translation from →Hintikka, 1996, p. 206. Bracketed in Ross (→Aristotle, 1950) as a possible alternative; see →Hintikka, 1996, p. 207, and →Coope, 2012, p. 273.

13 It is a further question how this fits with Aristotle’s commitment elsewhere to the fixity (“necessity”) of the past and present as opposed to the future; compare *De interpretatione* 9.

14 See →Benardete, 1964; →Moore, 2018.

15 See, for example, →Boolos, 1989; →Cartwright, 1994; and →Williamson, 2003.

16 Their treatments of the infinite are extensive and well discussed. See, for example, →Arthur, 1989 and 2013; →Mancosu, 1996 and 2009; →Knobloch, 1999 and 2011; and →Levey, 2015.

17 See →Hallett, 1984.

18 Cardinal Franzelin took issue with Cantor's argument here, pointing out that, at most, it shows that God is free to create an actual infinity, not that He is bound to do so by His very nature. Cantor seems to have accepted this, writing that "it was not my intention ... to speak of an objective, metaphysical necessity of the act of creation, to which God, the absolute Free, would have been subjected; on the contrary, I wanted to point to a certain subjective necessity for us to infer from God's benevolence and magnificence the creation" (→Cantor, 1932, p. 400, translated in →Jané, 2010, p. 213 note 45).

19 Likely recalling Leibniz's remarks (noted above) at AII2, p. 713, directly; Cantor quotes them in *Grundlagen* (→Cantor, 1883, Section 7.2).

20 Bracketed terms inserted by Cantor to "clarify the meaning" that "the relevant words have in the relevant places in St. Augustine," when he quotes *City of God*, XII.19 in a letter of February 28, 1886 to Eulenberg: "The infinity of number, though there is no [finite] number of infinite numbers, is yet not incomprehensible by Him

whose [absolute] understanding is without number. And thus, if everything comprehended by knowledge is made finite by the comprehension of the one who knows it, then all infinity is in some ineffable way made [de]finite [αφωρισμνον] to God, for it is not incomprehensible to His knowledge" (→Cantor, 1932, pp. 401–402n3, translated in Jané, 1995, p. 399). Cantor notes in the same letter: "das aktual Unendliche (αφωρισμνον)" (→Cantor, 1932, p. 401n2, translated in Jané, 1995, p. 399).

21 See Hallett's indispensable discussion in →Hallett, 1984.

22 For instance, →Cantor, 1932, p. 205n2, and p. 405. See →Hallett, 1984, pp. 43–44; and →Jané, 2010, p. 386, for discussion.

23 Some early formulations of the limitation of size conception notably include →Jourdain, 1904; →Russell, 1907 (as a skeptic); and →Mirimanoff, 1917. See also →von Neumann, 1925. For discussion, see →Hallett, 1984, pp. 176–185; and →Boolos, 1989.

24 See, for example, →Boolos, 1971. Even clearer is →Boolos, 1989, who insists that the generative language is for metaphorical purposes only. Contrast other developments of the iterative conception →Zermelo, 1930; Hellman, 1989; →Linnebo, 2013.

25 See →Velleman, 1993.

26 The discussion that follows is developed further in a companion piece to this article, "Potential infinity, paradox, and the mind of God: towards a resolution."

27 See e.g. →Weyl, 1921, p. 97, who suggests that a universal generalization over the potentially infinite sequence of natural numbers can be underwritten by its lying in “the essence of number” that each should have the relevant property, as well as →Fine (1994) for an account of the notion of essence.

27 Absolute Infinity, Knowledge, and Divinity in the Thought of Cusanus and Cantor

Anne Newstead

Abstract: This historical essay compares the views of Nicholas of Cusa ('Cusanus') and Georg Cantor on the topics of infinity, divinity, and mathematical knowledge. Echoing Nicholas and neo-Platonism, Cantor says in his *Grundlagen* (1883) that the transfinite sequence of all ordinals is a symbol of the absolutely infinite, which can only be acknowledged but never known. Moreover, Cantor envisions his transfinite set theory as providing the analytical methods and techniques necessary for a complete philosophy of nature. Cantor's novel mathematics is presented as part of a long tradition, to which Cusanus, Bruno, Spinoza, Leibniz and others belong, in which the infinite character of organic life forms is appreciated and is taken to be in some sense a mirror and symbol of the divine. The doctrine of symbolism present in both Cusanus and Cantor enables these thinkers to articulate a transcendental apophatic approach to divinity.

27.1 Introduction

What is the relationship between infinity and divinity, if any? Traditionally infinity was held to be a divine attribute, a property of the *ens realissimum*, the most perfect Being. Such a conception is present in Anselm's designation of God as "that than which nothing greater can be conceived." Similarly, God

has been described as “the Absolute Maximum,” “that than which there cannot be anything greater” and “beyond all we can conceive,” which as “all that can be” is “altogether actual” (DDI, Book I, ch.4; Cusanus, 1985 [1440], p. 53).[1] This conception breaks classical logic with its insistence on both the completeness and infinity of divine being. For if divine being is complete, how can it be infinite? And if divine being is infinite, how is it complete? One way out of the impasse is to separate the metaphysical, theological notion of completeness from the mathematical notion of infinity.

In this comparative essay, I focus on the thought of two German-speaking thinkers whose work spans the disciplines of mathematics, philosophy, and theology, and whose contributions to debates about infinity and divinity have been seminal: the fifteenth-century Cardinal Nicholas of Cusa (Nikolaos of Kues, ‘Cusanus’) and the nineteenth-century mathematician Georg Cantor (1845–1918). Both thinkers made epochal contributions. Cusanus is sometimes described as a precursor of Copernicus, who finally shattered the finitude of the Aristotelian cosmos and rejected geocentric cosmology. His most famous contribution may be to have taken a metaphor about God and to have applied it to the physical universe as whole, which is now described as “a sphere of which the centre is everywhere and the circumference nowhere.”[2] Although he distinguished between the absolute actual infinity of God and the boundless infinity of the universe, Cusanus definitely succeeded in removing infinity from being the sole provenance of divinity, and in so separating the attribute from substance, made it possible for mathematicians and logicians to think about infinity without doing theology. Centuries later, Cantor is appreciated as the key mathematician responsible for set theory, demonstrating in depth how to build a viable theory of infinite *numbers* (‘transfinite numbers,’ distinguishing clearly between

cardinal and ordinal number concepts) in an infinite set-theoretic universe. While both thinkers' contributions might be known in outline, their key texts and their epistemologies—*De Docta Ignorantia* and the *Grundlagen*—are known to a scholarly few, partially owing to their relative obscurity and the relative lack of availability of good English translations. An additional barrier is that the interpretation of both thinkers requires a sound knowledge of movements such as Platonism, Neo-Platonism, Aristotelianism, and Neo-Scholasticism. They are both, furthermore, part of a long partially forgotten tradition linking divinity and mathematical knowledge. Thierry of Chartres exemplifies this approach when he writes "The custom of the ancients was to teach first of all mathematics in order to approach knowledge of the divine."[3]

In an endnote to his *Grundlagen*, Cantor situates his own thought in relation to Cusanus' thought and Leibnizian metaphysics in a positive light as one might expect for a nineteenth-century German thinker. The full title of Cantor's work is *Grundlagen einer allgemeiner Mannigfaltigkeitslehre* (Foundations of a general theory of manifolds/multiplicities) (1883). The subtitle of this work, *Ein mathematisch-philosophischer Versuch in der Lehre des Unendlichen,* translates as "a mathematical philosophical investigation into the theory of the infinite." This mixed, interdisciplinary genre of investigation still flourished in nineteenth-century German-speaking lands, two other examples being the Bohemian Bernhard Bolzano's profound *Paradoxien des Unendlichen* (1851), and Constantin Gutberlet's (less mathematically successful) *Das Unendliche metaphysisch und mathematisch betrachtet* (1886). Cantor's text, no less than these other texts and indeed Cusanus' text as well, belong to a heady pre-modern tradition that mixes philosophy, theology, and mathematics into one discussion.

The endnotes to Cantor's *Grundlagen* are essential for understanding Cantor's orientation with regard to the western philosophical tradition. It is as though Cantor is laying down breadcrumbs that enable readers to recover his philosophical commitments. Some of the endnotes are quite substantial and amount to a reflection on the *Grundlagen* itself. Moreover, there is every reason to suppose that Cantor viewed these philosophical commitments as essential to his developing thought, an interpretation only strengthened by his subsequent petition to move from the mathematics faculty into the philosophy faculty at the University of Halle in 1884.

27.2 Cantor's Allusion to Cusanus in the *Grundlagen* and Philosophical Context

In endnote 2 to Section 4 of the *Grundlagen*, Cantor cites Eduard Zeller's 'great' work, *Die Philosophie der Griechen* (The Philosophy of the Greeks, 1844–52) to note that a Platonic conception of the infinite is very different from the Aristotelian one. It is clear from the main text of the *Grundlagen*, that Cantor regards the Aristotelian-scholastic tradition to have pushed the dismissal of the actual, proper (*eigentlich*) infinite too far in favor of only accepting the mathematical legitimacy of the potential, improper (*uneigentlich*) infinite. Nor can the Hegelian view that the actual infinite is the only genuine infinite be correct, as it ignores the role of the potential infinite in mathematics. Both types of infinity, Cantor maintains, have their place in metaphysics and mathematics, with the actual infinite being the ontological foundation that grounds statements about the potential infinite.[4]

In the *Grundlagen*, Cantor seems more sympathetic to Platonism as a metaphysics and epistemology than Aristotelian finitism. In Section 4 of the *Grundlagen*, Cantor argues that

Aristotelian arguments against actual infinities are circular and question-begging, presupposing the existence of only finite numbers by making the properties of finite numbers essential to all numbers. He writes:

> As is well known, throughout the Middle Ages the proposition "infinitum actu non datur," taken from Aristotle, was held to be incontrovertible by all the scholastics. But if one considers the reasons which Aristotle brought forward against the real existence of the infinite (see e.g., his *Metaphysics*, XI.10), it will be found that the main issue can be traced back to a presupposition which involves a *petitio principii*, namely, the presupposition that there are only finite numbers. (*Grundlagen*, Section 4, [9]; →Ewald, 1996, p. 889)

Cantor also likes the Pythagorean idea—the germ of which is present in Plato's *Philebus*—that the infinite can be made definite (and hence actual): the imposition of a boundary (*peras)* on the infinite (the *apeiron*) renders it into something both definite and delimited and actual, yet infinite.[5] Cantor thinks of his transfinite numbers as such definite infinities (*bestimmt unendlichen*). The contrast between the transfinite numbers and the indefinite, untamed (absolute) infinite could not be more dramatic.

In his endnote 2 to the *Grundlagen*, almost in passing, after considering the scale of all transfinite ordinal numbers, Cantor says he finds a point of contact between his philosophy of the infinite and the philosophy of Nicolaus Cusanus. He cites a study of Cusanus as a predecessor to Leibniz, and then mentions Giordano Bruno as the successor of Cusanus:

> Also, I find a point of contact for my conception [of the infinite] in the philosophy of Nicholas the Cusan. (See R. Zimmermann, *Der Cardinal Nicolaus von Cusa als Vorgänger Leibnizens* (Sitzungsberichte der Wiener Akademie der Wissenschaften, 1852.) Similarly for Giordano Bruno, the successor of Nicholas. See Brunnhofer, *Giordano Brunos Weltanschauung und Verhängnis*, Leipzig 1882. (→Cantor, 1932 [1883], endnote 2; →Ewald, 1996, p. 891)

What are we to make of these allusions in the *Grundlagen* to Cusanus, Bruno, Spinoza and Leibniz? This constellation of thinkers comprises a tradition that is very affirmative of infinities in nature. However, this tradition struggles, to some extent, to come to terms with the infinity of numbers and the idea of infinite quantity and number. Perhaps Cantor sees himself as coming to the rescue of this tradition. How so?

According to Euclid's Common Notion 5: "The whole is greater than the part." By the transitivity of size, it follows that the part is smaller than the whole of which it is a part. Obviously, in the case of infinite collections such as the collection *E* of all even numbers and the collection *N* of all natural numbers, this axiom is violated. The cardinality of *E* is equal to that of *N*, because there is a one-one pairing between members of *E* and members of *N*. Yet *E* is in a sense just a part—a proper subset—of *N*. Cantor introduces the all-important equicardinality principle and the idea of a 1:1 mapping between elements of collections as a criterion for sameness of cardinal number. Rather than abandon his important equicardinality principle, Cantor simply drops the part-whole axiom instead, noting that it does not govern transfinite numbers. Leibniz and the other philosophers, however, by and large stick with the part-whole axiom and so do not even think to break through to acknowledge the existence of transfinite numbers.[6] Dropping the part-whole axiom, however, is an incredibly bold move, one that moves the mathematical notion of a manifold, whole, or set (*Menge*) further away from the realm of appearances and sensory perception. Whenever we finite human beings perceive a whole, we see each visible part as smaller than the whole.

Cantor sees himself as developing the position that Leibniz should have held, taking Leibniz's view that there are actual infinities to its proper conclusion by supplying the appropriate rigorous mathematical theory and conception of number to

characterize the actual infinities in nature. The project of mathematising nature is one that Cantor adopts with enthusiasm, and as a kind of Leibnizian he does not see a conflict between a mathematical view of nature and a dynamic, organic explanation of nature. Organicism about nature was very much the flavor of the day—from perhaps the 1830s–1860s until Darwinism hit—as an alternative view to what some German thinkers perceived as the soul-less, mechanical philosophy descending from Newtonian physics. In fact, we know from his notebooks that Cantor attended F. A. Trendelenburg's lectures (on Spinoza's *Ethics*) in Berlin in 1871, and Cantor may have been to Hermann Lotze's lectures at the University of Göttingen, so he had ample exposure to the debate between philosophies of mechanism and teleology, as well as to the metaphysics of monism.[7]

Trendelenburg seems to have championed a teleological organicist world-view in his hefty tome, *Logische Untersuchungen*, while Lotze in his enormous *Mikrokosmus* (1856–64) tried to steer a middle course between mechanism and materialism, by holding that while mechanistic explanations were perfectly suitable for scientific explanation, they were in some sense 'subordinate' to the realm of purpose and value (→Beiser, 2014). In particular, Lotze adopted the idealistic Leibnizian view that matter and material bodies are "secondary derived realities" that are grounded in the reality of aggregates of points or forces (→Beiser, 2014). Thus, Lotzeans accepted mechanical forms of explanation in science without adopting a materialist metaphysics.[8]

Cantor's attendance of Lotze's lectures would have exposed him to Lotze's monism and his advocacy of Leibniz's modifications to Spinoza's system, with his insistence on immaterial points or forces in nature. It is partially in this context that we should understand Cantor's remarks in the *Grundlagen*

that he has introduced methods that will solve the problems that plague the systems of Spinoza and Leibniz, and in doing so will provide a mathematically rigorous yet organic explanation of nature.

> The chief difficulties in the systems of the last two-named thinkers [Spinoza and Leibniz] can, I believe, be brought to a solution by the methods I have introduced; indeed, some of the difficulties can already be satisfactorily solved and explained. These difficulties have understandably given rise to later criticism. But for all its merits this criticism has not, it seems to me, yielded an adequate substitute for the hampered development of the theories of Spinoza and Leibniz. For alongside of the mechanical explanation of nature … there has until now not been even the start of an organic explanation of nature that would attempt to go further and that would be armed with the same mathematical rigour; this organic explanation can, I believe, be initiated only by taking up afresh and continuing the works and endeavours of those thinkers [Spinoza and Leibniz]. (→Cantor, 1932, *Grundlagen*, p. 177; →Ewald, 1996, p. 892)

So, at a time when the metaphysics of Spinoza and Leibniz may have seemed obsolete and even bizarre, we find Cantor views himself and his set theory as providing the mathematical methods for an organic conception of nature. In fact, Cantor does not cleave to a metaphysics that is authentically Spinoza's, but rather to a German version of Spinozism and *Naturphilosophie*, perhaps deriving from Trendelenburg or Schelling. A key point of difference is that German 'organicists' with their vitalist monism were advocates of teleology, whereas Spinoza's own metaphysics has no room for final causes (see Appendix to *Ethics* I).

As Cantor came of age, the controversy over materialism in Germany into the 1860s threatened to upend conventional wisdom and conservative politics (→Beiser, 2014). The arrival of Darwin's theory of natural selection in his *On the Origin of Species* (1859) intensified the conflict between the old teleological

metaphysics and the new materialism. There were affinities between the politics of conservatives and theism, and the politics of leftists and materialism. It is clear from Cantor's writings from the *Grundlagen* onwards, that he increasingly aligns himself with theism and rejects materialism. He appears careful to avoid some of the modern 'errors' identified by Pope Pius IX in his 1864 *Syllabus Errorum*, where the very first few errors identified include pantheism, naturalism [materialism] and absolute rationalism.[9]

27.3 "Omnis determinatio est negatio": Negative Theology in Cantor's *Grundlagen*

As his many letters to theologians show,[10] it was important for Cantor to maintain some kind of orthodox approach to religion and to not overstep the bounds in his theorizing about infinity into offending religious authorities. Part of Cantor's strategy was to embrace *negative theology*. Cantor is persuaded by the contention of negative theologians that the nature of God cannot be determined and that strictly speaking one cannot assign determinations (or predications) to God. In the historical and philosophical sections of the *Grundlagen*, he finds this tradition appearing in the work of the founders of modern philosophy such as Descartes and Locke.

> However different the theories of these writers [Descartes and Locke] may be … they agree … that *the true infinite or Absolute, which is in God, permits no determination whatsoever* [emphasis added]. As to the latter point I fully agree, and cannot do otherwise; the proposition: "omnis determinatio est negatio" is for me entirely beyond question. (→Cantor, 1932 [1883]; →Ewald, 1996, pp. 890–891)

Cantor's quotation "*omnis determinatio est negatio*" is at once a reference to Spinoza (with whom the phrase originated) but also

reflects Cantor's reading of the history of modern philosophy which is shaped by Hegel and his influential history of philosophy. Spinoza notoriously defines God as an "absolutely infinite being," in which something is "absolutely infinite" (rather than infinite in its kind) whereby "whatever expresses essence and does not involve any negation belongs to its essence." (Spinoza, *Ethics* I, def. 6, explication, trans. Shirley 1992, p. 31)).

Briefly the argument from negative theology is that to determine an object—even in thought by means of concepts—would be to limit it in some way. If the Absolute or true infinite is unlimited, then it cannot be determined (cf. Descartes, *Principia* I, 27).[11] Spinoza relies on this type of reasoning in the *Ethics* (Book I, Ip5, Ip14) to argue that there can be exactly one 'absolutely infinite' substance (God), since if there were more than one, it would therefore be limited and not absolutely infinite. So, the unlimited nature of the absolute Infinite guarantees its uniqueness. Cantor respects the unicity of the absolute Infinite, which he identifies with God. Such a unique and absolute infinity, however, cannot be identified with the whole of the universe, as Spinoza appears to do in speaking indifferently of "Deus sive Natura."

In *Grundlagen* (Section 5), Cantor thinks that Spinoza does not properly relate the modes to the one absolutely infinite substance. Spinoza acknowledges in fact that there are infinite modes, such as the "face of the universe." But Cantor says Spinoza does not know how to relate finite modes to the infinite mode, and that his transfinite ordinal arithmetic laws show the proper relation (→Cantor, 1932, p. 177; →Ewald, 1996, p. 89). In the *Grundlagen*, Cantor assumes that all sets can be well-ordered and correspond to an ordinal number. Omega (ω) symbolizes the natural numbers in their usual order {1, 2, 3…}. For an ordinal number, the order of the elements is all-important, so that the

ordinal number corresponding to the sequence {ω, 1} is quite different from that pertaining to {1, ω}. These kinds of distinctions are completely lost before Cantor's distinction between cardinal number (or power of a set) and ordinal number (order type of a set). Indeed, the very distinction between cardinality (power, size) and ordinality (order type) is not really needed if one is only dealing with finite sets. Cantor evidently thinks this insight might solve Spinoza's problem of how an individual finite mode relates to an infinite mode, but it is likely that Cantor has simply changed the topic of discussion altogether. Importantly, none of these concerns of Cantor yet touch on the absolute infinite.

The absolute infinite, or 'absolute Maximum' plays a central role in Cusanus's metaphysics and is identified with God. Cusanus famously asserts that there can be no proportion between the finite and the infinite, and he agrees with the major premise of negative theology that one cannot predicate any attribute truly of God. In this respect, Cusanus often cites Dionysus the Areopagite with approval (see DDI, Book I, ch. 26). He sticks to the pre-modern view that *if* there were a true accurate description of God, consisting of a concatenation of infinitely many names, such a name would be ineffable (DDI, Book I, ch. 24; Cusanus, 1985 [1440], p. 81). So there remains a chasm between the finite descriptions that human beings employ and the truly infinite description of the Absolute. God, for Cusanus, has an infinite ineffable name. By modern lights, this would presumably have to be a countably infinite name, as if all the names could be arranged in a list. One is reminded here of Cantor's diagonal argument, which shows that even if—miraculously—were one to list all the real numbers, there is always a recipe to generate a real number not on the list. We can also thus use a diagonal argument to show that if we think we have a countably infinite series of names for God, this series

cannot be complete.[12] This amounts to an argument for ineffability, or as the theologians say, an apophatic approach to deity, in which God is described by what he is not.

To be sure, Cusanus writes without the advantage of Cantorian arguments, and cannot recognize a difference between countable infinities and uncountable infinities. God cannot have a proper (finite) name, for Cusanus, because it would offend God's nature as that which enfolds all things and is all in all. Cusanus concludes: "Sacred ignorance has taught us [the name of] God is ineffable. And by virtue of the fact … we speak of God more truly through removal and negation, as teaches the greatest Dionysus…" (DDI, ch. 26; Cusanus, 1985, p. 84).

There might seem so far to be an affinity between Cusanus and Cantorian metaphysics and theology. However, Cantor's position is quite different from Cusanus's position. Both Cusanus and Cantor (and Bruno as well) acknowledge that the universe is in some restricted sense infinite. Cusanus says that universe is "privatively infinite" and unbounded.[13] However, positive infinity is reserved for God alone. Cantor, however, does not accept that there are not positive actual infinities between the true infinite and the finite. Instead, Cantor posits the existence of a transfinite realm:

> What I assert and believe to have proved in this work as well as in my earlier writings is this: that following the finite there is a Transfinitum (which one could also call Suprafinitum)—that is, there is an unbounded step-ladder of determinate modes which in their nature are not finite but infinite, but which, just like the finite, can be determined by well-defined and distinguishable numbers. I am convinced that the domain of definable quantities is not exhausted by the finite quantities, and the bounds of our knowledge may accordingly be extended without violence to our nature. (→Cantor, 1932 [1883]; →Ewald, 1996, p. 891).

Thus, Cantor saw his transfinite numbers[14] as a kind of bridge from the finite into the true infinite or Absolute, which however, remained radically unknowable. The outlines of the transfinite realm were known, however, and Cantor was keen to point out that unlike earlier thinkers (Cusanus, Spinoza, Leibniz) who grappled unsuccessfully with the notion of 'infinite number,' he divided his transfinite numbers into number classes, ordered according to size: thus ordinal numbers are ordered in equivalence classes based on cardinality, starting with $\aleph_0$ (aleph-0), and continuing in order aleph-1, aleph-2, and so on ad infinitum.[15] Needless to say there is no last number class nor can one ask after the size of the universe of number-classes (sets) as a whole. For the whole is itself identified by Cantor with the Absolute, that is, God. Further echoing negative theology, in a very important endnote (endnote to Section 4, note 2) Cantor asserts:

> The absolute can only be acknowledged [*anerkannt*] but never known [*erkannt*]—and not even approximately known.
>
> [*Das Absolute kann nur anerkannt, aber nie erkannt, auch nicht annähernd erkannt werden*.]
>
> (→Cantor, 1932 [1883], p. 205; →Ewald, 1996, p. 891)

The language is important here. First, there is the allusion to "the Absolute," a common fixture in nineteenth-century German idealism, variously used to gesture towards God, infinite substance, and so on. Second, there is the contrast between acknowledgment (*Anerkenntnis*) and knowledge (*Erkenntnis*), with knowledge being strictly off limits in the case of the Absolute. The Absolute cannot be an object of knowledge. Third, not only can the Absolute not be known, it cannot be known "not even approximately." Interestingly, Cusanus uses the language and

metaphor of approximation to discuss how the mind may approach knowledge of the Maximum or true infinite (Absolute). In *Docta Ignorantia* I, ch. 3, Cusanus urges that the precise, exact truth about things eludes us, and that similar things could always be even more similar. The measure used (the mind or the ruler) and the measured (the object) will always remain different. “Hence the intellect never comprehends the truth so precisely that the truth cannot be comprehended infinitely more precisely. For the intellect is to truth as (an inscribed polygon) is to [the inscribing] circle” (DDI; Cusanus, 1985 [1440], p. 52) This geometrical image and metaphor is just one of many that Cusanus uses as part of his theological exercises for the mind. Cusanus is alive to the problem of the inexactness of approximating one kind of figure (a circle) with another (a polygon). This sensitivity is to be expected, perhaps, of a mathematician who expended energy trying to show—what we now know to be impossible—how to square the circle using a compass and straight-edge (see Cusanus’s *De Circuli Quadratura*, 1450).[16]

Human knowledge for Cusanus is *conjectural* and consists of conjectures (*coniecturae*), which is to say opinions based on partial information and taken from a given limited finite perspective.[17] We have conjectures about the Absolute, about God, and these are fallible proposals that may involve imagery and symbolism. Conjectures typically fall short of the knowledge that God has, and human beings must constantly strive to improve their conjectures. Still symbols and mathematical figures and signs are an important means of awareness of the divine for Cusanus, as they point beyond themselves and their contemplation prepares the mind to move beyond simple limited concepts towards a more inclusive synthesis that threatens to dissolve the concepts of individuals. For example, curves and straight lines lose their difference in the Absolute, as a curve can

be straightened out and stretched gradually until it coincides at the limit with a straight line (DDI, I Book I, ch. 13).[18]

In Cusa's epistemology we have the beginnings of a truly critical philosophy, even a fallibilist one, that is aware of the limits of human knowledge. We have no choice but to accept that the human mind is ignorant and missing much positive knowledge. However, it is not surprising that just for this reason, Cusanus was sometimes mischaracterized as a skeptic by the authors of textbooks in early modern philosophy, when in fact his position is affirmative of divinity and the world.[19] With some considerable tension, Cusanus claims that there must be both an affirmative and a negative theology.[20] So, what we find is that despite espousing negative theology, Cusanus goes on to affirm a great deal that cannot be known, but which he nonetheless believes to be true about God. He has intuitions about the qualities appropriate to divinity and finds that some are more appropriate than others (virtue rather than vice, justice rather than injustice). These qualities form the basis of worship and faith in God as Cusanus sees it.

27.4 Symbols, Mathematical Notation, and Contemplation of the Absolute

So, what can be said about the Absolute? Cantor says that: "The absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute." (Cantor 1932 (1883), endnote 2, Ewald 1996, p.79). Given the importance of symbols for someone like Cusanus, it is interesting to also see Cantor invoking the idea of using a symbol to attain some apprehension of the Absolute. Of course, neither Cusanus nor Cantor would allow that awareness of such symbolism and attentiveness to the number sequence as a symbol is sufficient for knowledge of God. Such knowledge cannot be had, and (as with Cusanus),

human beings must settle for something other than knowledge of God. This much is entirely consistent with the tradition of negative theology.

Moreover, initially in his mathematical studies Cantor did see himself to be initially dealing with mere (empty, formal) symbols, using a sideways '8' (Wallis's infinity symbol) initially for his infinite point sets. At a certain stage, certainly by around the time of his *Grundlagen* (1883), he took a more realistic view of these symbols. He introduced the symbol ω (omega) to mark this shift from empty symbolism to realism about the transfinite ordinal numbers. Much later on, in his *Beiträge* (1895), he uses the Hebrew aleph $\aleph_0$ to stand for the first transfinite cardinal number, and the sequence of alephs stands for all the cardinal numbers.[21] Of course, Cantor remained alive to the distinction between symbol (sign) and object. For him, the transfinite numbers were really existing definite numbers, and his symbols referred to these numbers, and in turn the sequence of all such numbers, the well-ordered number scale, pointed the mind to the Absolute, and as it were, functioned as a symbol or mirror of the Absolute. This conception echoes Cusanus's doctrine that the world is mirror or symbol of the Absolute, that "the universe … is a likeness of the Absolute" (DDI, Book II, ch. 4; Cusanus, 1985, p. 96).

27.5 Reason, the Law of Non-Contradiction, and Attempting to Think about the Absolute

If both Cantor and Cusanus are somewhat mystical about apprehension of the Absolute, they *nonetheless deploy logic and regard the faculty of reason differently.* A major difference between Cantor and Cusanus is, of course, their attitude as regards logic and the Aristotelian law of non-contradiction (LNC). Cusanus is rather unfazed by contradictions, whereas Cantor

accepts that contradictions have no place in mathematical knowledge. Cusanus's doctrine of the "coincidence of opposites" follows from his conception of the Absolute, also referred to by Cusanus in an Anselmian vein as "the Maximum," or "the absolute Maximum." Neglecting to distinguish between properties and predicates, objects and terms, Cusanus moves from the fact that we affirm contrary predicates of the Absolute, to the conclusion that the Absolute, containing all things, must include contrary properties. What he has in mind is analogous to the way in which although a straight-line segment and a curve are radically differently different in nature, infinitely many line segments might approximate a curve, or a curve might be straightened out into a line. Through infinite processes, at the limit, in the Infinite, distinct natures are dissolved in the One, and in this way, Cusanus holds that the law of non-contradiction (LNC) does not really apply to the Infinite/Absolute. This is a familiar mystical view, the doctrine of *coincidentia oppositorum*, but it does require limiting the hegemony of reason, logic, and the LNC (see DDI, I "Book I", ch. 4–5). For Cusanus, the absolutely Maximum "is beyond both affirmation and negation" and "transcends all our understanding . For our intellect cannot, by means of reasoning, combine contradictories... Our reason falls far short of this infinite power and is unable to connect contradictories, which are infinitely distant" (DDI, I Book I, ch. 4; Cusanus, 1985, p. 54). A truly thorough going rationalism, however, will not limit the applicability of the LNC. It is truly destabilizing to hold that individual objects lose their identity in 'the Absolute.'

Proponents of di-aletheism find in Cusanus's doctrine of the "coincidence of opposites" a kind of precedent for their view which needs correction (e.g., →Priest, 1995), as do Hegelians who suspend the law of non-contradiction (LNC).[22] Reason and LNC are subordinate in Cusanus' epistemology to other mystical

forms of awareness that are presumably not governed by such principles. The surrender of the LNC as a principle would seem to be a fairly desperate move, as it is bedrock fundamental law of thought and logic, and from a contradiction anything follows. A more appealing move, to my mind, is to view the appearance of contradictions not as an indication of di-aletheism, but rather simply as a sign of the limitations of human reason. This is a Kantian move instead of the Hegelian move. The Kantian move is to say that the antinomies of pure reason show that certain metaphysical concepts are being improperly applied by human reason to objects that transcend human experience. Cantor himself, rejecting Kantianism and the way it limits human knowledge to the realm of appearances and opens the room for skepticism, would have none of this Kantian position. Cantor was never skeptical and did not seem to doubt the existence of God.

Finally, we must acknowledge that positivists and atheists would simply regard the entanglement in contradictions as evidence of a concept with no corresponding reality. Kantians leave it open that there may be some *noumenon* (God, Absolute) with attributes corresponding to metaphysical concepts of infinity and omniscience and other divine attributes. But as the nineteenth century turned into the twentieth, and as Cantor continued to pursue his theistic views, elsewhere Friedrich Nietzsche published *Die fröhliche Wissenschaft* (1882) in which it is declared somewhat perversely that “God is dead, and we have killed him” (Section 125). The scene was set for a struggle between materialist atheism and immaterialist theism.

There can be no doubt about Cantor’s view. He is a theist, of course, but more than that he is confident that in the end logical and mathematical ideas and metaphysics will cohere. He speaks of “the unity of the All, to which we ourselves belong.” He shares Spinoza’s view that “the order of ideas and things is one and the same” (*Ethics* IIp7s). He values consistency in mathematics and

mathematical concepts and upholds the LNC without reservation. Cantor declares mathematics to be “free” and subject only to the constraint that its concepts are consistent (internally and with one another). The LNC governs all rational knowledge. While rejecting Aristotelian finitism, Cantor accepts the LNC and classical logic. He never attempts to resolve paradoxes in reasoning about the infinite by concluding that human reason or logical principles such as LNC are to blame. Nor does Cantor take refuge in the vagueness of statements about infinite collections or suspend the law of the excluded middle (LEM) as L. E. J. Brouwer and his followers would in the twentieth century.

27.6 Epistemic Humility and Epistemic Exuberance

The realm of divinity, which Cantor does believe in, is not a realm of *knowledge*. At best, there is deep *ignorance* about the Absolute. In this respect, he shares with Cusanus a kind of epistemic humility and reservation about the possibility of knowledge of the Absolute. But unlike Cusanus, Cantor is not a pessimist about the ability of the finite human mind to know all about infinity. Cusanus still held that between the finite mind and the infinite there could be no proportion, no measure, and therefore no knowledge, since he held that all knowledge was a kind of measuring. In a striking passage, Cusanus says;

> … it is evident that the unqualifiedly Maximum cannot be any of the things that we either know or conceive: when we set out to investigate the Maximum symbolically, we must leap beyond simple likeness. For since all mathematicals are finite and otherwise could not even be imagined: if we want to use finite things as a way of ascending to the unqualifiedly Maximum, we must first consider mathematical figures together with their characteristics and relations. Next, [we] apply these relations in a transformed way to the corresponding infinite mathematical figures. Third, thereafter in a still more highly transformed

> way, apply the relations of these infinite figures to the simple Infinite, which altogether independent even of all figure. At this point our ignorance will be taught incomprehensibly how we are to think more correctly and truly about the Most High as we grope by means of a symbolism. (DDI, Book I, ch. 12, 33; Cusanus, 1985, p. 63).

Thus, for Cusa, in seeking knowledge of the Absolute at best we can "grope by means of a symbolism" (DDI, I Book I, ch. 12). Cusanus takes symbols (*aenigmata*) to be the objects the mind contemplates that point beyond itself and enable the mind, however feebly, to apprehend dimly "incomprehensibly" the divine. Cusanus is acutely aware of the finitude of the human mind. His whole doctrine of "learned ignorance" radiates the virtue of epistemic humility, the awareness and willingness to admit one's own ignorance in matters of importance.

By contrast, Cantor instead rejects the very assumption that the capacity of the human mind is finite and *therefore* incapable of comprehending infinity in mathematics. Rather Cantor's own mathematical experience showed him that the transfinite symbols corresponded, he was convinced, to real actually infinite numbers. Consequently, he boldly abandons the traditional assumption that the human understanding is merely finite. On the contrary, like a demi-god, the human understanding is in a certain sense infinite:

> The finiteness of human understanding is often adduced as the reason why only finite numbers can be thought; but again I see in this assertion the vicious circle already mentioned. Namely, it is tacitly meant by 'finiteness of understanding' that its capacity with respect to the formation of numbers [*Zahlenbildung*] is confined to finite numbers. But if it turns out that the understanding can also in a determinate sense define and distinguish infinite, that is, superfinite [*überendliche*] numbers, then either the words 'finite understanding' must be given an extended meaning (from which that conclusion can then no longer be drawn); or the predicate 'infinite' must in certain respects be conceded to the human understanding. The latter is in my opinion the only correct course. The words 'finite understanding' which one hears so frequently are in my

> opinion not at all apt; however limited in truth human nature may be, still very much of the infinite adheres to it, and I even assert that if it were not itself in many respects infinite, the solid confidence and certainty in the being [Sein] of the Absolute, about which we know we all agree, would be inexplicable.
>
> In particular I contend that the human understanding has an unbounded capacity for the stepwise formation of classes of integers which stand in a determinate relationship to the infinite modes [*Modis*] and whose powers are of increasing strength. (*Grundlagen*, Section 5, par. 4, p. 176, →Ewald, 1996, p. 891, emphasis mine).

So here Cantor asserts (i) human understanding is infinite ("much of the infinite belongs to" human nature), and (ii) this explains our certainty in the existence of the Absolute. Unlike many modern thinkers after Descartes, Cantor is immune to skepticism, and was strongly of a theist persuasion.

While it would be easy to dismiss Cantor's remarks as handwaving, few theorists have done as much real foundational work in developing the mathematical theory of the infinite. Therefore, Cantor's claim that the human mind can grasp the infinite, so to speak, is in his mind substantiated by his actual mathematical experience in laying out the principles that govern the sequence of transfinite numbers, and thereby attaining some understanding (if not vision) of this sequence. It is not unusual to find pure mathematicians with such intensely spiritual inclinations.[23] Such subjective experiences do not prove anything objectively, but of course Cantor was convinced that the subjective validity of his experience corresponded to objective validity and reality. His basis for this confidence was his idea, derived from a Platonist realism and Spinoza's doctrine of parallelism, that the order of ideas and the order of things is one and the same (Spinoza's *Ethics* IIp7; *Grundlagen*, endnote 6). To be sure, realism about numbers is not an argument in itself for

theism, but rather seems to be part of a complete world-view, a mutually reinforcing package of ontological commitments.

Some might criticize Cantor for assuming that he could waive into existence numbers (including the new-fangled transfinite numbers) 'like a god,' but this was part of Cantor's pre-axiomatic method.[24] While his mathematics was not at all axiomatized like subsequent Zermelo-Fraenkel (ZF) set theory and subsequently ZFC (set theory plus the Axiom of Choice), it nonetheless did seem to follow from certain principles and laws of thought. Cantor's boldest move, no doubt, is simply *to posit* ω as the first ordinal number (*Anzahl*) to succeed all the natural numbers in their usual order {1, 2, 3, ..., ν, ...}, which pertain to the first number class (I):

> The sequence (I) of positive integers 1, 2, 3, ..., ν, ... has its ground of origin [Entstehungsgrund] in the repeated positing (Setzung) and uniting (Vereinigung) of underlying unities (Einheiten), which are regarded as alike; the number ν is the expression for a definite finite *Anzahl* of such positings following one another in a sequence; it is also the expression for the unification of the posited unities into a whole. However contradictory it might be to speak of a greatest number of class (I), there is on the other hand nothing offensive in thinking of a new number which we shall call ω at and which shall be the expression for the fact that the entire aggregate (I) is given in its natural succession according to a law. (→Cantor, 1932, *Grundlagen*, Section 11; →Ewald, 1996, p. 907)

The notion of *positing* is itself interesting: if we posit a unit, we assume its existence. So, it does seem that the existence enjoyed by a set (unity) of posited units is a hypothetical existence. *If* we may posit endlessly many units and *if* we may unite them into a unity in thought to form a set (*Menge*), then we have an actually infinite set. What guarantees that we can do so? Here indeed it might be said that Cantor *invokes realism and theism*, and much the same logical role is played in set theory by the Axiom of Infinity.

Another crucial law of thought evident in Cantor's *Grundlagen* is that all sets are well-ordered. This is known to be equivalent to the Axiom of Choice and greatly facilitates derivation of results. Again, one might ask why Cantor was so confident that sets could be well-ordered. On the one hand, he shows how to generate sets from generating principles that will ensure they will in fact be well-ordered. On the other hand, in his correspondence after the *Grundlagen*, he often resorts to an Augustinian or Christian Platonist view of the numbers.[25] There is a *prima facie* tension in Cantor's thought between his dynamic generating principles (evident in the *Grundlagen* with an emphasis on the operations such as +1 or Union or power set and taking the limit) and his repeated reference to Platonist epistemology and ideas (evident especially in the endnotes), suggesting that the Platonism may be overlaid on the original composition of the *Grundlagen.*

27.7 Conclusion

Human finitude and the limitations of the human mind do not prevent a glimpse of the quantitative infinite in mathematics. So, Cantor's epistemology departs in a major way from that of Cusanus, even though he is sympathetic to the Cardinal's view that the Absolute is not an object of knowledge. As we have seen Cantor thought that indefinite multiplicities or "absolutely infinite" collections were a symbol of the Absolute, and not an object of mathematical knowledge. Moreover, Cantor retains a reverential attitude towards the absolutely infinite in mathematics, as he regarded the number scales as providing a kind of ladder that drew the mind to move from contemplation of infinity to contemplation of the divine. The existence of the divine Absolute seems to be an unquestioned presupposition of Cantor's philosophy. Likewise, Cusanus also thought that

mathematical considerations pointed to the divine, although his primary focus in *De Docta Ignorantia* (Book I) was on geometrical transformations and the way in which individual figures lose their identity when certain transformative procedures were taken to the absolute limit. For both thinkers, the existence of the Absolute is not in question, but full knowledge of the Absolute is denied to the human mind in order to maintain the ontological primacy of the Absolute. Both thinkers represent views consistent with a Neo-Platonist realism that persists in conceptions of divinity and mathematics.

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Notes

1 Here and in the following, DDI refers to Nicholas of Cusa, *De Docta Ignorantia* (On Learned Ignorance). Translations are from Jasper Hopkins edition of Nicholas of Cusa (Cusanus), *On Learned Ignorance* (1985 [1440]).

2 Koyré (1991 [1957], p. 18). See also →Harries (2001). I would like to record an intellectual debt both to Karsten Harries' long running seminar at Yale, "Infinity and Perspective,"(see →Harries (2001)) and to Michael Della Rocca's Spinoza seminar.

3 Quotation of Thierry of Chartres taken from →Drozdek (1995, p. 134.). For the influence of Thierry of Chartres on Cusanus, see →Albertson (2014).

4 For discussion of this claim "the Domain Principle," see →Hallett (1984) and Priest (1995).

5 Cantor (1932 [1883]), endnote to Section 1, note 1, p. 204.

6 For discussion, see →Mancosu (2016, Chapter 3). Mancosu thinks that Cantor's theory was not inevitable.

7 For the depiction of Cantor as a 'late Romantic,' I have relied on Ferreiros (2009). Thanks also to Charles Barbour for impressing on me in conversation the influence of Adolf Trendelenburg in demolishing Hegelian logic.

8 So Cantor was exposed to plenty of advocates of teleology and monism in natural philosophy, and also received a holistic education (*Bildung*) that suggests it was the vocation of thinkers and scholars to develop a comprehensive world-view (*Weltanschauung*). In this respect, Cantor is "a typical representative of late Romanticism" (→Ferreiros, 2009).

9 For the general history of mathematics background during Cantor's time and after, see →Gray (2008). Also noteworthy is Ferreiros (1999). Finally, there is Dauben (1991) and →Hallett (1984).

10 Background history: →Tapp (2005).

11 Descartes (2017, Principle I, 27): "27. The difference between the indefinite and the infinite. The point of using 'indefinite' rather than 'infinite' is to reserve 'infinite' for God, because he's the only thing that our understanding positively tells us doesn't have any limits. The most we know about anything else is the negative information that we can't find any limits in it."

12 I first saw this idea, I believe, in Graham Priest's book, *Beyond the Limits of Thought* (→Priest, 1995). It contains a very useful short introduction to Cusanus and Cantor's diagonal arguments and displays the application of diagonal arguments and discusses antinomies. Priest, like Cusanus, rejects the law of non-contradiction (LNC) in some sense, and so must be a kindred spirit with Cusanus and Hegel. Cantor would be shocked at this application of his diagonal argument, I think, as he would not want to give up the LNC.

13 DDI, Book II; Cusanus (1985, p. 90).

14 For the Cantor enthusiasts, the transfinite numbers with which Cantor is concerned here are mainly the ordinal numbers, ordered by number classes of a given size.

15 The essential difference (with the conception of these predecessors Bruno, Cusanus, and Leibniz), Cantor says, is that he takes the actual infinite to be divided once and for all into number-classes (I) (the first transfinite number class), (II), (III), and so on, and only then does he look at the relationships between these infinite numbers not just as mathematical, but he also examines how they occur, can be proven, and can be pursued in nature.

16 For discussion of Cusa's constructions, see →Phillips (2011).

17 →Miller (2021).

18 Cusanus (1985, p. 63): "Indeed, in the maximum line curvature is straightness."

19 →Popkin (2003, p. 77).

20 See Cusanus, DI, Book I, ch. 24: "affirmative names befit Him [God] only in relation to created things" (Cusanus, 1985, p. 81).

21 Dauben (1979, p. 99).

22 It is debatable whether Hegel actually ever read Cusanus, but many philosophers have remarked on the affinities between their views, specifically as regards

their obsession with an Absolute One, and their willing suspension of the LNC. In any case, it would be normal for a professional German philosopher, especially one with theological training too, such as Hegel, to have such knowledge.

23 Jean Michel Kantor and Loren Graham in *Naming Infinity* demonstrate how Russian set theorist Nikolai Luzin (1883–1950), like Cantor, was drawn to the mystical neo-Platonism of Plotinus. See →Kantor and Graham (2009, p. 94ff.).

24 For Frege's often unfair criticism of Cantor, see →Tait (1997).

25 →Drozdek (1995).

VIII God and (Meta-)Mathematics

28 Reflection in Apophatic Mathematics and Theology

Neil Barton

Abstract: A long tradition in theology holds that the *divine* is in some sense incomprehensible, ineffable, or indescribable. This is mirrored in the set-theoretic literature by those who hold that the *universe of sets* is incomprehensible, ineffable, or indescribable. In this latter field, set theorists often study *reflection principles*; axioms that posit indescribability properties of the universe. This paper seeks to examine a *theological reflection principle*, which can be used to deliver a very rich ontology. I argue that in analogy with set-theoretic reflection principles, we should understand theological reflection via schematic commitment.

28.1 Introduction

Ineffability, indescribability, and incomprehensibility—the idea that the divine can neither be properly picked out by a concept, not described, nor known—is advocated on many conceptions of theology. A related idea is present within set theory and its philosophy—in particular one often assumes that the *universe of sets* is somehow ineffable.[1] This idea is often held to be captured by *reflection principles* which state that any property held by the universe of sets is held by some set of a particular kind. Such principles have many 'richness' consequences for the universe of sets, implying that various kinds of 'large cardinals' exist. Less explored, though not unexamined, is the idea that there might be a kind of *theological reflection*. Such a principle says that the ineffability of the divine can be explored via reflection-like ideas

—any property (or perhaps some analogue thereof) held by the divine is held by some creature.[2] Often such theological ideas are left somewhat underspecified and are *used* to motivate the standard set-theoretic reflection principles. However not much has been done in the other direction—using observations from mathematics and set theory to articulate the notion of theological reflection more fully. This paper seeks to (partly) fill this gap.

The idea of apophatic theology can be used to motivate theological reflection. But (as we'll see) there is an apparent tension here in that theological reflection itself can be viewed as a true claim about the divine. This is an instance of a common problem in apophatic theology, namely that the negative view seems self-undermining. I think that an approach to reflection principles in set theory—developed under a corresponding apophatic view in mathematics—can help to inform characterisations of theological reflection. In particular, I will argue that:

Main Claim. Taking inspiration from reflection in set theory, there is a formulation of theological reflection that is compatible with the apophatic stance. More specifically, we should view theological reflection as a *schematic* commitment.

Exactly what this means, and in particular the nature of theological reflection and schematic commitment, I hope to make clear in due course. For now, here's the plan for the article:

Section 28.2 will set up some theoretical background, including apophaticism/negative theology, and the ideas of incomprehensibility, ineffability, and indescribability. Section 28.3 will present the theological reflection principle and discuss some of its consequences. In particular, I note that the ontology thereby provided is very *rich*, but that there are possibilities for interpreting these somewhat counterintuitive ideas. Section 28.4 presents a possible problem (familiar in the literature on

negative theology) that the apophatic theologian contravenes their own doctrine. I'll adapt this problem to the specific case of theological reflection. Section 28.5 goes on to present what I call 'apophatic mathematics'; the view that only first-order claims about the universe of sets can be true. Section 28.6 then explains how the apophatic mathematician faces a similar problem concerning set-theoretic reflection as the apophatic theologian does concerning theological reflection. I'll explain how the apophatic mathematician can respond to this problem by using a notion of *schematic commitment*. Section 28.7 will then explain how theological reflection can similarly be understood via schematic commitment, dissolving the tension of Section 28.4. Section 28.8 provides some conclusions and open questions, in particular noting issues that need to be addressed by the proponent of theological reflection.

28.2 Apophatic Principles

Let's first set up some claims about divine incomprehensibility, indescribability, ineffability, and apophaticism. These details will be known to specialists in the philosophy of religion, but may be less familiar to philosophers of logic and mathematics. Moreover, the terminology in this literature can be somewhat fraught, so it will be helpful to lay down what we mean by various terms. We start with:

Definition 1. (Informal) Say that an entity[3] is incomprehensible to a community *C* given a class of properties *P*, if members of *C* are unable to grasp truth about whether properties from *P* hold of it.[4]

We can now lay down the following principle:

Definition 2. (Informal) *Incomprehensibility*. The divine is incomprehensible to the whole of humanity for a significant class of properties *P*.

Of course, the rub will come in specifying for what class of properties we take the divine to be incomprehensible. We'll examine some responses to this problem in a moment, once we have some other principles on the table. For now, we should note what it means for the divine to be *incomprehensible*—it is a claim about the extent to which properties held by the divine (a metaphysical fact) are (not) humanly graspable (an epistemic state of affairs). Closely linked is the idea of:

Definition 3. (Informal) We say that an entity is ineffable to a community *C* relative to some concepts *Co*, if there is no concept from *Co* that *C* could employ that would pick it out.

And again, we can identify:

Definition 4. (Informal) *Ineffability*. The divine is ineffable to the whole of humanity for a significant class of concepts.

Note that there is not a metaphysical component to *Ineffability* (as there was *with Incomprehensibility*). Rather *Ineffability* is a claim about what can be done with *concepts*. In this way it is a *metasemantic* claim about how our concepts relate to entities in the world. A final piece of our puzzle is:

Definition 5. (Informal) An entity is *indescribable* to a community *C* using language $\mathscr{L}$ if *C* is unable to pick it out using language $\mathscr{L}$.

Definition 6. (Informal) *Indescribability*. The divine is indescribable to humanity using human languages.

Indescribability (unlike *Ineffability* and *Incomprehensibility*) is a linguistic claim, concerning how our languages and language use relates to entities.

For the sake of brevity, I'll fix the following:

Definition 7. (Informal) I'll refer to *Indescribability, Ineffability*, and *Incomprehensibility* as the *3Is*.

Let's discuss the relationship between the *3Is*. First, we should note that these claims are often put forward by many theological standpoints. Ideas of ineffability appear in Hinduism

(→Priest, 2018, §2.3) and Buddhism (→Priest, 2014, §13.9). Within Christian thought the claim is repeatedly advanced, for example by Gregory of Nyssa, John Chrysostom, Pseudo-Dionysus, and Thomas Aquinas. The literature here is enormous, and so I will only discuss it insofar as it relates to the issues I'll consider below.[5]

However, we should also note that each is a separate claim, despite the fact that the *3Is* are often thrown together. An entity can be incomprehensible (given some properties), but neither ineffable nor indescribable (say because the community cannot grasp how the property corresponding to the concept picking out an entity applies to said entity). Similarly, ineffability entails neither incomprehensibility nor indescribability (say because (i) the properties that apply to it are graspable, but do not correspond to any possessed concept, and (ii) there is a piece of language that describes the entity, but not through any concept possessed by the community, e.g. in a case of deference to an expert outside the community). Finally, indescribability entails neither ineffability nor incomprehensibility (say if a property is graspable and a concept picks out an entity, but the community lacks the language to express claims involving the concept).

However, the *3Is* clearly *fit well together*, even if there are not *logical entailments* between them. If an entity is incomprehensible, then it is likely it is ineffable and indescribable too (since if its properties are not graspable, then it is unlikely that we have the linguistic/conceptual resources to pick it out). If an entity is one of ineffable or indescribable, and we hold a tight metasemantic link between language and concepts, then it will likely be the other too. And we might take both ineffability and incomprehensibility as indicative that the properties of the relevant entity are not graspable.

Second, we might think that the *3Is* are necessary features of the divine (at least for a broad class of possible worlds). As

Simon Hewitt puts it:

> With the proviso that the needed clarification will be provided, I think [*Indescribability*] is true. Nor is it true in a purely contingent way—it doesn't just so happen that God can't be described in words, but had our languages been more sophisticated or our brains bigger, things would have been different. The intended reading of [*Indescribability*] is one that could be appropriately interpreted in the familiar heuristic: at no (metaphysically) possible world can God be described in words. (→Hewitt, 2020, p. 13)

It thus is not merely that if we were just cleverer and/or could speak a richer language, we could grasp properties of the divine or use concepts that picked it out. Rather, it is of the divine's nature that it be always incomprehensible/ineffable/indescribable, at least insofar as we are human. (Perhaps the divine is self-comprehensible or can pick out itself in some sense, but for *human* possibility, it is necessarily incomprehensible/ineffable/indescribable.)

Often the *3Is* (and their necessity) are linked to the following aspect of the divine:

Definition 8. (Informal) *Transcendence*. The divine is radically dissimilar from every other entity.

Radical dissimilarity can be characterised in terms of logical space.[6] I can say that my couch is dissimilar from my table in various respects. But they are not *radically* dissimilar in virtue of being members of many of the same contrast classes (e.g. furniture, physical objects, etc.). Radical dissimilarity on the other hand occurs when entities cannot even be put into the same contrast class or region of logical space, there is no intelligible basis on which to contrast them. *Transcendence* is sometimes seen as the *metaphysical* correlate or motivation for the *epistemic 3Is*. If the divine is radically dissimilar from every other entity then it makes sense that its properties would not be graspable

to us, or that we couldn't pick it out with concepts/language. Since we primarily come into contact with creaturely entities, and our language is designed for talking about these, there is a gap between the tools we have and the nature of the divine.[7] The idea that the 3Is might hold links closely to the idea of:

> *Negative theology*/*Apophaticism*. For some class of propositions *P*, we can only *deny* sentences expressing propositions from *P* (or perhaps propositions from *P* are not truth-evaluable), and we cannot *truthfully assert* any proposition from *P* of the divine.

The motivation from the *3Is* and *Transcendence* is as follows. Given these assumptions, it's natural to think that given a property, concept, or description of the divine, it falls short, especially since the divine does not even occupy the same region of logical space. Therefore, we cannot truthfully assert that said property, concept, or description applies.

Apophaticism has been advanced variously throughout the literature. Famous examples include Nicolas de Cusa, Augustin, Karl Barth, Plotinus, and Pseudo-Dionysus, as well as more recent authors. Again the literature is large and others are better placed to assess it, so I'll say little about it here.[8]

Apophatic theology contrasts with *cataphatic* theology, on which we are able, for a given class of propositions, to assert them of the divine. But now a puzzle emerges. We'll term this difficulty, following Lebens (2014) (who in turn is drawing on →Plantinga, 2000), the problem of Incoherence. Suppose that we are committed to a strong form of the *3Is* and *Apophaticism*, on which we think that nothing can be truthfully asserted of the divine. But then we simply note that each of the *3Is* and *Apophaticism themselve*s make apparent (truthful) assertions about the divine. Thus (so the charge goes) *Apophaticism* must draw on cataphatic assertions in stating the position. So, by their

own lights, they contravene their own doctrine. The position is self-undermining.

Exactly how one cashes out this problem is something of a subtle issue (→Lebens, 2014, and →Hewitt, 2020, ch. 2, provide a thorough examination) but the above informal statement will be enough for present purposes. The problem has been around pretty much as long as *Apophaticism* itself, and again there's lots one might discuss here. We'll narrow down on one response to the problem and set some others aside. Since we will want to say that properties reflect from the divine to creatures, and motivate this on the basis of the *3Is* and *Apophaticism*, I'll want to:

1. be able to say that the *3Is* and *Apophaticism* are *correct* in an important sense (e.g. they are not merely false but heuristically useful claims), and
2. be able to say that there *are* similarities between the creaturely properties we're familiar with and ones held by the divine.

This will rule out some responses to *Incoherence* as unfit for what we'll do here (though they may be useful for other purposes). One could, for example, take a dialethist approach, accept the contradiction, and adopt a paraconsistent logic for reasoning about the divine. This is the strategy of →Beall (2021) and Priest (MS). Whilst interesting, I lack expertise here (and also I happen to not be a dialethist), so I'll set them to one side. A different approach is that of Lebens (2014) who (drawing on Wittgenstein) takes the *3Is* to be illuminating falsehoods (→Hewitt, 2020, suggests that we should take them instead as meaningless within Lebens' framework). This clearly contravenes (2). An alternative is to delineate some class of properties and hold that the *3Is* and *Apophaticism* lie outside this class.[9] One such is Hick's (1989) distinction between formal, non-formal, negative, and

positive properties. I find these distinctions hard to maintain and it's unclear to me that they can do the job one wants. I will simply refer the reader to Lebens (2014) for some criticism of this approach. Another is the 'fundamental properties' approach of →Jacobs (2015), on which we can say nothing about the divine's fundamental intrinsic nature, but can make other claims (the *3Is* and *Apophaticism* then become non-fundamental claims). This would be fiddly to operate with regarding reflection, as we'll constantly have to assess whether our predicates are fundamental or not, so again I'll shelve it for now. Another way out is provided by →Keller (2018), according to which knowledge of the divine should be understood as 'Franciscan' (i.e. personal) rather than propositional. Again this will create difficulties for reflection, and so I set it to one side.

Perhaps the above can be modified so as to work with reflection, but I'll leave this open. For now, we'll want something satisfying both (1) and (2). For this I will follow approaches that hold that the divine holds properties that are *analogical* to the ordinary creaturely ones that we're familiar with. Such an idea can be linked to the work of Aquinas and Anselm (see Chapter 65 of the *Monologian*), and include the apophatic approaches of Herbert McCabe and Alasdair MacIntyre, as well as the more recent Grammatical Thomism of Simon Hewitt.[10] Going through the many subtle details of these theological approaches would take us too far afield. However, what is common to all is that they hold that whilst we can't make true *univocal* predications of the divine, we can gain *partial* understanding of the divine's properties via *analogy*. Again there are various ways to do this. One can posit a realm of 'analogous' divine predicates/properties to which we don't have access. However, there are also more 'mundane' ways of understanding the distinction. For example, in Hewitt's (2020) Grammatical

Thomism, analogy should be understood via the extension of predicate use:

> Imagine that we know how to use a predicate *F* with respect to a certain class of entities. We know when to apply *F* to a member of the class, and we know when to withhold it. The know-how we thereby exercise may be vague or admit borderline cases. To know how to use an expression is to be initiated into a social practice, and these are often not precise affairs. Still, we know how to use *F* of a certain portion of reality. Now suppose we consider the members of some second disjoint class of entities. We wonder whether we ought to say (of some or all of them) that they are *F*. At least some of the rules of use for *F* with respect to the original class are not applicable to members of the new class. It nonetheless does not follow that we must refuse to predicate *F*-ness of any member of the new class. It may be that some of the rules for *F* are applicable within the new class, as when say of human bodies and dinners alike that they are healthy, and in both cases are prepared to infer that this is something desirable. Then again it may be that entities in the new class bear some interesting relation, perhaps causal, to the *F*-ness of entities in the old class, such that it seems appropriate to us to apply *F* to at least some of these entities. Again, this is the case with predications of healthiness: we call dinners healthy because they cause healthiness in human beings…
>
> In a case where use is extended in this sort of continuous way, or where variation in use could be illustrated by the supposition that it has been, I will term the relationship between the uses (and therefore the meaning) analogous. Moreover this is all I have in mind when talking about analogy: no high metaphysics, no systematic theory of predication, merely the observation that the use of words can be extended in certain ways. (→Hewitt, 2020, pp. 113–114)

So there are a great many ways that we might cash out this analogousness. I'll remain neutral on this score—I do not require much, I just need some way of there being a *resemblance* between creaturely properties and divine ones. And these approaches provide a way of getting as close to cataphatic statements as possible, whilst maintaining a degree of *Apophaticism* consistent with the *3Is*.

There is a question of whether or not they go too far, and lapse into full-blown cataphatic language. But there are scholars who hold that such use is consistent with *Apophaticism*. As MacIntyre writes:

> Theists in recognising that God exceeds the grasp of our understanding must also recognise that in trying to speak of God we are extending our use of words and the application of our concepts, so that we no longer understand what we mean when we talk about God to the same extent and in the same way that we do in our speech about finite beings. (→MacIntyre, 2009, p. 7)

So this talk of 'extending' is important, we simultaneously are able to make meaningful claims and have *partial* understanding, the view is apophatic in that we are unable to make *univocal* claims about the divine.

Interestingly several mathematical logicians have found *Apophaticism* attractive. Cantor writes:

> The absolute can only be acknowledged, but never known—and not even approximately. (Cantor, in →Ewald, 1996, p. 916)

Whilst Cantor proposes *Apophaticism* here, it is a very strong version and it is unclear whether this is compatible with analogousness accounts of divine properties.[11] Gödel, however, seems more sympathetic. He writes the following in his notebooks:

> It is clear that we have no adequate concept of God but merely approximations. (→Gödel, 2021, p. 252)

Gödel's talk of 'approximations' is highly suggestive of the analogousness account of divine properties. It is thus interesting to note that this kind of *Apophaticism* appears in the ideas of

mathematical logicians of the late nineteenth and early twentieth century, as well as the more traditional sources.

Perhaps the approach sketched here dances dangerously close to cataphatic theology. However, the ability to make kinds of predications will be needed for what follows, and I hope that the examination of theological reflection will actually help elucidate the apophatic view.[12]

28.3 A Theological Reflection Principle

Given the *3Is, Apophaticism*, and the characterization of divine talk via analogousness, we can come to a consideration of the idea of *theological reflection*, the main focus of this paper. In this section, I'll state the principle, discuss its historical roots, and explain some of its consequences.

Let's start with some motivation. Given that the *3Is* hold, we know that we fail to talk about the divine *univocally*. Instead our talk should be taken *analogically*. But it certainly seems that there are people who *have something in mind* when they *aim* to talk univocally about the divine. There are plenty of folks out there who really do think that there is a being who is all-powerful, where powerfulness should be understood as a property/predicate of the same logical kind as my ability to lift a certain amount of weight, but more. One way to interpret these folks is simply as confused, conflating univocal predicates/properties with their analogical counterparts (this is very much the default). Certainly, this has to be part of the story for the apophatic theologian. But perhaps we can be more charitable. Perhaps there is something in the world that they conflate with the divine because it very *closely resembles it*.

This view, at first blush, looks bizarre. Part of what I'll do in this section is formulate the principle and try to argue that it's

perhaps not as outlandish as it seems. To get going, let's formulate the principle a little more carefully:

Definition 9. (Informal) The *Theological Reflection Principle* (or *TRP*) is the claim that if the divine possesses a (possibly ungraspable) property *P*, then there is a creature that possesses a creaturely property *P*' (of which *P* is the divine analogue).

I make no secret of the fact that I've basically just extracted this formulation from set-theoretic reflection principles. If you replace 'creature' with 'set,' 'divine' with 'universe of sets,' and interpret 'analogue' via quantifier restriction, you'll get a set-theoretic reflection principle.

What is interesting is that (1) there are significant similarities between this principle and some things theologians have said, and (2) similar moves can be made in the case of theological reflection as set-theoretic reflection. The eagle-eyed reader may already see revenge problems lurking in this latter regard. One of the main points of this paper (as we'll see after this section) is that similar responses are available in each case. For now, let's examine some places where this kind of theological reflection is proposed.

van Atten (2009) and →Horsten (2016) note the following passage from Odo Reginaldus, as quoted by Côté:

> How can the finite attain [knowledge of] the Infinite? On this question some said that God will show Himself to us in a mediated way, and that he will show Himself to us not in His essence, but in created beings. This view is receding from the aula... (Odo Reginaldus, quoted in →Côté, 2002, p. 78; Horsten's (2016) translation)

van Atten claims the following regarding Reginaldus' remarks:

> From here it is only a small step to: "Suppose creature A has a perception of God. Then God is capable of making a creature B such that A's perception cannot distinguish between God and B." (→van Atten, 2009, footnote 84, p. 22)

Presuming (in line with Apophaticism) that we cannot distinguish a creaturely property from its divine analogue (call this idea (∗)) van Atten's principle closely relates to the *TRP*. By the *TRP*, given a property *P* possessed by the divine, there is a creature *C* possessing some *P*' of which *P* is an analogue. By (∗), we can't distinguish the divine from *C* using *P*'. Assuming that this goes for all creaturely perceptions of the divine (in line with the necessity of the *3Is*), given a creature *A* and a 'perception' of some property of the divine, we can always generate such a *C*. So even if not obviously equivalent, van Atten's principle looks motivated if we assume the *TRP*. We'll put this to one side since for the apophatic theologian, 'perception' of the divine is a tricky issue. The *TRP* is a little easier to state given what we have, but it is an interesting question whether there could be a 'perceptual' version of the principle.

Horsten (2016, pp. 114–115) conjectures that by "view that is receding from the aula" Reginaldus is referring to Philo of Alexandria, who says in "On Dreams":

> Thus in another place, when he had inquired whether He that is has a proper name, he came to know full well that He has no proper name, [the reference is to Exodus 6:3] and that whatever name anyone may use for Him he will use by licence of language; for it is not in the nature of Him that is to be spoken of, but simply to be. Testimony to this is afforded also by the divine response made to Moses' question whether He has a name, even "I am He that is (Exodus 3:14)." It is given in order that, since there are not in God things that man can comprehend, man may recognise His substance. To the souls indeed which are incorporeal and occupied in His worship it is likely that He should reveal himself as He is, conversing with them as friend with friends; but to souls which are still in the body, giving Himself the likeness of angels, not altering His own nature, for He is unchangeable, but conveying to those which receive the impression of His presence a semblance in a different form, such that they take the image to be not a copy, but that original form itself.

This, Horsten has argued in recent work, can be viewed as an endorsement of something like our *TRP*:[13]

> This is a reflection phenomenon not from the world to God (as with Augustin), but from God to the world. An 'angel' reflects the essence of God in the form of an image. But this angel-image is such a perfect copy that we cannot distinguish it from God in any way, so we humans tend to take such an 'angel' to be God himself. (Horsten, MS, p. 6)

So versions of the *TRP*, or perhaps *TRP*-like intuitions, are at play in the thought of historical figures as well as contemporary authors. But what might some *consequences* of the *TRP* be?

Well, it seems clear that if the divine satisfies the *TRP*, then the world is going to be *extremely* ontologically rich. Presumably, the divine is not finite (a core tenet of many religions). And presumably our grip on this kind of 'analogous' infinitude is sufficient to get us an infinite creature via the *TRP*.[14]

But standard reflection-style arguments are then available. Call our infinite creature above C_1. We can then observe (using the assumption that the divine explains the existence of every creature), argue that the divine is analogously-infinite and can analogously-see an infinite creature (namely C_1). We then (by the *TRP*) get a creature C_2 that reflects this property and can see C_1. Clearly this idea iterates to get any number of infinite creatures you like.

There are some similarities to Cantor's theologico-mathematical thought here. After remarking that the Absolute can never be known (and noting a similarity between his thought and the apophaticist Nicolas de Cusa) he provides a similar kind of reflection argument:

> For just as in numberclass (I) every finite number, however great, always has the same power of finite numbers greater than it, so every supra-finite number, however great, of any of the higher number-classes (II) or

> (III), etc. is followed by an aggregate of numbers and number-classes whose power is not in the slightest reduced compared to the entire absolutely infinite aggregate of numbers, starting with 1. As Albrecht von Haller says of eternity: 'I attain to the enormous number, but you, O eternity, lie always ahead of me.' The absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute; in contrast, the infinity of the first number-class (I), which has hitherto sufficed, because I consider it to be a graspable idea (not a representation [Vorstellung]), seems to me to dwindle into nothingness by comparison. (Cantor, in →Ewald, 1996, p. 916)

Although Cantor has the infinite cardinal/ordinal numbers in mind here, the intimate relationship he sees between the mathematical and theological absolute suggests a more theological reading of this idea, in line with our reasoning above.[15]

It is possible that we can extend even further. Above, we reflected *individual* properties to particular creatures. But the creatures we obtain can change as we consider different properties. What if we wanted an "angel-image" that is a "perfect copy" (as in Horsten's (MS) reading of Philo of Alexandria)? We would then need a creature that reflected every (analogous) property of the divine. We therefore define:

Definition 10. (Informal) The *Extended Theological Reflection Principle* (or *ETRP*) is the claim that there is a creature *C*, such that if the divine possesses a (possibly ungraspable) property *P*, then *C* possesses *P*' (of which *P* is the divine analogue).

This principle is stronger than the *TRP* due to the different scope of the quantifiers. Suppressing the talk of analogue vs. non-analogous properties: For the *TRP* we had that for any property *P*, there is some creature *C* possessing *P*, for the *ETRP* we have a single creature *C* that reflects every property *P*.

We can strengthen further by giving additional axioms that can be fed in to the *TRP*/*ETRP*. Note that we can view "*x* satisfies the *TRP*/*ETRP*" as a predicate itself (I've used this predicate

above). We can then ask whether there is a property corresponding to this predicate. Here's a way of making these claims:

Definition 11. (Informal) The *Double Reflection Axiom* (or *DRA*) states that there is both a creaturely and divine property corresponding to the TRP. The *Extended Double Reflection Axiom* (or *EDRA*) states that there is both a creaturely and divine property corresponding to the *ETRP*.

Adding the *DRA* or *EDRA* on top of the *TRP* or *ETRP* has some interesting consequences. Under both, we obtain a creature that satisfies some creaturely version of the *TRP*/*ETRP*, simply by using the *TRP*/*ETRP* to reflect the *divine version* of the *TRP*/*ETRP* to a creature with the *creaturely version* of the *ETRP*/*TRP*. Let's see just how strong this can be by adopting the *ETRP* and *EDRA*. Since we have the *ETRP* and *EDRA*, we can have a creature C_1 that is an almost perfect copy of the divine, up to and including the *ETRP* itself. But then, running our familiar reflection argument, we obtain a creature C_2 that can see C_1 and also satisfies the (creaturely version of) the *ETRP* and *EDRA*, and is also an almost perfect copy of the divine. Iterating this idea, we can generate any number we like of creatures that both reflect and closely resemble the divine.

There are affinities here with views of 'angelic orders' proposed by the likes of Pseudo-Dionysus. In *De Coelesti Hierarchia* he suggests that there are three hierarchies of angels, each of which is split into three orders. The theology proposed here suggests that Pseudo-Dionysus was on the right track but out by many infinite orders of magnitude. The also finds expression in art. Under the ETRP and EDRA, Francesco Botticini's "The Assumption of the Virgin" needs to be extended, fractal-like, into an infinite reflecting tower of angels. Gustav Doré's illustration of the abode of God in "Dante's Paradiso" gets closer to the mark, but he still ran out of space and ink.

The view of the divine and the creaturely proposed by the *TRP* is thus *exceedingly* rich. One might, at this point (especially given the comparison with angels) worry that it is just obviously wrong. All this talk of apparently rich and reflecting beings is surely too much for a respectable theology, no?

I am not sure, once one is engaging in talk of the divine and *Apophaticism*, that holding these kind of reflection 'axioms' is *substantially* worse. We are already engaging in philosophy on the verge of mysticism. Adopting theological reflection is a bit like taking one small step off the Apollo 11 lunar lander. Getting to space and landing on the moon whilst keeping the astronauts alive was the bigger step.

In any case, there is a dialectical point to be made here. Even if the view is *false*, if it can be made *coherent*, then this represents a win for apophatic theism. If we can argue that the view is coherent *even when augmented* with ludicrously strong principles like the *ETRP* and *EDRA*, then we can have an increased confidence in the view itself. One can never make a coherent position incoherent by taking away assumptions.

We should, however, try to make these reflection ideas somewhat more palatable. One point to note is that the kind of ontological richness postulated appears in the thought of Gödel, who writes in his notebooks:

> … in order to get to know an object *a*, I need to have: 1). certain fundamental mappings of certain empirical objects (possibly beginning with God = universal set; devil = an irrefutable wrong system (or empty set); human being = ω_1; animal = ω (or correct ethical and theoretical system); time = real number). (→ Gödel, 2021, pp. 246–247)

Gödel's remarks are somewhat cryptic, and it is a little unclear what epistemological point he is trying to uncover. However, it is clear that he has a picture on which entities can be correlated with cardinal numbers (or perhaps ordinals), and in particular

animals can be mapped to the countable, humans to the least uncountable cardinal, and the divine to the ċardinality' of the universe. An immediate question is then whether the proper class of cardinals above $\aleph_1 = \omega_1$ have corresponding entities. And a natural thought is that they can be, and these 'angels' can just be thought of as the entities that are so correlated. Assuming that this is at least on the right track, and Gödel had thoughts in line with the ontological richness yielded by the reflecting theology proposed here, we are at least in good company.[16]

However, more can be said. It is tempting, given the kinds of beings delivered by theological reflection, to think of these 'angels' as fantastical divine-like beings (with flaming swords and all the other clichés). But there is no *requirement* to think of them this way. Rather they may be conceived of as more abstract kinds of creature. For example, one could think of them as abstract possibilities for existence, that may or may not be physically instantiated. Though we think that some infinite cardinals are physically instantiated (e.g. it's a popular assumption to think that spacetime is continuum-sized) we are not required to think every cardinality is so instantiated. Similarly with these 'angels' delivered by reflection; they could be understood as abstract entities that do not interact with the physical realm directly and are not physically instantiated.

There is a certain affinity here with Spinozean metaphysics. Under one reading, Spinoza holds that the divine is a being with infinite attributes that exhibit an infinite diversity of modes. Any possible way of being is exemplified by these modes (to have anything less would be a limitation).[17] So one way of thinking of these 'angels' might be to view them as partial reflections of the entire structure of attributes. And to have less than what is given by the theological reflection would be a kind of limitation. (Of course, it is unclear whether the theological reflection can be

squared with Spinoza's monism, but we set this issue to one side.)

These observations indicate some possible ways out of viewing theological reflection as delivering a radically implausible ontology. But before moving on, I should emphasise that these are the mere germs of a response that would need working out more fully in order to be satisfactory. Moreover, a full defence of the coherence of theological reflection should really be obtained by formalising and providing a mathematical model for the 'axioms' I've proposed, and I don't have the space to do so here. My focus here is not to provide a complete response to these thorny issues, but rather indicate some affinities between theological reflection and reflection in set theory. And I do think (as we'll see) that the relationship between the two is sufficiently close to indicate that the former is coherent.

Before we move on to this relationship, I want to identify a nice feature delivered by theological reflection concerning epistemology of the divine. A salient problem given *Apophaticism* is that we cannot get knowledge of the divine. As noted earlier, a natural way to do so is via some understanding of analogousness. In this context, we gain partial understanding of the divine's properties via creatures. As Hewitt writes:[18]

> … all talk of God is licensed via more direct talk about creatures, … it arises out of radical questioning of more readily comprehensible creaturely reality… (→Hewitt, 2020, p. 78)

One question, given this perspective, is to what extent we can gain partial understanding of the divine via studying creatures. Given theological reflection, we have a tremendously positive outlook on this question. For, any property of the divine is instantiated by some similar property in the creaturely, and we

can have understanding of creatures. Theological reflection thus guarantees us the existence of the creatures needed to obtain partial understanding of the divine: we can substitute direct investigation of the divine with examining creatures (and their properties). Thus, while the theology proposed under the theological reflection, the *3Is*, and *Apophaticism* is negative, it is perhaps the most positive of such theologies. Our understanding is always partial, but can always be improved via a more expansive understanding of creaturely reality.[19], [20]

28.4 Tensions between Apophatic Theology and Reflection

We are now in a position where we have some characterisations of theological reflection on the table. We've seen that it whilst they appear to present a very strange pictures of the world, they can be made somewhat more palatable and have some attractive properties for the apophatic viewpoint. In this section, I want to raise a challenge for the advocate of the theological reflection. We'll then see in the rest of the paper how to resolve this tension via comparison with the set-theoretic case.

The core question is whether, by advocating the theological reflection, we thereby lapse into cataphatic theology. This is essentially the old problem of Incoherence mentioned above, but theological reflection presents an additional challenge.

The core issue is that to possess the *TRP*/*ETRP* is *itself* a property that the divine can satisfy (we seemed to trade on this when formulating the *DRA*/*EDRA*). But presumably the *TRP*/*ETRP* are understood as a non-analogous claim about the divine. If this is the case, however, then we seem to have contravened Apophaticism, we have predicated something of the divine *univocally*. The problem is brought into sharper focus if we consider one natural way of phrasing the *TRP*/*ETRP* via

quantification. We'll just consider the TRP, but it easily generalises to the *ETRP*. One natural way we might state the *TRP* is as follows:

Definition 12. (Informal) The *Quantificational Theological Reflection Principle* (or *QTRP*) holds that for any property *P*, if *P* is held by the divine and if *P* is the analogue of a creaturely property *P*', then there is a creature with *P*'.

The *QTRP* looks like it could be additionally problematic. The version of *Apophaticism* we're considering merely has a notion of analogous property, and a commitment that claims about the divine should be understood as partial understanding of analogous properties. But here we are quantifying over all properties, both analogous and non-analogous. How should we understand this? Is this not an attempt to predicate something univocally of the divine? Have we not lapsed into cataphatic theology?

The issue is especially troubling, as it presses a kind of revenge incoherence problem. Note that we motivated theological reflection on the basis of the *3Is* and *Apophaticism*. Perhaps this motivation can be questioned. But if it is accepted, then *even granting* a notion of analogousness for the apophatic theologian, there is a still a question of *Incoherence* using theological reflection. If *Apophaticism* and the *3Is* motivate theological reflection, but theological reflection is incoherent with *Apophaticism* and the *3Is*, then *Apophaticism* and the *3Is* are incoherent (at least insofar as the motivation is robust). Do we have a reductio of the apophatic viewpoint?

In the rest of the paper, I will show how this problem can be overcome. We will see that there is a version of mathematics that is apophatic in its own way, and that similar problems concerning reflection occur there. We will also see, however, that the set-theoretic case is perhaps more tractable and there is a

clear answer there, and the similar theological problem admits of a similar solution.

28.5 Apophatic Mathematics

An important distinction in set theory is the distinction between *proper classes* and *sets*. A *proper class*, loosely speaking, is an extensional class that does not form a set (on pain of contradiction). Familiar examples from set theory include the Russell class (the class of all non-self-membered sets), the class of all ordinals, and the universal class. There is a rich and detailed literature on how to handle talk of classes (and whether, for instance, we can think of every class as a set in an expanded domain).

For our purposes, we will want to consider an analogy with the absolute as it appears in theology, and the absolute as it appears in set-theoretic mathematics. For this reason we will consider the following position:

Definition 13. (Informal) *Universism* is the claim that there is a unique universe of set theory that contains every possible set.

Universism contrasts with multiversism (the position that there are multiple equally legitimate universes of set theory, with no maximal such), and potentialism (the position that the universe is modally indefinite).[21] In the context of set theory, one can view *Universism* as a little like (mono-)theism in theology: There is an absolute out there.

Given *Universism*, there is a question of explaining the nature of proper classes, given that they are also extensional collections. As Boolos writes:

> Wait a minute! I thought that set theory was supposed to be a theory about all, "absolutely" all, the collections that there were and that "set" was synonymous with "collection." (→Boolos, 1998, p. 35)

There are some responses out to Boolos' challenge. For instance, proper classes could be thought of as given by plural reference to sets or extensions of properties/predicates.[22] One option that is *not* available to the universist, however, is to simply view proper classes as sets in some expanded domain (since our universe provides a maximal domain). A different possibility is to hold that there are *no* proper classes. This meshes well with the following position:

Definition 14. (Informal) *Apophatic Mathematics* is the view that all talk of proper classes is meaningless (or perhaps false, where it involves existential quantification). The only legitimate proper class talk is that which can be paraphrased by first-order talk about sets.

The parallel with *Apophaticism* is that there is there is class of statements that we cannot truly assert about the absolute. Perhaps there are other assertions that we can make (e.g. *Universism* itself is a claim about the absolute), but for mathematical statements (set theory) or statements about divine properties (theology), we cannot make true assertions. Instead, all talk of the absolute can only be true insofar as they talk about sets (set theory) or the creaturely (theology). There is thus a clear parallel here between sets/creatures and the universe of sets/the divine, and how we talk about them.

This said, we do use proper classes in set theory (much as the apophatic theologian will often predicate things of the divine for ease of expression). So how exactly does one interpret proper class talk given *Apophatic Mathematics*? Before we get going, it will be useful to set up the following:

Definition 15. The *language of set theory*, or $\mathscr{L}_{\in}$, is the extension of the language of first-order logic (including identity) with a single non-logical dyadic predicate symbol "$\in$" (intended to denote membership) added.

Now, we can begin by noting that there are various proper classes that are *defined by formulas* in $\mathscr{L}_{\in}$. Good examples are the formulas "$x \notin x$" (the Russell class), "x is an ordinal" (the class of all ordinals), and "$x = x$" (the universal class). So one option is just to regard *all* talk of classes this way. Given a set x, and adding a parameter $\bar{x}$ for x to $\mathscr{L}_{\in}$, we can interpret claims like "x is a member of the Russell class" simply as "$\bar{x} \notin \bar{x}$." Whilst the former statement seems to talk about classes, the latter is simply a sentence in $\mathscr{L}_{\in}$ with $\bar{x}$ added. And this technique can be used for far more complicated sentences, extending to discussion even of large cardinals. This is perhaps the traditional approach that is used in many set theory textbooks (e.g. →Kanamori, 2009). And it provides the apophatic mathematician with a way of interpreting the (very useful) proper class talk in a manner more congenial to their viewpoint.

The approach is somewhat controversial. Many find more liberal interpretations of proper class talk preferable, on both philosophical and mathematical grounds.[23] But we will set aside these tricky issues in the philosophy of set theory—*Apophatic Mathematics* is clearly a popular and viable view. What is important for us is that the apophatic mathematician may want to use versions of *set-theoretic* reflection, and we will see similar problems as in the theological case.

28.6 Reflection for the Apophatic Mathematician

The basic idea of a reflection principle is that the universe of sets is so rich that parts of the universe resemble the universe itself. This motivation is somewhat controversial, but let's just assume it for now.[24] Reflection principles are often seen as some of the most attractive principles of set theory.[25] The relationship between theological reflection and set-theoretic reflection has

already been noted by →Welch and Horsten (2016) and Horsten (MS). Important for us will be a specific challenge in the case of the apophatic mathematician. To see this, let's discuss a little the role of reflection in set theory.

Insofar as the most widely accepted theory of sets (Zermelo-Fraenkel set theory with the Axiom of Choice or *ZFC*) is part of our conception of set, so is some kind of reflection. Within *ZFC* (indeed *ZF*)[26] the universe can be stratified into levels. In particular, one can define:

Definition 16. (*ZFC*) Let $\wp$(x) denote the *power set* of *x* (i.e. the set of all subsets of *x*). The *Cumulative Hierarchy of Sets* or *V* is defined as follows:[27]

1. $V_0 = \emptyset$;
2. $V_{\alpha+1} = \wp(V_\alpha)$, where α + 1 is a successor ordinal;
3. $V_\lambda = \bigcup_{\alpha<\lambda} V_\alpha$ (if λ is a limit ordinal).

We can also prove:

Theorem 17. (*ZFC*) For every set *x*, there is an ordinal α such that $x \in V_\alpha$.

One question that is often asked is whether there are levels that can resemble the universe (much as we asked earlier if there are creatures that can resemble the divine). The answer is again affirmative. Start with the following:

Definition 18. (*ZFC*) The *Lévy-Montague Reflection Principle*, is the following schema of assertions (where φ (*x*⃗) is a sentence in $\mathscr{L}_\in$ in the parameters *x*⃗):

φ(*x*⃗) → ∃αφ$^{V\alpha}$

Where φ$^{V\alpha}$ is the restriction of all quantifiers and parameters in φ to V_α.

We now note:

Theorem 19. *ZFC* proves every instance of the Lévy-Montague Reflection Principle.

Insofar as *ZFC* is justified then, so is some sort of reflection.

Interestingly, the generalisation of reflection to higher-order logics results in stronger principles. Introducing variables and quantifiers for (possibly proper) classes into $\mathscr{L}_{\in}$ yields a language we'll call $\mathscr{L}_{\in,2}$. Doing so allows us formulate theories of classes. The class theory GB (sometimes denoted *NBG* or *VNBG*) is the theory obtained by adding to all first-order axioms of *ZFC* second-order versions of the Axiom of Replacement and adopting a predicative comprehension axiom for classes. Predicative comprehension only asserts that there are classes corresponding to any formula where all class quantification is bounded. *GB* is consistent relative to *ZFC*, and so represents a very mild extension thereof. Kelley-Morse class theory *KM* is obtained by adding an *impredicative comprehension scheme* to *GB* where *arbitrary* formulas are allowed in class comprehension (sometimes a second-order version of the Axiom of Choice is included in *KM* too). The distinction is important for the apophatic mathematician, since given a model $M \models ZFC$, we know that the structure $(M, \in, Def(M))$ (i.e. the structure where the classes of M are interpreted as the *definable* classes) satisfies *GB*. An apophatic mathematician can then just view *GB* as giving us a nice way of talking about the definable classes the universe, but not to be taken ontologically seriously. Not so for *KM*, which implies the existence of a large number *of non-first-order definable* proper classes.[28]

Now we have second-order resources on the table, we can formulate second-order reflection principles:

Definition 20. (*GB*) The *second-order reflection principle* is the following schema of assertions in $\mathscr{L}_{\in,2}$ (given a formula φ in the possibly second-order parameters $x\vec{}$):

$\varphi(x\square) \rightarrow \exists\alpha(V_\alpha, \in, \wp(V_\alpha)) \vDash \varphi(x\square)^{V\alpha}$,

where $\varphi(x\square)^{V\alpha}$ is obtained by restricting all quantifiers and parameters in $\varphi(x\square)$ to V_α.

Second-order reflection is known to be quite strong, implying the existence of many cardinal numbers not provable to exist in *ZFC* (so-called 'large cardinals'). And, taking some care with the parameters allowed, we can generalise reflection to higher and higher orders.[29]

There is a challenge here however for the apophatic mathematician. Suppose that they accept that the universe is very rich, and wish to advocate reflection on this basis. Second-order reflection seems to make reference to classes, formulated as it is in *GB*, but as noted above *GB* can be viewed as a convenient way for talking about first-order definable classes, if so desired. However, we can now point out:

Fact 21. Over *GB*, second-order reflection implies every instance of the *impredicative* comprehension scheme.[30]

Given this (and noting that the impredicative comprehension scheme implies the existence of non-first-order-definable classes) one *cannot*, as an apophatic mathematician, accept the second-order reflection principle about the universe.

But can the apophatic mathematician *approximate* higher-order reflection principles? There is a trick one can pull here. The apophatic mathematician may accept the idea of reflection as a motivating (but non-formalisable) principle, but then find formal axioms that capture *something* of the idea.

We'll approach a full response slowly. We've already seen that the Lévy-Montague reflection principle is provable in *ZFC*. But it's also interesting to note that it can be used to *get* richness in the universe:

Theorem 22. Let *ZC*-Infinity be the axioms of *ZFC* with the Replacement Scheme and Axiom of Infinity removed. Then if we

add every instance of Lévy-Montague reflection to *ZC*-Infinity, we can prove Infinity and every instance of the Replacement Scheme.

I want the reader to note a couple of things about Lévy-Montague reflection at this point. The first, a theme that will recur repeatedly from now on, is that Lévy-Montague is a *scheme*, and cannot be given by a single axiom in *ZFC*.[31] Commitment by apophatic mathematicians to this kind of reflection is thus schematic, they commit to every instance of the scheme, but *not* an *individual sentence*.

Second, I want the reader to note some affinity with the *TRP*. Lévy-Montague reflection asserts that anything true in the universe is already true in some initial segment thereof. But this initial segment can change as we vary φ (much as the creature we reflected to can change under the *TRP*).[32] The question we must now ask ourselves is whether we can get something closer to the *ETRP*, where we reflect all sentences of the universe to a single initial segment. The answer is affirmative and again uses the notion of schematic commitment. Suppose we believe that the universe is so rich that there is some V_κ that satisfies all the same sentences of $\mathscr{L}_\in$ as the absolute universe of sets. The existence of such a V_κ will not be definable in *V* by Tarski's theorem. However, we can once again (via the use of parameters and schematic commitment) formulate a version of the principle:

Definition 23. Let $\mathscr{L}_{\in,\bar{\tau}}$ be the language obtained by adding a single additional predicate $\bar{\tau}$ into $\mathscr{L}_\in$. Let ZFC_τ be the following theory in $\mathscr{L}_{\in,\bar{\tau}}$:

1. Every axiom of ZFC (including instances of the Replacement Scheme in this new language)
2. The axiom "$\bar{\tau}$ is an ordinal"

3. The following axiom scheme:

$\varphi \leftrightarrow \varphi^{V_{\bar{\tau}}}$

for every formula φ of *ZFC*.

This move essentially copies a strategy from (→Feferman, 1969) to obtain a countable transitive model of *ZFC*.[33] ZFC_τ is known to be consistent (relative to very mild theories).[34] But ZFC_τ is just a first-order theory using one extra set parameter, and hence is acceptable to the apophatic mathematician.

Here we have something closely resembling the *ETRP*, in that ZFC_τ provides us with a V_τ that reflects every sentential truth of *V* in $\mathscr{L}_\in$. Of course, V_τ is only partial in other respects—if we have some sentence in an expanded language with a parameter $\bar{\theta}$ for some $\theta > \tau$, then V_τ doesn't say anything about θ. But for truth in $\mathscr{L}_\in$, V_τ agrees with *V*.

Let's now move on to higher-order reflection for the apophatic mathematician. Here we'll see that a similar move can be made, and again schematic commitment is vital. In order to see the trick, we'll need the *complexity hierarchy*. For natural numbers *m* and *n*, the complexity hierarchy classifies formulas as either $\sum_n^m$ or $\prod_n^m$. The superscript denotes the largest order of quantifiers appearing in the formula, starting with 0 for first-order, 1 for second-order etc. The subscript denotes how many quantifier alternations there are, starting with 0 for quantifier-free formulas, $\sum_1^m$ denoting a single bank of m-order existential quantifiers, and $\prod_1^m$ denoting a single bank of universal quantifiers, and then numbers increasing with alternations of quantifiers (so a $\prod_2^3$ formula starts with a bank of 4th-order universal quantifiers, then has a bank of 4th-order

existential quantifiers, followed by a quantifier-free formula). We can then define:

Definition 24. (*ZFC*) For Q = Σ or Q = Π and *m, n* ∈ ω, a cardinal *κ* is Q_n^m -indescribable iff for any formula φ of Q_n^m -complexity and parameter $A \subseteq V_\kappa$ (with *m*-order over quantifiers any V_α interpreted as ranging over $V_{\alpha+m}$) we have:

$(V_{\kappa+0}, ..., V_{\kappa+m}, A, \in) \vDash \varphi \rightarrow \exists \beta < \kappa (V_{\beta+0}, ..., V_{\beta+m}, V_\beta \cap A, \in) \vDash \varphi$.

We can generalise this to:

Definition 25. (*ZFC*) A cardinal κ is *totally indescribable* iff κ is Q_n^m -indescribable for both Q = Π, Q = Σ, and any *m, n* ∈ ω.

What a totally indescribable cardinal provides is effectively a set satisfying higher-order reflection of arbitrary order with second-order parameters. But now we can define:

Definition 26. Let $\mathscr{L}_{\in,\bar{\sigma}}$ be the language obtained by adding a single additional predicate $\bar{\sigma}$ into $\mathscr{L}_\in$. Let ZFC_σ be the following theory in $\mathscr{L}_{\in,\bar{\sigma}}$:

1. Every axiom of *ZFC* (including instances of the Replacement Scheme in this new language)
2. The axiom "$\bar{\sigma}$ is a totally indescribable cardinal"
3. The following axiom scheme:

$\varphi \leftrightarrow \varphi^{V_{\bar{\sigma}}}$

for every formula φ of *ZFC*.

This theory does exactly what we did before, but now σ is a totally indescribable cardinal rather than some ordinal or other. In this way ZFC_σ is something like having the *ETRP* with the *EDRA* added—V_σ resembles *V* for first-order truth *and* satisfies arbitrary amounts of higher-order reflection. And again, we can note that the theory ZFC_σ is a first-order theory with one parameter $\bar{\sigma}$ added for σ.

But once more, such a V_σ is only an approximation to V (in line with apophatic mathematics). Whilst V_σ and V satisfy all the same $\mathscr{L}_\in$ sentences, there are sets (and thus available set parameters) outside V_σ. In the end, V_σ is merely a *partial* approximation, even if a *very close* one. But importantly, we can learn about V via studying V_σ. And if some sentence φ from $\mathscr{L}_\in$ can be proved using reflection (of any order!) to hold in V_σ, then φ is true *simpliciter* (assuming that we think ZFC_σ is true). Much like Philo of Alexandria's image, V_σ provides a clear picture of the absolute, even if it would be confused to say that it is identical with it.

Does this kind of idea extend yet further? Suppose now one wants to consider a set x such that $x \notin V_\sigma$. Since every set is in some V_σ, we know that $x \in V_\gamma$ for $\gamma > \sigma$. Without loss of generality, we can make γ as large as we like. And, given our commitment to richness motivating reflection properties, we extend ZFC_σ with the following axioms:

Definition 27. Let $\mathscr{L}_{\in,\bar{\sigma},\bar{x},\bar{\gamma}}$ be the language obtained by adding two additional predicates $\bar{\gamma}$ and $\bar{x}$ into $\mathscr{L}_{\in,\bar{\sigma}}$. Let $ZFC_{\sigma,\gamma,x}$ be the following theory in $\mathscr{L}_{\in,\bar{\sigma},\bar{x},\bar{\gamma}}$:

1. Every axiom of *ZFC* (including instances of the Replacement Scheme in this new language)
2. The axiom "$\bar{\sigma}$ is a totally indescribable cardinal"
3. The axiom "$\bar{\gamma}$ is a totally indescribable cardinal"
4. The axiom "$\bar{x} \notin V_{\bar{\sigma}}$"
5. The axiom "$\bar{x} \in V_{\bar{\gamma}}$"
6. The following axiom schemes:
   $\varphi \leftrightarrow \varphi^{V_{\bar{\sigma}}}$
   for every formula of $\mathscr{L}_{\in,}$ and
   $\varphi \leftrightarrow \varphi^{V_{\bar{\gamma}}}$

for every formula of $\mathscr{L}_{\in,\bar{x},\bar{\sigma}}$ (i.e. just taking out any formulas containing the parameter $\bar{\gamma}$).

This theory axiomatises the idea that we now have a new totally indescribable cardinal γ above another totally indescribable cardinal σ, such that V_γ contains *x*, and V_γ and V_σ are elementary in *V* for first-order truth in $\mathscr{L}_\in$. Indeed, V_γ gets much more, capturing truth when we allow parameters for *x* and σ too. But V_γ is still only partial, and if we consider some $y \notin V_\gamma$ we may have to expand our language and axioms again to get a totally indescribable cardinal elementary in *V* that is also able to capture truth when we have a parameter for *y*.

What this shows is that there are two kinds of schematic commitment going on for the apophatic mathematician who holds that reflection cannot be truthfully stated about the universe, but nonetheless wants to capture the effect of reflection on it. The first is the use of schemes in the theories proposed. The second is more metatheoretic in nature: Such an apophatic mathematician has a commitment to accept that if you give me any set *z*, I will formulate a theory with a totally indescribable cardinal κ such that V_κ contains *z* and is $\mathscr{L}_{\in,z}$elementary in the universe. But any such *particular* theory always manifests a partial rather than full understanding, in line with *Apophatic Mathematics*.[35]

28.7 Resolving the Tensions?

The parallels between the versions of the theological reflection under *Apophaticism* and set-theoretic reflection for the apophatic mathematician are (by design) close. Moreover, we've seen that the apophatic mathematician can formulate versions of reflection that they find acceptable. But can we use the insights

from the apophatic mathematician to avoid the charge of *Incoherence*?

I think we can. The core point from the case of the apophatic mathematician is to note that reflection was formulated via *schematic* commitments (both in the object language and metatheoretically). And we can use a notion of schematic commitment under *Apophaticism* too.

The charge of Incoherence only bites if we think that theological reflection should be formulated as a propositional claim about the divine (in particular the *QTRP* looked like such a formulation). But we might rather formulate theological reflection *schematically* as follows. For the *TRP*, instead of holding that it is a proposition about the divine, we can take on a commitment such that, if we're given some creaturely property *P*, and think that an analogue of *P* holds of the divine, then we think that there's a creature with *P*. We treat talk of the relevant kind of *P* and its analogues as *schematic*, rather than *quantificational*. In the formulation of theological reflection, we treat mention of a divine property *P* as a kind of open variable for suitable properties. And indeed, this is what should want for apophatic theology. Predicating properties of the divine is tricky business (especially under *Apophaticism*), and we should not expect ourselves to be able to routinely quantify over divine properties. However, given some property *P*, we can consider whether it is plausible that the analogue of that property might hold of the divine, though our knowledge of said property is merely partial. And if we *do* think it's plausible that some divine property holds, then the *TRP* says that there is a creature possessing *P*.

Moreover, much as we did for V_τ, there's no obstacle to extending this strategy to the *ETRP*, even with the *EDRA* added. This informal theory produces divine-like entities that seem to reflect 'every' property of the divine, including the *ETRP* itself.

But just as with V_σ, we need not formulate this via quantification over properties. We can hold that there is a creature *C* such that *if we are given* a property *P*, if we think an analogue of *P* holds of the divine then *P* holds of *C*. Moreover, we can hold that the creaturely version of the *ETRP* holds of *C*, but here we really can think of this via quantification if we so desire. And anything we accept about creature *C* (without attending to its specific creaturely-ness) we will know provides an analogical resemblance to the divine. We have *negative* theology, but the most positive version thereof.

As we did before with V_γ, we can extend by reflecting on the creaturely nature of *C*. If we introduce a name $\bar{C}$ for *C*, we use the schematic commitment to the *ETRP* again to reflect a creature *C*' that can see that *C* is a mere image of the divine. But we have to *introduce a name for C* and assume it *denotes a creature* to do so, in a more restricted language, *C* is indistinguishable from the divine. And again this iterates; we can reflect on the creaturely nature of *C*' to get a *C*'', and so on. We obtain arbitrarily close but partial creaturely images of the divine, without ever committing to the idea that we have some univocal language in which we can distinguish the divine from creatures.

There is a similarity here with a particular view of negative theology proposed by Merlin Carl and Rico Gutschmidt.[36] They view Nicolas de Cusa's negative theology in line with Cantor's diagonal argument. They propose that both can be understood as a kind of schematic commitment to performatively undermine a given understanding of the divine/the universe of sets, given some candidate understanding. As they write:

> As we have argued… the *via negativa* is not committed to making a theoretical statement about the incomprehensibility of the absolute, which would be self-defeating, but can be—and usually is—understood as a practice that performatively undermines our putative understanding of the absolute. We also pointed out that this practice concerns not only the

> theological notion of God, but also, particularly in Neoplatonism, the philosophical problem of the totality of the world. In terms of the One, Plotinus performatively undermines our putative understanding of this totality and its origin. Similarly, we just argued that the method of diagonalization can be interpreted as performatively undermining the notion of absolutely everything. Thus, we think that this method constitutes a modern version of the *via negativa*. (Carl and Gutschmidt, MS, p. 19)

The present proposal meshes very well with this approach. Theological reflection presents a schematic commitment such that any creature, no matter how closely it resembles the divine, can be witnessed to be creaturely by some other creature. But this is a schematic commitment, the creature itself may even satisfy the *ETRP* and be indistinguishable from the divine, *until* its creaturely nature is reflected upon.

28.8 Conclusions and Open Questions

In this paper, I've proposed a particular kind of theological reflection principle. I've argued that whilst it is a somewhat strange idea, it has historical roots and can be given a more charitable interpretation.

Whilst we posed some initial questions about its coherence, I argued that they can be resolved in analogy with the apophatic mathematician's attitude to set-theoretic reflection. In particular, I argued that the theological reflection should be understood through schematic commitment and that, given the apophatic stance, is exactly how things should be.

I want to close with two open questions. The first concerns an underlying theme of this paper:

> **Question.** How similar are theological and set-theoretic reflection?

In particular, whilst we drew something of a tight analogy between the two in virtue of schematic commitment, there are differences. For the apophatic mathematician, talk of 'properties' of *V* can only be given in a first-order fashion. But 'first-order' here just means in terms of statements about sets, whereas the version of *Apophaticism* that makes a distinction between creaturely properties/predicates and their divine analogues, *does* have some notion of divine-but-not-creaturely property. There is also the point that reflection is a standard part of cataphatic *mathematics*, on which we do allow the use of non-first-order definable proper classes. This contrasts sharply with many cataphatic theologies, where, though *TRP* and *ETRP* can be formulated, they do not mesh well with several commitments normally associated with the view (e.g. the personhood of the divine).

The second question concerns an issue already given a cursory treatment in the text:

> **Question.** How should we understand the metaphysical picture proposed under the *TRP, ETRP, DRA*, and *EDRA*? In particular, what is the metaphysics of these "divine-like" creatures?

Whilst I did make some short remarks in this direction, linking their existence to 'abstract possibilities of existence' and Spinozean metaphysics, this is scarcely a full account. If we wish to really move forward using theological reflection, a better understanding is required. In line with apophaticism in both mathematics and theology, our knowledge of theological reflection remains incomplete.

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Notes

1 See Roberts (MS) for a survey.

2 Here, and throughout, I will follow much of the literature in using the term ‘creature’ to denote non-divine entities.

3 Note: I use the term ‘entity’ in a broad sense, and do not commit to whether or not entities are individuals, since the assumption that the divine is given by an individual is highly problematic in this context.

4 This is adapted from →Keller (2018). She uses the term ‘weak incomprehensibility’ but since the difference between strong and weak incomprehensibility won’t play a role in what follows, I’ve suppressed it.

5 Concise surveys are available as parts of →Keller (2018), Horsten (MS), and Priest (MS); see also Hewitt (2020) for a recent book-length treatment.

6 This is also the move of Hewitt (2020; e.g. p. 14).

7 Transcendence in turn is sometimes motivated by: “*Simplicity*. The divine is a simple (as opposed to complex) entity (i.e. the divine is not composed from many simples).” See, for example, Hewitt (2020) for an argument to this effect. I won’t use *Simplicity* (though it is in the background), so I relegate mention of it to this footnote.

8 For a survey of apophatic theology, see Hewitt (2020, Chapter 1), and for an introductory book-length treatment, see →Turner (1995).

9 There is something of a similarity with theories of truth, and responses to the liar here. For example, on the Tarskian account, paradox is avoided by having the truth-predicate for an object language not be within that language. Of course, in that context we get a hierarchy of truth predicates, which isn't such a common response for the apophatic theologian.

10 See →McCabe (1987), MacIntyre (2009), and Hewitt (2020).

11 See →Newstead (2009) for a discussion of Cantor and infinity regarding the divine mind.

12 There are still lots of objections to the *3Is* and *Apophaticism* in general that I haven't considered. In particular, Plantinga (2000) argues that there is a lack of argument for the position, it is too revisionist, and arrogant. Nothing I've said here has any bearing on those matters, and I set them to one side (though see →Lebens, 2014 for a rebuttal).

13 Horsten (MS) terms this 'ontological,' rather than 'theological' reflection.

14 This is a little controversial, since whether this analogue of 'infinite' is appropriately related the usual property *infinite* needs support. I think that it's natural to think that the analogue will deliver an infinite creature under

the *TRP*, so I'm happy to simply make this as an assumption.

15 See →Hauser (2013) for a further detailed discussion of the link between reflection, apophaticism, and absolute in Cantor's thought.

16 For some additional remarks on Gödel's theology in relation to set theory (and, indeed, some connections to Aristotelian thought) see Engelen (forthcoming).

17 This interpretation is controversial, but can be viewed as extending his remarks about the number of attributes in *Ethics*, VI: "By God (Deus) I understand a being absolutely infinite, that is, a substance consisting of infinite attributes, each of which expresses eternal and infinite essence. Explanation. I say absolutely infinite, but not in its kind. For of whatever is infinite only in its kind, we can deny infinite attributes; but to the essence of what is absolutely infinite there appertains whatever expresses essence and involves no negation." (transl. →Parkinson 2000) It is an interesting question whether ideas of *reflection* can also be extracted from Spinoza's work.

18 Here Hewitt is actually considering why questions concerning the existence of the divine are poorly posed, so is intended for a slightly different objective. However, it suffices to get the epistemic position of understanding of the divine via an understanding of creatures on the table.

19 Of course, cataphatic theology can hold versions of theological reflection too, it is just that it is less likely to

be motivated on apophatic grounds. Moreover, theological reflection seems to conflict with some other commitments that are normally accepted by cataphatic theologies. For example, it is unclear on this picture how the divine could be a *person* (in any reasonable sense of the term). I'll set this issue to one side, despite its interest.

20 This account of the epistemology of the divine under versions of theological reflection has some further nice consequences for questions about the consistency of the divine. Here, we have set dialethism to one side, but it bears mentioning that if theological reflection could be made to work in the dialethist context, we can address many questions regarding consistency. If the divine instantiates inconsistent properties, then this inconsistency should be reflected (by theological reflection) in creaturely reality. We thus provide a bridge (using theological reflection) between the usual theological questions concerning the consistency of the divine and dialethism regarding creaturely reality: If the divine is inconsistent, then there is a creature exemplifying a similar inconsistency that we can go and *look for*.

21 See →Barton (2021) for a survey.

22 See §1 of Barton and Williams (forthcoming) for a survey.

23 See here Hamkins et al. (2012), Barton and Williams (forthcoming).

24 See →Barton (2016) and →Incurvati (2017) for discussion of this issue.

25 See →Tait (2005), →Koellner (2009), and →Welch and Horsten (2016) among many others.

26 From now on I'll just assume the Axiom of Choice holds, and suppress details about the exact background theory.

27 For simplicity, I am giving the version for pure sets, if you want to include *Urelemente* then clause (i) should be replaced by $V_0 = \{x \mid \dot{x} \text{ is an urelement'}\}$, and (ii) should be replaced by $V_{\alpha+1} = \wp(V_\alpha) \cup V_\alpha$.

28 See here Kameryn J. Williams' thesis (→Williams, 2018) for details.

29 Reflection with arbitrary third-order parameters is inconsistent, see →Reinhardt (1974) and →Koellner (2009). Restricting to only second-order parameters but arbitrary orders of logics is widely believed to be consistent, and is relatively low in the large cardinal hierarchy. We'll see some discussion of this below with *totally indescribable cardinals*.

30 This observation was first communicated to me by Øystein Linnebo and Sam Roberts and I'm grateful for their input. *Proof.* If impredicative comprehension fails, then there is an instance φ witnessing such a failure. There is thus a $(V_\kappa, \in, \wp(V_\kappa))$ that violates φ. But this is impossible, every $(V_\kappa, \in, \wp(V\kappa))$ satisfies every instance of the impredicative comprehension scheme.This proof is highly generalisable, and applies to any sentence true

over every $(V_\kappa, \in, \wp(V_\kappa))$. For example, so long as we have *ZFC* in the universe, second-order reflection will reverse also to class choice (since we get class choice over every $(V_\kappa, \in, \wp(V_\kappa))$ by using set *AC* in the universe).

31 If it were, *ZFC* would be finitely axiomatizable, but this isn't possible by Gödel's Second Incompleteness Theorem and reflection.

32 The analogy is not perfect, since the closer analogy when talking about properties of the divine are *higher-order* versions of reflection, but we'll get to that in a second.

33 Feferman's axioms are basically the same, with axiom (ii) replaced by "τ is countable and transitive," and axiom scheme (iii) replaced by every instance of "$\varphi \leftrightarrow \varphi^\tau$."

34 For example, if we have a truth predicate, then one can prove that V is the union of a chain of such V_τ, all elementary in V. See Williams (2018).

35 One might wonder, can one produce such an axiomatization but with a proper class of totally indescribable cardinals, each elementary in *V*, using a scheme? This would capture any given set in some member of the proper class. The only way I know of doing so—communicated to me by Kameryn J. Williams—adds in a class predicate *D* and the schema that every $\gamma \in D$ reflects every formula. But the introduction of predicates for proper classes is unlikely to sit well with the apophatic mathematician.

36 See Carl and Gutschmidt (MS). I am very grateful to both Dr. Carl and Dr. Gutschmidt for several helpful discussions here.

29 Are Proofs of God’s Existence Impossible? A Critical Examination of Kant’s Critique

Meir Buzaglo

Abstract: In the chapter on The Ideal of Pure Reason in the *Critique of Pure Reason*, Kant attempts to disprove any possible proof for the existence of God. However, as I have demonstrated in a previous paper, this claim is not consistent with Gödel’s Second Theorem. In light of this, it is interesting to establish exactly where Kant went wrong in his discussion. A careful examination reveals three types of errors: a. Kant did not consider all the possible proofs for the existence of God, and in this respect he fell into the false dilemma fallacy; b. Kant’s objection to the cosmological argument is insufficient to show its absurdity; and finally c. It is possible to deal with Kant’s objections of the physico-theological argument by using Kantian assumptions.

29.1 Prologue

A good place to begin this paper on Kant’s refutation of proofs for God’s existence is with the following much quoted statement by Richard Feynman:

> It doesn’t seem to me that this fantastically marvelous universe, this tremendous range of time and space and different kinds of animals, and all the different planets, and all these atoms with all their motions, and so on, all this complicated thing can merely be a stage so that God can watch human beings struggle for good and evil—which is the view that

> religion has. The stage is too big for the drama. (*Viewpoint* Interview by Bill Stout, May 1, 1959 quoted in →Gleick (1992, p.372))

Human beings are minuscule creatures, infinitesimal by comparison to the dimensions of the universe. How is it conceivable that everything was created so that God could take an interest in creatures so limited in time and space? We have all read similar statements, but after a while second and third thoughts arise. Possibly God does not take an interest in humans, but the size of the stage and the minuscule dimensions of humans do not necessarily imply that they are insignificant. Feynman should have been among the first to realize this. In a generation where miniaturization has become the be-all and end-all, reduction in size does not entail a loss of significance.[1] Indeed, concepts of size have given way to concepts such as complexity, and as far as I am aware there is nothing in the natural world to rival the complexity of the human brain. Feynman's comments are a fitting background to Kant's statement concerning the value of man in the world:

> All these diverse creatures would exist for nothing if they did not include human beings (or some kind of rational beings) no matter how artfully devised these creatures may be... Without man all creation would be a mere wasteland, gratuitous and without a final purpose. (*Critique of Judgment*, Sec. 86, p. 442)

One way to describe the opposition between Feynman and Kant is that they agree on the facts but disagree in their assessment of the facts. For the former, the dimensions of the universe bear witness to the insignificance of human existence, which we only value because of narcissistic chauvinism. For the latter, who as an astrophysicist knew something about the size of the universe, there is no point to a universe containing methane planets, or even panda bears: it may be infinitely large, but no stage is too

large for human conscience and its concerns. We may attribute the difference between Feynman and Kant to personal taste, character, or to their answers to the question of whether they believe in God or not. We can also attempt to go further and try to make a definite decision in favor of one side or the other. In this case at least part of the discussion will be meta-ethical, concerning the objectivity of ethics and the ontological status of humanity as a creature with an obligation to values. Even if we unearth their presuppositions concerning ethics, we should lower expectations in the attempt to decide between Feynman and Kant's positions: we may hope that the debate is not merely a matter of taste and personal inclination, but since it belongs to metaphysics it cannot be determined by human reason as if it were a conventional scientific question.

At this point another characteristic of the Kantian position emerges, his pessimism in relation to speculative metaphysics according to which there is no rational criterion for a decision between the two views. This addition makes the whole matter more interesting, since Kant, who limited human reason, would not have left the debate between him and a naturalist like Feynman to a matter of taste. This is the point on which I would like to dwell. Kant's philosophical project was a systematic and comprehensive attempt to map the borders of reason with respect to metaphysical questions. This is a feature of all the foundations of his method and is clearly visible in his discussion of proofs of God's existence. Kant believed that the capabilities of human reason are limited, but this limitation can be exhaustively characterized by the same pure reason. While many believe that it is not possible to prove the existence of God by rational means, Kant believed that this is not only a question of faith: it is possible to prove that it is impossible to prove God's existence. According to this opinion, by making the transition into meta-theology one can formulate a stance maintaining that

with respect to the possibility of responding to metaphysical controversies, we are not in a position to know for certain that they are irresolvable. In a previous paper (→Buzaglo, 2019) I argued that formulating such positions contradicts Gödel's second theorem. In sufficiently powerful and consistent systems it is impossible to prove that it is impossible to prove a given proposition. Human reason is not capable of mapping itself.

We remain with the concrete aspect of the Kantian position as reflected in his text, and the question of whether and where he went wrong, as emerges from Gödel's second theorem. It appears that at first sight the problems in Kant's treatment can be divided into three:

1. Kant's classification of proofs is not exhaustive, and there is a concern that he committed the false dilemma fallacy.
2. Proofs that he analyzed, such as the cosmological proof, may be amended and made more interesting by adding assumptions external to the Kantian doctrine.
3. In relation to his critique of the physico-theological proof, it is possible to introduce an amendment that relies on Kantian tenets.

Firstly, I will make some preliminary remarks on the general question of the problem of the idea of proofs of the unprovability of specific propositions. I will conclude the article with a meta-philosophical remark concerning the way we should portrait the tension between the quotes from Feynman and Kant with which I opened.

29.2 Kant's Refutations: Reservations and Corrections

Kant's position on the ability of human reason to prove God's existence is unique in the history of philosophy. As stated, many have rejected ambitious attempts at a proof for various reasons, but Kant went one step further with the claim that he could prove that it is impossible to prove God's existence. Kant classified all possible proofs into three categories: proof of God's existence from the concept of God; proof of God's existence from the existence of the world; and proof of God's existence from the nature of the world. The first is the ontological proof, the second is the cosmological proof and the third is the physico-theological proof. Kant's achievement here is exceptionally original for his period. His strategy consists of blocking every path leading to a proof of God's existence, starting with the well-known objection to the assumption that existence is a predicate, which is at the basis of the ontological proof, and pointing out that the other proofs eventually lead back to the ontological proof.

However, as I demonstrated in my earlier paper, Kant's objective is in opposition to Gödel's second theorem. To be precise, we will never succeed in reproducing Kant's argument or similar arguments within a sufficiently rich axiom system. My paper includes a rigorous examination of this claim, but it can be explained in a brief informal fashion by pointing out that consistent axiom systems cannot prove their own consistency, but if we could prove that there is a proposition[2] that is not provable in the given axiom system, we would have *ipso facto* proved its consistency from within itself (since anything is provable in an inconsistent system).[3]

The *a priori* refutation of Kant's attempt to prove that the existence of God cannot be proven, naturally invites the search for fallacies in his discussion. As it turns out this inquiry is rewarding in ways that go beyond a completion of an examination of Kant's position.

29.2.1 False Dilemma

Kant claims that certain propositions are unprovable, but propositions are only provable in the context of a given system.[4] In order to prove that it is impossible to prove the existence of God, Kant had to show that there is no set of assumptions from which to prove the existence of God, but his division of all the proofs into three types does not really guarantee that he went through all the axiomatic systems.

Let us take for example the refutation of the ontological proof as an *a priori* proof from the concept of God. Kant argues that unless we assume that existence is a predicate it is impossible to prove the existence of God, however this does not imply that His existence cannot be proved from other assumptions. There are existence theorems that are provable (for example the existence of an infinite number of primes), without assuming that existence is a predicate. The lively discussion surrounding the ontological proof, with powerful logic deployed by distinguished logicians including Gödel shows that Kant's view is a long way from being the last word on the topic. Take a proof for God's existence like Spinoza's, which I will reproduce here in highly abbreviated form:

> If, then, no cause or reason can be given, which prevents the existence of God, or which destroys his existence, we must certainly conclude that he necessarily does exist. If such a reason or cause should be given, it must either be drawn from the very nature of God, or be external to him—that is, drawn from another substance of another nature. For if it were of the same nature, God, by that very fact, would be admitted to exist. But substance of another nature could have nothing in common with God (by Prop. ii.), and therefore would be unable either to cause or to destroy his existence. (*Ethics*, Part one proposition 11, trans. Samuel Shirley Spinoza 2002)

This is a proof that depends on a strong version of the principle of sufficient reason: things that do not exist have a reason for not existing, and we are not entitled to suggest a reason for God's non-existence. In this formulation the proof is perhaps unconvincing, but we would like to see a response to this unconvincing proof as part of Kant's treatment. It appears from a preliminary examination that he does not take it nor other similar proofs into account.

29.2.2 Completion by Non-Kantian Assumptions

The cosmological proof was rejected because Kant held that at some point in the proof we fall back on the ontological proof. But this can be easily refuted. The discussion of the cosmological view is being renewed nowadays, and in the spirit of the possibilities to restore it, I offer the formulation that avoids the assumption that existence is a predicate:[5]

1. Every contingent entity has a cause different from itself.
2. The set of all contingent entities is a contingent entity.
3. The cause of a contingent set is the cause of every member of the set.
4. The totality of all contingent entities C is a set.
5. There exists at least one contingent entity.

From 1 and 2 it follows that C has a cause G. Now, G cannot be contingent: if it were contingent it would be within C, which implies, contrary to 1, that G is the cause of itself.

Like the proof I quoted earlier from Spinoza's *Ethics*, assumption 4 here, which plays an important role, can be strengthened by a similar principle of sufficient reason, which in this case in fairly reasonable. In any case, whatever this proof is

worth, I have difficulty understanding from Kant how to reduce it to the ontological proof as he promises.

In Kant's discussion of proofs for God's existence he raises various criticisms, which show that even if we permit the proofs to start from some assumptions they fail, and he also adds the problems that such proofs have from an internal systematic perspective. It is clear that if we understand the term "cause" in a Kantian fashion, as a relation that only exists between phenomena, the cosmological proof does not even get off the ground (and I will return to this point towards the end of the article). This may be the basis for Kant's confidence that these proofs are worthless. If we understand existence as an empirical concept, as something that requires observational verification, then there is no room whatever for conceptual proofs of God's existence, and the same is true if one understands the concept of cause as a relationship which does not permit one to go beyond phenomena. The criticisms that I raise here relate partly to the refutations of proofs that Kant believed could be raised against those who present the proof, without relating to the essentials of his critique. But we are not discussing inessentials: a single valid proof of God's existence that does not depend on observation and does not get involved with antinomies would challenge Kant's critique of metaphysics, which is one of his chief concerns. If we have informative and non-analytical knowledge of the ideas of reason, Kant's whole transcendental dialectic is called into question.

29.2.3 Correction of the Physico-Theological Proof

Kant opens his discussion of the physico-theological proof with a brief claim that he finds convincing.[6] Investigation of the order and purposefulness of the world, however illuminating it may be,

cannot prove the existence of a being as sublime as God. In his own words:

> For how can any experience be given that is supposed to be adequate to an idea? For what is special about an idea is just that no experience can ever be congruent to it. (*Critique of Pure Reason*, A621, B649)

The harmony and purposefulness of the world are empirical findings.[7] How can one go from empirical observations to an idea which is not subject to experience? However instructive nature may be, it will never be sufficient to reach the sublime idea of God. Kant goes further and supports his claim with fairly reasonable assumptions that can be separated from Kantian jargon:

> The transcendental idea of a necessary all-sufficient original being is so overwhelmingly great, so sublimely high above everything empirical, which is at all times conditioned, that partly one can never even procure enough material in experience to fill such a concept, and partly if one searches for the unconditioned among conditioned things, then one will seek forever and always in vain, since no law of any empirical synthesis will ever give an example of such a thing, or even the least guidance in looking for it. (see above, A621, B649)

Yet, Kant does not hide his respect and appreciation for this proof. Unlike the other proofs, this one is capable of contributing both to science and to faith:

> This proof always deserves to be named with respect. It is the oldest, clearest and the most appropriate to common human reason. It enlivens the study of nature, just as it gets its existence from this study and through it receives ever renewed force. It brings in ends and aims where they would not have been discovered by our observation itself, and extends our information about nature through the guiding thread of a particular unity whose principle is outside nature. But this acquaintance also reacts upon its cause, namely the idea that occasioned it, and

> increases the belief in a highest author to the point where it becomes an irresistible conviction. (*Critique of Pure Reason* see above, A623-4, B651-2)

At the same time, in a step impressive for its integrity and restraint, he finds himself forced to limit this claim.[8] It is impossible to see here a rational proof for God's existence.

As with the cosmological proof, here too Kant takes two distinct steps. The first is a critique from the principles of the method. Limitation of causality to phenomena, and perhaps also the gap between idea and empirical experience, are internal aspects liable to weaken any physico-theological proof *a priori*. At the same time, there is a generous aspect in which he is willing to allow his opponent to claim, on the basis of analogy, that there is a wise creator that created the world with free will. However, and here we return to the point where we began, even when we permit this analogy we have no reason to identify this cause with God.

A comparison with a similar critique proposed by David Hume is useful here.[9] In his *Enquiry Concerning Human Understanding*, which is devoted to criticism of the proof of God's existence from nature, he begins by making the Kantian claim, but also sharpens the principle on which it depends:

> When we infer any particular cause from an effect, we must proportion the one to the other, and can never be allowed to ascribe to the cause any qualities, but what are exactly sufficient to produce the effect. A body of ten ounces raised in any scale may serve as a proof, that the counterbalancing weight exceeds ten ounces; but can never afford a reason that it exceeds a hundred, If the cause, assigned for any effect, be not sufficient to produce it, we must either reject that cause, or add to it such qualities as will give it a just proportion to the effect. But if we ascribe to it farther qualities, or affirm it capable of producing other effects, we can only indulge the license of conjecture, and arbitrarily suppose the existence of qualities and energies, without reason or authority. (Section 11 in *Enquiry Concerning Human Understanding*)

This formulation adduces Kant's essential critique of the physico-theological proof. As we shall see, there are other objections to the proof, but as long as we fail to address this objection, there will be no point in contending with the others.

29.2.3.1 A Kantian Reply

It turns out that if we study the proof and enrich it with an explicitly Kantian assumption, one that he does not share with Hume, we can defend the physico-theological proof from this criticism. At this point we should remind ourselves of an oft-quoted passage from the end of the *Critique of Practical Reason*, and which corresponds nicely to the one we opened with. After the brief and well-known introduction:

> Two things fill the mind with ever new and increasing admiration and reverence, the more often and more steadily one reflects on them: the starry heavens above me and the moral law within me. I do not need to search for them and merely conjecture them as though they were veiled in obscurity or in the transcendent region beyond my horizon; I see them before me and connect them immediately with the consciousness of my existence. (*Critique of Practical Reason*, 5:162)

Kant goes on to describe two kinds of infinity, firstly from humanity's place in the manifested world:

> The first begins from the place I occupy in the external world of sense and extends the connection in which I stand into an unbounded magnitude with worlds upon worlds and systems of systems, and moreover into the unbounded times of their periodic motion, their beginning and their duration. (see above)

This is the infinity of unlimited space and time, concerning which he continues:

> The first view of a countless multitude of worlds annihilates, as it were, my importance as an animal creature, which after it has been for a short time provided with vital force (one knows not how) must give back to the planet (a mere speck in the universe) the matter from which it came. (see above)

In his discussion of the physico-theological proof in the *Critique of Pure Reason*, he discusses the sublimity arising from contemplation of nature and in this respect expands on the infinity mentioned in the conclusion of the *Critique of Practical Reason*. The passage is worth quoting:

> even after we have attained to the highest summit of knowledge which our weak minds can reach, we find that language in the presence of wonders so inconceivable has lost its force, and number its power to reckon, nay, even thought fails to conceive adequately, and our conception of the whole dissolves into an astonishment without power of expression—all the more eloquent that it is dumb. (see above)

A naturalist like Hume, and as far as one can tell like Feynman, whom I quoted at the beginning of this paper, would stop here. But Kant goes on to define the second dimension, and contrasts it with the first dimension by using the phrase “true infinity”:

> The second begins from my invisible self, my personality, and presents me in a world which has true infinity (*wahre Unendlichkeit*) but which can be discovered only by the understanding, and I cognize that my connection with that world (and thereby with all those visible worlds as well) is not merely contingent, as in the first case, but universal and necessary. (see above)

Later he expands on this:

> The second, on the contrary, infinitely raises my worth as an intelligence by my personality, in which the moral law reveals to me a life independent of animality and even of the whole sensible world, at least so far as this may be inferred from the purposive determination of my

> existence by this law, a determination not restricted to the conditions and boundaries of this life but reaching into the infinite. (see above)

The infinity bestowed on humanity by the ethical dimension goes beyond space and time. It is important for Kant to clarify that the value of humanity does not stem from its cognitive understanding as a mere speaking animal:

> On the other hand, it is not by reference to man's cognitive power (theoretical reason) that the existence of everything else in the world first gets its value, i.e., it is not [because] (say) there is someone *to contemplate* the world. For if all this contemplation offered to man's presentation nothing but things without a final purpose, then the fact that the world is cognized cannot make its existence valuable; only if we presupposed that the world has a final purpose, could its contemplation itself have a value by reference to that purpose. (*Critique of Judgment*, p. 442)

Here appears the idea of freedom in the Kantian sense:

> Hence the only [thing] which can give man's existence an absolute value, and by reference to which the existence of the world can have a *final purpose*, is the power of desire. But I do not mean here that power of desire which makes man dependent on nature (through impulses of sense), i.e., not the one according to which the value of man's existence depends on what he receives and enjoys. I mean the value that he can only give himself, and that consists in what he does, how and on what principles he acts, not as a link in nature, but in the *freedom* of his power of desire; in other words, I mean a good will. (*ibid*, p. 443)

As long as we remain within the natural dimension, even if it extends to infinity, we cannot attain another infinity and the value that Kant terms absolute.

What difference does it make if we strengthen the physico-theological proof by including this creature among the entities that God created? Assuming that we permit ourselves to arrive at a reason for nature by means of analogy, have we remembered to include in the realm of existence, if not in the

realm of nature, a metaphysical dimension such as the ethical dimension inseparably linked to the concept of humanity?

In the first place we are entitled to relate to God as the reason for that infinite dimension in humanity, with a profound connection to what any religion worthy of its name attributes to God, i.e., a strong and intimate affinity to justice. Secondly, God as creator of humanity, achieves an infinite dimension of the type that Kant terms “true.” The second addition is particularly important and requires further clarification. When Kant distinguishes between the infinity of the universe and the infinity of conscience he seems to consider them to be comparable. However, these two infinities, of nature as a whole and of human freedom or the world in which this freedom dwells, cannot easily be compared. At the end of the *Critique of Practical Reason*, Kant turns to the relationship between the value of man as a free creature and animals without freedom to illustrate the infinity required for ethics, but this does not help much either. Infinity assists us in placing qualitatively different issues on the same scale: thus we can see a circle as an equilateral polygon with an infinite number of sides, or as the limit of a series of polygons with an increasing number of sides. But we do not have such a conceptual method which would be capable of defining the relationship between humans’ value as animals and their value as free beings. Man, from a Kantian point of view, is obviously the purpose of the evolution of the animal world, however, he is also a leap that is not on the same continuum.

It seems that it would be futile to reach man by a gradual development from the simple to the complex. And it is therefore impossible to understand his infinity by comparing him with animals. If we allow ourselves to be helped by the Bible, we may gain inspiration for an interesting move. Psalm 8 allows us to view this from a different angle. Humans were created in God’s

image and likeness, and the author of the book of Psalms gives a poetic description that recalls Kant's philosophical discussion:

> When I consider your heavens, the work of your fingers, the moon and the stars, which you have set in place, what is mankind that you are mindful of them, human beings that you care for them? You have made them a little lower than God[10] and crowned them with glory and honor. (Psalm 8: 3–5)

Human beings have two dimensions: they are infinitely small, but nevertheless they have a greatness that closely approaches the Divine. This affinity allows us to see in humanity and the potential it contains an existence that bears witness to God. The gap that Kant perceived between God and nature, a gap that does not permit a transition from nature to God, is significantly narrowed if we add humanity to nature and see God as the creator of humanity.

29.2.3.2 Kant's Second Opposition

We will return to this later, meanwhile the problems that Kant sees in the physico-theological proof are worthy of further investigation. Let us examine the following criticism by Kant:

> According to this inference, the purposiveness and well-adaptedness of so many natural arrangements would have to prove merely the contingency of the form, but not of the matter, i.e., of substance, in the world; for the latter would further require that it be able to be proved that the things of the world would in themselves be unsuited for such an order and harmony according to universal laws if they were not in their substance the product of a highest wisdom; but entirely different grounds of proof from those provided by the analogy with human art would be required for this. Thus the proof could at most establish a highest architect of the world, who would always be limited by the suitability of the material on which he works, but not a creator of the

> world, to whose idea everything is subject. (*Critique of Pure Reason* see above, A627, B655)

I will attempt to rephrase this complex paragraph in my own words. Kant limits the physico-theological proof in another respect. A proof that the order of things in the world is impressive in its purposefulness is adequate to prove the existence of a wise architect, an arranger, but does not succeed in demonstrating that matter is the product of wisdom that is responsible for its existence. Kant also goes into detail on what would be necessary to prove that God is not just an architect. We need to show that the harmony in nature would not be realizable without material totally devoted to this objective. Here he adds a note that in his opinion closes the book on the whole subject. The analogy between the creation of nature and the creation of tools is limited, since the creation of tools assumes pre-existing materials, and from here we can never make the step to the concept of God as creator.

If we understand the physico-theological proof as moving from the purposefulness of creation to God as organizer, the objection above takes on one meaning. If we understand physico-theology as derived from the rationality that we observe in the universe, this stronger position will be subject to another kind of objection. In the second case, modern physics may supply a preliminary answer through consideration of the order and harmony that we encounter in the very structure of matter. Molecules amazed Maxwell, who stood in awe of their identity. An interesting paper by Maxwell begins with an observation, trivial for a materialist scientist, but which he sees as no less than stunning:

> Each molecule, therefore, throughout the universe bears impressed on it the stamp of a metric system as distinctly as does the metre of the

> Archives at Paris, or the double royal cubit of the Temple of Karnac. (James C. Maxwell, “Molecules” in *Nature* 1873 see above)

From here he arrives at a conclusion concerning the limits of science:

> Thus we have been led, along a strictly scientific path, very near to the point at which Science must stop. Not that Science is debarred from studying the internal mechanism of a molecule which she cannot take to pieces, any more than from investigating an organism which she cannot put together. But in tracing back the history of matter Science is arrested when she assures herself, on the one hand, that the molecule has been made, and on the other, than it has not been made by any of the processes we call natural. (see above)

This rationality exists in elementary particles that reveal remarkable structures (groups etc.). Thanks to the centrality of these structures (and the connection between mass and energy, etc.) we avoid the identification of physical research with materialism and see it as physicalism. We cannot conceive that there exist electrons, protons and elementary particles and that God only organized the laws that apply to them. The forces and interactions between the particles are not arrangements by an architect, but are, at least in part, integral to their nature.[11] If the order and rationality controlling them are the product of an architect then we can conclude from certain perspectives on elementary particles and their properties that they owe their existence to the same “architect.”[12] In any case, Kant’s objection to the physic-theological proof rests on the false assumption that one can separate matter from the structures that are carried by it.

Another way to understand Kant is that he requires the purposefulness of nature from which we arrive at the idea of God to be sufficiently powerful that we can derive from it the actual properties of the matter comprising the world. The

phenomena of life determine the properties of matter and its actual existence. I believe that here too we can use the developments of physics to provide a response. Since we mentioned Maxwell's position concerning molecules I will use molecules as an example. Maxwell is correct that it is not possible to explain molecules as a consequence of a gradual process, but the evolution of life as we know it could not have taken place without indivisible molecules. Evolution requires precise and steady copying of genetic material which can be refined by mutations that benefit survival and the production of descendants. The genetic material must be stored in an unambiguous language, and molecules are a possible tool for the realization of evolution. If we understand that life is impossible without evolution, then the processes of life would not be possible without molecules. One could expand here on physical constants and the like. What we need to stress is that the objective that Kant set us in order to demonstrate that God is more than an architect can be addressed by invoking modern developments in natural science.

These supplements that we have added to strengthen the physico-theological argument are not patches to cover up Kant's problems. Explaining physical properties by the phenomena of life and adding the ethical dimension to the discussion are conceptually connected, since it is only with respect to living beings and not with respect to galaxies, however large, that human beings have ethical obligations. Life is a prerequisite for a world in which ethics are relevant, and the existence of a creature with obligations towards living beings is a more advanced stage in the process required for completing the whole picture.

There is still a gap between God as defined by Kant and the physico-theological proof. Kant's concept of the Transcendental Ideal speaks of the sublime concept of God and the proof

offered here falls short of that. In view of this, it would be interesting to amend the concept of God in the light of the proof so that it approaches it as closely as possible. It emerged from the discussion that one can defend God as creator of the world and, having placed humanity on its richly complex foundation, as having a conceptual connection to ethics.

29.3 Subjects for Further Study

In the previous discussion I studied Kant's refutations of the theoretical proofs for the existence of God, and I found inaccuracies as predicted by the general argument I raised against Kant which was based on Gödel's second theorem.

The possible link between the discussion here and an examination of the physico-theological and cosmological proofs requires an examination. Recall that the existence promised by the cosmological proof is not contingent, and as a result one may conclude that an understanding of the physico-theological proof in the light of the cosmological proof will propose that the entity which we have arrived at, a God who created the world and who maintains a connection to ethics, is not contingent. Indeed, if it were contingent we would be allowed to attach it to nature and apply the physico-theological proof to the attachment. Thus we can enrich our current concept of God and claim that He is the creator of the world who maintains a conceptual relation to morality and is a necessary being.

The second comment returns us to Kant's intellectual generosity that allowed us to maintain an analogy between the making of a tool for a given purpose and a wisdom that created the totality of creatures in the world. As will be recalled, Kant rejects this analogy for systematic internal reasons, but in the discussion of the current proof he is willing to assume it, with the clarification that even if we grant it to an opponent it is not

sufficient to lead to the existence of God. In the discussion above I have raised several objections to this claim of Kant using a Kantian assumption, but what about this assumption itself?

A physicist investigates the transformations of matter, and the investigation is subject to measuring devices, differential equations and technology. The origin of the universe and the cause of the universe are not within the parameters of his research: the claim that the universe has no cause, like the claim that the universe has a cause, with this cause considered to be God the Creator, is not part of his concern as a physicist. However, if one is thinking of a Theory of Everything, of the kind which is concerned with the universe, the living world, and human conscience, one must think as a meta-physicist. A radical materialist, who believes that all these phenomena can be explained in terms of material interactions, is not functioning as a regular scientist but has crossed the line into metaphysics. At this point one can think of a competing metaphysics. If we understand the term "cause of the universe" as an extension of the mundane concept of cause, another possibility is opened up for us. An analogy which creates unity between the phenomena I mentioned here—humans in their unique character as free agents, the living world, the characteristics of physical material and the purposefulness that they enable, is a seed of philosophical view. We do not postulate a cause for the universe as an unfounded assumption as Kant thought, but rather as an assumption in its own right that stands or falls on its coherence and fruitfulness.[13]

These comments apply to Kant's critique of the physico-theological proof as it appears in the *Critique of Pure Reason*. The proof reappears in the last chapter of the *Critique of Judgment* where he discusses the proof in the light of the ethico-theological proof. Since the amendment proposed here to the physico-theological proof took the proof from the physical

sciences to ethics, it is important, to sharpen the distinction between the position presented here and ethico-theology by pointing out two differences.[14] First, the ethico-theological proof purports *inter alia* to prove the existence of God in a stronger sense than that claimed here: It is intended to ensure the possibility of the highest good, and among other things to ensure a relation between moral human conduct and a promised bliss. The conclusion presented above is weaker. The God who creates humanity, as a creature with free will, expressed in its obligation to laws of ethics, must be connected to the concept of ethical obligations. The assumption that God is the creator of humanity is *ipso facto* more powerful than the God promised by the physico-theological proof, which does not take humanity into account. However, this does not lead us to a God who rewards or punishes or is attentive to human struggles. On the other hand, the validity of the proof here is objective and not limited to practical contexts—In this sense, the result of the correction above is stronger than in Kant's ethico-theology.

29.4 Conclusion

Let us return to Feynman's comment with which we began. As I explained in the last comment above, I have presumed to demonstrate here the existence of God with an interest in human behavior, and from that point of view we have not answered the question he raised. The dimensions of the universe and the smallness of humanity are not opposed to the possibility that there is a God with profound interest in human fate. The understanding raised here is not sufficiently strong to settle fundamental questions of this kind, but this does not mean that these questions should be left to subjective, psychological considerations. As far as Kant is concerned, a more comprehensive discussion is called for at this stage.

To someone who holds an atheist position and believes that the phenomena of life and moral obligation can be reduced to the language of physics, it will indeed seem strange that human behavior carries a more interesting dimension than a random wave in the ocean. This position is in fact a scientific program that if we only follow we will understand the phenomenon of life, morality and the development of conscience in terms of the natural sciences etc. The main arguments in its favor are its simplicity and its generality. At the same time, it seems to me that today we are in a phase of disillusionment with the reduction, and are beginning to see it as a temptation; we are more open to seeing the uniqueness of life, and are aware of the difficulties of explaining human morality in naturalistic terms. The discussion above joins the trends to search for a unified view of the world that does not fall into reductionism.

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Notes

1 Feynman merely assumed that the universe is too large relative to humans, but it is not at all clear that the development of humans is possible within smaller dimensions of space and time. Consider, for example, the fact that the dimensions in which the drama unfolds are not negligible in relation to the size of the universe; The age of Earth is about a third of the age of the universe. Is the time elapsed from the moment of Earth's formation to the creation of complexities, as we know them today, too long? This is a question we are grappling with today (see →Valiant, 2009), and at this moment, we do not know. If the evolution of Earth and the evolution of humans - what Feynman refers to as "the drama" - is on a scale of the stage, then the universe and its size are not necessarily excessively large. The question of whether the size of the universe is too large is reducible to the question of whether the rate of the universe's expansion is too large. However, stating that a rate is too large or too small is meaningless. In other words, the assertion that the universe's size in terms of space is too large for humans lacks a clear meaning.

2 I allow myself to speak freely here without distinguishing between sentences and propositions, predicate and properties.

3 The fact is that for exactly the same reason it is impossible to prove that God does not exist.

4 Any proposition is provable in a system which includes it as one of its axioms (I thank the late Mark Steiner for emphasizing this remark). In reviewing all the ways to prove the existence of God we should of course skip over systems where there are such trivial proofs for the existence of God. My argument is that even if we grant Kant the coherence of such a review there will still be problems as I show in the article.

5 Similar formulations are found in the literature, but those I am familiar with do not include step 4. It should be noted that if there exist contingent entities, there exist sets of contingent entities. The set of all contingent entities is equivalent to the universal set. If we accept Zermelo-Fraenkel set theory we now have a problem, since one can prove that there is no such set. One can invoke here von-Neumann set theory and view the universal set as a class. A better possibility would be to use a set theory with a universal set (see my paper in this collection *Spinoza and the Universal Set*). Noteworthy, the necessary Being that is guaranteed by the cosmological proof mentioned above is not included in the universal set. In this sense, we avoid the pantheistic world view that was defended in the paper on Spinoza.

6 A comprehensive study of the physico-theological proof would call for an examination of Kant's other writings, which include other criticism. See →Proops (2021, ch. 16).

7 In this paragraph Kant does not expand on the question of the perceptual sense of seeing harmony. Kant

distinguishes between the connection of purposefulness, beauty, and sublimity to human observation, but at this stage we will not get involved with the specific manner in which they are connected to experience.

8 This of course leads to Kant's interpretation of the complexities it involves. However, it is advisable to study the footnotes at the end of the *Critique of Pure Reason* on doctrinaire belief: A820, B848 to A831, B859.

9 On the relations between Kant and Hume on this point see →Proops (2021, §16.6).

10 The Hebrew word here is God (מיהולא). Some translations translate this as "angels." This is inconsistent with a worldview according to which man as a free-willed being is able to surpass the rank of angels.

11 This issue is of course connected to the question whether other laws of nature concerning the structure of elementary particles and so on are possible. See my paper on the topic, →Buzaglo (2010).

12 If the centrality of the structure of the world of particles is also a requirement to get rid of the idea of matter, as Heisenberg thought for example, we are left with a world of phenomena and with mathematical entities. The issue requires an extensive discussion that I cannot offer here.

13 In fact, Kant himself already raised the idea of causality of a different kind to deal with the possibility of freedom

in a world where phenomena are governed by causality (See, e.g. *Critique of Pure Reason*, A545, B573). What we propose here is an extension of this concept in such a way that it would also include the creation of the universe

14 See →Kravitz (2017).

30 Spinoza and the Universal Set: A Proposal

Meir Buzaglo

Abstract: According to the prevailing position these days in the discussion of infinity, one should distinguish and separate mathematical discussion from metaphysical-theological discussion. According to the position that emerges from Cantor's writing, these two fields should be distinguished, but not separated. Cantor was able to express his position in the context of a critique of Spinoza's pantheism. In this article I attempt to respond on behalf of Spinoza to Cantor's criticism. If one reads a Spinozist position into the realm of sets, a set theory emerges that includes a universal set. In this article I have taken the first steps towards such a theory of sets, and added comments on the cross-fertilization that exists between the metaphysical discussion and the mathematical discussion on the subject of infinity.

I remember that when I was studying Cantor's transfinite numbers with Spinoza's *Ethics* at the back of my mind, a question immediately arose concerning Spinoza's definition of God:

> By *God*, I mean a being absolutely infinite—that is, a substance consisting in infinite attributes, of which each expresses eternal and infinite essentiality. (*The Ethics*, Part I Definition VI trans. R. H. M. Elwes, 1883)

On reading this definition after Cantor, we are compelled to ask —what kind of infinite?! Like the natural numbers? Like the

number of points on a continuum? If we read the definition according to the ZF axiom system or according to Cantor's position, we arrive at a quandary concerning Spinoza's approach. As will be remembered, according to Spinoza we recognize two attributes: extension and thought, but we are not entitled to restrict ourselves to these attributes, which express human limitation rather than the nature of God (see →Melamed, 2018). According to Spinoza, such restriction limits God. This is why we refer to God as having infinite attributes or as "absolutely infinite," as the definition says. But here the quandary arises: If God consists "only" in $\aleph_0$ attributes, we have a limitation similar to that we saw when ascribing only two attributes to God. Furthermore, since for every transfinite cardinal there exists a greater transfinite cardinal, any ascription of a transfinite number of attributes to God will arrive at the same limitation that we encountered with the ascription of only two attributes. We are forced willy-nilly to ascribe to God an infinity that has no greater infinity. Here we arrive at Cantor's paradox: the concept of a greatest infinity is considered in set theories such as ZF to be internally inconsistent, since the power set of such an infinity is greater than it. Thus, random speculation following basic modern mathematics casts a shadow over the definition of God in Spinoza's *Ethics*.

Study of the history of set theory revealed that I had unintentionally stumbled upon a dispute between Cantor and Spinoza. Cantor was famously a faithful reader of Spinoza (→Bussotti and Tapp, 2009), and at the time that he was attacked by his teachers for the transfinites that he introduced, his most important allies were Spinoza's *Ethics* and *Letter on the Infinite*. On the other hand, Cantor also provides us with a critique of Spinoza's pantheism, which as a faithful Christian he found disturbing. The dialogue is passionate because of the forces of attraction and repulsion at work within it, and the thought with

which I opened this paper does form one of its stages. The dialogue between Cantor and Spinoza is the subject of a lively literature, which in my impression lays stress on the historical-conceptual. I will begin this paper with a short description of Cantor's objections to Spinoza's pantheism as they emerge from this literature. Following this, I will propose outlines for a Spinozist response rejecting this assumption. Incidentally to this will appear initial outlines for a Spinozist set theory which I propose here for mathematical research.

The article aims to show the bearing of the metaphysical-mathematical discourse on infinity. This is in contrast to distinguished experts in set theory who understand the discussion of infinity in mathematics as a mathematical internal issue (see e.g., →Kanamori, 2012). I therefore tried not to assume prior knowledge, to allow as many readers as possible to join the discussion.

30.1 Cantor's Challenge to Spinoza

Spinoza's pantheism concerned Georg Cantor throughout his life, as is clear from his familiarity with the *Ethics*, his careful study of the *Letter on the Infinite* and the critique of Spinoza he formulated. This dialogue interested Cantor not only as a mathematician but also as a philosopher with a distinct theological and metaphysical interest. Although Cantor's writings include differing formulations with no single consistent approach, he appears to be aligned with a concept connected to his distinction between three concepts of the actual infinite:

> The actual infinite was distinguished by three relations: first, as it is realized in the supreme perfection, in the completely independent, extra worldly existence, *in Deo*, where I call it absolute infinite or simply absolute; second to the extent that it is represented in the dependent, creatural world; third as it can be conceived *in abstracto* in thought as a

> mathematical magnitude, number or order type. In the latter two relations, where it obviously reveals itself as limited and capable for further proliferation and hence familiar to the finite, I call it *Transfinitum* and strongly contrast it with the absolute. (Cantor, 1986, p. 372)

When we refer to the natural numbers as being *potentially* infinite, we mean that any finite natural number has a number greater than it, but a set is *actually* infinite, if the number of its members is greater than any natural number. On the one hand, the transfinite numbers in mathematics are greater than any finite number, but on the other hand for any actual transfinite number a greater number can be conceived. From this point of view, they are no different from finite numbers.

In this paragraph, Cantor discusses the infinite in the light of the objects in which it is realized: God, nature and abstract thought. From this point of view, his discussion is similar to Spinoza's, which defines the infinite in the light of the metaphysical structure of existence: substance and its properties, with distinctions in the properties of perception, intellect, and imagination (in particular in the *Letter on the Infinite* mentioned earlier, and in expanded form in Proposition XV of Part 1 of the *Ethics*). However, although Cantor's discussion of the infinite is rich in metaphysical-theological concepts, his chief focus is mathematical. In this respect he differs from Spinoza. It is worth mentioning here that Cantor's proposal for three senses of the "actual infinite" emerged in a period in which mathematics was going through a process in which proofs and definitions were being made rigorous; this is also the period in which Karl Weierstrass was eliminating any reference to the actual infinite in the definition of limits and differentials through the use of epsilons and deltas. Cantor, a student of Weierstrass who proposed the concept of the actual infinite, undermined these developments, and to add to the drama Leopold Kronecker, another of Cantor's teachers, completely rejected the

concept of transfinites and declared war on his ideas as "corrupting the youth." Fortunately, the youths in question included mathematicians such as David Hilbert who were enchanted by the world that Cantor opened to them and aided in the acceptance of Cantor's ideas in spite of the opposition to them.

For our purposes, the important distinction is between the actual infinities believed to exist in nature—the infinite dimensions of space, or the number of points on a line or on a time continuum—and the infinities attributed to God. The qualitative difference perceived between them is the possibility of increase. While one can conceive without contradiction a number greater than any given finite number, and one can conceive an infinity greater than the universe, with respect to the infinity of God such a possibility leads to a contradiction. The absolute infinite is the perfect totality which cannot be increased.

This position has roots in theology, including such figures as Maimonides, and in Kabbala (see →Buzaglo, 2022, Chapter 11). The most frequently quoted source is presumably St. Anselm of Canterbury, who used it as the basis of a proof of God's existence. God is not just a great and powerful being, greater than any finite created entity: no greater perfection can be conceived. From this point of view, and to the best of my knowledge Cantor does not go beyond this comparison, the universal set represents the totality of God. The emphasis is on "represents": it is not identical (→Cantor, 1883, p. 205).[1]

Interest in absolute infinity undergoes a transformation when thanks to Cantor's theorem we can prove that absolute infinity includes a contradiction. There are a number of ways to arrive at a contradiction and we will restrict ourselves to Cantor's proof that the cardinality of a set is strictly less than the cardinality of its power set. If we apply this theorem to the set of

all sets we can conclude that there is a set larger than absolute infinity.

However, Cantor does not see this paradox as a problem.[2] For Cantor God and the absolute infinite are not a topic for mathematical research:

> The absolute can only be acknowledged [anerkannt] but never known [erkannt]—and not even approximately known. (Cantor, Gesammelte Abhandlungen, Grundlagen endnote 2, 205; *From Kant to Hilbert II*)

In a statement relevant to the question of the significance of paradoxes he sees a mathematical result in this contradiction:

> I venture to designate this theorem, which is completely secure and is proved from the definition of the "totality of all alephs," as the most important and noble theorem of set theory. But one must understand the expression "finished" correctly. I say of a set that it can be thought of as finished, and name such sets, if they contain infinitely many elements, "transfinite" … if it is possible without contradiction (as is the case for finite sets) to think of all their elements as existing together and hence the set itself as joined together as a thing by itself; or also, in other words, if it is possible to think of the set with the totality of its elements as presently (→Cantor, 1991, p. 390 trans. by D. E., →Rowe 2023, p.6).

The classification of three types of actual infinity and the inherent contradiction in the concept of the absolute infinite are the basis of Cantor's critique of Spinoza. In Cantor's opinion, the application of the concept of transfinites to the absolute leads to a contradiction, and this proves the transcendence of God. For Cantor, this contradiction only proves the fact that the universal set represents the infiniteness of God. Hallett made an important observation: for Cantor there is no need to "fix" set theory to prevent contradictions from arising. In his diagnosis the problems stem from collections so large that it is paradoxical to view them as a single unity. This leads to the refutation of the

radical rationalism that holds that it is possible to conceive of a perfect totality implied by the concept of God without contradiction.

This trend in criticism also arises from the distinction between the absolute infinite and the infiniteness of nature. The trend can be developed in a number of directions: one simple direction is to point out that nature contains a finished finiteness, but the absolute infinite is an infinity that cannot be conceived as a finished whole. Nature as finished infinite may be represented by the transfinite cardinals. The number of points on a continuum, the size of the universe: all these are conceived in mathematical terms, and from this perspective they differ from the absolute infinite. Indeed, when he was attacked by the Catholic Church on the grounds that by also allowing the ascription of infinity to nature he was impugning the uniqueness of God and leaning towards Spinoza's pantheism, Cantor had to make this distinction between infinities. The infiniteness of God is different in kind, since the human intellect is unable to grasp it as a finished entity. Cantor expands:

> Another common confusion is seen with the two forms of the actual infinite, in which namely the transfinite is mixed up with the absolute, while these concepts are strongly distinguished, insofar as the former is infinite, but can still be added to, while the later is essentially such that it cannot be added to, and therefore is not determinable by mathematical thought; we encounter this failure, for example, in pantheism, and it constitutes the Achilles' heel of Spinoza's *Ethics*, of which in fact F. H. Jacobi remarked, that it cannot be contradicted by principles of reason (*Vernunftgründen*). (→Cantor, 1962, p. 375, trans. →Newstead 2009, p.253)

God for Spinoza is substance—THE substance. His attributes, modes, and essence exist simultaneously as a perfect existence. In his writing Cantor saw this infiniteness as inconsistent, as a being that cannot be conceived as complete. A Spinozist is

unable to agree with this perspective, since it would require him to admit that God is not substance.

30.2 A Pantheistic Set Theory: The Concept

It is possible to attempt to reply to these difficulties in a number of ways. The most direct way is to mention the status of mathematical substances for Spinoza. While the geometric method was considered vitally important by Spinoza, numbers were considered to be objects with a low ontological status (→Melamed, 2000). In fact, Spinoza's interpreters are inclined to object to the mixture of approaches and to see every critique as an attempt to "cantorize Spinoza."[3] This critique generally leaves the discussion in the realm of interpretation, and without reaching a conclusion: the discussion is summed up by the claim that from Cantor's perspective there is a question, but from Spinoza's perspective there is no question. In other words, the discussion remains within each side's premises. Although the rejection of cantorizing Spinoza makes sense when aimed at contemporary interpreters wishing to understand or critique Spinoza, it is valueless in itself as an attempt to deflect Cantor's critique of Spinoza.

Similarly, it is doubtful whether this process can rescue the uses Spinoza makes of the idea of the infinite. The application of infinite attributes to God that I referred to at the beginning of this paper shows that infinity of attributes can be summed up not by understanding it as an uncontainable totality but rather as one for which a limitation to only two attributes is opposed to its nature.

Spinoza is forced to ascribe to God a number of attributes that cannot be grasped by human intellect. Another way to attempt to respond on behalf of Spinoza is by means of an axiom system permitting mathematical treatment of the set of

all sets. Von Neumann proposed a system that has since been improved and upgraded to NBG. In this system, a distinction is introduced between classes and sets: the former are too large to be considered sets. The term “large” is defined here in a reasonable fashion, as large as the class of all sets. Von Neumann’s diagnosis of Russell’s paradox, Burali-Forti’s paradox and Cantor’s paradox is based on the fact that they refer to groups that are “large” in this sense. However, this does not mean that they are objects that preclude mathematization: they can be given mathematical treatment on condition that we attend to the special logic that classes must conform to. Sets that are not large can belong to classes or to other sets, but classes do not belong to classes or to other sets.

However, it appears that this approach too will not really satisfy a Spinozist. From the perspective of Von Neumann’s system (and its various developments) a class such as the set of all sets and a class such as Russell’s class attain the same status. The fact is that as we described above it accepts Cantor’s approach concerning the sources of the paradoxes and it attempts to evade them instead of celebrating them as a theological discovery. It is also impossible to escape the impression that von Neumann’s axiom system gives ontological priority to sets over classes. Not only because of the naturalness with which quantification is treated over sets, but because the very diagnosis of the strangeness of classes as a result of their size makes them similar to Kant’s ideas, and deprives them of the status of complete wholes.

The third way to respond on Spinoza’s behalf is to develop a set theory based on Spinoza’s principles, not as a tactic to avoid paradoxes. By this I do not mean to examine the question through the Spinozist approach but to make use of the qualitative traits of Spinoza’s pantheism in a way that is not restricted to Spinoza’s approach and takes Cantor’s ideas into

account. To this end, I will sketch in general outline a pantheistic set theory in which Cantor's question does not arise, and which can have an independent mathematical value in the current debate. This pantheistic set theory is a theory that does not dismiss out of hand an at least partial representation of the infiniteness of the Spinozist substance—God. It seems to me correct at this stage to present three qualitative characteristics of Spinoza's method that may be considered the essence of the Spinozist version of pantheism:

1. The metaphysical characteristic: God is the substance which is the foundation of everything and everything is included in it/dependent on it in one way or another (I leave on one side here questions of interpretation of the relationship between attributes and modes).
2. The methodological-epistemological characteristic: the total intelligibility of substance, without which nothing can be fully understood.

In the spirit of these characteristics one can add a the principle of plentitude (which is similar to what is called in contemporary literature "maximal ontology") which carries implications for mathematics:

1. The non-existence of an object of a certain kind must be due to an absurdity in the concept of this object. If a concept is consistent then there is an object of which this concept is true.

> Of everything whatsoever a cause or reason must be assigned, either for its existence, or for its non-existence—e.g. if a triangle exist, a reason or cause must be granted for its existence; if, on the contrary, it does not exist, a cause must also be granted, which prevents it from existing, or annuls its existence. This reason or cause must either be contained in the nature of the thing in question, or be external to it. For instance, the

> reason for the non-existence of a square circle is indicated in its nature, namely, because it would involve a contradiction. On the other hand, the existence of substance follows also solely from its nature, inasmuch as its nature involves existence. (See Prop. vii.) (*The Ethics*, Part I, Proposition XI 1, trans. Elwes).

These are the foundations of the pantheistic approach in its Spinozist version, so it is clear why Cantor's claim that the absolute is not a complete and finished entity, and the paradox that this entails, challenge these two propositions of Spinoza. Like Spinoza, Cantor assumes that the absolute cannot be added to, and therefore this is not a dialogue of the deaf, but Cantor believes that this characteristic of the absolute is sufficient to formulate his paradox and refute Spinoza's pantheism. Starting from the metaphysical and epistemological conceptions of God is expressed by seeing these as presuppositions, such that any challenge to them will be dismissed.

30.3 A Spinozist Beginning

We begin by determining that there exists a set which is the set of all sets, and determine to eliminate any proposition that leads to a paradox emerging from the assumption that such a set exists. This is the starting point for a Spinozist response from a pantheistic perspective. At this stage, we adopt the claim that this infiniteness is a representation of totality, and accept that totality in itself is in no respect contradictory. With this axiom and with the decision to eliminate paradoxes arising from this assumption, we encounter Cantor's paradox. This will require us to pay a price: if we need to make over-artificial assumptions in order to reject the paradox, there will be a problem with this attempt at an answer to Cantor's challenge. Let us examine this closely. We will set down two required axioms.

Axiom 1: *V is a set*.

Axiom 2: *If x is a set then P(x) is a set*.

Theorem 1: *The power set of the set of all sets is identical to the set of all sets. In other words, P(V)=V.*

Proof: Every member of V is a subset of V. In fact, for all x, if $x \in V$, then if $y \in x$ then $y \in V$. This means that every member of x is a member of V, and therefore x is a subset of V. On the other hand, every subset of V is a member of V. From these two propositions we arrive at the following equality: P(V)=V.

Here we have assumed the axiom of extensionality, which we will call "Axiom 0." To these I will add an axiom of a totally different kind, which in some developments can be omitted. At this stage, I write these axioms and others that I will propose freely, under the understanding that at some point I will have to make my statements rigorous, confirm independence, etc.

Axiom 3*: I(x), the identity function, is a bijective function from V to P(V).*

At this point Cantor's diagonal is lying in wait, ready to create a paradox. We define the set R that corresponds to every injective function creating a problem: $R=\{x/x \notin I(x)\}$. But when I(x) is the identity function, $R=\{x/x \notin x\}$.

In other words, in order to create Cantor's paradox we have to assume that there is a set conforming to Russell's property. If we accepted a set that we knew was legitimate "in the light of reason," we would see it as problematical for our determination to inactivate Cantor's paradox. However, we have here accepted a set which is notorious independently of our whole discussion, and thus we affirm the decision to eliminate Cantor's paradox. This can be formulated as a theorem.

Theorem 2: *No set conforms to the attribute "$x \notin x$."*

Proof: Following what was stated earlier, V is a set, and therefore so is P(V), and the identity function. If a set conformed to this property, we would arrive at a contradiction.[4]

It is worth comparing this discussion with the discussion in the ZF system accepted by contemporary mathematicians. Here we begin by determining known and largely accepted axioms that include the axiom of separation. This replaces the axiom of comprehension which leads to excessive absurdity, and instead we adopt a weaker axiom: Given a set x and given any attribute A(x), there exists a set: {y|y∈x and A(y)}.

The idea is that from a "safe" set x it is possible to separate a subset that satisfies a particular property A(y). Now, Russell's predicate is used in ZF to prove that V is not a real set. If it were a set, we would derive Russell's paradox via the weak axiom of separation. When we accept V as a legitimate set we use Russell's paradox to reject the axiom of separation:

ZF states: *If we accept the axiom of separation, then the set V does not exist.*

Spinozist: *V exists, and therefore the axiom of separation in its general formulation is refuted*.

Spinoza's radical rationalism makes it possible to strengthen the position presented here.

Indeed, there is no need for the separation axiom to avoid Russell's paradox. If a concept is not coherent then there is no object that satisfies it. The property of the Russellian set contains a contradiction and with this the question of its existence is decided. At the same time, a Spinozist might refers to the principle of plentitude that she shares with Cantor according to which if a concept does not contain a contradiction then there is an object that satisfies it. After all, this was one of Cantor's proofs for the existence of the transfinite numbers (→Hallett (1984), p.20). Applying plentitude to the set of all sets, one can deduce its existence from its consistency.[5] Reading the discussion in a broad context may bring up a somewhat embarrassing situation. Thanks to the consistency of ZF it is possible to prove the consistency of the set of all sets (see e.g.,

Church 1974), and assuming the principle of plentitudes, one might conclude that ZF which does not acknowledge the existence of V is false. This is how the situation looks at least after the first round.

So far I have described our starting point. I will now attempt to delineate in a general and preliminary fashion several steps to complete the process. First we will include the process suggested above by limitation of the axiom of comprehension. Any bijective function—not only the identity function—between V and P(V) limits the axiom of comprehension, similarly to the removal of Russell's set.

Axiom 4: *Given a bijective function F from V to P(V) the predicate "x∉F(x)" does not define a set.*

From here a reasonably clear direction is defined. We will add axioms that will ensure the existence of certain sets, and we must remain alert that none of these sets contradict Axiom 4. If we extend the system so that we recognize set S, and bijective function G from the set of all sets to the set of its subsets, we must take care that there is no case in which S={x⁄x∉G(x)}.

This is possible by rejecting the existence of such sets S, or by rejecting such functions G (this is why I added Axiom 3). This procedure begins to be mathematically interesting as well, and it is reasonable to assume that it can be implemented in more than one way.

30.4 Upward Extensions

The recognition of V as a set has implications for the axiom system, but does not determine it from every angle. If we are interested in a richer set theory, we are entitled to fill it out in a consistent way. One method that suggests itself is to build upwards with a constructive axioms concerning sets. A number of axioms naturally suggest themselves inspired by ZF. Thus, if

we move downwards we arrive at what are normally called “classes” as legitimate sets, and dismiss problematical classes. On the other hand, moving upwards we can proceed by adding safe sets. I will permit myself to proceed freely here, leaving questions of consistency and independence on one side for advanced mathematical investigation.

Let us adopt the conventional definition of the empty set: Ø= {x/x≠x}, and add the following axiom:

Axiom 5: *There exists an empty set.*

If we employ the power set axiom stated as Axiom 2 on Axiom 5 we can prove the existence of sets such as:

{Ø}, {Ø,{Ø}} and so on.

The pairing axiom is called for here, and with it we arrive at the axiom of infinity, which requires the existence of an infinite set, in one of the accepted manners of formulating this axiom. These required steps are not problematic in any way that appears at this stage. The interesting challenge is to formulate weaker versions of the axiom of separation and the axiom of replacement. And there is more than one way to do this. I chose here one formulation which has a metaphysical implication.

One can find in Cantor an identification between the absolute infinite that nothing is larger than, and the absolute infinite that is larger than any transfinite number. It is possible to understand the source of this identification. “Larger than any transfinite number” leads to a contradiction, indicating that it is a whole which is “too big,” and there is no need to look for anything larger. However, once we have accepted that there is no contradiction in the idea of a set larger than all the transfinite numbers, it is interesting to investigate whether the converse is true: in other words, if there can be a set that is larger than all the transfinites but nevertheless smaller than the universal set. Here is a possible direction to consider. Hartog’s theorem famously proved in ZF that for every set S there is a cardinal

number H, such that H is greater than the power of S. The theorem is based on the axiom of replacement, which as we have explained above is not applicable to the set theory whose basis we have sketched here. The proposal of an updated formulation of the axiom of replacement would be helpful here.[6] I leave this question to mathematicians, but I make the following suggestion for consideration (and possibly refutation).

Axiom 6 (weak replacement): *If α is an ordinal, in von Neumann's sense, and F is a function, then F[α] is a set.*

To grasp the richness of this system, it would be interesting to look for possibilities that do not arise in ZF and other similar conservative systems. One such possibility is a set greater than all the transfinites and smaller that the universe. In other words, is it consistent to assume a set that reflects the relation of V to cardinal numbers? An attractive candidate that suggests itself naturally is the continuum: Is it consistent (with Axioms 0–6) to assume that P(N) is greater than every aleph? A positive answer might improve our understanding of the nature of the undecidability of the continuum hypothesis.[7] This also has a clear metaphysical implication: the idea of an entity that cannot be added to and increased is different from an entity that is greater than any transfinite number. These considerations, which arose from mathematical curiosity, may have a reciprocal influence on the metaphysical discussion.

I return to the question with which I opened the article. The infinity that characterizes the number of attributes of the divine is greater than any natural number and greater than any transfinite cardinal. This is the necessary update stemming from Cantor's expansion of the concept of numbers to transfinite numbers. Cantor's paradox can be quashed by using the principle of plentitude that is shared by him and by Spinoza. However, caution must be exercised to establish this as a defense of Spinoza's definition of the essence of God. If we

assume sets that include a number of members that is greater than any transfinite number, yet still smaller than the set of all sets, it is also possible to contemplate a substance with an infinity of attributes that goes beyond every aleph, and yet there remains a distinction between this substance and God as the perfect all-encompassing Being. In that case Spinoza's definition of God is not problematic because the number of attributes that it ascribes to God is "too big," but because this definition gives us a notion of God that is not big enough.

30.5 Conclusion

The well-known lesson from discussions of infinity, not to apply what is true for finite sets (e.g. that a proper subset is always less powerful than the set) to infinite sets, has its parallel here. In our case we maintain that for a large family of sets it is true that the power set is always greater than the set; but this is not true for the set of all sets. This can be stated parsimoniously in the form that it is a set in the same sense that the empty set is a set, but different principles apply to it. There is no reason why what is true for some sets must be true for all sets. In more general terms, the universe of sets is regulated by one logic, but the true principles of sets are not universal. Thus we limit the meaning of Cantor's finitism, which sees a strong analogy between finite sets and infinite sets.

From here one can understand the way this approach differs from von Neumann's set theory. In the case of the latter, we assume that the reason there are paradoxes in set theory is due to the fact that we have not distinguished between classes that are "too big" and small sets. In this approach we have no direct argument to think that very large sets are problematic, only an indirect one which stems from studying a number of paradoxes that we get rid of if we distinguish between classes and sets. If

we assume Spinoza's principle of plentitude as manifested in the third principle I listed above, we adopt the existence of the set of all sets and reject the axiom of comprehension. Notably, Russell's property fails to define a set not due to an overwhelming magnitude of sets satisfying its criteria, but rather due to its inherent contradiction with the principle of plentitude. A metaphysical principle that is shared by Cantor, Spinoza and others allows us to decide between their systems. What is obviously required here is a study of the system outlined here which include an evaluation of its relation to other set theories, and developing the potential inherent in it for fundamental questions in set theory.

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Notes

1 There is another way to represent God's relation to size, which is neither Cantorian nor pantheistic, which understands God's glory as going beyond the world of sets. See footnote 4 in "Are proofs for the existence of God impossible," in this collection.

2 There is an additional proposition connected to the Burali-Forti paradox that proves a similar absurdity, which Cantor was well acquainted with. In this paper I have restricted myself to Cantor's paradox and to Cantor's rejection of the axiom of comprehension—a rejection which has implications for the class of ordinals.

3 Following →Arthur (2001).

4 The history of this remark can be traced back to Russell's early position that Cantor's argument is simply wrong. Russell believed that Cantor's paradox is not a paradox but a mistake in Cantor's proof. However, after walking around for many months convinced that a mistake had been made here, he abandoned this direction and proceeded to develop his theory of types. A claim in a similar direction was made in Church (1974).

5 See →Buzaglo (2021) for a further discussion.

6 Church (1974) proposes a version of the axiom of replacement for what he called "low sets" – sets which have a one to one relation with a well-founded set. The possibility we are entertaining here does not exist in his system, since in his system Hartog's theorem is valid.

7 If, e.g., we view the mathematics of infinity as an expansion of finite mathematics we could understand transfinite numbers and the power set axiom as two forced external expansions (in the sense developed in Buzaglo 2002) that are indifferent to each other.

31 A Divine Consistency Proof for Mathematics

Harvey M. Friedman

Abstract: We present familiar principles involving objects and classes (of objects), pairing (on objects), choice (selecting elements from classes), positive classes (elements of an ultrafilter), and definable classes (definable using the preceding notions). We also postulate the existence of a divine object in the formalized sense of lying in every definable positive class. ZFC (even extended with certain hypotheses just shy of the existence of a measurable cardinal) is interpretable in the resulting system. This establishes the consistency of mathematics relative to the consistency of these systems. Measurable cardinals are used to interpret and prove the consistency of the system. Positive classes and various kinds of divine objects have played significant roles in theology.

Introduction

This work came about from our interactions at and reflection on two conferences hosted by the John Templeton Foundation. The first of these meetings was held in honor of the 100th birthday of Kurt Gödel, in Vienna, in 2006. The second was held in honor of the 100th birthday of Sir John Templeton, in Heidelberg, in 2012.

Interactions at the second of these meetings have previously led to →Friedman (2012b), where a basic mathematical property of a mathematical object in a mathematical structure, immediately suggested by familiar ideas about God, is shown to

hold within some artificially created mathematical structure. Specifically, there is a relational structure of finite relational type with a unique undefinable element, in the usual sense of first order predicate calculus with equality.

At the Vienna meeting, there was discussion of Gödel's formalization of earlier ontological arguments for the existence of God. This relied heavily on modal logic, but what we found particularly striking was the use of "positive properties," which has a substantial history and goes back at least to Leibniz.

From a theological standpoint, it is perhaps natural to view the attribute of positive as a facility that God has created and given the world access to, perhaps in order to help direct us into "positive behaviors."

It is clear that Gödel was using "positive properties" as, mathematically speaking, an ultrafilter on properties. In fact, at least implicitly, he was using "positive properties" as an ultrafilter on extensions of properties. I.e., whether a property is positive depends only on what objects it holds of. For discussion of ultrafilters and positivity, see Section 31.3 below.

It occurred to us that perhaps this highly intriguing ultrafilter, viewed as an ultrafilter on classes of objects, can be used to prove the consistency of mathematics. At the time, we just didn't see how to get such ambitious mathematical mileage out of this "positivity ultrafilter," at least in any simple basic conceptual way. This goal seemed particularly remote since the positivity ultrafilter, as discussed by Gödel and implicitly by others, is what is called a "trivial ultrafilter," i.e., an ultrafilter consisting merely of the classes containing some given special point.

In particular, following Gödel, and going back at least to Leibniz (see →Gödel, 1995, and also →Anderson, 1990; →Leibniz, 1923, 1956, 1969, 1996; →Look, 2006; →Oppy, 1995, 1996, 2006, 2012; →Sobel, 1987, 2004), a class of objects is positive if and

only if it contains God. And trivial ultrafilters are, as the name suggests, mathematically trivial. So it would appear that one cannot expect to do anything substantial, mathematically, with the positivity ultrafilter.

Interactions with scholars from theology at the second meeting rekindled our interest in trying again to do something powerful with this mathematically trivial positivity ultrafilter. It occurred to us that if we take God out of the class of all objects, treating God as exceptional, but keeping the positivity ultrafilter, pruned to be over the class of objects excluding God, then the positivity ultrafilter is no longer trivial. In fact, it is a nontrivial ultrafilter over the class of all objects without God.

It is well known that in the context of set theory, certain kinds of ultrafilters are enough to prove the consistency of mathematics, as formalized by the usual ZFC axioms. In particular, a nontrivial countably complete ultrafilter serves this purpose. However, to straightforwardly state that the positivity ultrafilter is a nontrivial countably complete ultrafilter, together with the mathematical infrastructure needed to state and make use of this, is far too technically brutal to be of fundamental philosophical or theological meaning.

So we encountered the following challenge: to find fundamental properties of the positivity ultrafilter which, in the context of a conceptually very minimal mathematical infrastructure, has sufficient power to construct a model of ZFC, and perhaps more.

This paper offers our response to this challenge. Here is a succinct six-part description of our framework.

1. Objects and classes of objects, linked by membership. This is arguably viewed as within our capacity to clearly imagine, as it is an accepted useful working component of modern mathematics. It should be noted that in modern

mathematics, objects are usually restricted to objects with a definite mathematical purpose, rather than a completely general notion of object. See Section 31.1.

2. Pairing of objects—a combining of any two objects into one. This is generally viewed as within our capacity to clearly imagine. E.g., the pair of two objects can be taken to be the idea of having the first followed by the second. In various guises, this is also an accepted useful working component of modern mathematics. See Section 31.1.
3. A choice operator CHO that picks an element out of each nonempty class of objects. Formally, $x \in A \rightarrow CHO(A) \in A$, where CHO(A) is read "chosen element from A." This appears to be beyond our capacity to clearly imagine. I.e., we don't have an understanding of how to go about making the choices. However, this continues to be an accepted useful working component of modern mathematics. We can relevantly view a CHO as given to us by God as a manifestation of his omnipotence. See Section 31.2.
4. The positivity attribute on classes of objects (the positivity ultrafilter). This seems definitely beyond our capacity to clearly imagine. I.e., we don't have an understanding of definite criteria for positivity. The notion of ultrafilter is also an accepted useful working component of modern mathematics. But a specific preferred ultrafilter, particularly on the entire universe of objects, has not appeared as a generally accepted component of modern mathematics. See Section 31.3.
5. The definability attribute on classes of objects, reflecting the standard notion of "class of objects definable from the preceding concepts 1–4 without parameters." We have a clear understanding of this notion, relative to the primitives to which it is being applied (here, in 1–4 above).

This is an essential feature of modern mathematical logic, and also has useful interactions with several branches of mathematics—especially real and complex algebraic geometry. It is exemplified by Tarski's formal treatment of truth in formalized languages. See Section 31.4.
Thus the positivity attribute is the one feature above that has such a deep philosophical and theological meaning. The additional axiom that creates the vast logical power, is the Divine Object axiom. Recall that there does not exist an object which lies in all positive classes, as we treat God as exceptional and outside the class of objects. This is reflected by the positivity ultrafilter being nontrivial. The crucial innovation is this: an object is *divine* if and only if it lies in all *definable* positive classes.

6. The Divine Object axiom. There is a divine object. See Section 31.5.

We can give the following natural theological interpretation of this development. God created a certain structure, with several components, which he gave the world intellectual access to. This intellectual access is reflected by the system T_5. Using T_5, man creates a mathematical universe, or model of the mathematical universe, through set theory, as presented in Sections 31.6 and 31.7 in this paper.

The axioms of T_5 are so simple that a promising research project is suggested: to analyze all such simple systematizations, and determine what their mathematical ramifications are. As we prove in Section 31.7, T_5 corresponds, logically, to something close to ZFM = ZFC + "there exists a measurable cardinal." See Section 31.5.

In modern set theory, there are developments that have led far beyond the scope of normal modern mathematics. One of these is the detailed study of ultrafilters on arbitrary sets (or the

closely related abstract measure theory). This has led to a study of certain set-theoretic hypotheses postulating the existence of ultrafilters with special properties. One of these set-theoretic hypotheses is “there exists a measurable cardinal.” The usual ZFC axioms augmented with this hypothesis is what we use to interpret our system with the Divine Object axiom. See Section 31.5.

We then show that we can interpret the usual ZFC axioms for mathematics in our system (Section 31.6). In fact, we interpret ZFC augmented with a set-theoretic hypothesis close to, but a little weaker, than “there exists a measurable cardinal” (Section 31.7).

In Sections 31.1–31.5, we gently build up to T_5 with a series of theories T_1, T_2, T_3, T_4. The impatient reader can safely begin with T_5 in Section 31.5.

After writing this paper, we have seen how to incorporate God as an object, as part of a wider supernatural world. In this second approach, we have both the “Real World” and the wider “Supernatural World.” Thus we have a more complex framework in that we have two sorts of objects. But a number of axioms are simplified. In particular, in this second approach, we do not need the positivity ultrafilter. So a tradeoff is emerging, where we use one sort of object and the positivity ultrafilter (this is what we do here), or two sorts of objects without the positivity ultrafilter (this is what we will do in →Friedman, 2013).

We view →Friedman (2011, 2012a, 2012b, 2013) as part of a wider program which we call *Concept Calculus*. In *Concept Calculus*, we aim to identify groups of informal concepts throughout the informal and semiformal intellectual landscape, and formulate fundamental transparent principles which are of sufficient power to interpret the usual formalizations for mathematics and beyond. These provide formal interpretations of mathematics, consistency proofs of mathematics, and relative

consistency proofs of mathematics, as in Theorems 6.27 and 7.34.

31.1 T_1: Objects, Classes, Pairing

In Sections 31.1–31.5, we present, in cumulative stages, the axioms of our theory T_5 that we use to interpret ZFC in Section 31.6. The reader can dispense with this background material and go directly to Section 31.5. T_1–T_4 admit familiar standard models constructed well within ZFC, but T_5 does not.

In this section, we focus on a particularly simple system T_1 that provides the basic apparatus needed to support the substantial and flexible use of objects and classes of objects required for the interpretation of ZFC.

The two sorted T_1 is very similar to the two sorted system Z_2 of second-order arithmetic. The main difference is that in Z_2, the objects are nonnegative integers, with arithmetic operations taken as primitives. In T_1, the objects are not constrained in any way, and are meant to include all objects whatsoever, as reflected in type 0 of the usual Russell theory of simple types (in modern formulation). The classes of objects are reflected in type 1 of the usual Russell theory of simple types (in modern formulation). For presentations of Z_2, see Simpson (1999, p. 4), and →Wikipedia (2023).

We follow the treatment in Simpson (1999) using the two sorted language for nonnegative integers and sets of nonnegative integers, with 0, 1, +,• , <, = on nonnegative integers, $\in$ between nonnegative integers and sets of nonnegative integers, set induction, and full comprehension.

The crucial issue is to determine just what should take the place, in T_1, of the arithmetic operations on the objects that are used in Z_2.

Our solution is to use a pairing function P. The Pairing axiom below is all that is required. We establish a detailed connection between T_1 and Z_2.

We first introduce the language L_1 of T_i, which has two sorts: objects and classes of objects (classes). L_1 has object variables v_i, $i \geq 1$, and class variables A_i, $i \geq 1$. L_1 uses a binary relation symbol $\in$ between objects and classes, a binary function symbol P on objects, and = between objects. (P is not used in the usual Russell theory of simple types, or in Z_2).

Here $v_i \in A_j$ means that the object v_i is a member of the class A_j. $P(v_i,v_j)$ is the object which is the ordered pair of v_i and v_j.

The object terms of L_1 are inductively defined by i. For all $i \geq 1$, v_i is an object terms of L_1; and ii. For all object terms s,t, P(s,t) is an object term of L_1.

The atomic formulas of L_1 are given as follows: i. For object terms s, t of L_1, s = t is an atomic formula of L_1; ii. For all $i \geq 1$ and object terms t of L_1, $t \in A_i$ is an atomic formula of L_1; where $i \geq 1$ and s,t are terms of L_1.

The formulas of L_1 are inductively defined as follows: i. Every atomic formula of L_1 is a formula of L_1; ii. For all formulas φ,Ψ of L_1, $\neg\varphi$, $\varphi \wedge \Psi$, $\varphi \vee \Psi$, $\varphi \rightarrow \Psi$ are formulas of L_1; and iii. For all $i \geq 1$ and formulas φ of L_1, $(\forall v_i)(\varphi)$, $(\exists v_i)(\varphi)$, $(\forall A_i)(\varphi)$, $(\exists A_i)(\varphi)$ are formulas of L_1.

We use $\leftrightarrow$ as an abbreviation. i.e., $\varphi \leftrightarrow \Psi$ abbreviates $(\varphi \rightarrow \Psi) \wedge (\Psi \rightarrow \varphi)$.

There are standard axioms and rules of inference for L_1, with a corresponding completeness theorem. The system T_1, in language L_1, has the following nonlogical axioms.

Nonlogical Axioms for T1:

Pairing:	$(\exists v_1,v_2)(v_1 \neq v_2)$, and
	$P(v_1,v_2) = P(v_3,v_4) \rightarrow v_1 = v_3 \wedge v_2 = v_4$.
L_1 Comprehension:	$(\exists A_1)(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, where φ is a formula of L_1 in which A_1 is not free.

We make some remarks concerning the axioms of T_1.

1. As we shall see, T_1 is mutually interpretable with Z_2. In fact, we establish that fragments of T_1 are mutually interpretable with fragments of Z_2. T_1 does not use Extensionality. Thus we can also think of the variables A_i as ranging over properties of objects, where $v_i \in A_j$ means that A_j holds at v_i. The matter of Extensionality is taken up in Sections 31.2 and 31.8. Specifically, we will be incorporating Extensionality in T_2. In Section 31.8, we discuss the avoidance of Extensionality.
2. Pairing. Given any two objects x,y, we can form the ordered pair P(x,y). We can raise the issue of just what P(x,y) really is. There is the conceptual idea of x followed by y. So we can define P(x,y) as the conceptual idea of x followed by y. Furthermore, P is definable, as we have just defined it. Also, the Pairing axiom, line 2, is immediate. There are of course many ways to justify the existence of at least two distinct objects. One way is to use the approach in the previous paragraph, where we take any object x, and consider the conceptual idea of x followed by x, and also the conceptual idea of x followed by x followed by x. Obviously we can argue that x cannot be the same as x followed by x, but perhaps more convincing is that x followed by x, and x followed by x followed by x are of

different lengths, and therefore are distinct. (The existence of at least one object x is normally considered to be a logical truth.)

3. L_1 Comprehension. This is the standard class existence principle. The objects and classes of objects, with L_1 Comprehension, constitutes the first two stages of a common version of the Russell theory of simple types. L_1 Comprehension alone has the trivial model with exactly 1 object and exactly 2 classes. The usual way of achieving strength is to add an axiom of infinity, involving the existence of an ordering on objects with a certain property. Or alternatively, to use the inclusion relation among classes as the ordering. Under these approaches, we need to use classes of classes of objects, or even classes of classes of classes of objects, or alternatively, binary relations on classes.

In contrast, here we use only objects and classes of objects, with the Pairing axiom doing not only the work of the axiom of Infinity, but also enough work to avoid having to use higher types or binary relations. Also, Pairing has the explicit interpretation as discussed above.

In Sections 31.6 and 31.7 we will need only L5 Comprehension where φ has only a few class quantifiers. In this paper, we will not go into just how many quantifiers are needed, but preliminary indications are that two suffice. There is also the matter of how many quantifiers over objects are needed, which is also beyond the scope of this paper.

We will consider fragments of L_1 Comprehension, and compare them with corresponding fragments of Z_2. These fragments of Comprehension are finitely axiomatizable.

The following result goes back to Tarski, but we include a proof here. It is of independent interest, and allows us to avoid a

number of details when establishing finite and non finite axiomatizability. However, in Section 31.3 we cannot easily take this route, and use another method to establish Theorem 3.6.

Lemma 1.1. Let S_1, S_2 be two theories with commuting interpretations π_1,π_2. I.e., for all sentences φ of S_1, $\varphi \leftrightarrow \pi_2(\pi_1(\varphi))$ is provable in S_1. Then S_1 is finitely axiomatizable if S_2 is finitely axiomatizable. If K is a finite axiomatization of S_1 then $\pi_1(K)$ is a finite axiomatization of S_2 (S_1).

Proof: Let S_1, S_2, π_1, π_2 be as given. Let Ψ be a sentence of S_2. We want $\Psi \leftrightarrow \pi_1(\pi_2(\Psi))$ provable in S_2. It suffices to show that $\pi_2(\Psi \leftrightarrow \pi_1(\pi_2(\Psi))) = \pi_2(\Psi) \leftrightarrow \pi_2(\pi_1(\pi_2(\Psi)))$ is provable in S_1. This follows from the hypothesis applied to $\pi_2(\Psi)$. Let K finitely axiomatized S_1. Let S_2 prove Ψ. Then S_1 proves $\pi_2(\Psi)$. Hence K proves $\pi_2(\Psi)$. Therefore $K \rightarrow \pi_2(\Psi)$ is provable from nothing. Hence $\pi_1(K) \rightarrow \pi_1(\pi_2(\Psi))$ is provable from nothing. Therefore $\pi_1(K)$ proves Ψ. QED

Lemma 1.2. T_1 is interpretable in Z_2.

Proof: We take the objects for T_1 to be Gödel numbers of terms in 0 and the binary function symbol P. We take the classes for T_1 to be the sets in Z_2. QED

Theorem 1.3. T_1 and Z_2 have commuting interpretations.

Proof: Let π_1 be the interpretation of T_1 in Z_2 given by the proof of Lemma 1.2. We now give the interpretation π_2 of Z_2 in T_1. We have to develop arithmetic in T_1. By the first line of Pairing, fix $a \neq b$. Set $0 = P(a,b)$. By the second part of Pairing, 0 is not of the form P(v,v). We say that A is good if and only if $0 \in A \wedge (\forall v \in A)(P(v,v) \in A)$. Let ω be any class consisting of the objects lying in every good A. Let 1 be P(0,0). Obviously $v,w \in \omega \rightarrow P(v,v) \neq 0 \wedge (P(v,v) = P(w,w) \rightarrow v = w)$. Also if $A \subseteq \omega$, $0 \in A$, and $(\forall v \in A)(P(v,v) \in A)$, then ω,A have the same elements (because A is good). So P(v,v), $v \in \omega$, serves as an inductive successor function

on ω. Using P, we have full access to ternary relations on ω, and hence also binary functions on ω, including comprehension involving quantification over them. So we can develop +,• , < and their standard properties in the usual way. For Z_2, the sets will be the subsets of ω in T_1. Set induction in Z_2 gets interpreted using the inductive successor function P(v,v) on ω in T_1. Set comprehension in Z_2 gets interpreted using L_1 Comprehension in T_1. To check that π_1,π_2 are commuting interpretations, we can view them as model constructions, where it suffices to show that if we compose the two model constructions, first from models M of Z_2 to models M'of T_1 via π_1, and second from models M'of T_1 to models M"of Z_2 via π_2, then we obtain a model M"of Z_2 that is isomorphic with M. So begin with a model M of Z_2. We build a model M'of T_1 within M, according to π_1. N hasn't really changed, but has merely been reassembled in the form of closed terms. Now build a model M"of Z_2 within M', according to π_2. The resulting (ω,0,S) can be determined from the point of view of M, and seen to be a second-order successor system, according to M. Therefore, the resulting model M"is isomorphic to M via an isomorphism that can be clearly defined in M. QED

Definition 1.1. Let n ≥ 0. T_1[n] is the subsystem of T_2, where in L_1 comprehension, the formula φ is required to start with at most n class quantifiers, followed by a formula without class quantifiers. Z_2[n] is the subsystem of Z_2, where in set comprehension, φ is required to start with at most n set quantifiers, followed by an arithmetic formula (i.e., a formula without set quantifiers). As in Simpson (1999, p. 16), ACA_0 is the fragment of Z_2 where comprehension is for arithmetic formulas (i.e., without set quantifiers), and $\prod^1_n$ -CA_0 is the fragment of Z_2 where comprehension is for $\prod^1_n$ formulas. These start with a

series of n alternating set quantifiers, starting with ∀, followed by an arithmetic formula.

Lemma 1.4. For n ≥ 0, $Z_2[n]$ is equivalent to $\prod^1_n$ -CA_0.

Proof: Obviously $\prod^1_n$ is a fragment of $Z_2[n]$. We show that $Z_2[n]$ is a logical consequence of $\prod^1_n$ -CA_0. Let φ be a formula for comprehension in $Z_2[n]$. In the series of at most n set quantifiers in front, we can collapse blocks of like set quantifiers to one, using manipulations already supported in ACA_0. We therefore obtain ≤ n alternating set quantifiers. We can obviously pad this with more alternating set quantifiers, if need be, to obtain exactly n alternating set quantifiers. If they begin with ∀, then we are done. If they begin with ∃, then we can take the negation, and obtain the complement of the set we need. But the existence of complements is obviously supported by ACA_0. QED

Lemma 1.5. For n ≥ 0, $T_1[n]$ is interpretable in $Z_2[n]$.

Proof: Let n ≥ 0. We use the construction in Lemma 1.1. Each class quantifier in T_1 is interpreted by a like set quantifier in Z_2. QED

Theorem 1.6. For n ≥ 1, $T_1[n]$, $Z_2[n]$, $\prod^1_n$ -CA_0 have commuting interpretations.

Proof: Let n ≥ 1. We interpret $Z_2[n]$ in $T_1[n]$. We use the same interpretation of Z_2 in T_1 presented in the proof of Theorem 1.3. We can prove the existence of an ω in $T_1[1]$, and use ω as a set parameter. We need to show that $T_1[n]$ proves the interpretation of $\{m: (\forall x_1)(\exists x_2)...(Qx_n)(\varphi(m))\}$ exists. But this gets interpreted as $\{m \in \omega: (\forall A_1 \subseteq \omega)(\exists A_2 \subseteq \omega)...(QA_n \subseteq \omega)(\varphi'(m))\}$ exists, where in φ′, all quantifiers are relativized to ω. This is provable in $T_1[n]$ by obvious quantifier manipulations. Also note that set induction

gets interpreted as $(\forall A \subseteq \omega)(0 \in A \wedge (\forall n \in \omega)(n \in A \rightarrow n+1 \in A) \rightarrow \omega \subseteq A)$, which is provable in $T_1[1]$. We can now give the model operation isomorphism argument that we gave in the proof of Theorem 1.3. QED

Theorem 1.7. T_1, Z_2 are not finitely axiomatizable. For all $n \geq 1$, $T_1[n]$, $Z_2[n]$ are finitely axiomatizable. $Z_2[0] = ACA_0$ is finitely axiomatizable.

Proof: Z_2 is not finitely axiomatizable, since Z_2 proves the consistency of each of its finite fragments (see →Simpson, 1999, section VII.7). For $n \geq 1$, $Z_2[n]$ is finitely axiomatizable, by standard normal form theorems. Also $Z_2[0] = ACA_0$ is finitely axiomatizable (see →Simpson, 1999, Lemma VIII.1.5). Now observe that in the proof of Theorems 1.2 and 1.5, we have actually given commuting interpretations. Now apply Lemma 1.8. QED

Theorem 1.8. $T_1[0]$ is interpretable in RCA_0.

Proof: Let P be one of the standard pairing functions (N,P) treated in →Tenney (1972, 1974). The first order theory of these (N,P) is shown to be just beyond elementary recursive (see →Ferrante and Rackoff, 1979, p. 163). We interpret the objects of $T_1[0]$ to be elements of N, and the classes of $T_1]0]$ to be the subsets of N first order definable over (N,P). L_1 comprehension is immediate in RCA_0. QED

Question 1.1. What is the interpretation power of $T_1[0]$? Is $T_1[0]$ finitely axiomatizable?

According to Ferrante and Rackoff (1979, Chapter 8), the first order theory of any pairing function (N,P) is at least just beyond elementary recursive, and according to →Tenney (1972, 1974), for various standard pairing functions P, the theory of (N,P) is at most just beyond elementary recursive. These considerations should be at least close to enough to answer Question 1.1.

Definition 1.2. The standard models of T_1 are of the form $(D,\aleph_1(D),\in,P)$, where $|D| \geq 2$, and $P: D^2 \to D$ is one-one.

31.2 T_2: Extensionality, Choice Operator

In this section, we extend L_1 and T_1 to L_2 and T_2. T_2 will also be mutually interpretable with Z_2.

The language L_2 extends L_1 with the unary function symbol CHO from classes to objects, and = between classes. Here CHO refers to a choice operator.

The object terms, atomic formulas and formulas of L_1 are extended in the obvious way to L_2, incorporating the new function symbol CHO, and = on classes.

The system T_2, in language L_2, has the following nonlogical axioms.

Nonlogical Axioms for T_2:

Pairing:	$(\exists v_1,v_2)(v_1 \neq v_2)$, and
	$P(v_1,v_2) = P(v_3,v_4) \to v_1 = v_3 \wedge v_2 = v_4$.
L_2 Comprehension:	$(\exists A_1)(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, where φ is a formula of L_2 in which A_1 is not free.
Extensionality:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \to A_1 = A_2$.
Choice Operator:	$v_1 \in A_1 \to CHO(A_1) \in A_1$.

We make some remarks concerning the axioms of T_2.

1. L_2 Comprehension. We again consider fragments of L_2 Comprehension here, and compare them with the corresponding fragments of Z_2. These fragments of Comprehension are shown to be finitely axiomatizable.
2. Extensionality. Classes (of objects) are normally considered identical if they have the same elements, whereas properties (of objects) are not considered identical just because they hold of the same objects. We could use properties (of objects) rather than classes (of objects), avoid the extensionality axiom, and define extensional equality as: holding of the same objects. We take this approach in Section 31.8. Extensional equality arises because the power of the Choice Operator for Section 31.6 rests on CHO(A) depending only on the objects in the class A, or only on the objects for which the property A holds.
3. Choice Operator. A Choice Operator operating locally—i.e., at a single class—can be given immediately. If A has an element, then CHO(A) is any element of A; otherwise CHO(A) is any object. We can view the (global) Choice Operator as an obvious extrapolation of the trivial local version achieved by an "infinite mind."

In fact, we can invoke the omnipotence of God—that God can do anything "logically possible." The logical possibility here should be manifested by the triviality of what a Choice Operator does locally.

In this connection, it is interesting to determine precisely what kind of simply described global tasks of this kind are "logically possible" as this can be viewed as achieving more understanding of the nature of God's omnipotence.

Lemma 2.1. T_2 is interpretable in Z_2.

Proof: We take the objects for T_2 to be the Gödel numbers of terms in 0 and the binary function symbol P. We take the classes

for T_1 to be the sets in Z_2. We interpret = between classes for T_1 as extensional equality for sets in Z_2. We interpret CHO in T_2 by CHO(A) = min(A) if A ≠ ∅; 0 otherwise. QED

Theorem 2.2. T_2, Z_2 have commuting interpretations.

Proof: We use the interpretation from the proof of Lemma 2.1 as π_1. We reuse π_2. We follow the argument given for Theorem 1.3. QED

Definition 2.1. For n ≥ 0, T_2[n] is the subsystem of T_2, where in L_2 comprehension, the formula φ is required to start with at most n class quantifiers, followed by a formula without class quantifiers.

Lemma 2.3. For n ≥ 0, T_2[n] is interpretable in Z_2[n].

Proof: Let n ≥ 0. We use the construction in Lemma 1.1. Each class quantifier in T_1 is interpreted by a like set quantifier in Z_2. QED

Theorem 2.4. For n ≥ 1, T_2[n], Z_2[n], $\prod^1_n$-CA_0 have commuting interpretations. T_2[0] is interpretable in RCA_0.

Proof: See Theorem 1.6. The interpretations given for Lemma 1.5 easily extend to accommodate Extensionality and Choice Operator. QED

Theorem 2.5. T_2 is not finitely axiomatizable. For all n ≥ 1, T_2[n] is finitely axiomatizable.

Proof: By Lemma 1.1 and Theorem 2.4. QED

Question 2.1. What is the interpretation power of T_2[0]? Is T_2[0] finitely axiomatizable?

Definition 2.2. The standard models of T_2 are of the form (D,ℵ(D),∈,P,CHO), where |D| ≥ 2, P:D^2 → D is one-one, and CHO:ℵ(D) → D, where A ≠ ∅ → CHO(A) ∈ A.

31.3 T_3: Positive Classes

The notion of “positive property (of objects)” is explicitly used in Gödel (1995, pp. 403–404). Gödel had an initial version in 1941. The informal theological idea of “positive property” goes back at least to Leibniz, and plays a significant role in Theology. See Gödel (1995, pp. 388–402) and also Anderson (1990), Leibniz (1923, 1956, 1969, 1996 [1704]), Look (2006), Oppy (1995, 1996, 2006, 2012), Sobel (1987, 2004).

In our framework, we can view the attribute of Positive as a facility that God has created and given the world access to, perhaps in order to direct us into “positive behaviors.” In our context, we are not incorporating any direct access to God, but we do have access to various of God’s creations that he has chosen to give us access to.

The attribute of Positive can be viewed as such a fundamental facility.

Gödel uses positive properties of objects, whereas we use positive classes of objects. Gödel rightly does not assume extensionality for properties of objects. This raises the possibility that our use of positive classes is a greater commitment than Gödel’s use of positive properties. However, this is not the case, as we now show. In addition, this is reflected in the development in Section 31.8.

Gödel assumes that

1. The conjunction of any two positive properties is a positive property.
2. Every property or its negation is positive, with exclusive or.
3. If God exists, then God possesses all positive properties.

But Gödel does not assume extensionality for properties. Thus the conjunction of any two properties is a property (not necessarily the only property) that holds of exactly the objects that the two properties hold of. The negation of any property is a

property (not necessarily the property) that holds of exactly the objects that the given property does not hold of.

It is clear that “every positive property holds of some object” is at least implicit in Gödel 1995, pp. 403–404. It follows from i-iii and “God exists,” which Gödel is striving to prove (prove is necessarily true).

It now follows that if properties P, Q hold of the same objects, then P is positive if and only if Q is positive. For suppose P, Q hold of the same objects, and P is positive. Suppose Q is not positive. Then the negation of Q is positive, and its conjunction with P is therefore positive. But this conjunction does not hold of any object. Contradiction.

This argument surfaces in the treatments that avoid extensionality in Section 31.8.

We now return to our context of classes of objects with extensionality, and the theory T_3. Since positivity is so very closely related to the purely mathematical notion of ultrafilter, we make a brief mathematical digression.

Definition 3.1. An ultrafilter over a set X is a $K \subseteq \wp(X)$ such that: i. $A, B \in K \rightarrow A \cap B \in K$, ii. $A \in K \leftrightarrow X \backslash A \notin K$, and iii. $\emptyset \notin K$.

Theorem 3.1. Let K be an ultrafilter over X. Then $X \in K$. If $A \subseteq B$ and $B \in K$, then $A \in K$. There is an ultrafilter over X if and only if X is nonempty.

Proof: Well known and straightforward. QED

Definition 3.2. A trivial ultrafilter over X is an ultrafilter of the form $\{A \subseteq X: x \in A\}$, for some $x \in X$. It is easy to see that there can be at most one such x. A nontrivial ultrafilter is an ultrafilter which is not trivial.

Theorem 3.2. Let K be an ultrafilter over X. The following are equivalent.

1. K is nontrivial.
2. Every element of K has at least two elements.

3. Every element of K is infinite.

There is a nontrivial ultrafilter over X if and only if X is infinite.

Proof: The equivalences are well known and straightforward. The last claim is a well-known application of Zorn's Lemma, and cannot be proved in ZF, even in the case X = N. QED

We can simplify the definitions of ultrafilter and nontrivial ultrafilter in the following uniform way.

Theorem 3.3. K is an ultrafilter over X if and only if

1. $X = A \cup B \rightarrow A \in K \vee B \in K$.
2. $A, B \in K \rightarrow |A \cap B| \geq 1$.

K is a nontrivial ultrafilter over X if and only if

1. $X = A \cup B \rightarrow A \in K \vee B \in K$.
2. $A, B \in K \rightarrow |A \cap B| \geq 2$.

Proof: Let $K \subseteq \wp(X)$. If K is an ultrafilter then i,ii follow immediately. If K is a nontrivial ultrafilter then iii, iv follow immediately by Theorem 3.2. Suppose i,ii hold. Suppose $A, B \in K$. Now $X = (A \cap B) \cup (X\backslash A \cup X\backslash B)$. Hence $A \cap B \in K \vee X\backslash A \cup X\backslash B \in K$. Hence $A \cap B \in K \vee X\backslash A \in K \vee X\backslash B \in K$. By ii, $A \cap B \in K$. Suppose $A \in K$. Then $X\backslash A \notin K$ by ii. Suppose $A \notin K$. Now $X = A \cup X\backslash A$. Hence $A \in K \vee X\backslash A \in K$. Therefore $X\backslash A \in K$. Clearly $\emptyset \notin K$, by ii. Now suppose iii,iv. By the preceding argument, K is an ultrafilter over X. By Theorem 3.2, we have only to show that $A \in K \rightarrow |A| \geq 2$. This follows from iv by setting B = A. QED

In this section, we extend L_2 and T_2 to L_3 and T_3. T_3 will also be mutually interpretable with Z_2.

L_3 extends L_2 by the unary relation symbol POS on classes. POS(A) is read "the class A is positive."

The object terms, atomic formulas and formulas of L_2 are extended in the obvious way to L_3, incorporating the new atomic

formulas $POS(A_i)$, $i \geq 1$.

The system T_3, in language L_3, has the following nonlogical axioms.

Nonlogical Axioms for T_3:

Pairing:	$P(v_1,v_2) = P(v_3,v_4) \rightarrow v_1 = v_3 \wedge v_2 = v_4$.
L_3 Comprehension:	$(\exists A_1)(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, where φ is a formula of L_3 in which A_1 is not free.
Extensionality:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \rightarrow A_1 = A_2$.
Choice Operator:	$v_1 \in A_1 \rightarrow CHO(A_1) \in A_1$.
Positive Classes:	$(\forall v_1)(v_1 \in A_1 \vee v_1 \in A_2) \rightarrow POS(A_1) \vee POS(A_2)$, and
	$POS(A_1) \wedge POS(A_2) \rightarrow (\exists v_1 \neq v_2)(v_1,v_2 \in A_1 \wedge v_1,v_2 \in A_2)$.

This completes the presentation of the nonlogical axioms of T_3.

Here is an alternative axiom group for Positive Classes.

Positive Classes Alternative:

$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \notin A_2) \rightarrow (POS(A_1) \leftrightarrow \neg POS(A_2))$,
$(\forall v_1)(v_1 \in A_1 \leftrightarrow (v_1 \in A_2 \wedge v_1 \in A_3)) \rightarrow (POS(A_2) \wedge POS(A_3) \rightarrow POS(A_1$, and
$POS(A_1) \rightarrow (\exists v_1 \neq v_2)(v_1,v_2 \in A_1)$.

Note that we have dropped the first line of Pairing. It can be easily recovered from Positive Classes.

Theorem 3.4. T_3 proves the existence of at least two objects. T_3 proves T_2. T_3 is logically equivalent to T_3 with Positive Classes replaced by Positive Classes Alternative. Let $(D,K,P,\in,CHO,POS)$ satisfy T_3, where $K \subseteq \aleph(D)$. Then POS is a nontrivial ultrafilter over D.

Proof: For the first claim, from Positive Classes we have that the class of all objects, V, is positive or the empty class is positive. The latter is impossible, since it has no elements. Hence there are at least two objects. The second claim is immediate from the first claim. For the third claim, we first derive Positive Classes Alternative. For the first line, it suffices to show that ¬(A is positive ∧ V\A is positive). This is clear from Positive Classes. For the second line of Positive Classes Alternative, we assume A,B are positive, and show $A \cap B$ is positive. Suppose $A \cap B$ is not positive. By line 1 of Positive Classes Alternative, $V\setminus A \cup V\setminus B$ is positive. By Positive Classes, V\A is positive or V\B is positive. These violate line 1 of Positive Classes Alternative. The third line of Positive Classes Alternative follows from the second line of Positive Classes by setting $A_2 = A_1$. We now derive Positive Classes from Positive Classes Alternative. Let $V = A \cup B$. Suppose A,B are not positive. By Positive Classes Alternative, V\A and V\B are positive, $V\setminus A \cap V\setminus B$ is positive, $V\setminus(V\setminus A \cap V\setminus B)$ is not positive, $A \cup B$ is not positive, V is not positive, $\emptyset$ is positive, violating line 3 of Positive Classes Alternative. Finally, Let A,B be positive. By Positive Classes Alternative, $A \cap B$ is positive, and so $A \cap B$ has at least two distinct elements. Let $(D,K,P,\in,CHO,POS)$ satisfy T_3, where $K \subseteq \aleph(D)$. POS is an ultrafilter over D since we have Positive Classes Alternative (this needs only $POS(A1) \rightarrow (\exists v_1)(v_1 \in A_1)$). Line 3 of Positive Classes Alternative and Theorem 3.2

guarantees that POS is a nontrivial ultrafilter. We can also use Theorem 3.3 and Positive Classes. QED

Definition 3.3. For $n \geq 0$, $T_3[n]$ is the subsystem of T_3, where in L_3 Comprehension, the formula φ is required to have at most n class quantifiers.

Theorem 3.5. T_3,Z_2 are mutually interpretable. For all $n \geq 1$, $T_3[n]$,$Z_2[n]$ are mutually interpretable. $T_3[0]$ is interpretable in ACA_0.

Proof: Recall the interpretation of T_1 in Z_2 given for Theorem 1.5. We now have to interpret POS using a defined nontrivial ultrafilter over N. This cannot be done in Z_2. However, we can use the well-known construction of the constructible hierarchy in Z_2. For a treatment within ATR_0 and its applications to Z_2 and strong fragments of Z_2 with and without choice principles, see sections VII.4–VII.6 in Simpson (1999). Thus we interpret the objects in T_3 as nonnegative integers, and the classes in T_3 as constructible subsets of N. For CHO, we can of course continue to use the least element construction. It is well known that we obtain the full comprehension of Z_2. Now it is easily proved in ACA_0 that for any nontrivial ultrafilter U on a countable Boolean algebra B of subsets of N, and $A \subseteq N$, $A \notin B$, there is a unique nontrivial ultrafilter on the Boolean algebra generated by $B \cup \{A\}$, with $B \in U$, extending U. This allows us to make an obvious transfinite construction along the constructible hierarchy of subsets of N that results in an explicitly definable ultrafilter. We use this ultrafilter to interpret POS. This establishes the first claim. For $n \geq 2$, this argument is robust enough to interpret $T_3[n]$ in $Z_2[n]$, as $\prod^1_2$ -CA_0 is rather substantial and well supports the constructible hierarchy for subsets of N. To interpret $T_3[1]$ in $Z_2[1]$, it is better to think in terms of the first ω hyperjumps. We build a nontrivial ultrafilter arithmetically in the first hyperjump

for sets recursive in the first hyperjump. We then extend this to a nontrivial ultrafilter arithmetically in the second hyperjump for sets recursive in the second hyperjump, etcetera. The classes in $T_3[1]$ are interpreted as the sets recursive in some iterated hyperjump. We have to be careful not to use more than set induction, so we don't have that for all n, the n-th hyperjump exists. Only for a cut of n. Then we build the nontrivial ultrafilter as indicated. Any single class quantifier statement in $T_3[1]$ becomes an arithmetic statement in the hyperjump of its class parameters under this interpretation. Thus the interpretation of single class quantifier comprehension in $T_3[1]$ is provable in $Z_2[1]$. To interpret $T_3[0]$ in $Z_2[0] = ACA_0$, we follow the above approach, thinking in terms of the first ω Turing jumps. Again we have to be careful not to use more than set induction, so we don't have that for all n, the n-th Turing jump exists. Again, only for a cut of n. We build the nontrivial ultrafilter as above. QED

Theorem 3.6. T_3 is not finitely axiomatizable. For $n \geq 1$, $T_3[n]$ is finitely axiomatizable.

Proof: Since we are not claiming that T_3,Z_2 have commuting interpretations, use a different kind of argument to show that T_3 is not finitely axiomatizable. Let K be a finite axiomatization of T_3. The interpretation of T_3 in Z_2 given in the proof of Theorem 3.5 shows that Con(K) is provable in Z_2. Let n be such that Con(K) is provable in $Z_2[n]$. Now $Z_2[n]$ is finitely axiomatizable, and interpretable in T_3. Let K'be a finite fragment of T_3 such that $Z_2[n]$ is interpretable in K'. Using the proof of K'in K, we see that $Z_2[n]$ proves Con(K'). Hence $Z_2[n]$ proves Con($Z_2[n]$), and therefore Z_2 is inconsistent. This is a contradiction. Let $n \geq 1$. To see that $T_3[n]$ is finitely axiomatizable, we argue directly. We obtain the expected normal form theorem for formulas in L3 without class quantifiers, via a formula beginning with 1 class quantifier followed by a formula with no class quantifiers. The

class quantifier can be taken to be either $\forall$ or $\exists$. This works even though POS is present. QED

Question 3.1. What is the interpretation power of $T_3[0]$? Is $T_3[0]$ finitely axiomatizable?

There are many ways to justify both parts of Positive Classes, or, alternatively, all three parts of Positive Classes Alternative.

A particularly straightforward way to motivate the first line of Positive Classes is to assume the existence of a perfect object v. Let $V = A_1 \cup A_2$. Then $v \in A_1 \vee v \in A_2$.

We claim that if $v \in A_1$ then A_1 is positive. To argue this, suppose A_1 is not positive. Then v would not belong to A_1, since if v is perfect, v would choose not to belong to A_1.

So we have argued that $v \in A_1 \rightarrow A_1$ is positive, and $v \in A_2 \rightarrow A_2$ is positive. Therefore A_1 is positive or A_2 is positive.

Unfortunately, if we admit a perfect object v, then the second part of Positive Classes fails. This is because the class consisting of exactly v must be positive; i.e., $v \in \{v\} \rightarrow \{v\}$ is positive. But {v} has exactly one element.

In order to reconcile the discussion with Positive Classes, we must deliberately exclude any perfect object from the totality of objects. E.g., redefine the objects to consist of the imperfect objects.

A very natural viewpoint is this: no object is perfect, but objects can "approximate perfection" without limit. I.e., any level of "approximate perfection" can be achieved by imperfect objects.

In particular, all objects have flaws. This is an interesting statement in its own right, formalized as follows.

Universal Fallibility: $(\forall v_1)(\exists A_1)(v_1 \in A_1 \wedge \neg POS(A_1))$.

Note that Universal Fallibility is provable in T_3 as follows. Let v be an object. Since positive classes have at least two distinct elements, {v} is not positive. Hence the complement of {v} is

positive (e.g., use the second line of Positive Classes), yet does not contain v.

We continue the argument for Positive Classes. Now suppose all objects lie in A_1 or A_2. If A_1,A_2 are not positive, then objects are certainly not approximating perfection without limit, in that every object is flawed even with respect to just A_1 and A_2. So this justifies the first part of Positive Classes.

For the second line of Positive Classes, let A_1,A_2 be positive classes. There must exist an object v in both A_1 and A_2 (since we can approximate perfection without limit). Suppose v is the unique object in both A_1 and A_2. By Universal Fallibility, let $v \notin A_3$, where A_3 is positive. Since we can approximate perfection without limit, let w lie in A_1,A_2,A_3. Then $v \neq w$, and we have the desired contradiction.

Definition 3.4. The standard models of T_3 are of the form (D, $\wp$(D), $\in$, P, CHO, POS), where P: $D^2 \rightarrow D$ is one-one, CHO:$\wp$(D) $\rightarrow$ D, $A \neq \emptyset \rightarrow CHO(A) \in A$, and POS is a nontrivial ultrafilter over D.

31.4 T_4: Definable Classes

We follow the usual convention in mathematical logic that "definable" means "definable with parameters," and "0-definable" means definable without parameters.

In this section, we extend L_3 to L_4 by the unary relation symbol DEF on classes. The intended meaning of DEF(A) is "the class A is L_3 0-definable." This is the same as "the class A is L_4 0-definable without DEF."

The object terms, atomic formulas and formulas of L_3 are extended in the obvious way to L_4, incorporating the new unary relation symbol DEF on classes.

The system T_4, in language L_4, has the following nonlogical axioms.

Nonlogical Axioms for T_4:

Pairing:	$P(v_1,v_2) = P(v_3,v_4) \rightarrow v_1 = v_3 \wedge v_2 = v_4$.
L_4 Comprehension:	$(\exists A_1)(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, where φ is a formula of L_4 in which A_1 is not free.
Extensionality:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \rightarrow A_1 = A_2$.
Choice Operator:	$v_1 \in A_1 \rightarrow CHO(A_1) \in A_1$.
Positive Classes:	$(\forall v_1)(v_1 \in A_1 \vee v_1 \in A_2) \rightarrow POS(A_1) \vee POS(A_2)$, and
	$POS(A_1) \wedge POS(A_2) \rightarrow (\exists v_1 \neq v_2)(v_1,v_2 \in A_1 \wedge v_1,v_2 \in A_2)$.
0-Definable Classes:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi) \wedge DEF(A_2) \wedge \ldots \wedge DEF(A_n) \rightarrow DEF(A_1)$, where φ is a formula of L_4 without DEF, with at most the free variables $v_1,A_2,\ldots,A_n$, $n \geq 1$.

Definition 4.1. For $n \geq 0$, $T_4[n]$ is the subsystem of T_4, where in L_3 Comprehension and 0-Definable Classes, the formula φ is required to have at most n class quantifiers.

Theorem 4.1. T_4,Z_2 are mutually interpretable. For all $n \geq 1$, $T_4[n]$,$Z_2[n]$ are mutually interpretable. $T_4[0]$ is interpretable in ACA_0.

Proof: This is immediate from Theorem 3.5, since we can interpret DEF(A) as A = A. We won't be able to interpret DEF(A) in this trivial way for T_5, since T_5 proves $\neg(\forall A)(DEF(A))$. QED

Theorem 4.2. T_4 is not finitely axiomatizable. For $n \geq 1$, $T_4[n]$ is finitely axiomatizable.

Proof: Suppose T_4 is equivalent to K, where K is finite. Now replace each $DEF(A_i)$ with $A_i = A_i$ in all axioms of T_4 and K,

obtaining T_4' and K'. Then T_4', K' are logically equivalent. But T_4' is logically equivalent to T_3. This contradicts that T_3 is not finitely axiomatizable. The argument given for the second claim of Theorem 3.6 adapts to this context. QED

Question 4.1. What is the interpretation power of $T_4[0]$? Is $T_4[0]$ finitely axiomatizable?

Definition 4.2. The standard models of T_4 are of the form $(D,\wp(D),\in,P,CHO,POS,DEF)$, where $P:D^2 \to D$ is one-one, $CHO:\wp(D) \to D$, $A \neq \emptyset \to CHO(A) \in A$, POS is a nontrivial ultrafilter over D, and DEF is the set of all subsets of D that are 0-definable over $(D,\wp(D),\in,P,CHO,POS)$.

We can formulate 0-Definable Classes in the following reduced form.

0-Definable Classes (binary): $(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi) \wedge DEF(A_2) \wedge \ldots \wedge DEF(A_n) \to DEF(A_1)$, where φ is a formula of L_4 without DEF, with at most the free variables $v_1,A_2,\ldots,A_n$, $1 \leq n \leq 3$.

Theorem 4.2. T_3 is logically equivalent to T_3 formulated with 0-Definable Classes (binary). This is also true for all $T_3[n]$, $n \geq 0$.

Proof: We work with 0-Definable Classes (binary), and the rest of T_3. We establish $(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi) \wedge DEF(A_2) \wedge \ldots \wedge DEF(A_n) \to DEF(A_1)$, where φ has no DEF, and at most the free variables $v_1,A_2,\ldots,A_n$, holds. We can assume that $n \geq 3$. Assume $(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, $DEF(A_2),\ldots,DEF(A_n)$. With n-2 uses of the binary form, we obtain $DEF(A_{n+1})$, with $A_{n+1} = \{P(x_2,\ldots,x_n): x_2 \in A_2 \wedge \ldots \wedge x_n \in A_n\}$, where here we have used P associated to the left. We replace the free occurrences of A_i in φ, $2 \leq i \leq n$, by $\{x_i: (\exists x_2,\ldots,x_{i-1},x_{i+1},\ldots,x_n)(P(x_2,\ldots,x_n) \in A_{n+1}\}$, and then simplify by removing these set abstraction terms in the usual way. Thus we have defined A_1 with the single parameter A_{n+1}, where $DEF(A_{n+1})$. Hence $DEF(A_1)$. Since this argument does not introduce any class quantifiers, the second claim is also established. QED

31.5 T_5: Divine Objects

Here we finally introduce the theory T_5 that we use in Section 31.6 to interpret ZFC.

The language of T_5 is the same as the language of T_4, and T_5 merely extends T_4 by the Divine Object axiom.

Since in Section 31.1, we suggested that the reader can safely start reading this section, we now give a fully self-contained presentation of T_5.

The language L_5 of T_5 has two sorts: objects and classes of objects (classes). L_5 has object variables v_i, $i \geq 1$, and class variables A_i, $i \geq 1$. L_4 uses a binary relation symbol $\in$ between objects and classes, a binary function symbol P on objects (pairing), = between objects, a unary function CHO from classes to objects (choice), a unary relation symbol POS on classes (positive classes), and a unary relation symbol DEF on classes (definability without DEF).

Note that L_5 is the same as the language L_4 presented in Section 31.4.

The object terms of L_5 are inductively defined by:

1. For all $i \geq 1$, v_i is an object term of L_5,
2. For all object terms s,t, P(s,t) is an object term of L_5 and iii. For all $i \geq 1$, CHO(A_i) is an object term of L_5.

The atomic formulas of L_5 are given as follows:

1. For all $i,j \geq 1$, $A_i = A_j$ is an atomic formula of L_5,
2. For object terms s,t of L_5, s = t is an atomic formula of L_5,
3. For all $i \geq 1$ and object terms t of L_5, $t \in A_i$ is an atomic formula of L_5,
4. For all $i \geq 1$, POS(A_i), DEF(A_i) are atomic formulas of L_5.

The formulas of L_5 are inductively defined as follows:

1. Every atomic formula of L_5 is a formula of L_5,
2. For all formulas φ,Ψ of L_5, $\neg\varphi$, $\varphi \wedge \Psi$, $\varphi \vee \Psi$, $\varphi \rightarrow \Psi$ are formulas of L5, and
3. For all $i \geq 1$ and formulas φ of L_5, $(\forall v_i)(\varphi)$, $(\exists v_i)(\varphi)$, $(\forall A_i)(\varphi)$, $(\exists A_i)(\varphi)$ are formulas of L_5.

We use $\leftrightarrow$ as an abbreviation. I.e., $\varphi \leftrightarrow \Psi$ abbreviates $(\varphi \rightarrow \Psi) \wedge (\Psi \rightarrow \varphi)$.

There are standard axioms and rules of inference for L_5, with a corresponding completeness theorem. The system T_5, in language L_5, has the following nonlogical axioms.

Nonlogical Axioms for T_5:

Pairing:	$P(v_1,v_2) = P(v_3,v_4) \rightarrow v_1 = v_3 \wedge v_2 = v_4$.
L_5 Comprehension:	$(\exists A_1)(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, where φ is a formula of L_5 in which A_1 is not free.
Extensionality:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \rightarrow A_1 = A_2$.
Choice Operator:	$v_1 \in A_1 \rightarrow CHO(A_1) \in A_1$.
Positive Classes:	$(\forall v_1)(v_1 \in A_1 \vee v_1 \in A_2) \rightarrow POS(A_1) \vee POS(A_2)$, and
	$POS(A_1) \wedge POS(A_2) \rightarrow (\exists v_1 \neq v_2)(v_1,v_2 \in A_1 \wedge v_1,v_2 \in A_2)$.
0-Definable Classes:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi) \wedge DEF(A_2) \wedge \ldots \wedge DEF(A_n) \rightarrow DEF(A_1)$, where φ is a formula of L_5 without DEF, with at most the free variables $v_1,A_2,\ldots,A_n$, $n \geq 1$.
Divine Object:	$(\exists v_1)(\forall A_1)(DEF(A_1) \wedge POS(A_1) \rightarrow v_1 \in A_1)$.

As discussed in Section 31.3, T_5 proves that there is no object that lies in every positive class. I.e., there is no perfect object. In Section 31.3, we discussed the informal principle that we can approximate perfection without limitation.

With this background, we say that an object is *divine* if and only if it is an element of all 0-definable positive classes. The Divine Object axiom asserts that there exists a divine object.

Divine objects turn out to be surprisingly powerful approximations to perfection. In Section 31.6, we use the existence of a divine object to give an interpretation of ZFC in T_5. This is extended in Section 31.7 by giving an interpretation of ZFC together with a rather substantial large cardinal hypothesis —the existence of arbitrarily large strong Ramsey cardinals. This is just short of the well-known large cardinal hypothesis that asserts the existence of a measurable cardinal. See Definition 7.18. These interpretations also provide consistency and relative consistency proofs in the usual way.

But how do we know that T_5 is consistent—i.e., free of contradiction?

We now present the standard models of T_5 using measurable cardinals from set theory.

Definition 5.1. In set theory, a von Neumann cardinal κ is measurable if and only if κ is uncountable, and there is nontrivial κ complete ultrafilter U over κ. I.e., a nontrivial ultrafilter U over κ such that the intersection of fewer than κ many elements of U is an element of U. This is equivalent to: the intersection of fewer than κ many elements of U is nonempty.

Definition 5.2. ZFM is the theory ZFC + “there exists a measurable cardinal.”

ZFM is a very well studied system which is very much stronger than ZFC alone. See, e.g., →Kanamori 1994, →Jech 2006.

Definition 5.3. The standard models of T_5 are $(\kappa,\wp(\kappa),\in,P,CHO,POS,DEF)$, where $P:\kappa^2 \rightarrow \kappa$ is one-one, $CHO:\wp(\kappa) \rightarrow \kappa$, $A \neq \emptyset \rightarrow CHO(A) \in A$, POS is a nontrivial κ complete ultrafilter over κ, and DEF is the set of all subsets of κ that are 0-definable over $(\kappa,\wp(\kappa),\in,P,CHO,POS)$.

Definition 5.4. EFA = exponential function arithmetic, is our notation for a system that later became known as $I\sum_0(\exp)$. See →Hájek and Pudlak (1993, p. 37).

The first claim of Theorem 5.1 is an interpretability result. The second claim is a consistency proof. The third claim is a relative consistency proof. All three of these results establish that T_5 is "all right" assuming that ZFM is "all right."

The main point of this paper goes the other direction. That ZFC is "all right" assuming T5 is "all right"—see Theorem 6.27. Moreover, that something just shy of ZFM is "all right" assuming T_5 is "all right"—see Theorem 7.34.

Theorem 5.1. ZFM proves that the structures in Definition 5.3 are models of T_5. T_5 is interpretable in ZFM. ZFM proves the consistency of T_5. EFA proves $Con(ZFM) \rightarrow Con(T_5)$.

Proof: All of the axioms of T_4 obviously hold in the $(\kappa,\wp(\kappa),\in,P,CHO,POS,DEF)$ of Definition 5.3. Since there are fewer than κ many (in fact, only countably many) subsets of κ that are 0-definable over $(\kappa,\wp(\kappa),\in,P,CHO,POS)$, the intersection of those that have POS (i.e., lie in the nontrivial κ complete ultrafilter) is nonempty (in fact, has POS). Any element of this intersection is therefore divine. This provides an interpretation of T_5 in ZFM. Also ZFM obviously proves the consistency of T_5 since ZFM proves the existence of a model of T_5. The last claim follows from the interpretability of T5 in ZFM. The third claim follows immediately from the fact that the first claim is provable in EFA. QED

Note that we have only used that the intersection of countably many elements of the ultrafilter over κ is nonempty. Thus we only need a weaker condition on the ultrafilter, called countable completeness.

Definition 5.5. A countably complete ultrafilter over a set D is an ultrafilter U where the intersection of any countable number of elements of U is an element of U. This is equivalent to: the intersection of any countable number of elements of U is nonempty.

It is well known that the existence of a nontrivial countably complete ultrafilter is equivalent to the existence of a measurable cardinal, over ZFC. See, e.g., Kanamori (1994, p. 23).

It is natural to expand our set-theoretic models of T_5 in the following way.

Definition 5.6. The preferred models of T_5 are $(D, \wp(D), \in, P, CHO, POS, DEF)$, where $P: D^2 \to D$ is one-one, $CHO:\wp(D) \to D$, $A \neq \emptyset \to CHO(A) \in A$, POS is a nontrivial countably complete ultrafilter over D, and DEF is the set of all subsets of D that are 0-definable over $(D, \wp(D), \in, P, CHO, POS)$.

31.6 Interpreting ZFC in T_5

In this section, we interpret ZFC in T_5. We work in T_5, developing the interpretation. We carry out the development in T_4 for as long as we can, before using the powerful Divine Object axiom of T_5.

Definition 6.1. V is the class of all objects. Relations on V are treated as classes using the pairing function P. Functions are treated as relations.

Definition 6.2. A well ordering is a pair (A,<), where A and $< \subseteq A^2$ are classes, such that < is irreflexive, transitive, connected, and where every nonempty $B \subseteq A$ has an R least element. Define

$x \leq y \leftrightarrow x < y \vee x = y$. Define $<[x] = \{y: y < x\}$, $\leq[x] = \{y: y \leq x\}$, $>[x] = \{y: x < y\}$, $\geq[x] = \{y: x \leq y\}$. x is a < limit point if and only if x is not < least and x has no immediate predecessor in <. If x is not < greatest, define x+1 to be the immediate successor of x in <. If x is not a limit point and not < least, define x-1 to be the immediate predecessor of x in (A,<).

Definition 6.3. $A \subseteq V$ is finite if and only if A is empty, or there is a well ordering (A,<) with a < greatest object and without a < limit point. A is infinite if and only if A is not finite.

Lemma 6.1. (T_1). Let (A,<) be a well ordering with a < greatest element and without a < limit point. Then < induction holds. I.e., suppose $0 \in B \wedge (\forall x \in A)(x \in B \rightarrow x+1 \in B)$, where 0 is < least. Then $A \subseteq B$.

Proof: Let A,<,0,B be as given. Let x be < least such that $x \notin B$. Now $x = 0 \vee$ x is a < successor $\vee$ x is a < limit point. None of these three disjuncts are possible. QED

Lemma 6.2. (T_1). Let (A,<) be a nonempty linear ordering (irreflexive, transitive, connected), with no < greatest element. Then A is infinite.

Proof: Let (A,<) be as given, and assume A is finite. Let (A,<') be a well ordering, with a <'greatest element, and with no <'limit points. We prove by <'induction (Lemma 6.1) that $(\forall x \in A)(\exists y \in A)(\forall z <' x)(z < y)$. This induction is clear using that there is no < greatest element. Now apply this quantified statement to the <' greatest element, resulting in a contradiction. QED

Lemma 6.3. (T_1). V is infinite.

Proof: Suppose V is finite. Let < be a well ordering of V with a < greatest object, and with no < limit points. We first prove by < induction on x that every one-one $f:\leq[x] \rightarrow \leq[x]$ is onto. This is obvious if x is < least. Suppose this is true for x. Let $f:\leq[x+1] \rightarrow \leq[x+1]$ be one-one. Let g be f restricted to the subdomain $\leq[x]$. Case 1. f(x+1) = x+1. By the induction hypothesis, rng(g) = $\leq[x]$. Therefore f is onto. Case 2. $f(x+1) \neq x+1$. Now $f(x+1) \notin$ rng(g). If

x+1 ∉ rng(g) then by the induction hypothesis, rng(g) = ≤[x], contradicting f(x+1) ∉ rng(g). If x+1 ∈ rng(g), then let g(b) = x+1. Let h be the same as g except h(b) = f(x+1). By the induction hypothesis, rng(h) = ≤[x]. Hence rng(g) = ≤[x+1]\{f(x+1)}. Hence rng(f) = ≤[x+1]. This completes the induction argument. By setting x to be the < greatest object, we see that every one-one f:V → V is onto. But let a ≠ b (since there are at least two objects). Then the function f(x) = P(a,x) is one-one, but does not achieve P(b,b). This is a contradiction. QED

Definition 6.4. We say that (A,R) is critical if and only if (A,R) is a well ordering, and for all x ∈ A, x = CHO({y: ¬R(y,x)}).

Lemma 6.4. (T_2). Let (A,R),(B,S) be critical. There is a longest common initial segment. (A,R) is an initial segment of (B,S) or (B,S) is an initial segment of (A,R).

Proof: Let A,B,R,S be as given. By well foundedness, there is a longest initial segment (C,T) of (A,R) that is an initial segment of (B,S). If C = A then (A,R) is an initial segment of (B,S). If C = B then (B,T) is an initial segment of (B,S), and so S = T and (B,S) is an initial segment of (A,R).

The remaining case is where C ≠ A ∧ C ≠ B. Since C ⊆ A ∩ B, let x be R least outside C, and y be S least outside C. Now the R predecessors of x and the S predecessors of x are the same (the elements of C). By the definition of critical, x = y. This contradicts that (C,T) is longest. QED

We refer to the second claim of Lemma 6.4 as critical comparability.

Lemma 6.5. (T_2). Let A be the union of the domains of the critical well orderings, and R be the union of the relations of the critical well orderings. Then (A,R) is critical.

Proof: Let A,R be as given. Obviously R ⊆ A_2 and (A,R) is irreflexive. To see that (A, R) is transitive, let x R y R z. Let (B,S), (C,T) be critical, where S(x,y) ∧ T(y,z). By critical comparability, T(x,z) ∨ S(x,z). Hence R(x,z). To see that (A,R) is connected, let x ≠

y, x,y ∈ A. Let x ∈ B, y ∈ C, (B,S),(C,T) critical. By critical comparability, T(x,y) ∨ T(y,x) ∨ S(x,y) ∨ S(y,x). Hence R(x,y) ∨ R(y,x). To see that (A,R) is well ordered, let E ⊆ A, E ≠ ∅. Let x ∈ E, x ∈ B, (B,S) critical. Let y be the S least element of E ∩ B. We claim that y is the R least element of E. To see this, let z R y, z ∈ E. Let T(z,y), (C,T) critical. By critical comparability, S(z,y). This contradicts the choice of y. To see that (A,R) is critical, let x ∈ A, x ∈ B, (B,S) critical. Then x = CHO(V\{a: a S x}). It suffices to prove that (∀y)(R(y,x) ↔ S(y,x)). Clearly S(y,x) → R(y,x). Suppose R(y,x). Let T(y,x), (C,T) critical. Since x,y ∈ B ∩ C, we have S(y,x). QED

Lemma 6.6. (T_2). There is a unique critical well ordering (V,<). (T_4). DEF(<) holds.

Proof: Let (A,R) be as given by Lemma 6.3. Suppose A ≠ V. Let x = CHO(V\A). Then we can extend (A,R) by putting x ∉ A on top. The result is a critical (A ∪ {x},R′), contradicting the definition of (A,R). Note that (A,R) has been explicitly defined without parameters using only ∈,P,CHO, and equality between objects and between classes. Hence DEF(<) by 0-Definable Classes. QED

Definition 6.5. A is negative if and only if A is not positive. DEF(A_1,...,A_n) ↔ DEF(A_1) ∧ ... ∧ DEF(A_n).

Lemma 6.7. (T_4). There is a well ordering (V,<*) with DEF(<*) such that each ≤*[x] is negative. Every finite set is negative.

Proof: Let < be as given by Lemma 6.6. Suppose some ≤[x] is positive. Let x be < least such that ≤[x] is positive. Then <[x] is positive and DEF({x}).

Let <* be the result of removing ≥[x] and putting it at the bottom. Then (V,<*) is a well ordering, and DEF(<*). Let y ∈ V. If y <* x then ≤[y] is negative, and so ≤*[y] is negative, since ≥[x] is negative. If y ≥ x then ≤*[y] ⊆ ≥[x], and so ≤*[y] is also negative. For the second claim, let A be finite. We can assume A ≠ ∅. Let (A,<) be a well ordering with a greatest element and no limit point. It suffices to show that for all x ∈ A, ≤[x] is negative. Let x

be the < least counterexample. Then x is not the < least element of A. Now ≤[x-1] is negative. Hence ≤[x] is negative, and we have a contradiction. QED

Definition 6.6. <* is as given by Lemma 6.7. 0 is the <* least element. 1 is the immediate successor of 0 in <*. x+1 is the immediate successor of x in <*, if it exists.

Lemma 6.8. (T_4). There is no <* greatest object. <* induction holds if <*[x] is finite. I.e., suppose $0 \in A \wedge (\forall x \in A)(<^*[x]$ finite $\rightarrow x+1 \in A)$. Then $(\forall x)(<^*[x]$ finite $\rightarrow x \in A)$.

Proof: Let x be <* greatest. Then ≤*[x] is negative by Lemma 6.7. But ≤*[x] = V, which is positive. There is no <* greatest object, and hence every x+1 exists. For the second claim, let A be as given, and let x be <* least such that $x \notin A$ and <*[x] is finite. Then $x \neq 0$ and x is not a <* successor. Hence x is a <* limit point, which contradicts that <*[x] is finite, by Lemma 6.2. QED

Definition 6.7. sup(A) is the <* least object x with $(\forall y \in A)(y <^* x)$, provided such an x exists. Otherwise, we take $\sup(A) = \infty$.

Definition 6.8. (x,f) is a reduction if and only if i. dom(f) = V ∧ sup(rng(f)) = x, ii. For all $y <^* x$, $f^{-1}(y)$ is negative, and iii. DEF(f). (x,f) is a semi reduction if and only if the above holds, with sup(rng(f)) ≤* x.

Definition 6.9. We use γ for the <* least object such that there is a reduction (γ,f), provided there is some reduction. Otherwise, we take $\gamma = \infty$, which is not considered to be an object. If $\gamma \neq \infty$ then we use J for a fixed function such that (γ,J) is a reduction. If $\gamma = \infty$ then we use J for the identity function on V. We extend <* to $V \cup \{\infty\}$. We continue to use letters x,y,z,w,u,v,... for objects (elements of V) only.

Lemma 6.9. (T_4). If (x,f) is a reduction then DEF({x}). If (x,f) is a semi reduction then (sup(rng(f)),f) is the unique reduction with second term f. If (x,f) is a semi reduction then γ ≤* x. Suppose $\gamma \neq \infty$. Then DEF({γ}), and γ is a <* limit point.

Proof: Let (x,f) be a reduction. Since x = sup(rng(f)), we have DEF({x}). The second claim is immediate. For the third claim, let (x,f) be a semi reduction. Then (sup(rng(f)),f) is a reduction, and so $\gamma \neq \infty$ and $\gamma \leq^* \sup(rng(f)) \leq^* x$. Now suppose $\gamma \neq \infty$. By the first claim, DEF({γ}). Obviously $\gamma \neq 0$. Let $\gamma = x+1$. Let $f(y) = H(y)$ if $J(y) <^* x$; 0 if $J(y) = x$. Note that $f^{-1}(0) = J^{-1}(0) \cup J^{-1}(x)$, which is negative. Also DEF(f), since DEF({γ},{x},f,{0}). Hence (x,f) is a semi reduction, $x <^* \gamma$. Hence there is a reduction (y,f), $y \leq^* x <^* \gamma$, by the previous claim. This is a contradiction. QED

Note that if $\gamma = \infty$, then we can still use γ as if DEF({γ}).

Lemma 6.10. (T_4). If $x <^* \gamma$ then $J^{-1}[<^*[x]]$ is negative. Let $f:<^*[\gamma] \rightarrow <^*[x]$ be partially defined, dom(f) is positive, $x <^* \gamma$, and DEF(f). Then some $f^{-1}(y)$ is positive.

Proof: If $\gamma = \infty$ then $J^{-1}[<^*[x]] = <^*[x]$, which is negative by Lemma 6.7. Assume $x <^* \gamma \neq \infty$. Suppose $J^{-1}[<^*[x]]$ is positive. Define $f:V \rightarrow <^*[x]$ by $f(y) = J(y)$ if $J(y) <^* x$; 0 otherwise. Since the default set is negative, we see that (x,f) is a semi reduction, $x <^* \gamma$. By Lemma 6.9, $\gamma \leq^* x$, which is a contradiction. For the second claim, let f be as given. We can assume that x is $<^*$ least such that $rng(f) \subseteq <^*[x]$. Extend f to $g:<^*[\gamma] \rightarrow <^*[x]$, where g is 0 off of dom(f). Then DEF(g). If all $g^{-1}(y)$ are negative, then (x,g) is a reduction. But this contradicts the definition of γ. QED

Lemma 6.11. (T_4). Let $x <^* \gamma$, $f:<^*[x] \rightarrow <^*[\gamma]$, and DEF(f). Then $\sup(rng(f)) <^* \gamma$.

Proof: let x,f be as given. Suppose $\sup(rng(f)) = \gamma$. Note that DEF({x}). Define $g:V \rightarrow <^*[x]$ by $g(z) = \min\{w <^* x: J(z) <^* f(w)\}$. Note that g is well defined since $\sup(rng(f)) = \gamma$. Also DEF(g).

Let $y <^* x$. We claim that $g^{-1}(y) \subseteq J^{-1}[<^*[f(y)]]$. To see this, suppose $g(z) = y = \min\{w <^* x: J(z) <^* f(w)\}$. In particular, $y \in \{w <^* x: J(z) <^* f(w)\}$. I.e., $J(z) <^* f(y)$, as required. By Lemma 6.10, $J^{-1}[<^*[f(y)]]$ is negative, since $f(y) < \gamma$. This establishes that (x,g) is a semi reduction, $x <^* \gamma$. This is a contradiction. QED

Definition 6.10. We treat f:A $\to$ $\wp$(B) as the corresponding class {P(x,y): y $\in$ f(x)}, which is also a relation on A $\cup$ B.

Lemma 6.12. (T_4). Let x <* y. There is no one-one f:<*[y] $\to$ $\wp$(<*[x]) with DEF(f). Let g:<*[y] $\to$ $\wp$(<*[x]) be partially defined and one-one, with DEF(g). Then dom(g) is a <* bounded subclass of <*[y].

Proof: For the first claim, let x <* y. Let f:<*[y] $\to$ $\wp$(<*[x]) be one-one, with DEF(f). We first define a special E $\subseteq$ <*[x]. For each w <* x, we look at {z: w $\in$ f(J(z))} and {z: w $\notin$ f(J(z))}. One of these is positive and the other is negative. E consists of the w for which the first of these two sets is positive. Obviously DEF(E). We now define g:V $\to$ <*[x]. Set g(z) = min(E Δ f(J(z))} if this exists; 0 otherwise. Clearly DEF(g). We show that (x,g) is a semi reduction. This is a contradiction since x < y. In this definition of g, the default applies for g(z) if and only if f(J(z)) = E (in which case, we are taking the min of $\emptyset$, which does not exist). I.e., when J(z) = f^{-1}[E]. Here we are inverting f at the point E, as f is one-one. (If f^{-1}(E) does not exist, or if it lies outside rng(J), then there is no default in the definition of g). Thus the default applies only for z in a single J^{-1}(u), u <* y, or not at all. In any case, the default applies only on a negative class of z's. It now suffices to show that for all w <* x, {z: min(E Δ f(J(z)) = w} is negative. This is because {z: min(E Δ f(J(z)) = w} is the same as g^{-1}(w) off of the negative default class of z's. Case 1. w $\in$ E. Then {z: min(E Δ f(J(z)) = w} $\subseteq$ {z: w $\notin$ f(J(z))}. If g(z) = w then w $\notin$ f(J(z)). Hence z lies in the negative set {z: w $\notin$ f(J(z))}. This set is negative because w $\in$ E. Case 2. w $\notin$ E. Then {z: min(E Δ f(J(z)) = w} $\subseteq$ {z: w $\in$ f(J(z))}. This set is negative because w $\notin$ E. This completes the proof of the first claim. For the second claim, let g:<*[y] $\to$ $\wp$(<*[x]) be partially defined, with DEF(g). Suppose dom(g) is <* unbounded in y. Let h be the unique order preserving comparison between (<*[y],<*) and (dom(g),<*). Then DEF(h). By

Lemma 6.11, it is clear that h is an isomorphism. Now let f:<*[γ] → ϰ(<*[x]), where f(z) = g(h(z)). Then DEF(f), and apply the first claim to f,x. QED

Lemma 6.13. (T_4). Suppose <*[x] is finite. Then DEF(x). Every A ⊆ <*[x] has DEF(A).

Proof: The first claim is by an obvious induction on the x <* γ with <*[x] finite (see Lemma 6.8). For the second claim, we prove by induction on the x <* γ with finite <*[x], that (∀A)(A ⊆ <*[x] ∧ A finite → DEF(A)). This is clearly true for x = 0. Suppose this is true for fixed x <* γ with finite <*[x]. Let A ⊆ <*[x+1] be finite. Then A\{x} is finite, and so by the induction hypothesis, DEF(A\{x}). Since DEF({x}), we have DEF(A). QED

We now use the Divine Object axiom for the first time.

Definition 6.11. x is divine if and only if (∀A)(DEF(A) ∧ POS(A) → x ∈ A). Note that by the Divine Object axiom in T_5, there is a divine object.

Lemma 6.14. (T_5). Let x be divine. (∀y)(DEF({y}) → y <* x). ¬DEF({x}) ∧ <*[x] is infinite. If γ ≠ ∞ then x >* γ. (∃z)(<*[z] is infinite). The class K of all divine objects has ¬DEF(K).

Proof: Let x is divine. Let DEF({y}). By Lemma 6.7, >*[y] is positive and DEF(>*[y]). Hence x ∈ >*[y], y <* x. From the first claim, ¬DEF({x}). Suppose <*[x] is finite. By the induction in Lemma 6.8, we see that DEF(<*[x]), which is a contradiction. Suppose γ ≠ ∞. Then DEF({γ}), DEF(<*[γ]), and by Lemma 6.7, ≤*[γ] is negative. Hence x ≥* γ. By the second claim, <*[x] is infinite. Hence (∃z)(<*[z] is infinite). If the class K of all divine objects has DEF(K), then let x be the <* least element of K. Clearly DEF({x}) and x is divine. This contradicts the second claim. QED

Definition 6.12. ω is the <* least object such that <*[ω] is infinite.

Lemma 6.15. (T5). DEF(ω). ω <* γ.

Proof: DEF(ω) by Definition 6.12. By Lemma 6.9, γ is a <* limit point. Hence <*[γ] is infinite. Therefore ω ≤* γ. Now suppose γ = ω. Then (ω,J) is a reduction. Let x be divine, and J(x) = n <* ω. Now $J^{-1}(n)$ is negative. By Lemma 6.13, DEF({n}), DEF($J^{-1}(n)$). Therefore $x \notin J^{-1}(n)$. But $x \in J^s(n)$, and we have a contradiction. Hence ω <* γ. QED

We now want to code the constructible hierarchy up through γ. For this purpose, we will need the "0-definable regularity" of γ afforded by Lemma 6.11.

Throughout this section, and even Section 31.7, we will operate under ignorance as to whether γ = ∞ or γ ≠ ∞. We are able to argue uniformly. We will also freely use ω, and so we will from now on just consider our claims to be proved in T_5.

We now sharpen Lemma 6.11.

Definition 6.13. A is a <* bounded subclass of <*[γ] if and only if sup(A) <* γ. B is a <* bounded subclass of $<^*[\gamma]^2$ if and only if there exists x <* γ such that for all (y,z) ∈ B, y,z <* x.

Lemma 6.16. Let f:V → <*[γ] be partially defined, where DEF(f). Then f maps <* bounded subclasses of <*[γ] onto <* bounded subclasses of <*[γ]. Let $g:V^2 \to$ <*[γ] be partially defined, where DEF(g). Then g maps <* bounded subclasses of $<^*[\gamma]^2$ onto <* bounded subclasses of <*[γ].

Proof: The first claim follows from the second easily by using g(x,x). For the second claim, let g be as given. Suppose this is false, and let x be <* least such that x <* γ and $g[<^*[x]^2]$ is <* unbounded in <*[γ]. Then DEF({x}). We claim that for all y <* γ, sup({g(y,z): z <* x}) <* γ. Suppose this is false, and let y be <* least such that y <* x and sup({g(y,z): z <* x}) = γ. Then DEF({y}), and we can apply Lemma 6.11 to the function g(y,z), with y fixed. Hence sup({g(y,z): z <* x}) <* γ. This contradicts the choice of y. We now define h:<*[γ] → <*[γ] by h(y) = sup({g(y,z): z <* x}) <* γ, using the claim in the previous paragraph. Then by Lemma 6.11,

sup(rng(h)) <* γ. But clearly sup(rng(h)) = sup({g(y,z): z <* x}) <* γ. This contradicts the choice of x. QED

The pairing function P is rather uncontrolled. E.g., P(0,0) might even be larger than γ (although of course this can't happen if γ = ∞). We need a pairing function that maps $<^*[\gamma]^2$ into <*[γ].

Definition 6.14. $(x,y) <' (z,w) \leftrightarrow \max(x,y) <^* \max(z,w) \vee (\max(x,y) = \max(z,w) \wedge (x,y) <_{lex} (z,w))$. Here max and $<_{lex}$ are with respect to <*.

Lemma 6.17. $(V^2,<')$ is a well ordering.

Proof: <' is obviously irreflexive. Let $(x,y) <' (z,w) \wedge (z,w) <' (u,v)$. Clearly $\max(x,y) \leq^* \max(z,w) \leq^* \max(u,v)$. If not all three of these max's are equal, then $(x,y) <' (u,v)$. So assume that all three max's are equal. Then $(x,y) <_{lex} (z,w) <_{lex} (u,v)$. Therefore $(x,y) <' (u,v)$. Let $(x,y) \neq (z,w)$. If $\max(x,y) \neq \max(z,w)$ then $(x,y) <' (z,w) \vee (z,w) <' (x,y)$. So assume that $\max(x,y) = \max(z,w)$. If $(x,y) <_{lex} (z,w)$ then $(x,y) <' (z,w)$. If $(z,w) <_{lex} (x,y)$ then $(z,w) <' (x,y)$. Let $A \subseteq V^2$ be nonempty. Cut down to the elements of A with <* least possible max. Then cut down to those with <* least possible first coordinate. Then pick the one with <* least second coordinate. This is the <' least element of A. QED

Definition 6.15. We say that f is a (<',x,y) function if and only if i. $f:<'[(x,y)] \rightarrow V$, and ii. For all $(z,w) <' (x,y)$, $f(z,w) = \sup(\{f(u,v): (u,v) <' (z,w)\})$.

Lemma 6.18. If f is a (<',x,y) function and g is a (<',z,w) function, then $f \subseteq g \vee g \subseteq f$. For all x,y <* γ, there is a unique (<',x,y) function. The range of this function is a <* bounded subset of <*[γ].

Proof: The first claim is easily proved by using the <' least (u,v) such that $f(u,v) \neq g(u,v)$. We now prove the remaining claims. Let (x,y) be <' least with x,y <* γ, such that there is no (<',x,y) function f with sup(rng(f)) < γ. Then x is unique such that

there exists y such that (x,y) is <′least with this property, and y is unique such that (x,y) is <′least with this property. Hence DEF({x}), and therefore DEF({y}). Clearly (x,y) is not <′least. I.e., (x,y) ≠ (0,0). Case 1. (x,y) is a <′immediate successor. Let (x,y) be the <′immediate successor of (z,w). Then z,w <* γ. Also DEF({z},{w}). Let f be the (<′,z,w) function. By the choice of (x,y), sup(rng(f)) <* γ. Also DEF(f). Extend f at (z,w) by f′(z,w) = sup(rng(f)). Then DEF(f′), and sup(rng(f′)) <* γ. This contradicts the choice of (x,y). Case 2. (x,y) is a <′limit point. Let g be the union of the various (<′,z,w) functions, where (z,w) <′(x,y). This union is a function by the first claim. Clearly DEF(g), and g is a (<′,x,y) function. Note that g is a partial function from ≤* $[\max(x,y)]^2$ into <*[γ]. By Lemma 6.16, sup(rng(g)) <* γ. This contradicts the choice of (x,y). Thus we have a contradiction. Hence For all (x,y) <* γ, there is a (<′,x,y) function f with sup(rng(f)) <* γ. By the first claim, f is unique. QED

Definition 6.16. We define P*(x,y) for x,y <* γ as follows. Let (x′,y′) be the <′immediate successor of (x,y). Let f be the unique <′(x′,y′) function. Define P*(x,y) = f(x′,y′). For other x,y, P*(x,y) is undefined.

Lemma 6.19. P*:<*$[\gamma]^2$ → <*[γ] is one-one with DEF(P*). x,y ≤* P*(x,y).

Proof: DEF(P*) by Definition 6.16. Now let P*(x,y) = P*(z,w). We want x = z ∧ y = w. By symmetry, assume (x,y) <′(z,w). Let (x′,y′),(z′,w′) be the <′immediate successors of (x,y),(z,w), respectively. Clearly x′,y′,z′,w′ <* γ, (x′,y′) <′(z′,w′). By Lemma 6.18, let f,g be the (<′,x′,y′),(<′,z′,w′) functions, respectively. By Lemma 6.18, f ⊆ g. Also P*(z,w) = g(z,w) = sup({g(u,v): (u,v) <′(z,w)}). Hence f(x,y) = g(x,y) = P*(x,y) <* f(z,w) = P*(z,w), which is a contradiction. For the second claim, suppose this is false. Let (x,y) be <′least such that x,y ≤* P*(x,y) is false. Then (x,y) ≠ (0,0). Suppose (x,y) is the <′immediate successor of (z,w). Clearly max(x,y) ≤* max(z,w)+1 ≤* P*(z,w)+1 = P*(x,y). Let (x,y) be a <′

limit point. For all $(z,w) <' (x,y)$, $z,w \leq^* P^*(z,w)$. Now $P^*(x,y) = \sup(\{P^*(z,w): (z,w) <' (x,y)\}) \geq^* \sup(\{\max(z,w): (z,w) <' (x,y)\}) \geq^* \max(x,y)$. Thus $x,y \leq^* P^*(x,y)$, which contradicts the choice of (x,y). This is a contradiction. QED

Definition 6.17. We extend P^* to any standard arity ≥ 2, left associatively, by $P^*(x_1,...,x_n) = P^*(...(P^*(x_1,x_2),...,x_n)$. We say that f is a finite sequence if and only if $f:<^*[x] \to V$, for some $x <^* \omega$. The length of f, lth(f), is x.

Definition 6.18. Let f be a finite sequence of length x. A P^* sequence for f is a finite sequence g of length x such that i. $0 <^* x \to g(0) = P^*(x,0,f(0))$, and ii. $(\forall y+1 <^* x)(g(y+1) = P^*(x,g(y),f(y+1)))$.

Lemma 6.20. Every finite sequence into $<^*[\gamma]$ has a unique P^* sequence into $<^*[\gamma]$.

Proof: By obvious induction arguments along lengths $x <^* \omega$. Use $x,y <^* \gamma \to P^*(x,y) <^* \gamma$. QED

Definition 6.19. Let f be a finite sequence of length x into $<^*[\gamma]$. y is a P^* code for f if and only if $y = g(x-1)$ if $x \neq 0$; $P(0,0,0)$ otherwise, where g is the P^* sequence for f. A P^* code is a P^* code for some finite sequence into $<^*[\gamma]$. B has a P^* code if and only if B is a finite sequence into $<^*[\gamma]$.

Lemma 6.21. Two finite sequences into $<^*[\gamma]$ have the same P^* code if and only if they are equal. $(\forall x <^* \gamma)(\exists y <^* \gamma)(\forall z,w <^* x)(P^*(z,w) <^* y)$.

Proof: Suppose x,y are finite sequences into $<^*[\gamma]$ with different lengths. Their P^* codes are of the form $P^*(lth(x),z,w)$ and $P^*(lth(y),u,v)$, respectively. Hence their P^* codes are not equal. We prove by induction on $x <^* \omega$ that any two finite sequences f,g, with lth(f) = x, and the same P^* code, are equal. The basis case x = 0 is immediate, as there is exactly one finite sequence of length 0. Suppose this is true for fixed $x \neq 0$. Let f',g' be the P^* sequences for f,g, respectively. We prove by backwards induction i = x-1,...,0, that f(i) = g(i). To see this, clearly f(x-1) =

g(x-1), as these are the P* codes for f,g, respectively. Suppose f(z) = g(z), z ≠ 0. Then P*(x,f(z-1),f(z)) = P*(x,g(z-1),g(z)), and so f(z-1) = g(z-1). Since f' = g', we see that for all 0 ≠ i <* x, f(i) = g(i), and also f(0) = g(0). Hence f = g. The second claim follows immediately from Lemma 6.16. QED

Lemma 6.22. Let x <* γ. The P* codes of finite sequences into <*[x] form a <* bounded subclass of <*[γ].

Proof: Suppose this is false, and let x <* γ be the <* least counterexample. Then DEF({x}). We first show that for all y <* ω, the P* codes of length y sequences into <*[x] form a <* bounded subclass of <*[γ]. Let y <* ω. We show by induction that for all i <* y, {g(i): g is a P* sequence for some finite sequence of length y into <*[x]} is <* bounded in <*[γ]. The basis case is obvious, and the induction step is clear using the second claim of Lemma 6.21. By setting i = y-1, we see that the P* codes of finite sequences of length y into <*[x] are bounded in <*[γ]. (The case y = 0 is trivial). Define g:<*[ω] → <*[γ] by g(y) = the sup of the P* codes of length y sequences into <*[x]. Then DEF(g). By Lemma 6.16, sup(rng(g)) <* γ. QED

Definition 6.20. Let x be a P* code. lh(x) is lth(f), where the P* code of f is x. x<y> is f(y), where the P* code of f is x. Note that x<y> is defined if and only if y <* lh(x).

We are now in a position to code the constructible hierarchy below γ. We freely assume standard Gödel numberings of formulas in the language LST of set theory, which is based on ∈,=, and using variables $v_1,v_2,...$

Definition 6.21. Let R ⊆ A^2. SAT(A,R,x,y) if and only if i. x <* ω is a Gödel number of a formula φ of LST, ii. y is the P* code of a finite sequence f into A, where all subscripts of variables appearing in φ are <* lth(f), and iii. (A,R) satisfies φ at the assignment f, where R interprets ∈.

Definition 6.19 is formalized in T_5 using Tarski's truth definition (satisfaction relation definition). This makes reference

to a provably unique class that forms the relevant satisfaction relation.

Definition 6.22. A wfes (well-founded extensional structure) is an (A,R), $R \subseteq A^2$, where $(\forall x,y \in A)((\forall z)(R(z,x) \leftrightarrow R(z,y)) \rightarrow x = y)$, every nonempty subset of A has an element with no R predecessor from A, and A is a <* bounded subclass of <*[y].

Definition 6.23. Let (A,R) be given, $R \subseteq A^2$. x is a basic code over (A,R) if and only if i. x is the P* code of a nonempty finite sequence f into A, ii. {u ∈ A: SAT(A,R,f(0),u,f(1),...,f(lth(f)-1))} is not of the form {a ∈ A: R(a,b)}, b ∈ A, and iii. {u ∈ A: SAT(A,R,f(0),u,f(1),...,f(lth(f)-1))} is not {u ∈ A: SAT(A,R,g(0),u,g(1),...,g(lth(g)-1))}, where the P* code of nonempty g is <* x.

x is a code over (A,R) if and only if x = P*(y,sup(A)), where y is a basic code over (A,R).

The last line of Definition 6.23 is to guarantee that the codes are higher than A in <*.

Definition 6.24. Let (A,R) be given, $R \subseteq A^2$. FODO(A,R) = (B,S), where B is A union the class of all codes over (A,R), and S(u,x) if and only if $R(u,x) \vee (u \in A \wedge SAT(A,R,(f(0),u,f(1),...,f(lth(f)-1))))$, where x = P*(y,sup(A)) and y is the P* code of f.

Definition 6.25. We say that (A,R) is an initial segment of (B,S) if and only if $R \subseteq A^2 \wedge S \subseteq B^2 \wedge A = B \cap <*[sup(A)] \wedge R = S \cap A^2$. The union of a family of (A,R)'s is (B,S), where B is the union of the A's and S is the union of the R's. sup(A,R) = sup(A).

Lemma 6.23. Every wfes (A,R) is a proper initial segment of the wfes FODO(A,R).

Proof: Let A,R be as given. The first component of FODO(A,R) is a <* bounded subclass of <*[y] by Lemma 6.21. Extensionality in FODO(A,R) is clear by the construction in Definition 6.25. That (A,R) is an initial segment of FODO(A,R) is also clear by the construction in Definition 6.25, as the new elements are ≥ sup(A)

using Lemma 6.19. For the wellfoundedness of FODO(A,R), let X be a nonempty subset of dom(FODO(A,R)). If $X \cap A \neq \emptyset$, then use an R least element of X. Otherwise, use any new element. It remains to show that FODO(A,R) is not (A,R). We have only to show that some (A,R) definable $B \subseteq A$ is not the set of R predecessors of any $x \in A$. Take $B = \{x \in A: \neg R(x,x)\}$. QED

Recall the convention in Definition 6.10.

Definition 6.26. f is an L function if and only if $f:<^*[x] \to \wp(<^*[y])$, where i. $x <^* y$, ii. $0 <^* x \to f(0) = (\emptyset,\emptyset)$, iii. $y+1 <^* x \to f(y+1) = FODO(f(y))$, and iv. Let $y <^* x$, and y be a $<^*$ limit point. $f(y) = \cup_{z<^*y} f(z)$.

In T_5, $f:V \to \wp(V)$ are interpreted as corresponding relations on V.

Lemma 6.24. If f,g are L functions then $f \subseteq g \vee g \subseteq f$. For all $x <^* y$ there exists a unique L function $f:<^*[x] \to \wp(<^*[y])$. This unique f has $\sup(\cup rng(f)) <^* y$. Furthermore, for all $y \leq^* z <^* x$, f(y) is an initial segment of f(z), and f(y) is a wfes.

Proof: Let f,g be L functions, and suppose $(\exists x)(f(x) \neq g(x))$. Let x be $<^*$ least such that $f(x) \neq g(x)$. Then $f(x) = g(x)$, which is a contradiction. Hence f,g are comparable. We now prove that for all $x <^* y$, there exists an L function $f:<^*[x] \to <^*[y]$ with $\sup(\cup rng(f)) <^* y$, and for all $y \leq^* z <^* x$, the wfes f(y) is a proper initial segment of the wfes f(z). Suppose this is false, and let x be the $<^*$ least counterexample. Obviously $x \neq 0$, and DEF(x). Case 1. x is a $<^*$ limit point. Let f be the union of the L functions whose domain is some $<^*[y]$, $y <^* x$. This union is a function by the first claim. Clearly $f:<^*[x] \to \wp(<^*[y])$. By Lemma 6.16, $\sup(rng(f)) <^* y$. Also, the initial segment property holds for f, since the initial segment property holds for the L functions whose domain is some $<^*[y]$, $y <^* x$. Case 2. $x = y+1$. Let f be the L function with $f:<^*[y] \to <^*[y]$. If $y = 0$ then extend f at 0 by $f'(0) = (\emptyset,\emptyset)$. If $y = z+1$ then extend f at y by $f'(y) = FODO(f(z))$. Since $\sup(\cup rng(f)) <^* y$, we see that $\sup(\cup rng(f')) <^* y$, by Lemma 6.22. Also the initial

segment property holds by Lemma 6.23. This contradicts the choice of x. Assume y is a <* limit point. Let f be the L function with f:<*[y] → <*[y]. Let (A,R) = $\cup_s$f(z). Since y <* x, sup(∪rng(f)) <* y. From the initial segment property for f, we see that (A,R) is a wfes. Hence FODO(A,R) is a wfes. Extend f at y by f'(y) = (A,R). Then f' is an L function with f':<*[x] → <*[y], sup(∪rng(f')) <* y, and we have the initial segment property. This contradicts the choice of x. Thus we have arrived at a contradiction, and so for all x <* y, there exists an L function f:<*[x] → <*[y] with sup(∪rng(f)) <* y, where for all y ≤* z <* x, f(y) is an initial segment of f(z), and f(y) is a wfes. QED

Definition 6.27. Let x <* y. (L[x],E[x]) = f(x), where f is the L function with domain <*[x+1].

Lemma 6.25. (L[0],E[0]) = (∅,∅). For all x <* y, (L[x+1],E[x+1]) = FODO(L[x],E[x]). For all <* limit points x <* y, L[x] = $\cup_{y<^*x}$L[y], E[x] = $\cup_{y<^*x}$E[y]. For all x <* y <* y, (L[x],E[x]) is an initial segment of (L[y],E[y]). For all x,y,z <* y, (E[x](y,z) → y <* z) ∧ (z ∈ E[y]\E[x] → sup(E[x]) <* z).

Proof: The first four claims follow easily from Lemma 6.24 using Definition 6.29. For the last claim, let x be the <* least counterexample. Clearly x ≠ 0. Let x = w+1. Since w has the property, x has the property by the FODO construction, and x,y ≤ P*(x,y) from Lemma 6.19. Let x be a <* limit point. Then x has the property, using that each y <* x has the property, and the initial segment claims in Lemma 6.24. We have contradicted the choice of x, and hence have a contradiction. QED

Definition 6.28. L = $\cup_{x<^*y}$L[x], E = $\cup_{x<^*y}$E[x].

Lemma 6.26. DEF(L,E). E(x,y) → x <* y. L ∩ <*[y] ⊆ L[y].

Proof: DEF(L,E) is clear from the definitions of L,E. The second claim is by Lemma 6.24 and the last claim of Lemma 6.25. For the third claim, let z ∈ L ∩ <*[y]. Let z ∈ L[w], z <* y. If w ≤* y

then z ∈ L[y]. If w >* y then by Lemma 6.24, (L[y],E[y]) is a proper initial segment of (L[w],E[y]). Hence z ∈ L[y]. QED

Theorem 6.27. T_5 proves (L,E) is a model of ZF. T_5 proves the consistency of ZFC, when formulated with standard formalized syntax. ZFC is interpretable in T_5. EFA proves Con(T5) → Con(ZFC).

Proof: In T_5, we can define the satisfaction relation over (L,E), no matter whether γ = ∞ or γ < ∞. The axioms of extensionality, pairing, union, infinity, foundation are easily verified. This leaves separation, power set, and replacement. Separation is verified by adapting the standard argument using Skolemization, in order to pull down a formula φ with unbounded quantifiers, and parameters from some L[x], to a formula Ψ with quantifiers bounded to some L[y], where φ,Ψ are equivalent for all parameters from some L[x]. Here Ψ depends only on φ, and y depends only on x,φ. In the sense internal to (L,E), the required set lies in L[y+1]. This pulling uses the regularity of γ given by Lemma 6.16. Replacement is established by first using the regularity of γ given by Lemma 6.16, and then applying separation. For power set, we show that (∀x)(∃y)(∀z)((L,E) satisfies z ⊆ x → z ∈ L[y]). This suffices, since we have already established Separation in (L,E). Suppose this is false, and let x be the <* least counterexample. Then DEF(x). We define partial f:<*[γ] → ℵ$_1$(<*[γ]) by f(z) = {w: E(w,z)} if (L,E) satisfies z ⊆ x; undefined otherwise. Then DEF(f). By extensionality in (L,E), f is one-one. Also for all z ∈ dom(f), f(z) ∈ ℵ$_1$(<*[x]) by Lemma 6.26. By Lemma 6.12, let y = sup(dom(f)). By Lemma 6.26, L ∩ <*[γ] ⊆ L[y]. Hence (∀z)((L,E) satisfies z ⊆ x → z ∈ L[y]). This contradicts the choice of x. Thus we have proved the first two claims. For the third claim, note that ZFC is interpretable in ZF. The fourth claim follows easily from the second claim. QED

31.7 Interpreting a Strong Extension of ZFC in T_5

In this section, we extend Theorem 6.26 to ZFC + “there are arbitrarily large strong Ramsey cardinals.” See Definition 7.18.

We use the development and notation from Section 31.6. The ideas are an adaptation of the well-known inner model construction L[U], where U is a nontrivial κ complete ultrafilter on a measurable cardinal κ, to the present context. See, e.g., Kanamori (1994, p. 261).

One difference in the present construction is that we are not sitting inside a model of ZFC above γ. Recall that we have only $\gamma \in V \vee \gamma = \infty$, and we are arguing uniformly under both cases. Thus we will not be getting a model of ZFC + “there exists a measurable cardinal.” We know that we cannot get such a model from T_5 because of the last claim in Theorem 5.1.

Another difference is that we cannot normalize the ultrafilter given by POS before starting the construction. Usually, in set theory, with L[U] type constructions, the filter is normalized first before constructing from it. We will, however, still get an internal γ complete ultrafilter, which, internally, can be normalized—even though we are not sitting within a model of ZFC. Then the normal γ complete ultrafilter is used in familiar ways to prove the existence of arbitrarily large strong Ramsey cardinals below γ. This will be enough to get a model of ZFC with arbitrarily large strong Ramsey cardinals.

We argue entirely in T5.

Definition 7.1. Let (A,T) be a well ordering. An end extension of (A,T) is a well ordering (A′,T′) such that i. $A \subseteq A' \wedge T \subseteq T'$, and ii. $y \in A \wedge T'(x,y) \rightarrow x \in A$. A proper end extension of (A,T) is an end extension which is not (A,T).

In this section, in a typical well ordering (A,T), A is a <* unbounded subclass of <*[γ], and (A,T) is much longer than (<*[γ],<*). The end extensions (B,W) of (A,T) will typically have many $x \in A$ and $y \in B \backslash A$ with $x >^* y$. This is in contrast with the initial segment notion in Definition 6.27, which is tied to <*.

Definition 7.2. We often consider $(A,T \cap A^2)$. It is convenient to define $(A,T/) = (A,T \cap A^2)$, as the expression for A may be long.

The following definition was made in Section 31.6 only for $<^*$ (Definition 6.14). Here we need the more general notion.

Definition 7.3. Let (A,T) be a wo. We define $<_{A,T}$ on A^2 as follows. $(x,y) <_{A,T} (z,w) \leftrightarrow T(\max(x,y),\max(z,w)) \vee (\max(x,y) = \max(z,w) \wedge (x,y) <_{lex} (z,w))$. Here max and $<_{lex}$ are with respect to (A,T).

Lemma 7.1. If (A,T) is a wo then $(A^2,<_{A,T})$ is a well ordering.

Proof: See Lemma 6.17. QED

Lemma 7.2. Let $A \subseteq {<^*}[\gamma]$ be $<^*$ unbounded in $<^*[\gamma]$, DEF(A). There is a unique isomorphism from $(A,<^*/)$ onto $({<^*}[\gamma],<^*/)$. This unique isomorphism f has DEF(f).

Proof: Let A be as given. First show that the isomorphisms from $(A \cap {<^*}[x],<^*/)$ onto some $({<^*}[y],<^*/)$ cohere. Then show that for all $x <^* \gamma$, there exists a unique isomorphism from $(A \cap {<^*}[x],<^*/)$ onto some $({<^*}[y],<^*/)$, using the regularity given by Lemma 6.11. Finally, take the union f of these coherent isomorphisms. By Lemma 6.11, f is onto. QED

Lemma 7.3. Let $A \subseteq {<^*}[\gamma]$ be $<^*$ unbounded in $<^*[\gamma]$, $0 \neq x \leq^* \gamma$, and DEF(A,{x}). There exists $f:A \to {<^*}[x]$ such that for all $y <^* x$, $f^{-1}(y)$ is $<^*$ unbounded in $<^*[\gamma]$, and DEF(f).

Proof: Let A,x be as given. In light of Lemma 7.2, it suffices to find $f:{<^*}[\gamma] \to {<^*}[x]$ such that for all $y <^* x$, $f^{-1}(y)$ is $<^*$ unbounded in $<^*[\gamma]$, and DEF(f). Take $f:{<^*}[\gamma] \to {<^*}[x]$ given by $f(y) = z$ if $(\exists w <^* \gamma)(z <^* x \wedge y = P^*(z,w))$; 0 otherwise. Then for all $z <^* x$, $f^{-1}(z) = \{P^*(z,w): w <^* \gamma\}$, which is unbounded in $<^*[\gamma]$ by Lemma 6.11 and that P^* is one-one. QED

Definition 7.4. The f constructed in the proof of Lemma 7.2 is called the x splitting of A.

Lemma 7.4. There exists $A \subseteq {<^*}[\gamma]$, where i. ${<^*}[\omega] \subseteq A$, ii. ${<^*}[\gamma] \backslash A$ is $<^*$ unbounded in $<^*[\gamma]$, iii. POS(A), and iv. DEF(A).

Proof: Let f be the 2 splitting of <*[γ]. Then POS($f^{-1}(0)$) ∨ POS($f^{-1}(1)$). Take A = $f^{-1}(0)$ ∪ <*[ω] if POS($f^{-1}(0)$); $f^{-1}(1)$ ∪ <*[ω] otherwise. We are using Lemma 6.7. Here 0 <* 1 <* 2 are the first three elements of <*[ω]. QED

Definition 7.5. Let X be the A constructed in the proof of Lemma 7.4. Let θ be the <* least element of <*[γ]\A. Clearly θ >* ω.

All of the well orderings that we will be now using will end extend X ∪ {θ}, with θ moved to just above X.

θ is very important to the rest of this section. Ultimately, θ will become an internal strongly inaccessible cardinal with large cardinal properties, and V(θ) will be a model of ZFC with arbitrarily large strong Ramsey cardinals.

Definition 7.6. A good wo is a well ordering (A,T) such that i. A ⊆ <*[γ], ii. <*[γ]\A is <* unbounded in <*[γ], and iii. (A,T) is an end extension of (X ∪ {θ},{(x,y): (x <* y ∧ x,y ∈ X) ∨ (x ∈ X ∧ y = θ)}).

We impose condition ii because we will want to be end extending good wo's, and we need some room.

Definition 7.7. (A,T) is an adequate wo if and only if i. (A,T) is a good wo, ii. (A^2,$<_{A,T}$) is order isomorphic to (A,T). $P_{A,T}$ is the unique order isomorphism from (A^2,$<_{A,T}$) onto (A,T).

Lemma 7.5. (X ∪ {θ},{(x,y): (x <* y ∧ x,y ∈ X) ∨ (x ∈ X ∧ y = θ)}) is a good wo whose two components have DEF. Let (A,T) be a good wo, DEF(A,T). There is an adequate wo (B,W) which is a proper end extension of (A,T), with DEF(B,W).

Proof: The first claim is immediate. For the second claim, we put ω successive copies of (A^2,$<_{A,T}$) on top of (A,T). We have to replace ordered pairs with points, and we also have to disjointify. For the n-th copy, n <* ω, use points from $f^{-1}(n+1)$, where f is the ω splitting of <*[γ]\A. Specifically, for each n <* ω, let g_n:<*[γ] → $f^{-1}(n+1)$ be the unique <* preserving bijection, and h_n:A^2 → f^{-}

$^{1}(n+1)$ be given by $h_n(y,z) = g_n(P_{A,T}(y,z))$. The n-th successive copy of $(A_2,<_{A,T})$ will be the push of h_n, so that its domain is a <* unbounded subclass of $f^{-1}(n+1)$. Note that $f^{-1}(0)$ remains unused. QED

An adequate wo (A,T) is a pretty good place to build hierarchies over its <* unbounded domain, because of its closure under the natural pairing function $P_{A,T}$. However, it is more convenient to have the stronger closure property in Definition 7.8.

Definition 7.8. f is special for (A,T) if and only if

1. (A,T) is an adequate wo.
2. dom(f) = A.
3. f(0) = (A,T).
4. If T(x,y) then f(x),f(y) are adequate wo's, and f(y) is a proper end extension of f(x).
5. ∪rng(f) is an adequate wo.

Lemma 7.6. Let (A,T) be an adequate well ordering, DEF(A,T). There is a special f for (A,T), DEF(f).

Proof: Let (A,T) be as given. We consider specific approximations to the desired f. These are based on the γ splitting F of <*[γ]\A. We require that f(0) = (A,T), and that f at limits in (A,T) be the union of the earlier values. At successors x' in (A,T), we require that f(x') be the end extension of f(x) constructed in the proof of Lemma 7.5, then trivially modified so that the new points lie in $F^{-1}(x')$. There is a longest such f. If dom(f) = A then we are done. Assume dom(f) ≠ A. Then DEF(f), and so we can extend f. Hence dom(f) = A. In the construction, we never use $F^{-1}(0)$, and so condition v holds. QED

Definition 7.9. (A,T) is a special wo if and only if (A,T) is an adequate wo which is order isomorphic to $(B,T \cap B^2)$, where B =

{x ∈ A: {y: T(y,x)} is closed under $P_{A,T}$}}. We write $P\#_{A,T}$ for this order isomorphism.

Lemma 7.7. Let (A,T) be an adequate wo, DEF(A,T). There is a special wo (B,W) which is a proper end extension of (A,T), with DEF(B,W).

Proof: Let (A,T) be as given. We construct (A,T) = (A_0,T_0), (A_1,T_1), ..., and functions $f_0,f_1,...$, such that each f_i is special for (A_i,T_i), and each (A_{i+1},T_{i+1}) is the union of the range of f_i. Specifically, each f_i is given by the explicit construction used in the proof of Lemma 7.6. The union (A',T') of the (A_i,T_i) meets all of the requirements of a special wo, except that <*[γ]\A' might be <* bounded in <*[γ]. This can be easily fixed using the 2 splitting of A'\(X ∪ {θ}), resulting in the desired (B,W), DEF(B,W). QED

We now develop the constructible hierarchy relative to POS, along any special wo (A,T). We can build this without assuming DEF(A,T).

Definition 7.10. Let (A,T) be a special wo, and $(D,T \cap D^2,R,U)$ be given, where D is a proper initial segment of A under T, X ∪ {θ} ⊆ D, R ⊆ D^2, U ⊆ D. Furthermore, we assume that (D,R) satisfies extensionality, and R(x,y) → T(x,y). We define the continuation of $(D,T \cap D^2,R,U)$ in (A,T) as the following quadruple $(D',T \cap D^2,R',U')$. We have D ⊆ D', and D' is a proper initial segment of A under T. The sets {y: R'(y,x)}, x ∈ D', comprise exactly the subsets of D that are first order definable over $(D,T \cap D^2,R,U)$, without repetition (extensionality). We accomplish this by assigning, to each x ∈ D'\D, one finite sequence $(\varphi,y_1,...,y_n)$, φ a formula in the language of $(D,T \cap D^2,R,U)$, with free variables assigned the elements $y_1,...,y_n$ ∈ D, with the free variable v of φ reserved to define the associated subset of D definable over $(D,T \cap D^2,R,U)$. The constraint is that extensionality is to be maintained. Suppose that we have used, in this way, all y above D but below a given x, under T. For x, we use the "least" finite

sequence that will maintain extensionality when used in this manner. (If we cannot maintain extensionality, then the construction has already been completed). Here “least” is in terms of the ordering T of finite sequence codes arising out of the pairing function $P_{A,T}$. We take U’to consist of all x ∈ D’such that {y: R’(y,x)} ⊆ X and POS({y: R’(y,x)}).

Definition 7.10 is a more careful form of Definitions 6.23, 6.24. Here we are working in an environment (A,T), of which there are many—rather than previously in one specific environment (<*[γ],<*). Here we retain (A,T) as part of a richer structure, that also incorporates a piece of POS. So we make sure that we use initial segments of (A,T) as sets of points. In Definition 6.29, we didn’t care about L[x] skipping over points in <*[γ].

Lemma 7.8. Definition 7.10 is well defined. (D,T ∩ D^2,R,U) is a proper initial segment of (D’,T’ ∩ D'^2.R’,U’) in the sense that (D’,T’ ∩ D'^2) is a proper end extension of (D,T ∩ D^2), R = R’ ∩ D^2, and U = U’ ∩ D.

Proof: For the first claim, the issue is whether there is enough room to go through the finite sequences from D in the manner stipulated. This follows from the adequacy of (A,T): specifically the closure of A under $P_{A,T}$. For the second claim, it suffices to show that some E ⊆ D that is (D,T ∩ D^2,R,U) definable if not the set of R predecessors of any x ∈ D. Take E = {x ∈ D: ¬R(x,x)}. QED

Definition 7.11. Let (A,T) be a special well ordering. For each x ∈ A, we define L[x,A,T,POS] to be a proper initial segment of A, E[x,A,T,POS] ⊆ T ∩ L[x,A,T,POS]2, POS*[x,A,T] ⊆ L[x,A,T,POS], as follows. We take L[0,A,T,POS] = X ∪ {θ}, E[0,A,T,POS] = T ∩ (X ∪ {θ})^2, POS*[0,A,T] = {θ}. If x is a limit point in (A,T), then L[x,A,T,POS] = ∪{L[y,A,T,POS]: T(y,x)}, E[x,A,T,POS] = ∪{E[y,A,T,POS]: T(y,x)}, POS*[x,A,T] = ∪{POS*(y,A,T): T(y,x)}. Let x’

be the immediate successor of x in (A,T). We take (L[x′,A,T,POS], T/,E[x,A,T,POS],POS*[x,A,T)) to be the continuation of (L[x,A,T,POS],T/,E[x,A,T,POS],POS*[x,A,T]) in (A,T), assuming L[x,A,T,POS] is a proper initial segment of A.

Definitions 7.10, 7.11 are formalized in T5 using approximating functions, as in Section 31.6.

Lemma 7.9. Definition 7.11 is well defined for all $x \in A$.

Proof: The issue is whether we run out of points in A during the construction. We must check that if x is a limit point in (A,T), and we have defined L[x,A,T,POS], E[x,A,T,POS], POS*[x,A,T), that we have not used up all of A. We use $P\#_{A,T}$ for this purpose. It is easy to show that for each $x \in A$, the construction can be made uniquely for all y with T(y,x), so that $L[y,A,T,POS] \subseteq P\#_{A,T}(2+y)$. Since (A,T) is a special wo, A is closed under $P\#_{A,T}$. QED

Definition 7.12. Let (A,T) be a special wo. $E[A,T,POS] = \cup_{x \in A} E[x,A,T,POS]$. $POS^*[A,T] = \cup_{x \in A} POS^*[x,A,T]$. Let M[A,T] = (A,T,E[A,T,POS],POS*[A,T]). For purely set-theoretic statements in M[A,T], quantifiers range over A, and $\in$ is interpreted as E[A,T,POS].

Lemma 7.10. Let (A,T) be a special well ordering. M[A,T] is well founded, and satisfies extensionality, pairing, union, infinity, foundation, Δ_0 separation, and "every set is in one-one correspondence with an ordinal." $E[A,T,POS](x,y) \rightarrow T(x,y)$. X forms a proper initial segment of the set-theoretic ordinals of M[A,T], with θ as the next set-theoretic ordinal of M[A,T]. The ordinals of M[A,T] under E[A,T,POS] are isomorphic to (A,T).

Proof: Left to the reader. T_5 is sufficient to formalize the statement that M[A,T] satisfies Δ_0 separation. QED

We want to work with an M[A,T] which satisfies "V(θ) exists." Some work is needed in order to find such M[A,T] within T_5. We need to develop canonical codes for some M[A,T].

Lemma 7.11. There is a sentence φ in ∈,=,POS such that the following holds. Let (A,T) be a special well ordering. M[A,T] is the unique expansion of (A,T) satisfying φ which interprets POS correctly, in the following sense. For any x ∈ A, POS(x) holds if and only if the ∈ predecessors of x forms a subset of X which actually has POS.

Proof: Left to the reader. QED

Definition 7.13. Let (A,T) be a special wo. For x ∈ A, define $M[A,T]_x = \{y: E[A,T,POS](y,x)\}$. B is M[A,T] internal if and only if $B = M[A,T]_x$ for some necessarily unique x ∈ A. $B <_{A,T,POS} C$ if and only if $(\exists x,y)(B = M[A,T]x \wedge C = M[A,T]_y \wedge T(x,y))$.

Definition 7.14. (A,T) is a great wo if and only if i. (A,T) is a special wo, ii. Some subset of X that is definable over M[A,T] with parameters from X, is not M[A,T] internal, and iii. ii holds for any special wo (B,W) such that (A,T) is an end extension of (B,W).

Lemma 7.12. Let (A,T) be a great wo. Some subset of X definable over M[A,T] without parameters, is not M[A,T] internal.

Proof: This amounts to being able to kill the parameters used for the subset of X. First note that θ is defined as the least ordinal with POS*[A,T]. Hence X is also defined without parameters. Now use T least counterexamples to successively kill the parameters. QED

Lemma 7.13. Let (A,T) be a great wo. Every element of A is definable over M[A,T] with parameters from X.

Proof: Let (A,T) be a great wo. By Lemma 7.12, let {x ∈ A: φ(x) holds in M[A,T]} be not M[A,T] internal, where φ has no parameters. Let B be the set of all elements of A which are definable over M[A,T] with parameters from X. Using T least examples, we see that the restriction of M[A,T] to B, M[A,T]|B, is an elementary substructure of M[A,T]. It is easy to see that $(B,T \cap B^2)$ is a special wo. Since it is a restriction of M[A,T] that contains X, it interprets POS correctly. By Lemma 7.11, M[A,T] satisfies the

sentence φ used there. Hence M[A,T]|B also satisfies this φ. Hence by Lemma 7.11, M[A,T]|B = M[B,T ∩ B^2]. Clearly {x ∈ A: φ(x) holds in M[A,T]} = {x ∈ A: φ(x) holds in M[B,T ∩ B^2]} is not M[A,T] internal and not M[B,T ∩ B^2] internal. Also (B,T ∩ B^2) is obviously not longer than (A,T). Suppose (B,T ∩ B^2) is shorter than (A,T). Then (B,T ∩ B^2) is isomorphic to a proper initial segment of (A,T). Therefore {x ∈ A: φ(x) holds in M[B,T ∩ B^2]} is M[A,T] internal. This is a contradiction. Thus (B,T ∩ B^2) and (A,T) are isomorphic, and therefore M[A,T] and M[B,T ∩ B^2] are isomorphic. The isomorphism must be the identity on X. Since every element of B is definable over M[A,T]|B = M[B,T ∩ B^2] with parameters from X, it is also the case that every element of A is definable over M[A,T] with parameters from X. We can also conclude that B = A (although this is not needed). QED

Definition 7.15. Let (A,T) be a great wo. CODE(A,T) is the set of all sentences with parameters from X that hold in M[A,T]. Using $P_{A,T}$, CODE(A,T) is viewed as a subset of X.

Lemma 7.14. Let (A,T),(B,W) be great wo's. We can explicitly recover an isomorphic copy of M[A,T] from CODE(A,T). The following are equivalent. i. CODE(A,T) = CODE(B,W), and ii. (A,T), (B,W) are isomorphic.

Proof: Let A,T,B,W be as given. Using CODE(A,T), we construct an isomorphic copy of M[A,T], and therefore of (A,T), as follows. First identify definitions of prospective points, with parameters from X, under the equivalence relation of equality. Then define the components of the prospective copy of M(A,T), acting on the equivalence classes. Then explicitly pick a unique representative from each equivalence class, using the explicit well ordering of X. By Lemma 7.13, this results in an isomorphic copy of M[A,T]. The second claim follows immediately from the first claim. QED

Lemma 7.15. Let (A,T),(B,W) be great wo's. If (A,T) is shorter than (B,W), then CODE(A,T) is M[B,W] internal. CODE(A,T) is not definable over M[A,T]. In particular, CODE(A,T) is not M[A,T] internal.

Proof: Any great wo (B,W) has easily enough power to internally construct CODE(A,T) for each proper initial segment (A,T) that is a great wo. Suppose CODE(A,T) is definable over M[A,T]. Then E = {(x,y): x codes the Gödel number of a formula φ and a finite sequence from X, and y ∈ X, and φ(x,y)} enumerates all subsets of X definable over M[A,T]. Since E can be read off of CODE(A,T), E is definable over M[A,T]. Therefore {x ∈ X: (x,x) ∉ E} is definable over M[A,T]. Fix x ∈ X such that {y: (x,y) ∈ E} = {x ∈ X: (x,x) ∉ E}. Then (x,x) ∈ E ↔ (x,x) ∉ E. QED

Lemma 7.16. There is a great wo, (A,T), with DEF(A,T).

Proof: We start with the good wo of Lemma 7.5, (X ∪ {θ}, {(x,y): (x <* y ∧ x,y ∈ X) ∨ (x ∈ X ∧ y = θ)}). The proof of Lemma 7.5, second claim, produces a specific adequate wo (A,T). The proof of Lemma 7.7 produces a specific proper end extension (B,W) of (A,T) which is a special wo. We claim that (B,W) is a great wo. To see this, we repeat these specific constructions internally in M[B,W]. We obtain $R \subseteq X^2$, definable over M[B,W], such that the R_x, x ∈ X, comprise exactly the M[B,W] internal subsets of X. {x: ¬R(x,x)} is therefore definable over M[B,W], yet not M[B,W] internal. Clearly DEF(B,W). QED

Lemma 7.17. Let (A,T) be a great wo, DEF(A,T). There is a great wo (B,W) which is a proper end extension of (A,T), with DEF(B,W).

Proof: Let A,T be as given. Apply the construction in the proof of Lemma 7.7 to (A,T), obtaining the special wo (B,W), DEF(B,W). As in the proof of Lemma 7.16, repeat this specific construction applied to (A,T), internally in M[B,W]. Argue as in the proof of Lemma 7.16 that there is a subset of X, definable over M[B,W], which is not M[B,W] internal. QED

Lemma 7.18. Let $x \in X$. There is a great wo (A,T) such that the following holds.

i. M[A,T] satisfies "V(x) exists and is in one-one correspondence with some $u \in \theta$."

ii. For any great wo (B,W) longer than (A,T), the V(x) of M[B,W] is the same as the V(x) of M[A,T].

Proof: Assume the hypothesis. Suppose this is false, and let $x \in X$ be a $<^*$ least counterexample. Clearly DEF({x}), and $x \neq 0$. Case 1. x is the immediate successor of y in X. Clearly DEF({y}). Let (A,T) be a great wo and $u \in X$ be $<^*$ least such that i. M[A,T] satisfies "V(x) exists and is in one-one correspondence with u." ii. For any great wo (B,W) longer than (A,T), the V(x) of M[B,W] is the same as the V(x) of M[A,T]. Then DEF({u}). We now show that there is a great wo (B,W) and $v \in X$ such that iii. M[B,W] satisfies "$\aleph_\zeta$(u) exists and is one-one correspondence with v."

iv. For any great wo (C,Y) longer than (B,W), the $\aleph_\zeta$(u) of M[C,Y] is the same as the $\aleph_\zeta$(u) of M[B,W]. To establish iii, iv, look at all of the M[B,W] internal subsets of u for the various great wo's (B,W), under the various $<_{B,W,POS}$. Suppose that the resulting length is at least that of $(<^*[\gamma],<^*)$. Then we can find a one-one $f:<^*[\gamma] \rightarrow \aleph_\zeta(<^*[u])$ with DEF(f), using that DEF({x},{y},{u}). This contradicts Lemma 6.12. Thus the resulting length is that of some $(<^*[v],<^*/)$, $v <^* \gamma$, DEF({v}). We now show that we can find a single special wo (B,W) long enough so that all of the internal subsets of u living in the various great wo's are M[B,W] internal.

Clearly $v \neq 0$. Suppose v is a successor in $<^*$. We use any great wo which has all of these subsets of u, internally. Since v is a successor, $\aleph_\zeta(u)$ exists internally in this great wo, and also the internal one-one correspondence with the analog of v in X. Thus we now assume that v is a $<^*$ limit point.

For each $w <^* v$, let h(w) be CODE(C,Y), for the shortest great wo (C,Y) whose internal subsets of u are of length at least $(<^*[w],$

<*/) under $<_{C,Y,POS}$. Note how the use of these codes, with Lemma 7.14, avoids use of any form of choice which is unavailable in T_5. From each h(w), we can recover a copy of the corresponding great wo with code h(w), using Lemma 7.14. We can then put these great wo's together to form a single long special wo, (B,W). It only remains to verify that (B,W) is a great wo. Obviously, if there is a longest h(w), w <* v, then there is nothing to prove, since h(w) is already great. So we assume that there is no longest h(w), w <* v. To verify this, we shift the work along X, using v'∈ X with the same position in X that v has in <* [γ]. Internally in M[B,W], define h'(w), w ∈ X, w <* v', as CODE(C,Y), for the shortest great wo (C,Y) whose internal subsets of u are of length at least the ordinal w. Since each code is a subset of X, and these codes are indexed by {w ∈ X: w <* v'}, we can join them together to form a subset of X that is definable over M[B,W]. But this join cannot be M[B,W] internal. If it is M[B,W] internal, then it is M[C,Y] internal, for some great (C,Y) that is a proper initial segment of (B,W), and CODE(C,Y) is M[C,Y] internal. This contradicts Lemma 7.15. So we have shown that M[B,W] is a great wo. But in order to get the desired contradiction, we need to properly end extend (B,W) to another great wo. Note that DEF(B,W). Hence we can apply Lemma 7.17. Case 2. x is a <* limit point in X. For each y <* x, y ∈ X, let f(y) be CODE(A,T), where (A,T) is a great wo of least length such that i. M[A,T] satisfies "V(y) exists and is in one-one correspondence with some u ∈ θ." ii. For any great wo (B,W) longer than (A,T), the V(y) of M[B,W] is the same as the V(y) of M[A,T]. We also let g(y) be the least u ∈ θ in clause i. Clearly DEF({x},f,g). By the regularity in Lemma 6.16, fix w to be the sup in X of the values of g. Note that DEF({w}). By Lemma 7.14, for each y <* x, y ∈ X, recover a copy of the corresponding great wo with code f(y), and then piece these together to form a single long special wo, (B,W), DEF(B,W). By redoing the construction of (B,W) within

M[B,W] as in case 1, we see that (B,W) is a great wo. By Lemma 7.17, let (C,Y) be a great wo which is a proper end extension of (B,W). Then M[C,Y] satisfies that V(x) exists. Also, by Lemma 7.10, in M[C,Y], V(x) is in one-one correspondence with an ordinal. Using w and the regularity in Lemma 6.16, we see that in M[C,Y], V(x) is in one-one correspondence with an ordinal < θ. Hence x is not a counterexample, and we have a contradiction. Since both cases lead to contradictions, x is not a counterexample, and the Lemma is proved. QED

Lemma 7.19. There is a great wo (A,T) such that the following holds: i. M[A,T] satisfies "V(θ) exists and is in one-one correspondence with θ," ii. For any great wo (B,W) longer than (A,T), the V(θ) of M[B,W] is the same as the V(θ) of M[A,T], iii. DEF(A,T), and iv. M[A,T] satisfies "θ is a strongly inaccessible cardinal."

Proof: First construct a great wo (A,T) with i,ii above, in a manner that is entirely analogous to how we argued case 2 in the proof of Lemma 7.18. I.e., for each x ∈ θ there exists a least code of a great wo that stabilizes V(x) in the sense of Lemma 7.18, with internal one-one correspondence with some u ∈ θ. Then piece these codes together to form a special wo (B,W) which stabilizes V(θ), and in light of Lemma 7.10, has V(θ) is one-one correspondence with θ. By redoing the construction inside (B,W), we see as before that (B,W) is a great wo. We will also have DEF(B,W) because of Lemma 7.14. For iv, use this M[B,W] with i-iii above, and Lemma 6.12 and 6.16. We can apply Lemmas 6.12, 6.16, since DEF[B,W]. QED

We now normalize POS.

Definition 7.16. We fix the great wo (A,T), built from CODE(A,T), given by Lemma 7.19.

For the remainder of this section, the only great wo that we use is (A,T). By "internal" we will always mean internal to M[A,T].

We will be making a number of arguments internally in M[A,T]. We do not have to worry about using choice within M[A,T], as all internal objects are given by points in A, and we have access to the well ordering T of A.

However, we have to be careful about the separation in M[A,T] that we use. We have V(θ) as internal in M[A,T]. However, we have only Δ_0 separation in M[A,T], and we have to be careful not to use separation in M[A,T] with unbounded quantifiers. Of course, we have access to POS internally.

We now normalize the ultrafilter POS in M[A,T].

Definition 7.17. f is internally adequate if and only if $f:\theta \rightarrow \theta$, f is internal, and for all $x \in \theta$, $\neg POS(f^{-1}(x))$. $f <\# g$ if and only if f,g are adequate and $POS(\{x: f(x) <^* g(x)\})$. We say that f is <# minimal if and only if f is internally adequate and there is no $g <\# f$.

Lemma 7.20. There exists f such that f is <# minimal and DEF(f).

Proof: Suppose there is no <# minimal f. Externally, we build internally adequate $f_0,f_1,...$ as follows. f_0 is the identity on θ. f_{n+1} is the M[A,T] least $f_{n+1} <\# f_n$. Let $F(n,x) = f_n(x)$, $x \in \theta$. We are not claiming that F is internal, but clearly DEF(F). For all $x \in X$, let h(x) be the least n such that $f_n(x) \in ... \in f_0(x)$ is false. $h:X \rightarrow <^*[\omega]$ by well foundedness. Clearly DEF(h). Since $\omega <^* \gamma$, by Lemma 6.10, some $h^{-1}(n)$ is positive. I.e., fix n such that $\{y \in \theta: f_n(x) \in ... \in f_0(x)$ is false$\}$ is positive. We claim that $\{y \in \theta: f_n(y) \in ... \in f_0(y)\}$ is positive. This is proved by straightforward induction up through n. This contradicts the choice of n. We have a contradiction. So there is a <# minimal f. Choose f to be the M[A,T] least <* minimal f. Then DEF(f). QED

Definition 7.18. Fix H to be the # minimal f constructed in Lemma 7.20. Define POS#(E) if and only if $E \subseteq \theta$ is internal, and $POS(H^{-1}[E])$.

Lemma 7.21. POS# is a nontrivial θ complete ultrafilter in the following sense: i. If $x \in \theta$ then ¬POS#({x}), ii. If $E \subseteq \theta$ is internal, then POS#(E) ↔ ¬POS#(θ\E), and iii. Let $f:\theta \to x$, $x \in \theta$, be partially defined and internal, where POS#(dom(f)). There exists $y \in x$ such that POS#($f^{-1}(y)$).

Proof: i follows from the internal adequacy of H. ii follows from the same statement for POS. Suppose iii is false, and let f be the M[A,T] least counterexample. Let x be least such that $f:\theta \to x$. Then DEF(f,{x}). Define $G:\theta \to x$ by $G(y) = f(H(y))$, $y \in \theta$. Then $H^{-1}[f^{-1}(y)] = G^{-1}(y)$. Clearly DEF(G). By Lemma 6.10, there exists $y \in x$ such that POS($G^{-1}(y)$), POS($H^{-1}[f^{-1}(y)]$), POS#($f^{-1}(y)$). QED

Lemma 7.22. POS# is normal in the following sense. Let $f:\theta \to \theta$ be internal and partially defined, where POS#({x: $f(x) \in x$}). There exists $c < \theta$ such that POS#($f^{-1}(c)$).

Proof: Suppose this is false, and let f be the M[A,T] least counterexample. Note that POS#(dom(f)). Extend f to $f':\theta \to \theta$ by setting f′ to be 0 on θ\dom(f). There is no $c \in \theta$ such that POS#($f'^{-1}(c)$). Hence there is no $c \in \theta$ such that POS#($f'^{-1}(c)$), since $f'^{-1}(c) = f^{-1}(c) \cup (\theta\backslash dom(f))$. Let $G(x) = f'(H(x))$, $x <^* y$. Then DEF(f′,{x},G), and G is internal. Case 1. G is not internally adequate. Let POS($G^{-1}(c)$). Now $G^{-1}(c) = \{y: f'(H(y)) = c\} = \{y: H(y) \in f'^{-1}(c)\} = H^{-1}(f'^{-1}(c))$. Hence POS($H^{-1}(f'^{-1}(c))$), and so POS#($f^{-1}(c)$). Case 2. G is internally adequate. Since POS#({x: $f(x) <^* x$}), we have POS({x: $f(H(x)) <^* H(x)$}). Hence $G <\# H$. But this contradicts that H is <# minimal. Since both cases lead to contradictions, the Lemma has been proved. QED

Lemma 7.23. POS# diagonal intersection. Let $R \subseteq \theta^2$ be internal, where for all $x \in \theta$, POS#(R_x). Then POS#({$x \in \theta$: $(\forall y \in x)(x \in R_y)$}).

Proof: Let R be as given. Suppose ¬POS#({$x \in \theta$: $(\forall y \in x)(x \in R_y)$}). Then POS#({$x \in \theta$: $(\exists y \in x)(x \notin R_y)$}). Let $f:\theta \to \theta$ be the partial function given by $f(x) = (\mu y \in x)(x \notin R_y)$. Then f is internal,

POS#({x: f(x) ∈ x}). Let c ∈ θ be such that POS#(f^{-1}(c)). Now f^{-1}(c) ⊆ θ\$R_c$. Hence POS#(θ\$R_c$)), contradicting that POS#(R_c). QED

Definition 7.19. We make the following definitions in set theory. $[E]^k$ is the set of all k element subsets of E. $[E]^{<\omega}$ is the set of all nonempty finite subsets of E. κ is a Ramsey cardinal if and only if κ is a cardinal such that the following holds. Let $f:[\kappa]^{<\omega} \to \alpha$, α < κ. There exists unbounded E ⊆ κ such that for all n ≥ 1, f is constant on $[E]^n$. The strong Ramsey cardinals are defined the same way with "unbounded" replaced by "stationary." I.e., "meets every closed unbounded subset of κ."

Lemma 7.24. Let $f:[E]^{k+1} \to x$ be internal, POS#(E), x ∈ θ. There exists internal E'⊆ E with POS#(E') such that the following holds. For each $x \in [E]^k$, f is constant on the x ∪ {y}, max(x) ∈ y ∈ E'.

Proof: Let f be as given. We define internal sets $C_x \subseteq (x,\theta)$, for x ∈ θ. Fix x ∈ θ. For each $y \in [E]^k$, there exists unique u ∈ x such that POS#({z: f(y ∪ {z}) = u}), by Lemma 7.21. Let B_y = {z: max(y) ∈ z ∧ f(y ∪ {z}) = u}. We have POS#(B_y). By Lemma 7.23, POS#(C_x), where C_x = {z ∈ θ: (∀y ∈ z)(z ∈ B_y)}. By Lemma 7.23, POS#(E'), where E'= {y ∈ E: (∀x ∈ y)(y ∈ C_x)}). Let $z \in [E]^k$, max(z) = x. Then f is constant on the z ∪ {y}, where x ∈ y ∈ E'. QED

We need the following refinement of Lemma 7.24.

Lemma 7.25. In Lemma 7.24, if the M[A,T] level of f is at most the limit point u ∈ A, θ ∈ u, then E'can be taken to be of M[A,T] level < u.

Proof: This is evident, as the constructions made in the proof of Lemma 7.24 are very explicit. QED

Lemma 7.26. Let k be such that the following holds. For all limit points u ∈ A, x ∈ θ ∈ u, every $f:[E]^k \to x$ of M[A,T] level < u, POS#(E), is constant on some E'⊆ E, POS#(E'), of M[A,T] level < u. Then this holds with k replaced by k+1.

Proof: Let k be as given. Let $u \in A$ be a limit point, $x \in \theta < u$, $f:[E]^{k+1} \to x$ of M[A,T] level < u, POS#(E). Let E′be as given by Lemma 7.24, E′of M[A,T] level < u. Let $g:[E']^k \to x$ be given by $g(y) = f(y \cup \{z\})$, for $\max(y) \in z \in E'$. Then g is of M[A,T] level < u. By hypothesis, let $E'' \subseteq E'$, POS#(E″), g constant on $[E'']^k$, where E″is of M[A,T] level < u. Note that f is constant on $[E'']^{k+1}$. QED

Lemma 7.27. Let k be a positive integer. Let $u \in A$ be a limit point, $x \in \theta \in u$, $f:[E]^k \to x$ be internal of level < u, POS#(E). There exists $E' \subseteq E$, POS#(E′), where f is constant on $[E']^k$, and of M[A,T] level < u.

Proof: By external induction on k. The basis case k = 1 is obvious using Lemma 7.21. The induction step is from Lemma 7.26. We use external induction since the induction hypothesis has unbounded quantification over $u \in A$. QED

Lemma 7.28. Let $f:[E]^{<\omega} \to x$ be internal, $x \in \theta$, $E \subseteq \theta$, POS#(E). There exists internal $E' \subseteq E$, POS#(E′), where f is constant on every $[E']^k$.

Proof: Let f,E,x be as given, where E is of M[A,T] level < u, $\theta \in u$, u a limit point. For all $k \geq 1$, let E_n be the M[A,T] least subset of E such that f is constant on $[E]^k$ and POS#(E_n). By Lemma 7.27, the E_n are of M[A,T] level < u, and so the sequence $E_1,E_2,...$ is M[A,T] internal. By Lemma 7.21, we see that $\cap_k E_k$ is internal, and POS#($\cap_k E_k$). QED

Lemma 7.29. Let $E \subseteq \theta$ be internal and internally closed and unbounded in θ. Then POS#(E). If $E \subseteq \theta \wedge$ POS#(E), then E is internally stationary in θ.

Proof: Let E be as given. Let $f:\theta \backslash E \to \theta$ be given by f(x) = the greatest $y \in x$ that lies in D if $0 \in x \in \theta$; 0 otherwise. By Lemma 7.21, if POS#(θ\E), some $f^{-1}(c)$ is unbounded in θ. Hence POS#(E). The second claim follows immediately. QED

Lemma 7.30. Let $f:\theta \to V(\theta)$ be internal, where for all $x \in \theta$, $f(x) \subseteq V(x)$. There exists internal $E \subseteq \theta$, POS#(E), such that for all

$x \in y \in E$, $f(x) = f(y) \cap V(x)$.

Proof: Let f be as given. For each $x \in V(\theta)$, let $g(x) = 1$ if POS#($\{y \in \theta: x \in f(y)\}$); 0 otherwise, and let $h(x) = \{y \in \theta: x \in f(y)\}$ if $g(x) = 1$; $\{y \in \theta: x \notin f(y)\}$ otherwise. For each $y \in \theta$, let $B_y = \cap_{x \in V(y)} h(x)$. By Lemma 7.21, for all $y \in \theta$, POS#(By). By Lemma 7.23, POS#($\{y \in \theta: y \in B_y\}$). If $y \in B_y \wedge z \in B_z \wedge y \in z$, then f(y),f(z) have the same elements from V(y). QED

Lemma 7.31. Let $E \subseteq \theta$ be internal, POS#(E). POS#($\{x \in \theta: E \cap x$ is internally stationary in x$\}$).

Proof: Note that the set in question involves only quantifiers ranging over the $V(\theta)$ of M[A,T]. Let E be as given. Suppose POS#($\{x \in \theta: E \cap x$ is not internally stationary in x$\}$). Then POS#(B), where $B = \{x \in \theta: x$ is an internal limit ordinal $\wedge$ $E \cap x$ is not internally stationary in x$\}$. (The set of limit ordinals $\in \theta$, both in the internal and external sense, must have POS# by Lemma 7.22). For each internal limit ordinal $x \in B$, let $C_x \subseteq x$ be the M[A,T] least internally closed and unbounded set in x that is disjoint from E. By Lemma 7.30, let $B' \subseteq B$ be internal, POS#(B′), where for all $x \in y$ from B′, $C_x = C_y \cap V(x)$. Then the union of the C_x, $x \in E'$, is internally closed unbounded in θ, and disjoint from E. This contradicts that POS#(E), by Lemma 7.29. QED

Lemma 7.32. POS#($\{x \in \theta: x$ is an internal strong Ramsey cardinal$\}$).

Proof: Suppose $B = \{x \in \theta: x$ is not an internal strong Ramsey cardinal$\}$ has POS#(E). Let $C = \{x \in \theta: x$ is an internal cardinal $\wedge$ x is not an internal strong Ramsey cardinal$\}$. Then POS#(C). For each $x \in C$, let f_x be M[A,T] least such that $f:[x]^{<\omega} \rightarrow x$, $\sup(\text{rng}(f)) \in x$, and for no $E \subseteq x$, E internally stationary in x, is it the case that for all k, f is constant on $[E]^k$. Then each $f_x \subseteq V(x)$, and so we can apply Lemma 7.30. We obtain internal $E' \subseteq \theta$, POS#(E′), such that the f_x, $x \in E'$, are comparable under $\subseteq$. Let $F:[\theta]^{<\omega} \rightarrow \theta$ be the union of the f_s, $x \in E'$. Then $\sup(\text{rng}(F)) = y \in \theta$. By Lemma 7.28,

let D ⊆ θ, POS#(D), where F is constant on every $[D]^k$. By Lemma 7.31, POS#({x ∈ θ: D ∩ x is internally stationary in x}). Let x ∈ θ, x an internal cardinal, D ∩ x is internally stationary in x. Clearly f_x is constant on every $[D \cap x]^k$. This contradicts the choice of f_x. QED

Lemma 7.33. The V(θ) of M[A,T] under E(A,T,POS) satisfies ZFC + "there are arbitrarily large strong Ramsey cardinals."

Proof: By Lemma 7.19, the V(θ) of M[A,T] under E(A,T,POS) satisfies ZFC. Now apply Lemma 7.32. QED

Theorem 7.34. ZFC + "there are arbitrarily large strong Ramsey cardinals" is interpretable in T_5. T_5 proves the consistency of ZFC + "there are arbitrarily large strong Ramsey cardinals," when formulated with standard formalized syntax. EFA proves Con(T_5) → Con(ZFC "there are arbitrarily large strong Ramsey cardinals"). T_5 is interpretable in ZFM. ZFM proves the consistency of T_5. EFA proves Con(ZFM) → Con(T_5).

Proof: By Theorem 5.1 and Lemma 7.33. Since the theories involved are not finitely axiomatizable, the EFA results do not immediately come from the existence of the interpretations. The simplest route is to verify that EFA is sufficient to prove that the interpretations work. QED

31.8 Without Extensionality

In Gödel (1995) positivity is used as an attribute on properties of objects, not on classes of objects. As discussed in Section 31.3, Gödel assumed that positivity behaves as an ultrafilter, and this leads to an easy derivation that positivity depends only on the extension of properties. I.e., if two properties hold of the same objects, then one is positive if and only if the other is positive. Adams (in →Gödel, 1995) also discusses the Leibniz view of positivity, in some detail, citing parts of Leibniz (1923, 1956, 1969).

This would suggest that we might be able to simply drop Extensionality from T_5, and be able to retain Theorem 7.34, or at least Theorem 5.1. However, we doubt this, as a difficulty arises in connection with Choice Operator.

We now present the system T_5(prop), where "prop" abbreviates "properties." For ease of comparison with T_5, we take the language L_5(prop) of T_5[prop] to be the same as the language L_5 of T_5, except we remove equality between class variables. Here $x \in A$ read "the property A holds of the object x." Extensionality is dropped, and the axioms that are modified are asterisked. The only substantial modification is to Choice Operator, where a second line is added.

We show that T_s(prop) proves the strongest form of extensionality that can be formulated without equality between classes. From this result, it is easy to conclude that Theorem 7.34 holds for T_5(prop).

Nonlogical Axioms for T5(prop):

Pairing:	$P(v_1,v_2) = P(v_3,v_4) \rightarrow v_1 = v_3 \wedge v_2 = v_4$.
L_5(Prop) COMPREHENSION:	$(\exists A_1)(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi)$, where φ is a formula of L_5(prop) in which A_1 is not free.
Augmented Choice Operator:	$v_1 \in A_1 \rightarrow CHO(A_1) \in A_1$.
	$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \rightarrow CHO(A_1) = CHO(A_2)$.
Positive Classes:	$(\forall v_1)(v_1 \in A_1 \vee v_1 \in A_2) \rightarrow POS(A_1) \vee POS(A_2)$.
	$POS(A_1) \wedge POS(A_2) \rightarrow (\exists v_1 \neq v_2)(v_1,v_2 \in A_1 \wedge v_1,v_2 \in A_2)$.
0-Definable Classes (prop):	$(\forall v_1)(v_1 \in A_1 \leftrightarrow \varphi) \wedge DEF(A_2) \wedge ... \wedge DEF(A_n) \rightarrow DEF(A_1)$, where φ is a formula of L_5(prop) without DEF, with at most the free variables $v_1,A_2,...,A_n$, $n \geq 1$.
Divine Object:	$(\exists v_1)(\forall A_1)(DEF(A_1) \wedge POS(A_1) \rightarrow v_1 \in A_1)$.

Here is the strongest form of extensionality in the language of L_5(prop), where we have dropped equality between classes.

ExtensionItiy without Class Equality:	$(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \rightarrow (\varphi \leftrightarrow \Psi)$, where φ,Ψ are formulas of L_5(prop), and Ψ is the result of replacing zero or more free occurrences of A_1 by A_2.

Theorem 8.1. T_5(prop) proves Extensionality Without Class Equality.

Proof: By standard predicate calculus manipulations, it suffices to prove this for atomic formulas φ,Ψ. Working in T_5(prop), assume $(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2)$. Case 1. s = t, where s,t are object terms. The relevant substitutions are of the form s′= t′,

where s′,t′ are obtained from s,t by replacing some A_1's by A_2, respectively. Now the A_1's appear only as part of subterms CHO(A_1), and Augmented Choice Operator proves CHO(A_1) = CHO(A_2). Therefore we obtain s = s′, t = t′ by the axioms of equality (for the equality that we do have, namely between objects). Hence we have s = t ↔ s′ = t′. Case 2. t ∈ A_i, where t is an object term. The relevant substitutions are of the form t′ ∈ A_j, where the reasoning in case 1 yields t = t′. Now if i = j then we are done. Otherwise, i = 1 and j = 2. But t ∈ A_1 ↔ t′ ∈ A_2 is provable, using t = t′. Case 3. DEF(A_i). We have only to prove DEF(A_1) ↔ DEF(A_2). By 0-Definable Classes (prop), we have the universal closure of $(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \wedge \text{DEF}(A_2) \rightarrow \text{DEF}(A_1)$ and therefore $(\forall v_1)(v_1 \in A_2 \leftrightarrow v_1 \in A_1) \wedge \text{DEF}(A_1) \rightarrow \text{DEF}(A_2)$. Hence we have DEF($A_1$) ↔ DEF($A_2$). Case 4. POS($A_i$). We have only to prove POS(A_1) ↔ POS(A_2). Assume POS(A_1). By L_5(prop) Comprehension, let A_3 be such that $(\forall v_1)(v_1 \in A_3 \leftrightarrow v_1 \notin A_1)$. By $(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2)$, we have $(\forall v_1)(v_1 \in A_2 \vee v_1 \in A_3)$. By Positivity, $\text{POS}(A_2) \vee \text{POS}(A_3)$. We claim that ¬POS($A_3$). Assume POS($A_3$). By Positivity, $(\exists v_1 \neq v_2)(v_1, v_2 \in A_1 \wedge v_1, v_2 \in A_3)$. Let v_1 be such that $v_1 \in A_1$, $v_1 \in A_3$. Then $v_1 \notin A_1$. Contradiction. By $\text{POS}(A_2) \vee \text{POS}(A_3)$, ¬POS($A_3$), we have POS($A_2$). We have established the universal closure of $(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2) \rightarrow (\text{POS}(A_1) \rightarrow \text{POS}(A_2))$ and therefore $(\forall v_1)(v_1 \in A_2 \leftrightarrow v_1 \in A_1) \rightarrow (\text{POS}(A_2) \rightarrow \text{POS}(A_1))$. Hence we have established POS(A_1) ↔ POS(A_2). QED

Theorem 8.2. T_5(prop) and T_5 are mutually interpretable. Theorem 7.34 holds for T_5(prop).

Proof: As in the proof of Theorem 7.34, we want to use that EFA proves that a particular interpretation of T_5 in T_5(prop) works. This will be clear below. Note that Theorem 8.1 is

provable in EFA. This will be enough to establish Theorem 7.34 for T_5(prop) based on Theorem 7.34 for T_5.

In an interpretation that does not treat equality as equality, we have to be particularly careful. Here we do interpret equality between objects as equality between objects, but we obviously don't interpret equality between classes as equality between classes, since we don't have equality between classes in the language. The simplest approach to interpretations is to use the associated theories with only relation symbols and no equality. Here all equality relations on sorts are viewed simply as binary relation symbols, all constant symbols are replaced by unary predicates, and all function symbols are replaced by relation symbols of one higher arity. In the associated theory, we include not only the modified form of the nonlogical axioms, but also the axioms for equality, now formulated in the modified language, and now viewed as nonlogical axioms. Thus an interpretation of T in T'is an interpretation of modified T in modified T'. We take this approach now in our interpretation of T_5 in T_5(prop). The axioms for equality can always be formulated in terms of equivalences between atomic formulas. With these caveats in mind, we now give the interpretation of T_5 in T_5(prop). We interpret the objects of T_5 as the objects of T_5(prop), the classes of T_5 as the classes of T_5(prop), = between objects of T_5 as = between objects of T_5(prop), the ternary relation P of T_5 as the ternary relation P of T_5(prop), $\in$ of T_5 as $\in$ of T_5(prop), DEF of T_5 as DEF of T_5(prop), POS of T_5 as POS of T_5(prop). We interpret $A_1 = A_2$ in T_5 as $(\forall v_1)(v_1 \in A_1 \leftrightarrow v_1 \in A_2)$ in T_5(prop). We interpret the binary relation CHO of T_5 as the binary relation CHO of T_5(prop).

We now have to show that the interpretation of all of the axioms of modified T_5, including all of the equality axioms in its language, are provable in modified T_5(prop). 1. Equality axioms.

The proof of Theorem 8.1 establishes this. 2. Nonlogical axioms. We have only to consider the nonlogical axioms of modified T_5 where = between classes occurs. The only such axiom is modified L_5 Comprehension. But its interpretation is clearly follows from modified L_5(prop) Comprehension. QED

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32 Naming God's Essence: Ineffability, Analogy and Set Theory

Claudio Ternullo

Abstract: The attribution of the same properties to God and the created beings is, in many respects, problematic for (Judeo-)Christian theology. As early as the fifth century AD, Pseudo-Dionysius maintained that no *positive* characterization of the divine essence was possible. However, later, in the thirteenth century, a more articulated (and optimistic) conception of the *analogy* between God and the created beings came to the fore, mainly thanks to Aquinas' work. In this paper, I will focus on two fundamental research strands in contemporary set theory (reflection principles and the set-theoretic multiverse) which provide us with conceptual tools able to express and understand these two different conceptions of analogy. I will, finally, make the case that, if Aquinas' conception is to be preferred, then the 'multiversist' option is more suitable.

32.1 Introduction

Christian theology (like Jewish theology beforehand), had to deal very soon with the issue of attributing to God *properties* which could also be attributed to God's created beings. A distinguished doctrine originating in the context of Neo-platonic philosophy, and appropriately modified to suit Christian dogma, denied that God could be attributed the same properties as those of His creatures (for that matter, *any* property). However, God could be characterized by what He was not, that is, by the properties that could, certainly, not be attributed to Him.

The history and development of this doctrine, called ‘negative way’ or ‘negative (*apophatic*) theology’ spans several centuries of Christianity, and, arguably, has its earliest, fundamental manifestation in Dionysius the Areopagite’s complex of works, globally known as *Corpus Areopagiticum*. The Dionysian *Corpus*’ impact, and enduring effects, on the subsequent Christian theology, philosophy, and mysticism can hardly be overestimated.[1]

However, a different doctrine, essentially due to St. Thomas Aquinas, gradually emerged and made its way to Christian thought in the thirteenth century. This is known as ‘doctrine of analogy.’ According to this doctrine, properties attributed to God and His creatures are predicated neither *univocally* nor *equivocally*, but *analogically*, which means that God’s properties are, in part, *synonymous* with, and in part, *non-synonymous* with, those predicated about created beings. Being able to spell out the exact level of synonymity of two analogous properties is one of the fundamental theoretical issues concerning the doctrine of analogy, since, by its own nature, analogy entails both some degree of *identity* and some degree of *difference* between analogous terms.

The doctrine of analogy is both *metaphysical* and *logical*. In fact, its roots lie more in logical and linguistic disputes than in metaphysical concerns. However, earlier speculations about analogy, for instance in the very context of the Areopagite’s theology, are significantly more metaphysical in character.

Overall, at the theological level, the two ways we have mentioned would seem to lead to opposite outcomes: the first one would imply that our thought and language are not able to faithfully describe God’s essence, whereas the second one would commit itself to the view that God’s essence can be characterized *positively*, but the full picture, as we shall see, is a lot more articulated.

The purpose of this paper is to bring the methods of contemporary set theory to bear on the interpretation of, and mutual comparison between, these two conceptions. The paper's methodology will consist, that is, in using concepts of set theory to deal with those of theology. It seems rather natural to conjecture that there exist *conceptual* similarities between set-theoretic and theological discourse, similarities also reflected by the underlying logic, but a more striking consequence of this kind of investigation could also be that set theory may provide theological modes of reasoning with more transparent conceptual tools. However, the broader issue of why, and in what sense, set theory and theology may be able to shed light on each other's subject matters and methods clearly lies beyond the scope of this paper.[2]

32.2 The Problem

32.2.1 Absolute Transcendence: Negative Theology

Negative theology holds that a characterization of God's essence transcends the resources of human language and thought. This view may be found in the work of both Jewish and Christian thinkers. A forceful manifestation of it is in the work of the character known, in the Middle Ages, as Dionysius the Areopagite, allegedly, a member of the Athenian Areopagus converted by the apostle Paul in the first century AD, in fact, as proved by his language and doctrines, a posterior Christian Neo-platonic, most probably active in the early decades of the fifth century AD, whose conceptions are strongly indebted to Proclus' thought.[3]

The work of Pseudo-Dionysius, as is also commonly known, that is, the *Corpus Areopagiticum*, represented the main source of

negative theology during early Christianity, but also exerted much influence on later Christian theologians.

For my purposes, I shall fundamentally focus on three main features of the conception expounded by Pseudo-Dionysius, as can be found in two of his works, *De divinis nominibus* and the *Theologia mystica*. The first one is:

> *Ineffability*. God's essence (properties) cannot be named, that is, they lie outside the realm of what is expressible with human language.

Ineffability has its roots in the Neo-platonic speculations on the nature of the One. By the Proclian (and beforehand, Plotinian) conception of the One, attempting to 'name,' describe (the properties of) the One is vain, as one is not really able to refer to the 'real' One in one's linguistic utterances. What is, then, left to do, with respect to the One, is to contemplate it as a 'nothing(ness),' the only intellectually accessible characterization of the First Principle.

It is fundamental to note that Ineffability may come in different degrees (and forms): one could say that the One (for the sake of our discourse, God) is, globally, ineffable, that His essence is ineffable, that His properties, are ineffable, or, possibly, all of these things. Now, when one utters a sentence referring to the 'One' or to a 'property of the One,' presumably one is, indeed, referring to something else, if language, as most Medieval philosophers and theologians thought, always *refers*. How should we construe, then, the *reference* of such linguistic expressions?

One option considered by (Christian) theologians is to think of such sentences as referring to *entities* which are *lesser* than God himself. In other terms, in order to prevent theological discourse from being vacuous, it is surmised that, for any property *P* predicated about God, there is some entity *x, lesser*

than God, which has that property, so it is *x*, ultimately, the actual referent of those sentences.

This conceptual strategy, was, indeed, mentioned by the Areopagite, and afterwards, albeit unsystematically, also by other theologians.[4] I will call this doctrine:

> *Reflection*. Properties predicated about God are, in fact, predicated about God's created beings.

The third, and last, feature of negative theology I shall be concerned with may be explained as follows. In his *De divinis nominibus*, the Areopagite hints at a different way in which a more positive characterization of God is, in principle, possible, although, as explained in the *Theologia mystica*, such a characterization is, by its own nature, always insufficient and, to some extent, even deceptive. But it is something, nonetheless. The Areopagite acknowledges that there must be some degree of (imperfect) analogy between say 'goodness,' as (1) referred to God and (2) as referred to created beings. Now, his contention is that (1) should be more carefully and adequately referred to as '*super*-goodness.' In general, for any property *P* referring to the created beings, if *P* may also be referred to God, then *P* should be taken to correspond to a 'super-*P*.' This leads us to identify the last fundamental doctrine of negative theology:

> *Super-Essentiality*. Properties attributed to God, which also happen to be attributed to created beings, are, in fact, 'super'-properties.

As we shall see (Section 32.4.2), Super-Essentiality plays a crucial role for the purpose of 'naming' God's essence through logical (set-theoretic) means.

32.2.2 Aquinas' Conception of Analogy

Using a logico-linguistic classification partly introduced by Aristotle, scholastic logicians referred to ‘synonymity’ or ‘non-synonymity’ of two terms as *univocity* (when the terms under consideration meant the same concept) or *equivocity* (when they didn’t).[5]

Now, the problem arose for them of whether predication about God and His creatures was univocal or equivocal. Simply put, the question was: when I say that “God is *X*” and also that “Socrates is *X*,” am I attributing one and the same *property*, ‘*X*-ness,’ both to God and Socrates? Now, if predication about God and His creatures is taken to be *univocal*, then God and Socrates share the *same X*-ness; if, on the contrary, it is *equivocal*, two different kinds of *X*-ness must exist, God’s and Socrates’. Both cases are problematic: the former, insofar as God’s essence is *radically different* from that of His creatures, and the latter, insofar as, at least to some extent, presumably Socrates’ *X*-ness *shares* some features of God’s *X*-ness.

Against this background, negative theology may be seen as holding that predication about God and creatures is uncontroversially equivocal; however, as said in the previous section, even in the *Corpus Areopagiticum*, one finds traces of a different conception, acknowledging some sort of similarity between expressions referring to God and created beings.

A ‘compromise’ solution, based on what we have already referred to as doctrine of analogy, emerged in the thirteenth century, and was, in particular, advocated by Aquinas.[6] According to this doctrine, God’s *X*-ness (X_G), is *different* from, but *analogous* to, Socrates’ *X*-ness (X_S), which means, in logical terms, that the predicates X_G and X_S, albeit non-equivalent, should be taken to have some mutual, specifiable, *relationship*.

In an intensely discussed passage of his *De veritate* which introduces the notion of analogy, Aquinas says that there are

two main ways to construe the notion: (1) as *proportion* in the most ordinary mathematical sense (e.g., 4 is *analogous to* 2, insofar as 4 is a *multiple* of 2), and (2) as *proportionality* (4 is *analogous to* 6, insofar as, for instance, 4:2 = 6:3).[7] It is worth reproducing the passage in full:

> But conformity (*convenientia*) according to a proportion can be twofold, and so a twofold community of analogy is to be taken account of. For there is a conformity between the things themselves which are proportioned to one another in having a determinate distance of some other relationship (*habitudinem*) to one another, e.g. (the number) 2 to unity, 2 being the double. But we also sometimes take account of conformity between two things which are not mutually proportioned, but rather there is a likeness between two proportions; e.g. 6 is conformed to 4 because as 6 is twice 3 so 4 is twice 2. (in → Bocheński, 1961, p. 178)

How should the passage above be interpreted? As pointed out by scholars, Aquinas does not provide us with further details.[8] In any case, what can be gleaned from Aquinas' relevant textual sources, and what scholars seem to agree upon is that, in Aquinas' view, the analogy between God and creatures would, at least, entail the following facts:

1. God is superior to any created being, so predication about Him is still *radically different* from predication about creatures (hence, there is a sense in which also Aquinas' conception subscribes to Ineffability);
2. Precisely because of such radical distance between God and His creatures, simple proportion (1) is not adequate to express the *similarity* between God and the created beings, as well as that between the corresponding forms of predication;
3. As a consequence, Aquinas resorts to a more complex form of proportion, that he calls *proportionality* (2), to express the idea of analogy, whereby there are two levels

of reality, God and creatures, and *similar* relationships between God and His properties, on the one hand, and creatures and their properties, on the other, hold;

4. Metaphysically, Aquinas' conception of analogy also implies some *structural similarity* between the essence of God and the essence of the created beings.

Overall, Aquinas' conception marks a significant departure from negative theology's (*viz*., the Areopagite's) conception. A quick comparison shows, in particular, that, while, as said, (1) God's Ineffability is still held by Aquinas, the *analogical* mode of predication he subscribes to allows one to concretely, non-vacuously refer to God's properties; (2) Reflection should be construed in terms of 'participation' (a central feature of Aquinas' metaphysics), of the created beings in God's essence; (3) Super-Essentiality does not seem to be required by Aquinas' conception of analogy, although, as is clear, by Aquinas' view, there is still a thick sense in which God's properties are 'super'-properties, as compared to those of the created beings.

A distinguished interpretation of Aquinas' doctrine of analogy, crucial to our purposes, has been formulated by Józef Bocheński, and we now proceed to examine it.

32.3 The Isomorphism Hypothesis

Let us go back, for a moment, to Aquinas' passage in *De veritat*e I have cited in the previous section. Bocheński (1961) comments (and elaborates on) the passage as follows:[9]

> The ruling idea is then of a relation of similarity between two relations. This relation between relations is such as to allow inference from what we know about one to something about the other, though at the same time we have the assertion: 'we cannot know what God is'. The apparent contradiction disappears when it is realized that we are dealing with

> *isomorphy*. For this does in fact allow one to transfer something from one relation to another, without affording any experience of the *relata*. (→Bocheński, 1961, p. 179; italics are mine)

In the passage above, Bocheński makes it clear that, as said in the previous section, positing an analogy between God and the created beings does not contradict God's Ineffability. In order to explain how this is concretely possible, Bocheński takes Aquinas's text (and conception) to be (informally) delineating an *isomorphism* between God and the created beings, so that any *relationship* holding between two aspects of God's essence is *isomorphically* reflected by a corresponding *relationship* holding between two aspects of the creaturely essence. I shall call Bocheński's interpretative hypothesis the 'isomorphism hypothesis.'

A simple, down-to-earth way of looking at the concept of isomorphism used by Bocheński is through the idea of 'substitution': that is, two mathematical structures are isomorphic if and only if the elements of one may be substituted with the elements of the other without affecting the structural relations holding among the elements of each structure. Now, it is precisely an essential aspect of an *isomorphism*, and one on which Bocheński lays a lot of emphasis, that one does not need to know what the elements of two isomorphic structures are like, to understand the isomorphism itself, and this is the basis for Bocheński's claim that, by his own interpretation of Aquinas' conception, one may know what relationships hold in the Godhead without really knowing what God is like.

It is fundamental, for our purposes, to inspect more closely the 'isomorphism hypothesis,' that is, to recall some technical features of isomorphisms. This can be done by using, in particular, *model-theoretic* concepts and definitions.

Given a language $\mathscr{L}$, and two models $\mathfrak{U}$ and $\mathfrak{B}$ of that language, an *isomorphism* between $\mathfrak{U}$ and $\mathfrak{B}$ is a 1–1 and onto map f: $A \rightarrow B$ (where A and B are the domains of, respectively, $\mathfrak{U}$ and $\mathfrak{B}$), such that for any function F and $a_1, \ldots, a_n \in A$:

$f(F^{\mathfrak{U}}(a_1, \ldots, a_n)) = F^{\mathfrak{B}}(f(a_1), \ldots, f(a_n))$

for any (n-place, with n, possibly, greater than 1) property P and $a_1, \ldots, a_n \in A$:

$P^{\mathfrak{U}}(a_1, \ldots, a_n)$ iff $P^{\mathfrak{B}}(f(a_1), \ldots, f(a_n))$

and for any constant c:

$f(c^{\mathfrak{U}}) = c^{\mathfrak{B}}$.

Moreover, an isomorphism between $\mathfrak{U}$ and $\mathfrak{B}$ also implies the *elementary equivalence* of $\mathfrak{U}$ and $\mathfrak{B}$, that is, the fact that, for any *sentence* (closed formula) φ of $\mathscr{L}$, φ is true in $\mathfrak{U}$ if and only if φ is true in $\mathfrak{B}$.

Now, on the grounds of the technical features of isomorphisms reviewed above, two main difficulties arise in connection with Bocheński's hypothesis.

One is that, if the analogy between God and the creatures really is to be interpreted in terms of a mathematical isomorphism, this will imply that 'God's structure' and the 'creaturely structure' will have the same *size* (set-theoretically, their domains will have the same *cardinality*), and this seems to violate our intuition, discussed in the previous sections, that God is *superior* to the created beings.

A second objection to the hypothesis may be formulated as follows. One fundamental feature of isomorphisms is that these postulate that two structures $\mathfrak{U}$ and $\mathfrak{B}$ are not *distinguishable* by any *property* expressible in $\mathscr{L}$ (as well as by any sentence of $\mathscr{L}$, since $\mathfrak{U}$ and $\mathfrak{B}$, as we have seen, must also be *elementarily equivalent*). Now, this does not seem to suit the present case where the structures involved are God and the created beings. After all, there must be properties of God such that the created

beings will not have the corresponding property (e.g., *omnipotence*), whilst, on the other hand, there must be properties of the created beings such that God will not have the corresponding property (e.g., *spatio-temporality*), and the same will apply to *sentences* expressing that those properties hold.

How could one respond to this second objection? A quick response would consist in asserting that the isomorphism under consideration, say, *f*, only applies to properties that one is already sure can be predicated about God and the created beings, so, in particular, one is not entitled to deducing further properties of God from the corresponding properties of the created beings, and *vice versa*, on the grounds of *f*. This quick solution, however, seems to be too drastic, insofar as it, ultimately, makes the postulation of an isomorphism between God and the created beings, on the whole, useless.

As far as the first objection is concerned, one possible fix would consist in positing the existence of a different, although very kindred, kind of mathematical *resemblance* between God and the created beings not implying that the 'God-structure' has the same size as the 'creaturely structure.'

In the next section, in the wake of Bocheński's work on the 'isomorphism hypothesis,' I shall not only show how one can produce accurate set-theoretic representations both of negative theology and Aquinas' conception, but also how these provide a refinement of the 'isomorphism hypothesis,' by responding to the aforementioned objections.

32.4 Enter Set Theory

The set-theoretic principles (and methodologies) I will introduce and discuss in this section are, on the one hand, the reflection principles (Sections 32.4.2 and 4.3), and, on the other, a specific conception of the set-theoretic multiverse, the *V*-logic multiverse

(Section 32.4.4). But before (Section 32.4.1), I will review some preliminary material concerning the rise of the idea of reflection in set theory (and more general connections between set theory and theology).

32.4.1 The Very Idea of Reflection

Fundamental theoretical connections between set theory and theology were drawn by Georg Cantor who, on the one hand, (1) identified his own Absolute Infinite with the divine infinite and, on the other, (2) suggested that the Absolute Infinite was, to some extent, unknowable and outside the scope of mathematical knowledge.[10]

Already Cantor (1883) explains that, alongside the *Transfinitum* (*transfinite*), that is, the infinite *ordinals* and *cardinals*, there exists one further manifestation of the actual infinite, the Absolute Infinite (or just the Absolute). Cantor explicitly says that the latter may be taken to be the 'infinity of God', as such, an infinity beyond human comprehension.[11] He, very evocatively, expresses this fact by appealing to Albrecht von Haller's words:

> I attain to the enormous number, but you, O eternity, lie always ahead of me.[12] (→Cantor, 1883, in →Ewald, 1996, p. 896)

In →Cantor (1885) and in several letters Cantor exchanged with theologians (some of which were subsequently collected in →Cantor (1887)), the Absolute Infinite is, again, taken by Cantor to be unknowable, beyond mathematical characterization.

However, in subsequent work, Cantor formulated a more articulated doctrine of the Absolute. As is known, he distinguished between transfinite (*consistent*) and absolutely infinite (*inconsistent*) multiplicities, the latter, now, being, to

some extent, mathematically describable.[13] He took the distinction to be the basis for his 'limitation of size doctrine,' whereby sets are identified with all those collections which are *not bijectable* with absolutely infinite collections (in contemporary language, *proper classes*).

Building upon Cantor's conception of the Absolute, Kurt Gödel drew even more explicit connections between theological and set-theoretic ideas. In particular, according to Wang (1996)'s account, Gödel came to think that the axioms of set theory should be reduced to the Reflection Principle (stated below), in turn motivated by the following principle (equivalent to Ineffability):

> (*Refl*). The Absolute is unknowable.

In order to see how (*Refl*) may motivate the Reflection Principle, we have to introduce another fundamental set-theoretic concept, that of the universe of sets (cumulative hierarchy) *V*, a *proper class* structure which contains all sets.[14] *V* is absolutely infinite, so it really is an embodiment of the Absolute. But if *V* is like the Absolute, then (*Refl*) implies that also *V* is unknowable. Now, by using ideas related to Reflection expounded in Section 32.2.1, one reaches the conclusion that whatever is predicated about *V* is, in fact, predicated about some set in *V*, and, finally, obtain the:

> *Reflection Principle* (*RP*). Any formula in the language of set theory true in *V* is already true in some set in (some 'portion' of) *V*.[15]

Now, Gödel is also credited by Hao Wang with maintaining that, through increasing the degree of 'unknowability' of *V*, one could produce increasingly stronger reflection principles equivalent, in

turn, to further stronger set-theoretic principles. In the next subsection, we shall see that Gödel's view fits into (and motivates) a specific strengthening of *RP* which has extremely powerful mathematical consequences.

Overall, Gödel's foray into theology (and metaphysics) was not disinterested. In fact, he thought that the iterated use (and strengthening) of *RP* would result in producing principles concretely able to reduce set-theoretic incompleteness. Those principles are *large cardinal axioms* (Gödel calls them "strong axioms of infinity"), not provable from the standard axioms of set theory ZFC.[16] Now, Gödel's program, as expounded in →Gödel (1947), is precisely the program of producing new axioms to solve open problems such as the Continuum Problem. Among such axioms, large cardinal axioms (and thus, the *RP*'s which motivate them) are seen by Gödel as being especially promising. So, in the end, Gödel's program, insofar as it is, to a significant extent, based on the idea of the 'unknowability of *V*,' and, as a consequence, on large cardinal axioms, may also be taken to be a *bona fide* theological project.[17] It is no wonder, then, that two influential large-cardinal theorists would subsequently refer to their subject as follows:

> The adaptation of strong axioms of infinity is thus a theological venture, involving basic questions of belief concerning what is true about the universe. (→Kanamori and Magidor, 1978, p. 104)

32.4.2 Reinhardt's Reflection

In Wang (1996), Gödel also refers to (*Refl*) as "Ackermann's Axiom." This is because Ackermann's axiomatic system (henceforth, A) in →Ackermann (1956) contained an axiom (Axiom γ), which may, again, be seen as a mathematical expression of (*Refl*). Let us see how.

A's objects are *classes*, some of which happen to be *sets*. The Axiom γ says that, if the only objects which satisfy some property expressible in the language of A (whose parameters are just *set-parameters*), are sets, then the collection of all such sets is also a set. A crucial feature of the Axiom γ is that it may not contain the sethood predicate: *'x* is a set.' Ackermann's reason for introducing such constraint was that he assumed that it was not *knowable, a priori*, what sets there are, and as a consequence, that sethood could not be characterized by any expressible set-theoretic property.[18]

Later, William Reinhardt used a variant of A, ZA, a class-set theory, to extend and strengthen *RP*'s so as to motivate very strong large cardinal axioms.[19] He proceeded as follows. Mathematical expressions of *RP* consider a *single* formula Φ of the (first- or higher-order) language of set theory (informally, a single 'property'), and state, as we have seen, that some set (some 'portion' of) *V* satisfies Φ. The reflection principle that Reinhardt wished to formulate was, on the contrary, based on the idea of a broader 'resemblance' between *V* and its initial segments V_α's, and, in order to implement such an idea, he used *elementary embeddings*. Elementary embeddings may be seen as a special kind of isomorphism, in particular:

Definition 1 (*Elementary Embedding*). Given two structures $\mathfrak{U}$ and $\mathfrak{V}$, *j* is an elementary embedding from $\mathfrak{U}$ to $\mathfrak{V}$ if and only if *j* is an *isomorphism* from $\mathfrak{U}$ to a substructure $\mathfrak{V}' \subset \mathfrak{V}$.

So, an elementary embedding is an isomorphism which is injective, but not surjective.[20] Now, Reinhardt's reflection requires, as anticipated, that not only *V*, but also its *properties*, as befits Gödel's elaboration on Ineffability we have reviewed in the previous subsection, are reflected downwards.

In order to attain this goal, Reinhardt defines an elementary embedding j from an initial segment V_α of V, to another one, V_β,

and also from an initial segment $V_{\alpha+1}$ to another one, $V_{\beta+1}$, with $\beta>\alpha$; members of V_α sent by j to members of V_β are sets, members of $V_{\alpha+1}$ sent by j to members of $V_{\beta+1}$ are 'properties' (classes). In Reinhardt (1974a), this reflection principle is formulated, in ZA, as follows:

$$(\forall x, y \in V)(\forall P \subseteq V)[\theta^{\wp(V)}(x, y, P) \leftrightarrow \theta(x, y, jP)], \qquad \text{(S4)}$$

where the variables x, y range over sets and jP's are the image, under j, of properties P's which belong to $\wp(V)$ (the $V_{\alpha+1}$ above) and θ is any formula of the language of ZA.[21]

Now, Reinhardt's motivating strategy for S4 is not exclusively based on Gödel's (*Refl*). At the beginning of →Reinhardt (1974b), Reinhardt hints at one further fundamental reason to support S4, which resembles very much Ackermann's motivation for the Axiom γ, that is, the irreducibly potential character of *V*, which makes it impossible to know *a priori* what sets there are.[22] But, later, in a revealing 'historical remark' Reinhardt explicitly recalls the issue of *analogical predication* in theology, and discusses a purely theological interpretation of S4 in line, this time, with Gödel's speculations. He says:

> Formal devices such as *j* and the idea of analogical predication are hardly new to the history of thought. The author recently stumbled on an early use of a j-like device by the writer known as Pseudo-Dionysius (fifth century (?)); Greek scholars can probably find earlier antecedents. According to the negative way of Proclus followed by Pseudo-Dionysius, created beings are (for example) wise or unwise (as the case may be), but the Creator is neither; he is super-wise. According to (S4), of course, ordinals are accessible or not, but Ω is *j* (accessible). (→Reinhardt, 1974a, p. 198)

So, Reinhardt construes *P*'s in S4 as properties of the created beings, whilst *jP*'s may be taken to be properties of God; moreover, for each *P, jP*, clearly, is the Areopagite's 'super-property.' Thus, Reinhardt is consciously suggesting that S4 may be seen as the mathematical expression of negative theology; but, insofar as S4 is based on elementary embeddings, which are, as we have said, a kind of isomorphism, it may also be able to express Aquinas' conception of analogy, in particular, it could count as a reformulation of Bocheński's 'isomorphism hypothesis.'

It would, then, be natural, to wonder how Reinhardt's reflection, qua reformulation of Bocheński's 'isomorphism hypothesis,' fares with the objections raised in Section 32.3 against the hypothesis itself.

As far as the first objection is concerned, it would seem that Reinhardt's S4 would be well-suited to respond to it, since elementary embeddings seem to be able to express the idea of a mathematical structure $\mathfrak{U}$'s being *less than* another structure $\mathfrak{V}$, that is, of $\mathfrak{U}$'s being isomorphic to a *smaller* structure $\mathfrak{V}'$ *inside* $\mathfrak{V}$ itself.

As far as the second objection is concerned, elementary embeddings still imply that $\mathfrak{U}$ and $\mathfrak{V}$ are *elementarily equivalent*, so the objection raised against the 'isomorphism hypothesis' may also extend to S4.

32.4.3 The Global Reflection Principle and Its Theological Implications

In recent years, Philip Welch has formulated a different strengthening of *RP* that he has called Global Reflection Principle (*GRP*). Again, as in the case of Reinhardt's reflection, the motivating idea, in the case of *GRP*, is that there is a broad

resemblance between *V* and a 'portion' of it, much broader than that allowed by *RP.*

However, globally, the set-theoretic conception underlying *GRP* is of a fairly different nature than that behind S4. Reinhardt had introduced, and defended, S4 on the grounds of a *potentialist* conception of *V*, whereas Welch now takes *V* to be an *actual, fully determinate* object.[23]

As a consequence, Welch explains the existence of classes in a different manner than how these are explained by Ackermann and Reinhardt. To this end, →Horsten and Welch (2016) adopts a *mereological* interpretation, that is, it takes classes to be 'parts' of *V* which *determinately* exist in *V*; they are given once and for all, and constitute a separate domain of objects, that the authors denote with $\mathscr{C}$.

Thus, *GRP* envisages the reflection of, respectively, (elements of) $\mathscr{C}$ (the classes) as well as of elements of *V* (the sets) to elements of initial segments of *V* (i.e., to other sets). Again, in order to attain the proposed degree of reflection, one uses an *elementary embedding*. *GRP* is, then, the statement of the existence of an elementary embedding:

$$\frac{j\colon (V_\kappa, \in, D) \to (V, \in, \mathscr{C})}{} \qquad (GRP)$$

where D is the collection of all subsets of V_κ, and j sends elements of $V_{\kappa+1}$ to proper classes in $\mathscr{C}$.[24]

Now, how is such a form of *resemblance* construed by its author in theological terms? (→Horsten, 2016) provides an extended examination of the issue.[25]

We have seen that *GRP* implies the reflection of both sets and classes. Now, as in Reinhardt's reflection, the latter are construed by Horsten (2016) as being properties of God, which

are reflected by the created beings (mathematically, the elements of $V_{\kappa+1}$, for some large cardinal κ). Again, the main theological thrust of Horsten's interpretation is negative theology, that is, Ineffability and Reflection.

But just how are such sets construed theologically? Here's the crux, and the innovative aspect of, Horsten's theological interpretation of *GRP*: the reflecting sets are taken to be created beings quite different from human beings, namely, *angels*. The main theological source of Horsten's interpretation is, this time, the Jewish philosopher Philo of Alexandria, in particular the latter's *De somniis*, where Philo affirms that 'properties of God' appear to humans through the mediation of *angels*. Horsten explains:

> [*GRP*] is the class-theoretic counterpart of the theological thesis that is defended by Philo of Alexandria. Just as in the theological context, there are 'angels' such that every humanly describable property of God also applies to them, in the class theoretic context there are some sets such that every property of the universe also holds when relativised to them. (→Horsten, 2016, p. 114)

Horsten (2016)'s reference to (and use of) angels in this passage will be crucial for the purpose of checking that *GRP* may also be taken to express Aquinas' conception of analogy via the 'isomorphism hypothesis.' In particular, *GRP* seems to be better suited than S4 to respond to the objections to the hypothesis.

To begin with, since *GRP*, like Reinhardt's S4, is hinged on the use of an elementary embedding, like Reinhardt's S4, it seems to express, more aptly than Bocheński's hypothesis, the kind of analogical relationship holding between God and the created beings.

Coming to the second objection, Horsten's interpretation of *GRP* as expressing a resemblance between *God and angels* provides Horsten with a forceful way to respond to it. The reason

is, angels are taken by the Judeo-Christian theological tradition as being *qualitatively* comparable to God; thus, it is not implausible to imagine that, if God makes himself accessible to humans through angels, then these may be seen as genuinely enjoying God's properties. So, it would not be unreasonable to imagine that angels may be *indistinguishable* from God.

On the other hand, by focusing on angels rather than human beings, *GRP*, as is clear, will not be able to explain analogical predication referring to *God and human beings*, and being able to make room for this case in one's account of analogical predication would be extremely desirable.

32.4.4 Analogy in the V-logic Multiverse

In contemporary set theory, the notion of 'set-theoretic multiverse' has recently emerged as a consequence of the independence phenomenon. Multiverse conceptions hold that there is no *single* universe of sets, but rather a *multiplicity* of universes, consisting of some (even *all*) models of some theory of sets *T* (mostly, ZFC).

Among such conceptions, I will focus on what was once called 'hyperuniverse conception,' and now may be called '*V*-logic multiverse conception.'[26] The *V*-logic multiverse consists of all models of ZFC which can be expressed in *V*-logic. This is an *infinitary* logic in the language $\mathscr{L}_{\kappa^+,\omega}$, where κ is an inaccessible cardinal, enriched with κ-many constants (one for each set in *V*).[27]

Now, the *V*-logic multiverse will, in particular, comprise 'outer models' of *V*, that is, models obtained through the technique of *forcing*,[28] and it can be proved that, for any outer model *W* of *V* satisfying some consistent first-order sentence φ, there exists a single sentence of *V*-logic which expresses that φ is true in *W*. In particular, the existence of the *V*-logic theory:

T = ZFC + "$W \models \varphi$"

is captured by a single V-logic sentence. The next couple of results are fundamental to our purposes:

Theorem 1 (*Barwise*). T above has a model if and only if V is countable.

Axiom 1 (*Multiverse Axiom Schema*). For all W, let T = ZFC + "$W \models \varphi$," then T is consistent in V-logic.

Axiom 1 guarantees that V-logic theories are consistent, that is, that they have *models*, and, in combination with Theorem 1, it also states that consistent V-logic theories are satisfied by *countable models*. Thus, any set-theoretic sentence φ, holding in an outer model of V, is concretely 'realized' in some countable model M, a member of the V-logic multiverse. Hence, the V-logic multiverse becomes a fully meaningful construct only in the presence of a countable V.[29]

Now, the complex mathematics of the V-logic multiverse suggests the following scenario with respect to our theological issue.

The relationship between V and the members of the V-logic multiverse will be taken to stand for the relationship between God and the created beings. This strikes an immediate disanalogy with S4 and *GRP*, since now, the created beings are, mathematically, very little entities (countable models), whilst God, as ever, is taken to be absolutely infinite.

It is of fundamental importance to understand how, mathematically, members of the V-logic multiverse are 'obtained from' V. We first note that V-logic theories such as T are first-order over HYP(V), the least admissible set over V.[30] Now, through an application of the Downwards Löwenheim-Skolem Theorem, one gets that those theories have countable models *elementarily equivalent* to V. Precisely on such grounds, one takes the V-logic multiverse to express, theologically, like S4 and *GRP*, that properties of God are 'reflected' by the created beings

(humans), although no actual Reflection Principle is in play in this case.

On this interpretation, the *V*-logic multiverse conception may, thus, stand out as yet another restatement of the 'isomorphism hypothesis,' since, again, we would find ourselves with an expression of resemblance between God and the created beings. How does this conception fare, then, with the objections discussed in Section 32.3?

The first objection seems to be very aptly dealt with by this conception: we know that *V* and countable models have different sizes, and the unfathomable distance between God and the created beings is very appropriately expressed by the radical distance between the absoluteness of *V* and the countability of members of the *V*-logic multiverse.

As far as the second objection is concerned, again, if one takes the *V*-logic multiverse to arise from applying the Downwards Löwenheim-Skolem Theorem, then the resulting degree of resemblance between God and the created beings might still be too high. However, the 'countabilization of *V*,' which is crucial to the purpose of yielding models of *V*-logic theories, suggests an alternative, more articulated, theological interpretation of the relationship between *V* and countable models. On this interpretation, it is not God's properties that are directly reflected by created beings, rather God's properties appropriately scaled down to the *level* of created beings, that is, the properties of a 'countable *V*'.

In sum, the *V*-logic multiverse conception seems to express very well Aquinas' conception of analogy, that is, both the idea of a radical distance between God and the created beings, and the proportionality between God's properties and those of the created beings. It might also be able to capture aspects of negative theology, Reflection, for instance, as expressed by the use of the Downwards Löwenheim-Skolem Theorem to obtain

countable models, and, to some extent, also Super-Essentiality, if one takes properties of 'real *V*,' as opposed to 'countable *V*,' to be 'super'-properties, although, arguably, S4 and *GRP* display a more direct connection with negative theology.

32.5 Summary: Analogy Reconsidered

We have seen that there are two main ways of conceiving of the relationship between God and the created beings, one based on negative theology, and one on a more articulated conception of *analogy*. Notwithstanding their mutual differences, both conceptions are based on, or are conducive to, the idea that there is a *resemblance* between God and His creatures which does not contradict God's ineffability. Moreover, we have seen that such a resemblance can be articulated in terms of the purely mathematical idea of 'isomorphism,' although this interpretative hypothesis is far from being unproblematic.

The first way, negative theology, has, to some extent, inspired the formulation of abstract principles in set theory which, mathematically, take the form of Reflection Principles. The second one, the actual 'way of analogy', seems to be more aptly reflected by a multiverse conception such as the *V*-logic conception for the reasons that have been pointed out in the previous section: the mathematical constructs involved (*V* and countable models), the kind of relationship proposed by the conception itself, and the subtle distinction this conception makes between God's 'real properties' and God's properties 'made accessible to humans.'

A few further concluding remarks are in order. It would be tempting to see the distinction between the two conceptions, and their emergence at different times, to reflect the historical development of Medieval theology itself, with the negative way being more characteristic of pre-Scholastic theology, still heavily

under the influence of late antiquity's Neo-platonism, and with the positive conception of *analogy* fully emerging in an era of profound intellectual changes and transformations like the thirteenth century.

But this picture would hardly be fully accurate. It is known that even thirteenth century's philosophers, such as Aquinas, were, at least, as much influenced by the mysticism of the Areopagite's conception as ninth century's Neo-platonically imbued theologians like John Scotus.[31] As a consequence, it would be more accurate to say that the two conceptions are closely intertwined manifestations of a unique, uniform, approach to the problem of the relationship between God and creatures, an approach founded on analogy.

By this picture, Aquinas' later conception does not really contradict the received view that God's properties are, ultimately, *ineffable*; it is just slightly more optimistic than negative theology in that it maintains that our logical and linguistic means are, at least, able to produce adequate approximations of these properties.

That the two conceptions, somehow, represent a seamless continuum in the Medieval theorizations on the relationship between God and His creatures also seems to be shown by the structural similarities among the different set-theoretic methodologies I have discussed in the previous sections. In particular, S4, *GRP* and the *V*-logic multiverse conception all commit themselves to the view that the resemblance between God and His creatures neither makes God equal His creatures, nor lifts the created beings to the level of God's essence. *GRP* offers a slightly more articulated position, insofar as it posits that God's properties are made accessible to humans through a distinct category of created beings, that is, angels.

Finally, a conspicuous difference among S4, *GRP*, on the one hand, and the *V*-logic multiverse conception, on the other, lies in

the latter's use of countable models, theologically interpretable as the created beings, a move which, as I have previously argued, makes the *V*-logic multiverse conception better suited to express, mathematically, Aquinas' conception.

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Notes

1 An English translation of the *Corpus Areopagiticum* in its entirety can be found in →Pseudo-Dionysius (1987). A concise overview of the history of the Areopagite's influence on Christian thought may be found in the same volume's introductory essays.

2 Thoughtful considerations on this issue may be found in the vast and stimulating →Heller and Woodin (2011) (cf., in particular, →Achtner, 2011).

3 For a careful examination of the Areopagite's Neoplatonism and its potential derivation from Proclus' doctrines, cf. →Klitenic Wear and Dillon (2007) and →Moran (2014).

4 See Section 32.4.1.

5 For a concise overview of the problem, see →Pinborg (1972). For Aristotle's conception, see Bocheński (1961) and →McInerny (1996, ch. 2).

6 For an exhaustive exposition of Aquinas' conception, cf. →McInerny (1971, 1996) as well as the less recent →Klubertanz (1960) and →Montagnes (1963).

7 *De veritate*, 2, 11, reproduced in Bocheński (1961, p. 178). For an exhaustive list of other salient passages, see again →McInerny (1996). For a concise, but exhaustive account of Aquinas' oscillation between alternative conceptions of analogy also cf. →D'Ettore and Ashworth (2021, pp. 15ff.).

8 Cf. →McInerny (1971, p. 1ff.) and →D'Ettore and Ashworth (2021, pp. 15–19).

9 Cf. Bocheński (1961, p. 179).

10 Fundamental details on the origin, development and theological connections of the Cantorian conception of the Absolute may be found in →Hallett (1984), →Jané (1995, 2010), →Tapp (2012), and →van der Veen and Horsten (2013).

11 Cantor (1883) in Ewald (1996, p. 891ff.). Among other things, Cantor says: "the true infinite, or the Absolute, permits no determination whatsoever" (→Ewald, 1996, p. 891).

12 Incidentally, also von Haller's quote seems to be reminiscent of the Areopagite's theology and, in particular, of Reflection. Among other theological passages manifestly referring to Reflection, one could cite Saint Gregory the Great's assertion in his *Homiliae in Ezechielem*: "No matter how far our mind may have progressed in the contemplation of God, it does not attain to what He is but only to what is beneath Him" (in →Potter, 2004, p. 223), or Odo Rigaldus' statement: "How can the finite attain [knowledge of] the Infinite? On this question some said that God will show Himself to us in a mediated way, and that he will show Himself to us not in His essence, but in created beings" (in →Horsten, 2016, p. 114).

13 The fundamental reference, here, is Cantor's 1899 letter to Dedekind, for which see Ewald (1996, pp. 931–935).

14 V comes in stages indexed by the ordinals, starting with $V_0 = \emptyset$, and then taking the power-set of the previous stage at stages indexed by successor ordinals and the union of all previous stages at stages indexed by limit ordinals.

15 Technically, it is said that it is true in some *rank initial segment* V_α of V.

16 For an overview of large cardinals, and their manifold connections to other mathematical areas, see →Kanamori (2009). For an overview of *RP*'s of different strength and motivation, cf. →Drake (1974) and →Koellner (2009).

17 Gödel's theological views were also extensively indebted to Leibniz, as he thought that "[his] theory was a monadology, with a central monad [namely, God]" (→Wang, 1996, p. 290). For this, cf. →van Atten (2009).

18 Cf. Ackermann (1956, pp. 336–337). It should be noted that Ackermann did not, in the least, attempt to motivate his axiom through appealing to any *theological* principle.

19 For an overview of Reinhardt's seminal contributions to the development of the large-cardinal program, see Kanamori (2009, ch. 23).

20 It should be noted that, in order to have the requisite large-cardinal strength, embeddings should not be trivial, that is, they should not be the *identity*.

21 Note that the language of ZA, in addition to the symbols used in ZF, also contains, among other things, a symbol for V. Moreover, in ZA j is treated as a unary function.

22 Reinhardt (1974b, p. 191). Reinhardt (1974a) even draws a distinction between ‘existing’ and ‘imaginary’ sets.

23 Cf. the discussion about this issue in Section 2 of →Horsten and Welch (2016, pp. 91–95).

24 Note that, formally, the theory in which GRP is more easily formulated is not ZFC, but a class theory such as NBG, cf. →Welch (2019, p. 94). Moreover, it is only when the level of elementarity of *j* is first-order ‘maximal’ ($\sum^0_\infty$-elementarity) that the principle above is called Global Reflection Principle.

25 Also see Horsten’s contribution in the present volume.

26 Cf. →Arrigoni and Friedman (2013), Antos et al. (2015), and then Antos et al. (2021).

27 Cf. Antos et al. (2015, p. 2486), and Antos et al. (2021, pp. 129ff.). For a general overview of infinitary logics, see, for instance, →Keisler (1974).

28 Forcing consists in extending a (countable) model *M* of (a fragment of) the axioms of set theory, which may be taken to be equivalent to *V*, by ‘adding’ to *M* a generic object *G*, a subset of a partially ordered set $\mathbb{P} \in M$. The resulting model, $M[G]$, may, then, express some additional property of *V*. For a classic overview of forcing, see →Kunen (2011).

29 The collection of all countable models of ZFC was originally called *hyperuniverse* in →Arrigoni and Friedman (2013), and this is the reason why the conception originally came to be known as 'hyperuniverse conception.'

30 For the concept of least admissible set, see →Barwise (1975, pp. 43ff.). HYP(*V*) does not literally exist, since, of course, *V* is not contained by any set; however, there are ways to make sense of HYP(*V*), for instance, if *V* is taken to be countable, or through coding it with a class (for the latter approach, see Antos et al. (2021, pp. 137ff)).

31 For the influence of Pseudo-Dionysius' thought on Aquinas' conceptions, and on the structure of *Summa Theologiae*, cf. →Corrigan and Harrington (2019, pp. 27ff.).

IX God and Mind

33 What's on God's Mind?

John Heil

Abstract: What might the mind of God be like? The answer is not obvious. God is said to be omniscient, omnipotent, and omnibenevolent. God is transcendent, outside space and time. The cosmos owes its being and character to God. What would the thoughts of such a divine being be like? What would they concern? Familiar accounts of meaning and reference would seem not to apply to God. Does this mean that it would be a mistake to ascribe thoughts to God? The paper addresses these and related matters, including God's relation to the cosmos. It proceeds, not by engaging with scholarly debates about God's nature, but by starting afresh.

33.1 Prologue

When Mirosław Szatkowski asked me whether I would be interested in contributing a discussion of the mind of God to this collection, my initial reaction was tepid. I work in down to earth secular metaphysics and, although I have published a paper on transubstantiation and another on the Trinity, these were purely metaphysical in character. What could I possibly have to say on the topic of God's mind? The seed had been planted, however, and eventually I came to the realization that I needed to find out for myself what I thought about the matter.

My philosophical mentor, C. B. Martin, liked to say that you did not really understand a position until you could 'live it.' By this he did not mean that you had to accept the position as true. Rather, you needed to understand it well enough to appreciate its attraction. Although I now regard this as the only way to

engage in metaphysics seriously, this is not something my training as a philosopher prepared me to do. I learned it from Martin by being in his presence when he was metaphysically engaged.

This paper, then, is an attempt to work my way into what is for me unfamiliar metaphysical territory and appreciate it on its own terms. I believe Martin would have approved. You the reader might be less enthusiastic. I have undoubtedly failed to acknowledge or consider influential positions that deserve acknowledgment and consideration. Still, my thoughts on the topic might lead readers with settled opinions to sharpen those opinions.

33.2 Plan of Attack

What might the mind of God be like? As noted above, I mean to approach this question not by engaging with centuries of scholarly work dedicated to the topic, but as a philosopher chiefly interested in metaphysics and, to a lesser extent, the philosophy of mind. The advantage of proceeding in this manner is that I am not encumbered by preconceptions concerning the range of acceptable answers. In my experience, preconceptions, 'what everyone knows,' are responsible for a good deal of flabby metaphysics. I accept that in starting afresh you risk reinventing the wheel. Not only that, you risk reinventing a wheel with well-documented defects. I would worry about this more were the stakes higher. As it stands, my being utterly off base would put me in excellent philosophical company.

I begin the discussion by laying out some ground rules concerning accounts of the nature of God, then turn to a consideration of implications of the divine nature and their bearing on the question whether an entity with this nature could entertain thoughts and what this might imply about the

character of the cosmos. The upshot is something that is, for me, surprising. Even if you have a less (or more) demanding conception of the divine nature, however, much of what I have to say about the mind of God could still be of interest.

33.3 The Divine Nature

For the question whether God has a mind and entertains thoughts to be interesting, God must be understood as something more than a superhuman agent who assembled and has subsequently presided over the cosmos, intervening in its operation from time to time. Marduk would be such an agent. The God I want to consider lacks human traits, or at any rate those that would be expressions of finitude. In the medieval-anchored western tradition to which I belong (through no choice of my own) God is an eternal, omniscient, omnipotent, omnibenevolent agent responsible for the existence and character of the cosmos. God is transcendent, subsisting outside space and time. God is not made up of whatever makes up the cosmos. (I prefer 'cosmos' to 'world' or 'universe.' If ours is simply one among many universes, the cosmos comprises the lot.)

Difficulties abound. What could it mean to say that God is eternal? An eternal being is a being that exists at all times, but it would be misleading if not downright incoherent to characterize a being residing outside space and time as eternal in this sense. Suppose an eternal being is a being for which the following is true: there is no time at which the being does not exist? This might be an improvement, but it retains the temporal awkwardness of the original. Whatever God's being eternal amounts to, it is not a temporal trait.

What of God's benevolence? Maybe benevolence springs from omniscience and omnipotence, so is not in fact an

additional trait: an omniscient omnipotent being is perforce benevolent. In any case, benevolence smacks of attitudes, states of mind. Including benevolence in the divine nature at the outset would require first establishing that God could have attitudes, the very matter at issue. Once you see this point, you can see that it extends to omniscience and perhaps to omnipotence as well. An omniscient being knows everything. But knowledge, however it is characterized, includes true beliefs or something in the neighborhood of true beliefs, what philosophers call propositional attitudes.

What of omnipotence? If being omnipotent is being able to do whatever you choose, then it too includes attitudes. What would it be for an omniscient being to choose? An omniscient being would know everything there is to know including what it would choose. Choices turn on the consideration of options, but how could an omniscient being have genuine options? If you were offered a choice between your favorite pudding and a long painful death by torture, you would have a choice in name only. If you know that a certain alternative is optimal, you do not need to assess options.

What these considerations show is not that one familiar conception of the divine nature is incoherent, but that it is a mistake to think that we have a very clear grasp of what the divine nature portends. Our conception of minds and their contents is tailored to finite beings and does not extend smoothly to the divine nature. Characterizing God as omniscient, omnipotent, and omnibenevolent without further ado threatens to muddy the water. Such a characterization is at best figurative and at worst utterly misleading, analogous to characterizing an infinite number as a surpassingly large number. Infinity must be approached in a different manner.

An infinite number is a number, but a number the character of which cannot be extrapolated from the character of finite

numbers. Consider: an infinite set is one in which there is a one–one mapping between members of the set and a proper subset of the set's members. There are infinitely many integers. If you subtract all the even integers, the infinity is not halved, not reduced. Could the cosmos accommodate an infinite number of electrons? If it could, if every electron were associated with an integer, annihilating all the even-numbered electrons would not diminish the total, bad news for the conservation of mass–energy. The cosmos is hostile to actual infinities.

You might doubt this. Space is infinitely divisible, space, and in fact any finite region of space, comprises an infinity of spatial points. Spatial points do not make up space as bricks make up a wall, however. You get the wall by assembling the bricks. The wall depends for its existence on the existence of the bricks, which in turn depend on the particles that make them up. In contrast, to get spatial points you need space. Spatial points and regions depend on space, not the other way round. The points and regions are abstractions from the whole, not parts making up the whole.

An infinitely powerful, knowledgeable, benevolent being would be a being, but not of the cosmos. If there is no room in the cosmos for an infinite being, where would such a being *be*? Were the being outside spacetime, the being would be nowhere. Being somewhere is to be somewhere in space and time, even if this somewhere is everywhere. Fields are everywhere, suffusing space, but they are still in space and time. To say that an infinite being is nowhere, however, is not to say that there is no such being, any more than saying that space itself is nowhere is to say that space does not exist.

What, then, can be said about the divine nature? Not much, but not nothing either. The divine nature does not lend itself to characterizations we understand because we understand their application to finite beings. Talk of God's wisdom, power, and

benevolence are profoundly misleading if taken literally. An infinite number differs not in magnitude but in kind from a finite number. An infinite being's wisdom, power, and benevolence differ not simply in magnitude but in kind from a finite being's wisdom, power, and benevolence.

33.4 The Divine Mind

To the extent that the nature of an infinite being differs in kind from the nature of a finite agent, the mind of an infinite being would differ in kind from the mind of a finite being. That being so, why imagine that it would be appropriate to think of an infinite being as having anything describable as a mind? One possibility is that an infinite being deserving to be called divine, despite being immutable, is not passive. A divine being acts, a divine being exhibits agency. Agents act for reasons. Agents form intentions to act based on what they believe and what they want. Unencumbered, agents do what they intend to do. An infinite divine agent, then, would have a mind.

What would such a mind contain? Would it have beliefs and desires and, on the basis of these form intentions and act intentionally? Again, in the case of the divine mind, talk of beliefs, desires, intentions, and actions is at best figurative.

Consider beliefs and their objects. Many philosophers accept that a belief's being about something, the moon, for instance, requires a causal connection between what the belief concerns, the moon, and the believer. The connection might be indirect, it might be founded on descriptions provided by others, but there must be a causal route between the object of the belief and the believer. False beliefs might lack such connections, but the idea is that false belief is possible only in the company of true beliefs that do enjoy causal connections to their objects.

False beliefs are not an issue for an omniscient believer, but causal connections of the familiar sort would be. Indeed, efficient causation has no application whatever to a divine being. An omniscient being's knowledge is not something that increases over time, it is wholly there if it is ever there. This is simply a reflection of the divine nature. The divine nature is unchanging, not dynamic. In that case, however, could a divine being entertain thoughts about anything? Ordinary beliefs, and thoughts generally, stand in need of causal connections, both between one another and between themselves and whatever they are directed on. But God's relation to the cosmos is nothing like this.

Omniscience cannot be understood independently of omnipotence, nor can omnipotence be understood independently of omniscience. Take God's creating the cosmos. God does not mull over the possibilities, form the intention to create the universe, then engage in an act of creation. God's omniscience obviates the need to consider options, and if creating the cosmos includes creating space and time, then there is no time prior to the appearance of the cosmos. We finite beings apprehend a dynamic cosmos that unfolds in space over time. God's perspective on the cosmos is four-dimensional, timeless, *sub specie aeternitatis*. God's unchanging nature includes an appreciation of the cosmos in its spatio-temporal entirety.

God's creating the cosmos would not be the result of a choice in anything like the ordinary sense. The idea that God deliberates, and subsequently freely chooses to bring the cosmos into being is at odds with the divine nature. If time begins with the cosmos, there was no time prior to its coming into being. God's creating the cosmos could not have been an instance of efficient causation. Indeed, talk of creation is, as are most commonplace characterizations of the deity, categorically

misleading. God makes it the case that the cosmos is, and is as it is, not by willing it into being. The cosmos is an expression of God's nature.

I recognize that this last comment is hopelessly vague, but I mean only to be gesturing at a dependence relation. If you have God, you have the cosmos as it is, and if you have the cosmos as it is, you have God. The dependence is asymmetrical, not in the way causal relations are thought to be asymmetrical, but because any accounting of why the universe is and is as it is rests with God's nature. Please understand that I am not arguing that this is how it is, only that given the character of the divine nature set out earlier, this is how it is.

All of this elides a difficulty mentioned earlier. How could God's thoughts be *about* anything? In the case of finite minds, many philosophers accept that it is at least a necessary condition on thoughts' being directed at particular things that the thinker be in causal contact with those things. I hope that by now it is clear that this way of accounting for the aboutness of thoughts cannot be extended to God. F. H. Bradley held that God's thoughts are to be identified with their objects (Bradley, 1930 [1893]). In that case, God's thoughts about the cosmos just are the cosmos. There is no causal or epistemological gap between what God thinks and what is the case. This would locate the cosmos in God's mind, a pleasing thought to some.

What are the alternatives? God's being omniscient means that any thoughts God might have could not fail to be true. This is not because God invariably gets things right. God gets things right because the things are as they are owing to God. God's thoughts do not conform themselves to the things, the things are a reflection of God's thoughts. There is no epistemic gap between God's thought and what there is. God's thinking that the cat is on the mat, whatever form this might take in God's

mind, is of a piece with what makes it the case that the cat is on the mat. *This* is its aboutness.

Omniscience and omnipotence alike are abstractions: different ways of considering the same thing. The distinction between omniscience and omnipotence is what medieval and early modern philosophers called a distinction of reasoned reason and philosophers today would call a conceptual distinction. Distinctions of reasoned reason were contrasted with real distinctions. Two things, *A* and *B*, are really distinct just in case they could exist apart and independently of anything else. Descartes held that minds and extended objects were, in this sense, really distinct.

33.5 A Humean Cosmos

I have been told that, when they were both at Adelaide University in the 1950s, C. B. Martin convinced J. J. C. Smart that prayer could not be efficacious. In due course, Smart gave up theism. Part of the problem is that a prayer's being heard and subsequently acted on would require causal interaction between a material being, Smart, and a transcendent nonmaterial being, God.

The real difficulty, however, does not center on the character of material-nonmaterial causal relations but on an incompatibility between the divine nature and efficient causation quite generally. God is unchanging, so the idea of our bringing it about that God intercedes temporally in the Cosmos on our behalf has no application. What is less readily appreciated is that an unchanging God requires an unchanging cosmos. How so? God's being omniscient means that God's perspective on the universe is timeless. God comprehends the cosmos in its spatial and temporal entirety at once. God's perspective on the cosmos

is something like your perspective on a chess board: it is all there in front of you at once.

The universe perceived by God *sub specie aeternitatis* is a four-dimensional whole. What finite agents experience as changes belong to the appearances, not to reality, not to the cosmos as it is. The cosmos as it is for God resembles David Lewis's Humean cosmos.

> [A]ll there is to the world is a vast mosaic of local matters of particular fact, just one little thing after another. (But it is no part of the thesis that these local matters are mental.) We have geometry: a system of external relations of spatio-temporal distance between points. Maybe points of spacetime itself, maybe point-sized bits of matter or æther or fields, maybe both. And at these points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point to be instantiated. For short: we have an arrangement of qualities. And that is all. There is no difference without difference in the arrangement of qualities. All else supervenes on that. (→Lewis, 1986, pp. ix–x)

What I am calling a Humean cosmos is one in which nothing affects or is affected by anything else. Everything simply is as it is timelessly. One billiard ball's striking another and the trajectories of both balls changing in consequence is in fact a matter of qualities in spacetime having one kind of distribution rather than another. The qualities do not interact. Interactions belong to the appearances, as does change. The four-dimensional array of qualities simply is what it is. Given this picture of the cosmos, God's making it the case that something happens is not a matter of God's intervening in the course of events thereby changing the cosmos, but God's making it the case that the cosmos as it is includes one pattern of qualities rather than another. God's answering a prayer would be God's making it the case that the cosmos timelessly includes you as you are alongside the object of your petition.

33.6 From the Sublime to the Ridiculous?

In metaphysics, one thing inevitably leads to another. An innocent-seeming starting point can set you on the path to an antecedently shocking destination. So it is here. You start with a conception of the divine nature as omniscient, omnipotent, and omnibenevolent and find that this leads you, one step at a time, to a Humean cosmos, a cosmos apparently hostile to all those values God is invoked to accommodate. In such a cosmos nothing is as it appears to be, all is illusion. Worse, such a cosmos is inimical to human freedom, a bridge too far for most of us. If everything we do is fixed timelessly, our sense that we are in control of our lives is specious.

I do not expect you to be with me on any of this, especially if you have theistic leanings. The route from a relatively uncontroversial, or at least familiar, starting point—omniscience, omnipotence, omnibenevolence—to a Humean cosmos required taking on board a bevy of controversial metaphysical theses, any of which could be rejected. Even if the reasoning proved valid, the conclusion would warrant a rejection of one or more of the premises: *modus tollens* to my *modus ponens*. My argument would in effect be a *reductio* of my conception of the divine nature. If, in contrast, you are skeptical of the possibility of a being with the divine nature, you might regard my arguments as supporting your skepticism: belief in God would require all manner of implausible assumptions and lead ineluctably to shocking conclusions, avoidable only by backtracking on one or more of the assumptions.

I do not share the antipathy of many toward the Humean cosmos. On the contrary, I believe such a cosmos could prove much more attractive than a competitor Aristotelian cosmos when both are scrutinized closely and appreciated for what they are (→Heil, 2021). In Section 33.7 I shall explain why a Humean

cosmos does not in fact have the dire consequences it might initially appear to have. First, however, I propose to address concerns about the argument leading up to this point.

You might think that my characterizing the foregoing as an argument is already to give it too much credit. I began with a portrayal of the divine nature as omniscient, omnipotent, and omnibenevolent. In the course of working out what these might come to I first raised doubts about whether omnibenevolence was really a distinct trait, whether it might not be encompassed in omniscience and omnipotence, then turned to difficulties in the ascription of propositional attitudes—beliefs, desires, intentions—to an omniscient omnipotent being. Our understanding of the attitudes, I maintained, is tied to their occurrence in finite minds. Essential features of the attitudes extend, at best, only figuratively to a divine mind.

That was the easy part. The much harder part included my efforts to say what omniscience and omnipotence would *be*. In the case of omniscience, I argued that God's conception of how things stand in the cosmos would be four-dimensional, *sub specie aeternitatis*. Omniscience includes a timeless appreciation of what was, is, and ever shall be. Given God's unchanging nature, whatever that appreciation amounts to, it is not a matter of God's harboring beliefs that stand in causal relations to their objects. Omniscience and omnipotence are intertwined. God's omniscient perspective is not true because it reflects how things stand in the cosmos, rather how things are in the cosmos is a reflection of God's perspective.

My suggestion was that God's knowledge of the cosmos and God's creating the cosmos are really distinct ways of considering the divine character, a distinction of reasoned reason, not a real distinction. There is no gap between God's omniscience and what there is. This takes you to what I have called the Humean cosmos. Given the divine nature and the relation of omniscience

and omnipotence, the cosmos must reflect the timeless, unchanging divine nature. I believe the Humean cosmos comes close to satisfying this requirement, or at least comes closer than its most obvious cosmological competitor, an Aristotelian cosmos.

But does it really? An Aristotelian cosmos is dynamic and highly interactive. In discussing finite minds I emphasized the significance of causal connections: reasons are causes, thoughts concern what they do owing to causal connections they bear to their objects. In a Humean cosmos nothing interacts with anything else. There is 'just one little thing after another.' In a Humean cosmos, past, present, and future are set in stone. This is not determinism: events do not necessitate subsequent events, because nothing *in* the cosmos necessitates anything else. The cosmos as a whole is what it is and, given the divine nature, could not have been otherwise.

33.7 Appearance and Reality

Am I asking you to swallow an outrageous picture of the cosmos or maybe engaging in a disingenuous attempt to undermine theism? I hope to convince you that I am doing neither, and to do so without resorting to special pleading. The points I want to make are perfectly general and neutral on the question of theism. I defend what I say here in much more detail in another place, so I will be brief (see →Heil, 2021).

We finite creatures rely on the sciences to provide accurate accounts of the cosmos and its contents. The cosmos as portrayed by the sciences often clashes with ways the cosmos appears to us. Quantum gravity and general relativity afford many examples, but it is easy to see the difficulty by considering an everyday case. Here is a ripe passion fruit. The passion fruit is purple. Physics tells us that the passion fruit is made up of

countless colorless particles. How could these combine to make up something purple?

It is no help to appeal to differences in the surfaces of objects that dispose them to reflect light radiation so as to appear colored. The question remains, how are we to account for colored appearances in light of our belief that all there is are colorless particles? Wilfrid Sellars distinguished what he called the manifest image and the scientific image.

> The philosopher is confronted not by one complex many-dimensional picture, the unity of which, such as it is, he must come to appreciate; but by *two* pictures of essentially the same order of complexity, each of which purports to be a complete picture of man-in-the-world, and which, after separate scrutiny, he must fuse into one vision. Let me refer to these two perspectives, respectively, as the *manifest* and the *scientific* images of man-in-the-world. (Sellars, 1963, p. 5)

The manifest image corresponds to what I have been calling the appearances. You could think of the scientific image as what emerges from physics, which, among the sciences, aims at the most fine-grained assay of what there is. The question, how is the manifest image related to the scientific image, is simply a version of the age-old question, how are the appearances related to reality.

Two ways of answering this question have impressive pedigrees. First, many tough-minded philosophers and scientists are happy to accept that the appearances are illusory, perhaps by virtue of being only in the minds of observers. The passion fruit's purpleness is not in the passion fruit, but in us. The appearances stand between us and our surroundings. Doubtless there is a kind of mapping between features of what appears and features of what there is, although when you think seriously about it, what reason do we have for thinking there is a reality independent of the experiences?

This thought leads to a second approach to the question, how are the appearances related to reality, how is the manifest image related to the scientific image, that takes the opposite tack. The appearances are what is real, the scientific image is a kind of construct, useful in negotiating the appearances perhaps, but with no pretense of revealing a reality hidden behind the appearances. This is how Berkeley saw it and how twentieth-century verificationists and instrumentalists understood the place of the sciences.

Questions about the relation minds would bear to the cosmos encompassed by physics aside, the divine nature as I have characterized it does not fit comfortably with either of these options. The second is clearly at odds with the divine nature, in particular with the identification of omniscience and omnipotence. The first option is equally unsatisfactory. It would, for instance, mean that God is bent on deceiving occupants of the cosmos: either the appearances are illusory, or we are deluded in supposing that our thoughts are directed on an observer-independent cosmos ‘out there.’

A more recent approach to the appearance–reality distinction is pleasingly ecumenical. Both the manifest image and the scientific image are veridical. The scientific image, what you have in physics, aims at describing a fundamental level of reality. The manifest image encompasses a non-fundamental higher level, or successively higher levels, of reality, each level dependent on but distinct from the next lower level until you reach the ground floor level of physics.

Despite its good press, this option, non-reductive physicalism, is beset by problems centering on the character of the relation levels bear to one another. I shall not pause here to rehearse those difficulties because I think there is a fourth much better option available, one that makes sense metaphysically and, as it happens, comports with the divine nature.

Simply put, the scientific image is what the manifest image is an image *of*. The passion fruit appears purple, but what is it to be purple, what is the nature of the truthmaker for 'the passion fruit is purple'? The appearances are not deceptive, but they leave open the nature of what appears. This is supplied by the sciences and ultimately by physics, the science that supplies our sharpest picture of what there is. On this view, the appearances are not deceptive. Truths—*truths*—at home in the manifest image have truthmakers, the nature of which lies within the purview of physics. The appearances take us *to* reality, but do not themselves reveal the nature of that reality.

Reality is not hidden behind a veil of appearances. Reality presents itself to us as it does owing to two factors: how reality is and how we are. Objects are colored, for instance. Physics tells us what it is for objects to be colored as they are. It turns out that the truthmaker for 'the passion fruit is purple' is a cloud of colorless particles or something equally exotic. If our cosmos is Humean, if it is a vast arrangement of qualities in spacetime, then the truthmaker for 'the ball rolls' is nothing that moves, and the truthmaker for 'the balls' striking one another caused them to change their trajectories' includes no interactions between distinct existences.

Talk of colors, motion, and causation is suited to the appearances, but not necessarily to reality. On this account it would be misleading to say that, in reality, nothing is colored, nothing moves, nothing interacts with anything else. You can know a lot about the cosmos without knowing much at all about the nature of the truthmakers for the truths you accept. The important point here is that truths of the manifest image are true. What is left to physics is an account of the character of the truthmakers. This is not a commitment to the idea that truths about colors and causal relations could be recast in the vocabulary of physics, it is, rather, to acknowledge that whatever

could be truly described as colored or causally engaged could be described as well in the manner of physics.

The point applies to ordinary assessments of the actions of intelligent creatures. When you act for reasons, your reasons are responsible for your actions. When you act freely, you act in a way that has what it takes for your act to be free. A specification of what it takes to act freely, the essence of free action, would be couched in a vocabulary suited to the manifest image. The mistake made again and again by philosophers who discuss agency and free will is to imagine that a specification of what it takes to act freely constrains the nature of the truthmakers for ascriptions of agency and free will.

Philosophers disagree on how best to characterize agency. Some hold that agency, or genuine agency, is incompatible with determinism, and, on that basis, conclude either that free actions are undetermined or that there are no free actions. By my lights this is to imagine that truths belonging to the manifest image constrain the character of the truthmakers for those truths as captured in the scientific image. There is no incompatibility between a Humean cosmos and free will.

At any rate, this is my story, and I am sticking to it. It provides what I regard as a satisfying way of reconciling the manifest and scientific images, the appearances and reality. Lest it be forgotten, the sciences, including physics, depend epistemologically on the appearances. Instruments deployed in laboratories, experimental manipulations, and observations of meter readings are all founded on the appearances. We expect these to supply reliable information about the cosmos. If I am right, if truths about the appearances leave open the nature of whatever makes them true, you can see how the appearances could be reliable without thereby being the last word on what there is. Our access to reality depends on how reality appears to

us, but this does not mean that we can read off from the appearances the nature of what appears.

33.8 An Impersonal Deity?

Suppose I am right or at least not altogether wrong about the divine nature. Would this excise God's personhood? Would we be marionettes? Would we be bit players on God's stage, dutifully reciting our lines before exiting—permanently—stage right when our part called for it?

That is not how I see it. When you reflect on your circumstances independently of theological considerations, you can see that you are made up of particles that do what they do owing to their natures. Your walking across the room to open a window is a matter of billions of particles altering their motions in a coordinated manner, the result being your walking across the room and opening the window.

Three points bear mention here. First, I am eschewing the kind of top-down causation promoted by some, Roger Sperry, for instance:

> Subjective mental phenomena are conceived to influence and govern the flow of nerve impulse traffic by virtue of their encompassing emergent properties. Individual nerve impulses and other excitatory components of a cerebral activity pattern are simply carried along or shunted this way or that by the prevailing overall dynamics of the whole active process (in principle—just as drops of water are carried along by a local eddy in a stream or the way the molecules or atoms of a wheel are carried along when it rolls down the hill, regardless of whether the individual molecules and atoms happen to like it or not). (→Sperry, 1969, p. 534)

As Jaegwon Kim notes: "an eddy is there because the individual water molecules constituting it are swirling around in a circular motion; in fact, an eddy is nothing but these water molecules engaged in this pattern of motion" (→Kim, 2000, p. 313).

This is not the place to investigate top-down causation, and its sister doctrine, emergence, but I consider that the burden of proof is on their proponents' shoulders. If ignoring physics is not an option, might there be some way for the mechanisms underlying free choices to find their way into physics? The prospects are not encouraging. Physicist, Carlo Rovelli, observes that

> There is nothing about us that can escape the norms of nature. If something in us could infringe the laws of nature, we would have discovered it by now. There is nothing in us in violation of the natural behavior of things. The whole of modern science—from physics to chemistry, and from biology to neuroscience—does nothing but confirm this observation. (→Rovelli, 2016, pp. 72–73)

A second point concerns this conception of the cosmos as comprising particles moving about and interacting. Such a conception is rife with efficient causation, which I have contended is at odds with the divine nature. If efficient causation belongs only to the appearances, not to reality, then your walking across the room and opening the window is not a matter of your being a participant in a complex causal sequence, top-down or otherwise. You and your actions comprise a Humean pattern of qualities distributed in spacetime. Where these are concerned, there is no interaction, no change, no motion.

You might doubt the plausibility of this kind of Humean picture of a once and for all cosmos of qualities distributed over spacetime. As far as anyone knows, however, the cosmos is in fact Humean, the particles belonging to the manifest image. A universe of fields in which particles turned out to be fuzzy concentrations of energy would be such a universe. The jury is still out, but the point to note is that whether the cosmos is

Humean or not, our self-conception as agents freely initiating actions would belong to the appearances, to the manifest image.

In the previous section I argued that to locate agency in the manifest image is not to make it out to be false or illusory. Truths embedded in the manifest image are nevertheless true. It is true that you acted freely in walking across the room and opening the window provided only that the conditions on your acting freely are satisfied. These conditions are to be spelled out in an idiom at home in the manifest image. What physics has to say about your nature is another matter. The point is perfectly general: the appearances take us to reality, but leave open the nature of what they take us to, the truthmakers for truths of the manifest image.

33.9 Speaking Personally

The time has come to pull all of this together and to consider its implications for conceptions of the divine persona. The picture of the divine nature as timeless and unchanging coupled with a four-dimensional Humean cosmos of qualities is apparently out of keeping with a caring, personal deity. This impression is reinforced when you consider my earlier insistence that attempting to characterize the divine nature using terms the truth conditions for which pertain to the appearances. How could a being with a timeless nature presiding over a once and for all cosmos be in any sense personal?

Consider petitionary prayer. A child, Lilian, secretly prays that she will be given a puppy for her birthday. Lilian’s birthday arrives and a homeless puppy appears at her front door apt for adoption. Lovely for Lilian, but how could this be considered anything more than a happy coincidence? How could it be the upshot of Lilian’s prayer. Lilian, her prayer, and the puppy are qualities distributed in spacetime. A being with the divine nature

viewing the cosmos *sub specie aeternitatis*, would apprehend all this at once. Such a being's answering Lilian's prayer would not be a matter of deciding, upon witnessing her petition, to manipulate the cosmos so as to bring it about that a puppy arrives on Lilian's doorstep at the appointed hour. A being with the divine nature would make it the case that the cosmos includes what it does: Lilian, her petition, her puppy. You could consider this as a potential truthmaker for the deity's responding to Lilian's petition.

The same line of thought would apply to the personhood of a being with the divine nature. The truthmaker for ascriptions of personhood to such a being would be a manifestly impersonal cosmos reflecting a manifestly impersonal deity. This is just to say that true ascriptions of personhood, divine or otherwise, could have what are, *sub specie tempori*, surprising truthmakers.

My argument has turned on a conception of the relation the appearances bear to reality that I regard as independently plausible. My aim has not been to advocate or confute theism, but simply to follow out the implications of conceptions of the divine nature according to which God is omniscient, omnipotent, and omnibenevolent coupled with a conception of the cosmos motivated by an interest in reconciling the appearances with reality. The idea is that a being with the divine nature as I have described it is at least consistent with, and perhaps even congenial to, a cosmos as I have described it. Such a being would not be remote from a created cosmos. On the contrary, there would be no distance at all between the being and the cosmos.

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34 How Phenomenal Consciousness Provides Evidence for God's Existence and Informs What It Means to Say God Is a Spirit

James P. Moreland

Abstract: There are two purposes for this chapter: (1) defend an argument for God from consciousness (hereafter, AC) and (2) provide a positive account of what it means for God to be a spirit/soul and defend this account against incoherency claims based on a purely *via negativa*. To accomplish these purposes, I address them in order. Regarding AC, I lay out important preliminary considerations, state three versions of AC and defend the premises of the deductive version, followed by a critique of the two dominant rivals to AC—naturalistic emergence and panpsychism. Regarding the presentation of God as a spirit, I defend the positive understanding of "spirit," followed by a critique of the claim that since "spirit" must be understood in purely negative terms, it is explanatorily impotent or, worse, incoherent.

34.1 Introduction

Thomas Nagel has reminded us that topics in the philosophy of mind are not local ones; they invade our understanding of the entire cosmos and its history. If irreducible consciousness and reason exist, we simply must ask what sort of reality could and, most plausibly, did give rise to them. Thus, says Nagel, one must

keep an eye on worldview implications of a position on a topic in philosophy of mind.[1]

Consciousness is among the most mystifying features of the cosmos. Geoffrey Madell opines that "the emergence of consciousness, then is a mystery, and one to which materialism signally fails to provide an answer."[2] Naturalist Colin McGinn claims that its arrival borders on sheer magic because there seems to be no naturalistic explanation for it: "How can mere matter originate consciousness? How did evolution convert the water of biological tissue into the wine of consciousness? Consciousness seems like a radical novelty in the universe, not prefigured by the after-effects of the Big Bang; so how did it contrive to spring into being from what preceded it?"[3] Finally, naturalist William Lyons notes that

> [physicalism] seem[s] to be in tune with the scientific materialism of the twentieth century because it [is] a harmonic of the general theme that all there is in the universe is matter and energy and motion and that humans are a product of the evolution of species just as much as buffaloes and beavers are. Evolution is a seamless garment with no holes wherein souls might be inserted from above.[4]

Lyons' reference to souls being "inserted from above" appears to be a veiled reference to the explanatory power of theism for consciousness, viz., that if "souls" exist, they would have to be "inserted from above" since natural processes by themselves are "seamless." More generally, some argue that, while certain features of finite mental entities may be inexplicable on a naturalist worldview, they are robustly explained by theism, thereby furnishing evidence for God's existence. And in light of comments like these, one has hope of developing and defending just such an argument for God based on either the existence of

finite conscious states or their regular correlation with physical states.

However, the nature of finite consciousness also provides resources for offering a positive account of what it means to say that God is a spirit. Among other things, if such an account is successful, then it furnishes a defeater for those who argue that the concept of a spirit is purely negative—an incorporeal or immaterial entity—and, thus, incoherent.

Accordingly, there are two purposes for this chapter: By employing the existence of irreducible, intrinsically characterized conscious properties and states, I will (1) present, clarify and defend an argument for God from consciousness (hereafter, AC) and (2) provide a positive account of what it means for God (angels, human/animal souls) to be a spirit/soul (using these interchangeably for my purposes), defend this account against incoherency claims based on a purely *via negativa*, and offer defeaters for the idea that a purely negative account of a spirit is all we have and need.

To accomplish these purposes, I will address them in order. Regarding AC, I will lay out important preliminary considerations, state three versions of AC and defend the premises of the deductive version, followed by a critique of the two dominant rivals to AC—naturalistic emergence and panpsychism. Regarding the presentation of what it means to say God is a spirit, I shall present and defend the positive understanding, followed by an examination and critique of the claim that since "spirit" must be understood in purely negative terms, it is explanatorily impotent or, worse, incoherent. Next, I will present and critique a case offered by Andrew Bailey to the effect that the *via negativa* is the correct way to describe a spirit and it is perfectly adequate to do the job. However, before we embark on this task, two items of clarification are needed.

34.2 Two Items of Clarification

It is important to get clear on two topics—the nature, extent and contingency of phenomenal consciousness, and the distinction between staunch and faint-hearted naturalism.

34.2.1 Phenomenal Consciousness

34.2.1.1 The Nature of Phenomenal Consciousness

Setting aside mere access consciousness, hereafter, I will use "consciousness" to mean "phenomenal consciousness." Our first task is to clarify "consciousness." At least five kinds of conscious states exist. A *sensation* is a state of awareness or sentience, e.g., a conscious awareness of sound or pain. Some sensations are experiences of things outside me like a tree or the color red. Others are awarenesses of states within me like pains. Emotions are a subclass of sensations and, as such, they are forms of awareness of things. I can be aware of something in an angry way. A *thought* is a mental content that can be expressed in an entire sentence. Some thoughts logically imply other thoughts. For example, "All dogs are mammals" entails "This dog is a mammal." If the former is true, the latter must be true. Some thoughts don't entail, but merely provide evidence for other thoughts. For example, certain thoughts about evidence in a court case provide grounds for the thought that a person is guilty. Thoughts are the sorts of things that can be true or false, reasonable or unreasonable. A *belief* is a person's view, accepted to varying degrees of strength, of how things really are. A *desire* is a certain felt inclination to do, have, or experience certain things or to avoid such. An *act of will* is a choice, an

exercise of power, an endeavoring to act, usually for the sake of some purpose.

Focusing on conscious states, there are three basic ways to characterize consciousness. First, one should *define* consciousness ostensively. Since we are assuming consciousness is genuinely and irreducibly mental, there is nothing more basic in terms of which consciousness or conscious states can be *defined*. An ostensive definition can be provided by describing a person regaining consciousness after being under deep anesthesia during surgery. Slowly the person comes to have sensations, thoughts, and so on, and we define conscious states as ones like *that*.

Besides offering a definition of a conscious state and setting aside intentionality for our purposes, there are two widely employed strategies for providing necessary, even essential, characteristics of a conscious state.

The Experiential Strategy: For all states x, x is a conscious state iff there is a what-it-is-like to be in x.

The Privacy Strategy: For all x, x is a conscious state iff the subject of x has private access to x.

The privacy strategy is often and erroneously taken to be an epistemological one, viz., it states a new way (via the first-person) of knowing what we already know in a third-person way. But as John Searle recognized, this is just a mistake.[5] The privacy strategy is straightforwardly ontological: It states that conscious states are of a unique sort compared to physical states in that the former but not the latter are capable of being related to a subject in a private sort of way. Conscious states are those kinds of things; privacy gives us insight about their unique nature.

34.2.1.2 The Extent of Consciousness

Regarding the extent of phenomenal consciousness, many limit it to sensations and, under a certain construal, desires. Left out are thinking and believing states. I demur and believe all five of the states listed above are states of phenomenal consciousness, including thinking and believing states. This broader view has been thoroughly defended elsewhere, and I will not do so here except to make two brief points.[6] For one thing, without a distinct, phenomenologically available what-it-is-like to one's thought and belief content, it would be impossible to distinguish the content of one thought/belief from another by simply attending to those contents, something we do all the time. For another, there is a clear phenomenological difference between reading a sentence with and without understanding, and that difference seems to be between the experience of thinking or entertaining the sentence's thought content vs. failing to have such an experience.

34.2.1.3 The Contingency of Consciousness

Besides the sheer existence of consciousness, another feature provides what seems to be a defeater for a naturalist worldview: the contingency of consciousness. Based on widely held, common sense intuitions, there are well-known thought experiments that appear to establish the metaphysical possibility of the relevant envisaged scenarios and, thus, the contingency of the link between mental and physical entities:

1. *Inverted Qualia*: Possibly, two people could satisfy the same functional role—scan a room with their eyes, sort the red objects from all the others, point to the red objects and say "These are the red objects."—while one sorts on the basis of having a sensation of red and the other on a sensation

of blue. Possibly, two people could be in exactly similar brain states while one experiences a sensation of blue and the other a sensation of red.

2. *Absent qualia*: The nation of China could satisfy the functional role of being in pain but there would be no felt experience of painfulness by the nation as a whole or in part. This could happen if all the Chinese people lined up from the western to the eastern border, someone outside the western boarder inserted a sword across the border, causing the chain of people to pass on "Our boundary was pierced with a sword." until the last person on the eastern boarder gets the message, shouts "Ouch!" and starts a chain of people who pass on the command "Go into a state of wanting soothing!" Here the functional role of being in pain would be satisfied by the nation of China, but the nation would have no felt sensation.
3. *Zombie worlds*: Possibly, there is a world in which we have exact physical and body-movement duplicates but which are unconscious zombies.
4. *Possibly, we exist disembodied*. Even if they are all false, descriptions of Near-Death Experiences are perfectly coherent and compossible. That is why people will let the evidence decide their veracity while an incoherent story—Jones found square circles in Montana—will not be investigated because it is not possible.

In sum, Jaegwon Kim said the following:

> The [property-dualist] case against qualia supervenience therefore is not conclusive, though it is quite substantial. Are there, then, considerations in favor of qualia supervenience? It would seem that the only positive considerations are broad metaphysical ones that might very well be accused of begging the question.[7]

Kim goes on to say that these broad metaphysical considerations amount to the assumption that physicalism *must* be true. He also claims, correctly in my view, that the corresponding dualist intuitions are not based on a prior commitment to property dualism but, rather, provide justification for property dualism.

34.2.2 Staunch vs. Faint-hearted Naturalism

As I will use the term, *staunch naturalism* is the view that the cosmos and all entities within it—e.g., particulars, properties, relations, events, processes—are and only are physical. *Strong physicalism* adopts the same ontology concerning consciousness, the human person, and other entities studied in the philosophy of mind. While he does not use the term, Galen Strawson's understanding of standard physicalism and his claim that it is the ubiquitously adopted view underscores my assertions about *staunch naturalism*: "I take [standard] physicalism to be the view that every real, concrete [i.e., spatio-temporally located] phenomenon in the universe is … physical" (brackets mine).[8] According to Strawson, standard physicalism eschews *sui generis*, irreducible, ineliminable emergent properties.[9] Staunch naturalists embrace structurally supervenient properties but not genuinely emergent ones. *Faint-hearted naturalism*[10] and *weak physicalism* differ only in that they embrace emergent entities, usually properties, but in a few cases, systems and even substances.

In the category of property, structurally supervenient properties are those that are merely a new relational combination of the parts and properties at the subvenient level —such as being H_2O. By contrast, *emergent properties* are completely unique, new kinds of properties different from those

that characterize their subvenient bases. Accordingly, and minimally, *emergent supervenience* (or simply, emergence) depicts the supervenient property as a simple, novel property different from and not composed of subvenient entities. Being painful is an example.

Mark Bedau identifies *Twin Hallmarks of emergence*. All emergence involves macrophenomena that (i) arise from and depend on more basic phenomena, and (ii) are simultaneously autonomous from the microlevel on which they depend. Consequently, a conception of emergence requires an interpretation of dependence and autonomy that is metaphysically coherent, without negative metaphysical implications, and that helps scientists understand some interesting aspect of reality. Bedau asserts that all the evidence shows that genuine emergence is scientifically irrelevant, and it straps us with a host of brute, inexplicable phenomena that should be avoided if at all possible. Moreover, it seems that all alleged cases of emergence can be explained away.[11]

Keep in mind that in the first half of the twentieth century, emergent properties were defined epistemologically:[12] Property P is an emergent property of some particular x at level l_n just in case P is a property of x at l_n, and no amount of knowledge of (or descriptive statements about) entities at subvenient levels below l_n would justify a prediction of (or logically entails a descriptive statement about) P at l_n. In this sense an emergent property (or a statement about it) is surprising, unexpected, and utterly brute (I use “brute” or “brute fact” to refer to a brute thing/entity and not a brute truth) relative to all entities at lower levels. Thus, the prior probability of emergent properties, given complete knowledge of lower levels, would be zero.

34.3 Stating and Defending AC

34.3.1 Two Issues in Scientific Theory Acceptance

34.3.1.1 Basicality

While theism and naturalism are broad worldviews and not scientific theories, three issues that inform the adjudication between rival scientific theories are relevant to our topic. The first issue involves deciding whether it is appropriate to take some phenomenon as *basic* such that only a description and not an explanation for it is required, or whether that phenomenon should be understood as something to be explained in terms of *more basic* phenomena. For example, attempts to explain uniform inertial motion are disallowed in Newtonian mechanics because such motion is basic on this view, but an Aristotelian had to explain how or why a particular body exhibited uniform inertial motion. Thus, what is basic to one theory may be derivative in another.

34.3.1.2 Naturalness

Issue two is the *naturalness* of a postulated entity in light of the overall theory of which it is a part. The types of entities postulated, along with the sorts of properties/powers they possess and the relations they enter should be "at home"—there should be a fittingness—with other entities in the theory. Some entity (particular thing, process, categorical or dispositional property, or relation) e is natural for a theory T just in case either e is a central, core entity of T or e bears a relevant similarity to central, core entities in e's category within T. If e is in a category such as individual, force, property, event, relation, or cause, e should bear a relevant similarity to central, core entities of T in that category. This is a formal definition and the

material content given to it will depend on the theory in question.

Moreover, given rivals R and S, the postulation of e in R is *ad hoc* and *question-begging* against advocates of S if e bears a relevant similarity to the appropriate entities in S, and in this sense, is at home in S, but fails to bear this relevant similarity to the appropriate entities in R. The notion of "being *ad hoc*" is notoriously difficult to specify precisely. It is usually characterized as an intellectually inappropriate adjustment of a theory whose sole epistemic justification is to save the theory from falsification. Such an adjustment involves adding a new supposition to a theory not already implied by its other features. In the context of evaluating rivals, R and S, the principle just mentioned provides a sufficient condition for the postulation of e to be *ad hoc* and question-begging.

The issue of naturalness is relevant to theory assessment between rivals in that it provides a criterion for advocates of a theory to claim that their rivals have begged the question against them or adjusted their theory in an inappropriate, *ad hoc* way. And though this need not be the case, naturalness can be related to basicality in this way: Naturalness can provide a means of deciding the relative merits of accepting theory R**,** which depicts phenomenon e as basic**,** vs. embracing S**,** which takes e to be explainable in more basic terms. If e is natural in S but not in R, it will be difficult for advocates of R to justify the bald assertion that e is basic in R and that all proponents of R need to do is describe e and correlate it with other phenomena in R as opposed to explaining e. Such a claim by advocates of R will be even more problematic if S provides an explanation for e.

By way of application, any account the origin and contingency of conscious states must be evaluated by its comportment with the broader worldview within which it is embedded. Now, according to classic Western theism, God is the

basic entity and He is a spiritual substance with a variety of conscious states. Thus, given theism, consciousness is basic. As I will develop in more detail later, it should be clear that the appearance of consciousness is natural in this theistic view but quite odd, unnatural and not basic in a naturalist worldview. Thus, it may very well be *ad hoc* and question-begging for someone to claim that the appearance of consciousness, perhaps as an emergent property, is "at home," or "natural" given naturalism.

34.3.2 The Nature of the Theistic Argument from Consciousness

34.3.2.1 Natural-Scientific Explanation vs. Personal Explanation

Personal explanation differs from realist scientific explanation understood as having two components: (1) Event covering-law explanations are employed almost universally in natural science. Associated with *event* causation is a covering law model of explanation according to which some event (the *explanandum*) is explained by giving a correct deductive or inductive argument for that event. Such an argument contains two features in its *explanans*: a (universal or statistical) law of nature *and* initial causal conditions. The ideal gas law is an example: PV=nRT. (2) A covering law explanation must be underwritten by some sort of realist model of the underlying entities that ground and explain the covering law by showing why it metaphysically or nomologically *has* to occur in nature. Such models are necessitating. The ideal gas model is an example.

By contrast, a *personal* explanation (divine or otherwise) of some state of affairs brought about intentionally by a person will

employ notions such as the intention of the agent and the relevant causal power of the agent that was exercised in causing the state of affairs. In general, a personal explanation of some basic result R brought about intentionally by person P, where this bringing about of R is a basic action A, will cite the intention I of P that R occur and the basic power B that P exercised to bring about R. P, I, and B provide a personal explanation of R: agent P brought about R by exercising power B in order to realize intention I as an irreducibly teleological goal.

34.3.2.2 Three Forms of AC

The argument for God's existence from the reality of finite consciousness seeks to show that, given consciousness, a theistic personal explanation is epistemically and explanatorially superior to a natural scientific explanation. The argument itself is best expressed in one of three forms: an inference to the best explanation (IBE), a Bayesian argument, or a straightforward deductive argument.

An IBE begins with certain data to be explained (e.g., the metaphysical entities constitutive of consciousness), assembles a pool of live options that explain the data, and usually on the basis of certain criteria—e.g., explanatory power by making the data more epistemically likely than rivals, naturalness, being less *ad hoc*—one option is chosen as the best explanation of the data. Applied to our case, the claim is made that, on a theistic metaphysic, one already has an instance of an unembodied mind in God. Therefore, it is hardly surprising that embodied or unembodied finite conscious agents should exist in the world. But on a naturalist view, mental entities are so strange and out of place that their existence (or regular correlation with physical entities) defies adequate explanation. There appear to be two

realms operating in causal harmony and theism provides the best explanation of this fact. It is hard to see how the combinatorial processes that constitute the naturalist creation account could give rise to emergent conscious properties.

The argument may also be construed in Bayesian terms along the following lines.

To see this, recall that:

$$\frac{P(h/e\&k)}{P(\neg h/e\&k)} = \frac{P(h)}{P(\neg h)} \times \frac{P(e/h\&k)}{P(e/\neg h\&k)}$$

Where e is the existence of the metaphysical entities constitutive of consciousness, ¬h=naturalism (technically, it is the negation of theism, but we can assume that theism and naturalism are the only live options for present purposes), and k=background knowledge. The first ratio is the prior probability of naturalism, given consciousness and background knowledge divided by the prior probability of theism, given consciousness and background knowledge.

These probabilities are hard to assign objectively, but surely the numerator is not ten times higher than the denominator. Given that these two probabilities are in the same ballpark, the really important ratio for the Bayesian argument is the second one: the “fittingness” (naturalness) of consciousness, given naturalism, divided by the “fittingness” of consciousness, given theism. The advocate of this form of the argument will claim that consciousness is natural and “fits” in a theistic worldview, but not in a naturalistic one due to two problems: (1) a proper understanding of scientific naturalism, the view rightly held by the vast majority of scientific naturalists; (2) genuinely emergent

properties are, in principle, unpredictable and, thus, there is no reason to think that they would exist, given naturalism.[13]

Finally, AC may be expressed as a straightforward deductive argument. It is this version of AC that I will now defend. I employ the deductive version simply to clarify the issues most effectively, but nothing dialectically hangs on the choice.

34.3.2.3 Statement and Defense of the Deductive Argument

As noted, AC may be expressed in deductive form. Here is one deductive version of AC:

1. Genuinely non-physical conscious states exist.
2. There is an explanation for the existence of conscious states.
3. Personal explanation is different from natural scientific explanation.
4. The explanation for the existence of conscious states is either a personal or natural scientific explanation.
5. The explanation is not a natural scientific one.
6. Therefore, the explanation is a personal one.
7. If the explanation is personal, then it is theistic.
8. Therefore, the explanation is theistic.

Theists such as Robert Adams[14] and Richard Swinburne[15] have advanced a slightly different version of AC which focuses on mental/physical correlations and not merely on the existence of mental states. In either case—the sheer existence of conscious states or their lawlike correlation with physical state—we have what appears to be a contingent fact(s).

Premises (2), (4), and (5) are the one most likely to come under attack. We are granting (1) for the sake of argument.

Outside of the results of a libertarian free act, contingent facts are poor candidates for bruteness. Calling them brute is merely to slap a label on them in a question-begging way if a rival theory has a good explanation for them. Contingent facts cry out for explanation.

(3) turns on the fact that personal explanation differs natural-science explanation constituted by event-causal covering law explanations underwritten by necessitating causal models that ground the covering laws. Advocates of AC employ the difference between these two modes of explanation to justify premise (3). Briefly, the argument is that given a defense of premise (4) and (5), there is no natural scientific explanation of conscious entities. Since both modes of explanation are properly used by people all the time, there is no reason to take mental entities as brute facts (especially since they are contingent) and there is precedent for proffering a personal explanation for them.

Premise (7) seems fairly uncontroversial. To be sure, Humean style arguments about the type and number of deities involved could be raised at this point, but these issues would be intramural theistic problems of small comfort to naturalists. That is, if the explanation for finite conscious minds is supernatural, then naturalism is false. (4) will be examined in the next section when I take up two versions of the most important rival to AC, especially to (4): naturalistic emergentism and some version of panpsychism.

That leaves (5). At least four reasons have been offered for why there is no natural scientific explanation for the existence of mental states (or their regular correlation with physical states):

(a) *The uniformity of nature*. Prior to the emergence of consciousness, the universe contained nothing but aggregates of particles/waves standing in fields of forces relative to each other. The story of the development of the cosmos is told in

terms of the rearrangement of micro-parts into increasingly more complex structures according to natural law. On a naturalist depiction of matter, it is brute mechanical, physical stuff. The emergence of consciousness seems to be a case of getting something from nothing. In general, physico-chemical reactions do not generate consciousness, not even one little bit, but they do in the brain, yet brains seem similar to other parts of organisms bodies (e.g., both are collections of cells totally describable in physical terms).

How can like causes produce radically different effects? The appearance of mind is utterly unpredictable and inexplicable. This radical discontinuity seems like an inhomogeneous rupture in the natural world. Similarly, physical states have spatial extension and location but mental states seem to lack spatial features. Space and consciousness sit oddly together. How did spatially-arranged matter conspire to produce non-spatial mental states? From a staunch-naturalist point of view, this seems utterly inexplicable.

(b) *Contingency of the mind/body correlation.* The regular correlation between types of mental states and physical states seems radically contingent. Why do pains instead of itches, thoughts or feelings of love get correlated with specific brain states? No amount of knowledge of the brain state will help to answer this question. For the naturalist, the regularity of mind/body correlations must be taken as contingent brute facts. But these facts are inexplicable from a robust naturalistic standpoint, and they are radically *sui generis* compared to all other entities in the naturalist ontology. Thus, it begs the question simply to announce that mental states and their regular correlations with certain brain states is a naturalistic fact. As naturalist Terence Horgan acknowledges, “in any metaphysical framework that deserves labels like ‘materialism’,

‘naturalism’, or ‘physicalism’, supervenient facts must be explainable rather than being *sui generis*.”[16]

Since on most depictions, the theistic God possesses libertarian freedom, God is free to act or refrain from acting in various ways. Thus, the fact that the existence of consciousness and its precise correlation with matter is contingent fits well with a theistic personal explanation that takes God’s creative action to have been a contingent one. God may be a necessary being, but God’s choice to create conscious beings and to correlate certain types of mental states with certain types of physical states were contingent choices**,** and this fits nicely with the phenomena themselves.[17]

(c) *Epiphenomenalism and causal closure*. Most naturalists believe that their worldview requires that all entities whatever are either physical or depend on the physical for their existence and behavior. One implication of this belief is commitment to the causal closure of the physical. On this principle, when one is tracing the causal antecedents of any physical event, one will never have to leave the level of the physical. Physical effects that have causes have only physical causes. Rejection of the causal closure principle would imply a rejection of the possibility of a complete and comprehensive physical theory of all physical phenomena—something that no naturalist should reject.

Thus, if mental phenomena are genuinely non-physical, then they must be epiphenomena—effects caused by the physical that do not themselves have causal powers. Thus most naturalists agree with Cosmides and Tooby who recognize that epiphenomenalism follows from faint-hearted naturalism: “The brain is a physical system whose operation is governed solely by the laws of chemistry and physics. What does this mean? It means that all of your thoughts and hopes and dreams and feelings are produced by chemical reactions going on in your head.”[18]

Further, James Rachels reminds us that naturalistic evolutionary theory alone undermines the reality of free will: "... aspects of our behavior which we previously thought were matters of free choice are really the products of deep, genetically controlled forces... Humans may fabricate all sorts of other reasons for what they do, but these are mere rationalizations. In reality their conduct is fixed by the territorial imperative built into their genes..."[19]

Unfortunately, epiphenomenalism is false. Mental causation seems undeniable and, thus, for the naturalist, the mental can be allowed to have causal powers only if it is in some way or another identified with the physical. The admission of epiphenomenal non-physical mental entities may be taken as a refutation of naturalism. As naturalist D. M. Armstrong admitted,

> I suppose that if the principles involved [in analyzing the single all-embracing spatio-temporal system which is reality] were completely different from the current principles of physics, in particular if they involved appeal to mental entities, such as purposes, we might then count the analysis as a falsification of Naturalism.[20]

(d) *The inadequacy of evolutionary explanations*. Naturalists are committed to the view that, in principle, evolutionary explanations can be proffered for the appearance of all organisms and their parts. It is not hard to see how an evolutionary account could be given for new and increasingly complex physical structures that constitute different organisms. However, organisms are black boxes as far as evolution is concerned. As long as an organism, when receiving certain inputs, generates the correct bodily outputs under the demands of fighting, fleeing, reproducing and feeding, the organism will survive.

What goes on inside the organism is irrelevant and only becomes significant for the processes of evolution when an output is produced. Strictly speaking, it is the output, not what caused it, that bears on the struggle for reproductive advantage. Moreover, the functions organisms carry out consciously *could just as well have been done unconsciously*. Thus, both the sheer existence of conscious states and the precise mental content that constitutes them is outside the pale of evolutionary explanation. As Howard E. Gruber explains:

> the idea of either a Planful or an Intervening Providence taking part in the day-to-day operations of the universe was, in effect, a competing theory [to Darwin's version of evolution]. If one believed that there was a God who had originally designed the world exactly as it has come to be, the theory of evolution through natural selection could be seen as superfluous. Likewise, if one believed in a God who intervened from time to time to create some of the organisms, organs, or functions found in the living world, Darwin's theory could be seen as superfluous. Any introduction of intelligent planning or decision-making reduces natural selection from the position of a necessary and universal principle to a mere possibility.[21]

We have looked at four reasons why many scholars, including most naturalists hold that robust naturalism requires the rejection of consciousness construed along dualist lines. Speaking of the conjunction of naturalism and evolution, naturalist Paul Churchland asserts:

> The important point about the standard evolutionary story is that the human species and all of its features are the wholly physical outcome of a purely physical process… If this is the correct account of our origins, then there seems neither need, nor room, to fit any nonphysical substances or properties into our theoretical account of ourselves. We are creatures of matter. And we should learn to live with that fact.[22]

Space considerations do not allow a more elaborate defense of AC here. I have done that elsewhere.[23] In closing out part I, it is time to turn to an interaction with the chief rival to AC.

34.3.3 The Chief Rival to AC: Two Versions of Emergentism

For those who accept the existence of genuine, irreducible, non-physical conscious states and, at the same time, desire to avoid AC, the most popular current strategy is to appeal to some sort on natural emergence. In the contemporary literature, this strategy takes two forms: a strictly naturalist account and some sort of panpsychism view. Let's examine these in order.

34.3.3.1 Naturalistic Emergence

34.3.3.1.1 The View

For those who accept the existence of genuine, irreducible, non-physical conscious states and, at the same time, affirm naturalism, the most popular current strategy is to adopt faint-hearted naturalism and weak (usually, non-reductive) physicalism, and appeal to some sort on natural emergence. According to the staunch-naturalist account, matter at the most basic level—the micro-physical level—is or will be completely describable within an ideal physics. Staunch naturalist David Papineau's view on this matter is canonical. While acknowledging that progress towards an ideal science may require categorial supplementation of some sort, he maintains the following: "What I do not believe is that they [the current categories of physics plus future supplementation] will need supplementation by psychological categories."[24] (brackets mine)

That ideal description will be able to capture the complete nature of basic matter, and it will entail that basic matter is bereft of any mental entities, including mental potentialities. So far, a faint-hearted naturalist would agree. But they go on to affirm that when matter reaches a certain, "appropriate" level of complexity, a completely new range of *sui generis* emergent properties, especially conscious properties, appears from a subvenient base wholly bereft of, say, consciousness or conscious potentialities.

34.3.3.1.2 Seven Criticisms of the View

For one thing, an appeal to emergent properties has always seemed suspect to us if it is offered as an explanation: *"emergence" is not a solution but a name for the problem to be solved*. It is a label and that's all. This is especially problematic in the current setting in which scientists and philosophers of science are increasingly elevating the importance of seeking mechanisms that explain how and why emergent properties appear under a specific set of base conditions.[25]

In a closely analogous context, Jaegwon Kim has convincingly made the same charge against the widely employed, but illicit inclusion of dependency in strong property supervenience.[26] He notes that such supervenience is merely a principle of property covariance and, as such, it is consistent with a number of disparate views on the mind/bod problem, including some forms of property or substance dualism. According to Kim, supervenience is a name that provides no explanation of the problem to be solved, viz., the dependency and determination of the mental on or by the physical.

Kim's solution to this problem is to adopt a version of physicalist functional realizationism, which entails that mental properties supervene on physical properties because physical properties realize mental properties. The point here is not to evaluate Kim's solution; rather, it is to acknowledge his charge that slapping the label "supervenience" on the problem of mental dependency on the physical is no solution at all. Some further theoretical work is needed to satisfy that problem-solving need.

We are making the same claim about the naturalist employment of "emergence." We recognize that supervenience *tout court* is not precisely analogous to genuine emergence, but we think that Kim's treatment of strong property supervenience *is* appropriately analogous to our claim about "emergence." In fact, employment of "emergence" is more egregious than using "supervenience" in the Kim discussion. Why? As Kim shows, at least the "supervenience" case allows room for the development of an explanatory theory that ground and explains why supervenient dependency is true.

But this is not possible for emergent properties. Recall that until the middle of the twentieth century, emergent properties were characterized epistemically, e.g., as being unpredictable from exhaustive knowledge of base entities. Genuine emergence is the Platonic Form of being a brute fact, and as such, there is *in principle* no deeper explanation of emergent properties on offer. Consequently, the charge that emergence is just a name for the problem and not a solution seems justified.

In the present context, while we do not agree with his overall position, it is instructive to notice an important aspect of Colin McGinn's Mysterian Naturalism.[27] According to McGinn, given that there is no knowable, current naturalistic solution to the origin of consciousness or its regular correlation with matter, a due to our cognitive limitations, he sets out two conditions for

any solution to these issues: (i) It must be a naturalistic solution. (ii) It must depict the emergence of consciousness and its regular correlation with matter as necessary and not contingent facts.

More specifically, there must be three kinds of unknowable natural properties that solve the problem. First, there must be some general properties of matter that enter into the production of consciousness when assembled into a brain. Thus, all matter has the potentiality to underlie consciousness. Second, there must be some natural property of the brain he calls C* that unleashes these general properties under the right conditions. Third, just as the brain must have a hidden unknowable structure that allows consciousness to emerge from it, so consciousness must have a hidden unknowable essence that allows it to be embedded in the brain.

The important point about McGinn's project is this: He recognized that a naturalist employment of "emergence" to solve the problems of the origin of consciousness and its regular correlation with matter is no explanation at all. It is what needs to be explained, and on this basis, McGinn offers a solution for the label. The simple fact that he does this demonstrated his recognition of the explanatory vacuity of the label "emergence."

Second, as a label, it leaves open other empirically equivalent views, viz., that the new kind of property may reside in a new substance (see below) or be due to God's regular intention that the property appear under the same set of circumstances. As Timothy O'Connor has noted, if the lawlike link between occurrent subvenient properties and their associated occurrent emergent properties is contingent, then the only adequate explanation for the link and the appearance of an emergent property is God's direct activity and stable intention that things be so.[28] Now it seems to many, perhaps most philosophers who do not start with a prior commitment to strong physicalism, that

the link is indeed contingent, as seen in several well-known thought experiments.

Third, emergence seems to be a case of getting something from nothing, a case of magic without a Magician. If matter is relevantly similar to what current physics and chemistry tell us, and it is bereft of any sort of mental entities, then matter does not have mental potentialities. Given this, it is hard to see how the mere spatial rearrangement of, say, atomic simples to form a more complicated spatial structure would be an adequate cause for bringing into existence *ex nihilo* a completely new sort of entity. This is the problem mentioned above that is allegedly solved by panpsychism.

Fourth, this "*ex nihilo*" problem generates another difficulty with emergent properties. To see this, consider the following tensed definition of coming-to-be:[29]

B_t "X comes into existence" = Df. There is a property P which is such that *x* has P, and there is no property Q which is such that *x* had Q.

Stated without tense, we have:

B_{nt} "X comes into existence at *t*" = Df. There is some property P which is such that P*x* at *t*, and for all times *t*' prior to *t* there is no property Q such that Q*x* at *t*'.

Now, consider two contrasting views of the composition of time:

Instantism: Temporal intervals are derived and not fundamental entities. Intervals are wholly grounded in dimensionless instants.

Intervalism: There are extended temporal intervals that are fundamental in that they are not wholly grounded in the existence of other entities.

B_t and B_{nt} are meant to capture the idea that coming-to-be takes place at an instant of time and is not an extended temporal

process or interval. For example, it is not a case of alteration or elanguescence. Further, these definitions, while consistent with Instantism, do not entail it. If, as seems likely, intervals have unextended boundaries at their beginning, then coming-to-be would occur at that boundary.

The important point about B_t and B_{nt} is the fact that coming into existence is not a process that could be governed by something, e.g., a (Non-Humean) law of nature. Thus, there is no naturalistic or scientific explanation whatsoever as to why one entity comes into existence at *t* as opposed to any other entity. Under the view being considered, emergence is a case of coming-to-be and, as such, it leaves as a bizarre brute fact that whenever matter is at such and such configuration, the same emergent property—e.g., being painful—always emerges. Besides the fact that the naturalist ontology is now chock full of emergent brute facts—an embarrassing state of affairs, given the naturalist's favorite trump card of ontological parsimony—there is also the improbable coincidence, bordering on a miracle, that the same emergent property always seems to show up when the same physical conditions obtain.

Galen Strawson has given considerable reflection to our last two points.[30] He notes that when it comes to liquidity or heat (the objective physical phenomena and not our experience of them), we have a good story to tell as to how and why these (structural) properties regularly appear under certain circumstances. But when it comes to genuinely emergent properties—he focuses on experiential phenomenal qualia—due to the nature of emergence, there is in principle *no* story to tell.[31] He argues that if it is actually the case that Y is emergent from X, then in the relevant sense, it must be the case that Y is wholly dependent on and only on X such that all the features of Y can be metaphysically intelligibly traced, in principle, back to X. If this is not true, then Y did not emerge solely from X.

Based on this, Strawson rightly asserts that emergence cannot be brute since, given the nature of brute emergence, there is absolutely nothing about the intrinsic or relational nature of X in virtue of which Y emerges. That's simply what it means for emergence to be *brute*. *Brutality rules out nothing.* Thus, Strawson concludes, emergence is a genuine miracle, not only because it is the appearance of something from nothing in virtue of which it appears, but given that whenever the "appropriate" physical conditions obtain, there is also the bizarre fact that the very same emergent property appears.

But this is utterly ridiculous and, indeed, incoherent. In principle, there is no reason why, says Strawson, a negative number couldn't emerge from the addition of certain positive numbers. We could not agree more with his summary warning: "If someone says he chooses to use the word "emergence" in such a way that the notion of brute emergence is not incoherent, I will know that he is a member of the Humpty Dumpty army and be very careful with him."[32]

Fifth, sorites problems are lurking in the neighborhood. Could this emergent property, say being painful, be instantiated with one less atomic part in the subvenient base when that base is the "right" level of complexity? Surely the answer is yes. How about two fewer atomic parts? And so on. *Sans* ontological vagueness, at some point, the emergentist must say that the subtraction of one small atomic part has a huge, disproportionate metaphysical effect on the disappearance of the emergent property or properties, e.g., all the irreducibly mental properties constituting consciousness.

But how can such a significant metaphysical effect be due to such an insignificant cause? And why was the "right" level of complexity one atomic part greater than the actual level? And the very notion of "the right level of complexity" is completely vacuous. Why? What is the right level? Answer: Whatever the

level is when the emergent property appears. When does the emergent property appear? When the right level of complexity is reached. There are no independent criteria for identifying the "right level of complexity" besides the appearance of the so-called emergent property.

Sixth, David Chalmers has raised an additional difficulty with SE, given some form of panpsychism or (we add) strict Microphysicalism—the Quality Combination Problem (viz., plausibly explaining how microqualities combine to yield macroqualities, e.g. phenomenal and moral ones?) of which one especially difficult version is the Palette Problem (viz., Given the highly likely fact that there is a limited number of microqualities but a large number of macroqualities, how do so many get yielded from so few resources?)[33]

Chalmers considers unfavorably two attempts to resolve the Palette Problem: *small-palette solutions* (somehow, all macroqualities can be generated from just a few microqualities, if we find the right ones with sufficient flexibility and generality) and *large-palette solutions* (the full range of macroqualities are included among the microqualities). Against the former, Chalmers argues that it is pure speculation and still faces the Quality Combination Problem (viz., How do microqualities combine to yield simple macroqualities?).

Against the latter, Chalmers argues that there are, in fact, only a small number of causally efficacious microqualities; the rest will be epiphenomenal. To deny this, one would have to deny the simplicity of physics with a small number of properties and laws. Says Chalmers, "Large-palette solutions seem once again to be stuck with a form of epiphenomenalism or radical revision of the fundamental dynamics of the physical world."[34]

Seventh, Ben Page has advanced a Bayesian argument to the effect that the obtaining of the correct conditions for the emergence of consciousness (and, presumably, moral

properties) in the actual world is substantially more likely, given theism compared to atheism.[35] Thus, these emergent properties (if there are such) confirm theism over atheism.

The prior probability of conscious (and intrinsically valuable) beings is higher on theism than on atheism. Why think this? Page argues that given theism, God would have several reasons for creating conscious (and intrinsically valuable) beings and bring about whatever conditions that would do the job: It seems likely that God would want to have love relationships with creatures, to share His knowledge and what it is like to experience various things with them; being creative, God desires to create all kinds of beings, those like human persons being one kind, and so on. But on atheistic hypotheses about the actual world, there is no such being who wants to bring about creatures like us. There is no teleological driving force behind the laws of nature that would be directed towards bringing about the conditions required for our coming-to-be.[36] Rather, on atheist worlds with laws like ours, it is extremely unlikely that those conditions would obtain.

34.3.3.2 Panpsychism

In light of these problems, especially the difficulty of getting something from nothing, some philosophers such as Thomas Nagel and David Skrbina have turned to a version of panpsychism to explain the origin of consciousness.[37] Roughly, panpsychism is the view that either all objects or all basic entities (those without proper parts) or some relevant range of basic entities have phenomenal consciousness, e.g., there is a what-it-is-like to be an electron (strong panpsychism) or an attenuated, incipient form of consciousness (weak panpsychism). When these relevant entities are brought together in the right way,

unified macro-consciousness as we find in humans and sentient animals emerges. It does seem that panpsychism solves the problem of getting something from noting since, in some way or another, macro-consciousness emerges from and is the actualization of micro-consciousness. Still, there are several problems that, taken together, justify a rejection of panpsychism.

First, as I observed earlier, the mental-physical link's contingency is a serious problem, one that panpsychism does not solve. Again, panpsychism is merely a label for and not an explanation of the phenomena to be explained. As Geoffrey Madell notes, "the sense that the mental and the physical are just inexplicably and gratuitously slapped together is hardly allayed by adopting … a pan-psychist … view of the mind, for [it does not] have an explanation to offer as to why or how mental properties cohere with physical."[38]

Second, as I have argued elsewhere, setting aside issues about the origin of macro-consciousness, theism is more rational than panpsychism.[39] If that is right, then since AC is intertwined with a superior theistic worldview when compared to contemporary versions of panpsychism, and since theism explains the contingency of the link between conscious and physical states while panpsychism does not, AC is preferable to panpsychism. Moreover, panpsychism faces the sorites problem regarding the precise conditions for macro-consciousness to appear.

Finally, it is hard to see how the holistic, non-atomistic unity of consciousness can be explained by the mere rearrangement of entities with attenuated or fully actualized consciousness. Surely such a process would produce an aggregate of independent conscious base entities such that there would be no unified whole "over and above" these entities that would have its own what-it-is-like, its own first-person point of view.

This issue is called the combination problem. This may be the most serious difficulty facing panpsychism and, in my view, panpsychists have failed to solve it.[40]

David Chalmers has identified seven different combination problems two of which are relevant to my present critique:[41] *The Unity Problem* (viz., How can myriads of micro entities—brute physical particles or microexperiences—combine to yield a unified consciousness?) and *The Grain Problem* (viz., How do micro entities—brute physical particles or microexperiences—combine to yield simple, homogeneous experiences of red (e.g., simple phenomenal and moral properties) instead of an enormous jagged array of distinct particles or qualities?).[42]

These combination problems stand unresolved as Chalmers admits: “It is fair to say that no proposed solution has yet gained much support…”[43] Again, “I conclude that both panpsychism and panprotopsychism suffer from serious combination problems.”[44]

In Section 34.3 , I have tried to present, clarify and defend an argument for God from consciousness. In Section 34.4, I seek to provide a positive account of what it means for God (angels, human/animal souls) to be a spirit.

34.4 Consciousness and God as a Spirit

34.4.1 Consciousness and a Positive Characterization of a Spirit

34.4.1.1 What Is a Spirit?

It is widely thought that the concept of a spirit—especially a pure spirit such as God—can only be characterized in a negative way as an immaterial or incorporeal substance and, as a result, the

concept is vacuous and unintelligible. Thus, Paul Churchland speaks for many when he laments that employing “nonphysical” in characterizing a soul “leaves us wanting to ask for something more in the way of a *positive* characterization of this proposed mind-stuff.”[45] Later, I will criticize the idea that substance dualists employ mind-stuff to assay a spirit/soul (I use these interchangeably). For now, the important aspect of Churchland’s complaint is the need for a positive characterization to make sense of the notion of mind, spirit, and so on.

In response to Churchland and his ilk, it may help to present a relevant discussion about the adequacy of a wholly negative characterization of a spirit offered by Joshua Hoffman and Gary S. Rosenkrantz.[46] They define a soul “as a non-spatial substance that has mental properties.” Next, they consider this objection that the very concept of a soul is unintelligible:

> A first argument protests that the nature of a soul cannot be fully explained without describing it in *negative terms*, i.e., as *unlocated*. We reply that there are intelligible *physical* entities whose nature cannot be fully explained unless they are described in negative terms. For example, to fully explain the nature of a photon, a photon must be described as having *zero* rest mass (i.e., as *not* having rest mass). Of course, the basic nature of a photon includes certain other, *positive* attributes. But the same is true of the basic nature of a soul. It includes such positive attributes as *being a substance* and *being conscious* (or *being capable of consciousness*). So, if it is argued that souls are unintelligible because their basic nature is *wholly* negative, then such an argument is **unsound**.[47] (italics and bold theirs)

Two points are important. (1) They seem to assume that if a concept (or entity?) is wholly negative, then it is unintelligible. They go on to say that souls are explained (described? conceptualized?) in positive terms and, thus, are at least partially positive. It is this positive characterization that renders the concept of a soul intelligible. (2) The content of that positive

conception captures the basic nature (kindedness) of a soul and includes the categorial feature of being a substance and the essential properties of mentality/consciousness (which are used interchangeably). In this way, we employ consciousness to define a spirit. Minimally,

> A spirit $=_{def.}$ a substance that is essentially characterized by the actual and dispositional properties of consciousness.

34.4.1.2 Responding to a Defeater for This Understanding of a Spirit

Ian Ravenscroft has raised a defeater for our response, or at least for a close cousin to it:

> It might be asserted that the nonphysical mind *just is* conscious, but now the substance dualist must make that claim plausible. In addition, if it's OK to assert without argument that the nonphysical mind just is phenomenally conscious, why can't the physicalist help herself to the same move? The brain *just is* phenomenally conscious and no explanation of that fact is needed.[48]

Here are two problems with Ravenscroft's claims. First, substance dualists have offered a number of powerful arguments for why one is warranted in believing there is a soul. And these arguments also provide an understanding of what sort of a think a soul is, viz., a substance that is (at least) essentially characterized by consciousness. So, substance dualists do not simply assert that the soul just is conscious without supporting arguments. Whether or not these arguments are successful is beside the present point. Their mere presence as a crucial aspect of the substance dualist's case is enough to refute Ravenscroft's charges.

Second, Ravenscroft's complaint against the substance dualist's (alleged) assertion "that the nonphysical mind *just is* conscious" is wide of the mark. Why? Using "soul" rather than "mind" for consistency, as we saw above, it is just false to claim that there is no positive characterization of a soul. Dualists hold that it is a substance essentially characterized by the actual and dispositional properties of consciousness (and, perhaps, life). Some would add the categorial feature of being a substance.

Ravenscroft is misled by his unfortunate view that substance dualists hold that the soul is a mass of negatively characterized stuff that *causes* conscious states. In fact, the substance dualist characterizes the soul as essentially *that which is* _____, where the blank is filled with (at least) these eight features which identify the essential desiderata for a theory of substance and a theory of SD (substance dualism) in particular:

1. The appropriate entity for exemplifying mental properties.
2. The appropriate entity for holistically unifying mental properties.
3. The appropriate entity for being an enduring mental continuant.
4. The appropriate entity for being the employer and referent of "I."
5. The appropriate entity for having an irreducible "First-Person Point of View."
6. The appropriate entity for being possibly disembodied as a unified center of consciousness.
7. The appropriate entity for ontologically grounding claims like "Necessarily, thoughts have thinkers."
8. The appropriate entity for exercising active power and teleologically guiding a deliberative process towards an end.

To unpack this last claim in more detail, let us begin with an analysis of the notion of a substance. For present purposes, let us grant that a substance is a primitive, underived unity of actual and potential properties or inseparable parts at a time, it sustains absolute sameness through accidental change, and so on regarding the list above, and it has an essence that answers the most fundamental question “What kind of thing is this?” and that grounds its membership in its *infimae species*.

Note that the characterization just given is a formal one. It merely states necessary conditions for anything to count as a substance without giving material content necessary to distinguish one kind of substance from another. When it comes to characterizing specific kinds of substances, it seems that there are only two things one can do: state the formal criteria and list a set of dispositions or potentialities as the material content for a specific kind of substance.

For example, a characterization of an atom of gold as a substance might go something like this: It is a primitive, underived unity of actual and potential properties or parts at a time (having the power to melt at such and such degrees, dissolving in *aqua regia*, having atomic number 79 which, in turn, is the potential to attract a certain number of electrons, to resist inertial changes to degree thus and so), it sustains absolute sameness through accidental change, and it has an essence (being gold) that answers the most fundamental question “What kind of thing is this?” and that grounds its membership in its *infimae species* (the natural kind *gold*).

Note that the formal aspects of the definition of gold (in italics) are the same as those for a substance in general and the material content is listed in parentheses. Note also that the various properties listed in parentheses that give material content to “being gold” so as to distinguish it from “being iron” are dispositions or potentialities, properties that are actualized

by a gold atom if certain conditions obtain (e.g., gold will dissolve in *aqua regia* if placed in it).

Substances are more than their potentialities; indeed, it seems reasonable to say that something can have potential properties only if it has actual properties: No potentiality without actuality. Unfortunately, it is very difficult to say what the essence of a substance is without referring to its potentialities. Thus, we say that gold is *that which* has the potential to do x, y, and z; iron is *that which* has the potential to do f, g, and h; and so on.

By way of application, it is hard if not impossible to characterize adequately a specific kind of substance without reference to the various potentialities it has by its very nature and which distinguish its essence from the essences of other kinds of substances. That's just the way it seems to be with substances. Still, it is important to realize that the dispositional nature of the material content of a substance is true of physical substances just as much as spiritual substances. The problem here resides in limitations of characterizing substances in general, and is not uniquely troublesome to souls. The important point is this: Whether they involve potential or actual properties, all of the eight items listed above imply that a soul or spirit is essentially characterized by consciousness and that is what constitutes the positive characterization of a soul/spirit.

34.4.2 Responding to the Explanatory Impotence of Substance Dualism and Soul Stuff

34.4.2.1 Substance Dualism and Explanatory Impotence

A frequently raised objection to SD is the claim that postulating a soul provides no explanation at all for the occurrence of

conscious states. Thus, Ian Ravenscroft claims that there are "… six general features of mental life which a good theory of mental states should be able to explain (or explain away)."[49] Here are the first three features:

1. Some mental states are caused by states in the world.
2. Some mental states cause actions.
3. Some mental states cause other mental states.

Ravenscroft concludes that SD is virtually silent in explaining (1)–(3) with no answer forthcoming. Along similar lines, against the charge that we have no clue as to how consciousness could arise in a physical object, Jaegwon Kim replies: "Is it any easier to understand how thoughts and consciousness can arise in an immaterial substance… Understanding how mentality can arise in something immaterial may be no easier than understanding how it could arise in a material system; in fact, it might turn out to be more difficult."[50] Finally, Eric Olson declares that

> [I]t may well be that we cannot conceive how a material thing could produce thought or consciousness. But then we can no more conceive how an *im*material thing could produce thought or consciousness; no matter how carefully we reflect on the notion of an immaterial substance, we shall never find anything that could account for thought or consciousness.[51]

34.4.2.2 Substance Dualism and Soul Stuff

Tim Bayne complains that with respect to a wide range of problems regarding consciousness, while physicalism has a solid track record of explaining such problems and offers the prospect of accounting for further difficulties, "[s]ubstance dualists offer no answer to these questions…"[52] This objection is often raised in conjunction with the notion that a soul is composed of some

sort of mind stuff which is virtually impossible to characterize and, if described, it is done so by an inadequate negative one (e.g., immaterial). To cite one example, Ravenscroft complains that SDs "… assert that nonphysical mental stuff is conscious; they do not tell us what it is about nonphysical stuff that facilitates [causes] consciousness."[53] (brackets ours) What should a SD say in response to these charges?

34.4.2.3 A SD Response to These Objections

These associated objections express a very deep and troubling confusion on the part of such physicalists. In short, when used as an explanatory postulate (and many SD's are such because they take themselves to be directly aware of their simple souls), SD is not intended to provide a *causal explanation* for the occurrence of conscious states; SD is an ontological theory as to what sort of thing must an entity be to capture adequately a number of features, not of mental states, but of the thing that *has* them. The eight desiderata mentioned above constitute (most of) those features.

Employing a longstanding, doggedly persistent notion in the history of philosophy—that of a classic substance—the substance dualist refuses to reduce or eliminate these eight features, and instead, uses that notion to explain and ontologically ground them in a fundamental substance. Whether correct or not, the substance dualist holds to a substantial soul because it is the best ontological account of these features, not because a soul best explains what causes conscious states. Thus, *the soul is essentially constituted by actual and dispositional powers of consciousness (and, for some, life) and is not the cause of mental states except in the case of certain agent-causal libertarian acts.* It follows that these complaints about explanatory impotence

address a strawman. It also follows that a soul's essential constituents are properties and property-instances, and not stuff. To our knowledge, the only contemporary substance dualist who embraced soul-stuff was Richard Swinburne, a position he has subsequently abandoned.[54]

34.4.3 A Response to Andrew Bailey's Contingent Physicalism and the *Via Negativa*

Andrew Bailey has defended two theses that provide defeaters for my account of a spirit: First, he advocates contingent physicalism such that a wholly material thinking object (e.g., a human animal) in this world could be a wholly immaterial thinking object (e.g., a spirit) in another world.[55] Second, he claims that by employing a *via negativa* and his particular set of categories, one can say what a spirit is without needed a positive characterization of it, e.g. in terms of consciousness.

34.4.3.1 Bailey's Position

Here are four main features of Bailey's approach.

1. Bailey on the Categories. In developing his set of ontological categories, Bailey locates himself squarely within "the grand tradition of ontological categorizing" (p. 20) and is "doing old-fashioned categorial ontologizing" (p. 21). Accordingly, he seeks a realist set of categories that provides "the highest level of abstraction how reality divides" (p. 5). The categories he employs are these: Level I: Thing; Level II: Abstract Property and Concrete Object; Level III: branching out of Concrete Object are Material Object and Immaterial Object. Level IV: branching out of

Material Object are Thinking Material Object and Unthinking Material Object; branching out of Immaterial Object are Thinking Immaterial Object and Unthinking Immaterial Object. Finally, a curious feature of Bailey's categories is that in at least some cases, members belong to their category contingently such that they could change categories. Specifically, items in Thinking Material Object in one world belong to Thinking Immaterial Object in another world.

2. Bailey's characterization of Wholly Material and Wholly Immaterial Objects. While acknowledging that he lacks "rigorous definitions of the key ideas here—mentality, material object, human person, spirit" he offers remarks about them he takes to be adequate for his purposes. For Bailey, x is a wholly material object iff at some level or other x is composed of items (separable parts?) with narrow physical properties (those figuring into fundamental physics) none of which thinks (pp. 10–11). As far as we can tell, Bailey almost entirely characterizes a spirit by way of the *via negativa*, viz., as an incorporeal or immaterial object.

   However, in one place he says that spirits are such that at every level they have parts (or are themselves things) that either think or are not characterized by fundamental physics. Since it seems that the only kind of part with which Bailey is familiar is a separable part, by granting the possibility that a spirit has parts, indeed, thinking parts, his view is at odds with most dualists, including us, about the simplicity of a spirit.[56] In any case, we don't know how important this depiction is because in virtually all other cases, he uses "immaterial" or "incorporeal" thing. In fact, on the same page as the depiction just mentioned, Bailey returns to his usually pattern and tells the reader that from

that point on, he will use “wholly immaterial” or “wholly incorporeal” thing for what it is to be a spirit.

3. Bailey’s Thought Experiment in defense of Contingent Physicalism: According to Bailey, our nature is highly contingent (p. 9) and, more specifically, in the actual world we are wholly material objects, yet there is a possible world in which we are wholly immaterial objects. He invites us to consider the following thought experiment (p. 14):
   Imagine that Bradley, a living organism, has at some very low level a multitude of tiny physical parts we’ll call atoms. Through a complex network of integrated and united causal dispositions, the atoms compose Bradley. In general, a sufficient condition for some atom x to be a part of a living organism O is for x to play a functional role in O’s complex network of causal relations.
   One day, God annihilates one of Bradley’s atoms and replaces it with an angelic surrogate that is functionally equivalent with the replaced atom and, thus, exhibits the integration and unity to count as a part of Bradley. God continually reiterates this process, atom by atom, angel by angel, until after a few months, the transformation is complete. Since angels are wholly immaterial objects, Bradley is now at some low level, entirely composed if immaterial parts, and thus, Bradley is a wholly immaterial object. Throughout the process, Bradley sustains absolute Leibnizian identity.
4. Bailey’s Assumptions: Bailey lays out four assumptions on which his argument rests (p. 14). Three of them are relevant to our discussion.
   1. Causal interaction makes for parthood. If something assumes an atom’s role in a complex network of causal relations, it becomes part of the greater whole just as the atom was.

2. If something is, at some very low level, composed entirely of wholly immaterial thinking beings, then it is itself wholly immaterial.
3. Something can survive the replacement of one part with a surrogate that exactly duplicates that part's causal role.

34.4.3.2 A Critique of Bailey's Position

What should we make of Bailey's argument? It is surely a fun, creative and interesting bit of philosophy. But in our view, there are several problems with it and, in the end, we judge it a failure to establish the truth or plausibility of contingent physicalism. Many of the objections raised above against Merricks's view apply with equal force to Bailey's contingent physicalism. We will not repeat them unless a unique aspect of Bailey's views presents an occasion to restate one of these arguments from a slightly different perspective. Further, while we want to gather into one place the general problems with Bailey's view, some of our points are developed more fully elsewhere in this book. In those cases, our remarks will be brief and we will direct the reader to other parts of the book where a fuller presentation is found. With that said, here are four general problems with Bailey's position some of which have a few distinct subpoints.

First, Bailey's depiction of living organisms, especially human persons, is question-begging and, most likely, false in the actual world. Bailey appears to be presenting a view of living organisms as they are in the actual world. We say this for two reasons. First, he claims that his categories, especially those with human persons as items, present truths about us (p. 5), specifically, contingent truths that are actually true of us even though they

could have been false (p. 6). And by pointing to a node, you say "that's our place in this world" (p. 6).

Second, Bailey's thought experiment is intended to show that a wholly material being could become a wholly immaterial being. Since he is a physicalist—specifically, an animalist of some sort—he holds that we are actually wholly material objects. And while his thought experiment is diachronic in that a wholly material object *becomes* a wholly immaterial object, given its speculative nature, we think the real point of the experiment, especially as a defeater of a modal argument for SD, is that just because there is a possible world in which we are wholly immaterial, it does not follow that in the actual world we are not wholly material.

If we are right about this, then Bailey begs the question against many dualists like us in his depiction of the soul or of biological organisms. Many (especially Cartesian) substance dualists do not grant that we are identical to living organisms. Rather, we are simple souls. Moreover, staunch hylomorphic dualists do not think that living organisms are composed of separable parts; rather, they are substances that have no such parts and, thus, a compositional, part-whole approach to organisms is wrong (see Chapter 7 for a presentation and defense of versions of staunch hylomorphism).

In fact, Bailey's depiction of living organisms appears to be an expression of some version of animalism or a close cousin (e.g., a modified form of constitutionalism), and we have subjected animalism to substantial criticism in Chapter 6. Bailey does not so much as mention in a footnote that there are problems with his depiction and alternatives to it. Thus, his appropriation of that depiction is question-begging and far too quick. We need an argument here, especially since his thought experiment and other relevant claims are presented as capturing the truth in the actual world.

Further, I have argue elsewhere and provide significant documentation that well-developed philosophical versions of staunch hylomorphism and scientific versions of organicism have greater explanatory power than does the depiction of organisms presented by Bailey and, thus, the latter is most likely false.[57]

Second, Bailey's living organisms are mereological aggregates (MAs). Accordingly, they inherit a number of successful defeaters for such aggregates and for his appropriation of them in building his case. In Chapter 7, we presented a thorough depiction of a MA, and we pointed out a number of problems with wholes construed as such. Recall that, setting aside eliminativism, a MA is a weakly unified collection of separable parts that stand in external relations to other such parts, they and their parts exhibit efficient causality and play no functional role (nor exemplify irreducible teleology) which is intrinsic to the wholes or their parts, and are depicted, at least in the actual world, as exhibiting part priority, usually, micro-physical part priority. By "weakly unified" we mean (i) synchronically, the parts exist and have their identity independently from the whole of which they are parts and, thus, the whole is merely a collection of parts standing in certain external (e.g., causal) relations. On a sliding scale from mere heaps (e.g., a heap of sand) to classic Aristotelean substances, mereological aggregates fall towards the heap end of the scale; (ii) diachronically, MAs are subject to mereological essentialism constraints regarding endurance and perdurance.

I take Bailey's characterization of living organisms to render them MAs, though I do not believe that was his intent. He would most likely reject my view on this. However, I claim that they are MAs because (i) Bradley is his paradigm case, and it is clear that from First Bradley (who is wholly material) to Final Bradley (who is wholly immaterial) the collection of parts throughout the

transformation involve separable ones; (ii) the causal relations must be external if the relata (the separable parts) retain their identity whether or not they are in the whole or standing in these relations; (iii) Bailey claims that the parts play a functional role, but his metaphysics of living organisms disallows them to do so literally. MA's are like artifacts: They (or their parts) may be described with functional concepts, but these are ways of *taking* the relevant entities, of describing them *as if* they exhibited functionality. The employment of such concepts does not literally ascribe intrinsic functional roles or teleology to the MAs or their parts. Remember, the relevant "complex network of causal relations" is constituted by and only by efficient causal relations. No part actually plays a role in such a network.

If this is correct, then Bailey's employment of his criterion for parthood (Causal interaction makes for parthood. If something assumes an atom's role in a complex network of (efficient) causal relations, it becomes part of the greater whole just as the atom was.) is not possible since there are no functional roles the parts literally play.

And even if somehow Bradley smuggles in literal, intrinsic functional roles/intrinsic teleology, not only would this appear to be an odd *ad hoc* move, it would also render it difficult to claim that living organisms *qua* MAs (or animals) are wholly material objects. We know of no one who treats intrinsic teleology as physical, and consequently, physicalists avoid attributing such to wholly material objects, preferring instead to give a physically acceptable reduction (e.g., aeteological) to teleology. And the paradigm cases of functional roles are cultural artifacts (being a quarterback, a dollar bill, etc.) that are constituted as such by the intention(s) of the artifact's inventor or by the culture in which the artifact is located and identified. Thus, it is hard to see how a functional role is physical. And if one responded by depicting a functional role in terms of intrinsic teleology, then such

teleology, and the metaphysical commitments within which it is plausible (e.g., substantial forms, inseparable parts) imply that living organisms are not, in fact, wholly material objects.

Third, several aspects of Bailey's thought experiment render it a failure. I offer four such aspects. To begin with, Bradley is a mereological aggregate (a collection of separable parts standing to each other in external relation-instances) and, as such is subject to mereological constraints. Since structure is constituted by an ordered aggregate of relation-instances, mereological constraints apply to structure, undermining the idea that somehow sameness of structure or "causal network" can ground endurance through part replacement. Thus, from the first part replacement to the wholly immaterial Bradley, we have a succession of different wholes which are not identical to the wholly material Bradley. So, the thought experiment is otiose.

In addition, it is far from clear that a wholly material object in this world could be identical to a wholly immaterial object in some possible world because the two objects have nothing whatsoever in common to ground or make sense of their identity. Rather than the two objects being exhaustively (wholly) constituted by their material or immaterial constituents, if "they" retain identity, it seems that their physicality and materiality are possessed by something contingently, and that something is what remains the same. It *has* physicality in one world and immateriality in another. Moreover, when one is dealing with a complex object, it is misguided to claim that identity is brute.

Bailey's thought experiment makes explicit why this is a problem. Remember, First Bradley is a wholly material object. How so? Well, given Bailey's compositional approach to analyzing Bradley, all of his separable parts are physical as are all the external causal relations exemplified between and among them. We may add physical properties that characterize these

parts and relations, and, perhaps, the whole they compose, and physical (efficiently causal) activities of the parts, as long as everything employed in the analysis is physical. Besides these constituents, there doesn't seem to be anything else to Bradley. Now think of Final Bradley, a wholly immaterial object. Every constituent of Final Bradley must be immaterial, e.g., his angelic parts, and the exertion of active power to push and pull other angels.[58]

Now ask: What possible entity do First and Last Bradley have in common that either grounds their identity or is the item that just is identical in the two stages? There is no answer to this question and, thus, no way of providing a metaphysical account of their identity. That identity is a mere assertion and, as noted, cannot be asserted as brute with propriety.

If I am correct about all this, then while genuine Aristotelean substances which have (i) inseparable parts/modes (but are not composed of separable parts) are the sorts of things that can literally be constituted by intrinsic teleology and have (ii) inseparable parts essentially characterized by their intrinsic functional roles in the whole, Bailey's living organisms *qua* mereological aggregates are capable of neither. They are like artifacts with *as if* and not real, intrinsic functional roles. This becomes clear when Bradley starts having more and more angelic parts. Note, that when an angel becomes a part of Bradley, it does not undergo substantial change and get its identity as an inseparable part from the teleological role it plays. Indeed, that role is not an intrinsic, constituting essence of the angel. Instead, the angel is *told what to do* and acts accordingly. The angel is not literally a functional entity whose role is part of its essence and identity. Rather, it functions as if it played a role.

This is importantly analogous to the way the parts of an artifact "function" in a whole, e.g. the parts of a car's engine. The engine's parts engage in mere efficient causation and do

not literally play any functional role. But the car's inventor designed the parts and by doing so, "told the parts," as it were, to behave via efficient causation in a way that an observer could act as if the part played a functional role. Maybe so in a loose and popular sense, but not in a strict, philosophical sense.

Further, the thought experiment is inconsistent with widely accepted Maximality Principles. The notion of maximality is an important one, and there seem to be two different, but closely related maximality principles relevant to assessing Bailey's thought experiment. We cite two physicalists who embrace one or both of them. The first is embraced by Hud Hudson and the second by Bailey himself.[59]

Individual Maximality (IM): x is an individual maximal object just in case, necessarily, x is an F and there are no proper parts p of x that are Fs.

Property Maximality (PM): A property P is maximal just in the case that necessarily, for any x that is F, there is no y such that x is a proper part of y and y is F.

IM is Hudson's way of stating the principle, and uses as substitutions for x human persons, persons, and living human organisms. For example, IM implies that if x is a person, then x has no proper parts that are persons. PM is stated precisely as Bailey presents it and his main example of a maximal property is being conscious. It follows that being conscious is maximal iff necessarily, for any x that is conscious, there is no other conscious particular of which x is a proper part.

It should be clear that all the iterated Bradleys after First Bradley violate IM (assuming Bradley is a person throughout the transformation process and that angels are persons, then the iterated Bradleys are persons (or the iterated Bradley is a person) and have persons as proper parts). The thought experiment also violates Bailey's own statement of PM which, remember, is a necessary truth. Assuming that Bradley is

conscious at every stage of the process, in all stages besides First Bradley, he is conscious and has proper parts (angels) that are also conscious. By violating IM and PM, it would seem that Bailey's thought experiment is in deep trouble, even by his own ontological commitments.

Finally, briefly put, Bailey's understanding of living organisms, especially as exhibited by the transformation of Bradley, will have serious difficulties accounting for the unity of consciousness (especially for Final Bradley) and the reality of evidential near-death experiments in the actual world.[60]

Fourth, Bailey defends an implausible view of categories, yet he needs this view to do important work in defending contingent physicalism. As noted above, in developing his ontological categories, Bailey locates himself squarely in the realist camp and claims to be doing old-fashioned categorial ontologizing in the grand tradition of such activity. However, there are two features of his categories that seem to be quite problematic: (1) the contingency of (at least some) items relative to their categories and (2) the identification of important nodes in purely negative terms. But of these features are important for Bradley's defense of contingent physicalism. Let's examine these in order.

Regarding (1), for various reasons, e.g., to defeat the modal argument for SD by claiming we are, possibly, disembodied, Bailey needs to employ contingent physicalism. He accomplishes this in two ways. First, his presentation of categories isolates their modality—the necessity/contingency of items belonging to their category—and he claims that contingency prevails, at least in the subcategories of Thinking Material Object and Thinking Immaterial Object. Second, his thought experiment about Bradley's transformation is used. If we understand him correctly, the contingency of the relevant subcategories is the ontological and epistemic grounds for the intelligibility of the thought experiment.

Unfortunately, categorial contingency is an extremely odd view that requires substantial warrant to be accepted. To our knowledge, no realist about categories in the history of philosophy up to the present has even considered, much less advanced such a view.[61] It is widely acknowledged that the great realists about the categories, e.g., Aristotle, Bergmann, Butchvarov, Chisholm, Grossmann, Loux/Crisp, Lowe, and Hoffman/Rosenkrantz to name a few, all held that members of categories necessarily belonged to them.[62]

I am not simply collecting votes here. Rather, these and all of the other realist philosophers whom I know are agreed on the very nature of what it was to develop a set of categories and what a category was: The goal was to discover a set of exhaustive, (often) mutually exclusive (except for the transcendentals), most basic divisions or reality into which all entities necessarily and essentially fell. A category was constituted by what its members shared essentially in common, and a category often provided essential or necessary and sufficient truth conditions for something to belong to it. Given this near unanimous understanding among categorial realists, as close to unanimity as one finds in philosophy, the idea of separating out a modal question would be otiose. The modality of membership-necessity—was already implicit in the very notion of what it was to be a category in the first place.

Bailey seems to be virtually alone in his approach. This does not automatically render his view of categories unjustified. But surely, if he is going to break with such a homogenous pack, he owes us at least a brief account of why his approach is justified and why the majority view can be abandoned. But Bailey fails to offer such an account. He simply asserts his categorial contingency thesis. This fact is made even more troubling by something Bailey himself affirms.[63] In discussing the development of categories as an aid in probing the metaphysics

of human nature and answering the question of what we are, Bailey rightly distinguishes the categorial approach from other approaches that provide true, but the wrong kind of answers to count as categorial, e.g., we are each no more than 4,300 years old, have at least one great-grandparent, and so on. These answers fail because they classify us in virtue of contingent features or features that don't get at the specific, deepest notion of what kind of thing we are. However, it seems that Bailey's contingent categorial membership fails in just this sort of way.

Regarding (2) (using purely negative descriptions of different subcategories), Bailey rejects an argument for SD based on the fact that we have a clear, adequate concept of being a soul or spirit, e.g., when we say that God is a spirit.[64] Briefly, the argument claims that we have clear concept of a soul/spirit, and for that to be the case, we need a distinct positive concept of such. If thinking matter is metaphysically possible, then the properties that constitute consciousness can characterize either a material object or a spirit. Thus, we cannot appeal to those properties for a distinct positive concept of either. Regarding a conscious material object, we have distinct positive concepts about what makes it physical, but we lack such for a spirit. Characterizing a spirit is either vacuous or by way of negation, and neither provides us with what we actually possess, viz., a distinct positive concept of a spirit. Thus, thinking matter is impossible.

At this point, it may be useful to provide a relevant discussion about the adequacy of a wholly negative characterization of a spirit offered by Joshua Hoffman and Gary S. Rosenkrantz.[65] To repeat a point noted earlier, they define a soul "as a non-spatial substance that has mental properties." Next, they consider this objection that the very concept of a soul is unintelligible:

> A first argument protests that the nature of a soul cannot be fully explained without describing it in *negative terms*, i.e., as *unlocated*. We reply that there are intelligible *physical* entities whose nature cannot be fully explained unless they are described in negative terms. For example, to fully explain the nature of a photon, a photon must be described as having *zero* rest mass (i.e., as *not* having rest mass). Of course, the basic nature of a photon includes certain other, *positive* attributes. But the same is true of the basic nature of a soul. It includes such positive attributes as *being a substance* and *being conscious* (or *being capable of consciousness*). So, if it is argued that souls are unintelligible because their basic nature is *wholly* negative, then such an argument is **unsound**.[66] (italics and bold theirs)

Obviously, if true, this claim would be bad news for Bailey. So, he provides defeaters for this sort of argument. Evaluating Bailey's counterargument is not relevant here. But one key defeater he raises is pertinent to our critique of (2). Bailey claims that the Positive Requirement (one needs a distinct positive concept of something to have a clear, adequate concept of it) is false and provides a counterexample to it. He claims that we have an adequate concept of the abstract, but no positive concept of it. Our concept is by way of the *via negativa*.

I have two responses to this counterargument. First, it is false. We do, in fact, have a positive concept of being abstract or, more importantly, of abstract things—being abstract is being necessary, being outside space and time, necessarily standing in such and such internal relations to certain entities (e.g., necessarily, redness stands in the internal relation brighter than to yellowness). And we can cite distinct positive features of abstract objects, e.g., the number two is essentially even, it is a whole integer that stands mathematically between the numbers one and three, and so on.

Second, the most employed strategy in describing abstractness is by way of listing examples of abstract objects, e.g., various logical relations, properties, numbers. Each member

of the list has positive characteristics that are the basis of satisfying the Positive Requirement.

Bailey also claims that we can easily say what, e.g., a material or immaterial object is even if it is characterized by way of the *via negativa*. How? According to the Chart Ontologist, one can say what something is by pointing to the relevant node on the chart of categories. Thus, to say what a spirit is requires only that one point to the node Thinking Immaterial Object.

Again, we think this solution fails. While we cannot enter into a discussion of the pros and cons of the *via negativa*, we note that in our view, one must know something positive about what an entity is before one can say what it is not. The *via negativa* fails in this regard and, without positive knowledge, it leads to skepticism about nature of the object characterized merely in a negative way.

Moreover, Bailey's strategy seems merely to kick the can down the road. In solving the problem of not being able to say what a spirit is without having a positive concept of a spirit, Bailey claims that by pointing to the node Thinking Immaterial Object, that adequately says what it is to be a spirit. But it seems clear, that the node exhibits the same problem of lacking a positive concept of "immaterial" as did the characterization of a spirit as an immaterial or incorporeal spirit that is not material. Unless we are missing something, we don't see a solution here to the initial concern. Until we do, we judge this strategy to be a failure.

Before leaving this topic, there is an additional consideration that we believe presents serious difficulties with Bailey's claim that pointing to a purely negative node—Thinking Immaterial Object—is all one needs to do to know what a soul/spirit is. Let us use TMO and TIO for Thinking Material Object and Thinking Immaterial Object, respectively. Adolf Reinach has observed that we often make these four judgments:[67]

1. Positive Judgment about a Positive State of Affairs: (It is the case that rose r is red.)
2. Negative Judgment about a Positive State of Affairs: (It is not the case that rose r is red.)
3. Positive Judgment about a Negative State of Affairs: (It is the case that rose r is non-red.)
4. Negative Judgment about a Negative State of Affairs: (It is not the case that rose r is non-red.)

In all four cases, a conscious side is in some way related to an objective side. Let's focus on (ii) and (iii). According to Reinach, there are two specific preconditions, two position-takings for being justified in asserting (ii). *The psychological precondition*: There is an intellectual position-taking about some other state of affairs (we apprehend r's being yellow) that is in conflict with the original state of affairs (r's being red). *The epistemic fundament*: In approaching the original state of affairs (r's being red) in light of the second state of affairs (r's being yellow) and grasping the conflict, the later appears in a new light, viz., as having an evidential character relative to the former. On this basis, we are justified in affirming (ii) (It is not the case that r is red.).

In order to avoid quantifying over negative states of affairs as in (iii), Stephen Mumford argues that in all cases of (iii), we reduce them to (ii), placing the negativity on the consciousness side.[68] Applied to Bailey, if we take TIO to be a negative judgment about a positive state of affairs (In some world w with thinking things, it is false that a material thing is thinking in w (or it is false that TMO has items in w.)), then we aren't capturing Bailey's view that pointing to a purely negative node—Thinking Immaterial Object—is all one needs to do to know what a soul/spirit is. As far as we can tell, Bailey's pointing strategy is not a mere expression of negativity, but a positive assertion if it is going to capture "what a spirit/soul is."

So, we need to examine (iii). For the sake of argument, let us grant the existence of negative states of affairs. Regarding positive judgments about negative states of affairs, Reinach claims that such judgments have two preconditions: (1) *a psychological precondition*: Focus on the negative state of affairs as such (3 is not smaller than 2) and put it into doubt. (2) *an epistemic fundament (foundation for the positive judgment)*: the apprehension of another, positive state of affairs (3 is larger than 2) such that the two states of affairs are necessarily "bound up" together and the latter is the epistemic foundation for the former.

If (iii) is what Bailey is expressing in his node-pointing strategy, then what is this positive epistemic foundation for the judgment "in w, it is true that there are thinking things that are non-material (or it is true that there are items in TIO)"? He needs a positive state of affairs to be "bound up" with "immaterial." But given his *via negativa* approach, this is precisely what Bailey cannot provide.

Bailey could respond that our argument begs the question against his approach. But even if the readers disagree with the details of Reinach's analysis, we believe we learn something from it: *Negative judgments and negative states of affairs are privations or absences, and one cannot understand or have an epistemic fundament for them without possessing an understanding, especially an evidential one, of the associated positivity*.

34.5 Conclusion

Phenomenal Consciousness has proven itself to be a recalcitrant fact for staunch and faint-hearted naturalism. And it provides the resources for proffering a positive description of a spirit/soul in general, and of what it is for God to be a spirit in particular. In

this chapter, by employing the existence of irreducible, intrinsically characterized, phenomenally conscious properties and states, I have (1) presented, clarified and defended an argument for God from phenomenal consciousness (AC) and (2) provided a positive account of what it means for God (angels, human/animal souls) to be a spirit/soul), defend this account against incoherency claims based on a purely *via negativa*, and offer defeaters for the idea that a purely negative account of a spirit is all we have and need. In this way, I have sought to contribute to a cumulative case for the existence of God and a clarification/defense of an essential feature of God, viz., His being a spirit. This is my contribution to debates about the ontology of divinity. I leave it to the reader to judge if, and to what extent, my contribution has been helpful.

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Notes

1 Th. →Nagel, 2012, Chapter 1.

2 →Madell, 1988, p. 141.

3 →McGinn, 1999, pp. 13–14.

4 →Lyons, 1995, p. lv. In context, Lyons remark is specifically about the identity thesis, but he clearly intends it to cover physicalism in general. Similarly, while he explicitly mentions an entity in the category of individual—the soul—the context of his remark makes clear that he includes mental properties and events among the entities out of step with scientific materialism.

5 →Searle, 2004, pp. 97–99.

6 →Pitt, 2004.

7 →Kim, 2006, p. 233.

8 →Strawson, 2006. The quote is from p. 3. Strawson's idiosyncratic use of "physicalism" includes irreducible experiential states as a kind of physical state, and he distinguishes his view from "physicSalism," the faith commitment that the nature or essence of all concrete reality can in principle be captured in the terms of physics. The definition of "physicalism" in the quote is not his own panpsychist one, but is more akin to physicSalism and what we mean by staunch naturalism.

9 For our purposes, we set aside the distinction between Global Naturalists (who deny the existence of abstract objects) and Local Naturalists (who accept abstract objects, usually with the proviso that the only abstract objects that are exemplified must be physical properties or relations.) For a good treatment of this distinction put in different terms and including a case that a naturalist ought to be Global, see →Grossmann, 1992, pp. 1–46; →Moreland, 2000.

10 I adapt the terminology of Robert Koons. See →Koons, 2014.

11 →Bedau, 2010.

12 For a classic comparison of the epistemic and ontological characterizations of emergent properties, see E. →Nagel, 1979, pp. 366–380.

13 Graham Oppy has objected to my Bayesian form of AC. See →Oppy, 2011; cf. →Oppy, 2006, pp. 382–400. For my response to Oppy, see →Moreland, 2011 and 2012b.

14 See →Adams, 1992.

15 See →Swinburne, 1979, Chapter 9; 1997, pp. 183–196; 1996, pp. 69–94; 1995.

16 →Horgan, 1993, pp. 313–314.

17 Daniel Lim has criticized my claim that the mental/physical correlation (or the brute appearance of consciousness) is contingent. See →Lim, 2011. For my response, see →Moreland, 2012a.

18 Cosmides and Tooby, 1997, p. 4.

19 →Rachels, 1990, pp. 73–74.

20 →Armstrong, 1978, p. 262.

21 →Gruber, 1974, p. 211.

22 →Churchland, 2013, p. 35.

23 See →Moreland, 2008.

24 →Papineau, 1993, p. 31.

25 See →Tabaczek, 2019, pp. 27–34.

26 →Kim, 1998, pp. 1–27. Kim employs nomological necessity in his understanding of the sort of modality implied by “strong.”

27 →McGinn, 1999, especially pp. 46–54, 66–76, 95–101. One of us has subjected McGinn’s view to a careful analysis and critique. See →Moreland, 2008, pp. 95–113.

28 See →O’Connor, 2000, footnote 8, 70–71. I have argued that the causal relation between mental and physical states is, in fact, due to God’s direct activity and stable intention that things remain a certain way. See →Moreland, 2008.

29 Chisholm, 1989, p. 56.

30 →Strawson, 2006, see especially pp. 12–21.

31 When John Searle developed his biological naturalism, he dismissed the problem of (genuinely) emergent

properties like those constituting consciousness by making a tight analogy between them and structurally supervenient properties, e.g., solidity, liquidity. This allowed him to find emergent mental properties to be ordinary physical, structural ones and their appearance to be unsurprising. See →Searle, 1992, pp. xii, 13–19, 25–28, 32, 56–57, 85–93, 95, 118–124. For a response to Searle, see →Moreland, 2008, Chapter 3.

32 →Strawson, 2006, p. 18.

33 →Chalmers, 2016, pp. 204–206.

34 →Chalmers, 2016, p. 206.

35 →Page, 2020, pp. 336–362.

36 Thomas Nagel sees the problem here and advances a non-theistic teleological view of our universe to overcome it. See Th. →Nagel, 2012. For a critique, see →Moreland, 2012c.

37 See Th. →Nagel, 2012; →Skrbina, 2005.

38 →Madell, 1988, p. 3.

39 See →Moreland, 2008, pp. 117–126.

40 David Chalmers does not solve the sorites problem, the contingency problem or the unity-of-consciousness problem since Chalmers' subject of consciousness is a composite object. (For more on this problem, see ibid., pp. 128–131.) Chalmers has offered a different solution to the origin of macro-phenomenal consciousness. See →Chalmers, 2016. Space considerations prevent me

from addressing Chalmers' quite nuanced view. It is clear, however, that he distinguishes his position (constitutive Russellian pansprotopsychism plus broad physicalism) from any form of emergentism. For those unfamiliar with his theory, the following (inadequate) precis will most likely be unhelpful. But for those who are, here are some critical bullet points: (1) It is highly speculative (see the source cited in note 5 above.) (2) It is hard to see how the simple spatial re-arrangement of entities with proto-consciousness (aka., conscious precursors, proto-phenomenal properties, preconscious properties) that lack what-it-is-like to be those entities can give rise to full-blown phenomenal consciousness. This difficulty is not due to an epistemic gap between narrow physicalism and phenomenal consciousness as Chalmers would have it. Rather, it arises from broad physicalism along with his characterization of the categorical quiddities in the terms just mentioned. (3) When we conceive of zombies, we do not conceive of structural zombies; rather, we conceive of broad physical zombies. That is, we conceive of zombies with the same physical structural features as we have, along with physical stuff playing the role of quiddities. We strongly conceive that such zombies could be conscious or unconscious *while leaving everything that is physical the same.*

41 →Chalmers, 2016, pp. 179–214.

42 →Chalmers, 2016, pp. 182–185.

43 →Chalmers, 2016, p. 180.

44 →Chalmers, 2016, p. 185.

45 →Churchland, 2013, p. 12.

46 Hoffman and Rosenkrantz, 2002, pp. 41–43.

47 Hoffman and Rosenkrantz, 2002, pp. 42–43.

48 →Ravenscroft, 2019, p. 272.

49 →Ravenscroft, 2005, p. 17.

50 →Kim, 2006, p. 46. Physicalists are often so confused about precisely what the issue is, that no less a philosopher that Kim conflates the problem of explaining how consciousness could arise in or be caused by an immaterial substance (as expressed in the quote) with the problem of how consciousness could "be possessed by" or "inhere in" a simple, immaterial substance. See Ibid. Clearly, these are *different* problems. As we will see, a failure to distinguish them has a number of philosophers to chase an unprofitable red herring.

51 →Olson, 2007, p. 153. See also →McGinn, 1999, pp. 94–95.

52 →Bayne, 2018, pp. 208–225. The quote is from p. 223.

53 →Ravenscroft, 2005, p. 17.

54 →Swinburne, 1986, pp. 153–155; cf. →Swinburne, 2013, pp. 33–38.

55 →Bailey, 2021. Subsequent page numbers within the text refer to this book.

56 In this regard, Bailey's view is a close cousin to classic Mormon anthropology. See →Moreland, 2002.

57 My use of "organicism" in these pages is quite different from Bailey's usage. For him, "organicism" seems to be equivalent to "animalism." See →Bailey, 2016, pp. 453–470. See →Moreland, 2018.

58 Bailey does qualify his views as follows: First Bradley is a wholly material object in that at some very low level he is composed entirely of physical atoms. At higher levels, he still has cells and organs. I assume these are physical as well. Final Bradley is wholly immaterial in that at some very low level he is composed entirely of wholly immaterial objects, viz., angels. But at higher levels, he retains cells and organs. I see how First Bailey is wholly material since his cells and organs are wholly physical, being composed entirely by wholly physical atoms. But if Final Bradley's cells and organs are like First Bradley's, then Final Bradley is not wholly immaterial, having physical cells and organs. Bailey recognized this problem and rightly point out that the cells and organs of Final Bradley are wholly immaterial, being entirely composed of wholly immaterial parts (angels).

59 See →Hudson, 2001, p. 121. Cf. pp. 26–27, 37–38, 101–105, 117, 121–122, 127–128. →Bailey, 2014, especially pp. 145, 151–156.

60 See →Habermas, 2018; Long and Perry, 2010; Rivas et. al., 2016.

61 For a possible exception to this claim, see →Westerhoff, 2005.

62 Aristotle, *Categories*; *Metaphysics IV*; →Bergmann, 1967, pp. 4–27, 101, 141; →Butchvarov, 1995; Chisholm, 1989, pp. 162–168; Chisholm, 1996; →Grossmann, 1983, pp. 3–18; →Grossmann, 1992, pp. 1–3, 46–47, 91; →Loux and Crisp, 2017, pp. 10–15; →Lowe, 2006, pp. 3–23; Hoffman and Rosenkrantz, 1994, pp. 14–21; Hoffman and Rosenkrantz, 1997, pp. 46–50. I am not settled on exactly where Peter van Inwagen comes down on this issue. Many things he says indicate he is a realist about the categories and holds that membership is necessary (though some categories are contingent in that they have members in some worlds and are empty in others). On the other hand, while he claims that ontological categories must in some sense be modally robust, he claims that he has yet to spell out precisely what this means. See →van Inwagen, 2014, Chapter 9.

63 →Bailey, 2014, p. 5.

64 See →Moreland, 2013. Cf. →Bailey, 2014, pp. 16–22.

65 Hoffman and Rosenkrantz, 2002, pp. 41–43.

66 Hoffman and Rosenkrantz, 2002, pp. 42–43.

67 →Reinach, 1982, especially pp. 315–354.

68 →Mumford, 2007, pp. 45–71.

35 God and Mind

Charles Taliaferro

Abstract: The philosophy of mind and the philosophy of God are deeply intertwined. The philosophy of mind today is rarely developed in a vacuum, but in the context of naturalism that is hostile to classical and contemporary concepts of God as a transcendent reality. There are, however serious objections to scientific naturalism, and more moderate forms of naturalism also face deep problems (the causal closure of the physical world leads to an unacceptable view of human agency). Once we are open to a non-naturalist ontology, the way is clear for multiple theistic and non-theistic concepts of the divine. As we become more aware of the plurality of views of God, mind, and values we can overcome stereotypes such as the alleged distinction between Western and Eastern philosophy.

35.1 Introduction

Issues in philosophy are often intertwined. If you are disposed to some form of behaviorism, you are not going to be impressed with any philosophical positions that appeal to subjective experience, introspection, first-person reports about sensations (sense-data or qualia), and so on. Similarly, a deeply entrenched scientific naturalist is not going to welcome theistic accounts in ontology or value theory. Fortunately, there is some evidence that the current intellectual climate is shifting from its prior more monolithic stages—logical positivism and Wittgenstein in the 1940s and 50s, the scientific naturalism heyday of the 1960s

through the 1990s—to a more pluralistic state of affairs. This is good news for the philosophy of God.

There are three sections in this essay. The first offers an overview of philosophical pluralism, and sketches the reasons why three ostensibly irrefutable, dominant philosophical models turned out to be quite vulnerable. Specifically, I will stress the problems facing scientific naturalism. The second highlights why philosophical theism has made a come-back philosophically over the past 40 years in the English-speaking world. The third proposes how current debate about the philosophy of God can thrive in a pluralistic context.

35.2 The Rise and Fall of Dominant Philosophies, Especially Scientific Naturalism

An intellectual context is philosophically pluralistic when incompatible philosophical methods and positions—Platonism, Spinozist monism, a Kalam cosmological theistic argument, a defense of the analytic-synthetic distinction, free will libertarianism, British Idealism, and so on—are deemed by significant numbers of philosophers to be worthy of debate, publication, dissertation topics, and not ruled out of philosophical bounds because of institutionally entrenched philosophical convictions. Let us consider three periods when the world of philosophy was not pluralistic the reign of logical positivism, Wittgenstein's hegemony, and scientific naturalism.

35.2.1 Logical Positivism

"Positivism" is a term introduced by Auguste Comte (1798–1857), a French philosopher who championed the natural and social sciences over against theology and the philosophical practice of metaphysics. The term "positivism" was used later (sometimes

amplified to *Logical Positivism* by A. J. Ayer) by a group of philosophers who met in Austria called the Vienna Circle from 1922 to 1938. This group, which included Moritz Schlick and Max Planck, advanced an empirical account of meaning, according to which for a proposition to be meaningful it needed either to be a conceptual or formal statement in mathematics or about analytic definitions ("triangles have three angles") or about matters that can be empirically verified or falsified. Ostensibly factual claims that do not make any difference in terms of our actual (or possible) empirical experience are void of meaning. A British philosopher, who visited the Vienna Circle, A. J. Ayer, popularized this criterion of meaning in his 1936 book, *Language, Truth, and Logic*. In it, Ayer argued that religious claims as well as their denial were without cognitive content. By his lights, theism, and also atheism and agnosticism, were nonsense, because they were about the reality (or unreality or unknowability) of that which made no difference to our empirical experience. How might one empirically confirm or disconfirm that there is an incorporeal, invisible God or that Krishna is an avatar of Vishnu? Famously, Antony Flew employed this strategy in his likening the God of theism to a belief that there is an undetectable, invisible gardener who could not be heard or smelled or otherwise empirically discovered (Flew, 1955). In addition to rejecting traditional religious beliefs as meaningless, Ayer and other logical positivists rejected the meaningfulness of moral statements. By their lights, moral or ethical statements were expressions of persons' feelings, not about values that have a reality independent of persons' feelings.

The logical positivist critique of religion is not dead. It can be seen at work in Herman Philipse's *God in the Age of Science; A Critique of Religious Reasons* (2012). Still, the criterion of meaning advanced by logical positivism faced a series of objections (for details see →Taliaferro, 2005b). But consider five objections that

were instrumental in the retreat of logical positivism from its position of dominance.

First, it was charged that logical positivism itself is self-refuting. Is the statement of its standard of meaning (propositions are meaningful if and only if they are about the relations of ideas or about matters that are subject to empirical verification or falsification) itself about the relations of ideas or about matters that are subject to empirical verification or falsification? Arguably not. At best, the positivist criterion of meaning is a recommendation about what to count as meaningful.

Second, it was argued that there are meaningful statements about the world that are not subject to direct or indirect empirical confirmation or disconfirmation. Plausible candidates include statements about the origin of the cosmos or, closer to home, the mental states of other persons or of non-human animals (for discussion, see →Taliaferro, 1994).

Third, limiting human experience to what is narrowly understood to be empirical seemed to many philosophers to be arbitrary or capricious. C. D. Broad and others defended a wider understanding of experience to allow for the meaningfulness of moral experience: arguably, one can experience the wrongness of an act as when an innocent person feels herself to be violated.

Fourth, Ayer's rejection of the meaningfulness of ethics seemed to cut against his epistemology or normative account of beliefs, for he construed empirical knowledge in terms of having the right to certain beliefs. If it is meaningful to refer to the right to beliefs, why is it not meaningful to refer to moral rights such as the right not to be tortured? And if we are countenancing a broader concept of what may be experienced, in the tradition of phenomenology (which involves the analysis of appearances) why rule out, as a matter of principle, the experience of the divine or the sacred?

Fifth, and probably most importantly in terms of the history of ideas, the seminal philosopher of science Carl Hempel (1905–1997) contended that the project of logical positivism was too limited (→Hempel, 1950). It was insensitive to the broader task of scientific inquiry which is properly conducted not on the tactical scale of scrutinizing particular claims about empirical experience but in terms of a coherent, overall theory or view of the world. According to Hempel, we should be concerned with empirical inquiry but see this as defined by an overall theoretical understanding of reality and the laws of nature. This was not *ipso facto* a position that favored the meaningfulness of religious belief, but Hempel's criticism of positivism removed their barrier for overall metaphysical accounts of reality, be these accounts theistic, pantheistic (roughly, God is everything), naturalistic, and so on. Moreover, the positivist critique of what they called metaphysics was attacked as confused as some metaphysics was implied in their claims about empirical experience; see the aptly titled classic *The Metaphysics of Logical Positivism* (1954) by Gustav Bergmann (1906–1987).

Let us now turn to Wittgenstein (1889–1951) and the philosophy of religion his work inspired.

35.2.2 The Hegemony of Wittgenstein

Wittgenstein's early work was interpreted by some members of the Vienna Circle as friendly to their empiricism, but they were surprised when he visited the Circle and, rather than Wittgenstein discussing his *Tractatus*, he read them poetry by Rabindranath Tagore (1861–1941), a Bengal mystic. In any case, Wittgenstein's later work, which was not friendly to their empiricism, was especially influential in post-World War II philosophy and theology.

In the *Philosophical Investigations* (published posthumously in 1953) and in many other works (including the publication of notes taken by his students on his lectures), Wittgenstein opposed what he called the picture theory of meaning. On this view, statements are true or false depending upon whether reality matches the picture expressed by the statements. Wittgenstein came to see this view of meaning as deeply problematic. The meaning of language is, rather, to be found not in referential fidelity but in its use in what Wittgenstein referred to as *forms of life*. As this position was applied to religious matters, D. Z. Phillips, B. R. Tilghman, and, more recently, Howard Wettstein, sought to displace traditional metaphysical debate and arguments over theism and its alternatives and to focus instead on the way language about God, the soul, prayer, resurrection, the afterlife, and so on, functions in the life of religious practitioners. For example, Phillips contended that the practice of prayer is best not viewed as humans seeking to influence an all-powerful, invisible person, but to achieve solidarity with other persons in light of the fragility of life. Phillips thereby sees himself as following Wittgenstein's lead by focusing, not on which picture of reality seems most faithful, but on the non-theoretical ways in which religion is practiced.

> To ask whether God exists is not to ask a theoretical question. If it is to mean anything at all, it is to wonder about praising and praying; it is to wonder whether there is anything in all that. This is why philosophy cannot answer the question "Does God exist?" with either an affirmative or a negative reply … "There is a God", though it appears to be in the indicative mood, is an expression of faith. (→Phillips, 1976, p. 181)

At least two reasons bolstered this philosophy of religion inspired by Wittgenstein. First, it seemed as though this methodology was more faithful to the practice of philosophy of religion being truly *about the actual practice of religious persons*

themselves. Second, while there has been a revival of philosophical arguments for and against theism and alternative concepts of God, significant numbers of philosophers from the mid-twentieth century onward have concluded that all the traditional arguments and counter-arguments about the metaphysical claims of religion are indecisive. If that is the case, the Wittgenstein-inspired new philosophy of religion had the advantage of shifting ground to what might be a more promising area of agreement.

While this non-realist approach to religion has its defenders today, especially in work by Howard Wettstein, many philosophers have contended that traditional and contemporary religious life rests on making claims about what is truly the case in a realist context. It is hard to imagine why persons would pray to God if they, literally, thought there is no God (of any kind).

Interestingly, perhaps inheriting the Wittgenstein stress on practice, some philosophers working on religion today place greater stress on the meaning of religion in life, rather than seeing religious belief as primarily a matter of assessing an hypothesis. Nonetheless the influence of Wittgenstein was not able to banish the nagging perennial questions of ontology and value. Is there a loving, Creator and Redeemer of the cosmos? Both theists and atheists revived debate on the problem of evil. If the God of Abrahamic faiths exists, why is there so much evil? Such a question demands a realist framework, whatever one's final verdict. Most agnostics assert that they are not certain that there is or is not a God, but they rarely contend this is a matter of forms of life and the meaning of religious practices.

35.2.3 The Dominance of Scientific Naturalism

According to scientific naturalism, all of reality is best described and explained by the natural or physical sciences (physics,

chemistry, and biology). In the late twentieth century Willard Van Orman Quine, for example, contended that we should not give primacy to philosophy. He endorsed a form of naturalism that involved the "abandonment of the goal of a first philosophy."

> [Naturalism] sees natural science as an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal, and not in need of any justification beyond observation and the hypothetical deductive method of doubt. (→Quine, 1981, p. 72)

The hypothetical deductive method involves constructing (or discovering) the laws of nature and being able to explain and predict events via a method in which implications are deduced from an hypothesis and an attempt is made to falsify or confirm the hypothesis. According to Quine, the methods employed in the sciences may be imperfect, but science itself is self-justifying and not in need of any deeper, philosophical justification.

A key element in all forms of naturalism is the rejection of theism or, in terms often employed by naturalists, supernaturalism.

> Naturalism on any reading is opposed to supernaturalism.... By 'supernaturalism' I mean the invocation of an agent or force that somehow stands outside the familiar natural world and whose doings cannot be understood as part of it. Most metaphysical systems of the past included some such agent. A naturalistic conception of the world would be opposed to all of them... Most philosophers for at least one hundred years have been naturalist in the nonsupernaturalist sense. (→Stroud, 2004, p. 23).

Most naturalists are physicalists. Here is a frequently cited account of physicalism, advanced by Daniel Dennett:

> There is only one sort of stuff, namely matter—the physical stuff of physics, chemistry and physiology—and the mind is somehow nothing but a physical phenomenon. In short, the mind is the brain ... we can (in

> principle!) account for every mental phenomenon using the same physical principles, laws, and raw materials that suffice to explain radioactivity, continental drift, photosynthesis, reproduction, nutrition, and growth. (→Dennett, 1991, p. 33)

Notice that acceptable explanations involve no ethical components. Radioactivity is not prompted by moral judgments. The rate of radionuclides decaying has nothing to do with matters of virtue and vice.

Scientific naturalism was completely embedded orthodoxy in many philosophy departments in the USA and UK for decades. It still thrives, and some form of physicalism is often considered the only live ontological position. But at least three challenges have led philosophers to modify their commitment to naturalism and adopt what is sometimes referred to as liberal naturalism.

As noted at the outset of this essay, if you are committed to behaviorism, you will not be impressed by appeals to subjective experience. But one of the most pressing objections to behaviorism and forms of naturalism that either deny the existence of subjective experience or radically explain it away in terms of non-conscious elements is that this move seems to deny the most evident fact to virtually all of us; we are conscious, experiencing agents who act on the basis of reason and desire.

Second, contra Quine, the success of the sciences, including the physical sciences depends on the reality and reliability of our conscious observations, our use of reason and our other mental powers. Dennett's examples of causation (continental drift, etc.) make no use of reason, experience, observation, and so on. But without the recognition of reason and its being trustworthy, there would be no science.

Third, while ideal explanations in scientific naturalism make no appeal to ethical values, it is hard to imagine any extant human life without recourse to virtue and vice, good and evil. The dangerous radiation of a region may be due to human error

or arrogance, climate change may be due to the decisions of industrialists, human reproduction may be due to loving, consensual sex or rape. matters of human digestion may not be understandable without taking into account the nature and value of agriculture and the use of non-human animals.

To many, but not all philosophers the inescapability of conscious, subjective experience, the evident reality and trustworthiness of reason, and the overwhelming role of appeals to justice, goodness, and virtue and the prohibited nature of injustice, evil, and vice, have led to the development of less extreme forms of naturalism or to the development of philosophical positions that deny naturalism altogether.

35.3 The Revival of Philosophical Theism

With the retreat of scientific naturalism, analytical philosophers such as Richard Swinburne, Alvin Plantinga, William Alston, Marilyn Adams, R. M. Adams, P. L. Quinn, and many others began publishing substantial articles and books on theistic ontology and values. The whole state of play in philosophy of religion shifted from a defensive mode (advancing reasons for thinking religious theistic language was meaningful over against the power of logical positivism, Wittgenstein, and scientific naturalism) to offering positive accounts of divine attributes, theistic arguments in natural theology, and even constructive philosophical work on Christiaan ideas about the incarnation and trinity.

An example of this shift may be illustrated by closely considering the following case for the incoherence of theism.

> It is perhaps barely possible to conceive of a disembodied spirit which is individuated not by having a body but by having an individual locus or viewpoint on the world. By this I mean that we imagine it as possessing information which, in the case of a normal embodied mind, would be

> available only from a particular point in space and time. This limited viewpoint would mark off an individual of this kind from other possible such disembodied entities. The viewpoint would thus find expression in the content of the thoughts entertained by such a being. The being could be tracked, one might say, as an information centre. Such a being would be something like a poltergeist or a tinkerbell. The intelligibility of the notion of pure spirit along this route seems to be in direct proportion to its triviality.
>
> Even if such a spirit is conceivable it will not help us in giving content to the notion of a God who is a non-embodied mind. For it was precisely the limitations in space and time that we imagined for such a being which made it possible to individuate it without a body. That is of no assistance towards conceiving of a personal God who is immaterial, ubiquitous and eternal. It is not just that we cannot know what thoughts are God's thoughts, but that there does not seem to be anything which would count as ascribing a thought to God in the way that we can ascribe thoughts to individual human thinkers. (→Kenny, 2004, p. 79)

This critique, advanced by Anthony Kenny (an adherent to Wittgenstein's view of the mental which is quasi-behavioristic), seems to be grounded in the idea that we individuate each other on the basis of our embodied, limited points of view. If God has no body (unlike Greek gods like Zeus), we have no way to identify and individuate God's reality or thoughts. But why privilege our embodiment and assume that all conceivable beings must have finite, circumscribable bodies? If it makes sense to think of you and I having different points of view, why would it be absurd to imagine a being who is omniscient and therefore knows the points of view of all persons? An omniscient being would know when our points of view are impaired, tarnished by racist bias and narcissism. The references to a poltergeist, tinkerbell, and triviality seem unaccountably off track. In relating to someone the concept of God in the Abrahamic faiths, would it be helpful to say that this God is like a poltergeist or tinkerbell, only omniscient, omnipotent, omnipresent, without beginning or end, unsurpassably good,

the creator and sustainer of all that there is? None of the two trillion galaxies would continue existing without the conserving, creative power of God who may be encountered through experience as we find recorded in the work of Julian of Norwich, among millions of others. Actually, what might be comparatively less interesting are tribal gods with finite minds and bodies, even if, like poltergeists, they are scary and uncanny or perhaps charming elvish fairies like tinkerbell.

The contemporary revival of theism has led to a renewal of *perfect being theology*, according to which God is understood to be maximally excellent or unsurpassable in greatness. This tradition was (famously) developed by Anselm of Canterbury (1033/4–1109). For a contemporary work offering an historic overview of Anselmian theism, see Yujin Nagasawa's *Maximal God: A New Defense of Perfect Being Theism* (2017). Divine attributes in this tradition have been identified by philosophers as those attributes that are the greatest compossible set of great-making properties; properties are compossible when they can be instantiated by the same being. Traditionally, the divine attributes have been identified as omnipotence, omniscience, perfect goodness, worthiness of worship, necessary or non-contingent existence, and eternality (existing outside of time or atemporally). Each of these attributes has been subject to nuanced different analysis, as noted below. God has also been traditionally conceived to be incorporeal or immaterial, immutable, impassable, omnipresent. And unlike Judaism and Islam, Christian theists conceive of God as triune (the Godhead is not homogenous but consists of three Persons, Father, Son, and Holy Spirit) and incarnate as Jesus of Nazareth (fully God and fully human).

The revival of theism has been marked by extraordinary diversity. Do you have difficulty reconciling divine attributes and theistic convictions, like the compatibility of God's omniscience

and human freedom? You now have many options: you might embrace open theism according to which God has maximal knowledge (and thus omniscience) but it is impossible for God or any intelligence to know about future free agents or you might defend the idea that because God is timelessly eternal, the future for us is not God's future. Theistic philosophers have crafted many relevant, useful notions here, such as middle knowledge, speculation on temporality, different concepts of freedom. All the classical theistic arguments have made a come-back, including the ontological argument. Inquiry into the problem of evil has led to nuanced reflection on the hiddenness of God, the Free Will Defense, the distinction between logical and empirical versions of atheistic arguments, skeptical theism (the idea that if there is an all good God we may expect not to know the reasons behind Cod's allowing evil), greater good theodicies.

Among all the topics related to philosophical theism today, perhaps the most energy has focused on the interrelationship of epistemology and metaphysics. The Enlightenment notion of reason and evidence found in Descartes, Locke, and Hume, among many others, has been challenged by alternative accounts of justification and warrant. At the heart of what is known as Reformed Epistemology is the thesis that if Christian theism is true, then one may be warranted in adhering to Christian theism even in the absence of evidential arguments (For an extensive, critical overview, see →Gutting, 2009).

35.4 Ontological Concepts of God in a Pluralistic Context

One of the benefits of contemporary philosophical reflection on God and ontology is that we now have an extensive literature on diversity in religion and philosophy (→Meister, 2010). Appreciating diverse concepts of God, mind, and value can assist

us in overcoming some stereotypes, such as the difference between the West and the Eastern philosophy.

One of the most outrageous and misleading, yet popular distinctions between so-called Eastern and Western philosophy is that the West is individualistic, materialistic (in the "popular" sense of consumerism and market exchanges), imperialistic, and laced with dualities or polarities of false or illusory oppositions such as male and female, reason and desire, justice and mercy, soul and body, etc. To sum up the West in a single (stereotype) term, it is dualistic ("dualistic" is probably the most abused term in English; it sometimes means philosophies involved in immoral, sexist, superstitious, clearly false and damaging concepts of human persons in which there is a pernicious distinction between the soul or mind on the one hand and the grotesque body on the other). By way of contrast, Eastern philosophy is viewed as relational, non-individualistic, spiritual, democratic, non-imperialistic, holistic (seeing mind and body, male and female, reason and desire, as compatible). And so on.

This portrait is false, but in the spirit of charity, it may be allowed that the above crude portrait has some *highly misleading* yet superficial truth. To offer an example: yes, if by the "West" we mean European and post-Columbian America and those cultures and persons impacted by them, then there has been more stress on individual rights—as found, for example, in the 1689 Bill of Rights—a bill that secured "the freedom of speech and debates in Parliament" and their protection from censure—than one finds during that period of time (the surrounding one hundred years) in China, India, Japan, or Korea. But the "East" has included philosophers who have embraced natural law (see, for example, Dang Zhangshu) and very vigorous ethical and social-political positions that give individual liberty an extraordinary wide scope. The East shares some of the Western background in terms of pre-linguistic, oral philosophy, and an

inheritance of stories of ostensibly supernatural (or at least not-mundane) gods and goddesses not completely different from the world of Homer and Hesiod; you can find philosophers in the East who (as in the West) have held that human beings are naturally oriented to the good (Mencius) or we are naturally drawn to less good, selfish ends (Xunit); there are cases of what some would call “dualism” (a term that should probably be banished from the language) in the sense that some Eastern philosophers distinguished matter and spirit (as one finds with Ishvarakrishna in which matter is prakriti and spirit is purusha); there are skeptical arguments that predate Descartes’ famous challenge when it comes to thinking about radical alternatives—how do you know that you are actually experiencing and interacting with the world as it is or simply dreaming or you are under the influence of an all-powerful, deceitful spirit? Some contemporary philosophers think that radical skepticism is an exclusively Western produce: this is demonstrably false. See the work of Nagarjuna.

You might think of Karma and reincarnation as distinctively “Eastern,” but Western thinkers such as Pythagoras and Plato may be attributed with belief in reincarnation that is determined (as with Karma) by merit. You may think that, after the advent of Christianity, the West is exclusively concerned with the existence or non-existence of God as a Creator and person-like as opposed to the monism (all is one) of the East. But the West has philosophers who embrace forms of monism (Spinoza), and while the East has its monists (Shankara), it has its theists or theistic-like counterparts (Ramanuja). In many areas, including logic, philosophy of mind, aesthetics, epistemology, metaphysics, political theory, you can find some parallels: for example, Machiavelli might be favorably compared with Han Feizi or Kautilya.

If you do a close study of the main currents of Eastern philosophy, you will cover much of the ground of philosophy in the West, and vice versa, but of course the history of culture and philosophy East and West has many significant, particular features that do not admit of translation or parallels. The history of philosophy and religion on the Korean peninsula is, for example, unique in world history. The Japanese drawing on (or importing of) Chinese philosophy and culture also seems unparalleled (it is not sufficiently similar to the Roman appropriation of Greek philosophy and culture for the comparison to be interesting).

In closing, I suggest that work on the ontology of God, mind, and values has provided a wider philosophical breadth of views than ever before. It enables us to do serious work on comparative treatments of good and evil, as in the six-volume *The History of Evil*. Just as important, an awareness of such diversity can help us to overcome simplistic stereotypes, as in the popular depictions of God, mind, and values in the West versus the East.

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