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FORTRAN SUBROUTINES FOR THE EVALUATION OF THE CONFLUENT HYPERGEOMETRIC FUNCTIONS

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In this report we list the Fortran subroutines for evaluating the confluent hypergeometric functions M(a,b;x) and U(a,b;x). These subroutines use the stable recurrence relations given e.g. in Wimp.

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Fortran Subroutines for the Evaluation of the Confluent Hypergeometric Functions

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Abstract

In this report we list the Fortran subroutines for evaluating the confluent hypergeometric functions M(a,b;x) and U(a,b;x). These subroutines use the stable recurrence relations given e.g. in Wimp.

Key words:
confluent hypergeometric functions
stable algorithm
Fortran subroutine
recurrence relation

Introduction

It is well known that the ordinary differential equation

$$x \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} - ay = 0$$

has a solution

$$y(x) = AM(a,1;x) + BU(a,1;x)$$

if a is <u>not</u> a negative integer.

This problem arises e.g. when solving the linearized shallow water equations with the full linear variation in depth included (see Williams, Staniforth and Neta, [1]).

The computation of the confluent hypergeometric functions is based on the Miller algorithm (see e.g. Wimp, [2]). In general, one has a second order difference equation

$$z(n) + a(n)z(n+1) + b(n)z(n+2) = 0, \quad n \geq 0, \quad b(n) \neq 0 .$$

If $b(n) = 0$ for some n, in some cases one can make a change of variable $Y(n) = \lambda(n)z(n)$ which will produce an equation of the desired type. Let $w(n)$ be a nontrivial solution and the sum of the normalizing series

$$S = \sum_{k=0}^{\infty} c(k)w(k) \neq 0$$

is known. Let N be a large integer and define $z_N(n)$, $0 \leq n \leq N+1$, by

$$z_N(n) = \begin{cases} 0 & n = N+1 \\ 1 & n = N \end{cases}$$

$$z_N(n) + a(n)z_N(n+1) + b(n)z_N(n+2) = 0, \quad n = N-1, \ldots, 1, 0 .$$

One can approximate $w(n)$ by $w_N(n)$

$$w_N(n) = Sz_N(n)/S_N$$

where

$$S_N = \sum_{k=0}^{N} c(k)z_N(k) .$$

The algorithm is said to converge if

$$w_N(n) \to w(n) \quad \text{as} \quad N \to \infty .$$

The function $M(a,b;x)$ satisfies the recurrence relation

$$(2n+b+2)(n+a)z(n) - (2n+b+1)\left\{(2a-b) + \frac{(2n+b)(2n+b+2)}{x}\right\}z(n+1)$$

$$- (2n+b)(n+b+1-a)z(n+2) = 0 .$$

The minimal solution is

$$w(n) = \frac{x^n (a)_n}{(b)_{2n}} M(a+n,2n+b;x)$$

where

$$(c)_n = \frac{\Gamma(n+c)}{\Gamma(c)} .$$

The normalization relationship used in our subroutine is

$$S = b-1 = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}(b-1)_k(b+2k-1)w(k) .$$

An obvious modification must be made if $b = 1$. The algorithm is not defined if b, $b+1-a$, a are negative integers or zero.

The function $U(a,b;x)$ satisfies the relationship

$$(n+a)(n+a+1-b)z(n) - (n+1)[2(n+a+1)+x-b]z(n+1)$$
$$+ (n+1)(n+2)z(n+2) = 0 .$$

The minimal solution is

$$w(n) = \frac{x^n (a)_n (a+1-b)_n}{n} U(a+n,b;x)$$

for $|\arg x| < \pi$. A normalization relation is

$$1 = \sum_{k=0}^{\infty} w(k) .$$

In the next section we give a listing of the Fortran subroutines.

```
      SUBROUTINE MILLER(N,ALPHA,BETA,X,S,SS,COEFF)
      INTEGER N
      REAL*8  ALPHA,BETA,X,SS
      REAL*8  S(0:1000)
      EXTERNAL COEFF
C     USES THE J.C.P. MILLER ALGORITHM TO COMPUTE
C     S(0:N).
C     BEGIN
         INTEGER NN,K
         REAL*8 T,D,EPS,A,B,C
         REAL*8 OLDS(0:1000)
         EPS = 0.000000001
C        INITIALIZE OLDS.
         DO 1 K = 0, 1000
            OLDS(K) = 0
   1     CONTINUE
C        CHOOSE INITIAL NN.
         NN = N + 2
C        INITIALIZE K, S AND T.
   2     K = NN
         S(K+1) = 0
         S(K)    =1
         CALL COEFF(K,ALPHA,BETA,X,A,B,C)
         T       = 2*C*S(K)
C        TAKE A BACKWARD RECURRENCE STEP AND UPDATE IT.
   3     K = K - 1
         CALL COEFF(K,ALPHA,BETA,X,A,B,C)
         S(K) = A*S(K+1) + B*S(K+2)
C        CHECK FOR OVERFLOW AND RESCALE IF NECESSARY.
         D= DABS(S(K))
         IF (D .GT. 1.D30) THEN
C        BEGIN
            CALL SCALE(K,NN,S,T,D)
         END IF
         IF (K .GT. 0) THEN
C        BEGIN
            T = T + 2*C*S(K)
            GO TO 3
         END IF
         T = T + C*S(0)
         DO 4 K = 0, N
            S(K) = S(K)/T
   4     CONTINUE
C        TEMPORARY PRINT STATEMENT.
C        PRINT*, S(0)
C        TEST FOR CONVERGENCE.
         D = 0
         DO 5 K = 0, N
   5        D = D + S(K)**2
         CONTINUE
         D = DSQRT(D)
         T = 0
```

```
         DO 6 K = 0, N
            T = T + (S(K) - OLDS(K))**2
   6     CONTINUE
         T = DSQRT(T)
C        TAKE ANOTHER STEP IF NO CONVERGENCE.
         IF (T .GT. EPS*D) THEN
C        BEGIN
            NN = 2*NN
            DO 7 K = 0, N
               OLDS(K) = S(K)
   7        CONTINUE
            IF(NN .LE. 1000) GO TO 2
         PRINT 999,NN,ALPHA,BETA,X,T
999      FORMAT(' ** NO CONVERGENCE ** NN AP CP X T ',I5,4E14.7)
        END IF
        SS=S(0)
        RETURN
      END
```

```
      SUBROUTINE COEFF(N,ALPHA,BETA,X,A,B,C)
      INTEGER N
      REAL*8 ALPHA,BETA,X,A,B,C
C     COMPUTES COEFFICIENTS USED BY J.C.P. MILLER ALGORITHM FOR
C     A CONFLUENT HYPERGEOMETRIC FUNCTION  M(a,b:x)
C     SEE JET WIMP. COMPUTATION WITH RECURRENCE RELATIONS.
C     PITMAN 1984 PP. 61-62
C     BEGIN
        INTEGER M,K
        REAL*8,T,U,V,W
        S = 2*ALPHA - BETA
        T = N + ALPHA
        M = 2*N
        U = M + BETA
        V = U + 1
        W = V + 1
        A = (S/W + U/X)*V/T
        B = (N + BETA - ALPHA + 1)*U/T/W
        T = 1
        IF (N .GT. 0) THEN
C       BEGIN
           S = BETA - 1
           DO 1 K = 1, N-1
              T = -T*(1+S/K)
    1      CONTINUE
           T = -T*(1+S/M)
        END IF
        C = T
        RETURN
      END




      SUBROUTINE SCALE(K,N,S,T,D)
      INTEGER N,K
      REAL*8 T,D
      REAL*8 S(0:1000)
C     BEGIN
         INTEGER J
         T = T/D
         DO 1 J = K, N
            S(J) = S(J)/D
    1    CONTINUE
         RETURN
      END
```

```
      SUBROUTINE COEFU(N,ALPHA,BETA,X,A,B,C)
      INTEGER N
      REAL*8 ALPHA, BETA,X,A,B,C
C     COMPUTES COEFFICIENTS USED BY J.C.P. MILLER ALGORITHM FOR
C     A CONFLUENT HYPERGEOMETRIC FUNCTION  U(a,b;x)
C     SEE JET WIMP, COMPUTATION WITH RECURRENCE RELATIONS,
C     PITMAN 1984 PP. 63-64
C     BEGIN
         INTEGER M,K
         REAL*8  S,T,U,V,W
         S =   ALPHA + QFLOAT(N)
         T = S + 1.DO
         U = S*(T - BETA)
         V = QFLOAT(N + 1)
         W = V + 1.DO
         A = (2*T + X - BETA)*V/U
         B = - V*W/U
         C = 1
         RETURN
      END
```

Remark: The program that calls Miller must supply as a last parameter either COEFF (for M) or COEFU (for U).

The subroutines are available on a diskette from either author upon request. These subroutines were tested extensively for various values of a, b and x.

<u>Remark</u>: If the parameter is a negative integer, the solution of the differential equation is

$$y = AL_n(x) + B\{\ln|x|L_n(x) + \sum_{m=0}^{\infty} \beta_m x^m\}$$

where n = -a.

$L_n(x)$ are Laguerre polynomials whose coefficients α_i satisfy

$$\alpha_i = \frac{i-n-1}{i^2}\,\alpha_{i-1}\ . \qquad i = 2.\ \ldots\ n\ .$$

$$\alpha_1 = -n\ .$$

The coefficients β_m satisfy

$$\beta_{m+1} = \frac{(m-n)\beta_m + \left(1 - \frac{2(m-n)}{m+1}\,\alpha_m\right)}{(m+1)^2} \qquad m = 1.\ldots.n-1$$

$$\beta_m = \frac{1}{(n+1)^2}\,\alpha_n \qquad m = n$$

$$\beta_m = \frac{m-n-1}{m^2}\,\beta_{m-1} \qquad m = n+1.n+2.\ldots$$

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2. J. Wimp. Computation with Recurrence Relations, Pitman Advanced Pub. Program, Boston, 1984.

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